KATHRIN KOSLICKI

ISOLATION AND NON-ARBITRARY DIVISION: *FREGE'S TWO CRITERIA FOR COUNTING*[1]

ABSTRACT. In §54 of the *Grundlagen*, Frege advances an interesting proposal on how to distinguish among different sorts of concepts, only some of which he thinks can be associated with number. This paper is devoted to an analysis of the two criteria he offers, isolation and non-arbitrary division. Both criteria say something about the way in which a concept divides its extension; but they emphasize different aspects. Isolation ensures that a concept divides its extension into discrete units. I offer two construals of this: isolation as *discreteness*, i.e. absence of overlap, between the objects to be counted; and isolation as the drawing of *conceptual boundaries*. Non-arbitrary division concerns the internal structure of the units we count: it makes sure that we cannot go on dividing them arbitrarily and still find more units of the kind. Non-arbitrary division focuses not only on *how long* something can be divided into parts of the same kind; it also speaks to the *way* in which these divisions are made, *arbitrarily* or *non-arbitrarily*, as well as to the *compositional structure* of the objects divided.

1. INTRODUCTION

In §54 of the *Grundlagen*, Frege advances an interesting proposal on how to distinguish among different sorts of concepts, only some of which he thinks can be associated with number. The kind of association with number Frege has in mind is that found, for instance, in certain kinds of questions beginning with the words "how many" (and their corresponding answers), e.g. "How many moons of Jupiter are there?". The question "How many moons of Jupiter are there?" associates the concept "moons of Jupiter" with the number four.[2] His proposal in §54 is as follows:

> Only a concept which isolates what falls under it in a definite manner, and which does not permit any arbitrary division of it into parts, can be a unit relative to finite Number. (Frege 1980, p. 66)[3]

Thus, concepts that can be associated with number must satisfy two criteria, which I shall label the "isolation criterion" and the "non-arbitrary division criterion". A concept passes the isolation criterion if it "isolates what falls under it in a definite manner". A concept passes the non-arbitrary division

Synthese **112:** 403–430, 1997.

criterion if it “does not permit any arbitrary division of [what falls under it] into parts”.

Frege’s proposal has been the topic of extensive philosophical discussion, particularly in relation to such issues as relative identity and how to conceive of domains of quantification.[4] However, despite the notable exceptions of Pelletier (1975) and Simons (1982), it has gone widely unnoticed that Frege’s proposal is also of direct relevance to the semantics of the mass/count distinction.[5]

The mass/count distinction is a grammatical distinction many languages exhibit. To illustrate, consider the role of “hair” in (1) and (2):

(1) There is hair in my soup.

(2) There is a hair in my soup.

In (1), “hair” has a *mass-occurrence*; in (2), a (singular) *count-occurrence*.[6,7] Very briefly, these two kinds of noun-occurrences can be marked off from each other largely on syntactic or morphological grounds, e.g. on the basis of facts concerning quantification and pluralization. Thus, when “hair” occurs with the plural suffix or next to quantifiers like “a”, “many”, “few”, and “three hundred”, it has a count-occurrence. When it occurs in the singular without a determiner or next to quantifiers like “much”, “little”, and “three pounds of”, it has a mass-occurrence.

Some nouns standardly have only mass-occurrences, e.g. “snow”, “wine”, “mud”, “gold”, “trash”, “gravel”, “clothing”, “furniture”, “music”, and “information”.[8] Some standardly have only count-occurrences, e.g., “thunderstorm”, “river”, “person”, “circle”, “molecule”, “word”, “line”, and “definition”. Others standardly have both kinds of occurrences, e.g. “hair”, “chicken”, “carrot”, “apple”, “proof” and “truth”.[9] In each group, there are some nouns which denote concrete things (e.g. “snow”, “river”, “chicken”) and some which denote abstract entities (e.g. “information”, “line”, “proof”).

Frege’s proposal translates straightforwardly into a semantic distinction between mass-occurrences and count-occurrences of nouns.[10] As its name indicates, the mass/count distinction is connected very intimately with our practices of counting. Nouns in their count-occurrences determine concepts that can be associated with number, e.g. “moons of Jupiter”, while nouns in their mass-occurrences determine concepts that cannot be associated with number, e.g. “sugar”. We can sensibly ask about the moons of Jupiter *how many* there are. But we cannot sensibly ask about the sugar in the bowl how many there are; we can only ask *how much* of it there is.[11] Some care is needed only in the case of nouns like “apple”, which standardly have both mass- and count-occurrences, as in (3) and (4):

(3) How many apples did you put in this fruit salad?

(4) How much apple did you put in this fruit salad?

In these cases, there is one noun, “apple”, but two concepts. The concept determined by the noun “apple”, in its count-occurrence in (3), can be associated with number; its mass-occurrence in (4), on the other hand, cannot be. Since this is Frege’s preferred way of speaking, I shall cast my discussion in terms of concepts.

This paper is devoted to an analysis of Frege’s two criteria. The isolation criterion encourages us to think of the objects we count along the lines of neatly separated parcels. I offer two ways of spelling out this image: isolation as *discreteness*, i.e. absence of overlap, between the objects counted; and isolation as the drawing of *conceptual boundaries*. Frege’s non-arbitrary division concerns the internal structure of these units: it makes sure that we cannot go on dividing the units arbitrarily and still find more of the same kind. Non-arbitrary division focuses not only on *how long* something can be divided into parts of the same kind;[12] it also speaks to the *way* in which these divisions are made, viz. *arbitrarily* or *non-arbitrarily*, as well as to the *compositional structure* of the objects divided.

2. ISOLATION AND NON-ARBITRARY DIVISION

2.1. *Frege’s Proposal*

Frege’s examples of concepts that satisfy both isolation and non-arbitrary division are “letters in the word ‘three”’ and “syllables in the word ‘three”’:

> The concept “letters in the word ‘three”’ isolates the “t” from the “h” from the “r”, and so on. The concept “syllables in the word ‘three”’ picks out the word as a whole, and as indivisible in the sense that no part of it falls any longer under that same concept. (Frege 1980, p. 66)

“Letters in the word ‘three’ ” passes isolation, because it isolates what falls under it in a definite manner: it marks off the “t” from the “h” from the “r” and so on. And it passes non-arbitrary division, because it does not permit any arbitrary division of what falls under it into parts: no proper part of the letter “t” is a letter in the word “three”.

Roughly, Frege’s picture of counting is this. When we count something, we determine what number belongs to a given concept. For example, when we count the moons of Jupiter, we determine that the number belonging to the concept "moons of Jupiter" is four. The concept "moons of Jupiter" is what stays the same as we move from one moon to the next. By contrast, the moons themselves must obviously be distinct, if there are to be more than one. The concept "moons of Jupiter" delineates the individual moons into four "units".[13]

Now, it is Frege's view that only concepts which satisfy isolation and non-arbitrary division can play the role of dividing up what falls under them into countable units. Both isolation and non-arbitrary division say something about *how* a concept divides up its extension; but they emphasize different aspects.

The job of isolation is to ensure that the concept divides its extension into discrete units (as opposed to, say, undifferentiated goo). These units, I take it, must be discrete both from each other as well as from everything else in the universe: to count something, we need to know *what* to count. Furthermore, what we are supposed to count cannot be like the contents of a lava lamp, a mushy substance whose components are constantly flowing into one another. To count the moons of Jupiter, each individual moon must be delineated from each other moon and from everything else in the universe.

Because Frege speaks of isolation "in a definite manner", it is very natural to read the isolation criterion as being primarily about vagueness. There are different kinds of vagueness (cf. Quine 1960, pp. 125ff). In particular, there is indeterminacy as to whether a given object o falls under a concept C. If it is indeterminate, whether o falls under C, then o is a borderline case of C. A concept that has borderline cases is vague. We might term this kind of vagueness "vagueness among concepts", because it concerns concepts with fuzzy boundaries. The paradigm example for vagueness among concepts is baldness.

But objects can also have fuzzy boundaries. We might term this kind of vagueness "vagueness among objects". Vagueness among objects consists in indeterminacy as to where one object ends and another one begins. This kind of vagueness thus has to do more generally with the preciseness of identity conditions. Mountains, for example, have fuzzy spatial boundaries. But the temporal boundaries of an object can also be fuzzy: that is, it might be indeterminate when an object has come into existence and when it has gone out of existence. A restaurant with a complicated history of changes in ownership might be an example of an object with fuzzy temporal boundaries (cf. Stalnaker 1988, pp. 350ff). Moreover, vagueness among objects also applies to the abstract case. For example, it might be indeterminate where the legacy of one influential figure ends and that of another begins.

But the job of isolation, in my view, is *not* to eliminate vagueness.[14] Vagueness does not distinguish between the two groups of concepts; nor does it differentiate between the objects that fall under them. Let me now say why isolation is not about vagueness. More detailed positive suggestions, as to what isolation *is* about, will follow in Section 2.2.

Like most ordinary concepts, “sugar” surely has some borderline cases where it is indeterminate whether something is still sugar. For example, suppose a quantity of sugar is combined with progressively more and more salt. Initially, when the sugar still by far outweighs the salt, what we have is a quantity of sugar containing a few grains of salt. When the salt by far outweighs the sugar, on the other hand, we have a quantity of salt which contains some sugar. But, in between, there will be stages in which we are torn between “both” and “neither”, because it is indeterminate whether the quantity before us is a quantity of sugar containing some salt or a quantity of salt containing some sugar.

But “moons of Jupiter” also has borderline cases. A natural satellite, I take it, must be sufficiently large in order to qualify as a moon. Thus, suppose an object of approximately the same surface area as Connecticut begins to rotate around Jupiter. Because an object of this size is, in astronomical terms, exceedingly small, the new satellite might constitute merely some debris in the orbit of Jupiter and not a fifth moon. If, on the other hand, the new satellite is the size of the earth’s moon, say, it would presumably be considered a new moon. But because there is no precise cut-off point as to how large exactly a natural satellite must be in order to count as a moon, there will be some indeterminacy. Thus, both “sugar” and “moons of Jupiter" have borderline cases. Fuzzy conceptual boundaries, therefore, cannot be the reason why “sugar” is not suitable for association with number, while “moons of Jupiter” is.

The group of concepts Frege is concerned to single out in §54 must have sharp boundaries, because *all* Fregean concepts do. The official Fregean doctrine is that there are no vague concepts. Vague predicates determine no referent: they are like would-be names that do not succeed in naming anything. Moreover, a Fregean concept is also applicable across the board. It delineates what falls under it both sharply and completely; that is, distinguishes it from everything else in the domain of quantification. Addition, for example, is defined not just for numbers, but even for such non-mathematical objects as the moon. Thus, both precise boundaries and universal applicability are built into the very notion of a Fregean concept from the outset, while isolation and non-arbitrary division are additional criteria imposed on concepts.

From our perspective, it might not secm desirable to place such strict requirements on concepts. But then Frege is in the business of constructing a perfectly precise, artificial language, designed explicitly for use in logical and mathematical reasoning. His definitions are stipulative and are not meant to reflect an independently existing practice. Where a predicate is not defined for all cases, it is merely a matter of arbitrary stipulation to fill in the gaps. Thus he seems to assimilate vagueness to incompleteness of definition.[15]

I assume that Frege would have much the same reaction to vagueness among objects: it is also one of the defects of natural language (in this case, having to do in particular with the reference of singular terms), to be avoided at all costs in a perfectly precise, artificial language. Vagueness among objects, like that among concepts, also poses a threat to classical two-valued logic. For suppose I pick out a region of space with precise boundaries on the map and I ask "Is the region of space occupied by Mt. Rainier identical to this region?". If I picked the region of space appropriately, the truth-value of the identity-statement may well be indeterminate.

But this kind of vagueness, again, does not serve to distinguish between concepts like "sugar" and those like "moons of Jupiter": it is no more or less prevalent among the objects that fall under the first group as under the second. Thus, the identity conditions of the pain I felt yesterday or the land I grew up on may be indeterminate in various respects; but the same may be true for mountains, restaurants and legacies. Conversely, the identity conditions of the furniture I own or the sugar I just added to this cake seem no more blurry than those of the moons of Jupiter. In short, if vagueness is relevant to association with number at all, it is only so because of general commitments Frege brings to the discussion, such as a disdain for the vagaries of ordinary language and a strong disposition towards a classical two-valued logic.

Let's now turn to Frege's second criterion. Non-arbitrary division states that no arbitrary part of something which falls under the concept in question is itself to fall under the concept. While isolation ensures that the concept delineates its extension into discrete units, non-arbitrary division concerns the *internal structure* of these units. Once we are down to the level of discrete units, non-arbitrary division tells us that we cannot go on dividing the original units arbitrarily and expect to find more units of the same kind. For example, the letter "t" falls under the concept "letters in the word 'three"'; but the letter "t" has no (proper) part which itself falls under the concept.

But what is the force of "arbitrary" in "does not permit any arbitrary division of [what falls under it] into parts"? Here, Frege must have roughly the following in mind. A building, for instance, can be made up of smaller buildings: proper parts of something that falls under the concept "building" can themselves fall under the concept. But not just any old part of a building will do: the windows will not, nor will the rooms, the doors, or the walls. Only certain very specific ways of dividing up a building will result in something that is itself a building.

Frege's example, in §54, of a concept that fails non-arbitrary division is "red":

> We can, for example, divide up something falling under the concept "red" in a variety of ways, without the parts thereby ceasing to fall under the same concept "red". To a concept of this kind no finite number will belong. (Frege 1980, p. 66)

"Red", he says, does not pass non-arbitrary division, because we can divide up a red thing "in a variety of ways" and still get a red thing. The "variety of ways" must reflect the arbitrariness of the division into parts. In the case of the red thing, we do not find the kinds of constraints that we do find in the case of the building that is made of smaller buildings.

The passage concerning "red" has generated some interesting commentary. In *Reference and Generality*, Geach remarks that what Frege should have said is not that no finite number belongs to "red" or "red thing" (i.e. that we cannot stop counting), but that no number at all belong to "red" (i.e. that we cannot even begin to count). Geach's diagnosis of what is wrong with the concept "red", such that no number belongs to it, is that it fails to supply us with a criterion of identity, a criterion by which we can tell whether something is the same red thing as something (Geach 1962, §30, p. 63; §92, p. 177).

Geach and Frege are in disagreement over how to conceive of domains of quantification. Geach's universe is one of undifferentiated goo. (Dummett calls this the "amorphous lump picture"; cf. Dummett, 1973, p. 563.) Frege's universe, on the other hand, is one that comes already divided into objects, and it does so in a fixed, non-sorted manner. However, Frege's main point, in the passage concerning "red", is to give an example of a concept that fails non-arbitrary division. He happens to pick the adjectival term "red". But I suspect that he would have been equally happy with "water" or "mud", in which case Geach's criticism would never have gotten off the ground. For Geach grants that "water" and "mud" determine a criterion of identity; what "water" and "mud" lack, according to Geach, is a criterion of individuation. This is how "water" and "mud" differ from "river" and "person".[16]

2.2. *Evaluation of Frege's Proposal – Isolation*

Is Frege's proposal successful? Let's consider the two criteria in turn. First, what work does isolation do in marking off concepts that can be associated with number from the rest? Isolation, to repeat, ensures that concepts delineate their extension into discrete units. "Moons of Jupiter" meets this condition, but "sugar" does not. But what exactly does it mean for a concept to delineate its extension in this way? Because Frege gives us very little detail, it will be necessary to try out two different construals of isolation to see which one works the best.

One attractive construal is to interpret isolation as ruling out overlap. This reading focuses primarily on the discreteness of the units to be counted:

> *ISOLATION-DISCRETENESS:*
> A concept C isolates what falls under it in a definite manner iff the objects falling under C do not overlap.

By "overlap" I mean simply the sharing of one or more common parts.

For concrete objects, a straightforward kind of overlap is spatial overlap. However, in what follows, I will not restrict myself to concrete objects, for I think this would not adequately reflect Frege's purposes, in the *Grundlagen*. He repeatedly insists on the wide applicability of the concept of number, as illustrated by the following passage:

> Not without reason do we feel it puzzling that we should be able to assert the same predicate of physical and mental phenomena alike, of the spatial and temporal and of the non-spatial and non-temporal. But then, this simply is not what occurs with statements of number any more than elsewhere; numbers are assigned only to the concept, under which are brought both the physical and mental alike, both the spatial and temporal and the non-spatial and non-temporal (Frege, 1980, §48, pp. 61–62).

And even independently of Frege's purposes, I see no reason to restrict ourselves to the concrete case: counting steps in a proof, movements in a symphony, amendments to the Constitution, and so on, does not seem any more difficult than counting the moons of Jupiter. Nor does the abstract case strike me as merely a metaphorical extension of the concrete case.

Does isolation, under this construal, adequately capture our practices of counting? No doubt, we *usually* avoid overlap when counting. The paradigm case of counting is surely one where the objects we count are like neatly separated parcels. For example, we might have trouble counting the branches on a tree or the waves in the ocean because there is too much overlap between them.

However, there are other cases where overlap does not inhibit our ability to count. Here are some examples. On many printers, an "i" occurring next to an "f" overlaps with the "f". In particular, the curl of the "f" may often obscure part of the dot of the "i". To avoid this, some printers treat the "f" and the "i" as one complex symbol: they create a ligature, to the effect that part of the curl of the "f" is simultaneously functioning as the dot of the "i"; and part of the bar of the "f" is simultaneously functioning as the top serif of the "i" Now consider the question "How many letters are there in the word 'first'?". Of course, as applied to letter-types, the answer is straightforwardly "five". But, arguably, the answer stays the same, even when we have in mind the physically inscribed letter-tokens: arguably, a token of the word "first" also contains five letter-tokens, despite the fact that the "f" and the "i" may have parts in common. In a similar vein, consider the question "How many 'O's are there in 'QUORUM'?"; if we consider concrete letter-tokens, the answer could arguably be "two", counting an "O" as part of a "Q".

Siamese twins are *two* human beings or persons, even if they share a liver or other body-parts. Furthermore, consider once more the building consisting of smaller buildings. It is quite possible that some of the smaller buildings share rooms, doors, corridors or wings and thus have overlapping parts. But the overlap need not inhibit our ability to count the smaller buildings.

A similar point is expressed in Simons (1982) by means of the illustration in Figure 1 (cf. Simons 1982, p. 171).[17] As Simons remarks, the question "How many squares are there in this figure?" has several possible answers, some of which are of course more plausible than others. For example, we could respond with "three", considering only squares A, B, and C; or with "five", considering squares A, B, C, D, and E. (Other combinations are possible as well.) But, either way, the fact that there is overlap does not seem to inhibit our ability to count the squares.

Similarly, we can count the strings in the alphabet {"a", "b"}, even though they are all built up from the same two components, "a" and "b". Thus, "a", "b", "ab", "aba", "abab", and so on, are so-and-so many different strings in the alphabet {"a", "b"}, even though they share parts (Cartwright 1994a, p. 76). Or consider sets. Suppose we are trying to determine how many people in this room are tall and how many are handsome. If John is lucky enough to be both tall and handsome, then he belongs both to the set containing all and only the tall people in the room and to the set containing all and only the handsome people in the room. The two sets thus have a member in common. However, the fact that we have to count John twice (once under the heading "tall" and once under the heading "handsome") should not obstruct our ability to carry out the initial task.

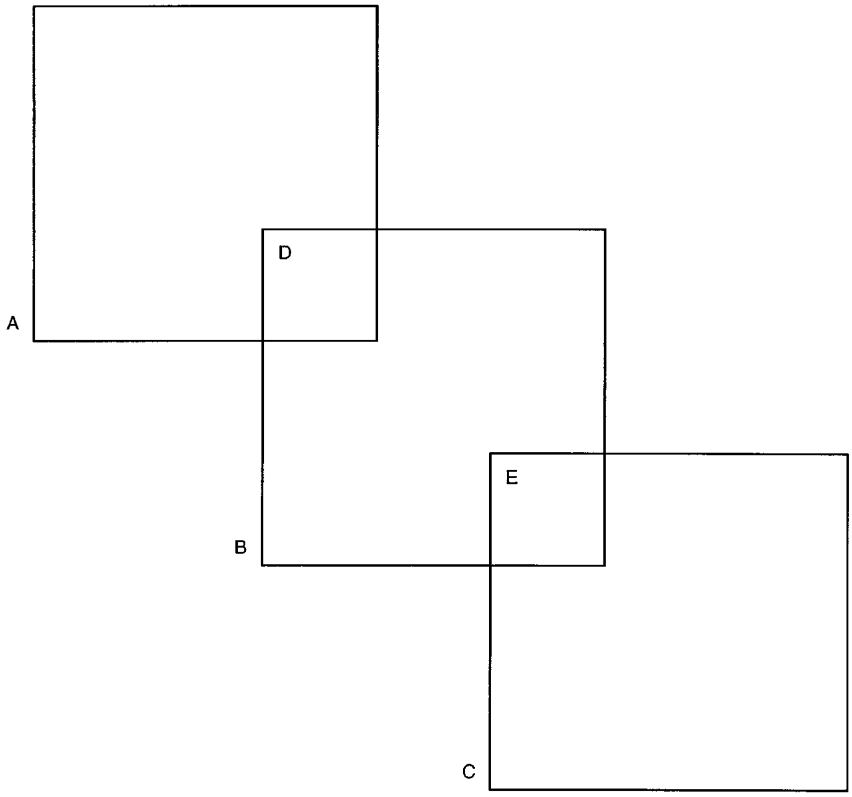

Figure 1. Depending on how we count, the question "How many squares are there in this figure?" could have different answers; but the fact that there is overlap does not seem to inhibit our ability to count the squares.

I have just presented the following examples: the branches on the tree; the waves in the ocean; the letters in the word "first"; the Siamese twins; the building consisting of smaller buildings; the overlapping squares; the strings in the alphabet {"a", "b"}; and the two sets that share a member. The branches on the tree and the waves in the ocean suggest that too much overlap can inhibit counting, while the rest indicate that overlap need not always have this effect.

It would seem, then, that isolation as discreteness, i.e. as the absence of overlap, does not successfully mark off concepts that can be associated with number from those that cannot. No doubt, isolation as discreteness has considerable intuitive force: the paradigm case of counting is surely one where the objects to be counted are non-overlapping. However, as my examples indicate, we often have no trouble counting things, even in situations where there is overlap, as long as it is clear what we are supposed to count.

But perhaps this construal takes the discreteness of the countable units too literally. It might be that what matters is not actual discreteness in the objects to be counted, but rather a kind of *conceptual* discreteness. What matters, on this view, is that the concepts under which we are counting delineate their extensions into conceptually discrete units, even if the objects themselves might be intertwined in certain ways. What counting requires, then, is that the concept draw precise boundaries around each object in its extension. These conceptual boundaries are like a kind of invisible line around the object. For them to be precise means that there be no fuzziness as to what is "inside" the boundary and what is "outside".[18] We might express this new construal of isolation in the following manner:

> *ISOLATION–BOUNDARY-DRAWING:*
> A concept C isolates what falls under it in a definite manner iff for any part p of an object o, such that o falls under C, it is definite whether p is inside the boundary drawn around o by C.

How this invisible conceptual line is drawn in each case depends on the concept in question. In general, it will be drawn in terms of considerations relevant to the identity of the objects falling under the concept.

Isolation as boundary-drawing allows for overlap. That is, it is permissible for two objects, o and o', both of which fall under C, to share a common part. The common part does not obstruct counting, as long as C still clearly differentiates o and o' as *two* different Cs, which only share a part. Thus, under the new construal, none of the examples mentioned above need to be considered counter-examples to isolation.

Under the new construal, the difficulty with the branches on the tree and the waves in the ocean, then, is not that there is too much overlap between them. Rather, the reason why we have a hard time counting the branches and the waves is because our concepts "branches on the tree" and "waves in the ocean" do not determine sufficiently precise boundaries: the concepts do not draw a clear invisible line around each branch and each wave, such that every part either definitely belongs inside or outside of it. As a result, we are unsure about where one branch or wave ends and another one begins.

This feature is strikingly absent from the other concrete examples: the letters in the word "first"; the Siamese twins; the building consisting of smaller buildings; and the overlapping squares. Despite the spatial overlap, the concept in each case still manages to delineate the objects to be counted clearly into conceptually discrete units. For example, the concept "letters in the word 'first"' draws the boundaries in such a way that certain parts count simultaneously as parts of the "f" and as parts of the "i", but the boundaries of both letters are nevertheless well-defined.

Let's now turn to the abstract cases, the "strings in the alphabet {"a", "b"}" and the two sets that share a member. In what sense, then, are "a", "b", "ab", "aba" and "abab" discrete units under the concept "string in the alphabet {"a", "b"}"? They are clearly distinguishable as different strings, even though they overlap in constituents. In fact, under some rules of composition, even "ab" and "ba" might count as different strings, despite the fact that they contain exactly the same constituents, "a" and "b". Moreover, their two constituents have the same number of occurrences, viz. one each. This suggests that overlap in constituents does not inhibit counting, because the concept "strings in the alphabet {"a", "b"}" determines additional criteria in terms of which the boundary is drawn around the different strings. Such additional criteria might include the order of the constituents, the number of occurrences, and perhaps also the rule(s) of composition under which the string was put together. These are all considerations relevant to the identity of strings: strings could not overlap in *all* respects relevant to their identity and nevertheless count as different strings.

The example concerning the two sets that share a member works in much the same way. The case of sets is interesting, because the identity of sets is determined exclusively in terms of the identity of their members. The boundaries drawn by the concept "set" are such that membership is the only consideration relevant to the identity of sets. The set containing all and only the handsome people in the room and the set containing all and only the tall people in the room still count as two discrete units under the concept "set", because the overlap in membership is only partial. If the sets shared all of their members, they would be the same set.

What all these examples have in common is that the concept in question draws precise conceptual boundaries around the objects in its extension. This allows for overlap in the objects to be counted in constituents, regions of space they occupy, or what have you. Of course, for them to count as discrete units under some concept C, they cannot overlap in all respects relevant to their identity.

An interesting case is the statue and the lump of clay that constitutes it. According to some views of constitution (e.g. Stone 1987; Johnston 1992), the statue and the clay are different objects, even though they overlap completely, during a certain period of time, with respect to the region of space they occupy. Of course, on such a view, the identity of the statue and the clay, and hence the boundary determined by the concept "material object", must be construed with respect to other considerations besides the region of space they occupy. Otherwise, they would count as the same object. There are different possibilities as to what these additional ingredients might be, e.g. their different histories, their different modal properties, the different sortal concepts under which they fall, etc. But the important feature of this example is that conceptual boundaries can be drawn in such a way that two material objects can count as different, even though they occupy exactly the same region of space.

Also, consider a modified version of the Siamese-twin example. Suppose, instead of sharing a liver, the Siamese twins share a single brain. Suppose further that we construe the concept "person" in terms of the *psychological* criterion of personal identity. The Siamese twins would then count as a single person (though, perhaps, two human beings), because what makes for personal identity, under the psychological criterion, is memories, character-traits, and so on. Since the brain happens to be the carrier of all these psychological traits, and the two twins share the same brain, the psychological criterion would predict that what we have is a single person. This is a case where the spatial overlap is only partial. However, the boundaries drawn by the concept "person", under the psychological criterion, are such that this partial overlap in space leads to complete conceptual overlap.

Isolation as boundary-drawing seems to have roughly the force of what is commonly known as *individuation* or *reference-dividing.* It has often been suggested that the crucial difference between concepts like "moons of Jupiter" and those like "sugar" is that the former possess, while the latter lack, "built-in modes [...] of dividing their reference" (cf. Quine 1960, p. 91; also Strawson 1953/54, especially p. 242; Geach 1962, pp. 63–64). Quine, in *Word and Object*, expresses this sentiment as follows:

> " . . . [C]onsider "shoe", "pair of shoes", and "footwear": all three range over exactly the same scattered stuff, and differ from one another solely in that two of them divide their reference differently and the third not at all" (Quine 1960, p. 91).[19]

This view is still fairly widespread today (cf. Bunt 1979, 1985; Simons 1982; et al.).

A built-in mode of dividing their reference, according to Quine, is something general terms possess and singular terms lack. What this means, for Quine, is that a competent speaker who has mastered the machinery of general terms will be able to tell not only "how much of what goes on counts as apple", but also "how much counts as *an* apple, and how much as another" (Quine 1960, p. 91). To have mastered the use of general terms is to be able to individuate, identify and contrast particular apples, as exemplified in the use of expressions like "an apple", "the apple", "that apple", "the same apple", "another apple" and "apples". For Quine, this is precisely what it means to have grasped "the scheme of enduring and recurrent physical objects" (Quine 1960, p. 92). in contrast, no such mastery of special devices is required for the use of singular terms: singular terms simply refer.[20]

According to Quine's view, "sugar" and the like do not just divide their reference *differently* from "moons of Jupiter" (in a way that is not suitable for association with number). Rather, "sugar" and the like do not succeed in delineating anything *at all*, unless we add appropriate reference-dividers, such as "is-a-cup-of", "is-a-cube-of", "is-a-spoonful-of", "is-a-bit-of", "is-a-packet-of" and the like. But then it is these reference-dividers which do all the work in singling out individual portions of sugar. "Sugar" cannot by itself function as a general term, unless we take it to be elliptical for a more complex expression containing an appropriate reference-divider. By itself, it functions as a singular term denoting a scattered object, the totality of the world's sugar.

Isolation as boundary-drawing is indeed very close to what Quine means by reference-dividing. Both notions agree that whatever work association with number requires is to be done by our concepts. Objects do not by themselves naturally fall into countable units. Quine's footwear-example illustrates this point nicely: "shoe", "pair of shoes" and "footwear" range over exactly the same objects, but they do so differently; the first two permit association with number, the third does not. Frege has many examples to the same effect. in fact, he also points out that "one pair of boots" and "two boots" point to a difference in number with no corresponding physical difference (cf. Frege 1980, §25, p. 33). Moreover, we can conceive of the Iliad as one poem, twenty-four books, or some large number of verses (Frege 1980, §22, p. 28). We can talk either of the leaves of a tree or of its foliage (Frege 1980, §22, p. 28). We can regard a pile of playing cards as either one complete pack, so-and-so many individual cards, or even so-and-so many points in a certain card game (Frege 1980, §22, p. 28). The very same "external phenomenon" can be described either as a copse or as five trees; as four companies or five-hundred men (Frege 1980, §46, p. 59). What changes, in all these cases, is nothing in the objects themselves individually or as a whole. The changes take place, as Frege puts it, in my "terminology", in the particular concept under which I choose to count (Frege 1980, §46, p. 59).

But there is one important respect in which it is wise to part ways with Quine. For Quine, expressions like "the water in this glass", "the furniture in this room" and "the music we heard this evening" are always elliptical for more complex expressions of the form "the . . . . . . of water in this glass", "the _____ of furniture in this room" and "the __ __ __ of music we heard this evening", where the blank is to be filled in by appropriate reference-dividers. This, I think, is a mistake. There is a difference between "the furniture in this room" and "the *pieces of* furniture"; and this difference is crucial. For we can count the pieces of furniture in this room, but we cannot count the furniture in this room. The concept "pieces of furniture in this room" draws a conceptual boundary around each item of furniture, the table, the chairs, the bookshelf, the dresser, and so on. The concept "furniture in this room", on the other hand, delineates its extension differently. As a matter of fact, the furniture in this room takes the form of tables, chairs, bookshelves, dressers, and so on. But the concept "furniture in this room" only picks out what all these pieces of furniture are pieces of, viz. furniture. Thus, suppose that someone requests that half the furniture in this room be carried into the next room. Of course, fulfilling this request will involve moving individual pieces of furniture. But it has not been said whether the pieces to be moved are chairs, tables, sofas, or bookshelves.

Similarly, "the water in this glass" cannot be understood as elliptical for an expression like "the *molecules of* water in this glass". We can count the molecules in this glass, but we cannot count the water in this glass. The concept "molecules of water" draws certain conceptual boundaries around parts of the water, while the concept "water in this glass" picks out what these partitionings consist of, viz. water. Music, as a matter of fact, takes the form of songs, symphonies, sonatas, piano concertos, operas and so on. But the concept "music we heard this evening" applies purely to *what* we heard, without specifying how the music was organized.

This, I take it, is Helen Cartwright's point, when she urges us to allow for more than one mode of reference-dividing (cf. Cartwright 1963, 1965, 1970; cf. also Laycock 1975, for discussion of related issues). According to Cartwright, to say that "sugar", "gold" and "snow" do not by themselves delineate their extension, in the absence of a reference-divider, would be like saying that "cat" or "apple" (in its count-use) do not by themselves isolate anything, until we add "breed of", "litter of", "crop of", "bushel of", or whatever the relevant phrase might be. But it makes perfect sense to talk simply of cats or apples without specifying whether it is litters or bushels we have in mind. Similarly, it makes perfect sense to talk of sugar or snow, as in "this snow", "the same snow", "last year's snow", "the snow I shoveled yesterday", without inserting reference-dividers like "falls of", "drifts of" or "expanses of".

In fact, talk of snow *simpliciter* accomplishes something that talk of drifts and expanses misses: talk of snow concerns what stays the same when some snow changes, as it might be, from a heap of snow to a drift, to an expanse. It is true that every time we talk of snow, a paraphrase containing a reference-divider can be found. But the paraphrase only tells us how the snow happens to be organized at this particular moment.

Thus, following Cartwright, the difference between "sugar" and "moons of Jupiter" is not that the first fails to delineate anything in the absence of a reference-divider. Rather, they both delineate, only they do so differently. "Cat" delineates its extension into individual cats, each of which is a cat. "Snow", in turn, delineates its extension into individual instances of snow, each of which is some snow. (The unstressed "some" plays the role of the indefinite article in the mass-system.)

To conclude, it is the job of isolation to ensure that concepts delineate their extension into discrete units. I have offered two ways of making sense of tins: isolation as discreteness, i.e. the absence of overlap; and isolation as conceptual boundary-drawing. The second construal explains why counting is possible even in cases of overlap. Isolation as boundary-drawing is quite close to the traditional notion of individuation or reference-dividing. However, I suggested that the two should part ways in one important respect. Any adequate characterization of our practices of counting ought to distinguish such pairs of concepts as "the furniture in this room" and "the *pieces of* furniture in this room"; the first is not merely an elliptical variant of the second.

2.3. *Evaluation of Frege's Proposal – Non-Arbitrary Division*

Let's now turn to Frege's second criterion. To repeat, isolation ensures that the concept divides its extension into discrete, parcel-like units. Non-arbitrary division concerns the internal structure of the things falling under a concept. Its point is to ensure that we cannot go on dividing these units arbitrarily and still expect to find more things of the same kind. Let's get clearer about this.

As the example involving buildings indicates, there are plenty of concepts which can be associated with number, even though some or even many proper parts of what falls under the concept themselves fall under it. We can count the clouds in the sky, at least on some days, even though clouds may consist of smaller clouds. Then there is Wiggins' famous example concerning the Pope's crown, made up of many smaller crowns. Still, the Pope's crown is *one* crown and it consists of so-and-so many smaller crowns (cf. Wiggins 1980, 73). Of course, not just any arbitrary part of a building, a cloud or the Pope's crown will itself count as a building, a cloud or a crown. So certainly "building", "cloud" and "crown" do not constitute counter-examples to non-arbitrary division.

But what is an *arbitrary* division into parts? We can distinguish a strong and a weak thesis. According to the strong thesis, if a concept (such as "red", in Frege's view) admits arbitrary divisions of what falls under it into parts, it must at least satisfy the following condition: every division of something falling under the concept must give us back something that itself falls under the concept. For example, every division of the red thing, according to the strong reading, must give us back a red thing. The strong construal of non-arbitrary division thus reads as follows

> *NON-ARBITRARY DIVISION – STRONG THESIS:*
> A concept C satisfies non-arbitrary division iff not every proper part of something that falls under C itself falls under C.

The negation of the strong thesis states that a concept admits arbitrary divisions (i.e. violates the non-arbitrary division criterion) just in case every proper part of something that falls under the concept itself falls under it.[21]

I suspect that there is more to Frege's non-arbitrary division criterion than what is captured in the strong thesis. Perhaps non-arbitrary division is intended to incorporate what we might call "compositional constraints", information concerning the way in which the proper parts of a thing are put together:[22]

> *NON-ARBITRARY DIVISION – REVISED STRONG THESIS:*
> A concept C satisfies non-arbitrary division iff (i) not every proper part of something that falls under C itself falls under C and (ii) C imposes compositional constraints on what falls under it.

We might say that a concept imposes no compositional constraints on what fails under it, if it imposes no constraints on how proper parts of something that falls under it can be arranged and rearranged (while remaining a thing of the same kind). Obviously, this would only be true of something that is truly continuous and has no internal structure (molecular or otherwise) to speak of. In contrast, a concept imposes compositional constraints on what falls under it if the proper parts of what falls under it can only be arranged and rearranged in certain patterns but not others. For example, the concept "furniture" imposes compositional constraints on what falls under it, because arbitrary arrangements of furniture-parts will usually not result in more furniture. But the concept "water" also imposes compositional constraints on what falls under it, because not every arrangement of hydrogen and oxygen atoms will result in water.[23]

There is some indication that Frege intended non-arbitrary division to be construed in this strong way. He does say that we can go on dividing the red thing "in a variety of ways" *forever* (no finite number belongs to "red") and still get back something red.[24] But the trouble with the strong reading of non-arbitrary division is that, with the possible exception of concepts like "space" and "time" (and only according to conceptions of space and time that view them as continuous), it seems to be true of few (if any) concepts that we can go on dividing what falls under them infinitely and always get back the same sort of thing. Non-arbitrary division, under the strong reading, only marks off things that are continuous, infinitely divisible and without compositional constraints from those that are not. But very few things, if any, are continuous, infinitely divisible and without compositional constraints.

In particular, the strong construal does not succeed in distinguishing concepts that can be associated with number from those that cannot. What falls under the concept "water" is not infinitely divisible into water, because individual hydrogen and oxygen atoms are not water. But, on a straightforward reading of the part/whole relation, they are parts of something that is water. The same holds for "mud", "stew" and "dirt", though for different reasons. Also consider "traffic", "mathematics", "tennis", "furniture", "candy", and "silverware". None of these concepts is properly associated with number. Yet what falls under them is not infinitely divisible. Moreover, many of these concepts impose compositional constraints on how the proper parts of what falls under them must be arranged. Silverware is certainly not infinitely divisible into silverware; and arbitrary arrangements of silverware-parts, more often than not, will not result in more silverware.[25]

Let's now consider the weaker reading of non-arbitrary division. I said earlier that, in the case of a building that is made up of smaller buildings, not just any old proper part of the building will itself count as a building; only certain very specific parts will. The question is, of course, how to understand "any old". The strong thesis construes "any old", as literally every proper part, including even the most minuscule constituents. In this sense, only something that is continuous, infinitely divisible and entirely without compositional constraints would admit arbitrary divisions.

We might attempt a weaker reading of "any old" or "arbitrary" as meaning something along the lines of: division *in a myriad of unprincipled ways* (though perhaps not absolutely every way). A concept would then satisfy the non-arbitrary division criterion if it does not permit such division of what falls under it:

> *NON-ARBITRARY DIVISION – WEAK THESIS:*
> A concept C satisfies non-arbitrary division iff C does not permit division of what falls under it in a myriad of unprincipled ways.

As it stands, it is of course not entirely obvious what is meant by division "in a myriad of unprincipled ways". Still, the general intuition underlying the weak thesis is reasonably clear. We can, for example, divide up a quantity of water in lots of ways and still get water back. Taking a spoonful of water from a glass of water, still leaves us with water; so does a dropful, a splashful, a handful, etc. Moreover, when we put the spoon in the glass of water, we do not need to be careful to avoid taking a part that is not water. Only a scientist, using special devices, could extract something from the glass of water that is not itself water, viz. individual hydrogen or oxygen atoms. In this respect, there is a striking contrast between "water" and "building".

The weak thesis roughly comes to this: a concept permits division of what falls under it in a myriad on unprincipled ways just in case *many* (though perhaps not all) proper parts of what falls under it themselves fall under the concept *and* we can pick these many proper parts *randomly* without any particular care. Of course, unless something is continuous, infinitely divisible and entirely without compositional constraints, there will be, among the many randomly picked proper parts, some that do not fall under the concept in question. But the force of the weak thesis is that these are insignificant as compared to myriad of parts that do fall under the concept.

For example, stew may contain pieces of carrot as parts, but a piece of carrot is not itself stew.[26] Taking a piece of carrot from a quantity of stew ought to count as one among the myriad of unprincipled divisions. Surely, the solitary carrot could be one of the many randomly picked proper parts. Suppose I reach into the stew with my spoon, without any particular plan in mind, and out comes the piece of carrot, all by itself. It seems that I have now randomly picked one of the many proper parts of the stew. But the proper part I picked is itself not stew. Of course, many unprincipled divisions will put stew on my spoon. In this respect, stew is quite different from, say, a potato: few unprincipled divisions of a potato will result in something that is itself *a* potato, though lots of them will result in potato.

Arbitrary division, as construed in this weaker way, holds more promise than the strong construal. At least, the weak construal does not single out merely what is continuous, infinitely divisible and entirely without compositional constraints. The trouble with the weak thesis is that it may not be strong enough to distinguish concepts that can be associated with number from the rest. On both sides, there are apparent counter-examples. On the one hand, the concepts "furniture" and "silverware" cannot be associated with number, but they also do not permit a myriad of unprincipled divisions of what falls under them into parts: only very few and principled divisions, if any, of furniture or silverware will result in more of the same. On the other hand, concepts like “pattern” and “line-segment” can be associated with number, but they do permit a myriad of unprincipled divisions.

To conclude, non-arbitrary division concerns the internal structure of the things falling under a concept: its point is that we cannot go on dividing them arbitrarily and still expect to find more things of the same kind. I distinguished a weak and a strong thesis. According to the strong thesis, non-arbitrary division only succeeds in singling out concepts that apply to what is continuous, infinitely divisible and entirely without compositional constraints. This does not effect the distinction we were initially looking for, between concepts like “moons of Jupiter”, on the one hand, and concepts like “sugar”, on the other. The weak thesis construes non-arbitrary division in terms of division in a myriad of unprincipled ways. This is intuitively attractive, because *many* divisions of sugar or water result in more sugar or water; moreover, we can be quite careless in making these divisions. The apparent counter-examples to the weak thesis indicate that the weak thesis may not deliver an exceptionless generalization which separates all and only the concepts that can be associated with number from the rest. However, this should not lead us to discard the weak thesis altogether. In many cases, the idea of a myriad of unprincipled divisions may nevertheless play an important role in our practices of counting.

Finally, let me make a suggestion as to why Frege speaks, in §54, specifically of association with *finite* number. Of course, some questions beginning with the words “how many” have, as their correct answers, “infinitely many”, e.g. “How many natural numbers are there?”. But the concept “natural number” is surely as suitable for counting as any concept can be. For this reason, Frege’s restriction, in §54, is somewhat puzzling. But Frege intends to rule out only certain ways in which a concept can be a unit relative to infinite number. The way in which the concept “natural number” can be such a unit is not one of the suspect ones. For there are infinitely many natural numbers, *not* because we can go on dividing arbitrarily each of the units determined by the concept “natural number” and still get back a natural number. Presumably, a natural number has no proper part that is itself a natural number. The correct answer “infinitely many” is generated because the concept “natural number” divides its extension into infinitely many discrete units; but none of these units has a proper part that itself falls under the concept. The suspect cases, I suggest, are these in which the infinite number is due to the fact that the concept in question does not pass the non-arbitrary division criterion. This, Frege seems to believe, is the situation with respect to the concept “red”: the infinite number, in this case, is due to the fact that we can go on dividing a red thing in a variety of ways, forever, and still get back something red. “Red” does not pass the non-arbitrary division criterion. “Natural number”, on the other hand, does and is therefore not among the suspect cases.

3. CONCLUSION

The subject of this paper has been Frege’s proposal, in §54 of the *Grundlagen*, on how to distinguish between concepts that can be associated with number and those that cannot. Frege offers two criteria, isolation and non-arbitrary division: only concepts that meet these two criteria are suitable for association with number. For a concept to be suitable for association with number, it must divide up what falls under it into countable units. Both isolation and non-arbitrary division say something about the way in which a concept divides its extension; but they emphasize different aspects. Isolation ensures that a concept divides its extension into discrete units. Non-arbitrary division concerns the internal structure of these units: it makes sure that we cannot go on dividing the units arbitrarily and still find more units of the same kind.

On the negative side, I suggested that isolation is not meant to rule out vagueness, although it is tempting to think so. Vagueness cannot serve to distinguish between the two groups of concepts Frege is attempting to separate; for there are borderline cases on both sides. Rather, vagueness comes into the picture only through some of Frege’s general commitments. Because his aim is the construction of a perfect artificial language, precise boundaries and universal applicability are built into the very notion of a Fregean concept.

On the positive side, I offered two construals of isolation: isolation as discreteness among the objects to be counted and isolation as the drawing of conceptual boundaries. The first construal already captures some of what is at work in our ordinary practices of counting. The paradigm case is surely one where the objects to be counted are non-overlapping. But although too much overlap among objects can inhibit counting, it need not always have this effect. This suggests that what matters in counting is not so much actual discreteness among the objects to be counted, but rather a kind of conceptual discreteness. According to this second construal, a concept isolates what falls under it into countable units just in case it draws a kind of invisible conceptual line around each object in its extension.

Isolation as conceptual boundary-drawing has some affinities to the traditional notion of individuation or reference-dividing. But I suggested, following Helen Cartwright, that any adequate characterization of our practices of counting ought to distinguish between such pairs of concepts as “the furniture in this room” and “the *pieces of* furniture in room”: the latter is not merely an elliptical variant of the former, as Quine would have it. For we can sensibly count the pieces of furniture in this room, but not the furniture.

Non-arbitrary division concerns the internal structure of the countable units: it ensures that we cannot go on dividing the original units arbitrarily and still find more units of the same kind. The challenge here is to find a sensible interpretation of the word “arbitrary”. Again I offer two construals. The strong thesis reads “arbitrary” as implying infinite divisibility and the absence of compositional constraints. But this seems too strong. The weak thesis takes “arbitrary” to mean a myriad of unprincipled divisions. Despite some apparent counter-examples, this construal is quite attractive. For it brings out why, compared to the many randomly picked proper parts of water that are themselves water, the single hydrogen or oxygen atoms are insignificant.

It is tempting to think that there is some underlying metaphysical fact or collection of facts that explains why we can count the moons of Jupiter, but not the sugar in this bowl. Perhaps, the difference is that sugar is a *stuff*, while moons are *things*. Quantities of stuff can be divided and combined in any number of ways, while remaining quantities of the same stuff; things cannot be. Half of the sugar in this bowl is still sugar; the mereological sum of the sugar in this bowl and sugar in that bowl is still sugar. In contrast, parts and sums of moons are typically not themselves moons.

But one of the most important lessons we derive from Frege’s approach to counting is that the distinction between what we count and what we do not count is drawn by our concepts. In fact, one of Frege’s main aims in the earlier parts of the *Grundlagen* is to show that, as he would put it, “number is not a property of external things”. We can describe the very same “external phenomenon” either as the leaves of a tree or its foliage. The former way of speaking admits of association with number; the latter does not. But nothing about the tree has changed.

The concepts “carrot” and “asparagus” also illustrate this point quite nicely. Why do speakers of English count carrots but not asparagus? There is no “deep” reason. The two kinds of vegetables are, it would seem, quite similar in shape and other physical characteristics; moreover, there are no dramatic differences in the role they play in our lives. There simply has not been any pressure to start using the word “asparagus” differently, to mean, for example, “spears of asparagus”. But this could easily change. Imagine the price of asparagus going up so radically (perhaps due to certain other changes in the world of asparagus-farming) that people with regular middle-class incomes could only afford to buy a few single spears of asparagus at a time, rather than the bunches we are currently accustomed to. In such a situation, it might be more convenient to start speaking of *an* asparagus, as opposed to *some* asparagus. It is my view, and perhaps Frege would agree, that the case of carrots and asparagus characterizes quite accurately our practices of counting in general.

NOTES

[1] As this paper grew out of my doctoral dissertation, I would first like to acknowledge the help of my thesis supervisor, Judith Jarvis Thomson, as well as the rest of my committee, Richard Cartwright, Robert Stalnaker, and the late George Boolos. I also profited greatly from discussion with Sylvain Bromberger, Richard Heck, Edward Johnson, Richard Larson, Mitzi Lee, Jim Mazoué, Ian Rumfitt, Jason Stanley and Zoltan Szabo. The chessboard example mentioned in note 17 was suggested to me by Alan Soble, himself an avid chess player. In addition, I would like to thank my anonymous referees for their many helpful comments. Finally, very special thanks are due to Graeme Forbes and Jim Stone, who carefully worked through several drafts of this paper; many of their comments made it into the final draft.

[2] In §44 of the *Grundlagen*, Frege says "What answers to the question How many? is number . . . " (Frege 1980, p. 57). For more discussion on questions beginning with the words "how many", see Richard Cartwright (1994, p. 67ff.).

[3] Frege speaks of associatiopn specifically with *finite* number. I will comment on this somewhat puzzling qualification below, in Section II.3. For now, we will simply ignore it.

[4] Cf. Geach 1962, 1973; Dummett 1973, 1981, 1991.

[5] There is a large body of literature on the mass/count distinction, beginning with the first serious attempt at systematic treatment in Quine (1960). A good bibliography of works written up until the mid-seventies can be found in Pelletier (1979, pp. 295–8). For more recent references, see e.g. Bunt (1985), Pelletier and Schubert (1989), Lønning (1987), Gillon (1992), Zimmerman (1995). Helen Morris Cartwright's work on the subject is particularly illuminating, cf. Cartwright (1963, 1965, 1970, 1975a, 1975b).

[6] Attempts have been made (e.g. Moravcsik 1973) to extend the mass/count distinction to syntactic categories other than nouns and noun-phrases. However, in what follows, I shall restrict myself to nouns and noun-phrases.

[7] A competing way to characterize (1) and (2) is that they exhibit a *lexical ambiguity.* The lexicon, according to this approach, contains two separate entries for "hair": the mass-noun, "hair", and the count-noun, "hair". For reasons I cannot discuss here, I favor the occurrences approach (for more discussion, see Pelletier and Schubert (1989) and Koslicki (1995, 1997a)). However, the choice between these two approaches is tangential to the issues discussed in this paper.

[8] Of course, even nouns that standardly only have mass-occurrences can always have count-occurrences read as "kind of . . . " or "kinds of . . . ", as in "The best wines in the world come from California".

[9] In the case of some nouns, it may not be immediately obvious to which category (if any) they belong, e.g. "groceries", "cattle", "spaghetti", "mashed potatoes" and "weather" (cf. Ware 1975). One might take examples such as these to indicate that the mass/count distinction is not an exhaustive classification of nouns.

[10] It has been pointed out that the distinction Frege is after may be that between *sortal* and

non-sortal concepts (cf. Pelletier 1975). In the use of Wiggins (1980), a sortal predicate (which denotes a sortal concept) is "[a]ny predicate whose extension consists [. . .] of all the particular things or substances of one particular kind, say horses, or sheep, or pruning knives . . . " (p. 7). The ontological classification Wiggins has in mind is intended to correspond to Aristotle's category of substance (as contrasting with quality, quantity, etc.). The match between the sortal/non-sortal distinction and the linguistic distinction between mass- and count-occurrences of nouns is not perfect. For example, the nouns "thing" and "quantity" standardly have count-occurrences, but they do not seem to determine sortal concepts. However, we can safely ignore these cases here.

[11] Of course, concepts, according to Frege, are the semantic values of predicates. It is an interesting question whether nouns in their mass-occurrences should be analyzed as playing the semantic role of predicates. While I in fact believe that they should be so analyzed, nothing I say here turns on it (for a defence of this view, cf. Koslicki 1997a). In other words, someone who believes that nouns in their mass-occurrences play the semantic role of a name or singular term can agree with my evaluation of Frege's proposal, while disagreeing with me on the semantic category to which nouns in their mass-occurrences should be assigned.

[12] I am here alluding to a property commonly known as "distributivity". A noun N is distributive just in case N applies to every part of something to which it applies. This property is usually mentioned together with "cumulativity": a noun N is cumulative just in case N applies to every (mereological) sum of things to which it applies. The conjunction of distributivity and cumulativity is known as "homogeneity". Quine (1960, p. 91, n. 3; p. 99, n. 4) attributes these properties to Goodman (1951), who has "dissective" instead of "distributive" and "collective" instead of "cumulative" (pp. 38–9). Ultimately, homogeneity surely derives from Aristotle's homoiomerous or uniform substances (e.g. flesh, milk, etc.). It is often said that singular count-occurrences are neither distributive nor cumulative; plural count-occurrences, cumulative but not distributive; and mass-occurrences, homogeneous, i.e. both cumulative and distributive. In addition, mass-occurrences and count-occurrences of nouns are also often contrasted semantically in terms of the atomicity of their extensions. The extensions of nouns in their count-occurrences (both singular and plural) are thought to be atomic (the atoms in the case of "person" and "people", for example, are the individual people), while those of nouns in their mass-occurrences are thought to be non-atomic. For more discussion of homogeneity and atomicity, cf. Koslicki (1997b).

[13] Frege also uses the term "unit", in a second sense, to apply to the concept itself: concepts are "units", not as units of measurement but as "units of counting". (When Frege speaks of concepts as "units relative to finite Number", it is this second sense of "unit" he has in mind.) But, to avoid confusion, I use the term "unit" only in the first sense, as applying to the objects counted.

[14] Here, I differ from Simons (1982), who does take isolation to be about vagueness, among other things (pp. 182ff).

[15] For more discussion of Frege on vagueness, see van Heijenoort (1986), Williamson (1994).

[16] Geach (1962, pp. 63–64). A criterion of identity is a criterion by which we tell whether a thing, a, is the *same* thing as a thing, b; a criterion of individuation is a criterion by which we tell whether something is a so-and-so. For insightful discussion of the passage concerning "red", see Dummett (1973, pp. 542–83; 1981, pp. 196–233; 1991, pp. 94–5). For more on Frege's conception of domains of quantification, sec also van Heijenoort (1986); for more on restricted versus unrestricted quantification, see also Richard Cartwright (1994b).

[17] For a similar figure, used for the same purposes, cf. Dummett (1973, p. 549). Another

good way to illustrate this point is by means of a chessboard: how many squares are there on a chessboard? There are all kinds of possible answers ranging from "one" to "two-hundred and four" (1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 = 204).

[18] Again, as I remarked earlier, vagueness does *not* serve to distinguish between concepts like "sugar" and those like "moons of Jupiter". We find vagueness among concepts as well as vagueness among objects on both sides. Thus, the core of isolation, under this construal, lies not in the preciseness of the boundaries. The need to be precise arises only because of Frege's general views on the defects of natural language. Fuzzy boundaries will inevitably lead to statements with indeterminate truth-values and thus threaten a classical two-valued logic. However, someone interested primarily in the analysis of ordinary language might well want to relax Frege's stringent requirements.

[19] Quine is traditionally read as assigning a dual semantic role to nouns in their mass-occurrences: that of a singular term, when they occur before the copula, denoting a scattered object; and that of a general term, when they occur after the copula, dividing its reference among portions of the scattered object ("excluding only the parts too small to count", p. 98). The passage cited in the text refers to the first of these two roles: it is in its role as a singular term, denoting the totality of the world's footwear, that "footwear" does not divide its reference; the general term, on the other hand, does divide its reference among (sufficiently large) portions of footwear. I in fact believe that there is another reading of Quine, according to which the name-like role of nouns in their mass-occurrences is basic and general terms are to be viewed as *elliptical* for a singular term plus an appropriate reference-dividing relation, e.g. "is-a-bit-of", etc. According to this second reading, then, all the reference-dividing work is done by this relation (for a statement and discussion of these two readings of Quine, see Koslicki (1997a)).

[20] In one sense, mastery of divided reference *is* required for mastery of the use of singular terms. Quine would probably say that a speaker who has not yet mastered the use of general terms, also cannot have mastered the use of singular terms. Such a speaker would not understand the *contrast* between singular terms and general terms. When Quine imagines a child to use "apple" in the same way as the child uses "mother", he does not seem to want to suggest that the child has already mastered the use of singular terms, but not yet that of general terms. Rather, he seems to want to suggest that the child is using both "apple" and "mother" in the manner of what Strawson would call a "feature-placing" expression. When the child utters "apple", it must be interpreted as saying that there is some applehood around. In this sense, mastery of the use of general terms, i.e. mastery of the scheme of divided reference, and mastery of the use of singular terms really go hand in hand. At the same time, singular terms themselves, according to Quine, do not divide their reference.

[21] According to the strong thesis, a concept that admits arbitrary divisions of what falls under it into parts satisfies the property known as distributivity (cf. n. 12 above).

[22] This further ingredient is hinted at in passing in Geach (1973, p. 291), as "combinability of parts".

[23] Frege's language is unfortunately not very explicit. This additional component might be read as entailing cumulativity. If so, non-arbitrary division entails both distributivity and cumulativity and hence captures all of homogeneity (cf. n. 12 above). For suppose a concept admits arbitrary divisions of what falls under it into parts. Then, we can go on dividing it *ad infinitum* and always get back something falling under the same concept. Furthermore, we can arrange the proper parts in any way we like and still get back something falling under the same concept. But this is only possible if every sum of something that falls under C itself falls under C (cumulativity).

[24] I actually find it somewhat puzzling why Frege says this. On the face of it, despite Frege's

claim to the contrary, it is simply not true that every proper part of something that is red is itself red. Since molecules have no color, dividing a red surface into ever smaller parts will eventually result in parts to which no color-attributes apply. And divisions may cease to result in further red things long before we reach the molecular level. For example, a red book-cover may only be red on the outside.

[25] Because the strong construal of non-arbitrary division entails distributivity, it also runs into what is known as "the problem of minimal parts". Quine is typically credited with having been the first to bring this problem to our attention, when he remarks that " . . . there are parts of water, sugar, and furniture too small to count as water, sugar and furniture" (Quine 1960, p. 99). Although I cannot imagine that Frege would be sympathetic to this kind of approach, it has been suggested (cf. Bunt 1979, 1985) that concepts like "water" and "furniture" allow us to speak of what falls under them *as if* their extension was continuous, infinitely divisible and entirely without compositional constraints, even if, as a matter of empirical fact, it is not. But this requires two levels of semantic theory: one level is, as Bunt would put it, "purely linguistic" and has nothing to do with the real-world referents of our words. (This is, of course, a common assumption in model-theoretic approaches.) The other incorporates facts concerning the actual referents, such as the fact that water actually consists of H_2O-molecules. Although I cannot properly state my reasons here, I find this "purely linguistic" level quite puzzling and am therefore not persuaded that Bunt's view provides an adequate solution to the problem of minimal parts (for more discussion, cf. Koslicki 1997b).

[26] Some may believe that a single piece of carrot, in this context (i.e. just having been removed from a bowl of stew), *is* stew, just as a single bean might be considered succotash (cf. Sharvy 1979, for discussion of this and similar examples). But consider the following examples. A single thread is not itself fabric, though it is part of fabric. A single small piece of rock is not itself soil, though a quantity of soil may contain small pieces of rock as parts. Or consider mixtures: whiskey by itself is not whiskey-and-water, but it is part of a quantity of whiskey-and-water.

REFERENCES

Bunt, H. C.: 1979, 'Ensembles and the Formal Semantic Properties of Mass Terms', in Francis Jeffry Pelletier (ed.), *Mass Terms: Some Philosophical Problems*, D. Reidel Publishing Co., Dordrecht, Holland, pp. 249–277.

Bunt, H. C.: 1985, *Mass Terms and Model-Theoretic Semantics*, Cambridge University Press, Cambridge.

Burge, T.: 1972, 'Truth and Mass Terms', *The Journal of Philosophy* **LXIX**, 263–82.

Burge, T.: 1975, 'Mass Terms, Count Nouns and Change', *Synthese* **31**, 459–78.

Cartwright, R.: 1994a, 'Notes on Aquinas' Metaphysics', unpublished manuscript, draft of September 1994.

Cartwright, R.: 1994b, 'Speaking of Everything', *Nous* **28**, 1–20.

Cartwright, H. M.: 1963, *Classes*, *Quantities and Non-Singular Reference*, unpubl. Ph.D. Thesis, University of Michigan, Ann Arbor.

Cartwright, H. M.: 1965, 'Heraclitus and the Bath Water', *Philosophical Review* **74**, 466–85.

Cartwright, H. M.: 1970, 'Quantities', *Philosophical Review* **79**, 25–42.

Cartwright, H. M.: 1975a, 'Some Remarks About Mass Nouns and Plurality', *Synthese* **31**, 395–410.

Cartwright, H. M.: 1975b, 'Amounts and Measures of Amounts', *Nous* **9**, 143–64.
Cheng, C. Y.: 1973, 'Response to Moravcsik', in J. Hintikka et al. (eds.), *Approaches to Natural Language*, D. Reidel Publishing Co., Dordrecht, Holland, pp. 286–88.
Cocchiarella, N.: 1976, 'On the Logic of Natural Kinds', *Philosophy of Science* **43**, 202–22.
Dummett, M.: 1973, *Frege: Philosophy of Language*, Harvard University Press, Cambridge, MA.
Dummett, M.: 1981, *The Interpretation of Frege's Philosophy*, Harvard University Press, Cambridge, MA.
Dummett, M.: 1991, *Frege: Philosophy of Mathematics*, Harvard University Press, Cambridge, MA.
Frege, G.: 1980, *The Foundations of Arithmetic*, transl. by J. L. Austin, Northwestern University Press, Chicago.
Geach, P. T.: 1962, *Reference and Generality*, Cornell University Press, Cornell, NY.
Geach, P. T.: 1973, 'Ontological Relativity and Relative Identity", in Milton Munitz (ed.), *Logic and Ontology*, NYU Press, pp. 287–302.
Gillon, B. S.: 1992, 'Towards a Common Semantics for English Count and Mass Nouns', *Linguistics and Philosophy* **15**, 597–639.
Goodman, N.: 1951, *The Structure of Appearance*, D. Reidel Publishing Co., Dordrecht, Holland.
van Heijenoort, J.: 1986, 'Frege and Vagueness', in L. Haaparanta and J. Hintikka (eds.), *Frege Synthesized*, D. Reidel Publishing Co., Dordrecht, Holland, pp. 31–45.
Higginbotham, J.: 1994, 'Mass and Count Quantifiers', *Linguistics and Philosophy* **17**, 447–80.
Johnston, M.: 1992, 'Constitution Is Not Identity', *Mind* **101**, 89–105.
Koslicki, K.: 1995, *Talk about Stuffs and Things: The Logic of Mass and Count Nouns*, unpubl. Ph.D. Thesis, MIT.
Koslicki, K.: 1997a, 'The Semantics of Mass-Predicates', unpubl. manuscript.
Koslicki, K.: 1997b, 'Homogeneity and the Problem of Minimal Parts', unpubl. manuscript.
Landman, F.: 1991, *Structures for Semantics*, Kluwer Academic Publishers, Dordrecht, Holland.
Laycock, H.: 1972, 'Some Questions of Ontology', *Philosophical Review* **81**, 3–42.
Laycock, H.: 1975, 'Theories of Matter', *Synthese* **31**, 411–42.
Leonard, H. S. and N. Goodman: 1940, 'The Calculus of Individuals and Its Uses', *Journal of Symbolic Logic* **5**, 45–55.
Link, G: 1983, 'The Logical Analysis of Plurals and Mass Terms: A Lattice-Theoretical Approach', in R. Bäuerle et al. (eds.), *Meaning, Use and Interpretation of Language*, De Gruyter, Berlin, pp. 302–23.
Lønning, J. T.: 1987, 'Mass Terms and Quantification', *Linguistics and Philosophy* **10**, 1–52.
Moravcsik, J.: 1973, 'Mass Terms in English', in J. Hintikka et al. (eds.), *Approaches to Natural Language*, D. Reidel Publishing Co., Dordrecht, Holland, pp. 263–85.
Pelletier, F. J.: 1975, 'Non-Singular Reference: Some Preliminaries', *Philosophia* **5**, 451–65.
Pelletier, F. J. (ed.): 1979, *Mass Terms: Some Philosophical Problems*, D. Reidel Publishing Co., Dordrecht, Holland.
Pelletier, F. J. and L. K. Schubert: 1989, 'Mass Expressions', in D. Gabbay and F. Guenthner (eds.), *Handbook of Philosophical Logic*, D. Reidel Publishing Co., Dordrecht, Holland, pp. 327–407.
Quine, W. v. O.: 1960, *Word and Object*, M.I.T. Press, Cambridge, MA.

Roeper, P.: 1983, 'Semantics for Mass Terms with Quantifiers', *Nous* **17**, 251–65.
ter Meulen, A.: 1981, 'An Intensional Logic for Mass Terms', *Philosophical Studies* **40**, 105–25.
Sharvy, R.: 1978, 'Maybe English Has No Count Nouns: Notes on Chinese Semantics; An Essay in Metaphysics and Linguistics', *Studies in Language* **2**, 345–65.
Sharvy, R.: 1979, 'The Indeterminacy of Mass Predication', in Francis Jeffry Pelletier (ed.), *Mass Terms: Some Philosophical Problems*, D. Reidel Publishing Co., Dordrecht, Holland, pp. 47–61.
Simons, P.: 1982, 'Three Essays in Formal Ontology', in Barry Smith (ed.), *Parts and Moments*, *Studies in Logic and Formal Ontology*, Philosophia Verlag, Munich, pp. 111–260.
Stalnaker, R.: 1988, 'Vague Identity', in D. F. Austin (ed.), *Philosophical Analysis*, Kluwer, Dordrecht, pp. 349–60.
Stone, J.: 1987, "Why Potentiality Matters', *Canadian Journal of Philosophy* **17**, 815–30.
Strawson, P. F.: 1953/54, 'Particular and General', *Proceedings of the Aristotelian Society* **LIV**, 233–60.
Ware, R. X .: 1975, 'Some Bits and Pieces', *Synthese* **31**, 379–93.
Wiggins, D.: 1980, *Sameness and Substance*, Harvard University Press, Cambridge, MA.
Williamson, T.: 1994, *Vagueness*, Routledge, London.
Zimmerman, D. W.: 1995, 'Theories of Masses and Problems of Constitution', *Philosophical Review* **104**, 53–110.

Department of Philosophy
University of New Orleans
New Orleans, LA 70148
U.S.A.