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# **ULTIMATE STRENGTH OF ECCENTRICALLY LOADED FILLET WELDED CONNECTIONS**

by  
DALE F. LESIK  
and  
D.J. LAURIE KENNEDY

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D. J. Laurie Kennedy

Department of Civil Engineering

University of Alberta

Edmonton, Alberta

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## ABSTRACT

Fillet welded connections are frequently loaded eccentrically in shear with the externally applied load in the same plane as the weld group. Some methods of analysis used in the past were basically incorrect because they mixed inelastic and elastic analyses. They also gave very conservative and variable margins of safety. Connection design is now customarily based on ultimate strength. A method is developed to predict the ultimate strength of eccentrically loaded fillet welded connections based upon load-deformation characteristics of fillet welds and the fulfilment of equilibrium and compatibility conditions. This method is compared with current design tables and with full-scale tests of others.

Even though the load-deformation characteristics of fillet welds are dependent on the direction of loading, current design procedures generally use a lower bound approach based on the longitudinal strength, regardless of the loading direction. An expression is developed for the ultimate strength of fillet welds loaded in shear for any loading direction. Using this expression, design tables for eccentrically loaded fillet welded connections are proposed. Recognition of the increased strength of fillet welds loaded other than longitudinally leads to more consistent margins of safety and greater economy for welded connections.

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## LIST OF SYMBOLS

$a$	parameter which defines eccentricity of a weld group
$A_w$	effective throat area of a fillet weld, $\text{mm}^2$
$A_\theta$	area of fracture surface for a fillet weld at any angle of loading, $\text{mm}^2$
$A_l$	area of fracture surface for a longitudinal fillet weld, $\text{mm}^2$
$C$	coefficient indicative of the ultimate strength of an eccentrically loaded fillet weld group
$d$	fillet weld leg size, mm
$D$	fillet weld group leg size, mm
$e$	eccentricity, mm
$F_x$	force in x-direction
$F_y$	force in y-direction
$k$	parameter which defines weld group geometry
$l$	length of fillet weld, mm
$L$	basic length of fillet weld group, mm
$M$	moment
$n$	sample size; weld element number
$P$	externally applied eccentric load = ultimate strength of a fillet weld group, kN
$P_\theta$	load acting on a fillet weld at any angle of loading, kN
$P_{u\theta}$	ultimate strength of a fillet weld loaded in shear at any angle of loading, kN

$Pu_l$	ultimate strength of a longitudinal fillet weld loaded in shear ( $\theta = 0^\circ$ ), kN
$Pu_t$	ultimate strength of a transverse fillet weld loaded in shear ( $\theta = 90^\circ$ ), kN
$r_o$	distance in x-direction which locates' the instantaneous centre of rotation, mm
$r, r_n$	radius of rotation = the distance from the instantaneous centre to the centroid of a fillet weld element, mm
$R$	nominal resistance
$R_n$	resistance of a fillet weld element, kN
$R_H$	horizontal component of $R_n$ , kN
$R_V$	vertical component of $R_n$ , kN
$S, S_i$	nominal load effect(s)
$V$	coefficient of variation
$V_r$	factored shear resistance of a longitudinal fillet weld, kN
$V_{r_\theta}$	factored shear resistance of a fillet weld at any angle of loading, kN
$V_u$	unit shear resistance of a longitudinal fillet weld, kN/mm/mm
$V_R$	coefficient of variation for ratio of mean-to-nominal resistance
$V_G$	coefficient of variation for measured-to-nominal ratio of effective weld throat area



$V_{M1}$	coefficient of variation for measured-to-nominal ratio of ultimate tensile strength of weld metal
$V_{M2}$	coefficient of variation for measured-to-nominal ratio of shear strength/ultimate tensile strength
$V_p$	coefficient of variation for test-to-predicted ratio
$x$	parameter which determines the location of the centre of gravity for a weld group in the x-direction
$x_n$	x-coordinate of a fillet weld element = the distance in the x-direction from the instantaneous centre to the centroid of the weld element, mm
$\bar{x}$	distance in x-direction to the centre of gravity of a weld group, mm
$X_u$	ultimate tensile strength of weld metal as given by the electrode classification number, MPa
$y$	parameter which determines the location of the centre of gravity for a weld group in the y-direction (only for weld groups not symmetrical about the x-axis)

$y_n$	y-coordinate of a fillet weld element = the distance in the y-direction from the instantaneous centre to the centroid of the weld element, mm
$\alpha$	fracture angle of a fillet weld loaded in shear, degrees
$\alpha_R$	coefficient of separation
$\beta$	reliability index
$\Delta, \Delta_n$	deformation of a fillet weld (element) at any angle of loading, mm
$\Delta_{\max}$	maximum deformation of a fillet weld (element) at any angle of loading, mm
$\Delta_{fr}$	deformation at fracture of a fillet weld (element) at any angle of loading, mm
$\Delta_{Pu}$	deformation at ultimate load $Pu_\theta$ of a fillet weld (element) at any angle of loading, mm
$\theta, \theta_n$	angle of loading for a fillet weld (element) = the angle between the direction of the load and the axis of the weld, degrees
$\lambda, \lambda_i$	load factor(s)
$\mu$	mean value
$\rho$	non-dimensional ratio of $\Delta/\Delta_{Pu}$
$\rho_R$	ratio of mean-to-nominal resistance
$\rho_G$	measured-to-nominal ratio of effective fillet weld throat area

$\rho_{M1}$	measure-to-nominal ratio of ultimate tensile strength of weld metal
$\rho_{M2}$	measured-to-nominal ratio of shear strength/ultimate tensile strength
$\rho_p$	test-to-predicted ratio
$\sigma$	standard deviation
$\sigma_u$	ultimate tensile strength of weld metal, MPa
$\tau_u$	ultimate shear strength of a longitudinal fillet weld, MPa
$\phi, \phi_1, \phi_2$	resistance factors
$\phi_w$	resistance factor for weld metal = 0.67
$\phi$	central safety factor



## 1. INTRODUCTION

### 1.1 General

Fillet welded connections are frequently loaded eccentrically in shear with the externally applied load in the same plane as the weld group. Examples are shown in Fig. 1.1. The method of analysis in which the shear stresses due to the concentric load are assumed to be uniformly distributed on the weld and the shear stresses due to the moment distributed as a function of the elastic section modulus is basically incorrect because it combines inelastic and elastic analyses. Furthermore, it gives margins of safety that are both very conservative and variable.

Connection design is now customarily based on ultimate strength. Butler et al (1972) developed a method of analysis for predicting the ultimate capacity of eccentrically loaded welded connections based upon load-deformation characteristics of the weld and fulfilment of equilibrium and compatibility conditions of the connection. The fillet weld group is considered to be divided into a discrete number of finite weld elements and the resistance of the weld group is taken as the sum of resistances of these elements.

For a given weld metal the ultimate strength of a fillet weld element loaded in shear is dependent upon the angle of loading. As shown by Butler and Kulak (1971) and Miazga and Kennedy (1986) among others, the strength

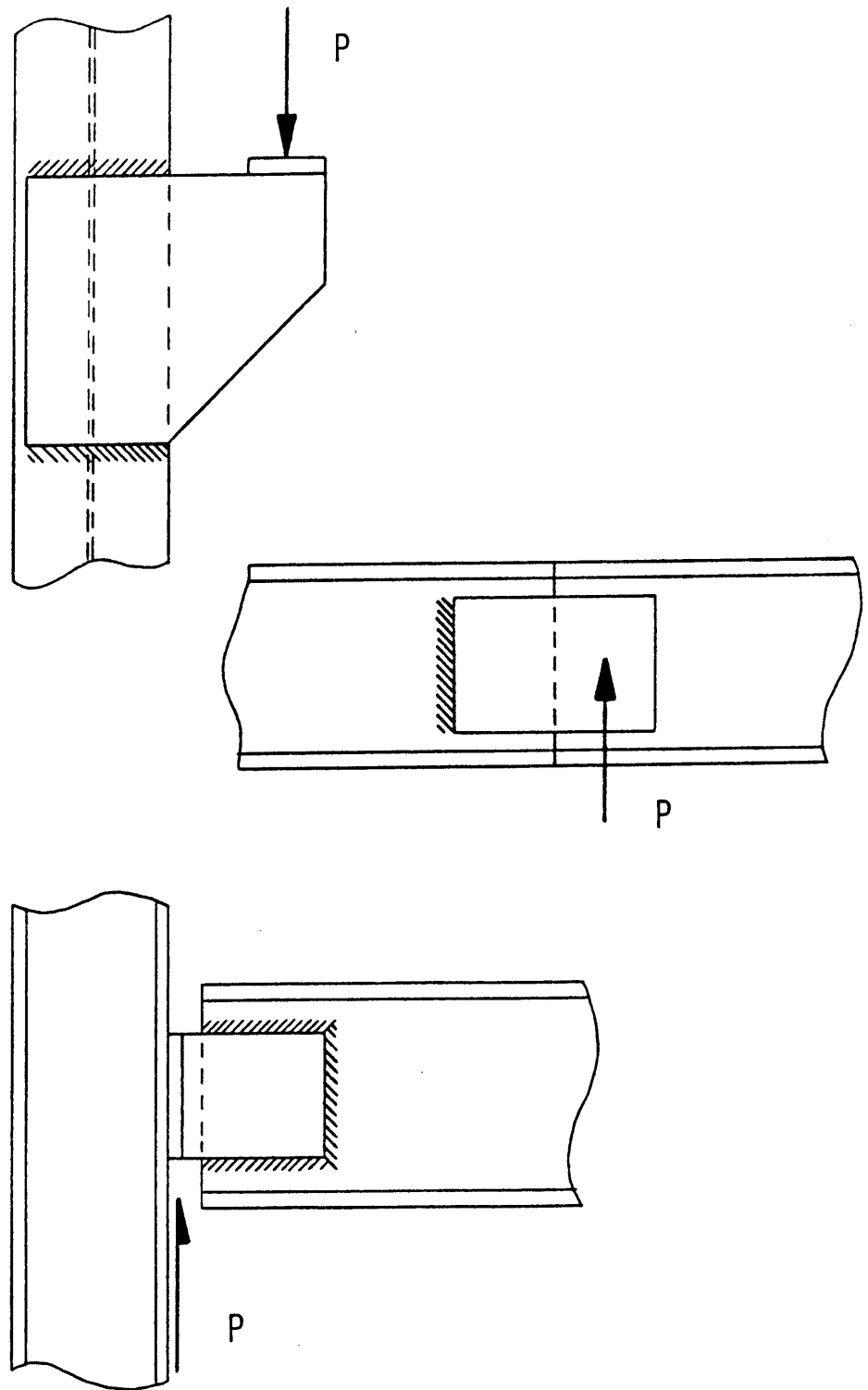


Figure 1.1 Eccentrically Loaded Welded Connections

increases as the orientation of loading changes from parallel to the axis of the weld (longitudinal fillet weld) to perpendicular to the axis of the weld (transverse fillet weld). Fillet welds loaded longitudinally are the weakest and thus provide a lower bound to fillet weld strength.

Current design standards such as CSA Standard CAN3-S16.1-M84 "Steel Structures for Buildings (Limit States Design)" (CSA 1984) and CSA Standard W59-M1984 "Welded Steel Construction (Metal Arc Welding)" (CSA 1984) base the strength of fillet welds subject to shear on the strength of longitudinal fillet welds regardless of the loading angle. This approach is conservative. However, these two standards state that the vector sum of factored longitudinal and transverse shears shall not exceed factored resistances based on the strength of longitudinal fillet welds unless an ultimate strength analysis is used instead although such a method of analysis is not prescribed.

Ultimate strength design tables for eccentric loads on weld groups presented in the CISC Handbook of Steel Construction (CISC 1985) are based on the work of Butler et al (1972). These tables will be compared with results derived from the ultimate strength analysis proposed herein.

## 1.2 Objectives

Miazga and Kennedy (1986) developed a theoretical analytical method for predicting the ultimate strength of fillet welds loaded in shear as a function of the loading angle. Based on the rationale proposed by them, the objectives of this study are:

1. to develop a method for predicting the ultimate strength of eccentrically loaded fillet weld groups loaded in shear,
2. to compare these results with current CISC Handbook design tables and with the full-scale test results of others, and
3. to make appropriate design recommendations.

Recognition of the increased strength of fillet welds loaded other than longitudinally should lead to more consistent margins of safety for welded connections.

## 1.3 Scope

An analytical method for predicting the ultimate strength of eccentrically loaded fillet welded connections is developed based on the work of Miazga and Kennedy. Results derived from this method are compared with the test results of others and with the design tables for eccentrically loaded weld groups given in the CISC Handbook of Steel Construction. Design recommendations are proposed.



## 2. THE INSTANTANEOUS CENTRE OF ROTATION METHOD

The analytical approach used to predict the ultimate strength of eccentrically loaded fillet weld groups loaded in shear is based upon load-deformation characteristics of the weld and fulfilment of equilibrium and compatibility conditions of the connection. The instantaneous centre of rotation method is used to satisfy the latter criterion. This method relies on the following assumptions:

1. Under an eccentric load, a fillet weld group tends, at any load level, to rotate about an instantaneous centre of rotation.
2. A fillet weld group is considered to be divided into a discrete number of finite weld elements. The resisting force of each elemental length of weld acts through its centroid. The resistance of the weld group to the external eccentric load is provided by the combined resistances of the weld elements.
3. The deformation of a weld element varies linearly with the distance from the instantaneous centre to the centroid of the weld element (radius of rotation) and acts in a direction perpendicular to the radius. The angle of loading is therefore known.
4. The resistance of a weld element is obtained from the average load-deformation response as determined empirically for the specific angle of loading.
5. The load-deformation response of a fillet weld loaded in compression-induced shear is the same as for a

similar weld loaded in tension-induced shear.

6. The ultimate strength of a weld group is obtained when the maximum deformation of some weld element is reached.

Although the method is general, the case presented here is that in which the weld group is symmetrical about a centroidal axis and the line of action of the external load is perpendicular to this same axis. For this case, the instantaneous centre lies on the centroidal axis of symmetry. Its location on the axis must be determined iteratively.

Consider a fillet weld group subjected to an external load,  $P$ , located at an eccentricity,  $e = aL$ , from the centre of gravity (c.g.) as shown in Fig. 2.1. A trial location of the instantaneous centre of rotation (i.c.) is chosen at a distance  $r_0$  from the vertical length of weld. Coordinate axes  $x$  and  $y$  are located as shown.

The radius of rotation of a weld element is

$$[2.1] \quad r_n = \sqrt{x_n^2 + y_n^2}$$

When the weld element under consideration is in a horizontal portion of the weld group, the angle which the resisting force of the weld element makes with the longitudinal axis of the weld element (angle of loading) is

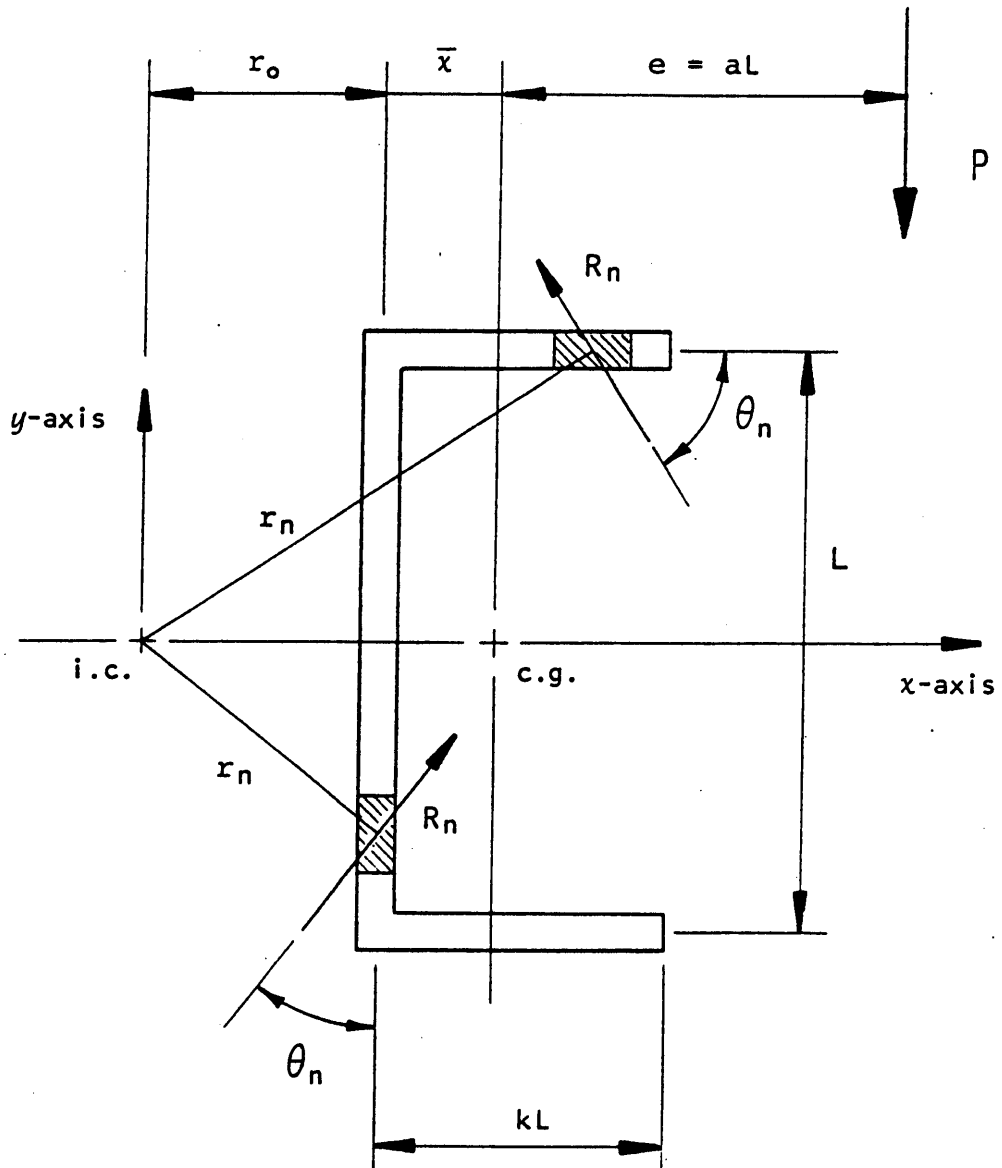


Figure 2.1 Eccentrically Loaded Weld Group

$$[2.2] \quad \theta_n = \tan^{-1} \left( \frac{x_n}{y_n} \right)$$

When the weld element is in a vertical portion of the weld group, the angle of loading is

$$[2.3] \quad \theta_n = \tan^{-1} \left( \frac{y_n}{r_o} \right)$$

The weld element which first reaches its maximum deformation, the critical weld element, must be located. In many situations, this is the weld element located furthest from the instantaneous centre. Mathematically, it is the element for which the ratio of maximum deformation to radius of rotation is a minimum [ie.  $(\Delta_{\max})_n/r_n$  is a minimum]. The maximum deformation of a weld element,  $(\Delta_{\max})_n$ , is a function of the loading angle,  $\theta_n$ , as discussed in Chapter 3. The deformation of any other weld element is

$$[2.4] \quad \Delta_n = r_n \left[ \frac{\Delta_{\max}}{r} \right]_{\text{critical}}$$

where  $\Delta_{\max}/r$  is the ratio for the critical weld element.

The resisting force,  $R_n$ , acting at the centre of the  $n^{\text{th}}$  weld element and at a loading angle  $\theta_n$ , is determined from the load-deformation characteristics of the weld for the value of  $\Delta_n$  calculated from [2.4]. The load-deformation characteristics of a fillet weld depend on the angle  $\theta_n$  as discussed in Chapter 3. The vertical

and horizontal components of the resisting force for each weld element are calculated from the geometry of the weld group. For elements in the horizontal portion of the weld group

$$[2.5] \quad (R_V)_n = R_n \sin \theta_n$$

$$[2.6] \quad (R_H)_n = R_n \cos \theta_n$$

and for elements in the vertical portion of the weld group

$$[2.7] \quad (R_V)_n = R_n \cos \theta_n$$

$$[2.8] \quad (R_H)_n = R_n \sin \theta_n$$

The equations of equilibrium

$$[2.9] \quad \Sigma F_x = 0$$

$$[2.10] \quad \Sigma F_y = 0$$

$$[2.11] \quad \Sigma M = 0$$

are checked for the assumed position of the instantaneous centre. For the case presented here, [2.9] is automatically satisfied because of the symmetry of the weld group and because there are no external forces in the

x-direction. For weld groups not symmetrical about the centroidal x-axis, satisfying [2.9] establishes the location of the instantaneous centre in the y-direction corresponding to the trial location of the instantaneous centre in the x-direction,  $r_o$ . From [2.11], the externally applied load,  $P$ , is found by taking the sum of moments about the instantaneous centre:

$$[2.12] \quad P (e + \bar{x} + r_o) - \sum_{n=1}^n (R_n \times r_n) = 0$$

Load  $P$  from [2.12] is used to check [2.10]:

$$[2.13] \quad P - \sum_{n=1}^n (R_V)_n = 0$$

If [2.13] is not satisfied, a new trial location of the instantaneous centre is chosen and the procedure repeated. When a value of  $r_o$  is found which satisfies the equations of statics, the value of  $P$  so obtained is the ultimate load which the weld group can sustain.

### 3. ANALYSES OF FILLET WELDS LOADED IN SHEAR

#### 3.1 General

The ultimate strength and maximum deformation of fillet welds loaded in shear are dependant upon the direction of the applied load. Miazga and Kennedy (1986) performed 42 tests on fillet weld specimens with seven angles of loading and two weld sizes. Their data, results, analytical observations and conclusions form the basis for sections 3.2 to 3.4 of this chapter.

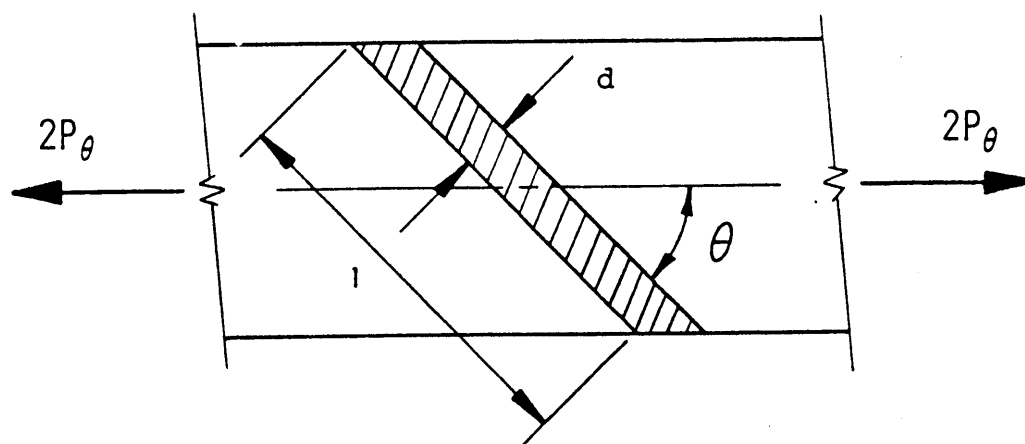
#### 3.2 Ultimate Strength

Miazga and Kennedy developed a rational analytical method for predicting the ultimate strength of fillet welds loaded in shear as a function of the loading angle that is consistent with both measured ultimate strengths and fracture observations. This method is based on an equilibrium analysis of a fillet weld loaded in shear and on the maximum shear stress failure criterion. The expression for ultimate load carried by a fillet weld loaded at any angle  $\theta$ , normalized by dividing by the ultimate load for  $\theta = 0^\circ$ , is

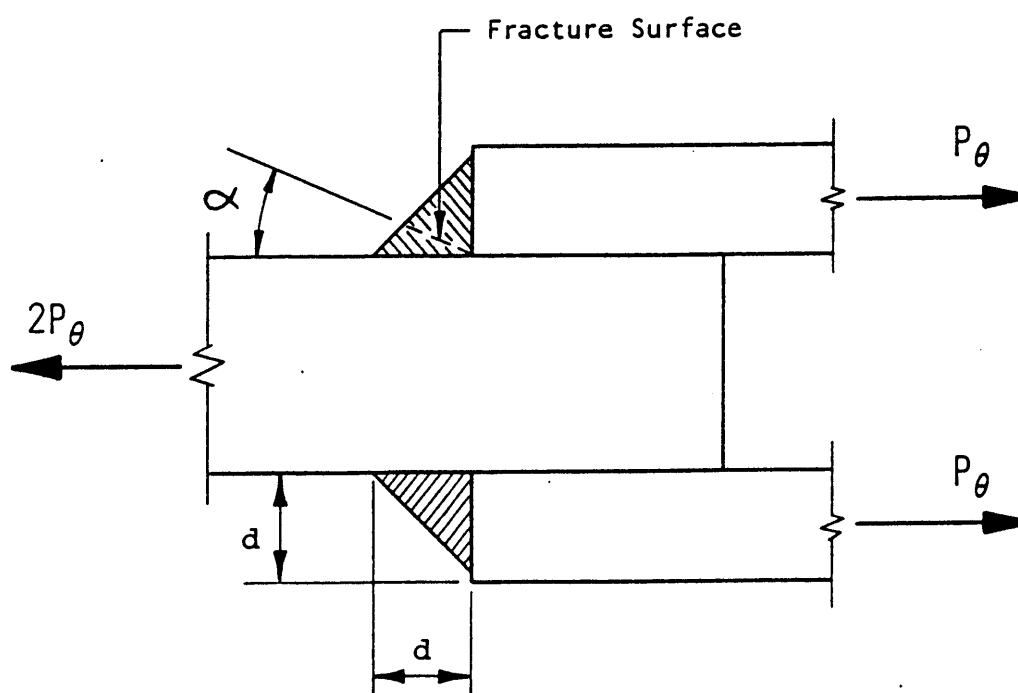
$$[3.1] \quad \frac{P_{u\theta}}{P_{u\lambda}} = \frac{A_\theta (1 + 0.155\sin\theta)}{A_\lambda [(\sin\theta\cos\alpha - 0.3\sin\theta\sin\alpha)^2 + \cos^2\theta]^{1/2}}$$

The fracture surface area,  $A_\theta$ , as shown in Fig. 3.1, is

$$[3.2] \quad A_\theta = \frac{dl \sin(\pi/4)}{\sin(\pi/4 + \alpha)}$$



a) Plan



b) Section

Figure 3.1 Fillet Weld Loaded at a General Angle



The fracture angle,  $\alpha$ , is a function of the loading angle. Values of  $\alpha$  for any value of  $\theta$  are determined from the data reported by Miazga and Kennedy or derived iteratively from [5.4] given by them that maximizes the average shear stress on the fracture surface with respect to  $\alpha$ . A simplified form of [3.1], giving the ultimate load as a function of the loading angle only, is

$$[3.3] \quad \frac{Pu_{\theta}/dl}{Pu_{\lambda}/dl} = \frac{Pu_{\theta}}{Pu_{\lambda}} = 0.5 \sin^{1.5} \theta + 1.0$$

Figures 3.2 and 3.3 compare this approximate relationship with the more accurate expression, [3.1], derived by Miazga and Kennedy. The curves are in good agreement.

Table 3.1 gives the test-to-predicted ratios for the ultimate strength of a fillet weld using [3.3] and the test data of Miazga and Kennedy. Ultimate test loads in kN per mm of fillet weld size per mm of weld length, also given in Table 3.1, were normalized by dividing the measured load by the actual average weld leg size,  $d$ , and weld length,  $l$ , for each test specimen. The ultimate loads are further normalized by dividing by the average longitudinal ultimate load,  $Pu_{\lambda} = 0.291$  kN/mm/mm, for the six specimens loaded at  $\theta = 0^{\circ}$ . The first two numbers in the test designation indicate the angle of loading for the specimen. The mean value, standard deviation and coefficient of variation of the test-to-predicted ratios for all 42 tests are given at the end of the table. The

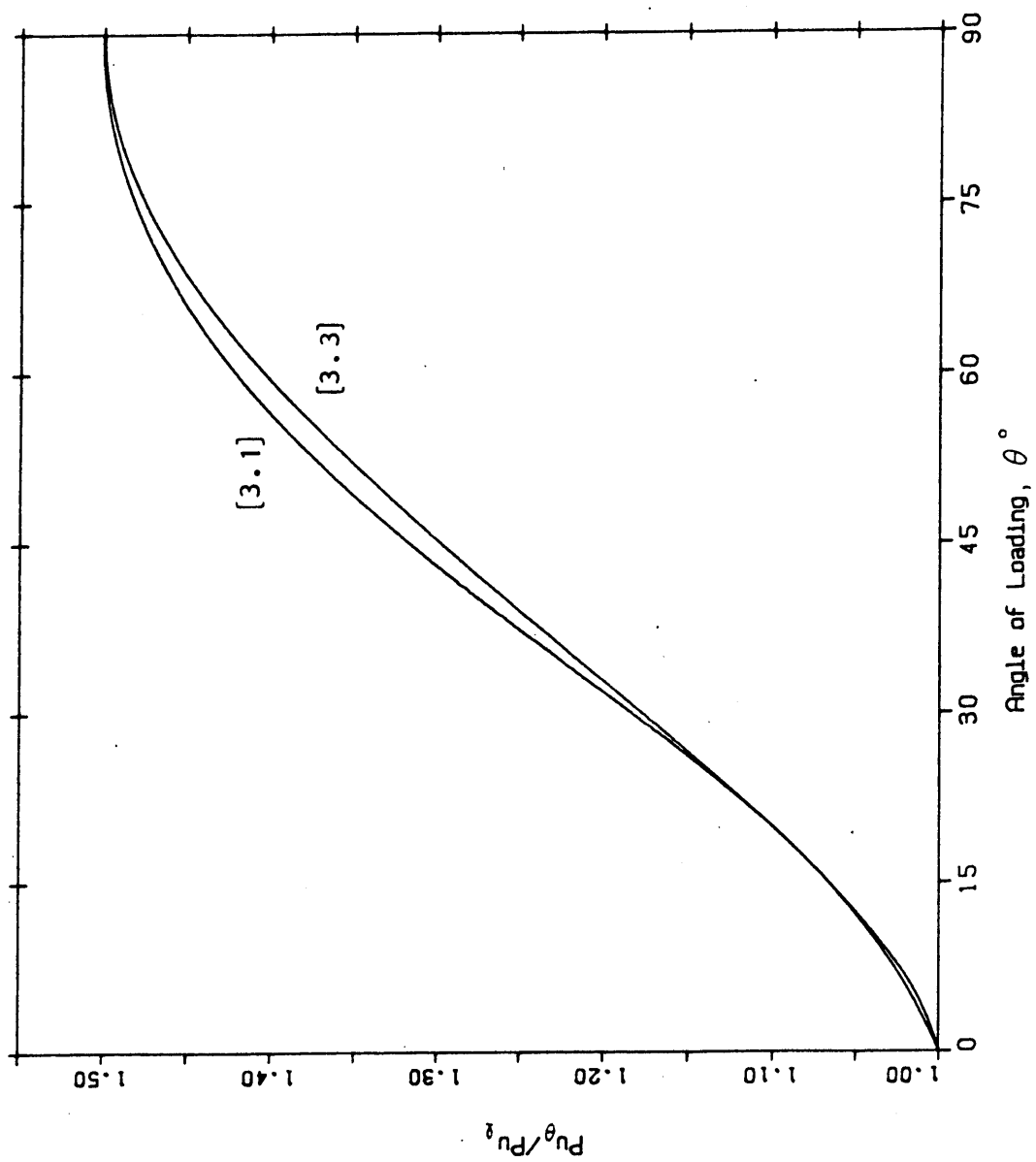


Figure 3.2 Normalized Ultimate Load

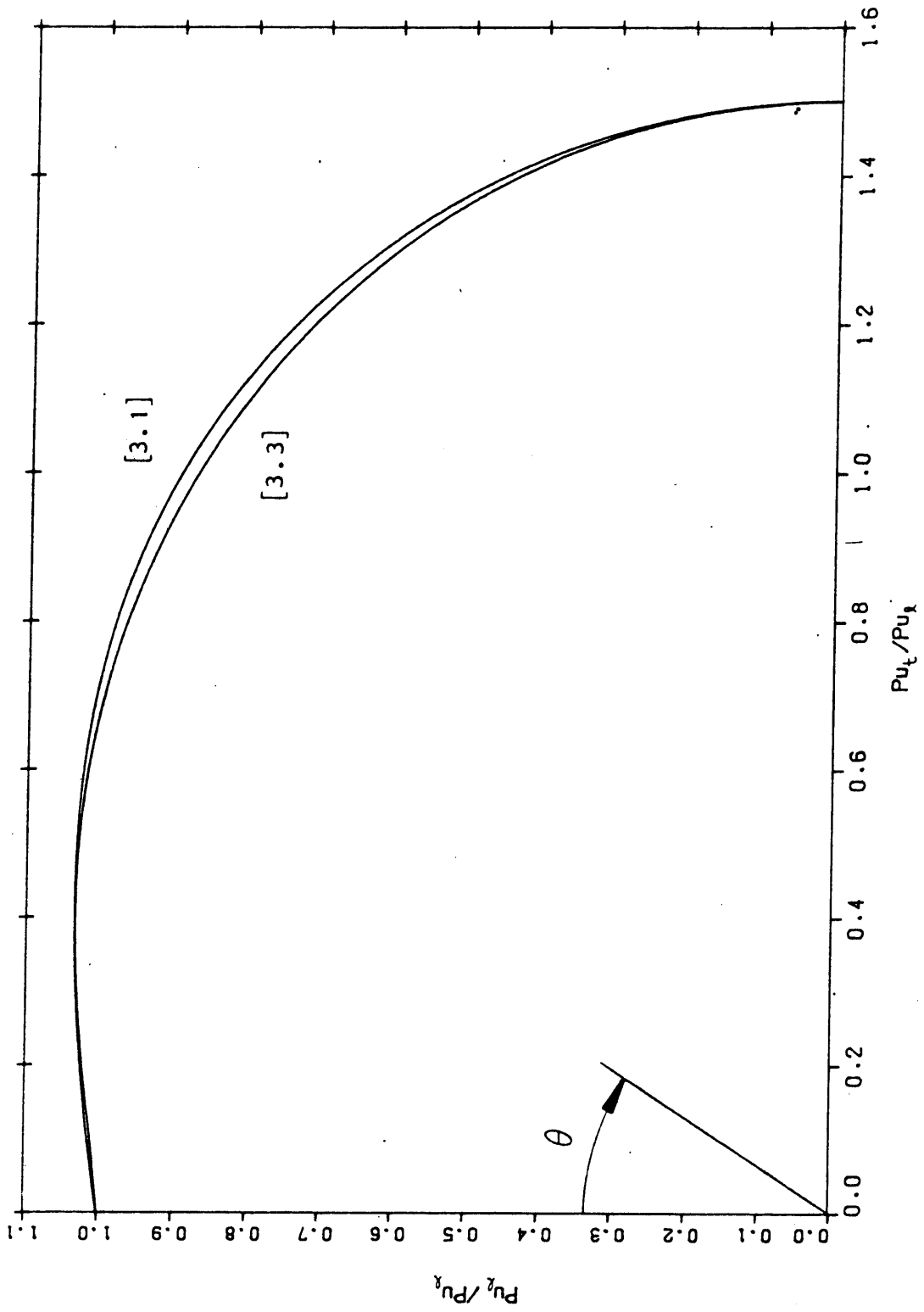


Figure 3.3 Ultimate Load Interaction Diagram

Table 3.1 Test-to-Predicted Ratios for Ultimate Strength

Test	Ultimate Load $Pu_{\theta}/dl$ (kN/mm/mm) [test]	Normalized Ultimate Load $Pu_{\theta}/Pu_{\lambda}$ [test]	Normalized Ultimate Load $Pu_{\theta}/Pu_{\lambda}$ [3.3]	Test Predicted
00.1	0.32825	1.12933		1.12933
00.2	0.30178	1.03826		1.03826
00.3	0.29744	1.02333	1.00000	1.02333
00.11	0.26383	0.90769		0.90769
00.12	0.28182	0.96959		0.96959
00.13	0.27083	0.93178		0.93178
15.1	0.30474	1.04844		0.98367
15.2	0.29641	1.01978		0.95679
15.3	0.30378	1.04514	1.06584	0.98058
15.11	0.29330	1.00908		0.94675
15.12	0.26692	0.91832		0.86159
15.13	0.30536	1.05057		0.98567
30.1	0.39370	1.35450		1.15102
30.2	0.37662	1.29574		1.10109
30.3	0.39163	1.34738	1.17678	1.14497
30.11	0.35181	1.21038		1.02855
30.12	0.35760	1.23030		1.04548
30.13	0.34521	1.18768		1.00926
45.1	0.40761	1.40236		1.08098
45.2	0.42553	1.46401		1.12851
45.3	0.41848	1.43976	1.29730	1.10981
45.11	0.32819	1.12912		0.87036
45.12	0.32559	1.12017		0.86346
45.13	0.33655	1.15788		0.89253
60.1	0.48259	1.66032		1.18344
60.2	0.48432	1.66628		1.18769
60.3	0.49012	1.68623	1.40296	1.20191
60.11	0.41985	1.44447		1.02959
60.12	0.40373	1.38901		0.99006
60.13	0.39650	1.36414		0.97233

Table 3.1. Test-to-Predicted Ratios for Ultimate Strength (cont.)

Test	Ultimate Load $P_{u\theta}/dl$ (kN/mm/mm) [test]	Normalized Ultimate Load $P_{u\theta}/P_{u\lambda}$ [test]	Normalized Ultimate Load $P_{u\theta}/P_{u\lambda}$ [3.3]	Test Predicted
75.1	0.42214	1.45235		0.98487
75.2	0.42764	1.47127		0.99770
75.3	0.43734	1.50464	1.47466	1.02033
75.11	0.42475	1.46133		0.99096
75.12	0.43163	1.48500		1.00701
75.13	0.41703	1.43477		0.97295
90.1	0.40059	1.37821		0.91881
90.2	0.40399	1.38991		0.92661
90.3	0.38274	1.31680	1.50000	0.87787
90.11	0.44095	1.51706		1.01137
90.12	0.43512	1.49701		0.99801
90.13	0.43202	1.48634		0.99089
			$\mu$	1.00961
			$\sigma$	0.09010
			V	0.08924
			n	42

proposed expression for the ultimate strength of a fillet weld loaded in shear, [3.3], shows excellent agreement with test results with a mean test-to-predicted ratio of 1.0096, a standard deviation of 0.090 and a coefficient of variation of 0.089.

### 3.3 Deformation

The maximum deformation of a fillet weld loaded in shear may be taken as either the deformation at weld fracture or the deformation at ultimate load  $Pu_\theta$ . Both cases are considered in Chapter 4. The test data indicate that the maximum deformation decreases as the angle of loading increases. Using a linear regression analysis, the exponential function developed for the normalized deformation at weld fracture as a function of the loading angle,  $\theta$ , is

$$[3.4] \quad \frac{\Delta_{fr}}{d} = 1.087 (\theta + 6)^{-0.65}$$

in mm/mm of fillet weld size. Similarly, the normalized deformation at ultimate load  $Pu_\theta$  is

$$[3.5] \quad \frac{\Delta_{Pu}}{d} = 0.209 (\theta + 2)^{-0.32}$$

The multiplicative coefficients in these expressions have been adjusted to give mean test-to-predicted ratios of 1.0.

Equations [3.4] and [3.5] are compared to average test data in Figs. 3.4 and 3.5, respectively. Tables 3.2 and 3.3 list test-to-predicted ratios computed using [3.4] and [3.5]. Test deformations listed in the tables, in mm/mm of fillet weld size, are an average measurement of four readings normalized by dividing the measured value by the actual average weld leg size,  $d$ , for each test specimen. The standard deviation and coefficient of variation for the test-to-predicted ratios are also given in each table. The curves of Figs. 3.4 and 3.5 and the tables indicate that the proposed expressions predict the average test data at each loading angle reasonably well with a coefficient of variation of about 17 to 19%.

### 3.4 Load-Deformation Response

After several trials, a polynomial was selected to model the load-deformation response of the 42 tests. The data for each test specimen loaded at an angle  $\theta$  were non-dimensionalized by dividing the normalized load,  $P_\theta/dl$ , by the normalized ultimate load,  $P_{u\theta}/dl$ , for that specimen and by dividing the corresponding normalized deformation,  $\Delta/d$ , by the normalized maximum deformation at ultimate load,  $\Delta_{pu}/d$ , for that specimen. Loads, in kN/mm/mm, were normalized by dividing the measured values by the actual average weld leg size,  $d$ , and weld length,  $l$ , for each test specimen. Deformations, in mm/mm, were normalized by dividing the measured values by the actual

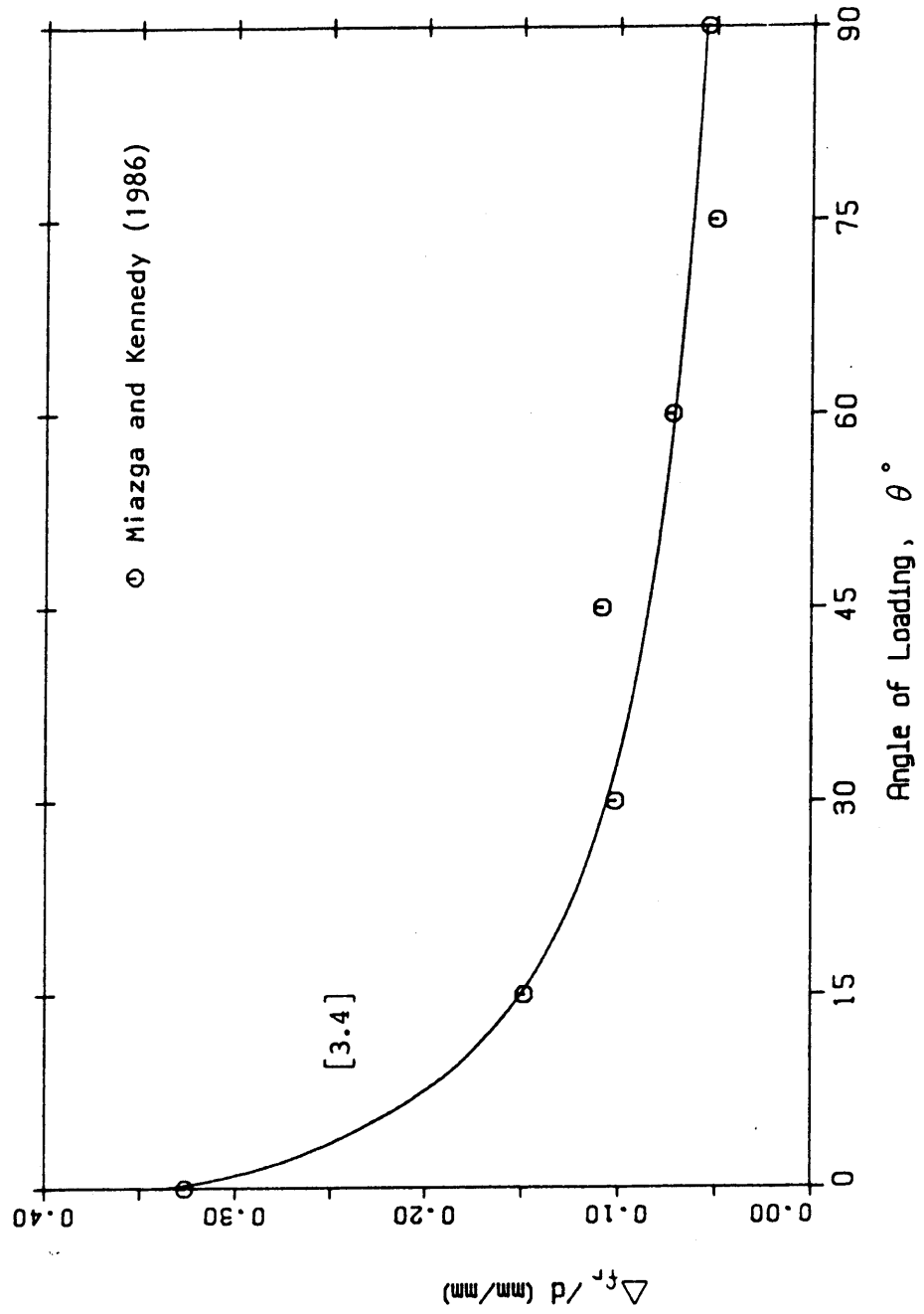


Figure 3.4 Normalized Deformation at Fracture



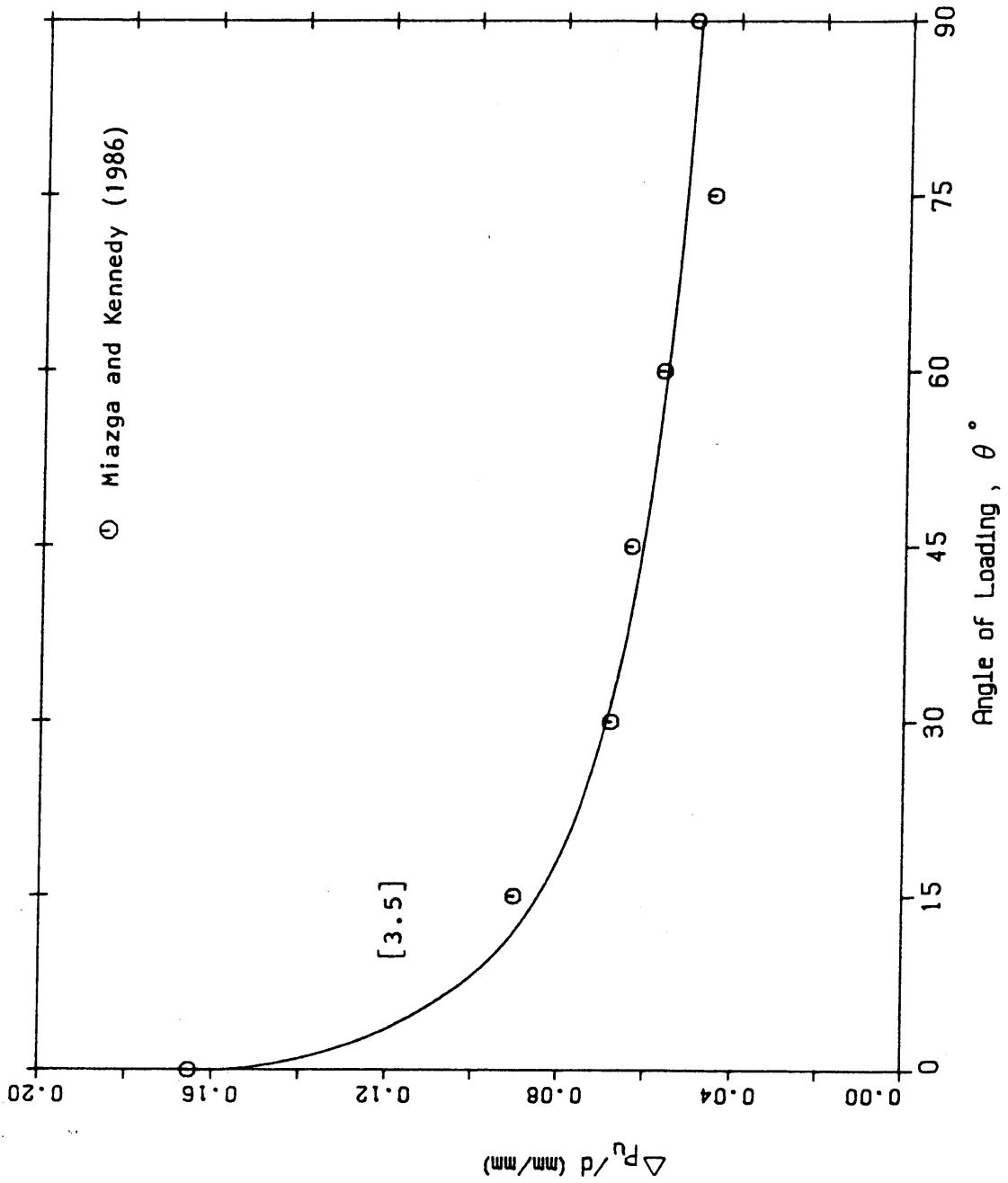


Figure 3.5 Normalized Deformation at Ultimate Load

Table 3.2 Test-to-Predicted Ratios for Normalized Deformation at Fracture

Test	Normalized Deformation at Fracture, $\Delta_{fr}/d$ (mm/mm) [test]	Normalized Deformation at Fracture, $\Delta_{fr}/d$ (mm/mm) [3.4]	Test Predicted
00.1	0.36689	0.33907	1.08205
00.2	0.34657		1.02212
00.3	0.31741		0.93612
00.11	0.31317		0.92361
00.12	0.32187		0.94927
00.13	0.29056		0.85693
15.1	0.13366	0.15019	0.88994
15.2	0.11719		0.78028
15.3	0.15257		1.01585
15.11	0.21131		1.40695
15.12	0.14706		0.97916
15.13	0.12904		0.85918
30.1	0.10937	0.10580	1.03374
30.2	0.11184		1.05709
30.3	0.11522		1.08904
30.11	0.08319		0.78629
30.12	0.08189		0.77401
30.13	0.10878		1.02817
45.1	0.08875	0.08437	1.05191
45.2	0.10754		1.27462
45.3	0.09158		1.08546
45.11	0.11868		1.40666
45.12	0.11544		1.36826
45.13	0.12848		1.52282
60.1	0.07718	0.07135	1.08171
60.2	0.07701		1.07933
60.3	0.08243		1.15529
60.11	0.06751		0.94618
60.12	0.06417		0.89937
60.13	0.06548		0.91773
75.1	0.04626	0.06246	0.74063
75.2	0.05468		0.87544
75.3	0.04716		0.75504
75.11	0.05660		0.90618
75.12	0.05743		0.91947
75.13	0.04114		0.65866

Table 3.2 Test-to-Predicted Ratios for Normalized Deformation at Fracture (continued)

Test	Normalized Deformation at Fracture, $\Delta_{fr}/d$ (mm/mm) [test]	Normalized Deformation at Fracture, $\Delta_{fr}/d$ (mm/mm) [3.4]	Test Predicted
90.1	0.05941		1.06222
90.2	0.03812		0.68157
90.3	0.06662	0.05593	1.19113
90.11	0.05796		1.03630
90.12	0.05152		0.92115
90.13	0.05547		0.99178
		$\mu$	1.0
		$\sigma$	0.19364
		V	0.19364
		n	42

Table 3.3 Test-to-Predicted Ratios for Normalized Deformation at Ultimate Load

Test	Normalized Deformation at Ultimate Load, $\Delta_{pu}/d$ (mm/mm) [test]	Normalized Deformation at Ultimate Load, $\Delta_{pu}/d$ (mm/mm) [3.5]	Test Predicted
00.1	0.18948	0.16750	1.13122
00.2	0.19455		1.16149
00.3	0.19043		1.13690
00.11	0.14181		0.84663
00.12	0.10797		0.64460
00.13	0.16769		1.00113
15.1	0.10839	0.08445	1.28348
15.2	0.09673		1.14541
15.3	0.09573		1.13357
15.11	0.07864		0.93120
15.12	0.08378		0.99207
15.13	0.07820		0.92599
30.1	0.08069	0.06898	1.16976
30.2	0.06938		1.00580
30.3	0.07681		1.11351
30.11	0.06402		0.92810
30.12	0.04926		0.71412
30.13	0.06815		0.98797
45.1	0.05726	0.06099	0.93884
45.2	0.05941		0.97409
45.3	0.06653		1.09083
45.11	0.06501		1.06591
45.12	0.05347		0.87670
45.13	0.08046		1.31923
60.1	0.06512	0.05582	1.16661
60.2	0.06349		1.13741
60.3	0.06794		1.21713
60.11	0.05442		0.97492
60.12	0.04041		0.72393
60.13	0.04870		0.87245
75.1	0.04627	0.05208	0.88844
75.2	0.05469		1.05012
75.3	0.04218		0.80991
75.11	0.04679		0.89843
75.12	0.04845		0.93030
75.13	0.03526		0.67704

Table 3.3 Test-to-Predicted Ratios for Normalized Deformation at Ultimate Load (continued)

Test	Normalized Deformation at Ultimate Load, $\Delta_{pu}/d$ (mm/mm) [test]	Normalized Deformation at Ultimate Load, $\Delta_{pu}/d$ (mm/mm) [3.5]	Test Predicted
90.1	0.05934		1.20610
90.2	0.03815		0.77541
90.3	0.06659	0.04920	1.35346
90.11	0.04920		1.00000
90.12	0.04708		0.95691
90.13	0.04151		0.84370
		$\mu$	1.0
		$\sigma$	0.17151
		V	0.17151
		n	42

average weld leg size for each specimen.

Coefficients for the polynomial were determined by using a computer program, adapted from Gerald (1978), to perform a non-linear regression analysis of all the non-dimensionalized data. Consideration was given to the declining number of data points beyond  $\Delta_{Pu}/d$  for test specimens loaded at angles greater than  $0^\circ$ . The polynomial expression developed from this analysis for the load-deformation response of a fillet weld loaded in shear at any angle  $\theta$  is

$$[3.6] \quad \frac{P_\theta/dl}{P_{u_\theta}/dl} = \frac{P_\theta}{P_{u_\theta}} = -13.29\rho + 457.32\rho^{1/2} - 3385.9\rho^{1/3} \\ + 9054.29\rho^{1/4} - 9952.13\rho^{1/5} + 3840.71\rho^{1/6}$$

when  $\rho > 0.0325$

and

$$[3.7] \quad \frac{P_\theta}{P_{u_\theta}} = 8.23384\rho$$

when  $\rho \leq 0.0325$

where  $\rho$  is

$$[3.8] \quad \rho = \frac{\Delta/d}{\Delta_{Pu}/d} = \frac{\Delta}{\Delta_{Pu}}$$

and  $\Delta p_u/d$  is given by [3.5].  $P_{u\theta}$  is determined from [3.3] when the value of the longitudinal ultimate strength,  $P_{u\lambda}$ , is known.

Equations [3.6] and [3.7] are shown in Fig. 3.6. The curve shows that after the ultimate load is reached, unloading occurs with increasing deformation until fracture occurs. The point of fracture moves up and to the left along the curve as the angle of loading,  $\theta$ , increases from  $0^\circ$  (longitudinal fillet weld) to  $90^\circ$  (transverse fillet weld).

Restating [3.6] and [3.7] as

$$[3.9] \quad P_\theta / P_{u\theta} = f(\rho)$$

and combining with [3.3] gives

$$[3.10a] \quad P_\theta = P_{u\theta} f(\rho) = P_{u\lambda} (0.5 \sin^{1.5} \theta + 1.0) f(\rho)$$

or in normalized form

$$[3.10b] \quad \frac{P_\theta}{dl} = \frac{P_{u\lambda}}{dl} (0.5 \sin^{1.5} \theta + 1.0) f(\rho)$$

The load sustained by a fillet weld loaded in shear at any angle  $\theta$ , for a known deformation, is predicted from [3.10] using the longitudinal ultimate strength,  $P_{u\lambda}$ , of the fillet weld. Using the average test value of  $P_{u\lambda}/dl = 0.291 \text{ kN/mm/mm}$  for specimens loaded

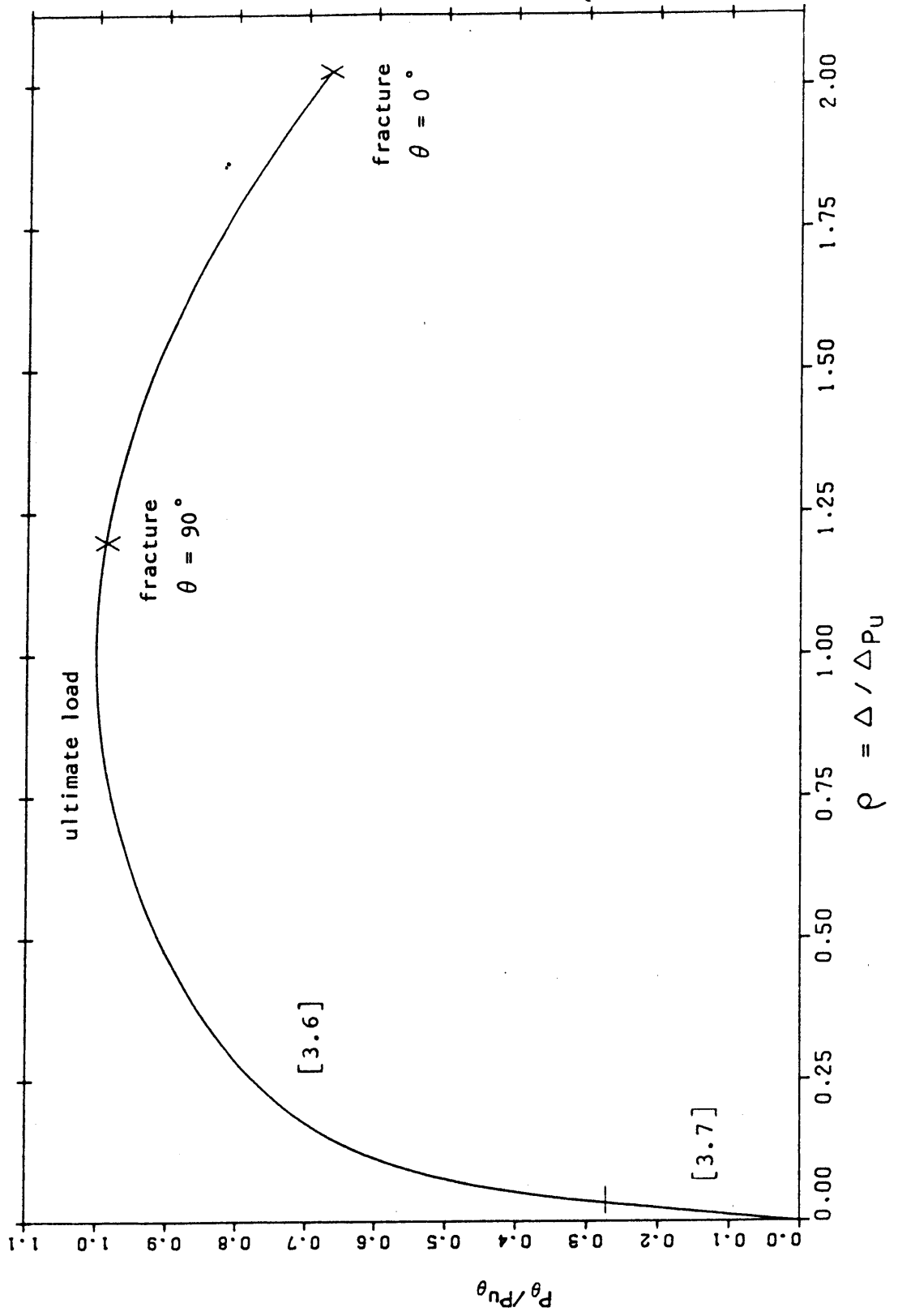


Figure 3.6 Non-Dimensionalized Load-Deformation Curve



longitudinally, [3.10] is compared to test data in Figs. 3.7 to 3.13 for seven angles of loading. The model predicts the load-deformation data reasonably well.

Equation [3.10] can be rewritten as ,

$$[3.11] \quad \frac{P_{\theta}}{P_{u_{\lambda}}} = (0.5 \sin^{1.5} \theta + 1.0) f(\rho)$$

Figure 3.14 compares [3.11] for seven angles of loading.

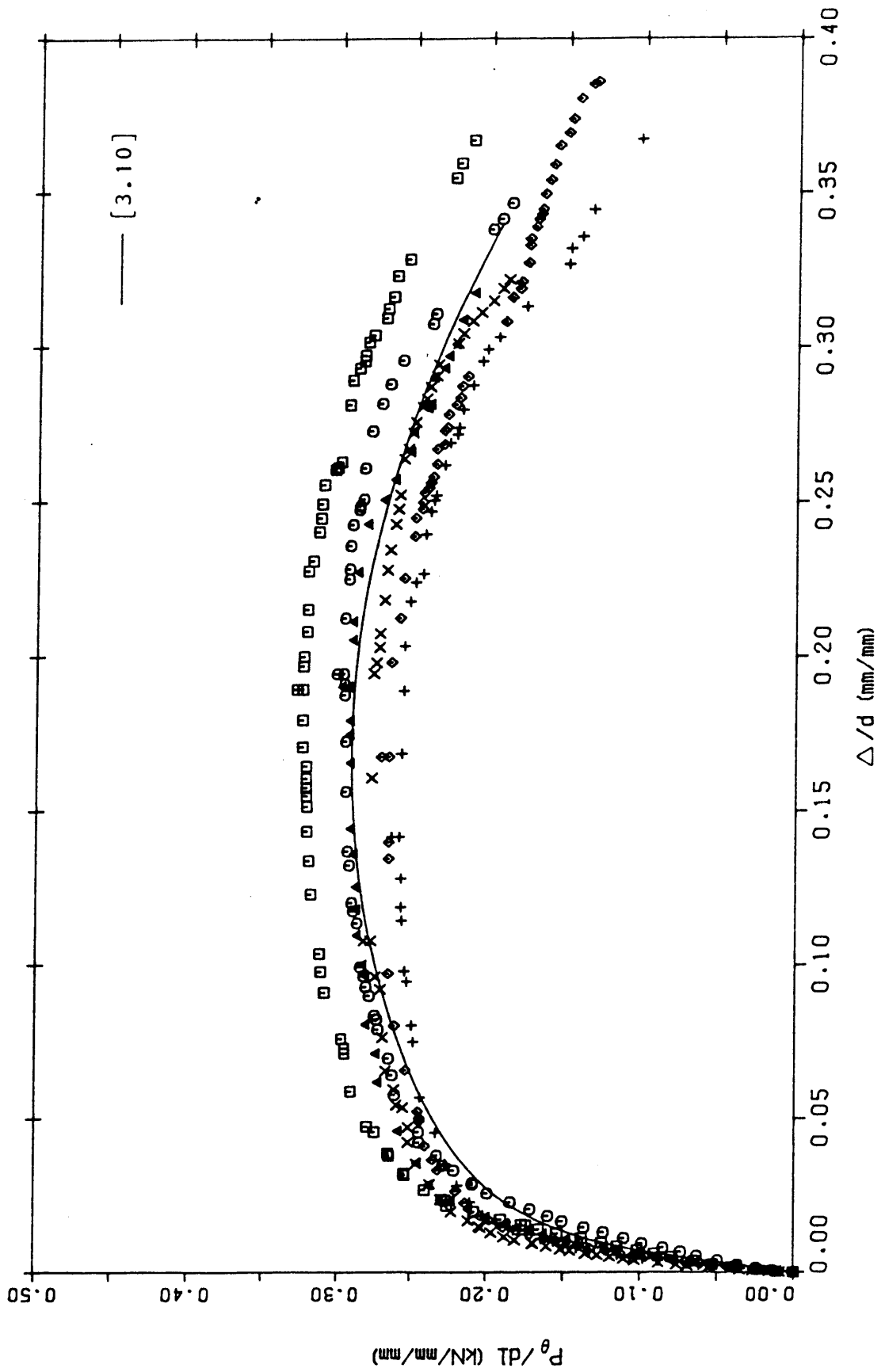


Figure 3.7 Load-Deformation Curve  $\theta = 0^\circ$

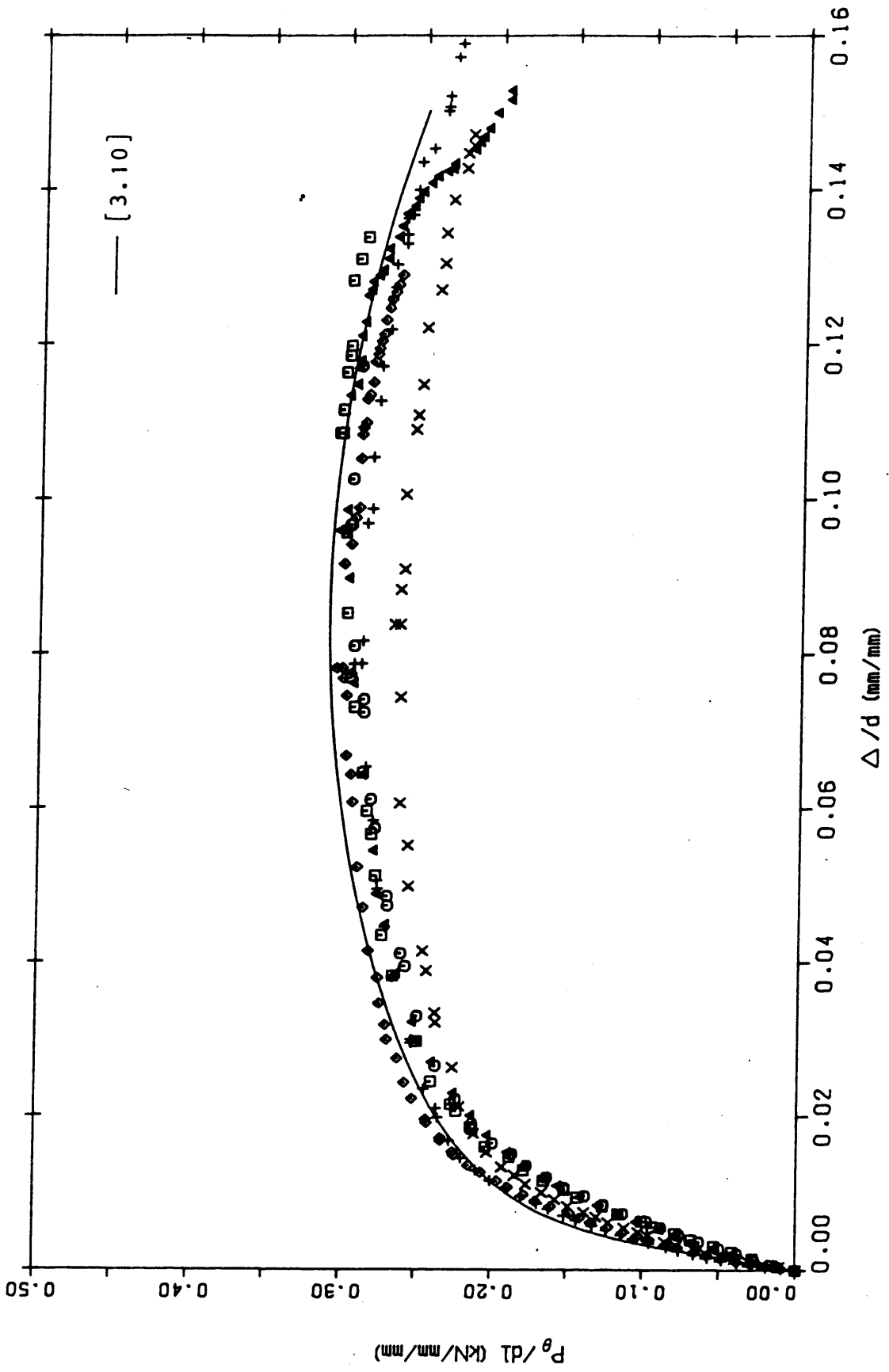


Figure 3.8 Load-Deformation Curve  $\theta = 15^\circ$

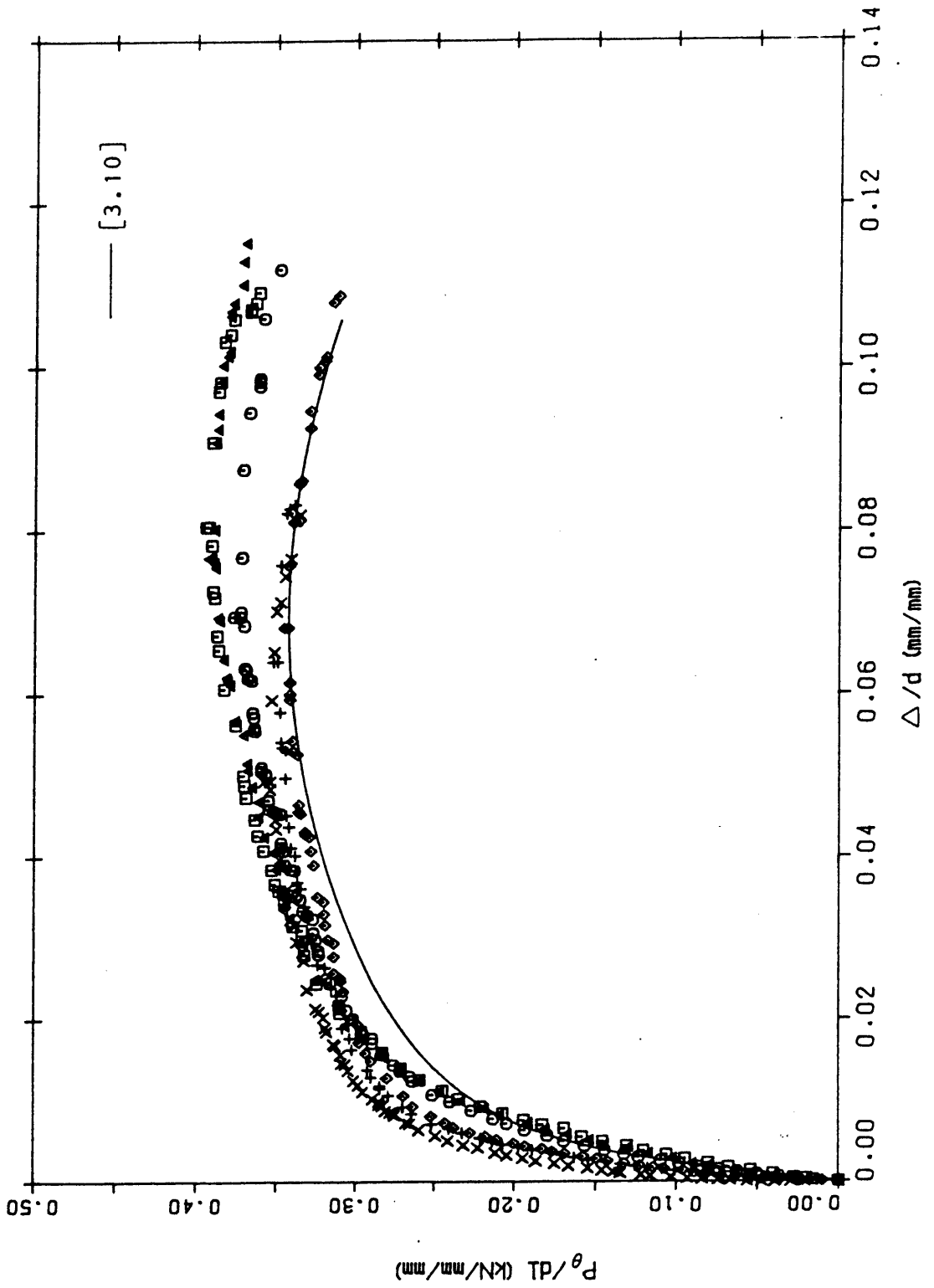


Figure 3.9 Load-Deformation Curve  $\theta = 30^\circ$

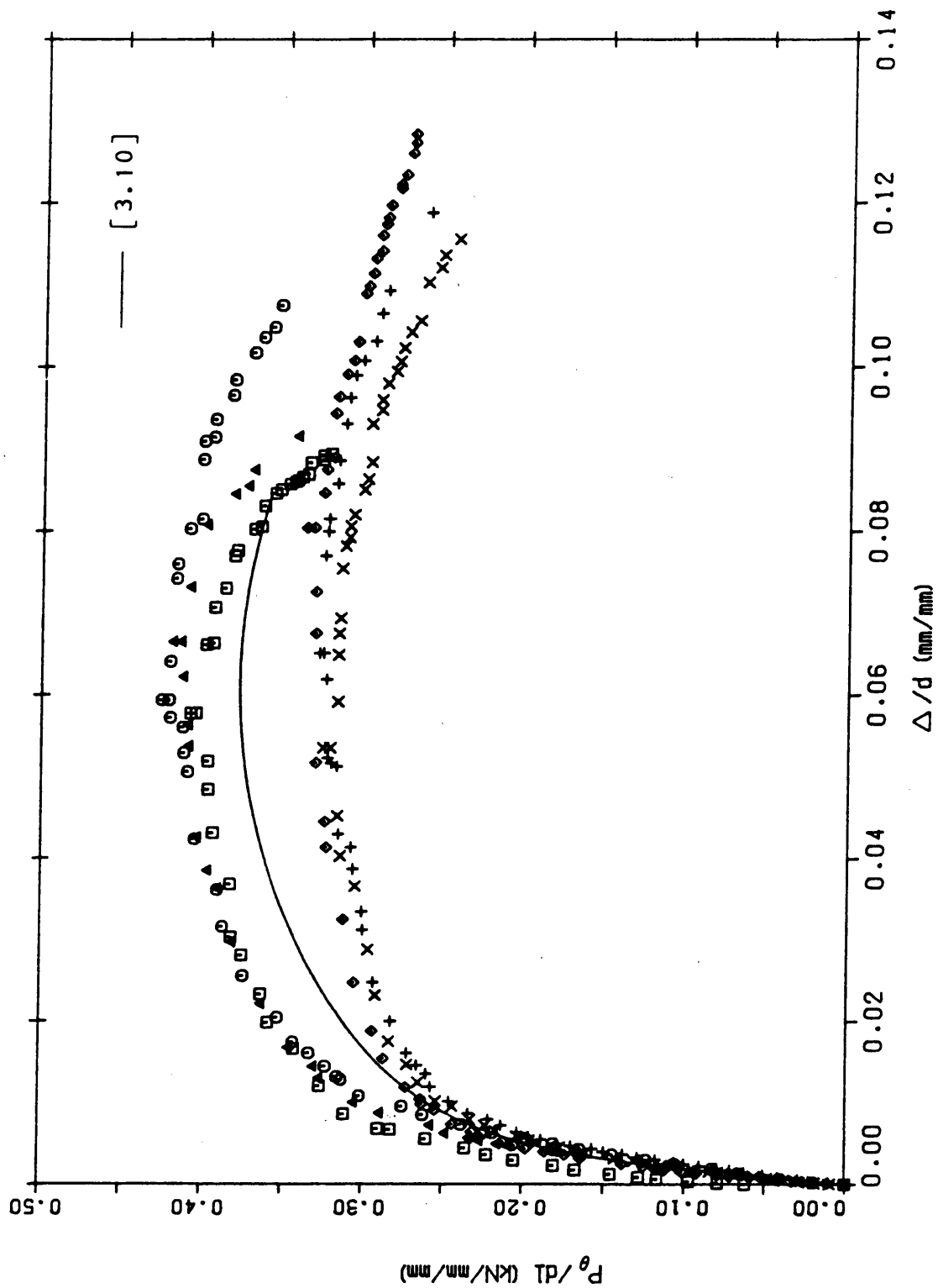


Figure 3.10 Load-Deformation Curve  $\theta = 45^\circ$

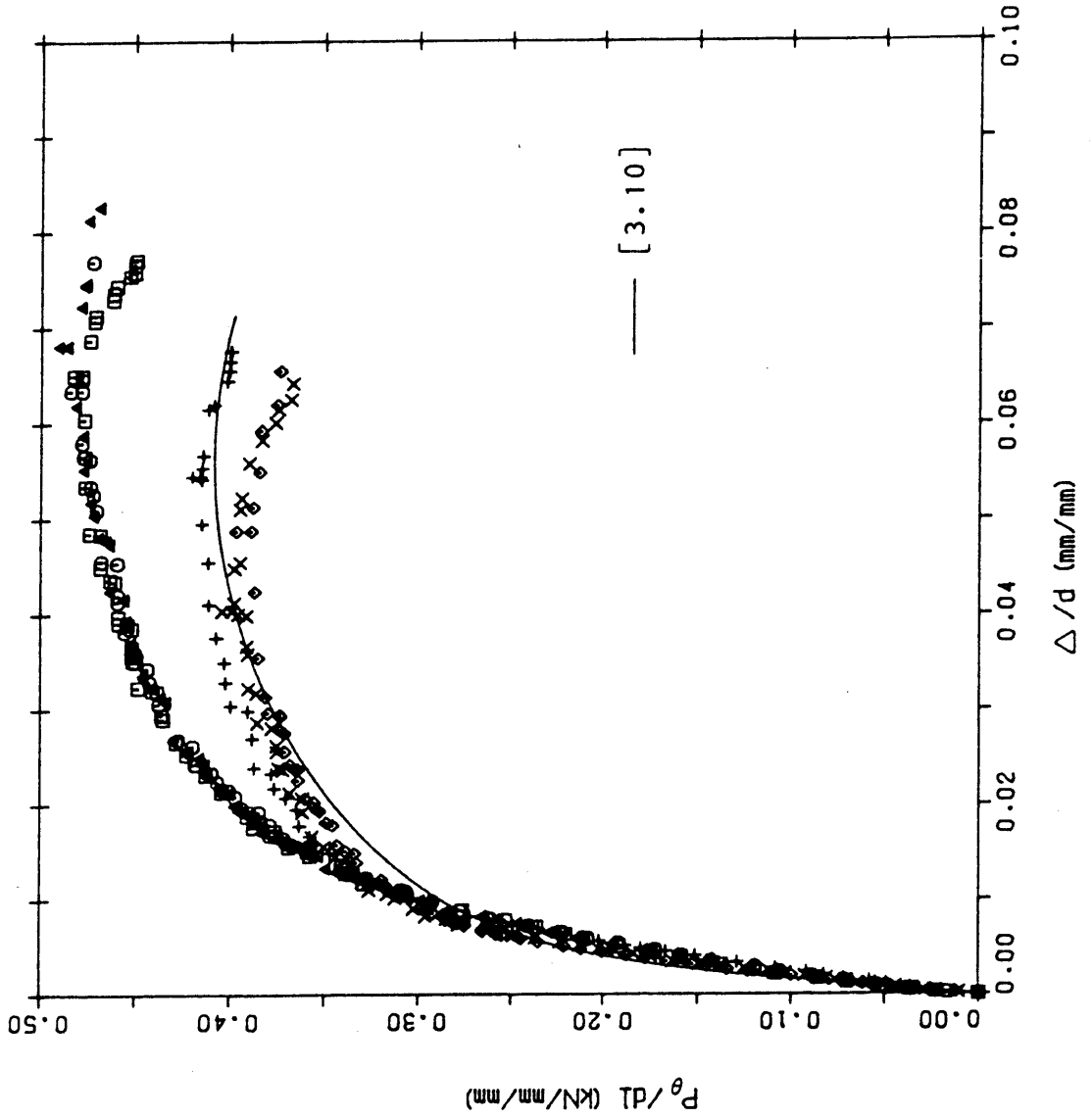


Figure 3.11 Load-Deformation Curve  $\theta = 60^\circ$

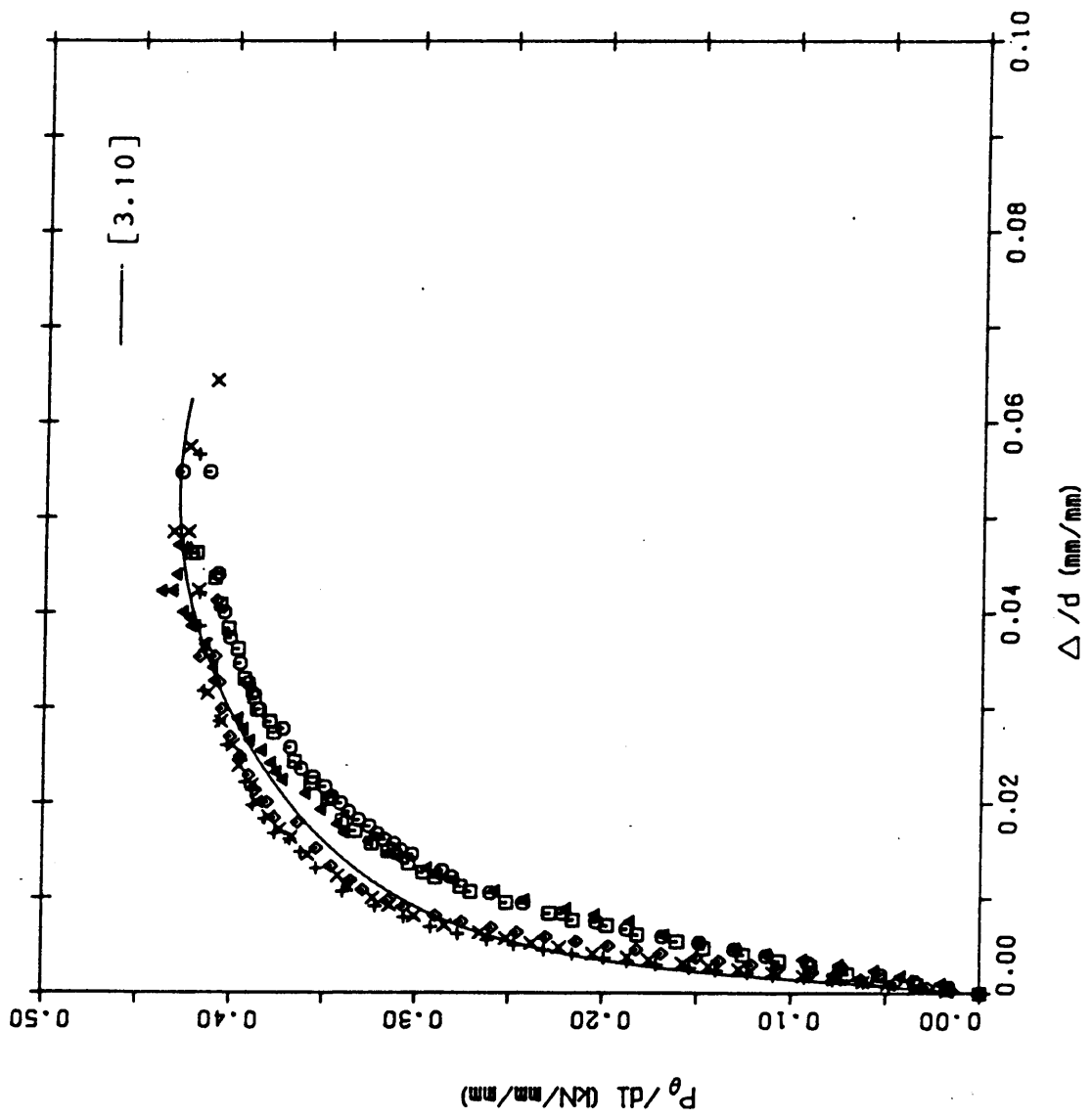


Figure 3.12 Load-Deformation Curve  $\theta = 75^\circ$

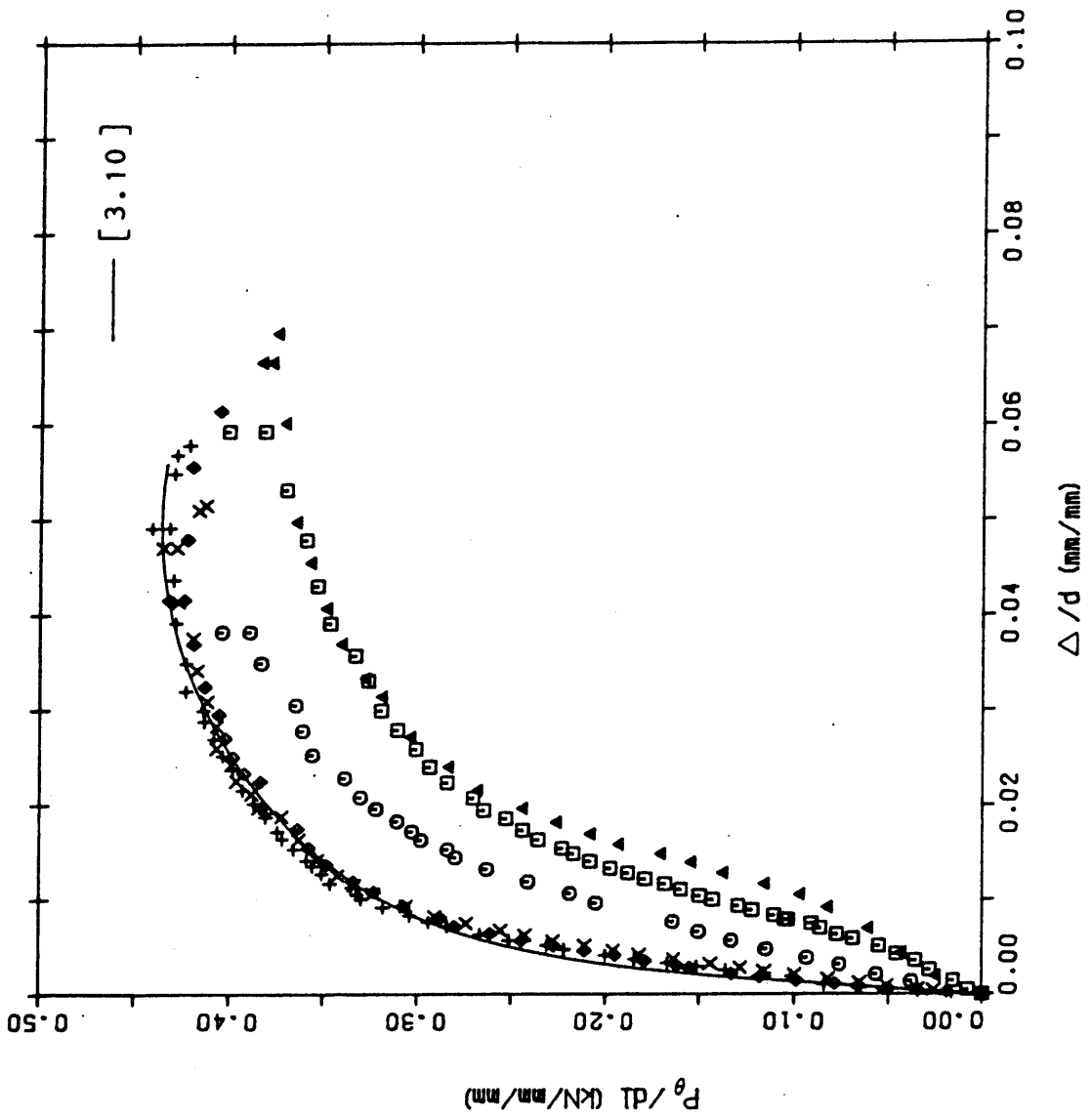


Figure 3.13 Load-Deformation Curve  $\theta = 90^\circ$



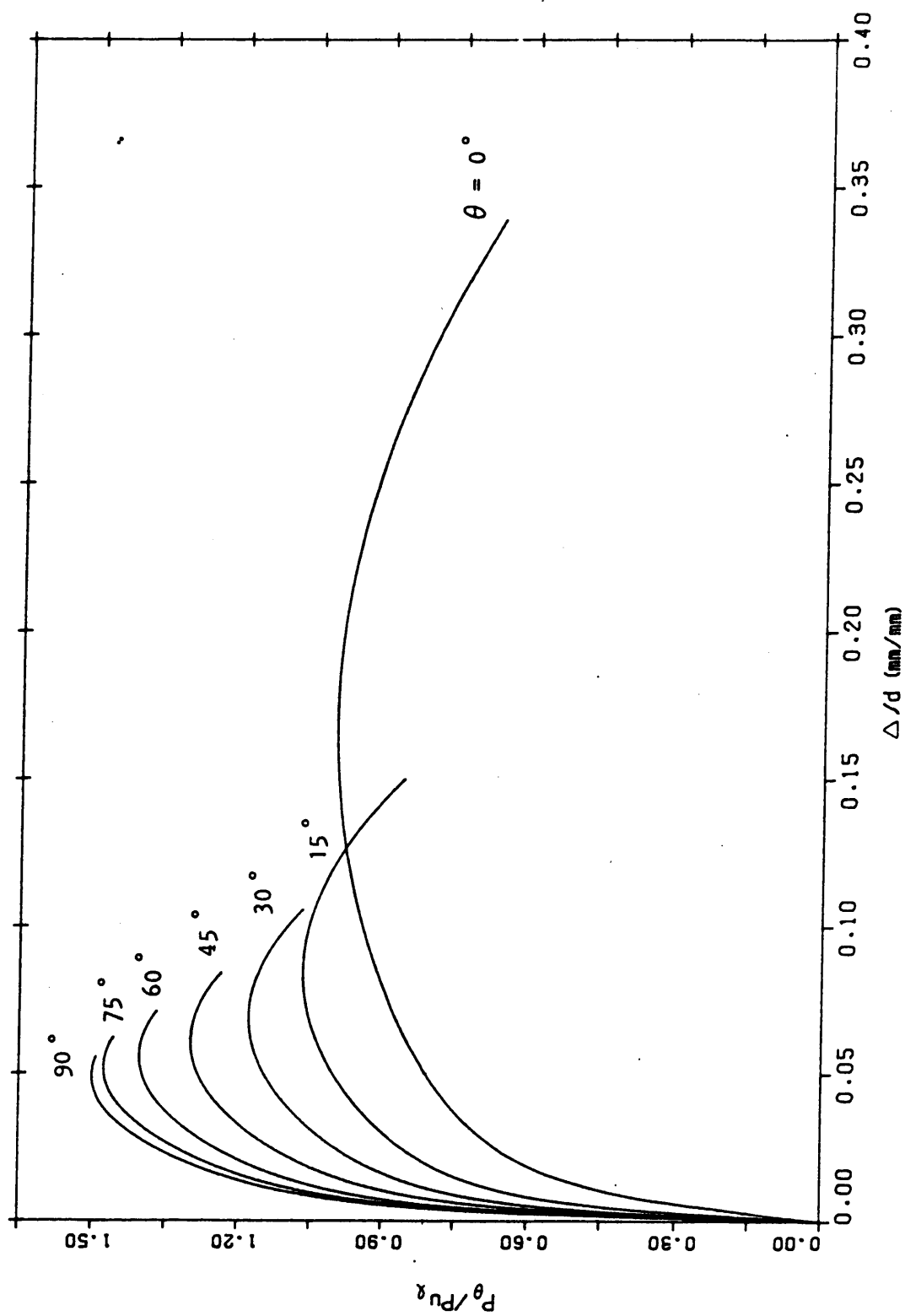


Figure 3.14 Normalized Load-Deformation Curves: Eqn. [3.11]

## 4. DISCUSSION

### 4.1 Coefficients 'C' for Different Weld Groups

The CISC Handbook of Steel Construction (CISC 1985) contains design tables for eccentric loads on weld groups. Tables are given for eight weld configurations of varying geometry and load eccentricity as shown in Fig. 4.1. The tables utilize a coefficient 'C' computed for various values of an eccentricity parameter 'a' and a geometric parameter 'k'. Values of C are used to determine the capacity of the eccentrically loaded weld group, P, as

$$[4.1] \quad P = C D L$$

where  $P$  = externally applied eccentric load, kN

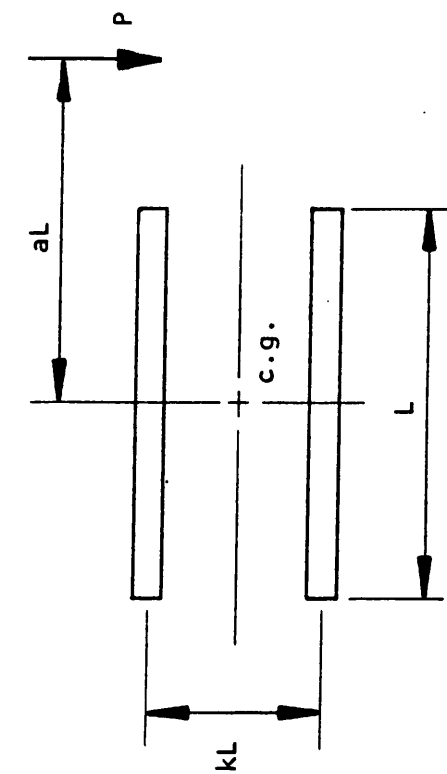
$D = d$  = fillet weld leg size, mm

$L$  = basic length of fillet weld group, mm (see Fig. 4.1)

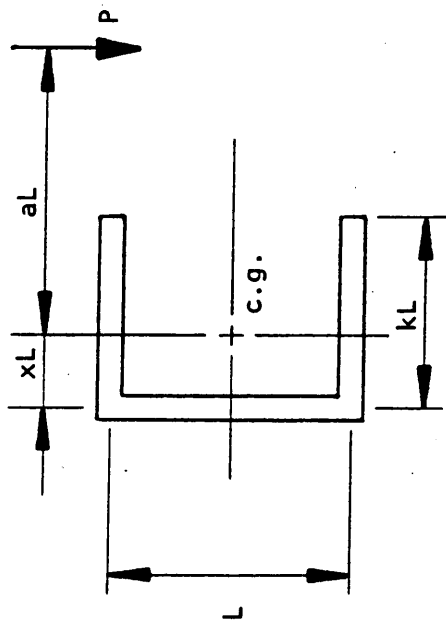
$C$  = tabulated coefficients

Design tables similar to those in the CISC Handbook were prepared for the same eight weld groups in Fig. 4.1 using a computer program to perform the iterative steps described in Chapter 2.

From the expressions developed for the load-deformation characteristics of fillet welds and using the instantaneous centre of rotation method, the ultimate strength of eccentrically loaded fillet welded connections

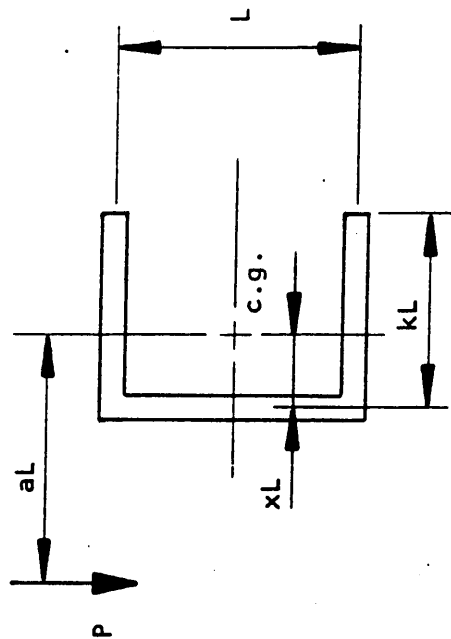


a) Two Vertical Welds



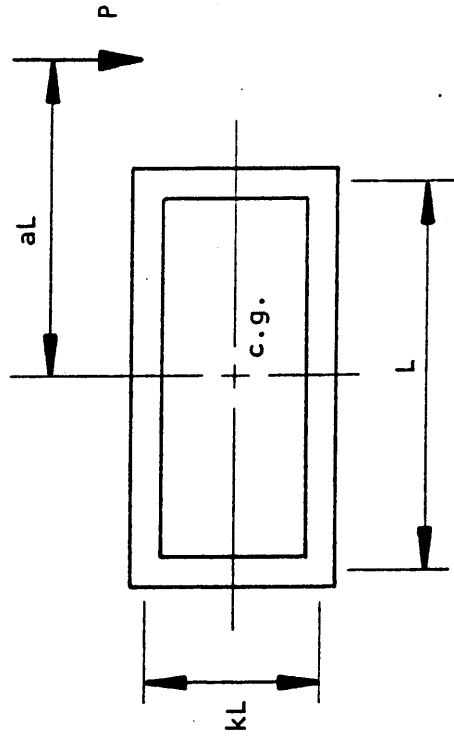
c) Channel Weld - Positive Eccentricity

b) Two Horizontal Welds

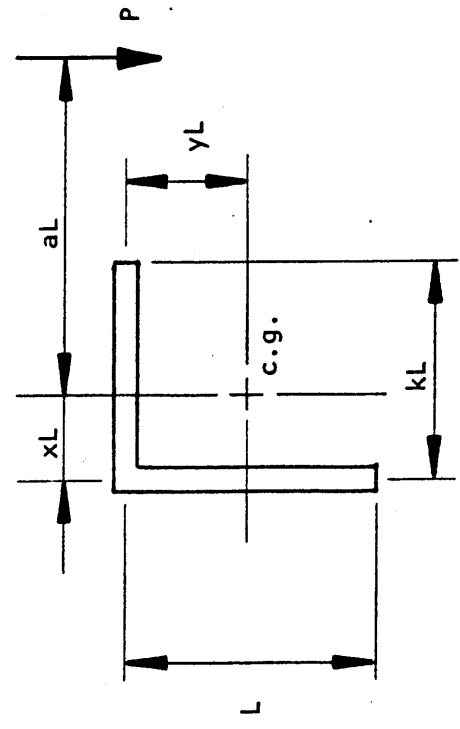


d) Channel Weld - Negative Eccentricity

Figure 4.1 Weld Group Types

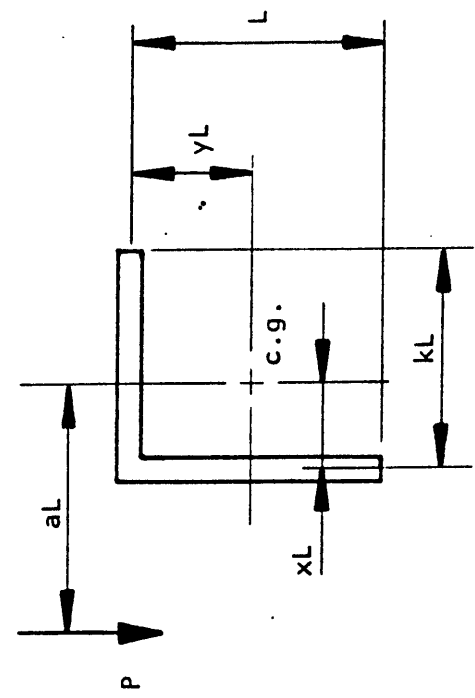


e) Vertical Box Weld



g) Angle Weld - Positive Eccentricity

f) Horizontal Box Weld



h) Angle Weld - Negative Eccentricity

Figure 4.1 continued

can be predicted. If  $d$ ,  $l$ , and  $\Delta$  in Equation [3.10] now represent the leg size, length and deformation of a fillet weld element, respectively, the resistance of the element is  $R_n = P_\theta$  and therefore, the externally applied eccentric load,  $P$ , is found using [2.12] and [2.13] for a specific weld group.

In order to compare the design tables using the method developed herein with those in the CISC Handbook, the longitudinal ultimate strength,  $Pu_\ell$ , in [3.10] is taken from CSA Standard CAN3-S16.1-M84 (CSA 1984) or from CSA Standard W59-M1984 (CSA 1984) as the factored longitudinal strength of a fillet weld loaded in shear, that is

$$[4.2] \quad Pu_\ell = V_r = 0.67 \phi_w A_w X_u$$

where

$$[4.3] \quad A_w = \frac{d}{\sqrt{2}} l = 0.7071 d l$$

Rearranging [4.2] gives

$$[4.4] \quad Vu = \frac{V_r}{dl} = 0.7071 \phi_w 0.67 X_u = 0.15236 \text{ kN/mm/mm}$$

for E480XX electrodes with  $X_u = 480$  MPa.  $Vu$  is the unit shear resistance of a longitudinal fillet weld in kN per mm of weld leg size per mm of weld length. Using [4.4] in

[3.10] gives  $P_\theta$  as

$$[4.5] \quad P_\theta = R_n = V_u d l (0.5 \sin^{1.5} \theta + 1.0) f(\rho)$$

The ultimate load computed from [2.12] and [2.13] is divided by  $d \times L$  to obtain the coefficient  $C$  for specific values of the parameters 'a' and 'k' for each weld group.

The eight weld groups were analyzed by considering the maximum deformation of the critical weld element to be either the deformation at fracture,  $\Delta_{\max} = \Delta_{fr}$ , or the deformation at ultimate load,  $\Delta_{\max} = \Delta_{pu}$ . In the latter case the effects of unloading are neglected. Tables 4.1 to 4.8 list values of  $C$  for the case when  $\Delta_{\max} = \Delta_{fr}$ , except for that noted below.

When the maximum deformation is taken as the deformation at ultimate load,  $\Delta_{\max} = \Delta_{pu}$ , the values of  $C$  are less than the values obtained when the maximum deformation at fracture is used except for the weld group consisting of two vertical welds, and even for this case values of  $C$  are less for lower values of the parameter  $k$ . The higher values of  $C$  obtained when the maximum deformation at ultimate load is used are listed in Table 4.1 for this weld group.

## 4.2 Comparison with CISC Handbook of Steel Construction

The values of  $C$  listed in Tables 4.1 to 4.8 are generally slightly lower than those tabulated in the CISC

Table 4.1 Coefficient C - Two Vertical Welds

	0.0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0.0	0.30472	0.30472	0.30472	0.30472	0.30472	0.30472	0.30472	0.30472	0.30472	0.30472	0.30472
0.10	0.30224	0.30280	0.30383	0.30361	0.30169	0.30094	0.29861	0.29806	0.29473	0.29246	0.29199
0.20	0.28795	0.28862	0.28633	0.28606	0.28336	0.28097	0.28071	0.27894	0.27672	0.27518	0.27417
0.30	0.25444	0.25430	0.25366	0.25470	0.25410	0.25338	0.25328	0.25307	0.25295	0.25152	0.25137
0.40	0.21957	0.21933	0.22044	0.22177	0.22356	0.22515	0.22657	0.22736	0.22872	0.22953	0.22966
0.50	0.18850	0.18842	0.19080	0.19379	0.19702	0.19974	0.20280	0.20499	0.20716	0.20920	0.21031
0.60	0.16417	0.16519	0.16717	0.17058	0.17430	0.17823	0.18218	0.18522	0.18838	0.19127	0.19411
0.70	0.14511	0.14556	0.14758	0.15161	0.15601	0.16085	0.16501	0.16883	0.17261	0.17596	0.17817
0.80	0.12865	0.12865	0.13216	0.13606	0.14079	0.14581	0.15037	0.15483	0.15846	0.16221	0.16560
0.90	0.11594	0.11662	0.11922	0.12318	0.12820	0.13317	0.13807	0.14238	0.14673	0.15020	0.15357
1.00	0.10513	0.10585	0.10845	0.11268	0.11740	0.12246	0.12782	0.13179	0.13614	0.13986	0.14375
1.10	0.09626	0.09699	0.09936	0.10373	0.10851	0.11359	0.11818	0.12238	0.12725	0.13106	0.13461
1.20	0.08870	0.08824	0.09192	0.09598	0.10056	0.10545	0.11012	0.11464	0.11896	0.12302	0.12684
1.30	0.08203	0.08282	0.08532	0.08905	0.09377	0.09842	0.10328	0.10785	0.11207	0.11583	0.11970
1.40	0.07621	0.07710	0.07960	0.08333	0.08788	0.09232	0.09710	0.10189	0.10583	0.10987	0.11326
1.50	0.07146	0.07210	0.07454	0.07827	0.08241	0.08704	0.09148	0.09583	0.09997	0.10395	0.10736
1.60	0.06701	0.06790	0.07022	0.07364	0.07778	0.08209	0.08647	0.09080	0.09487	0.09881	0.10232
1.70	0.06334	0.06396	0.06623	0.06952	0.07340	0.07776	0.08204	0.08630	0.09005	0.09401	0.09772
1.80	0.05981	0.06058	0.06273	0.06599	0.06977	0.07391	0.07790	0.08200	0.08590	0.08990	0.09343
1.90	0.05656	0.05734	0.05957	0.06266	0.06634	0.07038	0.07436	0.07831	0.08228	0.08579	0.08939
2.00	0.05382	0.05460	0.05665	0.05967	0.06323	0.06708	0.07109	0.07485	0.07854	0.08224	0.08561
2.10	0.05134	0.05206	0.05404	0.05698	0.06034	0.06408	0.06789	0.07169	0.07536	0.07876	0.08219
2.20	0.04900	0.04970	0.05153	0.05441	0.05772	0.06133	0.06508	0.06891	0.07243	0.07583	0.07919
2.30	0.04698	0.04771	0.04950	0.05219	0.05540	0.05879	0.06238	0.06600	0.06959	0.07290	0.07622
2.40	0.04504	0.04570	0.04752	0.05004	0.05314	0.05661	0.05999	0.06352	0.06705	0.07038	0.07364
2.50	0.04337	0.04385	0.04558	0.04804	0.05106	0.05445	0.05775	0.06134	0.06482	0.06782	0.07095
2.60	0.04166	0.04217	0.04389	0.04631	0.04914	0.05231	0.05565	0.05908	0.06244	0.06557	0.06856
2.70	0.04007	0.04063	0.04233	0.04462	0.04745	0.05064	0.05374	0.05701	0.06020	0.06338	0.06643
2.80	0.03866	0.03917	0.04074	0.04316	0.04581	0.04881	0.05202	0.05518	0.05827	0.06141	0.06433
2.90	0.03737	0.03787	0.03940	0.04157	0.04435	0.04723	0.05033	0.05336	0.05646	0.05946	0.06237
3.00	0.03613	0.03664	0.03813	0.04030	0.04267	0.04570	0.04873	0.05168	0.05473	0.05775	0.06062

Table 4.1 continued

	k									
	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
0.0	0.30472	0.30472	0.30472	0.30472	0.30472	0.30472	0.30472	0.30472	0.30472	0.30472
0.10	0.28855	0.28823	0.28841	0.28812	0.28759	0.28778	0.28724	0.28699	0.28674	0.28686
0.20	0.27166	0.27217	0.27128	0.27174	0.27227	0.27190	0.27233	0.27275	0.27280	0.27238
0.30	0.25166	0.25174	0.25272	0.25353	0.25444	0.25528	0.25610	0.25677	0.25795	0.25859
0.40	0.23150	0.23236	0.23371	0.23569	0.23711	0.23838	0.23992	0.24165	0.24281	0.24395
0.50	0.21209	0.21448	0.21661	0.21889	0.22134	0.22335	0.22506	0.22698	0.22860	0.23022
0.60	0.19576	0.19816	0.20113	0.20401	0.20683	0.20866	0.21120	0.21340	0.21548	0.21751
0.70	0.18113	0.18421	0.18710	0.19018	0.19335	0.19613	0.19876	0.20130	0.20359	0.20601
0.80	0.16788	0.17153	0.17494	0.17822	0.18174	0.18443	0.18721	0.18999	0.19274	0.19516
0.90	0.15577	0.16041	0.16398	0.16751	0.17080	0.17416	0.17698	0.17999	0.18261	0.18525
1.00	0.14892	0.15045	0.15438	0.15799	0.16131	0.16461	0.16771	0.17065	0.17364	0.17642
1.10	0.13827	0.14179	0.14555	0.14917	0.15293	0.15622	0.15932	0.16249	0.16554	0.16808
1.20	0.13013	0.13376	0.13767	0.14140	0.14507	0.14833	0.15153	0.15492	0.15771	0.16065
1.30	0.12301	0.12663	0.13062	0.13430	0.13779	0.14119	0.14461	0.14787	0.15078	0.15392
1.40	0.11682	0.12016	0.12417	0.12775	0.13142	0.13476	0.13805	0.14129	0.14452	0.14757
1.50	0.11136	0.11433	0.11824	0.12200	0.12543	0.12883	0.13215	0.13532	0.13848	0.14155
1.60	0.10580	0.10812	0.11286	0.11645	0.12005	0.12340	0.12667	0.12980	0.13298	0.13593
1.70	0.10119	0.10439	0.10793	0.11140	0.11516	0.11842	0.12162	0.12485	0.12780	0.13090
1.80	0.09666	0.10002	0.10351	0.10687	0.11038	0.11382	0.11707	0.12012	0.12326	0.12616
1.90	0.09271	0.09588	0.09915	0.10278	0.10615	0.10946	0.11270	0.11570	0.11887	0.12169
2.00	0.08806	0.09221	0.09538	0.09887	0.10213	0.10541	0.10867	0.11170	0.11465	0.11773
2.10	0.08542	0.08863	0.09187	0.09523	0.09850	0.10182	0.10490	0.10800	0.11087	0.11372
2.20	0.08238	0.08518	0.08851	0.09177	0.09500	0.09814	0.10129	0.10428	0.10724	0.11022
2.30	0.07944	0.08219	0.08532	0.08865	0.09187	0.09495	0.09805	0.10111	0.10391	0.10674
2.40	0.07658	0.07962	0.08251	0.08563	0.08887	0.09195	0.09503	0.09780	0.10076	0.10351
2.50	0.07394	0.07680	0.07974	0.08287	0.08596	0.08913	0.09208	0.09492	0.09765	0.10055
2.60	0.07159	0.07446	0.07718	0.08029	0.08329	0.08642	0.08929	0.09214	0.09492	0.09761
2.70	0.06926	0.07210	0.07486	0.07783	0.08089	0.08384	0.08663	0.08940	0.09228	0.09481
2.80	0.06717	0.06991	0.07254	0.07561	0.07847	0.08138	0.08417	0.08694	0.08976	0.09227
2.90	0.06516	0.06783	0.07051	0.07339	0.07625	0.07914	0.08180	0.08457	0.08735	0.08988
3.00	0.06330	0.06590	0.06839	0.07131	0.07413	0.07680	0.07955	0.08232	0.08501	0.08762



Table 4.2 Coefficient C - Two Horizontal Welds

k	h										
	0.0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0.0	0.45708	0.45708	0.45708	0.45708	0.45708	0.45708	0.45708	0.45708	0.45708	0.45708	0.45708
0.10	0.35548	0.35579	0.35788	0.36244	0.36945	0.40872	0.41788	0.42587	0.43187	0.43854	0.43982
0.20	0.25065	0.25339	0.26120	0.27278	0.28854	0.34474	0.36188	0.37628	0.38646	0.39808	0.40583
0.30	0.24142	0.24440	0.25135	0.26194	0.27445	0.28952	0.30775	0.32810	0.33989	0.35422	0.36701
0.40	0.20441	0.20643	0.21370	0.22268	0.23493	0.24805	0.26283	0.27894	0.28605	0.31243	0.32785
0.50	0.17584	0.17821	0.18415	0.19286	0.20354	0.21550	0.22852	0.24203	0.25768	0.27473	0.28033
0.60	0.15377	0.15529	0.16118	0.16936	0.17884	0.18982	0.20177	0.21457	0.22785	0.24271	0.25448
0.70	0.13622	0.13811	0.14332	0.15056	0.15911	0.16894	0.18001	0.19181	0.20350	0.21760	0.23023
0.80	0.12189	0.12379	0.12851	0.13511	0.14286	0.15240	0.16228	0.17273	0.18423	0.19614	0.20788
0.90	0.11018	0.11177	0.11619	0.12247	0.12874	0.13647	0.14557	0.15718	0.16750	0.17848	0.18950
1.00	0.10084	0.10219	0.10520	0.11175	0.11908	0.12651	0.13521	0.14418	0.15404	0.16383	0.17400
1.10	0.09244	0.09391	0.09745	0.10282	0.10942	0.11672	0.12458	0.13333	0.14185	0.15121	0.16078
1.20	0.08552	0.08686	0.09044	0.09521	0.10098	0.10818	0.11587	0.12358	0.13157	0.14031	0.14918
1.30	0.07947	0.08072	0.08391	0.08850	0.09438	0.10084	0.10788	0.11505	0.12301	0.13101	0.13950
1.40	0.07417	0.07519	0.07845	0.08288	0.08817	0.09412	0.10082	0.10788	0.11513	0.12272	0.13048
1.50	0.06980	0.07053	0.07355	0.07789	0.08298	0.08841	0.09479	0.10130	0.10804	0.11528	0.12286
1.60	0.06542	0.06634	0.06914	0.07313	0.07786	0.08339	0.08930	0.09553	0.10207	0.10885	0.11566
1.70	0.06175	0.06279	0.06531	0.06914	0.07385	0.07939	0.08442	0.09037	0.09668	0.10287	0.10972
1.80	0.05854	0.05940	0.06180	0.06541	0.06981	0.07457	0.08013	0.08573	0.09168	0.09778	0.10417
1.90	0.05559	0.05648	0.05877	0.06224	0.06632	0.07105	0.07619	0.08138	0.08729	0.09316	0.09905
2.00	0.05292	0.05384	0.05611	0.05925	0.06310	0.06786	0.07285	0.07777	0.08312	0.08873	0.09438
2.10	0.05044	0.05135	0.05352	0.05650	0.06024	0.06488	0.06925	0.07420	0.07945	0.08481	0.09024
2.20	0.04829	0.04914	0.05114	0.05411	0.05755	0.06177	0.06527	0.07108	0.07556	0.08110	0.08655
2.30	0.04621	0.04705	0.04890	0.05188	0.05530	0.05924	0.06352	0.06810	0.07295	0.07787	0.08292
2.40	0.04440	0.04515	0.04707	0.04981	0.05310	0.05680	0.06108	0.06544	0.06995	0.07470	0.07957
2.50	0.04258	0.04341	0.04523	0.04791	0.05107	0.05468	0.05883	0.06300	0.06736	0.07185	0.07681
2.60	0.04101	0.04189	0.04345	0.04610	0.04925	0.05271	0.05646	0.06089	0.06498	0.06932	0.07388
2.70	0.03958	0.04028	0.04185	0.04444	0.04740	0.05085	0.05438	0.05846	0.06253	0.06682	0.07122
2.80	0.03825	0.03881	0.04054	0.04293	0.04678	0.04895	0.05256	0.05643	0.06048	0.06484	0.06878
2.90	0.03691	0.03758	0.03813	0.04138	0.04430	0.04742	0.05088	0.05458	0.05848	0.06239	0.06635
3.00	0.03573	0.03636	0.03781	0.04014	0.04280	0.04588	0.04917	0.05275	0.05657	0.06040	0.06435

Table 4.2 continued

a	k									
	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
0.0	0.45708	0.45708	0.45708	0.45708	0.45708	0.45708	0.45708	0.45708	0.45708	0.45708
0.10	0.44240	0.44430	0.44615	0.44800	0.44735	0.44851	0.44854	0.45054	0.45150	0.45108
0.20	0.41312	0.41928	0.42411	0.42780	0.43093	0.43432	0.43852	0.43842	0.43974	0.44135
0.30	0.37787	0.38623	0.39332	0.40057	0.40636	0.41205	0.41548	0.41871	0.42208	0.42530
0.40	0.33985	0.35325	0.36093	0.37029	0.37820	0.38553	0.39187	0.39787	0.40255	0.40718
0.50	0.30551	0.31786	0.32889	0.34128	0.34857	0.35857	0.36591	0.37358	0.38075	0.38605
0.60	0.27307	0.28727	0.30005	0.31272	0.32321	0.33204	0.34184	0.34853	0.35587	0.36375
0.70	0.24589	0.25812	0.27355	0.28453	0.29608	0.30723	0.31828	0.32647	0.33402	0.34251
0.80	0.22041	0.23484	0.24846	0.25173	0.27251	0.28394	0.29409	0.30373	0.31183	0.32180
0.90	0.20120	0.21480	0.22734	0.24008	0.25154	0.26218	0.27338	0.28338	0.28298	0.30148
1.00	0.18497	0.19558	0.20782	0.22034	0.23192	0.24258	0.25287	0.26378	0.27391	0.28324
1.10	0.17061	0.18063	0.19197	0.20368	0.21501	0.22573	0.23594	0.24618	0.25525	0.26509
1.20	0.15834	0.16752	0.17807	0.18818	0.19813	0.20836	0.21888	0.22885	0.23885	0.24754
1.30	0.14774	0.15551	0.16554	0.17498	0.18551	0.19587	0.20572	0.21572	0.22528	0.23422
1.40	0.13848	0.14551	0.15557	0.16435	0.17341	0.18372	0.19244	0.20253	0.21089	0.21878
1.50	0.13045	0.13821	0.14584	0.15435	0.16251	0.17210	0.18087	0.19048	0.19894	0.20774
1.60	0.12280	0.13027	0.13782	0.14558	0.15342	0.16147	0.17049	0.17870	0.18813	0.19604
1.70	0.11640	0.12344	0.13049	0.13780	0.14476	0.15287	0.16107	0.16955	0.17818	0.18644
1.80	0.11054	0.11755	0.12385	0.13076	0.13780	0.14521	0.15273	0.16093	0.16895	0.17630
1.90	0.10528	0.11142	0.11794	0.12440	0.13104	0.13775	0.14485	0.15240	0.16055	0.16785
2.00	0.10025	0.10611	0.11274	0.11872	0.12479	0.13173	0.13844	0.14536	0.15225	0.16022
2.10	0.09504	0.10175	0.10757	0.11359	0.11977	0.12575	0.13192	0.13837	0.14486	0.15228
2.20	0.09174	0.09750	0.10303	0.10872	0.11457	0.12007	0.12635	0.13240	0.13908	0.14559
2.30	0.08805	0.09348	0.09867	0.10433	0.10991	0.11540	0.12125	0.12689	0.13295	0.13934
2.40	0.08474	0.08970	0.09507	0.10013	0.10555	0.11112	0.11633	0.12181	0.12800	0.13358
2.50	0.08148	0.08635	0.09130	0.09645	0.10142	0.10655	0.11195	0.11735	0.12255	0.12849
2.60	0.07849	0.08322	0.08801	0.09282	0.09795	0.10295	0.10792	0.11322	0.11810	0.12357
2.70	0.07578	0.08021	0.08501	0.08967	0.09441	0.09927	0.10431	0.10900	0.11400	0.11930
2.80	0.07320	0.07750	0.08195	0.08654	0.09121	0.09583	0.10071	0.10543	0.11031	0.11527
2.90	0.07075	0.07509	0.07935	0.08385	0.08817	0.09282	0.09705	0.10189	0.10658	0.11119
3.00	0.06850	0.07255	0.07690	0.08107	0.08545	0.08988	0.09413	0.09855	0.10295	0.10751

Table 4.3 Coefficient C - Channel Weld-Positive Eccentricity

a	k										
	0.0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0.0	0.18236	0.19807	0.24378	0.28949	0.33520	0.38090	0.42661	0.47232	0.51803	0.56374	0.60945
0.10	0.18112	0.19930	0.23033	0.27203	0.31557	0.35986	0.40363	0.44833	0.49323	0.53828	0.58275
0.20	0.14388	0.17857	0.21842	0.25555	0.28565	0.31675	0.34832	0.42084	0.46236	0.50484	0.54688
0.30	0.12722	0.15953	0.19368	0.22908	0.26503	0.30250	0.34019	0.37857	0.41705	0.45552	0.49333
0.40	0.10879	0.13881	0.16870	0.20051	0.23317	0.26613	0.30025	0.33434	0.36870	0.40378	0.44322
0.50	0.09475	0.12022	0.14722	0.17501	0.20411	0.23381	0.26453	0.29557	0.32782	0.36089	0.39588
0.60	0.08208	0.10487	0.12857	0.15385	0.17959	0.20679	0.23433	0.26295	0.29312	0.32400	0.35535
0.70	0.07255	0.09235	0.11385	0.13653	0.15968	0.18388	0.20945	0.23582	0.26358	0.29226	0.32186
0.80	0.06433	0.08204	0.10154	0.12183	0.14303	0.16520	0.18889	0.21277	0.23838	0.26458	0.29230
0.90	0.05787	0.07389	0.09128	0.10981	0.12968	0.14992	0.17128	0.19400	0.21728	0.24168	0.26806
1.00	0.05257	0.06705	0.08280	0.09880	0.11748	0.13682	0.15678	0.17740	0.19852	0.22233	0.24682
1.10	0.04813	0.06122	0.07574	0.09148	0.10815	0.12602	0.14402	0.16372	0.18427	0.20510	0.22850
1.20	0.04436	0.05638	0.06978	0.08431	0.09980	0.11628	0.13354	0.15175	0.17088	0.19174	0.21268
1.30	0.04102	0.05227	0.06450	0.07808	0.09275	0.10788	0.12403	0.14148	0.15972	0.17882	0.19888
1.40	0.03811	0.04868	0.06005	0.07288	0.08644	0.10084	0.11614	0.13200	0.14911	0.16728	0.18653
1.50	0.03573	0.04545	0.05622	0.06800	0.08079	0.09455	0.10908	0.12415	0.14026	0.15750	0.17538
1.60	0.03350	0.04258	0.05287	0.06386	0.07480	0.08617	0.10266	0.11700	0.13217	0.14856	0.16570
1.70	0.03157	0.04026	0.04983	0.06025	0.07173	0.08405	0.09704	0.11080	0.12515	0.14078	0.15882
1.80	0.02981	0.03802	0.04703	0.05685	0.06680	0.07851	0.09196	0.10888	0.11872	0.13443	0.14889
1.90	0.02828	0.03607	0.04459	0.05408	0.06439	0.07653	0.08748	0.09981	0.11288	0.12686	0.14171
2.00	0.02691	0.03424	0.04238	0.05140	0.06135	0.07198	0.08309	0.09508	0.10767	0.12081	0.13518
2.10	0.02557	0.03285	0.04035	0.04901	0.05847	0.06871	0.07935	0.09065	0.10276	0.11547	0.12915
2.20	0.02450	0.03113	0.03855	0.04683	0.05585	0.06575	0.07685	0.08868	0.09848	0.11044	0.12372
2.30	0.02349	0.02981	0.03693	0.04483	0.05346	0.06278	0.07275	0.08321	0.09422	0.10626	0.11846
2.40	0.02252	0.02859	0.03536	0.04286	0.05135	0.06030	0.06984	0.07981	0.09053	0.10186	0.11371
2.50	0.02168	0.02740	0.03398	0.04131	0.04928	0.05784	0.06701	0.07701	0.08702	0.09808	0.10924
2.60	0.02083	0.02637	0.03265	0.03970	0.04746	0.05583	0.06480	0.07400	0.08370	0.09444	0.10545
2.70	0.02004	0.02546	0.03147	0.03821	0.04574	0.05384	0.06230	0.07127	0.08081	0.09103	0.10171
2.80	0.01934	0.02448	0.03040	0.03688	0.04413	0.05192	0.06003	0.06887	0.07805	0.08782	0.09841
2.90	0.01869	0.02370	0.02933	0.03565	0.04259	0.05014	0.05812	0.06661	0.07542	0.08504	0.09504
3.00	0.01805	0.02287	0.02835	0.03444	0.04125	0.04846	0.05621	0.06436	0.07303	0.08216	0.09200
x	0.0	0.00433	0.02857	0.05525	0.08889	0.12500	0.16364	0.20417	0.24515	0.28929	0.33333

Table 4.3 continued

$\alpha$	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
0.0	0.65516	0.70086	0.74857	0.79228	0.83789	0.88370	0.92940	0.97511	1.02082	1.06653
0.10	0.62774	0.67243	0.71824	0.76238	0.80723	0.85086	0.89585	0.94188	0.98716	1.03076
0.20	0.58881	0.63148	0.67525	0.71711	0.76049	0.80289	0.84876	0.89973	0.93241	0.97656
0.30	0.53446	0.57880	0.61485	0.65554	0.69653	0.73763	0.77781	0.81866	0.86244	0.90374
0.40	0.48010	0.51738	0.55531	0.59396	0.63609	0.67418	0.71483	0.75505	0.79468	0.83464
0.50	0.43080	0.46884	0.50188	0.53872	0.57721	0.61486	0.65284	0.68183	0.72867	0.78000
0.60	0.38730	0.42181	0.45506	0.49048	0.52815	0.56146	0.59852	0.63660	0.67376	0.71129
0.70	0.35174	0.38321	0.41829	0.44876	0.48272	0.51672	0.55195	0.58753	0.62425	0.66055
0.80	0.32094	0.35025	0.38122	0.41280	0.44552	0.47758	0.51172	0.54508	0.58011	0.61525
0.90	0.29551	0.32306	0.35159	0.38185	0.41142	0.44230	0.47525	0.50762	0.53989	0.57504
1.00	0.27248	0.29810	0.32552	0.35344	0.38247	0.41209	0.44237	0.47348	0.50497	0.53749
1.10	0.25216	0.27732	0.30289	0.32839	0.35718	0.38515	0.41371	0.44389	0.47385	0.50483
1.20	0.23536	0.25857	0.28287	0.30830	0.33444	0.36121	0.38753	0.41878	0.44451	0.47468
1.30	0.22007	0.24184	0.26490	0.28885	0.31328	0.33817	0.36506	0.39180	0.41979	0.44744
1.40	0.20850	0.22732	0.24847	0.27152	0.29514	0.31995	0.34441	0.37012	0.39612	0.42341
1.50	0.19443	0.21453	0.23472	0.25528	0.27872	0.30226	0.32840	0.35018	0.37572	0.40123
1.60	0.18355	0.20242	0.22204	0.24251	0.26373	0.28546	0.30847	0.33224	0.35533	0.38123
1.70	0.17408	0.19178	0.21040	0.23000	0.25004	0.27103	0.29316	0.31850	0.33818	0.36221
1.80	0.16511	0.18208	0.19987	0.21855	0.23791	0.25793	0.27877	0.30051	0.32282	0.34824
1.90	0.15721	0.17356	0.19038	0.20814	0.22670	0.24643	0.26818	0.28848	0.30807	0.33026
2.00	0.14989	0.16535	0.18174	0.19875	0.21652	0.23501	0.25449	0.27407	0.29494	0.31830
2.10	0.14329	0.15832	0.17367	0.18982	0.20711	0.22495	0.24340	0.26274	0.28260	0.30285
2.20	0.13714	0.15141	0.16647	0.18220	0.19871	0.21612	0.23364	0.25219	0.27122	0.29102
2.30	0.13151	0.14518	0.15945	0.17502	0.19075	0.20736	0.22444	0.24230	0.26070	0.27982
2.40	0.12635	0.13967	0.15355	0.16841	0.18341	0.19833	0.21618	0.23315	0.25127	0.26961
2.50	0.12152	0.13450	0.14773	0.16189	0.17655	0.19164	0.20795	0.22504	0.24217	0.25975
2.60	0.11710	0.12953	0.14250	0.15593	0.17015	0.18519	0.20053	0.21686	0.23379	0.25103
2.70	0.11306	0.12494	0.13752	0.15078	0.16436	0.17854	0.19350	0.20835	0.22562	0.24243
2.80	0.10894	0.12054	0.13272	0.14551	0.15882	0.17281	0.18734	0.20262	0.21842	0.23466
2.90	0.10544	0.11673	0.12844	0.14056	0.15359	0.16715	0.18136	0.19623	0.21130	0.22707
3.00	0.10199	0.11283	0.12434	0.13624	0.14885	0.16177	0.17570	0.18991	0.20470	0.22014
$\infty$	0.37812	0.42353	0.46944	0.51578	0.56250	0.60952	0.65682	0.70435	0.75208	0.80000

Table 4.4 Coefficient C - Channel Weld-Negative Eccentricity

	K										
	0.0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0.0	0.15236	0.19807	0.24378	0.28948	0.33520	0.38090	0.42661	0.47232	0.51803	0.56374	0.60945
0.10	0.15112	0.19005	0.23078	0.27222	0.31380	0.35534	0.39675	0.43868	0.47927	0.52028	0.56168
0.20	0.14398	0.17774	0.21451	0.25178	0.28857	0.32503	0.36225	0.40381	0.44150	0.47801	0.51550
0.30	0.12722	0.15811	0.19080	0.22363	0.25885	0.29240	0.32815	0.36388	0.39839	0.43327	0.46762
0.40	0.10878	0.13771	0.16822	0.19538	0.22584	0.25788	0.28988	0.32116	0.35467	0.38728	0.42114
0.50	0.08475	0.11899	0.14421	0.17077	0.19760	0.22580	0.25593	0.28559	0.31635	0.34638	0.37732
0.60	0.06208	0.10365	0.12528	0.14877	0.17371	0.19943	0.22581	0.25398	0.28252	0.31173	0.34073
0.70	0.07285	0.09086	0.11048	0.13168	0.15481	0.17880	0.20332	0.22937	0.25584	0.28281	0.31777
0.80	0.06433	0.08102	0.09847	0.11818	0.13934	0.16105	0.18473	0.20837	0.23353	0.25854	0.28463
0.90	0.05787	0.07252	0.08881	0.10705	0.12630	0.14720	0.16853	0.19105	0.21428	0.23811	0.26272
1.00	0.05267	0.06571	0.08106	0.09763	0.11572	0.13470	0.15477	0.17579	0.19776	0.22039	0.24344
1.10	0.04813	0.06035	0.07420	0.08957	0.10632	0.12447	0.14288	0.16289	0.18330	0.20482	0.22705
1.20	0.04435	0.05547	0.06828	0.08256	0.09841	0.11500	0.13280	0.15155	0.17088	0.19087	0.21212
1.30	0.04102	0.05150	0.06326	0.07686	0.09182	0.10723	0.12389	0.14160	0.15975	0.17883	0.19880
1.40	0.03811	0.04782	0.05810	0.07183	0.08562	0.10031	0.11690	0.13249	0.15006	0.16785	0.18715
1.50	0.03573	0.04490	0.05540	0.06726	0.08012	0.09412	0.10907	0.12451	0.14112	0.15829	0.17641
1.60	0.03350	0.04216	0.05210	0.06326	0.07538	0.08873	0.10279	0.11770	0.13218	0.14869	0.16588
1.70	0.03187	0.03957	0.04817	0.05861	0.07122	0.08377	0.09715	0.11139	0.12616	0.14180	0.15814
1.80	0.02991	0.03755	0.04647	0.05641	0.06762	0.07951	0.09203	0.10577	0.11997	0.13443	0.15029
1.90	0.02828	0.03559	0.04412	0.05345	0.06403	0.07560	0.08753	0.10040	0.11406	0.12812	0.14312
2.00	0.02691	0.03383	0.04200	0.05081	0.06107	0.07207	0.08353	0.09584	0.10864	0.12228	0.13668
2.10	0.02567	0.03222	0.04004	0.04868	0.05828	0.06870	0.07963	0.09152	0.10374	0.11695	0.13073
2.20	0.02450	0.03083	0.03825	0.04647	0.05570	0.06569	0.07625	0.08754	0.09947	0.11199	0.12538
2.30	0.02349	0.02951	0.03662	0.04451	0.05339	0.06284	0.07306	0.08394	0.09517	0.10729	0.12029
2.40	0.02252	0.02827	0.03506	0.04275	0.05120	0.06029	0.07013	0.08039	0.09141	0.10292	0.11536
2.50	0.02168	0.02711	0.03369	0.04103	0.04921	0.05792	0.06735	0.07738	0.08801	0.09911	0.11061
2.60	0.02083	0.02612	0.03245	0.03948	0.04738	0.05579	0.06489	0.07453	0.08474	0.09551	0.10702
2.70	0.02004	0.02514	0.03123	0.03808	0.04565	0.05366	0.06251	0.07185	0.08158	0.09204	0.10309
2.80	0.01934	0.02429	0.03020	0.03670	0.04406	0.05190	0.06033	0.06929	0.07878	0.08892	0.09965
2.90	0.01869	0.02347	0.02912	0.03545	0.04256	0.05008	0.05832	0.06701	0.07617	0.08580	0.09632
3.00	0.01806	0.02265	0.02816	0.03433	0.04112	0.04854	0.05644	0.06485	0.07362	0.08301	0.09313
X	0.0	0.06833	0.02857	0.05625	0.08889	0.12500	0.16364	0.20417	0.24615	0.28929	0.33333

Table 4.4 continued

	K									
	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
0.0	0.85515	0.70086	0.74857	0.79228	0.83798	0.88370	0.92940	0.97511	1.02082	1.06653
0.10	0.60160	0.64260	0.68342	0.72485	0.76452	0.80451	0.84586	0.88505	0.92507	0.96908
0.20	0.55531	0.59294	0.63171	0.67028	0.70836	0.74793	0.78790	0.82439	0.86413	0.90231
0.30	0.50313	0.53932	0.57849	0.61393	0.65010	0.68800	0.72821	0.76378	0.80273	0.84043
0.40	0.45489	0.48964	0.52475	0.55948	0.59374	0.62948	0.66458	0.70036	0.73858	0.77863
0.50	0.40908	0.44214	0.47359	0.50768	0.54184	0.57648	0.61217	0.64806	0.68127	0.71818
0.60	0.37082	0.40178	0.43276	0.46474	0.49705	0.53034	0.56311	0.59818	0.63162	0.66830
0.70	0.33814	0.36780	0.39745	0.42791	0.45944	0.49035	0.52281	0.55498	0.58810	0.62143
0.80	0.31126	0.33897	0.36728	0.39595	0.42578	0.45578	0.48827	0.51733	0.54995	0.58181
0.90	0.28821	0.31368	0.34085	0.36825	0.39536	0.42503	0.45448	0.48420	0.51475	0.54584
1.00	0.26750	0.29220	0.31794	0.34378	0.37014	0.39736	0.42570	0.45448	0.48333	0.51328
1.10	0.24943	0.27308	0.29703	0.32184	0.34705	0.37322	0.40014	0.42783	0.45555	0.48438
1.20	0.23387	0.25576	0.27873	0.30245	0.32678	0.35181	0.37728	0.40389	0.43084	0.45779
1.30	0.21933	0.24083	0.26250	0.28481	0.30787	0.33203	0.35862	0.38178	0.40759	0.43428
1.40	0.20686	0.22698	0.24775	0.26919	0.29132	0.31430	0.33813	0.36168	0.38696	0.41264
1.50	0.19502	0.21480	0.23448	0.25541	0.27622	0.29864	0.32127	0.34398	0.36815	0.39203
1.60	0.18491	0.20352	0.22284	0.24225	0.26292	0.28376	0.30651	0.32758	0.35074	0.37436
1.70	0.17586	0.19319	0.21185	0.23073	0.25015	0.27041	0.29118	0.31291	0.33486	0.35734
1.80	0.16704	0.18428	0.20181	0.22008	0.23881	0.25810	0.27836	0.29910	0.32027	0.34251
1.90	0.15920	0.17537	0.19281	0.21029	0.22850	0.24714	0.26658	0.28646	0.30682	0.32815
2.00	0.15228	0.16777	0.18429	0.20122	0.21845	0.23644	0.25547	0.27483	0.29450	0.31493
2.10	0.14628	0.16076	0.17653	0.19281	0.20980	0.22700	0.24531	0.26377	0.28258	0.30292
2.20	0.13927	0.15418	0.16939	0.18533	0.20135	0.21841	0.23556	0.25366	0.27241	0.29147
2.30	0.13359	0.14816	0.16268	0.17793	0.19374	0.21009	0.22695	0.24414	0.26225	0.28066
2.40	0.12830	0.14235	0.15644	0.17119	0.18677	0.20217	0.21881	0.23580	0.25302	0.27105
2.50	0.12358	0.13689	0.15081	0.16511	0.18011	0.19533	0.21128	0.22727	0.24406	0.26165
2.60	0.11903	0.13192	0.14549	0.15940	0.17381	0.18846	0.20380	0.21989	0.23613	0.25303
2.70	0.11510	0.12734	0.14028	0.15387	0.16783	0.18244	0.19728	0.21257	0.22873	0.24494
2.80	0.11090	0.12283	0.13563	0.14894	0.16240	0.17654	0.19075	0.20588	0.22114	0.23746
2.90	0.10720	0.11892	0.13110	0.14386	0.15707	0.17073	0.18507	0.19948	0.21442	0.22893
3.00	0.10378	0.11505	0.12690	0.13954	0.15246	0.16565	0.17928	0.19358	0.20807	0.22323
X	0.37812	0.42353	0.46844	0.51579	0.56250	0.60852	0.65682	0.70435	0.75208	0.80000

Table 4.5 Coefficient C - Vertical Box Weld

	k										
	0.0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0.0	0.30472	0.35043	0.39514	0.44165	0.48755	0.53327	0.57887	0.62468	0.67038	0.71610	0.76181
0.10	0.30224	0.34047	0.37845	0.41880	0.45719	0.49371	0.52972	0.56595	0.60246	0.63878	0.67488
0.20	0.28795	0.32084	0.35558	0.39229	0.42875	0.46509	0.50106	0.53745	0.57428	0.61141	0.64884
0.30	0.25444	0.28523	0.31810	0.35137	0.38573	0.41986	0.45320	0.48688	0.52036	0.55441	0.58844
0.40	0.21957	0.24807	0.27817	0.30870	0.34064	0.37347	0.40602	0.43887	0.47428	0.50844	0.54471
0.50	0.18950	0.21428	0.24133	0.27053	0.30061	0.33119	0.36258	0.39448	0.42645	0.45859	0.49313
0.60	0.15417	0.18684	0.21138	0.23844	0.26539	0.29506	0.32523	0.35494	0.38528	0.41719	0.44925
0.70	0.14511	0.16487	0.18707	0.21163	0.23811	0.26531	0.29309	0.32175	0.35062	0.38027	0.41078
0.80	0.12885	0.14683	0.16700	0.18964	0.21431	0.24014	0.26822	0.29863	0.32860	0.35872	0.38816
0.90	0.11594	0.13183	0.15055	0.17167	0.19475	0.21980	0.24378	0.26863	0.29450	0.32160	0.34882
1.00	0.10513	0.11966	0.13688	0.15644	0.17828	0.20080	0.22401	0.24769	0.27255	0.29762	0.32433
1.10	0.09526	0.10980	0.12538	0.14370	0.16392	0.18487	0.20738	0.22878	0.25309	0.27718	0.30171
1.20	0.08870	0.10088	0.11570	0.13265	0.15172	0.17165	0.19284	0.21404	0.23816	0.26312	0.28248
1.30	0.08203	0.09337	0.10722	0.12323	0.14105	0.16051	0.18026	0.20025	0.22105	0.24276	0.26508
1.40	0.07621	0.08705	0.09987	0.11524	0.13216	0.15024	0.16871	0.18781	0.20800	0.22811	0.24879
1.50	0.07146	0.08138	0.09360	0.10808	0.12415	0.14102	0.15828	0.17681	0.19562	0.21540	0.23584
1.60	0.06701	0.07643	0.08803	0.10167	0.11682	0.13290	0.14958	0.16739	0.18493	0.20281	0.22288
1.70	0.06334	0.07204	0.08286	0.09589	0.11033	0.12570	0.14180	0.15838	0.17579	0.19326	0.21210
1.80	0.05981	0.06813	0.07863	0.09085	0.10438	0.11907	0.13445	0.15038	0.16686	0.18380	0.20165
1.90	0.05656	0.06469	0.07453	0.08629	0.09921	0.11329	0.12777	0.14304	0.15876	0.17522	0.19221
2.00	0.05382	0.06149	0.07105	0.08218	0.09460	0.10781	0.12177	0.13661	0.15161	0.16748	0.18355
2.10	0.05134	0.05852	0.06775	0.07836	0.09021	0.10285	0.11653	0.13063	0.14514	0.16018	0.17588
2.20	0.04900	0.05603	0.06479	0.07493	0.08627	0.09866	0.11145	0.12526	0.13902	0.15347	0.16867
2.30	0.04698	0.05349	0.06200	0.07174	0.08260	0.09450	0.10698	0.11990	0.13359	0.14742	0.16213
2.40	0.04504	0.05134	0.05949	0.06885	0.07934	0.09059	0.10265	0.11525	0.12823	0.14172	0.15600
2.50	0.04337	0.04942	0.05704	0.06606	0.07627	0.08710	0.09870	0.11073	0.12355	0.13655	0.15005
2.60	0.04185	0.04748	0.05501	0.06366	0.07337	0.08398	0.09516	0.10675	0.11887	0.13165	0.14490
2.70	0.04007	0.04570	0.05306	0.06133	0.07072	0.08083	0.09177	0.10310	0.11479	0.12702	0.13983
2.80	0.03868	0.04406	0.05111	0.05925	0.06825	0.07815	0.08861	0.09955	0.11081	0.12290	0.13517
2.90	0.03737	0.04250	0.04939	0.05727	0.06607	0.07558	0.08558	0.09631	0.10739	0.11890	0.13087
3.00	0.03613	0.04121	0.04781	0.05536	0.06382	0.07309	0.08278	0.09319	0.10384	0.11505	0.12673







Table 4.7 continued

	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
0.0	0.40376	0.42861	0.44847	0.47232	0.48517	0.51803	0.54088	0.55374	0.56659	0.56945
0.10	0.38327	0.40558	0.42828	0.45071	0.47267	0.49535	0.51799	0.54045	0.55255	0.55509
0.20	0.35855	0.38003	0.40065	0.42158	0.44388	0.46504	0.48686	0.50811	0.52981	0.55160
0.30	0.32253	0.34319	0.36351	0.38382	0.40419	0.42502	0.44574	0.46705	0.48828	0.50984
0.40	0.28783	0.30832	0.32857	0.34824	0.36885	0.38825	0.40866	0.42823	0.44708	0.46831
0.50	0.25608	0.27683	0.29226	0.31060	0.32976	0.34858	0.36805	0.38889	0.40815	0.43026
0.60	0.22808	0.24898	0.26347	0.28098	0.29855	0.31639	0.33761	0.35672	0.37626	0.39834
0.70	0.20647	0.22332	0.23876	0.25607	0.27348	0.29159	0.30980	0.32826	0.34715	0.36544
0.80	0.18740	0.20282	0.21840	0.23462	0.25112	0.26821	0.28585	0.30319	0.32149	0.33983
0.90	0.17184	0.18559	0.20032	0.21551	0.23122	0.24771	0.26442	0.28165	0.29907	0.31661
1.00	0.15789	0.17084	0.18501	0.19956	0.21414	0.22943	0.24585	0.26205	0.27893	0.29548
1.10	0.14582	0.15848	0.17166	0.18537	0.19905	0.21380	0.22831	0.24448	0.26082	0.27713
1.20	0.13550	0.14741	0.15971	0.17265	0.18616	0.20024	0.21451	0.22948	0.24484	0.26018
1.30	0.12677	0.13765	0.14843	0.16156	0.17455	0.18775	0.20172	0.21684	0.23030	0.24552
1.40	0.11865	0.12914	0.14031	0.15182	0.16412	0.17690	0.18994	0.20360	0.21757	0.23189
1.50	0.11151	0.12152	0.13209	0.14304	0.15476	0.16690	0.17862	0.19251	0.20576	0.21966
1.60	0.10520	0.11489	0.12477	0.13543	0.14634	0.15807	0.17006	0.18240	0.19529	0.20862
1.70	0.09867	0.10879	0.11831	0.12832	0.13881	0.15007	0.16146	0.17342	0.18568	0.19869
1.80	0.09464	0.10322	0.11235	0.12195	0.13224	0.14266	0.15367	0.16526	0.17694	0.18834
1.90	0.09004	0.09809	0.10699	0.11624	0.12600	0.13600	0.14683	0.15754	0.16926	0.18084
2.00	0.08578	0.09374	0.10205	0.11090	0.12024	0.13006	0.14018	0.15082	0.16177	0.17311
2.10	0.08185	0.08956	0.09758	0.10623	0.11507	0.12443	0.13435	0.14443	0.15508	0.16587
2.20	0.07834	0.08571	0.09341	0.10165	0.11028	0.11921	0.12872	0.13846	0.14887	0.15914
2.30	0.07518	0.08223	0.08966	0.09762	0.10574	0.11449	0.12368	0.13309	0.14295	0.15302
2.40	0.07218	0.07894	0.08613	0.09361	0.10172	0.11019	0.11889	0.12809	0.13747	0.14735
2.50	0.06944	0.07592	0.08286	0.08927	0.09601	0.10309	0.11045	0.11888	0.12681	0.14199
2.60	0.06693	0.07307	0.07989	0.08703	0.09439	0.10231	0.11046	0.11888	0.12801	0.13694
2.70	0.06447	0.07052	0.07715	0.08399	0.09116	0.09885	0.10667	0.11489	0.12346	0.13234
2.80	0.06239	0.06820	0.07447	0.08114	0.08808	0.09544	0.10321	0.11123	0.11942	0.12785
2.90	0.06029	0.06605	0.07196	0.07853	0.08525	0.09223	0.09985	0.10753	0.11549	0.12403
3.00	0.05831	0.06367	0.06973	0.07590	0.08257	0.08941	0.09666	0.10405	0.11192	0.12014
x	0.28609	0.32727	0.36739	0.40833	0.45000	0.49231	0.53518	0.57857	0.62241	0.66666
y	0.23810	0.22727	0.21739	0.20833	0.20000	0.19231	0.18519	0.17857	0.17241	0.16667



Table 4.8 continued

x	y										
	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00	
0.0	0.40378	0.42881	0.44947	0.47232	0.48817	0.51803	0.54088	0.56374	0.58859	0.60845	
0.10	0.36178	0.37886	0.39516	0.41344	0.43047	0.44835	0.46841	0.48885	0.50342	0.52188	
0.20	0.32447	0.34111	0.35784	0.37551	0.39284	0.41057	0.42888	0.44880	0.46539	0.48396	
0.30	0.28188	0.30815	0.32508	0.34168	0.35841	0.37554	0.39421	0.41217	0.43072	0.44888	
0.40	0.25175	0.27810	0.29521	0.31218	0.32884	0.34551	0.36393	0.38123	0.39819	0.41728	
0.50	0.23608	0.25310	0.26938	0.28505	0.30222	0.31879	0.33591	0.35350	0.37058	0.38858	
0.60	0.21378	0.23048	0.24628	0.26245	0.27875	0.29485	0.31191	0.32806	0.34550	0.36232	
0.70	0.19505	0.21048	0.22668	0.24250	0.25782	0.27373	0.28978	0.30614	0.32252	0.33907	
0.80	0.17845	0.19347	0.20887	0.22387	0.23850	0.25448	0.26983	0.28581	0.30197	0.31782	
0.90	0.16443	0.17843	0.19300	0.20785	0.22260	0.23746	0.25258	0.26779	0.28327	0.29808	
1.00	0.15191	0.16544	0.17839	0.19351	0.20818	0.22221	0.23685	0.25147	0.26648	0.28172	
1.10	0.14136	0.15383	0.16712	0.18084	0.19502	0.20918	0.22301	0.23738	0.25110	0.26582	
1.20	0.13197	0.14384	0.15655	0.16967	0.18315	0.19663	0.21014	0.22376	0.23768	0.25174	
1.30	0.12356	0.13487	0.14708	0.15963	0.17215	0.18555	0.19876	0.21178	0.22521	0.23881	
1.40	0.11614	0.12703	0.13853	0.15045	0.16278	0.17537	0.18824	0.20070	0.21380	0.22705	
1.50	0.10983	0.11976	0.13075	0.14223	0.15388	0.16536	0.17886	0.19084	0.20305	0.21588	
1.60	0.10384	0.11320	0.12390	0.13486	0.14621	0.15776	0.16998	0.18187	0.19394	0.20600	
1.70	0.09813	0.10763	0.11765	0.12807	0.13820	0.15035	0.16183	0.17357	0.18520	0.19701	
1.80	0.09333	0.10235	0.11187	0.12197	0.13260	0.14312	0.15443	0.16556	0.17714	0.18873	
1.90	0.08883	0.09768	0.10671	0.11648	0.12633	0.13682	0.14780	0.15876	0.16943	0.18085	
2.00	0.08481	0.09318	0.10204	0.11112	0.12091	0.13102	0.14128	0.15207	0.16289	0.17344	
2.10	0.08118	0.08912	0.09751	0.10653	0.11591	0.12555	0.13556	0.14590	0.15623	0.16667	
2.20	0.07774	0.08537	0.09360	0.10223	0.11128	0.12050	0.13007	0.14021	0.15031	0.16038	
2.30	0.07451	0.08194	0.08991	0.09814	0.10674	0.11568	0.12500	0.13471	0.14465	0.15458	
2.40	0.07166	0.07887	0.08647	0.09438	0.10284	0.11153	0.12033	0.12956	0.13927	0.14916	
2.50	0.06910	0.07589	0.08324	0.09097	0.09900	0.10735	0.11591	0.12485	0.13423	0.14375	
2.60	0.06684	0.07308	0.08028	0.08780	0.09560	0.10364	0.11194	0.12058	0.12948	0.13888	
2.70	0.06420	0.07066	0.07740	0.08475	0.09234	0.10000	0.10795	0.11638	0.12507	0.13418	
2.80	0.06201	0.06831	0.07483	0.08183	0.08921	0.09680	0.10440	0.11267	0.12114	0.12974	
2.90	0.05998	0.06604	0.07250	0.07932	0.08630	0.09354	0.10103	0.10883	0.11711	0.12572	
3.00	0.05810	0.06397	0.07016	0.07677	0.08355	0.09050	0.09783	0.10556	0.11348	0.12190	
x	0.28808	0.32727	0.36738	0.40833	0.45000	0.49231	0.53518	0.57857	0.62241	0.66586	
y	0.22810	0.22727	0.31738	0.20833	0.20000	0.19231	0.18518	0.17857	0.17241	0.16667	

Handbook, with a maximum difference of about 18%. However, for smaller values of 'a' (smaller eccentricities) and for smaller values of the parameter k (see Fig. 4.1) the values given in Tables 4.1 to 4.8 tend to be higher. These differences vary for each weld group. For one weld group, Table 4.7, about half the values are actually slightly higher than those in the handbook. On the average, the CISC values are about 1.05 times the values reported herein. There appear, however, to be two minor inconsistencies in the values given in the CISC Handbook.

— First, the maximum values of C in the handbook tables are limited to that corresponding to a fillet weld loaded longitudinally even though the tables intrinsically recognize that welds loaded transversely are stronger. This limitation is not justified as the increased strength when transversely loaded is taken into account for most of the values in the tables. This limitation was not considered in developing Tables 4.1 to 4.8. The effect of the limitation is especially evident for weld groups where a portion of the group is loaded transversely. Tables 4.1 to 4.8 give values of C as much as 1.5 times those in the CISC Handbook under these circumstances. The values in the handbook where this limitation has been applied are those above the horizontal lines in the tables.

The second inconsistency relates to the derivation of the handbook values from the strengths obtained by tests.

The tables in the CISC Handbook are based upon empirical relationships derived by Butler et al (1972). The ultimate strength,  $Pu_{\theta}$ , is given as

$$[4.6] \quad Pu_{\theta} = \frac{10 + \theta}{0.92 + 0.0603\theta}$$

in kips/inch of weld length for a 1/4 inch fillet weld using E60XX (E410XX) electrodes. The longitudinal unit shear resistance,  $Vu$ , from equation [4.6] when  $\theta = 0^{\circ}$ , is

$$[4.7] \quad Vu = \frac{Pu_l}{dl} = \frac{10/0.92}{0.25 \times 1.0} = 43.5 \text{ kips/inch/inch,}$$

or 
$$Vu = 0.300 \text{ kN/mm/mm}$$

The values given in the handbook, except for those above the horizontal lines in the tables, were obtained from this value by multiplying by the product of the resistance factor,  $\phi_w = 0.67$ , the assumed ratio of shear strength to ultimate tensile strength of 0.67 and the ratio of electrode strengths for which the tables are produced to that used by Butler et al, 70/60, that is, by 0.524 to give a longitudinal unit shear resistance of 0.157 kN/mm/mm. Actually, to make the test results of Butler et al fit the shear strength for longitudinal welds given in CSA Standard S16.1 or W59 (Equation [4.2]), the values from [4.6] should have been divided by 0.300 and then multiplied by the factored value from [4.4] of 0.152.

Thus the results in the tables are too large and should be reduced by  $0.152/0.157 = 0.968$ . The procedure proposed herein simply uses the test data to establish the strength of fillet welds at various angles of loading relative to that of longitudinal welds.

#### 4.3 Comparison of Full-Scale Tests of Others

Predictions based on the ultimate strength model developed herein are compared with ultimate loads for the full-scale tests of Butler et al (1972), Swannell and Skewes (1979) and Kulak and Timler (1984) in Tables 4.9 to 4.11. The actual value of the ultimate longitudinal unit shear resistance,  $V_u = P_{u\ell}/d\ell$ , as reported for each test series, was used in [4.5]. Only weld groups similar to those shown in Fig. 4.1 were investigated and the maximum deformation was taken as that at fracture.

Tables 4.9 and 4.10 give mean test-to-predicted ratios of the ultimate strength of 0.977 and 1.008 and coefficients of variation of 0.128 and 0.083, respectively. The model predicts the ultimate strength reasonably well. On the otherhand the mean test-to-predicted ratio for the test data of Kulak and Timler is 1.282 although the coefficient of variation is still only 0.081. The reason for this discrepancy has not been determined.

Table 4.9 Test-to-Predicted Ratios for Ultimate Loads of Full-Scale Test Specimens: Butler, Pal and Kulak (1972).

Specimen Number	Ultimate Load, P, (kN)		Test
	Test	Predicted	Predicted
V1	560.5	534.8	1.04806
V2	1014.2	1043.8	0.97164
V3	418.1	454.4	0.92011
V4	1383.4	1445.9	0.95677
V5	1036.4	1155.6	0.89685
V6	720.6	802.2	0.89828
V7	1450.1	1621.6	0.89424
V8	1058.7	1069.6	0.98981
C1	1321.1	1391.2	0.94961
C2	1000.8	1219.1	0.82093
C3	1401.2	1044.0	1.34215
C4	1338.9	1325.6	1.01003
C5	1294.4	1299.2	0.99631
		$\mu$	0.97652
		$\sigma$	0.12525
		V	0.12826
		n	13



Table 4.10 Test-to-Predicted Ratios for Ultimate Loads of Full-Scale Test Specimens: Swannell and Skewes (1979)

Specimen Number	Ultimate Load, P, (kN)		<u>Test</u> <u>Predicted</u>
	Test	Predicted	
1	306.0	291.6	1.04938
3	179.3	160.7	1.11574
4	98.0	91.3	1.07338
5	220.0	240.9	0.91324
6a	180.6	184.5	0.97886
7	211.3	202.1	1.04552
8	218.6	228.7	0.95584
10a	196.6	226.9	0.86646
11	235.3	218.5	1.07689
		$\mu$	1.00837
		$\sigma$	0.08391
		V	0.08321
		n	9

Table 4.11 Test-to-Predicted Ratios for Ultimate Loads of Full-Scale Test Specimens: Kulak and Timler (1984)

Specimen Number	Ultimate Load, P, (kN)		<u>Test Predicted</u>
	Test	Predicted	
1	612.3	518.1	1.18182
2	464.9	364.8	1.27440
3	499.6	359.6	1.38932
$\mu$			1.28185
$\sigma$			0.10395
V			0.08109
n			3

#### 4.4 Design Application

From [3.3] the ultimate strength of a fillet weld loaded in shear at any angle of loading  $\theta$  as a function of the longitudinal strength is

$$[4.8] \quad P_{u\theta} = P_{u\lambda} (0.5 \sin^{1.5} \theta + 1.0)$$

Equation [4.8] is then used in [3.10] for determining the ultimate strength of an eccentrically loaded fillet welded connection. Similar to [4.2] for the longitudinal ultimate strength  $P_{u\lambda}$ , [4.8] can be written as

$$[4.9] \quad V_{r\theta} = \phi_1 A_w 0.67 X_u (0.5 \sin^{1.5} \theta + 1.0)$$

where the resistance factor,  $\phi_1$ , must be determined.

##### 4.4.1 Resistance Factor

The limit states design criterion for ultimate strength is

$$[4.10] \quad \phi R > \lambda S = \sum \lambda_i S_i$$

which states that the factored resistance must be greater than or equal to the effect of factored loads. Using a first-order probabilistic design procedure, Galambos and Ravindra (1973) have shown that the resistance factor can be written as

$$[4.11] \quad \phi = \rho_R \exp(-\beta \alpha_R V_R)$$

Ravindra and Galambos (1978) and Fisher et al (1978) propose that the reliability index,  $\beta$ , be taken as 4.5 for connections to ensure that the probability of failure of the connector is less than that of the member as a whole for which a value of  $\beta = 3.0$  is commonly used for building structures. Galambos and Ravindra (1973) also have proposed a value of  $\alpha_R = 0.55$ . To establish the resistance factor, values of  $\rho_R$  and  $V_R$  are needed.

The ratio of the mean-to-nominal resistance is

$$[4.12] \quad \rho_R = \rho_G \rho_{M1} \rho_{M2} \rho_P$$

and the associated coefficient of variation is

$$[4.13] \quad V_R^2 = V_G^2 + V_{M1}^2 + V_{M2}^2 + V_P^2$$

Using fillet weld leg dimensions for 42 test specimens as reported by Miazga and Kennedy (1986) the mean value of the measured-to-nominal ratio of effective weld throat area,  $A_w$ , is  $\rho_G = 1.034$  with a coefficient of variation  $V_G = 0.026$ . Two material parameters are needed to account for the variation of material strength. First the variation of the ultimate tensile strength of the weld metal as compared to the electrode classification must be determined. Second the ratio of the shear strength as a

function of the weld metal ultimate tensile strength to that assumed in the design expression must be determined. The mean value of the measured-to-nominal ratio of ultimate tensile strength,  $\rho_{M1}$ , defined as  $\sigma_u/X_u$ , is given in Table 4.12 for tests of others. From this data the mean value is 1.123 with a coefficient of variation  $V_{M1} = 0.077$ . Table 4.13 gives for a total of 126 tests a mean value for the measured shear strength/ultimate tensile strength ratio all divided by the nominal value of 0.67, of  $\rho_{M2} = 1.118$  with a coefficient of variation  $V_{M2} = 0.121$ . The variability associated with the ultimate strength model is given by test-to-predicted ratios,  $\rho_p$ . Test-to-predicted ratios are given in Table 3.1 for the test data of Miazga and Kennedy and in Tables 4.9 to 4.11 for the full-scale test results of others. These values are summarized in Table 4.14. From this data  $\rho_p$  has a mean value of 1.015 with a coefficient of variation  $V_p = 0.111$ .

Using these values in [4.12] and [4.13] gives

$$\rho_R = 1.034 \times 1.123 \times 1.118 \times 1.015 = 1.318$$

$$V_R^2 = 0.026^2 + 0.077^2 + 0.121^2 + 0.111^2 = 0.183^2$$

Table 4.12 Measured-to-Nominal Ratios for Ultimate Tensile Strength of Weld Metal

Source	Sample Size n	Nominal Tensile Strength, $X_u$	Mean Tensile Strength, $\sigma_u$	Measured Nominal [ $\sigma_u/X_u$ ]	Standard Deviation	Coefficient of Variation
Miazga and Kennedy (1986)	3	480 MPa	537.7 MPa	1.12014	0.01577	0.01408
Gagnon and Kennedy (1987)	10	480 MPa	579.9 MPa	1.20813	0.04284	0.03546
Swannell and Skewes (1979)	2	410 MPa	538.8 MPa	1.31415	0.02621	0.01994
Fisher et al (1978)	127	60 ksi	66.0 ksi	1.10000	0.04267	0.03879
	138	70 ksi	74.9 ksi	1.07000	0.03814	0.03564
	136	80 ksi	87.9 ksi	1.09875	0.05425	0.04937
	16	90 ksi	100.2 ksi	1.11333	0.04800	0.04311
	72	110 ksi	116.9 ksi	1.06273	0.04255	0.04004
	128	70 ksi	85.4 ksi	1.22000	0.06814	0.05585
	40	70 ksi	86.8 ksi	1.24000	0.14114	0.11382
				$\mu$	1.12344	
				$\sigma$	0.08688	
				V	0.07733	
				n	672	

Table 4.13. Measured-to-Nominal Ratios for Shear Strength/Ultimate Tensile Strength

Source	Sample Size, n	Mean $\tau_u/\sigma_u$	Measured Nominal $\frac{\tau_u/\sigma_u}{[ \frac{\tau_u/\sigma_u}{0.67} ]}$	Standard Deviation	Coefficient of Variation
Miazga and Kennedy (1986)	6	0.76451	1.14106	0.09246	0.08103
Swannell and Skewes (1979)	5	0.68641	1.02449	0.02963	0.02892
Kato and Morita (1969)	11	0.65561	0.97852	0.07531	0.07696
Ligtenburg (1968)					
Appendix I-2	18	0.73895	1.10291	0.16817	0.15248
I-3	17	0.80863	1.20691	0.10806	0.08953
I-4	18	0.72796	1.08651	0.13476	0.12403
I-6	18	0.74253	1.10825	0.13414	0.12104
I-7	17	0.81041	1.20957	0.09260	0.07656
I-9	16	0.74065	1.10545	0.11506	0.10408
$\mu$		0.74901	1.11792		
$\sigma$		0.09059	0.13521		
V		0.12095	0.12095		
n	126		126		

Table 4.14. Test-to-Predicted Ratios for Ultimate Strength Model

Source	Sample Size, n	Mean Test Predicted	Standard Deviation	Coefficient of Variation
Miazga and Kennedy (1986) (Table 3.1)	42	1.00961	0.09010	0.08924
Bulter, Pal, and Kulak (1972) (Table 4.9)	13	0.97652	0.12525	0.12826
Swannell and Skewes (1979) (Table 4.10)	9	1.00837	0.08391	0.08321
Kulak and Timler (1984) (Table 4.11)	3	1.28185	0.10395	0.08109
$\mu$		1.01521		
$\sigma$		0.11238		
$V$		0.11070		
$n$		67		



and substituting these values of  $\rho_R$  and  $V_R$  into [4.11] gives

$$\phi_1 = 1.318 \exp (-4.5 \times 0.55 \times 0.183)$$

$$\phi_1 = 0.838 \approx 0.84$$

Although [4.11] allows  $\phi_1$  to be computed separately and similar expressions allow load factors to be computed separately (Ravindra and Galambos, 1978), load and resistance factors are not independent as indicated by [4.10]. Specifically, if a set of load factors are computed for a particular value of  $\beta$ , then resistance factors should be computed for the same value of  $\beta$ . In this case, load factors corresponding to  $\phi_1$  should be based on a value of  $\beta = 4.5$ , however, load factors currently used for building structures were computed using a value of  $\beta = 3.0$ . Rather than using different load factors the effect of the different  $\beta$ 's can be imposed on the value of  $\phi$  to be used. Equation [4.10] can be rewritten as

$$[4.14] \quad R > \phi S$$

where  $\phi = \lambda/\phi$  is the central safety factor corresponding to a particular value of  $\beta$ . An adjusted resistance factor,  $\phi_2$ , to be used with load factors based on  $\beta = 3.0$

can be computed such that the central safety factor for  $\beta = 4.5$  is maintained. That is,

$$[4.15] \quad \phi_2 = \phi_1 \frac{\lambda_{\beta=3.0}}{\lambda_{\beta=4.5}}$$

such that

$$[4.16] \quad \frac{\lambda_{\beta=3.0}}{\phi_2} = \phi_{\beta=4.5} = \left[ \frac{\lambda}{\phi_1} \right]_{\beta=4.5} = 4.5$$

Fisher et al (1978) have shown that the ratio of load factors in equation [4.15] can be taken as 0.88 for fillet welded connections. Therefore, the adjusted resistance factor becomes

$$\phi_2 = 0.84 \times 0.88 = 0.739 \approx 0.74$$

The ultimate strength design expression from [4.9], giving the factored shear resistance of a fillet weld at any angle of loading, becomes

$$[4.17] \quad V_{r\theta} = \phi_2 A_w 0.67 X_u (0.5 \sin^{1.5} \theta + 1.0)$$

with  $\phi_2 = 0.74$

#### 4.4.2 Proposed Design Tables

The values of coefficient C listed in Tables 4.1 to 4.8 were computed using a longitudinal ultimate strength

based on a resistance factor  $\phi_w = 0.67$ . However, these values of C can be increased by the ratio of  $\phi_2/\phi_w = 1.104$  for use when designing eccentrically loaded fillet welded connections and still maintain a reliability index of 4.5.

When these increased values are compared to the current CISC Handbook design tables, it is then established that on the average they are about 1.05 times the current CISC values.

## 5. SUMMARY AND CONCLUSIONS

### 5.1 Summary and Conclusions

1. A method has been developed to predict the ultimate strength of eccentrically loaded fillet welded connections. It is based upon load-deformation characteristics of fillet welds obtained by Miazga and Kennedy and upon fulfilment of equilibrium and compatibility conditions of the connections using the instantaneous centre of rotation method.
2. A simplified expression has been established for predicting the ultimate strength of fillet welds loaded in shear at any angle of loading. This expression models the theoretical expression developed by Miazga and Kennedy reasonably well and is in good agreement with test data with a mean test-to-predicted ratio of 1.0096.
3. The deformation capacity of fillet welds decreases as the angle of loading increases. Expressions have been developed to predict maximum deformation at ultimate load and maximum deformation at fracture.
4. The load-deformation curve for fillet welds loaded in shear at any angle of loading are represented by one non-dimensionalized polynomial expression. This curve models the test data of Miazga and Kennedy reasonably well and shows that unloading occurs before the weld fractures.

5. Design tables have been prepared for eight weld groups. The analysis was performed by considering the maximum deformation of fillet weld elements to be either the deformation at fracture or the deformation at ultimate load. Results show that values of the coefficient C, indicative of the weld group strength, are generally larger when the deformation at fracture is used as the critical deformation.
6. Tabulated values of the coefficient C for the eight weld groups are on the average about 5% lower than those listed in the CISC Handbook of Steel Construction.
7. The method developed predicts the full-scale test results of others reasonably well with a mean test-to-predicted ratio for two sets of data of 0.990 with a coefficient of variation of 0.110. The greater test-to-predicted ratio for one set of tests (Kulak and Timler) remains unexplained.
8. The factored shear resistance of fillet welds at any angle of loading can be written as

$$Vr_{\theta} = 0.67 \phi_2 A_w X_u (0.5 \sin^{1.5} \theta + 1.0)$$

with  $\phi_2 = 0.74$  to provide a reliability index of 4.5. Using this expression, values of the coefficient C are on the average 5% higher than those given in the CISC Handbook of Steel Construction.

## 5.2 Further Study

1. A sufficient quantity of statistical data for fillet welds is required to establish the validity of the resistance factor determined for the ultimate shear strength of fillet welds.
2. The reliability index used for connections could be re-evaluated as the difference between this value and that used for members as a whole may be unnecessarily too large.

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