injection rates and low mobility ratios. As these increase, the oil recoveries from isolated and modified patterns become identical.

The numerical computations show that even at high mobility ratios, the oversweep of isolated patterns persists. Although the absolute value of the amount of oil recovery from outside of the pattern decreases, the ratio of this amount to the amount of oil recovery from a confined pattern remains a constant for different mobility ratios.

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PATTERN OVERSWEEP IN FOUR AND SEVEN-SPOT PATTERN WATERFLOODS.

by

© Viviane C. Blond

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN PETROLEUM ENGINEERING.

DEPARTMENT OF MINERAL ENGINEERING

EDMONTON, ALBERTA

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THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled " Pattern Oversweep in Four and Seven-spot Pattern Waterfloods " submitted by Viviane S. Blond in partial fulfilment of the requirements for the degree of Master of Science in petroleum engineering.

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Date November 28, 1980

ABSTRACT

An experimental study of the oversweep in four- and seven-spot pattern waterfloods was conducted. The porous medium consisted of a pack of glass beads between two lucite plates.

The effect of water injection rate and mobility ratio on the oil recovery was examined, as well as the result of modifying the isolated pattern by surrounding it with a ring of like patterns.

Waterflood recovery data were also computed using a two-dimensional, two-phase flow numerical model. A larger range of mobility ratios was studied using this model. Oil recoveries from isolated and confined pattern waterfloods were compared.

An increased injection rate leads to lower oil recoveries from isolated four- and seven-spot pattern waterfloods. The effect is similar for modified pattern waterfloods until the critical rate is attained. Above this rate the oil recovery is independent of the injection rate.

The effect of the water-oil ratio on the oil recovery is studied. A decrease of the oversweep is observed as the mobility ratio is increased.

The oversweep in a four-spot pattern waterflood is much more extensive than in a seven-spot pattern. It may be limited in either pattern by surrounding the pattern with a ring of similar patterns. This is effective only at low

List of Tables

Table of Contents

Chapter Page

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List of Figures

1. INTRODUCTION

Pilots waterfloods are used commonly to investigate the production behaviour of petroleum reservoirs. They are carried out on a representative portion of a reservoir and serve to indicate the ultimate oil recovery which can be expected from a field-size waterflood. Oil recovery data obtained from these pilots may be far too optimistic or pessimistic. Many such examples have been reported in the literature (1, 2, 3, 4, 5, 6,.).

The amount of oil recovery from an unconfined pilot involving only one or a few patterns is not the same as from a large-scale flood. The perimeter of the pilot does not act as an effective boundary and the fluids are free to move into or out of the pilot area.

Previous authors, at the University of Alberta (7, 8), were interested in the oversweep in five and nine-spot pattern pilots. They reported recoveries from experimental waterfloods equal to five or six times the amount of oil initially in place in the pattern.

The five-spot pattern is the most commonly used in waterflooding operations, mainly because in many fields wells are drilled on a square grid and, when the decision to start a waterflood is taken, some wells are simply converted from producers to injectors.

However, some interest has been shown in the seven-spot pattern, particularly in steamfloods, although a seven-spot pattern waterflood requires a non-standard drilling pattern and thus, the drilling of extra wells. It was then decided to carry out an experimental study of seven-spot pattern waterfloods. The aim of this work was to show the extent of the oversweep and to determine if, for some combination of injection rate and mobility ratio, the confinement of the pattern could be attained.

Laboratory experiments are one way of studying the effects of parameters such as injection rate or mobility ratio on a waterflood. Another way is to use a numerical model. The physical processes taking place during a waterflood can be described by a set of mathematical equations. This mathematical model must include the influence of gravitational, viscous and capillary forces on the displacement. This set of equations is then solved numerically. It was decided to use such a model to obtain oil recovery curves for a seven-spot pattern waterflood and then compare the results with the experimental ones.

2. LITERATURE REVIEW

2.1 Performance of Confined Laboratory Pilots.

Many investigators studied the effects of parameters such as water injection rate, injection pressure, and water-oil mobility ratio on oil recovery from pilot waterfloods. In many of these studies, physical models were used. These models were made of artificial porous media, consolidated or unconsolidated (9, 10, 11, 12, 13). They comprised one whole five-spot pattern or sometimes only an element of symmetry of the pattern. The four sides of the porous media were sealed and the models were thus mechanically confined.

Besides the physical models, other types of models have been used, such as electrolytic and potentiometric models. Considering the fact that a field-size waterflood is necessarily confined, most laboratory models were designed to reproduce these confinement conditions.

The results of these studies indicate that in a confined five-spot pattern waterflood, the water-oil mobility ratio is one of the most critical factors. The pattern sweep efficiency is strongly dependent upon the mobility ratio. The breakthrough sweep efficiency decreases with increasing mobility ratio (16). At favourable mobility ratios, the areal sweep efficiency is high and, after breakthrough the water-oil ratio remains low. Most of the oil in place can be recovered before a high water-cut makes the production uneconomical. At unfavourable mobility ratios, the opposite happens. The breakthrough sweep efficiency is low and the water-oil ratio increases rapidly after breakthrough (17, 18, 19). Only a small fraction of the oil in place may be recovered before the production must be stopped.

2.2 Pattern Oversweep in Unconfined Pilots.

As mentioned above, most laboratory models used for studying pattern waterfloods comprised only one pattern or an element of symmetry of a pattern and were confined. Several studies have been conducted at the University of Alberta on a larger model in order to determine the degree of confinement of the pattern during a waterflood. This was done in order to help in the interpretation of oil recovery data obtained from pilot waterfloods, which comprise only one or a small number of patterns.

Jain (20) reported oil recoveries from an isolated five-spot pattern as high as five times the initial oil in place, the total recovery continuously increasing as the water injection continues. He examined the effect of surrounding the basic pattern with a ring of eight like patterns. This pattern arrangement will be refered later as a "modified" five-spot pattern. This resulted in limiting the oversweep but was not sufficient to obtain effective pattern confinement. His results also show that a variation of the injection pressure had no effect on the oil recovery, within the range of pressures studied.

Peters (21) completed Jain's study of the five-spot and extended it to the nine-spot pattern. Qualitatively, the results for both patterns were similar. The oversweep of an isolated pattern decreases as the injection rate and/or the water-oil mobility ratio increases. Some oversweep persists for any combination of these parameters. A similar effect is observed when the pattern of interest is surrounded with a ring of eight like patterns. However, in this case, for a given mobility ratio, there is a value of the injection rate above which the pattern is confined. The recovery is no longer dependent on the injection rate and the waterflood is said to be stabilized. The value of the injection rate necessary to obtain a stabilized flood decreases as the mobility ratio increases. Furthermore, the oil recovery from modified patterns is always lower than the one from isolated patterns.

2.3 Performance of Seven- and Four-spot Patterns.

A seven-spot pattern represents a compromise between the five- and nine-spot patterns. A normal seven-spot with an injection to production well ratio of two will be preferred to a five-spot in case of low permeability formation or low injectivity. An inverted seven-spot (or four-spot pattern) will be preferred to a five-spot if the displacing fluid injectivity is high. The reduction in the number of injection wells necessary will then improve the economics of the project. These results are summarized below.

Normal 5-spot	normal 7-spot	normal 9-spot
I/P=1	I/P=2	I/P=3

-->

Preferred in case of low permeability formation or low injectivity.

inverted 5-spot	inverted 7-spot	inverted 9-spot
I/P=1	I/P=1/2	I/P=1/3

-->

Preferred in case of high injectivity.
More economical.

Although the five-spot pattern is the most popular, seven-spot pilots have been employed, particularly in steamfloods (22).

The performances of seven- and four-spot pattern waterfloods have received very little attention in the literature.

Muskat(23) presented analytical expressions for the pressure distribution and sweep efficiency of a seven-spot pattern at steady state (when the total produced volume is equal to the total injected volume). He obtained the following expression for the areal sweep efficiency for a unit mobility ratio:

$$E = \frac{\mu q}{khA} \int_0^{S_t} \frac{dS}{\frac{\partial P}{\partial S}}$$

where

μ is the viscosity in centipoises

q is the production rate in cubic feet per second

k is the absolute permeability in millidarcies

h is the thickness of the formation in feet

A is the network element area associated with an injection well, in square feet.

dS denotes an element of length along the streamline

S_t is the total path length between the injection and producing well.

The integral is evaluated graphically, and the result is an areal sweep efficiency of 74 percent at breakthrough. Thus, at a mobility ratio of one, the breakthrough sweep efficiency of a seven-spot pattern is approximately equal to that of a five-spot pattern.

Wyckoff et al conducted an experimental study using a blotting-paper electrolytic model (24). This procedure is equivalent to a waterflood under conditions of a unit-mobility ratio. They report a breakthrough sweep efficiency of 82 percent for the inverted seven-spot pattern and 80 percent for the normal seven-spot pattern. The fact that they found about the same value for the two patterns is to be expected. For a mobility ratio of one, the pressure distribution and the streamlines are the same in the inverted and in the normal seven-spot patterns. The direction of flow is simply reversed. Thus, the same value of the breakthrough sweep efficiency is expected.

The latest results were presented by Guckert (25). He conducted a detailed experimental study of the normal and inverted seven-spot patterns on a laboratory model representing one twelfth of the basic pattern. His conclusions were three-fold:

1) The breakthrough sweep efficiencies of the normal and inverted seven-spot patterns are equal for the same value of the mobility ratio over the range of mobility ratios from 0.24 to 4.0.

2) Recovery decreases as mobility ratio increases, varying from a breakthrough recovery of 90 percent for a mobility ratio of 0.25 to 65 percent for a mobility ratio of four.

3) The area swept after breakthrough per unit pore volume injected is slightly greater for the inverted than for the normal seven-spot pattern, at the same value of the mobility ratio.

3. STATEMENT OF THE PROBLEM

It was shown in the literature review that very little research has been done on seven-spot pattern waterfloods. As this pattern becomes more popular, it appears important to study it more thoroughly. In order to help in the interpretation of oil recoveries from seven-spot pilots, it would be of particular interest to know what is the extent of the oversweep in these pilots.

The oversweep in pilot waterfloods has already been studied at the University of Alberta. The previous authors were interested in the five- and nine-spot patterns. It was decided to extend this study to the four- and seven-spot patterns. For this purpose a laboratory model was built. It was intended to study experimentally the effect of increasing water injection rate and increasing oil viscosity on the pattern oversweep, as well as to compare oil recoveries from pilots of increasing complexity.

The scaling of the model was necessary in order to relate experimental data to field conditions.

Laboratory waterfloods are limited by physical factors such as the maximum injection rate and the maximum injection pressure available with the injection system. The most limiting factor is the viscosity of the oil, as highly viscous oils cannot be injected in the model. Numerical models do not have this limitation. Thus, it was decided to use a two-phase, two-dimensional numerical model to simulate waterflood experiments. This model could be used to determine the extent of oversweep in isolated pattern waterfloods for high values of the water-oil mobility ratio. Even for low mobility ratios, it was hoped that numerical computations could replace time-consuming experiments.

The simplest flooding patterns used in pilots are isolated patterns. An isolated seven-spot pattern comprises six injectors around one production well. An isolated four-spot pattern is composed of three injectors and one producer. An isolated pattern is situated in the middle of a field and its boundaries are open to flow.

To approximate the extent of the oversweep in an isolated pattern waterflood, the amount of oil recovery must be compared to the amount recovered from a confined pattern. A basic pattern (four-spot, seven-spot) is confined when it is situated in the middle of a fully developed field, and surrounded by several rings of like patterns. In this case no fluid from outside can enter the pattern. In numerical computations, a confined pattern may be represented by a basic pattern whose boundaries are closed.

As it is recognized that the oversweep in isolated pattern pilot waterfloods may be extensive, modified flooding patterns are often used in pilots in order to obtain better confinement. A modified pattern is a basic pattern (four-spot, seven-spot) surrounded by one ring of like patterns, i.e. a seven-spot pattern surrounded by six seven-spots or a four-spot surrounded by three four-spots.

The external boundaries of a modified pattern are open to flow. The aim of the experimental study was to determine if the confinement of the central pattern of these arrangements could be attained.

4. MODEL SCALING

Laboratory models used in the study of the displacement of oil by water in a waterflood must be scaled. The experimental results can then be used to predict the behavior of field-size waterfloods. The scaling procedure may be divided into two steps: geometrical scaling and scaling of forces acting during the flood: viscous forces, capillary and gravity forces. The second step requires a study of the equations governing the flow.

Using dimensional analysis, Engelberts and Klinkenberg (26) obtained the conditions required for the scaling of two-phase flow experiments in porous media. Rapoport (27) derived general scaling laws by application of inspectional analysis.

A review of these works and others dealing with the scaling of laboratory models may be found in Reference 28. Only the results will be presented here.

4.1 Two-dimensional Horizontal Flow. No Gravity.

The equation describing the displacement of oil by water in a porous medium is obtained by a combination of Darcy's equation with the law of conservation of matter. The dimensionless form of the displacement equation for two-phase, two-dimensional flow with no gravity effect is:

$$\frac{df}{dS}\left(\frac{\partial S}{\partial X_d}+\frac{\partial S}{\partial Y_d}\right)+\frac{\sigma\cos\theta\sqrt{K\phi}}{\mu_w u^* L}\,\frac{1}{\lambda}\left(\frac{\partial}{\partial X}\left(\psi\overline{Pc}'\frac{\partial S}{\partial X_d}\right)+\frac{\partial}{\partial Y_d}\left(\psi\overline{Pc}'\frac{\partial S}{\partial Y_d}\right)\right)=-\frac{\partial S}{\partial t_d} \qquad (4\text{-}1)$$

where u^* is a characteristic velocity, equal to the superficial velocity for linear floods.

The flooding behaviour is controlled by the similarity group:

$$C_1=\frac{\mu_w u^* L}{\sigma\cos\theta\sqrt{K\phi}} \qquad (4\text{-}2)$$

A field waterflood will be properly scaled if, in the model and the prototype, the oil viscosity, the relative permeability curves, the dimensionless capillary pressure curve, and the scaling factor C_1 are the same.

4.2 Scaling of Seven-spot Pattern Waterfloods.

The scaling coefficient C_1 was obtained for linear waterfloods. L is the length of the system and U^* is the superficial velocity. When considering the scaling of pattern waterfloods, problems arise as to the choice of the characteristic length and the characteristic velocity .

Rapoport and Leas (29) suggested the approximation of a five-spot pattern waterflood by a linear waterflood in the direction of the predominant streamlines in a confined flood. In the present work a similar approximation is used for a seven-spot pattern. The characteristic length L is taken as the distance between injector and producer.

The scaling coefficient proposed by Rapoport, Carpenter and Leas (30) was used.

$$C_2 = \frac{q\mu_w}{\sigma\sqrt{K\phi}} \qquad (4-3)$$

C_2 is equivalent to C_1 with $u = q/L$. The production rate q is equal to twice the injection rate for a confined seven-spot waterflood and to half the injection rate for a confined four-spot watesflood.

4.3 Stabilized Flood.

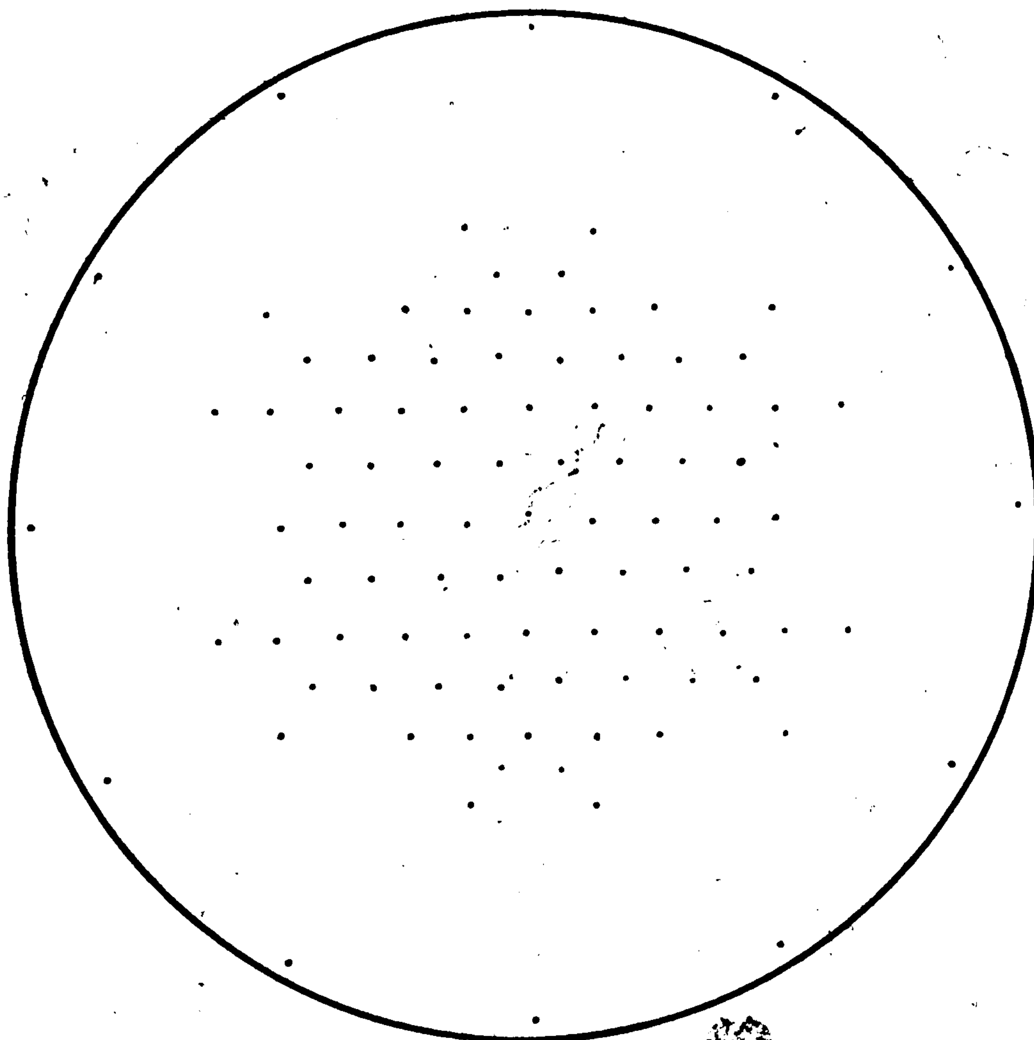
From Equation 4-1, one may observe that, as the scaling coefficient increases, the importance of the capillarity term decreases. At high fluid velocities, the effect of the capillary forces on the displacement process is negligible. The waterflood performance is then independent of the injection rate and the flood is said to be stabilized.

5. EXPERIMENTAL PROCEDURE.

5.1 Equipment.

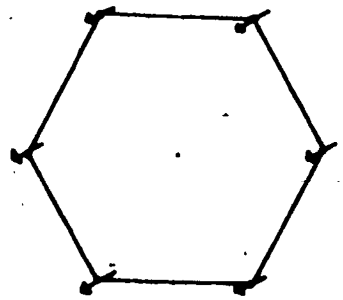
The model consisted of two circular transparent lucite plates having each a diameter of 1.8 meters. Between them glass beads of regular diameter were packed to obtain an isotropic and homogeneous porous medium. The thickness of the porous medium was 6.4 millimeters. This value was very small compared to the diameter of the model and we may then consider that the displacement was not affected by the gravity and was two-dimensional. The medium had 95 fully penetrating wells, set on an hexagonal matrix, as shown on Figure 1. This arrangement of wells permitted the study of various combinations of four- and seven-spot injection patterns. The following patterns were used in the course of this study: isolated seven-spot, modified seven-spot (normal seven-spot surrounded by a ring of six seven-spot patterns), isolated four-spot, four-spot surrounded by three four-spot patterns, and a four-spot surrounded by 12 four-spot patterns. These patterns are reproduced on Figure 2. Properties of the model and the glass beads are summarized in Table 1.

The injection system consisted of a Ruska proportioning pump and 40 injection cylinders actuated by a double-acting master cylinder. This permitted injection through up to 39 injection lines at a constant rate. The injection rate was limited to 540 cc/hr.line. This material was previously used

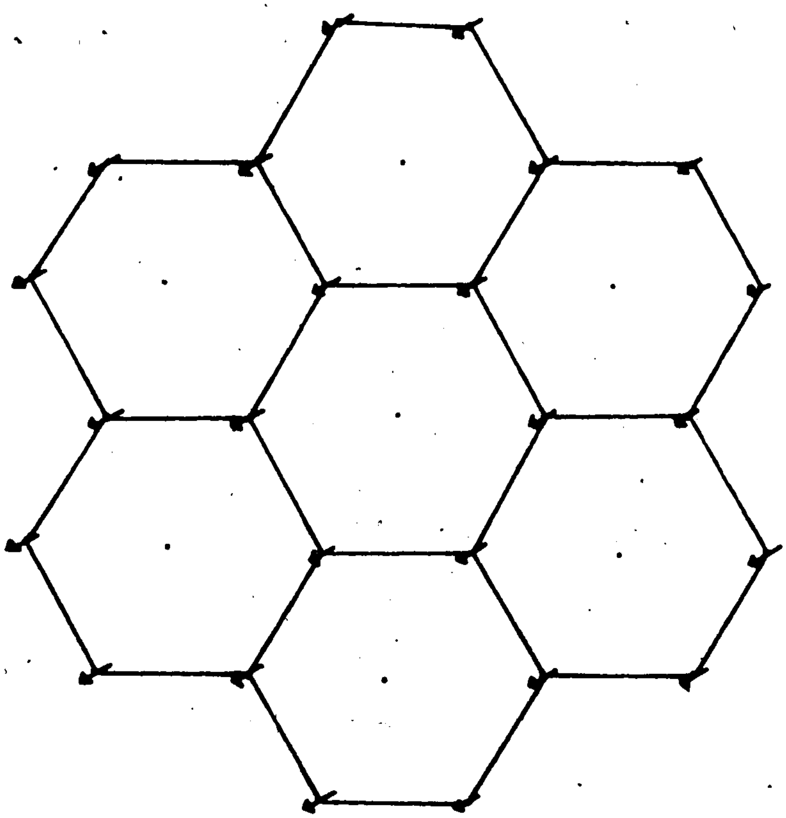


Valve
Scale 1/12th

Figure 1
Valves set-up

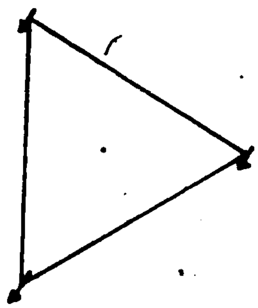


Isolated Seven-spot

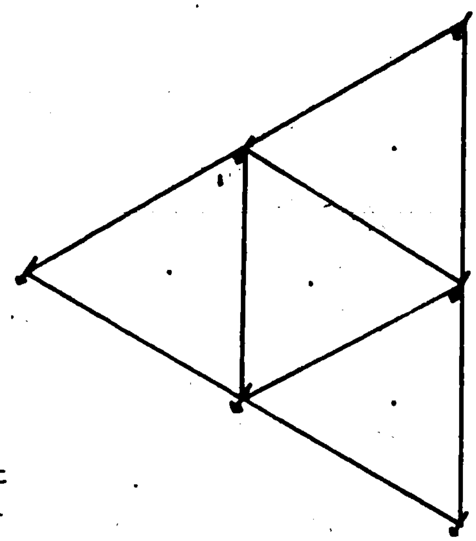


Modified Seven-spot

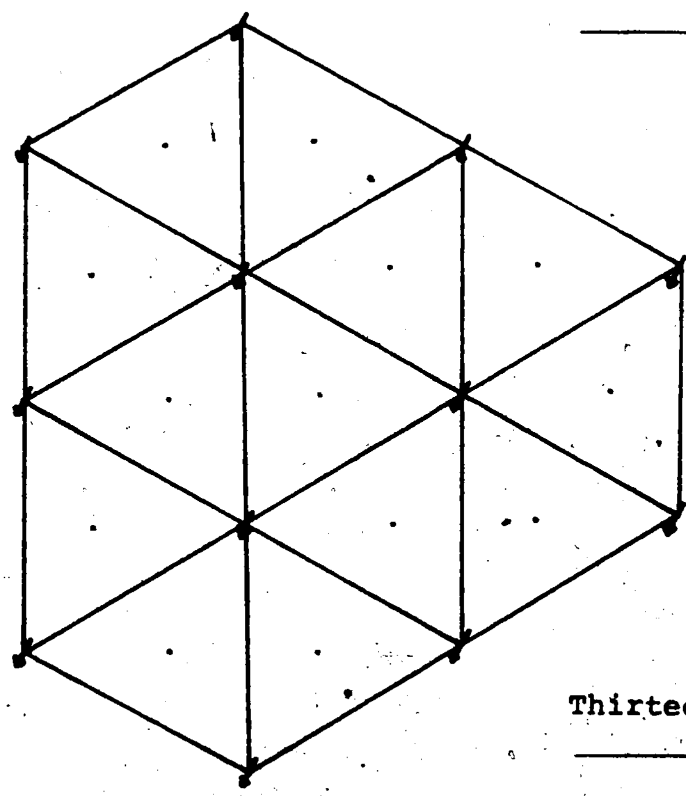
Figure 2 : Flooding Patterns.



Isolated Four-spot



Modified Four-spot



Thirteen (Four-spot)

Figure 2 (continued)

Flooding Patterns.

by Jain and Peters. A detailed description and a flow-chart may be found in Reference 19.

5.2 Preparation of the Porous Medium.

Prior to packing, the beads were soaked in a solution of household detergent, rinsed several times with distilled water and dried. A preferentially water-wet medium was then obtained. This was necessary for easy cleaning of the model after a run. The beads were weighed before packing for calculation of the porosity. Two electric vibrators were used for a total duration of five hundred hours to help pack the model. The porous medium was also alternatively saturated with water and dried; this was found to improve the packing operation. The result was a uniform packing with a porosity of 36.2 percent and a permeability of 6.8 darcies. Details on porosity and permeability calculations are given in Appendix A.

5.3 Fluid Properties.

Distilled water coloured with sodium fluoresceine was injected in the central pattern of the arrangement studied. By means of ultra-violet lighting it was then possible to take photographs of the water-invaded zone during a flood. Distilled water dyed with a red food colour was injected into the surrounding patterns . This allowed visual observation of the flood front displacement.

The oils used were kerosene and mixtures of kerosene and lube oil. The kerosene was mixed with some lube oil to obtain a desired viscosity. The viscosities were limited by experimental reasons; the viscosity had to be low enough to allow filling of the model in a reasonable amount of time. The properties of the fluids are summarized in Table 2. The viscosities were measured with a Cannon-Fenske viscometer and the interfacial tensions with a Du Nouy tensiometer. The end-point relative permeabilities to oil and water were determined at residual water saturation and residual oil saturation respectively.

The mobility ratio was defined as

$$M = \frac{k_{wro}\mu_o}{k_{ocw}\mu_w}$$

where μ_o and μ_w are the oil and water viscosities,

k_{ocw} , the effective permeability to water at residual oil saturation, and

k_{wro} , the effective permeability to oil at residual water saturation.

Details on the measurement of the relative permeabilities are given in Appendix A.

6. EXPERIMENTAL RESULTS

Thirty-three runs were made with the three types of oils previously described (Table 2): 19 runs at a mobility ratio of 0.83 (oil:kerosene), eight runs at a mobility ratio of 1.36 (oil:75 % kerosene, 25 % lube oil), and six runs at a mobility ratio of 2.70 (50 % kerosene, 50 % lube oil). Injection rates ranged from 53 cc/hr.well to 537 cc/hr.well.

All experimental data are given in Appendix B in tabular form. They are presented graphically on Figures 3 through 24. The cumulative oil recovery (percentage of initial oil in place) is plotted as a function of the cumulative liquid production (pattern pore volumes produced). The experimental conditions prevailing for each run are shown in Table 3 .

Typical photographs of the water-front at successive stages of a run are shown in Appendix G.

Each run was arbitrarily terminated when six to seven pattern pore volumes had been produced. In the range of mobility ratios and injection rates studied, the oil recovery continuously increased as the water injection was continued. This can be observed on all recovery profiles (Figures 3 through 24) but particularly on Figure 3 which corresponds to Run 29. To show the increase in oil recovery, this run was continued till 14 pattern pore volumes had been produced. After production of two pore volumes the recovery curve is almost linear, the slope of the curve decreases two sets of runs, before a new oil was used, the model was flushed with a solution of household detergent and rinsed with distilled water in order to eliminate all residual oil.

5.4 Procedure

The model was placed in the vertical position and distilled water was injected through the three lower wells till total saturation of the porous medium was reached. When all the air had been eliminated, oil was injected through the upper wells while the water was allowed to drain through the lower well. Oil injection was continued till no more water was produced. This resulted in a residual water saturation of 40 to 42 percent, the value depending on the kind of oil displaced. Then, the model was placed in the horizontal position and the wells of the selected pattern were opened to stabilize the pressure. The injection rate was selected and injection was started. The oil and water produced from the central pattern were collected in a bank of graduated test tubes. The production of all surrounding patterns was collected in the same vessel. Photographs of the waterfront were taken at regular intervals during the run . A black and white, Tri-X, 400 ASA film was used. The experiment was terminated when six to seven pattern-pore-volumes had been produced. The model was flushed with about ten pore-volumes of distilled water, injected through the lower wells. No more oil was displaced after. Although the porous medium was preferentially water-wet there was a residual oil saturation varying between three and eight percent. The porous medium was then resaturated with oil for the next run. A good reproducibility of the medium properties was obtained using the above procedure. Between

TABLE 1.

Summary of Model Properties.

Diameter		182.9	cm
Average thickness		0.643	cm
Injector-producer distance		10.25	cm
Pattern pore volume	4-spot	31.6	cm^3
	7-spot	63.2	cm^3
Average diameter of beads		0.05	cm
Porosity		36.2	%
Absolute permeability		6.8	D

Table 2.

Fluids Properties
at 22 Degrees C.

Fluid	density (g/cm^3)	viscosity (cp)	viscosity ratio	mobility ratio	int. tension (dynes/cm)
water	0.996	0.99			
kerosene	0.799	1.38	1.39	0.83	32.7
25 % kerosene 75 % lube oil	0.827	2.40	2.42	1.36	35.7
50 % kerosene 50 % lube oil	0.839	4.47	4.52	2.70	38.0
lube oil	0.856	30.20	30.50		40.1

Table 3.

Summary of Experimental Results:Percentage of Initial Oil in Place Recovered after Injection of 2 and 5 Pattern Pore Volumes.

Pattern Type	Mobility Ratio	Run #	Q cc/hr	%IOIP 2 PV inj.	%IOIP 5 PV inj.
isolated 4-spot	0.83	1	53	331	738
		2	102	335	636
		3	183	289	520
		4	389	215	367
	1.36	20	183	210	363
		21	537	166	239
	2.70	27	389	109	156
4 (4-spot)	0.83	5	102	228	425
		6	183	139	259
		7	389	136	234
		8	537	101	188
	1.36	22	183	126	215
		23	537	122	172
	2.70	29	389	104	147
13 (4-spot)	0.83	9	102	278	
		10	183	258	380
		11	537	126	161
	2.70	30	537	101	124
isolated 7-spot	0.83	12	53	270	478
		13	183	126	238
		14	183	132	221
		15	389	92	129
	1.36	24	183	127	204
		25	537	77	101
	2.70	31	183	72	102
		32	389	76	94
7 (7-spot)	0.83	16	53	205	350
		17	183	116	167
		18	389	91	112
		19	537	91	112
	1.36	26	183	130	169
		27	537	78	109
	2.70	33	389	68	86

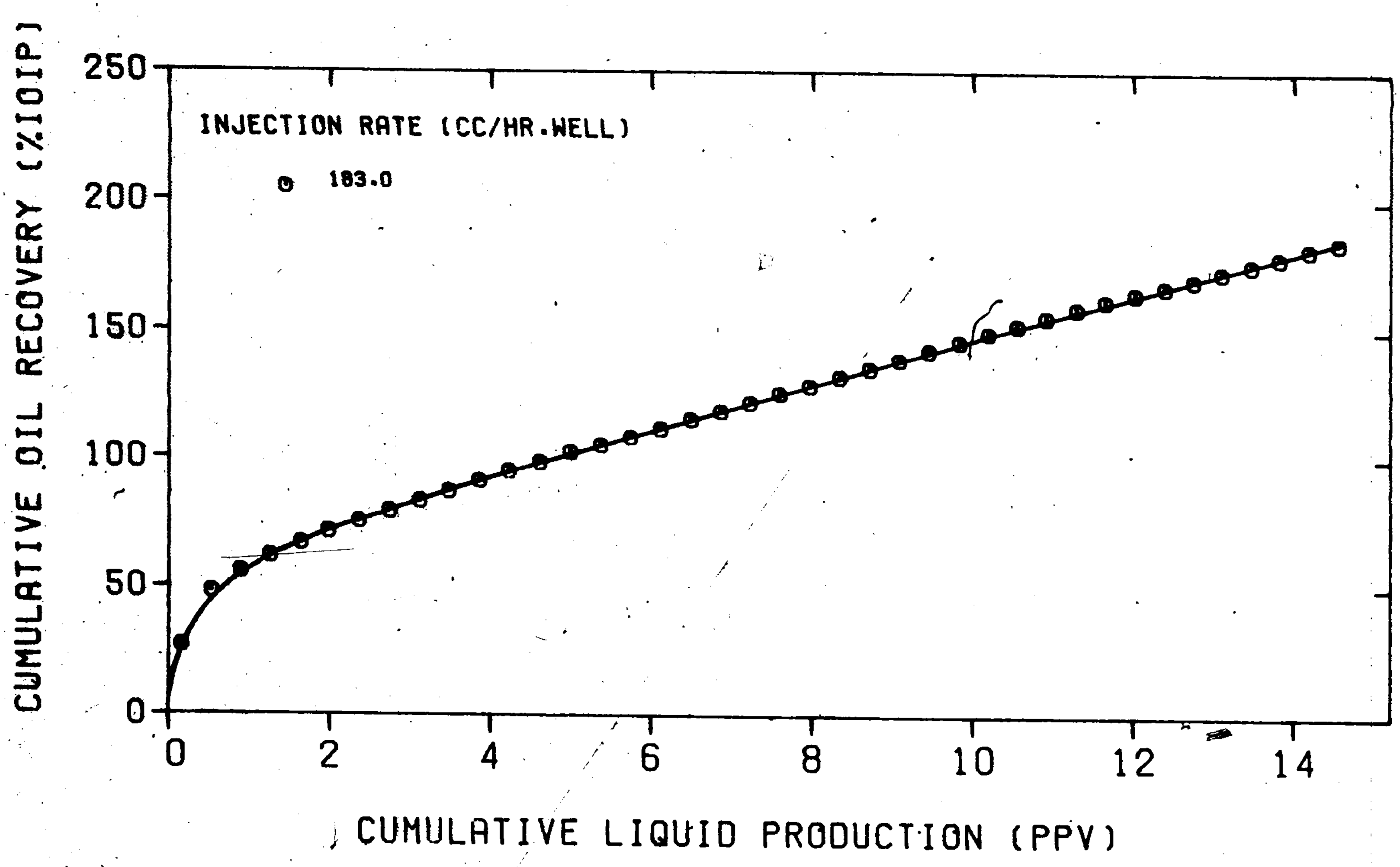


FIGURE 3 : INCREASE IN OIL RECOVERY WITH CONTINUOUS WATER INJECTION. ISOLATED 7-SPOT PATTERN. M=2.70.

only very slightly.

The same experimental conditions were maintained for Runs 13 and 14 in order to check the reproducibility of the experiments. The recovery profiles for these two runs are plotted on Figure 4: It may be observed that the oil recoveries do not differ by more than a few percent.

Four attempts were made to study the confinement of the central pattern in a 13(four-spot) pattern arrangement (cf Figure 2). No useful results were obtained, because the production of the surrounding patterns was not uniform and the production rates of the 12 producers around the pattern of interest had to be continuously adjusted. Figures 5 and 6 are comparisons of the recovery profiles from a four(four-spot) pattern and a thirteen(four-spot) pattern, at a mobility ratio of 0.83 and injection rates of 102 and 537 cc/hr.well. Only four runs were conducted using the 13(four-spot) pattern arrangement. In each case an irregular pattern development was observed, due to partially plugged wells. For cumulative productions lower than four pattern-pore-volumes, the oil recovery from the 13(four-spot) pattern is higher than the one from the four(four-spot) pattern. The opposite would be expected as an increase in the number of patterns involved in the waterflood should lead to a better confinement of the central pattern and thus to smaller oil recoveries. However, for higher cumulative productions, the 13(four-spot) seems to be more confined than the four(four-spot) pattern. After

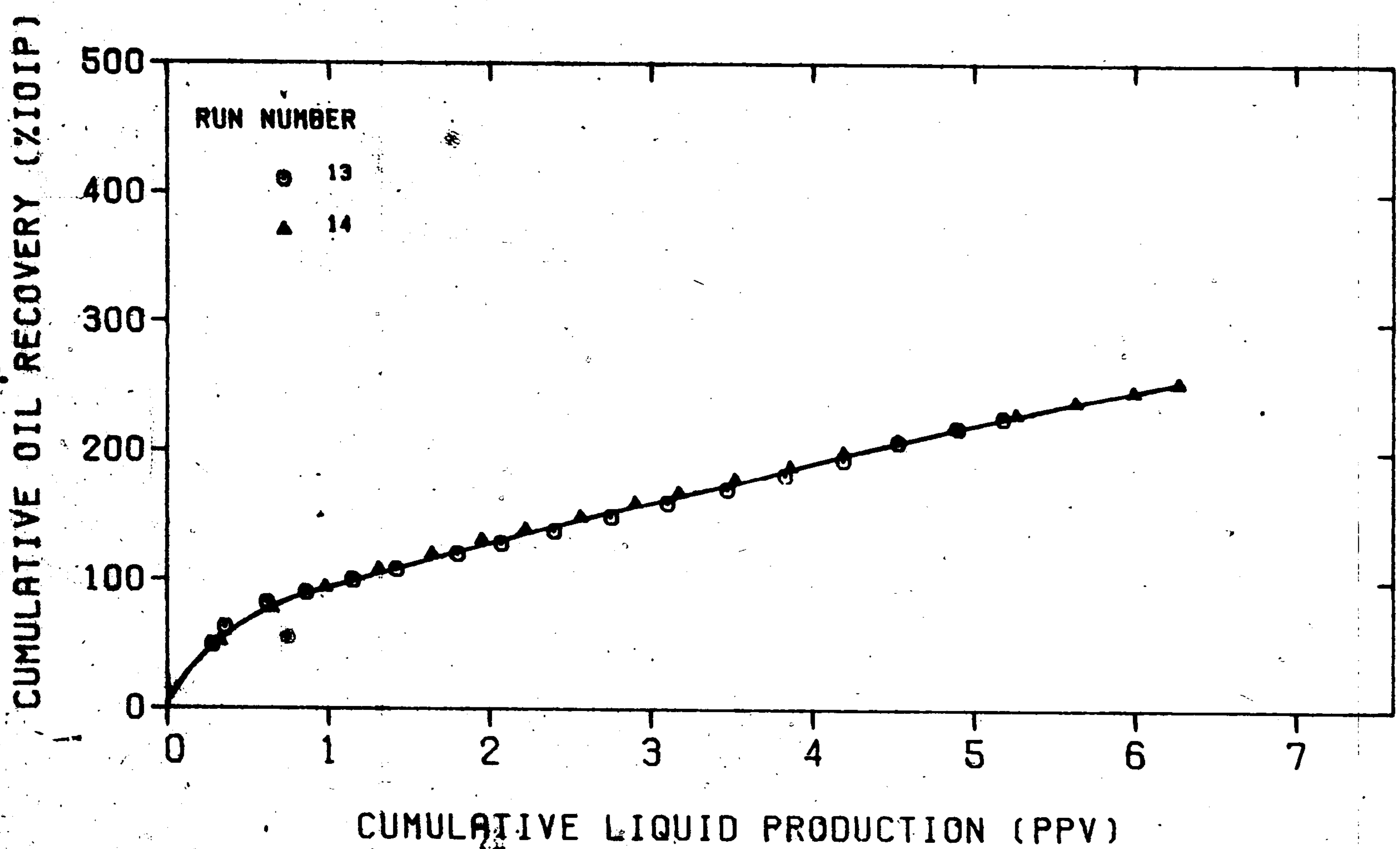


FIGURE 4: REPRODUCIBILITY OF EXPERIMENTS.
ISOLATED 7-SPOT PATTERN. Q=183 CC/HR. WELL. M=0.83.

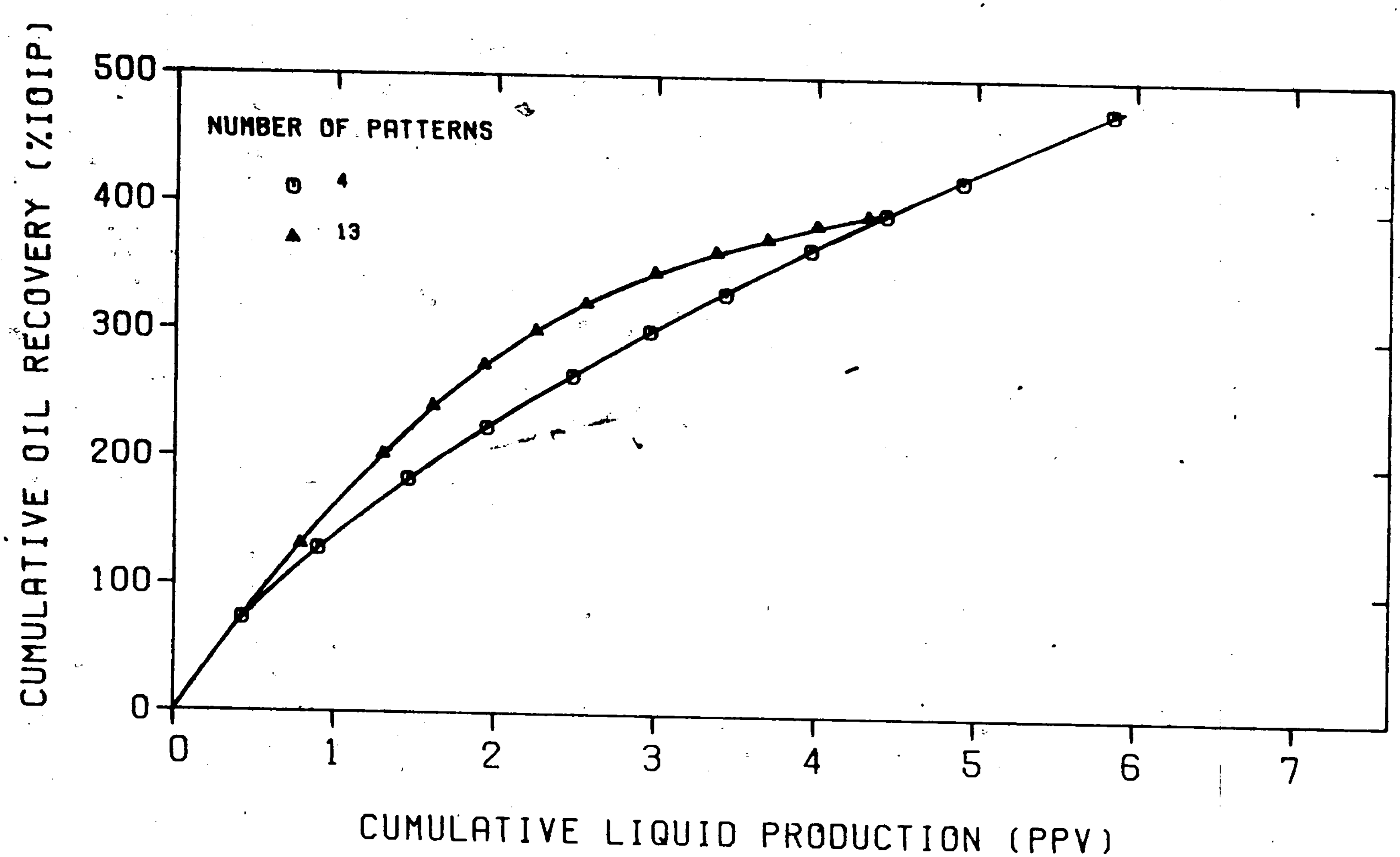


FIGURE 5: EFFECT OF PATTERN ARRANGEMENT ON CONFINEMENT.
4-SPOT PATTERN. M=0.83. Q=102 CC/HR.WELL.

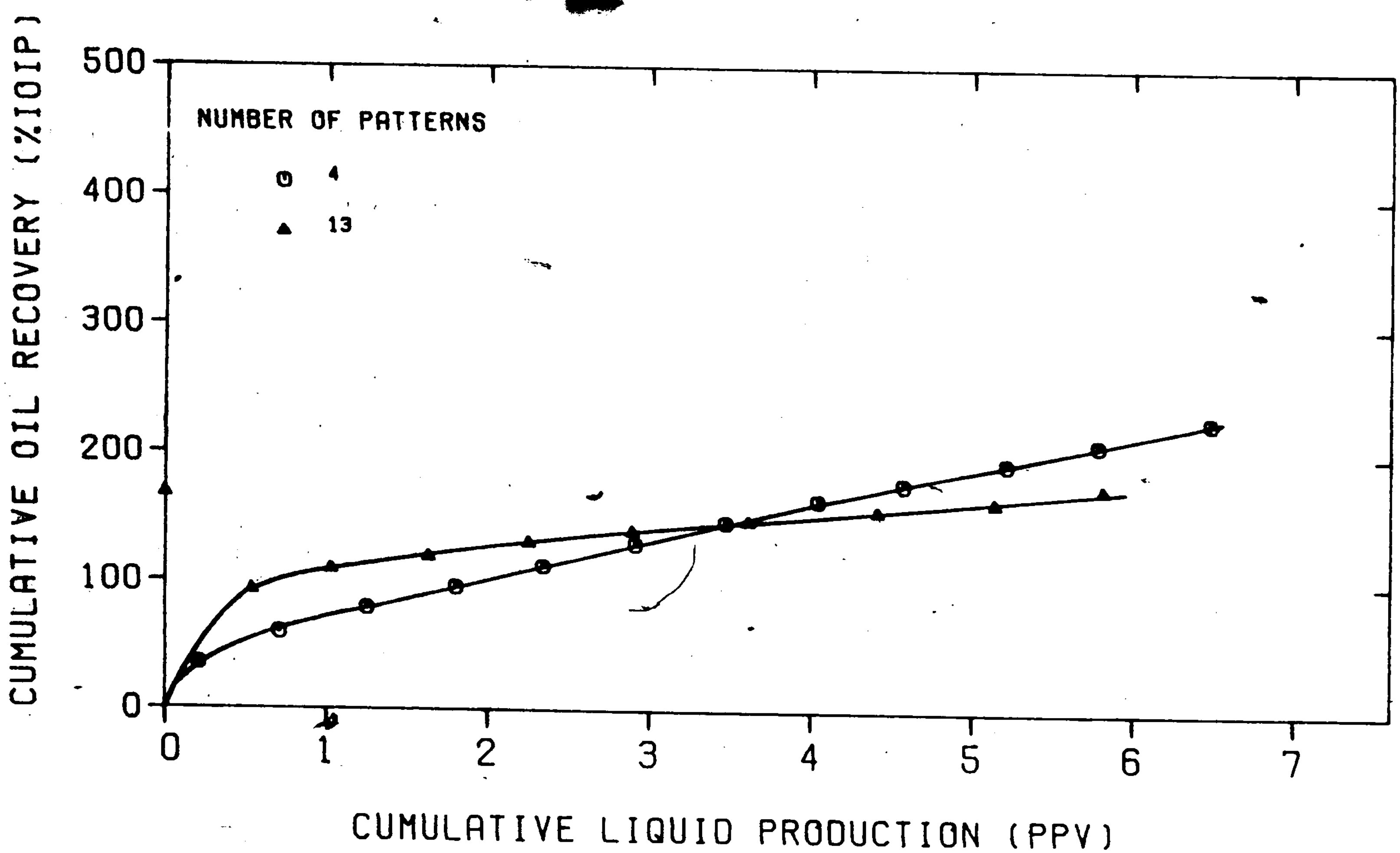


FIGURE 6:EFFECT OF PATTERN ARRANGEMENT ON CONFINEMENT.
4-SPOT PATTERN.M=0.83.Q=537 CC/HR.WELL.

more than four pattern-pore-volumes have been produced the slope of the oil recovery curve is much smaller. These results are not conclusive and it was then decided to exclude from the discussion the results of these four runs. As a result, in this work now, the term "modified" four-spot pattern will refer to a four-spot pattern surrounded by three like patterns.

Previous experimentors noted a time interval between the water arrival at the pilot producer and water production. This fact was attributed to the capillary end effect. Nothing similar was noticed during these experiments. However, as this phenomenon lasts a very short time, it may have been overlooked.

6.1 Oil Recovery from Isolated Patterns. (Mobility Ratio:0.83).

Figure 7 and 8 present the oil recovery profiles for isolated four-spot and seven-spot patterns respectively, at various injection rates. A significant pattern oversweep is observed, particularly at low injection rates. At the lowest rate, the recovery from a four-spot pattern exceeds five times the initial oil in place after three pore volumes have been injected. The oversweep of the four-spot pattern is much more extensive than the oversweep of the seven-spot, as may be observed from Figure 9.

For both patterns, the oil recovery decreases with increasing injection rate. At the highest injection rate the

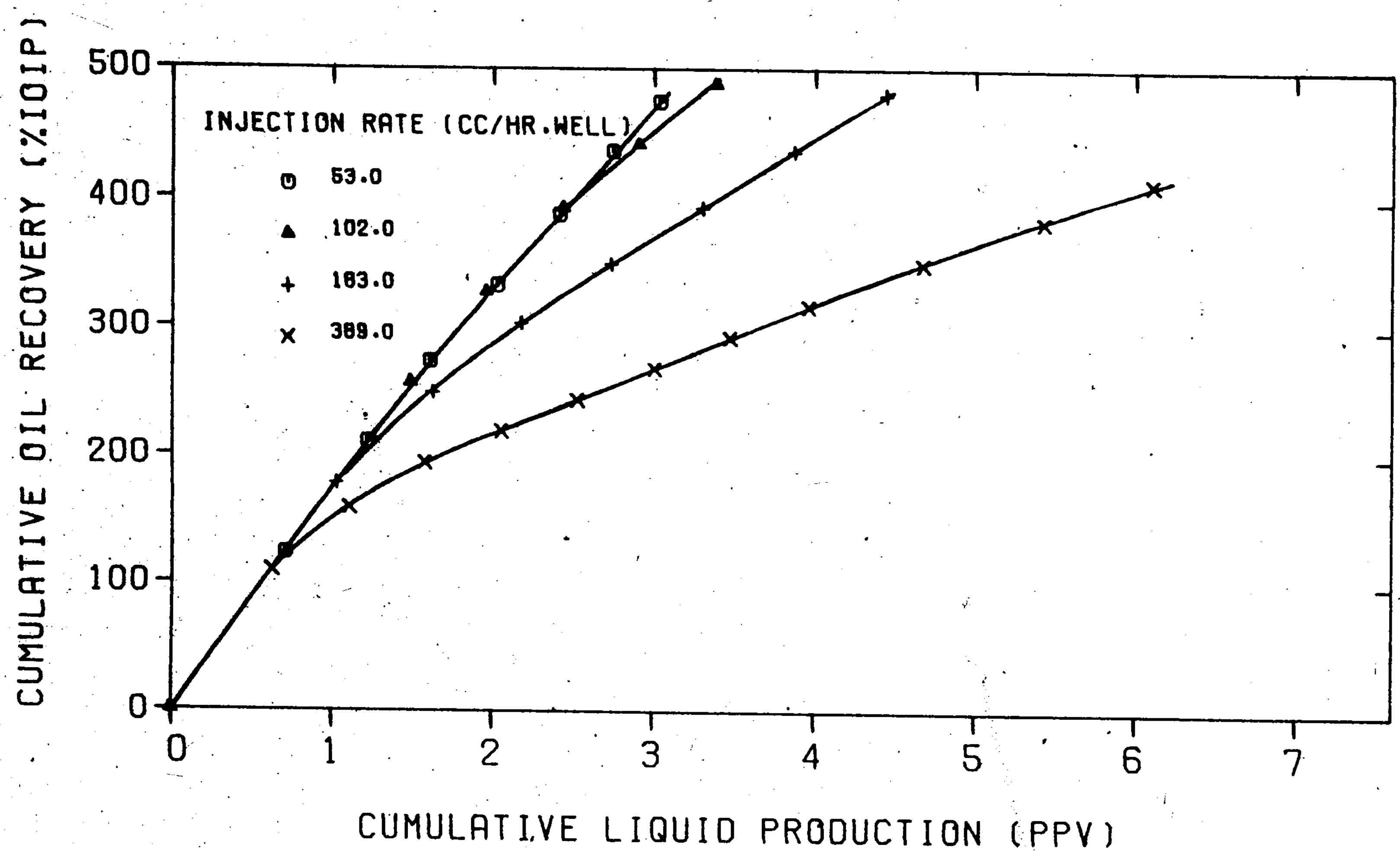


FIGURE 7: EFFECT OF INJECTION RATE ON OIL RECOVERY ISOLATED 4-SPOT PATTERN. M=0.83.

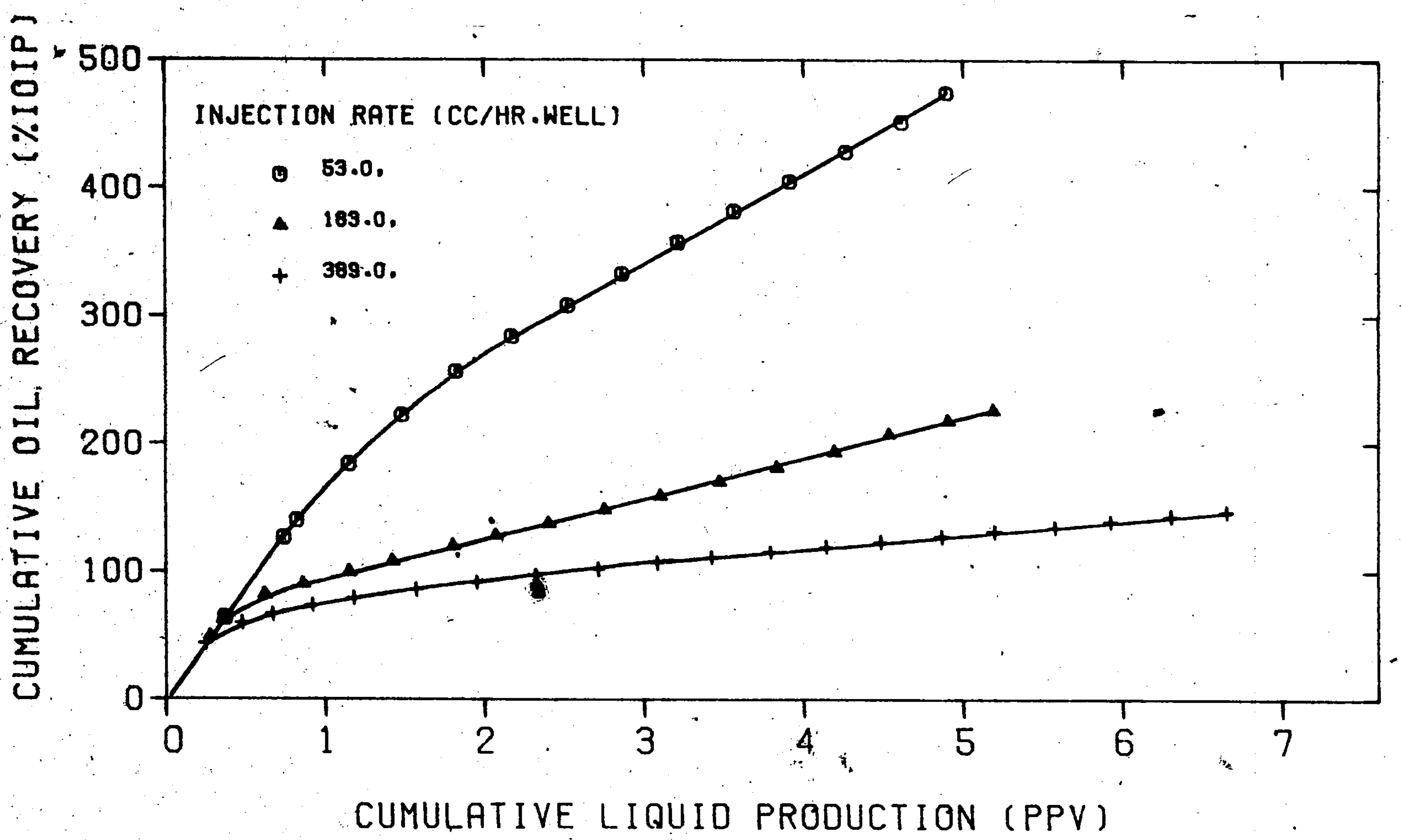


FIGURE 8: EFFECT OF INJECTION RATE ON OIL RECOVERY.
ISOLATED 7-SPOT PATTERN. M=0.83.

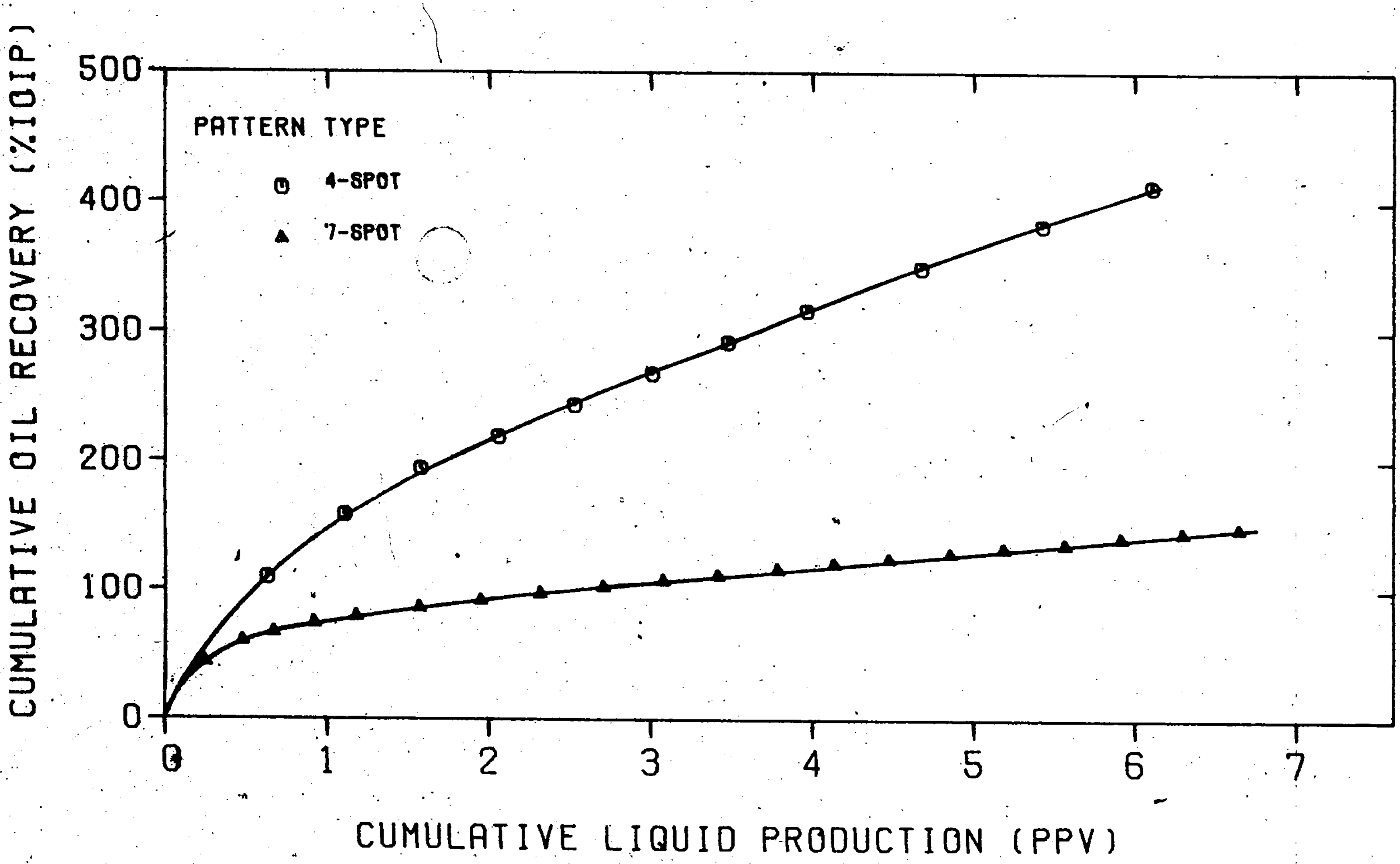


FIGURE 9:COMPARISON OF OIL RECOVERIES FROM 4 AND 7-SPOT PATTERNS. M=0.83. Q=389 CC/HR.WELL.

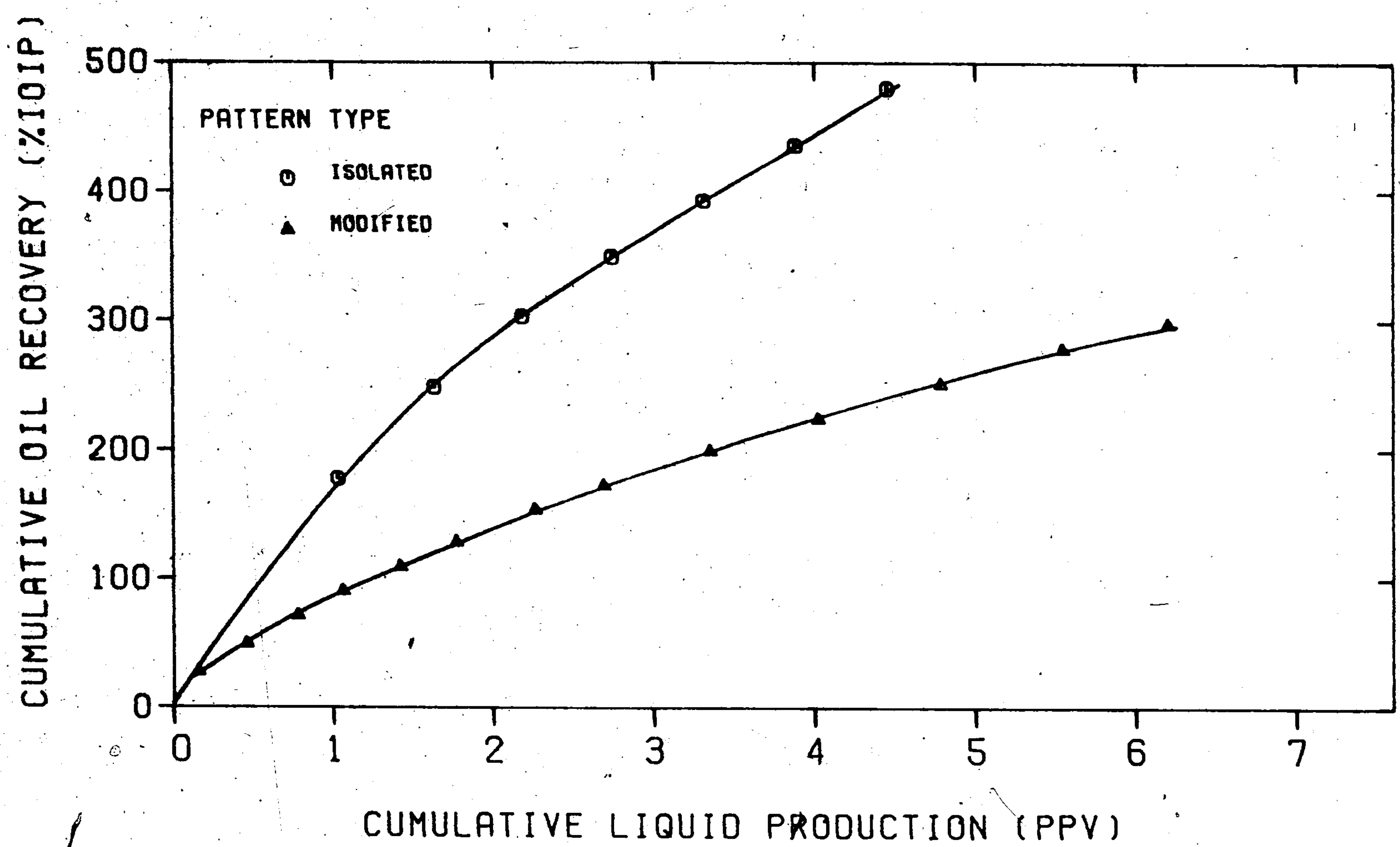


FIGURE 12:COMPARISON OF OIL RECOVERIES FROM THE ISOLATED AND MODIFIED 4-SPOT PATTERNS.M=0.83.Q=183 CC/HR.WELL.

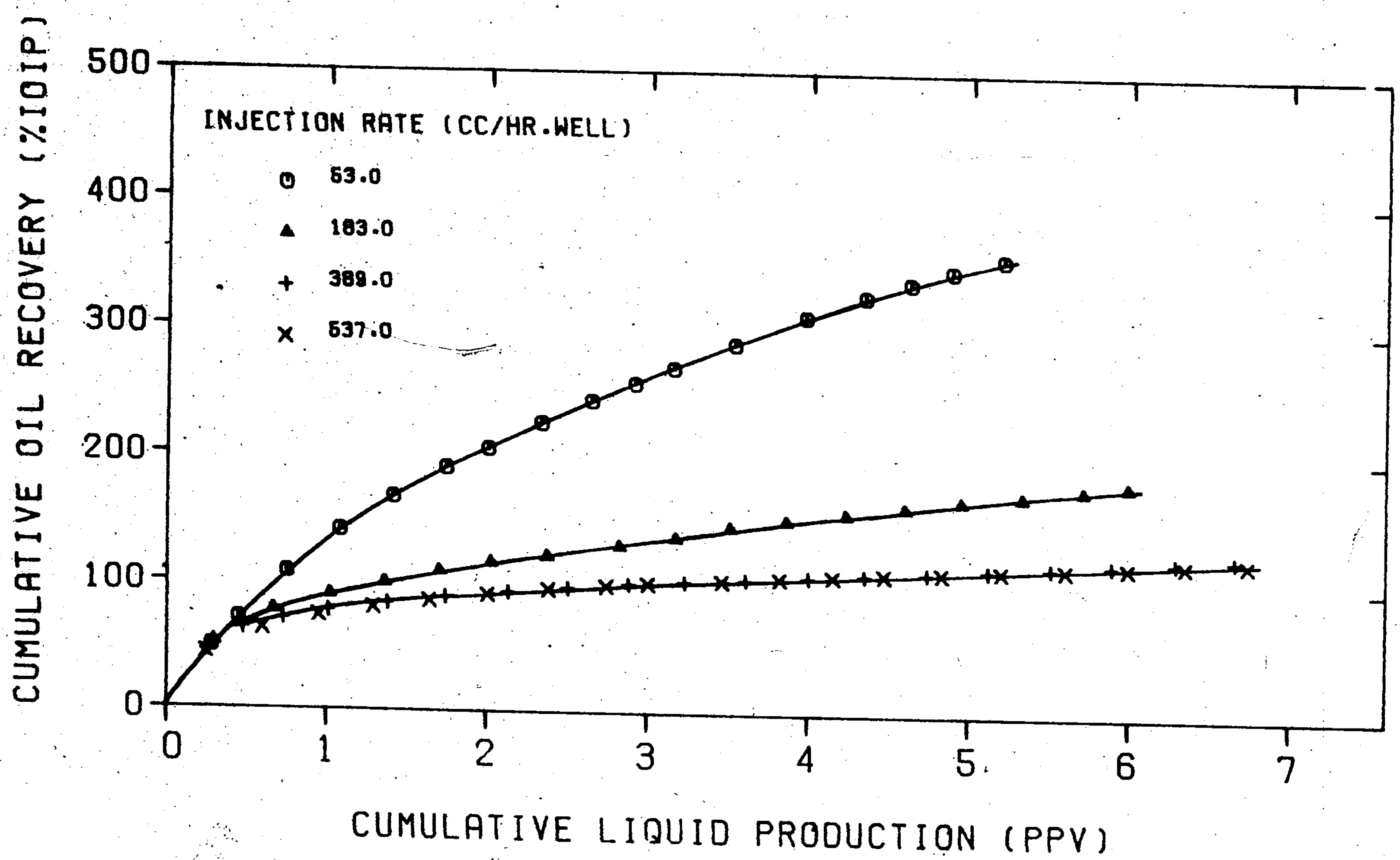


FIGURE 11:EFFECT OF INJECTION RATE ON OIL RECOVERY. MODIFIED 7-SPOT PATTERN. M=0.83.

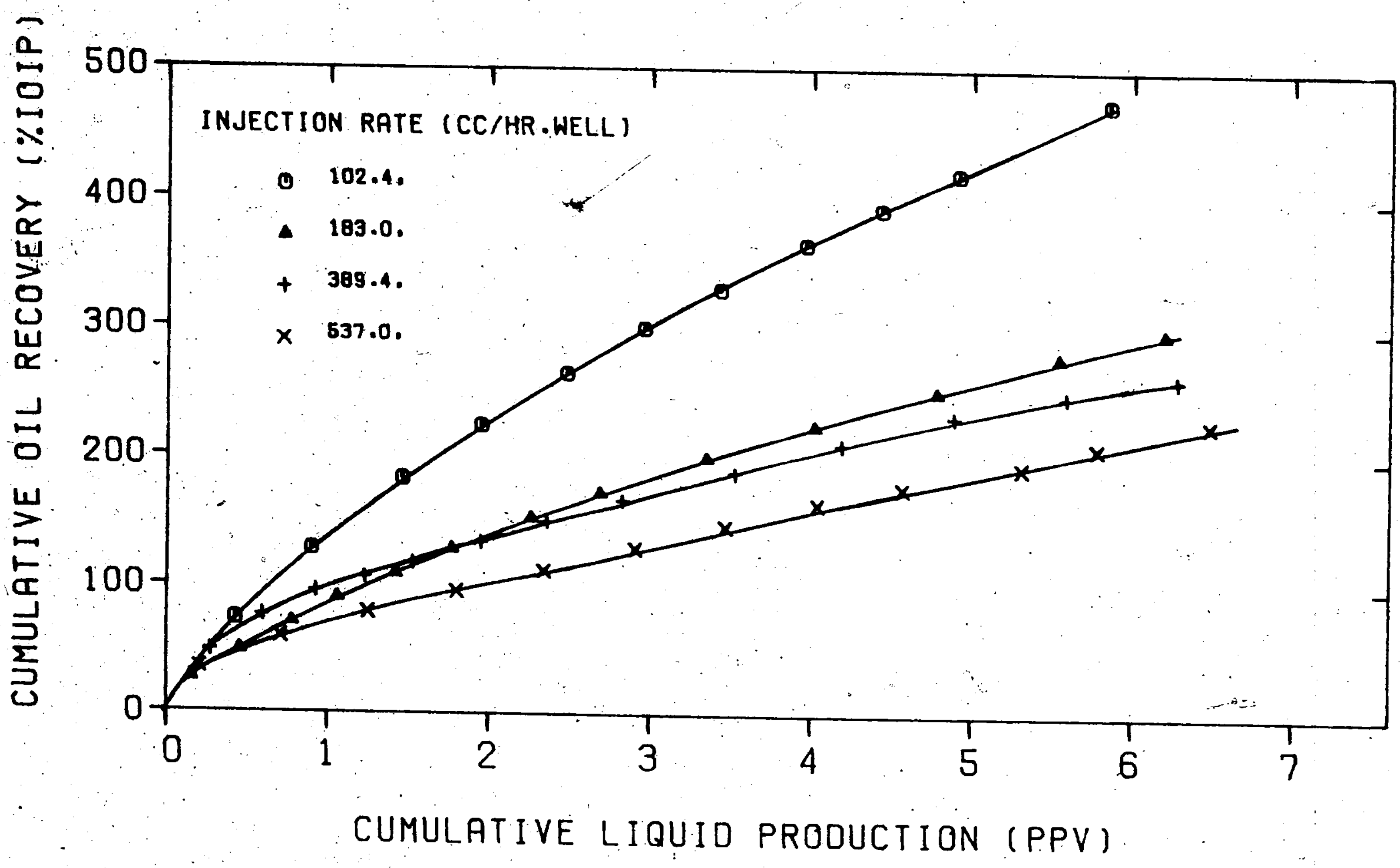


FIGURE 10:EFFECT OF INJECTION RATE ON OIL RECOVERY.
FOUR 4-SPOT PATTERN. M=0.83.

oversweep of the seven-spot pattern is small (120 % of recovery after four pore volumes have been injected).

6.2 Oil Recovery from Modified Patterns.(Mobility Ratio:0.83).

Figures 10 and 11 present the recovery profiles for the modified patterns. As already noted in the case of the isolated patterns, the oil recovery decreases as the injection rate increases. For the modified four-spot, the oversweep is still significant, even at the highest injection rate (Figure 10). For the modified seven-spot, the recovery curves at the two highest injection rates are identical and the oversweep is very limited (Figure 11).

At any injection rate, the oil recovery from a modified pattern is lower than the recovery from the corresponding isolated pattern. The difference is much larger for the four-spot than for the seven-spot (cf Figures 12 and 13). Furthermore, the difference tends to vanish with an increasing injection rate as can be observed when comparing Figures 13 and 14.

6.3 Recovery Profiles at Higher Mobility Ratios.

Three mobility ratios were studied, one favorable (less than one) and two unfavorable(greater than one).

Eight runs were made at a mobility ratio of 1.36: two runs at the injection rates of 183 and 537 cc/hr.well for

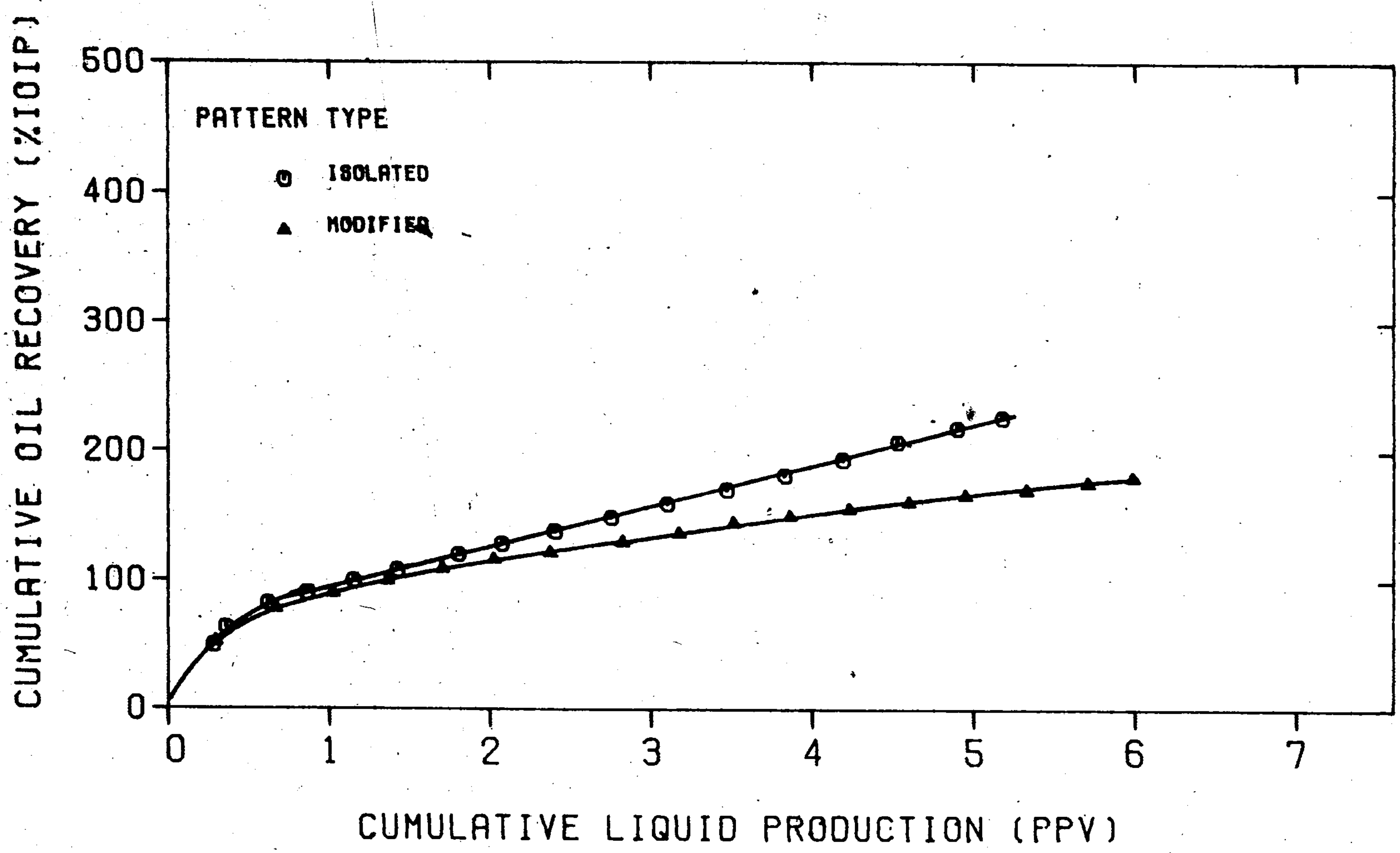


FIGURE 13: COMPARISON OF OIL RECOVERIES FROM THE ISOLATED AND MODIFIED 7-SPOT PATTERNS. M=0.83. Q=183 CC/HR.WELL.

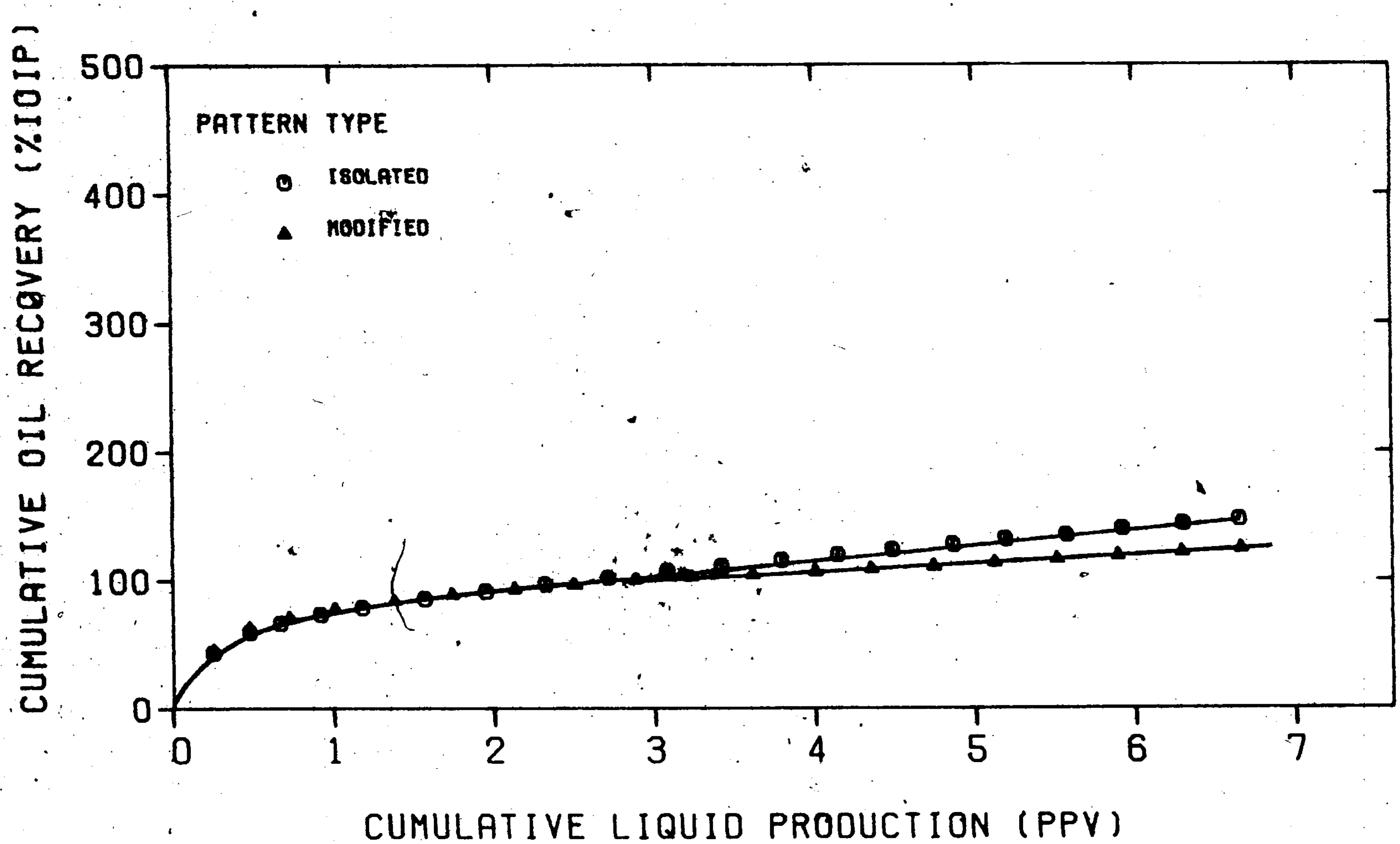


FIGURE 14: COMPARISON OF OIL RECOVERIES FROM THE ISOLATED AND MODIFIED 7-SPOT PATTERNS. M=0.83. Q=389 CC/HR.WELL.

each of the four patterns arrangements considered. Figures 15 and 16 present the recovery profiles for the isolated and modified four-spot patterns. As observed previously, the oversweep is much more significant for the isolated pattern than for the modified pattern, the oil recovery decreases as the injection rate increases and, the effect of the injection rate is less for the modified pattern.

Similar observations are valid for the seven-spot pattern (Figures 17 and 18). At the higher injection rate (537 cc/hr.well) the oil recoveries from the isolated and modified patterns are identical (Figure 19).

Seven runs were made at a mobility ratio of 2.70. The results reinforce what has been previously stated. Figure 20 is a plot of recovery curves from an isolated seven-spot pattern. The oil recovery increases only slightly with the injection rate. At the highest rate (389 cc/hr.well), the oil recovery is still less than 100 percent after five pore volumes have been injected but the cumulative recovery is still increasing with continuing water injection. It may be observed from Figure 21 that, at the highest injection rate, the recovery profiles from the isolated and modified seven-spot patterns are almost identical.

The effect of mobility ratio on oil recovery is shown on Figures 22 and 23. For the isolated seven-spot (Figure 22), the decrease in recovery with increasing mobility ratio is significant. The effect is much less for the modified seven-spot (Figure 23).

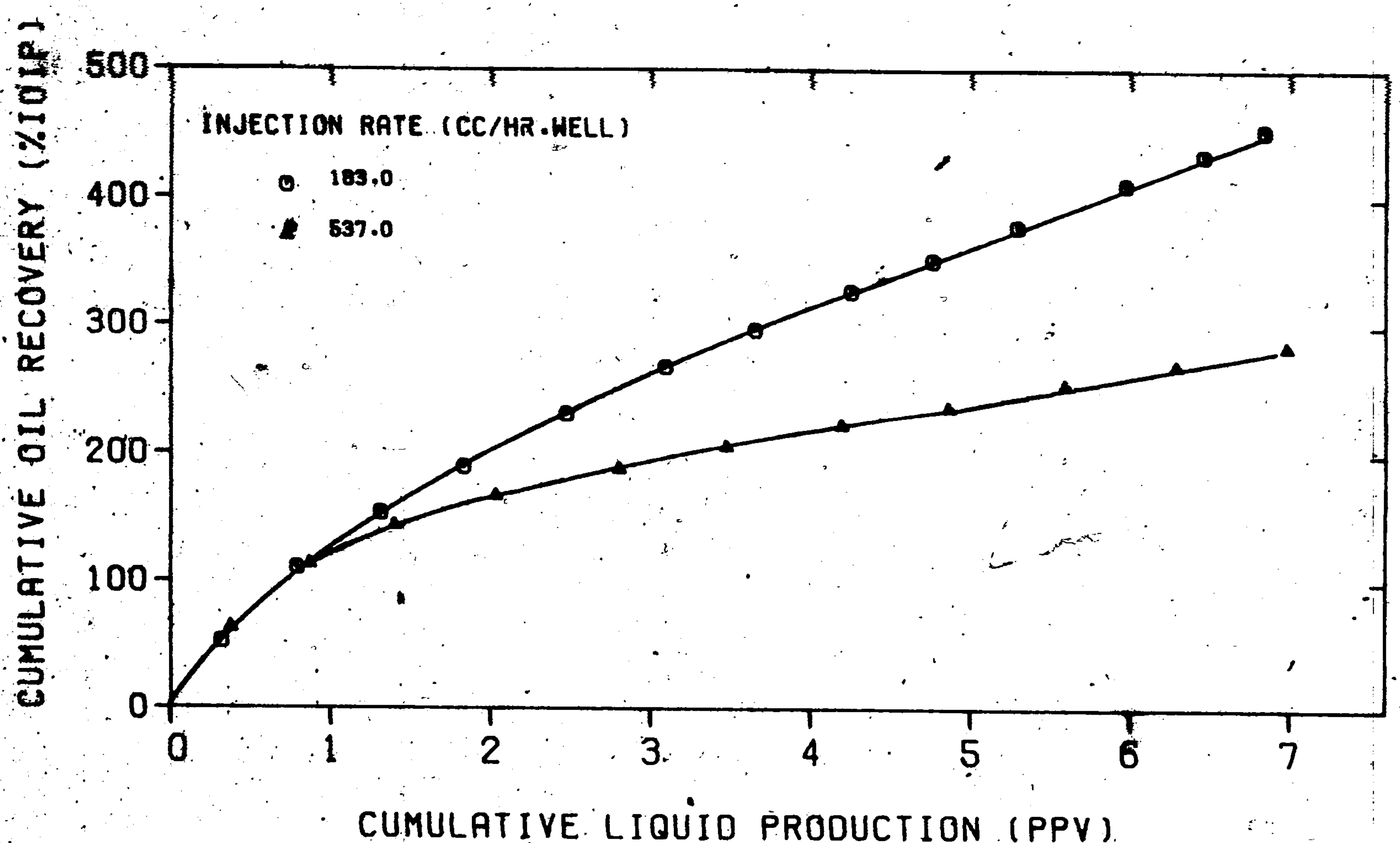


FIGURE 15: EFFECT OF INJECTION RATE ON OIL RECOVERY.
ISOLATED 4-SPOT PATTERN. M=1.36.

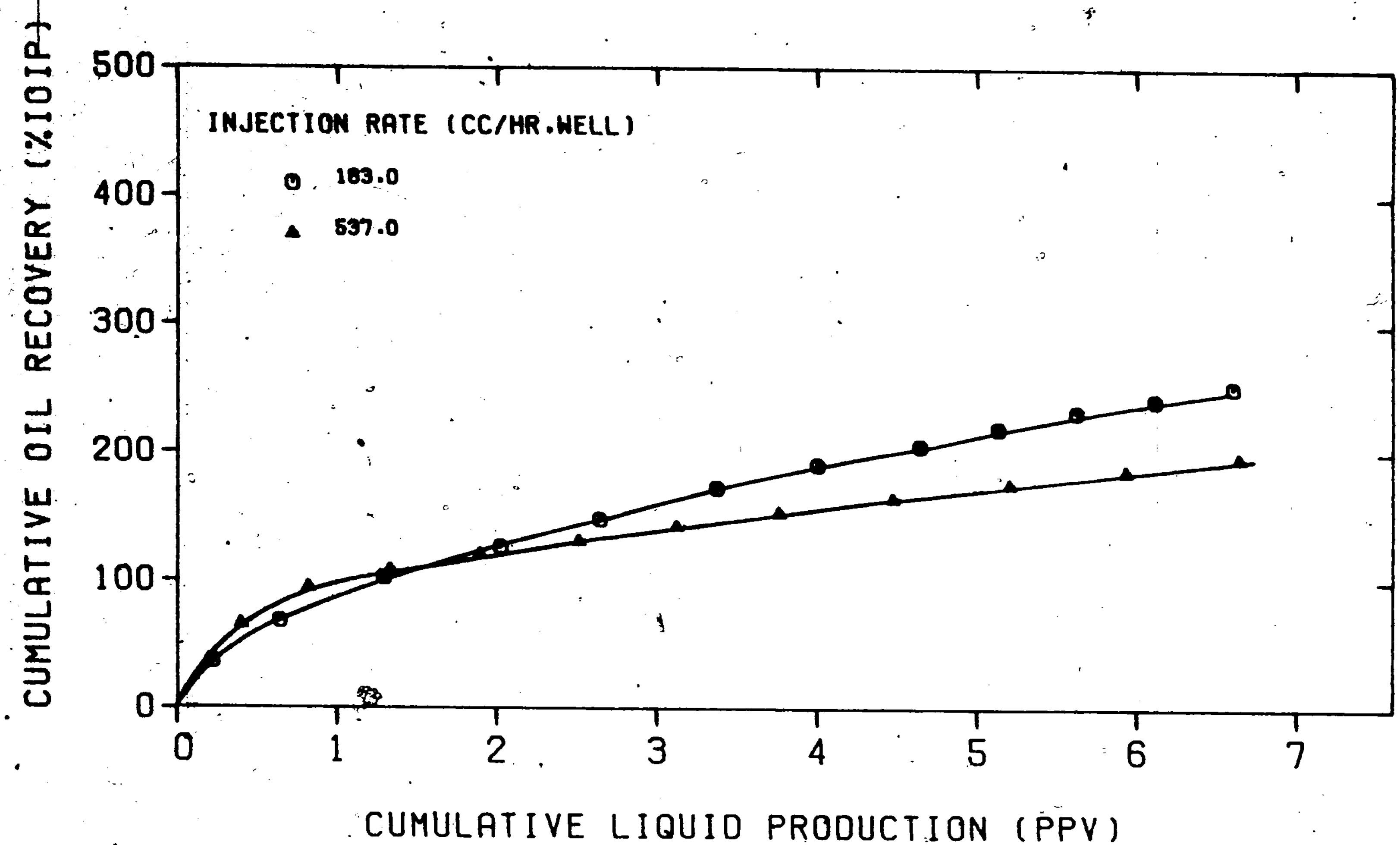


FIGURE 16: EFFECT OF INJECTION RATE ON OIL RECOVERY.
MODIFIED 4-SPOT PATTERN. M=1.36.

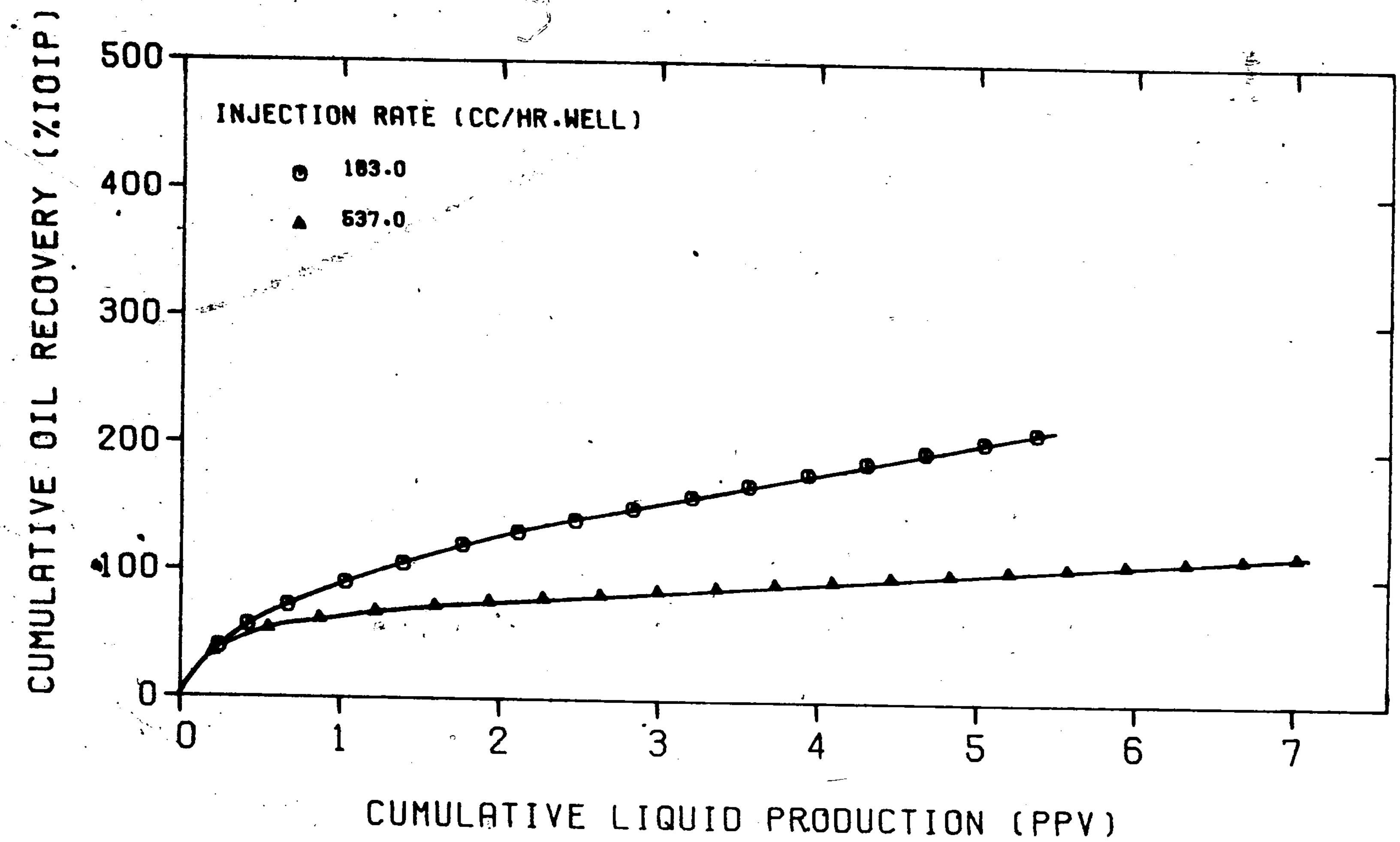


FIGURE 17: EFFECT OF INJECTION RATE ON OIL RECOVERY. ISOLATED 7-SPOT. M=1.36.

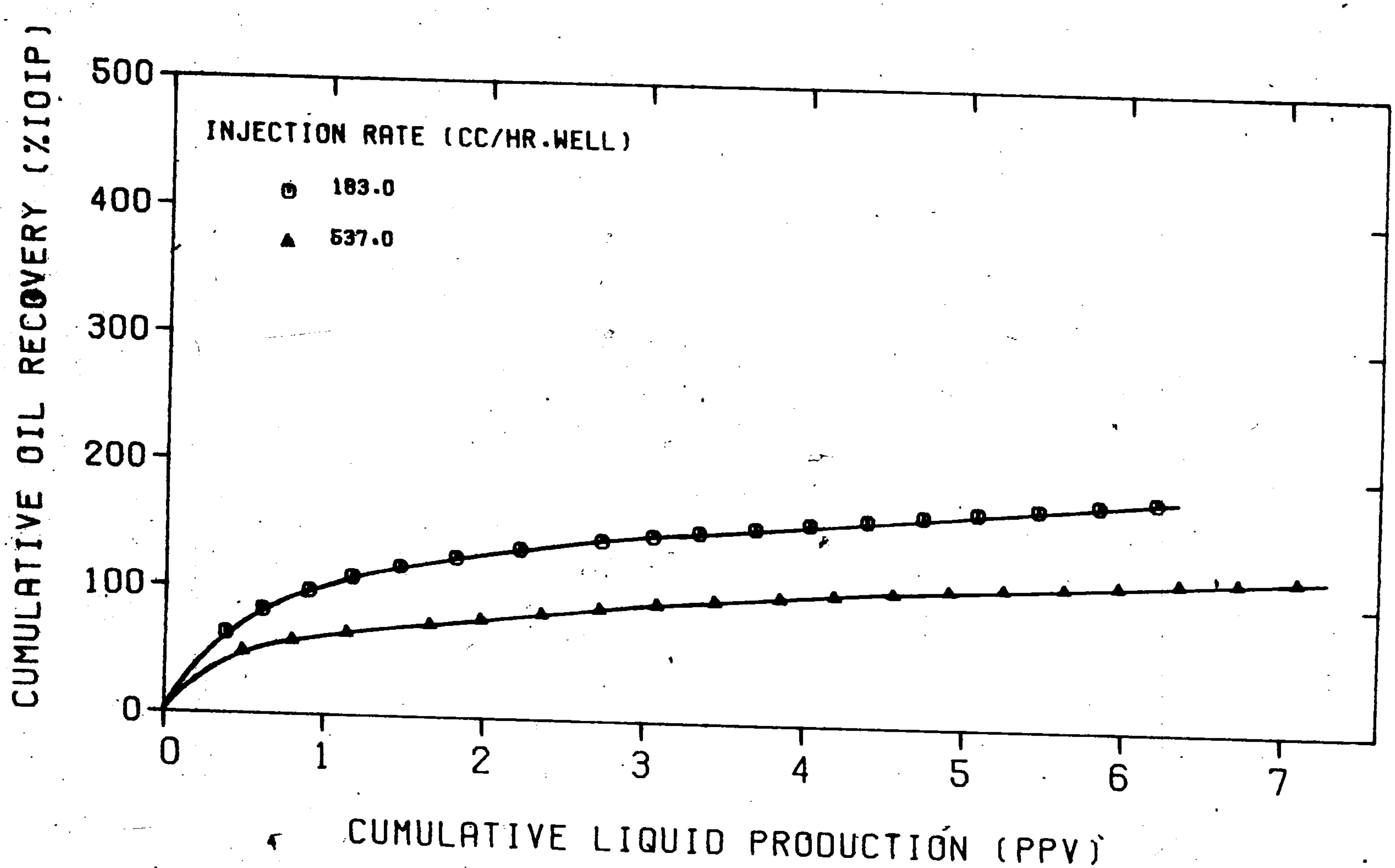


FIGURE 18:EFFECT OF INJECTION RATE ON OIL RECOVERY.
MODIFIED 7-SPOT.M=1.36.

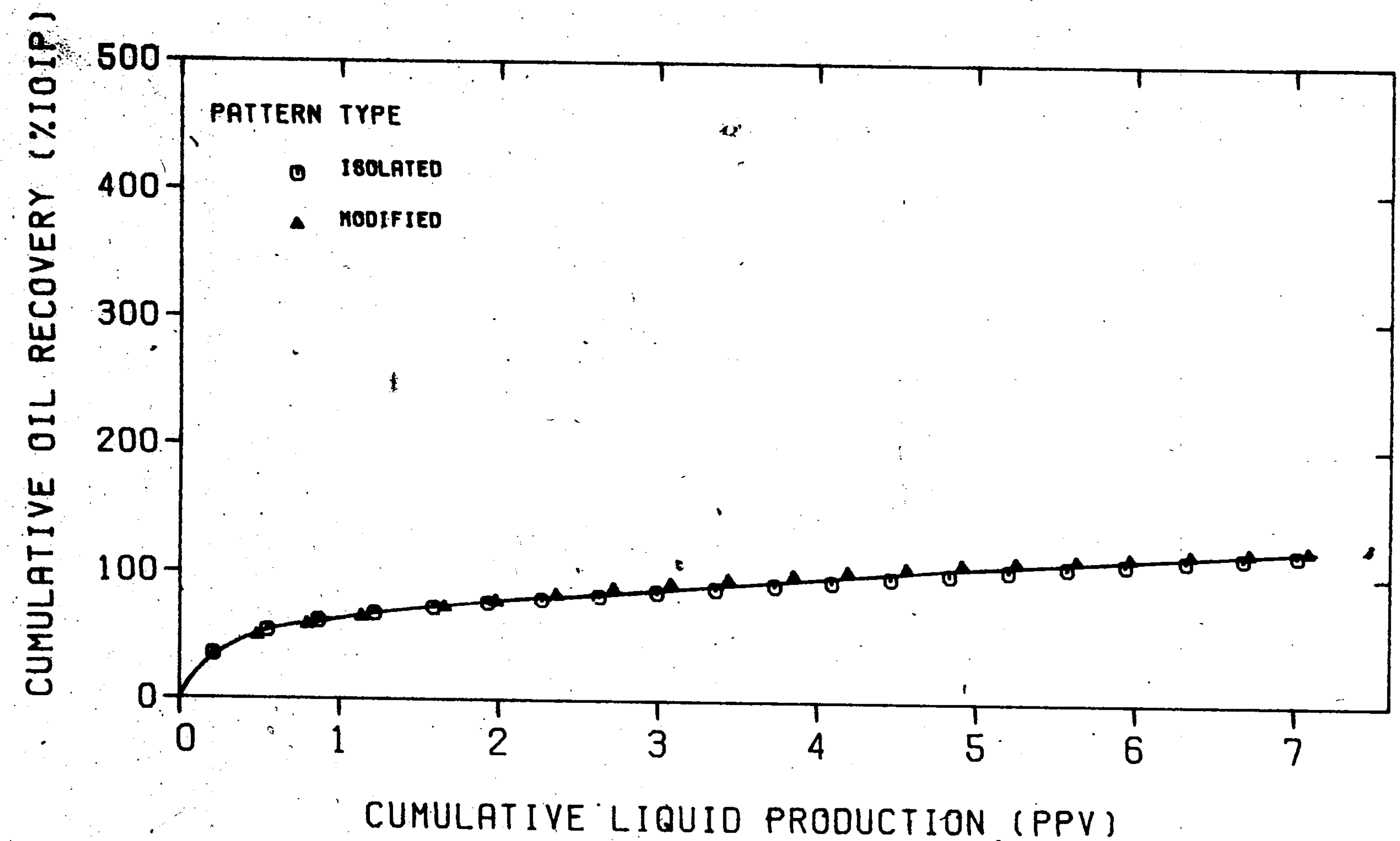


FIGURE 19:COMPARISON OF OIL RECOVERIES FROM ISOLATED AND MODIFIED 7-SPOT PATTERNS.M=1.36.Q=537 CC/HR.WELL.

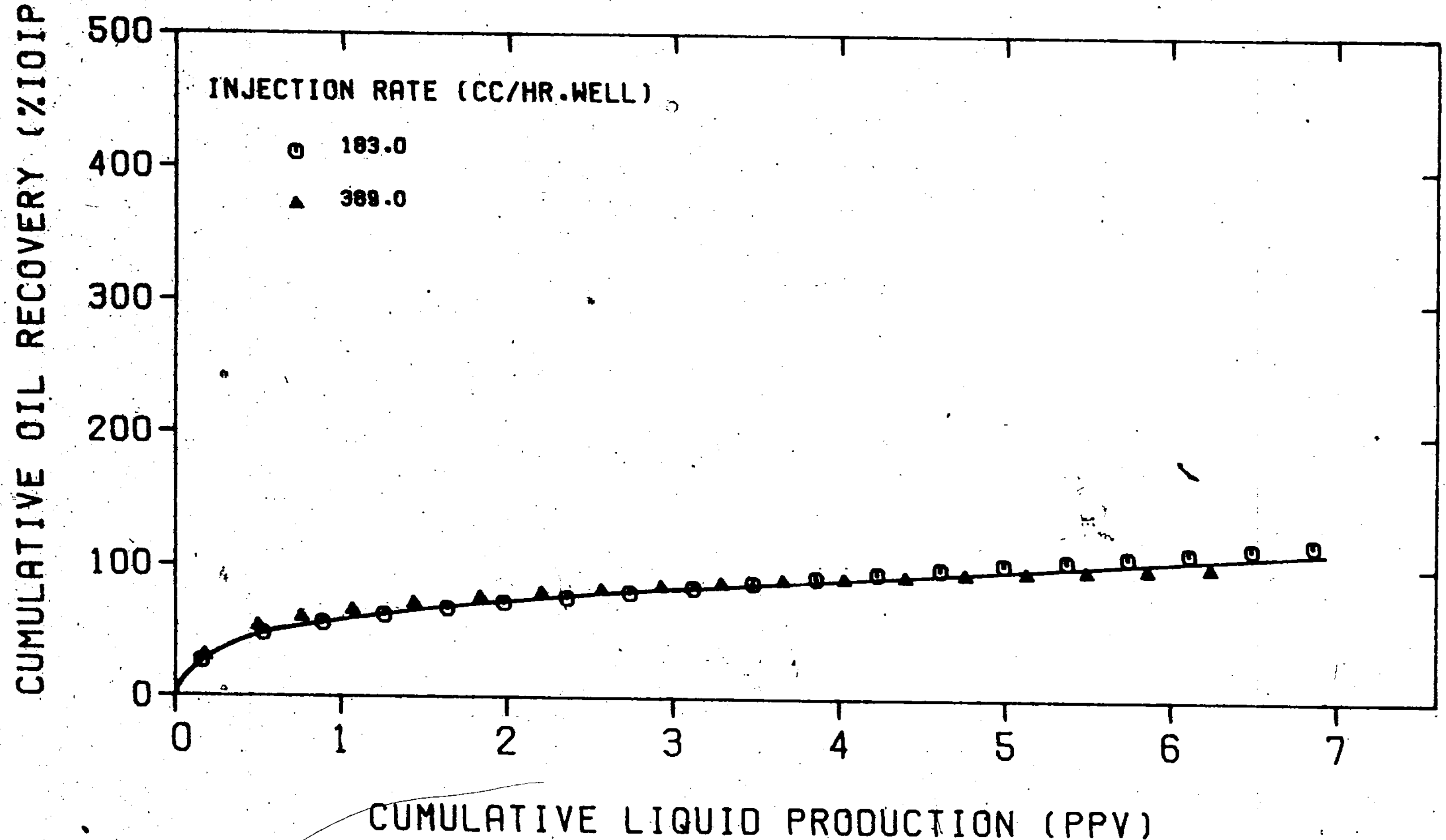


FIGURE 20:EFFECT OF INJECTION RATE ON OIL RECOVERY.
ISOLATED 7-SPOT PATTER'.M=2.70.

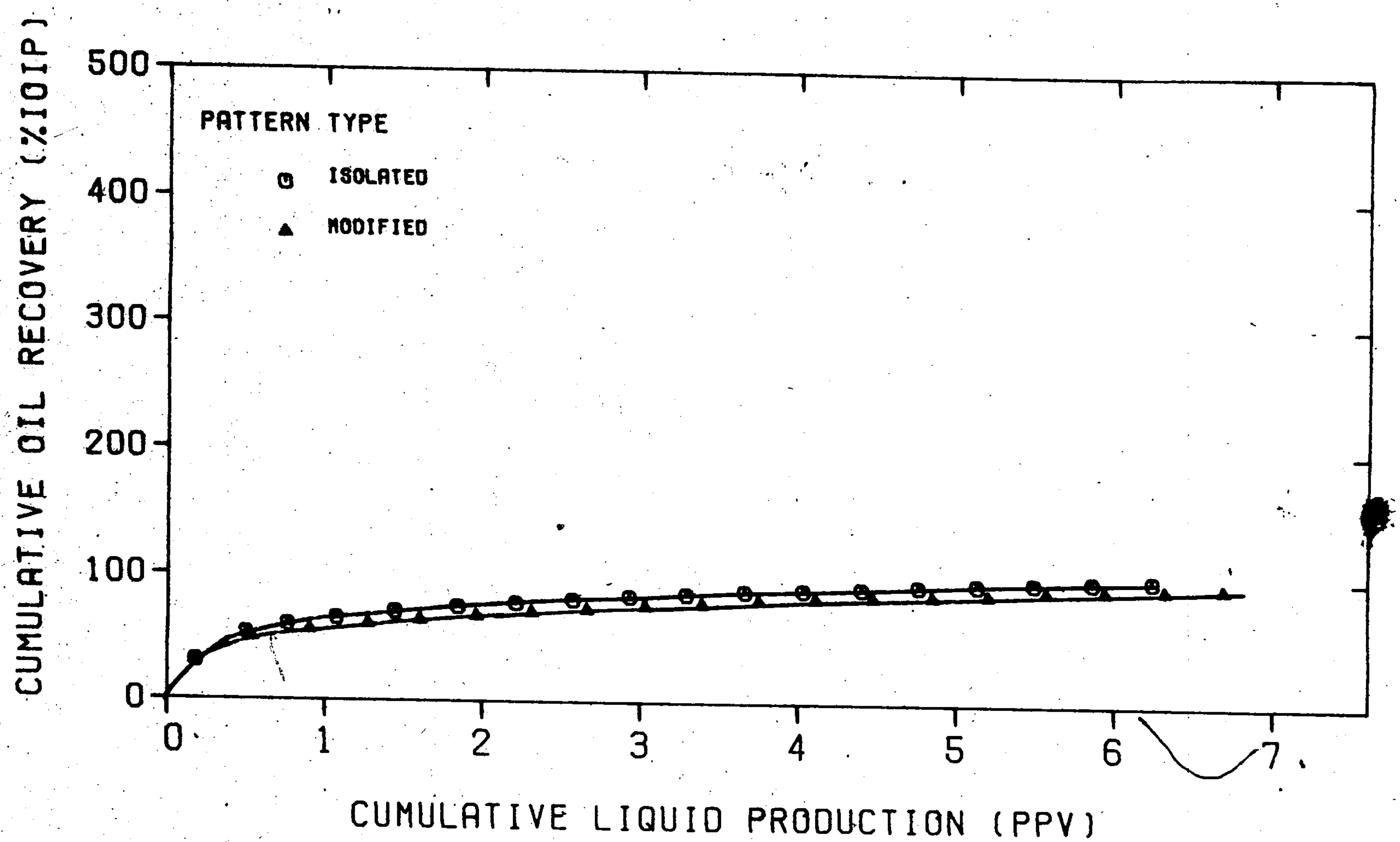


FIGURE 21:COMPARISON OF OIL RECOVERIES FROM ISOLATED AND MODIFIED 7-SPOT PATTERNS.M=2.70.Q=389 CC/HR.WELL.

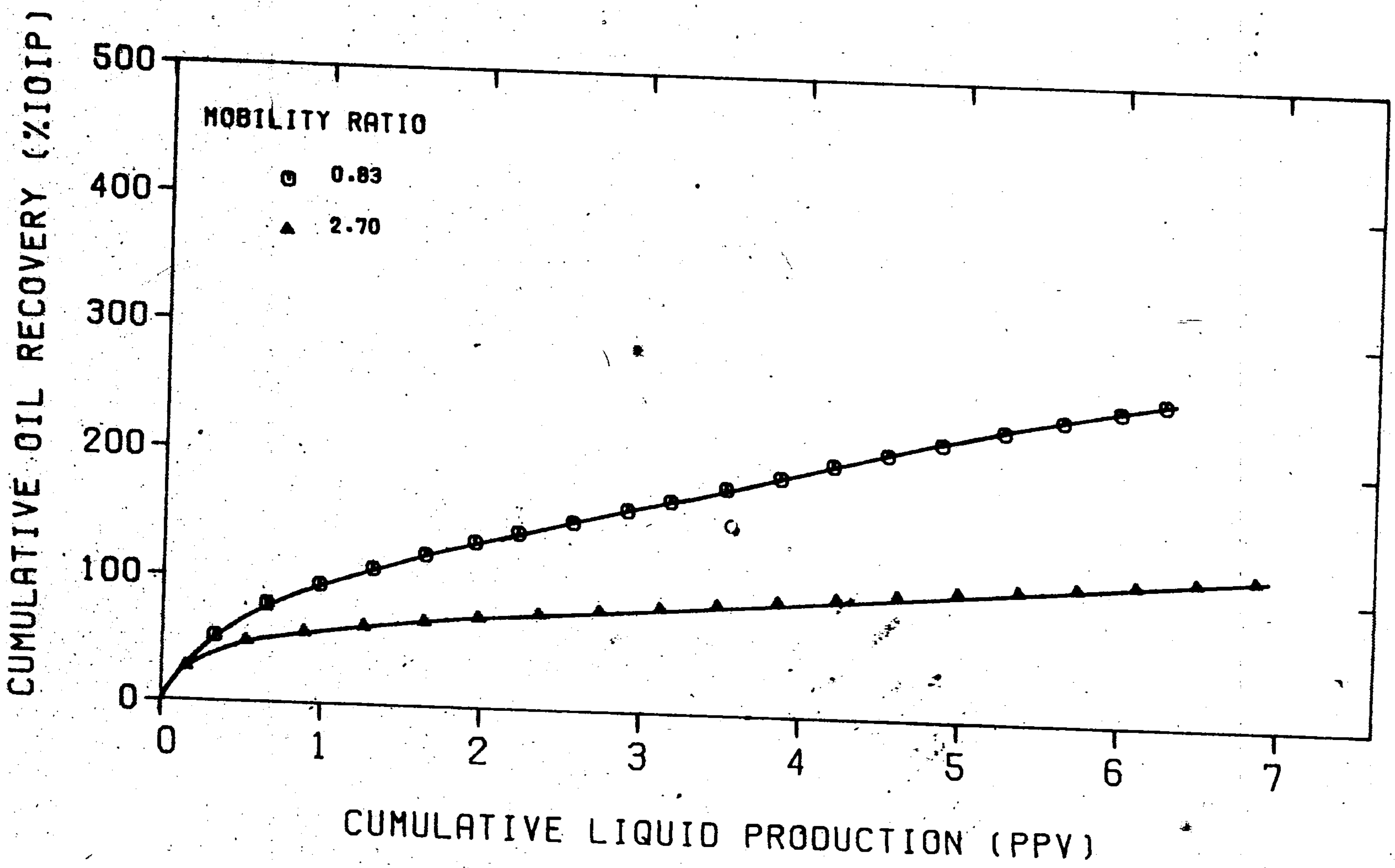


FIGURE 22:EFFECT OF MOBILITY RATIO ON OIL RECOVERY.
ISOLATED 7-SPOT PATTERN.Q=183 CC/HR.WELL.

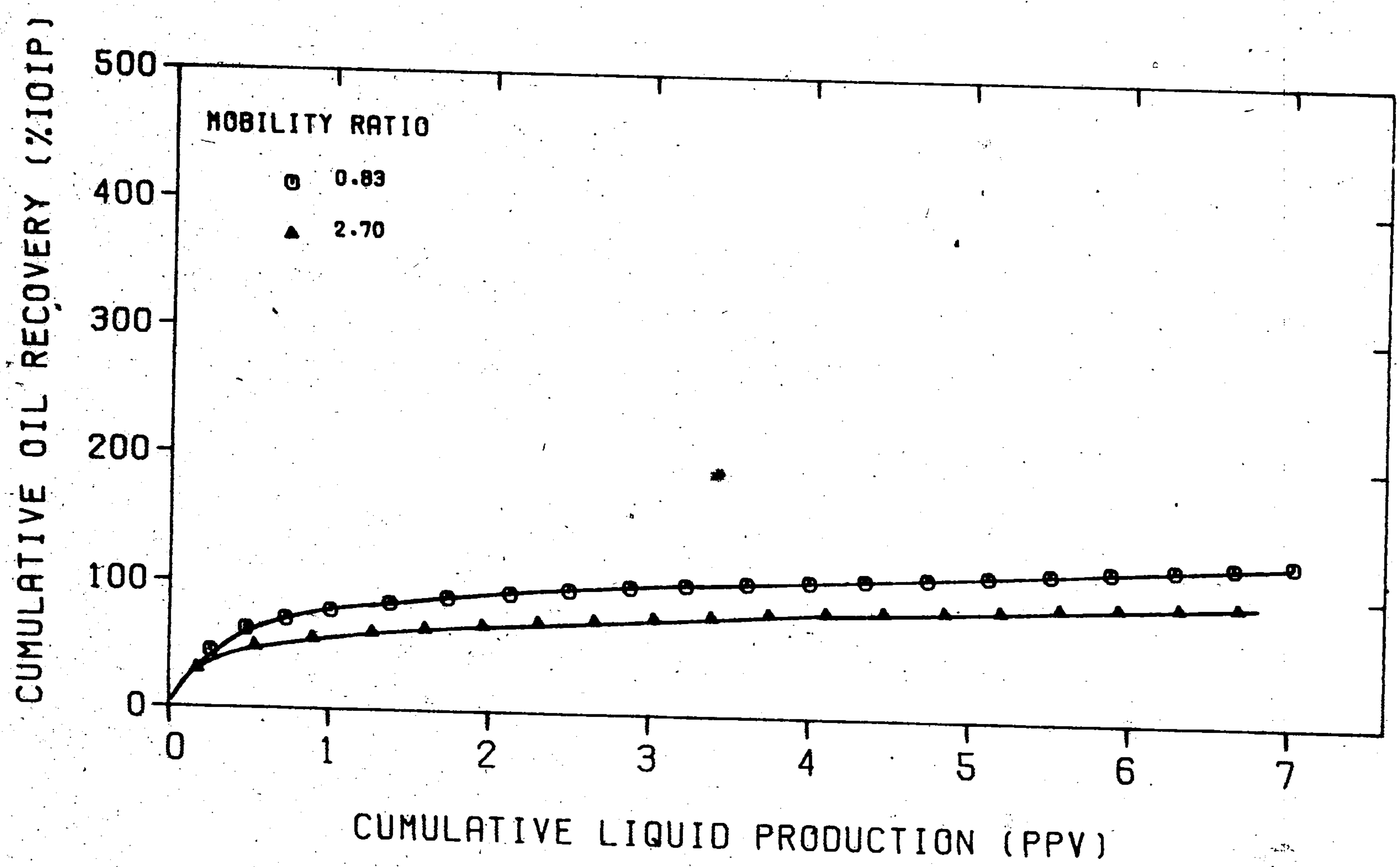


FIGURE 23:EFFECT OF MOBILITY RATIO ON OIL RECOVERY. MODIFIED SEVEN-SPOT PATTERN.Q=183 CC/HR.WELL.

7. MATHEMATICAL MODEL

The displacement process taking place during a waterflood may be described by a set of differential equations. These equations must account for gravity, capillary, and viscous forces. The differential equations are obtained by combining Darcy's equation with the law of conservation of matter for each phase.

These equations form a mathematical model. In a second step, the reservoir is divided into discrete blocks. Pressures and saturations are defined only at the centers of these blocks. The differential equations are replaced by a set of finite difference equations. This set of equations may be solved either by direct or iterative methods to obtain the pressure and saturation changes at the centers of the blocks during successive time intervals.

In this work, a two-dimensional, water-oil simulator was used to compute oil recovery data from a seven-spot pattern waterflood.

The oil and water equations were derived using the IMPES procedure (Implicit Pressure, Explicit Saturation, (31)) In this method, two differential equations containing two dependent variables pw and Sw are obtained. Then, they are combined to eliminate time derivatives of saturations. The new equation is approximated by a difference equation where the water pressure at an advanced time level is the unknown. This difference equation is written for each block of the grid. A set of equations is thus obtained which can be solved numerically to obtain the pressure distribution at the new time. Then the new pressures are used in a difference approximation of the original water equation to yield the water saturation distribution. This is an iterative process which is repeated till the values of pressures and saturations have converged.

The sets of difference equations were solved using the Douglas-Rachford's ADIP method(Alternating Direction Implicit Procedure, (32)).

This procedure was previously used to simulate five-spot pattern waterflood experiments (33). Good agreement was obtained between experimental and numerical results.

The derivation of the equations and the solution procedure are detailed in Appendix D. The model was simplified in order to reduce computing time. Conditions close to the ones prevailing in the laboratory during the experiments were retained for the numerical runs. The fluids were assumed to be incompressible, the reservoir pressure was close to the atmospheric pressure and thus the oil and water formation volume factors were equal to one. Capillary forces were also neglected. This last simplification will be justified later.

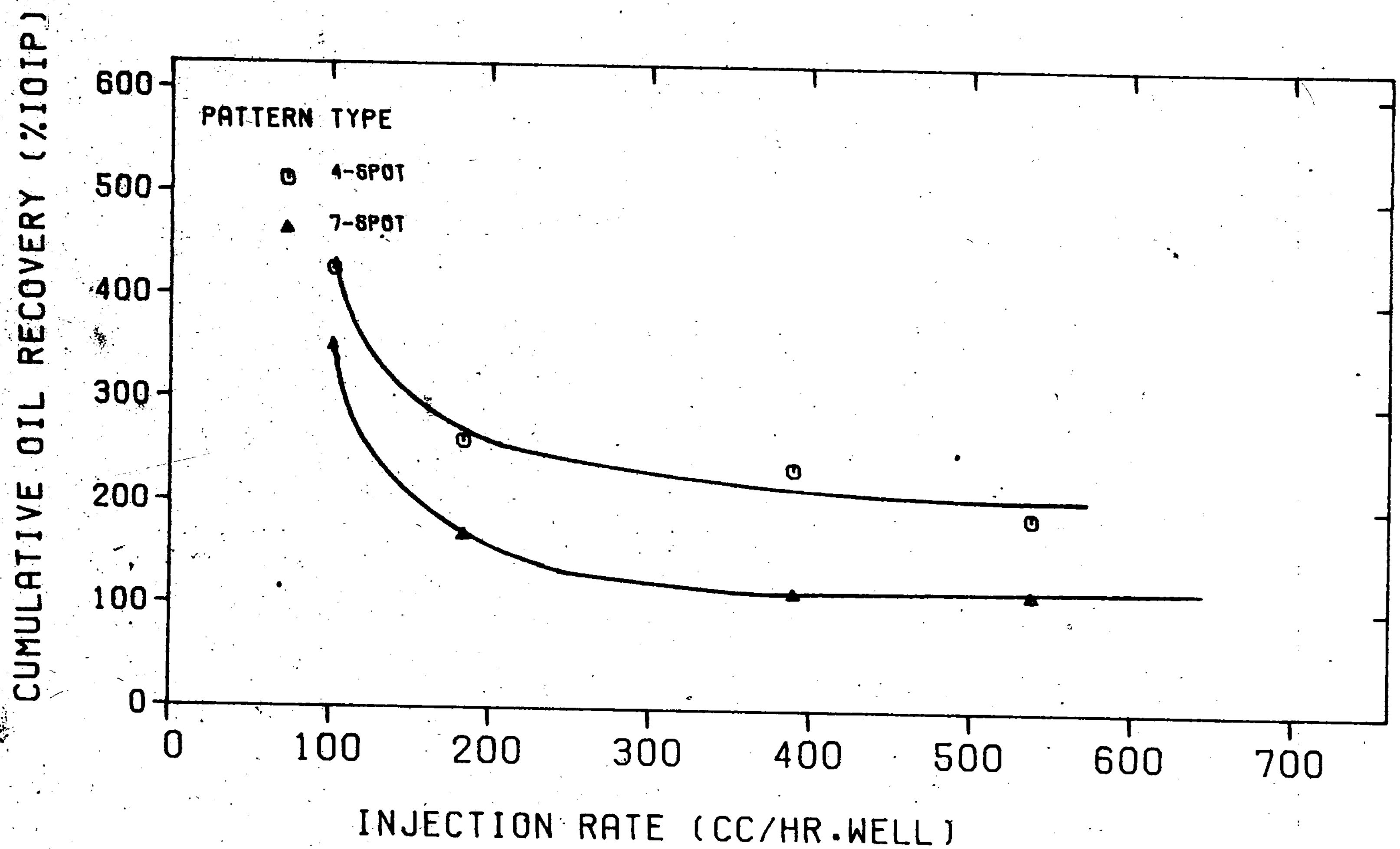


FIGURE 24:EFFECT OF INJECTION RATE ON OIL RECOVERIES FROM MODIFIED 4 AND 7-SPOT PATTERNS.M=0.83.CUMULATIVE INJECTION:5 PPV.

6.4 Model Scaling

The effect of injection rate on oil recovery from the modified four- and seven-spot patterns is plotted on Figure 24.

For the four-spot pattern, the oil recovery is continuously decreasing with increasing injection rate. The injection rates available on the pump were not high enough to obtain a stabilized flood. This was also true at higher mobility ratios.

On the other hand, the recovery from the modified seven-spot pattern is constant for injection rates greater than 300 cubic centimetres per hour per well. Thus for high injection rates, the flood is stabilized and the capillary effects are negligible.

Using this rate of 300 cubic centimetres per hour per well, the value of the scaling factor is 0.507. The details of the calculations are given in Appendix C.

A comparison with a field having average properties was made to understand the meaning of this value. The permeability of the formation was 100 millidarcies, the porosity 20 percent, and the thickness 10 metres. For such conditions, a production rate of two m^3/day was necessary to obtain the same value of the scaling factor. This is less than the production rates commonly used in waterflood operations.

Insufficient data were available to calculate the scaling factor at higher mobility ratios.

7.1 Grid Size

The grid is a rectangular, block-centred grid. A confined seven-spot pattern was represented by a 13 by 7 grid. The grid is reproduced on Figure 25. This ratio of the number of rows over the number of columns was necessary for the following reasons: the hexagonal geometry of the seven-spot pattern must be respected, the six injectors must be at the vertices of the hexagon and the producer at the centre, furthermore, the wells must be at the centre of a block.

Symmetry of the pressure and saturations distribution was obtained using this grid. During a run the oil saturations at the different injectors did not differ by more than two percent. The oil recovery curve was not modified if a finer grid was used, as can be seen on Figure 26. This figure is a comparison of recovery curves obtained using the above grid and a finer grid comprising 21 columns and 11 rows of blocks, the area of the pattern remaining the same.

The block size was chosen so as to have the distance between producer and injector equal to 10.25 centimetres, which was the distance in the physical model.

An isolated pattern consists of a seven-spot pattern in the middle of a large field. The 21 by 11 grid shown on Figure 25 was used to simulate isolated pattern waterfloods but two rows of blocks of larger dimensions had been added all around so as to obtain a 25 by 15 grid with a pore

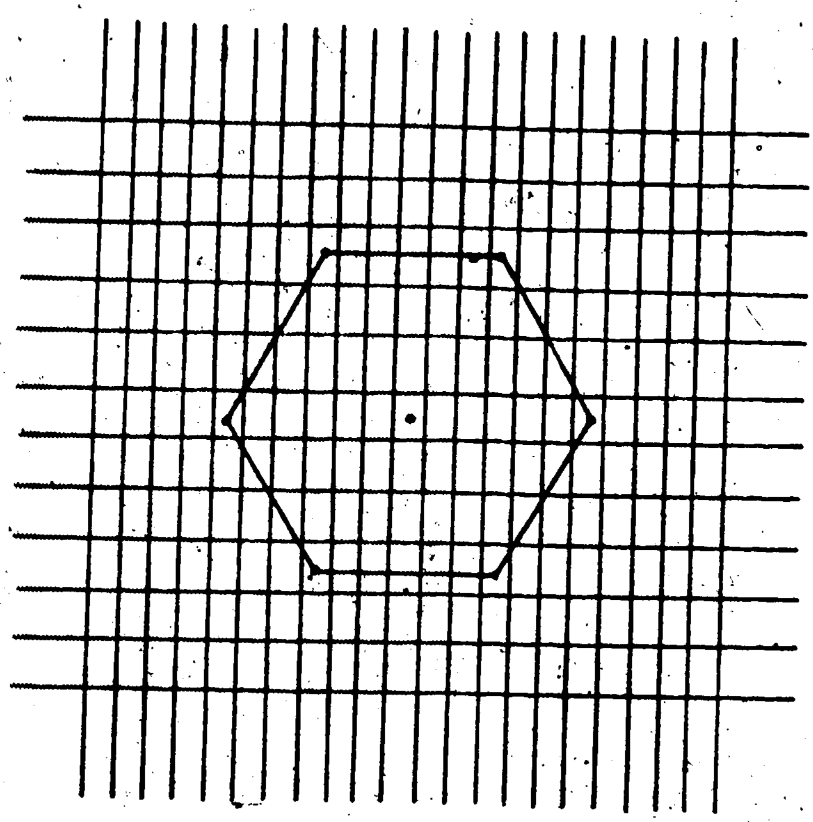


Figure 25.

Grid Used in Numerical Model.

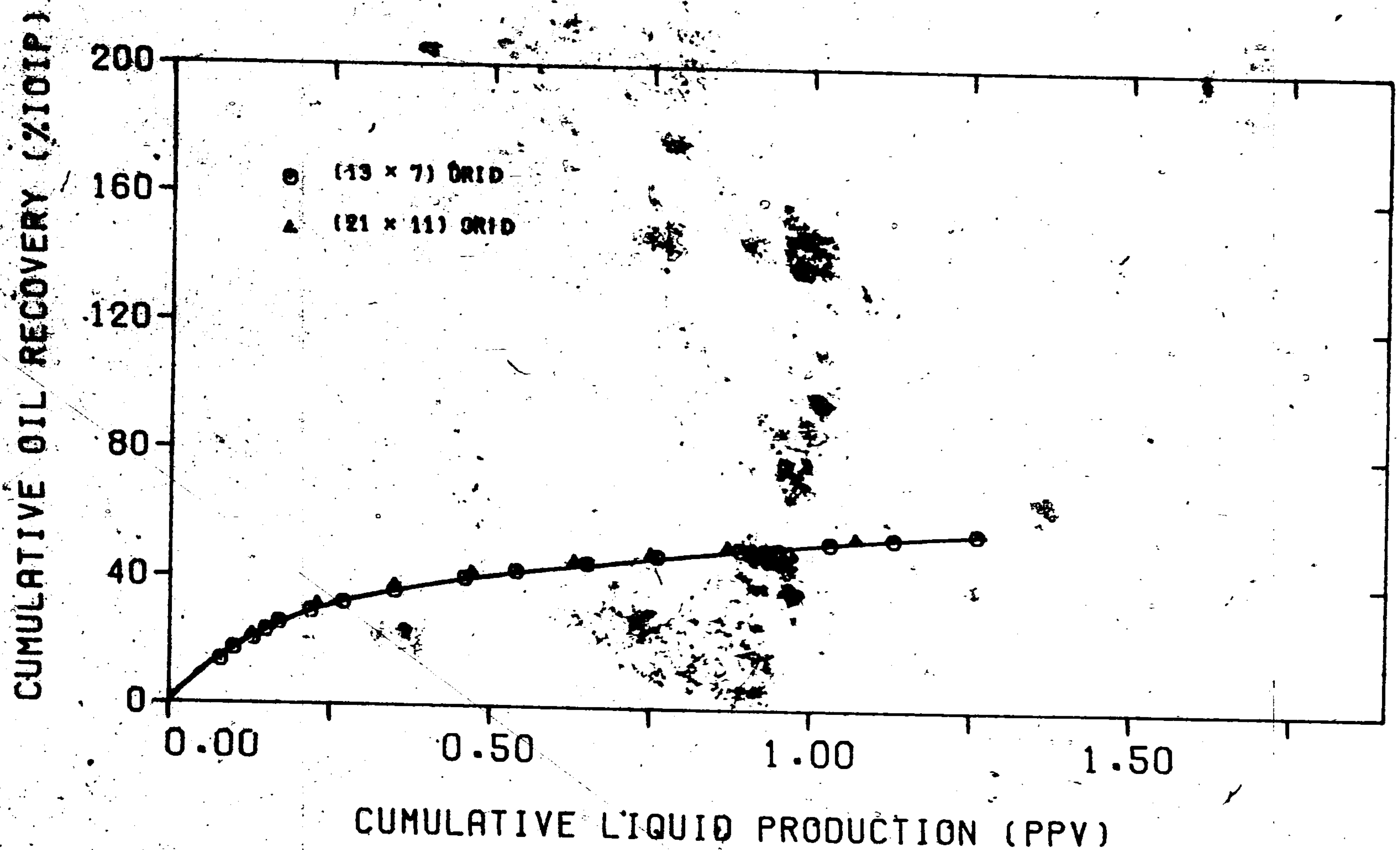


FIGURE 26 : EFFECT OF GRID SIZE ON OIL RECOVERY
NUMERICAL MODEL. M=2.70.

volume equal to 45 times the pattern-pore-volume.

7.2 Boundary Conditions

No flow occurs accross the boundaries. The normal component of the velocity at the boundary of the reservoir is zero. Numerically, this is accomplished by setting the values of the transmissibilities from one block outside of the grid to one block inside the grid equal to zero.

In the case of the isolated seven-spot, the area covered by the grid was large enough so that the oil and water recoveries from the central pattern were not affected by the boundary conditions.

7.3 Permeability Data

Relative permeability curves are given on Figure 27. The end-point permeabilities correspond to the ones measured on the physical model with the most viscous fluid.
The residual saturations are equal to the experimental values:

$$S_{or}=8\%$$

$$S_{wr}=40\%$$

The shape of the two curves is arbitrary but a few preliminary runs showed that the oil recoveries were not sensitive to the shape of the relative permeability curves.

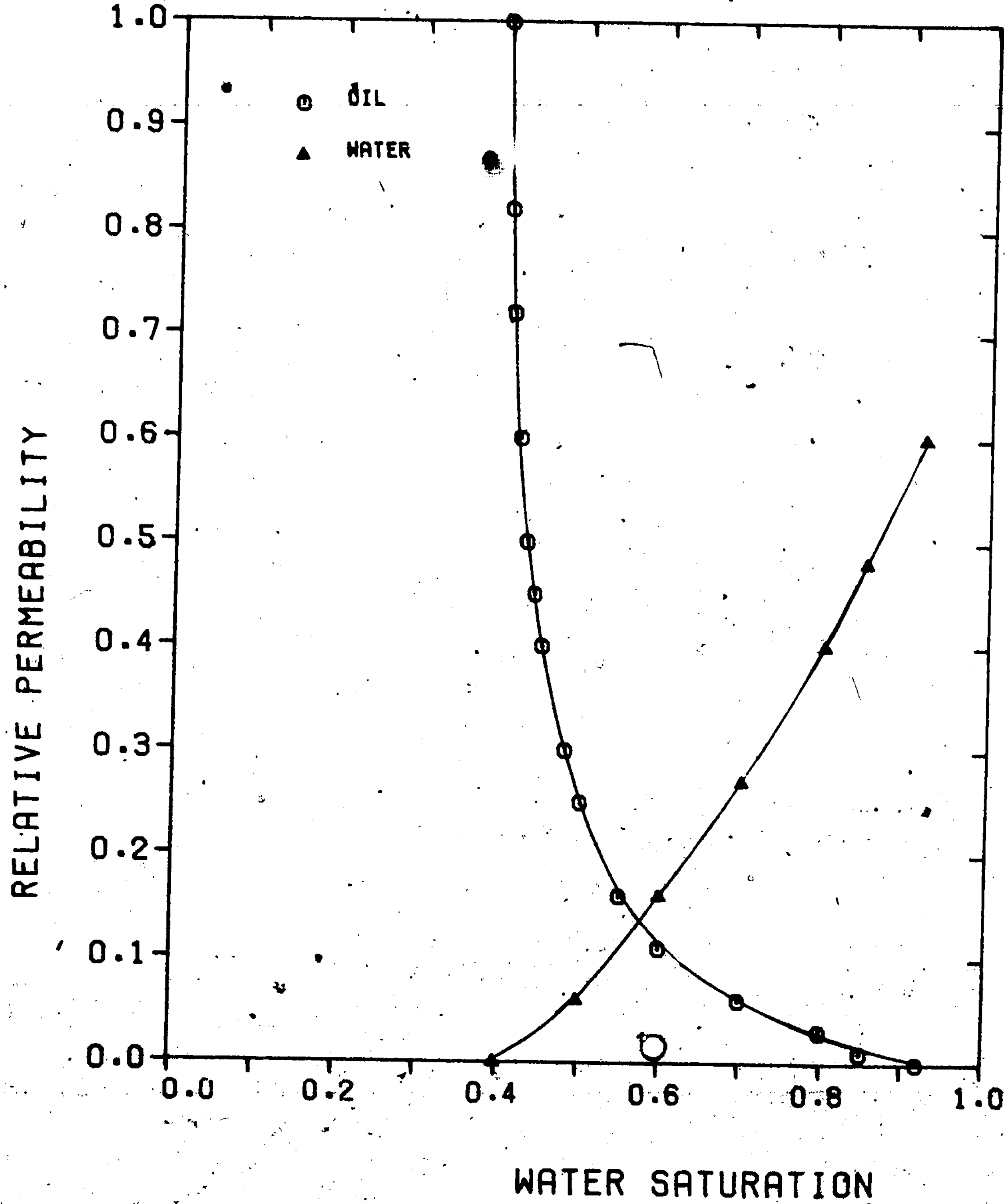


FIGURE 27: RELATIVE PERMEABILITY CURVES
USED IN THE NUMERICAL MODEL

7.4 Capillary Pressure

A waterflood displacement is dominated by viscous forces, particularly for high oil viscosities. The experimental results presented previously show that at sufficiently high injection rates, the capillary forces do not affect the displacement process. As the effect of the injection rate on the oil recovery has been studied experimentally, it did not seem necessary to include it in the numerical study. The capillary forces were then neglected. This reduced the computing time by a factor three to four and made easier the comparison of the results from the different runs. Hence oil recovery data from numerical runs correspond to the case of a stabilized flood.

7.5 Initial Pressure

At pressures close to the atmospheric pressure, the fluid viscosities and densities may be considered to be constant. The oil recovery is not affected by the absolute pressure level but by the pressure difference between injector and producer. The uniform initial pressure was assumed to be one atmosphere except at the injection wells where the initial pressure was higher, increasing with the oil viscosity.

7.6 Time Step

Very small time steps had to be used to ensure stability of the system. The volume of the water injected into the block containing one injector had to be less than five percent of the block pore volume, because the amount of liquid produced during a time step cannot correspond to a too large fraction of the pore volume of the block containing the producer. This would give rise to instabilities in the solution procedure. It must be recalled that this amount is equal to six times the amount of water injected in one block containing an injector.

8. NUMERICAL RESULTS

The advantage of the numerical model was the ability to cover a wider range of mobility ratios than was possible with the physical model. There were no limitations on the pressure or the oil viscosity. However, only isolated and confined pattern waterfloods could be simulated. Partial confinement conditions, such as those occurring in a modified pattern waterflood, could not be reproduced because the relative production rates of the different producers were not known.

Although the conditions used for the numerical runs were close to the ones prevailing during the experiments, no attempt was made to match the results exactly. Figures 28 and 29 show the difference between the experimental and numerical results in the case of isolated 7-spot pattern waterfloods. They correspond to mobility ratios of 0.83 and 2.70. The difference is rather large at a mobility ratio of 0.83 but the results are almost identical at a mobility ratio of 2.70. This difference is due to the experimental error and to the facts that the capillary forces were neglected, and also, only end-point relative permeabilities were available.

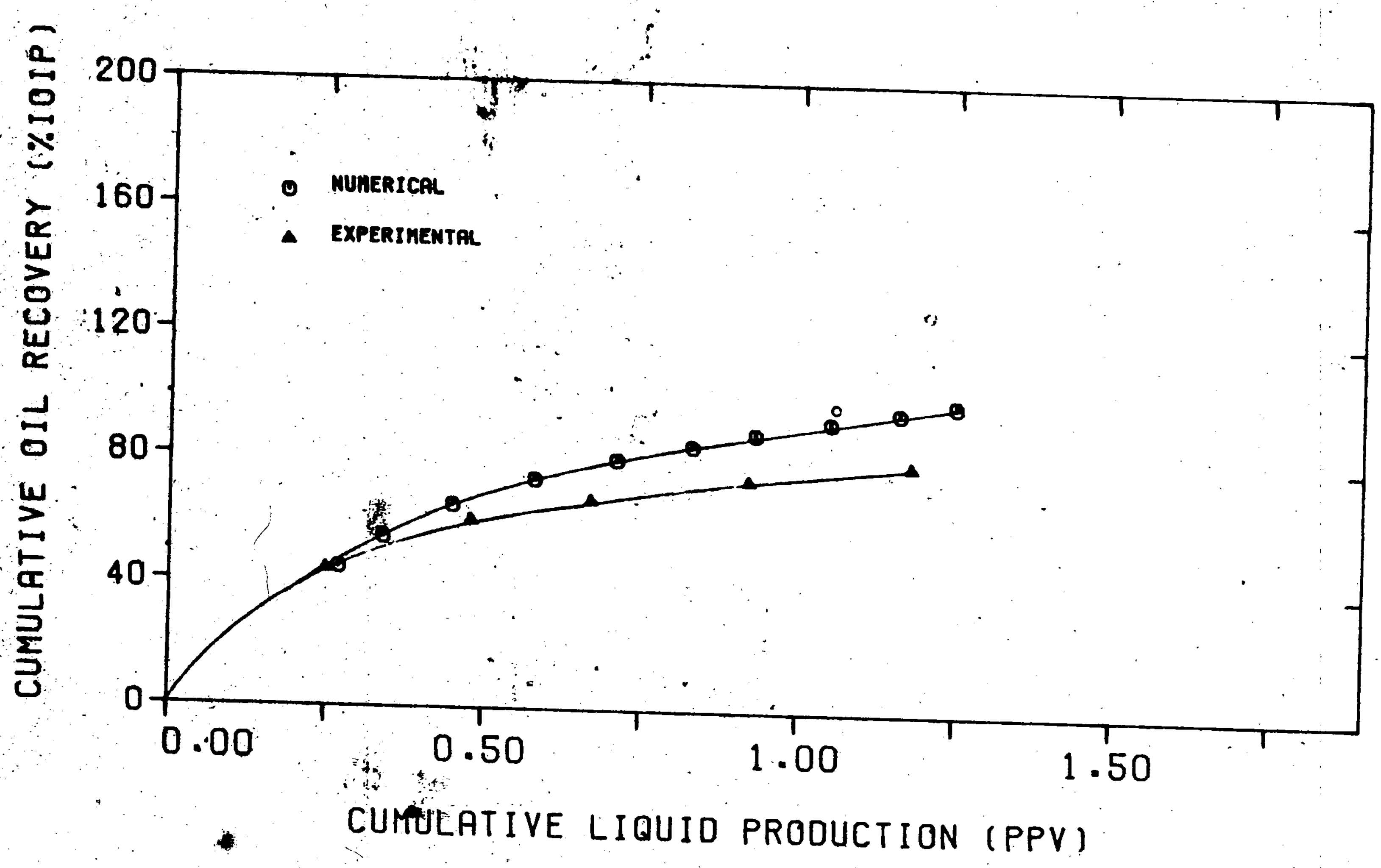


FIGURE 28 :COMPARISON OF NUMERICAL AND EXPERIMENTAL RESULTS.
ISOLATED 7-SPOT.M=0.83.Q=389 CC/HR.WELL(EXP.)

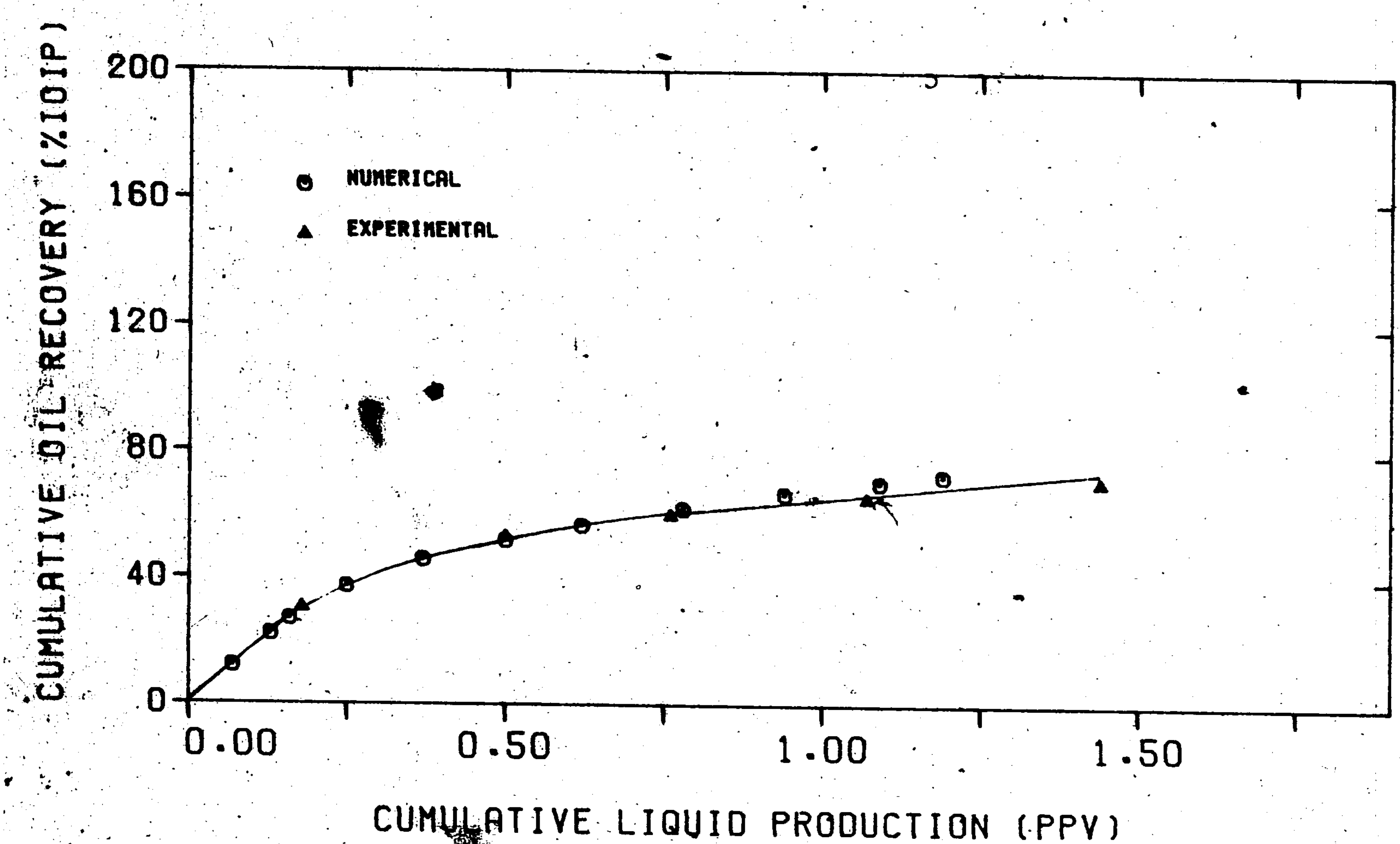


FIGURE 29: COMPARISON OF NUMERICAL AND EXPERIMENTAL RESULTS. ISOLATED 7-SPOT. M=2.70. Q=389 CC/HR. WELL (EXP.)

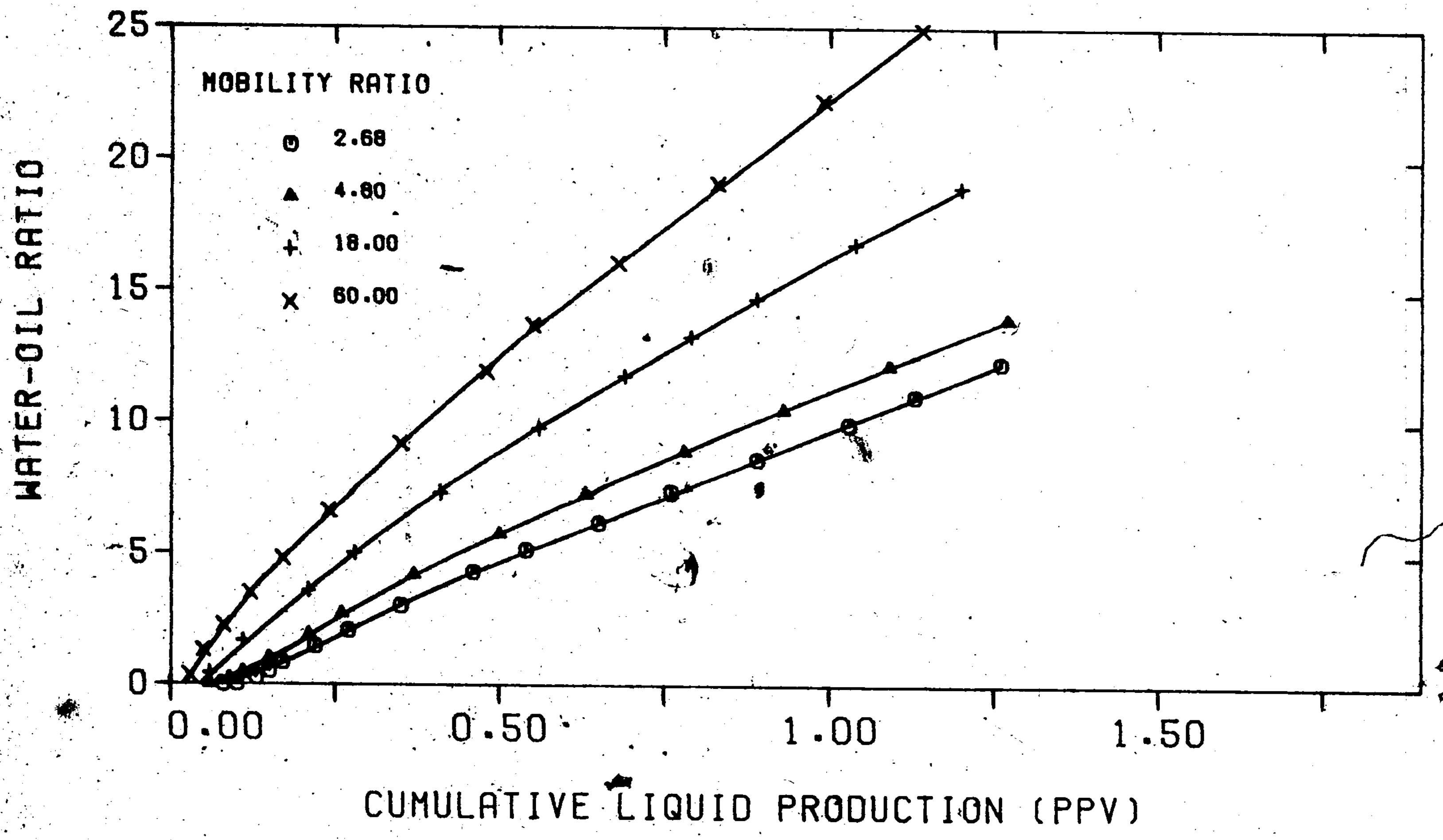


FIGURE 31: EFFECT OF MOBILITY RATIO ON PRODUCING WATER-OIL RATIO. CONFINED SEVEN-SPOT PATTERN. NUMERICAL RESULTS.

distance between the two curves of each figure shows the extent of the oversweep.

From a comparison of Figures 32, 33 and 34, it may be observed that the amount of recovery due to the oversweep of the pattern decreases with increasing mobility ratios. The difference between the recovery curves from isolated and confined patterns decreases. For a mobility ratio of 18 and a cumulative production of one pattern- pore-volume, the difference in recovery is only eight percent of the oil initially in place. However the absolute value of the oil recovery also decreases and the ratio of the difference between recoveries to the oil recovery in a confined pattern is almost constant.

Figures 35 and 36 show the variation of the water-oil ratio during isolated and confined pattern waterfloods. As opposed to the water-oil ratio in a confined pattern waterflood which increases linearly during the flood, the water-oil ratio from an isolated pilot waterflood tends to a constant value. Some oil is continuously coming from outside of the pilot area and this results in much smaller values of the producing water-oil ratio.

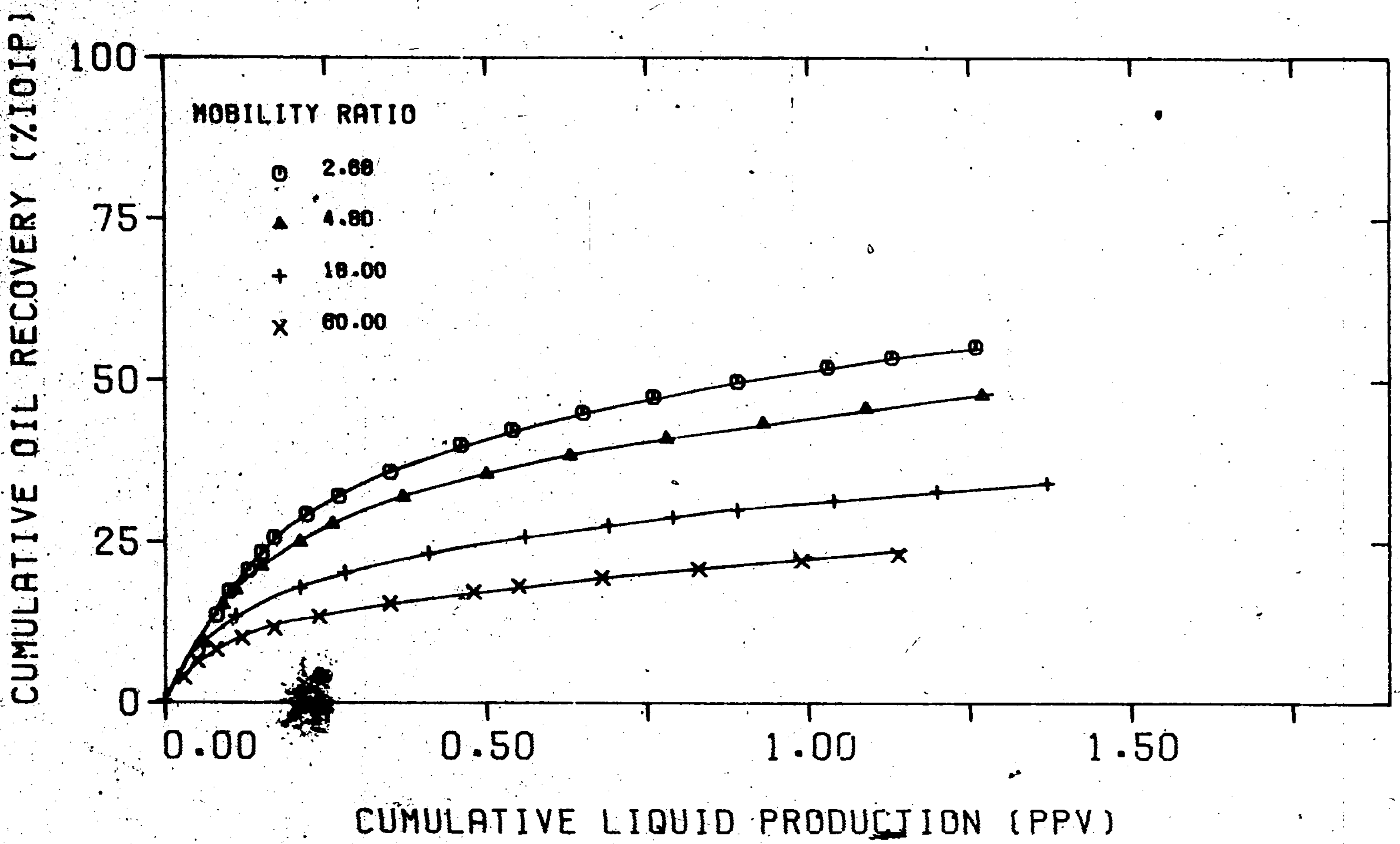


FIGURE 30: EFFECT OF MOBILITY RATIO ON OIL RECOVERY. CONFINED SEVEN-SPOT PATTERN. NUMERICAL RESULTS.

8.1 Oil Recovery from Confined Patterns

Oil recovery curves were computed for mobility ratios ranging from 2.70 to 60; the lowest value corresponding to the highest mobility ratio used during the experiments. These mobility ratios correspond to viscosity ratios ranging from 4.47 to 100.

Time steps of 0.8 second at lower viscosity ratios, and 0.5 second at higher viscosity ratios were used and 700 to 800 time steps were necessary to simulate about 10 minutes of an experimental waterflood.

Figure 30 shows the decrease in oil recovery with increasing mobility ratio. Similar results were observed during the experimental study. The oil recovery is very small for highly viscous oils. For an oil viscosity of 100 centipoises, less than 25 per cent of the oil in place has been produced after one pore volume has been injected.

Figure 31 shows the variation of the producing water-oil ratio during a waterflood. It may be noted that the variation of the water-oil ratio with the cumulative production becomes linear at an early stage. High water-oil ratios are attained for very low cumulative productions. The increase in water-oil ratio with mobility ratio is very important.

8.2 Comparison of Isolated and Confined Patterns.

Oil recoveries from isolated patterns are larger than recoveries from confined patterns (Figure 32,33,34) The

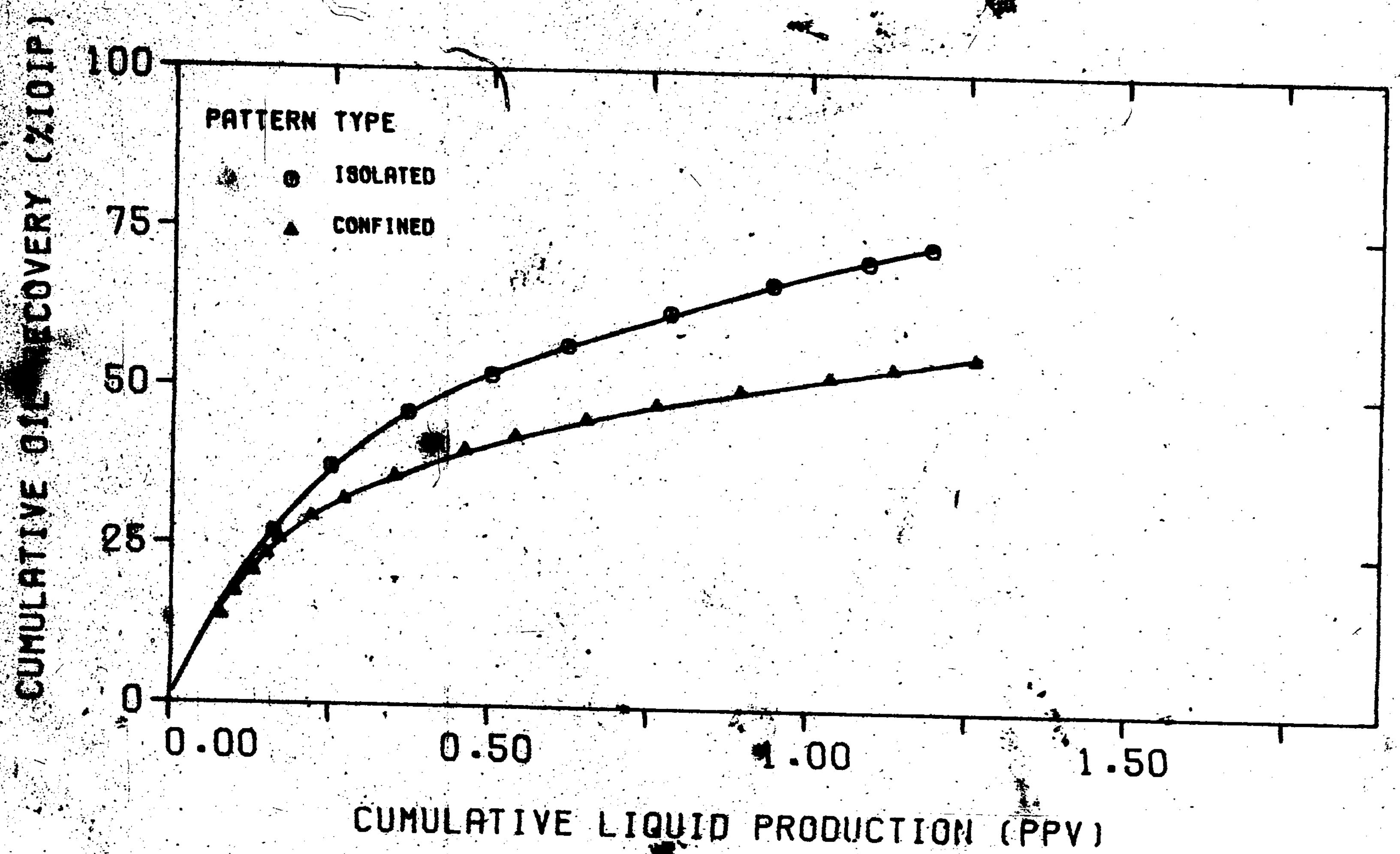


FIGURE 32: COMPARISON OF OIL RECOVERIES FROM AN ISOLATED AND CONFINED 7-SPOT PATTERN. NUMERICAL RESULTS. M=2.70.

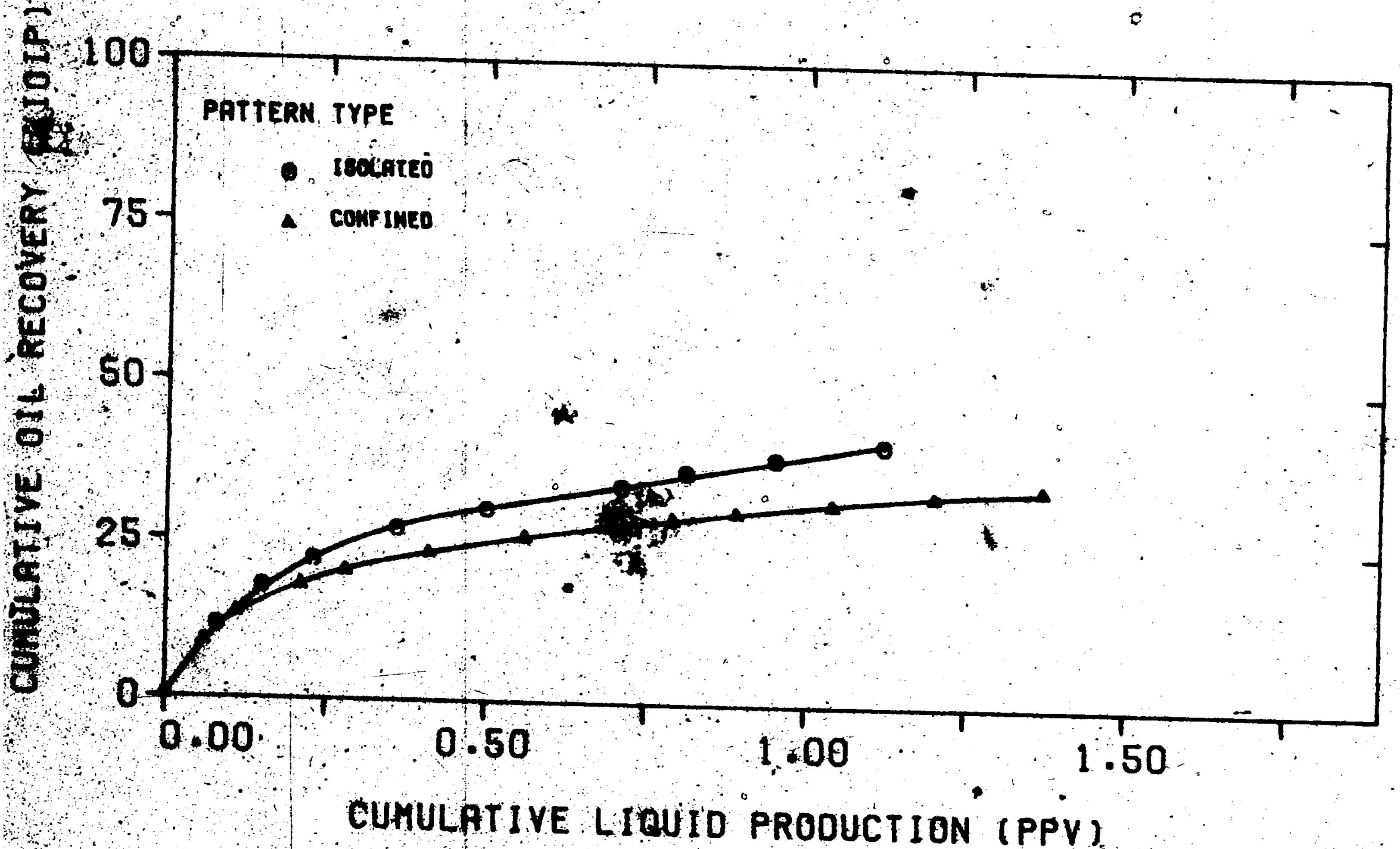


FIGURE 33: COMPARISON OF OIL RECOVERIES FROM AN ISOLATED AND CONFINED 7-SPOT PATTERN. NUMERICAL RESUL[illegible]D.

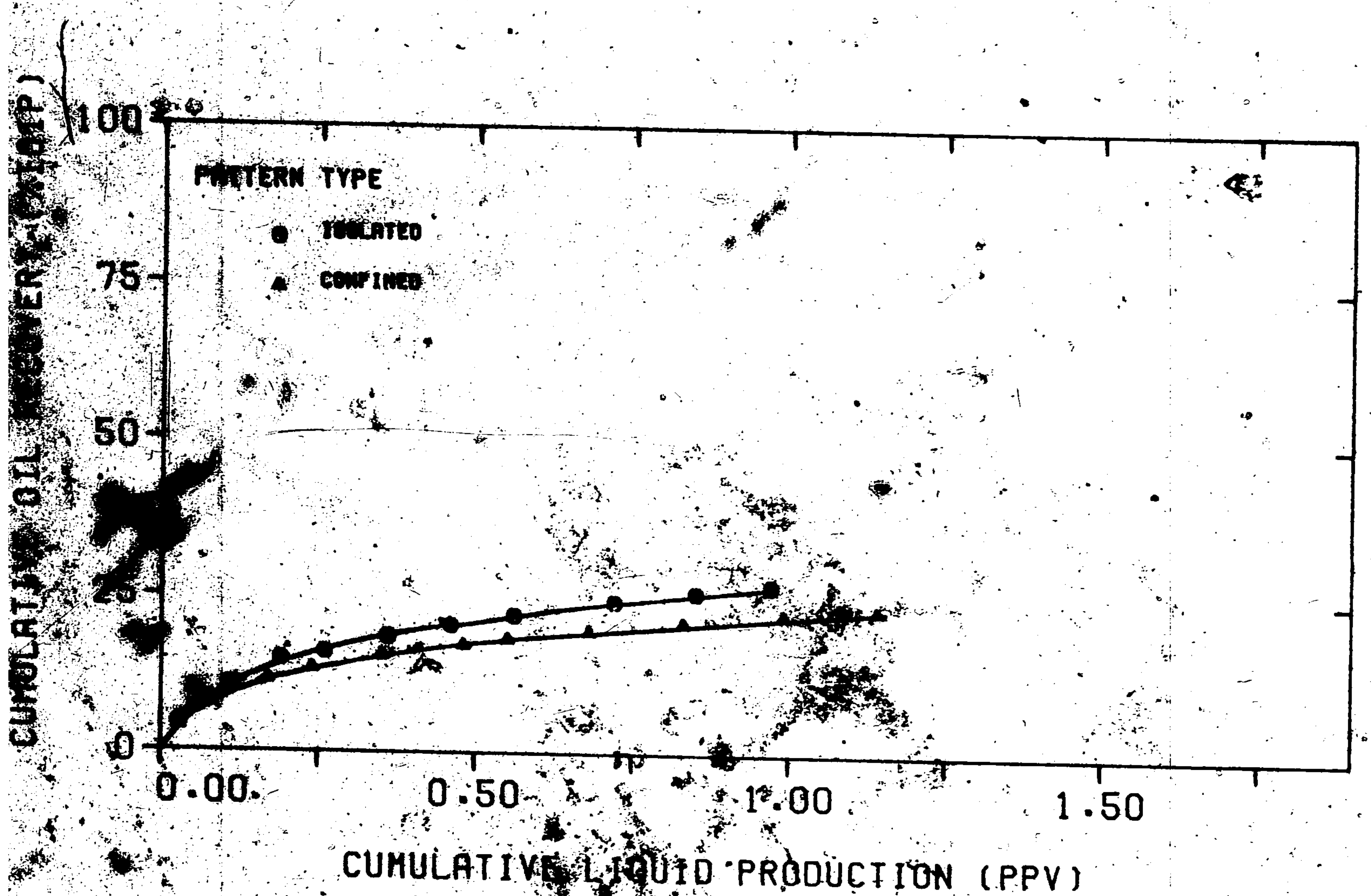


FIGURE 34: COMPARISON OF OIL RECOVERIES FROM AN ISOLATED AND CONFINED 7-SPOT PATTERN NUMERICAL RESULTS. M=60.0.

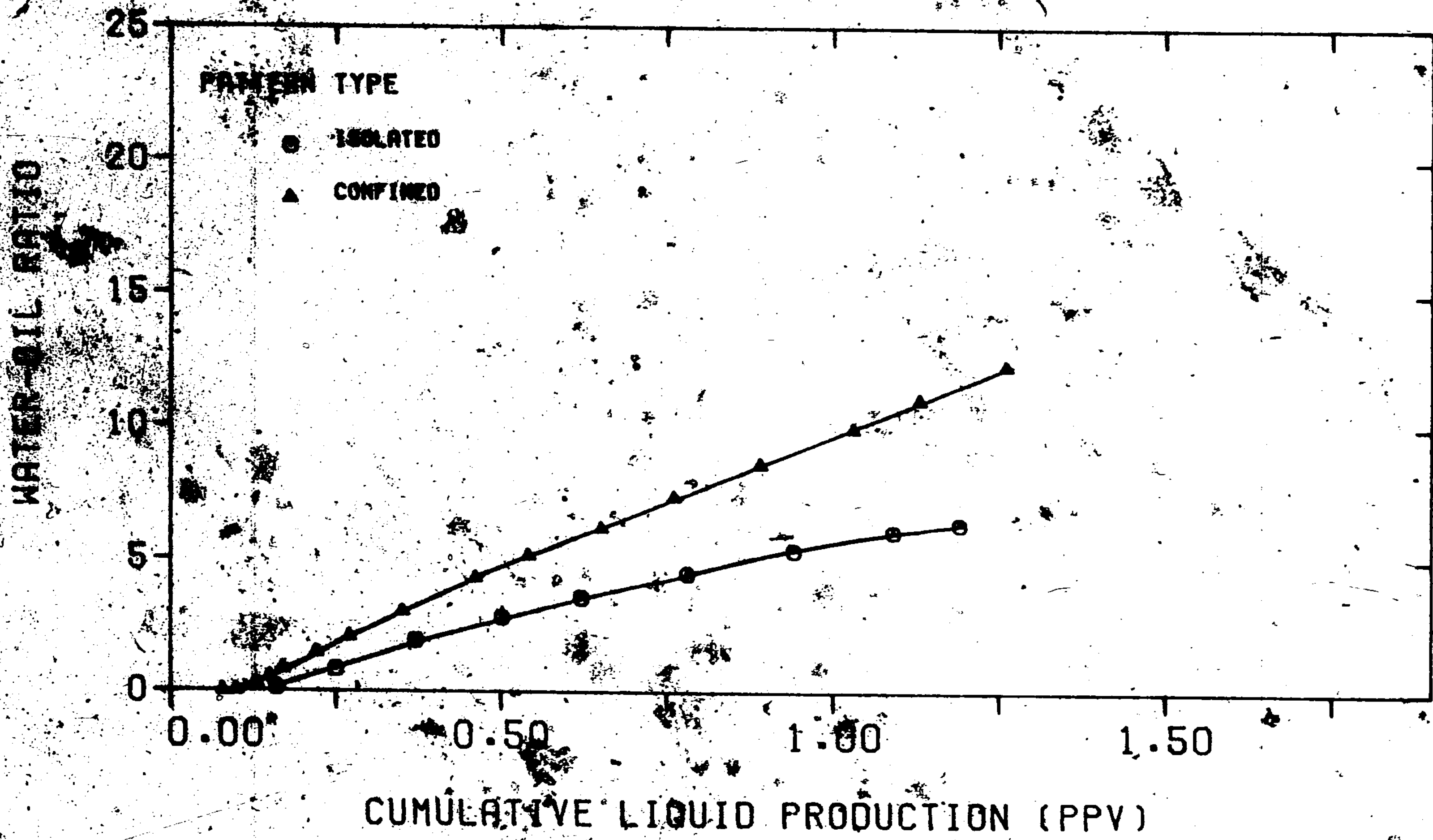


FIGURE 35: COMPARISON OF THE PRODUCING WATER-OIL RATIOS FROM AN ISOLATED AND CONFINED 7-SPOT PATTERN. NUMERICAL RESULTS. M=2.70.

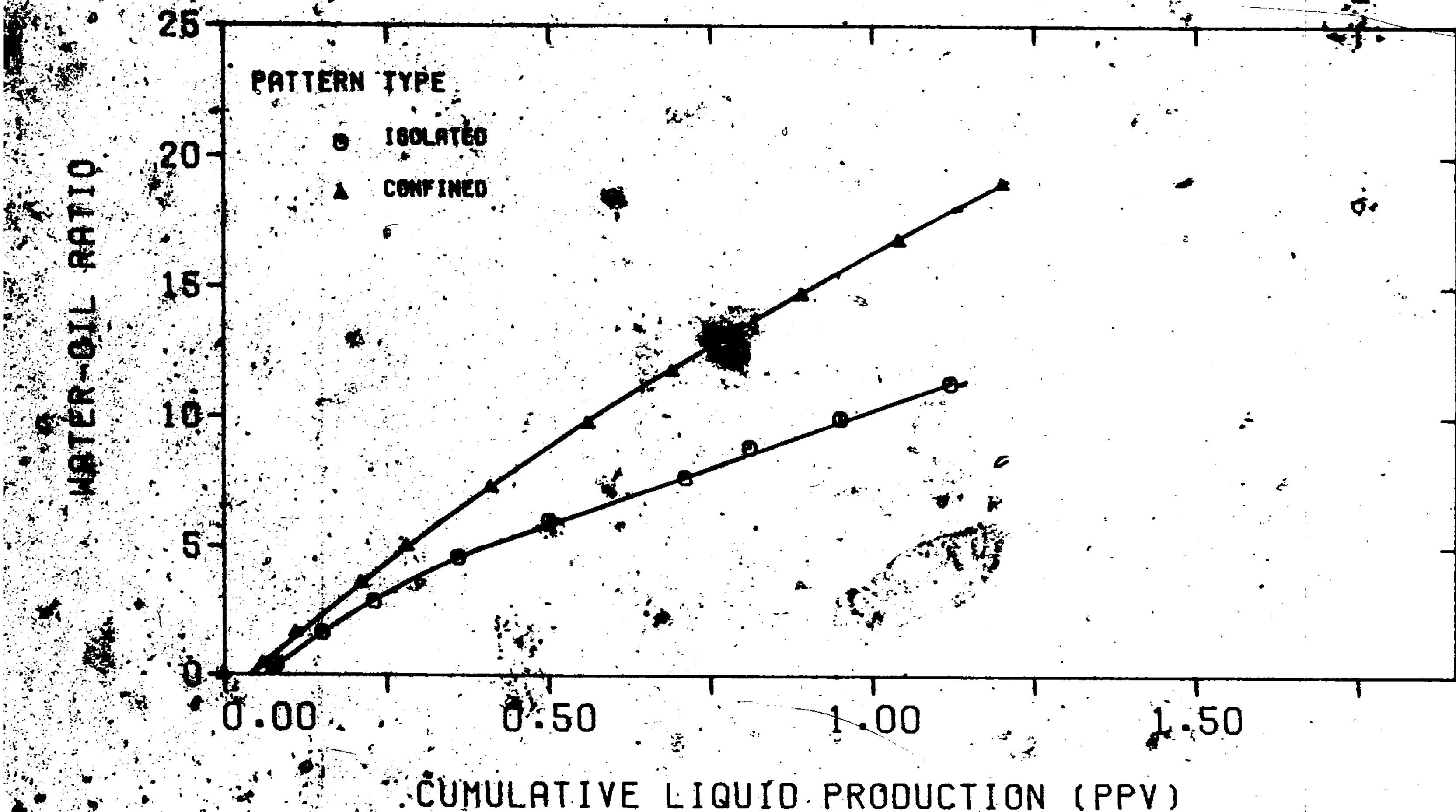


FIGURE 36: COMPARISON OF THE PRODUCING WATER-OIL RATIOS FROM AN ISOLATED AND CONFINED 7-SPOT PATTERN. NUMERICAL RESULTS. M=18.0.

9. DISCUSSION OF RESULTS.

The experimental and numerical results illustrate the three different ways of limiting the oil recovery from a pilot waterflood : the oil recovery decreases as the injection rate or the water-oil mobility ratio increases or when the basic pattern is surrounded by a ring or more of identical patterns.

9.1 Effect of Injection Rate

As the injection rate becomes larger, the importance of the capillary pressure term in the equation describing the displacement process (4-1) decreases. As a result, the flowing water-cut increases and the oil recovery will be lower. In a water-wet medium, the capillary forces tend to oppose the flow of water by causing the imbibition of the injected water into the smaller pores, thus displacing the oil in place. However, as the initial water saturation obtained in the physical model, as well as the porosity of the medium were very high, the extent of the imbibition phenomenon could not be significant.

The decrease in oil recovery observed with increasing injection rates, could be explained by a decrease in areal sweep efficiency. However, when comparing the photographs taken during different runs, no observation supporting this contention could be made. The area of the pattern ultimately covered by the injected water did not seem to decrease as the injection rate was increased. So, if the areal sweep efficiency is constant, the saturation distribution during a run depends on the injection rate. To study this point, a means of measuring the oil and water saturations at any point in the porous medium would be necessary.

9.2 Scaling

When the velocity of the fluids is high enough so that the capillary term in the flow equation becomes so small that it can be neglected, the flow is said to be stabilized. Not enough experimental results were available to calculate the critical rates for all the types of pattern studied. However it was shown that the critical rate for the modified seven-spot pattern was far below the usual injection rates used in waterflooding operations. Thus the effects of capillary forces in field waterfloods are small, particularly for viscous oils.

9.3 Effect of Mobility Ratio

The effect of mobility ratio on oil recovery from pattern waterfloods is identical for all types of patterns. As the mobility ratio increases, the oil recovery decreases in a four- or a seven-spot pattern waterflood. Peters observed the same effect in five- and nine-spot pattern waterfloods. The decrease in oil recovery is due partly to a decrease in areal sweep efficiency. Channels of preferential flow are formed from the injector to the producer. Most of the injected water flows along these channels without contacting new parts of the reservoir and displacing new quantities of oil.

The decrease in oil recovery is also due to a decrease in the displacement efficiency, characterized by higher residual oil saturations as the mobility ratio increases.

It is difficult to compare the effect of mobility ratio on oil recovery from four- and seven-spot pattern waterfloods, respectively. The injection rate enters as an additional parameter and complicates the interpretation of the results. The same injection rate used with both patterns does not correspond to the same velocity of the fluid front and thus, the results cannot be compared as the oil recovery depends on the injection rate and, thus, on the velocity of the water-front.

9.4 Comparison of Isolated and Modified Patterns

At the lowest mobility ratio and the lowest injection rates used in this study, the oil recovery from a modified pattern is much lower than the recovery from an isolated pattern. In the isolated pattern waterflood, the injected water always contacts new areas of the reservoir and displaces the oil. One ring of patterns around the central pattern is effective in preventing the oil from outside from entering the pilot area. This oil is produced by the additional producers around the basic pattern.

At high injection rates and high mobility ratios, the modification of the pattern has no effect. Oil recoveries from modified and isolated patterns are identical. At high mobility ratios, the oversweep from an isolated pattern is limited to a narrow ring of the porous medium around the pattern. The oil initially in this portion of the reservoir will be produced by the central producer. When the pattern is modified, there is competition between the different producers for the oil in place in the portion of the reservoir separating them. If the oil viscosity is high each producer can only attract the oil in place in a small portion of the reservoir next to it. The central pattern behaves as if it were isolated. The addition of a ring of patterns around it has no effect.

9.5 Comparison of Isolated and Confined Patterns

The numerical results show that even at high mobility ratios, an isolated pattern pilot is never totally confined. For any value of the mobility ratio some oversweep exists. However the amount of oil produced from outside the pilot becomes very small.

It was said in the above chapter that, at high mobility ratios, the same amount of oil is recovered during a waterflood, whether the pattern is an isolated or a modified one. As isolated pattern waterfloods are not confined, even for viscous oils, modified pattern waterfloods are not confined either. A percentage of the oil recovery (25 to 30 percent in this study) is due to the oversweep of the pattern. More than one ring of patterns around the basic pattern would be necessary to obtain confinement.

the same pressure. To ensure this the porous medium must be perfectly homogeneous and isotropic and plugging problems must be solved.

As it was mentionned, the injection rates available on the pump were not high enough to obtain a stabilized flood, in the case of the four-spot pattern. Higher rates should be used in order to obtain a value of the critical rate for this pattern.

If one wants to match the experimental results with the numerical ones, the relative permeability curves should be adjusted, either by an experimental determination of the actual curves or by a trial-and-error procedure. However, at the mobility ratios which pertained in this study, the choice of relative permeability curves is not important. Furthermore, the capillary pressures could be included in the numerical model to see if this would result in a better match with the favourable mobility ratio displacement.

It was said that the numerical model could not be used to compute oil recovery curves from modified pattern waterfloods. To calculate the production rates at the different producers, their pressures must be known. It was thought that imposing the pressures would be equivalent to imposing the production rates and thus predetermining the result. However this part deserves further study.

NOMENCLATURE

A : Area perpendicular to direction of flow

B_o : Oil formation volume factor

B_w : Water formation volume factor

C_1, C_2 : Scaling coefficients

d : Distance between injector and producer

f : Fraction of fluid flowing

g : Acceleration due to gravity

h : Thickness of the porous medium

IOIP : Initial Oil In Place

k : Absolute permeability

k_{ocw} : Effective permeability to oil at residual water saturation

k_r : Relative permeability

k_{wor} : Effective permeability to water at residual oil saturation

$k_x k_y$: Absolute permeabilities in the X and Y-directions.

L : Characteristic length of the system

M : Mobility ratio

10. CONCLUSIONS

As a result of the experimental and numerical studies, the following conclusions can be drawn.

1) The pattern oversweep in seven and four-spot waterfloods is extensive, particularly at favorable mobility ratios and low injection rates.

2) The oversweep decreases as the injection rate increases till the critical rate is attained.

3) Above the critical rate the oil recovery is no longer rate dependent.

4) The oversweep from a four-spot pattern is much larger than that from a seven-spot pattern.

5) The oversweep can be limited by surrounding the basic pattern with a ring of like patterns but cannot be completely eliminated by this means.

6) At higher mobility ratios the oil recoveries from the isolated and the modified patterns are identical.

10.1 Recommendations

Further research would be necessary to complete this study. Particularly the following points should be examined.

Waterfloods using a 13(four-spot) pattern arrangement were difficult to carry out experimentally because the pressures at the producing wells could not be monitored. It is important that all the producing wells of each ring be at

N_X, N_Y : Number of blocks in the X and Y-directions.

p : Pressure

p_c : Capillary pressure

$\bar{p}_c$: Dimensionless capillary pressure

PPV : Pattern Pore Volume

Q : Total injection rate per unit reservoir thickness

q : Injection rate per well

r_w : Wellbore radius

S : Saturation

T : Transmissibility

T' : $T' = T \cdot \rho \cdot g$

u : Characteristic superficial velocity

V_b : Block volume

μ : Viscosity

Φ : Potential

ϕ : Porosity

ρ : Density

σ : Interfacial tension

ψ : Kro times fractional flow of displacing fluid

<u>Subscripts:</u>

i,j are indexes indicating the position of [illegible]n the grid

o : Oil

w : Water

x,y refer to the direction

<u>Superscripts :</u>

n and n+1 ref[illegible]ctively to present time level and advanced time l[illegible].

m refers to values at the beginning of iteration number m.

m+1 refers to updated values at the end of iteration number m.

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Appendix B

Experimental Results.

1) Runs 1 to 19

Oil type:kerosene.

Residual water saturation:42.0%

Residual oil saturation:3.0%

2) Runs 20 to 27

Oil type:75% kerosene,25% lube oil.

Residual water saturation:40.0%

Residual oil saturation:8.0%

3) Runs 28 to 33

Oil type:50% kerosene,50% lube oil.

Residual water saturation:40.0%

Residual oil saturation:8.0%

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APPENDIX A

Porosity and Permeability Calculations

1)Calculation of the Porosity.

To calculate the porosity, the bulk volume of the model, the weight of glass beads necessary for packing, and the density of the beads were determined.

The results are as follows:

[1]Bulk volume of the model:	16900 cc
Total weight of beads:	2[illegible]180 g
Density of glass beads:	2.242 g/cc
Pore volume:	6115 cc
porosity:	36.2 %

2)Determination of the Absolute Permeability

Muskat's seven-spot formula was used to calculate the absolute and effective permeabilities at the end-point saturations.

$$k = \frac{Q\ \mu\ \ln(d/r_w) - 0.5691}{0.14876\ .\ h\ .\ \Delta p}$$

where k is the permeability (mD),

Q is the total injection rate (cc/hr),

μ the fluid viscosity (cp),

d, the distance between injector and producer (cm)

r_w, the well-bore radius (cm),

h, the thickness of the porous medium (cm),

Δp, the pressure drop between injector and producer (kPa)

The model was fully saturated with distilled water and then, water was injected through a seven-spot pattern at various rates.

A plot of P versus Q is a straight line with a slope m. The variables have the following values:

d=10.25 cm

r =0.16 cm

h=0.643 cm

=0.99 cp

From Figure 37 the value of the slope is equal to 0.00547.

The absolute permeability is equal to:

$$k = \frac{(0.992 \cdot \ln(10.25/0.16) - 0.5691) \cdot 1000}{0.14876 \cdot 0.643 \cdot 5.47}$$

= 6827 mD = 6.8 Darcies

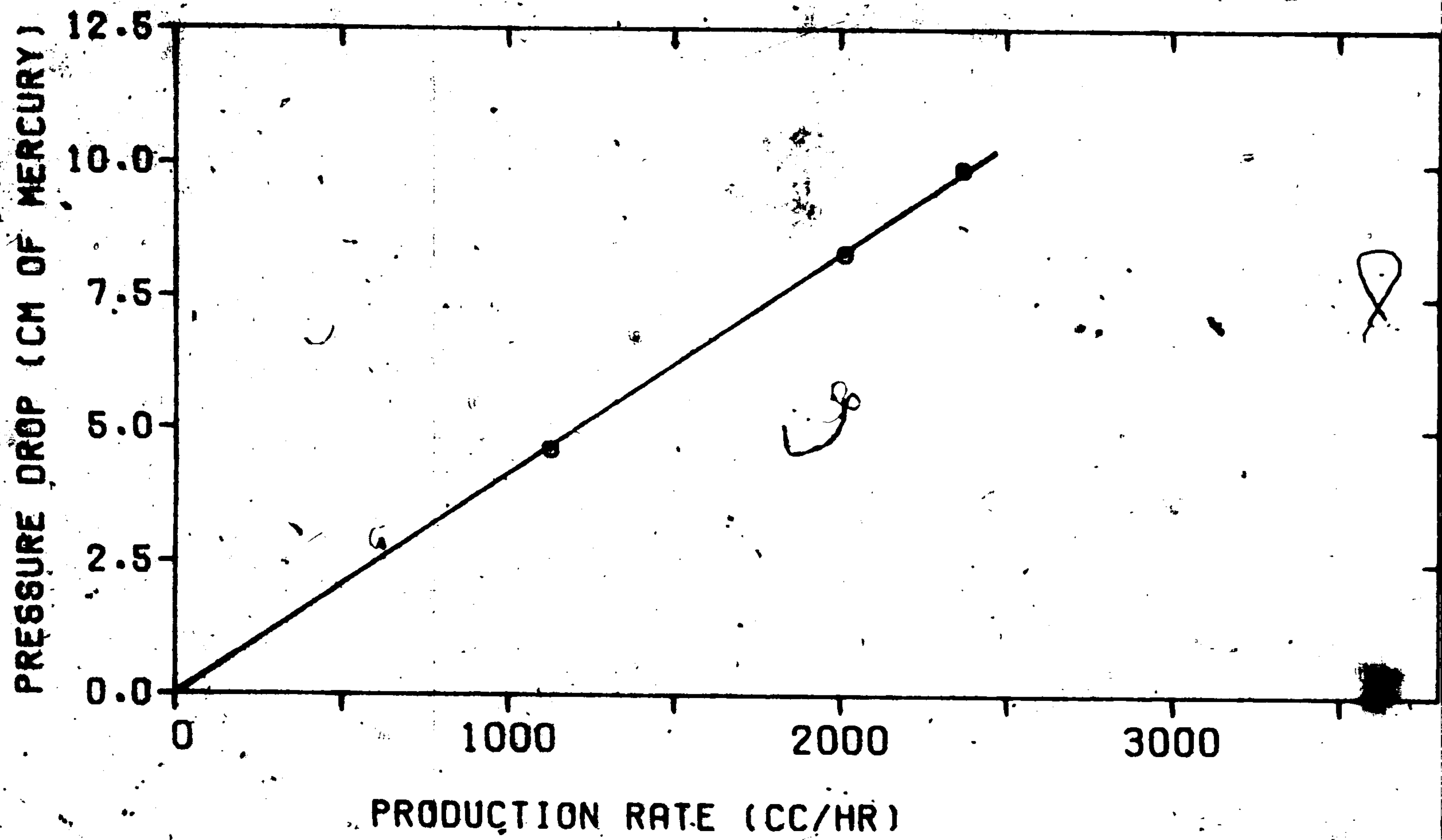


FIGURE 37 : MEASUREMENT OF THE ABSOLUTE PERMEABILITY.

3)Determination of the Effective Permeabilities and Mobility Ratios

The end-point effective permeabilities were determined using Muskat' seven-spot formula.

The mobility ratio is defined as

$$M = \frac{k_{wro} \cdot \mu_o}{k_{ocw} \cdot \mu_w}$$

where and are the oil and water viscosities,

k_{wro} , the effective permeability to water at residual oil saturation,

k_{ocw} , the effective permeability to oil at residual water saturation.

The results are:

fluid type	μ	k_{wro}	k_{ocw}	M
kerosene	1.38	3.7	6.2	0.83
75% kerosene 25 %lube oil	2.40	3.15	5.6	1.36
50% kerosene 50% lube oil	4.47	3.3	5.5	2.70

ISOLATED FOUR-SPOT PATTERN

RUN # 3

INJECTION RATE(CC/HR):183.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
32.5	0.0	32.5	1.03	177.3
13.0	6.0	19.0	1.63	248.3
10.0	7.5	17.5	2.18	302.8
8.5	9.0	17.5	2.74	349.2
8.0	10.0	18.0	3.31	392.8
8.0	10.0	18.0	3.88	436.5
8.0	10.0	18.0	4.45	480.1
7.5	10.5	18.0	5.02	521.1
7.0	10.5	17.5	5.57	559.3
14.0	20.0	34.0	6.65	635.8

ISOLATED FOUR-SPOT PATTERN

RUN # 4

INJECTION RATE(CC/HR):389.4

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
20.0	0.0	20.0	0.63	109.1
9.0	6.0	15.0	1.11	158.2
6.5	8.5	15.0	1.58	193.7
4.5	10.5	15.0	2.06	218.2
4.5	10.5	15.0	2.53	242.8
4.5	10.5	15.0	3.01	267.4
4.5	10.5	15.0	3.48	291.9
4.5	11.0	15.5	3.97	316.5
6.0	16.5	22.5	4.68	349.2
6.0	17.5	23.5	5.43	381.9
5.5	16.0	21.5	6.11	411.9
6.0	17.0	23.0	6.84	444.7
5.5	16.0	21.5	7.52	474.7
11.5	26.5	38.0	8.72	537.4

FOUR FOUR-SPOT PATTERN

RUN # 5

INJECTION RATE(CC/HR):102.4

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
13.5	0.0	13.5	0.43	73.7
10.0	5.0	15.0	0.90	128.2
10.0	7.5	17.5	1.48	182.8
7.5	8.0	15.5	1.95	223.7
7.5	9.5	17.0	2.48	264.6
6.5	8.5	15.0	2.96	300.1
5.5	9.5	15.0	3.43	330.1
6.5	10.0	16.5	3.96	365.6
5.0	10.0	15.0	4.43	392.8
5.0	10.0	15.0	4.91	420.1
10.0	20.0	30.0	5.85	474.7

FOUR FOUR-SPOT PATTERN

RUN # 6

INJECTION RATE(CC/HR):183.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
5.0	0.0	5.0	0.16	27.3
4.0	5.5	9.5	0.46	49.1
4.0	6.0	10.0	0.78	70.9
3.5	5.5	9.0	1.06	90.0
3.5	8.0	11.5	1.42	109.1
3.5	7.5	11.0	1.77	128.2
4.5	11.0	[illegible]	2.26	152.8
3.5	10.0	13.5	2.69	171.9
5.0	[illegible]	[illegible]	3.36	199.1
4.5	16.5	21.0	4.02	223.7
5.0	19.0	24.0	4.78	251.0
5.0	19.0	24.0	5.54	278.3
3.5	17.5	21.0	6.20	297.4
3.5	18.5	22.0	[illegible].90	316.5

FOUR FOUR-SPOT PATTERN

RUN # 7

INJECTION RATE(CC/HR):389.4

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
9.0	0.0	9.0	0.28	49.1
5.0	5.0	10.0	0.60	76.4
3.5	7.0	10.5	0.93	95.5
2.0	7.5	9.5	1.23	106.4
2.0	7.5	9.5	1.53	117.3
3.0	10.0	13.0	1.95	133.7
3.0	10.0	13.0	2.36	150.0
3.0	12.0	15.0	2.83	166.4
4.0	18.0	22.0	3.53	188.2
4.0	17.0	21.0	4.19	210.1
4.0	18.0	22.0	4.89	231.9
3.0	19.0	22.0	5.59	248.3
2.5	19.5	22.0	6.28	261.9
2.5	19.5	22.0	6.98	275.5
2.5	17.5	20.0	7.61	289.2

ISOLATED FOUR-SPOT PATTERN

RUN #1

INJECTION RATE(CC/HR): 53.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
22.5	0.0	22.5	0.71	122.8
16.0	0.0	16.0	1.22	210.1
11.5	1.0	12.5	1.61	272.8
11.0	2.0	13.0	2.03	332.8
10.0	2.0	12.0	2.41	387.4
9.0	2.0	11.0	2.75	436.5
7.0	2.0	9.0	3.04	474.7
9.5	2.5	12.0	3.42	526.5
9.5	2.5	12.0	3.80	578.4
9.5	2.5	12.0	4.18	630.2
9.5	2.5	12.0	4.56	682.0
9.0	3.0	12.0	4.94	731.1
4.0	2.0	6.0	5.13	752.9

ISOLATED FOUR-SPOT PATTERN

RUN # 2

INJECTION RATE(CC/HR):102.4

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
47.0	0.0	47.0	1.49	256.4
13.0	2.0	15.0	1.96	327.4
12.0	3.0	15.0	2.44	392.8
9.0	6.0	15.0	2.91	441.9
8.5	6.5	15.0	3.39	488.3
8.5	8.5	17.0	3.92	534.7
9.5	8.5	18.0	4.49	586.5
8.5	6.5	15.0	4.97	632.9
8.5	6.5	15.0	5.44	679.3

FOUR FOUR-SPOT PATTERN

RUN # 8

INJECTION RATE(CC/HR):537.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
6.5	0.0	6.5	0.21	35.5
4.5	11.5	16.0	0.71	60.0
3.5	13.5	17.0	1.25	79.1
3.0	14.5	17.5	1.80	95.5
3.0	14.0	17.0	2.34	111.9
3.2	14.6	17.8	2.91	129.3
3.2	14.8	18.0	3.47	146.8
3.2	14.8	18.0	4.04	164.2
2.3	14.3	16.6	4.57	176.8
3.0	17.3	20.3	5.21	193.1
2.8	15.0	17.8	5.78	208.4
3.4	19.0	22.4	6.48	227.0
2.2	15.6	17.8	7.05	239.0
3.2	18.3	21.5	7.73	256.4
3.3	16.2	19.5	8.34	274.4
2.8	16.5	19.3	8.96	289.7
2.5	16.0	18.5	9.54	303.4
1.4	15.6	17.0	10.08	311.0

THIRTEEN FOUR-SPOT PATTERN

RUN # 9

INJECTION RATE(CC/HR):102.4

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
24.0	1.0	25.0	0.79	130.9
13.0	3.0	16.0	1.30	201.9
7.0	3.0	10.0	1.61	240.1
6.0	4.0	10.0	1.93	272.8
5.0	5.0	10.0	2.25	300.1
4.0	6.0	10.0	2.56	321.9
4.5	9.0	13.5	2.99	346.5
3.0	9.0	12.0	3.37	362.8
2.0	8.0	10.0	3.69	373.7
2.0	8.0	10.0	4.00	384.7
1.5	8.5	10.0	4.32	392.8

THIRTEEN FOUR-SPOT PATTERN

RUN #10

INJECTION RATE(CC/HR):183.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
24.5	0.0	24.5	0.78	133.7
10.5	3.0	13.5	1.20	191.0
8.0	6.5	14.5	1.66	234.6
6.0	8.5	14.5	2.12	267.4
4.0	10.5	14.5	2.58	289.2
4.0	12.0	16.0	3.09	311.0
3.5	12.0	15.5	3.58	330.1
4.0	15.5	19.5	4.19	351.9
4.0	16.0	20.0	4.83	373.7
3.0	16.0	19.0	5.43	390.1
3.0	16.5	19.5	6.04	406.5
3.0	16.5	19.5	6.66	422.9
3.0	19.0	22.0	7.36	439.2

THIRTEEN FOUR-SPOT PATTERN

RUN #11

INJECTION RATE(CC/HR):537.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
17.0	0.0	17.0	0.54	92.8
3.0	12.5	15.5	1.03	109.1
1.9	17.2	19.1	1.63	119.5
2.0	17.4	19.4	2.25	130.4
1.6	18.8	20.4	2.89	139.1
1.5	21.3	22.8	3.61	147.3
1.5	23.5	25.0	4.41	155.5
1.3	22.0	23.3	5.14	162.6
1.0	20.2	21.2	5.81	168.0
0.0	0.0	0.0	5.81	168.0
1.0	22.0	23.0	6.54	173.5
0.8	18.6	19.4	7.16	177.9
1.1	21.6	22.7	7.87	183.9
0.5	14.3	14.8	8.34	186.6

ISOLATED SEVEN-SPOT PATTERN

RUN # 12

INJECTION RATE(CC/HR): 53.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
23.5	0.0	23.5	0.37	64.1
23.0	0.0	23.0	0.74	126.9
5.0	0.0	5.0	0.82	140.5
16.0	5.0	21.0	1.15	184.2
14.0	7.0	21.0	1.48	222.4
12.5	9.0	21.5	1.82	256.5
10.0	12.0	22.0	2.17	283.8
9.0	13.0	22.0	2.52	308.3
9.0	13.0	22.0	2.86	332.9
9.0	13.0	22.0	3.21	357.4
9.0	13.0	22.0	3.56	382.0
8.5	13.5	22.0	3.91	405.2
8.5	13.5	22.0	4.26	428.4
8.5	13.5	22.0	4.61	451.6
8.5	13.5	22.0	4.95	474.8
8.0	14.0	22.0	5.30	496.6
4.0	7.0	11.0	5.48	507.5

ISOLATED SEVEN-SPOT PATTERN

RUN # 13

INJECTION RATE(CC/HR):183.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
18.0	0.0	18.0	0.28	49.1
5.0	0.0	5.0	0.36	62.8
7.0	9.0	16.0	0.62	81.9
3.0	12.5	15.5	0.86	90.0
3.5	14.5	18.0	1.15	99.6
3.0	14.0	17.0	1.42	107.8
4.5	20.0	24.5	1.80	120.1
3.0	14.0	17.0	2.07	128.2
3.5	17.0	20.5	2.40	137.8
4.0	18.5	22.5	2.75	148.7
4.0	18.0	22.0	3.10	159.6
4.0	19.0	23.0	3.47	170.5
4.0	19.0	23.0	3.83	181.4
4.5	18.0	22.5	4.19	193.7
5.0	17.0	22.0	4.53	207.3
4.0	19.0	23.0	4.90	218.2
3.0	15.0	18.0	5.18	226.4

ISOLATED SEVEN-SPOT PATTERN

RUN #14

INJECTION RATE(CC/HR):183.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
19.0	2.0	21.0	0.33	51.8
9.5	10.5	20.0	0.65	77.8
6.0	15.0	21.0	0.98	94.1
5.0	15.5	20.5	1.31	107.8
4.5	16.5	21.0	1.64	120.1
4.0	15.5	19.5	1.95	131.0
3.0	14.0	17.0	2.22	139.2
3.5	18.0	21.5	2.56	148.7
4.0	17.5	21.5	2.90	159.6
3.0	14.5	17.5	3.17	167.8
4.0	18.0	22.0	3.52	178.7
3.5	18.0	21.5	3.86	188.3
4.0	17.0	21.0	4.19	199.2
3.5	18.0	21.5	4.53	208.7
3.5	17.5	21.0	4.87	218.3
4.0	21.0	25.0	5.26	229.2
3.5	20.0	23.5	5.63	238.7
3.0	19.5	22.5	5.99	246.9
2.5	15.0	17.5	6.27	253.8

ISOLATED SEVEN-SPOT PATTERN

RUN #15

INJECTION RATE(CC/HR):389.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
16.0	0.0	16.0	0.25	43.7
6.0	8.5	14.5	0.48	60.0
2.5	9.5	12.0	0.67	66.8
2.5	13.0	15.5	0.92	73.7
2.0	14.5	16.5	1.18	79.1
2.5	22.0	24.5	1.57	85.9
2.0	22.0	24.0	1.95	91.4
2.0	21.5	23.5	2.32	96.9
1.8	22.7	24.5	2.71	101.8
1.8	21.7	23.5	3.08	106.7
1.5	20.8	22.3	3.43	110.8
1.8	21.0	22.8	3.79	115.7
1.5	20.5	22.0	4.14	119.8
1.4	20.3	21.7	4.48	123.6
1.5	22.0	23.5	4.86	127.7
1.5	19.5	21.0	5.19	131.8
1.5	22.5	24.0	5.57	135.9
1.5	21.0	22.5	5.92	140.0
1.5	22.0	23.5	6.30	144.1
1.3	21.2	22.5	6.65	147.6

MODIFIED SEVEN-SPOT PATTERN

RUN #16

INJECTION RATE(CC/HR): 53.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
18.0	0.0	18.0	0.28	49.1
8.0	2.5	10.5	0.45	70.9
13.5	5.5	19.0	0.75	107.8
12.0	9.0	21.0	1.08	140.5
9.5	11.0	20.5	1.41	166.4
8.5	12.5	21.0	1.74	189.6
5.5	11.0	16.5	2.00	204.6
7.5	13.0	20.5	2.33	225.1
6.5	13.5	20.0	2.64	242.8
5.0	12.0	17.0	2.91	256.5
4.5	10.5	15.0	3.15	268.8
7.0	17.0	24.0	3.53	287.9
8.0	20.0	28.0	3.97	309.7
6.0	17.0	23.0	4.34	326.1
4.0	14.0	18.0	4.62	337.0
3.5	13.0	16.5	4.88	346.5
3.5	16.5	20.0	5.20	356.1

MODIFIED SEVEN-SPOT PATTERN

RUN #17

INJECTION RATE(CC/HR):183.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
19.0	0.0	19.0	0.30	51.8
9.5	14.0	23.5	0.67	77.8
4.5	17.5	22.0	1.02	90.0
3.5	18.0	21.5	1.36	99.6
3.5	18.0	21.5	1.70	109.1
2.5	17.5	20.0	2.02	116.0
2.0	20.0	22.0	2.37	121.4
3.0	26.0	29.0	2.82	129.6
2.5	19.0	21.5	3.17	136.4
3.0	19.0	22.0	3.51	144.6
2.0	20.0	22.0	3.86	150.1
2.0	21.5	23.5	4.23	155.5
2.0	21.0	23.0	4.60	161.0
2.0	20.0	22.0	4.95	166.4
1.5	22.5	24.0	5.33	170.5
2.0	22.0	24.0	5.71	176.0
1.5	16.5	18.0	5.99	180.1

MODIFIED SEVEN-SPOT PATTERN

RUN #18

INJECTION RATE(CC/HR):389.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
16.5	0.0	16.5	0.26	45.0
6.5	7.5	14.0	0.48	62.8
3.0	12.5	15.5	0.73	70.9
2.5	15.5	18.0	1.01	77.8
2.3	21.0	23.3	1.38	84.0
1.8	20.7	22.5	1.74	88.9
1.5	23.0	24.5	2.13	93.0
1.3	22.2	23.5	2.50	96.6
1.3	23.2	24.5	2.88	100.1
0.8	21.2	22.0	3.23	102.3
0.8	23.2	24.0	3.61	104.5
0.8	23.5	24.3	4.00	106.7
0.8	21.6	22.4	4.35	108.9
0.7	24.0	24.7	4.74	110.8
1.0	23.0	24.0	5.12	113.5
1.0	23.5	24.5	5.51	116.2
1.0	23.0	24.0	5.89	119.0
1.0	24.0	25.0	6.29	121.7
1.0	22.5	23.5	6.66	124.4
1.0	22.5	23.5	7.03	127.1

MODIFIED SEVEN-SPOT PATTERN

RUN #19

INJECTION RATE(CC/HR):537.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
16.0	0.0	16.0	0.25	43.7
7.0	15.0	22.0	0.60	62.8
4.0	18.0	22.0	0.95	73.7
2.5	18.8	21.3	1.29	80.5
2.0	20.4	22.4	1.64	85.9
1.8	20.6	22.4	2.00	90.9
1.6	22.8	24.4	2.38	95.2
1.1	21.4	22.5	2.74	98.2
0.8	15.5	16.3	3.00	100.4
1.0	28.5	29.5	3.46	103.1
0.8	22.0	22.8	3.82	105.3
0.7	20.0	20.7	4.15	107.2
0.7	19.3	20.0	4.47	109.1
0.6	22.4	23.0	4.83	110.8
0.8	22.5	23.3	5.20	113.0
0.8	24.2	25.0	5.60	115.1
0.8	24.2	25.0	5.99	117.3
0.8	21.7	22.5	6.35	119.5
0.7	24.3	25.0	6.74	121.4
0.5	18.5	19.0	7.04	122.8

ISOLATED FOUR-SPOT PATTERN

RUN #20

INJECTION RATE(CC/HR):183.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
10.0	0.0	10.0	0.32	52.7
11.0	4.0	15.0	0.79	110.8
8.2	8.3	16.5	1.31	154.0
6.8	9.6	16.4	1.83	189.9
7.8	12.2	20.0	2.47	231.0
7.0	12.6	19.6	3.09	267.9
5.5	12.4	17.9	3.65	296.9
5.7	13.3	19.0	4.25	327.0
4.6	11.4	16.0	4.76	351.3
5.1	11.7	16.8	5.29	378.2
6.3	15.0	21.3	5.97	411.4
4.4	11.3	15.7	6.46	434.6
3.7	8.3	12.0	6.84	454.1

ISOLATED FOUR-SPOT PATTERN

RUN #21

INJECTION RATE(CC/HR):537.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
12.0	0.0	12.0	0.38	63.3
9.5	6.0	15.5	0.87	113.4
5.8	11.0	16.8	1.40	144.0
4.4	15.3	19.7	2.03	167.2
4.1	20.3	24.4	2.80	188.8
3.2	18.0	21.2	3.47	205.7
3.2	19.6	22.8	4.19	222.6
2.6	18.7	21.3	4.86	236.3
3.3	19.6	22.9	5.59	253.7
2.9	19.4	22.3	6.29	269.0
2.9	18.8	21.7	6.98	284.3
2.9	19.7	22.6	7.70	299.6
2.8	20.5	23.3	8.43	314.3

FOUR FOUR-SPOT PATTERN

RUN #22

INJECTION RATE(CC/HR):183.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
7.0	0.0	7.0	0.22	36.9
6.0	7.3	13.3	0.64	68.6
6.4	14.0	20.4	1.29	102.3
4.5	18.5	23.0	2.02	126.1
4.1	15.5	19.6	2.64	147.7
4.7	18.6	23.3	3.37	172.5
3.4	16.3	19.7	4.00	190.4
2.8	17.5	20.3	4.64	205.2
2.6	13.0	15.6	5.13	218.9
2.5	13.0	15.5	5.62	232.1
1.8	13.5	15.3	6.11	241.6
2.0	13.5	15.5	6.60	252.1
2.3	14.5	16.8	7.13	264.2

FOUR FOUR-SPOT PATTERN

RUN #23

INJECTION RATE(CC/HR):537.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
12.5	0.0	12.5	0.40	65.9
5.4	8.0	13.4	0.82	94.4
2.6	13.4	16.0	1.33	108.1
2.3	15.5	17.8	1.89	120.3
2.0	17.5	19.5	2.51	130.8
2.2	17.3	19.5	3.12	142.4
2.0	18.0	20.0	3.76	153.0
2.1	20.5	22.6	4.47	164.0
2.1	20.9	23.0	5.20	175.1
2.0	21.0	23.0	5.93	185.7
2.0	20.5	22.5	6.64	196.2
2.0	19.8	21.8	7.33	206.8
1.8	17.2	19.0	7.93	216.2
2.0	23.0	25.0	8.72	226.8

ISOLATED SEVEN-SPOT PATTERN

RUN #24

INJECTION RATE(CC/HR):183.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
15.0	0.0	15.0	0.24	39.6
6.6	5.2	11.8	0.42	57.0
5.8	9.7	15.5	0.67	72.3
6.9	15.8	22.7	1.03	90.5
5.7	17.3	23.0	1.39	105.5
5.7	17.5	23.2	1.76	120.5
3.9	18.3	22.2	2.11	130.8
3.7	18.9	22.6	2.47	140.6
3.7	19.3	23.0	2.83	150.3
3.6	19.4	23.0	3.20	159.8
3.6	19.4	23.0	3.56	169.3
3.5	19.9	23.4	3.93	178.6
3.5	20.0	23.5	4.30	187.8
3.3	20.1	23.4	4.67	196.5
3.1	20.0	23.1	5.04	204.7
2.9	18.0	20.9	5.37	212.3

ISOLATED SEVEN-SPOT PATTERN

RUN #25

INJECTION RATE(CC/HR):537.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
13.2	0.0	13.2	0.21	34.8
7.3	14.0	21.3	0.55	54.1
2.9	17.8	20.7	0.87	61.7
2.2	20.0	22.2	1.22	67.5
1.8	21.0	22.8	1.59	72.3
1.5	20.0	21.5	1.93	76.2
1.0	20.7	21.7	2.27	78.9
1.2	21.5	22.7	2.63	82.0
1.3	21.7	23.0	2.99	85.5
1.1	22.4	23.5	3.36	88.4
1.2	22.1	23.3	3.73	91.5
1.0	21.6	22.6	4.09	94.2
1.2	22.3	23.5	4.46	97.3
1.1	22.2	23.3	4.83	100.2
1.2	22.0	23.2	5.20	103.4
1.1	22.2	23.3	5.57	106.3
1.1	22.7	23.8	5.94	109.2
1.1	22.4	23.5	6.32	112.1
1.0	22.3	23.3	6.68	114.7
1.1	20.4	21.5	7.02	117.6

MODIFIED SEVEN-SPOT PATTERN

RUN #26

INJECTION RATE(CC/HR):183.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
24.0	0.0	24.0	0.38	63.3
7.0	7.5	14.5	0.61	81.8
5.9	12.4	18.3	0.90	97.3
4.3	13.0	17.3	1.17	108.7
3.5	15.5	19.0	1.47	117.9
3.1	18.6	21.7	1.82	126.1
3.0	22.5	25.5	2.22	134.0
3.0	29.5	32.5	2.73	141.9
1.7	18.0	19.7	3.05	146.4
1.4	17.4	18.8	3.34	150.1
1.6	20.4	22.0	3.69	154.3
1.6	19.7	21.3	4.03	158.5
1.4	21.1	22.5	4.39	162.2
1.5	20.7	22.2	4.74	166.2
1.4	20.4	21.8	5.08	169.9
1.4	22.4	23.8	5.46	173.6
1.5	22.9	24.4	5.84	177.5
1.3	21.0	22.3	6.20	180.9

MODIFIED SEVEN-SPOT PATTERN

RUN #27

INJECTION RATE(CC/HR):537.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
18.5	12.5	31.0	0.49	48.8
3.5	16.3	19.8	0.80	58.0
2.5	18.5	21.0	1.14	64.6
3.0	30.0	33.0	1.66	72.5
1.8	18.6	20.4	1.98	77.3
2.1	21.7	23.8	2.36	82.8
1.9	21.0	22.9	2.72	87.8
1.6	21.1	22.7	3.08	92.1
1.4	21.4	22.8	3.44	95.7
1.3	24.4	25.7	3.85	99.2
1.2	20.7	21.9	4.19	102.3
1.3	21.6	22.9	4.56	105.8
1.1	21.1	22.2	4.91	108.7
0.7	20.8	21.5	5.25	110.5
0.9	23.0	23.9	5.63	112.9
0.7	20.9	21.6	5.97	114.7
0.9	23.0	23.9	6.35	117.1
0.8	22.7	23.5	6.72	119.2
0.7	22.9	23.6	7.09	121.1
0.5	23.0	23.5	7.46	122.4

ISOLATED FOUR-SPOT PATTERN

RUN #28

INJECTION RATE(CC/HR):389.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
8.0	0.0	8.0	0.25	42.2
6.3	8.7	15.0	0.73	75.4
4.1	17.7	21.8	1.42	97.0
2.8	19.8	22.6	2.13	111.8
2.5	20.7	23.2	2.87	125.0
2.3	21.7	24.0	3.63	137.1
2.0	24.0	26.0	4.45	147.7
2.0	21.4	23.4	5.19	158.2
2.2	22.5	24.7	5.97	169.8
1.8	21.8	23.6	6.72	179.3
1.9	21.5	23.4	7.46	189.3
1.7	21.6	23.3	8.20	198.3
1.8	21.7	23.5	8.94	207.8

FOUR FOUR-SPOT PATTERN

RUN #29

INJECTION RATE(CC/HR):389.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
5.5	0.0	5.5	0.17	29.0
7.2	7.6	14.8	0.64	67.0
4.1	17.2	21.3	1.32	88.6
2.8	17.5	20.3	1.96	103.4
2.4	20.0	22.4	2.67	116.0
2.3	20.2	22.5	3.38	128.2
1.6	20.4	22.0	4.08	136.6
1.6	21.7	23.3	4.81	145.0
2.0	24.0	26.0	5.64	155.6
1.6	23.1	24.7	6.42	164.0
1.6	23.2	24.8	7.20	172.5
1.6	22.7	24.3	7.97	180.9
1.3	17.4	18.7	8.56	187.8

THIRTEEN FOUR-SPOT PATTERN

RUN #30

INJECTION RATE(CC/HR):537.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
13.7	0.0	13.7	0.43	72.3
2.2	9.2	11.4	0.79	83.9
1.6	12.0	13.6	1.22	92.3
1.3	15.4	16.7	1.75	99.2
0.8	15.2	16.0	2.26	103.4
0.9	19.1	20.0	2.89	108.1
1.2	18.6	19.8	3.52	114.5
0.8	17.2	18.0	4.09	118.7
0.7	16.8	17.5	4.64	122.4
0.7	18.1	18.8	5.24	126.1
0.7	16.3	17.0	5.78	129.7
0.8	17.7	18.5	6.36	134.0

ISOLATED SEVEN-SPOT PATTERN

RUN #31

INJECTION RATE(CC/HR):183.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
10.2	0.0	10.2	0.16	26.9
8.0	15.0	23.0	0.53	48.0
3.0	20.0	23.0	0.89	55.9
2.3	21.2	23.5	1.26	62.0
1.9	22.0	23.9	1.64	67.0
1.7	20.1	21.8	1.98	71.5
1.5	22.0	23.5	2.36	75.4
1.5	22.5	24.0	2.74	79.4
1.5	22.5	24.0	3.12	83.3
1.4	21.9	23.3	3.48	87.0
1.5	22.3	23.8	3.86	91.0
1.4	22.2	23.6	4.23	94.7
1.3	22.7	24.0	4.61	98.1
1.5	22.5	24.0	4.99	102.1
1.0	22.5	23.5	5.37	104.7
1.2	22.6	23.8	5.74	107.9
1.3	21.7	23.0	6.11	111.3
1.4	22.6	24.0	6.49	115.0
1.2	22.4	23.6	6.86	118.2
1.3	21.6	22.9	7.22	121.6
1.4	22.0	23.4	7.59	125.3
1.2	22.4	23.6	7.97	128.4
1.3	22.0	23.3	8.34	131.9
1.3	22.3	23.6	8.71	135.3
1.3	22.0	23.3	9.08	138.7
1.4	22.1	23.5	9.45	142.4
1.3	22.9	24.2	9.83	145.9
1.3	21.5	22.8	10.19	149.3
1.1	21.3	22.4	10.55	152.2
1.2	22.0	23.2	10.91	155.4
2.5	44.1	46.6	11.65	161.9
2.0	44.5	46.5	12.39	167.2
2.2	43.0	45.2	13.10	173.0
2.2	42.9	45.1	13.82	178.8
2.2	44.6	46.8	14.56	184.6

ISOLATED SEVEN-SPOT PATTERN

RUN #32

INJECTION RATE(CC/HR):389.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
11.5	0.0	11.5	0.18	30.3
8.6	11.4	20.0	0.50	53.0
2.6	13.9	16.5	0.76	59.9
2.1	17.6	19.7	1.07	65.4
2.0	21.3	23.3	1.44	70.7
1.6	23.4	25.0	1.84	74.9
1.2	22.3	23.5	2.21	78.1
1.1	21.8	22.9	2.57	81.0
1.1	21.8	22.9	2.93	83.9
0.8	21.8	22.6	3.29	86.0
1.0	22.5	23.5	3.66	88.6
0.5	22.5	23.0	4.03	89.9
0.6	23.0	23.6	4.40	91.5
0.8	22.0	22.8	4.76	93.6
0.5	22.8	23.3	5.13	95.0
0.6	22.4	23.0	5.49	96.5
0.5	23.0	23.5	5.86	97.9
0.6	23.4	24.0	6.24	99.4

MODIFIED SEVEN-SPOT PATTERN

RUN #33

INJECTION RATE(CC/HR):389.0

OIL (CC)	WATER (CC)	TOTAL (CC)	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
11.5	0.0	11.5	0.18	30.3
7.0	15.5	22.5	0.54	48.8
2.6	20.4	23.0	0.90	55.7
2.0	21.0	23.0	1.27	60.9
1.3	20.0	21.3	1.60	64.4
1.2	21.3	22.5	1.96	67.5
1.2	21.1	22.3	2.31	70.7
0.9	20.9	21.8	2.66	73.1
1.1	22.2	23.3	3.03	76.0
0.8	22.1	22.9	3.39	78.1
1.2	21.6	22.8	3.75	81.2
0.7	22.0	22.7	4.11	83.1
0.5	22.5	23.0	4.47	84.4
0.5	23.5	24.0	4.85	85.7
0.5	21.5	22.0	5.20	87.0
1.0	22.5	23.5	5.57	89.7
0.5	23.0	23.5	5.94	91.0
0.5	23.0	23.5	6.32	92.3
0.5	23.0	23.5	6.69	93.6

APPENDIX D

Derivation of the Oil and Water Equations in Two Dimensions. Implicit Pressure-Explicit Saturation Procedure.

The equations describing the displacement process are obtained from a combination of Darcy's equation and the law of conservation of matter for each phase.

For the oil phase:

$$\frac{\partial}{\partial X}\left(\frac{k_x k_{ro}}{\mu_o B_o} \cdot \frac{\partial \Phi_o}{\partial X}\right) + \frac{\partial}{\partial Y}\left(\frac{k_y k_{ro}}{\mu_o B_o} \cdot \frac{\partial \Phi_o}{\partial Y}\right) + q_o^{*} = \frac{\partial}{\partial t}\left(\phi \frac{S_o}{B_o}\right) \qquad \text{(D-1)}$$

where q_o^{*} is the injection or production rate in m^3 per meter and per second, and is positive for injection, negative for production.

Equation D-1 can be written for each block i,j of the grid. After multiplying both sides by the block volume, the oil equation becomes:

$$\frac{\partial}{\partial X}\left(\frac{A_x k_{ro}}{\mu_o B_o} \cdot \frac{\partial \Phi_o}{\partial Y}\right)_{i,j} \Delta X + \frac{\partial}{\partial Y}\left(\frac{A_y k_y k_{ro}}{\mu_o B_o} \cdot \frac{\partial \Phi_o}{\partial Y}\right)_{i,j} \cdot \Delta Y$$

$$+ q_{o_{i,j}} = V_b \frac{\partial}{\partial t}\left(\phi \frac{S_o}{B_o}\right)_{i,j} \qquad \text{(D-2a)}$$

where

$$V_{b_{i,j}} = \Delta X_{i,j} \cdot \Delta Y_{i,j} \cdot h_{i,j}$$

Similarly for the water phase:

$$\frac{\partial}{\partial X}\left(\frac{A_x k_x k_{rw}}{\mu_w B_w} \cdot \frac{\partial \phi_w}{\partial X}\right)_{i,j} \cdot \Delta X + \frac{\partial}{\partial Y}\left(\frac{A_y k_y k_{rw}}{\mu_w B_w} \cdot \frac{\partial \phi_w}{\partial Y}\right)_{i,j} \cdot \Delta Y$$

$$+ q_{w_{i,j}} = V_b \frac{\partial}{\partial t}\left(\phi \frac{S_w}{B_w}\right)_{i,j} \qquad \text{(D-2b)}$$

The right-hand side terms in equations D-2a and D-2b are the accumulation terms . They are approximated as follows:

$$\frac{\partial}{\partial t}\left(\phi \frac{S_o}{B_o}\right)_{i,j} \equiv \Delta_t \left(\phi \frac{S_o}{B_o}\right)_{i,j} \qquad \text{(D-3)}$$

The operator Δ_t performs the following operation:

$$\Delta_t \left(\phi \frac{S_o}{B_o}\right) = S_o^{n+1} \phi^{n+1} \left(\frac{1}{B_o}\right)' \frac{\delta p_o}{\Delta t} + S_o^{n+1} \left(\frac{1}{B_o}\right)^n \phi' \frac{\delta p_o}{\Delta t}$$

$$+ \left(\frac{\phi}{B_o}\right)^n \frac{\delta S_o}{\Delta t} \qquad \text{(D-4)}$$

where

- superscripts refer to time levels n and n+1
- $\delta p_o = p_o^{n+1} - p_o^n$
- $\delta S_o = S_o^{n+1} - S_o^n$

We substitute in the expression of the scaling coefficient to obtain:

$$C_2 = \frac{2.592.10^{-5}.10^{-3}}{3.27.10^{-2}.\ 6.8.10^{-12}.0.36} = 0.5067$$

<u>Conversion to Field Conditions.</u>

Typical values of reservoir properties were chosen for this calculation.

k=100 mD

ϕ =20 percent

The values of interfacial tension and water viscosity were the same as in the model.

The scaling coefficient can be expressed as a function of the production rate:

$$C_2 = \frac{q.10^{-3}}{3.27.10^{-2}.\ \sqrt{.0.2.10^{-13}}} = 2.1624.10^5\ .\ q$$

To obtain a value of C_2 equal to 0.5067, the production rate must be:

$$q = \frac{0.5067}{216240} = 2.343.10^{-6}\ m^3/s.m$$

$$= 0.202\ m^3/day.m$$

This value is much lower than the usual rates of injection maintained during a waterflooding operation which are of the order of 6 m^3 per day and per meter of sand thickness.

APPENDIX C

<u>Calculation of the Scaling Coefficient.</u>

The injection rate necessary to obtain stabilization in a modified seven-spot pattern waterflood was found to be equal to 300 cc/hr.well (cf Figure 24), when the displaced fluid was kerosene. The scaling coefficient proposed by Rapoport, Carpenter, and Leas was used (ref 30).

$$C_2 = \frac{q\ \mu_w}{\sigma\ \sqrt{K\phi}}$$

where:

q is the production rate per unit of pay equal to twice the injection rate.

is the interfacial tension.

k is the absolute permeability.

is the porosity.

$$q = \frac{300.10^{-6}\ .\ 2}{0.00643.3600} = 2.592.10^{-5}\ m^3/s.m$$

$$\mu_w = 10^{-3}\ Pa.s$$

$$\sigma = 32.7\ \text{dynes/cm} = 3.27.10^{-2}\ N/m$$

$$k = 6.8.10^{-12}\ m^2$$

$$\left(\frac{1}{B_o}\right)' = \frac{\left(\frac{1}{B_o}\right)^{n+1} - \left(\frac{1}{B_o}\right)^{n}}{P_o^{n+1} - P_o^{n}}$$

$$\phi' = \frac{\phi^{n+1} - \phi^{n}}{P_o^{n+1} - P_o^{n}}$$

Approximation of the Left-hand Side Terms.

The oil transmissibility terms are equal to:

$$\frac{\partial}{\partial X}\left(\frac{A_x K_x K_{ro}}{\mu_o B_o} \cdot \frac{\partial \phi_o}{\partial X}\right) . \Delta X \equiv \Delta_x \left(T_{ox}^{n} \Delta_x P_o^{n+1} - T_{ox}^{n'} \Delta_x D \right),$$

(D-5)

where

$$\Delta_x \left(T_{ox}^{n} \Delta_x P_o^{n+1} \right) = T_{o_{i+\frac{1}{2},j}} \left(P_{o_{i+1,j}}^{n+1} - P_{o_{i,j}}^{n+1} \right)$$

$$- T_{o_{i-\frac{1}{2},j}}^{n} \left(P_{o_{i,j}}^{n+1} - P_{o_{i-1,j}}^{n+1} \right) \qquad \text{(D-6)}$$

and $T_{ox_{i+\frac{1}{2},j}}^{n}$ is the transmissibility in the X-direction from block(i,j) to block(i+1,j)

$$T_{ox_{i+\frac{1}{2},j}}^{n} = \left.\frac{A_x K_x K_{ro}}{\Delta X \mu_o B_o}\right|_{i+\frac{1}{2},j}^{n}$$

$$= \frac{A_{x_{i+1,j}} K_{x_{i+1,j}} A_{x_{i,j}} K_{x_{i,j}}}{A_{x_{i+1,j}} K_{x_{i+1,j}} \Delta X_{i,j} + A_{x_{i,j}} K_{x_{i,j}} \Delta_{x_{i+1,j}}} \cdot$$

$$\frac{1}{(\mu_o B_o)_{i+1,j}} \cdot \frac{1}{(\mu_o B_o)_{i,j}} \cdot K_{ro_{ups}} \qquad \text{(D-7)}$$

$$K_{ro_{ups}} = K^n_{ro_{i,j}} \quad \text{if} \quad \Phi_{o_{i,j}} \geq \Phi_{o_{i+1,j}}$$

$$= K^n_{ro_{i+1,j}} \quad \text{if} \quad \Phi_{o_{i,j}} < \Phi_{o_{i,j}}$$

$$T^{n'}_{ox_{i+\frac{1}{2},j}} = T^n_{ox_{i+\frac{1}{2},j}} \cdot \rho^n_o \cdot g \qquad \text{(D-8)}$$

Replacing both sides by the equivalent expressions, the oil equation D-2a becomes:

$$\Delta_x (T^n_{ox} \Delta_x P^{n+1}_o) - \Delta_x (T^{n'}_{ox} \Delta_x D) + \Delta_y (T^n_{oy} \Delta_y P^{n+1}_o)$$

$$- \Delta_y (T^{n'}_{oy} \Delta_y D) + q_o = \frac{V_b}{\Delta t} (S^{n+1}_o \phi^{n+1} (\frac{1}{B_o})' \cdot \delta P_o$$

$$+ S^{n+1}_0 \phi' \frac{1}{B^n_o} \delta P_o + (\frac{\phi}{B_o})^n \delta S_o) \qquad \text{(D-9)}$$

where the subscripts i,j have been omitted.

The saturation term in equation D-9 is:

$$\frac{V_b}{\Delta t} (\frac{\phi}{B_o})^n \delta S_o$$

Similarly, the water equation D-2b becomes:

$$\Delta_x (T_{wx}^{n} \Delta_x P_w^{n+1}) - \Delta_x (T_{wx}^{n'} \Delta_x D) + \Delta_y (T_{wy}^{n} \Delta_y P_w^{n+1})$$

$$- \Delta_y (T_{wy}^{n'} \Delta_y D) + q_o = \frac{V_b}{\Delta t} (S_w^{n+1} \phi^{n+1} (\frac{1}{b_w})' \delta P_w$$

$$+ S_w^{n+1} \phi' \frac{1}{B_w^n} \delta P_w + (\frac{\phi}{B_w})^n \delta S_w) \qquad \text{(D-10)}$$

The saturation term is:

$$\frac{V_b}{\Delta t} (\frac{\phi}{B_w})^n \delta S_w$$

We have two equations (water and oil equations D-9 and D-10) and four unknowns po, pw, So, Sw. We can write the oil equation in terms of water pressure and water saturation using the capillary pressure and saturation relationships D-11 and D-12.

$$P_{cow} = P_o - P_w \qquad \text{(D-11)}$$

$$S_o + S_w = 1 \qquad \text{(D-12)}$$

Then

$$\Delta P_o^{n+1} = \Delta P_w^{n+1} + \Delta P_{cow}^{n+1}$$

$$= \Delta \delta P_w + \Delta P_w^n + \Delta P_{cow}^{n+1} \qquad \text{(D-13)}$$

$$\delta S_o = -\delta S_w \tag{D.14}$$

Substituting D-13 and D-14 into D-9, the oil equation becomes:

$$\Delta_x (T^n_{ox}\Delta_x\delta P_w) + \Delta_x (T^n_{ox}\Delta_x P^n_w) + \Delta_x (T^n_{ox}\Delta_x P^{n+1}_{cow})$$

$$- \Delta_x (T^{n'}_{ox}\Delta_x D) + \Delta_y (T^n_{oy}\Delta_w\Delta_y P^n_w) + \Delta_y (T^n_{oy}\Delta_y P^{n+1}_{cow})$$

$$\Delta_y (T^{n'}_{oy}\Delta_y D) + q_o = \frac{V_b}{\Delta t} (S^{n+1}_o \phi^{n+1} (\frac{1}{B_o}) \delta P_w$$

$$+ S^{n+1}_o \phi^{n+1} (\frac{1}{B_o})' \delta P_{cow} + S^{n+1}_o \phi' \frac{1}{B^n_o} \delta P_w + \delta P_{cow}$$

$$- (\frac{\phi}{B_o})^n \delta S_w) \tag{D-15}$$

The saturation term is:

$$-\frac{V_b}{\Delta t} (\frac{\phi}{B_o})^n \delta S_w$$

Now we multiply the oil equation D-15 by Bo and the water equation D-10 by Bw, in order to eliminate the saturation terms when adding both equations. Thus we obtain 1 equation with 1 unknown, Pw. The resulting equation is

$$B^n_o\Delta_x (T^n_{ox}\Delta_x\delta P_w) + B^n_o\Delta_x (T^n_{ox}\Delta_x P^n_w)$$

$$+ B^n_o\Delta_x (T^n_{ox}\Delta_x P^{n+1}_{cow}) - B^n_o\Delta_x (T^{n'}_{ox}\Delta_x D)$$

$$+ B_o^n \Delta_y (T_{oy}^n \Delta_y P_w) + B_o^n \Delta_y (T_{oy}^n \Delta_y P_w^n)$$

$$+ B_o^n \Delta_y (T_{oy}^n \Delta_y P_{cow}^{n+1}) - B_o^n \Delta_y (T_{oy}^{n'} \Delta_y D) + B_o^n q_o^n$$

$$+ B_w^n \Delta_x (T_{wx}^n \Delta_x \delta P_w) + B_w^n \Delta_x (T_{wx}^n \Delta_x P_w^n)$$

$$- B_w^n \Delta_x (T_{wx}' \Delta_x D) + B_w^n \Delta_y (T_{wy}^n \Delta_y \delta P_w)$$

$$+ B_w^n \Delta_y (T_{wy}^n \Delta_y P_w^n) - B_w^n \Delta_y (T_{wy}' \Delta_y D) + B_w^n q_w$$

$$= \frac{V_b}{\Delta t} (B_o^n S_o^{n+1} \phi^{n+1} (\frac{1}{B_o})' \delta P_w + B_o^n S_o^{n+1} \phi^{n+1} (\frac{1}{B_o})' \delta P_{cow}$$

$$+ S_o^{n+1} \phi' (\delta P_w + \delta P_{cow}) + B_w^n S_w^{n+1} \phi^{n+1} (\frac{1}{B_w})' \delta P_w$$

$$+ S_w^{n+1} \phi' \delta P_w \qquad \text{(D-16)}$$

We substitute the expressions of the oil transmissibility terms D-6 into Equation D-16. The resulting equation can take the following form:

$$B_{i,j} \delta P_{w_{i,j-1}} + D_{i,j} \delta P_{w_{i-1,j}} + E_{i,j} \delta P_{w_{i,j}}$$

$$+ F_{i,j} \delta P_{w_{i+1,j}} + H_{i,j} \delta P_{w_{i,j+1}} = Q_{i,j} \qquad \text{(D-17)}$$

where:

$$B_{i,j} = B_{o_{i,j}}^n \cdot T_{oy_{i,j-\frac{1}{2}}} + B_{w_{i,j}}^n \cdot T_{wy_{i,j-\frac{1}{2}}}$$

$$D_{i,j} = B_{o_{i,j}}^n \cdot T_{ox_{i-\frac{1}{2},j}} + B_{w_{i,j}}^n \cdot T_{wx_{i-\frac{1}{2},j}}$$

$$F_{i,j} = B^n_{o_{i,j}} \cdot T_{ox_{i+\frac{1}{2},j}} + B^n_{w_{i,j}} \cdot T_{ox_{i+\frac{1}{2},j}}$$

$$H_{i,j} = B^n_{o_{i,j}} \cdot T_{oy_{i,j+\frac{1}{2}}} + B^n_{w_{i,j}} \cdot T_{oy_{i,j+\frac{1}{2}}}$$

$$E_{i,j} = -B_{i,j} - D_{i,j} - F_{i,j} - H_{i,j} - \frac{\Gamma_{i,j}}{\Delta t}$$

$$Q_{i,j} = \frac{V_b}{\Delta t} \Big(B^n_o S^{n+1}_o \phi^{n+1} \left(\frac{1}{B_o}\right)' \delta P_{cow_{i,j}}$$

$$+ S^{n+1}_o \phi' \delta P_{cow_{i,j}} - B^n_o q_{o_{i,j}} - B^n_w q_{w_{i,j}}$$

$$- B^n_o \Delta_x (T^n_{ox} \Delta_x P^n_w) - B^n_o \Delta (T^n_{ox} \Delta_x P^{n+1}_{cow}) + B^n_o \Delta_x (T^{n'}_{ox} \Delta_x D)$$

$$- B^n_w \Delta_y (T^n_{wy} \Delta_y P^n_w) + B^n_w \Delta_x (T^{n'}_{wx} \Delta_x D) \;\; B^n_w \Delta_y (T^n_{wy} \Delta_y P^n_w)$$

$$- B^n_o \Delta_y (T^n_{oy} \Delta_y P^n_w) \cdot B^n_o \Delta_y (T^n_{oy} \Delta_y P^{n+1}_{cow}) + B^n_o \Delta_y (T^{n'}_{oy} \Delta_y D)$$

$$+ B^n_w \Delta_y (T^{n'}_{wy} \Delta_y D) \Big)$$

$$\Gamma_{i,j} = V_b \Big(B^n_o S^{n+1}_o \phi^{n+1} \left(\frac{1}{B_o}\right)' + S^{n+1}_o \phi'$$

$$+ B^n_w S^{n+1}_w \phi^{n+1} \left(\frac{1}{B_w}\right)' + S^{n+1}_w \phi \Big)$$

Solution Procedure

DouglasRachford's A.D.I.P.

The equation to solve has the form:

$$B_{i,j}\delta P_{w_{i,j-1}} + D_{i,j}\delta P_{w_{i-1,j}} + E_{i,j}\delta P_{w_{i,j}}$$

$$+ F_{i,j}\delta P_{w_{i+1,j}} + H_{i,j}\delta P_{w_{i,j+1}} = Q_{i,j} \qquad \text{(D-17)}$$

where

$$\delta P_{w_{i,j}} = P^{n+1}_{w_{i,j}} - P^{n}_{w_{i,j}}$$

is the pressure difference during 1 time step.

Let us split the term $E_{i,j}$ into 3 parts

$$E_{i,j} = E_{x_{i,j}} + E_{y_{i,j}} - \frac{\Gamma}{\Delta t}$$

where

$$E_{x_{i,j}} = -D_{i,j} - F_{i,j}$$

is the opposite of the sum of transmissibility terms in the X-direction

$$E_{y_{i,j}} = -B_{i,j} - H_{i,j}$$

is the opposite of the sum of transmissibility terms in the Y-direction

The solution to equation D-17 is obtained by an iterative procedure in two time steps.

Iteration number m:

$$\delta P^{(m)} \xrightarrow{\text{step 1}} \delta P^{*} \xrightarrow{\text{step 2}} \delta P^{(m+1)}$$

The iterations are stopped when convergence is obtained:

$$\max_{i,j} \left| \delta P^{(m+1)} - \delta P^{(m)} \right| < \varepsilon_p .$$

Step 1. $\delta P^{(m)} = P^{(m)} - P^{n} \implies \delta P^{*} = P^{*} - P^{n}$

P* are intermediate values of pressures with no physical meaning which arise from the solution procedure itself. Let us rewrite Equation D-17 so that the left-hand-side is composed of the terms corresponding to the X-direction and the right-hand-side of all the other terms with their values at the iteration level m (known values).

$$D\delta P^{*}_{i-1,j} + E_x \delta P^{*}_{i,j} + F\delta P^{*}_{i+1,j} = -B\delta P^{(m)}_{i,j}$$

$$- H\delta P^{(m)}_{i,j+1} + \frac{\Gamma}{\Delta t} \cdot \delta P^{*}_{i,j} + Q_{i,j} - E_y \delta P^{(m)}_{i,j}$$

$$+ \alpha_m \sum_4 T(\delta P^*_{i,j} - \delta P^{(m)}_{i,j}) \qquad \text{(D-18)}$$

where

. α_m is an iteration parameter which will speed the convergence of the iteration process.

. $\sum_4 T = B + D + F + H$ is the sum of transmissibility terms around the block.

Writing equation D-18 for j=1 and i=1, 2...Nx, we obtain a set of tridiagonal equations where the unknowns are $\delta P^*_{i-1,j}$, $\delta P^*_{i,j}$, $\delta P^*_{i+1,j}$.

This system is solved by Thomas's algorithm (36) and the procedure repeated for j=2, 3...until all the grid is covered. Thus we have the values of δP^* for each block of the grid.

<u>Step 2.</u> $\delta P^* = P^* - P^n \Longrightarrow \delta P^{(m+1)} = P^{(m+1)} - P^n$

Similarly to what was done in step 1, Equation D-17 is written with the terms in the y-direction implicit.

$$\delta P^{(m+1)}_{i,j-1} + E_y \delta P^{(m+1)}_{i,j} + H\delta P^{(m+1)}_{i,j+1} =$$

$$-D\delta P^*_{i-1,j} - E_x \delta P^*_{i,j} - F\delta P^*_{i+1,j} + \frac{\Gamma}{\Delta t} \cdot \delta P^{(m+1)}_{i,j}$$

$$+ q_{i,j} + \alpha_m \sum_4 T(P^{(m+1)}_{i,j} - P^{(m)}_{i,j}) \qquad \text{(D-19)}$$

Equation D-19 is written for i=1 and j=1, 2...The set of equations obtained is solved by Thomas's algorithm, and then the procedure repeated for i=2, 3...

At the end of step 2 we have values of $\delta p^{(m+1)}$ for each block of the grid. The convergence criterion is applied to decide if another iteration is needed.

Iteration Parameter.

Iterations parameters are necessary to speed the convergence of iterative processes. For an areal grid (two-dimensional, horizontal grid) the parameter described here can be used (35). This parameter is cyclic, after five iterations it's value comes back to the original value. The five values it may take are all comprised between zero and one. The iteration parameter remains constant during the two steps of an iteration.

Let us define $\rho_1 = \left[\frac{K_y}{K_x} \cdot \frac{\Delta X}{\Delta Y}\right]^2$ and $\rho_2 = \left[\frac{K_x}{K_y} \cdot \frac{\Delta Y}{\Delta X}\right]^2$

$$\alpha_{max} = 1 - \min(i,j) \quad \frac{\pi^2}{2N_x^2(1+\rho_1)} \; , \; \frac{\pi^2}{2N_y^2(1+\rho_2)}$$

$$\alpha_m = 1 - (1-\alpha_{max})^{\frac{m}{c-1}} .$$

where m= 1, 2, 3, 4, 5.

NUMERICAL RESULTS
CONFINED SEVEN-SPOT PATTERN

RUN # 2

Conditions identical to run 1 but a 21 by 11 grid was used.

OIL VISCOSITY(CP): 4.47
MOBILITY RATIO: 2.70

CUMULATIVE OIL PRODUCTION (CC)	CUMULATIVE WATER PRODUCTION (CC)	WATER-OIL RATIO	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
9.58	0.13	0.11	0.13	21.63
13.88	2.81	0.77	0.23	31.33
16.59	9.13	3.15	0.35	37.45
18.49	16.44	4.58	0.47	41.74
20.25	25.92	6.19	0.63	45.71
21.41	33.80	7.42	0.75	48.33
22.44	42.13	8.71	0.87	50.65
23.80	55.26	10.70	1.07	53.72

NUMERICAL RESULTS
CONFINED SEVEN-SPOT PATTERN

RUN # 3

OIL VISCOSITY(CP): 8.00
MOBILITY RATIO: 4.80

CUMULATIVE OIL PRODUCTION (CC)	CUMULATIVE WATER PRODUCTION (CC)	WATER-OIL RATIO	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
4.22	0.00	0.00	0.06	9.53
6.74	0.21	0.20	0.09	15.21
7.76	0.55	0.44	0.11	17.52
9.36	1.68	0.99	0.15	21.13
11.05	4.08	1.87	0.21	24.94
12.29	6.93	2.69	0.26	27.74
14.15	13.25	4.17	0.37	31.94
15.76	21.18	5.74	0.50	35.58
17.03	29.45	7.26	0.63	38.44
18.23	39.15	8.87	0.78	41.15
19.26	49.03	10.45	0.93	43.48
20.26	60.30	12.15	1.09	45.73
21.19	72.45	13.94	1.27	47.83

APPENDIX E

NUMERICAL RESULTS
CONFINED SEVEN-SPOT PATTERN

RUN # 1

OIL VISCOSITY(CP): 4.47
MOBILITY RATIO: 2.70

CUMULATIVE OIL PRODUCTION (CC)	CUMULATIVE WATER PRODUCTION (CC)	WATER-OIL RATIO	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
6.04	0.00	0.00	0.08	13.63
7.68	0.01	0.01	0.10	17.34
9.13	0.33	0.27	0.13	20.61
10.30	0.78	0.51	0.15	23.25
11.37	1.48	0.82	0.17	25.67
12.97	3.29	1.45	0.22	29.28
14.21	5.47	2.05	0.27	32.08
15.91	9.73	3.01	0.35	35.91
17.75	16.41	4.28	0.46	40.07
18.80	21.32	5.12	0.54	42.44
19.96	27.83	6.17	0.65	45.06
21.06	35.25	7.34	0.76	47.54
22.10	43.58	8.57	0.89	49.89
23.10	52.80	9.91	1.03	52.14
23.77	59.80	11.00	1.13	53.66
24.52	68.42	12.28	1.26	55.35

<u>Water Equation Solved for Water Saturation.</u>

The water equation (10) may be rewritten

$$\frac{V_b}{\Delta t} \cdot \left(\frac{\phi}{B_w} \right)^n \cdot \delta S_w = \Delta_x \left(T_{wx}^n \Delta_x P_w^{n+1} \right) - \Delta_x \left(T_{wx}^{n'} \Delta_x D \right)$$

$$+ \Delta_y \left(T_{wy}^n \Delta_y P_w^{n+1} \right) - \Delta_y \left(T_{wy}^{n'} \Delta_y D \right) + Q_w$$

$$- \frac{V_b}{\Delta t} \left(S_w^{n+1} \phi^{n+1} \left(\frac{1}{B_w} \right)' \delta P_w + S_w^{n+1} \phi' \frac{1}{B_w^n} \delta P_w \right) = A$$

Thus

$$\delta S_w = \frac{A \Delta t B_w^n}{V_b \phi^n}$$

Saturation changes are calculated after each iteration and the convergence is checked. Iterations are stopped when

$$\max_{i,j} \left| \delta P^{(m+1)} - \delta P^{(m)} \right| < \varepsilon_p$$

and

$$\max_{i,j} \left| \delta S_w^{(m+1)} - \delta S_w^{(m)} \right| < \varepsilon_s$$

where ε_p and ε_s are chosen values.

NUMERICAL RESULTS
CONFINED SEVEN-SPOT PATTERN

RUN # 4

OIL VISCOSITY(CP): 30.00
MOBILITY RATIO: 18.00

CUMULATIVE OIL PRODUCTION (CC)	CUMULATIVE WATER PRODUCTION (CC)	WATER-OIL RATIO	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
0.14	0.0	0.0	0.00	0.31
3.90	0.32	0.41	0.06	8.80
5.96	2.34	1.65	0.11	13.45
7.89	7.24	3.56	0.21	17.81
8.93	11.65	5.00	0.28	20.16
10.27	19.86	7.28	0.41	23.18
11.42	29.61	9.73	0.56	25.78
12.23	38.34	11.74	0.69	27.61
12.79	45.28	13.25	0.79	28.87
13.29	52.28	14.70	0.89	30.00
13.94	62.53	16.77	1.04	31.47
14.58	73.89	18.95	1.20	32.91
15.17	85.83	21.14	1.37	34.24

NUMERICAL RESULTS
CONFINED SEVEN-SPOT PATTERN

RUN # 5

OIL VISCOSITY(CP):100.00
MOBILITY RATIO: 60.00

CUMULATIVE OIL PRODUCTION (CC)	CUMULATIVE WATER PRODUCTION (CC)	WATER-OIL RATIO	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
1.74	0.13	0.32	0.03	3.93
2.89	1.02	1.29	0.05	6.52
3.66	2.38	2.24	0.08	8.26
4.46	4.65	3.48	0.12	10.07
5.13	7.39	4.79	0.17	11.58
5.92	11.81	6.61	0.24	13.36
6.83	19.07	9.14	0.35	15.42
7.63	27.47	11.89	0.48	17.22
8.05	32.76	13.67	0.55	18.17
8.62	41.22	16.04	0.68	19.46
9.23	51.86	19.09	0.83	20.84
9.79	63.48	22.21	0.99	22.10
10.23	74.02	24.93	1.14	23.09

NUMERICAL RESULTS
ISOLATED SEVEN-SPOT PATTERN

RUN # 6

OIL VISCOSITY(CP): 1.38
MOBILITY RATIO: 0.83

CUMULATIVE OIL PRODUCTION (CC)	CUMULATIVE WATER PRODUCTION (CC)	WATER-OIL RATIO	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
19.73	0.10	0.07	0.27	44.54
24.11	1.04	0.40	0.34	54.42
28.71	4.62	1.19	0.45	64.81
32.49	10.66	2.00	0.58	73.34
35.38	17.38	2.64	0.71	79.86
37.48	23.45	3.12	0.83	84.60
39.31	29.59	3.58	0.93	88.74
41.05	36.24	4.05	1.05	92.66
42.56	42.70	4.50	1.16	96.07
43.78	48.43	4.89	1.25	98.83

NUMERICAL RESULTS
ISOLATED SEVEN-SPOT PATTERN

RUN # 7

OIL VISCOSITY(CP): 4.47
MOBILITY RATIO: 2.70

CUMULATIVE OIL PRODUCTION (CC)	CUMULATIVE WATER PRODUCTION (CC)	WATER-OIL RATIO	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
5.32	0.0	0.0	0.07	12.01
9.79	0.02	0.02	0.13	22.10
11.89	0.17	0.12	0.16	26.84
16.52	2.09	0.82	0.25	37.29
20.34	7.26	1.91	0.37	45.91
23.02	13.58	2.79	0.50	51.96
25.13	20.26	3.53	0.62	56.73
27.52	29.73	4.44	0.78	62.12
29.60	39.91	5.32	0.94	66.82
31.22	49.13	6.05	1.09	70.47
32.23	55.49	6.53	1.19	72.75

NUMERICAL RESULTS
ISOLATED SEVEN-SPOT PATTERN

RUN # 8

OIL VISCOSITY(CP): 30.00
MOBILITY RATIO: 18.00

CUMULATIVE OIL PRODUCTION (CC)	CUMULATIVE WATER PRODUCTION (CC)	WATER-OIL RATIO	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
5.13	0.45	0.43	0.08	11.58
7.85	3.18	1.66	0.15	17.72
9.73	7.44	2.88	0.23	21.96
11.79	15.07	4.58	0.36	26.61
13.34	23.32	6.02	0.50	30.11
15.01	34.74	7.69	0.67	33.88
16.07	43.50	8.84	0.81	36.28
17.04	53.37	9.94	0.95	38.46
18.12	64.20	11.31	1.12	40.90

NUMERICAL RESULTS
ISOLATED SEVEN-SPOT PATTERN

RUN # 9

OIL VISCOSITY(CP):100.00
MOBILITY RATIO: 60.00

CUMULATIVE OIL PRODUCTION (CC)	CUMULATIVE WATER PRODUCTION (CC)	WATER-OIL RATIO	CUMULATIVE PRODUCTION (PPV)	CUMULATIVE OIL RECOVERY (%IOIP)
2.02	0.11	0.25	0.03	4.56
4.79	3.30	2.29	0.11	10.81
6.60	7.14	3.80	0.19	14.90
7.08	11.74	5.22	0.26	15.98
8.19	18.56	7.11	0.36	18.49
9.00	24.90	8.58	0.46	20.32
9.70	31.36	9.97	0.56	21.90
10.72	42.61	12.18	0.72	24.20
11.38	51.15	13.71	0.85	25.69
11.96	59.60	15.13	0.97	27.00

APPENDIX F

```
C      WATERFLOOD SIMULATION. MAIN PROGRAM.
C      THIS PROGRAM COMPUTES OIL AND WATER RECOVERY DATA
C      FOR ISOLATED AND CONFINED PATTERN WATERFLOODS.
C      ALL DATA MUST BE IN METRIC UNITS.
C
C
       DIMENSION SO1(15),SW1(15),
      1XKRO(15),BKRO(15),CKRO(15),DKRO(15),
      2XKRW(15),BKRW(15),CKRW(15),DKRW(15),
      3PEROIL(25,15),PERWAT(25,15),
      4QOIL(25,15),QWATER(25,15),Q(25,15),
      5SON(25,15),SWN(25,15),SOM(25,15),SWM(25,15),
      6PON(25,15),POM(25,15),
      7TRANSX(25,15),TRANSY(25,15),TOX(25,15),
      8TWX(25,15),TOY(25,15),TWY(25,15),
      9B(25,15),D(25,15),F(25,15),H(25,15),EX(25,15),
      1EY(25,15),SIGMA4(25,15),B1(600),D1(600),F1(600),
      2,H1(600),E1(600),
      3 RESIDU(25,15),RESID1(600),G(600),W(600),
      4VB(25,15),DPSTAR(25,15),DP2STA(25,15),
      5DELTAX(25),DELTAY(15),INGRID(25,15),
      6AA(25,15),BB(25,15)
       COMMON/ING/NX,NY,INGRID
       COMMON/GRI/PAY,PORI,PERM,DELTAX,DELTAY,SAT
       COMMON/PROP/VISOIL,VISWAT,SOR,SWR,PHI
       COMMON/OUTPUT/SO1,SW1,XKRO,XKRW
       COMMON/VAR/IKRO,IKRW
       COMMON/RESU/DELTAT
C
C
C      TO READ TABULATED DATA:POROSITY,RESIDUAL SATURATIONS
C      AND RELATIVE PERMEABILITIES:
       CALL READ
C
C      TO DEFINE GRID AND READ FORMATION PROPERTIES:


       CALL GRID
C    CALCULATE PRESSURES AND SATURATIONS AT EACH BLOCK
C    AT TIME T.
C    USE DOUGLAS-RACHFORD'S ITERATIVE SOLUTION PROCEDURE.
C
C
C    GENERATE ITERATION PARAMETER;
C                  CYCLIC VARIATION;5 PARAMETERS IN A CYCLE.
C
       PI=2.*ARSIN(1.)
       ISTORE=1
```

```
      DO 1 J=1,NY
      DO 1 I=1,NX
      IF(INGRID(I,J).EQ.0) GO TO 1
      BA=DELTAX(I)/DELTAY(J)
      RHO1=BA*BA
      RHO2=1./RHO1
      AMIN=PI*PI/(2.*NX*NX*(1.+RHO1))
      A2=PI*PI/(2*NY*NY*(1+RHO2))
      IF(A2.LE.AMIN) AMIN=A2
      IF(ISTORE.EQ.1) STORE=AMIN
      IF(AMIN.LE.STORE) STORE=AMIN
      ISTORE=ISTORE+1
    1 CONTINUE
C
      CALL SPLINE(IKRO,SO1,XKRO,BKRO,CKRO,DKRO)
      CALL SPLINE(IKRW,SW1,XKRW,BKRW,CKRW,DKRW)
      READ(5,999) DELTAT,TIME
      CUMOIL=0.
      CUMWAT=0.
      READ(5,1002) NINJ,QINJ,PINJ
      READ(5,1000) IPRO,JPRO
C
C   CALCULATE TERMS CONSTANT FOR EACH BLOCK IN
C   THE EXPRESSION OF TRANSMISSIBILITIES.
C   STORE IN ARRAYS: TRANSX(I,J)
C                    TRANSY(I,J)
C
      DO 12 I=1,NX
      DO 12 J=1,NY
      IF(INGRID(I,J).EQ.0) GO TO 11
      IF(I-NX) 3,2,3
    2 TRANSX(I,J)=0.
      GO TO 6
    3 IF(INGRID(I,J)-INGRID(I+1,J)) 4,5,4
    4 TRANSX(I,J)=0.
      GO TO 6
    5 AXI=DELTAY(J)*PAY*PERM
      AAX=AXI*AXI
      ABX=AXI*(DELTAX(I)+DELTAX(I+1))
      TRANSX(I,J)=AAX/ABX
    6 IF(J-NY) 8,7,8
    7 TRANSY(I,J)=0.
      GO TO 12
    8 IF(INGRID(I,J)-INGRID(I,J+1)) 9,10,9
    9 TRANSY(I,J)=0.
      GO TO 12
   10 AYI=DELTAX(I)*PAY*PERM
      AAY=AYI*AYI
      ABY=AYI*(DELTAY(J)+DELTAY(J+1))
      TRANSY(I,J)=AAY/ABY
      GO TO 12
   11 TRANSX(I,J)=0.
      TRANSY(I,J)=0.
   12 CONTINUE
```

```
C
C   INITIALIZE OIL PRESSURE AND OIL SATURATION
C   BEFORE FIRST TIME STEP
C
 13  DO 14 J=1,NY
     DO 14 I=1,NX
     PON(I,J)=PORI
     SON(I,J)=SAT
     VB(I,J)=DELTAX(I)*DELTAY(J)*PAY
 14  CONTINUE
C
C
C   CALCULATE OIL AND WATER PROPERTIES FOR EACH BLOCK,
C   AT TIME LEVEL N.
C
C   PEROIL=RELATIVE PERMEABILITY TO OIL
C   PERWAT=RELATIVE PERMEABILITY TO WATER
C   THE ABOVE VALUES ARE INTERPOLATED FROM TABULATED
C   DATA USING THE SUBROUTINE SPLINE AND FUNCTION SEVAL.
C   VISOIL=OIL VISCOSITY
C   VISWAT=WATER VISCOSITY
C
C
C   READ PRODUCTION AND INJECTION RATES.
C   NINJ=NUMBER OF INJECTORS.
C   QINJ=INJECTION RATE (M3/S)
C   PINJ=INITIAL INJECTION PRESSURE.
C   IPRO,JPRO=COORDINATES OF PRODUCER.
C
     DO 15 J=1,NY
     DO 15 I=1,NX
     IF(INGRID(I,J).EQ.0) GO TO 15
     QWATER(I,J)=0.
     QOIL(I,J)=0.
 15  CONTINUE
     DO 16 L=1,NINJ
     READ(5,1000) I,J
     PON(I,J)=PINJ
 16  QWATER(I,J)=QINJ
     SUM=0.
     DO 17 L=1,NY
     DO 17 K=1,NX
     IF(INGRID(K,L).EQ.0) GO TO 17
     SUM=SUM+QWATER(K,L)
 17  CONTINUE
C
C
C   CALCULATE FLUIDS PROPERTIES.
C
     N=1
C---------------
C   NEW TIME STEP.
C---------------
 18  DO 24 J=1,NY
```

```
      DO 24 I=1,NX
      SWN(I,J)=1.-SON(I,J)
      IF(SON(I,J)-SOR) 19,19,20
 19   PEROIL(I,J)=0.
      GO TO 21
 20   PEROIL(I,J)=SEVAL(IKRO,SON(I,J),SO1,XKRO,BKRO,CKRO
     *,DKRO)
 21   IF(SWN(I,J)-SWR) 22,22,23
 22   PERWAT(I,J)=0.
      GO TO 24
 23   PERWAT(I,J)=SEVAL(IKRW,SWN(I,J),SW1,XKRW,BKRW,CKRW
     *DKRW)
 24   CONTINUE
C
C   FOR A PRODUCING WELL,OIL AND WATER PRODUCTIONS ARE
C   PROPORTIONAL TO THE OIL/WATER MOBILITY RATIO.
C   CALCULATE WATER PRODUCTION RATE AND TOTAL OIL AND WATER
C   PRODUCTIONS DURING A TIME STEP.
      OIL=0.
      WATER=0.
      RATIO=PERWAT(IPRO,JPRO)*VISOIL/(PEROIL(IPRO,JPRO)*
     *VISWAT)
      QOIL(IPRO,JPRO)=-SUM/(RATIO+1.)
      QWATER(IPRO,JPRO)=QOIL(IPRO,JPRO)*RATIO
      OIL=QOIL(IPRO,JPRO)*DELTAT
      CUMOIL=CUMOIL+OIL
      WATER=QWATER(IPRO,JPRO)*DELTAT
      CUMWAT=CUMWAT+WATER
C
C   CALCULATE TRANSMISSIBILITIES AT TIME LEVEL N
C   IF WATER SATURATION OF UPSTREAM BLOCK
C   OR TWY =0.
C   IF OIL SATURATION OF UPSTREAM BLOCK
C   OR TOY =0.
C
      DO 30 J=1,NY
      DO 30 I=1,NX
      IF(TRANSX(I,J)) 25,25,26
 25   TOX(I,J)=0.
      TWX(I,J)=0.
      GO TO 27
 26   AB=2/VISOIL
      UPS=PEROIL(I,J)
      IF(PON(I+1,J).GT.PON(I,J)) UPS=PEROIL(I+1,J)
      TOX(I,J)=TRANSX(I,J)*AB*UPS
      AB=2/VISWAT
      UPS=PERWAT(I,J)
      IF(PON(I+1,J).GT.PON(I,J)) UPS=PERWAT(I+1,J)
      TWX(I,J)=TRANSX(I,J)*AB*UPS
 27   IF(TRANSY(I,J)) 28,28,29
 28   TOY(I,J)=0.
      TWY(I,J)=0.
      GO TO 30
 29   AB=2/VISOIL
```

```
      UPS=PEROIL(I,J)
      IF(PON(I,J+1).GT.PON(I,J)) UPS=PEROIL(I,J+1)
      TOY(I,J)=TRANSY(I,J)*AB*UPS
      AB=1/VISWAT
      UPS=PERWAT(I,J)
      IF(PON(I,J+1).GT.PON(I,J)) UPS=PERWAT(I,J+1)
      TWY(I,J)=TRANSY(I,J)*AB*UPS
   30 CONTINUE
C
C     CALCULATE COEFFICIENTS FOR EACH BLOCK:B,D,E,F,H.
C
      DO 38 J=1,NY
      DO 38 I=1,NX
      IF(INGRID(I,J).EQ.0) GO TO 38
      IF(I-1) 31,31,34
   31 D(I,J)=0.
      IF(J-1) 32,32,33
   32 B(I,J)=0.
      GO TO 37
   33 B(I,J)=TOY(I,J-1)+TWY(I,J-1)
      GO TO 37
   34 D(I,J)=TOX(I-1,J)+TWX(I-1,J)
      IF(J-1) 35,35,36
   35 B(I,J)=0.
      GO TO 37
   36 B(I,J)=TOY(I,J-1)+TWY(I,J-1)
   37 F(I,J)=TOX(I,J)+TWX(I,J)
      H(I,J)=TOY(I,J)+TWY(I,J)
      EX(I,J)=-D(I,J)-F(I,J)
      EY(I,J)=-B(I,J)-H(I,J)
      SIGMA4(I,J)=B(I,J)+D(I,J)+F(I,J)+H(I,J)
   38 CONTINUE
C
C     CALCULATE Q(I,J).
C
      DO 39 J=1,NY
      DO 39 I=1,NX
      IF(INGRID(I,J).EQ.0) GO TO 39
      B0=-QOIL(I,J)
      B2=-QWATER(I,J)
      B3=-DELTA(TOX,PON,1,I,J)
      B4=-DELTA(TWX,PON,1,I,J)
      B5=-DELTA(TOY,PON,2,I,J)
      B6=-DELTA(TWY,PON,2,I,J)
      Q(I,J)=B0+B2+B3+B4+B5+B6
   39 CONTINUE
C
C     ITERATION #M
C
C     INITIALIZE PRESSURE AND SATURATION
C
      M=1
      DO 40 J=1,NY
      DO 40 I=1,NX
```

```
      POM(I,J)=PON(I,J)
      SOM(I,J)=SON(I,J)
      SWM(I,J)=SWN(I,J)
      POM(I,J)=PON(I,J)
   40 CONTINUE
C
C     START ITERATION #M;CALCULATE ITERATION PARAMETER ALPHA.
C
   41 MM=MOD(M,6)
      IF(MM.EQ.0) MM=MM+1
      ALPHA=1-STORE**(MM*    5)
C
C                *CALCULATE RESIDUAL
C
      DO 48 I=1,NX
      DO 48 J=1,NY
      IF(INGRID(I,J).EQ.0) GO TO 48
      IF(J-1) 43,42,43
   42 C1=0.
      GO TO 44
   43 C1=B(I,J)*(POM(I,J-1)-PON(I,J-1))
   44 C2=(EY(I,J)+ALPHA*SIGMA4(I,J))*(POM(I,J)-PON(I,J))
      IF(J-NY) 46,45,46
   45 C3=0.
      GO TO 47
   46 C3=H(I,J)*(POM(I,J+1)-PON(I,J+1))
   47 RESIDU(I,J)=Q(I,J)-C1-C2-C3
   48 CONTINUE
C
C
C              STAGE 1.WRITE EQUATIONS WITH TERMS IN X
C     DIRECTION UNKNOWN.SOLVE FOR PRESSURE CHANGES USING
C     THOMAS'S ALGORITHM.
C
      DO 58 J=1,NY
      ISTART=0
      DO 49 I=1,NX
      ISTART=ISTART+1
      IF(INGRID(I;J).NE.0) GO TO 50
   49 CONTINUE
   50 E1(1)=EX(ISTART,J)-ALPHA*SIGMA4(ISTART,J)
      D1(1)=D(ISTART,J)
      F1(1)=F(ISTART,J)
      RESID1(1)=RESIDU(ISTART,J)
      IS=ISTART+1
      II=1
   51 DO 53 I=IS,NX
      IF(INGRID(I,J).NE.0) GO TO 52
      ISTOP=I-1
      GO TO 55
   52 II=II+1
      E1(II)=EX(I,J)-ALPHA*SIGMA4(I,J)
      D1(II)=D(I,J)
      F1(II)=F(I,J)
```

```
      RESID1(II)=RESIDU(I,J)
   53 CONTINUE
      CALL THOMAS(D1,E1,F1,RESID1,G,W,II)
      JJ=1
      DO 54 I=ISTART,NX
      DPSTAR(I,J)=G(JJ)
   54 JJ=JJ+1
       GO TO 58
   55 CALL THOMAS(D1,E1,F1,RESID1,G,W,II)
      JJ=1
      DO 56 I=ISTART,ISTOP
      DPSTAR(I,J)=G(JJ)
      JJ=JJ+1
   56 CONTINUE
      ISTART=ISTOP+1
      II=0
      DO 57 I=ISTART,NX
        IS=I
      IF(INGRID(I,J).NE.0) GO TO 51
   57 CONTINUE
   58 CONTINUE
C
C              STAGE 2.Y DIRECTION
C
      DO 65 I=1,NX
      DO 65 J=1,NY
      IF(INGRID(I,J).EQ.0) GO TO 65
      IF(I-1) 60,59,60
   59  R1=0.
      GO TO 61
   60 R1=D(I,J)*DPSTAR(I-1,J)
   61 R2=EX(I,J)*DPSTAR(I,J)
      IF(I-NX) 63,62,63
   62 R3=0.
      GO TO 64
   63 R3=F(I,J)*DPSTAR(I+1,J)
   64 R4=ALPHA*SIGMA4(I,J)*(POM(I,J)-PON(I,J))
      RESIDU(I,J)=Q(I,J)-R1-R2-R3-R4
   65 CONTINUE
      DO 75 I=1,NX
      JSTART=0
      DO 66 J=1,NY
      JSTART=JSTART+1
      IF(INGRID(I,J).NE.0) GO TO 67
   66 CONTINUE
   67 E1(1)=EY(I,JSTART)-ALPHA*SIGMA4(I,JSTART)
      B1(1)=B(I,JSTART)
      H1(1)=H(I,JSTART)
      RESID1(1)=RESIDU(I,JSTART)
      JS=JSTART+1
      II=1
   68 DO 70 J=JS,NY
      IF(INGRID(I,J).NE.0) GO TO 69
      JSTOP=J-1
```

```
      GO TO 72
   69 II=II+1
      E1(II)=EY(I,J)-ALPHA*SIGMA4(I,J)
      B1(II)=B(I,J)
      H1(II)=H(I,J)
      RESID1(II)=RESIDU(I,J)
   70 CONTINUE
      CALL THOMAS(B1,E1,H1,RESID1,G,W,II)
      JJ=1
      DO 71 J=JSTART,NY
      DP2STA(I,J)=G(JJ)
   71 JJ=JJ+1
      GO TO 75
   72 CALL THOMAS(B1,E1,H1,RESID1,G,W,II)
      JJ=1
      DO 73 J=JSTART,JSTOP
      DP2STA(I,J)=G(JJ)
      JJ=JJ+1
   73 CONTINUE
      JSTART=JSTOP+1
      II=0
      DO 74 J=JSTART,NY
      JS=J
      IF(INGRID(I,J).NE.0) GO TO 68
   74 CONTINUE
   75 CONTINUE
      DO 76 I=1,NX
      DO 76 J=1,NY
      IF(INGRID(I,J).EQ.0) GO TO 76
      IF(DP2STA(I,J).EQ.0) DP2STA(I,J)=DPSTAR(I,J)
   76 CONTINUE
C
C     CALCULATE PRESSURES AND SATURATIONS AND CHECK FOR
C     CONVERGENCE. MAXIMUM OF 20 ITERATIONS
C
      CHECK1=0.
      CHECK2=0.
      DO 77 J=1,NY
      DO 77 I=1,NX
      IF(INGRID(I,J).EQ.0) GO TO 77
      PO=POM(I,J)-PON(I,J)
      SW=SWM(I,J)-SWN(I,J)
      POM(I,J)=PON(I,J)+DP2STA(I,J)
      G1=DELTA(TWX,POM,1,I,J)
      G2=DELTA(TWY,POM,2,I,J)
      G3=(VB(I,J)*PHI)/(DELTAT)
      DELSW=(G1+G2+QWATER(I,J))/G3
      DIF1=ABS(DP2STA(I,J)-PO)
      IF(DIF1.GE.CHECK1) CHECK1=DIF1
      SWM(I,J)=SWN(I,J)+DELSW
      IF(SWM(I,J).GT.(1-SOR)) SWM(I,J)=1-SOR
      DELSW=SWM(I,J)-SWN(I,J)
      IF(SWM(I,J).LT.SWR) SWM(I,J)=SWR
      SOM(I,J)=1-SWM(I,J)
```

```
      DIF2=ABS(DELSW-SW)
      IF(DIF2.GT.CHECK2) CHECM2=DIF2
   77 CONTINUE
      IF(CHECK1.LT.100.AND.CHECK2.LT.0.0005) GO TO 78
      M=M+1
      IF(M.LT.21) GO TO 41
C
C    CALCULATE MATERIAL BALANCE BEFORE PROCEEDING TO  NEXT
C    TIME STEP
C
   78 S1=0.
      S2=0.
      DO 79 J=1,NY
      DO 79 I=1,NX
      IF(INGRID(I,J).EQ.0) GO TO 79
      S1=S1+VB(I,J)*PHI*(SOM(I,J)-SON(I,J))/DELTAT
      S2=S2+QOIL(I,J)
   79 CONTINUE
      WOR=WATER/OIL
      BALOIL=S1/S2
      CALL RESULT(N,BALOIL,POM,SOM,OIL,WATER,
     1QOIL,QWATER,CUMOIL,CUMWAT)
C
C    STOP IF W.O.R GREATER THAN WOR MAXIMUM .
C    W.O.R=TOTAL PRODUCED WATER/TOTAL PRODUCED OIL.
      IF(WOR.GE.50.) STOP
      DO 311 J=1,NY
      DO 311 I=1,NX
      IF(INGRID(I,J).EQ.0) GO TO 80
      PON(I,J)=POM(I,J)
      SON(I,J)=SOM(I,J)
   80 CONTINUE
      IF(N*DELTAT.GE.TIME) STOP
      N=N+1
      GO TO 18
      STOP
999   FORMAT(3F12.5)
1000  FORMAT(2I5,2F15.2)
1001  FORMAT(20I2)
1002  FORMAT(I5,2F15.2)
      END
```

```
      SUBROUTINE READ
C    READS TABULATED DATA.
C       PHI=POROSITY
C       SWC=CONNATE WATER SATURATION.
C       VISOIL=OIL VISCOSITY (PA.S)
C       VISWAT=WATER VISCOSITY (PA.S)
C       SOR=RESIDUAL OIL SATURATION.
C       SWR=RESIDUAL WATER SATURATION.
C       XKRO=RELATIVE OIL PERMEABILITY.
C       XKRW=RELATIVE WATER PERMEABILITY.
C
      DIMENSION SO1(15),SW1(15),XKRO(15),XKRW(15)
      COMMON/OUTPUT/SO1,SW1,XKRO,XKRW
      COMMON/VAR/IKRO,IKRW
      COMMON/PROP/VISOIL,VISWAT,SOR,SWR,PHI
      READ(5,1000) PHI,SWR,SOR,VISOIL,VISWAT
C
C    READ PERMEABILITY DATA.
C       IKRO=NUMBER OF DATA POINTS FOR OIL CURVE.
C       IKRW=NUMBER OF DATA POINTS FOR WATER CURVE.
   12 IKRO=0
      IKRW=0
   13 IKRO=IKRO+1
      READ(5,1000) SO1(IKRO),XKRO(IKRO)
      IF(SO1(IKRO).EQ.0.AND.XKRO(IKRO).EQ.0.) GO TO 14
      GO TO 13
   14 IKRW=IKRW+1
      READ(5,1000) SW1(IKRW),XKRW(IKRW)
      IF(SW1(IKRW).EQ.0.AND.XKRW(IKRW).EQ.0.) GO TO 15
      GO TO 14
   15 CONTINUE
      IKRO=IKRO-1
      IKRW=IKRW-1
 1000 FORMAT(5F15.5)
      RETURN
      END
```

```
      SUBROUTINE GRID
C     DEFINES GRID AND READS FORMATION PROPERTIES
C     NX,NY:NUMBER OF BLOCKS IN X OR Y DIRECTION
C
      DIMENSION DELTAX(25),DELTAY(15),INGRID(25,15)
      COMMON/ING/NX,NY,INGRID
      COMMON/GRI/PAY,PORI,PERM,DELTAX,DELTAY,SAT
C   PORI=UNIFORM INITIAL PRESSURE.
C   INGRID=INDEX
C      INGRID(I,J)=0 :BLOCK(I,J) IS OUTSIDE GRID
C      INGRID(I,J)=1 :BLOCK(I,J) IS INSIDE GRID
C
      READ(5,1000) NX,NY,PORI,SAT,PERM,PAY
      DO 1 J=1,NY
    1 READ(5,1001) (INGRID(I,J),I=1,NX)
      READ(5,999) (DELTAX(I),I=1,NX)
      READ(5,999) (DELTAY(J),J=1,NY)
  999 FORMAT(5F18.4)
 1000 FORMAT(2I4,F10.0,F5.2,E12.5,F10.0)
 1001 FORMAT(30I2)
      RETURN
      END
```

```
      FUNCTION DELTA(T,A,INDEX,I,J)
C
C    EVALUATES THE EXPRESSION:DELTA(T*DELTA(A))
C    WHERE T IS AN ARRAY OF TRANSMISSIBILITIES,
C      IF INDEX=1,CONSIDERS X DIRECTION
C      IF INDEX=2,CONSIDERS Y DIRECTION
C          A IS AN ARRAY OF EITHER PRESSURES OR DEPTH,
C
      DIMENSION T(25,15),A(25,15),INGRID(25,15)
      COMMON/ING/NX,NY,INGRID
      IF(INDEX.EQ.2) GO TO 8
      IF(I-1) 4,3,4
  3   DELTA=T(I,J)*(A(I+1,J)-A(I,J))
      RETURN
  4   IF(I-NX) 6,5,6
  5   DELTA=-T(I-1,J)*(A(I,J)-A(I-1,J))
      RETURN
  6   DELTA=T(I,J)*(A(I+1,J)-A(I,J))-T(I-1,J)*(A(I,J)
     *-A(I-1,J))
  7   RETURN
  8   IF(J-1) 10,9,10
  9   DELTA=T(I,J)*(A(I,J+1)-A(I,J))
      RETURN
 10   IF(J-NY) 12,11,12
 11   DELTA=-T(I,J-1)*(A(I,J)-A(I,J-1))
      RETURN
 12   DELTA=T(I,J)*(A(I,J+1)-A(I,J))-T(I,J-1)*(A(I,J)
     *-A(I,J-1))
      RETURN
      END
```

```
      SUBROUTINE SPLINE(N,X,Y,B,C,D)
C
C THE COEFFICIENTS B(I),C(I),D(I),I=1,2,...,N ARE
C COMPUTED FOR A CUBIC INTERPOLATING SPLINE
C S(X)=Y(I)+B(I)*(X-X(I))+C(I)*(X-X(I))**2+D(I)*(X-X(I))**3
C FOR X(I).LE.X.LE.X(I+1)
C INPUT
C N=THE NUMBER OF DATA POINTS OR KNOTS (N.GE.2)
C X=THE ABSCISSAS OF THE KNOTS IN STRICTLY INCREASING ORDER
C Y=THE ORDINATES OF THE KNOTS
C OUTPUT
C B,C,D=ARRAYS OF SPLINE COEFFICIENTS AS DEFINED ABOVE.
C
C USING P TO DENOTE DIFFERENTIATION,
C Y(I)=S(X(I))
C B(I)=SP(X(I))
C C(I)=SPP(X(I))/2
C D(I)=SPPP(X(I))/6  (DERIVATIVE FROM THE RIGHT)
C
C THE ACCOMPANYING FUNCTION SUBPROGRAMM SEVAL CAN BE USED
C TO EVALUATE THE SPLINE
C
      DIMENSION X(N),Y(N),B(N),C(N),D(N)
      NM1=N-1
      IF(N.LT.2) RETURN
      IF(N.LT.3) GO TO 50
C
C   SET UP TRIDIAGONAL SYSTEM
C   B=DIAGONAL,D=OFFDIAGONAL,C=RIGHT HAND SIDE
      D(1)=X(2)-X(1)
      C(2)=(Y(2)-Y(1))/D(1)
      DO 10 I=2,NM1
         D(I)=X(I+1)-X(I)
         B(I)=2.*(D(I-1)+D(I))
         C(I+1)=(Y(I+1)-Y(I))/D(I)
         C(I)=C(I+1)-C(I)
   10 CONTINUE
C
C   END CONDITIONS.THIRD DERIVATIVES AT X(1) AND X(N)
C   OBTAINED FROM DIVIDED DIFFERENCES
      B(1)=D(1)
      B(1)=-D(1)
      B(N)=-D(N-1)
      C(1)=0.
      C(N)=0.
      IF(N.EQ.3) GO TO 15
      C(1)=C(3)/(X(4)-X(2))-C(2)/(X(3)-X(1))
      C(N)=C(N-1)/(X(N)-X(N-2))-C(N-2)/(X(N-1)-X(N-3))
      C(1)=C(1)*D(1)**2/(X(4)-X(1))
      C(N)=-C(N)*D(N-1)**2/(X(N)-X(N-3))
C
C   FORWARD ELIMINATION
   15 DO 20 I=2,N
         T=D(I-1)/B(I-1)
```

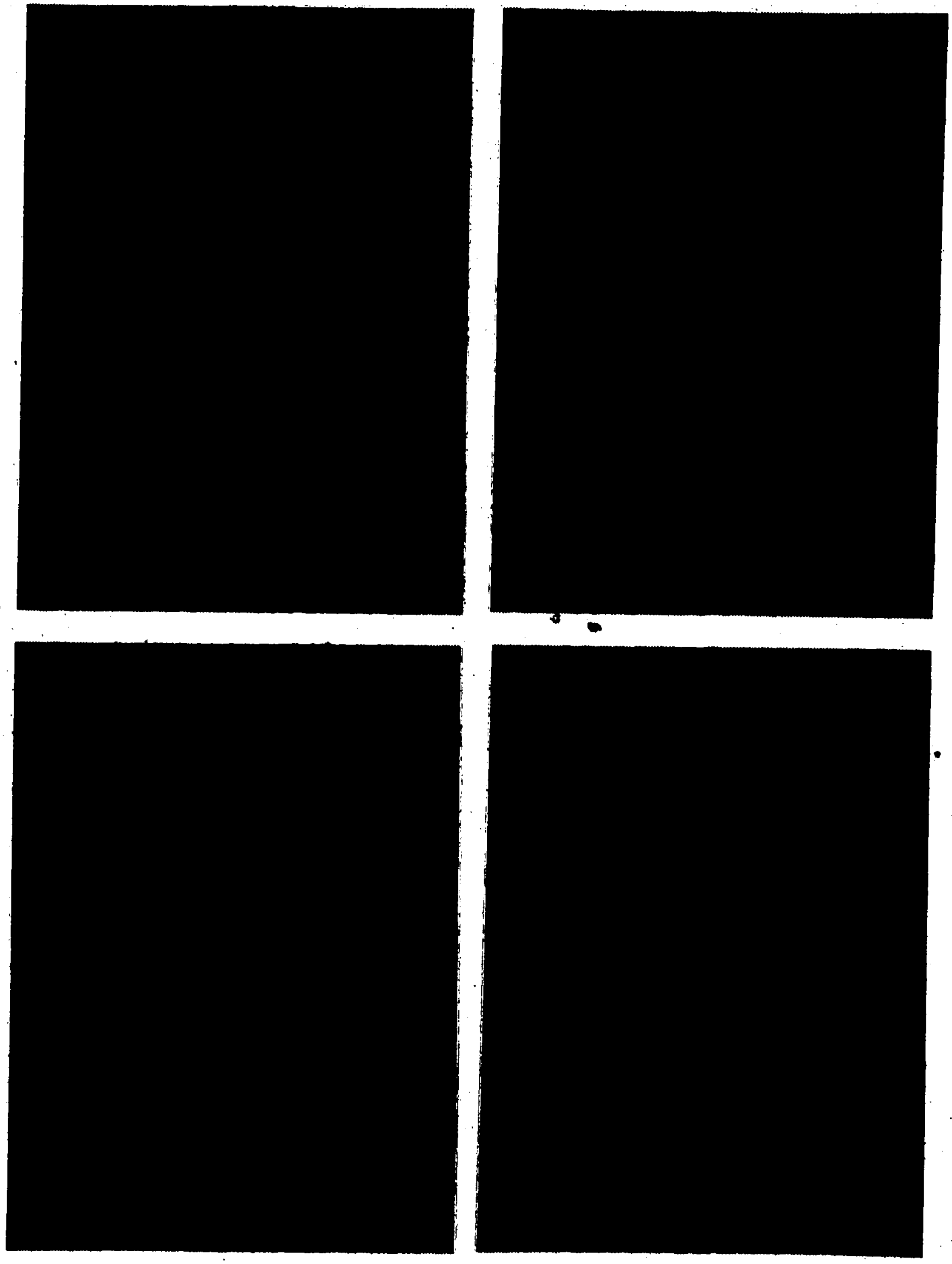
```
         B(I)=B(I)-T*D(I-1)
         C(I)=C(I)-T*C(I-1)
   20  CONTINUE
C
C    BACK SUBSTITUTION
       C(N)=C(N)/B(N)
       DO 30 IB=1,NM1
          I=N-IB
          C(I)=(C(I)-D(I)*C(I+1))/B(I)
   30  CONTINUE
C
C    COMPUTE POLYNOMIAL COEFFICIENTS
       B(N)=(Y(N)-Y(NM1))/D(NM1)+D(NM1)*(C(NM1)+2.*C(N))
       DO 40 I=1,NM1
          B(I)=(Y(I+1)-Y(I))/D(I)-D(I)*(C(I+1)+2.*C(I))
          D(I)=(C(I+1)-C(I))/D(I)
          C(I)=3.*C(I)
   40  CONTINUE
       C(N)=3.*C(N)
       D(N)=D(N-1)
       RETURN
   50  B(1)=(Y(2)-Y(1))/(X(2)-X(1))
       C(1)=0.
       D(1)=0.
       B(2)=B(1)
       C(2)=0.
       D(2)=0.
       RETURN
       END
```

```
      FUNCTION SEVAL(N,U,X,Y,B,C,D)
C
C THIS SUBROUTINE EVALUATES THE CUBIC SPLINE FUNCTION.
C SEVAL=Y(I)+B(I)*(U-X(I))+C(I)*(U-X(I))**2+D(I)*(U-X(I))**3
C WHERE X(I).LT.X(I+1),
C IF U.LT.X(1) THEN I=1 IS USED
C IF U.GE.X(N) THEN I=N IS USED
C INPUT
C N=NUMBER OF DATA POINTS
C U=THE ABSCISSA AT WHICH THE SPLINE IS TO BE EVALUATED
C X,Y=THE ARRAYS OF DATA ABSCISSAS AND ORDINATES
C B,C,D=ARRAYS OF SPLINE COEFFICIENTS COMPUTED BY SPLINE.
      DIMENSION X(N),Y(N),B(N),C(N),D(N)
      I=1
      J=N+1
    1 K=(I+J)/2
      IF(U.LT.X(K)) J=K
      IF(U.GE.X(K)) I=K
      IF(J.GT.I+1) GO TO 1
      DX=U-X(I)
      SEVAL=Y(I)+DX*(B(I)+DX*(C(I)+DX*D(I)))
      RETURN
      END
```

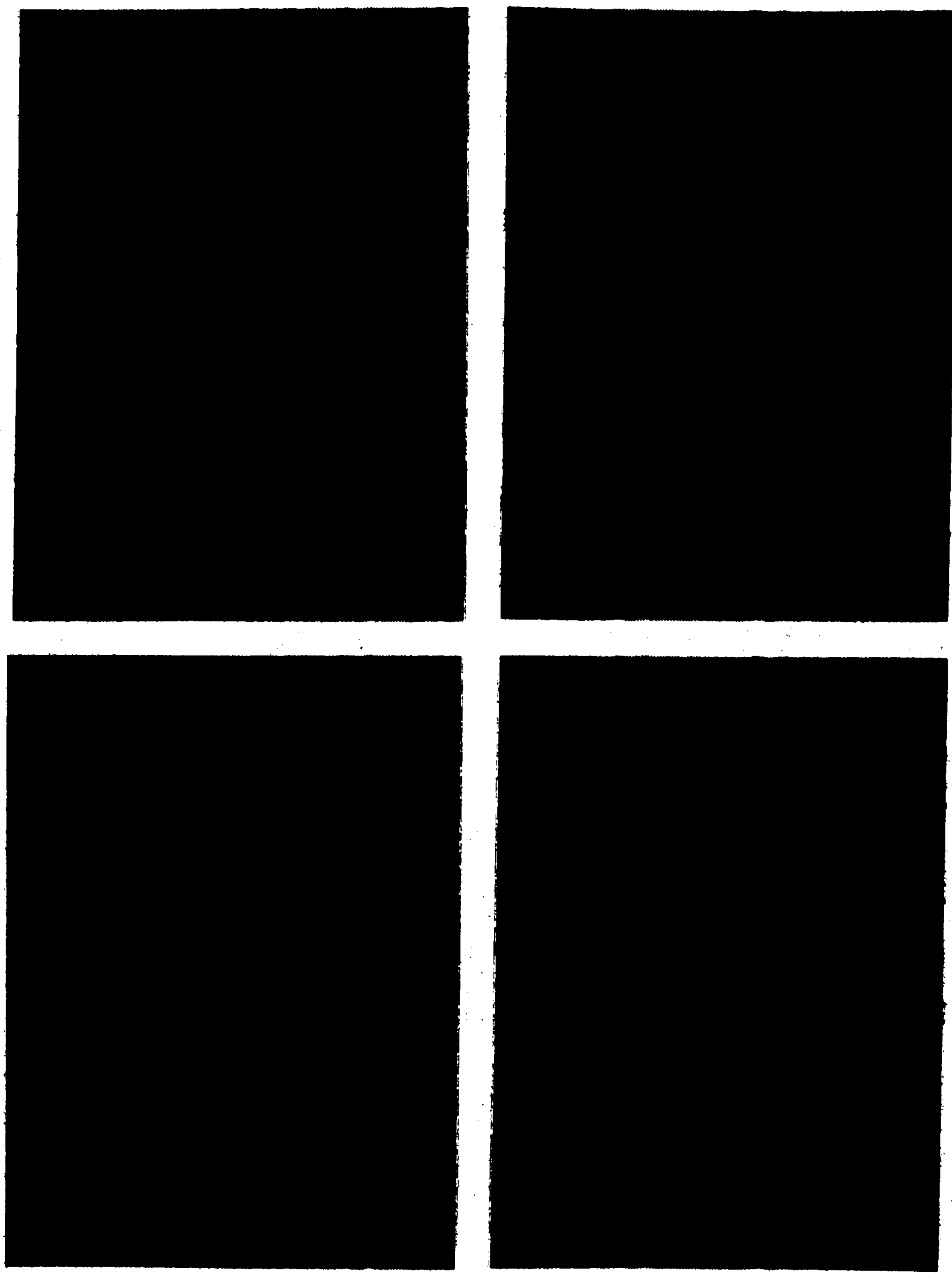
```
      SUBROUTINE THOMAS(A,B,C,D,G,W,NEQU)
C
C   SOLVES A TRIDIAGONAL SYSTEM OF NEQU EQUATIONS,
C   USING THOMAS'S ALGORITHM.
C   A,B,C ARE THE ARRAYS OF L.H.S COEFFICIENTS,
C   D IS THE ARRAY OF R.H.S COEFFICIENTS.
C   THE SOLUTION VECTOR IS RETURNED IN ARRAY G.
C
      DIMENSION A(NEQU),B(NEQU),C(NEQU),D(NEQU),W(NEQU),
     *G(NEQU)
      IF(NEQU.NE.1) GO TO1
C
C   NEQU=1,SINGLE EQUATION:
      G(1)=0.
      RETURN
    1 IF(NEQU.NE.2) GO TO 2
C
C   NEQU=2
      DEN=B(2)*D(1)-A(2)*C(1)
      G(1)=(D(1)*B(2)-D(2)*C(1))/DEN
      G(2)=(D(2)*B(1)-A(2)*D(1))/DEN
      RETURN
C
    2 W(1)=C(1)/B(1)
      G(1)=D(1)/B(1)
      JK=NEQU-1
      DO 3 I=2,JK
      DEN=B(I)-A(I)*W(I-1)
      W(I)=C(I)/DEN
    3 G(I)=(D(I)-A(I)*G(I-1))/DEN
      G(NEQU)=(D(NEQU)-A(NEQU)*G(JK))/(B(NEQU)-A(NEQU)*
     **W(JK))
      DO 4 I=1,JK
      II=NEQU-I
    4 G(II)=G(II)-W(II)*G(II+1)
      RETURN
      END
```

```
      SUBROUTINE RESULT(N,BALOIL,POM,SOM,OIL,WATER,QOIL,
     1QWATER,CUMOIL,CUMWAT)
C
C     PRINTS PRESSURE AND SATURATION DISTRIBUTIONS AT END
C     OF EACH TIME STEP.
C     PRINTS MATERIAL BALANCE,OIL AND WATER PRODUCTIONS.
C
      DIMENSION POM(25,15),SOM(25,15),POOUT(25,15),
     1SOOUT(25,15),QOIL(25,15),QWATER(25,15),INGRID(25,15)
      COMMON/RESU/DELTAT
      COMMON/ING/NX,NY,INGRID
      TOTAL=N*DELTAT/60
      WRITE(6,2000) N,TOTAL,BALOIL
      W=ABS(WATER)*1E+06
      O=ABS(OIL)*1E+06
      CW=ABS(CUMWAT)*1E+06
      CO=ABS(CUMOIL)*1E+06
      WRITE(6,2030) W,O,CW,CO
      IF(MOD(N,10).NE.1) GO TO 7
      DO 1 I=1,NX
      DO 1 J=1,NY
      IF(INGRID(I,J).EQ.0) GO TO 8
      POOUT(I,J)=POM(I,J)/1000.
      SOOUT(I,J)=SOM(I,J)
      GO TO 1
    8 POOUT(I,J)=0.
      SOOUT(I,J)=0.
    1 CONTINUE
      IF(NX.GT.15) GO TO 5
      WRITE(6,2021)
      WRITE(6,2024) (I,I=1,NX)
      DO 2 J=1,NY
    2 WRITE(6,2025) J,(POOUT(I,J),I=1,NX)
      WRITE(6,2023)
      WRITE(6,2024) (I,I=1,NX)
      DO 4 J=1,NY
    4 WRITE(6,2026) J,(SOOUT(I,J),I=1,NX)
      RETURN
C
C     FOR LARGER GRIDS ,PRINT ONLY PRESSURES AND SATURATIONS
C     AT INJECTION AND PRODUCING WELLS
    5 WRITE(6,2040)
      DO 6 I=1,NX
      DO 6 J=1,NY
      IF(INGRID(I,J).EQ.0) GO TO 6
      IF(QOIL(I,J).EQ.0.0.AND.QWATER(I,J).EQ.0.) GO TO 6
      WRITE(6,2050) I,J,POOUT(I,J),SOOUT(I,J)
    6 CONTINUE
    7 CONTINUE
 2000 FORMAT(1H0,19X,'END OF TIME STEP',I3,
     1/,20X,'TOTAL TIME ELAPSED (MN):',
     2F6.2,/,20X,'MATERIAL BALANCE (1 FOR PERFECT BALANCE):
     3',/,30X,'OIL BALANCE:',F7.3)
 2021 FORMAT(1H0,20X,'OIL PRESSURE DISTRIBUTION',/)
```

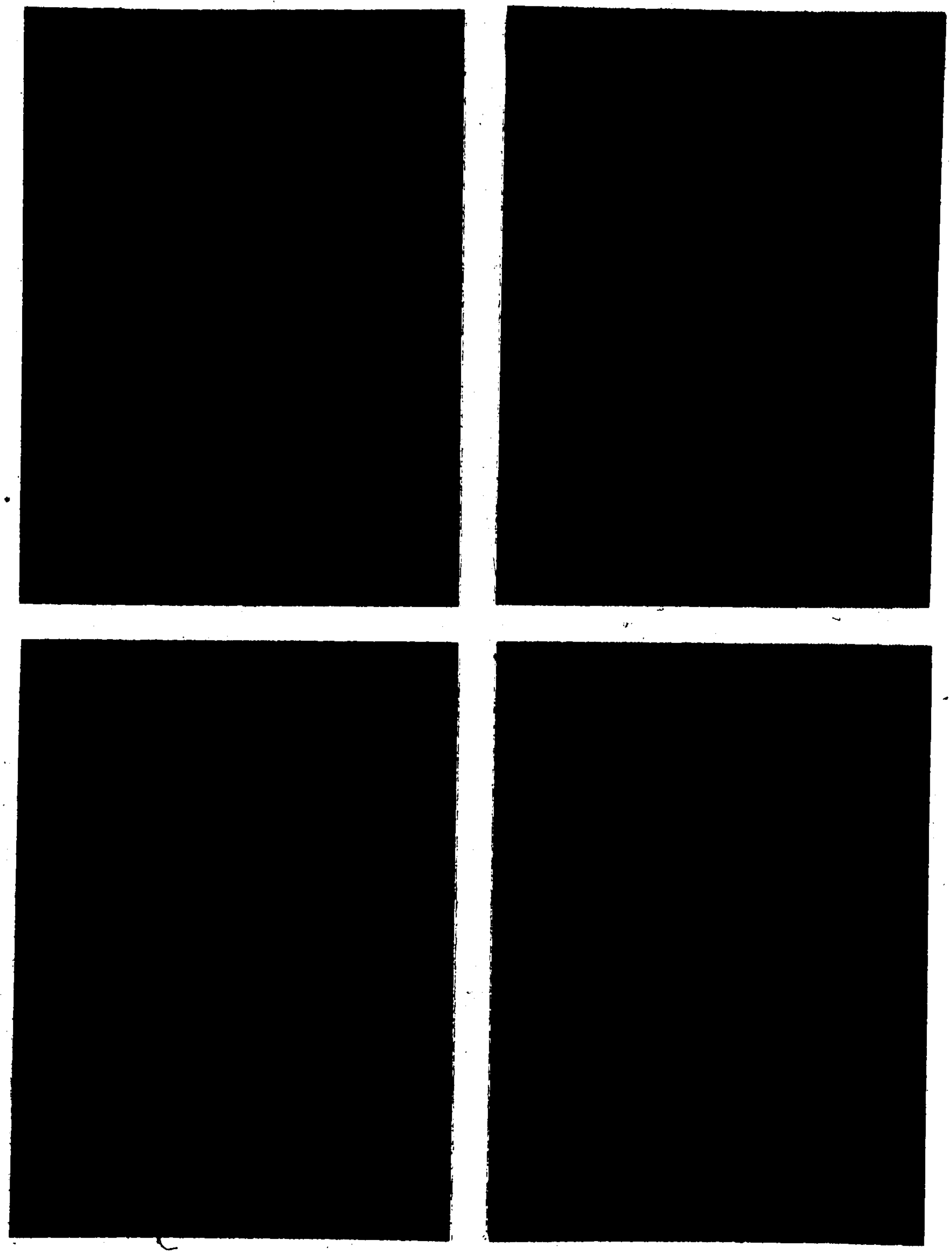
```
2023 FORMAT(1H0,20X,'OIL SATURATION DISTRIBUTION',/)
2025 FORMAT(1H ,3X,I2,3X,15(F6.0,2X))
2026 FORMAT(1H ,3X,I2,3X,15(F5.3,3X))
2030 FORMAT(1H0,19X,' WATER PRODUCTION DURING TIME STEP
    1 (CM3):',E12.5,/,20X,' OIL PRODUCTION (CM3):',E12.5,
    2/,20X,'CUMULATIVE WATER PRODUCTION (CM3):',E12.5,
    3/,20X,'CUMULATIVE OIL PRODUCTION (CM3):',E12.5)
2040 FORMAT(1H0,20X,'I',7X,'J',7X,'POM',5X,'SOM'
    1,//)
2050 FORMAT(1H ,19X,I2,6X,I2,4X,F6.0,3X,F5.3)
     RETURN
      END
```



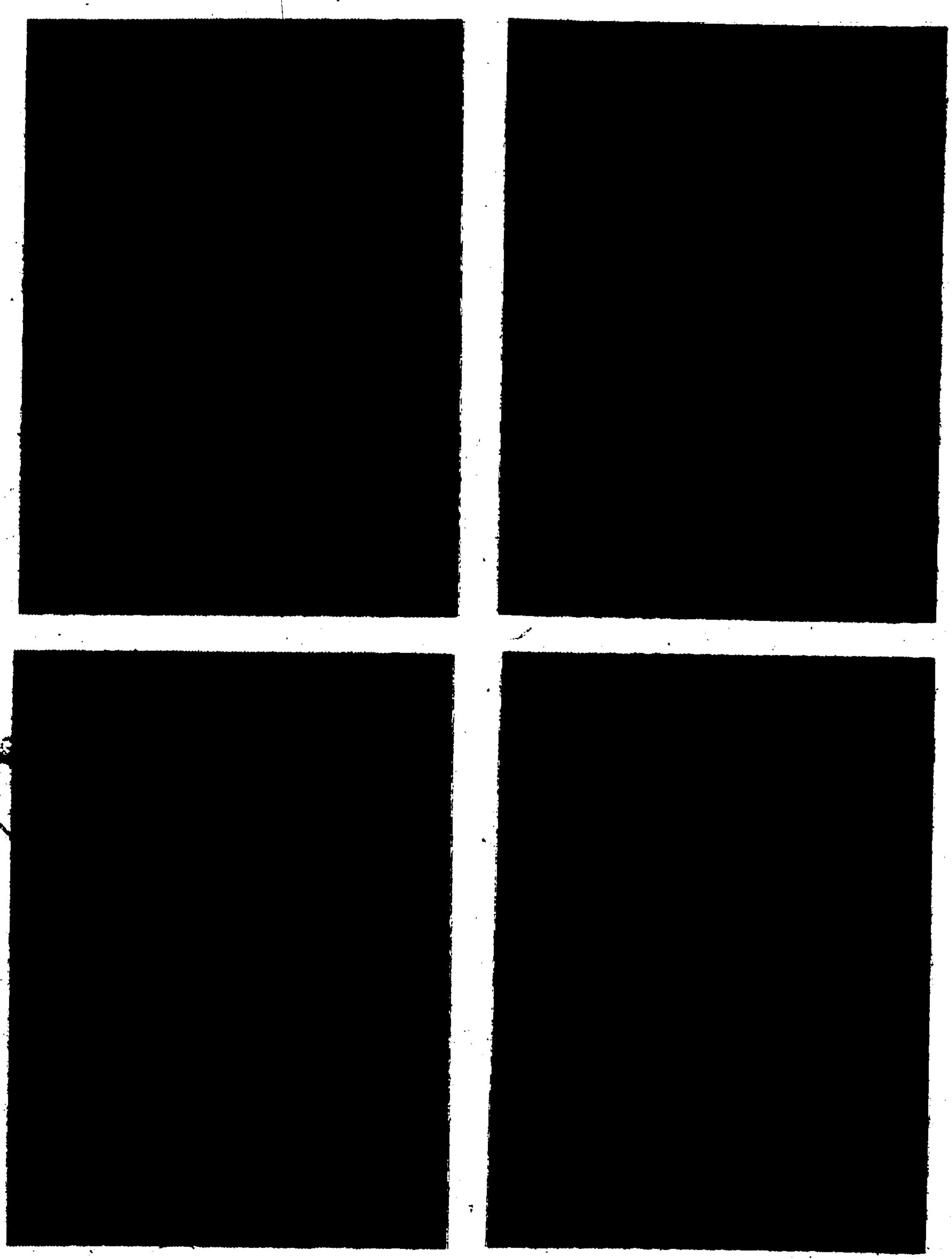
Isolated Four-spot Pattern Waterflood



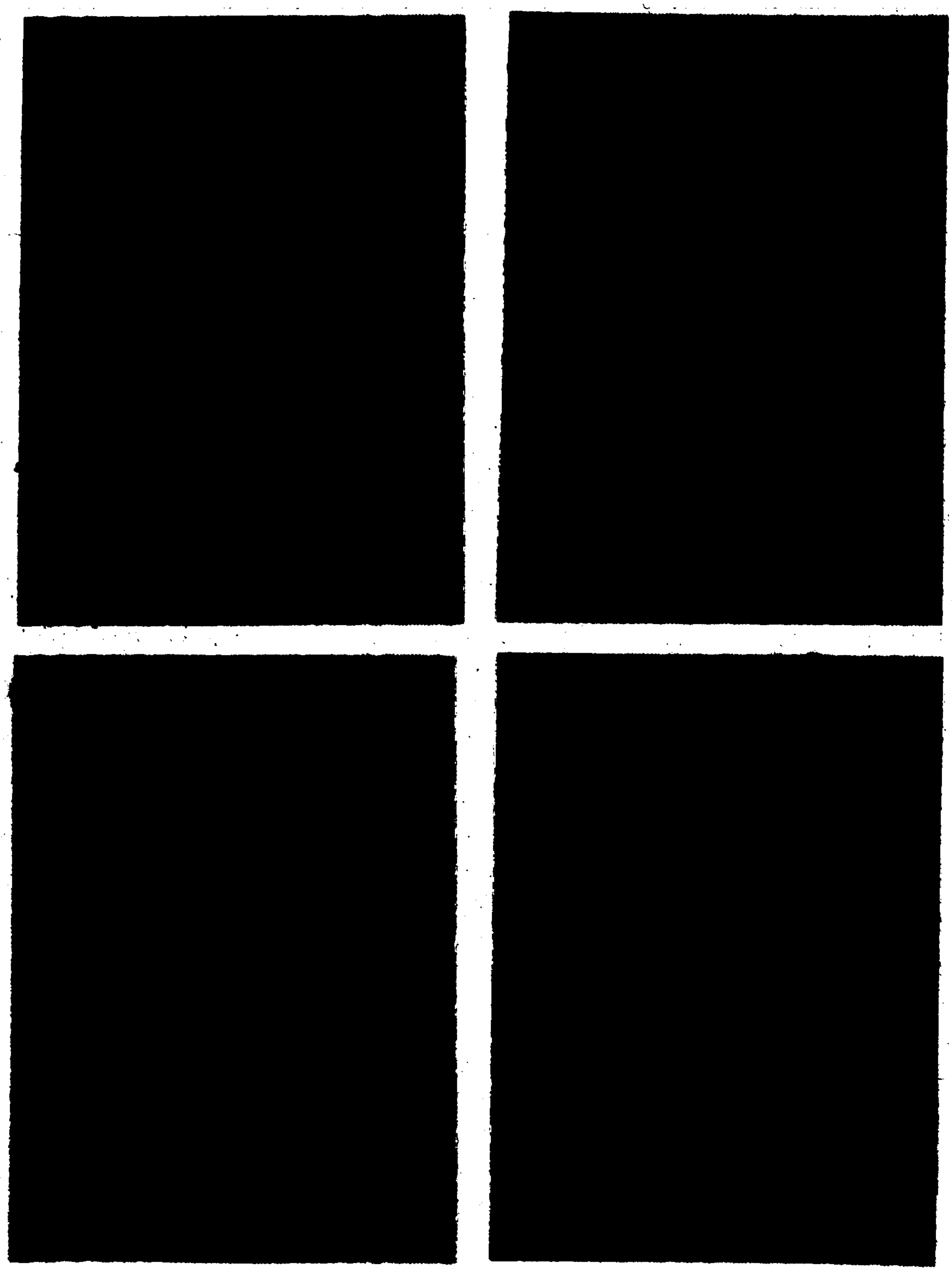
Modified Seven-spot Pattern Waterflood



Isolated Seven-spot Pattern Waterflood



Modified Four-spot Pattern Waterflood



Thirteen Four-spot Pattern Waterflood