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Choice and Temporal Welfare Impacts: Dynamic GEV Discrete Choice Models

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Abstract

Welfare economics is often employed to measure the impact of economic policies or externalities. When demand is characterized by discrete choices, static models of consumer demand are employed for this type of analysis because of the difficulty in estimating dynamic discrete choice models. In this paper we provide a tractable approach to estimating dynamic discrete choice models of the Generalized Extreme Value (GEV) family that addresses many of the problems identified in the literature and provides a rich set of parameters describing dynamic choice. We apply this model to the case of recreational fishing site choice, comparing dynamic to static versions. In natural resource damage assessment cases, static discrete choice models of recreational site choice are often employed to calculate welfare measures, which will be biased if the underlying preferences are actually dynamic in nature. In our empirical case study we find that the dynamic model provides a richer behavioral model of site choice, and reflects the actual choices very well. We also find significant differences between static and dynamic welfare measures. However, we find that the dynamic model raises several concerns about the specification of the policy impact and the subsequent welfare measurement that are not raised in static cases.

Key Words: Dynamic choice, GEV choice models, welfare measurement

1.0 Introduction

Welfare economics is often employed to measure the impact of economic policies or externalities, typically using static models of consumer demand. However, if demand has dynamic properties, perhaps characterized by habits, learning, state dependence, or consumption inertia, the impacts of policies or externalities will necessarily have temporal dimensions. Heckman (1981), for example, describes the issue of economic policies to alleviate unemployment, and their potential to reduce long term unemployment by having a long-lasting effect on future unemployment rates. In an environmental context, consider the impact of a chemical spill on beach recreation that may drive recreationists away from the affected site and change their visitation pattern so that they do not return to the affected site even long after the physical impact has disappeared. In transportation research, assessing the impacts of tolls, new alternatives (e.g. light rail transit systems) or congestion also involves dynamic elements. Users of transport systems are influenced by their experiences and may exhibit habit persistence or variety-seeking in transportation mode choice and shopping destination selection.

Economic assessments of policy options that do not take these dynamic behavioral factors into account will likely be biased. Economic policies like taxes or fees may not initially have an impact on demand as the inertia associated with consumption continues to "fuel" demand until the consumers adjust their expectations about the costs associated with the good or service. While casual observation suggests that such dynamic effects occur, there has been relatively little empirical implementation of dynamic welfare analysis in the economics literature, especially where demand involves discrete choices. To a large degree this arises because of the difficulty in estimation of utility-consistent dynamic discrete choice models of consumer choice.

Heckman (1981) provides a summary of the key issues involved in dynamic discrete panel data: *state dependence* (current preferences being affected by previous choices), *habit persistence* (current preferences being affected by previous preferences), *initial conditions* (lack of knowledge about preferences before the observation period available to the analyst), and *taste heterogeneity* (taste differences between individuals and over time). The literature on dynamic choice model estimation has progressed in multiple directions since Heckman's contribution. Several researchers have explored the use of "previous choice" within static discrete choice model structures in a relatively *ad hoc* fashion (among others, Morikawa 1994 and Adamowicz 1994). However, these methods suffer from the "initial conditions" problem, which renders estimates inconsistent. Methods that account for state dependence and heterogeneity and address the confounding associated with heterogeneity and unknown initial conditions include Heckman (1981), Hensher et al. (1992), Rossi et al. (1996) and Degeratu (1999). Heckman originally proposed a number of solutions to the initial conditions problem, the most tractable of which is the approximation of the initial time period's probability distribution by a model estimated on the first period's choices. Degeratu (1999) refers to this general approach as the correlated initial conditions approach. A variety of approaches have been presented to address this issue, including Keane (1997), Degeratu (1999), and Roy et al. (1996). Keane, for example, alludes to the difficulties associated with the initial conditions problem by saying that "… [the initial value problem] is a problem of awesome computational and economic complexity." (Keane 1997, p. 315) Thus, it is not surprising to note that a thread running through much of this literature is the difficulty in developing a model that is utility-consistent, addresses state dependence and initial conditions (as well as other structural aspects of dynamic models), and yet is tractable.

An alternative approach to the entire issue of dynamic choice is to represent the decision maker as formally solving a dynamic programming problem and to embed the statistical estimation process within this dynamic programming framework. Examples of uses of discrete choice dynamic programming methods are discussed in Wolpin (1996), Erdem and Keane (1996) and Rust (1987). Applications include replacement of capital equipment (Rust, 1987), recreational fishing site choice (Provencher and Bishop 1997), and brand choice in consumer packaged goods (Erdem and Keane 1996). These are structural modeling approaches to the dynamic choice problem that examines hypothesized underpinnings of the dynamic choice process rather than simpler reduced forms. The main advantage of structural models is that they address the so-called "Lucas critique," since the parameters generated in these models are constructed to be unaffected by policy changes, and thus produce unbiased assessments of the response to policy change (Erdem and Keane 1996; Lucas 1976). While appealing in theoretical form, they are relatively difficult to estimate and at times produce results that appear counter-intuitive (e.g. Provencher and Bishop, 1997).

In this paper we focus on an application of discrete choice modeling to environmental valuation, specifically to modeling recreation site choice. It is somewhat surprising, though understandable given the associated challenges, that there have not been more applications involving temporal aspects in the environmental economics literature, given the importance of dynamics, learning, state dependence and habit persistence in activities related to natural resources and the environment. In the literature on environmental valuation the applications of dynamic choice models include Adamowicz (1994); McConnell et al. (1990) and Provencher and Bishop (1997). Smith (1997), reviewing the literature in this area, discusses the importance of temporal dimensions in environmental valuation. The issues are perhaps best addressed by an example. Much of the recreation valuation literature focuses on developing welfare estimates associated with damages to the natural environment – with the use of these models aimed at natural resource damage assessment (see Chapman et al. 1998 and Dunford 1999 for a discussion on one such case – the American Trader). In these cases the welfare impacts associated with the damages are usually assessed using static models, and then the impacts on aggregate visitation rates are examined in separate models that examine time series of recreational site use. If consumers exhibit explicit dynamic elements like habit persistence, then welfare estimates from the static models will be significantly biased. For this reason we turn our attention to the development of a dynamic model of choice, and compare welfare measures from this model with those of a static model.

In this paper we provide a tractable approach to estimating dynamic discrete choice models of the Generalized Extreme Value (GEV) family that addresses many of the problems identified in the literature with extant approaches and provides a rich set of parameters describing dynamic choice. The model is shown to be consistent with stochastic utility maximization through the conditions of the Generalized Extreme Value Theorem (McFadden, 1978; Maddala, 1983). The modeling approach allows for habit persistence, state dependence, alternative specifications of initial conditions, future expectations, time-varying tastes, as well as time-varying covariance structures. The specification of initial conditions as a parameter estimated in the model provides information to address the initial conditions problem (Heckman, 1981) and allows untangling its effect from that of state dependence. Time-varying covariance structures, including changes in both diagonal and off-diagonal elements, which lead to dynamic heteroscedasticity and cross-substitution effects, are features that have received relatively little attention in the dynamic choice modeling literature. The proposed class of models is consistent with the GEV structure, and thus can be (1) empirically tested for consistency with stochastic utility maximization and (2) estimated in a relatively straightforward fashion. A specific form of the model is applied to a case of recreational fishing in which expected environmental attribute levels (catch rates) vary over time. Welfare measures for policy simulations, including changes to access costs and changes in environmental quality, are examined and compared for static and dynamic models. The dynamic models illustrate the persistence of perceived damages beyond the time period of the actual "physical" impact and show a corresponding welfare impact that is larger than that generated from static models.

The paper is structured as follows: the next section introduces the proposed class of GEV dynamic choice models, after which we introduce the data utilized in the empirical study and discuss model estimation results; following, several typical policy scenarios are contrasted using a standard model and the richer dynamic GEV model derived from our approach; the paper concludes with a summary of our contribution and suggestions for future research.

2.0 Dynamic Choice Models

Heckman (1981) presents a general structure for discrete panel data models in which the latent element y_t, a continuous variable (utility) underlying the choice of a particular alternative whenever a threshold is crossed, is expressed as a function of attributes of the alternative facing the consumer in the current time period (Z_t), utilities (or attributes) from previous time periods, decisions made in previous time periods, and an error term. The utilities from previous time periods reflect habit persistence, while the decisions made in previous time periods reflect state dependence. This general structure (as presented by Heckman 1981) can be written as

$$y_{it} = Z_{it}\,\beta + \sum_{j=1}^{\infty} \delta_{t-j,t}\, d_{i,t-j} + \sum_{j=1}^{\infty} \lambda_{j,t-j} \prod_{l=1}^{j} d_{i,t-l} + G(L)\, y_{it} + \varepsilon_{it} \quad . \qquad (1)$$

When y crosses a specified threshold, the consumer chooses that alternative (or, as Heckman states, "the event occurs") and d, the indicator of the event or choice, will equal 1, otherwise zero. Z is a vector of exogenous variables, (β, λ, δ) are parameters and $G(L)$ is a lag operator. Thus, the "utility" associated with alternative i in time period t is decomposed into a random component (ε) and a component containing exogenous and endogenous variables. These variables are exogenous "attributes" (that may include past, present and expectations of future exogenous elements), previous choices (state dependence), and previous utilities (habit persistence).

In our formulation of this dynamic model, we imbed a dynamic expression of the form above in a Generalized Extreme Value (GEV) structure. The reason for this approach is that McFadden (1978) showed that if a probabilistic structure satisfies the conditions of the GEV theorem, it is consistent with stochastic utility maximization. This tie-in permits the empirical testing of consistency with stochastic utility maximization. Although our empirical analysis employs the more complex Tree Extreme Value (TEV) models, to aid in presentation we first develop the model for the multinomial logit (MNL) case (thus restricting the covariance matrix); subsequently we relax this and other restrictions to examine the case of any member of the GEV family.

2.1 MNL Panel Data Model

Let the GEV generation function be

$$G(y_t \mid I_t) = \sum_{j=1}^{J} \prod_{s=0}^{t} \left(\alpha_{js} y_{jt-s}\right)^{\mu_t} , \qquad (2)$$

where t=1,…,T, is a discrete time point, j is one of J alternatives, I_t is the information set at time t (as noted above, this included past choices, past and future utilities, current and past attributes, etc.), $y_t = (y_{1t},...,y_{Jt})$, and unknown parameters are

Initial utilities: $y_0 = (y_{10},...,y_{J0})$ (3a)

Past dependence: $0 \le \alpha_{js} \le 1,\ \alpha_{j0} \equiv 1, \forall s,\ j = 1,...,J, 0 \le s \le t,$ (3b)

Scale factors: $\mu_t \ge 0, t = 1,...,T.$ (3c)

This describes a set of utilities (y) linked through time by dependence on previous utilities and initial utilities, and associated with a set of time-varying scales (or inverse of variance) terms. The initial utilities are also parameters estimated in the model and are essentially averages over the sample of the expected initial utility.[1] We now show that this is a valid GEV generating function and derive the probability expressions arising from this form.

Proposition 1:

Function (2) is a valid GEV generating function that satisfies the conditions of the GEV Theorem[2] if $\mu_t \ge 0, t = 1,...,T$.

Proof.

(2) is μ_t-homogenous, as demonstrated below for any $\delta \ge 0$:

$$G(\delta y_t \mid I_t) = \sum_{j=1}^{J} \prod_{s=0}^{t} \left(\alpha_{js} \delta y_{jt-s}\right)^{\mu_t} = \delta^{\mu_t} \sum_{j=1}^{J} \prod_{s=0}^{t} \left(\alpha_{js} y_{jt-s}\right)^{\mu_t} = \delta^{\mu_t} G(y_t \mid I_t) .$$

Further, the sign requirements in the GEV Theorem will be met for any non-negative values of μ_t. This restriction is determined from the second-order partial derivative of $G()$ with respect to distinct y_{jt} and y_{kt} in the current time period:

$$\frac{\partial^2 G}{\partial y_{jt}\partial y_{kt}} = \frac{\partial}{\partial y_{kt}}\left[\frac{\partial G}{\partial y_{jt}}\right] = \frac{\partial}{\partial y_{kt}}\left[\mu_t y_{jt}^{\mu_t - 1}\left(\prod_{s=1}^{t}\left(\alpha_{js} y_{jt-s}\right)^{\mu_t}\right)\right] = 0 .$$

All higher-order derivatives will be zero also, hence the alternating sign requirements of the GEV Theorem are trivially satisfied.

Q.E.D.

Given that the form presented satisfies the requirements of the GEV theorem, we now turn to the development of the choice probabilities.

Proposition 2:

Choice probabilities resulting from GEV function (2) are

$$P(i \mid I_t) = \frac{\exp(\mu_t \tilde{V}_{it})}{\sum_{j=1}^{J}\exp(\mu_t \tilde{V}_{jt})}, \tag{4a}$$

where

$$\tilde{V}_{it} = \sum_{s=0}^{t}\left(V_{it-s} + \ln \alpha_{is}\right), i = 1,\dots, J, t = 1,\dots,T , \tag{4b}$$

$y_{it}=\exp(V_{it})$.

<u>Proof.</u> Substitute (2) and its first-order derivative with respect to y_{it} into the expression for GEV choice probabilities; suitable rearrangement will result in (4a,b).

Q.E.D.

Proposition 2 shows that the probability expressions arising from the GEV model are relatively straightforward, with a meta-utility $\tilde{V}_{it}$ that is a function of utilities in current and previous periods, initial utilities, as well as past-dependence parameters. Depending on the choice of the functional form for the meta-utility, a number of special cases arise. In the empirical analysis presented later we employ a simple geometric decay formulation[3] of the model, as follows: let $\alpha_{js} = \rho_j^s, 0 \leq \rho_j \leq 1, j = 1,..., J, s = 0,..., t$; then the pseudo-utility function (4b) becomes simply

$$\tilde{V}_{it} = \sum_{s=0}^{t} \left(V_{it-s} + s \ln \rho_i \right), i = 1,..., J, t = 1,..., T, \tag{5}$$

and choice probabilities are given by (4a) using utilities (5). Note that the decay parameters estimated provide information on the degree to which previous utilities affect current choices, with large values of ρ corresponding to long "memories" or high degrees of habit persistence. Note also that the decay factors are alternative-specific, allowing for differing degrees of decay across alternatives.

2.2 General GEV Panel Data Models

The MNL Panel Data Model presented above is actually a special case of a class of GEV panel data models. Let

$$\tilde{y}_{it} = \left(\prod_{s=1}^{\infty} \gamma_{is} y_{it+s} \right) \left(y_{it} \right) \left(\prod_{s=1}^{t-1} \alpha_{is} y_{it-s} \right) \left(\alpha_{it} y_{i0} \right), \tag{6}$$

where all quantities are as previously defined. The terms on the right-hand side of (6) are, respectively, from left to right,

1. discounted expectations of future utilities (with discount factors γ_{is});
2. current utility;
3. discounted past utilities within the observation window (discount factors α_{is}); and
4. discounted utility of observations before the observation window (discount factors α_{it}).

Redefining the first term, let

$$w_i = \left(\prod_{s=1}^{\infty} \gamma_{is} y_{it+s} \right). \tag{7}$$

Essentially, we subsume all effects of future expectations on current utility into w_i. Thus,

$$\tilde{y}_{it} = w_i (y_{it}) \left(\prod_{s=1}^{t-1} \alpha_{is} y_{it-s} \right) (\alpha_{it} y_{i0}) = w_i \left(\prod_{s=0}^{t} \alpha_{is} y_{it-s} \right) \tag{8}$$

where we require, as before, that $0 \le \alpha_{is} \le 1,\ \alpha_{i0} \equiv 1, \forall i, 0 \le s \le t.$ This simplification leads to utility functions of the form

$$\tilde{V}_{it} = \ln w_i + \sum_{s=0}^{t} (V_{it-s} + \ln \alpha_{is}),\ i = 1,\dots, J,\ t = 1,\dots,T. \tag{9}$$

Unfortunately, this parameterization results in confounding between w_i and alternative-specific constants in V_{it}.

This leads to suggesting an alternative parameterization. Consider that the term in (6) capturing future expectations can be expressed as a power of current utility, like so:

$$y_{it}^{\phi_{it}} = \left(\prod_{s=1}^{\infty} \gamma_{is} y_{it+s} \right), \tag{10}$$

for some $\phi_{it} \geq 0$. Therefore, (6) can be rewritten as

$$\tilde{y}_{it} = y_{it}^{1+\phi_{it}} \left(\prod_{s=1}^{t} \alpha_{is} y_{it-s} \right), \tag{11}$$

for which meta-utility functions are given by

$$\tilde{V}_{it} = (1 + \phi_{it}) V_{it} + \sum_{s=1}^{t} (V_{it-s} + \ln \alpha_{is}),\ i = 1,\dots, J,\ t = 1,\dots,T. \tag{12}$$

This parameterization makes the hypothesis of the impact of future expectations testable: if $(1 + \phi_{it})$ is not significantly different from one, future expectations do not impact current utilities at time period t. Estimating a large number of ϕ's by alternative and time period is impractical, so empirical applications are likely to be more parsimoniously specified by applying the restriction that the ϕ's vary only by alternative.

We can now consider any μ_t-homogenous GEV generating function $H(\tilde{y}_t \mid I_t)$, where $\tilde{y}_{it}$ is given by (11). This leads to a generalization of the GEV family from cross-sectional to conditional time-series models that permit modeling

1. Initial conditions (y_{i0}),
2. Future expectations (ϕ_{it}),
3. Temporal dependence on past utilities, or habit persistence and variety-seeking (α_{js}),
4. State dependence (via inclusion of past choices in the information set I_t),
5. Time-varying scales and covariance structures (by making certain parameters specific to H() be time-varying), and
6. Time-varying tastes (identification restrictions apply, however).

With respect to point (5), note that meta-utilities (12) can be used in *any* GEV model. We have shown a more restricted version of (12) applied to the MNL model (see 4a,b), but as shown in Table 1, one can also develop dynamic Tree Extreme Value (TEV — see McFadden 1981) and Nested MNL models,[4] with and without time-varying covariance structures, as well as other GEV model forms (to cite a few, Ordered GEV, Small 1987; Paired Combination Logit, Chu 1989, Koppelman and Wen 2000; Cross-Nested Logit, Vovsha 1998; Choice Set Generation Logit, Swait 2000).

--- Insert Table 1 ---

Since these models are developed within the GEV framework, empirical assessment of consistency with stochastic utility maximization is possible. Some members of the GEV family, made dynamic through the use of meta-utilities (12), are shown in Table 1. Note that in the 2-level TEV with a constant tree structure, the parameters θ_k do not change over time, implying that the covariance matrix is fixed over time, whereas the 2-level TEV with time-varying structure allows the covariance matrix to change over time, thus exemplifying point (5) above. Figure 1 outlines the general structure for such 2-level TEV models, with an overall “root” node, followed by nodes for time periods, which are themselves followed by the usual tree structure represented by clustering or nesting of elemental alternatives; the tree structure is repeated under each time period node.

--- Insert Figure 1 ---

Table 1 also presents the conditions for each model to be consistent with stochastic utility maximization, which require that (1) the scale terms in the tree structure be greater than the time scale terms μ_t for all time periods, and (2) the time period scales be greater than the overall scale μ. Basically, lower level scale terms must be greater than scale terms higher up the tree.

Estimation requires decomposition of the observation’s likelihood into a product of conditional probabilities. Assuming individual parameters stacked in ψ_n, the likelihood of a choice sequence for individual n over T time periods is simply

$$\begin{aligned} P_n(\delta_{nt}, \delta_{nt-1}, ..., \delta_{n1} \mid \psi_n) &= \int_\psi \left(P_n(\delta_{nt} \mid \delta_{nt-1}, ..., \delta_{n1}, \psi_n) P_n(\delta_{nt-1}, ..., \delta_{n1}, \psi_n) \right) f_t(\psi) d\psi \\ &= \int_\psi \left(\prod_{t=1}^{T} P_n(\delta_{nt} \mid I_{nt}, \psi) \right) f_t(\psi) d\psi \end{aligned} \qquad (13)$$

where δ_{nt} is a vector of indicator variables for the chosen alternative in period t=1,…,T, person n, I_{nt} is the information set at period t, and $f_t(\psi)$ is the time-varying distribution describing the occurrence of parameters in the population. For a random sample of N decision makers, maximization of the log likelihood

$$L(\psi_n) = \sum_{n=1}^{N} \ln\left(\int_{\psi} \left(\prod_{t=1}^{T} P_n(\delta_{nt} \mid I_{nt}, \psi) \right) f_t(\psi) d\psi \right) \tag{14}$$

will yield consistent and unbiased estimates of the taste vector and dynamic process parameters included in ψ.

Consistency with stochastic utility maximization also allows the computation of welfare measures based on the theory of welfare in GEV models. Choi and Moon (1997) provide an overview of GEV models and their corresponding welfare measures. In our case this allows for the computation of welfare measures that contain habit persistence, state dependence and future expectations as arguments. Many policy-relevant applied analyses have been conducted without consideration of dynamic elements because of the difficulties in developing tractable models of choice and welfare in a dynamic context. Several authors (see the review in Smith, 1997) have discussed the importance of this omission. Thus, development of models that permit welfare analysis in a dynamic context will represent a significant contribution to the literature and to policy analysis.

We now turn to an application of this model to a case of recreational fishing in Western Australia.

3.0 An Application to Recreational Fishing Site Choice

Dynamic behavior is observed in many markets and activities. Recreational fishing choices are examined in this paper because of the inherent dynamics in the situation, and because of the policy relevance of the examination of welfare impacts of policy or environmental changes. In many cases of environmental impacts on water quality, recreational angling is one of the key activities affected and is often analyzed for the determination of damages or welfare losses. If an impact is experienced at a specific site, a key question is whether the impact is felt only during the duration of the physical impact, or if the welfare impact persists over some time period. In our empirical analysis we develop a dynamic model of recreational fisheries choice that allows us to investigate this issue.

3.1 Description of Data

Data for the application was collected by issuing a log book to 68 shore anglers who reside in the metropolitan area of Perth, capital city of Western Australia. Survey participants were asked to record details of their fishing trips over a period of 20 weeks, the last 19 of which are used in our analysis. Details included their trip destination, trip costs, catch expectations prior to going fishing (by three target fish types) and actual, ex post catches. While it would have been ideal to elicit catch expectations for every site in the choice set each time a trip was made, this was not practical owing to the excessive burden it would have placed on respondents. Instead, expectations were only elicited for the respondent's intended destination. These data were then used to estimate an "expectations function" which was used to predict weekly catch rate expectations for each site in the choice set. Respondent perceptions about the size and diversity of fish at each site were collected at the commencement of the survey period. A copy of the survey instrument is contained in van Bueren (1999).

Complete records for 671 trips were obtained from the returned log books. For the purposes of estimating the choice models presented in this paper, three sites were specified which comprised an aggregate of individual fishing locations. Site 1 attracted the majority of trips (55%), whilst fewer trips were taken to Sites 2 and 3 (35% and 10% respectively) owing to their greater distance from the residence of most respondents in the sample. Catch rates were highly variable over the survey period, reflecting the migratory and feeding behavior of the fish types involved. The abundance of Fish Type 1 tended to increase towards the end of the season at most fishing locations, while catches of Fish Type 2 declined. This variability strengthens the case for specifying a choice model that allows the spatial distribution of catch expectations to change through time.

3.2 Estimation Results

Five models are estimated on the panel data described above and presented in Table 2. All models are TEV variants, with the tree/covariance structure reflecting a separate "Go Fishing" versus "Stay Home" partition, with the fishing sites (1, 2 and 3) being the elemental alternatives under the "Go Fishing" branch; the elemental "Stay Home" alternative is indexed "4" in Figure 2, which presents the entire model structure graphically.

--- Insert Table 2 ---

--- Insert Figure 2 ---

The first model is a simple cross-sectional TEV model that includes no dynamics in the sense that the meta-utilities are restricted to be equal to current utilities: there are no previous utilities considered, and the time-varying scale parameters are restricted to unity, though the model does include state dependence through the "Last Site Fished?" dummy.[5] The second model includes time-varying scales and inclusive value parameters for the "Go Fishing" nest, which makes it equivalent to a series of TEV models that differ through period-specific covariance matrices (both variances and covariances change in this model). The third model adds to the second dynamics (initial conditions, past utilities, etc., as per meta-utility (12)), but no future expectations. The fourth model adds this latter dynamic feature, while the final model applies certain restrictions to the fourth model (subsequently explained). Note that all models except the first assume that the identifiable tree scale factors ($\theta_{GF,t}$ in Figure 2) are time-varying. These 5 models are referred to as:

1. TEV
2. TEV + Cov(t)
3. TEV + Cov(t) + Dynamics
4. TEV + Cov(t) + Dynamics + Expectations
5. TEV + Cov(t) + Dynamics + Expectations (Restricted)

The results for the basic TEV model show that angler site choice is affected by the length of coast at that site, costs of access, angler perceptions of fish size, and if the site was visited on the previous trip (i.e. first-order state dependence, which is the only dynamic feature of this simplest model). Note that expected catch rates appear to not strongly influence site choice, as measured through the significance of the respective parameters. The tree scale parameter for "Go Fishing" versus staying at home is highly significant and approximately equal to $\hat{\theta}_{GF} \approx \exp(1.9285) \approx 6.88$ (all scale parameters estimated must actually be exponentiated to yield the estimate of scale; see notes in Table 2). Note that this scale parameter and the commonly estimated inclusive value parameter are simply inverses one of another. Hence, the usual stochastic utility consistency tests which require that inclusive value parameters be between zero and one, and more generally, that they decrease as one goes deeper into the tree, are simply inverted to require that scale factors increase monotonically down the tree. (See Borsch-Supan 1990 and Herriges and Kling 1996 for further details on testing stochastic utility maximization consistency.) For Model 1, as we stated above, the tree scale is approximately 6.88, so the inclusive value term is about 0.145;

since the root scale and inclusive value are unity, we can consider Model 1 to be consistent with utility maximizing behavior.

Continuing our examination of Model 1, note that the number of weeks that the angler does not go fishing has a quadratic relationship with probability of site choice. Initially, the longer an angler stays at home, the less likely he/she is to go fishing the next week. However, after some time staying home (estimated to be about 10 weeks in this data) the probability of continuing to stay home decreases, implying an increase in the likelihood of going fishing. This effect of weeks without fishing is strongly significant in the static model, but as we shall see, it becomes less relevant in the dynamic versions of the model. Note that state dependence (introduced via the dummy variable "Last Region Fished?") is significant and distinguishable, in the "Stay Home" alternative, from the effect of weeks without fishing.

The Retiree and Fishing Club Member dummies indicate that being in these categories influences the probability of choosing any fishing site (versus choosing to not go fishing). Retirees are more likely to go fishing while Fishing Club Members, somewhat surprisingly, are not as likely to go fishing. A possible explanation for this result is that club members were found to have higher income, which is negatively correlated with fishing activity participation.

Model 2 enhances the prior model by permitting the covariance matrix (i.e. both heteroscedasticity and inter-alternative substitutability) to vary over time. Thus, scale factors for each time period are estimated, as are "Go Fishing" node scale factors within each period. (The corresponding "Stay Home" scale factors are not identified, of course, since only one alternative is present in that nest. Accordingly, in Model 2, as well as subsequent models, those scale factors are set equal to the scale of their parent node in the tree as a normalizing condition – see Figure 2.) This model, when compared to Model 1, generates a chi-squared statistic of 220.2 with 34 degrees of freedom, which is statistically significant at the 95% confidence level. Much of the improvement seems to be due to the highly significant variation in time period and "Go Fishing" scale factors, since there is very little change in the parameters of the utility function (except for linear scaling effects due to time-varying scales). Looking ahead, it should be noted that the introduction of the time-varying covariance matrix is the single most important improvement in the goodness-of-fit of the models tested in this exercise. Finally, observe that Model 2 is consistent with utility maximization, as evidenced through a period-by-period comparison of the tree node and time period scale factors.

Model 3 generalizes Model 2 through the inclusion of dynamics, in the form of (1) memory decay, or habit persistence, effects and (2) initial utilities. This addition is highly significant: the chi-squared statistic is 69.3 with 6 degrees of freedom. In Model 3 the weeks without fishing parameters are no longer significant, as they were in the first two models, neither of which included true dynamic effects. This indicates that when dynamics are explicitly included in the model, *ad hoc* "dynamicizing" measures are likely to become less significant. Despite the estimation of initial utilities in this model, note that the last region fished variable is still significant, indicating a degree of state dependence. The parameters on the initial utilities are significant and indicate that the "Stay Home" alternative has the highest overall sample average initial utility, while the fishing sites have very similar, but lower, initial utilities. In general, the significance of the initial utility estimates is likely to be a function of the length of the observation window: the longer the window, the less significant these parameters should be. Finally, this model is *not* consistent with utility-maximizing behavior since the tree scale in period 13 (exp(0.8739)≈2.40) is smaller than the corresponding period scale (exp(1.1177)≈3.06). In addition, the scale factors for periods 16 and 17 are both smaller than the unit-valued root scale factor.

The decay factor parameters estimated in Model 3 are all highly significant. (Due to identification restrictions, the factor for one alternative must be held constant. In this model, the decay factor of “Stay Home” was established by the optimization process, then held constant to permit identification of remaining parameters.) The parameters in Table 2 imply decay factors of about 0.66 for all four alternatives (see Note 3 of Table 2), which means that after about 8 periods the impact of a period’s utility is about 4% of its original value. This is a relatively strong persistence effect, particularly given the short duration of the fishing season in this data.

The addition of future expectations to Model 3, resulting in Model 4 – the full model, again produces a significant improvement (chi-squared of 26.9 with 4 degrees of freedom) over and above the inclusion of “backward-looking” dynamics. Initial utilities are similar to those in Model 3, but there is now better differentiation between the values for the three fishing sites. The expectation weights are representations of how much weight is placed on future conditions when making current decisions. The estimation results show expectation weights of 2.85, 1.46, 2.62, 3.31, for Sites 1, 2, 3, and staying home, respectively (in this list, only the first value is statistically different from unity at the 90% confidence level). Scale factors indicate violation of consistency with utility maximization in exactly the same pattern as evidenced in Model 3.

Model 5 is a restriction of Model 4 that (1) imposes constraints on several time period scale factors and period 13 “Go Fishing” tree node scale factor, and (2) removes the weeks without fishing variable from the utility function for the “Stay Home” alternative (see Table 2 for implementation details). Using a likelihood ratio test, the hypothesis for the restrictions cannot be rejected based on the calculated chi-squared value of 6.1, with 6 degrees of freedom. Under this somewhat simplified model, behavior is found to be consistent with utility maximization.

To contrast these five models, we present in Figure 3 a comparison of the predicted correlation between alternatives in the “Go Fishing” cluster over time. Models 2 through 5, all of which permit the covariance matrix to vary over time, generally predict correlations that are higher than Model 1, which we consider a default, or base, specification. The only significant exception to this is in period 13, where several of the models predict a *negative* correlation within the cluster. This, of course, is a direct result of violating the GEV conditions for utility maximizing behavior in Models 3 and 4. Thus, in the final model (5), the net effect of the restrictions on the scale factors is to produce a correlation of 0 in period 13, which is to say, the observed behavior is consistent with Independence of Irrelevant Alternatives in that period. Examination of the raw data and other sources has not helped to elucidate this peculiar feature of the data, particularly noticeable since periods 14-19 seem in line with those preceding 13.

--- Insert Figure 3 ---

Figure 4 displays simulations of the predicted probabilities for each week within the sample for two of the five models (base Model 1 and Model 5), plus the observed shares for the “Stay Home” alternative and Site 1. Note how the model with dynamics appears to fit the sample well in the early periods (due to initial utility estimates) and in the later periods (as future expectations of the end of the season begin to affect behavior). The static model seems to reflect a "smoothing" of the variability in week-to-week site choice variation by anglers, rather than capturing the variation. Overall, Model 5 appears to track the observed data quite well, especially at the endpoints of the series.

--- Insert Figure 4 ---

3.3 Welfare and Policy Analysis

As described above, one of the most important uses of choice models of this type is the assessment of environmental impacts or policy changes in terms of welfare and/or changes in behavior (demand). The theory of welfare measurement for cross-sectional GEV models has been well documented by Choi and Moon (1998). In this case we assume that there are no income effects, and apply the standard analytical form for the expected value of compensating variation. We shall employ two models in our comparisons: Model 1 (base cross-sectional TEV) and Model 5 (all dynamic components TEV with restrictions).

We examine next simulations of impacts of four different scenarios or policies, to wit:

Policy 1: An increase in the price (access cost) of 100% for all angling alternatives starting in week 4 and proceeding throughout the observation period;

Policy 2: The closure of Site 2, beginning in week 4 and continuing until week 7 (a total of one month);

Policy 3: Deterioration of fishing quality (movement to the lowest level of the scale) at Site 1, again for weeks 4 to 7; and

Policy 4: Deterioration of fishing quality at Site 1 for weeks 13 to 19, which is simply Policy 3 delayed for 9 more weeks into the season.

Policy 1: Price Increase

Policy 1 results in welfare losses for all weeks starting with week 4, with the dynamic model producing much larger per trip and cumulative welfare losses (area under the curves in Figure 5). In fact, Model 5's welfare impacts are on the order of 8 times larger than those of Model 1's, accumulated over the observation period. The negative impacts of the price increase are compounded in the dynamic model since previous utilities affect current utilities, making the utility of staying home much more attractive than would otherwise have been the case. These results are not unexpected since the angler's habits associated with fishing have been negatively affected by the price increase. Note how Model 1, omitting dynamics, predicts that the impact of the policy is uniform throughout the final 16 weeks of the observation period; in contrast, Model 5 predicts a pattern of worsening welfare losses through about week 12, and after that a gradual flattening of the loss at about $1.75/trip.

--- Insert Figure 5 ---

Policy 2: Site Closure for 4 Weeks

Policy 2 provides much more interesting dynamics, and also presents interesting conceptual problems to be simulated. Beach closures or fishing area closures due to chemical spills or other similar incidents are often the focus of natural resource damage assessment (NRDA) cases. In this simulation, Site 2 is closed for 4 weeks, that is, it is made unavailable to anglers for this period. To simulate this closure effect, two approaches were taken: (a) fuel costs for Site 2 were made extremely large during the affected period, effectively creating large disutilities for the location, which then were used in subsequent periods as part of determining current preferences; (b) Site 2 was removed from the choice set during the affected weeks, but the utility of the site during those weeks was impacted by increasing fuel costs significantly (though not nearly as much as in method (a)). The fuel cost increase in method (b) is intended to associate some disutility with the closure itself, which one would reasonably expect to occur. What should the magnitude of this disutility be? Clearly, the optimal solution would have been to observe closures in the data itself, and estimate (for example) dummies reflecting the disutility. In the absence of such information, this impact must become a parameter of the simulation process that the analyst must establish through other means than the model.

Figure 6 shows how the two methods compare in terms of their predicted share of trips to Site 2, over time. Model 1 predicts that the impact of closure is felt only during the weeks in question; the predictions of Model 5 are a function of the method of implementing the closure. Specifically, under method (a) the share of visits to Site 2 remains zero from week 4 onwards, due to the heavy penalty imposed on the utility of that site during the affected period and the memory process captured by the dynamics of Model 5. Under method (b) the disutility associated with the closure was assumed to be equivalent to a 400% increase in fuel price for trips to Site 2; this more reasonable disutility creates the depicted pattern of recuperation of trips to the site following re-opening, which is between that of Model 1 and the somewhat heavy-handed predictions of Model 5 with method (a). The greater the penalty factor used in method (b), the more nearly like method (a) will be the ensuing trip pattern after week 7. Thus, the impacts predicted under method (b) are bounded by those of Model 1 and of Model 5, method (a).

--- Insert Figure 6 ---

Returning to Figure 5, the graphs therein illustrate that the dynamic model predicts a much larger and longer lasting welfare impact than the static model. Note that the per trip welfare measures arising from the dynamic model (method a) do eventually become smaller, but it takes much of the time period for the closure effect to dampen (note also that towards the end of the season angler participation in the activity is naturally decreasing anyway, so an alternative explanation is not that the closure effect is disappearing, but that the desire to go fishing is decreasing for other reasons). In the static model the impact of closure is felt only for the 4 weeks that the site is unavailable, and then there are no further negative impacts of the closure. Again, the dynamics in Model 5 imply the anglers retain the negative effect of the closure in future periods. Note again that the parametrization of the fuel penalty in simulating Policy 2 by methods (a) and (b) imply that an envelope enclosing the welfare impacts can be derived, as was suggested in Figure 6.

A key issue in the simulation of site closure is the fact that the observation of site closure is seldom present in data. Thus, the issue of most common concern in natural resource damage assessment is difficult to assess using the dynamic model because the determination of the impact of closure on "memory" is not well defined without observations of closures. Static models do not reflect this difficulty since no "memory" is required for these models – the consumer simply returns to the original preference structure after the damage has been removed.

Policy 3: Deterioration of Fishing Quality (early in season)

Policy 3 causes Site 1 to suffer a fishing quality deterioration to the lowest level for a four week period, beginning the 4th week. Figure 5 provides details on the welfare loss dynamics of this case, while Figure 7 shows simulated choice probabilities for Site 1 and staying home under this policy. The probability of choosing Site 1 (displayed in the lower panel for Figure 7) shows that the static model actually responds more significantly to the change in quality, reducing choice of Site 1 to a lower level for the 4 week period of the quality deterioration. The dynamic model shows inertia or habit persistence because choice of Site 1 declines, but not to the degree shown in the static model. However, the experience of the decline in fishing quality carries on in the dynamic model, as the probability of choosing the location is below the static model. This illustrates that the dynamic model tends to dampen the initial effect of a reduction in quality, but carries the effect on for a longer period. The overall welfare impact is, consequently, significantly larger in the dynamic model. The graph of the probability of not going fishing under this policy (see Figure 7) shows that the dynamic model predicts a persistence of the quality impact that increases the probability of anglers staying home, relative to the static model.

--- Insert Figure 7 ---

Policy 4: Deterioration of Fishing Quality (late in season)

Policy 4 causes the same impact (fishing quality deterioration) as Policy 3, but it is assumed to occur later in the season (beginning in week 14). A similar pattern to Policy 3 arises (Figure 5), but the welfare impacts are smaller (per trip) than Policy 3 since the impact occurs later in the season and the dynamics are shifting more probability to staying home, rather than choosing other angling sites. This suggests that not only are the dynamics important in identifying welfare impacts, but specifying the seasonal nature of the activity and the time within that season that the impact occurs is important in the determination of "damages."

These simulations emphasize the very real and high value of representing the dynamics of choice by capturing habit persistence and state dependence, initial utilities and so forth. Extending choice models from the GEV family to straightforwardly incorporate these features should help improve the diffusion of the improvements made possible by this research into practical policy cases (including natural resource damage assessment) since the theory for welfare impact assessment has already been extended to this family of models.

4.0 Conclusion and Future Research

We have proposed an extension of the GEV family of choice models that incorporates a number of dynamic features: initial utility estimation, habit persistence, state dependence and future expectations are among them. Additionally, depending upon the member of the GEV family that is employed, it is possible to straightforwardly allow for time-varying covariance matrices, thus capturing both heteroscedasticity and inter-temporal differences in cross-substitutions among alternatives. For example, if the MNL model is used with our approach, the resulting model can be temporally heteroscedastic, though all alternatives will have identical error variances at a given time period and all covariances will be zero; if a TEV or NMNL model is specified, however, the covariances identified through the tree structure will be estimable (either constant or varying across time, as we have done in our empirical application; see Table 1).

In practice, GEV models, particularly its more standard members (MNL, TEV, NMNL) are relatively easy to estimate via standard methods. The challenges imposed by the GEV extensions we have proposed are relatively minor, particularly when compared to extant dynamic alternatives. We particularly see the TEV and NMNL extensions as being useful tools to support welfare assessment efforts in practice.

The use of the dynamic model for simulation and welfare measurement has raised several issues that are not apparent when using static models. Some of these issues include the specification and interpretation of time-varying covariance structures, and the interpretation of hierarchies of scale (inclusive value) parameters. However, the most challenging issue arising from this study is the specification and interpretation of the welfare measures in the case of dynamic demand. Policy changes or environmental changes will have temporal impacts in dynamic models. Thus, certain changes (e.g. a loss in environmental quality) will remain in the memory of the consumer for several periods and will affect choice and welfare for several periods, even after the effect has been eliminated. This is particularly challenging for the case of site closures due to environmental damage. Unless there have been observations of closure, and the impact of closures modeled explicitly, it is difficult to specify exactly what the dynamic impact will be. A static model treats this issue simplistically – individuals return to their original utility levels after the site has been re-opened. However, dynamic models require that the "memory" of the disutility associated with the closure be reflected in previous period utility weights when specifying the utility in the current period. This issue opens a wide variety of research questions, including the assessment of closures of different types (environmental damage, administrative closures, etc.) and the appropriate modeling of each when they are observed. Certainly it would seem recommendable that the decay, or memory, factors proposed in the models here become a function of the nature of the closure.

This paper presents a tractable dynamic discrete choice model, but the empirical work presented does suffer from several shortcomings. We have not included taste heterogeneity in our empirical work, but it is straightforward to do so in these dynamic extensions of the GEV family, as shown in the log likelihood expression (14), particularly using simulated maximum likelihood methods (e.g. McFadden and Train 1997, Brownstone and Train 1999). This topic also brings up a shortcoming of our current work, which is the omission of serial correlation from the dynamics. This omission can be partially accounted for through the inclusion of parameter heterogeneity in models, but future research should extend the suggested approach to include serial correlation.

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Figure 1: Dynamic Nest Structure of a 2-Level TEV Model with Time-Varying Covariance Matrix

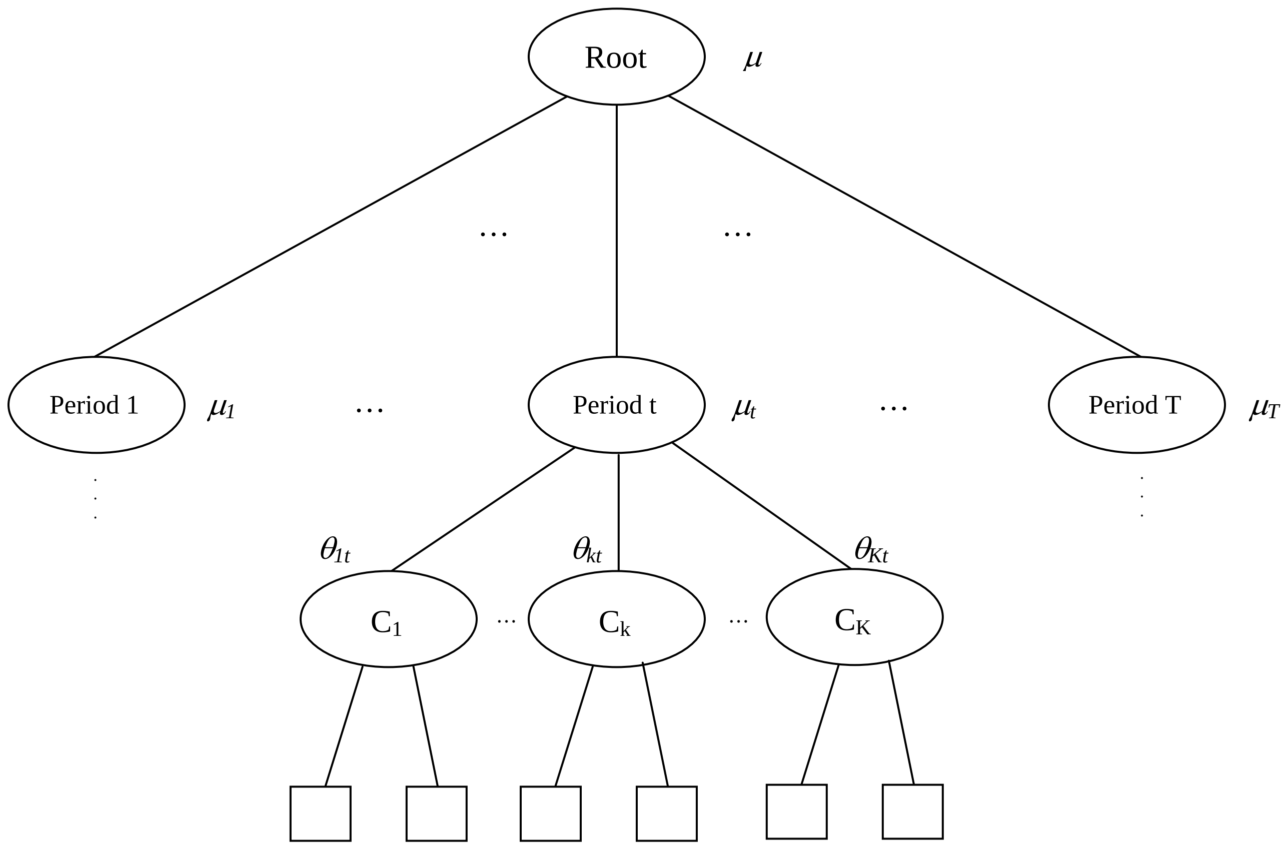


Figure 2: Dynamic Nest Structure for Recreational Fishing Application

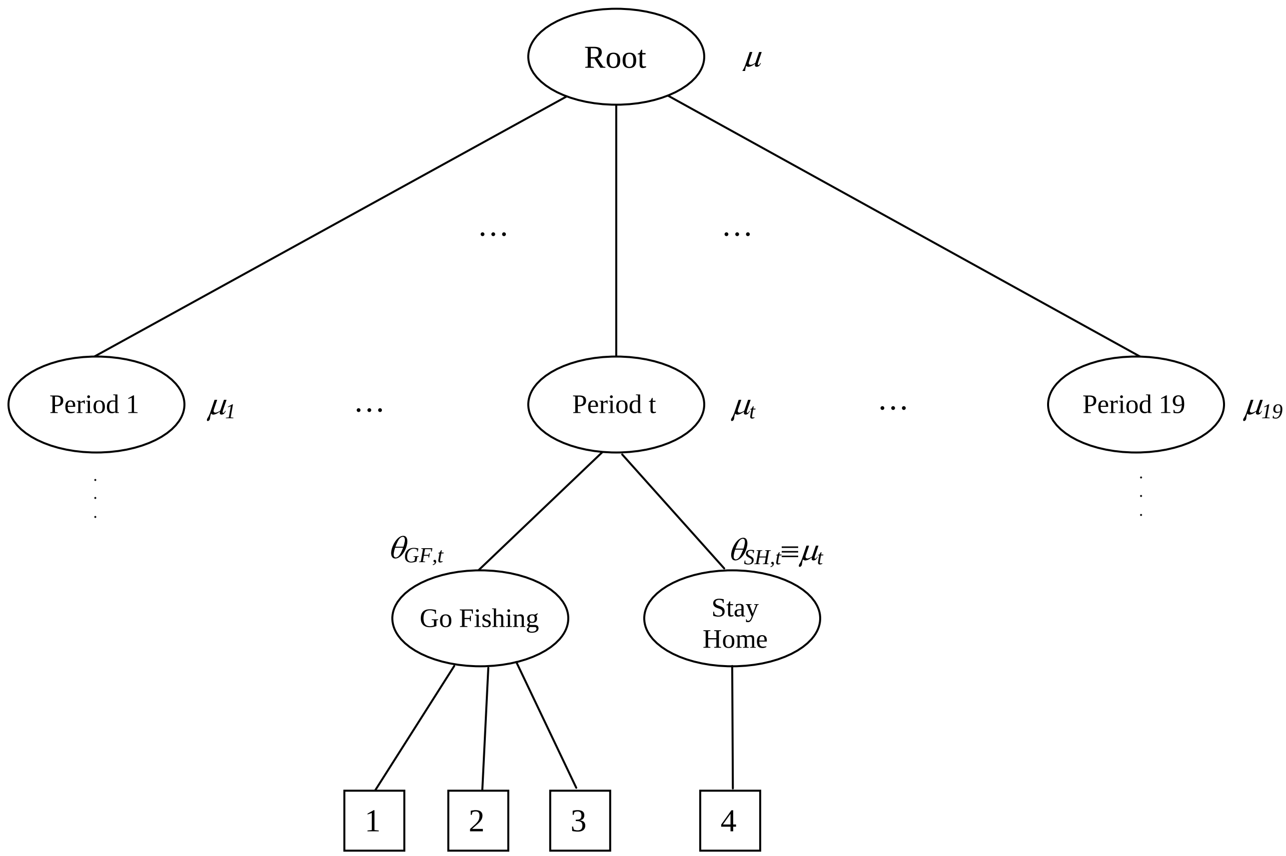


Figure 3: "Go Fishing" Cluster (Sites 1, 2 and 3) Correlations (Covariance Matrix Estimates)

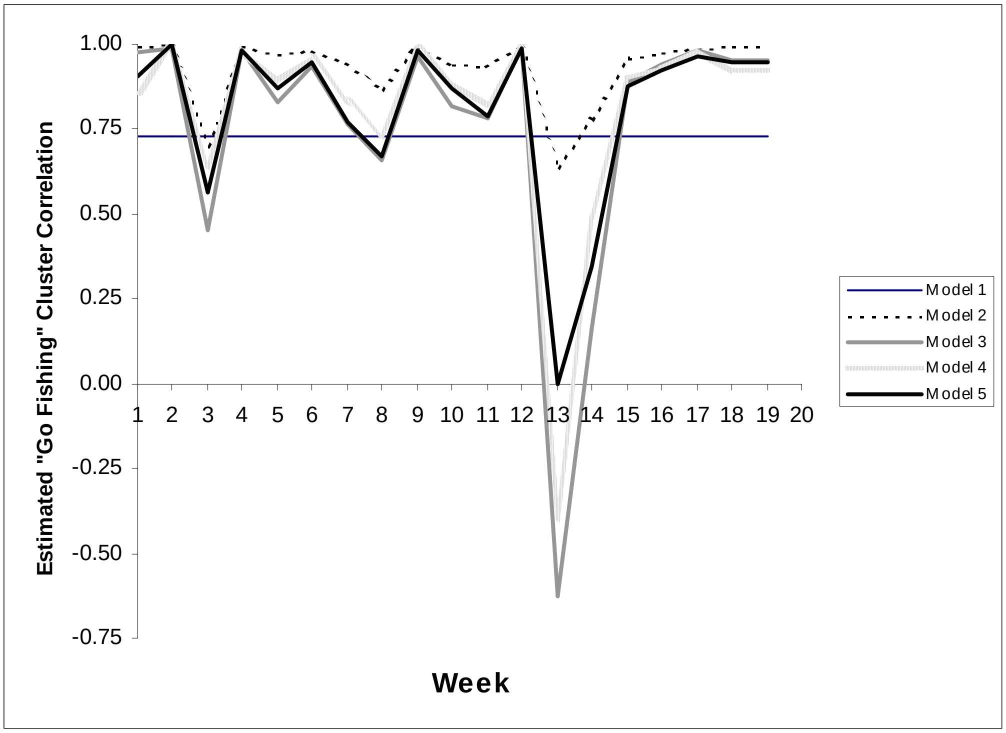
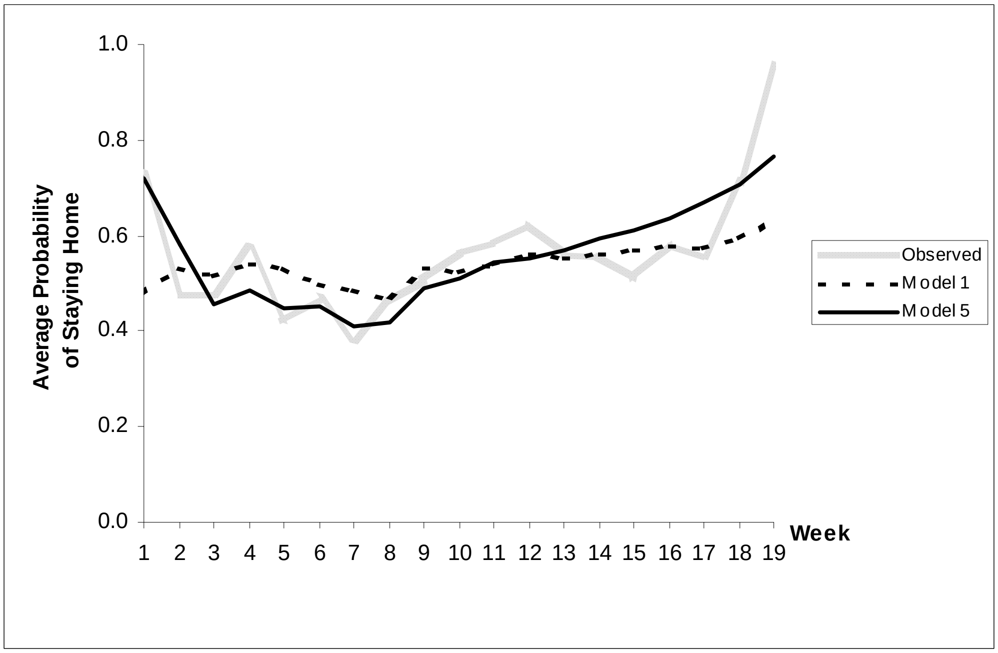


Figure 4: Base Simulations of Probabilities of "Stay Home" and Choosing Site 1



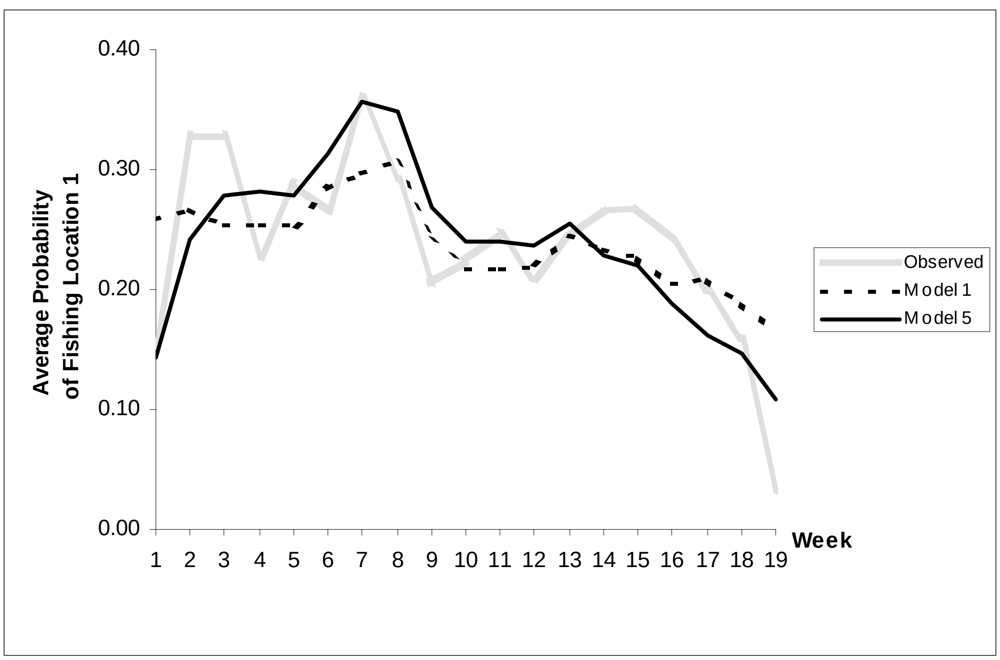
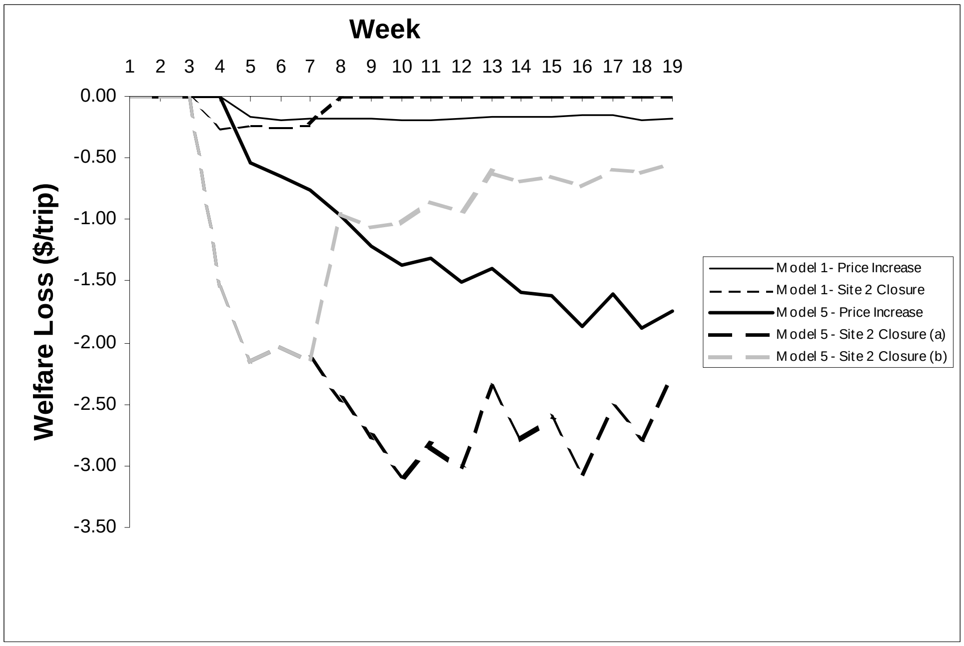
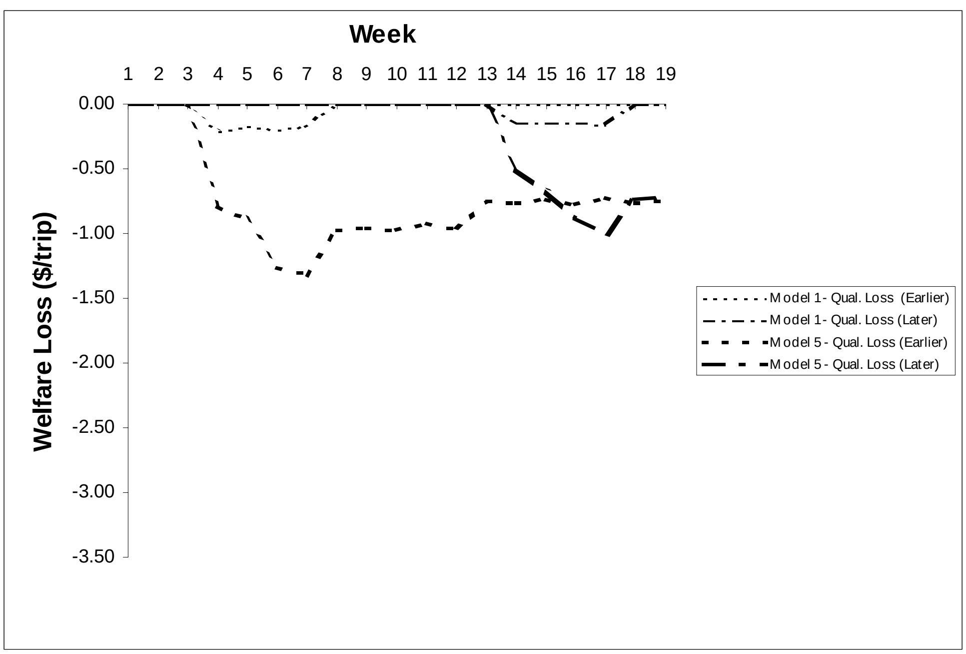


Figure 5: Welfare Measures, TEV Models, for 4 Simulated Policy Changes





Legend: Policy 1 – Doubled cost for all sites, beginning with week 5. Policy 2a,b – Closure of location 2, weeks 4-7. Policy 3 – Fishing quality deterioration, location 1, weeks 4-7. Policy 4 – Fishing quality deterioration, location 1, weeks 13-19.

Figure 6: Simulations of Probabilities of Choosing Site 2 Under Policy 2 – Site 2 Closure, Weeks 4-7

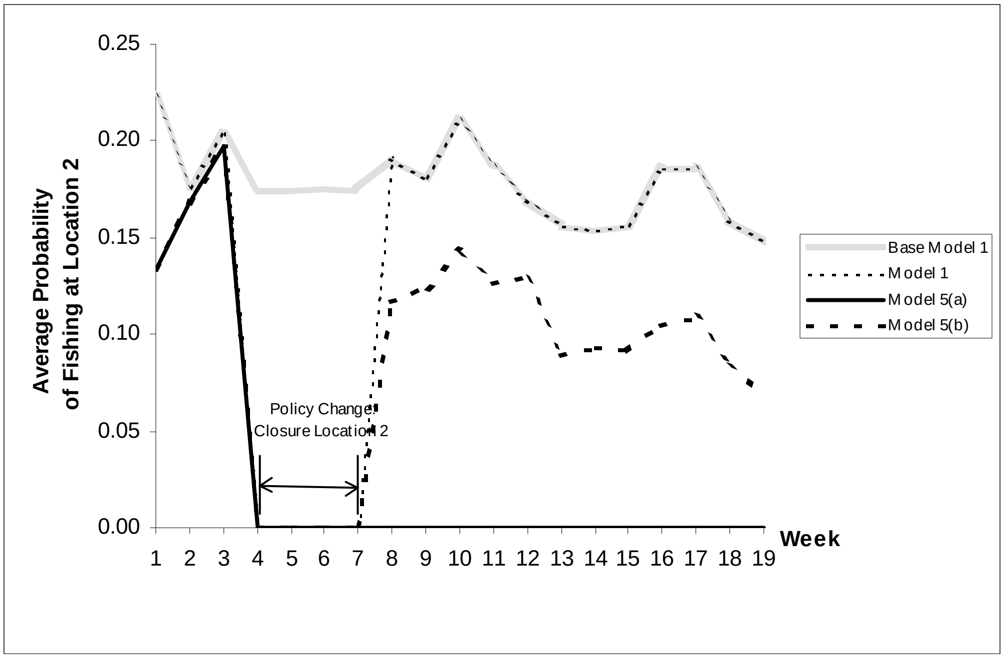
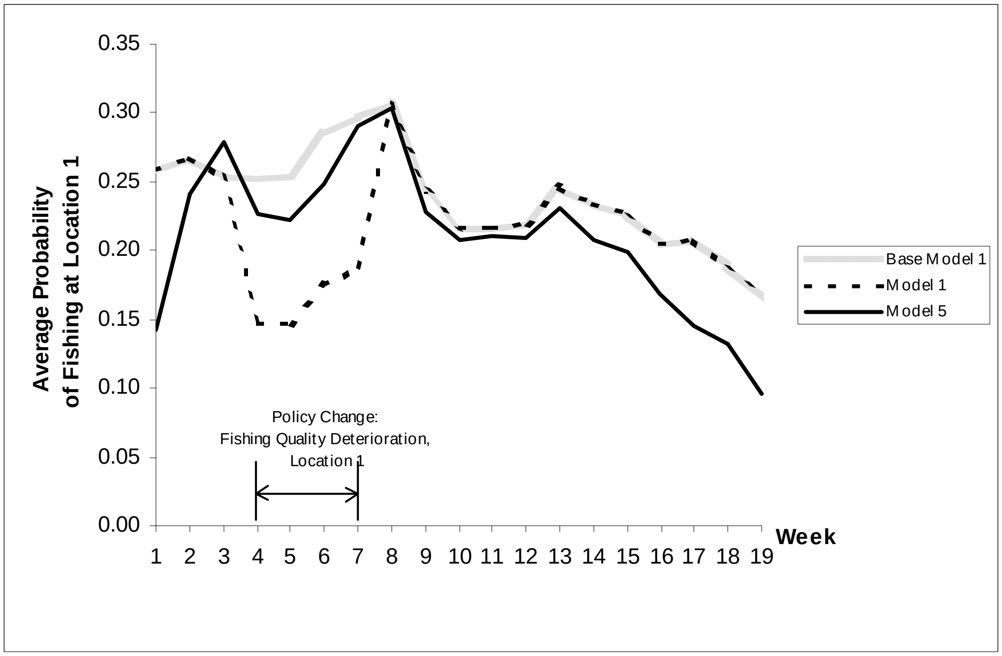


Figure 7: Simulations of Probabilities of Choosing Site 1 and "Not-Going" Under Policy 3 – Fishing Quality Deterioration, Site 1, Weeks 4-7



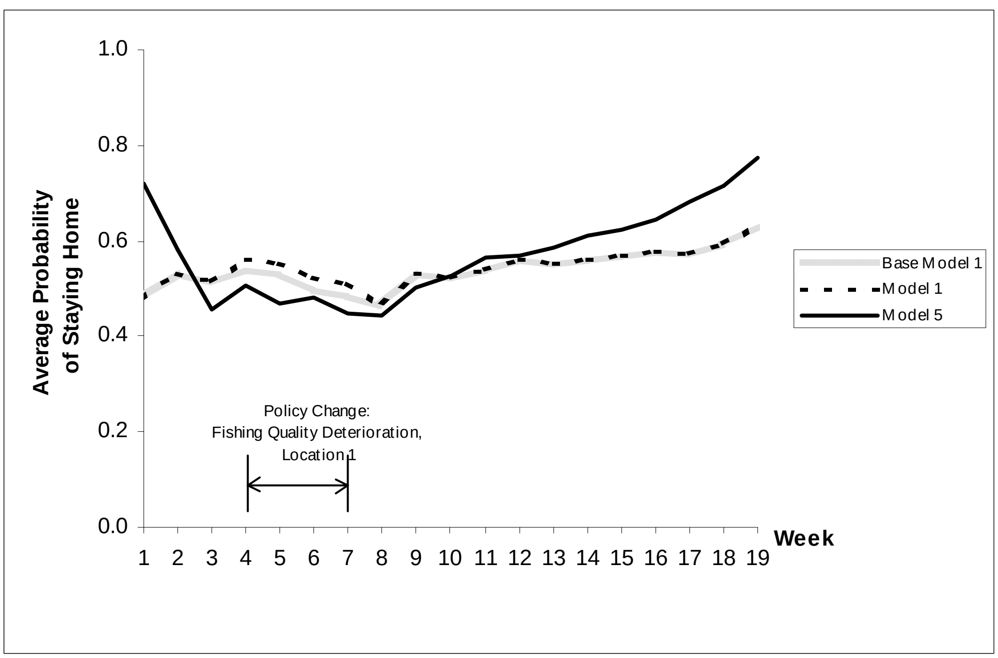


Table 1: Summary of Certain GEV Models, Generator Functions and Stochastic Utility Maximization Consistency Conditions

Description	$H(\tilde{y}_t \mid I_t)$	$P(i \mid I_t)$	**Conditions for Consistency with Stochastic Utility Maximization**
MNL Panel Data Model	$\sum_{j=1}^{J} \tilde{y}_{jt}^{\mu_t}$	$\left(\frac{\exp(\mu_t \tilde{V}_{it})}{\sum_{j=1}^{J} \exp(\mu_t \tilde{V}_{jt})} \right)$	$\mu_t \geq \mu \geq 0,\ t = 1,...,T$
Tree Extreme Value 2-Level Panel Data Model (time-invariant tree structure)	$\sum_{k=1}^{K} \left(\sum_{j \in C_k} \tilde{y}_{jt}^{\theta_k} \right)^{\frac{\mu_t}{\theta_k}}$, K is the number of nests.	$\left(\frac{\exp(\theta_k \tilde{V}_{it})}{\sum_{j \in C_k} \exp(\theta_k \tilde{V}_{jt})} \right) \left(\frac{\exp(\mu_t \tilde{I}_{(i)t})}{\sum_{k=1}^{K} \exp(\mu_t \tilde{I}_{kt})} \right)$ *where* $\tilde{I}_{kt} = \frac{1}{\theta_k} \ln\left(\sum_{j \in C_k} \exp(\theta_k \tilde{V}_{jt}) \right)$	$\theta_k \geq \mu_t \geq \mu \geq 0,$ $k = 1,..., K,$ $t = 1,..., T$
Tree Extreme Value 2-Level Panel Data Model (time-varying tree structure)	$\sum_{k=1}^{K} \left(\sum_{j \in C_k} \tilde{y}_{jt}^{\theta_{kt}} \right)^{\frac{\mu_t}{\theta_{kt}}}$, K is the number of nests.	$\left(\frac{\exp(\theta_{kt} \tilde{V}_{it})}{\sum_{j \in C_k} \exp(\theta_{kt} \tilde{V}_{jt})} \right) \left(\frac{\exp(\mu_t \tilde{I}_{(i)t})}{\sum_{k=1}^{K} \exp(\mu_t \tilde{I}_{kt})} \right)$ *where* $\tilde{I}_{kt} = \frac{1}{\theta_{kt}} \ln\left(\sum_{j \in C_k} \exp(\theta_{kt} \tilde{V}_{jt}) \right)$	$\theta_{kt} \geq \mu_t \geq \mu \geq 0,$ $k = 1,..., K,$ $t = 1,..., T$
GenL Choice Set Generation Logit[1] (time-invariant scales)	$\sum_{s=1}^{S} \left(\sum_{j \in C_s} \tilde{y}_{jt}^{\theta_s} \right)^{\frac{\mu_t}{\theta_s}}$, S is the number of possible choice sets.	$\left(\frac{\exp(\theta_s \tilde{V}_{it})}{\sum_{j \in C_k} \exp(\theta_s \tilde{V}_{jt})} \right) \left(\frac{\exp(\mu_t \tilde{I}_{(i)t})}{\sum_{s=1}^{S} \exp(\mu_t \tilde{I}_{st})} \right)$ *where* $\tilde{I}_{st} = \frac{1}{\theta_s} \ln\left(\sum_{j \in C_k} \exp(\theta_s \tilde{V}_{jt}) \right)$	$\theta_s \geq \mu_t \geq \mu \geq 0,$ $s = 1,..., S,$ $t = 1,..., T$

Notes:
1. See Swait (2000).

Table 2 – Panel Tree Extreme Value Model Estimation Results

Variables	❶ Cross-Sectional TEV	❷ ❶+Time-Varying Tree and Period Scales	❸ ❷+ Dynamics	❹ ❸+Expectations	❺ ❹ Restricted
	Parameter Estimates (t-stats)				
Utility Function					
Length of Coast @ Location	0.0492 (3.56)	0.0020 (2.46)	0.0037 (0.81)	0.0118 (2.44)	0.0093 (2.35)
Fuel Cost to Access Location	-0.2026 (-4.8)	-0.0056 (-2.41)	-0.0183 (-2.41)	-0.0248 (-3.71)	-0.0234 (-3.00)
Size Reliability @ Location	0.1242 (5.09)	0.0042 (2.85)	0.0116 (3.32)	0.0100 (3.71)	0.0095 (4.17)
Retiree (Base=stay home)	0.6535 (4.47)	0.0281 (3.33)	0.0337 (3.56)	0.0347 (4.17)	0.0303 (3.87)
Fishing Club Member (Base=stay home)	-0.8280 (-4.85)	-0.0236 (-2.79)	-0.0274 (-2.69)	-0.0252 (-3.33)	-0.0256 (-3.16)
Expected Catch Rate Type 1	0.0201 (0.74)	0.0074 (1.28)	-0.0118 (-0.69)	-0.0076 (-1.25)	-0.0096 (-1.02)
Expected Catch Rate Type 2	-0.1401 (-2.01)	-0.0113 (-1.69)	0.0022 (0.15)	0.0190 (1.45)	0.0149 (1.22)
Expected Catch Rate Type 3	-0.0255 (-0.55)	-0.0020 (-0.27)	0.0263 (1.29)	0.0120 (0.91)	0.0154 (1.11)
Last Site Fished?	0.2347 (4.38)	0.0541 (3.30)	0.1110 (4.46)	0.0896 (4.44)	0.0892 (7.97)
Weeks w/o Fishing (stay home)	0.4279 (7.79)	0.0250 (2.63)	-0.0038 (-0.27)	0.0039 (0.37)	0 (—)
(Weeks w/o Fishing)^2 (stay home)	-0.0209 (-3.96)	-0.0019 (-2.02)	0.0002 (0.11)	-0.0002 (-0.17)	0 (—)
Initial Utilities[1]					
Location 1	0 (—)	0 (—)	-0.1233 (-3.37)	-0.3692 (-2.94)	-0.3252 (-3.27)
Location 2	0 (—)	0 (—)	-0.1173 (-3.30)	-0.2345 (-2.53)	-0.2068 (-2.77)
Location 3	0 (—)	0 (—)	-0.1207 (-3.01)	-0.4312 (-2.74)	-0.3170 (-2.61)
Stay Home	0 (—)	0 (—)	0 (—)	0 (—)	0 (—)
Tree Root Scale					
$\ln(\mu)$	0 (—)	0 (—)	0 (—)	0 (—)	0 (—)
Time Period Scales					
$\ln(\mu_1)$	$\equiv \ln(\mu)$	3.7693 (7.26)	3.2503 (6.04)	2.6865 (5.58)	2.7748 (6.00)
$\ln(\mu_2)$	$\equiv \ln(\mu)$	2.2907 (6.08)	2.2550 (7.13)	1.1505 (2.92)	1.3793 (4.43)
$\ln(\mu_3)$	$\equiv \ln(\mu)$	2.2828 (6.71)	2.0395 (6.44)	1.3682 (4.08)	1.5402 (5.27)
$\ln(\mu_4)$	$\equiv \ln(\mu)$	1.1815 (2.79)	0.8185 (1.80)	0.6370 (1.69)	0.7466 (2.11)
$\ln(\mu_5)$	$\equiv \ln(\mu)$	0.8280 (1.73)	0.9092 (2.42)	0.5363 (1.37)	0.6834 (2.09)
$\ln(\mu_6)$	$\equiv \ln(\mu)$	1.0272 (2.80)	0.8777 (2.59)	0.6190 (1.85)	0.7323 (2.36)
$\ln(\mu_7)$	$\equiv \ln(\mu)$	1.2853 (3.95)	1.2882 (4.51)	1.0719 (3.83)	1.2043 (4.69)
$\ln(\mu_8)$	$\equiv \ln(\mu)$	1.3508 (4.41)	1.1491 (4.09)	1.0182 (3.75)	1.1185 (4.44)
$\ln(\mu_9)$	$\equiv \ln(\mu)$	0.2119 (0.47)	0.2225 (0.60)	-0.0015 (-0.01)	$\equiv \ln(\mu)$
$\ln(\mu_{10})$	$\equiv \ln(\mu)$	0.9167 (2.84)	0.7626 (2.55)	0.5691 (1.88)	0.6845 (2.56)
$\ln(\mu_{11})$	$\equiv \ln(\mu)$	1.0211 (3.33)	0.8950 (3.25)	0.8158 (3.06)	0.9294 (3.90)
$\ln(\mu_{12})$	$\equiv \ln(\mu)$	0.5349 (1.62)	0.2307 (0.74)	0.1038 (0.34)	0.2024 (0.69)
$\ln(\mu_{13})$	$\equiv \ln(\mu)$	1.2483 (4.16)	1.1177 (3.98)	1.0605 (3.90)	1.0560 (5.87)
$\ln(\mu_{14})$	$\equiv \ln(\mu)$	1.0180 (3.33)	0.7440 (2.57)	0.6318 (2.34)	0.7491 (2.66)
$\ln(\mu_{15})$	$\equiv \ln(\mu)$	0.6216 (1.98)	0.1904 (0.64)	0.1555 (0.55)	0.2709 (1.06)
$\ln(\mu_{16})$	$\equiv \ln(\mu)$	0.2690 (0.79)	-0.132 (-0.41)	-0.1643 (-0.52)	$\equiv \ln(\mu)$
$\ln(\mu_{17})$	$\equiv \ln(\mu)$	0.3897 (1.23)	-0.2569 (-0.80)	-0.3200 (-1.02)	$\equiv \ln(\mu)$
$\ln(\mu_{18})$	$\equiv \ln(\mu)$	$\equiv \ln(\mu)$	$\equiv \ln(\mu)$	$\equiv \ln(\mu)$	$\equiv \ln(\mu)$
$\ln(\mu_{19})$	$\equiv \ln(\mu)$	$\equiv \ln(\mu)$	$\equiv \ln(\mu)$	$\equiv \ln(\mu)$	$\equiv \ln(\mu)$

(continued)

Table 2 – (continued)

Variables	❶ Cross-Sectional TEV	❷ ❶+Time-Varying Tree and Period Scales	❸ ❷+ Dynamics	❹ ❸+Expec-tations	❺ ❹ Restricted
	Parameter Estimates (t-stats)				
Tree Scales[2]					
$\ln(\theta_{1,1})$	1.9285 (9.36)	6.3427 (13.59)	5.1151 (10.16)	3.6557 (5.71)	3.9689 (6.07)
$\ln(\theta_{1,2})$	1.9285 (—)	5.8637 (6.93)	4.4799 (6.87)	7.0160 (5.67)	4.8250 (1.42)
$\ln(\theta_{1,3})$	1.9285 (—)	2.8837 (7.30)	2.3413 (8.49)	1.8746 (5.75)	1.9572 (7.14)
$\ln(\theta_{1,4})$	1.9285 (—)	3.6947 (7.88)	2.7703 (6.41)	2.7235 (5.44)	2.8102 (7.39)
$\ln(\theta_{1,5})$	1.9285 (—)	2.5638 (6.72)	1.7933 (5.68)	1.6590 (4.87)	1.6970 (6.47)
$\ln(\theta_{1,6})$	1.9285 (—)	3.0033 (7.76)	2.2622 (6.72)	2.1944 (6.38)	2.1931 (9.66)
$\ln(\theta_{1,7})$	1.9285 (—)	2.7808 (6.92)	2.0167 (5.78)	1.9564 (5.48)	1.9458 (6.24)
$\ln(\theta_{1,8})$	1.9285 (—)	2.3431 (6.30)	1.6864 (5.62)	1.6740 (5.28)	1.6772 (6.62)
$\ln(\theta_{1,9})$	1.9285 (—)	2.6421 (6.66)	1.9194 (5.45)	2.1325 (5.85)	2.0933 (6.43)
$\ln(\theta_{1,10})$	1.9285 (—)	2.3341 (5.92)	1.6168 (4.78)	1.6520 (4.82)	1.7063 (5.86)
$\ln(\theta_{1,11})$	1.9285 (—)	2.3843 (5.66)	1.6570 (4.38)	1.6848 (4.47)	1.7005 (6.10)
$\ln(\theta_{1,12})$	1.9285 (—)	3.3484 (5.75)	2.7011 (4.87)	2.3153 (4.31)	2.4723 (4.50)
$\ln(\theta_{1,13})$	1.9285 (—)	1.7480 (4.75)	0.8739 (2.70)	0.8924 (2.82)	$\equiv \ln(\mu_{13})$
$\ln(\theta_{1,14})$	1.9285 (—)	1.7601 (4.61)	0.8340 (2.26)	0.9672 (2.70)	0.9636 (3.09)
$\ln(\theta_{1,15})$	1.9285 (—)	2.2141 (5.17)	1.2877 (3.03)	1.2995 (3.05)	1.3083 (3.56)
$\ln(\theta_{1,16})$	1.9285 (—)	2.0987 (4.91)	1.2959 (3.21)	1.1647 (3.04)	1.2980 (2.79)
$\ln(\theta_{1,17})$	1.9285 (—)	2.5747 (5.20)	1.7565 (3.71)	1.6581 (3.86)	1.6915 (3.14)
$\ln(\theta_{1,18})$	1.9285 (—)	2.4363 (4.76)	1.5394 (2.82)	1.2778 (3.09)	1.4499 (3.13)
$\ln(\theta_{1,19}) \equiv \ln(\theta_{1,18})$	1.9285 (—)	2.4363 (—)	1.5394 (—)	1.2778 (—)	1.4499 (—)
Decay Factors[3,4]					
Location 1	$-\infty$	$-\infty$	0.6626 (72.45)	0.6574 (67.96)	0.6623 (107.5)
Location 2	$-\infty$	$-\infty$	0.6660 (72.01)	0.6545 (59.00)	0.6615 (86.99)
Location 3	$-\infty$	$-\infty$	0.6726 (65.21)	0.6619 (53.52)	0.6692 (74.17)
Stay Home	$-\infty$	$-\infty$	0.7044 (—)	0.7044 (—)	0.7044 (—)
Expectation Weights[5]					
Location 1	$-\infty$	$-\infty$	$-\infty$	0.6174 (1.48)	0.5908 (1.43)
Location 2	$-\infty$	$-\infty$	$-\infty$	-0.7813 (-0.71)	-0.8540 (-0.69)
Location 3	$-\infty$	$-\infty$	$-\infty$	0.4823 (0.97)	0.1667 (0.20)
Stay Home	$-\infty$	$-\infty$	$-\infty$	0.8373 (1.84)	0.6473 (1.70)
Compatible with Utility Maximization?	✓	✓	✗	✗	✓
Goodness-of-Fit					
Number of Parameters	12	46	52	56	50
Log Likelihood @ Convergence (LL(0)=-2084.43)	-1182.38	-1072.29	-1037.62	-1024.18	-1027.22
$\bar{\rho}^2$	0.427	0.464	0.477	0.482	0.483

Notes: 1. An identification restriction requires that at least one of the initial utilities be zero.
2. All models constrain $\theta_{2,t}$, t=1,…,19, to be equal to the period scale μ_t, t=1,…,19, for identification purposes.
3. Decay factors defined as $\rho=[1+\exp(-\gamma)]^{-1}$, where γ is value shown.
4. Decay factor for Location 4 held constant due to lack of variability in data, which required imposition of this condition to permit identification of remaining parameters. This identification condition is not theoretically required by the model.
5. Expectation weights defined as $(1+\varphi)$, $\varphi=\exp(\gamma)$, where γ is value shown.

ENDNOTES

[1] Many treatments of the initial conditions problem employ information on the sample of individuals to address the fact that unobserved heterogeneity essentially "causes" the bias arising from the unknown initial utilities. In our approach we estimate sample average initial utilities; however, these could easily be augmented to depend on characteristics of the sample or could be estimated as random parameters and thus would include heterogeneity considerations in the determination of initial utilities.

[2] GEV Theorem: Suppose $G(y_1,\ldots,y_J)$ is a non-negative, homogenous-of-degree-μ (where $\mu \geq 0$) function of $(y_1,\ldots,y_J) \geq 0$. Suppose lim $G() = \infty$ as $y_i \to \infty$, for $i=1,\ldots,J$. Suppose for any distinct $\{i_1,\ldots,i_k\} \subseteq \{1,\ldots,J\}$, $\partial^k G()/\partial y_{i1} \cdots \partial y_{ik} \geq 0$ if k is odd and ≤ 0 if k is even. Then, $P(i) = y_i G_i(y_i,\ldots,y_J)/\mu G(y_i,\ldots,y_J)$, where $G_i = \partial G/\partial y_i$. The corresponding multivariate CDF of the error vector ε_{Jx1} is given by $F(\varepsilon) = \exp[-G(\exp(-\varepsilon_1),\ldots,\exp(-\varepsilon_J))]$, $-\infty < \varepsilon < \infty$. See McFadden (1978), Ben-Akiva and Francois (1983).

[3] There are many specifications of decay that can be employed. The geometric decay employed here is a special case of the Schmidt Decay Model. Let $\alpha_{js} = \rho_j^s (s+1)^{\delta_j/(1-\delta_j)}$, $0 \leq \rho_j \leq 1$, $0 \leq \delta_j < 1$, $j=1,\ldots,J$, $s=0,\ldots,t$. Utilities (4b) become

$$\tilde{V}_{it} = \sum_{s=0}^{t} \left(V_{it-s} + s \ln \rho_i + \frac{\delta_i}{1-\delta_i} \ln(s+1) \right), i = 1,\ldots,J, t = 1,\ldots,T \text{ ,}$$

and choice probabilities are given by (4a) using the utility function above. This model has as a special case the Geometric Decay Model, when $\delta_j \equiv 0$, $\forall j$. In the Schmidt Decay Model, past utilities can have a greater impact than current utilities, so it can be useful to capture inertia in behavior, habit, etc.

[4] NMNL models are simply TEV models in which construct node scales are constrained to be equal at the same level of the tree.

[5] Note that all scale factors reported in Table 2 are actually the logarithm of the scale factors. This transform was employed to guarantee non-negativity of scale estimates.