UNIVERSITY OF ALBERTA

INTERACTIVE TEACHING IN MATHEMATICS

by

GEORGE R. DAWKINS ©

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ABSTRACT

The study investigated a North American designed instructional model for effective high school mathematics instruction applied to a Jamaican high school. The four month case study research was conducted in Spanish Town, Jamaica.

Based on Good, Grouws, and Ebmeier (1986) revised by Sigurdson (1989), and Kabaroff (1992), the model was further revised using Kabaroff (1992) and Sigurdson (1995) to include cooperative/collaborative and individual learning.

A grade eight class of average mathematical ability was compared with two control group classes. Data collection included observations, interviews, journals, pre and post unit tests, and a student attitude questionnaire.

The treatment teacher satisfactorily implemented the model in his classroom. Specifically, approaches to homework, oral work, daily review, lesson development, teaching for meaning and group work were considered effective. The treatment group significantly outperformed the control group on the achievement test results and showed a significant difference in attitude towards mathematics.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

THE PROBLEM

INTRODUCTION

Donna: Hey, what do you think of our new math teacher?

Eudeen: He seems very good.

Donna: Yes, he seems to do good planning and preparation for class. You know, I like teachers who know their 'stuff' and are able to present it in a creative way.

Eudeen: You know what, he is quite friendly and he gives us opportunity to ask questions and discuss math problems in class.

Donna: Did you notice how he helped Marcia to solve the problem by asking her lots of questions?

Eudeen: Man, that was something! I think he asked her about ten questions in a row.

Donna: Let us see if we can act it out. I will be Sir, and you will be Marcia.

Eudeen: O.K.

> Problem:
> A triangular plot of land has AB = 16 m, AC = 14 m, angle ABC = 40 degrees, and angle ACB = 60 degrees. Calculate the length of BC.

Marcia: Sir, why did you use the cosine rule to solve for BC and not the sin, cos or tan ratio?

Sir: What type of triangle is it?

Marcia: It looks like a right-angled triangle.

Sir: What makes a triangle a right-angled triangle?

Marcia: One of the three interior angles must be equal to 90 degrees.

Sir: Examine all three angles. Is any interior angle equal to 90 degrees?

Marcia: Let me see, 40 degrees plus 60 degrees equals 100 degrees and 180 degrees minus 100 degrees equals 80 degrees. No, Sir!

Sir: Therefore ...

Marcia: It is not a right-angled triangle.

Sir: Very good, Marcia. What conditions are present in this non-right angled triangle?

Marcia: There are two given sides and the included angle, which is 80 degrees.

Sir: What rule or formula can we use to find BC?

Marcia: Cosine rule.

Sir: Class, use the cosine rule to determine BC. Marcia, I will have a look at your solution when you are finished finding BC.

The initial conversation between these two students is familiar. This type of conversation occurs daily as students try to make sense of mathematical instruction and in the process of doing so, assess the teaching methodology of their teachers. I overheard this conversation between two students several years ago. I have paraphrased parts of it but it essentially conveys the contents of the conversation between Marcia and Eudeen.

The dialogue between the teacher and Marcia illustrates a practical example of one aspect of interactive teaching in a mathematics classroom. This interactive teaching approach consists of teacher-directed teaching, seatwork, collaborative learning, and a combination of assignments, homework, and worksheets.

This teacher's approach is in keeping with the view expressed by Leinhardt (1988) in his description of expert teachers. He suggested that effective teachers manage classroom discussions, demonstrations, seatwork, and independent practice effectively, while at the same time presenting concepts clearly. Lampert (1988) in commenting on the nature of teachers as a representation of what it means to know mathematics, says,

> For students to see what sorts of knowing mathematics involves, the teacher must make explicitly the knowledge she is using to carry on an argument with them about the legitimacy or usefulness of a solution strategy. She/he needs to follow students' arguments as they wander around in various mathematical terrain and must provide evidence as appropriate to support or challenge their assertions and then support students as they attempt to do the same thing with one another's assertions. (p. 152)

PERSONAL PERSPECTIVE

Traditionally, Jamaica's education structure is a model of the British system in which there are great demands and strict accountability on the part of teachers and students to fulfill the requirements of prescribed curricula. As such there is great emphasis on behavioural objectives and the learning of content in preparation for examinations at various levels. This stress on achievement tends to neglect the affective and social aspects of the curriculum even when the curriculum explicitly states these goals.

However, during the decades of the 80s and 90s there have been moves to change this situation in schools, to emphasize 'meaning' in the teaching of mathematics. There has been a noticeable change in construction of mathematics curricula to include social and affective goals. Teaching strategies and models such as guided discovery, laboratory, and problem solving approaches are common features of the current mathematics curriculum.

Significant too, is a change from the British General Certificate of Education (GCE) to the Caribbean Examination Council (CXC). The CXC examination is intended to reflect the culture and social orientation of Caribbean students. Some of the good features of this examination are school-based assessment as well as a final examination component. Also each examination certificate includes a profile of each student's performance which provides a useful indicator of students' competence and special skills to prospective employers.

Presently, the elementary schools reflect, to some degree, aspects of interactive teaching in mathematics. This is possible since teachers are given some autonomy to adapt mathematics curricula to reflect students' personal experiences and practical realities. Also examinations are flexible and are determined to a large extent by teacher choices. *Mathematics for Primary School Teachers* (1995) supports the relative autonomy and flexibility given to primary school teachers by stating that the process of instruction should vary with context and should include:

1. Appropriate projects, or assignments.
2. Exposition by a teacher, or instructional guide.
3. Practice of techniques and methods.
4. Group and individual activities.
5. Discovery through investigations.
6. Discussions and problem solving.
7. Collaborative tasks.
8. Concept formation with little emphasis placed on symbol manipulation (p. 3).

However the situation is quite different at the secondary level. Most, if not all, high schools have specific schedules for examinations especially in English and Mathematics. These are unit tests, usually every three weeks, end of term tests, and an external examination (CXC) for Grade 11 students. End of year examinations are used to determine future employment and access to higher education, and so it is perhaps logical that parents exert great pressure on school administrators and teachers to improve students' achievement in school mathematics.

Teachers are in a dilemma. Some teachers want to engage students in interactive mathematics teaching but they think they are limited to a strictly direct teaching approach because of pressure from school administrators and parents for students to perform in mathematics. A teacher's reputation is not based on how effective and creative he/she is but almost entirely on students' achievement in examinations, especially the CXC.

Consequently, most teachers adopt a direct teaching approach since they perceive this method as being most efficient in terms of covering the content of the syllabus and lesson preparation is not as time consuming as for other methods like cooperative learning and guided discovery.

This situation is surprising as past and present curricula have stressed skills and attitudes in addition to behavioural objectives. For example the *Teachers' Guide Grades 7-9 Mathematics* (1992) stresses the skills/attitudes for the *Number* topic. These are:

1. Comparing
2. Ordering
3. Computing accurately
4. Drawing
5. Investigating number patterns
6. Approximating
7. Appreciating the need for set language in everyday situations. Developing a 'feel' for numbers
9. Appreciating the reasonableness of answers (p. 19).

These skills/attitudes clearly indicate a methodology of teaching mathematics in which meaning is emphasized. However, the reality is that in most schools, teachers do not emphasize skills and attitudes, but tend to use cognitive objectives instinctively. This is done with, perhaps, legitimate reasons as most examination questions emphasize these cognitive objectives.

The researcher has had experience with a type of interactive teaching as a mathematics lecturer at a teachers' college in Jamaica. It is obvious that since the goal is training teachers of mathematics, the emphasis on meaning and active mathematics learning is recommended. The researcher has had considerable success using this method as evidenced by student achievement of 100% pass rate in most cases for the past five years on the Joint Board of Teacher Education Examination (JBTE). The researcher has also noticed improvement in students' attitudes and motivations toward mathematics using this interactive approach.

Perhaps the most astounding evidence and positive effects of interactive mathematics teaching occurred between 1991 and 1993 at three eight-week summer programs sponsored by the United Nations Development Program (UNDP). The researcher was one of ten lecturers selected to teach this program. Lecturers were selected on the basis of their teaching experience and academic qualification. Two hundred students were selected each summer. These were primary school teachers who have failed mathematics repeatedly at teachers' college level. The average pass rate was 91.3% (Packer, 1993).

The researcher does not believe that this high success rate was due only to selected aspects of interactive teaching. There are other factors such as teacher effectiveness, financial incentives provided for teachers, and, to a lesser extent, the teachers' attitudes and motivations. However, the interactive teaching methodology may have contributed to the achievement of students in mathematics particularly as it relates to teacher effectiveness. This idea of teacher effectiveness in student achievement is supported by research (Doyle, 1983, Tobin and Fraser, 1987, Good, Grouws, and Ebmeier, 1986).

As a result of my practical and personal experience of teaching mathematics over the past twelve years, and experimenting with various methodologies, I became interested in interactive teaching when I encountered the idea in a mathematics course at the University of Alberta in Canada. This interest grew as I conversed with my advisor and other faculty members. As a result of these experiences with the interactive teaching approach, I decided to conduct a case study to investigate the effectiveness of interactive mathematics teaching in a selected secondary school in Jamaica.

PURPOSE OF THE STUDY

The primary purpose of this study was to investigate the effectiveness of interactive teaching in mathematics at a selected high school in Jamaica. The model used was an adaptation of the Missouri mathematics program described by Good, Grouws, and Ebmeier, in *Active Mathematics Teaching* (1986).

A second purpose of the study was to test the suitability of interactive teaching in a Jamaican setting by observing a teacher and students in a mathematics classroom for a period of four months. Furthermore, students' attitudes toward and responses to this program were noted.

The study was based on the philosophy that no single or best way of teaching mathematics exists, but that experienced mathematics teachers should continually seek ways of improving their practice of mathematics teaching.

SIGNIFICANCE OF THE STUDY

In the context of our modern technological world it is necessary for teachers to find efficient and meaningful means of helping students understand mathematics, while simultaneously offering them mathematics which is enjoyable, creative, and stimulating. Thus enhancing their individual mathematics potential as they become active constructors of their own mathematics reality.

This study proposes a model which seeks to teach mathematics efficiently and with meaning and understanding. This model could be particularly useful in a Jamaican context where many teachers are beset and burdened with numerous classroom and logistic problems. Some of these include inadequate classroom facilities, high student-teacher ratio, long hours of teaching, and an examination driven curriculum.

CHAPTER 2

REVIEW OF RELATED LITERATURE

INTRODUCTION

This chapter examines the principles and essential elements of interactive teaching which is based largely on the Missouri Model. (Goods, Grouws and Ebmeier, 1986). Also, there is relevant literature on teacher-directed instruction, seatwork, homework, and cooperative/collaborative learning all of which are key ingredients in the Interactive Teaching Model.

INTERACTIVE TEACHING

The interactive teaching approach had its beginning in the teacher-directed instruction of the 1970s and 80s. The *teacher effectiveness* research culminated, in mathematics education, in the active teaching lesson format of Good and Grouws (1986).

Sigurdson and Olson (1989), indicate achievement gains from their study of the use of the interactive lesson format in grade eight. The results of this study have led to several refinements of the lesson format including its name, Interactive Teaching.

The principles of the Interactive Teaching Model include:

1. Teacher as the source of pace, energy and focus. The teacher is the centre of the classroom, especially in initial learning stages. In large classes which include students with a wide range of ability, the mediating influence of the teacher is essential to efficient and effective learning. The goal is to develop autonomous learners of mathematics. This indeed is the challenge of any teacher and no less those using interactive teaching. Individual work, the focus of the seatwork and homework, is an important opportunity for students to develop independence of thought.
2. Success, not motivation, is important. Students gain their enthusiasm for mathematics primarily from being successful. Of course interesting, practical, motivational activities which are appropriate to what is to be learned are a bonus.
3. Distributed practice. The model allows many opportunities for distributed practice. The initial review, homework accountability, beginning the lesson with a variety of mathematical activities, controlled practice in the development, selected seatwork and homework also give the students opportunities to use their mathematics. All of these

times, distributed throughout the lesson, are opportunities for students to be actively engaged in mathematics.

4. Interactive sessions. Because of the teachers' prominence in the lesson at every stage, sensitive interactiveness is essential. The model can easily be abused by an overbearing teacher. Interactiveness also implies that students listen to each other. The modified interactive model which includes cooperation/collaboration will allow for the interactiveness of students.
5. Teacher responsibility. Many senior secondary school teachers do not see it their role to look after every student. Interactive teaching implies not simply explaining or providing activities, it is the process of engaging students in learning mathematics. Very few students come to school knowing why they should learn mathematics, how they should learn mathematics, or if they should learn it. Mathematics is important to all of these students, and the teacher's job is to inspire all of them.
6. Meaning in mathematics. Although capability acquisition is the central task of the mathematics program in secondary schools, the meaning of mathematics, and more specifically the understanding of mathematics is essential to this acquisition and subsequent use.
7. Seatwork is special. It provides students with an opportunity for immediate and successful practice after the completion of the development segment of the lesson.

Finn (1994) expresses a genuine concern of teachers.

> It's vital to ask of any new approach being thrust upon the education world whether it has been fully tested with students to insure that it yields the desired results. (p. 37)

New ideas, theorems, and teaching approaches must be tested before they are recommended for use by teachers. Some studies using the general principles of interactive teaching have been done by researchers. These results will be discussed now.

The Interactive Teaching Model presented in this review is an adaptation of an Active Teaching Model developed and revised by Good & Grouws (1975, 1977, 1979, 1981). The following review relates specifically to the Missouri Mathematics Effectiveness Project cited in *Active Mathematics Teaching*.

This project, conducted in the mid 1970s and early 1980s, is based upon a decade of naturalistic and experimental study of mathematics classrooms in intermediate elementary grades and in junior high school classrooms in the United States of America. The Active Teaching Model consisted of daily review, development, seatwork, homework assignment, and special review.

An interesting finding of the Missouri project was that students who were classified as independent learners did relatively poorly academically in non-structured environments. Similar results were reported by Whitzel & Winne (1976) and Peterson (1977). Solomon and Kendall (1976) suggest that children characterized as social, independent, motivated, and direct (but not academically oriented) did better in traditional classes.

A second general finding was that students labelled as high achieving profited most from classrooms characterized as fast-paced, teacher-centered, structured, and demanding. Again, there is consistent support for this conclusion from a number of sources.

In a study of sixth grade mathematics achievement, Whitzel and Winne (1976) concluded that high achievers performed better in traditional classrooms because of the faster pace and increased competition. Also, Peterson (1977), Bennett (1976), and Solomon and Kendall (1976) report that students described as prior high achievers functioned best in highly structured, high participation, fast-paced, and formal classrooms.

One likely explanation of this situation could be the natural tendency of high achievers to quickly adjust to a variety of learning conditions so that these conditions have a lesser effect on them than other types of students. Thus it is likely that if teachers can make academic goals reasonably clear and provide appropriate opportunities, high ability students will profit from most instructional programs.

Peterson, Janicki, and Swing (1981) found that prior high achieving students did better in a variation of the Active Teaching Model that allowed more independence and choice of homework, within a small group setting. They point out the possible importance of adapting the model for high achievers. Also, related research has illustrated that under certain conditions peer instruction in small groups can facilitate achievement (Webb, 1982; Peterson & Wilkinson, 1982).

A third major finding suggests that students characterized as dependent function best with sympathetic teachers in a rather teacher-centered classroom and with teachers who employ techniques typically suggested by the active teaching model. In contrast, dependent students and extremely low achievers do poorly academically with teachers who are subject oriented and emphasize accomplishment over social concerns. In reports which suggest appropriate classroom environments for students with varying characteristics, the most frequent recommendation is that dependent students need structured, teacher-centered classrooms in which there is opportunity for frequent teacher-student interaction and where rules and objectives are clearly delineated (Solomon & Kendall, 1976; Bennett 1976).

These observations and findings suggest that achievement in mathematics is related to students characteristics, mode of teaching, and, to a lesser extent, teacher qualities. These factors will be taken into consideration in my observations and conversations with teacher and students.

Finally a major finding, and one of great potential significance for those charged with implementing educational programs, involved the interaction between teacher types and the treatment program. Good, Grouws, and Ebmeier (1986) and Peterson, Janicki, and Swing (1981) suggest that teachers who implement the model get good results, yet some teacher types choose to use more facets of the model than other teacher types. They feel that this is probably a result of the different ways teachers perceive their roles, their previous experience, and the expectation of their particular schools. For instance, people will more likely adapt and internalise ideas that are consonant with their existing beliefs. This view, expressed by Peterson, Janicki, and Swings (1981), presents a real challenge to researchers of the Interactive Teaching Model to select teachers whose philosophy of mathematics teaching is consonant with the Interactive Teaching Model.

TEACHER-DIRECTED INSTRUCTION

Although my focus here is to examine direct instruction in mathematics teaching in terms of what a teacher can do to maximize the likelihood that students will learn mathematics, I do make certain assumptions about how people learn. These learning theories are elaborated in detail elsewhere (Engelmann & Carine cited in Silbert, Carine, & Stein, 1982) and will not be summarized here.

A theoretical framework, by Resnick (1983), does provide a general notion about these assumptions. He is of the view that students seek meaning from learning situations. The concepts and skills possessed by a teacher cannot be transmitted to a passive learner. The learner is continually trying to "make sense" out of what a teacher says and does. The way in which teachers organize the learning environment determines how successful learners will be in constructing the meaning that teachers intend to convey.

> We err when we slight the acquisition of facts, specific knowledge and simple skills, both as building blocks of more complex intellectual structures and as potential motivations. (Kelly, 1994, p. 13)

Kelly highlights the need for specific formative knowledge as a basis for further intellectual growth. Direct instruction is often criticized as failing to relate symbols and mathematical theories to students existing knowledge, often overlooking individual differences and presenting content too quickly to be understood (Baroody and Ginsberg. 1990). However, direct instruction provides a comprehensive set of guidelines for organizing instruction so that students acquire, retain, and generalize new learning in as humane, efficient, and effective a manner as possible. Direct instruction attempts to provide a comprehensive analysis of the instructional variables for which teachers are responsible: program design, presentation techniques, and organisation of instruction, particularly amount of teaching time.

These three variables are essential ingredients for a successful mathematics program using the direct teaching approach. It is apparent to many teachers that no single type of instructional program, presentation technique, or classroom organization is appropriate at all times. Direct instruction, therefore, takes on different characteristics depending on the type of student being taught and the objectives under consideration. Direct instruction, when used with intermediate grade level students at average or above average skill levels, is characterized by a heavy emphasis on student-directed independent work. On the other hand, the use of direct instruction with primary level students or with intermediate level students who have encountered difficulty in earlier grades is characterized by a more structured, more teacher-directed environment. Teachers work with students in small groups, monitor students performance carefully, provide immediate feedback and corrections, and praise frequently (Silbert, Carine and Stein, 1982).

Originally, reviewers cautiously asserted that direct instruction was effective for increasing student achievement of lower level skills in reading and mathematics, but that the effects on higher level skills were less certain (Rosenshine, 1979). More recently however, reviewers appear to be advocating direct instruction for teaching higher level skills in reading and mathematics. For example, Brophy and Good (1986), concluded that the research shows "students learn more efficiently when their teachers first structure new information for them and help them relate it to what they already know, then monitor their performance and provide corrective feedback during recital, drill, practice, or application activity" (p. 138). Drill and practice of mathematics algorithms spoken of by Brophy and Good (1986) has been viewed in a negative way by constructivists, but earlier Sueltz (1967) stated useful purpose served by drill procedures in the teaching of mathematics. He relates the following example to strengthen this view. A four-year-old student was stacking blocks of various sizes into a column which repeatedly fell down after he had placed a few blocks upon each other. He tried holding the blocks with one hand but as soon the hand was removed the blocks fell. He discovered that straightness or perpendicularity was a factor and tried to arranged them accordingly and met with better success. Then his teacher showed him how to place larger blocks at the bottom. The boy tried again and met with more success and showed the glow of accomplishment. Note that this experience involves not only drill or practice but also elements of discovery. The boy is an active participant. He uses a combination of mental, visual, and manual avenues of learning. He has used drill-experience in each of these avenues of learning.

Peterson (1988) suggests that with more direct teaching, students tended to perform slightly better on achievement tests, but not as well on tests of more abstract thinking, such as creativity and problem solving. Conversely, with less direct, more open approaches students performed more poorly on achievement tests but tended to do better on creativity and problem solving.

> Diverse classroom strategies should be welcomed-so long as solid learning occurs. (Kelly, 1994, p. 14)

This view indicates the open minded approach mathematics teachers should have as they seek to improve their mathematics teaching through practice. The direct instruction method is only one strategy that can be used by teachers, or it may be a part of a general strategy as suggested by the adapted Interactive Teaching Model proposed in this research.

Sigurdson and Olson (1989) conducted an experimental study related to meaning in mathematics teaching. Their sample included fifty-four eighth grade mathematics teachers who taught for a six-month period using one of these four approaches:

1. Direct instruction using the Missouri model,
2. Direct instruction with emphasis on meaning,
3. Direct instruction with emphasis on meaning and problem solving,
4. Conventional text book instruction.

A summary report of the study supported the effectiveness of the Missouri Model using direct instruction with emphasis on meaning.

Seatwork

Seatwork refers to practice that students complete individually at their desks. Seatwork assignments allow students to practice, on their own, problems and principles that were actively taught. Seatwork provides students with an opportunity for immediate and successful practice. This practice experience allows students to achieve increased proficiency and to consolidate learning (Good, Grouws and Ebmeier, 1986).

Rosenshine (1979) examined the effect that time-on-task has on academic achievement. Rosenshine commented that seatwork is of considerable importance in how much students achieve. Quality, focus, and time spent on seatwork are likely to make a major difference in the level of achievement that a teacher can expect from his or her students.

Anderson (1981), in her research on students responses to seatwork, concluded that while doing seatwork, first-grade students perceive purpose in terms of doing the work and progressing through a book rather than understanding the specific content-related purposes of assignments. She goes on to say that we can only speculate about reasons for this pattern of student response. Certainly, the age and development level of the children should be taken into account, in that one would not expect a first grader to give answers that would suggest a grand scheme for organizing reading skills, or to have a firm set of concepts for thinking about their own learning processes.

Anderson (1981) also says that observation of teachers and their presentation of assignments have led her to consider an additional hypothesis. That is, students' perceptions of the purpose of seatwork may be related to the information that they receive from teachers about their work. She goes on to say that although systematic analysis has not been carried out yet, the impression at this point is that very few teacher presentations include specific statements about the content-related purposes of assignments. Instead they consist mostly of procedural statements. In addition, teacher feedback following completion of work often consists of statements about the correctness of answers and direction for what to do next, but not reminders about the purpose of the content that has been encountered by student.

It seems obvious from these observations about seatwork that teachers should have clear and specific objectives to optimize the effectiveness of seatwork. Good, Grouws, and Ebmeier (1986) provide some principles for achieving this.

1. The first principle, *momentum*, means 'keep the ball rolling' without any sharp break in teaching activity and in student involvement. Teachers can achieve momentum by ending the development portion of the lesson by working problems similar to the ones that students are asked to work individually and by starting students on individual work with a simple and direct statement.
2. *Alerting* is a second principle to observe during seatwork. Alerting behaviours tell students that they will be held accountable for their work. Often students engage in off-task behaviour because they are not alerted to the fact that they will have their work checked at a specific point in time. If students are assigned seatwork that won't be

checked until the following day (or when it is not checked at all), students are not likely to be highly engaged in seatwork. A statement like, "we'll check the work at the end of the period," alerts students to the fact that there is reason to engage in productive work immediately.

3. *Accountability* is the third principle to observe during seatwork. Alerting, as we noted, is a signal to students that their work will be checked. Accountability is the actual checking of the work. It is important that your accountability efforts do not interrupt the seatwork behaviour of students.

Homework

Mathematics homework is written work done by students outside the mathematics class period. It is usually done at home; thus, it is distinctly different from seatwork, which is done within mathematics class time.

As far back as the 1950s, homework was of concern to parents, teachers, educators, and administrators. One of the most widely read and quoted articles concerning homework appeared in the *New York Times* in 1952. Dr. Benjamin Finc, Times Education editor, discussed the pros and cons of homework in general for school subjects. The arguments offered in favour of homework were that the school day was not long enough to allow the student to do the necessary studying, the parent was brought into closer contact with the school, the child learned self-study, and the child developed a sense of responsibility while being kept out of mischief by the demands of homework. The arguments against homework were that homework took up too much of the child's after-school time, the home did not always provide a suitable setting, parents often did their children's homework, emotional tensions sometimes developed, and homework was often used as punishment. No data from research were given in support of the statement favouring or opposing homework.

Good, Grouws, and Ebmeier (1986) feel that in some schools, homework is never given or so few problems are assigned that an excellent opportunity for distributed practice is wasted. While there is obvious misuse of homework, they think that homework can be an important part of mathematics learning if certain guidelines are followed. They say that giving homework to students on a regular basis may increase achievement and improve attitude towards mathematics. Short assignments have been found to be more effective and some variety in the type of homework is helpful.

Because of the important role homework will play in the adapted Missouri model of teaching, the following recommended directions to teachers by Good, Grouws, and Ebmeier (1986) were used in implementing this model.

1. At the very end of the mathematics class period on Monday through Thursday, give a homework assignment which is due at the beginning of the class period the following day.
2. Each assignment should require about 15 minutes of outside class time.
3. The primary focus for an assignment should be on the major ideas discussed in class that day. Also each assignment given on Tuesday and Wednesday should include one or two review problems from the current week's work.
4. Each assignment given on Thursday should be primarily devoted to review problems from the current week's work. In order for sufficient practice to be given on the material discussed on Thursday, this assignment will be a bit longer than assignments for other days and will probably take about 20 minutes for most students to complete.
5. Typically, each assignment should be scored (number correct) by another student. Papers should then be returned to their owners for brief examination. Finally, papers should be passed forward so that the scores can be recorded in the grade book.
6. The assignments given should be recorded daily in the teacher's log.

COOPERATIVE/COLLABORATIVE LEARNING

In numerous contexts, the terms cooperation and collaboration may be used interchangeably. Compton and Hord, cited in Hodgson-Ward (1991), however, identify some important distinctions between the two processes and their outcomes. Compton states that the main difference is that cooperation involves at least two people, neither of whom "can actively oppose the efforts of the other. By contrast, collaboration implies the parties share responsibility and autonomy for basic decision making" (p.2). Hord (1986) says, "collaboration is not possible without cooperation, but the inverse is not true. Collaboration requires more effort, but ideally yields more" (p. 2-3).

She describes collaboration as the more complex of the forms of interaction. She lists ten "demands of collaboration" (p. 26).

1. Mutual sense of gain.
2. Requires effort to reach out and take action.
3. Greater time commitment than for cooperation.
4. Frequent interaction and sharing needed.
5. Worthwhile rewards or outcome for both parties.

6. Promotion of similar activities between parties.
7. Willingness to relinquish personal control and assume more risk will create a more flexible environment.
8. Empathy for others' point of view.
9. Strong enthusiastic leadership.
10. Simple patience, persistence and willingness to share (p. 26).

"Collaboration is a valuable means of interacting with peers who have similar commitment to personal growth and excellence. As they discuss and make plans, students practice not only their linguistic and cognitive skills, but their social skills as well" (Gilles & Vandover, 1988, p. 26). The same authors list these four generalizations from their observations of collaborative classrooms.

1. Collaboration works best when students are given a real problem to solve.
2. A collaborative environment grows slowly.
3. Collaboration isn't a panacea. Problems may occur; working through these problems may help to strengthen the sense of community.
4. Students do not have to be directly involved in the collaboration to learn from it. It is possible for children to observe and overhear others' learning and, in turn, learn new ideas themselves (pp. 29-34).

The Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) proposes not only content changes to reform school mathematics, but also identifies new societal goals for schooling. It says that schools, a product of the industrial age, are not addressing today's needs. The new societal goals for education should provide for

1. mathematically literate workers,
2. lifelong learning,
3. opportunity for all, and
4. an informed electorate.

In expanding the definition of mathematical literacy, the *Curriculum Standards* emphasizes developing the student's ability to work with others when solving problems. Their goal is simply to meet the needs of an increasingly technological and information-based society. The *Curriculum Standards* subscribes to the view that learning is an active process in which students bring prior knowledge to the new situation and construct their own meanings. This is a social process. To this end, therefore, instruction should vary and include opportunities for group and individual assignments, and for discussion between teacher and students and among students.

Davidson (1990) argues that the NCTM Standards of mathematical communication, logical reasoning, problem solving, and making mathematical connections are enhanced by cooperative structures. In addition, cooperative learning provides:

1. social support for the discussion of mathematical ideas,
2. opportunities for all students to succeed,
3. a forum for group discussion and the resolution of mathematical problems,
4. a vehicle for the exploration of alternative approaches,
5. an atmosphere in which students learn through discussing ideas, listening, and teaching others,
6. opportunities for creative and critical thinking through the exploration of non-routine problems that may be beyond a single individual's ability, and
7. opportunities for the mastery of basic skills in a context.

Johnson, Johnson, and Johnson-Holubec (1986) distinguish among competitive, individualistic, and cooperative structures of classroom instruction. They emphasize that for a cooperative learning lesson to be effective in enhancing learning it should contain the following five elements: positive interdependence established through student discussion enabling students to assist each other to understand the material and encouraging each other to work hard, face-to-face interaction, individual accountability, interpersonal and small group skills, and group processing of those social skills.

The importance of peer relationships in enhancing learning is supported by Doyle and Sanford (1985) and Prawat (1989). Prawat (1989) maintains that communicating ideas helps to clarify them and that the explanation of ideas to others allow them to assimilate those ideas into their schemata. Discussion engenders new ideas, and the sharing of different standpoints promotes understanding. He uses the term "negotiation" to describe the process in which learners, through the discussion of differing or parallel ideas, reach consensus, and construct knowledge consistent with the accepted view in the discipline. Doyle and Stanford (1985), in their studies of academic work, argue that group work reduces the anxiety and risks individual students experience in tackling novel tasks; and that anxiety diminishes understanding and limits higher mental activity.

The positive effects of cooperative learning have been well documented in research. Johnson, Johnson, and Johnson-Holubec (1986) report a meta-analysis of 122 studies in which cooperative learning is shown to result in higher academic achievement, enhanced problem solving, and greater retention of the material. These studies were conducted across all ages and in all subject areas. Slavin (1990a) analysed 60 studies in which cooperative learning strategies and more traditional methods of instruction were employed. The results indicate that positive gains in achievement occurred when the teacher emphasized both individual and group goals. Students gained in mutual respect, self esteem, time on task, and attendance.

The model of cooperative/collaborative learning most similar to the one proposed for this study is Student Teams Achievement Divisions (Slavin, 1990b). In this model, after the teacher completes the development segment of the lesson, the students work in preassigned learning groups that are composed heterogeneously in terms of achievement level and sex. The aim is to facilitate students sharing mathematical knowledge and experience in solving mathematics problems and general assigned tasks.

SUMMARY

According to Lambert (1988) research on effective mathematics instruction indicates that students learn better and produce higher mathematics achievement in an interactive teacher-directed classroom setting. In this context teachers deliver a curriculum that addresses student questions, values their opinions and thinking, works from the premise that students are actively forming their representations of mathematical ideas, and provides appropriate experiences to motivate students to learn. Sigurdson (1995) says, "learning is active because only the learner can learn, the teacher cannot do the learning for the student" (p. 18).

The reality of mathematics classroom practice in Jamaica dictates that teachers must move entire classes through prescribed curricula. Within this context, systematic approaches to instruction are required (Brophy, 1986). Johnson (1982) suggests approaches for effectively using time in classrooms for active instruction in presenting material, having students practice, monitoring progress with appropriate questioning, providing feedback and trying to get improved responses and performance from students.

The Interactive Teaching Model used in this study incorporates teacher-directed instruction, individual seatwork, cooperative/collaborative learning, and homework. It is hoped that this model will be considered and possibly implemented by mathematics teachers, with appropriate refinement if needed, as we strive to find better and more efficient ways of teaching mathematics.

CHAPTER 3

METHODOLOGY

OVERVIEW

This study was primarily a case study. The essential procedure of the case study method is to take account of all pertinent aspects of one thing or situation, employing as the unit for study an individual, an institution, a community, or any group considered as a unit (Good & Scates, 1954, p. 726).

The case study enabled the researcher to look at the problem in greater depth, use a variety of qualitative and quantitative data collection techniques, and better understand the problem in relation to the participants in question (Borg and Gall 1989). Merriam's (1988) comments lend support as she states "case study research is an ideal design for understanding and interpreting observations of educational phenomena" (p.2).

The purpose of this study was to determine the appropriateness and suitability of an instructional model designed for effective high school mathematics instruction in a Jamaican context. The study involved several phases:

1. The development of the model;
2. The model's implementation and interpretation by the classroom teacher (researcher);
3. The testing for achievement gains; and
4. The analysis of students attitudes towards mathematics.

The preparatory phase of the study involved the development of a model for effective mathematics teaching. The design of the interactive teaching model for effective mathematics teaching is based on specific instructional strategies proposed by Good, Grouws, and Ebmeier (1986) and selected elements of collaborative/cooperative learning. Some modification of the original model was done by Sigurdson and Olson (1989) and Kabaroff (1992) as a result of their research experience with the original model.

DESIGN OF THE INTERACTIVE TEACHING MODEL

The following is an outline of the basic lesson format used in the teaching model. It is structured for a duration of 60 minutes.

Lesson format

Daily review and homework check—10 minutes

- Deal with homework.
- Begin with oral work that reviews and reinforces previous lesson' skills and knowledge and sets the scene for day's lesson.

Development—25 minutes

- Place concept to be taught in context of past student knowledge and future problem.
- Emphasize meaning.
- Introduce novel problems, when appropriate, to underscore process.
- Monitor student understanding through active questioning.
- Reinforce concepts through controlled practice.

Practice/individual practice—25 minutes

- Provide opportunities in alternate lessons for group and individual practice.
- Provide opportunity for successful practice.
- Include word problems and applications related to the lesson.
- Assign more difficult questions in this segment rather than for homework to take advantage of peer and teacher support.
- Encourage active discussion.
- Keep individuals accountable.

Homework

- Assign homework relevant to the lesson.
- Ensure that the questions assigned can be completed successfully by the majority of students working independently.
- Include a review question.

DISCUSSION OF THE MODEL

The lesson structure is an adaptation of a set of key instructional strategies for effective mathematics instruction proposed by Good, Grouws, and Ebmeier (1986). This approach, originally based on teacher-directed, whole-class instruction, has been modified to emphasize student-student interaction and discussion about the mathematics presented, and to focus on mathematical meaning (see Sigurdson & Olson, 1989; Kabaroff, 1992). Throughout the lesson the teacher is expected to provide opportunities to actively engage the students in their own learning by using activity based lessons and group work.

Oral work at the beginning of the lesson is intended to emphasize the importance of the first few minutes of the class. Rather than focusing on routine clerical matters, all students will engage in relevant and meaningful mathematical activities that will set the tone for the rest of the period. Less time than teachers routinely spend is devoted to addressing problems with the previous day's homework. Problems from assigned work, will be dealt with in a whole class setting, and are limited to those which concern the majority of the class. By restricting the time spent on taking up homework, more time can be devoted to the presentation and discussion of new material.

The development segment of the lesson is that part of the period devoted to actively involving students in developing their understanding of skills and concepts. Meaning is established by relating the content to previous knowledge, by placing the concepts in the context of the students' own experience, by modelling everyday situations, by focusing on application and by using concrete representations when applicable. Through questioning, the teacher monitors student understanding and ensures that students are held accountable. During this stage of the lesson, the number of questions posed to students may increase as the teacher begins to assess comprehension and provides them an opportunity to model processes already demonstrated, and to verbalize the understanding they developed. During this phase the teacher may decide that further explanations and demonstrations are necessary or decide that controlled practice is appropriate since students seem to understand what they are doing.

In the cooperative/collaborative practice segment, students worked in groups of six. This number was based on a class size of forty-nine students. These groups are heterogeneous by sex and achievement. The intent of the groups is to provide student support when they are working on questions assigned based on the material presented in the lesson development. In this context, both group and individuals are held accountable for the work to be completed in class.

SAMPLE

The original plan was to train two teachers to use the Interactive Teaching Model to teach two grade eight classes. Unfortunately, this plan was not possible as the two teachers originally identified were no longer available because of changes in their personal circumstances. I sought to get two other teachers but this did not materialise because of practical circumstances and divergence of personal philosophy of mathematics teaching. In the end, I had to assume the dual role of participant and researcher. Specifically, I taught one treatment and one control group (Grade 8) at a traditional high school in Jamaica. In addition to the one control group that was taught by the researcher, a second control group was used. It was taught by a Grade 8 teacher using a *traditional* model. This was done to neutralise the possible interference between researcher's methodology with the control group and the treatment. The researcher examined the lesson format used by the teacher of the other control group and used it to teach his control group.

In the traditional model, the teacher:

- gives a stimulating introduction to the lesson;
- presents a problem;
- shows how the problem is done but questions students in the process to ensure their participation and proper development of concept(s); and
- students replicate the problem and the solution and do similar problems.

The model for the treatment group was used by the researcher to teach five consecutive units namely:

1. Properties of Arithmetic
2. Integers
3. Sets
4. Basic Algebra
5. Measurement

This is a grade eight course of study for a period of one term (12 weeks). Thirty lessons developed by the researcher were taught to the treatment group for this period of time. (A copy of these lessons is included in Appendix 1). The same number of lessons with content prepared by the researcher and the other control teacher was taught to the control groups.

CHAPTER 4

THE INSTRUMENTS

INTRODUCTION

This section provides information regarding the instruments used to gather data. These are pre- and post-tests, unit tests, teacher and student interviews, student attitude questionnaire, classroom observation scale, teacher/researcher journal, and conversations with students and teacher.

PRE- AND POST-TESTS

The pre-test was used to determine the difference in mathematics achievement, if any, between the control groups and the treatment group. Twenty multiple-choice test items based on the Grade 7 mathematics syllabus were prepared and administered to students. This test is included in Appendix 2. The test was piloted prior to the implementation of the model with a group of Grade 8 students. Items were revised based on the following criteria:

1. At least 8% of the students chose each distractor.
2. At least 35% of the students answered each question correctly. As a result selected items were revised or replaced.

The post-test used was based on the following units completed. These are:

1. Properties of Arithmetic
2. Integers
3. Sets
4. Basic Algebra
5. Measurement

This post-test was used to determine the effect of the treatment on the treatment group, and to determine if there was any significant difference in achievement between the treatment group and the two control groups. A sample of the post-test is included in Appendix 2. The content validity of the post-test was addressed by asking teachers who participated in the study to assist in the construction of the test. Their suggestions and revisions were noted and used in its construction.

In order to measure the reliability of the pre- and post-test, the *Tukey-B reliability analysis* was done. The reliability coefficients for the pre and post-test were -0.0652 and

0.4957 respectively. This indicates that the reliability of the pre-test is essentially zero while there is low reliability for the post-test.

UNIT TESTS

Five unit tests were administered to the treatment and control groups over the four-month period to determine the level of performance of both groups, and if there are real differences in achievement between the two groups. All five unit tests were prepared by teachers of Grade 8 mathematics classes which included the teacher of one control group and the teacher (researcher) of the treatment group and the other control group. These tests were prepared collaboratively with Grade 8 mathematics teachers.

The *Tukey-B reliability analysis* was used to determine the reliability coefficients of the five Unit Tests. These are summarised in Table 4.1 below.

Table 4.1
Reliability Coefficients of Unit Tests

TESTS	RELIABILITY COEFFICIENTS	INTERPRETATION
Unit 1	0.3014	low reliability
Unit 2	0.1695	low reliability
Unit 3	0.0979	low reliability
Unit 4	0.0485	low reliability
Unit 5	0.8358	high reliability

Table 4.1 indicates low reliability for unit tests 1, 2, 3, and 4 and a high reliability for Unit test 5. One likely reason for the low reliability coefficients of the tests (with the exception of Unit 5) could be the essay type structure of the tests and the relatively small number of questions (N=4).

STUDENT INTERVIEWS

Eight students in the treatment group were selected for interviews during the research. These students represented high, low and average ability students in the class, four of which were boys and four were girls. These students were asked to give their impressions of the lesson format, style of presentation and group work, the idea being to determine the effectiveness and workability of the model in a local setting. Students were interviewed in December near the end of the study. Each student was asked the same questions.

Student Interview Protocol

During this term, you have done five units. These have been taught to you with a particular lesson design that may or may not differ from the style of instruction that you are use to in mathematics. I will be asking you several questions which relate to the instructional approach I have used during this term in teaching the following units: Properties of Arithmetic, Integers, Sets, Basic Algebra and Measurement. Your answers are important to me in deciding the appropriateness of this approach to teaching mathematics. Your answers will be kept in strict confidence. Please answer each question as accurately as possible.

Student Interview Questions

1. What were the main differences in the way the lessons were taught this term compared to previous terms?
2. Do you think those differences have helped you to learn mathematics better? Explain.
3. What are your impressions about working together with other students on mathematics problems?
4. Have you been doing more mathematics in class? Why or why not?
5. Has it been easier to understand the material you have been taught? Why or Why not?
6. How useful to you is the mathematics you have been taught this term?
7. Have you been completing your homework assignments?
8. What things can a teacher do to make it easier for you to learn mathematics?

STUDENT ATTITUDE QUESTIONNAIRE

Twenty-five closed-ended items comprised the attitude to mathematics student survey. It was administered to students at the end of the study to determine the students' perception of the relevance of mathematics to their daily lives and the appropriateness of the instructional model. A number of these items were selected from published attitude surveys (Tobin and Fraser, 1987; Sigurdson and Olson, 1989) and adapted by Kabaroff (1992).

Some minor modifications were done to the items to reflect the context and culture of the Jamaican educational setting and the specific goals of the study. The attitude survey model focused on eight factors namely:

1. Interest in Mathematics
2. Cooperation
3. Teacher Dependence
4. Homework and Relevance of Mathematics
5. Task Orientation
6. Teacher Assistance
7. Independence
8. Perception Towards Problem Solving and Doing Mathematics

The attitude Questionnaire is shown in Appendix 3.

TEACHER INTERVIEW

The teacher of the control group was interviewed to provide the researcher with information on the methodology he used, and reasons for using it. The researcher also interviewed three teachers of Grade 8 about the methodology they observed the researcher using with the treatment group. These interviews were based on their weekly observations of the researcher using this model with the treatment group and their evaluation of the researchers lesson plans.

Each of the three teachers who observed the researcher using the Interactive Teaching Model were interviewed at the end of the study. The interview examined their impressions of the model, their observations of student reaction, and the elements of the treatment program that they would incorporate into teaching of their mathematics classes.

Teacher Interview Protocol

This interview seeks to obtain information from you about your observation of the Interactive Teaching Model. Please be as honest as possible in answering the following questions. Any information given to me will be strictly confidential and will only be used for research purposes.

Teacher Interview Questions

1. How does the Interactive Teaching Model differ from how you have traditionally taught?

2. What are the advantages if any, with beginning the lesson with oral work?
3. How do you view the idea of giving homework at the end of each class, and do you think it is a necessary part of students' learning of mathematics?
4. Do you think the interactive lesson takes more time for preparation than the traditional methods? Why?
5. Is teaching for 'meaning' difficult? Why?
6. How do you reconcile teaching for meaning with teaching for skills?
7. Based on your observation of the class, do you think students were comfortable with the arrangement of the class into groups for the cooperative/collaborative segment of the lesson?
8. What difficulties do you see with "collaborative/cooperative practice"?
9. What are your views on the strength of the Interactive Teaching Model that has been used in this study?
10. What are your views on the weaknesses of the Interactive Teaching Model that has been used in this study?
11. Can you make some general comments about the model, and possible modification of it, that would be suited to our Jamaican educational context?

In the light of the fact that the researcher was the one who taught the treatment group and with a sincere and genuine interest to be as objective as possible, I decided to ask an independent person not involved in the research to interview me using questions used by Kabaroff (1992) in his study using the Interactive Teaching Model. The interview was audio recorded.

Teacher Interview Questions

1. How does the Interactive Teaching Model differ from how you have traditionally taught?
2. What are the advantages, if any, with beginning the lesson with oral work?
3. How have you been dealing with homework during this study?
4. Have you been satisfied with this approach to homework? Why?
5. Have you been spending more time in preparing lessons? Why?

6. Is teaching for 'meaning' difficult? Why?
7. How do you reconcile teaching for meaning with teaching for skills?
8. How did you select the group for the collaborative/cooperative practice segment?
9. How have the students accepted this arrangement?
10. What difficulties or advantages do you see in collaborative/cooperative practice?
11. How does the homework you have been assigning differ from what you had normally done in the past?
12. What are the students' impressions of the Interactive Teaching Model?
13. Do you feel the students are learning more or less mathematics under this system? Why?
14. How do you think students learn mathematics?
15. What are important considerations a teacher should take into account to enhance his/her students' learning mathematics?
16. What are your impressions of the strengths of the Interactive Teaching Model?
17. What are your impressions of the weaknesses of Interactive Teaching Model?
18. Do you feel your teaching will change after the completion of the study? Why?
19. What aspects of the Interactive Teaching Model will you continue after the completion of this study? Why?
20. What aspects will you not continue? Why?
21. Do you feel the students learned more or less mathematics under this system? Why?
22. What are your views on the times allotted to the various segments of the lesson format?

CLASSROOM OBSERVATION

The researcher who was the teacher of the treatment group was present at all classes and used the teaching format in teaching this class. The researcher used a structured observational scale to determine the extent to which the teacher adhered to the model. This classroom observation scale (Figure 4.1) was modified by the researcher from a similar instrument developed by Sigurdson and Olson (1989) and later revised by Kabaroff (1992). The objective of the scale was twofold:

1. To record the times the teacher spent on oral work, review of homework, lesson development, and collaborative/cooperative practice; and, at the end of each lesson, the length of time, students would require to complete homework.
2. To score each lesson segment on a four-point implementation scale, according to the extent each matched the expectations of the Interactive Teaching Model.

Table 4.1

Classroom Observation Scale

Date:________ Topic: __________________________ Lesson #:__

Segments of lesson	Time (minutes)
1. Oral work	1.
2. Review	2.
3. Homework	3.
4. Development	4.
5. Collaborative/cooperative practice	5.
6. Homework	6.
Implementation of Model	**Problem Solving Scale (PS)**
0 - Not attempted	0 - No PS activity
1 - Some implementation	1 - PS given
2 - Implementation incomplete	2 - Teacher solution
3 - Fully implemented	3 - Solved interactively
Meaning Scale	**Comments**
0 - Not attempted	
1 - Some attempted	
2 - attempted, but not complete	
3 - Completed effectively	

At the end of each lesson, lesson segments were assigned a score from 0 through 3 based on the level to which the teacher taught for meaning rather than merely emphasizing an algorithmic approach. This scale, like the Implementation Scale, ranged from 0 to 3 where 0 represented no attempt on the teacher's part to teach understanding and 3 represented a sincere and effective approach to teach for meaning. This includes the use of real world examples, using models of concrete representations, mathematical reasoning and justification, teaching with mathematical contexts, mathematical investigations, teaching using applications, and engaging students in dialogue through effective questioning techniques.

In addition to the structured observation rating scale (modified from Kabaroff, 1992), two Grade 8 mathematics teachers attended my class once per week at a scheduled time when they were not teaching their individual classes. They were to record the time spent on each lesson segment, make observations of the teaching process using the Interactive Teaching Model, and to refer to these in our weekly conversation segment.

JOURNAL/FIELD NOTES

I kept a journal where daily observations and reports were recorded. Unlike specific instruments that have been described previously, entries in this journal were done on a daily basis and reflected the entire spectrum of school and classroom observations including interactions with students and teachers.

The idea of journal keeping as a powerful way for individuals to give an account of their experience is supported by Simpson and Wheellock (1982). Connelly and Clandinin (1988) see journals as a method of creating field texts which allows for ongoing records of practices and reflections on practices.

The researcher was closely related to participants in a kind of ethnographic setting (Van Mananen, 1988; Sanjek, 1990). Field notes were used as a kind of field text to record events happening in the classroom. The researcher also used a portable tape recorder from which selected transcriptions were done to fill in missing information in field records.

CONVERSATION

The nature of the relationship between researcher and participants encouraged oral conversation between teacher and students. These conversations between teachers and students provided valuable insights into how students learn mathematics and general educational experiences. Conversation was used as a personal experience method (Oakley, 1992). Connelly and Clandinin (1988) view conversation as entailing listening where the listener's response may constitute a probe into experience that takes the representation of experience far beyond what is possible in an interview. There is probing in conversation, in-depth probing, but it is done in a situation of mutual trust, listening and caring for the experience described by the other (Connelly & Clandinin, 1988). This idea reflected the centrality of the relationship between the researcher and the participants. Dodworth (1994) feels that researcher and participant collaboration is a way of negotiating entry into the classroom in ways that are participatory and respectful of the learning community.

CHAPTER 5

QUALITATIVE RESULTS

INTRODUCTION

This chapter contains the qualitative results of the study and addresses the treatment teacher and his students' perceptions of the effectiveness and appropriateness of the Interactive Teaching Model. Data were gathered and verified through classroom observation, personal interviews with participating teachers, student interviews, the teacher's daily journal, and conversation with participating teachers and students in the treatment group.

PERSONAL BACKGROUND AND POSSIBLE SOURCE OF BIAS

This study was conducted while I was on four months study leave from work as a mathematics lecturer. My education experience includes six years of teaching mathematics at senior high school and six years of teaching mathematics at a teachers' training college in Jamaica.

My professional experience includes working with the Ministry of Education and Culture in developing the *Scope and Sequence* mathematics curriculum for grades 7-9. Subsequently, I was asked by the Ministry of Education and Culture to evaluate the *Grades 7-9 Teachers' Guide* which replaced the *Scope and Sequence*. I am a member of the Joint Board of Teacher Education (JBTE), which is a professional body comprising lecturers from teachers' colleges and external examiners from the University of the West Indies and representatives from the Ministry of Education. This body is responsible for certifying teachers who graduate from these teachers' colleges. Some of our major responsibilities include: drafting and grading examination papers, assessing teacher trainees on teaching practice and evaluating students' final year research projects. I have also been involved in teaching experimental programmes in mathematics, the most recent one being the JBTE/UNDP summer program for primary school teachers.

As teacher of the treatment group and researcher I found myself in a position of great responsibility and trust which required me to participate in this study in a fair and impartial manner. I have endeavoured to present the findings of the research in an objective manner by simply reporting the views, feelings, ideas, and thinking of the participants.

During the study, I was teaching mathematics at a teachers' college located in urban Jamaica. I have been teaching mathematics for twelve years. My qualifications include a three year Diploma in Mechanical Technology and a Bachelor of Education degree. I am an active member of college staff participating in extra curricular activities such as the Shortwood Teachers' College Staff Association and I am the coordinator of several clubs and societies. I am also very active in my community where I am presently the President of the Citizens' Association.

I have a reputation as an excellent mathematics teacher and I enjoy the respect and confidence of my colleagues and students. I am hard working and I provide motivation for students with positive thinking strategies and frequent 'pep' talk. I believe that all students are capable of learning mathematics at their own pace. Differences in rate of learning, I think, are due in part to variation in students environment, general mathematical experiences, and orientation towards the subject. I am a great believer in using concrete representations, models, diagrams, specially designed teaching aids, real life applications, and personal mathematical experiences to demonstrate concepts in mathematics. My lessons are well planned and my wide range experiences in science and engineering enable me to draw on these experiences to add meaning and practicality to mathematics teaching. My students relate well to my style. My temperament and educational background are well suited to teaching mathematics. In addition my Christian convictions influence my caring attitude and concern for students which I think motivate them to do well in mathematics.

VIEWS OF THE RESEARCHER

The Grade 8 mathematics class, in which I implemented the Interactive Teaching Model, consisted of forty-nine students. The class met three times per week; each class period was sixty minutes in length. The treatment began with the introduction of the unit on Properties of Arithmetic and ended four months later with the completion of the unit on Measurement. Throughout the study I made a concerted effort to apply both the lesson format and the spirit and essence of the treatment.

During a conversation with members of the mathematics department two weeks after the study began, I remarked that the Interactive Teaching Model is not a radical departure from my usual way of teaching mathematics, but in the past I have not been as mindful of time scheduling for different parts of the lesson as I am now doing with this Model.

> I usually do not limit myself to a specific time for homework. I am usually guided by students' needs. There are times when it may be necessary to spend

> up to fifteen minutes on home work and there are other times when five minutes is all that is needed. Now I am much more organized and I keep within the prescribed time for homework. This allows me to teach much more efficiently as I am able to time my lessons and cover planned work that I would not have otherwise completed normally.

As a part of the treatment, I divided the class into seven groups of six and one group of seven. I used the students' grades which reflected their achievement on the pre-test and their average mark in Grade 7 as a guide for the grouping of students. For each group I chose a high-achieving student, a low achiever and four students with average marks. Students were distributed equitably in groups by sex. In addition, every effort was made to avoid placing more than one poor attendee in a given group. For the most part, these groups remained the same except in two instances where two students were reassigned to other groups due to specific social problems.

Prior to implementing the model, I discussed my expectations with the class. I asked the students to maintain orderly notebooks, with homework assignments clearly dated and referenced to their mathematics textbook where appropriate. The students were made aware that they would be held accountable for their homework. Students were told that their group work was to be graded as well. Each member of the group was to be responsible for recording the work that the group was asked to complete. The class was told that any member of the group could be called on to hand in his/her work and that all members of the group would receive the same grade. These expectations were summarized on a handout and the students were asked to retain it for reference.

I noted in my journal a few 'teething pains' associated with the Model on the completion of Unit 1.

> I tend to be over concerned with completing each stage of the lesson on time and indeed the entire lesson. This seems to result in a kind of mechanical and artificial approach to teaching which sometimes contributed to the neglect of students questions, discussions and general input.

Another of my initial concerns was the time taken in lesson preparation; however I believe that the time spent in preparation is worth it and it may become more feasible if planned cooperatively with teachers of the same grade level. Indeed I noted later that it became a workable routine after eight weeks of planning these lessons.

From the observation of the two teachers who visited my class on a weekly basis during the first four weeks, it was noted that I began the lessons with daily reviews and homework check. The solutions were written on the blackboard quickly. Questions with which the students had difficulty, were discussed, and time allowed for the students to correct their written work and ask questions. Periodically, students were asked to hand in homework questions for grading.

I limited the amount of homework I assigned to what I estimated the majority of the students could complete successfully in fifteen minutes. I did not include difficult questions in the homework; non routine and challenging questions were reserved for group discussion. There was excellent compliance to the homework with an average of ninety percent of students doing their homework. This is in marked contrast to students in the control group who had a compliance of an average of seventy percent. Interviews and comments from students suggest that homework was valuable to them as a way of assessing their own learning, and hence the motivation to complete their homework. However, students generally felt that homework should not be too difficult as this could lead to discouragement and frustration. One student made a most profound comment about homework. He said,

> I don't mind doing homework as it helps me to see where I am going wrong and helps me to always review for my tests. But I do not like it when teachers give homework that is difficult and I can't do it. When this happens I get discouraged and cry sometimes.

During the lessons observed, it was reported that I followed the homework with oral exercises. These oral activities served a dual role; setting the scene for the day's lesson and also reviewing key concepts of the previous lesson. For example in the lesson on *simplifying like and unlike algebraic terms*, I reviewed orally the key elements of the previous class on *integers*. Specifically I reviewed:

1. Adding a positive integer to a positive integer
2. Adding a negative integer to a negative integer
3. Adding a positive integer to a negative integer
4. Adding a negative integer to a positive integer

These properties were prerequisites for the current day's lesson. Subsequently, students were able to do a question like $3a + 4a - 2a + 7a - 11a$ without great difficulty.

When interviewed about my impressions of oral work, I replied:

> It gets them 'going' and focuses attention to the task at hand which is the day's lesson. It provides an excellent link between the last lesson and the present lesson since it usually takes the form of review of essential elements of previous lessons.

Within the development segment of the lessons, I focused on teaching with meaning and for understanding by referring to real-life and practical examples when appropriate. This was noticeable when I taught the topic on *Commutative and Associative properties of Arithmetic*. The lesson began with a novel introductory example in which I used a chart which displayed two columns of instructions as shown below.

A	B
1. Put on your socks and then put on your shoes.	Put on your shoes and then put on your socks.
2. Kill the snake and pick it up.	Pick up the snake and kill it.
3. Walk six paces East and then three paces North.	Walk three paces north and then six paces East.
4. Add 21 and 32.	Add 32 and 21.
5. Divide 8 by 3.	Divide 3 by 8.

Students were asked to examine each problem and to say if the results are the same in column A and column B. I then discussed the results with the students. At this stage I introduced the term *commutative property* and gave the evolving definition to students. That is, *an instruction is called commutative if changing the order gives the same result.*

Selected students were then asked to tell which of the above examples were commutative giving reasons for their choice of answer. I then proceeded to develop the *commutative property of addition* by referring to the check out counter of a supermarket where the cashier checks out grocery items randomly but the total is the same, irrespective of the order in which he/she checks out the items.

I commented in my journal on the idea of teaching for meaning and for student understanding. I said,

> teaching with meaning is an essential ingredient of any classroom practice and even more so in mathematics where students often do not see the connection between mathematics and reality. It is imperative that mathematics teachers teach with meaning and for understanding so as to facilitate long term learning, higher order learning skills, and to foster positive attitudes in students for mathematics.

Expanding my idea of real life applications, I see it as important to teaching with meaning and this should be incorporated into mathematics lessons when appropriate. Included in my journal is another account of a particular lesson in which I said students were extremely motivated and ecstatic about the way the introductory lesson on integers related to their every day lives. I remarked that several students told me how interesting and motivational that lesson was for them.

The introductory part of the lesson began with identifying integers as positive and negative whole numbers including zero. This was followed by an oral discussion between George and students on the importance of integers to our every day lives. Some of the findings generated from the discussions are listed below:

1. Temperature as either positive or negative. For example the average temperature in Jamaica is between +28 to +32 degrees Celsius whereas in Alberta, Canada it ranges from about -30 degrees Celsius to +10 degrees Celsius in winter.
2. Distance as positive or negative. For example above or below sea level. Heights above ground level as positive and depths below ground level as negative. The idea of upstairs and basement came into the conversation as well as forward and backward and East versus West.
3. Banking—deposits as positive and withdrawals as negative. Also saving as positive and a loan as negative. We also examined key words that imply positive and negative respectively. These are surplus versus deficit, prepayment versus arrears, gain versus loss, up versus down, acceleration versus retardation, positive growth versus negative growth (see lesson # 7 in Appendix 1).

This kind of oral introductory activity, shows how real life applications could be used to enhance teaching mathematics with meaning. This could provide motivation for many students who see mathematics as distant from reality.

After the completion of the unit on Basic Algebra, in the ninth week of the programme, I made a most revealing comment in my journal which initially seems to contradict my former view on teaching with meaning. On close scrutiny, however, it seems to be a balanced view on teaching with meaning and essentially places into context and perspective the idea of teaching with meaning. I said,

> On one hand, to teach mathematics with meaning is desirable and commendable. However, there are times when it seems that meaning has to give way to the development of skills and procedures as this is simply the most efficient way to teach certain topics in mathematics. I have observed that high ability students appreciate teaching with meaning more than low ability students. Low ability students often get frustrated with teaching with meaning and would rather stick to prescribed procedures.

The following day I recorded a similar argument in my journal. This indicated an obvious sustained reflection on the development of skills and its possible connection with teaching for meaning. I said,

> I wonder if there is a connection between learning algorithms and mathematical understanding, and if so, which should come first. I would like to suggest that the convention of mathematical understanding preceding algorithms and the learning of skills may not be feasible and practical in many Jamaican classrooms where there are fifty students in a small overcrowded classroom facing an overworked teacher. In this context it is imperative that the teacher begins by focusing on specific skills and possibly incorporate meaning when algorithm proficiency is achieved. Starting the lesson with teaching for meaning may frustrate and lead to confusion and discouragement of weaker students.

I feel that there must a realistic balance between teaching with meaning and the teaching of skills and procedures. Drill and practice should not be viewed negatively as these are essential ingredients for skill development. Activities with meaning should be followed with practice exercises for skill development. However once skills are in place the teacher should provide students with the opportunity to explore the material further to foster a deeper understanding of concepts. Also, students who are weak in skills may initially have difficulty in abstracting meaning, but may develop this once algorithmic proficiency is in place (see Sigurdson, 1995). I feel that examinations should reflect not only algorithmic expertise but also understanding. My enthusiasm for mastery of content through understanding is tempered by the realization that, for some students, full understanding of mathematical concepts or procedures develop over time.

I pointed out a situation where the teaching of skills was necessary with relatively small amount of meaning. This lesson was on *Substitution in Algebraic Expressions*. Apart from a general introduction in which the students and teacher discussed the meaning of the word substitution as applied in every day life, and its relationship to this topic, the remainder of the class and succeeding classes were spent on developing the skill of substitution (see lesson # 20, appendix 1).

After the development segment of alternate lessons, the students worked in groups to which they were assigned. I either handed each group their assignment or in some cases wrote it on the chalkboard. I moved around each group to observe students working together, sometimes offering hints and suggestions when necessary and appropriate.

I noted some initial difficulties with the group work and commented extensively on these in my journal. I expressed the view that:

> students seemed not to appreciate working in groups, they would rather work as individuals. I gather from my conversations and interviews with them that over the years they have been socialized to work as individuals. One student remarked, "I was flogged by the teacher for asking my friend in class to help me with my maths problem. She said I was 'taking copy.' I should work on my own." Other students tell me that most of their maths teachers tell them to work math problems on their own, because in the test and examination they will not get any help from other students. In fact one student said one teacher told her that, "every tub will have to sit on its own bottom." This I gather means independent effort in achieving academic goals.

I tried to reeducate them of the need to work together especially in the light of our modern world where dialogues, conversations, and committee meetings are the norm in the work environment. I pointed out to them that often great ideas are born when people work together and pool ideas. I told them about my experience of working on various committees and the benefits of sharing ideas as well as listening to other peoples' ideas. I recorded in my journal that there was some improvement in terms of cooperation/ collaboration in the groups but not to the level desired. Added to this problem was the large number of students in the class and in each group.

It was noticeable that some members of the group tended to contribute much more than others and that students who were not contributing as much as they should to the group effort were often reprimanded by other members of the group. This resulted in greater compliance and more dedication to the group work by individual students.

My overall impression of the Interactive Teaching Model was positive. Students were generally more actively involved in the lessons than in other traditional methods. There was greater efficiency in terms of students and teacher staying on task and trying to get through the specific content for each class. However, there is a level of artificiality and prescriptiveness to this model which does not cater to the dynamics of classroom teaching. In particular, although I felt that I would continue the ten-minute oral review and the strategies for handling homework, I would not necessarily limit, as rigidly, the time allocated for addressing students difficulties on assignments. I intend to use group activities regularly, but not necessarily on a daily basis or in alternate classes. This is because of content, curriculum constraints, and the preparation time necessary. I attributed the relative success of the group work to the receptiveness and positiveness of the class. I feel that group work with cooperative/collaborative elements may not work in many typical Jamaican schools where there are space limitations and a large number of students in classes.

Although I considered the times recommended for the lesson segments reasonable, I consider them confining. I would prefer more flexibility in setting my own agenda for the class.

The experience of planning several lessons, made lesson preparation easier, but nonetheless time consuming.

VIEWS OF THE STUDENTS

Towards the end of the study, that is after twelve weeks into the programme, I noted from interviews with students that they were getting accustomed to working in groups and seemed to be finding it rewarding and fulfilling. However there were some students who found it inefficient and ineffective for learning mathematics. This initial observation was later confirmed during interviews with a sample of ten students. They essentially conceded that although they found the cooperative/collaborative segment unfamiliar and uncomfortable to them at first, they later found it quite useful in learning mathematics and developing positive attitudes towards the subject. But it did not allow them to work many problems quickly.

This fact is best illustrated by reporting parts of transcripts of students interviewed about group work.

Interview 1

Researcher: What are your impressions of working together with other students in groups on mathematics problems?

Chris: I find working in groups with other students not my style. This is because they are not cooperative and they idle my time. I like working alone because I get through more work.

Researcher: Do you think that there are any benefits to be had from working on math problems in groups?

Chris: Yes, I think there could be benefits if students are cooperative and work together. There were times that we worked together and it was fun.

Researcher: Could you give me an example of the time when you had fun working together.

Chris: Let me see. Can I use my notebook to look at the problem?

Researcher: Certainly, go ahead use your book.

Chris: Yes, this was on Integers. "A traveller in Jordan descended 500 m from a hill 430 m above sea level. What was his/her height above sea level?" Each member in my group worked the problem alone then we discussed our answer. Two of us got *positive 70 m* for our answer and the other members of my group got *negative 70 m*.

Researcher: Interesting go on ...

Chris: They explained with a drawing of the hill of 500 m and the distance 430 m above sea level. And instead of 500 m – 430 m = 70 m, it is 430 m – 500 m = -70 m. We also discussed the use of the terms *above sea level* as against *below sea level*. If it was below sea level then it would have been 500 m – 430 m = 70 m.

Researcher: Do you think you could have learned the same concept if you were taught by a teacher.

Chris: Maybe, but in this situation, I was a part of the learning and that made it fun for me, and I think I will always remember this situation.

This interview with Chris shows the negative and positive sides of the cooperative/collaborative group work. Some of the major difficulties apart from the undesirable large size of the groups, had to do with the slow progress of work covered and students not staying on task and actively contributing to the group work. However, in the same group there were instances in which Chris found the group work beneficial and actually contributing to real learning of mathematics.

In an interview with another student, Sandra, some interesting comments were made by her which could point to one of the key advantages of group work.

Interview 2

Researcher: What are your impressions of working together with other students to solve problems in mathematics?

Sandra: Well, group work is something new to me. Not many of my teachers give us group work. I found the group work to be very beneficial to me.

Researcher: In what ways was it beneficial to you?

Sandra: I learn better from other students as I am more acquainted with them and when some of us in the group don't understand the problem, getting the solution approach from a student will help me to understand it better.

Researcher: What other benefit did you get from working with the group?

Sandra: I get to hear how other students express themselves about mathematics and how they are thinking about the solutions to the maths problems. Sometimes I learn better when my classmates explain things to me than listening to a teacher.

This interview with Sandra seems to point to the potential benefit that could be gained from students listening to each other explain how they would solve a particular problem and generally explore different solution strategies and share mathematical experiences with peers as a way of adding meaning to the learning of mathematics.

I commented in my journal about how teachers need to imagine and think about the problems students have in learning mathematics. My comments seem to be based on my observation of how students learn mathematics in the cooperative/collaborative setting.

> I wonder if we as teachers really understand how students learn mathematics. I think there is a real danger that, because we have been teaching mathematics for a number of years and have become competent with the content, we may be blinded to students' difficulties and realities in understanding and interpreting mathematics. We sometimes feel that students should think like we do and possess the same mathematical maturity and understanding.

At the end of study, nine students from the class were selected from those who volunteered to be interviewed. I identified three high achievers, three average achievers and three low achievers, with a view to obtaining a wide range of opinions.

The first question asked for the differences they observed in the way the lessons were taught during the treatment as opposed to what had been done in class prior to the study. Table 5.1 shows how the students responded to this question. The table indicates that at least 78% of students saw differences in lesson organisation, group work and review but little difference on homework.

Table 5.1

Number of Students Identifying Differences in Treatment From Regular Classroom Routine

Difference	Treatment class (N = 9)
Lesson Organization and Presentation	9
Group work	9
Homework	3
Review	7

The following are comments from students when interviewed about their impressions of the Interactive Teaching Model. One student remarked,

> I think this Interactive Teaching Model is a good one as it makes mathematics classes fun. Plus I get a chance to share my ideas with other students and I get help when I need it. Also I am getting to see why we do mathematics and how it can be useful to me.

Another student commented that,

> The class is well organized and Sir makes the class interesting by using charts and practical examples, and shows us how mathematics can be useful to us. Homework and group work help me to check if I am learning. Because sometimes I think I understand when I am listening to Sir but when I can solve the problems then I am certain that I understand.

A third student criticized the group work. He said,

> Although I can see where the group work can be useful in terms of sharing knowledge and ideas, it was not very beneficial to me. Firstly the students in my group were uncooperative and sometimes did not do any work and chatted about other things. As a result I did not get through with as much work as I do when I work by myself.

One student made a most pragmatic comment about group work which touches the root of the problem of teaching with meaning and group work in a Jamaican context. He said,

> I like the group work, it is a wonderful opportunity to learn mathematics with friends. But I have a problem with this method, it tends to be a slow way of

> learning and when we get tests, we are examined only on calculations and not on some of the things we do in groups.

One student made a most revealing comment about her new found confidence in mathematics as a result of her experience with the Interactive Teaching Model and particularly her experience with my personal way of teaching mathematics. She said,

> I used to hate mathematics because I found it difficult to understand and I get easily frustrated when this happens. As a result I have been failing my mathematics tests. Now because of Sir and his interesting way of teaching, I have grown to like mathematics, and more than that, I have passed all my mathematics tests this term. Also, Sir is very caring and always talks to me and encourages me to try my best in my studies.

Seven of the nine students felt that the content taught in the five units was easier to understand and with more meaning than the content taught previously. These students gave the following reasons for this achievement:

1. The greater depth to which content was taught,
2. Feedback in group work and daily homework assignment,
3. Quality of teacher explanations,
4. Daily reviews,
5. Teaching with meaning,
6. Use of concrete representations and real life applications, and
7. Teacher's caring attitude which proved motivational to students.

VIEWS OF THE TEACHERS

At the end of the study, three teachers who visited my classes were interviewed about their impressions of the Interactive Teaching Model (ITM). This is based on their one visit per week observation of my class in which I used this model. The interview examined their impressions of the model, their observations of students reaction, and the elements of the treatment program that they would incorporate into teaching of their mathematics classes (see questions used for the interview on pages 26–27). A summary of their views is included in Table 5.2.

Table 5.2

Teachers' Observation of Interactive Teaching Model

Question	Attribute	Ann's Views	Sharon's Views	Don's Views
1	ITM vs. traditional method	Difference	Considerable difference	Little difference
2	Oral work	Possible advantages, but situational	Advantages	Distinct Advantages
3	Homework	Strongly recommended	Strongly recommended	Strongly recommended
4	Time preparation of ITM vs. Traditional	ITM time consuming, and unrealistic for most teachers.	ITM definitely time consuming, but workable.	ITM time consuming but possible.
5	Teaching for meaning	Teaching for meaning difficult but necessary at times.	Teaching for meaning is challenging and situational.	Teaching for meaning is recommended but often not practical and possible.
6	Teaching for meaning vs. teaching for skills	Teaching for meaning should be related to teaching of skills.	Both are necessary for complete understanding but often not easy to do.	Teaching for meaning should precede teaching of skills.
7	Cooperative/ collaborative practice	Students not entirely comfortable with this method of instruction.	Students made good attempt to work in groups. Some students seem to have enjoyed group work especially the more extroverted students.	Students tried to work in groups. Girls were generally more cooperative and seemed to have benefited more from group work.
8	Possible difficulties with Cooperative/ collaborative practice	Unaccustomed teaching strategy but could be useful. Students seem not to have preferred this method.	Students are not socially oriented towards group work but made good adjustment to the group work. Excellent supervision and monitoring of groups by teacher.	A good method of teaching but works best with ideal classroom conditions, unlike our typical Jamaican situation.
9	Strengths of ITM	It has a good blend of traditional method and constructivist elements.	It seems to be an efficient and organized way of teaching mathematics.	Well planned and structured way of teaching mathematics. It keeps teachers and students accountable.
10	Weaknesses of ITM	It is prescriptive and requires time for preparation.	It is restrictive and inflexible.	It requires above average time for planning and preparation.

Table 5.2 does provide a good indication of how the three teachers felt about the Interactive Teaching Model. However there are other views expressed by these teachers that cannot be appropriately summarized in Table 5.2. These will be dealt with now.

Ann is of the view that the Interactive Teaching Model is prescriptive and may not work in all situations, and that some aspects of it such as cooperative learning are not appropriate to the typical Jamaican setting. She acknowledged that it seemed to have worked for the treatment teacher mainly because of great effort on his part and his exceptional teaching skills and experience. This could be quite different with relatively inexperienced teachers and older teachers who are entrenched in their own ways of teaching.

Sharon agrees with the basic principles of the Interactive Teaching Model but says that she would not accept it "wholesale" but she would incorporate aspects of it that can work in her classes. For example homework, individual/cooperative seatwork, and oral reviews are things that she has used in the past and will continue to use as teaching situations dictate.

Don feels that the interactive teaching methodology is a good one since it incorporates teaching for understanding while at the same time considering skills acquisition. It also has the cooperative/collaborative learning segment which facilitates students discussion on problem solving type mathematics.

However he feels that this model is not entirely suitable for a Jamaican setting in which there is an average of fifty students to one teacher. This makes it difficult for grouping and supervision of students. He also says that most of the affective skills that are emphasized in group work and teaching with meaning are not tested in most examinations, especially external ones. Consequently students are oriented towards algorithm practice and focus almost exclusively on this aspect of learning. He feels that this methodology would be better suited for primary schools and Junior High schools where most of the examinations are internal and teachers have greater autonomy in interpreting and enriching their teaching.

SUMMARY

The Interactive Teaching Model is being used by some teachers in Jamaica but not to the extent and details specified by this model. Participating teachers find the time limits a hindrance to the free flow of classroom dynamics particularly as it relates to students and teacher behaviours, but they acknowledge that it does provide a worthwhile guideline for teachers to achieve greater efficiency in teaching mathematics.

On the basis of the comments from participating teachers and students the model was successfully implemented. Students interest was sustained throughout the four-month period with oral activities and novel introductions coupled with the interactive activity based lesson development. The lesson development which incorporated a good blend of real-life applications, integration across the curriculum, concrete representations of mathematical concepts, problem solving in cooperative/collaborative group situations were hallmarks of the model which enhanced meaning and depth in mathematics.

Cooperative/collaborative group work though not fully endorsed by students in the initial stage of the study, gradually found favour with the majority of students who later testified of its contribution to their learning and a manifest desire to work cooperatively in future classes if the opportunity is given to them by their teachers.

Finally participating teachers and students are supportive of this model but express reservation of accepting it "wholesale." Indeed changes would of necessity be needed, to take into consideration local curriculum design and orientation, cultural and social practices in schools, and school climate and environment.

CHAPTER 6

QUANTITATIVE RESULTS

OVERVIEW

The quantitative data for the study was obtained over a twelve-week period during the first term of the 1995-1996 school year. The results presented in this chapter are based on classroom observation, pre- and post-testing, and an attitude questionnaire. The data was obtained from three grade eight mathematics classes—one treatment group and two control groups. These classes were taught by one teacher (researcher) and by one teacher who continued to use his regular classroom routine. The teacher of one of the control groups has been teaching mathematics for the past seven years and holds a B.Sc. degree in mathematics/science.

Three research questions are addressed in this chapter:

1. To what extent did the treatment teacher implement the Interactive Teaching Model, and was he doing something different from the control teacher?
2. Did the treatment class achieve higher on the unit tests and post-test than the control classes?
3. Did the Interactive Teaching Model affect students' attitude to learning mathematics?

CLASSROOM OBSERVATION

The researcher taught the treatment group and Control Group B; the other control group was taught by Mr. Daley, a teacher at this school. Observation of Mr. Daley's class was done three times during the first week prior to the commencement of study. The objective was to determine if there is a real difference in teaching between the traditional method and the Interactive Teaching Model.

The fact that the teacher of the treatment group was also the researcher meant that the daily observation schedule had to be done by the teacher. This was done in as objective and unbiased way as possible. Also the researcher evaluated each class shortly after completing each lesson to assess whether or not the model was followed.

In order to be as objective as possible, three teachers who visited the researcher's class once per week were asked to assess the class by using an observation rating scale. In addition, they were asked to record the time the classes spent on oral work, daily review homework, lesson development, and individual/cooperative practice, and an estimate of the expected time students would require to complete homework assigned.

Each of the lesson segments was scored on a four-point implementation scale, according to the extent each matched the expectations of the Interactive Teaching Model.

Implementation Scale

0 Not attempted
1 Some implementation
2 Implementation incomplete
3 Fully implemented

At the end of each lesson, the extent to which the teacher had attempted to develop student understanding through combinations of worked examples, real-world applications, concrete representations, and problem solving activities was scored on a meaning scale.

Meaning Scale

0 Not attempted
1 Some attempt
2 Attempt but not complete
3 Completed effectively

Tables 6.1 and 6.2 illustrate the classroom practice of two control teachers based on ten classroom visits to each teacher's class. It is to be noted that even though the teachers did not use the Interactive Teaching Model, they were nonetheless competent, experienced and well qualified to teach mathematics.

The control teachers varied daily in their classroom activities; although most classes began with a discussion of previously assigned work, algorithmic approach to the lesson development, followed by individual seatwork and ended with homework. Those students who did not finish their assignments within the class time were expected to complete them as homework. Neither of the two control teachers used oral work or cooperative/collaborative practice.

On the implementation scales for Review, Homework, Development, and Meaning, the control teachers were rated between " Not attempted" and "Attempted but not complete."

Table 6.1

Time (Minutes) Spent on Each Segment by Control Teachers—10 Lessons Observed

Teacher	Oral Work	Review	Home-work	Develop-ment	Cooperative Practice	Homework Assigned
Group B Mean	0.0	7.2	13.0	37.8	0.0	10.4
Group C Mean	0.0	8.0	12.0	39.5	0.0	12.9

Table 6.2

Implementation Scale for Lesson Segments for Control Teachers—10 Lessons Observed

Teacher	Oral Work	Review	Home-work	Develop-ment	Cooperative Practice	Homework Assigned
Group B Mean	0.0	0.60	1.65	1.25	0.0	1.56
Group C Mean	0.0	0.69	1.61	1.20	0.0	1.65

Table 6.3 and 6.4 illustrate the classroom practice of the treatment teacher during the implementation phase of the study. The teacher implemented the Interactive Teaching Model.

Table 6.3

Time (Minutes) Spent on Lesson Segments by Treatment Teacher—10 Lessons Observed

Teacher	Oral Work	Home-work	Review	Develop-ment	Cooperative Practice	Homework Assigned
Treatment Mean	5.82	4.14	3.63	26.1	23.0	15.0

Table 6.4

Implementation Scale for Lesson Segments for Treatment Teacher—10 lessons Observed

Teacher	Oral Work	Review	Home-work	Develop-ment	Cooperative Practice	Meaning
Treatment Mean	2.62	2.26	2.45	2.86	2.10	2.98

The treatment teacher structured his lessons closely to the recommended times for different parts of the lesson. This is indicated by the mean times for each part of the lessons in Table 6.3. The treatment teacher was rated between "Attempted but not complete" and

"Fully implemented" based on the Implementation Scale from 0 to 3. This is shown in Table 6.4.

Table 6.5 demonstrates the differences between the treatment teacher, and the control teachers based on the Implementation Scale.

Table 6.5
Comparison Between Treatment and Control Teachers on the Implementation Scale

Group	Oral Work	Review	Home-work	Develop-ment	Cooperative Practice	Meaning
Treatment Mean	2.62	2.26	2.45	2.86	2.10	2.98
Control B Mean	0.00	0.65	1.67	1.23	0.00	1.61
Control C Mean	0.00	0.58	1.23	1.56	0.00	1.05

The comparative results from Table 6.5 and Figure 6.1 indicate that the treatment teacher was rated between "Attempted but not complete" and " Completed effectively." The implementer had not used cooperative practice or oral work as a part of his daily routine prior to using the model.

The control teachers did not change their classroom practice during the study. None used oral work or cooperative practice as defined in this model. On the Implementation Scale for review, homework, development, and meaning, their classroom practice was rated, on average, between "Not attempted" and "Attempted but not complete."

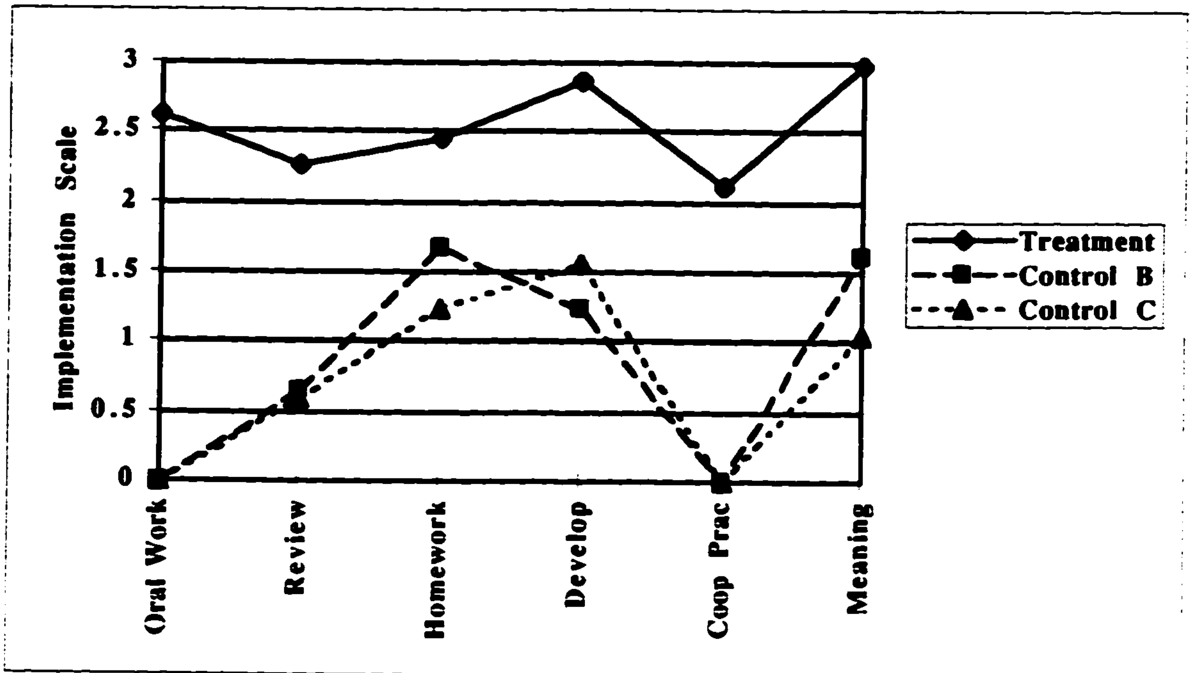
TEST RESULTS

This section deals with the achievement differences between students in the treatment group and the control groups.

At the beginning of the study, a twenty-item multiple choice survey test was administered to the treatment class and the two control classes. The intent of this pre-test was to determine the equivalence of the treatment group and control groups. The items on the pretest were based on the Grade 7 mathematics curriculum which students would have been taught prior to the implementation of the treatment programme.

Figure 6.1

Comparison Between Treatment and Control Teachers on Implementation Scale



In addition, five unit tests were administered to these groups during the four month period to check students' achievement. These tests were done by all Grade 8 students, and were prepared collaboratively by Grade 8 teachers (including the researcher). Also, a post-test based on the content of the course was administered to these groups.

A *One-Way Analysis of Variance (F-Tests)* was employed to determine significant differences between and within the means of the treatment group and the control groups on the pre-test, unit tests and the post-test.

Table 6.6 shows the results for the pre-test.

Table 6.6

A comparison of Pre-Test Scores for Treatment Group and Control Groups

Group	N	Mean	Standard Deviation	Standard Error	95% Confidence Interval for Mean
Treatment	49	54.3878	10.9274	1.5611	51.249 to 57.527
Control B	49	53.4694	12.4668	1.7810	49.889 to 57. 050
Control C	48	53.8542	11.0241	1.5912	50.653 to 57.055
Total	146	53.9041	11.4243	0.9455	52.035 to 55.773

$F = 0.0788$; $p < 0.9242$, hence null hypothesis is tenable.

Table 6.6 supports the null hypothesis that, on the basis of the pre-test scores, no two groups are significantly different at the 0.05 level.

Table 6.7

A Comparison of Unit Test #1 Scores for Treatment and Control Groups

Group	N	Mean	Standard Deviation	Standard Error	95% confidence Interval for Mean
Treatment	49	57.65	10.9735	1.5676	54.50 to 60.81
Control B	49	49.74	7.1123	1.0160	47.70 to 51.79
Control C	48	50.52	9.9861	1.4414	47.62 to 53.42
Total	146	52.65	10.0868	0.8348	51.00 to 54.30

$F = 10.3001$; $p < 0.0001$, hence the null hypothesis is not tenable.

Table 6.7 rejects the null hypothesis. That is, on the basis of scores, there was a significant difference in performance between the treatment group and the two control groups (B & C). The treatment group outperformed both control groups. There was no significant difference in performance between the two control groups.

Table 6.8

A Comparison of Unit Test #2 Scores for Treatment and Control Groups

Group	N	Mean	Standard Deviation	Standard Error	95% confidence Interval for Mean
Treatment	49	56.4286	8.0526	1.1504	54.1156 to 58.7415
Control B	49	55.6633	6.4907	0.9272	53.7989 to 57.5276
Control C	48	56.3021	8.9768	1.2957	53.6955 to 58.9087
Total	146	56.1301	7.8514	0.6498	54.8459 to 57.4144

$F = 0.1319$; $p < 0.8765$, hence null hypothesis is tenable.

Table 6.8 supports the null hypothesis that, on the basis of the pre-test scores, there was no significant difference between and within the three groups on Unit Test 2.

Table 6.9

A Comparison of Unit Test #3 Scores for Treatment and Control Groups

Group	N	Mean	Standard Deviation	Standard Error	95% confidence Interval for Mean
Treatment	49	59.0306	5.5859	0.7980	57.4262 to 60.6351
Control B	49	66.5816	6.7449	0.9636	64.6443 to 68.5190
Control C	48	65.1596	6.1105	0.8913	63.3655 to 66.9537
Total	146	63.5690	6.9581	0.5778	62.4268 to 64.7111

$F = 20.6845$; $p < 0.0000$, hence the null hypothesis is not tenable.

Table 6.9 rejects the null hypothesis. That is on the basis of the scores, there was a significant difference in performance between the treatment group and each of the control groups in favour of the control groups. There was, however, no significant difference in performance between the two control groups.

Table 6.10

A Comparison of Unit #4 Test Scores for Treatment and Control Groups

Group	N	Mean	Standard Deviation	Standard Error	95% confidence interval for mean
Treatment	49	67.1939	6.5278	0.9325	65.3189 to 69.0689
Control B	49	64.9494	6.6436	0.9491	63.0407 to 66.8573
Control C	48	60.5208	4.9989	0.7215	59.0693 to 61.9724
Total	146	64.2466	6.6719	0.5522	63.1552 to 65.3379

F= 14.9484; $p < 0.0000$, hence the null hypothesis is not tenable.

Table 6.10 rejects the null hypothesis. On the basis of the scores, there was a significant difference in performance between and within the three groups. The mean for the treatment group was higher than for the two control groups.

Table 6.11

A Comparison of Unit #5 Test Scores for Treatment and Control Groups

Group	N	Mean	Standard Deviation	Standard Error	95% confidence Interval for Mean
Treatment	49	71.5816	11.3960	1.6280	68.3083 to 74.8549
Control B	49	64.1837	8.6961	1.2423	61.6859 to 66.6815
Control C	48	57.9688	9.5880	1.3839	55.1847 to 60.7528
Total	146	64.6233	11.3538	0.9396	62.7661 to 66.4805

F = 22.7217; $p < 0.0000$, hence null hypothesis is not tenable.

Table 6.11 rejects the null hypothesis. That is, on the basis of the scores, there was a significant difference in performance between and within the three groups. The differences were in favour of the treatment group.

Table 6.12

A Comparison of Post-Test Scores for Treatment and Control Groups

Group	N	Mean	Standard Deviation	Standard Error	95% confidence Interval for Mean
Treatment	49	66.8980	12.3495	1.7642	63.3508 to 70.4452
Control B	49	51.1837	7.9783	1.1398	48.8920 to 53.4753
Control C	48	51.1837	7.4832	1.0801	49.5354 to 53.8812
Total	146	51.7083	11.9741	0.9910	54.6715 to 58.5888

$F = 42.7770$; $p < 0.0000$, hence null hypothesis is not tenable

Table 6.12 compares the relative performance of the treatment group and control groups on the post-test. It indicates significant difference in performance between the treatment group and the control groups in favour of the treatment group. There was, however, no significant difference in performance between the two control groups.

Table 6.13 provides a summary of Tukey-B tests for pre- and post-tests and unit tests. It indicates that there were significant differences in performance at the 0.05 level on all tests except the pre-test and Unit Test #2.

ATTITUDE QUESTIONNAIRE

This section addresses the issue of attitude differences to mathematics, if any, between the treatment group and the control groups.

At the end of the study, a twenty-five item Likert-style questionnaire was administered to all students in both the treatment class and control classes. Each item consisted of a statement followed by four possible levels of agreement. Each item was scored from 1 to 4 on the following basis:

Never = 1, Seldom = 2, Often = 3, and Always = 4.

Eight factors were obtained from combinations of those items on the questionnaire which were most closely related and which were applicable to specific components of students' attitude towards mathematics. Table 6.14 outlines these factors, and the items which comprise them.

Table 6.13

Tukey-B Test Summary for Groups

(A = Treatment; B = Researcher Taught Control; C = Other Control)

Tests	Groups (N = 146)	F Ratio	F Prob.	Significant Difference	No Significant Difference
Pre-test	A and B				✓
	A and C	0.0788	0.9242		✓
	B and C				✓
Unit 1	A and B			✓	
	A and C	10.3001	0.0001	✓	
	B and C				✓
Unit 2	A and B				✓
	A and C	0.1319	0.8765		✓
	B and C				✓
Unit 3	A and B			✓	
	A and C	20.6845	0.0000	✓	
	B and C				✓
Unit 4	A and B			✓	
	A and C	14.9484	0.0000	✓	
	B and C			✓	
Unit 5	A and B			✓	
	A and C	22.7217	0.0000	✓	
	B and C			✓	
Post-test	A and B			✓	
	A and C	42.7770	0.0000	✓	
	B and C				✓
Total				12	9

A one-way *Tukey-B* Analysis of Variance was used to determine if there were significant differences in performance between the treatment group and control groups with respect to the eight factors previously identified. Table 6.15 indicates these results.

Table 6.14

Factors Obtained from Attitude Questionnaire

Factor	Description	Questionnaire Items
1	Interest in mathematics	2 5 13 14 20
2	Cooperation	6 7 8
3	Teacher dependence	11 12
4	Homework and relevance of mathematics	1 23
5	Task orientation	15 22 24 25
6	Teacher assistance	9 10
7	Independence	18 19
8	Perception towards problem solving and doing mathematics	3 4 16 17 21

Table 6.15

A Comparison of Students Attitude Scores for Treatment and Control Groups

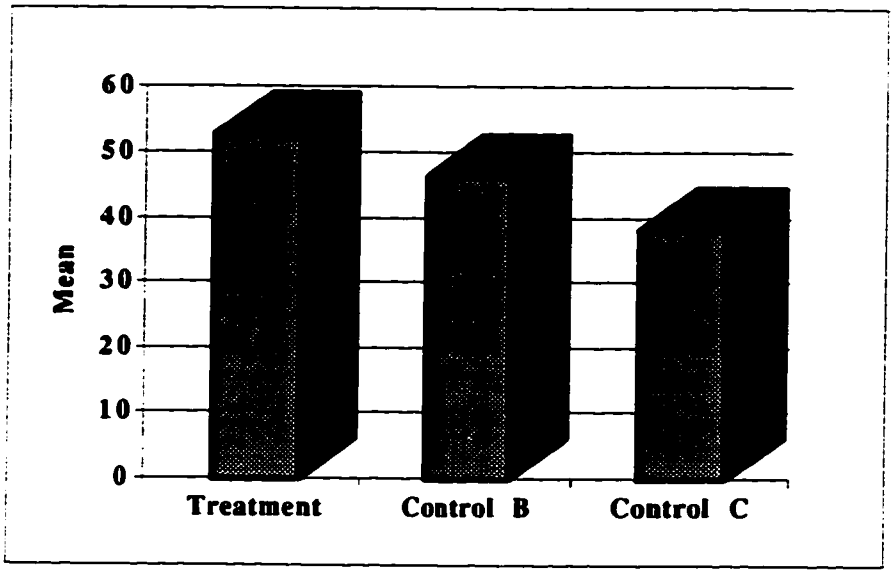
Group	N	Mean	Standard Deviation	Standard Error	95% Confidence Interval for Mean
Treatment	49	52.7347	4.3146	0.6164	51.4954 to 53.9740
Control B	49	46.1837	4.5491	0.6499	44.8770 to 47.4903
Control C	48	38.1250	4.2307	0.6107	36.8965 to 39.3535
Total	146	45.7329	7.3899	0.6116	44.5241 to 46.9417

$F = 136.0322$; $p < 0.0000$, hence the null hypothesis is not tenable

Table 6.15 indicates that there was a significant difference in performance among the three groups. The treatment group and Control Group B differ significantly, the treatment group and Control Group C differ significantly, and both control groups differ significantly.

Figure 6.2 shows the difference in means for the treatment and control groups in which the treatment group had a higher mean than the two control groups

Figure 6.2
Student Attitude Results



SUMMARY

On the basis of the analysis in this chapter, the treatment teacher implemented the Interactive Teaching Model. This implementation was a significant departure from the regular classroom routine and differed from the control teachers in areas of cooperative/collaborative practice, oral work, review, development and teaching for meaning.

The treatment group outperformed the control groups on unit tests 1, 4, and 5 and on the post-test. However, the two control groups outperformed the treatment group on unit test 3, whereas there was no significant difference in performance on the pre-test and on unit test 2. The difference in performance between the treatment group and the control groups was significant at the 0.05 level.

On the attitude scale there was significant difference between the three groups at the 0.05 level. On the basis of the higher means for the treatment group, it is assumed that the Interactive Teaching Model improved students attitude to mathematics in this group. The significant difference in means between the two control groups could possibly be a result of the mediating influence and teacher characteristics of the treatment teacher teaching Control Group B.

CHAPTER 7

SUMMARY AND CONCLUSIONS

INTERACTIVE TEACHING MODEL

Homework

During implementation the treatment teacher used the first ten minutes of class for homework check and introduction of the lesson with an oral review of the previous lesson. Some strategies that were used for homework included displayed worked solution on a side blackboard, collecting assigned work, having students mark other students homework from a teacher prepared answer key, and oral check of homework. Students having special problems with homework were assigned to competent students for extra assistance. Regardless of the method used for homework check, the students were held accountable by keeping a record of homework grades. Students, generally, completed their homework for the most part and indicated interest and benefited from engaging in homework. However, there was the feeling that although fifteen minutes was a reasonable time for homework check; there will be times when this time will be inadequate if students experienced difficulties with homework. In this case teachers will need to resolve these difficulties before commencing the day's lesson.

All teachers agreed with the general principles governing homework and saw it as a very potent form of student evaluation of their own learning and it also provides teachers with a continuous assessment of students mathematical progress. The fact that students were held accountable for completing homework seems to have significantly influenced students to complete homework.

Oral work

The treatment teacher connected the oral work at the beginning of the class to the review, using this as an opportunity to review prerequisite concepts and key elements of the previous lesson. This provided an excellent link with the lesson development. All teachers felt oral review provided a means for engaging the class quickly, getting them to think about mathematics, and providing an effective transition to the lesson development segment of the lesson.

Development

The lesson development was conducted in a whole class setting. After the lesson's objective was stated, the teacher generated discussion of new content through active questioning and discussion. Skill development was monitored and reinforced through controlled practice. Teaching for meaning was developed through the following strategies:

1. Incorporating real life applications derived from students' and teacher's every-day experiences to relevant mathematical concepts;
2. Approaching problem solving situations with a variety of solution alternatives;
3. Using concrete representations and appropriate teaching models, and
4. Emphasizing mathematical patterns, investigations, illustrations and mathematical proof.

The lesson development segment seemed to have presented the most challenge to the treatment teacher who found the time limit confining. This was especially true as non-routine or non-algorithmic examples, by their nature, are time consuming. Furthermore in an effort to foster student understanding, he would have preferred that lesson development was longer or at least more flexible with respect to time allotment. Also, some mathematics topics at high school level require more time to address adequately.

It was the consensus of teachers that although teaching for meaning within the development segment is desirable, it is not always possible, especially in a local Jamaican context with problems such as classroom environment and unsuitable student teacher ratio.

It was the feeling of participants that the success of this approach depends on the philosophy of individual teachers, their beliefs about how students learn, and how they perceive the balance between teaching for meaning and teaching for skills. They also felt that teaching for meaning cannot be completely successful unless the fundamental structure of examinations is revised to reflect not only algorithm but conceptual and affective skills.

COOPERATIVE/COLLABORATIVE LEARNING

This phase of the lesson involved students working in groups to discuss and solve problems in mathematics. Some initial problems were experienced with group work as students were unaccustomed to this method of instruction and took some time to be oriented to group work. However, after much effort from the teacher, the cooperative/collaborative segment was implemented successfully.

Students reaction to group work was generally positive except for selected students who found difficulties with this method. Some of these difficulties were the large number of students in classes, the physical size of the classroom and other environmental factors.

However, students identified some benefits of group work. These included student-to-student interaction as motivating, enhancing self-esteem, providing immediate feedback, providing the opportunity to positively interact with others, and to view alternative mathematical perspectives.

The success of group work was due, in part, to the classroom management of the teacher. He continually monitored the groups ensuring that students stayed on task, collected and graded work, and offered assistance and encouragement when required.

In spite of the apparent success of the cooperative practice, the treatment teacher and teachers who visited the classroom weekly felt that the cooperative practice which was done in alternate classes should not be followed rigidly as this method may not be best suited for certain lessons. The general idea is that it should be used with discretion by teachers.

The cooperative grouping facilitated the integration of problem solving activities into this segment of the lesson. Also, the lesson development segment facilitated the integration of problem solving, with class discussion of heuristics and alternative perspectives embedded within the lesson.

As a consequence of working with the Interactive Teaching Model for four months and listening to the feeling and opinions of participants in this research, I propose the following modified Interactive Teaching Model which may be more relevant to the Jamaican context.

PRINCIPLES OF INTERACTIVE TEACHING

Daily review and homework check: 10–15 minutes

- Deal with homework
- Students having problems with homework should be assigned to competent students for help
- Begin with oral review that reviews and reinforces previous lesson's skills and knowledge and sets the scene for the day's lesson
- Consider integrating homework with oral review if appropriate
- Oral review should usher in the development phase

Development: 25–30 minutes

- Place concepts to be taught in context with students knowledge and experience, and future problems
- Emphasize meaning through patterns, mathematical proofs, concrete representations, models, pictures, and personal mathematical experiences of students and teachers
- Introduce novel problems, when appropriate, to underscore process
- Monitor student understanding through active questioning
- Reinforce concepts and skills through controlled practice

Cooperative/collaborative practice: 15–20 minutes

- Provide opportunities in class for group and individual practice as seen fit by teacher
- Provide opportunity for successful practice
- Include problem solving and applications related to the lesson
- Assign more difficult questions in this segment rather than for homework
- Encourage active discussion
- Keep individuals accountable
- Provide opportunity for mental computation

Homework

- Assign homework relevant to the lesson
- Ensure that questions assigned can be completed successfully by the majority of students working independently
- Include a review question

Evaluation

- Unit tests, term and annual examinations
- Cumulative worksheets
- Mathematical investigations and projects
- Field trip reports
- Students' journals
- Student interviews

TEACHER CHANGE

Elements that the researcher intends to continue after the conclusion of the study include:

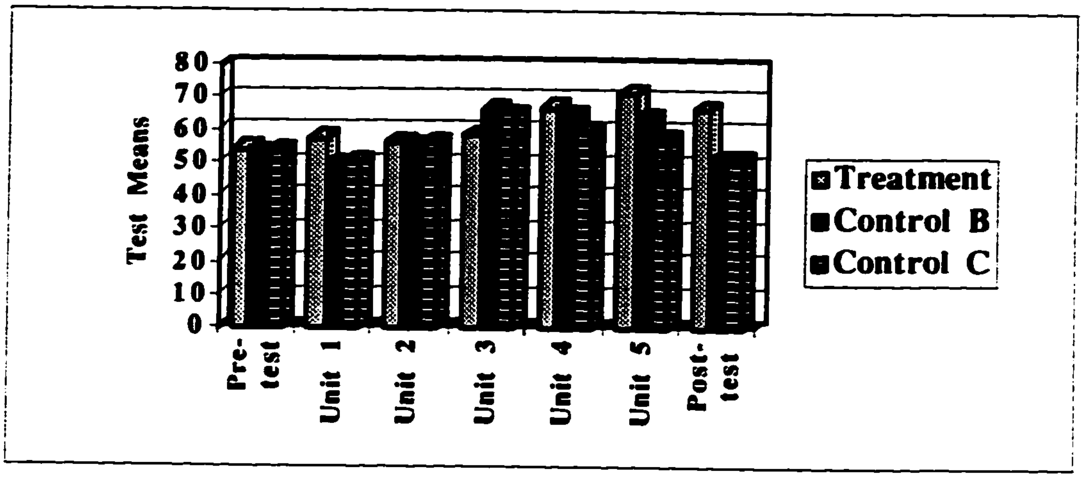
1. Strategies for dealing with homework
2. Oral work
3. Teaching for meaning—real-life applications, concrete representations, and problem solving
4. Cooperative/collaborative practice on a regular basis
5. Giving more time to practice concepts independently and within groups
6. Lesson sequence

STUDENT ACHIEVEMENT

Three types of tests were used to assess students achievement. These were a pre-test, five unit tests, and a post-test. The results of the pre-test indicated no significant difference in performance between the control groups and the treatment group. However the results of unit tests 1, 4, and 5 indicated a significant difference in performance at the 0.05 level in favour of the treatment group. The analysis of Unit Test 3 showed a significant difference in favour of the control groups. Also, the post-test indicated a significant difference in performance at the 0.05 level in favour of the treatment group. Figure 7.1 shows a comparative illustration of the treatment and control students' achievement on pre-test, unit tests, and post-test.

Figure 7.1

Comparison of Test Means of Treatment Group and Control Groups



STUDENT PERCEPTION

Seven of the nine students interviewed felt that the program enhanced their mathematical understanding and helped them learn mathematics. They identified teacher interaction, individual/cooperative practice, and homework as contributing factors. Students of the treatment group responded positively to the Interactive Teaching Model as evidenced by their scores on the attitude questionnaire.

LIMITATIONS OF THE RESEARCH

One of the weaknesses of this study was the dual role that the researcher had to assume, that is as both researcher and treatment teacher. This situation resulted when pre-selected teachers were no longer available for this research, and subsequent difficulty the researcher experienced in finding suitable replacement teachers to be trained using the Interactive Teaching Model. If the research were repeated, adequate numbers of teachers would be selected prior to the study and trained to implement the Interactive Teaching Model. Monetary incentive might be provided for participating teachers who would be asked to sign a contractual agreement.

Another weakness of the study was the relatively low reliability measures of the test instruments used in the research. For a future study, consideration should be given to pilot testing the unit tests and post-test.

IMPLICATIONS FOR RESEARCH

The results of the study suggests that the Interactive Teaching Model, even with its limitations, should be considered as one of the teaching strategies that could enhance the efficient and effective teaching of mathematics in Jamaican secondary high schools. Students' positive attitude and response to the model is of significance as components of the model such as cooperative learning and problem solving are being stressed by the recently developed *ROSE program* designed for Grades 7-9 students in Jamaican High Schools (see *Mathematics Curriculum Guide, Grade 7-9)*.

A second study involving more teachers and students to determine if results can be replicated, and indeed if the result is a function of the treatment or could it be related to the teacher's competence, enthusiasm and hard work. Also, it may be of interest to investigate if the achievement gains for the five units of study and the positive student perception and attitude of the model could be sustained over the entire course.

A follow-up study would be valuable to ascertain those elements of the model the treatment teacher continues to use regularly as part of his general teaching strategy. What implications does this have for teacher inservice training? To what extent do teachers change their teaching practice when presented with alternatives? Also, can Jamaican teachers successfully implement and use this model in the light of large class sizes, undesirable classroom conditions and the heavy work load of the average teacher.

Considering the variable and undesirable classroom conditions in many schools in Jamaica, it would be advisable for future studies using this model to assess the classroom environment of these schools. It is suggested that the Classroom Environment Scale shown in Table 7.1 be considered or a possible refinement of it.

Table 7.1

Classroom Observation Scale

Classroom Observation Scale

School _____________ Teacher _____________ Grade/Class _____ Observer _________________

I. Class enrollment: _______

II. Class configuration:

Attendance: [Number of students] ________

III. Classroom:
Multi-subject ___
Room partitioned by board ___
Room dedicated to subject ____
Room dedicated to subject/resource room/lab ___
Other ___________________

IV. Materials used by teacher:
Textbooks ____
Worksheets ____
Paper and pencil ____
Calculators (N = ____)
Hands-on materials _____

V. Displays and resources:
Printed posters: Number: ___
Types: Motivational/attitudinal: ___; Career: ________ Education: ___
Related to Subject: ___ Describe: (or teacher-made)

Student work posted: None: ___ A few students: ___ Many students: ___ Group work: ___
Observable classroom resources:
Amount of class shelves/types of books: None _____ Some ________
Describe:
Resource center/station: None ____ Yes ___ Describe:
Classroom closets/cabinets: None ___ Yes ____ Describe:

VI. Noise level:
Relatively quiet: __ Occasionally disruptive: __ Continuous disruptive: ___ Productive/busy:____
Generated by: Class: ___ Other classes __ Other external: ____

VII. Disruption due to observation:
For students: None to slight: ________ Moderate: _____ Considerable: _________
For the teacher: None to slight: ______ Moderate: _____ Considerable: _________

VIII. Irregularities/special circumstances (Describe):

(ROSE Programme, 1996)

Of note is the fact that this study was conducted with grade eight students who are not as hard pressed with preparation for the CXC examination in mathematics as the grade ten and eleven students. Could the Interactive Teaching Model work with students who are highly motivated and examination conscious?

The treatment teacher who participated in this study was well experienced with the benefit of good training and years of successful practice. He brought to the model a mature perspective that facilitated its implementation. Would less experienced, less competent, or novice teachers implement this program successfully?

Of continuing interest to teachers, school administrators, and educators are instructional strategies for enhancing student performance and attitude towards mathematics. It is possible that this research could serve to stimulate other researchers to conduct similar studies.

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APPENDIX 1

UNIT PLAN

and

LESSON PLANS

GRADE 8 COURSE OUTLINE

TERM 1 - SEPTEMBER–DECEMBER

Unit 1: Properties of arithmetic

Unit 2: Integers (Directed numbers)

Unit 3: Sets (worded equations involving 2 sets) Venn diagrams

Unit 4: Algebra—simplification, expansion, substitution in algebraic expressions

Unit 5: Measurement—non-standard and standard measures; distance, time, money, mass/weight, area, perimeter

TWELVE WEEK UNIT PLAN

Time	Topics	Outcomes	Concept	Activities	Evaluation
Two weeks	**Properties of arithmetic** - Commutative - Associative - Distributive	1. Describe and give examples of commutative, associative and distributive properties 2. Use of these properties in providing quick methods of calculations—e.g. ready reckoners.	**Commutative-** *changing the order of operation always gives the same result.* **Associative-** *when instruction contains the same operation twice, and it does not matter which one is done first.* **Distributive-** a(b+c) = ab + ac and, a(b-c) = ab-ac.	Students will examine novel problem situation to develop the meaning and concept of commutative property. E.g. shopping list, ready reckoner tables etc.	1. Individual practice 2. Collaborative group practice 3. Teacher directed questions to students 4. General observation 5. Homework 6. Two weekly tests

Time	Topics	Outcomes	Concept	Activities	Evaluation
Two weeks	**Integers** - addition - subtraction - identities	1. Use number line to provide a concept of integers 2. Use number line to demonstrate addition and subtraction of integers 3. Show practical applications of integers. E.g. thermometer. height above and below sea level etc.	Integers are positive and negative whole numbers. i.e (. . . -3, -2 , -1, 0, 1, 2, 3, . . .)	Walk forward and backward to demonstrate negative and positive numbers. Use a adding and subtracting machine to perform addition and subtraction of integers Use number line to illustrate addition and subtraction of integers	1. General observation 2. Individual seatwork 3. Collaborative practice 3. Oral questions 4. Unit test 5. Home work
Two weeks	**Sets** - sub-set. membership - set notation - Venn diagrams - interpreting Venn diagrams - logic	1. Identify sets and subsets in every day life. 2. Use set notations to describe sets in ordinary English and visa versa. 3. Use Venn diagrams to represent sets 4. Interpret Venn diagrams 5. Use knowledge of set to develop logical and illogical deductions	A set is a *collection of similar objects, things that are well described.*	List sets and subsets in the classroom. Identify sets and subsets in social life. Relate set notations to conversations in ordinary English. Deducing true and false conclusions from popular every day arguments by using sets and logic.	Same as Integers

Time	Topics	Outcomes	Concept	Activities	Evaluation
Three weeks	**Basic Algebra** Simplifying algebraic terms of the form 2a +3a +6a - 4a and 7xy + 4yx - 5xy etc. Simple multiplication with letters Expansion and simplifying of binomial expressions Substitution in algebraic expressions	1. Simplify simple algebraic terms 2. Perform simple multiplication and simplify algebraic terms 3. Expand and simplify binomial expressions 4. Substitute real values in algebraic expressions	Algebra is the use of symbols letters or. generally. variables to represent numbers	Use algebra tiles and concrete representations to denote variables and numbers. Use appropriate analogies to represent variables i.e a = number of apples etc. Use labeled rectangle to develop the idea of expansion of binomials	1. Individual practice 2. Collaborative practice 3. Quiz 4. Cumulative review 5. Homework
Three Weeks	**Measurement** Distance/ length -Time -Money -Mass/weight	1. State reasons for measuring things 2. Identify and use non-standard units of measure 3. Differentiate between standard and non-standard units of measure 4. Name standard metric units of distance/length. time. temperature. mass/weight	Measurement could be defined as the numerical quantity assigned to an attribute by making comparison with a known attribute.	Use skit of every day applications and situations to develop non-standard units. Use oral questioning and discussion to develop the awareness of the need for standard units of measurement	1. Discussion 2. Questioning 3. Short quiz 4. Individual practice 5. Collaborative practice 6. Homework
	-Area -Perimeter	5. Develop the concept of area and perimeter showing their differences.	Area—the measure of a surface Perimeter-measure of distance	Covering surfaces with tiles. Counting grids of surfaces etc.	1. Discussion 2. Questioning 3. Individual practice 4. Collaborative practice

UNIT 1

Topic: Properties of Arithmetic

Lesson #1: Commutative property of addition and subtraction

Introduction: Oral work (10 minutes)

- Present a novel example to illustrate the concept of commutative property.

  Question: Are the results of the instruction in the two columns the same?

	A	B
1.	Put on your socks and then your shoes.	Put on your shoes and then your socks.
2.	Kill the snake and then pick it up.	Pick up the snake and then kill it.
3.	Walk six paces East and then three paces North.	Walk three paces North and then six paces East.
4.	Add 21 and 32.	Add 32 and 21.
5.	Divide 8 by 3.	Divide 3 by 8.

- Observation: In some cases, the information in each column gives the same result. In other cases, the results are different.

<u>Commutative</u>: *An instruction is called commutative if changing the order always gives the same result.*

Development: 25 minutes

- Use shopping example to illustrate commutative property of addition.
- Example - mother went to the supermarket to buy the following grocery items:

Grocery Items	Price List
1 tin milk	Tin Milk $25.00
1 tin Milo (1500 g)	Milo $67.50
5 lb. bag rice	5 lb. Rice. $60.00
5 lb. bag sugar	5 lb. Sugar................ $25.00
3 lb. Chicken	3 lb. Chicken.............$120.00

- Add five items in different order, and notice the result. Is it the same each time?

Yes, it is, therefore the order of addition does not affect the result.

- *Conclusion: addition is commutative*

Additional Examples

a) Is 54 + 37 = 37 + 54?

b) Is 3.5 + 2.1 = 2.1 + 3.5?

c) Is 8 + 6 + 2 = 2 + 6 + 8?

d) Is $a + b = b + a$?

- Examine subtraction to see if it is commutative

  Calculate:

  a) 5 - 3
  b) 6 - 4
  c) 8 - 2
  d) 4.1 - 3.1

- Reverse the order for each, does it give the same result?

  Observation: a) Is 5 - 3 = 3 - 5
  b) Is 6 - 4 = 4 - 6
  c) Is 8 - 2 = 2 - 8
  d) Is 4.1 - 3.1 = 3.1 - 4.1

- *Conclusions: 1. Addition is commutative*
  2. Subtraction is non commutative

Seatwork—Individual practice: 25 minutes

Which of the following are commutative?

a. Wash the shirt and then iron it.

b. Fetch the water and then chop the wood.

c. Boil the bananas and then take off the skin.

d. Saw the wood and then plane it.

e. Put the stamp on the envelope and then address it.

Homework

J.S.P Caribbean Math. Textbook 2; p.18, Ex. 2a #2,3.

Topic: Properties of Arithmetic

Lesson #2: Commutative Property of Multiplication and Division

Introduction: Oral work (10 minutes)

- Review homework.
- Examine the following examples to determine if the result is the same in each case.
  - The diagram below shows a crate of D & G drinks arranged in a rectangular array.

O	O	O	O
O	O	O	O
O	O	O	O
O	O	O	O
O	O	O	O
O	O	O	O

- Using the diagram to refer to rows and columns, check by counting bottles to verify that the products give the same result.

  i.e. 4 times 1 = 1 times 4

  4 times 2 = 2 times 4, etc.

- Use the idea of 4 times 1 as 4 groups of ones and 1 times 4 as one group of four etc.

Development: 25 minutes

- Copy the following problems and multiply each pair to see if the result is the same.

  a) 4 times 3 and 3 times 4

  b) 3.1 times 4.1 and 4.1 times 3.1

  c) ½ times 6 and 6 times ½

  d) a times b times c and c times a times b

  What conclusion can you draw from the above calculations?

- *Conclusion: multiplication is commutative*
- Copy and complete the following exercise and calculate each to see if the result is the same in each case.

  a) 3 divide by 8 and 8 divide by 3

  b) 3.5 divide by 0.5 and 0.5 divide by 3.5

  c) a divide by b and b divide by a

  Is the result the same?

- *Conclusion: division is not commutative*

Seatwork: Cooperative/collaborative practice (25 minutes)

1. a) Copy and complete

   i) 121 x 867 ii) 867 x 121

   Which is easier? Why?

   b) Suppose that you were asked to find the product of 7 and 238. Would you multiply 7 x 238 or 238 x 7? Why? What property are you using?

2. a) Is 12 divided by 6 = 6 divided by 12?

   b) Find the following and write down your observation.

   i) 2½ divided by 1 2/3 ii) 1 2/3 divided by 2½

   c) If you change the order in a division, do you always get the same result?

   d) Is division commutative?

- *Conclusion: division is not commutative*

Homework: JSP Textbook 2. p.18-19; ex 2a, # 4; ex 2b #1 f, h; #2 a, h.

Topic: Properties of Arithmetic

Lesson #3: Associative property

Introduction: Oral Work (10 minutes)

- Review Homework

Consider the following example:

1. For his supper, Karl is given Chicken and Rice and Coca-Cola. Is there any difference if:
   (a) he eats the chicken, and takes a mixture of Rice and Coca-Cola with it?
   Is (Chicken + Rice) + Coca-Cola = Chicken + (Rice + Coca-Cola)?
2. Lily is making a Fruit Salad from Pineapple, Bananas, and Oranges. Does it make any difference if:
   (a) she mixes Pineapple and Bananas and then mixes in the Oranges; or
   (b) she mixes the Pineapple with a mixture of Oranges and Bananas?
   Is (Pineapple + Bananas) + Oranges = Pineapple + (Bananas + Oranges)?

- *Conclusion: When an instruction contains the same operation twice, and it does not matter which one is done first, we say that the operation is Associative.*

Development: 25 minutes

Given: i) 59 + 7 + 3

ii) 27 + 159 + 1

iii) 5/8 + 3/8 + 11/3

iv) 8 - 3 - 4

v) $1\frac{1}{2} - \frac{1}{2} - 4$

vi) 4 + 1 - 2

Consider:

i)	(59 + 7) + 3	vs. 59 + (7 + 3)
ii)	(27 + 159) + 1	vs. 27 + (159 +1)
iii)	(5/8 + 1 1/3) + 3/8	vs. 5/8 +(11/3 +3/8)
iv)	(8 - 3) - 4	vs. 8 - (3 - 4)
v)	(11/2 - $\frac{1}{2}$) - 4	vs. 11/2 - ($\frac{1}{2}$ - 4)
vi)	(4 +1) - 2	vs. 4 + (1 - 2)

1. Did you get the same result in each case?
2. Which is easier to calculate?
3. Does the associative property apply to addition?
4. Does the associative property apply to subtraction?

Seatwork: Individual practice (25 minutes)

1. Calculate the following using a quick method (use brackets wherever necessary).

    a) 4 + 69 + 16

    b) 74 + 26 + 214

    c) 5 x 71 x 2

    d) 4 x 67 x 25

    e) 7 3/4 + 5 1/8 + 3 1/4

    f) 3.5 x 4.5 x 2.0

    g) 5 7/8 x 11/3 x 8

2. Work out:

    a) (12 divide by 3) divide by 2 and 12 divide by (3 divide by 2)

    b) (3 divided by 3/8) divided by 3/4 and 3 divided by (3/8 divided by 3/4)

- Consider:
    1. Does it matter where the brackets are placed?
    2. Is it easier to calculate if brackets are determined by choice are selected arbitrarily.
    3. Is multiplication associative?
    4. Is division associative?
- *Conclusion: addition and multiplication are associative whereas subtraction and division are not associative.*

Homework:

JSP Textbook 2; P.P. 20, Speed Test 1.

Topic: Properties of Arithmetic

Lesson #4: Distributive Property

Introduction: Oral work 10 minutes

- Review Homework
- Review main points of commutative and associative properties.
    1. Addition and multiplication are commutative.

        i.e. a + b = b + a and, ab = ba.
    2. Subtraction and division are non-commutative.

        i.e. a - b ≠ b - a, and a divided by b is not equal to b divided by a.
    3. Addition and multiplication are associative

        i.e. a + b) + c = a + (b + c), and (ab) c = a (bc).
    4. Subtraction and division are not associative

        i.e. (a - b) - c is not equal to a- (b - c)

        and (a divided by b) divided by c is not equal to a divided by (b divided by c).

Development: 25 minutes

A. This example will be used to develop the concept of the distributive property.

ABCDEFGHIJKL MNOPQRSTUVWXYZ
ABCDEFGHIJKL MNOPQRSTUVWXYZ
ABCDEFGHIJKL MNOPQRSTUVWXYZ
ABCDEFGHIJKL MNOPQRSTUVWXYZ
ABCDEFGHIJKL MNOPQRSTUVWXYZ

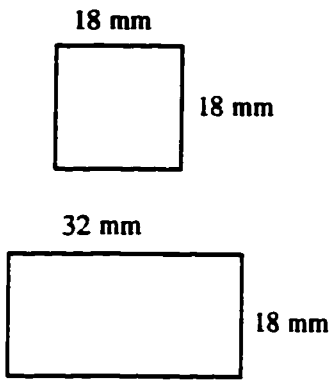
How many letters are in the five rows of letters of the alphabet?

- Examine several methods of arriving at a solution.
- Emphasize a quick method of arriving at a solution applying the distributive property.

i.e. suggested method is $5(12 + 14)$

$= 5(26) = 130.$

B. What is the total area of the two rectangles?



A quick method is $(18 \times 18) + (18 \times 32)$, i.e $18 \times 50 = 900\ mm^2$

Therefore, $a(b + c) = ab + ac$ (Distributive property for addition),

also, $a(b - c) = ab - ac$

Seatwork: Cooperative/collaborative practice (25 minutes)

A. Use a quick method to calculate the following:

a) $4 \times 7 + 4 \times 13$

b) $26 \times 5 - 11 \times 15$

c) $2 \times 8 + 3 \times 8$

d) $5 \times 9 - 3 \times 9$

e) $6 \times 21 + 6 \times 28$

f) $7 \times 48 - 7 \times 28$

B. (a) Two buckets, one holding 12 litres and the other 8 litres, were used to fill a tank. Both buckets were used 48 times. How many litres did the tank hold?

(b) Find the profit made by buying 12 pairs of shoes at $470.50 each and selling them at $670.50 each.

Homework

JSP Caribbean Math. Textbook, p. 23

Speed Test 4; exercise 2d # 6.

Topic: Properties of Arithmetic

Lesson #5: Applying the distributive property to Ready Reckoner calculations.

Introduction: Oral work (10 minutes)

- Review homework
- Introduce a novel situation by making reference to a Ready Reckoner table to calculate costs of items.

Number of patties	Cost
1	$15.00
2	$30.00
3	$45.00
4	$60.00
5	$75.00
6	$90.00
7	$105.00
8	$120.00
9	$135.00
10	$150.00

- Use the ready reckoner table to determine the following costs of patties.

  (a) 4 patties (b) 15 patties (c) 200 patties

  The solution for (a) is a straight forward observation; i.e refer to row 4 in the table which is $60.00.

  The solution for (b) involves use of expanded form. That is, (10 + 5), the corresponding costs are in row 10 and row 5. The result gives $150.00 + $75.00 = $225.00.

  The solution for (c) involves the use of the distributive property. That is,. 10(20) which is 10(10 + 10), data from table gives 10($150.00 +$150.00) = 10($300.00)

  = $3000.00

Development: 25 minutes

- Use the ready reckoner table to calculate the following costs.

A READY RECKONER TABLE FOR COSTS

	PRICE PER ARTICLE				
Number of Articles	$17.60	$17.70	$17.80	$17.90	$18.00
1	17.60	17.70	17.80	17.90	18.00
2	35.52	35.40	35.60	35.80	36.00
3	52.80	53.10	53.40	53.70	54.00
4	70.40	70.80	71.20	71.60	72.00
5	80.80	88.50	89.00	89.50	90.00
6	105.60	106.20	106.80	107.40	108.00
7	123.20	123.90	124.60	125.30	126.00
8	140.80	141.60	142.40	143.20	144.00
9	158.40	159.30	160.20	161.11	162.00
10	176.00	177.00	178.00	179.00	180.00
20	352.00	354.00	356.00	358.00	360.00
30	528.00	531.00	534.00	537.00	540.00
40	704.00	708.00	712.00	716.00	720.00
50	880.00	885.00	890.00	895.00	900.00
60	1 056.00	1 062.00	1 068.00	1 074.00	1 080.00
70	1 232.00	1 239.00	1 246.00	1 253.00	1 260.00
80	1 408.00	1 416.00	1 424.00	1 432.00	1 440.00
90	1 584.00	1 593.00	1 602.00	1 611.00	1 620.00
100	1 760.00	1 770.00	1 780.00	1 790.00	1 800.00
200	3 520.00	3 540.00	3 560.00	3 580.00	3 600.00
300	5 280.00	5 310.00	5 340.00	5 370.00	5 400.00
400	7 040.00	7 080.00	7 120.00	7 160.00	7 200.00
500	8 800.00	8 850.00	8 900.00	8 950.00	9 000.00
1 000	17 600.00	17 700.00	17 800.00	17 900.00	18 000.00
2 000	35 200.00	35 400.00	35 600.00	35 800.00	36 000.00
3 000	52 800.00	53 100.00	53 400.00	53 700.00	54 000.00
4 000	70 400.00	70 800.00	71 200.00	71 600.00	72 000.00
5 000	88 000.00	88 500.00	89 000.00	89 500.00	90 000.00
10 000	176 000.00	177 000.00	178 000.00	179 000.00	180 000.00

1. (a) 4 articles at $17.60 per article
   (b) 20 articles at $18.00 per article
   (c) 100 articles at $17.70 per article
   (d) 2 000 articles at $17.60 per article
   (e) 15 000 articles at $17.70 per article

- Suggested solutions:
  (a) row 4, column $17.60 gives $70.40
  (b) row 20, column $18.00 gives $ 360.00
  (c) row 100, column $17.70 gives $1700.00
  (d) row 2000, column $17.60 gives $3 520.00
  (e) row 15 000 imply (10 000 + 5 000)
  i.e. row 10 000, column $17.70 gives $177 000.00 and row 5,000 column $17.70 gives $88 500.00, therefore the cost is $177 000.00 + $88 500.00 = $265 500.00
- Verify results by direct multiplication.

Seatwork: **Individual** practice—25 minutes

- Use the ready reckoner table to calculate the following costs:
  1. 7 articles at $17.80
  2. 60 articles at $17.70
  3. 400 articles at $18.00
  4. 5000 articles at $ 17.80
  5. 20000 articles at $ 18.00
  6. 196 articles at $18.00

Homework

Use the ready reckoner table to calculate the following costs:

(a) 3 articles at $18.00 (b) 700 articles at $18.00
(c) 93 articles at $17.78 (d) 793 articles at $17.77

Topic: Properties of Arithmetic

Lesson #6: Ready Reckoner Table for Percentages

Introduction: Oral work (10 minutes)

- Review homework as a mean of reviewing the main points of the previous lesson. i.e. direct table observation for costs; e.g. 3 articles and indirect table calculation for costs; e.g. 700 articles which implies (600 + 100) etc.

Development: 25 minutes

- Use (ready reckoner table on p.27 of JSP Caribbean Mathematics Textbook 2) to calculate:
  (a) 7½% of $6.31 (b) 6 3/8% of $6.32
  (c) 27% of $6.34 (d) 56½% of $6.35
  NB.(Give your answer to the nearest cent)

Example:

7½% implies (7% + ½%). From the table, 7% gives 0.442 and ½% gives 0.032

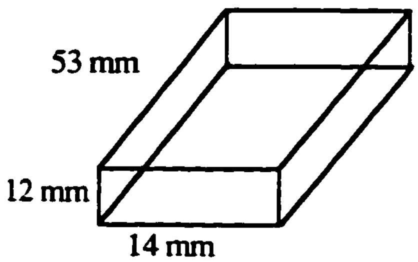
i.e. 0.442 + 0.032. Answer = $0.474 = $0.47 (2 d.p.)

Seatwork: Cooperative/collaborative practice—25 minutes

1. Use ready reckoner table on p. 27 of textbook to calculate (to the nearest cent):
   (a) 3 3/4 % of $6.34
   (b) 85 % of $6.32
   (c) 12 1/4 of $6.33
2. A certain savings account pays 8Ω % interest per annum. What is the annual interest on a sum of $634.00. (Hint: use table on p. 27 of textbook)
3. A library had 63 500 books in it, of these, 27% were non-fiction. How many non-fiction books were there in the library? (hint: use table on p. 27) NB: Optional question for students who are above average ability in class).

Homework

1. The volume of a cuboid (rectangular box) measuring 3 cm x 4 cm x 5 cm is (3 x 4 x 5) cm^3.
   What does the fact (3 x 4) x 5 = 3 x (4 x 5) = (3 x 5) x 4 tell you about the volume? [review question]
2. What area of card would be required to make the match box cover shown below? (Ignore the flap which sticks the box together.)



UNIT 2

Topic: Integers

Lesson #7: Introduction to Integers

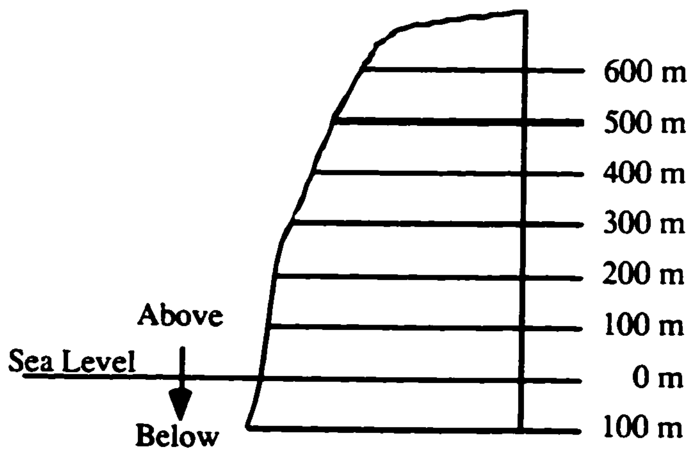
Introduction: Oral work (10 minutes)

- Discuss homework orally with class
- Focus students on the new unit-integers
- Use number line to introduce students to the set of integers

  {. . . -3, -2 , -1 , 0, 1, 2, 3, . . . }
- Emphasize negative and positive integers and their relative position on the number line.

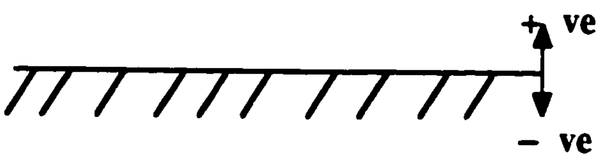
Development: 25 minutes

- Examine practical applications of integers:
  1. Temperature between 0 degrees Celsius and 100 degrees Celsius and temperature below 0 degrees Celsius (below freezing point of water). For example, temperature in Alberta, Canada in winter ranges from about -30 degrees Celsius to +10 degrees Celsius.
  2. Height above or below sea level.



  3. Distance (as negative and positive).

     a) Height above ground level and depth below ground level.



b) Distance from a point O. East of O denotes positive; West of O denotes negative.

Example:

A man is walking along a straight road running E-W. The point O is on a road, we can measure his distance from O at any time. If he is EAST of O, we can say his distance is positive. If he is WEST of O his distance from O is negative. The following table records one man's walk.

Time (h)	0	1	2	3	4	5
Distance (km)	7	5	3	1	-1	-3

i) In which direction is he walking?

ii) How fast is he walking?

iii) When does he pass O?

4. Acceleration (driving a car) in forward motion or reverse from rest.

Seatwork: Cooperative/collaborative practice (25 minutes)

Students will work in groups after which the teacher will discuss results orally with students.

1. a) The temperature of a liquid is 8°C. If the temperature drops by 10°C what is the new temperature?

   b) A woman stands on a hill which is 80 m above sea level. She climbs down 100 m. What is her height now relative to sea level?

   c) Zero hour on a certain day was 12 noon. What time was 4 a.m?

   d) A man is 5 km East of a town. He walks 8 km West. How far East of the town is he?

2. a) A traveler in Jordan descended 500 m from a hill 430 m above sea level. What was her height above sea level?

   b) She then climbed 50 m. What was her height above sea level then?

   c) Finally she climbed another 100 m. What was her height above sea level at the end of her journey?

Homework

JSP Textbook 2; p. 60, ex 5d # 6,8.

Topic: Integers

Lesson #8: Formal addition and subtraction of Integers—1

Introduction: Oral Work: (10 minutes)

- Oral review of homework, and quick solution on board
- Use number line to perform addition and subtraction of positive and negative integers.

Examples:

1 a) 5 + 7
  b) 7 - 5
  c) 5 - 7
  d) -5 - 7
  e) -7 - 5
  f) - 7 - 5

Development: 25 minutes

- Examine patterns in addition and subtraction of integers (directed numbers) to develop convenient and practical rules. Refer students to number line to verify examples.
- *Adding a positive integer to a positive integer*

1. a) 5 + 1 = 6   b) 5 + 4 = 9
  c) 3 + 2 = 9   d) 5 + 2 = 7
  e) 3 +1 = 4

- Possible evolving rule: *add numbers arithmetically and retain sign that is present, i.e. (+) sign.*
- Observation: *resulting number is larger when added.*
- *Adding a negative integer to a negative integer*

2. a) -5 + -1 = -6   b) -5 + -4 = -9   c) -3 + -2 = -5
  d) -5 + -2 = -7   e) -3 + -1 = - 4

- Evolving possible rule: *add numbers arithmetically and retain sign that is present. i.e. (-) sign.*
- Observation: *resulting number is smaller when added.*

Seatwork: Individual practice—25 minutes

Add the following integers using a method that works for you. (Hint: you could use the number line to check your answer).

1 a) 1 + 4
  b) 10 + 7
  c) 8 + 4
  d) -3 + (-5)
  e) -9 + (-1)

Homework:

1. Calculate the following:
   a) 1 + 9
   b) 9 + 1
   c) -2 + (-8)
   d) -8 + (-2)
   e) -5 + (-5)
   f) 5 (- 5)
2. What arithmetic property is present in question? [review question]

Topic: Integers

Lesson #9: Formal addition and subtraction of Integers—2

Introduction: Oral work (10 minutes)

- Review homework assignment as a basis for lesson

Development: 25 minutes

- Adding a negative integer to a positive integer

Examples:

a) -5 + 5
b) -5 + 6
c) -5 + 7
d) 7 + (-5)
e) 8 + (-3)
f) 10 + (-5)
g) 8 + (-8)

Evolving rule: $x + -y = x - y$

- Subtracting a negative integer from a positive integer

Example:

Calculate:

(a) 5 - (-5)   (b) 6 - (-5)   (c) 10 - (-2)   (d) 1 - (-2)

Evolving rule: $-(-x) = x$ *and* $x - (-y) = x + y$

- Subtracting a positive integer from a negative integer

Example

Calculate:

(a) -5 (- 5)   (b) -5 (- 6)   (c) -2 - 1   (d) -2 - 10

Evolving rule: $-x - y = -(x + y)$

Seatwork: Cooperative/collaborative practice (25 minutes)

Compute:

1. (a) 7 - -2
   (b) 2 - (-7)
   (c) 31 - (-21)
   (d) 21- (-31)
   (e) 0 - (-5)
2. Mr. Johnson has a bank account. His deposits and withdrawals (in dollars) during one month were as follows. A deposit is given by a positive number, and a withdrawal by a negative number.

   600.00 -400.00 163.00 -250.00 4.65 -27.80 -181.83 234.20

   His balance at the beginning of the month was $50.00. Find his balance after each of the transactions shown above, and state his balance at the end of the month. What does it mean if his balance is negative?

Homework: Speed Test #6, p.66.

Topic: Integers

lesson #10: Multiplication and division of integers

Introduction: Oral work (10 minutes)

- Oral review of general rules governing addition and subtraction of integers

Development: 25 minutes

- Provide examples to show multiplication and division of integers

<u>Multiplication</u>

i. -4 x 1 = -4	iv) 4 x -1 = -4
ii) -4 x 2 = -8	v) 4 x -2 = -8
iii) -4 x 3 = - 12	vi) 4 x -3 = -12

- Evolving rule: *a positive number multiplied by a negative number gives a negative product and visa versa. [generally, (-) x (+) = (-), and (+) x (-) = (-)].*

Also: -4 x -3 = + 12
-2 x -1 = + 2
-7 x -4 = + 28 etc.

- *Evolving rule: a negative number multiplied by a negative number gives a positive product. [generally,(-) x (-) = (+)].*

Division

-3 divided by 1 = -3

-4 divided by -4 = +1

-2 divided by 2 = -1

-10 divided by 5 = -2., etc.

- *Evolving rule: when dividing, numbers with like signs give a positive quotient and numbers with unlike signs give a negative quotient.*

The rule may be summarized as follows:

1. (+) divided by (-) = (-),

2. (-) divided by (+) = (-),

3. (-) divide by (-) = (+).

Seatwork: Individual practice—25 minutes

Find the values of the following:

1) 7 x 6
2) 7 x (- 6)
3) (-6) x 7
4) (-7) x -6
5) (-2) x (-3) x 5
6) 6 divided by (-3)
7) (-3) divided by (-3)
8) (-3) divided by (6)
9) 28 divided by 7
10) - 28 divided by - 7

Homework

1) 18 divided by 6
2) 6 divided by -3
3) (-6) divided by 2
4) (-2) x (-4) x (-6)
5) 3 x (-5) x (-2) x 4

UNIT 3

Topic: Sets

Lesson #11: Sets and subsets

Introduction: Oral work (10 minutes)

- Discuss the idea of set with a view to arriving at a workable definition.

  Generally a set is called *a collection of things or objects.*

Qualities of a set are:

(1) well defined

(2) elements of a set are clearly identified

(3) elements possess the same attribute

For example, the set {tall girls at St. Catherine High School} does not meet all the qualities of a set listed above. But the set {girls over 5 feet 8 inches at St. Catherine High School} conforms to these criteria.

<u>Examples of Sets</u>

1. The set of teachers at St. Catherine high school.
2. The set of letters in the alphabet.
3. The set of planets in the solar system.
4. The set of girls who live in Spanish Town.
5. The set of students whose parents are medical doctors.
6. The set of positive integers between -1 and 6

Development: 25 minutes

- Develop the idea of subset by using a novel example (story).

<u>Story</u>

One day the social studies teacher asked for volunteers to assist in planning the Heroes Day function at school. Ten students out of a class of 50 students volunteered. We could say that the ten students could be considered a subset of the class of 50 students.

<u>Evolving definition of a subset</u>

A. *Set A is called a subset of a set B when all the members of A are also members of B.*

The symbol $\subset$ is use to denote a subset i.e. $A \subset B$

Suggest some subsets of {students in your class}. e.g.

{students who wear a watch} is a subset of {students in your class} etc.

Can a subset have a subset? If so, suggest some in your class.

B. Give some examples of subsets of the following sets. List two or more members of each subset, if possible.

(a) {whole numbers}

(b) {quadrilaterals}

(c) {1,2,3,4,5}

(d) {months of the year}

C. {Jamaicans} is a subset of {West Indians}. This means, "all Jamaicans are West Indians." In a similar way, rewrite the following in ordinary English.

(a) {Jamaica, Barbados, Trinidad & Tobago } is a subset of {West Indian countries}

(b) {horses} is a subset of {animals}

(c) {4, 6, 8, 9} is not a subset of {prime numbers}

(d) { pentagon } is a subset of {polygon}

Seatwork: Individual practice—25 minutes

A. Here are some sets. Name them and give the attribute you use for naming each.

1. {Monday, Tuesday, ... Sunday} Set of _______ of the week.
2. {2, 4, 6, . . . }. Set of _________ numbers
3. {1, 3, 5, 7, . . .}. Set of ________ numbers
4. {3, 6, 9, . . .}. Set of multiples of ________.

B. 1. Write three other subsets from Set A = {the days of the week}.

2. Given Set A = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, write three subsets of A.

Homework

1. Let U = {Barbados, Belize, Cuba, Costa Rica, Dominion Republic, El Salvador, Haiti, Guyana, Jamaica, Mexico, Nicaragua, Trinidad & Tobago}.

Find the following subsets:

(a) {Islands}

(b) {countries in Central America}

(c) {countries where English is the official language}

(d) {countries whose population is less than one million}

Topic: Sets

Lesson #12: Set notation

Introduction: Oral work (10 minutes)

- Review two symbols we have encountered so far in sets, namely: { } and $\subset$.
- Quick review of sets and subsets in terms of definitions.
- Review of homework.

Development: 25 minutes

- Introduce the concepts of membership and its notation.

  The symbol $\in$. means "is a member of or belongs to"

- Provide example to show the difference between subset and membership

  April $\in$ { months of the year},

  {April, June} $\subset$ { months with 28 days }

- Clarify possible confusion between $\subset$ and $\in$. The symbol $\subset$ connects two sets. The symbol $\in$ connects a member and its set.

  For example {April} $\subset$ {months}, but April $\in$ {months}.

  Also, {months } $\supset$ { April} and Wednesday $\notin$ {Alphabet}.

  NB. The set with fewer members goes at the closed end of the subset sign.

<u>Examples</u>

1. Choose the correct symbol, $\in$, $\notin$ or $\subset$, to complete the following:
   (a) {2, 4, 6, 8} ______ {whole numbers}
   (b) 18 _______ {multiples of 5}
   (c) Earth ______ {Planets}
   (d) {birds} ______ {animals}
2. Write the following statements in set notation.
   (a) Kerosene belongs to the set of fuel oils.
   (b) The set of cars is a subset of the set of vehicles.
   (c) The set of even numbers is the same set as the set of multiples of 2.
   (d) The set consisting of 1/2, 2/3, 3/4, 4/5 and so on, is a subset of the set of proper fractions.

Seatwork: Cooperative/Collaborative practice—25 minutes

1. Choose the correct symbol, $\subset$, $\supset$, $\notin$, $\not\subset$ or = to complete the following. If none of these is correct, say so.
   (a) factors of 36} ___ {3, 6, 9}
   (b) 39 ___ {prime numbers}
   (c) squares ___ {quadrilateral}
   (d) { -2, 0, 1} ___ {Integers}
   (e) my dog ___ {boxes}
   (f) chalkwood ___ {objects made of wood}
2. Rewrite the following in ordinary English.
   (a) { French, Dutch, Spanish} ___ {languages spoken in the Carribean}
   (b) {girls who like music} ___ {girls in my form}
   (c) Walter Boyd ___ {Jamaica's national football squad}

Homework:

JSP Carribean Textbook 2, p.11, ex 1b # 7,8.

Topic: Sets

Lesson #13: Venn Diagrams

Introduction: Oral review of homework (10 minutes)

- General importance of diagrams to represent and illustrate approaches to solving mathematical problems. Include the saying that "a diagram is worth a thousand words."
- Discussion of importance of Venn Diagram developed by John Venn to show relationship between sets.
- Use this example to illustrate use of Venn diagram.

  If U = {months}, J = {months whose names begin with J}

U

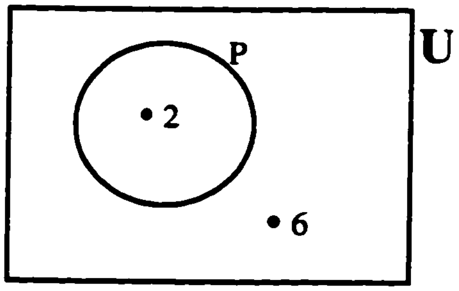
JAN	FEB	MAR	APRIL
MAY	JUN	JUL	AUG
SEPT	OCT	NOV	DEC

We know that $J \subset U$.

- Use Venn diagrams to illustrate membership and subsets.

EG.1

In the Venn diagram to the right, U = {whole numbers less than 10}, and P = {prime numbers less than 10}



Choose the correct symbols to complete the following:

(a) 2 ___ p

(b) 6 ___ p

(c) p ____ u

(d) U ____ p

(e) 8 ___ p

(f) 11 ____ p

EG. 2

Draw a Venn diagram of the following sets.

U = {geometrical shapes}

P = {polygons}

Q = {quadrilaterals}

Mark *s* for a square and *t* for a triangle. Copy and complete the following:

(a) *s* ___ Q

(b) *t* ___ p

(c) p ___ Q

(d) p ___ U

Seatwork: Individual practice—25 minutes

1. Draw a Venn diagram of the following sets.

U = {students in your school}

F = {students in your form}

Represent yourself by a point *y* on your diagram. Mark a point *z* for your head student.

Copy and complete the following:

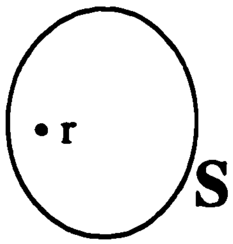
(a) F ____ U

(b) y ____ F

(c) z ____ F

(d) z ____ U

2. What information is given by the Venn diagram shown below?



r = Water
S = {Drinks}

- Use set notation to show the set relationship between water and drinks.
- Use ordinary English to describe the set notation in (a).

Homework

JSP Carribean Textbook p.17, ex 1d #1

Topic: Sets

Lesson #14: Two set intersection

Introduction: 10 minutes

- Quick review of homework
- Brief review of concept of subset stressing the operative word "all".
- Develop the idea of intersection by making reference to intersection of streets and avenues. The idea of a common region will be emphasized.
- Refer to practical examples in which the word "some" is emphasized.

<u>Example</u>: Grade 8 students have the choice of joining the Red Cross Society (**R**) and the Key Club (**K**). Some students are members of only one club while others are members of both clubs. Also, there are students who are not members of either clubs.

This information can be illustrated by the Venn diagrams shown below.

Development—25 minutes

- Examine examples of two set intersections

<u>EG. 1</u>

If A= {1,3, 5, 7, 9} and B = {2, 3, 4, 5, 6}, is there any number that is common to both sets? List these numbers.

Illustrate this set relationship by using a Venn diagram.

EG. 2

Let A = {2, 4, 6, 8, 10, 12} B = {2, 3, 5, 7, 11, 13}.

(a) Find A ∩ B

(b) Draw a Venn diagram to illustrate the set relationship between A and B.

Observation: *The intersection of A and B is the set which contains all the elements which are common to both A and B.*

- Examine an example of disjointed sets.

If A = { 2, 4, 6, 8, 10}, and B = {1, 3, 5, 7, 9, 11}, is there any number that is common to both sets? If no, illustrate this relationship by using a Venn diagram (disjointed sets).

Seatwork: Cooperative/Collaborative Practice—25 minutes

1. Let X = {apple, mango, orange, pineapple},and
   Y = {cherries, melon, guava, Jackfruit, apple}
   (a) Find X ∩ Y
   (b) Draw a Venn diagram to show X ∩ Y
2. A Club has 25 members who all participate in two games, football and cricket. Each member participates in at least one game. If 18 participate in football and 15 participate in cricket. How many members participate in both games?

Homework

1 If A = {1,6, 7, 9, 10}, and B = {2, 4, 7, 9, 12}
   (a) Find A ∩ B
   (b) Draw a Venn diagram to show A ∩ B
2. If F= {factors of 12}, and P = {prime numbers between 1 and 12}
   (a) Find F ∩ P
   (b) Draw a Venn diagram to show F ∩ P

Topic: Sets

Lesson #15: Interpreting Venn diagrams

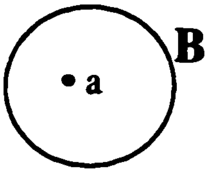
Introduction: 10 minutes

- Quick review of homework
- Discuss the use of diagrams including Venn diagrams as a way of communicating information and ideas.

Example

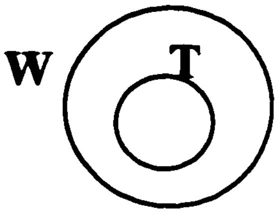
What information is given by the diagrams below? Write your answer in ordinary English.

A.



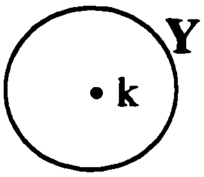
a = * this book
B = {math textbooks}

B.



T = {Trinidadians}
W = {West Indians}

C.



k = Kevin
Y = {People who like football}

Suggested answer (A): "This book is a mathematics textbook."

Suggested answer (B): "Trinidadians are West Indians."

Suggested answer (C): "Kevin likes football."

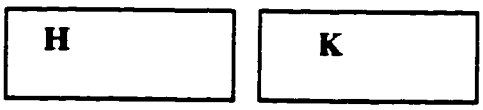
Development—25 minutes

- Examine Venn diagrams as a means of communicating in ordinary English.

(a)

P = figure PQRS
S = {Squares}

Answer. "The figure PQRS is a Square."



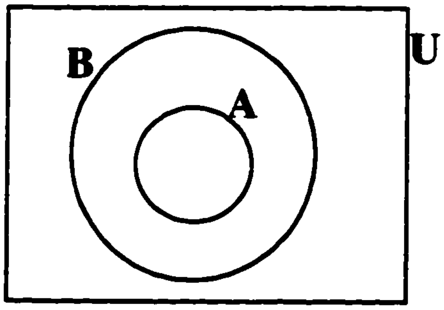
H = {even numbers}
K = {odd numbers}

Answer. "Even numbers are not odd numbers."

(b) U = {students at King's College}

A = {students in Form 1}

B = {those who like Coca-Cola}



Answer. "Form 1 students at King's College who like Coca-Cola."

- Draw additional Venn diagrams to illustrate ordinary English:

(a) All my friends go to Jamaica College.

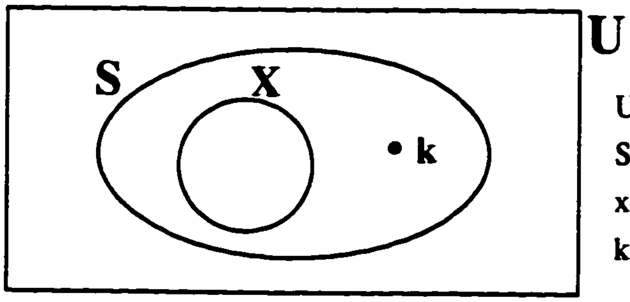
(b) Ben Johnson comes from Jamaica.

(c) Sharlene is not my friend.

(d) None of the students at St. Catherine High School comes from India.

Seatwork: Cooperative/Collaborative Practice (25 minutes)

1. Draw a Venn diagram to illustrate each of the following:
   (a) Carol is my friend.
   (b) All my friends go to church on Sundays.
   (c) I never go to church on Sundays.
   (d) April Allen passed her exams last year.
2. From the diagram below, write down as much information as you can about the following:



U = { Schools in Jamaica }
S ={secondary schools}
x = {co-educ. schools}
k = {Kingston college}

(a) Kingston college

(b) Jamaican schools in general

Homework:

JSP Caribbean Textbook 2

p. 16, ex 1f # 1, 3, 4, 5. (review questions)

UNIT 4

Topic: Basic Algebra

Lesson #16: Simplifying like terms

Introduction: Oral Work—10 minutes

- Review of homework.
- Comparing specific operations in arithmetic with operations in algebra. For example, in arithmetic specific numbers are used while in algebra, letters are used for numbers. In arithmetic 4 + 4 + 4 can be written as 3 x 4. Similarly, in Algebra a + a + a = 3 x a which is 3a.

  In the expression 3a, 3 is said to be the coefficient of a, and it tells how many a's are added.
- *Like terms*

  The sum of 5 apples and 8 apples is 13 apples. This can be written as 5a + 8a = 13a.

Development: 25 minutes

- Provide examples to practice addition and subtraction of like terms.

Simplify:

1) 2x + x　　2) 6d + d

3) a + 8a　　4) 9b + 3b

5) 3m + 5m　　6) 8e - 3e

7) 13ab - 12ab　　8) 2bc - 3bc + 4bc + 9bc

9) 11mn + 12mn - 2mn + 3nm

NB. nm = mn (commutative property)

Seatwork: Individual practice—25 minutes

Simplify, if possible:

1) 3xy + 4xy

2) 9ab - 5ab

3) 8uv + 7vu

4) 17hk - 9kh

5) 10ab + 7ba

6) 8bc + 6bc - 4bc + 5cb - cb

Homework

Simplify:

1) 8bc - 9bc　　2) 10 st + 4st - 5 ts

3) - 4at - 2at + 3at　　4) ab + 2ab - 4ab + ab

Topic: Basic Algebra

Lesson #17: Simplifying unlike terms

Introduction: Oral Work (10 minutes)

- Brief review of homework.
- Review the method of adding like terms.

Development: 25 minutes

Unlike terms

- Concept: If we have 3 apples + 4 apples + 6 bananas, the result can be written as 3a + 6a + 4b = 9a + 4b.
- Provide examples to practice addition and subtraction of unlike terms.

Simplify, if possible:

1) a + b + 2b + 5a
2) - 3c + d + 4d - 6c + 10c
3) 5ab - 2ab + 3bc - ba
4) 4ab + 10bc - ab - 7cb
5) 5uv - 2vw - 3vu + 4vw - uv
6) 3ab - bc + 5cd + 3cb - 4dc
7) 7fg + 8gf - 9fg + 3gh

Seatwork: Cooperative/collaborative practice—25 minutes

Simplify, if possible:

1) 3a - 4a + 5a + 10 b + 15b
2) 3b + 4c - 5b + 4b + c
3) 3x + y + 5x + 3y - 10x + 5x
4) 4ab - 5xy + 10xy + ab - 2ab
5) 4xyz + 5xyz - 3yzx + 2zyx

Homework

Simplify, if possible:

1) 3b + 4c - 2b +3c
2) 3a + 5b + 6b -2a
3) 3ab - 2ba + 3st + 2ts

Topic: Basic Algebra

Lesson #18: Simple multiplication with letters

Introduction: Oral Work (10 minutes)

- Review of homework

- Comparing addition and multiplication arithmetically. That is, multiplication is a short way of adding. For example, the statement 3 + 3 + 3 + 3 = 15 can be expressed briefly as 3 x 5 = 15. Similarly, the statement $4a + 4a + 4a = 12a$ can be shortened to $4a \times 3 = 12a$, so that $4a \times 3 = 3a \times 4$. In fact the terms 3, 4 and a may be multiplied together in any order.

  i.e. 3a x 4 = 4 x 3a = 4a x 3 = 3 x 4a = 4 x 3 x a, etc.

Development—25 minutes

- Perform simple multiplication to simplify algebraic expressions

Simplify if possible:

1) 5a + 5a + 5a + 5a, i.e. 4 x 5a = 20a
2) 3 x 5y
3) 3y x 3
4) 32x divided by 4
5) 17b - 5b x 2
6) 1/3 of 15bc
7) 4g + 5g x 4
8) m x 5 x m x 3
9) 3a + 4b
10) 5 x 6d - 4d x 0

Seatwork: Individual practice (25 minutes)

Simplify, if possible:

1) 5m + 5m + 5m
2) 4 x 8a
3) 7h x 3
4) 4 x 5p - 3p
5) 4a x 3 + 5
6) 8m x 0
7) 4 x 8y - 7y x 3
8) 4n x 7n
9) m x 5 x n
10) 28xy divided by 4x

Homework

1) 3w x 5 + 4w
2) 28xy divided by 9
3) 4y x 3s x 2
4) 5 x 3t - 4t x 2

Topic: Basic Algebra

Lesson #19: Expansion and simplifying of Binomial Expressions

Introduction: 10 minutes

- Brief review of homework
- Expansion and simplification of terms. That is, 4(y + 2) means 4 x (y + 2) which is 4y + 8, and 3(3a + 2) + 2(2a - 4), which gives 9a + 6 + 4a - 8. Simplifying gives 13a - 2.

Development—25 minutes

- Removal of brackets and simplification of algebraic terms

EG.1

3(a + 2b), gives 3a + 6b

EG.2

3(a - 2b) + 5(2a - 3b), gives 3a - 6b + 10a - 15b which is 13a - 21b.

Seatwork: Cooperative/collaborative practice (25 minutes)

Remove the brackets and simplify:

1) 5(2a - 3a)

2) 4(5p - 3q)

3) -(p - q)

4) 3(a - 2b) + 5(2a - 3b)

5) -(p + 2q) - (3p + 5q)

6) 2(3m + 5n) - 3(2m - 5n) + 3(m - 2) + 6(2m +3)

Homework

Remove the brackets and simplify:

1) 3(x + y) + 2(x + 2y)

2) 2(x + y) - (x - 2y)

3) (2x - y) - (2x + y) + 5(x + 6y) - (x - 3y)

Topic: Basic Algebra

Lesson #20: Substitution in Algebraic Expressions

Introduction: Oral Work (10 minutes)

- Quick review of homework
- Develop the concept of substitution by referring to every day examples eg. filing income tax return from computer software, substituting a football player on a team because of injury, etc.
- Relate the idea of substitution to algebraic expressions. i.e. finding the value of 2x + 5 if x = 5.

Development—25 minutes

- Perform examples involving substitutions in algebraic expressions

EG.1

Find the value of $2a - 9$ if $a = 7$.

If $a = 7$, then $2(7) - 9$

$= 14 - 9 = 5$.

EG.2

Find the value of $d^2 - 2d + 5$ if $d = 3$.

then $(3)^2 - 2(3) + 5 = 9 - 6 + 5$

$= 9 + 5 - 6$

$= 14 - 6 = 8$

NB. $d^2 = d \times d$ or $(d)(d)$, and not $2 \times d$

EG.3

If $x = 5$, and $y = 2$, find the value of $4x - 3xy + 6y$.

If $x = 5$, and $y = 2$, then $4(5) - 3(5)(2) + 6(2)$

$= 20 - 30 + 2$

$= 32 - 30 = 2$.

EG.4

If $m = 2$, $n = 4$, $u = 1$, and $v = 3$, find the value of $3m - 2n - u + v$

Substituting the values of m, n, u, and v in the expression gives

$3(2) - 2(4) - (1) + (3)$

$= 6 - 8 - 1 + 3$

$= 6 + 3 - 8 - 1$

$= 9 - 9 = 0$.

Seatwork: Individual Practice (25 minutes)

1) If $a = 4$, find the value of:
   - a) $3a + 2$ b) $4a - 2$
   - c) $5 + 2a$ d) $9a - 5a$
2) If $m = 5$, and $n = 2$, find the value of:
   - a) $2m + 3n$ b) $2m - 3n$
   - c) $34 - 3mn^2$ d) $4mn^2 - 7m^3 + 3n$

Homework

If $a = 3$, $b = 2$, find the value of:

- a) $3a + 2$ b) $3a - 4$
- c) $6a - 2b$ d) $a - 3b$
- e) $2a - 3b2$

UNIT 5

Topic: Measurement

Lesson #21: Introduction to measurement/historical development of measurement

Introduction: Oral Work (10 minutes)

- Oral questioning and discussion to develop the idea and concept of measurement.

Launch Questions

1. Who is the tallest person in your family?
2. Who is the heaviest person in your family?
3. Which holds more a mug or a coffee cup?
4. What time of the day is usually the hottest?
5. How many books are in the library at your school?
6. If you are feeling very warm, do you have a fever?

Questions to consider

How did you get answers to the questions above? Did you use any instrument? Did you line up in order of height and compare with each other?

- Evolving definition: *Measurement is the quantifying of attributes, or some aspects of an object by making comparisons with some known attribute or object size.*

Development: 25 minutes

- Discuss with students some problems ancient man had and how they coped and devised ways of measuring certain attributes.
- Review non-standard measuring system for distance developed by Babylonians, and Egyptians, followed by the Greeks and Romans.

These measurements were based on dimensions as it related to the human body. The units of length used are shown in the table below.

Fathom	The distance between a man's hands with arms outstretched.
Cubit	The distance from the elbow to the tip of the middle finger.
Digit	The width of the index finger.
Foot	The distance from the heel to the tip of the big toe.
Span	The distance between the thumb and little finger, when the fingers are fully extended.
Yard	The distance from the tip of the nose to the tip of the middle finger, when the arm is extended at right angle to the head.
Hand	The width of the hand.

Seatwork: Cooperative/collaborative practice (25 minutes)

Copy and complete the table below. Record your own measurements, then compare measurements other members of your group.

DISTANCE MEASURED	Fathom	Cubit	Yard	Feet	Digit
1. Width of your desk					
2. With of chalkboard					
3. Length of a window					
4. Thickness of a book					
5. Length of classroom					

- Question for contemplation: For each distance you measured, did you get the same result as other members in your group.
- Evolving conclusions:

1. When we use units based on parts of the body of different persons the measurement may vary with individuals, because we differ in size.
2. Need for standard measures.

Homework

1. Write a conversation between two persons measuring 10 yards of cloth (use the non-standard unit for yard). They are much different in size, and there is a resulting dispute about the difference in measurement of the ten yards.
2. Provide brief advice to them for solving this problem.

Topic: Measurement

Lesson #22: Standard units of measurement

Introduction: Oral Work (10 minutes)

- Oral review to include examination of homework to lay the foundation for the days' lesson. Specifically the need for standard units of measurement.

Development: 25 minutes

- Examine attributes of measures with a comparison of non-standard and standard units.
- Discuss and compare metric and imperial standard units.

Attributes of Measures	Non-standard Units	Standard Units
Time	Time Span between two: (a) drips from a faucet (b) taps of a pencil	Hour, seconds, minute, day. month, year, decade. century.
Money	Bartering of goods	Dollar, yen, lira, etc.
Distance	Length of a string, length of one's step.	Metre, kilometre, etc.
Mass/weight	Mass/weight of stone, piece of wood, bag of sand.	Kilogram, pound, ounce, gram. tonne
Volume	Handful, heap	Cubic centimetre
Capacity	Bottle cap, jug, bucket	litre, millilitre, quart, pint.

Seatwork: Individual Practice (25 minutes)

1. Construct a table with two columns (A and B). Put each of the following units of measurements into either Group A or Group B: Spans, handful, metre, mass of wood, length of string, bag of sand, gram, pint, bucket of water, furlong, decimetre, tonne.

A **Standard Units**	B **Non-standard units**
____________	______________, etc.

2. Match units in column A with their measure in Column B by putting the correct letter from Column A in the appropriate space in Column B

Column A	Column B
(a) dollars	Length ___
(b) degrees	Time ___
(c) rod	Temperature ___
(d) litre	Mass ___
(e) centimetre	Capacity ___
(f) kilogram	Money ___
(g) hour	
(h) pint	
(i) pound	

Homework

1. With reference to any four attributes measured, explain why it is necessary for standard units of measures to be used.

OR

2. Develop a conversation between yourself and a seller from whom you are purchasing an item, but the seller is not using a measuring instrument.

Topic: Measurement

Lesson #23: Measuring distance

Introduction: Oral Work (10 minutes)

- Homework check. Read a few samples of good responses to homework questions.
- Use launch questions to stimulate oral discussion on standard measuring units and methods of measurement. Example: The Police Academy requires male/female recruits of heights no less than 160 cm. Do you qualify? How can you tell for anyone?

Development: 25 minutes

- Examine with students practical ways of measuring distances.

1. Measuring heights of students
2. Measuring length of classroom, buildings, playing field, etc.
3. Measuring width of classroom, playing field, etc.

- Examine instruments used to measure metric distances.
  These are: micrometers, Vernier caliper, 10-centimetre ruler, metre ruler, measuring tapes (50 m, 100 m, etc.), etc.
- Demonstrate the use of these instruments mentioned above with particular emphasis on the metre rulers.

Seatwork: Cooperative/collaborative practice (25 minutes)

- Provide activities to practice measuring distances.
  Measure each of the following distances using appropriate measuring instruments and state the unit of measurement and symbol in each case.

Distance	Metric Unit	Symbol
a) The thickness of this book		
b) The length of a new pencil		
c) The length of the classroom		
d) The height of the school gate		
e) The length of the Netball court		
f) The width of the football field		

Homework

1. Make a list of objects and measure them in mm and cm.
2. Measure the length and width of your house in metres. Why would you not measure the length of your house in kilometres?
3. How many metres equal one kilometre?

Topic: Measurement

Lesson #24: Measuring time

Introduction: Oral Work (10 minutes)

- Quick homework check
- Oral review of main points in previous lesson
- Oral discussion with students on the use of the clock, or watch and calendar as two of the more popular ways of measuring time.
- Examine the relationship between the following units of measure for time.

60 seconds	1 minute
60 minutes	1 hour
24 hours	1 day
7 days	1 week
12 months	1 year

Development: 25 minutes

- Examine the reading of the 24 hour clock. That is, 2400 gives 12 p.m., 0600 gives 6 a.m. etc.
- Examine time schedules of traveling itinerary, Also, television schedules and calendar to do practical calculations involving the measurement of time.

Seatwork: Individual Practice (25 minutes)

The Television schedule for JBC TV and CVM TV on December 14,1995 is shown below:

TV GUIDE

JBC- TV	CVM-TV
4:00 Cartoons	4:00 Cartoons
5:00 Sesame Street	5:00 School Challenge Quiz
6:15 Music	6:00 The Young and Restless
6:30 Lime Tree lane	7:00 News/Weather/Sports
7:00 News/Weather/Sports	8:00 Pointman
7:30 Comedy	9:00 Oprah Winfield Show
8:00 Evening Magazine	10:00 Early movie
9:30 Dallas	11:30 CNN News
10:30 Early Movie	12:00 Late night Movie
12:00 Late night Movie	1:30 Sign off
1:30 Sign off	

Problem #1:

Karl is allowed two hours of TV this evening and one hour of homework assignment. He, however has to keep within the bed time schedule of 9:00 p.m. He has to watch the news. What other TV programmes can he view?

Problem #2:

Sandra is cooking a special dinner for her family. She is serving roast chicken with rice, spicy carrots and beans, followed by banana pie and ice cream. Work out a plan to show when she must start each stage so that dinner will be ready at 1930. What is the latest time she must start preparation? Remember she can only do one thing at a time.

Times required

Boil rice	**15 min**	**Roast chicken**	**90 min**
Peel carrots	**10 min**	**Make pastry**	**20 min**
Cook carrots	**15 min**	**Prepare bananas**	**20 min**
Wash beans	**5 min**	**Make pie**	**15 min**
Cook beans	**7 min**	**Cook pie**	**35 min**

Homework

Here is part of an air time-table. It shows the times of departure and arrival of 6 flights from Jamaica. All planes depart from Jamaica.

Depart	Arrive	Destination
0630	0930	Puerto Rico
0815	1105	Nassau
0930	1005	Barbados
0745	1235	Toronto
1245	2255	London
0400	0930	New York

(a) Find out how long each flight takes?

(b) Which of these flights take the longest time?

(c) Copy the time-table but write the times using a.m. and p.m.

N.B Disregard the time zone differences

Topic: Measurement

Lesson #25: Mass/weight

Introduction: Oral Work (10 minutes)

- Homework check and oral review.
- Display samples of products sold at the Supermarket to see the weights of each, and their respective units of measurement.
- Identify and differentiate between imperial units and metric units.

Development: 25 minutes

- Illustrate the difference between weight and mass.
- Measure weights of objects in grams and kilograms using a scale which has an accuracy of within 1 gram.
- Discuss the different metric units and symbols used to measure weight/mass. That is,

  1000 mg = 1 g

  1000 g = 1 kg

  1000 kg = 1 tonne

- Perform conversion from one unit to the others, etc.
- Develop a table that shows appropriate units to use for different weights.

Weight	Unit	Symbol
tin milk	grams	g
piece of Yam	kilogram	kg
one capsule	milligram	mg
a truck	tonne	tonne
a small tin of Milo	grams or kilograms	g or kg

Seatwork: Cooperative/collaborative practice (25 minutes)

1. Provide each group with objects of different weights. Ask them to give an estimate of each and write this down in a table. They are to measure these objects using a scale provided for each group and record their weights beside the estimated weights. Determine the accuracy of your estimates by comparing estimated weights with actual weights.

Table

Objects	Estimated Measurement	Actual Measurement	Difference
1.			
2.			
3.			
4.			
5.			

2. Convert the following to kg:
   a) 345 g
   b) 6789 g
   c) 89 g

3 Convert the following to g:
   a) 56 kg
   b) 2.9 kg
   c) 523 kg

Homework

1. Convert the following to grams:
   a) 1 kg
   b) 5 kg
   c) 4.8 kg

2. Convert the following to kg:
   a) 500 g
   b) 3000 g
   c) 5400 g

Topic: Measurement

Lesson #26: Perimeter

Introduction: 10 minutes

- Homework check
- Oral review to include a summary of basic principles of conversions, i.e. converting from a smaller unit to a bigger unit you divide by the appropriate multiple of 10 and converting from a bigger unit to a smaller unit you multiply by the appropriate multiple of 10.

Quick oral review:

1. About the basic units of length
2. How to use a ruler, tape measure, and metre stick
3. What are plane shapes
4. How to estimate lengths

Development: 25 minutes

- Launch Activity:
- Consider which is longer:

1. The distance, in one lane, around a normal athletics running track, <u>or</u> the distance, along the outer edge, around the same track.
2. Identify and discuss with students real-life situations in which we measure distances (linear measurement).

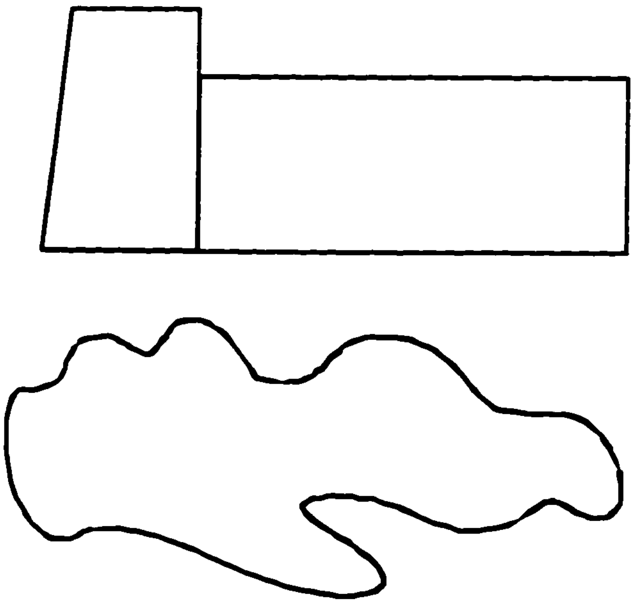
- Possible suggestions:
  - fencing a garden, yard etc.
  - putting ropes around the boundary of a play field
  - the length of a belt around you waist, etc.
- Demonstrate measurement of plane shapes in the classroom such as desks, tables, etc. With a measuring tape (unit of measurement cm/m).
- Discuss that measuring distances is really measuring the perimeter of these plane shapes.
- Measure irregular plane shapes with string and compare with metre rule.

Seatwork: Cooperative/collaborative practice (25 minutes)

Students will be working inside and outside the classroom to measure the perimeter of classroom, playing field, doors, windows, and the outer boundary of an irregular plane surface on the playing field.

Homework

Use a string with your ruler to measure the perimeter of the following plane shapes.



Topic: Measurement

Lesson #27: Perimeter

Introduction: Oral Work (10 minutes)

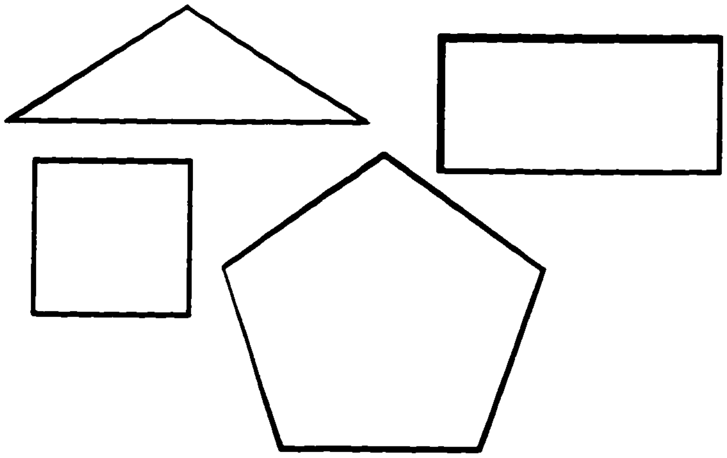
- Quick check of Homework
- Oral review to include the concept of perimeter, the length of the boundary of any plane figure.

Development: 25 minutes

- Derive formulas for common plane shapes such as a square, a rectangle, and regular polygons.
- Provide examples of each plane shape with specified measurements in which the perimeters will be measured with a ruler.
- Verify the answers by using the appropriate formulas.

Seatwork: Individual practice: 25 minutes

Measure the length of sides of each polygon below and complete the table which follows:



Polygon	Number of Sides	Length of sides	Perimeter
1.			
2.			
3.			
4.			
5.			

Questions to consider:

1. What do you notice about the sides of some of the polygons
2. Could you find the perimeter of a polygon with all sides equal, if you knew only the length of one of the sides?
3. Can you derive and state a rule for finding the perimeter of a regular?

Homework

Calculate the perimeter of the following regular polygons:

1. with 4 sides, each equal to 5 cm.
2. with 7 sides, each equal to 3 cm.
3. with 8 sides, each equal to 2.6 cm.

Topic: Measurement

Lesson #28: Area

Introduction: Oral Work (10 minutes)

- Homework check
- Quick oral review of perimeter as a measure of distance and derive rules for specific regular plane shapes.

- Discuss the basic concept of area as measuring the surface of an object (not only plane shapes.
- Discuss practical applications of area. i.e. painting walls, paving the surface of a driveway, cutting the grass on the lawn, tiling the surface of floor, etc.

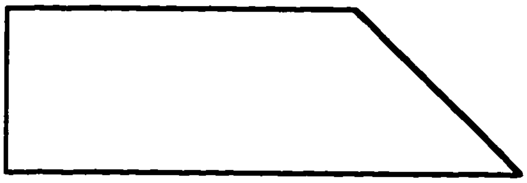
Development: 25 minutes

- Explore the covering of plane surfaces with circular stoppers, leaves, dominoes, and lastly with squares to demonstrate the idea of tessellation. (Complete surface coverage.)
- Discuss which one of these methods covers the surface completely. Expected answer: Squares.
- Conclusion: Square units are most appropriate for measuring area as it covers the surface completely, hence the standard unit of area is square unit.

Seatwork: Cooperative/collaborative practice (25 minutes)

<u>Student Activity</u>:

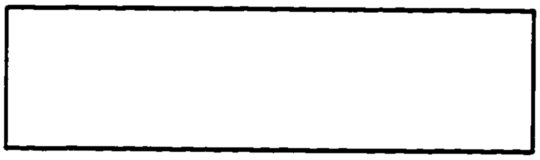
Use square tiles of dimensions 1 cm x 1 cm to find estimates of area for the following shapes.

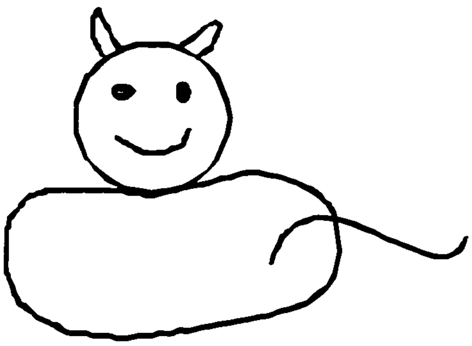


How did you deal with the surfaces on which the (1 cm x 1 cm) squares did not fit exactly?

Homework

Use 1 cm by 1 cm squares to fit into the following plane shapes and determine each area in squares units.





Topic: Measurement

Lesson #29: Area

Introduction: Oral Work (10 minutes)

- Oral review of homework to form the basis of today's lesson. Confirmation of determining the area of plane shapes by physically counting the square units and estimating fractional squares for surfaces on which the squares will not fit exactly.
- Discuss with students the difficulties with this method and human errors with estimation of fractional squares.

Development: 25 minutes

<u>Area of rectangle</u>

- Show how the area of a rectangle 5 cm by 4 cm can be determined by counting the squares in the rows and columns, i.e. 5 rows and 4 columns of squares or 4 columns and 5 rows of squares which gives 20 squares. Therefore, the area is 5 x 4 = 4 x 5 = 20 squares units
- Examine a few examples to develop a pattern.

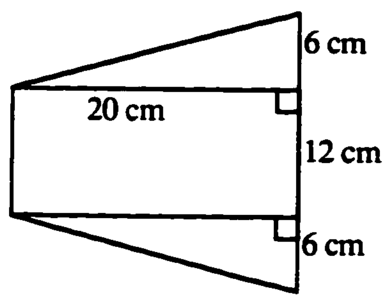
  By examining the pattern, derive a rule to find the area of a rectangle. If rows represent length and columns represents width then Area of rectangle = length x width or

  $$\boxed{A = L X W}$$

- Use derived rule to work two examples to find the area of a rectangular shapes. Verify the result in each case by counting the squares.

Seatwork: Individual Practice: (25 minutes)

1. Use the formula for the area of a rectangle to calculate the area of the following rectangles:
   a) length = 6 cm, width = 3 cm
   b) length = 5 cm, width = 7 cm
   c) drawing each rectangle on graph paper and count the squares to verify that the calculated area for each rectangle is the same.
2. Calculate the area of the figure drawn below.



Topic: Measurement

Lesson #30: Area

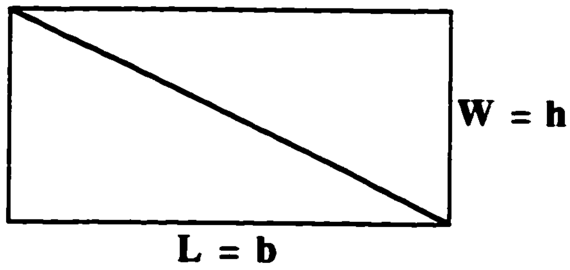
Introduction: 10 minutes

- Quick review of Homework.
- Oral review of rule relating to the area of rectangle.

Development: 25 minutes

- Develop the rule for area of triangles in which the perpendicular heights and bases are specified.
- Develop the relationship between a rectangle and a triangle (assuming same base and height for both rectangle and triangle).

  The diagram below shows that a diagonal bisects the rectangle into halves.



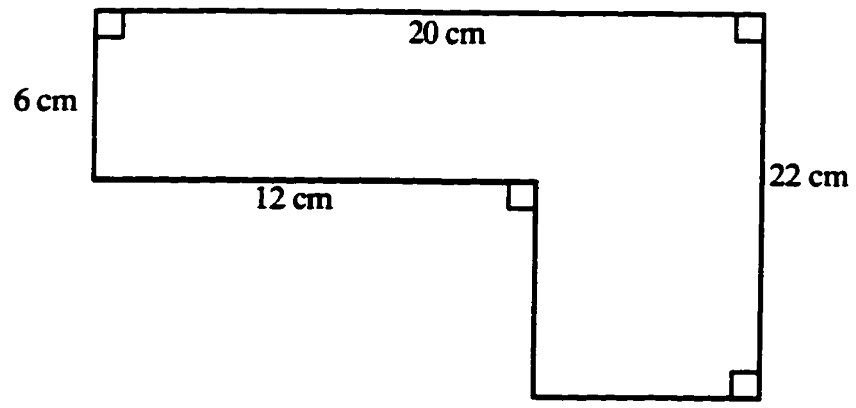
Therefore, the length of the rectangle becomes the base of the triangle. That is, L = b. and the width of the rectangle is the height of the triangle. That is, W = h. It Is also observed that the diagonal bisects the rectangle into two halves, therefore the area of this right angled triangle is half the base of the rectangle, hence the rule area of triangle.

Area $= \frac{1}{2}$ base x height.($\frac{1}{2}$ b x h)

- Work with students examples of finding the area of triangles given bases and perpendicular heights.

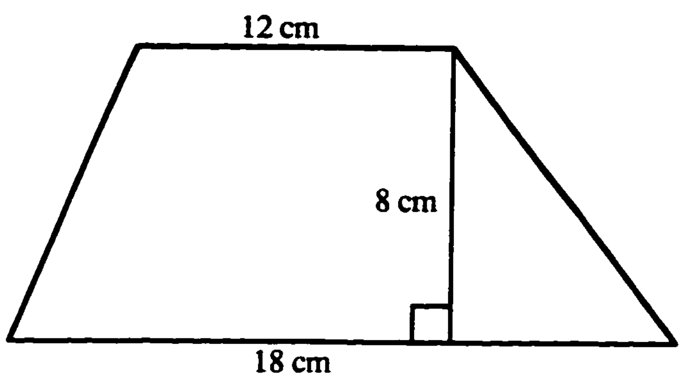
Seatwork: Cooperative/collaborative practice (25 minutes)

1. Calculate the area of the following triangles:
   a) Length = 6 cm and height = 4 cm
   b) Length = 9 cm and height = 5 cm
2. Calculate the area of the figure shown below:



Homework

Calculate the area of the figure shown below:



APPENDIX 2

PRE- AND POST TESTS

Survey test ---- Pre-test

GRADE 8 **SEPTEMBER, 1996.**

Instructions: Underline the correct answer from the responses A, B, C, and D for each question. Time given for this test is one hour.

Example

The next number in the sequence 1, 2, 3, 4, __, __ is

A. 6

B. 7

C. 5

D. 8

---

1. Which of the following numbers is an even number?
   A. 7
   B. 9
   C. 2
   D. 1
2. Which of the following numbers is an odd number?
   A. 4
   B. 5
   C. 6
   D. 18
3. Which of the following numbers is a prime number?
   A. 1
   B. 2
   C. 9
   D. 25
4. The value of 2^5 is
   A. 32
   B. 16
   C. 10
   D. 3

5. The LCM of 3, 4 and 9 is
   A. 36
   B. 24
   C. 18
   D. 12
6. The HCF of 15 and 25 is:
   A. 3
   B. 5
   C. 15
   D. 25
7. The next two number in the sequence 1, 4, 9, 16, ... is:
   A. 25, 36
   B. 20, 28
   C. 18, 24
   D. 32, 64
8. The height of a building is measured in
   A. m
   B. m^2
   C. litres
   D. kg
9. Which of the following numbers is the smallest:
   A. ½
   B. 0.6
   C. 3/8
   D. 7/16
10. The value of (3/4 x 1 1/3) - 8/15 is
   A. 17/60
   B. 7/15
   C. 11/20
   D. 3/5

11. The ratio of Marcia's share to Carla's share in a business is 5:4. If the total profit is $4500.00, then Marcia's share is

A. $900.00

B. $2000.00

C. $2250.00

D. $2500.00

12. 9/10 expressed as a percentage is

A. 0.9%

B. 9%

C. 90%

D. 99%

The following information is to be used in questions 13 and 14.

$E = \{1, 2, 3, 4, 5, 6, 7\}$

$X = \{1, 2, 6, 7\}$

$Y = \{2, 3, 4, 5, 6\}$

$Z = \{4, 5, 6, 7\}$

13. $X \cap Y$ is

A. $\{2, 6\}$

B. $\{2, 3, 7\}$

C. $\{3, 4, 5\}$

D. $\{1, 2, 3, 4, 5, 6, 7\}$

14. $X \cup Z$ is

A. $\{2, 6\}$

B. $\{4, 5, 6, 7\}$

C. $\{3, 4, 5\}$

D. $\{1, 2, 3, 4, 5, 6, 7\}$

The following information is to be used to answer questions 15 and 16.
Students were interviewed to find out which of four sports they preferred. The results were as follows:

SPORTS	CRICKET	FOOTBALL	SWIMMING	ATHLETICS
NUMBER OF STUDENTS	480	800	400	320

15. The most popular sport is

    A. swimming

    B. football

    C. athletics

    D. cricket

16. The total number of persons interviewed is

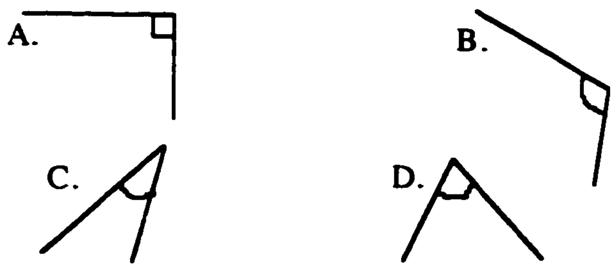
    A. 2000

    B. 800

    C. 320

    D. 1280

17. Which of the following represents an obtuse angle?



18. The line AB shown below measures

    A. 5.1 cm

    B. 5.0 cm

    C. 5.6 cm

    D. 6.0 cm

19. The volume of a box (cuboid) with length = 65 cm, width = 25 cm and height = 20 cm is

A. 110 cm^3

B. 32 500 cm^3

C. 3250 cm^3

D. 1625 cm^3

20. A quadrilateral with four axes of symmetry is a

A. rectangle

B. parallelogram

C. equilateral triangle

D. square

End of Test

Survey Test ---- Post-Test

GRADE 8 **DECEMBER, 1996**

Instructions:

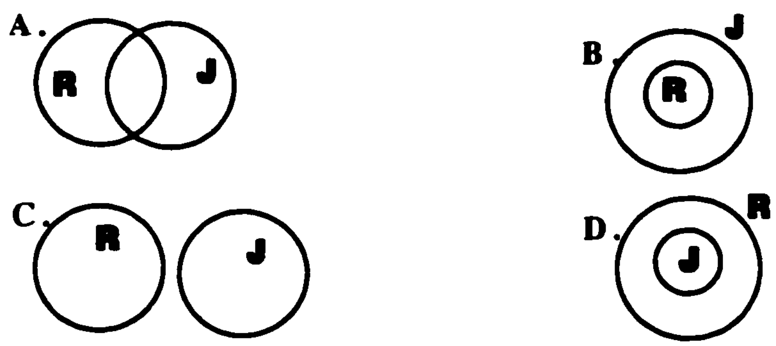
In section 1 select and circle the letter A, B, C, or D that best represents the correct answer. In section 2 answer all questions and show all working. Time allowed for the test is 90 minutes.

SECTION 1

1. 34 x 43 = 43 x 34 is an example of:
   A. Commutative Property
   B. Associative Property
   C. Distributive Property
   D. Addition Property
2. Which diagram best describes "Some students who enjoy Reggae also enjoy Jazz"?

   R = {students who enjoy Reggae music}

   J = {Students who enjoy Jazz music}



3. The value of 3 + 4 x 2 + 5 is
   A. 13
   B. 11
   C. 49
   D. 16

4. 20 - 35 + 15 =

A. 30

B. 0

C. -30

D. 35

The following information is to be used to answer questions 5 through 7.
In a certain form, the average height of the girls was 152 cm. The teacher wrote down how much taller or shorter (cm) each girl was than the average; she wrote:

-12 15 8 -5 -18 7 13 -8

16 -15 -10 -13 3 0 -7 3

5. The tallest girl was:

A. 168 cm

B. 134 cm

C. 149 cm

D. 175 cm

6. The shortest girl was:

A. 134 cm

B. 149 cm

C. 155 cm

D. 152 cm

7. How many girls were more than 152 cm in height?

A. 17

B. 9

C. 8

D. 7

8. A plot of land measures 37 m by 58 m. The area in square metres is

A. 5080

B. 2146

C. 1946

D. 176

9. Kasoon buys gifts costing $412.75, $122.09 and $321.25. The change from $1000.00 will be

A. $143.91

B. $856.09

C. $567.09

D. $278.00

10. The value of 1½ divided by 1/3 is

A. 2/9

B. ½

C. 2

D. 4½

11. The capacity of a teaspoon is likely to be about:

A. 0.5 g

B. 0.05 l

C. 5 ml

D. 5 mg

12. Kim's house is 11 minutes from the bus station. She is 8 minutes early for a 10:15 a.m. bus. At what time did she leave her house?

A. 9:56 a.m.

B. 10:04 a.m.

C. 10:07 a.m.

D. 10:34 a.m.

13. A cup holds 250 ml of juice. A jug holds 1.5 litres of juice. How many cups of juice can I get from the jug of juice?

A. 6000

B. 600

C. 250

D. 1500

SECTION 2

14. State which of the following are commutative and give a reason for your choice of answer for each.

    A. Wash your skirt then iron it

    B. Have a bath then comb my hair

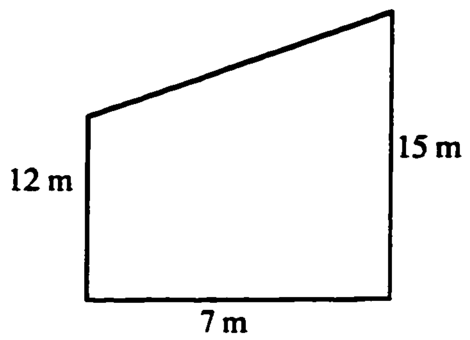
    C. Put the stamp on the envelope then mail it

    D. Put the stamp on the envelope then address it

    E. Boil the water in a pot then put salt in it

    F. 23 + 12

15. The diagram below shows a rectangular shaped cow's pen (not drawn to scale).



    A. Calculate the area of the cow's pen.

    B. If the cow's pen is to be fenced with 5 rows of wire, calculate the total length of wire that would be needed to do this job.

16. A refrigerator is switched on at 2 p.m., and the temperature inside it is then 28 degrees Celsius. At 3 p.m. the temperature has fallen to 9 degrees.

    A. What is the fall in temperature?

    B. At 4 p.m. the temperature is -2 degrees Celsius, and at 5 p.m. it is -5 degrees Celsius. Find the fall in temperature between 3 and 4 p.m. and between 4 and 5 p.m.

17. Your are invited to give a fifteen minute address to the Mathematics Club at your school on the topic: "The importance of Measurement to the Society." Write your presentation notes. Hint: include non-standard, standard units, metric units, S.I. units. (Approximately 150 words.)

End of Test

APPENDIX 3

STUDENT ATTITUDE QUESTIONNAIRE

STUDENT ATTITUDE QUESTIONNAIRE

Answer all questions on the answer sheet provided.

Circle the response that you agree with most.

A = always B = often

C = seldom D = never

There are no right answers; please indicate what you think.

---

1. I can see how the mathematics in this class can be applied outside the classroom.
2. I am interested in the work we do in this mathematics class.
3. It is important to learn the material taught in this class.
4. What I learn in mathematics will help me when I have a job.
5. I find mathematics class interesting.
6. I like working with the other students in this class on mathematics.
7. Students help each other learn mathematics in this class.
8. I like to discuss problems in mathematics with other students in this class.
9. I like it when my teacher does a few examples before I am asked to do a question on my own.
10. I need my teacher to learn mathematics.
11. When I have difficulty with a question in mathematics, I ask my teacher.
12. I prefer to have my teacher help me with a difficult problem rather than try it on my own.
13. This class is well organized.
14. I like the way the material is presented in this class.
15. I spend most of the time in each class doing mathematics.
16. Getting a certain amount of class work done is very important in this class.
17. Doing challenging, thinking questions is an important part of mathematics.
18. There is more than one way to do most mathematics problems.
19. I like to do mathematics problems my own way.
20. Mathematics problems are interesting.
21. Doing mathematics problems is a good way to learn mathematics.
22. It is important to complete my homework in this class.
23. I can do most of the questions assigned for homework in this class.
24. Doing homework makes it easier to get better marks in this class.
25. Doing homework for this class makes it easier to learn mathematics.