University of Alberta

ROBUSTNESS OF LORD'S FORMULAS FOR ITEM DIFFICULTY AND DISCRIMINATION CONVERSIONS BETWEEN CLASSICAL AND ITEM RESPONSE THEORY MODELS

by

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DEDICATION

This work is dedicated to Mark and Margaret Dawber. To Mark for always modeling integrity before self. To Margaret for her vitality and energy, and continuing commitment to family.

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Table of Contents

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1 INTRODUCTION

The field of psychometrics encompasses different models that offer alternative frameworks for performing test and item analyses. The purpose of any model is to describe how inferences from examinee item responses and/or test scores can be made about unobservable examinee characteristics or traits that are measured by the test (Hambleton & Swaminathan, 1985). The classical test score theory (CTST) model, the foundation of which was provided by Charles Spearman in 1904, is the traditional means of conducting item and test analyses. The family of item response theory (IRT) models, first introduced by Lord in 1952 for dichotomously scored items, was developed to circumvent the limitations of CTST. However, under certain conditions Lord (1980; also see Lord & Novick, 1968) proposed formulas that link the item difficulty and item discrimination indices of the two models.

Lord (1980, pp. 33-34) stipulated that under certain conditions the difficulty and discrimination indices derived from the two measurement frameworks are connected. For item discrimination, to the extent that number correct score x is a measure of ability (θ), the biserial correlation between the item and test scores (ρ'_{ix}) is an approximation to the correlation between the item and ability estimates ($\rho_{i\theta}$). The association yields a conceptually illuminating relationship between the conventional biserial item-test correlation and the IRT discrimination index (a_i). The proposed relationship is:

$$a_i \cong \frac{\rho'_{ix}}{\sqrt{1-\rho'^{\,2}_{ix}}} \quad .$$

Therefore, the IRT item discrimination parameter and the biserial correlation are approximately monotonic increasing functions of each other. Lord stated that the relationships are "valid only for the case where θ is normally distributed and there is no guessing" (p. 33). However, Lord qualified that the approximations are crude and may fall short because (1) the test score x contains errors of measurement while θ does not, and (2) x and θ have differently shaped distributions, since the relation between x and θ is nonlinear.

Similarly, there is a monotonic relation between the IRT difficulty index (b_i) and the classical difficulty index (π_i) when all items are equally discriminating. The relationship between the difficulty indices is described as $b_i \cong \frac{y_i}{\rho'_{ix}}$. The difficulty parameter b_i is proportional to y_i, the cut point on the continuous normal distribution underlying the binary item that separates the proportion of incorrect answers ($1 - \pi_i$) and the proportion of correct answers (π_i). To determine y_i, it is necessary to find the cut point between q and p so that when the item is easy, y_i will be negative, and when the item is difficult, y_i will be positive. Both b_i and y_i decrease as π_i increases.

Lord and Novick (1968, p. 375) first introduced the formulas with the restriction that the ability of examinees is normally distributed, although they did not specify the IRT model. Urry (1974), Schmidt (1977), Jensema (1976), and Ree (1979) investigated the formulas using the three-parameter model. General findings indicated that the estimated b-parameters derived from the heuristic method (i.e., Lord's formula) were highly correlated ($r \geq 0.95$) with true or maximum likelihood (ML) estimates of b-values. The correlations for the estimated a-parameters were more variable and ranged from 0.35 to 0.89. Subsequently, Lord (1980) clarified the situation for which the formulas were relevant by restating that examinee ability is normally distributed, and specifying that the two-parameter IRT model be used and that all items are equally discriminating for the difficulty formula. Interestingly, no further work in which the formulas were examined has been conducted.

Purpose

To date no one has investigated the veracity of item parameter estimates derived from Lord's formulas for situations congruent with and incongruent with the conditions Lord prescribed in 1980. Therefore, the purposes of the present study were to document (1) the accuracy of the two formulas within the context Lord (1980) proposed, and (2) the robustness of the formulas beyond the initial and restrictive conditions identified by Lord. The conditions he specified included that ability (θ) is normally distributed with a mean of 0 and unit variance, and that there is no guessing, implying that the two-parameter IRT model be used. An additional condition, specified for the difficulty parameter, was that all items be equally discriminating. Lord proposed his formulas for dichotomously scored items in the context of norm-referenced testing. Therefore, the focus of the present study was on dichotomously scored items within a norm-referenced framework. The appropriateness of the formulas was investigated as a function of the shape of the ability distribution, sample size, test length, and item characteristics.

Definition of Terms

Dichotomously scored item. A dichotomously scored item is an item that is scored on a two-point scale. Typically, one point is awarded for the correct response or best answer and no point is awarded for all other responses. A multiple-choice (M-C) item is an example of a dichotomously scored item (Lord & Novick, 1968).

Norm-referenced testing. The meaningfulness of an individual's test score is derived by comparing the individual's performance with the performance of others in the same population (Crocker & Algina, 1986).

Organization of Thesis

The introduction of Lord's formulas, a brief summary of the research findings using the formulas, and presentation of the specific research questions is found in Chapter 1. Chapter 2 presents and discusses item analyses from the perspective of the classical test model and the item response models, followed by the review of the studies investigating Lord's formulas. Chapter 3 describes the procedures used for this study and the procedures involved in analyzing the data. The results and discussion of the results are found in Chapter 4. Chapter 5 consists of the summary of the study, conclusions, recommendations for practice, and recommendations for future research.

CHAPTER 2 REVIEW OF THE LITERATURE

Chapter 2 is organized into two main sections. Section 1 consists of an overview of the CTST model, in which conventional item analysis is rooted, followed by an overview of IRT models, in which modern item analysis is rooted. The underlying assumptions and item analyses procedures for both models are described. Section 2 comprises a review of the studies that utilized the formulas in the context proposed by Lord and Novick (1968).

Description of the CTST and IRT Models

The purposes of conducting item and test analyses are to assess the characteristics of the items and the composite test. Mathematical models include a set of assumptions about the data to which the model applies and specify the relationships among observable and unobservable constructs described in the model (Hambleton & Swaminathan, 1985). Often the assumptions of the models can be broken down into components whose validity easily can be examined. Some models, based on weak assumptions, have applicability in many, if not all, situations. The CTST model is an example of a weak true-score model. Other models require assumptions that are more difficult to satisfy, and therefore are relevant in more limited situations. IRT models are examples of strong true-score models (Lord & Novick, 1968). The assumptions and relationships among observable and unobservable constructs described in each model will be discussed, followed by a description of the item difficulty and item discrimination parameters.

Classical Test Score Theory

Classical test score theory (Gulliksen, 1950) is an early example of a mathematical approach to educational measurement, and has maintained a strong influence among testing practitioners. Between 1904 and 1913 Charles Spearman provided the foundation when he published logical and mathematical arguments that test scores are imperfect measures of human traits, and that the correlation between test scores is lower than the correlation between their true values (Crocker & Algina, 1986).

Assumptions of the CTST Model

The CTST model introduces two unobservable constructs: a true score and an error score. The true score for an examinee is defined as his or her expected test score over repeated administrations of the test or parallel forms. The error score is the difference between the true score and an observed score. Error scores are considered random or unsystematic, and may be large or small, and positive or negative. Due to extraneous factors, an individual may correctly answer an item that he or she does not know, or incorrectly answer an item that he or she does know (Gulliksen, 1950). Positive errors of measurement contribute to an overestimate of an individual's true score, whereas negative errors of measurement contribute to an underestimate of an individual's true score.

The definition of true score indicates that the true score is a theoretical construct since the administration of many parallel forms of a test to the same individual is not possible. To obtain estimates of true score and error of measurement, measurement specialists turned to the variation among examinees within the population of interest. Six assumptions underlie the application of the CTST model. The first three pertain to performance at the individual level; the second three pertain to the performance of a group of examinees.

The first assumption is that the three scores are linearly related such that the observed score is the sum of the true score and the error score:

$$X_{jf} = \tau_j + \varepsilon_{jf},$$

where X_{jf} is the observed score of person *j* on form *f*,

τ_j is the true score of person *j* on the construct being measured, and

ε_{jf} is the error score of person *j* on form *f*.

The definition of the true score for person *j* is the mean of the theoretical distribution of the observed scores that would result from repeated testing of a person with parallel forms of the test:

$$\xi(X_{jf}) = \tau_j,$$

where ξ is the expectation taken across an "infinite" number of parallel forms. Furthermore, it is assumed that testings are independent, such that each testing has no influence on subsequent testings. As indicated earlier, the true score remains a theoretical entity because a lack of independence among testings is not feasible and an infinite number of testings is not available (Allen & Yen, 1979).

Following from the first assumption, the second and third assumptions are that the error of measurement of person *j* on form *f* across an infinite number of interchangeable forms is distributed normally and independently with mean zero and variance error of measurement $\sigma^2_{\varepsilon_j}$:

$$\varepsilon_{jf} \cong NID(0, \sigma^2_{\varepsilon_j}).$$

The next three assumptions relate to a population of examinees administered a single form of a test. The fourth assumption is that over a sufficiently large number of examinees, the average random error will be zero:

$$\xi(\varepsilon_f) = 0,$$

where the expectation is taken over examinees, $N \rightarrow \infty$. The fifth assumption concerns independence of error scores. It is assumed that error scores on two parallel tests are uncorrelated in the population:

$$\rho_{\varepsilon_1 \varepsilon_2}.$$

The sixth assumption pertains to the relationship between true scores and error scores. By definition, if errors correlate with true scores, they are not random. Thus, true scores and error scores obtained from a population of examinees on one test are assumed to be uncorrelated:

$$\rho_{\tau\varepsilon} = 0.$$

Gulliksen (1950) pointed out that the assumptions do not hold true unless the number of examinees is very large. In practice, it is customary to assume that they hold for any given set of test data.

Item Analyses with the CTST Model

The test item is the building block of which the composite test is constructed. Statistical characteristics of the total test rely entirely on the statistical characteristics of the items contained within. Therefore, the importance of item analysis arises from the impact item properties have on the characteristics of the composite test (Lord & Novick, 1968). Although numerous item parameters have been proposed for describing the properties of test items, Lord and Novick (1968) state their criterion for selecting an item parameter: "The basic requirement of an item parameter is that it have a definite (preferably a clear and simple) relationship to some interesting total-test-score parameter" (p. 328). The item parameters used in the classical model are difficulty and discrimination.

Item difficulty. The mean item score of a dichotomously scored item communicates the proportion of examinees correctly responding to an item (Lord, 1980). The population difficulty of an item is conveyed by π_i, whereas the sample difficulty of an item is conveyed by p_i and is referred to as the *p*-value. Difficulty values range from 0, when no one answered an item correctly, to 1, when everyone answered an item correctly. The higher the *p*-value, the easier the item is for that group of examinees. The item characteristic p_i is considered an estimate, unbiased over random sampling of examinees, of the population item difficulty π_i. The population test score is equal to the sum of the population item difficulties over *n* items: $\mu_X = \sum_{i=1}^{n} \pi_i$ (Lord & Novick, 1968, p. 328).

In a norm-referenced framework, the *p*-value is an important item parameter, fulfilling the criterion that it has a definite and simple relationship to the total test score. In constructing a test from a pool of items for a group of examinees with similar characteristics to the field test group, the predetermined desired test mean, μ, will guide the test developer to select items with a certain difficulty level to achieve the test's target mean (Lord & Novick, 1968).

Item discrimination. Item discrimination refers to the effectiveness of an item to discriminate or order examinees along a continuum from high to low ability (Lord & Novick, 1968, p. 331). The purpose of norm-referenced tests is to provide information about individual differences on the construct purportedly measured by the test or on an external criterion that test scores are expected to predict. In the absence of an external criterion, the total test score serves as an internal criterion of the examinee's standing on the construct of interest. Items should discriminate or differentiate examinees that know the material from those who do not know the material (Crocker & Algina, 1986). In large-scale testing programs, item discrimination is usually determined by the point-biserial item-test correlation, represented as ρ_{ix} or ρ_{pb} for a population and r_{ix} or r_{pb} for a sample, or the biserial item-test correlation, represented as ρ'_{ix} or ρ_b for a population and r'_{ix} or r_b for a sample (Lord, 1980).

The Pearson product moment correlation between a dichotomous variable (i.e., item score) and a continuously distributed variable (i.e., total test score) is the point-biserial correlation coefficient (Lord & Novick, 1968). The point-biserial correlation coefficient indicates how closely performance on an item score is related to performance on the total test score. The computational formula for the point-biserial correlation is:

$$\rho_{pb} = \frac{\mu_{+} - \mu_X}{\sigma_X}\sqrt{\pi / (1-\pi)},$$

where μ_{+} is the mean criterion score for the population of examinees who answered the item correctly,

μ_X is the mean criterion score for the full population of examinees,

σ_X is the standard deviation of the criterion score,

π is the proportion of examinees in category 1 on the item, and

1 - π is the proportion of examinees in category 0 on the item (Crocker & Algina, 1986).

If the mean of the scores for examinees passing the item is greater than the mean of the scores for examinees failing the item, the correlation will be positive. For a perfect point-biserial correlation between the scores on the dichotomous item and the scores on the continuous distribution, there must be no overlap between the two groups of scores on the continuous distribution. All test scores for examinees in category 1 must lie above the mean and all test scores for examinees in category 0 must lie below the mean. If there is no relation between the continuous variable and the dichotomous variable, the means of the test scores for both groups will be the same and equal to the mean of the test (Magnusson, 1967).

In contrast to the point-biserial correlation, the biserial correlation is not a product moment correlation. The biserial correlation, first derived by Karl Pearson in 1909, is obtained by hypothesizing that examinees' true scores for an item are normally distributed along the response continuum (Crocker & Algina, 1986). The distribution with two categories will then be the result of splitting a normal distribution into two parts: examinees who are above the pass level on the performance continuum are in one category, and examinees who are below the pass level on the performance continuum are in the other category. The computational formula for the biserial correlation is:

$$\rho_b = \frac{\mu_+ - \mu_X}{\sigma_X}\sqrt{\pi / u},$$

where u is the standard normal density associated with the z-score above which p cases fall, and

μ_+, μ_X, σ_X, and π are previously defined (Crocker & Algina, 1986).

The relation between the values obtained by the two correlation methods is:

$$\rho_b = \frac{\sqrt{\pi(1-\pi)}}{y}\rho_{pb},$$

where y is the value of the ordinate on a normal curve at the z-score associated with the p-value for the item.

The biserial correlation is equal to the point-biserial correlation multiplied by a factor that depends only on item difficulty. Since the value of the y is always less than $\sqrt{\pi(1-\pi)}$, the value of the biserial will be at least one-fifth greater than the value of the point-biserial correlation (Lord & Novick, 1968). Englehart (1965) found the linear relationship between the biserial and point-biserial correlations to be 0.98 and 0.99 for two parallel 60-item high school graduation exams. To further explore the relation, Magnusson (1967) graphically depicted the ratio of ρ_b to ρ_{pb} across all possible values of p. The magnitude of the ratio remained fairly constant for item difficulties within the range of 0.25 and 0.75. As the p–values reached the extremes, the ratio of ρ_b to ρ_{pb} increased sharply such that the value of the biserial correlation was up to four times greater than the value of the point-biserial correlation.

Lord and Novick (1968) state that for all other things being equal, the higher the item-test correlation, the higher coefficient α (Cronbach, 1951) fulfilling the criterion that the item discrimination parameter has a definite and simple relationship to a total test score parameter. Very difficult and very easy items, which have low item variances, contribute little to the discriminating power of a test, except for examinees near the extremes of the score range. For very difficult M-C items, the biserials and point-biserials will both be low, which is what is expected from items that can be answered correctly by guessing. If examinees respond to an item more or less at random, then the item score cannot be highly correlated with any criterion (Lord & Novick, 1968, p. 342-343).

Sample Dependency of CTST Item Statistics

One shortcoming of the CTST model is the dependence of item statistics on the sample of examinees that respond to the items. The average level and range of examinee ability influence the values of the item statistics. For example, *p*-values will be higher when examinee samples are of higher ability than when examinee samples are of lower ability. In addition, the point-biserial correlations will be higher when estimated from an examinee sample that is heterogeneous in ability than when estimated from an examinee sample that is homogenous in ability, due to the effect of group heterogeneity on correlation coefficients (Lord & Novick, 1968). Therefore, if the examinee sample does not closely match the population for whom the test is intended, item statistics obtained from the sample are limited in their usefulness. The dependency of item statistics on the sample of examinees led to the introduction of IRT (Lord, 1952; Hambleton, 1989; Hambleton, Jones, & Rogers, 1993).

The exception to sample dependency of item parameters estimated from the classical model is the biserial coefficient because it has been observed to be more stable across samples than the point-biserial coefficient (Englehart, 1965; Lord & Novick, 1968). Lord and Novick (1968, p. 341-343) state that the biserial tends to be more stable from group to group compared to the point-biserial correlation for any criterion positively correlated with the items. A low point-biserial correlation obtained from one group does not indicate that the item will have a low point-biserial correlation for other groups of examinees; however, a low biserial correlation does indicate that the item will have low biserial correlations for other groups of examinees.

Item Response Theory Models

Hambleton and Swaminathan (1985) noted that Binet and Simon in 1916 may have been the first psychologists to plot level of performance on a task against an independent measure when they considered the performance of children of increasing age on a variety of cognitive tasks. The plots, referred to as Item Characteristic Curves, are fundamental to IRT models whereby examinee item performance is related to level of an unobservable ability or latent trait. The work of Lord in 1952 and 1953 is generally regarded as the origin of IRT, likely because he was the first to develop an item response model, methods for parameter estimation, and the expertise to apply the model to achievement test data (Hambleton & Swaminathan, 1985).

Assumptions of the IRT Models

Central to IRT is the assumption that a mathematical model can portray the relationship between the observable examinee test performance and the unobservable ability or latent trait assumed to account for test performance. Although multidimensional IRT models exist, only the unidimensional models are of relevance to the present study and will be discussed. In its most common and popular form, item response theory postulates that (a) underlying examinee performance on a test is a single ability or trait, represented by θ, and (b) the relationship between examinee item performance and the trait assumed to be influencing item performance may be depicted by a monotonically increasing function. The S-shaped function, called an item characteristic function or an item characteristic curve (ICC), provides the probabilities of examinees at various ability levels successfully answering an item. Examinees with more ability have greater expected probabilities for answering an item correctly than examinees with lower ability. By specifying the assumptions one is willing to make about the test data, different item response models are formed. The ICCs for the unidimensional models are typically described by one-, two-, or three-item parameters (Hambleton, 1989; Hambleton & Swaminathan, 1985).

There are two primary advantages of item response models:

(1) Assuming the existence of a large pool of items all measuring the same trait, an examinee's ability estimate is independent of the sample of test items administered to the examinee. An examinee has the same ability across various samples of test items, even though estimates may vary because of measurement error. This property permits examinees to be compared even though they may not have taken an identical set of test questions.

(2) Assuming the presence of a large population of examinees, the characteristics of a test item are independent of the sample of examinees drawn for the purpose of calibrating the item.

These two properties give rise to what are referred to as item-free ability estimates and sample-free item parameter estimates, respectively. The extent to which the advantages are realized in an application of an item response model relies on the degree of fit between a set of test data and the model (Hambleton, 1989; Hambleton & Swaminathan, 1985).

Justification of an item response model is predicated on the satisfaction of the underlying assumptions of that model. Determining the adequacy with which a test data set fits the set of model assumptions is imperative when selecting a model. When the assumptions of a model cannot be satisfied, the model-data fit will likely be poor, and the model will be of questionable utility in its application. Four common assumptions to be met include unidimensionality, local item independence, nonspeededness, and monotonicity (Hambleton & Swaminathan, 1985). In addition, the assumption that guessing has minimal influence on examinee performance is required because the two-parameter IRT model does not include a guessing parameter, as is the case for Lord's formulas.

Unidimensionality. For most IRT applications it is assumed that only one trait or ability is sufficient to account for examinee performance. The assumption of a unidimensional latent space is common for test developers to make because they usually want to construct unidimensional tests to enhance the interpretability of test scores. Since the θ estimates provided by the models are an estimate of each student's latent trait, it is important that the items are measuring a single trait. The assumption cannot strictly be met because of several cognitive, personality, and test-taking factors (i.e., level of motivation, test anxiety, speed of performance, test sophistication, reading skills) that impact examinee performance to some extent (Hambleton, 1989).

Although counting only one dominant ability violates Lord and Novick's (1968) definition of unidimensionality, it is commonly accepted that in order to apply the IRT unidimensional models it is sufficient to show that there is one dominant ability underlying examinees' responses to the test items (Hambleton, 1989; Hambleton, Swaminathan, & Rogers, 1991). Essential unidimensionality is achieved when the data are dominated by one dimension, and other dimensions contribute weaker influences (Stout, 1987). If this assumption is met, the ICC may be represented by a simple curve drawn in two-dimensional space (McKinley & Mills, 1989).

Factor analysis has been used to evaluate the dimensionality of a given set of data (Hambleton & Swaminathan, 1985). Researchers have concluded that an examination is essentially measuring a unidimensional trait if there is a dominant first factor, indicated by a high ratio between the first and second eigenvalues in comparison to the ratio between other successive pairs of eigenvalues (Gorsuch, 1983; Reckase, 1979). In addition, the scree plot can be used to confirm the results by graphically examining the distribution of eigenvalues (Cattell, 1966).

Local independence. The assumption of local independence implies that an examinee's responses to the items in a test are statistically independent. The concept of local independence is fundamental to IRT because it permits the responses of a single individual to different stimuli to be modeled separately (Lord, 1980; McKinley & Mills, 1989). To be satisfied, an examinee's performance on one item must not affect his or her responses to other items on the test (Hambleton & Swaminathan, 1985). The assumption specifies that only an examinee's ability and the characteristics of the test item have an effect on performance.

When the assumption is satisfied, the probability of any response pattern for an examinee is equal to the product of the probabilities associated with the examinee's responses to the items. Consider U_i as a random variable that designates the response of a randomly chosen examinee to item i (i = 1, 2, 3, …, n), such that $U_i = 1$ if the examinee answers the item correctly and $U_i = 0$ if the examinee answers the item incorrectly. P_i represents the probability that the examinee answers the item correctly and Q_i represents the probability that the examinee answers the item incorrectly. P_i and Q_i depend on an examinee's ability. In mathematical notation, local independence is conveyed by the following equation:

$$\mathrm{Prob}(U_1 = u_1, U_2 = u_2, \ldots, U_n = u_n \mid \theta) = \prod_{i=1}^{n} P_i^{u_i} Q_i^{1-u_i} \, .$$

In practice, the probabilities of various response patterns based upon the assumption of local independence can be computed and compared to the occurrence rates of the same response patterns in a sample of examinees at the same ability levels (Hambleton, 1989). The assumption of local independence does not imply that test items are uncorrelated over the total group of examinees; rather, item scores are uncorrelated for examinees of a given ability level (Lord, 1980; Lord & Novick, 1968).

Nonspeededness. An assumption relevant to all commonly used IRT models is that the test is not administered under speeded conditions. Nonspeededness refers to the observation that the majority of the examinees have sufficient time to attempt all questions. Incorrect responses or nonresponses are due to lack of knowledge rather than lack of time to complete the exam. When almost all examinees complete almost all the items, speed is considered to be unimportant to test performance (Hambleton, Swaminathan, & Rogers, 1991). It is suggested that if 95 percent of examinees complete the last three items, then the speed of which examinees answered the questions is considered to be unimportant in test performance (Lord, 1980). Nonspeededness can be assumed if the assumption of unidimensionality is met because a speeded test would require a second factor to account for the dimension of speed (Hambleton & Swaminathan, 1985).

Monotonicity. An ICC is a mathematical function that relates the probability of success on an item to the ability measured by the test. The primary difference between IRT models is the mathematical form of the ICC. The ICC is the nonlinear regression of item score on ability, indicating that the curve does not increase at the same rate and ideally never decreases (Hambleton & Swaminathan, 1985).

Lack of guessing. Since the two-parameter IRT model does not include a guessing parameter, the assumption that guessing has minimal influence on test performance is required. Hambleton et al. (1991) suggested that the assumption be evaluated by examining the performance of low achieving examinees on the most difficult items. If their performance is close to zero, the assumption is viable. Low achieving examinees were defined as examinees who obtained a total score at or below the chance level. The chance level was calculated using the following formula:

$$Y = X_c + 2SE_c,$$

where

$$X_c = n_i / k,$$

$$SE_c = \sqrt{n_i p_i q_i}\text{, and}$$

Y is the calculated chance level score given two standard errors of chance,

X_c is a score obtained by random guessing on a set of M-C items,

n_i is the number of test items,

k is the number of response options on a test item,

SE_c s the standard error of chance,

p_i is the probability of guessing a correct answer given *k*, and

q_i is the probability of guessing an incorrect answer given *k*.

When examinee performance of the full sample is compared to examinee performance of low achieving examinees for the three most difficult items, the assumption of lack of guessing is tenable when the *p*-values are close to zero for the low achieving group.

Description of the IRT Models

Lord (1952) proposed a two-parameter model in which ICCs took the form of the cumulative normal distribution, also called the normal ogive. In 1957 and 1958, Birnbaum substituted more tractable logistic curves for the normal ogive curves, thus providing the necessary developments for the logistic models to facilitate the use of IRT models. Today the normal ogive is more of historical interest since the logistic models are used for computation (Hambleton, 1989; Hambleton & Swaminathan, 1985).

Different IRT models involve different numbers of item parameters. The most popular of these for analyzing dichotomous data include the one-, two-, and three-parameter logistic models, referred to as the 1PL, 2PL, and 3PL models, respectively. These models are described, starting with the 2PL model.

Two-parameter dichotomous item response model. The two-parameter dichotomous item response model is defined by the logistic probability function in which $P_i(\theta)$ is given by:

$$P_i(\theta) = \frac{1}{1 + e^{-Da_i(\theta - b_i)}},$$

where $P_i(\theta)$ is the probability that a randomly selected examinee with ability θ answers item i correctly,

b_i is the difficulty index and represents a point on the ability scale at which an examinee has a 50 percent probability of answering item i correctly,

a_i is the item discrimination index and is proportional to the slope of $P_i(\theta)$ at the point $\theta = b_i$, and

D is a scaling factor to bring the interpretation of the parameters of the logistic model in line with those of the normal ogive model. When D is equal to 1.7, the values of $P_i(\theta)$ for the two-parameter normal ogive and the two-parameter logistic models differ in absolute value by less than 0.01 for all values of θ.

In the two-parameter model, higher b-values represent items that are more difficult than items with lower b-values. Items with higher a-values indicate ICCs that that are steeper and more discriminating than items with lower a-values. It is assumed that examinees cannot get the items correct by guessing, since the probability of a correct response to the item decreases to 0 as ability decreases (Hambleton, 1989; Hambleton & Swaminathan, 1985). Examples of ICCs for the two-parameter model are shown in Figure 1.

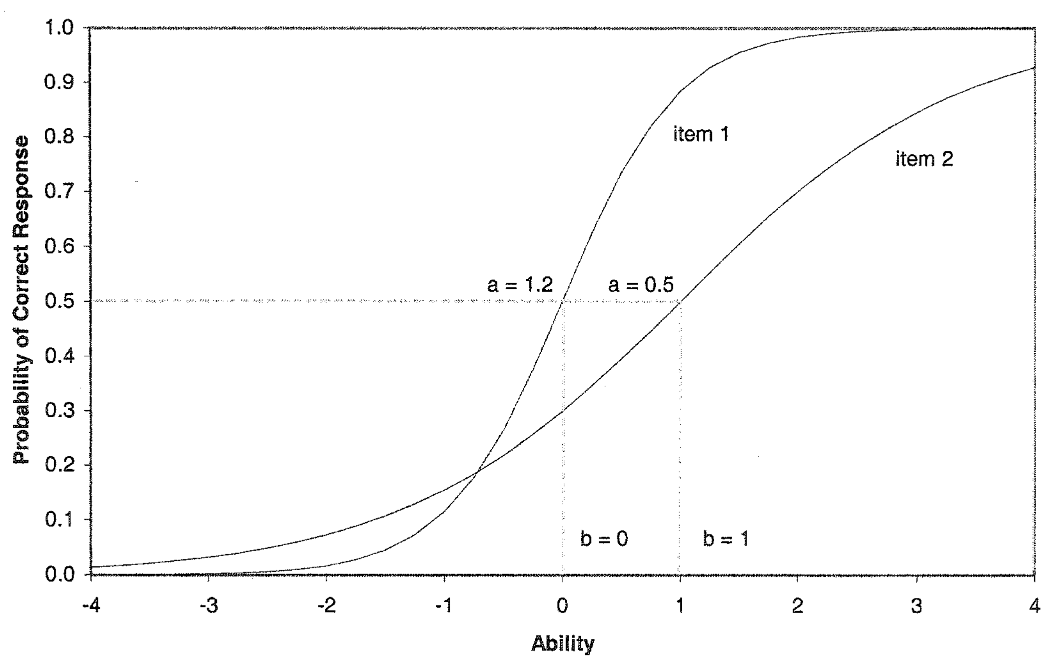


Figure 1. Sample item characteristic curves of the two-parameter IRT model. Adapted from Hambleton et al. (1991, p. 16).

As illustrated in Figure 1, the two ICCs have different slopes, indicating that the a-parameters are different. Item 1 has a higher discrimination than item 2, as evident by the steeper slope. Item 2 is the more difficult item since the 0.50 probability of correctly answering the item falls at an ability level of 1, compared to an ability level of 0 for item 1. Each curve has a lower asymptote of zero, reflecting the assumption of the two-parameter model that items may not be correctly answered by guessing. Examinees with very low ability have a zero probability of correctly answering the item.

Three-parameter dichotomous item response model. The three-parameter dichotomous response model is defined by the logistic probability function in which $P_i(\theta)$ is given by:

$$P_i(\theta) = c_i + \frac{(1-c_i)}{1+e^{-Da_i(\theta-b_i)}},$$

where $P_i(\theta)$ is the probability that a randomly selected examinee with ability θ answers item i correctly,

b_i is the item difficulty parameter where the probability of correct response is equal to $(1+c_i)/2$,

a_i is the item discrimination parameter,

D is a scaling factor equal to 1.7, and

c_i is the lower asymptote of the ICC, representing the probability of examinees with low ability correctly answering item i.

The c-parameter is included in the model to account for item response data from low ability examinees, where guessing may be a factor in test performance. Hambleton et al. (1991) note that the assumption of no guessing is most plausible with free-response items, but can often be met when tests containing dichotomously scored items are administered to students following effective instruction. Parameter c is referred to as the pseudochance level or pseudoguessing parameter and represents the probability that a person completely lacking in ability $(\theta \rightarrow -\infty)$ will answer the item correctly. If an item cannot be answered correctly by guessing, then $c_i = 0$ (Lord, 1980). Generally, c_i assumes values that are smaller than would be expected if examinees of low ability were to guess randomly due to the ability of item writers to construct distracters that are attractive to low ability examinees (Hambleton, 1989). Examples of ICCs for the three-parameter model are depicted in Figure 2.

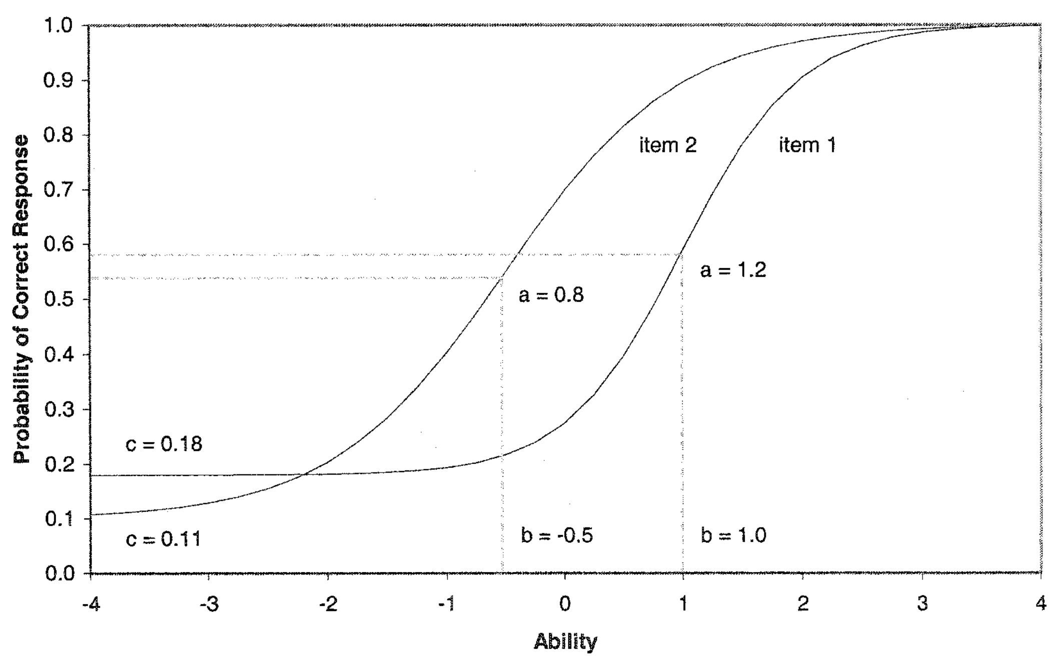


Figure 2. Sample item characteristic curves of the three-parameter IRT model. Adapted from Hambleton et al. (1991, p. 18).

As can be seen in Figure 2, the two ICCs display different discrimination and difficulty values. Item 1 is more discriminating and more difficult than item 2. Unique to the three-parameter model is the presence of the c-parameter, which alters the shape of the ICCs. A comparison of the lower asymptotes indicates that examinees with very low ability have an 18% chance of correctly answering item 1, but only an 11% chance of correctly answering item 2.

One-parameter dichotomous item response model. The Rasch model, proposed by Georg Rasch in the early 1960's, was developed independently from the other IRT models (Hambleton & Swaminathan, 1985). The Rasch model may be viewed as an item response model in which the ICC is a one-parameter logistic function. This model assumes that all items are equally discriminating and guessing is minimal. The one-parameter dichotomous response model is defined by the logistic probability function in which $P_i(\theta)$ is given by:

$$P_i(\theta) = \frac{1}{1+e^{-Da(\theta-b_i)}},$$

where $P_i(\theta)$ is the probability that a randomly selected examinee with ability θ answers

item i correctly,

b_i is the item difficulty index,

a is the common level of discrimination for all the items, and

D is the scaling factor (Hambleton & Swaminathan, 1985).

Examples of ICCs for the one-parameter model are depicted in Figure 3.

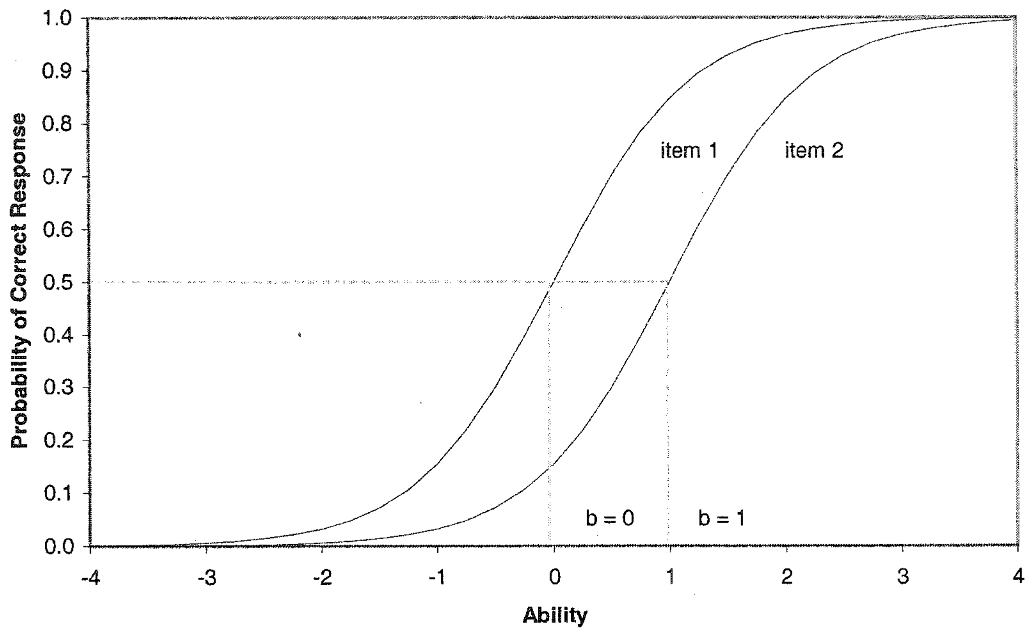


Figure 3. Sample item characteristic curves of the one-parameter IRT model. Adapted from Hambleton et al. (1991, p. 14).

Figure 3 illustrates two sample ICCs for the one-parameter model. The curves differ only by their location on the ability scale. Item 1 is the easier item. Examinees with an ability level of 0 have a 50% chance of answering item 1 correctly, but only a 15% chance of answering item 2 correctly. Examinees with an ability level of 1 have a 50% chance of answering item 2 correctly, and an 85% chance of answering item 1 correctly. In the one-parameter model, it is assumed that item difficulty is the only item characteristic that influences examinee performance. Therefore, all items are considered equally discriminating and examinees with low ability are considered to have a zero probability of successfully responding to the item by guessing.

Item Analyses for the Two-Parameter IRT Model

Since the focus was on Lord's formulas, the two-parameter model was used. Item difficulty and discrimination parameters are described. Following this, the relevance of the item difficulty and discrimination parameters is discussed in terms of their contribution to item information and test information.

Item difficulty. When the ability distribution for a group of examinees is transformed so that its mean is 0 and standard deviation is 1, Hambleton (1989) states that generally b-values vary from –2 to 2. Values of b close to –2 indicate that the items are very easy for the group of examinees, and few examinees unsuccessfully answer the items. Values of b close to 2 indicate that the items are very difficult for the group of examinees, and few examinees successfully answer the items (Hambleton, 1989). In the two-parameter model, the b_i for an item is the point on the ability scale where the probability of a correct response is 0.50. The b-value indicates the position of the ICC with respect to the ability scale. The greater the value of the b_i parameter, the greater the ability required for an examinee to have a 50% chance of correctly responding to the item (Hambleton et al., 1991).

IRT discrimination. The item discrimination parameter, a_i, indicates how well an item discriminates among different levels of θ. The a-parameter is proportional to the slope of the curve at the inflexion point and theoretically can assume values between $-\infty$ and $+\infty$. However, values typically fall between 0 and 2 for the correct option. Higher a-values result in steeper ICCs, demonstrating a greater change in the probability of a correct response for a given change in θ. Lower a-values translate to ICCs that increase gradually as a function of ability. Items with steep slopes are more useful for differentiating examinees into different ability groups than items with gradual slopes. Items with low positive values within this range or negative values are discarded from the test because it is undesirable to have items that poorly discriminate examinees at different ability levels or items where the probability of correctly answering decreases as ability increases (Hambleton, 1989; McKinley & Mills, 1989).

Item information. One of the most important statistics yielded by IRT is the item information, which indicates the degree to which a particular item provides useful information about an examinee's θ. Item information is computed as a function of the slope of the ICC at θ. When the ICC is steep at a given value of θ, the item is contributing useful information. When the ICC is relatively flat at a given value of θ, the item does not contribute much information about θ. Therefore, in areas of the scale where the ICC is flat, big changes in θ are accompanied by very small changes in the probability of a correct response, indicating the item does not help discriminate among examinees with relatively large differences in θ. Inspection of an item information curve reveals how well an item will function, and for which respondents it will function well and for which respondents it will not function well (McKinley & Mills, 1989).

Item information plays an important role in test development because it communicates each item's contribution to ability estimation at points along the ability continuum. The extent to which each item's contribution is realized depends on an item's discriminating power and difficulty location (Hambleton et al., 1991; McKinley & Mills, 1989). Figure 4 illustrates item information functions for two items at various points along the θ scale. A comparison of these two curves indicates that item 1, with an a_i of 1.50, yields higher information than item 2, with an a_i of 1.00, and that item 1 has a lower location parameter ($b_i = -1.00$) compared to item 2 ($b_i = 1.00$).

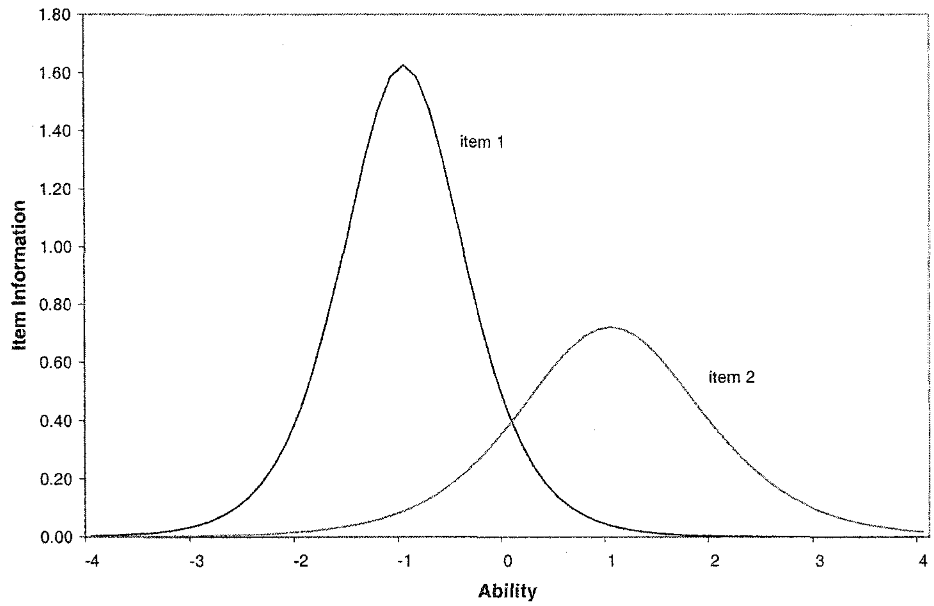


Figure 4. Sample item information curves for the two-parameter IRT model.

Test information. For any value of θ, the information provided by the test can be obtained by computing the item information at that value of θ for each item, and summing across items. Therefore, each item contributes independently to the test information function. Because of this summative property, it is possible to use the test information curve as a guide in selecting items to yield tests with specific measurement properties. Test information is considered a measure of accuracy of the measurement of θ, and an indication of the quality of a test for assessing different ranges of the θ-scale (Hambleton et al., 1991; McKinley & Mills, 1989). Figure 5 shows the test information for two tests: Test 1 is relatively flat, indicating that the test would yield fairly good measurement across the scale, and Test 2 is more peaked and narrow indicating the test would measure well in the middle range of the scale, but not well for the extremes.

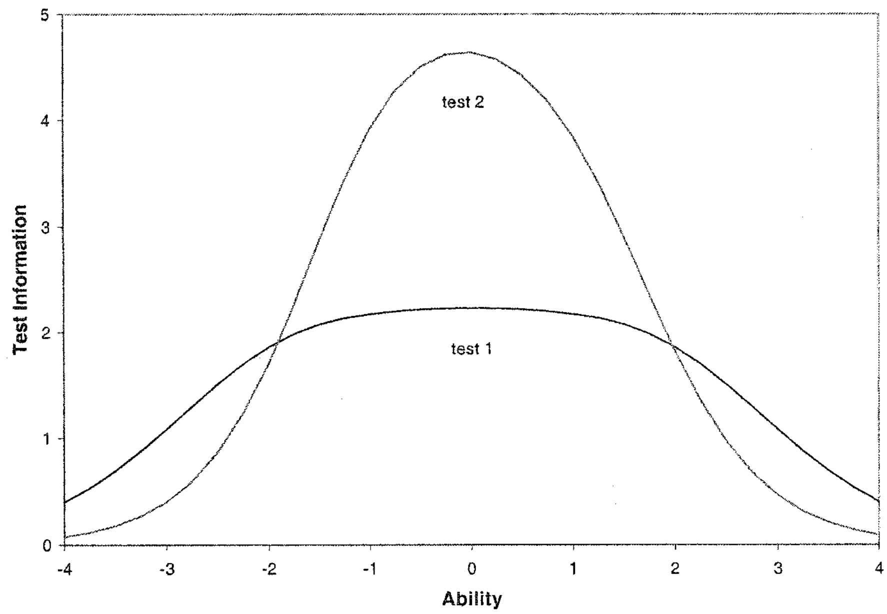


Figure 5. Sample test information curves for the two-parameter IRT model.

Literature Review of Studies Using Lord's Formulas

The formulas provided by Lord (1980) were first presented in Lord and Novick (1968). Even though the relationships described are the same, the only qualifying condition in the earlier writing was that θ be normally distributed with a mean of zero and unit variance. Several studies in the 1970's were conducted using the formulas, subsequently referred to as the heuristic method, within the framework of the three-parameter model. In 1980, Lord added the stipulations that the formulas were only applicable for the 2PL model and the difficulty formula was only applicable when all items were equally discriminating. Despite the use of an incorrect IRT model, the following studies provide insight into how the formulas may function in the intended context.

Description of the Studies

Using the formulas proposed by Lord and Novick (1968), Urry (1974) developed a graphical method to estimate the IRT parameters from Lord's formulas. Urry's intention was that the approximations be used to (a) screen items for the purpose of tailored testing, (b) select items as a bank for tailored testing, (c) provide starting values for item parameter estimation in the ML equations, and (d) inexpensively obtain item parameter estimates for direct use in tailored testing. He endorsed the use of the heuristic method for the first three purposes, but the efficacy of the fourth required further investigation.

The figures involve a grid system in which the *a*- and *b*-parameters are mapped onto a coordinate system. The coordinate system is defined by the population point-biserial correlation, rather than the biserial correlation, along the ordinate and the population proportion *p*-value along the abscissa. In each figure, the estimated *a*-values were vertically arranged and the estimated *b*-values were horizontally arranged on the grid system. By plotting the data points for a given item using the CTST indices, the values of $\hat{a}_i$ and $\hat{b}_i$ may be interpolated from the grid system. Each graph was unique for a specified level of c_i. The *c*-values depicted were 0.0, 0.20, 0.25, 0.33, and 0.50, which correspond to the free-response, 5-, 4-, 3-alternative options, and true-false item types. When there is no guessing, the graph is symmetric. When there is guessing, the graph is displaced to reflect inflation in the proportion passing the item and attenuation in the point-biserials through error due to guessing. The distortion becomes more pronounced as c_i increases.

Urry (1974) warned that four conditions needed to be met for effective application of his method. Urry's criteria were: (1) the latent trait is normally distributed; (2) the classical indices are based on large samples ($n_s = 2{,}000$) in order to approximate the set of parameters; (3) the items in the test must be homogenous (K-R 20 $\geq$ 0.90); and (4) the items in the test must be of sufficient number ($n_i = 80$), so that the point-biserial correlation between item and total test score bears a close relationship to the correlation between the item and the latent ability measured by the test.

The grid approximations proved fairly successful judging from the correlations. The data employed consisted of 4,950 examinee responses to 98 unscreened mathematics items from the Washington Pre-College Test Battery, a highly reliable test (K-R 20 = 0.93). Correlations between the estimated *a*- and *b*-parameters derived from the grid system and their ML estimates were 0.89 and 0.97 respectively. Urry concluded that the correlation coefficients indicated a strong degree of accord between the heuristic approximations and the ML estimates.

As a follow up to Urry's work, Schmidt (1977) proposed that the graphical procedure tended to systematically underestimate a_i and overestimate $|b_i|$ and the variance of b_i. The reason is that the point-biserial correlation between the item score and the estimated latent trait (i.e., total test score), $r_{i\hat{\theta}}$, is taken as an estimate of the point-biserial correlation between the binary item and the perfectly reliable latent trait, $\hat{\rho}_{i\theta}$. Values of $r_{i\hat{\theta}}$ are attenuated because of guessing on item *i*, and the unreliability of $\hat{\theta}$. Although Urry suggested four criteria for the total test score to be an estimate of the latent trait score, $\hat{\theta}$, Schmidt argued that the criteria would minimize, rather than eliminate, the effect. Higher values of the correlation, as would be the case using the biserial coefficient, would result in larger $\hat{a}_i$ and smaller $|\hat{b}_i|$.

The heuristic method has also been used with simulation data. Jensema (1976) simulated data adhering to a variety of conditions and compared the parameter estimates set during the data generation phase to the estimates derived from the heuristic method and ML estimation. In order for the mathematical relationships to be applicable in practical situations, the following assumptions were specified: reasonably good estimates of c_i are made, the proportion of the population passing item i is an estimate of the probability of a correct response, $P_i(\theta)$, and the total test score is a measure of true ability θ.

Forty-eight data sets were created with a total of 2,800 items and 44,000 simulated examinees. The true abilities of examinees were normally distributed. The simulation design consisted of a 5 (sample size) by 3 (test length) by 4 (item discrimination) fully crossed design. The data sets varied in number of examinees (i.e., 250, 500, 750, 1,000, 2,000), number of items (i.e., 25, 50, 100), true item discrimination values (i.e., 0.5, 1.0, 1.5, 2.0). The *c*-parameter values were set to 0.2 for all items. The *b*-parameters ranged from -2.4 to 2.4 at intervals of 0.2, yielding 25 values for *b*. Hence, all *b*-values were represented once for the 25 item tests, twice for the 50 item tests, and four times for the 100 item tests. Parameter values derived from the heuristic method were used as starting values for the ML procedure.

The mean correlations between the true and heuristic estimates were 0.80 and 0.96 for the *a*- and *b*-values respectively, while the overall correlations between the true and ML estimates were 0.86 and 0.97 for the *a*- and *b*-values, respectively. Jensema concluded that the heuristic estimates were "surprisingly accurate" (p. 713). The correlations revealed that the true *a*-parameters and the corresponding *a*-values derived from Lord's formula increased with larger sample sizes ($r = 0.69$ for sample size of 250; $r = 0.85$ for sample size of 2,000) and increased slightly with greater number of test items ($r = 0.77$ for 25 items; $r = 0.82$ for 100 items), as proposed by Urry. The correlations revealed that the true *b*-parameters and the corresponding *b*-values derived from Lord's formula did not show a difference across sample size ($r = 0.94$ for sample size of 250; $r = 0.96$ for sample size of 2,000) or test length ($r = 0.96$ for all test lengths). Jensema suggested that the heuristic method may be used as a convenient technique for examining the worth of an item pool for tailored testing.

Ree (1979) also conducted a simulation study to assess the effectiveness of the heuristic method. Four procedures of parameter estimation were compared: the parameter estimates derived from the heuristic method and the ML estimates derived from three common computer programs (i.e., ANCILLES, LOGIST, OGIVIA). The particular processes underlying the computer programs were not specified. Using the 3PL model, data were generated for an 80-item test for three groups of 2,000 simulated examinees, where θ was normally distributed, positively skewed, and uniformly distributed. The true item parameters represented real examination data and were normally distributed: *a*-parameters had a mean of 0.95 ($SD = 0.28$), *b*-parameters had a mean of 0.16 ($SD = 0.93$), and *c*-parameters had a mean of 0.20 ($SD = 0.05$). For the heuristic method, a constant value of 0.20 was used for the *c*-parameter. The estimated item parameters derived from the heuristic method and the separate computer programs were correlated with the true parameters for each data set.

The correlations between the estimated and true parameters revealed the *b*-values yielded higher correlations than the *a*-values. For the heuristic method, the correlations of the difficulty parameters for the normal, skewed, and uniform data sets were 0.97, 0.92, and 0.96, respectively. In comparison, the correlations of the difficulty parameters ranged from 0.90 to 0.98 for the computer programs. For the heuristic method, the correlations of the discrimination parameters for the skewed, normal, and uniform data sets were 0.32, 0.35, and 0.59, respectively. Likewise, the correlations of the discrimination parameters for the computer programs depended on the distribution of θ. The lowest correlations, which ranged from 0.44 to 0.57, were observed for the skewed data, whereas high correlations were observed for the normal distribution (range of 0.83 to 0.84) and the uniform distribution (range of 0.87 to 0.90). In short, the heuristic procedure fared well in estimating the b-parameters, but fared less well in estimating the a-parameters using the 3PL model.

Contributions and Shortcomings of the Studies

Even though the three-parameter model was used with Lord's formulas, the results of the four studies suggest that the transformation procedures may provide some promise as a heuristic technique under certain conditions. All of the empirical studies indicated that the estimated b-values derived from the heuristic method were highly correlated ($r > 0.90$) with the true or ML estimates of b-values. There was some discrepancy in the findings for the correlations of the discrimination parameter. With normally distributed ability scores, Urry and Jensema obtained strong correlations ($r \geq 0.80$) for discrimination parameters. Ree (1979) observed correlations in the moderate range for normal as well as non-normal ability distributions.

All the researchers used correlations as the dependent measure to assess the concordance between the heuristic estimation of IRT item parameters and the true or ML values. One advantage of correlations is that variables of different metrics can be correlated to provide evidence of a linear relationship, but the data were in the same metric for the Urry, Jensema, and Ree studies. One disadvantage of correlations is that they only reflect the rank ordering of the variables being correlated, and therefore do not guarantee that the true and estimated parameters are close in value (Harwell, Stone, Hsu,

& Kirisci, 1996). Ree (1981) noted that a correlation between a set of parameters and estimates of the parameters would be misleading if systematic bias was evident such that the set of parameters was consistently over- or under-estimated. In addition, the assumptions underlying valid interpretations of correlations (e.g., linearity, homoscedasticity, no truncation or outliers) may not be satisfied (Harwell et al., 1996).

Parameter recovery is best assessed by evaluating the difference between the parameter and an estimate of that value (Harwell et al., 1996). Measures such as bias, Mean Absolute Deviation (MAD), and Root Mean Squared Error (RMSE), indicate the degree of agreement between the true and estimated values (Stone, 1992). Only by utilizing these measures will the degree of accuracy of the formulas be known.

When Lord (1980) presented the formulas, he suggested that the relationships fall short of accuracy because the test score contains measurement error. Three of the four articles reviewed alluded to this fact. Jensema commented that a weakness of the heuristic method is that the item-excluded test score is not the same as the latent ability, whereas Urry and Schmidt highlighted the importance of test reliability and its implications for the formulas to accurately reproduce the IRT parameters. In addition, Urry noted that longer tests and larger samples, which tend to increase variability in test scores, are necessary for successful application of the heuristic method.

The formulas were employed with the three-parameter model, violating the condition specified by Lord in 1980 that the IRT model not include guessing. As noted by Urry and Schmidt, with the 3PL model the estimated *a*-parameters will be attenuated and the estimated *b*-parameters will be augmented due to guessing. Without guessing as a factor, the *a*- and *b*-parameters may be more accurately estimated. In 1974 Urry pointed out that the size of the efficacy loss resulting from use of his item parameter approximations in comparison to more exact estimates or true parameter values was an open empirical question. To date this question has yet to be answered.

As noted in Chapter 1, Lord (1980) provided formulas that permitted prediction of the IRT discrimination parameter from the CTST biserial correlation, and the IRT difficulty parameter from the CTST *p*-value and the biserial correlation. More specifically, for item discrimination, the *a*-parameter is a function of the biserial correlation (ρ'_{ix}) between the item and the total test score. The following formula was proposed that associates the biserial correlation and the a_i parameter when θ is normally distributed and there is no guessing:

$$a_i \cong \frac{\rho'_{ix}}{\sqrt{1-\rho'^{2}_{ix}}} \quad .$$

For the difficulty indices, there is a monotonic relation between b_i and π_i when all items are equally discriminating: $b_i \cong \frac{y_i}{\rho'_{ix}}$. That is, the difficulty parameter b_i is proportional to y_i, the normal curve deviate corresponding to the proportion of correct answers (π_i).

CHAPTER 3 METHOD

Lord's formulas were investigated using simulated data and actual achievement data. The simulated data were used to examine the behaviours of the formulas under different experimental conditions, where the population parameters were known. To delineate realistic test and item characteristics, reference was made to researchers working with achievement data and modeling achievement data for simulation purposes. The achievement data were used to examine the extent to which the simulation results were generalizable to real achievement data.

Simulation Study

A Monte Carlo (MC) approach was used to investigate how closely the estimated *b*-parameters computed from the *p*-values and the estimated *a*-parameters computed from the biserial correlations corresponded to the parameters used to develop the response data matrices. Harwell et al. (1996) state that "MC studies are in many ways mirror images of empirical studies with one key difference: The data are simulated using a computer program" (p. 102). MC studies are conceptualized as statistical sampling experiments whose results are used to answer research questions (Harwell et al., 1996). According to the publication policy of *Psychometrika* (Psychometric Society, 1979), "Monte Carlo studies should be employed only if the information cannot reasonably be obtained in other ways" (p. 133).

For the given research questions, the most compelling reasons to use the MC technique are the capability of delineating and manipulating parameter values and observing the effects of several factors at one time. However, the usefulness of the results depends on how realistic the conditions are modeled (Ansley & Forsyth, 1985; Harwell et al., 1996). The research questions should determine the independent variables and suggest values for these variables. In addition, researchers need to consider the relationship between the number of independent variables and the interpretability of the results (Harwell et al., 1996). Naylor, Balintfy, Burdick, and Chu (1968) also cautioned that the inclusion of too many variables may hinder the ability to elucidate their joint effect.

Research Design

The research design encompassed independent variables relating to the population and samples and to the test and item characteristics. The design was a 3 (population ability distribution) x 3 (sample size) x 3 (test length) x 2 (item discrimination) fully crossed design, comprising 54 cells. The research design was a mixed model, like most models in education (Wilcox, 1988). Item parameters represented as equally spaced values across a fixed range or as estimates from a previously calibrated test are considered fixed effects. Random sampling of *a*- and *b*-parameters from specified distributions implies item parameters are random effects. An advantage of randomly selecting model parameters is that generalizability is achieved (Harwell et al., 1996).

The present research design consisted of fixed factors (i.e., ability distribution, sample size, test length) and one random factor (i.e., item difficulty). Item discrimination consisted of one condition that was fixed (i.e., unity) and one condition that was random (i.e., variable). Different sets of item parameters were selected for each experimental condition, which eliminates the dependency of the responses (Harwell et al., 1996). The conditions were designed to represent realistic response data. Descriptions of the conditions are discussed in light of this primary goal.

Population Ability Distributions

Lord (1980) stipulated that the formulas relating IRT item indices to conventional item indices are relevant when the underlying ability (θ) is normally distributed. Given that ability is likely not normally distributed for most groups of examinees (Lord, 1980, p. 20), two skewed distributions were modeled as well as the normal distribution.

Although most simulated studies assume an underlying normal ability distribution, researchers have modeled non-normal distributions using varied methods. Skewness and kurtosis are denoted by γ_1 and γ_2, respectively (Glass & Hopkins, 1996). For example, Yen (1987) generated three non-normal distributions: positively skewed ($\gamma_1 = 0.4$, $\gamma_2 = -0.1$), negatively skewed ($\gamma_1 = -0.4$, $\gamma_2 = -0.1$), and approximately symmetric but platykurtic ($\gamma_1 = 0.1$, $\gamma_2 = -0.4$), by combining two normal distributions with different mean abilities, standard deviations, and contributing proportions to form the final sample. Seong (1990) derived positively skewed ($\gamma_1 = 1.0$) and negatively skewed ($\gamma_1 = -1.0$) distributions by adapting the χ^2 distribution ($df = 8$). Stone (1992) generated positively skewed ($\gamma_1 = 0.75$, $\gamma_2 = 0.00$) and symmetric but platykurtic ($\gamma_1 = 0.0$, $\gamma_2 = -1.0$) distributions using a power method that involved transforming a standard normal deviate.

In the present study, the skewed ability distributions were generated using the beta probability density function. Beta distributions are a family of probability densities of continuous random variables taking on values in the interval ($0 < x < 1$). The function represents a two-parameter family of densities and can assume a variety of different shapes. The beta parameters *a* and *b*, often called "shape parameters," determine the shape of the distribution. When *a* is greater than *b*, the distribution is negatively skewed;

when b is greater than a, the distribution is positively skewed. If a random variable X is said to have a density given by:

$$f_x(x) = f_x(x;a,b) = \frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1} I_{(0,1)}(x),$$

where $a > 0$ and $b > 0$, then X is defined to have a beta distribution (Mood, Graybill, & Boes, 1974, p. 115). The beta function, denoted by $B(a,b)$, is defined by:

$$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}dx$$

for $a > 0$ and $b > 0$. $B(a,b)$ is the notation for the definite integral (Mood et al., 1974, p. 535).

For a given beta distribution, the expected mean is defined as $\xi(\mu) = \dfrac{a}{a+b}$ and the expected variance is defined as $\xi(\sigma^2) = \dfrac{ab}{(a+b+1)(a+b)^2}$. The third property, skewness, and fourth property, kurtosis, are calculated by:

$$\xi(\gamma_1) = \frac{2(-a+b)\sqrt{1+a+b}}{\sqrt{a}\sqrt{b}(2+a+b)} \text{ and}$$

$$\xi(\gamma_2) = \frac{3(1+a+b)(ab(-6+a+b)+2(a+b)^2)}{ab(2+a+b)(3+a+b)},$$

respectively (Wolfram, 1991).

Admittedly, it is difficult to generalize regarding the skewness and kurtosis of the distributions likely encountered in practice, although the skewness and kurtosis of theoretical distributions are well-known (Glass, Peckham, & Sanders, 1972). Lindquist (1953) described moderate skewness as 0.50 (kurtosis of 0.70) and extreme skewness as 1.00 (kurtosis of 0.80). Pearson and Please (1975) defined "typical" non-normality as having skewness greater than zero and less than 0.80 and kurtosis between –0.60 and 0.60 for their study of assessing the robustness of simple test statistics. Considering these guidelines and the values adopted by other researchers, the positively skewed distribution, defined as beta (2.9, 5.7), achieved an expected skewness of 0.40 and an expected kurtosis of -0.30; the negatively skewed distribution, defined as beta (5.7, 2.9), achieved an expected skewness of -0.40 and an expected kurtosis of -0.30. The beta distributions were rescaled so that the mean and standard deviation of the θ distributions were 0 and 1, respectively.

Population/Sample Size

Since the purpose of the study was to test the robustness of Lord's formulas, it was of interest to examine recovery of the population parameters with sample sizes relevant to practical use. A population of 10,000 examinees was generated and random samples drawn. Estimates of the minimal sample size required for the 2PL model include 500 (Hulin, Lissak, & Drasgow, 1982), 700 (McKinley & Mills, 1989), and 1,000 (Ree & Jensen, 1980). However, rules of thumb are not absolute and there is need for further research (Hambleton, 1989).

Harwell et al. (1996) explored the conditions under which MC techniques have been used for IRT research. During the years 1981 and 1991 inclusive, 26 published studies appeared in *Applied Psychological Measurement*, *Psychometrika*, and *Journal of Educational Measurement* that pertained to parameter estimation. Although Harwell et al. did not summarize the findings by IRT model, the following information for sample size and number of studies was documented: samples of 100 examinees or less (n = 9), samples of 150 to 200 (n = 5), samples of 300 to 500 (n = 13), samples of 900 to 1,000 (n

= 11), and samples of over 1,000 (n = 7). In the present study, random samples of 1,000, 500, and 250 are drawn from the simulated population.

Test Length

Questions about the lengths of tests needed for proper parameter estimation are difficult to answer because a number of factors come into play. The suitability of IRT models is dependent on the choice of IRT model, the parameter estimation method, the distribution of examinee ability, whether item parameters and/or ability parameters are being estimated, and the intended application with consideration given to acceptable errors (Hambleton, 1989). Generally fewer items are needed to calibrate IRT models with fewer parameters and more items are needed to calibrate IRT models with more item parameters. A minimum test length of 30 items is recommended for the 2PL model (Hulin et al., 1982).

Three test lengths were employed: a short exam of 20 items, a moderate exam of 40 items, and a long exam of 80 items. The short and moderate exam lengths are consistent with that frequently found in psychological and educational applications (e.g., Seong, 1990; Yen, 1987) and with that chosen by other researchers simulating data (e.g., Liou & Yu, 1991; MacDonald & Paunonen, 2002). The longest exam is consistent with Urry's requirement for the item-test biserial correlation to be a close approximation to the item-latent trait correlation. Specifically, he proposed that the test be of sufficient length (n_i = 80), the items homogenous (K-R 20 $\geq$ 0.90), and the sample size large (n_s = 2,000).

Item Difficulty

In the Harwell et al. (1996) review article, b-parameters were selected from normal distributions (n = 10), uniform distributions (n = 9), fixed values (n = 2), and by other means (n = 1). In simulation studies, it is common to have the item difficulty values centred at 0 and range from –2.0 to 2.0 (e.g., Ansley & Forsyth, 1985; Hambleton et al., 1993; Lautenschlager & Park, 1988; Maranon, Garcia, & Costos, 1997; Park & Lautenschlager, 1990; Ree, 1981), although a broader interval of -3.0 to 3.0 has been used (e.g., Veerkamp & Berger, 1997).

From the CTST perspective, Lord and Novick (1968) point out that when a test is built from pretested items, items with high variance are often chosen in preference to items with low variance. For binary items, the variance is largest when $p_i = 0.50$. Items with p-values close to zero or one contribute little to the overall test-score variance. From the IRT perspective, Hambleton and Swaminathan (1985) note that when a test is designed for norm-referenced testing, items are generally chosen to have a narrow range and medium level of difficulty, such as b-values between -2 to 2. This strategy will differentiate examinees on their competence in a given subject area, producing a broad range of scores that maximize discrimination among examinees. However, Drasgow (1989) suggested that parameter estimation in simulation studies would be improved by more closely matching the distribution of difficulty parameters to the ability distribution. To observe the effect of the full range of b-parameters on the formulas, b-values were selected from a normal distribution (μ = 0, σ = 1) in the present study.

Although other distributions have been modeled for b-parameters, Yen (1987) observed that uniform distributions of b-parameters are not frequently found in practice. The choice of a normal distribution of b-parameters is consistent with the selection process used in simulation research conducted at ETS (D. L. Henderson-Montero, personal communication, May 2, 2003).

Item Discrimination

Like the modeling of difficulty values, there is no one accepted method of selecting discrimination values for simulation studies. In the Harwell et al. (1996) review article, several selection methods for item discrimination values were documented: uniform distribution ($n = 9$), fixed values ($n = 6$), and another selection strategy ($n = 3$).

Two conditions of item discrimination were investigated. One condition maintained a constant value for a-parameters, which adheres to Lord's stipulation that there is a monotonic relation between b_i and π_i when items are equally discriminating. Selection of true discrimination values equal to one is consistent with other researchers employing the 1PL model (e.g., Hambleton et al., 1993; MacDonald & Paunonen, 2002; Yen, 1981, 1984).

Traub (1983) commented on the appropriateness of the assumption that all item discrimination parameters are equal. Considering the abundance of empirical evidence, he stated: "The fact that otherwise acceptable achievement items differ in the degree to which they correlate with the underlying trait has been observed so very often that we should expect this kind of variation for any set of achievement items we choose to study" (p. 64). Therefore, variable discrimination values were modeled. A log normal distribution ($\mu = 0$, $\sigma = 0.4$) was chosen, given it is the default distribution for slopes in BILOG (Mislevy & Bock, 1990) and has been selected by other researchers modeling achievement data (e.g., D. L. Henderson-Montero, personal communication, May 2, 2003; Seong, 1990). The specified log normal distribution was positively skewed with the following expected descriptive statistics: $\mu = 1.08$, $\sigma^2 = 0.20$, $\gamma_1 = 1.32$, and $\gamma_2 = 9.26$.

Procedure

Data Generation

The item response generation technique described by Harwell et al. (1996) was used to create the data. The procedure entailed four steps. Step 1 involved the generation of true ability scores (θ). In Step 2, tests were created according to the test specifications (test length, *a*-parameters, and *b*-parameters). For each cell in the experimental design, a unique test was created by deriving new sets of *a*- and *b*-parameters consistent with the specifications for that cell. In Step 3, response probabilities of a population of 10,000 examinees to the n_i (20, 40, 80) items based on the 2PL IRT model were determined, producing a 10,000 x n_i matrix. In Step 4, the matrix of response probabilities was translated into a 10,000 x n_i data matrix containing 0/1 responses. Each response probability was compared to a random number drawn from a uniform distribution of values in the closed interval [0,1]. If the response probability was equal to or greater than the random number, a '1' was assigned for that item score; if the response probability was less than the random number, a '0' was assigned for that item score.

Computer Programs

Mathematica for Students (Version 4, Wolfram, 2000) was used to generate the response data matrices. Appendix A contains the program for a 20 item test. LERTAP5 (Nelson, 2000), an Excel application, was employed to obtain the classical item analyses and Lord's estimation of the *a*- and *b*-parameters. BILOG (Version 3.2, Mislevy & Bock, 2000) was employed to determine the population IRT item indices for the real achievement data. Random sampling from the 10,000 examinees was performed with an Excel (Version 5) macro program, a copy of which appears in Appendix B.

Replications

The number of replications in a MC study corresponds to sample size in empirical studies; therefore, the criteria used to guide sample size selection in empirical studies apply to MC studies. In IRT research, the number of replications hinges on the purpose of the MC study, the need to minimize the sampling variance of the estimated parameters, and the need for the outcome measures to have adequate power to detect differences of interest (Harwell et al., 1996).

The number of replications influences the precision of the estimated parameters, such that the greater the number of replications, the less sampling variance inherent in the estimates. Statisticians typically use several thousand replications in order to minimize sampling variance, in direct contrast to IRT-based MC research where some studies have utilized no replications. In the Harwell et al. (1996) review, 16 studies performed only one replication, 3 studies executed between 2 and 10 replications, 3 studies used 11 to 50 replications, one study ran between 51 and 100 replications, and 2 studies executed more than 100 replications.

The benefits of replicated over non-replicated IRT MC studies are the same as that observed in empirical studies; aggregating results over replications generates more stable and reliable findings. The folly of not using replications lies in the fact that the sampling variance may be large enough to question the reliability of parameter estimates. Therefore, increasing the number of replications is an attractive technique for reducing the variance of the estimated parameters (Harwell et al., 1996). In the present study, 100 random samples were drawn from each population for a given experimental condition, following the suggestion that generating a large number of responses and selecting random samples from the population is a means of minimizing random error (Harwell et al., 1996).

Diploma Examination Study

Lord's formulas were applied to actual achievement data sets. These data sets consisted of the item scores obtained on provincial examinations by examinees who wrote the biology exam ($N = 9{,}030$), representing the sciences, and the English exam ($N = 13{,}375$), representing the humanities (Alberta Learning, 1999a, b). The exams are high school graduation examinations, which contribute 50% towards examinees' final course grades. Only the M-C components of the exams were used, which comprised 48 items for the biology exam and 70 items for the English exam. The biology exam consisted of stand-alone items, except for one testlet of three items. The English exam consisted of seven testlets of 8 to 12 items.

Psychometric Properties of the Exams

Descriptive statistics for the total test and ability distributions, and item level information are presented in Table 1. The observed score distributions for both exams were negatively skewed and platykurtic. The classical test score item information reveals that the items were of moderate difficulty, as judged by the mean p-values. Item requirements for the exams include minimum and maximum acceptable difficulty levels 0.30 and 0.85, respectively, and a minimum acceptable point-biserial correlation of 0.20 (Alberta Education, 1999), which approximately corresponds to a biserial correlation of between 0.25 and 0.30. The mean biserial correlations met the criterion of 0.40 to be considered high (Nelson, 2001).

Examination of the IRT data reveals that the ability distributions for both exams were positively skewed and leptokurtic. The IRT item data indicates that the mean of the estimated difficulty and discrimination parameters for the achievement data are lower than that modeled in the simulation study.

Table 1

Psychometric Properties of the Biology and English Exams Using CTST and IRT Analyses

	CTST		IRT	
	Biology	English	Biology	English
Test Level Information				
Mean	33.48	44.26	0.08	0.03
Median	34	44	-0.10	-0.08
SD	7.83	11.08	1.21	1.09
Reliability (α)	0.86	0.89		
Skewness	-0.37	-0.13	0.78	0.66
Kurtosis	-0.52	-0.69	0.53	0.35
Item Level Information				
M Difficulty	0.70	0.63	-1.12	-0.87
M Discrimination	0.44[a] (0.32[b])	0.41[a] (0.32[b])	0.56	0.47

[a] r_b. [b] r_{pb}.

Statistical Analyses

The estimated item parameters derived from Lord's formulas were compared to the parameters used to generate the data matrices for the simulated data. Since the true item parameters for the achievement exams were not known, the item parameters derived from BILOG using the two-parameter model were investigated as possible values that could be used to evaluate the formulas.

Dependent Variables

Multiple criterion variables are desirable because they can provide additional evidence of the effect of an independent variable (Harwell et al., 1996), although too many outcome measures may decrease the efficiency of the study and increase the incidence of chance differences (Naylor et al., 1968). The importance of interpreting the dependent measures together was underscored by Ree (1981), who noted that the correlation of a parameter and an estimate of that parameter would be misleading if a constant bias was evident. Ree advocated measures of bias and variability to interpret parameter recovery. Two indices of estimation success were employed in the present study. In the formulas presented, *i* references the item and *r* references the replications.

Bias

Parameter recovery is assessed by comparing an item parameter estimate and the corresponding parameter value (Harwell et al., 1996). Estimation bias is defined as the mean difference between the estimated and true parameter value for an item across 100 replications. Bias in each $\hat{a}_i$ was calculated by:

$$Bias\ \hat{a}_i = \sum_{r=1}^{100} (\hat{a}_{ir} - a_i) / 100 .$$

Bias in $\hat{b}_i$ was calculated similarly. Smaller differences indicate the estimates are closely aligned to the parameter values compared to larger differences. By maintaining the valence of the difference it is possible to determine whether the estimates systematically overestimate (positive bias) or underestimate (negative bias) the parameter values. Examining the nature of the bias is particularly important in light of Schmidt's (1977) contention that Lord's formulas tend to systematically underestimate a_i and overestimate $|b_i|$.

Standard Errors

Gifford and Swaminathan (1990) presented the following formula for variance of the estimates across replications:

$$\hat{\sigma}_{a_i}^{\ 2} = \frac{\sum_{r=1}^{100} (\hat{a}_{ir} - \bar{\hat{a}}_i)^2}{100},$$

where $\hat{\sigma}_{a_i}^{\ 2}$ is the sampling variance of the estimated item discrimination for item i and $\bar{\hat{a}}_i$ is the mean of the estimated a-parameters for item i across 100 replications. The sampling variance for b_i was calculated similarly. Smaller values of sampling variance suggest that the estimates are fairly stable and reliable, whereas larger values indicate the estimates may be unreliable.

Empirical standard errors were calculated from the sampling variances by taking the square root of the mean of the sampling variance across items for each condition. The standard error (SE) for item discrimination is presented in the following formula:

$$SE_a = \sqrt{\frac{\sum_{i=1}^{n} \hat{\sigma}_{a_i}^2}{n}}$$

The SE for item difficulty is calculated similarly. The standard errors were used to construct 95% confidence intervals (CI) around the bias of zero, and was used to determine whether bias was significant or not. When the item bias was within two standard errors (2SE) of the bias of zero, the item was considered to be only randomly different from zero (i.e., not significantly different from zero), and therefore well estimated. When the item bias exceeded 2SE of the bias of zero, the item was considered to be significantly different from zero, and therefore poorly estimated.

CHAPTER 4 RESULTS AND DISCUSSION

Chapter 4 contains the results and discussion of the results of the simulated and real achievement data. The results are presented using two formats. First, scatter plots display the patterns of bias across the range of classical parameter values. Second, the observed biases, organized in terms of intervals of the classical test score statistics, are summarized in a table that accompanies the scatter plots. The simulated data results are presented first, followed by the achievement data results. A discussion of the results accompanies each presentation.

Simulation Study Results

Preliminary Considerations

Several considerations need to be addressed prior to presenting the results of the simulation study. To illustrate the values of the estimates obtained, the results for one condition, a 20 item test with unit discrimination, sample size of 1,000 examinees, and a population of examinees with normally distributed ability, are presented in Table 2. The information provided in the table, moving from left to right, includes the item number, *p*-value (p), biserial correlation (r_b), true item difficulty (b_τ), BILOG item difficulty (b_B), LERTAP item difficulty (b_L), mean LERTAP item difficulty computed from 100 replications ($\bar{\hat{b}}_L$), sampling variance for LERTAP item difficulty computed from 100 replications ($\hat{\sigma}_b^2$), true item discrimination (a_τ), BILOG item discrimination (a_B), LERTAP item discrimination (a_L), mean LERTAP item discrimination computed from 100 replications ($\bar{\hat{a}}_L$), and sampling variance for LERTAP item discrimination computed from 100 replications ($\hat{\sigma}_a^2$).

Table 2

Item Indices for the Conditions of 20 Items, Normal Ability Distribution, Unit Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	081	074	-120	-125	- 116	-116	00054	100	098	111	113	00101
Q2	060	077	-036	-038	- 034	-035	00027	100	099	121	124	00098
Q3	041	075	030	034	030	030	00027	100	095	114	115	00059
Q4	014	069	151	156	154	155	00112	100	097	095	095	00059
Q5	077	075	-103	-106	- 096	-096	00031	100	097	113	115	00075
Q6	070	075	-075	-079	- 071	-072	00036	100	095	114	114	00092
Q7	061	077	-041	-040	- 036	-035	00021	100	097	119	122	00090
Q8	055	075	-019	-020	- 018	-019	00022	100	093	115	114	00063
Q9	055	076	-020	-019	- 017	-018	00030	100	096	117	118	00065
Q10	072	076	-082	-085	- 077	-076	00038	100	098	116	117	00089
Q11	046	075	013	013	012	012	00020	100	093	113	113	00066
Q12	019	071	127	131	126	126	00058	100	098	101	102	00050
Q13	060	077	-039	-037	- 034	-034	00020	100	098	120	121	00084
Q14	041	075	033	033	030	030	00025	100	097	115	116	00067
Q15	087	072	-164	-167	- 158	-159	00098	100	096	104	104	00080
Q16	025	074	095	095	090	090	00042	100	099	109	109	00061
Q17	067	077	-061	-065	- 058	-059	00030	100	098	119	120	00081
Q18	020	070	118	126	120	121	00077	100	093	098	098	00069
Q19	075	076	-096	-099	- 091	-090	00041	100	100	117	120	00126
Q20	029	072	078	084	078	079	00044	100	092	104	104	00061

Note. Data are presented without decimal points. All entries are given to the second decimal place except for the sampling variances, $\hat{\sigma}_b^2$ and $\hat{\sigma}_a^2$, which are given to the fourth decimal place. For example, b_τ of -120 implies -1.20, and $\hat{\sigma}_b^2$ of 00054 implies 0.0054.

The item index values are given to two decimal places except for the sampling variances, which are given to four decimal places. For example, as shown in Table 2, the fixed parameters for question 1 are $p = 0.81$, $r_b = 0.74$, $b_B = -1.25$, $b_L = -1.16$, $a_B = 0.98$, and $a_L = 1.11$ based on the population of 10,000 examinees. Random sampling of 1,000 examinees over 100 replications resulted in $\bar{\hat{b}}_L = -1.16$, $\hat{\sigma}_b^2 = 0.0054$, $\bar{\hat{a}}_L = 1.13$, and $\hat{\sigma}_a^2 = 0.0101$. The parameter values used to generate the data were $b_\tau = -1.20$ and $a_\tau = 1.00$. Bias values are not provided, but may be computed from the information provided in the table.

As can be seen in Table 2, the mean of Lord's item difficulty and discrimination values across 100 replications (i.e., $\bar{\hat{b}}_L$, $\bar{\hat{a}}_L$) closely approximated the corresponding values obtained for the population (i.e., b_L, a_L). To discuss the population and sample means separately would be redundant. As indicated in Chapter 3, the mean sample estimates were used to evaluate the closeness of the estimated values to the corresponding true values. Tables containing the same item level information as that reported in Table 2 are provided in Appendix C for the other 53 cells in the design.

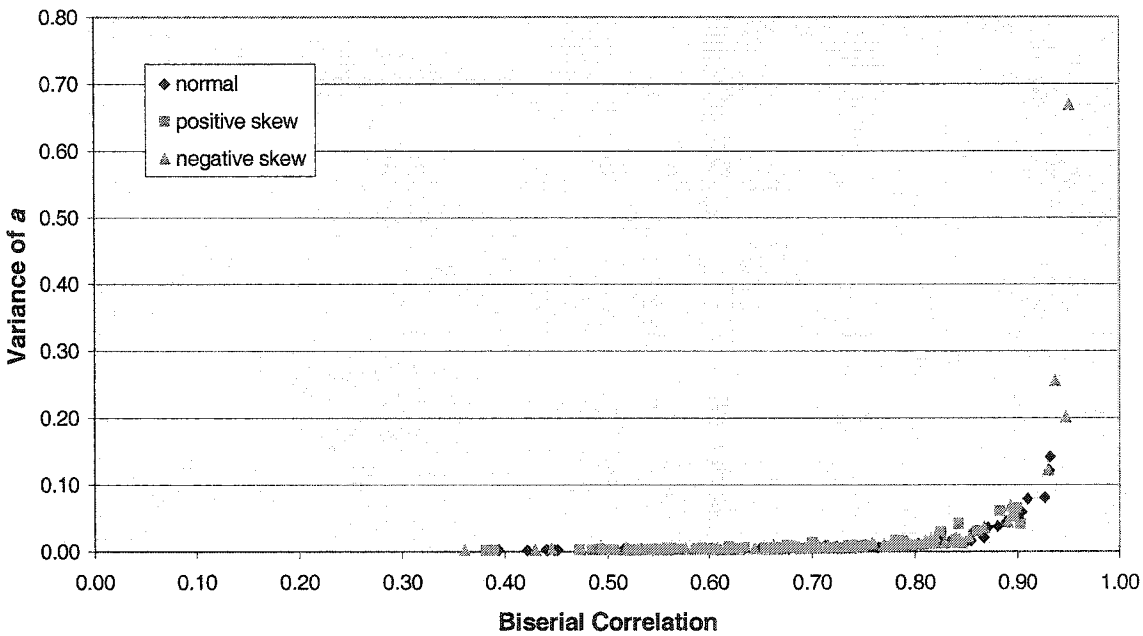
The first preliminary consideration that needed to be addressed related to two situations for which estimation of Lord's parameter values is not possible. For item discrimination, the denominator of the formula is undefined when the biserial correlation equals or exceeds one (i.e., $r_b \geq 1.00$). Replications with such biserials were deleted. For item difficulty, the *b*-parameter formula is undefined when the *p*-value equals 0 or 1 because it is not possible to compute a *z*-score that is required for the numerator, and a biserial correlation that is required for the denominator. Replications with such *p*-values were deleted. Consequently, the means and sampling variances of Lord's estimates for a few items were determined on fewer than 100 replications, as footnoted in the tables.

Second, although test length initially was an independent variable, the patterns of bias and sampling variance were consistent across the 20, 40, and 80 item tests. Similarly, MacDonald and Paunonen (2003) found that test length did not influence the correlations between classical and IRT item information in their simulated study. Therefore, the 80 item tests were used to illustrate the results because the patterns are more clearly delineated with more items. The other independent variables (i.e., sample size, shape of the ability distribution, and unit or variable discrimination) were related to the patterns of bias and/or sampling variance. These factors are discussed in the main text.

Standard Errors

The third preliminary consideration that needed to be resolved concerned the range of p and r_b values to use to compute the corresponding SEs. As will be shown, the SEs were a function of sample size for the discrimination estimates, and a function of sample size and nature of the item discrimination (i.e., unit, variable) for the difficulty estimates.

Item discrimination. The distribution of the sampling variances of the item discrimination estimates is illustrated in Figure 6 as a function of the biserial correlation. As shown for the sample size of 1,000 examinees, the formula for the a-parameter is an asymptotic function that yields high a_i estimates as the biserial correlation approaches one. For example, Lord's formula with biserial correlations of 0.94, 0.96, and 0.98 yield $\hat{a}_i$ values of 2.76, 3.43, and 4.92, respectively. Small fluctuations in high biserial values from sample to sample resulted in dramatic differences in the $\hat{a}_i$, which is reflected in the variance of $\hat{a}_i$. Consequently, to avoid this sampling behaviour, mean item variances of $\hat{a}_i$ were computed from items with biserial coefficients less than 0.90 to prevent unrealistically high SEs.



Note: One item from the normal ability distribution (r(bis) = 0.98, a = 2.92) was omitted from the scatter plot due to an extreme variance of 37.88.

Figure 6: Variance of the discrimination estimates as a function of the biserial correlation for the 80 item test, variable discrimination, and sample size of 1,000.

The mean sampling variances of item discrimination estimates for the conditions of variable discrimination and 80 items are reported in Table 3. The first column contains the mean sampling variances of $\hat{a}_i$ for the sample sizes of 1,000 examinees. The mean sampling variances of $\hat{a}_i$ for the normal, positively skewed, and negatively skewed ability distributions were 0.0084, 0.0081, and 0.0086, respectively. Consequently, given the similarity of these variances, the SE was obtained by taking the square root of the overall mean variance, 0.0084. Rounding to two decimal places, the SE of 0.09 was used to construct the 95% CI of +/- 0.18 around a bias of zero. The same procedure was followed to calculate the mean sampling variance for the samples of 500 and 250, resulting in CIs of 0.30 and 0.42, respectively.

Table 3

Mean Sampling Variance of the Item Discrimination Estimates for the 80 Item Tests with Variable Discrimination (r_b < 0.90)

	Sample Size		
Ability Distribution	1,000	500	250
Normal	0.0084	0.0232	0.0421
Positively Skewed	0.0081	0.0225	0.0508
Negatively Skewed	0.0086	0.0228	0.0398
Mean Variance	0.0084	0.0228	0.0442
Standard Error	0.0915	0.1511	0.2103
Interval Width (+/-)	0.18	0.30	0.42

Although it was expected that the width of the CI would increase as the sample size decreased, it was not expected that the number of items with biased discrimination indices would be noticeably different. Table 4 contains the numbers of discrimination estimates identified outside of the CI for the 80 item tests using the CI of 0.18 for the sample of 1,000 examinees and the sample-specific CIs for the smaller sample sizes (0.30 for n_s = 500, and 0.42 for n_s = 250). Reading across the columns for the last two rows (i.e., n_s = 250), 13, 35, and 20 items were identified as biased when the CI of 0.18 was applied, respectively, to the normal, positively skewed, and negatively skewed ability distributions. Using the CI of 0.42 with the same data, only 3, 12, and 6 items were identified as biased. Hence, to avoid the erroneous conclusion that smaller sample sizes resulted in fewer items with biased discrimination indices, the CI associated with the sample of 1,000 examinees was applied to the smaller sample sizes.

Table 4

Number of Discrimination Estimates Identified Outside of the Confidence Interval for the 80 Item Tests Using the Criteria for the Sample of 1,000 and the Given Sample Size

n_s	CI	Normal	Positive Skew	Negative Skew
1,000	0.18	15	20	17
500	0.18	13	29	25
	0.30	5	11	16
250	0.18	13	35	20
	0.42	3	12	6

Item difficulty. The distribution of the sampling variances of the item difficulty estimates is illustrated in Figure 7 as a function of classical difficulty for the conditions of variable discrimination and sample of 1,000 examinees. The sampling variances increased as the p-values became more extreme. The numerator of Lord's formula for $\hat{b}_i$ is the z-score associated with the p-value. The z-score changes more rapidly as the two limits, 0 and 1, are approached than when the p-values are moderate because there are fewer examinees with extreme score values. The result is greater fluctuations from sample to sample, which serves to increase the variance. Consequently, to avoid unrealistically high SEs, the mean variances of the estimated item difficulties were computed from items within the range of $0.05 < p < 0.95$.

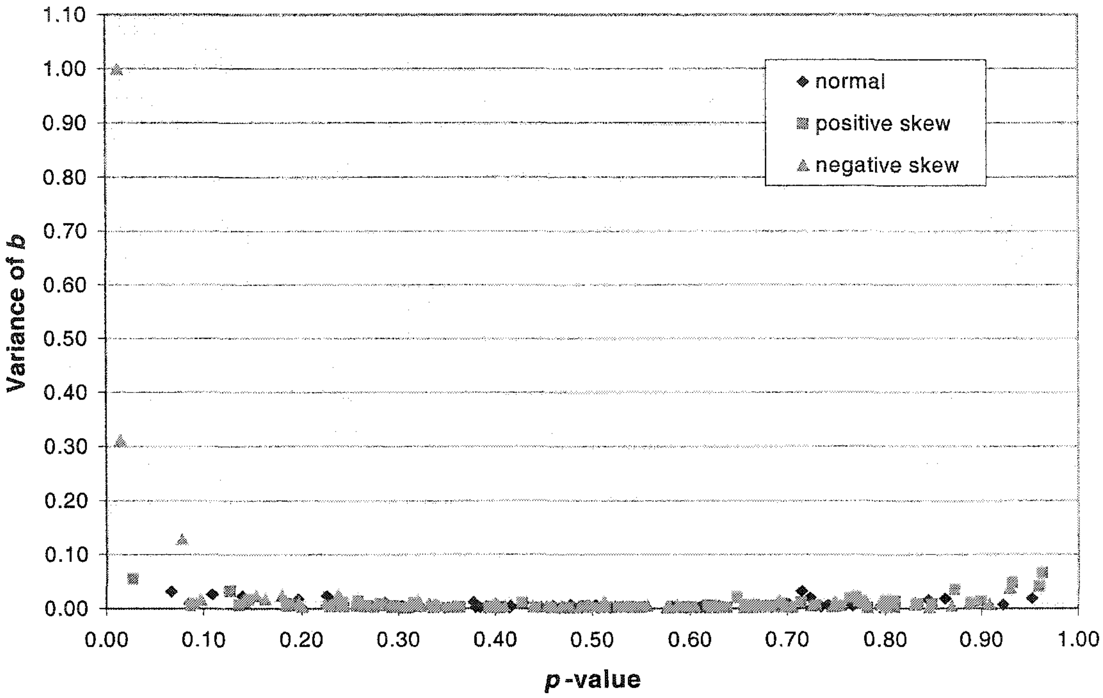


Figure 7. Variance of the difficulty estimates as a function of classical difficulty for the 80 item test, variable discrimination, and sample size of 1,000.

The mean sampling variances of the item difficulty estimates for the unit and variable discrimination conditions are provided in Table 5. The first column contains the mean sampling variances for the sample size of 1,000 examinees. The mean sampling variances of $\hat{b}_i$ for the normal, positively skewed, and negatively skewed ability distributions for the condition of unit discrimination were 0.0061, 0.0063, and 0.0069, respectively. Given the similarity of these variances, the SE was obtained by taking the square root of the overall mean variance, 0.0064. Rounding to two decimal places, the SE of 0.08 was used to construct the 95% CI of +/- 0.16 around a bias of zero. The same procedure was followed to calculate the mean sampling variances for the samples of 500 and 250, resulting in CIs of 0.22 and 0.36, respectively, for the unit discrimination condition. The mean sampling variances of $\hat{b}_i$ for the normal, positively, and negatively skewed ability distributions for the condition of variable discrimination were 0.0067, 0.0075, and 0.0084, respectively, resulting in a mean variance of 0.0075, and a SE of 0.0867. Rounding to two decimal places, the SE of 0.09 was used to construct the 95% CI of +/- 0.18 around a bias of zero. Therefore, when the mean item difficulty estimates were within 2SE of the true value (i.e., 0.16 for unit and 0.18 for variable discrimination), Lord's estimate of the item difficulty was considered to be unbiased. The CIs were consistently 0.02 wider for the variable discrimination conditions compared to the unit discrimination conditions of the same sample size.

Table 5

Mean Sampling Variance of the Item Difficulty Estimates for the 80 Item Tests (0.05 < p < 0.95)

	Sample Size		
Unit Discrimination	1,000	500	250
Normal	0.0061	0.0136	0.0323
Positively Skewed	0.0063	0.0128	0.0319
Negatively Skewed	0.0069	0.0117	0.0316
Mean Variance	0.0064	0.0127	0.0319
Standard Error	0.0803	0.1127	0.1787
Interval Width (+/-)	0.16	0.22	0.36
Variable Discrimination			
Normal	0.0067	0.0169	0.0363
Positively Skewed	0.0075	0.0121	0.0365
Negatively Skewed	0.0084	0.0156	0.0403
Mean Variance	0.0075	0.0149	0.0377
Standard Error	0.0867	0.1219	0.1941
Interval Width (+/-)	0.18	0.24	0.38

Like item discrimination, the CIs increased as the sample size decreased, resulting in fewer biased items for the smaller sample sizes. Table 6 contains the numbers of difficulty estimates identified outside of the CI for the 80 item tests using the CI for the sample of 1,000 examinees and the sample-specific CIs for the smaller sample sizes. Reading across the columns for the last two rows of the unit discrimination conditions (n_s = 250), there were 4, 11 and 12 items identified as biased when the CI of 0.16 was applied to the normal, positively skewed, and negatively skewed ability distributions, respectively. Using the CI of 0.36 with the same data, only 1, 6 and 6 items were identified as biased. In an effort to avoid the erroneous conclusion that smaller sample sizes resulted in fewer items with biased difficulty indices, the CI associated with the sample of 1,000 examinees was applied to the smaller sample sizes.

Table 6

Number of Difficulty Estimates Identified Outside of the Confidence Interval for the 80 Item Tests Using the Criteria for the Sample of 1,000 and the Given Sample Size

Discrimination	n_s	CI	Normal	Positive Skew	Negative Skew
Unit	1,000	0.16	3	5	9
	500	0.16	4	3	3
		0.22	4	3	2
	250	0.16	4	11	12
		0.36	1	6	6
Variable	1,000	0.18	2	6	6
	500	0.18	6	4	4
		0.24	6	4	3
	250	0.18	3	7	10
		0.38	0	2	5

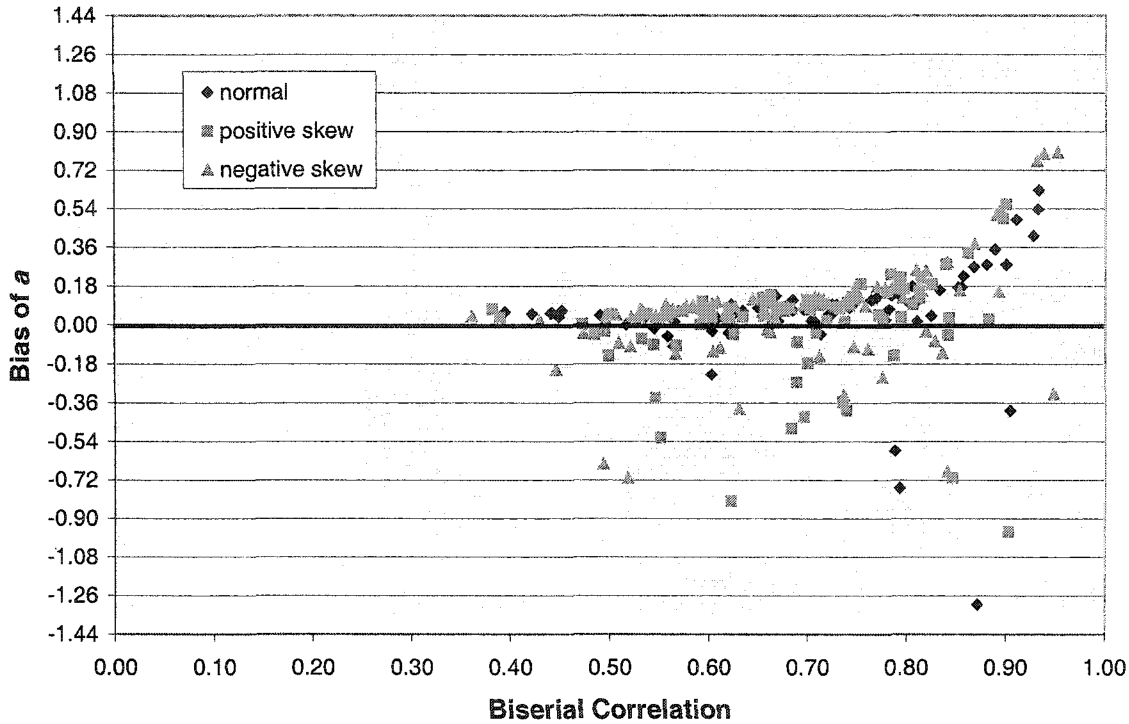
Presentation of Bias

The results for the 80 item tests are discussed with respect to the sample sizes of 1,000, 500, and 250. Although it was noted earlier that the mean estimated values across replications were very similar to the population parameter values regardless of sample size, it is important to document the consistency of the findings across the three sample sizes.

In the interest of space, only the data for the samples of 1,000 are illustrated in the scatter plots. The classical item indices are documented on the horizontal axis and bias is documented on the vertical axis. Bias is presented in increments of 2SE. In some instances, extreme bias values are not shown on the figure. In order to accommodate these values, the vertical scale would have to be altered in such a way that the patterns of the remaining items would be obscured. The information for these items is documented in a footnote to the figure.

Item Discrimination

Evaluation of Lord's discrimination formula is considered with respect to the variable discrimination condition. The patterns of bias were consistent across sample sizes; therefore, the percents of items with unbiased or biased estimates are considered after collapsing across sample size. However the patterns were not consistent across the three ability distributions. Consequently, the bias patterns for the normal, positive, and negatively skewed ability distributions are illustrated as a function of the biserial correlation in Figure 8 for the sample of 1,000 examinees. The items with discrimination estimates that were within 2SE of their true values were considered well estimated ($\bar{\hat{a}}_i = a_\tau$). Items with discrimination estimates that overestimated their corresponding true values ($\bar{\hat{a}}_i > a_\tau$) possessed positive bias, while items with discrimination estimates that underestimated their corresponding true values ($\bar{\hat{a}}_i < a_\tau$) possessed negative bias.



Note: One item from the normal distribution (r(bis) = 0.98, p = 0.39, a = 2.92) was omitted from the scatter plot due to an extreme bias of 4.22.

Figure 8. Bias of the discrimination estimates as a function of the biserial correlation for the 80 item test, variable discrimination, and sample size of 1,000.

Table 7 provides the numbers of well estimated, and positively and negatively biased items, where the values of the *a*-parameter estimates are considered with regard to four intervals of biserial correlations: $r_b < 0.60$, $0.60 \leq r_b \leq 0.69$, $0.70 \leq r_b \leq 0.79$, and $r_b \geq 0.80$. The selection of these categories was based on determining a means of describing the biserial coefficients that are likely encountered in the practical setting (e.g., r_b within the 0.6 range, r_b within the 0.7 range), and on the observation that there were approximately equal numbers of items within the four categories.

Table 7

Number of Biased and Unbiased Discrimination Estimates Categorized by the Biserial Correlation

		Biserial Interval											
		$r_b < 0.60$			$0.60 \leq r_b \leq 0.69$			$0.70 \leq r_b \leq 0.79$			$r_b \geq 0.80$		
n_s	Distribution	Unbiased	+ Bias	- Bias	Unbiased	+ Bias	- Bias	Unbiased	+ Bias	- Bias	Unbiased	+ Bias	- Bias
1,000	Normal	16/16	0	0	18/19	0	1	21/23	0	2	10/22	10	2
		(100%)			(94.7%)			(91.3%)			(45.5%)		
	Pos Skew	20/22	0	2	15/18	0	3	18/25	4	3	7/15	6	2
		(90.9%)			(83.3%)			(72.0%)			(46.7%)		
	Neg Skew	21/24	0	3	19/20	0	1	16/18	0	2	7/18	9	2
		(87.5%)			(95.0%)			(88.9%)			(38.9%)		
500	Normal	13/14	0	1	23/25	0	2	19/23	0	4	12/18	1	5
		(92.9%)			(92.0%)			(82.6%)			(66.7%)		
	Pos Skew	9/10	0	1	13/16	0	3	25/31	3	3	4/23	15	4
		(90.0%)			(81.3%)			(80.6%)			(17.4%)		
	Neg Skew	17/19	0	2	14/16	0	2	18/19	0	1	6/26	19	1
		(89.5%)			(87.5%)			(94.7%)			(23.1%)		
250	Normal	13/13	0	0	24/25	0	1	20/23	0	3	10/19	8	1
		(100%)			(96.0%)			(87.0%)			(52.6%)		
	Pos Skew	17/18	0	1	6/9	0	3	19/30	7	4	3/23	18	2
		(94.4%)			(66.7%)			(63.3%)			(13.0%)		
	Neg Skew	19/23	0	4	23/24	0	1	14/18	1	3	4/15	11	0
		(82.6%)			(95.8%)			(77.8%)			(26.7%)		

The results shown in Figure 8 and reported in Table 7 indicate that discrimination was best estimated when the biserial correlations were lower, regardless of the shape of the ability distribution and sample size. A high percentage of $\bar{\hat{a}}_i$ for items with biserial coefficients less than 0.60 were within 2SE of their true values (i.e., 42/43, 97.7% for normal; 46/50, 92.0% for positively skewed; and 57/66, 86.4% for negatively skewed, collapsing across sample sizes). Similarly, a high percentage of $\bar{\hat{a}}_i$ for items with biserial coefficients between 0.60 and 0.69 inclusive were well estimated (i.e., 65/69, 94.2% for normal; 34/43, 79.1% for positively skewed; and 56/60, 93.3% for negatively skewed, collapsing across sample sizes). Lord's formula yielded negatively biased estimates of the true a-value for the remaining items with biserial correlations less than 0.70 (i.e., $\bar{\hat{a}}_i < a_\tau$).

As the value of the biserial correlation increased, the percentages of items with unbiased estimates of a_τ tended to decrease, particularly for r_b values greater than or equal to 0.80. In contrast to the case where the values of the biserials were less than 0.70, both positive and negative bias estimates occurred, with the number of positive estimates increasing as the value of r_b increased. Furthermore, the influence of the shape of the ability distribution became more noticeable.

When the biserial correlation was between 0.70 and 0.79 inclusive, a high majority of the discriminations were well estimated, particularly for the normal and negatively skewed ability distributions. The sample discrimination estimates for 87.0% (60 of 69) of the items from the normal ability distribution and 87.3% (48 of 55) of the items from negatively skewed ability distribution were well estimated, whereas only 72.1% (62 of 86) of the items from the positively skewed ability distribution were well estimated. Of the biased items, the discriminations of all of the items from the normal ability distributions and most of the items from the negatively skewed ability distributions (6 of 7, 85.7%) were underestimated, whereas the majority of the discriminations of the items (14 of 24, 58.3%) from the positively skewed ability distributions were overestimated.

Lord's discrimination formula performed most poorly when the values of the biserial coefficients were equal to or greater than 0.80. The mean sample discrimination index was within 2SE of their corresponding true values for approximately half of the items (54.2%) from normal ability distribution conditions, and only about a quarter of the items from the skewed ability distribution conditions (23.0% for positive, 28.8% for negative). Of those items with biased discrimination estimates, most were positively biased (70.4% for the normal, 83.0% for the positive, and 92.9% for the negatively skewed ability distributions).

The largest values of bias occurred for biserials close to 1. For example, one item from the normal ability distribution with the sample size of 1,000 examinees (r_b = 0.98, p = 0.39, a_τ = 2.92) had a bias of 4.22, whereas one item from the positively skewed distribution (r_b = 0.98, p = 0.24, a_τ = 2.53) and one item from the negatively skewed distribution (r_b = 0.97, p = 0.05, a_τ = 2.26) both with the sample sizes of 500 examinees were overestimated by 2.73 and 2.71, respectively.

Summary. Taken together, these results suggest that Lord's formula for predicting $\hat{a}_i$ from r_b works very well for items with biserial coefficients less than 0.70, and works reasonably well for items with biserial coefficients between 0.70 and 0.79 inclusive, regardless of the shape of the ability distribution. However, the formula tends to overestimate the a-parameter for biserials 0.80 and greater, and the accuracy of the formula was poorer for the skewed ability distributions than for the normal distribution.

Item Difficulty

The results of the assessment of Lord's formula for converting a classical test score p-value to an IRT difficulty estimate for the two-parameter model is assessed first for the condition of equal item discrimination that Lord stipulated for the formula, followed by the assessment of Lord's formula when the item discrimination is variable. Bias patterns will be discussed and illustrated as a function of the p-value. The patterns of bias were consistent across sample sizes; therefore, the percents of items with unbiased or biased estimates are considered after collapsing across sample size. For all ability distributions, the relationships between bias and p-value were curvilinear. The conditions of unit and variable discrimination yielded very similar results.

Unit discrimination. The bias patterns for the estimated b-parameter as a function of the p-value are illustrated in Figure 9 for the three ability distributions. Overall, the graph indicates that the difficulty parameter is well estimated for a wide range of p-values. Without exception, when biases were observed, the b-parameter was overestimated for difficult items, and underestimated for easy items.

Lord's formula accurately predicted the true b-parameter for p-values between approximately 0.10 and 0.90 for the normal distribution. When p-values were less than 0.10, positive bias was observed; when p-values were greater than 0.90, negative bias was observed. However, biases were differentially affected by the direction of skewness of the ability distributions, such that the patterns of bias for the skewed distributions were mirror images of each other. Bias estimates generally were found at only one end of the difficulty scale for the skewed ability distributions. For the positively skewed ability distribution, mean item sample difficulties were well estimated for p-values between 0 and 0.85, but were underestimated for p-values greater than or equal to 0.85. The best fitting curve tended to arc downwards for the easier items, such that easiest item had the most extreme bias. In contrast, the mean item sample difficulties were well estimated for p-values of 0.15 and higher for the negatively skewed ability distribution, but were overestimated for p-values less than 0.15. The best fitting curve tended to arc upwards for the more difficult items, such that the most difficult item had the most extreme bias.

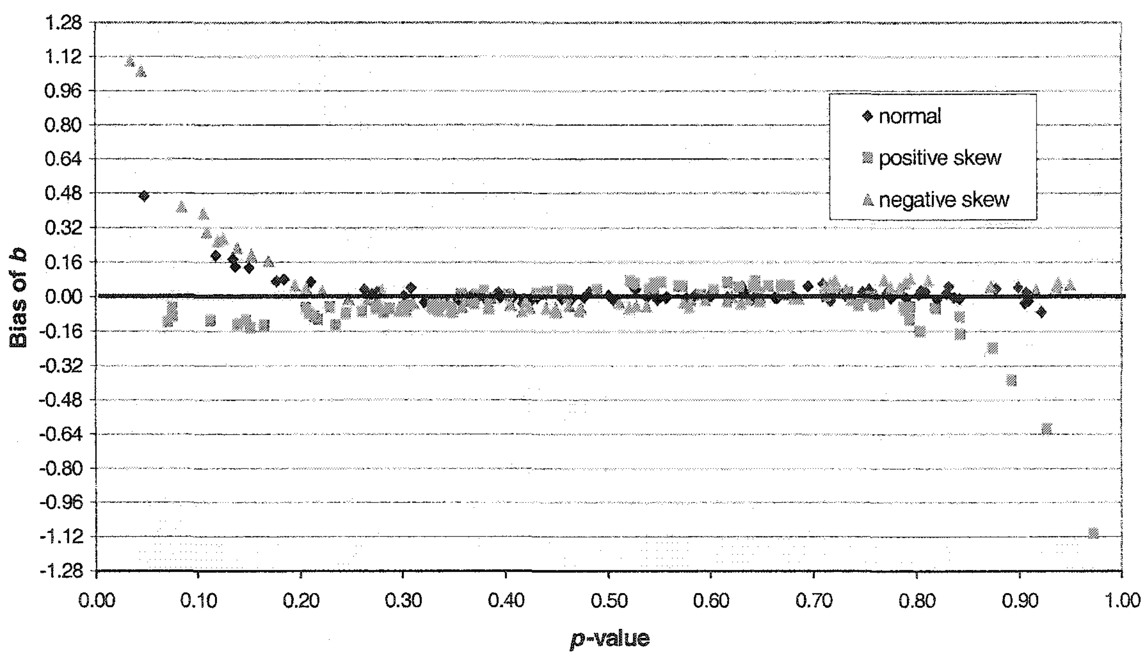


Figure 9. Bias of the difficulty estimates as a function of classical difficulty for the 80 item test, unit discrimination, and sample size of 1,000.

Table 8 provides the number of items with difficulty estimates within 2SE of their corresponding true values, and the numbers of items with positively and negatively biased estimates. Biases were considered with regard to the three intervals of p-values: p

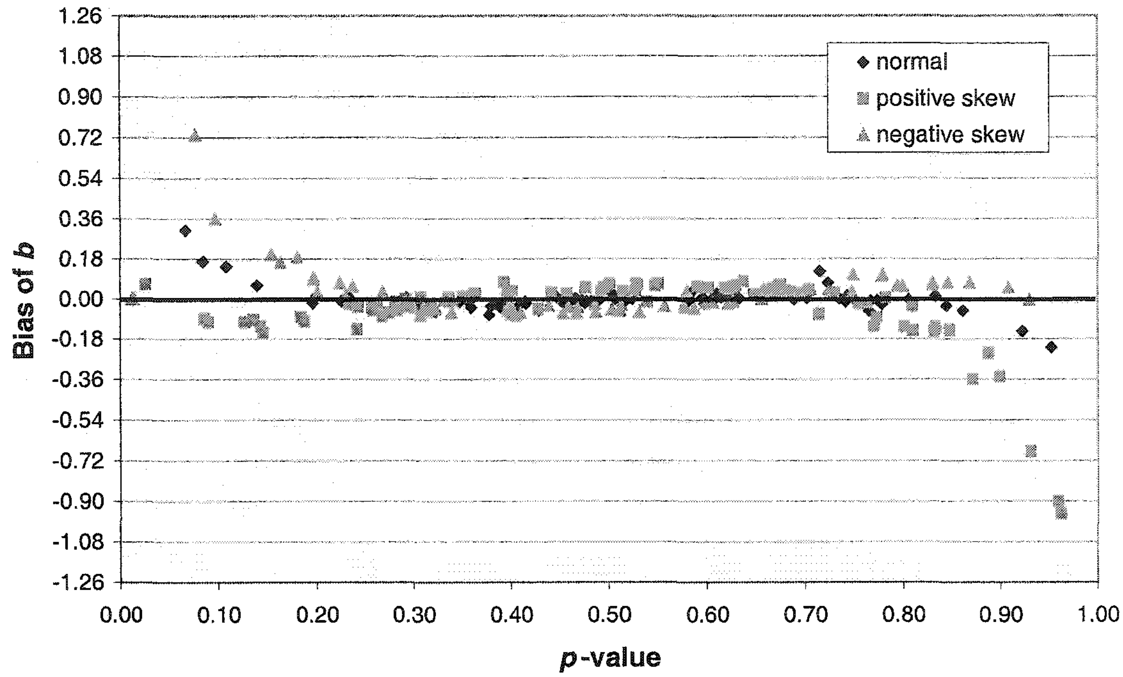
≤ 0.15, $0.15 < p < 0.85$, and $p \geq 0.85$. These cut-scores generally coincided with the natural breaks between the items with biased and unbiased difficulty indices, $\hat{b}_i$. What the table does not communicate is where the biased difficulty items are located within the interval. Unlike the bias values for item discrimination estimation, the biases occurred in a much more predictable manner, and were close to the chosen cut points of 0.15 and 0.85. For example, there were five items with p-values less than or equal to 0.15 for the normal ability distribution with sample size of 1,000, two of which were unbiased and three of which were biased. The two unbiased items had higher p-values ($p = 0.15$, bias = 0.13; $p = 0.14$, bias = 0.14), and therefore were closer to the cut point of 0.15 than the three biased items ($p = 0.05$, bias = 0.47; $p = 0.12$, bias = 0.19; $p = 0.13$, bias = 0.17). Likewise, when there were biased items within the interval of $0.15 < p < 0.85$, the p-values were close to the cut point of 0.15 for the positively biased items and close to the cut point of 0.85 for the negatively biased items. For example, there was one negatively biased item between $0.15 < p < 0.85$ for the positively skewed ability distribution and sample of 1,000 that was very close to the 0.85 cut point ($p = 0.84$, bias = -0.17).

Variable discrimination. The bias patterns for the estimated b-parameter as a function of the p-value for the condition of variable discrimination are illustrated in Figure 10 for the three ability distributions. Overall, the patterns of bias are similar to that observed for the unit discrimination conditions. The difficulty parameter is well estimated for a wide range of p-values. When biases were observed, the b-parameter was overestimated for difficult items, and underestimated for easy items.

Table 8

Number of Biased and Unbiased Difficulty Estimates Categorized by p-value for the Unit Discrimination Condition

		p-value Interval								
		$p \leq 0.15$			$0.15 < p < 0.85$			$p \geq 0.85$		
n_s	Distribution	Unbiased	+ Bias	- Bias	Unbiased	+ Bias	- Bias	Unbiased	+ Bias	- Bias
1,000	Normal	2/5	3	0	69/69	0	0	6/6	0	0
		(40.0%)			(100%)			(100%)		
	Pos Skew	6/6	0	0	69/70	0	1	0/4	0	4
		(100%)			(98.6%)			(0%)		
	Neg Skew	0/9	9	0	67/67	0	0	4/4	0	0
		(0%)			(100%)			(100%)		
500	Normal	3/6	3	0	69/69	0	0	4/5	0	1
		(50%)			(100%)			(80%)		
	Pos Skew	6/6	0	0	71/72	0	1	0/2	0	2
		(100%)			(98.6%)			(0%)		
	Neg Skew	0/3	3	0	72/72	0	0	5/5	0	0
		(0%)			(100%)			(100%)		
250	Normal	7/9	2	0	63/63	0	0	6/8	0	2
		(77.8%)			(100%)			(75.0%)		
	Pos Skew	4/4	0	0	65/67	0	2	0/9	0	9
		(100%)			(97.0%)			(0%)		
	Neg Skew	0/8	8	0	66/69	3	0	2/3	0	1
		(0%)			(95.7%)			(66.7%)		



Note: Two items from the negatively skewed ability distribution (p = 0.01, r(bis) = 0.49, b = 2.88; p = 0.01, r(bis) = 0.52, b = 2.73) were omitted from the scatter plot due to extreme biases of 1.97 and 1.61, respectively.

Figure 10. Bias of the difficulty estimates as a function of classical difficulty for the 80 item test, variable discrimination, and sample size of 1,000.

Table 9 provides the number of items with difficulty estimates, $\bar{\hat{b}}_i$, within 2SE of the true values, and the number of items with positively and negatively biased estimates. Biases were considered with regard to the three intervals of *p*-values: $p \leq 0.15$, $0.15 < p < 0.85$, and $p \geq 0.85$. For items within the middle range ($0.15 < p < 0.85$), all of the items from the normal ability distribution, and the vast majority of items from the positively and negatively skewed distributions (206 of 210, 98.1%; (209 of 211, 99.1%) were well estimated. For the most difficult items ($p \leq 0.15$), all of the items from the positively skewed distribution were estimated within 2SE of true values except one (16 of 17, 94.1%), all of the items from the negatively skewed distribution overestimated the true value, and the majority of items from the normal ability distribution (10 of 17, 58.8%)

were unbiased. For the easiest items ($p \geq 0.85$), all but one of the items from the positively skewed distribution (12 of 13, 92.3%) underestimated the true values, all but two of the items (11 of 13 or 84.6%) from the negatively skewed distribution were estimated within 2SE of true values, and the majority of items (17 of 21, 81.0%) from the normal ability distribution were unbiased.

Summary. Taken together, these results suggest that Lord's formula for predicting $\hat{b}_i$ from the p-value and r_b works very well for items with a broad range of difficulty values ($0.15 < p < 0.85$). Item discrimination, whether unit or variable, had little effect on the results, which implies the formula is robust to Lord's requirement of equal item discrimination. Without exception, when biases were observed, $\hat{b}_i$ was overestimated for difficult items, and underestimated for easy items. The bias patterns for the skewed distributions were differentially affected by the direction of skewness, and were mirror images of each other: the easier items ($p \geq 0.85$) were underestimated for the positively skewed ability distributions, whereas the more difficult items ($p \leq 0.15$) were overestimated for the negatively skewed ability distributions.

Table 9

Number of Biased and Unbiased Difficulty Estimates Categorized by p-value for the Variable Discrimination Condition

		p-value Interval								
		$p \leq 0.15$			$0.15 < p < 0.85$			$p \geq 0.85$		
n_s	Distribution	Unbiased	+ Bias	- Bias	Unbiased	+ Bias	- Bias	Unbiased	+ Bias	- Bias
1,000	Normal	3/4 (75.0%)	1	0	73/73 (100%)	0	0	2/3 (66.7%)	0	1
	Pos Skew	7/7 (100%)	0	0	66/66 (100%)	0	0	1/7 (14.3%)	0	6
	Neg Skew	0/5 (0%)	5	0	70/71 (98.6%)	1	0	4/4 (100%)	0	0
500	Normal	3/7 (42.9%)	4	0	62/62 (100%)	0	0	9/11 (81.8%)	0	2
	Pos Skew	5/6 (83.3%)	1	0	71/71 (100%)	0	0	0/3 (0%)	0	3
	Neg Skew	0/4 (0%)	4	0	72/72 (100%)	0	0	4/4 (100%)	0	0
250	Normal	4/6 (66.7%)	2	0	67/67 (100%)	0	0	6/7 (85.7%)	0	1
	Pos Skew	4/4 (100%)	0	0	69/73 (94.5%)	0	4	0/3 (0%)	0	3
	Neg Skew	0/7 (0%)	7	0	67/68 (98.5%)	1	0	3/5 (60.0%)	0	2

Achievement Test Results

In addition to the simulation data, actual achievement data were used to evaluate the robustness of the formulas in an actual test administration. In the absence of knowledge of the true parameter values for the real achievement data, BILOG parameter estimates were considered as possible population values to which the parameter estimates derived from Lord's formulas could be compared.

Preliminary Considerations

Bias of BILOG Estimates

The purpose of assessing bias in the BILOG estimates of the difficulty and discrimination indices with the simulation data was to determine if BILOG values could be used to evaluate Lord's formulas for the achievement data. Therefore, BILOG estimates obtained for the population of 10,000 examinees were compared to the corresponding true values used to generate this population. The same rules for evaluating the estimates yielded by Lord's two formulas for the simulation data were retained to evaluate the BILOG estimates. As can be seen in Figure 11, the BILOG estimates of the population item discrimination indices were within 2SE of true values (i.e., +/- 0.18) for all items for the normal and negatively skewed distributions, and for all but one item from the positively skewed distribution (r_b = 0.62, a_τ = 1.62, bias = -0.25).

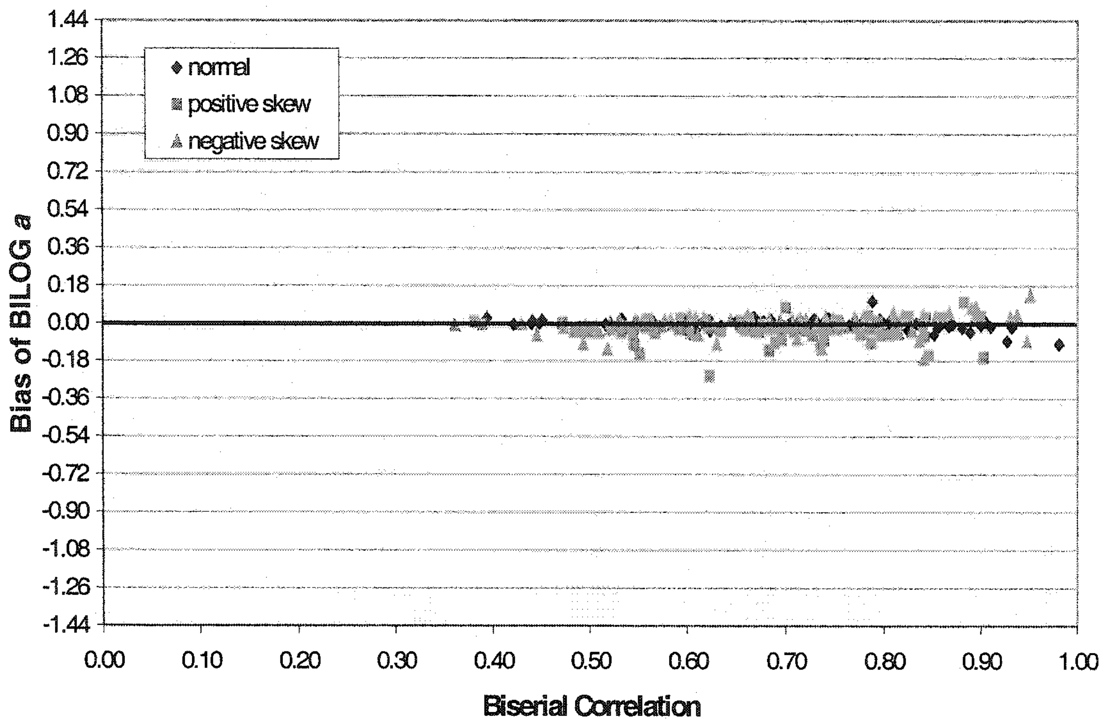


Figure 11. Bias of the BILOG discrimination values as a function of the biserial correlation for the 80 item tests with variable discrimination.

Figure 12 illustrates the patterns of bias in the BILOG difficulty estimates as a function of the p-value. For the condition of variable discrimination, all of the BILOG difficulty estimates were within 2SE (i.e., +/- 0.18) of their corresponding true values with the exception of two items from the positively skewed ability distribution. Both items had high p-values ($p = 0.93$, $r_b = 0.55$, $b_\tau = -2.05$; $p = 0.96$, $r_b = 0.62$, $b_\tau = -1.91$), and were underestimated by 0.19 and 0.20, respectively. Given the small number of biases, it was concluded that the BILOG estimates could be used as population values for the English and biology achievement tests.

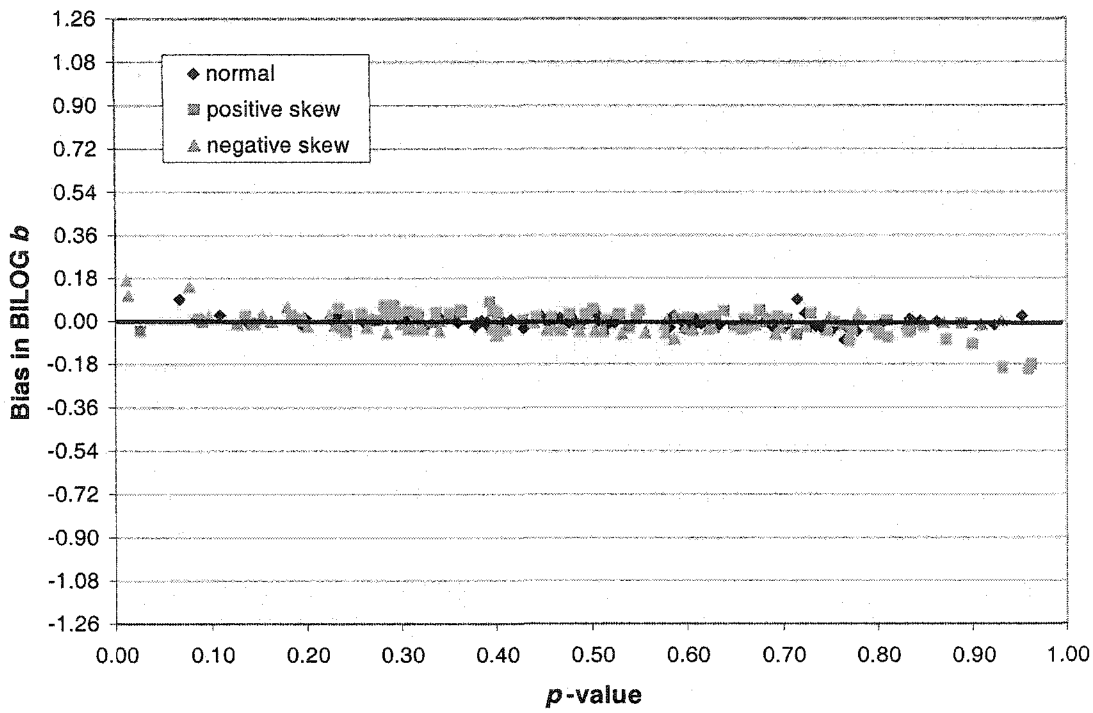


Figure 12. Bias of the BILOG difficulty values as a function of classical difficulty for the 80 item tests with variable discrimination.

Assumptions of the Two Parameter IRT Model

The item response model considered in the present study is predicated on four basic assumptions that underlie the unidimensional IRT models. Before the evaluation of Lord's two formulas were completed for two achievement tests, the tenability of the four assumptions - unidimensionality, local independence, non-speededness, and minimal guessing - were evaluated for both sets of diploma examination data. The results of the model data fit analyses are presented below.

Unidimensionality. To determine the dimensionality of the Alberta Learning achievement data sets, the following procedures were employed. First, a principal components extraction was conducted. It was expected that 1) the first component would account for a large proportion of the variance; 2) the difference between the first and second components would be large enough to support the inference that the first component is the dominant component, and 3) the differences among adjacent pairs of components, beginning with the second component, would be irrelevant (Gorsuch, 1983). Second, scree plots were utilized to verify the results from the principal components analysis (Gorsuch 1983).

A principal component factor analysis of the correlation matrix for the English exam yielded 14 components with eigenvalues greater than 1.0. The eigenvalue for the first component was 8.37, and accounted for 12% of the variance. The difference between the first and second eigenvalues was 7.01, while the differences between successive eigenvalues were small (0.11, 0.05, 0.04, 0.04). Further, the ratio between the first and second eigenvalues was 6.16, while the ratios of the remaining successive eigenvalues were close to 1 (i.e., < 1.10). The results suggested that there was one dominant first principal component for the English exam, which was confirmed by viewing the scree plot illustrated in the left panel of Figure 13.

A principal component factor analysis of the correlation matrix for the biology exam yielded seven components with eigenvalues greater than 1.0. The eigenvalue for the first component was 6.85, and accounted for 14% of the variance. The difference between the first and second eigenvalues was 5.42, while the differences between successive eigenvalues were small (0.29, 0.01, 0.08, 0.03). Furthermore, the ratio between the first and second eigenvalues was 4.76, while the ratios of the remaining successive eigenvalues were close to 1 (i.e., ≤ 1.25). The results suggested that there was one dominant first principal component, which was confirmed by viewing the scree plot illustrated in the right panel of Figure 13. Based on these results, it was determined that the English and biology exams met the unidimensionality criterion necessary to justify the use of a unidimensional IRT model.

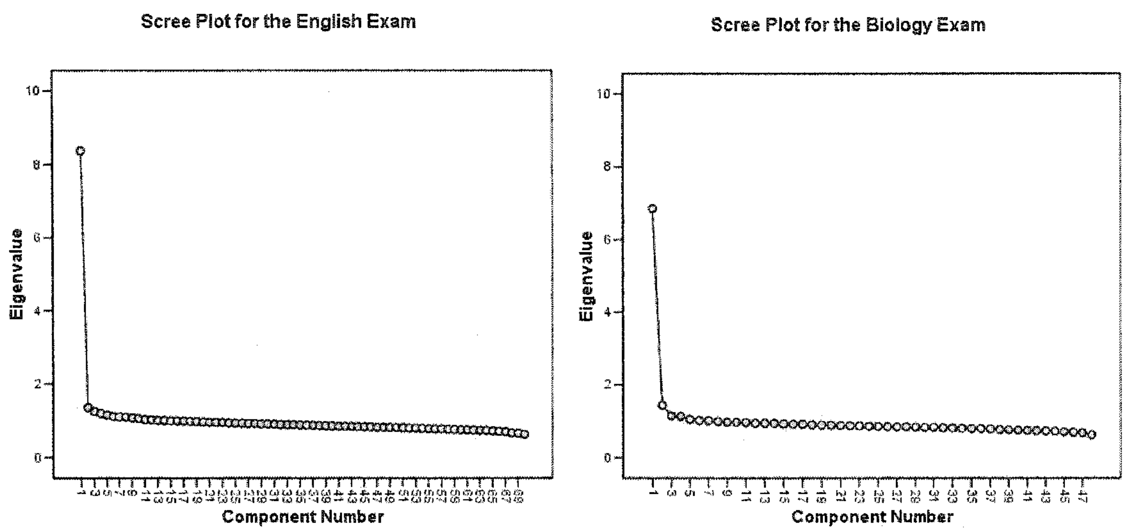


Figure 13. Scree plots for the English and biology exams.

Local independence. Given that the assumption of essential unidimensionality was met for the English and biology exams, the assumption of local independence for both exams was tenable (Hambleton, et al., 1991; Lord, 1980).

Non-speededness. The non-response rates for both exams were very small at the individual item level, ranging from 0 to 31 (< 0.24%) for the 13,375 examinees who wrote the English exam, and from 0 to 14 (< 0.16%) for the 9,030 examinees who wrote the biology exam. There were 24 examinees (0.18%) writing the English exam who did not respond to the last three items, and only one person (0.01%) writing the biology exam who did not respond to the last three items. Therefore, more than 99.8% of examinees completed the last three exam items for both exams. It was determined that all of the M-C items were attempted by essentially all of the examinees, and the assumption of non-speededness was tenable for both exams.

Lack of guessing. The performance of low-achieving examinees on the three most difficult items was examined to test the assumption of non-guessing. Low achieving examinees were defined as those examinees achieving at or below two standard errors above chance level, which translates to a score of 21 on the English exam (n = 215, 1.6%) and a score of 15 on the biology exam (n = 117, 1.3%). The p-values for these examinees ranged from 0.07 to 0.54 (M = 0.27) for the English exam and from 0.09 to 0.50 (M = 0.28) for the biology exam. Table 10 presents the p-values of the three most difficult items for the full sample and the low achieving examinees. In all cases, the p-values for the full sample exceeded the p-values for the low achieving examinees; however, the p-values for the low achieving examinees were not small enough to warrant the conclusion that guessing was not a factor in test performance.

Table 10

P-values for the Three Most Difficult Items for the Full Sample and Low Achieving Examinees on the English and Biology Exams

English			Biology		
Item	Full Sample	Low Achieving	Item	Full Sample	Low Achieving
Q16	0.35	0.12	Q46	0.39	0.23
Q10	0.39	0.11	Q27	0.39	0.13
Q29	0.42	0.17	Q12	0.42	0.09

Summary. The results of evaluating the assumptions of the two-parameter IRT model indicate that three of the four assumptions were met. The English and biology exams were found to be essentially unidimensional, and consequently, local independence was obtained for both examinations. Judging from the low non-response rates, it was determined that speed was not a factor affecting examinee performance. However, the assumption of non-guessing was not tenable for both examinations.

The item parameters and Lord's estimates of the item parameters for the achievement data are provided in Table 11 for the English exam and Table 12 for the biology exam. Bias estimates, equal to the difference between the estimate produced using Lord's formula and the BILOG parameter value, may be computed for each item from the information provided in these tables.

Standard Errors

The procedure followed in the simulation study was used to determine an estimate of the SEs of the item discrimination and difficulty indices obtained using Lord's formulas. SEs for the achievement items were calculated from the sampling variances of 100 random samples of 1,000 examinees.

The distribution of the sampling variances for the discrimination estimates is illustrated as a function of the biserial correlation in Figure 14. The range of biserials is less than that observed in the simulation study. Since no extreme biserial coefficients were encountered, mean item variances were computed using all of the items. None of the variances exceeded 0.01. As is evident in Figure 14, the sampling variances tended to increase as the biserials increased. In the case of English exam, the item with the lowest biserial, 0.21, had a variance of 0.0018, and the item with the highest biserial, 0.67, had a

Table 11

Item Indices for the English Exam

Item	p	r_b	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	082	039	-282	- 236	- 237	01038	035	042	043	00038
Q2	061	044	-070	- 062	- 063	00107	043	049	049	00022
Q3	065	052	-078	- 072	- 071	00085	056	061	062	00023
Q4	080	048	-188	- 175	- 175	00360	051	055	056	00043
Q5	061	054	-056	- 051	- 051	00062	059	065	065	00022
Q6	070	054	-101	- 094	- 095	00102	061	065	066	00030
Q7	074	033	-244	- 196	- 196	01154	027	035	035	00031
Q8	065	053	-079	- 073	- 075	00087	057	062	062	00026
Q9	081	046	-209	- 191	- 193	00348	047	052	052	00030
Q10	039	041	069	066	068	00128	040	046	045	00023
Q11	057	033	-065	- 052	- 057	00199	028	035	034	00016
Q12	061	052	-057	- 051	- 051	00068	056	061	061	00024
Q13	065	043	-098	- 087	- 089	00117	042	048	048	00022
Q14	083	054	-186	- 175	- 175	00174	060	064	065	00038
Q15	075	049	-151	- 141	- 141	00197	051	056	056	00028
Q16	035	044	086	084	085	00128	045	049	049	00022
Q17	047	047	012	013	012	00080	049	054	054	00019
Q18	049	044	002	004	004	00064	044	049	049	00016
Q19	056	052	-031	- 027	- 028	00066	056	062	062	00023
Q20	067	034	-161	- 134	- 135	00339	029	036	036	00020
Q21	075	053	-135	- 127	- 128	00150	058	062	063	00034
Q22	070	039	-154	- 135	- 138	00318	036	042	042	00021
Q23	053	052	-018	- 015	- 015	00053	055	061	061	00024
Q24	062	040	-087	- 076	- 075	00159	037	044	044	00019
Q25	080	057	-150	- 144	- 144	00176	068	070	071	00046
Q26	069	039	-152	- 130	- 130	00311	035	042	042	00022
Q27	073	032	-238	- 189	- 191	00587	026	034	034	00018
Q28	046	039	027	026	026	00116	036	042	042	00021
Q29	042	047	044	044	044	00088	048	053	053	00027
Q30	055	052	-026	- 022	- 022	00054	056	061	062	00022
Q31	066	029	-186	- 144	- 144	00679	022	030	030	00018
Q32	058	040	-060	- 051	- 050	00103	037	044	044	00015
Q33	070	033	-191	- 159	- 161	00580	028	035	035	00019
Q34	070	048	-119	- 109	- 108	00128	050	055	056	00026
Q35	077	059	-128	- 123	- 124	00144	071	073	074	00054
Q36	054	045	-026	- 022	- 021	00075	044	050	051	00020
Q37	057	036	-060	- 051	- 049	00160	032	039	040	00016
Q38	079	044	-207	- 186	- 187	00466	044	049	050	00032
Q39	062	041	-085	- 074	- 074	00118	038	045	046	00021
Q40	060	048	-061	- 054	- 054	00085	048	055	055	00028

Table 11 (cont'd)

Item	p	r_b	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	043	056	031	032	033	00041	064	067	067	00032
Q42	063	052	-069	- 063	- 062	00065	057	061	062	00024
Q43	053	045	-019	- 015	- 014	00097	043	050	050	00023
Q44	065	062	-068	- 064	- 065	00057	075	078	079	00026
Q45	053	051	-017	- 013	- 013	00051	054	059	059	00029
Q46	059	040	-066	- 055	- 055	00122	036	044	044	00022
Q47	075	047	-156	- 142	- 142	00188	047	053	053	00029
Q48	051	051	-008	- 005	- 005	00064	053	059	058	00021
Q49	060	049	-058	- 052	- 053	00090	050	056	056	00024
Q50	067	047	-105	- 095	- 096	00140	047	053	053	00020
Q51	058	037	-064	- 054	- 054	00133	033	039	040	00016
Q52	043	052	033	034	035	00065	057	061	061	00022
Q53	048	021	038	029	029	00324	014	022	022	00018
Q54	061	043	-074	- 066	- 066	00106	041	047	048	00021
Q55	059	058	-042	- 038	- 037	00045	068	071	073	00034
Q56	073	046	-145	- 134	- 132	00211	047	052	053	00029
Q57	086	049	-241	- 220	- 221	00648	051	056	056	00045
Q58	064	046	-084	- 076	- 078	00096	045	051	050	00021
Q59	068	042	-129	- 112	- 110	00190	039	046	047	00024
Q60	071	044	-139	- 125	- 125	00203	042	048	049	00023
Q61	068	042	-125	- 112	- 112	00174	041	047	047	00021
Q62	063	053	-071	- 065	- 065	00057	057	062	063	00026
Q63	046	054	015	017	017	00057	061	064	065	00021
Q64	061	040	-083	- 072	- 073	00145	037	044	044	00020
Q65	053	067	-014	- 010	- 009	00030	089	091	091	00047
Q66	077	050	-155	- 145	- 144	00169	054	058	060	00028
Q67	075	054	-132	- 125	- 126	00117	060	064	064	00033
Q68	052	035	-016	- 012	- 013	00125	031	037	037	00016
Q69	075	056	-125	- 119	- 119	00123	065	068	069	00038
Q70	061	038	-088	- 073	- 074	00154	034	041	041	00015

Note. Data are presented without decimal points. All entries are given to the second decimal place except sampling variances, $\hat{\sigma}_b^2$ and $\hat{\sigma}_a^2$, which are given to the fourth decimal place.

Table 12

Item Indices for the Biology Exam

Item	p	r_b	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	063	046	-079	-071	- 072	00109	044	051	052	00023
Q2	053	049	-015	-013	- 013	00063	051	057	057	00022
Q3	064	048	-080	-073	- 076	00111	048	054	055	00021
Q4	071	049	-119	-114	- 113	00173	053	057	057	00030
Q5	045	054	025	025	026	00058	059	063	064	00026
Q6	054	050	-021	-018	- 018	00056	051	058	058	00017
Q7	083	042	-244	-233	- 230	00889	043	046	048	00039
Q8	072	057	-110	-105	- 104	00081	064	069	070	00030
Q9	066	044	-101	-092	- 090	00110	043	049	051	00025
Q10	062	051	-064	-059	- 059	00063	053	059	059	00023
Q11	083	062	-153	-154	- 155	00159	081	080	080	00073
Q12	042	048	047	043	047	00081	047	055	054	00021
Q13	065	055	-076	-071	- 072	00076	061	066	065	00028
Q14	081	060	-151	-150	- 150	00125	074	074	075	00043
Q15	081	066	-136	-134	- 133	00085	088	087	089	00058
Q16	066	056	-078	-074	- 074	00049	064	068	069	00021
Q17	076	033	-237	-213	- 217	00970	031	035	035	00027
Q18	055	030	-048	-041	- 042	00177	025	031	031	00018
Q19	070	051	-108	-102	- 102	00124	054	059	059	00024
Q20	082	049	-188	-185	- 188	00308	056	057	056	00033
Q21	083	051	-183	-182	- 182	00280	060	060	060	00039
Q22	080	044	-195	-187	- 189	00423	047	050	050	00028
Q23	078	073	-107	-105	- 105	00062	109	108	109	00093
Q24	068	068	-073	-068	- 068	00036	087	092	093	00055
Q25	062	034	-106	-093	- 094	00242	030	036	037	00022
Q26	088	051	-223	-231	- 233	00633	062	059	059	00047
Q27	039	038	087	076	077	00189	034	041	041	00019
Q28	079	048	-176	-172	- 174	00297	052	054	054	00030
Q29	057	035	-058	-050	- 051	00139	030	037	037	00017
Q30	068	044	-121	-107	- 107	00130	041	049	049	00021
Q31	081	053	-167	-164	- 163	00199	061	063	064	00036
Q32	075	045	-160	-149	- 150	00357	045	050	051	00036
Q33	075	053	-128	-126	- 125	00129	061	063	064	00035
Q34	072	055	-109	-104	- 103	00106	062	066	067	00039
Q35	081	059	-151	-150	- 151	00135	074	074	075	00046
Q36	064	050	-076	-072	- 071	00092	054	058	059	00026
Q37	072	062	-098	-094	- 093	00053	074	078	079	00036
Q38	083	050	-193	-191	- 188	00287	057	058	059	00028
Q39	080	060	-143	-140	- 139	00150	073	074	076	00046
Q40	069	061	-085	-081	- 080	00052	072	077	077	00036

Table 12 (cont'd)

Item	p	r_b	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	076	059	-120	-117	- 118	00102	071	074	073	00047
Q42	082	054	-171	-168	- 168	00209	063	064	064	00035
Q43	078	043	-195	-182	- 183	00469	044	048	048	00029
Q44	077	037	-222	-199	- 204	00968	035	040	040	00029
Q45	066	051	-086	-079	- 079	00065	053	060	060	00026
Q46	039	036	090	080	082	00213	033	039	039	00019
Q47	083	061	-159	-158	- 159	00183	076	077	077	00057
Q48	062	037	-092	-079	- 080	00172	032	040	040	00016

variance of 0.0047. For the biology exam, the item with the lowest biserial, 0.30, had a variance of 0.0018, and the item with the highest biserial, 0.73, had a variance of 0.0093. The mean item variances of the *a*-parameter estimates were 0.0026 and 0.0033 for the English and biology exams, respectively. The square root of the mean variances resulted in SEs of 0.0505 for the English exam and 0.0577 for the biology exam. Rounding to two decimal places, the SE of 0.05 was used to construct the 95% CI, +/- 0.10, around a bias of zero for the items in both examinations.

The distribution of the sampling variances for the difficulty estimates is illustrated as a function of the *p*-value in Figure 15. Like the case for the discrimination estimates, the range of *p*-values is less than that observed in the simulation study. Since no extreme *p*-values were encountered, mean item variances were computed using all of the items. As is evident in Figure 15, the general trend was that the sampling variances tended to increase as the *p*-value increased. For example, in the English exam, the item with the lowest *p*-value, 0.35, had a variance of 0.0128, and the item with the highest *p*-value, 0.86, had a variance of 0.0648. In the biology exam, the item with the lowest *p*-value, 0.39, had a variance of 0.0213, and the item with the highest *p*-value, 0.88, had a variance

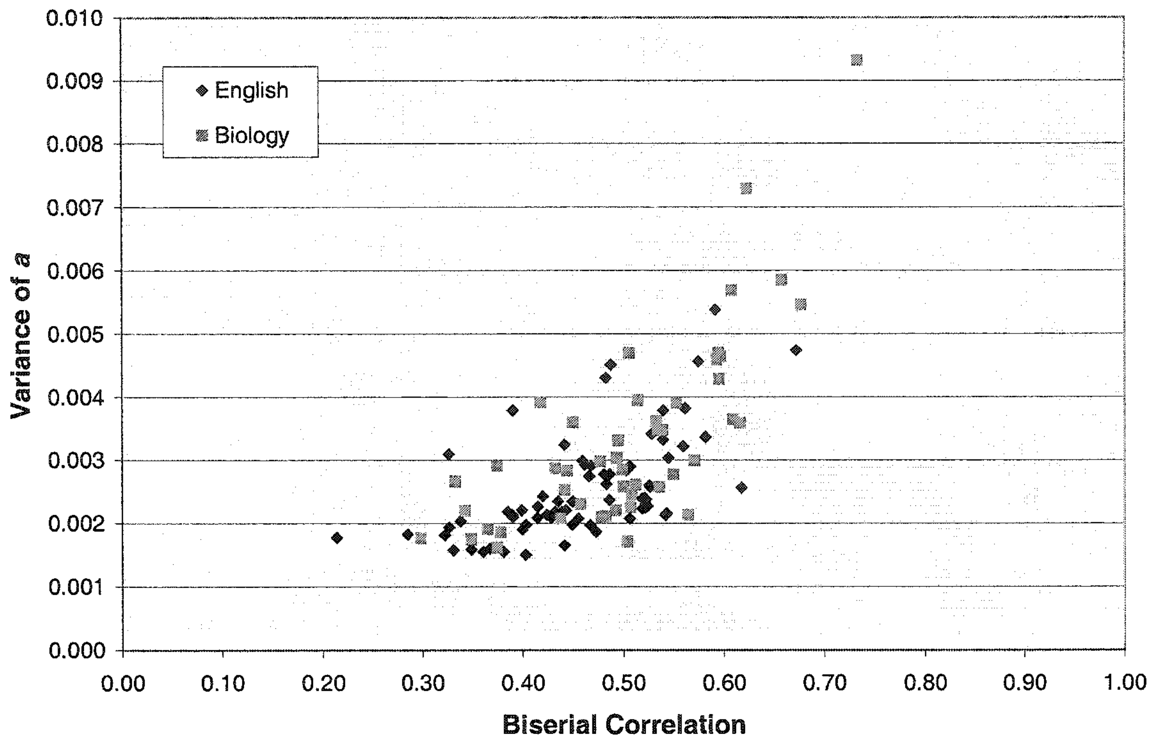


Figure 14. Variance of the discrimination estimates as a function of the biserial correlation for the English and biology exams.

of 0.0633. The mean item variances of the *b*-parameter estimates were 0.0193 and 0.0213 for the English and biology exams, respectively. The square root of the mean variances resulted in SEs of 0.1391 for the English exam and 0.1461 for the biology exam. Rounding to two decimal places results in SE of 0.14 for the English exam and 0.15 for the biology exam. Using the more conservative value of 0.14, the 95% CI of +/- 0.28 was used for both examinations.

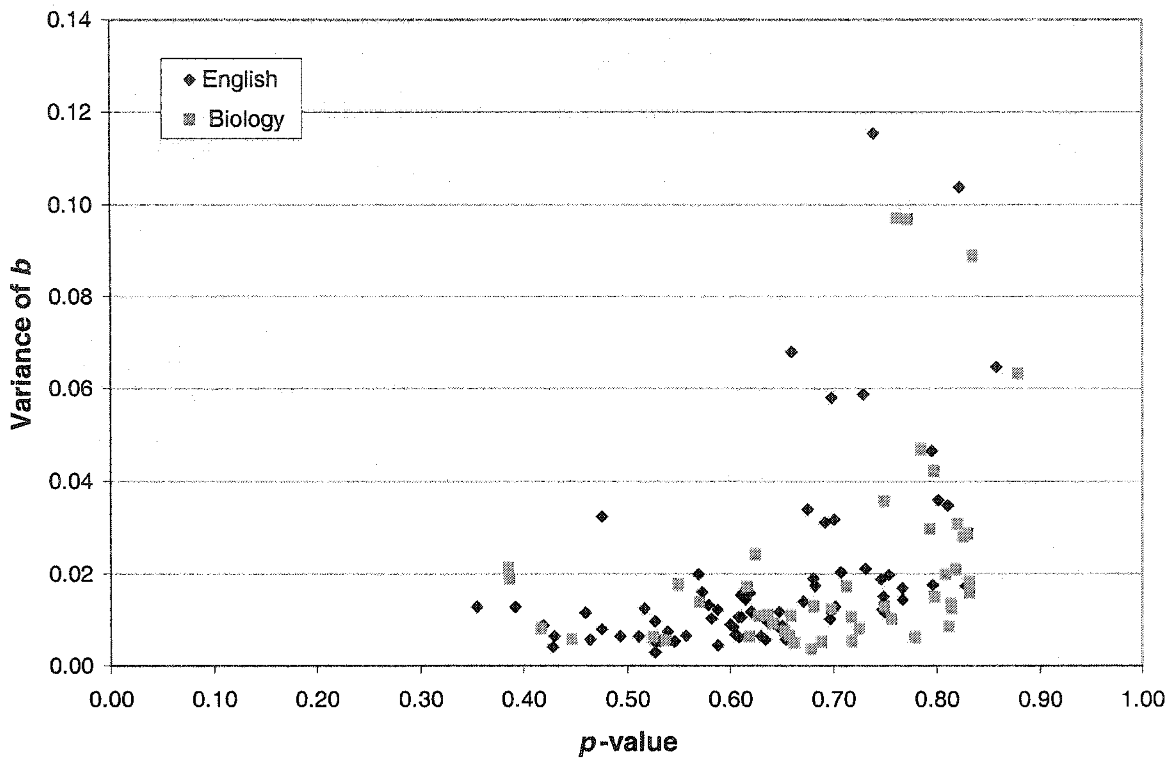


Figure 15. Variance of the difficulty estimates as a function of classical difficulty for the English and biology exams.

Presentation of Bias

Since extreme bias values were not encountered with the real data, the presentation of bias for the achievement data is in increments of 1SE. The classical item indices appear on the horizontal axis and bias appears on the vertical axis.

Item Discrimination

Figure 16 illustrates the distributions of bias of Lord's *a*-parameter estimates as a function of the biserial correlation for the English and biology exams. The estimated *a*-values for both examinations were within 0.10, or 2SE, of their corresponding BILOG population values. The biases for the English exam ranged between 0.0155 and 0.0806 and the biases for the biology exam ranged between –0.0350 and 0.0798.

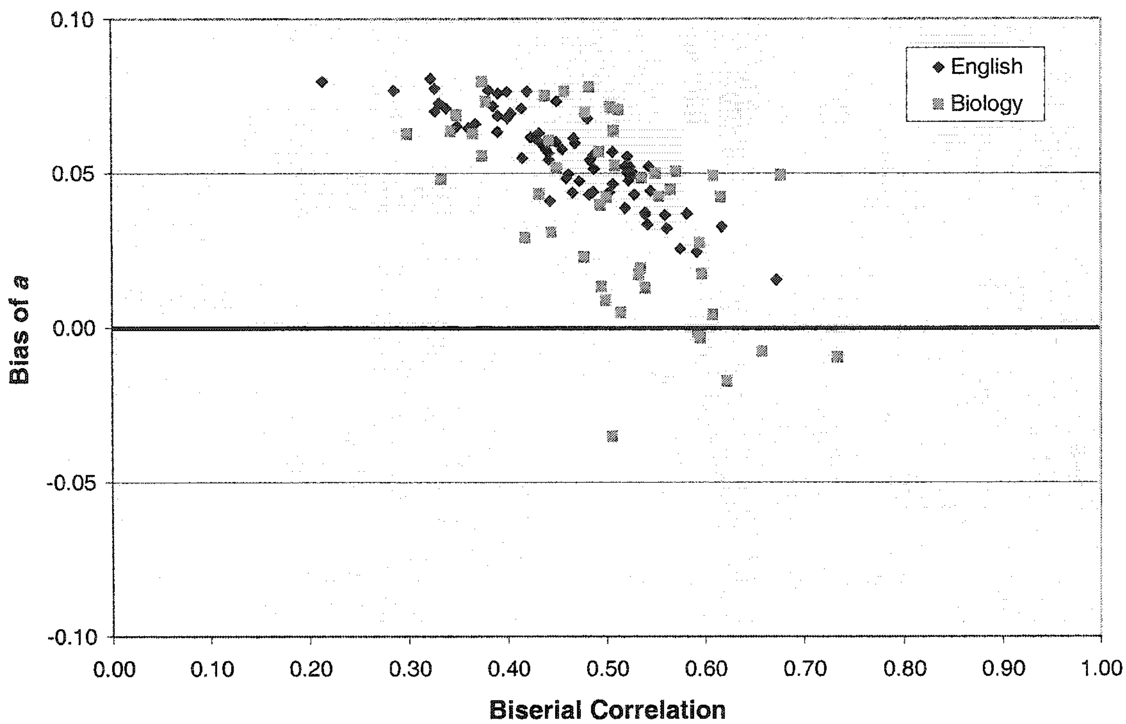


Figure 16. Bias of the discrimination estimates as a function of the biserial correlation for the English and biology exams.

Item Difficulty

Figure 17 illustrates the distributions of bias of Lord's *b*-parameter estimates as a function of classical difficulty for the English and biology exams. All of the estimated *b*-parameters were within 0.28, or 2SE, of their corresponding BILOG population values for the English exam, except for five items (7.1%) that demonstrated positive biases. All of the estimated *b*-parameters were within the 2SE of the BILOG difficulty population values for the biology exam.

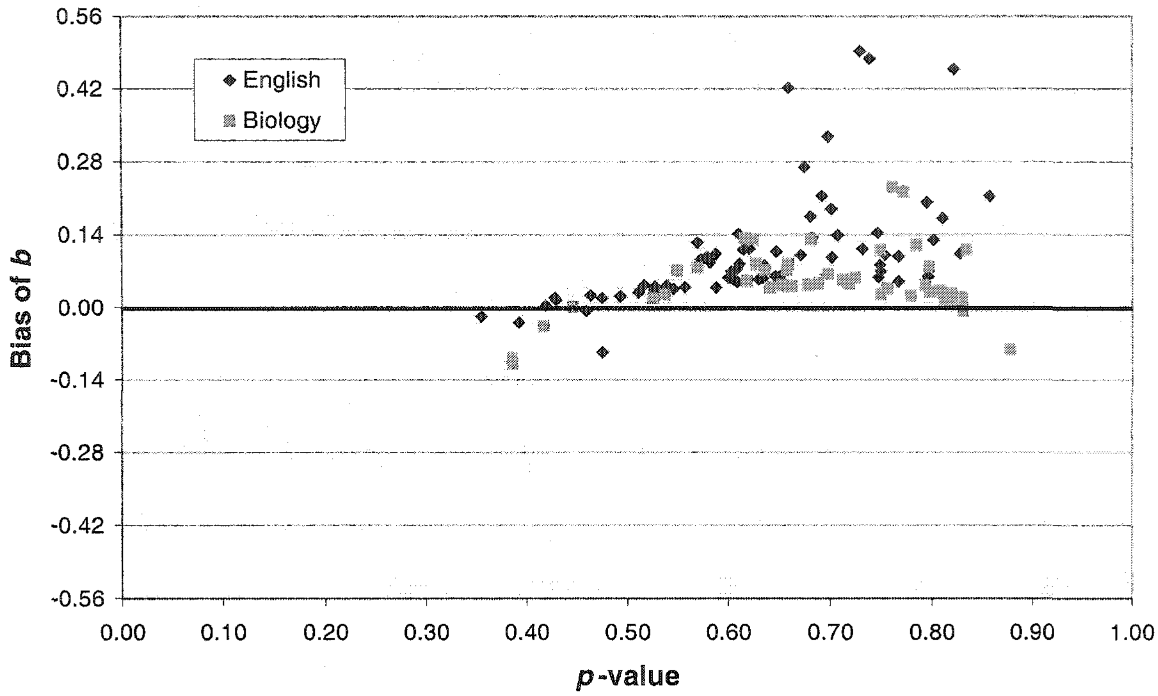


Figure 17. Bias of the difficulty estimates as a function of classical difficulty for the English and biology exams.

The common characteristic of the five English items that exhibited bias is their low biserial correlations, as shown in Figure 18. Of the five items with the lowest biserials ($r_b \leq 0.33$), four were positively biased. The biserial coefficient for the remaining biased item was also low ($r_b = 0.39$). The two items with the highest biases share similar item statistics: $p = 0.73$, $r_b = 0.32$, $b_B = -2.38$, bias = 0.49; and $p = 0.74$, $r_b = 0.33$, $b_B = -2.44$, bias = 0.48. Although the items with biased difficulty estimates were among the items with the lowest biserial coefficients, not all the items with low biserial coefficients had biased difficulty estimates.

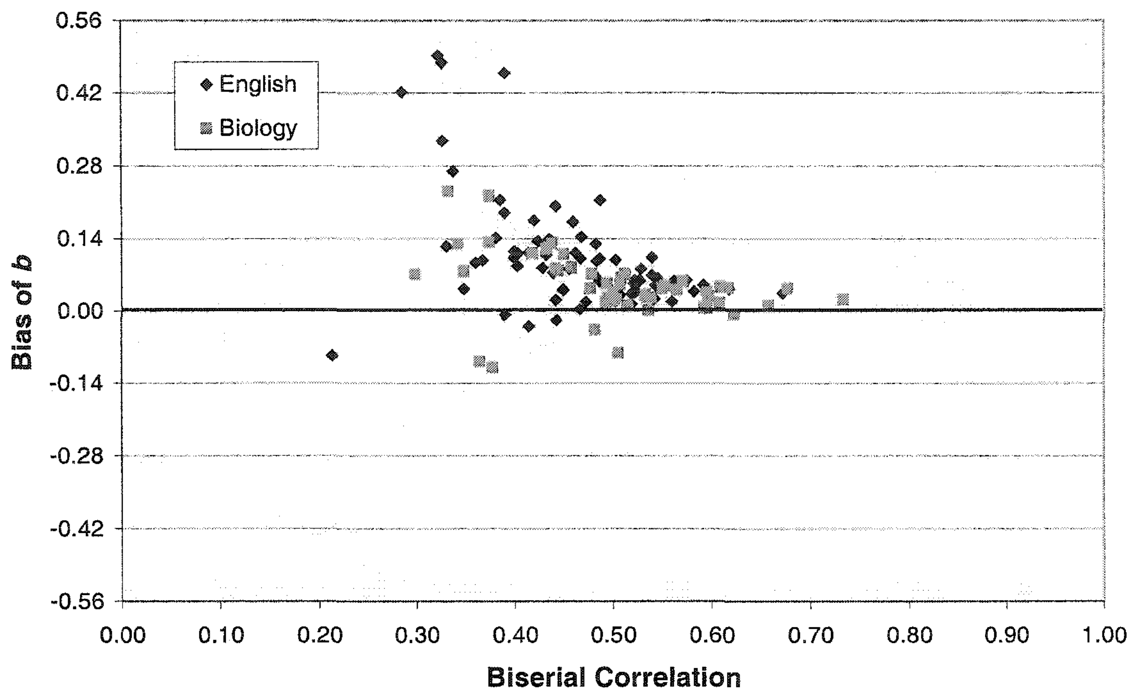


Figure 18. Bias of the difficulty estimates as a function of the biserial correlation for the English and biology exams.

CHAPTER 5 SUMMARY AND CONCLUSIONS

The research questions and a brief description of the methods used in this study are presented first. The findings are then highlighted, followed by the limitations of the study. Results are discussed in light of the previous research. The chapter concludes with recommendations for practice and recommendations for future research.

Summary of Research Questions and Methods

The CTST and IRT models offer alternative frameworks for performing test and item analyses. However, Lord (1980) stipulated that under certain conditions the item difficulty and discrimination indices derived from the two measurement frameworks are connected. For item discrimination, Lord proposed a relationship between the biserial item-test correlation (ρ'_{ix}) and the IRT discrimination index (a_i). The proposed relationship is:

$$a_i \cong \frac{\rho'_{ix}}{\sqrt{1-\rho'^{2}_{ix}}} \quad .$$

Therefore, the two indices are approximately monotonic increasing functions of each other. Lord stated that the relationships are "valid only for the case where θ is normally distributed and there is no guessing" (p. 33).

Similarly, a monotonic relation between the IRT difficulty index (b_i) and the classical difficulty index (π_i) was proposed, with the additional requirement that all items are equally discriminating. The relationship between the difficulty indices is described as $b_i \cong \frac{y_i}{\rho'_{ix}}$, such that the difficulty parameter b_i is proportional to y_i, the cut point on the continuous normal distribution underlying the binary item, divided by the biserial correlation (ρ'_{ix}).

Lord and Novick (1968, p. 375) first introduced the formulas with the single restriction that the ability of examinees is normally distributed. Subsequently, Urry (1974), Schmidt (1977), Jensema (1976), and Ree (1979) investigated the formulas using the three-parameter model. Correlations were presented as evidence of the accuracy of Lord's formulas to reproduce the true or ML estimates of the item parameters. General findings indicated that the correlations for the *b*-parameters were high and the correlations of the *a*-parameters were more variable and generally lower. However, correlations only indicate the extent to which the sets of values are linearly related, and may be misleading if systematic bias is present, such as consistent over- or under-estimation (Ree, 1981).

In 1980, Lord clarified the circumstances for which the formulas were relevant by stating that θ is normally distributed and there is no guessing, implying that the two-parameter IRT model be used. An additional condition, specified for the difficulty parameter, was that all items are equally discriminating. In 1974, Urry pointed out that the efficacy of the formulas was an open empirical question. To date, the accuracy of Lord's formulas has not been evaluated. Therefore, the following two research questions were addressed in this study:

1. What is the veracity of the two formulas within Lord (1980) proposed?
2. What is the robustness of the formulas beyond the initial and restrictive conditions identified by Lord?

In order to assess the accuracy of the formulas, simulated and real achievement data sets were used. The purpose of the simulated data was to examine the behaviours of the formulas under different experimental conditions, where the population parameters are known. The conditions were designed to represent realistic response data. The research design was a 3 (ability distribution) x 3 (test length) x 3 (sample size) x 2 (item discrimination) fully crossed design.

A population of 10,000 examinees was generated for each cell in the design, and random sample sizes of 1,000, 500, or 250 were drawn. From these data, two dependent variables were calculated. Estimation bias was defined as the mean difference between the estimated and true parameter value for an item. Empirical standard errors were computed from the mean item variances to construct 95% confidence intervals around the bias of zero. When the estimated value was within 2SE of the true value, the item was considered to be well estimated.

Lord's formulas also were applied to actual achievement data sets to determine the generalizability of the simulation results to real data. These data sets consisted of the item scores obtained on provincial examinations by examinees who wrote the English exam (N = 13,375) and the biology exam (N = 9,030) (Alberta Learning, 1999a, b). The exams are high school graduation examinations, which contribute 50% towards examinees' final course grades. Only the multiple-choice components of the exams were used, which comprised 70 items for the English exam, and 48 items for the biology exam.

The assumptions underlying the two-parameter IRT model, unidimensionality, local independence, non-speededness, and no guessing, were assessed. All assumptions were met, with the exception of no guessing. The IRT ability distributions for both exams were positively skewed and leptokurtic. The mean IRT difficulty values, approximately -1.00, and mean IRT discrimination values, approximately 0.50, for both exams were lower than the values modeled in the simulation study. Since the true item parameters for the achievement exams were not known, the item parameters derived from BILOG using the two-parameter model were used to evaluate the formulas.

The data from the simulation study and the achievement data sets were presented in scatter plots to display the patterns of bias across the range of classical parameter values. The findings of the simulation and real achievement data were used to propose guidelines for the practical application of the formulas.

Summary of Findings

The following summary presents the highlights of the results. First, two general findings from the simulations are noted. The specific findings related to item discrimination estimation and item difficulty estimation follow, with reference made to the simulation results before the real achievement results.

Findings specific to the simulation study

1. Test length had no bearing on the patterns of bias and sampling variance.
2. Sample size had no bearing on the patterns of bias, but sampling variance increased as sample size decreased, resulting in wider confidence intervals. Refer to Tables 3 and 5.

Item discrimination

1. Results from the simulation study suggest that Lord's formula for predicting $\hat{a}_i$ works well when the biserial correlations were 0.70 or less, regardless of the shape of the ability distribution. When estimation errors occurred, the item discrimination values were underestimated (i.e., $\bar{\hat{a}}_i < a_\tau$). Refer to Figure 8 and Table 7.

2. Item discrimination values were well estimated for the majority of items (87% for the normal and negatively ability skewed distributions, and 72% for the positively skewed ability distributions) when the biserial correlations were between 0.70 and 0.79. The shape of the distribution was related to the direction of bias. When the items displayed bias, all of the items from the normal and the majority of the items (86%) from the negatively skewed ability distributions were underestimated, whereas the majority of the items (58%) from the positively skewed ability distributions were overestimated.
3. Items with biserial correlations of 0.80 and above were the least well estimated, and the accuracy of the formula was poorer for the skewed ability distributions (approximately 25% accurate) compared to the normal distribution (54% accurate). Of those items with biased discrimination estimates, the majority was positively biased.
4. The largest values of bias occurred for biserials close to 1. Refer to Figure 8.
5. Results from the diploma exams were consistent with the results of the simulation study. Since the biserial coefficients ranged between 0.30 and 0.70, all discrimination values were well estimated. Refer to Figure 16.
6. Results from the simulation diploma exam data sets indicate that the item discrimination formula is robust to the violation of normal ability distribution for biserial coefficients of 0.70 or less.
7. Results from the diploma exams indicate that the item discrimination formula is robust to the violation of non-guessing.

Item difficulty

1. The results from the simulation study suggest that Lord's formula for predicting $\hat{b}_i$ works very well for items with a broad range of difficulty values ($0.15 < p < 0.85$), regardless of the shape of the ability distribution. When estimation errors occurred, the item difficulties were underestimated (i.e., $\bar{\hat{b}}_i < b_\tau$) for easy items ($p \geq 0.85$) and overestimated (i.e., $\bar{\hat{b}}_i > b_\tau$) for difficult items ($p \leq 0.15$). Refer to Tables 8 and 9 and Figures 9 and 10.
2. The bias patterns were similar for the conditions of constant and variable item discrimination. Lord's requirement of equal item discrimination appears to be not necessary.
3. The bias patterns for the skewed ability distributions were mirror images of each other: the easier items ($p \geq 0.85$) were underestimated for the positively skewed ability distribution, and the more difficult items ($p \leq 0.15$) were overestimated for the negatively skewed ability distribution.
4. The largest biases occurred for items with p-values that approached 0 or 1.
5. The difficulty values for the diploma exams were well estimated for the vast majority of items across the spectrum of p-values observed ($0.30 < p < 0.90$). Refer to Figure 17.

6. Results from the simulation and diploma exam data sets indicate that the difficulty formula is robust to the violations of normal ability distribution and equal item discrimination.
7. Unlike the simulation results for the positively skewed ability distributions, the easiest items on the English exam were overestimated rather than underestimated.
8. Results from the diploma exams indicate that the item difficulty formula is robust to the violation of non-guessing.

Limitations of the Study

The study is limited by the population and test characteristics explored. The use of simulated data permits infinite possibilities for modeling examinee, test, and item characteristics. Following a review of the literature, the work of other researchers was used as a guide to set examinee ability distributions and item and test characteristics so that the simulated data sets would represent realistic data. However, not all possible conditions were considered. Consequently, the results are limited in generalizability to the examinee population and test and item characteristics of the simulated and real achievement data sets studied in this research.

Discussion

There are several differences between the present research and the earlier studies that prevent direct comparison of the findings. The IRT model used for the present study was the two-parameter model, whereas the earlier studies used the three-parameter model. Secondly, the biserial correlation was used in the present study, while the point-biserial correlation was used by the researchers of the previous studies. The third difference is that the performance of the formulas were evaluated using different dependent variables. Estimation bias and sampling variances were calculated in the present study; correlational techniques were employed in the earlier studies. The fourth difference relates to the psychometric frame of reference. The present researcher considered the estimated IRT parameters in relation to the classical item indices, while the previous researchers worked exclusively within an IRT framework. Hence, the results are discussed in light of these differences.

Item Discrimination

The results of the simulation study suggested that Lord's item discrimination formula performed very well for biserial correlations of 0.70 and less and results of the real achievement data confirm this finding. Despite the fact that the real achievement data represented a greater departure from normality than that modeled in the simulation study, the results were more similar to the data from the normal ability distribution where all items with biserial coefficients less than 0.70 were well estimated. Another difference between the simulated and real achievement data was the kurtosis of the ability distributions. The ability distributions of the achievement data sets were leptokurtic whereas the skewed ability distributions in the simulation study were platykurtic. Therefore, it appears that Lord's discrimination formula is robust to the violation of normal ability distribution when biserial coefficients are 0.70 or less.

The findings from the present study were not consistent with the opinions put forth by Schmidt (1977). In the context of Urry's graphical procedure that used the point-biserial correlation and the three-parameter IRT model, Schmidt proposed that $\hat{a}_i$ would be systematically underestimated. He reasoned that the point-biserial correlation between the item score and the estimated latent trait (i.e., total test score), $r_{i\hat{\theta}}$, is taken as an estimate of the point-biserial correlation between the item score and the perfectly reliable latent trait, $\hat{\rho}_{i\theta}$. Values of $r_{i\hat{\theta}}$ will be attenuated because of guessing on item *i*, and the unreliability of $\hat{\theta}$. Schmidt pointed out that increased values of the correlation, as would be the case when using the biserial correlation, imply larger $\hat{a}_i$. No subsequent work has verified this criticism of Urry's research. However, the results of the present study suggest that the discrimination parameters were not systematically underestimated, and were indeed well estimated, in the context Lord intended: using the biserial correlation and the two-parameter IRT model.

Urry (1974) found the correlation between the *a*-parameters derived from his graphical procedure and their ML estimates was 0.89, while Jensema (1976) found correlations in the 0.80 range. Ree observed lower correlations of 0.32, 0.35, and 0.59 for the skewed, normal, and uniform data sets, respectively, with relatively high discriminating items ($M_a = 0.95$). Since none of the researchers illustrated the relationships graphically, it is unknown whether there were curvilinear relationships between the true *a*-values and Lord's estimated *a*-values like the relationships observed between the biserial correlations and Lord's estimated *a*-values in the simulation study. Again, limited information may be gleaned from the correlations, since the assumptions underlying the valid interpretation of the correlations were not documented.

Item Difficulty

Schmidt (1977) contended that $|b_i|$ derived from Lord's formula would be systematically overestimated in the context of Urry's work. Results from the present study suggest that this is not the case when the biserial correlation and the two-parameter IRT model were used. The results of the simulation study suggested that Lord's formula for item difficulty performed quite well for p-values between 0.15 and 0.85, regardless of the shape of the ability distribution. The patterns of bias in the b-estimates observed for the conditions of variable discrimination were comparable to the patterns of bias in the b-estimates observed for unit discrimination. Seemingly, Lord's restrictions of equal item discrimination and normal ability distribution are not required for a broad range of difficulty values (i.e., $0.15 < p < 0.85$).

When biased difficulty estimates occurred, they were differentially affected by the direction of the skewness. Bias was most pronounced and negative for the easy items in the positively skewed distribution and most pronounced and positive for the difficult items in the negatively skewed distribution. These results may be explained by examining the nature of the skewed distributions. Fewer examinee ability values were observed in the non-tailed region compared to the tailed region. There were few ability values at the high end of the ability scale ($\theta > 2.20$) for the negatively skewed population and there were few ability values at the low end of the ability scale ($\theta < -2.20$) for the positively skewed population. The result is floor and ceiling effects, respectively. The effect on b-parameter estimation is dramatic. The numerator of the formula is a z-score, which changes more rapidly as p-values reach very high and very low levels, driving up the absolute value of the z-score. As a consequence, b_i is overestimated when most examinees answer incorrectly and b_i is underestimated when most examinees answer correctly. Although high p-values (i.e., $p \geq 0.85$) and low p-values (i.e., $p \leq 0.15$) are not desirable item characteristics, the findings highlight the limitation of the formula to accurately predict IRT difficulty parameters in such circumstances.

Difficulty estimates using the real achievement data sets suggested that the formulas performed very well. The b-parameters were estimated within 2SE of BILOG values for all items on the biology exam and all but five of the easier items on the English exam. The bias for the easy items was positive, rather than negative, as found in the simulation study for the positively skewed ability distributions. Two possible explanations were investigated. First, the placement of these items within the English examination was considered. The English examination consisted of seven testlets determined by the nature of the prose the examinees were to read and answer questions (e.g., poems, short story, excerpts from a play or essay). It was hypothesized that the first items in the testlets may be easy and correspond to the biased items. However, only 1 of the 5 overestimated items was the first position.

The second possible explanation related to the violation of the assumption of non-guessing. Hambleton et al. (1991) state that the assumption of no guessing is most plausible with constructed-response items, but may be met approximately with multiple-choice items when a test is not too difficult for the examinees. For example, they suggest this assumption may be met when tests are given to students following effective instruction. Guessing was a factor in test performance because the low achieving students had some success in answering the most difficult items.

Upon review of the item content, some items were deemed to be susceptible to testwiseness. Testwiseness has been defined as a person's ability to utilize the characteristics and formats of the test and/or the test-taking situation to improve his/her test score. If an examinee possesses relevant partial knowledge of the content area and knowledge of testwiseness strategies, and if the test contains testwise-susceptible items, then the combination of these elements may result in a higher test score (Millman, 1966; Rogers & Bateman, 1991). Examinees may eliminate incorrect options and select among the remaining choices, thereby increasing their chances of success on these items. The five biased items also exhibited low biserial correlations (refer to Figure 18). As Lord and Novick (1968) pointed out, this is what is expected from items that can be answered correctly by guessing because the item score cannot be highly correlated with any criterion. A low biserial correlation would contribute to the overestimation of b_i, given the biserial is the denominator of the formula.

The correlations between the true or ML estimates of *b*-values and *b*-values estimated from Lord's formula resulted in near perfect correlations in the work of previous researchers. Urry (1974) and Jensema (1976) reported values of 0.97 and 0.96, respectively, using normal ability distributions. Ree (1979) reported values of 0.97, 0.92, and 0.96 for the normal, skewed, and uniform data sets, respectively. Although the correlations are consistently strong, it is impossible to tell whether the true *b*-values are the same as the *b*-values estimated from Lord's formula, or whether the sets of values are only linearly related.

To conclude, Hambleton (1989) commented on problems associated with the use of Lord's formulas. Most notably the relationships between the IRT parameters and CTST item statistics are based on highly restrictive assumptions, and the usefulness of the parameter estimates is reduced if the assumptions made are not met. Results from the present study suggest that violations of the prescribed conditions, normally distributed ability, equal item discrimination, and non-guessing, appear not to have a detrimental effect on parameter estimates using Lord's formulas.

Recommendations for Practice

The rationale for the present study is predicated on the notion that Lord's formulas may provide a simple conversion of classical item statistics to IRT item statistics, providing item characteristics that are independent of the sample of examinees. The context the formulas were initially investigated was obtaining item parameters to screen items for tailored testing (Jensema, 1977; Ree, 1979; Urry, 1974). The following guidelines are prescribed for tests with high reliability, as was the case for the simulated data ($\alpha > 0.95$) and the diploma examination data ($\alpha > 0.85$). With this in mind, the findings suggest that the formulas work well for: (1) samples with a minimum of 1,000 examinees; (2) items with biserials of 0.70 or less for the discrimination formula; and (3) items with p-values of between 0.15 and 0.85 for the difficulty formula.

With respect to the sample size, greater variance of the item parameter estimates was observed for the sample sizes of 500 and 250. Item variances for these sample sizes were not depicted in figures, but were provided in Tables 3 and 5 for item discrimination and item difficulty, respectively. Higher variance implies that any given replication may yield a parameter estimate that departs substantially from the mean estimated value over 100 replications.

With respect to the ranges of classical item indices recommended, Harwell et al. (1996) noted that estimation bias and sampling variance are correlated in the sense that parameter estimates showing little variance over replications would be close to the true parameter values, and parameter estimates showing larger variance over replications would not be close to the true parameter values. The results of the present study are consistent with this view. Greater variance and estimation bias was found for high biserial correlations ($r_b > 0.70$) and p-values close to 0 ($p \leq 0.15$) and 1 ($p \geq 0.85$). Hence, it is not recommended to use the formulas with classical item characteristics in such ranges.

In practice, classical item characteristics tend to be within the recommended ranges to use Lord's formulas. For example, the requirements for the Alberta Learning exams include minimum and maximum acceptable difficulty levels 0.30 and 0.85, respectively, and a minimum acceptable point-biserial correlation of 0.20 (Alberta Education, 1999).

Recommendations for Future Research

Based on the findings of the current study, there are a number of recommendations for future research. These are proposed as follows.

The results from the simulation study indicate that the formula does not perform well for more discriminating items (i.e., $r_b \geq 0.70$). Whether this finding will hold for actual achievement test items with higher biserial correlations than those observed in the two examinations needs to be verified.

An additional characteristic of tests not addressed in the current study was test reliability. Urry suggested that the use of these formulas is best done when the items in the test are homogenous (K-R 20 $\geq$ 0.90). The simulated and real achievement data all obtained high reliability coefficients. The performance of the formulas has not been assessed with tests of lower reliability, where greater error is present in test scores.

Work is needed to understand the influence of guessing on the estimates of b_i produced by the formula. No attention was given to the cognitive processes used by examinees in the test-taking situation. Protocol analysis (Ericsson & Simon, 1993) would enable researchers to identify the thought processes the examinees employed when making their decisions. This would be particularly relevant for investigating how the low achieving examinees respond to the difficult items, thereby enhancing their chances of success.

Hambleton (1989) also pointed out that parameter estimates produced by the heuristic method do not have known sampling distributions, and therefore the standard errors associated with the estimates are unknown. Although standard errors were empirically derived for the present study, theoretically derived standard errors have yet to be established.

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APPENDIX A

Mathematica Program to Simulate Item Responses for a Population of Examinees

Sample Theta from ND, convert to 1/0 and write to file
Mod. Date: March 6/03
Steve Hunka

Required

```
<< Statistics`NormalDistribution`
<< Statistics`DescriptiveStatistics`
<< Graphics`Graphics`
SetDirectory[
 "C:\\Documents and Settings\\tdawber\\My Documents\\Tess_CRAME\\Mathematica\\data runs"]
C:\Documents and Settings\tdawber\My Documents\Tess_CRAME\Mathematica\data runs
```

Function Description

sampND: main function to generate and write results to a file
ndistrn: takes random sample (thetas) from ND(a,b)
getitmp: gets 0/1 response vector from sampled values
tobin: contains rule for generating 0/1 response
psigmd: 2-P IRT logistic function
wrbf: writes binary strings to user named file; user prompted for unique file name
chkFname: checks if a user named file already exists
Note: in the function wrbf, if the file name already exists the user is prompted for another file name. If a file name entered is the word quit, no recording is made.

Functions

sampND[a,b,n,s,d]: gets n random numbers from Normal Distribution ND(a,b), calculates binary response vector (string) for each examinee, and writes binary response vectors to user named file.
a: mean of Normal Distribution
b: std. dev. of Normal Distribution
n: sample size
s: slopes; vector of i elements for i items
d: difficulties; vector of i elements for i items
Calls: ndistrn, getitmp, wrbf, psigmd, chkFname, tobin
Rqd: Statistics`NormalDistribution`, Statistics`DescriptiveStatistics`, Graphics`Graphics`
Note: to read complete file recorded use ReadList["filename",String]
Mod. Date: Mar7/03

```
sampND[a_, b_, n_, s_, d_] := Module[{rn, b10},
   rn = ndistrn[a, b, n];
   b10 = getitmp[s, d, rn];
   wrbf[b10];

  ];
```

ndistrn[a,b,n] obtains Normal random sample of size n from ND(a,b)
a: mean of ND from which sample is taken
b: std dev of ND from which sample is taken
n: sample size
Rqd: Statistics`NormalDistribution`, Statistics`DescriptiveStatistics`, Graphics`Graphics`

```
ndistrn[a_, b_, n_] := Module[{ndist, sk, k, rn, obs, obk},
  Print["Input(a,b)=", {a, b}];
  Print["Sample Size=", n];
  ndist = NormalDistribution[a, b];
  sk = Skewness[ndist];
  Print["Expected sk=", N[sk]];
  k = Kurtosis[ndist];
  Print["Expected k=", N[k]];
  rn = RandomArray[ndist, n];
  Histogram[rn];
  Print["obsvd Mean=", Mean[rn]];
  Print["obsvd Var=", VarianceMLE[rn]];
  obs = Skewness[rn]; Print["Obsvd skewness=", obs];
  obk = Kurtosis[rn]; Print["Obsvd kurtosis=", obk];
  Return[rn];
 ]
```

chkFname[f] checks whether filename f exists by using results from FileNames[]. f is a string, i.e., "f"
Returns:
{1,filename} if f exists; if filename starts with a colon as produced by V3.0, the colon is removed;
{len,filename} if f does not exist filename is an empty list and len=0; if f produces an ambiguous result filename contains a list of file names matching f, e.g., "tst*" results in all files starting with "tst", then len is the length of this list.
Mod. Date: Oct 3/97

```
chkFname[f_] := Module[{fnf, len},
        fnf = FileNames[f];
        len = Length[fnf];
        If[len == 1,
         If[False == LetterQ[StringTake[Part[fnf, 1], 1]],
            fnf = StringDrop[Part[fnf, 1], 1]];
            Return[{1, fnf}], (*else*)
            Return[{len, fnf}]];
        ];
```

```
chkFname["tes2"]
{0, {}}
```

wrbf[b] writes binary vector of strings b to user named file. If user named file already exists, a prompt for another file name is made. If the file name is quit, exit is made from the function and no recording is made. The location (path) of the recorded file can be obtained by executing Directory[ ]
Calls: chkFname
Mod. Date: Mar 7/03

```
wrbf[b_] := Module[{fn, r, ferr = 1},
   While[ferr == 1,
    fn = InputString["Type file name"];
    If[fn == "quit", Break[]];
    r = chkFname[fn];
    If[0 == Part[r, 1],
     Export[fn, b, "Lines"];
     Print["File ", fn, " recorded"];
     ferr = 0;, (*else*)
     Print["File ", fn, " already exists"];
    ];(*end If*)
   ]; (*end While*)
  ];
```

psigmd[a_,b_,t_]; return probability of correct response given discrimination parameter **a**, difficulty parameter **b**, and ability parameter **t** (theta) using the logistic function.
Mod. Date: May 4/96

$$\text{psigmd[a\_, b\_, t\_]} := \frac{1}{1 + E^{-1.7 a (t-b)}}$$

tobin[p] compares random number with prob in p; If p>= rn return 1, else 0
p: vector of prob of length i items
Returns: binary vector

```
tobin[p_] := Module[{rn, bin},
   rn = Table[Random[], {Length[p]}];
   bin = MapThread[#1 ≥ #2 &, {p, rn}] /. {True → 1, False → 0};
   Return[bin];
  ];
```

getitmp[sl,dif,th] calculates 1/0 item response
sl: vector of slopes for i items
dif: vector of difficulties for i items
th: vector of thetas for examinees
Calls: psigmd, tobin
Returns: 1 or 0 for all items for each examinee as a vector of strings, e.g., {10110, 01111,....} where each element is a string.
Mod. Date: Mar 6/03

```
getitmp[sl_, dif_, th_] := Module[{p, b},
   p = Map[psigmd[sl, dif, Table[#, {Length[sl]}]] &, th];
   (*Print["p=",p];*)
   b = Map[tobin[#] &, p];
   b = Map[ToString[#] &, b];
   (*remove commas spaces and { } *)
   b = Map[StringReplace[#, {", " → "", "{" → "", "}" → ""}] &, b];
   Return[b];
  ];
```

Test Data

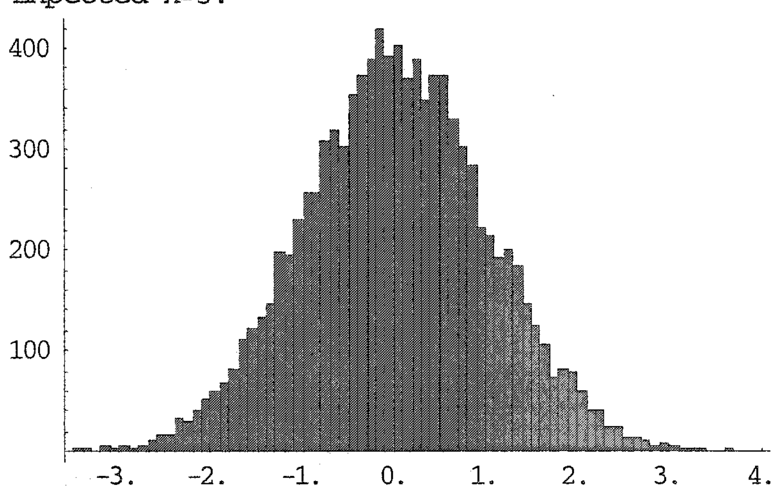
```
slopes = {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
  1, 1, 1, 1, 1, 1, 1, 1, 1, 1}

{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}
```

```
df = {-1.0901713383663854`, 0.414552331612241717`, 2.21406657187409`, -1.175473964502976`,
  -0.18701335930623822`, -0.400393497561208`, -0.69050360165592`, -0.0504820867267184`,
  -0.2732206412786494`, 0.13833532322252765`, -1.6409964988127774`, -0.6570910417317192`,
  -0.1598142870629618`, -0.10273931223856195`, -0.5118520048483698`, -2.4131082990722925`,
  0.5399217046983914`, 0.46570710220763534`, -0.32869591551077215`, 1.0190300935912813`,
  -0.5245806491569329`, 1.9954952330860605`, 1.1465576665117125`, -0.37869206321769516`,
  0.575702024453694`, -0.805652663810747`, 1.2645365838501412`, 0.21584879013545757`,
  0.41023665021854`, 0.7276023616715221`, -0.9201210028620739`, -0.08868321405496524`,
  -0.033796324220952766`, -0.7554682206611526`, 1.2252641546817105`, -0.35868616058217123`,
  0.41859534645537992`, 0.5912674244109616`, 0.1031824327831062`, 1.3591861675164174`}
```

```
{-1.09017, 0.414552, 2.21407, -1.17547, -0.187013, -0.400393, -0.690504, -0.0504821,
 -0.273221, 0.138335, -1.641, -0.657091, -0.159814, -0.102739, -0.511852, -2.41311,
 0.539922, 0.465707, -0.328696, 1.01903, -0.524581, 1.9955, 1.14656, -0.378692,
 0.575702, -0.805653, 1.26454, 0.215849, 0.410237, 0.727602, -0.920121, -0.0886832,
 -0.0337963, -0.755468, 1.22526, -0.358686, 0.418595, 0.591267, 0.103182, 1.35919}
```

Example 1

```
sampND[0, 1, 10000, slopes, df]
Input(a,b}={0, 1}
Sample Size=10000
Expected sk=0.
Expected k=3.
```




```
obsvd Mean=0.020048
obsvd Var=1.03108
Obsvd skewness=0.0227625
Obsvd kurtosis=3.01799
```

APPENDIX B

Excel Macro Program to Perform Random Sampling from the Population of Examinees

```
Sub Macro1()

Dim objExcel As Object
Dim rowObj As Object
Dim ranArray(1 To 1000) As Integer
Dim ranNum As Integer
Dim element As Integer
Dim found As Boolean
Dim runs As Long
Dim globalColPos As Integer
```

Creates new worksheets called AData and BData.

```
 Randomize
 runs = 1
 globalColPos = 1
 Set objExcel = Sheets.Add
 objExcel.Name = "AData"
 Set objExcel = Sheets.Add
 objExcel.Name = "BData"
```

Randomly selects 1,000 examinees from the population of 10,000.

```
 Do
   element = 1
   ranArray(element) = Int(10000 * Rnd) + 1

   Do
     found = False
     ranNum = Int(10000 * Rnd) + 1
     For j = 1 To element
       If ranNum = ranArray(j) Then
         found = True
       End If
     Next j
     If found = False Then
       ranArray(element + 1) = ranNum
       element = element + 1
     End If
   Loop Until element = 1000
```

```
Set objExcel = Sheets.Add
objExcel.Name = "Data"
```

Copies randomly selected examinees to the temporary worksheet AMasterData.

```
For i = 1 To element
   Sheets("AMasterData").Select
   Sheets("AMasterData").Rows(ranArray(i)).Select
   Selection.Copy
   Sheets("Data").Select
   Sheets("Data").Rows(i).Select
   ActiveSheet.Paste
Next i

Rows("1:1").Select
Selection.Insert Shift:=xlDown
Rows("1:1").Select
Selection.Insert Shift:=xlDown
If runs = 1 Then
   Application.Run ("Lertap5.xls!RunCCs")
End If

Sheets("Sub1").Select
Application.Run ("Lertap5.xls!ElmillonDirect")
```

Copies the column vector of a-parameters to AData

```
Sheets("Stats1b").Select
Sheets("Stats1b").Columns(10).Select
Selection.Copy
Sheets("AData").Select
Sheets("AData").Columns(globalColPos).Select
Sheets("AData").Paste
```

Copies the column vector of b-parameters to BData

```
Sheets("Stats1b").Select
Sheets("Stats1b").Columns(9).Select
Selection.Copy
Sheets("BData").Select
Sheets("BData").Columns(globalColPos).Select
Sheets("BData").Paste
```

```
        Application.DisplayAlerts = False
        Sheets("Data").Delete
        If runs = 1 Then Sheets("Freqs").Delete
        Sheets("Scores").Delete
        Sheets("Stats1f").Delete
        Sheets("Stats1b").Delete

        globalColPos = globalColPos + 1

        runs = runs + 1

    Loop Until runs = 101 ' always one more than the number of loops

End Sub
```

APPENDIX C

Item Level Information for the Simulated Data Conditions

Table C1

Item Indices for the Conditions of 20 Items, Normal Ability Distribution, Variable Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	066	066	-071	-071	- 061	-062	00054	073	069	087	087	00044
Q2	042	092	023	025	022	022	00016	205	190	243	245	00821
Q3	064	066	-061	-063	- 054	-054	00041	071	070	089	089	00040
Q4	009	058	228	232	230	233	00484	077	073	071	071	00059
Q5	066	075	-060	-063	- 056	-056	00040	095	091	114	115	00086
Q6	061	088	-031	-034	- 030	-031	00020	157	145	185	184	00289
Q7	078	083	-097	-102	- 094	-093	00036	134	129	150	152	00186
Q8	031	046	126	136	109	111	00153	041	038	051	052	00024
Q9	020	068	124	131	123	124	00073	091	086	094	093	00045
Q10	019	080	102	110	108	109	00042	150	140	133	133	00100
Q11	032	070	073	077	069	069	00032	086	082	097	097	00047
Q12	084	072	-146	-150	- 139	-139	00080	096	093	103	104	00079
Q13	072	068	-092	-097	- 085	-084	00038	078	074	092	093	00051
Q14	071	084	-069	-071	- 064	-064	00030	129	126	155	158	00301
Q15	059	095	-027	-028	- 025	-025	00016	235	209	321	339	07909
Q16	039	072	037	043	038	037	00027	088	085	103	105	00069
Q17	027	068	095	102	093	092	00043	085	080	092	093	00046
Q18	050	061	002	001	001	001	00041	063	059	076	077	00031
Q19	081	073	-128	-134	- 122	-122	00065	097	093	107	108	00085
Q20	059	075	-032	-033	- 029	-028	00035	092	091	115	114	00050

Table C2

Item Indices for the Conditions of 40 Items, Normal Ability Distribution, Unit Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}^2_b$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}^2_a$
Q1	079	071	-109	-130	-112	-112	00047	100	089	102	103	00058
Q2	039	075	041	043	039	039	00028	100	089	112	114	00071
Q3	006	065	221	236	233	233	00338	100	095	086	087	00085
Q4	080	071	-118	-135	-117	-117	00055	100	089	100	101	00063
Q5	056	075	-019	-027	-020	-021	00025	100	089	113	113	00069
Q6	062	075	-040	-051	-041	-042	00040	100	091	113	112	00086
Q7	068	073	-069	-078	-064	-064	00030	100	087	106	108	00067
Q8	052	075	-005	-010	-006	-006	00026	100	088	113	112	00087
Q9	058	075	-027	-035	-027	-027	00031	100	090	114	116	00061
Q10	047	076	014	010	011	011	00026	100	091	117	118	00084
Q11	087	069	-164	-184	-167	-167	00118	100	091	095	095	00063
Q12	069	075	-066	-078	-065	-065	00033	100	092	112	113	00097
Q13	055	076	-016	-022	-016	-016	00022	100	091	116	115	00054
Q14	053	075	-010	-016	-011	-011	00026	100	089	114	115	00088
Q15	064	074	-051	-060	-048	-049	00032	100	089	111	112	00089
Q16	095	063	-241	-265	-267	-264	00641	100	097	082	085	00095
Q17	036	075	054	054	048	048	00032	100	092	115	115	00082
Q18	037	074	047	049	043	044	00034	100	088	110	111	00067
Q19	060	075	-033	-042	-033	-033	00023	100	090	113	113	00068
Q20	024	073	102	110	097	098	00052	100	090	106	106	00074
Q21	065	074	-052	-063	-051	-052	00033	100	089	111	112	00074
Q22	009	068	200	208	200	200	00185	100	096	092	092	00078
Q23	021	072	115	124	111	110	00066	100	089	103	104	00057
Q24	061	074	-038	-049	-038	-039	00028	100	087	109	110	00075
Q25	035	075	058	059	053	053	00032	100	091	114	115	00087
Q26	072	074	-081	-096	-080	-080	00037	100	090	109	111	00070
Q27	019	072	126	133	120	119	00062	100	093	105	105	00074
Q28	045	073	022	019	018	018	00027	100	084	107	108	00064
Q29	038	073	041	045	041	041	00036	100	085	106	106	00068
Q30	032	074	073	072	064	063	00034	100	091	111	113	00084
Q31	074	073	-092	-106	-089	-090	00038	100	092	108	109	00056
Q32	053	076	-009	-015	-010	-010	00021	100	091	117	118	00069
Q33	051	076	-003	-007	-004	-005	00023	100	090	116	117	00093
Q34	071	074	-076	-090	-075	-076	00040	100	091	110	111	00076
Q35	020	070	123	132	119	119	00066	100	087	099	099	00065
Q36	060	075	-036	-043	-034	-034	00026	100	089	112	112	00063
Q37	039	074	042	042	037	037	00030	100	088	111	111	00065
Q38	034	075	059	061	054	053	00035	100	092	114	117	00094
Q39	047	074	010	008	009	008	00032	100	086	110	112	00091
Q40	018	071	136	146	132	132	00080	100	090	100	100	00072

Table C3

Item Indices for the Conditions of 40 Items, Normal Ability Distribution, Variable Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	061	066	-046	-049	- 043	-043	00043	079	068	087	087	00042
Q2	034	088	047	058	046	046	00022	164	142	182	186	00289
Q3	075	069	-098	-112	- 097	-098	00057	091	078	096	095	00055
Q4	028	064	091	110	090	090	00042	076	068	083	084	00039
Q5	027	078	080	095	078	078	00029	126	106	127	128	00107
Q6	063	095	-037	-041	- 036	-036	00014	231	185	292	302	03258
Q7	063	067	-051	-057	- 049	-050	00030	082	070	090	091	00046
Q8	063	065	-054	-061	- 053	-053	00032	079	066	085	086	00041
Q9	046	086	013	017	011	012	00018	148	125	170	171	00197
Q10	061	085	-034	-035	- 032	-031	00024	136	117	159	160	00189
Q11	058	043	-044	-052	- 044	-043	00080	041	036	048	049	00026
Q12	057	078	-023	-025	- 024	-023	00016	108	097	127	128	00093
Q13	027	053	126	145	119	120	00111	058	050	062	063	00026
Q14	076	074	-093	-109	- 096	-095	00040	110	093	111	112	00086
Q15	037	089	040	048	037	037	00020	163	143	192	194	00317
Q16	031	079	066	076	061	061	00022	121	104	128	129	00112
Q17	010	067	178	203	189	189	00168	104	092	089	090	00056
Q18	021	071	116	133	113	112	00046	099	089	101	103	00056
Q19	048	048	007	013	008	009	00073	050	041	055	055	00020
Q20	027	073	087	100	082	083	00039	106	091	108	108	00058
Q21	049	047	007	008	003	003	00073	047	039	053	054	00027
Q22	089	075	-154	-177	- 165	-165	00093	133	114	113	114	00121
Q23	088	068	-171	-192	- 177	-177	00153	100	088	092	093	00071
Q24	015	053	183	220	193	194	00309	069	056	063	063	00037
Q25	075	079	-087	-098	- 086	-086	00031	125	108	130	132	00116
Q26	027	079	078	092	075	076	00036	124	108	130	132	00109
Q27	024	077	095	109	091	091	00034	121	103	119	120	00078
Q28	052	076	-007	-004	- 006	-005	00026	107	089	118	119	00053
Q29	069	084	-059	-066	- 058	-057	00023	138	121	153	156	00170
Q30	046	066	018	020	014	013	00029	082	069	088	088	00034
Q31	057	051	-033	-042	- 036	-036	00067	051	045	060	060	00031
Q32	037	086	040	049	037	037	00020	151	130	172	176	00246
Q33	059	082	-030	-032	- 029	-029	00026	129	108	143	144	00136
Q34	057	052	-038	-039	- 034	-033	00054	050	046	061	062	00027
Q35	049	096	003	006	002	002	00015	235	183	325	348	05179
Q36	054	061	-013	-015	- 015	-014	00033	065	060	078	079	00034
Q37	051	070	-004	001	- 002	-001	00027	087	075	097	098	00039
Q38	079	080	-101	-113	- 100	-100	00045	126	113	132	133	00140
Q39	084	078	-122	-139	- 125	-124	00042	129	113	126	128	00101
Q40	076	061	-113	-132	- 115	-114	00079	076	064	078	079	00039

Table C4

Item Indices for the Conditions of 80 Items, Normal Ability Distribution, Unit Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	077	072	-105	-109	- 105	-106	00077	100	096	104	103	00101
Q2	080	071	-120	-124	- 119	-118	00053	100	095	101	103	00095
Q3	034	070	059	060	059	058	00035	100	097	099	099	00054
Q4	056	074	-018	-019	- 019	-018	00022	100	100	110	110	00079
Q5	069	073	-074	-073	- 069	-069	00039	100	097	108	110	00083
Q6	084	072	-136	-141	- 137	-137	00073	100	099	103	104	00089
Q7	042	073	033	031	029	030	00032	100	100	105	106	00063
Q8	060	073	-034	-035	- 034	-034	00023	100	098	108	109	00062
Q9	045	072	017	017	016	016	00027	100	098	105	106	00062
Q10	012	062	170	177	190	189	00188	100	094	080	081	00042
Q11	037	072	048	048	047	047	00036	100	099	102	102	00055
Q12	027	069	086	088	087	088	00059	100	096	095	095	00062
Q13	090	070	-186	-189	- 188	-189	00225	100	101	097	098	00135
Q14	018	065	129	133	138	137	00117	100	095	086	088	00053
Q15	038	072	044	044	042	042	00028	100	100	104	105	00064
Q16	063	073	-050	-047	- 045	-045	00026	100	098	108	109	00070
Q17	058	073	-029	-030	- 029	-029	00025	100	097	107	109	00055
Q18	053	072	-012	-009	- 009	-009	00027	100	095	104	105	00053
Q19	075	072	-098	-099	- 095	-095	00055	100	097	105	106	00081
Q20	073	072	-089	-090	- 086	-086	00048	100	095	103	105	00093
Q21	030	071	074	075	074	074	00035	100	100	100	100	00057
Q22	092	068	-201	-207	- 208	-208	00269	100	098	092	094	00107
Q23	088	071	-166	-168	- 164	-162	00112	100	099	100	102	00087
Q24	037	071	050	050	048	049	00031	100	099	102	102	00054
Q25	042	073	030	030	028	029	00026	100	101	106	107	00058
Q26	035	070	055	056	054	054	00031	100	095	098	099	00044
Q27	083	071	-136	-139	- 134	-134	00102	100	097	101	103	00121
Q28	050	072	000	001	000	000	00021	100	096	104	104	00062
Q29	079	071	-114	-118	- 113	-115	00072	100	095	102	101	00079
Q30	050	072	000	-001	- 002	-001	00031	100	094	102	102	00054
Q31	071	072	-081	-079	- 076	-076	00042	100	096	105	105	00076
Q32	073	073	-084	-088	- 084	-084	00032	100	098	107	108	00098
Q33	055	073	-015	-017	- 016	-016	00025	100	097	106	107	00062
Q34	013	063	157	164	174	174	00182	100	095	082	083	00044
Q35	063	075	-047	-047	- 045	-045	00025	100	102	113	114	00088
Q36	046	074	012	013	012	012	00024	100	101	109	109	00055
Q37	021	067	114	118	120	121	00079	100	096	090	090	00039
Q38	083	074	-135	-135	- 130	-130	00063	100	104	109	110	00103
Q39	090	071	-185	-181	- 179	-180	00151	100	103	101	101	00080
Q40	081	071	-124	-127	- 122	-122	00073	100	096	102	104	00086

Table C4 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	064	073	-050	-052	- 050	-050	00032	100	098	108	109	00065
Q42	014	064	157	160	170	171	00177	100	097	084	084	00048
Q43	026	069	089	092	092	092	00047	100	098	095	096	00052
Q44	054	074	-013	-013	- 013	-013	00025	100	100	109	110	00058
Q45	091	069	-192	-194	- 194	-194	00234	100	099	094	095	00108
Q46	048	074	009	009	008	009	00027	100	101	109	108	00050
Q47	082	072	-126	-132	- 127	-127	00067	100	098	103	105	00094
Q48	056	074	-019	-019	- 019	-019	00017	100	100	110	111	00069
Q49	068	073	-064	-066	- 063	-063	00026	100	098	108	109	00065
Q50	018	066	133	135	140	140	00094	100	098	088	088	00041
Q51	005	056	244	256	296	291	00653	100	091	068	070	00054
Q52	039	072	036	040	038	038	00026	100	100	105	107	00059
Q53	043	073	028	028	026	027	00026	100	100	107	109	00059
Q54	066	074	-057	-060	- 057	-058	00034	100	100	110	112	00113
Q55	073	073	-088	-089	- 085	-085	00036	100	096	106	106	00075
Q56	032	071	067	067	065	065	00038	100	101	102	102	00044
Q57	078	072	-106	-110	- 106	-105	00057	100	096	103	104	00083
Q58	062	074	-040	-041	- 040	-040	00022	100	099	110	111	00066
Q59	028	068	085	087	087	087	00056	100	094	092	093	00061
Q60	060	073	-036	-036	- 035	-035	00030	100	098	108	108	00082
Q61	065	073	-051	-054	- 052	-051	00027	100	097	107	109	00075
Q62	084	070	-143	-148	- 144	-144	00136	100	094	097	099	00091
Q63	066	073	-058	-060	- 057	-058	00028	100	098	108	108	00059
Q64	027	069	088	089	089	090	00048	100	096	095	095	00041
Q65	091	070	-190	-189	- 188	-188	00179	100	102	098	100	00111
Q66	015	066	144	150	157	158	00169	100	099	087	088	00068
Q67	058	073	-030	-030	- 029	-029	00032	100	098	108	108	00064
Q68	041	072	034	034	032	034	00026	100	100	105	104	00048
Q69	039	072	037	039	037	037	00030	100	101	105	107	00047
Q70	058	072	-026	-027	- 026	-026	00024	100	096	105	106	00081
Q71	031	070	068	073	071	073	00041	100	098	099	099	00047
Q72	077	073	-105	-107	- 102	-103	00064	100	098	106	107	00096
Q73	075	073	-093	-095	- 091	-091	00054	100	099	107	109	00107
Q74	065	073	-053	-055	- 052	-052	00029	100	095	105	106	00072
Q75	035	070	056	055	053	055	00048	100	096	099	098	00059
Q76	080	073	-121	-123	- 118	-118	00058	100	100	106	107	00086
Q77	048	073	005	008	007	007	00029	100	098	106	106	00047
Q78	072	074	-076	-081	- 078	-077	00048	100	099	109	109	00079
Q79	063	075	-047	-046	- 045	-045	00028	100	102	113	114	00080
Q80	043	072	025	026	024	024	00028	100	098	104	105	00060

Table C5

Item Indices for the Conditions of 80 Items, Normal Ability Distribution, Variable Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	095	079	-192	-190	- 211	-214	00192	186	197	128	128	00163
Q2	020	053	163	161	160	161	00172	061	063	063	063	00036
Q3	073	078	-081	-083	- 080	-081	00035	119	119	125	126	00073
Q4	032	072	069	067	064	064	00039	100	099	105	105	00080
Q5	050	044	-005	-002	- 002	-002	00066	044	044	049	049	00025
Q6	052	079	-006	-006	- 006	-007	00018	116	115	130	130	00104
Q7	045	088	014	016	015	015	00022	159	157	186	187	00379
Q8	048	085	009	008	007	007	00018	146	141	162	163	00159
Q9	026	078	085	085	083	083	00043	122	123	124	124	00109
Q10	042	061	037	037	035	035	00051	073	070	077	076	00029
Q11	075	081	-085	-088	- 085	-087	00043	135	132	138	137	00145
Q12	020	090	092	094	094	093	00030	259	260	211	219	00588
Q13	052	067	-005	-006	- 006	-007	00034	078	081	090	092	00042
Q14	074	074	-090	-091	- 088	-088	00047	108	104	109	110	00071
Q15	077	071	-103	-106	- 103	-104	00047	099	097	100	101	00058
Q16	068	093	-056	-055	- 052	-052	00018	208	200	247	249	00808
Q17	060	079	-030	-033	- 031	-032	00026	114	114	128	129	00101
Q18	063	077	-045	-047	- 044	-045	00022	109	109	121	122	00089
Q19	026	067	097	098	095	095	00051	089	088	090	090	00038
Q20	038	072	043	043	041	041	00026	099	096	104	104	00056
Q21	060	089	-028	-029	- 027	-028	00018	164	160	194	199	00446
Q22	014	056	190	189	194	197	00233	072	072	067	067	00036
Q23	058	063	-031	-034	- 032	-032	00043	076	075	082	082	00042
Q24	023	057	129	132	129	130	00139	067	067	069	068	00041
Q25	009	079	156	157	173	173	00098	207	206	130	131	00103
Q26	083	071	-137	-136	- 136	-136	00071	107	104	102	102	00085
Q27	039	065	047	045	043	043	00039	078	079	085	086	00046
Q28	058	062	-034	-034	- 032	-032	00030	070	072	079	080	00038
Q29	048	042	013	015	013	012	00072	042	042	046	047	00015
Q30	011	056	206	209	219	221	00265	079	076	068	069	00036
Q31	046	049	022	023	021	020	00054	052	050	056	057	00031
Q32	063	068	-051	-051	- 049	-050	00047	083	084	092	092	00038
Q33	036	067	056	058	055	054	00040	086	085	090	091	00049
Q34	029	052	110	113	109	108	00116	061	058	061	062	00037
Q35	076	054	-127	-135	- 132	-133	00131	067	063	065	066	00030
Q36	035	090	044	046	043	043	00017	184	183	207	212	00507
Q37	067	074	-061	-062	- 059	-059	00035	099	102	111	111	00070
Q38	041	085	026	027	025	025	00018	148	145	165	165	00165
Q39[a]	039	098	031	031	028	029	00018	292	282	506	714	378799
Q40	062	093	-034	-035	- 033	-033	00016	198	196	258	261	01418

Table C5 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	049	081	006	005	004	005	00023	119	120	136	137	00093
Q42	061	083	-036	-037	- 035	-035	00021	134	134	150	150	00143
Q43	043	044	047	044	041	042	00104	043	044	049	049	00020
Q44	077	053	-141	-144	- 141	-142	00160	061	061	063	063	00031
Q45	069	072	-071	-071	- 068	-067	00041	098	097	105	107	00061
Q46	051	091	-002	-002	- 002	-002	00015	176	174	219	225	00789
Q47	063	086	-041	-041	- 038	-038	00019	146	144	166	168	00287
Q48	052	068	-004	-006	- 006	-005	00028	083	085	094	095	00045
Q49	068	074	-066	-067	- 064	-064	00039	104	103	111	112	00052
Q50	070	093	-061	-061	- 058	-058	00018	210	210	257	264	01210
Q51	086	062	-173	-173	- 176	-178	00182	083	083	079	079	00053
Q52	092	087	-149	-151	- 163	-164	00074	308	308	177	178	00362
Q53	069	062	-079	-082	- 079	-080	00065	076	075	080	081	00050
Q54	038	045	076	074	069	069	00122	044	045	051	051	00024
Q55	036	073	053	052	049	049	00034	097	099	107	107	00048
Q56	084	060	-167	-167	- 169	-171	00162	076	077	075	075	00045
Q57	029	082	066	069	066	066	00027	143	140	146	148	00176
Q58	007	060	217	226	249	248	00319	099	094	075	076	00046
Q59	059	062	-039	-037	- 035	-035	00042	073	072	079	080	00035
Q60	034	081	052	054	052	051	00036	130	129	139	141	00113
Q61	045	073	018	017	016	016	00030	096	095	105	105	00049
Q62	071	039	-159	-150	- 144	-147	00322	037	039	043	043	00023
Q63	061	068	-043	-044	- 041	-041	00031	088	085	094	094	00049
Q64	049	087	004	004	003	003	00018	148	147	174	175	00211
Q65	023	052	147	147	146	146	00230	060	060	060	060	00042
Q66	052	062	-009	-010	- 009	-009	00045	075	072	080	080	00034
Q67	061	070	-043	-042	- 040	-040	00028	091	089	097	098	00051
Q68	070	056	-095	-098	- 094	-095	00079	064	063	068	068	00038
Q69	078	070	-107	-112	- 109	-110	00063	096	096	099	098	00070
Q70	047	076	009	010	009	009	00021	107	107	119	119	00062
Q71	038	074	044	044	041	041	00025	103	103	111	112	00081
Q72	030	066	080	080	077	076	00053	089	085	089	090	00045
Q73	041	080	031	031	029	028	00023	126	121	135	136	00115
Q74	061	078	-035	-036	- 034	-035	00020	114	114	127	128	00110
Q75	074	067	-097	-100	- 097	-098	00068	086	086	089	089	00055
Q76	061	074	-036	-038	- 036	-037	00034	099	099	109	109	00081
Q77	081	060	-144	-145	- 143	-145	00108	078	075	076	075	00043
Q78	047	081	010	011	010	010	00017	125	123	138	138	00118
Q79	048	080	007	006	006	006	00020	115	117	131	131	00091
Q80	072	045	-140	-137	- 132	-133	00204	047	047	050	050	00022

[a] $n = 94$; 6 sample values removed due to biserials ≥ 1.

Table C6

Item Indices for the Conditions of 20 Items, Normal Ability Distribution, Unit Discrimination, Sample Size of 500

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	087	074	-161	-162	- 152	-156	00187	100	100	110	111	00251
Q2	056	078	-024	-022	- 020	-020	00062	100	100	123	124	00211
Q3	078	076	-109	-112	- 102	-102	00099	100	099	116	119	00236
Q4	090	071	-184	-192	- 182	-185	00477	100	095	101	102	00303
Q5	070	076	-076	-077	- 070	-070	00077	100	097	118	122	00268
Q6	044	077	021	022	020	021	00036	100	102	122	120	00164
Q7	079	074	-112	-116	- 106	-107	00121	100	095	112	111	00192
Q8	041	076	033	033	030	030	00050	100	098	116	119	00169
Q9	057	077	-025	-024	- 022	-024	00048	100	099	122	121	00175
Q10	040	075	034	035	032	032	00051	100	097	114	114	00161
Q11	076	076	-098	-100	- 091	-093	00098	100	099	119	118	00187
Q12	093	069	-214	-222	- 216	-214	00607	100	095	095	102	00394
Q13	040	075	036	038	035	035	00062	100	095	113	112	00136
Q14	062	077	-045	-044	- 040	-042	00049	100	099	121	122	00209
Q15	052	077	-011	-008	- 007	-007	00053	100	100	122	123	00203
Q16	058	078	-029	-029	- 027	-028	00049	100	103	126	126	00205
Q17	011	067	172	176	179	181	00256	100	098	091	091	00117
Q18	041	076	031	032	029	029	00062	100	099	117	118	00161
Q19	024	073	098	100	095	096	00102	100	101	108	108	00122
Q20	007	063	221	226	239	242	00770	100	095	081	082	00126

Table C7

Item Indices for the Conditions of 20 Items, Normal Ability Distribution, Variable Discrimination, Sample Size of 500

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	031	069	075	079	070	070	00066	083	081	096	098	00113
Q2	008	066	207	211	219	216	00362	097	100	087	091	00118
Q3	023	069	114	115	106	105	00161	085	086	096	098	00121
Q4	074	086	-078	-083	- 076	-076	00065	145	138	166	168	00736
Q5	079	076	-119	-118	- 107	-108	00153	105	101	118	120	00235
Q6	076	053	-142	-154	- 129	-128	00279	055	049	063	065	00063
Q7	045	059	023	025	021	020	00096	056	056	073	073	00072
Q8	082	074	-133	-136	- 124	-125	00182	097	095	109	111	00205
Q9	077	082	-097	-099	- 090	-092	00090	121	120	142	143	00412
Q10	064	083	-048	-050	- 045	-045	00067	126	117	146	148	00399
Q11	083	088	-109	-115	- 108	-108	00093	178	166	184	193	01946
Q12	063	060	-066	-064	- 054	-055	00109	060	057	075	076	00080
Q13[a]	044	095	017	019	016	016	00041	284	244	320	379	24505
Q14	056	054	-034	-034	- 028	-027	00100	049	048	064	064	00054
Q15	030	054	110	116	098	099	00197	055	051	065	066	00070
Q16	029	074	075	081	074	073	00077	106	098	111	113	00103
Q17	026	058	118	126	110	110	00201	064	059	072	072	00064
Q18	032	089	052	055	052	052	00050	190	186	192	201	01014
Q19	061	089	-033	-036	- 033	-033	00043	159	153	192	197	00794
Q20	040	056	054	056	046	046	00097	048	051	067	068	00061

[a] $n = 98$; 2 sample values removed due to biserials ≥ 1.

Table C8

Item Indices for the Conditions of 40 Items, Normal Ability Distribution, Unit Discrimination, Sample Size of 500

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	054	074	-014	-016	- 012	-012	00058	100	088	110	111	00123
Q2	055	074	-017	-022	- 016	-015	00045	100	088	111	114	00118
Q3	042	073	028	030	026	026	00067	100	083	107	110	00132
Q4	055	075	-022	-023	- 018	-017	00072	100	087	112	116	00182
Q5	033	073	061	066	058	059	00069	100	086	108	109	00121
Q6	005	061	241	275	273	277	01567	100	088	077	079	00155
Q7	042	075	029	032	028	029	00069	100	091	114	115	00126
Q8	024	073	097	105	094	096	00127	100	095	108	110	00197
Q9	025	070	098	113	097	098	00123	100	080	098	099	00135
Q10	023	072	106	114	103	104	00109	100	094	104	106	00127
Q11	054	075	-017	-015	- 014	-012	00059	100	093	115	115	00154
Q12	079	073	-118	-135	- 111	-111	00133	100	088	106	106	00169
Q13	055	075	-020	-023	- 017	-015	00055	100	089	113	112	00119
Q14	064	075	-056	-059	- 049	-048	00053	100	088	112	110	00147
Q15	047	074	006	010	009	010	00048	100	086	110	112	00143
Q16	063	075	-050	-057	- 045	-044	00070	100	091	114	112	00175
Q17	054	075	017	-020	- 014	-013	00050	100	087	113	114	00163
Q18	017	068	133	156	138	138	00212	100	080	094	096	00119
Q19	059	074	-034	-035	- 032	-030	00058	100	083	109	111	00193
Q20	081	073	-120	-141	- 119	-119	00137	100	092	106	109	00166
Q21	081	073	-122	-140	- 119	-121	00215	100	091	106	105	00198
Q22	072	074	-081	-094	- 077	-076	00087	100	090	109	111	00159
Q23	043	074	023	023	023	024	00057	100	087	110	112	00135
Q24	085	070	-154	-171	- 150	-149	00215	100	089	099	101	00142
Q25	018	070	135	141	131	133	00207	100	092	099	101	00121
Q26	062	074	-043	-052	- 042	-042	00065	100	088	111	111	00150
Q27	023	073	105	115	102	103	00126	100	092	107	109	00156
Q28	076	072	-098	-112	- 096	-096	00124	100	088	105	105	00171
Q29	053	074	-009	-011	- 009	-009	00057	100	084	111	111	00192
Q30	085	070	-150	-165	- 147	-146	00222	100	091	099	102	00240
Q31	059	075	-033	-038	- 031	-030	00062	100	093	115	115	00128
Q32	067	074	-066	-073	- 061	-060	00078	100	091	108	110	00140
Q33	055	074	-020	-022	- 018	-018	00056	100	082	109	111	00149
Q34	053	076	-016	-014	- 011	-010	00055	100	094	118	120	00163
Q35	014	069	154	172	156	158	00332	100	089	096	097	00170
Q36	064	074	-051	-057	- 048	-047	00048	100	092	110	112	00124
Q37	012	067	173	191	178	179	00377	100	090	090	092	00157
Q38	076	072	-103	-114	- 098	-097	00106	100	086	103	104	00162
Q39	034	072	058	066	057	058	00065	100	086	105	108	00118
Q40	020	071	124	133	121	124	00193	100	090	100	103	00133

Table C9

Item Indices for the Conditions of 40 Items, Normal Ability Distribution, Variable Discrimination, Sample Size of 500

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	061	074	-037	-045	- 038	-038	00064	103	088	110	109	00119
Q2	066	068	-066	-071	- 060	-059	00056	085	074	094	096	00094
Q3	043	047	040	043	035	035	00180	047	040	053	053	00045
Q4	072	079	-078	-088	- 074	-075	00057	120	104	130	130	00241
Q5	062	089	-035	-040	- 034	-034	00042	168	143	192	201	00779
Q6	028	068	090	100	087	088	00112	085	079	092	093	00108
Q7	061	042	-072	-082	- 066	-066	00214	040	035	046	047	00041
Q8	063	042	-090	-102	- 081	-083	00357	041	035	047	047	00053
Q9	040	078	035	038	032	031	00053	117	101	125	125	00198
Q10	030	055	101	113	097	097	00193	060	054	066	066	00066
Q11	053	060	-012	-016	- 014	-014	00063	064	058	074	075	00056
Q12	003	069	233	249	271	273	01156	145	142	097	099	00221
Q13	015	079	129	142	134	134	00113	154	136	127	129	00174
Q14[a]	084	094	-103	-120	- 104	-106	00052	279	223	272	282	07143
Q15	065	067	-065	-070	- 059	-059	00087	081	072	091	091	00097
Q16	071	073	-079	-090	- 076	-077	00090	095	086	107	109	00137
Q17	041	067	035	040	034	033	00074	078	072	089	090	00067
Q18	018	051	178	204	179	181	00748	057	051	059	060	00078
Q19	018	045	212	231	202	211	01045	047	043	050	048	00054
Q20	041	076	029	034	029	030	00066	103	095	117	116	00139
Q21	052	074	-008	-009	- 007	-008	00053	094	088	111	111	00110
Q22	075	065	-110	-121	- 103	-105	00094	078	069	085	083	00069
Q23	022	069	113	126	112	112	00111	096	086	096	096	00090
Q24	070	063	-090	-101	- 086	-087	00089	070	065	080	080	00081
Q25	032	077	062	071	062	061	00069	119	104	122	123	00170
Q26	042	072	028	032	027	027	00067	093	083	104	105	00101
Q27	029	071	082	092	080	080	00099	094	086	100	099	00109
Q28	038	087	036	042	035	036	00052	157	141	173	174	00529
Q29	046	087	012	015	012	011	00039	160	137	178	183	00426
Q30	090	068	-189	-210	- 191	-193	00337	097	088	093	093	00184
Q31	085	060	-170	-192	- 169	-171	00460	076	066	075	076	00108
Q32	031	060	090	097	084	086	00148	069	063	076	076	00077
Q33	077	082	-093	-103	- 089	-089	00067	137	117	144	145	00335
Q34[b]	063	095	-039	-043	- 036	-037	00038	228	192	307	327	06534
Q35	022	069	112	127	114	113	00149	096	086	095	097	00137
Q36	005	076	189	209	223	224	00437	214	184	118	120	00270
Q37[a]	074	096	-070	-078	- 067	-070	00036	253	207	322	390	51815
Q38	082	071	-127	-147	- 128	-128	00138	104	088	102	104	00177
Q39	078	070	-117	-129	- 111	-112	00134	089	081	097	098	00115
Q40	053	054	-018	-015	- 013	-012	00075	055	049	064	065	00052

[a] $n = 99$; 1 sample value removed due to biserial ≥ 1.
[b] $n = 98$; 2 sample values removed due to biserials ≥ 1.

Table C10

Item Indices for the Conditions of 80 Items, Normal Ability Distribution, Unit Discrimination, Sample Size of 500

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	033	072	064	065	062	062	00095	100	099	105	108	00142
Q2	069	071	-073	-075	- 072	-070	00076	100	095	101	102	00140
Q3	066	072	-058	-061	- 059	-058	00069	100	097	104	105	00131
Q4	054	072	-014	-016	- 015	-015	00050	100	095	105	107	00148
Q5	040	074	035	035	033	033	00047	100	101	110	112	00120
Q6	056	074	-022	-022	- 021	-021	00058	100	099	109	110	00175
Q7	073	072	-084	-086	- 083	-084	00088	100	100	105	106	00182
Q8	086	068	-153	-154	- 156	-156	00236	100	098	093	093	00117
Q9	062	074	-040	-045	- 043	-042	00071	100	100	109	111	00134
Q10	093	060	-212	-225	- 245	-241	00773	100	091	076	079	00131
Q11	063	074	-048	-048	- 046	-045	00058	100	102	111	113	00140
Q12	054	072	-015	-014	- 013	-012	00056	100	094	103	103	00098
Q13	039	071	039	041	039	039	00062	100	092	100	101	00133
Q14	074	071	-090	-093	- 090	-090	00087	100	097	101	103	00120
Q15	058	073	-028	-028	- 027	-026	00062	100	097	107	108	00130
Q16	089	066	-177	-178	- 184	-188	00463	100	097	087	087	00121
Q17	054	073	-013	-013	- 012	-012	00049	100	098	108	109	00151
Q18	059	074	-032	-033	- 032	-032	00063	100	099	109	108	00164
Q19	046	073	016	016	015	015	00071	100	097	107	108	00137
Q20	052	073	-006	-006	- 006	-006	00052	100	096	106	107	00127
Q21	077	070	-109	-108	- 106	-107	00129	100	096	098	098	00095
Q22	085	068	-146	-153	- 154	-156	00200	100	097	092	092	00104
Q23	058	075	-026	-029	- 027	-026	00050	100	102	113	116	00146
Q24	081	068	-121	-128	- 127	-131	00176	100	092	092	091	00107
Q25	044	075	022	020	019	019	00054	100	103	113	114	00131
Q26	077	071	-104	-104	- 102	-100	00113	100	099	102	103	00150
Q27	039	073	041	040	038	039	00055	100	099	108	109	00118
Q28	027	071	087	088	086	087	00087	100	099	102	102	00134
Q29	032	072	065	069	066	067	00096	100	098	104	105	00158
Q30	068	071	-066	-069	- 066	-066	00077	100	095	102	102	00110
Q31	083	067	-133	-141	- 141	-142	00239	100	092	090	091	00123
Q32	052	074	-008	-006	- 006	-006	00066	100	100	111	111	00141
Q33	033	072	062	062	059	060	00090	100	099	105	107	00171
Q34	026	071	090	092	090	090	00121	100	099	101	102	00184
Q35	022	071	110	108	107	108	00122	100	103	102	104	00139
Q36	058	073	-027	-028	- 027	-027	00067	100	098	107	108	00123
Q37	056	074	-020	-020	- 019	-018	00048	100	099	109	111	00216
Q38	041	073	031	034	032	034	00069	100	099	108	110	00125
Q39	047	072	013	012	011	010	00057	100	095	104	107	00097
Q40	015	068	149	150	153	155	00246	100	101	094	095	00142

Table C10 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	053	074	-007	-010	- 009	-010	00062	100	099	109	111	00197
Q42	053	073	-010	-010	- 009	-008	00065	100	097	107	107	00148
Q43	061	074	-041	-041	- 039	-039	00062	100	099	109	109	00146
Q44	016	067	141	144	146	147	00260	100	095	090	092	00156
Q45	032	073	066	067	064	065	00088	100	100	106	107	00134
Q46	049	074	002	004	004	004	00039	100	100	110	111	00144
Q47	062	074	-045	-044	- 042	-041	00050	100	102	111	110	00168
Q48	057	073	-024	-024	- 023	-022	00052	100	097	107	109	00151
Q49	041	074	030	034	032	032	00068	100	101	110	110	00123
Q50	012	065	167	174	181	178	00386	100	095	085	089	00126
Q51	083	068	-134	-138	- 138	-140	00200	100	096	094	094	00137
Q52	025	070	099	101	098	100	00115	100	095	097	097	00085
Q53	067	072	-059	-062	- 060	-059	00082	100	097	105	107	00150
Q54	028	071	084	084	082	082	00097	100	098	102	104	00145
Q55	037	074	045	046	044	046	00064	100	102	110	111	00110
Q56	060	074	-038	-035	- 034	-033	00066	100	102	112	111	00173
Q57	027	071	088	088	085	086	00097	100	099	102	102	00130
Q58	005	061	241	245	274	281	01347	100	099	076	077	00128
Q59	017	070	136	134	135	139	00206	100	102	097	096	00162
Q60	068	072	-067	-069	- 066	-066	00069	100	097	104	105	00119
Q61	018	068	131	134	134	137	00191	100	098	094	094	00119
Q62	088	067	-168	-170	- 175	-175	00340	100	098	090	090	00158
Q63	051	073	-005	-005	- 005	-004	00055	100	096	107	109	00165
Q64	053	075	-013	-009	- 008	-007	00049	100	103	114	116	00183
Q65	033	073	061	063	060	061	00087	100	099	106	106	00119
Q66	008	062	201	208	223	224	00645	100	094	079	081	00141
Q67	070	072	-074	-075	- 072	-073	00120	100	096	102	101	00127
Q68	035	075	056	055	052	053	00070	100	105	113	114	00145
Q69	037	075	048	047	044	045	00056	100	103	112	113	00155
Q70	015	067	146	150	152	155	00256	100	097	091	091	00155
Q71	042	073	030	030	028	029	00070	100	099	108	109	00154
Q72	068	071	-065	-068	- 066	-065	00076	100	094	100	101	00143
Q73	006	062	227	231	254	257	01265	100	098	078	081	00188
Q74	059	073	-032	-033	- 031	-030	00078	100	097	107	108	00151
Q75	076	071	-100	-100	- 098	-098	00082	100	097	100	099	00129
Q76	034	071	055	059	056	058	00082	100	096	102	104	00138
Q77	072	073	-083	-083	- 080	-080	00084	100	103	108	108	00148
Q78	073	071	-088	-090	- 087	-089	00103	100	098	102	101	00126
Q79	073	071	-086	-088	- 085	-087	00100	100	097	102	102	00137
Q80	054	073	-017	-016	- 015	-014	00053	100	098	108	109	00101

Table C11

Item Indices for the Conditions of 80 Items, Normal Ability Distribution, Unit Discrimination, Sample Size of 500

Item	p	r_b	b_T	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_T	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	030	084	065	065	063	063	00054	148	148	155	156	00322
Q2	075	073	-093	-096	- 092	-092	00116	103	103	108	110	00180
Q3	040	067	036	038	037	037	00078	085	083	090	090	00105
Q4	036	088	040	040	039	040	00039	175	172	189	199	00926
Q5	026	064	104	101	099	102	00151	079	080	083	082	00077
Q6	008	067	190	197	210	207	00505	110	105	089	093	00163
Q7	091	075	-171	-175	- 177	-178	00260	127	125	114	116	00276
Q8	067	081	-057	-056	- 053	-052	00040	130	130	140	141	00276
Q9	033	090	050	051	050	051	00038	185	187	201	211	01056
Q10	054	055	-016	-018	- 017	-016	00090	060	060	066	067	00051
Q11	061	064	-046	-046	- 043	-043	00085	076	077	084	084	00080
Q12	039	085	030	033	032	032	00054	146	146	158	164	00355
Q13	003	074	213	215	246	250	00619	185	172	109	111	00248
Q14	087	086	-130	-134	- 133	-133	00120	183	174	166	172	01556
Q15	070	065	-085	-085	- 081	-080	00114	077	079	085	085	00078
Q16	077	081	-093	-097	- 093	-092	00071	132	133	138	142	00420
Q17	095	079	-194	-198	- 204	-209	00632	160	153	128	128	00620
Q18	079	077	-105	-107	- 103	-103	00078	116	118	121	123	00266
Q19	016	050	196	199	202	207	00927	059	058	057	057	00064
Q20	084	082	-122	-125	- 122	-123	00103	139	143	142	143	00395
Q21	073	072	-091	-090	- 087	-087	00077	099	100	105	104	00126
Q22	048	063	009	010	010	010	00069	075	074	081	082	00065
Q23	041	052	045	047	044	044	00115	055	055	061	061	00052
Q24	014	080	129	128	132	133	00099	155	155	136	138	00281
Q25	082	068	-131	-135	- 132	-134	00184	092	093	094	095	00135
Q26	074	044	-152	-158	- 151	-157	00563	046	045	049	048	00046
Q27	021	076	104	105	105	105	00091	121	121	119	119	00201
Q28	037	077	044	043	042	041	00059	116	112	120	122	00191
Q29	094	060	-249	-250	- 261	-262	01682	085	086	076	079	00235
Q30	032	088	053	053	052	052	00042	177	174	186	192	00758
Q31	091	088	-148	-150	- 151	-153	00150	215	217	188	191	01177
Q32	053	068	-012	-012	- 010	-009	00077	088	085	093	093	00090
Q33	082	048	-188	-194	- 191	-190	00712	055	054	055	056	00058
Q34	056	086	-020	-020	- 019	-018	00035	150	151	168	176	00520
Q35	028	063	097	094	092	094	00158	077	077	081	082	00089
Q36	033	063	076	074	072	071	00094	077	076	081	083	00076
Q37	085	064	-162	-160	- 159	-161	00306	085	084	084	082	00140
Q38	087	090	-124	-127	- 126	-126	00102	233	225	212	224	03730
Q39	097	057	-286	-302	- 334	-346	0525	096	089	070	071	00366
Q40[a]	032	097	048	049	048	049	00030	346	320	401	455	51463

Table C11 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	004	063	253	246	279	285	01410	105	110	081	082	00179
Q42	037	085	040	040	039	039	00045	146	150	162	163	00389
Q43	059	079	-031	-031	- 029	-029	00050	120	119	129	129	00160
Q44	053	065	-013	-013	- 012	-010	00070	076	077	085	085	00068
Q45	080	063	-134	-136	- 132	-132	00230	081	079	081	083	00128
Q46	060	073	-036	-038	- 036	-036	00058	102	100	108	110	00102
Q47	066	058	-073	-074	- 070	-070	00118	066	065	071	071	00065
Q48	027	077	081	081	080	082	00076	117	117	119	120	00220
Q49	017	068	138	139	140	141	00210	101	097	093	095	00153
Q50	071	062	-095	-096	- 091	-093	00134	073	074	078	079	00067
Q51	072	066	-088	-092	- 088	-088	00116	084	083	088	088	00084
Q52	002	061	260	281	335	351	05654	133	117	077	079	00348
Q53	080	052	-164	-166	- 162	-164	00446	059	059	061	061	00070
Q54	031	077	067	067	066	066	00069	115	117	122	123	00220
Q55	046	063	021	017	017	018	00055	075	074	081	083	00077
Q56	075	082	-083	-085	- 082	-082	00048	139	136	142	141	00240
Q57	002	075	224	222	262	265	00593	198	212	113	114	00236
Q58	083	078	-121	-126	- 122	-122	00144	127	123	123	126	00307
Q59	069	074	-071	-071	- 068	-067	00071	099	105	111	112	00151
Q60	085	065	-159	-158	- 156	-159	00252	084	087	086	086	00108
Q61	055	077	-017	-018	- 016	-017	00040	114	109	119	120	00140
Q62	044	043	039	040	037	036	00210	042	041	047	047	00054
Q63	039	067	045	042	041	042	00068	083	084	090	090	00074
Q64	012	067	172	172	179	181	00422	102	101	090	091	00161
Q65	054	055	-013	-020	- 018	-018	00099	062	059	065	065	00082
Q66	038	068	049	048	047	048	00073	088	086	092	093	00080
Q67	070	057	-099	-096	- 091	-093	00217	063	064	069	069	00080
Q68	074	052	-131	-127	- 122	-121	00291	056	057	062	064	00063
Q69	050	086	001	-002	- 001	-001	00038	147	150	168	168	00320
Q70	081	075	-118	-122	- 119	-118	00121	115	111	113	116	00228
Q71	055	076	-016	-019	- 017	-017	00049	112	107	116	119	00145
Q72	026	082	081	080	078	078	00066	142	141	143	147	00400
Q73	025	070	095	096	095	094	00097	093	095	097	100	00130
Q74	085	077	-137	-138	- 136	-136	00163	123	124	122	125	00378
Q75	026	074	086	087	085	085	00062	108	107	109	113	00141
Q76	053	069	-011	-011	- 010	-010	00052	091	088	096	097	00097
Q77	095	073	-210	-219	- 231	-235	00705	132	129	106	112	00546
Q78	022	055	136	141	140	141	00371	067	065	066	067	00122
Q79	049	056	004	005	005	005	00121	058	060	067	067	00049
Q80	047	074	010	012	012	013	00052	102	100	109	111	00107

[a] $n = 93$; 7 sample values removed due to biserials ≥ 1.

Table C12

Item Indices for the Conditions of 20 Items, Normal Ability Distribution, Unit Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\overline{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\overline{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	063	076	-052	-050	- 045	-045	00100	100	094	116	116	00290
Q2	058	078	-033	-031	- 027	-027	00092	100	100	124	128	00441
Q3	045	074	012	019	017	018	00141	100	091	112	112	00345
Q4	072	076	-089	-087	- 078	-080	00139	100	097	117	118	00429
Q5	035	074	055	058	053	052	00112	100	095	111	113	00284
Q6	058	076	-028	-028	- 025	-024	00118	100	095	118	119	00368
Q7	020	072	121	123	118	118	00273	100	098	103	107	00346
Q8	072	075	-080	-085	- 076	-078	00166	100	094	113	115	00261
Q9	068	075	-070	-069	- 061	-062	00120	100	093	114	116	00314
Q10	033	074	061	064	058	060	00126	100	096	111	111	00358
Q11	076	076	-101	-101	- 091	-091	00177	100	099	116	121	00520
Q12	056	077	-025	-024	- 021	-021	00080	100	097	121	124	00371
Q13	079	076	-120	-117	- 107	-108	00207	100	102	117	120	00442
Q14	039	076	040	040	036	036	00106	100	097	116	119	00409
Q15	088	072	-168	-171	- 162	-164	00580	100	097	103	105	00428
Q16	026	073	089	094	088	088	00174	100	097	107	111	00263
Q17	058	077	-030	-029	- 025	-025	00092	100	098	121	127	00362
Q18	048	077	004	007	006	006	00098	100	097	119	119	00353
Q19	062	078	-044	-042	- 038	-037	00074	100	100	124	126	00356
Q20	027	073	084	091	085	085	00232	100	095	106	111	00363

Table C13

Item Indices for the Conditions of 20 Items, Normal Ability Distribution, Variable Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}^2_b$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}^2_a$
Q1	027	071	095	095	086	088	00245	088	085	102	101	00331
Q2	051	077	-004	-003	- 002	-003	00095	096	095	119	119	00305
Q3	057	077	-024	-027	- 023	-024	00117	103	098	122	126	00361
Q4	085	078	-133	-138	- 132	-133	00238	126	121	123	129	00538
Q5	080	072	-126	-129	- 119	-122	00402	100	091	103	105	00387
Q6	035	080	053	054	049	050	00138	118	112	135	143	00659
Q7	046	092	010	011	010	010	00082	187	170	228	246	04450
Q8	067	081	-058	-059	- 053	-053	00104	118	117	140	148	00773
Q9	086	071	-157	-162	- 154	-156	00457	098	097	100	104	00332
Q10	002	078	216	221	259	267	02519	260	264	123	127	00660
Q11	025	061	122	128	112	114	00444	065	062	076	076	00212
Q12	055	070	-020	-022	- 019	-019	00127	080	078	099	102	00314
Q13	076	081	-089	-093	- 086	-087	00133	131	123	138	144	00658
Q14	035	089	045	047	044	042	00096	170	161	200	208	02219
Q15	032	088	055	057	053	053	00091	159	158	186	187	01226
Q16	066	063	-072	-075	- 064	-065	00217	067	064	081	084	00211
Q17	016	068	158	157	148	150	00491	082	085	092	093	00338
Q18	076	066	-110	-119	- 106	-109	00417	084	076	089	089	00232
Q19	042	065	032	035	030	029	00167	068	066	084	087	00185
Q20	030	068	082	086	076	075	00155	080	075	092	094	00242

Table C14

Item Indices for the Conditions of 40 Items, Normal Ability Distribution, Unit Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}^2_b$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}^2_a$
Q1	079	072	-113	-124	- 110	-112	00253	100	090	103	108	00279
Q2	087	070	-164	-176	- 162	-160	00495	100	092	097	103	00481
Q3	041	074	029	036	030	032	00124	100	089	111	114	00282
Q4	023	073	107	120	104	103	00171	100	090	106	110	00348
Q5	078	071	-109	-120	- 106	-108	00390	100	088	102	106	00485
Q6	062	075	-042	-047	- 041	-042	00117	100	091	113	116	00317
Q7	020	072	121	132	115	116	00225	100	091	104	105	00299
Q8	061	074	-040	-042	- 036	-036	00142	100	087	109	114	00279
Q9	050	075	000	001	000	001	00133	100	089	112	113	00359
Q10	072	073	-086	-093	- 081	-082	00191	100	090	108	109	00356
Q11	028	074	081	091	079	080	00195	100	094	112	115	00441
Q12	021	071	116	132	115	114	00220	100	087	100	102	00248
Q13	056	074	-023	-023	- 020	-020	00139	100	087	109	112	00319
Q14	037	074	048	053	045	046	00125	100	090	111	114	00328
Q15	067	073	-066	-071	- 062	-060	00137	100	089	108	110	00222
Q16	062	073	-041	-046	- 040	-041	00135	100	087	108	110	00306
Q17	043	074	023	028	024	026	00105	100	088	111	115	00336
Q18	005	063	238	262	256	266	02664	100	090	082	084	00390
Q19	021	073	116	128	111	113	00227	100	093	107	110	00342
Q20	035	073	057	061	052	051	00153	100	088	108	110	00285
Q21	012	069	171	189	174	173	00608	100	093	095	099	00297
Q22	044	074	020	025	021	020	00106	100	089	112	115	00308
Q23	086	071	-156	-166	- 151	-152	00453	100	093	100	101	00288
Q24	057	073	-028	-029	- 025	-027	00111	100	086	108	111	00235
Q25	085	069	-146	-163	- 147	-149	00485	100	088	096	098	00311
Q26	013	071	165	174	158	162	00497	100	095	100	100	00322
Q27	059	075	-033	-035	- 030	-030	00121	100	092	115	120	00307
Q28	031	074	069	078	066	066	00113	100	089	109	114	00360
Q29	040	073	036	042	035	035	00103	100	086	108	112	00236
Q30	075	073	-097	-106	- 093	-095	00254	100	091	106	105	00272
Q31	077	072	-107	-117	- 103	-102	00206	100	091	105	112	00404
Q32	022	072	112	123	107	107	00229	100	091	105	107	00261
Q33	038	074	045	051	043	043	00131	100	089	111	112	00292
Q34	045	075	017	019	015	016	00107	100	090	113	117	00302
Q35	049	075	007	006	005	004	00096	100	090	115	119	00276
Q36	040	074	036	042	035	034	00123	100	088	110	114	00383
Q37	056	075	-022	-023	- 020	-022	00084	100	089	112	115	00297
Q38	097	065	-271	-280	- 284	-292	02520	100	101	084	088	00495
Q39	081	071	-123	-136	- 122	-120	00235	100	091	102	110	00574
Q40	082	070	-132	-147	- 131	-133	00473	100	086	097	100	00341

Table C15

Item Indices for the Conditions of 40 Items, Normal Ability Distribution, Variable Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}^2_b$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}^2_a$
Q1	042	084	022	030	023	023	00088	139	117	152	156	00731
Q2	043	068	026	035	028	029	00122	083	072	092	095	00206
Q3	033	086	051	063	052	053	00103	165	138	172	175	01716
Q4	059	069	-034	-037	-032	-032	00136	088	075	096	097	00215
Q5	072	077	-079	-087	-075	-077	00181	111	096	120	120	00413
Q6	087	088	-122	-141	-128	-128	00203	218	191	185	200	02154
Q7	040	070	037	047	038	036	00141	090	077	097	100	00230
Q8	059	079	-031	-034	-029	-030	00108	117	099	128	134	00621
Q9	035	071	054	067	055	055	00181	096	082	102	105	00336
Q10	032	063	072	090	074	076	00320	079	064	081	081	00189
Q11	068	078	-061	-069	-059	-059	00126	115	098	124	128	00498
Q12	072	081	-074	-084	-072	-072	00119	132	113	141	151	00716
Q13	041	080	026	034	027	027	00100	125	105	134	135	00397
Q14	048	059	010	010	007	008	00151	065	056	073	075	00140
Q15	085	066	-162	-180	-160	-157	00439	090	078	087	092	00269
Q16	025	060	114	136	114	118	00616	074	062	075	074	00178
Q17	091	068	-182	-213	-198	-199	00959	109	092	093	098	00480
Q18	041	089	024	033	026	027	00072	172	143	194	200	01950
Q19	072	060	-102	-116	-098	-102	00454	072	060	075	075	00177
Q20	055	077	-018	-018	-016	-018	00090	115	095	123	125	00385
Q21	097	060	-270	-301	-309	-316	05815	101	090	075	081	00667
Q22	007	069	195	220	214	220	01350	122	108	095	096	00496
Q23	042	066	031	040	031	032	00174	083	068	088	088	00187
Q24	065	064	-065	-072	-061	-060	00209	074	065	083	086	00144
Q25	055	064	-019	-022	-019	-020	00123	075	064	083	083	00160
Q26[a]	002	069	236	277	314	310	02568	185	167	095	102	00373
Q27	042	068	030	037	029	030	00125	082	072	092	092	00194
Q28	053	052	-018	-019	-016	-017	00223	053	046	061	064	00115
Q29[b]	072	092	-065	-075	-065	-066	00092	207	174	228	253	10795
Q30	067	087	-050	-058	-050	-050	00094	161	135	178	191	02369
Q31	009	053	242	275	256	269	02710	070	062	063	063	00205
Q32	037	087	037	047	038	039	00096	158	135	176	178	01411
Q33	090	054	-239	-259	-234	-249	03469	064	059	065	065	00269
Q34	028	087	067	081	068	069	00096	164	145	175	173	00997
Q35	077	077	-099	-111	-096	-097	00184	119	103	122	126	00563
Q36	023	082	088	106	091	091	00175	143	124	141	147	00707
Q37	086	059	-180	-208	-185	-196	01581	076	065	072	072	00298
Q38	018	067	132	157	137	141	00404	093	080	089	090	00244
Q39	015	062	160	188	167	168	00883	082	071	078	081	00195
Q40	055	074	-019	-020	-018	-019	00102	103	086	110	112	00294

[a] $n = 98$; 2 sample values removed due to biserials ≥ 1.
[b] $n = 99$; 1 sample value removed due to biserial ≥ 1.

Table C16

Item Indices for the Conditions of 80 Items, Normal Ability Distribution, Unit Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	049	073	003	005	004	003	00135	100	097	106	109	00263
Q2	033	071	061	063	060	060	00173	100	096	102	105	00325
Q3	089	065	-175	-183	-186	-185	00716	100	092	085	091	00313
Q4	097	063	-289	-271	-308	-319	06628	100	112	081	086	00539
Q5	039	072	041	041	038	041	00168	100	097	105	104	00379
Q6	058	073	-030	-030	-029	-029	00116	100	098	107	111	00326
Q7	059	074	-032	-031	-030	-029	00121	100	100	110	113	00317
Q8	059	074	-031	-031	-030	-029	00120	100	100	110	116	00419
Q9	071	073	-081	-081	-078	-078	00177	100	100	105	110	00270
Q10	038	074	041	043	040	042	00129	100	103	111	114	00362
Q11	014	068	156	157	161	162	00465	100	100	092	096	00427
Q12	011	066	178	180	189	192	00829	100	100	087	091	00245
Q13	068	073	-066	-066	-063	-063	00185	100	098	105	108	00311
Q14	016	069	142	142	144	146	00457	100	101	095	097	00275
Q15	042	074	029	030	028	027	00153	100	100	110	111	00208
Q16	077	073	-106	-103	-100	-102	00215	100	102	106	111	00357
Q17	065	073	-053	-053	-051	-052	00128	100	099	107	112	00291
Q18	041	073	032	035	033	032	00121	100	098	106	108	00266
Q19	036	071	050	054	051	050	00133	100	095	102	105	00259
Q20	090	067	-185	-183	-189	-186	00921	100	100	091	097	00329
Q21	036	073	050	052	049	050	00132	100	099	106	108	00223
Q22	059	074	-035	-033	-032	-033	00126	100	100	110	111	00279
Q23	061	074	-042	-041	-039	-040	00139	100	101	110	113	00395
Q24	092	065	-198	-204	-214	-215	01909	100	096	085	089	00382
Q25	051	074	-004	-003	-004	-003	00121	100	100	110	115	00349
Q26	053	073	-007	-009	-009	-010	00131	100	096	105	109	00358
Q27	008	065	192	199	212	215	01272	100	101	086	087	00267
Q28	032	072	067	067	064	067	00208	100	098	103	102	00218
Q29	045	074	016	018	017	018	00154	100	101	110	112	00336
Q30	092	066	-207	-204	-214	-216	01077	100	101	088	092	00340
Q31	086	069	-157	-159	-160	-166	00776	100	099	094	094	00372
Q32	030	071	074	076	073	074	00175	100	097	102	105	00291
Q33	068	074	-065	-065	-062	-063	00158	100	101	108	115	00339
Q34	023	071	107	108	106	107	00337	100	100	100	102	00336
Q35	064	074	-053	-051	-049	-049	00126	100	100	109	112	00327
Q36	010	067	184	179	188	198	01036	100	104	090	090	00262
Q37	009	066	191	189	199	206	01532	100	102	088	089	00305
Q38	044	073	021	021	020	021	00116	100	098	107	111	00318
Q39	076	071	-099	-100	-097	-096	00214	100	097	102	106	00351
Q40	050	073	-002	002	002	002	00106	100	098	108	111	00328

Table C16 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	079	070	-117	-119	- 117	- 115	00340	100	096	098	102	00273
Q42	074	071	-091	-093	- 090	- 090	00226	100	097	102	107	00306
Q43	059	074	-031	-030	- 029	- 029	00123	100	099	109	108	00289
Q44	080	071	-122	-121	- 119	- 118	00252	100	100	100	107	00411
Q45	060	074	-035	-034	- 033	- 033	00121	100	100	110	112	00359
Q46	078	072	-112	-111	- 109	- 109	00208	100	101	103	105	00249
Q47	014	067	151	155	158	159	00607	100	099	091	096	00375
Q48	052	074	-006	-005	- 005	- 006	00133	100	099	109	112	00233
Q49	022	070	113	112	111	113	00287	100	100	099	100	00299
Q50	041	072	029	035	032	031	00122	100	096	104	106	00289
Q51	012	068	168	167	173	178	00777	100	102	092	093	00279
Q52	068	072	-066	-066	- 063	- 062	00118	100	097	105	111	00290
Q53	056	073	-024	-023	- 022	- 022	00141	100	098	107	109	00311
Q54	016	068	135	143	144	147	00571	100	098	092	094	00306
Q55	056	074	-022	-022	- 021	- 020	00128	100	098	109	110	00317
Q56	050	074	-001	000	000	- 002	00131	100	100	110	111	00302
Q57	040	074	034	035	033	033	00121	100	100	109	113	00425
Q58	054	073	-013	-015	- 014	- 014	00129	100	098	108	111	00325
Q59	046	074	014	014	013	013	00104	100	100	109	112	00315
Q60	034	073	062	061	058	058	00176	100	101	107	110	00327
Q61	033	073	063	064	061	061	00153	100	101	107	109	00237
Q62	059	075	-033	-033	- 031	- 032	00101	100	104	114	115	00395
Q63	061	073	-041	-042	- 040	- 040	00133	100	098	107	110	00309
Q64	089	069	-173	-173	- 177	- 176	00936	100	102	095	099	00401
Q65	065	072	-054	-054	- 052	- 052	00106	100	096	105	106	00189
Q66	021	070	115	116	115	116	00296	100	099	097	099	00352
Q67	030	071	075	076	073	073	00172	100	097	102	104	00240
Q68	050	074	001	001	001	001	00122	100	099	109	112	00329
Q69	056	073	-026	-023	- 023	- 021	00119	100	096	105	108	00365
Q70	056	073	-024	-023	- 022	- 022	00130	100	097	106	110	00352
Q71	018	068	129	132	132	135	00490	100	096	093	093	00231
Q72	048	073	007	007	006	006	00110	100	097	106	108	00251
Q73	079	070	-114	-114	- 112	- 111	00235	100	098	099	103	00344
Q74	066	073	-061	-061	- 059	- 059	00166	100	098	106	111	00266
Q75	014	067	154	160	164	166	00659	100	098	090	093	00255
Q76	031	072	071	070	068	070	00176	100	100	105	106	00303
Q77	071	073	-078	-077	- 074	- 074	00219	100	100	107	108	00258
Q78	005	061	239	238	265	286	04539	100	099	077	076	00387
Q79	086	070	-153	-153	- 154	- 158	00632	100	101	097	098	00269
Q80	066	073	-058	-058	- 056	- 055	00145	100	098	106	109	00293

Table C17

Item Indices for the Conditions of 80 Items, Normal Ability Distribution, Unit Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}^2_b$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}^2_a$
Q1	066	075	-054	-057	- 055	- 057	00159	107	106	113	117	00578
Q2	035	074	054	055	053	052	00166	106	102	110	112	00294
Q3	061	061	-048	-051	- 048	- 049	00226	072	070	076	078	00150
Q4	052	091	-006	-006	- 006	- 007	00066	191	185	221	239	04949
Q5	022	078	099	100	098	097	00150	133	127	126	128	00438
Q6	014	059	176	181	183	189	01223	075	075	072	072	00150
Q7	092	075	-178	-180	- 190	- 196	00629	138	141	115	116	00593
Q8	072	083	-070	-073	- 070	- 072	00123	144	140	149	155	01030
Q9	086	083	-127	-130	- 131	- 134	00241	168	164	148	153	00858
Q10[a]	030	093	061	059	057	057	00085	233	232	252	275	08949
Q11	082	078	-119	-120	- 119	- 121	00245	124	126	123	125	00384
Q12	091	076	-167	-169	- 177	- 182	00541	143	141	118	122	00649
Q13	069	081	-061	-063	- 061	- 062	00112	131	128	138	140	00618
Q14	024	068	104	104	101	102	00282	090	092	094	096	00225
Q15	022	060	134	131	129	129	00587	075	075	075	078	00205
Q16	020	063	136	135	134	136	00457	082	082	082	083	00241
Q17	057	075	-021	-024	- 023	- 025	00110	101	102	112.	110	00264
Q18	057	083	-021	-021	- 020	- 022	00058	136	135	151	155	00754
Q19	062	070	-043	-045	- 043	- 046	00116	091	090	097	097	00230
Q20	029	047	116	123	118	121	00629	051	049	053	053	00099
Q21	043	065	028	028	026	027	00134	078	079	086	087	00231
Q22	038	051	066	066	062	061	00319	055	054	059	059	00110
Q23	050	078	005	002	001	000	00070	110	112	123	128	00812
Q24	071	080	-068	-070	- 068	- 068	00156	125	124	132	137	00553
Q25	031	052	097	099	095	099	00555	056	056	060	059	00109
Q26	093	057	-248	-249	- 267	- 284	05701	078	080	068	069	00316
Q27	050	082	002	000	000	- 001	00096	128	127	142	144	00661
Q28	040	064	044	044	042	042	00145	075	076	083	084	00179
Q29	058	070	-029	-030	- 028	- 029	00115	089	090	098	102	00351
Q30	059	062	-036	-038	- 036	- 039	00225	072	072	080	080	00163
Q31	059	069	-032	-036	- 034	- 034	00153	090	087	095	096	00250
Q32	062	045	-069	-072	- 068	- 073	00488	047	045	050	051	00092
Q33	045	059	026	024	022	023	00174	069	067	074	074	00110
Q34	023	069	109	111	108	108	00262	093	093	095	096	00247
Q35	029	059	095	095	091	093	00445	068	069	073	073	00139
Q36	045	067	019	018	017	014	00160	085	082	090	091	00199
Q37	013	062	179	179	184	189	00999	084	086	080	081	00281
Q38	087	064	-171	-172	- 175	- 181	00982	089	089	084	084	00267
Q39	023	074	102	103	102	099	00186	120	111	110	113	00327
Q40	069	069	-070	-073	- 070	- 074	00210	096	090	096	092	00217

Table C17 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	051	057	-006	-007	- 006	- 009	00151	063	063	070	070	00138
Q42	003	077	219	215	244	254	01336	196	202	119	120	00718
Q43	052	074	-004	-008	- 008	- 009	00116	103	100	110	111	00357
Q44	051	085	-003	-003	- 003	- 004	00081	144	143	161	164	00887
Q45	018	066	137	136	135	133	00449	089	090	089	092	00289
Q46	035	065	064	061	058	060	00255	083	079	086	087	00242
Q47	024	078	093	090	088	088	00183	126	125	127	129	00500
Q48	086	071	-152	-152	- 153	- 159	00505	108	109	102	101	00339
Q49	053	085	-009	-011	- 010	- 012	00089	142	141	158	160	00942
Q50	012	075	157	154	158	163	00441	126	129	115	115	00379
Q51	085	077	-129	-133	- 134	- 139	00410	132	129	121	122	00643
Q52	078	090	-088	-090	- 088	- 091	00107	197	198	202	207	02028
Q53	075	066	-101	-105	- 102	- 107	00359	085	084	087	087	00176
Q54	076	075	-093	-099	- 096	- 099	00143	112	109	112	112	00354
Q55	037	038	094	093	087	089	01001	034	037	041	041	00080
Q56	068	081	-056	-059	- 057	- 059	00104	131	126	137	141	00508
Q57	065	086	-046	-047	- 045	- 046	00093	158	154	172	181	01988
Q58	044	057	031	030	029	029	00159	061	063	070	072	00133
Q59	041	085	030	028	027	025	00074	150	148	164	170	01451
Q60	057	063	-027	-028	- 027	- 029	00134	073	074	081	080	00126
Q61	066	069	-064	-062	- 059	- 060	00133	091	088	094	098	00244
Q62	054	089	-011	-011	- 010	- 012	00077	174	171	199	206	02364
Q63	016	082	124	121	121	122	00177	148	155	142	150	00746
Q64	055	068	-017	-019	- 018	- 019	00148	083	084	092	093	00182
Q65	055	091	-012	-014	- 013	- 014	00060	184	180	213	229	03118
Q66	031	064	080	078	075	077	00268	076	078	083	084	00161
Q67	074	070	-091	-096	- 093	- 091	00219	093	093	097	103	00295
Q68	030	067	085	083	080	081	00287	083	086	090	091	00203
Q69	041	069	038	037	035	032	00139	092	088	096	100	00250
Q70	014	083	129	129	131	132	00280	173	169	147	150	00944
Q71	078	048	-159	-162	- 158	- 161	01183	055	053	055	057	00114
Q72	058	045	-044	-048	- 045	- 048	00417	048	045	051	051	00090
Q73	008	067	196	199	211	216	01556	112	109	091	093	00470
Q74	072	080	-071	-075	- 072	- 075	00134	127	125	132	133	00644
Q75	043	067	027	028	027	025	00134	082	084	091	093	00173
Q76	053	075	-009	-010	- 009	- 012	00098	106	102	112	112	00321
Q77	060	075	-037	-037	- 035	- 037	00112	103	103	113	116	00287
Q78	032	068	071	071	068	066	00186	085	086	092	096	00210
Q79	016	075	130	132	132	129	00257	125	120	113	117	00319
Q80	042	072	028	029	028	027	00149	099	097	105	108	00316

[a] $n = 99$; 1 sample value removed due to biserial ≥ 1.

Table C18

Item Indices for the Conditions of 20 Items, Positively Skewed Ability Distribution, Unit Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}^2_b$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}^2_a$
Q1	006	074	228	224	206	207	00256	100	102	111	113	00201
Q2	065	074	-059	-056	- 051	-051	00032	100	097	109	110	00055
Q3	065	074	-059	-057	- 052	-052	00037	100	098	109	110	00048
Q4	026	080	092	093	082	082	00039	100	103	133	133	00113
Q5	021	080	113	113	099	099	00047	100	103	133	136	00216
Q6	092	059	-198	-220	- 237	-237	00297	100	085	073	073	00037
Q7	079	068	-117	-121	- 119	-120	00072	100	092	092	092	00037
Q8	034	080	056	057	051	050	00025	100	103	135	135	00133
Q9	007	076	222	216	197	197	00192	100	106	117	119	00313
Q10	074	071	-093	-093	- 090	-089	00053	100	095	100	101	00055
Q11	040	079	033	036	033	033	00027	100	102	130	133	00124
Q12	046	077	010	013	012	013	00030	100	098	122	123	00059
Q13	064	073	-058	-054	- 049	-049	00033	100	095	108	109	00047
Q14	069	072	-072	-072	- 067	-067	00036	100	097	105	105	00053
Q15	027	080	083	085	075	077	00041	100	102	132	132	00164
Q16	019	079	122	126	110	110	00041	100	103	131	132	00195
Q17	045	078	014	017	016	016	00029	100	100	125	126	00128
Q18	043	078	023	026	024	024	00022	100	098	124	126	00105
Q19	020	079	116	119	105	105	00033	100	101	130	133	00153
Q20	020	080	119	119	105	105	00031	100	104	133	134	00165

Table C19

Item Indices for the Conditions of 20 Items, Positively Skewed Ability Distribution, Variable Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	034	069	066	068	059	060	00039	077	075	096	096	00064
Q2	041	063	041	042	037	037	00045	063	063	081	081	00031
Q3	056	088	-023	-020	- 016	-016	00020	179	169	186	183	00217
Q4	063	062	-059	-062	- 056	-056	00051	071	066	078	078	00026
Q5	037	059	059	064	054	055	00046	057	056	074	074	00032
Q6	031	090	060	061	055	055	00017	144	146	210	210	00464
Q7	043	095	013	018	018	018	00014	210	201	300	313	04256
Q8	056	078	-023	-021	- 018	-017	00020	107	109	125	125	00081
Q9	075	055	-132	-133	- 122	-121	00121	060	058	066	066	00028
Q10	015	085	137	135	120	121	00049	119	127	163	168	00512
Q11	014	063	201	191	168	166	00185	065	069	082	084	00077
Q12	028	073	089	093	081	081	00035	082	083	107	108	00081
Q13	027	088	078	079	070	071	00024	130	135	189	193	00549
Q14	034	077	061	062	055	056	00024	092	093	120	120	00084
Q15	056	062	-027	-027	- 023	-023	00031	066	063	078	078	00033
Q16	015	081	149	147	130	131	00061	111	109	137	140	00233
Q17	087	060	-162	-176	- 187	-187	00140	097	086	076	076	00027
Q18	061	051	-070	-067	- 056	-054	00061	046	045	059	060	00020
Q19	052	068	-012	-009	- 007	-007	00034	079	075	092	094	00043
Q20	022	051	180	184	151	154	00196	046	045	059	058	00034

Table C20

Item Indices for the Conditions of 40 Items, Positively Skewed Ability Distribution, Unit Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	040	077	034	039	034	036	00029	100	091	121	120	00082
Q2	031	078	070	078	065	067	00040	100	092	124	124	00128
Q3	031	077	069	078	066	067	00030	100	090	121	120	00098
Q4	022	079	111	121	100	101	00038	100	097	128	131	00161
Q5	025	079	096	104	086	087	00033	100	097	130	135	00145
Q6	024	078	103	109	091	092	00039	100	094	124	126	00129
Q7	055	074	-023	-024	- 018	- 017	00023	100	091	111	111	00045
Q8	029	079	080	085	071	072	00034	100	097	130	133	00135
Q9	025	078	097	106	088	088	00042	100	092	123	125	00126
Q10	056	075	-026	-026	- 020	- 020	00027	100	094	114	115	00064
Q11	047	075	005	010	011	012	00021	100	090	115	115	00070
Q12	029	078	080	088	073	075	00033	100	092	123	124	00117
Q13	030	078	074	082	069	069	00030	100	093	124	126	00104
Q14	085	063	-145	-164	- 165	- 166	00134	100	087	081	081	00033
Q15	034	078	057	063	053	055	00032	100	094	126	126	00118
Q16	073	068	-089	-099	- 090	- 090	00054	100	088	094	094	00045
Q17	050	075	-002	-001	001	002	00026	100	090	114	114	00055
Q18	073	069	-088	-096	- 087	- 085	00045	100	090	096	097	00049
Q19	062	071	-046	-049	- 042	- 040	00033	100	086	101	102	00050
Q20	094	054	-219	-257	- 287	- 292	00434	100	081	064	064	00032
Q21	076	068	-104	-112	- 105	- 103	00060	100	090	092	093	00042
Q22	089	060	-171	-198	- 207	- 207	00253	100	085	074	074	00042
Q23	078	065	-111	-124	- 116	- 117	00078	100	084	087	086	00036
Q24	065	071	-060	-064	- 055	- 056	00027	100	087	100	100	00040
Q25	083	062	-135	-160	- 156	- 156	00149	100	081	079	079	00032
Q26	045	076	012	016	015	017	00022	100	092	118	118	00075
Q27	095	053	-234	-272	- 312	- 317	00799	100	083	062	062	00036
Q28	018	076	136	148	123	123	00068	100	090	116	117	00134
Q29	088	060	-168	-188	- 193	- 194	00177	100	085	076	075	00030
Q30	008	073	214	224	195	196	00208	100	096	108	109	00150
Q31	022	078	109	119	099	100	00047	100	095	126	127	00151
Q32	024	078	104	112	093	094	00038	100	093	124	126	00157
Q33	077	066	-105	-119	- 110	- 110	00070	100	084	088	088	00054
Q34	029	079	079	082	069	069	00031	100	095	127	127	00109
Q35	029	077	075	084	071	071	00029	100	092	121	122	00091
Q36	068	071	-069	-074	- 065	- 064	00034	100	091	102	101	00036
Q37	088	060	-172	-194	- 200	- 204	00221	100	084	074	073	00036
Q38	071	069	-085	-090	- 080	- 081	00044	100	087	095	096	00047
Q39	004	069	262	279	254	258	00591	100	097	096	098	00195
Q40	031	077	069	079	066	067	00036	100	090	120	119	00092

Table C21

Item Indices for the Conditions of 40 Items, Positively Skewed Ability Distribution, Variable Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	077	083	-093	-099	- 087	- 087	00031	151	133	147	149	00192
Q2	053	082	-016	-014	- 011	- 011	00018	123	108	143	143	00137
Q3	084	057	-160	-190	- 177	- 178	00170	077	065	069	069	00035
Q4	009	068	208	215	201	201	00267	096	097	093	095	00109
Q5	042	068	031	039	031	031	00031	081	071	092	092	00048
Q6	068	077	-068	-073	- 062	- 061	00029	118	099	120	120	00074
Q7	067	061	-079	-087	- 073	- 072	00051	070	061	076	077	00032
Q8	044	062	028	028	022	021	00036	067	061	079	081	00035
Q9	063	072	-050	-054	- 045	- 046	00029	093	082	102	104	00056
Q10	038	065	045	055	045	045	00035	074	067	086	087	00043
Q11	043	074	023	030	025	024	00025	093	085	110	112	00060
Q12	082	048	-190	-216	- 195	- 198	00391	057	048	055	054	00031
Q13	029	085	070	080	067	067	00027	132	126	160	162	00166
Q14	077	070	-105	-117	- 104	- 104	00048	104	086	098	099	00058
Q15	047	052	013	017	013	013	00050	051	046	060	060	00019
Q16	071	075	-075	-085	- 073	- 073	00032	114	094	112	114	00073
Q17	056	071	-026	-025	- 020	- 020	00026	088	079	101	102	00056
Q18	052	059	-009	-008	- 007	- 007	00050	063	056	073	074	00032
Q19	074	053	-125	-140	- 121	- 121	00143	058	052	062	063	00035
Q20	060	086	-034	-035	- 028	- 028	00017	149	129	168	172	00207
Q21	084	060	-152	-179	- 166	- 166	00174	084	070	074	075	00037
Q22	029	078	076	083	069	069	00029	107	100	126	127	00142
Q23	002	070	256	263	306	307	00523	197	221	097	098	00068
Q24	031	071	073	083	069	069	00027	088	079	100	100	00047
Q25	032	063	082	090	075	076	00072	070	065	081	081	00051
Q26	067	083	-060	-065	- 055	- 054	00021	151	119	146	149	00176
Q27	051	057	-003	-002	- 003	- 003	00039	061	054	070	071	00032
Q28	050	082	-007	-002	- 001	000	00019	120	108	142	143	00157
Q29	033	088	052	061	051	050	00019	150	141	187	192	00379
Q30	041	074	029	038	031	030	00023	093	085	110	111	00082
Q31	046	089	007	013	012	012	00015	153	137	193	197	00463
Q32	029	040	157	172	136	137	00268	035	033	043	044	00021
Q33	071	071	-083	-090	- 077	- 077	00037	098	085	100	101	00066
Q34	098	056	-240	-299	- 349	- 349	01260	127	098	068	070	00075
Q35	095	062	-199	-245	- 268	- 273	00488	137	103	079	078	00049
Q36	039	081	033	041	034	033	00024	119	105	137	138	00109
Q37	032	078	062	073	061	061	00023	106	097	123	123	00112
Q38	076	092	-082	-088	- 076	- 076	00025	259	207	237	241	01536
Q39	060	061	-046	-047	- 039	- 039	00046	067	061	077	078	00036
Q40	078	057	-130	-150	- 132	- 130	00115	072	060	070	071	00032

Table C22

Item Indices for the Conditions of 80 Items, Positively Skewed Ability Distribution, Unit Discrimination, Sample Size of 1,000

Item	p	r_b	b_T	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_T	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	084	065	-144	-138	- 154	- 153	00113	100	106	086	087	00035
Q2	080	064	-117	-125	- 133	- 133	00098	100	095	083	084	00033
Q3	076	068	-098	-101	- 103	- 102	00063	100	100	092	093	00049
Q4	040	075	031	033	033	032	00023	100	099	115	116	00068
Q5	015	073	153	153	144	142	00096	100	100	107	109	00113
Q6	008	071	213	208	203	204	00227	100	105	100	100	00113
Q7	073	069	-092	-090	- 090	- 090	00046	100	101	095	097	00055
Q8	071	068	-082	-082	- 082	- 082	00045	100	097	094	093	00037
Q9	028	077	082	081	076	075	00043	100	103	119	121	00102
Q10	053	074	-015	-011	- 008	- 009	00029	100	099	110	112	00083
Q11	079	065	-117	-117	- 124	- 123	00077	100	096	085	086	00036
Q12	037	077	042	045	043	043	00034	100	104	121	121	00099
Q13	072	068	-087	-086	- 086	- 085	00050	100	098	093	095	00040
Q14	065	071	-055	-056	- 053	- 053	00029	100	099	101	101	00046
Q15	038	077	037	042	040	039	00030	100	104	122	124	00099
Q16	027	075	085	087	080	080	00045	100	098	112	111	00077
Q17	022	076	114	111	103	103	00049	100	104	117	118	00093
Q18	089	058	-173	-182	- 213	- 212	00198	100	093	071	072	00025
Q19	066	072	-061	-059	- 056	- 056	00032	100	102	103	104	00045
Q20	046	076	011	014	015	014	00026	100	103	118	120	00072
Q21	021	075	115	117	109	109	00057	100	103	115	116	00120
Q22	079	065	-115	-118	- 125	- 126	00066	100	097	086	086	00033
Q23	036	077	047	051	048	047	00031	100	102	119	122	00089
Q24	044	077	021	021	021	022	00026	100	104	120	120	00096
Q25	054	073	-022	-018	- 015	- 015	00030	100	098	107	109	00051
Q26	007	070	219	215	211	207	00284	100	103	097	100	00108
Q27	079	065	-118	-117	- 123	- 122	00075	100	095	085	086	00030
Q28	076	067	-104	-104	- 107	- 107	00066	100	099	091	092	00035
Q29	011	072	180	177	168	169	00148	100	102	104	105	00136
Q30	014	074	158	156	147	146	00081	100	103	109	111	00100
Q31	033	076	063	062	058	057	00040	100	101	118	119	00088
Q32	049	076	000	002	004	003	00022	100	104	118	120	00088
Q33	026	077	090	090	084	083	00036	100	104	120	121	00101
Q34	021	076	113	113	105	104	00039	100	103	116	119	00083
Q35	082	065	-135	-130	- 140	- 140	00106	100	100	085	085	00034
Q36	062	071	-048	-045	- 041	- 041	00035	100	097	102	104	00052
Q37	068	069	-070	-068	- 066	- 065	00039	100	097	097	099	00046
Q38	055	074	-024	-020	- 017	- 017	00031	100	099	108	109	00063
Q39	064	070	-056	-054	- 051	- 052	00038	100	096	099	099	00035
Q40	034	076	060	060	056	056	00030	100	100	116	118	00097

Table C22 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	055	074	-023	-021	- 017	- 017	00028	100	102	111	113	00065
Q42	074	068	-091	-092	- 093	- 092	00044	100	098	093	094	00041
Q43	038	075	040	041	039	039	00033	100	099	114	115	00074
Q44	057	075	-029	-028	- 024	- 025	00030	100	104	112	114	00068
Q45	031	075	071	073	068	067	00034	100	098	114	114	00105
Q46	079	065	-120	-118	- 125	- 125	00071	100	099	087	088	00039
Q47	015	074	154	149	140	139	00089	100	101	109	110	00095
Q48	097	050	-271	-270	- 382	- 381	02327	100	101	058	059	00062
Q49	084	063	-143	-144	- 160	- 160	00131	100	098	081	081	00038
Q50	067	070	-067	-065	- 062	- 062	00032	100	099	099	100	00048
Q51	044	076	017	019	020	019	00028	100	102	117	121	00085
Q52	029	075	080	081	075	074	00044	100	098	113	115	00075
Q53	087	061	-164	-164	- 188	- 188	00199	100	097	076	077	00033
Q54	036	075	048	052	049	049	00028	100	097	112	113	00057
Q55	046	076	009	012	013	012	00025	100	104	118	120	00080
Q56	036	075	046	049	046	046	00030	100	099	115	115	00077
Q57	054	073	-019	-018	- 015	- 016	00038	100	099	108	109	00057
Q58	037	076	047	046	044	043	00035	100	101	118	118	00100
Q59	064	072	-058	-054	- 051	- 051	00039	100	102	104	105	00053
Q60	074	066	-096	-097	- 099	- 100	00050	100	093	087	088	00035
Q61	046	076	012	012	013	012	00024	100	102	117	117	00082
Q62	093	055	-206	-214	- 266	- 267	00438	100	092	065	065	00039
Q63	054	075	-017	-016	- 013	- 013	00031	100	104	114	116	00075
Q64	033	076	060	062	058	058	00030	100	102	119	120	00099
Q65	021	075	116	117	109	109	00065	100	100	112	114	00126
Q66	039	077	037	039	038	037	00023	100	105	122	123	00086
Q67	057	075	-028	-026	- 023	- 023	00033	100	104	112	113	00089
Q68	043	076	022	023	023	023	00026	100	101	117	117	00069
Q69	036	077	048	048	046	046	00028	100	102	120	121	00095
Q70	008	071	210	208	204	205	00231	100	105	100	101	00108
Q71	059	072	-035	-035	- 032	- 034	00034	100	096	102	102	00055
Q72	052	075	-015	-010	- 007	- 008	00023	100	102	113	114	00064
Q73	060	072	-040	-040	- 036	- 037	00028	100	098	103	104	00040
Q74	063	070	-051	-049	- 046	- 047	00039	100	095	099	099	00052
Q75	075	068	-100	-098	- 100	- 100	00039	100	102	094	094	00039
Q76	016	075	144	140	130	131	00076	100	104	114	112	00096
Q77	038	075	039	043	041	040	00030	100	099	115	116	00080
Q78	025	077	097	097	090	090	00049	100	104	120	120	00103
Q79	023	075	104	107	099	100	00062	100	101	114	112	00092
Q80	023	075	109	104	096	096	00047	100	101	115	116	00106

Table C23

Item Indices for the Conditions of 80 Items, Positively Skewed Ability Distribution, Variable Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	035	080	047	051	048	048	00020	118	113	132	135	00099
Q2	031	090	055	059	056	056	00021	157	159	203	206	00508
Q3	093	055	-205	-225	- 272	- 273	00485	099	089	065	065	00033
Q4	019	070	136	137	126	126	00086	090	089	098	100	00092
Q5	024	086	085	088	081	082	00030	138	140	169	171	00306
Q6	030	065	085	088	082	082	00049	076	077	087	088	00053
Q7	032	050	101	104	096	095	00102	053	051	058	058	00025
Q8	062	060	-055	-054	- 051	- 050	00047	073	071	076	076	00031
Q9	028	066	092	094	087	087	00059	077	077	087	088	00050
Q10	050	068	-007	-002	000	000	00027	086	084	093	094	00051
Q11	053	071	-016	-013	- 010	- 009	00021	093	092	101	103	00052
Q12	096	055	-227	-245	- 322	- 323	00661	119	105	066	067	00038
Q13	089	070	-152	-153	- 175	- 176	00109	139	131	097	096	00051
Q14	032	075	065	067	063	062	00038	098	099	115	117	00091
Q15	070	079	-071	-071	- 068	- 067	00029	142	133	128	129	00090
Q16	077	048	-140	-149	- 152	- 153	00219	060	056	055	056	00027
Q17	063	074	-050	-048	- 045	- 044	00024	109	106	109	110	00066
Q18	043	038	052	053	048	048	00106	035	035	041	042	00017
Q19	053	070	-013	-012	- 010	- 009	00026	092	090	099	099	00057
Q20	027	078	082	083	076	076	00027	105	108	126	128	00110
Q21	081	054	-146	-153	- 161	- 160	00139	075	070	065	066	00029
Q22	078	090	-086	-086	- 086	- 086	00018	308	292	210	212	00409
Q23	083	069	-127	-131	- 141	- 139	00094	122	113	095	096	00044
Q24	096	062	-191	-211	- 280	- 281	00411	162	137	079	080	00046
Q25	083	074	-116	-121	- 131	- 131	00077	145	135	109	110	00088
Q26	036	084	039	043	042	041	00021	130	129	154	158	00175
Q27	049	056	000	003	003	003	00055	062	060	067	068	00029
Q28	003	070	273	269	276	279	00554	115	123	098	097	00135
Q29	051	074	-008	-007	- 005	- 004	00024	103	100	111	112	00067
Q30	044	079	016	017	018	018	00020	113	113	130	132	00104
Q31	067	084	-056	-056	- 052	- 051	00019	162	156	156	158	00132
Q32	028	057	103	109	101	101	00083	064	062	070	070	00031
Q33	066	053	-076	-079	- 077	- 076	00070	060	059	062	063	00024
Q34	026	054	127	130	121	122	00133	059	058	064	065	00034
Q35	071	049	-109	-115	- 114	- 116	00117	060	055	057	057	00021
Q36	066	062	-070	-069	- 067	- 067	00044	078	076	079	080	00026
Q37	015	062	183	182	170	168	00166	073	074	079	081	00071
Q38	077	053	-133	-137	- 141	- 141	00128	069	065	063	063	00022
Q39	014	068	170	169	158	158	00118	086	086	092	093	00090
Q40	065	039	-101	-103	- 098	- 099	00208	039	038	042	042	00017

Table C23 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	090	068	-151	-161	- 187	- 185	00144	143	130	093	095	00064
Q42	055	060	-027	-023	- 021	- 020	00031	070	068	075	076	00038
Q43	068	059	-082	-083	- 081	- 079	00057	074	071	073	074	00034
Q44	081	085	-100	-099	- 104	- 103	00030	230	215	159	159	00123
Q45	013	056	215	214	204	205	00325	064	066	067	068	00051
Q46	068	062	-080	-076	- 074	- 074	00051	077	077	079	079	00038
Q47	014	082	142	144	133	133	00062	131	134	146	150	00296
Q48	053	081	-015	-012	- 009	- 008	00020	128	123	136	137	00096
Q49	073	062	-101	-097	- 098	- 098	00060	084	082	080	080	00034
Q50	048	075	003	006	008	008	00023	101	100	112	113	00059
Q51	054	059	-015	-018	- 016	- 017	00037	069	068	074	074	00034
Q52	076	047	-147	-149	- 152	- 149	00200	054	054	054	055	00025
Q53	036	073	047	051	049	049	00034	098	092	106	107	00057
Q54	026	079	087	090	083	083	00045	113	110	128	129	00152
Q55	063	072	-050	-048	- 045	- 045	00031	103	100	104	105	00051
Q56	040	070	035	039	037	038	00031	088	086	098	100	00048
Q57	029	067	081	088	081	081	00044	084	080	091	092	00053
Q58	080	057	-136	-142	- 149	- 149	00149	079	074	069	069	00031
Q59	034	079	053	056	053	053	00025	111	112	130	133	00095
Q60	085	074	-125	-127	- 139	- 139	00082	149	141	110	110	00093
Q61	009	088	163	164	154	154	00060	189	199	188	191	00606
Q62	068	071	-070	-070	- 067	- 067	00031	105	101	100	101	00044
Q63	024	059	132	128	118	118	00111	064	066	074	075	00050
Q64	075	069	-098	-099	- 100	- 100	00053	104	102	095	096	00039
Q65	060	077	-039	-037	- 033	- 034	00024	117	115	122	122	00069
Q66	039	081	026	034	033	033	00021	120	120	140	142	00112
Q67	050	081	-003	-001	002	002	00023	124	123	139	140	00141
Q68	029	090	062	064	060	060	00022	159	162	206	215	00652
Q69	062	053	-060	-059	- 056	- 055	00058	060	058	063	064	00031
Q70	046	067	014	016	016	017	00033	084	080	090	091	00042
Q71	055	081	-019	-018	- 014	- 014	00018	129	126	139	142	00126
Q72	018	078	122	125	115	114	00053	111	112	125	127	00161
Q73	009	084	169	169	159	159	00086	156	158	156	159	00418
Q74	059	078	-034	-033	- 029	- 029	00025	121	116	123	123	00063
Q75	087	050	-196	-204	- 228	- 232	00356	071	067	058	057	00028
Q76	064	079	-053	-049	- 045	- 045	00021	128	128	131	131	00085
Q77	040	078	029	033	032	032	00024	111	108	125	126	00114
Q78	069	055	-095	-094	- 092	- 093	00096	065	064	066	066	00023
Q79	027	066	101	101	093	093	00053	075	078	088	089	00038
Q80	023	063	119	124	115	117	00076	077	074	082	081	00057

Table C24

Item Indices for the Conditions of 20 Items, Positively Skewed Ability Distribution, Unit Discrimination, Sample Size of 500

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}^2_b$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}^2_a$
Q1	071	072	-084	-086	- 078	-077	00081	100	091	105	108	00133
Q2	043	079	021	025	023	023	00057	100	103	129	131	00261
Q3	056	077	-025	-024	- 021	-021	00041	100	099	123	123	00201
Q4	042	079	029	028	026	027	00035	100	103	129	130	00190
Q5	055	076	-022	-020	- 017	-017	00040	100	095	118	119	00164
Q6	082	069	-133	-140	- 134	-136	00175	100	089	095	094	00124
Q7	026	075	092	095	086	088	00099	100	097	115	114	00200
Q8	079	070	-119	-122	- 115	-116	00130	100	091	099	098	00122
Q9	065	074	-058	-059	- 052	-052	00062	100	092	110	113	00138
Q10	055	077	-023	-020	- 017	-017	00044	100	096	120	121	00195
Q11	022	077	113	111	102	103	00080	100	106	121	123	00206
Q12	024	077	100	099	090	092	00095	100	103	120	120	00221
Q13	051	076	-011	-005	- 004	-004	00049	100	094	119	119	00138
Q14	064	075	-053	-052	- 046	-046	00062	100	095	115	116	00178
Q15	038	079	040	044	040	040	00060	100	102	127	125	00175
Q16	096	057	-245	-279	- 310	-317	03078	100	086	069	071	00161
Q17	054	078	-018	-015	- 012	-012	00045	100	100	124	127	00169
Q18	064	074	-055	-053	- 047	-047	00051	100	092	110	110	00116
Q19	008	070	204	198	196	197	00530	100	106	099	102	00248
Q20	082	070	-127	-134	- 128	-130	00148	100	093	099	099	00135

Table C25

Item Indices for the Conditions of 20 Items, Positively Skewed Ability Distribution, Variable Discrimination, Sample Size of 500

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	064	056	-074	-075	- 062	-064	00122	057	052	068	067	00060
Q2	023	079	104	106	095	095	00076	105	105	129	131	00299
Q3	051	086	-006	-005	- 002	-002	00045	136	129	166	170	00454
Q4	052	092	-014	-010	- 006	-006	00041	186	173	231	234	01754
Q5	032	075	069	071	062	063	00065	087	089	113	114	00198
Q6	030	088	060	064	058	059	00044	142	141	186	190	00956
Q7	046	050	024	026	021	020	00146	042	042	058	058	00057
Q8	052	058	-010	-011	- 009	-008	00079	057	054	071	073	00074
Q9	058	069	-034	-037	- 031	-030	00072	082	076	095	096	00087
Q10	058	088	-032	-028	- 023	-023	00044	164	157	188	195	00828
Q11	071	082	-074	-072	- 067	-067	00057	142	135	143	146	00237
Q12	017	093	107	109	100	102	00064	212	213	259	268	04902
Q13	007	077	203	199	194	197	00381	120	127	120	122	00429
Q14	057	064	-037	-036	- 029	-029	00052	068	066	083	083	00066
Q15	093	057	-219	-239	- 256	-255	00890	089	081	069	071	00102
Q16	048	073	008	006	007	006	00068	082	082	105	105	00112
Q17	091	057	-199	-226	- 231	-235	00937	090	076	070	071	00110
Q18	028	069	097	097	085	086	00076	077	077	096	097	00133
Q19	071	066	-090	-093	- 083	-084	00117	081	076	089	090	00074
Q20	066	062	-072	-077	- 067	-069	00113	071	066	079	079	00069

Table C26

Item Indices for the Conditions of 40 Items, Positively Skewed Ability Distribution, Unit Discrimination, Sample Size of 500

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	096	052	-243	-289	- 336	- 345	03164	100	082	060	061	00100
Q2	031	078	072	079	065	064	00060	100	091	124	128	00305
Q3	011	075	179	196	165	166	00288	100	094	112	115	00327
Q4	090	058	-180	-211	- 221	- 229	00743	100	083	072	071	00077
Q5	088	058	-169	-202	- 206	- 206	00558	100	079	071	073	00069
Q6	056	073	-024	-025	- 019	- 019	00065	100	085	107	108	00149
Q7	014	075	160	172	144	146	00215	100	093	114	117	00312
Q8	015	076	148	163	135	135	00181	100	094	118	122	00423
Q9	052	076	-012	-009	- 006	- 006	00049	100	090	116	118	00129
Q10	067	070	-067	-074	- 064	- 063	00095	100	086	098	101	00108
Q11	087	060	-165	-189	- 190	- 193	00463	100	083	076	076	00066
Q12	038	077	043	048	041	041	00045	100	090	121	122	00154
Q13	060	073	-037	-043	- 035	- 036	00047	100	088	107	109	00142
Q14	044	077	018	022	019	019	00052	100	090	120	122	00173
Q15	036	077	047	054	045	045	00063	100	090	122	126	00309
Q16	019	076	125	139	114	112	00113	100	090	118	126	00447
Q17	015	074	150	170	140	143	00235	100	088	111	112	00327
Q18	060	073	-042	-043	- 035	- 035	00048	100	087	106	108	00127
Q19	023	079	104	115	094	095	00097	100	096	127	129	00284
Q20	016	075	148	161	133	132	00152	100	092	115	118	00292
Q21	063	072	-053	-056	- 047	- 047	00061	100	087	103	103	00092
Q22	058	074	-029	-033	- 026	- 026	00058	100	088	109	111	00123
Q23	032	078	062	071	059	060	00058	100	093	126	129	00301
Q24	059	072	-037	-038	- 031	- 031	00065	100	085	104	106	00127
Q25	036	077	048	055	046	046	00050	100	091	122	125	00299
Q26	036	076	048	057	047	048	00050	100	088	117	118	00205
Q27	029	078	077	088	072	073	00064	100	092	123	125	00240
Q28	043	077	020	025	022	022	00049	100	091	120	122	00162
Q29	073	069	-087	-098	- 088	- 088	00116	100	088	094	096	00079
Q30	067	069	-065	-075	- 064	- 065	00075	100	083	096	096	00095
Q31	027	078	084	093	077	077	00076	100	093	127	130	00246
Q32	062	074	-045	-048	- 040	- 040	00061	100	091	109	112	00180
Q33	035	077	053	060	050	050	00057	100	090	122	126	00238
Q34	063	073	-050	-054	- 044	- 045	00053	100	088	105	106	00117
Q35	082	067	-131	-143	- 138	- 137	00205	100	094	090	092	00083
Q36	040	075	032	040	034	034	00058	100	085	113	116	00158
Q37	064	072	-055	-058	- 049	- 049	00066	100	088	104	103	00106
Q38	043	075	018	025	022	021	00056	100	087	114	115	00168
Q39	049	075	-002	003	005	004	00044	100	088	115	117	00168
Q40	045	076	013	018	016	017	00046	100	090	119	119	00152

Table C27

Item Indices for the Conditions of 40 Items, Positively Skewed Ability Distribution, Variable Discrimination, Sample Size of 500

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	070	073	-077	-084	- 071	- 071	00075	103	090	106	108	00100
Q2	078	065	-119	-132	- 117	- 118	00168	085	076	086	086	00097
Q3	096	056	-231	-281	- 304	- 314	02639	107	083	067	068	00138
Q4	011	086	151	160	144	145	00143	182	177	170	172	00651
Q5	024	067	110	122	103	105	00251	082	073	091	091	00111
Q6	072	069	-088	-098	- 084	- 084	00088	098	082	095	098	00127
Q7	050	057	-003	-001	000	001	00106	060	054	070	069	00064
Q8	068	073	-068	-075	- 063	- 064	00073	104	091	107	108	00153
Q9	013	089	128	141	124	126	00092	217	188	195	195	00992
Q10	073	072	-089	-098	- 085	- 087	00108	107	090	103	103	00136
Q11	054	091	-018	-016	- 010	- 011	00040	178	161	221	227	01424
Q12	035	086	045	052	046	045	00052	136	127	169	170	00762
Q13	091	060	-190	-222	- 220	- 224	00694	097	080	075	074	00091
Q14	027	068	094	108	091	092	00153	081	073	091	092	00120
Q15	070	065	-088	-096	- 083	- 084	00146	083	072	085	085	00076
Q16	030	055	099	114	094	095	00204	056	051	065	065	00059
Q17	005	071	229	248	238	237	00617	127	114	100	103	00248
Q18	064	077	-053	-057	- 047	- 047	00057	113	097	120	119	00189
Q19	089	065	-177	-194	- 191	- 191	00392	101	091	085	086	00112
Q20	048	067	001	005	006	007	00067	081	071	091	093	00146
Q21	047	077	007	011	011	011	00054	100	095	121	121	00215
Q22	036	074	047	055	047	048	00078	096	086	110	111	00138
Q23	066	060	-075	-079	- 066	- 067	00138	071	061	075	075	00048
Q24	027	054	131	137	114	115	00304	055	051	064	065	00068
Q25	082	088	-102	-114	- 103	- 103	00082	249	203	185	198	04101
Q26	061	071	-047	-050	- 041	- 040	00066	094	082	101	104	00139
Q27	019	069	138	150	128	127	00219	085	079	095	097	00134
Q28	069	075	-071	-077	- 065	- 066	00068	107	097	113	114	00111
Q29	065	050	-086	-095	- 077	- 078	00183	049	044	057	057	00045
Q30	090	065	-172	-200	- 198	- 199	00354	111	093	086	086	00093
Q31	033	088	048	057	050	049	00055	158	140	190	190	00690
Q32	025	070	107	113	096	097	00137	084	080	099	098	00139
Q33	048	041	008	017	014	015	00230	037	033	045	045	00039
Q34	065	080	-054	-058	- 047	- 048	00052	126	111	134	137	00232
Q35	028	089	069	077	066	066	00067	160	142	191	191	00865
Q36	064	054	-070	-082	- 067	- 068	00138	059	051	065	065	00068
Q37	060	073	-042	-043	- 034	- 034	00061	094	085	106	106	00107
Q38	039	049	062	072	059	059	00191	047	043	057	056	00059
Q39	091	063	-179	-215	- 214	- 214	00474	110	089	081	083	00098
Q40	052	080	-010	-009	- 005	- 005	00047	119	105	135	136	00220

Table C28

Item Indices for the Conditions of 80 Items, Positively Skewed Ability Distribution, Unit Discrimination, Sample Size of 500

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	064	072	-056	-055	- 051	- 052	00074	100	103	104	105	00116
Q2	050	075	-002	-002	001	000	00054	100	100	113	114	00178
Q3	082	064	-131	-131	- 142	- 143	00173	100	099	083	084	00072
Q4	061	073	-044	-043	- 039	- 039	00060	100	103	107	108	00141
Q5	069	068	-075	-075	- 073	- 073	00102	100	096	094	095	00085
Q6	006	069	232	230	223	228	00810	100	101	097	096	00272
Q7	041	075	029	031	031	031	00057	100	096	112	115	00191
Q8	048	074	005	005	007	008	00056	100	098	111	113	00201
Q9	071	068	-083	-084	- 083	- 084	00117	100	096	092	093	00088
Q10	036	077	050	050	048	046	00063	100	101	119	122	00174
Q11	050	074	-001	-001	001	000	00071	100	097	109	110	00159
Q12	072	068	-087	-086	- 085	- 087	00101	100	097	092	091	00091
Q13	035	076	052	055	051	050	00069	100	098	116	115	00167
Q14	084	061	-143	-148	- 164	- 166	00322	100	095	077	077	00058
Q15	062	070	-047	-049	- 045	- 045	00089	100	095	098	099	00098
Q16	067	070	-068	-064	- 061	- 060	00078	100	099	098	100	00106
Q17	029	075	074	079	073	074	00080	100	096	113	114	00222
Q18	076	065	-099	-103	- 106	- 107	00142	100	094	086	087	00076
Q19	030	076	078	076	070	071	00081	100	099	117	117	00253
Q20	047	076	006	007	009	008	00056	100	102	117	119	00211
Q21	066	072	-060	-061	- 058	- 058	00071	100	103	103	104	00103
Q22	081	063	-127	-130	- 141	- 141	00266	100	096	082	082	00094
Q23	044	075	015	018	019	019	00053	100	098	113	113	00177
Q24	047	075	007	009	011	010	00046	100	100	114	115	00148
Q25	034	076	053	057	053	054	00067	100	099	118	115	00186
Q26	024	076	102	103	094	094	00088	100	101	119	121	00289
Q27	075	065	-100	-102	- 104	- 105	00175	100	093	085	086	00090
Q28	053	074	-014	-013	- 009	- 009	00045	100	100	110	110	00140
Q29	026	077	087	090	082	082	00069	100	102	122	125	00258
Q30	073	066	-088	-090	- 090	- 090	00112	100	094	089	090	00089
Q31	074	068	-094	-096	- 097	- 099	00120	100	100	092	091	00102
Q32	048	075	002	003	005	004	00059	100	100	113	114	00147
Q33	064	070	-057	-055	- 051	- 052	00076	100	097	099	099	00130
Q34	045	074	015	015	016	017	00072	100	097	111	113	00151
Q35	049	076	000	000	003	003	00055	100	104	118	119	00204
Q36	057	072	-030	-028	- 024	- 024	00064	100	097	104	103	00108
Q37	067	071	-064	-064	- 061	- 062	00102	100	102	101	103	00133
Q38	074	067	-094	-095	- 096	- 099	00125	100	098	091	092	00080
Q39	017	075	138	140	128	127	00150	100	100	115	119	00418
Q40	025	077	096	097	089	088	00082	100	102	120	121	00306

Table C28 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	029	077	077	077	071	071	00064	100	101	120	121	00185
Q42	095	051	-225	-244	- 314	- 322	02469	100	090	060	060	00084
Q43	024	077	101	100	092	094	00102	100	100	119	120	00262
Q44	033	076	059	061	057	056	00083	100	098	116	121	00306
Q45	032	077	062	064	060	059	00074	100	101	120	122	00269
Q46	034	076	060	060	056	056	00061	100	099	117	118	00222
Q47	022	076	114	112	103	103	00088	100	100	117	118	00210
Q48	047	074	008	008	010	009	00053	100	098	112	114	00174
Q49	011	073	179	180	168	169	00357	100	099	106	106	00298
Q50	066	070	-060	-061	- 057	- 058	00102	100	097	098	099	00104
Q51	060	072	-040	-038	- 034	- 035	00069	100	099	104	103	00073
Q52	047	074	003	007	009	009	00076	100	098	111	111	00155
Q53	029	077	075	080	074	074	00096	100	101	120	122	00225
Q54	039	076	038	039	038	037	00069	100	099	116	117	00189
Q55	013	073	159	164	152	152	00245	100	098	107	111	00351
Q56	051	073	-008	-007	- 004	- 003	00051	100	096	107	109	00122
Q57	060	073	-041	-038	- 034	- 034	00074	100	100	105	108	00106
Q58	059	072	-036	-036	- 032	- 031	00073	100	098	104	105	00105
Q59	047	075	007	008	010	009	00057	100	099	113	115	00175
Q60	015	074	150	154	142	144	00172	100	099	110	110	00197
Q61	016	076	143	144	132	132	00157	100	103	116	121	00274
Q62	060	072	-037	-039	- 035	- 036	00067	100	099	104	105	00156
Q63	017	074	137	138	127	130	00166	100	099	112	112	00320
Q64	012	073	179	175	163	166	00387	100	100	107	107	00357
Q65	026	077	088	089	082	082	00085	100	101	119	121	00328
Q66	018	076	134	132	120	121	00171	100	101	116	117	00342
Q67	024	076	098	102	093	093	00119	100	099	116	121	00371
Q68	038	076	040	041	040	038	00060	100	101	118	123	00195
Q69	024	078	100	099	090	090	00084	100	105	125	129	00341
Q70	043	078	022	022	022	022	00055	100	105	123	124	00161
Q71	006	069	231	232	228	234	00948	100	102	095	094	00286
Q72	029	076	076	080	074	074	00088	100	099	118	120	00195
Q73	022	077	113	111	101	100	00112	100	102	120	123	00301
Q74	031	077	068	069	064	064	00061	100	100	119	121	00209
Q75	091	056	-187	-197	- 236	- 231	00647	100	092	068	071	00076
Q76	069	069	-076	-074	- 071	- 073	00099	100	097	095	096	00115
Q77	028	077	081	081	074	074	00079	100	102	122	124	00240
Q78	047	076	006	009	011	012	00051	100	102	116	119	00163
Q79	063	073	-052	-051	- 047	- 048	00078	100	104	106	107	00122
Q80	056	073	-024	-024	- 020	- 022	00071	100	099	108	107	00095

Table C29

Item Indices for the Conditions of 80 Items, Positively Skewed Ability Distribution, Unit Discrimination, Sample Size of 500

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	062	070	-048	-046	- 043	- 044	00058	093	093	098	099	00108
Q2	079	072	-110	-107	- 110	- 108	00099	117	114	104	106	00130
Q3	033	085	055	055	052	053	00043	136	135	161	163	00584
Q4	072	071	-087	-084	- 083	- 083	00088	105	103	101	101	00105
Q5	049	062	000	003	003	004	00054	070	071	079	080	00080
Q6	098	064	-209	-224	- 312	- 320	02231	186	165	083	083	00127
Q7	079	063	-125	-124	- 129	- 129	00212	089	088	082	083	00082
Q8	083	073	-122	-123	- 132	- 130	00147	138	127	106	109	00123
Q9	021	062	136	137	128	130	00239	071	073	079	080	00083
Q10	045	077	014	016	016	016	00039	106	107	122	126	00237
Q11	030	082	063	068	063	064	00048	117	121	141	142	00276
Q12	075	051	-134	-133	- 135	- 137	00376	060	059	059	060	00065
Q13	048	078	002	005	006	007	00045	111	109	124	128	00213
Q14	064	055	-069	-065	- 063	- 064	00118	060	062	066	067	00055
Q15	064	082	-050	-047	- 043	- 042	00058	138	134	141	142	00278
Q16	064	081	-049	-047	- 043	- 042	00048	131	130	136	139	00246
Q17	026	076	085	090	084	085	00073	103	104	117	118	00194
Q18	034	075	053	057	054	055	00068	095	099	113	114	00157
Q19	071	038	-145	-146	- 143	- 148	00635	039	039	041	042	00045
Q20	066	051	-087	-086	- 084	- 085	00228	057	055	059	060	00056
Q21	039	051	055	061	056	057	00129	056	053	060	061	00049
Q22	080	079	-105	-103	- 107	- 107	00087	159	150	128	129	00216
Q23	046	073	011	014	014	014	00061	096	096	108	110	00192
Q24	006	081	191	191	193	194	00356	175	186	140	144	00525
Q25	006	082	185	184	184	186	00290	180	187	145	148	00548
Q26	004	074	232	223	233	241	00875	133	146	111	110	00407
Q27	073	070	-087	-087	- 086	- 085	00093	103	100	098	100	00107
Q28	030	088	062	065	060	060	00040	146	149	181	183	00475
Q29	058	070	-036	-032	- 030	- 030	00075	094	091	098	101	00142
Q30	046	039	029	031	028	029	00249	039	037	043	043	00048
Q31	041	079	026	030	029	029	00051	113	113	130	131	00312
Q32	083	063	-143	-140	- 151	- 150	00282	095	094	082	083	00090
Q33	070	068	-083	-079	- 078	- 078	00104	094	093	093	094	00109
Q34	024	092	082	083	077	077	00052	176	190	239	257	03519
Q35	002	065	283	291	324	332	02663	133	123	085	086	00225
Q36	071	080	-076	-072	- 070	- 069	00070	140	138	133	133	00230
Q37	064	065	-059	-059	- 057	- 056	00088	086	082	086	087	00085
Q38	064	071	-056	-054	- 052	- 051	00059	102	097	101	102	00113
Q39	073	072	-087	-086	- 086	- 086	00100	112	106	102	103	00113
Q40	015	072	147	150	142	144	00186	102	100	104	106	00170

Table C29 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	092	056	-205	-212	-251	-252	00858	092	088	067	068	00088
Q42	060	070	-043	-039	-037	-036	00065	089	091	097	099	00109
Q43	048	070	000	005	006	005	00049	088	087	097	098	00131
Q44	052	076	-013	-008	-006	-006	00044	105	105	117	119	00205
Q45	056	073	-027	-025	-022	-021	00062	099	099	108	110	00146
Q46	076	077	-095	-092	-093	-093	00073	134	131	120	120	00133
Q47	067	076	-060	-062	-059	-059	00075	120	114	116	118	00181
Q48	049	087	-003	001	003	004	00034	147	148	176	183	00920
Q49	075	074	-093	-091	-092	-092	00077	121	117	110	111	00157
Q50	061	086	-039	-036	-032	-030	00047	154	154	168	173	00564
Q51	018	080	128	126	117	118	00169	115	121	131	136	00323
Q52	024	072	101	104	096	098	00120	095	094	104	103	00148
Q53	040	056	040	049	046	048	00112	062	060	068	068	00046
Q54	028	082	074	076	070	070	00061	120	121	142	146	00330
Q55	054	076	-019	-017	-014	-013	00054	107	107	118	118	00209
Q56	039	077	036	040	038	039	00064	100	104	120	122	00220
Q57	033	087	051	054	051	052	00046	151	146	180	186	00949
Q58	096	069	-174	-193	-255	-258	00659	228	181	095	095	00100
Q59	058	084	-031	-027	-023	-023	00041	143	140	154	157	00365
Q60	042	061	033	037	034	035	00084	069	069	078	078	00076
Q61	037	064	056	057	052	054	00120	075	073	082	081	00096
Q62	032	048	108	105	097	098	00315	048	049	055	056	00058
Q63	025	083	087	090	083	084	00062	132	130	149	151	00484
Q64	045	060	019	022	021	022	00073	067	068	076	076	00075
Q65	078	086	-090	-088	-088	-089	00054	199	201	169	171	00352
Q66	079	071	-110	-109	-113	-112	00124	121	113	102	104	00109
Q67	054	061	-019	-017	-016	-014	00065	069	069	076	077	00068
Q68	025	084	086	086	079	080	00067	128	132	155	157	00421
Q69	030	081	069	070	065	065	00058	113	117	137	137	00426
Q70	056	055	-030	-029	-028	-029	00099	061	060	066	066	00067
Q71	046	075	010	012	013	014	00042	101	101	115	115	00131
Q72	065	075	-057	-053	-051	-051	00060	109	109	113	114	00124
Q73	081	060	-134	-139	-148	-146	00281	090	082	075	076	00083
Q74	017	070	141	144	135	134	00150	092	091	097	098	00093
Q75	061	064	-050	-046	-043	-043	00089	081	079	084	085	00076
Q76	079	089	-092	-089	-090	-089	00062	263	247	194	201	00958
Q77	037	084	039	042	040	040	00052	127	129	155	156	00405
Q78	017	085	120	122	113	113	00073	149	151	163	169	00617
Q79	062	066	-050	-050	-047	-047	00105	086	084	089	089	00108
Q80[a]	024	098	076	079	072	074	00043	253	271	460	526	60490

[a] $n = 88$; 12 sample values removed due to biserials ≥ 1.

Table C30

Item Indices for the Conditions of 20 Items, Positively Skewed Ability Distribution, Unit Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	048	078	006	005	007	008	00104	100	098	123	125	00512
Q2	094	061	-218	-236	- 251	-269	03179	100	091	078	076	00306
Q3	027	078	085	084	077	080	00139	100	102	125	128	00588
Q4	019	076	127	125	116	116	00225	100	103	119	123	00399
Q5	088	065	-168	-185	- 182	-181	00866	100	087	085	088	00244
Q6	031	078	067	068	062	063	00113	100	100	124	129	00600
Q7	054	077	-017	-018	- 013	-012	00091	100	097	121	124	00411
Q8	060	076	-040	-041	- 034	-034	00142	100	096	117	122	00348
Q9	040	079	035	034	033	035	00103	100	102	129	136	00492
Q10	080	071	-123	-127	- 119	-119	00302	100	096	102	103	00241
Q11	063	076	-052	-051	- 043	-043	00149	100	096	116	118	00370
Q12	079	071	-119	-120	- 112	-113	00301	100	094	101	104	00339
Q13	041	080	034	031	030	030	00103	100	106	134	138	00512
Q14	074	072	-099	-100	- 090	-090	00184	100	091	103	105	00266
Q15	043	079	023	021	021	022	00097	100	101	127	130	00429
Q16	041	078	027	030	029	031	00106	100	100	126	127	00366
Q17	009	071	202	196	191	197	01060	100	104	102	108	00620
Q18	074	072	-093	-099	- 089	-089	00217	100	091	103	106	00376
Q19	024	077	101	098	090	093	00179	100	102	122	123	00374
Q20	072	072	-085	-089	- 079	-080	00198	100	090	105	106	00280

Table C31

Item Indices for the Conditions of 20 Items, Positively Skewed Ability Distribution, Variable Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	025	076	098	099	090	088	00165	096	098	115	119	00338
Q2	067	066	-074	-080	- 067	-070	00216	076	070	087	089	00244
Q3	046	070	015	013	013	013	00141	076	075	098	098	00193
Q4	067	079	-061	-063	- 054	-056	00121	114	106	131	133	00516
Q5	024	065	123	126	110	113	00404	070	069	085	089	00304
Q6	059	084	-032	-032	- 026	-026	00072	128	124	158	165	01006
Q7	035	082	045	050	046	046	00112	115	114	142	147	00572
Q8	060	073	-043	-041	- 034	-033	00154	090	084	107	111	00306
Q9	092	078	-150	-171	- 176	-178	00448	197	152	126	132	00637
Q10	074	084	-083	-087	- 078	-079	00143	151	134	155	159	01112
Q11	033	057	095	093	077	078	00272	052	052	069	071	00145
Q12	035	077	057	056	051	051	00127	096	098	122	124	00485
Q13[a]	069	097	-059	-056	- 050	-052	00068	337	295	384	414	43003
Q14	074	063	-116	-118	- 101	-103	00364	069	067	082	083	00164
Q15	078	079	-102	-109	- 098	-099	00157	130	115	130	134	00608
Q16	025	088	079	081	076	076	00076	165	169	187	200	02497
Q17	016	072	148	149	138	140	00336	093	095	105	107	00373
Q18	086	076	-136	-151	- 143	-147	00307	143	117	118	120	00429
Q19	040	067	044	045	039	039	00155	070	069	090	091	00191
Q20	010	066	209	204	194	198	00959	085	087	089	091	00330

[a] $n = 91$; 9 sample values removed due to biserials ≥ 1.

Table C32

Item Indices for the Conditions of 40 Items, Positively Skewed Ability Distribution, Unit Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	063	073	-051	-055	- 045	- 046	00133	100	088	106	109	00244
Q2	093	057	-207	-245	- 260	- 268	02741	100	082	069	070	00194
Q3	095	055	-226	-266	- 296	- 305	03473	100	085	066	069	00236
Q4	050	075	-007	-003	000	- 001	00125	100	088	113	116	00347
Q5	053	076	-016	-013	- 009	- 010	00090	100	091	116	117	00317
Q6	033	078	058	065	055	055	00124	100	096	126	126	00384
Q7	020	077	118	128	108	109	00204	100	095	119	125	00542
Q8	054	075	-019	-017	- 012	- 014	00125	100	090	114	115	00253
Q9	078	067	-110	-126	- 114	- 116	00323	100	083	089	091	00229
Q10	063	073	-053	-054	- 045	- 046	00113	100	089	108	110	00199
Q11	044	077	015	021	019	018	00094	100	091	119	120	00353
Q12	017	072	139	156	134	137	00480	100	089	105	105	00413
Q13	078	067	-112	-126	- 115	- 118	00324	100	086	090	093	00173
Q14	036	077	048	054	046	048	00138	100	091	120	121	00430
Q15	051	076	-009	-008	- 005	- 006	00097	100	092	117	118	00320
Q16	041	078	032	034	030	030	00111	100	094	123	128	00522
Q17	039	076	034	041	035	037	00143	100	088	115	119	00317
Q18	037	076	044	049	042	042	00114	100	091	118	122	00356
Q19	058	075	-037	-036	- 029	- 028	00170	100	091	112	112	00318
Q20	060	074	-040	-043	- 035	- 036	00103	100	090	111	112	00283
Q21	055	075	-019	-020	- 015	- 016	00112	100	090	114	117	00284
Q22	010	073	189	194	172	178	00596	100	100	108	111	00575
Q23	073	071	-087	-097	- 086	- 085	00150	100	090	100	104	00222
Q24	080	065	-122	-141	- 130	- 136	00367	100	083	086	087	00145
Q25	094	056	-224	-263	- 286	- 303	05054	100	083	067	068	00227
Q26	069	071	-079	-083	- 072	- 075	00222	100	086	100	100	00239
Q27	024	074	101	114	096	097	00194	100	088	111	110	00292
Q28	018	075	133	145	123	125	00251	100	092	112	113	00405
Q29	052	075	-013	-012	- 008	- 010	00081	100	089	113	114	00266
Q30	012	071	172	192	169	171	00610	100	091	101	104	00462
Q31	040	077	035	039	034	034	00133	100	092	121	122	00478
Q32	032	077	065	072	061	061	00139	100	092	121	126	00528
Q33	057	075	-028	-029	- 023	- 022	00133	100	093	115	116	00260
Q34	069	071	-071	-079	- 068	- 070	00154	100	088	101	102	00149
Q35	019	075	128	140	119	120	00250	100	092	113	116	00428
Q36	062	074	-052	-050	- 041	- 042	00114	100	091	110	114	00340
Q37	050	076	-005	-004	- 001	- 001	00111	100	090	116	119	00331
Q38	013	074	161	173	151	157	00849	100	096	108	109	00609
Q39	070	070	-079	-087	- 076	- 076	00234	100	086	099	102	00329
Q40	076	069	-101	-115	- 104	- 106	00220	100	087	094	097	00190

Table C33

Item Indices for the Conditions of 40 Items, Positively Skewed Ability Distribution, Variable Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}^2_b$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}^2_a$
Q1	027	071	089	099	085	085	00199	091	084	102	106	00248
Q2	022	078	103	113	099	100	00162	121	110	127	130	00588
Q3	027	077	084	094	081	084	00224	110	100	121	123	00549
Q4	017	062	166	176	156	161	00786	074	070	079	080	00267
Q5[a]	001	064	286	300	370	383	07886	195	180	084	086	00374
Q6	025	089	076	085	074	074	00127	180	160	193	214	10715
Q7	030	082	067	073	064	064	00133	127	117	145	150	00596
Q8	076	074	-102	-113	- 097	- 097	00239	115	096	112	118	00530
Q9	052	064	-013	-012	- 009	- 009	00182	072	065	084	084	00184
Q10	030	050	116	129	107	111	00553	049	045	058	059	00112
Q11	061	078	-042	-046	- 036	- 037	00124	110	097	124	127	00324
Q12	021	078	108	117	103	103	00181	115	107	123	129	00437
Q13	097	077	-174	-218	- 238	- 245	02395	304	192	121	125	00855
Q14	087	075	-142	-165	- 153	- 157	00483	142	112	112	115	00432
Q15	032	054	095	106	088	086	00364	056	050	064	066	00145
Q16	080	071	-115	-138	- 121	- 123	00263	112	089	100	103	00367
Q17	062	075	-046	-050	- 040	- 039	00109	106	089	113	114	00320
Q18	062	068	-049	-054	- 043	- 042	00142	082	073	092	095	00223
Q19	092	056	-219	-256	- 250	- 259	03790	084	070	067	070	00261
Q20	070	063	-090	-100	- 084	- 087	00283	078	066	081	082	00172
Q21	059	088	-034	-035	- 026	- 026	00075	158	139	188	203	02656
Q22	080	079	-108	-123	- 108	- 109	00159	137	117	129	133	00462
Q23	055	070	-021	-024	- 018	- 020	00124	085	076	098	100	00200
Q24	095	063	-199	-254	- 261	- 265	01795	134	097	082	085	00313
Q25	091	062	-185	-230	- 222	- 229	01561	107	081	078	079	00290
Q26	067	058	-080	-090	- 075	- 073	00278	063	057	071	073	00123
Q27	056	072	-028	-027	- 020	- 020	00113	091	082	104	103	00217
Q28	052	075	-011	-011	- 007	- 007	00138	098	087	113	114	00324
Q29	089	055	-204	-236	- 222	- 232	02348	075	065	066	067	00201
Q30	057	060	-031	-039	- 030	- 030	00132	064	058	075	077	00134
Q31	054	051	-026	-029	- 022	- 022	00316	050	044	059	059	00117
Q32	081	076	-118	-132	- 116	- 119	00321	126	107	118	122	00541
Q33	049	092	-004	-001	002	002	00074	187	156	232	255	05865
Q34	026	072	099	102	089	090	00187	092	087	105	108	00292
Q35	055	069	-021	-023	- 017	- 017	00117	083	073	094	096	00266
Q36	043	092	014	020	019	019	00089	186	161	234	258	05699
Q37	071	088	-067	-077	- 063	- 062	00085	179	149	186	193	01944
Q38	070	062	-090	-103	- 086	- 085	00293	076	065	080	082	00175
Q39	074	074	-090	-103	- 088	- 089	00181	108	094	111	115	00397
Q40	056	069	-028	-027	- 021	- 019	00149	087	075	096	100	00188

[a] $n = 91$; 9 sample values removed due to p-value = 0.

Table C34

Item Indices for the Conditions of 80 Items, Positively Skewed Ability Distribution, Unit Discrimination, Sample Size of 250

Item	p	r_b	b_T	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_T	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	071	067	-083	-084	- 083	- 086	00237	100	092	091	090	00176
Q2	051	074	-008	-004	- 002	- 001	00095	100	099	110	111	00292
Q3	017	074	140	137	129	129	00310	100	102	109	117	00530
Q4	006	068	233	233	234	246	01933	100	105	093	095	00510
Q5	093	055	-213	-223	- 274	- 282	02394	100	091	066	066	00158
Q6	050	075	-003	-001	001	002	00127	100	100	112	113	00365
Q7	052	074	-013	-010	- 008	- 009	00084	100	099	109	109	00218
Q8	081	066	-129	-127	- 136	- 140	00323	100	099	087	086	00195
Q9	037	076	043	047	045	045	00098	100	101	116	118	00443
Q10	064	072	-054	-052	- 049	- 050	00163	100	099	103	104	00228
Q11	024	076	099	099	092	096	00176	100	103	116	115	00350
Q12	055	074	-021	-020	- 017	- 017	00111	100	101	111	113	00231
Q13	038	076	040	044	042	045	00112	100	100	115	117	00399
Q14	037	075	044	047	044	046	00139	100	100	115	119	00483
Q15	029	076	079	076	071	072	00134	100	103	118	123	00602
Q16	031	075	067	070	065	065	00111	100	100	115	117	00398
Q17	073	070	-091	-087	- 087	- 088	00206	100	100	097	098	00187
Q18	020	074	123	125	116	116	00263	100	100	110	113	00453
Q19	018	074	129	131	123	124	00251	100	101	109	113	00483
Q20	087	060	-161	-170	- 193	- 198	01244	100	092	074	075	00189
Q21	097	049	-262	-277	- 370	- 470	385168	100	089	056	058	00370
Q22	025	074	096	098	091	091	00189	100	099	111	115	00312
Q23	052	076	-011	-009	- 006	- 008	00117	100	106	118	118	00334
Q24	064	071	-052	-052	- 049	- 049	00121	100	096	100	100	00200
Q25	017	075	137	138	129	131	00296	100	104	112	116	00521
Q26	067	068	-066	-068	- 066	- 067	00178	100	092	094	096	00219
Q27	064	072	-055	-053	- 050	- 051	00179	100	101	105	105	00299
Q28	054	073	-019	-016	- 013	- 012	00099	100	097	106	106	00262
Q29	086	063	-158	-155	- 173	- 175	00685	100	097	080	082	00206
Q30	029	075	079	080	075	074	00140	100	100	114	120	00550
Q31	034	076	055	059	055	055	00107	100	101	116	118	00393
Q32	080	066	-122	-119	- 125	- 127	00307	100	098	088	088	00150
Q33	080	064	-119	-123	- 129	- 130	00341	100	093	084	085	00171
Q34	064	071	-055	-052	- 050	- 050	00146	100	098	101	103	00245
Q35	039	075	039	040	038	039	00119	100	099	114	114	00358
Q36	063	072	-053	-049	- 046	- 047	00165	100	098	103	105	00250
Q37	074	070	-096	-093	- 094	- 096	00181	100	103	098	099	00231
Q38	038	077	041	043	041	042	00083	100	104	119	121	00308
Q39	052	074	-011	-009	- 007	- 007	00113	100	099	110	110	00301
Q40	089	061	-173	-174	- 201	- 203	00916	100	099	077	078	00158

Table C34 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	016	072	139	144	135	136	00304	100	098	105	107	00357
Q42	028	077	081	082	076	078	00137	100	106	121	130	01134
Q43	014	072	157	157	149	152	00429	100	102	105	108	00485
Q44	084	063	-144	-146	- 159	- 164	00611	100	095	081	081	00157
Q45	077	068	-106	-104	- 107	- 107	00254	100	100	093	095	00231
Q46	082	065	-128	-129	- 138	- 139	00347	100	098	086	088	00184
Q47	028	075	085	086	079	082	00233	100	100	114	115	00419
Q48	015	072	147	151	141	142	00357	100	099	105	110	00508
Q49	051	073	-009	-006	- 004	- 005	00117	100	097	107	108	00249
Q50	058	073	-035	-032	- 029	- 029	00146	100	100	108	108	00262
Q51	076	068	-103	-103	- 106	- 106	00297	100	100	093	094	00214
Q52	073	068	-090	-090	- 091	- 091	00222	100	096	093	095	00223
Q53	053	074	-017	-014	- 012	- 012	00083	100	101	111	111	00303
Q54	029	076	078	079	073	076	00191	100	102	117	116	00413
Q55	048	075	005	006	008	007	00107	100	101	113	114	00404
Q56	054	073	-018	-017	- 014	- 015	00096	100	099	108	108	00293
Q57	058	073	-033	-031	- 028	- 028	00135	100	099	106	107	00316
Q58	086	063	-151	-155	- 173	- 176	00815	100	098	080	081	00215
Q59	011	070	176	181	173	174	00771	100	096	097	101	00483
Q60	067	071	-063	-063	- 061	- 061	00115	100	100	101	102	00220
Q61	053	074	-015	-013	- 011	- 011	00130	100	099	109	111	00227
Q62	028	075	079	082	076	077	00158	100	099	114	115	00360
Q63	047	075	009	009	010	010	00120	100	100	113	118	00274
Q64	041	075	031	031	030	031	00107	100	100	115	115	00315
Q65	037	075	042	045	042	042	00135	100	099	114	120	00467
Q66	020	074	120	120	112	113	00282	100	100	110	117	00677
Q67	056	072	-026	-023	- 020	- 021	00129	100	097	105	106	00276
Q68	035	076	052	053	050	050	00099	100	103	118	121	00421
Q69	088	060	-170	-175	- 200	- 207	00810	100	093	075	075	00136
Q70	066	071	-063	-062	- 059	- 058	00145	100	100	102	105	00207
Q71	034	075	055	058	054	055	00130	100	098	113	114	00323
Q72	028	074	083	085	079	080	00214	100	098	111	115	00500
Q73	025	076	093	096	089	091	00215	100	102	116	123	00750
Q74	083	063	-137	-141	- 153	- 158	00696	100	094	081	080	00200
Q75	089	058	-168	-180	- 207	- 209	00868	100	090	072	073	00151
Q76	062	071	-047	-045	- 042	- 044	00163	100	097	102	102	00247
Q77	076	067	-105	-104	- 107	- 110	00346	100	096	090	088	00174
Q78	061	072	-042	-041	- 038	- 036	00145	100	098	103	107	00299
Q79	050	075	-003	000	001	002	00106	100	102	114	120	00381
Q80	089	059	-170	-180	- 207	- 211	01402	100	091	073	074	00184

Table C35

Item Indices for the Conditions of 80 Items, Positively Skewed Ability Distribution, Unit Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}^2_b$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}^2_a$
Q1	065	075	-058	-055	- 052	- 050	00107	120	110	114	120	00371
Q2	029	058	099	105	097	099	00333	068	064	072	072	00156
Q3	054	056	-022	-019	- 018	- 016	00183	063	061	068	069	00098
Q4	075	074	-093	-091	- 092	- 092	00226	121	121	111	110	00193
Q5	041	075	026	031	030	031	00114	101	097	113	115	00563
Q6	064	075	-056	-052	- 048	- 048	00172	113	108	112	116	00345
Q7	050	057	-004	001	001	001	00189	064	062	069	070	00128
Q8	010	082	167	167	160	162	00615	145	147	141	149	01468
Q9	036	081	043	046	043	043	00106	114	117	140	146	00599
Q10	057	058	-033	-030	- 029	- 030	00153	067	065	071	072	00172
Q11	056	072	-029	-022	- 020	- 022	00102	094	095	105	106	00277
Q12	043	058	029	032	030	029	00222	063	063	071	072	00150
Q13[a]	029	094	060	064	058	059	00087	184	189	282	347	30004
Q14	026	087	077	083	075	075	00116	147	145	179	196	02492
Q15	072	087	-072	-071	- 069	- 069	00106	206	194	174	184	01292
Q16	074	091	-077	-074	- 072	- 072	00096	296	289	222	243	05321
Q17	034	080	052	056	051	053	00120	111	111	131	134	00584
Q18	043	071	024	026	024	026	00102	089	088	100	102	00301
Q19	066	075	-060	-059	- 055	- 058	00101	114	112	114	116	00313
Q20	067	055	-079	-082	- 080	- 081	00273	065	063	066	069	00149
Q21	038	070	047	048	045	045	00143	083	085	097	098	00256
Q22[b]	017	093	110	113	103	104	00130	204	205	249	273	11536
Q23[c]	041	096	020	024	024	023	00047	208	201	325	346	23390
Q24	037	072	049	051	047	048	00182	090	089	103	106	00374
Q25	045	061	015	021	020	020	00162	068	069	077	080	00150
Q26	055	076	-020	-018	- 015	- 016	00116	108	105	116	118	00327
Q27	084	054	-168	-170	- 185	- 191	01341	073	073	064	066	00162
Q28	043	074	024	027	025	026	00118	092	095	110	112	00351
Q29	009	071	194	192	185	188	01096	103	104	102	105	00592
Q30	036	087	040	045	042	042	00074	144	139	177	191	01827
Q31	048	086	002	006	007	008	00107	139	139	169	177	02135
Q32	044	091	014	016	017	017	00065	160	163	217	251	61340
Q33	032	046	104	106	098	105	00814	045	046	052	051	00106
Q34	017	084	122	124	114	116	00177	140	141	156	161	01494
Q35	021	067	132	128	119	119	00371	082	082	090	091	00251
Q36	092	059	-187	-195	- 236	- 239	02229	111	102	073	075	00189
Q37	027	074	092	091	083	083	00154	090	095	109	113	00291
Q38	050	079	-004	-002	000	000	00097	116	113	129	136	00573
Q39	042	075	023	028	027	026	00129	097	099	114	118	00497
Q40	018	083	116	118	108	109	00230	126	133	151	159	01156

Table C35 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\hat{\bar{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\hat{\bar{a}}_L$	$\hat{\sigma}_a^2$
Q41	035	073	059	059	055	055	00120	092	092	107	108	00368
Q42	057	073	-029	-028	- 025	- 025	00095	101	099	107	110	00290
Q43	058	053	-045	-039	- 036	- 036	00221	055	056	062	062	00109
Q44	055	078	-022	-020	- 017	- 017	00122	112	112	125	129	00582
Q45	051	077	-010	-007	- 004	- 005	00109	110	107	121	124	00375
Q46	089	065	-151	-159	- 185	- 185	00601	129	117	086	089	00199
Q47	082	037	-215	-233	- 243	- 259	05009	045	042	040	040	00089
Q48	009	072	195	195	188	190	00845	103	108	104	109	00823
Q49	023	072	111	110	101	102	00225	089	093	105	109	00393
Q50	067	044	-094	-101	- 099	- 101	00938	048	046	049	051	00123
Q51	031	092	059	060	055	056	00086	166	167	233	263	12146
Q52	059	067	-040	-035	- 033	- 034	00136	086	083	090	092	00176
Q53	028	060	109	106	097	099	00321	063	067	076	075	00123
Q54	084	063	-140	-143	- 156	- 165	00812	104	095	081	079	00185
Q55	057	083	-029	-024	- 021	- 020	00085	134	132	148	149	00626
Q56	059	040	-063	-062	- 059	- 060	00607	040	039	044	046	00107
Q57	051	089	-008	-006	- 003	- 004	00067	165	160	197	214	02471
Q58	060	077	-037	-037	- 033	- 034	00117	117	114	122	123	00371
Q59	041	057	041	042	039	039	00163	061	062	070	072	00150
Q60	071	083	-071	-069	- 066	- 066	00121	166	160	151	156	00686
Q61	055	067	-026	-022	- 020	- 019	00130	087	083	091	091	00171
Q62	081	050	-158	-163	- 173	- 178	01213	065	062	058	059	00111
Q63	086	072	-130	-135	- 152	- 153	00421	157	141	103	105	00171
Q64	082	073	-116	-117	- 126	- 127	00306	142	130	106	108	00212
Q65	039	080	037	039	036	036	00106	109	111	132	134	00639
Q66	083	074	-117	-121	- 131	- 132	00270	154	142	110	112	00222
Q67	025	049	143	143	134	138	01242	052	052	057	057	00157
Q68	043	050	041	037	034	036	00258	048	050	057	057	00128
Q69	009	078	181	181	175	177	00493	135	134	126	133	00725
Q70	031	078	069	071	065	065	00148	104	105	123	124	00489
Q71	071	073	-080	-078	- 076	- 078	00132	111	112	108	112	00367
Q72	083	068	-129	-130	- 141	- 143	00459	114	110	091	092	00221
Q73	041	071	032	034	032	031	00114	088	088	101	107	00276
Q74[d]	031	090	058	061	056	056	00089	150	154	202	219	04094
Q75	047	081	004	008	009	009	00086	120	118	139	143	00706
Q76	082	075	-111	-112	- 120	- 124	00292	154	141	114	114	00295
Q77	061	085	-040	-037	- 033	- 034	00090	146	147	160	168	01346
Q78	037	042	093	090	082	087	00834	040	040	046	045	00102
Q79	035	062	067	069	063	065	00304	068	069	079	081	00238
Q80	038	087	034	038	036	036	00093	139	140	177	184	01907

[a] $n = 96$; 4 sample values removed due to biserials ≥ 1.
[b] $n = 97$; 3 sample values removed due to biserials ≥ 1.
[c] $n = 93$; 7 sample values removed due to biserials ≥ 1.
[d] $n = 99$; 1 sample value removed due to biserial ≥ 1.

Table C36

Item Indices for the Conditions of 20 Items, Negatively Skewed Ability Distribution, Unit Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	041	076	037	036	032	031	00026	100	102	116	115	00062
Q2	070	080	-073	-074	- 065	-066	00036	100	099	133	134	00168
Q3	085	080	-149	-148	- 129	-129	00062	100	103	135	138	00331
Q4	092	080	-210	-200	- 177	-179	00126	100	107	132	132	00296
Q5	070	081	-072	-073	- 064	-065	00030	100	103	138	139	00141
Q6	056	079	-018	-019	- 018	-019	00021	100	102	128	129	00120
Q7	036	073	058	053	049	048	00024	100	098	108	109	00042
Q8	016	063	143	154	160	160	00147	100	089	081	080	00033
Q9	091	080	-196	-187	- 164	-167	00167	100	105	132	134	00431
Q10	055	078	-014	-015	- 015	-016	00021	100	100	125	125	00086
Q11	039	074	044	043	039	038	00033	100	098	110	110	00049
Q12	060	079	-035	-033	- 030	-031	00024	100	101	130	131	00111
Q13	097	076	-296	-274	- 257	-259	00641	100	114	116	121	00504
Q14	089	078	-180	-179	- 156	-158	00142	100	100	126	127	00235
Q15	068	080	-066	-067	- 059	-059	00022	100	102	136	138	00167
Q16	046	075	018	017	014	014	00025	100	095	113	114	00052
Q17	080	081	-118	-118	- 102	-101	00029	100	101	137	140	00223
Q18	087	081	-161	-157	- 137	-138	00064	100	105	137	137	00299
Q19	052	078	-004	-006	- 007	-007	00024	100	100	124	126	00078
Q20	046	077	017	016	013	013	00023	100	101	119	119	00049

Table C37

Item Indices for the Conditions of 20 Items, Negatively Skewed Ability Distribution, Variable Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	043	075	029	027	023	023	00033	095	091	113	115	00069
Q2	047	080	015	013	010	009	00024	106	104	133	132	00071
Q3	040	068	048	044	038	038	00038	075	074	092	094	00044
Q4	071	088	-067	-072	- 063	-063	00020	140	135	181	185	00337
Q5	051	078	004	-002	- 002	-001	00024	096	097	125	125	00069
Q6	003	053	258	300	345	351	01510	097	082	062	062	00062
Q7	053	081	-007	-011	- 011	-010	00019	106	106	138	140	00125
Q8	058	094	-017	-023	- 021	-021	00017	174	175	282	283	02262
Q9	076	080	-099	-101	- 089	-089	00039	111	108	133	135	00126
Q10	042	074	032	031	027	027	00027	094	089	110	111	00051
Q11	059	084	-027	-031	- 028	-028	00020	116	117	157	161	00233
Q12	056	077	-016	-021	- 018	-018	00024	095	093	122	123	00104
Q13	012	062	179	190	190	191	00216	088	082	079	079	00036
Q14	034	076	060	059	053	053	00038	102	098	116	116	00075
Q15	052	069	-004	-007	- 006	-006	00028	074	073	094	095	00043
Q16	041	087	034	031	026	026	00021	149	141	175	179	00285
Q17	079	063	-147	-152	- 131	-131	00123	064	066	080	081	00058
Q18	069	080	-065	-069	- 060	-061	00032	103	103	134	134	00148
Q19	089	071	-191	-186	- 173	-173	00150	093	095	101	102	00091
Q20	009	063	179	201	211	212	00235	112	092	081	081	00044

Table C38

Item Indices for the Conditions of 40 Items, Negatively Skewed Ability Distribution, Unit Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	049	076	007	006	003	003	00030	100	089	116	115	00064
Q2	057	074	-020	-029	- 025	- 024	00031	100	083	111	110	00090
Q3	022	067	110	124	113	116	00092	100	085	090	089	00045
Q4	058	077	-028	-032	- 027	- 028	00027	100	090	121	120	00092
Q5	053	077	-009	-012	- 011	- 011	00027	100	091	122	123	00073
Q6	027	069	093	104	091	093	00061	100	085	094	094	00051
Q7	057	077	-020	-025	- 022	- 022	00027	100	090	119	119	00087
Q8	090	074	-180	-197	- 170	- 171	00119	100	097	109	109	00114
Q9	029	069	080	091	079	080	00043	100	085	096	097	00034
Q10	088	074	-177	-186	- 159	- 160	00152	100	094	109	109	00176
Q11	058	076	-025	-030	- 026	- 027	00033	100	087	117	117	00070
Q12	019	065	125	144	136	135	00096	100	086	086	087	00047
Q13	098	065	-298	-309	- 304	- 305	00938	100	100	085	087	00131
Q14	062	078	-041	-047	- 039	- 039	00025	100	091	123	124	00098
Q15	057	077	-019	-025	- 022	- 021	00025	100	090	121	122	00101
Q16	056	078	-016	-021	- 019	- 018	00022	100	093	124	124	00080
Q17	056	077	-016	-021	- 019	- 018	00024	100	092	123	124	00077
Q18	055	077	-014	-017	- 016	- 015	00022	100	090	120	120	00074
Q19	048	075	011	012	008	008	00021	100	088	114	114	00064
Q20	094	069	-238	-251	- 228	- 226	00224	100	097	096	099	00110
Q21	053	077	-005	-009	- 009	- 009	00022	100	092	121	122	00082
Q22	082	076	-136	-145	- 121	- 121	00069	100	094	118	118	00134
Q23	044	074	025	028	021	022	00034	100	087	109	108	00060
Q24	071	078	-075	-084	- 070	- 069	00029	100	093	123	122	00101
Q25	020	066	122	136	126	128	00096	100	084	087	086	00048
Q26	064	077	-049	-056	- 046	- 046	00025	100	090	121	122	00084
Q27	025	068	096	109	097	098	00055	100	086	093	093	00047
Q28	085	075	-150	-162	- 136	- 135	00078	100	092	113	115	00118
Q29	060	077	-033	-040	- 034	- 033	00025	100	091	123	124	00086
Q30	068	079	-064	-074	- 061	- 061	00034	100	094	128	128	00146
Q31	005	054	228	270	297	302	00591	100	081	065	064	00033
Q32	009	058	186	220	231	234	00313	100	084	072	072	00044
Q33	034	072	061	066	055	057	00035	100	088	104	104	00053
Q34	070	078	-077	-083	- 069	- 068	00038	100	093	125	125	00108
Q35	038	074	049	052	042	043	00039	100	090	110	110	00049
Q36	075	078	-094	-103	- 085	- 084	00029	100	094	125	128	00146
Q37	065	077	-051	-058	- 049	- 048	00026	100	091	122	124	00111
Q38	028	069	086	095	083	083	00049	100	085	095	095	00054
Q39	045	075	021	022	016	018	00036	100	088	112	111	00070
Q40	025	067	099	112	101	103	00061	100	083	090	089	00038

Table C39

Item Indices for the Conditions of 40 Items, Negatively Skewed Ability Distribution, Variable Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	038	083	043	045	036	036	00023	148	130	151	151	00101
Q2	062	055	-063	-066	-055	-055	00057	055	050	065	065	00027
Q3	046	079	017	017	012	011	00024	109	102	128	130	00097
Q4	064	073	-051	-055	-048	-048	00037	088	082	107	109	00056
Q5	012	062	168	188	194	195	00185	106	091	078	079	00045
Q6	077	066	-120	-136	-114	-114	00086	079	069	088	089	00070
Q7	032	071	071	077	066	067	00037	102	091	101	101	00045
Q8	097	068	-281	-296	-281	-281	00849	101	099	092	094	00187
Q9	080	070	-131	-140	-118	-118	00060	083	077	098	098	00058
Q10	037	083	047	051	041	041	00021	152	130	147	148	00097
Q11[a]	086	095	-127	-137	-116	-116	00035	218	205	304	334	09224
Q12	092	085	-181	-191	-167	-166	00079	145	145	164	169	00495
Q13	042	046	049	057	045	045	00066	042	039	051	052	00017
Q14	032	057	086	097	084	083	00063	067	060	070	071	00026
Q15	061	076	-037	-043	-038	-038	00029	098	089	116	117	00071
Q16	027	051	130	142	124	123	00144	057	050	059	059	00024
Q17	070	082	-069	-076	-065	-066	00029	112	104	141	141	00165
Q18	076	087	-091	-098	-083	-083	00033	130	125	177	179	00386
Q19	027	061	100	111	098	099	00074	076	070	078	078	00034
Q20	044	053	038	038	030	030	00055	051	049	062	063	00025
Q21	053	082	-005	-008	-009	-010	00022	119	110	143	144	00119
Q22	049	074	009	007	003	003	00023	095	085	109	109	00069
Q23	088	056	-229	-242	-209	-211	00462	060	058	068	068	00066
Q24	059	068	-032	-039	-034	-035	00035	083	072	094	094	00051
Q25	089	084	-167	-175	-149	-150	00071	130	125	154	156	00317
Q26	049	082	009	009	005	004	00019	124	113	144	144	00119
Q27	060	072	-031	-040	-035	-036	00029	090	080	104	103	00052
Q28	082	058	-175	-188	-157	-157	00142	061	058	072	073	00039
Q29	088	073	-172	-186	-158	-158	00108	091	087	106	107	00110
Q30	080	084	-110	-118	-099	-100	00044	121	114	153	154	00254
Q31	017	054	166	184	175	174	00238	070	063	064	065	00032
Q32	067	065	-075	-082	-069	-070	00050	070	066	087	086	00041
Q33	037	087	047	049	040	040	00015	178	158	177	178	00229
Q34	034	071	060	066	056	056	00040	103	089	102	103	00050
Q35	058	057	-039	-040	-035	-036	00051	058	054	070	070	00026
Q36	009	052	210	243	254	257	00461	083	070	061	061	00031
Q37	017	047	198	223	208	207	00325	056	050	053	054	00020
Q38	059	085	-028	-029	-027	-027	00022	126	117	160	160	00160
Q39	028	069	084	093	082	082	00058	102	088	096	096	00048
Q40	053	075	-008	-011	-011	-011	00025	095	087	113	114	00079

[a] $n = 99$; 1 sample value removed due to biserial ≥ 1.

Table C40

Item Indices for the Conditions of 80 Items, Negatively Skewed Ability Distribution, Unit Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}^2_b$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}^2_a$
Q1	033	071	066	063	060	060	00038	100	099	100	102	00050
Q2	019	065	127	123	131	132	00092	100	098	087	086	00043
Q3	065	077	-050	-053	- 049	- 050	00033	100	105	122	121	00082
Q4	038	072	046	044	041	041	00030	100	099	104	103	00055
Q5	068	077	-062	-064	- 060	- 061	00037	100	104	120	120	00081
Q6	031	070	073	073	071	071	00044	100	097	097	097	00043
Q7	058	076	-025	-027	- 027	- 027	00028	100	103	118	117	00072
Q8	045	074	024	020	017	017	00028	100	100	109	109	00057
Q9	011	060	177	175	204	207	00222	100	098	076	075	00035
Q10	065	076	-050	-053	- 049	- 050	00030	100	102	118	119	00091
Q11	087	071	-166	-167	- 160	- 161	00130	100	100	101	101	00096
Q12	052	075	-002	-006	- 007	- 007	00029	100	100	113	113	00077
Q13	042	074	032	029	026	027	00036	100	103	111	110	00062
Q14	035	071	059	057	055	054	00042	100	098	100	101	00050
Q15	077	076	-103	-104	- 096	- 096	00049	100	105	119	119	00095
Q16	025	069	101	097	099	100	00056	100	101	095	094	00044
Q17	004	051	254	267	358	364	02096	100	093	059	059	00062
Q18	079	074	-119	-119	- 110	- 111	00064	100	101	112	112	00100
Q19	072	076	-084	-083	- 077	- 077	00049	100	103	117	118	00081
Q20	067	077	-059	-060	- 056	- 058	00034	100	105	121	120	00084
Q21	045	074	023	020	018	017	00033	100	099	109	108	00070
Q22	068	077	-061	-066	- 061	- 061	00032	100	105	121	122	00088
Q23	005	050	232	261	337	337	01264	100	086	058	059	00048
Q24	057	075	-022	-025	- 025	- 025	00024	100	100	114	114	00081
Q25	073	075	-085	-089	- 082	- 082	00045	100	100	114	116	00080
Q26	075	074	-094	-099	- 091	- 094	00052	100	098	110	108	00068
Q27	081	073	-129	-129	- 120	- 122	00086	100	100	108	108	00089
Q28	076	074	-098	-104	- 096	- 097	00045	100	098	109	111	00075
Q29	054	076	-008	-011	- 012	- 013	00034	100	104	119	118	00085
Q30	033	071	062	062	060	059	00028	100	101	102	102	00062
Q31	044	075	026	023	020	020	00030	100	103	112	112	00071
Q32	015	061	147	151	167	166	00161	100	092	078	079	00042
Q33	062	075	-038	-041	- 039	- 040	00030	100	099	114	114	00076
Q34	028	070	087	085	085	086	00053	100	101	097	097	00054
Q35	062	075	-042	-043	- 041	- 042	00022	100	099	115	114	00078
Q36	094	070	-224	-219	- 218	- 220	00367	100	108	097	098	00106
Q37	040	073	037	036	033	033	00029	100	100	107	106	00071
Q38	038	072	046	044	041	041	00032	100	099	104	105	00065
Q39	064	076	-045	-048	- 046	- 046	00026	100	103	119	120	00092
Q40	033	070	067	066	064	063	00036	100	096	098	098	00052

Table C40 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	047	073	014	011	009	009	00027	100	097	108	108	00064
Q42	035	071	061	058	055	055	00040	100	099	102	102	00053
Q43	053	075	-005	-008	- 009	- 009	00026	100	100	113	114	00088
Q44	092	070	-201	-201	- 197	- 198	00175	100	104	097	098	00073
Q45	095	066	-253	-241	- 247	- 247	00485	100	105	089	090	00120
Q46	012	061	168	168	193	193	00218	100	098	077	078	00046
Q47	046	075	016	015	013	012	00021	100	103	114	114	00064
Q48	094	068	-232	-227	- 227	- 225	00275	100	102	092	095	00096
Q49	047	074	016	012	010	009	00024	100	100	111	111	00087
Q50	077	075	-107	-109	- 101	- 102	00055	100	100	112	111	00102
Q51	042	072	037	032	030	030	00039	100	096	103	102	00052
Q52	036	072	056	054	051	051	00036	100	099	103	102	00046
Q53	071	076	-078	-079	- 073	- 073	00034	100	103	118	119	00135
Q54	051	076	-001	-002	- 003	- 004	00030	100	104	118	118	00049
Q55	022	067	113	110	115	115	00061	100	098	090	089	00031
Q56	065	075	-049	-054	- 050	- 050	00025	100	099	114	116	00064
Q57	030	071	082	077	076	075	00050	100	101	100	100	00060
Q58	009	058	198	196	237	240	00335	100	097	071	071	00039
Q59	042	072	030	031	028	028	00031	100	096	104	104	00063
Q60	017	064	133	137	149	149	00145	100	098	084	085	00061
Q61	014	061	158	159	178	180	00152	100	093	077	077	00035
Q62	038	072	045	045	042	041	00042	100	098	103	103	00069
Q63	034	069	061	062	060	059	00038	100	094	096	096	00050
Q64	069	075	-064	-069	- 064	- 065	00037	100	100	114	115	00090
Q65	035	071	062	058	055	055	00044	100	098	101	101	00050
Q66	078	074	-112	-115	- 106	- 108	00060	100	099	109	109	00098
Q67	030	070	078	074	073	072	00049	100	100	099	099	00049
Q68	030	070	078	076	075	075	00038	100	099	098	099	00053
Q69	079	074	-115	-117	- 108	- 109	00052	100	101	111	111	00096
Q70	035	071	060	059	056	056	00041	100	097	100	100	00053
Q71	072	077	-080	-080	- 074	- 074	00041	100	105	121	121	00095
Q72	027	069	092	090	091	091	00073	100	099	095	094	00050
Q73	059	076	-031	-033	- 032	- 032	00031	100	101	116	116	00070
Q74	063	074	-043	-047	- 044	- 046	00027	100	097	112	112	00096
Q75	045	072	024	020	018	017	00026	100	096	105	107	00067
Q76	028	067	085	087	087	088	00053	100	092	090	090	00031
Q77	021	067	120	117	123	125	00079	100	100	090	089	00040
Q78	013	062	161	163	186	188	00165	100	098	078	078	00038
Q79	058	075	-023	-027	- 027	- 027	00028	100	099	113	113	00077
Q80	011	059	176	182	212	215	00346	100	094	073	073	00043

Table C41

Item Indices for the Conditions of 80 Items, Negatively Skewed Ability Distribution, Variable Discrimination, Sample Size of 1,000

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	026	076	085	082	083	083	00044	129	128	117	118	00084
Q2	040	067	044	041	039	039	00036	088	085	090	089	00043
Q3	050	066	004	-001	- 001	- 002	00029	076	080	088	088	00046
Q4	067	055	-083	-084	- 077	- 079	00071	060	059	067	066	00038
Q5	016	052	173	173	189	190	00173	071	068	061	061	00025
Q6	083	076	-133	-135	- 126	- 126	00075	109	108	116	118	00108
Q7	075	077	-089	-093	- 086	- 087	00041	104	106	121	121	00091
Q8	031	083	064	063	060	060	00024	156	153	148	149	00128
Q9	040	056	049	046	044	044	00061	062	064	068	068	00030
Q10	040	066	045	043	040	040	00036	083	082	087	087	00033
Q11	034	055	077	078	077	077	00080	061	062	065	065	00026
Q12	066	066	-066	-066	- 062	- 062	00045	075	078	088	089	00042
Q13	046	066	020	017	016	016	00029	083	080	087	087	00040
Q14	031	095	061	058	053	054	00017	332	324	297	300	02005
Q15	018	051	162	168	180	181	00237	068	064	059	060	00028
Q16	075	081	-086	-089	- 082	- 083	00039	125	121	141	141	00185
Q17	060	084	-028	-030	- 030	- 030	00026	126	130	155	155	00206
Q18	053	071	-005	-011	- 011	- 011	00025	087	089	100	100	00047
Q19	008	045	250	265	319	323	01284	070	065	050	050	00039
Q20	093	066	-226	-226	- 222	- 227	00376	091	094	088	088	00090
Q21	032	067	073	070	069	069	00048	088	088	089	091	00040
Q22	058	075	-022	-027	- 026	- 026	00024	097	099	113	114	00074
Q23	079	072	-122	-124	- 115	- 116	00078	091	093	102	102	00084
Q24	020	074	108	109	116	116	00063	141	129	109	109	00052
Q25	024	047	149	146	151	154	00254	057	055	054	053	00025
Q26	078	056	-146	-145	- 136	- 136	00112	061	063	068	068	00037
Q27	066	058	-075	-077	- 071	- 071	00061	065	064	072	073	00035
Q28	067	061	-078	-078	- 072	- 072	00058	066	068	077	076	00042
Q29	001	052	273	284	423	435	03110	131	119	061	060	00051
Q30	062	089	-033	-036	- 035	- 035	00020	148	155	195	199	00434
Q31	040	065	041	041	039	039	00028	081	081	086	087	00040
Q32	001	049	288	305	457	485	09989	121	111	057	056	00069
Q33	032	043	114	110	109	110	00150	045	045	048	047	00018
Q34	075	056	-131	-130	- 120	- 120	00128	059	061	067	068	00041
Q35	070	082	-071	-071	- 066	- 066	00029	118	121	143	143	00123
Q36	028	053	111	106	107	108	00102	060	062	063	063	00026
Q37	066	059	-070	-073	- 068	- 068	00052	066	066	074	074	00037
Q38[a]	085	095	-115	-115	- 107	- 107	00021	248	261	309	328	06688
Q39	083	068	-149	-150	- 140	- 142	00102	085	087	094	093	00064
Q40	027	071	085	086	087	088	00046	115	107	101	100	00049

Table C41(cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	033	052	085	085	084	084	00091	058	058	061	062	00023
Q42	023	078	094	093	095	096	00042	146	142	123	122	00073
Q43	091	082	-168	-169	- 162	- 163	00080	148	150	143	145	00221
Q44	059	053	-037	-045	- 041	- 041	00059	055	055	063	063	00029
Q45	040	085	036	032	028	029	00019	149	152	163	165	00216
Q46	034	055	076	077	075	076	00076	062	063	066	065	00030
Q47	047	060	016	014	013	013	00041	065	069	075	076	00034
Q48	045	069	022	019	017	016	00040	088	089	096	096	00042
Q49	061	075	-034	-038	- 036	- 036	00027	102	099	114	115	00089
Q50	020	084	098	095	100	100	00031	223	206	155	155	00163
Q51	087	089	-134	-134	- 126	- 126	00047	185	188	198	201	00677
Q52	056	064	-018	-023	- 022	- 022	00052	073	075	084	085	00043
Q53	066	087	-047	-049	- 046	- 047	00020	137	142	174	175	00360
Q54	024	075	096	092	094	094	00049	123	124	112	113	00074
Q55	058	058	-038	-037	- 035	- 034	00035	061	063	071	070	00036
Q56	020	060	132	133	141	142	00132	088	084	076	076	00033
Q57	078	060	-142	-139	- 129	- 131	00136	066	068	074	074	00039
Q58	041	073	037	034	031	031	00035	100	101	107	108	00054
Q59	044	072	024	024	021	021	00032	094	096	104	104	00057
Q60	051	036	-007	-011	- 009	- 010	00132	035	034	039	039	00019
Q61	049	080	010	006	004	005	00023	120	118	132	133	00085
Q62	063	089	-035	-038	- 036	- 036	00020	150	158	200	204	00565
Q63	066	058	-076	-078	- 072	- 073	00050	062	064	072	071	00031
Q64	058	093	-018	-020	- 021	- 021	00017	183	186	254	259	01219
Q65	068	081	-062	-063	- 058	- 059	00032	112	117	138	137	00153
Q66	069	051	-102	-108	- 099	- 100	00143	054	052	059	059	00032
Q67	022	061	116	119	124	124	00092	088	083	077	077	00029
Q68	051	074	001	-003	- 004	- 004	00025	099	097	109	109	00059
Q69	070	078	-069	-072	- 066	- 067	00034	106	108	124	123	00081
Q70	049	074	007	006	005	005	00024	097	100	110	110	00059
Q71	030	084	066	065	063	062	00025	169	160	152	155	00150
Q72	072	071	-088	-090	- 084	- 084	00051	089	090	101	101	00069
Q73	032	066	077	073	072	072	00045	090	087	088	088	00038
Q74	015	057	161	164	180	181	00231	082	079	069	069	00037
Q75	080	094	-095	-098	- 090	- 090	00026	204	208	269	284	02547
Q76	010	063	172	174	206	208	00160	120	110	081	081	00032
Q77	040	049	059	053	051	052	00089	054	053	057	057	00025
Q78	067	063	-074	-076	- 071	- 071	00054	072	072	080	081	00034
Q79	034	062	073	069	067	067	00053	073	076	078	078	00029
Q80	047	074	017	013	011	011	00031	100	101	111	111	00061

[a] $n = 99$; 1 sample value removed due to biserial ≥ 1.

Table C42

Item Indices for the Conditions of 20 Items, Negatively Skewed Ability Distribution, Unit Discrimination, Sample Size of 500

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	022	069	115	118	114	117	00150	100	093	095	095	00091
Q2	053	079	-007	-010	- 011	-010	00038	100	101	129	129	00229
Q3	057	079	-021	-025	- 024	-023	00035	100	100	129	133	00253
Q4	003	055	255	285	339	346	02583	100	092	065	066	00116
Q5	021	068	116	121	117	120	00171	100	092	094	094	00107
Q6	013	065	162	171	178	183	00316	100	095	085	085	00112
Q7	060	080	-034	-035	- 032	-033	00052	100	101	132	131	00216
Q8	039	076	045	044	038	040	00058	100	100	116	117	00105
Q9	071	080	-076	-076	- 068	-067	00062	100	102	134	136	00311
Q10	065	080	-055	-055	- 049	-048	00053	100	102	134	133	00308
Q11	087	079	-165	-158	- 143	-144	00227	100	108	128	127	00312
Q12	049	078	009	006	003	004	00058	100	101	126	127	00219
Q13	052	079	-002	-004	- 005	-004	00040	100	101	129	130	00259
Q14	062	079	-038	-044	- 040	-038	00053	100	100	130	132	00233
Q15	091	075	-193	-191	- 176	-176	00313	100	102	114	116	00266
Q16	048	078	012	010	007	009	00056	100	100	124	125	00149
Q17	046	076	021	018	014	016	00045	100	097	118	119	00192
Q18	093	072	-220	-219	- 206	-208	00540	100	100	105	106	00348
Q19	067	080	-060	-061	- 055	-054	00050	100	100	131	131	00253
Q20	080	081	-118	-117	- 104	-103	00098	100	107	136	140	00334

Table C43

Item Indices for the Conditions of 20 Items, Negatively Skewed Ability Distribution, Variable Discrimination, Sample Size of 500

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	074	071	-106	-105	- 091	-092	00093	076	080	100	103	00172
Q2	090	086	-154	-155	- 149	-149	00150	177	175	165	173	00881
Q3	026	090	076	073	071	072	00046	247	220	203	205	01061
Q4	050	065	-002	-002	- 002	-002	00081	065	065	085	085	00087
Q5	083	050	-212	-226	- 188	-199	01076	050	046	057	056	00094
Q6	042	060	043	040	033	033	00110	062	057	074	076	00068
Q7	072	094	-064	-067	- 063	-063	00040	205	218	282	311	09799
Q8	069	056	-103	-109	- 090	-093	00180	053	051	068	067	00080
Q9	097	075	-255	-246	- 260	-264	00975	133	150	112	115	00340
Q10	016	071	132	140	138	140	00138	120	104	102	102	00083
Q11	044	061	031	032	027	027	00088	061	059	077	078	00077
Q12	024	064	118	122	110	113	00153	075	072	084	084	00088
Q13	043	091	028	022	018	018	00040	184	175	217	224	01918
Q14	031	064	088	088	077	077	00093	072	068	083	084	00071
Q15	055	069	-023	-022	- 019	-018	00068	074	073	096	097	00157
Q16	072	078	-084	-086	- 076	-077	00074	095	099	123	124	00247
Q17	047	089	015	010	008	008	00034	149	151	190	192	00715
Q18	038	077	046	044	039	039	00060	107	100	121	122	00188
Q19	059	073	-033	-036	- 032	-032	00060	083	083	107	108	00173
Q20	006	052	239	290	298	311	02042	083	066	061	060	00086

Table C44

Item Indices for the Conditions of 40 Items, Negatively Skewed Ability Distribution, Unit Discrimination, Sample Size of 500

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	030	069	076	085	076	077	00076	100	088	095	096	00080
Q2	039	071	043	046	038	038	00091	100	086	102	101	00114
Q3	065	078	-050	-056	- 048	- 048	00062	100	093	125	129	00255
Q4	019	064	126	142	139	138	00192	100	086	083	083	00059
Q5	062	078	-041	-044	- 039	- 039	00052	100	095	125	128	00260
Q6	002	046	294	355	459	487	15508	100	081	051	051	00144
Q7	080	077	-122	-133	- 110	- 110	00113	100	092	122	127	00376
Q8	059	078	-029	-031	- 028	- 028	00060	100	095	125	127	00223
Q9	061	076	-035	-042	- 036	- 037	00059	100	088	115	116	00140
Q10	038	073	051	050	042	042	00075	100	093	108	108	00112
Q11	087	079	-161	-170	- 142	- 144	00203	100	100	127	128	00404
Q12	029	069	078	087	078	078	00101	100	088	095	098	00086
Q13	063	077	-045	-050	- 043	- 042	00075	100	090	121	122	00234
Q14	011	059	170	197	208	211	00478	100	085	073	073	00064
Q15	034	071	066	069	060	060	00100	100	089	100	100	00098
Q16	085	076	-152	-164	- 136	- 137	00201	100	091	117	121	00441
Q17	091	075	-202	-212	- 181	- 182	00319	100	097	114	118	00441
Q18	072	078	-082	-090	- 075	- 075	00090	100	094	126	128	00324
Q19	044	073	028	027	021	021	00077	100	089	107	107	00107
Q20	080	077	-119	-128	- 107	- 107	00104	100	093	122	126	00339
Q21	051	074	001	-001	- 003	- 002	00073	100	087	110	111	00110
Q22	072	077	-083	-093	- 077	- 076	00075	100	091	122	124	00211
Q23	084	078	-144	-154	- 128	- 129	00117	100	096	126	129	00411
Q24	073	077	-084	-096	- 080	- 080	00072	100	090	119	120	00262
Q25	082	077	-130	-141	- 116	- 118	00086	100	093	122	125	00318
Q26	094	073	-235	-248	- 217	- 223	00752	100	097	106	108	00502
Q27	053	076	-008	-009	- 010	- 010	00055	100	091	117	120	00207
Q28	035	072	057	062	053	053	00075	100	093	105	105	00079
Q29	094	072	-231	-249	- 218	- 221	00651	100	093	103	105	00406
Q30	011	060	173	196	209	211	00773	100	089	074	075	00089
Q31	055	076	-012	-017	- 016	- 017	00057	100	092	118	119	00173
Q32	082	077	-136	-146	- 121	- 121	00138	100	093	121	123	00344
Q33	011	060	173	190	202	205	00356	100	090	076	076	00063
Q34	034	071	058	067	058	057	00075	100	090	101	102	00148
Q35	068	077	-065	-073	- 061	- 061	00073	100	091	122	126	00289
Q36	012	059	170	192	202	204	00697	100	085	073	073	00081
Q37	051	076	001	-003	- 004	- 005	00051	100	091	115	117	00139
Q38	057	076	-022	-027	- 024	- 024	00050	100	089	117	116	00165
Q39	061	076	-036	-043	- 037	- 038	00054	100	089	117	117	00163
Q40	051	075	002	-002	- 004	- 004	00058	100	091	115	117	00190

Table C45

Item Indices for the Conditions of 40 Items, Negatively Skewed Ability Distribution, Variable Discrimination, Sample Size of 500

Item	p	r_b	b_τ	b_B	b_L	$\hat{\bar{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\hat{\bar{a}}_L$	$\hat{\sigma}_a^2$
Q1	040	060	048	052	043	043	00104	148	061	074	076	00078
Q2	072	089	-072	-082	- 067	- 067	00047	055	123	198	213	01944
Q3	049	076	011	009	004	004	00058	109	091	116	117	00164
Q4	056	081	-018	-020	- 019	- 019	00052	088	100	137	138	00214
Q5[a]	060	097	-021	-026	- 025	- 026	00030	106	183	403	508	92189
Q6	024	047	140	169	146	149	00307	079	046	054	054	00043
Q7	067	087	-054	-061	- 051	- 051	00039	102	115	175	183	01119
Q8	077	072	-115	-128	- 103	- 105	00116	101	077	105	106	00216
Q9	047	084	014	016	010	009	00041	083	124	158	162	00414
Q10	057	081	-023	-024	- 022	- 023	00047	152	102	139	144	00212
Q11	072	080	-082	-089	- 073	- 073	00057	218	095	135	135	00506
Q12	022	051	143	167	151	152	00302	145	054	060	059	00039
Q13	025	034	218	241	198	212	04333	042	029	037	036	00042
Q14	096	068	-256	-284	- 250	- 258	02523	067	088	093	096	00423
Q15	041	052	053	055	044	045	00156	098	047	060	061	00052
Q16	047	061	020	018	013	013	00081	057	060	078	077	00080
Q17	035	044	097	112	090	090	00240	112	038	049	050	00039
Q18[b]	073	100	-068	-076				130	188			
Q19	096	078	-240	-251	- 219	- 221	00749	076	119	126	133	01122
Q20[c]	080	094	-102	-112	- 089	- 089	00040	051	143	264	312	16108
Q21	079	070	-125	-140	- 113	- 114	00135	119	074	099	101	00165
Q22	019	061	132	149	143	143	00214	095	080	078	078	00056
Q23	034	078	056	062	052	051	00045	060	116	124	126	00129
Q24	026	069	092	103	093	093	00093	083	093	095	096	00086
Q25	081	086	-115	-126	- 101	- 100	00068	130	114	171	190	01626
Q26	059	092	-020	-024	- 023	- 023	00034	124	143	229	248	02586
Q27	065	082	-050	-055	- 047	- 047	00055	090	101	145	151	00509
Q28	029	061	091	103	090	090	00145	061	067	076	077	00066
Q29	053	071	-006	-009	- 010	- 011	00051	091	077	101	105	00107
Q30	003	058	199	244	314	321	01432	121	132	072	072	00072
Q31[d]	085	100	-118	-134				070	214			
Q32	082	062	-161	-181	- 145	- 146	00309	070	061	080	082	00167
Q33	087	077	-163	-183	- 149	- 150	00216	178	093	121	126	00480
Q34	024	074	092	101	095	094	00071	103	122	111	112	00097
Q35	058	084	-021	-024	- 023	- 024	00055	058	109	153	157	00450
Q36	037	083	046	048	038	038	00056	083	138	151	154	00246
Q37	088	075	-177	-191	- 156	- 155	00210	056	087	113	118	00361
Q38	065	064	-064	-071	- 058	- 057	00090	126	062	084	085	00100
Q39	053	059	-017	-015	- 014	- 015	00089	102	056	074	075	00062
Q40	006	044	270	318	349	356	03468	095	060	049	050	00068

[a] $n = 92$; 8 sample values removed due to biserials ≥ 1.
[b] $r_b = 1$; population Lertap values not reported and sample Lertap values not computed due to 54 biserials ≥ 1.
[c] $n = 98$; 2 sample values removed due to biserials ≥ 1.
[d] $r_b = 1$; population Lertap values not reported and sample Lertap values not computed due to 50 biserials ≥ 1.

Table C46

Item Indices for the Conditions of 80 Items, Negatively Skewed Ability Distribution, Unit Discrimination, Sample Size of 500

Item	p	r_b	b_T	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}^2_b$	a_T	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}^2_a$
Q1	033	071	068	066	064	064	00090	100	099	100	102	00130
Q2	059	076	-027	-031	- 029	- 029	00059	100	101	117	120	00191
Q3	078	073	-107	-112	- 103	- 105	00137	100	098	108	110	00202
Q4	022	067	111	109	113	111	00136	100	098	091	093	00120
Q5	032	071	074	070	067	069	00083	100	100	102	100	00097
Q6	084	072	-142	-147	- 138	- 139	00143	100	100	104	105	00211
Q7	058	077	-028	-028	- 027	- 028	00057	100	103	119	121	00184
Q8	043	074	030	026	023	023	00060	100	099	109	110	00152
Q9	030	071	079	075	074	073	00088	100	101	101	102	00102
Q10	027	068	090	091	091	092	00126	100	097	094	094	00117
Q11	041	074	035	033	030	030	00064	100	100	108	109	00131
Q12	056	076	-017	-022	- 021	- 021	00054	100	101	117	120	00209
Q13	043	073	029	026	023	023	00057	100	098	108	110	00161
Q14	061	074	-036	-040	- 038	- 038	00072	100	094	109	110	00186
Q15	069	077	-069	-068	- 063	- 063	00078	100	104	120	122	00220
Q16	070	076	-074	-074	- 068	- 068	00060	100	102	118	120	00187
Q17	036	072	056	053	050	050	00088	100	100	104	105	00125
Q18	045	073	020	019	017	017	00060	100	097	107	108	00134
Q19	071	074	-077	-081	- 075	- 076	00083	100	098	111	113	00229
Q20	063	075	-045	-048	- 045	- 045	00082	100	098	114	115	00174
Q21	018	066	137	132	142	145	00267	100	099	087	087	00094
Q22	019	065	124	126	133	136	00229	100	094	085	083	00086
Q23	026	068	098	093	094	095	00131	100	098	094	094	00097
Q24	038	071	045	045	042	042	00055	100	094	100	101	00106
Q25	035	072	059	056	053	053	00075	100	099	103	104	00144
Q26	021	066	119	119	125	127	00205	100	096	087	087	00086
Q27	065	075	-054	-054	- 050	- 049	00061	100	099	115	117	00168
Q28	086	072	-157	-158	- 150	- 150	00221	100	103	105	107	00239
Q29	080	075	-118	-119	- 110	- 108	00146	100	104	114	119	00222
Q30	013	061	161	164	186	185	00341	100	096	077	079	00087
Q31	069	076	-069	-072	- 066	- 067	00081	100	101	116	117	00213
Q32	018	065	131	132	143	143	00271	100	100	086	088	00108
Q33	046	075	021	018	015	015	00063	100	100	112	114	00199
Q34	027	070	091	088	088	088	00115	100	100	097	097	00130
Q35	078	075	-109	-112	- 103	- 104	00119	100	101	113	114	00191
Q36	030	069	078	076	075	074	00077	100	096	096	097	00100
Q37	064	076	-047	-050	- 046	- 047	00060	100	101	118	120	00212
Q38	035	072	059	056	053	054	00069	100	100	104	104	00139
Q39	049	074	008	006	004	003	00058	100	099	111	113	00184
Q40	068	075	-065	-068	- 062	- 062	00060	100	099	114	117	00221

Table C46 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	055	076	-014	-018	- 018	- 017	00040	100	101	117	118	00216
Q42	040	072	041	038	035	036	00054	100	096	103	105	00130
Q43	030	070	079	076	074	074	00104	100	099	098	100	00126
Q44	009	058	185	195	231	236	00901	100	094	070	071	00097
Q45	042	074	032	029	026	027	00059	100	101	111	112	00173
Q46	031	070	073	072	070	070	00087	100	097	098	099	00102
Q47	030	070	078	077	076	076	00085	100	097	097	099	00087
Q48	032	071	071	068	066	066	00111	100	100	101	103	00110
Q49	014	063	152	153	171	172	00480	100	100	081	082	00097
Q50	049	075	008	003	002	001	00060	100	100	113	112	00146
Q51	082	075	-132	-132	- 123	- 123	00132	100	105	113	114	00206
Q52	031	070	073	073	072	073	00116	100	097	098	097	00125
Q53	076	074	-102	-104	- 096	- 097	00140	100	099	110	112	00191
Q54	086	072	-156	-156	- 147	- 145	00190	100	101	104	110	00226
Q55	048	074	013	010	008	008	00059	100	097	109	109	00129
Q56	049	074	009	007	005	004	00088	100	099	111	114	00165
Q57	077	074	-106	-107	- 099	- 100	00128	100	100	111	112	00175
Q58	040	073	039	038	035	034	00052	100	099	107	106	00109
Q59	061	079	-037	-039	- 036	- 037	00060	100	109	129	131	00292
Q60	076	075	-097	-099	- 092	- 093	00114	100	103	115	116	00247
Q61	035	073	061	058	055	054	00079	100	103	106	108	00129
Q62	041	072	037	033	030	030	00051	100	097	105	106	00144
Q63	034	071	063	060	057	057	00064	100	097	100	101	00100
Q64	061	076	-034	-038	- 035	- 036	00057	100	099	116	116	00220
Q65	048	075	012	008	006	005	00056	100	100	113	113	00148
Q66	065	076	-053	-056	- 052	- 052	00065	100	101	118	120	00179
Q67	053	075	-009	-011	- 011	- 011	00058	100	098	112	113	00209
Q68	074	074	-093	-095	- 088	- 088	00086	100	098	111	114	00158
Q69	086	071	-162	-162	- 154	- 154	00273	100	099	101	103	00208
Q70	079	075	-117	-115	- 106	- 107	00113	100	103	114	116	00271
Q71	087	071	-169	-168	- 161	- 162	00250	100	101	101	103	00194
Q72	086	071	-154	-159	- 150	- 150	00232	100	099	102	105	00239
Q73	065	076	-053	-053	- 049	- 049	00058	100	101	117	119	00231
Q74	060	077	-034	-036	- 034	- 034	00055	100	102	119	118	00139
Q75	045	074	024	019	017	016	00068	100	100	111	111	00147
Q76	031	069	071	072	070	069	00087	100	095	096	098	00111
Q77	028	070	086	083	082	083	00102	100	099	097	098	00102
Q78	061	075	-038	-041	- 038	- 038	00045	100	097	113	115	00166
Q79	036	071	058	051	049	048	00074	100	097	102	104	00140
Q80	039	073	045	043	039	039	00077	100	099	106	107	00127

Table C47

Item Indices for the Conditions of 80 Items, Negatively Skewed Ability Distribution, Unit Discrimination, Sample Size of 500

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	041	094	031	029	024	025	00044	208	212	270	293	08314
Q2	041	061	043	041	039	040	00095	073	072	077	077	00089
Q3	039	082	038	038	034	034	00058	135	132	142	143	00188
Q4	053	093	-002	-007	- 009	- 008	00034	177	177	253	268	06251
Q5	056	089	-013	-015	- 016	- 015	00035	149	151	197	204	01137
Q6	056	084	-016	-019	- 019	- 018	00051	128	127	154	156	00455
Q7	076	072	-106	-107	- 098	- 098	00113	091	094	105	107	00239
Q8	061	086	-031	-033	- 031	- 031	00052	133	133	168	174	00718
Q9	038	066	050	048	046	047	00072	085	084	088	088	00096
Q10	055	053	-022	-024	- 022	- 021	00127	056	056	063	063	00058
Q11	029	065	085	084	084	086	00136	090	088	087	086	00068
Q12	067	080	-058	-061	- 056	- 056	00051	111	112	133	134	00223
Q13	022	059	120	125	131	132	00240	081	079	073	074	00070
Q14	064	090	-039	-043	- 040	- 039	00041	160	152	204	207	01151
Q15	089	085	-152	-152	- 145	- 145	00157	169	173	163	169	00756
Q16	015	063	144	145	162	164	00217	105	101	082	083	00067
Q17	057	086	-016	-021	- 021	- 020	00047	135	136	168	169	00541
Q18	057	056	-028	-034	- 031	- 032	00111	059	060	067	068	00070
Q19	025	058	115	113	117	122	00177	074	074	071	069	00074
Q20	025	052	129	125	128	132	00297	060	062	061	061	00052
Q21	059	071	-032	-034	- 032	- 031	00086	089	087	100	101	00138
Q22	049	063	005	003	003	002	00078	070	072	081	083	00081
Q23	045	081	021	018	015	016	00068	121	121	137	136	00213
Q24	061	069	-036	-043	- 040	- 040	00089	086	084	096	097	00115
Q25	077	077	-102	-104	- 095	- 095	00107	105	107	120	121	00297
Q26	014	045	201	213	238	245	01514	063	058	051	050	00067
Q27	048	072	009	008	006	006	00053	091	092	102	103	00135
Q28	054	058	-013	-017	- 015	- 014	00081	066	064	072	072	00084
Q29	067	055	-090	-088	- 081	- 081	00193	058	059	066	066	00069
Q30	073	033	-195	-195	- 182	- 187	01824	032	032	036	037	00052
Q31	060	066	-037	-041	- 038	- 038	00091	080	076	087	089	00137
Q32	019	068	121	120	130	133	00172	114	114	094	093	00091
Q33	052	088	-003	-004	- 006	- 005	00037	141	149	188	193	01038
Q34	087	071	-169	-167	- 158	- 159	00295	094	097	100	102	00211
Q35	064	084	-044	-045	- 041	- 041	00045	122	125	154	159	00482
Q36	042	075	035	031	028	029	00060	105	106	114	114	00129
Q37	046	061	020	017	016	016	00065	069	069	076	075	00062
Q38	064	054	-066	-071	- 066	- 066	00159	057	057	064	064	00075
Q39	027	082	076	075	074	076	00075	164	158	141	141	00224
Q40	049	077	007	004	002	002	00049	108	107	121	123	00166

Table C47 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	039	074	041	040	037	038	00068	107	105	111	113	00126
Q42	040	068	043	041	038	040	00080	087	087	092	094	00118
Q43	034	051	083	084	083	086	00171	059	057	059	059	00053
Q44[a]	075	094	-078	-080	- 073	- 073	00046	189	197	286	378	197333
Q45	061	092	-028	-033	- 031	- 030	00035	157	163	233	253	04200
Q46	064	091	-040	-043	- 040	- 039	00038	157	156	214	218	01824
Q47	033	049	096	093	092	094	00231	054	054	056	056	00051
Q48	043	078	029	027	024	025	00053	113	113	124	125	00251
Q49	058	070	-031	-032	- 030	- 028	00066	084	087	099	100	00122
Q50	035	078	057	053	050	051	00061	126	123	126	127	00249
Q51	043	073	028	026	024	023	00078	101	097	105	106	00121
Q52	030	058	094	090	091	093	00185	073	072	072	071	00062
Q53	064	070	-050	-053	- 049	- 050	00066	088	087	099	100	00128
Q54	066	068	-066	-064	- 059	- 060	00096	077	081	092	093	00137
Q55	015	058	157	159	178	184	00420	092	086	072	070	00056
Q56	078	079	-103	-106	- 097	- 098	00113	115	114	130	130	00307
Q57	028	058	099	099	100	101	00219	077	073	072	073	00066
Q58	062	044	-072	-074	- 067	- 067	00246	042	043	049	047	00039
Q59	082	063	-152	-154	- 144	- 146	00297	075	074	080	081	00146
Q60[b]	057	100	-014	-016				257	256			
Q61	031	077	069	066	064	065	00060	126	124	122	124	00186
Q62	036	075	052	049	046	047	00087	107	109	113	114	00196
Q63	077	074	-106	-109	- 100	- 101	00110	094	097	110	110	00273
Q64	052	089	-001	-005	- 007	- 006	00049	150	151	193	192	00858
Q65	036	071	057	053	051	051	00076	104	099	101	103	00118
Q66	054	082	-009	-011	- 012	- 010	00040	119	121	144	149	00430
Q67	092	082	-179	-176	- 173	- 172	00211	152	167	142	148	00462
Q68	085	064	-170	-169	- 160	- 162	00325	077	079	083	084	00144
Q69	055	087	-013	-015	- 016	- 015	00046	138	142	179	182	00937
Q70	043	080	026	024	021	021	00057	124	120	133	134	00269
Q71	070	050	-114	-116	- 107	- 104	00284	050	051	057	058	00057
Q72	069	085	-057	-062	- 057	- 057	00051	130	130	161	163	00614
Q73	009	057	196	194	235	238	00528	098	096	070	070	00065
Q74	053	066	-007	-011	- 011	- 010	00086	080	078	088	087	00084
Q75	061	069	-042	-044	- 040	- 040	00071	083	083	095	096	00108
Q76	040	056	045	045	044	046	00125	063	062	067	067	00055
Q77[c]	075	097	-074	-076	- 069	- 069	00043	226	227	421	497	134760
Q78	020	075	106	104	111	114	00127	139	137	112	112	00146
Q79	071	067	-089	-090	- 083	- 081	00132	076	081	091	092	00108
Q80	046	085	022	016	013	013	00063	143	139	160	161	00509

[a] $n = 96$; 4 sample values removed due to biserials ≥ 1.

[b] $r_b = 1$; population Lertap values not reported and sample Lertap values not computed due to 63 biserials ≥ 1.

[c] $n = 90$; 10 sample values removed due to biserials ≥ 1.

Table C48

Item Indices for the Conditions of 20 Items, Negatively Skewed Ability Distribution, Unit Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\overline{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\overline{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	059	079	-030	-033	- 030	-030	00118	100	100	128	131	00638
Q2	066	078	-055	-058	- 052	-054	00149	100	099	125	127	00559
Q3	060	078	-033	-036	- 033	-031	00089	100	097	124	127	00682
Q4	057	078	-023	-025	- 023	-023	00107	100	099	126	131	00549
Q5	044	076	026	024	020	019	00129	100	097	118	121	00325
Q6	012	061	169	189	195	198	01148	100	084	077	079	00206
Q7	040	074	043	040	035	035	00122	100	092	111	111	00308
Q8	049	077	007	005	003	004	00107	100	096	120	123	00394
Q9	057	079	-019	-025	- 023	-025	00099	100	100	127	130	00543
Q10	038	075	049	046	041	040	00130	100	097	114	119	00413
Q11	035	074	060	058	052	050	00131	100	095	110	114	00390
Q12	080	077	-116	-120	- 108	-110	00251	100	100	120	122	00520
Q13	027	071	090	091	084	084	00182	100	092	101	104	00254
Q14	054	078	-009	-013	- 012	-013	00108	100	099	125	128	00419
Q15	037	075	049	049	043	043	00131	100	094	112	113	00203
Q16	056	078	-018	-022	- 020	-021	00117	100	099	126	129	00640
Q17	085	077	-153	-150	- 137	-141	00305	100	106	119	123	00597
Q18	085	077	-153	-150	- 137	-139	00291	100	108	120	124	00621
Q19	017	067	135	143	142	144	00397	100	092	090	093	00192
Q20	058	079	-023	-026	- 024	-024	00113	100	099	127	130	00549

Table C49

Item Indices for the Conditions of 20 Items, Negatively Skewed Ability Distribution, Variable Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	068	062	-085	-090	- 075	-079	00307	062	061	078	078	00159
Q2	081	073	-129	-133	- 122	-122	00273	097	099	108	111	00390
Q3	031	049	119	126	101	102	00480	047	041	056	056	00106
Q4	058	088	-022	-026	- 023	-024	00071	151	147	189	198	01937
Q5	042	081	028	027	023	023	00093	113	107	139	145	00998
Q6	077	088	-085	-090	- 084	-086	00138	182	177	181	190	01515
Q7	056	075	-020	-023	- 020	-022	00113	093	088	113	118	00357
Q8	066	073	-057	-064	- 055	-056	00142	090	084	105	109	00221
Q9	024	087	085	090	081	078	00089	167	144	173	187	01750
Q10	072	080	-077	-082	- 073	-076	00148	119	116	136	143	00840
Q11	019	078	114	123	112	113	00208	122	109	124	128	00637
Q12	015	077	135	144	136	136	00410	128	113	120	126	00969
Q13	018	071	135	142	129	130	00345	101	089	102	103	00307
Q14[a]	028	094	071	069	061	059	00081	225	200	267	359	144610
Q15	029	063	098	103	087	088	00230	066	064	082	084	00224
Q16	066	071	-063	-067	- 057	-059	00234	079	079	100	102	00283
Q17[b]	002	057	273	327	375	405	16371	123	094	069	072	00499
Q18	023	066	123	132	115	116	00372	078	071	087	087	00192
Q19	028	083	083	080	071	069	00103	119	120	147	161	01418
Q20	046	071	018	018	015	014	00114	083	078	101	103	00280

[a] $n = 97$; 3 sample values removed due to biserials ≥ 1.
[b] $n = 99$; 1 sample value removed due to p-value = 0.

Table C50

Item Indices for the Conditions of 40 Items, Negatively Skewed Ability Distribution, Unit Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}^2_b$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}^2_a$
Q1	057	075	-019	-023	- 022	- 020	00104	100	087	114	116	00295
Q2	036	072	055	058	048	049	00142	100	090	104	108	00243
Q3	008	057	199	223	241	243	01116	100	087	070	072	00135
Q4	068	078	-064	-073	- 061	- 061	00106	100	092	126	132	00707
Q5	085	078	-150	-161	- 133	- 132	00313	100	095	124	132	00902
Q6	079	079	-116	-122	- 100	- 101	00197	100	096	131	133	00833
Q7	050	076	004	004	000	002	00118	100	090	115	119	00331
Q8	043	074	029	033	025	024	00102	100	090	109	115	00269
Q9	026	068	093	104	094	097	00284	100	089	093	093	00166
Q10	073	079	-087	-094	- 079	- 076	00126	100	095	131	142	00956
Q11	018	064	132	146	142	144	00455	100	090	084	086	00137
Q12	072	077	-083	-091	- 076	- 075	00198	100	089	121	127	00706
Q13	050	075	005	004	000	001	00118	100	089	115	116	00364
Q14	064	078	-046	-051	- 044	- 043	00089	100	092	125	127	00353
Q15	062	079	-039	-042	- 037	- 036	00086	100	095	128	135	00544
Q16	005	054	229	262	305	310	03701	100	088	063	065	00133
Q17	057	076	-023	-024	- 023	- 022	00099	100	089	118	127	00514
Q18	067	078	-058	-066	- 056	- 055	00127	100	092	125	130	00535
Q19	033	069	069	076	065	065	00194	100	086	096	100	00213
Q20	060	077	-033	-038	- 034	- 034	00102	100	091	123	130	00531
Q21	077	079	-102	-113	- 093	- 092	00180	100	093	128	131	00562
Q22	094	075	-227	-236	- 204	- 212	00886	100	100	112	113	00785
Q23	050	075	004	003	- 001	000	00123	100	089	114	121	00295
Q24	073	077	-085	-093	- 077	- 077	00150	100	090	122	125	00746
Q25	082	077	-131	-144	- 119	- 116	00225	100	091	122	130	01179
Q26	019	064	122	142	136	136	00469	100	086	083	085	00182
Q27	041	074	036	038	029	030	00134	100	091	110	113	00205
Q28	076	077	-098	-110	- 091	- 091	00161	100	089	121	124	00435
Q29	040	073	042	045	035	035	00147	100	089	106	110	00207
Q30	096	070	-255	-273	- 243	- 255	02663	100	092	097	101	00889
Q31	073	078	-086	-095	- 079	- 077	00093	100	092	125	131	00661
Q32	063	079	-045	-048	- 042	- 040	00116	100	095	129	134	00671
Q33	029	070	081	089	078	079	00189	100	090	097	100	00275
Q34	068	079	-063	-069	- 058	- 057	00144	100	092	127	131	00456
Q35	071	078	-075	-086	- 072	- 071	00118	100	091	125	133	00667
Q36	081	077	-130	-138	- 114	- 115	00223	100	091	122	124	00632
Q37	035	070	060	066	055	057	00157	100	087	099	102	00165
Q38	034	070	061	071	060	060	00180	100	086	098	101	00231
Q39	081	077	-127	-138	- 114	- 115	00304	100	092	122	126	00877
Q40	024	067	105	117	107	106	00272	100	088	089	093	00151

Table C51

Item Indices for the Conditions of 40 Items, Negatively Skewed Ability Distribution, Variable Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	056	066	-021	-027	- 024	- 024	00169	077	089	089	090	00226
Q2	074	061	-118	-134	- 108	- 112	00625	064	083	077	077	00263
Q3	062	069	-047	-054	- 045	- 047	00163	081	085	096	100	00367
Q4	055	053	-027	-028	- 024	- 024	00171	053	090	063	064	00116
Q5	074	069	-099	-112	- 091	- 095	00301	079	091	095	095	00318
Q6	052	072	-004	-007	- 008	- 009	00127	090	085	105	109	00431
Q7	025	050	139	155	138	141	00883	056	090	057	058	00122
Q8	059	049	-049	-059	- 048	- 049	00267	047	097	056	057	00107
Q9	009	059	182	204	232	236	01299	114	085	073	073	00127
Q10	067	080	-056	-066	- 055	- 056	00112	106	094	134	138	00618
Q11	003	052	244	287	370	397	12949	122	087	060	059	00172
Q12	047	086	014	013	008	006	00076	151	086	170	177	01013
Q13	024	059	111	129	119	123	00443	083	100	074	074	00110
Q14	056	074	-023	-023	- 021	- 021	00117	088	091	110	111	00389
Q15	053	064	-012	-013	- 012	- 012	00172	073	090	084	087	00238
Q16	067	081	-061	-068	- 056	- 057	00082	108	093	137	145	00673
Q17	038	057	058	064	053	055	00233	063	092	070	071	00133
Q18	046	062	019	019	015	015	00145	071	090	079	080	00155
Q19[a]	092	094	-163	-178	- 147	- 150	00213	212	088	273	313	30061
Q20[b]	084	090	-129	-140	- 112	- 115	00145	142	097	211	238	10093
Q21	053	077	-003	-007	- 008	- 011	00091	104	092	119	124	00435
Q22	057	084	-017	-022	- 021	- 022	00096	122	094	152	157	00841
Q23	012	051	193	221	225	232	01850	074	087	060	059	00134
Q24	063	067	-057	-060	- 050	- 050	00153	071	093	090	092	00206
Q25	027	053	118	131	116	117	00488	062	084	063	064	00091
Q26	050	069	002	001	- 002	- 004	00135	085	090	096	099	00308
Q27[a]	083	095	-113	-124	- 099	- 104	00169	179	086	317	325	30026
Q28	024	060	117	128	119	117	00376	078	092	075	078	00133
Q29[c]	066	090	-049	-056	- 047	- 049	00079	146	091	209	236	04676
Q30	050	072	001	004	001	001	00130	096	094	104	108	00336
Q31	060	072	-034	-041	- 035	- 035	00138	084	081	103	102	00275
Q32	046	091	017	018	010	010	00068	195	084	216	226	02655
Q33[d]	073	088	-078	-086	- 070	- 070	00108	134	088	183	202	04030
Q34	057	071	-022	-026	- 023	- 024	00132	085	093	102	105	00278
Q35	064	086	-044	-049	- 042	- 043	00086	129	090	171	180	02015
Q36	044	079	025	027	020	019	00107	124	094	129	136	00471
Q37	086	073	-161	-182	- 148	- 152	00645	090	091	106	111	00822
Q38	074	085	-083	-093	- 076	- 076	00095	123	085	162	181	02879
Q39	071	077	-076	-087	- 071	- 073	00171	096	088	121	125	00649
Q40	093	071	-228	-248	- 211	- 219	01796	094	083	100	105	01027

[a] $n = 86$; 14 sample values removed due to biserials ≥ 1.
[b] $n = 96$; 4 sample values removed due to biserials ≥ 1.
[c] $n = 99$; 1 sample value removed due to biserial ≥ 1.
[d] $n = 98$; 2 sample values removed due to biserials ≥ 1.

Table C52

Item Indices for the Conditions of 80 Items, Negatively Skewed Ability Distribution, Unit Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	059	076	-031	-032	- 032	- 033	00126	100	101	116	116	00317
Q2	047	073	014	011	009	008	00136	100	097	107	106	00237
Q3	050	075	005	003	000	000	00123	100	101	112	115	00384
Q4	076	076	-098	-099	- 092	- 093	00236	100	102	117	122	00644
Q5	036	072	055	053	050	049	00166	100	101	103	105	00259
Q6	073	075	-089	-090	- 084	- 086	00154	100	099	113	115	00478
Q7	027	068	090	089	090	090	00214	100	098	094	094	00173
Q8	030	068	079	079	078	080	00188	100	095	093	096	00205
Q9	013	060	162	166	190	190	01088	100	094	075	078	00215
Q10	033	070	065	067	065	064	00176	100	097	097	097	00181
Q11	068	076	-067	-065	- 061	- 062	00139	100	100	116	120	00737
Q12	018	063	130	133	144	147	00560	100	095	082	083	00183
Q13	064	077	-050	-047	- 045	- 047	00121	100	103	120	121	00420
Q14	083	075	-136	-136	- 127	- 127	00232	100	105	115	122	00548
Q15	076	076	-101	-101	- 093	- 093	00281	100	104	118	127	00779
Q16	047	074	013	011	008	007	00159	100	101	111	112	00255
Q17	082	075	-134	-134	- 124	- 127	00378	100	102	112	116	00592
Q18	011	060	171	176	206	210	01186	100	098	075	076	00175
Q19	097	066	-287	-271	- 282	- 308	07940	100	107	087	091	00782
Q20	019	066	125	124	132	136	00335	100	100	088	087	00123
Q21	040	071	042	040	037	037	00161	100	096	102	101	00294
Q22	028	068	086	085	085	086	00213	100	097	094	095	00230
Q23	077	074	-107	-107	- 099	- 100	00245	100	098	111	117	00597
Q24	049	073	009	006	004	004	00153	100	096	106	107	00275
Q25	052	074	-003	-004	- 006	- 005	00108	100	098	110	111	00415
Q26	073	077	-087	-087	- 081	- 084	00232	100	105	121	124	00700
Q27	053	075	-005	-008	- 010	- 012	00117	100	101	114	116	00390
Q28	060	075	-033	-033	- 032	- 034	00136	100	100	115	118	00462
Q29	011	059	176	178	206	215	01107	100	094	073	073	00151
Q30	079	076	-117	-117	- 109	- 112	00301	100	103	116	121	00714
Q31	075	077	-094	-093	- 086	- 090	00242	100	104	120	122	00489
Q32	072	078	-082	-080	- 074	- 076	00154	100	106	123	126	00490
Q33	044	074	027	023	020	020	00142	100	103	111	115	00359
Q34	062	076	-040	-043	- 041	- 042	00147	100	102	118	120	00356
Q35	064	074	-049	-050	- 047	- 048	00119	100	097	112	111	00352
Q36	008	057	201	203	247	248	01690	100	097	070	072	00151
Q37	034	071	065	059	056	055	00170	100	101	102	104	00254
Q38	027	068	090	091	091	092	00272	100	098	093	094	00295
Q39	040	073	042	037	033	032	00154	100	102	108	109	00292
Q40	078	076	-106	-108	- 100	- 104	00224	100	103	116	117	00595

Table C52 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	042	074	036	032	029	027	00109	100	102	109	112	00260
Q42	080	075	-122	-121	- 112	- 116	00310	100	101	114	116	00859
Q43	072	077	-081	-081	- 075	- 078	00151	100	104	121	124	00511
Q44	030	070	082	077	076	075	00209	100	102	099	100	00223
Q45	064	077	-048	-047	- 045	- 048	00153	100	104	121	122	00480
Q46	063	076	-043	-046	- 044	- 046	00147	100	101	116	119	00401
Q47	045	074	024	020	017	017	00132	100	101	110	112	00252
Q48	057	076	-020	-024	- 024	- 025	00116	100	102	117	122	00445
Q49	016	062	143	146	161	166	00834	100	093	079	079	00167
Q50	084	074	-146	-143	- 133	- 133	00369	100	102	110	117	00701
Q51	012	061	170	169	195	197	01116	100	098	076	078	00180
Q52	003	049	267	284	396	455	27082	100	095	056	055	00305
Q53	002	046	280	310	431	454	22501	100	085	051	054	00214
Q54	067	075	-059	-060	- 057	- 057	00142	100	100	115	120	00450
Q55	076	074	-105	-104	- 096	- 098	00283	100	098	111	113	00565
Q56	060	076	-031	-036	- 035	- 035	00155	100	101	116	122	00439
Q57	016	061	146	149	166	172	00893	100	093	078	076	00169
Q58	017	065	140	137	150	153	00384	100	100	084	082	00108
Q59	053	073	-006	-009	- 011	- 011	00115	100	094	106	106	00308
Q60	018	064	130	131	142	144	00527	100	097	083	085	00243
Q61	024	066	101	104	107	108	00406	100	094	087	088	00217
Q62	056	075	-019	-021	- 021	- 022	00105	100	101	115	118	00257
Q63	062	075	-042	-043	- 041	- 043	00123	100	100	115	119	00473
Q64	047	074	014	012	009	009	00150	100	101	111	111	00266
Q65	028	069	088	085	085	086	00228	100	098	094	096	00203
Q66	011	059	175	181	211	216	01332	100	093	072	073	00175
Q67	019	064	123	127	136	141	00434	100	095	083	083	00157
Q68	034	071	061	061	059	058	00151	100	101	101	105	00318
Q69	083	075	-138	-134	- 125	- 129	00350	100	105	115	117	00743
Q70	065	076	-053	-052	- 049	- 051	00128	100	100	116	120	00409
Q71	037	073	051	047	044	043	00137	100	102	106	108	00233
Q72	021	065	118	120	126	129	00412	100	096	086	085	00175
Q73	070	076	-069	-073	- 068	- 070	00186	100	101	116	121	00442
Q74	087	075	-162	-157	- 147	- 149	00555	100	108	114	126	03072
Q75	018	065	128	130	140	143	00501	100	098	085	085	00166
Q76	045	074	020	019	016	016	00128	100	100	109	108	00308
Q77	061	076	-035	-038	- 036	- 039	00116	100	101	116	119	00488
Q78	089	072	-178	-176	- 167	- 169	00419	100	103	105	109	00462
Q79	064	077	-049	-048	- 045	- 047	00118	100	103	119	123	00560
Q80	049	075	006	004	002	000	00112	100	101	112	115	00389

Table C53

Item Indices for the Conditions of 80 Items, Negatively Skewed Ability Distribution, Unit Discrimination, Sample Size of 250

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q1	038	066	050	048	045	046	00173	081	081	087	090	00191
Q2	041	094	033	031	025	025	00082	212	210	268	307	27424
Q3	058	068	-028	-031	- 030	- 031	00129	084	083	094	096	00284
Q4	017	045	198	202	213	226	02058	055	053	050	050	00110
Q5	027	073	086	086	085	087	00190	119	110	108	110	00285
Q6	017	051	175	178	188	187	01151	067	064	060	063	00138
Q7	009	058	195	201	234	245	02028	095	092	072	072	00190
Q8	084	077	-138	-138	- 132	- 134	00425	116	120	120	125	00779
Q9	033	084	059	056	052	053	00097	154	148	155	161	00771
Q10	057	066	-027	-029	- 027	- 025	00171	080	077	087	089	00204
Q11	027	061	103	100	100	100	00321	075	076	077	080	00142
Q12	056	074	-017	-019	- 019	- 018	00097	096	096	110	112	00241
Q13	068	083	-056	-059	- 055	- 056	00130	130	130	148	155	01014
Q14	097	071	-235	-239	- 257	- 264	01869	141	139	100	101	00422
Q15	019	078	110	109	112	114	00241	162	147	127	131	00514
Q16[a]	052	095	000	-003	- 005	- 004	00090	207	210	308	404	108013
Q17	035	049	084	083	080	081	00460	052	052	056	059	00098
Q18	083	083	-120	-122	- 116	- 117	00212	141	146	147	155	01444
Q19	050	072	002	000	- 001	- 001	00129	096	094	104	107	00271
Q20	066	061	-067	-073	- 068	- 070	00209	070	069	076	077	00171
Q21	037	080	050	046	043	044	00098	123	123	132	138	00661
Q22	053	079	-007	-010	- 011	- 010	00097	113	113	130	136	00698
Q23	021	062	121	126	129	136	00637	085	082	078	078	00177
Q24	021	076	104	102	104	108	00221	131	130	119	119	00391
Q25	008	062	186	189	223	232	01279	110	107	080	081	00241
Q26	083	057	-162	-171	- 165	- 176	01055	071	068	070	069	00206
Q27	027	057	105	105	105	106	00367	071	069	070	071	00135
Q28	082	060	-165	-162	- 155	- 160	01049	068	071	074	075	00238
Q29[b]	056	096	-012	-014	- 015	- 015	00079	219	218	339	391	46052
Q30	062	071	-045	-046	- 044	- 043	00134	091	090	101	102	00262
Q31	010	053	204	209	238	240	01944	079	077	063	065	00143
Q32	076	087	-083	-085	- 080	- 080	00103	149	160	179	190	03077
Q33	040	085	036	034	030	030	00096	140	142	159	166	01041
Q34	094	069	-218	-223	- 229	- 237	01543	113	111	094	096	00384
Q35	020	070	113	115	119	118	00287	112	107	098	101	00231
Q36	056	063	-026	-027	- 026	- 026	00220	076	073	082	083	00184
Q37[c]	001	056	241	284	453	467	35626	205	161	067	071	00337
Q38	030	054	101	100	098	101	00374	063	062	064	066	00149
Q39	044	067	027	024	022	023	00162	081	081	089	090	00183
Q40	053	064	-006	-012	- 012	- 011	00186	074	073	083	085	00183

Table C53 (cont'd)

Item	p	r_b	b_τ	b_B	b_L	$\bar{\hat{b}}_L$	$\hat{\sigma}_b^2$	a_τ	a_B	a_L	$\bar{\hat{a}}_L$	$\hat{\sigma}_a^2$
Q41	046	069	019	016	014	014	00154	085	085	094	095	00210
Q42	028	072	084	082	080	080	00166	102	102	102	105	00302
Q43	038	074	047	045	042	041	00115	107	103	111	118	00472
Q44	009	059	184	193	222	224	01406	097	093	073	075	00165
Q45	050	049	006	001	001	002	00231	049	050	056	058	00111
Q46	031	087	065	062	058	057	00092	169	170	177	190	01443
Q47	042	053	041	041	039	041	00250	057	057	062	061	00111
Q48	044	061	029	026	025	024	00182	071	071	077	080	00139
Q49	060	061	-043	-043	- 040	- 039	00221	065	068	076	078	00163
Q50	037	059	060	058	056	057	00269	072	069	074	073	00146
Q51	052	083	-003	-006	- 007	- 007	00086	128	128	150	154	00590
Q52	030	068	078	078	075	077	00206	093	090	093	095	00197
Q53	059	060	-034	-041	- 038	- 038	00234	063	067	075	074	00119
Q54	060	057	-042	-047	- 043	- 044	00228	063	062	069	069	00113
Q55	059	057	-039	-041	- 038	- 040	00263	063	061	069	069	00150
Q56	032	057	085	086	084	089	00273	065	066	069	070	00146
Q57	040	059	047	047	044	045	00215	069	067	073	074	00144
Q58	064	069	-053	-054	- 051	- 051	00219	084	086	096	095	00171
Q59	082	078	-121	-121	- 115	- 119	00237	117	121	125	127	00711
Q60	020	053	158	154	160	163	01156	066	065	063	065	00188
Q61	045	065	025	021	019	021	00171	078	077	085	085	00162
Q62	066	059	-073	-077	- 071	- 073	00235	067	065	073	074	00149
Q63	086	075	-144	-148	- 143	- 144	00438	112	115	114	118	00548
Q64	019	071	118	117	121	119	00287	117	112	102	104	00309
Q65	094	072	-207	-206	- 211	- 216	00857	121	124	103	106	00517
Q66	085	082	-127	-134	- 129	- 130	00221	146	147	143	146	00716
Q67	019	067	125	124	129	132	00355	103	098	090	093	00224
Q68[d]	002	053	267	284	394	468	97398	119	108	062	062	00347
Q69	077	077	-100	-102	- 096	- 098	00220	108	111	119	120	00424
Q70	024	053	135	133	135	141	00735	062	062	062	061	00125
Q71	014	056	175	179	196	199	01164	079	076	067	069	00179
Q72	028	059	101	098	098	097	00341	075	072	073	075	00154
Q73	016	076	121	124	132	129	00247	153	140	116	125	00783
Q74[e]	052	096	001	-002	- 005	- 005	00073	217	221	343	401	43704
Q75	034	069	063	061	058	056	00148	091	090	095	099	00236
Q76	077	063	-121	-127	- 120	- 123	00476	081	076	081	081	00182
Q77	065	065	-063	-065	- 061	- 062	00159	079	076	085	084	00154
Q78	031	069	076	074	072	071	00157	093	094	096	102	00263
Q79	074	086	-076	-078	- 073	- 074	00101	149	152	172	181	02313
Q80	053	082	-004	-007	- 008	- 009	00116	120	124	143	145	00456

[a] $n = 94$; 6 sample values removed due to biserials ≥ 1.
[b] $n = 90$; 10 sample values removed due to biserials ≥ 1.
[c] $n = 70$; 30 sample value removed due to p-values = 0.
[d] $n = 98$; 2 sample value removed due to p-values = 0.

[e] $n = 89$; 11 sample values removed due to biserials ≥ 1.