

**University of Alberta**

Three Essays on Monetary and Financial Economics

by

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## **Abstract**

This thesis contains three chapters on financial and macroeconomics.

Chapter 1 is an empirical study on what is referred to in the finance literature as “pairs trading”. Pairs trading involves simultaneous trades in two equity securities that have been identified as being very highly correlated historically. The idea is to trade the pairs when their prices diverge from another and to unwind the trade when their prices (hopefully) converge. The contribution of chapter 1 is to rigorously examine alternative techniques for identifying stock pairs. I consider two main techniques: a “distance” approach and cointegration. Each of these techniques is evaluated when pairs are selected within the same industry (“restricted pairs”) and when pairs are selected from the broad universe of stocks (“unrestricted pairs”). The main findings are that unrestricted pairs are preferred to restricted pairs for the distance approach and that restricted pairs work better for the cointegration approach, especially for the services, financial and retail trade sectors. In addition, the cointegration approach yields a higher excess return than the distance approach. Nevertheless, more risk-averse investors might prefer the distance approach based on my analysis of information ratios for the two approaches.

Chapter 2 is an empirical study of monetary policy in China. The main focus is identifying the effectiveness of alternative monetary instruments in affecting real economic activity. This chapter employs a structural vector autoregression (SVAR) methodology that is tailored to specific characteristics of the environment faced by Chinese policymakers—namely, exchange rate

targeting, capital flow restrictions, and sterilization of the buildup of foreign exchange reserves. Briefly, we find that the money supply is an effective monetary instrument, while the interest rate is not.

Chapter 3 contains a theoretical model of bank runs. The main contribution is to show that bank runs—more broadly interpreted as financial instability—can arise purely from the joint interaction of business cycle fluctuations and ordinary consumption smoothing by households. To highlight this, chapter 3 shows that, in addition to classic panic-based bank runs, bank runs can be caused by a decrease in aggregate labor income, i.e., a recession.

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## Table of Contents

<b>Chapter 1:</b>	<b>Stock-Price Pairs Arbitrage</b>	1
1.1	Introduction	1
1.2	Literature review	3
1.3	Pairs trading method	9
1.3.1	The distance approach	9
1.3.2	The cointegration approach	10
1.3.3	Opening a pairs position	11
1.3.4	Closing a pairs position	12
1.3.5	One day later rule	12
1.3.6	Transaction cost approach	13
1.3.7	Calculation of returns	13
1.4	Empirical results	14
1.4.1	The distance approach with transaction cost adjustment	15
1.4.1.1	Profitability of the strategy	15
1.4.1.2	Information ratio	18
1.4.2	Distance approach with one day later rule	19
1.4.2.1	Profitability of the strategy	19
1.4.3	Cointegration Approach	23
1.4.3.1	Profitability of the strategy	23
1.4.3.2	Information ratio	26
1.4.4	Distance approach vs. cointegration approach	27
1.4.5	What determines the returns in the distance approach?	28
1.5	Conclusion	30
	Bibliography	31
<b>Chapter 2:</b>	<b>Measuring Monetary Policy in China</b>	33
2.1	Introduction	33
2.2	The Chinese monetary policy framework	34
2.2.1	History	34
2.2.2	Exchange rate	34
2.2.3	Capital movements	35
2.2.4	Targets of monetary policy	35

2.2.5	Monetary policy instruments .....	35
2.2.5.1	Open market operations .....	36
2.2.5.2	Reserve requirements.....	36
2.2.5.3	Interest rate.....	36
2.2.5.4	Rediscount rate .....	37
2.2.6	The realities of the Chinese policy framework.....	37
2.3	Literature review .....	38
2.3.1	Monetary policy.....	38
2.3.2	Model development.....	39
2.3.3	China monetary policy .....	40
2.4	Models and estimations .....	41
2.4.1	VAR model.....	41
2.4.2	SVAR approach.....	43
2.4.2.1	SVAR model .....	43
2.4.2.2	Identification .....	45
2.4.2.3	Data.....	51
2.4.2.4	Impulse response analysis from the SVAR estimation.....	52
2.4.3	FASVAR approach .....	56
2.4.3.1	FASVAR model .....	56
2.4.3.2	Data.....	57
2.4.3.3	FASVAR estimation.....	58
2.4.4	Robustness.....	63
2.4.5	Variance decomposition.....	64
2.5	Conclusion .....	65
	Bibliography.....	66
	Appendix A: Variables detail .....	71
	Appendix B: Econometric tests.....	75
	1. Unit root test.....	75
	2. Lag selection and cointegration test.....	76
	Appendix C: Data description .....	89
<b>Chapter 3:</b>	<b>Business Cycles, Consumption Smoothing, and Bank Runs .....</b>	<b>92</b>
3.1	Introduction.....	92
3.2	Literature review .....	95

3.3	Basic model.....	100
3.3.1	Long-lived agents' problem.....	102
3.3.2	Bank's problem.....	103
3.3.3	Early withdrawal and bank runs .....	104
3.4	Extended model.....	106
3.4.1	Bank's problem.....	108
3.4.2	Early withdrawal.....	109
3.4.2.1	Type 1 agents withdraw first.....	109
3.4.2.2	Type 2 agents withdraw first.....	110
3.5	Conclusion .....	113
	Bibliography.....	114
	Appendix D: Proofs .....	116
1.	Long-lived agent's problem-basic model.....	116
2.	Early withdrawal-basic model.....	116
3.	Early withdrawal-extended model.....	117

## List of Tables

Table 1-1: Optimal excess return - distance approach with transaction cost.....	16
Table 1-2: Optimal excess return - distance approach with one day later rule.....	20
Table 1-3: Optimal excess return – cointegration approach with transaction cost .....	24
Table 2-1: Fraction of variance explained by money supply shock and interest rate shock .....	64



## List of Figures

Figure 1-1: Optimal excess return - distance approach with transaction cost .....	18
Figure 1-2: Information ratio - distance approach with transaction cost.....	19
Figure 1-3: Optimal excess return - distance approach with one day later rule.....	22
Figure 1-4: Information ratio - distance approach with one day later rule.....	22
Figure 1-5: Optimal excess return - cointegration approach with transaction cost.....	26
Figure 1-6: Optimal information ratio- cointegration approach with transaction cost.....	27
Figure 1-7: Excess return for 500, 1000, and 2000 draws.....	29
Figure 1-8: Information ratio for 500, 1000, and 2000 draws .....	29
Figure 2-1: SVAR response to money supply shock .....	53
Figure 2-2: SVAR response to interest rate shock.....	54
Figure 2-3: FASVAR response to money supply shock .....	59
Figure 2-4: FASVAR response to interest rate shock.....	61
Figure 2-5: FASVAR response to foreign reserve shock.....	62
Figure 2-6: Credit growth rate vs. M2 growth rate .....	63
Figure 3-1: Greek unemployment rate .....	93
Figure 3-2: Greek deposits .....	93
Figure 3-3: Time line - basic model.....	102
Figure 3-4: Time line - extended model.....	108
Figure 3-5: Panic-based bank runs.....	112

## Abbreviations

ADF	Augmented Dickey-Fuller
AIC	Akaike Information Criterion
BIS	Bank for International Settlements
CPI	Consumer Price Index
CRSP	Center for Research in Security Price
DD	Diamond-Dybvig
DF-GLS	Generalized Least Squares Dickey-Fuller
EX	Exchange rate
FASVAR	Factor Augmented Structural Vector Autoregression
FDI	Foreign Direct Investment
FPF	Final prediction error
FR	Foreign Exchange Reserves
GDP	Gross domestic product
HQ	Hannan Quinn
IMF	International Monetary Fund
INF	Inflation rate
INT	Interest rate
LR	Likelihood-Ratio test
MFS	Market Factor Spread
MR	Minimum Reserve Requirement
MS	Money Supply
NBSC	National Bureau of Statistics of China
OMO	Net Securities held by the PBC
PBC	People's Bank of China
PP	Phillips-Perron
RMB	Renminbi
RR	Rediscount Rate
SAFE	State Administration of Foreign Exchange
SC	Schwarz Criteria
SSD	Sum of Squared Differences
SVAR	Structural Vector Autoregression
USEUGDP	Sum of European Union and U.S. real GDP
VAR	Vector Autoregression
Y	Output growth rate

# Chapter 1: Stock-Price Pairs Arbitrage

## 1.1 Introduction

Pairs trading is widely used by hedge funds and investment banks because of its easy conceptualization. The idea is simple: find two stocks that have similar price paths; monitor the spread between them; when the spread between them is large enough, long the loser and short the winner; unwind the position when the two stocks converge. The strategy is, however, more complicated in practice than in principle. The biggest practical challenge is to identify pairs. The literature provides two main approaches to selecting pairs: the so-called distance approach and the cointegration approach.

The distance approach is based on a conceptually simple statistical method: the co-movement in pairs is measured by “distance”, defined as the sum of squared differences between two normalized price series. In effect, this method looks for two stocks that have the closest historical normalized prices. This approach is based on the simple rule of “law of one price” proposed by Ingersoll (1987), who states that “two investments with the same payoff in every state of nature must have the same current value.” In practice, even though prices may diverge in matched pairs temporarily because of market inefficiency, arbitrage should cause the prices to converge. This approach is normative, easily implemented, economics free, and it avoids some possible mis-specification problems in regression analysis. A potential problem with this approach is that, being non-parametric, the strategy lacks forecasting power in pairs spread. Put differently, one is never really sure why the statistical relation exists, and thus one cannot be certain when it will end: for every divergence, one is not sure if it is because of the market inefficiency or because the relationship no longer exists, in which case the divergence of price paths is permanent.

The cointegration approach looks for stocks that share the same stochastic trends, so that a linear combination of the two-stock prices is a stationary mean-reverting time series. One advantage of the cointegration approach is that the

relation is not based on pure statistical arguments—common stochastic trends deriving from common fundamentals drive the value of the assets. Vidyamurthy (2004), for example, relates the cointegration model to the Arbitrage Pricing Theory. A problem with this approach is that, because it is parametric, it may be prone to errors from mis-specification. These estimation errors may result in spurious estimates. Another shortcoming of the cointegration model is that it is not well suited for automated computer pair matching using simple algorithms because of its increased complexity.

An important question that arises from the pairs-matching process is whether the pairs should be selected from the same sectors or simply from the universe of stocks.<sup>1</sup> Stocks in the same sectors may have common factor exposure, which may increase the likelihood of finding matched pairs. Stocks from the same sectors may be subject to less cross-sector variance in shocks by construction, and a close price path may arguably make economic sense for such stocks. For  $N$  stocks,  $\frac{N \times (N-1)}{2}$  possible pairs need to be compared. If we can limit the potential matches to stocks within the same sector, the process is computationally simpler than if we do not limit the stocks. On the other hand, selecting pairs from a larger set may yield a better match. The best pairs are those that continuously repeat the process of diverging and converging with a high spread and quick reversal. The literature contains little work on the topic of pairs matching source. Most papers either choose securities from all sectors or else choose industry-restricted pairs. Only three papers were found that discuss this issue. Gatev, Goetzmann and Rouwenhorst (2006) and Cummins (2010) argue that no difference exists between the profitability of industry-restricted pairs and unrestricted pairs. Do and Faff (2010) argue that industry-restricted pairs are more profitable than unrestricted pairs. However, all these papers use the distance approach, and their trading strategy is predetermined, so nothing guarantees that the operating return is optimal.

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<sup>1</sup> Restricted pairs can dominate unrestricted pairs because the latter will increase the probability of spurious correlation, which may cause substantial loss. The literature includes several papers comparing pairs from same sectors and from all universes.

In this chapter, we offer a more comprehensive analysis of the pairs-matching problem—where the pairs should be chosen from. We compare the unrestricted and industry-restricted pairs from both the distance approach and the cointegration approach. When comparing the pairs-matching strategy, we consider the optimal trading strategy that will yield the highest return for the selected pairs.

The rest of chapter is organized as follows. Section 1.2 is the literature review. Section 1.3 introduces pairs trading methods. Section 1.4 is the estimation results and section 1.5 concludes.

## 1.2 Literature review

The pairs-trading strategy has been widely used since mid-1980s, when Nunzio Tartaglia led his quantitative team at Morgan Stanley to uncover arbitrage opportunities in the equities markets. One of the techniques the team used was to trade pairs of securities. The important process before trading was identifying securities pairs with high co-movement of prices. The team traded pairs with the idea that any divergence between them would finally converge. This activity was the beginning of pairs trading. Although pairs trading has become more popular in the financial industry, few academic studies have been published. The most well-known works are by Gatev et al. (2006) and Vidyamurthy (2004).<sup>2</sup> The former paper examines pairs trading empirically using the distance approach. Gatev et al. (2006) use daily U.S. stock price data from 1962 to 2002 and find that pairs trading generates an excess return of 11% per year and a monthly sharp ratio six times larger than that of the overall market.<sup>3</sup> They also show that pairs-trading returns have high risk adjusted Jensen alphas, are low exposure to common measures of systematic risk, cover reasonable transaction costs, and do not come from short-term return reversals mentioned by Lehmann (1990). Gatev et al. (2006) find the excess returns from pairs trading have declined over time, which

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<sup>2</sup> Gatev et al.'s work was published in 2006. However, the first draft appeared as an unpublished working paper in 1999, which used data from 1962 to 1998. After the first draft, the authors use the sample period 1999-2002 as an out-of-sample test of their strategy.

<sup>3</sup> Sharp ratio measures the excess return per unit of standard deviation.

they attribute to pairs trading strategies becoming more common (i.e., increased competition).

Vidyamurthy (2004) discusses pairs trading using the cointegration approach. He motivates his approach by appealing to the Arbitrage Pricing Theory, and adopts Engle and Ganger's two-step approach (Engle and Granger, 1987) to first test for cointegration and second estimate an ARMA process to look for mean reversion of the difference in normalized prices of the pairs.

More recently, a number of papers have considered pairs trading. One group of papers focuses on the distance approach used by Gatev et al. (2006). These studies include Nath (2003); Papadakis and Wysocki (2007); Ehrnrooth (2007); Engelbert, Gao, and Jagannathan (2009); Perlin (2008); Plater and Nisar (2010); Do and Faff (2010); Bolgun, Kurun and Guven (2010); Cummins (2010); and Broussard and Vaihekoski (2010). Nath (2003) examines the reward of pairs trading in the secondary market for U.S. Treasury securities. The research finds that the pairs-trading strategy outperforms most of the benchmarks. Papadakis and Wysocki (2007) examine the impact of accounting information events on the profitability of pairs trading strategies. They find that earning announcements and analyst forecasts can cause drift in relative prices, which often trigger the opening of pairs trading. But since the divergence is caused by the under-reaction/over-reaction of investors, such event-triggered pairs trading is less profitable compared to non-event-triggered one. Ehrnrooth (2007) examines the pairs-trading strategy on the Helsinki stock exchange and find that the strategy works even better on the Helsinki stock than on the New York stock exchange. Engelbert et al. (2009) investigate how information and liquidity influence the profitability of the pairs trading strategy. These researchers find that profit is lower when the news is specific to only one stock in the pairs. The idiosyncratic news increases the divergence risk and horizon risk. When the news affects both stocks in the pairs and sluggish response for one stock exists, pairs trading will earn a high return. They also find that trading on large and liquid pairs tend to outperform trading on smaller and less liquid pairs because liquid pairs have a higher probability of opening a position and usually converge faster after initial

divergence. Perlin (2008) researches the performance of pairs trading in the Brazilian market. The researcher finds that pairs trading generates positive excess returns and high frequency (daily) data yields better returns than weekly and monthly data. Plater and Nisar (2010) implement the pairs trading strategy in non-equity assets—price indexes, commodities, and currencies. They find this strategy produces an excess return of 1.6% every six months and a Sharpe ratio almost doubles sharp ratio of the benchmark portfolio. Do and Faff (2010) take the exact same pairs trading algorithm of Gatev et al. (2006). These researchers find a higher excess return, higher volatility and superior Sharpe ratio when pairs trading is operated in a bear market. They argue the declining trend in pairs-trading profitability in a bull market is because of the higher arbitrage risk, not the increasing market efficiency.<sup>4</sup> Bolgun et al. (2010) test the pairs trading strategy for the Istanbul stock market. They find that a pairs-trading portfolio outperforms the market portfolio. Cummins (2010) tests the pairs-trading strategy in the U.S., Japan, Hong Kong, and China mainland markets. The author finds excess returns in the Japan and U.S. markets, but no significant excess returns in the Hong Kong and China markets. Like Do and Faff (2010), Cummins (2010) finds a better performance for pairs-trading strategy during the global financial crisis. Broussard and Vaihekoski (2010) study the pairs-trading strategy for the Finland stock market, a market with less liquidity than the U.S. market. They find that pairs trading produces an excess return of 14.99% in Finland market, which is higher than excess return in the U.S. market.

A second group of papers studies the cointegration approach detailed by Vidyamurthy (2004). These papers include Agarwal, Madhogaria, and Narayanan (2004); Lin, Mccrae, and Gulati (2006); Mavrakis and Alexakis (2011); and Kim (2011). Agarwal et al. (2004) find that pairs trading based on the cointegration approach is profitable. Lin et al. (2006) apply the cointegration approach with a

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<sup>4</sup>The arbitrage risks include fundamental risk, noise-trade risk and synchronization risk. Fundamental risk refers to the possibility of an unexpected disruption in the relative relationship between paired securities. Noise-trader risk comes from irrational trading of noise traders, which will deter the convergence. Synchronization risk is risk that other arbitrageurs will also exploit the mispricing.

minimum profit constraint. The empirical results show that their method does not reduce absolute profits compared with the original method. Mavrakis and Alexakis (2011) examine the pairs-trading performance in the German and Greek stock markets. These researchers find that mean-reversion of the spread in the pairs' prices is more likely to hold with moderate overall market performance than with other types of performance.<sup>5</sup> They suggest the pairs-trading strategy should be used cautiously when large movements in all prices occur, because the long-term relation may be changed in this case. Kim (2011) examines the pairs-trading strategy in the Korea stock market with high frequency data. The researcher finds positive return in all market conditions with superior performance in bear markets. Kim (2011) also finds the performance of the strategy is related to the market entry timing. The superior performance is found for trades originated around the opening and closing of the daily market.

The other papers study some new approaches. Huck (2007, 2010) develops a methodology that combines the forecasting techniques and multi-criteria decision making method. The researcher ranks the assets according to the expected return and pairs the assets with the highest over-valuations and under-valuations. The empirical result shows that this approach is successful in generating positive returns. Elliott, Der, and Malcolm (2005) propose a mean reverting Gaussian Markov chain model. They use a Gaussian noise process to predict the spread between pairs.<sup>6</sup> When the subsequent observation of the spread is larger than the predicted spread, these researchers open the pairs position by longing the stock with the lower price and shorting the one with the higher price. When the observation of the spread is smaller than the predicted spread, they do the opposite operation to close the position. Hong and Susmel (2003) study the pairs trading strategy by longing the Asian share and shorting corresponding American Depositary Shares. These researchers find that the strategy generates significant profit. Perlin (2007) proposes a new multivariate approach to replace

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<sup>5</sup> The period of moderate market performance is the period in which the market experiences more than 50% down returns.

<sup>6</sup> Gaussian noise is a statistical noise that its probability density function is equal to the normal distribution.



traditional one-by-one pairs trading. The researcher suggests for a particular asset, pairs can be built with the information of  $m$  ( $m > 1$ ) assets. Baronyan, Boduroglu and Sener (2010) examine the pairs-trading strategy by combining the distance approach, the cointegration approach, and the stochastic spread approach. They find that pairs-trading strategy works better under severe market conditions.

Most papers use the distance approach to select the pairs from the universe of stocks, i.e., without an industry constraint. An exception is Engelbert et al. (2009), who limit the pairs matching to stocks within the same industry. They use the Fama-French twelve-industry classification scheme. Most papers use the cointegration approach to select pairs from the stocks within the same industries. Lin et al. (2006) use two Australia bank stocks (the Australia New Zealand Bank and the Adelaide Bank) to test the cointegration-based procedure. Mavrakis and Alexakis (2010) only apply pairs-trading strategy to Bank stocks. Kim (2011) considers the pairs that are selected in the same industry groups. The researcher classifies the groups according to FnGuide Industry Group Classification Standard. Only Agarwal et al. (2004) (with the cointegration approach) do not limit their pairs to the same industry. However, the method these researchers use to implement the trading of pairs is extremely simple. They only consider the correlation between the residuals from the regression lines, which is arguably a problematic method.

The practical reason why studies using the distance method typically use unrestricted pairs is that the distance approach is computationally very simple: the only step is to calculate the “distance” of prices of a pair. On the other hand, in the cointegration approach, the matching process is more complicated, so often an industry restriction is used. An economic argument advanced by some authors for using an industry restriction is that using industry-restricted pairs avoids risk due to different relative shocks to different industries.

I found only three papers that mention the choice between the unrestricted pairs and industry-restricted pairs. The earliest one is by Gatev et al. (2006). After testing the behavior of unrestricted pairs, they use four broad industries classified

by Standard and Poor's to form restricted pairs: utility, transportation, financial, and industrials. These researchers find that pairs trading is profitable in restricted pairs and especially high in the utility and financial sectors. However, they find no difference between the profitability of the industry-restricted pairs and unrestricted pairs. Do and Faff (2010) test the restricted pairs with the same sector classification as that used by Gatev et al. (2006). In a cross-sectional analysis that regresses pairs returns on a time trend, the sum of squared differences (SSD), the square of SSD, the "crossing rate" of the pairs, an industry dummy, industry volatility, and the square of industry volatility, Do and Faff (2010) find that pairs of stocks within the same industry perform better than pairs in different industries. However, the  $R^2$  value is only 0.009, which raises doubts about their conclusion. Cummins (2010) uses nine industry sectors specified by Bloomberg. Unlike Gatev et al. (2006) and Do and Faff (2010), Cummins (2010) finds utilities is the worst performing among all sectors. The researcher also finds no superior results for industry-restricted pairs when compared to unrestricted ones. Even though restricted pairs have a higher returns it comes with the cost of higher variance. Cummins (2010) argues that unrestricted pairs can benefit from the diversification effect. All these three papers use the distance approach. No paper using the cointegration approach compares restricted and unrestricted pairs. Moreover, in all papers that do consider restricted and unrestricted pairs, the same trading strategy is used for the compared pairs (e.g., same trading sign, same trading period). Given that the variance for unrestricted pairs is smaller according to Cummins (2010), the optimal signals for the opening pairs trading strategies for restricted pairs and unrestricted pairs might be different. As well, for different industries, the optimal interval for a pair relation to exist could be different.

There is no comprehensive analysis of pairs trading using both the distance and cointegration approaches and considering both restricted and unrestricted pairs. This chapter fills this gap in the literature.

## 1.3 Pairs trading method

Pairs trading consists of two stages. The first stage is the formation period, where pairs of stocks are selected according to the historical data. The second stage is the trading period, where trades are made on the chosen pairs if trading conditions are met.

### 1.3.1 The distance approach

The first step in the distance approach is to normalize the price of each stock to a unity value at the beginning of the formation period. The reason to make such a transformation is straightforward: the distance calculated based on the raw prices could be misleading, because two stocks can move together but have a high squared distance between them. After the normalization, all stocks will have the same standard unit and this permits a quantitatively fair formation of pairs.

Let  $T_f$  denote the number of trading days in the formation period. The normalized price of each stock at the end of each trading day  $t, t = 1, 2, \dots, T_f$  is

$$P_t^i = \prod_{\tau=1}^t 1 \times (1 + r_\tau^i), \quad (1)$$

where  $P_t^i$  is stock  $i$ 's normalized price at the end of the trading day  $t$ ,  $\tau$  is the index for all the trading days from the first trading day to the trading day  $t$ , and  $r_\tau^i$  is the stock's daily return (inclusive of dividends) for stock  $i$  on trading day  $\tau$ . The distance between two stocks over the formation period is calculated as

$$D_{i,j} = \frac{\sum_{t=1}^{T_f} (P_t^i - P_t^j)^2}{T_f}, \quad (2)$$

where  $P_t^i$  and  $P_t^j$  are the normalized prices for stock  $i$  and stock  $j$  respectively on trading day  $t$  in the formation period. For  $N$  stocks available for consideration, we need to compute  $\frac{N \times (N-1)}{2}$  distances. Then we rank the candidate pairs from lowest to highest according to the distance and take only top pairs that have the smallest

distance. The standard deviation of the squared normalized price difference can be calculated as

$$\text{Std}D_{i,j} = \sqrt{\frac{1}{T_f-1} \sum_{t=1}^{T_f} [(P_t^i - P_t^j)^2 - D_{i,j}]^2}. \quad (3)$$

### 1.3.2 The cointegration approach

The first step is to test each series individually for their order of integration. We use Augmented Dickey-Fuller (ADF) tests to divide the stocks into sub-samples with same orders of integration, because only two series that are integrated of the same order can have a cointegrating relationship.

The second step is to calculate the price ratio of two stocks in the possible pairs.  $PR_t^{ij} = \log P_t^i - \log P_t^j$ , where  $PR_t^{ij}$  is the price ratio for stock  $i$  and  $j$ ,  $P_t^i$  and  $P_t^j$  are the prices for stock  $i$  and  $j$  on trading day  $t$  in the formation period. Then we use ADF test to test for the mean-reversion characteristics of the spread. That is, regress the difference of the price ratio  $\Delta PR_t^{ij}$  on the lagged value of  $PR_{t-1}^{ij}$  (i.e.,  $\Delta PR_t^{ij} = \gamma PR_{t-1}^{ij} + \varepsilon_t$ ) and test the null hypothesis that  $\gamma = 0$ . If the null hypothesis can be rejected, it indicates that the price ratio is following a weak stationary process and therefore the spread mean reverting. Herlemont (2004) suggests a confidence level of 99%. He argues that if the confidence level is lower, the pairs' mean-reversion property will be less certain and thus the profitability of the pairs trading strategy may be weakened.

The third step is to test for cointegration. According to common trends models (Stock and Watson, 1988), any time series can be expressed as a simple sum of two component time series: a stationary component and a non-stationary one. Vidyamurthy (2004) states that if two time series are cointegrated, the cointegrating linear composition can nullify the non-stationary components and leave the stationary part. In this chapter, we use the Johansen cointegration test to find those pairs with cointegration characteristics.

The fourth step is to use Granger causality tests to determine whether stock prices within the same pairs can informationally lead each other. Granger causality does not indicate causality in the logical sense. “A Granger causes B” only means the former can be used to predict the latter. A two-way Granger causality is stronger than one-way Granger causality. A pair selected based on a two-way relation is less likely to experience permanent divergence caused perhaps by a structural breakdown in the pairs relationships. Therefore, in this chapter, we only consider the pairs with two-way relation.

After these four steps, if there is still a very large number of pairs left, we consider a fifth step—the Market Factor Spread (MFS). A pairs-trading strategy is in some sense a market-neutral strategy. Even though not all pairs trading are 100% market neutral, we prefer those pairs with less systematic risk. This is done by picking pairs that have highly similar market exposures. The closer the market exposures are, the better the market risk hedging is. The market factor spread is calculated as  $MFS = |\beta_i - \beta_j|$ , where  $\beta_i$  and  $\beta_j$  are the market factors for stock  $i$  and stock  $j$  calculated in the Capital Asset Pricing Model.  $\beta_i = \frac{\text{cov}(R_i, R_m)}{\text{var}(R_m)}$ , where  $R_i$  is the return of the stock  $i$ ,  $R_m$  is the return of the market (measured by the Standard and Poor’s 500 Index). We rank the pairs from the lowest to highest based on MFS, and choose the top ones with the lowest spread.

### 1.3.3 Opening a pairs position

After the “formation period”, we track the behavior of the chosen pairs over the “trading period.” For each pair, there is a threshold for trading, which is named as “trading sign”. The “trading sign” is defined as the scaled standard deviation of the pairs spread calculated in the formation period. Specifically, for a pair with stock  $i$  and  $j$ ,  $\text{Trading\_Sign}^{ij} = n \times \text{std}D_{ij}$ , where  $n$  is the multiplier of the standard deviation, and  $\text{std}D_{ij}$  is the standard deviation of divergence of stock  $i$  and stock  $j$ . Gatev et al. (2006) use a multiplier of two times the standard deviation as a benchmark. In this chapter, we will try different multipliers.

In the beginning of the trading period, again, we re-normalize the stock prices to equal unity, and track the normalized price spread. When  $P_t^i - P_t^j \geq \text{Trading\_Sign}^{ij}$ , we open a pair position by longing the stock with the relatively lower price and shorting the stock with the relatively higher price.<sup>7</sup> Here we assume one dollar long-short position; i.e., we spend one dollar in buying the “cheap” stock and short sell one dollar of the “expensive” stock.

### **1.3.4 Closing a pairs position**

After the pairs-trading position is open, it will be held until the prices of the stocks converge during the trading period. If the pairs position remains open at the end of the trading period, the position is automatically closed and profit or loss will be calculated based on the closing stock prices on the last day of the trading period. For any pairs that have been closed without convergence, further trades are prohibited until the pairs spread equals zero again.

### **1.3.5 One day later rule**

Gatev et al. (2006) apply the “one day later rule”; i.e., open the position one day after the day the stock spread exceeds the trading sign and close the position one day after the day the normalized price paths cross (i.e., converge). The reasoning behind this rule is to minimize the effects of bid-ask bounce associated with using daily closing stock prices from the Center for Research in Security Price (CRSP) database. CRSP uses the average bid-ask closing price as the index of the daily stock price. The excess return calculated from these prices could be biased upwards, because in practice when we open the position we buy at the ask (higher) and sell at the bid (lower) prices. The opposite is also true when closing the position. However, applying the one day later rule may cause a downward bias to excess returns if the mean reversion characteristic is very strong; i.e., market effect will rapidly adjust any divergence in prices of pairs.

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<sup>7</sup> We assume that traders can long and short securities in the market without any restrictions. We do not consider options in this chapter.

Therefore, in this chapter, we also consider an alternative to the one day later rule called the “transaction cost approach.”

### 1.3.6 Transaction cost approach

The transaction cost approach explicitly accounts for the bid-ask spread in return calculations from pairs trading. Gatev et al. (2006) estimate the effective spread is 81bp, i.e., a transaction cost of 162bp per pair per round trip. Peterson and Fialkowski (1994) find that the average effective spread for a stock in the CRSP is 37bp. Bessembinder (2003) studies the bid-ask spreads on the New York Stock Exchange and National Association of Securities Dealers Automated Quotations market, and finds that the average spreads (for all stocks) are 0.486 and 0.739 percent of the share price respectively. For large stocks the spreads are 0.212 and 0.238 percent. Given this range of bid-ask spread calculations in the literature, we assume a spread of 50bp, i.e., a 1% transactions cost adjustment per pairs trade from market entry to position clearing.

### 1.3.7 Calculation of returns

We use the same method to calculate portfolio return as in Gatev et al. (2006). For pair  $p^i$ ,  $p^i = (l^i, s^i)$  indicates it is composed by the longed stock  $l^i$  and shorted stock  $s^i$ . Let  $d^i$  indicates the most recent day of divergence for pair  $p^i$ .  $R_t(l^i)$  and  $R_t(s^i)$  respectively represent the return on stock  $l^i$  and stock  $s^i$  in day  $t$ . The return for  $p^i$  in day  $t$ ,  $R_t(p^i)$  is

$$R_t(p^i) = R_t(l^i) - R_t(s^i). \quad (4)$$

The return on a portfolio of  $N$  pairs on day  $t$  is

$$R_t(\text{Portfolio}) = \sum_{i=1}^N W_t^i R_t(p^i), \quad (5)$$

where the weight  $W_t^i = \omega_t^i / \sum_{j=1}^N \omega_t^j$ , captures the compound effect.  $\omega_t^i = [1 + R_{t-1}(p^i)] \times [1 + R_{t-2}(p^i)] \times \dots \times [1 + R_{d+1}(p^i)]$ , for  $t \geq d^i + 2$  and  $\omega_t^i = 1$ , for  $t \geq d^i + 1$ . In words, we use the  $N$  open pairs that are held in the portfolio on day  $t$  to calculate the daily return of the portfolio, which is equal to

the weighted average return of the  $N$  pairs. The weight given to a pair is determined by its cumulative return relative to the sum of cumulative returns of all pairs in the portfolio. Thus, the excess return per month for the portfolio in the trading period is  $E[R_t(\text{Portfolio})] = \frac{\sum_T R_t(\text{Portfolio})}{M}$ , where  $T$  is the number of trading days in the trading period, and  $M$  is the number of months included in the trading period. The return after considering transaction costs is  $E[R_t(\text{Portfolio})](1 - C)$ , where  $C$  is the transaction cost in percentage. Because the strategy is based on a long-short position of one dollar, the return of the portfolio has the interpretation of excess return; i.e., the net investment in a pair is zero.

## 1.4 Empirical results

We use CRSP daily data from Jan 2005 to Dec 2012 in this chapter. Like Gatev et al. (2006), we consider only common stocks (stocks with share code 10 or 11) and filter out stocks that have either no trading data or invalid return data for one or more days. Unlike Gatev et al. (2006), who assign the securities to four major industry groups, we divide the securities into seven groups according to the Standard Industrial Classification: 10-14 for mining (104 stocks), 20-39 for industry (1153 stocks), 40-49 for transportation & public utilities (245 stocks), 50-51 for wholesale (102 stocks), 52-59 for retail trade (181 stocks), 60-67 for financial (535 stocks), and 70-89 for services (450 stocks). This is a total of 2770 stocks generates 3,835,065 possible pairs. For each stock, we use the total return index, which includes dividends, instead of the regular stock price. As mentioned above, the optimal trading strategy for different groups could be different. We try several different opening signs and different formation periods and treat the ones with the best results as the optimal results for that group. Do and Faff (2010) find a declining trend in the profitability of pairs trading, which could occur because the time period that the co-movement of pairs lasts has shortened over time. We begin by using Gatev et al.'s (2006) strategy with a 12-month formation period; we also try two shorter formation periods, 9 months and 6 months. The trading period is 6 months because we find that a shorter trading period may cause many



pairs either to be unclosed or inactive at the end. For the trading sign, we consider multipliers of {0.3, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0}. We compare unrestricted pairs and industry-restricted pairs with the optimal strategies for each group.

In this chapter, we use an “overlapping method” as in Gatev et al. (2006): the implementation periods are staggered by one month; i.e., the first implementation period begins on the first trading day of Jan. 2005, the second period begin on the first trading day of Feb. 2005, and both formation and trading periods roll forward by one month. There are 55 trading intervals for the 12-6 strategy (12 formation months and 6 trading months), 58 trading intervals for 9-6 strategy and 61 trading intervals for 6-6 strategy.

## **1.4.1 The distance approach with transaction cost adjustment**

### **1.4.1.1 Profitability of the strategy**

Table 1-1 lists the excess return for both unrestricted and industry-restricted pairs net of the transaction cost. The six-month excess return for the top 5 pairs is the highest in the unrestricted pairs, at 15.13%.<sup>8</sup> The profits for the industry groups are somewhat lower: service 3.94%, financial 11.01%, retail trade 10.45%, wholesale 7.09%, transportation & public utilities 10.2% and mining - 0.42% for top 5 pairs. Pairs from the “industry” sector have the highest top 5 excess return, 16.17%, but they underperform unrestricted pairs in the top 10, top 20, top 100 and top 200. The distribution of excess returns of the unrestricted pairs and pairs in services, financial, and wholesale are all skewed right and exhibit positive excess kurtosis relative to a normal distribution. This result indicates pairs trading in these groups is profitable. Diversification benefits from combining multiple pairs in a portfolio. As the number of pairs increases, the portfolio standard deviation falls, the minimum realized return increases, and the maximum realized return either remains stable or decreases. Figure 1-1 shows a more apparent profitability comparison of the different matching strategies. The unrestricted strategy outperforms the restricted pairs strategy. For industry-

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<sup>8</sup> The top pairs in the distance approach are the pairs with the lowest distance. The top pairs in the cointegration approach are the pairs with the lowest market factor spread.

restricted strategies, greater profit occurs in the financial, transportation & utilities and industry sectors, possibly because these industries might arguably contain more common shocks to firms within these industries than some of the other industries. Except for the financial sector pairs, almost all pairs trading are less profitable as more pairs are added to the portfolio, because as the number increases, more imperfectly matched pairs are added into portfolios. The reason for the gradual increase in profitability for the financial sector pairs is not obvious.

Table 1-1: Optimal excess return - distance approach with transaction cost

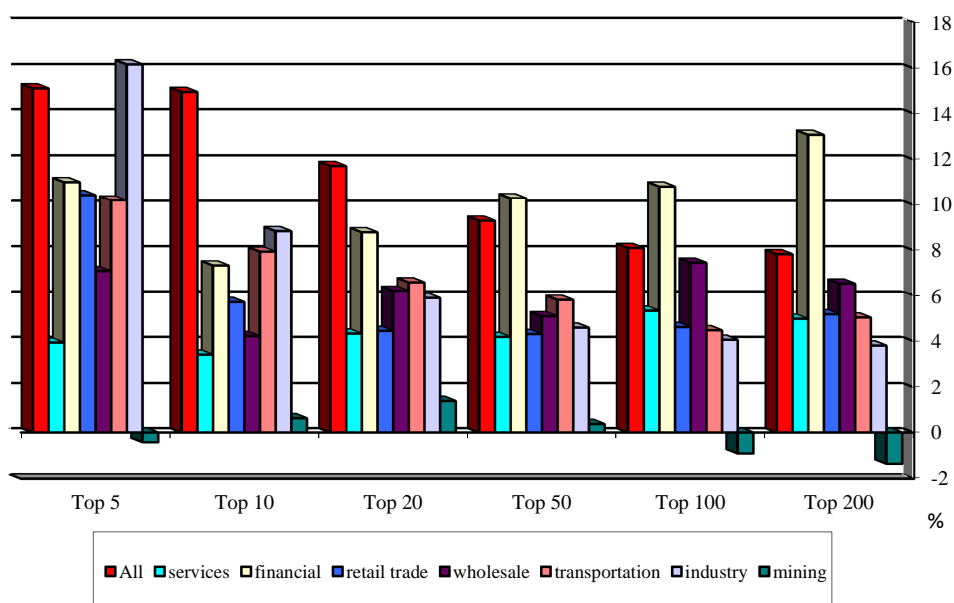
	Top 5	Top 10	Top 20	Top 50	Top 100	Top 200
<b>All_9 months formation period</b>						
	<b>multiplier=2.5</b>					
<b>Mean excess return</b>	<b>0.151325</b>	<b>0.149779</b>	<b>0.11719</b>	<b>0.093074</b>	<b>0.081049</b>	<b>0.078339</b>
Standard deviation	0.184608	0.145373	0.094095	0.069152	0.062082	0.058261
t statistics	6.242739	7.846555	9.485023	10.25036	9.942384	10.24036
skewness	0.957081	0.867872	0.893416	0.740391	1.129754	1.438632
kurtosis	4.76004	3.98539	3.566246	2.915612	4.568457	6.103986
minimum	-0.23885	-0.14543	-0.04574	-0.03039	-0.01992	-0.00912
maximum	0.745525	0.617023	0.378038	0.26158	0.285433	0.312087
median	0.119756	0.139242	0.106836	0.083058	0.077233	0.067826
Positive return (%)	89.65517	87.93103	94.82759	96.55172	94.82759	98.27586
<b>Services_12 months formation period</b>						
	<b>multiplier=3</b>					
<b>Mean excess return</b>	<b>0.039418</b>	<b>0.034189</b>	<b>0.043333</b>	<b>0.041869</b>	<b>0.05329</b>	<b>0.04979</b>
Standard deviation	0.150619	0.096298	0.114644	0.117751	0.119574	0.108633
t statistics	1.940867	2.633009	2.803173	2.636978	3.305169	3.399105
skewness	1.295792	0.98366	0.778276	1.299095	1.159552	0.897722
kurtosis	6.295921	4.190884	3.194666	5.26218	4.278759	3.777015
minimum	-0.23064	-0.1375	-0.13048	-0.13145	-0.13047	-0.1291
maximum	0.646256	0.359602	0.350149	0.478188	0.426597	0.343312
median	0.017292	0.024292	0.018061	0.018464	0.03066	0.028735
Positive return (%)	54.54545	61.81818	60	54.54545	63.63636	65.45455
<b>Financial_12 months formation period</b>						
	<b>multiplier=2</b>					
<b>Mean excess return</b>	<b>0.110146</b>	<b>0.073358</b>	<b>0.087962</b>	<b>0.103085</b>	<b>0.108239</b>	<b>0.130765</b>
Standard deviation	0.192771	0.11136	0.11084	0.089523	0.084216	0.094161
t statistics	4.237497	4.885368	5.885433	8.539684	9.531736	10.2991
skewness	2.054923	0.456084	1.323321	0.927749	1.005383	1.183497
kurtosis	8.3077	4.265768	8.186205	5.839534	3.831044	4.416179
minimum	-0.19204	-0.21152	-0.20314	-0.12882	-0.03649	-0.02865
maximum	0.894792	0.403563	0.566373	0.439297	0.348671	0.445246
median	0.062556	0.065045	0.06511	0.094006	0.093903	0.116748
Positive return (%)	70.90909	80	90.90909	92.72727	94.54545	96.36364

<b>Retail trade_12 months formation period</b>					<b>multiplier=2.5</b>	
<b>Mean excess return</b>	<b>0.104478</b>	<b>0.057075</b>	<b>0.044488</b>	<b>0.043107</b>	<b>0.046166</b>	<b>0.051733</b>
Standard deviation	0.164144	0.126986	0.107424	0.093347	0.099382	0.106149
t statistics	4.720411	3.333269	3.071291	3.424701	3.445081	3.614344
skewness	-0.35101	-0.16392	-0.50545	-0.1334	-0.04246	0.692948
kurtosis	4.621086	4.048228	3.998834	3.093731	3.3509	4.832801
minimum	-0.47313	-0.33403	-0.31962	-0.20352	-0.19766	-0.19043
maximum	0.435235	0.336075	0.277522	0.238219	0.309885	0.419667
median	0.096275	0.045097	0.060006	0.029601	0.031899	0.038755
Positive return (%)	72.72727	74.54545	65.45455	67.27273	72.72727	69.09091
<b>Wholesale_6 months formation period</b>					<b>multiplier=3</b>	
<b>Mean excess return</b>	<b>0.070913</b>	<b>0.042237</b>	<b>0.062161</b>	<b>0.050875</b>	<b>0.07457</b>	<b>0.06537</b>
Standard deviation	0.16733	0.142196	0.131889	0.124383	0.145298	0.155417
t statistics	3.309933	2.319923	3.681095	3.194515	4.008406	3.285063
skewness	0.169042	0.170074	0.263742	0.30029	0.974189	1.19911
kurtosis	3.221392	3.548609	3.276593	2.57606	4.839371	4.768334
minimum	-0.34127	-0.34731	-0.2787	-0.1865	-0.16844	-0.17027
maximum	0.50573	0.429121	0.412375	0.391284	0.617548	0.552508
median	0.065517	0.029009	0.024002	0.027906	0.074339	0.055093
Positive return (%)	63.93443	59.01639	67.21311	55.7377	67.21311	63.93443
<b>Transportation &amp; public utilities_12 months formation period</b>					<b>multiplier=3</b>	
<b>Mean excess return</b>	<b>0.102105</b>	<b>0.079449</b>	<b>0.065902</b>	<b>0.058029</b>	<b>0.044775</b>	<b>0.050317</b>
Standard deviation	0.132295	0.09593	0.08721	0.087262	0.081259	0.08223
t statistics	5.72382	6.142055	5.604234	4.931716	4.086432	4.538049
skewness	-0.612	0.35076	0.370821	0.91717	1.230081	0.533082
kurtosis	4.059427	2.639143	2.57349	3.348892	3.962147	4.562054
minimum	-0.31247	-0.1469	-0.09985	-0.06984	-0.05901	-0.18929
maximum	0.397306	0.312738	0.275224	0.281903	0.270293	0.280195
median	0.119273	0.068027	0.056706	0.040547	0.025896	0.046445
Positive return (%)	83.63636	74.54545	74.54545	70.90909	63.63636	70.90909
<b>Industry_12 months formation period</b>					<b>multiplier=2</b>	
<b>Mean excess return</b>	<b>0.161729</b>	<b>0.088472</b>	<b>0.058879</b>	<b>0.045915</b>	<b>0.040634</b>	<b>0.038116</b>
Standard deviation	0.168643	0.100765	0.080013	0.067251	0.063152	0.059244
t statistics	7.112185	6.511453	5.457325	5.063329	4.771825	4.771401
skewness	0.278216	0.478246	1.034168	0.203504	-0.07011	0.071209
kurtosis	4.2175	2.86531	6.433886	2.988659	2.253894	2.576673
minimum	-0.36952	-0.11274	-0.08074	-0.11001	-0.08512	-0.08418
maximum	0.599594	0.332986	0.381283	0.213578	0.172286	0.173306
median	0.116886	0.077	0.057548	0.046704	0.048622	0.041625
Positive return (%)	92.72727	80	78.18182	78.18182	67.27273	72.72727
<b>Mining_12 months formation period</b>					<b>multiplier=0.3</b>	
<b>Mean excess return</b>	<b>-0.0042</b>	<b>0.006204</b>	<b>0.01368</b>	<b>0.003692</b>	<b>-0.00909</b>	<b>-0.01355</b>
Standard deviation	0.125906	0.10562	0.106839	0.070375	0.056442	0.042196
t statistics	-0.24718	0.435618	0.949619	0.389035	-1.1942	-2.38217

skewness	0.529758	0.535305	2.071174	0.693943	-0.62351	-0.52503
kurtosis	4.366451	3.477833	10.12309	3.891229	6.114867	4.407482
minimum	-0.32511	-0.21712	-0.13286	-0.14957	-0.22928	-0.16162
maximum	0.392604	0.293058	0.515907	0.235599	0.115357	0.077302
median	-0.01261	-0.00042	-0.00409	-0.00839	-0.01447	-0.00992
Positive return (%)	45.45455	49.09091	47.27273	43.63636	38.18182	36.36364

Summary statistics for the excess return distribution for pairs trading from the distance approach over the six-month trading period. Pairs-trading portfolios include all stocks and stocks from different sectors. Here, we choose the optimal strategy that will get the highest excess return for different sectors. We trade according to the rule that opens a position in a pair at the end of the day when prices of the stocks in the pair diverge by multiplier-historical standard deviation. The “top  $n$ ” portfolios include the  $n$  pairs with the least distance measures.

Figure 1-1: Optimal excess return - distance approach with transaction cost

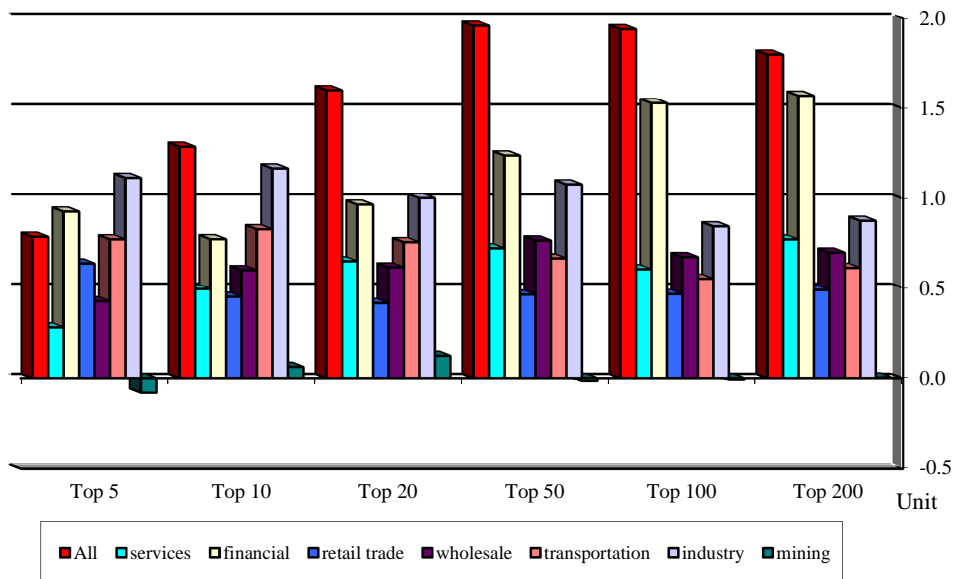


#### 1.4.1.2 Information ratio

Given that excess return does not consider the risk of pairs trading, we next compare the information ratios for the various portfolios of pairs trades.<sup>9</sup> Figure 1-2 reveals that, except for the top-5 pairs portfolio, the financial and industry sectors have higher information ratios, and that unrestricted pairs have superior information ratios.

<sup>9</sup> The information ratio is defined as the active return divided by the tracking error. The active return is the difference between the return of the security and the return of a selected benchmark index. The tracking error is the standard deviation of the active return.

Figure 1-2: Information ratio - distance approach with transaction cost



In summary, for the distance approach with transaction cost adjustment, the unrestricted pairs are preferred to the industry-restricted pairs.

## 1.4.2 Distance approach with one day later rule

### 1.4.2.1 Profitability of the strategy

Table 1-2 lists the excess return for both unrestricted and industry-restricted pairs with the one day later rule. The highest return in the unrestricted pairs, service, and financial sectors are 6.16%, 5.98%, and 6.14%, respectively, with the top-5 pairs portfolio. The highest return in the wholesale sector is 8.92% with the top-20 pairs portfolios. The excess returns from the remaining sectors are lower. Figure 1-3 shows that in the case of the one day later rule, the wholesale sector is more profitable than the rest of the restricted sectors and the unrestricted one. For the transportation & utility, financial, and industry sectors, we find no difference between the profitability of the industry-restricted pairs and unrestricted pairs. Our finding is consistent with that in Gatev et al. (2006). Figure 1-4 presents the information ratio for all strategies. Here, we find that the restricted pairs of the wholesale sector are superior to the unrestricted pairs, but at the cost of higher volatility. When we consider the risk, industry-restricted pairs

do not improve the result. This conclusion is consistent with Cummins' (2010) findings.

Table 1-2: Optimal excess return - distance approach with one day later rule

	<b>Top 5</b>	<b>Top 10</b>	<b>Top 20</b>	<b>Top 50</b>	<b>Top 100</b>	<b>Top 200</b>
<b>All_6 months formation period</b>						
	<b>multiplier=2.5</b>					
<b>Mean excess return</b>	<b>0.06158</b>	<b>0.023669</b>	<b>0.033209</b>	<b>0.021089</b>	<b>0.01836</b>	<b>0.017613</b>
Standard deviation	0.112389	0.118943	0.071422	0.048536	0.04860	0.047737
t statistics	4.279348	1.554215	3.631504	3.393572	2.95056	2.881702
skewness	3.092615	-2.76480	0.690146	0.897702	1.12942	1.096447
kurtosis	14.90088	15.11907	4.228017	4.158217	6.333785	4.567541
minimum	-0.18124	-0.60770	-0.13012	-0.07133	-0.08849	-0.07325
maximum	0.573708	0.234617	0.258429	0.181319	0.201981	0.160155
median	0.032069	0.026035	0.027714	0.016176	0.01221	0.007191
Positive return (%)	96.72131	83.60656	67.21311	65.57377	62.29508	59.01639
<b>Services_12 months formation period</b>						
	<b>multiplier=3</b>					
<b>Mean excess return</b>	<b>0.059791</b>	<b>0.033163</b>	<b>0.028616</b>	<b>0.019448</b>	<b>0.014297</b>	<b>0.029470</b>
Standard deviation	0.121576	0.130735	0.129752	0.09872	0.091695	0.088840
t statistics	3.647261	1.881259	1.635575	1.461015	1.156312	2.460118
skewness	-0.65663	0.489551	0.796893	0.44194	0.076517	0.269349
kurtosis	5.509536	2.345294	3.180786	2.341783	3.551352	2.810223
minimum	-0.40142	-0.20270	-0.17452	-0.15664	-0.26529	-0.16257
maximum	0.330702	0.325092	0.373814	0.228149	0.210578	0.229376
median	0.068303	-0.0029	-0.00107	0.003175	0.011059	0.025022
Positive return (%)	67.27273	49.09091	47.27273	54.54545	58.18182	60
<b>Financial_9 months formation period</b>						
	<b>multiplier=2.5</b>					
<b>Mean excess return</b>	<b>0.061398</b>	<b>0.024173</b>	<b>0.017805</b>	<b>0.012366</b>	<b>0.013542</b>	<b>0.012486</b>
Standard deviation	0.124714	0.103154	0.106698	0.09416	0.081035	0.075913
t statistics	3.749353	1.78471	1.270851	1.000205	1.272734	1.252585
skewness	1.469379	0.263324	-0.66447	-0.63495	-0.90748	-1.70069
kurtosis	6.736418	3.019042	3.872238	5.139388	4.728667	7.203019
minimum	-0.15095	-0.19593	-0.29795	-0.30273	-0.23512	-0.27507
maximum	0.574463	0.319291	0.23408	0.288826	0.190682	0.167715
median	0.048304	0.016768	0.027807	0.023824	0.022324	0.030585
Positive return (%)	70.68966	58.62069	63.7931	70.68966	68.96552	70.68966
<b>Retail trade_6 months formation period</b>						
	<b>multiplier=3</b>					
<b>Mean excess return</b>	<b>0.044836</b>	<b>0.0279</b>	<b>0.025712</b>	<b>0.003877</b>	<b>0.007969</b>	<b>0.002211</b>
Standard deviation	0.135951	0.168668	0.151479	0.150072	0.129099	0.110384
t statistics	2.575807	1.291946	1.325693	0.201773	0.482132	0.156420
skewness	0.139832	-0.73209	0.638782	-0.92158	-0.44309	-0.83314
kurtosis	2.696495	4.542034	4.711404	6.430516	5.185738	6.793764
minimum	-0.25369	-0.58139	-0.3045	-0.57236	-0.3702	-0.37519
maximum	0.347685	0.386521	0.553696	0.401797	0.376814	0.325727
median	0.036328	0.027641	0.013669	0.016388	0.015547	0.014171

Positive return (%)	62.29508	60.65574	55.7377	52.45902	59.01639	57.37705
<b>Wholesale_12 months formation period</b>				<b>multiplier=3</b>		
<b>Mean excess return</b>	<b>0.048789</b>	<b>0.087202</b>	<b>0.089173</b>	<b>0.054051</b>	<b>0.048246</b>	<b>0.033399</b>
Standard deviation	0.12282	0.163437	0.161949	0.163589	0.172328	0.166797
t statistics	2.945997	3.956923	4.083534	2.450345	2.076292	1.485009
skewness	0.842231	0.795845	0.740469	-0.86754	-1.57977	-0.10070
kurtosis	4.083817	3.330537	5.213779	4.568042	9.029469	5.606054
minimum	-0.19066	-0.19097	-0.25797	-0.4816	-0.72684	-0.44206
maximum	0.459146	0.579005	0.654135	0.378142	0.393619	0.586485
median	0	0.067157	0.084626	0.061717	0.067382	0.053557
Positive return (%)	49.09091	60	72.72727	72.72727	72.72727	63.63636
<b>Transportation &amp; public utilities_9 months formation period</b>				<b>multiplier=1</b>		
<b>Mean excess return</b>	<b>0.00825</b>	<b>0.016778</b>	<b>0.011614</b>	<b>0.022373</b>	<b>0.022166</b>	<b>0.017038</b>
Standard deviation	0.165284	0.097808	0.066917	0.068819	0.05277	0.042488
t statistics	0.380116	1.306394	1.321778	2.475858	3.199086	3.054064
skewness	-2.40491	-0.29207	0.245212	1.655394	0.428014	0.177985
kurtosis	10.27356	4.090069	4.769709	8.526889	3.603749	4.387659
minimum	-0.69528	-0.25992	-0.15752	-0.13985	-0.09321	-0.10769
maximum	0.352303	0.302428	0.235196	0.317097	0.18639	0.157070
median	0.042379	0.028148	0.016288	0.013813	0.020893	0.016427
Positive return (%)	72.41379	62.06897	63.7931	58.62069	70.68966	67.24138
<b>Industry_12 months formation period</b>				<b>multiplier=2.5</b>		
<b>Mean excess return</b>	<b>0.043152</b>	<b>0.030146</b>	<b>0.015471</b>	<b>0.015853</b>	<b>0.012434</b>	<b>0.026282</b>
Standard deviation	0.17725	0.144071	0.131397	0.082789	0.076024	0.07129
t statistics	1.805507	1.551784	0.873219	1.420128	1.212976	2.734108
skewness	2.241819	-1.42528	-0.27154	0.303015	-0.2684	0.228436
kurtosis	14.27191	9.772576	6.330285	3.522392	3.000956	3.185933
minimum	-0.49249	-0.62668	-0.46764	-0.16401	-0.16669	-0.12054
maximum	0.835523	0.355549	0.404421	0.261959	0.18063	0.232487
median	0.016841	0.031496	0.029528	0.012077	0.018541	0.027053
Positive return (%)	70.90909	65.45455	61.81818	58.18182	61.81818	63.63636
<b>Mining_9 months formation period</b>				<b>multiplier=3</b>		
<b>Mean excess return</b>	<b>-0.04511</b>	<b>-0.03187</b>	<b>0.006831</b>	<b>0.012004</b>	<b>0.016086</b>	<b>0.008727</b>
Standard deviation	0.174261	0.210738	0.16228	0.149676	0.160304	0.156778
t statistics	-1.97138	-1.15159	0.320599	0.610767	0.764243	0.423932
skewness	-0.37955	-1.20854	-0.27353	-0.08664	-0.1911	0.198206
kurtosis	3.393552	8.077303	3.756918	3.017167	4.337181	3.786185
minimum	-0.51741	-0.95646	-0.49187	-0.36009	-0.46365	-0.40874
maximum	0.284998	0.444167	0.358468	0.329985	0.447454	0.424603
median	-0.04652	-0.02436	-0.00038	-0.01042	-0.00125	-0.01623
Positive return (%)	37.93103	43.10345	48.27586	46.55172	50	44.82759

Summary statistics for the excess return distribution for pairs trading from the distance approach over the six-month trading period. Pairs-trading portfolios include all stocks and stocks from different sectors. Here, we choose the optimal strategy that will get the highest excess return for different sectors. We trade according to the one day later rule, which opens a position in a pair at

the end of the next day when prices of the stocks in the pair diverge by multiplier-historical standard deviation. The “top n” portfolios include the n pairs with the least distance measures.

Figure 1-3: Optimal excess return - distance approach with one day later rule

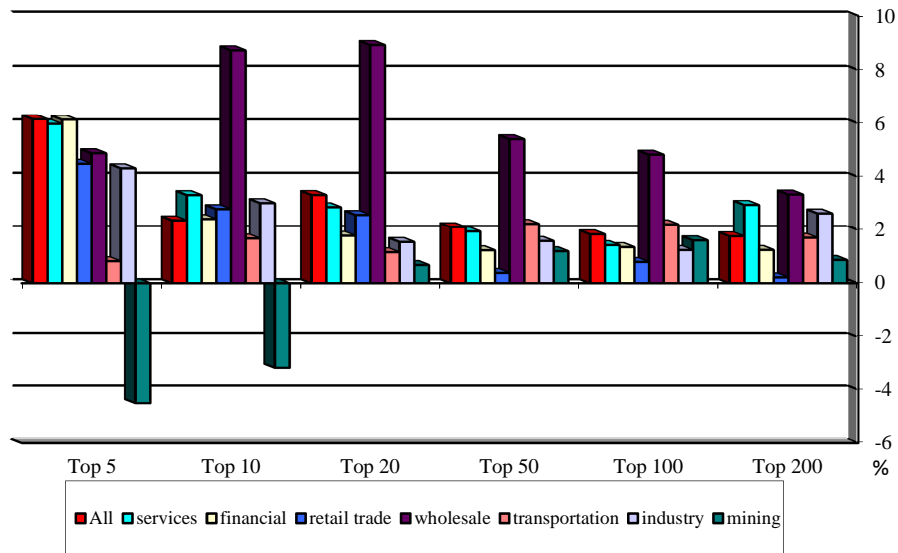
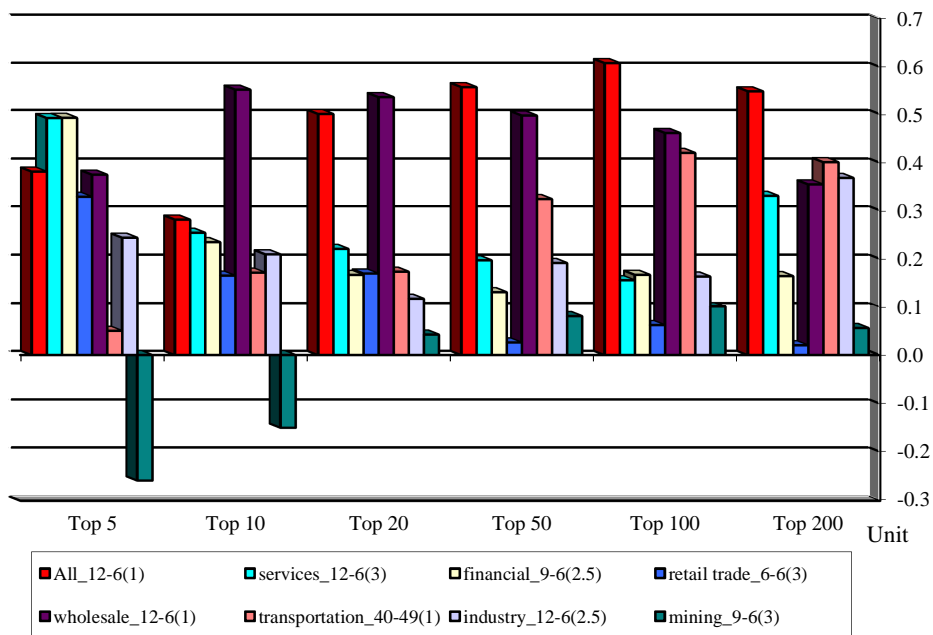


Figure 1-4: Information ratio - distance approach with one day later rule





### **1.4.3 Cointegration Approach**

#### **1.4.3.1 Profitability of the strategy**

Table 1-3 lists the excess return from the cointegration approach. Because the pair-matching process of the cointegration approach is more complicated and strict than the process of the distance approach, we do not have enough matching pairs for the multiple portfolios in the retail, wholesale, transportation & public utilities, and mining sectors. For example, for the optimal strategy, in the retail trade, there are 58 trading intervals, the maximum number of matched pairs for all intervals is 28, and there are 13 intervals without matched pairs; in the wholesale sector, there are 58 trading intervals, the maximum number of matched pairs is 16, and there are 30 intervals that have no matched pairs; in the transportation and public utilities sector, there are 61 trading intervals, the maximum number of matched pairs is 92, and there are 9 intervals that do not have matched pairs; in the mining sector, there are 55 trading intervals, the maximum number of matched pairs is 26, and there are 29 intervals that have no trading at all. The largest six-month excess return for the unrestricted pairs is 10.5% with the top-5 portfolio, which is much lower than that for the industry-restricted pairs in financial sector (20.03%), retail trade (11.44%), and industry (11.28%). The unrestricted pairs are skewed left in the top 5, top 20, and top 50 portfolios. Except for the top 10 pairs in the service sector and the top 200 pairs in the financial and industry sectors, all other portfolios are skewed right in the industry-restricted pairs. This result indicates that an industry-restricted strategy is more profitable than the unrestricted strategy. Also, if we omit those sectors that do not have enough matched pairs, the return under the unrestricted strategy is lower than that for the industry-restricted strategy. Figure 1-5 clearly shows that the restricted pairs are more profitable than the unrestricted pairs in the financial, service, retail trade and industry sectors.

Table 1-3: Optimal excess return – cointegration approach with transaction cost

	Top 5	Top 10	Top 20	Top 50	Top 100	Top 200
<b>All_12 months formation period</b>						
	<b>multiplier=2.5</b>					
<b>Mean excess return</b>	<b>0.105056</b>	<b>0.102324</b>	<b>0.067041</b>	<b>0.049833</b>	<b>0.060319</b>	<b>0.075945</b>
Standard deviation	0.367261	0.354729	0.278509	0.200062	0.181359	0.219699
t statistics	2.121418	2.139246	1.785182	1.847288	2.466578	2.563599
skewness	-0.06243	0.145608	-0.40019	-0.18295	0.372338	1.516868
kurtosis	6.428431	3.005645	3.201761	2.480704	3.271509	7.839868
minimum	-1.23336	-0.78597	-0.79057	-0.4282	-0.3537	-0.40685
maximum	1.272326	0.947453	0.55635	0.448469	0.545968	0.997854
median	0.076462	0.055182	0.071896	0.087203	0.069381	0.037344
Positive return (%)	61.81818	60	58.18182	65.45455	56.36364	61.81818
<b>Services_6 months formation period</b>						
	<b>multiplier=2</b>					
<b>Mean excess return</b>	<b>0.084672</b>	<b>0.097589</b>	<b>0.172498</b>	<b>0.174216</b>	<b>0.184672</b>	<b>0.189482</b>
Standard deviation	0.347628	0.295194	0.31302	0.288863	0.283712	0.290889
t statistics	1.902339	2.582002	4.304048	4.710452	5.083812	5.087516
skewness	1.151584	-0.2986	0.506826	0.115144	0.08838	0.295842
kurtosis	9.17643	5.116686	3.967204	4.345623	4.534133	5.119902
minimum	-0.83333	-0.97674	-0.61892	-0.76233	-0.76233	-0.76233
maximum	1.667571	0.946302	1.166839	0.946541	0.946541	1.099006
median	0.089227	0.089904	0.143718	0.147352	0.148842	0.152196
Positive return (%)	63.93443	59.01639	70.4918	78.68852	78.68852	78.68852
<b>Financial_12 months formation period</b>						
	<b>multiplier=3</b>					
<b>Mean excess return</b>	<b>0.200361</b>	<b>0.16937</b>	<b>0.17593</b>	<b>0.141803</b>	<b>0.164912</b>	<b>0.175603</b>
Standard deviation	0.429031	0.341256	0.271705	0.229366	0.220485	0.199621
t statistics	3.463427	3.680764	4.802031	4.584983	5.546958	6.523884
skewness	1.610963	1.07796	1.127974	1.008875	0.60753	-0.12014
kurtosis	5.875998	4.592	4.100001	4.736559	5.645096	2.748774
minimum	-0.40707	-0.4969	-0.22663	-0.3444	-0.47557	-0.29786
maximum	1.829784	1.305147	1.050971	0.940475	0.979477	0.570283
median	0.021898	0.087557	0.11477	0.130477	0.134305	0.183201
Positive return (%)	58.18182	69.09091	74.54545	74.54545	81.81818	83.63636
<b>Retail trade_9 months formation period</b>						
	<b>multiplier=2</b>					
<b>Mean excess return</b>	<b>0.114392</b>	<b>0.11442</b>	<b>0.104505</b>	<b>0.116137</b>	<b>n/a</b>	<b>n/a</b>
Standard deviation	0.419655	0.430105	0.369345	0.371284	n/a	n/a
t statistics	2.075957	2.026006	2.154868	2.382203	n/a	n/a
skewness	1.726993	3.126973	2.439702	2.32116	n/a	n/a
kurtosis	9.276938	16.24472	11.2462	10.72947	n/a	n/a
minimum	-0.9896	-0.59517	-0.59517	-0.59517	n/a	n/a
maximum	1.773688	2.358099	1.665744	1.665744	n/a	n/a
median	0	0	0	0	n/a	n/a
Positive return (%)	46.55172	48.27586	44.82759	46.55172	n/a	n/a
<b>Wholesale_9 months formation period</b>						
	<b>multiplier=1</b>					
<b>Mean excess return</b>	<b>0.044397</b>	<b>0.061929</b>	<b>0.057202</b>	<b>n/a</b>	<b>n/a</b>	<b>n/a</b>

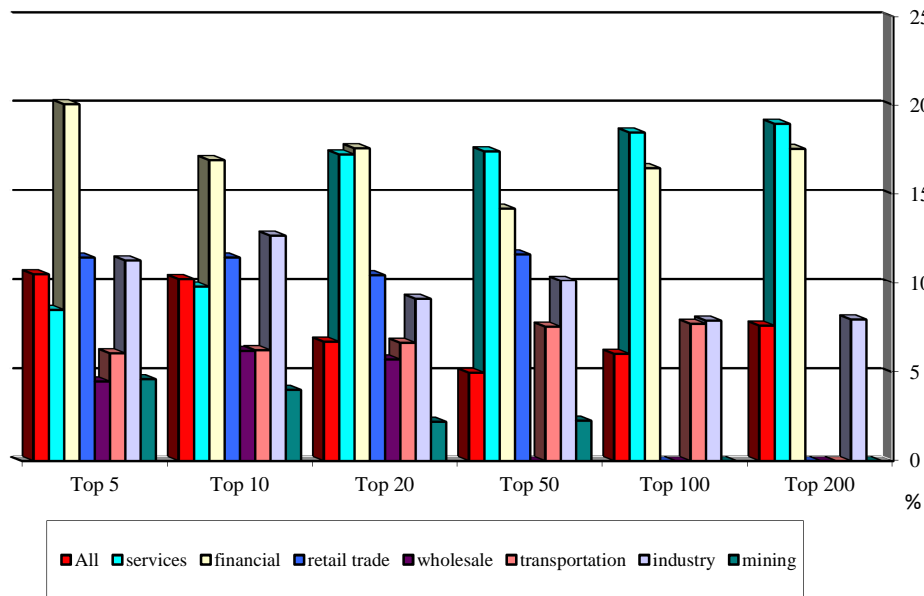
Standard deviation	0.203355	0.175216	0.17616	n/a	n/a	n/a
t statistics	1.662676	2.69174	2.472957	n/a	n/a	n/a
skewness	1.172411	2.057009	2.092997	n/a	n/a	n/a
kurtosis	6.341134	9.124349	9.620181	n/a	n/a	n/a
minimum	-0.46198	-0.32085	-0.32085	n/a	n/a	n/a
maximum	0.821447	0.833588	0.854671	n/a	n/a	n/a
median	0	0	0	n/a	n/a	n/a
Positive return (%)	29.31034	34.48276	32.75862	n/a	n/a	n/a
<b>Transportation &amp; public utilities_6 months formation period</b>				<b>multiplier=3</b>		
<b>Mean excess return</b>	<b>0.060618</b>	<b>0.06235</b>	<b>0.066427</b>	<b>0.075343</b>	<b>0.076951</b>	<b>n/a</b>
Standard deviation	0.335916	0.275197	0.228535	0.242846	0.241595	n/a
t statistics	1.40941	1.769524	2.270144	2.423143	2.487677	n/a
skewness	2.542797	2.170044	1.09379	1.852981	1.874017	n/a
kurtosis	13.92577	9.931383	5.591196	8.42646	8.538671	n/a
minimum	-0.73848	-0.45868	-0.38072	-0.33557	-0.33557	n/a
maximum	1.723844	1.284018	0.903103	1.09502	1.09502	n/a
median	0	0.001372	0.046654	0.051878	0.051878	n/a
Positive return (%)	47.54098	50.81967	54.09836	55.7377	55.7377	n/a
<b>Industry_9 months formation period</b>				<b>multiplier=1</b>		
<b>Mean excess return</b>	<b>0.112776</b>	<b>0.126663</b>	<b>0.090845</b>	<b>0.101259</b>	<b>0.078732</b>	<b>0.079375</b>
Standard deviation	0.348099	0.292303	0.181082	0.134369	0.105956	0.111148
t statistics	2.467335	3.300114	3.820668	5.739173	5.659	5.438697
skewness	1.966238	1.50226	1.158053	0.509371	0.018404	-0.05324
kurtosis	10.17629	6.552514	4.925604	3.165071	2.447389	4.007446
minimum	-0.50763	-0.40135	-0.32324	-0.14455	-0.17274	-0.19226
maximum	1.758226	1.306812	0.715919	0.483831	0.321565	0.431479
median	0.073911	0.040712	0.070248	0.080389	0.076307	0.081918
Positive return (%)	56.89655	62.06897	68.96552	77.58621	72.41379	79.31034
<b>Mining_12 months formation period</b>				<b>multiplier=3</b>		
<b>Mean excess return</b>	<b>0.045731</b>	<b>0.039736</b>	<b>0.021902</b>	<b>0.022521</b>	<b>n/a</b>	<b>n/a</b>
Standard deviation	0.28254	0.311543	0.247889	0.248032	n/a	n/a
t statistics	1.200375	0.945912	0.655239	0.673387	n/a	n/a
skewness	2.102152	2.162191	2.228962	2.217557	n/a	n/a
kurtosis	8.717843	9.302693	8.890309	8.847474	n/a	n/a
minimum	-0.50474	-0.52699	-0.3442	-0.3442	n/a	n/a
maximum	1.105169	1.357857	0.968155	0.968155	n/a	n/a
median	0	0	0	0	n/a	n/a
Positive return (%)	23.63636	20	18.18182	18.18182	n/a	n/a

Summary statistics for the excess return distribution for pairs trading from the cointegration approach over the six month.<sup>10</sup> Pairs-trading portfolios include all stocks and stocks from different sectors. Here, we choose the optimal strategy that will get the highest excess return for different

<sup>10</sup> We get an extremely high excess return for pairs from all universes with the strategy of 6 months formation period and a 3 multiplier of the standard deviation. The top 5 pairs return is 44.65%, while for the rest of the multiple portfolios, the returns are all lower than 9%. We think this extremely high return is caused by a one-time event.

sectors. We trade according to the rule that opens a position in a pair at the end of the day when prices of the stocks in the pair diverge by multiplier-historical standard deviation. The “top n” portfolios include the  $n$  pairs with the least market factor spread.

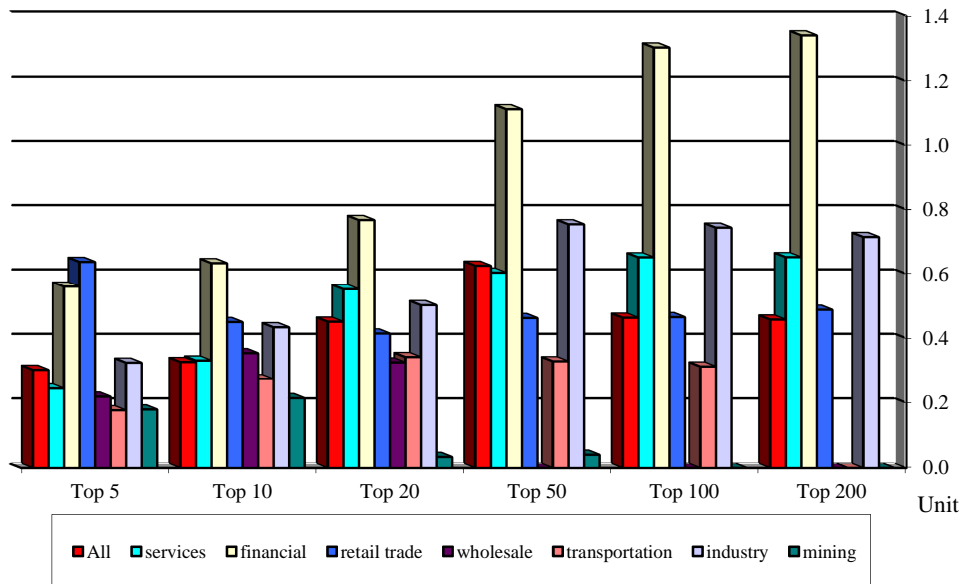
Figure 1-5: Optimal excess return - cointegration approach with transaction cost



### 1.4.3.2 Information ratio

The industry-restricted strategy provides a higher return. This result makes us wonder if this higher profit occurs at the cost of higher risk. Figure 1-6 shows the information ratios for different sectors. We find that the pairs from the services, financial, retail trade, and industry sectors have higher information ratios than those for the unrestricted pairs.

Figure 1-6: Optimal information ratio- cointegration approach with transaction cost



In summary, when we consider both return and risk, restricted pairs are better than unrestricted for the cointegration approach with transaction cost adjustment. This result also occurs when we apply the one day later rule.

#### 1.4.4 Distance approach vs. cointegration approach

Here, we simply compare the distance approach and the cointegration approach. When we do the comparing, we just consider the unrestricted pairs in the distance approach and the restricted pairs in the cointegration approach net of transaction cost. From Table 1-1 and Table 1-3, we can compare the excess return from the two approaches. The pairs-trading return from the cointegration approach in the financial and services sectors are higher than the return from the distance approach. But in the cointegration approach, the standard deviation is much higher than the one in the distance approach, which indicates that higher profit in the cointegration approach is at the cost of higher volatility. Comparing Figure 1-2 and Figure 1-6, higher information ratio is found from the distance approach compared to the cointegration approach. Therefore, for those who only focus on higher excess return, and consider using different portfolios to diversify the risk, the cointegration approach is a better choice. And for those looking for

higher risk adjusted returns and simply apply the pairs-trading strategy, the distance approach a better choice.

#### **1.4.5 What determines the returns in the distance approach?**

From the distance approach, after considering the transaction costs, we find that unrestricted strategy with 2770 candidate stocks is better than the restricted strategy. In all restricted pairs, pairs in financial and industry sectors, with 535 stocks and 1153 stocks respectively, perform better in both excess return and information ratio. For the remaining sectors (which do not perform well), fewer candidate stocks exist. This raises the question as to whether the number of candidate stocks is related to the performance of pairs trading in the distance approach. If a positive relationship exists, then in the distance approach the more stocks that are included the higher the profit should be. We randomly draw 500, 1000, and 2000 stocks from 2770 stocks, 5 times each and compare the pairs-trading results. To simplify the comparison, we apply the 12 formation period, 6 trading period and 2 multiplier trading strategy used by the Gatev et al. (2006). The excess returns are presented in Figure 1-7. The orange bars represent the 500 draws of pairs; the purple and blue bars represent the 1000 draws; and the green and yellow bars represent the 2000 draws. The interesting result is that the larger draws have the higher excess return for the top 5, top 10, and top 20 pairs portfolios, while the lower draws have a slightly higher return for the top 50, top 100, and top 200 pairs portfolio. The higher returns for the top 5 and top 10 pairs indicate better matched pairs when more stocks are added in as candidates. The higher return for the top 100 and top 200 pairs comes from the higher standard deviations that result from the imperfectly matched pairs, which may create higher return once two stocks converge. Figure 1-8 reveals the information ratio of the different draws. Clearly higher draws have a higher information ratio for top 5, top 10, top 20, top 50 and top 100 pairs. Overall, the random draw results show a positive relationship exists between the profit and the number of candidate stocks.

Figure 1-7: Excess return for 500, 1000, and 2000 draws

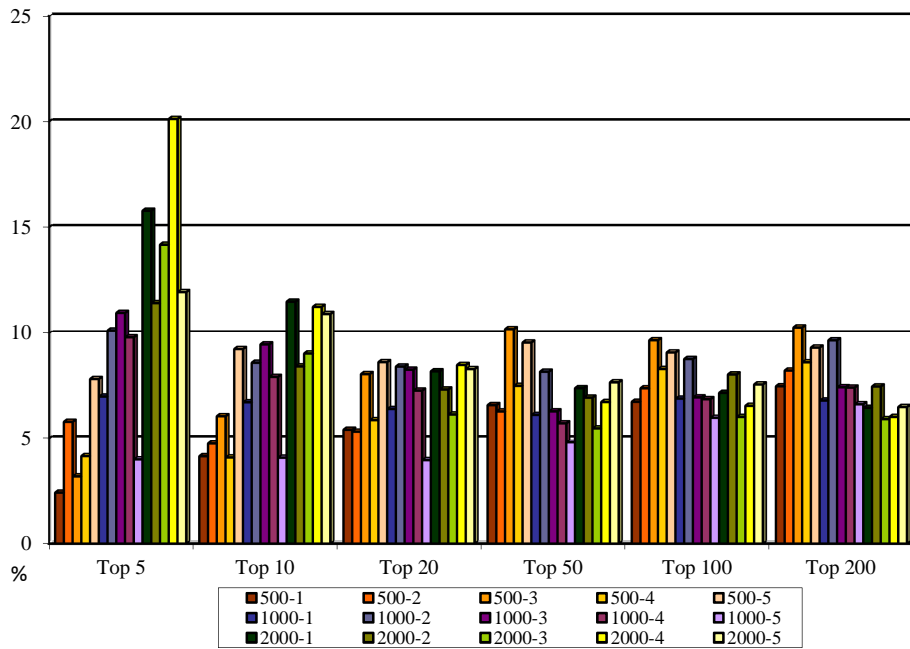
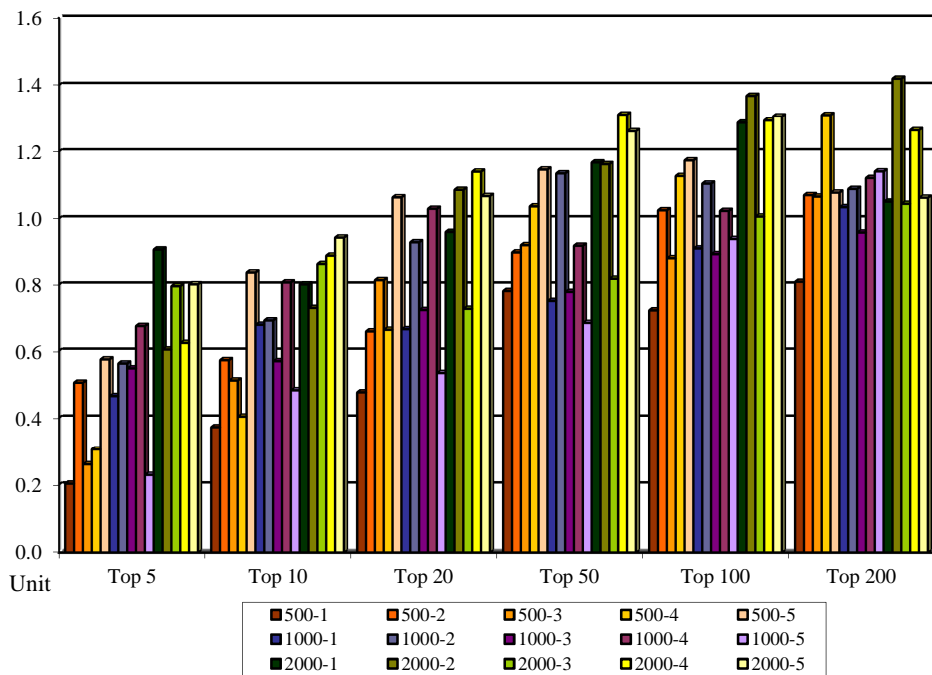


Figure 1-8: Information ratio for 500, 1000, and 2000 draws



## 1.5 Conclusion

This chapter studied alternative techniques for identifying stock pairs in a pairs-trading strategy. We considered two main techniques: the distance approach and the cointegration approach. Each of these techniques was evaluated when pairs were selected within the same industry and when pairs were selected from the broad universe of stocks. We found that for the distance approach, unrestricted pairs were preferred to restricted pairs and that for cointegration approach, restricted pairs worked better, especially for the service, financial, and retail trade sectors. The cointegration approach yields a higher excess return than the distance approach at the cost of high volatility. Therefore, the more risk-averse investors might prefer the distance approach, and the less risk-averse investors might like the cointegration approach. In addition, we found that for distance approach, a positive relationship exists between the profitability and the number of candidate stocks.

Future studies can improve upon this study in the following ways: first, as in other studies, we ignore possible restrictions and the costs of shorting stocks. This factor can be expected to reduce the candidate stocks and lower the returns from pairs trading. Second, using higher frequency data would provide more accurate results. Third, finding the optimal trading strategy is also important for pairs trading.



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## **Chapter 2: Measuring Monetary Policy in China**

### **2.1 Introduction**

According to the “impossible trinity”, an independent monetary policy cannot be pursued in an open economy with a fixed exchange rate and perfect capital mobility. The feasible policies are any two combinations of the three conditions—free capital mobility, fixed exchange rate, and independent monetary policy. It is probably fair to say that opting for a flexible exchange rate, relatively free capital mobility, and an independent monetary policy has become increasingly fashionable. The notable exceptions include China, which is the world’s second-largest economy. Guided partly by the desire to maintain some control over monetary policy and partly by the desire to target the exchange rate, the Chinese authorities have relied on a complicated mix of controls over international capital movements. Nevertheless, some types of capital flows, e.g., inward and outward foreign direct investment (FDI) as well as some categories of portfolio investment, remain considerably mobile. In this context, a natural question is how much control does China in fact have over domestic monetary policy? In a broad sense, the answer to this question is the subject of this chapter.

The goal of this chapter is to construct a scheme for measuring of monetary policy shocks and their real effects by paying particular attention to the broader policy framework in place in China. We propose a structural specification scheme that centered on the central bank’s balance sheet and the money supply system in China. The questions addressed include whether China has an effective monetary policy and which monetary instruments are effective.

The main analytical framework used in this chapter is that of the structural vector autoregression (SVAR) model and the factor augmented structural vector autoregression (FASVAR) model. The SVAR model is based on the simpler vector autoregression (VAR) framework. In the standard VAR model, the residuals are often contemporaneously correlated across equations in the VAR. This feature presents a well-known problem when conducting impulse response

analysis—the effects of a shock to the money supply—because the results can depend greatly on the order of the variables in the VAR. A technique that overcomes this problem is SVAR: a structure is imposed on the reduced-form residuals in order to identify structural (i.e., orthogonal) innovations to the variables in the VAR. We also go a step further by estimating a factor-augmented SVAR (referred to as “FASVAR”). This fairly recent idea incorporates a factor into the SVAR framework where the factor summarizes information contained in a large set of economic variables. The motivation for doing so is that, in determining monetary policy, central bankers often consider a very wide range of economic variables. The FASVAR is a parsimonious method to capture this idea in a SVAR framework.

The rest of the chapter is organized as follows. Section 2.2 introduces China’s economy. Section 2.3 is the literature review. Section 2.4 introduces the VAR model, SVAR model, and FASVAR model and estimation results. Section 2.5 concludes.

## **2.2 The Chinese monetary policy framework**

### **2.2.1 History**

Before 1978, China’s production and resource allocation were planned by the government. The deficient resources were imported and the exports of excess goods were used to finance the imports. The domestic economy was largely dominated by state-owned companies. Banks made financial decisions based on government orders rather than profit. In 1978, economic reforms started and China underwent substantial decentralization in both industry and banking. Now, a market-oriented system has replaced the central planning system.

### **2.2.2 Exchange rate**

China has an inflexible (nominal) exchange rate policy. From 1994 to 2005, the Renminbi (RMB) was pegged to the US dollar. After the Asian financial crisis, the daily range of the exchange rate fluctuation was narrowed to

the extent of a virtual fixed exchange rate. In 2005, China announced that it would peg the RMB against a basket of unannounced currencies, apparently with the aim of allowing more flexibility in the exchange rate.

### **2.2.3 Capital movements**

International capital movements in China are restricted by the State Administration of Foreign Exchange (SAFE). SAFE is under the leader of both the People's Bank of China (PBC) and State Council. The capital controls in China are aimed at the quantitative limitation and the direction of financial flows between China and foreign countries. The restrictions are mainly focused on capital outflow, although significant restrictions are made on some types of inflows. But in recent years, given the high foreign exchange reserves, the capital account has been gradually liberalized.

### **2.2.4 Targets of monetary policy**

In 1994, the PBC announced three money supply indicators: M0, M1 and M2. In 1996, the PBC announced M2 as the main intermediate target. The theoretical assumption about China's monetary policy is that the GDP growth rate and the inflation rate are correlated with the money supply and that the money supply is determined to a large extent by the PBC. Since 1994, Chinese monetary policy has had three ultimate targets: "inflation target, economic growth target and exchange rate target" (Geiger, 2006); i.e., the PBC has aimed for a policy that realizes price stability, promotes economic growth and maintains the de facto pegged exchange rate system.<sup>11</sup>

### **2.2.5 Monetary policy instruments**

"The major monetary policy instruments include: open market operation, reserve requirement, interest rate policy, re-lending and rediscount, and credit policy." (PBC, n.d. (b))

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<sup>11</sup> "The objective of the monetary policy is to maintain the stability of the value of the currency and thereby promote economic growth." (PBC, n.d. (a)).

### **2.2.5.1 Open market operations**

In 1993, Open market operations were introduced as a monetary policy instrument to adjust the money supply. Except for trading government securities, the PBC also issues its own bonds.<sup>12</sup> The PBC withdraws base money by issuing central bank bonds and selling government securities held by the PBC, and increases base money by undertaking the opposite transactions.

### **2.2.5.2 Reserve requirements**

Since 1984, the PBC has used minimum reserve requirements to adjust financial sector liquidity. Recently, the changes in the reserve requirement have been aimed mainly at fighting inflation. Both the minimum and excess reserves held at the PBC are interest-bearing. The PBC argues that the interest rate on reserves is helpful in constituting a lower limit for the money market rate (Xie, 2004). Substantial excess reserves are on deposit at the PBC. The PBC alters the minimum reserve requirement to influence the interest rate and money multiplier.<sup>13</sup> However, compared to the other instruments, reserve requirements are seldom used.

### **2.2.5.3 Interest rate**

The interest rate here refers to the deposit and lending rates. In China, interest rate liberalization began in 1993 and has not been completely finished yet. Liberalization is closer to complete in wholesale transactions that include the bond and interbank markets. In retail transactions, a floor exists for the lending rate, and a ceiling exists for the deposit rate. The benchmark lending and deposit rates of the PBC represent the administrative orders from the monetary authority. Through these benchmark rates, the PBC controls the credit flows to the non-banking sector and adjusts the tempo in the real sectors. Although commercial

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<sup>12</sup> Once the PBC has determined to issue bonds, it will ask commercial banks to report on demand for those bonds.

<sup>13</sup> The money supply is the product of the base money and money multiplier. Theoretically, money multiplier =  $\frac{(1+c)}{(c+r)}$ . We assume that people hold a constant fraction of their deposits as cash;  $c$  is the currency-deposit ratio, and  $r$  is the reserve requirement rate.

banks can set the rate according to their own assessment, the rates must not breach the benchmark rates.

#### **2.2.5.4 Rediscount rate**

The rediscount rate was first introduced in 1988. From 1988 to 1997, the rediscount rate was set within a floating range of 5 to 10 percent below the same year benchmark lending rate. Since 2004, the rediscount rate was installed as the reference of the benchmark lending rate; i.e., the PBC sets the benchmark lending rate within an upward floating range above the rediscount rate.

The rediscount rate operates as the lending rate to commercial banks, and it influences the base money as well as the money multiplier. However, in China, because of the undeveloped commercial paper market, the real effect of the rediscount rate is limited.

### **2.2.6 The realities of the Chinese policy framework**

Under the relatively rigid exchange rate policy practiced by China, when domestic interest rates diverge from foreign rates, a tendency for capital flows occurs. Given the low interest rate in China, the capital control policy aims to prevent large private capital outflows from China. However, limiting private capital outflows, along with the enormous trade surplus for many years, has led to soaring official foreign exchange reserves. Moreover, especially since 2005 when the exchange rate regime was weakened somewhat, a sharp rise has occurred in foreign capital inflows, a significant portion of which has been labeled “hot money.”<sup>14</sup> In order to hold down the value of the RMB, the imbalance of payments requires the PBC to intervene almost daily by purchasing foreign currency. These large interventions in the foreign exchange market have given the PBC the enormous challenge of sterilizing the associated rise in the money supply. Without sterilization, the growth in the money supply would fuel an

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<sup>14</sup> “Hot money” has no formal definition. It usually refers to the flow of capital from one country to another in order to earn a short-term profit on interest rate differences or anticipated exchange rate shifts.

explosion of bank credit and broad monetary aggregates, the dangers of which include a surge in non-performing bank loans as well as domestic inflation.

## **2.3 Literature review**

### **2.3.1 Monetary policy**

Early notable studies in monetary policy include Thornton (1802), Bagehot (1873), Wicksell (1907), Fisher (1920, 1926), Simons (1936), and Friedman (1948, 1960). Barro and Gordon (1983) study activist rules that allow the policy instrument to be set conditional on the state of the economy. In their model, the practice of “discretion” with period-by-period re-optimization is preferred to a “rule”. The practical application of a discretionary policy is complicated by difficulties in defining the nature and magnitude of shock or the target value. Taylor (1993) argues that we should distinguish “rule-like” behavior from discretionary behavior in practice. “Rule-like” indicates that the central bank policy committee will enrich its consideration by accounting for the instrument setting in the formula instead of following the literal formula. McCallum (1993) argues that the monetary authority must consider the private sector’s expectation and response when designing the rule. Taylor (1983, 1993) and McCallum (1995, 1997) argue that an independent central bank can freely set the monetary policy instrument.

From the monetary-policy perspective, both moving peg and narrow band exchange rate policies should be categorized as fixed exchange rate regimes. The selection of level of the fixed exchange rate depends on the advantages and disadvantages of the exchange rate level. Based on the example of the European monetary union, Bruno (1993) argues that there are macroeconomic advantages of fixed exchange rates. McCallum (1999) states a fixed exchange rate policy is more suitable for small economies.



### 2.3.2 Model development

The VAR has been widely used as a standard instrument for policy analysis in econometrics since the critique of Sims (1980). The reduced form relates endogenous variables to lagged endogenous variables and exogenous variables. These models assume a causal ordering in the dynamic response functions: the latter variables in the ordering are assumed to have no immediate effect on the earlier variables.<sup>15</sup> The assumed-order makes the relationship between endogenous variables predetermined. Because of the ordering issue, a VAR is structurally fragile and is therefore deficient as a tool for estimating the effects of monetary policy. Brunner (2000) proposes transforming the reduced-form model (VAR) into a “structural VAR” (SVAR), which allows for contemporaneous interaction between the endogenous variables. The major advantage of SVAR is that it can deliver empirical analysis of the dynamic response of key economic variables without requiring a complete structural model of the economy. One problem of the SVAR model is that it has limited variables, whereas central banks and financial market participants may have to make decisions based on hundreds of data series. Bernanke, Boivin and Eliasch (2005) argue that the sparse information in SVAR will cause at least two potential sets of problems. First, since the information normally used is not completely reflected in the SVAR analysis, the estimation of monetary policy innovations may be incorrect. Second, SVAR’s use of a specific variable to represent a general economic concept is usually arbitrary to some degree. Moreover, SVAR assumes all the variables, are observable and usually, the “observable” measure is likely to be contaminated by collection or measurement errors. Factor Augmented SVAR (FASVAR) offers a possible solution to these problems: the idea is to pool the information of many time series, averaging away idiosyncratic variation in the individual series. The FASVAR was first used by Stock and Watson (2002) to develop a dynamic factor model to summarize the information in large data sets

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<sup>15</sup> For example,  $Y_t = [\text{GDP}, \text{CPI}, \text{MS}, \text{INT}, \text{EX}]$ . In the VAR model, the consumer price index (CPI) has no immediate effect on GDP, the money supply (MS) has no immediate effect on the CPI, the interest rate (INT) has no immediate effect on the MS, and the exchange rate (EX) has no immediate effect on the INT.

for the purpose of forecasting. Bernanke et al. (2005) show that FASVAR can improve the analysis of monetary policy.

### **2.3.3 China monetary policy**

At a very general level, two opposite schools of thoughts about China's monetary policy exist. One school argues that China has a largely independent monetary policy and there is little difficulty for the PBC to control excess liquidity (Anderson, 2004; Green, 2007). The other school argues that the pegged exchange rate has greatly reduced the ability to conduct an independent monetary policy and China should increase currency flexibility (Goldstein and Lardy, 2006; Lardy, 2006; Prasad, Rumbaugh, and Wang, 2005).

The literature contains limited works of China's monetary policy, especially works in English. Gong and Gao (2008) postulate a theoretical model of the Chinese economy under the assumptions of an open capital account and a pegged exchange rate. In the empirical literature, Xiu and Luo (2002) argue that a Taylor Rule describes fairly well the monetary policy in China. However, subsequent studies (Lu and Zhang, 2003; Liu, 2003; Ban, 2006; Zhang and Zhang, 2007) reach varying conclusions. A number of studies (Yin, Zhao, and Zhan, 2001; Xiang and Yuan, 2004; Yuan, 2006; Song and Li, 2007), in Chinese, analyze the McCallum rule for the Chinese economy. None of these studies allow for a dynamic response among variables.

Liu and Zhang (2007) evaluate China's monetary policy framework with a three-equation New Keynesian model, i.e., a Phillips curve, an IS curve, and a monetary policy reaction function. They argue that a hybrid of an interest rate rule and a money supply rule best describe monetary policy in China. Kong (2008) studies the monetary policy rule for China during 1994-2006 with modified Taylor and McCallum rules. He finds that Taylor rules are better than McCallum rules in explaining China's monetary policy.<sup>16</sup> Sun (2009) examines the autonomy

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<sup>16</sup> Taylor rule stipulates changes of the nominal interest rate in response to changes in the inflation and output. McCallum rule specifies a target for the money base given the desired rate of inflation and the growth rate of real GDP.

and effectiveness of monetary policy in China with monthly data from 1998 to 2005 using a VAR model. The researcher argues that no Granger causality exists between the foreign exchange reserves and the money supply, which indicates an autonomous monetary policy; and there is no Granger causality from the interest rate (or the money supply) to the price level and output, which indicates an ineffective monetary policy. Lauren and Maino (2007) use a VAR model to assess the effect of monetary policy on output, the exchange rate and prices. They find that changes in the interest rate have little impact on economic variables. Dichinson and Liu (2007) examine whether institutional changes have affected interactions between the real economy and monetary policy in China. They find increasing interest rate effects on output over 1984 to 1997 and non-state owned enterprises increasingly respond to monetary policy innovation. None of these papers clearly examine how economic variables (e.g., output, inflation, exchange rate etc.) respond to monetary policy shocks.

There are other papers that discuss monetary policy in China in general, but do not formulate empirical models (Goldstein and Lardy, 2004, 2006, 2007; Anderson, 2004; Prasad, Rumbaugh, and Wang, 2005; Lardy, 2006; Geiger, 2006; Goodfriend and Prasad, 2006; Gu and Zhang, 2006; Green, 2007).

## **2.4 Models and estimations**

### **2.4.1 VAR model**

The VAR model is a basic framework for measuring monetary policy shocks and their effects on macroeconomic variables. VAR is a model in which  $K$  endogenous variables over the same sample period are a linear function of  $p$  of their own lags,  $p$  of the other  $K-1$  variables and possible additional exogenous variables. The set of lagged variables is assumed to be a good proxy for the information set available to economic agents at the beginning of period  $t$ .

Let  $y_t = (y_{1t}, \dots, y_{Kt})'$  be a  $K \times 1$  vector of endogenous variables. A  $p$ -th order VAR, written VAR( $p$ ) is defined as

$$y_t = v + A_1 y_{t-1} + \cdots + A_p y_{t-p} + B_0 x_t + \cdots + B_l x_{t-l} + u_t, \quad (1)$$

where  $v$  is a  $K \times 1$  vector of parameters,  $A_i$  are  $K \times K$  coefficient matrices,  $x_t$  is  $S \times 1$  vector of exogenous variables,  $B_i$  are  $K \times S$  matrices of coefficients, and  $u_t$  is a  $K$ -dimensional process assumed to be white noise.

We can represent the VAR(p) process as a VAR(1) process by redefining the dependent variables:

$$m_t = \alpha + A m_{t-1} + B n_t + \mu_t, \quad (2)$$

$$\text{where } m_t = \begin{bmatrix} y_t \\ \vdots \\ y_{t-p+1} \end{bmatrix}, \alpha = \begin{bmatrix} v \\ 0 \\ \vdots \\ 0 \end{bmatrix}, A = \begin{bmatrix} A_1 & A_2 & \cdots & A_{p-1} & A_p \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} B_0 & \cdots & B_l \\ 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}, n_t = \begin{bmatrix} x_t \\ \vdots \\ x_{t-l} \end{bmatrix}, \mu_t = \begin{bmatrix} u_t \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

where the dimensions of the vectors  $m_t$ ,  $\alpha$ ,  $n_t$ , and  $\mu_t$  are  $KP \times 1$ , the dimension of the matrix  $A$  is  $KP \times KP$ , and the dimension of the matrix  $B$  is  $KP \times SL$ . For a given sample of the endogenous variables and sufficient pre-sample values, we can use least squares to efficiently estimate the model. Once VAR(p) has been estimated, we can do further analysis.

If the VAR is stable, i.e., if  $|\lambda| < 1$  for  $\det(A - \lambda I_{kp}) = 0$ , we can rewrite  $y_t$  as

$$y_t = \omega + \sum_{i=0}^{\infty} D_i x_{t-i} + \sum_{i=0}^{\infty} \Phi_i u_{t-i}, \quad (3)$$

where  $\omega$  is the  $K \times 1$  time-invariant mean of the process.  $D_i$  and  $\Phi_i$  are the  $K \times S$  and the  $K \times K$  matrices of parameters.  $\Phi_0 = I_k$  and  $\Phi_M$  can be computed recursively according to  $\Phi_M = \sum_{j=1}^M \Phi_{M-j} A_j$ , where  $A_j = 0$  for  $j > p$ . Equation (3) is the moving average decomposition for the stable VAR(p). This equation states that the process by which the variables in  $y_t$  fluctuate about the time-invariant mean  $\omega$  is dependent on the parameters of  $D_i$  and  $\Phi_i$ , the past history of

the exogenous variables  $x_t$ , and the innovations  $u_t, u_{t-1}, \dots$ .  $D_i$  is the dynamic-multiplier function, and  $\Phi_i$  is the simple impulse response function.

## 2.4.2 SVAR approach

### 2.4.2.1 SVAR model

SVAR is based on the VAR framework. A structure is imposed on the reduced-form residuals in order to identify structural (i.e., orthogonal) innovations to the variables in the VAR. The structure imposed on the reduced form residuals is based on economic theory and/or the realities of the economic and policy framework in a country. Usually, the estimation of a SVAR model requires two steps. The first one is to estimate the VAR model. The residuals from the VAR (the  $u$ 's) are referred to as “innovations.” Second, the innovations are regressed on themselves, by using one of several statistical procedures, and given a structural interpretation (i.e., identification) (Brunner, 2000). The identification is important. First, it has to provide enough restrictions. Second, it should reflect the actual macroeconomic reality being studied. Third, it is not necessarily based on practical theories but it should have a reasonable explanation. Theoretically, SVAR identification is achieved by imposing contemporaneous restrictions on both the structure of the economy and the stochastic structure of the model. The details of this process are discussed next.

A structural VAR with  $p$  lags is

$$C_0 y_t = \gamma + C_1 y_{t-1} + \dots + C_p y_{t-p} + E_0 x_t + \dots + E_l x_{t-l} + G \varepsilon_t, \quad (4)$$

where  $\gamma$  is a  $K \times 1$  vector of parameters,  $C_i$  are  $K \times K$  coefficient matrices,  $x_t$  is a  $S \times 1$  vector of possible exogenous variables,  $E_i$  are  $K \times S$  matrices of coefficients,  $G$  are  $K \times K$  coefficient matrices, and  $\varepsilon_t$  is a  $K \times 1$  vector of the error terms that are assumed to be white noise. By pre-multiplying the SVAR with the inverse of  $C_0$ , we get the reduced form of the VAR model:

$$y_t = C_0^{-1} \gamma + C_0^{-1} C_1 y_{t-1} + \dots + C_0^{-1} C_p y_{t-p} + C_0^{-1} E_0 x_t + \dots + C_0^{-1} E_l x_{t-l} + C_0^{-1} G \varepsilon_t. \quad (5)$$

Let  $C_0^{-1}\gamma = v$ ,  $C_0^{-1}C_i = A_i$  for  $i = 1, \dots, p$ ,  $C_0^{-1}E_j = B_j$  for  $j = 1, \dots, l$  and  $C_0^{-1}G\varepsilon = u_t$ , we obtain equation (1).

As discussed above, we can easily estimate (3), but to get (4) from the reduced form (3), we must solve a problem of identification. Here, we consider the reduced form residuals  $u_t$  as "innovations" of the economy for the current period. Being innovations, these reduced-form residuals are observable, but these observable innovations are the result of the unobservable shocks  $\varepsilon_t$  that happened in the current period. The unobservable shocks can affect more than one of the observable innovations; one innovation can be tracked from more than one of the shocks. Therefore, determining the underlying shocks is difficult because the innovation can be the result of different sets of shocks.

Consider a SVAR model of the vector  $y_t$ , which is different from equation (4):

$$C_0^* y_t = \gamma^* + C_1^* y_{t-1} + \dots + C_p^* y_{t-p} + E_0^* x_t + \dots + E_l^* x_{t-l} + G^* \varepsilon_t. \quad (6)$$

Multiply both sides of (6) by  $(C_0^*)^{-1}$  to obtain

$$y_t = (C_0^*)^{-1}\gamma^* + (C_0^*)^{-1}C_1^* y_{t-1} + \dots + (C_0^*)^{-1}C_p^* y_{t-p} + (C_0^*)^{-1}E_0^* x_t + \dots + (C_0^*)^{-1}E_l^* x_{t-l} + (C_0^*)^{-1}G^* \varepsilon_t. \quad (7)$$

Assume there exists a  $K \times K$  orthogonal matrix  $P$  such that  $C_0 = PC_0^*$ ,  $\gamma = P\gamma^*$ ,  $C_i = PC_i^*$  for  $i = 1, \dots, p$ ,  $E_j = PE_j^*$  for  $j = 1, \dots, l$  and  $G = PG^*$ . From equation (5) we get

$$y_t = (PC_0^*)^{-1}(P\gamma^*) + (PC_0^*)^{-1}(PC_1^*)y_{t-1} + \dots + (PC_0^*)^{-1}(PC_p^*)y_{t-p} + (PC_0^*)^{-1}(PE_0^*)x_t + \dots + (PC_0^*)^{-1}(PE_l^*)x_{t-l} + (PC_0^*)^{-1}(PG^*)\varepsilon_t. \quad (8)$$

Since  $(PC_0^*)^{-1} = (C_0^*)^{-1}P^{-1}$ , we have

$$y_t = (C_0^*)^{-1}(\gamma^*) + (C_0^*)^{-1}(C_1^*)y_{t-1} + \dots + (C_0^*)^{-1}(C_p^*)y_{t-p} + (C_0^*)^{-1}(E_0^*)x_t + \dots + (C_0^*)^{-1}(E_l^*)x_{t-l} + (C_0^*)^{-1}(G^*)\varepsilon_t, \quad (9)$$

which is same as equation (7). The second moment of the reduced-form VAR of the model is

$$Eu_t u_t' = \Sigma = C_0^{-1} G \Lambda G' (C_0')^{-1} = (P C_0^*)^{-1} P G^* \Lambda (P G^*)' (P C_0^*)'^{-1} = C_0^{*-1} G^* \Lambda G^{*'} (C_0^{*'})^{-1}, \quad (10)$$

where  $E \varepsilon_t \varepsilon_t' = \Lambda$ . Therefore, both structural models yield the same reduced-form representation; i.e., without identification, we are not sure which set is the optimal structural parameter.

The identification is important. Theoretically, SVAR identification is achieved by imposing contemporaneous restrictions on both the structure of the economy and the stochastic structure of the model. According to equation (4), the restrictions are placed on the elements of  $C_0$  and  $G$ :

$$Eu_t u_t' = C_0^{-1} G \Lambda G' (C_0')^{-1} = \Sigma, \quad (11)$$

where  $\Lambda = E \varepsilon_t \varepsilon_t'$  is a real symmetric matrix with a rank of  $K$ . An estimate of  $\Sigma$  can be obtained by estimating the VAR model. We have  $\hat{u}_t \hat{u}_t' = \hat{\Sigma}$ , where  $\hat{u}_t$  is the vector of the residuals obtained from estimating equation (1). At most  $\frac{K(K+1)}{2}$  unique, non-zero parameters are present in  $\hat{\Sigma}$ , while  $K^2$  parameters are in  $C_0$ ,  $K^2$  parameters in  $G$ , and  $K$  parameters are in  $\Lambda$ ; i.e., there are  $2K^2 + K$  structural parameters. Therefore, at least  $\frac{3K^2 + K}{2}$  restrictions have to be imposed on  $C_0$  and  $G$  for identification.<sup>17</sup>

#### 2.4.2.2 Identification

The SVAR model in this chapter contains nine endogenous variables: M2 growth rate (MS), foreign exchange reserves (FR), minimum reserve requirement (MR), rediscount rate (RR), net securities held by the central bank (OMO), real GDP growth rate, inflation rate (INF), nominal effective exchange rate (EX) and

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<sup>17</sup> In a basic SVAR model, it is usually assumed that  $G = I$  and that the diagonal elements of  $C_0$  are equal to unity. The left  $\frac{K(K-1)}{2}$  restrictions come from the assumption that  $C_0$  is a lower triangular.

the 6-month to 1-year loan rate (INT).<sup>18</sup> The reduced-form residuals are denoted  $\{u_{MS}, u_{FR}, u_{MR}, u_{RR}, u_{OMO}, u_{GDP}, u_{INF}, u_{EX}, u_{INT}, \}$ .

We choose M2 and a 1-year loan rate as the indicators of monetary policy.<sup>19</sup> We use USEUGDP, the sum of the European Union and U.S. real GDP as an exogenous variable.<sup>20</sup> Because the European Union and U.S. are the top two targets of China's exports and also are sources of foreign direct investment. We also use the U.S. Treasury bill rate and U.S. bond rate as exogenous variables.

The identification scheme in this chapter uses information from the items on the balance sheet of the PBC and money supply system in China. The money supply is important because it has served as an intermediate target in China. From the well-known relation, money supply = money multiplier  $\times$  base money. The money multiplier is a function of the minimum reserve requirement set by the PBC. According to the PBC balance sheet: base money = foreign exchange reserves (denominated in domestic currency) + private domestic credit + government securities held by the central bank - securities issued by the PBC. Here, private domestic credit reflects the refinance business, which is affected by the PBC rediscount rate. Net value of government securities held by the central bank and securities issued by the central bank reflects open market operations. Therefore, the innovation of money supply (MS) in China is affected by the innovation in the minimum reserve requirement (MR), foreign exchange reserves (FR), rediscount rate (RR), and open market operations.<sup>21</sup> We write the associated identity as

$$(a) u_{MS} = \delta_{FR}u_{FR} + \delta_{MR}u_{MR} + \delta_{RR}u_{RR} + \delta_{OMO}u_{OMO} + v_{MS},$$

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<sup>18</sup> Net securities held by the central bank = government securities held by the central bank - securities issued by the PBC. When OMO is negative, the value of government securities held by the central bank is less than the value of bonds issued by the PBC.

<sup>19</sup> The literature reveals a debate about whether the credit supply or money supply should be used as the indicator of monetary policy in China. So far, no consensus on this issue has been reached. Because we cannot obtain credit supply data prior to 2010, we choose to use the money supply as the indicator in this chapter.

<sup>20</sup> European Union real GDP is based on the 2005 European Union price index. U.S. real GDP is based on the 2005 U.S price index.

<sup>21</sup> Given that the loan rate is also considered a factor that affects the money supply, we have tried to use the loan rate to replace the rediscount rate. This method yields similar results to those obtained by using the rediscount rate.



$v_{MS}$  denotes the “structural” money supply shock, and the  $u$ ’s are the reduced-form innovations from the VAR. Equation (a) describes a key relationship to identify. To precisely trace the effect of monetary policy in China, we need more conditions to account for the behavior of the external sector of the economy and markets for the items in the money supply system.

According to SAFE, foreign exchange reserves mainly derive from current account and capital account balances. The main component of the current account is international trade (i.e., net exports) and the main component of the capital account is FDI. The exchange rate has two effects on China’s international trade. The first one is the direct effect: there is a negative causality from the exchange rate (US dollar per RMB) to exports—an appreciation of the RMB will reduce foreign demand for Chinese goods. Thorbecke and Zhang (2009) argue that an appreciation of the RMB will reduce exports of labor-intensive goods from China. However, how significant the effect of an appreciation of the RMB is controversial. It is possible that the effect of the appreciation can be offset to some extent by a reduction in costs of import inputs. Marquez and Schindler (2007) and Thorbecke and Smith (2010) find that an RMB appreciation causes a larger decline in ordinary exports than in processed exports.<sup>22</sup> Given the large share of processed exports in total exports, a revalued RMB may not lead to a significant drop in exports. The second effect of the exchange rate lies in its rigidity rather than in its movements; i.e., a pegged exchange rate reduces exchange rate risk. China has one of the most stable exchange rates in the world. Low exchange rate volatility may attract foreign corporations.

The literature reveals wide discussion about the determinants of FDI. Generally speaking, there are four key factors: GDP, inflation, exchange rate and interest rates. GDP as the determinant for the market-seeking behavior of multinational corporations is one of the most important factors that attract large FDI inflows to China. Choe (2003) finds there is strong causality from economic growth to FDI. Basu, Chakraborty and Reagle (2003) find a bidirectional

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<sup>22</sup> Ordinary exports are products produced with local inputs.

causality between GDP and FDI for more open economies, and a unidirectional causality from GDP to FDI for less open economies. Hsiao and Hsiao (2006) discover that GDP strongly contributes to FDI inflows to China. High inflation may increase the cost of capital and reflects a country's macroeconomic instability, which may also indicate a political instability. An appreciation of the RMB will reduce the relative wealth of foreign investors and increase the relative labor costs in China, which may cause a reduction in inward FDI. Kok and Ersoy (2009) find that per capita GDP growth has a positive influence on FDI, while inflation has a negative influence. Majeed and Ahmad (2009) argue that GDP and GDP growth have significantly positive effects on FDI and the exchange rate and inflation rate have significantly negative effects on FDI. China has a relatively low lending rate, which is an attractive factor for foreign investors, given the higher financing cost in some other countries.

According to interest rate parity, interest rate differentials are offset by expected exchange rate changes. However, in China, because of the inflexible exchange rate, the local interest rate is an important factor in determining capital inflows. The PBC has sought to keep local interest rates low in order to limit such inflows. Recently, higher interest rates and appreciation of the RMB have raised the issue of the inflow of hot money. From 2005 to 2008, the RMB has appreciated in value by approximately 22%. Li (2008) argues that "hot money" speculators can obtain over 10% profit with little risk from these hot money flows into China. Overall, the innovation in foreign exchange reserves in China is affected by innovations in the GDP, inflation rate, exchange rate, and the interest rate. So we write the following condition for foreign exchange reserves:

$$(b) u_{FR} = \alpha_{GDP}u_{GDP} + \alpha_{INF}u_{INF} + \alpha_{EX}u_{EX} + \alpha_{INT}u_{INT} + v_{FR},$$

where  $v_{FR}$  is the foreign exchange market structural shock.

China's monetary policy has three final targets: economic growth, inflation, and the exchange rate. Therefore, the minimum reserve requirement, rediscount rate and open market operations as the instruments of the monetary policy should be the response to the shocks in inflation, GDP and exchange rate:

$$(c) u_{MR} = \Phi_{GDP}v_{GDP} + \Phi_{INF}v_{INF} + \Phi_{EX}v_{EX} + v_{MR},$$

$$(d) u_{RR} = \beta_{GDP}v_{GDP} + \beta_{INF}v_{INF} + \beta_{EX}v_{EX} + v_{RR},$$

$$(e) u_{OMO} = \rho_{GDP}v_{GDP} + \rho_{INF}v_{INF} + \rho_{EX}v_{EX} + v_{OMO},$$

where  $v_{MR}$ ,  $v_{RR}$ , and  $v_{OMO}$  denote structural monetary policy shocks.

GDP can be affected by many factors. Given the endogenous variables used in this model, we consider four factors: money supply, inflation, exchange rate, and interest rates. The motivation for using these variables is as follows. First, Burdekin and Siklos (2008) find that the money supply in China is related to GDP growth. Second, the Phillips curve implies a positive, short-term relationship between inflation and output. Recent works, primarily in growth economics, suggest that inflation may be harmful to output growth at longer horizons (Barro, 1991; Fischer, 1993; Bullard and Keating, 1995; Gylfason, 1998; Michener, 1998). In addition, several papers have documented a “threshold effect”: inflation rates below the threshold are neutral or possibly beneficial to output growth, whereas inflation rates over the threshold are harmful to output growth (Bruno and Easterly, 1998; Mubarik, 2005; Khan and Senhadji, 2001). Khan and Senhadji (2001) find that the average level of threshold for developing countries is about 11 percent. Third, in China, the exchange rate affects GDP through net exports. In 1996, exports amounted to 17% of GDP. In 2005, the number increased to 37% of GDP. Even in 2009, given the worldwide recession, exports still amounted to 23% of GDP in China. From 1980 to 2008, Chinese exports have increased at an average rate of 12.4%, while during the same period, global exports only expanded at 4.0%. Fourth, the interest rate is widely used as a monetary policy instrument because of its effect on inflation and output. Different from other monetary policy instruments in China, the interest rate affects GDP both directly and indirectly through the money supply. In sum, the innovation in GDP is affected by innovations in the money supply, inflation rate, exchange rate, and the interest rate:

$$(f) u_{GDP} = \lambda_{MS}u_{MS} + \lambda_{INF}u_{INF} + \lambda_{EX}u_{EX} + \lambda_{INT}u_{INT} + v_{GDP},$$

where  $v_{GDP}$  is the structural demand shock.

It is widely known that high rates of inflation are caused by excessive growth of the money supply. Geiger (2006) states that the PBC manipulates the monetary growth rate to influence the inflation rate. Also according to New Keynesian theory, when GDP grows too fast, the inflation rate will accelerate as suppliers increase their prices. Therefore, we have the identity condition for innovations in inflation rate:

$$(g) u_{INF} = \theta_{MS}u_{MS} + \theta_{GDP}u_{GDP} + v_{INF},$$

where  $v_{INF}$  denotes the structural cost push shock.

China adopts a pegged exchange rate. Therefore for the nominal exchange rate, the most important factor is the inflation rate. Even though China's exchange rate is still pegged, it is now more flexible than before. The exchange rate had substantial movements in the past few years and the PBC has the incentive to adjust the nominal exchange rate in response to the changes in the inflation rate. Researchers argue that China can increase the exchange rate to lower the inflation rate. A strong local currency reduces the cost of imported products, which has a negative effect on inflation. Also because of international competitions, domestic producers may cut their prices in response to an appreciation in RMB. Thus, we write

$$(h) u_{EX} = \gamma_{INF}u_{INF} + v_{EX},$$

where  $v_{EX}$  denotes structural shocks to the currency peg.

The interest rate is a monetary policy instrument. Its innovation should respond to the shocks to GDP, inflation rate and the exchange rate. Since we use the nominal interest rate, it should also be affected by the innovation in inflation rate. Therefore, we have the identity condition for innovations in interest rate:

$$(i) u_{INT} = \kappa_{GDP}v_{GDP} + \kappa_{INF}v_{INF} + \kappa_{EX}v_{EX} + \kappa_{INF2}u_{INF} + v_{INT},$$

where  $v_{INT}$  is a structural monetary policy shock.

In a matrix form, equation (a)-(i) can be expressed as

$$\begin{bmatrix}
1 & -\delta_{RF} & -\delta_{MR} & -\delta_{RR} & -\delta_{OMO} & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -\alpha_{GDP} & -\alpha_{INF} & -\alpha_{EX} & -\alpha_{INT} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-\lambda_{MS} & 0 & 0 & 0 & 0 & 1 & -\lambda_{INF} & -\lambda_{EX} & \lambda_{INT} \\
-\theta_{MS} & 0 & 0 & 0 & 0 & -\theta_{GDP} & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\gamma_{INF} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\kappa_{INF2} & 0 & 1
\end{bmatrix}
\times
\begin{bmatrix}
u_{MS} \\
u_{FR} \\
u_{MR} \\
u_{RR} \\
u_{OMO} \\
u_{GDP} \\
u_{INF} \\
u_{EX} \\
u_{INT}
\end{bmatrix}$$

$$=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & \Phi_{GDP} & \Phi_{INF} & \Phi_{EX} & 0 \\
0 & 0 & 0 & 1 & \beta_{GDP} & \beta_{INF} & \beta_{EX} & 0 \\
0 & 0 & 0 & 0 & 1 & \rho_{GDP} & \rho_{INF} & \rho_{EX} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & \kappa_{GDP} & \kappa_{INF} & \kappa_{EX} & 1
\end{bmatrix}
\times
\begin{bmatrix}
v_{MS} \\
v_{FR} \\
v_{MR} \\
v_{RR} \\
v_{OMO} \\
v_{GDP} \\
v_{INF} \\
v_{EX} \\
v_{INT}
\end{bmatrix}. \tag{12}$$

### 2.4.2.3 Data

In this chapter, we use quarterly data from 1994:Q1 to 2010:Q3. These data were collected from the National Bureau of Statistics of China, the PBC, the SAFE, the Bank for International Settlements, the International Monetary Fund, Eurostat, and Federal Reserve Economic Data. We set 1994 as the base year to calculate the inflation rate and real GDP. All data are detailed in Appendix A. All tests are presented in Appendix B. First, we test for a stationary process by using the ADF test, Generalized Least Squares Dickey-Fuller (DF-GLS) test and Phillips-Perron (PP) test. Except for the inflation rate, we cannot reject a unit root at the 1% significance level and differences of these variables are found to be stationary. Sims (1980) and Sims, Stock, and Watson (1990) recommend against differencing to induce a stationary process if variables are co-integrated. These researchers argue that the purpose of the estimation is to examine the interrelationships among the variables through the impulse response functions, rather than the significance of individual coefficients. If the related variables are not co-integrated, then using the first difference is preferable. We find co-integration between variables, but the AR roots table shows that if all variables

are in levels, the SVAR will not satisfy the stability condition. This problem may cause invalid results. Therefore, we use the GDP growth rate and M2 growth rate instead of levels. The Schwarz Criteria (SC) suggests one lag; the Likelihood-ratio test (LR), Final prediction error (FPF), and Hannan Quinn (HQ) suggest two lags; and Akaike Information Criterion (AIC) suggests four lags should be used in the SVAR model. According to the test of the serial autocorrelation and normality of the residuals, we use one lag in the SVAR model.

#### **2.4.2.4 Impulse response analysis from the SVAR estimation**

We use the impulse response function to evaluate monetary policy. The impulse response function represents the dynamic response of a particular variable in the system to a positive shock (error) in one of the equations. Any variable can be expressed as a combination of the current and all past errors in the equations, with weights given by the impulse responses.

Figure 2-1 presents the impulse response of the variables to a one-standard-deviation money supply shock.<sup>23</sup> The shock causes a short-term hike in GDP growth rate with a one-period-response lag. For a one-standard-deviation shock in M2 growth rate, which is about 1%, the GDP growth rate rises by about 0.15%. Cochrane (1994) shows that for the U.S., a one-standard-deviation shock in M2 (0.5%) causes the GDP to rise by about 0.5%. The relatively small effect found for China may be explained by China's high M2/GDP ratio. In 2009, China's M2/GDP ratio was 1.78, while the United States' ratio was only 0.59.<sup>24</sup> The increase in foreign exchange reserves peaks at 0.5% in the sixth period, after that foreign exchange reserves increase at a decreasing rate. This result can be simply explained by the response path of exchange rate. After the money supply shock, exchange rate decreases. The lower exchange rate will attract more foreign direct investment and increase the trade surplus, as imports decrease and exports increase. The effect on hot money is uncertain. The shock could decrease the hot money inflow by undermining speculators or it could attract more speculators, as

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<sup>23</sup> Figure shows the percentage changes of variables in response to the money supply shock.

<sup>24</sup> 0.59 is a very high ratio in U.S. history. Because of the recession in 2008, the United States expanded its money supply to stimulate the economy in 2009.

they will expect that after the shock, the exchange rate will gradually go back to its original level. The response of the inflation rate is puzzling. After the money supply shock, inflation rate decreases.

Figure 2-1: SVAR response to money supply shock

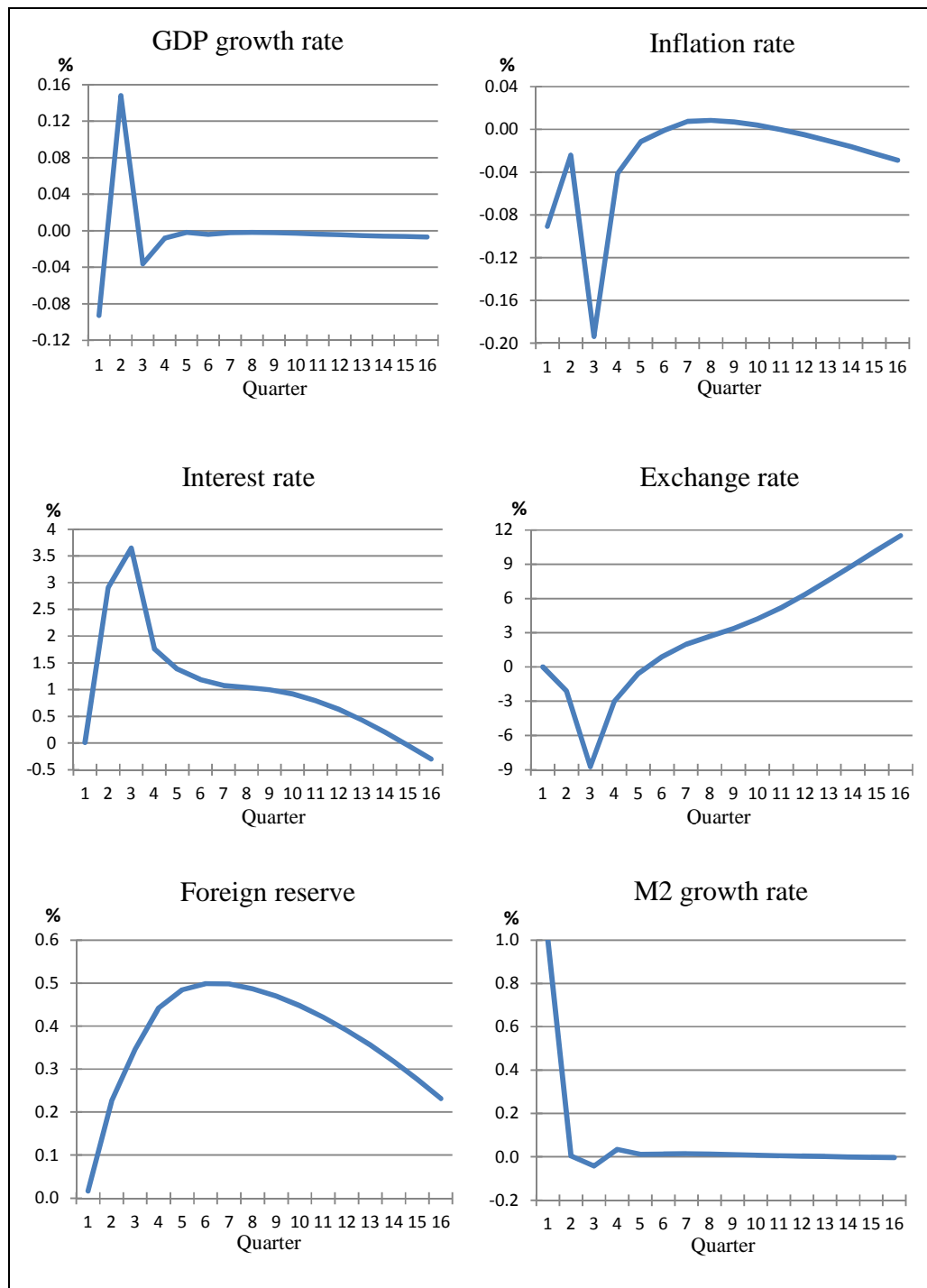
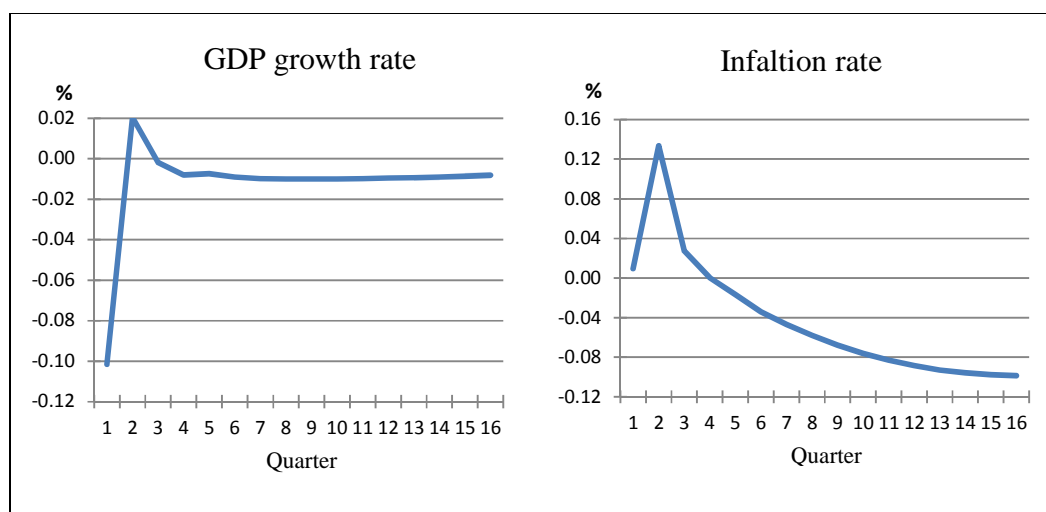
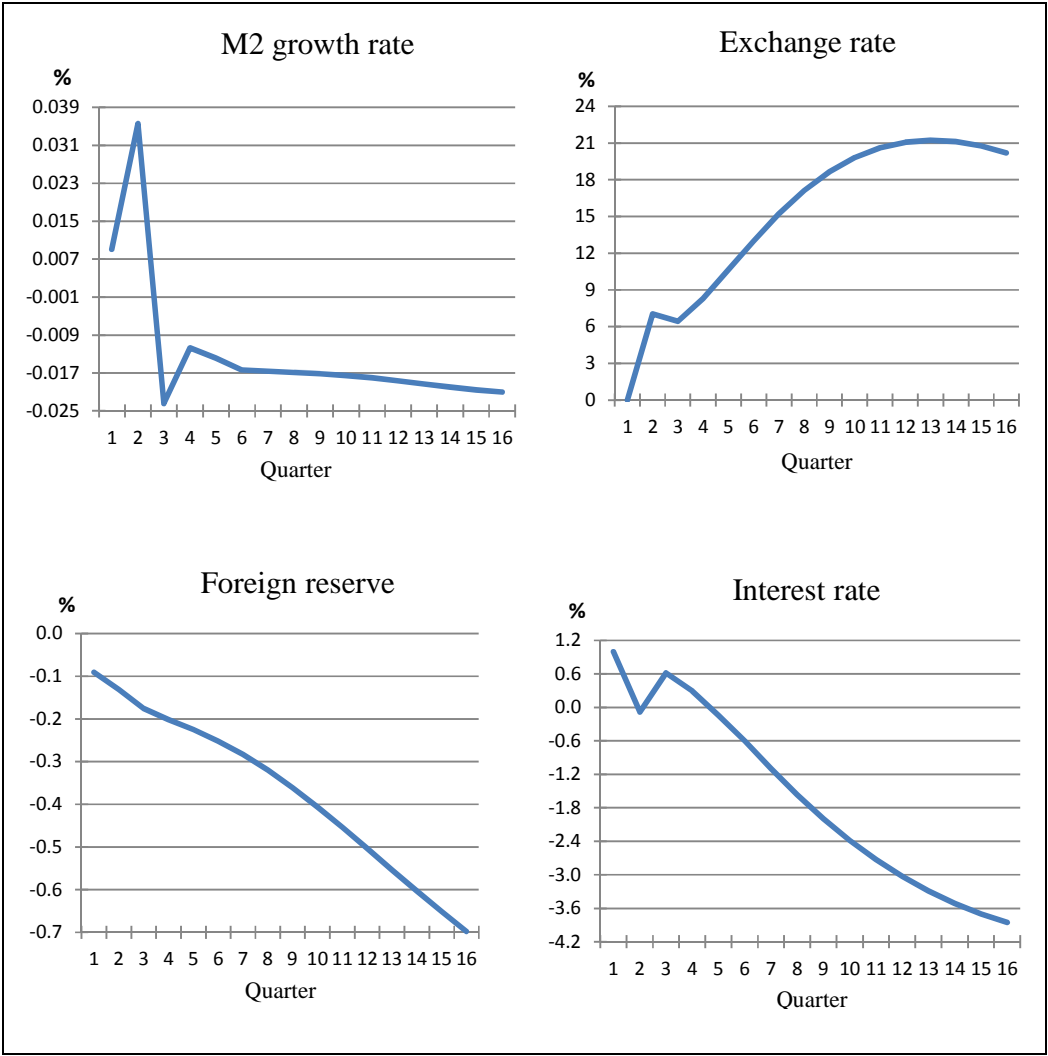


Figure 2-2 shows the response of the variables to a one-standard-deviation interest rate shock, which is about 1%. An unexpected rise in the interest rate is followed by a 2-period rise in the money supply, peaking at 0.036% in the second period and then followed by a protracted decrease in the money supply. After the interest rate shock, a 4-period rise occurs in the inflation rate, peaking at 0.13% in the second period and then followed by a protracted decrease. A long-term increase in the exchange rate occurs in response to the interest rate shock. The movements in the exchange rate are much larger than we expected for, it peaks at 21%. Bernanke et al. (2005) shows that for the U.S., a one-standard-deviation shock in the federal fund rate (0.1%) causes the exchange rate to rise by about 0.1%. GDP growth rate falls in the initial period, but quickly rebounds. Foreign exchange reserves have a protracted decline after the interest rate shock. We can explain these responses as follows. As the interest rate increases, the exchange rate rises. A stronger RMB raises the price of China's exports, diminishes China's attractiveness as a destination for FDI, and results in a slowdown of the inflow or even the outflow of hot money. All of these results cause a decline in the foreign exchange reserves and a decrease in GDP growth. However, as reveals, after the initial tumble, GDP growth rate almost recovers in the third period to its original level.

Figure 2-2: SVAR response to interest rate shock







## 2.4.3 FASVAR approach

### 2.4.3.1 FASVAR model

FASVAR combines the SVAR analysis with factor analysis. FASVAR uses an estimated factor to summarize the information in a variety of economic variables by pooling the information to average away idiosyncratic variation in the individual series.<sup>25</sup> In this chapter, we follow the method introduced by Stock and Watson (1998, 1999, 2002), which estimates the factors by using static principal components. It is a two-step procedure. In the first step, the augment factors are estimated by using the first principal components of the unobserved variables. In the second step, we do SVAR with the unobserved variables replaced by the corresponding augment factors. The FASVAR model is the SVAR model with some compressed data, which reduces the number of dimensions, hopefully without much loss of information.

Let  $m_t$  be a vector of the economic variables, with dimension  $N \times 1$ .  $m_t$  contains perhaps many economic time series.  $y_t$  is an  $M \times 1$  vector of the observable macroeconomic variables. Some unobservable fundamental forces affect the dynamics of  $m_t$ , which can be represented by a  $Q \times 1$  vector of factors  $f_t$ , such that

$$m_t = \Lambda f_t + e_t, \quad (13)$$

where  $\Lambda$  is a  $N \times Q$  matrix of factor loadings, and  $e_t \sim i.i.d N(0, R')$  is a  $N \times 1$  vector of the error term. Take a partition of  $m_t$ , denoted  $m_t^1, m_t^2, \dots, m_t^l$ , where  $m_t^i$  is a vector with dimension  $N_i \times 1$  and  $\sum_i N_i = N$ . Each of the vectors  $m_t^i$  is explained by only some of the elements of vector  $f_t$ . That is, a partition of  $f_t$ , say  $f_t^1, f_t^2, \dots, f_t^l$  occurs, where  $f_t^i$  is a vector with dimension  $Q_i \times 1$  and  $\sum_i Q_i = Q$ ,  $Q_i < N_i$ . Therefore, we can transfer (13) into

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<sup>25</sup> Here, like Belviso and Milani (2006), we assume that each set of series is explained by exactly one factor.

$$\begin{bmatrix} m_t^1 \\ \vdots \\ m_t^I \end{bmatrix} = \begin{bmatrix} \Lambda^1 & 0 & \cdots & 0 \\ 0 & \Lambda^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \Lambda^I \end{bmatrix} \times \begin{bmatrix} f_t^1 \\ \vdots \\ f_t^I \end{bmatrix} + \begin{bmatrix} e_t^1 \\ \vdots \\ e_t^I \end{bmatrix}. \quad (14)$$

Equation (13) states that the large number of variables in  $m_t$  are affected by some common factors  $f_t$ . To simplify the problem, in this chapter, we assume that  $m_t^i$  is explained by only one factor; i.e.,  $Q_i = 1$  for all  $i$ . Thus,  $f_t$  become a vector with dimension  $I \times 1$ , and  $\Lambda$  becomes a  $N \times I$  matrix. Then the FASVAR model becomes

$$\begin{bmatrix} m_t \\ y_t \end{bmatrix} = \begin{bmatrix} \Lambda & 0 \\ 0 & I_M \end{bmatrix} \times \begin{bmatrix} f_t \\ y_t \end{bmatrix} + \begin{bmatrix} e_t \\ 0 \end{bmatrix}, \quad (15)$$

and

$$Z \begin{bmatrix} f_t \\ y_t \end{bmatrix} = \Delta(L) \begin{bmatrix} f_{t-1} \\ y_{t-1} \end{bmatrix} + \tau_t, \quad (16)$$

where  $Z$  is a  $(M + I) \times (M + I)$  matrix,  $\Delta(L)$  is a  $(M + I) \times (M + I)P$  matrix,

$$\text{such that } \Delta(L)_j = \begin{bmatrix} \Delta_j^1 \\ \Delta_j^2 L^{-1} \\ \Delta_j^3 L^{-2} \\ \vdots \\ \Delta_j^p L^{-p+1} \end{bmatrix} \text{ and } \tau_t \sim i.i.d N(0, R'') \text{ is a } (M + I) \times 1 \text{ vector. } e_t$$

and  $\tau_t$  are independent.

#### 2.4.3.2 Data

In the FASVAR model, we assume three unobserved variables: money supply growth rate (MS), output growth rate (Y), and inflation (INF). We also assume five observed variables: foreign exchange reserve (FR), minimum reserve rate (MR), rediscount rate (RR), net securities held by the central bank (OMO), loan rate (INT) and nominal effective exchange rate (EX). We use USEUGDP, the U.S. bill rate, and the U.S. government bond rate as the exogenous variables. According to the method proposed by Stock and Watson (2002),  $m_t$  should be  $I(0)$ . Thus, all series in the unobserved variable data set are first transferred to

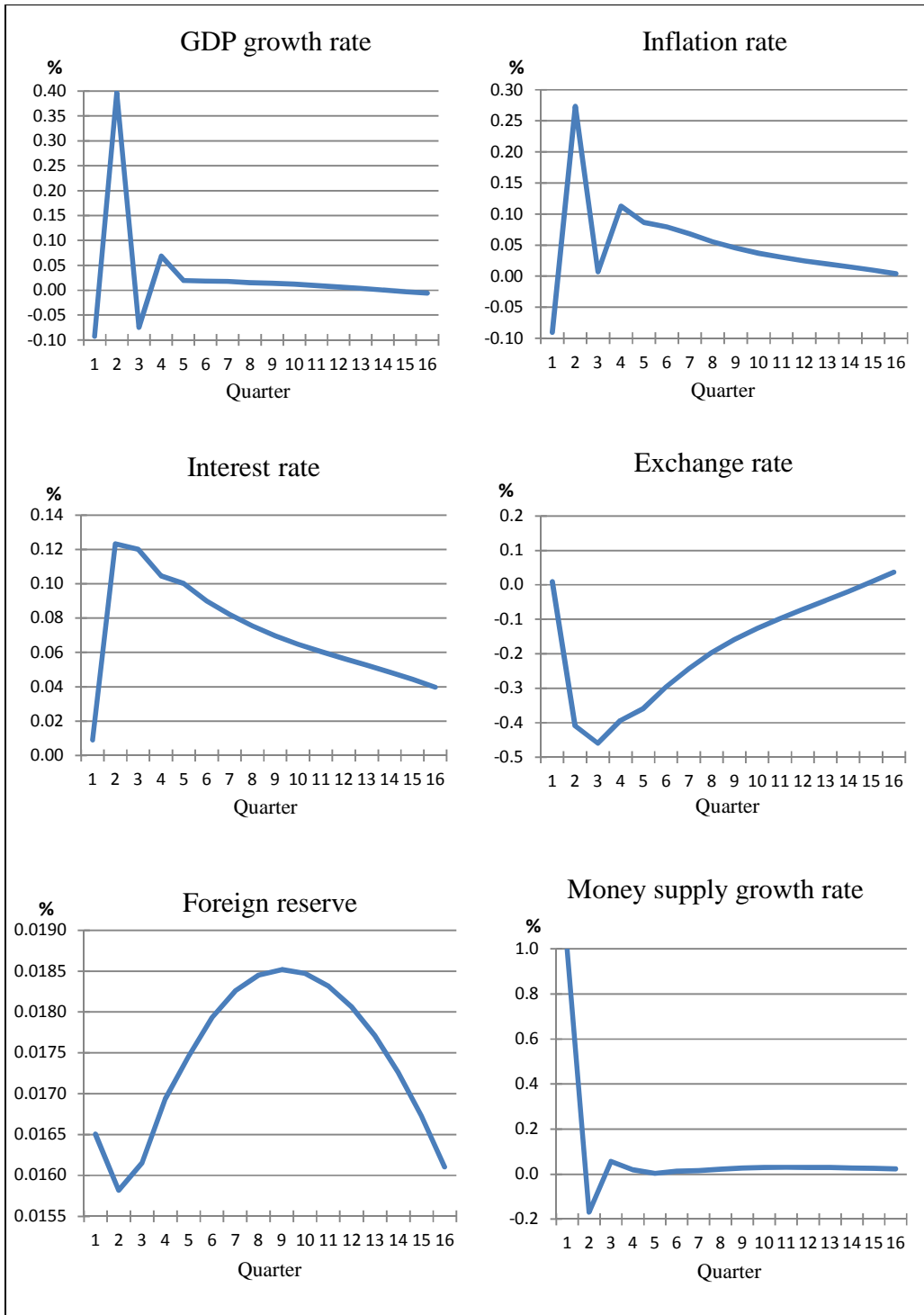
induce stationary process. The descriptions of the series in the data set and the transformation are listed in Appendix C.

All tests are presented in Appendix B. The Johansen co-integration test indicates that co-integration occurs between all variables and that the FASVAR satisfies the stability condition. SC and HQ suggest one lag; LR and FPF suggest two lags; and AIC suggests four lags should be used. Based on the test of the serial autocorrelation and the normality of the residuals, we use one lag in the FASVAR model.

#### **2.4.3.3 FASVAR estimation**

Figure 2-3 presents the impulse responses of the variables to a one-standard-deviation money supply shock. We get response paths similar to those in the SVAR model. An exception is the inflation rate, which increases after the money supply shock with one-period lag, while inflation rate decreases in the SVAR model. In the FASVAR model, a money supply shock has a stronger influence on GDP growth rate and inflation rate, and a weaker effect on the interest rate, exchange rate and foreign reserves. Overall, money supply is effective as a monetary instrument. After the money supply shock, just as we expected, the minimum reserve rate and rediscount rate are both hiked, and the net securities held by the central bank fall. With all these changes, the money supply growth rate immediately decreases and almost returns to its original level in the second period. We can conclude that the minimum reserve rate, rediscount rate and open market operation can be effective as instruments for adjusting the money supply.

Figure 2-3: FASVAR response to money supply shock



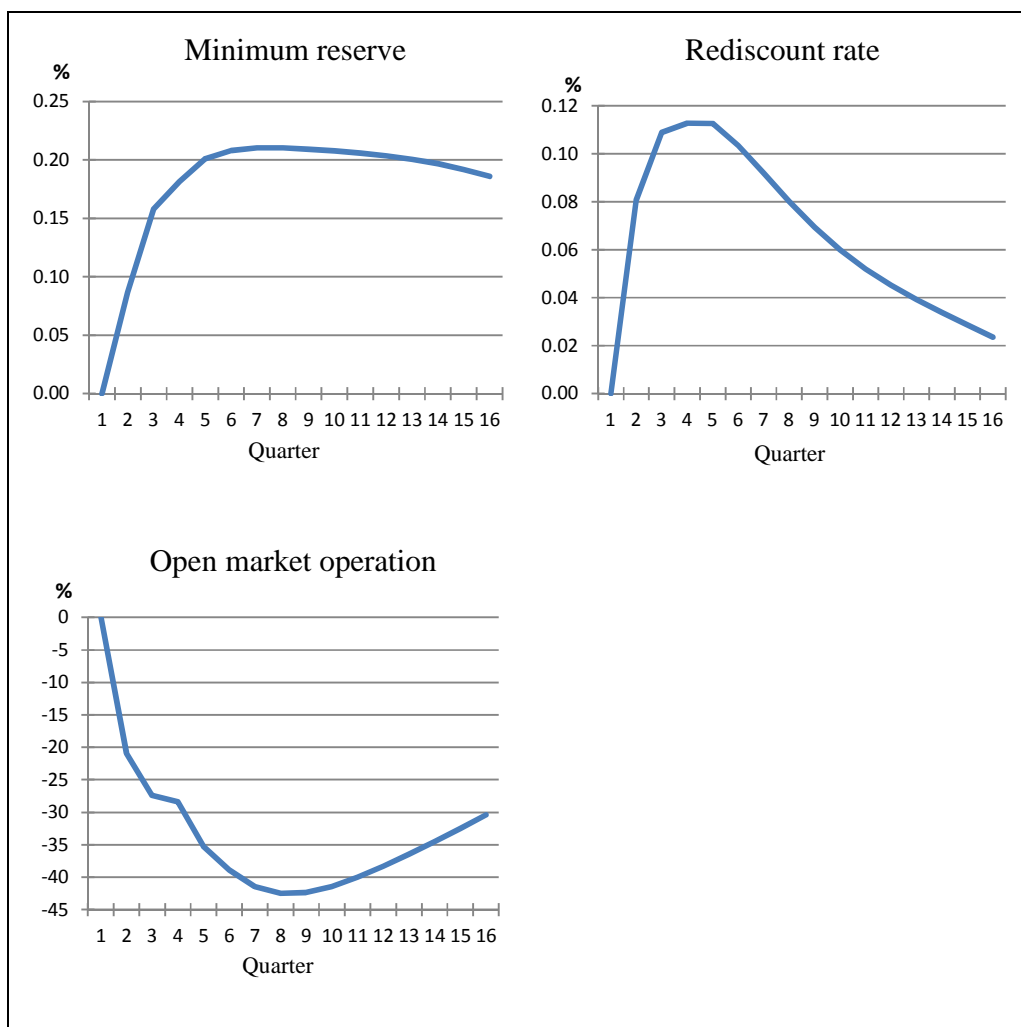


Figure 2-4 shows the impulse responses of variables to a one-standard-deviation interest rate shock. From the FASVAR model, we get an improved result. As in the SVAR model, after the shock, the exchange rate increases, the foreign exchange reserves decrease and the money supply growth falls. Unlike the GDP growth in the SVAR model, the GDP growth in the FASVAR model displays a protracted decrease after the interest rate shock. To stem the decline in the money supply, the PBC increases the net securities held through open market operations. In the FASVAR model, the response function completely fits the theory we mentioned in the SVAR model.

Figure 2-4: FASVAR response to interest rate shock

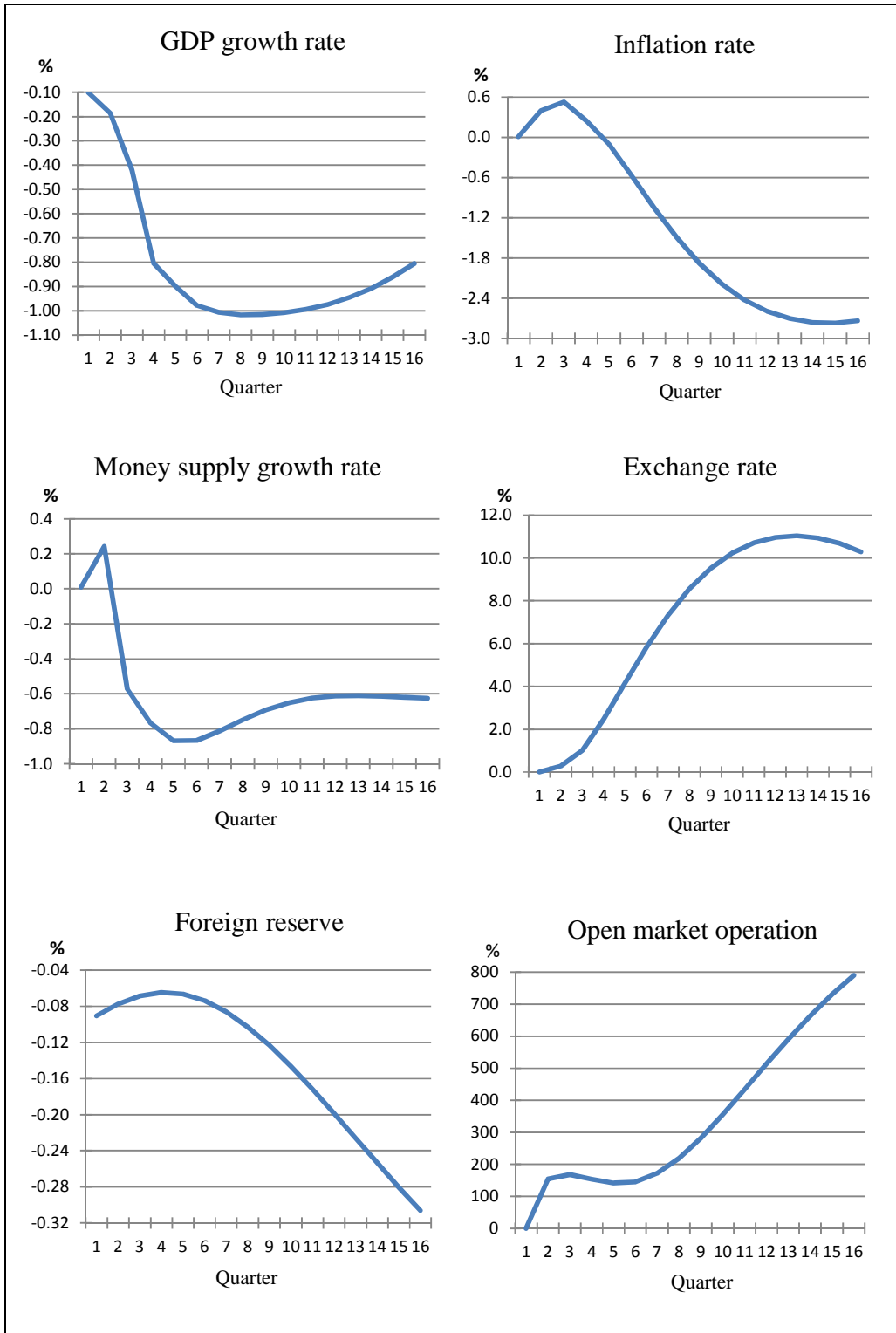
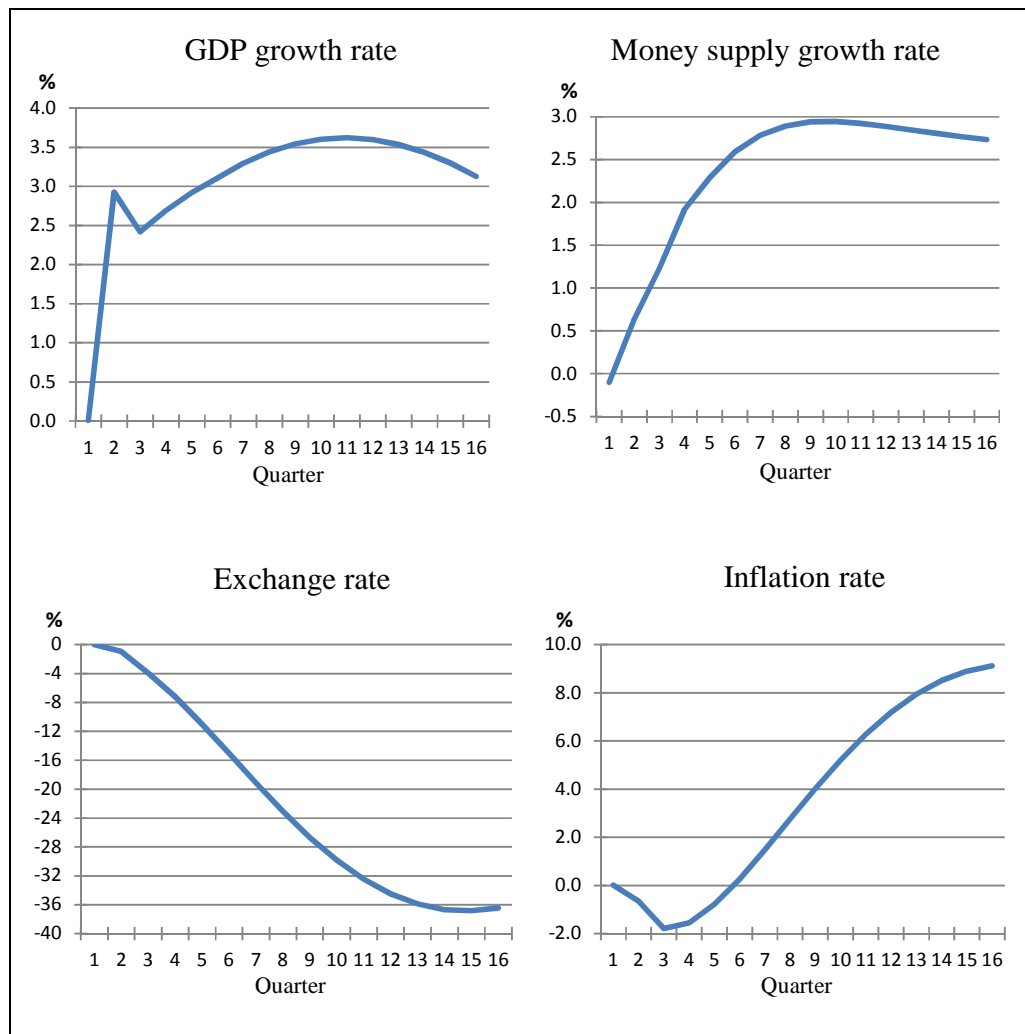


Figure 2-5 shows the response of variables to a one-standard-deviation foreign exchange reserve shock. According to PBC balance sheet, an increase in foreign exchange reserve means more money is put into the market. The excess money supply causes downward pressure on the exchange rate. The lower exchange rate attracts more foreign investors and increases exports. This results in an increase in aggregate demand and GDP growth rate.

Figure 2-5: FASVAR response to foreign reserve shock

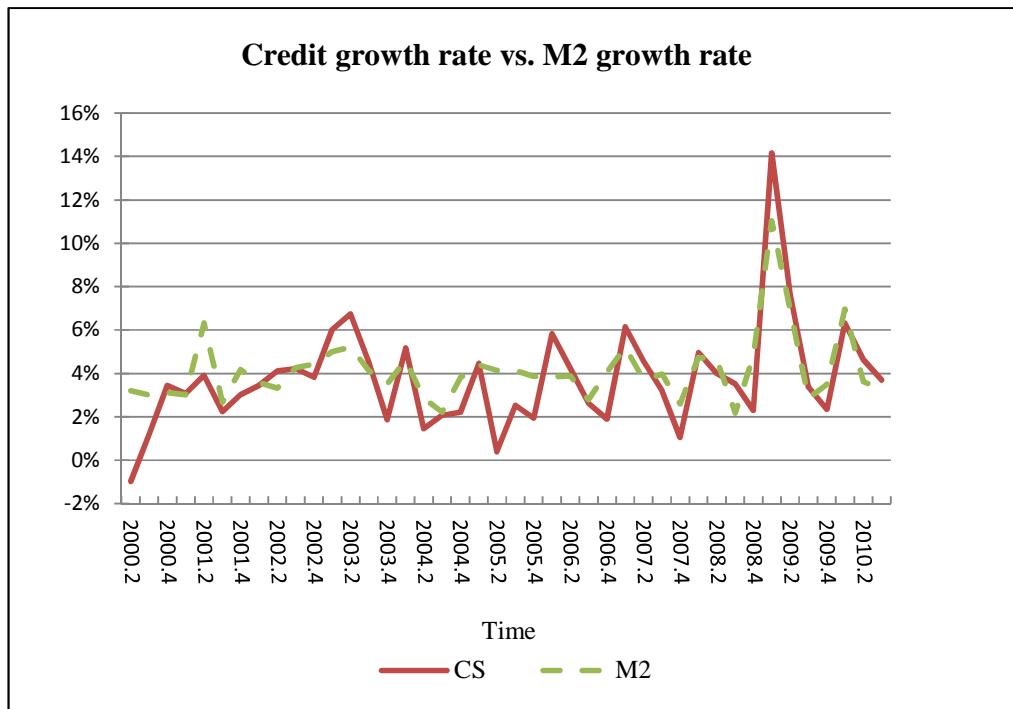




## 2.4.4 Robustness

Considering the sterilizing done by the PBC, we wonder whether the money supply shock in our model is the result of monetary policy or simply of excess liquidity. Given that the PBC also puts direct administrative limits on loan growth to assist in controlling the money supply, we compare the credit supply growth rate and money supply growth rate to see where our shock comes from. Figure 2-6 reveals that the movements of the credit supply and money supply are fairly consistent. We conclude that the money supply shock is the result of the monetary policy. For the FASVAR model, we also check for the robustness by altering the number of factors used in the augment factor. Our results are robust to these changes. Also, we have tried to use the RMB/US exchange rate and real effective exchange rate to replace the nominal effective exchange rate. We get very similar results.

Figure 2-6: Credit growth rate vs. M2 growth rate



## 2.4.5 Variance decomposition

Table 2-1 reports the variance decomposition results. The first two columns list the contribution of the money supply shock to the variance of the forecast error at the 8 and 16-quarters horizons. The last two columns list the contribution of the interest rate shock to the variances. The money supply shock explains less than 10 percent of the forecast variance of all variables, while the interest rate shock explains more than 10 percent of the forecast variance in inflation, exchange rate and interest rate at 8-quarters horizons. Overall, the interest rate shocks play a larger role than the money supply in explaining fluctuations in real activity. However, both shocks appear to explain a relatively small part of the variation. This result is not surprising. It is consistent with Lagana and Mountford's (2005) statement that "monetary policy shocks account for only a small proportion of the total variation in the data set." That said, we note that the interest rate shock appears to have a larger contribution in China than in other countries.<sup>26</sup>

Table 2-1: Fraction of variance explained by money supply shock and interest rate shock

Variables	Money supply (%)		Interest rate (%)	
	8	16	8	16
MS	2.521572	0.919275	9.066248	6.114929
FR	0.048399	0.037742	1.038110	3.443538
Y	0.243657	0.097482	6.412048	6.432771
INF	0.453680	0.022599	16.21956	10.17766
EX	0.052047	0.007358	11.37525	8.725609
INT	0.734165	0.039677	32.26618	9.339253

<sup>26</sup> See Banbura (2010).

## 2.5 Conclusion

This chapter used SVAR and FASVAR models to identify the effectiveness of alternative monetary instruments in affecting real economic activity in China. The SVAR methodology is tailored to the specific economic characteristics faced by Chinese policy makers: namely, exchange rate targeting, capital flow restrictions, and sterilization of foreign exchange market intervention. Both the SVAR and FASVAR approaches showed that the money supply is an effective monetary instrument in China and interest rate is not an effective instrument. Facing the enormous challenge of sterilizing the excess money supply, the PBC cannot raise the interest rate to contract the money supply and reduce the inflation rate, because the higher interest rate will cause upward pressure on the exchange rate and substantially impair China's economy. To minimize the interest rate effect on the exchange rate, the PBC has to restrict interest rate fluctuation in a small range. This restriction limits the effectiveness of the interest rate as a monetary policy instrument. From the FASVAR model, we also found that a positive foreign exchange reserve shock would benefit real activity in China.

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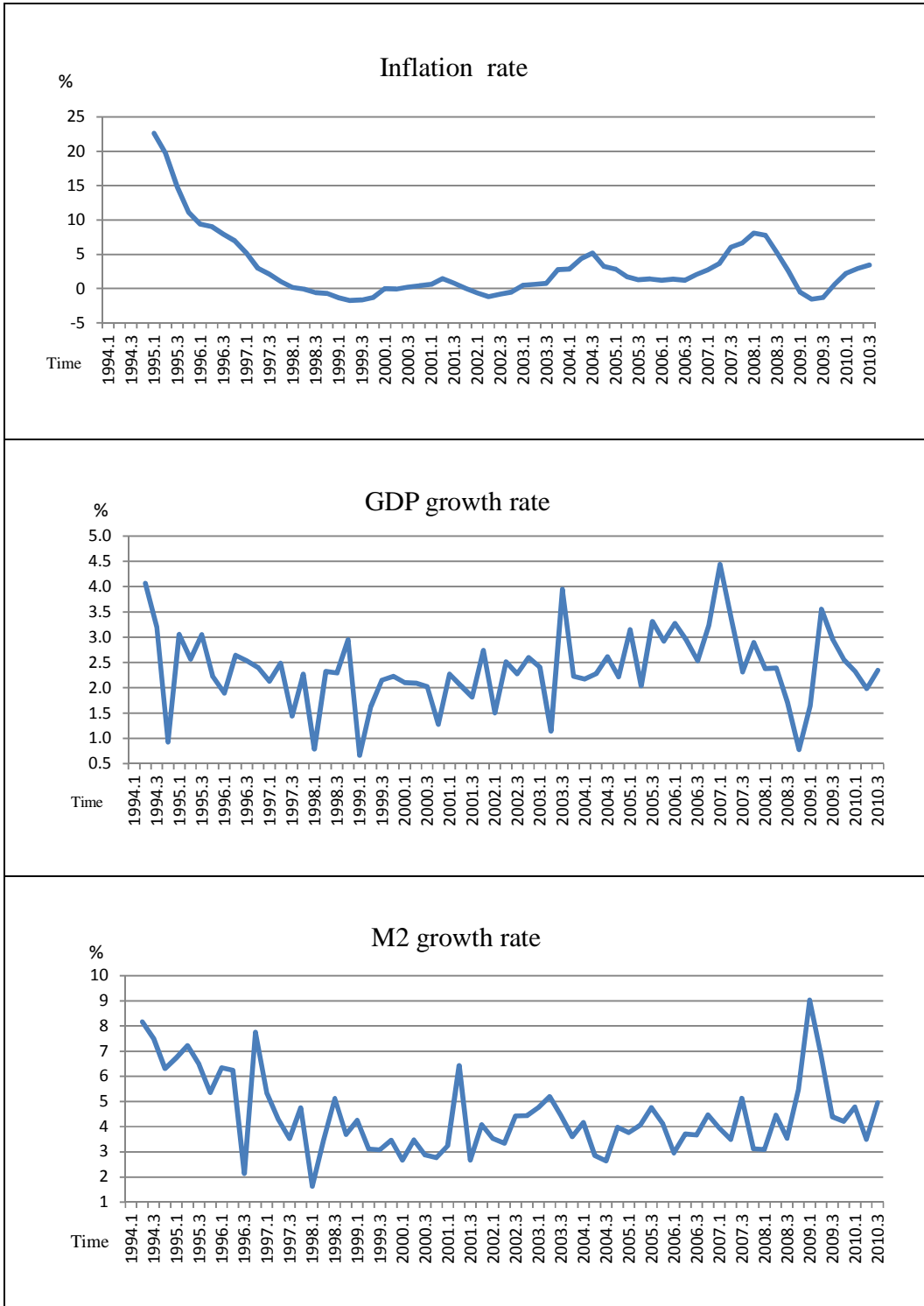
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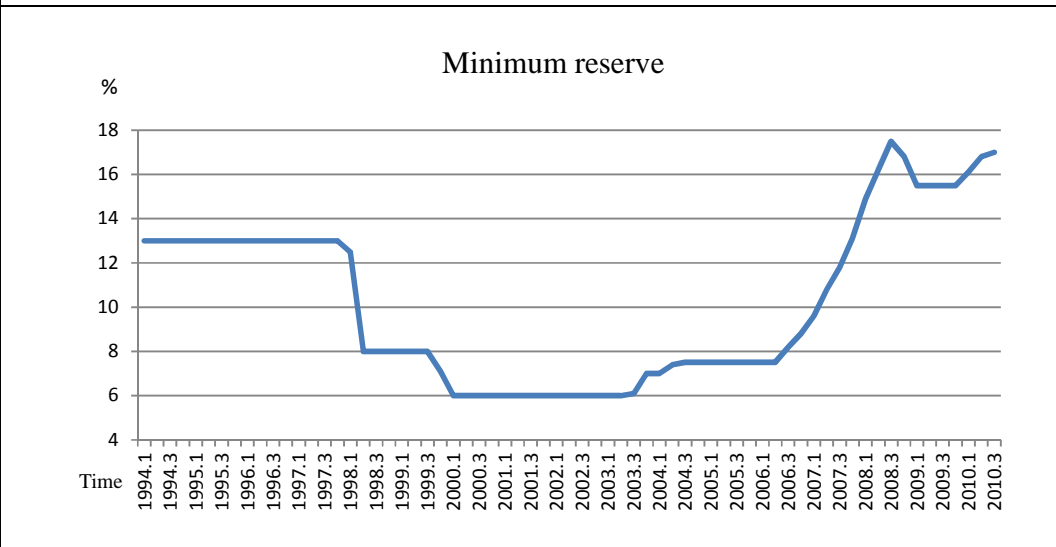
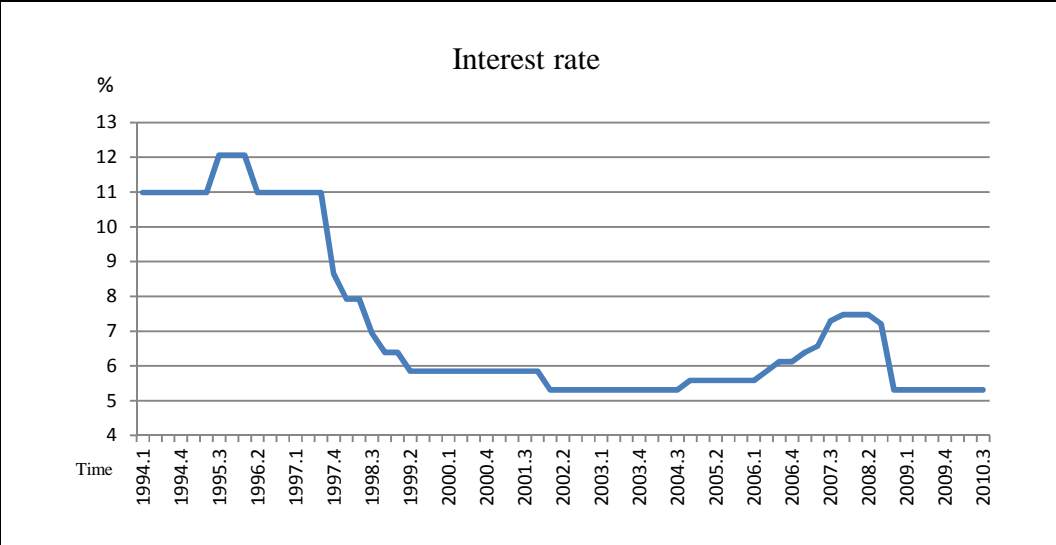
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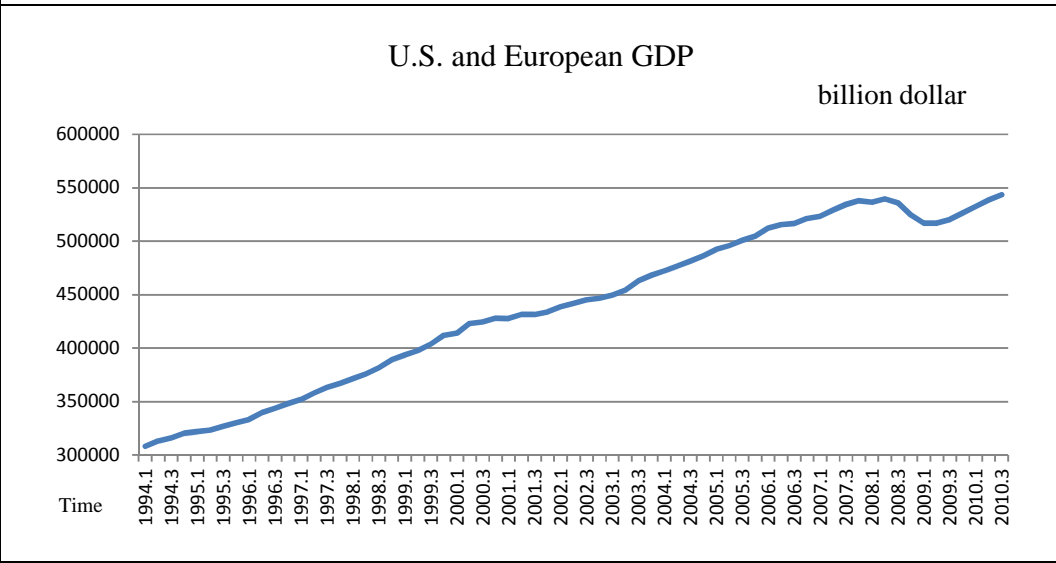
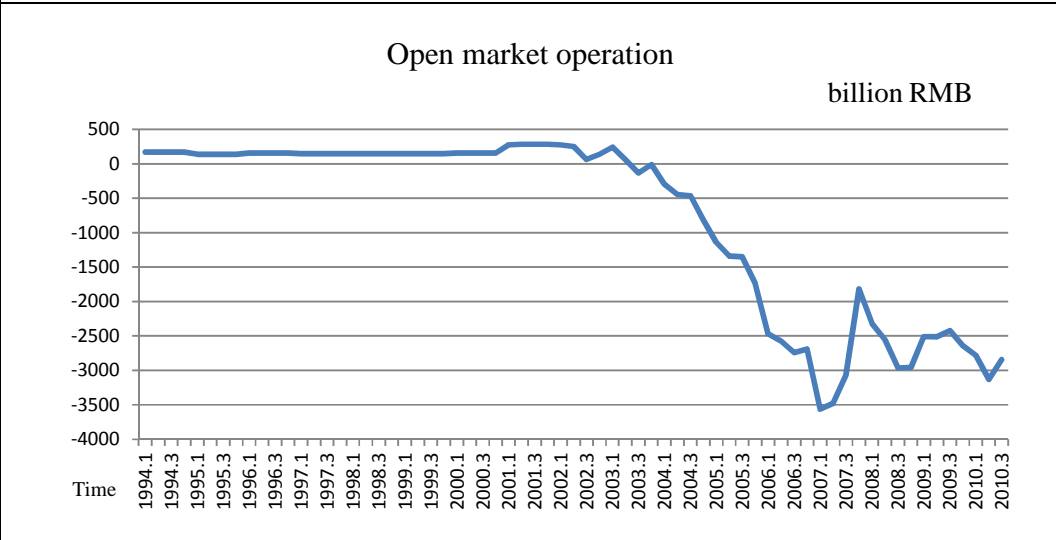
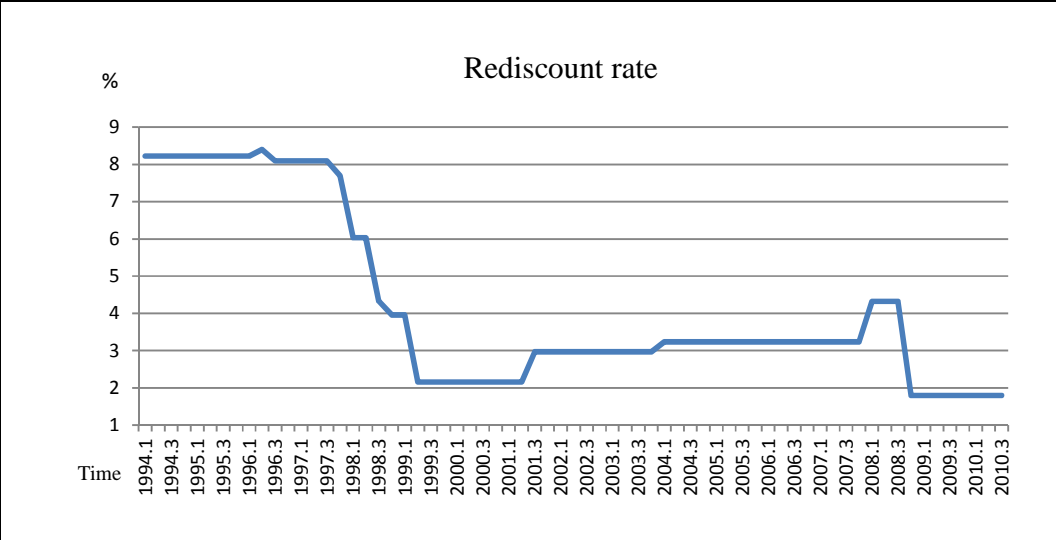
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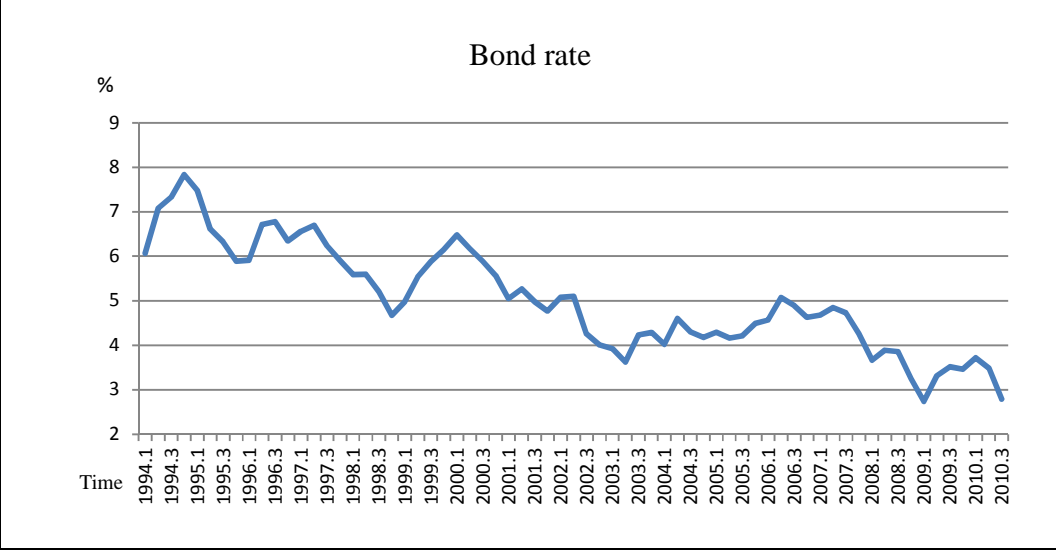
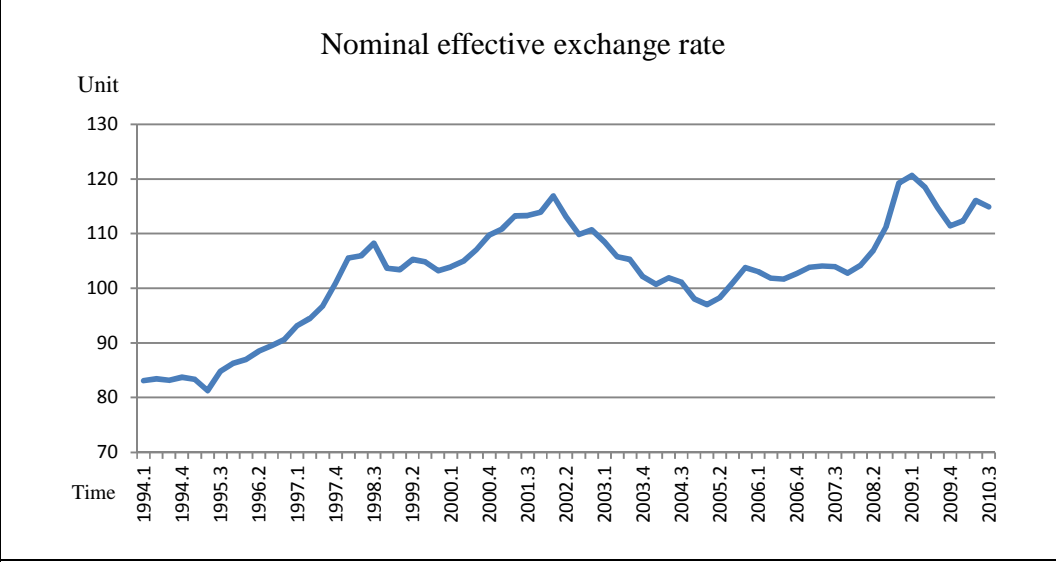
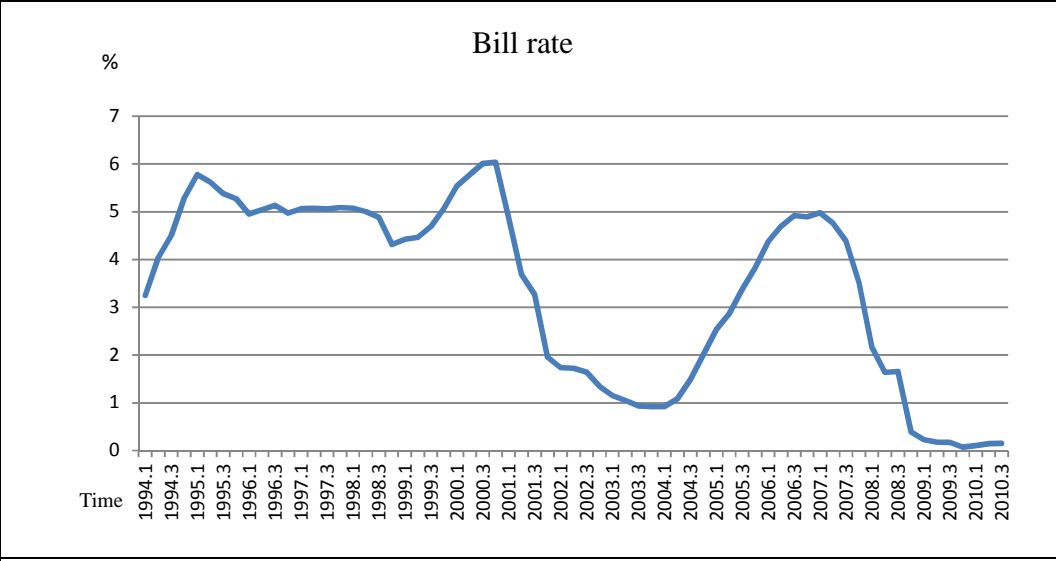


## Appendix A: Variables detail









## Appendix B: Econometric tests

### 1. Unit root test

Variable	ADF	DF-GLS	PP
INF	-3.24**	-3.22***	-2.98**
GDP	0.98	0.44	0.89
M2	-2.15	2.64	-1.42
INT	-1.48	-0.20	-1.52
EX	-1.94	-0.63	-1.68
D(GDP)	-7.21***	-2.14**	-7.22***
D(M2)	-5.40***	-1.03	-5.41***
D(INT)	-6.59***	-6.58***	-6.62***
D(EX)	-5.33***	-5.36***	-5.38***
FR	-0.29	1.21	-1.33
MR	-1.92	-0.48	-1.75
RR	-1.40	-0.10	-1.45
OMO	-0.51	-0.23	-0.82
D(FR)	-4.06***	-2.07**	-3.87***
D(MR)	-3.72***	-3.658745***	-3.645754***
D(RR)	-7.663551***	-7.63***	-7.72***
D(OMO)	-12.12***	-12.08***	-12.88***
M1	-0.21	1.64	-0.57
M0	-0.92	1.64	-1.22
D(M1)	-6.92***	-4.52***	-6.90
D(M0)	-9.36***	-0.06	-9.33***
INF-MF	-3.85***	-4.20***	-2.15
INF-TF	-2.31	-1.89	-2.04
INF-HF	-2.79*	-2.35**	-2.66*
INF-REF	-3.17**	-3.12***	-2.52
INF-PPF	-1.85	-1.86*	-3.02**
INF-EPF	-2.00	-2.08**	-2.84*
INF-NRF	-2.81*	-2.86***	-2.81*
INF-FF	-3.00**	-2.98***	-2.73*
INF-CF	-2.15	-2.17**	-2.19
INF-ESF	-2.04	-2.02**	-1.88
D(INF-TF)	-6.22***	-6.27***	-6.22***
D(INF-CF)	-8.09***	-8.15***	-8.09***
D(INF-ESF)	-2.94**	-2.99***	-7.33***
GDP-FI	0.80	1.28	0.43
GDP-SI	0.26	1.06	0.30
GDP-TI	-1.07	-0.23	1.22
D(GDP-FI)	-3.30**	-1.13	-10.36***
D(GDP-SI)	-8.90***	-6.77***	-8.89***
D(GDP-TI)	-8.70***	-6.55***	-8.77

\*\*\*indicates significant at 1%, \*\* indicates significant at 5%, \* indicates significant at 10%.

## 2. Lag selection and cointegration test<sup>27</sup>

### 2.1 SVAR: $y_t = [M2, FR, MR, RR, OMO, GDP, INF, EX, INT]$ —GDP, M2 in level

#### *Lags test*

The SC suggests using the model with only one lag. The LR test and FPF suggest using of three lags. AIC and the HQ suggest the use of four lags.

Lags	LR	FPF	AIC	SC	HQ
1	837.4403	9.50e-13	-2.195318	1.784790*	-0.629922
2	154.4233	3.59e-13	-3.390310	3.345256	-0.741179
3	117.9700*	2.07e-13*	-4.505444	4.985582	-0.772577
4	84.34637	2.35e-13	-5.601248*	6.645236	-0.784646*

#### *AR roots table*

All VAR(1), VAR(3) and VAR(4) specifications do not satisfy the stability conditions.

#### *VAR(1)*

Root	Modulus
1.006025	1.006025
0.931834 - 0.121652i	0.939741
0.931834 + 0.121652i	0.939741
0.927377	0.927377
0.794098 - 0.205461i	0.820247
0.794098 + 0.205461i	0.820247
0.476465	0.476465
0.061222 - 0.111887i	0.127541
0.061222 + 0.111887i	0.127541

#### *VAR(3)*

Root	Modulus
1.008887	1.008887
0.960243	0.960243
0.930022 - 0.168846i	0.945224
0.930022 + 0.168846i	0.945224

<sup>27</sup> We first do the lag length test to determine the possible lag orders. Enders (2004) states that the lag length can be examined regardless of whether the variables in question are stationary or integrated. Second, we use the Johansen cointegration test to determine whether we should use the variables in level or in first difference. Finally, we do the portmanteau autocorrelation test to determine the best lag order.

0.846166 - 0.286692i	0.893414
0.846166 + 0.286692i	0.893414
0.357890 - 0.730677i	0.813618
0.357890 + 0.730677i	0.813618
-0.521023 + 0.617298i	0.807788
-0.521023 - 0.617298i	0.807788
0.567732 - 0.500180i	0.756637
0.567732 + 0.500180i	0.756637
0.214965 + 0.699013i	0.731320
0.214965 - 0.699013i	0.731320
-0.602481 - 0.382839i	0.713828
-0.602481 + 0.382839i	0.713828
0.342911 + 0.595507i	0.687180
0.342911 - 0.595507i	0.687180
-0.327931 - 0.540745i	0.632411
-0.327931 + 0.540745i	0.632411
-0.170370 - 0.581059i	0.605521
-0.170370 + 0.581059i	0.605521
0.578103	0.578103
-0.567797	0.567797
-0.455498	0.455498
0.049226 - 0.290356i	0.294500
0.049226 + 0.290356i	0.294500

**VAR(4)**

<b>Root</b>	<b>Modulus</b>
0.996736 + 0.207913i	1.018189
0.996736 - 0.207913i	1.018189
0.987100 + 0.007912i	0.987131
0.987100 - 0.007912i	0.987131
-0.911282	0.911282
-0.453883 + 0.786398i	0.907982
-0.453883 - 0.786398i	0.907982
0.122797 + 0.882964i	0.891462
0.122797 - 0.882964i	0.891462

0.467730 + 0.755586i	0.888640
0.467730 - 0.755586i	0.888640
0.767377 + 0.440898i	0.885018
0.767377 - 0.440898i	0.885018
0.779374 + 0.186498i	0.801377
0.779374 - 0.186498i	0.801377
-0.712068 + 0.346043i	0.791698
-0.712068 - 0.346043i	0.791698
0.214343 - 0.727909i	0.758811
0.214343 + 0.727909i	0.758811
0.487786 - 0.555023i	0.738909
0.487786 + 0.555023i	0.738909
0.515673 + 0.515747i	0.729324
0.515673 - 0.515747i	0.729324
-0.535812 + 0.492508i	0.727777
-0.535812 - 0.492508i	0.727777
0.050589 + 0.688679i	0.690535
0.050589 - 0.688679i	0.690535
-0.150353 + 0.667274i	0.684003
-0.150353 - 0.667274i	0.684003
-0.668682	0.668682
-0.526926 + 0.018966i	0.527267
-0.526926 - 0.018966i	0.527267
-0.164940 - 0.420764i	0.451938
-0.164940 + 0.420764i	0.451938
-0.301866	0.301866
-0.115297	0.115297

## 2.2 SVAR: $y_t = [M2, FR, MR, RR, OMO, GDP, INF, EX, INT]$

### *Lags test*

The SC suggests using the model with only one lag. The LR test, FPF and the HQ suggest the use of two lags. AIC suggests the use of four lags.



Lags	LR	FPF	AIC	SC	HQ
1	699.0205	5.48e-13	-2.748691	1.265418*	-1.172650
2	153.0569*	2.06e-13*	-3.962211	2.830896	-1.295066*
3	84.02327	3.39e-13	-4.059736	5.512369	-0.301486
4	88.55100	3.04e-13	-5.471878*	6.879225	-0.622523

\* indicates optimal lag order.

### *Johansen cointegration test*

We reject the null hypothesis for all model specifications. It is better to use the variables in level.

Lags	Trace	5%Critical Value	Max-Eigen	5%Critical Value
1	378.6615	197.37	91.32224	58.43
2	407.3688		114.3333	
4	710.1023		199.5091	

Ho: no cointegration.

### *AR roots table*

All VAR(1), VAR(2) and VAR(4) specifications satisfy the stability condition.

#### *VAR(1)*

Root	Modulus
0.951972 - 0.088714i	0.956097
0.951972 + 0.088714i	0.956097
0.821268 - 0.125319i	0.830775
0.821268 + 0.125319i	0.830775
0.527991 - 0.060988i	0.531501
0.527991 + 0.060988i	0.531501
0.294955	0.294955
-0.070938 - 0.238832i	0.249144
-0.070938 + 0.238832i	0.249144

#### *VAR(2)*

Root	Modulus
0.952153 + 0.077830i	0.955328
0.952153 - 0.077830i	0.955328
0.896877 + 0.305032i	0.947330
0.896877 - 0.305032i	0.947330
0.373021 + 0.734886i	0.824137
0.373021 - 0.734886i	0.824137
-0.533121 + 0.107807i	0.543912
-0.533121 - 0.107807i	0.543912
-0.424434 - 0.326574i	0.535532
-0.424434 + 0.326574i	0.535532

$0.482823 + 0.193014i$	0.519974
$0.482823 - 0.193014i$	0.519974
$-0.002951 - 0.473715i$	0.473724
$-0.002951 + 0.473715i$	0.473724
$0.370225 + 0.269733i$	0.458064
$0.370225 - 0.269733i$	0.458064
$-0.004713 + 0.340128i$	0.340161
$-0.004713 - 0.340128i$	0.340161

**VAR(4)**

<b>Root</b>	<b>Modulus</b>
$0.994962 + 0.079414i$	0.998127
$0.994962 - 0.079414i$	0.998127
$-0.979583$	0.979583
$0.916460 + 0.255723i$	0.951469
$0.916460 - 0.255723i$	0.951469
$0.919002$	0.919002
$0.504077 - 0.757293i$	0.909717
$0.504077 + 0.757293i$	0.909717
$-0.448507 + 0.789438i$	0.907949
$-0.448507 - 0.789438i$	0.907949
$0.142855 + 0.864900i$	0.876618
$0.142855 - 0.864900i$	0.876618
$0.616865 - 0.600846i$	0.861126
$0.616865 + 0.600846i$	0.861126
$-0.778391 + 0.362804i$	0.858789
$-0.778391 - 0.362804i$	0.858789
$0.708378 - 0.412315i$	0.819635
$0.708378 + 0.412315i$	0.819635
$0.236777 + 0.744329i$	0.781082
$0.236777 - 0.744329i$	0.781082
$-0.290004 - 0.675063i$	0.734719
$-0.290004 + 0.675063i$	0.734719
$-0.179319 - 0.670252i$	0.693825
$-0.179319 + 0.670252i$	0.693825
$0.354717 - 0.587790i$	0.686528
$0.354717 + 0.587790i$	0.686528
$-0.544782 + 0.378153i$	0.663164
$-0.544782 - 0.378153i$	0.663164
$-0.453061 + 0.386562i$	0.595563
$-0.453061 - 0.386562i$	0.595563
$0.103454 - 0.573129i$	0.582392
$0.103454 + 0.573129i$	0.582392
$-0.551178 - 0.146256i$	0.570252
$-0.551178 + 0.146256i$	0.570252
$0.383961$	0.383961

-0.069030	0.069030
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***Lagrange Multiplier autocorrelation test (p-value)***

To determine how many lags should be used in the SVAR model, we first run the Lagrange Multiplier (LM) test to test the no serial autocorrelation assumption. At conventional significant levels (5% or 10%), the specifications with one and two lags outperform the four lags in the terms of absence of autocorrelation.

<b>Lags</b>	1	2	3	4	5	6	7	8	9	10	11
<b>VAR(1)</b>	0.000	0.0	0.90	0.60	0.8	0.98	0.8	0.99	0.99	0.9	0.1
<b>VAR(2)</b>	0.000	0.5	0.81	0.37	0.5	0.89	0.1	0.62	0.68	0.4	0.3
<b>VAR(4)</b>	0.15	0.9	0.000	0.00	0.4	0.00	0.3	0.78	0.95	0.5	0.0

Ho: no serial correlation.

***SVAR residual normality test  $\chi^2$***

Second, we try to determine if the residuals of the specifications can be considered to be normally distributed. VAR(2) and VAR(4) specifications outperform the VAR(1) specification.

	<b>VAR(1)</b>	<b>VAR(2)</b>	<b>VAR(4)</b>
<b>M2</b>	6.44E+11*	7.25E+10*	8.33E+09*
<b>FR</b>	1.74E-08	3.04E-06	6.71E-08
<b>MRR</b>	0.001353	2.75E-05	8.50E-06
<b>RD</b>	0.115570	0.017222	0.000469
<b>OMO</b>	6.45E+17*	7.26E+16*	8.35E+15*
<b>GDP</b>	7.23E-07	6.83E-08	1.06E-08
<b>INF</b>	4.22E-08	1.58E-09	8.98E-16
<b>EX</b>	14.31149*	1.903054	0.118334
<b>INT</b>	0.096618	0.014896	0.000571

Ho: residuals are multivariate normal. \* indicates reject at 5% significant level, \*\* indicates reject at 10% significant level.

**Portmanteau test –p value**

Third, we use the portmanteau test to further investigate the autocorrelation problem. The VAR(1) specification performs better than the VAR(2) and VAR(4).

Lags	VAR(1)	VAR(2)	VAR(4)
1	NA	NA	NA
2	0.0000	NA	NA
3	0.0001	0.0000	NA
4	0.0007	0.0000	NA
5	0.0059	0.0000	0.000
6	0.0530	0.0002	0.000
7	0.1268	0.0004	0.000
8	0.4208	0.0022	0.000

Ho: no residual autocorrelation up to lag “h”.

**Correlation of VAR**

Finally, we check the correlation matrix of the residuals. The VAR(1) and VAR(2) specifications have lower correlation between residuals than the VAR(4) specification.

**VAR(1)**

	M2	FR	MRR	RD	OMO	GDP	INF	EX	INT
M2	1								
FR	0.272	1							
MRR	0.113	0.099	1						
RD	0.029	0.114	0.252	1					
OMO	-0.224	-0.271	-0.056	0.074	1				
GDP	0.112	-0.023	-0.056	0.277	0.019	1			
INF	0.032	-0.191	0.105	0.144	-0.080	0.152	1		
EX	0.015	-0.151	-0.133	-0.430	-0.153	-0.404	-0.250	1	
INT	0.026	0.147	0.279	0.500	0.0001	0.106	0.134	-0.297	1

**VAR(2)**

	M2	FR	MRR	RD	OMO	GDP	INF	EX	INT
M2	1								
FR	0.275	1							
MRR	0.185	0.094	1						
RD	0.032	0.052	0.352	1					
OMO	-0.331	-0.275	-0.025	-0.055	1				
GDP	0.147	0.008	0.256	0.278	-0.117	1			
INF	-0.023	-0.360	0.109	0.191	-0.193	0.197	1		
EX	0.044	-0.052	-0.365	-0.342	-0.232	-0.297	-0.109	1	
INT	0.089	0.142	0.335	0.521	-0.014	0.034	0.221	-0.255	1

**VAR(4)**

	<b>M2</b>	<b>FR</b>	<b>MRR</b>	<b>RD</b>	<b>OMO</b>	<b>GDP</b>	<b>INF</b>	<b>EX</b>	<b>INT</b>
<b>M2</b>	1								
<b>FR</b>	0.229	1							
<b>MRR</b>	0.398	0.270	1						
<b>RD</b>	0.163	0.177	0.578	1					
<b>OMO</b>	-0.418	-0.410	0.024	-0.050	1				
<b>GDP</b>	0.381	0.189	0.347	0.267	-0.217	1			
<b>INF</b>	-0.117	-0.300	-0.221	-0.268	0.323	-0.013	1		
<b>EX</b>	0.099	0.015	-0.522	-0.308	-0.363	-0.142	-0.075	1	
<b>INT</b>	0.308	0.257	0.753	0.526	0.166	0.255	-0.064	-0.466	1

Overall, the VAR(1) specification performs better in the autocorrelation test. The Portmanteau test shows low correlation between variables. Although it underperforms in the normality test, the non-normality is less important. Here, we use one lag in the SVAR model.

**2.3 FASVAR  $y_t = [MS, FR, MR, RR, OMO, GDP, INF, EX-NE, INT]$** **Lags test**

The HQ and the SC suggest using the model with only one lag. The LR test and FPF suggest using three lags. The AIC suggests using four lags.

<b>Lags</b>	<b>LR</b>	<b>FPF</b>	<b>AIC</b>	<b>SC</b>	<b>HQ</b>
1	685.1904	0.527346	24.84418	28.85829*	26.42022*
2	137.3662	0.292987	24.02293	30.81603	26.69007
3	103.8252*	0.254933*	23.28663	32.85873	27.04488
4	83.01296	0.294579	22.12622*	34.47732	26.97557

\* indicates optimal lag order.

**Johansen cointegration test**

For all specifications, we reject the null hypothesis of no cointegration.

<b>Lags</b>	<b>Trace</b>	<b>5%Critical Value</b>	<b>Max-Eigen</b>	<b>5%Critical Value</b>
1	374.8852	197.37	87.68519	58.43
3	429.7870		104.4745	
4	761.9170		208.2618	

Ho: no cointegration.

**AR roots table**

All VAR(1), VAR(3) and VAR(4) specifications satisfy the stability conditions.

**VAR(1)**

Root	Modulus
0.942124 - 0.073212i	0.944964
0.942124 + 0.073212i	0.944964
0.851889 - 0.181575i	0.871024
0.851889 + 0.181575i	0.871024
0.443145 - 0.086092i	0.451431
0.443145 + 0.086092i	0.451431
0.407542	0.407542
-0.201700 - 0.159520i	0.257157
-0.201700 + 0.159520i	0.257157

**VAR(3)**

Root	Modulus
0.914582 + 0.354730i	0.980965
0.914582 - 0.354730i	0.980965
0.971070 + 0.083029i	0.974614
0.971070 - 0.083029i	0.974614
0.251457 - 0.793632i	0.832516
0.251457 + 0.793632i	0.832516
-0.593160 + 0.528080i	0.794171
-0.593160 - 0.528080i	0.794171
0.430876 - 0.664669i	0.792110
0.430876 + 0.664669i	0.792110
0.774079	0.774079
-0.268822 - 0.718368i	0.767019
-0.268822 + 0.718368i	0.767019
0.304841 - 0.702124i	0.765445
0.304841 + 0.702124i	0.765445
-0.755972	0.755972
-0.583552 - 0.255215i	0.636921
-0.583552 + 0.255215i	0.636921
-0.363425 - 0.467184i	0.591894
-0.363425 + 0.467184i	0.591894
0.426973 + 0.322990i	0.535377
0.426973 - 0.322990i	0.535377
0.106603 - 0.485800i	0.497359
0.106603 + 0.485800i	0.497359
0.401172	0.401172
-0.320172 + 0.048700i	0.323855
-0.320172 - 0.048700i	0.323855

**VAR(4)**

<b>Root</b>	<b>Modulus</b>
0.989699 - 0.155271i	1.001805
0.989699 + 0.155271i	1.001805
0.856857 - 0.516157i	1.000311
0.856857 + 0.516157i	1.000311
0.963643	0.963643
0.484181 + 0.781186i	0.919066
0.484181 - 0.781186i	0.919066
0.223255 + 0.886193i	0.913882
0.223255 - 0.886193i	0.913882
-0.683527 + 0.584820i	0.899569
-0.683527 - 0.584820i	0.899569
-0.468297 + 0.755481i	0.888849
-0.468297 - 0.755481i	0.888849
-0.808082 + 0.300515i	0.862152
-0.808082 - 0.300515i	0.862152
0.057528 - 0.829626i	0.831619
0.057528 + 0.829626i	0.831619
-0.812911 + 0.088662i	0.817732
-0.812911 - 0.088662i	0.817732
0.781482 - 0.113045i	0.789615
0.781482 + 0.113045i	0.789615
0.643736 + 0.425394i	0.771594
0.643736 - 0.425394i	0.771594
-0.610549 + 0.432788i	0.748382
-0.610549 - 0.432788i	0.748382
0.312533 - 0.674572i	0.743455
0.312533 + 0.674572i	0.743455
-0.072813 + 0.720795i	0.724463
-0.072813 - 0.720795i	0.724463
-0.223844 - 0.679276i	0.715208
-0.223844 + 0.679276i	0.715208
0.360394 + 0.489168i	0.607593
0.360394 - 0.489168i	0.607593
-0.515503	0.515503
0.129225	0.129225
0.039887	0.039887

***LM autocorrelation test (p-value)***

To determine how many lags should be used in the FASVAR model, we first run the LM test to test the no serial autocorrelation assumption. At conventional significant levels (5% or 10%), the specification with one lag outperforms the other two specifications.

Lags	1	2	3	4	5	6	7	8	9	10	11
VAR(1)	0.02	0.09	0.60	0.69	0.87	0.93	0.97	0.71	0.97	0.24	0.25
VAR(3)	0.04	0.04	0.003	0.002	0.07	0.77	0.28	0.82	0.35	0.16	0.08
VAR(4)	0.26	0.76	0.10	0.33	0.01	0.63	0.05	0.97	0.30	0.04	0.009

Ho: no serial correlation.

***SVAR residual normality test  $\chi^2$***

Second, we try to determine if the residuals of the specifications could be considered to be normally distributed. The VAR(3) and VAR(4) specifications outperform the VAR(1) specification.

	VAR(1)	VAR(3)	VAR(4)
M2	6.10E+11*	5.18E+10*	2.25E+09*
FR	0.000309	9.55E-06	4.94E-08
MRR	0.012985	6.09E-06	1.84E-05
RD	0.030704	0.006674	0.000396
OMO	6.16E+17*	5.35E+16*	2.23E+15*
GDP	10.47590*	0.000598	0.011911
INF	528.2223*	0.212854	1.91E-06
EX	8.176691*	0.693401	0.188370
INT	0.027993	0.005500	0.000256

Ho: residuals are multivariate normal. \* indicates reject at 5% significant level, \*\* indicates reject at 10% significant level.



**Portmanteau test –p value**

Third, we use the portmanteau test to further investigate the autocorrelation problem. The VAR(1) specification performs better than the VAR(3) and VAR(4) specifications.

Lags	VAR(1)	VAR(3)	VAR(4)
1	NA	NA	NA
2	0.0000	NA	NA
3	0.0002	NA	NA
4	0.0032	0.000	NA
5	0.0127	0.000	0.000
6	0.0562	0.000	0.000
7	0.1883	0.000	0.000
8	0.3065	0.000	0.000

Ho: no residual autocorrelation up to lag “h”.

**Correlation of VAR**

Finally, we check the correlation matrix of the residuals. The VAR(1) specification has a lower correlation between residuals than the VAR(3) and VAR(4) specifications.

**VAR(1)**

	M2	FR	MRR	RD	OMO	GDP	INF	EX	INT
M2	1								
FR	0.314	1							
MRR	0.048	0.130	1						
RD	-0.030	0.088	0.233	1					
OMO	-0.184	-0.277	-0.032	0.080	1				
GDP	0.055	-0.044	-0.062	0.264	0.006	1			
INF	-0.360	-0.237	0.015	0.109	-0.071	0.156	1		
EX	0.007	-0.114	-0.065	-0.415	-0.194	-0.407	-0.186	1	
INT	0.061	0.091	0.175	0.500	0.032	0.104	0.127	-0.251	1

**VAR(3)**

	M2	FR	MRR	RD	OMO	GDP	INF	EX	INT
M2	1								
FR	0.106	1							
MRR	0.302	0.184	1						
RD	0.169	0.251	0.297	1					
OMO	-0.230	-0.382	-0.044	-0.089	1				
GDP	0.444	0.320	0.222	0.217	-0.225	1			
INF	-0.032	-0.051	-0.046	0.086	-0.296	0.007	1		
EX	-0.126	-0.128	-0.331	-0.293	-0.253	-0.311	-0.002	1	
INT	0.196	0.200	0.229	0.574	0.064	0.175	0.122	-0.291	1

**VAR(4)**

	<b>M2</b>	<b>FR</b>	<b>MRR</b>	<b>RD</b>	<b>OMO</b>	<b>GDP</b>	<b>INF</b>	<b>EX</b>	<b>INT</b>
<b>M2</b>	1								
<b>FR</b>	0.104	1							
<b>MRR</b>	0.338	0.392	1						
<b>RD</b>	0.068	0.301	0.428	1					
<b>OMO</b>	-0.247	-0.438	-0.032	-0.037	1				
<b>GDP</b>	0.398	0.256	0.213	0.206	-0.230	1			
<b>INF</b>	-0.024	-0.073	-0.033	0.099	0.341	0.101	1		
<b>EX</b>	-0.165	-0.070	-0.443	-0.323	-0.380	-0.240	-0.351	1	
<b>INT</b>	0.081	0.255	0.419	0.442	0.134	0.167	0.324	-0.409	1

Overall, the VAR(3) and VAR(4) specifications perform better in the normality test. VAR(1) does better in the LM autocorrelation test and Portmanteau test. Given that the non-normality is not important and we use one lag in the SVAR approach, we choose to use one lag in the FASVAR model.

## Appendix C: Data description

Source: National Bureau of Statistics of China (NBSC); The People's Bank of China (PBC); State Administration of Foreign Exchange (SAFE); Bank for International Settlements (BIS); International Monetary Fund (IMF).

Transform: 1 no transformation; 2 first difference; 3 logarithm; 4 first difference of logarithm; 5 seasonal adjustment; 6 first difference of seasonal adjusted series; 7 logarithm of seasonal adjusted series; 8 first difference of logarithm of seasonal adjusted series.

### *Money supply*

Mnemonic	Description	Source	Transform
M2	Money stock: m2	NBSC & IMF	8
M1	Money stock: m1	NBSC & IMF	8
M0	Money stock: m0	NBSC & IMF	8

### *Foreign reserve*

Mnemonic	Description	Source	Transform
FR	Foreign reserve dominated by RMB	NBSC	7

### *Minimum reserve rate*

Mnemonic	Description	Source	Transform
MR	Minimum reserve rate	PBC	1

### *Rediscount rate*

Mnemonic	Description	Source	Transform
RR	Rediscount rate	PBC	1

### *Open market operation*

Mnemonic	Description	Source	Transform
OMO	Claims on central government - bond issues by the central bank	PBC	1

### *Real output*

Mnemonic	Description	Source	Transform
GDP	Real GDP total base year 1994	NBSC	8

GDP-FI	Real first industry output base year 1994	NBSC	8
GDP-SI	Real second industry output base year 1994	NBSC	8
GDP-TI	Real third industry output base year 1994	NBSC	8

### *Inflation*

<b>Mnemonic</b>	<b>Description</b>	<b>Source</b>	<b>Transform</b>
INF	National average consumer price index compared to same period last year, base year 1994	NBSC	1
INF-MF	National medical treatment care & individual articles price index compared to same period last year, base year 1994	NBSC	1
INF-TF	National transport & communications price index compared to same period last year, base year 1994	NBSC	2
INF-HF	National housing price index compared to same period last year, base year 1994	NBSC	1
INF-REF	National recreation, education, cultural article & services price index compared to same period last year, base year 1994	NBSC	1
INF-PPF	Average purchase price of raw materials fuel and power compared to same period last year, base year 1994	NBSC	1
INF-EPF	National average ex-factory price index of industrial products compared to same period last year, base year 1994	NBSC	1
INF-NRF	National average retail sale price, base year 1994	NBSC	1
INF-FF	National food price index compared to same period last year, base year 1994	NBSC	1
INF-CF	National average retail sale price compared to same period last year, base year 1994	NBSC	2
INF-ESF	National home equipment & services price index compared to same period last year, base year 1994	NBSC	2

*Exchange rate*

<b>Mnemonic</b>	<b>Description</b>	<b>Source</b>	<b>Transform</b>
EX	Nominal effective exchange rate	SAFE	1

*Interest rate*

<b>Mnemonic</b>	<b>Description</b>	<b>Source</b>	<b>Transform</b>
INT	Benchmark loan rate for loan six months to one year	PBC	1

## **Chapter 3: Business Cycles, Consumption Smoothing, and Bank Runs**

### **3.1 Introduction**

Recently, bank runs in Greece have attracted considerable attention. These runs are arguably not classic Diamond-Dybvig (DD) panic-based runs in which withdrawal decisions are based solely on the belief that widespread withdrawals will occur, and thus a depositor “place in line” will determine how much they will be able to withdraw before the bank is declared insolvent if it must arrange a fire sale of its illiquid assets. Surely some, perhaps many, of the withdrawals were motivated by the desire to accumulate euros based on the fear that if Greece leaves the euro, bank deposits in Greece might be converted into a newly issued Greek currency. However, surely another factor is also at work in these bank runs. Withdrawals in Greece have coincided with a massive adverse shock to the real economy, including large-scale job losses. Greece has been in recession since 2008, and the crisis deepened beginning in 2010. Figure 3-1 reveals that according to data from Eurostat, a substantial increase in the Greek unemployment rate has occurred since 2010. In April 2012, Greece’s unemployment rate was more than double the euro zone average. According to Greece’s statistics service, 1.075 million Greeks were out of work in March 2012 (the labor force in Greece was 7.128 million). “Christina Tsakalou, 40, who lost her job as a store manager, said, ‘My unemployment benefit is 360 euros (\$450) a month and will run out in four months.’” (Reuters, 2012). It is reasonable to believe that as more and more people in Greece lose their jobs, withdrawals from banks will intensify. Figure 3-2 (Durdan, 2012), based on data from the Bank of Greece, shows that between Jan 2010 and Jan 2012, nearly a third of Greece’s bank deposits were withdrawn.

Figure 3-1: Greek unemployment rate

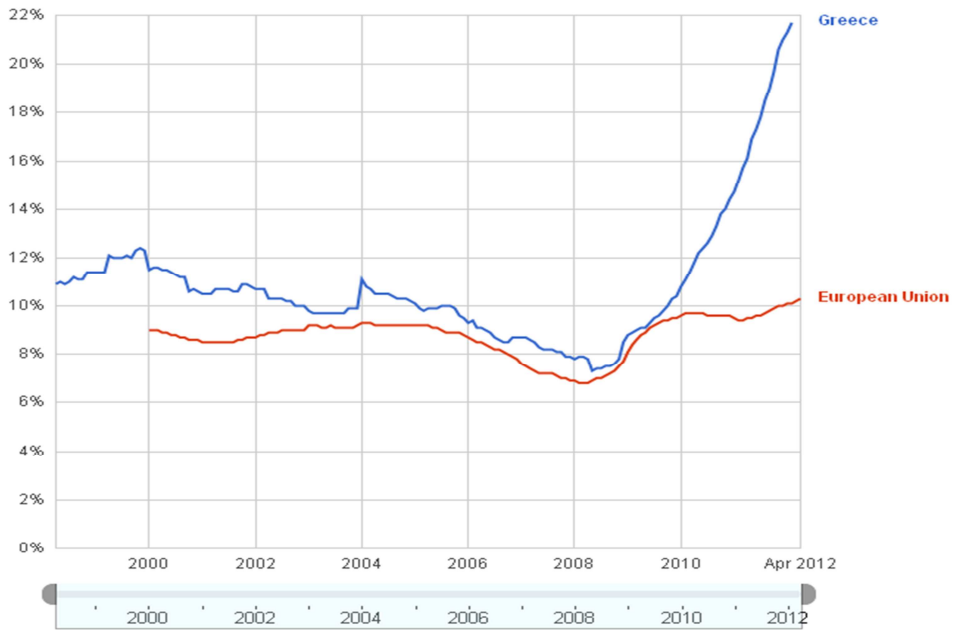
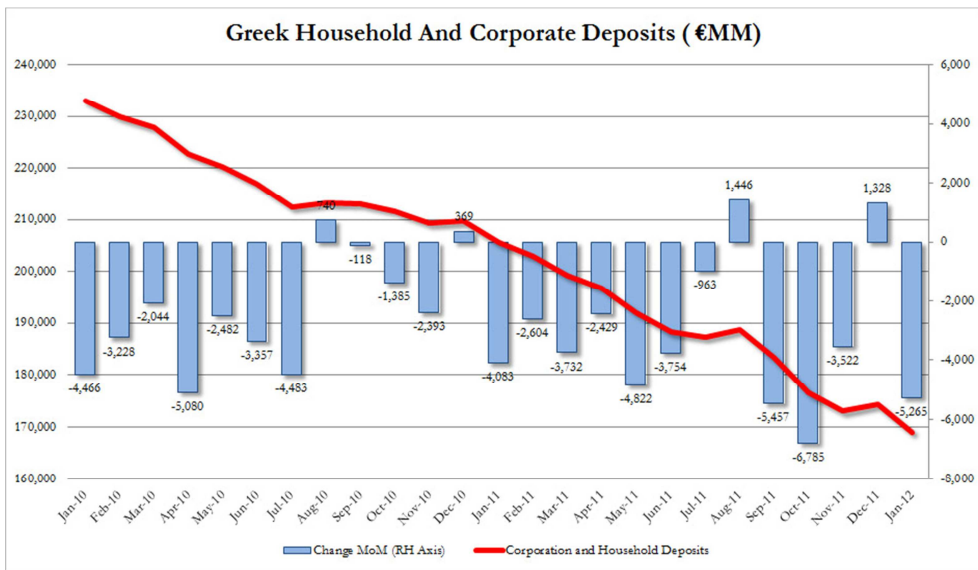


Figure 3-2: Greek deposits



The link between the real economy and bank runs is the main focus of this chapter. The principal modification is that depositors in banks have uncertain labor income. In addition to standard panic-based runs (which also exist in our model), bank runs—which might be better interpreted as financial fragility—can occur if enough aggregate variation exists in labor income. Uncertain labor income in an environment where households seek to smooth consumption translates into uncertainty about withdrawals. In short, bank runs can occur as an equilibrium phenomenon without the traditional elements of bank-run models.

This chapter supports the view that an economic slowdown can cause banking crises. During an expanding economy, both the employment rate and average income are high, whereas during economic contractions, the reverse happens. Motivated by the desire to smooth consumption, in an economic slowdown some agents will choose to withdraw money early from banks, and this behavior can have nothing to do with their beliefs about other agents' withdrawal intentions. This view is broadly consistent with that of Allen and Gale (1998, 2000), who argue that bank runs are in fact responses to macroeconomic fundamental shocks. Mason (2003) argues that economic fundamentals explain most of the bank failures prior to 1933.

This chapter considers an environment similar to the one in Diamond and Dybvig (1983). However, unlike Diamond and Dybvig (1983), we assume that long-lived agents' consumption preferences are smooth, as described in Debreu (1972), with strictly positive utility for both consumption periods; we assume that, along with the deposits in banks, labor income is also a source of wealth and consumption. In our model, we show that even if a bank sets an optimal contract to maximize depositors' expected utility, the uncertain labor income tied to business cycles will create a positive probability of bank runs.

By analyzing bank runs from a different perspective, this chapter makes three contributions to the literature. First, we offer an alternative explanation for bank runs that are caused only by large withdrawals made by those who need money for consumption smoothing. Second, by adding smooth consumption



preferences and labor income, this chapter extends and complements the standard bank runs model. Third, this chapter also exhibits strategic complementarity in a run-proof bank contract. When setting a deposit contract, the bank manager needs to consider not only the traditional element that may cause bank runs, but also the one that may trigger large withdrawals caused by the consumption needs of agents.

The rest of chapter is organized as follows. Section 3.2 is the literature review. Section 3.3 introduces the basic model. Section 3.4 presents the extended model. And section 3.5 concludes.

## **3.2 Literature review**

Diamond and Dybvig (1983) is the cornerstone of the bank-run literature. These researchers assume banks offer a demand deposit contract to depositors who are *ex ante* unsure about their preference, and show that in a competitive market bank demand deposit contracts could improve welfare compared to the no bank case. They show, however, that the banking equilibrium is prone to panic-based bank runs.

The implication that bank runs are caused by pure panics has been widely questioned in subsequent literature. Gorton (1988) studies bank runs during the U.S. National Banking Era and states that bank runs are affected by the fundamental factors. He argues that if bank panics are random events, the relations of nominal variables at non-panic dates should not be able to describe a bank's characteristics at panic dates, an implication that is against his empirical results. Calomiris and Mason (2003) study the causes of bank distresses during the Great Depression and find that bank failure is related to international, national, and state level fundamental factors.

In the literature, bank-run models can be divided into two categories. First, information-based bank runs, in which bank runs are driven by pessimistic information about a bank's financial situation. If the information is clear and accurate, information-based bank runs could be efficient. Most models of

information-based bank runs assume that uncertainty about banks' assets exists. Jacklin and Bhattacharya (1988) introduce an exogenous bank assets' return distribution into the bank-run model and allow a proportion of depositors to obtain clear information about the prospects of the bank without observing each other's behavior. They show that there exists a threshold for bank runs to happen. When the expected return of bank assets is lower than the threshold, bank runs will happen and the threshold is inversely related to the dispersion of the return. Similarly, Morris and Shin (2000) and Goldstein and Pauzner (2005) study models that allow depositors to receive a noisy signal that is related to banks' asset return distribution. Both researchers find a threshold, below which bank runs will occur. Empirically, Loewy (1998) finds that the bank runs triggered by pessimistic information confirm certain evidence about bank runs during the 1929-1933 period. Schumacher and Liliana (2000) find evidence of information-based bank runs in Argentina.

The second category is panic-based bank runs, that is, bank runs driven by some random events, as in DD. Unlike information-based bank runs, panic-based bank runs will cause large welfare loss, so government intervention is called for. Chari and Jaganathan (1988) study a model that allows depositors to observe other depositors' behaviors. They argue that if agents misinterpret liquidity shocks as a sign of pessimistic information of future asset returns, bank runs could happen. Chen and Hasan (2008) argue that depositors' expectations about the quality of the information can affect the occurrence of bank runs. Bank runs can take place if agents believe that banks will only reveal noisy information instead of precise information. Bank runs also can be the reconciliation of both panic and information. Nikitin and Smith (2008) study a model that allows depositors to purchase the information about fundamentals (verification option). Agents withdraw funds only if they verify the inefficient banks and all other agents do the same thing.

Bank runs can also be categorized into non-contagious bank runs and contagious bank runs. Contagious bank runs are those triggered by runs of other banks. Empirically, Saunders and Wilson (1996) find evidence of contagion

effects of bank failures. They analyze the behavior of deposit flows in a sample of failed national banks and control banks during 1929 to 1933. They find that the control banks' deposit flow is positively related to the matched failing bank's deposit flow. The literature contains several explanations for the contagion effect. One explanation is information asymmetry that uninformed depositors observe the behavior of informed agents. Chen (1999) argues that when information acquisition is asymmetric, uninformed depositors may treat failures of other banks as a sign of weakness of their own banks and make early withdrawals. Vaugirard (2007) studies the cross-country spread of bank runs through an information channel. He argues that bank panics in one country will induce lenders to downgrade yields in other countries and bid lower prices to new debt issued by banks. As a result, banks become illiquid and prone to runs. The second explanation is a wealth effect that, with a decreasing absolute risk aversion, reductions in wealth will make agents become more risk averse and increase their incentives to withdraw their deposits. Kumar and Persaud (2002) argue that contagion can be explained by a reduction in investors' appetite for risks. They use the currency market as an example to show that changes in risk appetite occur and they are negatively related to the investment return. Kyle and Xiong (2001) study a model in which financial contagion is caused by the wealth effect. Goldstein and Pauzner (2004) find that a crisis in a particular country will reduce the wealth of the investors, which results in a more risk-averse attitude of investors. This increases the probability of crises in other countries. A third theory involves balance sheet connections or clearinghouse arrangements. A clearing house arrangement can solve liquidity shock problems between banks, but it makes banks prone to contagious bank runs. Aghion, Bolton and Dewatripont (2000) suggest that in a clearinghouse system if one member is insolvent, the public will treat it as a signal of an aggregate liquidity shortage and facilitate runs on the entire system. Dasgupta (2004) suggests that with inter-bank deposits, once a creditor bank fails, it will lead to runs on debtor banks. Skeie (2004) argues that bank runs can propagate through the aggregate price level. When interbank lending breaks down, one bank's bankruptcy can cause price deflation. The

deflation propagates liquidity shortages and can cause contagion of bank runs. Diamond and Rajan (2005) argue that bank failures can trigger a liquidity problem by shrinking the common pool of liquidity and creating aggregate liquidity shortages.

In reality, bank runs often cause large wealth losses and deepen economic recessions. The cost of cleaning up a banking crisis can be large—“with fiscal costs averaging 13% of GDP and economic output losses averaging 20% of GDP for important crises from 1970 to 2007” (Laeven and Valencia, 2008). In the literature, several papers discuss whether bank runs can be prevented and how to prevent bank runs. DD state that bank runs can be efficiently prevented by deposit insurance and suspension of convertibility (in the case of knowing the normal volume of withdrawals). Alonso (1996) argues that when bank runs are triggered by negative signals about banks’ investments, banks can use their knowledge about the distribution of signals to design a demand deposit contract to prevent bank runs. Cooper and Ross (1998) argue that banks can hold more liquid investments to stop bank runs, if the probability of occurrence is fixed. Alonso (1996) and Cooper and Ross (1998) argue that though a run-proof contract can forbid runs, the contract does not necessarily maximize depositors’ ex-ante utility. Smarith (2003) studies a model that compares the run-proof contract to a contract that allows bank runs. He finds that the run-proof contract is only welfare superior when the liquidation value of long-term assets is lower than the return of the bank’s assets and the probability of a low return is above a threshold. Skeie (2004) states that the chance of bank runs can be reduced in a modern banking system: if withdrawn currency is re-deposited into other banks, with an efficient interbank lending system, bank runs will be harmless. Goldstein and Pauzner (2005) argue that a demand deposit contract that efficiently reduces the probability of bank runs will increase the cost of bank runs once it occurs. Miller (2008) argues that if the government cares about solvency and stability of a currency peg, medium size banks will not experience bank runs; if foreign exchange reserves are ample or the costs of printing money are small, all banks will be immune to runs. Wanger (2009) finds that if bank owners purchase put

options on their own bank, capital financing can reduce the inefficient banks runs and also disciplines bankers.

In the literature, most of the papers rely on models in which depositors, who are not in need of funds, panic and cause bank runs. Agents withdraw money early from banks and store the money to consume in the next period, because they are afraid that bank assets will yield a lower return in the next period or that banks will go bankrupt soon. Bank runs occur just because of the uncertainty of the number of early withdrawers who are really in need of money. One explanation for aggregate uncertainty is that there are finite numbers of agents instead of infinite. When the number of agents is infinite, according to the large sample theory, knowing each agent's probability of being a certain "type", banks can know the proportion of patient and impatient agents. However, when the number of agents is finite, the proportion is uncertain. Carmona (2007) states that under a finite number of consumers, with the optimal contract that maximize depositors' expected utility, banks had a positive probability of failure. Green and Lin (2003) prove that in a finite-depositor model, a flexible demand deposit contract can solve the bank-run problem. Peck and Shell (2003) demonstrate that even with a flexible contract there still exists a bank-run equilibrium. Both Green and Lin (2003) and Peck and Shell (2003) treat the bank as a social planner that maximizes depositors' expected utility. Andolfatto and Nosal (2008) treat banks as self-interested agents and analyze flexible contracts. Their finding supports Green and Lin (2003)'s conclusion that a flexible contract could solve the bank-run problem. No matter how banks are operated, there exists a truth-telling equilibrium. The problem is that such sophisticated contracts are not observed in practice, because the flexible contract may enable banks to lie about their circumstances and pay less to investors.

This chapter offers another explanation for bank runs that are caused by large withdrawals made by illiquid depositors. The aggregate uncertainty over the number of early withdrawals could be driven by consumption smoothing with response to business cycles, whether or not there is a finite number of agents. The inspiration comes from Carmona (2007), where he suggests that we could

interpret the uncertainty as reflecting business cycle conditions. Fundamental shocks lead to a large number of early withdrawers to smooth their consumption. Carmona (2007) suggests that the number of people who need short-term funding might be influenced by the unemployment rate. However, he does not explicitly model how aggregate uncertainty was related to business cycle. In this chapter, we model aggregate uncertainty affected by business cycle through the introduction of labor income. During a recession, agents receive a lower income. To smooth consumption, they demand early withdrawals from their banks. In this case, even if a bank's long-term investment return is not affected by the business cycle, a positive probability of bank runs still exists. We also further supplement the model of illiquid withdrawal to show that except for the DD equilibriums three more Nash equilibriums exist.

Like Jacklin and Bhattacharya (1988) and Alonso (1996) modeling smooth consumption preferences, we assume that long-lived agents have positive utility functions for consumption in both periods.<sup>28</sup> To maximize utility, agents will seek to smooth consumption across time. This is different from most papers, in which an agent has only one utility function; i.e., long-lived agents only care about the sum of the two periods' consumption. In this case, smooth consumption is not necessary. Jacklin and Bhattacharya (1988) and Alonso (1996) assume that banks would offer contracts that allow depositors to make two periods' regular withdrawals, that is, agents could withdraw money in both periods.

### **3.3 Basic model**

Consider an economy populated by a continuum of ex-ante identical agents of measure one. There are three periods,  $t = 0, 1, 2$ . Each agent is born in period 0 and lives for, at most, three periods. Agents can be two different types. They do not know their own types until the beginning of period 1. With

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<sup>28</sup> Jacklin and Bhattacharya (1988) and Alonso (1996) use two-period utility functions for both short-lived and long-lived agents. Here, to simplify the problems, we assume only the long-lived agents have two-period utility functions. Allowing two-period utility functions for both agents will not change the key findings, because the key to determine bank runs are behaviors of impatient agents.

probability  $\alpha$ , an agent will be a short-lived agent (will die at the end of period 1) and will derive utility only from period 1 consumption; with probability  $1 - \alpha$ , an agent will be a long-lived agent (will die at the end of period 2) and will derive utility from both period 1 and period 2 consumption. Agents' types are independent and identically distributed. Let  $I$  be the type of agent. The utility of each agent can be presented as follows:

$$U(c_1, c_2, I) = \begin{cases} u(c_1) & I = \text{Short Lived} \\ u(c_1) + u(c_2) & I = \text{Long Lived} \end{cases} \quad (1)$$

where  $u(x) = \frac{x^{1-\rho-n}}{1-\rho}$ ,  $\rho \geq 1, n \geq 0$ .<sup>29</sup> When  $x \geq (1/n)^{\frac{1}{\rho-1}}$ , agents will have nonnegative utilities. Agents with zero consumption will have infinite negative utility.

Two technologies are involved: a riskless short-term project and a long-term project. The riskless short-term project is a storage technology. For one unit investment, it has a unit return in the next period. The long-term project is a riskless project with a fixed return  $R$ .<sup>30</sup> Banks can invest in the long-term project directly, but agents can invest in the long-term project only indirectly by depositing money into banks. Banks will offer an ex ante deposit contract  $(C_1, C_2)$  allowing depositors to choose either withdrawal  $C_1$  in period 1 or withdrawal  $C_2$  in period 2. If the long-term investment is interrupted in period 1, it will yield  $L$  per unit of period 0 investment. Here, we assume  $L < \min\{C_1, 1\}$ .<sup>31</sup> Thus, banks will put part of their resources in the storage technology to satisfy early withdrawals, since liquidation of long-term projects has a lower return than the storage technology. Banks satisfy withdrawals on a first-come, first-serve basis, and agents can observe the withdrawal behavior of others.

Agents have two sources of income. One is a unit endowment received at the beginning of period 0. The second is labor income  $k$  received in period 1. Let

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<sup>29</sup> Auerbach and Kotlikoff (1987) show that empirical estimate of  $\rho$  lies between 1 and 10.

<sup>30</sup> The fixed return assumption rules out any possible bank runs caused by failed investment. Adding the risk factor into the long-term project will not change the key findings in this chapter.

<sup>31</sup> In the case of  $L > C_1$ , bank runs will never happen. Banks can satisfy the withdrawals by liquidating their long-term projects.





The solution is:

$$\text{when } C_2 \leq k, x = \frac{k+C_2}{2}, \quad (3)$$

$$\text{when } C_2 > k, x = k. \quad (4)$$

That is, a long-lived agent will store some labor income in period 1 if and only if the labor income is higher than the bank payment in period 2.

On the other hand, long-lived agents may receive labor income low enough that they choose to withdraw from banks in period 1. If long-lived agents make an early withdrawal, then by the nature of the bank deposit contract, they will consume in period 2 only the amount they store in period 1. Thus, in this scenario, long-lived agents must decide in period 1 how much to store in the short-term project, which we denote by  $x$ :

$$\text{Max: } u(C_1 + k - x) + u(x), \text{ s. t. } k < x < C_1 + k. \quad (5)$$

Solving the problem, we get

$$x = \frac{(k+C_1)}{2}. \quad (6)$$

That is, if long-lived agents decide to make an early withdrawal, they will consume half of the money in period 1 and store the remaining half in the storage project.

### 3.3.2 Bank's problem

The bank is assumed to maximize expected utility of agents in period 0. We assume that the bank does not factor labor income into the design of the deposit contract. We make this assumption because it is both realistic and greatly simplifies the bank's problem.<sup>32</sup> Thus, the bank solves the following problem:

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<sup>32</sup> One could of course solve the more complicated problem in which the bank designs a deposit contract taking into account labour market outcomes. This is a much more complicated problem, but such a model would have the same qualitative predictions as the simpler framework considered here.

$$\text{Max: } \alpha \int u(k + C_1) dk + (1 - \alpha) \times \left\{ \int_0^{C_2} [u(k) + u(C_2)] dk + \int_{C_2}^R \left[ 2u\left(\frac{k+C_2}{2}\right) dk \right] \right\}, \quad (7)$$

s.t.

$$(1 - \alpha)C_2 = R(1 - \alpha C_1), \quad (8)$$

$$\int_0^{C_2} [u(k) + u(C_2)] dk + \int_{C_2}^R 2u\left(\frac{k+C_2}{2}\right) dk \geq \int 2u\left(\frac{k+C_1}{2}\right) dk. \quad (9)$$

The objective function here is the expected utility from the population of agents in period 0 under the assumption that only type-1 agents withdraw early.  $\alpha$  is the proportion of agents that will be short-lived agents and have the expected utility  $\int u(k + C_1) dk$  in period 1.  $1 - \alpha$  is the proportion of agents that will be long-lived agents and have expected utility  $\int_0^{C_2} u(k) dk + \int_{C_2}^R u\left(\frac{k+C_2}{2}\right) dk$  in period 1 and expected utility  $\int_0^{C_2} u(C_2) dk + \int_{C_2}^R u\left(\frac{k+C_2}{2}\right) dk$  in period 2. Condition (8) is the resource constraint. It shows that banks store part of deposits  $\alpha C_1$  in the short-term project to satisfy the liquidity need in period 1 and invest the rest of the money in the long-term project. Condition (9) states that in designing a deposit contract the bank must ensure long-lived agents have higher expected utility making withdrawals late than they would if they withdrew early. The left-hand-side of (9) is the maximum utility of the long-lived agents if they make a withdrawal in period 2. The right-hand-side is the maximum utility of the long-lived agents if they make an early withdrawal. It can be shown that (9) implies  $C_2 > C_1$ , as in other bank run models.

### 3.3.3 Early withdrawal and bank runs

In the basic model, bank runs are defined as a situation in which all long-lived agents attempt to withdraw in period 1. This could occur for two reasons. First, this model always admits a standard DD panic-based Nash equilibrium. That is, because the liquidation of long-term assets by the bank is sufficiently costly, if the long-lived agents simply believe that everyone will run, it is optimal

for all long-lived agents to run as they will receive nothing if they do nothing. In the remainder of this chapter we do not study this equilibrium, as it is well known in the literature.

The second possibility for early withdrawal by long-lived agents is when labor income is sufficiently low. Long-lived agents will make an early withdrawal if and only if they can get a higher utility by doing so. Let  $M(k)$  denote the difference between the utility of a late withdrawal and an early withdrawal given the labor income  $k$ . Bank runs will occur if  $M(k) < 0$ .  $M(k)$  can be expressed as follows:

$$M(k) = \begin{cases} 2u\left(\frac{C_2 + k}{2}\right) - 2u\left(\frac{C_1 + k}{2}\right), & \text{when } C_2 \leq k, \\ u(k) + u(C_2) - 2u\left(\frac{C_1 + k}{2}\right), & \text{when } C_2 > k. \end{cases} \quad (10)$$

Equation (10) is the case where labor income is higher than the late payment. Because  $C_2 > C_1$ , it is easy to prove that in this case the value of  $M(k)$  is always larger than zero; i.e., early withdrawal will never occur. Equation (11) is the case where labor income is lower than the late payment. We prove that in this case there exists a threshold of labor income  $\hat{k}$ , where  $\hat{k} < C_1$ . When labor income in period 1 is lower than this threshold, early withdrawal will happen. The proof is provided in the Appendix D.

Since  $k$  is determined by only the state of the economy, we can express the threshold as  $\hat{\theta} = K^{-1}(C_1, R, \alpha, \rho)$ . Once the long-lived agents observe that the state of the economy  $\theta$  is worse than the threshold  $\hat{\theta}$ , all agents will run on the bank. We get proposition 1:

**Proposition 1:** The basic model has a Nash equilibrium in which long-lived agents will run if the state of the economy is below the threshold  $\hat{\theta}$ .

Proposition 1 is similar to a finding in Goldstein and Pauzner (2005), who also state that a threshold exists in terms of the fundamental for bank runs. However, the threshold here will cause long-lived agents' early withdrawal, even

though banks' investments are not affected by the fundamental. Unlike other researchers who assume that long-lived agents will save any withdrawals until the final period, we assume that these agents will consume part of their withdrawals during the first period. In other words, long-lived agents withdraw deposits because their labor income is low. The bank runs take place because long-lived agents need to consume in period 1 but the labor income itself cannot satisfy the desired smooth consumption profile. From Proposition 1, we have Corollary 1:

**Corollary 1:** The threshold  $\hat{\theta}$  is increasing in  $C_1$ .

Corollary 1 states that as the early payment increases, the threshold for bank runs increases. It indicates that a lower early payment will reduce the chance of bank runs caused by an economic recession. This result occurs because, first, a low  $C_1$  will reduce the incentive of long-lived agents to make early withdrawal. Second, a low  $C_1$  will reduce the difference between  $C_1$  and the liquidation value of long-term projects. It is therefore easier for banks to satisfy early withdrawals. Therefore, reducing the early payment is a feasible way to prevent bank runs. This corollary is consistent with a theorem in Goldstein and Pauzner (2005), but in a different context.

### 3.4 Extended model

Next, we drop the assumption that all agents receive the same labor income. In reality, in a recession period the average income will be lower than income in an expansion period, but not all agents' income will be affected. To this end, we assume that fraction  $\beta$  of agents labor income will be affected by the economy factor  $\theta$ , while for the rest of agents  $1 - \beta$ , their income is fixed at  $\tilde{K} = K(0.5)$ .  $\beta$  is affected by the state of economy,  $\theta$ :

$$\beta = \begin{cases} 1 - \theta & \text{if } \theta \in [0, 0.5) \\ \theta & \text{if } \theta \in [0.5, 1]. \end{cases} \quad (12)$$

Equation (12) indicates that in an expansion period,  $\theta \in [0.5, 1]$ , the better the economy is, the more agents will get a higher income; in a recession period,  $\theta \in [0, 0.5)$ , the worse the economy is, the more agents will get a lower

income. Let us call long-lived agents with flexible income as type 1 agents and long-lived agents with fixed income as type 2 agents. Assume that in period 1 agents learn their type: they learn their time preference for consumption and whether their income is fixed or not, and if not, they learn whether they are in a recession or an expansion period. Type 1 agents will know the state of the economy by the level of income they received. Type 2 agents have no clear information about  $\theta$ .

In this imperfect information framework, four bank run equilibriums are possible. The first is the classic DD panic equilibrium in which all agents withdraw in period 1 simply because they expect everyone else to. We focus on the other three possible equilibriums. The first possibility is a bank run with the same basic features as the one discussed in the basic model. The other two possibilities are panic-based bank runs caused by the imperfect information of the subset of type-2 agents that cannot observe  $\theta$ . The basic idea is that these type 2 agents may make panic withdrawals in period 1 rather than wait to make a decision after observing others' behavior in period 1. For these panic-based bank runs we require an additional assumption on the liquidation value:<sup>33</sup>

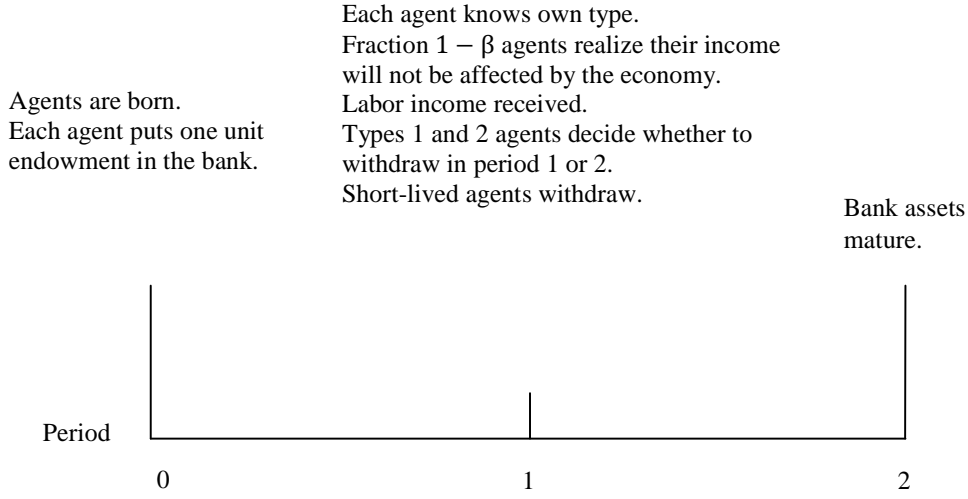
$$L < \min \left\{ \frac{0.5(1-\alpha)R}{R(1-\alpha C_1') - 0.5(1-\alpha)}, C_1', 1 \right\}. \quad (13)$$

Figure 3-4 shows the timing of events in the extended model.

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<sup>33</sup> If  $L > \frac{0.5(1-\alpha)R}{R(1-\alpha C_1') - 0.5(1-\alpha)}$ , purely panic bank runs (defined in the following paragraphs) will not occur. If  $L > C_1'$ , bank runs will never occur.

Figure 3-4: Time line - extended model



### 3.4.1 Bank's problem

In the extended model, the bank's problem is:

$$\begin{aligned} \text{Max: } & \alpha \int \{ \beta \int u[K(\theta) + C_1'] dK(\theta) \} d\theta + (1 - \alpha) \int \{ \beta \int_0^{C_2} [u(K(\theta)) + \\ & u(C_2)] dK(\theta) + \beta \int_{C_2}^R \left[ 2u \left( \frac{K(\theta) + C_2'}{2} \right) \right] dK(\theta) \} d\theta + \alpha(1 - \\ & \beta)u(\tilde{K} + C_1') + \\ & (1 - \alpha)(1 - \beta) \max \left\{ u(\tilde{K}) + u(C_2'), 2u \left( \frac{\tilde{K} + C_2'}{2} \right) \right\}, \end{aligned} \quad (14)$$

s.t.

$$(1 - \alpha)C_2' = R(1 - \alpha C_1'), \quad (15)$$

$$\int_0^{C_2} [u(k) + u(C_2')] dk + \int_{C_2}^R \left[ 2u \left( \frac{k + C_2'}{2} \right) \right] dk \geq \int 2u \left( \frac{k + C_1'}{2} \right) dk. \quad (16)$$

The objective function here is as follows:  $\alpha \int \{ \beta \int u[K(\theta) + C_1'] dK(\theta) \} d\theta$  is the expected utility of short-lived agents whose labor income is uncertain;  $(1 - \alpha) \int \{ \beta \int_0^{C_2} [u(K(\theta)) + u(C_2)] dK(\theta) + \beta \int_{C_2}^R \left[ 2u \left( \frac{K(\theta) + C_2'}{2} \right) \right] dK(\theta) \} d\theta$  is the expected utility of long-lived agents whose labor income is uncertain;  $\alpha(1 - \beta)u(\tilde{K} + C_1')$  is the expected utility of short-lived agents whose labor income is

fixed; and  $(1 - \alpha)(1 - \beta) \max \left\{ u(\tilde{K}) + u(C_2'), 2u\left(\frac{\tilde{K} + C_2'}{2}\right) \right\}$  is the expected utility of long-lived agents whose labor income is fixed. The two constraints have the same interpretations as in the basic model.

### 3.4.2 Early withdrawal

#### 3.4.2.1 Type 1 agents withdraw first

First, we consider the case in which type 2 agents wait and make decisions after observing type 1 agents' behavior. As in the basic model, in the extended model, long-lived agents whose income is affected by the economy will make an early withdrawal if their labor income is lower than the threshold  $\hat{k}$ ; i.e., if the fundamental is worse than the threshold level  $\hat{\theta}$ . In this case, to satisfy the extra early withdrawals, banks will liquidate some of their long-term investments. Here, three possibilities exist:

$$\text{Case1: } L < \beta C_1', \quad (17)$$

$$\text{Case2: } L > \beta C_1' \text{ and } \frac{R \left[ 1 - \alpha C_1' - \frac{(1-\alpha)\beta C_1'}{L} \right]}{(1-\beta)(1-\alpha)} < C_1', \quad (18)$$

$$\text{Case3: } L > \beta C_1' \text{ and } \frac{R \left[ 1 - \alpha C_1' - \frac{(1-\alpha)\beta C_1'}{L} \right]}{(1-\beta)(1-\alpha)} > C_1'. \quad (19)$$

In case 1, banks will go bankrupt because they cannot satisfy the withdrawal requirements in period 1.  $L$  is the liquidation value of the long-term investment, and  $\beta C_1'$  is the amount withdrawn by the type 1 agents.

In case 2, bank runs will also occur. Although banks can satisfy the withdrawal requirement of type 1 agents, the runs of type 2 agents will cause the banks to go bankrupt. When the economy is sufficiently bad that type 1 agents, who are substantial in proportion, make an early withdrawal, it will reduce the

late payment  $\frac{R \left[ 1 - \alpha C_1' - \frac{(1-\alpha)\beta C_1'}{L} \right]}{(1-\beta)(1-\alpha)}$ , so that the late payment is less than the early

payment  $C_1'$ . In such a situation, type 2 agents will also rush to the bank to make early withdrawals.

In case 3, bank runs will not occur. The amount of type 1 agents is small enough that after type 1 agents withdraw the money in period 1, the late payment for type 2 agents is still higher than the early payment. Therefore, type 2 agents will choose to wait for the second period and after liquidating some long-term investments, banks can continue to operate.

Since  $\beta$  is determined by the state of the economy, we conclude that an information-based bank-run equilibrium exists: bank runs will occur if the state of the economy is below the threshold:

$$\widehat{\theta}_l = \min \left\{ \widehat{\theta}, \frac{R}{R-L} - \frac{RL(1-\alpha C_1')}{(1-\alpha)C_1'(R-L)} \right\}. \quad (20)$$

### 3.4.2.2 Type 2 agents withdraw first

Now consider the case in which type 2 agents decide to withdraw immediately in period 1. If type 2 agents know that they are in a recession period, they might make an early withdrawal in period 1. Knowing that in a recession period, type 1 agents are likely to withdraw, under the first-come-first-serve policy, the best choice for type 2 agents may be to withdraw (hopefully) first. In this situation, the payment for type 1 agents in period 2 will be

$$\frac{R \left[ 1 - \alpha C_1' - \frac{(1-\beta)(1-\alpha)C_1'}{L} \right]}{\beta(1-\alpha)}.$$

If the late payment is lower than the early payment, i.e.,  $\frac{R \left[ 1 - \alpha C_1' - \frac{(1-\beta)(1-\alpha)C_1'}{L} \right]}{\beta(1-\alpha)} < C_1'$ , all agents will panic. We name this situation pure panic bank runs. That is, type 2 agents, who are substantial in proportion, make early withdrawals, it reduces the late payment for type 1 agents so that the late payment become less than the early payment. Because we assume that the number of type 2 agents is positively related to the state of economy in a recession period, pure panic bank runs are more likely to happen in the beginning of a recession



period. That is, bank runs will occur when  $0.5 > \theta > \frac{RL(1-\alpha C_1')}{(1-\alpha)(R-L)} - \frac{L}{R-L}$ . Our finding is consistent with that in Gorton (1988), who finds that bank runs tend to occur after business cycle peaks. The pure panic bank runs require that the liquidation value,  $L$  is small enough that  $L < \frac{0.5(1-\alpha)R}{R(1-\alpha C_1') - 0.5(1-\alpha)}$ .

If the late payment is higher than the early payment, bank runs still can occur: after type 2 agents make panic withdrawals, type 1 agents may also withdraw. We name this situation panic-information bank runs. Panic-information bank runs have a threshold  $\check{k}$  for the flexible income, i.e.,  $\check{\theta}$  for the economy. Bank runs happen only when the economy is worse than the threshold  $\check{\theta}$ .

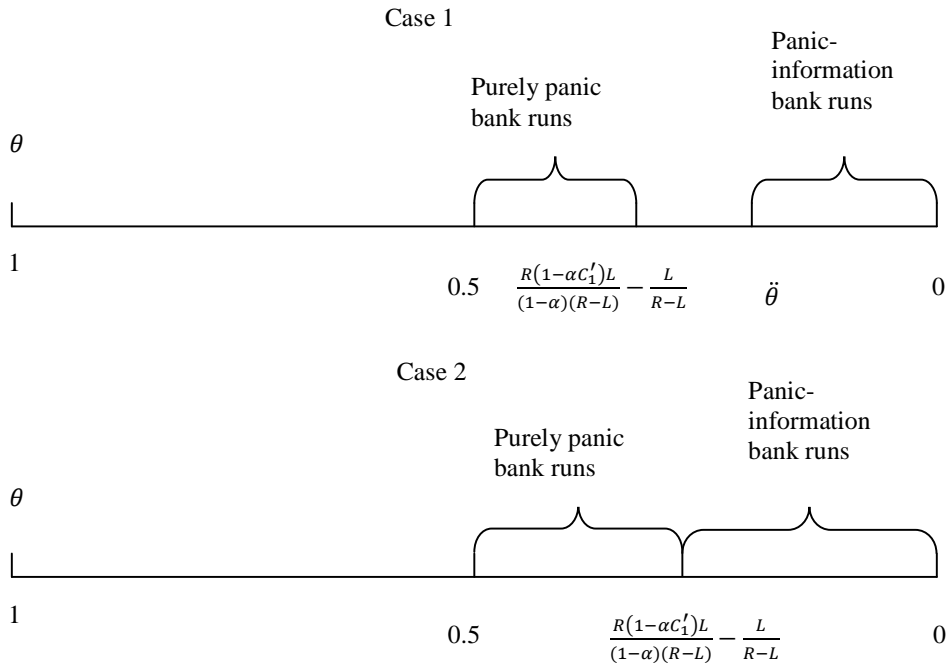
Based on the analysis above, we conclude that when type 2 agents withdraw first, two possible equilibriums exist. Bank runs can occur if the state of the economy  $\theta$  satisfies either inequality (21) or (22):

$$0.5 > \theta \geq \frac{R(1-\alpha C_1')L}{(1-\alpha)(R-L)} - \frac{L}{R-L}, \quad (21)$$

$$\theta < \min \left\{ \frac{R(1-\alpha C_1')L}{(1-\alpha)(R-L)} - \frac{L}{R-L}, \check{\theta} \right\}. \quad (22)$$

When the fundamental satisfies inequality (21), pure panic bank runs will occur; when it satisfies inequality (22), panic-information bank runs will occur. These two equilibriums will exist in different stages of a recession period, which are detailed in Figure 3-5. In case 1,  $\frac{R(1-\alpha C_1')L}{(1-\alpha)(R-L)} - \frac{L}{R-L} > \check{\theta}$ , in a recession period, there is a stage in which bank runs will not occur, even though type 2 agents will make panic withdrawals in period 1. In case 2,  $\frac{R(1-\alpha C_1')L}{(1-\alpha)(R-L)} - \frac{L}{R-L} \leq \check{\theta}$ , in a recession period, bank runs are always possible.

Figure 3-5: Panic-based bank runs



In summary, we get proposition 2.

**Proposition 2:** In the extended model, except for the DD equilibrium, there exist three more possible Nash equilibriums: information-based bank runs, pure panic bank runs, and panic-information bank runs.

In the extended model, the information-based bank-run equilibrium is determined by the state of the economy and the liquidation value of bank assets. As the economy turns bad, type 1 agents make withdrawals in period 1. If the liquidation value of bank assets is high enough, the withdrawals will not cause bank runs; if the liquidation value is low, the withdrawals will lower the late payment in period 2 sufficiently to cause the rest of the long-lived agents to rush to withdraw.

Both pure panic bank runs and panic-information bank runs are caused by type 2 agents' panic withdrawal. Due to the absence of clear information on the fundamentals, the fear of low payment drives type 2 agents to withdraw money in the beginning of period 1. Their withdrawals can be followed by the panic

withdrawals of type 1 agents, which are defined as pure panic bank runs, or by the illiquid withdrawals of type 1 agents, which are defined as panic-information bank runs. In the extended model, all bank runs will cause welfare losses. Purely panic bank runs can be prevented by setting a lower than optimal early payment, while for the information-based bank runs and panic-information bank runs, the lower early payment can only reduce the probability of bank runs.<sup>34</sup> To prevent bank runs, government intervention such as suspending withdrawals and deposit insurance may be useful as in other models. In our model, another effective method is unemployment insurance.

### **3.5 Conclusion**

This paper presented a theoretical model of bank runs from a new perspective. The paper showed that bank runs can arise purely from the joint interaction of business cycle fluctuations and consumption smoothing by households. By introducing labor income to reflect the business cycle, we showed that along with the DD Nash equilibrium, three more equilibriums are possible: information-based bank runs, pure panic bank runs and panic-information bank runs.

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<sup>34</sup> An optimal payment is the payment that maximizes the depositors' utilities.

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## Appendix D: Proofs

### 1. Long-lived agent's problem-basic model

(1) Max:  $u(x) + u(k - x + C_2)$ , s. t.  $x \leq k$

$$\text{Foc: } (x)^{-\rho} + (k - x + C_2)^{-\rho} = 0$$

$$\text{we get } x = \frac{(k+C_2)}{2}.$$

$$x \leq k \Rightarrow k - x = \frac{(k-C_2)}{2} \geq 0 \Rightarrow k \geq C_2$$

(2) Max:  $u(x) + u(k + C_1 - x)$ , s. t.  $k < x < k + C_1$

$$\text{Foc: } (x)^{-\rho} + (k + C_1 - x)^{-\rho} = 0$$

$$\text{we get } x = \frac{(k+C_1)}{2}.$$

$$x > k \Rightarrow \frac{(k+C_1)}{2} > k \Rightarrow C_1 > k.$$

Therefore, when  $k < C_1$ , early withdrawal will occur  $x = \frac{(k+C_1)}{2}$ .

When  $k \geq C_1$ , early withdrawal will not happen: if  $C_1 \leq k < C_2$ ,  $x = k$ ; if

$$k \geq C_2, x = \frac{(k+C_2)}{2}.$$

### 2. Early withdrawal-basic model

#### Proposition 1:

$$C_1 < k < C_2 \Rightarrow M(k) > 0$$

$$C_1 \geq k \Rightarrow \frac{\partial M(k)}{\partial k} \geq 0, \text{ that is, the value of } M(k) \text{ is strictly increasing in } k.$$

Because when  $k = C_1$ ,  $M(k) > 0$  and when  $k = 0$ ,  $M(k) < 0$ , there must exist a threshold  $\hat{k}$ , that when labor income is below this value, long-lived agents will prefer an early withdrawal.

**Corollary 1:**

$$M(k) = 0 \Rightarrow u(\hat{k}) + u(C_2) = 2u\left(\frac{C_1 + \hat{k}}{2}\right)$$

Take total differential:

$$u'(\hat{k})d\hat{k} + u'(C_2)\frac{-\alpha R}{1-\alpha}dC_1 = u'\left(\frac{C_1 + \hat{k}}{2}\right)(d\hat{k} + dC_1)$$

$$\left[u'(\hat{k}) - u'\left(\frac{C_1 + \hat{k}}{2}\right)\right]d\hat{k} = \left[u'\left(\frac{C_1 + \hat{k}}{2}\right) + u'(C_2)\frac{\alpha R}{1-\alpha}\right]dC_1$$

$$\text{We get } \frac{d\hat{k}}{dC_1} = \frac{\left[u'\left(\frac{C_1 + \hat{k}}{2}\right) + u'(C_2)\frac{\alpha R}{1-\alpha}\right]}{u'(\hat{k}) - u'\left(\frac{C_1 + \hat{k}}{2}\right)}$$

$$C_1 > \hat{k} \Rightarrow u'(\hat{k}) - u'\left(\frac{C_1 + \hat{k}}{2}\right) > 0 \Rightarrow \frac{d\hat{k}}{dC_1} > 0$$

$$\text{That is, } \frac{d\hat{k}}{dC_1} = \frac{dK(\hat{\theta})}{d\hat{\theta}} \times \frac{d\hat{\theta}}{dC_1} > 0.$$

$$\frac{dK(\hat{\theta})}{d\hat{\theta}} > 0 \Rightarrow \left[\frac{d\hat{\theta}}{dC_1}\right] > 0$$

**3. Early withdrawal-extended model****Proof of existence of  $\hat{k}$ :**

Let  $\dot{M}$  denotes the difference between the utility of a late withdrawal and an early withdrawal. Bank runs will occur if  $\dot{M}(k) < 0$ .  $\dot{M}$  can be expressed as follows:

$$\dot{M}(k) =$$

$$\begin{cases} 2u\left(\frac{C_2' + k}{2}\right) - 2u\left(\frac{C_1' + k}{2}\right), & \text{when } C_2' \leq k \\ u(k) + u(C_2') - 2u\left(\frac{C_1' + k}{2}\right), & \text{when } C_2' > k \end{cases}$$

$$C_1' < C_2' \leq k \Rightarrow \dot{M}(k) > 0$$

$$C_1' < k < C_2' \Rightarrow \dot{M}(k) > 0$$

$C_1' \geq k \Rightarrow \frac{\partial \dot{M}(k)}{\partial k} \geq 0$ , that is the value of  $\dot{M}(k)$  is strictly increasing in  $k$ .

Because when  $k = C_1'$ ,  $\dot{M}(k) > 0$  and when  $k = 0$ ,  $\dot{M}(k) < 0$ , there must exist a threshold  $\check{k}$ , that when labor income is below this value, type 1 agents will prefer an early withdrawal.

**Proof of existence of  $\check{k}$ :**

Let  $\ddot{M}$  denote the difference between the utility of a late withdrawal and an early withdrawals. Bank runs will happen if  $\ddot{M}(k) < 0$ . Let the late payment

$\frac{R \left[ 1 - \alpha C_1' - \frac{(1-\beta)(1-\alpha)C_1'}{L} \right]}{\beta(1-\alpha)}$  denotes as  $C'$ ,  $\ddot{M}$  can be expressed as follows:

$$\ddot{M}(k) = \begin{cases} 2u\left(\frac{k+C'}{2}\right) - 2u\left(\frac{k+C_1'}{2}\right), & \text{when } C' \leq k \\ u(k) + u(C') - 2u\left(\frac{k+C_1'}{2}\right), & \text{when } C' > k \end{cases}$$

$$C_1' < C' \leq k \Rightarrow \ddot{M}(k) > 0$$

$$C_1' < k < C' \Rightarrow \ddot{M}(k) > 0$$

$C_1' \geq k \Rightarrow \frac{\partial \ddot{M}(k)}{\partial k} \geq 0$ , that is the value of  $\ddot{M}(k)$  is strictly increasing in  $k$ .

Because when  $k = C_1'$ ,  $\ddot{M}(k) > 0$  and when  $k = 0$ ,  $\ddot{M}(k) < 0$ , there must exist a threshold  $\check{k}$ , that when labor income is below this value, type 1 agents will prefer an early withdrawal.