Security Constrained Transmission Expansion Planning by Accelerated Benders Decomposition

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Abstract—**This paper formulates the security constrained transmission expansion planning (SCTEP) into a standard two-stage stochastic programming (SP) problem with complete recourse, which is then tackled by Benders decomposition (BD) due to its special decomposable structure, additionally, three improvements are also employed to accelerate the classical BD: valid inequality, multicut strategy, and optimal precondition. The performance of the improved BD is demonstrated by massively case studies on three classical benchmarks: the Garver 6-bus system, the IEEE 24-bus system, and the IEEE 118-bus system. Significant reduction in both execution time and the number of iterations are achieved for all acceleration strategies.**

Index Terms—**Benders decomposition, multicut strategy, security constrained transmission expansion planning, stochastic programming.**

I. Introduction

Due to the continuous increase in power consumption and generation, new circuits and transformers should be constructed to relieve the critically burdened power system, which comprises the transmission expansion planning (TEP) problem. Initially, emphasis was laid on the economic profits, to minimize the total investigation and operation cost. Reliability and environmental concerns were added subsequently due to several blackouts worldwide, such as the "Northeast blackout of 2003", evolving the TEP into the security constrained TEP (SCTEP). SCTEP will continue to assume great importance in the current context of smart grid, where large amounts of distributed generators collecting renewable energy need to be integrated, bringing in a lot of variation and uncertainty.

Mathematically, SCTEP is in general a large-scale, mixed-integer, nonlinear, non-convex, and NP-hard [1] problem, which presents a major challenge to all known optimization approaches, ranging from analytical method to heuristic and meta-heuristic algorithms [2], [3]. Heuristic algorithms can find the global optimal or suboptimal solution in a short time with limited computational effort, but the performance is problem dependent and would be poor for large-scale systems; meta-heuristic algorithms are robust, with the capability of solving problems regardless of their size and whether they are convex or non-convex, continuous or discrete, however the computation is more intensive than heuristic algorithms, and the optimality of the final solution can not be validated. Generally, analytical methods can guarantee a global optimal solution based on rigorous mathematical derivation, which will converge into global optimal solution in a finite number of iterations, and the optimality could be indicated by the lower and upper bounds at each iteration. However, the three most widely employed analytical methods in TEP solution also exist some drawbacks:

- linear programming (LP) is just suitable for the transportation model (a relaxed version of DC model with Kirchhoffs voltage law (KVL) constraints eliminated);
- nonlinear programming (NLP) technique for large and complex systems is still not robust and reliable (especially for non-convex problems);
- branch and bound (BB) utilizes selective/partial enumeration, which can run out of the memory for large-scale problems, and the high execution time remains another serious problem.

Therefore, a lot of decomposition strategies are investigated to separate the complex full problem into several simple subproblems, among which Benders decomposition (BD) [4] has received much attention in the literature.

One of the pioneering works which introduced BD into TEP solving is [5], where a minimum value of the disjunctive parameter is derived to eliminate the numerical difficulties when utilizing the disjunctive model (an equivalent version of DC model with integer decision variables are represented by binary bits); however, the security constraints have not been considered. Since then, several investigations have sought to improve the performance of BD from two aspects: master problem and subproblem. Generally, the master problem of SCTEP is integer programming (IP) or mixed-integer programming (MIP), which is the most time consuming part for large-scale problems since the solution techniques (e.g., branch and reduce) are not as efficient as these (e.g., interior point) for LP, thus several efficient strategies have been investigated to relieve the complexity and search space of MIP and IP [6]–[8]. On the other hand, much effort has also been poured into generating more effective cuts (feasibility and optimality) from subproblem [7]–[9], to eliminate regions that contain suboptimal or non-improving solutions [10], thus reducing the total number of iterations required for the whole algorithm.

Instead of IP and MIP solution, [6] introduced a local search

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procedure to solve the master problem, with considerably decreased computational time reported, and the global optimality was also preserved; however, the classical Benders cuts were not enhanced. [7] introduced a lot of interesting improvements employed by a practical project with textual description, such as, inexact master problem resolution mechanism, semi-relaxed cuts for discrete decision variables, and combination of monocut and multicut, etc.; nevertheless, a few technical and mathematical details were exhibited. [8] introduced two valid inequalities to reduce the search space of the master problem, as well as multiple generation cuts and strong high density cut to boost the convergent efficiency. [9] proposed a set of appropriate Benders cuts specifically tailored for the binary decision variables, and the effectiveness of standard and modified disjunctive model has also been studied.

Although remarkable advances have been made in BD for finding the global optimal solution of SCTEP, some beneficial endeavours still need to be conducted. In this work, we first modeled the SCTEP into a standard two-stage stochastic programming (SP) problem with complete recourse in Section II; then Section III illustrates the improvements on BD for problem solution, including valid inequality, multicut strategy, and optimal precondition; comprehensive comparison based on three benchmark systems have been carried out in Section IV to illustrate the performance of each strategy; finally conclusions and future work are highlighted in Section V.

II. Problem Formulation

Although SCTEP has been modeled as a problem of SP by the previous works [6]–[9], some details still have not been revealed, of which at least one (non-anticipativity constraints) is of key importance.

Generally, the two-stage SP is modeled as follows [11]:

$$\min_{x,y_s} \quad c^T x + \sum_{s\in S} p_s q_s^T y_s \tag{1}$$

$$s.t. \quad Ax \le b, \tag{2}$$

$$T_s x + W_s y_s \le h_s, \tag{3}$$

$$x \in X,\ y_s \ge 0.$$

where c and q_s are the cost vectors for the first and second stages; p_s is the probability for each scenario s; x is a vector that denotes the first stage decision variables restricted by an integer set X; y_s are the continuous second stage decisions for each scenario s; A and b are parameter matrix and vector independent of the scenarios; T_s, W_s, and h_s are parameter matrices and vector for each scenario $s \in S$.

In order to formulate SCTEP into (1), scenario dependent parameters T_s, W_s, and h_s should be determined. However, this process is either not mentioned or replaced by another straightforward but may not rigorous method. For example, in [9], scenario s is represented by $\bar{x}_s$, which is a revised solution by reducing one corresponding circuit from the first stage solution $\bar{x}$. This process is performed by matrix $L1$ in that paper, resulting in $\bar{x}_s$ which is different for each scenario that violates the non-anticipativity constraints [12] of SP.

In the following proposed SCTEP model (4), only fixed parameters T_s, W_s, and h_s are used to construct scenario s, therefore the unique first stage solution of each iteration can be directly utilized by every scenario without any variation, which means non-anticipativity constraints are satisfied.

$$\min \sum_{k=1}^{K} \sum_{(i,j)\in\mathcal{C}} c_{ij} n_{ij}^k + \sum_{s=1}^{|\mathcal{S}|} \left(p_s \sum_{i=1}^{|\mathcal{N}|} r_i^{(s)} \right), \tag{4}$$

subject to:

$$n_{ij}^{k+1} \le n_{ij}^k,\ k = 1, ..., K-1, \qquad ij \in \mathcal{C} \tag{5}$$

$$\sum_{ij\in\mathcal{E}} f_{ij}^{0(s)} + \sum_{k=1}^{K} \sum_{ij\in\mathcal{C}} f_{ij}^{k(s)} + g_i^{(s)} + r_i^{(s)} = d_i, \quad i \in \mathcal{N} \tag{6}$$

$$f_{ij}^{0(s)} - \gamma_{ij} n_{ij}^{0(s)} \left(\theta_i^{(s)} - \theta_j^{(s)}\right) = 0, \qquad ij \in \mathcal{E} \tag{7}$$

$$|f_{ij}^{k(s)} - \gamma_{ij} \left(\theta_i^{(s)} - \theta_j^{(s)}\right)| \le M_{ij} \left(1 - n_{ij}^k\right), \quad ij \in \mathcal{C} \tag{8}$$

$$|f_{ij}^{0(s)}| \le \bar{f}_{ij} n_{ij}^{0(s)}, \qquad ij \in \mathcal{E} \tag{9}$$

$$|f_{ij}^{k(s)}| \le \bar{f}_{ij} n_{ij}^{k}, \qquad ij \in \mathcal{C} \tag{10}$$

$$0 \le g_i^{(s)} \le \bar{g}_i, \qquad i \in \mathcal{N} \tag{11}$$

$$0 \le r_i^{(s)} \le d_i, \qquad i \in \mathcal{N} \tag{12}$$

$$n_{ij}^k \in \{0,1\},\ \theta_i^{(s)} \text{ and } \theta_j^{(s)} \text{ unbounded},\ s \in \mathcal{S}.$$

where the objective function (4) comprises of construction cost and load curtailment penalty; $\mathcal{C}$, $\mathcal{N}$, and $\mathcal{S}$ represent the set of all candidate circuits, bus nodes, and security contingencies (scenarios) respectively; K is the maximum number of circuits that can be added in parallel in each candidate corridor; n_{ij}^k is a binary decision variable that denotes the decision of whether to build the kth duplicate line for candidate line $i-j$ or not, accompanied by an investment cost of c_{ij} if constructed; p_s is the penalty factor; $r_i^{(s)}$ is the loss of load for bus i, where the superscript $^{(s)}$ represents scenario s, and hence forth. Equation (5) restricts the selection of each duplicate candidate circuit should be in an increasing order, i.e., select n_{ij}^1 before n_{ij}^2, which makes the final solution unique, otherwise there might be large numbers of optimal solutions since $[n_{ij}^1\ n_{ij}^2] = [1\ 0]$ is equivalent with $[n_{ij}^1\ n_{ij}^2] = [0\ 1]$ in the logical and mathematical sense. Equation (5) is the valid inequality, whose effectiveness will be presented in the case studies. Equation (6) is the load balance constraint (Kirchhoff's current law, KCL) for bus i; $f_{ij}^{0(s)}$ and $f_{ij}^{k(s)}$ are the power flow of the original and the kth duplicated circuit $i-j$; $g_i^{(s)}$ and d_i are the amount of generation and load for bus i. Equation (7) and (8) are DC power flow KVL constraints; $\mathcal{E}$ is the set of all existing circuits; $n_{ij}^{0(s)}$ and γ_{ij} are the initial number and susceptance of circuit $i-j$; $\theta_i^{(s)}$ is the voltage angle of bus i; M_{ij} is a large number representing the disjunctive parameter, which can be valued as $M_{ij} = 2\pi/\gamma_{ij}$ as shown in [13]. Equation (9) and (10) are power flow limits. Equation (11) and (12) indicate the restrictions on generation and load curtailment. It should be noted that $f_{ij}^{0(s)}$, $f_{ij}^{k(s)}$, and $\theta_i^{(s)}$ may get negative values in the above formulations, which is slightly different from SP (1) with all nonnegative variables. Actually, translation is also easy by adding more decision variables, for example, set $\theta_i^{(s)} = \theta_i^{(s)+} - \theta_i^{(s)-}$, where both $\theta_i^{(s)+}$ and $\theta_i^{(s)-}$ are nonnegative.

In order to save computational time, the contingency set $\mathcal{S}$ is defined as small as possible, given by:

$$\mathcal{S} = \{ij | n_{ij}^0 + n_{ij}^1 > 0, \, ij \in \mathcal{C}\}. \tag{13}$$

For each scenario $s \in \mathcal{S}$, one of the original n_{ij}^0 or added $n_{ij}^k (k = 0, 1, ..., K)$ circuits will out of service due to some unpredicted reasons, consisting the $N-1$ security criteria. In order to keep the solution $n_{ij}^k (k = 0, 1, ..., K)$ generated from the first stage the same for each scenario in the second stage, i.e., to meet non-anticipativity constraints, the contingency is formulated from two aspects: if $n_{ij}^0 > 0$, then set $n_{ij}^{0(s)} = n_{ij}^0 - 1$; otherwise ($n_{ij}^1 > 0$ according to (13)), replace the constraint (8) when $k = 1$ by $f_{ij}^{1(s)} = 0$, which means $n_{ij}^1 = 1$ has been logically invalidated while the solution is kept the same. The smallest contingency set $\mathcal{S}$ defined in (13) is a subset of $\mathcal{C}$, however, it should be pointed out that the contingency formulation method described above could still be valid even when the largest contingency set $\mathcal{S} = \mathcal{C}$ is employed.

Converting the SCTEP (4) into SP (1) is relatively straightforward, where constraints (5) and (6)-(12) belong to (2) and (3), respectively. It should be noted that, from the point of practical operation, if any loss of load $r_i^{(s)}$ is positive, the solution is invalid; however, mathematically speaking, with the appearance of $r_i^{(s)}$, the system is always feasible, i.e., any solution of $\bar{n}_{ij}^k$ won't violate any constraint since there always has at least a scheme of $r_i^{(s)} \le d_i$ to compensate the load imbalance in (6). Therefore, the SP generated from SCTEP is a complete recourse problem, i.e., any first stage solution is feasible in the second stage.

III. Solution Methodology

BD can be advantageous in situations where the number of variables that link the two stages is small or if the master problem and the subproblem have a different nature [7]. Both conditions are possessed by SCTEP, the only variables that link the two stages are n_{ij}^k, which are relatively few when compared with the second stage variables $f_{ij}^{0(s)}$, $f_{ij}^{k(s)}$, $g_i^{(s)}$, $r_i^{(s)}$, $\theta_i^{(s)}$, and $\theta_j^{(s)}$; in addition, the master problem and the subproblem are MIP and LP respectively. In this section, a standard BD is introduced first, followed by three acceleration strategies.

A. Classical Benders Decomposition

Two stage SP shown in (1) is employed for the illustration of deriving standard BD. The full problem of (1) is MIP, after decomposition, the MIP master problem can be given as:

$$\min_x \quad c^T x + Q \tag{14}$$

$$s.t. \quad Ax \le b, \, x \in X \tag{15}$$

where $Q = \sum_{s \in S} p_s Q_s$ is the weighted sum of objective function values of LP subproblem [SP] for each scenario s.

$$\text{[SP]} \quad \min_{y_s} \quad q_s^T y_s \tag{16}$$

$$s.t. \quad W_s y_s \le h_s - T_s \bar{x}, \, y_s \ge 0. \tag{17}$$

And the dual problem [DP] of [SP] is also given as:

$$\text{[DP]} \quad \max_{u_s} \quad (h_s - T_s \bar{x})^T u_s \tag{18}$$

$$s.t. \quad W_s^T u_s \le q_s, \, u_s \le 0. \tag{19}$$

As this is a complete recourse problem, [SP] is always feasible, thus [DP] is bounded, and the objective function value of [DP] provides a valid lower bound for [SP] according to dual theory, therefore an optimality cut could be generated:

$$Q_s \ge (h_s - T_s x)^T \bar{u}_s, \tag{20}$$

where $\bar{u}_s$ is the optimal solution of [DP].

Due to complete recourse, feasibility cut does not need to be generated, then the master problem [MP] can be expressed as:

$$\text{[MP]} \quad \min_x \quad c^T x + Q \tag{21}$$

$$s.t. \quad Ax \le b, \, x \in X, \tag{22}$$

$$Q \ge \sum_{s \in S} p_s (h_s - T_s x)^T \bar{u}_s^i, \, i = 1...N \tag{23}$$

where N is the current iteration number.

The implementation framework of standard BD is given as follows:

- **Step 1.** Set $i = 1$, $LB = -\infty$ and $UB = \infty$, solve [MP] without cuts. Denote the objective function value as LB' and the optimal solution of first-stage decision variable as $\bar{x}$. If $LB' > LB$, set $LB = LB'$.
- **Step 2.** Solve [DP] for each scenario s based on $\bar{x}$ from [MP]. Mark the optimal solution as $\bar{u}_s$, and compute $UB' = c^T \bar{x} + \sum_{s \in S} p_s (h_s - T_s \bar{x})^T \bar{u}_s$. If $UB' < UB$, set $UB = UB'$.
- **Step 3.** If $UB - LB \le \varepsilon$ (e.g., 10^{-6}), stop and output final solution $\bar{x}$; otherwise generate optimality cut (20) with $\bar{u}_s$ from Step 2, solve full [MP] with cuts to generate new solution $\bar{x}$, then set $i = i + 1$, and go to Step 2.

It should be noted that LB' is actually not required in Step 1, since the [MP] is continuously restricted by the newly added cuts, which automatically makes the objective function value LB' larger, thus LB' is monotonically increasing. On the contrary, the UB' is demanded in Step 2 to make the upper bound decrease continually, however, note that whether adding or leaving out UB' does not impact the convergent characteristic of the whole problem solution.

B. Valid Inequality

Although valid inequality is usually problem dependent that may not suitable for other types of practical application, they are always the first avenue to accelerate the convergence of BD. By using these inequalities, many infeasible or non-improvement solutions may be eliminated during or even before the search process of the algorithm [8]. Therefore, the search space is reduced, as well as the total iterations.

The circuits built in parallel are alike, thus there will be several equivalent optimal solutions, which may introduce complexity during the solution process. In order to make the optimal solution logically unique, valid inequality (5) is employed. Suppose $K = 4$, take n_{12}^k as an example, if 2 links should be constructed on corridor $1-2$, then there will be $C_4^2 = 6$ equivalent solutions, such as $[n_{12}^1\ n_{12}^2\ n_{12}^3\ n_{12}^4] = [0\ 1\ 0\ 1]$; however, if valid inequality (5) is utilized, there will be only one valid solution $[n_{12}^1\ n_{12}^2\ n_{12}^3\ n_{12}^4] = [1\ 1\ 0\ 0]$. It can be easily derived that the total number of equivalent first-stage solutions is:

$$\prod_{ij\in\mathcal{C}} C_K^{\sum_{k=1}^{K}\bar{n}_{ij}^k} \gg 1, \tag{24}$$

where C is the combinatorial number operator with $C_m^n = \frac{m!}{n!(m-n)!}$. Reducing (24) into 1 saves much effort solution searching, which will be discussed in Section IV.

C. Multicut Strategy

Instead of returning only one cut at each iteration to [MP] as in the classical BD, a multicut strategy is employed to enhance the convergence efficiency by generating one cut from each scenario, i.e., multiple cuts are introduced for each iteration.

The process is performed by decomposing the variable Q into Q_s, resulting in the following multicut master problem:

$$\min_{x} \quad c^T x + \sum_{s\in S} p_s Q_s \tag{25}$$

$$s.t. \quad Ax \le b,\ x \in X, \tag{26}$$

$$Q_s \ge (h_s - T_s x)^T \bar{u}_s^i,\ i = 1...N. \tag{27}$$

This is a variant of the classical BD method with faster convergence rate, which has been demonstrated and proved in [11] and [14].

D. Optimal Precondition

One of the preconditions for the optimality of SCTEP solution is the sum of objective function values of subproblem Q equals to zero, i.e., no loss of load is tolerable. Since p_s is positive, then $Q = 0$ is equivalent to $Q_s = 0$ for all $s \in \mathcal{S}$. Therefore, this optimality precondition can be embedded into the Benders cut generation (20) by force $Q_s = 0$, thus a new multicut master problem can be expressed as:

$$\min_{x} \quad c^T x \tag{28}$$

$$s.t. \quad Ax \le b,\ x \in X, \tag{29}$$

$$0 \ge (h_s - T_s x)^T \bar{u}_s^i,\ i = 1...N. \tag{30}$$

Different from the MIP characteristic of (25), the new master problem is a purely IP problem.

E. Accelerated Benders Decomposition

In our computational experiments described in Section IV, we provide insights into algorithmic performance by comparing alternative versions of accelerated BD given in Table I.

TABLE I
ALTERNATIVE VERSIONS OF ACCELERATED BD ALGORITHMS

Algorithms	***Alg.1***	***Alg.2***	***Alg.3***	***Alg.4***
Valid inequality	×	√	√	√
Multicut strategy	×	×	√	√
Optimal precondition	×	×	×	√

TABLE II
BASIC INFORMATION OF BENCHMARK TEST SYSTEMS

Items	**6-bus**	**24-bus**	**118-bus**
No. of buses	6	24	118
No. of existing branches	6	38	186
No. of candidate branches	15	41	186
Load demand (MW)	760	8,550	3,733
Generation capacity (MW)	1,110	10,215	8,270
Maximum No. of circuits K	4	2	1
No. of scenarios	15	41	186
No. of binary variables	60	82	186
No. of continuous variables	2,610	14,022	226,176
References with detailed data	[15]	[16]	[17]

IV. CASE STUDIES

In this section, three classical benchmark systems of different scales are employed: the Garver 6-bus system, the IEEE 24-bus system, and the IEEE 118-bus system, where the former two systems are used to validate the effectiveness since the global optimal solution is available, the last one is utilized to exemplify the potential on large-scale systems. The basic information of benchmark systems is given in Table II.

We programmed *Alg.1* – *Alg.4* using Matlab 2015b in concert with MIP solver *lp_solve 5.5.2.0* [18]. All runs are made on a Windows desktop with an Intel Xeon E5-2620 CPU at 2.10GHz with 32GB RAM.

A. Garver 6-bus System

In order to fully compare the performance of each improvement strategy, all four algorithms in Table I are conducted on the Garver 6-bus test system, and basic results are reported in Fig. 1. All these four algorithms get a final cost of 180M$ under generation re-dispatch and $N-1$ security criteria, which is identical with the global optimal solution of [19]; thus the capability of global convergence for each algorithm is verified. When it comes to the performance, classical BD (*Alg.1*) arrives at the global optimal after 685 iterations, however, that number is improved to 127 by *Alg.2*, which indicates that the valid inequality played a major role in the convergence process. As for execution time, *Alg.1* is 68.5 times slower than *Alg.2*. Although *Alg.2* gains a huge improvement on *Alg.1*, it is still not efficient enough with a running time of 22.66s. After introducing the multicut strategy, the execution time decreases to as small as 1.39s, which is just 6.13% of the former. Additionally, the number of iteration is reduced from 127 to 18. Finally, the newly added optimal precondition gains an improvement of 21.6% and 22.2% for execution time and total

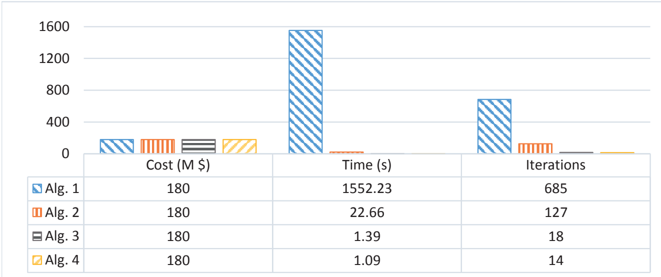

Fig. 1. Simulation results for the Garver 6-bus test system.

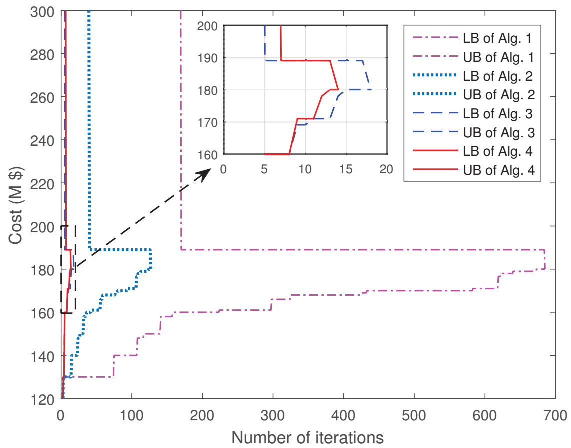

Fig. 2. Behaviour of algorithm convergence for the Garver 6-bus test system.

iterations respectively over the already acceptable performance of *Alg.3*.

The convergence process of different algorithms is illustrated in Fig. 2. From the behaviour of *Alg.2* – *Alg.4*, it can be seen that the upper bound drops sharply at the beginning and stay flat for a long time, while the lower bound always increases incrementally to meet with upper bound. The total number of iterations closely depends on the rate of increase of the lower bound, which is determined by the quality of cuts generated by subproblems. The slow convergence of *Alg.1* is due to its large unrestricted search space.

B. IEEE 24-bus System

Since *Alg.1* and *Alg.2* are 1117 and 16 times slower respectively than *Alg.3* in the small-scale Garver 6-bus system, we will not consider them in the medium- and large-scale systems for saving computational time. A convergence criterion (bound gap, shown in (31)) introduced by [20] is utilized to show the characteristic of convergence. Another idea of reducing the search space of binary variables proposed in [6] is also employed to release the complexity.

$$Bound\ gap = \frac{2(UB - LB)}{UB + LB}. \tag{31}$$

The results generated from *Alg.3* and *Alg.4* for IEEE 24-bus system are reported in Table III. Both algorithms arrive at a

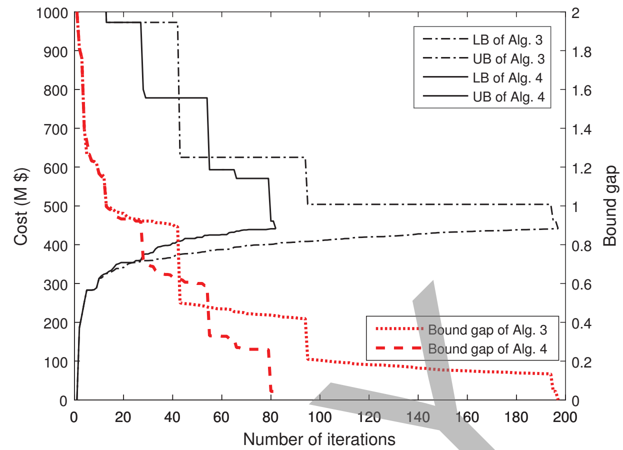

Fig. 3. Behaviour of algorithm convergence for the IEEE 24-bus test system.

TABLE III
SIMULATION RESULTS FOR THE IEEE 24- AND 118-BUS TEST SYSTEMS

	Results	24-bus	118-bus
Alg.3	Cost (M $)	441	–
	CPU time (s)	12,129.79	–
	No. of iter.	197	–
Alg.4	Cost (M $)	441	1,133.40
	CPU time (s)	607.64	13,433.64
	No. of iter.	82	38

– : No result reported after 70,000s of run time.

final cost of 441M$, which is coincident with [6] and [19]. Similar with before, *Alg.4* performs much better than *Alg.3*, especially for the execution time, a saving of 94.99% is gained. Fig. 3 illustrates the convergence behaviour of *Alg.3* and *Alg.4*. *Alg.4* converges faster than *Alg.3* almost all of the time except during the interval of iterations 43 to 54, which is probably because *Alg.3* occasionally finds a good local solution, which satisfies large numbers of scenarios, therefore the penalty is small, and then the upper bound is reduced heavily.

C. IEEE 118-bus System

Fig. 4 shows the single line diagram of IEEE 118-bus system. In coordinate with [9], the maximum capacity of each line has been reduced to 40% of the capacity given originally to increase the computational complexity of the problem. It should be mentioned that the original data set of [17] does not contain the price information, in this work, an assumption of $c_{ij} = 1000\gamma_{ij}$ is adopted for each branch, since circuit cost is related to length, and length has a positive correlation with line reactance.

After a predefined maximum running time of 70,000s, only *Alg.4* can get solution with the bound gap equal to 0, shown in Table IV. The detailed data and convergence tendency are depicted in Table III and Fig. 5 respectively. It should be noticed that, although the total numbers of iteration is only 38 (far less than that of the IEEE 24-bus system), however

TABLE IV
RESULT OF INVESTMENT PLAN FOR THE IEEE 118-BUS TEST SYSTEM

Branch	No.	Branch	No.	Branch	No.	Branch	No.
01 – 03	1	03 – 05	1	08 – 05	1	11 – 12	1
02 – 12	1	07 – 12	1	19 – 20	1	08 – 30	1
29 – 31	1	45 – 46	1	51 – 52	1	60 – 61	1
74 – 75	1	77 – 78	1	79 – 80	1	88 – 89	1
94 – 95	1	105 – 106	1	12 – 117	1		

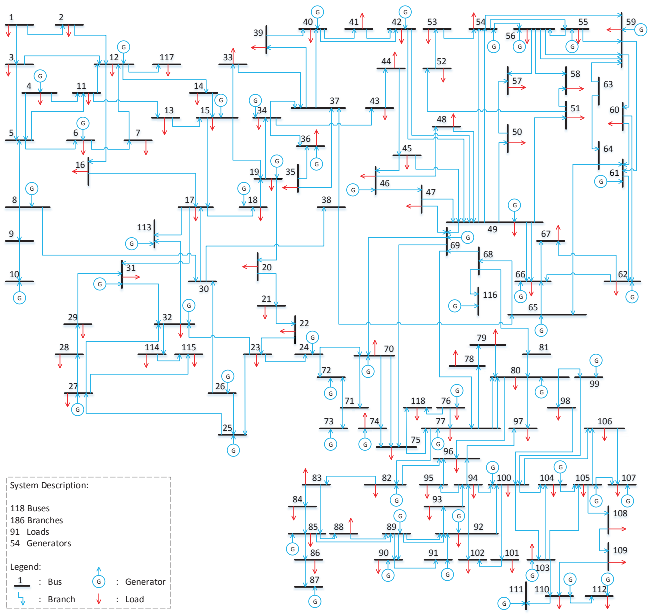

Fig. 4. Single line diagram of the IEEE 118-bus test system.

the execution time is much longer. That is due to the large amount of decision variables and scenarios.

V. Conclusion

In this paper, three improvements are implemented on the classical Benders decomposition: valid inequality, multicut strategy, and optimal precondition, to tackle the security constrained transmission expansion planning with $N-1$ security criteria and generation re-dispatch. Three case studies are investigated to validate the performance of the proposed algorithms. The improvements of execution time as well as the total number of iterations are reported for each strategy. For the future work, two directions of parallel computing will be considered: 1) using SIMD (single instruction, multiple data) framework to handle each scenario subproblem in parallel; 2) utilizing efficient parallel algorithm to tackle the large scale MIP master problem.

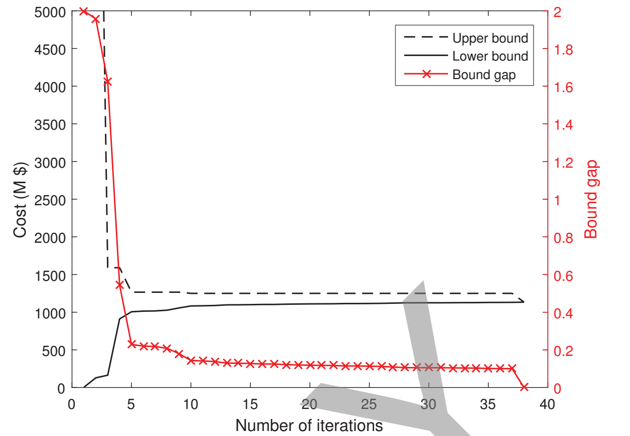

Fig. 5. Behaviour of algorithm convergence for the IEEE 118-bus test system.

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