Structural Engineering Report No. 118

EFFECTIVE LENGTHS OF LATERALLY UNSUPPORTED STEEL BEAMS

by
CALVIN D. SCHMITKE
D. J. LAURIE KENNEDY

October, 1984

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and

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October 1984

Abstract

The stability of laterally unsupported steel beams is examined with consideration given to the effects of the type of load, point of application of the load, and the type of end restraints supplied. The amount of end restraint provided is a function of the type of connections used and of the lateral continuity of the beam. Effective lengths can be used to model the effect of the end restraints on the beam's stability. Effective length factors are given for single span and cantilever beams. For continuous beams, methods for determining effective length factors are described.

Acknowledgements

This report is a reprint of the thesis "Effective Lengths of Laterally Unsupported Beams" submitted by Calvin D. Schmitke in partial fulfillment of the requirements for the degree of Master of Science. Dr. D. J. Laurie Kennedy was the supervisor for the thesis. Some modifications to the thesis have been made for this report.

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List of Symbols

A = area enclosed by middle of tube wall

a_1 = variable defined by Eqn. [6.10], used in Eqn. [6.8]

C = coefficient for top or bottom flange loading

C_w = warping torsional constant

d = depth of beam section

E = modulus of elasticity

F_y = yield strength

G = shear modulus of elasticity

$G_{A/B}$ = joint stiffness ratio (subscripts apply for respective ends of beam)

G_t = shear modulus of elasticity of material used for tubular device

h = distance between centroids of beam flanges

I_x = moment of inertia about x-axis (strong axis)

I_y = moment of inertia about y-axis (weak axis)

J = St. Venant's torsion constant

J_t = St. Venant's torsion constant of tubular device

K = elastic rotational stiffness

k = effective length factor

k_C = effective length factor of critical segment

k_R = effective length factor of restraining segment

k_y = effective length factor with respect to lateral bending

k_z	=	effective length factor with respect to torsion
L	=	laterally unsupported length
L_c	=	length of critical segment
L_e	=	equivalent length of beam segment
L_R	=	length of restraining segment
M_{cr}	=	critical buckling moment
M_F	=	moment at subassemblage buckling
M_f	=	factored bending moment
M_{f1}	=	smaller of factored end moments
M_{f2}	=	larger of factored end moments
M_i	=	inelastic buckling moment
M_{im}	=	modified inelastic buckling moment
M_{max}	=	maximum bending moment acting on a beam
M_p	=	plastic moment
M_r	=	factored moment resistance
M_u	=	elastic lateral-torsional buckling moment resistance
M_{ur}	=	reduced value of M_u
M_y	=	yield moment
M_o	=	end moment
M_{ocr}	=	critical end moment
$M_{1,2,3,4,5}$	=	bending moments acting at quarter points of a beam
m	=	equivalent moment factor
n	=	factor used in Eqns. [5.2], [5.10], [5.13], [5.14]

P	= concentrated load
R	= reaction
S	= length of perimeter of tube
T_o	= end torque
t	= wall thickness
t_f	= thickness of beam flange
u	= displacement in x-direction
u', u'', u^{iv}	= derivatives of u with respect to z
v	= displacement in y-direction
v', v''	= derivatives of v with respect to z
V	= coefficient of variation
V_D	= coefficient of variation of discretization factor
V_{EM}	= coefficient of variation for errors in measurement
V_G	= coefficient of variation of geometric properties
V_M	= coefficient of variation of material properties
V_P	= coefficient of variation of test to predicted strength
V_R	= coefficient of variation of resistance
V_o	= coefficient of variation prior to reduction due to errors in measurement
w	= uniformly distributed load
x	= coordinate axis
y	= coordinate axis

Z	=	plastic section modulus
z	=	coordinate axis
α	=	coefficient of seperation
a_C	=	stiffness of critical segment
a_R	=	stiffness of restraining segment
β	=	safety index
γ	=	constant used in Eqn. [3.3]
δ	=	lateral buckling coefficient, used in Eqn. [3.3]
θ	=	rotation about z-axis
θ', θ'', θ^{iv}	=	derivatives of β with respect to z
K	=	beam parameter
κ	=	ratio of end moments
λ	=	moment factor, ratio of critical moment to applied moment
λ_C	=	moment factor of critical segment
λ_F	=	moment factor of critical segment at subassemblage buckling
λ_{Fn}	=	most recent value of λ_F
λ_R	=	moment factor of restraining segment
λ_{Rmin}	=	lowest moment factor of two restraining segments
μ_P	=	mean value of test to predicted ratios
ρ	=	ratio of measured to nominal property
ρ_D	=	mean value of the discretization factor
ρ_G	=	ratio of measured to nominal geometric propeties

ρ_M = ratio of measured to nominal material properties

ρ_P = ratio of test to predicted strength

ρ_R = ratio of mean to nominal resistance

τ = shear stress

ϕ = resistance factor

ψ = lateral-torsional buckling parameter

Ω^2 = shape parameter

ω = equivalent moment factor

1. INTRODUCTION AND SCOPE

One limit state of a laterally unsupported beam is that associated with lateral instability. Failure of this kind occurs either by elastic or inelastic lateral-torsional buckling. Laterally unsupported beams with sufficeintly short spans reach their ultimate moments, either the plastic or yield moments, before instability occurs. This report deals with beams that are not short enough to reach the plastic or yield moments and thus fail by lateral-torsional buckling. The capacity of these beams depends in part on the geometric and material properties of the beam. Other important factors include the shape of the bending moment diagram, the location of application of the load on the beam cross-section - whether above, at or below the shear center - and the restraints at the ends of the beam. These last three factors are dealt with in this report with particular emphasis on the effect of end restraints and how their influence can be modelled by effective length factors.

The basic assumptions involved in the derivation of the elastic critical buckling moment equation and how the shape of the bending moment diagram, point of load application and effect of end restraints are reviewed. Examination of the pertinent equations show how they model the various influencing factors. The three types of laterally unsupported beams treated are single span, cantilever, and continuous beams.

For single span beams a variety of analytical procedures with various strengths and weaknesses exist in the literature. Effective length factors can be used to reflect the degree of end restraint provided and practical methods for achieving these end conditions are given.

The special category of cantilever beams is examined with a review of the classical solutions as well as code requirements. Effective lengths corresponding to various conditions of end restraint at each end of the cantilever are given.

The term laterally continuous is used to refer to a beam which is continuous in the lateral plane through a number of discrete lateral supports. Several methods for the analysis of these beams are discussed.

Finally, examples given in the appendix illustrate some of the design procedures described in the text.

2. BACKGROUND AND THEORY

2.1 Classical Solution

2.1.1 The Critical Moment Equation

Consider a beam with a doubly symmetric cross-section such as the I-shaped beam in Fig. 2.1. The coordinate system is defined as shown in the figure. Let the symbols u, v, and θ represent the deflection of the beam in the x-direction, deflection in the y-direction and rotation about the z-axis respectively. Primed symbols denote differentiation with respect to z. The load is assumed to act on the beam in the y-z plane. The beam has a greater flexural rigidity in the vertical plane than in the horizontal plane ($I_x > I_y$). Assume also, that the beam has pinned supports at each end with respect to lateral bending and with respect to torsion as given by Eqns. [2.1] and [2.2] respectively.

[2.1] $$u = u'' = 0 \quad @ \quad z = 0, L$$

[2.2] $$\theta = \theta'' = 0 \quad @ \quad z = 0, L$$

No support is provided between the two ends. If the beam is acted upon by equal and opposite end moments such as those shown in Fig. 2.2 the differential equations for elastic lateral-torsional buckling are (Galambos 1968):

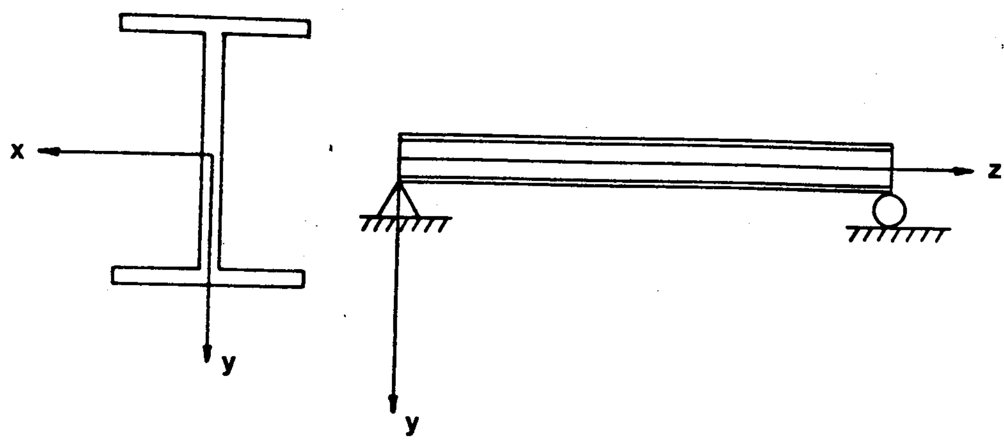
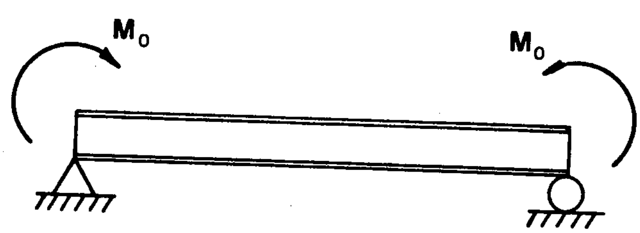


Figure 2.1 Coordinate System



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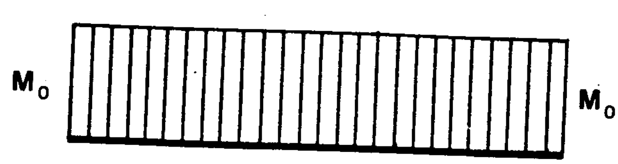


bending moment diagram

Figure 2.2 Beam Loaded by Equal and Opposite End Moments

[2.3] $$EI_y u^{iv} + M_0 \theta'' = 0$$

[2.4] $$EC_w \theta^{iv} - GJ\theta'' + M_0 u'' = 0$$

The solution of Eqns. [2.3] and [2.4] results in the following expression for the elastic critical buckling moment of the beam (Timoshenko and Gere 1961; Galambos 1968):

[2.5] $$M_{ocr} = \frac{\pi}{L}\sqrt{EI_y GJ + \frac{\pi^2 E^2 I_y C_w}{L^2}}$$

If the applied end moments reach the value given by Eqn. [2.5] the beam is in a state of unstable equilibrium and can fail by lateral-torsional buckling.

Two types of deformations are involved in the lateral-torsional buckling of a beam. The top flange, stressed in compression, has a tendency to buckle laterally (in the x-direction). The bottom flange, stressed in tension, has no tendency to buckle and therefore tends to restrain the beam from deflecting laterally. The combination of the tendency to buckle and of restraint in the same beam results in a twisting action (McGuire 1968). The resulting buckled position is shown in Fig. 2.3 (Adams, Krentz and Kulak 1979). The deflection, v, is due to the vertical loads and occurs prior to buckling.

When a beam buckles, the deformations, u and θ, can become extremely large. This results in considerable

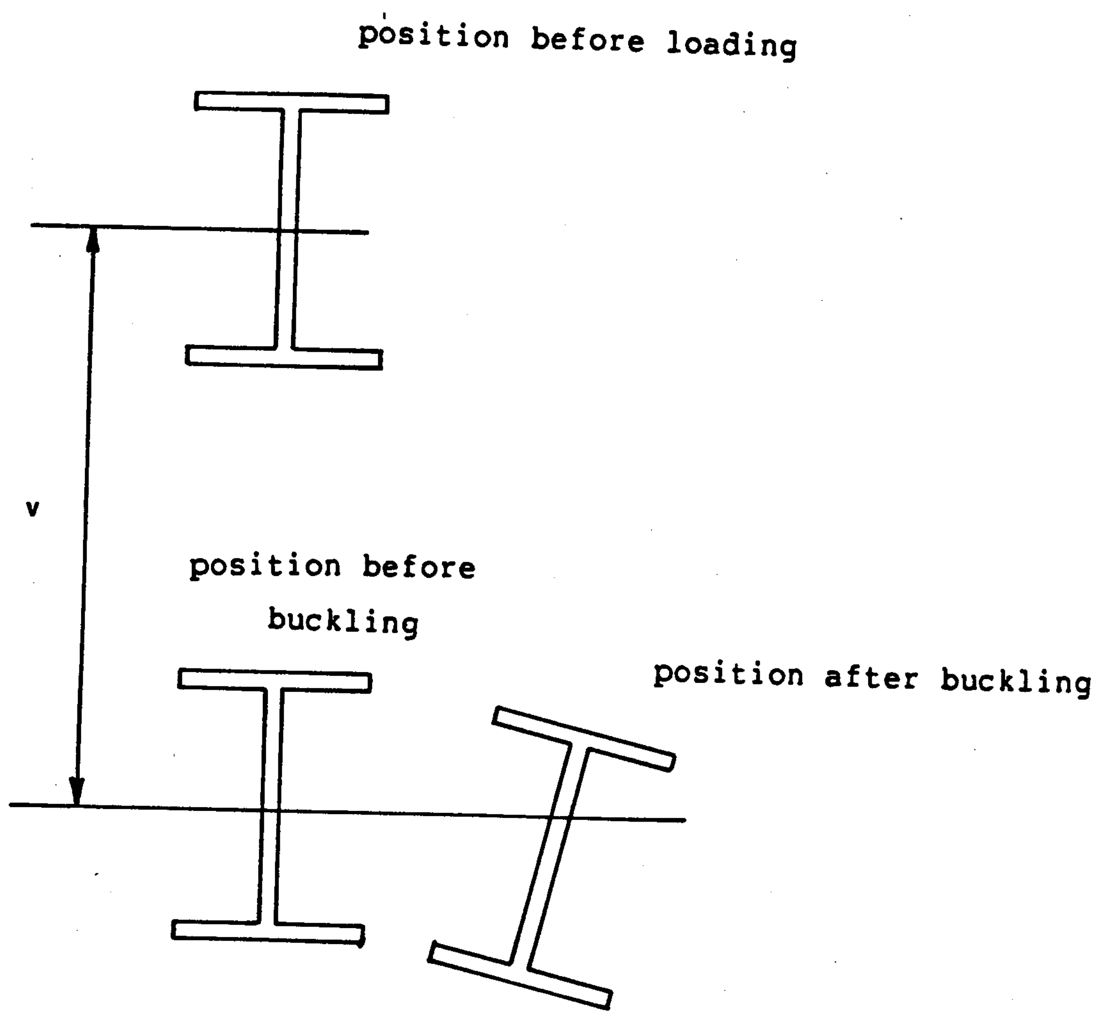


Figure 2.3 Buckled Position of Beam Cross-Section
(Adams et. al. 1979)

"softening" of the beam making it incapable of carrying additional load (Galambos 1968). It has reached its ultimate limit state.

2.1.2 The Effect of End Restraints

In the derivation of Eqn. [2.5] it was assumed that the beam considered was laterally simply supported. Equations [2.1] and [2.2] described this condition in mathematical terms. Not all beams have simple supports. For example, the boundary conditions of a laterally fixed-ended beam are described by the following equations:

[2.6] $$u = u' = 0 \quad @ \quad z = 0, L$$

[2.7] $$\theta = \theta' = 0 \quad @ \quad z = 0, L$$

Considering a beam loaded by equal and opposite end moments and using the above boundary conditions the governing differential equations (Eqns. [2.3] and [2.4]) can be solved once again to yield the following expression for the critical buckling moment (Chajes 1974):

[2.8] $$M_{ocr} = \frac{\pi}{0.5L}\sqrt{EI_yGJ + \frac{\pi^2E^2}{(0.5L)^2}I_yC_w}$$

Clearly, the type of end restraint exerted on a beam affects its stability. The following sections describe various types of boundary conditions and how they are achieved with commonly used connections.

2.1.2.1 Torsional Restraint

A beam is said to have a torsionally pinned end condition if, at the support being considered, twisting about the longitudinal axis is completely restrained and warping of the flanges is permitted. This type of support was described mathematically by Eqn. [2.2] for a beam with torsional pins at both ends. Figures 2.4(a) and 2.4(c) show idealized supports which provide torsionally pinned boundary conditions.

Warping is the type of deformation in which the flanges bend about the y-axis. The top and bottom flange each bend in the opposite direction to the other (Kirby and Nethercot 1979). A torsionally pinned support offers no resistance to this type of deformation. Resistance to warping, therefore, must be developed by longitudinal stresses in the flanges along the length of the beam. The term EC_w is a measure of this resistance. Figure 2.5 shows a beam which is torsionally pinned. Only torsional deformations are shown (no lateral deflections are included). For clarity, the web is not shown.

Figure 2.6(a) shows a double angle shear connection. This is an example of a connection that can be assumed to provide a torsionally pinned support condition (Galambos 1968). The restraint against twisting at the ends is almost complete ($\theta \simeq 0$) and because nothing bears against the flanges (Fig. 2.6(b)) there is little restraint against

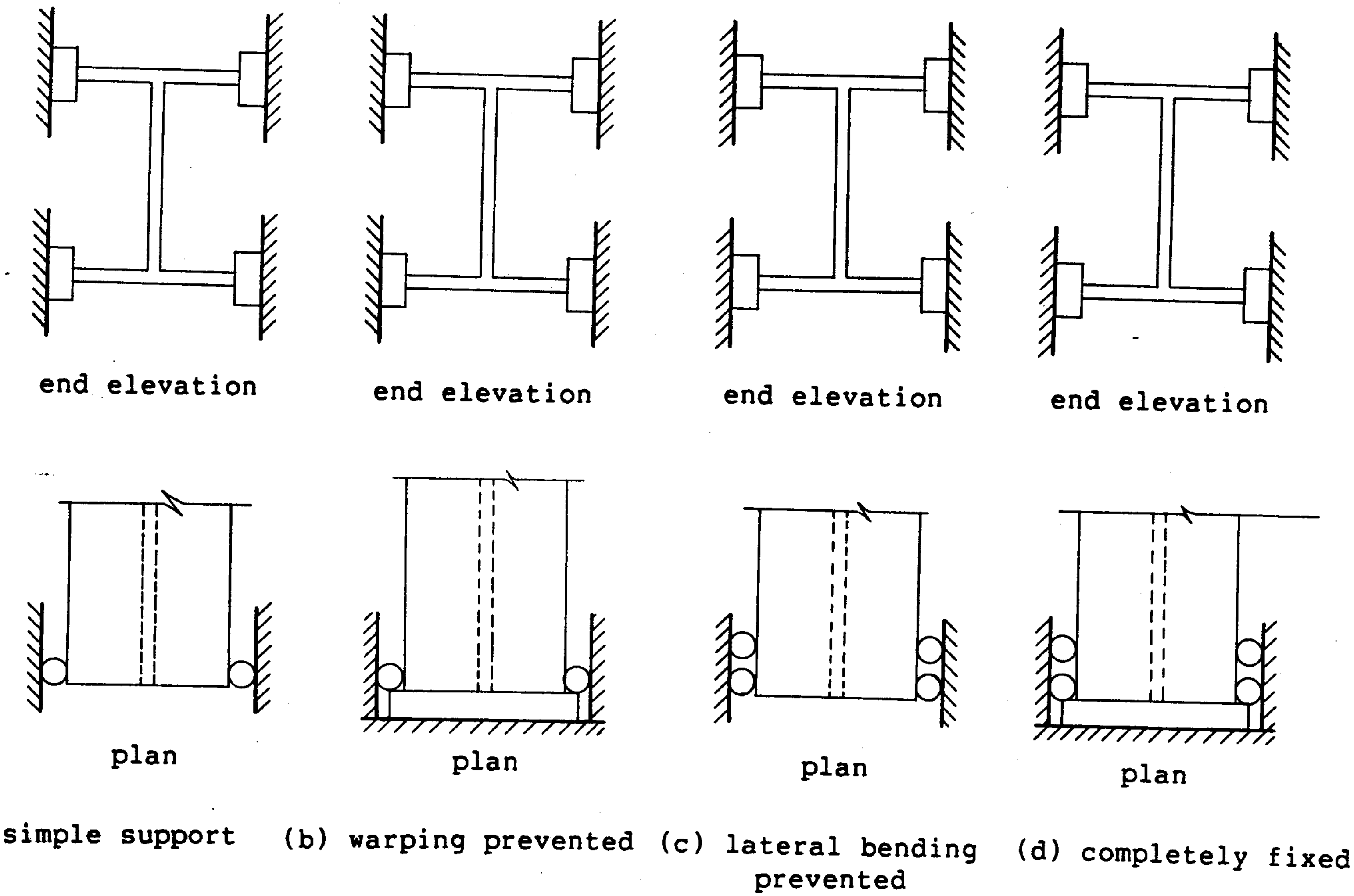


Figure 2.4 Idealized End Restraints With Respect to Lateral and Torsional Movements (Nethercot and Rockey 1971,1973)

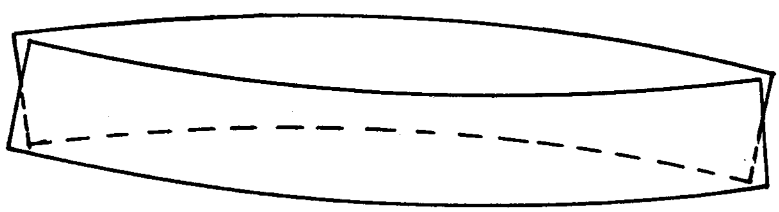
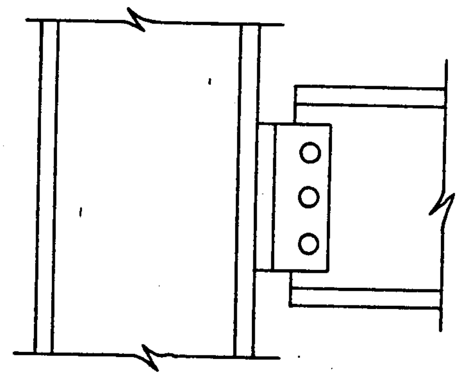
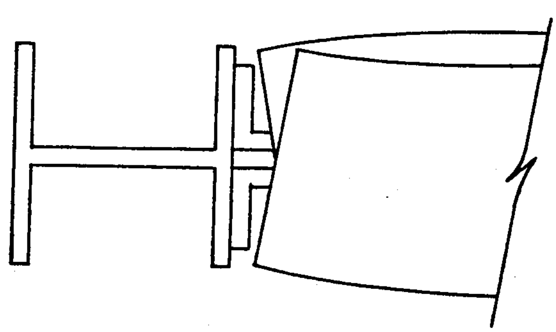


Figure 2.5 Warping Deformations of Beam Flanges



(a) side view



(b) plan

Figure 2.6 Double Angle Shear Connection warping ($\theta'' \simeq 0$).

The end plate connection shown in Fig. 2.7 can also be idealized as a torsionally pinned connection. The restraints are the same as the double angle connection. Rotation of the end of the beam about the longitudinal axis is approximated as zero. Warping deformations are unrestricted. A torsionally fixed end condition exists if rotation of the beam about its longitudinal axis is completely prevented and warping of the flanges is not allowed. Equation [2.7] is the appropriate mathematical expression for the boundary conditions of a beam torsionally fixed at each end. The first requirement, θ equal to zero, is the same as that for a torsional pin. The second requirement, θ' equal to zero, results in greater stability for a given beam.

Figure 2.8 shows an idealization of a torsionally fixed end. The beam is acted upon by a concentrated torque at some distance from the support. At the support forces are developed that resist warping of the section. These forces set up shear stresses in the flanges. The shear stresses acting in the flanges produce a couple (Fig. 2.9) which resists the applied torque (McGuire 1968). A welded connection of a rigid frame (Fig. 2.10) can be considered as providing fixed boundary conditions (Galambos 1968). Twisting is almost completely restrained at the ends ($\theta \simeq 0$) and warping deformations are impeded ($\theta' \simeq 0$) by the horizontal stiffeners bearing against the column flange.

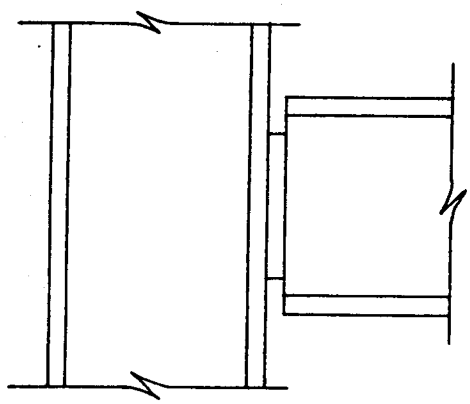
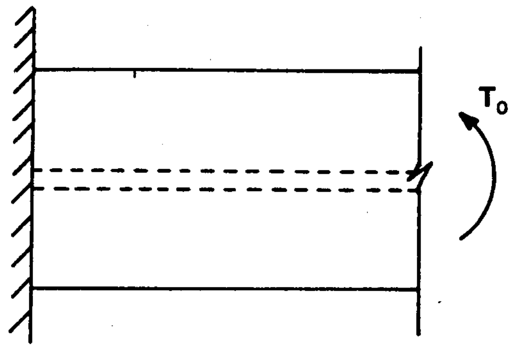
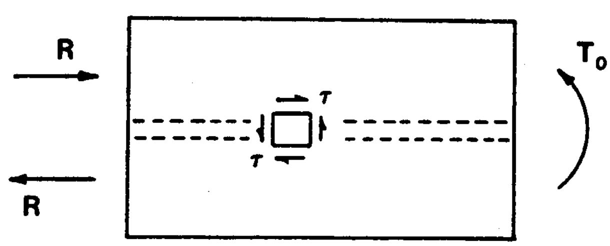


Figure 2.7 End Plate Shear Connection



idealized structure



free body diagram

Figure 2.8 Torsionally Fixed End

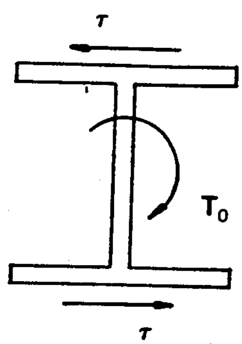


Figure 2.9 Torque Resisted by Flange Shears
(McGuire 1968)

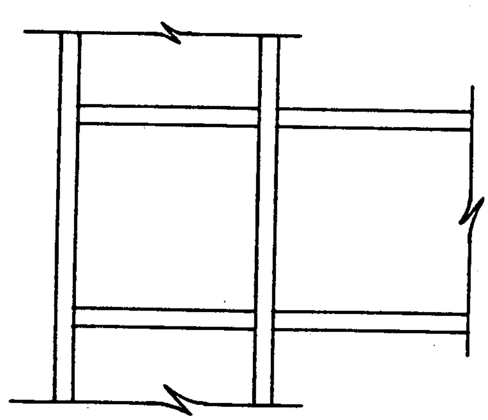


Figure 2.10 Rigid Connection

2.1.2.2 Lateral Restraint

A laterally pinned connection exhibits the same respective characteristics as a connection pinned in the vertical plane. Lateral deflection of the supported beam is prevented at the joint while rotation about the vertical axis is unrestrained. Equation [2.1] is the mathematical expression representing the boundary conditions of a laterally pin-ended beam. Figures 2.4(a) and 2.4(b) show idealizations of lateral pins.

The double angle connection of Fig. 2.6 is an example of a commonly used connection that provides approximately a laterally pinned boundary condition (Galambos 1968). The angles provide nearly full restraint to lateral deflection ($u \simeq 0$) while offering very little resistance to rotation about the vertical axis ($u'' \simeq 0$).

A beam is laterally fixed at a joint if rotation about the vertical axis as well as lateral deflections are completely restrained. Equation [2.6] mathematically describes the boundary conditions for a laterally fixed-ended beam. The supports shown in Figs. 2.4(c) and 2.4(d) provide full lateral fixity.

The rigid frame connection of Fig. 2.10 can be idealized as a torsionally fixed end (Galambos 1968). It almost completely restricts lateral deflections ($u \simeq 0$) and rotations about a vertical axis ($u' \simeq 0$).

2.1.2.3 Effective Length Factors

The form of Eqn. [2.8] suggests that the use of effective length factors may be appropriate for modelling the effects of restraints on the end of a beam. This situation is analagous to the column buckling problem where effective lengths are also used. In Eqn. [2.8] the term, 0.5L, outside the radical represents the effective length with respect to lateral bending. The 0.5L term inside the radical represents the torsional effective length. A large number of effective length factors may be used to account for the various conditions of end restraint (Vlasov 1961; Galambos 1968). Alternatively, only a few, more general factors may be incorporated (Galambos 1968; Structural Stability Research Council 1976), resulting in greater convenience but less precision. Effective length factors have also been used to account for the shape of the bending moment diagram and the effect of the level of application of the load (Trahair 1963).

2.1.3 The Effect of the Shape of the Moment Diagram

Equation [2.5] was derived for the case of a beam acted upon by equal and opposite end moments (Fig. 2.2). Obviously, other types of loading conditions are possible but basing the derivation on this particular load case does have advantages (Kirby and Nethercot 1979). The first one is convenience. The differential equations of lateral-torsional buckling are most easily derived and solved if equal and opposite end moments act on the beam. Secondly, the case considered is the most severe loading condition that can be applied to the beam. Equation [2.5] will yield conservative buckling moments when other shapes of the moment diagram exist. Making appropriate modifications to Eqn. [2.5] for various loading conditions will result in the calculation of larger critical moments.

Figure 2.11 shows a beam acted upon by two end moments, not necessarily of the same magnitude or sign. The term κ is equal to the ratio of the smaller to the larger end moment. A value of κ equal to -1.0 represents the severe loading case of Fig. 2.2. The severity of this case is exemplified by the the bending moment diagram which shows the maximum bending moment to be acting along the entire length of the beam. Most loading conditions found in practice will result in the maximum bending moment acting at only one, two, or three points along the beam. The lateral-torsional buckling formula (Eqn. [2.5]) can be modified by an equivalent moment factor (Trahair 1977), m, which accounts for variations in the shape of the moment diagram:

[2.9] $$M_{ocr} = \frac{m\pi}{L}\sqrt{EI_yGJ + \frac{\pi^2E^2I_yC_w}{L^2}}$$

where

[2.10] $$m = 1.75 + 1.05\kappa + 0.3\kappa^2 \leq 2.3$$

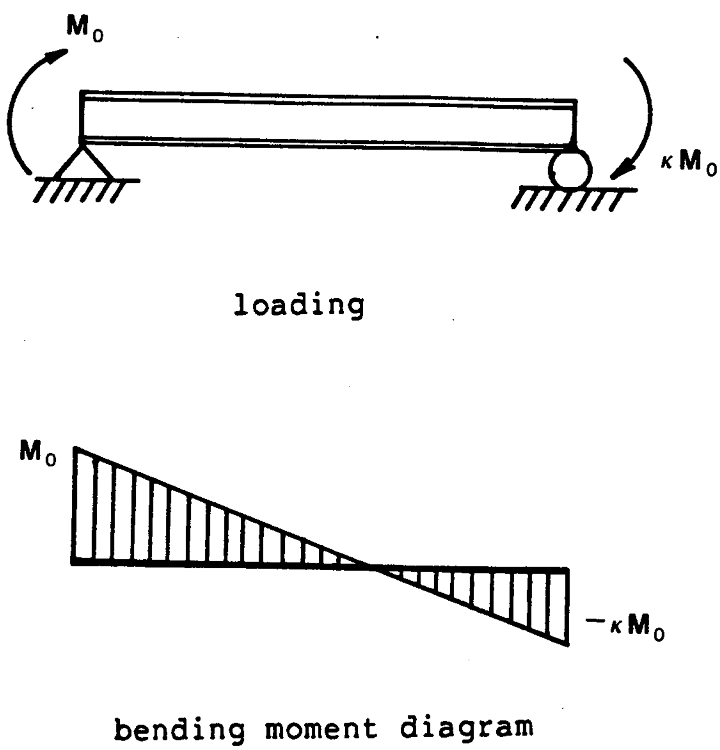


Figure 2.11 Beam With Moment Gradient

A good approximation in lieu of Eqn. [2.10] is:

[2.11] $$\frac{1}{m} = 0.6 - 0.4\kappa \geq 0.4$$

Values of the equivalent moment factor are tabulated (Structural Stability Resarch Council 1976; Trahair 1977; Kirby and Nethercot 1979) for load cases in which Eqns. [2.10] and [2.11] cannot be used (i.e. a beam loaded by a uniformly distributed load). The tables, however are not exhaustive. Equivalent moment factors are not readily available for some unusual shapes of bending moment diagrams. In these situations Kirby and Nethercot (1979) have proposed the following formula:

[2.12] $$\frac{1}{m} = \frac{3M_2 + 4M_3 + 3M_4 + 2M_{max}}{12M_{max}}$$

The variables in Eqn. [2.12] are defined in Fig. 2.12.

2.1.4 The Effect of the Level of Application of the Load

It would be unusual, in practice, to find a beam loaded by equal and opposite end moments. In most situations encountered transverse forces are applied to a beam. The level of application of these loads on the cross-section of the beam has an influence on its stability (Timoshenko and Gere 1961). A load applied to the top flange results in decreased stability since a torsional force develops as the

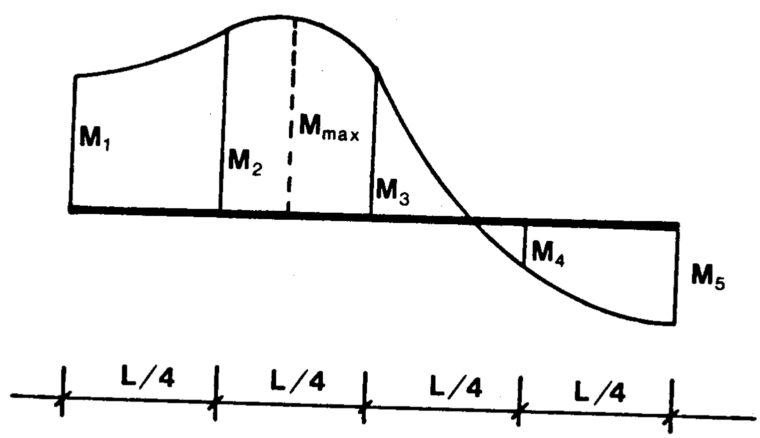


Figure 2.12 Arbitrary Bending Moment Diagram

beam begins to twist. This torsion causes even more twisting to take place. A load on the bottom flange increases the stability since the torsional force developed acts to resist further twisting. The critical buckling moment formula as it appears in Eqn. [2.9] is accurate for loads applied at the level of the shear center. For doubly symmetric sections the shear center coincides with the centroid. Equation [2.9] does not, however, account for the destabilizing effect of loads applied to the top flange nor the increased stability resulting from loads applied to the bottom flange. The following equation (Structural Stability Research Council 1976) accounts for these effects by including the constant, C:

[2.13] $$M_{ocr} = \frac{m\pi}{L} \sqrt{EI_yGJ + \frac{\pi^2E^2}{L^2}I_yC_w(1 + C^2)} \pm \frac{\pi EI_yhC}{2L^2}$$

Values of C based on the work of Clark and Hill (1962) are tabulated by the Structural Stability Research Council. The last term in the expression is positive for loads applied at the bottom flange and negative for loads applied at the top flange. For loads applied at the shear center, C is zero, and Eqn. [2.13] reduces to Eqn. [2.9].

2.2 Provisions in the Canadian Standard

2.2.1 Elastic Lateral-Torsional Buckling

Clause 13.6 of CSA Standard CAN3-S16.1-M78(Canadian Standards Association 1978) states that for a laterally unsupported, doubly symmetric beam the factored moment resistance may be taken as

[2.14] $$M_r = \phi M_u$$

where

[2.15] $$M_u = \frac{\pi}{\omega L}\sqrt{EI_yGJ + \frac{\pi^2E^2}{L^2}I_yC_w}$$

provided that

[2.16] $$M_u \leq \frac{2M_p}{3}$$

for class 1 and 2 sections, or

[2.17] $$M_u \leq \frac{2M_y}{3}$$

for class 3 and 4 sections. The requirements of Eqn. [2.16] and [2.17] ensure that yielding of the cross-section has not yet started and thus, the beam fails by elastic lateral-torsional buckling. The 2/3 factor is an allowance for the premature yielding of the cross-section caused by the presence of residual stresses. Once this yielding begins, failure of the beam occurs by inelastic lateral-torsional buckling. Inelastic buckling will be discussed in more detail later.

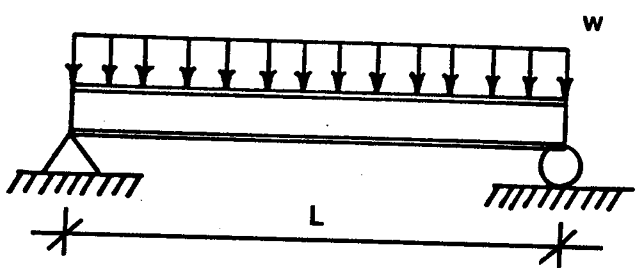
In equation [2.15] the term ω is the equivalent moment factor. It is identically equal to 1/m in Eqn. [2.11]. For members bent in single curvature

$$\omega = 0.6 + 0.4\frac{M_{f1}}{M_{f2}} \qquad [2.18]$$

For members bent in double curvature

$$\omega = 0.6 - 0.4\frac{M_{f1}}{M_{f2}} \geq 0.4 \qquad [2.19]$$

M_{f1}/M_{f2} is the ratio of the smaller to the larger end moment acting on the beam. The clause states that if at any point in the span the bending moment is greater than M_{f2}, ω is taken as equal to 1.0. This results in some conservatism. For instance a uniformly distributed transverse force (Fig. 2.13) would render an equivalent moment factor of 1.0. The Commentary (Canadian Institute of Steel Construction 1980) to S16.1-M78 mentions the tabulated values of equivalent moment factors by the Structural Stability Research Council. Using these, a value of 0.88 is selected for the above case.



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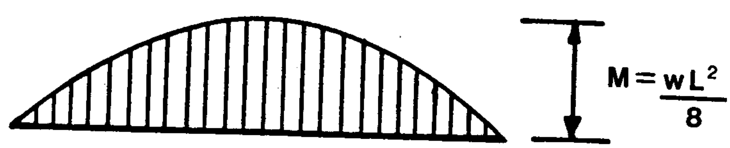


bending moment diagram

Figure 2.13 Beam Loaded by a Uniformly Distributed Load

Equation [2.15] ignores the effects (positive or negative) of loads applied at levels other than the shear center. The equation, as it is, implicitly assumes the load is applied at the shear center. In a situation where a beam is loaded on the bottom flange, the code requirements are conservative but for a beam loaded on its top flange the Standard is unconservative in its provisions. The Commentary makes no mention of the position of the load and its effects on stability. It is suggested that if a designer faces a situation where a beam is loaded on the top flange or if he wishes to take advantage of the stabilizing effect of bottom flange loading, Eqn. [2.13] should be used.

Another alternative appears in the 1969 version of the BSI Standard (British Standards Institution 1969). The requirement is that the effective length of a beam loaded on its top flange be increased by 20%. This empirical modification was based on the work of Kerensky, Flint, and Brown (1956). They showed that for short, deep beams the critical buckling moments corresponding to top flange loading were up to 30% less than for beams acted upon by equal and opposite end moments. They found that increasing the effective length by 20% would compensate for this loss of stability.

The Canadian Standard bases its requirements on the buckling of a simply supported beam. That is, the assumed end conditions are described by Eqns. [2.1] and [2.2]. In many cases more end restraint is provided than that of a simple pinned connection. S16.1-M78 does not account for this increased stability.

Although increased stability resulting from more rigid end conditions is conservatively ignored, provision has been made for the reverse situation. If less than simple support ($u \neq 0$, $\theta \neq 0$) is provided at one end of the beam the critical moment is reduced. Equation [2.15] can be modified for this effect by setting the equivalent moment factor, ω, equal to 1.0. If, however, the beam is acted upon by a uniform bending moment (Fig. 2.2) ω would be equal to 1.0 anyway. The end condition would not be accounted for. In this case, Eqn. [2.15] appears to be unconservative. Fortunately, the case of a uniform bending moment is unusual and the situation described is unlikely to occur.

In S16.1-M78, Eqn. [2.15] is taken to apply to HSS's as well as I-shaped members. Lateral-torsional buckling is not likely to be a problem for an HSS beam since an HSS section has high torsional rigidity. Hence, an HSS beam would have to be extremely long for lateral-torsional buckling to be a limiting factor in design. A square HSS beam will not buckle at any length since it has equal rigidities in each of the x-z and y-z planes (see Section 2.1.1. regarding assumptions).

2.2.2 Inelastic Lateral-Torsional Buckling

Beams in which partial yielding of the cross-section precedes buckling are said to fail by inelastic lateral-torsional buckling (Timoshenko and Gere 1961; Galambos 1968). Of course, if enough lateral support is provided or the member is sufficiently short, the plastic or yield moments can be reached and no lateral-torsional buckling will occur. With the onset of yielding the section is weakened and the elastic properties of Eqn. [2.15] are no longer applicable. In S16.1-M78 the relevant empirical equations are as follows:

[2.20] $$M_r = 1.15\phi M_p\left(1 - \frac{0.28M_p}{M_u}\right) \leq \phi M_p$$

provided

$$M_u \geq \frac{2M_p}{3}$$

for class 1 and 2 sections. For class 3 and 4 sections

[2.21] $$M_r = 1.15\phi M_y\left(1 - \frac{0.28M_y}{M_u}\right) \leq \phi M_y$$

provided

$$M_u \geq \frac{2M_y}{3}$$

Equations [2.20] and [2.21] are approximations of the inelastic buckling moments of laterally unsupported beams.

Closed form solutions to determine inelastic buckling moments taking into account the presence of residual stresses are not known to exist. Equations [2.20] and [2.21] represent curves extending from the range of laterally unsupported lengths where the plastic or yield moments can be reached to the range where elastic lateral-torsional buckling governs. Figure 2.14 shows the relationship between the unsupported length, L, and the factored moment resistance, M_r, for a class 1 or 2 section.

It can be seen that the inelastic buckling equation depends on the value of the elastic buckling moment, M_u. In this report methods for improving the calculated value of M_u are presented. It is suggested that a more accurate estimate of M_u will, in turn, result in a more accurate estimate of the inelastic buckling moment.

2.2.3 Approximate Nature of the Canadian Standard

The requirements made by S16.1-M78 with regard to lateral-torsional buckling of laterally unsupported beams neglect the effects of loads applied above or below the shear center. In Section 2.2.1. it was recommended that the SSRC equation (Eqn. [2.13]) be used to deal with this situation.

The provisions of S16.1-M78 also ignore the effects of beams having other than pinned end connections. This conservatism leads to designs which are safe but in some cases unnecessarily expensive. If the effects of end

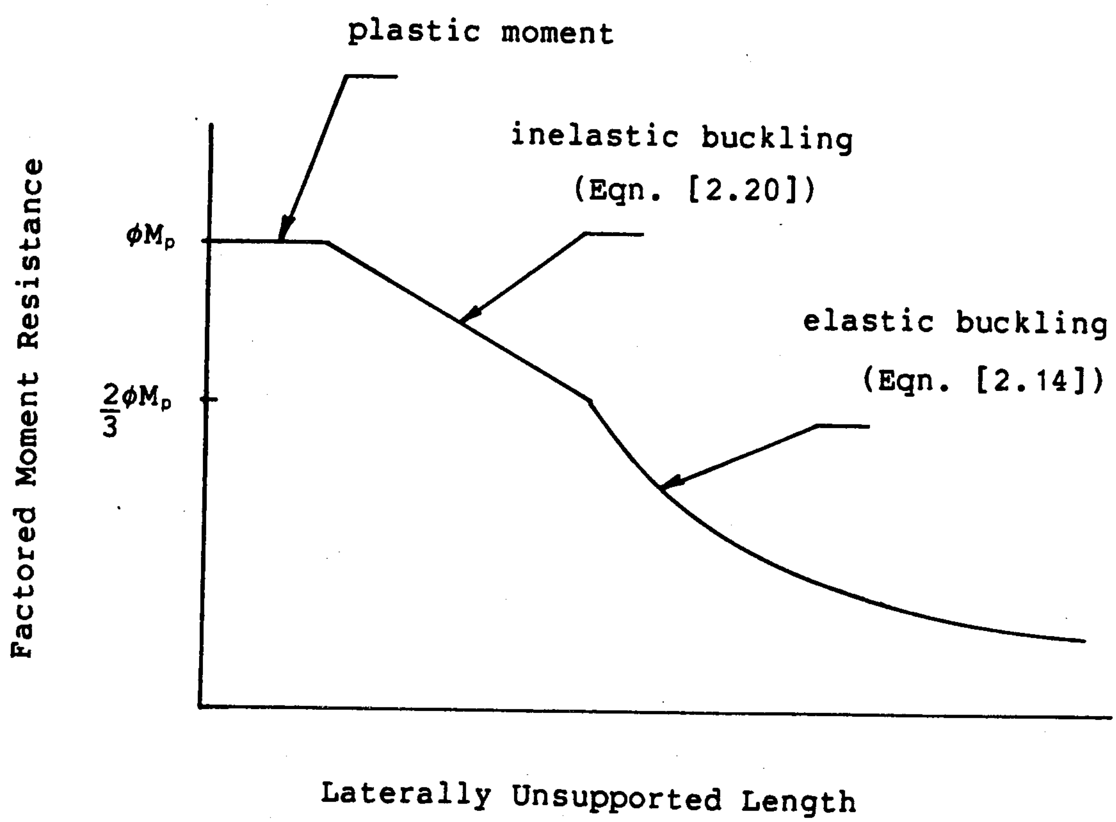


Figure 2.14 Factored Moment Resistance as a Function of the Laterally Unsupported Length

restraints are properly modelled, designs which are more economical, yet safe, can be achieved.

3. SINGLE SPAN BEAMS

3.1 Solutions in the Literature

A number of authors (Austin, Yegian and Tung 1957; Trahair 1963,1965,1966; Nethercot and Rockey 1971,1973) have provided methods for obtaining the elastic lateral-torsional buckling load of a single span beam. Each method attempts to account accurately for the effects of the variation in bending moments along the length of the beam, the level of application of the load with respect to the shear center, and the effect of end restraints. Some procedures are more suitable for design than others.

The critical buckling moment formula can be written in the following form:

[3.1] $$M_{cr} = \psi \frac{\sqrt{EI_yGJ}}{L}$$

The term ψ is called the lateral-torsional buckling parameter. Austin et. al. (1957) have provided tables of critical load parameters which can be easily converted to lateral-torsional buckling parameters. The tables are for beams with variable end restraint about each of the strong (x) and weak (y) axes. The critical load parameters were generated by a computer. End restraints ranging from unrestricted rotation to complete fixity about the vertical (y) axis were considered. It was assumed that the supports completely restricted lateral movement as well as twisting about the longitudinal (z) axis:

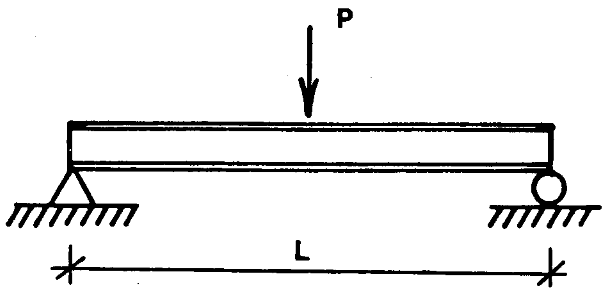
$$u = 0 \quad @ \quad z = 0, L$$

$$\theta = 0 \quad @ \quad z = 0, L$$

The results were based on the assumption that the torsional fixity is related to the weak axis restraint. It will be seen later that this is not necessarily true but in most cases is a reasonable approximation. Taking account of strong axis fixity is an indirect means of modelling the variation of moments along the length of the beam and thus, an equivalent moment factor is not needed in this procedure. The restraints were assumed to be symmetric about the mid-span of the beam (the same restraint at each end).

Only two types of loading were considered. They were a uniformly distributed load along the length of the beam (Fig. 2.5) and a concentrated load at mid-span (Fig. 3.1). The loads were applied at the top flange, shear center, and the bottom flange with seperate tables generated for each.

The method is not particularily suitable as a design tool. The critical load parameters are provided for a variety of connection stiffnesses. Unfortunately, the designer does not often know the connection stiffness to the same degree of precision as the tables provide solutions for. The tables only cover two types of loading conditions and require symmetry of the end restraints. To be used in



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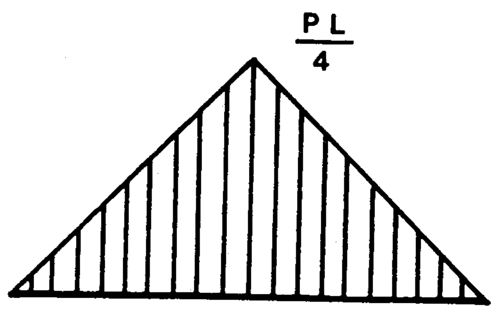


bending moment diagram

Figure 3.1 Beam Acted Upon by a Concentrated Load at Mid-Span design, the method would require more tables which could be used for unsymmetric end conditions and a greater variety of load types. This, however, would result in a larger number of tables and their use would be cumbersome.

The results obtained by Austin et. al. are useful for comparing with the results of other proposed methods (Trahair 1965).

Trahair (1963) presented values of effective length factors for simply supported beams. They were to be used in conjunction with the British Standard (British Standards Institution 1969). The factors were functions of the shape of the moment diagram, the level of application of the load with respect to the shear center, and the type of end restraints. An effective length factor was given for each combination of the above variables.

In two later papers (1965, 1966), Trahair produced tables of critical load parameters that, like Austin et. al.'s, are easily converted to lateral-torsional buckling parameters. He analyzed similar support conditions to those of Austin et. al. Varying degrees of restraint about the strong and weak axes were examined. Torsional fixity, assumed to be independent of the weak axis restraint, was also considered. In the 1965 paper, the effect of twisting of the support was analyzed. Again, symmetry of the end restraints was assumed.

The loading conditions examined were a uniformly distributed load, a concentrated load at mid-span, and equal and opposite end moments. Seperate tables were generated for loads applied to the top flange, shear center, and bottom flange.

As in the method of Austin et. al., a numerical value must be determined for each type of restraint (vertical, lateral and torsional). With these one can find the critical buckling moment of the beam from the appropriate table.

Using this procedure in routine design would not be practical for the same reasons that prohibit the use of the method of Austin et. al.

The shape parameter, Ω^2, is defined by the following equation:

$$\Omega^2 = \frac{L^2GJ}{EC_w} \qquad [3.2]$$

In Figs. 3.2 and 3.3 plots of ψ versus Ω^2 show that the results obtained by Timoshenko and Gere, Austin et. al., and Trahair are in close agreement.

Nethercot and Rockey (1971,1973) proposed a method, based on a finite element solution (Barsoum and Gallagher 1970) which does not require extensive use of tabulated numerical results nor does it require symmetry of the end conditions. A variety of loading conditions can be analyzed when the end conditions are symmetric (1971). Forces can be applied to the top flange, shear center or bottom flange. For beams with mixed end conditions (1973) only end moments can be imposed for the method to be applicable.

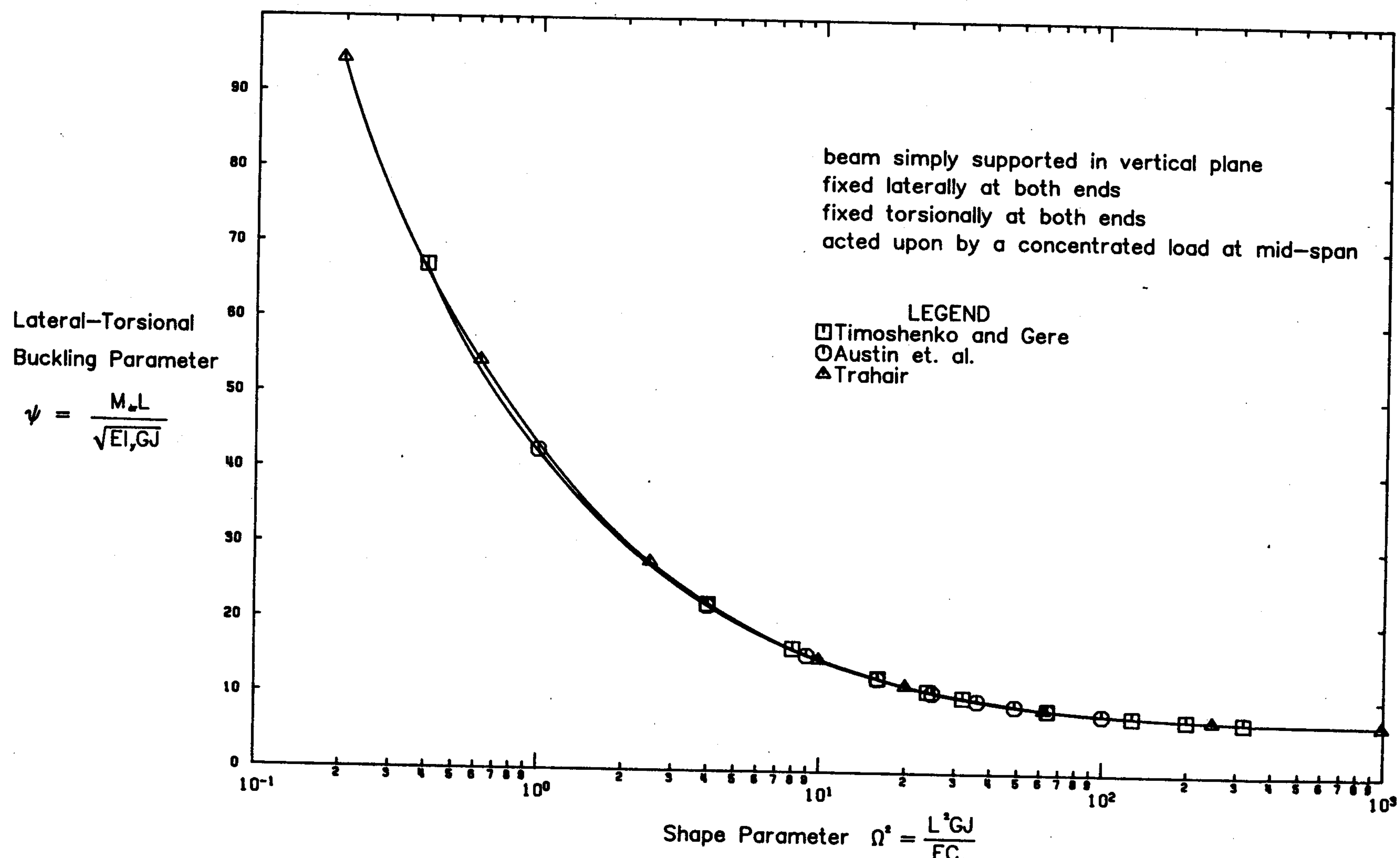


Fig. 3.2 Lateral–Torsional Buckling Parameters for Beam Carrying a Concentrated Load at Mid–Span

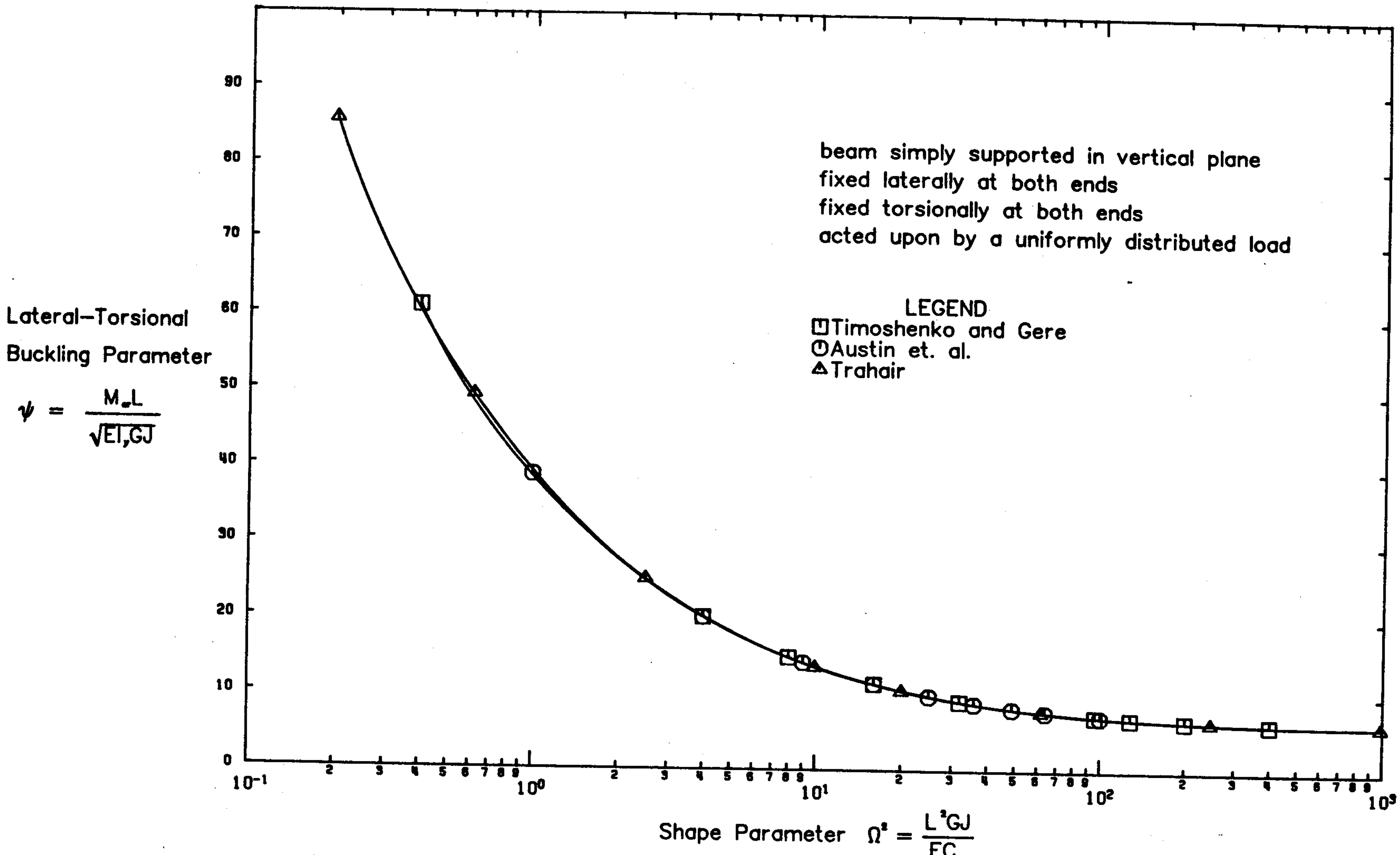


Fig. 3.3 Lateral–Torsional Buckling Parameters for Beam Carrying a Uniformly Distributed Load

Four types of end restraint are considered. They are shown in Fig. 2.4.

The basic equation used in the procedure is:

[3.3] $$M_{cr} = \delta\gamma\sqrt{EI_yGJ}$$

where

[3.4] $$\gamma = \frac{\pi}{L}\sqrt{1 + \frac{\pi^2}{\Omega^2}}$$

The term δ, called the lateral buckling coefficient accounts for the type of load, level of application of the load, and the type of support provided at the ends of the beam. Considering Eqn.[3.1] it can be seen that

$$\delta\gamma = \frac{\psi}{L}$$

A value of δ is evaluated for each combination of load type and support condition. For example, if a uniformly distributed load is applied to the shear center along the length of a beam which is restrained at each end by supports of the type shown in Fig. 2.4(c), then

$$\delta = 1.9 - \frac{1.184}{\Omega^2} + \frac{0.02}{\Omega}$$

Using the appropriate beam properties, the critical buckling moment can then be evaluated. Other equations for δ, corresponding to different conditions of restraint and loading, were presented in the original papers. The equation for δ is shown here to demonstrate the degree of complexity involved with the method. For a beam with simple supports and equal and opposite moments acting at each end, δ is 1.0, and Eqn. [3.3] reduces to Eqn. [2.5].

Nethercot and Rockey compared their method to other solutions (Winter 1943; Schrader 1943; Horne 1954; Kerensky et. al 1956; Timoshenko and Gere 1961) and found the results were indistinquishable.

The following equation can be used to calculate elastic critical buckling moments of I-beams:

[3.5] $$M_{cr} = \frac{m\pi}{k_y L}\sqrt{EI_y GJ + \frac{\pi^2 E^2}{(k_z L)^2} I_y C_w}$$

It is a simplified form of an equation presented by Galambos (1968). Equation [3.5] is similar to the one used in S16.1-M78 (Eqn. [2.15]) except for the inclusion of two effective length factors, k_y and k_z. The terms k_yL and k_zL are the effective lengths. Here they model the effect of end restraints only . Thus, seperate terms account for the type of loading and the support conditions while the level of application of the load on the cross-section is ignored.

The two different effective lengths represent the two types of deformations involved in lateral-torsional buckling. The term k_yL is the effective length with respect to lateral deflection, u, while the term k_zL is the effective length with respect to twisting, θ, about the longitudinal axis. Vlasov (1961) and Galambos (1968) have tabulated values of k_y and k_z for the same type of boundary conditions considered by Nethercot and Rockey (1971,1973). It is not necessary that the supports be the same at each end of the beam. The results of Vlasov and Galambos show that k_y is dependent upon both the lateral and the torsional restraint while k_z depends only upon the torsional restraint.

Equation [3.5] is ideally suited to be used in conjunction with S16.1-M78. To take advantage of the beneficial effects of end restraint it is only necessary to use the correct values of k_y and k_z in the equation. The following sections will examine the limiting conditions of end restraint and which effective length factors are appropriate for each.

3.2 Effective Lengths

3.2.1 Effective Lengths With Respect to Torsion

In Section 2.1.2.1 it was stated that double angle shear connections and end plate connections provide simple support with respect to torsion. These connections allow warping to take place while preventing twisting of the cross-section about its longitudinal axis. Throughout this report, with the exception of cantilevers it will be assumed that twisting is prevented at the ends of the beam.

$$\theta = 0 \quad @ \quad z = 0, L$$

This is an assumption that is commonly made (Austin et. al. 1957; Trahair 1966; Nethercot and Rockey 1971,1973). S16.1-M78 also makes this assumption (see the Commentary). Experimental results of Bennetts, Thomas, and Grundy (1982) showed that some connections do not completely prevent end twisting. Since real connections are not perfectly rigid they undergo some rotation about the longitudinal (z) axis. The Australian Standard (Standards Association of Australia 1981) makes provisions for the effects of partial restraint against twisting by stipulating an increase in the effective length of the beam by 20% (Bennetts et. al. 1982). As stated before (Sec. 2.2.1) the Canadian Standard accounts for this by requiring that ω be set equal to 1.0.

Schmidt (1965) has shown that if the torsional stiffness of the support is greater than 20 times the torsional stiffness, GJ/L, of the supported beam, then end twisting is effectively prevented. Nethercot and Rockey (1971) stated that most connections used in practice (the double angle and end plate connections are examples) will likely have a torsional stiffness of at least 20GJ/L and thus prevent twist. A double angle shear connection with a depth of the angle approaching the depth of the beam web should meet this requirement. A beam with no restraint against end twisting is unstable and cannot carry any load.

A finding of Bennets et. al. was that, in addition to rotation of the connection, web distortion can occur (Fig. 3.4). Another example is shown in Fig. 3.5 (Kirby and Nethercot 1979). If, at the end of a beam, the web distorts, twisting is not completely prevented. Web distortion and the reduction in beam stability caused by it can be prevented by vertical stiffeners (Kirby and Nethercot 1979).

It was previously stated (sec.2.1.2.1) that welded connections in rigid frames (Fig. 2.10) can be idealized as torsionally fixed supports. It was mentioned that warping deformations are restricted by the horizontal stiffeners bearing against the column flange. If the stiffeners are not provided, the connection is not completely fixed with respect to torsion although some resistance to warping will be provided by the column flange. The degree of restraint provided lies somewhere between that of a pin and that of a fixed connection. A conservative solution would be to consider it as providing no restraint against warping (Vacharajittiphan and Trahair 1974).

Warping can be prevented by a device of the type shown in Fig. 3.6 (Ojalvo and Chambers 1977). Channels or angles can be used to form a tubular shape that extends from the top flange to the bottom flange of the beam. An HSS can also be used if a slit is cut down one of its walls. Then the HSS can be slid into place from the end of the beam. Figure 3.7 shows this type of device used with an unstiffened seated

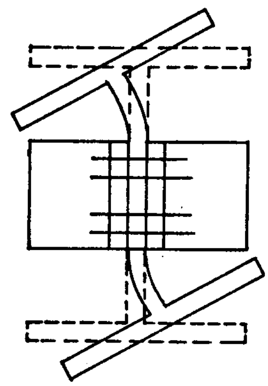


Figure 3.4 Web Distortion (Bennets et. al. 1982)

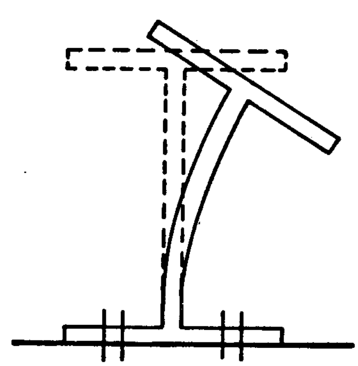


Figure 3.5 Web Distortion (Kirby and Nethercot 1979)

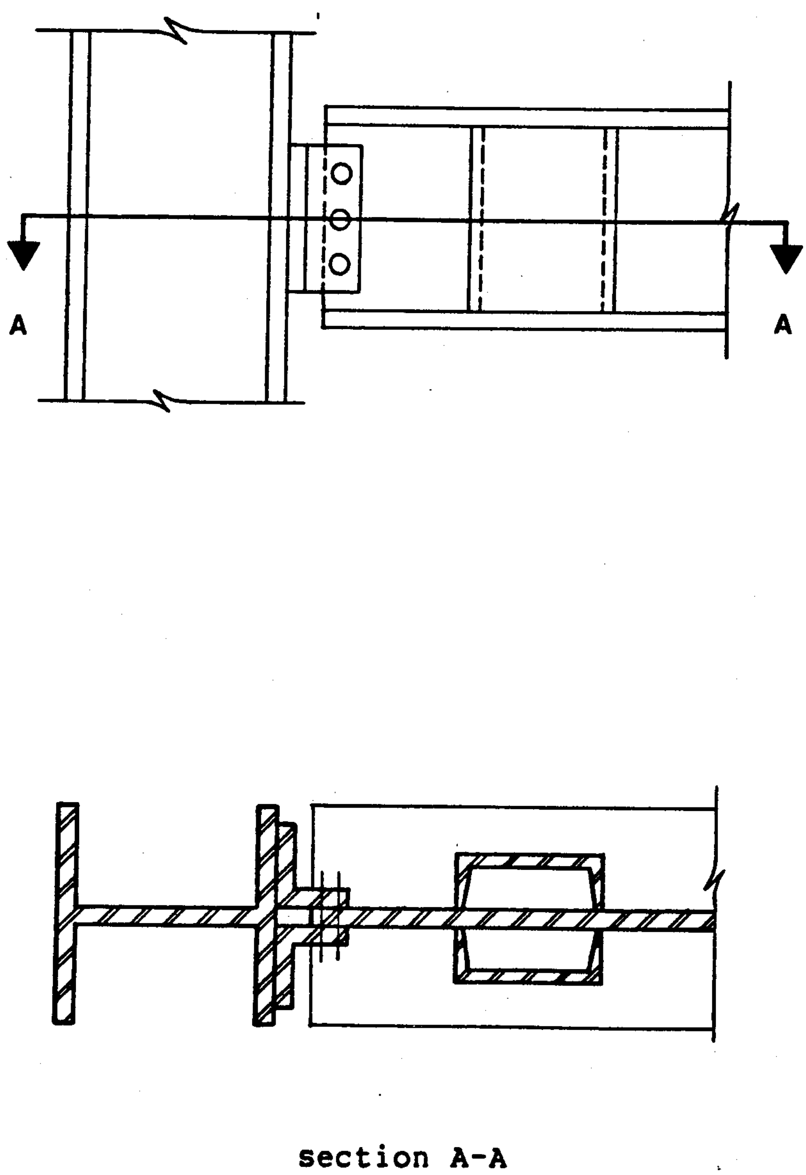
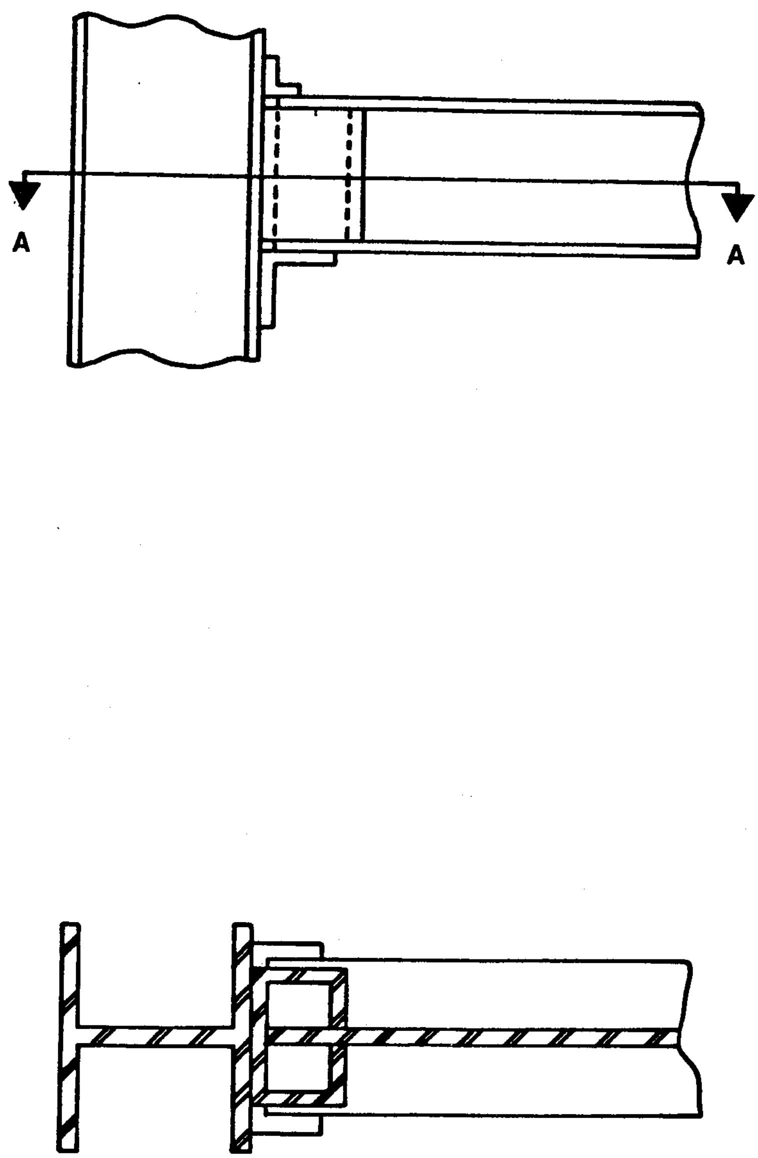


Figure 3.6 Channels Used to Restrain Warping



section A-A

Figure 3.7 HSS Used to Restrain Warping

beam shear connection. Warping resistance is provided by the tube which acts in torsion (Fig. 3.8). The situation is analagous to a beam which is subjected to equal and opposite twisting moments at each end. In the analogy the tube is represented by the beam. It spans from flange to flange. The warping moments acting on the flanges are represented by twisting moments at each end of the beam. Since closed tubular shapes provide high resistances to torsion it can be seen intuitively how connections of this type would be effective in resisting warping. The use of this type of device together with the double angle connection of Fig. 2.6 is a means by which torsional fixity can be achieved without providing restraint to lateral bending. Thus, weak axis and warping restraints are not necessarily correlated as Austin et. al. assumed (Sec. 3.1)

An analytical study by Ojalvo and Chambers indicated that a tube with a diameter equal to the width of the flange and with a wall thickness equal to 6.4 mm (1/4 inch) will practically prevent warping. Vacharajittiphan and Trahair (1974) developed a mathematical expression that can be used to evaluate whether or not warping is prevented by a tubular device. For warping to be effectively restrained

[3.6] $$K > \frac{5EI_yR}{\tanh(LR/2)}$$

where

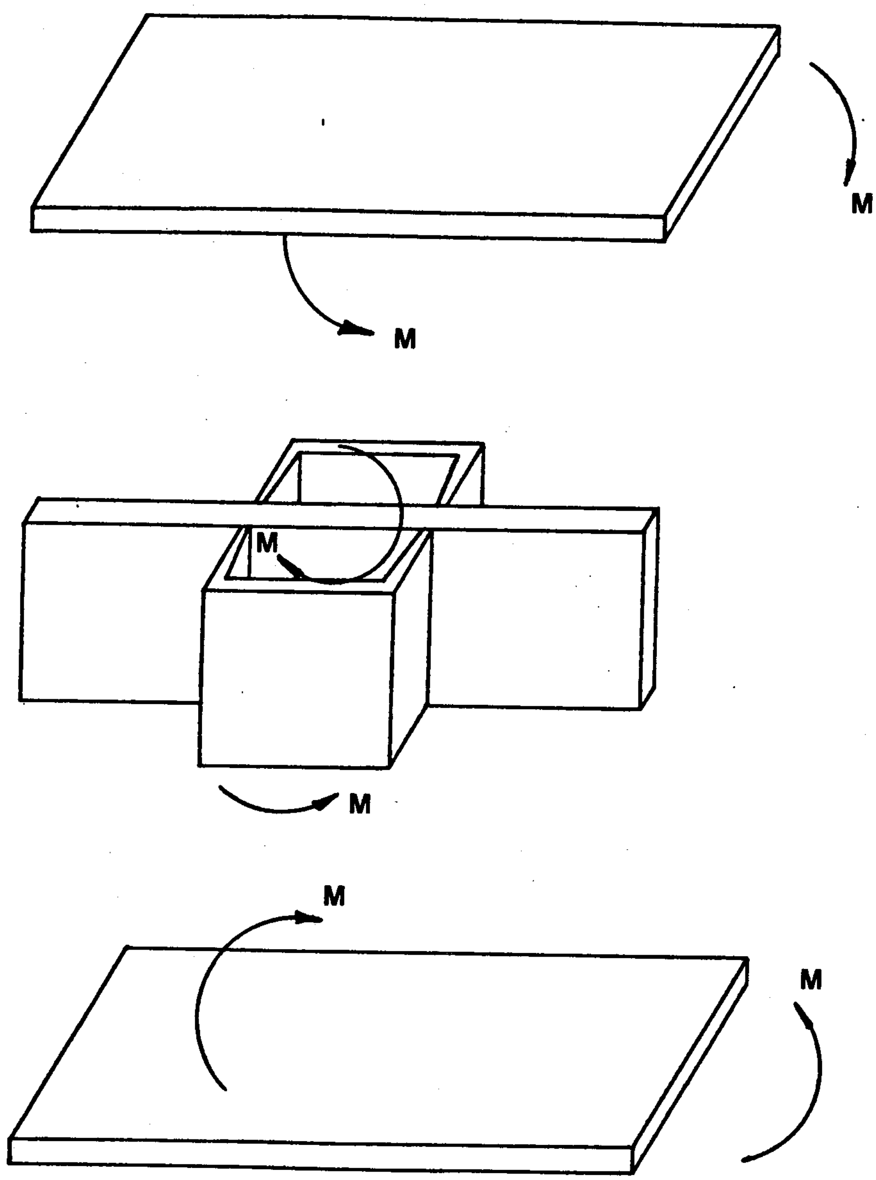


Figure 3.8 Transfer of Warping Moments Through Tubular Device (Ojalvo and Chambers 1977)

[3.7] $$R = \sqrt{\frac{GJ}{EC_w}}$$

K is called the elastic rotational stiffness of the element. For a tubular element (Ojalvo 1975)

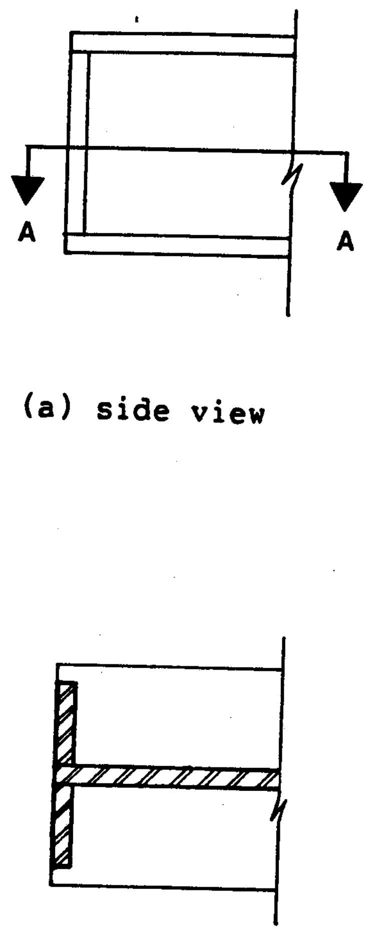
[3.8] $$K = \frac{2G_tJ_t}{h}$$

where h is the distance between the centroids of the beam flanges. The torsional stiffness parameter, G_tJ_t, is that of the tube. An example of the use of Eqns. [3.6] and [3.8] to evaluate the warping rigidity of a boundary condition is contained in Appendix A.1.

For the tube to be effective the following requirements should be met (Ojalvo and Chambers 1977):

1. The tube must be welded to each flange. This ensures that the warping moments in the flanges are transferred into the tube.
2. The channel, angle, or HSS should be welded to the web of the beam. This ensures that a closed section is formed.
3. In cases where cyclic loading is involved, consideration must be given to the possibility of a fatigue failure at the weld between the tube and the tension flange of the beam.

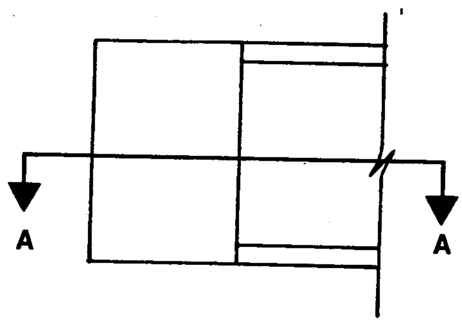
Other types of devices have been considered for use as warping restraints. Vertical web stiffeners (Fig. 3.9 are not effective in preventing warping (Vacharajittiphan and Trahair 1974). Additional web plates (Fig. 3.10) have also



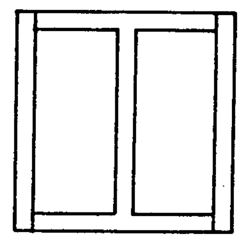
(a) side view

(b) section A-A

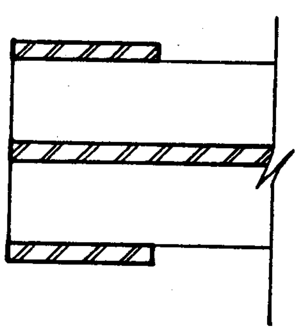
Figure 3.9 Vertical Web Stiffeners



(a) side view



(b) end elevation



(c) section A-A

Figure 3.10 Additional Web Plates

been studied (Vacharajittiphan and Trahair 1974). These too, have been shown to be inadequate (Vacharajittiphan and Trahair 1975; Ojalvo and Chambers 1977). The reason for the ineffectiveness of these devices can be seen in Figs. 3.9(b) and 3.10(c). The top views show that these devices make up open sections and, therefore, are weak torsionally. They cannot transfer the warping moments developed in the beam flanges. Another method used to prevent warping is to use the combination of stiffeners shown in Fig. 3.11. Ojalvo and Chambers reported that this device is not as effective as a tube. It requires more material and a greater length of weld.

For a beam with both ends torsionally pinned (Eqn. [2.2]) the torsional effective length factor is given by Eqn. [3.9] (Vlasov 1961; Galambos 1968):

[3.9] $$k_z = 1.00$$

This is a reasonable value for use in design. It is the same as the recommended value found in Appendix B of S16.1-M78 for the design of columns which are approximated as pin-ended.

A fix-ended beam (Eqn. [2.7]) has a smaller torsional effective length factor:

$$k_z = 0.492$$

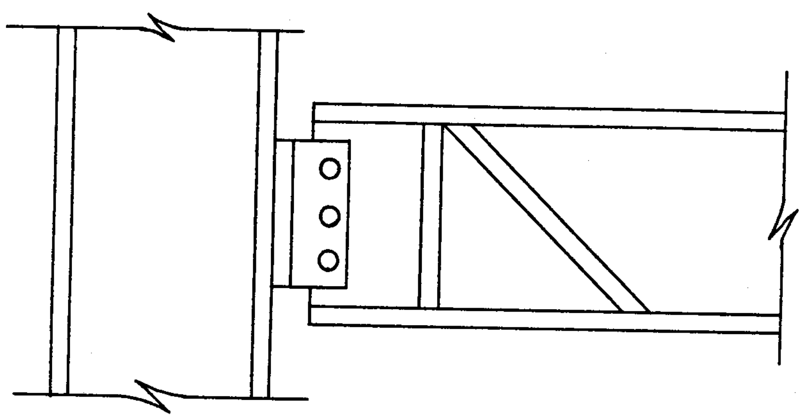


Figure 3.11 Stiffener Configuration Used to Restrain Warping

Galambos suggests that a value of 0.50 is an adequate approximation. For use in design, a slightly larger value should be used:

$$k_z = 0.65 \qquad [3.10]$$

Again, this is based on the recommendation in Appendix B of S16.1-M78 for the design of columns. The larger value of 0.65 is used because the smaller value of 0.50 applies for a perfectly fixed condition. In practice this ideal cannot be achieved. Thus, the effective length factor is pro-rated.

The effective length factor for a beam torsionally pinned at one end and torsionally fixed at the other is

$$k_z = 0.693$$

An approximation of 0.70 is suggested by Galambos. For use in design:

$$k_z = 0.80 \qquad [3.11]$$

As before, this is the same value used for a column with similar boundary conditions. It is increased above the ideal value less than in Eqn. [3.10] since a fixed condition is assumed at only one end of the beam.

3.2.2 Effective Lengths With Respect to Lateral Restraint

As previously described, a lateral pin prevents lateral deflection of the supported beam at the connection. Also, it allows unrestricted rotation about a vertical axis. The first requirement above is commonly assumed to be fulfilled (Austin et. al. 1957; Trahair 1965,1966; Nethercot and Rockey 1971,1973). There are situations, however, in which lateral defections do take place. Figure 3.5 shows such a situation. Partial restraint against lateral deflection can be accounted for in the same way as is partial restraint against twisting(see sec.3.2.1).

A beam laterally pinned at both ends will have a lateral effective length factor, k_y, ranging from 0.88 to 1.00, depending on the degree of torsional restraint provided. Galambos suggests that the following value be used for all cases of laterally pin-ended beams regardless of the torsional restraint provided:

[3.12] $$k_y = 1.00$$

This value would seem reasonable for design. The corresponding effective length factor for a pin-ended column is also 1.00.

For a beam with laterally fixed ends k_y ranges from 0.43 to 0.49. Again, the range is due to different degrees of torsional restraint. The suggested value by Galambos is:

$$k_y = 0.50$$

Using the corresponding column effective length factor for design purposes (Canadian Standards Association 1978):

[3.13] $$k_y = 0.65$$

A beam which is laterally pinned at one end and laterally fixed at the other will have a lateral effective length factor in the range of 0.61 to 0.69. Nethercot and Rockey (1973) have pointed out that Galambos did not include an effective length for the case where one end is pinned laterally and fixed torsionally

$$u = u'' = \theta = \theta' = 0$$

and the other is fixed laterally and pinned torsionally

$$u = u' = \theta = \theta'' = 0$$

Vlasov did, however, include this case. His results show k_y is equal to 0.61. Vlasov neglected to include factors for beams in which

$$u = u'' = \theta = \theta' = 0$$

at one end and

$$u = u' = \theta = \theta' = 0$$

at the other. It seems strange that Galambos, obtaining his values from Vlasov's, did give an effective length factor of 0.61 for this case. Regardless, Galambos's suggested values of 0.70 appears to be a reasonable approximation for all cases of laterally fixed - laterally pinned beams. Again,

[3.14] $$k_y = 0.80$$

should be used in design.

It should be noted that the design effective length factors for lateral fixity are identical to those for torsional fixity.

Figures 3.12 and 3.13 show plots of lateral-torsional buckling parameters obtained using Galambos's effective length factors. Also shown are parameters obtained by the finite element method of Barsoum and Gallagher. The curves indicate that Galambos's effective length factors are safe. However, the effective length factors are unconservative if the support conditions are poorly matched to the loading condition. For example, consider a beam with a large positive bending moment at the left end and a smaller negative bending moment acting at the right end (Fig. 3.14). If a laterally and torsionally fixed support was at the left end and a laterally and torsionally pinned support was at the right end the loading condition and the supports would

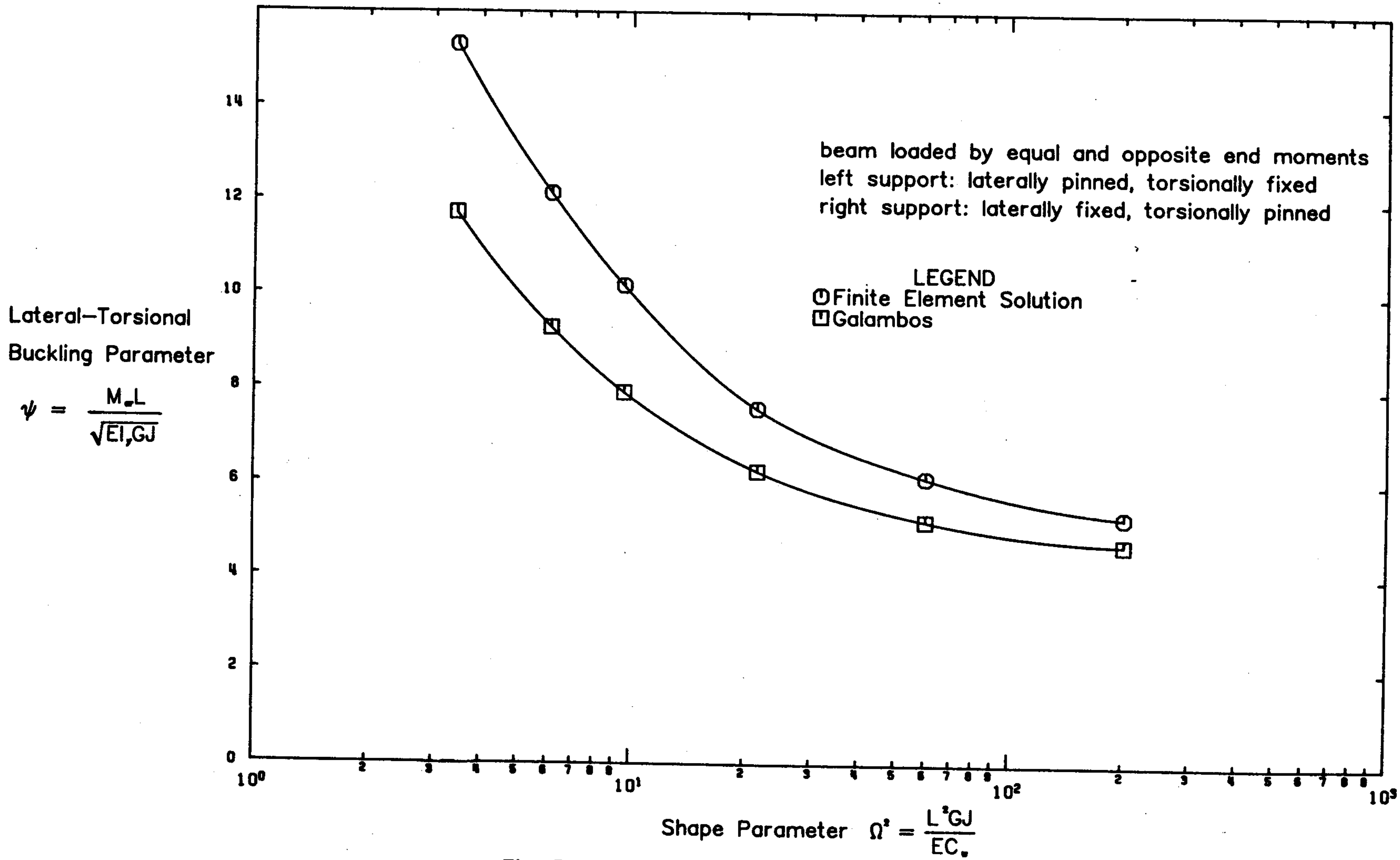


Fig. 3.12 End Restraints Modelled by Effective Lengths

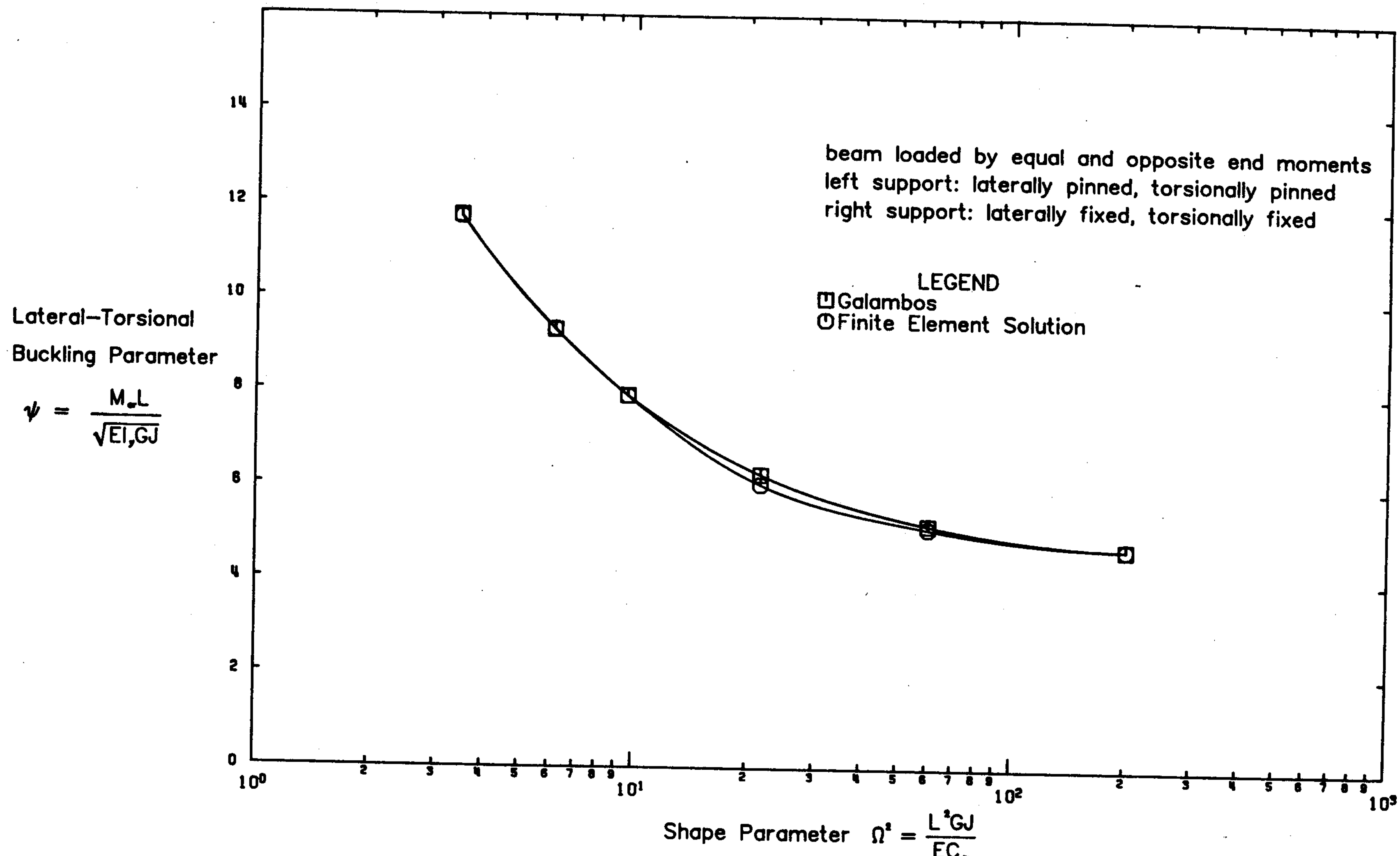
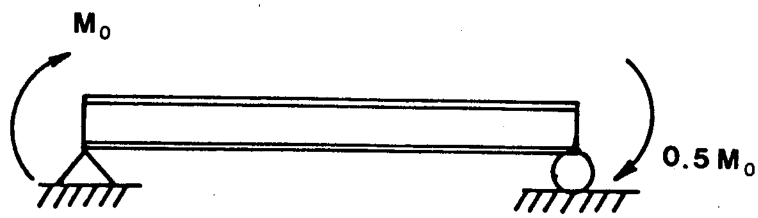


Fig. 3.13 End Restraints Modelled by Effective Lengths



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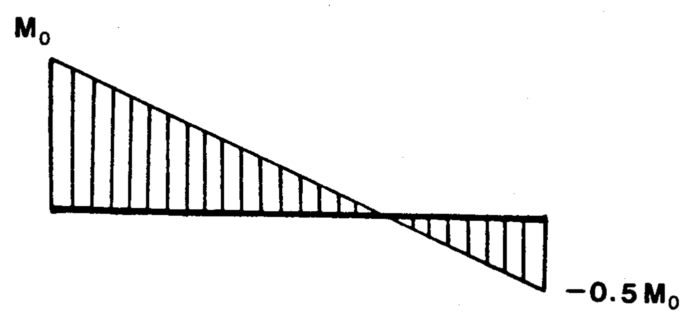


bending moment diagram

Figure 3.14 Beam With Large Moment at One End and Small Moment at the Other End be advantageously matched. This is because the larger moment acts near the stronger support. Conversely, if the supports were reversed they would not be well matched with the load. Figures 3.15 and 3.16 show that the effective length factors of Galambos yield satisfactory results for well matched beams but not for poorly matched beams.

3.3 Summary and Recommendations

The use of effective length factors to model the effects of end restraints on single span laterally unsupported beams is not a complicated process. Equation [2.15] is modified only slightly to become Eqn. [3.5]. Only two new constants need to be known to evaluate M_u. Although a variety of effective length factors are possible, the analysis is greatly simplified if just three are used. The same three factors can be used for both lateral effective lengths and torsional effective lengths. The effective length factors proposed for use with beams are completely analagous to those used in the design of columns. No additional tables, graphs, or equations are required. Only two limiting conditions of end restraint (fixed and pinned) for each of the torsional and the lateral deformations involved in beam buckling were considered. Obviously, many intermediate situations can exist. It is up to the designer to use his judgement to decide which idealized support condition can be safely used to approximate reality. If more precision is desired other

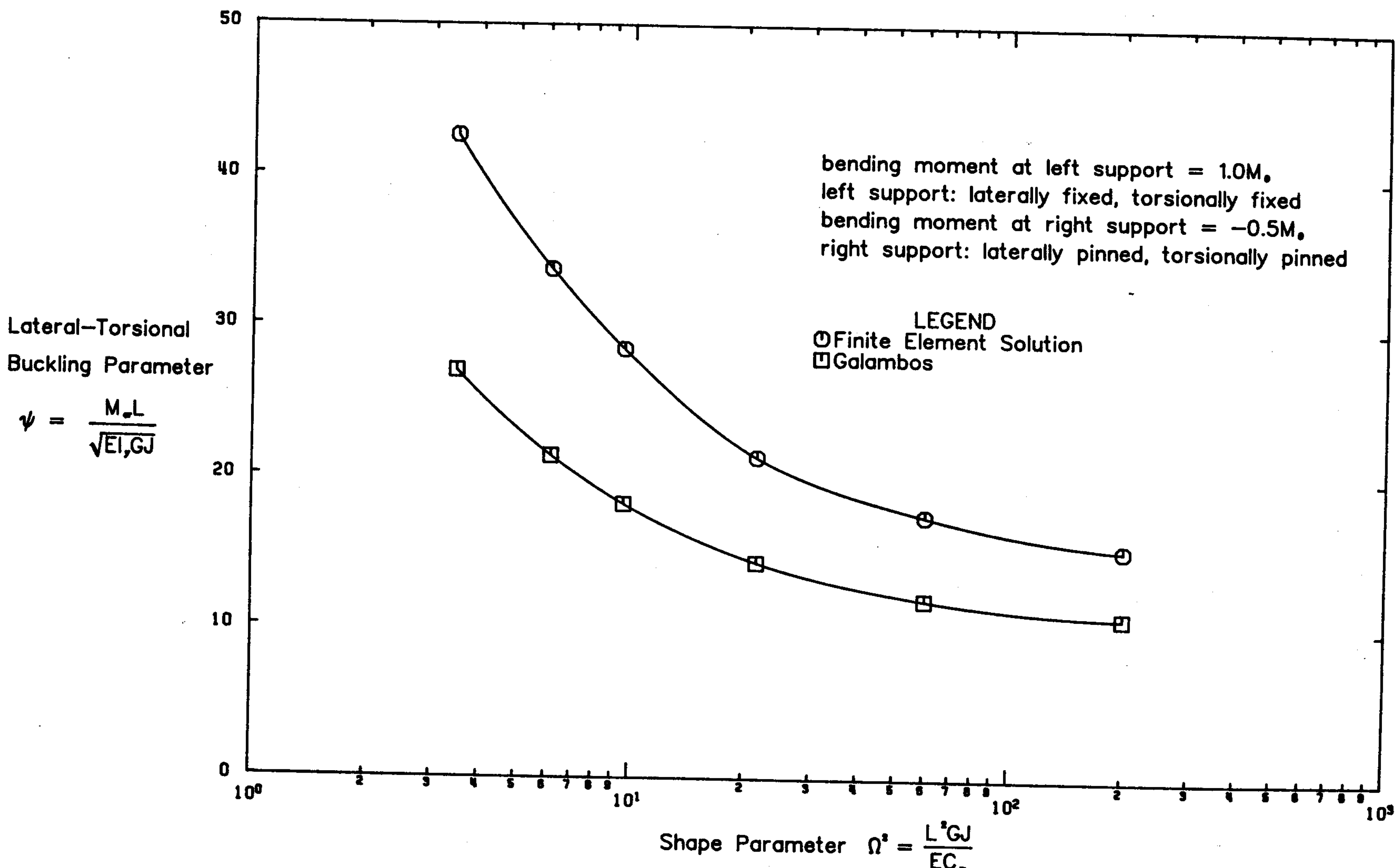


Fig. 3.15 Beam With Good Match Between Load and Supports

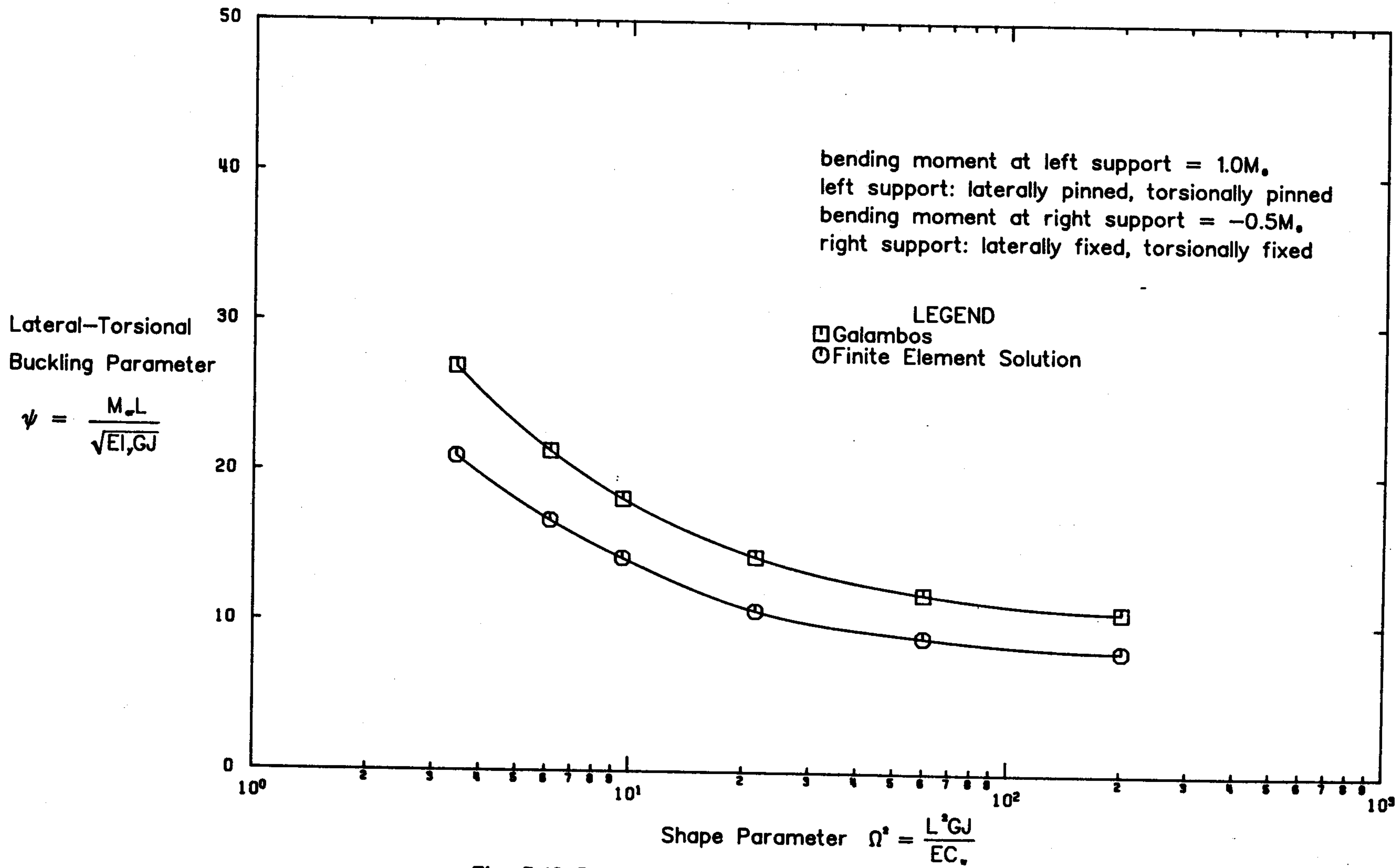


Fig. 3.16 Beam With Poor Match Between Load and Supports

methods (Austin et. al. 1957; Trahair 1965,1966) can be used at the expense of speed and simplicity.

In the design of a structural system involving a laterally unsupported beam, one should try to match the end conditions with the applied load. This results in a more efficient structure. If a beam and the loads applied to it are poorly matched the effective length factors by Galambos should not be used.

The application of effective length factors with Eqn. [2.15] of S16.1-M78 will result in elastic critical buckling moments that are equal to or greater than those calculated using the present method. Beams which buckle inelastically will also have computed strengths which are equal to or greater than those calculated without effective length factors. This is because the inelastic buckling strength, as calculated in S16.1-M78 (Eqn. [2.20] or [2.21]), is a function of the elastic buckling strength. These increases in calculated strengths should give a more accurate representation of the real situation. The increased accuracy will result in less conservatism and a saving in cost. An example of the use of effective length factors is presented in Appendix A.1.

4. CANTILEVERS

The cantilever beam of Fig. 4.1 is completely fixed at its support and acted upon by a moment, M_o, at its free end or tip. The expression for the critical buckling moment of the beam is (Trahair 1977)

[4.1] $$M_{ocr} = \frac{\pi}{2L}\sqrt{EI_yGJ + \frac{\pi^2E^2}{(2L)^2}I_yC_w}$$

Equation [4.1] implies that the effective length factor of the beam is 2.0. Figure 4.2 shows the same beam except that a concentrated force, instead of a moment, acts at the free end. The force is applied at the level of the shear center. The critical moment equation is given by Eqn. [3.1] (Timoshenko and Gere 1961).

[3.1] $$M_{cr} = \psi\frac{\sqrt{EI_yGJ}}{L}$$

The dimensionless lateral-torsional buckling parameter, ψ is a function of the shape parameter, Ω^2.

[3.2] $$\Omega^2 = \frac{L^2GJ}{EC_w}$$

Timoshenko and Gere tabulated values of Ω^2 and ψ. as given in Table 4.1. Alternatively, they propose the following approximate equation: (only for large values of Ω)

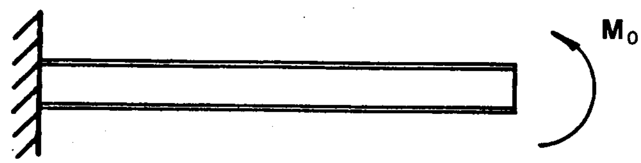


Figure 4.1 Cantilever Acted Upon by an End Moment

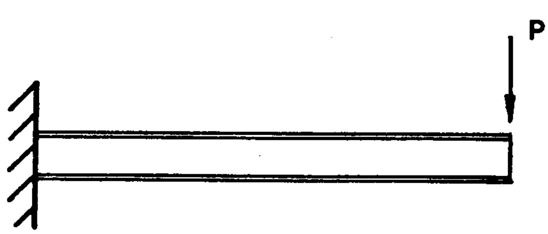


Figure 4.2 Cantilever Acted Upon by a Concentrated Load

Table 4.1 Lateral-Torsional Buckling Parameters for Tip Loaded Cantilevers (Timoshenko and Gere 1961)

Ω^2	ψ
0.1	44.3
1	15.7
2	12.2
3	10.7
4	9.76
6	8.69
8	8.03
10	7.58
12	7.20
14	6.96
16	6.73
24	6.19
32	5.87
40	5.64

[4.2] $$\psi = \frac{4.013}{(1 - 1/\Omega)^2}$$ (only for large values of Ω)

4.1 Provisions in S16.1-M78

The Canadian Standard deals with the stability of cantilevers in an indirect manner. Clause 13.6 states:

> "ω = 1.0 ... when there is no effective lateral support for the compression flange at one of the ends of the unsupported length"

This describes the condition at the free end of a cantilever. Thus, the critical moment of a cantilever beam is calculated using Eqn. [2.15] with ω equal to 1.0 and L equal to the length of the beam. The beam is designed as if it is simply supported at each end and acted upon by equal and opposite end moments. The effective length factor is 1.0. It turns out that this simply supported case is more severe than the cantilever and thus safe designs will usually result. The exception being unconservative results are obtained when the beam is loaded on the top flange (Nethercot 1973).

4.2 Recommendations by the SSRC

The SSRC (1976) recommendations also make use of equivalent moment factor to model the stability of a cantilever. If Eqn. [2.9] is used, conservative results will be obtained if m is set equal to 1.3 for a concentrated force on the free end. If a uniformly distributed load acts on the cantilever, m can be conservatively taken as 2.05. Again, the effective length factor is 1.0. The above cases are for loads applied at the shear center. The SSRC does not describe a procedure to account for loads applied above or below the shear center. Clark and Hill (1962) suggest that Eqn. [2.13] can be used in these situations. For a concentrated force acting at the end, C can be taken as 0.64. C is negative for top flange loading and positive for bottom flange loading. The equivalent moment factor remains equal to 1.3. A value for C was not given for a uniformly distributed force.

4.3 Effective Lengths by Nethercot

4.3.1 General Description

Nethercot (1973) has provided a set of effective length factors to be used for various types of cantilever beams. They are detailed in Sections 4.3.2 and 4.3.3. As in the two previous papers by Nethercot and Rockey (1971 and 1973) the values are based on results obtained from finite element solutions. They are valid for beams behaving in the elastic range. The effective length factors model the effects of the type of loading, the level of application of the load and the type of end restraint provided.

Two types of loading conditions are considered. They are a concentrated load acting at the free end and a uniformly distributed force acting along the length of the cantilever. Nethercot recommends the use of just one set of effective length factors for all loading conditions. This set is based largely on the case of a concentrated force acting at the free end. It is the more critical of the two cases and therefore designs based on this situation will be safe. Employing just one set is simpler than having one for every possible situation. For the most severe case in which an end moment acts on the cantilever Eqn. [4.1] should be used.

Nethercot's effective length factors account for end restraints at each end of the cantilever. At the support two types of boundary conditions are considered. They are either a completely fixed end or else a support over which the cantilever beam is continuous. At the tip various combinations of lateral and/or torsional restraints can be applied with seperate effective length factors corresponding to each.

4.3.2 Simple Cantilevers

As previously stated, simple cantilevers have completely fixed supports. This fixity can be achieved by making rigid connections (Fig. 2.10) or by running the beam into concrete or masonry walls.

4.3.2.1 Free Ended Simple Cantilevers

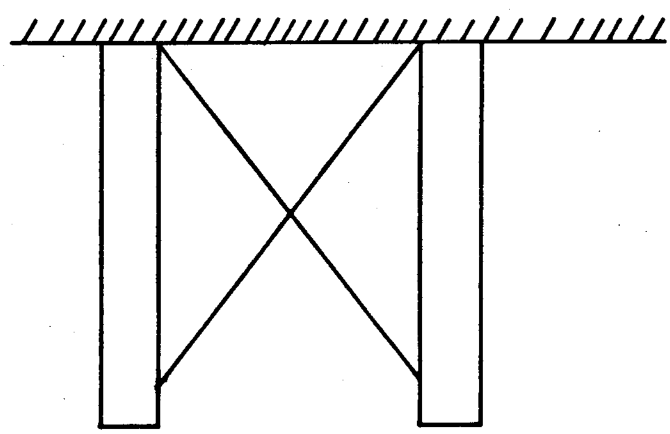
It is not likely that a cantilever will have a completely free end since usually the loading device will provide some kind of restraint. Where negligible restraint is provided a free end should be assumed. Effective length factors of 0.75 for shear center loading and 1.40 for top flange loading are suggested by Nethercot.

4.3.2.2 Lateral Deflections Prevented at the Tip

Figure 4.3 shows a plan view of two cantilevers restrained laterally by cross-bracing. All other displacements at the tips are unrestricted. For maximum lateral stability of the beams the braces should be attached to the compression flanges. The effective length factors for this situation are 0.65 for shear center loading and 1.40 for top flange loading.

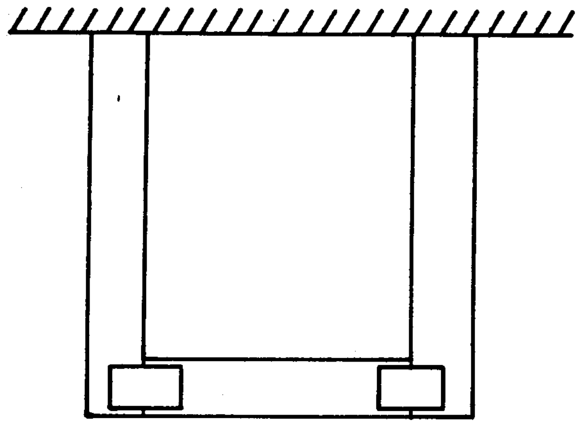
4.3.2.3 Twisting Prevented at the Tip

In Fig. 4.4 a plan view of a structure which has cantilever beams with the tips restrained from twisting is shown. Twisting action in the cantilevers is prevented by the strong axis rigidity of the cross-beam. The connections must be able to transfer these moments. An example of the type of connection needed to facilitate this is also shown. The effective length factor for these beams should be taken as 0.55. This is for load applied at any level. When the load is applied at a point where twisting is prevented additional torsional forces cannot be produced. Therefore, no increase nor reduction in stability occurs due to the level of application of the load. (see sec.2.1.4).

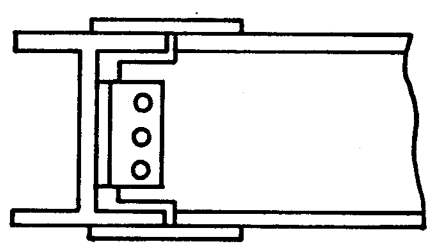


Plan View

Figure 4.3 Cantilevers Restrained Laterally at Tips



Plan View



Connection Detail

Figure 4.4 Cantilevers Restrained From Twisting at Tips

4.3.2.4 Lateral Deflection and Twisting Prevented at the Tip

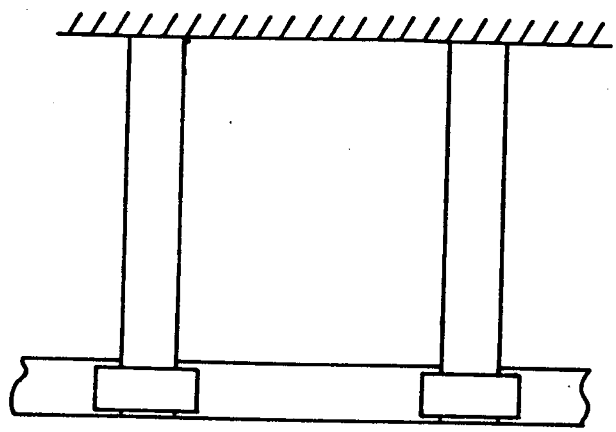
Figure 4.5 shows a structure in which the tips of the cantilevers are prevented from twisting by the strong axis rigidity of the cross-beams. Again, the connections must be able to transfer the bending moments developed at the points of intersection. Lateral deflection of the tip is prevented by the axial rigidity of the cross-beams. At some point in the structure there should be a reaction point for this axial force. The effective length factor for these cantilevers is 0.45.

4.3.2.5 Completely Fixed at the Tip

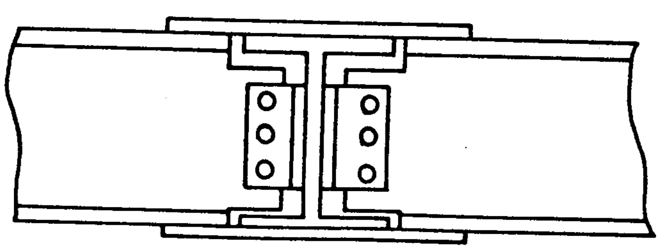
The cantilevers in the structure shown in Fig. 4.6 are completely fixed (except vertically) at their free end. Lateral deflection is prevented by the cross-bracing. Lateral bending is prevented by the use of heavy cover plates to facilitate weak axis moment transfer between the cantilevers and the cross-beam. Twisting is also prevented by the cross-beam. The tubular device (see connection details) prevents warping in the cantilever. The effective length factor for these cantilevers is 0.35.

4.3.3 Cantilevers Continuous at the Support

Figure 4.7 shows a cantilever which has a continuous support. The far left end is called the root, the continuous support is the fulcrum, and the far right end is the tip. Nethercot found that the vertical support condition at the root has very little effect on the stability of the

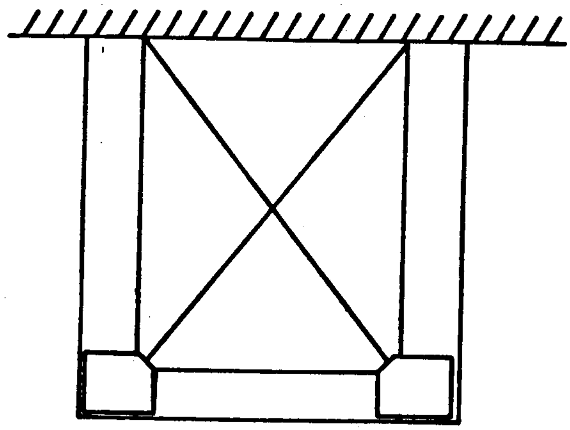


Plan View

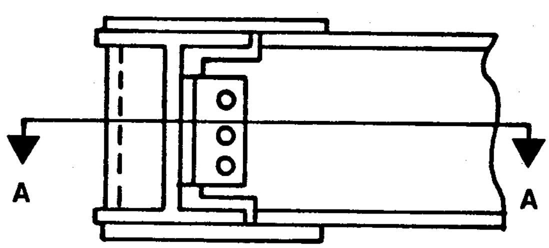


Connection Detail

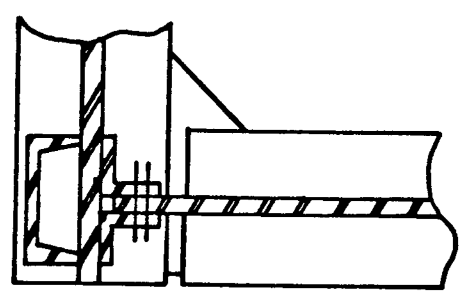
Figure 4.5 Cantilevers with Twisting and Lateral Deflection Prevented at Tips



Plan View



Connection Detail



section A-A

Figure 4.6 Cantilevers with Tips Fixed Laterally and Torsionally

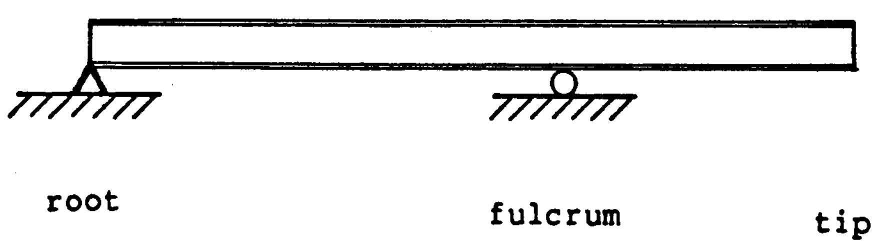


Figure 4.7 Cantilever with Continuous Support

cantilever beam. He also found that if twisting at the fulcrum is prevented the lateral support condition at the root is not influential. Two sets of effective length factors were derived. For beams in which twisting at the fulcrum was prevented the effective length factors are 1.0 and 2.6 for shear center and top flange loading respectively. If, at the fulcrum, twisting is not prevented the appropriate values are 2.5 and 7.0. Twisting can be prevented at the fulcrum by using vertical stiffeners (Fig. 4.8) and ensuring the bottom flange is rigidly attached to its support. If, at the tip, a brace is placed, preventing lateral displacements and twist, the cantilever is laterally continuous. Such a beam can be analyzed using one of the methods described in Chapter 5 or 6 for the design of laterally continuous beams.

4.3.4 Additional Comments

Nethercot's effective length factors are given for loads applied to the top flange and for loads applied to the shear center. No provisions are made for loads applied to the bottom flange. This is unfortunate since bottom flange loading is a common occurance in practice. It would be useful to have a way of accounting for this beneficial effect. Timoshenko and Gere (1961) do not address this problem for I-shaped beams. Only Clark and Hill's method can be used but that procedure is limited to simple cantilevers with free ends.

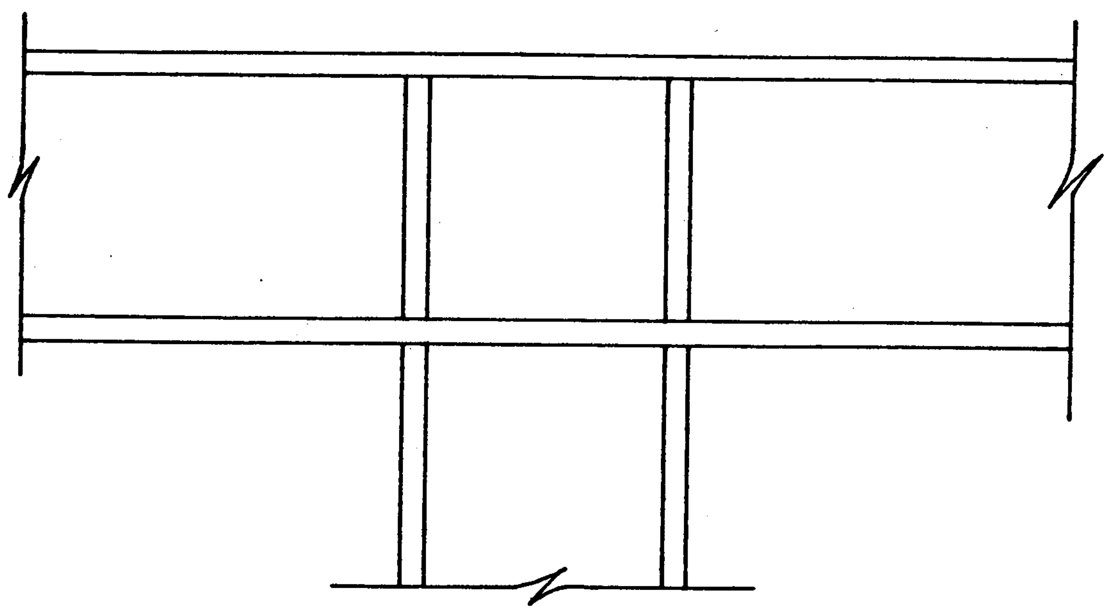


Figure 4.8 Vertical Stiffeners Used to Prevent Twisting

The use of effective length factors to model the effects of the type of load and the level of its application as well as the end restraints provided is philosophically different from other procedures recommended in this paper. In the other chapters effective length factors are used only to modify buckling equations for various types of end restraints. These factors are easily incorporated into existing design equations. For cantilevers, however, it seems necessary to account for all three influences with a single factor. This is because the equivalent moment factor, as found in its present form (Eqns. [2.10], [2.11] or [2.18] and [2.19]), is not applicable to cantilevers. S16.1-M78, by requiring that the equivalent moment factor be set equal to 1.0, ensures a safe solution. Setting ω equal to 1.0 is an indirect way of accounting for an effective length factor which, if modelling end restraints only, is actually greater than 1.0. The equivalent moment factor, ω, if modelling load type only, is actually less than 1.0. Thus, as presently used in S16.1-M78 for the design of cantilevers, ω is actually an empirical constant employed to yield safe answers. The use of Nethercot's effective length factors for cantilevers is no more irrational than the method presently used. In addition, Nethercot's factors reflect the destabilizing effects of loads applied to the top flange. They also provide more accurate solutions. Figure 4.9 shows plots of the lateral buckling parameter as a function of Ω^2 for a tip loaded simple cantilever (Fig. 4.2). It can be

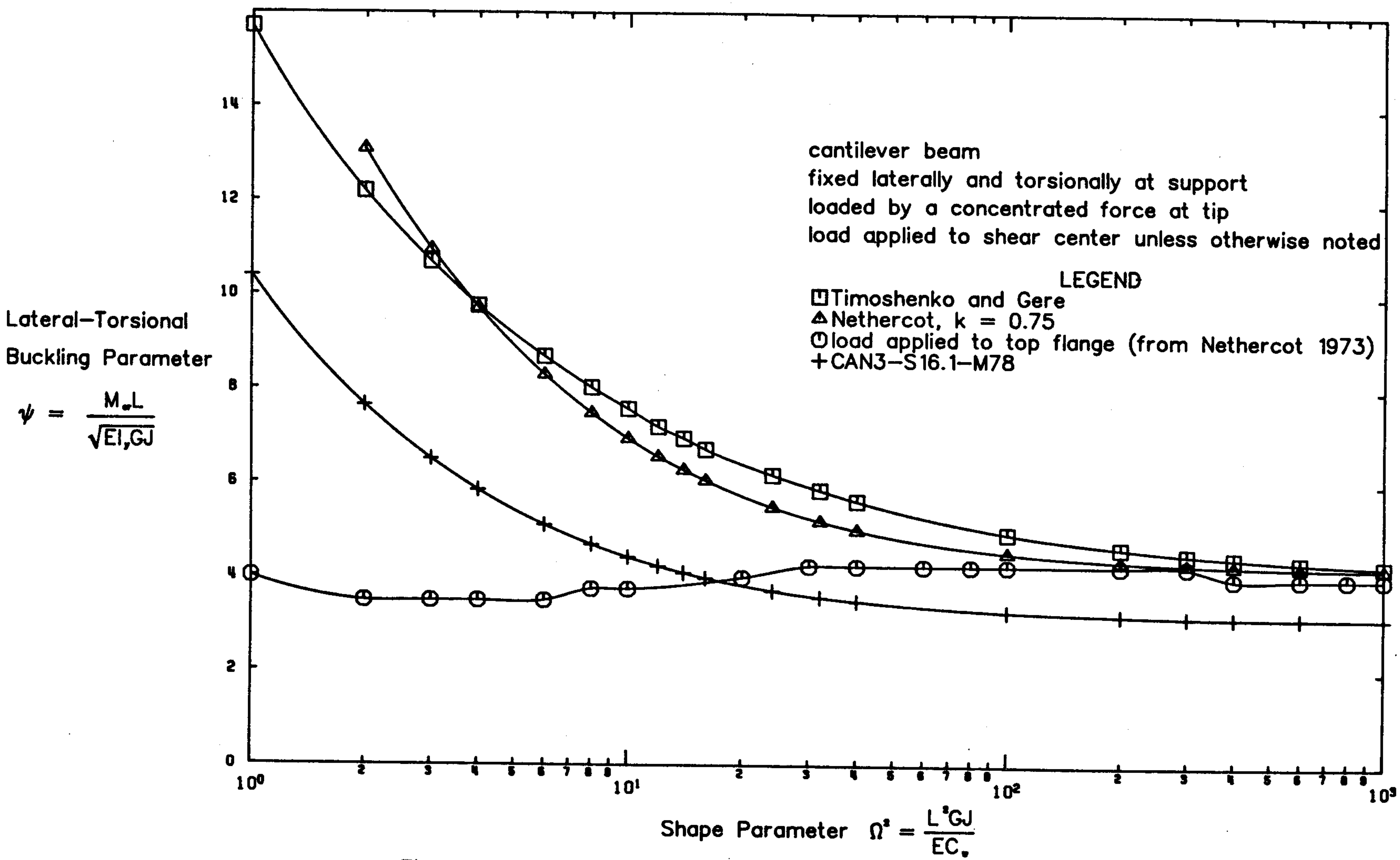


Fig. 4.9 Lateral-Torsional Buckling Parameters for Tip Loaded Cantilever

seen that strengths calculated using Nethercot's effective lengths are much closer to Timoshenko and Gere's solution than are those by S16.1-M78. In the range where Ω^2 is less than 4 Nethercot's method gives higher strengths than Timoshenko and Gere's. Beams that fall in this range will likely fail by inelastic buckling or reach their plastic moment capacity (Nethercot and Rockey 1971). Nethercot's effective length factors should not be used for inelastic beams.

4.4 Summary and Recommendations

Nethercot's effective length factors appear to be the best solution available since they yield more accurate results and provide a way of accounting for loads applied to the top flange. However, they do have some important limitations. They can only be used for elastic beams and they do not account for loads applied to the bottom flange. Improvements in these areas would be advantageous.

If an expression for an equivalent moment factor for cantilevers were developed a more rational approach to the stability of these beams could be used. Effective lengths could then be based solely on end restraints. For simple cantilevers, Galambos's effective length factors (1968) might be applicable while continuous cantilevers would be treated in a manner similar to continuous beams. It would not seem unreasonable to extend these procedures to be used for inelastic cantilevers.

5. CONTINUOUS BEAMS

5.1 Interaction Buckling

The preceeding chapters showed how end conditions influence the stability of a beam. It was shown that different types of connections provided various types of boundary conditions. In this chapter the focus is upon laterally continuous beams where the end conditions provided are a result of the continuity of the beam.

Consider a simply supported beam, continuous over three spans. Assume that it carries load in the vertical plane - say concentrated forces at the middle of each span. The beam is shown in Fig. 5.1. Assume, also, that each support provides simple support with respect to lateral-torsional buckling (i.e. each support prevents twisting and lateral displacement but not warping). Each length of beam between two consecutive supports is called a segment. Figure 5.2 schematically shows the plan view of the beam. Under the action of the applied forces the beam deflects vertically in the shape shown in Fig. 5.3. On either side of each of the interior supports are points of contraflexure which define the points of zero moment. At these points the stresses in the top flange change from compression to tension. Since compression is inherent to buckling problems it may seem reasonable to use these points of zero moment to define the effective length of a segment in a laterally unsupported beam. That, however, is an incorrect conclusion. Figure 5.4

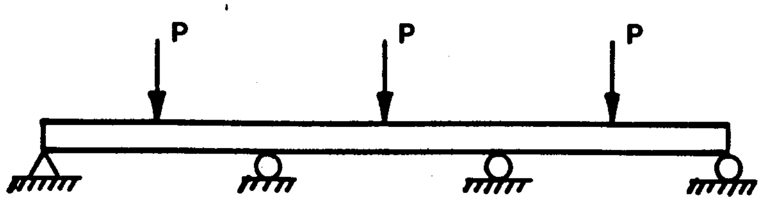
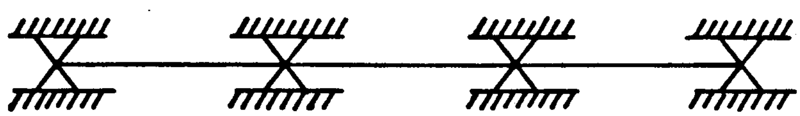


Figure 5.1 Continuous Beam



Plan View

Figure 5.2 Laterally Continuous Beam

Figure 5.3 Deflection in Vertical Plane of Continuous Beam

Plan View

Figure 5.4 Buckled Shape of Laterally Continuous Beam shows the plan view of the buckled shape (Trahair 1977; Kirby and Nethercot 1979). Notice that there is only one point of contraflexure near each interior support. The points of contraflexure in the vertical plane are quite different from the points of contraflexure in the lateral plane. It is the latter that define the effective lengths of the segments.

Figure 5.4 shows how the entire structure buckles when the critical moment of a segment is reached. This is called interaction buckling (Trahair 1968b). Some segments provide restraint for others. This kind of restraint is called positive restraint. The same segments that provide restraint for the others are themselves weakened by this interaction. They are provided with negative restraint by adjacent segments. In Fig. 5.4 the inner segment restrains the two outer segments. It provides positive restraint while the outer segments provide negative restraint. The effective length factor of the inner segment is greater than one while the outer segments have effective length factors less than one.

Figure 5.5 shows a simply supported beam with two forces applied at braced points. The braces are assumed to completely prevent lateral deflection and twisting of the beam. That the loads are applied at braced points is not an unreasonable assumption since the framing through which the loads are applied usually act as braces as well (Kirby and Nethercot 1979). It should be noted that the level of

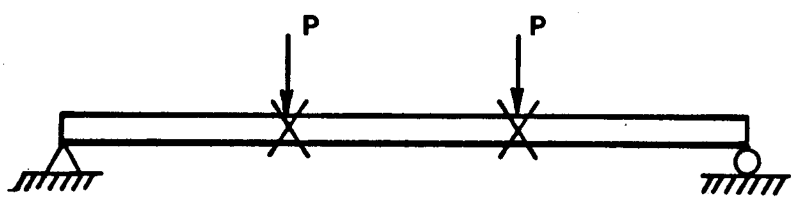


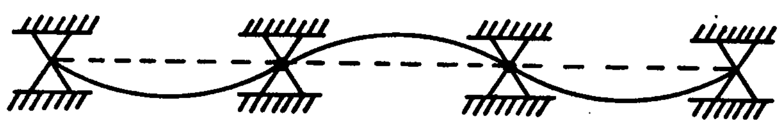
Figure 5.5 Simply Supported Braced Beam

application of the load with respect to the shear center is irrelevant when the loads are applied at braced points. The effect of the braces is to make the beam continuous in the lateral plane. Figure 5.2 is the plan view of the laterally braced beam shown in Fig. 5.5. A beam, therefore, does not necessarily have to be continuous in the vertical plane to be continuous in the lateral plane. The concept of interaction buckling is equally applicable to this type of beam as it is to a vertically continuous beam.

5.2 Solutions in the Literature

Once the buckling behaviour of a laterally continuous beam is understood the task is to calculate the locations of the points of lateral inflection. This has proven to be a difficult task.

Salvadori (1955) developed a lower bound solution by assuming that no interaction takes place between adjacent segments. The effective length factor of each segment is taken as one. The assumed buckled shape appears in Fig. 5.6. Note that the inflection points are at the supports. Since the restraint provided by adjacent segments is ignored the method gives lower bound solutions for segments which are actually provided with positive restraint. However, the method gives upper bound solutions for segments which, in reality, receive negative restraint (Trahair 1968a). The design of the beam is based on the lower bound critical moment. Salvadori's method provides the exact solution when



Plan View

Figure 5.6 Salvadori's Assumed Buckled Shape the loading and the span lengths are such that all the segments buckle independently (Kirby and Nethercot 1979). In this case there is no interaction between segments - as Salvadori assumed. This is called zero interaction buckling. For the maximum buckling strength of the beam to be utilizied the braces should be positioned so as to facilitate zero interaction buckling.

The Canadian Standard, S16.1-M78, incorporates the Salvadori approximation in its provisions for the design of laterally unsupported beams. The Commentary acknowledges that this is a conservative solution but states that due to the difficulty involved in evaluating boundary conditions no allowances are made for the effects of lateral continuity.

The tables of critical load parameters produced by Austin et. al. (1957) and Trahair (1965,1966) can be used for the design of continuous beams. This requires that the end restraint supplied by adjacent segments can be evaluated. Although the stiffness of an unloaded segment or a buckled segment can readily be evaluated (Trahair 1966), quite often a segment is loaded to a point between these two extreme conditions. It is the determination of the restraint povided by such a segment that is difficult. Trahair (1968b,1969,1977) developed a method for estimating approximate critical loads by utilizing the already existing tables. The procedure involves producing an approximate interaction diagram from which critical load combinations can be determined. For instance, if a two span continuous beam (Fig. 5.7) is loaded by a concentrated force on each segment the actual interaction diagram would look like the one shown by the solid line in Fig. 5.8. Trahair's approximate interaction diagram (boken line) would be produced by connecting straight lines between points 1, 2 and 3. Points 1 and 2 represent the situations where one span is loaded and the other is unloaded. These points can be found by using existing formulae (Trahair 1966) to evaluate end restraints and using the results to obtain critical loads from the critical load parameter tables. Point 3 is the zero interaction point. At this point both spans buckle without interaction. It is found by a trial and error procedure using formulae developed by Trahair (1968,1977) along with the critical load parameter tables. Any combination of loads falling within the interaction diagram can be safely carried by the beam. Any combination falling outside of the interaction diagram will cause lateral-torsional buckling to occur. Salvadori's solution is shown by the broken-dotted line (Trahair 1968).

Experimental results confirmed that the method has reasonable accuracy (Trahair 1969). Unfortunately, it is an unsuitable design tool for the same reasons that prohibit the use of tables by Austin et. al. and Trahair for the design of single span beams (Trahair 1977). The tables are somewhat inconvenient to use and lack generality.

Computer programs (Barsoum and Gallagher 1970; Powell and Klinger 1970) which utilize the finite element method

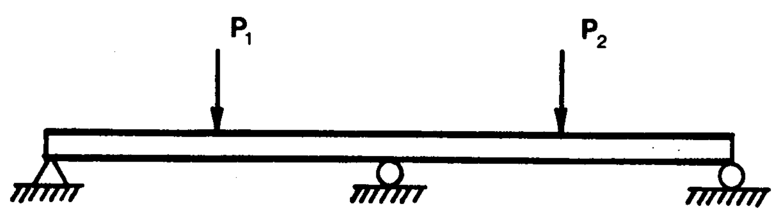


Figure 5.7 Two Span Continuous Beam

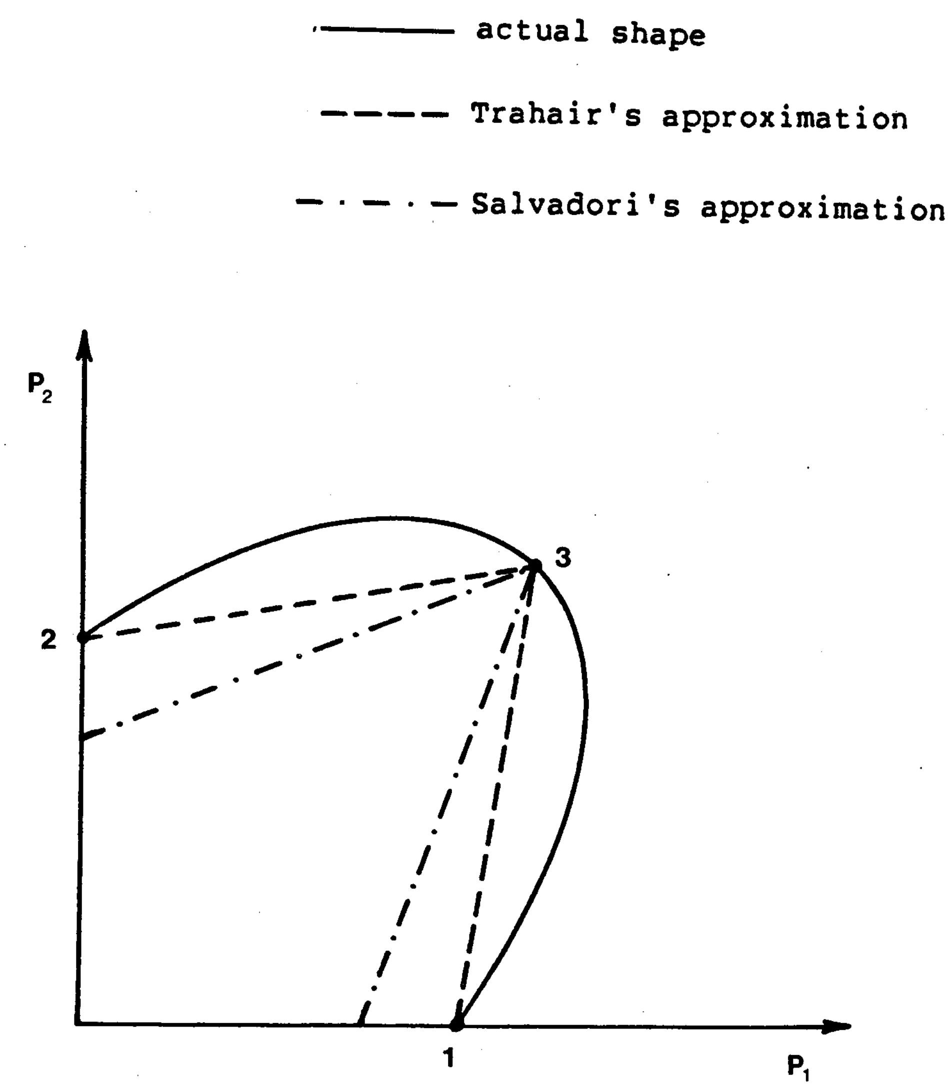


Figure 5.8 Buckling Interaction Diagram

have been used to analyze the buckling of continuous beams. Very good accuracy has been reported but not every designer has easy access to such programs. The programs are useful for researchers who test the accuracy of proposed methods.

5.3 Method of Nethercot and Trahair

Nethercot and Trahair (1976b) developed a procedure for calculating effective length factors for continuous beams. In it, a single effective length factor is calculated to account for both warping and weak axis bending interactions between segments.

The first part of the procedure is identical to the practice presently used in Canada. It begins by using the Salvadori approximation to calculate critical buckling moments for each segment. If the design is in accordance with the current Canadian Standard each of these critical moments are compared to the actual applied moments. Since the calculated resistance of the segment which governs the design will be lower bound the solution of the continuous beam will be lower bound. The Nethercot and Trahair procedure, attempting to improve on the accuracy to the Salvadori solution, takes the analysis a step further. The ratio of the elastic critical moment, M_u, to the applied moment, M_f, is calculated for each segment. The moment factor, λ, is thus defined as:

[5.1] $$\lambda = \frac{M_u}{M_f}$$

The critical segment is then identified. It is the segment with the lowest moment factor, denoted λ_C. The segments on either side of the critical segment are called restraining segments. The moment factor of a restraining segment is denoted as λ_R. The critical segment and the two restraining segments make up a subassemblage (Dux and Kitipornchai 1982).

The moment factors of the segments making up the subassemblage are then used to calculate the stiffness, α_R, for each of the restraining segments.

[5.2] $$\alpha_R = n\left(\frac{EI_y}{L}\right)_R \left(1 - \frac{\lambda_C}{\lambda_R}\right)$$

Equation [5.2] is based on an analogy made between the buckling of laterally continuous beams and the buckling of columns within a framework. The term $n(EI_y/L)_R$ is the stiffness of a restraining column where the value of n depends on the support condition at its far end. The far end is the end not adjacent to the column which is being restrained. If the far end is continuous, n is 2. If it is pinned, n is 3 and if it is fixed, n is 4. For interaction buckling of continuous beams the the same values can be used. Figure 5.9 shows restraining segments with each of the three types of end conditions described. In the figure the left end is the far end while the critical segment is adjacent to the right end. As the loads on restraining columns approach the buckling load the restraint provided

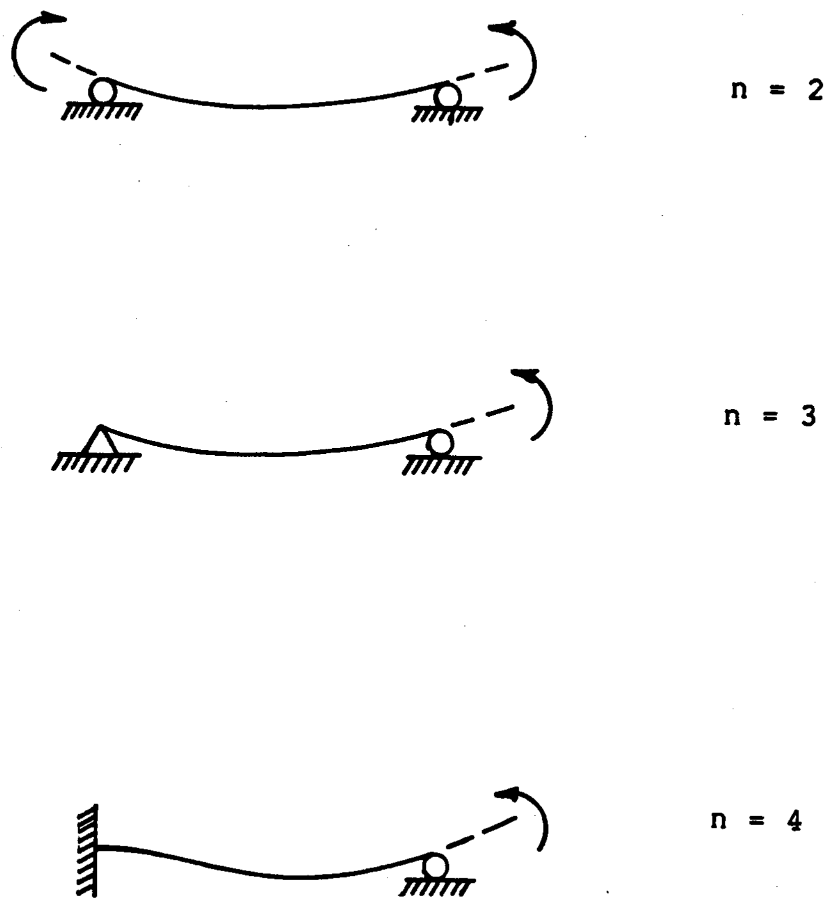


Figure 5.9 Restraining Segments

approaches zero. The same is true for laterally continuous beams. The term $(1 - \lambda_C/\lambda_R)$ approximates this effect. It accounts for the reduction in the stiffness of the restraining member due to the loads applied directly to it. The term reduces to zero when both the critical segment and the restraining segment buckle without interaction. The stiffness of the critical segment is given by

[5.3] $$a_C = 2\left(\frac{EI_y}{L}\right)_C$$

Equation [5.3] gives the stiffness of a sway prevented compression member. Nethercot and Trahair have extended it to apply to beams. The sway prevented case is used since it has been assumed that twisting and lateral deflection are prevented at the ends of the segment. The stiffnesses are calculated using lateral bending stiffness terms, EI_y, but without warping stiffness terms, EC_w. This is because the assumption was made that the two are closely related and that a reduction in warping stiffness can be adequately approximated by reducing the lateral bending stiffness. Austin et. al. (1957) made the same assumption.

The next step is to calculate the stiffness ratio, $G_{A/B}$, for each end of the critical segment

[5.4] $$G_{A/B} = \frac{a_C}{a_R}$$

The two stiffness ratios are then used with the sway prevented nomograph (Structural Stability Research Council 1976; Canadian Institute of Steel Construction 1980) (Fig. 5.10) used in column design to find the effective length factor, k, of the critical segment (Nethercot and Trahair used a chart which was equivalent to the sway prevented nomograph.) Just one effective length factor is used instead of two as was previously used for single span beams. This is because of the assumption regarding the similarities between warping and lateral bending interactions. Finally, the effective length factor is used in calculating the elastic buckling moment of the beam.

Results obtained using Nethercot and Trahair's procedure have been compared to solutions obtained by finite element methods (Fig. 5.11). The finite element methods were considered to be extremely accurate (Nethercot 1972). Nethercot and Trahair's method gave "exact" solutions for beams which were acted upon by a uniform bending moment (Fig. 2.2). For other cases the method was generally conservative except for a few instances. Unconservative solutions resulted for beams which were acted upon by loadings which produce high moment gradients ($\kappa \simeq 1.0$). The beam parameter, K, is a measure of the resistance to lateral-torsional buckling developed by warping stresses through the length of the beam.

[5.5] $$K = \sqrt{\frac{\pi^2 E C_w}{L^2 G J}}$$

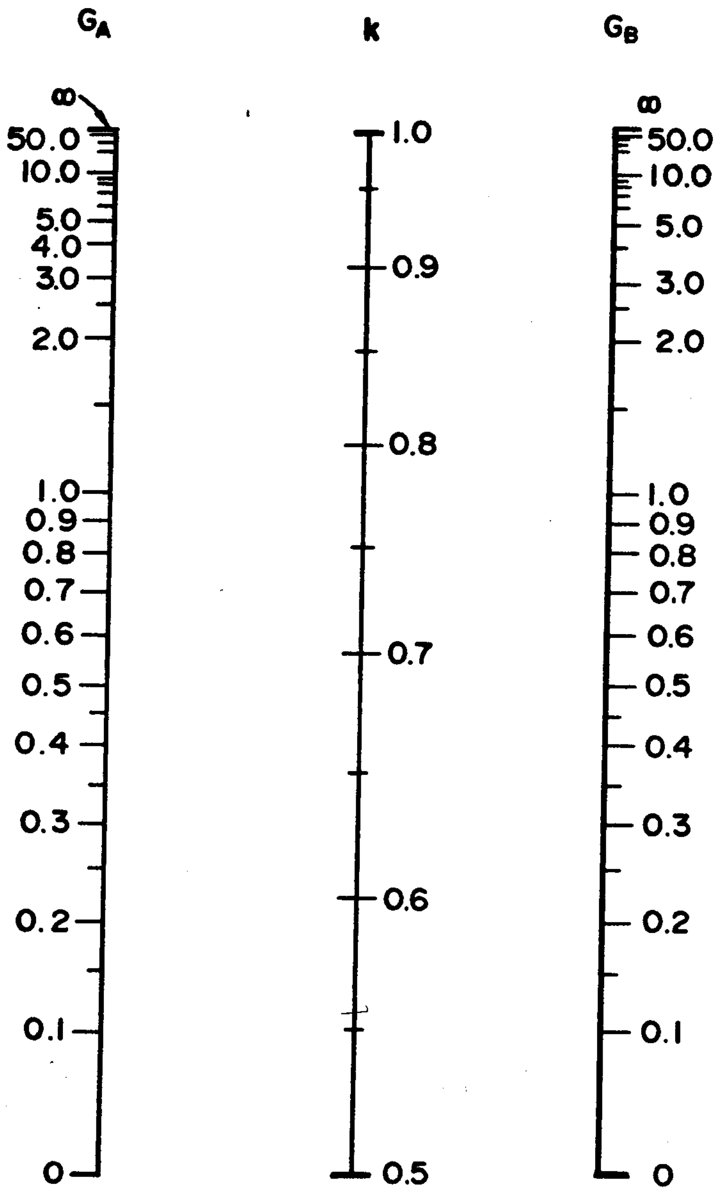


Figure 5.10 Effective Length Nomograph (Structural Stability Research Council 1976; Canadian Institute of Steel Construction 1980)

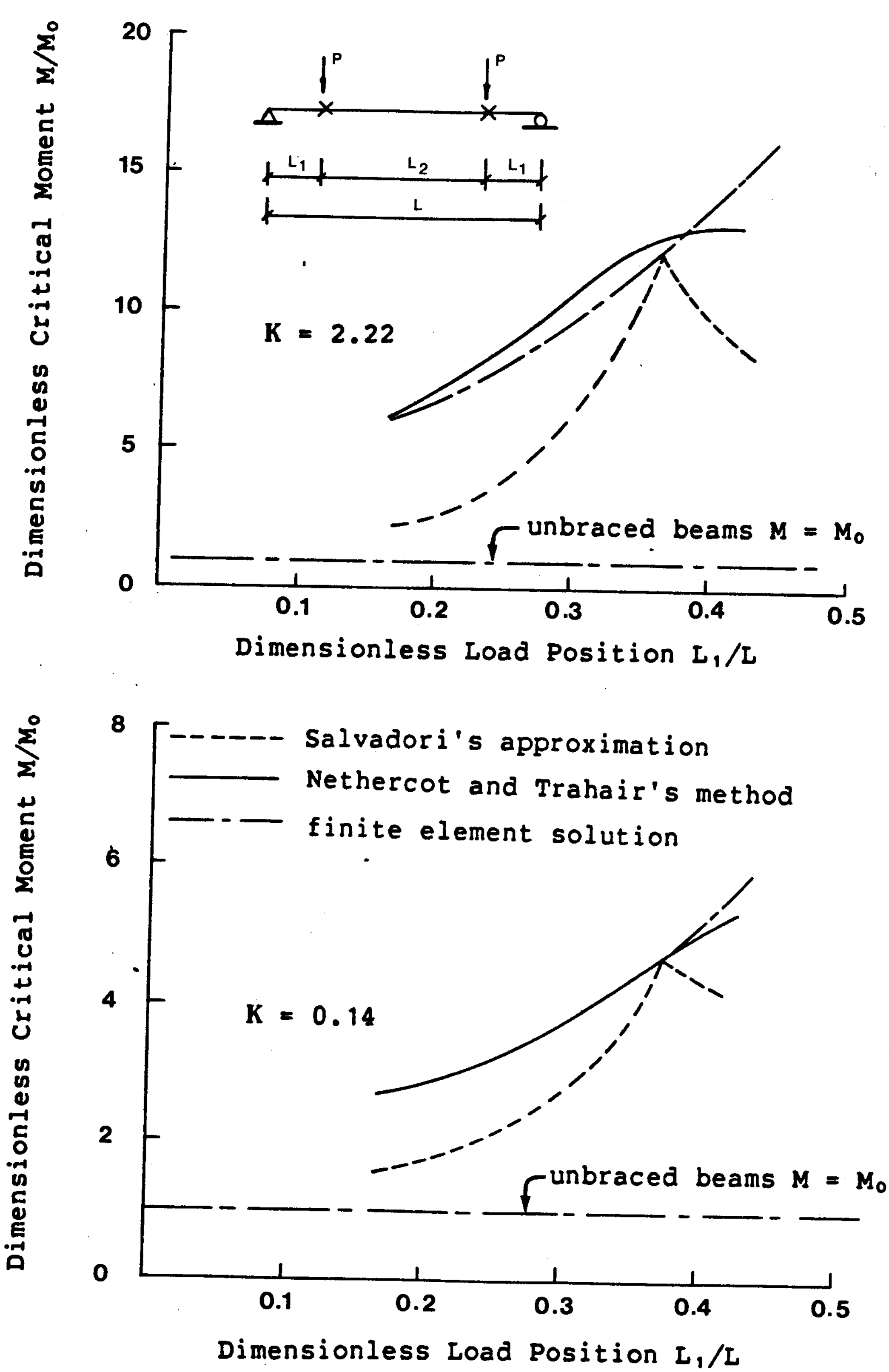


Figure 5.11 Accuracy of Nethercot and Trahair Procedure (Nethercot and Trahair 1976b)

It is related to the shape parameter defined in Sec. 3.1. Nethercot and Trahair's procedure yielded unconservative solutions for beams with high beam parameters ($K \simeq 3.0$). Unconservative results were also obtained for beams with high end restraints ($G_{A/B} \simeq 0$).

5.3.1 Inelastic Buckling

The method described in the previous section can be extended to inelastic buckling by making a few simple modifications (Nethercot and Trahair 1976a). As before, the first step is to calulate the Salvadori critical buckling moment for each segment. Since partial yielding of the beam cross-section occurs the inelastic buckling moment equation must be used. For class 1 and 2 sections

[5.6] $$M_i = 1.15M_p\left(1 - \frac{0.28M_p}{M_u}\right)$$

This equation is identical to Eqn. [2.20] except that M_i replaces M_r/ϕ. The resistance factor is not included until the final calculation of the beam moment resistance is made. In the original work by Nethercot and Trahair a different inelatic buckling equation was used:

[5.7] $$M_i = M_p\left[0.70 + 0.30\left(\frac{1 - 0.70M_p/M_u}{0.61 - 0.30\kappa + 0.07\kappa^2}\right)\right]$$

For design in accordance with S16.1-M78 Eqn. [5.6] should be used instead.

Moment factors are calculated using the inelastic critical buckling moment.

[5.8] $$\lambda = \frac{M_i}{M_f}$$

The procedure is then the same as for elastic beams until the calculation of the segment stiffness is made. At this step partial yielding of the beam must be accounted for in the calculation of the stiffness. The reduced stiffness of the critical segment is approximated by:

[5.9] $$a_C = 2\left(\frac{EI_y}{L}\frac{M_i}{M_u}\right)_C$$

Similarily for the restraining segments:

[5.10] $$a_R = n\left(\frac{EI_y}{L}\frac{M_i}{M_u}\right)_R\left(1 - \frac{\lambda_C}{\lambda_R}\right)$$

From these, one calculates the stiffness ratios and the effective length factor in the same way as for elastic beams. Finally, the moment resistance of the beam is calculated using the effective length factor with the performance factor also included. See Appendix A.2 for an example of the use of Nethercot and Trahair's method.

In a later paper (Nethercot and Trahair 1977) the calculation of the segment stiffnesses was modified. It was found that in many cases the stiffnesses estimated by Eqn. [5.10] were much lower than in the real situation. This was because quite often the value of M_u was much greater than M_i. This resulted in very low reduced stiffnesses of the restraining segments. To correct this the value of M_i is modified by multiplying it by λ_C/λ_R.

[5.11] $$M_{im} = M \frac{\lambda_C}{\lambda_R}$$

If the value of M_{im} is less than or equal to $2/3M_p$ the restraining segment behaves elastically and its stiffness is calculated by Eqn. [5.2]. If the value of M_{im} is greater than $2/3M_p$ the restraining segment is in the inelastic range. A new value of M_u is calculated by substituting M_{im} into the inelastic buckling equation and solving for M_u. The new, reduced value of M_u is denoted M_{ur}.

[5.12] $$M_{ur} = \frac{0.28M_p}{1 - M_i/1.15M_p}$$

Equation [5.12] is simply a rearrangement of Eqn. [5.6] with M_{ur} replacing M_u. M_{ur} and M_{im} are then used to calculate the reduced stiffness of the restraining segment

[5.13] $$\alpha_R = n\left(\frac{EI_y}{L}\,\frac{M_{im}}{M_{ur}}\right)_R \left(1 - \frac{\lambda_C}{\lambda_R}\right)$$

The calculation of the reduced stiffness of the critical segment, a_c, is not changed in the modified method.

The modified method seems to be very tedious. In some cases it is unconservative. Not including the changes will yield results at least as safe as those with the changes included. The results will be closer to the "correct" solution than those obtained using the Salvadori lower bound approach. Thus, for sake of simplicity, one might not include the modifications to the restraining segment stiffness calculations.

5.3.2 Refinements by Dux and Kitipornchai

Dux and Kitipornchai (1980,1982) proposed a method which was a refinement of the one by Nethercot and Trahair. It increased the accuracy of the technique but at the cost of more complexity.

They pointed out that the effective length factor, k, is a function of the restraint parameters, $G_{A,B}$, the moment gradient, κ, and the beam parameter, K. Nethercot and Trahair recognized all these factors but did not incorporate all of them. The restraint parameters were accounted for in the effective length factor chart. The chart, however, is only "exact" for cases in which κ is equal to -1.0. In this case the effective length factor is independent of K. For all other situations the effective length factor is is also a function of K. Dux and Kitipornchai made provisions for this in their refined procedure. They also recognized and accounted for the fact that the restraint provided is more important at the end of the beam which has the larger bending moment (see Sec. 2.3). Again, the chart used in the Nethercot and Trahair method does not account for this. Dux and Kitipornchai asserted that the chart is only valid for cases when κ is equal to -1.0 but that it yields satisfactory results as long as the two restraint parameters are approximately equal and κ is less than zero.

The method proceeds as follows. The Salvadori lower bound buckling moment for each segment is calculated along with the moment gradient κ, and the beam parameter, K. In order for κ to be determined the moment gradient must be constant - otherwise the method cannot be used. The critical segment is identified and the moment factors, λ_C for the critical segment, and λ_R for each of the two restraining segments, are calculated. A value of λ_F, the moment factor at subassemblage buckling is then assumed. The value of λ_F lies between λ_C and the lower of the two values of λ_R.

$$\lambda_C < \lambda_F < \lambda_{Rmin}$$

The next step is to calculate the stiffnesses of the segments. The critical segment stiffness is calculated from Eqn. [5.3]. The equation for calculating the restraining segment stiffness is slightly different from that used by Nethercot and Trahair.

$$[5.14] \qquad \alpha_R = n\left(\frac{EI_y}{L}\right)_R \left[1 - \left(\frac{\lambda_F}{\lambda_R}\right)^2\right]$$

Using the segment stiffnesses, the stiffness ratios are calculated using Eqn. [5.4].

The stiffness ratios are used to determine the effective length factor but Dux and Kitipornchai employ a number of charts instead of the single chart used by Nethercot and Trahair. There are seperate charts for various values of κ and within each chart variations in K are accounted for. In the original paper 19 charts were presented (Dux and Kitipornchai 1980). After obtaining the effective length factor a new buckling moment, M_F, is calculated from Eqn. [5.5]. From this, a new value of λ_F is computed.

$$[5.15] \qquad \lambda_{Fn} = \left(\frac{M_F}{M_u}\right)_C \lambda_C$$

Note that M_u and λ_C are the values obtained by the Salvadori lower bound approach. Next, compare λ_F and λ_{Fn}. If the two are in close enough agreement the process stops. If the two differ significantly the procedure is repeated using the new value of λ_F. Usually 2 or 3 cycles are sufficient.

Dux and Kitipornchai pointed out that there may be some difficulty in identifying the true critical segment. If, when calculating the moment factors, λ, some segments have similar values, the critical segment may not be the one with the lowest moment factor. If one segment has very stiff adjacent segments it may not be critical even if it has the lowest moment factor. In this situation, one should check each of the segments in question.

The results obtained using the refinements proposed by Dux and Kitipornchai have been compared to finite intergral solutions and finite element solutions. Good agreement was found (Fig. 5.12).

The method of Dux and Kitipornchai gives more accurate results. However, the larger number of charts that it requires make it awkward to use. Also, these charts are not easily accessable for Canadian designers at the present time. For these reasons it is not likely that the method will gain widespread acceptance.

Dux and Kitipornchai suggested that a cyclic process, like that used in their method, can be incorporated into the Nethercot and Trahair procedure. This would result in reduced estimates of the buckling moments and hopefully eliminate unconservative solutions. However, as they point out, there is no guarantee that this will yield a safe result and it will make already conservative results even more conservative.

5.4 Summary

Methods for determining buckling loads of laterally continuous, laterally unsupported beams have been described. The Salvadori lower bound approach is safe but considered to be too conservative. Trahair's method (1968b, 1969, 1977) is too involved and lacking in generality. The method of Nethercot and Trahair (1976a, 1976b) is simple and rational.

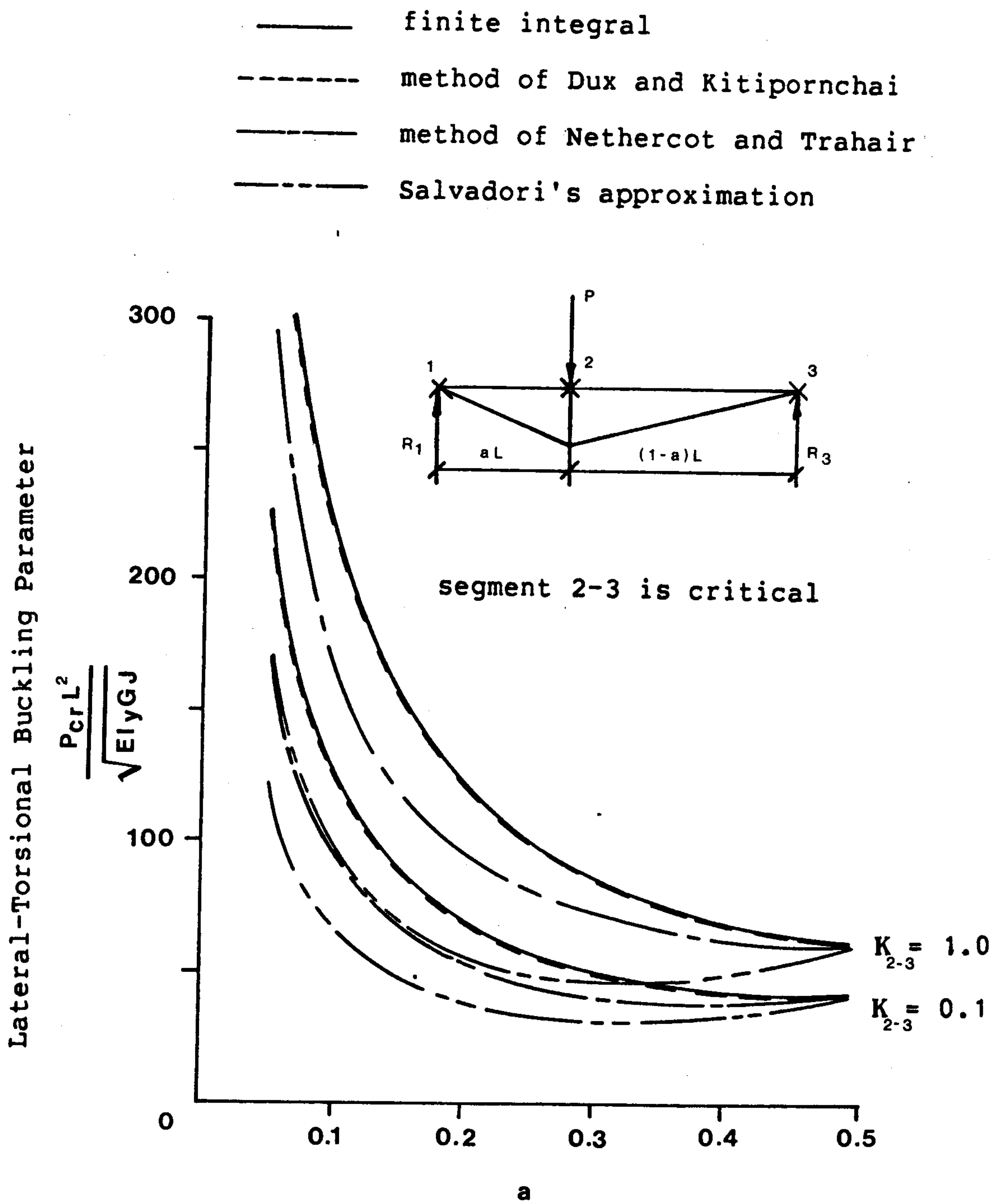


Figure 5.12 Accuracy of Dux and Kitipornchai Procedure (Dux and Kitipornchai 1982)

It conveniently makes use of the nomograph which is widely used in the design of columns. The refinements suggested by Dux and Kitipornchai (1980, 1982) improve on the accuracy obtained but greatly increase the complexity of the procedure.

6. NEW METHODS FOR THE ANALYSIS OF CONTINUOUS BEAMS

Two new methods are developed for the calculation of effective lengths of laterally continuous, laterally unsupported beams. In each procedure a single effective length factor is determined to account for both lateral and torsional restraint.

6.1 Iterative Method

The iterative method can be divided into two sub-methods. In one, the equivalent moment factor, ω, is kept constant. In the second, the equivalent moment factor is modified with each iteration. In each of the iterative methods the objective is to find effective lengths for each segment, with some effective lengths greater than the actual segment lengths and others less, such that all segments have the same ratio of critical to applied moments. The respective procedures are described in the following sections.

6.1.1 Constant Equivalent Moment Factor

STEP 1: The routine, like Nethercot and Trahair's, begins with the calculation of the Salvadori lower bound elastic critical buckling moments for each segment of the beam. Equation [2.15] is used.

STEP 2: The critical segment is identified. It is the one with the lowest moment factor found from Eqn. [5.1] or [5.8] as appropriate. In situations where M_f is the same for each beam segment Eqn. [5.1] is sufficient and thus M_i need not be computed.

STEP 3: The critical segment provides adjacent segments with negative restraint while the adjacent segments, acting to restrain the critical segment, provide positive restraint. Since the critical segment is restained by the other segments its effective length factor, k_C, is less than 1.0.

$$k_C < 1.0$$

Conversely the restraining segments have effective length factors, k_R, greater than 1.0.

$$k_{R1} > 1.0$$

$$k_{R2} > 1.0$$

Knowing this, the designer can assume a value of either the critical effective length or values of the two restaining effective lengths. The quantity which is assumed is used in the following equation to calculate the other:

[6.1] $$L = k_C L_C + k_{R1} L_{R1} + k_{R2} L_{R2}$$

where L_C, L_{R1}, L_{R2} are the lengths of the critical and the bounding restraining segments respectively and

[6.2] $$L = L_C + L_{R1} + L_{R2}$$

The effective length factor should not be less than 0.5. A value of 0.5 represents a fixed-ended beam.

STEP 4: With the effective lengths determined in step 3 the designer calculates new critical buckling moments.

STEP 5: Steps 2 through 4 are repeated until the moment factors are identical for each segment. The iteration is also terminated if the critical effective length factor becomes 0.5 and is tending to go below 0.5.

An example of the iterative procedure is presented in Appendix A.3.

6.1.2 Modified Equivalent Moment Factor

This method is the same as the one described in the preceeding section except that the equivalent moment factor, ω, is modified with each iteration. A modified value of ω is calculated in step 3 after the estimation of the effective lengths. Since each segment has a new effective length, a different portion of the bending moment diagram corresponds to that effective length. A modified value of ω is calculated from each new effective length. It is likely that the new portion of the moment diagram will have an odd shape. If so, it may be neccessary to use Eqn. [2.12] to calculate ω.

See Appendix A.4 for a demonstration of this method.

6.2 Equivalent Beam Method

The equivalent beam procedure involves transforming the actual beam to an "equivalent beam" which is easier to analyze. The method is as follows:

STEP 1: The Salvadori lower bound elastic critical buckling moments are found for each segment.

STEP 2: The "equivalent length", L_e, of each segment is found. It is the length which will result in an identical buckling moment as that found in step 1 with ω set equal to 1.0.

$$L_e = \frac{\pi\sqrt{EI_yGJ}}{(\sqrt{2})M_u}\sqrt{1 + \sqrt{1 + \frac{4M_u{}^2C_w}{I_y(GJ)^2}}} \qquad [6.3]$$

Equation [6.3] was derived by Nethercot (1973). Now the equivalent beam has a length equal to the sum of the lengths of the equivalent segments. The equivalent beam is acted upon by equal and opposite end moments resulting in a value of 1.0 for ω.

STEP 3: Since the equivalent moment factor, the section properties, and the material properties are the same for each segment and since the whole beam buckles interactively, each segment must buckle at the same critical moment. Each segment, therefore, must have the same effective length. The critical effective length is calculated by dividing the total equivalent length by the total number of segments. The effective length factor for an individual segment is computed by dividing the segment equivalent length by the critcal effective length. The effective length factor should not be less than 0.5.

STEP 4: The elastic critical buckling moment, M_u, is then calculated using the effective length calculated in step 3.

Appendix A.5 has an example of the use of this method.

6.3 Comparison of Methods

6.3.1 Dibley's Tests

An experimental study was undertaken by Dibley (1969) in which beams of the type shown in Fig. 6.1 were tested. Point loads were applied at braced points. The beams were continuous in the lateral plane. The test set-up was such that a uniform bending moment acted over the central segment. Nominal section properties of the beams tested are shown in Table 6.1. Table 6.2 shows the measured values.

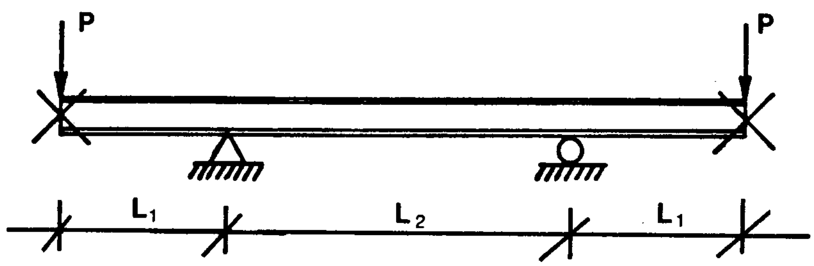
In this report 6 different methods of evaluating the effective lengths of Dibley's beams are compared. They are:

1. An effective length factor of 0.5 is used. This represents the extreme case of a fixed-ended beam.

2. The iterative procedure with a constant equivalent moment factor.

3. The equivalent beam method.

4. The iterative procedure with a modified equivalent moment factor.



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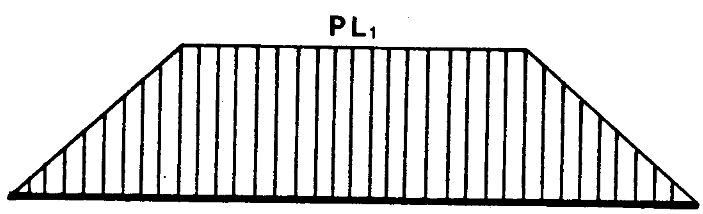

bending moment diagram

Figure 6.1 Dibley's Test Set-Up

Table 6.1 Nominal Cross-Section Properties of Dibley's Test Specimens (Baker and Kennedy 1984)

Section	I_y ($\times 10^6 mm^4$)	J ($\times 10^3 mm^4$)	C_w ($\times 10^9 mm^6$)	Class
203.2 x 133.4 UB25	3.1	61.1	29.6	3
203.2 x 203.2 UC86	31.4	1400	320	2
304.8 x 101.6 UB28	1.58	75.7	35.5	2
152.4 x 152.4 UC30	5.58	105	30.6	3
254.0 x 101.6 UB20	1.23	43.4	18.8	2

Table 6.2 Results From Dibley's Tests (Baker and Kennedy 1984)

Test	Section	F_y (MPa)	E (MPa)	S ($\times 10^3 mm^3$)	Z ($\times 10^3 mm^3$)	Failure Moment (KNm)
1	UB25	505	202700	246	277	91.0
2	UB25	505	202700	249	280	84.5
3	UB25	505	202700	248	278	103.5
4	UB25	505	202700	248	278	102.5
5	UC86	457	220600	880	1017	438.1
6	UC86	457	220600	880	1017	435.3
7	UB28	516	205500	369	429	105.9
8	UB28	516	205500	369	429	96.8
9	UB28	516	205500	369	429	118.5
10	UB28	516	205500	369	429	125.3

continued ...

Table 6.2 continued

Test	Section	F_y (MPa)	E (MPa)	S ($x10^3mm^3$)	Z ($x10^3mm^3$)	Failure Moment (KNm)
11	UB25	505	202700	248	278	131.1
12	UB25	505	202700	248	278	130.6
13	UB28	516	205500	368	427	190.0
14	UB28	516	205500	368	427	180.8
15	UB28	516	205500	368	427	217.4
16	UB28	516	205500	368	427	223.3
17	UB28	516	205500	368	427	204.6
18	UB28	516	205500	368	427	235.6
19	UB25	505	202700	246	280	138.3
20	UB25	505	202700	246	280	130.7

continued ...

Table 6.2 continued

Test	Section	F_y (MPa)	E (MPa)	S ($x10^3 mm^3$)	Z ($x10^3 mm^3$)	Failure Moment (KNm)
21	UC30	580	210700	242	271	153.8
22	UC30	470	210000	229	257	127.3
23	UC30	459	199500	229	257	94.1
24	UC30	459	199500	231	258	84.7
25	UC86	457	220600	870	1007	464.7
26	UC86	457	220600	870	1007	473.6
27	UC30	468	195000	230	257	102.5
28	UC30	566	189000	243	273	114.5
29	UB25	309	207000	275	308	78.5
30	UB20	463	200000*	238	274	68.3

* nominal value

5. Nethercot and Trahair's method.

6. The Salvadori lower bound approach.

The ultimate strengths as predicted by each of these methods were computed. Ratios of test to predicted strengths are shown in Table 6.3. The table also indicates which mode of failure is predicted by the method in question. It can be seen that the fixed-ended beam approach, giving the shortest effective lengths, predicts more beams will fail by reaching their plastic or yield moments. Using effective length factors of 0.5 resulted in the prediction of inelastic action in all 30 beams. Conversely, the Salvadori approach predicted more beams to behave elastically.

Table 6.3 also gives mean values of the test to predicted ratios for each method as well as the corresponding coefficients of variation. The mean value for the Nethercot and Trahair procedure is within 2 per cent while the proposed methods give mean values ranging from within 4 to 6 per cent. These values are satisfactory. The fixed-ended beam approach has a somewhat lower mean value while the conservative Salvadori method has a very high mean value. The coefficients of variation are reasonably good for the Nethercot and Trahair procedure as well as for the proposed methods but are considered too high in the Salvadori method.

Table 6.3 Ratios of Test to Predicted Strengths for Dibley's Tests

Test	k = 0.5	Iterated k Constant	Equivalent Beam	Iterated k Modified	Nethercot & Trahair	Salvadori
1	0.899	0.899	0.899	0.899	1.006	2.157#
2	0.831	0.831	0.831	0.831	0.932	2.013#
3	0.878	0.912	0.948	0.953	1.022	1.657#
4	0.869	0.904	0.939	0.944	1.012	1.641#
5	0.943*	0.943*	0.943*	0.943*	0.943*	0.958
6	0.937*	0.937*	0.937*	0.937*	0.937*	0.952
7	0.696	0.810#	0.920#	0.937#	1.125#	2.121#
8	0.637	0.741#	0.841#	0.854#	1.028#	1.939#
9	0.680	0.736	0.804#	0.822#	1.000#	1.937#
10	0.719	0.778	0.850#	0.869#	1.057#	2.048#
11	1.047*	1.054	1.078	1.082	1.272	1.483

* beam failure predicted by M_p or M_y

beam failure predicted by M_u

continued ...

Table 6.3 continued

Test	k = 0.5	Iterated k Constant	Equivalent Beam	Iterated k Modified	Nethercot & Trahair	Salvadori
12	1.043*	1.050	1.074	1.078	1.123	1.477
13	0.866	0.896	0.919	0.925	0.970	1.479#
14	0.824	0.853	0.875	0.880	0.923	1.408#
15	0.987*	0.987*	0.987*	0.987*	0.987*	0.992
16	1.013*	1.013*	1.013*	1.013*	1.014*	1.019
17	0.929*	0.929*	0.929*	0.929*	0.929*	1.084
18	1.069*	1.069*	1.069*	1.069*	1.069*	1.249
19	1.113*	1.113*	1.113*	1.113*	1.113*	1.174
20	1.052*	1.052*	1.052*	1.052*	1.052*	1.110
21	1.096*	1.096*	1.096*	1.096*	1.096*	1.144
22	1.183*	1.183*	1.183*	1.183*	1.183*	1.183*

* beam failure predicted by M_p or M_y

\# beam failure predicted by M_u

continued ...

Table 6.3 continued

Test	k = 0.5	Iterated k Constant	Equivalent Beam	Iterated k Modified	Nethercot & Trahair	Salvadori
23	0.924	0.924	0.924	0.924	0.983	1.330
24	0.894	0.894	0.894	0.894	0.933	1.563#
25	1.010*	1.010*	1.010*	1.010*	1.010*	1.010*
26	1.029*	1.029*	1.029*	1.029*	1.029*	1.029*
27	0.963	0.963	0.963	0.963	1.004	1.336#
28	0.887	0.887	0.887	0.887	0.939	1.487#
29	1.006	1.006	1.006	1.006	1.007	1.866#
30	0.706	0.706	0.724	0.743	0.941#	2.003#
μ_P	0.924	0.940	0.958	0.962	1.016	1.462
V	0.149	0.125	0.107	0.102	0.069	0.269

* beam failure predicted by M_p or M_y

beam failure predicted by M_u

6.3.2 Resistance Factors

Resistance factors were calculated for each of the methods used to predict the strenths of Dibley's beams. The resistance factor, ϕ, can be calculated from the following expression:

$$[6.4] \qquad \phi = \rho_R \exp(-\beta\alpha V_R)$$

where the coefficient of seperation, α, is taken as 0.55 (Galambos and Ravindra 1974) and the safety index, β, is set equal to 3.0. The ratio of the mean to nominal resistance, ρ_R, is found from

$$[6.5] \qquad \rho_R = \rho_P \rho_M \rho_G$$

where ρ_P is the ratio of the test to predicted strength, ρ_M is the ratio of the measured to nominal material properties, and ρ_G is the ratio of the measured to nominal geometric properties. The coefficient of variation of the resistance, V_R, is found from

$$[6.6] \qquad V_R^2 = V_P^2 + V_M^2 + V_G^2$$

where the quantities V_P, V_M, and V_G are the coefficients of variation of the test to predicted strengths, material properties, and geometric properties respectively. The values of all the statistical parameters relating to geometric and material properties have been determined previously by Baker and Kennedy (1984) for rolled shapes and Kennedy and Baker (1984) for welded shapes so it was only neccessary to determine the parameters relating to the test to predicted strengths. From these, ρ_R and V_R were calculated and used to find resistance factors. An additional factor, the discretization factor, must also be included when considering rolled shapes. This factor is introduced by the designer when he selects a beam size equal to or greater than that needed. Baker and Kennedy (1984) suggest a mean value, ρ_D, of 1.059 and a coefficient of variation, V_D, of 0.039 be used.

Errors in measurement (Mirza and MacGregor 1982) were accounted for by reducing the coefficients of variation. The coefficient of variation for the errors in measurement varied linearly from 0.00 to 0.04 as the coefficient of variation for the measured property varied linearly from 0.00 to 0.06. A maximum value of the former quantity was set at 0.04 if the latter quantity increased above 0.06. The reduced coefficient of variation, V, was found from

[6.7] $$V^2 = V_o^2 - V_{EM}^2$$

where V_o is the initial (not reduced for errors in measurement) coefficient of variation and V_{EM} is the coefficient of variation for the errors in measurement.

In the inelastic range the significant statistical parameters of a laterally unsupported beam change with the length of the beam. For example, with long beams the modulus of elasticity is important, while for short beams the yield strength is more important. In this report the resistance factors were calculated based on the statistical parameters for short beams as a study showed little variation over the inelastic range when these changes were considered. The error associated with this approximation is not appreciable. The geometric property used was the section modulus (plastic or elastic as appropriate) while the material property used was the yield strength.

In the elastic range the significant geometric property is the moment of inertia while the significant material property is the modulus of elasticity.

The statistical parameters needed for the above calculations are listed in Tables 6.4 and 6.5. The results of the analysis are shown in Tables 6.6 for rolled shapes and 6.7 for welded shapes. It can be seen that the resistance factors for the fixed-ended beam method are inconsistent and low while for the Salvadori method they are inconsistent and high. For the method in which the effective length is iterated and the equivalent moment factor is kept constant the resistance factors are somewhat inconsistent and in the case of elastic beams considerably different from the value of 0.90 currently used in S16.1-M78. Nearly identical results were obtained for the equivalent

Table 6.4 Statistical Parameters Needed for Calculating Resistance Factors for Beams

Quantity	Mean to Nominal Ratio, ρ	Coefficient of Variation, V	Coefficient of Variation as Reduced for Errors in Measurement
yield strength			
rolled shapes	1.07	0.065	0.051
welded shapes	1.10	0.110	0.102
modulus of elasticity	1.02	0.016	0.016
elastic section modulus			
rolled shapes	0.99	0.021	0.021
welded shapes	1.02	0.010	0.010
plastic section modulus			
rolled shapes	0.99	0.038	0.038
welded shapes	1.02	0.015	0.015

Table 6.5 Strength Statistical Parameters for Calculating Resistance Factors Based on Dibley's Tests

Quantity	Mean to Nominal Ratio, ρ_P	Coefficient of Variation, V_P	Coefficient of Variation as Reduced for Errors in Measurement
k = 0.5			
inelastic range			
class 2	0.870	0.170	0.165
class 3	0.979	0.108	0.100
elastic range	-	-	-
iterated k			
constant ω			
inelastic range			
class 2	0.914	0.127	0.121
class 3	0.985	0.104	0.0960
elastic range	0.776	0.0629	0.0485
equivalent beam			
inelastic range			
class 2	0.949	0.0987	0.0902
class 3	0.992	0.103	0.0949
elastic range	0.854	0.0557	0.0423

continued ...

Table 6.5 continued

Quantity	Mean to Nominal Ratio, ρ_P	Coefficient of Variation, V_P	Coefficient of Variation as Reduced for Errors in Measurement
iterated k			
modified ω			
inelastic range			
class 2	0.951	0.0930	0.0840
class 3	0.994	0.103	0.0949
elastic range	0.871	0.0557	0.0415
Nethercot & Trahair			
inelastic range			
class 2	0.981	0.0490	0.0368
class 3	1.035	0.0750	0.0634
elastic range	1.030	0.0662	0.0527
Salvadori			
inelastic range			
class 2	1.037	0.0921	0.0830
class 3	1.272	0.124	0.117
elastic range	1.777	0.157	0.152

Table 6.6 Resistance Factors for Calculating Strengths of Laterally Unsupported Beams Based on Dibley's Tests - Rolled Shapes

Method	Inelastic Range		Elastic Range
	Class 2	Class 3	
k = 0.5	0.72	0.90	-
iterated k, constant	0.81	0.91	0.75
equivalent beam	0.88	0.92	0.84
iterated k, modified	0.89	0.92	0.85
Nethercot & Trahair	0.96	1.00	1.00
Salvadori	0.97	1.14	1.48

Table 6.7 Resistance Factors for Calculating Strengths of Laterally Unsupported Beams Based on Dibley's Tests - Welded Shapes

Method	Inelastic Range		Elastic Range
	Class 2	Class 3	
k = 0.5	0.71	0.87	-
iterated k, constant	0.79	0.88	0.78
equivalent beam	0.85	0.88	0.86
iterated k, modified	0.86	0.89	0.88
Nethercot & Trahair	0.92	0.95	1.04
Salvadori	0.94	1.10	1.48

beam method and the iterated effective length / modified equivalent moment factor method. Reasonable consistency was obtained. A value of about 0.85 is a good representative figure for ϕ for both rolled and welded shapes. It is reasonably close to the value used in S16.1-M78. The results obtained using Nethercot and Trahair's procedure indicate that this method can be satisfactorily used with the present S16.1-M78 value of 0.90 used for the resistance factor.

6.4 Summary and Recommendations

Reanalysis of Dibley's test data (1969) has confirmed that the Salvadori lower bound approach, currently used in S16.1-M78, is very conservative. At the other extreme, considering a laterally continuous beam as fixed at both ends is unsafe and should not be used. It is also considered that the iterative method, proposed herein, with a constant equivalent moment factor is also unsuitable. However, by modifying the equivalent moment factor with each iteration the method's accuracy is improved. Based on the data available from Dibley's experiments, the latter procedure can be used in design with a value of 0.85 used for the resistance factor. Similarily, the same tests indicate that the equivalent beam method can also be employed with a resistance factor of 0.85. A resistance factor of 0.90 is recommended for the Nethercot and Trahair procedure. The refinements suggested by Dux and Kitipornchai (1980, 1982) improve on the accuracy obtained but greatly increase the complexity of the procedure. The resistance factors recommended above apply to both rolled and welded shapes.

Most experiments on laterally unsupported beams have been for those with constant moment gradients over a central segment (Fukumoto and Masahiro 1977). For this reason it is difficult to make a full evaluation, based on tests results, of proposed methods. Experimental studies on laterally continuous beams with a variety of loading conditions would be useful.

7. SUMMARY AND RECOMMENDATIONS

Although the requirements set by the Canadian Standard (CAN3-S16.1-M78 Canadian Standards Association 1978) for the design of laterally unsupported beams leads to designs which are safe, it is considered that in some cases the provisions are too conservative. Exceptions are beams in which the load is applied above the shear center. For these beams the Standard does not make allowance for the decreased stability that is inherent. The use of effective length factors to model the stabilizing effects of different degrees of lateral and torsional fixity at the ends of beams leads to designs which are less conservative and, therefore, more economical.

Galambos (1968) has provided effective length factors which can be used for single span beams. These are particularily useful since they can easily be used with the present design equations in S16.1-M78. The effective length factors are analagous to those used in the design of columns. With an understanding of the concepts of warping and lateral bending, the designer can determine what kind of restraints a particular connection provides and then choose an appropriate effective length factor. Galambos's effective length factors should not be used when the end restraints provided are poorly matched with the applied loads.

Effective length factors which eliminate the use of an equivalent moment factor have been suggested by Nethercot for the design of cantilevers. This is philosophically different from the notion of having seperate factors to account for loading and end restraints. Unfortunately, no equivalent moment factor which can be used for cantilevers has been developed. If one were developed, it is considered that effective length factors similar to those used for single span beams could be employed for simple cantilevers while the methods used for continuous beams could be applied to continuous cantilevers. Lacking the above, Nethercot's effective length factors give results that are safe and more economical than those obtained using the provisions of S16.1-M78.

End restraints for segments in a continuous beam are provided by the adjacent segments. The degree of restraint provided is a function of the applied loads and the span lengths of each segment. The design procedure prescribed by the Canadian Standard conservatively ignores the end restraints that are supplied.

Resistance factors have been derived for various methods of analyzing laterally continuous beams. Factors were derived for both rolled and welded shapes. It was found that the difference between the two sets was not large and thus one set of resistance factors is recommended for both rolled and welded shapes.

Nethercot and Trahair (1976a,1976b) have developed a simple method which can be used to calculate the amount of restraint provided by adjacent segments. From these values an effective length factor can be determined. The method is not difficult to use. The procedure conveniently utilizes the nomograph normally applied to column design. Unfortunately, under certain conditions the method gives unconservative results. An awareness of these conditions is necessary for the safe application of the procedure. Dux and Kitipornchai (1980,1982) refined the method, improving its accuracy but also making it more cumbersome to use. The refined method requires an extensive number of charts. For reasons of simplicity the unrefined method of Nethercot and Trahair seems more suitable for routine design. A resistance factor of 0.90 is recommended for use with this design procedure.

A procedure which involves successively adjusting assumed effective length factors has been developed in this report. It was found that the accuracy of this method is improved if the equivalent moment factor is also successively adjusted. If the latter routine is followed a resistance factor of 0.85 should be used. The method is not recommended if the equivalent moment factor is not modified.

In another method developed for the analysis of laterally continuous, laterally unsupported beams the real beam is replaced by an equivalent beam which is easier to analyze. A resistance factor of 0.85 should be used with this method.

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APPENDIX

A.1 Example 1

The following example demonstrates the use of effective length factors for single span beams. Also included is a check on the warping stiffness of a tubular device made from a pair of channels.

A laterally unsupported beam spans 10.0 m and supports a 90 kN load (factored) acting at mid-span (Fig. A.1). The beam is supported by double angle connections (Fig. 2.6) at each end.

$$M_f = 225 \text{ kNm}$$

The load is applied to the beam at the level of the shear center. The beam is a W460x82 section; a class 2 section.

$$M_p = 549 \text{ kNm}$$

The properties of the beam are found in the Handbook of Steel Construction (Canadian Institute of Steel Construction 1980):

$$I_y = 18.6 \times 10^6 \text{ mm}^4$$

$$J = 691 \times 10^3 \text{ mm}^4$$

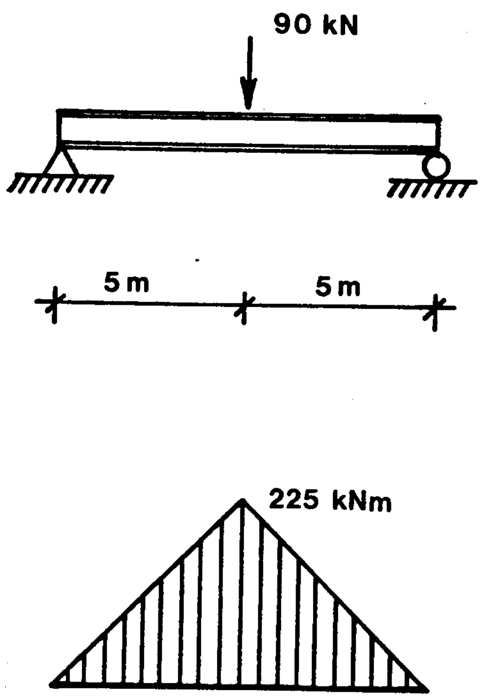

Figure A.1 Loading and Bending Moment Diagram for Example 1

$$C_w = 918 \times 10^9 \ mm^6$$

$$E = 200 \times 10^3 \ MPa$$

$$G = 77 \times 10^3 \ MPa$$

The beam can be assumed to be pinned with respect to both lateral bending and torsion.

[3.12] $k_y = 1.00$

[3.9] $k_z = 1.00$

From the Structural Stability Research Council (1976):

$$m = 1.35$$

or

$$\omega = 0.74$$

Using Eqn. [3.5] and replacing M_{cr} by M_u:

$$M_u = 219 \ kNm$$

This is less than $2/3M_p$. Thus, the factored moment resistance is equal to the elastic buckling moment multiplied by the resistance factor.

$$M_r = \phi M_u$$
$$= (0.9)(219 \text{ kNm})$$
$$= 197 \text{ kNm}$$

The factored resistance is less than the moment due to the factored loads. The structure is inadequate.

To improve the stability of the structure, pairs of channels are attached at each end to form tube shaped devices (Fig. 3.6). The channels used are C310x45's. The flange widths, b, are 80 mm. The distance between the beam web and the beam flange is 90.6 mm. This leaves enough room to place the channel and weld it to the beam. The web depth, d, of the channel is 305 mm. The torsional constant of the tube must be computed. From McGuire (1968):

$$J_t = \frac{4A^2t}{S}$$

where A is the area enclosed by the middle of the tube wall, t is the wall thickness, and S is the length of the perimeter of the tube. For the C310x45's:

$$t = 13 \text{ mm}$$

$$A = (305 \text{ mm} - 13 \text{ mm})[2(80 \text{ mm}) - 13 \text{ mm}]$$
$$= 42924 \text{ mm}^2$$

$$S = 2(305 \text{ mm} - 13 \text{ mm}) + 2[2(80 \text{ mm}) - 13 \text{ mm}]$$

$$= 878 \text{ mm}$$

$$J_t = \frac{4(42924 \text{ mm}^2)^2(13 \text{ mm})}{878 \text{ mm}}$$

$$= 109.1 \times 10^6 \text{ mm}^4$$

The distance between the centroids of the beam flanges is calculated:

$$h = d - t$$

$$= 460 \text{ mm} - 16 \text{ mm}$$

$$= 444 \text{ mm}$$

Using Eqn. [3.8] the elastic rotational stiffness of the tube is calculated:

[3.8] $$K = \frac{2G_tJ_t}{h}$$

$$= \frac{2(77000 \text{ MPa})(109.1 \times 10^6 \text{ mm}^4)}{444 \text{ mm}}$$

$$= 37.8 \times 10^9 \text{ Nmm}$$

Now, Eqn.[3.6] must be satisfied:

[3.6] $$K > \frac{5EI_yR}{\tanh(LR/2)}$$

where

[3.7] $$R = \sqrt{\frac{GJ}{EC_w}}$$

Using the following relationship the above expression can be evaluated:

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Substituting the appropriate values:

$$\frac{5EI_yR}{\tanh(LR/2)} = 10.1 \times 10^9 \text{ Nmm}$$

This is less than the tube stiffness. It can, therefore, be assumed that warping is prevented. A new torsional effective length factor can be used:

[3.10] $$k_z = 0.65$$

A new value of M_u is computed using the new value of k_z:

$$M_u = 254 \text{ kNm}$$

This is less than $2/3M_p$. The factored moment resistance is calculated:

[2.14] $$M_r = \phi M_u$$

$$= (0.9)(254 \text{ kNm})$$

$= 229$ kNm

The moment resistance is greater than the moment due to the factored loads. The use of the tubular device to resist warping has allowed the use of a beam that otherwise would not be adequate.

A.2 Example 2

The following example demonstrates the unmodified Nethercot and Trahair procedure for calculating effective length factors for continuous beams.

A laterally unsupported beam spans 17.0 m. It is loaded at braced points. The braces prevent twisting and lateral deflection of the beam. The beam is shown in Fig. A.2. The loads shown are the factored loads. The beam is a W460x82 section. The properties of this section are given in Example 1.

The braces divide the beam into three segments numbered 1 to 3 from left to right. The first step in the analysis is to calculate the Salvadori lower bound estimates of the segment buckling moments.

segment 1:

$L = 4000$ mm

$\omega = 0.6$

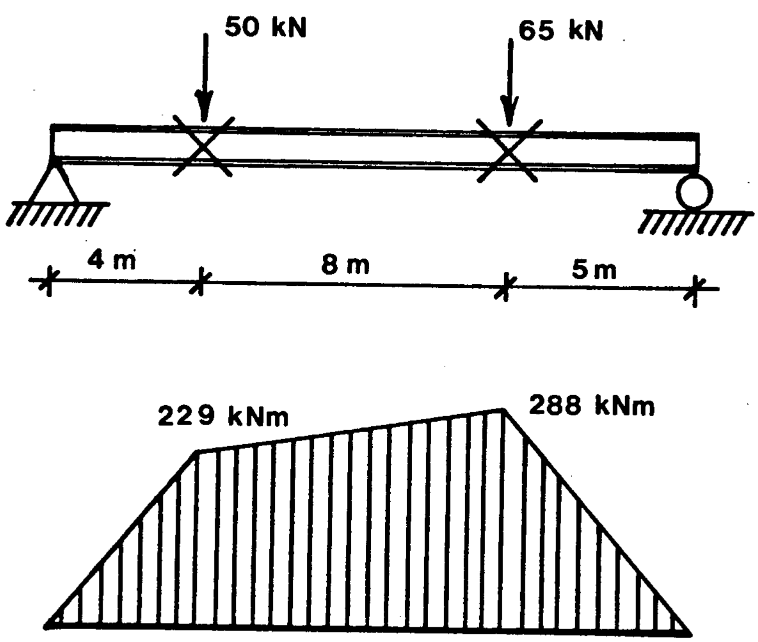

Figure A.2 Loading and Bending Moment Diagram for Example 2 using Eqn. [2.15]

$$M_u = 1030 \text{ kNm}$$

this is greater than $2/3M_p$ (recall that M_p = 549 kNm)

[5.6] $$M_i = 1.15M_p\left(1 - \frac{0.28M_p}{M_u}\right)$$

$$= 537 \text{ kNm}$$

segment 2:

$$L = 8000 \text{ mm}$$

$$\omega = 0.6 + 0.4\left(\frac{229 \text{ kNm}}{288 \text{ kNm}}\right)$$

$$= 0.92$$

$$M_u = 235 \text{ kNm} < 2/3M_p$$

segment 3:

$$L = 5000 \text{ mm}$$

$$\omega = 0.6$$

$$M_u = 716 \text{ kNm} > 2/3M_p$$

$$M_i = 496 \text{ kNm}$$

If this beam was being designed according to the specifications of the present Canadian Standard the above values of critical moments would be multiplied by the performance factor to get the moment resistance of the beam. Segment 2 would not be adequate. Continuity of the beam makes it stronger than the above calculations indicate. Continuing on using Nethercot and Trahair's procedure:

Note that segments 1 and 3 are in the inelastic range while segment 2 is in the elastic range. The moment factors for each segment are now determined.

segment 1:

[5.8]
$$\lambda = \frac{M_i}{M_f} = \frac{537 \text{ kNm}}{229 \text{ kNm}} = 2.34$$

segment 2:

[5.1]
$$\lambda = \frac{M_u}{M_f} = \frac{235 \text{ kNm}}{288 \text{ kNm}} = 0.82$$

segment 3:

[5.8] $$\lambda = \frac{M_i}{M_f}$$

$$= \frac{496\ \text{kNm}}{288\ \text{kNm}}$$

$$= 1.72$$

Segment 2 is the critical segment. Segments 1 and 3 are restraining segments.

segment 1:

$$\lambda_R = 2.34$$

segment 2:

$$\lambda_C = 0.82$$

segment 3:

$$\lambda_R = 1.72$$

The stiffnesses of the segments are calculated:

segment 1:

$$n = 3$$

[5.10]
$$a_R = n\left(\frac{EI_y}{L}\,\frac{M_i}{M_u}\right)_R\left(1 - \frac{\lambda_C}{\lambda_R}\right)$$
$$= 3(9.3 \times 10^8\ \text{Nmm})\left(\frac{537\ \text{kNm}}{1030\ \text{kNm}}\right)\left(1 - \frac{0.82}{2.34}\right)$$
$$= 945 \times 10^6\ \text{Nmm}$$

segment 2:

[5.3]
$$a_C = 2\left(\frac{EI_y}{L}\right)_C$$
$$= 2(4.65 \times 10^8\ \text{Nmm})$$
$$= 930 \times 10^6\ \text{Nmm}$$

segment 3:

$$n = 3$$

[5.10]
$$a_R = n\left(\frac{EI_y}{L}\,\frac{M_i}{M_u}\right)_R\left(1 - \frac{\lambda_C}{\lambda_R}\right)$$
$$= 3(7.44 \times 10^8\ \text{Nmm})\left(\frac{496\ \text{kNm}}{716\ \text{kNm}}\right)\left(1 - \frac{0.82}{1.72}\right)$$
$$= 809 \times 10^6\ \text{Nmm}$$

The stiffness ratios are determined.

For the left end:

$$G_A = \frac{a_C}{a_R}$$

$$= \frac{930 \times 10^6 \text{ Nmm}}{945 \times 10^6 \text{ Nmm}}$$

$$= 0.98$$

for the right end:

$$G_B = \frac{a_C}{a_R}$$

$$= \frac{930 \times 10^6 \text{ Nmm}}{809 \times 10^6 \text{ Nmm}}$$

$$= 1.15$$

The nomograph in Fig. 5.10 is used to calculate the effective length factor.

$$k = 0.78$$

Using this effective length factor a new critical moment is calculated for the critical segment:

$$M_u = 333 \text{ kNm} < 2/3M_p$$

The moment resistance of the beam is determined:

[2.14] $$M_r = \phi M_u$$

$$= (0.9)(333 \text{ kNm})$$

$$= 300 \text{ kNm}$$

This is greater than the maximum moment due to the factored loads. The beam is adequate.

It was noted in Chapter 5 that the Nethercot and Trahair procedure was unconservative if a few conditions were present. One of these was the presence of high end restraints on the critical segment ($G_{A/B} \simeq 0$). That is not a factor in this example. High moment gradients were also found to lead to unconservative results. For this example the critical segment is acted upon by a nearly uniform bending moment distribution. Finally, results from beams with high beam parameters are suspect. A value of K in the order of 3.0 or higher should cause some concern. The value of K in this case turns out to be 0.73. Based on these considerations it can be assumed that the analysis was reasonably accurate.

A.3 Example 3

In this example the iterative procedure for determining effective length factors is used. The equivalent moment factor, ω, is not modified.

Dibley's test beam no. 13 is analyzed. The test set up is shown in Fig. 6.1. The length of each outer segment is 1050 mm while the inner segment spans 2090 mm giving a total length of 4190 mm. The material and geometric properties of the beam can be obtained from Tables 6.1 and 6.2.

$$C_w = 35.5 \times 10^9 \text{ mm}^6$$

$$J = 75.7 \times 10^3 \text{ mm}^4$$

$$I_y = 1.58 \times 10^6 \text{ mm}^4$$

$$E = 205500 \text{ MPa}$$

$$G = 79000 \text{ MPa}$$

$$F_y = 516 \text{ MPa}$$

The section is class 2.

$$Z = 427 \times 10^3 \text{ mm}^3$$

$$M_p = 220 \text{ kNm}$$

The moment diagram is shown in Fig. 6.1. The Salvadori lower bound moments are calculated for each segment.

segment 1:

$$L_1 = 1050 \text{ mm}$$

$$\omega_1 = 0.6$$

using Eqn. [2.15]

$$M_{u1} = 759 \text{ kNm}$$

segment 2:

$$L_2 = 2090 \text{ mm}$$

$$\omega_2 = 1.0$$

$$M_{u2} = 128 \text{ kNm}$$

segment 3: same as segment 1

The maximum applied moments are identical for each segment. Thus, at failure, the critical buckling moment for each span will also be identical. Since the calculated elastic buckling moment of the outer segments is larger than the calculated elastic buckling moment of the inner segment the outer segments restrain the inner segment. An estimate of the effective length factors is made. Assume the effective length factor of the inner segment, k_2, is 0.65. Thus:

$$k_2L_2 = (0.65)(2090 \text{ mm})$$
$$= 1359 \text{ mm}$$

The effective lengths of the outer two segments are identical to each other.

$$k_1L_1 = k_3L_3$$

$$k_1L_1 = \frac{L - k_2L_2}{2}$$

$$= \frac{4190 \text{ mm} - 1359 \text{ mm}}{2}$$

$$= 1416 \text{ mm}$$

$$k_1 = k_3$$

$$k_1 = \frac{k_1L_1}{L_1}$$

$$= \frac{1416 \text{ mm}}{1050 \text{ mm}}$$

$$= 1.35$$

The elastic buckling moments are recalculated using the new effective lengths.

segments 1 and 3:

$$k_1L_1 = 1416 \text{ mm}$$

$$M_{u1} = 431 \text{ kNm}$$

segment 2:

$$k_2L_2 = 1359 \text{ mm}$$

$$M_{u2} = 279 \text{ kNm}$$

Again, it is seen that the buckling moment of the outer spans is greater than that of the inner span. The inner span is restrained by the outer spans. New effective length factors are assumed.

try $k_2 = 0.55$

this leads to

$$k_1 = k_3 = 1.44$$

segments 1 and 3:

$$M_{u1} = 381 \text{ kNm}$$

segment 2:

$$M_{u2} = 377 \text{ kNm}$$

These values are reasonably close. Thus, the elastic critical buckling moment of the beam is:

$$M_u = 377 \text{ kNm}$$

This is greater than $2/3M_p$. The unfactored moment resistance is calculated using Eqn. [2.20].

$$\frac{M_r}{\phi} = 1.15M_p\left(1 - \frac{0.28M_p}{M_u}\right)$$

$$= (1.15)(220\text{kNm})\left[1 - \frac{0.28(220\text{ kNm})}{377\text{ kNm}}\right]$$

$$= 212\text{ kNm}$$

In Dibley's test the beam failed at a moment of 190 kNm. Thus, the test to predicted ratio is

$$\frac{190\text{ kNm}}{212\text{ kNm}} = 0.90$$

A.4 Example 4

Dibley's test beam no. 13 is reanalyzed using the iterative procedure but with the equivalent moment factor modified with each iteration.

The first iteration is exactly as in Example 3. For the second iteration, assume an effective length factor of 0.65 for the inner segment. This is the same value as assumed in Example 3. Again, the effective lengths will be 1359 mm and 1416 mm for the inner and outer segments respectively. Figure A.3 shows the bending moment diagram matched against the new effective lengths. The equivalent moment factors are calculated from the parts of the bending moment diagram which correspond to their respective effective lengths.

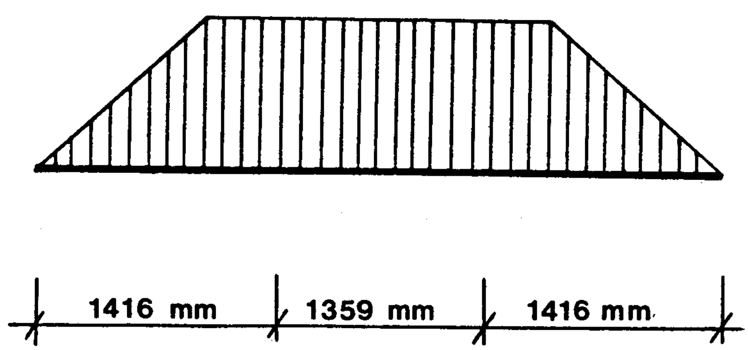

Figure A.3 Bending Moment Diagram and Assumed Effective Lengths

Referring to Fig. A.4 and using Eqn. [2.12]

$$\omega_1 = \omega_3$$

[2.12] $$\frac{1}{m} = \frac{3M_2 + 4M_3 + 3M_4 + 2M_{max}}{12M_{max}}$$

$$\omega_1 = \frac{3(0.377) + 4(0.674) + 3(1) + 2(1)}{12(1)}$$

$$= 0.73$$

The inner segment equivalent moment factor remains equal to 1.0.

segment 1 and 3:

$$\omega_1 = 0.73$$

$$k_1L_1 = 1416 \text{ mm}$$

$$M_{u1} = 354 \text{ kNm}$$

segment 2:

$$\omega_2 = 1.0$$

$$k_2L_2 = 1359 \text{ mm}$$

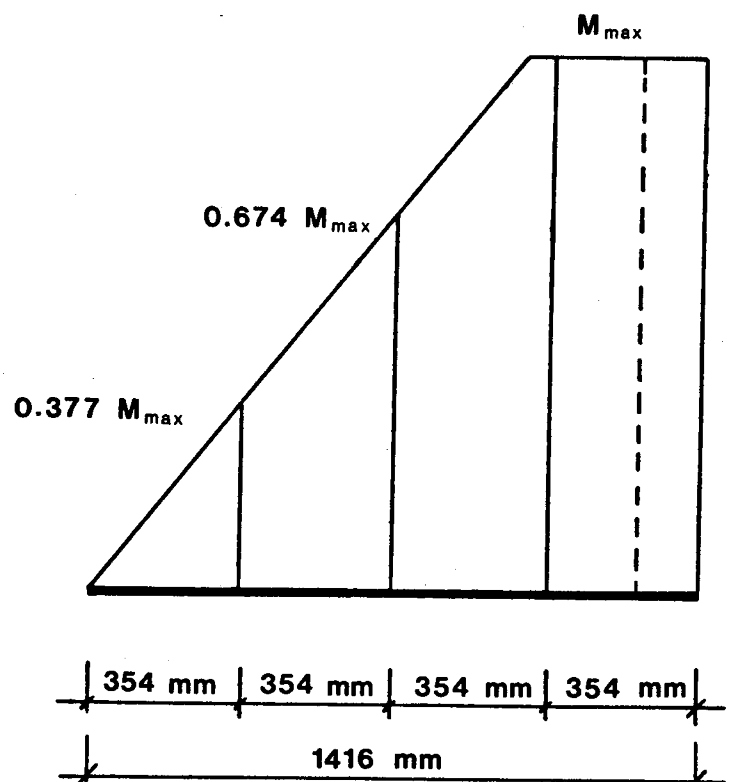

Figure A.4 Bending Moment Diagram Corresponding to Effective Length of Segment 1

$$M_{u2} = 279 \text{ kNm}$$

The iterative process continues until the elastic buckling moments are the same for each segment. This occurs when the effective length factors are 0.60 and 1.40 for the inner and outer segments respectively.

$$M_u = 326 \text{ kNm}$$

This leads to

$$\frac{M_r}{\phi} = (1.15)(220 \text{ kNm}) \left[1 - \frac{0.28(220 \text{ kNm})}{326 \text{ kNm}} \right]$$

$$= 205 \text{ kNm}$$

The test to predicted ratio is

$$\frac{190 \text{ kNm}}{205 \text{ kNm}} = 0.93$$

A.5 Example 5

The equivalent beam method is now demonstrated. Again, Dibley's test beam no. 13 is analyzed.

As before, the Salvadori lower bound elastic critical moments are found for each segment:

$$M_{u1} = M_{u3} = 759 \text{ kNm}$$

$$M_{u2} = 128 \text{ kNm}$$

Equation [6.3] is used to find the equivalent length of each segment.

[6.3] $$L_e = \frac{\pi\sqrt{EI_yGJ}}{(\sqrt{2})M_u}\sqrt{1 + \sqrt{1 + \frac{4M_u^2C_w}{I_y(GJ)^2}}}$$

segment 1 and 3:

$$L_e = 806 \text{ mm}$$

segment 2:

$$L_e = 2090 \text{ mm}$$

Now, the equivalent beam is as shown in Fig. A.5. The total length of the equivalent beam is 3702 mm. There are three segments. The critical effective length is

$$(kL)_{cr} = \frac{3702}{3} \text{ mm}$$

$$= 1234 \text{ mm}$$

The effective length factors are checked to see that they are at least 0.50.

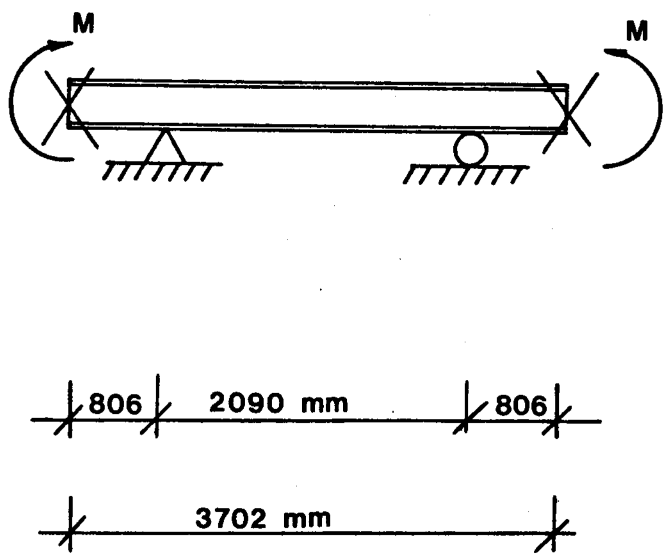

Figure A.5 Equivalent Beam for Example 5

$$k_1 = k_3 = \frac{1234 \text{ mm}}{806 \text{ mm}}$$

$$= 1.53 \qquad \text{OK}$$

$$k_2 = \frac{1234 \text{ mm}}{2090 \text{ mm}}$$

$$= 0.59 \qquad \text{OK}$$

M_u is calculated

$$\omega = 1.0$$

$$(kL)_{cr} = 1234 \text{ mm}$$

$$M_u = 335 \text{ kNm}$$

Finally

$$\frac{M_r}{\phi} = (1.15)(220 \text{ kNm})\left[1 - \frac{0.28(220 \text{ kNm})}{335 \text{ kNm}}\right]$$

$$= 207 \text{ kNm}$$

The test to predicted ratio is

$$\frac{190 \text{ kNm}}{207 \text{ kNm}} = 0.92$$