University of Alberta

Probabilistic Approach for MPC Performance Assessment

by

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in

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dedicated to

the first teachers in my life
my parents

Abstract

The performance of the MPC controllers can be improved by increasing the degrees of freedom (DoF) for control purposes. The degrees of freedom can be increased by relaxing the constraints or by reducing the variance of the process variables. The Linear-Quadratic (LQ) optimization method can be used for providing guidelines for increasing the DoFs for a controller. As the LQ optimization considers mean operating point and the processes do not always operate on but around the mean operating point, it is essential to take into account the variability or distribution. Due to the presence of variability the process variables have the probabilities to be inside and outside the constraint limits. In this thesis, the Bayesian method is utilized to take into account these probabilities for the assessment of the decisions related to increasing the controller DoFs. The algorithm, for Bayesian analysis, discussed in this thesis can also be used to obtain the guidelines for increasing the controller DOFs to achieve certain level of performance. By extending this idea a probabilistic optimization technique is also introduced in this thesis. The optimization function defined in probabilistic optimizer (PO) takes into consideration the probabilities for the data distribution and the profit/loss terms associated with the distribution. The PO can be used to obtain the constraint tuning guidelines for the controllers. Extending the idea of applying Bayesian methods for LQ optimization, a tool is developed for assessing the decisions, based on PO approach, for increasing controller DoF.

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List of Symbols

α_i	: *Linear Coefficient for* y_i
α_j	: *Linear Coefficient for* u_j
β_i	: *Quadratic Coefficient for* y_i
β_j	: *Quadratic Coefficient for* u_j
Δ_i	: *Range of operation for* y_i
Δu_j	: *Change in* u_j *operating point*
Δy_i	: *Change in* y_i *operating point*
ΔJ_E	: *Optimal yield without tuning the controller*
ΔJ_I	: *Ideal yield of the controller*
ΔJ_T	: *Optimal yield after tuning the controller*
η_E	: *Economic performance index*
η_T	: *Theoretical economic performance index*
μ_i	: *Target operating point for* y_i
ν_j	: *Target operating point for* u_j
σ_i	: *Standard deviation for* y_i
σ_{i0}	: *Base case standard deviation for* y_i
Ω^i	: y_i *state matrix*
$B_{u_j}^H$	: *Maximum limits change (as percentage of original range) for* u_j *high limit*
$B_{u_j}^L$	: *Maximum limits change (as percentage of original range) for* u_j *low limit*
$B_{y_i}^H$	: *Maximum limits change (as percentage of original range) for* y_i *high limit*
$B_{y_i}^L$	: *Maximum limits change, as percentage of original range, for* y_i *low limit*
C_{i1}	: *Penalty for operating* y_i *in Zone 1*
C_{i6}	: *Penalty for operating* y_i *in Zone 6*
Ch	: *Child Node*
$E(R_0)$	: *Expected Returns for the base case condition*
$E(R_c)$	: *Expected Returns potential by adjusting the constraints*
$E(R_p)$	: *Expected Returns potential for the base case condition*
$F_i(z)$	: *Cost factor of* Z^{th} *state of* y_i
H_{ky_i}	: *High zone-limit of the* k^{th} *zone for* y_i
H_{u_j}	: *High limit for* u_j
H_{y_i}	: *High limit for* y_i
J	: *Objective function*
J_{ik}	: *Value of objective function for* y_i *to be in Zone k*

K	: *Steady state process gain matrix*
K_{ij}	: *Process gain for* y_i and u_j
L_{ky_i}	: *Low zone-limit of the* k^{th} *zone for* y_i
L_{u_j}	: *Low limit for* u_j
L_{y_i}	: *Low limit for* y_i
m	: *Number of ouput variables*
n	: *Number of input variables*
N	: *Total number of changeable CVs and MVs*
$P(\neg A)$	: *Probability for A not to be true*
$P(A\|B)$	: *Probability for A to be true given B*
$P(A)$	: *Probability for A to be true*
Pa	: *Parent Node*
P_{pk}	: *Probability for a CV to be in* k^{th} *zone for* p^{th} *case*
$P_{ci}(x)$	: *Cumulative probability for* $y_i \leq x$
$P_i(z)$	: *Probability for* y_i *to be in* z^{th} *state*
$P(y_i \in \Omega_k)$	: *Probability for* y_i *to be in Zone k*
q	: *Number of quality variables*
$\hat{r}$	: *Constraint tuning guidelines*
$\hat{r}_{u_j}$	: *Constraint tuning guidelines for* u_j
$\hat{r}_{y_i}$	: *Constraint tuning guidelines for* y_i
r_{u_j}	: *Limits change (as percentage of original range) for* u_j
r_{y_i}	: *Limits change (as percentage of original range) for* y_i
$r_{u_j}^H$	: *Optimum limits change (as percentage of original range) for* u_j *high limit*
$r_{u_j}^L$	: *Optimum limits change (as percentage of original range) for* u_j *low limit*
$r_{y_i}^H$	: *Optimum limits change (as percentage of original range) for* y_i *high limit*
$r_{y_i}^L$	: *Optimum limits change (as percentage of original range) for* y_i *low limit*
R_j	: *Quarter of range for* u_j
R_{j0}	: *Base case quarter of range for* u_j
S_{u_j}	: *Percentage change in base case* R_j
S_{y_i}	: *Percentage change in base case* σ_i
U	: *Utility Node*
$\bar{u}_j$	: *Optimized mean operating point for*u_j
$\bar{y}_i$	: *Optimized mean operating point for* y_i
$\bar{u}_{j0}$	: *Base case mean operating point for* u_j
$\bar{y}_{i0}$	: *Base case mean operating point for* y_i
u_{holj}	: *Half of limits for* u_j
y_{holi}	: *Half of limits for* y_i
u_j	: j^{th}*Input Variable (MV) for MPC*
y_i	: i^{th} *Output Variable (CV) for MPC*
y_{ip}	: *Optimum operating point for* y_i *for* p^{th} *case*

1 Introduction

1.1 Motivation

Performance assessment is a systematic observation of actual performance and the rating of that performance according to an established performance criterion. Controller performance assessment is the action of evaluating statistics reflecting the control performance at a certain point in time. Other terms that can be used interchangeably with assessment are monitoring and auditing.

Malfunctioning in the control loops and process model mismatch are some of the reasons for poor performance of Model Predictive Controllers (MPCs). The steps involved in controller performance assessment are (Jelali 2006):

1. Determination of the control loop to be assessed.
2. Selection and design of benchmark for performance assessment.
3. Assessment and detection of poor performing loops.
4. Diagnosis of the underlying causes.
5. Suggestions of improvement measures.

For a chemical process industry, process control is an engineering activity that is concerned with the systematic adjustment of the material and energy balance, around a process unit, necessary to keep it on course. A process is said to be 'in control' if the adjustments are made such that the sensors measuring the material and energy balance track their target value within some acceptable engineering tolerance (Moore 1988). The performance of a control system relates to its ability to deal with the deviations between the controlled variables and their desired values. These deviations are generally quantified by a *performance index.* The most widely used criterion for the performance assessment is *variance.* Performance of the control loop is said to be unacceptable, if the variance of the controlled variables exceeds a certain critical value. Different research groups have defined different performance indices, like Harris Index, Extended Horizon Performance Index etc. that can be used for performance assessment of the controllers (Jelali 2006).

The MPCs control the process by repeatedly solving both the online and the optimal control problem simultaneously. This is performed by optimizing the defined optimization function, over a future horizon (control and prediction horizon), for which constraints are defined by the limits for the Controlled Variables (CVs) and the Manipulated Variables (MVs). The future predictions for the process variables are made in accordance with the process model. In other words, the MPC systems rely upon generating the values of the process inputs, as the solution to online optimization problem. The process model, constraint limits and the process measurements together define the constraints for the optimization problem. As the online optimization defines the control action for the controller, the behavior of the controller can be quite complicated; but with the advancement in the field of computation techniques and use of computers the problem has been solved for all classes of system. The MPC control is very similar to a chess game, where the positions of the chess pieces define the state of the chess board, which is to be taken into consideration by the player according to the prediction of the future evolution of the game, i.e., the expected moves made by the opponent. The objective function for the problem is to obtain a checkmate and the constraints are set by the rules specified for making the moves on the chess board.

The MPCs move the process MVs in order to satisfy one or more of the following practical performance criteria (Garcia *et al.* 1988):

1. *Economic*: The economic criteria is associated with either maintaining process variables at the target values dictated by the optimization or dynamically minimizing the cost function.

2. *Safety and Environment*: Some process variables must never violate the specified constraints/bounds for safety reasons and/or environmental policies and regulations.

3. *Equipment*: The control system should not drive the process outside the physical limitations of the equipment.

4. *Product Quality*: The product specifications demanded by the consumer must always be satisfied.

5. *Human Preferences*: The operating personnel should be comfortable with the control system.

Thus, the MPC control system can very well ensure safe operation of the process equipment and the operating personnel by taking care of the constraints while exploiting the maximum production capacity of the process plant and minimizing the production cost. Researchers have focused mainly on the development of MPC control techniques. The outcome of research in the field of MPC (Rawlings and Muske 1993) has helped in building a strong conceptual and practical framework for control theory developers, implementers and end users. While several aspects of the controller are still to be explored, the framework provides a strong foundation for the development of the control system. With most of the research work focused on methods to identify the system and develop the MPC controllers little work has been done in the field of assessing the MPC performance and to identify the methods to improve the controller performance. Current methods available for improving the controller performance are:

1. Changing the structure of the MPC controller.

2. Tuning the controller.

3. Developing the control algorithms for obtaining the numerical solutions to the MPC online optimization problem, thus enabling the formulation of more realistic optimization problem and therefore improved performance.

It can be seen from the above cited methods for improving the controller performance that the methods need a thorough knowledge of the process and the control philosophy and thus are not recommended to be changed very often. However, it may be noted that other than the above mentioned methods there are other parameters like CV and MV constraint limits that also affect the controller performance. Providing the controller with wrong constraint limits can restrict the controller from achieving its maximum potential. Thus, it is essential that the constraint limits be provided appropriately, so that the maximum can be extracted from the controller for the given set of conditions.

The current online steady state economic optimization for the MPC controller is defined as a linear-quadratic problem, where the linear and quadratic coefficients define whether the process variable is to be maximized or minimized and the importance for them to be maintained at a certain target value. The linear-quadratic optimization problem is based upon the mean operating point for the process variables; however, the variability and the constraint limits for the process variables define how far or close the process variables will operate, relative to the optimum operating point. The solutions thus obtained are also based upon the mean operating points, variability and the constraint limits.

The work by Xu *et al.* (2007) focuses on the MPC performance assessment using deterministic linear-quadratic optimization and obtaining the constraint tuning guidelines for the controller. As linear-quadratic optimization function is based upon the mean operating point of the process variables, the MPC controller performance is evaluated based upon the mean operating point of the process variables. However, the real process operations are associated with variability; they seldom operate precisely on the mean operating point, but around the mean operating point. The process variables thus have probability of violating the constraint limits. Thus, it is essential that these probabilities be taken into account while making any decisions related to the MPC controller tuning by adjusting the controller constraint limits. The literature cites the qualitative effect of adjusting the constraint limits of the process variables but do not quantify the effect. Therefore, it is essential to develop a methodology to assess the effect of implementing the changes in constraint limits on the overall controller performance. Also, at times in industries it is essential to make decisions so as to achieve target return or profits from the process. Thus, it is required to develop tools that will help in making decisions for achieving target return. These need to have a methodology to assess the effect of changes in constraint limits and also to get some means to obtain guidelines for decision making to achieve target profit levels have motivated our research, with an objective to develop algorithms for the same.

The algorithms discussed in the thesis have been used to develop applications using MATLAB for the following reasons:

1. MATLAB being a programming language, with debugging facility, that allows easy coding.
2. MATLAB is supported by various toolboxes like statistics, and also other toolboxes like SeDuMi, BNT are available that can be used for the purpose.
3. MATLAB codes are easy to comprehend.
4. MATLAB is widely used for educational and research purposes.

For developing the algorithms for the Bayesian applications, discussed in Chapter 3 and 5, Bayes Net Toolbox (BNT) developed by Kevin Murphy (http://bnt.sourceforge.net) has been used.

1.2 Optimization and Bayesian Analysis

The commercially available controllers (Qin and Badgwell 1997) utilize one or the other form of the linear-quadratic optimization function of steady state variables. The optimization function is determined by the steady state mean operating point and the linear-quadratic coefficients for the process variables. The linear-quadratic optimization coefficients define the contribution of each process variable to the objective function, and the need to maintain them at desired operating points. By function of minimization, the higher the linear coefficient for a process variable, the more desirable it is to minimize the process variable; and the more negative it is, the more desirable that the process variable be maximized. The quadratic coefficient defines the need to maintain the process variable at a target value. The higher it is, the higher is the desire to maintain the process variable at the target value. Due to the presence of disturbances and plant model mismatch, the process variables are operated at some distance from the constraints. This is a trade-off between avoiding constraint violation and concession on the profit from the process. The problem in this case is identification of the optimal operating point, taking into consideration the process interactions. The closer the process operates to the optimum operating point, the better is the performance of the controller. Thus, it is essential to assess the performance of the controller as an estimate to how far or close, to the optimum operating points, the process is operating. The performance assessment technique, Linear Matrix Inequality Performance Assessment (LMIPA), discussed in Xu *et al.* (2007) is based upon the linear-quadratic optimization. The algorithm provides with the base case return potential without any tuning, the ideal case return and the constraint tuning guidelines for the process variables in the MPC controller, so as to obtain a target return defined as a percentage of the ideal return.

Using previous methods, two case studies are carried out, in this thesis, on a simulated distillation column (Volk *et al.* 2005) and an industrial distillation column. In Chapter 2, case studies based on LMIPA are performed. The case studies brought about not only the advantages of the LMIPA approach but also the shortcomings of the performance assessment using deterministic linear-quadratic optimization. The constraint tuning guidelines obtained are at times observed to be unrealistically high, which are practically impossible to implement, e.g. in some cases the constraint limit is suggested to be increased by 100% and even more. Also, sometimes the suggestions made are obtained for the variables for which it is not desired to change the limits e.g. quality CVs. Thus, there is a need to obtain more realistic tuning guidelines for the variables for which it is possible to change the constraints and which could be implemented for practical purposes. Also, since the LMIPA approach is a deterministic approach for performance assessment, and as the real world, processes are never deterministic, it would be appropriate to take into consideration the uncertainties associated with the process variability.

The work by Rahim and Shaibu (2000) proposed the use of probabilities for estimating the expected return and optimal target values for product qualities. The methodology proposed by them involved the probabilities for product qualities to be inside and outside the specifications and the costs/profits associated with them. The methodology considered the product qualities only and did not take into account the interaction of various process variables. The same idea will be extended to assess the performance of MPC controllers using Bayesian technique (Korb and Nicholson 2004, Murphy 2001, Tan 2001, Charniak 1991).

Since constraint change can affect the MPC performance, a Bayesian analysis method is thus developed to assess the performance of the controller when decisions to apply constraint changes are made. The performance assessment thus done will help evaluating the decision. To achieve this objective, a Bayesian Network is created for the controller and the decision made is provided as the evidence for the Network. The analysis then provides with the probabilities for the process variables to be within and outside the specification and the expected return are then estimated. Comparison of the expected return thus obtained with the current value of expected return will provide the assessment of the performance of the controller if the changes are made in the controller. The Bayesian Network can also be used to obtain the guidelines for the decisions to be made for achieving the target return from the process. The states of process variables, for the target return, are estimated and these states act as evidence for the analysis purposes for which the maximum a posteriori estimate of the states for the decision of the parent nodes is evaluated.

In Chapter 3, the Bayesian analysis application uses probabilities for evaluating the decisions; however, the optimization performed is based on the deterministic approach, linear-quadratic optimization. In Chapter 4, extending the idea of the Bayesian analysis using probabilities, for the process variables to be within and outside the specifications, a Probabilistic Performance Assessment (PPA) application is developed. The objective function defined for the PPA involves the probabilities for the process variables to be within and outside the specifications and the profit/loss associated with them to be inside and outside the limits. Unlike deterministic linear-quadratic optimization, which tends to drive the mean operating point to the optimal operating point, the PPA tends to identify the optimum operating point for the process variables so that its probability to be within the specified constraints is maximized and thus the return from the process are also maximized. Besides this the PPA can also be provided with the information about which process variables are available for making constraint changes and what is the maximum change that can be made in their constraints. The solution optimizes the objective function and provides the tuning guidelines for the process variables so that the maximum change of the limits for the constraint are not violated. Thus, the results obtained from the PPA are more realistic and more practical to implement.

As in the case of Bayesian analysis for the MPC controller using linear-quadratic optimization, in Chapter 5, an algorithm is developed using similar approach for the PPA. The Bayesian Network is created for the process in the same manner as for the Bayesian technique for linear-quadratic optimization. The decisions for making the changes in the constraints are provided, which are the evidence for the analysis purposes. The analysis estimates the probabilities for the process variables to be inside and outside the specifications, using probabilistic optimization function. The expected value of the return is then estimated and this can be used to compare with the controller performance with the existing constraint limits or the tuning. The same network can also be used to obtain the guidelines for tuning the controller constraint limits so that the target return is achieved.

As the processes seldom operate precisely on the mean operating point but around the mean operating point, estimating the return by only taking into account the mean operating point cannot be a true representation of the controller performance. It is therefore essential that the return be estimated by taking into account the probability for the process data to be inside and outside the specifications. The Bayesian technique using linear-quadratic optimization estimates the return from the controller using these probabilities. Thus the performance assessment made and the tuning guidelines obtained through this approach are more realistic than the simple linear-quadratic performance assessment.

Using probabilities in the optimization objective function is a closer approximation of the process than the Bayesian technique using linear-quadratic optimization. In this approach the controller is optimized using the optimization function for PPA. The tuning guidelines thus obtained are more practical to be implemented.

1.3 Thesis outline

The optimization problem discussed in the work by Xu *et al.* (2007) has been an important work in the field of MPC controller performance assessment and tuning the controller with the constraints. Chapter 2 of the thesis discusses two case studies using the existing methods. The first case study is carried out on the binary distillation column described in Volk *et al.* (2005) and the second case study is carried out on an industrial distillation column.

The tuning guidelines obtained from the analysis are critically analyzed for their effect on the overall process, when implemented. Chapter 3 describes the proposed Bayesian methods and its application for MPC controller performance assessment using linear-quadratic optimization. The algorithm is explained with illustrations using the binary distillation column discussed in Volk *et al.* (2005) and an industrial distillation column. A new proposed probabilistic performance assessment technique and its application are discussed in Chapter 4. The application of probabilistic optimizer is explained again with two applications, one with the binary distillation column and second with the industrial distillation column. The Bayesian performance assessment of the MPC controller using optimization function used for PPA is explained and discussed in Chapter 5. Finally the conclusion of all the work is summarized in chapter 6 of the thesis.

This thesis has been written in a paper-format in accordance with the rules and regulations of the Faculty of Graduate Studies and Research, University of Alberta. In order to link the different chapters, there is some overlap and redundancy of material. This has been done to ensure completeness and cohesiveness of the thesis material and help the reader understand the material easily.

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2

MPC economic performance assessment revisit and case studies - Deterministic approach

Abstract: For a distillation column, which separates the feed mixture into its components, Model Predictive Controller (MPC) has been developed and implemented. The MPC has been established as an effective and efficient control strategy that deals with economic optimization of the process control objective. The performance of the MPC controller needs to be evaluated and if necessary needs to be re-tuned. The performance assessment method to be used in this chapter is based on mean operating points. This study aims at assessing the application of performance of MPC controller for a distillation column and evaluating the suggestions made for limits tuning for the MPC controller. Through the case studies, advantages and disadvantages of the existing approach are discussed. The objective of this chapter is therefore 1) to review the existing approach and concepts through case studies, 2) to identify shortcoming of the existing approach, and 3) to build connection with the following chapters.

2.1 Introduction

With the increase in competitive pressures on commercial and technical fronts, process control has evolved and is still to reach its peak. Process control ensures most profitable production, leading to greater production of consistent quality products, with reliability and at minimum operational cost (Xu *et al.* 2007). Over the years Model Predictive Control

(MPC) has evolved as a popular control strategy (Morari and Lee 1999, Qin and Badgwell 2003). A number of commercial MPC controllers are available with different optimization objective functions. For example, Dynamic Matrix Control (DMC^{TM}) integrates the Linear Programming (LP) for estimating the optimum steady state (Sorensen and Cutler 1998) whereas, the Robust Model Predictive Control Technology (RMPCT^{TM}) also includes Quadratic Programming (QP) for the optimization of the economic function (Krishnan *et al.* 1998).

As distillation is the most common unit operations in a chemical process plant, two distillation operations have been chosen for the purpose of the case study. The purpose of the distillation column is to separate the components in the feed stream. In a petroleum refinery, the crude oil, a mixture of several hydrocarbons, needs to be fractionated into useful product streams. The fractionation of crude oil into various product streams is a complex process, thus it needs a proper control strategy. Model Predictive Controller (MPC) has emerged as a successful control strategy for a distillation column. The success of MPC can be rendered to factors such as (Pannocchia and Rawlings 2002):

1. Ability to operate on multivariable systems.
2. Ability to directly handle the input and output process constraints, using quadratic programming.

The process data from the two distillation columns, controlled by their respective MPC system, is considered for the purpose of carrying out the study. The main objective behind carrying out this study in this chapter are to:

1. Establish the utility of the algorithm developed by Xu *et al.* (2007),
2. Assess the practicality of implementing the results obtained from the analysis,
3. Suggest modifications, if required, in the optimization problem for making the results more pragmatic and acceptable.

The practical implications of results, for the analysis of the data from the two distillation column processes, are presented and discussed in this chapter. The effect of implementing the suggested tunings on each CV and MV is discussed individually, to bring about the significance of each suggestion made by the algorithm. The chapter is organized as follows. Section 2.2 discusses the fundamentals of the Linear Matrix Inequality based Performance Assessment (LMIPA). Section 2.3 describes the binary distillation column (Volk *et al.* 2005) for which the LMIPA study is carried out and provides the analysis of the results. Section 2.4 describes an industrial distillation column MPC application for which the LMIPA analysis is done followed by analysis of the results obtained. Section 2.5 provides the conclusion of the analysis.

2.2 LMIPA analysis of the process

Linear Matrix Inequality based Performance Assessment (LMIPA) is a tool for performance assessment of MPCs (Xu *et al.* 2007). This tool, when provided with the process data in the required format and the plant model, will do the benefit estimate analysis for all the process variables.

For an $m \times n$ system with n inputs and m outputs, having steady state process gain matrix, K, which is controlled by an MPC controller, let ($\bar{y}_{i0}$, $\bar{u}_{j0}$) be the current mean operating points for i^{th} output and j^{th} input variable and be referred to as the base case operating points. Also, let L_{y_i} and H_{y_i} be the low and the high limits for y_i, and L_{u_j} and H_{u_j} be the low and the high limits for u_j, respectively. If ($\bar{y}_i, \bar{u}_j$) are the operating points for y_i and u_j, respectively, then the economic objective function for the system can be defined as a quadratic function:

$$J = \sum_{i=1}^{m} \left(\alpha_i \times \bar{y}_i + \beta_i^2 \left(\bar{y}_i - \mu_i \right)^2 \right) + \sum_{j=1}^{n} \left(\alpha_j \times \bar{u}_j + \beta_j^2 \left(\bar{u}_j - \nu_j \right)^2 \right) \tag{2.1}$$

where, μ_i and ν_j are the target values for i^{th} CV (y_i) and j^{th} MV (u_j) respectively, α_i and β_i are the linear and quadratic coefficients for y_i, α_j and β_j are the linear and quadratic coefficients for u_j.

For the defined system the assessment of yield is done for various cases described in detail below:

1. Assessment of ideal yield,

2. Assessment of optimal yield without tuning the controller,

3. Assessment of improved yield by reducing variability,
4. Assessment of improved yield by constraint relaxation and
5. Constraint tuning for desired yield.

1. *Assessment of ideal yield*:

   For assessing the ideal yield, steady state operations are considered with no variability in both y_i and u_j. Under this scenario the optimization problem for the system is defined as:

$$\min_{\bar{y}_i, \bar{u}_j} J \tag{2.2}$$

subject to:

$$\Delta y_i = \sum_{j=1}^{n}[K_{ij} \times \Delta u_j] \tag{2.3}$$

$$\bar{y}_i = \bar{y}_{i0} + \Delta y_i \tag{2.4}$$

$$\bar{u}_j = \bar{u}_{j0} + \Delta u_j \tag{2.5}$$

$$L_{y_i} \leq \bar{y}_i \leq H_{y_i} \tag{2.6}$$

$$L_{u_j} \leq \bar{u}_j \leq H_{u_j} \tag{2.7}$$

where, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

2. *Assessment of optimal yield without tuning the controller*:

   The assessment of the optimal yield of the controller without tuning means to assess the yield that should be obtained from the controller for the given constraints and the existing variability in the base case operations. This scenario considers moving the actual operating point of y_i to its optimal operating conditions, as close as possible, subject to the constraints. Under this scenario the optimization problem for the system is the same as defined in equation (2.2) subject to equalities defined in equations (2.3), (2.4) and (2.5); however, the inequalities are according to equations (2.8) and (2.9):

$$L_{y_i} + 2 \times \sigma_{i0} \leq \quad \bar{y}_i \quad \leq H_{y_i} - 2 \times \sigma_{i0} \tag{2.8}$$

$$L_{u_j} + 2 \times R_{j0} \leq \quad \bar{u}_j \quad \leq H_{u_j} - 2 \times R_{j0} \tag{2.9}$$

where, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$; σ_{i0} and R_{j0} are the base case standard deviation and the quarter of the range for y_i and u_j, respectively. The inequalities defined for the problem allow 5% constraint limit violation i.e. 95% of the operation is within two standard deviations (Latour *et al.* 1986, Martin *et al.* 1991).

3. *Assessment of improved yield by reducing variability*:
   This case involves tuning of the control system such that the variability of one or more output variables can be reduced. Reducing the variability provides the opportunity to push the operating points closer to the optimum and thus improve the yield. Practically, the reduction in variance of one variable (say quality variable) is transferred to the variability of some other variables, such as constrained variables. Since constraint variables do not directly affect the profit, their variability is not of concern, as long as they are maintained well within the set limits. Thus, variability of a quality variable can be reduced by transferring it to that of the constraint variables. For assessing the improved yield by variability reduction the optimization problem and its equality conditions are the same as defined in equations (2.2) and [(2.3), (2.4) and (2.5)] respectively; however the inequalities are changed and are defined in equations (2.10) and (2.11):

$$L_{y_i} + 2 \times \sigma_{i0} \times (1 - S_{y_i}) \leq \quad \bar{y}_i \quad \leq H_{y_i} - 2 \times \sigma_{i0} \times (1 - S_{y_i}) \tag{2.10}$$

$$L_{u_j} + 2 \times R_{j0} \times (1 - S_{u_j}) \leq \quad \bar{u}_j \quad \leq H_{u_j} - 2 \times R_{j0} \times (1 - S_{u_j}) \tag{2.11}$$

where, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$; S_{y_i} and S_{u_j} are the percentage reduction in the base case variability of y_i and u_j. S_{y_i} and S_{u_j} are obtained from Multi-Variable Performance Assessment (MVPA) (Xu *et al.* 2007). The inequalities defined for the problem allow 5% constraint limit violation (Latour *et al.* 1986, Martin *et al.* 1991).

4. *Assessment of improved yield by relaxing constraints*:

   Relaxing the constraints for one or more process variables in an MPC controller can help moving the operating points so as to increase the yields. Relaxing the limits for the constraint variables can help in moving the quality variables closer to the optimum operating points, improving the yield. Thus, the optimization problem for this case can be defined by equation (2.2), the equality conditions are defined by equations (2.3), (2.4) and (2.5); however, the inequalities defining the constraints are considered in equations (2.12) and (2.13):

$$L_{y_i} + 2 \times \sigma_{i0} - y_{holi} \times r_{y_i} \leq \quad \bar{y}_i \quad \leq H_{y_i} - 2 \times \sigma_{i0} + y_{holi} \times r_{y_i} \tag{2.12}$$

$$L_{u_j} + 2 \times R_{j0} - u_{holj} \times r_{u_j} \leq \quad \bar{u}_j \quad \leq H_{u_j} - 2 \times R_{j0} + u_{holj} \times r_{u_j} \tag{2.13}$$

   where, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$; r_{y_i} and r_{u_j} are the user specified percentage relaxation in the limits for y_i and u_j. The inequalities defined for the problem allow 5% constraint limit violation (Latour *et al.* 1986, Martin *et al.* 1991).

5. *Constraint tuning for desired yield*:

   The constraint tuning guidelines for achieving the target value of return or yield from the system can be obtained by performing optimization defined for this case. If the ratio of target yield and the ideal yield is R_c then the constraint tuning guidelines, $\hat{r}_{y_i}$ and $\hat{r}_{u_j}$ for y_i and u_j, respectively, can be obtained as a solution to the optimization problem discussed here. The optimization for the scenario can be defined as:

$$\min_{\bar{y}_i, \bar{u}_j, \hat{r}_{y_i}, \hat{r}_{u_j} \hat{r}} r \tag{2.14}$$

   subject to,

$$L_{y_i} + 2 \times \sigma_{i0} - y_{holi} \times \hat{r}_{y_i} \leq \bar{y}_i \leq H_{y_i} - 2 \times \sigma_{i0} + y_{holi} \times \hat{r}_{y_i} \tag{2.15}$$

$$L_{u_j} + 2 \times R_{j0} - u_{holj} \times \hat{r}_{u_j} \leq \bar{u}_j \leq H_{u_j} - 2 \times R_{j0} + u_{holj} \times \hat{r}_{u_j} \tag{2.16}$$

$$0 \leq \hat{r}_{y_i}, \hat{r}_{u_j} \leq 1 \tag{2.17}$$

   where, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. The equality constraints for the problem remain the same as in equations (2.3), (2.4) and (2.5).

The economic performance assessment of the controller can now be done using the information obtained after performing the optimizations discussed above. Two terms, *economic performance index* (η_E) and *theoretical economic performance index* (η_T), are defined to assess the economic performance.

$$\eta_E = \frac{\Delta J_E}{\Delta J_I} \tag{2.18}$$

$$\eta_T = \frac{\Delta J_T}{\Delta J_I} \tag{2.19}$$

where, ΔJ_E is the optimal yield without tuning of the controller, as obtained from case-2, ΔJ_I is the ideal yield, as obtained from case-1 and ΔJ_T is the upper bound of the theoretical yield that can be achieved through minimum variance control plus steady state optimization, as obtained in case-3 for minimum variance control. By comparing the values for η_E and η_T the following relation holds:

$$0 \leq \eta_E \leq \eta_T \leq 1 \tag{2.20}$$

2.3 LMIPA analysis for a Binary Distillation Column

2.3.1 Description of the Binary Distillation Column

The column (fig- 2.1) is a binary distillation column which is used to separate light petrol and the heavy petrol from the petrol obtained from an upstream desulphurization unit (Volk *et al.* 2005).The light petrol comprises of the components in the boiling range of 30 to 65^oC and the heavy petrol has the hydrocarbon components in the boiling range of 65 to 180^oC.

The feed to the column is heated by steam and flashed into the column. The lighter fractions of the hot feed vaporize and are collected as the top product in the overhead vessel and the heavier components are obtained at the bottoms of the column. The vapors from the top of the column are cooled and condensed by air fin coolers. A part of the condensed overhead vapors are sent back into the column as the reflux and the rest is drawn, under a level control, as the top product i.e. the light petrol. The reflux helps in maintaining the column top temperature and helps in maintaining the vapor liquid traffic in the reflux zone of the column, which helps in better fractionation. The column operating pressure is maintained by a split range control which adjusts the cooling of the overhead vapors and if required, vents off the incondensable components from the column, as the off gas. The heavier components in the feed do not vaporize and are obtained from the column bottoms. A part of the bottom stream is sent to the reboiler, where it is reboiled by a heating medium using a duty controller. The heating medium is heated in the reboiler furnace, whose duty is required to be kept constant. The vapors from the reboiler are sent back into the column and they assist in stripping off any lighter components that could not flash into the flash zone of the column and are carried to the bottom of the column. The balance of the heavier components is drawn from the column as the heavy petrol, under a bottom level controller. The column has the following basic control strategies:

1. *Reflux Control:* The basic control is a PI control loop which controls the reflux flow to the column.

2. *Pressure Control:* The pressure controller is a split range controller. It takes care of the column pressure by adjusting the cooling of the vapors from the column top and if required by venting off any non-condensable. At 50% the cooling is at its maximum value by adjusting the air fin cooler, a further decrease in pressure is done by venting off the off gas to the flare by operating the flare valve, which is undesirable.

3. *Feed Temperature Control:* This controls the temperature of the feed to the column by controlling the steam flow in the feed heater. Increasing the feed temperature also helps in reducing the duty consumption in the reboiler.

4. *Reboiler duty:* The reboiler duty can be controlled by adjusting the flow of the heating medium.

5. *Reflux drum level control:* The level in the reflux drum is controlled by controlling the light petrol flow.

6. *Reboiler level control:* The level in the reboiler is maintained by controlling the flow of the heavy petrol.

2.3.2 MPC control strategy and objectives

The MPC controller designed for the system described above has 4 input variables and 10 output variables. The main control objectives for the MPC controller are to set the range under consideration and to minimize the variability. The main process parameters and

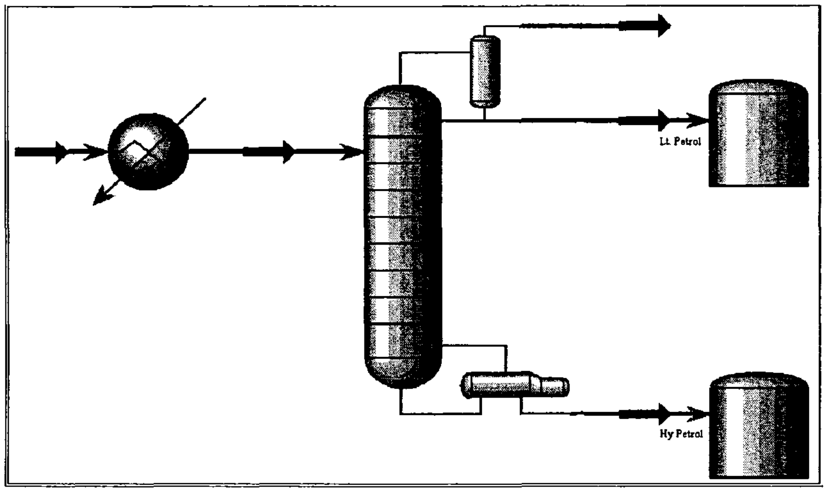

Figure 2.1: Binary Distillation Column

variables to be optimized are the light petrol Final Boiling Point (FBP), column bottom Pressure Compensated Temperature (PCT), reboiler furnace duty and column pressure. The constraint variables that are to be maintained within the set constraints while the above mentioned parameters are optimized are the feed temperature, top PCT, reflux flow, pressure valve position, feed temperature valve position, bypass valve position.

1. *Reflux Flow:* The reflux flow is essential to be maintained within a certain limits as having less reflux into the column will reduce the vapor-liquid traffic in the reflux zone of the column, which will have undesirable effect on the fractionation.

2. *Light Petrol FBP:* The FBP of the light petrol is essential to be maintained as it defines the quality of the main product from the column. The FBP is the boiling temperature of the product, if 99% of the product has been evaporated.

3. *Top PCT:* Top PCT is another indication of the top product quality. It shows a change in the distillate quality earlier than the measured FBP.

4. *Pressure Valve Position:* The pressure valve position is always required to be less than 50% as for position greater than 50% the off gas flare valve will open, which is undesirable.

5. *Column Bottom PCT:* PCT is a cheaper alternative to the bottom product quality measurements. PCT reflects the pressure and temperature measurement at bottom of the column.

6. *Column Pressure:* The column pressure is to be kept minimum so as to reduce the energy consumption in the reboiler furnace.

7. *Feed Temperature:* The column feed temperature is desired to be maintained as close as possible to the flash tray temperature and can be increased to reduce the reboiler furnace duty.

8. *Reboiler Furnace Duty:* The heating oil in the reboiler is heated in the reboiler furnace, whose duty is required to be kept constant. This can be achieved by adjusting the bypass flow of the heating oil, so as to maintain constant reboiler pressure drop. Maintaining constant pressure drop, across the bypass valve, ensures constant furnace load.

9. *Duty:* The duty of the reboiler is to be maintained, to ensure some minimum flow through the reboiler furnace.

10. *Bypass Valve Position:* The bypass valve maintains the bypass to the reboiler, in order to maintain a constant pressure drop across the reboiler and thus constant reboiler furnace duty. The valve position is required to be maintained in a range so that hardware constraints of the valve are never violated.

The input variables or the MVs that act as the handles to optimize and maintain the above discussed controlled variables are:

1. *Reflux Flow Controller Set Point:* The reflux flow helps in maintaining the column top temperature and it helps maintaining adequate liquid traffic in the reflux zone of the column to ensure proper separation between the light and heavy petrol. The minimum limits are to be set to ensure that the reflux pump has minimum flow required and that the trays do not run dry. The maximum limits are to be set to ensure no flooding in the trays of the column.

2. *Column Pressure Controller Set Point:* The column pressure is an important handle as it affects all the Controlled Variables. This needs to be minimized to reduce the energy consumption. The minimum constraints are to be set so as to avoid jet flooding in the column. The high limit is to be set so as to ensure perfect separation and to ensure that the column is operated well below the column safety valve setting.

Table 2.1: LQ optimization coefficients

	CV#										MV#			
	1	2	3	4	5	6	7	8	9	10	1	2	3	4
Linear Coef	0	2	0	0	1	1.5	0	1.75	0	0	0	0	0	0
Quad Coef	0	0	0	0	0	0	0	0	0	0	0	0	0	0

3. *Feed Temperature Control Valve Position:* This valve controls the steam flow through the feed heater. The minimum and maximum constraints for this MV define the hardware limitation of the control valve.

4. *Duty Valve Position:* The minimum constraints for this MV are to be set to ensure some minimum furnace circulation and to protect it from film evaporation in case of failure of bypass valve and to keep minimum heat transfer coefficient. The maximum constraints are set to avoid meeting hardware constraints of the valve.

The steady state gain matrix for the process can be given as:

$$K=\begin{bmatrix} 0.9500 & 0 & 0 & 0 \\ -0.0469 & -4.9852 & 0.1341 & 0.3324 \\ -0.0600 & -6.3750 & 0.1715 & 0.4250 \\ 0 & -25.000 & 0 & 0 \\ -0.0170 & -8.8125 & 0.1930 & 0.5100 \\ 0 & 0.3750 & 0 & 0 \\ 0 & 0 & 0.2680 & 0 \\ 0 & 0 & -0.0700 & 0 \\ 0 & 0 & 0 & 0.1700 \\ 0 & 0 & 0 & 0.6650 \end{bmatrix}$$

2.3.3 Case study

The study was carried out for all the five scenarios, discussed in section 2.2, for the distillation plant controlled by an MPC. Based on the control objective mentioned in Volk *et al.* (2005), the linear quadratic objective function coefficients required to carry out the LMIPA analysis are defined as given in table- 2.1. The base case mean operating points and standard deviation for the output variables are listed in table- 2.2. Table- 2.3 lists the mean operating points and the quarter of the range for the input variables.

Table 2.2: Base case mean operating point and Std Dev of output variables

CV#	1	2	3	4	5	6	7	8	9	10
$\bar{y}_{i0}$	322.39	68.00	60.99	20.11	85.81	0.16	70.09	-500.03	4.75	47.50
Std Dev, σ_{i0}	11.40	0.13	0.35	0.75	0.57	0.05	1.12	3.13	0.12	0.01

Table 2.3: Base case mean operating point and Std Dev of output variables

MV#	1	2	3	4
$\bar{u}_{j0}$	497.16	0.19	50.32	81.00
Qtr of Rng, R_{j0}	33.14	0.01	5.00	0.03

Based on the information provided above, the optimization function can be defined as equation (2.21) and the constraints can be defined by substituting K and the values provided in tables 2.1, 2.2 and 2.3.

$$J = \sum_{i=1}^{10} \left(\alpha_i \times \bar{y}_i + \beta_i^2 (\bar{y}_i - \mu_i)^2\right) + \sum_{j=1}^{4} \left(\alpha_j \times \bar{u}_j + \beta_j^2 (\bar{u}_j - \nu_j)^2\right) \tag{2.21}$$

The constraints for the problem are defined using the steady state gain, K, and the information given in tables above.

The results obtained for the five cases discussed in section-2.2 are discussed below:

1. *Assessment of ideal yield*: The assessment of ideal yield for the process was done by the optimization problem given by equation (2.2) subject to equations (2.3) to (2.7). The ideal yield for the process is assessed to be 28.48 units.

2. *Assessment of optimal yield without tuning the controller*: To assess the optimal yield from the process, without tuning the controller, the optimization problem is given by equation (2.2) subject to equations (2.3) to (2.5) and equations (2.8) and (2.9). The optimal yield from the the process without tuning the controller is assessed to be 20.24 units.

3. *Assessment of improved yield by reducing variability*: To improved yield by reducing the variability is assessed by the optimization problem given by equation (2.2) subject to equations (2.3) to (2.5) and equations (2.10) and (2.11). The performance indices

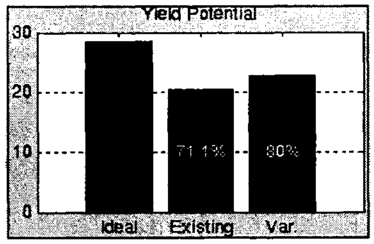

Figure 2.2: Yield pattern

obtained from MVPA with MVC as benchmark are used to assess the improved yield by reducing the variability. The improved yield is assessed to be 22.78 units.

4. *Assessment of improved yield by constraint relaxation*: This case provides with the sensitivity of the process yield to constraint relaxation. The optimization problem for this case is defined by equation (2.2) subject to equations (2.3) to (2.5) and equations (2.12) and (2.13). The analysis shows that the process yield is most sensitive to CV5 constraint changes.

5. *Constraint tuning for desired yield*: Setting the target yield as 85% of the ideal yield the tunign guidelines are obtained from the optimization problem defined in equation (2.14) subject to equations (2.3) to (2.5) and equations (2.15) to (2.17). The tuning guidelines thus obtained are given in table-2.4 and shown in fig-2.3.

The results from the analysis showed that under ideal conditions the return are expected to be 28.48 units; however, the optimum yield without the controller tuning is about 20.24 units (fig- 2.2). As for the optimum yield estimation, the variability and the constraints for the controller are not changed, a high value for the optimum yield indicates that currently the controller is running at points far from the optimal constraints. This also indicates that there is one or more process variables that has high variance. This goes in accordance to the data used for the analysis which shows that CV1 has a very high variance of 11.40 and this restricts the controller from operating at points close to optimum operating points.

A high value for the optimum yield, without tuning of the controller, also indicates that with the current controller the plant is operated at points far from the optimum values.

Table 2.4: Suggested constraint relaxation(%age) for the MPC controller

	CV#										MV#			
	1	2	3	4	5	6	7	8	9	10	1	2	3	4
Constraint relaxation	0	0	0	0	6.75	0	0	0	6.75	0	6.75	0	6.75	0

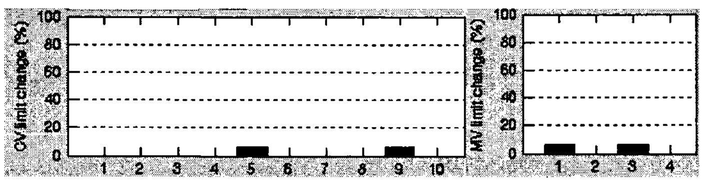

Figure 2.3: Suggested constraint tuning

This is also reflected by the mean operating points for CV2, CV5, CV6 and CV8. These CVs are operated at 68.00, 85.81, 0.16 and -500.03, while their operating limits are (40, 70), (80, 120), (0.1, 0.4) and (-1000, 0), respectively. For the assessment of optimal yield without tuning of the controller the limits for the CVs, estimated using equations (2.8 and 2.9), are given in table- 2.5. Thus, with reference to the LQ optimization coefficients, the optimum operating points for the CV2, CV5, CV6 and CV8 are 40.25, 81.15, 0.21 and -993.73 respectively. It can be seen that the controller is currently operating at points far away from the optimum.

Setting a target yield as 85% of the ideal yield (24.21 units), the constraints tuning guidelines for the controller are obtained. Since return equivalent to 20.24 units can be achieved by optimal operations of the controller under the base case conditions, it is only a small fraction of the return which have to be extracted by constraint tuning. Thus, the controller tuning will contribute to the balance $24.21 - 20.24 = 3.97$ units of the return. The constraint relaxations suggested by the algorithm are listed in table- 2.4 and shown in fig- 2.3, the effect of suggested constraint tunings is discussed below.

Table 2.5: CV Limits considered for assessing optimal yield without tuning

CV#	1	2	3	4	5	6	7	8	9	10
Low Lt	22.79	40.35	40.40	1.50	81.15	0.21	62.86	-993.73	4.24	0.02
High Lt	627.21	69.75	69.30	58.49	118.85	0.29	92.76	-6.27	5.25	94.98

1. *MV3, Feed Temperature Control Valve Position:* The constraint limits set for MV3 are (40, 60) and the mean operating point is 50.32. The operating range for this MV is also (40, 60). Thus, relaxing its constraint limits will assist in giving the controller more degrees of freedom to manipulate the feed temperature. The controller can increase the feed temperature and thus can reduce the furnace duty. However, this will also increase the light petrol FBP and the column bottom PCT (CV2 and CV5) but these can be taken care of by increasing the reflux and reducing the column operating pressure.

2. *MV1, Reflux Flow Controller Set Point:* Increasing the reflux to the column helps in better fractionation in the reflux zone of the column and also has a cooling effect on the column. For the data set provided the column reflux is operated in the range (433, 565). With this range of operations, the constraints need to be relaxed, so that the furnace duty is minimized to achieve the target return of 20.80 units. Relaxing the constraints for the column reflux will enable the controller to increase the reflux so as to minimize the effect of the increase in feed temperature on the top and the bottom temperatures and product qualities.

3. *CV5, Column Bottom PCT:* The bottom PCT is an indicative for the cut between the top and the bottom product. The hard limits in which, this CV is to be maintained are (80, 120) and the mean operating point is 85.81 units. As this CV is also to be minimized, the value of 81.15, the low limit for the current operating conditions, for this CV can be taken as the optimum operating point (table- 2.5). Since the current operating point is close to the optimum operating point, any action taken by the controller to lower the CV2 value will also decrease the CV5 value. Thus, it is essential that the constraint limits for CV5 be reduced so that CV2 can be reduced to its optimum operating point. Therefore, there is a suggestion for relaxing the constraints for CV5.

4. *CV9, Duty:* Relaxing the constraints for this CV will provide more room for the duty valve, MV4, to operate and thus CV2, CV3 and CV5 can much better be optimized.

The results show that the ΔJ_I=28.48 and ΔJ_E=20.24. The performance indices from MVPA with MVC as the benchmark are given in fig- 2.4 and accordingly, the ΔJ_T=22.78, The *economic performance assessment*, η_E, of the process is calculated as 71.1% and the *theoretical economic performance assessment*, η_T, is 80.0%. Thus, there exists a large scope

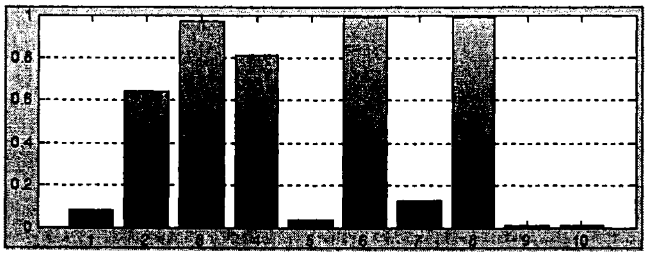

Figure 2.4: Performance Indices for CVs

in improving the economic performance of the controller.

Section-2.4 (pages 27(part), 28 to 32 and 33(part)) has been removed from this thesis for proprietary reasons.

2.5 Conclusion

The LMIPA performance assessment methodology is important work in the field of MPC economic performance assessment. It is based upon the mean optimization values of the process variables and shows that benefit potential can be achieved by constraint tuning of process variables, which will enable the process to reach optimum operating conditions.

The LMIPA performance assessment takes into consideration the mean operating point for the analysis and optimization purposes. However, the mean operating point is not the best representation of actual process operations. Considering the data distribution determined by the mean and the standard deviation is a method to be presented next. Thus, the benefit estimation will be done taking into consideration of the probabilities of the CVs.

Beyesian methods will be proposed as a tool for taking into consideration these probabilities and providing the tuning guidelines for MPC.

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3

MPC constraint tuning - Bayesian approach via deterministic LQ objective function

Abstract: Performance assessment of Model Predictive Control (MPC) systems has been focused on evaluation of variability-based performance with minimum variance or LQG/MPC simulation as benchmarks. These previous studies are mainly concerned with the dynamic disturbance regulatory performance of MPC. However, the benefit of MPC is largely attributed to its capability for steady-state economic optimization. The steady state economic performance, on the other hand, also depends on the variability reduction achieved through dynamic control. There is a need to assess MPC performance by considering steady state economic performance, variability and their relations. One of the best indications for this relation is the constraint setup or tuning. In practical MPC applications, the constraint setups are important when an MPC is commissioned, and constraint adjustments are not uncommon even when the MPC is already on-line. Thus the questions to ask are whether the constraint can be adjusted, which constraints should be adjusted, and what is the benefit to do so. By investigating the relation between variability and constraints, the problems of interest are solved under Bayesian Statistics framework: namely decision evaluation and decision making. The decision is referred to whether to adjust constraint to achieve optimal economic MPC performance and which constraints to adjust. Detailed case study for a distillation column MPC application is provided to illustrate the proposed MPC performance assessment and tuning methods.

3.1 Introduction

With globalization, there is an increase in the commercial and technical competitive pressures on the industries. This has driven the industries to improve their manufacturing efficiency, continually reduce their production costs and losses, maintain product qualities and reduce off-spec product generation. Process Control can help in improving the manufacturing efficiency by ensuring reliable operations and consistent production of quality products. According to Benson (1997), "Process Control ensures that the plant operates predictably in the most profitable range, leading to greater output of consistent product, reliability, yield and quality using less energy".

Model Predictive Control (MPC) is an advanced method of Process Control, which relies on the model of the process. It predicts the behavior of the Controlled Variables (CVs) based upon the Manipulated Variables (MVs) to compute a cost minimizing control. A multivariable MPC also takes into consideration the effect of change in one process variable on the other process variables, due to interaction between them. Thus, MPC provides a better control of the process operations as generally the variables in process plants are highly interactive. But having a multivariable controller is not just enough to serve the purpose. They need to be tuned properly with a proper understanding of the process behavior and the control philosophy adopted for the MPC. There are different commercial controllers available in the market that adopt different control philosophies (Qin and Badgwell 1997) but they all need to be tuned at the design and engineering level.

MPC controller tuning is primarily being restricted to the tuning of the penalty matrix on the output error and/or control moves so as to minimize the squared error of the controller output over the control horizon (Drogies 1999, Ou and Rhinehart 2002). This tuning approach is subjective in nature as the design of the penalty matrix for tuning is guided by the fact as to what is more important, the tampering of Manipulated Variable (MV) actions or to maintain the CVs within the constraints. The other tuning parameters involved are, for example, the prediction horizon and the control horizon. Tuning of MPC controllers with these parameters is done at the engineering level and requires a thorough understanding of the process and the control philosophy of the MPC application used. Though these are the key tuning parameters for an MPC controller, there are also some other factors that contribute to the performance of an MPC controller, such as, the CV/MV constraints and CV/MV variability. The CV/MV constraints should be carefully chosen as giving wrong constraint limits in CVs or MVs can lead to poor performance of the controller.

In an MPC controller, CVs generally reflect the desired product qualities, subject to the constraints, which are to be considered for optimization and the MVs are the handles available for controlling and optimizing the CVs within the set constraints. Thus, it is essential to provide the controller with proper constraint limits for CVs and MVs. The MVs are always within the range specified; however, the CVs can be under-spec (values less than the low limit), over-spec (values greater than the high limit) or in-spec (values within the limits). Having an in-spec value for the CVs cannot always be called a good control as within the limits there also exists an optimum operating point, at which the performance of the MPC controller is maximized under the given set of conditions. Due to the presence of variability in the process, CVs have probabilities associated with them to be under-spec, in-spec or over-spec. Also, the variability associated with the CVs defines how far or close the CVs are to the optimum operating point, which typically is located at the constraint limits. Thus depending upon the probabilities of CVs, to be under-spec, in-spec or over-spec, the expected return (Rahim and Shaibu 2000) from the controller can be estimated.

This work is driven by industrial needs and motivated by the following facts: 1) When commissioning an MPC and determining the constraint limits, it is highly desirable to have a tool to estimate the impact (or sensitivity) of the constraints on the performance, thus reducing the conservativeness when setting the constraints. 2) Even during operations of an MPC controller it is at times required to change the constraints of one or more CVs and MVs. This affects the performance of the controller. Some changes can improve the performance while others may have little effect on the performance. 3) Profit or return of advanced process control is often estimated based on average operating conditions. In Xu *et al.* (2007) the following problem has been considered: Assume that constraint limits or the variability of certain CVs and MVs can be adjusted and tuning of MPC is limited to the adjustment of the constraint limits and/or variability only. Upon adjustment of the constraint limits, CVs/MVs operating points move to their optimum. The new operating points are determined through steady state optimization according to an economic objective function and the return (profit), by adjusting the constraint limits, can be calculated accordingly. However, this procedure only considers the average operation and return is calculated based on the average response. The actual response can be different from the optimum owing to the variability. This study is thus to provide a probabilistic method to evaluate the effect of the change in the constraints, of the process variables, on the overall economic performance of the MPC controller, by taking into account the uncertainties introduced by the variability. The study also aims at providing the maximum a posteriori (MAP) explanation for the decisions to be made for achieving target value of return. The algorithm exploits the Bayesian methods for making the decisions to achieve the target value of return. Bayesian Statistics is a branch of statistical inference technique that deals with probabilities of occurrence of certain events given certain set of conditions or observations (Charniak 1991).

To make Bayesian inference and for decision making, what we need in the proposed approach is the plant routine operating data, the plant steady state gains, and other related process information such as, which CVs and MVs are allowed to change their limits and what is the preference to make any change. The algorithm performs optimization on the various combinations of the constraint changes on these CVs and MVs to establish the new optimum operating points under changed conditions and then to establish the new under-spec, in-spec and over-spec probability of the CVs. The new probability is then used to estimate the expected return through Bayesian methods.

A Bayesian network can be developed for the system with the CVs and the MVs, for which the constraints can be changed, as the parent node. All the quality CVs are the child nodes. The new probabilities estimated through the optimization are used to form the Conditional Probability Distribution Tables (CPT or CPD) for the network (Korb and Nicholson 2004, Pearl and Russel 2000). Once the Bayesian network for the system has been created, the network can then be provided with various cases of evidences to evaluate the probabilities for CVs to be in various states and thus evaluates the return.

The contribution of the chapter can be summarized as: 1) A systematic approach to Bayesian analysis of the decisions related to the constraint changes for CVs and MVs. 2)

Guidelines for constraint changes are derived according to the statistical inferences based upon the target value of return anticipated from the controller. The remainder of the chapter is organized as follows: Section 3.2 explains the preliminary concepts, where the problem is defined and some basic concepts of Bayesian methods are explained. Section 3.3 derives the Bayesian methods for MPC constraint analysis and tuning followed by an illustration in section 3.4. Section 3.5 and 3.6 provide case studies on a binary distillation column and an industrial distillation column, respectively. The conclusions are presented in section 3.7.

3.2 Preliminaries

3.2.1 Defining the problem

For illustration purposes, consider the process data as shown in fig- 3.1. As can be seen from the figure that even though the mean values for this set of data is within the specified limits, there are instances when the process values lie outside the limits. And if this data set represents a quality variable then having its value outside the set limits is undesirable as it can render the product unmarketable.

Decisions to change the constraints for an MPC controller, obtained by the LMIPA method, as discussed in Chapter 2, are based upon the mean operating point. As stated above, even when the mean operating point is within the constraint limits, at instances, data may lie outside the limits. Thus, it is essential that the decisions obtained from LMIPA method be analyzed taking into account the probabilities associated with the data to lie inside and outside the limits. The probabilities can be used to estimate significant relation between relaxation of particular set of constraints and the final plant performance.

The limits for the CVs and the MVs and their variability determine the optimum operating point for the MPC controller. Relaxing the limits for one or more process variables provides the controller with increased degrees of freedom and thus may help in improving the expected return even if there is no reduction in the variability of the variables. This is indicated in fig- 3.2.

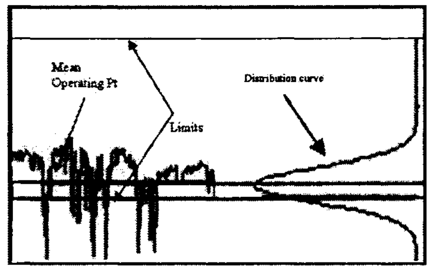

Figure 3.1: Base case data and distribution for a process variable

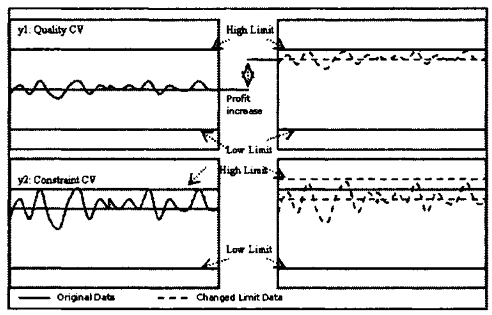

Figure 3.2: Effect of change in limits

As can be seen from fig- 3.2 that even though y_1 is far from the optimum operating point, the high limit, it may not be further raised to optimum value as y_2 is hitting the constraints at the high limit. Relaxing the constraint for y_2 on the high limits may provide the controller with additional degrees of freedom and y_1 may be raised to a point closer to the optimum operating point, leading to improvement of the controller performance. Thus it is essential to quantify the interaction between them in order to analyze the effect of change in constraints of one variable on the other variables. Using Bayesian methods, the effect can be quantified in terms of probabilities for the quality variables to be within the limits or outside the limits.

Since the limits for CVs/ MVs for an MPC controller cannot be changed dramatically, in this study, 10% constraint relaxation is considered.

3.2.2 Bayesian Statistics

Named after Thomas Bayes, Bayesian analysis is a branch of statistical inference that can be applied for decision making and statistical analysis using knowledge of prior events to predict future events. The Bayes theorem forms the backbone of Bayesian analysis. It enables calculating conditional probabilities for a hypothesis (Korb and Nicholson 2004, Tan 2001) and is also known as the principle of inverse probability.

Probability for a hypothesis A conditional on a given evidence B is the ratio of probability of the conjunction of A and B to the probability of B (Korb and Nicholson 2004) i.e.

$$\begin{aligned} P(A|B) &= \frac{P(A,B)}{P(B)} \\ &= \frac{P(B|A) \times P(A)}{P(B|A) \times P(A) + P(B|\neg A) \times P(\neg A)} \end{aligned} \tag{3.1}$$

where, $P(A)$ is the prior probability of occurrence of hypothesis A, also known as priori. $P(B|A)$ is the likelihood of obtaining evidence B given hypothesis A is true and $P(A|B)$ is the posterior probability of A to be true given the evidence B is obtained. $\neg A$ represents the case when the hypothesis A is not true.

3.2.3 Bayesian networks

For a system comprising of more random variables than A and B, a network connecting all variables can be built. This network representing the relationship between the various random variables is called a Bayesian network. A Bayesian network is defined by Korb and Nicholson (2004) as "a graphical structure that allows us to represent and reason about an uncertain domain. The nodes in the network represent a set of random variables." A pair of nodes are connected through directed arcs that represent a relationship between the nodes. The node through which the arc originates is called *parent node* and the node where it terminates is called *child node*. The nodes in a Bayesian network are the variables of interest and the link between them represents the probabilistic dependencies among the nodes (Pearl and Russel 2000). To specify the probability distribution of a Bayesian network, prior probabilities are to be defined for the root nodes i.e. the nodes with no

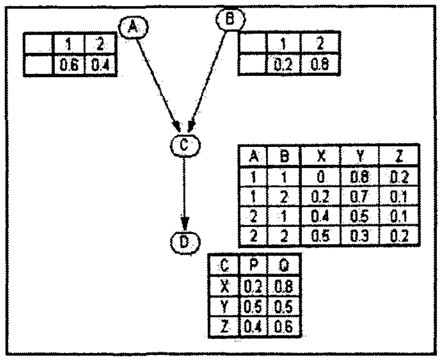

Figure 3.3: Typical Bayesian network

predecessor and the Conditional Probability Distribution Table (CPD or CPT) is defined for all non-root nodes, for all possible combinations of their direct predecessors (Charniak 1991). The CPT quantitatively represents the relationship between the parent and the child nodes. A typical Bayesian network is shown in fig- 3.3. Node C has two parent nodes A and B and one child node D. Nodes A and B have two states $(1, 2)$, node C has three states (X, Y, Z) and node D has two states (P, Q). The tables beside node A and B are their prior probabilities and those besides node C and D represent their CPT.

A Bayesian network cannot have directed cycles, i.e. a node cannot be reached again by following the directed arcs from the child nodes directly. Thus Bayesian networks are also called Directed Acyclic Graphs (DAGs). Through DAGs the parameters can be represented as nodes or random variables and be associated with a prior distribution. DAGs which include decision and utility nodes as well as chance nodes are known as *influence diagrams* or *decision networks* and be used for optimal decision making (Murphy 2001*b*).

The Conditional Probability Distribution Tables quantify the dependencies between the nodes. The CPD's are probability distribution $P(x_i|Pa_i)$, where x_i is the i^{th} node and Pa_i represents all of its Parent nodes (Murphy 2001b, Murphy 2004). There are three types of nodes, two of which are used in this paper:

1. *Chance Nodes*: These nodes represent random variables and are associated with CPT.
2. *Utility Nodes* or *Value Nodes*: These nodes represent the value of the utility function (benefit function). The parents for these nodes are the nodes whose outcome directly

affects the utility. These nodes are associated with utility table, with the value for each possible instantiation of its parents perhaps including an action taken (Korb and Nicholson 2004).

Thus, if x_j is the set of observed variables, x_i is the set of variables whose values we are interested in estimating, x_k is the set of variables in the system, not included in x_i and x_j, then inference in a Bayesian analysis means to compute:

$$\begin{aligned} P(x_i = a | x_j = b) &= \frac{P(x_i = a, x_j = b)}{P(x_j = b)} \\ &= \frac{\sum_{x_k,(k \neq i,j)} P(x_i = a, x_j = b, x_k)}{\sum_{x_k, x_i,(k \neq i,j)} P(x_i, x_j = b, x_k)} \end{aligned} \quad (3.2)$$

where, $P(x_i = a|x_j = b)$ is the probability for node x_i to take value a provided that node x_j takes the value b, and $P(x_j = b)$ is the probability for node x_j to take value b and $P(x_i = a, x_j = b)$ is the probability of conjunction of x_i and x_j.

This can be illustrated by the following example (Wikipedia n.d.). Suppose that a test for a particular disease has a very high success rate if a tested patient has the disease, the test accurately reports 'positive' with 99% probability and if a tested patient does not have the disease, the test accurately reports 'negative' with 95% probability. Suppose also, however, that only 0.1% of the population has that disease. We now have all the information required to use Bayes's theorem to calculate the probability that, given the test is positive, the test is a false positive.

Let A be the event that the patient has the disease, and B be the event that the test return a positive result. Then, using the Bayes's theorem the probability of a true positive is

$$P(B) = P(B|A) \times P(A) + P(B|\neg A) \times P(\neg A) \quad (3.3)$$

$P(B)$ is the probability that a given person tests positive. This depends on the two populations: those with the disease (and correctly test positive 0.99×0.001) and those without the disease (and incorrectly test positive 0.05×0.999).

$$\begin{aligned}
P(A|B) &= \frac{P(A,B)}{P(B)} \\
&= \frac{P(B|A) \times P(A)}{P(B|A) \times P(A) + P(B|\neg A) \times P(\neg A)} \\
&= \frac{0.99 \times 0.001}{0.99 \times 0.001 + 0.05 \times 0.999} \\
&= 0.0198
\end{aligned}$$

and hence the probability that a positive result is a false positive is about $(1 - 0.0198) = 0.9802$.

Despite the apparent high accuracy of the test, the incidence of the disease is so low (one in a thousand) that the vast majority of patients who test positive (98 in a hundred) do not have the disease. It should be noted that this is quite common in screening tests. It is therefore, more important to have a very low false negative rate than a high true positive rate.

The probability distributions form the basis for the statistical analysis of the data. There are a number of probability distributions functions, in this work, the data is assumed to be Gaussian distribution. Mathematically Gaussian distribution is represented as:

$$p(x) = \frac{1}{\sqrt{2\Pi\sigma^2}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}} \tag{3.4}$$

where, x is the data with mean $\bar{x}$ and standard deviation σ.

$p(x)$ plotted against x gives the probability density function and integral of $p(x)$ in the range $-\infty$ to x gives the cumulative distribution function. Cumulative distribution function can mathematically be represented as:

$$cdf(x) = \frac{1}{2}\left(1 + erf\left(\frac{x-\bar{x}}{\sigma\sqrt{2}}\right)\right) \tag{3.5}$$

$$erf(x) = \frac{1}{\sqrt{\Pi}} \int_0^x e^{-t^2}\, \mathrm{d}x \tag{3.6}$$

3.3 Bayesian methods for MPC constraint analysis and tuning

For an MPC application with n inputs and m outputs, let K be the steady state gain matrix and $(\bar{y}_{i0}, \bar{u}_{j0})$ be the current mean operating points, which are defined as the base case operating points (Xu *et al.* 2007). Also let the number of CVs for which it is allowed to change (by relaxation) their constraint limits be a and the number of MVs for the same be b. Thus a total of $N = a+b$ variables are available for which it is possible to make the change. With *yes* and *no* as the options for applying the limit change to these N variables, there are 2^N combinations for applying the constraint changes. Each change combination will have a specific optimal return, which can be obtained through optimization of the operating point, and thus will affect the MPC performance.

The optimization is carried out for each possibility, with the real time CV/MV data collected. The objective function is the economic benefit function and the constraints are CV and MV constraint limits, by taking into account the variability and the steady state gain relations. For simplicity of the presentation, only CVs have been considered as the quality variables that affect the economic benefit function. Without the loss of generality, we assume first q CVs, $y_1, \dots, y_q$, as quality variables. Thus, the optimization problem can be defined as the linear-quadratic function (Xu *et al.* 2007):

$$J = \sum_{i=1}^{q} \left(\alpha_i \times \bar{y}_i + \beta_i^2 \left(\bar{y}_i - \mu_i \right)^2 \right) \tag{3.7}$$

where, $\bar{y}_i$ and μ_i are the mean operating point and the target operating point for y_i, respectively. α_i and β_i are the linear and quadratic coefficients for y_i.

With $(\bar{y}_{i0}, \bar{u}_{j0})$ as the base case mean operating point, $(\bar{y}_i, \bar{u}_j)$ as the optimum operating point, when the base case operating points are moved by $(\Delta y_i, \Delta u_j)$, the equality constraints to be satisfied for the economic objective function are (Xu *et al.* 2007):

$$\begin{aligned} \Delta y_i &= \sum_{j=1}^{n} [K_{ij} \times \Delta u_j] && (3.8) \\ \bar{y}_i &= \bar{y}_{i0} + \Delta y_i && (3.9) \\ \bar{u}_j &= \bar{u}_{j0} + \Delta u_j && (3.10) \end{aligned}$$

Considering that upto 5% constraint violation is acceptable (Latour *et al.* 1986, Martin *et al.* 1991) for the output variables, a set of inequalities can be defined, which also need to be satisfied while optimizing the objective function defined in equation (3.7).

For the change of the constraint limits case, the inequalities for the objective function are defined by equations (3.11) and (3.12). These inequalities define the new limits for the CVs and MVs (Xu *et al.* 2007):

$$L_{y_i} + 2 \times \sigma_{i0} - y_{holi} \times r_{y_i} \leq \quad \bar{y}_i \quad \leq H_{y_i} - 2 \times \sigma_{i0} + y_{holi} \times r_{y_i} \tag{3.11}$$

$$L_{u_j} + 2 \times R_{j0} - u_{holj} \times r_{u_j} \leq \quad \bar{u}_j \quad \leq H_{u_j} - 2 \times R_{j0} + u_{holj} \times r_{u_j} \tag{3.12}$$

where, σ_{i0} is the base case standard deviation for y_i, R_{j0} is the base case quarter of range for u_j. L_{u_j} and H_{u_j} are the low and the high limits for u_j respectively. L_{y_i} and H_{y_i} are the low and the high limits for y_i respectively. r_{y_i} and r_{u_j} are the allowable change (percentage of the range) in the constraint limits for the process variables i.e. 10% for changeable constraints and 0% for others, y_{holi}, u_{holj} are half of the limits for y_i and u_j respectively.

Thus, the economic objective function for each of the constraint tuning case can be specified as (Xu *et al.* 2007):

$$\min_{\bar{y}_i, \bar{u}_j} J \; \textit{subject to} \; (3.8), (3.9), (3.10), (3.11), (3.12) \tag{3.13}$$

Thus, 2^N optimum operating points are obtained for each y_i, for the constraint change case. Superimposing the 2^N optimum operating points with the base case variability and assuming the data to be Gaussian distributed, the probability distribution for the data is obtained.

The Bayesian network is now created with N parent nodes, q child nodes and one utility node. For the limit change case, the parent nodes have two states (*yes, no*) where *yes* means *to change the limits* and *no* means *not to change*. By changing the constraint limits, the quality CVs are optimized to be operating as close as possible to their maximum return. In the same time, other non-quality CVs are also moved due to the interaction. Each optimal values taken by the CVs is called its state and it can take any value within the operaing range and is continuous. The expected return from the process can therefore be calculated as:

$$E(R) = \int_{y_1,\dots,y_q} p(y_1,\dots,y_q)F(y_1,\dots,y_q)dy_1\dots dy_q \tag{3.14}$$

where $p(y_1,\dots,y_q)$ is a probability density function, and $F(y_1,\dots,y_q)$ is a profit function. In general q is a small integer, and often there is only one quality variable ($q = 1$), for example, purity of the product, that reflects economic return while others are controlled variables, which greatly simplifies the computations. However, for $q \neq 1$, with linear or quadratic economic objective function, $F(y_1,\dots,y_q)$ has the following additive form:

$$F(y_1,\dots,y_q) = \sum_{i=1}^{q} F^{(i)}(y_i) \tag{3.15}$$

Therefore,

$$E(R) = \int_{y_1,\dots,y_q} p(y_1,\dots,y_q)(\sum_{i=1}^{q} F^{(i)}(y_i))dy_1\dots dy_q \tag{3.16}$$

Each CV can be discretized into a finite number of operating zones, in this chapter, 6 zones (*Zone 1, Zone 2, Zone 3, Zone 4, Zone 5, Zone 6*), defining the range in which the value of the output variables will lie, illustrated in fig- 3.4 (the number of zones can be increased depending upon the resolution required). *Zone 1* and *Zone 6* represent the region below and above the limits, respectively, and *Zone 2* to *Zone 5* represent the four zones defined within the limits. Thus, if L_{y_i} and H_{y_i} are the low and the high limits for a particular CV (y_i) then Δ_i is the span of the range in which its value is to be maintained i.e.

$$\Delta_i = H_{y_i} - L_{y_i}$$

Then the six zones for the states of the CV can be defined as:

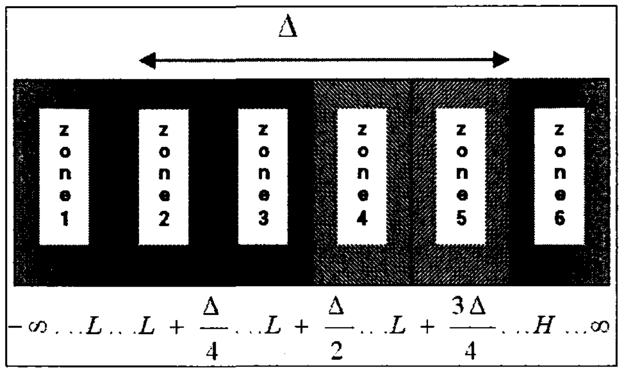

Figure 3.4: Division of zones for a child node

$$
\begin{aligned}
Zone\,1 &= -\infty \; to \; L_{y_i} \\
Zone\,2 &= L_{y_i} \; to \; \left(L_{y_i} + \frac{\Delta_i}{4}\right) \\
Zone\,3 &= \left(L_{y_i} + \frac{\Delta_i}{4}\right) \; to \; \left(L_{y_i} + \frac{\Delta_i}{2}\right) \\
Zone\,4 &= \left(L_{y_i} + \frac{\Delta_i}{2}\right) \; to \; \left(L_{y_i} + \frac{3\Delta_i}{4}\right) \\
Zone\,5 &= \left(L_{y_i} + \frac{3\Delta_i}{4}\right) \; to \; H_{y_i} \\
Zone\,6 &= H_{y_i} \; to \; \infty
\end{aligned}
\tag{3.17}
$$

The probability distribution for each y_i to be in the six zones is obtained for all the 2^N values obtained from optimization. For this purpose, the optimization results are superimposed with the base case variability and the data is assumed to be Gaussian distributed. Fig-3.5 shows the probabilities for i^{th} output variable to be in any of the six zones defined for CVs. Based on the probabilities thus obtained for each case and for each output variable, the CPT is created.

A simple discretized version of equation (3.14) can then be written as

$$E(R) = \sum_{y_1,\ldots,y_q} P(y_1,\ldots,y_q)F(y_1,\ldots,y_q) \tag{3.18}$$

where $P(y_1,\ldots,y_q)$ is now a probability function. Furthermore, if the uncertainties associated with each of $y_1,\ldots,y_q$ are mutually independent[1], and the profit function is additive

[1]Independence of $y_1,\ldots,y_q$ in terms of their uncertainties is possible since the properly selected quality

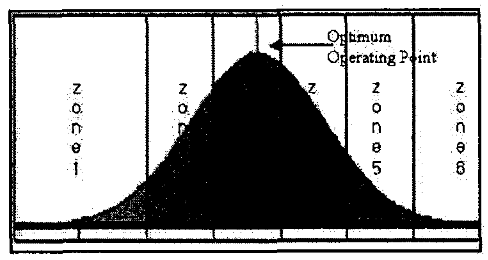

Figure 3.5: Probability for y_i to be in the six zones

shown in equation (3.15), then equation (3.18) can be further simplified to

$$E(R) = \sum_{i=1}^{q} \sum_{k=1}^{6} P(y_i) \times F^{(k)}(y_k) \tag{3.19}$$

or

$$E(R) = \sum_{i=1}^{q} \left(C_{i1} \times P(y_i \in \Omega_1) + \sum_{k=2}^{5} J_{ik} \times P(y_i \in \Omega_k) + C_{i6} \times P(y_i \in \Omega_6) \right) \tag{3.20}$$

where, $\Omega = \begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \\ \Omega_4 \\ \Omega_5 \\ \Omega_6 \end{pmatrix} = \begin{pmatrix} Zone\ 1 \\ Zone\ 2 \\ Zone\ 3 \\ Zone\ 4 \\ Zone\ 5 \\ Zone\ 6 \end{pmatrix}$

J_{ik} is the value of the objective function for i^{th} output variable to be in the k^{th} zone, C_{i1} and C_{i6} might be the penalty values set for i^{th} output variable to be in *Zone 1* and *Zone 6* respectively, and $P(y_i \in \Omega_k)$ is the probability for y_i to be in *Zone k*. J_{ik} can be calculated as:

$$J_{ik} = \frac{1}{H_{ky_i} - L_{ky_i}} \int_{L_{ky_i}}^{H_{ky_i}} \left(\alpha_i \times y_i + \beta_i^2 (y_i - \mu_i)^2 \right) \mathrm{d}y_i \tag{3.21}$$

where, H_{ky_i}, L_{ky_i} are the high and the low limits set for k^{th} zone for the i^{th} output variable and can be identified as given in equation (3.17).

In general, the probability for the CVs to be in each of the 6 zones is estimated by assuming the data to be Gaussian distributed with mean $\bar{y}_i$ and covariance of quality variables. For the constraint change only, the covariance remains the same as that of the base

variables typically reflect distinct aspects of the process.

case operation since the dynamic control is not touched. Optimizing the objective function subject to the new constraints provides the optimum operating points that will be the new mean operating points $\bar{y}_i$. The new mean operating points and base-case covariance, based upon the multivariate Gaussian distribution, are used to obtain the probabilities for the CVs to be in each of the six zones defined for them. For the simpler case where the uncertainties associated with the quality variables are mutually independent or there is only one economic quality variable, equation (3.22) can be applied. For computation tractability, when performing optimizations this thesis only considers the case of either q=1 or the selected quality CVs are independent.

$$P\left(y_i \in \text{Zone k}\right) = \frac{1}{\sigma_{i0}\sqrt{2\Pi}} \int_{L_{ky_i}}^{H_{ky_i}} exp\left(-\frac{\left(y_i - \bar{y}_i\right)^2}{2\sigma_{i0}^2}\right) \mathrm{d}x \tag{3.22}$$

where, σ_{i0} is the standard deviation of y_i in base case operation, $i = 1, 2, ..., q$, and $k = 1, 2, ..., 6$.

For q quality variables affecting the objective function there will be 6^q return values of the utility nodes. The values for the utility node are provided by the mean values taken by the economic objective function in all the 6 zones for each q quality variable.

The prior probability or the priori for the parent nodes can be user defined or obtained from the historical data. It indicates the preference *to change* or *not to change* the limits. For example, if a parent node has a priori of 0.8 for *making a change* to the limits. This means that the constraint limits for this variable has 80% tendency to change and 20% not to change.

Fig- 3.6 shows the Bayesian network thus created which can be used for decision evaluation and decision making purposes. Pa in the figure represents the parent nodes, Ch in the figure represents the child nodes and U is the utility node representing the benefit function.

1. *Decision evaluation* i.e. to infer the expected return (or the objective function values), if certain decisions regarding to make or not to make the change are made. For the decision evaluation, the decision whether to change or not to change the limits is provided. This is equivalent to the evidence for the Bayesian network conditional on

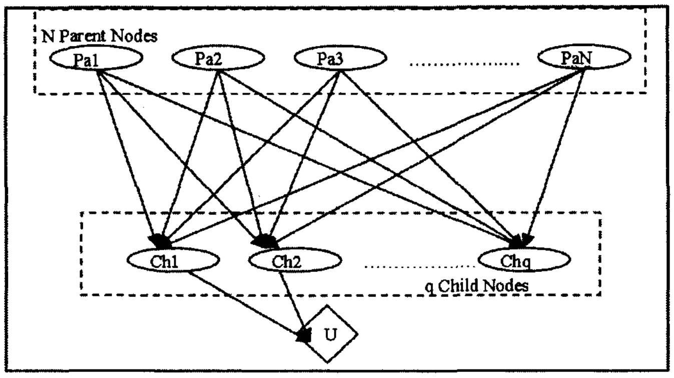

Figure 3.6: Bayesian network for $m \times n$ plant

which, the probabilities of locations of the CVs are then estimated. Thus the expected return can be evaluated using the relation specified in equation (3.20).

2. *Decision making* i.e. to obtain the maximum a posteriori explanation for decision making that will help to achieve a target value of expected return. For decision making purposes the target expected return are provided and the corresponding states for the CVs affecting the benefit function are then read from the utility node table. The states thus obtained from the table are the evidences for the Bayesian network and the maximum a posteriori estimate of the states of the parent nodes (i.e. change the limits or not) can be made.

3.4 Illustration on building a Bayesian network

To illustrate the computation procedure adopted for the proposed method consider four variables of a 2×2 system, i.e. 2 CV and 2 MV system, for which the steady state gain matrix, K, is represented as:

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

where, CV1 is a constraint variable and CV2 is a quality variable. The linear and quadratic coefficients for the output and the input variables considered in the application and whether they are allowed to change the limits are listed in table- 3.1 and table- 3.2 respectively.

Table 3.1: LQ Coef & Change allowed for CVs

CV	L Coef	Q Coef	Allowed to change
1	α_1	β_1	Yes
2	α_2	β_2	No

Table 3.2: LQ Coef & Change allowed for MVs

MV	L Coef	Q Coef	Allowed to change
1	α_1	β_1	No
2	α_2	β_2	Yes

As there are two variables available for making the changes the Bayesian network created, for the said system, will have two parent nodes: (MV2 and CV1*) and one child node: (CV2).

For the limits change, the parent and child nodes along with their states are described in table- 3.3 and table- 3.4 respectively.

For illustration purposes the prior probability for both MV2 and CV2*, is taken as 0.5, which means that there is no preference to make or not to make a change, i.e. both the process variables are equally likely to have the change. Table- 3.5 defines the prior probabilities for the parent nodes.

For the said system, there will be $2^2 = 4$ cases for which the optimizations are carried out with these combinations of limits changes for (MV2, CV1*): (*No, No*), (*No, Yes*), (*Yes, No*), (*Yes, Yes*). The optimizations are carried out for the optimization problem defined by equation (3.13).

Table 3.3: States for parent nodes for limit change

Parent Nodes	MV2	CV1*
States	(Yes,No)	(Yes,No)

Table 3.4: States for child node for limit change

Child Node	CV2
States	(Zone 1, Zone 2, Zone 3, Zone 4, Zone 5, Zone 6)

Table 3.5: Prior probability for making limits changes for parent nodes

Parent Node	Change	Do not change
MV2	0.5	0.5
CV1*	0.5	0.5

The optimum operating points are then obtained for each case of the change limits (table- 3.7). In table- 3.7, y_{ip} is the optimal operating point for i^{th} CV and the p^{th} case. However, due to variability, only average operating point can be at the optimal. The actual data can be distributed in any of the six zones. The probability for i^{th} CV to be in each of the six zones is estimated assuming the data to be Gaussian distributed with mean as the optimal operating point calculated from optimization and variance as the variance calculated from base operation. These probabilities form the CPT for the network (table- 3.8), where, P_{pk} is the probability for CV2 to be in k^{th} zone for p^{th} case.

In this example, as the profit function is defined only for the CV2, it is the only variable affecting the value for the expected return. The return values taken by the value node in the six zones or states are specified in table- 3.9.

The expected return for the existing system is calculated using equation (3.20), according to the values given in table- 3.9 and the probabilities for the variables to be in six zones

Table 3.6: Possible cases for applying limit changes

Parent Node	Case 1	Case 2	Case 3	Case 4
MV2	No	No	Yes	Yes
CV1*	No	Yes	No	Yes

Table 3.7: Optimum operating point for each identified case

	Case 1	Case 2	Case 3	Case 4
CV1	y_{11}	y_{12}	y_{13}	y_{14}
CV2	y_{21}	y_{22}	y_{23}	y_{24}

Table 3.8: Conditional probabilities for the child node

	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Case 1	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}
Case 2	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}
Case 3	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}	P_{36}
Case 4	P_{41}	P_{42}	P_{43}	P_{44}	P_{45}	P_{46}

Table 3.9: Utility table with profit values

CV2 state	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
U Node	U_1	U_2	U_3	U_4	U_5	U_6

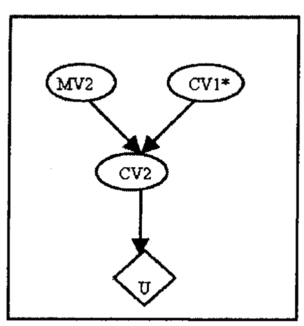

Figure 3.7: Bayesian network for 2×2 plant

Table 3.10: Probabilities of child nodes in its states

Node	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
CV2	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}

(table- 3.8). With CV2 as child node, MV2 and CV1* as parent nodes, priori as mentioned in table- 3.5, and the CPT for CV2 as given in table- 3.8, the Bayesian network for the system is created as shown in fig- 3.7. The network is now ready for performing decision evaluation and decision making.

1. *Decision evaluation*: For the purpose of decision evaluation the network is provided with the decision to be taken, which is equivalent to the evidence in the network. The evidence is then utilized to estimate the probabilities for the child node to have values in the six zones, which are then used to estimate the expected return.

   If the decision is made to change the limits for MV2, then the probability of CV2 to be in any of the six zones is estimated using Baye's Theorem. For illustration purposes one calculation is shown below:

$$
\begin{aligned}
P_{21} &= P(CV2 = Zone1|MV2 = Yes) \\
&= \frac{P(CV2 = Zone1|MV2 = Yes)}{P(MV2 = Yes)} \\
&= \frac{\sum_{CV1^*} P(CV1^*, CV2 = Zone1, MV2 = Yes)}{\sum_{CV1^*,CV2} P(CV1^*, CV2, MV2 = Yes)}
\end{aligned}
$$

   where according to fig- 3.7, the joint probability $P(CV1^*, CV2, MV2)$ can be calculated as:

$$P(CV1^*, CV2, MV2) = P(CV1^*)P(MV2)P(CV2|CV1^*, MV2)$$

   The probabilities for CV2 to be in any of the six zones are similarly estimated and listed in table- 3.10. The value of the expected return is then estimated using equation (3.20). Comparing to the nominal expected return the effect of the decision to change the limits for MV2 can be anticipated.

2. *Decision Making*: For the decision making purposes, the network is provided with the target value of the expected return. Thus reading from the table for the utility node, the value closest to the target value is selected and the corresponding states of the

child nodes act as the evidence for the analysis. If the target is set to have expected return of U units, such that $U_2 < U < U_3$ and U is closer to U_2 than it is to U_3, then from the table- 3.9, the state for CV2 to be in *Zone 2* is the evidence for the evaluation purposes. Thus, with CV2 to be in *Zone 2* as evidence the maximum a posteriori explanation for decision to be made is to change the limits for parent node CV1*.

3.5 Case study of a Binary Distillation Column

Consider the simulated binary distillation column MPC application, discussed in section- 2.3 of the thesis, with 10 Controlled Variables (outputs, y), 4 Manipulated Variables (inputs, u).

As CV2 and CV8 govern the overall economics of the operations of the column, they are chosen as the quality CVs here. The correlation coefficient between these two CVs is 0.0049 and they can be considered as independent. The economic coefficients for these two CVs can be determined through a profit function to be discussed in detail in Chapter-4. The profit function can be transferred to the following LQ cost function for the sake of applying the LMIPA algorithm.

$$J = \sum_{i=2,8} \left(\alpha_i \times \bar{y}_i + \beta_i^2 \left(\bar{y}_i - \mu_i \right)^2 \right) \tag{3.23}$$

where, (α_2, β_2) and (α_8, β_8) are (0.2364,0) and (0.1714,0), respectively. The list of CVs and MVs for the MPC application and whether or not they are available for limit change is given in table- 3.11 and 3.12 respectively. (C_{i1}, C_{i6}) have been assigned a value of (65.00, 0) and (200.00, 0) for CV2 and CV8 respectively and (0, 0) for rest of the CVs and MVs.

Based on the information provided in table- 3.11 and table- 3.12, a total of $2^7 = 128$ optimizations were carried out. The Bayesian network was created for the system defined with 7 parent nodes, 2 child nodes and 1 utility node. Table- 3.13 give the prior probabilities for making the limit changes on the parent nodes. The optimization results are used to create the CPT for the child nodes.

Table 3.11: Change allowed for CVs

CV	Description	Change Lt
1	Ref Flow	No
2	Lt Petrol FBP	Yes
3	Top PCT	No
4	Pr Vlv OP	No
5	Bttm PCT	Yes
6	Col Pr	Yes
7	Feed Temp	Yes
8	Reb Furnace Duty	No
9	Duty	No
10	Bypass Vlv OP	No

Table 3.12: Change allowed for MVs

MV	Description	Change Lt
1	Ref Flow SP	Yes
2	Col Pr SP	Yes
3	Feed Temp Vlv OP	Yes
4	Duty Vlv OP	No

Table 3.13: Prior probability for making limit changes for parent nodes

Parent Node	Change	Do not change
MV1	0.5	0.5
MV2	0.5	0.5
MV3	0.5	0.5
CV2*	0.5	0.5
CV5*	0.5	0.5
CV6*	0.5	0.5
CV7*	0.5	0.5

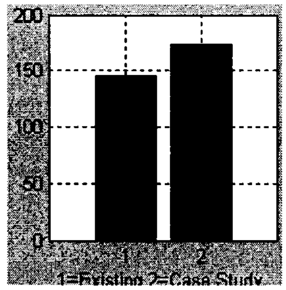

Figure 3.8: Comparison of expected return

The expected return for the system was estimated using equation (3.20)and (3.21), the information provided in the table- 3.11, table- 3.12 and table- 3.13 and the conditional probabilities of the CVs in the six zones. For the existing system setup the expected return for the operation are estimated to be 145.36 units.

Once the Bayesian network has been created for the system it can then be used for either of the two purposes previously discussed, namely, Decision evaluation and Decision making.

1. *Decision Evaluation*: For the purpose of inferencing the expected return for the system when a decision is made with regards to changing the constraint limits of the process variables in an MPC controller, the Bayesian network can be utilized.

   If the decision is to be made to increase the reflux flow (MV1) set point then the maximum a posteriori estimate of state of CV2 and CV8 are *Zone 3* and *Zone 4* and the expected return are 173.27 units. The comparison of the expected return of the controller before and after the decision is made and shown in fig- 3.8.

   Thus, it can be inferred that the decision to increase the limits set for the reflux flow to the column will increase the yield of operations.

2. *Decision Making*: This aspect of the algorithm can be utilized to help in decision making if the target to increase the expected return to a certain value is set. This helps in providing the directions for constraint changes for the MPC controller. Once the targets are set, the states for the CVs are determined from the table of the utility node and the states thus obtained are the evidence for the decision making.

   If the target is set to increase the return to 165.00 then the states for all child nodes

Table 3.14: State of child nodes

CV#	state
CV2	Zone 1
CV8	Zone 4

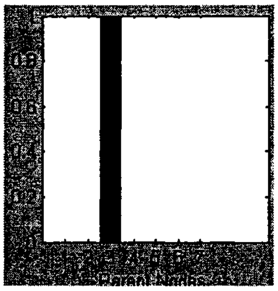

Figure 3.9: Decision for changing the constraint limits

i.e. CV2 and CV8 for this plant, are determined and listed in table- 3.14. With these states as evidence, the maximum *a posteriori* for the state of the parent nodes that are to have their limits changed are calculated.

For the case when the objective function value is targeted to change 165.00 units, parent node 3 i.e. MV3 is expected to have its limits changed (fig-3.9).

Section-3.6 (pages 60(part), 61, 62 and 63(part)) has been removed from this thesis for proprietary reasons.

3.7 Conclusion

The methodology that takes into consideration the process variability to tune the MPC controller for limit has been developed. The proposed method gives the constraint tuning guidelines by performing the Bayesian analysis of the process variables in the MPC controller.

As the real world is associated with uncertainty, the Bayesian approach of analysis is an appropriate tool that takes into consideration the probabilities for CVs to lie in the different zones defined. The results thus obtained from the analysis are more realistic than commonly used simple deterministic profit calculations. The Bayesian network built can also be used to assist in decision making when the set target value for objective function is defined.

The case studies provided in the chapter explain the industrial utility of the tool. The results from the study illustrate its significance and utility for the process engineer for day to day maintenance of the MPC controllers in the plant. The tuning guidelines obtained from the tool can be applied to improve the controller performance.

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4

MPC economic performance assessment - Probabilistic approach

Abstract: Advanced Process Control (APC), in particular Model Predictive Control (MPC), has emerged as the most effective control strategy in process industry, and numerous applications have been reported. Nevertheless, there are several factors that limit the achievable performance of MPC. One of the limiting factors considered in this chapter is the presence of constraints. To exploit optimal control performance, continuous performance assessment with respect to constraints of MPC is necessary. MPC performance assessment has received increasingly interest both in academia and in industries. This chapter is concerned with a practical aspect of performance assessment of industrial MPC by investigating the relationship among process variability, constraints, and probabilistic economic performance of MPC. The proposed approach considers the uncertainties induced by process variability and evaluates economic performance through probabilistic calculations. It also provides the guideline for the constraint tuning so as to improve MPC performance.

4.1 Introduction

Advanced Process Control (APC) applications such as Model Predictive Control (MPC) was emerged in mid-seventies of the twentieth century. Since then they have been regarded as the most popular industrial advanced multivariable control strategy. The main objective of the MPC controller is to calculate the control signals and to minimize the sum of the squares of the error between the reference signal and the output signal within a given future horizon (Bars and Haber 2006). On the top of dynamic control objective, an economic objective function is utilized to optimize steady state economic performance, subject to process constraints. The economic optimization drives the controlled variables (CVs) and manipulated variables (MVs) to their steady state optimum, thus generating economic benefits.

Despite great success of MPC, there are certain limitations on achievable performance. For example, performance of practical MPC applications can be limited by model uncertainty (Clarke 1998, Morari and Lee 1999, Singh and Seto 2002), as well as possible conservative tuning such as conservativeness in setting up constraints. The latter is not uncommon and has been the motivation of this work. Many commercial controllers are now available that are able to improve MPC controller performance by resolving one or the other of the practical issues (Qin and Badgwell 1997, Qin and Badgwell 2003). With most of the research being focused on the areas of developing MPC controllers, relatively less work has been reported in the field of evaluation and the practical tuning of MPC controllers. Conventional tuning for the MPC controller involves tuning the controller with respect to the prediction and control horizon, linear and quadratic objective functions (Maurath *et al.* 1988, Madar *et al.* 2005). Guidelines are available in literature, which provide qualitative relations between these parameters; however the quantitative relations differ from one system to another. Tuning of the controllers through these parameters is generally done at the design stage and is not advised to be changed on a day to day basis. To change these parameters it is essential to have thorough process knowledge, a deep understanding about the MPC control algorithm and the effect of these changes on the MPC controller performance. In daily operations, on the other hand, it is not uncommon to change some of the CV and MV constraint limits. This action is simple but affects the operating points of the CVs and the MVs. It can sometimes improve the controller performance and other times may have little effect. Thus, understanding of the relation among constraints, process variability, and economic objective functions is important to guide the constraint tuning.

Change of constraints is a decision making process in the economic optimization layer. In words of Hummel *et al.* (1991), "Plant optimization can be defined as the maximization of the profit function describing the economics of the plant. This function contains terms with product values, feedstock process, operational costs etc". Prett and Garcia (1988) states that the decision making is a multilayered process, involving measurement, controls, optimization and logistics. Measurement is gathering of information of the process measurements. Control is the manipulation of process degrees of freedom for satisfying the operation criteria. Optimization is defined as plant optimization which is a technique of manipulating the process degrees of freedom to satisfy the plant economic objectives. Logistics is the high level scheduling to respond to external market changes for profit maximization. The optimization layer contributes the most to the profit improvement. This layer receives the inputs of economic targets from the logistics layer and then passes the optimum operating target to the control layer for realization.

The previous work in the direction of performance assessment with respective to constraints analysis has focused primarily on evaluating the deterministic linear and quadratic objective function (Xu *et al.* 2007) subject to process variability. In this early work the economic function was evaluated by considering the mean operating points of the process variables subject to the constraints on the CVs/MVs, the process variance, and the steady state process gains. However, due to the presence of the disturbances, there is a variability associated with the process operating points, and consequently process variables do not always operate at the mean operating points. It is necessary to consider probabilistic objective function involving the uncertain operating points of the CVs and MVs, the profits associated with each operating points, and the probabilities for CVs and MVs to be in each operating points. The problem thus formulated will be nonlinear and the approach taken in this chapter is a step towards more realistic assessment of MPC control performance.

The contributions of this chapter can be summarized as: 1) a probabilistic approach for MPC performance assessment via constraint analysis is proposed; 2) guidelines for optimal constraint tuning are derived; 3) detail simulated as well as industrial case studies are presented. The chapter is organized as follows. Section 4.2 introduces the problem formulation. The formulation of the probabilistic objective function and the algorithms used for the Probabilistic Performance Assessment (PPA) are discussed in section 4.3. The section 4.4 illustrates a application for a simulated binary distillation column, and application to an industrial distillation column is discussed in section 4.5, followed by conclusion in section 4.6.

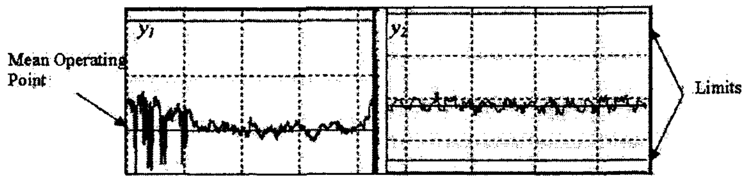

Figure 4.1: True process data

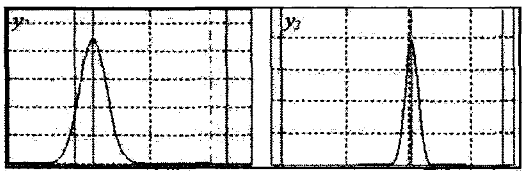

Figure 4.2: Process data distribution

4.2 Problem description

4.2.1 Mean operating point

The previous MPC performance assessment with respect to constraint analysis calculates economic optimum based on the mean operating point (Xu *et al.* 2007). As the plant seldom operates precisely on the mean operating point, it is essential to consider the data distribution while evaluating economic objective function. Thus, the probabilistic performance assessment method needs to be developed, which takes into consideration the probability distribution.

To illustrate this, consider two outputs, y_1 and y_2 of a system. Let y_1 be the quality variable and y_2 be a constraint variable. The current operating data, defined as base case operating data, for y_1 and y_2 are shown in fig- 4.1. Fig- 4.2 shows the probability distribution for the base case data when the Gaussian distribution is to be approximated.

As can be seen from fig- 4.1 and fig- 4.2 that even though the mean operating points for y_1 and y_2 are within the constraint limits throughout the period for which the data are collected, the data trend shows that most of time they are not on the mean operating

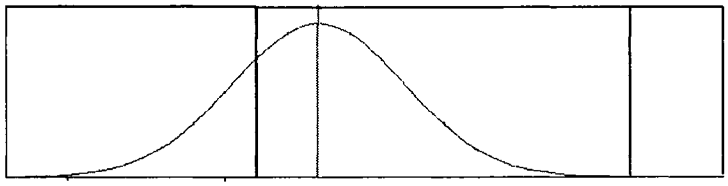

Figure 4.3: High probability of violating Low Limit

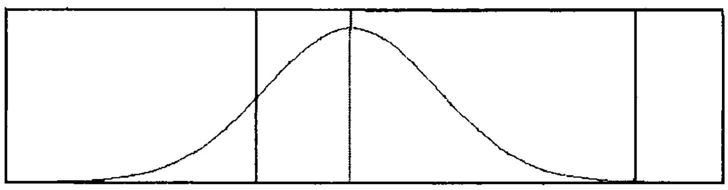

Figure 4.4: Low probability of violating Low Limit

points, and sometime even outside the constraint limits. Since y_1 is a quality CV, having its value outside the set limits is highly undesirable as it can render the product unmarketable. Considering only the mean operating point for performance assessment, the performance of the controller for the above data set can be referred to as satisfactory, but when also considering the data distribution the above performance may not necessarily be satisfactory. There exists an optimum operating point at which the value of the probabilistic expected economic objective function can be optimized, by taking into account data distribution. This can be explained as follows: assume that the target operating point for y_1 is at its low limit. Now moving its mean operating point closer to the low limit will not necessarily increase the profit margins. This is so, because due to the presence of variance, moving the operating point closer to the low limit will also increase the probability of violating the low limits and thus a tradeoff has to be considered while moving closer to the low limit (fig- 4.3, 4.4).

4.2.2 Effect of changing constraints

The effect of relaxing the constraint limits provides the controller with some additional degrees of freedom to operate, and thus improve the controller performance (section 3.2.1 of the thesis). This study is therefore to investigate the potential of increasing the expected return by changing the operating points or changing constraints on selected CVs/MVs. The optimal operating conditions are identified by optimizing the probabilistic economic objective function subject to the constraints and variability (Latour *et al.* 1986, Martin *et al.* 1991).

4.2.3 Problem formulation

For an $m \times n$ system with n inputs and m outputs, let K be the steady state gain matrix and $(\bar{y}_{i0}, \bar{u}_{j0})$ be the current mean operating points for i^{th} CV and j^{th} MV, i.e. the base case mean operating points. Let $(\bar{y}_i, \bar{u}_j)$ be the corresponding optimal operating points to be determined.

For the sake of simplicity of the presentation, in the sequel, we assume that the profit depends on CVs only, i.e. CVs are quality variables. The results can be easily extended to include MVs as quality variables. Given the mean operating points and the variance of the process data, the expected return, for a process with q quality variables, can be calculated as equation (3.14). If $F(y_1, \ldots, y_q)$ takes additive form, equation (3.15), then the expected return can be estimated as equation (3.16).

Each specific value that CV and MV (y_i, u_j) takes is called a state, and the state can be any value within the operating region and is continuous. For computation tractability, we discretize operating region of each CV into a finite number of operating zones. For illustration, we consider 6 equal spaced zones for each CV (Agarwal *et al.* 2007); thus each CV has a corresponding state space of dimension 6. If a higher resolution is required, the number of zones can be increased and this can be done easily in the computation program. As discussed previously, in section- 3.3 (fig- 3.4), discretizing each CV in 6 zones, the state space Ω^i for the CV can be written as:

$$\Omega^i = \begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \\ \Omega_4 \\ \Omega_5 \\ \Omega_6 \end{pmatrix} = \begin{pmatrix} Zone\ 1 \\ Zone\ 2 \\ Zone\ 3 \\ Zone\ 4 \\ Zone\ 5 \\ Zone\ 6 \end{pmatrix}$$

Thus discretizing the operating region for each CV, equation (3.16) can also be discretized (equation 3.18). Furthermore, if the uncertainties associated with each of $y_1, \ldots, y_q$

are mutually independent, and the profit function is additive shown in equation (3.15), then equation (3.18) can be further simplified to

$$E(R) = \sum_{i=1}^{q} \sum_{k=1}^{6} P(y_i \in \Omega_k) \times F(y_i \in \Omega_k) \tag{4.1}$$

The probability for the CVs to be in one of the six zones is estimated according to the distribution of the data. For a process comprising of one quality variable (CV) or having mutually independent uncertainties associated with the quality variables, the probability for each zone can be determined according to the mean $\bar{y_i}$ and standard deviation σ_i, where the standard deviation is calculated from the base operating data. Thus, $P(y_i \in \Omega_k)$ is

$$P\left(y_i \in \Omega_k\right) = \frac{1}{\sigma_i\sqrt{2\Pi}} \int_{L_{ky_i}}^{H_{ky_i}} exp\left(-\frac{\left(y_i - \bar{y_i}\right)^2}{2\sigma_i^2}\right) \mathrm{d}x \tag{4.2}$$

where, $k = 1, 2, ..., 6$ and $i = 1, 2, ..., q$. L_{ky_i} and H_{ky_i} are the low and the high zone-limits of the k^{th} zone for the variable y_i and can be determined by equation- 3.17. However, for a process comprising of more then one quality variables (CVs), with non-independent uncertainties, multivariate probability distribution has to be considered.

The return for y_i to be in each of the six zones can be user specified according to economic data. The minimum return (loss relative to the return of in-spec) is assigned for the products to be under-spec (in *Zone 1*) or over-spec (in *Zone 6*). The maximum return is obtained when the process operates at the maximum-return operating points (zones), and the maximum-return points typically lie in one of the constraint limits. Thus the maximum return can be either at the high constraint limit or at the low constraint limit. Given the maximum return and the minimum return, the profit for y_i in the in-spec zones can be estimated through interpolation and be calculated as given below.

Let F_k denote the return for the process variable y_i to operate at the k^{th} zone. If y_i is to be maximized, i.e. the maximum return is at the high constraint limit i.e. $F_{max} = F_5$, $F_{min} = F_1$, then

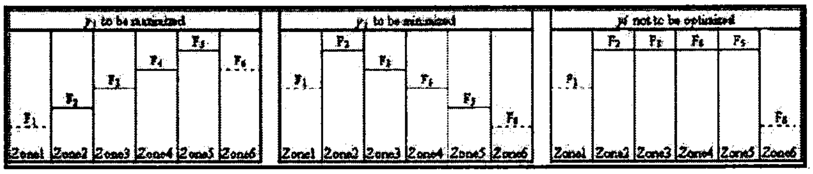

Figure 4.5: Profits for y_i to be in the six zones

$$\begin{aligned} \Delta F &= \frac{1}{4}(F_5 - F_1) \\ F_2 &= F_1 + \Delta F \\ F_3 &= F_1 + 2 \times \Delta F \\ F_4 &= F_1 + 3 \times \Delta F \end{aligned} \tag{4.3}$$

This calculation is illustrated in the left panel of fig- 4.5.

If y_i is to be minimized, i.e. the maximum return is at the low constraint limit i.e. $F_{max} = F_2$, $F_{min} = F_6$, then

$$\begin{aligned} \Delta F &= \frac{1}{4}(F_2 - F_6) \\ F_5 &= F_6 + \Delta F \\ F_4 &= F_6 + 2 \times \Delta F \\ F_3 &= F_6 + 3 \times \Delta F \end{aligned} \tag{4.4}$$

This calculation is illustrated in the middle panel of fig- 4.5.

However, if y_i is neither to be maximized or minimized then

$$F_2 = F_3 = F_4 = F_5 = F \tag{4.5}$$

This calculation is illustrated in the right panel of fig- 4.5.

F_1 and F_6 are shown as dotted lines in fig- 4.5, which represents that these can have values less than, greater than or equal to the return in *Zone 2* and *Zone 5* respectively, depending on specific processes.

Not all processes operate at their optimum even if there is potential to do so. Thus, it is possible to increase the profit by simply adjusting the operating points without doing any other tunings. There is a need for assessment of performance potential. The potential for expected return $E(R_p)$ by simply adjusting the operating points, without any other tuning such as constraint limits relaxation, is then estimated through the maximization of the expected economic objective function (equation-4.1). Thus, the objective function for the optimization can be specified as:

$$J = \sum_{i=1}^{q} \sum_{k=1}^{6} P(y_i \in \Omega_k) \times F(y_i \in \Omega_k) \tag{4.6}$$

With $(\bar{y}_{i0}, \bar{u}_{i0})$ as the base case mean operating point, $(\bar{y}_i, \bar{u}_i)$ as new mean operating point when the base case operating points are adjusted by $(\Delta y_i, \Delta u_i)$, then the above maximization should be subject to the following equality constraints:

$$\Delta y_i = \sum_{j=1}^{n} [K_{ij} \times \Delta u_j] \tag{4.7}$$

$$\bar{y}_i = \bar{y}_{i0} + \Delta y_i \tag{4.8}$$

$$\bar{u}_j = \bar{u}_{j0} + \Delta u_j \tag{4.9}$$

where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Considering that up to 5% constraint violation is acceptable for the output variables (Latour *et al.* 1986, Martin *et al.* 1991) a set of inequalities, defining the limits for y_i and u_j, can be identified. The following inequalities are required to be satisfied while optimizing the objective function defined in equation (4.1):

$$L_{y_i} + 2 \times \sigma_{i0} \leq \quad \bar{y}_i \quad \leq H_{y_i} - 2 \times \sigma_{i0} \tag{4.10}$$

$$L_{u_j} + 2 \times R_{j0} \leq \quad \bar{u}_j \quad \leq H_{u_j} - 2 \times R_{j0} \tag{4.11}$$

where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. σ_{i0} is the standard deviation for the base operation data y_i, R_{j0} is the quarter of the existing operating range for the base operation data u_j. L_{u_j} and H_{u_j} are the low and the high constraint limits for u_j and L_{y_i} and H_{y_i} are the low and the high constraint limits for y_i, respectively. Thus, the performance assessment problem can be specified as:

$$\max_{\substack{\bar{y}_1,\ldots,\bar{y}_m \\ \bar{u}_1,\ldots,\bar{u}_n}} \quad J \text{ subject to } (4.7), (4.8), (4.9), (4.10), (4.11) \tag{4.12}$$

The value of J thus estimated through the objective function defined in equation (4.6), subject to the constraints defined by relations defined in (4.7), (4.8), (4.9), (4.10) and (4.11), gives the potential to improve return by simply adjusting the operating points.

In practice, owing to some conservativeness or lack of information on the potential economic impacts of certain constraints when setting up their limits, the constraint limits for some CV and MV are often readjusted during process operations. Therefore, there is an incentive to determine the best adjustment of constraint limits according to the base case operation data. The results obtained can serve as a powerful tool set for process control engineers when they make adjustments of the constraints. On the other hand, even if there is no intention to change the constraints, the results can also provide an understanding of the sensitivity of the constraint limits of each variable in terms of economic return. These problems are considered next.

Let B be the maximum allowable change in the constraint limits that can be made for a CV or MV, and r ($r \leq B$) be the adjustment of the limits for that CV or MV that will yield the maximum return.

Under this scenario, the expected return $E(R_c)$ is given by the objective function (4.6) subject to the equalities specified in (4.7), (4.8) and (4.9); however the inequalities (4.10) and (4.11) change to (4.13) and (4.14) respectively.

$$L_{y_i} + 2 \times \sigma_{i0} - y_{holi} \times r^L_{y_i} \leq \quad \bar{y}_i \quad \leq H_{y_i} - 2 \times \sigma_{i0} + y_{holi} \times r^H_{y_i} \tag{4.13}$$

$$L_{u_j} + 2 \times R_{j0} - u_{holj} \times r^L_{u_j} \leq \quad \bar{u}_j \quad \leq H_{u_j} - 2 \times R_{j0} + u_{holj} \times r^H_{u_j} \tag{4.14}$$

$$0 \leq \quad r^L_{y_i} \quad \leq B^L_{y_i} \tag{4.15}$$

$$0 \leq \quad r^H_{y_i} \quad \leq B^H_{y_i} \tag{4.16}$$

$$0 \leq \quad r^L_{u_j} \quad \leq B^L_{u_j} \tag{4.17}$$

$$0 \leq \quad r^H_{u_j} \quad \leq B^H_{u_j} \tag{4.18}$$

where, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ and y_{holi}, u_{holj} are half of the range of the constraint limits for y_i and u_j; $B^L_{y_i}$, $B^H_{y_i}$, $B^L_{u_j}$, $B^H_{u_j}$ are the maximum allowable change (percentage of the range specified) in the limits for the CVs and MVs respectively; $r^L_{y_i}$, $r^H_{y_i}$, $r^L_{u_j}$, $r^H_{u_j}$ are the actual limit changes for the CVs and MVs respectively. Subscript 'y_i' and 'u_j' represent the the i^{th} output and j^{th} input; the superscripts 'L' and 'H' represent low and the high limits respectively. Thus, the performance assessment problem can be specified as:

$$\max_{\substack{\bar{y}_1, \ldots, \bar{y}_m \\ \bar{u}_1, \ldots, \bar{u}_n \\ r^L_{y_1}, \ldots, r^L_{y_m} \\ r^H_{y_1}, \ldots, r^H_{y_m} \\ r^L_{u_1}, \ldots, r^L_{u_n} \\ r^H_{u_1}, \ldots, r^H_{u_n}}} \quad J \textit{ subject to } (4.7), (4.8), (4.9), (4.13) - (4.18) \tag{4.19}$$

4.3 Algorithm

The procedure discussed in the previous section provides the solutions to the problem by constraint limits relaxation for the CVs and the MVs. The objective function defined for this purpose is nonlinear whereas the constraint equalities and the inequalities are linear.

For the purpose of explaining the calculations involved for the optimization, the terms involved in the objective function, equation (4.6), can be restated as: $F(y_i \in \Omega_k)$ is estimated using equations (4.3), (4.4) or (4.5), and the probabilities are given by:

$P(y_i \in \Omega_1) = P(y_i < L_{y_i})$,

$P(y_i \in \Omega_6) = P(y_i \geq H_{y_i})$, and

$P(y_i \in \Omega_k) = P(L_{ky_i} \leq y_i < H_{ky_i})$

where, $k = 2, 3, 4, 5$; L_{y_i} and H_{y_i} are the low and the high constraint limit for y_i; L_{ky_i} and H_{ky_i} are the low and the high limits for the k^{th} zone of y_i. As the process data is Gaussian distributed the above mentioned probabilities can be written as equation (4.2), or in the case of the cumulative probability function, it can be written as:

$$\begin{aligned}
P(y_i < L_{y_i}) &= \frac{1}{2}\left(1 + erf\left(\frac{L_{y_i} - \bar{y}_i}{\sigma_i\sqrt{2}}\right)\right) \\
&= P_{ci}(L_{y_i}) \\
&= P_{ci}(H_{1y_i}) & (4.20) \\
P(y_i \geq H_{y_i}) &= 1 - \frac{1}{2}\left(1 + erf\left(\frac{H_{y_i} - \bar{y}_i}{\sigma_i\sqrt{2}}\right)\right) \\
&= 1 - P_{ci}(H_{y_i}) \\
&= 1 - P_{ci}(L_{6y_i}) & (4.21) \\
P(L_{ky_i} \leq y_i < H_{ky_i}) &= P(y_i \leq H_{ky_i}) - P(y_i < L_{ky_i}) \\
&= P_{ci}(H_{ky_i}) - P_{ci}(L_{ky_i}) \\
&= P_{ci}(L_{(k+1)y_i}) - P_{ci}(L_{ky_i}) & (4.22)
\end{aligned}$$

where, $P_{ci}(x)$ is the probability for y_i to take value less than x, and $erf(x)$ is defined as

$$erf(x) = \frac{2}{\sqrt{\Pi}} \int_0^x \exp(-t^2)\, \mathrm{d}t \qquad (4.23)$$

Substituting equation (4.20), (4.21) and (4.22) in the objective function equation (4.6), it can be further written as equation (4.24). For simplicity of the terms, $F(y_i \in \Omega_k)$ has been replaced by F_{ky_i}.

$$
\begin{aligned}
J &= \sum_{i=1}^{q} (P(y_i < L_{y_i}) \times F_{1y_i} + P(L_{2y_i} \leq y_i \leq H_{2y_i}) \times F_{2y_i} \\
&\quad + P(L_{3y_i} \leq y_i \leq H_{3y_i}) \times F_{3y_i} + P(L_{4y_i} \leq y_i \leq H_{4y_i}) \times F_{4y_i} \\
&\quad + P(L_{5y_i} \leq y_i \leq H_{5y_i}) \times F_{5y_i} + P(y_i \leq H_{y_i}) \times F_{6y_i}) \\
&= \sum_{i=1}^{q} (P_{ci}(L_{2y_i}) \times F_{1y_i} + (P_{ci}(L_{3y_i}) - P_{ci}(L_{2y_i})) \times F_{2y_i} \\
&\quad + (P_{ci}(L_{4y_i}) - P_{ci}(L_{3y_i})) \times F_{3y_i} + (P_{ci}(L_{5y_i}) - P_{ci}(L_{4y_i})) \times F_{4y_i} \\
&\quad + (P_{ci}(L_{6y_i}) - P_{ci}(L_{5y_i})) \times F_{5y_i} + (1 - P_{ci}(L_{6y_i}) \times F_{6y_i})) \\
&= \sum_{i=1}^{q} (P_{ci}(L_{2y_i}) \times (F_{1y_i} - F_{2y_i}) + P_{ci}(L_{3y_i}) \times (F_{2y_i} - F_{3y_i}) \\
&\quad + P_{ci}(L_{4y_i}) \times (F_{3y_i} - F_{4y_i}) + P_{ci}(L_{5y_i}) \times (F_{4y_i} - F_{5y_i}) \\
&\quad + P_{ci}(L_{6y_i} \times (F_{5y_i} - F_{6y_i}) + F_{6y_i}) \qquad (4.24)
\end{aligned}
$$

Also the equality constraints for the problem, defined by equation (4.7), (4.8) and (4.9) can be simplified as:

$$
\begin{aligned}
\bar{y}_i &= \bar{y}_{i0} + \sum_{j=1}^{n} K_{ij} \times \Delta u_j \\
&= \bar{y}_{i0} + \sum_{j=1}^{n} K_{ij} \times (\bar{u}_j - \bar{u}_{j0}) \\
&= \left(\bar{y}_{i0} - \sum_{j=1}^{n} K_{ij} \times \bar{u}_{j0} \right) + \sum_{j=1}^{n} K_{ij} \times \bar{u}_j \qquad (4.25)
\end{aligned}
$$

where, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

The inequalities specified by equations (4.13) and (4.18) can be reformulated in the $Ax \leq b$ format as:

$$
\begin{aligned}
-y_{holi} \times r_{y_i}^{L} - \bar{y}_i &\leq -L_{y_i} - 2 \times \sigma_{i0} \qquad (4.26) \\
y_{holi} \times r_{y_i}^{H} + \bar{y}_i &\leq H_{y_i} - 2 \times \sigma_{i0} \qquad (4.27) \\
& \qquad (4.28)
\end{aligned}
$$

$$-u_{holj} \times r^L_{u_j} - \bar{u}_j \leq -L_{u_j} - 2 \times R_{j0} \quad (4.29)$$

$$u_{holj} \times r^H_{u_j} + \bar{u}_j \leq H_{u_j} - 2 \times R_{j0} \quad (4.30)$$

$$-r^L_{y_i} \leq 0 \quad (4.31)$$

$$r^L_{y_i} \leq B^L_{y_i} \quad (4.32)$$

$$-r^H_{y_i} \leq 0 \quad (4.33)$$

$$r^H_{y_i} \leq B^H_{y_i} \quad (4.34)$$

$$-r^L_{u_j} \leq 0 \quad (4.35)$$

$$r^L_{u_j} \leq B^L_{u_j} \quad (4.36)$$

$$-r^H_{u_j} \leq 0 \quad (4.37)$$

$$r^H_{u_j} \leq B^H_{u_j} \quad (4.38)$$

$$(4.39)$$

The optimization problem is now defined as equation (4.24), the equality constraint (4.25) and the inequalities (4.27) to (4.39) for constraint limits relaxation case. The decision variables for the objective function are $[r^L_{y_i}, r^H_{y_i}, r^L_{u_j}, r^H_{u_j}, \bar{y}_i, \bar{u_j}]$, where $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$. The values of these decision variables can now be calculated to optimize the objective function while satisfying the constraints.

The objective function defined is non-linear, and the constraints are linear. Sequential Quadratic Programming (SQP) is employed for solving the problem. In the SQP method the non-linear optimization function is quadratically approximated, by its Taylor series, and then the approximated function is optimized (Dixon 1975, Wismer and Chattergy 1978, Jacobs 1986, Berghen 2004).

4.4 Case study of a Binary Distillation Column

The binary distillation column (Section-2.3, Fig- 2.1) used to fractionate the petrol, obtained from an upstream desulphurization unit, into light petrol and the heavy petrol is considered. The controller designed for the column has 10 CVs and 4 MVs. The list of CVs along with the profit associated with them to be under-spec (u/s), in-spec(maximum) and over-spec (o/s) and their control objective are listed in table- 4.1. The list of MVs is given in table-

Table 4.1: Control objective and the profits for the CVs

CV	Description	Objective	**Profit**		
			u/s	Max	o/s
1	Reflux Flow	Constraint	0	0	0
2	Lt Petrol FBP	Minimize	65	65	0
3	Top PCT	Constraint	0	0	0
4	Pr Vlv OP	Constraint	0	0	0
5	Bttm PCT	Minimize	0	0	0
6	Col Pr	Minimize	0	0	0
7	Feed Temp	Constraint	0	0	0
8	Reboiler Duty	Minimize	200	200	0
9	Duty	Constraint	0	0	0
10	Bypass Vlv OP	Constraint	0	0	0

Table 4.2: List of MVs

MV	Description
1	Reflux SP
2	Col Pr SP
3	Feed Temp Vlv OP
4	Duty Vlv OP

4.2. As it has been discussed in Chapter-3, CV2 and CV8 are two independent quality variables.

As can be seen from the table- 4.1 that CV2 and CV8 are the quality variables to be minimized, the profits associated with these CVs to be in the six zones can be identified using equation (4.4).

For the optimization of the process with regards to the constraint limits change and the assumed maximum constraint change allowed for the process variables as listed in table- 4.3,

Table 4.3: Maximum constraint change allowed

	In Low Lt	In High Lt
CV1	0	0
CV2	10	10
CV3	0	0
CV4	0	0
CV5	10	0
CV6	10	0
CV7	10	0
CV8	0	0
CV9	0	0
CV10	0	0
MV1	10	10
MV2	10	10
MV3	10	50
MV4	0	0

the optimization problem can be defined as equation 4.40.

$$\max_{\substack{\bar{y}_1,\ldots,\bar{y}_{10}\\ \bar{u}_1,\ldots,\bar{u}_4\\ r^L_{y1},\ldots,r^L_{y10}\\ r^H_{y1},\ldots,r^H_{y10}\\ r^L_{u1},\ldots,r^L_{u4}\\ r^H_{u1},\ldots,r^H_{u4}}} \quad J = \sum_{i=2,8} \sum_{k=1}^{6} P(y_i \in \Omega_k) \times F(y_i \in \Omega_k) \tag{4.40}$$

subject to equations (4.7) to (4.9) and (4.13) to (4.18).

The expected return estimated for the base case operation, potential by adjusting operating points and potential by constraints relaxation are calculated using the proposed Probabilistic Performance Assessment (PPA) method. The results obtained are listed in table- 4.4. The results for the three scenarios of the PO method are shown in fig- 4.6.

Table 4.4: Comparison of the three scenarios

Scenario	Expected return PO
Base Case	145.36
Base Case Potential	190.64
Constraint Adjustment	192.38

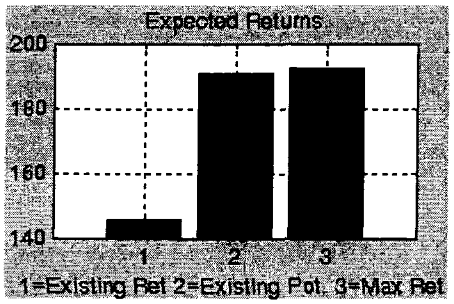

Figure 4.6: Comparison of the expected return

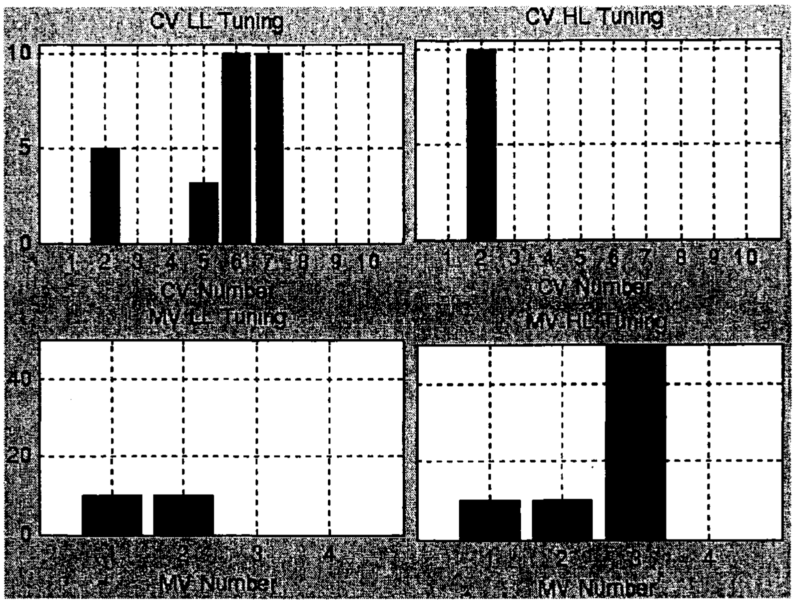

Figure 4.7: Constraint tuning guidelines (%age)

Fig- 4.6 shows that the expected return of base case operation is 145.36 units, whereas the potential return by adjusting operating points is 190.64 units, and the potential return by adjusting constraints is 192.38 units. To achieve the expected return of 192.38 units, the optimizer has suggested constraint relaxations for certain variables, listed in table- 4.5 and shown in fig- 4.7.

Section-4.5 (pages 84(part) and 85 to 89) has been removed from this thesis for proprietary reasons.

4.6 Conclusion

A performance assessment and constraint tuning method for the MPC controllers, which takes into account the distribution of the process data and the profits associated with the product quality, has been developed in this chapter. As the real processes always involve uncertainties, it is pertinent to develop probabilistic approach for MPC performance assessment. The developed method calculates expected return of base case operation, potential improvement of the expected return through adjusting the operating points as well as through adjusting constraints. It furthermore provides tuning guideline and determines which variables need to be adjusted and by how much. Two case studies, one for a simulated binary distillation column and the other for an industrial distillation column, have been provided, which illustrate the industrial utility of the proposed algorithm. The results from the studies show the feasibility of the proposed algorithm and also bring forth its the utility for the process control engineers for day to day maintenance of MPC controllers in a plant and for economic assessment of the performance of the controller.

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5

MPC constraint tuning - Bayesian approach via probabilistic opimization

Abstract: Any change in the operating conditions for an MPC application can affect the controller performance in one way or the other. The changes can be made either by changing the constraint limits for the CVs and the MVs of the controller among others. The tuning guidelines for the MPC controller can be obtained by optimizing the controller performance, either using probabilistic optimization function or a deterministic linear-quadratic optimization function. The probabilistic optimization function uses the probabilities and the profits associated with the process variables, whereas the linear-quadratic optimization function uses mean values of the process variables. As probabilistic optimization function is more realistic than the mean-value based linear-quadratic optimization function, the tuning guidelines thus obtained are more pragmatic. However, to seek the answers to questions such as: For which variables can the constraints be adjusted? Should the constraints for a particular variable be changed or not? Bayesian Statistics can be used. A detailed case study on a simulated distillation column MPC application is also provided to illustrate the proposed performance assessment method.

5.1 Introduction

Process control has helped industries in improving the efficiency of the operations and reducing the losses. A control system for any process is required to satisfy three general classes of needs (Stephanopoulos 1997). Firstly, to suppress the influence of external disturbances, secondly, to ensure stability of the process and lastly, to optimize the performance of the process. To satisfy the process specifications, is the key operational objective for any process. Once the objective is satisfied, the next operational goal is to optimize and to make the operations more profitable. As the process operations, to a great extent, are governed by the market forces of demand and profits, the operating conditions are required to be changed with the change in the demands and the product price patterns. However, the changes are required to be made in such a manner that the economic objective function is always optimized.

The Model Predictive Control (MPC) system performs the job of economic objective function optimization, on top of the dynamic optimization. Success of MPC system relies upon the accuracy of the model of the process for a good control. Using the process model, the controller predicts the behavior of the dependent variables (the output variables or the Controlled Variables (CVs)) of a dynamic system with respect to the changes in the independent process variables (the input variables or the Manipulated Variables (MVs)). The current process values and the process model are used to predict the future movements of the MVs that will result in the operation of the process honoring all the constraint limits defined for the CVs and the MVs (Wikipedia n.d.). Today, a lot of commercial controllers are available, each with its own control philosophy (Qin and Badgwell 1997, Qin and Badgwell 2003) and economic objective function. For example, Dynamic Matrix Control (DMC^{TM}) uses linear objective function through linear programming for optimizing the process operations (Sorensen and Cutler 1998) whereas, Honeywell's Robust Model Predictive Control Technology (RMPCT^{TM}) uses quadratic function along with the linear objective function to optimize and maintain the process variables at the desired values (Krishnan *et al.* 1998).

Having an MPC controller installed for a process, does not guarantee optimization of the process operations. For optimization, the controller is required to be tuned. The handles available to tune the controller are the control and prediction horizon, the step size for the moves on the MVs etc. Tuning of the controller, with these parameters, require a thorough understanding of the behavior of the process and the control philosophy of the MPC application being used. The controllers are tuned with these parameters at the design stage and it is not recommended to change them on day to day basis.

The performance of the controllers is also affected by the constraint limits given to the CVs and the MVs. Providing the controller with wrong and/or conflicting constraint limits is a main factors which can prevent the controller from operating the process at the optimum operating point. Since, in daily operations of the process plant, it is common to change the constraints, the controller performance is definitely affected when they are changed. Thus, it becomes imperative to assess the effect of any decision made with regards to constraint limits change. A methodology is therefore required to assess the change in the controller performance due to these changes. A statistical technique such as Bayesian Statistics can thus be used for the purpose.

The Bayesian analysis of the controller performance, as discussed in (Agarwal *et al.* 2007) is based on the linear-quadratic optimization (Xu *et al.* 2007), which is based upon the mean operating points. However, in reality the processes do not operate precisely on the mean operating point but around the mean operating point. Due to this, the process variables have probabilities associated with them to be inside and outside the constraint limits. Thus, it is essential that the objective function should also take into account the data distribution. An objective function taking into account the data distribution has been discussed in the previous chapter. Thus, extending the idea of Bayesian analysis for performance assessment of the MPC controller optimized through linear-quadratic optimization function (Agarwal *et al.* 2007) a similar analysis is proposed for the MPC performance assessment for a controller optimized with the probabilistic optimization function.

As discussed in the Chapter 3 of the thesis, for Bayesian analysis purposes, the following information is required: the routine operating data; process steady state gains; the profit/loss terms associated with the CVs to be within and outside the constraint limits and other related process information such as which CVs and MVs are allowed to change their limits and the preference to change them (prior probabilities). The algorithm then performs optimization on the various combinations of making the changes to obtain new optimum operating points. The optimization results are then used to establish the subsequent Bayesian decision making process.

A Bayesian network for the MPC application, under consideration, is then created with all CVs and MVs for which the changes can be made and all the quality variables. The probabilities estimated through the various optimization purposes are used to form the Conditional Probability Distribution Table (CPD or CPT) for the network (Korb and Nicholson 2004, Pearl and Russel 2000). The decisions regarding the changes are the evidences for the analysis. The analysis then provides the probabilities for the CVs to be inside and outside the specifications, which are then used for making the assessment of the performance of the controller. The network can also be used for the obtaining guidelines for making the decisions so as to obtain the desired performance level of the controller.

The contribution of this chapter can be summarized as: 1) A systematic approach for Bayesian analysis of the decisions related to the constraint changes for the CVs and the MVs, using a probabilistic optimization function. 2) Guidelines for making constraint changes, to achieve target controller performance, are derived according to the statistical inferences. As the theoretical concepts utilized for developing this algorithm have been discussed in detail in Chapter 3 of this thesis, they are not discussed here. The remainder of the chapter is organized as follows: Section 5.2 derives the Bayesian methods for MPC constraint analysis and tuning followed by case studies, on a simulated binary distillation column and an industrial distillation column, in section 5.3 and 5.4 respectively. Conclusion is given in Section 5.5.

5.2 Bayesian methods for MPC constraint analysis and tuning

As discussed previously, in detail in Chapter 3 of the thesis, for a $m \times n$ system, with the steady state gain matrix, K, and $(\bar{y}_{i0}, \bar{u}_{j0})$, the base case mean operating point of y_i and u_j respectively, a Bayesian network can be prepared that can be used to evaluate the decisions related to constraint changes for the controller. If a and b represent the number of CVs and MVs that are available for making constraint change, respectively, then $N = a + b$ is the total number of process variables available for making the changes. Thus, the Bayesian network will have N parent nodes. If the system has q quality variables, then the network will comprise of q child nodes. In continuation with the previous chapter, the algorithm has been developed for processes with CVs as the quality variables.

The options available for applying the constraint change are *yes* and *no* (*yes* means to make the changes and *no* means not to make the changes in the constraints). Thus, there are 2^N combinations for applying the constraint change. The optimal operating point corresponding to each application of the change can be obtained from the optimization function according to equation (4.19) and constraints defined for the case. The optimal expected return can then be estimated, using equation (3.14) for the optimal operating points. For $q = 1$, $F(y_1)$ is the return from the variables; however for $q > 1$, $F(y_1, \dots, y_q)$ has the additive form (equation- 3.15). Therefore, the expected return can be estimated as equation (3.16).

In continuation with the discussions in previous chapters, the continuous operating region for the CVs is discretized into 6 zones (Agarwal *et al.* 2007). Thus, each CV has a corresponding state space Ω of dimension 6 (*Zone 1, Zone 2, Zone 3, Zone 4, Zone 5, Zone 6*) (see section-3.3, fig- 3.4), which can be written as:

$$\Omega = \begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \\ \Omega_4 \\ \Omega_5 \\ \Omega_6 \end{pmatrix} = \begin{pmatrix} Zone\ 1 \\ Zone\ 2 \\ Zone\ 3 \\ Zone\ 4 \\ Zone\ 5 \\ Zone\ 6 \end{pmatrix}$$

If the uncertainties associated with each of $y_1, \dots, y_q$ are mutually independent, and the profit function is additive shown in equation (3.15), then equation (3.16) can be further simplified to

$$E(R) = \sum_{i=1}^{q} \sum_{k=1}^{6} P\left(y_i \in \Omega_k\right) \times F\left(y_i \in \Omega_k\right) \tag{5.1}$$

$P(y_i \in \Omega_k)$ is the probability for y_i to be in the k^{th} zone. $F(y_i \in \Omega_k)$ is the profit/loss for y_i to be in the k^{th} zone. For $q = 1$ univariate probability distribution function can be used, whereas, for $q > 1$ multivariate probability distribution function is to be used for estimating the probabilities for them to be in the various zones. However, in continuation with Chapter 4, the scope of the algorithm discussed here has been restricted to MPC applications with either one quality variable or quality variables that have mutually independent uncertainties. The probabilities for one or mutually independent quality variables to be in the six zones can be estimated using equation (5.2).

For *Zone k*, $P(y_i \in \Omega_k)$ is:

$$P(y_i \in \Omega_k) = \frac{1}{\sigma_i\sqrt{2\Pi}} \int_{L_{ky_i}}^{H_{ky_i}} exp\left(-\frac{(y_i - \bar{y}_i)^2}{2\sigma^2}\right) \mathrm{d}y_i \tag{5.2}$$

where, $i = 1, \ldots, q$ and $k = 1, 2, ..., 6$. L_{ky_i} and H_{ky_i} are the low and the high limits for y_i in the k^{th} zone (equation-3.17)

The return for the quality variables to be in each of the six zones are user specified according to the economic data. The profit/loss is usually assigned for the products to be under-spec (in *Zone 1*), in-spec (in *Zone 2 to 5*) and over-spec (in *Zone 6*). If F_1 and F_6 are the profits associated with y_i to be in *Zone 1* and *Zone 6*, respectively and F is the maximum return associated with it when the process operates in the maximum return zone i.e. the optimum operating zone, which typically lie at the constraint limits. Thus, the maximum return, with the constraint limit, will be in either *Zone 2* or *Zone 5*. Then assuming F to be the profit for optimum operating zone, the profit for y_i in the other 3 in-spec zones can be estimated through interpolation and be calculated using equation (4.3), (4.4), (4.5), as the case may be.

The optimization function can now be defined as equation-5.3.

$$J = \sum_{i=1}^{q}\sum_{k=1}^{6} P(y_i \in \Omega_k) \times F(y_i \in \Omega_k) \tag{5.3}$$

With $(\bar{y}_{i0}, \bar{u}_{j0})$ as the base case mean operating point, $(\bar{y}_i, \bar{u}_j)$ as the optimum operating point, when the base case operating points are moved by $(\Delta y_i, \Delta u_j)$, the equality constraints to be satisfied for the economic objective function are as defined in the previous chapter as:

$$\Delta y_i = \sum_{j=1}^{n}[K_{ij} \times \Delta u_j] \tag{5.4}$$

$$\bar{y}_i = \bar{y}_{i0} + \Delta y_i \tag{5.5}$$

$$\bar{u}_j = \bar{u}_{j0} + \Delta u_j \tag{5.6}$$

where, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Considering the acceptable limit for constraint violation, for the output variables, to be 5% (Latour *et al.* 1986, Martin *et al.* 1991), a set of inequalities can be defined which also need to be satisfied while optimizing the objective function defined in equation (5.3). The inequalities for the objective function are defined by equations (5.7) and (5.8). These inequalities define the constraint limits for the CVs and MVs:

$$L_{y_i} + 2 \times \sigma_{i0} - y_{holi} \times r_{y_i} \leq \quad \bar{y}_i \quad \leq H_{y_i} - 2 \times \sigma_{i0} + y_{holi} \times r_{y_i} \tag{5.7}$$

$$L_{u_j} + 2 \times R_{j0} - u_{holj} \times r_{u_j} \leq \quad \bar{u}_j \quad \leq H_{u_j} - 2 \times R_{j0} + u_{holj} \times r_{u_j} \tag{5.8}$$

where, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ and y_{holi}, u_{holj} are half of the range for y_i and u_j; r_{y_i}, r_{u_j} are the percentage change in the limits for the process variables. Subscript 'y_i' and 'u_j' represents the variable for the i^{th} output and j^{th} input variables, respectively. Since the constraint limits for any CV or MV are not changed randomly by large numbers, for illustration purposes, it is assumed that a change of 10% of the existing limit range can be made for the changeable variables. Thus, for the constraint limit change case the optimization problem can be defined as equation (5.9):

$$\max_{\substack{\bar{y}_1, \ldots, \bar{y}_m \\ \bar{u}_1, \ldots, \bar{u}_n}} \quad J \ \textit{subject to} \ (5.4), (5.5), (5.6), (5.7), (5.8) \tag{5.9}$$

The data and the optimization results thus obtained for all the cases of the limit change can now be used to build the Bayesian network for the process. The network comprises of the N variables available for making the changes as the parent nodes, q child nodes and one utility node, which represents the value of the benefit function for the process. For the constraint limits change case, the parent nodes have two states (*change limits, do not change limits*). The child nodes in the network have six states (*Zone 1, Zone 2, Zone 3, Zone 4, Zone 5, Zone 6*).

The prior probability or the priori define the preference for making or not making the change in the limits for the parent nodes. These can either be user defined or can be obtained from the historical data of the process. The Conditional Probability Table (CPT) state the probabilities for the child nodes to be in each of the six zones or the states. The optimization results for y_i, when combined with the standard deviation for the case and using Gaussian distribution, provides its probabilities for y_i to be in each of the six zones, which are used to make the CPTs for the child nodes.

For q quality variables affecting the economic objective functions, the utility node will have a total of 6^q values.

The Bayesian network thus created can now be used for decision evaluation and decision making purposes.

5.3 Case study of a Binary Distillation Column

Consider the MPC application for the simulated binary distillation column, discussed in section-2.3. The MPC controller designed for the process has 10 Controlled Variables (CVs) and 4 Manipulated Variables (MVs). The list of the CVs and their control objective and the profits associated with them to be under-spec (u/s), in-spec (i/s) and over-spec (o/s) are listed in table- 5.1 and the list of MVs if given in the table- 5.2. As it has been discussed in Chapter-3, CV2 and CV8 are two independent quality variables. As can be seen from the table- 5.1 that CV2 and CV8 are the quality variables to be minimized, the profits associated with these CVs to be in the six zones can be identified using equation (4.4).

The process variables that are available for making the constraint are listed in table- 5.3 and the optimization problem can be defined as equation (5.10).

$$\max_{\substack{\bar{y}_1,\ldots,\bar{y}_{10}\\ \bar{u}_1,\ldots,\bar{u}_4}} \quad J = \sum_{i=2,8} \sum_{k=1}^{6} P(y_i \in \Omega_k) \times F(y_i \in \Omega_k) \tag{5.10}$$

subject to, equations (5.4) to (5.8).

Based on the information provided in table- 5.3, the Bayesian network for the system can be created with 7 parent nodes (MV1, MV2, MV3, CV2*, CV5*, CV6* and CV7*), 2 child nodes (CV2 and CV8), and one utility node. The prior probabilities for the parent nodes were user defined and are listed in table- 5.4. Corresponding to 7 parent nodes for the network with two limit change states, $2^7 = 128$ optimizations were carried out. The

Table 5.1: Control objective and the profits for the CVs

CV	Description	Objective	**Profit**		
			u/s	Max	o/s
1	Reflux Flow	Constraint	0	0	0
2	Lt Petrol FBP	Minimize	65	65	0
3	Top PCT	Constraint	0	0	0
4	Pr Vlv OP	Constraint	0	0	0
5	Bttm PCT	Minimize	0	0	0
6	Col Pr	Minimize	0	0	0
7	Feed Temp	Constraint	0	0	0
8	Reboiler Duty	Minimize	200	200	0
9	Duty	Constraint	0	0	0
10	Bypass Vlv OP	Constraint	0	0	0

Table 5.2: List of MVs

MV	Description
1	Reflux SP
2	Col Pr SP
3	Feed Temp Vlv OP
4	Duty Vlv OP

Table 5.3: Variables available for change

	Change Lt
CV1	No
CV2	Yes
CV3	No
CV4	No
CV5	Yes
CV6	Yes
CV7	Yes
CV8	No
CV9	No
CV10	No
MV1	Yes
MV2	Yes
MV3	Yes
MV4	No

Table 5.4: Prior probability for making constraint changes for parent nodes

	Change Limits	Do not Change Limits
MV1	0.5	0.5
MV2	0.5	0.5
MV3	0.5	0.5
CV2*	0.5	0.5
CV5*	0.5	0.5
CV6*	0.5	0.5
CV7*	0.5	0.5

128 optimization results obtained for all the child nodes were used to create the CPT for the child nodes.

As for the process under consideration CV2 and CV8 are the two variables affecting the overall economic performance of the operations, the utility node for the process will have $6^2 = 36$ values for various combinations of the states of these CVs. The expected return from the process is 145.36 units and this will be used as the basis for evaluating the decisions made for applying limits change to the controller. The network was then used for decision evaluation and decision making purposes.

1. *Decision Evaluation*: For the said system if, the decision is made to change the limits of MV1, then the maximum a posterior estimate of the states of CV2 and CV8 are *Zone 4* and *Zone 3* respectively. For this decision, the expected return is estimated to be 176.61 units. The comparison of the expected return of the controller before and after the decision is made is shown in fig- 5.1. Thus, it can be inferred that the decision to increase the limits set for the MV1 will increase the expected return of the operations.

2. *Decision Making*: For the process under consideration, if the targets are set to increase the expected return from 145.36 units to 190.00 units, then the states (or the locations) of all the key process variables, i.e. the CVs affecting the utility node (CV2 and CV8) are determined (table- 5.5). With these states as the evidence, the maximum

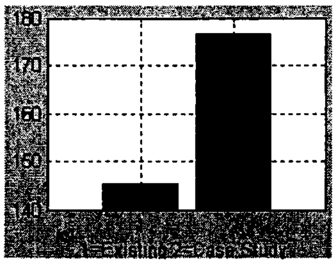

Figure 5.1: Comparison of the expected return

Table 5.5: States for key child nodes

CV#	State
CV2	Zone 4
CV8	Zone 3

a posteriori states for the parent nodes that are to have their limits changed are calculated.

For the case when the expected return are targeted to change from 145.36 units to 190.00 units, the parent node 1 and 3 i.e. MV1 and MV3 are expected to have their constraint limits changed by 10%, the same is shown in fig- 5.2.

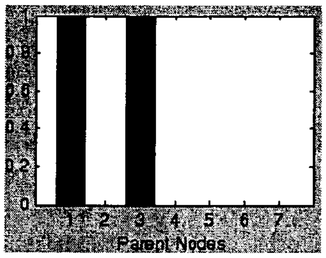

Figure 5.2: Tuning guidelines for limits change case

Section-5.4 (pages 104(part), 105 to 108 and 109(part)) has been removed from this thesis for proprietary reasons.

5.5 Conclusion

This chapter has brought about the use of Bayesian analysis for performance assessment of MPC controllers using probabilistic optimization function with regards to constraints tuning. The optimizer performs the optimization of the process for the controller taking into account the process variability and the same is being exploited by the above discussed methodology for evaluating the decisions regarding constraint changes for an MPC controller and for obtaining the guidelines for changing the constraint limits for the controller, so as to achieve the target return from the process.

Case studies have also been discussed those bring about the utility of the tool in process industry. The results obtained from the process can be used for day to day monitoring of the MPC controllers and for getting guidelines of making decisions for constraint changes for the controller. The decision guidelines obtained from the algorithm can be applied to improve the overall controller performance and thus improve the expected return from the process.

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6
Conclusion and Future Work

6.1 Conclusion

This thesis has proposed a probabilistic approach for MPC controller performance assessment and to obtain guidelines for tuning the controller. The approach recommends to take into consideration the process data variability to assess the performance of the controller. Taking into consideration the variability for the controller performance evaluation gives a more realistic assessment. Thus, the constraint tuning guidelines obtained by the use of probabilities is more practical and realistic.

Chapter 2 of the thesis provides two case studies on the Linear Matrix Inversion Performance Assessment (LMIPA). The first case study is on a simulated distillation column, while the second case study is performed on a distillation column from industry. The process data for these plants and the steady state gain models for the process were provided to the LMIPA for carrying out the analysis. The performance assessment for the controllers brought about some shortcomings about the LMIPA method. As, for performance assessment, the LMIPA method takes into consideration the mean operating points for the process variables while, in reality, the processes seldom operate precisely on the mean operating point, the controller tuning guidelines, provided by the algorithm, are function of the mean operating points only. Since the data trends show that even if the mean operating points are within the specifications, the data can be outside the specified limits at times, thus it is recommended that to obtain the controller tuning guidelines, data distribution be taken into account. A Bayesian approach is thus proposed for assessing and obtaining the controller tuning guidelines.

A Bayesian approach for controller performance assessment and constraint tuning has been provided in chapter 3 of the thesis. The approach utilizes linear-quadratic optimization function to evaluate the controller performance, for the various possible combination of tuning arrangements. It then provides the tuning decision analysis and the tuning guidelines, when provided with the target values of the controller performance assessment. Since the Bayesian analysis involves the use of probabilities to provide the controller tuning, the tuning guidelines provided by the controller are closer to reality and thus more realistic than using mean operating points for controller performance assessment.

The Bayesian algorithm discussed in the chapter 3 of this thesis utilized probabilities for decision analysis and to obtain the guidelines for making the decisions; however, the objective function used for the optimization purposes was based upon deterministic mean operating points for the process variables. Therefore, as an attempt to make more realistic optimization function, the idea of using probabilities, as in Bayesian approach of chapter 3, has been extended to the optimization function. A probabilistic optimization function, involving the probabilities, for Probabilistic Performance Assessment (PPA), has been introduced in chapter 4 of this thesis. Since the objective function used for controller performance assessment involves the probabilities, the controller performance is more realistic. The algorithm also facilitates the user to provide the maximum constraint change that is acceptable for each variable. It then provides with the optimal tuning of the process variables that can be made subject to the maximum change provided. Since the objective function involves the use of the in-spec and off-spec probabilities, it is more realistic and so are the tuning guidelines provided by it. It has also been demonstrated that LMIPA performance assessment is a special case of the probabilistic performance assessment.

Extending the idea of using Bayesian analysis for controller performance assessment, through LMIPA method, an algorithm has been developed for Bayesian analysis of the MPC controllers using Probabilistic Optimization method (chapter 5). Just as discussed for chapter 2, this algorithm also provides controller tuning decision analysis and the controller tuning guidelines for achieving the target value of the return. However, for this algorithm, since the optimization is carried out using the probabilistic objective function, the results are more pragmatic and realistic for practical applications.

6.2 Future work

The applications discussed in this thesis have been developed by assuming the data to be Gaussian Distributed and by discretizing the operating regions, of the CVs, into six zones. However, in reality, the process data is not necessarily be Gaussian Distributed and also, the process variables can assume values in continuous fashion. Thus, scope exists for developing similar algorithms that performs optimization and the analysis by estimating the true distribution for the data and for continuous regions of operations.

Also, for performing Bayesian analysis of the controllers, currently, processes with either one quality variable or with q quality variables with independent uncertainties have been considered and a 10% change from the original tuning values has been assumed. Scope also exists for developing algorithm that considers multivariate probability distribution and can provide the optimum tuning required for the changeable variables, by performing Bayesian analysis.

Appendices

A

GUIs for the algorithms developed

A.1 Bayesian-LMIPA

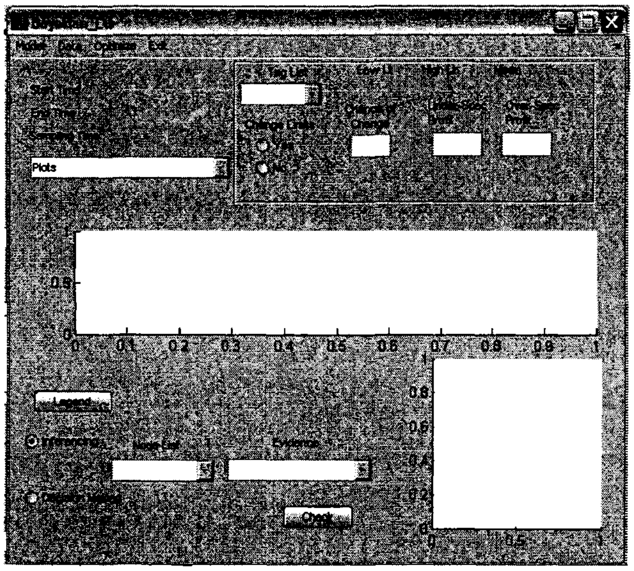

Figure A.1: Bayesian-LMIPA for limits change

Steps involved in using the GUI:

1. Load MPC model and the base case process data.

2. From the 'TagList' select the tags that are available for making the limit change and provide their probabilities for making the changes (Default value = 0.5).

3. Provide the profit/loss associated with the process variables to be under-spec and over-spec (Default value = 0).

4. Click 'Optimize' to start the optimization and to build the Bayesian Network for the process.

5. For 'Decision Evaluation', select 'Inferencing' radio button and select the parent node from the node list and provide the decisions for limits change as evidence. Click 'Check' to perform the evaluation.

6. For 'Decision Making', select 'Decision Making' radio button and provide the target Expected Returns for the process in the space that will appear on selecting this mode. Click 'Check' to obtain the guidelines for making the decisions.

A.2 Probabilistic Performance Assessment (PPA)

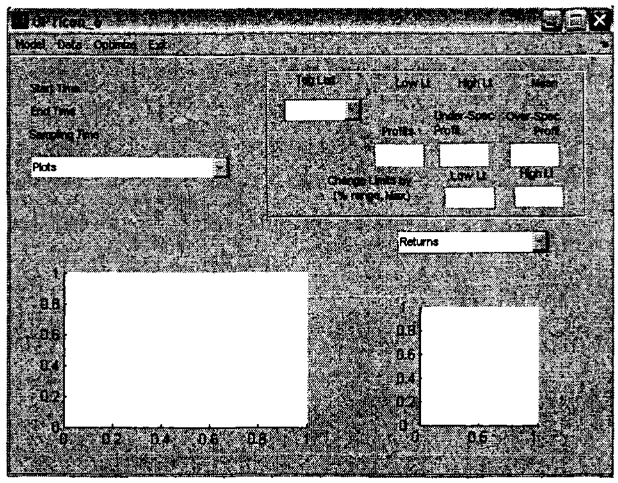

Figure A.2: Probabilistic Performance Assessment (PPA)

Steps involved in using the GUI:

1. Load MPC model and the base case process data.
2. From the 'TagList' select the tags that are available for making the limits change and provide the maximum allowable limits change that can be made to their low and the high limits (Default value = 10%).
3. Provide the profit/loss associated with the process variables to be under-spec, in-spec and over-spec (Default value = 0).
4. The left hand side plot area shows the process data trend, probability distribution and the cumulative probability distribution for the tag selected from the tag list.
5. Click 'Optimize' to start the optimization.
6. The right hand side plot area will show the base case expected returns, base case potential and the maximum expected returns that can be obtained after suggested constraint tunings are made. Suggested tuning guidelines can also be obtained from this plot area by selecting appropriate item from the drop down of 'Returns'.

A.3 Bayesian-PPA

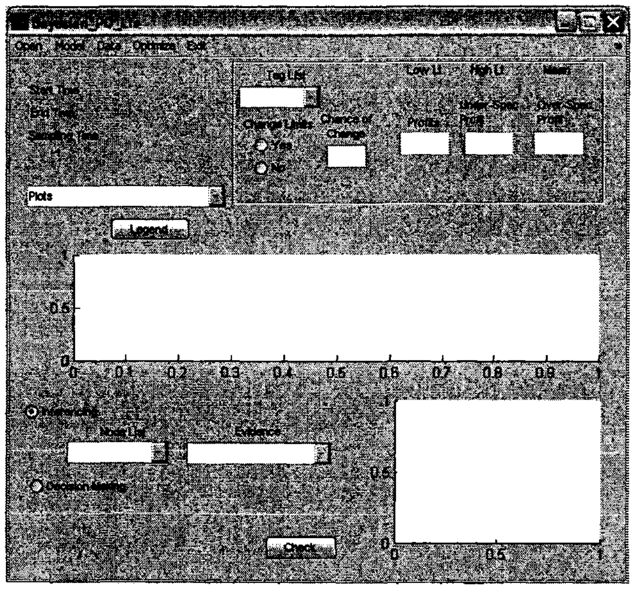

Figure A.3: Bayesian-PPA for limits change

Steps involved in using the GUI:

1. Load MPC model and the base case process data.

2. From the 'TagList' select the tags that are available for making the limits change and provide their probabilities for making the changes (Default value = 0.5).

3. Provide the profit/loss associated with the process variables to be under-spec, in-spec and over-spec (Default value = 0).

4. The upper plot area shows the process data trend, probability distribution and the cumulative probability distribution for the tag selected from the tag list.

5. Click 'Optimize' to start the optimization and to build the Bayesian Network for the process.

6. For 'Decision Evaluation', select 'Inferencing' radio button and select the parent node from the node list and provide the decisions for limits change as evidence. Click 'Check' to perform the evaluation.

7. For 'Decision Making', select 'Decision Making' radio button and Provide the target Expected Returns for the process in the space that will appear on selecting this mode. Click 'Check' to obtain the guidelines for making the decisions.