THE UNIVERSITY OF ALBERTA

A RESISTANCE ANALOG STUDY OF DRAINAGE DEPTH AND SPACING IN A GLACIAL TILL SOIL

by

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A THESIS

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ABSTRACT

An electrical resistance network analog was used to investigate drain depth and drain spacing in a glacial till soil. The "building block" approach was followed to represent the soil profile by electrical resistances. An impermeable layer was assumed to exist at a depth of five feet from the soil surface. The hydraulic conductivity of the glacial till soil simulated varied from 0.15 inches per hour to 0.45 inches per hour. Three drain depths of 3.0, 4.0 and 4.5 feet were simulated for each of twelve drain spacings which ranged from 10 to 120 feet in successive intervals of 10 feet. The following conclusions were drawn from this study.

1. Soil hydraulic conductivity is a key factor in enabling the resistance network analog to give comparable results to the field data.
2. The drain flow rate as found from the resistance network analog was approximately 3.5 to 4.5 times lower than that from the field data. The conclusion drawn was that either the soil hydraulic conductivity simulated was erroneous or that the soil was not isotropic as assumed.
3. The drain flow rate was found to be dependent upon the drain depth and drain spacings. As the drain depth was increased from 3.0 to 4.5 feet there was an increase in the drain flow rate. But, as the drain spacings were increased from 10 to 120 feet, there was a decrease in the drain flow rate.

4. The streamline distribution showed that the area contributing to 95 percent of the flow did not increase correspondingly with an increase in drain spacing.
5. It was concluded that an optimum depth and spacing of the drain in a glacial till soil was 4.5 feet and between 70 and 75 feet respectively.

ACKNOWLEDGEMENT

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Chapter 1

INTRODUCTION

With the increasing demand for food and fibre by a rising population, crop production will have to be increased. To increase production, either new land has to be developed or cultivation intensified. The amount of arable land in the world is limited so,to intensify cultivation, irrigation is essential. This is where the problem arises. With irrigation,there is a human tendency to apply an excessive amount of water. If the soil is impermeable or slowly permeable quite near to the surface, the excess water causes the water table to rise and with it the soil salinity increases. This excess amount of water and salts have a damaging effect on plant growth. To remove this excess water and to leach the salts, thus creating a proper environment for plant growth, drainage is the solution.

Drainage of agricultural land is the removal of the excess water by means of open or covered drains, the excess water being either on the soil surface or in the root zone. The excess water on the surface soil, which is a problem of surface drainage, can be remedied by any method of removing the surface water. The subsurface drainage problems are concerned with excess water in the root zone usually caused by a high water table. In some areas the water table can be lowered by controlling the sources of excess water. But, where an impermeable or very slowly permeable layer of soil is quite near the surface, the lowering of the water table can only be carried out by providing drains.

The purpose of drainage is to improve soil conditions by providing an environment in the root zone suitable to crop growth. Adequate drainage improves the soil structure thereby sustaining good crop yields over a long period of time. Other benefits derived from adequate drainage are numerous. For example, it facilitates early tillage and planting, lengthens the growing season, provides more available soil moisture and plant nutrition by increasing the depth of the root zone, improves soil aeration, favours the growth of bacteria, leaches excess salts and assures higher soil temperatures (13).

Considerable portions of the irrigated areas of Alberta and Saskatchewan, Canada, consist of coarse to medium textured soils underlain at shallow depths by slowly permeable glacial till. Under intensive irrigation the slowly permeable glacial till contributes to temporary water tables being formed, which are well within the root zone. There also is a salt concentration in the upper two feet of soil. Thus for satisfactory growth of crops, drainage is of utmost importance in these regions.

Determining the proper depth and spacing of drains is one of the most perplexing problems faced by a drainage engineer. The optimum depth and spacing of drains for a drainage system can be evaluated by;

1. Field experiments,
2. Laboratory studies involving tank models,
3. Analytical and numerical analysis, and
4. Analog systems such as an electrical resistance network.

Field experiments are accurate to a larger degree but they are time consuming and are a costly venture. Laboratory studies are also accurate if the soil conditions can be properly simulated. Numerical analyses are tedious, time consuming and occasional convergence difficulties are encountered but they have the advantage of simplicity and requiring no special equipment. The electrical analog, on the other hand, produces the same degree of accuracy as the others, requiring less time and having the advantage of relaxing the entire network.

There are two types of electrical analogs:

1. those utilizing the flow of electricity through sheets of electrical conducting paper, and
2. those using a network of resistors.

An electrical resistance network analog consists basically of a network of resistors mounted on a panel, a direct current (D.C.) source and a voltmeter. The operation is based on the analogy that exists between Darcy's Law and Ohm's Law. The reciprocal of hydraulic conductivity is represented by resistances, hydraulic heads (pressure head plus elevation head) by voltages and the rates of flow by currents.

The electrical resistance network is more flexible than the conducting paper because homogeneous or stratified, saturated or unsaturated, isotropic or anisotropic soil conditions can be simulated. An electrical resistance network model is well suited for studying drainage problems as it will give the desired accuracy in a shorter span of time and solutions can be obtained any time of the year when suitable field data are available.

The main objectives of this research were:

1. To simulate field soil conditions under tile drainage.
2. To determine the most suitable tile drain depth and spacing and compare these with experimental data.

3. Evaluation of maximum flow for different depth and spacing of tile drains.
4. Evaluation of the potential distribution in the draining soil.

It is to be noted that the term 'glacial till soil' implies a coarse- to medium- textured soil underlain at shallow depths by slowly permeable glacial till. The description and analytical data of such a typical soil have been given in the Appendix.

Chapter 2

REVIEW OF LITERATURE

The determination of depth and spacing of drains is still an empirical exercise. However, research workers have formulated a number of formulas based on a systematic and scientific approach for determining the depth and spacing of drains. Since soil characteristics are quite unpredictable because of the many variables involved, no satisfactory formula has yet been derived which will take into consideration the many variables and the soil differences.

Electrical analogs are a useful tool for studying drain depth and spacing, if intensive measurement of the hydraulic conductivity has been carried out. The advantages of electrical analogs are that they are relatively easy to construct and to operate, and the solutions can be obtained in a relatively short time.

2.1 Theory

There is an analogy between Ohm's Law and Darcy's Law which forms the basis of the application of electrical models in the investigation of the ground water flow problems. Analogy in simple terms means "similarity of properties or relations".

The fundamental equation for the flow of electric current is represented by the Ohm's Law, which can be expressed by an equation,

$$I = \frac{E}{R} \quad \ldots \quad (1)$$

where,

I = current in amperes, a quantity of flow,

E = pressure in volts, a potential function,

R = resistance, in ohms.

This equation could be given in terms of specific conductance instead of the resistance. The relationship is as follows,

$$\frac{1}{R} = K = k'\frac{A}{L} \quad \ldots \quad (2)$$

where,

K = conductance,

k'= specific conductance,

A = area,

L = length.

Equation (1) could be rewritten as,

$$I = k'\frac{E}{L}A. \quad \ldots \quad (3)$$

Darcy's Law can be expressed in terms of the following equation,

$$Q = k\frac{H}{L}A. \quad \ldots \quad (4)$$

where,

Q = the quantity of flow per unit time,

k = the hydraulic conductivity,

$\frac{H}{L}$ = the hydraulic gradient,

A = the cross sectional area.

It can be seen from the above two equations of Ohm's Law (equation 3) and Darcy's Law (equation 4), that I, the quantity of flow, k', the specific conductivity, E/L, the voltage gradient, A, the flow area of the Ohms Law equation is equivalent to Q, the quantity of flow, k, the hydraulic conductivity, H/L, the hydraulic gradient and A, the cross sectional area of the Darcys Law equation.

Darcy, in 1856, showed that the rate of flow of water through sand was proportional to the hydraulic gradient. Slichter (25) showed that a combination of Darcy's Law and the Law of Conservation of Mass would result in Laplace's equation. Slichter was also the first to recognize the similarity of Ohm's Law to Darcy's Law.

2.2 Mathematical Solution

A number of mathematical relationships for the movement of water into and through porous soil media have been evaluated by different workers. Hall (10) and Muskat (22) have used Darcy's Law and Laplace's equation to describe steady saturated flow. Darcy's Law has been used to describe steady unsaturated flow with some modifications by a number of investigators (1,16,24).

Mathematical solutions were obtained by Kirkham (15) for problems involving flat water tables or water ponded on the soil surface. In drainage the ponded condition is only temporary, but the data are of engineering importance as it gives the maximum flow into the drain per unit time and per unit length.

Luthin and Gaskel (18), using a numerical method, also known as the relaxation method, found an approximate solution to Laplace's equation. The numerical method was used for studying tile drainage in layered soil. This method is of little value, as tedious arithmetic calculations are involved, especially for complex boundary conditions.

2.3 Resistance Analog

The similarity of Darcy's Law to the Ohm's Law in the classic paper of Slichter (25) has simulated the development of electrical models for studying drainage and other problems by a number of research workers. Childs, cited by Luthin (19), made extensive use of this relationship in his work on a wide variety of ground water problems.

He used an electrical analogue made by soaking sheets of filter paper in graphite. Analogs constructed of continuous sheet conducters (presently a conductive paper invented by Western Union is available commercially under the name of Teledeltos) are also described by Muskat (22) and Karplus (14). This type of analog is useful for studying steady saturated flow in a homogeneous and isotropic media, but it is difficult or impossible to use for studying unsaturated flow or flow in a non-uniform soil. A number of investigators (6,8,21) have used this analog for studying two-dimensional flow through porous media.

An electrical analog which uses a network of variable resistors to simulate the distributed properties of the original field is known as an electrical resistance network. The theory and technique have been well developed by Luthin (19), Liebman (17) and Karplus (14). The electrical resistance network is more flexible than the conducting paper analog - first developed by De Pack (7) - and can be used to simulate flow conditions in soils that are homogeneous or stratified, saturated or unsaturated, isotropic or anisotropic. The resistance network has an important advantage of instantaneously relaxing the network, a procedure which requires many hours of tedious work with numerical analysis.

An infinite number of boundary conditions can be simulated and the conductivity of field varied at will by the use of variable resistors. Luthin (19) has discussed the methods of selecting different boundary conditions.

Craig (6) built a resistance network based on Luthin's principles to determine conditions necessary for a unique solution to a non-homogeneous, anisotropic flow system and to simulate relative conductivities of a specific soil profile. Hendricks (12) also used a variable resistance network for studying flow through saturated and anisotropic soil.

Worstell and Luthin (30) used a resistance network for studying seepage problems. Brutsaert, Taylor and Luthin (3) found that the electrical resistance network as described by Worstell and Luthin (30) was particularly useful for solving saturated flow problems where the potential distribution and total flux are required.

The electrical resistance analog has been used by Thiel, Vimoke et. al. (27) for studying moisture flow problems for steady state lateral flow and saturated flow of water in a three layered soil. Vimoke and Taylor (29) presented a method known as the "building block" method for calculating and assembling network resistances to represent water flow in soils.

Fitzsimmons and Corey (9) used an electrical resistance network for determining equivalent conductivities of soils for steady flow conditions. Corey and Fitzsimmons (5) also constructed the electrical resistance network for studying saturated flow in irrigation furrows.

Bouwer (2) used the resistance network for solving ground water problems. He found that predicting the behavior of actual ground water systems with a resistance analog requires adequate information on field hydraulic conductivities, water tables and boundary conditions. Conversely, the use of resistance analog permitted more complete utilization of complex field data than any other electric or physical model.

2.4 Accuracy of Electrical Analog

Luthin (19) compared the electrical resistance network solution with the exact solution obtained by numerical analysis. The resistance network gave values of hydraulic head that were identical to the numerical values to two places of accuracy. The comparison of the network solutions with the analytical solutions of Kirkham (15) indicated that the greatest error occurred at points near the drain where the hydraulic head was changing rapidly. This error was less than one percent in the interior of the region and was up to seven percent at points adjacent to the drain.

Vimoke et al (28) compared the drain flow rates in homogeneous medium as evaluated by the resistance network with the analytical solutions of Kirkham (15). It was found that the network generally deviates less than two percent if the logarithmic expression was used to calculate drain resistors while deviations were as high as 30 percent or more when a linear relationship was used.

An electrical resistance analog was constructed by Harper and Hammond (11) and was used to simulate water flow in two Florida soils. The results compared well with field data and were found to be similar.

Mein and Turner (20) used an electrical network for study of drainage in irrigated sand dunes. They found that errors arose due to contact resistance, finite mesh size, setting of resistors and variation in the input current sources. This analogue solution was thought to be accurate within four percent.

2.5 Computor model

A computor model was used by Taylor and Luthin (26) to solve for the hydraulic head potential for ponded flow in a stratified soil.

The procedure followed was essentially that reported by Luthin and Gaskel (18), the difference being that a high speed computor was used. A comparison was made with the exact analytical solution of Kirkham (15). The accuracy increased linearly with the reduction in mesh size and the deviations of computor results from those calculated ranged between one and two percent.

Chapter 3

METHODS AND PROCEDURES

3.1 Electrical Resistance Network Analog

The principle of operation of an electrical resistance network analog is based on the analogy of Darcy's and Ohm's Law as discussed previously. The resistance network analog consists basically of a network of resistors mounted on a panel or a board, a D.C. source and a voltmeter. The resistors are set to a proper resistance value to simulate field soil conditions.

An electrical resistance network is useful for studying drainage problems and mapping of flow nets. The problem under study here is determination of the drain flow rate at different depths and spacings of the drain in a glacial till soil.

3.2 Instrumentation

The basic instrumentation that is required for an electrical resistance network for the solution of a drainage or flow problem are the resistors, a network board, a power supply and a voltmeter/ohmeter for measuring voltage at different nodes and adjusting the resistors to a desired value.

3.2.1 Power Supply

The power supply was obtained from a EU-40A Solid State High Voltage Power Supply manufactured by Heath/Malmstadt Enke Instrumentation Laboratory (figure 1). It provided regulated D.C. voltages in the range of 50 to 300 volts up to 20 milliamperes. The D.C. output was regulated within one percent. The specification of the power supply was

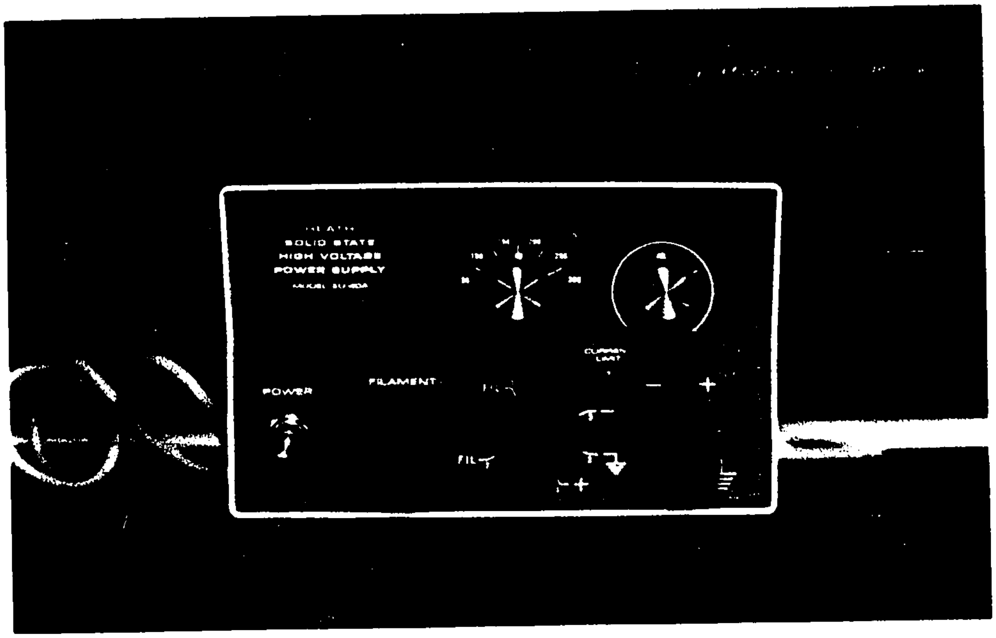


Figure 1: Power supply

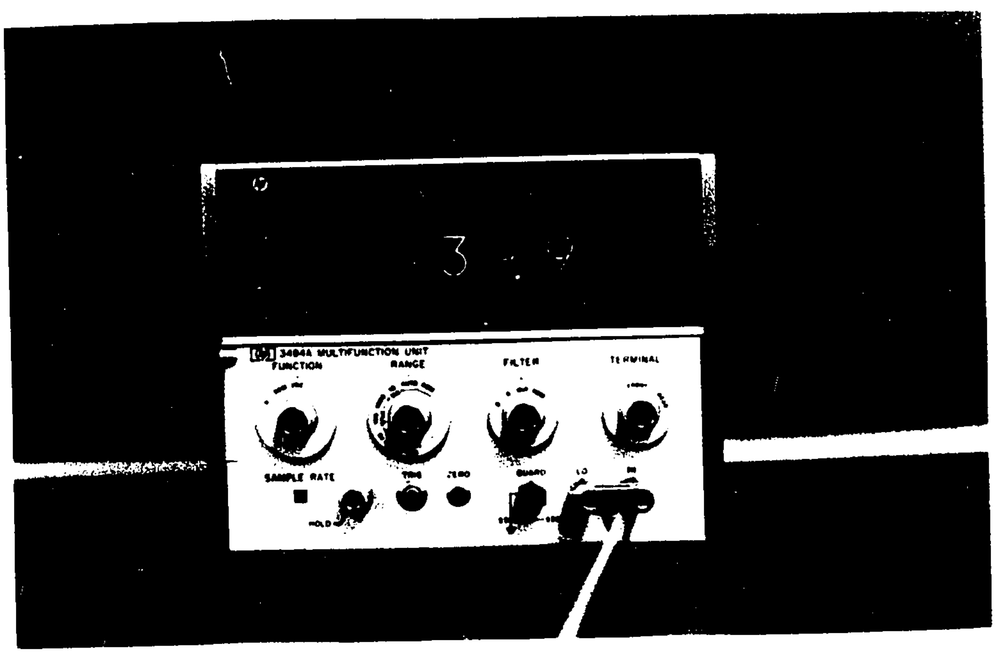


Figure 2: Digital voltmeter

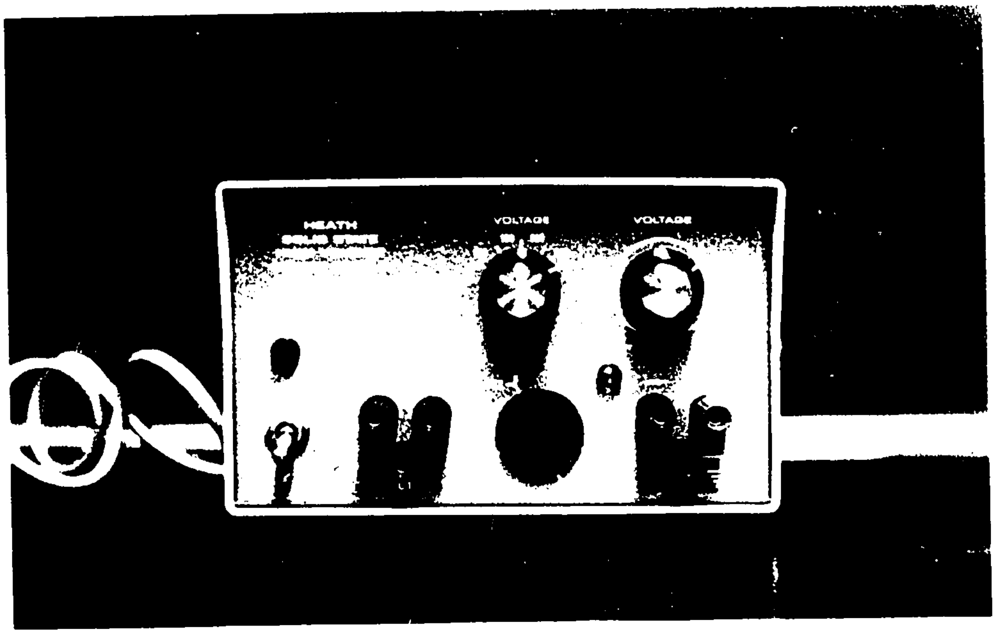


Figure 1: Power supply

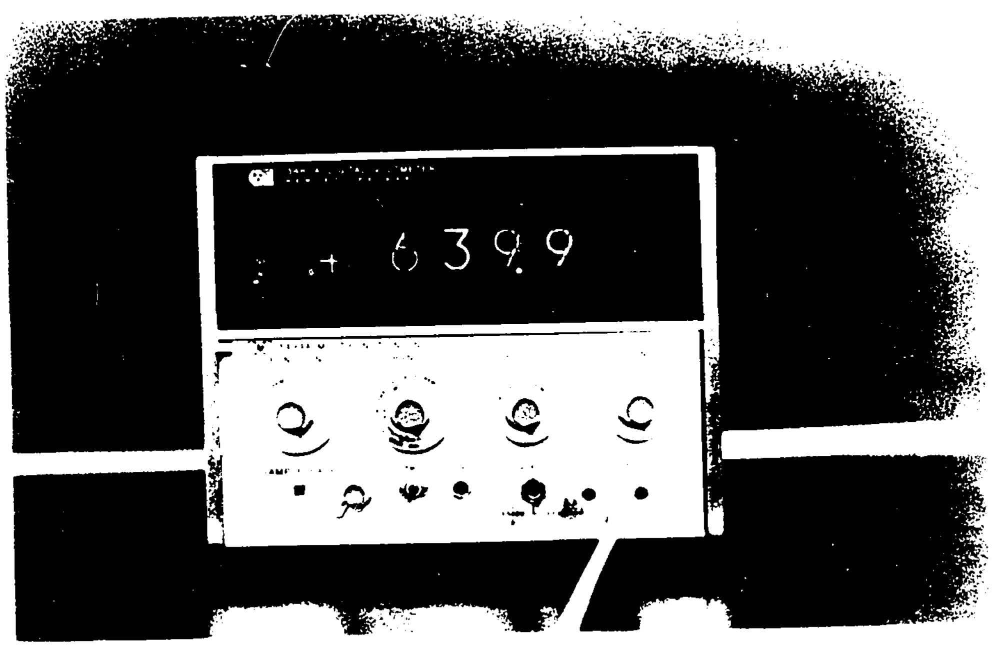


Figure 2: Digital voltmeter

as follows:

Voltage - 50 - 300 volts D.C. in 50 volt steps, each step continuously variable over 50 volt range.

Current - 0 – 20 mA regulated

25 mA with output shorted.

Regulation - 1% no load to full load.

0.5% with line voltage between 105 to 125 volts.

3.2.2 D.C. Voltmeter

A digital voltmeter Model 3480A manufactured by Hewlett Packard, together with a Model 3484A Multifunction Unit was used (figure 2). With this digital voltmeter,resistance settings could be adjusted to a desired value and voltages could be read at different nodes on the network.

The Model 3480A digital voltmeter makes positive or negative voltage measurements to four significant digits. Polarity selection and display are automatic. The sample rate is variable and by means of the front panel control in the plug-in unit 1 to 25 samples per second could be adjusted.

The specification of the voltmeter is as follows:

D.C. Volts Function:

Full Scale Voltage Ranges - 100 mV, 1000 mV, 10V, 100V, 1000V.

Voltage Accuracy - $\pm$(0.01% of reading +0.01% of range)

Input Resistance - 100mV, 1000 mV, 10V ranges: Greater than 10^{10} ohms.

100V, 1000V: 10 Megohms $\pm$ 0.1%.

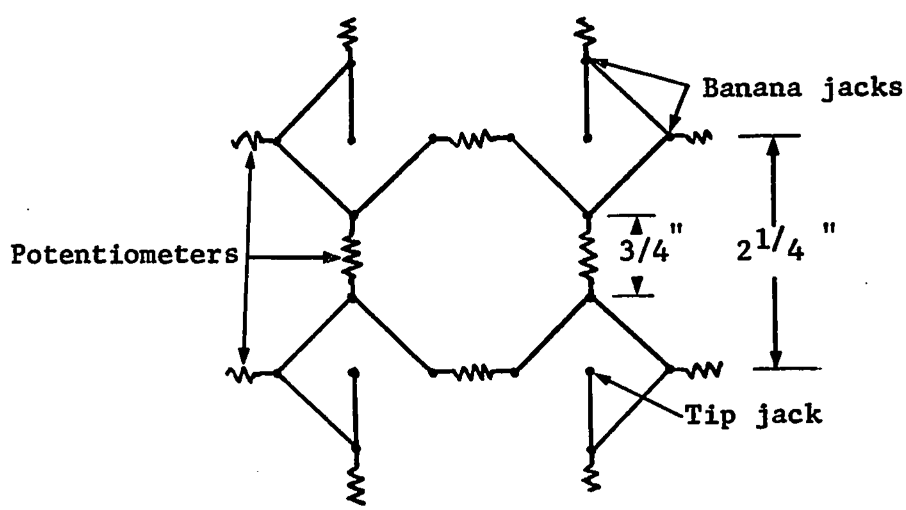
Ohms Function:

Full Scale Range	- 100 ohms, 1000 ohms, 10 kilohms.
	100 kilohms, 1000 kilohms, 10 megohms.
Measurement Accuracy	- 1000 ohms thru 1000 kilohms.
	±(0.01% of reading +0.01 % of range).

Voltage Across Unknown Resistance - 1V at full scale at all ranges.

3.2.3 Resistance Mounting Panel

Holes, 5/16 inch in diameter and 3/4 inch apart, were drilled in a masonite composition board 0.25 inch by 6 feet by 8 feet. A rectangular or a square block of soil is represented by a mesh or group of four resistors. A schematic diagram is shown below.



Banana jacks were installed in the drilled holes. A square mesh of banana jacks were shorted (as shown in the above diagram) to form a node or a junction which is further connected to the tip jack in the centre. This tip jack was useful in the measurement of the potentials at the nodes. The banana jacks were 3/4 inch apart to accommodate the banana plugs of the potentiometer. A full view of the board is shown in figure 3.

3.2.4 Potentiometers

A potentiometer together with its parts are shown in figure 4. Resistors with a range of 0 to 5 megohms were used in making the potentiometer. The potentiometer banana plugs were 3/4 inch apart so that they could be plugged into the board as well as on to the digital voltmeter for adjusting the resistance to the desired value.

3.3 Methods

The method adopted to represent the soil by a network of resistors is commonly known as the "building block" method employed by Vimoke and Taylor (29). The soil profile is considered to be composed of discrete rectangular blocks of soil. Each block of soil is represented by a mesh of four resistors and the meshes are joined to form a network.

3.3.1 Representing a Block of Soil

A square of conducting paper can be used to represent any square dimension of uniform soil which has a unit thickness. The same information can be obtained by using a group of resistors if the conditions that exist inside the boundaries of a square conducting paper are of no significance.

A square piece of conducting paper can be represented by a mesh of four resistors in different ways as shown in figure 5, the best and most convenient being as shown in figure 5 (b).

A rectangular block of soil 'a' by 'b' can be represented by resistive paper as shown in figure 6. If this paper is cut horizontally, then each section can be represented by a resistor Ra. Similarly, when

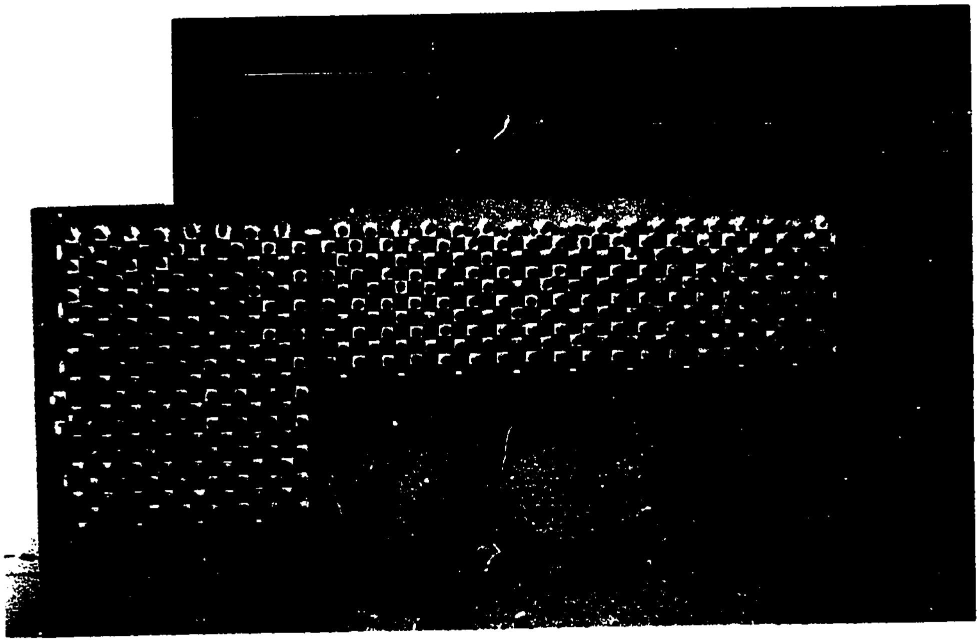


Figure 3: Resistance network board.

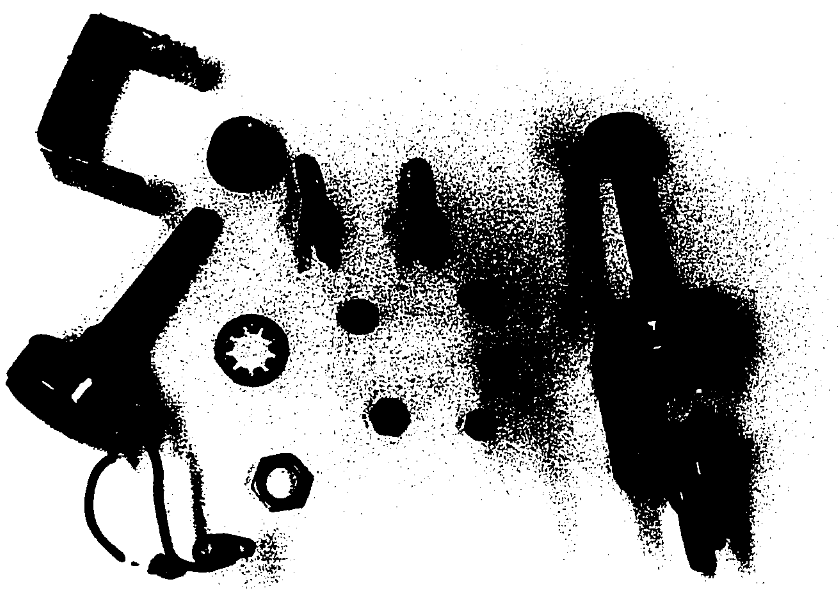


Figure 4: Potentiometer and its parts.

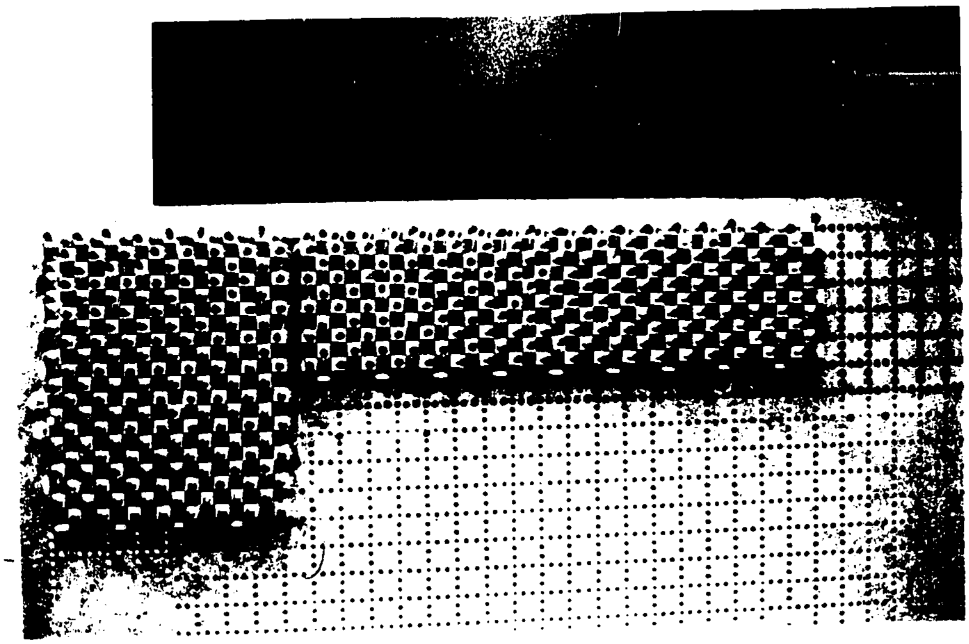


Figure 3: Resistance network board.

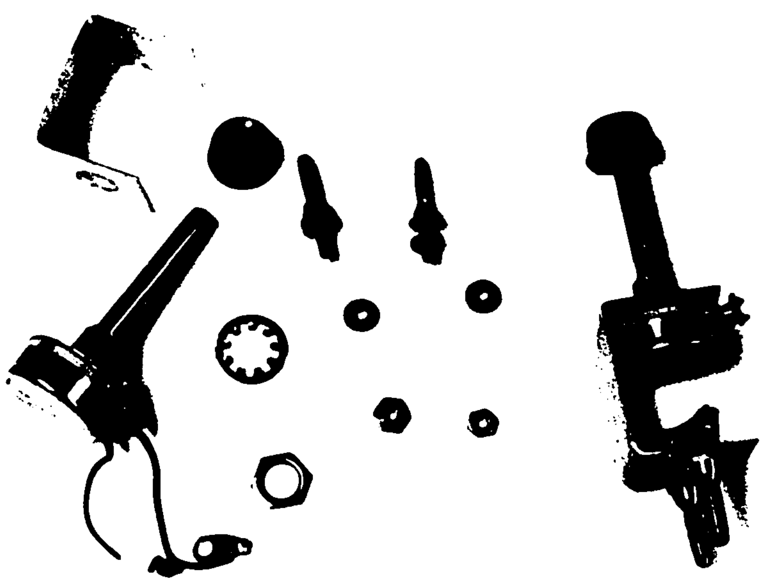


Figure 4: Potentiometer and its parts.

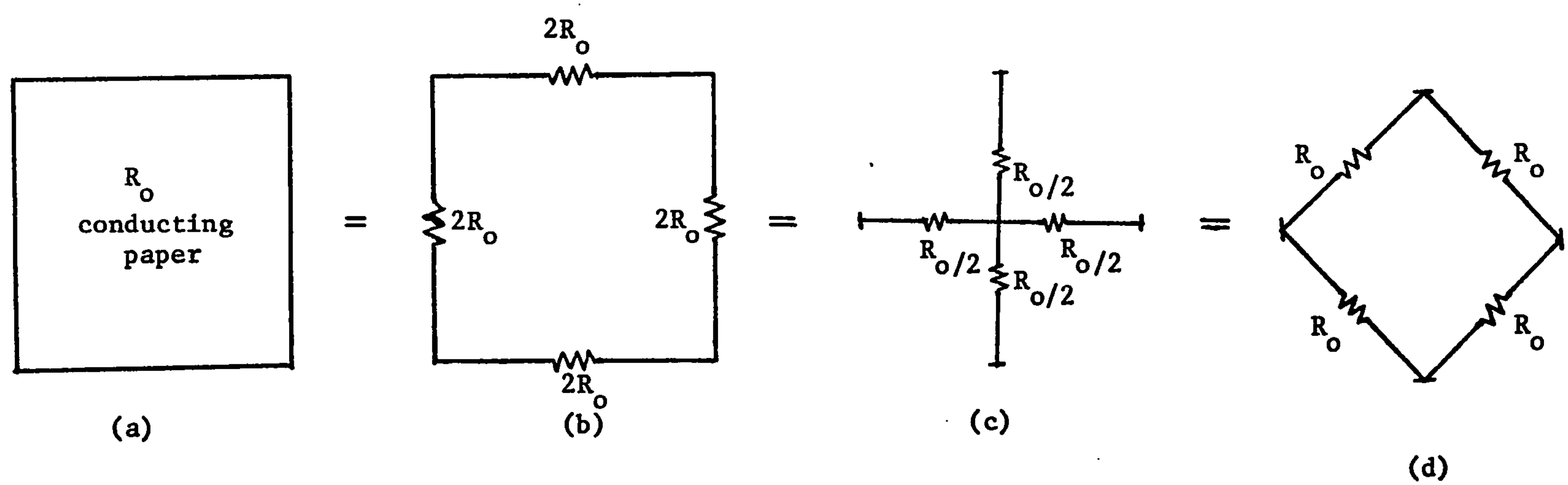


Figure 5. Conducting paper represented by a group of resistors.

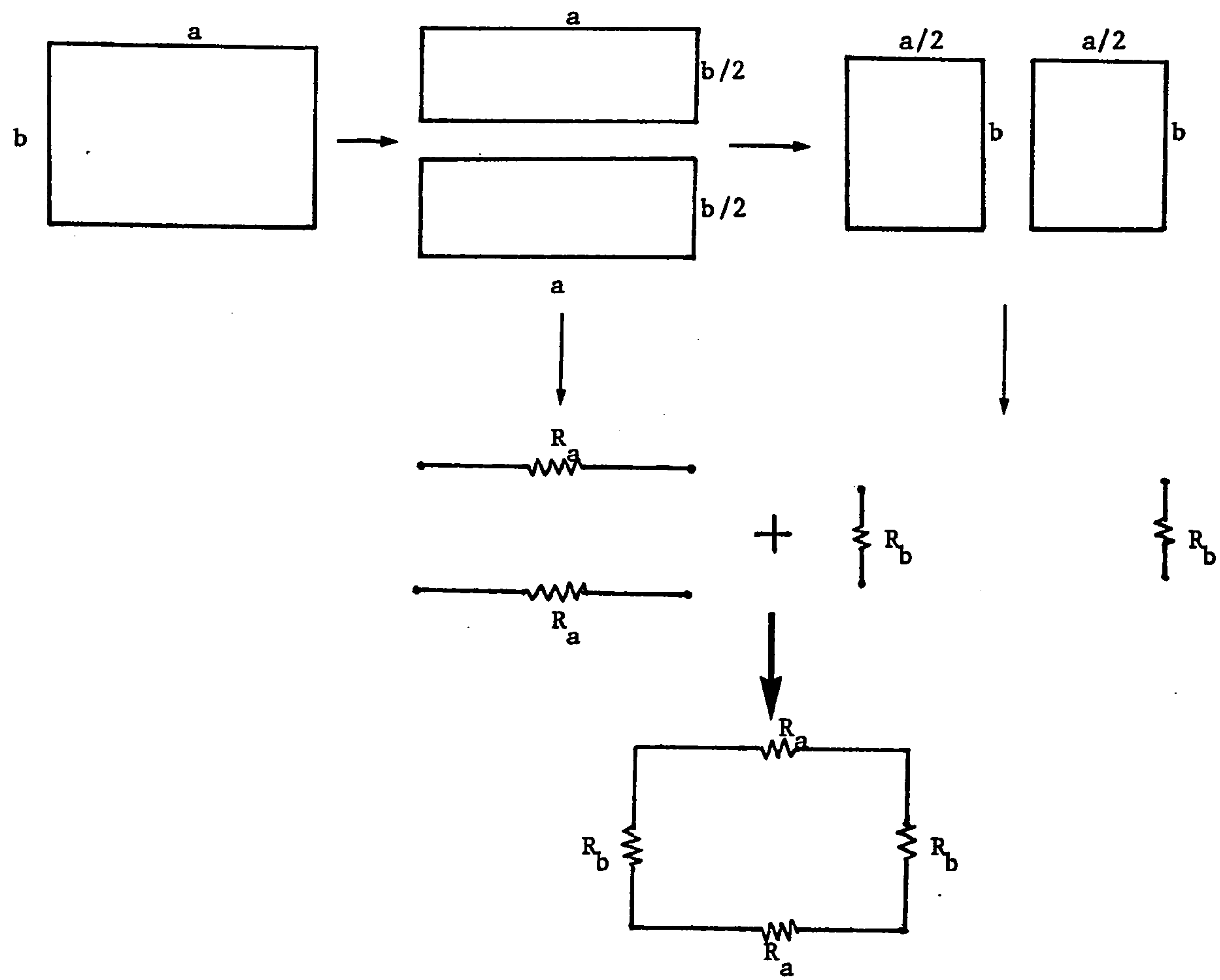


Figure 6. Representing a rectangular block of soil by four resistors.

the resistive paper is cut vertically, each section can be represented by a resistance Rb. The value of these resistances are as given below.

$$Ra = \frac{\text{length}}{\text{width}} \; Ro = \frac{a}{b/2} \; Ro = \frac{2a}{b} \; Ro$$

and,

$$Rb = \frac{\text{length}}{\text{width}} \; Ro = \frac{b}{a/2} \; Ro = \frac{2b}{a} \; Ro$$

where Ro is the characteristic resistance of the conducting paper.

To check the validity, if 'a' is equal to 'b', that is, in the case of a square block, then the boundary resistors become equal to 2Ro which is in agreement as shown in figure 5 (b).

3.3.2 Representing Drain by Network Resistances

Drain resistances were calculated using the method of Vimoke and Taylor (29). This method of calculating drain resistances takes into consideration the curvature of the drain. A network of fine mesh of resistances around the drain was used because of the greater potential drop here.

The mesh around the drain is as shown in figure 7. Mathematically, the resistance Rd is given as (29, pp. 24-25):

$$Rd = \frac{8\ Zo}{Z'o} \; Ro$$

where,

Ro = characteristic resistance of a square sheet of conducting paper.

Zo = characteristic impedance of a transmission line.

Z'o= characteristic impedance of free space.

For evaluating Zo and Z'o the following equations have been given:

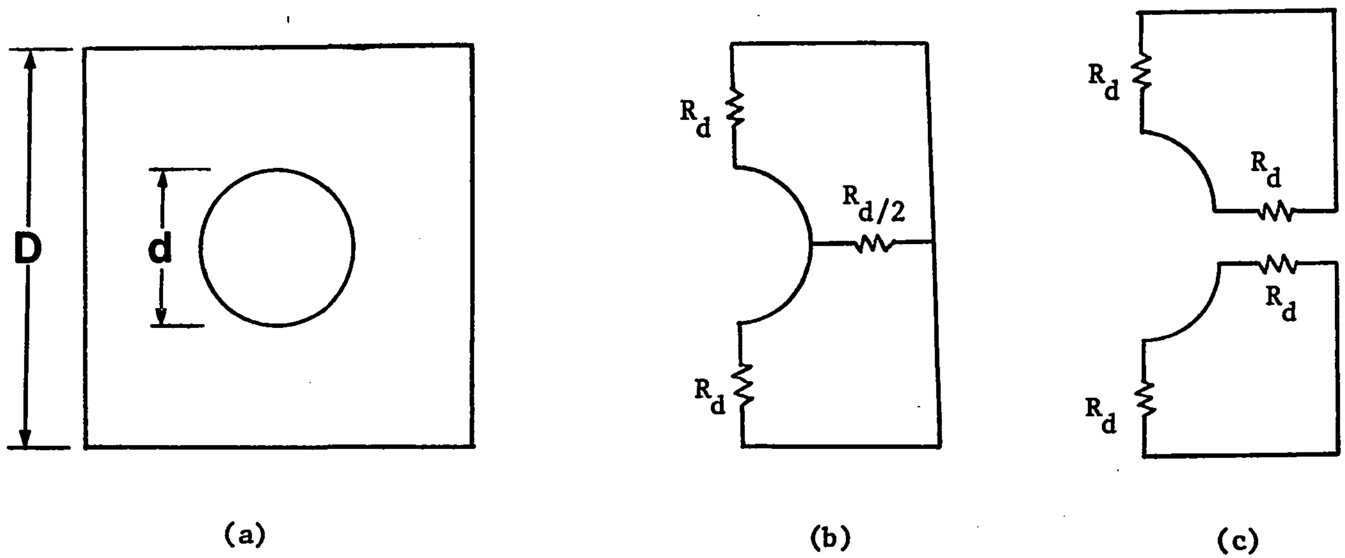


Figure 7. Representing drain by resistors.

$$Zo \simeq 138 \log_{10} p + 6.48 - 2.34A - 0.48B - 0.12C$$

where,

$$p = \frac{D}{d} \quad \text{(Figure 7a)}$$

$$A = \frac{1 + 0.405p^{-4}}{1 - 0.405p^{-4}}$$

$$B = \frac{1 + 0.163p^{-8}}{1 - 0.163p^{-8}}$$

$$C = \frac{1 + 0.067p^{-12}}{1 - 0.067p^{-12}}$$

and, $Z'o = \sqrt{\frac{\mu o}{\varepsilon o}} \simeq 120\pi = 376.7$ ohms.

where,

μo = permeability of free space

εo = permitivity of free space.

3.3.3 Calculating and Measuring Drain Flow Rate

The drain flow rate for different depths and spacings of tile drain have been calculated by the following formula (29, p. 30). The drain flow rate is given in cu ft/ft drain-hr

$$Q = 2K \frac{(d + t - r)}{Rn} Ro$$

where,

Q = drain flow in cu ft/ft drain-hr.

K = hydraulic conductivity of soil in which drain is embedded, ft/hr..

d = vertical distance between drain center and the soil surface, ft.

t = depth of ponded water, ft.

r = radius of drain, ft.

Ro = characteristic resistance of soil in which drain is embedded, ohms.

Rn = total resistance between top boundary of the network and the drain terminal, ohms.

The above formula is convenient in that the flow is independent of the applied voltage and the current. The only value that has to be known is the resistance, Rn, across the network which can be measured with a greater degree of accuracy, thus avoiding the errors introduced by the measurement of voltage and current in other techniques.

For measuring the resistance, Rn, the digital voltmeter's Multifunction Unit was used. Of the two terminals, one is connected to the drain terminal and the other to the top and furthermost corner of the network. The resistance measured between these two points is the resistance, Rn, of the network for the particular depth and spacing. The other terms, i.e. K, d, t and r, are known and the drain flow rate can be evaluated from the above formula. The value of Ro to be used in the formula is the characteristic resistance of the hydraulic conductivity of that layer of soil in which the drain is embedded.

3.3.4 Resistance Settings on the Potentiometer

The digital voltmeter was used to adjust the setting of a potentiometer to the desired value. The potentiometer was plugged on to the digital voltmeter's Multifunction Unit and by turning the knob it was set to give the desired value. The voltage passing across the unknown resistance was one volt. The measurement accuracy of this instrument was 0.01 percent of reading plus 0.01 percent of range.

A fine mesh of resistors was used around the drain as the greatest potential drop occurs around this region. In the experimental work carried out, a fine mesh of 4 feet by 5 feet was used with a mesh size of 6 inches by 6 inches, i.e. the soil simulated by a group of four resistors is a block 6 inches by 6 inches, for all depths and spacings of the drain. The remainder was a coarse mesh and its size depended upon the spacing of the drain.

3.3.5 Potential Measurements for Equipotential and Streamlines

Potential distribution can be obtained for a given set of boundary conditions. From this potential distribution equipotential or streamlines can be sketched so as to obtain the characteristic movement of water in the soil under study.

For obtaining the potential distribution for equipotential lines, the top boundary is shorted and an e.m.f. is applied, the drain terminal being connected to the ground as shown in figure 8 (a). The digital voltmeter was used to measure the potential at each node of the network. From this potential distribution, equipotential lines were drawn by interpolation.

To obtain the potential distribution for stream or flow lines, the boundaries of the network were reversed as shown in figure 8 (b). An e.m.f. was applied to previously unconnected boundaries and potential measurements were again taken at all the nodes of the network. From this potential distribution streamlines were drawn from which the direction of water flow can be determined.

3.4 Procedure

The procedure adopted for setting up the resistance network and the evaluation of the data was as follows:

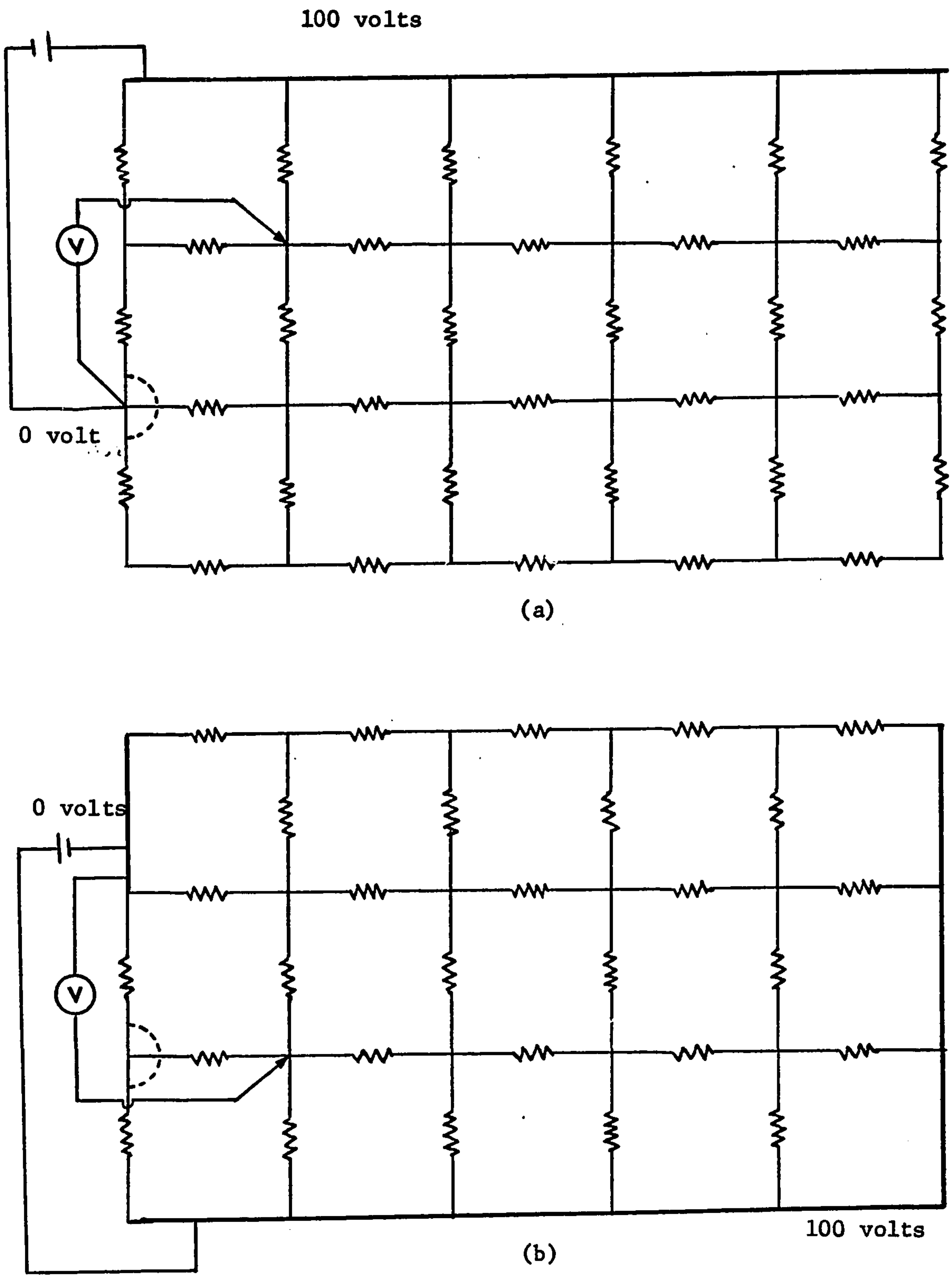


Figure 8. Boundary conditions for measuring potentials for
(a) Equipotential distribution, and
(b) Streamline distribution.

1. The hydraulic conductivity of the glacial till soil for different layers was taken from the work by Rapp (23). Table 1 gives the average hydraulic conductivity of different soil layers.
2. A characteristic resistance of 0.5 megohms was assumed for a hydraulic conductivity of 0.20 inches per hour. With the above assumption, the characteristic resistance of other layers was calculated and tabulated (table 1).

TABLE 1: HYDRAULIC CONDUCTIVITY AND CHARACTERISTIC RESISTANCE FOR DIFFERENT DEPTH RANGES.

Range of depth, ft.	Average hydraulic conductivity, inches/hr.	Characteristic resistance, kilohms
0 - 0.5	0.45	222
0.5 - 1.0	0.35	288
1.0 - 1.5	0.26	385
1.5 - 2.0	0.18	555
2.0+	0.15	666
0 - 1.0	0.40	250
1.0 - 2.0	0.22	454
2.0+	0.15	666

3. Having calculated the characteristic resistances, the resistances of the entire network were calculated.
4. With the drain diameter assumed as four inches, the resistances around the drain were evaluated.

5. The variables chosen in the experiment were depth and spacing. Three depths and twelve spacings were selected to be simulated on the resistance network.
6. The resistances were adjusted to the proper value with the digital voltmeter and then plugged on to the resistance network board.
7. For the measurement of flow, the resistance between the top furthermost corner of the network and the drain terminal was measured. Using this resistance value for Rn in the drain flow formula, the drain flow was evaluated.
8. For finding the potential distribution for equipotential and streamlines, an e.m.f. of 100 volts was applied to a set of boundary conditions as shown in figure 8 (a) and (b). The potentials at all the nodes were determined with the digital voltmeter. Equipotential and streamlines were then drawn by interpolation.

3.5 Assumptions Made

The following assumptions were made in the resistance network study of drainage of a glacial till soil.

1. An impermeable layer at a depth of five feet.
2. The soil was assumed to be isotropic.
3. The soil was assumed to be stratified into different layers according to the hydraulic conductivity of the soil at different depths.
4. The drain diameter of four inches is constant for all depths and spacings simulated.

5. The soil is assumed to be saturated and ponded flow was assumed to exist.

6. In the drain flow formula used, the drain was assumed to be flowing full with no back pressure and the drain was completely permeable to water.

7. A characteristic resistance of 0.5 megohms was assumed for a hydraulic conductivity of 0.20 inches per hour.

4. RESULTS AND DISCUSSION

4.1 Simulation of Field Soil Condidions Under Tile Drainage

One of the objectives of this study was the simulation of field soil conditions in a glacial till soil under a system of tile drainage. The procedure for simulation has been explained in the chapter on "Methods and Procedure."

A soil depth of 5 feet was simulated. The soil was considered to be stratified into definite layers and the hydraulic conductivity of each soil layer was known. The soil was considered to be isotropic, i.e., the hydraulic conductivity was the same in the horizontal direction as in the vertical direction. The hydraulic conductivity of the different soil layers was taken from a curve of estimated and measured variation of the hydraulic conductivity with depth (23). Hydraulic conductivity measurements in the field were made by the auger-hole method and the piezometer method.

A fine network mesh was installed around the drain region. A fine mesh meaning that the block of soil simulated was smaller in size. A fine mesh around the drain is used because the potential drop around the drain is greater as compared to the rest of the network. For all depths, a fine mesh of 4 feet by 5 feet was used with a mesh size of 6 inches by 6 inches. Because of the similar conditions existing on either side of the drain, only half the spacing of the drain was simulated.

The drain resistances were calculated according to the procedure given in chapter 3.3.2. Drain resistances calculated by the above procedure takes the curvature of the drain into consideration.

4.2 Maximum Drain Flow Rate at Different Depths and Spacings

For the evaluation of the drain flow rate for different depths and spacings, the formula given in section 3.3.3 was used, which is as follows:

$$Q = 2K \frac{(d + t - r)}{Rn} Ro$$

where,

Q = drain flow rate, cu ft/ft drain-hr.

K = hydraulic conductivity of the soil in which the drain is embedded, ft /hr.

d = vertical distance between drain center and the soil surface, ft.

t = depth of ponded water, ft.

r = radius of drain, ft.

Ro = characteristic resistance of soil in which the drain is embedded, ohms.

Rn = total resistance between the top boundary of the network and the drain terminal, ohms.

With the resistance network analog simulating a particular drain depth and spacing, the depth d and the radius r of the drain are known. The thickness or depth of the water applied was assumed to be one inch for all spacings and drain depths. The resistance Ro and the hydraulic conductivity K for the layer of soil in which the drain is embedded must be known for evaluating the drain flow rate. The resistance Rn is the total resistance which is measured between the top boundary of the network and the drain terminal.

As an example for calculating drain flow rate for a drain spacing of 30 feet and a drain depth of 3 feet, the hydraulic conductivity K of the soil layer in which the drain is embedded is 0.15 inches per hour. The characteristic resistance of that layer was 666 kilohms, the radius of the drain was 2 inches and the total resistance measured across the network was 2215 kilohms.

The drain discharge or flow rate is

$$Q = 2 \times \frac{0.15}{12} \; \frac{\left(\frac{36 + 1-2}{12}\right)}{2215} \; 666$$

$$= 21.924 \times 10^{-3} \text{ cu ft/ft drain-hr.}$$

The drain flow rate at different spacings and depths have been tabulated in table 2.

4.3 Comparison of the Drain Flow Rate

The drain flow rate at 30,60,90 and 120 foot spacings as computed from the resistance network analog was compared with the field experimental results as reported by Rapp (23). The maximum drain flows have been taken from tables (23, pp. 60-63) of experimental work.

The computation of the drain flow rate from the resistance network analog is the maximum for the soil conditions simulated. This can only be compared with the maximum drain flow of the experimental data, although there are losses due to consumptive use, deep percolation and lateral flow. The drain flow rate computed from the resistance network analog and field experimental results have been tabulated in table 3.

A study of table 3 shows a large difference in the drain flow rates. The maximum drain flow rates as evaluated from the field experimental data are approximately four times higher than those found

TABLE 2: DRAIN FLOW RATE AT DIFFERENT DEPTHS AND SPACINGS - CU FT/ FT DRAIN-HR x 10^{-3}.

Drain spacing, ft.	Drain depth, ft. 3.0	4.0	4.5
10	38.358	45.794	46.749
20	28.119	35.345	37.083
30	21.924	28.060	30.027
40	17.493	22.455	24.150
50	15.020	19.419	21.064
60	14.187	18.200	19.784
70	12.957	16.768	18.275
80	11.402	14.936	16.342
90	10.372	13.493	14.910
100	9.316	11.988	13.361
110	8.315	10.969	12.061
120	7.789	10.185	11.075

from the resistance network analog, which is quite contrary to expectations.

The maximum drain flow rates as abstracted from the field experimental data are for transient flow conditions, whereas the drain flow rates calculated from the resistance network analog are for a steady state ponded condition. Therefore, the drain flow rates as computed using the resistance network analog should be greater than those calculated from the field experimental data.

TABLE 3: COMPARISON OF DRAIN FLOW RATE.

Spacing, ft	Drain flow rate, cu ft/ft/drain-hr.		Ratio of Differences
	Resistance Network Analog	Field Experimental Data	
30	21.924	89.11	4.06
60	14.187	65.26	4.59
90	10.372	25.37	2.45
120	07.789	29.13	3.73

Another reason why the drain flow rates from the resistance network analog should be higher is because of the boundary conditions. In the resistance network analog all the flow has to pass through the drain, because of the boundary conditions, which is not the case with the field experimental work, where water is lost due to deep percolation, evaporation and consumptive use.

A conclusion can be reached that the values used to simulate field soil conditions are erroneous. By inspection of the formula for the calculation of discharge or drain flow rates, discrepancies could arise from the soil hydraulic conductivity values used, and also from the total resistance Rn across the resistance network.

The soil hydraulic conductivity values are possibly the greatest source of variance in the differences found in comparing the drain flow rates. The soil hydraulic conductivity for the stratified soil simulated was evaluated from the estimated variation of the hydraulic conductivity graph (23, pp. 58). The hydraulic conductivity for the estimated variation was measured in the field by the auger-hole method and the piezometer method. The differences in the drain flow rate are dependent on the estimated variation of the hydraulic conductivity which, in this case, was not truly representative of the field soil conditions.

The soil simulated was considered to be isotropic, i.e. the hydraulic conductivity is the same in all directions. This is not true with the glacial till soil. It is quite possible that the hydraulic conductivity in the horizontal direction may be four or more times greater than that in the vertical direction. If the soil is not isotropic, then the resistance settings of the potentiometer for the soil simulated will be different. This, in turn, will affect the total resistance Rn, which in turn will affect the computed drain flow rate.

The second factor that could explain the lower drain flow rates from the resistance network is the total resistance Rn. Studying the drain flow rate formula, it is seen that the higher the total resistance Rn, the lower is the drain flow rate. However, this is dependent upon the resistance settings of the resistors which were adjusted according to the soil hydraulic conductivity values used. The total resistance Rn across the resistance network analog was taken twice, once before and once after taking the potential readings at different nodes to check for any error.

The conclusion of the above discussion is that either the hydraulic conductivity of the soil simulated was erroneous or that the assumption made for an isotropic soil is not true.

4.4 Comparison of Drain Flow Rate and Depth of Drain for Constant Spacing

The drain flow rate at three different depths for each spacing are shown in figure 9. The family of curves indicate that as the depth of the drain is increased, there is a corresponding increase in the drain flow rate. For example, in the case of a 60-foot drain spacing, as the drain depth was increased from 3.0 to 4.0 to 4.5 feet, the drain flow rate increased from 14.2 to 18.2 to 17.8 cu ft/ft drain-hr. (10^{-3}) respectively. The same conclusions were obtained by Kirkham (15) in a study of flow through soil having ponded water on the surface.

The curves also indicate that at close drain spacings the rate of increase of drain flow rate is curvilinear, but as the drain spacings increase the drain flow rate increase with depth becomes more linear. This means that the rate of flow to the drain increases with an increase in depth. As the depth of drain is increased, there is a corresponding increase in the area contributing to the drain flow rate. This possibly could be the reason for a higher drain flow rate with a corresponding increase in drain depth.

The hydraulic conductivity of the soil from two feet depth onwards to the impermeable layer was the same, having a value of 0.15

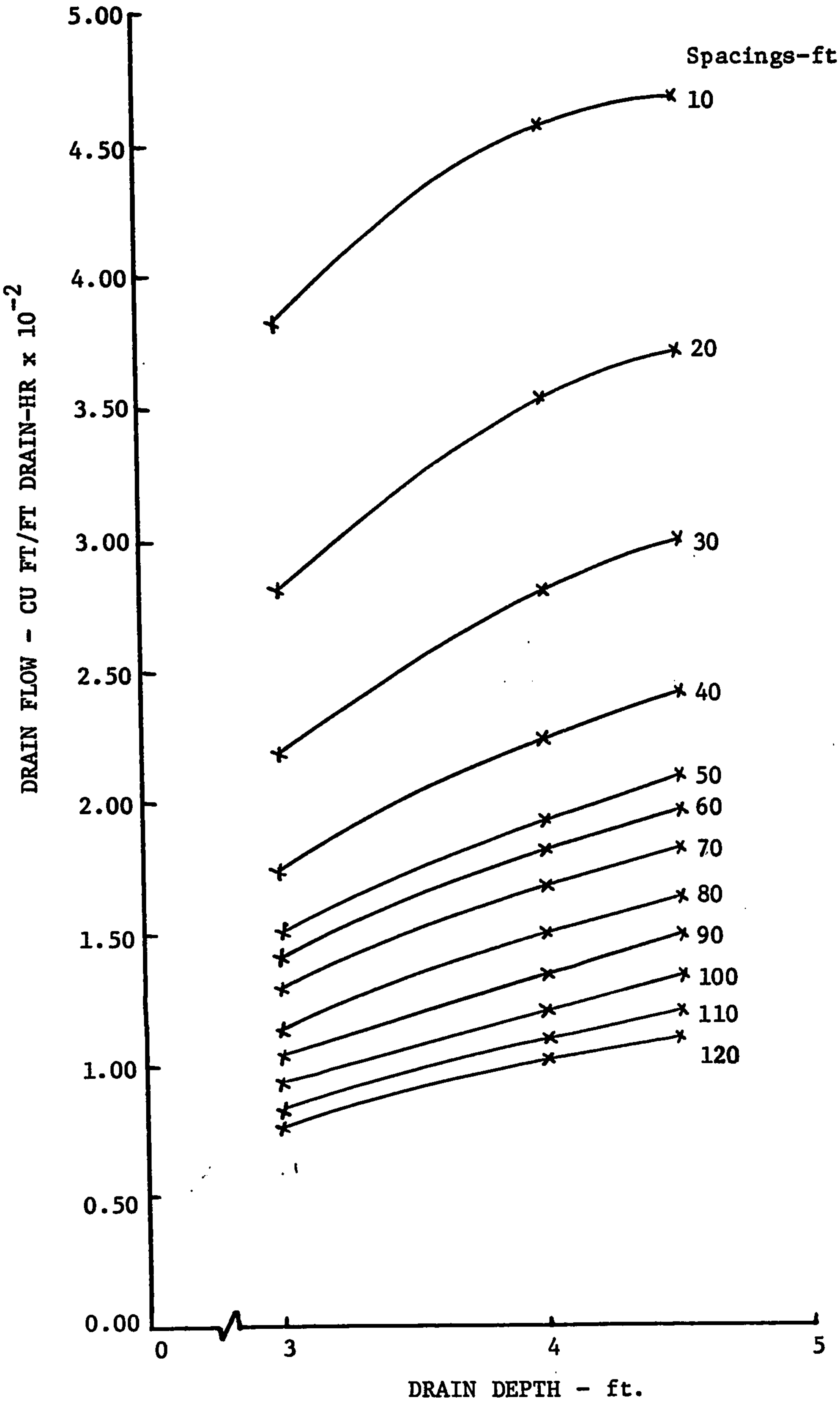


Figure 9. Comparison of drain flow rate and drain depth.

inches per hour. An impermeable layer was considered to be a layer through which no water could infiltrate and this was considered to be at a depth of 5 feet. Therefore, by increasing the drain depth, a higher drain flow rate can be achieved within the same soil layers.

It can also be observed that the closer the drain to the impermeable layer the higher the drain flow rate. This increase in the drain flow rate with depth could be the result of the boundary conditions simulated.

4.5 Comparison of Drain Flow Rate and Drain Spacings

A comparison of drain flow rate and drain spacings is shown in figure 10. The drain flow rate at close spacings is relatively high. At drain spacings from 10 to 40 feet the drain flow rate decreases rapidly. With drain spacings greater than 40 foot, the decrease in drain flow rate is more uniform and the curves become asymptotic.

Figure 10 also shows that the decrease in drain flow rate with increasing drain spacings approaches constant limits. Extrapolating these curves would indicate that with increasing spacings, there would be only a negligible decrease in the drain flow rate. Table 4 shows that the differences in drain flow rate at successive spacings of the drain reveal decreasing values to 60 foot spacings, a small increase in values to 80 foot spacings and steadily decreasing values to 120 foot spacings.

These observations indicate that in this experiment the drain flow rate is dependent upon the spacings of the drain, which is contrary to Kirkham's (15) solution of ponded water flow to a drain. Kirkham indicated that the flow rate into a drain is independent of the spacing of the drain, when the drains are more than 20 foot apart.

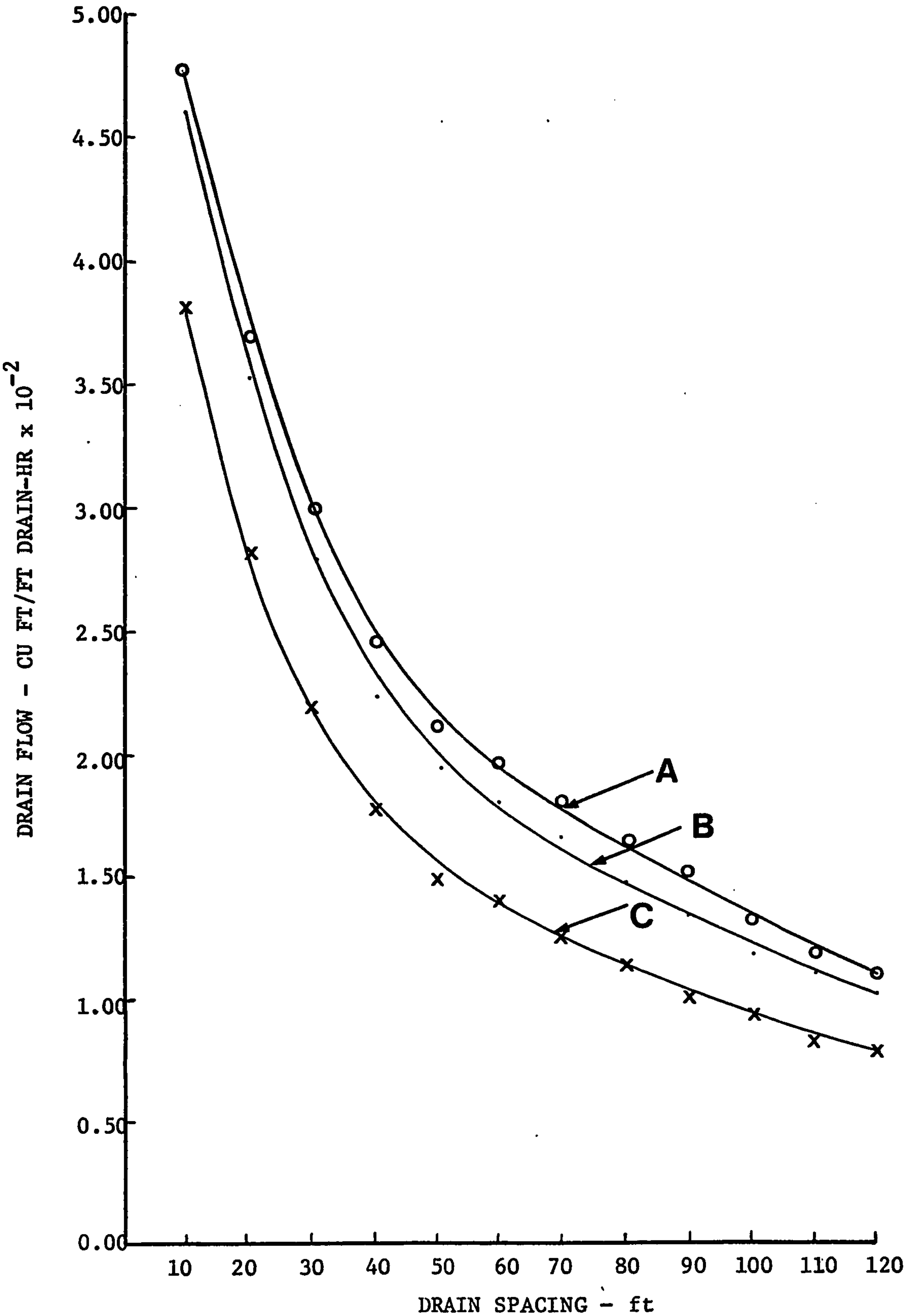


Figure 10. Drain flow rate and drain spacings (A) 4.5 ft depth, (B) 4.0 ft depth, and (C) 3.0 ft depth.

TABLE 4: DIFFERENCES IN DRAIN FLOW RATE AT SUCCESSIVE SPACINGS - CU FT/ FT DRAIN-HR x 10^{-3}.

Drain spacings, ft	Drain depth-ft.		
	3.0	4.0	4.5
10 - 20	10.239	10.449	9.666
20 - 30	6.195	7.285	7.056
30 - 40	4.431	5.605	5.877
40 - 50	2.473	3.036	3.086
50 - 60	0.833	1.219	1.280
60 - 70	1.230	1.432	1.509
70 - 80	1.555	1.832	1.933
80 - 90	1.030	1.443	1.432
90 -100	1.056	1.505	1.549
100 -110	1.001	1.019	1.300
110-120	0.526	0.784	0.986

4.6 Drain Flow Rate at Different Spacing/Depth Ratios

For comparison of drain flow rates, the ratio of spacing and depth of a drain were formed into dimensionless numbers. The drain flow at the various dimensionless numbers are tabulated in table 5, being extrapolated from table 2.

For the resistance network analog constructed for tile drainage in a glacial till soil the spacing/depth ratios plotted against the drain flow rate is as shown in figure 11.

A study of this plot reveals that the drain flow rate shows a rapid decrease to a spacing/depth ratio of 7.5, the rate of decrease being approximately linear. From a spacing depth ratio of 7.5 to 40, the decrease in the drain flow rate assumes a curvilinear relationship.

This graph is useful in interpolating the drain flow rate for any particular dimensionless number or spacing/depth ratio. A limitation is that the depth of the drain has to be within the limits of the boundary conditions simulated.

4.7 Flow Patterns by the Resistance Network Analog

The procedure for obtaining equipotential and streamlines with an electrical resistance network analog has been explained in the chapter on "Methods and Procedures". For obtaining the potential distribution, an e.m.f. of 100 volts was applied. Potential measurements were then taken at each node, the position of each node being known.

Figures 12 to 15 show the equipotential distribution for 30,60, 90 and 120 foot spacing of the drain. Each of the above figures show the equipotential distribution for 3.0, 4.0 and 4.5 foot depth of

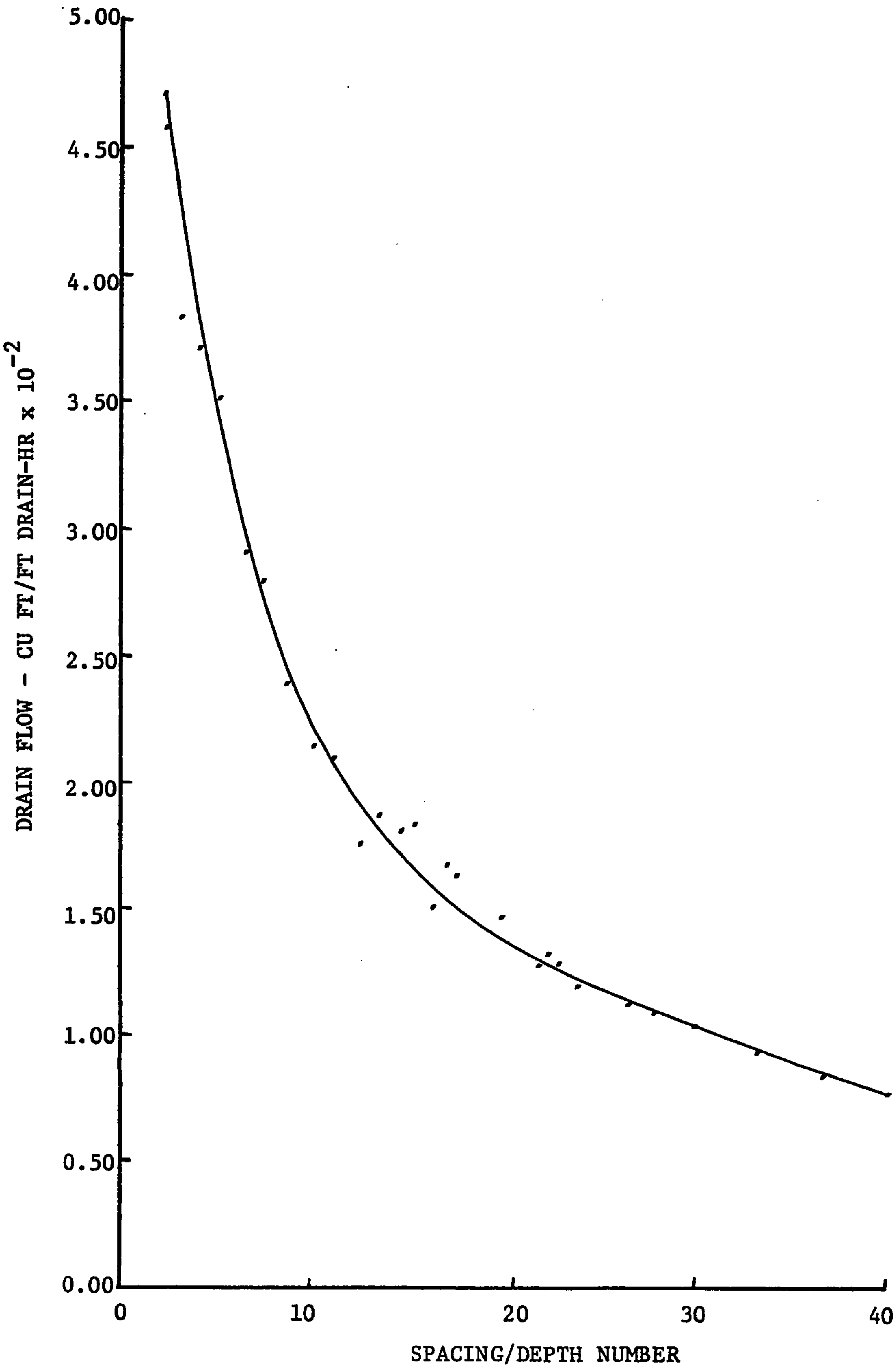


Figure 11. Drain flow rate and spacing/depth ratio.

TABLE 5: DRAIN FLOW RATE AT DIFFERENT SPACING/DEPTH RATIOS.

No.	Spacing/ depth ratio	Drain flow cu ft/ft drain-hr x 10^{-3}	No.	Spacing/ depth ratio	Drain flow cu ft/ft drain-hr x 10^{-3}
1	2.22	46.749	16	17.50	16.768
2	2.50	45.794	17	17.77	16.342
3	3.33	38.358	18	20.00	14.678
4	4.44	37.083	19	22.22	13.361
5	5.00	35.345	20	22.50	13.493
6	6.66	29.073	21	23.33	12.957
7	7.50	28.060	22	24.44	12.061
8	8.88	24.150	23	25.00	11.988
9	10.00	22.073	24	26.66	11.238
10	11.11	21.064	25	27.50	10.969
11	12.50	17.419	26	30.00	10.278
12	13.33	18.638	27	33.33	9.316
13	15.00	18.200	28	36.66	8.315
14	15.55	18.275	29	40.00	7.789
15	16.66	15.020			

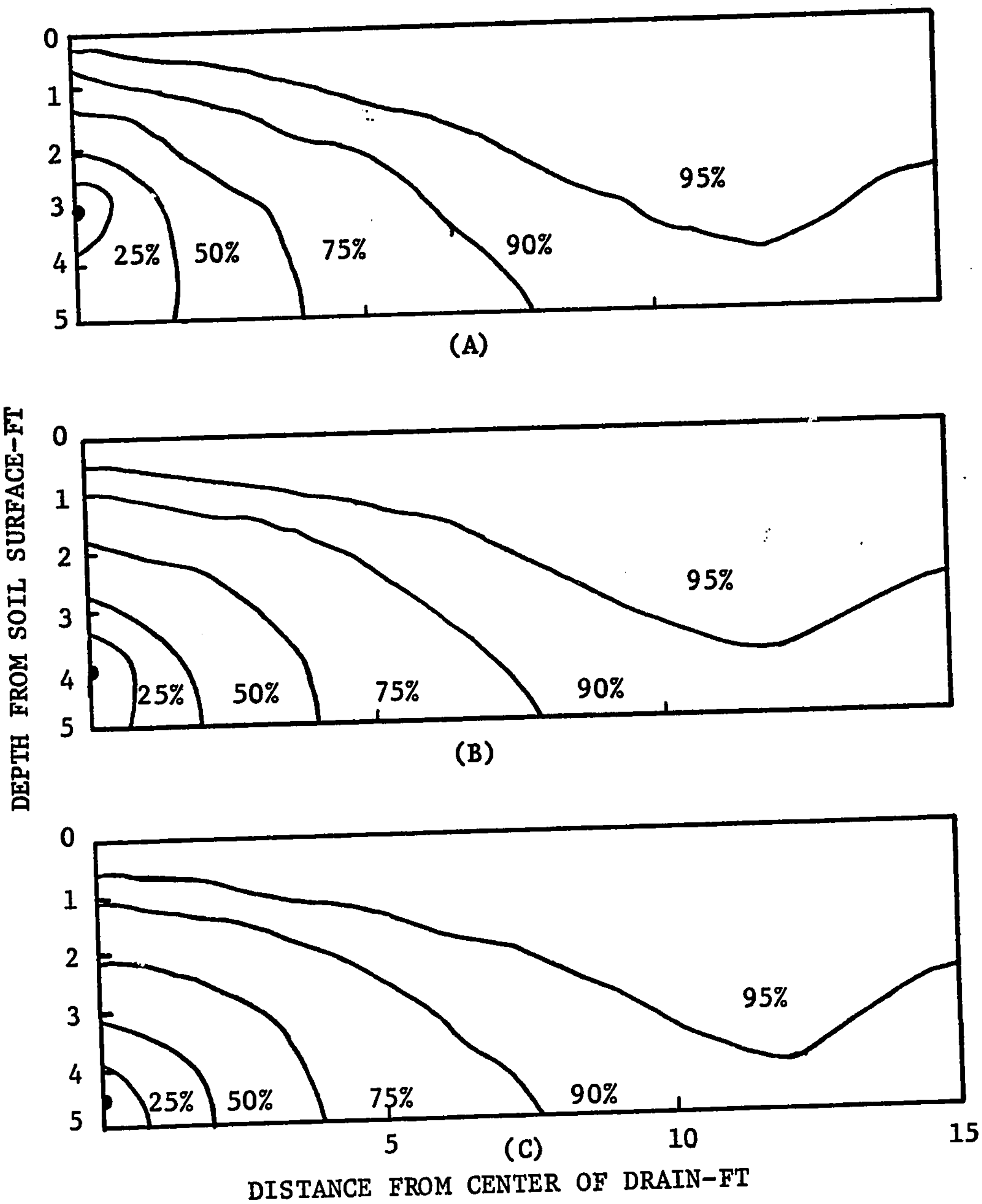


Figure 12. Equipotential distribution for 30 foot spacing (A) 3.0 foot depth, (B) 4.0 foot depth, (C) 4.5 foot depth.

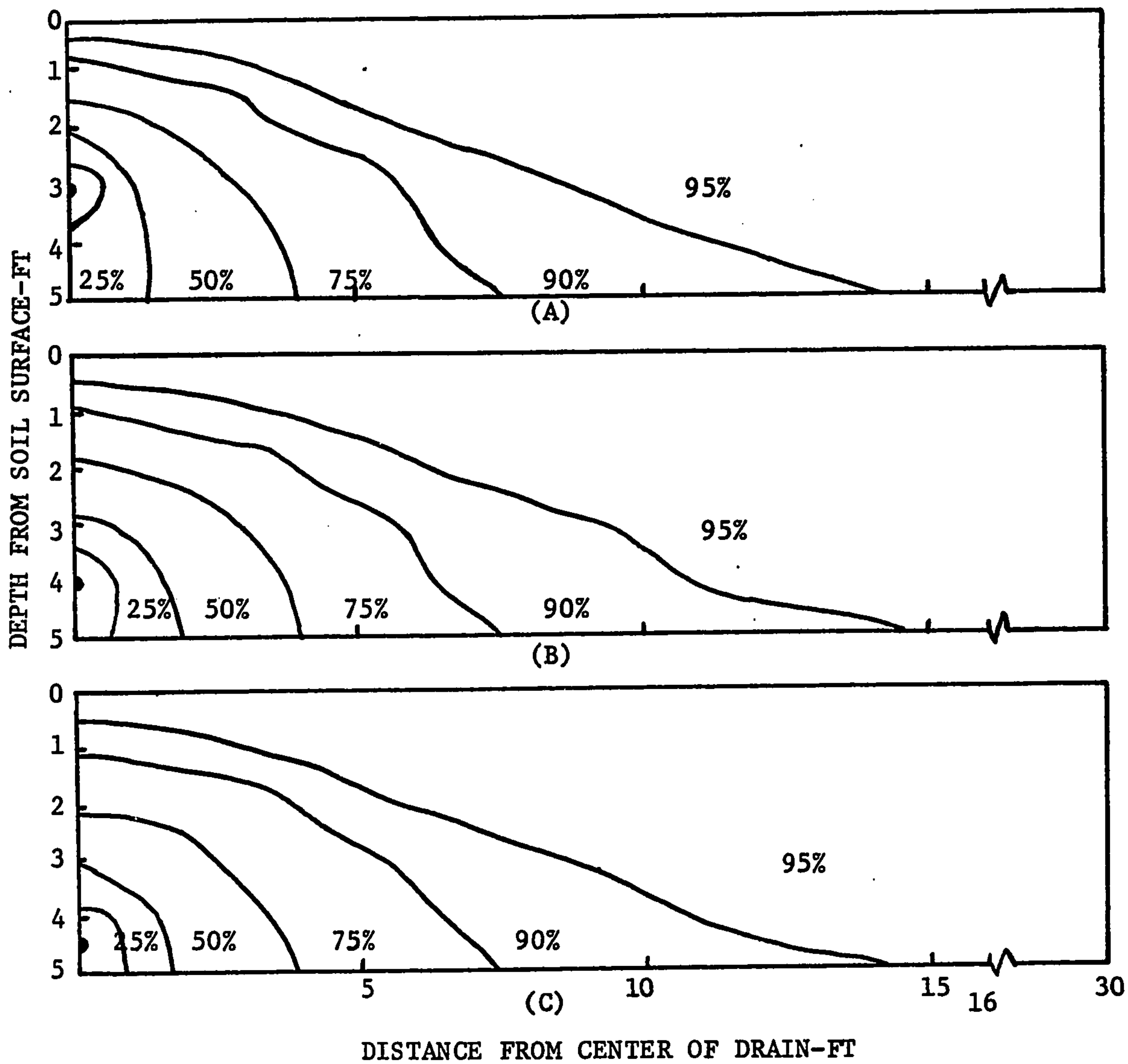


Figure 13. Equipotential distribution for 60 foot spacing (A) 3.0 foot depth, (B) 4.0 foot depth, (C) 4.5 foot depth.

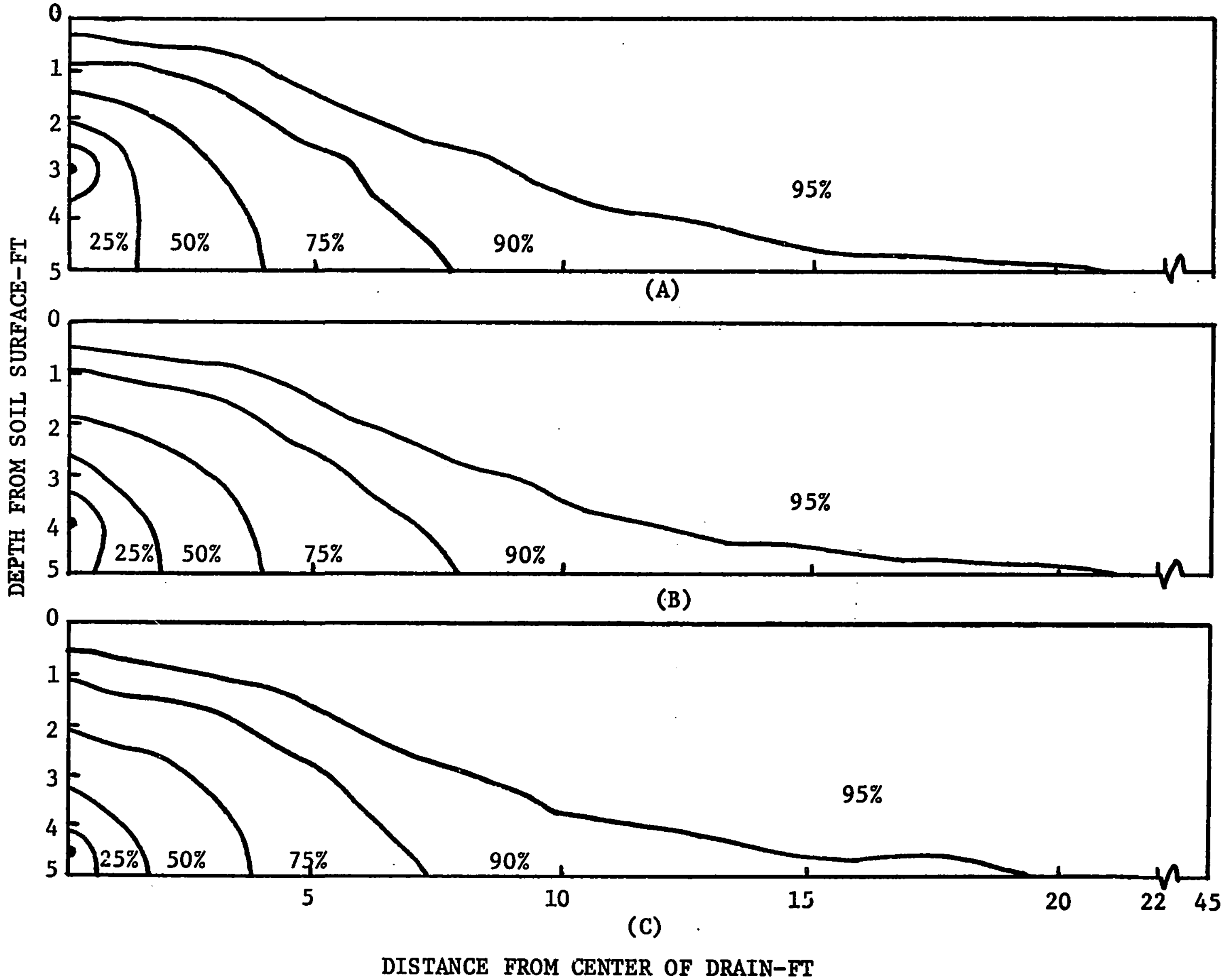


Figure 14. Equipotential distribution for 90 foot spacing (A) 3.0 foot depth, (B) 4.0 foot depth, (C) 4.5 foot depth.

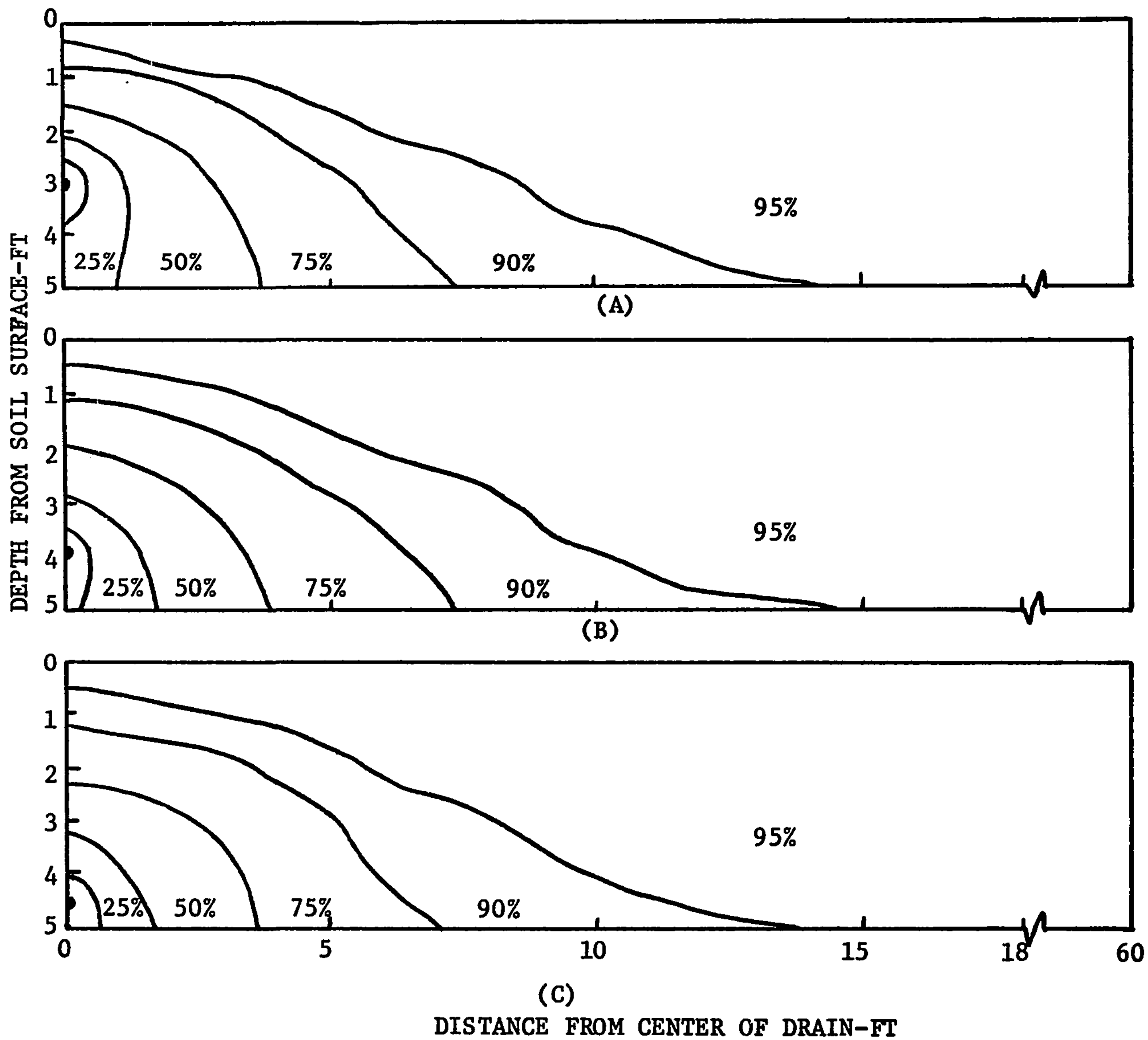


Figure 15. Equipotential distribution for 120 foot spacings (A) 3.0 feet depth, (B) 4.0 foot depth, (C) 4.5 foot depth.

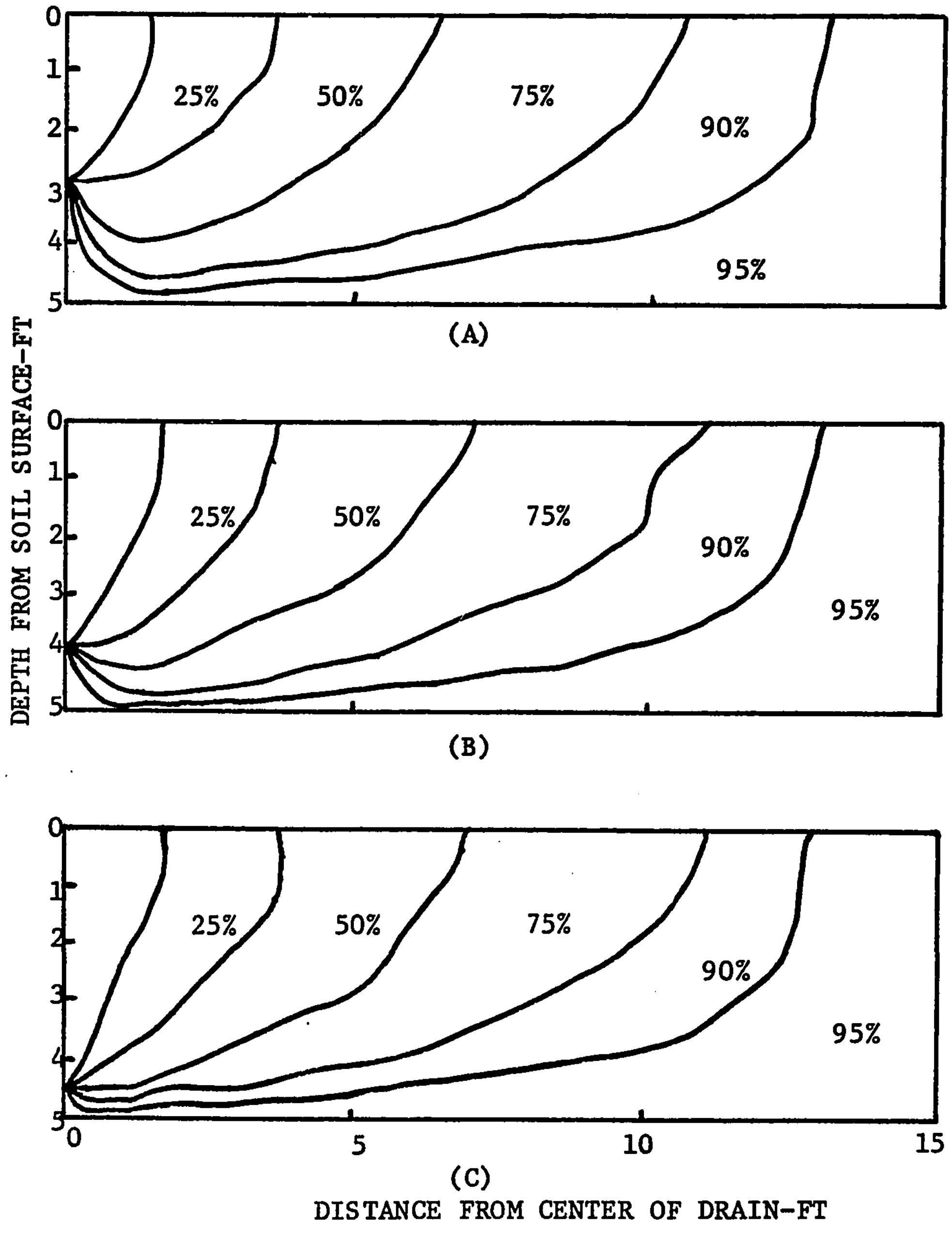


Figure 16. Streamline distribution for 30 foot spacing (A) 3.0 foot depth, (B) 4.0 foot depth, (C) 4.5 foot depth.

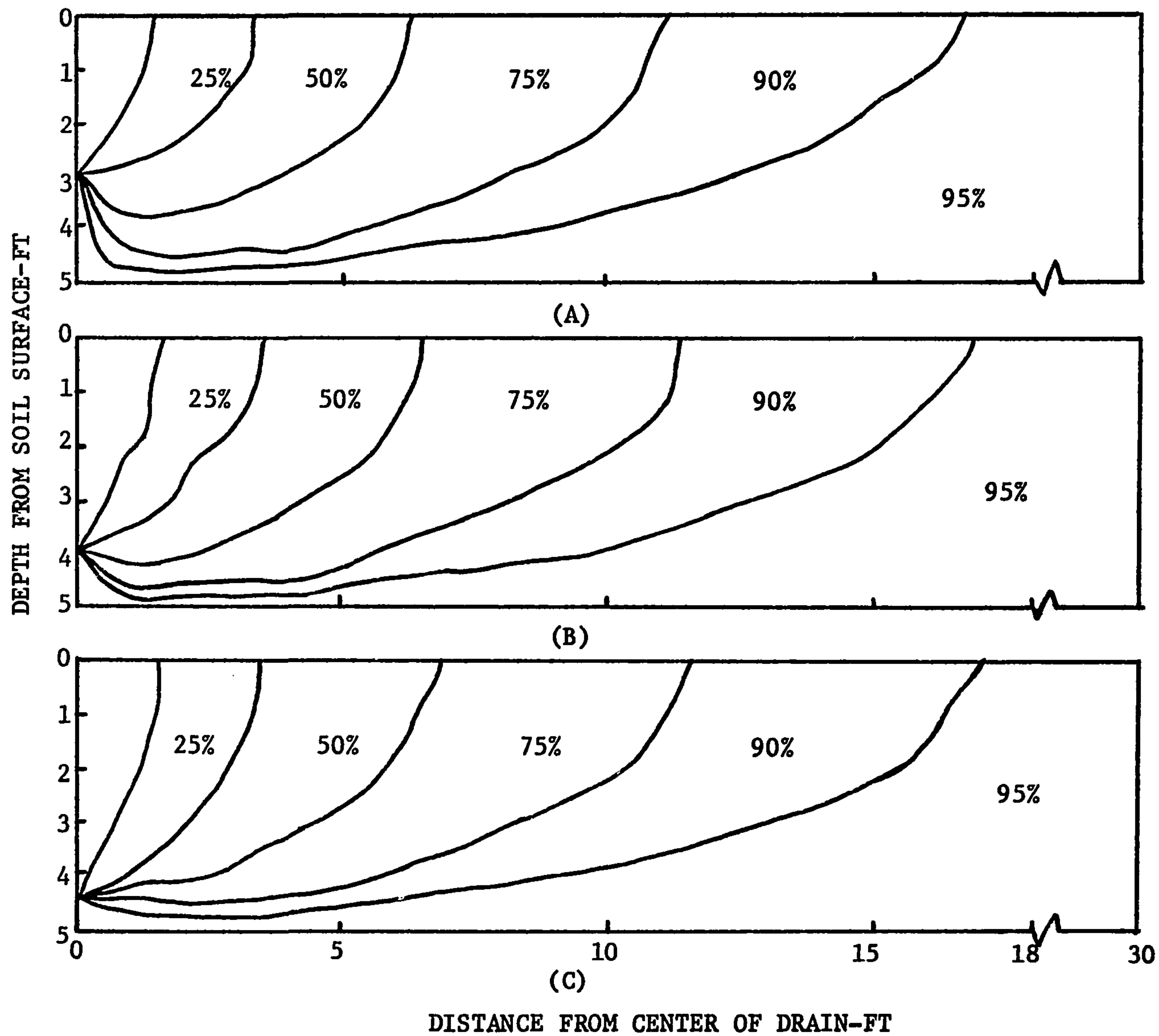


Figure 17. Streamline distribution for 60 foot spacing (A) 3.0 foot depth, (B) 4.0 foot depth, (C) 4.5 foot depth.

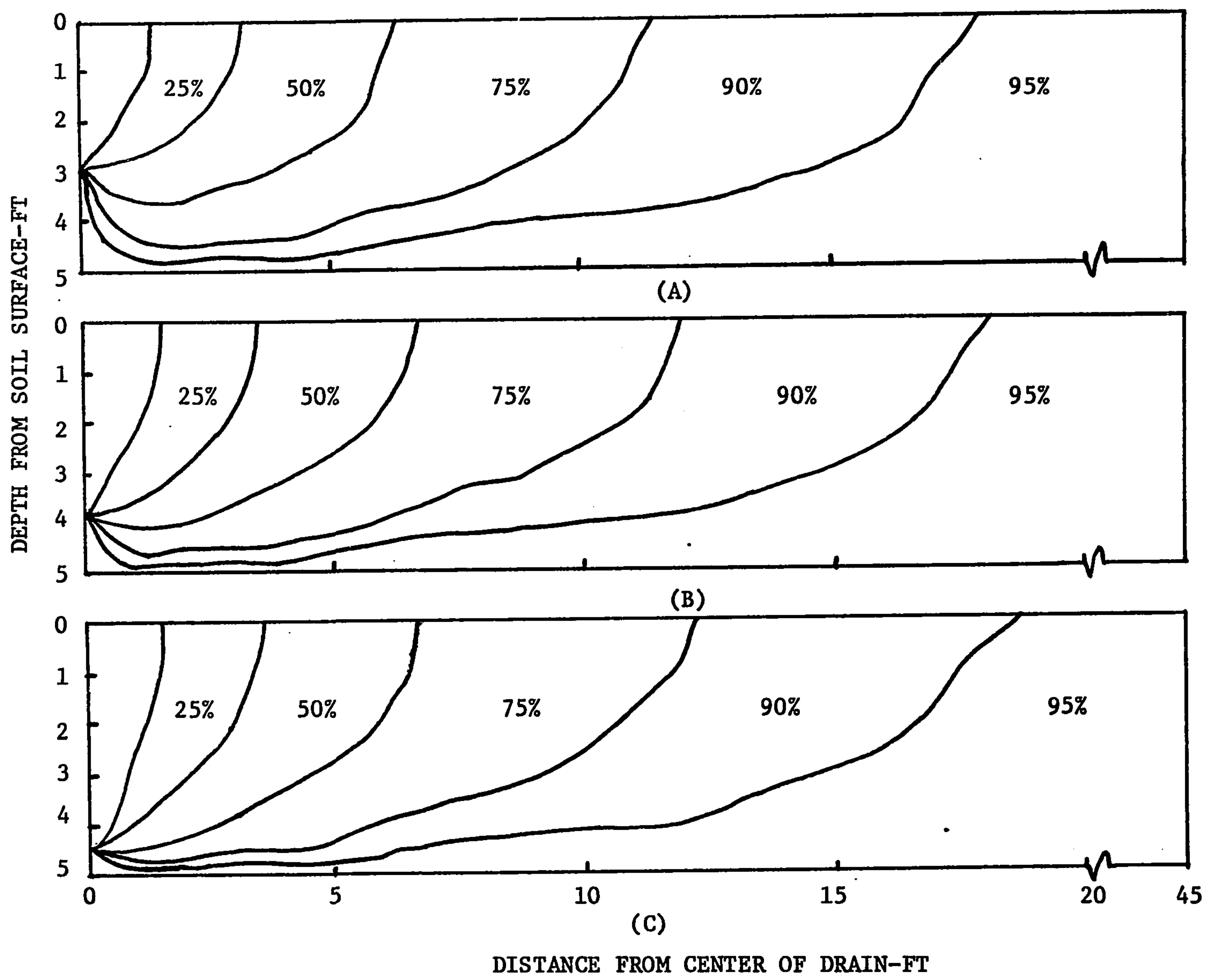


Figure 18. Streamline distribution for 90 foot spacing (A) 3.0 foot depth, (B) 4.0 foot depth, (C) 4.5 foot depth.

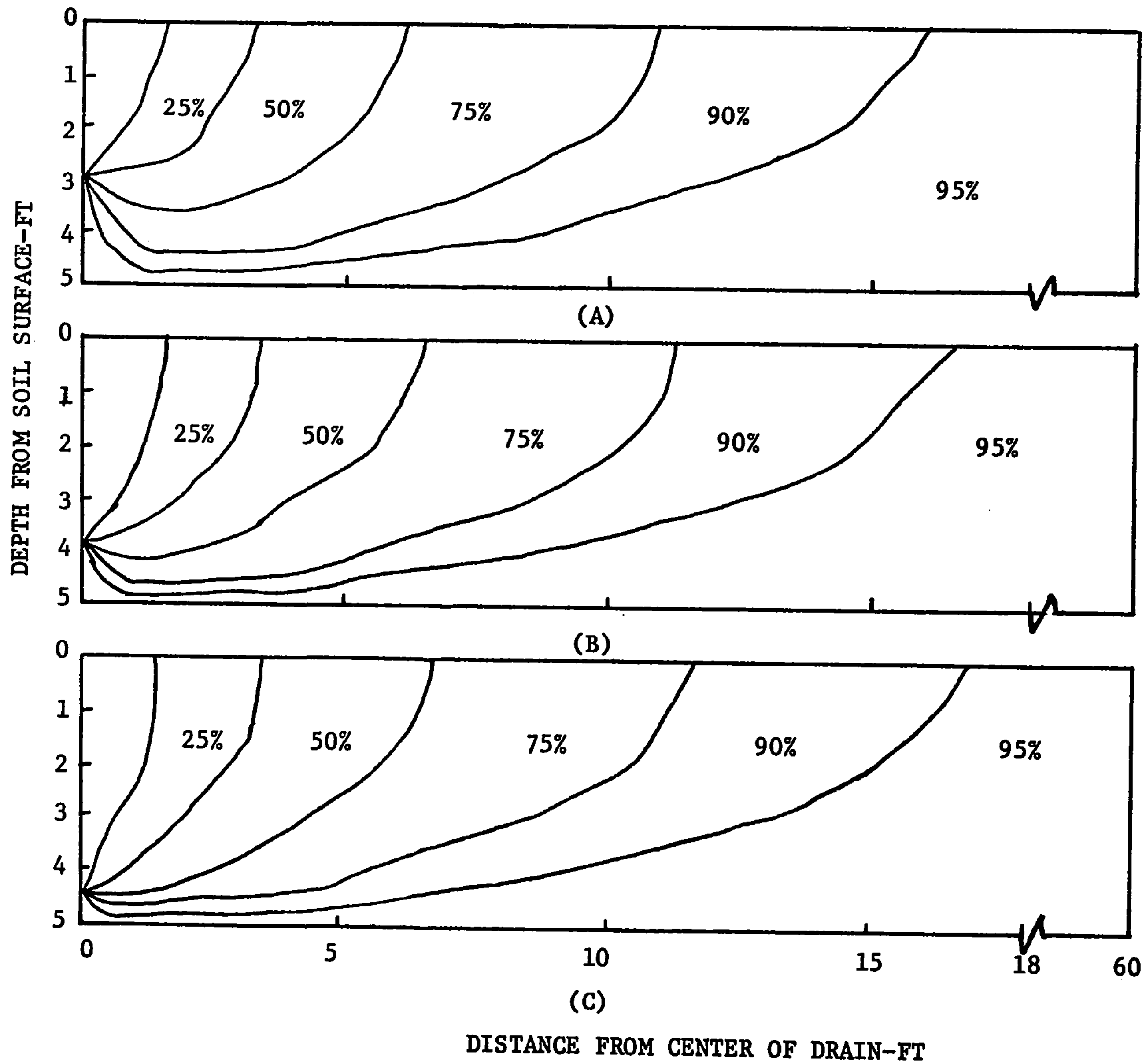


Figure 19. Streamline distribution for 120 foot spacing (A) 3.0 foot depth, (B) 4.0 foot depth, (C) 4.5 foot depth.

drain. Similarly figures 16 to 19 show the streamline distribution for 30, 60, 90 and 120 foot spacings. These are typical for the soil conditions simulated on an electrical resistance network analog. Each figure represents half the drain spacing because of the similar conditions existing on either side of the drain.

It can be seen that as the drain spacings are increased there is not a corresponding increase in the area contributing to the drain flow. Study of the figures 16 to 19 show that 95 percent of the drain flow for half the drain spacing was contributed by an area between the drain and a distance of approximately 13 feet on either side of the drain for 30 foot spacing, 17 feet on either side of drain for 60 foot drain spacing, 19 feet on either side of drain for 90 foot drain spacing, and only 17 feet on either side of drain for 120 foot drain spacing. Further observation reveals that the drain depth does not have an appreciable affect on the area contributing to the drain flow. A possible cause could be the boundary conditions simulated.

A comparison of the equipotential and streamline distribution with field or analog simulation could not be carried out as a study of literature revealed little or no interpretation of the flow pattern.

4.8 Optimum Depth and Spacing of Drain

The factors governing the depth and spacing of a tile drain system are the use to be made of the land and the time in which the excess water is to be removed. Various mathematical formulae based mainly on tests of hydraulic conductivity have been suggested to estimate the correct depth and the spacing between drains. The soil type and system of land use still provides the best guide to the optimum depth and spacing of the drain.

One of the objectives of the study undertaken was to evaluate an optimum depth and spacing of the drains in a glacial till soil. Three depths 3.0, 4.0 and 4.5 feet and twelve spacings 10 to 120 feet with intervals of 10 feet were simulated on an electrical resistance network analog. Drain flow rates were estimated and potential distribution was measured from which an optimum depth and spacing was to be evaluated.

The drain depth should be such that suitable growth of crop can be achieved. The deeper the drain the better as it facilitates greater drain spacings, but there is a limit due to the cost and also due to the presence of an impermeable layer of the soil. The drain flow rate should be such that it dewaters the soil faster. Considering the above factors, 4.5 foot depth of the drain will give a higher flow rate as compared to a 3.0 or 4.0 foot drain depth. Thus an optimum drain depth of 4.5 feet is recommended.

A study of the flow patterns figures 12 to 19 obtained from the resistance network analog was useful for obtaining an optimum spacing. The study reveals that the distance contributing to 95 percent of the drain flow rate was not appreciably greater for a 60 and 90 foot spacing. A distance of 17 feet on either side of the drain contributed to the 60 foot spacing, and a distance of 19 feet on either side of the drain contributed to the 90 foot spacing. Thus an optimum spacing lies between these two limits.

There seems to be some break-even point in the plot of the spacing/depth number and the drain flow rate (figure 11). A closer examination reveals that the graph could be divided into two straight lines. The point of intersection of these lines could be extrapolated to give the optimum spacing.

By extrapolation of the intersection of the straight lines of the graph (figure 11) gives a spacing/depth ratio of approximately 16. As the optimum depth of the drain was 4.5 feet, the spacing of the drain is 16 x 4.5 = 72 feet. Thus an optimum drain spacing in a glacial till soil is between 70 and 75 feet.

Chapter 5

SUMMARY AND CONCLUSIONS

1. The purpose of this research was to study the drain depth and spacing in a glacial till soil by simulating the field conditions on an electrical resistance network analog.
2. The drain flow rate was calculated by the electrical resistance network analog and compared with the field data. Potential distribution was also evaluated from the electrical resistance network analog.
3. Soil hydraulic conductivity is a key factor enabling the proper soil profile to be simulated by the resistance network analog. Soil hydraulic conductivity should be known in both the horizontal and vertical directions if the soil is anisotropic. The soil hydraulic conductivity for glacial till soil as measured in the field (23) by auger-hole and piezometer method was simulated on the resistance network analog.
4. A comparison of the drain flow rate as evaluated by the resistance network analog and that from the field experimental data showed a large difference. The drain flow rate as found by field experimental work was approximately 3.5 to 4.5 times higher than that calculated by resistance network analog for 30, 60, 90 and 120 foot spacing with a drain depth of 3.0 feet. The conclusion drawn was that either the soil hydraulic conductivity simulated was erroneous or that the soil was not isotropic as assumed.
5. The comparison of the drain flow rate and the depth of the drain for constant spacings showed that as the depth of the drain was

increased there was a corresponding increase in the drain flow rate. For example, for the 30-foot spacing, as the depth of the drain was increased from 3.0 to 4.0 to 4.5 feet, the drain flow rate increased from 0.0219 to 0.0280 to 0.0302 cu ft/ft drain-hr. respectively. As the impermeable layer simulated was at a distance of 5.0 feet from the surface, it was observed that the closer the drain to the impermeable layer the higher the drain flow rate.

6. A comparison of the drain flow rate and drain spacing showed that the drain flow rate was dependent upon the spacing of the tile drain, which was contrary to Kirkham's finding (15). It was observed that the drain flow rate was higher at close drain spacings. For example, the drain flow rate for a 4.0 foot depth and 20-foot spacing was 0.0353 cu ft/ft drain-hr., whereas the drain flow rate for 4.0 foot depth and 90-foot spacing was 0.0135 cu ft/ft drain-hr.

7. A graph was plotted for the drain flow rate and spacing/depth ratios. This graph was used in evaluating an optimum spacing/depth ratio for the glacial till soil simulated.

8. Flow patterns were drawn for some of the drain spacings simulated. It was observed by studying the streamline distribution that the area contributing to the flow did not increase proportionally as the drain spacing was increased. It was seen that 95 percent of the drain flow was contributed by an area between the drain and a distance of approximately 13 feet on either side of the drain for 30-foot spacing, 17 feet on either side for 60-foot spacing,

19 feet on either side for 90-foot spacing and only 17 feet on either side for 120-foot spacing.

9. An optimum depth and spacing for tile drains in a glacial till soil as concluded from this study was a 4.5 foot depth and a spacing between 70 and 75 feet.

Chapter 6

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APPENDIX

The following description and analytical data pertaining to the soils underlain by glacial till have been taken from the report "Land Classification of the Bow River Project" by the Committee of the Canada Department of Agriculture, P.F.R.A., Regina, 1960.

Physical and Chemical Characteristics

Chin Series

Order: Chernozemic

Sub Group: Orthic Brown

Texture: Silt Loam to very fine sandy loam

Deposition: Alluvial lacustrine overlying glacial till; in a few places it overlies sand

Topography: Level to gently rolling

Parent Material: Brown fairly uniform very fine sandy load to silt loam with medium lime content. The underlying glacial till is mainly of Belly River formation origin.

Types: Very fine sandy loam
Light loam
Loam
Silt loam

Phases: Chin over sand
Shallow Chin

The depth to glacial till usually varies from 20 inches to 6 feet from the surface. When the underlying glacial till occurs within 36" of the surface the word Shallow is prefixed to the soil series name.

This depth has been set tentatively until more information is available. A till-like material at from 45 to 60" appears to be resorted and is somewhat more friable than the underlying till. It varies in thickness from a few inches to over two feet.

PHYSICAL AND CHEMICAL CHARACTERISTICS OF TYPICAL DRY LAND CHIN

Horizon	Depth	Description	Sat.%	pH	E.C. mmhos /cm	H.C. in./hr	SAR	ESP	App. Density	Available Moisture in./ft.
A	0-5"	Brown to greyish brown 10YR 5/2-5/3 very fine sandy loam to silt loam, mildly prismatic, firm.	35-40	6.7-7.9	<2	0.7-0.3	<5	<8	1.3-1.4	1.2-1.8
B	5-18"	Brown to dark brown 10YR 4/3-5/3 silt loam to clay loam prismatic, very slightly stained.	40-50	6.9-8.0	<2	0.5-1.5	<5	<8	1.3-1.5	1.2-2.0
Ca	18-30"	Light brownish grey, 10YR 6/2, fine sand loam to loam, medium to heavy lime carbonate accumulation.	40-55	7.9-8.3	<4	0.3-0.1	<5	<8	1.3-1.5	1.5-2.0
C	30-45"	Yellowish brown silt, 10YR 5/4 loam, contains varying amounts of lime and some salts.	40-55	7.8-8.2	<8	0.3-0.1	<8	<12	1.4-1.5	1.5-2.5
*D_1	45-60"	Greyish brown 10YR 5/2 loam to sandy clay loam, friable, flecks of iron, coal, lime and gypsum.	40-55	7.8-8.2	5-10	0.2-0.8	5-12	10-15	1.5-1.6	1.5-2.5
D_2	60-120"	Greyish brown 2.5Y 5/2 sandy clay loam to clay loam, compact, iron stone, flecks of coal lime and gypsum.	40-55	7.5-8.0	5-15	0.2-0.6	8-15	10-20	1.5-1.7	2.0-3.0
D_3	120+	Till	45-55	7.5-8.0	5-10	0.1-0.6	8-15	10-20	1.5-1.7	2.0-3.0