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Maintenance Planning and Cost Effective Replacement Strategies

by

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Abstract

The research of this thesis deals with facility maintenance management strategies. Quantitative approaches are employed in maintenance decision making to realize the minimum maintenance cost per unit time.

An optimal preventive maintenance (PM) and repair model for a deteriorating system with a bathtub-shaped failure rate function is developed to determine optimal replacement time and maximum allowed failure rate. PM is performed whenever the system reaches its maximum allowed major failure rate. If the system fails, either a minor repair or a major repair is carried out on it. The optimal system replacement time is the last major failure time.

Joint optimization of block replacement (BR) and periodic-review spare parts provisioning policy with used items is developed for a series system. The system BR time is coupled with the spare parts provisioning period. Spare parts are ordered and inventoried according to the demand for a BR as well as random failures before recycling the used items. The used items are selected from the immediately previous BR activity and recycled in the later part of the BR period.

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I would like to dedicate this work to my son Darren and my wife Zhidan whose love and whole-hearted support helped me to bring this work and my degree to completion. I would also like to give thanks to my father, mother, sister and brother for encouraging me in pursuing my education.

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Nomenclature

c_r	replacement cost
c_{minor}	cost of a minor failure repair
c_{major}	cost of a major failure repair
c_p	cost of an instance of preventive maintenance
$C.R.$	cost rate of the system
W	the length of a life cycle of the equipment
X_n	the time interval between the $(n-1)$th major failure and the nth major failure
h_{major}	the major failure rate function of the system
$h_{major}^{(n)}$	the major failure rate function of the system after the nth major failure repair
h_{minor}	the minor failure rate function of the system
η_n	the adjustment factor for the slope of major failure rate function whenever the nth major failure happens
γ_n	the reduction factor for the virtual age reduction of the system whenever the nth major failure is repaired
t_n	the hypothetical point in time at which the major failure rate function reaches ξ
Ω_n	the time interval between successive PMs in the nth major repair cycle

N	the number of major failures experienced by the system in order for a replacement to be warranted
ξ	the specified level of the major failure rate that triggers a PM activity
V_n	the number of PMs conducted within the nth life cycle, X_n
Φ_n	the time between the last PM in X_n and the $(n+1)$th major failure
c	replacement cost connected with a random failure
K	setup cost of placing an order
p	cost of a PR for a component
s	price of one new spare part
h	cost of holding one new spare part per unit time
z	cost per unit time due to one spare part's being unavailable when needed
T	time interval between two consecutive PRs
τ	lead time for new spare parts procurement
δ	time interval for performing failure correction with used spare parts
S	the maximal stocking level of new spare parts during a PR period
S_1	the maximal stocking level of new spare parts immediately after a PR action
S_2	the maximal stocking level of used spare parts during a PR period
$F(t)$	the probability distribution function associated with a component's lifetime
$f(t)$	the probability density function associated with a component's lifetime
$H(t)$	the renewal function of a new spare part

$h(t)$	$d\mathrm{H(t)}/dt$, the renewal density function of a new spare part
$H_2(t)$	the renewal function of a used spare part
$h_2(t)$	$dH_2(t)/dt$, the renewal density function of a used spare part
$g_1(\cdot)$	the probability density function of demand for new spare parts
$g_2(\cdot)$	the probability density function of demand for used spare parts
$E(x_n)$	average demand for new spare parts in a PR period
$Var(x_n)$	variance of demand for new spare parts in a PR period
$E(y_n)$	average demand for used spare parts in a PR period
$Var(y_n)$	variance of demand for used spare parts in a PR period

Chapter 1

Introduction

1.1 Motivation

Many facilities in manufacturing and processing industry are subject to deterioration and random failures related to usage and age. This may lead to a higher risk of production loss and to safety problems. Maintenance activities are performed on these facilities to enhance or restore efficiency and to alleviate risks before or after breakdown; however, traditional maintenance activities are rarely scheduled in an optimal manner with a systematic approach. The approach, if there is one, is likely slight and merely quantitative (*A.K.S. Jardine* [1]). As a result, maintenance activities upon components and systems may be insufficiently scheduled, or too frequently repeated. Due to lack of a systematic approach to scheduling maintenance activities, maintenance may be erroneously carried out on like/similar units. Improper maintenance could result in a large waste of resources and manpower in heavy industries, such as refineries, power plants, chemical plants, mines, and airports, for which the cost of maintaining and replacing facilities constitutes a significant portion of annual operational budgets.

In addition, due to globalized competition and re-engineering management, companies in heavy industry are facing escalating challenge to reduce their operation cost. One important aspect of the challenge in reducing these costs is keeping the cost of maintenance down while maximizing the availability of the physical assets. Traditional research on maintenance policy seldom considers spare parts provisioning and inventory policies. To prevent loss through unexpected equipment breakdown, spare parts are often overstocked for possible maintenance activities. Sometimes, this may result in a large volume of particular spare parts being held in warehouses to ensure assets being kept in operation, because equipment breakdown would incur huge production losses. If the spare parts are expensive, a large amount of capital may be tied up due to overstocking (*Khailil F. Matta* [35]).

The purpose of this research is to address the above two aspects of maintenance management of plants and equipment. Stochastic mathematical models are developed to improve system reliability, prevent unexpected system failures and reduce maintenance costs.

1.2 Background

1.2.1 Maintenance Management

Over the past several decades, failure mechanisms and failure diagnostic methods for equipment have been extensively investigated. This allows maintenance activities to be performed in many ways. Correspondingly, various maintenance policies are developed to deal with different maintenance requirements. From the viewpoint of management, however, all maintenance policies commonly used in heavy industry can be categorized into only three maintenance management approaches (*Yam et al* [2]):

1. Failure-driven maintenance (FDM),

2. Time-based maintenance (TBM), and

3. Condition-based maintenance (CBM)(see Figure 1.1).

Failure-driven maintenance (FDM) is a reactive maintenance approach to equipment breakdown or production interruption. The target of this approach is reducing maintenance costs as much as possible by avoiding maintenance activities. Due to the unpredictability of failures, such maintenance practice is often ineffective. It may prove extremely expensive in the continuous-process heavy industry. Thus this maintenance approach is undesirable in modern industry.

Time-based maintenance (TBM), which is also known as periodic preventive maintenance (PPM), is a planned maintenance approach to performing maintenance at fixed intervals regardless of equipment condition. Such maintenance practice assumes the mean time between function failures (MTBF) during normal usage has been statistically or experientially made known (*Gertsbakh* [3]). This maintenance approach is very effective if there are a large number of identical units. Thus this management approach to maintenance is still commonly used in many companies.

Condition-based maintenance (CBM) involves maintenance actions based on actual equipment condition (objective determination of need) obtained from in-situ, non-invasive tests, operating and condition evaluation (*Bengtsson* [4]). Condition-based maintenance can be planned or on request. Its management is flexible and controllable; its maintenance practice is highly effective. As a result, such maintenance management is very popular now. The success of CBM lies in the accuracy of measuring or monitoring indicator of equipment

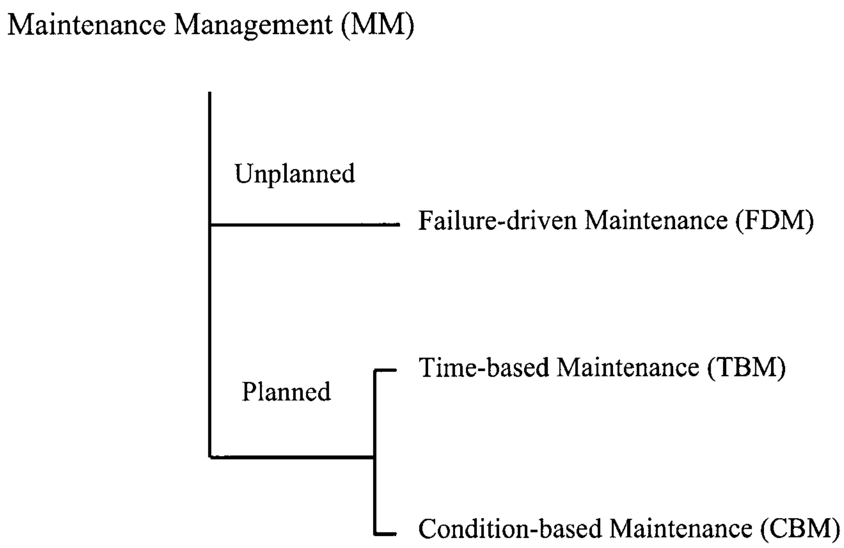

Figure 1.1: Three commonly used approaches to maintenance management

condition which is associated with some failure mechanisms.

Maintenance activities can also be categorized as corrective maintenance (CM) or preventive maintenance (PM). CM is the maintenance that occurs when a system fails. Some researchers refer to CM as repair, which is usually driven by the failure of a component or system. PM is the maintenance that occurs when a system is operating. It can be performed in a time-based or condition-based manner. In this thesis, maintenance is a general term and may represent either CM or PM. Replacement is a perfect maintenance. *Repair* and *CM* will be alternatively used throughout this thesis.

1.2.2 Reliability Concepts in Maintenance

One of the objectives of this research is to use the quantitative approach in maintenance decision-making using known facts to reduce reliance on subjective judgement in maintenance management. Research has shown that reliability centered maintenance (RCM) could offer the most systematic and efficient process for the optimization of plant and equipment maintenance (*Deshpande & Modak* [5]). In reliability theory, stochastic models are applied to maintenance problems to obtain optimal decisions for scheduling PMs and cost-effective system replacement. Significant reliability terms used in this research are *system*, *series system*, *reliability*, *failure rate* and *failure mode*. Their definitions are as follows:

1. ***System***: the overall plant or equipment that provides a specific function.

2. ***Series system***: system has multiple subsystems (components) with a series configuration when failure of any one subsystem results in the failure of the system.

3. ***Reliability***: the probability that an item (system) will function satisfactorily when used according to specified condition for a specified time interval. It is denoted by R(t).

4. ***Failure rate***: also called ***hazard rate***, this is determined as follows:

$$h(t) = lim_{\triangle t \to 0}\{\frac{R(t) - R(t + \triangle t)}{\triangle t \cdot R(t)}\} \tag{1.1}$$

where

$h(t)$ is the failure rate,

$R(t)$ is the reliability at time t, and

$\triangle t$ is the time interval.

The term failure rate designates equipment mortality. The study of failure rates gives us insight into the behavior of equipment failures, and enables us to make predictions about future performance (*Narayan* [6]). To some extent, it represents the health condition of the equipment in a specified time interval (*Jardine et al* [7]). Three typical failure rate curves of deteriorating systems are shown in Figure 1.2: bathtub curve, linearly increasing curves, and nonlinearly increasing curves.

5. ***Failure mode***: the condition or state which is the end result of a particular failure mechanism. A failure mode can be managed if the failure mechanism is understood (*August* [8]).

1.3 Problems in Maintenance Management

The primary function of maintenance management is to control the condition of equipment in order to obtain maximum availability and minimum operational cost. In this research, we deal with the following issues:

- Time intervals for performing preventive maintenance
- Failure repair policies
- Condition of equipment (characterized by failure rate)
- Replacement policies
- Reliability considerations
- Spare parts provisioning rules

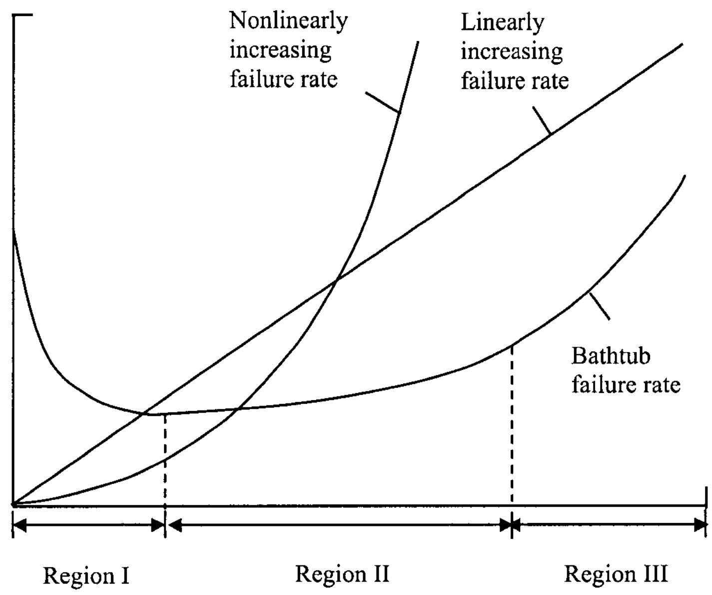

Figure 1.2: Typical failure rate curves. Region I: Infant mortality period, Region II: Normal period, Region III: Wear-out period

- Spare parts inventory

Steps in the application of the quantitative approach to maintenance management problems usually include setting objectives, modeling the behavior of the specific system and solving the model. The objective may be to maximize the availability of the system or to minimize the cost rate of the system operating under some constraints. Once the objective is formulated, an evaluative mathematical model can be constructed to determine the best way of operating the system to achieve the defined objective. Then the model is solved by an analytical technique or a numerical procedure. The results obtained can be used to guide the maintenance management.

1.4 Research Goal

The goal of this research is to develop new mathematical models for assisting management decision-making on the maintenance of equipment according to known facts. The purpose of this research is to improve maintenance function. The results of the research can address the maintenance management problems presented in the last section (see page 7). Considering the scenario of the maintenance problems, two research topics based on two different technical systems are studied.

The first is to model maintenance of a system with a bathtub-shaped failure rate and to find an optimal criterion for conducting condition-based PM and an optimal time for performing replacement so as to minimize the cost rate of the system (total operation cost/unit lifetime).

The second is to develop a model for optimizing both preventive maintenance and spare parts provisioning policy for a series system with used items. Decision variables include the optimal time to conduct periodic PM, the optimal time to reuse unfailed items, and the optimal number of new spare parts ordered in one PM period. Optimal solutions to the model can be obtained by minimizing the operational cost rate of the system.

1.5 Scope of Research

Facilities are subject to deterioration and random failure with usage and aging. Any random failure of equipment may stop production and incur severe economical loss. Planned maintenance activities are often carried out at the time that maintenance will have the minimum impact on production; therefore, the cost of planned maintenance is always much less than the economic loss of maintenance in response to unexpected failure.

This study is expected to have applications in the maintenance management of most sophisticated technical systems. They may include:

- facilities in paper mills, refineries and chemical plants
- assembly lines in the manufacturing industry
- steel fabrication
- complex electro-mechanical medical equipment.

1.6 Organization of Thesis

This chapter provides an introduction to the motivation of the present research, the background of maintenance management and the reliability concepts associated with maintenance. Chapter 2 introduces the literature on the basic models of the effects of failure and maintenance on deteriorating systems and reviews some previous optimal maintenance models which are relevant to our present research. In Chapter 3, we develop an optimal repair and PM model for a system with a bathtub-shaped failure rate function and two failure modes. Details are given explaining how the condition-based PMs are scheduled and the model is formulated. Numerical solutions to the model, obtained by Monte Carlo simulation, are provided. In Chapter 4, we develop a joint optimal model of preventive replacement and periodic spare parts provisioning policy with recycling used items for a series system. The designed inventory strategies for the new spare parts and the used spare parts are described. We also provide an algorithm to solve the model and a numerical solution example. Finally, research summaries and recommendations for future research are provided in Chapter 5.

Chapter 2

Literature Review

2.1 Introduction

A significant part of the recent literature on maintenance management is related to the deteriorating technical systems which are ubiquitous in industry. Typically such a system exhibits an increasing failure rate (IFR) curve or bathtub-shaped failure rate curve in its life cycle (see Figure 1.2). The failures due to deterioration usually accelerate the system's degradation. Maintenance is employed to prevent failure or slow the degradation of the system's operational condition. Research on the process of deterioration and its associated failure mechanisms helps maintenance engineers carry out maintenance effectively in many ways. Various optimal maintenance models have been developed to determine optimal maintenance strategies according to a system's characteristics. In the next section, we briefly describe system failure mechanisms and maintenance impact on the failure rate function of the system. In Section 2.3, we survey previous relevant optimal maintenance policies for both single unit systems and multi-unit systems.

2.2 Deteriorating Systems: Failure Rate and Maintenance

A sophisticated technical system often degrades with age and usage. Failures due to degradation are inevitable if the system operates long enough. On failure, one of two possible actions can be taken: repair or complete replacement of the failed system. To reduce the frequency of failure, PMs are carried out on a system between two consecutive failures and between the last repair and complete replacement. In practice, maintenance managers or engineers have to face many management problems if they are to make maintenance activities effective and keep maintenance costs low. Their problems may include when to perform a PM, how often to conduct PMs and when to carry out a complete replacement of a system. In order to address these problems, it is imperative to understand the nature of failure and the characteristics of the deteriorating system.

2.2.1 System Health Condition and Failure Rate Function

As we have mentioned before, condition-based maintenance management is highly effective and is preferred by most industries. The key to its success lies in accurate and timely measurement of the condition of the system. In reliability theory, the general health condition of a system is described by its failure rate function and its effective age. The failure rate function can be estimated from the measured data. The higher the value of failure rate function, the worse the system's health condition (*Lin, Zuo and Yam* [12]).

Xie & Lai [10] propose an additive Weibull model to characterize a deteriorating system with a bathtub-shaped failure rate function, which is expressed as the sum of two failure rate functions of Weibull form. Usually there are different failure modes associated with a system. The system fails because of the occurrence of a failure mode. *Xie & Lai* assume that the system is affected by two major failure modes, each corresponding to a Weibull distributed failure time, but with different parameters. Thus their model can flexibly characterize the failures of a system. For example, the initial failures are usually caused by design faults and initial problems, which lead to a decreasing failure rate. The late part of the bathtub-shaped failure rate is usually caused by material fatigue or component aging, and this corresponds to an increasing failure rate. The system failure rate function is the sum of the two failure rate functions which are associated with the two different failure modes.

Wang et al [11] describe a sophisticated system whose degradation comes from four major failure mechanisms, (1) random failure: failure incurred due to intrinsic weakness or/and sudden change in environmental conditions; (2) cumulative damage: failure induced by deterioration of the strength of the system due to continuously applied stress; (3) man-machine interface: the result of interaction between human learning and system failure behavior; and (4) adaptation: the process of the condition of the system as mating components or subsystems adapt to each other. *Wang et al* assume that each failure mechanism has its own independent failure rate function and that the failure rate function of the system is a summation of all failure rate functions based on its failure mechanisms. This presents a bathtub-shaped curve. The explicit form of this bathtub-shaped failure rate function is rather complicated, therefore the assumption mentioned above is too ideal to be realistic.

2.2.2 Effects of PM and Repair on Deteriorating Systems

The action of repairing a failed system is often passive and thus is hard to control or plan in advance. In maintenance management, this situation is not desired. PM is introduced to prevent or decrease random failures. To understand the effects of PM and repair on the health condition of the system, researchers have investigated them and have produced cornerstone results which are extensively applied in designing maintenance policies.

Lie & Chun [14] and *Nakagawa* [15] introduce the concept of adjustment factors in hazard rate function and effective age in modeling the effects of PM. *Nakagawa* [16] proposes two PM models assuming that the required time for PM is negligible. Later *Lin, Zuo and Yam* [12] and *Monga* [17] extended the two PM models in their maintenance strategies. The following is a brief description of the two PM models.

1. *Hazard Rate PM Model*: With the hazard rate concept, a PM restores system performance to some extent; However, after each additional PM the slope of the failure rate function increases. This effect can be expressed in the following explicit form. During the ith PM interval, the hazard rate of the system is $h_i(t)$:

$$h_i(t) = \Theta_i \cdot h_{i-1}(t) \tag{2.1}$$

   where $h_0(t)$ is the original failure rate function of the system and Θ_i is the *failure rate deterioration factor* of the deteriorating system due to PM action. Θ_i must satisfy the following two conditions:

   - $\Theta_1 = 1$
   - $\Theta_{i+1} \geq \Theta_i$, where $i = 1, 2, \cdots$.

2. *Age Reduction PM Model*: With the virtual effective age concept, a PM action reduces the effective age of the system to some extent. Assume

that the system's current effective age is T_j; if a PM is conducted at this moment the system's effective age, T_j, is reduced to αT_j, where α is *the age reduction factor* of the deteriorating system due to the PM, such that, $0 \leq \alpha < 1$.

In many practical instances, the deteriorating system is degrading with usage even though the PMs improve its operational condition to some extent. Based on this observation, *Lin, Zuo, and Yam* [12] propose a *hybrid PM model* that combines the advantages of the *age reduction PM model* and the *hazard rate PM model*. In the Hybrid PM Model, the effects of each PM on the system's operational condition are dual: the system's effective age is reduced immediately but, on the other hand, the deteriorating process is accelerated when the equipment is put into use again. With this Hybrid PM Model, the failure rate function after the ith PM can be written as

$$h_i(T_i + t) = \Theta_i h(\alpha_i T_i + t) \tag{2.2}$$

where T_i is the time interval between the (*i-1*)th PM and ith PM, $0 \leq \alpha_i \leq 1$, and $1 \leq \Theta_i < \infty$.

Kijima [18] develops two general repair models for a repairable system by using the idea of the virtual age process of the system. The models characterize the effects of the repair action on the system by adjusting the virtual age of the system. They are described as follows. Let A_n be the degree of the nth repair effect on the system and take a value between 0 and 1. Two models are constructed depending on how the repair activities affect the virtual age process, $\{V_n\}_0^\infty$.

Model I: The nth repair cannot remove the damage incurred before the $(n-1)$th repair. It reduces the additional age X_n to $A_n X_n$. Accordingly the virtual age after the nth repair becomes

$$V_n = V_{n-1} + A_n X_n. \tag{2.3}$$

Model II: At the nth failure, a virtual age of $V_{n-1} + X_n$ has accumulated. The nth repair affects the virtual age so that

$$V_n = A_n(V_{n-1} + X_n). \tag{2.4}$$

In both models, if $A_n = 0$ for all $n \geq 1$ then one has a perfect repair model; whereas, if $A_n = 1$ for all $n \geq 1$, then it agrees with a minimal repair model. A_n has an average value at the nth repair for specific equipment in the long run. Some applications and modifications of the models can be found in *Jiang et al.* [19], *Stadje & Zuckerman* [20] and *Zhang et al.* [36].

2.2.3 Concluding Remarks

This section has described the potential failure mechanisms of a deteriorating sophisticated system and its bathtub-shaped failure rate function. The failure rate of the system may be associated with different failure modes. Each failure mode may be induced by one or more different failure mechanisms. Each failure mode affects the system in different ways. Research on failure modes and their related failure mechanisms helps us understand the effects of PM and repair on a system's health condition. The effect of a PM on a system's health condition is modeled by using the virtual age reduction concept or hazard rate concept or their hybrid. The effect of a general repair on the system's condition is characterized by using the concept of accumulated previous damage or the concept of accumulated virtual age reduction.

2.3 Optimal Maintenance Policies

Research on maintenance problems has generated many models of optimal maintenance policies. All of them can be categorized into two classes: optimal maintenance models for a single unit system and optimal maintenance models for a multi-unit system. This classification is amenable to current theoretical development and helps practitioners select the most appropriate model for their maintenance problems (*Wang* [9]). The classification can also serve as guidance for us in defining our present research on maintenance management problems.

2.3.1 One-unit System Maintenance

The major maintenance management problems associated with a single unit system mainly include how to schedule the PMs and when to perform a replacement to ensure minimum system operational cost. For these purposes, the optimal maintenance models are developed to determine such decision variables as the operational time, the accumulated failure number, or the total PMs that have been performed on the system.

Lam [21] & [22] introduces the geometric process to characterize the behavior of a repairable deteriorating system. Two optimal replacement policies are proposed: one is based upon the accumulated working time and the other, on the accumulated failure number of the system. Explicit expressions of the long-run average cost per unit time under each replacement policy are derived. Under some mild conditions, *Lam* has proved that the optimal policy based upon the number of failures is better than the optimal policy based upon the operational time. Afterward, *Zhang et al* [23] incorporated preventive maintenance (PM) activities into the failure-number-based replacement model. The PMs are carried out at a fixed time interval. The effect of PM is to restore the system to the same condition just after the last failure repair action, but it does not restore the system to its condition at time zero. A numerical example with a constant failure rate function is given to demonstrate the feasibility of this model.

Park et al [24] propose a periodic PM and minimal repair model for a repairable system. Under this policy, each PM releases stress temporarily and hence slows the rate of system degradation. The PMs are performed at fixed time intervals. The system is replaced at the Nth PM time point. If the system fails between PMs, it undergoes only minimal repairs, i.e., the failure rate of the system remains the same as before the minimal repair. The optimal PM number and the optimal interval of performing PM are the decision variables that minimize the system operational cost per unit time over an infinite time span. Explicit solutions for the optimal periodic PM are given for the Weibull distribution case.

Unlike the periodic PM policy, a system is preventively maintained at unequal time intervals under a sequential PM policy. The time interval for performing PMs becomes shorter and shorter as time passes because most systems need more frequent maintenance as their ages increase. *Nguyen et al* [25] introduce the sequential PM policy concept. If no failure occurs by some specific time, t_i, a PM is performed. The specific time, t_i, is the maximum time that the system should be left running without maintenance after the last repair or replacement action. Under this policy, the system is repaired at the time of failure or at age t_i, whichever occurs first, and it is replaced at the kth repair. *Lin et al* [12] build a hybrid imperfect PM model, which uses both effective age reduction and acceleration of deterioration after each PM to characterize the effect of PM on a system. They also introduce two new failure modes for a complex system: maintainable failure mode and nonmaintainable failure mode. PMs are performed at sequential intervals and can change the hazard rate of only the maintainable failure mode. With a fixed level of failure rate, which is used to indicate the health condition of the system, the optimal PM policy is determined by minimizing the average cost rate with respect to the total number of PMs being performed on the system.

In Chapter 3, we will extend the work of *Lin et al* [12] by considering a system with a bathtub-shaped failure rate function. The failures taking place during its life cycle will be classified into minor failures and major failures in terms of their effects on the health condition of the system after corrective actions. Periodic PMs are conducted between two consecutive major failures and the PMs' frequency depends on the current health condition of the system. The decision variables considered will include the total number of major failures and a specified critical level of the major failure rate.

2.3.2 Multi-unit System Maintenance

Research on multi-unit system maintenance policies has been gaining increased attention recently, because multi-unit systems are widely used. Examples of such systems include chemical processing facilities, power plants and production lines. For multi-unit system, the cost of unavailability (one-time unplanned shutdown of the system) is often much higher than the maintenance cost; therefore, performing maintenance on several subsystems jointly requires less money and/or time than would each subsystem separately. It is determining the optimal PM and the most cost-effective replacement schedules that constitute the primary maintenance management problems for the multi-unit system.

One of the important PM strategies for this system is block replacement policy. Block replacement policy derives its name from the commonly employed practice of replacing a block or group of units in a system at predetermined times, kT, $(k = 1, 2, \cdots)$, independent of the failure history of the system. This policy can address very well the economic dependency problem in multi-unit system maintenance activities; however, it may be rather wasteful since sometimes almost new units are replaced at the instant of performing block replacement. To alleviate the drawback of this policy, it has been extended to allow for reusing unfailed units which were taken out as parts from previous preventive replacements (see *Tango* [30]). Under *Tango*'s policy, block replacements are performed by using new items at times kT, $k = 1, 2, \cdots$. If a failure occurs in the time interval $[(k-1)T, kT-\delta)$, $0 \leq \delta \leq T$, the failed item is replaced by a new one. If a failure occurs in the time interval $[\ kT-\delta, kT)$, the failed item is replaced by a used one. The objective function of this model can be minimized by searching for optimal values for (T, δ). *Murthy & Nguyen* [29] modify *Tango*'s model by considering the fact that reused items may have different ages. The modification makes use of all used items and can improve *Tango*'s model to some extent. *Kadi & Cleroux* [28] append an idle period to *Tango*'s model to delay the block replacement time. During the idle time, the system still works but may not work as efficiently as before the idle period. *Sheu & Griffith* [31] apply the block replacement policy involving used items to a shock model. It is worth mentioning that all these block replacement policies assume spare parts are available whenever needed. There is no limit on the number of new spare parts and used spare parts being supplied when performing random failure replacement and block replacement. In practice, there is usually a large demand for spare parts during block replacement and between two consecutive block replacements. Inappropriate spare parts provisioning and inventory policy may delay block replacement or extend system unexpected shutdown time. Both will lead to heavy operational losses.

Chelbi & Ait-Kadi [26] propose a model that can jointly optimize block replacement and spare parts provisioning over an infinite time span. The system is made up of n identical components. The block preventive replacements are performed at predetermined instants, T, $2T$, ..., regardless of the age and the state of the components. If a failure occurs between two consecutive block preventive replacements, the failed component is replaced by a new one immediately if the latter is available. An (R, s) inventory control strategy is used in their spare parts management. The total operational cost is the sum of the block replacement cost and the spare parts inventory management cost during a block replacement period. The minimal expected cost rate can be obtained by searching for the optimal block replacement interval, T; the optimal replenishment cycle, $R = kT$, $(k = 1, 2, ...)$; and the minimum ordering spare parts level, s. Under this spare inventory policy, a reorder is placed as long as the stocking level of the spare parts is less than or equal to s. *Brezavscek & Hudoklin* [27] report a model similar to that in *Chelbi & Ait-Kadi* [26]. The only difference is that it is the (R, S) inventory control policy that is used in *Brezavscek & Hudoklin* [27]. Under this policy, at each review time, a sufficient quantity is ordered to bring the level of the available inventory up to the maximum inventory level, S. By letting R be equal to T, *Brezavscek & Hudoklin*'s model can closely couple the periodic PMs and the spare-provisioning policies. In practical applications, the (R, S) policy is the most commonly used periodic review policy. Its operation is simple. It results in a predictable work load on the purchasing or production scheduling departments, in contrast to the (R, s) policy where the number of orders released at a review time fluctuates depending on the relative position of the available inventory with respect to the order point. In addition, the computation of the controls, R and S, is simpler than the computation of R and s (see *Hax & Candea* [39]).

In Chapter 4, we will report an improved model based on *Brezavscek and Hudoklin*'s work, one which recycles the unfailed spare parts selected from the immediately previous block replacement. An (R, S_2, S) inventory control policy will be proposed. The value of the maximum reused spare parts inventory level, S_2, is determined by the block replacement interval, T, and the time interval of employing the used spare parts, δ. The decision variables of the new model will include T, δ and the maximal stocking level of new spare parts, S.

Chapter 3

An Optimal Preventive Maintenance And Repair Model For a Single Unit[1]

3.1 Introduction

In this chapter, we propose a preventive maintenance and repair model for a single unit system with a bathtub-shaped failure rate function. There may be different causes of system failures. A system failure can be ascribed to either the result of the adaptation of the system or interaction between the system, its subsystems and its working circumstance, or the result of accumulated damage due to aging (*Xie and Lai* [10]). Taking into account improvement in adaptation and increment in accumulated damage as the system's operational time increases, failures during the system life cycle are classified as being either of minor failure mode or of major failure mode. The minor failures are described by the minor failure rate function, which monotonically decreases with time. The major failures are characterized by the major failure rate function, which monotonically increases with time. The two types of failure produce completely different impacts on the performance of the system. An imperfect

[1]Two versions of this Chapter have been submitted for publication (see *Zhang et al* [36] & [37])

repair is carried out to restore function to the system as soon as a major failure occurs. A minimal repair is conducted to restore the system to the state just prior to the failure when a minor failure occurs. Since the major failure rate describes the system wear-out failures, which predominate in the later period of a life cycle, its level indicates the system health condition. PM is performed when the major failure rate reaches a specified critical level so as to improve the health condition of the system and, hence, to extend its total lifetime.

In this new model, the whole cost rate is formulated as a function of both the number of major failures that the system has experienced in a life cycle and the specified critical level of major failure rate. The optimal policy may be obtained by minimizing the whole cost rate with respect to the number of major failures that the system experiences in a life cycle and the specified critical level of major failure rate. Finally, the Monte Carlo method is employed to conduct a simulation study of the model. Numerical results reveal that this model is effective and feasible for maintenance decision-making.

The rest of this chapter is organized as follows. Section 3.2 describes the characteristics of the system and its failure rate behavior. Section 3.3 describes the repair policy and the PM policy. Section 3.4 builds up the optimization model for the system. Monte Carlo simulation is conducted in Section 3.5 and a numerical example illustrating the procedure is provided as well. Concluding remarks are presented in Section 3.6.

3.2 System Characteristics

The failure rate function of a complex system, $h(t)$, often exhibits a bathtub shape and is referred to as a bathtub-shaped curve. Failure rate is an important concept in reliability analysis since it represents the instantaneous probability of failure over the next instant of time of a system. Traditionally, the bathtub-shaped failure rate is divided into three different phases, namely, the infant period, the normal period, and the wear-out period. Most researchers have explored the three phases independently; however, in many practical situations, it is necessary to consider the whole life span including the three phases, simultaneously.

In the additive Weibull model (*Xie and Lai* [10]), a bathtub-shaped failure rate function is formulated as

$$h(t) = at^{b-1} + ct^{d-1},$$

where $a > 0$, $b > 1$, $c > 0$, and $0 < d < 1$ (see Figure 3.1).

Taking into account whether or not some damage caused by a failure remains after a repair, we classify all failures of the system into two categories: major failures and minor failures. The major failure rate is defined as

$$h_{major}(t) = at^{b-1}, \tag{3.1}$$

where $a > 0$ and $b > 1$. The minor failure rate is defined as

$$h_{minor}(t) = ct^{d-1}, \tag{3.2}$$

where $c > 0$ and $0 < d < 1$.

As shown in Figure 3.1, the minor failure rate decreases with time, while the major failure rate increases with time. Consider a newly installed electro-mechanical system, such as a medical Computed Tomography (CT) machine. During the first few weeks or months of use, the supplier is often called in

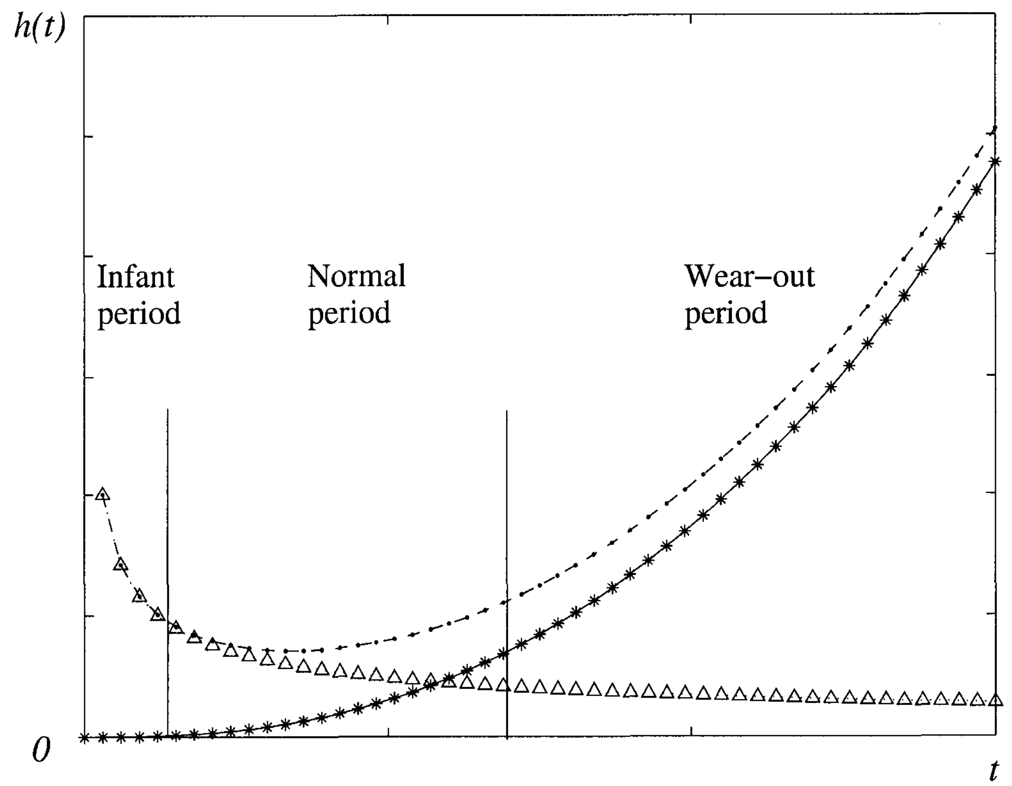

Figure 3.1: The bathtub-shaped failure rate (the point line) as a sum of the major failure rate (the star line) and the minor failure rate (the triangle line)

because of some unexpected electrical or display faults. These faults are usually due to operating mistakes of the novice, calibration error, or inadequate burn-in time. The average repair costs of such faults are relatively low and, after such service calls, the equipment can be regarded as being as good as it was before the minor failure. That is, after repair the minor failure rate remains the same as before the minor failure. As the equipment enters its stable operation period, such faults and operating mistakes occur less frequently; however, with equipment usage and aging, major faults in its machinery or its critical components start to appear. The average repair costs of such faults are usually high. In addition, the condition of the repaired equipment is no longer as good as it was just before the major failure. Hidden damage due to such failures accumulate and make the equipment deteriorate faster. As a result, it is the major failure rate that determines the health condition of the system.

After a major failure, if no replacement is warranted, a major repair has to be carried out. A major failure and its repair produce the following effects illustrated in Figure 3.2.

1. The slope of the major failure rate is steepened due to the damage which remains after each major failure.

2. The major failure rate drops immediately by a certain amount due to the reduction of the system's virtual age which results from the repair of a major failure.

3. The capacity for reduction of virtual age of the system decreases as the number of major failures increases.

At the beginning, the system deteriorates slowly, and the damage done by major failures due to such deterioration is small. With usage and aging,

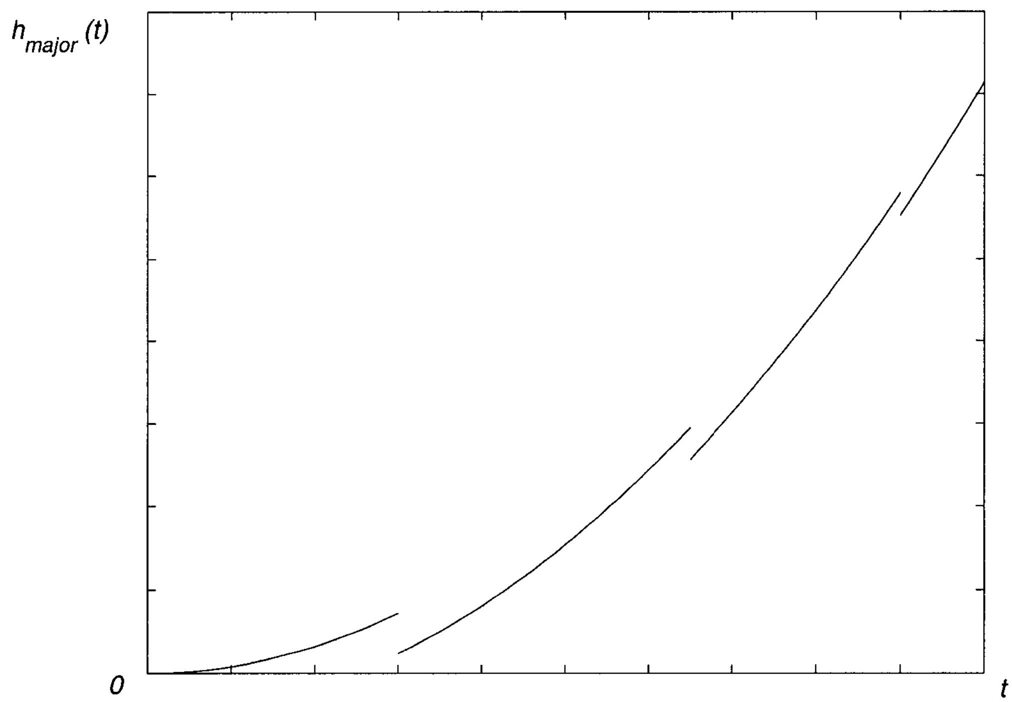

Figure 3.2: The effects of a major failure repair on the major failure rate function

the speed of deterioration accelerates and the corresponding damage due to random major failure increases.

Similarly, the system's maintainable capacity, which to some extent causes a reduction in the virtual age of the system, is high during the earlier life of the system. With increased deterioration of the system, this capacity becomes progressively smaller.

During the earlier stage of the system's life, major failures occur less frequently. The total operating cost is low. As the equipment ages, the number of major failures increases and the accumulated cost of major repairs increases. The total operating cost will eventually become excessive and the system will have to be replaced. As can be seen, there is a trade-off between the number of major failures that the system experiences and the cost of its replacement.

3.3 Preventive Maintenance

To a large extent, the major failure rate function determines the general health condition of a piece of equipment. Its value depends on its effective age. For deteriorating equipment, the higher the effective age, the worse the health condition. In practice, PM is often performed to improve equipment's health condition; it plays an important role in decreasing the effective age and slowing down the rate of deterioration. As stated earlier, minor failures do not leave equipment damaged after minimal repairs. The minor failure rate tends to be small and stable as the equipment enters the period of normal use. Thus, a PM has little impact on the minor failure rate. As a result, we ignore this effect. PM does, however, influence the major failure rate, which increases monotonically.

Too few PMs will not effectively slow down the rate of equipment deterioration. On the other hand, higher operational cost will be incurred if too many PMs are performed, therefore, it is important to determine an optimal PM policy, one capable of reducing the total operating cost of the system. In our model, whenever the major failure rate function reaches a specified value, ξ, the equipment is maintained preventively. Thus, there is a trade-off between the specified failure rate level, ξ, and the total cost of the system over its life cycle.

Further, it is assumed that, in the sense of failure rate, performing a PM between the $(n-1)$th and the nth major failure can restore the health condition of the system to the state just after the repair of the $(n-1)$th major failure, but never better than that because its maintainable capacity is determined by the $(n-1)$th major failure and its repair. In fact, a major repair always results in an overhaul of the system, hence it determines the maintenance capacity and health condition of the system. The possible effect and history of PMs combined with major failure repairs are shown in Figure 3.3.

Now, we are ready to present the PM and repair model.

3.4 System Costs and Problem Formulation

Assumptions:

1. The planning horizon is infinite.

2. When the nth major failure occurs, the damage incurred accelerates system deterioration and hence makes the major failure rate curve steeper. After the nth major failure repair, the system's virtual age is reduced to some extent, and the major failure rate function of the system is (also

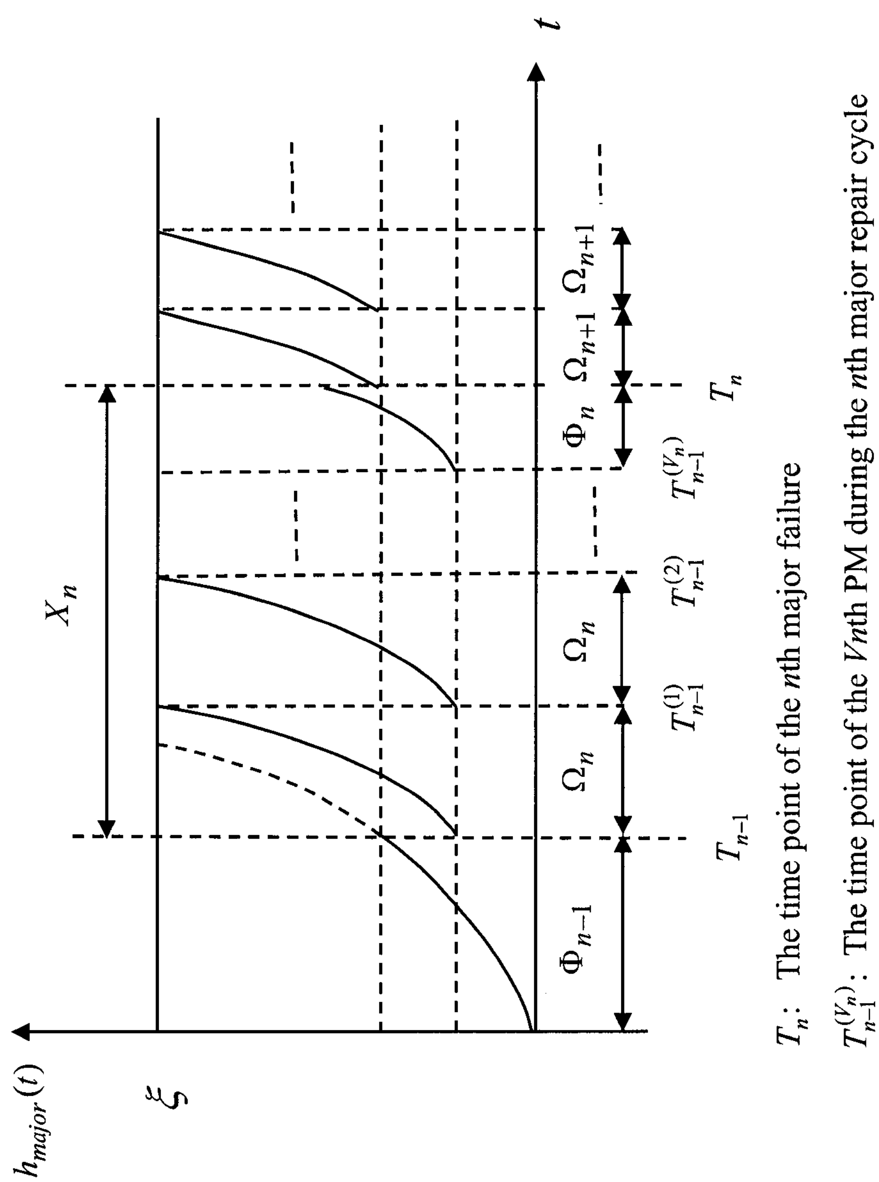

Figure 3.3: Effect of PMs between the $(n-1)$th major failure and the nth major failure

refer to Figure 3.2)

$$h_{major}^{(n)}(t) = h_{major}(\eta_n(t - \gamma_n \Phi_n)), \quad n = 1, 2, \ldots, \tag{3.3}$$

where

η_n is the adjustment factor for the slope of major failure rate function whenever the nth major failure happens,

γ_n is the reduction factor for the virtual age reduction of the system whenever the nth major failure is repaired,

Φ_n is the time between the last PM in X_n and the $(n+1)$th major failure (refer to Figure 3.3).

3. The system is to be replaced at the Nth major failure.
4. The cost of a minor repair is smaller than that of a major repair, which in turn is smaller than that of a replacement.
5. Whenever the major failure rate function reaches the specified level, ξ, PM is performed.
6. The time needed for PM, minor failure repair, major failure repair, or replacement is negligible.
7. Let X_n be the time interval between the $(n-1)$th major failure and the nth major failure. Each PM within X_n restores the health condition of the system to the state just after the $(n-1)$th major failure repair.

According to the above assumptions, as a random variable, the length of the life cycle of the system is

$$W = \sum_{n=1}^{N} X_n, \tag{3.4}$$

where X_n is the interval time between the $(n-1)$th major failure and the nth major failure (refer to Figure 3.3).

$$X_n = V_n \Omega_n + \Phi_n, \quad n = 1, 2, \ldots, N \tag{3.5}$$

where Ω_n is the time interval between successive PMs in the nth major repair cycle, and V_n is the number of PMs conducted within the nth major repair interval X_n.

The replacement cost in a life cycle is c_r. The major repair cost is c_{major} and the total major repair cost in a life cycle is $c_{major}(N-1)$. Since PM and imperfect repairs do not affect the minor failure rate function, the expected number of minimal repairs over a life cycle is

$$H_{minor}(W) = \int_0^W h_{minor}(t)dt = \int_0^W c\,t^{d-1}dt = \frac{c}{d}W^d.$$

Let c_{minor} be the minimal repair cost. The total cost of minimal repairs in a life cycle is then $c_{minor}H_{minor}(W)$. Let c_p be the average cost of each PM. The total cost of PMs in a life cycle is $c_p \sum_{n=1}^{N} V_n$. Thus, the cost rate over a life cycle is

$$f(N,\xi) = \frac{E\left[c_r + c_{minor}H_{minor}(W) + c_{major}(N-1) + c_p \sum_{n=1}^{N} V_n\right]}{E(W)}.$$

As usual, $f(N,\xi)$ serves as the objective function to be minimized by searching optimal decision variables N and ξ.

After the nth major failure and its repair, the major failure rate function could be formulated as follows:

$$h^{(n)}_{major}(t_n) = a\left[\eta_n\left(t_n - \gamma_n\Phi_n - \sum_{i=1}^{n-1}\Phi_i\right)\right]^{b-1}, \quad n = 1,2,\ldots,N \tag{3.6}$$

where t_n is the hypothetical time point for the major failure rate function to reach ξ.

If no major failure occurs again after the nth one, then a PM is performed whenever the major failure rate function, $h_{major,n}(t)$, reaches ξ. As a result, we have

$$t_n = \left(\frac{\xi}{a}\right)^{\frac{1}{b-1}}\frac{1}{\eta_n} + \gamma_n\Phi_n + \sum_{i=1}^{n-1}\Phi_i, \tag{3.7}$$

and

$$\Omega_n = t_n - \sum_{i=1}^{n}\Phi_i = \left(\frac{\xi}{a}\right)^{\frac{1}{b-1}}\frac{1}{\eta_n} + \gamma_n\Phi_n - \Phi_n, \quad n = 1,2,\ldots,N. \tag{3.8}$$

In summary, we have the following optimization model:

$$f(N,\xi) = \frac{c_r + c_{minor}E(W^d) + c_{major}(N-1) + c_p\sum_{n=1}^{N}E(V_n)}{E(W)}, \tag{3.9}$$

where c_r, c_{minor}, c_{major}, c_p and $\xi > 0$, W, X_n and Ω_n are determined by (3.4), (3.5) and (3.8), respectively.

Since a closed form of $E(W^d)$ in (3.9) is not available; in the next section, Monte Carlo simulation will be employed to evaluate $E(W^d)$.

3.5 Monte Carlo Simulation

Due to the ongoing development of computer power, Monte Carlo simulation has become a powerful tool for performing realistic reliability and availability analysis of complex systems (*Labeau and Zio* [32]). It is relatively straightforward to use Monte Carlo simulation to propagate uncertainty in the values of random variables of earlier events to current event.

The following outlines the procedure of Monte Carlo simulation for evaluating $E(W)$ and $E(W^d)$ in equation (3.9).

1. Set $\eta_n = g^{n-1}$, $\gamma_n = u^n$ and ξ.

2. Given a, b, g, u and ξ,

3. Set $n = 1$ and $\xi_n = \xi$.

4. Find t_n by setting $h^{(n)}_{major}(t)$ in equation (3.7) equal to ξ_n.

5. Find Ω_n by setting $h^{(n)}_{major}(t)$ in equation (3.8) equal to ξ_n.

6. Generate a random number denoted by R that follows the Weibull distribution with the failure rate function given in equation (3.6). If $R > \Omega_n$, another random number is generated in the same way until we obtain a random number that is less than or equal to Ω_n. Let i denote the number of such random numbers generated and let the sequence of random

numbers generated be denoted by R_1, R_2, $\ldots$, and R_i. Then, we have $V_n = i - 1$, $\Phi_n = R_i$, and $X_n = V_n\Omega_n + \Phi_n$.

7. Set $n = n+1$ and $\xi_n = \xi_{n-1} - h_{major}^{(n-1)}(\Phi_{n-1} - \gamma_{n-1}\Phi_{n-1})$. If $\xi_n \leq 0$, go to the next step; otherwise go to the previous step.

8. The simulation procedure is complete. We have obtained V_1, V_2, $\ldots$, V_{n-1}, $W = X_1 + X_2 + \ldots + X_{n-1}$, and W^d. The complete procedure can be repeated, say, M times. We would then have M observations of V_1, V_2, $\ldots$, V_{n-1}, W, and W^d. From these observations, we can find the averages $\bar{V}_1$, $\bar{V}_2$, $\ldots$, $\bar{V}_{n-1}$, $\bar{W}$, and $\bar{W}^d$, which can be used to approximate $E(V_1)$, $E(V_2)$, $\ldots$, $E(V_{n-1})$, $E(W)$, and $E(W^d)$.

In order to demonstrate the robustness of our model, several different sets of the parameters are tested. The number of simulation iterations, M, is set at 2000; and let $c_r = 2000$, and $c_{major} = 100$. Thirty five simulation runs are conducted. The numerical results are listed in Table 3.1. The optimal cost rate is the average of the 35 cost rates obtained from the simulation runs. The confidence interval, γ, indicates that there is a 95% confidence that the real cost rate is within $\pm\gamma$ of the average cost rate.

In Table 3.1, as can be seen in cases 1, 2, 3 and 4, the optimal cost per unit of working time, $C.R.^*$ and decision variables N^* and ξ^* vary with the different sets of g and u. As g increases while u decreases, the maintenance cost rate, $C.R.^*$, increases. Meanwhile, the optimal decision variables N^* and ξ^* become smaller. This tells us how the magnitude of the damage caused by each major failure and the reduction of virtual age due to major failure repair affect the lifetime and the minimal operational cost rate of the system. Cases 5, 6 and 7 demonstrate that the values of the optimal decision variables vary also with the change in shape of the bathtub curve.

Based upon the data set in case 4, the cost per unit of working time as a function of ξ and N is plotted in Figure 3.4. For a specified failure rate level, ξ, the value of the objective function varies with the number of major failures allowed, N, and reaches a minimum value at the optimal value of N. For example, for a specified ξ value of 0.20, when N takes the value of 7, the cost rate reaches its minimum of 88.239 (see Table 3.2). For a specified N, the value of the objective function varies with the value of ξ and reaches a minimum value at the optimal value of ξ. For example, for a specified N value of 16, when ξ takes the value of 0.50, the cost rate reaches its minimum of 89.557 (see Table 3.3). When both N and ξ take their optimal values of 10 and 0.35, respectively, the value of the objective function reaches the minimum of 86.026. Therefore, Figure 3.4 gives us a guide for making decisions on how to choose the failure rate level when conducting PMs and when we should replace the system.

Cases	a	b	c	d	g	u	c_{minor}	c_p	N^*	ξ^*	Optimal Cost Rate	Confidential Interval(95%), γ
1	1.3	6.5	2	0.3	1.006	0.97	10	50	30	0.7	73.9900	0.0134
2	1.3	6.5	2	0.3	1.007	0.96	10	50	26	0.65	75.0647	0.0181
3	1.3	6.5	2	0.3	1.008	0.95	10	50	21	0.60	76.0222	0.0229
4	1.3	6.5	2	0.3	1.009	0.94	10	50	19	0.55	76.9032	0.0235
5	3	4.5	2	0.3	1.009	0.9	50	30	13	0.5	78.8619	0.0386
6	1	5.5	4	0.7	1.007	0.9	50	30	14	0.3	61.7509	0.0490
7	1.2	5.5	2	0.5	1.01	0.9	10	50	20	0.6	81.6053	0.0255

Table 3.1: Optimal cost rate and decision variables with different parameters

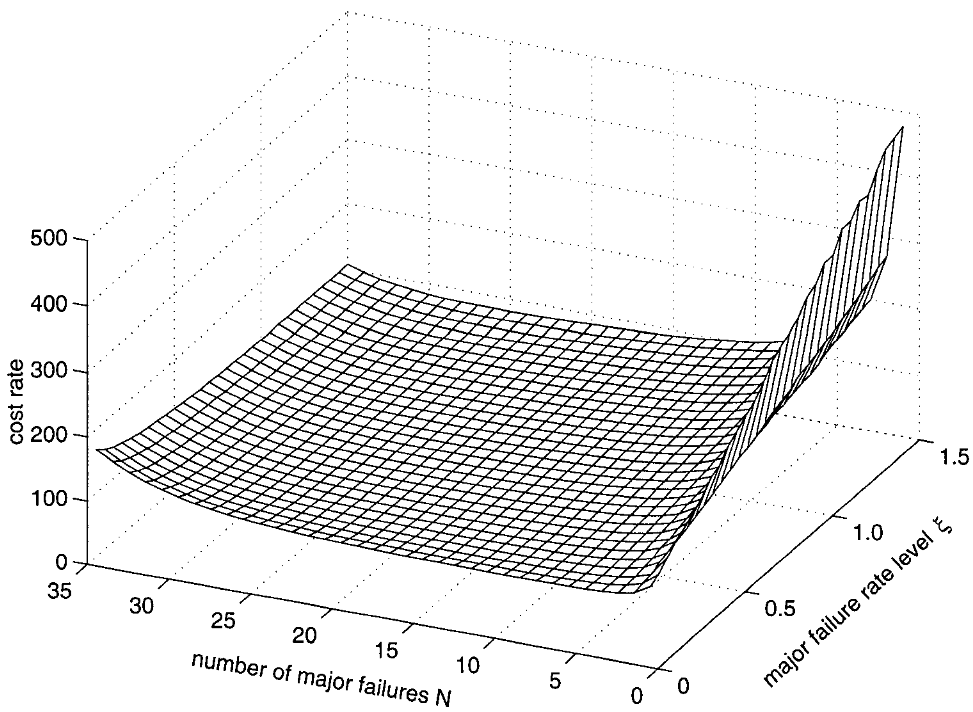

Figure 3.4: Cost rate against the number of major failures and the major failure rate level

Table 3.2: Cost rate with each specified ξ

case	specified ξ	N^*	cost rate
1	0.10	5	95.208
2	0.15	7	90.783
3	0.20	7	88.239
4	0.25	8	86.918
5	0.30	10	86.317
6	0.35	10	86.026
7	0.40	10	86.085
8	0.45	11	86.684
9	0.50	12	86.989
10	0.65	13	87.493
11	0.60	12	88.431
12	0.65	13	89.422
13	0.70	13	90.113

Table 3.3: Cost rate with some specified N

case	specified N	ξ^*	cost rate
1	5	0.20	90.701
2	6	0.25	88.581
3	7	0.30	87.313
4	8	0.25	86.638
5	9	0.35	86.115
6	10	0.35	86.026
7	11	0.40	86.253
8	12	0.40	86.754
9	13	0.40	87.442
10	14	0.50	88.024
11	15	0.50	88.777
12	16	0.50	89.557
13	17	0.60	90.515

3.6 Concluding Remarks

This chapter presents a new preventive maintenance and repair model for a deteriorating system with a bathtub-shaped failure rate function. By considering the minor failure mode and the major failure mode of the system, the cost rate of the life cycle is formulated as a function of the total number of major failures that the system is allowed to experience before being replaced, and a specified critical level of the major failure rate. The minor failure rate becomes lower and lower and the major failure rate becomes larger and larger as the system ages and deteriorates. After a minor failure repair, the system regains the same condition it had before the minor failure. If the system experiences a major failure, system deterioration is accelerated. After a major failure repair, the major failure rate is reduced to some extent. Thus the system health condition is determined by the level of its major failure rate. Performing PMs cannot, however, reduce the failure rate of the system to a state lower than its state after the last major failure repair. As a result, the frequency with which PMs are performed is determined by the health condition of the system. The optimal strategy is to minimize the whole cost rate with respect to the time to replace the system and the critical level of major failure rate. Finally, Monte Carlo method is employed to conduct a simulation study of the model. The numerical results reveal that the model proposed is effective and feasible.

In the next chapter we focus on building an optimal maintenance policy considering the spare parts provisioning strategies for a series system, a typical multi-unit system. The model will consider the salvage value of the used spare parts.

Chapter 4

Periodic Preventive Replacement and Spare Parts Provisioning Policy For a Series System[1]

4.1 Introduction

It is of great importance to prevent the failure of a system during actual operation when such an event is costly and/or dangerous. In practical situations, one important area of interest in reliability theory is the study of various maintenance policies in order to reduce operating cost and the risk of a catastrophic breakdown. One of the well-known PM policies is periodic preventive replacement (PR)(also called block replacement). Under such a policy, an operating system is preventively replaced by a new one at time $k \cdot T$, $k = 1, 2, \cdots$ regardless of its operational history. This policy is commonly used when there are a large number of similar systems in service; however, it cannot completely eliminate random failures between two consecutive PRs. To ensure availability of the system and reduce its operational cost, an optimal spare parts provisioning strategy within the maintenance-scheduling policy is imperative. Researchers

[1] A version of this Chapter has been submitted for publication (*Zhang et al* [38]).

have developed several maintenance and inventory policies, but relatively little effort has been expended on their joint optimization.

Block replacement policy enhances system reliability and prevents excessive unexpected failures, but, on the other hand, it cost a great deal if spare parts are expensive. As Section 2.3 of Chapter 2 indicates, sometimes almost new units are replaced and discarded when performing a block replacement. To alleviate this drawback, one of the widely used strategies in industry is to reuse unfailed units which were taken out in previous preventive replacements. Several papers have addressed this maintenance management problem. All of them assume that both new spare parts and used spare parts are available as soon as they are needed and that there is no limit on the number of used spare parts being supplied, even though these used spare parts are presumably selected from the previous block replacements. This is apparently unrealistic in practice.

This chapter extends *Brezavscek and Hudoklin*'s model [27], providing a new model of which can jointly optimize block replacement strategy and periodic review spare parts provisioning policy by recycling used items. Section 4.2 summarizes the assumptions, the model, and the equations reported in *Brezavscek and Hudoklin* [27] because they will be used or extended in later sections. The model this thesis proposes is presented in Section 4.3. In order to make the problem clear, the new model has been decomposed into two sub-models. One is the new spare parts inventory model which is described in Subsection 4.3.1. The other is the used spare parts inventory model which is described in Subsection 4.3.2. Section 4.4 presents the algorithm developed in order to solve the proposed equation of the problem. Numerical illustrations for the new model are given and discussed in Section 4.5. The concluding remarks for this chapter are then presented in Section 4.6.

4.2 Brezavscek and Hudoklin's Model (2003)

The system considered is made up of n independent and identical components. The components are subject to wear-out failure, so the component failure rate increases with time. The downtime of any component due to shortage of spare parts represents a loss of the system availability.

Maintenance of the system is performed according to block replacement (BR) policy. Under a BR policy, all components of the system are replaced at predetermined time intervals of length T. Failed components between two consecutive BRs are replaced immediately by spares, if they are available. The replacement time is negligible. If spares are not available, failed components are replaced as soon as the inventory of spares is replenished. The downtime of any n components due to shortage of spares (time between the moment of the component failure and the next order arrival) represents a loss of the system's operational time.

The inventory of spares is replenished at a single time point according to the periodic review (R, S) inventory policy. The reordering time point of spares is chosen at time instants $i \cdot T - \tau$, $i = 1, 2, \cdots$, where τ is the lead time of spare part procurement. At each reorder point, enough spares are ordered to bring the inventory level up to S.

Assumptions Used in This Paper Include:

1. The procurement lead time, τ, is constant and shorter than the PR interval, T.

2. The procurement lead time, τ, is independent of the quantity of spares being ordered.

3. There is no quantity discount; thus the spare part cost does not depend on the ordered quantity.

4. The inventory of spares for a given component type is replenished entirely independently of inventories of spares for other types of system components.

5. The holding cost in a given cycle is proportional to the part of this cycle where the inventory-level > 0, and to the average inventory on hand during this cycle (that is, the average number of spares physically located in the inventory during the cycle).

6. The shortage cost in a given cycle is proportional to the cumulative downtime due to shortage of spares during this cycle.

7. During the cycle, the inventory level decreases linearly.

Model Formulations:

Let x_n be the number of component corrective repairs that the system experiences during time interval T. Then x_n is a random variable with a probability density function (pdf) of $g(x_n)$. The $g(x_n)$ could be approximated by a normal pdf with mean

$$E(x_n) = n \cdot E(x) \tag{4.1}$$

and standard deviation

$$\sqrt{Var(x_n)} = \sqrt{n \cdot Var(x)} \tag{4.2}$$

where x is a random variable representing the number of corrective repairs if the system has only one component. The random variable x as a function of T can be described by the ordinary renewal process. A renewal process can be characterized by a renewal function $H(t)$, which is equal to the mean number of failures that occur up to the instant t (*Gnedenko et al.* [33]). $E(x)$ is equal to the value of $H(t)$ at time point T; however, for any type of pdf, the computation of $H(t)$ in the wear-out period is rather tedious.

In this paper, $H(t)$ is calculated by using a discrete approach according to the recurrence relation (*Jardine* [1]).

$$H(T) = \sum_{i=0}^{T-1} [1 + H(T-i-1)] \cdot \int_{i}^{i+1} f(t)dt, \tag{4.3}$$

where $f(t)$ is the probability density function of a given component failure time; $T = 1, 2, ...; H(0) = 0$.

Choosing an appropriate unit of T can make the length of each interval $(i, i+1)$, $i = 0, ..., T-1$, in equation (4.3) sufficiently short so that no more than "1 component failure" occurs within any interval. Calculation of $H(T)$ according to equation (4.3) is not difficult, irrespective of the type of $f(t)$, and of the values of the pdf parameters (integer or not).

The variance, $Var(x)$, which is used to determine the standard deviation of $g(x_n)$, can be calculated as follows (*Gnedenko et al* [33]):

$$Var(x) = H(T) - H(T)^2 + 2\int_0^T H(T-u) \cdot h(u)du \tag{4.4}$$

where $h(u)$ is the renewal density and defined as $\frac{dH(t)}{dt}$. It is obtained by numerical differentiation of the discrete function (4.3). The definite integral in (4.4) is solved numerically using the trapezoidal rule.

Model Development

System operational costs include cost of maintenance, of spare parts ordering and of inventory management. The objective function of the model represents the total operational cost of the system per unit time.

$$C_{total} = C_r + C_s \tag{4.5}$$

$$C_m = \frac{C_{total}}{T} \tag{4.6}$$

where C_{total} is the total cost in a PM cycle, C_m is the cost rate, C_r represents system maintenance costs, and C_s represents spare parts ordering and inventory management costs.

Let p be the preventive replacement cost of a component and c be the failure repair cost of a component. Then the maintenance cost of a system with n components is

$$C_r = n \cdot p + E(x_n) \cdot c = n \cdot p + n \cdot H(T) \cdot c. \tag{4.7}$$

The spare parts ordering and inventory management costs, C_s, consist of the ordering cost, C_o; holding cost, C_h; and shortage cost, C_{sh}. They are determined by equations (4.8), (4.9) and (4.10) respectively.

$$C_o = K + s \cdot (n + E(x_n)) = K + s \cdot n \cdot (1 + H(T)) \tag{4.8}$$

where K is the setup cost for placing an order and s is the cost of purchasing a spare part.

Immediately after the ordered spare parts arrive, n spare parts are used for the BR of the operating components. These spares cause no holding cost, therefore, in calculating the expected holding cost in a cycle, only spares needed for the corrective repair are considered. Assume that the maximal inventory level is S and the expected inventory level just after realization of a planned BR is S'. Considering assumption (7), Figure 4.1 shows that when there is no shortage of spares, the average inventory on hand during the cycle is $S' - (x_n/2)$. From Figure 4.2, it can be derived that in a shortage of spare parts, $T_1 = (S'/x_n \cdot T)$, and the average inventory on hand during the cycle is $S'/2$. Therefore, considering both possible variants of a cycle, C_h can be written as:

$$C_h = h \cdot T \cdot [\int_0^{S'} (S' - \frac{x_n}{2}) \cdot g(x_n)dx_n + \int_{S'}^{\infty} \frac{S'^2}{2x_n} \cdot g(x_n)dx_n]. \tag{4.9}$$

Figure 4.2 shows the shortage of spare parts during the cycle. Considering assumption (6), the shortage cost depends on the cumulative downtime due to shortage of spare parts during the cycle. From Figure 4.2, the average value for the cumulative downtime is

$$T_2 \cdot (\tfrac{x_n - S'}{2}) = (\tfrac{x_n - S'}{x_n} \cdot T) \cdot (\tfrac{x_n - S'}{2}).$$

The C_{sh} in a cycle, allowing for both possible variants of a cycle, is expressed as:

$$C_{sh} = \{ \begin{array}{ll} 0 & \text{if } x_n \leq S' \\ z \cdot T \cdot \int_{S'}^{\infty} \frac{(x_n - S')^2}{2x_n} \cdot g(x_n)dx_n & \text{otherwise} \end{array} \tag{4.10}$$

where z represents the cost of downtime due to a shortage of spare parts per

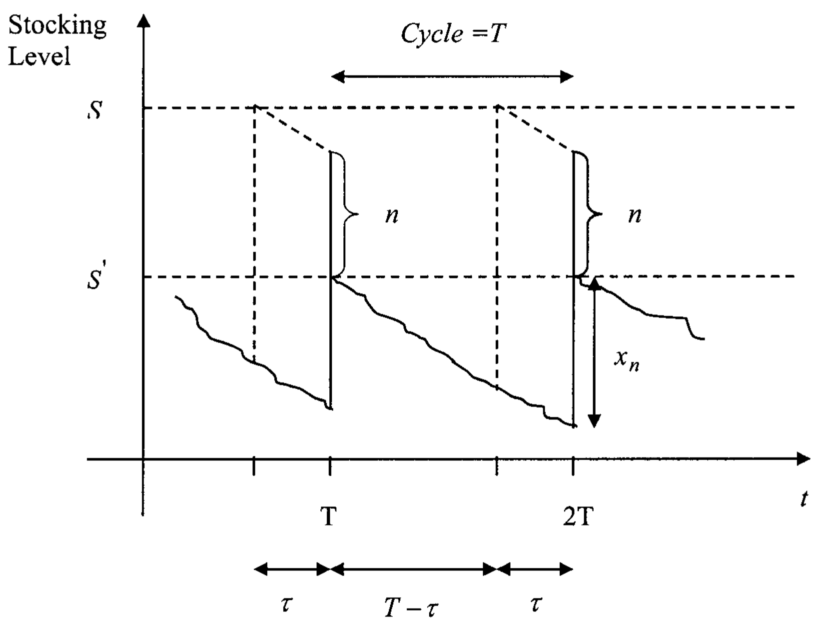

Figure 4.1: Excess of spare parts during a cycle

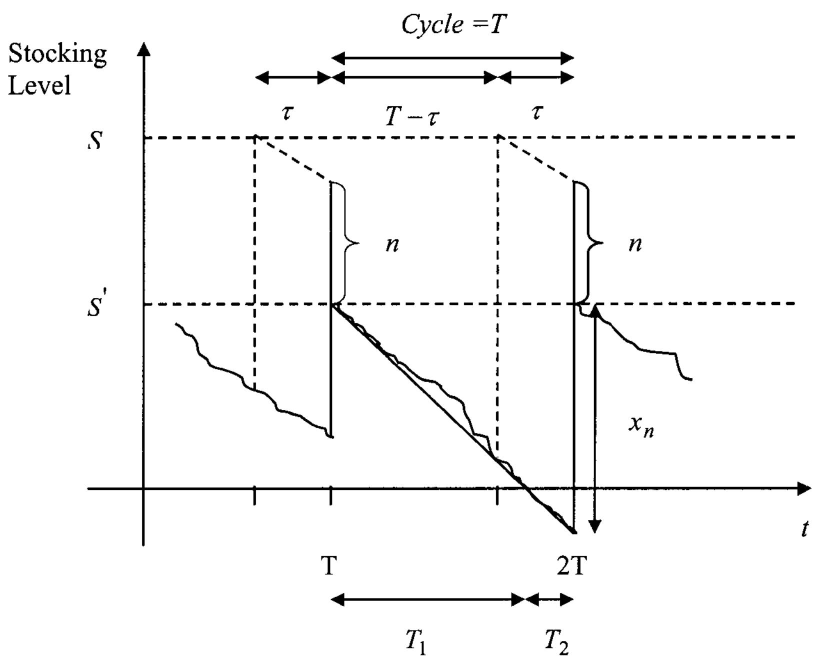

Figure 4.2: Shortage of spare parts during a cycle

component per unit time.

According to equation (4.5), the objective function of the model is expressed as

$$C_{total} = C_r + C_o + C_h + C_{sh}. \tag{4.11}$$

$$\begin{aligned} C_m = & \frac{1}{T} \cdot [n \cdot [p + H(T) \cdot c + (1 + H(T)) \cdot s] + K + \\ + & h \cdot T \cdot (\int_0^{S'} (S' - \frac{x_n}{2}) \cdot g(x_n)dx_n + \int_{S'}^{\infty} \frac{S'^2}{2x_n} \cdot g(x_n)dx_n) \\ + & z \cdot T \cdot \int_{S'}^{\infty} \frac{(x_n - S')^2}{2x_n} \cdot g(x_n)dx_n]. \end{aligned} \tag{4.12}$$

$$S' = S - n - n \cdot (H(T) - H(T - \tau)) = n \cdot H(T). \tag{4.13}$$

Optimal values for the decision variables T and S can be obtained by minimizing the expected total system maintenance cost per unit time.

Numerical Results

Brezavscek and Hudoklin [27] develop an iterative procedure to calculate the decision variables (DV), T and S, and the cost rate, C_m. In order to verify our basic program, we try to duplicate the results reported in their paper. The programs are coded in Matlab language. The results are listed in Figure 4.3. The following discussions are based on the assumption that the results reported in *Brezavscek and Hudoklin* [27] are accurate.

Around the local optimal points, the results produced by our program are quite close to the results reported in *Brezavscek and Hudoklin* [27]. Let S^* be a local optimal decision variable for a specific T. When $S > S^*$, with S

increasing, the deviation errors between the results produced by our program and theirs are very small; however, when $S < S^*$, with S decreasing, this deviation gets large gradually. One major reason is that the shortage cost used for model testing is much higher than other costs incurred by maintenance management. When there is a shortage of spare parts, the calculated cost is very sensitive to the values of the mean and the standard deviation of the probability density function in the shortage equation (4.10). Nevertheless, both the mean and the standard deviation are calculated according to the numerical methods. The error cannot be totally eliminated if the values of the mean and the standard deviation produced by our program are not identical with those produced by the original author's program.

4.3 The BR and Hybrid Spare Parts Inventory Model

In many practical situations, spare parts are expensive. Block replacement policy may result in almost new components being replaced if they are installed a short time before BR actions (*Wang* [9]). Sometimes this is rather wasteful. To reduce the operational cost of the equipment, *Brezavscek and Hudoklin*'s model [27] is extended by recycling the unfailed spare parts obtained from a previous BR activity. Under the new BR policy, the time interval T is divided into two time subintervals. If random failures occur during the first time subinterval $[(k-1)T, kT-\delta)$, $k = 1, 2, \cdots$, the failed components are replaced by new spares as long as they are available in the warehouse. If the random failures take place during the second time subinterval $[kT-\delta, kT)$, the failed components are replaced by used ones. Reemploying the used items too early may, however, result in a high frequency of unexpected failures in the time subintervals $[kT-\delta, kT)$. This will increase operational costs. Clearly,

S\T	12		13		14		15		16		17		18	
	B&H	Dupli	B&H	Dupli	B&H	Dupli	B&H	Dupli	B&H	Dupli	B&H	Dupli	B&H	Dupli
120	2462.1194	4729.014												
121	1115.8922	1348.500												
122	1046.9491	1053.262	1115.3621	1009.585	1175.4073	1042.170	1314.1411	1233.287	1923.8374	1719.150	1923.8374	2703.9490		
123	1046.6183	1045.975	978.2488	974.436	929.4614	919.703	908.9967	898.452	933.3759	966.407	1028.609	1253.0333		
124	1047.2161	1046.383	974.5089	974.245	914.8204	914.324	866.7016	865.795	832.8391	836.401	822.7651	866.1575	1230.6002	1072.5936
125			975.0715	974.708	915.0662	914.705	865.1456	864.754	824.1101	823.931	792.533	797.3276	857.0822	819.0567
126							865.699	865.224	824.26	823.766	790.3396	790.1483	775.5949	769.3049
127										824.265	790.7931	790.1758	763.9509	763.1660
128												790.6895	763.3577	763.1376
129													763.8839	763.6534
130														

S\T	19		20		21		22		23		24	
	B&H	Dupli	B&H	Dupli	B&H	Dupli	B&H	Dupli	B&H	Dupli	B&H	Dupli
124	1584.346	1707.338										
125	971.1222	1012.704										
126	788.4	798.819	1216.411	1017.434	1658.8874	1768.488						
127	748.3502	750.039	862.6882	799.033	1052.0736	1098.977						
128	742.4809	742.422	755.482	739.784	814.6946	831.182	1432.5877	1270.081				
129	742.3607	742.032	730.7411	727.861	738.956	743.526	981.5812	911.513	1341.5162	1623.893		
130	742.9057	742.516	726.8427	726.478	719.9047	720.776	796.5431	771.063	963.5263	1099.1225		
131			726.8502	726.823	716.5099	716.477	732.8639	725.198	798.0829	854.6562	1336.6449	1486.627
132					716.4768	716.261	714.8972	713.130	735.7406	755.8894	983.9158	1059.715
133					716.9993	716.740	711.2306	710.898	715.9461	721.8834	817.5241	851.483
134							710.996	710.976	710.9493	712.2321	748.3556	761.659
135							711.4671	711.489	710.2583	710.2668	723.4124	727.868
136									710.6095	710.3243	715.8777	717.066
137											714.2379	714.390
139											714.3075	714.177
												714.602

B&H: data abstracted from orignal paper
Dupli: duplicated data by our own Matlab program

Figure 4.3: Duplication of the original data with our own program

finding the optimal time to start reusing the unfailed items, that is, finding the optimal δ value, is highly desirable.

Figure 4.4 shows the possible spare parts inventory level behavior, according to (T, δ, S) control strategy with a procurement lead time of $\tau \leq \delta$. In this case, since the time point of reordering spare parts takes place after the period during which the new spare parts are used, the demand for the ordered spare parts during $[(k-1)T, kT-\delta)$ is explicitly determined. The total number of reordered spare parts is clear and is equal to the sum of component corrective repairs (CRs) during $((k-1)T, kT-\delta)$ and the spare parts required for a BR.

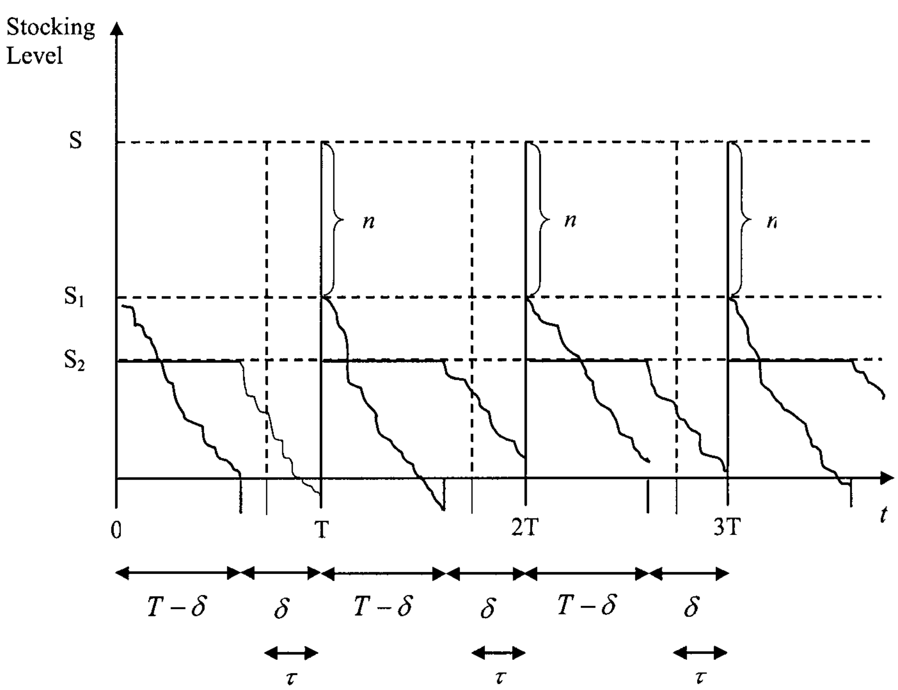

Figure 4.4: Stocking level behaviors according to (T, δ, S) control strategy with a procurement lead time of $\tau \leq \delta$

Figure 4.5 shows the possible behavior of the spare parts inventory level according to (T, δ, S) control strategy with a procurement lead time of $\tau \geq \delta$. In this case, an order is placed before the end of corrective repairs utilizing new spare parts; hence, the demand for new spare parts during the time interval $[kT-\tau, kT-\delta)$ has to be estimated if an appropriate order is to be placed. The total number of reordered spare parts includes the spare parts used in a BR, the corrective repairs during $((k-1)T, kT-\tau)$ and the estimated corrective repairs during $[kT-\tau, kT-\delta)$.

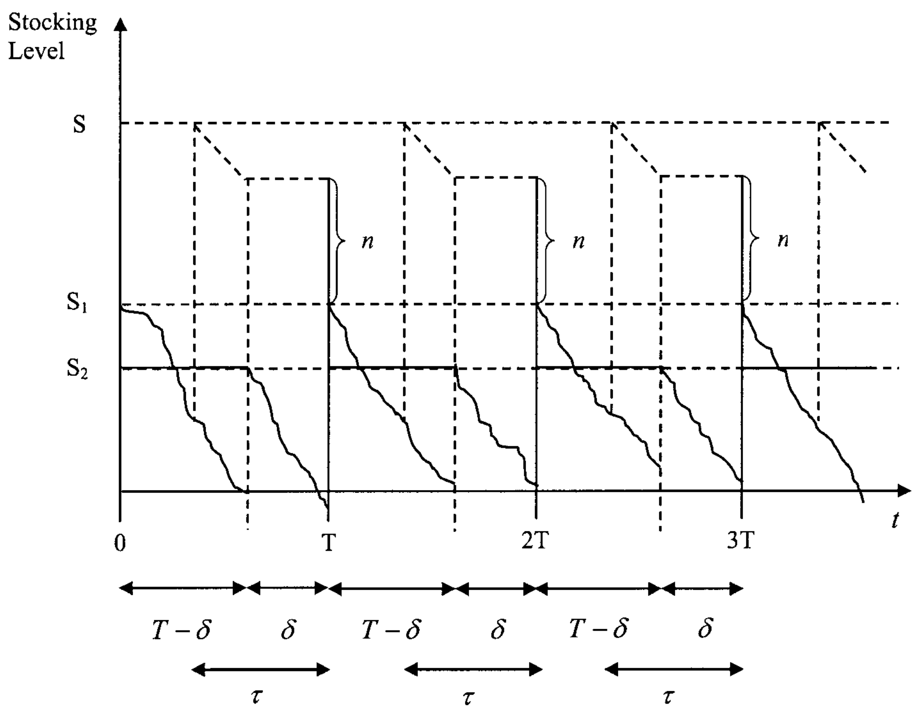

Figure 4.5: Stocking level behaviors according to (T, δ, S) control strategy with a procurement lead time of $\tau \geq \delta$

At each time point, kT, enough spare parts ordered brings the inventory level of new spare parts up to S. The n of them are used immediately, due to the BR. As a result, the inventory of new spare parts decreases to S_1. At the same time, some unfailed spare parts are selected from among the replaced components and are kept in inventory. Let S_2 be the inventory level of the used spare parts. An S_2 starts at the end of the immediately previous PR. It is drawn upon during $[kT - \delta, kT)$. When a new PR is completed, the inventory of used spare parts is refilled to S_2. Since S_1 and S_2 are refilled independently and are used in different periods, their inventory level behaviors are independent and can be modeled separately. In Subsection 4.3.1, the new spare parts inventory model is presented. In Subsection 4.3.2, the used spare parts inventory model is developed. The final objective function of our maintenance and spare parts inventory model is the combination of these two models.

4.3.1 The New Spare Parts Inventory Model

For simplicity, let us investigate only the inventory behavior of spare parts during the time interval $[T, 2T)$. Under the new BR policy, the ordered spare parts are used only in the time interval $[T, 2T-\delta)$. After the block replacement at time point T, there is only $S_1 = S - n$ new spare parts in stock for corrective repairs during the time interval $[T,2T - \delta)$. The behavior of the new spare parts inventory level is shown in Figure 4.6.

For the new spare parts inventory model, the assumptions in this thesis are the same as those of *Brezavscek and Hudoklin* 's model. As a result, most formulas can be obtained by simply substituting T by $T - \delta$ and S' by S_1 in their derived equations; however, an additional holding cost for new spare parts during $[2T - \delta, 2T)$ has to be considered if some new spare parts have not been used during the time interval $[T, 2T - \delta)$. These new parts have to

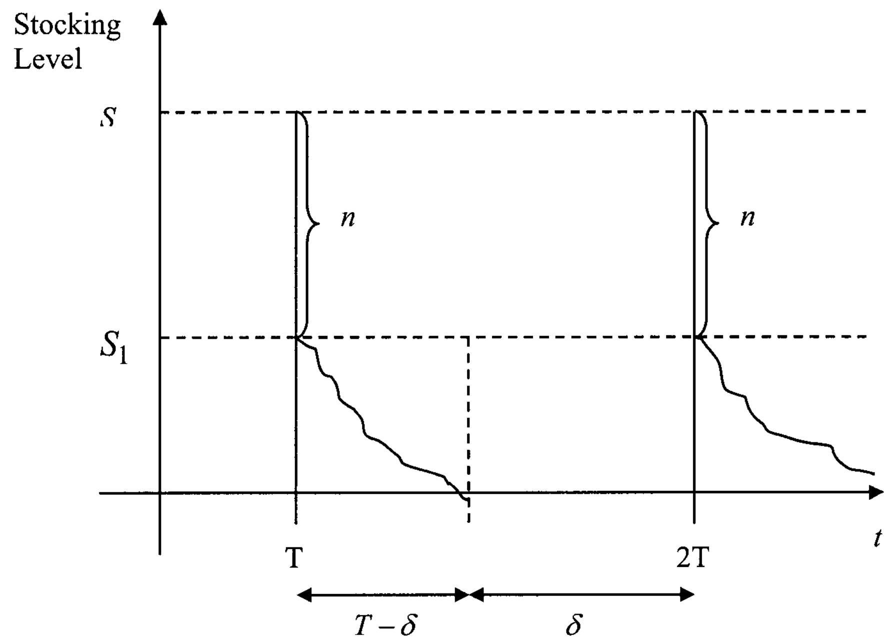

Figure 4.6: Spare parts stocking level behavior

be kept in inventory for the next BR cycle. Let c_{h1} be the holding cost of the new spare parts incurred during $[2T-\delta, 2T)$. It can be easily derived as

$$c_{h1} = \begin{cases} h \cdot \delta \cdot \int_0^{S_1} (S_1 - x_n) \cdot g_1(x_n) dx_n & \text{if } x_n < S_1 \\ 0 & \text{otherwise} \end{cases}$$

Since performing corrective repairs with the used spare parts is assumed to strictly start at $2T-\delta$, other cost items remain the same as those from *Brezavscek and Hudoklin* 's model. Thus the following expression of the cost rate related to the brand new spares inventory model is obtained.

$$\begin{aligned} C_{new} = & \frac{1}{T} \cdot [n \cdot [p + H(T-\delta) \cdot c + (1 + H(T-\delta)) \cdot s] + K + \\ + & h \cdot (T-\delta) \cdot (\int_0^{S_1} (S_1 - \frac{x_n}{2}) \cdot g_1(x_n) dx_n + \int_{S_1}^{\infty} \frac{S_1^2}{2x_n} \cdot g_1(x_n) dx_n) \\ + & h \cdot \delta \cdot \int_0^{S_1} (S_1 - x_n) \cdot g_1(x_n) dx_n \\ + & c_{sh} \cdot (T-\delta) \cdot \int_{S_1}^{\infty} \frac{(x_n - S_1)^2}{2x_n} \cdot g_1(x_n) dx_n]. \end{aligned} \tag{4.14}$$

4.3.2 The Used Spare Parts Inventory Model

In order to prevent almost new components being replaced by BR activity at time $2T$ and to decrease the quantity of new spare parts ordered, some unfailed spare parts are selected from the BP activity at time T and stocked for performing corrective repairs during the time interval $[2T-\delta,\ 2T)$. Figure 4.7 shows the different situations involving performing corrective repairs during $[2T-\delta,\ 2T)$. The behavior of the used spare parts inventory level is shown as Figure 4.8.

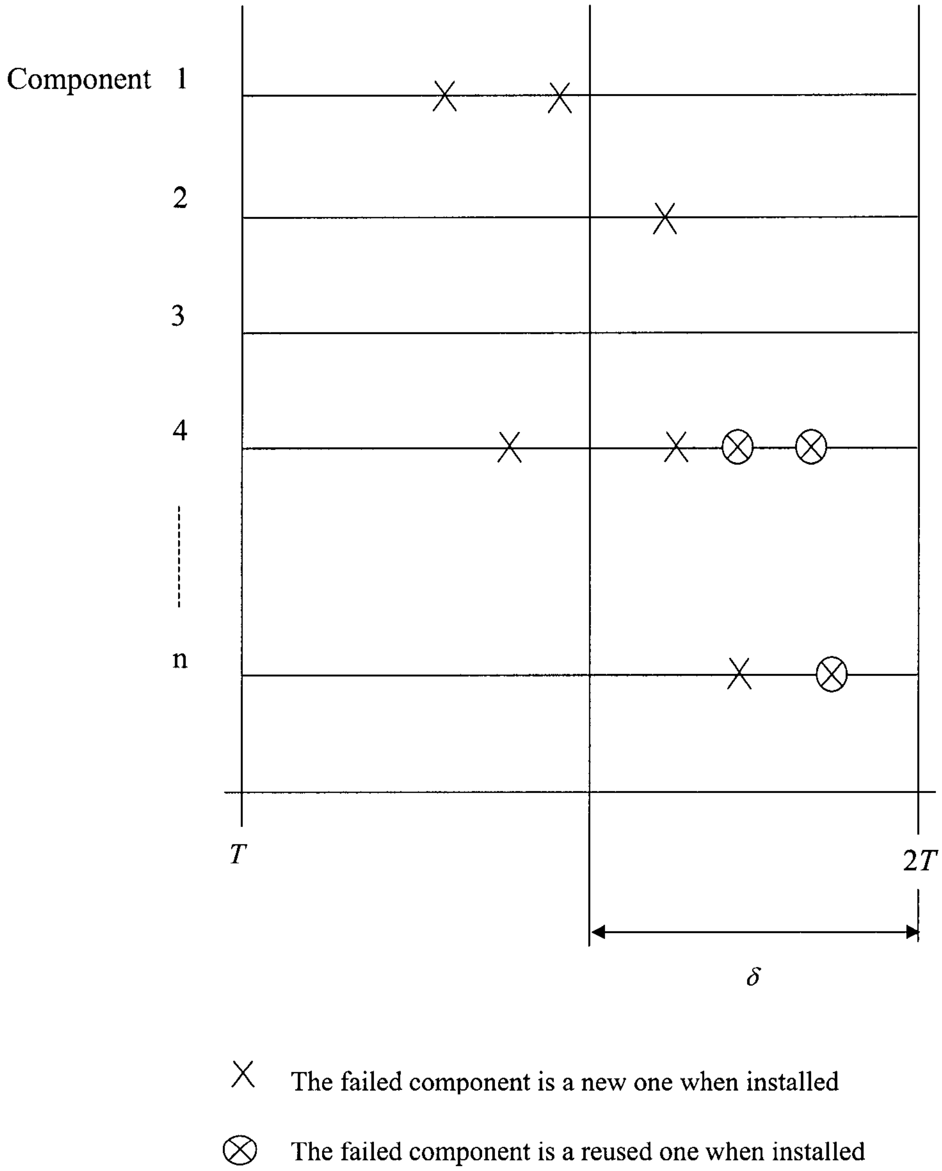

Figure 4.7: Replacement of failed components during one BR interval, with regard to used spare parts

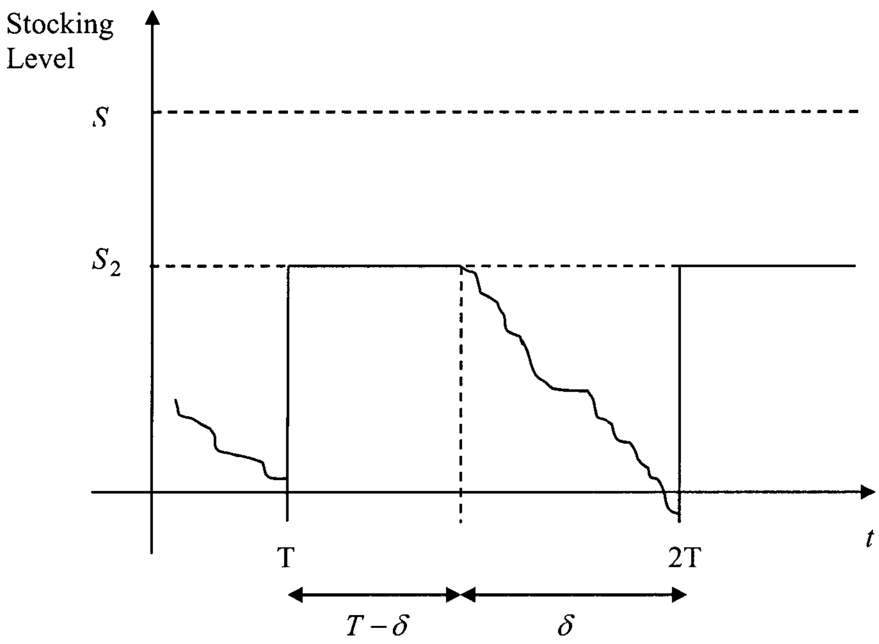

Figure 4.8: Used spare parts stocking level behavior

Assumptions for Used Spare Parts

The assumptions for the used spare parts inventory model include the assumptions for the new spare parts inventory model and the followings.

1. The planning horizon is infinite.

2. Spare parts for re-use are selected from the immediately previous preventive replacement activity; all unfailed spare parts that have not been used more than once are inventoried for re-use.

3. Old spare parts which are not reused during the immediately prior BR period are not inventoried for the next BR period.

Formulations of the Used Spare Parts Inventory Model:

Suppose that y is a random variable representing the number of corrective repairs during a time interval, $[2T - \delta, 2T)$, when the system has only one component. Unlike previous sections, y as a function of δ cannot be directly described by the ordinary renewal process. To calculate the expected value, $E(y)$, and variance, $Var(y)$, of y, we must consider two types of component failures during the time interval $[2T - \delta, 2T)$ (see Figure 4.7). The first type is caused by a new component. It is replaced immediately by a used spare part. After the first failure replacement, any failures before time point $2T$ are considered to be due to the reused components. The failure replacements of the reused components can be described by a new ordinary renewal process.

Let $H_2(T-z)$ be the renewal function of a used component during the time interval $[2T - \delta, 2T)$ and z be the instant of the first failure during the time interval $[2T - \delta, 2T)$. Then the average number of renewals in this interval is $1 + H_2(T - z)$ (see *Daoud Ait Kadi and Robert Cleroux* [28]). The expected number of failures during $[2T - \delta, 2T)$ can be calculated by

$$E(y) = \int_{T-\delta}^{T} [1 + H_2(T - z)]\psi(z)dz \tag{4.15}$$

where $\psi(z)$ is the probability density function of a used component failure at time z, and $\psi(z)dz$ is the probability of a used component failure between z and $z + dz$. Let $y = 1 + H_2(T - z)$. The variance of y is calculated by (see *Gnedenko*[33])

$$Var(y) = E(y^2) - E(y)^2 \tag{4.16}$$

where

$$E(y^2) = \int_{T-\delta}^{T} [1 + H_2(T - z)]^2\psi(z)dz. \tag{4.17}$$

Now let us find the expression of $\psi(z)dz$. Suppose that

- E_1 = {the event that the first failure of a new component installed at $t = 0$ occurs in $(z, z + dz)$ for $z \in [2T - \delta, 2T)$};

- E_2 = {the last renewal in $[T, 2T - \delta)$ occurred in $(v, v + dv)$ and the new component installed at that time fails between z and $z + dz$, where $z \in [2T - \delta, 2T)$}.

We have

$$\psi(z)dz = Pr(E_1) + Pr(E_2) \tag{4.18}$$

where $Pr(E_1) = f(z)dz$, $f(z)$ is the pdf for a new component failure time. $Pr(E_2) = \int_0^{T-\delta} h(v)f(z - v)dvdz$, $h(v)$ is the new component renewal density. Then

$$\begin{aligned} E(y) &= \int_{T-\delta}^{T} [1 + H_2(T-z)] f(z) dz \\ &+ \int_{0}^{T-\delta} \int_{T-\delta}^{T} [1 + H_2(T-z)] h(v) f(z-v) dz dv. \end{aligned} \qquad (4.19)$$

$$\begin{aligned} E(y^2) &= \int_{T-\delta}^{T} [1 + H_2(T-z)]^2 f(z) dz \\ &+ \int_{0}^{T-\delta} \int_{T-\delta}^{T} [1 + H_2(T-z)]^2 h(v) f(z-v) dz dv. \end{aligned} \qquad (4.20)$$

In that case, according to equation (4.16), the variance, $Var(y)$, is obtained.

Suppose that y_n is the number of corrective repairs during the time interval δ. Then y_n is a random variable with pdf $g(y_n)$. With $E(y)$ and $Var(y)$, $g(y_n)$ can be approximated by the normal pdf with mean

$$E(y_n) = n \cdot E(x) \qquad (4.21)$$

and standard deviation

$$\sqrt{Var(y_n)} = \sqrt{n \cdot Var(y)}. \qquad (4.22)$$

Under our maintenance policy, the failed components in [2T-δ, 2T) are immediately replaced by used spare parts. For simplicity, we assume that all used spare parts being selected have a common age, T, and hence the density of the residual life length is $f_T(t) = f(t+T)/[1 - F(T)]$ (see *Ait Kadi et al.*, 1988). In fact, a used spare part may have an age varying from δ to T. Nevertheless, evaluating the used spare parts demand with an assumption of a common age

T can guarantee a slightly higher inventory level than the actual demand. If the shortage cost is very high, a safety margin for the inventory level is always desirable.

To Determine The Used Spare Part Inventory Level S_2

Our principle of selecting used spares is as follows: according to assumption ♯2, any component, as long as it fails during the time interval $[2T-\delta, 2T)$, must be scrapped regardless of whether it is functional or not at time point $2T$. Only the components which do not fail during $[2T-\delta, 2T)$ are recycled for the next BR interval.

Let $\phi(z)dz$ be the probability of a component's surviving (having never failed) at the instant $z \in [2T-\delta, 2T)$, and let Pr represent the probability that a component does not fail during time interval $[2T-\delta, 2T)$. This probability can be calculated by

$$\begin{aligned} Pr &= \int_{T-\delta}^{T} \phi(z)dz \\ &= 1 - [\int_{T-\delta}^{T} f(z)dz + \int_{0}^{T-\delta}\int_{T-\delta}^{T} h(v)f(z-v)dvdz]. \end{aligned} \tag{4.23}$$

Let d_n be the total number of recyclable spare parts from a block replacement. Then its expected value, $E(d_n)$, can be calculated by (see *Shirmohammadi et al.* [34])

$$E(d_n) = n \cdot Pr = n \cdot \int_{T-\delta}^{T} \phi(z)dz. \tag{4.24}$$

When T and δ are fixed, $E(d_n)$ is explicitly determined. Let S_2 be the expected inventory level of used spare parts just after completion of a planned BR. According to assumption ♯3, S_2 is equal to the recycled spare parts from the immediately previous BR, i.e., $S_2 = n \cdot Pr$.

The Objective Function:

The operational cost rate of the used spare parts, C_{used}, is given as follows:

$$C_{used} = \frac{C_{ur} + C_{us}}{T} \tag{4.25}$$

where C_{ur} represents the system maintenance costs during $[2T - \delta, 2T)$ and C_{us} represents the used spare parts ordering and inventory management costs.

The maintenance cost $[2T-\delta, 2T)$ is only induced by random failure repairs. Assume that the cost of a corrective repair involving a used spare part is the same as that involving a new component. Then the maintenance cost of the system with n components during $[2T - \delta, 2T)$ is

$$C_{ur} = E(y_n) \cdot c = n \cdot E(y) \cdot c. \tag{4.26}$$

The spare part ordering and inventory management cost, C_{us}, consists of an ordering cost of C_{uo}, a holding cost of C_{uh} and a shortage cost of C_{ush}. All these are determined by equations (4.27), (4.28) and (4.29) respectively.

There is no ordering cost for the used spare parts; therefore, we have

$$C_{uo} = 0. \tag{4.27}$$

The holding cost of the used spare parts includes the cost of stocking them during the time intervals $[T, 2T - \delta)$ and $[2T - \delta, 2T)$. In our policy, we recycle all the reusable spare parts. Thus, their holding cost according to equation

(4.9) is given as

$$\begin{aligned} C_{uh} &= h \cdot [(T-\delta) \cdot S_2 \\ &+ (\delta) \cdot \int_0^{S_2} (S_2 - \frac{y_n}{2}) \cdot g_2(y_n) dy_n + \int_{S_2}^{\infty} \frac{S_2^2}{2y_n} \cdot g_2(y_n) dy_n]. \end{aligned} \quad (4.28)$$

The shortage cost of the used spare parts according to equation (4.10) is given as

$$C_{ush} = \begin{cases} 0 & \text{if } y_n \leq S_2 \\ c_{sh} \cdot (\delta) \cdot \int_{S_2}^{\infty} \frac{(y_n - S_2)^2}{2y_n} \cdot g_2(y_n) dy_n & \text{otherwise.} \end{cases} \quad (4.29)$$

According to equation (4.25), the system operational cost rate during $[2T-\delta, 2T)$ is expressed as

$$\begin{aligned} C_{used} = & \frac{1}{T} \cdot [n \cdot E(y) \cdot c + h \cdot (T-\delta) \cdot S_2 \\ + & h \cdot \delta \cdot (\int_0^{S_2} (S_2 - \frac{y_n}{2}) \cdot g_2(y_n) dy_n + \int_{S_2}^{\infty} \frac{S_2^2}{2y_n} \cdot g_2(y_n) dy_n) \\ + & z \cdot \delta \cdot \int_{S_2}^{\infty} \frac{(y_n - S_2)^2}{2y_n} \cdot g_2(y_n) dy_n] \end{aligned} \quad (4.30)$$

where $S_2 = n \cdot Pr$.

Combining the new spare parts inventory model and the used spare parts inventory model, we obtain a hybrid spare parts inventory model for maintenance management. The inventory level of the used spare parts may be higher or lower than the inventory level of the new spare parts. Their inventory level behaviors are shown in Figure 4.9 or Figure 4.10.

It is apparent that the new spare parts inventory and the used spare parts inventory are independent; therefore, the total operational cost rate for the

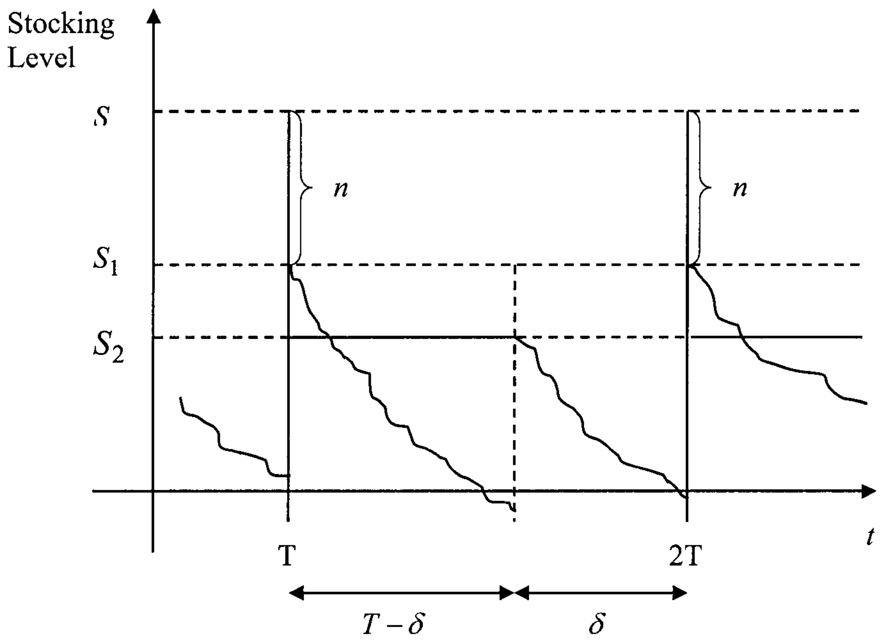

Figure 4.9: Stocking level behavior in a hybrid spare parts model ($S_1 \geq S_2$)

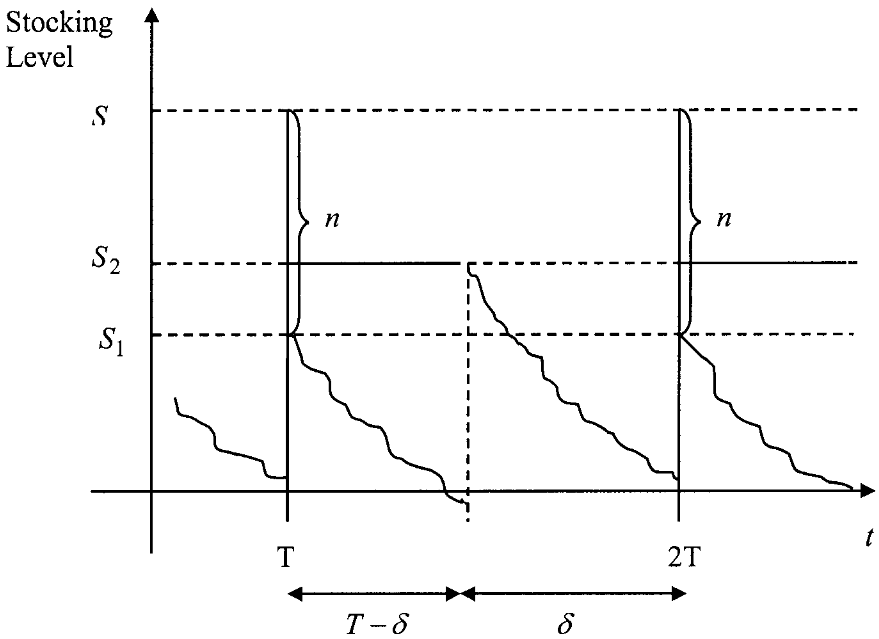

Figure 4.10: Stocking level behavior in a hybrid spare parts model ($S_1 \leq S_2$)

system in any specific block replacement period is given as

$$C_m = C_{new} + C_{used}. \tag{4.31}$$

The objective function is explicitly expressed as

$$\begin{aligned} C_m = & \frac{1}{T} \cdot [n \cdot [p + H(T-\delta) \cdot c + (1 + H(T-\delta)) \cdot s] + K + \\ + & n \cdot E(y) \cdot c + h \cdot (T-\delta) \cdot n \cdot Pr \\ + & h \cdot (T-\delta) \cdot (\int_0^{S_1} (S_1 - \frac{x_n}{2}) \cdot g_1(x_n) dx_n + \int_{S_1}^{\infty} \frac{S_1^2}{2x_n} \cdot g_1(x_n) dx_n) \\ + & h \cdot \delta \cdot \int_0^{S_1} (S_1 - x_n) \cdot g_1(x_n) dx_n \\ + & h \cdot \delta \cdot (\int_0^{S_2} (S_2 - \frac{y_n}{2}) \cdot g_2(y_n) dy_n + \int_{S_2}^{\infty} \frac{S_2^2}{2y_n} \cdot g_2(y_n) dy_n) \\ + & z \cdot (T-\delta) \cdot \int_{S_1}^{\infty} \frac{(x_n - S_1)^2}{2x_n} \cdot g_1(x_n) dx_n \\ + & z \cdot \delta \cdot \int_{S_2}^{\infty} \frac{(y_n - S_2)^2}{2y_n} \cdot g_2(y_n) dy_n] \end{aligned} \tag{4.32}$$

where

$$S_1 = \begin{cases} S - n - n \cdot (H(T-\delta) - H(T-\delta-(\tau-\delta))) = n \cdot H(T-\delta) & \text{if } \delta < \tau < T \\ S - n = n \cdot (H(T-\delta)) & \text{if } 0 \le \tau \le \delta \end{cases}$$

and $S_2 = n \cdot Pr$.

Optimal values for the decision variables, T, δ and S, can be obtained by minimizing the expected total system maintenance cost per unit time.

In *Brezavscek and Hudoklin* (2003), the authors develop an iterative procedure to solve the objective function (4.12).

This thesis also uses an iterative procedure to solve the objective function (4.32). The algorithm for this is described in the next section.

4.4 Algorithm

Calculation procedure is designed (see Figure 4.11) and described as follows:

1. Let $S = n + i$, where $i = 0, 1, 2, \cdots$

2. The procedure is initiated with a low value of δ and with $T = T_0$ ($\delta < T_0$). δ is then incremented by $\Delta\delta$ until it reaches a certain predetermined value, δ_c.

3. At each iteration, μ_1 and σ_1 are computed using equations (4.1) and (4.2). μ_2 and σ_2 are computed using equations (4.21) and (4.22). Meanwhile, $E(d_n)$ is calculated by (4.24).

4. Then C_{new} is obtained according to (4.14) and C_{used} according to (4.30). The total cost rate is $C_m = C_{new} + C_{used}$.

5. Find the local minimal C_m and write down its optimal value (δ^*, S^*).

6. Increment T with ΔT until T reaches a certain predetermined value, T_c; then find out the optimal value, T^*. Letting $i = i + 1$, iterate steps 1, 2, 3, 4, and 5.

4.5 Numerical Results

We employ the model parameters listed in *Brezavscek and Hudoklin* (2003) to test our model except for increasing the unit purchasing price. This is because it is more reasonable to assume that the spare parts are expensive when considering whether to reuse the old but unfailed spare parts.

The example that *Brezavscek and Hudoklin* (2003) uses for model testing is comprised of 30 electric locomotives of the same type, working in a similar

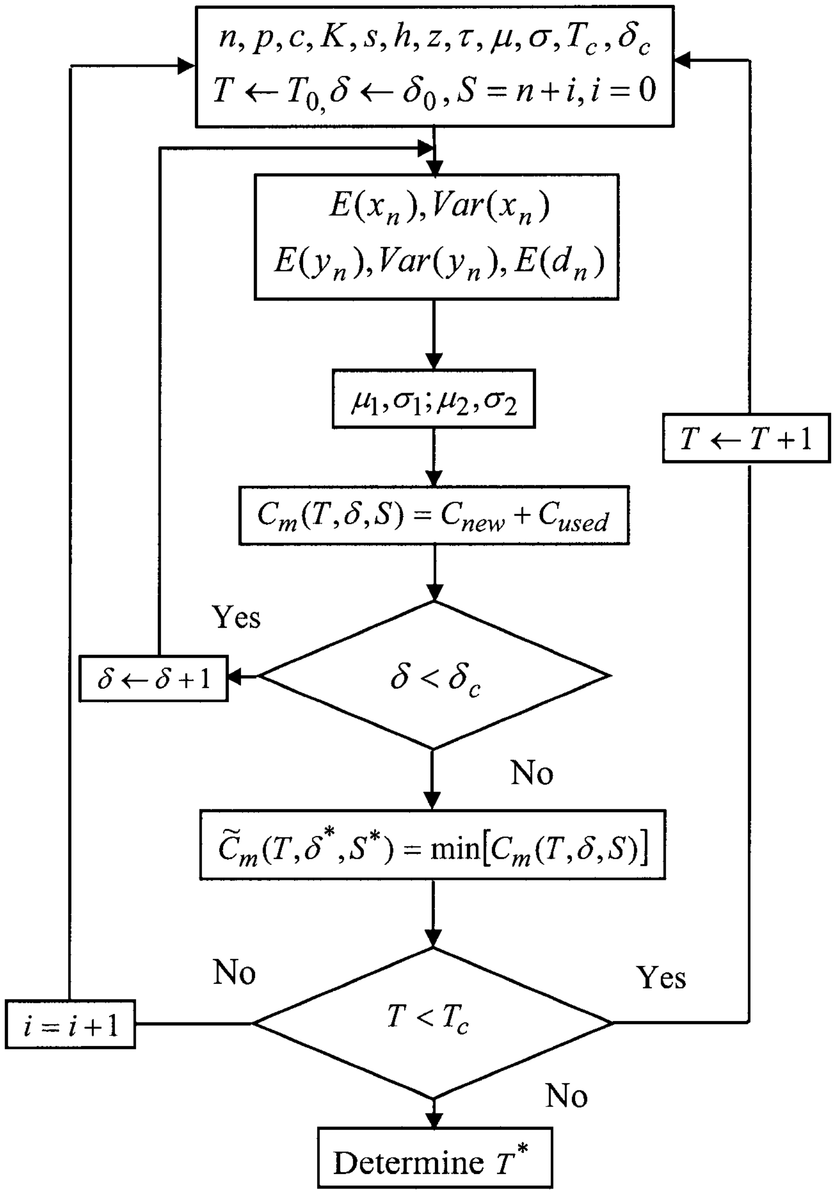

Figure 4.11: Computation procedure

regime. Each locomotive contains 4 identical arcing chambers operating in series. All 30 locomotives must be in operational condition. A failure of any arcing chamber leads to a degradation failure of the system and causes a heavy financial loss.

The values for the model parameters are listed as follows:

- Total number of the components of the system: $n = 120$
- $f(t)$ is the normal probability density function. The values of the parameters of $f(t)$ are $\mu = 44$ weeks and $\sigma = 12$
- The procurement lead time: $\tau = 12$ weeks
- The cost of performing component replacement: $p = 58.2$ units
- The cost of performing component corrective maintenance: $c = 800.5$ units
- Set-up cost for placing an order: $K = 20$
- The holding cost per spare part per unit time: $h = 0.6$ units/week
- The downtime cost due to shortage of spare parts per component per unit time: $z = 5196$ units/week
- Unlike *Brezavscek and Hudoklin* (2003), this thesis sets the cost of a spare part purchased at $s = 1800$ units.

The optimal values for the decision variables, T, δ and S, are computed by the algorithm outlined in the last section. The C_m is calculated according to the objective function (4.32); the results listed in the following tables are obtained according to different values for decision variables.

Table 4.1: Cost rates with the number of ordered spare parts, S, increasing

T	δ	S	C_m
37	6	150	8171.9139
37	6	151	8106.8759
37	6	152	8069.3986
37	6	153	8049.0347
37	6	154	8038.7124
37	6	155	8033.9503
37	6	156	8032.0901
37	**6**	**157**	**8031.6559**
37	6	158	8031.8809
37	6	159	8032.3924
37	6	160	8033.0208
37	6	161	8033.6941
37	6	162	8034.3836

Table 4.1 shows that when the block replacement period, T, and the time interval, δ, are fixed, the system's operational cost rate, C_m, reaches the minimum value as long as an appropriate inventory level of the new spare parts, S, is selected.

Table 4.2 presents how the system's cost rate varies with the length of the period, T, when the time interval, δ, and the inventory level, S, are predetermined. The system cost rate increases very quickly as T increases when T is greater than its optimal value, 37. As T increases, $T - \delta$ will increase correspondingly, resulting in a greater demand for the new spare parts for corrective repairs. Since S is fixed, the shortage of new spare parts is aggravated as T increases.

Table 4.3 shows that we can find an optimal value for the decision variable δ which will minimize the system's operational cost rate when T and S are fixed. From the data given in Table 4.3, we can easily conclude that both excessive shortness and excessive length of δ result in a large system operational cost

Table 4.2: Cost rates with the period of BR, T, increasing

δ	S	T	C_m
6	157	31	8380.0988
6	157	32	8270.7643
6	157	33	8183.5851
6	157	34	8116.9682
6	157	35	8070.3352
6	157	36	8041.9318
6	**157**	**37**	**8031.6559**
6	157	38	8049.0194
6	157	39	8215.3777
6	157	40	9064.2311
6	157	41	11582.5871
6	157	42	14623.6217
6	157	43	20952.6674

rate. The reason is that an overly short δ causes a shortage of new spare parts and increases the cost of holding recycled spare parts; while an overly long δ results in a shortage of used spare parts. A shortage of spare parts means that the system is in downtime. A long downtime usually results in a heavy economic loss.

Table 4.4 compares the minimum system operational cost rate, Cm, and the optimal values for corresponding decision variables, T and S, under various values for the time interval, δ. When $\delta = 0$, our model is identical to *Brezavscek and Hudoklin*'s model (2003). It is noteworthy that the data in Table 4.4 are obtained by assuming the price of purchasing a spare part, $s = 1800$. It is apparent that the system has a higher optimal operational cost rate with $\delta = 0$ in comparison with those cases where recycling the used spare parts has $\delta \leq 12$. Thus, our maintenance management policy is more effective than the policy proposed by *Brezavscek and Hudoklin* in 2003. When, however, $\delta \geq 13$, the system operational cost rate is higher than is the case when $\delta = 0$. Thus,

Table 4.3: Cost rates with the time interval, δ, increasing

T	S	δ	C_m
37	157	1	32136.5697
37	157	2	18987.0079
37	157	3	11491.3828
37	157	4	8643.5344
37	157	5	8084.4646
37	**157**	**6**	**8031.6559**
37	157	7	8036.8281
37	157	8	8063.1699
37	157	9	8109.8975
37	157	10	8174.8692
37	157	11	8257.7256
37	157	12	8351.20259
37	157	13	8464.54337

it is imperative to determine an optimal time interval, δ, for implementing the new maintenance management policy if one wishes to decrease the operational cost rate of the system.

4.6 Concluding Remarks

In this chapter, we extended the model of joint optimization of block replacement and periodic review spare parts inventory policy, proposed by *Brezavscek and Hudoklin* (2003), by taking into account recycling the used spare parts. The performance of the new model is evaluated in terms of the total average cost per time unit over an infinite horizon. In the new model, the period of performing block replacement is divided into two time subintervals. New spare parts are used in the block replacement and corrective repairs during the first time interval. Used spare parts, which are selected from the immediately previous block replacement activity, are used for corrective repairs during the

Table 4.4: Comparison of minimum cost rates under the different δs

T	S	δ	Cm
36	188	0	8407.9587
35	176	1	8357.1934
36	175	2	8240.8865
36	169	3	8155.8414
36	163	4	8094.1277
36	158	5	8055.6220
37	**157**	**6**	**8031.6559**
37	152	7	8033.8084
38	151	8	8056.6575
38	146	9	8095.3563
38	143	10	8152.9809
39	141	11	8229.0886
39	147	12	8315.9110
39	143	13	8417.7158
40	143	14	8533.8285
40	140	15	8657.3896

second time interval. The model takes into account the costs incurred in performing maintenance and managing the spare parts inventory. An iteration computation procedure is developed to search for the optimal values for the block replacement period, T^*, and the time interval for performing corrective repairs with the used spare parts, δ^*, and the new spare parts inventory, S^*, in order to minimize the operational cost rate of the system.

By model testing and comparison, we can draw the conclusion that the new model is more economical and practical in guiding maintenance management than is *Brezavscek and Hudoklin* (2003)'s model when the price of one unit is expensive.

Chapter 5

Conclusions and Further Research

This chapter highlights the goals that were accomplished in the research for this thesis. A summary of the overall results is covered in the conclusion. Some suggestions are made for future research based on current developed models.

5.1 Conclusion

Maintenance and replacement activities on a deteriorating system aim to improve its availability and reduce failure frequency during its service. Unappropriate maintenance activities on the system may, however, increase the system's operating cost while not effectively improving the system's reliability. This research, by studying stochastic behavior of the system under various maintenance policies, develops optimal maintenance models for providing support to maintenance management decision-making. The formulations presented in this research incorporate the concepts of condition-based maintenance(CBM), random failure corrective repair (CR), and block replacement (BR) into a comprehensive model which calculates the optimal quantitative health condition indicator of the system in order to guide PM policy making, especially with regard to optimal PM intervals and optimal system replacement time. To make the developed models best fit the maintenance problems, this research takes into account the difference of emphasis between maintaining single-unit systems and maintaining multi-unit systems.

For a single-unit system, our research focuses on determining the optimal system replacement time and optimal level of critical failure rate, where a PM action is triggered whenever the failure rate reaches this level. According to system's failure mechanisms, failures taking place during system's life cycle are classified into two failure modes: minor failures and major failures. Their corresponding failure rates are defined as the major failure rate and the minor failure rate. Minor failures predominate in the infant period of a system and do not cause it to deteriorate as long as they are totally corrected. As a result, the minor failure rate decreases as system operating time increases. Major failures predominate in the wear-out period of the system. They result from the degradation of the system's performance and usually expedite the degradation rate of the system. That is why the major failure rate indicates the health condition of the system. It will increase as the system ages. A bathtub-shaped failure rate function of a system can be flexibly decomposed into these two types of failure rates. PMs are purposely performed in order to improve the health condition of the system. Therefore, PMs aim to affect the major failure rate. They are performed according to the optimal level of major failure rate for reducing the frequency of a system's major failures in operation. With on-going performance degradation, PMs are not as effective as before but they increase a system's operational cost. That is why a system is replaced at the N^{th} major failure where the system's operational cost is minimal.

For a multi-unit system, block replacement is regarded as one of the best

PM strategies for decreasing operational cost. To improve the availability of the system, maintenance management considers not only its PM policy but also its spare parts provisioning strategy. In this research, we extend *Brezavscek and Hudoklin*'s model(2003) achieving joint optimization of BR and periodic review spare-provisioning policy, by recycling used spare parts selected from the immediately previous BR. Under the new maintenance policy, the BR interval, T, is divided into two periods. In the first period, the new spare parts provisioning policy is designed to meet demand created by regularly scheduled BR as well as by random failures of units. In the second period, selected unfailed spare parts are reused to meet the demand for spare parts created by random failures. The used spare parts are kept in inventory during the first period and are reemployed during the second period to avoid wasting expensive new spare parts. The new model includes three decision variables, the BR interval, T; the time interval for replacement with used spare parts, δ; and the maximum inventory level of the ordered spare parts, S. By finding optimal values for T, δ, and S, the operational cost rate of the system can be minimized.

5.2 Future Research

Along the research line of this thesis, further work can be done in the following directions.

In our PM and repair model for a single-unit system, we consider only two failure modes during the system's life cycle. Further research may consider three or more failure modes (see *Wang et al*(2002)). By understanding the physical mechanisms of these failure modes, the most effective repair action on each of them can be determined. These different maintenance effects could be incorporated into the extant model, possibly making the model more accurate in calculating decision variables. Another extension along this line of research line could modify the extant model, making it suitable for designing warranty policy by taking into consideration the expected sales function of the system according to marketing needs.

Our maintenance management model for a series system can be extended to other types of maintenance polices and to different system configurations. One possibility is the incorporation of minimal repair or imperfect repair into the model. This could further reduce the demand for ordered spare parts and decrease the inventory level of new spare parts, thereby decreasing the total operation cost of the system. Another possible direction for research is to extend the current model to a general k-out-of-n system.

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