A Comparison of Implicit and Explicit Methods for Contingency Constrained Unit Commitment

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***Abstract*—Reliability management is of great importance for the secure and sufficient operation of power systems, thus the $n - K^G - K^L$ contingency constrained unit commitment (CCUC) problem is determined for investigation in this paper. In order to reveal the capability of different methods on the solution of CCUC, both explicit and implicit decomposition frameworks have been investigated, as well as their inner feedback strategies, such as Benders decomposition and column-and-constraint generation (CCG) algorithm. In addition, sensitivity analysis, multi-cut strategy, and parallel implementation have also been analyzed and discussed. The comparison between nine explicit and implicit methods - all of them are deterministic with different configurations and the global optimal can be guaranteed in a finite number of iterations - is carried out on the IEEE 24-bus system, resulting in several interesting conclusions. Finally, the IEEE 118-bus test system is employed to explore the potential on the large-scale instance.**

***Index Terms*—Benders decomposition, contingency constrained unit commitment, mixed-integer linear programming, robust optimization.**

I. Introduction

Reliability and security are crucial for modern power systems. Unexpected outage of power grid components, which is mainly triggered by extreme weather events or other random factors, can result in dramatic electricity shortages or even large-scale blackouts. For the purpose of withstanding a specified level of destruction, the $n - K^G - K^L$ contingency constrained unit commitment (CCUC) problem was proposed, which furnishes the system (n components) with the capability of surviving from the sudden unavailability of K^G generation units and K^L transmission lines.

Two types of methodologies have been intensively investigated for the solution of CCUC: 1) deterministic methods that have explicit formulations and can lead to exact solutions, and 2) meta-heuristic methods such as particle swarm optimization, simulated annealing, and genetic algorithm. Although meta-heuristic methods are relatively straightforward and capable of addressing nonlinear, non-convex, combinatorial, and even NP-hard problems, the identification of the optimality of the final obtained solution is yet to be established. On the other hand, the global optimal solution is guaranteed in a finite number of iterations for the deterministic methods, thus, they are designated for investigation in this paper.

Reserve adjustment method [1] is widely utilized in the power industry due to its easy application [2], however, it is usually criticized for economic inefficiency or even inadequacy. Stochastic programming technique is authoritative in the formulation and solution of problems with uncertainty, which assures that the optimal solution is feasible for all (or almost all) of the possible realizations. Nevertheless, how to identify an accurate probability distribution of different types of uncertainties remains a great challenge for practical application. Instead of the hard-to-obtain probability distribution function, only the bounds of uncertainty are required for the robust optimization (RO) [3]–[8] approach. The obtained solution is global optimal and feasible for the worst case of uncertainty set, as well as all the other possibilities. Unfortunately, the RO approach might lead to a higher cost (the "price of robustness") since it protects against the most severe event regardless of its low probability.

Although RO is far from impeccable, it gained close attention in the solution of CCUC since the reliability has a conservatism nature, i.e., the final solution should withstand the worst circumstance. CCUC was first solved by [5] with RO under $n - K$ contingency criteria, however, neither the transmission capacity constraints nor the transmission line contingencies were considered. This work was extended by [6] with full consideration of transmission constraints and contingencies, but a heuristic was introduced to reduce the solution time, which made the optimality of the final solution difficult to identify. In [7], both generation unit and transmission line contingencies were investigated; nevertheless, this study was restricted on single-period CCUC (one-time decision rather than 24-hours successive decision), i.e., time-coupling constraints were omitted. Reference [8] enhanced this work from the consideration of day-ahead schedule.

RO employs a two-stage decomposition framework for the solution of CCUC, which divides the whole problem into two stages or problems as shown in Fig. 1. The master problem aims to minimize the total costs while satisfying the constraints from pre-contingency and post-contingency (cuts or constraint sets). At each iteration, the master problem is solved first to generate an intermediate solution (unit commitment schedule), which is then validated by subproblems to find the most violated scenario from all realizations of uncertainty set. If there exists a violated contingency scenario, one or more cuts will be generated and included in the master problem, and it goes to the next iteration; otherwise, the solution process is terminated since all realizations are satisfied. In terms of how

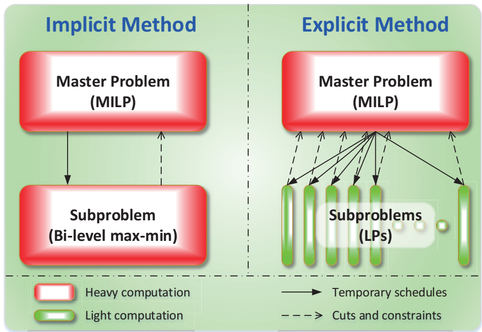

Fig. 1. Implementation frameworks of implicit and explicit methods.

to identify the worst case in the second stage, two schemes are illustrated in Fig. 1, i.e., implicit and explicit method.

To distinguish the most serious situation, the implicit method resorts to the bi-level max-min programming, which will be exemplified in Section 2. The $\max - \min$ problem in the solution approach is NP-hard, and is usually converted into a bilinear $\max - \max$ problem according to the strong duality theory, which is then relaxed and linearized based on the outer approach [9] or disjunctive constraints [7], resulting into a mixed-integer linear programming (MILP) problem [10], [11]. On the contrary, the enumeration strategy is adopted by the explicit method to determine the worst scene.

The explicit method was conventionally criticized since the number of subproblems presents an exponential dependence with n, K^G, and K^L. However, some promising features are overlooked: 1) each subproblem of characteristic is linear programming (LP), which is much easier to solve than MILP; 2) the subproblems are independent from each other, thus they can be solved simultaneously, i.e., suitable for parallel computing; 3) not only the most violated scenario can be found, but also all the other violated contingencies can also be identified, therefore, more cuts can be generated to enhance the convergence; 4) more information on the violated scenarios is beneficial for the releasing of the conservatism.

In order to intensively explore the full potential of these two methods and other constructive techniques that have been integrated within the two-stage decomposition framework for performance enhancement, such as Benders decomposition (BD) and column-and-constraint generation (CCG) [12] algorithm, extensive comparisons have been carried out between these two methods and commercial MILP solver Cplex. Finally, several instructive conclusions and concerns are developed based on the revealed pros and cons. Furthermore, the sensitivity analysis on contingency parameter K^G and K^L is also performed, as well as the potential exploration for large-scale instance.

The remainder of this paper is structured as follows. CCUC mathematical formulation is described in Section II. In Section III, both explicit and implicit decomposition algorithms are illustrated. Numerical case studies and discussions are reported in Section IV. Finally, conclusions and future works are provided in Section V.

II. Problem Formulation

Based on [6], [7], and [13], the CCUC can be given as:

$$\min_{\substack{v_g, c_g^u, c_g^d, Q^w, c_g^{p(0)},\\ r_i^{(0)}, p_g^{(0)}, \theta_i^{(0)}, f_{ij}^{(0)}}} \sum_{t\in T}\sum_{g\in G}\left(c_g^{p(0)}(t)+c_g^u(t)+c_g^d(t)\right) + Q^w \tag{1}$$

$$\begin{aligned} s.t.\ & -v_g(t-1)+v_g(t)-v_g(h)\le 0,\\ & \qquad \forall g\in G,\ \forall t\in T,\ \forall h: 1\le h-(t-1)\le T_g^U \end{aligned}\tag{2}$$

$$\begin{aligned} & v_g(t-1)-v_g(t)+v_g(h)\le 1,\\ & \qquad \forall g\in G,\ \forall t\in T,\ \forall h: 1\le h-(t-1)\le T_g^D \end{aligned}\tag{3}$$

$$c_g^u(t)\ge C_g^U\left(v_g(t)-v_g(t-1)\right),\ \forall g\in G,\ \forall t\in T \tag{4}$$

$$c_g^d(t)\ge C_g^D\left(v_g(t-1)-v_g(t)\right),\ \forall g\in G,\ \forall t\in T \tag{5}$$

$$c_g^{p(0)}(t)=A_g v_g(t)+B_g p_g^{(0)}(t),\ \forall g\in G,\ \forall t\in T \tag{6}$$

$$c_g^u(t),\ c_g^d(t)\ge 0,\ v_g(t)\in\{0{,}1\},\ \forall g\in G,\ \forall t\in T \tag{7}$$

$$\left(p_g^{(0)},\ f_{ij}^{(0)},\ r_i^{(0)},\ \theta_i^{(0)}\right)\in X^{(0)} \tag{8}$$

$$Q^w=\max_{s\in S}\left\{\min_{p_g^{(s)},\ f_{ij}^{(s)},\ r_i^{(s)},\ \theta_i^{(s)}}\sum_{t\in T}\sum_{i\in I}Pr_i^{(s)}(t)\right\} \tag{9}$$

$$\left(p_g^{(s)},\ f_{ij}^{(s)},\ r_i^{(s)},\ \theta_i^{(s)}\right)\in X^{(s)},\ \forall s\in S \tag{10}$$

where

$$X^{(s)}=\left\{\left(p_g^{(s)},\ f_{ij}^{(s)},\ r_i^{(s)},\ \theta_i^{(s)}\right):\right.$$

$$z_g^{(s)}v_g(t)\underline{P}_g\le p_g^{(s)}(t)\le z_g^{(s)}v_g(t)\bar{P}_g,\ \forall g\in G,\ \forall t\in T \tag{11}$$

$$-z_{ij}^{(s)}\bar{f}_{ij}\le f_{ij}^{(s)}(t)\le z_{ij}^{(s)}\bar{f}_{ij},\ \forall (i{,}j)\in L,\ \forall t\in T \tag{12}$$

$$f_{ij}^{(s)}(t)=\frac{z_{ij}^{(s)}}{x_{ij}}(\theta_i^{(s)}(t)-\theta_j^{(s)}(t)),\ \forall (i{,}j)\in L, \forall t\in T \tag{13}$$

$$\begin{aligned} & p_g^{(s)}(t)-p_g^{(s)}(t-1)\le (2-v_g(t-1)-v_g(t))\underline{P}_g\\ & \qquad +(1+v_g(t-1)-v_g(t))R_g^U,\ \forall g\in G,\ \forall t\in T \end{aligned}\tag{14}$$

$$\begin{aligned} & p_g^{(s)}(t-1)-p_g^{(s)}(t)\le (2-v_g(t-1)-v_g(t))\bar{P}_g\\ & \qquad +(1-v_g(t-1)+v_g(t))R_g^D,\ \forall g\in G,\ \forall t\in T \end{aligned}\tag{15}$$

$$\begin{aligned} & -r_i^{(s)}(t)\le \sum_{\forall j\in L_{(\cdot,i)}} f_{ji}^{(s)}(t)-\sum_{\forall j\in L_{(i,\cdot)}} f_{ij}^{(s)}(t)\\ & \qquad +\sum_{g\in G_i} p_g^{(s)}(t)-D_i(t)\le r_i^{(s)}(t),\ \forall i\in I,\ \forall t\in T \end{aligned}\tag{16}$$

$$\left. p_g^{(s)}(t),\ r_i^{(s)}(t)\ge 0,\ \forall i\in I,\ \forall g\in G,\ \forall t\in T\right\}. \tag{17}$$

where S, G, G_i, I, L, and T represents the set of indices of the scenarios, generate units, bus i possessed generators, buses, transmission lines, and time periods; Subscript/Superscript g, i, ij, (t), and (s) are the indicator of specified generator, bus, branch, time period, and scenario; c_g^p, c_g^d, and c_g^u are the production, shutdown, and startup costs; f, p, r, x, and θ describes the power flow, power output, loss of load, resistance, and phase angle; $\bar{f}$, $\bar{P}$ and $\underline{P}$ are the limits on power flow and power output; v and z are binary decision variable with '1' and '0' stands for the adequacy and outage; A_g and B_g are parameters for piecewise linear function; C^D/C^U, R^D/R^U, and T^D/T^U are fixed shutdown/startup costs, rampdown/ramp-up rate limit, and minimum down/up time; D, M, P, and Q are the load demand, disjunctive parameter, penalty factor, and the worst system operation cost.

The minimizing objective function (1) comprises of pre-contingency operation cost and post-contingency power imbalance penalty. Pre- and post-contingency constraints are modeled by (2)–(8) and (9)–(10) respectively. Constraints (2) and (3) represent the restrictions on the minimum up and down time for units. Start-up and shut-down costs are modeled in (4) and (5) respectively. Nonnegative and binary constraints are stated in (7). Economic dispatch (ED) problems for pre- and post-contingency are given by (8) and (10). (9) indicates that Q^w is the penalty for the worst-case of contingency.

For a fixed unit commitment decision $v_g(t)$ under any contingency scenario s, the ED is formulated as (11)–(17), where constraints on unit generation limit (11), transmission line capacity (12), power flow (13), ramping up/down limit (14)–(15), nodal power balance (16) are considered. (17) indicates the nonnegative constraints.

For simplicity, the piecewise linear approximation of the quadratic production cost function (18) is represented by (6), where parameters A_g and B_g are defined by (19) and (20).

$$c_g^p(t) = a_g v_g(t) + b_g p_g(t) + c_g p_g^2(t),\ \forall g \in G,\ \forall t \in T, \quad (18)$$

$$A_g = a_g - c_g \underline{P}_g \bar{P}_g,\ \forall g \in G, \quad (19)$$

$$B_g = b_g + c_g \bar{P}_g + c_g \underline{P}_g,\ \forall g \in G. \quad (20)$$

The contingency set S corresponding to the $n - K^G - K^L$ contingency criterion is defined as,

$$S = \left\{ (z_g, z_{ij}) \in \{0,1\} \;\middle|\; \begin{array}{l} \sum_{g \in G} z_g \geq |G| - K^G, \\ \sum_{(i,j) \in L} z_{ij} \geq |L| - K^L \end{array} \right\}. \quad (21)$$

III. Solution Methodology

A. Decomposition Framework

The specific CCUC problem (1)–(10) can be reformulated as a compact form (22), with $\boldsymbol{c}^T\boldsymbol{x} + P\sum_{s=1}^{|S|}\boldsymbol{q}_s^T\boldsymbol{y}_s$, $\boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{b}$, and $\boldsymbol{T}_s\boldsymbol{x} + \boldsymbol{W}_s\boldsymbol{y}_s \leq \boldsymbol{h}_s$ corresponds to (1) and (9), (2)–(8), and (10), respectively. Where $\boldsymbol{x}$ and $\boldsymbol{y}_s$ are the pre- and post-contingency decision variables; $\boldsymbol{c}$, $\boldsymbol{q}_s$, $\boldsymbol{A}$, $\boldsymbol{b}$, $\boldsymbol{T}_s$, $\boldsymbol{W}_s$, and $\boldsymbol{h}_s$ are coefficient matrices and vectors. The objective function (22) should be $\min_{\boldsymbol{x},\boldsymbol{y}_s}\left\{\boldsymbol{c}^T\boldsymbol{x} + P\max_{s\in S}\boldsymbol{q}_s^T\boldsymbol{y}_s\right\}$ in its original form, which is not suitable for the off-the-shelf MILP solver due to the $\min - \max$ programming. Since the final objective is eliminating the power imbalance, i.e., pursuing $\min\left\{\max_{s\in S}\boldsymbol{q}_s^T\boldsymbol{y}_s\right\} = 0$, which is equivalent with seeking $\min\left\{\sum_{s=1}^{|S|}\boldsymbol{q}_s^T\boldsymbol{y}_s\right\} = 0$ as $\boldsymbol{q}_s^T\boldsymbol{y}_s \geq 0$. Therefore, MILP problem (22) is deduced.

$$\begin{array}{llllllll} \min\limits_{\boldsymbol{x},\boldsymbol{y}_s} & \boldsymbol{c}^T\boldsymbol{x} & + & P\boldsymbol{q}_1^T\boldsymbol{y}_1 + P\boldsymbol{q}_2^T\boldsymbol{y}_2 \cdots + P\boldsymbol{q}_s^T\boldsymbol{y}_s & & & & (22) \\ s.t. & \boldsymbol{A}\boldsymbol{x} & & & & \leq & \boldsymbol{b}, & \\ & \boldsymbol{T}_1\boldsymbol{x} & + & \boldsymbol{W}_1\boldsymbol{y}_1 & & \leq & \boldsymbol{h}_1, & \\ & \boldsymbol{T}_2\boldsymbol{x} & + & \quad \boldsymbol{W}_2\boldsymbol{y}_2 & & \leq & \boldsymbol{h}_2, & \\ & \vdots & + & \qquad \cdots & & \leq & \vdots & \\ & \boldsymbol{T}_s\boldsymbol{x} & + & & \boldsymbol{W}_s\boldsymbol{y}_s & \leq & \boldsymbol{h}_s. & \end{array}$$

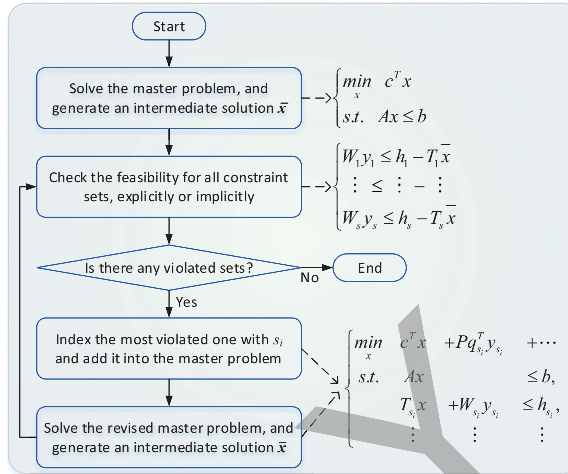

Fig. 2. Decomposition framework for the solution of CCUC with CCG.

Although (22) is applicable for available solver, it may present great challenges or even be intractable when faced with large-scale systems due to large numbers of decision variables and constraints. Therefore, decomposition strategy is usually employed to reduce the problem size. Fig. 2 presents a general decomposition framework suitable for both explicit and implicit methods.

B. Explicit Method

1) Subproblems: Explicit method formulate each realization of the uncertainty set S into a subproblem (23), where $z_g^{(s)}$ and $z_{ij}^{(s)}$ are fixed. By solving all of them, the most violated scenario can be determined.

$$\min \sum_{t\in T}\sum_{i\in I} Pr_i^{(s)}(t)\ s.t.\ (p_g^{(s)},\ f_{ij}^{(s)},\ r_i^{(s)},\ \theta_i^{(s)}) \in X^{(s)}. \quad (23)$$

In order to deduce Benders cuts, the corresponding dual problem (24)–(29) should be generated.

$$\max \quad Q_{it}^{(s)}(\beta,\ \tau,\ \zeta,\ \eta,\ \xi \mid v^*) \qquad \text{see (24)}$$

$$s.t. \quad \beta_g^{t+} - \beta_g^{t-} + \eta_g^{t-} - \eta_g^{(t+1)-} + \eta_g^{(t+1)+} - \eta_g^{t+} + \sum_{G_i \ni g}\xi_i^{t+} - \sum_{G_i \ni g}\xi_i^{t-} \leq 0,\ \forall g \in G,\ \forall t \in T \quad (25)$$

$$\tau_{ij}^{t+} - \tau_{ij}^{t-} + \zeta_{ij}^t - \sum_{i\in(i,j)}\xi_i^{t+} + \sum_{j\in(i,j)}\xi_j^{t+} + \sum_{i\in(i,j)}\xi_i^{t-} - \sum_{j\in(i,j)}\xi_j^{t-} = 0,\ \forall (i,j) \in L,\ \forall t \in T \quad (26)$$

$$\sum_{j\in L_{(i,\cdot)}}\frac{z_{ij}^{(s)}}{x_{ij}}\zeta_{ij}^t - \sum_{j\in L_{(\cdot,i)}}\frac{z_{ji}^{(s)}}{x_{ji}}\zeta_{ji}^t = 0,\ \forall i \in I,\ \forall t \in T \quad (27)$$

$$-\xi_i^{t+} - \xi_i^{t-} \leq P,\ \forall i \in I,\ \forall t \in T \quad (28)$$

$$\beta^{\pm},\ \tau^{\pm},\ \eta^{\pm},\ \xi^{\pm} \leq 0,\ \zeta \text{ unrestricted.} \quad (29)$$

where the subscript $_{it}$ represents the number of iteration; β^-/β^+, τ^-/τ^+, and ξ^-/ξ^+ are dual variables for the left/right hand side of constraints (11), (12), and (16); ζ, η^-, and η^+

$$
\begin{aligned}
Q_{it}^{(s)}(\beta,\,\tau,\,\zeta,\,\eta,\,\xi \mid v^*) = \sum_{t\in T}\Bigg[& \sum_{g\in G}\beta_g^{t+} z_g^{(s)} v_g(t)\bar{P}_g - \sum_{g\in G}\beta_g^{t-} z_g^{(s)} v_g(t)\underline{P}_g + \sum_{(i,j)\in L}\tau_{ij}^{t+} z_{ij}^{(s)}\bar{f}_{ij} + \sum_{(i,j)\in L}\tau_{ij}^{t-} z_{ij}^{(s)}\bar{f}_{ij} \\
& + \sum_{i\in I}\xi_i^{t+} D_i(t) + \sum_{g\in G}\eta_g^{t-}\left((2 - v_g(t-1) - v_g(t))\underline{P}_g + (1 + v_g(t-1) - v_g(t))R_g^U\right) \\
& - \sum_{i\in I}\xi_i^{t-} D_i(t) + \sum_{g\in G}\eta_g^{t+}\left((2 - v_g(t-1) - v_g(t))\bar{P}_g + (1 - v_g(t-1) + v_g(t))R_g^D\right)\Bigg].
\end{aligned}
\tag{24}
$$

are dual variables for constraints (13), (14), and (15), respectively. Constraints (25), (26), (27), and (28) correspond to the variables $p_g^{(s)}(t)$, $f_{ij}^{(s)}(t)$, $\theta_i^{(s)}(t)$, and $r_i^{(s)}(t)$ in (11)–(16). The upper script * represents the fixed value, and henceforth.

For the worst scenario and all the other violated scenarios, a Benders optimality cut can be generated, which is shown in (30). The Benders feasibility cut is omitted since the subproblem is always feasible due to the slack variable $r_i^{(s)}(t)$.

$$
Q^w \geq Q_{it}^{(s)}(v \mid \beta^*,\,\tau^*,\,\zeta^*,\,\eta^*,\,\xi^*),\ \forall s \in S. \tag{30}
$$

2) Master Problem: Take the optimality cuts (30) into consideration, the master problem is described as follows:

$$
\min_{v_g} \quad \sum_{t\in T}\sum_{g\in G}\left(c_g^{p(0)}(t) + c_g^u(t) + c_g^d(t)\right) + Q^w \tag{31}
$$

$$
s.t. \quad \text{constraints (2)–(8) and (30).} \tag{32}
$$

3) Bounds: The lower bound LB is the objective function value of master problem (31), and the upper bound UB is,

$$
\begin{aligned}
UB = & \sum_{t\in T}\sum_{g\in G}\left(c_g^{p(0)*}(t) + c_g^{u*}(t) + c_g^{d*}(t)\right) \\
& + \max_{s\in S}\left\{Q_{it}^{(s)}(v^*,\,\beta^*,\,\tau^*,\,\zeta^*,\,\eta^*,\,\xi^*)\right\}.
\end{aligned}
\tag{33}
$$

If the optimality criteria is met, i.e., $|UB - LB| \leq \epsilon$, then stop the process; otherwise, start a new iteration to solve the master problem.

C. Implicit Method

1) Subproblem: Implicit decomposition considers all the contingency scenario into one whole subproblem, where $z_g^{(s)}$ and $z_{ij}^{(s)}$ are binary decision variables. The primal $\max$–$\min$ subproblem is as follows:

$$
Q^w = \max_{z_g, z_{ij}}\left\{\Delta,\ s.t.(z_g,\, z_{ij}) \in S,\right. \tag{34}
$$

$$
\left.\Delta = \min\left[\sum_{t\in T}\sum_{i\in I} Pr_i(t),\ s.t.\text{(11)–(17) without }^{(s)}.\right]\right\} \tag{35}
$$

The inner problem (35) has the same form with (23), therefore, its dual problem is similar with (24)–(29). By replacing (35) with (24)–(29), the $\max$–$\min$ subproblem turns into $\max$–$\max$, which can be equivalently rewritten as a maximizing problem. However, several bilinear terms emerged in objective function (24) and constraints (27) since z_g and z_{ij} are also decision variables, such as $\beta_g^{t+} z_g$. In order to linearize the problem, the bilinear term will be replaced with new variables, i.e., $B_g^{t+} = \beta_g^{t+} z_g$, and the following disjunctive constraints should be added.

$$
-(1 - z_g)M \leq B_g^{t+} - \beta_g^{t+} \leq (1 - z_g)M, \tag{36}
$$

$$
-z_g M \leq B_g^{t+} \leq z_g M. \tag{37}
$$

2) Master Problem and Bounds: By solving the MILP subproblem, the worst contingency can be identified, then Benders cuts (30) can be generated. The remaining steps corresponding to the master problem and bounds are the same with the explicit method.

D. Acceleration Strategies

1) Parallel Computing: For the explicit method, all subproblems are independent and of the same scale, if parallel computing is utilized, the solution time for subproblems is almost inversely proportional to the number of processors [14]. On the other hand, there is only one MILP subproblem in the implicit method, which is not suitable for parallel implementation. In terms of MILP master problem, the parallel implementation can be worthwhile only if the calculation time for the subproblems can be reduced to the point where the master problem time becomes a significant fraction of the whole [15].

2) Multi-cut Strategy: As shown above, each contingency scenario can generate one Benders cut and one constraint set, containing information of the solution space, which benefits for the convergence process. If the complexity of the master is not considered, more cuts and constraints mean faster convergence. Therefore, a π-cut strategy can be proposed for the explicit method, which determines the most violated π realizations and adds corresponding cuts and constraints into the master problem.

IV. Numerical Results and Discussion

In this section, all the numerical tests are performed on AMPL IDE 3.1.0 using CPLEX 12.6.3 solver on a PC running on 64-bit Windows 8.1 operating system, with quad-core Intel Xeon E5-2620 v2 CPU (2.1 GHz) and 32 GB RAM. Parameters P, M and ϵ are valued as 10^3, 10^3 and 10^{-3}.

A. Benchmark Systems

Extensive performance evaluation and sensitivity analysis are implemented on the IEEE 24-bus test system [16] to illustrate the efficacy of explicit and implicit methods. The resilience against multiple contingencies is increased by adding 4 generators and 5 transmission lines. As a consequence, the

TABLE I
SCALES AND COMPLEXITY OF BENCHMARK TEST SYSTEMS UNDER $n-1-1$ CONTINGENCY CRITERION

Items	24-bus system	118-bus system
No. of buses:	24	118
No. of generators:	14	56
No. of transm. lines:	46	189
No. of scenarios:	645	10,585
No. of bin. variables:	336	1,344
No. of cont. variables:	1,889,233	136,422,169
No. of constraints:	3,378,347	230,945,121

TABLE II
ALTERNATIVE VERSIONS OF ALGORITHMS

Algorithms	Description
Alg.E.1/Alg.I.1:	Explicit/Implicit method with Benders cuts only.
Alg.E.2/Alg.I.2:	Explicit/Implicit method with constraint sets only.
Alg.E.3/Alg.I.3:	Explicit/Implicit method with both constraint sets and Benders cuts.
Alg.E.4:	***Alg.E.2*** with its subproblems solved in parallel by 24 threads.
Alg.E.5:	***Alg.E.2*** with multi-cut strategy.
Alg.C.6:	Commercial MILP solver CPLEX 12.6.3.

system is able to withstand the $n-3-0$ and $n-2-1$ contingency criteria. The IEEE 118-bus test system [17] is introduced to show the potential under large-scale circumstances, where 2 generators and 3 circuits are enhanced to meet the $n-1-1$ contingency criterion. Table I gives an overview of the scales and complexity of both systems.

B. Performance Evaluation

In order to extensively compare the explicit and implicit methods, several algorithms are separated and shown in Table II. The convergence behavior and time consumption is illustrated in Fig. 3 and Fig. 4, where a cutoff time of 1000s is employed. The global optimal objective value is 952,164$.

1) Performance Evaluation of Benders Cuts and Constraint Sets: According to Fig. 4, both ***Alg.E.1*** and ***Alg.I.1*** cannot

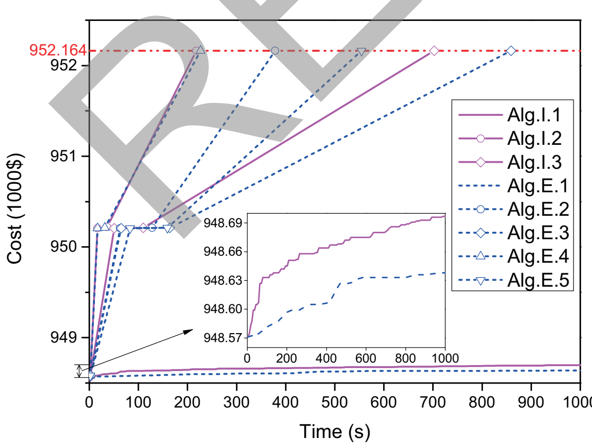

Fig. 3. Behavior of convergence for different algorithms.

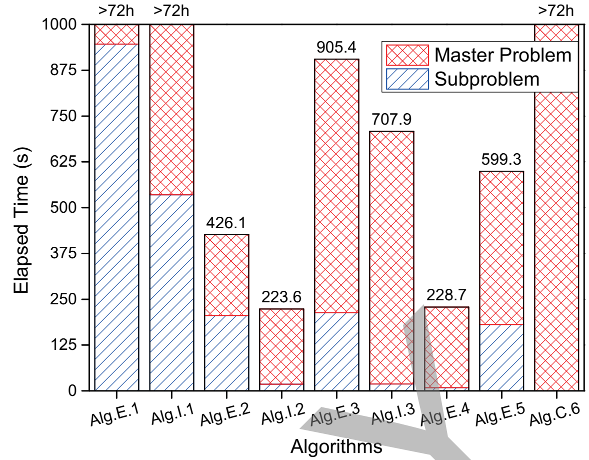

Fig. 4. Behavior of time consumption for different algorithms.

terminate after a run time of 1000s, while ***Alg.E.2*** and ***Alg.I.2*** converge at 426.1s and 223.6s, indicating that CCG is more efficient than Benders cuts. On the other hand, ***Alg.E.3*** and ***Alg.I.3*** are even slower than ***Alg.E.2*** and ***Alg.I.2*** with both CCG and Benders cuts involved, which means the introduction of Benders cuts even drags the solution efficiency. The reason is that a slack variable $r_i^{(s)}(t)$ is required during the solution process of Benders decomposition, which expands the scale for both master problem and subproblems.

2) Performance Evaluation of Parallel Implementation: It is noticeable in Fig. 4 that the time consumed by subproblem is much higher in the explicit method, while the master problem consumes almost the same amount of time for both methods, especially for ***Alg.E.2*** and ***Alg.I.2***. Therefore, the parallel implementation is introduced in ***Alg.E.4*** based on ***Alg.E.2***. The solution time is reduced from 426.1s to 228.7s, which is comparable to ***Alg.I.2*** with 223.6s; thus the performance improvement gained by parallel computing is promising.

3) Performance Evaluation of Multi-cut Strategy: In ***Alg.E.5***, two sets of constraints are added in each iteration, i.e., $\pi = 2$. However, it spends more time than ***Alg.E.2*** although their subproblems spend the same time. Which means adding one more set of constraints only increases the size of the master problem, but does not enhance the convergence, i.e., the benefit of multi-cut strategy is not significant. One reason is that the two most violated scenarios are similar and one of them is redundant. Another reason lies in the small number of iterations before termination, which can be seen from Fig. 3.

4) Performance Comparison between Explicit and Implicit Methods with MILP Solver: If the cutoff time of 1000s is replaced by 72h, the algorithms ***Alg.E.1*** and ***Alg.I.1*** end with a gap of 8.3% and 1.6% respectively; however, no valuable solution was reported by ***Alg.C.6***. Therefore, two advantages can be drawn from decomposition methods in comparison to direct MILP solution: 1) the solution process is observable, i.e., each intermediate solution can be output and its quality can also be identified by gap, and 2) the decomposition strategy makes the large-scale problem tractable in terms of

TABLE III
COMPUTATIONAL RESULTS FOR DIFFERENT K^G AND K^L VALUES

K^G	K^L	Cost ($)	Time (s)	K^G	K^L	Cost ($)	Time (s)
0	0	948,582	2.328	2	0	953,380	267.375
0	1	948,932	11.939	2	1	956,259	4,336.830
0	2	Infeasible	——	2	2	Infeasible	——
1	0	949,810	58.266	3	0	958,028	3,655.560
1	1	952,164	223.592	3	1	Infeasible	——
1	2	Infeasible	——	4	0	Infeasible	——

execution time and memory resources.

C. Sensitivity Analysis for K^G and K^L

Table III summarizes the total cost and execution time in terms of different values of K^G and K^L. As can be seen, the cost goes higher as K^G and K^L increase, since more units should be turned on to compensate for the failure of components. Comparing the most severe contingency with the sufficient one, only 1% additional cost is introduced; however, the solution time increases heavily from 2s to more than 1h, showing that the complexity of the problem is exponentially related with K^G and K^L. It is also noticeable that the system is more capable of surviving the loss of generator than the outage of circuits. The reason is that generator is much easier to be substituted by others if the network is still sufficient, while the loss of circuits usually results in node isolation.

D. Potential Exploration for Large-scale Implementation

The comprehensive case study and discussion on IEEE 24-bus test system reveals that the implicit method ***Alg.I.2*** and parallel explicit method ***Alg.E.4*** are superior to other algorithms and solvers. Their potential on the large-scale instance is validated by the IEEE 118-bus test system in this section. Fig. 5 depicts the convergence behavior of ***Alg.I.2***, which terminates at 3h after 3 iterations, while ***Alg.C.6*** runs out of memory at 1.2h. The solution process of ***Alg.E.4*** is similar with that of ***Alg.I.2*** illustrated in Fig. 5 except for the solution time of subproblems. Finally, ***Alg.E.4*** takes 3.79h in total for solution. Although ***Alg.I.2*** is faster, it may be intractable for larger systems since all the contingency scenarios are included in one MILP subproblem. On the other hand, the solver's limits on the solution of ***Alg.E.4***'s individual LP subproblem are far from being reached.

V. CONCLUSION

Both explicit and implicit decomposition frameworks have been investigated for the solution of $n - K^G - K^L$ CCUC problem. Except for the validation of conventional finding that the CCG dominates on Benders cuts, several other conclusions are made: 1) the introduction of Benders cuts may even drag the solution efficiency of CCG; 2) the parallel implementation of explicit method is proportionate with the implicit method; 3) the benefit of multi-cut strategy is not significant; 4) the decomposition framework is superior over commercial solver for this kind of problem; and 5) the system is more capable of surviving the loss of generator than the outage of circuits.

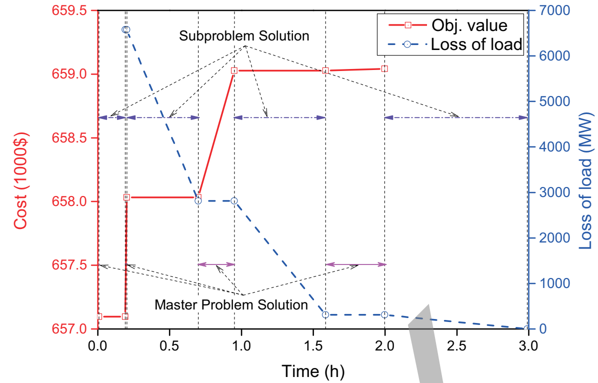

Fig. 5. Behavior of convergence of ***Alg.I.2*** for IEEE 118-bus test system.

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