Simulated Learning Model for Mineable Reserves Evaluation in Surface Mining Projects

by

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Abstract

The amount of information available for characterizing the geology of a deposit increases over time due to the continuous acquisition of data during mining. Throughout the lifetime of a mining project, the block model and the mining sequence are periodically updated to account for this new data. The acquisition of additional data increases the accuracy of the block model and clarifies the optimal mining sequence. There has been extensive research on mine planning, but current techniques do not consider the decrease in uncertainty as additional information becomes available. Conventional paradigms assume either 1) the kriged model is correct and uncertainty due to multiple realizations does not change the mining sequence, or 2) the mining sequence is unrealistically adapted to each realization.

A new paradigm is proposed for evaluating minable reserves of surface mining projects. This new paradigm accounts for the effects of the continuous acquisition of additional information during the mining of the deposit. In the implementation, multiple scenarios characterizing the dynamic nature of mining and data collection are generated. Each scenario accounts for how the mine may develop over time as new information is acquired. This provides a more realistic framework for evaluating mineable reserves with an appropriate level of uncertainty. The new paradigm can be used to evaluate infill drilling. The acquisition of additional information increases the revenue of the mining sequence as the block model becomes progressively more accurate. However, this increment in the revenue comes at the cost of implementing the infill program. In the new paradigm, the infill drilling strategies are evaluated in terms of their contribution to profit, difference between increment in revenue and cost of infill drilling.

The design of the mining sequence of the long-term plan may be problematic as each scenario has its own version of the mining sequence. To overcome this problem, the mining sequences of the scenarios are condensed into a few representative mining sequences by implementing customized clustering techniques. These few representative mining sequences can be used to design the mining sequence of the long-term plan along with contingency plans.

Preface

Chapter 2 of this thesis has been published as Cuba, M. A., Boisvert, J. B., & Deutsch, C. V. (2013). Simulated learning model for mineable reserves evaluation in surface mining projects. *SME transactions*, 527-534. I was responsible for the design of the proposed methodology, the data analysis, and the manuscript composition. Dr. Clayton V. Deutsch and Dr. Jeffery B. Boisvert were the supervisory authors and were involved with the concept formation and manuscript edits.

To my grandmother, my parents, and my brother, Thank you for supporting me and believing in me from the beginning.

To Roadrunner,

Thank you for cheering on me and brightening my days.

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List of Symbols, Nomenclature, and Abbreviations

CAP	capital expenses
CD	cost of infill drilling
CE	capital expenses without cost of infill drilling
CF	cash flow
CF'	cash flow based on executed production
EVPI	expected value of perfect information
EVSI	expected value of sample information
ISFC	indexed search floating cone
MAEP	mean absolute production error
MT	metric ton
NCF	net cash flow
NPV	net present value
SCD	sum of discounted cost of infill drilling
SCE	sum of discounted capital expenses other than infl drilling
SCF	sum of discounted cash flows
SLM	simulated learning model
SNCF	sum of discounted net cash flows
SPG	sum of production gap
USD	united states dollar

B_v	economic value of a block
c_{ore}	cost of mining ore material
c_{waste}	cost of mining waste material
cnt	standardized minimum distance to the center of the deposit
col	standardized minimum distance to drillhole collar
dc	drilling coverage
D	additional information available
m_p	metal price
r	period interest rate
seq	one minus standardized distance to mining region
T_p	block tonnage
z	block metal grade
P_r	planned production
P_t	executed production
d_0	average discretized distance
d_V	region distance
V	mining region
ΔD	additional information acquired
ΔSCF	incremental sum of discounted cash flows
$\Delta SNCF$	incremental profitt due to the acquisition of additional
η	penalty factor due to production excess

Chapter 1

Introduction

The evaluation of mineable reserves is critical in the assessment of the economic potential of mining projects. Because of the large amounts of money invested in the development of a mining project, an over-estimation of its economic potential may lead to significant economic losses. Similarly, an under-estimation of its economic potential may result in a sub-optimal design or a missed profitable bussiness opportunity. Typically, the evaluation of mineable reserves consists of the construction of a block model to characterize the relevant geology of the deposit and the design of a long-term mine plan that aims to maximize the profit of mining the deposit. However, not all mining projects are driven by the maximization of profit. An example are government-owned mining companies in which social and equity responsabilities are needed to be fulfilled (Gillis, 1982). This thesis focuses on mining projects that aim to maximize profit. Two types of block models can be built with Geostatistical techniques: 1) estimated and 2) simulated. An estimated model typically consists of one kriged block model where the estimate aims to minimize the estimation variance. A simulated model consists of a set of equally probable realizations of the deposit, that is, multiple block models that aim to reproduce the spatial variability of the deposit. Based on these two types of block models, it is assumed that either: 1) the kriged model is correct and uncertainty due to multiple realizations does not change the long-term mine plan, or 2) the long-term mine plan is unrealistically adapted to each realization. In this context, the block model considered, either estimated or simulated, is assumed to be static throughout the lifetime of the mining project.

In practice, the amount of information available to characterize the geology of the deposit increases over time due to the continuous acquisition of additional information. This additional information is collected from different sources, including geologic mapping, production data, and infill drilling. Thus, throughout the lifetime of the mining project, the block model and the long-term plan are periodically updated. In general, the acquisition of additional information increases the accuracy of the block model, reduces uncertainty in ore/waste limits, and clarifies the optimal mining sequence. As conventional paradigms do not anticipate the additional information that will become available, their estimates of the profitability of the mining project tend to be biased, either optimistic or pessimistic. In this thesis, a new paradigm for evaluating mineable reserves that accounts for the dynamic acquisition of additional information in the design of the long term mine plan and the evaluation of mineable reserves. The scope of this research is limited to the evaluation of open pit mining projects.

This chapter is organized as follows. In Section 1.1, the conventional paradigms for evaluating mineable reserves are described and compared. The potential shortcomings of conventional paradigms due to not considering the acquisition of additional information are dicussed. In Section 1.2, the impact of additional information on the evaluation of mineable reserves is discussed. A framework that considers additional information permits a more realistic evaluation of mineable reserves than conventional paradigms. In Section 1.3, the mining of the deposit is discussed as a dynamic process. The dynamic behavior of the mining of the deposit makes the acquisition of additional information also dynamic, thus, the design of the long-term plan and the calculation of mineable reserves are required to be updated accordingly. In Section 1.4, the problem to be addressed in this thesis is presented. In Section 1.5, general aspects of the design of the long-term plan are summarized. Two processes are described: 1) conventional, and 2) computer aided. An overview of the computational algorithms used in the computer aided design process of the long-term plan is presented.

1.1 Motivation

In mining, three types of studies are typically implemented in the evaluation of exploration and mining projects: 1) scoping, 2) pre-feasibility, and 3) feasibility (Gentry and O'Neil, 1992; Hustrulid and Kuchta, 1995; Dominy et al., 2002). These three stages are also referred to as: conceptual, preliminary, and definite, respectively (Dominy et al., 2002). The level of detail of each stage increases as the evaluation progresses. In the scoping stage, the drill spacing is too wide and only allows a global characterization of the deposit. This information only permits a rough assessment of the economic potential of the deposit. In the pre-feasibility stage, the level of geologic information defines the areas of the deposit that can be extracted economically based on preliminary operating parameters of the mining method considered. The calculated economic potential of the deposit is not yet suitable for investment decisions. In the feasibility study, the geologic information available permits an adequate geologic characterization of the deposit at a local scale. The outlining of the mineable region in the deposit is more detailed than in previous stages as the aspects of the mining method to be implemented are fully investigated. The calculated economic potential of the deposit provides a base for investment decisions.

1.1.1 Mineable Reserves

In the evaluation of mining projects, mineable reserves are calculated both in the prefeasibility and feasibility stages (Dominy et al., 2002). Steffen (1997) and Dominy et al. (2002) considered that mineable reserves are also calculated at different time scales. Dominy et al. (2002) referred to the mineable reserves calculated based on the long- and short-term plans to as 'local reserve' and 'production grade control reserve', respectively. This thesis is focused in the mineable reserves calculated in the feasibility stage. The potential of an open pit mining project, from a business perspective, is characterized by its mineable reserve. The reliable calculation of the mineable reserve is critical because of the considerable amounts of capital required to be invested. The mineable reserve is considered to be exploitable with existing operating conditions (Blondel and Lasky, 1956). The details of a mineable reserve, including the economic potential, mineral inventory, and geometric configuration of the regions to mine throughout time, depend on the long-term plan. In mining literature, the term 'mineable reserve' is also referred to as simply 'reserve', 'ore reserve', and 'recoverable reserve' (Hustrulid and Kuchta, 1995). There is extensive literature regarding mineable reserves, including Grace (1983), Noble (1992), and Hustrulid and Kuchta (1995).

The calculation of mineable reserves involves many factors that are unknown at the time of the evaluation of the mining project (Lane, 1999). Dominy et al. (2002) generalized some of the global factors involved. The global factors change continuously throughout the lifetime of the mining project (Blondel and Lasky, 1956). There are various authors that proposed models to forecast some of these factors, for example metal price, including Dooley and Lenihan (2005), Lemelin et al. (2007), and Ahti (2009). However, Journel and Kyriakidis (2004) referred to most of these factors as unpredictable. In practice, different assumptions are made to forecast the factors involved (Blondel and Lasky, 1956). These considerations introduce a degree of subjectivity in the calculation of mineable reserves. In this thesis, the global factors are also assumed fixed.

In the evaluation of mineable reserves, the geology of the deposit is characterized as a block model. Geostatistics is a standard practice for constructing the block model of the deposit (Sinclair and Blackwell, 2002). The block model is built at a resolution that approaches the smallest volume at which ore can be segregated from waste based on mining specifications (Journel and Kyriakidis, 2004). Two types of block models are typically built: 1) estimated and 2) simulated. In the estimated block model, the variables are characterized by conditional means. These models are constructed by implementing kriging techniques, which are widely discussed by many authors, including David (1977), Journel and Huijbregts (1978), and Isaaks

and Srivastava (1989). The use of conditional means introduces a smoothing effect that leads to a reduction in the variability of the variables modeled. The smoothing effect results in an under-estimation of high value regions and an over-estimation of low value regions, respectively (Journel and Kyriakidis, 2004). In practice, this problem is managed by tuning estimation parameters, however, this practice tends to introduce conditional bias in the estimates. In the simulated block model, a set of equally-probable realizations of the deposit are generated. Each of the realizations of the deposit reproduce the variability of the variables modeled. The simulated model permits a more complete evaluation of uncertainty than the estimated model as conditional distributions and the global distribution are characterized (Journel and Kyriakidis, 2004). The most popular technique implemented in mining software is sequential Gaussian simulation. The construction of simulated models is widely discussed by many authors, including Deutsch and Journel (1998), Goovaerts (1997), and Chiles and Delfiner (1999). Based on these two types of block models, three paradigms are typically considered for evaluating mineable reserves. Paradigm 1 relies on an estimated block model. Paradigm 2 relies on both an estimated and a simulated block models. Paradigm 3 relies on a simulated block model.

1.1.2 Paradigm 1 - Estimation

In this paradigm, the geology of the deposit is characterized by a kriged estimate model and the mining of the deposit is defined by one mining sequence (Figure 1.1). This paradigm precedes early developments of geostatistics. A shortcoming in this paradigm is that the long-term plan is potentially sub-optimal as the estimated model used often presents a smoothed and biased representation of the deposit (Journel and Kyriakidis, 2004). Moreover, the direct calculation of the economic block values based on the estimated variables may introduce error in the calculation of the profit as the economic functions, used to calculate the economic block model, are not linear. The profit and mineral inventory are reported as a single value without reference to the uncertainty associated (Dominy et al., 2002; Journel and Kyriakidis, 2004).



Figure 1.1: Sketch of the evaluation of mineable reserves in Paradigm 1. The input is an estimated model and the output is one mining sequence.

Although this paradigm is widely popular in the mining industry, its reliance on only the estimated model makes it a limited alternative for evaluating mineable reserves as the associated uncertainty cannot be quantified. Journel and Kyriakidis (2004) discussed several factors related to the geologic characterization of the deposit with estimated models that cannot be properly addressed in this paradigm, including data sparcity, support effect, and information effect.

1.1.3 Paradigm 2 - Estimation with Uncertainty

This paradigm is an extension of Paradigm 1. In this paradigm, a set of simulated realizations of the deposit are generated and used to evaluate the performance of the long-term plan based on Paradigm 1 (Figure 1.2). All realizations are independently processed through the single long-term plan to evaluate uncertainty in profit and production. The potential production variability due to uncertainty can be assessed. The inclusion of production variability in the calculation of profit is a more realistic assessment of the economic potential of the mining project in comparison to Paradigm 1. This assessment of uncertainty is pessimistic because the mine plan does not adapt to any of the realizations; in reality, however, the mine plan will adapt to discoveries of where the deposit is better or worse than expected.

1.1.4 Paradigm 3 - Simulation

In this paradigm, the deposit is characterized by a set of multiple realizations. A set of mining sequences are generated, one for each realization (Figure 1.3). A



Figure 1.2: Sketch of the evaluation of mineable reserves in Paradigm 2. The performance of the mining sequence of Paradigm 1 is evaluated based on a set of realizations of the deposit.

shortcoming in this paradigm is that, unlike Paradigms 1 and 2, because of the large amount of realizations to process, the operating design of the mining sequences cannot be implemented. Although useful information about the possible future mining path is obtained, it is difficult for mining engineers to know how to manage the multiple mine sequences. This means that there is no clear indication of how to proceed with the mining of the deposit to achieve the profit assessed. Moreover, the generation of realizations and optimization of the mining sequence on each realization is computationally expensive. The uncertainty in the mineable reserves is approached by the histogram of realized profits and mineral inventory values.

1.1.5 Comparison of Conventional Paradigms

These three paradigms are not directly comparable for two main reasons: 1) differences in the type of mining sequence considered and 2) difference in the type



Figure 1.3: Sketch of the evaluation of mineable reserves in Paradigm 3. The deposit is characterized by a set of realizations. One mining sequence is generated for each realization

of profit calculated. In the case of the type of mining sequence considered, the mining sequence used in Paradigms 1 and 2 includes an operating design of the mine, while in the case of Paradigm 3, the calculated block extraction sequence is assumed as the final mining sequence in each realization. For comparison purposes, it is considered that, as in the case of Paradigm 3, the mining sequence of Paradigms 1 and 2 is the calculated block extraction sequence without operating design. In the case of the types of profit calculated, two types of profit are considered in the three conventional paradigms: 1) planned and 2) realized. The planned profit is calculated in Paradigm 1 from planning on the estimated model. The realized profits are calculated in Paradigm 2 from processing the set of realization through the long-term plan of Paradigm 1. The planned profit represents the expected profit under the conditions of the input model. The realized profit represents an equally probable profit scenario that could be achieved if the mining sequence is strictly im-

plemented. In the case of Paradigm 3, as each mining sequence is calculated based on a realization of the deposit, the planned and realized profits are equal.

Paradigm 1 can yield a wide range of planned profit values depending on how the construction of the estimated model is tuned. The profit of Paradigm 1 can be either larger or smaller than the realized profits of Paradigms 2 and 3 because the tuning process is not part of the construction of the simulated model. In practice, the estimated model is often optimistic (Journel and Kyriakidis, 2004). Paradigm 2 resembles an out-of-sample evaluation of the mining sequence of Paradigm 1. In the out-of-sample evaluation, real or simulated information from a comprehensive model is used to measure the performance of a decision model in terms of real-world outcomes (Conejo et al., 2010). In Figure 1.4, a schematic comparison of the three paradigms is presented.



Figure 1.4: Sketch of comparison of conventional paradigms in terms of their respective profit calculated. In Paradigms 1 and 2, the planned and realized profits are calculated, respectively. In Paradigm 3, the planned and realized profits are identical.

In this comparison, the realized profit is preferred over the planned profit because it accounts for the peformance of the mining sequence. In this respect, only Paradigms 2 and 3 are compared. The profit of Paradigm 3 is larger than Paradigm 2. In each realization, the profit of Paradigm 3 is calculated based on the optimal mining sequence for each realization, while the profit in Paradigm 2 is calculated based on the mining sequence of Paradigm 1, which is sub-optimal for any given realization. In Figure 1.5, an example of the comparison between Paradigms 2 and 3 in terms of the profit of the ultimate-pit is presented. The example consists of simulating one hundred realizations of a deposit, conditioned to a synthetic dataset of twenty-eight drillholes, and calculating the profit of the realizations in two scenarios: A) with respect to an optimal ultimate-pit calculated based on an estimated model, and B) with respect to the corresponding optimal ultimate-pits of each realization. The first and second scenarios represent Paradigms 2 and 3, respectively. Since the profits calculated in Scenario B are optimal for each realization, no profit calculated in Scenario A can be larger that of Scenario B. In this example, the ultimate-pit is calculated for simplicity and illustration purposes.



Figure 1.5: Example of comparison of ultimate pit profits based on Paradigms 2 and 3. A: Scatter plot of ultimate-pit profits calculated in Paradigm 2 and 3. B: Box plot of profits calculated in Paradigm 2 and 3.

The difference in the calculated profits, between Paradigms 2 and 3, is due to the assumptions about uncertainty in the block model of the deposit that are made during the design of the long-term plan. In Paradigm 2, it is assumed the long-term mine plan is designed based on the available geologic information and it will not change throughout the lifetime of the mining project. The long-term plan is subject to errors as the information available is limited. In Paradigm 3, it is assumed there is access to perfect knowledge of the deposit, even before mining the deposit. Therefore, there is no error during the mining of the deposit. In practice, additional information is collected throughout the lifetime of the mining project, which improves progressively the performance of the block model and the mining sequence over time. Thus, the profit of the mining project is larger than calculated in Paradigm 2, but smaller than calculated in Paradigm 3 as perfect knowledge of the deposit cannot be achieved. Paradigms 2 and 3 present two extreme cases in the evaluation of mineable reserves. Paradigm 2 is pessimistic as it assumes the initial state of uncertainty in the deposit will remain static throughout the lifetime of the mining project. Paradigm 3 is optimistic as it assumes there is access to perfect knowledge of the deposit even before mining the deposit.

1.2 Effect of Additional Information

The geologic information available at the time of the evaluation of mineable reserves does not remain static throughout the lifetime of the mining project. In practice, additional information is acquired as it is budgeted as part of the design of the long-term plan (Hustrulid and Kuchta, 1995). In Paradigm 2, the cost of additional information is budgeted but not used. In Paradigm 3, although the cost of additional information is budgeted, it is not necessary as it is assumed there is access to perfect knowledge of the deposit. As the calculation of profit depends on the accuracy of the block model (Froylan et al., 2004), the inclusion of additional information has an impact on the profit of the mining project.

In Figure 1.6, the example discussed in Figure 1.5 is continued to illustrate the effect of additional information. A new scenario that considers additional information in the form of an infill campaign is discussed. In this new scenario, an infill campaign is sampled from each realization of the deposit and added to the initial dataset, thus generating a set of realizations of the acquisition of additional information. As in the case of Paradigm 2, in the additional information scenario, the profit of the ultimate pits is calculated based on the simulated model. In the example, the budget allocated to collect extra information permits drilling eighteen additional

drillholes. The use of the additional information results in an improvement with respect to Paradigm 2. This improvement in the profit is sensitive to the spatial configuration of the additional drillholes.



(B) Comparison of information scenarios

Figure 1.6: Example of impact of additional information in the calculation of ultimate-pit profits. 1A, 2A, and 3A represent scenarios of information available, Paradigm 2, Additional Information, and Paradigm 3, respectively, to calculate the ultimate-pit of the deposit. B: Box plot of profits calculated in Paradigm 2, Additional Information, and Paradigm 3 scenarios. Notice the estimated profit of the Additional Information scenario is in between the estimated profits of Paradigms 2 and 3.

A common problem in the evaluation of mineable reserves is the information effect (Chiles and Delfiner, 1999; Journel and Kyriakidis, 2004; Isaaks, 2005). The

information effect considers the negative impact of the prediction error of the block model used to design the mining sequence. At the time of production, this error is measured as the difference between the long- and short-term block model. An important contribution of the simulation of additional information is that it permits the assessment of the impact of the information effect on the evaluation of mineable reserves (Journel and Kyriakidis, 2004). In Paradigm 2, the profit considering only having access to the initial information is calculated. In the additional information scenario, the benefits of considering the additional information can be assessed by comparing the correspoding profit to Paradigm 2. Paradigm 3 is an unrealistic scenario where it is assumed to have access to perfect knowledge of the deposit.

The evaluation of mineable reserves can be studied in the context of decision theory, since the choices made in the design of the long-term plan, including characterization of ore and waste, specifying plant production capacity, and acquisition of additional infill drillholes, have economic consequences (Journel and Kyriakidis, 2004). The most important aspect of decision theory used in this thesis is the economic valuation of additional information. A formal evaluation of the effect of upgrading the characterization of the geology of the deposit is carried out by assigning value to the additional information collected (Steffen, 1997). The difference between the expected revenues of Paradigms 2 and 3 are referred to as expected value of perfect information (EVPI) (Dimitrakopoulos and Ramazan, 2008). The EVPI is the contribution to the revenue if the acquisition of perfect knowledge of the deposit would be possible. Similarly, the difference between expected revenues of the additional information scenario and Paradigm 2 is referred to as expected value of sample information (EVSI). The EVSI is the contribution to the revenue due to the collection of additional information. The EVSI is positive and always less or equal than the EVPI (Eislet and Sandblom, 2010), that is, the additional information has a positive contribution to the revenue. Froylan et al. (2004) considered a similar framework to quantify the economic value of additional infill drilling. The relationship between EVSI and EVPI is valid if the decision model remains static in any state of information. Fenwick et al. (2008) discussed the effects of modifying the decision model as a result of additional information. In practice, mining parameters are adapted as the global factors vary over time.

In the context presented, the economic potential of the mining project is assessed in terms of the realized profit. Alternatively, the estimated profit can be also used. In this case, the initial information scenario is Paradigm 1. The collection of additional information adds uncertainty to the single profit of Paradigm 1 and makes it move towards Paradigm 3. As the profit of Paradigm 1 is a single value that can be either larger or smaller than the expected profit of Paradigm 3, the additional information scenario accounts for how the extra dataset reduces the bias of the estimated profit in Paradigm 1. The contribution of the additional information is difficult to interpret because the source of the bias in the planned profit is mainly the tuning of the estimation parameters. In Figure 1.7, a sketch of the contribution of additional information based on estimated and realized profits is presented.

In this section, it is assumed that the additional information is collected once during the lifetime of the mining project. The impact of additional information in this context has been discussed by some authors. Froylan et al. (2004) proposed a methodology to value the contribution of infill campaigns. Khosrowshahi et al. (2004) considered the acquisition of of blasthole information with different degrees of errors. Journel and Kyriakidis (2004) discussed the effect of blasthole information in the evaluation of mineable reserves. Aspects including sampling error and classification error are covered. Boucher et al. (2005) proposed a methodology to quantify the contribution of infill drilling to the profit of the mining project. Jewbali and Dimitrakopoulos (2009) proposed a methodology to account for the effect of blasthole information in the calculation of the profit of the mining project. However, this assumption is simplistic as in practice the additional information is collected continuosly throughout the lifetime of the mining project.



Figure 1.7: Sketch of comparison of contribution of additional information based on estimated (top) and realized (bottom) profits. The Additional Information scenario is represented as shaded regions to denote that the range of profits depends on the amount of additional information acquired.

1.3 Dynamic Mining of the Deposit

From a global perspective, the main activities that take place in each period can be classified into two groups: 1) design of the mine plan and 2) execution of the mine plan. The design of the mine plan consists of: construction of the block model, design of the mining sequence, and calculation of the mineable reserves. The execution of the mine plan consists of: targeting of the next region of the mining sequence, design of medium- and short-term plans to adapt the targeted region of the mining sequence to meet planned production objectives at a smaller time scale (Jewbali and Dimitrakopoulos, 2009). At the end of the lifetime of the mining project, the mining sequence that is executed consists of all the regions mined in each period, and the profit of the mining project is a function of the executed net cash-flows of each period. The point at which the planned profit matches the executed profit depends both on the design of the long-term plan and the accuracy of the block model.

The assumption that the long-term plan is static throughout the lifetime of the mining project would be reasonable if the geologic characterization of the deposit is also static. However, as part of the execution of the mine plan, additional information is collected from different sources, including surface mapping, drilling, geophysical and geochemical surveys, and rock mechanic studies (Erickson and Padgett, 2011). The periodic acquisition of additional information makes the geologic characterization of the deposit evolve accordingly (Dominy et al., 2002). Thus, the long-term plan should adapt following the changes in the block model. In this context, the long-term plan depends only on the block model as the global factors are assumed to be fixed throughout the lifetime of the mining project. A proper framework for evaluating mineable reserves that accounts for the acquisition of additional information should consider the dynamic behavior of the mining of the deposit. This is the focus of this thesis.

1.4 Problem Statement

The conventional paradigms are different in terms of the assumptions regarding the geologic characterization of the deposit. In Table 1.1, a comparison of these paradigms is presented. The advantage of Paradigms 2 and 3 over Paradigm 1 is the possibility of quantifying uncertainty in the mineable reserves because the inclusion of simulated models in the evaluation. However, the profit values calculated are biased. In expected value terms, Paradigm 2 yields a pessimistic profit as it considers that no additional information will be collected in the future and the current state of uncertainty of the deposit will remain static throughout the lifetime of the mining project, while Paradigm 3 yields an optimistic profit as it considers that there is access to perfect information of the deposit even before mining the deposit. There is no direct relatonship between the profit of Paradigm 1 and the expected profits of Paradigms 2 and 3 because the tuning of the estimation model is not considered in the simulated model.

	Paradigm 1	Paradigm 2	Paradigm 3
Computational	low	moderate	high
requirement			
Reports	no	yes	yes
uncertainty			
Operating	yes	yes	no
design for im-			
plementation			
Bias in the	Profit can be	Profit is	Profit is
estimation of	over or	underestimated.	overestimated.
profit	underestimated		
	depending on		
	the tuning of		
	estimation		
	parameters.		

Table 1.1: Comparison of conventional paradigms.

Most likely, the 'correct' profit of the mining project is in between Paradigms 2 and 3. The acquisition of additional information, budgeted in the long-term plan, contributes positively to the profit with respect to Paradigm 2. As the mining progress, the acquisition of additional information reduces the uncertainty in the block model and improves the efficiency of the long-term plan to forecast the cashflow and mineral production. The profit of Paradigm 3 is unrealistically high because it considers error free mining conditions.

Thesis statement: The inclusion of the dynamic acquisition of additional geologic information in the evaluation of mineable reserves yields a more realistic assessment of the economic potential of the mining project than current conventional paradigms that do not account for it.

1.5 Long-Term Planning

The design of the long-term plan follows the design of the final pit (Steffen, 1997; Whittle and Whittle, 1999; Asa, 2002). The final pit represents the operating mineable limits of the deposit under present conditions. The design of the final pit, also referred to as life-of-mine planning (Steffen, 1997), is a two-step process: 1) calculation of the base framework, and 2) operating design. The base framework is the ultimate pit, which is calculated in the block model of the deposit as a shell that represents the economic mineable limits at block resolution. The final pit is designed by outlining the ultimate pit shell considering operating conditions, including final ramps, mineable widths, and operating slopes. The objectives of the design of the final pit are: 1) definition of the global inventory of mineable ore reserve within assumed economic parameters, 2) definition of the production capacity for the life of mine, 3) definition of infrastructure requirements, and 4) assessment of fixed capital costs (Steffen, 1997).

The long-term plan is a roadmap of how the deposit will be mined on a period basis. The duration of a period depends on company policies, typically one year. In the long-term plan, different aspects of the mineable reserves are determined, including mining objectives as scheduled regions, mineral inventory as production targets, and economic potential as cash flow.

1.5.1 Conventional Design of the Long-Term Plan

The design of the long-term mine plan consists of three steps: 1) sequence planning, 2) mine scheduling, and 3) production scheduling (Mathieson, 1982). An additional stage that consists of designing contingency plans to deal with potential problems due to various aspects, including poor block model estimation, tonnage and/or grade loss, and slope instability, can be also considered (Mathieson, 1982). The objectives of long-term planning are: 1) maximize value for investors, 2) minimize risk for investors, and 3) maximize life of the mining project (Steffen, 1997). In Figure 1.8, the workflow of the conventional process to design the long-term plan is presented.



Figure 1.8: Sketch of the conventional workflow to design the long-term plan.

In sequence planning, the final pit is divided into a set of mining stages commonly called expansions, phases, working pits, or pushbacks (Mathieson, 1982). The mining stages provide an initial scheme of the mining path to develop the final pit. As in the case of the design of the final pit, the most common technique consists of two steps: 1) calculation of the base framework, and 2) operating design. The base framework consists of a set of nested pits that are generated by varying the metal price (Hustrulid and Kuchta, 1995). The mining stages are designed based on the operating outline of the nested pits (Steffen, 1997). The mining stages are designed so that the mineability of the final pit is guaranteed (Mathieson, 1982). In mine scheduling, the final pit is also divided in a set of regions, however, unlike the mining stages, each of these regions are meant to be mined in one period (Mathieson, 1982). These regions are collectively referred to as mining sequence. The mining stages have variable lifetime because they are designed based on operating constraints, while in the case of the mining sequence, the regions are designed based on scheduling constraints (Mathieson, 1982). The mining sequence is designed based on the configuration of the mining stages (Osanloo et al., 2008), and consists of determining: 1) the operating geometry of the regions to mine, and 2) the timing in which these regions are mined (Whittle and Whittle, 1999; Halatchev, 2002). In production scheduling, the performance of the mining sequence is fine tuned. Alternative mining sequences are explored by considering variable production rate and cut-off grade strategies (Mathieson, 1982). These alternatives are discriminated based on an economic evaluation and comparison of their economic performance.
1.5.2 Design of the Long-Term Plan Aided by Programming Models

Since the 1960s, mathematical programming algorithms have been introduced to aid in the design of the final pit and the long-term plan. These algorithms consider largescale optimization problems that aim to maximize the profit of the mining project subject to a set of specified constraints and assumptions (Osanloo et al., 2008). The output of these algorithms is a block model in which the extraction period of the blocks is indicated. In this thesis, this block model will be referred to as the calculated mining sequence. The calculated mining sequence is not operational as it does not account for real mining situations, but provides an initial guideline to design the operating mining sequence (Mathieson, 1982). In the design of the long-term plan, the inclusion of mathematical programming algorithms changes the conventional workflow. Mathematical programming algorithms reduce the number of stages of the conventional process to design the long-term plan (Gaupp, 2008). In Figure 1.9, the workflow of the process to design the long-term plan aided by mathematical programming models is presented.



Figure 1.9: Sketch of the workflow to design the long-term plan aided by mathematical programming models.

It is debatable whether conventional or programming-model-aided workflow produces better long-term plans. For example, Gaupp (2008) states that making separate decision in each stage of the conventional workflow compromises the optimality of the mining sequence, and Lane (1999) states that mathematical programming models cannot account for all the operating conditions that are considered in the conventional workflow as planning is a creative activity whereas optimization is analytical. In practice, mining sequences generated based on programming models permit mine planners to quickly evaluate a wider range of alternatives with different mining parameters than in the conventional workflow.

1.5.3 Computational Algorithms Used in Long-Term Planning

In this section, an overview of the computational algorithms used to aid in the design of the final pit and the long-term plan is presented. The techniques discussed are either analytically optimal or heuristic. The analytically optimal techniques yield a global optimal solution whereas the heuristic techniques approximate the optimal solution, but are much less computationally demanding and easier to implement.

The different techniques to calculate the ultimate pit shell are extensively reviewed by many authors, including Wright (1990), Gaupp (2008), and Newman et al. (2010). The two most popular techniques, based on its implementation in commercial computer software, are: 1) Lerchs-and-Grossmann and 2) floating-cone. The graph theory based algorithm proposed by Lerchs and Grossmann (1965) calculates an ultimate pit shell that maximizes the undiscounted profit. This technique and its extensions appear very often in literature (Newman et al., 2010). The floating-cone technique proposed by Pana (1965) is the most representative heuristic technique for calculating the ultimate pit shell. However, Kim et al. (1987) mentioned that the floating-cone technique was initially developed by Kennecott Copper in 1961. The sub-optimality of floating-cone is more evident when implemented in precious metal deposits with erratic distributions (Kim et al., 1987). A comparison of three techniques: 1) Lerchs-and-Grossmann, 2) maximum network, and 3) floating-cone, conducted by Kim et al. (1987) showed that the sub-optimality of the floating-cone technique, with respect to Lerchs-and-Grossmann and maximum network, is of the order of 1%.

There is a wide range of techniques to calculate the mining sequence at block resolution. These techniques are extensively reviewed by many authors, including Gaupp (2008), Osanloo et al. (2008), and Newman et al. (2010). Gaupp (2008) classified the mine sequencing techniques into two categories: 1) ultimate pit based and 2) comprehensive techniques. In the ultimate pit based category, the ultimate pit is calculated before the mining sequence. These techniques tend to produce sub-optimal results as they depend on the ultimate pit, which is calculated aiming to maximize the undiscounted profit. The profit value used to characterize the economic potential of the mining project is discounted. Some authors that proposed techniques in this category are: Gershon (1987), Ramazan (2007), and Sattarvand (2009). In the comprehensive category, the ultimate pit and the mining sequence are generated simultaneously. These techniques aim to maximize the discounted profit. Even when comprehensive techniques do not require the previous calculation of the ultimate pit, they still require the input parameters defined during the design of the final pit, such as global mineral inventory, production capacity, and required infrastructure. Some authors that proposed techniques in this category are: Sevim and Lei (1998), Ramazan and Dimitrakopoulos (2004a), and Froylan et al. (2004).

Osanloo et al. (2008) classified the mine sequencing techniques into two groups: 1) deterministic and 2) uncertainty based. In the deterministic techniques, the inputs are assumed to be fixed and known. The impact of uncertainty is indirectly accounted for by classifying blocks in the model of the deposit into: measured, indicated, and inferred. Typically, in the calculation of the mining sequence, inferred blocks are not considered and measured and indicated blocks are assumed to be uncertainty free. Some authors that proposed techniques in this category are: Sevim and Lei (1998), Ramazan (2007), and Sattarvand (2009). In the uncertainty based techniques, the effect of uncertainty in the inputs are considered. The goal of these techniques, along with the maximization of the profit, is the reduction of uncertainty in the inputs to calculate the profit. Some authors that proposed techniques in this category are: Khosrowshahi et al. (2004), Dimitrakopoulos and Ramazan (2008), and Lamghari and Dimitrakopoulos (2012).

1.6 Dissertation Outline

In this thesis, a new paradigm that incorporates the dynamic acquisition of additional geologic information in the evaluation of mineable reserves is proposed. In Chapter 2, the details of the proposed paradigm are covered. Different implementation aspects, including simulation of the acquisition of different types of additional geologic information, simulation of the dynamic features of the mining process, and calculation of the profit of the mining project, are detailed. The scope and limitations of this new paradigm are discussed. In Chapter 3, it is described how the proposed paradigm is used as a tool for evaluating infill drilling strategies. The evaluation is carried out in terms of: 1) impact on the reduction of planned production variability, and 2) impact on the economic performance of the mining sequence. A methodology for evaluating cost-efficient infill programs is proposed. In Chapter 4, the set of realizations of the mining sequence are summarized into a few representative alternatives that permit the operating design of the long-term plan. A methodology based on clustering techniques to find major patterns in the mining paths of the mining sequences generated is proposed. In Chapters 5, 6, and 7, examples that illustrate the implementation of Chapters 2, 3, and 4, respectively, are discussed. Finally, in Chapter 8, the conclusions of this thesis and future research ideas based on the proposed paradigm are presented.

Chapter 2

Simulated Learning Model Paradigm

In Chapter 1, the motivation for incorporating the periodic acquisition of additional information in the evaluation of mineable reserves is discussed. The acquisition of additional information is an inherent part of the mining process that it is not accounted for in conventional paradigms. The lack of this aspect in the conventional paradigms yields an incorrect calculation of the mineable reserve. In this chapter, a new paradigm to evaluate the mineable reserve that accounts for the periodic acquisition of additional information is proposed. This proposed paradigm is called the Simulated Learning Model (SLM) because of the consideration of the mining of the deposit as a computational learning process.

The uncertainty in the block model depends on the geologic information available and affects negatively the performance of the mine plan in that the decisions made to mine the deposit have to be adjusted to meet production targets. These adjustments come at an increase in the mining costs, thus, reducing the profit margin of the mining project. The conventional paradigms present extreme cases in the calculation of the mineable reserve. In Paradigm 1, uncertainty in the block model is not accounted for. In Paradigm 2, uncertainty in the block model remains static during the lifetime of the mining project. In Paradigm 3, there is no uncertainty in the block model as it is assumed that perfect knowledge of the deposit is accessible before mining the deposit. The SLM paradigm provides an intermediate and more realistic framework, in which, during the lifetime of the mining project, the uncertainty in the block model is reduced progressively because of the acquisition of additional information.

This chapter is organized as follows. In Section 2.1, the framework of the SLM paradigm is introduced. The inclusion of both mining and data acquisition strategies in the evaluation of mineable reserves is discussed. In Section 2.2, the negative effect of uncertainty in the calculation of the profit of the mining project is discussed. The production variability is used as a metric to account for the effect of uncertainty in the performance of the mining sequence. In Section 2.3, the workflow of the event model used in the SLM paradigm to simulate the mining of the deposit is described. The assumptions considered and implementation aspects are detailed. In Section 2.4, the SLM paradigm and conventional paradigms are compared. The comparison is made based on the assessment of profitability of the mining project. Finally, in Section 2.5, the limitations of the SLM paradigm are discussed.

2.1 SLM Framework

In conventional paradigms, the mine plan considers parameters and conditions related to the extraction and processing of the mineral in the deposit. For example, configuration of mining regions, economic feasibility, and capacity of the processing plant. In this thesis, these specifications are referred to as mining strategy and are represented by a mining sequence algorithm. There are a wide variety of mining sequence algorithms designed based on specific mining conditions. For example, Wang and Sevim (1995) considers the closeness of the targeted regions, Akaike and Dagdelen (1999) considers the use of dynamic cutoff and stockpiles, and Ramazan and Dimitrakopoulos (2004b) considers aspects related to equipment access, blending, and production capacity. These algorithms aim to maximize the profit of the mining project based on their specific conditions, thus, they result in different calculations of the mineable reserve.

In conventional paradigms, the block model and the mining sequence remain static during the lifetime of the mining project. However, in practice, the amount of information available for characterizing the deposit increases over time due to the continuous acquisition of additional information. As a consequence, the block model and the mining sequence are periodically updated to account for the new data. The acquisition of additional information increases the accuracy of the block model, reduces uncertainty, and clarifies the optimal mining sequence. As conventional paradigms do not account for the continuous acquisition of additional information, the calculation of the mineable reserve is not realistic. In the proposed SLM paradigm, the effect of the acquisition of additional information in the mine plan is accounted for. The calculation of the mineable reserve considers the generation of multiple scenarios in which the dynamic nature of the mining and data acquisition are characterized. In the proposed methodology, the mine plan depends on both mining and data acquisition strategies.

2.1.1 Production Variability

The performance of the mining sequence depends on the geologic characterization of the deposit. Ideally, in each period, the mining sequence is executed and the planned production and economic targets are met. However, in practice, at the end of each period, the geometry of the planned and executed regions are different, as over the course of the period, the targeted mining region is adjusted by shortterm mine plans to meet planned production targets (Jewbali and Dimitrakopoulos, 2009). The geometric adjustment of the targeted mining region is due to the difference between the geologic characterization and the real geology of the deposit. Although the planned production targets are closely met, there is a reduction in the planned profit due to the extra expenses incurred in the adjustment of the targeted mining region. In conventional paradigms, the geometric correction of the targeted mining region to meet the planned production targets is not characterized because of its complexity. A set of different approaches that account for the reduction of the planned profit are proposed by many authors. These approaches consider that the mining sequence is strictly executed and the reduction of the planned profit is calculated as a function of discrepancy in production. Leite and Dimitrakopoulos (2007) considers the deviations of ore and waste from production targets. Boucher et al. (2005) considers the cost of block misclassification. Ramazan and Dimitrakopoulos (2007), Dimitrakopoulos and Ramazan (2008) and Lamghari and Dimitrakopoulos (2012) consider variability in ore tonnage, metal grade, and net metal quantity.

Two types of production are considered to report mineable reserves, 1) planned and 2) executed. The planned production is calculated from the input block model, which is used to calculate the mining sequence. The executed production is calculated from a realization of the deposit, at the resolution of input block model, within the limits of the targeted regions of the mining sequence. The conventional paradigms consider different assumptions to calculate the mineable reserve. In Paradigm 1, the mining sequence is calculated based on a estimated block model and the mineable reserve is reported in terms of planned production. In Paradigm 2, the mining sequence is calculated based on a estimated block model and the mineable reserve is reported in terms of planned production. In Paradigm 3, a set of mining sequences are calculated based on the realizations of a simulated block model and the mineable reserve is reported either in terms of planned or executed production as both are identical.

The difference between the planned and executed production is used to quantify the performance of the mining sequence. Many authors, including Dimitrakopoulos and Ramazan (2004), Khosrowshahi et al. (2004), and Leite and Dimitrakopoulos (2007), presented this discrepancy as production risk. In this thesis, this difference is referred to as production variability. In Paradigm 1, the production variability cannot be calculated as only the planned production is considered. In Paradigm 2, the production variability is the result that the existing information remains invariant during the lifetime of the mining project. This consideration is unrealistic as in practice, additional information is collected dynamically during the lifetime of the mining project. The additional information improves the performance of the mining sequence in reducing the production variability. Thus, the real production variability is expected to be smaller than in Paradigm 2. In Paradigm 3, the production variability is zero as it is considered that perfect knowledge of the deposit is accessible before mining the deposit. This consideration is unrealistic as the presence of geologic uncertainty ensures that the deposit cannot be mined perfectly. The proposed SLM paradigm aims to account for the impact of the acquisition of additional information to correct the production variability of Paradigm 2. The characterization of production variability in conventional paradigms is unrealistic. The SLM paradigm aims to characterize a more realistic production variability based on the mining and data acquisition strategies considered. A correct characterization of the production variability is important as it affects how the mineable reserve is reported.

2.1.2 Effect of the Dynamic Acquisition of Additional Information

The SLM paradigm accounts for the effect of the acquisition of additional information, along with the mining strategy, in the calculation of the mineable reserve. In the SLM paradigm, three sources of information are considered: 1) exploratory, 2) infill, and 3) blasthole drilling. These sources contribute differently to the geologic characterization of the deposit as they have different scales and sampling errors associated. The exploratory information is collected before mining the deposit. The infill and blasthole information are collected during the lifetime of the mining project. The existing information depends on the development stage of the mining project. If the deposit has not been mined, the existing information consists of exploratory drilling. If the the deposit is already in production stage, the existing information consists of all three drilling sources.

In the SLM paradigm, the lifetime of the mining project is divided into periods, which can be annual or semi-annual, depending on company policies that dictate the frequency in which the mine plan is required to be updated. In each period, information from infill and blasthole drilling are collected following specific conditions. The infill drilling information is collected to improve the geologic characterization of the deposit for different purposes, including exploration, mine planning, or a combination of them (Sinclair and Blackwell, 2002). The infill drilling information might be of different types, e,g, diamond-core and air-reverse. The selection of the type of infill drilling depends on many aspects, including implementation time, quality of information, and cost of drilling. Metz (1992) discussed various types of drilling used in exploration and mine development. In each period, the blasthole drilling is implemented to fragment material and help in ore control during the mine operations (Sinclair and Blackwell, 2002).

The dynamic behavior of the mining process resembles a computational learning process. In computational or machine learning, a typical learning process consist of two steps: 1) remembering, and 2) adapting (Marsland, 2009). A prediction model is calculated to progressively better predict unsampled outcomes after each iteration. In the beginning, the prediction model is calculated based on an initial training dataset. After a set of outcomes are predicted, their real values are collected and the system learns from the experience. The remembering step consists of including the real values of the predicted outcomes of the training dataset. The adapting step re-calculates the prediction model based on the updated training dataset. As the updated training dataset contains more information than the initial training dataset, the performance of the updated prediction model improves progressively.

In the mining case, the training dataset is the available geologic information and the prediction model is the mining sequence. The mining sequence is used to target mining regions and to forecast cash-flow throughout the lifetime of the mining project. In each period, the next mining region of the mining sequence is targeted for extraction. At the end of the period, the executed mining region and cash-flow are different from what were calculated in the mining sequence, thus, there is a prediction error. This difference is because the information available, used to calculate the mining sequence, is limited. The learning process begins after the additional information is collected during the period. In the remembering step, the collected additional information is added to existing dataset. In the adapting step, the estimated block model and the mining sequence are updated with the new dataset. As mining progresses, the performance of the mining sequence to predict the mining regions and cash flows improves.

Although the additional information considers both blasthole and infill drilling sources, only the implementation of infill drilling can be planned. In the design of the mine plan, a budget is allocated for the implementation of infill campaigns during the lifetime of the mining project. This budget limits the amount of infill drilling information that can be acquired. Three aspects are considered in the planning of infill drilling: 1) spatial configuration of infill drillholes, 2) number of infill drillholes, and 3) timing of drilling. In the case of blasthole drilling, the specifications of the implementation and costs depend entirely on the mining operations. In this thesis, the planning of infill drilling is considered in the data acquisition strategy and is represented by an infill program for the duration of the lifetime of the mining project.

The future additional information cannot be considered directly in the calculation of the mineable reserve as it is not accessible at present time. In the SLM paradigm, a stochastic approach is implemented to simulate the dynamic acquisition of future additional information. The simulation of data acquisition generates a set of equally probable scenarios of the mining of the deposit. In each mining scenario, the data acquisition and mining strategies interact together in the generation of a unique mining sequence. Each mining scenario accounts for how the future geology of the deposit might reveal itself and its effects in the performance of the mining sequence. In the SLM paradigm, the term 'existing information' is used to refer to the real information available at the moment of the calculation of the mineable reserve and 'additional information' is used to refer to the simulated future information that is collected during the lifetime of the mining project. In Figure 2.1, a sketch of the SLM framework is presented. In all the mining scenarios, the mining regions of the first period are identical because there is no additional information, thus, the first period relies only on the existing information.

The large majority of mining sequence algorithms are designed to maximize the profit of the mining project in the framework of the conventional paradigms. These algorithms consider that the estimated block model is static throughout the lifetime of the mining project. As in the SLM paradigm, the estimated block model is



Figure 2.1: Sketch of the SLM framework to generate scenarios of the mining of the deposit. The simulated future additional information is indicated by s.

dynamic, the maximization of profit after implementing these algorithms is not guaranteed. The SLM paradigm can be seen as a case of adaptive models in stochastic integer programming. Dimitrakopoulos and Ramazan (2008) discussed the challenges of optimizing adaptive model problems in mine planning. The main difficulty is that, in each period, the optimal decision has to be determined based on a estimated block model built with limited available information. This available information is updated as the mining progress and it is not accessible in time to improve the efficiency of the decisions to mine the deposit.

Unlike conventional paradigms, where only the mining strategy is specified, the SLM paradigm requires the input of both mining and data acquisition strategies to calculate the mineable reserve. In the SLM paradigm, different mining sequence algorithms and data collection programs have to be explored together aiming to maximize the profit of the mining project. The SLM paradigm allows assessing how specific mining and data acquisition strategies would perform in a specific deposit. As in the case of Paradigms 2 and 3, the mineable reserve of the SLM paradigm is reported based on executed production. The SLM paradigm and Paradigms 2 and 3 are comparable if the same mining strategy is considered. In this comparison, the effect of the specified data acquisition strategy in the calculation of the mineable reserve can be quantified.

2.2 Calculation of the Profit of the Mining Project

In the SLM framework, the profit of the mining project is measured in terms of the net present value (NPV). In each period, the net cash-flows are calculated as the difference between the cash-flow and the capital expenses. The calculation of the net cash-flow as discussed by Hustrulid and Kuchta (1995) is considered as a reference. In this thesis, the cost of infill drilling is considered to be part of the capital expenses. Thus, to account for the acquisition of infill information, the capital expenses is split in: 1) budget allocated to implement the infill campaign and 2) sum of the rest of the items. The net cash-flow is defined as:

$$NCF(D) = CF(D) - CAP(\Delta D),$$

= $CF(D) - \{CD(\Delta D) + CE\},$ (2.1)

where, NCF is the net cash-flow, CF is the cash-flow, CAP is the capital expenses, CD is the cost of infill drilling, CE is the capital expenses without the cost of infill drilling, D denotes the additional information available at the beginning of the period, and ΔD denotes the additional information acquired over the course of the period. The CF term is expressed as a function of D to indicate the mining sequence depends on the information available. The CAP and CD terms are expressed as a function of ΔD to indicate that these two terms depend on the amount of infill information collected. The cost of blasthole drilling is included in the CF term as it is part of the operating cost.

In practice, because of the presence of uncertainty, the planned profit is not achieved, but the executed profit, which depends on the real geology of the deposit (Froylan et al., 2004). In each period, the adjustment of the planned mining regions results in an increase in the estimated mining cost. For example, if less ore is found, non-scheduled regions are targeted for extraction to compensate the production gap. In an opposite case, if more ore is found, the surplus ore has to be stored in stockpiles or the mining region is modified to achieve the production target. In the SLM paradigm, the negative impact of the additional cost is accounted for as a function of the production variability. The negative impact of the production variability on the profit of the mining project is discussed by many authors, including Ramazan and Dimitrakopoulos (2007), Dimitrakopoulos and Ramazan (2008), and Lamghari and Dimitrakopoulos (2012).

In the SLM paradigm, the production variability affects the individual net cashflows. The impact of the additional cost on the CF term is accounted for based on two cases: 1) surplus, and 2) insufficient production. The effect of the additional cost is equivalent to penalize the income of the material produced as a function of the excess production. In the case of insufficient production, it is considered that the effect of additional cost is equivalent to the missing portion of the cash-flow due to not reaching the production target. The cash-flow is calculated as:

$$CF(D) = \begin{cases} CF'(D) - \eta (Pr - Pt) & \text{; if } Pr > Pt \\ CF'(D) & \text{; if } Pr \le Pt \end{cases},$$
(2.2)

where, Pt and Pr are the planned and executed production, respectively, CF' is the cash-flow calculated based on the executed production, η is a penalty factor due to production excess. The difference between Pr and Pt depends on the current information available D. The simulation of the executed production is conditioned to the current available information at the end of each period. In the scheme presented, the effect of production variability on the cash-flow is independent in each period. A more complex approach might consider the effect of compensating under-production with over-production in consecutive periods.

In Figure 2.2, the production variability of a mining sequence is illustrated in terms of extracted and planned ore material. In Equation 2.2, the CF term is calculated as a function of one aspect of production, for example, ore tonnage or net-metal content. In a more detailed evaluation, the calculation of the CF term may include more than one aspect of production and different metal attributes. The problem with the calculation of the CF term is the difficulty of the estimation of the penalty factors for each aspect of production. Lamphari and Dimitrakopoulos (2012) presented an example where the penalty factors of ore tonnage and net metal content are calculated from historic data.



Figure 2.2: Sketch of the comparison between planned and executed production in terms of ore tonnage.

The production variability affects negatively the performance of the mining sequence in terms of how much adjustment of the mining sequence would be necessary to meet the production targets. The scheme presented is an approximate way to account for the additional cost due under- and over-production. In practice, it is very unlikely the mine would operate below capacity and incur the cost described above. Nevertheless, this scheme permits the assessment of value of the additional information.

The profit of the mining project is calculated as the sum of the discounted net

cash-flows of all the periods. Based on Equation 2.1, the profit of the mining project can be expressed as:

$$SNCF(D_t) = SCF(D_t) - \{SCD(D_t) + SCE\}, \qquad (2.3)$$

where,

$$SCF = \sum_{i=1}^{np} \frac{CF_i(D_i)}{(1+r)^i},$$

$$SCD = \sum_{i=1}^{np} \frac{CD_i(\Delta D_i)}{(1+r)^i},$$

$$SCE = \sum_{i=1}^{np} \frac{CE_i}{(1+r)^i},$$

SNCF is the sum of discounted net cash-flows, SCF is the sum of discounted cash-flows, SCD is the sum of discounted infill drilling costs, SCE is the sum of discounted capital expenses other than infill drilling, D_t denotes the total additional information collected during the lifetime of the mining project, i is the period index, np is the number of periods, and r is the interest rate.

2.3 Simulation of Mining Scenarios

In the SLM paradigm, the mining of the deposit is simulated as a set of consecutive periods in which a set of three events occur (see Figure 2.3). The workflow of events accounts for the activities during the mining of the deposit that are relevant to the calculation of the mineable reserve. The proposed representation of the mining of the deposit allows including the mining and data acquisition strategies in the simulation of the mining scenarios. The order of the three events that occur in each period is:

Event 1: Consolidation of existing information.

Event 2: Design of the mining sequence.

Event 3: Acquisition of additional information.



Figure 2.3: Sketch of the event-based representation of the mining of the deposit implemented in the SLM paradigm.

The implementation aspects and details of the three events are described as follows:

2.3.1 Event 1: Consolidation of Existing Information

This event consists of two steps: 1) consolidation of existing information, and 2) construction of the estimated block model. In the first step, at the beginning of the period, all the existing data is consolidated into one dataset. The consolidated dataset grows periodically as extra information is simulated to be collected in each period. In the first period, the consolidated dataset consists of only the existing dataset. In the second step, an estimated block model of the deposit is built conditioned to the consolidated dataset in the first step. For practicality, the variography is calculated based on the existing dataset and remains static during the lifetime of the mining project.

2.3.2 Event 2: Design of the Mining Sequence

In this event, the mining sequence is calculated based on the estimated block model built in Event 1. The calculation of the mining sequence considers the implementation of a mining sequence algorithm that accounts for the characteristics and conditions of the specified mining strategy. In Section 1.5.3, the availability of a wide range of mining sequence algorithms is discussed. Any of these algorithms can be implemented. The region of the next period in the mining sequence is targeted for extraction. This region is considered to be the best decision to mine the deposit based on the information available. The mining sequence of the mining scenario consists of all the regions targeted for extraction in each period. The continuous acquisition of additional information makes the mining strategy to progressively make more informed decisions as the mining of the deposit progress.

2.3.3 Event 3: Acquisition of additional information

This event consists of two steps: 1) simulation of mining the region targeted for extraction, and 2) simulation of the acquisition of additional information. In practice, these two activities interact together and occur simultaneously. For simplicity, these two activities are simulated to occur one after another. In the first step, the region targeted for extraction is mined. In practice, this region is only a guideline and the final region extracted is based on the short term plan. A set of short-term and medium-term mine plans detail and adapt the mining of the deposit to achieve the planned production targets, thus, the difference between planned and executed production is small; ignoring unexpected downtime, the plant is fed continuously. The adjustment of the region targeted for extraction is highly difficult to automate and it is skipped. The mining of the current period is simulated by updating the current topography, at the beginning of the period, so that the region targeted for extraction is above the new topographic surface.

In the second step, the additional information consists of simulated blasthole and infill drilling data. This activity is implemented in two steps: 1) calculation of the configuration of samples, and 2) simulation of sample values. The spatial configuration of the blasthole samples depends on several factors, including mining conditions, rock type, and plant requirements. For example, ore material requires different fragmentation than waste material to be mined. For practicality, the spatial configuration of blastholes is assumed to be a regular grid with generic operating dimensions for each bench of the region targeted for extraction. The dimensions of the grid are specified based on site specific conditions. The spatial configuration of the infill drillholes depends on the data acquisition strategy. In Figure 2.4, a sketch of three potential objectives of data acquisition strategies is presented. The first case aims to aid the mine plan in the medium term by targeting regions in the following periods. The second case aims to aid the mine plan in the long term by targeting regions within the limits of the final pit. The third case aims to aid the geologic exploration by targeting regions beyond the limits of the final pit. The calculation of the position and configuration of the infill drillholes is automated by implementing an algorithm that considers the objective of the infill campaign and operating aspects. The objective of the infill campaign considers individual or a combination of drilling objectives. For example, 35% of medium-term, 40% of long-term, and 25% of exploration. The amount of infill drilling to be collected is specified in the budget allocated for infill drilling as part of the project costs.



----- Exploratory drillhole Infill drillhole

Figure 2.4: Sketch of three infill drilling objectives.

In the simulation of the sample values, the characteristics of the two sources of additional information are aimed to be reproduced so that their different contributions in the construction of the estimated model are accounted for. The contribution of infill drilling is higher than blasthole drilling because of the significant difference in sampling error. Moreover, the contribution of infill drilling is variable as there are different types of drilling that can be implemented. For example, diamond drilling has less sampling error than reverse circulation drilling, thus, its impact on updating the estimated model is higher. The sample values are simulated conditioned to the current consolidated dataset. The simulation of additional information has been discussed by different authors. Knudsen (1995), Journel and Kyriakidis (2004), Khosrowshahi et al. (2004), and Ortiz et al. (2011) considered the simulation of blasthole data in terms of true value and sampling error. Jewbali (2006) considered the use of co-simulation to generate realizations of blasthole data. Boucher et al. (2005) considered the simulation of infill data based on sampling realizations of the deposit.

2.4 Comparison of Paradigms

In this section, the SLM and conventional paradigms are compared in terms of how the mineable reserve is reported. The SLM paradigm is considered directly comparable to Paradigms 2 and 3 as the mineable reserve is reported in terms of executed production. This is not the case for Paradigm 1 as the mineable reserve is reported in terms of planned production. It is considered that all the paradigms share the same mining strategy. The executed production is calculated based on a set of simulated mining scenarios. The SLM paradigm and Paradigms 2 and 3 account for uncertainty in different ways. In the SLM paradigm, in each mining scenario, the initial uncertainty, based on the existing information, is reduced progressively because of the periodic acquisition of additional information. In Paradigm 2, a common mining sequence is considered for all the mining scenarios, meaning the initial uncertainty remains static during the lifetime of the mining project. In Paradigm 3, no uncertainty is considered as the mining sequence in each mining scenario is executed perfectly.

In this comparison, the profit of the mining project in the SLM paradigm and Paradigms 2 and 3 is calculated considering Equations 2.1 and 2.2. In these three paradigms, the profit of the mining project accounts for uncertainty as it is expressed as a function of the production variability. In Paradigm 2, the production variability depends only on the existing information and the mining strategy implemented. In the SLM paradigm, the production variability depends on the existing information and the mining and data acquisition strategies. The acquisition of additional information has an effect on reducing the production variability, with respect to Paradigm 2, thus, contributing to the increase in profit. In Paradigm 3, there is no production variability as it is assumed the deposit can be mined perfectly. In Figure 2.5, a comparison of the production variability of the SLM paradigm and Paradigms 2 and 3 is presented. In this comparison, to summarize the information of all the simulated mining scenarios, the production variability is measured, as a metric of prediction error, in expected value terms, for example, mean-absolute-error or mean-squared-error.



Figure 2.5: Sketch of production variability in Paradigm 2, SLM paradigm, and Paradigm 3

The magnitudes of the production variability are reflected in how the profit of the mining project is calculated. The profit of Paradigm 2 is pessimistic as the negative effect of production variability is exaggerated. The profit of Paradigm 3 is optimistic as the negative effect of production variability is not considered. The profit of the SLM paradigm presents an intermediate case that is more realistic because the effect

of the acquisition of additional information in reducing the production variability is accounted for. As the profit of Paradigm 1 is calculated based on planned production, it cannot be directly included in the comparison. This profit can be greater or lesser than the profit of the other three paradigms, depending on the tuning of the estimation parameters during the construction of the estimated block model. In Figure 2.6, a sketch of the comparison of the paradigms in terms of calculated profit is presented. The difference between the profit of the SLM paradigm and Paradigm 2 is due to the contribution of the acquisition of additional information. The two sources of additional information, blasthole and infill drilling, contribute differently in the reduction of the production variability. The contribution of the blasthole source is fixed while the contribution of the infill source depends on the data acquisition strategy. The SLM paradigm and Paradigm 2 cannot yield the same results as the minimum contribution is due to acquisition of blasthole information. The maximum contribution is due to the combined effect of the blasthole drilling and an optimal infill program. The optimality of the infill program refers to the best data acquisition strategy subject to the budget that allows acquiring infill information in each period. The difference between the profit of Paradigm 3 and the SLM paradigm represents the cost due to not having access to perfect knowledge of the deposit. This gap is due to the negative effect of production variability and the optimality of each of the mining sequences of Paradigm 3. The SLM paradigm cannot close this gap because the production variability cannot be eliminated. The acquisition of even more additional information does not necessarily help close this gap as it comes at an extra cost.

In the comparison, the profit of the mining project is related to the degree of uncertainty considered. This may not the case for the rest of the characteristics of the mineable reserve. The other characteristics, including ore tonnage, net metal quantity, and average metal grade, do not necessarily behave like the profit of the mining project. For example, the ore tonnage in Paradigm 2 can be either greater or smaller than Paradigm 3. The ore tonnage may not be proportional to the profit of the mining project as the corresponding amount of waste material to be removed



Figure 2.6: Sketch of comparison of SLM and Paradigms 1 and 2 in terms of calculated profit

could be greater to a point of reducing the profit margin.

2.5 Limitations

The evaluation of mineable reserves involves many other factors besides the geologic characterization of the deposit, including metal prices, mining and metallurgical technologies available, and local and international political environments. Because of the extensive lifetime of mining projects, these factors are subject to variability as a function of time. In conventional paradigms, these factors are assumed fixed due to the inability to properly forecast them. In the SLM paradigm, these factors are also assumed fixed.

In practice, additional data is collected from different sources, including surface mapping, drilling, geophysical and geochemical surveys, rock mechanic studies. The infill and blasthole sources are only part of this vast sources of information collected. These sources, other than infill and blasthole drilling, are considered to have little impact on the updating of the estimated block model. The SLM paradigms provides a simplified characterization of the evolution of the estimated block model throughout the lifetime of the mining project. Although the SLM paradigm accounts for the evolution of the estimated block model during the lifetime of the mining project, it is assumed that modeling parameters, including variography, definition of domains, and geologic structure of the deposit, are static over time. This is due to real additional data is not accessible at the time of the evaluation of mineable reserves. Thus, the reference of how the modeling parameters adjust over time cannot be replicated. As a consequence, in the SLM paradigm, the acquisition of simulated additional information makes that the presence of uncertainty decreases monotonically. In practice, as more information is available, the geologic understanding of the deposit can either improve or worsen. In Figure 2.7, profiles of the simulated and real reduction of uncertainty due to acquisition of additional information are presented.



(B) Real evolution of uncertainty

Figure 2.7: Sketches of SLM (top) and real (bottom) profiles of evolution of geologic uncertainty.

The implementation of the SLM paradigm is computationally more expensive than that of conventional paradigms. Most of the computational work corresponds to the updating of the model and the calculation of the mining sequence. In the simulation of the mining scenarios, these two tasks are repeated in each period during the lifetime of the mining project.

2.6 Remarks

In this chapter, a new paradigm for evaluating mineable reserves termed the SLM paradigm is presented. In this paradigm, the static behaviour of the estimated block model and the mining sequence considered in conventional paradigms is replaced by a dynamic behaviour. The continuous adaptability of the mining sequence due to the acquisition of additional information is characterized as a computational learning process. The SLM paradigm provides a more realistic framework for evaluating mineable reserves when compared to conventional paradigms.

As only the existing data is accessible at the time of the evaluation of the mineable reserves, the acquisition of future additional information is simulated. Each realization of the additional information leads to the generation of an equally probable scenario of the mining of the deposit. The simulation of the acquisition of future additional information helps to account for how the future geology of the deposit may reveal itself and how it affects the mining sequence.

The majority of the existing mining sequence algorithms are designed to maximize the profit of the mining project in the context the conventional paradigms. However, as the data acquisition strategy is not considered in the design these algorithms, the performance of the resulting mining sequences is not necessarily optimal in the context of accounting for the negative effect of presence of uncertainty. The implementation of the existing mining sequence algorithms, in the context of conventional paradigms, results either in an under- or over-estimation of the profit of the mining project.

Chapter 3

Evaluation of Infill Drilling

In Chapter 2, it is discussed that the direct implementation of a long-term plan leads to production variability, with respect to the planned targeted production, because of the presence of uncertainty in the block model. In practice, medium- and short-term plans adjust the long-term plan to eliminate the production variability so the targeted production is achieved. The extent of the adjustments depend on the magnitude of the production variability. In economic terms, the adjustments result in an increase in the planned mining costs, which reduces the profit of the mining project. The continuous acquisition of additional information leads to the periodic updating of the long-term plan and consequently a progressive reduction of the production variability.

In the SLM paradigm, the dynamic behaviour of the long-term plan and its associated production variability are accounted for by simulating the mining of the deposit and the periodic acquisition of additional information over a set of scenarios of the mining of the deposit. The SLM paradigm considers two sources of additional information: 1) blasthole and 2) infill drilling. From the perspective of long-term planning, although both sources contribute to improving the efficiency of the longterm plan, only the data collection configuration of infill drilling can be customized based on different strategies. In the case of blasthole drilling, the configuration of the samples follows strictly the mined regions. In this chapter, the SLM paradigm is used as a framework for evaluating the impact of different infill strategy alternatives as part of the design process of an infill program. Two metrics are discussed: 1) reduction of the production variability and 2) impact on the profit of the mining project.

This chapter is organized as follows. In Section 3.1, the definition of infill programs used in the SLM framework is described. This definition consists of the infill objective and the amount of drilling in each period. In Section 3.2, the effect of infill drilling on reducing of the production variability is discussed. The effect of production variability is measured period-by-period and globally for each simulated scenario of the mining of the deposit. In Section 3.3, the impact of infill drilling on the profit of the mining project is discussed. The contribution to the profit is split into: 1) contribution to the revenue, and 2) cost of infill program. It is shown that the contribution to the profit provides a more reasonable metric for evaluating the effect of infill drilling than the reduction of production variability. In Section 3.4, aspects of the evaluation of infill strategies based on their contribution to profit are discussed.

3.1 Definition of Infill Programs in the SLM Paradigm

In practice, the design of an infill campaign is a very difficult process as several aspects are involved, including geotechnical, geologic, and operating. The positioning of the infill drillholes is restricted to available regions within the mining operations. For example, it is not practical to position drillholes in unstable zones, final walls, or main road accesses. Metz (1992) discussed that the method to establish a drilling plan depends on various aspects, including type of deposit, stage of the project, and implementation costs. Shaddrick (1987) discussed the impact of geologic characteristics, including rock quality, openness, and water content, on the quality of the drilling samples.

The design of individual infill campaigns can be viewed either as a static or a dynamic problem. In static problems, the configuration of all the drillholes is set at the same time, while in dynamic problems, the drillholes are positioned sequentially, considering previous sampling results. The design of the infill program is entirely a dynamic problem, as each infill campaign depends on the previous collection of information. Bickel and Smith (2006) and Smith and Thompson (2008) proposed methodologies to design optimal oil exploration campaigns in a dynamic decision framework. However, because of the large amount of additional information used in mining and the complexity of the mining process, these methodologies cannot be easily implemented in mining cases. In dynamic frameworks, the construction of adequate models that characterize the learning process over time is important (Bickel and Smith, 2006). The SLM paradigm accounts for the learning aspect of the mining process, by simulating the adaptability of the mine plan as additional information is collected.

In the SLM paradigm, individual infill campaigns are specified based on three aspects: 1) objective, 2) amount, and 3) timing of drilling (see Section 2.3.3). From the perspective of long-term planning, infill campaigns are not specified individually, but jointly for the duration of the mining project. In the definition of infill programs, the three aspects of the infill campaigns are set globally. The timing aspect defines how the objective and amount aspects vary over time. In this thesis, the drilling objectives and specifications of amount of drilling of all the infill campaigns are referred to as infill strategy and configuration of amount of drilling, respectively. For simplicity, the infill drilling only considers one type of data, otherwise, the proportions of each type must be specified per period. In Figure 3.1, an example of two infill programs, A and B, is presented. In the example, the configurations of amount of drilling of the two infill programs are different, and, although the two infill programs consider similar objectives: exploration, long-term and medium-term planning, their infill strategies are also different as the range of periods in which they are implemented vary as a function of time.

The performance of a particular infill program depends on the mining strategy considered. For example, an infill program that focus on medium-term planning performs differently for a mining strategy that considers an uncertainty-based mining sequence algorithm than for a deterministic mining sequence algorithm. The



Definition of Infill Program A

Figure 3.1: Sketch of the definition of two infill programs.

former attempts to reduce uncertainty in the next regions and the latter is only focused on maximizing profit. In the design of an infill program, it is required to consider aspects of the mining strategy implemented.

The design of an infill program as an optimization problem is complex because of the objectivity in the definition of the infill strategy. The infill strategy may consists of several objectives. In practice, the decision of the infill strategy is made with the participation of different departments, including mine planning, mining operations, and geology, because of the many different factors involved. The configuration of amount of drilling is relatively simplier to define compared to the infill strategy. In the configuration of amount of drilling, it is specified the amount of infill drilling data that is to be collected in each period, throughout the lifetime of the mining project. In this chapter, the SLM paradigm is used as a framework to evaluate different alternatives of specified infill strategies. In each infill strategy, the configuration of

amount of drilling that maximizes the profit of mining project is explored. In the following section, the contribution of implementing an infill program is discussed based on two metrics: 1) reduction of production variability, and 2) increment to the profit of the mining project.

3.2 Infill Drilling and Production Variability

The implementation of an infill program improves the accuracy of the block model to characterize the geology deposit, thus, the production variability of the long-term plan is reduced. In this chapter, the performance of an infill program to reduce the production variability is quantified in terms of its improvement with respect to a base case scenario. In the base case scenario, no additional information is collected throughout the lifetime of the mining project. The effect of reduction of production variability is discussed at two time scales: 1) period-by-period, and 2) global.

3.2.1 Production Variability: Period-by-Period

The profile of production variability quantifies the effect of uncertainty in each simulated mining scenario by period. The production variability of the simulated scenarios is represented by a profile of the mean absolute production error (MAEP). In Figure 3.2, a sketch of the profiles of MAEP of three infill programs is presented. The difference of the three profiles of MAEP, with respect to the base case, depends on how the individual infill campaigns perform. Infill program 1 starts moderate in the first periods and then it becomes more aggressive. Infill program 2 is moderate for most of the periods. Infill program 3 is aggressive for most of the periods. In a preliminary evaluation, the profile of MAEP helps to identify periods in which more or less infill drilling would be required.

3.2.2 Production Variability: Global

For a global evaluation, the profile of MAEP of a mining scenario is summarized into a single value, the sum of production gap (SPG). The SPG is calculated as



Figure 3.2: Sketch of the effect of three infill programs on the reduction of the profile of production variability.

the sum of the individual mean absolute production error of each period. In Figure 3.3, a schematic representation of the relationship between the SPG and the total amount of infill data is presented. An example of two infill programs in which a deposit is mined in six periods is considered for illustration purposes. The SPG after implementing infill programs fall within a bounded region. The upper bound corresponds to the contribution of the blasthole source only. This means the infill drillholes have no effect on the mining sequence. For example, the infill drillholes are far from the area of influence of the mining sequence, drilled in the last period, or very close to existing drillholes. This condition is very unlikely and is only considered to set a reference to quantify the effect of the infill source. For any specific total amount of infill data, there is range of average SPG values, where the minimum and maximum values correspond to the most effective and ineffective infill strategies, respectively. The most ineffective infill strategy has no contribution in the reduction of the average SPG. In the example, one infill drillhole is collected per period. In infill strategy A, the six infill drillholes are positioned at the empty regions near to the middle of the deposit. In infill strategy B, the six infill drillholes are positioned very close to existing drillholes. In terms of effectively reducing the SPG, the drilling configuration of infill strategy A is more efficient than infill strategy B. The lower bound of the SPG region corresponds to the case when the configuration of amount of drilling, for a given total infill data, is optimal in terms of reducing geologic uncertainty. There is a limit to what any amount of infill data can contribute. This lower bound represents the largest contribution for different total amounts of infill data based on a specified mining strategy. The minimum SPG is achieved if the entire deposit is sampled as additional information and is the MAEP of the first period as it cannot be eliminated. The lower bound as a function of the total amount of infill drilling has an accelerated decrease that slows down progresively as more infill information is collected.



Figure 3.3: Sketch of the relationship between the configuration of amount of drilling and the infill strategy in terms of SPG.

In the case of an infill strategy, the corresponding SPG is a sub-region within the

global SPG region. The lower bound corresponds to its maximum performance as a function of the total amount of infill data. As in the case of the global SPG region, the minimum SPG of the infill campaign is the MAEP of the first period. The rate at which the lower bound converges towards the minimum SPG is specific for each infill strategy. However, the comparison of different infill strategies in terms of their lower bounds is not straighforward, as their convergence rates vary. In many cases, it is not possible to decide which infill strategy is more efficient without specifying a range of total amount of infill data. In Figure 3.4, the lower bounds of the average SPG sub-regions of two infill strategies are compared.



Figure 3.4: Sketch of the comparison of average *SPG* sub-regions of two infill strategies.

In Figure 3.5, the impact of the configuration of amount of drilling and infill strategy aspects is illustrated in a two-part example. A synthetic deposit of resolution 15×15 cells is considered. In the exercise, the acquisition of one additional sample in addition to an existing dataset that consists of nine samples is evaluated. The mining of the deposit consists of extracting regular panels of 5×5 cells. The configuration of the existing dataset considers one sample at the center of each

panel. The panels are extracted sequentially in descending order based on the average grade of the panels. The evaluation of the data acquisition is carried out based on one hundred simulated mining scenarios. The first part of the example is focused on the configuration of amount of drilling aspect (see Figure 3.5-left). In the two cases considered, the additional sample is collected at the same location, but in different periods. The sample collected earlier, case 1, has more impact on reducing the production variability than the sample collected later, case 2. In case 1, the profile of MAEP improves from the second period onwards, while, in case 2, the profile of MAEP improves from the seventh period onwards. The second part of the example is focused on the infill strategy aspect (see Figure 3.5-right). In the two cases considered, the additional sample is collected in the first period, but at different locations. In case 1, the additional sample is positioned in between two existing samples. In case 2, the additional sample is positioned in between four existing samples. As the sampling position of case 2 covers a larger unsampled region than of case 1, the sampling position of case 2 is more efficient in reducing the production variability. In the example presented, it is shown that for a specific total amount of infill data there is a wide range of possible scenarios for reducing the production variability because of the large number of parameters involved in the design of the infill program.

In this section, it is shown that analyzing the configuration of amount of drilling as an optimization problem, only aiming to minimize the production variability, is inadequate. The solution would have to consider implementing an exhaustive sampling of the entire deposit in the first period. Alternatively, the comparison of infill strategies based on their efficiency to reduce production variability is also inadequate, since the problem in this case consists of determining the range of the total amount of infill data at which the infill strategies are evaluated. Although the reduction of production variability is important to improve the efficiency of the longtern plan, economic aspects are also involved in the evaluation. The total amount of infill data is associated with a cost, which affects the calculation of the profit of the mining project. The cost of implementing the infill program prevents the sampling of



Effect of Configuration of Amount of Drilling and Infill Strategy on SPG

	Amount	Strategy
Base case:	2.72	
Case 1:	2.61	2.61
Case 2:	2.68	2.59

Figure 3.5: Example of effect of the impact of configuration of amount of drilling (left) and infill strategy (right) on profile of MAEP and average SPG. The units of MAEP and SPG are units of mass.

the whole deposit as a feasible alternative. The inclusion of economic considerations in the evaluation of infill strategies provides a more realistic cost-benefit framework to measure the effects of implementing infill programs.

3.3 Infill Drilling and Profit of the Mining Project

A proper design of infill drilling programs is important in the evaluation of mineable reserves, as the cost of implementation is in the order of millions of dollars (Boucher et al., 2005). The selection of an inappropriate infill drilling program has a negative impact, as a denser or a more sparse than necessary configurations result in a reduction of the potential optimal profit margin of the mining project (Metz, 1992).

In this section, the effect of implementing an infill program is measured in terms of how it affects the profit of the mining project. To quantify the impact of the infill program, the base case scenario is considered as a reference. Based on Equation 2.3, the profit of the base case scenario is expressed as:

$$SNCF(0) = SCF(0) - SCE, \qquad (3.1)$$

where, the D_t parameter is presented as zero to denote that neither blasthole nor infill drilling information is collected. As no infill information is collected, the SCD(0)term is zero.

The effect of acquiring additional information D_t on the profit of the mining project is calculated as the difference in profit after collecting additional information and the base case scenario. It is expressed as:

$$\Delta SNCF(D_t) = SNCF(D_t) - SNCF(0), \qquad (3.2)$$

where, $\Delta SNCF(D_t)$ is the incremental profit due to the acquisition of additional information. $\Delta SNCF(D_t)$ is used as the metric of performance. The acquisition of additional information has two components that contribute positively and negatively to the profit of the mining project (Boucher et al., 2005). The positive component is due to the reduction of the costs associated to the production variability. The negative component is due to the cost of implementing the acquisition of additional
information. To account for both components, Equation 3.2 is expressed as:

$$\Delta SNCF\left(D_{t}\right) = \Delta SCF\left(D_{t}\right) - SCD\left(D_{t}\right), \qquad (3.3)$$

where, $\Delta SCF(D_t)$ is the incremental sum of discounted cash-flows, $SCF(D_t) - SCF(0)$.

The $\Delta SCF(D_t)$ term quantifies the total increment in the sum of cash-flows due to the reduction production variability, that is, the benefits of acquiring additional information D_t . The infill program is labeled as efficient if $\Delta SCF(D_t) > SCD(D_t)$. In this case, the benefits of implementing the infill program exceed the cost of implementation. Thus, increasing the profit of the mining project. In case $\Delta SCF(D_t) < SCD(D_t)$, the infill program is labeled as inefficient as its implementation reduces the profit margin of the mining project.

For a specific total amount of infill data there is range of $\Delta SCF(D_t)$ in which the minimum and maximum values correspond to sub-optimal and optimal infill strategies, respectively. The variability in the corresponding $SCD(D_t)$ is mainly due to the discounting of the infill campaign costs in each period. In Figure 3.6, the $\Delta SCF(D_t)$ and $SCD(D_t)$ components of $\Delta SNCF$ are presented as a function of the total amount of infill data. The variability of $\Delta SCF(D_t)$ is represented as a region in which the contribution of the infill data is bounded by two curves. The lower bound corresponds to the contribution of only blasthole data. The upper bound corresponds to the maximum contribution that can be obtained as a function of the total amount of infill data. The upper bound has an accelerated growth and slows down as the amount of total infill data increases. The upper bound of $\Delta SCF(D_t)$ shares similar features as the lower bound of $SPG(D_t)$ because $\Delta SCF(D_t)$ is directly affected by production variability. The global maximum $\Delta SCF(D_t)$ is reached only if the entire deposit is sampled as additional information, thus, the negative effect of producion variability only affects the first period. The variability of $SCD(D_t)$ as a function of the amount of infill data is small compared to the variability of $\Delta SCF(D_t)$ that $SCD(D_t)$ is presented as an straight line. The $SCD(D_t)$ line marks the limit between efficient and inefficient infill strategies. The infill programs with $\Delta SCF(D_t)$ above the $SCD(D_t)$ line are the only ones that result in a positive contribution to the profit of the mining project.



Figure 3.6: Sketch of effect of total amount of infill data on the components of the profit contribution.

The $\Delta SNCF(D_t)$ region is calculated based on its components, $\Delta SCF(D_t)$ and SCD, as specified in Equation 3.3. In this case, the efficient and inefficient infill strategies are delimited by the zero profit threshold (Figure 3.7). The upper bound of the $\Delta SNCF$ region has a concave downward shape because of the different growth rates of the $\Delta SCF(D_t)$ and SCD components.

The exploration of infill strategies in the context of $\Delta SNCF$ may be intractable because of the difficulty of the parameterization of infill strategies. The $\Delta SNCF$ region can be used to be used to assess the economic performance of specified infill strategies. This information can be later used to decide the infill strategy to implement in the design of the infill program. The performance of a particular infill strategy is a sub-region within the global $\Delta SNCF(D_t)$ region. This sub-region represents the potential contribution of implementing the infill strategy subject to the



Figure 3.7: Sketch of effect of total amount of infill data on the profit contribution.

mining strategy considered. In Figure 3.8, the upper bounds of the $\Delta SNCF$ regions of two infill strategies are compared. In the example, the maximum contribution of infill strategy 2 M2 is larger than infill strategy 1 M1.

The evaluation of infill strategies based on $\Delta SNCF(D_t)$ is more reasonable than considering SPG. For each infill strategy evaluated, the problem is focused on finding the best configuration of amount of drilling that results in the largest $\Delta SNCF(D_t)$. The calculation of the maximum $\Delta SNCF(D_t)$ is difficult because of the dimensionality of the configuration of amount of drilling. To implement the evaluation, it is necessary to reduce the dimensionality of the configuration of amount of drilling.

3.4 Simplification of the Configuration of Amount of Drilling

The number of parameters of the configuration of amount of drilling is the number of periods in which the infill program is implemented. For each infill strategy evaluated,



Figure 3.8: Sketch of the comparison of profit contribution sub-regions of two infill strategies.

exploring the configuration of amount of drilling in such a high dimensional space, mapping the $\Delta SNCF(D_t)$ region, is impractical as it is computationally expensive. To overcome this problem, the parameterization of the configuration of amount of drilling is simplified based on two variables: 1) initial, and 2) final number of drillholes, within a range of periods. The amount of infill data to collect in the intermediate periods is set by the slope between the initial and final amounts.

In Figure 3.9, an example of the two-dimensional parameterization is presented. For practicality, in each period, the amount of infill data is expressed in terms of the number infill drillholes.

The proposed parameterization covers the most representative cases that would be considered from a practical perspective: 1) gradually increasing, 2) gradually decreasing, and 3) constant amount per period. The performance of the infill strategy is presented in the form of surfaces $SCD(D_t)$, $\Delta SCF(D_t)$, and $\Delta SNCF(D_t)$. In Figure 3.10, a schematic representation of these surfaces is presented.

For practicality, the mapping of the the $\Delta SNCF(D_t)$ surface can be initialized



Figure 3.9: Sketch of the parameterization of the configuration of amount of drilling. The configuration of amount of drilling is defined based on an initial and final number of drillholes for a range of periods.

with a coarse grid to outline the limits of the efficient region. Then, subsequently make the sampling configuration finer to search for the region with the largest contribution. The region surrounding the largest contribution is taken as a reference to explore more custom scenarios of the configuration of amount of drilling. For example, drilling lengths can be tuned to improve the profit contribution.

3.5 Remarks

The SLM paradigm is used to evaluate the economic impact of infill drilling on the profit of the mining project. The evaluation of infill drilling is more realistic in the context of the SLM paradigm than in conventional paradigms as the dynamic interaction between the mining and data acquisition strategies are accounted for. In this chapter, the mining strategy is considered fixed. The infill program may not necessarily perform efficiently with different mining strategies.



Figure 3.10: Sketch of the surfaces of economic metrics of an infill strategy.

In practice, infill campaigns may consists of different types of drillholes, where each of them have particular sampling errors and drilling costs associated. For example, diamond core drilling has less sampling error than air-reverse drilling, but it is more expensive. The implementation of less expensive type of drilling permits to cover larger unsampled regions at the cost of reducing the quality of the samples. This decision may be adequate for certain types of deposits depending on the mining strategy considered. Although the framework presented in this chapter considers one type of infill drilling, the evaluation of infill strategies can be extended to account for different types of infill data. However, the implementation of the SLM paradigm grows in complexity. In this case, the proportions of each type of infill drilling are specified as part of the definition of the infill strategy. In this chapter, an important part of the design of the infill program consists of evaluating a set of specified infill strategies. These infill strategies account for operating aspects of the mining of the deposit. For each infill strategy, the most efficient configuration of amount of drilling, that aims to maximize the profit of the mining project, is explored. The infill strategy that yields the largest profit contribution is proposed for implementation. It is not the goal of this chapter to aid in the design the optimal infill program that maximizes the profit of the mining project. The optimization of the infill strategy. A simplified definition of the infill strategy may result in an infill program that cannot be implemented because of operating restrictions.

Chapter 4

Clustering of Simulated Mining Sequences

In the SLM paradigm, a set of calculated mining sequences that are subject to specified mining and data acquisition strategies are generated. These calculated mining sequences cannot be used in operating design because they present a large number of mining alternatives to evaluate. In practice, the design of the operating mining sequence could only consider a few mining alternatives. Planning on one model is common, but it does not account for the variability in the evolution of the mining of the deposit due to geologic uncertainty.

In this chapter, the simulated mining sequences are summarized in the form of a decision network from which a reduced set of representative mining sequences can be identified. The representative mining sequences correspond to the branches of the decision network that are more likely to occur. These few representative mining sequences can be evaluated to design the operating mining sequence. The construction of the decision network of the mining sequences relies on the implementation of adapted hierarchical clustering techniques.

This chapter is organized as follows. In Section 4.1, the decision network of the mining sequences is described. The branches of the decision network represent schematic alternative directions that mining of the deposit could take. The nodes of the decision network are calculated by clustering the mining regions of the simulated mining sequences. The connections between nodes are calculated in terms of the frequency of associations between simulated mining regions in consecutive periods. In Section 4.2, aspects of clustering the mining regions of the simulated mining sequences are discussed. The clustering of mining regions is not straightforward because mining regions cannot be easily represented as point-like information. The main challenge of clustering mining regions is to find an appropriate metric of comparison. In Section 4.3, the process of identifying representative mining sequences from the decision network is described. The representative mining sequences are identified from the branches of the decision network that are more likely to occur. In Section 4.4, the simulated mining sequences are analyzed at a larger time scale to identify large scale mining paths. The mining sequences at larger time scales are specified by grouping consecutive periods so that the mining of the deposit only has a few stages. The large scale mining paths provide supplementary information of the representative mining sequences that can be used in the operating design of the mining sequence.

4.1 Framework to Condense SLM Mining Information

In this section, a framework to summarize the calculated mining sequences of the SLM paradigm is proposed. In Figure 4.1, an example of two 2D mining sequences is presented. Each mining sequence is considered as a set of consecutive decisions to mine a deposit. The decisions to mine Period 1 and Periods 4 to 6 are similar, while there are two equally probable paths to mine Periods 2 and 3. This information can be better presented in the form of a decision network. In operation research, decision networks are extensions of bayesian networks (Koller and Milch, 2003) that are graphical representations of uncertain decisions in the form of a directed graph network (Shachter, 1986). The nodes represent alternatives of decisions and the connections between nodes represent probabilistic dependences. Decision networks are useful as they are compact representations of complex data and are intuitive



(Shachter, 1986; Detwarasiti and Shachter, 2005).

Figure 4.1: Example of a decision network representation of two mining sequences. A: Cross section of two mining sequences. B: Representation of mining sequences as individual sequence of decisions. C: Representation of the two mining sequences as a decision network.

In cases where the mining sequences are compared directly, the dissimilarity would be expressed as a single value, thus possibly hiding similar features within certain periods. The representation of the calculated mining sequences in the form of a decision network provides a picture of how the mining sequences are related on a period basis. In the proposed framework, the set of mining sequences are condensed in the form of a decision network, where the nodes represent potential regions that can be mined in each period, and the connections are order associations between mining regions (see Figure 4.2). In this representation, the mining of the deposit moves throughout the network, starting from the first to the last period, choosing between different mining region alternatives that are available in each period. Each branch of the decision network has an associated mining sequence. The representative mining sequences are identified from the branches that are more likely to occur. As in the SLM paradigm the decision to mine the first period consists of only one alternative, there is only one node at the beginning of the decision network.



Figure 4.2: Sketch of a decision network representation of the first seven periods of simulated mining sequences. The size of the circles represents the occurrence of the mining regions. The bold connected branches represent the branches that are more likely to occur.

The nodes of the decision network consist of clusters of similar simulated mining regions. Each cluster has an associated representative mining region. Cluster analysis is often used to classify datasets into meaningful groups by capturing similarities present in the data (Duda et al., 2001; Tan et al., 2006). In this chapter, clustering analysis is implemented to classify the whole set of simulated mining regions in each period into a reduced set of paths or trajectories. The connections between nodes of the decision network are calculated as the frequency of associations between simulated mining regions in consecutive periods.

The main characteristic of the decision network is that it needs to be simple, i.e., having a small number of nodes in each period. The presence of a large number of nodes increases the number of potential combinations of the associations in the decision network. In this case, the purpose of summarizing the set of simulated mining sequences is lost because the evaluation of the representative mining sequences tends to be similar to evaluating the whole set of simulated mining sequences. The complexity of the decision network depends on the degree of variability of the simulated mining sequences.

The variability of the simulated mining sequences depends on two aspects: geologic uncertainty and mine planning parameters. These two aspects cannot be controlled at the time of the construction of the decision network. The uncertainty of the deposit depends on the existing dataset. The mine planning parameters are specified as part of the design of the long-term mine plan. In the presence of high geologic uncertainty, the scenarios of the evolution of the block model are very different from each other, thus, the simulated mining sequences likely take very different paths. In presence of little geologic uncertainty, the block models are similar, thus, the simulated mining sequences tend to follow similar mining paths. As more real information is added to the dataset, the geologic uncertainty of the deposit is reduced and the simulated mining sequences converge towards a unique mining sequence. The mining strategy affects the variability of the mining sequences because it defines the geometric characteristics of the mining regions, including shape, fragmentation, and position. For example, the simulated mining sequences are less variable if mining conditions do not permit sub-regions or force that consecutive mining regions are closer to each other.

4.2 Clustering of Mining Regions

The nodes of the decision network are identified by clustering the mining regions of the simulated mining sequences, in each period. Among the different clustering techniques, hierarchical clustering is implemented mainly because the number of clusters is not an input parameter. The clusters are identified by providing a clustering threshold value that limits the maximum dissimilarity that is allowed between data elements in a cluster. In the literature, hierarchical techniques are divided into agglomerative or divisive. In the agglomerative techniques, each data element is considered to be one cluster, and data elements are merged until one cluster is formed. In the divisive techniques, all elements start in one cluster, which is divided until each data element is in its own cluster (Kaufman and Rousseeuw, 2005). The use of agglomerative clustering is discussed.

In the case of clustering point information datasets, each cluster can be represented by a data point elements. In each cluster, the representative data point element of continuos attributes can be calculated by averaging the point data element in the cluster (Tan et al., 2006). This representative point data element is referred to as cluster prototype. However, as the mining regions are not point data, they cannot be averaged directly to calculate the prototype. The proposed approach to calculate the mining region prototypes consists of calculating their upper and lower surfaces. The upper surface is the average of all the initial surfaces of the clustered mining regions. The lower surface is the average of all the final surfaces of the clustered mining regions. The blocks between the upper and lower surfaces are considered as the mining region prototype. The prototypes are later refined by removing very small sub-regions based on a minimum volume threshold.

4.2.1 Example of Hierarchical Clustering

An example of agglomerative hierarchical clustering is presented in Figure 4.3. The dataset consists of point data elements of two attributes. The relationships between the data elements are presented in the form of a dendrogram that is a tree-like plot that presents how the data elements are related to each other (Fielding, 2007). The dataset is classified into three clusters based on a clustering threshold of 0.30. The clustering threshold produces an outlier near Cluster 1, which is a data element that is not considered in any of the three clusters. The definition of the number of clusters is subjective. Milligan and Cooper (1985) reviewed over thirty techniques for defining the number of clusters and concluded that the performance of any criteria depends on the nature of the dataset. A small clustering threshold results in a small

dispersion of the data elements within the clusters, and a large clustering threshold increases this dispersion, thus, making the clusters more generic.



Figure 4.3: Example of agglomerative hierarchical clustering. A: Data elements of the example dataset. B: Classification of the data elements in three clusters. Each cluster is represented by a data element called prototype. The size of the prototypes is exaggerated to indicate the number of data elements in each cluster. C: Agglomerative dendrogram of the data elements and clustering at 0.30 threshold.

The definition of clusters depends on the way data elements are compared. In the previous example, the metric of comparison used is the Euclidean distance between data points. Fielding (2007) discussed several other measures of distance, including Chebychev, City Block, and Mahalanobis, that are also considered in most statistical packages. These distances measure how different two data elements are. However,

as the clusters start to form, the comparison between clusters is not straightforward because clusters comprise more than one data element. Three typical approaches to compare clusters are: 1) MIN, 2) MAX, and 3) group average. These approaches are also referred to as graph-based definitions of cluster proximity (Tan et al., 2006). In each of these types of comparison, the distance between clusters is calculated in terms of the pairwise comparison of their individual data elements. MIN considers the two most similar data elements of the two clusters. MAX considers the two most different data elements of the two clusters. The group average comparison considers the averaged distance between all the data elements of the two clusters. The group average comparison is preferred as the average of the distances does not introduce bias in the comparison.

4.2.2 Importance of Metric of Comparison for Mining Regions

The majority of the clustering techniques require that the data elements are multidimensional points in the Euclidean space (Halkidi and Vazirgiannis, 2008). The majority of comparison metrics are designed for this type of data. However, as the mining regions are non-point data elements, but volumetric objects, the conventional comparison metrics cannot be implemented directly. Thus, it is necessary to find a metric of comparison that accounts for the geometric festures of the mining regions. The geometry of the mining regions in each period is quite variable, as it may consist of one region or a set of sub-regions (see Figure 4.4). The complexity in the geometry of the mined regions depends on the constraints of the mine sequencing algorithm implemented. For example, aspects of the mining regions that are parameterized are: maximum fragmentation of the mining region, maximum number of benches that can be mined, and minimum mining width.

The classification of volumetric datasets is carried out in applications such as medical imaging, molecular biology, and metereology, where the comparison accounts for the geometric shape of the objects (Ankerst et al., 1999). In these applications, the comparison is focused on finding similarities between data elements, for which aspects including translation, rotation, and local detail of shapes, are consid-



Figure 4.4: Two mining regions, A and B, with different geometric configuration in the same period. The current topography is presented in gray and the mining regions in red. The mining region A consists of two sub-regions. The mining region B consists of one region.

ered as sampling errors that are to be corrected (Ankerst et al., 1999). However, in the case of clustering mining regions, it is not only important to consider aspects related to shape but also to position. Thus, these techniques cannot be directly implemented.

In clustering analysis, the metrics of comparison considered have to satisfy specific conditions. In the case of a metric of dissimilarity, four conditions have to be met to make the metric licit: 1) the metric is nonnegative, 2) the magnitude of the metric is independent of direction, 3) the metric is zero when two elements are identical, and 4) when comparing three element objects, the magnitude of the metrics should behave as the sides of a triangle. For practicality, semi-metrics can be used instead of full metrics (Fielding, 2007; Goshtasby, 2012). Semi-metrics satisfy only the first three conditions. In the case of the mining regions, the fourth condition is difficult to meet. A large part of the research on hierarchical clustering is focused on the derivation of the metrics of comparison, as they have a large impact on the clustering performance (Castro et al., 2004).

4.2.3 Region Distance

In this section, a semi-metric of dissimilarity called region distance is presented. This semi-metric is calculated in terms of the average distance between the discretized points of one mining region to the other mining region. The region distance is a semi-metric of dissimilarity that is the distance from each region to the other. As part of the calculation, the mining regions are discretized based on their respective blocks. In each part, the distance from one region to the other region is calculated as the average of the minimum distances from the discretized locations of the initial region to the other region. The region distance, d_r , is then calculated as the average of the distances between the two regions. This semi-metric is expressed as:

$$d_r (V_a, V_b) = \frac{d_{ab} + d_{ba}}{n_a + n_b},$$
(4.1)

where,

$$d_{ab} = \sum_{i=1}^{n_a} \min_{\mathbf{v} \in V_b} \|\mathbf{u}_i - \mathbf{v}\|, \ \forall \ \mathbf{u} \in V_a,$$
$$d_{ba} = \sum_{j=1}^{n_b} \min_{\mathbf{u} \in V_a} \|\mathbf{v}_j - \mathbf{u}\|, \ \forall \ \mathbf{v} \in V_b,$$

 V_a and V_b are the two mining regions compared, **u** and **v** are the locations of discretized points within V_a and V_b , respectively, and n_a and n_b are the number of discretized locations in V_a and V_b , respectively.

The region distance allows accounting for the difference in the shape and position of two mining regions. When the mining regions overlap but their shapes are different, the region distance is greater than zero as the discretized distances from the non-overlapped discretized locations add up. The region distance is zero when the two mining regions completely overlap and have identical shapes. However, when any of the two mining regions consist of small sub-regions, the region distance may tend to underestimate the dissimilarity. In Figure 4.5, an example of the effect of fragmentation in the region distance is illustrated. The region distance is used to calculate the dissimilarity of two cases: without sub-regions and with subregions. In the case with sub-regions, the small sub-regions are positioned within the perimeter of the other mining region. This particular configuration of the small sub-regions means that the discretized distances are calculated within the perimeter of each large sub-region, thus, underestimating the dissimilarity. The measure of dissimilarity in the fragmented case with small sub-region drops 75% with respect to the non-fragmented case. In practice, it is expected that mining regions are moderately fragmented.



Figure 4.5: Comparison of the performance of the region distance semi-metric d_r in non-fragmented (top) and fragmented (bottom) cases. The two regions to compare are coloured as gray and black.

A comparison between the region distance d_r and an expression based on the average of discretized distances between the two mining regions d_0 is presented in Figure 4.6. In the comparison, four configuration cases for two types of regions, non-fragmented and fragmented, are considered. Both the regiond distance d_r and expression d_0 account for the magnitude of separation between the two regions. However, the expression d_0 fails to account for the case when both regions overlap perfectly because the dissimilarity is greater than zero. This aspect invalidates the requirements of licit semi-metrics and has negative consequences during the clustering of regions because the dissimilarity between elements is exagerated. The clustering of regions with expression d_0 would unnecessarily require a larger threshold than if the region distance d_r is used. Despite potential shortcomings of semi-metric d_r when comparing fragmented cases, it performs reasonably well when the fragmentation of the regions is moderate.

4.2.4 Selection of Clustering Threshold

Along with the clustering threshold, another parameter to define clusters of mining regions is the minimum number of data elements in each cluster. This parameter is used to separate outliers from clusters. The clusters with fewer number of mining regions than this parameter are set as outliers. A value of 10% of the total number of mining sequences generated seems reasonable. In the construction of the decision network, the outliers are not considered as nodes.

The clustering threshold is inversely proportional to the number of nodes identified. The simplification of the decision network, by increasing the clustering threshold, tends to reduce the performance to capture the occurrence of representative mining sequences because the comparison becomes more generic. In Figure 4.7, an example of clustering non-fragmented regions is presented. A clustering threshold of 200 yields three clusters. If the clustering threshold is reduced to 100, the comparison between regions is more strict and Cluster 1 is split into two clusters. Similarly, increasing the clustering threshold to 400 results in a more generic comparison, in which Clusters 1 and 2 are merged.

In the case of clustering of mining regions, the clustering threshold is specified as a function of the maximum clustering error allowed. In K-Means clustering of continuous variables, the clustering error of a point data element is the distance to its prototype (Tan et al., 2006). In this context, the clustering threshold forms a circular perimeter around the prototype. Thus, the maximum separation distance



Performance of region distance d_r

	Non-Fragmented		Fragmented	
	d _r	d _o	d _r	d _o
Case 1:	0.000	0.353	0.000	0.486
Case 2:	0.174	0.584	0.122	0.655
Case 3:	0.622	1.039	0.402	1.045
Case 4:	1.114	1.526	0.855	1.524

Figure 4.6: Four cases of configurations of two types of regions, non-fragmented (top-left) and fragmented (top-right). Comparison of region distance d_r and average discretized distance d_0 (bottom) for the four cases presented.

between cluster elements is twice the clustering threshold, that is, the diameter of the circular perimeter around the prototype (see Figure 4.8). The clustering threshold is specified in terms of the maximum separation distance between cluster elements.

The maximum clustering error of a mining region with respect to its prototype is half of the average spacing of the existing drilling campaign. As the maximum separation between mining regions in a cluster is twice the maximum clustering



Figure 4.7: Example of clustering of non-fragmented regions. Configuration of 2D regions forming three clusters (top). Dendrogram of 2D regions considering the region distance to measure dissimilarity and clustering threshold of 200 (bottom).

error, the clustering threshold is set as the average drilling spacing. Wilde (2010) discussed the calculation of drilling spacing for two- and three-dimensional regions. To be able to use the clustering threshold in the dendrogram, it is necessary to calculate the equivalence of the average drilling spacing in terms of the region distance semi-metric. The clustering threshold is calculated by selecting a set of mining regions from the simulated mining sequences. Then, each mining region is duplicated and separated, with respect to their centers of mass, a distance equal to the average drilling spacing. The clustering threshold is calculated as the average region distance of the selected mining regions and their duplicates. The calculated value represents the maximum clustering threshold that can be used in the dendrograms.



Figure 4.8: Example of implementation of a cluster threshold for a 2D dataset. A set of fifty point elements and their corresponding prototype are presented. The cluster error of the 13th data element is indicated. The clustering threshold is specified as a maximum cluster error allowed of 1.5. The maximum cluster error is the radius of the cluster threshold perimeter.

The practical clustering threshold that is finally used is calculated after inspecting and tuning the maximum clustering threshold in all the dendrograms.

The reduction of the clustering threshold is beneficial as it reduces the clustering error in the clusters identified. The adjusted clustering thresholds in each dendrogram are expected to be similar to try to maintain a constant average clustering error in all the nodes of the decision network. This aspect will aid in the construction of the mining sequences of the branches because all the mining region prototypes are similarly representative of the mining regions in the clusters. However, adjusting the clustering thresholds should not significantly increase the number of clusters or the number of outliers in each period.

4.3 Selection of Representative Mining Sequences

The representative mining sequences correspond to the branch with the greatest occurrence. The number of representative mining sequences to be identified from the decision network depends on the number of alternatives that can be evaluated during the operating design of the mining project. Considering the different aspects involved in the operating design of the mining project, including design of mining operations, deployment of electrical infrastructure, and construction of transportation routes, it is considered that a range of three to five of the most representative mining sequences is a practical number of mining alternatives to be evaluated.

The occurrence of a branch is calculated by adding up the occurrences of the connections between nodes along the branch. The main branch is identified as the branch with the largest occurrence. The derived branches are identified by ruling out specific connections of the main branch. The derived branches correspond to the branches that are most likely to occur under the consideration that the removed connections do not exist. The representative mining sequences are not identified based on the ranking of branches in terms of occurrences because the difference between two representative mining sequences could be small and irrelevant. For example, two mining sequences that are different only in the last period are not significantly different.

As the decision metwork is a directed graph, the major mining patterns can be identified by implementing the Dijkstra algorithm (Dijkstra, 1959) to find the branch with the largest cummulative occurrence between the nodes. The Dijkstra algorithm is typically implemented on directed graph-networks to solve the problem of finding the shortest path between nodes. In the case of the decision network, the occurrences of the node connections are multiplied by -1 and considered as the distance between nodes. In this context, the main branch is identified by direct implemention of the Dijkstra algorithm on the decision network. The derived branches are identified by switching on and off specific node connections of the main branch and implementing the Dijkstra algorithm in the modified decision network. The mining sequence of a branch is built by putting together the mining region prototypes of their nodes. In Figure 4.9, a sketch of two representative branches and their respective mining sequences is illustrated. The mining sequences of the branches share the same periodic production and net cash-flow estimates that were calculated in the SLM paradigm. This is because the mining sequences were calculated under the same mining conditions.



Figure 4.9: Sketch of two representative branches and corresponding mining sequences. Decision network of simulated mining sequences and two representative branches (top). The mining sequences of each branch are calculated from the prototypes of the corresponding nodes (bottom).

4.4 Identifying Large Scale Mining Paths

A complex decision network can be simplified by merging consecutive periods. For example, in Figure 4.1, if the second and third periods are merged, the two mining sequences become identical. As periods are merged, the number of branches tend to decrease and their corresponding mining sequences consists of fewer steps. The merging of consecutive periods comes at the cost of making the decision network more generic, thus, the identified mining sequences may not have the level of detail necessary to be used in the operating design of the mining sequence. In this section, generic mining sequences that consists of four to six merged periods are referred to as large scale mining paths and can be used as supplementary information in the operating design of the mining sequence.

As the mining region of the first period is identical for all the simulated mining sequences, the merging of periods is carried out from the second period onwards. The new resulting decision network is much simplier. In Figure 4.10, a sketch of the decision network of simulated mining sequences with merged periods and two large scale mining paths are presented. The two large scale mining paths represent different mining directions that the set of simulated mining sequences could take. The large scale mining path 1 mines the deposit vertically. The large scale mining path 2 mines the deposit horizontally.



Figure 4.10: Sketch of two large scale mining paths. Decision-network of simulated mining sequences with merged periods (top). The two large scale mining paths present different mining directions (bottom).

4.5 Remarks

In the operating design of the mining sequence, only a few mining alternatives can be evaluated. In practice, the operating design of the mining sequence is obtained by implementing either Paradigms 1 or 2 as only one mining sequence is calculated. Paradigm 3 is not implemented for this purpose because of the difficulty of having to evaluate a large set of simulated mining sequences. The SLM paradigm also generates multiple mining sequences. In this chapter, a methodology to condense the simulated mining sequences of the SLM paradigm is presented, so this information can be used in the operating design of the mining sequence.

The set of simulated mining sequences are summarized in the form of a decision network. A few representative mining sequences are indentified from the branches that are more likely to occur. The few representative mining sequences are suitable to be used in the operating design of the mining sequence. The construction of the decision network relies on the implementation of hierarchical clustering techniques on the set of simulated mining sequences to calculate the decision nodes. As conventional metrics of comparison only deal with point-like elements, a semi-metric of dissimilarity called region distance is presented. The connections of the nodes are calculated in terms of the frequency of associations between mining regions in consecutive periods. The decision network is expected to be simple in the sense that it effectively condenses the information from the simulated mining sequences so a few representative mining sequences can be identified. If the variability in the simulated mining sequences is too high, it may not be possible to build a simpler decision network. The proposed methodology aims to find a small number of representative mining sequences only if they exist.

The decision network can be also used as a reference to design cost-effective infill drilling programs. An infill drilling program can be planned to confirm or invalidate the occurrence of a specific branch or branches. In this context, the infill drilling program does not aim to reduce geologic uncertainty in the deposit, but to reduce variability in the simulated mining sequences.

Chapter 5

Example of Implementation of the SLM Methodology

In this chapter, implementation aspects of the SLM paradigm, presented in Chapter 2, are described. For illustration purposes, an exercise that consists of calculating the mineable reserve of a synthetic deposit is discussed. The exercise is designed to focus only on the effects of the specified mining and data acquisition strategies. The mining project is in the feasibility stage and the existing information from the deposit consists of an exploratory drilling campaign. The mining strategy considers the operating aspects to mine the deposit. The data acquisition strategy aims to reduce uncertainty on a medium-term range. The pros and cons of the SLM paradigm are examined in the context of a comparison of how the mineable reserve is reported against conventional paradigms.

This chapter is organized as follows. In Section 5.1, aspects of the exercise to calculate the mineable reserve are detailed. The characteristics of the deposit, existing drilling campaign, mining parameters and data acquisition specifications are described. In Section 5.2, the implementation details of the SLM paradigm are discussed. This section consists of two parts: 1) description of the generation of a mining scenario, and 2) reporting of the mineable reserve. In Section 5.3, the SLM and conventional paradigms are compared in terms of the mineable reserve. In the comparison, three aspects of the mineable reserve are considered: 1) ore tonnage, 2) metal tonnage, and 3) profit.

5.1 Description of the Exercise

The exercise consists of evaluating the mineable reserves of a deposit in the feasibility stage of a project. The dimensions of the deposit are $400m \times 240m \times 160m$ in x, y, and z, respectively. An initial topographic surface is used to represent the original state of the deposit before the mining. The existing information collected from the deposit consists of an exploratory drilling campaign of twenty-eight vertical drillholes in a regular drilling pattern of $50m \times 50m$. The elevation of the collar of the drillholes depends on the initial topography. The drillholes are drilled down to the vertical limit of the deposit (Figure 5.1). The resolution of the block model is $100 \times 60 \times 40$ blocks in x, y, and z, respectively, with a block size of $4m \times 4m$ $\times 4m$. A constant block tonnage of 1MT per block is assumed constant for the deposit. In case a block is intersected by the initial topographic surface, the block tonnage is calculated based on the proportion of the block below the surface.



Figure 5.1: Existing exploratory campaign (black dots) and initial topography (surface).

The exploratory drilling campaign is sampled from an unconditional realization

of the deposit. The attribute simulated is metal grade in percentage units. The metal content is the only attribute to calculate the mining sequence. A constant value of 4.5% is added to the unconditional realization to make all the values positive. Thus, the global distribution of the simulated attribute is normal with mean 4.5% and standard deviation 1.0%. This distribution is considered to eliminate the effect of outliers in the construction of the estimated block model. The variogram model used to generate the unconditional realization is a spherical model with an isotropic range of 80m.

Two destinations of mined material are considered: 1) ore blocks are sent to the processing plant and 2) waste blocks are sent to the waste dump. The destination of each block is calculated based on a cut-off grade of 4.5%. The plant capacity in each period is 2500MT of ore material. The total project expenses without considering the cost of the infill program is 100000 USD. This amount is spent at the beginning of the mining project. The budget dedicated to implement the infill program is 20000 USD. This amount is spent during the first twelve periods of the lifetime of the mining project. The total project expenses is 120000 USD.

The mnining sequence algorithm searches regions in the deposit that satisfy the plant requirements without considering the use of stockpiles. A customized GsLib program described in Appendix A is used in the implementation. The geometric configuration of the mining regions is constrained to have at most two sub-regions. There is no restriction regarding the position of the sub-regions. The mining slope of the mining regions is set to 45° . There are no restrictions regarding the tonnage of mined waste material. In each period, the algorithm selects sequentially the mining regions as long as the minimum planned ore production is above 1000MT and the profit of the mining regions is positive. The revenue of a mining region is calculated by adding up the economic values of the blocks, based on simulated attributes, within the mining limits. The cash-flow is calculated as a function of revenue and corresponding production variability. The economic value of each block Bv in USD

units is calculated as:

$$Bv_i = \begin{cases} (m_p \times z_i \times 22.046 - c_{ore}) \times Tp_i & ; \text{ if } z_i \ge 4.5\% \\ -c_{waste} \times Tp_i & ; \text{ if } z_i < 4.5\% \end{cases}$$

where, m_p is the metal price in USD/pound units, z is the block grade in percentage units, c_{ore} and c_{waste} are cost of mining ore and wate material in USD/MT units, respectively, and Tp is the block tonnage in MT units, based on the proportion of the block below the current topographic surface. The letter *i* denotes the index of the block. The metal price considered is 1USD/pound. The value 22.046 pound/MT represents the conversion from pounds to MT divided by 100 to convert metal grade percentage to proportion. The cost of mining ore and waste material is 48USD/MT and 45USD/MT, respectively. To account for production variability in the calculation of the cash-flow, the penalty for over-production is 500USD per 1MT of produced metal. In the case of under-production, it is considered that the penalty is the lost portion of the cash-flow due to not reaching the production target. For illustration purposes, no discount rate is considered. A summary of the mining specifications is presented in Table 5.1.

The budget of the infill program is distributed from Period 1 to 12 to implement the following configuration of amount of drilling: eight drillholes from Period 1 to 5, six drillholes from Period 6 to 9, and three drillholes from Period 10 to 12. In each period, the infill campaign aims to reduce uncertainty in the medium term range by targeting the most unsampled regions within an area of influence of the following mining region. Along with the infill drilling campaign, blasthole drilling data is also considered.

5.2 Implementation of SLM Paradigm

This section describes implementation aspects of the SLM framework and how multiple mining scenarios are generated. Each mining scenario is a realization of how the deposit might be mined based on the specified mining and data acquisition strate-

Specification	Description		
Project extent	$400 \text{m} \times 240 \text{m} \times 160 \text{m}$ in x, y, and z directions.		
Block model resolution	$100 \times 60 \times 40$ blocks in x, y, and z directions.		
Block size	$4m \times 4m \times 4m$ in x, y, and z directions.		
Block tonnage	1MT.		
Cut-off grade	4.5%.		
Plant capacity	2500MT.		
Waste dump capacity	infinite		
Total project expenses	100000 USD.		
w/o infill drilling			
Infill program budget	20000 USD.		
Mining slope	45°		
Metal price	1USD/pound		
Cost of mining ore	48USD/MT		
Cost of mining waste	45USD/MT		
Penalty for	500USD/MT		
overproduction			

Table 5.1: Exercise specifications.

gies. The mineable reserve is calculated based on one hundred mining scenarios.

5.2.1 Simulation of Mining Scenarios

Each mining escenario is constructed sequentially based on the simulation of the mining activities that occur in each period. These activities take place at the beginning, during, and at the end of each period. For illustration purposes, the generation of one mining scenario is described.

In the first period, the conditioning information available to build the estimated block model consists of only the existing exploratory campaign. The estimated block model is built by implementing simple kriging, using GsLib KT3D (Deutsch and Journel, 1998). The stationary mean considered is 4.5%. In Figure 5.2, the existing exploratory campaign and the starting estimated block model are presented.

The mining sequence is calculated based on the current estimated block model and specified mining conditions. This mining sequence is also the mining sequence of Paradigms 1 and 2. According to the current mining sequence, the lifespan of the mining project is thirteen periods. The mining region of the next period is targeted



Figure 5.2: Estimated block model, conditioned to the existing exploratory campaign, used to plan the first period. A and C: Vertical east-west (north 144m) and south-north (east 200m) cross sections of the estimated block model and drilling information. B: Conditioning information of the estimated block model. D: Topographic surface at the beginning of the first period and references of the vertical cross sections A (red line) and C (blue line).

for extraction. From this region, the planned ore material to produce is 2486MT of with an average grade of 5.44%. The estimated metal production and cash-flow are 135.27MT and 175434USD, respectively. In Figure 5.3, the calculated mining sequence and the region targeted for extraction in the first period are presented.

After identifying the region to mine, the mining of the deposit and the acquisition of additional information are simulated. In practice, the targeted region for extraction is mined by continuously adapting the mine plan to reach the estimated production target. The region mined ends up being different from the targeted region. The degree at which the mine plan is adapted depends on the precision of the estimated block model within the limits of the targeted region. The adaptation of the mine plan comes at an extra cost, which reduces the cash-flow of the current period. For simplicity, it is assumed that the targeted region is strictly mined. Thus, the estimated production is not reached. The extra costs due to adapting the mine



Figure 5.3: Calculated mining sequence based on existing exploratory campaign. A and C: Vertical east-west (north 144m) and south-north (east 200m) cross sections of the calculated mining sequences. B: Current ultimate-pit in the first period. D: Region to be mined in the first period (red region) and references of the vertical cross sections A (red line) and C (blue line).

plan are assessed based on the production gap.

The mining of the deposit is simulated by calculating the updated topographic surface. Each mining region has two associated topographies: 1) initial and 2) final. The initial topography is the topographic surface at the beginning of the period. The final topography, at the end of the period, is the updated initial topography after considering the mining region is already extracted.

The executed production is calculated based on a realization of the deposit within the limits of the targeted region. In this period, the realization is generated conditioned to the existing exploratory campaign, using GsLib SGSIM (Deutsch and Journel, 1998). The simulated ore material produced is 2422MT with an average grade of 5.78%. The corresponding simulated metal production and revenue are 140.22MT and 186537USD, respectively. As there is a metal production surplus of 4.96MT, the cash-flow of the first period is calculated by penalizing the revenue as a function of the excess in metal production. The corresponding penalty is 2478USD

and the cash-flow is 184059USD.

The additional information is collected in the form of blasthole and infill drilling. In the case of blasthole drilling, for simplicity, four data points are sampled randomly from the assumed-true realization within the mining region limits. In this exercise, this amount of sampled data points has been tuned to have a moderate impact on the increment of the profit of the mining project that resembles blasthole drilling data. Alternative and more complex approaches to simulate blasthole data are listed in Section 2.3.3. The implementation of these approaches requires having access to the blasthole sampling error. The implementation of these approaches would require the use of estimation and simulation techniques that are able to integrate data from different sources. In the case of infill drilling, the configuration of the infill campaign is calculated sequentially, one infill drillhole at a time. The position of each drillhole is calculated by identifying the areas with least drilling coverage. The drilling coverage is expressed as:

$$dc(\mathbf{u}) = col(\mathbf{u}) \times seq(\mathbf{u}) \times cnt(\mathbf{u})$$

where, dc is the drilling coverage, col is the standardized minimum distance to the closest drillhole collar, seq is one minus the standardized minimum distance to mined region, cnt is one minus the standardized distance to the center of the deposit, and **u** denotes the position in terms of x and y coordinates. The col term accounts for the existing drillholes. The seq term makes sure the next drillhole is located near the mined region. The cnt term tries to avoid positioning the next drillhole near the edge of the deposit. The position of the next drillhole, \mathbf{u}_{ddh} , is the location at which the drilling coverage dc is the maximum. The elevation of the drillhole collar is the elevation of the initial topographic surface at location \mathbf{u}_{ddh} . The data values of the new drillhole is added to the exploratory drilling campaign. In the next iteration, the calculation of the position of the second new drillhole considers the existing exploratory drilling campaign along with the first new drillhole. In Figure

5.4, the calculation of the positions the first two new infill drillholes and the entire infill campaign of the first period are presented.

In the second period, the conditioning information available to build the block model consists of the existing exploratory campaign and the simulated additional information in the previous period. The estimated block model is built below the updated topographic surface after mining the region targeted in the first period. In Figure 5.5, the estimated block model of the deposit, used to plan the second period and its conditioning information are presented.

The mining sequence is re-calculated based on the new estimated block model and specified mining conditions. The lifetime of the mining project is still thirteen periods as there are twelve periods scheduled in the updated mining sequence. The mining region of the next period is targeted for extraction. The geometric configuration of the targeted region is different than the second mining region of the first mining sequence, but positioned closely. This is because the simulated additional information, collected at the end of first period, confirmed some of the features of the first estimated block model. In other mining scenarios, the realizations of the simulated additional information may invalidate these features and make the geometric configuration of the targeted region significantly different in geometry and position. From this region, it is expected to mine 2513MT of ore material with an average grade of 5.47%. The estimated metal production and revenue are 137.64MT and 179536USD, respectively. In Figure 5.6, the calculated mining sequence and the region targeted for extraction in the second period are presented.

The simulated production is calculated conditioned to the available information at the beginning of the second period, existing exploratory campaign and simulated additional information in the first period. The simulated ore material produced is 2480MT with an average grade of 5.68%. The simulated metal production and revenue are 140.86MT and 186703USD, respectively. As there is a metal production surplus of 3.20MT, the cash-flow of the second period is calculated by penalizing the revenue as a function of the excess in metal production. The corresponding penalty is 1601USD and the cash-flow is 185101USD.



(A) Corresponding drilling coverage maps and collar positions of first (top) and second (bottom) infill drillholes.



(B) Infill campaign of the first period and updated topographic surface.

Figure 5.4: Calculation of geometric configuration of first infill campaign.


Figure 5.5: Estimated block model, conditioned to the existing exploratory campaign and simulated additional data, used to plan the second period. A and C: Vertical east-west (north 144m) and south-north (east 200m) cross sections of the estimated block model and drilling information. B: Conditioning information of the estimated block model. D: Topographic surface at the beginning of the second period and references of the vertical cross sections A (red line) and C (blue line).

The acquisition of additional information is simulated as in the case of the first period. The simulation of the mining of the following periods is repeated until no more than 1000MT of ore material can be planned. The mining sequence of this mining scenario consists of all the regions targeted for extraction in each period. In this mining scenario, the deposit is mined in twelve periods.

The mineable reserve is calculated based on the simulated production of each period. The presence of geologic uncertainty does not permit one to fully rely on the estimated production to calculate mineable reserves. In Figures 5.7-A and -B, the estimated and simulated ore tonnage and metal tonnage per period are compared. Although in practice the estimated producion is achieved by fixing the mine plan in each period, the extra costs incurred due to fixing the mine plan reduce the profit margin of the mining project. This extra cost is accounted for in each simulated cashflow by penalyzing the revenue as a function of the variability between estimated



Figure 5.6: Calculated mining sequence based on existing exploratory campaign and simulated additional data. A and C: Vertical east-west (north 144m) and south-north (east 200m) cross sections of the calculated mining sequences. B: Current ultimate-pit in the second period. D: Region to be mined in the second period (red region) and references of the vertical cross sections A (red line) and C (blue line).

and simulated metal production (Figure 5.7-C). The mineable reserve values of the simulated mining scenario are summarized in Table 5.2. The profit of the mining project is calculated by substracting the project expenses from the sum of cash-flows. For simplicity, no reserve classification scheme is implemented. A classification of blocks into proven, probable, and possible, to calculate the mining sequence, adds more complexity to the problem. All the blocks in the deposit are considered in the calculation of the mining sequence.

	\mathbf{Ore}	\mathbf{Metal}	\mathbf{Profit}
	(MT)	(\mathbf{MT})	(USD)
Estimated	29563	1562.53	1274351
Simulated	28126	1530.37	1193597

Table 5.2: Comparison between reported estimated and simulated mineable reserves of one simulated mining scenario of the SLM paradigm. The reported mineable reserve corresponds to the simulated production.



Figure 5.7: Summary of estimated and simulated production per period. A: Profile of estimated and simulated ore tonnage. B: Profile of estimated and simulated metal tonnage. C: Profile of estimated and simulated cash-flow.

5.2.2 Reporting Mineable Reserve Based on Simulated Scenarios

The one hundred mining scenarios allow for the assessment of uncertainty of different aspects of the mineable reserve. In Figure 5.8, histograms of ore tonnage, metal tonnage, and profit are presented. The reported mineable reserve depends on the specified mining and data acquisition strategies. A different combination leads to a different set of results, even when their implementation costs may be similar. For example, by instructing the mining sequence algorithm to also reduce production variability, the mineable reserves may suffer from not maximizing the global profit, but benefit from reducing production gaps. The modification of the infill strategy also affects the mineable reserve.

The lifetime of the mining project is on average thirteen periods and fluctuates



Figure 5.8: Histograms of mineable reserve values calculated in the SLM paradigm. A: Ore tonnage. B: Metal tonnage. C: Profit.

from eleven to fifteen periods. In Figure 5.9, a histogram of total number of periods of the simulated mining scenarios is presented. To report the mineable reserve, it is considered the lifetime of the mining project is thirteen periods.



Figure 5.9: Histogram of number of periods of one hundred mining scenarios.

5.3 Comparison to Conventional Paradigms

The SLM and conventional paradigms are compared in terms of reported mineable reserve. All the paradigms share the same mining strategy and project expenses. This means that a similar infill drilling budget is considered in all the paradigms. The comparison of the SLM paradigm against Paradigms 2 and 3 is made based on a simulated block model of one hundred realizations. In the case of Paradigm 1, only one scenario based on an estimated block model is considered.

In paradigm 1, the mining sequence is calculated based on the estimated block model conditioned to the existing exploratory drilling campaign. The mineable reserve of Paradigm 1 is reported in terms of estimated production (see Table 5.3). As the mining strategy tries to maximize cash-flow in each period, regions with little proportion of waste material tend to be targeted first. However, these regions do not reproduce the spatial variability of the metal attribute because of the smooth characteristics of the estimated block model. This yields a high production variability.

	Ore	\mathbf{Metal}	Profit
	(MT)	(MT)	(USD)
Paradigm 1	32581	1681.68	1374789

Table 5.3: Reported mineable reserve in Paradigm 1, calculated based on estimated production.

The mineable reserve of Paradigm 2 is reported in terms of the simulated production of the mining sequence calculated in Paradigm 1. In contrast to the SLM paradigm, the deposit is mined without collecting any additional information. Thus, the production variability in Paradigm 2 is higher than the production variability in the SLM paradigm. In Figure 5.10, histograms of ore tonnage, metal tonnage, and profit calculated in Paradigm 2 are presented.

The mineable reserve of Paradigm 3 is reported in terms of the simulated production of a set mining sequences, where each mining sequence is calculated based on a realization of the simulated block model. The set of realizations are generated conditioned to the existing exploratory drilling campaign. In this context, the de-



Figure 5.10: Histograms of mineable reserve values calculated in the Paradigm 2. A: Ore tonnage. B: Metal tonnage. C: Profit.

posit is mined considering perfect knowledge of the deposit. Thus, the acquisition of additional information has no effect on reducing production variability as the deposis is mined perfectly. In Figure 5.11, histograms of ore tonnage, metal tonnage, and profit calculated in Paradigm 3 are presented. The mineable reserve reported is unrealistic as achieving perfect mining of the deposit is impossible.

In Figure 5.12, a comparison of production variability of Paradigm 2 and the SLM paradigm, in terms of mean-absolute-error of produced metal tonnage, is presented. In these paradigms, the production variability negatively affects the performance of the mining sequence to achieve the estimated production. The SLM paradigm has less production variability than Paradigm 2 because the simulation of additional information is considered on top of the existing drilling campaign. In Paradigm 3, there is no production variability as perfect knowledge of the deposit



Figure 5.11: Histograms of mineable reserve values calculated in the Paradigm 3. A: Ore tonnage. B: Metal tonnage. C: Profit.

is considered before mining the deposit. Among these paradigms, the mineable reserve reported in the SLM paradigm is more realistic because it accounts for the acquisition of additional information, which is part of the mining of the deposit.

In Figure 5.13, the paradigms are compared in terms of three characteristics of the reported mineable reserves, ore tonnage, metal tonnage, and profit of the mining project. The mineable reserve reported in Paradigm 1 cannot be compared directly to the rest of the paradigms as it is the only one reported in terms of estimated production. Depending on how the estimated block model is tuned, the reported mineable reserve of this paradigm could be greater or smaller than the mineable reserves reported in the rest of the paradigms. In this case, Paradigm 1 over-estimates the potential of the deposit. The paradigms that report the mineable reserve in terms of simulated production, Paradigm 2, SLM paradigm, and



Figure 5.12: Production variability of Paradigm 2 and SLM paradigm in terms of mean-absolute-error of metal production.

Paradigm 3, present three cases of how the deposit might be mined based on the geologic information available: 1) only initial information, 2) initial plus additional information, and 3) complete information, respectively. The profit of the mining project increases as the production variability decreases. This behavior is because the mining sequence algorithm makes more precise decisions, to maximize the profit of the mining project, as the uncertainty in the estimated block model is reduced. In this exercise, the other characteristics of the mineable reserve also follow this behavior. However, this is circumstantial as no restrictions regarding the amount of waste material are specified. The production characteristics of the mineable reserve are not necessarily related to the uncertainty in the estimated block model.

5.4 Remarks

In this chapter, an exercise that illustrates the calculation of the mineable reserve in the context of the SLM paradigm is presented. In this paradigm, the mine plan consists of a mining and a data acquisition strategies as opposed to conventional paradigms, where only the mining strategy is considered. In the present exercise, the mining strategy is specified as a mining sequence algorithm that targets the most profitable regions. The data acquisition strategy is specified as a customized algorithm that calculates the position of the infill drillholes with respect to how



Figure 5.13: Comparison of reported mineable reserves in the SLM and conventional paradigms. A: Ore tonnage. B: Metal tonnage. C: Sum of net cash-flow. In the box plot legend, the letters q and n stand for quantile and number of samples, respectively.

the deposit is being mined focusing on reducing uncertainty on the medium term range. The mining and data acquisition strategies interact together to simulate how uncertainty in the deposit evolves periodically.

The SLM paradigm and Paradigms 2 and 3 calculate the mineable reserve in terms of simulated production. The impact of uncertainty on the calculation of the mineable reserve is measured in terms of the production variability, which has negative effects in the performance of the mining sequence. Paradigm 2 has the largest production variability as only the existing information is considered. The SLM paradigm has less production variability than Paradigm 2 as both the existing information and additional simulated information are considered. Paradigm 3 is unrealistic as it is considered that there is no variability in production. These three paradigms present different versions of how the mineable reserve of a mining project is reported. The SLM paradigm considers a more realistic aspects of the mining of the deposit, thus, the reported mineable reserve characterizes better the potential of the deposit.

Chapter 6

Example of Evaluation of Infill Drilling

In this chapter, implementation aspects of the methodology to evaluate the acquisition of future infill information, described in Chapter 3, are discussed. The exercise presented in Chapter 5 is continued considering the infill program. In this exercise, an infill program that considers an infill strategy which aims to reduce uncertainty in the medium term is evaluated. The evaluation consists of finding the amount and timing parameters that for the specified infill strategy results in a cost effective infill program that aims to maximize the profit of the mining project.

This chapter is organized as follows. In Section 6.1, the exercise to find the design parameters of the infill program is described. The exercise consists of evaluating a set of configurations of amount of drilling to find the one that aims to maximize the profit contribution. In Section 6.2, aspects of the estimation of the profit contribution are discussed. The profit contribution is estimated based on two components, revenue contribution and drilling cost. The profit contribution is estimated for a range of parameters under study. In Section 6.3, the selection of the best configuration of amount of drilling, from a set of candidates, is described.

6.1 Description of the Exercise

The present exercise is prepared on the basis of the exercise discussed in Chapter 5. This exercise consists of defining the infill program of the mining project based on an specified infill strategy. The goal of this exercise is to find the corresponding configuration of amount of drilling that yields the most profit. The specified infill strategy focuses on reducing uncertainty in the medium-term as the mining progress. A time range of twelve periods is considered based on the average lifetime of the mining project of the initial exercise. As in the case of the initial exercise, for illustration purposes, the effect of the interest rate in the calculation of the profit of the mining project is not considered.

As the infill program is to be implemented during the first twelve periods, the configuration of amount of drilling is defined by the number of infill drillholes to be collected in the first and twelfth periods. The amount of infill drillholes of the intermediate periods is specified based on the linear projection between these two parameters. For simplicity, all the infill drillholes are drilled down to the vertical limit of the deposit. The sampling scheme consists of nine cases that are considered for both of the first and twelfth periods: 0, 2, 4, 6, 8, 10, 15, 20, and 40 drillholes. Thus, a total of 81 configurations of amount of drilling are initially evaluated. In each mining scenario, the cost of infill drilling is calculated based on the amount of the infill program as a function of the amount of drilling. The cost of infill drilling is assumed to be 10USD for each 4m drilled.

6.2 Calculation of Economic Metrics

The infill program is evaluated in terms of the increase in the profit of the mining project. The profit contribution $\Delta SNCF$ is calculated based on two components: 1) contribution to the sum of cash-flow ΔSCF and 2) cost of infill drilling SCD. For each of the 81 configurations of amount of drilling, these two components are calculated in terms of the average of the SCF and SCD of the one hundred mining scenarios considered.

6.2.1 Contribution to the Sum of Cash-Flow

The contribution to the sum of cash-flow ΔSCF is calculated as the difference of the SCF after implementing a specified infill program and the SCF without collecting any additional information, which is also the SCF of Paradigm 2. The characterization of the progressive growth of the SCF due to the acquisition of more infill information is sensitive to the amount of mining scenarios considered. The use of a small number of mining scenarios might introduce extra variability to the SCFestimates. In Figure 6.1, the increment of the SCF is presented considering four cases of number of mining scenarios: 25, 50, 75, and 100. The first three cases do not characterize the relationship between the SCF and the amount of drilling because of the limited number of mining scenarios. The use of one-hundred mining scenarios results in a fair characterization of the behavior of the SCF as a function of infill information acquired. The increment of the SCF considers implicitly the acquisition of blasthole information along with infill information. In the amount of drilling configuration (0,0), zero drillholes in the first period and zero drillholes in the twelfth period, the ΔSCF increment is due to only blasthole information. This configuration of amount of drilling yields the minimum SCF of the mining project.

The scheme of the 81 configurations of amount of drilling emphasizes the evaluation of cases with moderate number of drillholes. The cases with moderate number of drillholes are of more interest because of their reduced implementation cost. The cases with a large amount of drillholes are sparsely sampled and evaluated for referential purposes. To analyze all the possible configurations of amount of drilling, based on the two parameters for the first twelve periods, a surface of the ΔSCF component is fitted based on the ΔSCF estimates of the 81 configurations of amount of drilling (Figure 6.2). This ΔSCF surface is used to analyze the intermediate configurations of amount of drilling that are not contemplated initially in the sampling scheme. The fitted ΔSCF surface also aims to filter out the extra variability from the ΔSCF estimates because of the use of a limited number of mining scenarios.



Figure 6.1: Growth of SCF as a function of infill information acquired. The SCF of Paradigms 2 and 3, based on one-hundred mining scenarios, are referential (dashed lines). A: Case with 25 mining scenarios. B: Case with 50 mining scenarios. C: Case with 75 mining scenarios. D: Case with 100 mining scenarios.

The growth of the ΔSCF surface starts accelerated and gradually slows down until it reaches a plateau. The ΔSCF surface is bounded by the ΔSCF that corresponds to the case of having access to perfect information minus the cost of uncertainty of the first period.

The ΔSCF surface characterizes the response of configurations of amount of drilling within the region of parameters under study. The form of the ΔSCF surface depends both on the specified drilling strategy and the mining strategy considered. In the ΔSCF surface, the configurations of amount of drilling that prioritize the acquisition of drillholes in early periods perform better than the configurations of amount of drilling that collect more drillholes in late periods. This is because the



Figure 6.2: Fitted SCF contribution ΔSCF surface over region of parameters under study. Left: Region of configurations of amount of drilling under study and corresponding ΔSCF surface. The 81 configurations of amount of drilling are represented as black dots. Right: Vertical cross section that corresponds to configurations of amount of drilling in which a constant number of drillholes are collected per period. The intersection of the ΔSCF surface is represented as a solid line. The dashed line represents the ΔSCF component considering perfect information of the deposit (Paradigm 3).

collection of drillholes in early periods tends to affect more mining regions than drillholes in late periods. Infill program designs with more acquisition of infill drillholes in late periods are not efficient.

6.2.2 Cost of Infill Drilling

The cost of drilling SCD represents the expected cost of implementing an infill program with a specified configuration of amount of drilling. Although blasthole drilling is implemented, the cost associated is not charged in the SCD component because it is already considered as part of operating expenses. As in the case of the ΔSCF component, the SCD component is also analyzed by considering a surface that maps the region of parameters under study (Figure 6.3). The SCD surface is fitted based on the SCD of the 81 configurations of amount of drilling. The SCDestimates present much less variability than the ΔSCF estimates because the SCDis implemented within a pre-defined range of periods.

The growth of the SCD surface is linear as it is proportional to the amount



Figure 6.3: Fitted cost of drilling SCD surface over region of parameters under study. Left: Region of configurations of amount of drilling under study and corresponding SCD surface. The 81 configurations of amount of drilling are represented as black dots. Right: Vertical cross section that corresponds to configurations of amount of drilling in which a constant number of drillholes are collected per period. The intersection of the SCD surface is represented as a solid line. The intersection of the ΔSCF surface is represented as a dotted line for reference. The dashed line represents the ΔSCF component considering perfect information of the deposit (Paradigm 3).

of infill information collected. In contrast to the ΔSCF surface, the SCD quickly reaches the ΔSCF and keeps growing. In Figure 6.3-left, a comparison of these two surfaces, for configurations of amount of drilling with constant number of drillholes per period, is presented. The region of configurations of amount of drilling in which the SCD is greater than the ΔSCF is considered profitable.

6.2.3 Contribution to Profit

The profit contribution $\Delta SNCF$ quantifies the economic impact of implementing the infill program. As in the case of its components, ΔSCF and SCD, the $\Delta SNCF$ is analyzed in the form of a surface for the region of parameters under study (Figure 6.4). The $\Delta SNCF$ surface is calculated by subtracting the SCD from the ΔSCF surface. The $\Delta SNCF$ surface has a concave downward shape because of the growth characteristics of the ΔSCF and SCD surfaces. Based on a zero contribution threshold, the $\Delta SNCF$ surface can be split in two sub-regions, positive and negative profit contribution. The positive contribution sub-region classifies the configurations of amount of drilling that potentially yield an economic return after implementation. The positive contribution sub-region is of main interest during the evaluation of the acquisition of future infill information as the search of the most cost efficient configuration of amount of drilling in focused in this sub-region. The negative contribution sub-region classifies the configurations of amount of drilling which implementation potentially result in economic losses.



Figure 6.4: Profit contribution $\Delta SNCF$ surface over region of parameters under study. Left: Region of configurations of amount of drilling under study and corresponding $\Delta SNCF$ surface. The 81 configurations of amount of drilling are represented as black dots. Right: Vertical cross section that corresponds to configurations of amount of drilling in which a constant number of drillholes are collected per period.

The sampling scheme of the 81 configurations of amount of drilling provides a good characterization of the $\Delta SNCF$ surface. Thus, no additional configurations of amount of drilling are needed to be evaluated to improve the characterization of the current $\Delta SNCF$ surface. In case additional configurations of amount of drilling were needed, they would target the region with high $\Delta SNCF$ to improve the surface. The evaluation of additional configurations of amount of drilling would involve repeating the calculation of the ΔSCF and SCD surfaces as discussed in previous sections.

6.3 Selection of Configuration of Amount of Drilling

The configuration of amount of drilling of the infill program is selected from an analysis of the $\Delta SNCF$ surface. A set of candidate configurations of amount of drilling are identified by specifying a region of interest in the $\Delta SNCF$ surface. The region of interest is defined by a threshold value with respect to the maximum $\Delta SNCF$ of the surface. A threshold of 95% of the maximum $\Delta SNCF$ is considered in the analysis. The maximum $\Delta SNCF$ in the surface is 29395 USD and the threshold for defining the region of interest is set to 28000 USD. This threshold is slightly greater than 95% of the maximum $\Delta SNCF$. The selection of the configuration of amount of drilling is carried out by analyzing the targeted region and the configurations of amount of drilling in it (Figure 6.5). There are five configurations of amount of drilling in the region of interest: (8,8), (10,6), (10,8), (10,10), and (15,4).



Figure 6.5: Profit contribution $\Delta SNCF$ surface divided in regions and selection region used to identify candidate configurations of amount of drilling. Left: Profit contribution regions greater or equal than 0, 10000, 20000, 24000, and 28000 USD. Right: Profit contribution region greater or equal than 28000 USD and corresponding parameters of configurations of amount of drilling (black dots).

An aspect to consider in the selection of the configuration of amount of drilling is the reduction of the SCD. For example, if there are two configurations of amount of drilling with different SCD that yield very similar $\Delta SNCF$, the one that is less expensive is preferred for implementation. This selection criteria is applied to all the candidate configurations of amount of drilling in the targeted region. Among the five candidates, the two less expensive alternatives of configurations of amount of drilling are (8,8) and (10,6). Their *SCD* are 30723 USD and 31349 USD, respectively. Any of these two configurations of amount of drilling are good alternatives for implementation. In Chapter 7, the mining sequence of the deposit is analyzed considering the (8,8) configuration of amount of drilling.

6.4 Remarks

In this chapter, the performance of a specified infill strategy based on the reduction of uncertainty in the medium term is evaluated in terms of the contribution to the profit of the mining project. The evaluation is focused on finding the best cost effective configuration of amount of drilling that aims to maximize the profit of the mining project. The performance of the infill strategy is analyzed based on three response surfaces: 1) contribution to profit, 2) contribution to revenue, and 3) drilling cost. A set of 81 configurations of amount of drilling are considered to calculate the response surfaces. A subset of candidate configurations of amount of drilling is identified by specifying a region around the maximum value of the profit contribution surface. The preferred configuration of amount of drilling, among the candidate alternatives, is selected considering a reduction in the cost of implementation.

A significant number of mining scenarios, in this case one hundred, are needed to properly estimate the responses of the configurations of amnount of drilling. The response surfaces are fit based on the estimated responses of the 81 configurations of amount of drilling. The response surfaces filter the remaining variability due to having access to only a limited number of mining scenarios.

Additional infill strategies can also be evaluated following the analysis presented in this chapter. The performance of these infill strategies would be measured in terms of their respective profit contribution surfaces. Aspects to consider in the comparison of infill strategies would include: shape of the profit contribution surface, configurations of regions around maximum profit contribution, and cost of drilling. The proposed methodology for evaluating infill strategies provides a rich source of information for designing the infill program to implement.

Chapter 7

Example of Clustering of Mining Scenarios

In this chapter, implementation aspects of the methodology to summarize the mining sequences generated by the SLM paradigm framework, described in Chapter 4, are discussed. The mining sequences of the case selected in the exercise of Chapter 6, eight infill drillholes per period, are considered as input. The goal of this exercise is to identify a reduced set of mining sequences that can be used in the operating design of the mining sequence.

This chapter is organized as follows. In Section 7.1, the steps to construct the decision network are described. In each step, the implementation aspects, including selection and interpretation of parameters, are described. In Section 7.2, aspects of identifying major mining sequences are discussed. For illustration purposes, two major mining sequences are evaluated. In Section 7.3, aspects of identifying large scale mining paths are discussed.

7.1 Construction of the Decision Network

Step 1 - Selection of Range of Periods and Calculation of Dendrograms

The duration of the major mining sequences is set to fourteen periods because the majority of the simulated mining sequences, 83%, span fourteen periods. This decision affects the design of the operating mining sequence as it establishes the lifetime of the mining project.

For a duration of fourteen periods, a set of thirteen dendrograms, one per period, is calculated. The first period does not require a dendrogram as all the mining regions are identical. The calculation of the set of dendrograms requires the calculation of matrices of dissimilarity of the mining regions in each period. The matrices of dissimilarity are computed considering the region distance semi-metric. As soon as clusters begin to form, the dissimilarity between forming clusters is calculated by considering the group average comparison, that is, the dissimilarity between forming clusters is the average of the region distances between their mining regions. The dendrogram is a structured representation of the matrix of dissimilarity.

Step 2 - Selection of Clustering Threshold and Definition of Clusters

The clustering threshold is used to classify the mining regions into clusters based on the structured representation of the dissimilarities in the dendrogram. The selection of the clustering threshold consists of: 1) calculation of the maximum clustering threshold and 2) global adjustment of the clustering threshold.

The maximum value of the clustering threshold is the equivalent average drilling spacing in terms of the region distance. The average drilling spacing is 50m as the existing drillholes are positioned over a regular grid pattern of $50m \times 50m$. The equivalent region distance of the average drilling spacing is calculated based on sixty mining regions selected randomly from the simulated mining sequences, five mining regions per period, from Period 2 to 13. Each mining region is duplicated and separated a specified distance with respect to its center of mass. Then the corresponding region distance is calculated. For illustration purposes, the selected mining regions are evaluated for a range of separation distances from 0 to 80m. In Figure 7.1-A, for each mining region, the region distance is presented as a function of the separation distance. The variability in the region distance curves is due to the differences in the geometry of the mining regions, including fragmented and non-fragmented configurations. The equivalent region distance is calculated based

on the average of the region distance curves. The equivalent region distance of 50m is 18.46. Along with the selection of the clustering threshold, another parameter needed to define clusters is the minimum number of mining regions. This value is the 10% of the total number of simulated mining sequences. Clusters with less than ten mining regions are labeled as outliers.

After inspecting the maximum clustering threshold in the set of dendrograms, it is found that the clustering threshold can be reduced up to 10 without increasing significantly the overall number of clusters identified for the majority the periods. The reduction of the clustering threshold aids in reducing the dispersion of mining regions within each cluster. However, in the case of periods 12 and 13, the dendrograms show that the cluster definition is highly variable with respect to the rest of the periods. The reduction of the clustering threshold causes 45% of the mining regions to be classified as outliers. The reduction of the clustering threshold is accepted because of the positive effect in the majority of the periods evaluated. Based on the region distance curves, the equivalent separation distance of a region distance of 10 is 33.6m. In Figures 7.1-B and C, the cross-sections of two mining regions and their respective duplicates, separated by 32m, are presented. These mining regions and their duplicates provide an initial idea of the maximum expected dispersion of the mining regions of a cluster.

In Figure 7.2, six out of thirteen dendrograms with their classification schemes based on a clustering threshold of 10 are presented. Additionally, the clustering threshold can be tuned by individually reviewing the cluster definition in each of the thirteen periods. To maintain a standard dispersion of mining regions in each cluster, the individual clustering thresholds cannot be widely variable. Maintaining a standard dispersion of mining regions is important for assembling the major mining sequences. In this exercise, a clustering threshold of 10 is kept and no local refinement is implemented.

The maximum region distance between mining regions in a cluster can be slightly greater than the clustering threshold specified because the group average comparison is implemented. This discrepancy can be even larger if the MIN comparison is



Figure 7.1: Hierarchical clustering threshold as a function of fluctuation of mining regions to prototypes. A: Average (black) and individual (gray) equivalent region distance of fluctuation of mining regions to prototypes. B and C: Cross sections of mining regions and their duplicates separated a distance of 32m.

implemented. The maximum region distance in a cluster is equal to or less than the clustering threshold only if the MAX comparison is implemented. However, the group average comparison is implemented as it avoids bias in the forming of clusters. In Figure 7.3, the total dispersion and the dispersion of the three clusters identified in Period 5 are compared. The three clusters identified significantly reduce the total dispersion of the mining regions. Notice some of the dissimilarities between mining regions in clusters A5 and B5 are slightly greater than the clustering threshold specified.

After identifying the clusters of mining regions, the decision network can be assembled. The clusters are the nodes of the decision network. The connections between nodes represent the frequency of the associations between mining regions from period to period. In Figure 7.4, the decision network of the one hundred simulated mining sequences is presented. The definition of clusters is consistent for most of the lifetime of the mining project, except for Periods 12 and 13, where 44% of the mining regions are considered outliers. This condition is accepted because the cluster definition for the majority of the periods is consistent.

Up to this point, the decision network alone cannot be used to evaluate mining



Figure 7.2: Dendrograms for periods 2, 4, 6, 8, 10 and 12 and their classification schemes based on a clustering threshold of 10.



Figure 7.3: Comparison of total dispersion of mining regions and dispersion of mining regions in the three clusters identified in Period 5. A: Dendrogram of Period 5 and definition of clusters. B: Boxplot of dissimilarities between all the mining regions and mining regions within each cluster in Period 5.



Figure 7.4: Decision network of one hundred simulated mining sequences. The nodes are identified by a letter that denotes the cluster id in the period and a number that denotes the period. The size of the nodes is related to the number of mining regions in the cluster. At the bottom, for each period, it is specified the number of mining regions considered in the valid clusters. The thickness of the connection lines denote the frequency of the number of associations between mining regions in the clusters. Connection lines with less than five lines are drawn as dashed lines.

alternatives because representive mining sequences of the branches cannot yet be calculated. To assemble mining sequences of the branches, it is necessary to calculate the representative mining regions of each node.

Step 3 - Calculation of Mining Region Prototypes

The mining region prototype of a cluster is a mining region that is representative of all the mining regions of a cluster. It is calculated as the intermediate region between the averaged initial and final topography surfaces of all the mining regions in a cluster. As averaged surfaces are used, there is the risk of finding very small fragmented sub-regions in the intermediate region. To deal with this problem, a volume threshold that controls the minimum volume of fragmented sub-regions is specified. The volume threshold is set to 150 blocks, that is, sub-regions with volumes less than 9600m³ are not considered part of the mining region prototype. The average number of cells of the mining regions is 3900. Specifying a volume threshold of 150 blocks means removing fragments of approximately 4% of the average volume of the mining regions. The volume threshold is specified by inspecting the mining region prototypes of all the dendrograms. It is recommended that the maximum volume threshold is 5% of the average volume of the mining regions. In Figure 7.5, the mining region prototypes of the three clusters identified in Period 5 are presented.

At the time of assembling the mining sequences of branches of the decision network, the mining region prototypes may not fit perfectly. The degree of overlapping and/or gaps between mining region prototypes depends on the dispersion between mining regions in the clusters.

7.2 Selection of Major Mining Sequences

The major mining sequences are identified from the most dominant branches of the decision network, in terms of the cumulative occurrence of the connections. The first dominant branch is identified by implementing the Dijkstra algorithm to find the branch with the largest cumulative occurrence between the first and the last period. This branch is referred to as the first dominant branch (Figure 7.6).



Figure 7.5: Mining region prototypes of clusters identified in Period 5. The mining regions of each cluster identified in the dendrogram, A5, B5, and C5, are in red and the end surfaces are in gray.

A set of additional dominant branches can be identified by disabling connections focusing on evaluating different mining alternatives. The number of dominant branches to identify depends on the number of alternatives that can be evaluated in the operating design of the mining sequences. A second dominant branch is identified by evaluating connections in the periods where the decision network starts to branch. In Period 3, the decision network splits in two directions. To identify the second dominant branch, the connection that corresponds to the first dominant branch, B3 to B4, is disabled. The second dominant branch is identified by implementing the Dijkstra algorithm in the modified decision network to find the new the branch with the largest cumulative occurrence (Figure 7.7). This branch is different from the first dominant branch in the interval between Periods 4 and 7.

The major mining sequences are built by assembling the mining region proto-



Figure 7.6: Branch with largest cumulative occurrence in the decision network. The branch is highlighted with bold dashed lines as connections. The nodes that do not belong to the branch are presented as empty circles with their connections removed. The numbers of mining regions per period that support the main mining pattern are indicated at the bottom.

types of the set of nodes along each branch. These mining sequences are named after their respective branches. The mining sequences of the first and second dominant branches are referred to as first and second major mining sequences, respectively. In Figure 7.8, vertical cross-sections of the first and second major mining sequences are presented. For comparison purposes, the range of periods that is different in the two major mining sequences is coloured.

The dissimilarity between two mining sequences can be quantified in terms of the sum of the region distances between their mining regions, in each period. In this exercise, the dispersion of a major mining sequence is quantified as the average of the dissimilarities between the major mining sequence and the simulated mining sequences. The smaller the dispersion the more representative the major mining sequence of the simulated mining sequences. In Figure 7.9, three branches are compared in terms of the dispersion of their mining sequences: 1) first dominant branch,



Figure 7.7: Branch with largest cumulative occurrence after disabling connection B3 to B4 of the first dominant branch. The branch is highlighted with bold dashed lines as connections. The nodes that do not belong to the branch are presented as empty circles with their connections removed. The numbers of mining regions per period that support the main mining pattern are indicated at the bottom.

2) second dominant branch, and 3) most unlike branch. The most unlike branch is identified as the branch with the smallest cummulative occurence of connections. For comparison purposes, the mining sequences of the three branches are compared in the range of periods from 1 to 13 as the minimum number of periods of the mining sequences is thirteen. The first major mining sequence is more representative than the second major mining sequence. The mining sequence of the most unlike branch is by far less representative than the first and second major mining sequences.

More simulated mining sequences are similar to the first major mining sequence than to the second major mining sequence. The minimum dissimilarities of the first and second major mining sequences are 28.1 and 69.5, respectively. In the interval of dissimilarities between 0 and 100, 35% correspond to the first major mining sequence and only 5% to the second major mining sequence. In the case of the mining sequence of the most unlike branch, there are no simulated mining



Figure 7.8: Mining sequences of first (top) and second (bottom) dominant branches. A and B are the east-west and north-south cross sections of the mining sequence of the first dominant branch. C and D are the east-west and north-south cross sections of the mining sequence of the second dominant branch.

sequences in the range of dissimilarities between 0 and 100.

7.3 Identifying Large Scale Mining Paths

The large scale mining paths are identified in the same way as the major mining sequences, except that consecutive periods are merged to reduce the number of stages in which the deposit is mined. To cover the range of fourteen periods, four period intervals are defined: A) Period 1, B) Period 2 to 5, C) Period 6 to 9, and D) Period 10 to 14. The first stage only considers Period 1 as it is common for all the mining sequences. The next two stages consists of four periods each. The last stage consists of five periods to include the last remaining periods.

The decision network of the mining paths is built by clustering the merged mining regions of each stage and calculating their respective connections. In each dendrogram, the merged mining regions are less variable than the mining regions of



Figure 7.9: Comparison of the average dissimilarity between the mining sequence of three branches and the simulated mining sequences. A: First dominant branch. B: Second dominant branch. C: Most unlike branch. Left: Dissimilarities between the mining regions of the mining sequence of a branch and the simulated mining sequences in each period. Right: Dissimilarities between the mining sequence of a branch and the simulated mining sequences.

individual periods as they tend to intersect more often. The maximum clustering threshold for a separation distance of 50m, average drilling spacing of the existing drilling campaign, is 9.49. After inspection of the dendrograms, the clustering threshold can be reduced up to 2 as the stages are closer to each other (Figure 7.10). The resulting decision network of the merged periods only has two branches (Figure 7.11).



Figure 7.10: Dendrograms for stages 2, 3, and 4 and their classification schemes based on a clustering threshold of 2.

The mining sequences of the branches of the decision network represent the large scale mining paths. In Figure 7.12, the vertical east-west cross sections of the two large scale mining paths are presented. In mining path A, the deposit is mined from east to west and then vertically. In mining path B, the deposit is mined vertically. Based on the ocurrence of the branches, mining path A (69%) is more likely to occur than mining path B (10%).



Figure 7.11: Decision network or merged mining regions to identify large scale mining paths. The decision network consists of two branches.



Figure 7.12: Two identified large scale mining paths. A: east-west cross section of first mining path. B: east-west cross section of second mining path.

7.4 Remarks

In this chapter, an example that illustrates the implementation aspects of the proposed methodology to identify major mining sequences is discussed. The input dataset consists of the simulated mining sequences of the case identified in Chapter 6. The output consists of a few mining sequences that are representative of the input dataset. The output mining sequences are named major mining sequences and account for the variability in the simulated mining sequences due to presence of geologic uncertainty under specified mining and data acquisition strategies.

In the example presented, the simulated mining sequences are able to be condensed in the form of a decision network that permits identifying major mining sequences. The calculated clustering threshold is 18.46. In the majority of the periods, the configuration of mining regions allows for a reduction in the clustering threshold to 10. However, in Periods 12 and 13, the variability of the mining regions is high and does not permit a consistent clustering. In these two periods, the clusters identified are supported by only fifty six mining regions. This high level of variability is expected at later time periods. If this variability were encountered at early time periods, then the proposed methodology may not work well. Then, it would be necessary to reduce the geologic uncertainty by acquiring real additional information, for example, additional exploratoty drilling campaigns.

In addition to the major mining sequences, large scale mining paths are also identified as supplementary information that can be used in the operating design of the mining sequence. The large scale mining paths are identified following the same process to identify the major mining sequences, except the lifetime of the mining project is divided into four stages by merging consecutive periods. The period merging scheme significantly reduces the variability of the mining regions in each stage, with respect to the mining regions in each period. The decision network of the large scale mining paths consists of only two branches. Alternatively, the reduction of variability due to merging periods can be exploited to help identifying major mining patterns of highly variable simulated mining sequences. A moderate period merging scheme can be implemented to reduce variability of problematic periods.

Chapter 8

Conclusions

The three conventional paradigms to evaluate mineable reserves tend to provide an unrealistic estimate of the economic potential of the deposit. Paradigm 1 does not account for uncertainty in the mineable reserve and is very sensitive to conditional bias, which potentially leads to either under- or over-estimate the economic potential of the deposit. Paradigms 2 and 3 rely on a simulated model to report uncertainty in the mineable reserve. However, in terms of estimating the economic potential of the deposit, Paradigm 2 is pessimistic as it assumes the mining sequence does not improve throughout the lifetime of the mining project as additional information is acquired. Paradigm 3 is optimistic as it assumes the mining sequence is able to mine the deposit perfectly. In practice, additional information is collected periodically resulting in an improvement in the performance of the mining sequence with respect to Paradigm 2. A more correct estimate of the economic potential of the deposit is higher than calculated in Paradigm 2, but smaller than calculated in Paradigm 3.

The SLM paradigm accounts for the acquisition of additional information, along with the mining strategy, to evaluate the mineable reserve. This characteristic allows for a more correct estimate of the economic potential of the deposit with respect to the conventional paradigms. The SLM paradigm accounts for uncertainty in the mineable reserve based on a set of mining scenarios in which the acquisition of additional information, blasthole and infill drilling, is simulated. In these mining scenarios, how the mining sequence adapts and improves as the mining progress is
accounted for. To design the operating mining sequence, the set of generated mining sequences is summarized into a few representative alternatives. Additionally, these reduced number of mining sequences can be also used to design contingency plans.

8.1 Contributions

A set of new approaches were developed or existing ones were adapted to aid in the implementation the SLM paradigm. A list of these approaches is presented as follow:

Event-based representation of the mining of the deposit: To simulate the mining of the deposit, from the perspective of long-term planning, the lifetime of the mining project is divided into time periods in which a set of events occur. This modular representation of the mining of the deposit gives flexibility to analyze different mining situations by considering customized events based on the aspects under study.

Two-dimensional parameterization of amount of drilling of infill programs: The number of parameters to define the amount of drilling of an infill program is the number of periods in which the infill program is implemented. Thus, infill programs would have to be evaluated in high-dimensional spaces. From a practical perspective, the amount of drilling of an infill program can be defined by two parameters, number of drillholes in the initial and final periods. The intermediate amount of drilling is calculated based on a linear projection between the initial and final number of drillholes. This two-dimensional parameterization permits to evaluate the infill program based on surfaces of the response variables, including revenue contribution, cost of drilling, and profit contribution.

Region distance metric to measure dissimilarity: A geometric comparison of mining sequences is carried out to asses the magnitude of the difference in their mining directions. As the geometric configurations of mining regions often consists of complex volumes, conventional point-based metrics of comparison cannot be easily implemented. The region distance metric is designed to compare mining regions specifically accounting for their geometric configurations.

Representation of mining sequences as a decision network: A mining sequence can be seen as a set of consecutive decisions made in each period. In this context, a set of simulated mining sequences can be represented as a decision network by analyzing dissimilarities in their mining regions, on a period basis. The decision network significantly reduces the complexity of analyzing all the simulated mining sequences at the same time.

Identification of patterns in mining sequence decision networks: Although a decision network summarizes a set of simulated mining sequences, it does not directly provide information regarding major patterns. The major patterns are identified by implementing the Dijkstra algorithm on the decision network to target branches with the largest frequency of connections under specified conditions. The mining sequence of the major pattern consists of the prototypes of mining regions of the targeted branch.

8.2 Future Work

The development of a methodology to identify regions in the block model that after updated drive changes in the direction of the mining sequence would permit to design efficient infill programs that aim to reduce uncertainty in the mining sequence. These regions are not necessarily single variations in the block model but a combination of them that are positioned at different locations and occur in different periods. These regions also depend on the characteristics of the mining strategy implemented.

The inclusion of other sources of additional information, including geologic mapping, short-term drilling, and different types of infill drilling, in the implementation of the SLM paradigm adds an additional degree of realism in the evaluation of the mineable reserve. In practice, depending on the type of deposit, these other sources play an important role in the calculation of the block model of the deposit.

8.3 Final Remarks

The SLM paradigm relies on the implementation of estimation and simulation techniques to update the block model, to simulate mined regions, and to simulate the acquisition additional information. The two types of attributes that characterize the geology of the deposit, continuous and categorical, are treated differently. In the case of continuous attributes such as metal grades, standard kriging and simulation techniques can be directly implemented or adapted to characterize the spatial distribuition of the attributes. In the case of categorical attributes such as rock type, kriging and simulation of indicators can be adapted to characterize the geology of the deposit. However, the use of indicator techniques limits the implementation of the SLM paradigm to cases of deposits with consistent geologic structures. The ocurrence of mid-size fractures, dikes, and faults, increases the complexity to characterize the geology of the deposit. The use of other customized techniques may have to be considered to evaluate deposits with complex geologic configurations.

In the SLM paradigm, the mining strategy is implemented as a mining sequence algorithm that accounts for operating aspects to mine the deposit. Two types of mining sequence algorithms can be implemented, mathematically optimal or heuristic. A shortcome of mathematically optimal algorithms is that they tend to be computationally expensive. These algorithms are designed in the context of conventional paradigms, thus, the optimality of their results in the context of the SLM paradigm is not guaranteed. It is recommended the mining strategy is implemented in the form of a heuristic algorithm. Heuristic algorithms produce results that are close to mathematically optimal and have the advantage that they are less computationally demanding. The practicality of heuristic algorithms permit the evaluation of different mining strategies during the evaluation of mineable reserves.

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Appendix A

Mining Programs for the SLM Paradigm

A set of three programs suited for implementing the SLM paradigm are discussed in this section. These programs rely on an algorithm called Indexed Search Floating Cone (ISFC) that is a variation of the conventional Floating Cone algorithm. These three programs are for calculating: 1) the mineable limit, 2) the ultimate-pit, and 3) the mining sequence to design a long-term mine plan.

A.1 Indexed Search Floating Cone

In the conventional Floating Cone algorithm, the search and extraction of cones is done following specified directions. In the ISFC algorithm, the search and extraction of cones consists of three steps: 1) construction of an inventory of available cones, 2) targeting and extraction of a candidate cone, and 3) updating the inventory of cones. The inventory of available cones is constructed by indexing the cones that can be extracted from the deposit within geometric limits. The searching of the candidate cone is done by evaluating all the cones in the inventory that satisfy specified mining conditions. The extraction of the targeted cone consists of removing it from the inventory. The updating of the cone inventory is necessary as cones share blocks.

The programs based on the ISFC algorithm are described in the following sec-

tions. The variables used in the parameter files are listed in Table A.1.

Variable	Description
INPUT_TOPO	Filename of the input topographic 2D map.
INPUT_MODEL	Filename of the input 3D block model.
OUTPUT_TOPO	Filename of the output topographic 2D map(s). The
	$program MININGSEQUENCE_BC$ generates more than
	one surface.
OUTPUT_MODEL	Filename of the output 3D block model.
ID_TOPO	Column id of the topographic data used in
	INPUT_TOPO.
ID_USDB	Column id of dollar/block data used in INPUT_MODEL.
ID_TONB	Column id of tonnage/block data used in
	INPUT_MODEL.
ID_MATT	Column id of material type data used in
	INPUT_MODEL.
CODE_WST	Code of material type that denotes 'waste' material.
XNUM, YNUM, ZNUM	Number of blocks in x, y, and z directions.
XINI, YINI, ZINI	Origin of blocks in x, y, and z directions.
XSIZ, YSIZ, ZSIZ	Size of blocks in x, y, and z directions.
M_BASE	Minimum radius of mining base at the bottom of
	cones.
M_SLOPE	Minimum pit slope.
ORE_TRG	Ore tonnage target per period.
ORE_TOL	Approximation tolerance of ORE_TRG.
NUM_FRG	Number of fragments of cones.
NUM_REFP	Optional parameter. Number of refining periods.
	The default value is 10. The 'NUM_PERIODS:' tag is
	required to identify this parameter in the 'extra'
	section.
NUM_REFF	Optional parameter. Number of refining fragments.
	The default value is 3. the 'NUM_FRAGMTS:' tag is
	required to identify this parameter in the 'extra'
	section.

Table A.1: List of variables used in the parameter files.

A.1.1 Program 1: Mineable Limits

This program identifies the blocks that cannot be extracted from the deposit due to the geometric limits of the project and geotechnical constraints. The geometric limits of the project restrict the region where the mining operations can take place. The set of geotechnical constraints, such as minimum pit slope, restrict the region within the limits of the project that could be mined regardless of its economic value.

The input data consists of a 2D map with topographic elevations. The output information consists of a 3D block model, where the non-mineable blocks are coded with 1, and the rest of the blocks are 0. The parameter file template is presented in Figure A.1.

1	MineableLimits	
2		
3	[input-output]	
4	<input_topo></input_topo>	*topographic 2D surface model
5	<output_model></output_model>	*block model with mineable limits data
6		
7	[run-options]	
8	<id_topo></id_topo>	*column of topographic variable
9	<xnum> <ynum> <znum></znum></ynum></xnum>	*number of blocks in x, y, and z directions
10	<xini> <yini> <zini></zini></yini></xini>	*origin of blocks in x, y, and z directions
11	<xsiz> <ysiz> <zsiz></zsiz></ysiz></xsiz>	*size of blocks in x, y, and z directions
12	<m_base></m_base>	*mining base radii
13	<m_slope></m_slope>	*mining slope

Figure A.1: Parameter file template of MINEABLELIMITS.

An example is presented in Figure A.2. Two vertical cross-sections of the mineable limits of a deposit are presented. The black cells represent the blocks that cannot be mined regardless of their economic metal content. The gray cells represent the mineable region of the deposit below the surface where the ultimate pit and its corresponding mining sequence are calculated.

The algorithm searches and extracts all the cones that satisfy the specified conditions such as the overall pit slope and the minimum radius of the mining base. To satisfy the condition to not to mine beyond the limits of the project, the cones with blocks below the surface that expand beyond the vertical projection of the limits are rejected. The mineable limits consist of all the blocks that were not extracted by the algorithm. The mineable limit narrows the search region to calculate the ultimate-pit. In the implementation of the SLM paradigm, this module helps in the evaluation of additional drilling by identifying sampling regions that are not relevant to the mining sequence. The mineable limits can be also implemented along with other algorithms such as Lerchs-and-Grossmann or Floating Cone to reduce



Figure A.2: Cross sections, east-west (top) and north-south (bottom), of mineable limits calculated based on initial topography and geometric mining constraints.

the computation time required to calculate the ultimate-pit.

A.1.2 Program 2: Ultimate-Pit

This program calculates the ultimate-pit. The calculation of an initial ultimate-pit consists of an iterative process, where the algorithm searches and extracts cones with the largest revenue taking into account the specified mining conditions. The algorithm stops when there are no more cones with positive revenue. In this module, the use of large volume cones reduces the flexibility extracting complex geometric mining regions because the geometry of the mining cuts is mainly dominated by one large single cone. To deal with this problem, a second algorithm is implemented to refine the current ultimate-pit. The refining algorithm aims to identify the regions that do not contribute to the overall revenue.

The input data consists of a 2D map with topographic information, and a 3D block model with dollar/block values. The dollar/block values need to be calculated considering the entire block, and not only the proportion below the topographic

surface. The program internally weights the dollar/block values according to the initial topographic surface. The program is designed this way to be able to handle different initial topographic surfaces without having to calculate the dollar/block values multiple times. The output information consists of a 3D block model, where the blocks within the ultimate-pit are coded as 1 and the rest of the blocks are 0. Also, a 2D map with the topographic information after mining the ultimate-pit is generated. The parameter file template is presented in Figure A.3.

```
1
    UltimatePit
2
3
   [input-output]
   <INPUT TOPO>
                                  *initial topographic 2D model
4
5 <INPUT MODEL>
                                 *3D block model
   <OUTPUT TOPO>
                                  *end topographic 2D model
6
7
    <OUTPUT_MODEL>
                                  *block model with ultimate pit data
8
9 [run-options]
10 <ID TOPO>
                                 *topo column id (topographic model)
   <ID USDB>
                                  *dollar/block column id (economic model)
12
   <XNUM> <YNUM> <ZNUM>
                                  *number of blocks in x, y, and z directions
13
   <XINI> <YINI> <ZINI>
                                  *origin of blocks in x, y, and z directions
14
    <XSIZ> <YSIZ> <ZSIZ>
                                  *size of blocks in x, y, and z directions
    <M BASE>
                                  *mining base radii
15
16
    <M_SLOPE>
                                  *mining slope
17
   [extra]
18
19 num_periods: <NUM_REFP>
                                  *number of refining periods
20 num_fragmts: <NUM_REFF>
                                  *number of refining fragments
```

Figure A.3: Parameter file template of ULTIMATEPIT.

An example is presented in Figure A.4. The black blocks represent the ultimatepit and the gray blocks represent the region that is within the preliminary limits but is rejected in the refining process. The inclusion of the gray blocks as part of the ultimate pit leads to a reduction of the overall revenue of the project.

Unlike the conventional Floating Cone algorithm that extracts cones in specified directions and requires multiple passes to increment its efficiency, the module to calculate the ultimate-pit limits searches and extracts cones based on a unique indexed search, thus requiring only one pass.



Figure A.4: Cross sections, east-west (top) and north-south (bottom), of unrefined and refined preliminary ultimate-pit.

A.1.3 Program 3: Mining Sequence

This program calculates the mining sequence based on the the ultimate-pit. The input information consists of a target ore tonnage and ore type to be mined in each period along with the operating specifications. The number of periods is obtained by dividing the tonnage of ore material in the ultimate-pit by the targeted ore tonnage per period. The calculation of the mining cut of a period consists of evaluating different geometric configurations of mining cuts that satisfy the specified mining conditions. Among all the alternatives, the mining cut that yields the maximum revenue is selected as the mining cut for the current period. This evaluation process is repeated until the ultimate-pit is mined. The geometry of the mining region can consist of complex shapes and sub-regions. The fragmentation of the mining regions is controlled by a parameter.

The input data consists of a 2D map with topographic information, and a 3D block model with dollar/block, block density, and material type information. The

dollar/block values should be calculated considering the entire block. The output information consists of a 3D block model. The blocks are coded based on the period number in which they are expected to be mined. Also, a set of 2D maps with topographic information at the end of each period is calculated. The parameter file template is presented in Figure A.5.

```
1
    MiningSequence BC
 2
 3 [input-output]
 4 <INPUT_TOPO>
                                 *initial topographic 2D model
 5 <INPUT MODEL>
                                *3D block model
 6 <OUTPUT TOPO>
                                *end topographic 2D model/period
7 <OUTPUT_MODEL>
                                *block model with mining sequence data
9 [run-options]
10 <ID TOPO>
                                *topo column id
11 <ID_USDB> <ID_TONB> <ID_MATT> *dollar/block, tonnage/block, and material type column id

    ccode_WST>
    *code of waste in material type variable

    13
    <XNUM> <YNUM> <ZNUM>

                                *number of blocks in x, y, and z directions
14 <XINI> <YINI> <ZINI>
                                *origin of blocks in x, y, and z directions
15 <XSIZ> <YSIZ> <ZSIZ>
                               *size of blocks in x, y, and z directions
16 <M BASE>
                                *mining base radii
17
   <M SLOPE>
                                 *mining slope
18 <ORE_TRG> <ORE_TOL>
                            *ore target and ore tolerance by period
19 <NUM FRG>
                                *number of fragments
20
21
   [extra]
22 num_periods: <NUM_REFP> *number of refining periods
23 num fragmts: <NUM REFF>
                                 *number of refining fragments
```

Figure A.5: Parameter file template of MININGSEQUENCE_BC.

In Figure A.6, two cross sections of a mining sequence are presented. The resulting scheduling of the ore material satisfies the ore tonnage constraint per period. This mining sequence serves as a reference to design the operational mining sequence, where the proportions of ore and waste over the lifetime of the project are kept consistent within period intervals so that the loading and hauling equipment can handle the mining properly.

A.2 Comparison of Algorithms Based on Ultimate-Pit Revenue

For verification purposes, the ultimate-pit program based on the ISFC algorithm is compared against the versions based on conventional algorithms, Lerchs-and-



Figure A.6: Cross sections, east-west (top) and north-south (bottom), of mining sequence.

Grossmann and Floating Cone. The ultimate-pit programs based on the conventional algorithms are from the commercial software package MineSight. The conventional algorithms are considered referential to measure the performance of ISFC algorithm to calculate the ultimate-pit. In the comparison, 25 block models at a resolution of $100 \times 60 \times 40$ blocks are generated based on unconditional simulated realizations. An initial topographic surface is used to set the initial state of the deposits before the mining takes place and a constant density of 1MT/m3 is assigned to all the material below the topographic surface. A pit slope of 45 is considered as the mining geometric constraint.

To assess the performance of the ISCF ultimate-pit, its revenue is compared to the revenues of the Floating Cone and Lerchs-and-Grossmann ultimate-pits. The comparison focus on finding the gap of the heuristic algorithms with respect to the optimal results. For each of the 25 realizations, proportions of revenue of ISCF and Floating Cone ultimate-pits are calculated with respect to Lerchs-and-Grossmann revenue. On average, the ISFC algorithm performs bettern than Floating Cone. In the example (Figure A.7), the revenue of the ISFC algorithm is 97.4% of the Lerchs-and-Grossmann revenue compared to 92.1% for Floating Cone. The ISFC algorithm also provides a smaller variability in the proportions of revenues compared to Floating Cone.



Figure A.7: Comparison between ISFC and Floating Cone algorithms in terms of approximation to optimal revenue.

The performance of the ISFC algorithm depends on the refining parameters. The number of refining periods is important as it has a direct effect in the revenue of the ultimate-pit. To illustrate the effect of increasing the number of refining periods in the performance of the ISFC algorithm, six values are tested, from 10 to 20 at intervals of 2. In Figure A.8, a boxplot of the proportions of ISFC revenues with respect to Lerchs-and-Grossmann revenues is presented. The groups of proportions of ISFC revenues are labelled from ISFC10 to ISFC20, where the last two digits indicate the value of the number of refining periods considered. The proportion of revenues of the initial ultimate-pit is labelled as Pre-ISFC. On average, even the initial ISFC ultimate-pit performs better than Floating Cone ultimate-pit, however, the variability of the revenues is higher. The implementation of the refining algorithm results in an improvement in the performance of the ISFC algorithm. The effect in the improvement of the performance of ISFC algorithm is not linear. It slowly approaches to the optimal revenue and reduces the variability of the revenues. For the 25 realizations, a preferable value for the number of refining periods is 14. Beyond this point, incrementing the number of refining periods does not produce significant improvement in the revenues.



Figure A.8: Comparison of Floating Cone and ISFC with different number of refining periods.

In terms of the computation time required, the ISFC algorithm using 20 refining periods is slightly faster than Floating Cone but much faster than Lerchs-and-Grossmann.