**University of Alberta**

# Authoring Themselves as Mathematical Learners: Students' Experiences of Learning to Learn High School Mathematics

by

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A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

Department of Secondary Education


Spring 2014
Edmonton, Alberta

## Abstract

High school mathematics students often complete homework and study for unit tests without support to consider how these actions could contribute to their mathematical learning. However, students can, through the process of learning to learn mathematics, to bring into view how they learn mathematics. Mathematics class is an interesting context to study the ways in which students could improve their approaches to learning because of the compulsory nature of course enrolment and the contentious nature of the content. This dissertation responds to the research question: How can we understand students' learning as they actively develop their processes of learning mathematics?

Constructivist grounded theory, repositioned in symbolic interactionism and constructivism, framed this interpretive inquiry. Thirteen grade 12 students from a Mathematical Learning Skills class, taken concurrently with an academic mathematics class, volunteered to co-construct data with the researcher over a four-month period. Data included interactive writing, small group sessions, interviews with students and the teacher, student working papers, and researcher field notes. Sensitizing concepts of intention/al/ity, voice, (re)forming identity, and relationships with sources of knowledge informed a comprehensive coding process. Categories of analysis were developed through prototypical exemplars and their integration resulted in theorizing about learning with the metaphor of authoring.

Students inquired into systemically defined and externally imposed tasks as they participated in learning-based conversations. They engaged in *becoming*

*aware*, *incorporating suggestions*, *verbalizing possibilities*, and *(re)forming intentions* as ways of learning to learn mathematics. Viewed as dynamic and authentic, the processes for learning mathematics students developed included examples like "creating summary sheets" and "formulating verbal explanations." The students also developed and verbalized mechanisms for making sense of mathematical ideas, component elements within the processes for learning mathematics. The mechanisms included: *breaking down*, *putting together*, *connecting*, and *writing down*. As the students were learning to learn, they were *authoring*. Authoring, as a metaphor for learning, is a generative activity of making meaning of experiences and interactions which shapes self and the world. I use the metaphor of authoring to draw together the complex experiences of the students' learning within the context of mathematics, as an abstract interpretive understanding. Students were authoring processes for learning, authoring mathematical ideas, and self-authoring as they began to see themselves as mathematical learners.

# Acknowledgements

I wish to thank Dr. Elaine Simmt, my supervisor, for providing space and support throughout the program and in the undertaking of this research project. For my supervisory committee – Dr. Lynn McGarvey and Dr. Anna Kirova – I appreciate their careful reading and comments to improve the document. And for my examining committee – Dr. Florence Glanfield, Dr. Ingrid Johnston, and Dr. Mellony Graven – I value their insightful comments and thoughtful questions that have encouraged me to imagine where I might take this work in the future.

I am greatly indebted to the students and teacher who participated in this study. Their acceptance of the invitation to support my learning was both courageous in that it gave me glimpses into the personal nature of learning, and generous in that they shared of themselves and their time. I was fascinated to notice their growth. Their contributions to my own growth as an inquirer are expressed in the pages of this dissertation. I appreciate their investment in making a difference for future students in mathematics classrooms.

My learning has been enriched and enlivened by the innumerable conversations with colleagues over the years. I have learned through interactions with my Canadian mathematics education community, and professors and peers in my doctoral program. For those inhabitants of Room 948, past and present, you have shaped my thinking and your participation in conversations echo in the pages of the document. I hold dear my friendship with Jody, for her understanding of the challenges and significance of the journey. I am extremely thankful for a conversation partner in Ralph, whose unwavering belief in my capability and enthusiasm for my learning was a sustaining force.

I gratefully acknowledge the financial support I received for my doctoral program. The Social Sciences and Humanities Research Council of Canada provided the opportunity for me to maintain a strong focus on my learning and research. I appreciate the financial support of the University of Alberta and the Department of Secondary Education throughout my program.

And finally, I am amazed by the support from my family – Jason, Rayna, and Caris – for your willingness to allow me to pursue this program. I am overwhelmed with gratitude. My special and deepest thanks for their patience and understanding of the intensity of my learning.

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# Chapter 1
# Inquiring into Learning to Learn Mathematics

*It's quiet in the classroom. Small desks are in rows, a child sitting in a chair behind each one. Lying open on each desk, an identical mathematics workbook. Pencils are in hand. Some children are writing down numbers. Others are counting on their hands. Still others are distracted by looking around the windowless room. The teacher looks over the class, from her desk at the front of the room.*

*The instructions were to complete the page of subtraction questions, so I work diligently. I check to make sure my pencil is sharp. I count on my fingers precisely. I want to be sure that each answer is correct. There are many questions on the page, so I persist. I have a sense of accomplishment when each line is filled. I look at my page, and then around the classroom. I begin to think about the classroom I am inhabiting.*

*I thought mom said that grade one would be like Sunday school. But what of this is Sunday school? Sitting individually, instead of grouped around a table. Quietness all around, instead of bustling noise. Questions to answer, instead of engaging stories. The authoritative gaze, instead of an embracing smile. Skills and concepts to remember, instead of experiencing life together. Sterile and static, instead of relevant and dynamic. How is this learning?*

*Twenty-six and a half years later, I find that mathematics book among a collection of old school books and papers. There it is! The pelican on the post, just as I had remembered it. Written on each blank is a numerical answer. Where is the reasoning and communication that could have come from interacting with others? My inquiry into learning has taken me on a journey into a doctoral program. I enter the program with intentions for my own learning, and engage in courses where I continue to wonder: how is this learning?*

*Through my experiences as a child in school, as a high school teacher in the area of mathematics, as a university student, as a pre-service teacher educator, as an emergent researcher and scholar listening care-fully to high school students, my questions around learning become more complex. How are moments of learning occasioned by a teacher? How do students take up those moments? How do all participants, teacher and students, make sense of these learning experiences? And then the focusing question emerges: how are students learning to learn mathematics? The authenticity of this learning opportunity is rich and compelling.*

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I begin this dissertation with a narrative illustrating a prominent feature of my experiences in school – attending to and trying to come to understand *learning*. While I have distinct remembrances as a student and a teacher of attending to my learning, it was not until I began to reflect on my experiences both in and out of school during my doctoral program that I began to see how my
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interest in learning had permeated many moments of my life. Because of my stance as a subjective researcher, I have chosen in this opening section to describe some key experiences as a student, teacher, and emergent researcher that led me to identify a research focus and to develop a question for this research project. I then survey some of the guiding concepts in the research question to locate the research within the discipline of mathematics education and explore constructs which supported the inquiry.

## My Experiences that Led to this Inquiry

As early as grade one and up to my current studies in the doctoral program, I have attended to learning as it is occurring in the classroom. Earlier, it was an awareness of my own learning and how my actions as a student influenced my success. For example, in high school I would independently create a variety of study strategies that would help me succeed on final examinations. These strategies often aimed at constructing the essential ideas within units of study and developing connections across the ideas through various verbal representations. A shift in focus occurred in my undergraduate mathematics education courses, where interacting with classmates about their learning provided opportunities to incorporate into my view the learning of others around me. It was not until my doctoral studies did I find ways to express through writing the importance of inquiring into my own learning which prompted me to engage in sustained scholarship related to learning.

As a teacher, I listened to the students who were in my classroom – as students, learners, and persons. It was a stance fostered in my undergraduate mathematics education courses, and it resonated with who I was – a learner and a teacher, an inquirer. The use of inquiry-based learning activities allowed me to notice students' learning in action, where their learning was the focus in the classroom in contrast to a focus on a teacher's thinking in a didactic approach. A listening-based stance provided opportunities to hear the uniqueness of how each student approached her or his learning. I used processes like interactive writing and portfolios to open space up for students to talk about their mathematical thinking and learning. I encouraged them to consider the nature of mathematics, to set goals, to shape learning strategies, to see the tasks offered (whether homework, in-class learning activities or otherwise) as learning opportunities. This stance was also well-supported by curricular shifts, both locally (Manitoba Education and Training, 1998; Manitoba Education, Training and Youth, 2002) and more broadly (National Council of Teachers of Mathematics [NCTM], 2000; Western and Northern Canadian Protocol [WNCP], 2008).

What I realized, though, was that students struggled to say things about themselves, their thinking, and their learning. In my master's thesis (McFeetors, 2003, 2006), I noticed that when students accepted invitations to be in conversation about their thinking and learning, their voices emerged. Many of the students' particular successes featured noticing and saying things about how and what they were thinking and learning. The experience of students coming to say things about their learning occurred across various mathematics courses I taught. As I have moved into teaching undergraduate education courses, I continue a similar focus on learning by inviting pre-service teachers to talk about their learning and plan for learning as they enter the field. For example, in a mathematics curriculum and instruction courses at the elementary and secondary levels, I encouraged pre-service teachers to write learning-based outcomes in lesson plans. The lesson plans were also to explain how they would occasion learning opportunities by stating what students would learn during elements of a lesson, rather than what students would be doing.

My interest in how students learn high school mathematics has informed my research projects. Ralph Mason and I conducted a five-year study, *Trajectories of Students Learning High School Mathematics and Science*. The study explored students' choices of courses and how they engaged in the challenges of learning within the courses. Following a cohort of students through grades 9 to 12, we conducted online surveys and had informal interviews twice a year. Repeatedly, students shared with us their desire to succeed by learning mathematics meaningfully and improving at the processes of learning mathematics, but received little or no support from their teachers (Mason & McFeetors, 2007). Even with the lack of support, there were students who, independently or with peers, were developing learning strategies in order to succeed in learning mathematics. The students also demonstrated that their experiences in high school mathematics were shaping who they were. The research I am reporting in this dissertation moves from a distant positioning in the *Trajectories* study to the intimacy of a classroom as I further explored the qualities of students' learning as they improved their processes of learning mathematics.

Through interpreting my experiences that led to this inquiry, I understand my way of being and becoming in the world as *learning as an ontological orientation*. I appreciate the distinction Packer (1999) makes in moving from an epistemologically-focused perspective of learning to the recognition that learning involves aspects of shaping who we are in the world. He, along with Goicoechea (2000) further developed a nondualist ontology of social constructivism which collapses the separation of knower and known to recognize that learning is not separate from being. In a similar way, Cobb and Yackel (1996/2004) coordinate interactionism and psychological constructivism which opens up space for considering how learning is an ontological endeavor (the shaping of one's self – the process of becoming). This reminds me of Aoki's living in between (1986/1991/2005) and that what emerges from each of these is a difference space that is not defined by the compromise of dualities in order to understand the richness and complexity of learning.

In this way, learning is being and becoming and moves beyond the epistemological. Bateson (1994) asserts that "learning is a way of life" (p. 211) and that re-visioning schools requires a "sense of ourselves as learning beings" (p. 212). These perspectives emphasize learning as fundamental to life and school. My ontological orientation is more than attending to learning around me; it is also a deeply personal way of being in the world. It was in my search for authenticity in this research process that I came to see that it is about *my* learning – the shaping of myself as I engaged in the opportunities to learn more about issues in mathematics education.

## Learning as an Ontological Orientation

At the outset of the inquiry, I reflected on what learning as an ontological orientation might mean for me as a learner. There were three perspectives that were forming in my thinking. I briefly explain these three perspectives in the following paragraphs, and then return to this notion with the voices of the students in the inquiry informing a more nuanced understanding of learning as an ontological orientation.

Learning cannot be an activity solely to reach a desired end to prepare for the future, but it is what it means to live the best way possible in that particular moment. Dewey (1938/1997) notes the orientation of a progressive education as “making the most of the opportunities of present life” (pp. 19-20). Extending this stance is to see every moment and encounter as having the possibility of being educative – there is something of value to learn and to grow. “We can carry on the process of learning in everything we do” (Bateson, 1994, p. 9). This perspective shaped the focus of my inquiry in moving from the mathematical cognition of students to attend to how students are learning more than mathematics in their endeavour to understand concepts and develop skills.

Learning is a way of being in relationship with others, with an openness to be shaped through interactions and to be implicated (acknowledging the responsibility) in the shaping of others. It is a certain kind of thoughtfulness, for me it is a caring relation (Noddings, 1984), which supports educative interactions. There is a sense of interconnectedness, as “all those others present with me are a source of identity” (Bateson, 1994, p. 75) and each person is “shaped and reshaped by learning” (p. 64). Dewey (1938/1997) perceives that “all human experience is ultimately social” (p. 38) and that experience “modifies the one who acts and undergoes” (p. 35) as well as the context of the experience. I am continually (re)forming my identity as I learn through my experiences. This perspective shaped my inquiry in wondering how students learn to learn mathematics as they are in conversation with others, and the reciprocity of shaping self and relationships with others.

Learning is a way of attending to others and to a context, to wonder about the actions and interactions and to try to make sense of why and for what effect we engage in learning. Bateson (1994) identifies that “*insight* ... refers to that depth of understanding that comes by setting experiences, yours and mine … side by side, letting them speak to one another” (p. 14). I view the notion of insight similar to a way of coming to understand through deep and active engagement. Dewey (1938/1997) refers to this thoughtfulness as “genuine reflection … used to organize what has been gained in periods of activity” (p. 62). This perspective informed my intention to understand students’ learning experiences and how they develop thoughtfulness about how they are learning mathematics. I find the perspectives and words of both Dewey and Bateson inspiring, as individuals and scholars who inquire into how people are in the world. They provide, for me, a sense that learning through mathematics in school is part of the process of becoming for each student. In each of the three perspectives, learning is both a way of being and a way of becoming for me.

As I consider learning as an ontological orientation for me and the connection between learning and becoming represented by Dewey and Bateson, I wondered about the meaning of the experiences of students learning high school mathematics. In this, my attention shifted from my own learning to the ways in which students are learning high school mathematics. As I viewed learning as a malleable process, I wondered at the beginning of the study whether students could improve not only in their mathematical competencies in mathematics class, but that they could also improve in their learning processes. In learning to learn mathematics, I also wondered if students' attentions could also shift toward wondering about their own learning. I wondered about the potency of learning as an ontological orientation for them.

I came to this research study with the intention to examine more closely ways in which students can learn (how) to learn mathematics in high school. As I discuss in the following section in this chapter, much work in mathematics education has been done and reported around students' cognition and other psychological, social, or cultural issues which can impact student learning. This inquiry explores the learning experiences of a small group of high school mathematics students and then considers more broadly the meaning of learning and what it means for them to be, as learners, and become through their learning experiences. In examining how students learn to learn mathematics, my intention in the inquiry was to come to understand their experiences.

Specifically, my research question was: **How can we understand students' learning as they actively develop their processes of learning mathematics?**

## Surveying the Research Question

Mathematics class is an interesting context to study the ways in which students learn to learn because of the compulsory nature of course enrolment and the contentious nature of the content. The compulsory characteristic of mathematics education, shaped by societal norms and the gate-keeping function of mathematics (Ma & Klinger, 2000; Madison & Hart, 1990), necessitates success for *all* students (Sliver, 1994). Students' beliefs about their capabilities as learners affect and are affected by issues of identity, voice, and intentionality (Nicol, Li-Liang, & Gaskell, 2004). However, students do not always succeed in learning mathematics as they struggle to see themselves as capable of learning. Nor do they perceive that their voices are heard as they fail to persist. D'Ambrosio (1999, 2007, 2008) calls on the field of mathematics education to develop mathematics curricula which values the dignity of each person. It is because mathematics class is the site of struggle and because students are being shaped by their experiences in mathematics class that I engaged in this research project. The ontological stance of high school students' experiences in learning mathematics, opening up opportunities for students to think about how they are learning mathematics, and incorporating multiple perspectives (especially the students') provided the framing for a unique study in which I theorize about students' learning experiences as they learn to learn mathematics. In this section, I explore concepts related to the research question to situate the study within mathematics education and to explain the focus of the research.

**Focusing on Students' Learning**

In order to respond to the challenges facing mathematics education as a discipline and the daily struggles of mathematics teachers and students, I have suggested a shift in research focus in this study by carefully examining students' learning. Recent mathematics education reforms have established better teaching practices (NCTM, 2000; WNCP, 2008). More significantly than shaping pedagogical approaches, mathematics education reforms have also emphasized students' personal development of mathematical ideas (e.g., Davis, 1986; Pirie & Kieren, 1989, 1994; Skemp, 1976/2006). Mathematics educators have attended to the qualities of students' sense-making of mathematics, noticing ways in which students come to understand mathematical concepts and the complexity of their mathematical cognition (e.g., Borasi, 1992; Confrey, 1998; Mason, 2002; Watson, 2008).

The focus on mathematical content and students' mathematical cognition has limited the attention paid to the processes of learning. To clarify the distinction: mathematical cognition attends to the thinking and sense-making of students (students as cognizers), while mathematical learning incorporates the orientations and actions of students to make possible the engagement in thinking mathematically (students as learners and persons being shaped by their experiences). Novak and Gowin (1984) help tease out the distinction between "learning and knowing. They are not the same. Learning is personal and idiosyncratic; knowing is public and shared. … Educating is the process by which we actively seek to change the meaning of experience" (p. 5). Dewey (1938/1997) concurs: "It is not the subject *per se* that is educative or that is conducive to growth" (p. 46). For Dewey, educative experiences are moments in which students have opportunities for growth and moments which sponsor further growth – the dynamic of *growing*. Shifting the focus is challenging work. Even though the aim of Johanning and Keusch (2004) is to explore how mathematics students "learn to learn" (p. 107), mathematical cognition – the variety of ways that students made sense of a mathematical task and engaged in reasoning – remained central, noticing students as mathematical thinkers. Remaining unexplored in this context are strategies for learning mathematics, approaches to learning mathematics, and students' epistemological stances. Watson (1994) makes an important shift in her exploration of student learning of mathematics,

> A teacher cannot assume that pupils automatically possess a range of ways of working or ways of thinking. At some stage these have to be offered and valued … It is a part of teaching to be explicit about useful learning

> practices and stratagems … Pupils should be given opportunities to develop their learning skills. (p. 57)

Situated within learning particular mathematical ideas, Watson also values the ways in which students are learning, enough to encourage mathematics teachers to teach students to shape and use a variety of strategies within their processes of learning.

**Learning to Learn Mathematics**

Watson's encouragement is, in my view, a call to occasion experiences for students to learn to learn mathematics. When I refer to students' learning in my research question and throughout this dissertation, I am pointing to students learning mathematics and students learning to learn mathematics – perhaps notating it as "learning (to learn) mathematics." Moreover, the students' learning and my theorizing about their learning in this dissertation can be seen as *learning within the context of mathematics*. What I anticipated at the beginning of the study was that with the active support of a mathematics teacher, students could learn about how they are learning mathematics. In other words, I saw the possibility that students could improve in their learning processes by examining their approaches to learning and interrogating the strategies they use, and then act on these reflective inquiries to construct intentions to learn through the development and shaping of specific strategies.

Fischer (1992) identifies "observing our learning process" (p. 16) as a way to humanize learning mathematics and also advocates for a more active stance in talking about "the situation of learning and its relation with the subject matter" (p. 17). In this way, the tacit knowing (Polanyi, 1966) of how students learn mathematics becomes a process in the classroom that is attended to and changed through conversation. In a course where learning strategies were explicitly taught at the college level, Smith (1999) noted that students "were empowered as a result of strategy instruction" (p. 796). Dahl (2004) concurs in the results of her study of successful high school mathematics students who displayed understanding of their own personal learning processes. While she did not explore learning to learn mathematics, she supposed that "since the pupils can verbalise and reflect upon their learning process, there is a potential for improving their learning" (p. 153). Accepting the invitation to further explore how students engage in learning about their own mathematical learning, my dissertation research examined students' improvement in their learning process for mathematics.

Some researchers have taken on the term "metalearning" to refer to aspects of students thinking about their learning. Novak and Gowin (1984) defined metalearning as "learning that deals with the nature of learning, or learning about learning" (p. 8). Goodchild (2001), in his study of students' learning goals, defined metalearning as "students' beliefs about mathematics, learning and classroom activity" (p. 223) and realized its impact on students' formation of learning processes. In addition, my exploration of metacognition in mathematics education led to me agree with the distinction between metacognition and metalearning which Goodchild (2001) and Novak and Gowin (1984) discuss, recognizing the unsuitability of metacognition in supporting the potency of students learning to learn mathematics. Rather than adopting labels, the focus of this inquiry was to understand what it means for students to learn to learn mathematics. I anticipated that students might: 1) shift stances in their learning processes toward orientations that help them develop deep and rich understanding of mathematics; 2) develop (through creating or adopting, and then refining or shaping) learning strategies; and 3) come to see themselves differently, to (re)form their identity, as they are personally implicated in their learning.

**Learning Processes**

In considering learning to learn mathematics, at least two dynamic elements are interesting, namely learning processes and learning strategies. In my research question, "processes of learning" points to something students are developing as a way to improve their learning of mathematics. In fact, *processes* have received emphasis in curriculum development broadly, where Costa and Liebmann (1997) advocate for a curriculum with a "shift [in] focus from the *what* of knowledge (content) to the *how* of learning (processes)" (p. xxi). I see learning processes as interconnected with orientations to the nature of mathematics and coming to understand mathematics. Indeed, Op't Eynde, De Corte, and Verschaffel (2006) found that students' epistemic beliefs were domain-specific within mathematics. For example, if mathematics is seen as a set of discrete skills, students would approach learning by memorizing and practicing in order to do; whereas if mathematics is seen as a complex discipline of concepts, students would approach learning by developing connections through multiple representations in order to understand. At the outset of this research project, I saw learning processes as general approaches to learning mathematics and I anticipated refining my understanding by interpreting students' experiences.

In designing the research question, I chose to omit "learning strategies" because I see them as narrow in focus. While discussions about learning strategies rarely appear in mathematics education literature, psychologists researching within mathematics education consider learning strategies to be "behaviours and thoughts" (Anthony, 1996, p. 23) or "tactics" (Jones, Estell, and Alexander, 2008, p. 2) which are connected to "age, gender, and socio-economic background" (Leutwyler, 2009, p. 112) and impact students' motivation and affect which in turn influences the acquisition of content. These researchers focus on psychological aspects of students, at the expense of recognizing the reflexivity of students and strategies.

Pragmatically, books written for teachers, parents, and/or students present prescribed methods of how to superficially complete homework and studying (e.g., Coles, White, & Brown, 2003; Cooper, 2007; Hellyer, Robinson, & Sherwood, 2001; Johnson & Johnson, 2001; Ooten & Moore, 2010; Peltz, 2007; Scaddan, 2009), without addressing how students could learn for understanding through the personal development of these strategies. The distinction is not necessarily in the type of learning strategies – which could include thoughtful ways of doing homework, re-viewing notes, studying for tests, developing connections among ideas and skills – but the intentions for which they are used and the qualities of the reflections on the effectiveness of the strategies for learning mathematics. Additionally, these how-to books are didactic in their approach and divorced from the particular contexts of learning mathematics, rather than working from individual students' current intentions and processes for learning mathematics. At the beginning of the research project, I saw learning strategies as specific procedures, developed within learning processes and aimed at particular goals to support success in learning mathematics.

**Experience**

Learning to learn mathematics could take place as students actively engage in learning experiences. The word "experiences" can be taken up in understanding learning because it emphasizes an educative way of students being in the classroom. Dewey (1938/1997) saw experience as the essence of education, where the aim of education is growing. At the same time, he asserted that an experience is not automatically educative, but that it "depends upon the *quality* of the experience" (p. 27). The quality of an experience can be determined through the two characteristics of continuity and interaction which mark an experience. Continuity considers each experience as growing out of another experience and also leading toward further experiences. Interaction involves the interplay between the internalization of an individual and the situation (both context and other people), and the reciprocal nature of shaping both individual and situation. There is an active quality to experience, where an activity occurs in which the individual acts and interacts. However, the activity is not the only element of an experience, but to transform the activity into an experience there must be a reflective act by the individual.

The reflective act, then, becomes the crucial element to transform an event into an experience. Dewey (1910) defines reflective thought as "*active, persistent, and careful consideration of any belief or supposed form of knowledge*" (p. 6) and that this thought is an interpretive act (in ascribing meaning to the event). In relation to educative experiences, Dewey (1938/1997) notes,

> Keeping track is a matter of reflective review and summarizing, in which there is both discrimination and record of the significant features of a developing experience. To reflect is to look back over what has been done so as to extract the net meanings which are the capital stock of intelligent dealing with further experiences. It is the heart of intellectual organization of the disciplined mind. (p. 87)

I see reflection as a way in which occasions arise for students to learn how they are learning mathematics. The activity which precedes the reflection is engagement in learning mathematics (for example, through a particular strategy like re-viewing class notes). As a student reflectively thinks about the learning activity, he or she makes sense of the learning in a way that might improve the strategy and impact further learning.

Dewey's notion of reflection as an important part of having an experience has been explored by mathematics educators. Mason, Burton, and Stacey (1985) describe reflection as "possibly the most important activity for improving mathematical thinking" (p. 42). This active positioning of thinking has improvement as its core action. Skovsmose (1992) and Goodchild (2001) both connect reflective knowing and thinking with the interpretation and meaning making of learning mathematics. Cobb, Boufi, McClain, and Whitenack (1997) see reflection as explicit consideration of preceding mathematical activity, and that the "reflective discourse constitutes the conditions for the possibility of learning, but it is the students who actually do the learning" (p. 272). However, in each of these examples, the experience was of learning mathematics, whereas my interest is primarily in the experiences students have when they are learning to learn mathematics. Because both contexts are aimed at being educative, I see significant parallels in the types of experience.

**Conversations**

Conversational spaces hold the possibility for students to engage in reflections on their learning processes. In other words, students could actively develop their processes of learning mathematics through conversation. Polanyi (1966) raises a tension for learning to learn mathematics through reflection on processes when he states, "*we can know more than we can tell*" (p. 4). I am aware of this tension in my research, but my response is that the use of reflection on action invites students to attend to their processes of learning – and that through these reflective acts students' learning is transformed from a form of tacit knowing to an intentional process. Additionally, the use of conversation as a way of learning mathematics is well-supported in the literature, both in oral communication (e.g., Gresalfi, Martin, Hand, & Greeno, 2009; Herbel-Eisenmann & Cirillo, 2009; Pimm, 1984) and in written communication (e.g., Masingila & Prus-Wisniowska, 1996; Morgan, 1998; Pugalee, 2004).

In a survey of mathematics education literature, uses of "conversation" pointed to a particular form of communication in which there is an interpersonal and intimate nature of turning round ideas for the purposes of growth, as well as a way of being in the world. Across readings that were influential in my thinking (Bauersfeld, 1995; Cobb, Boufi, McClain, & Whitenack, 1997; Davis, 1996; Ernest, 2003; Gordon Calvert, 2001), I constructed five features of conversation which I see as important orientations to conversations in classrooms that could support learning to learn mathematics. The features include: (1) withness, where the interpersonal is valued in a way where a mutual sense of trust and equality is negotiated among members; (2) listening, where each member is present, open, and responsive to the other for sense-making; (3) dynamic, where there is a fluidity, flexibility, and anticipation of the unexpected and an energetic investment by each member; (4) uncertainty, of where the conversation leads members and how the conversation is understood afterwards (exophoric orientation in interpreting the conversation); and (5) form, where conversations could be both oral and written.

The forms of conversations in this research included whole-class (during direction of learning strategies), small groups with classmates (during small group conversations or during unstructured times in class), one-on-one occasions (during interactive writing or speaking with the teacher, me or their classmates), and self-talk (this can only be reported by the student during other data construction elements). Cobb (in Sfard, Nesher, Streefland, Cobb, & Mason,

1998) provides a caution for mathematics education researchers to keep student learning as the core of their inquiries, rather than researching at the periphery on structures of conversation in the classroom. To this end, I view conversations as the context in which the learning experiences might occur, and I want to inquire into the *learning experiences*.

Coming to understand students' experiences of learning (how) to learn high school mathematics is the focus of the research I report on in this dissertation. My own experiences of learning and teaching brought me to this inquiry because of the complexity of what it means to learn and the potential for the growth of each person in a mathematics classroom as they come to understand their learning and who they are as learners. In addition to exploring my experiences that brought me to shaping this inquiry, this chapter surveyed underlying concepts which informed the posing of a particular research question. The complexity of the question, and therefore the inquiry, is highlighted in the provisional nature of the underlying concepts which will further be explored as the voices of the students who participated in the inquiry are incorporated.

## A Preview to the Dissertation

Two significant and interconnected components in the document guided my authoring of the dissertation: a story of my learning throughout the inquiry and a record of the research project. In the first instance, I viewed the inquiry as an opportunity for me to learn. In the second instance, the storying of the research study emphasizes the way in which I carried out the inquiry, showcasing the data as integral to the acts of theorizing which resulted in an interpretive understanding of students' learning. Both the relating of the learning journey and the research study are interwoven throughout the document explicitly, foregrounding at different moments my learning story or the research story. What strongly ties both of these elements together is their process-based nature and the inquiry into students' processes of coming to understanding mathematics in high school.

One of the characteristics of a narrative is the temporal element in how it unfolds, and because of this characteristic I chose to represent my learning and inquiry as a trajectory which traces the shifts in my thinking and growth as an inquirer. In choosing to have this dissertation document my learning journey through the research project, as there is a kind of temporality to the representation of the research project. These first two chapters inform my early stance with the inquiry and important ideas before I began the inquiry. The third chapter is temporally transitional, in laying out how the research was carried out. The fourth through seventh chapters develop my theorizing of the ways in which students were learning to learn mathematics, further abstracting as the chapters progress. The eighth chapter brings a sense of closure to this inquiry by imagining a continuity of interaction. In the remainder of this section, I highlight the main ideas of each of the remaining chapters in the dissertation.

Chapter 2, ***Navigating the Learning Opportunity: Methodological Considerations***, describes the methodological framing that supported how I learned throughout the inquiry. Constructivist grounded theory (CGT) was developed as a re-grounding of the first formulation of grounded theory, returning to the symbolic interactionist roots and using a constructivist framing. This provides a foundation for an interpretive methodology, through which the research aims to theorize. Rather than selecting a theoretical framework with which to view the data, I explicate four sensitizing concepts, or ideas I am drawn to attend to in data. The sensitizing concepts include: *intention/al/ity*, *voice*, *(re)forming identity*, and *relationships with sources of knowledge*.

Chapter 3, ***Engaging in the Opportunity to Learn***, builds on the theoretical focus of the previous chapter to detail the implementation of the inquiry. The research project took place in a grade 12 "Mathematical Learning Skills" course which students were taking concurrently with an academic mathematics course. Thirteen students, a teacher, and myself as participant-inquirer took part in the research project. The data were constructed over approximately four months of time in the school. The six forms of data include: field notes and research process journal, interactive journal writing, small group sessions, interviews with learner-participants, learner-participants' working papers, and conversations with the teacher-participant. Descriptions of the forms of data illustrate the complexity and depth of the data through which I engaged in theorizing. In learning with and through the data, I conclude the chapter by describing my data analysis processes of coding and categorizing which supported my theorizing explicated in the remainder of the document.

While the first three chapters in the dissertation describe and situate the research study, in the next four chapters (Chapters 4 through 7, inclusive) I build on my data analysis to portray several different interpretive moments with the data. The interpretive moments can be seen as a zooming in and out, to use different magnifications for understanding the learner-participants' learning within the inquiry. In Chapter 4, I take a broad view to explain features of conversational opportunities through which the learner-participants were shaping learning processes. In Chapter 5, I zoom into the conversational context to explicate the learner-participants' forms of engagement in developing learning processes for mathematics. In Chapter 6, I zoom in further, to examine within the learning processes the mechanisms by which the learner-participants were making sense of mathematical ideas. Finally, in Chapter 7, I integrate the interpretive moments to theorize the learner-participants' learning through the metaphor of authoring. The occurrence of multiple interpretative moments highlights the characteristics of provisionality, indeterminacy, and tentativeness of theorizing.

Chapter 4, ***Providing Opportunities for Learning-Based Conversations in High School Mathematics***, explores the context in which the learner-participants in the study were engaged in learning to learn mathematics. By viewing and interpreting their interactions with me and with each other, I noticed that they were engaging in opportunities for learning-based conversations. These conversations had as their focus the ways in which learner-participants were learning within the context of mathematics. I developed four features of these conversational opportunities: *preparation*, *presence*, *mode*, and *pace*. Through the use of a diagram, I draw together the four features and suggest the conversational moments could be placed within the diagram to define the space. This chapter is the first interpretive moment through which I offer an understanding of a particular way the learner-participants were going learning to learn mathematics.

Chapter 5, ***Developing Ways of Learning Academic Mathematics***, explicates the ways in which the learner-participants were shaping approaches to learning mathematics. This second interpretive moment zooms into particular ways of engaging in develop learning processes that were situated in the conversations described previously. The learner-participants inquired into the systemically defined and externally imposed tasks (such as doing homework or taking notes) which they perceived as static and superficial. As they did so, they were *becoming aware*, *incorporating suggestions*, *verbalizing possibilities*, and *(re)forming their intentions* to develop personal processes for learning mathematics. The four forms of engagement in developing learning processes illuminates the complexity of learning to learn mathematics. The learner-participants provide, through their data, several examples of learning processes (such as identifying key mathematical ideas or creating summary sheets) which they saw as dynamic and authentic.

Chapter 6, ***Making for Themselves Mathematical Ideas***, explores mechanisms the learner-participants used in making sense of the mathematical ideas presented to them in mathematics class. Zooming further into the learning processes developed by the learner-participants, this chapter represents a third interpretive moment through the data which is a great magnification of the learner-participants' learning. Understanding, described by the learner-participants, focused on why procedures they were required to learn worked to solve problems. Within the processes for learning mathematics the learner-participants developed, the learner-participants expressed using mechanisms to make sense of mathematical ideas that were common across learning processes. In particular, the learner-participants were *breaking down*, *putting together*, *connecting*, and *writing down* as they aimed to fulfill their evolving intentions to understand the mathematical content for themselves.

Chapter 7, ***Authoring Ways of Being in Mathematics Class***, articulates an abstract interpretive understanding of the complexity of the learner-participants' learning – and through that learning, being and becoming – in relation to mathematics. While it is a fourth interpretive moment, I see this as a culminating idea because of the abstraction of the theorizing and the integration of the previous interpretive moments within a significant idea. Rather than zooming in further, the abstraction calls for a lessening of the magnification or a zooming out. The metaphor of *authoring* integrates the various enactments of the learner-participants' learning: about learning processes for mathematics (Chapter 5), about mathematical ideas (Chapter 6), and about themselves as mathematical learners. Authoring is explicated as a generative activity of meaning-making, which shapes self and the world. I offer it as a novel metaphor for understanding what it means to learn (how to learn) within the context of mathematics. In positioning *authoring* as a metaphor for learning in mathematics class, it attends to and illuminates the important process of the learner-participants' (re)forming of their identity as emerging mathematical learners through self-authoring.

Chapter 8, ***Drawing the Inquiry to a Close***, takes from my learning and theorizing throughout the dissertation to provide some suggestions for future conversations and research related to my work. Beyond acknowledging an interpretive understanding of the learner-participants' learning as a scholarly contribution to the mathematics education community, I imagine how mathematics teachers and mathematics educators might attend differently to learning in mathematics classrooms through a range of learning experiences which sponsor students to author themselves as capable of learning mathematics. I close with a reflection on my own learning trajectory.

# Chapter 2
# Navigating the Learning Opportunity: Methodological Considerations

I used constructivist grounded theory [CGT] to come to understand students' learning as they learn to learn mathematics through conversation. As a methodology that I have not previously used, I learned much about the processes suggested by researchers who shaped CGT and found it potent for theorizing. I first came to grounded theory as a possible methodological framing for this study; it is an approach that resonates with my sense of how we come to understand experience. For me, CGT has a coherence with the processual nature of the phenomena I inquired into through this research project.

In this chapter, I provide an account of the development of grounded theory, a description of the particularities of CGT, and its use in mathematics education research. I then address the methodological appropriateness for my research and learning intentions. Last, I share the sensitizing concepts with which I began the research. The chapter will be a methodological examination, theoretical in focus. It will provide a foundational explanation for the specific research design in chapter 3. A thorough methodological exploration is provided for three reasons. First, it represents the expansion of my methodological repertoire first through scholarly learning. Second, because the methodology framed my decision making in both carrying out the research project and in authoring this dissertation, this chapter may be of use to readers of the dissertation. Third, recognizing that CGT is not a commonly used methodology in mathematics educational research, I wanted to develop its theoretical suitability explicitly as preparation for future academic discourses.

## The Development of Grounded Theory

The synthesis of my scholarly exploration below describes some of the fundamental influences on the initial formulation of grounded theory methodology, to support a focus on constructivist grounded theory and how the methodological framework could be useful in problematizing fundamental concepts and how it supported my research project. I document the historical development of grounded theory, conceptualized and written with minimal explicit references to the documentation but represents my holistic understanding of the development (Annells, 1996; Bryant, 2002; Bryant & Charmaz, 2007a, 2007b; Charmaz, 2000, 2006, 2008; Clarke, 2005; Corbin & Strauss, 2008; Covan, 2007; Glaser & Strauss, 1967; Strauss & Corbin, 1990, 1994).

### The Early Moments of Grounded Theory

*The Discovery of Grounded Theory*, published by Barney Glaser and Anselm Strauss in 1967, announced to the scholarly communities of sociology and of qualitative methodology a systematic way of inquiring into social experiences that would produce explanations that were abstract, yet were generated from the close attention to data collected. Through research projects in the early to mid-1960s, Glaser and Strauss had been attending to the way in which they were working together in analyzing data (e.g., Glaser & Strauss, 1965). The research community of sociology at the time was characterized by an entrenchment in quantitative methodologies which focused largely on the validation of grand theories produced by renowned sociologists like Marx, Durkheim, Mannheim, and Weber. Glaser and Strauss emphasized three characteristics for grounded theory: (1) the importance of large amounts of empirical data (as opposed to "arm chair" theorizing); (2) generating rather than validating theory; and (3) the use of qualitative rather than quantitative data.

The scholarly lineages of Glaser and Strauss influenced their shaping of a methodology that encompassed the three shifts in researching social phenomena. Here, *scholarly lineage* refers to both the scholars whom Glaser and Strauss can trace back their academic training and thinking to, and also the ideas those scholars generated that were influential in Glaser's and Strauss' thinking. The importance of the exploration of lineages arises from my own postmodernist influences – that the shaping of the methodology takes up the experiences of the two scholars and that their perspectives are embedded in the early formulation of grounded theory methodology.

Glaser completed his doctorate in sociology at Columbia University. He came from a strong positivist tradition, with Lazarsfeld (his supervisor) having developed a reputation for systematizing quantitative research methods so that they were rigorous, could claim an objective researcher stance, and were defensible as predictors of future social behaviour. This influence is apparent in the systematization of qualitative methods in grounded theory. Merton, a theorist situated at Columbia, developed notions around the creation of middle-range theories (in comparison to grand theories). This influence is apparent in the development of substantive theories located within a particular context in grounded theory.

Strauss completed his doctorate in sociology at the University of Chicago. Renowned for the "Chicago school," Strauss studied under Blumer and was influence by scholars such as Mead, Park, Pierce, and Dewey. Sociology, at the Chicago school, was influenced philosophically by American Pragmatism and was immersed in the development and use of symbolic interactionism as a way to understand human experience. The influences of constructivism and symbolic interactionism are apparent in grounded theory methodology through the notion of multiple interpretations of experiences, the socially-mediated creation of these meanings, the pragmatic desire of a theory to both fit the data and to be useful, the active stance of both researcher and participants, and the interest in exploring process, rather than static states.

The two distinct scholarly lineages caused me to wonder what had brought these two sociologists together: the focus on empirical data was important for both scholars. For Glaser, the focus grew out of a scientistic background, and for Strauss, out of the notion of people being active agents and inquiring into their experiences. As significant as the study of particular social contexts and experiences – through the collection and analysis of large bodies of data – was to both Glaser and Strauss, it appears that this was not enough to hold the two scholars together. In fact, it was their different philosophical perspectives which are highlighted by the diverse scholarly lineages of the two scholars that led toward a seemingly inevitable rift. In part because they were initially working within a positivist tradition (Charmaz (2000) also points to the positivism evident even in Strauss' work), the objectivist orientation the tradition encapsulated did not lead Glaser and Strauss to inspect their experiences and consider how their experiences impacted their shaping of grounded theory methodology. As I read *The Discovery of Grounded Theory* (1967), I noted an absence of description of their philosophical (epistemological and ontological) underpinnings – an observation confirmed by Bryant (2002).

### Branches of Grounded Theory

Substantial branches of grounded theory have developed. Tracing these developments might be an interesting scholarly task, but the task of selecting a methodological framing that supports my particular research project does not justify an extensive search along each branch. As I read along each branch, I engage in a process Mills, Bonner, and Francis (2006) recommend:

> Researchers, who first identify their ontological and epistemological position, are able to choose a point on the methodological spiral of grounded theory where they feel theoretically comfortable, which, in turn, will enable them to live out their beliefs in the process of inquiry. (p. 32)

The Glaserian school of grounded theory (Glaser, 1992) retained the objectivist focus of the role of the researcher, while at the same time defending an emergence of theory from data. Strauss worked with Juliet Corbin (Corbin & Strauss, 2008; Strauss & Corbin, 1990) in developing a Straussarian school of grounded theory which produced more specific analytic procedures such as axial coding and conditional matrices as prescriptions. Adele Clarke (2005, 2009) has shaped situational analysis, a postmodern and feminist theory formulation of grounded theory which attends to broader social worlds and discourse materials to construct situational mappings. Hermeneutic grounded theory, shaped by David Rennie (1998), uses hermeneutical thinking to align with the Glaserian school, collapsing dualities (such as realism/relativism, inductive/abductive) and validating grounded theories within their generation. However, it is Kathy Charmaz's (2006, 2009) formulation of constructivist grounded theory that resonated most with my epistemological orientation and the focus and assumptions of my research project.

## A Particular Way to Learn within the Research: Constructivist Grounded Theory

In this section, I explore CGT as a specific form of grounded theory that has been relocated within a postmodernist approach to research. Charmaz recognized that both branches of the two originators' methodologies (Corbin & Strauss, 2008; Glaser, 1992) were positivistic and objectivist in nature.

Dissatisfied ontologically and epistemologically, Charmaz (2000, 2005, 2006, 2009; Bryant & Charmaz, 2007a, 2007b) searched within the foundational orientations of grounded theory to "reposition [grounded theory methodology] in the light of the current philosophical and epistemological landscape" (Bryant & Charmaz, 2007b, p. 50). Symbolic interactionism and constructivism, as theoretical frames, were frameworks familiar to the early beginnings of grounded theory yet their re-inspection provided a new way of forming a methodology. Essentially, CGT responds to many of the criticisms raised about classic grounded theory, positioning the researcher with the setting, data, and themes differently and providing a coherence of theoretical foundation and data methods. Rather than a "revision" (Charmaz, 2009, p. 129) or "renewal and revitalization" (p. 135), I agree with Bryant's (2002) notion of a "re-grounding" – where CGT is not a subset of grounded theory but a substantial repositioning of a methodology as an interpretive endeavour. After describing the two areas of re-grounding, I will apply this re-grounding to specific research concepts.

**Symbolic Interactionism**

Because of Strauss' contribution to shaping grounded theory, re-turning to symbolic interactionism as an ontological orientation within CGT is consistent. Atkinson and Housley (2003) describe two traditions of interactionism, and CGT takes up "the tradition that stems from George Herbert Mead (1932, 1934, 1938) that was codified by Herbert Blumer (1962, 1969)" (p. 3). Creating a list of the main elements of symbolic interactionism is difficult because of the deeply intertwined nature of the ideas. Beginning with the label, however, offers some insights. Symbols are defined as "abstract representations of social objects that enable people to communicate both verbally and nonverbally and understand each other's intentions and actions" (Milliken & Schreiber, 2012, p. 686). Symbols are created out of the meanings – which are the results of interpretive acts – that people ascribe to "objects, events, and actions" (Prasad, 2005, p. 21). In this way, symbols are abstractions, rather than the objects and actions themselves. People engage in interpretation in a social or interactive manner, and so the symbols are not only created in interaction but are also modified through interaction. Meaning, then, is fluid and malleable rather than static. This informs a view of reality as "a social production" (Denzin, 1974, p. 269). At the same time, meanings of objects and actions also shape interactions between individuals (Milliken & Schreiber, 2012).

The aim in research, from a symbolic interactionist perspective, is to "seek an intimate understanding of social situations largely from the standpoint of participants themselves" (Prasad, 2005, p. 23). The implication for researchers with a symbolic interactionist framing is that the researcher needs to be reflexive within the study, the interpretations and perspectives of research participants are important as a place to begin analysis, the research results are contingent on the context and provisional, and the study of process (the actions of individuals and groups) is of central concern.

**Constructivism**

While I separate symbolic interactionism and constructivism for the practicalities of writing, it does misrepresent that the development of each orientation was continually informing the other (e.g., Denzin, 1992). Constructivism informs the epistemological orientation of CGT, with a strong connection to American Pragmatism through Dewey. In light of Gubrium and Holstein's (2008) view of constructivism as "a highly variegated mosaic of itself" (p. 4), I have chosen to use Creswell's (1998) epistemological emphasis on the relationship between the researcher and the focus of the research. In this way, constructivism recognizes that the researcher and "the object of investigation are assumed to be interactively linked so that the 'findings' are literally created as the investigation proceeds" (Guba & Lincoln, 1994, p. 111). Within CGT literature, Charmaz (2000) states, "Constructivism assumes the relativism of multiple social realities, recognizes the mutual creation of knowledge by the viewer and the viewed, and aims toward interpretive understandings of subjects' meanings" (p. 510). She also acknowledges "a reflexive stance on modes of knowing and representing studied life" (2008, p. 206). Reflexivity, for the researcher, extends beyond the notion of reflection to mean a recognition of self in the research context and how the researcher shapes the context through action (Bloor & Wood, 2006).

Packer and Goicoechea (2000) identify that researchers working from a constructivist stance primarily hold to epistemological concerns and do not necessarily explore issues of ontology. They encourage a reconciliation of sociocultural and constructivist perspectives in which a nondualist ontology is taken up in educational research. A nondualist ontology – the collapsing of dualities such as knower/known, mind/body, subject/environment – can be traced back to Hegel, who in turn informed Dewey's thinking and other scholars within American Pragmatism (Packer & Goicoechea, 2000). A nondualist ontology also recognizes a realist stance, where individuals make meaning of the world that exists around them through social interactions. In this research project, I hold a nondualist ontology that can be seen as consistent with a social constructivist perspective.

The implications for researchers with a constructivist approach includes acknowledging multiple perspectives, developing intimacy with the research context, interacting with participants while co-constructing one's understanding of the participants' experiences, valuing the perspectives of participants, and amplifying their voices in communicating resulting understandings from the study. It also demands that researchers move beyond issues of coming to know (epistemological concerns) and wrestle with what exists and the meaning of existence (ontological concerns). In education research, in particular, this means attending to learning in a way that notices the ways in which learners are being and becoming through their experiences in school. In the generation of theory, a constructivist orientation means that the theory is constructed by the researcher on a provisional basis and contingent to the context.

**Interpretive Research Tradition**

Constructivist grounded theory is located within an interpretive tradition. The salient features of interpretive methodologies include: (1) the social construction of reality, through the development of subjective meanings of objects and actions; (2) understanding the processes by which subjective meanings are developed in everyday contexts; (3) meaning-making is done in interaction with others, and this interpretation is the basis of developing knowledge; and (4) the centrality of interpretations as they are reified through use (Andrade, 2009; Prasad, 2005; Rowlands, 2005; Schwandt, 1994). The re-grounding of CGT in symbolic interactionism and constructivism demonstrates a similar orientation as interpretive research, where each of the features listed above is present. Prasad (2005) further connects American Pragmatism – and thus, symbolic interactionism and constructivism – back to Husserl's phenomenology, the genesis of the interpretive research tradition.

**Constructivist Grounded Theory Perspective on Research Concepts**

In the remainder of this section, I return to the particularities of CGT which have been re-grounded in symbolic interactionism and constructivism to acknowledge the impact of these perspectives on the purpose of CGT research, the type of phenomena studied, the roles of the researcher and participants, what constitutes data, and the function of research procedures in CGT. Charmaz (2000, 2006) has described CGT as a methodological framing for an interpretive process for inquiring into dynamic phenomena. The purpose, drawn from the previous definition of CGT, is to develop and explicate "understandings of research participants' actions and meanings, offer abstract interpretations of empirical relationships, and create conditional statements about the implications of their analyses" (Charmaz, 2005, p. 508). This meaning-making results in an interpretive theory that communicates what and how the researcher has come to understand the phenomenon under study through contextual and connected knowing. The focus of inquiry for studies using CGT is an "emphasis on processes, making the study of action central" (Charmaz, 2006, p. 9) and "giving close attention to empirical realities and our collected renderings of them" (Charmaz, 2005, p. 509). This processual focus attends to the dynamics of experience, recognizing that shifts in people's actions signify growth within the people and their interactions. Researchers using CGT view the mode of inquiry as both a process and a product – the grounding-of-theory is the process enacted by the researcher and the result of this process is the development of a grounded theory which explicates an understanding of human experience.

Within CGT, the researcher is a subjective knower (interpreter), whose previous experiences inform how he or she attends to the research setting. Sensitizing concepts (Blumer, 1954) are used to explicate the subjectivity of the researcher. The researcher is also a situated knower. Being immersed in the research setting, the researcher is actively co-constructing data with the participants – highlighting the interpretation of lived experiences as the process of data construction. Charmaz (2006) implicitly points to the importance of building relationships with the participants as she suggests that researchers "establish rapport with [participants]" (p. 19). The participants' interpretations, the meanings they make of objects and actions in their experiences, are valued as important perspectives of the researched phenomena. At the same time, Charmaz emphasizes that CGT researchers "try to understand but do not necessarily adopt or reproduce [participants'] views as our own; rather we interpret them" (p. 19). The intersubjectivity of understanding participants' experiences is challenging work that relies on the reflexivity of the researcher who simultaneously engages in interpretation to make meaning of the experiences and amplifies the voices of participants through using large portions of data.

Data in CGT studies are typically qualitative in form. Rather than focusing on the forms of data, though, CGT suggests tools for researchers to focus on thinking about and with the data from the research context, or "processes of conceptualization" (Morse, 2009, p. 18), indicating fluidity rather than rigid steps implemented mechanically. Data is simultaneously constructed and analyzed, each action informing the other. Charmaz (2006) reminds researchers that "data are never entirely raw. Recording data alone confers interpretations of them because we place a conceptual frame on them through our use of language and understandings about the world" (p. 40). So, rather than data being collected, data are interpretive (re)constructions of lived experiences. This is a consistent perspective in CGT, where Charmaz states, "*people* construct data" (p. 16). Constructivist grounded theory is open to any form of data constructed from the research context. Most often focusing on interviews, Charmaz encourages forms of data that reach deeply into experiences and broadly across experiences. Whatever the form of data, Charmaz encourages the gathering of *rich* data, where "rich data are detailed, focused, and full. They reveal the researcher's interpretation of the participants' views, feelings, intentions, and actions as well as the contexts and structures of their lives. Obtaining rich data means seeking 'thick' description (Geertz, 1973)" (p. 14). In general, substantial data with depth and scope are useful in developing categories and should have the "suitability and sufficiency" (p. 18) to support description of categories and provide illuminating examples of the theory constructed.

The researcher moves from rich empirical data through levels of abstraction toward developing a mid-range interpretive theory. Charmaz (2006) describes fluidity within the data analysis process when she writes, "grounded theory methods consist of systematic, yet flexible guidelines for collecting and analyzing qualitative data to construct theories 'grounded' in the data themselves. The guidelines offer a set of general principles and heuristic devices rather than formulaic rules" (p. 2). These guidelines included coding, memoing, categorizing, theoretical sampling, saturating, and sorting and support "analysis as created from shared experiences and relationships with participants and other sources of data" (p. 130). A detailed description of the guidelines and how they supported my theorizing will be woven into the explanation of how I carried out the research design and in the presentation of analysis in future chapters.

**Constructivist Grounded Theory in Mathematics Education**

Relatively new as a research methodology, CGT has not been widely taken up within the field of mathematics education. A brief exploration of the use of CGT in mathematics education provides a contextualization for my use and results, and points to a relatively uncharted territory of methodological consideration. I chose to search in research reports published as peer-reviewed journal articles. A search of seven mathematics education journals (including *Canadian Journal of Science, Mathematics and Technology Education*, *Educational Studies in Mathematics*, *For the Learning of Mathematics*, *Journal for Research in Mathematics Education*, *Journal of Mathematical Behavior*, *Journal of Mathematics Teacher Education*, and *Mathematical Thinking and Learning*) revealed only four articles that specifically used CGT. I refined the search to uses of “Charmaz” or “constructivist grounded theory,” as searches for “grounded theory” resulted either in reports of the use of “constant comparative method” for analyzing data from Glaser and Strauss within another research methodology or other formulations of grounded theory. An example of an article that uses grounded theory, yet contains a brief mention of Charmaz, is Bonner and Adams’ (2012) explanation of their development of a *Culturally Responsive Mathematics Teaching* theory. They point out that their “theory is an initial attempt to inform educators” (p. 35), where they have explored the interrelationships among their categories, and do not claim the theory is generalizable but could instead inform teaching practices which “are flexible and continually evolving” (p. 35). Additionally, two doctoral dissertation studies I am aware of which used CGT I was only able to locate in conference proceedings (Bruce, 2007; Preciado, 2010) and have not used them for this exploration.

Each of the four examples are interesting in degree to which they use CGT as a methodological framing, the choices of structure and content of the writing, and whether the results of the study are explicated as a theory. Liljedahl (2010) uses CGT in the analysis phase of an emergent research project, which resulted in his construction of “five distinct mechanisms of change” (p. 422) for teachers’ practice. These mechanisms represent the categories constructed, are illustrated by narratives, and are tentative in their invitation for further development. Preciado-Babb and Liljedahl (2012) build on three different task-design projects to create a framework which incorporates three categories which emerged from open and focused coding. The voices of participants are illuminated and the framework is presented across the three cases. Wager (2012) creates a framework from one phase of a study using four codes/categories (the terms are used interchangeably). The framework is offered “for teachers to consider ways to approach incorporating students’ out-of-school experiences” (p. 21) in their pedagogy. In each of these three examples, the authors do not name their results as the development of a theory or engaging in the process of theorizing, but invite others to consider how their structured analyses might inform an understanding of other contexts. In this way, generalization is not assumed while at the same time abstracted processes or principles are offered to readers.

The fourth example, Walter and Hart (2009), follows closely the processes of CGT. The structure of reporting in a journal article also represents CGT processes. The example warrants an extended review and explanation of the researchers' work compared to the other examples. The authors use a traditional research report format, but rather than presenting "related literature" or "theoretical framework" they demonstrate their understanding of the research area by a section entitled "Evident needs in research on student motivation" (p. 163). They provide a detailed description of how they coded data and their movement from open codes to focused codes to categories in their analysis. The resulting theory, *Contextual Motivation Theory*, "offers a lens for understanding the complexities of student motivations in mathematical learning within particular, contextual conditions" (p. 170). The interrelationship of the two categories they constructed is presented with data excerpts as they aimed to represent understanding of the phenomena (student motivation) as an act of theorizing. I found this example helpful because the authors demonstrated strong similarity to Charmaz's (2006) description of CGT.

As I consider my learning about paradigms of research methods and orientations within those paradigms, I have been fascinated with the scholarly work in reading and synthesizing literature. The section above represents a survey of the reading I have engaged in, and even that vast reading left me to realize the tracing of ideas and scholars gave me the opportunity to have fresh insights into CGT while coming to understand areas of re-grounding. The connectedness of the developing areas of study, and surprises along the way such as Denzin working from a symbolic interactionist perspective, caused me to appreciate the ways in which the ideas developed and how that shaped not only research paradigms but the researchers themselves. I have a renewed sense of interest in further explorations within interpretive research and how that can shape our understanding of students' learning in classrooms and the field of mathematics education.

## Theorizing through Constructivist Grounded Theory

Among the numerous reasons I held in selecting CGT as a methodological framing for this research project, the notion of a methodology supporting the process of theorizing was compelling in the choice. Examining the notions of "theory" and "theorizing" is crucial when it is an explicit aim in a research study, especially in light of the various understandings researchers have of these notions. In this section, I present how theory and theorizing are viewed from a CGT perspective, followed by a glimpse of how theory and theorizing are viewed within the field of mathematics education.

### Theory and Theorizing in Constructivist Grounded Theory

An understanding of theory has been re-grounded in CGT compared to the early ideas in Glaser and Strauss' (1967) grounded theory, with a more concerted focus on theorizing. Charmaz (2006) emphasizes the entire process as "creating abstract interpretive understandings of the data" (p. 9) through the use of CGT methods. In more expansive terms, Charmaz (2006) problematizes theory, and explains:

> An alternative definition of theory emphasizes *understanding* rather than explanation. Proponents of this definition view theoretical understanding as abstract and interpretive; the very understanding gained from the theory rests on the theorist's interpretation of the studied phenomenon. Interpretive theories allow for indeterminacy rather than seek causality and give priority to showing patterns and connections rather than to linear reasoning. … This type of theory assumes emergent, multiple realities; indeterminacy; facts and values as inextricably linked; truth as provisional; and social life as processual. … knowledge—and theories—are situated and located in particular positions, perspectives, and experiences. (pp. 126-127)

Charmaz's definition of theory, upon careful examination, contains the central characteristics of a theory generated through CGT methods. As I used the central characteristics as my understanding of theory and to guide my theorizing, I highlight these central characteristics by italicizing specific words or phrases from the above quote alongside my understanding. A theory is *interpretive*, explicating an *understanding* of the researched phenomenon. Through the explication of the theory, the researcher seeks to communicate the meaning of the phenomenon which moves beyond portrayal or description. This meaning is *abstract*, insofar as what is represented is an abstraction and not the objects, events or actions themselves that are the focus of an inquiry – as is consistent with a symbolic interactionist perspective. In fact, the things themselves have undergone many moments of interpretation, from the recording as data to the analysis of the data to the representation of the meaning. The meanings and labels given to the objects, events, and actions from the research context are only one of many meanings possible. Because of this multiplicity of available meanings and the idea that *theories—are situated and located*, a theory is not (necessarily) generalizable to other contexts – it cannot exist as a rule or prescription for all, but offered so that others can be invited to interpret their experiences from a different perspective than they had otherwise held.

When multiple meanings of the researched phenomenon are possible, a theory *rests on the theorist's interpretation*. As the theory is constructed by a particular researcher, the ways in which the researcher views the world, the processes the researcher uses, and the reflexivity of the researcher throughout the project needs to be acknowledged and described alongside representing the theory. The theory is understood within the context of the theorizer. The *indeterminacy* of the theory points to its provisional nature, where the theory is an understanding for this moment and contingent on this context, not seeking to be determinative, but tentative. The *connections* developed across interpretive codes (abstract labels) and categories create relationships that connect theoretical understanding with lived experiences. The participants' experiences, as a focus of the inquiry, are processes. A *processual* focus extends beyond the researched phenomenon to the theorizing itself and to the theory as it is offered up to be modified and shaped as others consider how it might help them understand their context and phenomenon within their context.

However, to focus solely on theory in CGT would be insufficient. The process-based nature of CGT extends beyond the phenomenon studied and to the methodology as a whole. “My preference for theorizing—and it is for theorizing, not theory—is unabashedly interpretive. Theorizing is a *practice*” (Charmaz, 2006, p. 128). In this way, Charmaz collapses the dichotomy of theory-practice when she continues, “[Theorizing] entails the practical activity of engaging the world and of constructing abstract understanding about and within it” (p. 128). I would extend this notion of practical activity to say that the theory itself is also practical insofar that it generates further inquiry. Kieren (1997) points to “theories for” as practical in the way it provides teachers different ways of listening to and observing students in their classrooms in order to shape their practice. Charmaz (2006) offers credibility, originality, resonance, and usefulness as criteria to evaluate a grounded theory.

**Theory and Theorizing in Mathematics Education**

My theorizing is contextualized within the field of mathematics education. Sriraman and English (2010) and Lerman, Zu, and Tsatsaroni (2002) published meta-analyses of theories of and in mathematics education where the role of theory within the discipline is portrayed. Both publications address mathematics educators taking up existing theories, rather than a call for generating theory. Sriraman and English (2010) implicitly conceptualize *theory* as grand theories, viewing them in a positivistic manner to verify, hypothesize and impose. This especially becomes a concern as Lerman *et al*. (2002) point to a lack of critical thinking about the theories taken up in mathematics education research. This opens up space, not only for the contribution of theory through my theorizing in this research, but also providing an alternative stance which makes visible the potency of theorizing in mathematics education that produces mid-range theories attentive to lived experiences of student learning.

Although Mason and Johnston-Wilder (2004) describe some of the formal theories commonly taken up in mathematics education, they also recognize the complexity of “theory” as a construct by describing personal theories – those theories from which people act. Vergnaud (1998) refers to these as “theorems-in-action” (p. 227). Theorizing, in these conceptualizations, is a way in which people make sense of and act in their world, a philosophical shift from the meta-analyses described above. Hiebert (1998) also takes an interpretive turn with theories and theorizing in mathematics education. He states,

> the primary goal of research in mathematics education should be to understand what we study … we need to think of theories not as grand global theories that unify the elements in mathematics education but rather as the products of making explicit our hypotheses and hunches about how things work. Theories result from making public our private intuitions about the important elements and relationships of the situation. … They demonstrate that we are seeking to understand. In this sense, theory

> building is a natural aspect of all research that aims to understand. (pp. 141-142)

His focus on coming to understand is in synchrony with both with my aims in this study and with the process that CGT methods support. As I considered my research question throughout the project, my aim was to engage in theorizing as a way to understand the experiences of the students as they developed ways of learning mathematics. The interpretive acts I engaged in as a researcher were informed by the lived experiences of the students and shaped by my sensitizing concepts.

## Attending to the Learning Opportunity: Sensitizing Concepts

Constructivist grounded theory recognizes the subjectivity of the researcher. This subjectivity means that the researcher comes to a research project and context with her or his experiences, including scholarly reading and previous research. Charmaz (2000) suggests that sensitizing concepts can be "those background ideas that inform our overall research problem. Sensitizing concepts offer ways of seeing, organizing, and understanding experience. … We may use sensitizing concepts *only* as points of departure from which to study the data" (p. 515). Subjectivity is acknowledged and brought forward in the construction, analysis, and interpretation of data. In this way, data interpretation can draw on the researcher's experiences without imposing a particular theoretical model.

### The Nature of Sensitizing Concepts

Blumer (1954) developed the notion of sensitizing concepts to "merely suggest direction along which to look" (p. 7) when interpreting data, recommending that empirically-based research is an opportunity to refine sensitizing concepts, moving toward definitive concepts. Although Blumer's writing opens up the possibility of the subjective stance of a researcher, he does not explicitly claim such a stance. He recognizes that, "this is a matter of filling out a new situation or of picking one's way in an unknown terrain. The concept sensitizes one to this task, providing clues and suggestions" (p. 8). However, the focus remains on the *research context*, and not on the *researcher*. For me, the researcher's sensitivities highlight how and what data the research constructs, as Mason (2002) supports when he encourages researchers "to notice something [they] are already professionally sensitized to" (p. 31). In addition, rather than refinement of sensitizing concepts as a research purpose, I came to understand students' learning through the constructing of a grounded theory.

As a particular example, I return to the project I conducted for my master's thesis. I had prepared myself with six different theoretical frameworks in order to support my meaning-making of students' experiences without imposing any one particular framing. As I described, "At the outset of the inquiry, I did not anticipate that the six interpretive frames would directly correspond to the experiences of the learner-participants, but that they would be used as a place to begin and to inform an interpretation of success" (McFeetors, 2003, p. 75). In coming to understand the nature of students' success in non-academic mathematics, I developed the notion of the emergence of voice where students spoke with an emergent voice. Only two theoretical frameworks mentioned voice in passing, with no frameworks focusing specifically on the concept of voice. Through the study, I came to be sensitized to notice issues of voice for high school students and I am sensitized to notice students' voice in interpreting data. Similarly, I began this study with a set of sensitizing concepts and represent them as temporally located at the outset, rather than expressing the sensitizing concepts I now hold as I consolidate my learning through writing this dissertation.

Using sensitizing concepts provided the opportunity for me to express my subjective stance as a researcher and to acknowledge the ways in which I attend to experiences before undertaking the project. My research question framed an aspect of mathematics education which I am deeply interested in coming to understand, so sensitizing concepts allowed me to remain open to a personal learning experience. Selecting and writing about sensitizing concepts as theoretical ideas allowed me to collect these notions conceptually. The descriptor "sensitizing" also works within a different metaphoric space than a visual or directional (lens or orientation) or combination of visual and directional metaphor (perspective, standpoint).

**Shaping and Explicating My Sensitizing Concepts**

Three experiences, in particular, have shaped my sensitizing concepts. First, selecting the wording for my research question identified several sensitizing concepts. These concepts are ones that supported my exploration within the boundaries set out by the research question. Second, the explication of the nature of students' success in Consumer Mathematics in my master's research project (McFeetors, 2003) brought forward several sensitizing concepts. While the particular success of many of the students was their identification of becoming mathematical thinkers and learners, it was through the emergence of voice that the students were able to notice their success. And, it was through this voicing that I was able to see their (re)formation of identity. The growth I noticed in the students could also be a part of the learning experiences I currently wish to explore. Third, the interpretive work I did within the *Trajectories of Students Learning High School Mathematics and Science* research project highlights notions that continually arise in my thinking. Students involved in *Trajectories* described their experiences across their high school mathematics courses, often expressing a desire to improve at learning mathematics.

The four sensitizing concepts are: intentions, identity, voice, and relationships with sources of knowledge. Figure 2.1 contains a diagram to represent the four sensitizing concepts. The sensitizing concepts guided what and how I attended to the students' actions and words in the data construction process. At the same time, this attending to extended beyond the construction of the data and was also taken up in the interpretation of the data. The sensitizing concepts informed the theory generation as I made meaning of the students' lived experiences. In the following sections, I provide a provisional synopsis of what each concept meant to me as a researcher at the beginning of the research project.

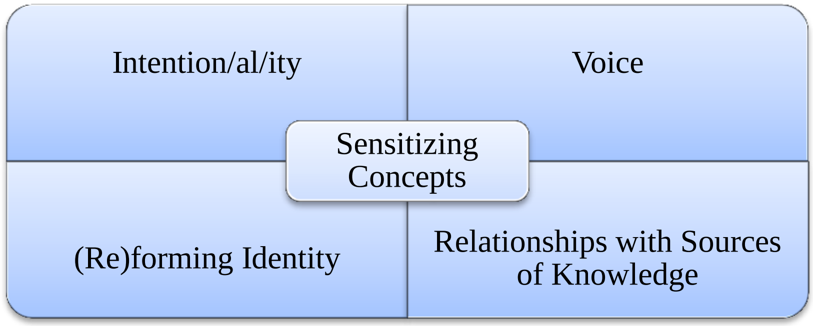


*Figure 2.1*. Diagram of sensitizing concepts.

**Intention/al/ity**. Intentionality became a sensitizing concept through my interpretation of students' success in my master's research. As students began to speak with an emergent voice, they developed intentions of affecting themselves (their identity), their learning, and their relationship with their teacher with their words. The shift toward students using their words intentionally was the dynamic element that I was drawn to see as their voices emerged. The intentions which I interpreted in their lived experiences were expressed indirectly as statements of the results of actions. In this way, I view intentions as internal constructs which give meaning to actions. These thoughts and desires arise from a kind of attention to previous experiences and to the consequences of actions, often through reflection. When students are intentional, they are acting with the intentions they have formed and hold, to move toward a particular aim. This aim, as an end-in-view, is fluid (as compared to a "goal" that is predetermined and fixed) and might also be described as an orientation. The behaviours of acting with intention might not be explicitly planned in advance, that the method of moving toward the aim contains ambiguity. Intentions point to what students want to do or achieve and a notion of how they might go about doing. So, intentions both mark an aim and a process.

My interest in attending to intentions is not unique in mathematics education. For example, Kember (1996) found that Asian students' underlying intention to understand mathematics supported sense-making through memory-focused learning. His study raises two important ideas for me: intentions are important in shaping students' learning and observable actions cannot always display the intentions which students hold. Houssart (2001) refers to "perspectives" and "cultures" (p. 2) in conflict in a class between inquiry-oriented students and a traditionally-oriented teacher; however, it could be that the perspectives informed intentions that were acted on by the individuals. In close relation to my research study, Alrø and Skovsmose (2002) explore the construct "intentions-in-learning" (pp. 156-163). They see students' dispositions and notions of consequences as shaping intentions, that "actions are constituted by intentions" (p. 156), and that intentions are malleable within the context they are formed and carried out. Intentions, then, hold a place of primacy in learning. Their linkage between intentions and learning could support my interpretation of students' learning.

Even in naming this sensitizing concept, I struggle with what words might point to this particular form of thought which precedes action. Literature contains a range of words that seek to understand a similar phenomenon. Dewey (1910, 1938/1997) wrote about *purpose* as a way of thinking which develops a plan which looks to the future and makes meaning of the context and previous experiences to inform actions. Schwab (1969/1978) wrote about *deliberation* as "a complex, fluid, transactional discipline aimed at identification of the desirable and at either attainment of the desired or at alteration of desires" (p. 291) and *intentions* as the values which guide curricular objectives. In fact, Null (2006) believes Schwab "replace[d] Dewey's *growth* with the practice of *deliberation*" (p. xx). Searle (1983) wrote about *intentionality* as a mental state that affects actions. Reuter (1999) describes Merleau-Ponty's understanding of *operational intentionality* as acting in a directed sense in the world. Finally, Bereiter and Scardamalia (1989) turned intentions directly toward learning in refining *intentional learning* to incorporate students' active role in shaping meaningful learning.

**Voice**. I interpreted the experiences of students' success in my master's research as the emergence of voice – becoming sensitized to issues of student voice. I explicated that emergent voice had three characteristics. Students were being vocal when they could say things out loud. Students were being verbal when they could name particular elements of thinking, learning or their identity. Students were being intentional when they said things to affect themselves or to affect the relationship with others, specifically the teacher. The process of the emergence of voice was tentative and emergent voice was not necessarily autonomous (needed support and was not consolidated). These crucial moments of growth were not described by any frameworks, but rather described an absence of voice or adults speaking with a mature voice (Baxter Magolda, 1992; Belenky, Clinchy, Goldberger, & Tarule, 1997). It was the emergence of voice, the dynamic of growth, which captured my interest. For me, voice points toward having space and confidence to say things and to do so, a reflective stance to make sense of experience through conversation, and being deeply implicated in actively shaping oneself. Voice is dynamic concept, one in which a student's voice is continually being refined through experience and through the voicing of the experience and growth of self.

A range of intensity with regard to student voice is evident in mathematics education literature. Leron and Hazzan (1997) view voice as students' words in mathematics class with acknowledgement of Confrey's (1999) student voice as articulation of thinking and responsibility of the audience to interpret students' perspectives through careful listening. The purpose of voicing perspectives tends to be to inform teachers of mathematical cognition (Confrey, 1998) or support the shaping of teaching practices (McFeetors, 2008). Student voice has as its focus mathematical ideas or classroom processes, rather than implicating the students in their growth through talk about their personal learning. When students' perspectives are valued, the students' voices carry an implied authority with the teacher and peers (Civil & Planas, 2004). At the same time as validating students' voices, it is at the discretion of the teacher and/or researcher to "give voice" to the students, rather than the students gaining voice.

Related to voice, agency is a concept which assumes a more active stance. Wagner (2007) describes agency as "control over the way the mathematics is done and expressed" (p. 36), characterized as acting on the world through initiative and participation – more than an authorization of students' perspectives. At the same time, agency was related to mathematical cognition rather than to students' positioning with learning mathematics. In my previous research, I have left the notion of agency relatively unexplored but recognized the extending of voice in this direction.

**(Re)forming Identity**. Inquiring further into emergent voice, my sensitizing concept of (re)forming identity was brought forward as an interpretation of the emergence of voice. By the end of my master's study, the students talked about themselves in a different way as mathematical thinkers and learners. Identity arose again as a key feature of students' experiences of grade 10 mathematics in the pilot project of the *Trajectories* study. Students' views of their capabilities and who they were as students informed their choices of mathematics courses and by the end of grade 10 could talk about how their relative success in mathematics class influenced how they saw themselves differently (Mason & McFeetors, 2007). For both of these groups of students, an ontological shift – a process of becoming – through their experiences in mathematics class has prompted me to reconsider the rationale for the compulsory nature of high school mathematics, that perhaps further interrogation could provide a more significant reason for inviting students to learn within the context of mathematics. For me, identity is an understanding or sense of self. It is a dynamic process, where the (re)forming of identity is continually undertaken through experiences and relating with others. While occasionally marked by large shifts, (re)forming identity is more often seen as shaping a way of being in the world and understanding that way of being. Rather than stating membership within particular groups (e.g., gender, race, social class) as an identification, shaping an identity is the ongoing negotiation of a student's relationship with mathematics, learning, schooling, others – identity is malleable and complex.

Because of my perspective that identity is not a fixed state and my interest in students' becoming through learning mathematics, I wondered about how shifts in identity happen, what they signify, and how those shifts might be explained to a broader audience. Looking at support from literature, I attended to the use of verbs around statements about identity to interrogate the metaphoric space occupied and how that could help me understand shifts in identity. The range of verbs (or their nominalization) include: "development" (Graven, 2003, p. 33), "created and recreated" (Sfard & Prusak, 2005, p. 15), "formation" (Belenky *et al*., 1997, p. 48), "establishing" (Chickering & Reisser, 1993, p. 173), "discover and recover" (Chambers, 2003, p. 244), "negotiated" and "recalibrated" (Mason & McFeetors, 2007, pp. 293 & 310, respectively), "produced and reproduced", "constructed and deconstructed", "fabrication and elaboration" (Britzman, 1994, pp. 54, 55, & 61, respectively). In addition, scholars use other terms to point to the notion of identity. Other terms to mark identity include: “sense of self” and “self-definition” (Bateson, 1994, pp. 73 & 79, respectively), “self-understanding” (Carr, 1986, p. 78), “view themselves” (Allen, 2004, p. 233), “stories about persons” (Sfard & Prusak, 2005, p. 14), “becoming” (Graven, 2003, p. 33), “conceptualize the self” (Belenky *et al*., 1997, p. 31). For each of these scholars, the metaphoric space within which the words could reside marks their understanding of identity and (re)formation of identity.

Dewey (1938/1997) substantiates (re)forming identity as a sensitizing concept in his assertion that the purpose of education is growth of a person. He demonstrates synchrony with symbolic interactionism when he states, “the educative process can be identified with growth when that is understood in terms of the active participle, *growing*. Growth, or growing as developing, not only physically but intellectually and morally” (p. 36). Having (re)forming identity, similar to Dewey’s concept of growing, as a sensitizing concept is to attend to what I perceive as the core of the endeavour of education – the personal shaping of each student through experiences in the context of school (and, for me, learning to learn mathematics). Bateson (1994) sees learning as pervading all of life as part of “personal growth” (p. 44) and “that education is not just about literacy and numeracy, that it has always been contested ground, the stuff of power and identity” (p. 211). While Dewey and Bateson philosophically explore the connections between learning and forming identity, Sfard and Prusak (2005) support the use of identity as an analytic construct in coming to understand students’ experiences in mathematics class.

**Relationships with Sources of Knowledge**. One of the ideas arising from the students’ descriptions, in the *Trajectories* study, of how they were learning in mathematics was their relationship to their teacher, peers, and occasionally with support materials (examples include textbooks and websites) or tutors. Each of these could be sources of knowledge with whom/which students take particular stances. The relationships could include dependence, independence/autonomy, and interdependence. The students did not explicitly state their epistemological stances through a particular relational orientation, but through specific examples illustrated where authority in mathematical knowledge lay and often through the learning processes they used. I have tentatively named this sensitizing concept *relationships with sources of knowledge*, to emphasize the students’ epistemological stance. Dependence is shown through a sole reliance on external forms for authority on mathematical content or learning strategies. Independence is represented by accepting only ideas or strategies developed internally. Interdependence involves the negotiating between external authority and self in the development of knowing. Each of the three stances point to beliefs about knowing and coming to know which are inextricably connected to the experiences of learning mathematics.

I have also considered Chickering and Reisser’s (1993) vector of “moving through autonomy toward interdependence” (pp. 115-144) which demonstrates both categories of relationship and movement across various positionings. The model’s vector moves from dependence through independence to interdependence, where mature development integrated external and internal sources of knowledge and a situatedness in community. Whereas Chickering and Reisser saw this vectoring as a part of psychosocial development, both Belenky *et al*. (1997) and Baxter Magolda (1992) viewed similar stances as indicating distinct epistemological stances – drawing into focus those whom students considered to be expert sources of knowing and how the students came to know. At the same time as not claiming stage-based models, these researchers used different names for similar types of knowers and saw a progression of relating to sources of knowing. The development of these models relied on research with adults, rather than high school students, and needs to be adapted or used simply as guiding concepts.

## Methodological Appropriateness for Research Intentions

Constructivist grounded theory provided a methodological fit with my intentions within this research study: for the domain within which the research intentions resides, for the particular research focus, and for myself as a researcher. As was seen in the unpacking of the research question, the focus of the study is located within mathematics education. While few published mathematics education studies have used CGT, there is a possibility that offering more studies like this one with a CGT framing could respond to the invitation for the development of theories in mathematics education held by the community of researchers. Theoretically, constructivism is a dominant epistemological orientation to the teaching and learning of mathematics (Bishop, 1985; Borasi, 1992; Davis, Maher & Noddings, 1990), often used in conjunction with symbolic interactionism for mathematics education research (Cobb & Bauersfeld, 1995; Pollard, 2004; Sierpinska, 1998; Voigt, 1994). Constructivist grounded theory has fit philosophically with the field of mathematics education.

Constructivist grounded theory provided support in exploring the focus of this research project: the learning experiences of students as they learn to learn mathematics. The active, process-based nature of learning is a phenomenon that can be explored through CGT as it focuses on the study of actions. More than attending to the learning of students, CGT supported the development of interpretive understandings in the construction of theory. As I sought the perspectives of the students and teacher, as well as my own as researcher, of the students' learning, CGT respects the multiple interpretations of experience. Constructivist grounded theory has fit conceptually with the research focus.

Finally, I found synchrony with CGT in my orientation to research and who I see myself as a researcher. As a subjective knower, sensitizing concepts provided a starting place to inquire into an authentic wondering where I had opportunities to listen carefully to students and their meaning-making of experiences. The processual nature of CGT supported my interest in how students engage in learning and processes of growth. It opened up space for my exploration into how learning is more than cognitive change, but also forms who the learner is (a nondualist ontological perspective within social constructivism).

Theorizing, as a form of coming to understand and explicating an interpretive understanding, is how I make sense of my world and my interactions with the world. As a researcher, I have further formed my identity as a theorizer through this project. Constructivist grounded theory supported my intention to theorize rigorously and with transparency. Because of the methodological reflexivity that is part of CGT, one of my intentions in this dissertation journey had been to learn about the value of CGT in educational research. I wanted to learn how CGT might support scholars in coming to understand students' learning and how theorizing might be used to communicate effective teaching and learning practices and generate further study in mathematics classrooms. There is an ontological fit with me as a researcher and scholar.

# Chapter 3
# Engaging in the Opportunity to Learn

My inquiry into students' learning to learn high school mathematics was situated in a particular context. The context was within learning high school mathematics. In the first two parts of the chapter, I describe the setting and the participants. In CGT, "we first aim to see this world as our research participants do—from the inside. … Seeing research participants' lives from the inside often gives a researcher otherwise unobtainable views" (Charmaz, 2006, p. 14). My aim in the chapter is to provide a glimpse into the participants' world in order to situate later theorizing.

In the third part of the chapter, I describe the processes of constructing data for the research project. Some data construction processes are common to interpretive methodologies, like one-on-one interviews, while other processes, like small group sessions, were shaped for the particularities of this project. In providing descriptions of the data-constructing processes, my intention is to support understanding of specific empirical examples used and the theorizing, itself. In the fourth part of the chapter, I describe the CGT-framed analytic processes I used. Additionally, I have interspersed throughout the chapter moments where I recognized important learning I was engaged in as a researcher through the implementation and data analysis of the inquiry. These are examples of the reflexivity expected of researchers carrying out a research project with a CGT framing.

In grounded theory, it is the detailed description of context and of the data construction processes that grounds both the interpretation of empirical examples and theorizing. There are at least two forms of theory, substantive theories and formal theories, that can be developed through the use of grounded theory. Glaser (2010) conceptualized the development of formal grounded theories by reaching across varied contexts in which similar social phenomena are enacted. In earlier work with Strauss (1967), grounded theory initially produced mid-range theories which were abstract understandings of processual phenomena but did not claim generalizability. Also called substantive theories, they "refer to an empirical area of sociological inquiry and *is* specific to groups and place" (Lempert, 2010, p. 246). My theorizing in this dissertation aims for a substantive theory, as are many grounded theories "because they address delimited problems in specific substantive areas" (Charmaz, 2006, p. 8). Further, Charmaz (2006) warns against decontextualized theorizing:

> Grounded theorists may produce decontextualized analyses when they disattend to context or are unaware of or unclear about it. Such analyses mask the significance of constructivist elements in grounded theory. Objectivist grounded theorists strive to attain generality and decontextualization typically results. (pp.133-134)

In heeding this warning, I describe the context of this inquiry so that the empirical data presented in the ensuing theory can be situated and the analyses of the students' experiences of learning to learn mathematics finds resonance. For this study, context refers to the course and classroom in which the study was located, to the participants who shared their perspectives and experiences, and to my interpretive actions aimed at capturing perspectives and experiences.

## Setting for the Research Project

The setting for this inquiry was in a learning skills support class for grade 12 Pure Mathematics within an academically-focused high school program. The setting is unique. Often it is difficult to carry out research with grade 12 mathematics students because of the reality of provincial examinations. Even though I was not present in the grade 12 mathematics class, I was still able to conduct the research project in a way that maintained a view of students' learning in grade 12 academic mathematics. Mathematical Learning Skills course, the specific course context, is not a commonly offered course. The course was an interesting site of activity and provided flexibility to incorporate the data construction elements of the research design smoothly. In this section, I move from a broad context, explained briefly through the school program and Pure Mathematics course, to the particularities of the Mathematical Learning Skills course.

### The School Program

The Mathematical Learning Skills course was offered within an academically-focused program in a large urban school division in a city in western Canada. The culture in the grade 10 to 12 high school program tended to be achievement focused, where students defined educational success as attaining high marks. Earning high marks was necessary for post-secondary entrance, where "now it's learn it to get the marks so you can go into school" (Teresa[1]). It was within this achievement-focused culture that the students in Mathematical Learning Skills lived – and also with the reality that their marks in mathematics class were not up to the standard expected by themselves and their families.

### Pure Mathematics 30

Three different mathematics courses were offered at the grade 12 level in the academic school program: an honours mathematics course, Mathematics 31 (Calculus) (Alberta Education, 1995), and Pure Mathematics 30 (Alberta Learning, 2002). The school program did not offer an Applied Mathematics course, even though a provincial course existed. Each of the learner-participants was enrolled in a Pure Mathematics 30 course. At the end of the course, all students write a diploma examination – a provincial standards examination – which counts for half of their course mark. In this section, I provide a brief description of the provincial aims and requirements of the Pure Mathematics course and then characterize the nature of the implementation of the course from the learner-participants' perspectives.

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[1] All participants are referred to by pseudonyms.

**Provincial Curriculum**. Pure Mathematics is a high school mathematics course which was designed for students intending to pursue mathematics or sciences at a post-secondary level. In the program of studies, the focus of the course is described as:

> Pure mathematics emphasizes mathematical theory and the testing of mathematical hypotheses. The pure mathematics approach, which is often deductive and symbolic, endeavours to show that concepts are valid all the time, or valid within a well-defined set of restrictions. Real-life problems are then presented in order for students to apply previously learned mathematical concepts and procedures. Students will make use of algebra and graphing to solve problems. (Alberta Learning, 2002, p. 1)

The Pure Mathematics course was designed within a constructivist framing, where the course was to be taught from the perspective that "students learn by attaching meaning to what they do; and they must be able to construct their own meaning of mathematics" (Alberta Education, 1998, p. 2). The program of studies lays out the expectation for teachers to incorporate seven mathematical processes – communication, connections, estimation and mental mathematics, problem solving, reasoning, technology, and visualization – in order to support students' development of the nature of mathematics that includes characteristics such as change, constancy, dimension, number, pattern, quantity, relationships, shape, and uncertainty. The content units in the course included: transformations of functions; exponents, logarithms and geometric series; trigonometry; conic sections; permutations and combinations; and statistics.

**The Students' Mathematics Class**. Pure Mathematics was taught by several teachers, each of whom followed the same order of units of content for Pure Mathematics. However, the length of each unit differed slightly among the classes and each teacher authored her or his own unit test and scheduled their tests themselves. Over the three and a half months I was in the school, the students covered the following units: conics, logarithms and exponents, combinatorics, and statistics. The supporting resources used varied across the classes, but included materials such as authorized textbooks (*Mathematics 12* (Alexander & Kelly, 1999) and *Mathpower 12* (Knill et al., 2000)), a locally/provincially designed workbook (*Pure Math 30 Workbook* (Appleby, Letal, & Ranieri, 2004)), and internet resources such as MathPure30.com. Having visited most of the mathematics classrooms, I noted that there was a common arrangement in the room with either a SMARTboard or overhead projector and screen as a focal point at the front of the room, a desk for the teacher, and rows of desks with attached chairs for students.

At the beginning of the first interview with the students who participated in the study, I asked them what they had been learning recently in mathematics class and how they had gone about that learning. Their responses provided an image of their mathematics classes, and are used extensively, verbatim, in the description below. To maintain flow in the description, the quotes from the learner-participants in this section are indicated only by having been italicized. The different classes had a common sequencing of lessons, and a standard malegendered pronoun is used for ease of reading. I prioritized the learner-participants' perspectives because it was from these perspectives that they were enacting and shaping ways of learning mathematics that are of central concern in this inquiry.

A lesson typically began when the teacher *tells what to highlight, what's important,* about the topic by using the workbook and *reads it out and we fill it in*. These worked solutions would *jump from this answer to this answer*. Sometimes *he asks people for the answers* and then *we just talk out loud*. *If somebody doesn't get it, then he explains it*. In classes where the workbook was not used, *you take the notes*. The teachers *word it in their kind of way*. As well, *he'll explain as much as he can, but I always have a need to know more, and I don't think there's a further reason why. I think that's just how it is.* With the notes consisting of little more than worked solutions to example questions, the notes *don't say what does it mean* and it would help *if it talked about the topic* and then *explained why we did it* with words. What was helpful was *his examples are laid out from what I find in his notes from the most simple and they get a lot more complex to the point where his usually last examples are diploma questions*. Aside from the *mindless writing* of notes, it is important that I'm *listening in class*. *It's hard to listen at the same time; so I think you just have to listen and then write, listen and write. You just kind of have to find a way to go between the two, quickly. For me I find that he talks really fast, and everybody else gets what he's saying, but I need somebody to go really slow and he doesn't do that*.

In the *last ten minutes he hands out the homework, it's kind of repetitive. It's usually a worksheet and there's ten questions on it. And we're expected to do it*. The questions are often, *the same, but different in different ways, right? 'Cause some of them have to take an extra step*. The teacher *gives us the end of the class to work on the assignment*. We can go *up to him to ask questions*, and *he's completely open to answering and helping*. If the homework assignment is not finished in class, then *you usually take that home* and complete it for the next period. The homework can help you *start to get something, understand something, and then you just keep doing it*. In order to independently correct the assignment, *we have the answers for them, like, on the back of the page*. W*ho checks* the homework*? Actually, no one really does*.

There were common evaluation pieces in Pure Mathematics classes. *We just had a quiz* that is in the middle of the unit. *The quizzes are extremely similar to what's seen in the notes. The questions are similar, the way that you do the questions it's all very similar*. After completing the unit, *he usually gives us that unit assignment at the end of every unit*. *We go through them in class so I can see where I made mistakes* and there is *the same process to answer questions as we did in the homework*. At the end of the unit is a *unit exam* or *test*, where *they're going to be testing you* and so you must *have that down pat in order to know what you're doing on a test*. At *the end of the year, the time comes for the diploma* which has *a lot of difficult higher level thinking questions*.

**Mathematical Learning Skills**

Within the academically demanding program, the course "Mathematical Learning Skills" was offered for students who struggled in their Pure Mathematics class. The intention of the course was to provide support for students who were not achieving as they wanted in their mathematics course. The Mathematical Learning Skills course was based on a course developed by the school division, which had an unpublished program of studies document and was approved by the province as a high school course for which students could receive credit. Mathematical Learning Skills counted as a full course, being timetabled in the same way as other full courses in the school and assigned a grade for each student. The students in the course would often call the class "Learning Skills," or more often simply "Skills."

The school division's program of studies for the course is a guide for teachers arranged around what students should know, understand, and be able to do in relation to learning skills by the end of the course. The aim of the course, explained in the program of studies, is to support students in becoming better students by developing positive attitudes, effective strategies, and knowledge about how to learn. A linear progression is outlined for students to follow in the course. First, students ascertain the type of learner they are by completing various inventories. Second, students review how to make decisions and then set goals. The goals to be set range from achievement for specific courses (short-range) to career planning (long-range), and students are required to identify supports to reach each goal and how to measure the achievement of each goal. Third, students focus on how to reach their goals primarily by evaluating and improving their health and mental well-being. A short section addresses students' anxiety around evaluation by encouraging daily "course maintenance," researching study skills, and acknowledging test anxiety. Fourth, students study how to respond to challenges by focusing on ways of communicating, beginning with positive self-talk and moving toward communicating with others through sharing circles and co-operative learning. Finally, students evaluate their goals and learning strategies by reflecting on the effectiveness and engage in career planning. The document ends with suggestions for student evaluation including checklists, student self-evaluations, and rubrics.

I entered the Mathematical Learning Skills class just over half way through the school year, finding the classroom norms well established. At the beginning of each period, Mrs. Finley (the teacher) would greet students personally as they entered the room and then would make a brief remark to the whole class about the progression of content and upcoming evaluations in the Pure Mathematics classes. Students were given freedom with the balance of the class to use the time at their discretion. Beyond interacting with individual students, Mrs. Finley would circulate around the classroom at least twice per period to monitor and to interact with each of the students. Most of the students arrived for class on time, and overall the attendance was strong in the class. Mrs. Finley saw attendance as an indicator of course success.

The environment of the class was relaxed and the use of the majority of the class time in Learning Skills was left up to each student. A few students worked individually, focused on school work. The rest of the students in the class interacted in pairs or small groups. They moved between school work and chatting with their classmates. When students engaged in school work, mathematics content received about the same amount of attention as the other courses the students were taking. Students who had Pure Mathematics class immediately before Learning Skills often worked on mathematics homework. When working on mathematics content, students readily asked Mrs. Finley, classmates or me for help. The questions I was asked were primarily about specific mathematics practice questions. I was rarely asked about learning strategies. In the first interviews with the learner-participants, I inquired into what they saw themselves doing on a daily basis. The accounts were frank as they described sometimes using the class to complete tasks related to mathematics class, to do homework or study for other courses, and often to relax and be with their peers. The learner-participants contrasted a period in Learning Skills class to be more focused and work-oriented compared to a period they could have spent in the cafeteria with minimal school work being accomplished.

Once every four classes, the class met during the first period of the day and Mrs. Finley had organized a “breakfast club” for that period. Students were free to choose whether they met in the classroom or in the cafeteria for the first half of the period. Mrs. Finley explained that breakfast club was a mechanism for students to engage in community-building – that a safe community was important – and as a reward for regularly attending a course perceived as an elective (not required) course. I would accompany Mrs. Finley to the cafeteria on these days, where she would check for attendance and discuss with students what homework they had. Only a few students would spend this time in the classroom, with all the students arriving for the second half of the period.

There were several times where Mrs. Finley would engage the class as a whole in a specific task. Near the beginning of my time in the classroom, the students were required to fill in a goal setting sheet that had space for short- and long-range goals, along with the materials and other supports needed to achieve the goals. Prior to final examinations, Mrs. Finley had the students complete a study schedule that apportioned time for each course as well as appropriate sleep time and relaxation. Mrs. Finley also invited into the class the university-college students who interned at the school as holistic health specialists. The interns presented a range of services they offered at the school and made health milk shakes with the class to promote strategies for well-being during examination preparation time.

There were a few mathematical-content related learning strategies addressed as a whole class, as well. In our second informal interview, Mrs. Finley reported making a mind map as a class for the transformation unit to summarize content. Mrs. Finley and one learner-participant also mentioned completing an “outcomes frame” in the early part of the school year, where for each specific learning outcome in the program of study for a unit, students would be required to create and complete an illuminating example and then provide a word-based explanation. Students were also given a choice, at the end of the exponents and logarithms unit, of two different structures to tie together the ideas in the unit and hand in a completed assignment sheet. At the end of each mathematics unit of content, Mrs. Finley would distribute extra practice questions for the students from resources to which they did not necessarily have access.

## Participants in the Research Project

From within this particular context of a strong academic program and the Mathematical Learning Skills course, I invited a teacher and students from a single course section to participate in the research project. In this section, I will outline the process I used to invite the students to participate, and then briefly describe the students who chose to participate and the teacher-participant. I conclude the section by describing my role in the classroom, as I have already addressed my subjective stance as a researcher in the overall framing of the project.

### Invitation for Students to Participate

After receiving the consent of the principal and willingness of the teacher, I introduced the research project to the Mathematical Learning Skills class. I talked about my teaching background and being interested in how students were learning. I described what I had learned through the *Trajectories of Students Learning High School Mathematics and Science* study (refer back to Chapter 1 for a description), that students were developing their own ways to learn mathematics and desired support from their mathematics teachers. Moving toward this research project, I explained how my intention was to learn how students could get better at the processes of learning mathematics and that the course they were in was a strong match to my learning goals. I invited them to participate and explained how they might benefit by being able to talk in a more focused way in order to get better at learning mathematics, to improve their learning strategies, and to experience a research project. I described what would be expected with each of the forms of data. As I handed out the consent and assent letters, Mrs. Finley reiterated the research process, demonstrating to the students her support. Over the next few classes, students returned the appropriate forms to be involved in the project.

### Learners-Participants

Thirteen out of the 29 students from the Mathematical Learning Skills class volunteered to participate in all aspects of the research study. I have struggled with an appropriate label to point to these individuals I was learning alongside during the study which captured their actions, intentions for actions, and roles in relation to others. “Participants” points to who they were in relation to the study, but they were not the only participants (as the teacher also participated and I view myself as a participant-inquirer). “Learners” points to who they were as individuals engaging in acts of learning. While they engaged in acts of learning frequently – learning a variety of things like how to act in a school, how to learn, how to think mathematically, how to live in relation to others and the world, etc. – it assumes a consolidated identity as mathematical learners that was absent or nascent for many of them at the beginning of our learning together. “Students” points to a particular positioning of individuals within the system of school, requiring submissiveness to authority as an expectation and intention for acting. Other labels like “adolescents,” “individuals” or “persons” are too generic to be meaningful. I have chosen the label “learner-participants” to refer to those students who participated in the study. While this phrasing might be cumbersome, it invites the awareness that the students who participated in the study were complex in their relationality, intentionality, and activity. Additionally, it allows the label of “students” to refer to the rest of the students in the Mathematical Learning Skills course or students in general.

All but one of the learner-participants was in grade 12; the exception was Teresa who was taking some grade 12 courses in her grade 11 year. There was a broad range of achievement in the Pure Mathematics class the learner-participants were taking, from several who were failing to those whose marks were in the 80 percent range. Most of the learner-participants were excelling in their other courses, and found mathematics class to be a singular site of difficulty. I found the learner-participants to be willing and able to verbalize their perspectives on school and mathematics. Even as I challenged them to say things to me about their learning that were not well-formed, they tried to put to words their nascent ideas for perhaps one of the first times.

The learner-participants came to the Mathematical Learning Skills course with a range of personal intentions and pathways, but similar in their desire to improve their achievement in mathematics class. Of the students participating in the study, eleven had taken the course from the beginning of September. One student joined partway through the fall replacing Mathematics 31 (Calculus) with Mathematical Learning Skills, and one enrolled in the course at the beginning of February. Rather than providing portraits of each of the 13 learner-participants here, I will provide sufficient descriptions when I include excerpts from data to illustrate what I have come to learn through this study. What I found as I began my work in analyzing and interpreting the data the learner-participants and I had constructed was that my understanding of each of them and what they had said to me had evolved into a rich, nuanced understanding of who they were.

In the second interview with the learner-participants, I asked about their reasons for becoming involved in the project. Some responses began with the novelty of the experience, as being “pretty cool” (Grace), “it’d be fun” (Teresa), and “it was kind of interesting” (Kylee). Other responses began with a sense of responsiveness to an invitation to be included, “I’m not one to not get involved” (Nadia), “you gave me a prompt, and I went with it” (Shane), and “I wanted to try different things” (Vanessa). Almost all the learner-participants identified being helpful as an important reason for being involved in the research project. For some, the helpfulness was directed toward me and my work: “You need students to help you; so I might as well” (Jocelyn) highlighted this sentiment. Some anticipated that their involvement would benefit their learning: “I just feel that it would be interesting if we could actually work something out and figure out how everything worked for me” (Laurel). One of the interesting occurrences was for the learners to quickly move from their initial reason to how they saw their involvement impacting their learning or possibly the learning of future mathematics students. And some students recognized that their involvement would "in the future, like, help kids a lot" (Grace), including impacts on teacher education where "there's new people becoming teachers, it's like they need to know these changes" (Teresa).

**Teacher-Participant**

I carried out my research project alongside Mrs. Finley, an experienced mathematics teacher who had a rich understanding of mathematical ideas and a genuine concern for her students. We had met a couple years earlier at a provincial in-service for mathematics teachers, and Mrs. Finley made it apparent that she was continually growing as a professional and shaping her teaching to support students' learning. Her understanding of the nature of high school mathematics focused on the processes of reasoning and communicating, and on the process of students making connections across topics not only within a unit but across units of study. Her goal was that students understand the big ideas in each lesson and unit.

The students recognized Mrs. Finley's expertise as a teacher in her ability to explain mathematical ideas when they approached her with questions. Mrs. Finley would spend considerable time one-on-one with a student in response to a request for her help. The students' conversations with Mrs. Finley occasionally extended beyond mathematical content toward learning skills. She noticed shifts in students in her classroom in their relationship with mathematics and their efforts to learn. Mrs. Finley also sought to develop relationships with each of her students and foster a sense of community in her classrooms. Whenever we spoke of specific students in the class, Mrs. Finley often filled in information about the student's home life as well as their friendships at school and attitude about their learning. Mrs. Finley sought to relate to her students through conversations about graduation outfits or balancing part-time jobs with homework.

**My Role as Participant-Inquirer**

I was a participant-inquirer in the context of the Mathematical Learning Skills class for this research project. The name of inquirer highlights the interpretive orientation of attending to and wondering about the events in the class periods I attended. I found that negotiating the opportunity to inquire in a classroom relies mostly on the openness of a teacher, and willingness of the students to interact with the inquirer. Developing a participatory stance as an initially external person in the community is challenging and nuanced work. I highlight some of the important elements that allowed me to take on a participatory role in the classroom.

Mrs. Finley supported my presence in the classroom, introducing me to the students as a person willing to help and interested in how they were learning. As I attended class each day, the majority of my time was spent filtering around the room and helping students with their mathematics questions. I developed credibility through my ability to explain mathematical concepts and steps, in addition to the students' knowledge that I had taught students high school mathematics in the past. Rather than a distant observer, I developed relationships with all the students as I had informal conversations with them and also engaged in interactive journal writing (described below) with all of them. With the learner-participants, in particular, the conversational opportunities of the small group sessions and the one-on-one interviews allowed me to participate actively in their shaping of learning strategies. Some learner-participants reported in the first interview that I was the only individual they had talked about how they learned mathematics with during the year.

Becoming a participant-inquirer in a high school mathematics classroom contributed to my own learning to carry out research in another teacher's classroom. As I recorded my thinking and learning about the research project in my research process journal daily, I considered how I was negotiating participation in the Mathematical Learning Skills class and with the students. My offers to help within the classroom were guided by the norms I observed as having been established earlier in the school year as I worked at fitting in to the particular context. I had noticed the significance of students having time and space to choose what to do at a relaxed pace, compared to the intensity with which I often interact in one-on-one teaching. The research process journal describes specific interactions with students which, when read chronologically, makes apparent how I learned to balance the intentions of the students with the intentions of the research project with the classroom context. The relationships I developed with the participants contributed to the interpretation of learner-participants' experiences in learning to learn mathematics represented in this dissertation, where I have been able to understand in a nuanced way because of the personal interactions with the participants.

## Forms of Data in the Research Project

Consistent with the constructivist and symbolic interactionist orientations of CGT, I see data as inherently interpretive – that every act of observing, every act of electronic recording, every act of re-presenting observations or things said are *interpretations* of the lived experiences of individuals within a particular setting. Because of this perspective, instead of referring to "collecting data" which has a connotation of taking from what is "out there" I refer to the process of "generating data" or "constructing data." These verb-phrases acknowledge the presence of the person(s) responsible for what becomes data.

Charmaz (2006) describes at least three criteria for the quality of data as including "usefulness for developing core categories … suitability and sufficiency for depicting empirical events" (p. 18). Each of these criteria assumes the use of rich data, where the richness is related to the variety of data generated over time and embedded deeply in a context. This quality of data is necessary in order to support the descriptions of categories and to theorize. The generating of data focuses on re-presenting participants' experiences so that they contribute to the ongoing interpretation of the phenomenon under study. For this research project, I constructed multiple forms of data to elicit a variety of perspectives on the learner-participants' learning (mine, the students, and the teacher), constructed with varied temporalities (in the moment and retrospective), in varied settings (immersed in and with distance from the classroom), and with varied repetition (from daily to bi-weekly to bi-monthly). Suddaby (2006) states, "most high-quality grounded theory research arises from an extensive and ongoing commitment to a line of theoretical research and an empirical site" (p. 640). The multiple and long-term forms of data in this study represent a situatedness of data that supported the development of an interpretive theory of students' learning to learn mathematics.

Within CGT studies, the data are typically qualitative and rich in their descriptive reconstruction of the experiences of participants. In the following sections, I will describe each of the forms of data construction I engaged in throughout the research project. I generated field notes and a research process journal. The learner-participants and I co-constructed interactive journal writings, small group sessions, and one-on-one interviews. The learner-participants constructed working papers as products of learning processes. Finally, Mrs. Finley, the teacher-participant, and I co-constructed conversations (informal interviews). Figure 3.1 below contains a graphic organizer of the forms of data.

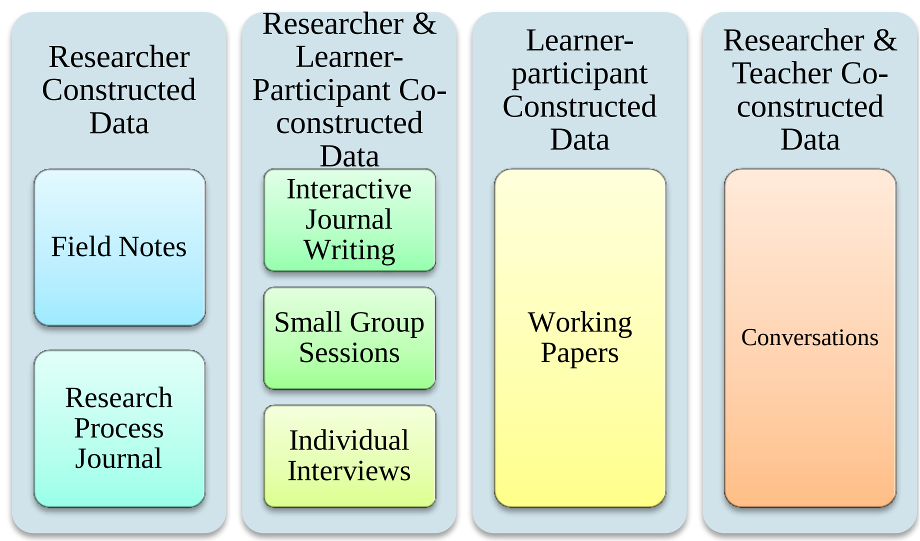



*Figure 3.1*. Forms of data constructed in the research project.

**Field Notes and Research Process Journal**

Field notes have been described as aiding in the documenting of what the researcher notices including the setting, chronological events, key statements made by individuals, and also emergent interpretive elements (Hesse-Biber & Leavy, 2011; Krathwohl, 2009; Lofland & Lofland, 1999). Angrosino and Mays de Pérez (2003) describe an interpretivist turn of participant observation toward a subjective and situated orientation that is responsive to the dialogic reality of researcher and participants in the research site. Clandinin and Connelly (2000) describe the sufficiency of field notes for constructing representations of daily experiences in a research context.

I recorded my classroom observations each day after class in my field notes in an electronic format to enable searching for analyzing data. My intention was to use the field notes as a way to capture the daily events occurring in the classroom, written from my perspective. Primarily containing my account of (Mason, 2002) students' actions, I recorded what was occurring across the whole classroom, within small clusters of students as they interacted, and as students worked individually. I also included contextual elements such as whole-school events, details of the flow of the class, and Mrs. Finley's actions within the class. When I interacted with students or with Mrs. Finley, I reconstructed those exchanges in the field notes as well. Later in the study, I found the field notes were a helpful record of the students' activities in class to personalize general questions for the one-on-one interviews with students. As I entered the analysis and interpretation phase of the project, I relied on the field notes to provide a context for data of other forms.

In addition to the field notes, I kept a research process journal. Charmaz (2006) points to an obligation of a researcher using CGT to be reflexive, including not only the researcher's subjectivity but also "the researcher's scrutiny of his or her research experience, decisions, and interpretations" (p. 188). The research process journal responds to this requirement. Similar to the field notes, the research process journal was written and stored electronically. Each time I worked on any aspect of the research project I recorded what I was working on, the process I was using, the intentions in my decision-making, and any emergent thinking related to the research. I also used the research process journal as a place to record my initial impressions of each of the interviews and small group sessions.

**Interactive Journal Writing**

Interactive writing is a particular form of journaling where a teacher provides a writing prompt, the students write a response, and then the teacher writes a reply to the students. The prompts could include invitations to write about learning goals, study skills and their effect on marks and learning, uses of mathematical processes, learning through activities, and specific mathematical content. Students write with the intent to respond, report, reflect, and/or relate (Mason & McFeetors, 2002). As the teacher replies, she interacts with the ideas, modeling thinking about mathematics and learning, and fostering a relationship through the interaction. As an instructional strategy, interactive writing directs students' attention toward learning. Students' reflections on learning (to learn) mathematics creates an experience (Dewey, 1938/1997) – as students write they can attach meaning to the lived events in the classroom. As they find ways to talk about their learning there are also occasions for them to improve in learning mathematics.

I negotiated with Mrs. Finley to engage all the students in the class interactive writing every other week. She, in turn, offered to also use the interactive writing as part of the evaluation for the course, in part to communicate the importance to the whole class in participating in the process of writing. I authored interactive journal writing prompts that were responsive to the students' experiences in the Mathematical Learning Skills and in their mathematics classes. The day before the interactive writing was scheduled, I would discuss the general direction of the prompt with Mrs. Finley and then would shape the wording of the prompt. Appendix A contains the prompts for each of the interactive journal entries.

After students responded to the prompt in class, I would collect the pieces and write a reply to each one. To satisfy students' desire for immediacy in reading my reply, I would hand back the interactive journal entry the following day to let the students read my reply and then would re-collect them for Mrs. Finley to read. I retained photocopies of the learner-participants' journals and also typed them up in order to store digitally.

In the past I have primarily used interactive journal writing as part of a pedagogical process, both with high school mathematics students and with pre-service education students. In this research project, while I was an adult in the classroom I was not the students' teacher and formed a different relationship without the authority of one assigning marks. So, I found the interactive journal writing to be a legitimate research process because of the potency of the data that it generated in this study. These research texts fit what Charmaz (2006) calls "elicited texts." She explains that "elicited texts work best when participants have a stake in the addressed topics, experience in the relevant areas, and view the questions as significant" (p. 37). The interactive journal writing contains all of these characteristics.

The ideas the learner-participants explored in the writing gave me specific starting places on which to follow up in interviews and added another layer of data for the process of coding and interpreting learners' experiences. As a researcher in another teacher's classroom, I also learned that the interactive writing provided an authentic piece that Mrs. Finley and I could shape together and helped us develop common intentions in supporting the students' growth as learners.

**Small Group Sessions**

The small group sessions provided opportunities for the learner-participants to collaborate on shaping a specific learning strategy. Starting with a focus on a mathematical topic and one learning strategy, the groups worked on developing the learning strategy to make sense of the selected mathematical topic. The process of developing the learning strategy included conversation about the mathematical topic, trying the strategy, making suggestions, listening to peers, and refining the strategy, as a range of examples. For each session, the learner-participants recorded the mathematical ideas they made sense of and felt were important on a sheet of paper. In the last part of the session I prompted a reflective turn toward consideration of the shaping of the learning strategy. I saw reflective statements made by the learner-participants as part of the experience-making of learning to learn mathematics. In this way, the small group discussions were an instructional strategy similar in nature to the interactive writing.

The small group sessions provided a different form and level of intimacy in discussions about learning (to learn) mathematics. Depending on the small group and how many times we met, I scaffolded the learning and encouraged interaction among the learner-participants. The data construction element of the small group discussions was similar to Fern's (2001) focus groups which encourage active participation, characterized as being dynamic and relational. Consistent with an exploratory focus group, learner-participants' talk about learning (to learn) mathematics were recorded and identified in order to "generate theory from focus group observations" (p. 7). Consistent with an experiential focus group, "understanding of individuals' language, knowledge, and experience" (p. 7) was developed through the shared statements of the learner-participants as they shaped a learning strategy. Rather than responding to questions in the classic sense of a focus group, the experiential focus group constructed data as we interacted around mathematical content and a learning strategy.

Each of the small group sessions was held in an alternate classroom, meeting during the Mathematical Learning Skills class period. The sessions were audio-recorded, using two recorders because of the size of the groups. I transcribed the recordings immediately afterwards and could usually identify the speaker for each turn in the discussion. In the remainder of this section, I describe how I formed each of the three small groupings which the learner-participants were members of in the research project. For each grouping, I provide an explanation of the focus at the outset and how I formed the focus from the learner-participants' previous data. Then I detail the membership in the group and the general aim of each of the sessions in which we met.

With 13 learner-participants, I decided that I would create three or four small groups to have approximately three to five individuals per group. I began the process of forming the groups by reviewing each person's interactive writings (the first two were completed) and my field notes. The focus of each small group was to be a particular learning strategy that the learner-participants could shape and refine together. With this focus in mind, I looked in the data for learning strategies each learner-participant was already using and what they identified as wanting to improve during the remainder of the course. I also considered, in a pedagogic fashion, what I had noticed in their approaches to learning mathematics in the Learning Skills class and how they might be invited to inquire into and refine the approaches. I took, together, the learner-participants' perspectives and my own interpretive understanding of their actions to tentatively make three groups. I sought out Mrs. Finley's perspective on the groupings, and organized the learner-participants into their groups, each with a learning strategy focus described below.

**Big Ideas Small Group**. I gave the Big Ideas Small Group the naming when I created the group and identified a focus, but over the small group sessions came to see that the label did not represent the thinking in the group. Initially the Big Ideas Small Group was intended to look back at a completed homework assignment and generate the big ideas or skills from the lesson. Jocelyn, Laurel, Nadia, Grace, and Elise were members of the small group. Each of the five learner-participants had indicated in their first journal entries that while they had some ways of learning mathematics under control – like homework and studying – they wanted to get more out of the processes so they could understand the mathematical ideas. In particular, Laurel's work for Chemistry had included making a quick notes page for each unit and Grace's desire to identify types of questions led me to consider identifying big ideas from completed homework as a way to attend to mathematical ideas. Mrs. Finley voiced a strong sense of synchrony with the focus of this small group because of her emphasis on explicating the big ideas in a unit as part of the school division's professional development initiative. She anticipated that "it'll morph into something hopefully that works for them." The Big Ideas Small Group met three times and the prompts for the small group sessions can be found in Appendix B.

Of the three small groups, this group was the most difficult to coordinate according to a mathematical topic. By this time in the school year, the content units were not aligned across the different classes of Pure Mathematics. For each of our sessions, I negotiated with the learner-participants a mathematical topic from earlier in the year and provided a matching textbook assignment prior to a session. The topics, in order of the sessions, were graphs of exponential and logarithmic functions, reflections of functions, and using special triangles to find exact values of trigonometric functions. Although committing to completing questions in advance, the learner-participants ended up using the first half of the session to work on the questions together and often required many probing questions and reminders of steps that had the composition of a tutoring session.

In the second half of each session, I invited the learner-participants to create a record sheet of mathematical ideas that came up for them as they completed the assignment. In each of the three sessions, terms, concepts, symbols, and steps were all mentioned as being important. Each of the record sheets was personalized as they kept track of individual trouble spots. The learner-participants appreciated the opportunity to understand the mathematical content, but also recognized a significant limitation of the time it took to create the "big ideas" sheet on top of completing an assignment. I also encouraged them to compare their record sheet to their notes, in order to develop awareness around what their homework assignment had helped them learn. It was in the second session that Elise and Grace provided a shift in the group's focus and challenged my perspective. In our conversation that reflected back on the "big ideas sheet" they had created, Elise noted that what the sheet allowed them to do was it "broke down what it actually meant." Grace echoed the idea and added that "it's easier to understand." This small group made apparent that the students needed the opportunity to see and make sense of all the component parts of a lesson, ascertaining these for themselves, before they built the parts back up into big ideas for a single lesson.

**Summary Sheets Small Group**. Rather than learning within a unit of content as the other two small groups did, the Summary Sheets Small Group explored the possibility of learning mathematical content as they prepared for a unit test. The focus of the strategy the learner-participants were developing and refining was to look back over a unit and select the important mathematical concepts and skills, and then to make connections across discrete ideas that had been presented in lessons. The resulting document was referred to as a "summary sheet" or "cheat sheet" or "review sheet," with no one label being prioritized over another. Ashley, Shane, Chelsea, Kylee, and Danielle were members of this small group. In their first journal entry, all five wrote about improving their end-of-unit studying so that it was "effective" and "productive" (terms they used, ambiguously). Danielle had already begun to use summary sheets, and I wondered if this approach would move Ashley toward thoughtful engagement with content instead of copying notes multiple times. I also wondered if Kylee would extend her use of cue cards toward the creation of summary sheets. Mrs. Finley also saw the possibility of "asking for help" for both Shane and Chelsea could occur within this small group because of the membership of the group and the focus of looking back on a unit of content. While the naming of the groups indicated a singular focus, Mrs. Finley's comment reminded me that there were complex intentions in forming the small groups. The Summary Sheets Small Group met five times and the prompts for the small group sessions can be found in Appendix C.

The Summary Sheets Small Group sessions differed from the Big Ideas because the content was familiar and the focus was creating a summary sheet. The flow of each of the small group sessions was similar, where the learner-participants worked on the learning strategy and then were invited to reflect on the activity. In the first session, each person shared how they had studied for a recent exponents and logarithms unit exam, where Danielle, Shane, and Ashley had created a summary sheet. Instead of working on the summary sheet, the learner-participants suggested a variety of ways to go through resources like notes and textbook and then how to structure a record sheet.

In the second session, the small group started a new summary sheet for the first unit in the course, transformations. They spent this session and the next adding to and refining the transformations summary sheet. Shane proposed three major sections in the unit to organize the content and the small group used this to systematically work through their resources. Rather than writing everything down, the learner-participants evaluated what they knew with fluency and excluded these ideas, while recording new ideas in the unit. Danielle inspired the group to use sticky notes for recording ideas so that they could be moved when needed to connect mathematical ideas; at the same time, each person used the sticky notes in slightly different ways. They wrestled with finding a balance between recording ideas/steps and illuminating examples of those ideas/steps. I also noticed their attention to structure, where Ashley explicitly chose a concept web and Danielle, Chelsea, and Shane had a linear approach (Kylee was absent for these sessions). In both the third and four sessions, Danielle and Chelsea expressed a desire to continue the summary sheet independently and to meet again – with these actions indicating a shift to me in their deepening engagement in a process to learn mathematics.

By the fourth session, the learner-participants demonstrated self-direction as they immediately began a summary sheet for the trigonometry unit. They had developed ways of using resources, collaborating with each other, and structuring their summary sheet that enabled their engagement with minimal prompting from me. The way in which the learner-participants went about their creation of the summary sheet and their interactions indicated to me the possibilities of the summary sheet as a learning process and the small group as a mechanism for students to interdependently learn to learn mathematics. In our fifth session, the learner-participants continued to work on their trigonometry summary sheet and we briefly talked about sharing the summary sheet strategy with the whole Learning Skills class the following day.

The Summary Sheets Small Group was the only group to explicitly share their process of learning with the entire class. Near the end of the study, Danielle, Shane, Elise, and Chelsea showed the class their summary sheet for the Transformations unit. (Elise was not part of the small group, but had developed summary sheets after watching Danielle do so.) They shared their common belief that the summary sheets learning process could be an important addition in mathematics classes to support students' learning and as effective end-of-unit studying. As well, they each shared with their peers what feature of the summary sheet impacted their learning most, including selecting big ideas, connecting ideas with sticky notes, deciding on a structure for recording ideas, and developing a toolbox of key ideas. After their presentation, each presenter led a small group in the class in trying to create a summary sheet for the first time. Their leadership ranged from providing a closer view of their personal summary sheet to filtering around the group and encouraging personal decision-making for content and form. Each student in the class began their own summary sheet for the transformations unit.

**Transition Small Group**. The Transition Small Group was intended to shape a strategy students could use to develop a transition from the notes they copied out in mathematics class to starting the homework assignment independently. Teresa, Robyn, and Vanessa were members of this small group. In journal 1, Vanessa pointed directly to her difficulty with homework – "I find that when I try to do the homework and understand the notes, I've already forgotten how and what to do by the end of the day" – while Robyn had trouble completing "the daily given math homework" and Teresa expressed in class how difficult it was to begin homework. Mrs. Finley suggested the idea of "transition" for Robyn and "getting started on homework" for Teresa in our first informal interview. We also agreed that this small group added a necessary process, one that was temporally different in comparison to the Big Ideas and Summary Sheets small groups, but shared their general goal of developing a process for understanding mathematical ideas. The Transition Small Group met two times and the prompts for the small group sessions can be found in Appendix D.

In our first session, a lesson on geometric series was a focal point. As they went through their class notes, Teresa and Vanessa suggested discrete skills and I asked them each time if they wanted to write the idea on the record sheet they were keeping. The "transition page" contained the ideas they felt important to record, and they left the session confident they could begin their homework assignment by looking back at the sheet. In our second session, we used notes from a lesson on the fundamental counting principle as a starting place. Teresa and Vanessa had difficulty picking out key ideas from the worked examples in their notes, although Teresa explained to Vanessa the notation and steps for factorial which had not been covered in Vanessa's class. Hoping to stimulate more conversation, I invited the girls to exchange notes to find any missed mathematical ideas which generated a couple more details to write on their record sheet.

The Transition Small Group differed from the other two groups in tone and number of session we had. Robyn did not attend any of the Transition Small Group sessions because of absences from the Learning Skills class, despite having received invitations and reminders about the sessions. Teresa and Vanessa both demonstrated a reliance on external forms of mathematical authority and hesitated in recognizing their own mathematical thinking. The reliance on external authority was apparent in the scarcity of their conversational turns. Neither showed me further progress with finding ways to transition from class notes to beginning homework. This group had the fewest sessions, in part because of the appearance of a lack commitment to the development of a learning process and the lack of conversational engagement in the group to contribute to data construction in the project. Figure 3.2 contains a diagram to summarize the three small groups.

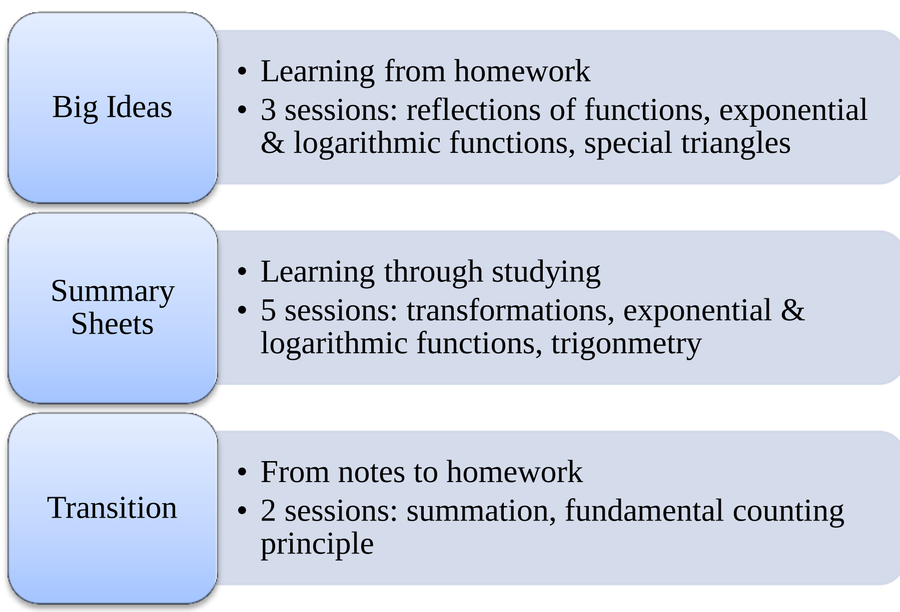



*Figure 3.2*. Summary of the three small groups.

**Small Group Sessions as Constructing Data**. The data constructed through the small group sessions illuminates the co-constructed nature of data, that the learner-participants and I were active together in the creation of the data. We could tease out distinctions and refine our understanding of each other's perspective throughout the sessions. The small group sessions also contributed to a unique form of data in the study, as I had access to in-the-moment developing of three specific learning strategies and then the reflections on the process immediately afterwards. One of my intentions in the project was to have an intimate, close-up view of the shaping and use of learning strategies and the small group sessions were a form of data that informed a nuanced understanding of the shaping of learning strategies. Data of the small group sessions also contributed to my interpretive thinking as I developed understanding of the learner-participants' learning to learn mathematics, and I was able to refine my interpretive thinking as I used excerpts from small group sessions in the one-on-one interviews.

In this project that focuses on learning to learn mathematics – the ways in which students come to make sense of mathematical ideas and how that affects them as learners – I am aware that the mathematical cognition of the learner-participants is often subordinated. At the same time, the mathematical context is important to my work, and the small group sessions gave me glimpses into the ways learner-participants were making sense of mathematical ideas. The examples range across most of the units of content and will be used as examples in future chapters of the dissertation. What I found interesting to consider was that it was the shaping and use of the learning strategy which occasioned moments to make sense of mathematical ideas. In the case of some learner-participants, like Elise and Grace, the product of their learning strategies became a record of the mathematical ideas they had rendered sensible.

The small group sessions were a new form of data construction for me, as a researcher. As I consider my learning about implementing a research project, I found the small group sessions are an illustrative example of a responsiveness to the context. I could only formulate the function of the small groups after spending time in the classroom; and, it required listening closely to the learner-participants and considering Mrs. Finley's perspective on their learning to create the groups and introduce the strategy they would focus on during the sessions. At the same time as responding to the learner-participants and context, I was able to maintain the intentions I had for the small group sessions in the generation of data for the project. Small groups of participants are complex in their interactions and I found myself figuring out how I would take less of a lead in the conversation space and encourage interactions among the learner-participants. In the case of the Big Ideas Small Group, student-student interaction occurred most often as they helped each other with mathematical steps and also as they found agreement in their perspective on the record-keeping. The learner-participants in the Summary Sheets Small Group became interdependent as each member shared their approach to summarizing the mathematical content from a unit and openly took on other members' suggestions. I will explore more deeply the conversational space that made these exchanges possible in Chapter 4.

**Individual Interviews with the Learner-Participants**

I conducted two individual interviews with the learner-participants over the course of my time in the school. The informal interviews can be characterized as semi-structured (Hesse-Biber & Leavy, 2011), where I began with a common set of questions to prompt conversation, I explored learner-participant's statements with fluidity, and I listened carefully as I sought to understand the learner-participant's perspective. The individual interviews fit what Charmaz (2006) describes as "intensive interviewing" (p. 28). The interviewing processes is characterized as "open-ended yet directed, shaped yet emergent, and paced yet unrestricted" (p. 28) and is viewed as the primary form of constructing data in CGT studies. I lived out the tensions Charmaz raises in the dualities listed which resulted in rich data that informs my research question. The interpretive form of the interview primarily focused on the learner-participant's understanding of her or his learning. Cook-Sather (2002) states, "Authorizing student perspectives means ensuring that there are legitimate and valued spaces within which students can speak, re-tuning our ears so that we can hear what they say, and redirecting our actions in response to what we hear" (p. 4). The learner-participants were descriptive in their responses to my prompts as they shared how they were learning mathematics and shaping their learning strategies.

For both interviews, I composed a set of questions that were similar across students yet personalized through my observations, the interactive writing, and the small group discussions. I worked from particular examples in previously constructed data and topics and events in the Mathematical Learning Skills class and the Pure Mathematics classes. These gave learner-participants a specific place to begin sharing their perspectives and modeled how to use examples to explain their ideas about how they were improving in learning academic mathematics. Within the interview I rarely referenced the interview guide, as I knew the prompts and the learner-participant's specific data intimately. The questions tended to flow from one to the next, although I was flexible about using prompts in alternate orders as learner-participants brought up different ideas in response to my questions. I probed for meanings through open and ambiguous follow up questions. Appendix E contains the prompts for both interviews. All the learner-participant interviews were audio-recorded and transcribed.

The first interview with the learner-participants occurred just over half way through the data construction time. All the learner-participants participated in the interviews. There were four broad topics I planned my prompts around: 1) teasing out the Pure Mathematics and Learning Skills classes as contexts for learning; 2) orienting the learner-participant to thinking about learning and using learning-based languaging; 3) eliciting the learner-participant's beliefs on the nature of mathematics; and 4) informing an interpretation of our shared experiences around learning strategies. In each of the broad topics, I listened to and probed the learner-participant's perspective and how that was informing her or his ways of learning mathematics. Most of the learner-participants expressed surprise at the length of the list of learning strategies, although as they described strategies they relied on I found there was more depth to the strategies than the labels implied. Even when the prompts seemed challenging, the learner-participants persisted in responding with emerging ideas about how they were going about learning mathematics.

The second interview with the learner-participants was at the end of my time in the school. The richness of the conversations in the first interview made authoring prompts for the second interview more challenging. One decision I made in creating prompts was to frame my wondering about their improvements in learning mathematics by looking forward, to the immediate future of the Pure Mathematics final examination and to the following year in post-secondary studies. As the year was drawing to a close quickly, the grade 12 students were occupied with what would come next for them. Also, we had already focused on a retrospective look in the previous interview. Speaking toward their anticipated post-secondary experiences seemed to allow the learner-participants to move beyond the specificity of a learning strategy to consider how our experiences had shaped them as learners. A second important decision was to choose a specific starting place to highlight a learning strategy and a significant moment in the project for them. While their response to the related prompts was informative, so was what they chose as a starting place. Similar to the first interview, there was richness to the ways learner-participants' spoke about their learning strategies which was occurring outside of class time and made the interviews an important data construction element for the research project. Finally, as I built on the interpretive work I was engaged in while coding previous data, I sought their perspective on my emerging interpretation of their learning to learn mathematics.

The data generated from the two interviews with each learner-participant formed the core of the analysis represented in this dissertation. Distinct from the interactive writing, small group sessions, and the working papers, the informal interviews were focused conversations between the learner-participant and me. The focus afforded me the opportunity to probe carefully for the meanings of each learner-participant's ideas. Through this process I was able to negotiate and check my understanding of the learning experiences of the learner-participants and of the shifts in the ways they were learning mathematics and how they saw themselves as learners.

**Learner-Participants' Working Papers**

Through my interactions with the learner-participants in class and in the above data construction processes, I invited the learner-participants to create written records of their processes of learning and to use specific examples to illustrate their perspectives on their learning (to learn) mathematics. Working papers included examples like notes from their mathematics class, records from our small group sessions (big idea sheets, summary sheets, transition sheets, and worked solutions to textbook questions). After the small group sessions and after a learner-participant introduced a specific example into a research-based element, I requested a copy of the working paper. The working papers represent a limited range as I faced challenges in the gap between learner-participant's willingness to bring in a document and actually remembering to do so. When I received a working paper, I photocopied it and returned the original to the learner-participant. These working papers are helpful in illustrating a product of the learner-participants' experiences of shaping learning processes.

**Conversations with the Teacher-Participant**

I engaged in two types of conversations with Mrs. Finley during the research project. The first type of conversations were daily informal interactions which occurred before class, during quiet moments in the class, and after class. The focus of these interactions, aside from relating with each other, ranged from events within the school to inquiries about specific students in the class to my research processes. Because Mrs. Finley included the interactive writing as part of the evaluation in the Learning Skills course, she would read through all of the interactive writings and we would talk about what captured our attention. Mrs. Finley also demonstrated interest in what was happening in the small groups outside of the classroom, and after each session I would give her a quick summary to which she would respond. These brief encounters helped me in my initial thinking and then planning for future interactive journal writings and small group sessions. For the entirety of the data construction time, I recorded ideas that surfaced in the informal conversations with Mrs. Finley in my field notes.

For my own learning as a researcher, I found these daily interactions to be important. This is the first research project I have carried out in another teacher's classroom and at the outset of the project I looked forward to learning how to negotiate this form of research. I found that our brief daily interactions provided opportunities to develop a relationship with Mrs. Finley. I felt I was genuinely working alongside Mrs. Finley when I shared the interactive journal writing with her, as we simultaneously circulated in the class helping students, and as we figured out together how to support particular students' efforts to learn. Mrs. Finley's interest in having a small group share their learning strategy with the whole class indicated to me that she saw the research project as a contribution to the Learning Skills course. At Mrs. Finley's invitation, I also got involved in the department's professional development gatherings as they collaborated with mathematics teachers across the school division. I saw growth in myself as I lived alongside Mrs. Finley in her classroom.

The second type of conversations were audio-recorded informal interviews. I viewed these conversations as "unstructured" (Fontana & Frey, 2003) or "in-depth" (Hesse-Biber & Leavy, 2011) interviews, where there is a mutual coming to understand of the focus topic through a researcher's active listening and openness to emergent strands of conversation. My intentions were to use the conversations as one way to develop a relationship with Mrs. Finely, to orient me to the classroom where she would add contextual richness to my understanding of the class and the learner-participants, and to examine my perspective of the learner-participants' learning with her perspective. Appendix F contains the prompts for both of the conversations. The prompts were a starting place, and the conversations were fluid as I posed follow-up questions. Below I describe the two informal interviews I had with Mrs. Finley.

In our first conversation, about one month into my time in the school, I asked Mrs. Finley for her perspective on the first two interactive journal writings and for her help in forming the small groups. Reflecting on the interactive journal writing, Mrs. Finley added her perspective on the students' current ways of going about learning mathematics as she interpreted their writing. She also imagined possibilities for improvements in learning mathematics that was personal to each learner-participant. In forming the small groups, Mrs. Finley encouraged looking beyond the words of the learner-participants and helped place students together that would support their learning. Throughout the conversation, Mrs. Finley confirmed my interpretive thinking with the learner-participants' interactive writing and my observations of them in class.

In our second conversation, about a month later, I invited Mrs. Finley to talk about the two most recent interactive journal writings and to support my understanding of some of the small group sessions. Corroborating the learner-participants' reflections on improving a learning strategy, Mrs. Finley provided additional specific examples of her interactions with the students in class. One of the things that struck me in the conversation was Mrs. Finley's ability to consider a learner-participant's experience of learning mathematics, provide an interpretation, and move directly to imagine how that informs her practice. As I shared my struggle with making sense of the "big ideas" and "breaking down" in the Big Ideas Small Group, Mrs. Finley helped me view the learner-participants' descriptions in a new way as she wondered aloud. Mrs. Finley continued to elaborate on the context of the Mathematical Learning Skills class, especially filling in events from earlier in the school year.

Although in the rest of the dissertation I seldom refer explicitly to Mrs. Finley's contributions to the construction of data, her perspective was helpful in filling in the context of the study and in deepening my understanding of each learner-participant as I interacted with them in the classroom.

**Research Processes as Pedagogic Acts**

The processes of constructing data were also pedagogical in nature, in the sense that they invited learner-participants to attend to and improve their processes of learning in ways other than they had been doing previously. I did not attempt to make comparisons among the students participating or not participating in the study, nor to other similar classes in the school in which I did not interact. What is apparent is that the interactive journal writing, the small group sessions, and the one-on-one interviews had not been a part of the Mathematical Learning Skills course previously, and the learner-participants did not demonstrate hesitation in being involved. Through these interactions with me, they were considering and shaping their process of learning mathematics. Anderson, Nashon, and Thomas (2009) have also wrestled with the notion of interviews in their metacognition research as being impactful experiences for participants, and resolve that what is of more importance is for the researcher to acknowledge that "the notion of impact of method and researcher on the phenomenon can rightly be seen as a virtue" (p. 192).

Neither is my intent in this research project to provide a prescription for methods to use with high school students to support their improvement of personal learning processes. In this way, the issue of the degree of intervention in the research site is a moot point because the learner-participants' experiences existed in the confluence of the Mathematical Learning Skills course as a particular context and the data construction activities. Instead, what I provide in chapter 4 is an explanation of the qualities of these conversational opportunities which learner-participants noted as supporting their learning to learn mathematics. Additionally, throughout the research project I did not treat the forms of data construction with the learner-participants in a mutually exclusive manner, artificially separating the purposes of different interactions as pedagogical or as research processes. I had complex intentions in the use of different forms of interaction which generated data which I could then use to interpret the experiences of learner-participants' learning to learn mathematics.

## Processes for Analyzing Data

The purpose of this section is to examine my processes in analyzing the data that led to the development of the following four chapters (Chapters 4 through 7). I am purposefully being explicit about my process because this is an element of conducting research that is often omitted, ignored or deemed not relevant by researchers in shorter reports of research, such as articles or book chapters. I offer this examination to depict the challenges of data analysis which can be fruitfully resolved, to demonstrate my reflexivity as a researcher using CGT, and to explicate the processes out of which my theorizing developed. This examination includes the guidelines describe in CGT methodology together with my experiences of coding and categorizing. I relate moments of my dissonance in using CGT procedures and the emerging resolution through methodological readings and consideration of the data.

My theorizing in each of Chapters 4 through 7 arose with relative similarity. A brief reminder of the overview of Chapters 4 through 7 given near the beginning of the dissertation aids in the use of specific examples below. In viewing the context of learner-participant's learning to learn mathematics, Chapter 4 contains a description of their opportunities for learning-based conversations through four features of *preparation*, *presence*, *mode*, and *pace*. Within the learning-based conversations, learner-participants where developing personal processes for learning mathematics. The focus of Chapter 5 is an explanation of the four forms of engagement through which learner-participants developed learning processes: *becoming aware*, *incorporating suggestions*, *verbalizing possibilities*, and *(re)forming intentions for learning*. The learner-participants were using the learning processes they developed in order to learn specific mathematical content. In Chapter 6 I look within the learning processes and generate four mechanisms the learner-participants point to through which they were making sense of mathematical ideas: *breaking down*, *putting together*, *connecting*, and *writing down*. To integrate these interpretive moments, in Chapter 7 I theorize by using the metaphor of authoring to understand the learner-participants' learning through: *authoring processes for learning mathematics*, *authoring mathematical ideas*, and *self-authoring*. I have stated in Chapter 1 that I view these as interpretive moments, attempts to develop in multiple ways an understanding of a rich and complex phenomenon – developing learning processes within the context of mathematics.

Throughout this section, it is important to note that in Charmaz's (2006) methodological writing, she offers up procedures like coding or categorizing as "a set of principles or practices, not as prescriptions or packages" (p. 9). As I analyzed the data in the study, I lived in tension with the recommendation of guidelines and using the explicit procedures as a first immersion in a new methodology. I found it challenging to make decisions about when to persist with an unfamiliar procedure and when to rely on my experience as a researcher to leave behind the procedures and build on my experience in analyzing data from previous studies. I have chosen to call the processes I describe below as “analysis” of data because I view all the acts within the research project as interpretive.

**Coding Data**

In CGT, coding data is the first phase of data analysis, meant to be “the process of defining what the data are all about” (Charmaz, 2006, p.43). In this section, I describe the multiple attempts I took in coding the data, during data construction, shortly after completing data construction, and later a comprehensive coding process. I describe the first two briefly, with more description in the comprehensive coding turn as this processing supported my analysis and interpretation for this dissertation.

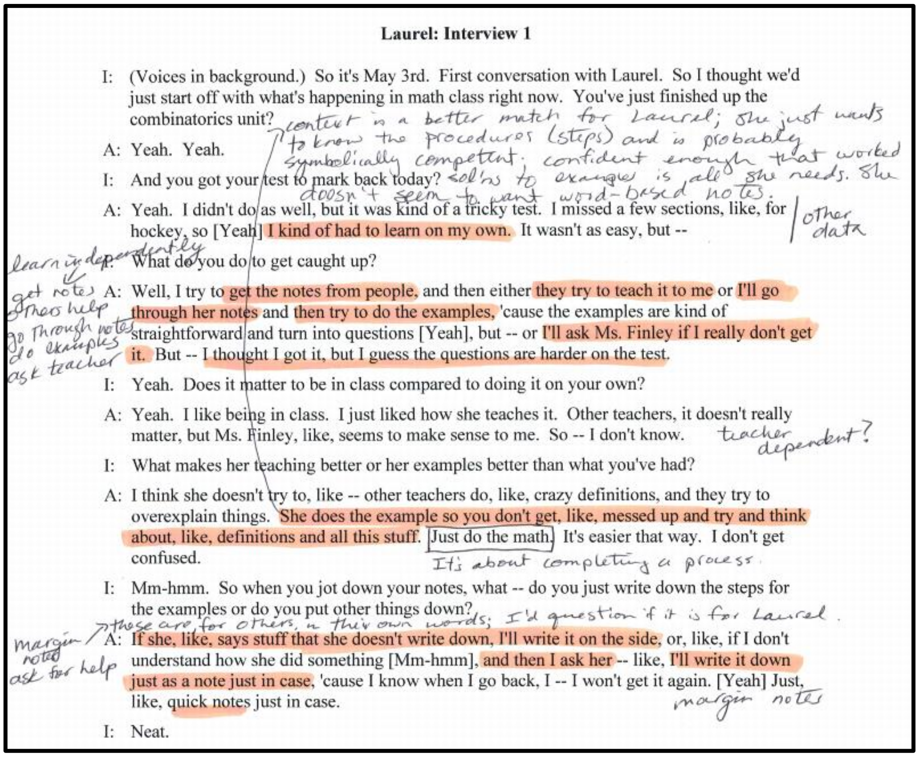
**Coding During Data Construction**. During the data construction phase of the inquiry, I began to code the interactive writing and first small group sessions. Charmaz (2006) notes that data construction and coding are done simultaneously, in part because coding informs further data construction. I found the coding to be unrealistic in terms of time commitment to immersion in the research context and inauthentic in its fragmentation of the data in light of my interpersonal commitments with the learner-participants. I abandoned the process at the time. I continued throughout the data construction time to read and re-read data, to write memos in my research process journal about the research-based experiences, and to engage in thinking about and with the data. As an example, in preparation for the second one-on-one interview with the learner-participants, I developed a theme for each one to invite his or her perspective on my emerging interpretation. Analysis of the data, although not in coding form, was ongoing throughout the data construction period.

**Coding After Data Construction**. After the data construction was complete, I returned to the data to begin anew the coding process. I re-read the chapter on coding in *Constructing Grounded Theory* and noted that Charmaz (2006) stated, “Coding means categorizing segments of data with a short name that simultaneously summarizes and accounts for each pieces of data. Your codes show how you select, separate, and sort data to begin an analytic accounting of them” (p. 43). Moving from this definition, Charmaz describes two phases of the coding: 1) “an initial phase” (p. 46) where coding words are created from the sensitivities of the researcher or from the participant’s words (*in vivo* codes) and pursue a wide range of theoretical possibilities; and, 2) “a focused, selective phase” (p. 46) where codes with high frequency from the initial phase are used to go through the data again, bringing coherence and organization to the data and synthesis to the theoretical thinking. Along with guiding prompts, Charmaz advocates for paying attention to actions through the use of gerunds for codes and move with rapid pace through the data. The unit of analysis could differ from words to phrases (lines of data) to incidents.

As I used these guiding principles for coding the data, I was challenged with the volume and complexity of the constructed data. I structured my line-by-line coding going through all the learner-participants' data within a form of data, and then shifted to working with one learner-participant's data at a time. Working in electronic files, I tried a variety of approaches like using different colours and styles of fonts, highlighting in different colours, and placing phrases beside each line of data to build a multi-layered analysis. The variation in the font only lead to a spreadsheet that was more efficient to scan, but did not communicate with any more depth the meaning of each line of data. Highlighting segments only called to attention those portions of data, without a word-based explanation of the perceived significance of the piece of data. And, often multiple phrases could be placed beside each line of data to describe what was happening. There was something that seemed forced about the process, and I was uncomfortable by the lack of ways to represent what I knew about the learner-participants and our experiences together through the coding. I was also concerned by the atomistic approach to the data, where fragmenting the data into small pieces that are then simplistically labeled, even as gerunds, does not capture the fullness of the moment of data. While persisting in coding the data, and often shifting approaches, I eventually abandoned this process because it was not provoking generative thought about the learner-participants' experiences in learning to learn mathematics. I perceived the coding as pulling me far away from the research question and the possibility of a response to it.

**Comprehensive Coding Process**. On returning to the data later with a renewed commitment to coding, I decided not to read any methodological writing but proceeded in working and thinking with the data. Earlier in my thinking with the data, I had considered a five-layered approach which began with the identification of learning strategies, what learning-participants had been doing with the learning strategies during the study (examples could include adding on, minor revisioning, restructuring, deleting, naming, substituting, etc.), the mechanism(s) by which they went about shaping the learning strategies, the intentions with which they used and shaped learning strategies, and how learner-participants' shifted their views of the nature of mathematics, learning, and who they were as learners. In the context of this thinking, I decided to attend to the learning strategies that the learner-participants were talking to me about or demonstrating in the data, with the intention of being able to identify what learning strategies the learner-participants were using and shaping throughout my time in the school.

I worked through one learner-participant's data at a time, and within this set of data grouped together all the common forms of data: interactive writing, field notes, small group sessions, and one-on-one interviews. I made three separate passes through the data. First, I worked with hard copies of the data, highlighting instances of learning strategies. In focusing on the learning strategies, I did not limit the coding to these instances but also wrote comments about the codes and data in the margins, as well as noting data that might be important later on or may inform other concepts besides learning strategies in particular. Figure 3.3 contains a copy of excerpt from Laurel's data in this initial processing for Interview 1.



**Laurel: Interview 1**

I: (Voices in background.) So it's May 3rd. First conversation with Laurel. So I thought we'd just start off with what's happening in math class right now. You've just finished up the combinatorics unit?

A: Yeah. Yeah.

I: And you got your test to mark back today?

A: Yeah. I didn't do as well, but it was kind of a tricky test. I missed a few sections, like, for hockey, so [Yeah] I kind of had to learn on my own. It wasn't as easy, but --

I: What do you do to get caught up?

A: Well, I try to get the notes from people, and then either they try to teach it to me or I'll go through her notes and then try to do the examples, 'cause the examples are kind of straightforward and turn into questions [Yeah], but -- or I'll ask Ms. Finley if I really don't get it. But -- I thought I got it, but I guess the questions are harder on the test.

I: Yeah. Does it matter to be in class compared to doing it on your own?

A: Yeah. I like being in class. I just liked how she teaches it. Other teachers, it doesn't really matter, but Ms. Finley, like, seems to make sense to me. So -- I don't know.

I: What makes her teaching better or her examples better than what you've had?

A: I think she doesn't try to, like -- other teachers do, like, crazy definitions, and they try to overexplain things. She does the example so you don't get, like, messed up and try and think about, like, definitions and all this stuff. Just do the math. It's easier that way. I don't get confused.

I: Mm-hmm. So when you jot down your notes, what -- do you just write down the steps for the examples or do you put other things down?

A: If she, like, says stuff that she doesn't write down, I'll write it on the side, or, like, if I don't understand how she did something [Mm-hmm], and then I ask her -- like, I'll write it down just as a note just in case, 'cause I know when I go back, I -- I won't get it again. [Yeah] Just, like, quick notes just in case.

I: Neat.

*Figure 3.3*. Coded segment from Laurel's interview 1.

Second, as I read through the data again I created a cover sheet for each form of data for the learner-participant consisting of a list of the codes I had written in the margins of the data. The cover sheet was created at the end of a pass through one form of data for an individual learner-participant. I refined some of the codes at this point as I was able to see the whole of the data in the pass. The refining included consolidating some codes that were similar under one code or grouping similar codes but leaving the initial labels. Within grounded theory methodology, the process of refining codes could also be called the "constant comparative method" (Glaser & Strauss, 1967, p. 102). In this case, the constant comparison occurred within a single form of data for a participant. As the comparing and refining of codes occurred, I made changes to the labels directly on the data. Figure 3.4 contains a copy of the cover sheet for Laurel's data for Interview 1.

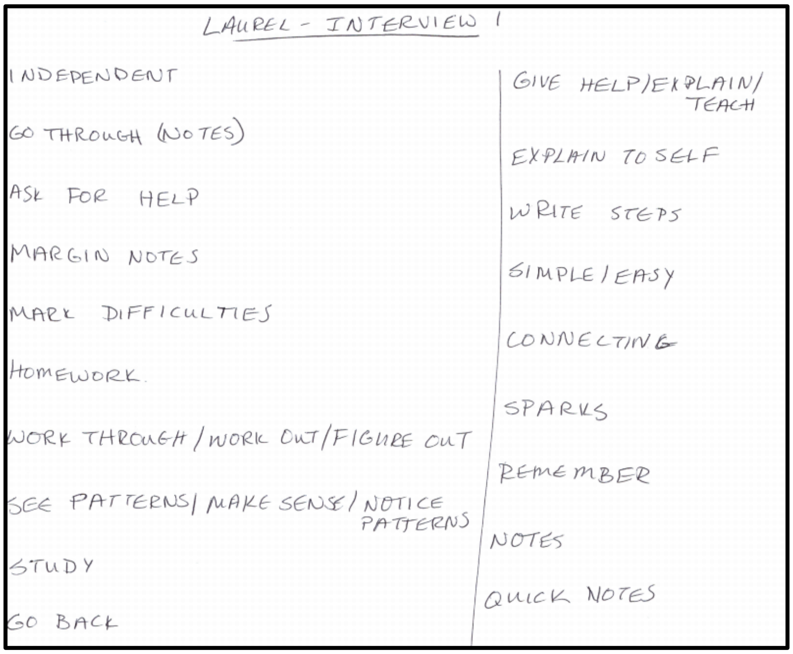



*Figure 3.4*. Cover sheet with codes from Laurel's interview 1.

Third, I created an individual electronic word processing file of the codes for each learner-participant. The electronic file was organized by form of data. I used the codes from the cover sheet to begin the file, typing in capitals and bold font as a heading. I then proceeded to read through the data as a third pass. The third pass occurred after I had completed all the forms of data for an individual learner-participant. While I read, I copied all the quotes of the instances I had labeled with the particular code and pasted it into the Learning Strategies file for each learner-participant. In this third pass, I fine-tuned the codes. Again, the constant comparative approach was used, although the comparisons were now occurring across all the forms of a single learner-participants' data. I also sometimes found elements of data that I had missed in coding through the first two passes that I included in the Learning Strategies file. In addition to excerpts of data, I occasionally added my own comments about a particular piece of data right after the quote in the file. Figure 3.5 contains an excerpt from Laurel's data for Interview 1. The files ranged from 10 to 34 pages long (most over 20 pages) among the learner-participants and can be seen as a distillation of the data. The organization of the files and the electronic format made searching for specific examples relatively efficient.

**Laurel – Learning Strategies from Interview #1**

**INDEPENDENT**
--" Well, I try to get the notes from people, and then either they try to teach it to me or I'll go through her notes and then try to do the examples, 'cause the examples are kind of straightforward and turn into questions [Yeah], but -- or I'll ask Ms. Finley if I really don't get it." (p. 1) – she missed classes because of basketball
--" Ms. Finley's a great teacher too, but [Yeah] it's definitely the home stuff." (p. 4)

**ASK FOR HELP**
--" or I'll ask Ms. Finley if I really don't get it" (p. 1) – when catching up independently
--" if I don't understand how she did something [Mm-hmm], and then I ask her" (p. 1)
--" she's here to ask questions or" (p. 2) – the possibility of asking for help, more than asking for help itself
--" once I started doing my homework, I started asking, like, more questions" (p. 4)
--" Or if we don't understand, we ask Ms. Finley, and then she teaches both of us." (p. 9)
--" there's teachers around to ask." (p. 10)
--" you can ask her stuff." (p. 10)

**MARGIN NOTES**
--" If she, like, says stuff that she doesn't write down, I'll write it on the side" (p. 1) – for others, this is in their own words, but I don't have information on that detail from Laurel
--" I'll write it down just as a note just in case, 'cause I know when I go back, I -- I won't get it again. [Yeah] Just, like, quick notes just in case" (p. 1) – this is after she's asked for help, she'll put information from the help she received in the margin

*Figure 3.5*. Excerpt from electronic codes file for Laurel's interview 1.

Rather than limiting the codes I constructed to follow the guidelines Charmaz (2006) recommended, I wrote what first seemed to fit. As Charmaz suggested, I worked with relative speed through the data so that I would not linger on the choice of codes. I used *in vivo* codes and my own languaging to describe what I understood to be occurring in the data. I treated this process as provisional and that orientation opened up space for me to begin and sustain the process of coding. The codes were the product of the coding process; however, the brevity of the labels is a limited representation of the thinking that produced the codes. Through the coding, I developed a sense of flow through each of the learner-participant's set of data and developed a familiarity with the particularities of the data. A challenge I found in deciding on codes is that often there were as many meanings or specific processes for a code (e.g., "doing homework" or "taking notes") as there were learner-participants in the study. The challenge arose in the tension between using small enough units of analysis to code and a broad enough view.

The constant comparative method was also used across the learner-participants. After completing the three passes through the data in coding, I transferred all the codes to a spreadsheet file. Each worksheet in the file contained all the codes for a particular form of data. The first column listed the codes in alphabetical order and the rest of the columns represented each learner-participant. I placed a code in a specific cell for a learner-participant if the code occurred in that person's data. I could make note of any differences or overlap in the labels for the code and I used the cells as a mapping tool to efficiently locate excerpts from data linked to the code. Figure 3.6 contains a portion of the spreadsheet containing the codes from Interview 1. For Interview 1 alone, I had

130 different codes; the other forms of data had fewer codes. The vast quantity of codes for one form of data looked daunting and it was difficult to determine where to begin working with the codes.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | | Jocelyn | Laurel | Ashley | Nadia |
| 2 | Answer in class | | | | Answer in class |
| 3 | Apply | | | Seeing patterns/Making patterns/ Bring together/Pic | Apply |
| 4 | Ask questions/Ask for help/Seek help/Ask Why | Ask questions/Ask for help | Ask for help | Ask for help | Ask for help/Ask questions/Gettin |
| 5 | Big ideas sheet/Main concepts | | | | Big ideas sheet/Main concepts |
| 6 | Break down | | | | Break down |
| 7 | Building/Adding on | | | | Building/Adding on |
| 8 | Cheat sheet | | Cheat sheet | | |
| 9 | Cheating | | | Cheating | |
| 10 | Clear up | | | | |
| 11 | Concentrate | | | | |
| 12 | Connecting | | Connecting | Seeing patterns/Making patterns/ Bring together/Pice together/Connecting/Applying | |
| 13 | Continue on | | | | Continue on |
| 14 | Copy assignments | | | | |
| 15 | Cue cards/Q-cards | | | Cue cards | |
| 16 | Distilling | | Distilling (key ideas or known/unde | Rewrite notes/Condensing (?) | |
| 17 | Do examples | | | | Do examples |
| 18 | Do question during notes | | | | Do question during notes |
| 19 | Explain to self/Self-talk | | Explain to self | | |
| 20 | Explain to others | | | | Explain to others |
| 21 | Figure out/Work through/Work out | | Figure out/Work through/Work out | | Figure out/Engage/See for yoursel |
| 22 | Flip through | | | | |
| 23 | Floating | | | | Floating |
| 24 | Focus/Sit down | | Focus | | Focus/Sit down (sit there as contr |
| 25 | Forget | Forget | | | |
| 26 | Form question | | | Form question | |
| 27 | Get answer | | | | Get answer |
| 28 | Get hang of it | | | | |
| 29 | Get/have down pat | | | | Get/have down pat |
| 30 | Go back/Look back/Go to | | Go back | Go back/Look back/Go to | Go back/Look back |
| 31 | Go through/Go over | Go through | Go through/Go over | Go through/Go over | Go through |
| 32 | Highlight | | | | |
| 33 | Homework | Homework | Homework | Practice/Homework | Hoemwork |
| 34 | Identify content | Identify content | | Identify content | Identify content |
| 35 | Identify misunderstanding/difficulties | | | Identify misunderstanding | |

*Figure 3.6*. Excerpt from codes spreadsheet for interview 1.

To begin purposeful work with the codes, I separated them into three groupings: 1) codes related to the actions of the learner-participants in relation to ways of learning; 2) codes that indicated learner-participants' working on mathematics content (evident primarily in the small group sessions form of data); and, 3) codes that identified ideas that were non-actions, including referring to mathematics content, representations or products of learning strategies, stance or orientation of the learner-participant, and contextual/other. Some codes, like "homework" were placed in both the action and non-action (product) groupings.

**Looking Differently at Data Analysis**

After completing the coding, I wondered how the codes would coalesce into categories and what the labels for the categories might be in the ensuing data analysis. I returned to *Constructing Grounded Theory* (Charmaz, 2006) to pay attention to the guidelines for the process of categorizing. Charmaz builds on Glaser and Strauss' (1967) definition of a category as the "conceptual element in a theory" (p. 37). The purpose of categories is to "explicate ideas, events, or processes in your data … A category may subsume common themes and patterns in several codes" (Charmaz, 2006, p. 91). As deep as my familiarity with the data and codes was, they did not necessarily point to particular themes or patterns. My thinking with the data had resulted in the five-layered approach (see "Comprehensive Coding Process" section above), but patterns within the layers were not apparent. Charmaz's advice to "raise [focused codes] to conceptual categories" (p. 92) left me with little guidance on what it means to "raise" codes to develop "sharp, clear categories" (p. 92). Even the three loose groupings of codes had not created clear categories because some codes belonged to more than one grouping.

At the same time, I was in the midst of authoring proposals for academic conferences. As I considered smaller pieces of the dissertation to begin presenting, I thought about different elements which would allow me to understand the learner-participants' learning in a complex manner. As a specific example, I thought about the context in which the learner-participants engaged in order to use and shape their learning strategies – a conversational context (related more fully in Chapter 4). Using my rich sense of the data and my experiences with the learner-participants in the class, I became aware of how much the learner participants valued a relaxed pace within the Mathematical Learning Skills class. This realization, which became the feature of *pace* for learning-based conversational opportunities, caused me to look differently at the data, searching for what I saw as "informative moments." With this understanding, I returned to my experiences with the learner-participants and the data to develop more features of the conversational opportunities.

I conceptualized categories by looking beyond the codes and making meaning of experiences and data. The categories were formed from my thinking and searching out the breadth of qualities with the magnification related to the interpretive moment and what the learner-participants indicated (through actions and words) were impactful in the conversations. In the languaging of CGT, the four features of opportunities for learning-based conversations (Chapter 4), the four forms of engagement in developing processes for learning mathematics (Chapter 5), the four mechanisms for making meaning of mathematical ideas (Chapter 6) could each been called categories. I have given the various sets of categories meaningful labels – features, forms, and mechanisms – in order to mark them in relation to particular interpretive moments with differing magnification, and to provide some distinctions among the chapters and interpretive moments. It is in the acknowledgement that these interpretive moments were each opportunities for coming to understand learning occurring within the study in different ways which highlights the complexity of the learner-participants' learning. In fact, they could have each been a separate constructivist grounded theory study.

After generating each of the sets of categories and substantiating them with specific examples from the data, I considered how the interpretive moments could be integrated into a further piece of theorizing. The metaphor of authoring, explicated in Chapter 7, integrates the sets of categories from Chapters 4, 5, and 6 into an interpretive understanding of the learner-participants' learning throughout the inquiry.

I questioned whether I was engaged in the processes of CGT or whether I was doing a CGT study because it seemed to me that this was not the way it was supposed to work – that the codes should lead directly to categories instead of a disjunction between the codes and categories. The categories I was constructing were formed through deep and careful consideration of the data as an empirical emphasis in the analytic work rather than the collection of codes under category labels. The sorting of codes that was implied in Charmaz's (2006) description was not evident as I examined the processes I had used. My concern led to me to more methodological reading, but this time more broadly in grounded theory rather than remaining with Charmaz's formulation of CGT.

**Interrupting Methodological Preconceptions: Categorizing**

Several chapters in *The SAGE Handbook of Grounded Theory* (Bryant & Charmaz, 2010) explore coding and categorizing data within grounded theory studies from a variety of perspectives. Rather than exploring the author's orientation and recommendations in each of these chapters, I will highlight the work of Dey (2010) because his chapter interrupted my methodological preconceptions of coding and categorizing. Even though he focuses on categorizing, Dey addresses coding throughout the chapter from his opening position, that "coding does not exhaust the analytic process; one can even question whether it is integral to it" (p. 167) to elaborating caveats to the process of coding. My readings in CGT and my engagement with data had focused largely on coding, and Dey's statements had dissonance with the readings and synchrony with my experience of coding in this research project. He interrogates the metaphors of codes and categories (Lakoff's work is central in Dey's argument) and the "classical model" (p. 169) of categorizing to demonstrate deficiencies such as rule-driven, definitive, and descriptive aspects of categories. He supports a repositioning through a synthesis of current thinking in the field.

Dey (2010) establishes a distinct perspective of categories and categorizing, divergent from Charmaz (2006), Kelle (2010), and Holton (2010). Dey (2010) begins his alternative view by claiming that "categories emerge initially from a close engagement with data … later fleshed out by identifying and analyzing in detail their various properties and relations" (p. 168). Perhaps relying on emergence can be taken as ambiguous and as ill-defined as Charmaz's (2006) raising of codes to category labels. At the same time as refraining from being prescriptive, Dey (2010) offers: "often categorization does not proceed through the invocation of rules at all but through comparison with recalled or prototypical exemplars" (p. 169). Stern (2010) describes these prototypical exemplars as "cream in the data" (p. 118). I understand the prototypical exemplars to be those moments in data that a researcher repeatedly finds herself or himself narrating in response to queries about the research project. This is what I had meant by "informative moments" (see the above section). The recollection of exemplars rests on a researcher's deep engagement with data. As richly familiar a researcher is with the data, Dey (2010) points out that "we need to give as much attention to [the category's] theoretical provenance as to their empirical base" (p. 177). Sensitizing concepts represent such theoretical bases for the development of categories.

The result of the process of categorizing are categories of analysis. Dey (2010) explains that categories "lack clear boundaries defined by an unambiguous set of criteria; categories are fuzzy and category membership is a matter of degree" (p. 170). In this way, Dey resists over-simplification of categories and over-structuring in their connections. Categories are not simple, but provisional concepts with the complexity inherent in meaning-making. Rather than over-structuring through hierarchies, categories could be viewed through illustrations such as Venn diagrams. In each of the interpretive moments contained in Chapters 4 through 7, I view the categories I developed as fuzzy because of their overlapping conceptually. Also, the same data could be placed not only within more than one category in an interpretive moment (represented by a singular chapter) but could also be placed within several chapters. The interpretive moments viewed the same phenomenon, but at different magnifications (zooming in and out). Drawing the categories together at the end of each chapter speaks to connections among the categories, and then the integration of the interpretive moments occurs through the metaphor of authoring in Chapter 7. As my theorizing does not lend itself to a neat diagram, it highlights the complexity of categories within interpretive moments and across the interpretive moments. Dey sees the function of categories in at least two ways: 1) as a way to construct concepts of the researched phenomena and to communicate this conceptualization meaningfully; and, 2) as further sensitizing concepts to inform the researcher's thinking. It is apparent that Dey has reconceptualized categorizing and categories for grounded theory.

What I learned through my experience in comprehensively coding the data and then beginning to develop categories related to various elements of the research question is that ***processing*** data is more important than coding or categorizing. In fact, coding and its products – the codes – do not need to be the intended focus of working with data during analysis. Rather, it is about a rich engagement with the data where the researcher is immersed in a conversation with the data and with the participants through the data. I formed a sense of intimacy, as a researcher, with the data. This intimacy – knowing moments in the data to recollect with fluency, developing awareness of how the data fit together as a whole (as opposed to fragmentation), and emerging sense-making of elements of data – contributes to theorizing. The processing of the data, more than coding or codes, was one of the facets that supported my shift to conceptualizing the meaning of the data. And so, coding is not necessarily meant to create codes that can then be categorized, but as way to invite researchers to engage with the data. As I returned to the CGT literature, I found Charmaz (2006) asserting that "we learn through studying our data" (p. 46). While her intention in the statements seems to be learning about the data itself, making sense of it to develop understanding, I had the opportunity to do that through studying my data. I also learned more about the quality of engaging with data. Seeing categories and coding from a different vantage point in the methodological literature affirmed that I was still engaged in the process of making sense of my data within a CGT approach and that I could simultaneously examine CGT as a methodological process.

### A Word about Memoing

Memoing is an important process in CGT for making sense of data. The purpose of memos is to "chart, record, and detail a major analytic phase" (Charmaz, 2006, p. 72). More specifically, Lempert (2010) describes memoing as "a private conversation between the researcher and his/her data. ... And all memos are partial and provisional" (p. 251). At times, I found the memoing to also be a space for self-talk as a researcher, pursuing a conversation with myself around moments in the data. I have not described writing memos separately from coding or categorizing data because it was imbedded in my research process journal which I kept throughout the inquiry. The comments I added in each learner-participant's Learning Strategies electronic file are brief memos. Similarly, Dey (2010) views memos "mainly as adjuncts to codes and catalysts for their further development" (p. 187). As I authored chapters in this dissertation, I was also writing-to-think and writing-to-learn, the intention I perceive for memoing, in a way that ideas from memos were interwoven into the writing itself.

**Theorizing Arising from the Analytic Process**

The focus of this inquiry was on the learner-participants' learning as they were developing processes for learning mathematics – or learning to learn within the context of mathematics. My intention was to engage in theorizing to offer an abstract interpretive understanding of the phenomenon. Because of the complexity of learning, the analytic process of coding and categorizing resulted in multiple ways of understanding learning to learn mathematics.

The complexity of both the phenomenon and my theorizing is represented in multiple interpretive moments. Each interpretive moment is explicated within a single chapter, for the following four chapters in the dissertation. Taking on the naming of "interpretive moments" is appropriate within CGT, as Charmaz's (2006; Bryant & Charmaz, 2007b) re-grounding places the methodology within an interpretive research tradition.

Each interpretive moment views the learner-participants' experiences as constructed through the data at different magnifications – zooming in and out to develop an insightful understanding of learning to learn mathematics. In the first interpretive moment, I zoom into the contextual situation to take a broad view to explain features of conversational opportunities through which the learner-participants were shaping learning processes (Chapter 4). For the second interpretive moment, I zoom into the conversational context to explicate the learner-participants' forms of engagement in developing learning processes for mathematics (Chapter 5). For the third interpretive moment, I zoom in further, to examine within the learning processes the mechanisms by which the learner-participants were making sense of mathematical ideas (Chapter 6). As a fourth interpretive moment, I come to a provisional-yet-closing abstract interpretation. In this, I integrate the previous interpretive moments to theorize the learner-participants' learning through the metaphor of authoring (Chapter 7). This closing act of theorizing could be seen as the most abstract of the interpretive moments. For each of these interpretive moments, categories of analysis were developed within the interpretive moment and described in the corresponding chapter.

Engaging in and then describing different interpretive moments allows me to offer up in one moment a particular way to understand the learning occurring within the inquiry, and then in another moment offering another way of understanding. This approach preserves the complexity, the all-at-onceness, of the learning occurring through the inquiry yet working with a reasonable unit of analysis. Using magnification of the learner-participants' experiences as an approach to understanding my acts of interpretation and theorizing offers a way to hold on to the complexity of the learning and inquire into the nuances of the learner-participants' growth. What is offered, then, in the following chapters is an account of my theorizing. My understanding of theorizing is drawn from Charmaz's (2006) explanation, described in Chapter 2, as an abstract interpretive understanding. My theorizing is context-dependent, provisional, and marked by multiplicity of understandings.

# Chapter 4
# Providing Opportunities for Learning-Based Conversations in Mathematics

In the previous three chapters, I described background and methodology for the study and situated the study in a specific setting with particular students and a teacher. Beginning with this chapter and continuing until Chapter 7, I build on my data analysis to portray several different interpretive moments with the data. This process highlights the provisional nature of interpreting data and the complexity of the data that is portraying the learner-participants' experiences within the inquiry.

This chapter provides the first of four interpretive moments as I engage in theorizing with the data. Using a large magnification – or a broad view – I represent the result of data analysis which brought into view the learner-participants' various opportunities to be in conversation about how they were learning mathematics.

Looking across the data for the conversational opportunities was done with the understanding that this was only one way to look across the data and that the provisionality of my interpretation is represented by the notion that I can, for this moment, see the providing of conversational opportunities in the data and then in another moment come to understand another facet of learner-participants' learning to learn mathematics. The chapter concludes as I draw together the four features, offering a possible visualization.

## Features of Opportunities for Learning-Based Conversations

Communication has been identified as a crucial process within mathematics classrooms and substantial work has been done in the field of mathematics education research around conversation and discourse. Mathematics educators have invested significant energy into researching various aspects of mathematical communication (Chronaki & Christiansen, 2005; Elliott & Kenney, 1996), emphasizing effective instructional strategies for increased student understanding (Nathan & Knuth, 2003). As described in chapter 1, I explored the use of the word "conversation" in mathematics education literature and the philosophical orientations of the research contributions. This led me to view conversation as a type of communicative act that could support a thoughtful attention to learning and to (re)forming identity within a deeply personal exploration of turning round processes of learning. In this way, conversation takes up a sense of withness and listening in a dynamic process and with an uncertainty in destination and understanding. Davis (1996) recognizes that a conversation can only be realized retrospectively, "when self and other have been altered" (p. 28), and so through my engagement with data I recognized that I had learned more about the qualities of the opportunities for students to engage in conversations which would support their learning to learn mathematics.

The conversations within this work had a focus on the ways in which students were learning; they were learning-based conversations. As well, the learner-participants were accepting invitations to participate actively in a variety of conversational opportunities. I use the word “invitations” intentionally, as I see this as explanatory of how I extended offers to be in conversation with the learner-participants. Because I did not have the same authoritative relationship with the learner-participants as a teacher who compels action with marks, I was able to ask – invite – the learner-participants into conversations without an authoritative expectation of participation. The learner-participants were free to choose – to accept the invitation or not – whether to engage in the learning-based conversations.

Within the research project, I engaged in conversations with the learner-participants that focused on mathematical content, as well as relationship building. I also observed the range of conversations they had in Learning Skills class, which included topics like school activities, their lives, other courses, and mathematics. For the scope of this chapter, I will focus on learner-participants’ conversations which explored their learning of mathematics. I refer to these conversations as learning-based because the talking and wondering about the learning of mathematics was foregrounded – with elements of relationship-building and mathematics still present – but the intention was to consider the ways in which the learner-participants were going about learning mathematics. Often, within a conversation there was fluidity between a learning focus and a mathematical focus. A narrowing to focus on examples of learning-based conversations does not negate the importance of all the other conversational foci, taken together, but I can speak through my work uniquely to the features of conversational opportunities for students to talk about and improve their processes of learning high school mathematics.

In this section, I explain four features of opportunities for learning-based conversations, including: *preparation*, *presence*, *mode*, and *pace*. These four features represent qualities of providing opportunities for students to talk about and improve their learning strategies. It is more explanatory of the occasioning of learning-based conversations, rather than of the qualities of conversations themselves. As such, it demonstrates an understanding of the ways in which teachers could provide opportunities for students to talk about how they learn. These features were created by looking at the range of examples from the study and attending to what the learner-participants emphasized when they identified conversations about their learning and what I noticed in their conversations through observations in the classroom and interpretation of data that the learner-participants did not explicitly identify as conversational moments. Placing these various forms of data together through interpretation informs the way in which opportunities for learning-based conversations could be provided to students.

**Preparation**

*Preparation* as a feature of opportunities for learning-based conversations points to the varying degrees of advanced planning that took place in providing opportunities for the learner-participants to attend to their learning. This feature has a temporal dimension, from spontaneous to deliberate interactions. In this section, I describe the scope of *preparation* in learning-based conversations, supported by specific examples, and then address intentionality in conversation.

Spontaneous conversations around learning mathematics arose in the Mathematical Learning Skills class as I walked around the class each day answering mathematics questions. For example, I recorded in my field notes from early in the study that Teresa and I could explore "what it means to ask for help and what kind of help to ask for," as I found her frequently asking me for help with specific mathematical steps. Later in the study, I recorded another interaction in class where Teresa "asked me if it was like a question in her notes … I encouraged her that she had found a similar question in her notes, and that was a great strategy" for getting unstuck when working on homework. Our one-on-one conversations in class highlight the fluidity of these moments, where Teresa would often ask for help with specific questions in a homework assignment and I would shift the conversation toward thinking about approaches to learning mathematics which arose in the moment. Even when learner-participants were studying for other courses, like Grace studying for biology with a content map, I recorded in my field notes how I wondered "about thinking about that for learning math. I told her about Ashley's layering of the examples on her summary sheet, and Shane's layering of the concepts and then examples on his summary sheet. She thought those were neat ideas." I found myself alert to opportunities of voicing wonderings which brought learning into view for the learner-participants. These spontaneous conversations are like what Gordon Calvert (2001) perceived as "improvisation" highlighted by being "spontaneous and unpredictable … [yet] by no means random" (p. 87). The metaphor of improvisation captures the idea that the conversations were at once fluid and meaningful, unanticipated yet intentional.

The Mathematical Learning Skills class itself rarely had deliberate opportunities for conversations about learning. I understand the notion of "deliberate" to mean a systematic shaping, in advance, of the focus of a conversation. Through the data construction elements of interactive writing, small group conversations, and one-on-one interviews I added deliberate interactions with the learner-participants focused on approaches to learning mathematics. Reviewing the interactive writing prompts found in Appendix A, all prompts invite students to consider their learning processes. In the first small group sessions for all three groups, I prepared for actively constructing a learning strategy. However, without explicit prompts to guide learner-participants to consider the ways they had been learning this was not addressed. For the next sessions, I included prompts which directly addressed learning with the specific strategy (prompts can be reviewed in Appendices B, C, and D).

The one-on-one interviews with each learner-participant contained a considerable amount of advanced preparation as I authored questions that would provide opportunities for learner-participants to attend to their learning and learning processes. Examples include "In the last two weeks, who have you talked with about how you learn math? Are you getting better at learning math? In what ways?" (Interview 1) and "In the last interview I asked you if you were getting better at learning math. Do you think you've become more aware of how you learn math? What have you done to figure out how you learn?" (Interview 2). When I personalized the interview guides for each participant-learner, I deliberately selected examples that would be generative in thinking about mathematical learning and demonstrate to each learner-participant a deliberate turning round of specific ideas within a relational space. Grace, upon looking at her list of learning strategies in our first interview exclaimed, "Wow, that's a lot! … Oh, I thought I only had two or three ways to learn math, kind of thing. Just never really think about it. It's like, 'Oh, I just do this to study math,' kind of thing." The deliberate preparation for our conversations about learning mathematics meant that I could deeply explore the processes and meaning for the learner-participants.

Frequently in class and through other research processes, I was providing opportunities for learning-based conversations. However, there are examples of learner-participants also initiating learning-based conversations. When I asked Kylee in our first interview about the development of her cue cards for learning mathematics, she described that in the stationary store where she worked, "one year we had a display, and they just showed us all these little notes. And I was like, 'You know what? That's a really good idea.'" After successfully using cue cards for biology, Kylee considered, "it was like, 'Okay, well maybe this will be useful in math because there's a hundred and ten examples here {little laugh} but I only really need to know two of them.' Right?" The self-talk Kylee reports began as she thought about adding on to her set of strategies for learning. As another example, Danielle initiated a fifth small group session by requesting that the group meet again the following week. The session marked a shift both in the learner-participants' learning together and in requiring less planning on my part to provide the opportunity. There was a degree of spontaneity in the learner-participants' initiating conversations about how they would improve their approaches to learning mathematics.

Regardless of the degree of advanced preparation for the conversational opportunities, the learning-based conversations were immersed in the intention of improving processes of learning mathematics. The intention, taken as the foundation for the conversations, was present for me as I formed prompts to pose to the learner-participants. As I inquired into how the learner-participants were improving in their learning processes for mathematics I would ask questions that would direct their attention toward learning, draw out their awareness of their learning, probe for the meaning of their explanations, and inquire into the processes themselves. These four guiding areas were helpful in framing questions both spontaneously and with deliberation and invited learner-participants to engage in an opportunity to explore their personal processes of learning mathematics through conversation.

**Presence**

Another feature of opportunities for learning-based conversations is the individuals who are present in the conversation. *Presence* refers both to the members of a learning-based conversation and to the composition of members. As with the range that existed in preparing for conversations, there was a range in the composition of members for learning-based conversations. The learner-participants demonstrated a value for different perspectives on specific learning strategies from a variety of individuals, while not viewing the perspectives as prescriptions. This openness to considering different approaches of learning mathematics, yet maintaining responsibility to personally shape suggestions from others, suggests that learner-participants were not looking for experts to tell them how to learn but were responsive to turning round ideas in conversation with others who were fellow inquirers. In this section, I explain possible members of learning-based conversations, explore self-talk as an individual conversational act, and explain the composition of groupings.

When I asked the learner-participants in the first interview about whom they had talked with about their mathematical learning recently, their responses included teachers (Mrs. Finley and myself) and peers – at least, for those learner-participants who even saw themselves talking about learning. The learner-participants saw opportunities for conversations existing within the interactions I prompted through research processes. Only a few learner-participants gave examples of talking about learning strategies outside of class with friends. Nadia recollected that “people ask me, ‘How did you prepare for this?’ And then I ask them, ‘How did you prepare--, how are you getting this ninety on this test?’” to find out that repetitive practice was a study approach for high achieving students. Parents as conversation partners did not come up very often in our interviews; however, Chelsea mentioned talking about learning with her parents and noted that her dad “tries to motivate me and tells me different ways” to study. Interestingly, some learner-participants indicated conversing with themselves as they thought about how to improve their mathematical learning.

Just as Kylee’s example of reported self-talk was described in the previous section, there were other examples of learner-participants who engaged in self-talk. Bakhtin’s (1986) notion of the dialogic reality supports self-talk as a conversational space:

> However monological the utterance may be (for example, a scientific or philosophical treatise), however much it may concentrate on its own object, it cannot but be, in some measure, a response to what has already been said about the given topic, on the given issue, even though this responsiveness may not have assumed a clear-cut external expression. … The utterance is filled with *dialogic overtones*, and they must be taken into account in order to understand fully the style of the utterance. After all, our thought itself—philosophical, scientific, and artistic—is born and shaped in the process of interaction and struggle with others’ thought, and this cannot but be reflected in the forms that verbally express our thought as well. (p. 92)

An utterance, which for Bakhtin can be written or oral, may appear to be singular when an individual thinks or says it aloud, but it is still responsive to and interacts with what has come before.

In many of the interviews and small group sessions, learner-participants described how they would state mathematical procedures (usually aloud) to learn from their homework. Grace explained that if she was not completing homework with her friends, “I even talk to myself” about the mathematical procedures. While self-talk was primarily focused on mathematical thinking, Danielle described creating the idea of summary sheets when she was “sitting on the bus, and I was thinking … how would I be able to separate my ideas and stuff, but then at the same time, I know why they go together in one lesson or something.” Even when Elise interacted with a text (described below in the section “Mode”), she indicates self-talk in her recounting. In each of the cases, the way in which a learner-participant recounted the moment was in a conversational way, talking to herself or himself, just as someone would recount dialogue with another person. The opportunity to engage in self-talk focused on possibilities for improved learning processes and was conversational in nature as the learner-participant was turning round ideas about her own mathematical learning.

When learner-participants had opportunities to interact with others about their mathematical learning, the groupings of conversational members ranged from one-on-one to small groups of students. Mrs. Finley recounted several examples of conversations around learning strategies learner-participants had been working on during small groups or in response to interactive journal writing. As I described in the above section, spontaneous conversations were often one-on-one in class, as were the conversations in interactive writing and interviews I had with each of the learner-participants. While I coached learner-participants during individual conversations, the small group sessions contained dynamic conversation as the students suggested and considered different approaches to learning mathematics. Ashley and Danielle, who exhibited several examples during the Summary Sheet Small Group sessions, took the opportunity to explore ideas like how to structure the summary sheets and how to connect mathematical ideas. Learner-participants did not mention whole-class conversations, and the didactic approach to the few whole-class elements I observed in the Learning Skills class did not open up opportunities for conversation. In the second session for the Summary Sheets Small Group, Ashley wished mathematics teachers would “do one entire mind map of the chapter on that big poster board with the class,” as she imagined the possibility of a whole-class grouping as an opportunity for learning-based conversation. This example expresses a common sentiment that learner-participants valued the opportunities to attend to their processes of learning with others who were oriented to listening.

Opportunities for learning-based conversations were composed of individuals who were inquiring into the ways in which learner-participants were learning mathematics and how they were improving their personal processes of learning mathematics. The conversations emerged from interactions among individuals with diverse approaches and ideas for learning. There was a genuine interest in coming to understand how the conversational partner(s) was/were learning mathematics in order to engage in thoughtful turning round of ideas. In the sixth interactive journal writing, Elise recognized her improvement in learning mathematics “by working with other students to gather ideas and collaborate … by talking and studying with others, I have learned and created different ways to study.” Collaborating, through learning-based conversations, with peers in small groups was viewed as one of the most important conversational opportunities by many of the learner-participants.

**Mode**

The *mode* through which the opportunity for conversation exists is another feature of learning-based conversational opportunities. This feature highlights the form in which the conversation takes place, usually among two or more people. Primacy is given to words in marking the conversation, especially in opening up the opportunity for conversation. Words can be used in either an oral (speaking aloud) or written manner to prompt conversation, where the words direct the focusing on learning mathematics. Although other modes of conversation are not necessarily excluded, such as gestural communicative acts (Gordon Calvert, 2001) or students' actions, the turning round of ideas related to learning mathematics is difficult to ascertain in these fleeting actions. In this section, I describe oral/aural conversations, conversations through textual artifacts, and the hybridity of the two modes of conversation.

A conventional mode of conversation is the speaking and listening for meaning that occurs between two or more people, an oral/aural mode of conversation. Quite often explorations of the notion of conversation focus on the speaker's contribution, but there was also an active stance in listening that was demonstrated as the conversational participants came to understand processes for learning mathematics. The oral/aural mode of conversation occurred within the Mathematical Learning Skills class, in the small group session, and in the one-on-one interviews. Nadia's conversation with a friend (described above) is an example of an oral/aural conversation which occurred outside of class.

Opportunities for learning-based conversations also occurred in a textual form during the interactive journal writing. There is strong support from the field of mathematics education that students engage in writing to learn (e.g., Borasi & Rose, 1989; Masingila & Prus-Wisniowska, 1996; McIntosh & Draper, 2001). As learner-participants were invited to consider in a focused manner their mathematical learning the conversations in the interactive writing supported their learning to learn mathematics. Learner-participants would usually respond directly to the prompt I provided and my reply was addressed to them and interacted with their ideas and themselves as learners. Consider the following example as a conversational exchange between Laurel and I in the first two interactive writings (students labeled them as "journals").

Journal #1: Laurel

> A1: Throughout the school year I have been focusing on completing all my assigned homework on a daily basis along with joining in on class discussions to further my learning and boost my confidence in the subject. Finally, asking for necessary help and helping others broadens my learning experience greatly.
>
> A2: Firstly I want to learn how to organize my busy life schedule around studying to find a happy medium, and secondly, different approaches to studying that may suit me better.

*Laurel,*

*It sounds like you have made a lot of progress already this year in your approach to learning math. You've identified many ways of supporting learning – homework, participation in discussions, asking and giving help. I wonder if we can work together on ways to get the most out of what you are already doing, instead of overloading your schedule. You mentioned that helping others benefits you. I wonder if you can incorporate explaining ideas into how you do your homework. This might only take a few extra minutes: after completing an assignment, look over it, and then in a few sentences explain what the important ideas are in the assignment. It gives you a chance to explain, and you make some unit review materials at the same time. Janelle*

Journal #2: Laurel

This week I am going to begin creating notes on what I complete in each of my homework assignments along with beginning to look back to the start of the year to begin review for diplomas. By making quick notes on each section on key points hopefully it will help to spark my memory on specific topics.

*Laurel,*

*I remember noticing last week that you had begun a similar process with making quick notes for chemistry. Your self-discipline to start this early getting ready for diplomas is amazing! I wonder if, while making the math notes, you'll notice connections across units. These connections could help you remember topics in a more lasting way. How might you keep track of these connections you are making through your notes reviews? Janelle*

It would have been difficult, and outside of the classroom norms, to invite students into learning-based conversations which gave students time to pause and consider deeply their approaches to learning mathematics. The process of interactive writing provided space for intimate conversations to occur between the learner-participants and me, a space that was safe for them to share their emergent thoughts about their learning and a space where I could scaffold learning about learning in specific ways for each particular learner.

On occasion, the interactive writing led to a conversation among the textual artifact, the learner-participant, and me. This interaction can be seen as a hybridity of modes where the opportunity for the learning-based conversation arises out of making sense collaboratively of a text. As an example, upon returning the first interactive journal writing, Kylee exclaimed that the suggestion in shaping her cue card strategies was helpful. Her engagement with the journal led to a conversation among the two of us and the text that foregrounded shaping an existing learning strategy of cue cards where she developed a stronger rationale for their use. There were other instances of the hybridity of modes when learner-participants and I were in conversation with each other and textual artifacts. Mainly, this opportunity arose in the small group sessions as I invited the group members to reflect on their record sheets and the process we had developed. At the end of the first Transition Small Group session, Vanessa, Teresa and I compared the class notes with their transition record sheet. Teresa commented that, “if you look at notes and it doesn’t really tell you, you can’t really figure it out;” whereas Vanessa explained that the record sheet was “less intimidating, when you see your own writing.” The conversation among the individuals and several texts provided an opportunity to consider the qualities of a transition sheet – in particular, putting mathematical ideas in your own words – in order to improve the learner-participants’ approach to learning through homework. Much of the students’ work in the Pure Mathematics class was textually-based and dependent on words and abstract symbols, so opportunities for a hybridity of modes was important.

There are at least two special cases that occurred in learning-based conversations with respect to modality. The two cases are connected because the conversation went on as a learner-participant was thinking, and not necessarily speaking aloud or recording on paper the conversation. It is impossible to know if these conversations were word-based because the learner-participants do not provide a detailed image of the conversations. One special case is when learner-participants engaged in internal self-talk, as described in the previous section. Another special case is when there was a conversation between student and text. Elise, in our first interview, explained the process of shaping her summary sheet approach:

> I don’t really talk about it. I just kind of look, and I’m like, “Oh, well, that could work,” and then I kind of just had put it how it could work for me, because -- I just noticed Danielle’s just putting sticky notes. Like, I don’t know how she’s putting them on and, what kind of way she’s doing it. But I know she used sticky notes and just put them on sheets. So I was like, “Oh, well, that’s a good idea.” So I just did it my way.

Rather than talking with Danielle, Elise observed Danielle’s summary sheet and engaged in a conversational manner. While these two cases have an ephemeral quality, the impact of the internal conversations can be seen in the creating and/or inclusion of new processes of learning and in the way in which the learner-participants viewed themselves as mathematical learners.

**Pace**

The final feature of opportunities for learning-based conversations is the intensity of content and teaching. The *pace* feature refers to the rate at which learner-participants perceived content to be unfolding with in a course. Different from the ranges that exist within each of the above three features of opportunities for learning-based conversations, the provision of time to make choices about how to learn mathematics mattered to the learner-participants. In this section I will describe the milieu of the Learning Skills class contrasted with mathematics class, portray the learner-participants’ perspectives, and highlight students’ choice as what is opened up in a less intense class.

My observations of the Learning Skills class revealed a class with a relaxed and flexible milieu. As I had informal conversations with Mrs. Finley, I came to see that she fostered an environment where students did not feel pressured by a fast pace, but had time to engage in learning mathematics. The students in the Mathematical Learning Skills class were left to be independent in deciding what courses they would address and how they would go about learning the content they selected. In these ways, they had time to decide what to learn and they had time to figure out how to learn. While learner-participants acknowledged that they did not always use the class time in productive ways, as Robyn admitted, “sometimes I do nothing,” they also valued the opportunity to learn “how to do my homework and how to ask questions and how to kind of feed off each other” as Jocelyn explained. Ashley commented to me in our first interview that, “in [math] class it’s limited because you have to move on and it’s a very fast paced environment. Here [in Learning Skills class] you can sit down and slowly work though everything.” In this way, the learner-participants juxtaposed the relaxed environment of Learning Skills with the speed their mathematics class moved, both within a class as teachers rapidly explained and from class-to-class as there was a new topic each day. When Shane explains that “sometimes I just think about how I learn” during our first interview, it was in the context of having time that this thinking occurred. The slower rhythm of the Learning Skills class provided opportunities for students to learn, both mathematics and learning strategies.

Within the relaxed pace of the Mathematical Learning Skills class there were opportunities for conversations about learning. The small group sessions, while a research process, became a part of the learner-participants’ experiences of the course. Ashley identified “the [small] groups that we’re doing, it’s mostly concentrated there” for where conversations about learning to learn mathematics occurred for her. Near the end of the first Summary Sheets Small Group session, Chelsea pointed out that, “I feel like when I write a summary sheet, I actually can think about what I’m writing” because she was “doing it step-by step slowly.” The development of summary sheets, as a new learning strategy for Chelsea, occurred within the conversational context of the small groups. The complexity of the opportunity in the small group is highlighted because there was time for the development of a new learning strategy along with reflective conversation within a relational space.

The different intensity, explained by the learner-participants as a change in pace, provided opportunities for students to choose to engage in learning to learn mathematics through conversations. When I joined the class just over half way through the course, the learner-participants pointed to conversational opportunities in their first journal. Chelsea identified “ask the people around me … ask teachers about what they think is a productive way to study and understand math” and Danielle agreed that the course would be “a way for me to ask how to remember a concept or how I should look at certain topics” in relation to their goals of improving learning strategies. Because the learner-participants had already experienced the structure of the course, their choices for ways to improve their learning strategies by prompting learning-based conversation was situated within the pace of the Learning Skills course. When I offered, many of the learner-participants chose to accept invitations to engage in conversations about their mathematical learning.

## Drawing the Features Together

Considering the features of opportunities for learning-based conversations does not portray the complexity of the opportunities as there is a singularity of view in examining each feature individually. In order to understand the complexity of the learner-participants' opportunities to be in conversation with each other and their teachers about their learning processes, attention needs to be given to how the features interact. In developing connections across categories, Charmaz (2006) suggests that "diagrams can offer concrete images of our ideas. The advantage of diagrams is that they provide a visual representation of categories and their relationships" (p. 117). With the benefit of drawing together the descriptions of each of the features of learning-based conversational opportunities, the diagram below is meant as a provocation for further thinking rather than a focal point. In this section, I offer a diagram which draws together the four features for consideration.

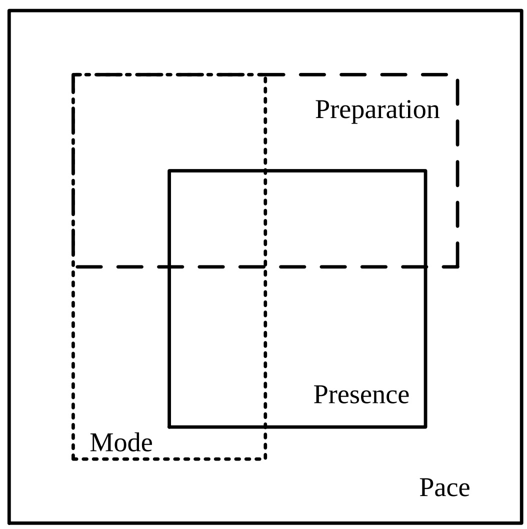



*Figure 4.1*. Diagram to draw together the four features of opportunities for learning-based conversations.

The diagram depicted in Figure 4.1 represents the four features of analysis and their interaction as it informs ways in which the learner-participants had opportunities to talk about their learning strategies and thereby refine their learning of high school mathematics. Preparation, presence, and mode are represented by the interior rectangles. Pace, as the fourth feature diagrammed as the exterior square, situates the other three features in a particular moment that is characterized by a relaxed intensity as described above. The four features in the diagram represent a space highlighting the complexity of learner-participants' opportunities to be in conversation about their learning.

In fact, each of the conversational moments within the inquiry could be placed within the space created by the rectangles. The placing of the conversational moments is what creates the space in which the learner-participants in this study were talking about their learning and shaping their learning strategies through conversation. While a diagram is somewhat limiting to represent complex interactions, the overlapping of features captures some of the complexity. For the learner-participants, the opportunities for many different kinds of conversations was important in their shaping of personal processes of learning mathematics.

# Chapter 5
# Developing Ways of Learning Academic Mathematics

The previous chapter contained my first interpretive moment in coming to understand the learner-participants' experiences of learning to learn mathematics. The learner-participants were engaging in learning-based conversations, and opportunities for those conversations could be understood through the four features of *preparation*, *presence*, *mode*, and *pace*. In particular, it is an interpretation of the context of the learner-participants' learning, that their learning can be understood as occurring as they engaged in conversations which were focused on the ways in which they were learning mathematics.

Beginning with learning-based conversations as a context in view, I zoom in – increasing in magnification – to attend to how the learner-participants were developing their own ways of learning mathematics within the conversational space. Rather than viewing these interpretive moments as separate phenomenon – as they were happening all at once for the learner-participants – it is only for the examination and explication of my interpretation that they are separated. This chapter provides a second interpretive moment with the data for the intention of understanding how the learner-participants were developing learning processes.

I return to my research question: *How can we understand students' learning as they actively develop their processes of learning mathematics?* As I re-consider my guiding question in light of the data and my understanding of the data, I have come to see that the learner-participants' engagement in developing mathematical learning processes speaks to the ways they were learning to learn mathematics. Explicating an understanding of the learner-participants' experiences is an interpretive undertaking, rather than an account of the data. In this way, the four forms of engagement I explain below are interpretive constructs that represent my meaning-making of students' processes of learning to learn mathematics. The resulting interpretive understanding illustrates the ways in which I noticed the learner-participants actively engaged in developing personal processes of learning mathematics.

The chapter begins with an exploration of tasks as those ways in which students were told to learn mathematics by their mathematics teachers. By contrast, the focus of the chapter is on four forms of engagement through which students were developing ways of learning academic mathematics, namely *becoming aware*, *incorporating suggestions*, *verbalizing possibilities*, and *(re)forming intentions for learning*. To highlight the growth of the learner-participants and their ways of learning, the chapter ends with descriptions of ways of learning mathematics that the learner-participants developed.

## Tasks for Students from Mathematics Class

Tasks for students were ways in which students were told by their mathematics teachers to work on mathematics either in class or independently.

Such tasks are commonly assigned by high school mathematics teachers across many school sites as observed within the *Trajectories of Students Learning Mathematics* research project (Mason & McFeetors, 2007). Often the students' implementation of tasks were assumed in the routines of school. For example, students indicated they are expected to copy the teacher's notes, presumed to do homework, and assumed to prepare for unit tests. When tasks were assigned, they were perceived as labels that did not probe the steps students would need to take to enact the tasks. Students were left on their own to carry out tasks for which they had no basis of action or had no opportunity for conversation about the tasks.

In the learner-participants' first interactive writing, they listed ways they were told to learn mathematics, such as study, review, copy notes, work with others, and do homework. Grace explained that her teacher "just hands out the homework" at the end of each mathematics class. Questions like the one Shane posed in his interactive writing, "Is there a way to do homework well?" demonstrate his lack of understanding of the processes the task entailed. In this section, I explore the term "task" to justify its use. I then describe two characteristics of tasks for students in order to portray the kinds of engagement learner-participants had with mathematical work before they shaped their own processes of learning mathematics.

**Exploring "Task" as an Appropriate Naming**

How-to books on ways of learning mathematics privilege the use of "skills" (e.g., Nolting, 2002; Ooten & Moore, 2010) as specific steps in ways to learn content. Nisbet and Shucksmith (1986) demonstrate the limits of skills in learning to learn, and instead advocate for strategies to learn as "the processes that underlie performance on thinking tasks" (p. 24). During the research study, the learner-participants tended to use the word "strategy" to point to the work they were doing for mathematics class. In the first interview, each of the learner-participants explored with me the labels of "learning strategies" and "learning processes." There was little consensus on what the phrases might point to in relation to their experiences in the Learning Skills class. Later, as I interpreted the learner-participants' experiences, I found the word "task" to be the strongest match in naming what students were required to do for mathematics class. Landers (2013), too, uses the term "task" in this way. He notes that mathematics homework assigned to students which does not necessarily hold the intention of meaning-making is a task.

The term "mathematical task" is frequently used in the mathematics education literature as a question posed to students for the purpose of focusing their learning on a particular mathematical idea (e.g., Boston & Smith, 2009; Doerr, 2006; Henningsen & Stein, 1997). (See Watson & Mason (2007) for an extended exploration of mathematical tasks.) Liljedahl, Chernoff, and Zazkis (2007) explore the notion of "'good' mathematical tasks" that support understanding of mathematical ideas, while the *Principles and Standards* document (NCTM, 2000) qualify such mathematical tasks as "worthwhile" (p. 19). In both cases, a task is viewed not as necessarily inherently valuable but the activity in and around the task is what provides opportunities for rich learning.

Christiansen and Walther (1986) provide an extended discussion about the distinction between task and activity. They see a task as “the assignment set by the teacher [which] becomes the object for the student’s activity” (p. 260). The authority of the teacher to designate specific assignments often results in an “overemphasis on the products to the detriment of the process in mathematics learning” (p. 246). In much the same way, students are often assigned tasks such as doing homework or copying notes by a teacher in order to be productive. Tasks, without moving to student activity, “is predominantly limited to drill and practice in relation to previously described concepts and procedures” (p. 245). These mathematics education researchers recognized the impact of the nature of tasks on students’ learning. They used “task” to point to what teachers assign for students to do, and if remaining at a general level of implementation leads to instrumental learning of mathematical ideas. In this way, the use of “task” aligns with naming what students are often told to do to succeed in mathematics.

Etymologically, the word “task” points to “the work appointed or assigned to one as a definite duty” (“Oxford English Dictionary Online”, 2013). The assignation quality of task, then, is seen both in its use in mathematics education and in an etymological tracing. In fact, the work-based metaphor was prevalent in the learner-participants’ talk about what they needed to do in order to succeed in mathematics class. For example, Nadia referred to work multiple times in our first interview in explaining that the *work*book “is where we do all our work in” inclusive of notes and home*work* practice, and that she was aware that she needed “to be more organized and to get the work done.” Teresa saw learning mathematics as a matter of needing to “put a lot of work and effort into it” and that learning occurred by “listening and having to go through the work.” Jocelyn went further in de-emphasizing learning by discussing how much time she would need to study for the final examination, which I coded as “doin’ time.” The focus was on putting in time rather than on the qualities of studying or learning.

Furthermore, learner-participants reported both what they needed to “do” and what they had completed. For Robyn, the goal was to “do math” in order to check it off a list. The reference of this generic “do” is reminiscent of a “to do” list which draws attention to completion and the resulting product, rather than the processes used to engage in learning mathematics. The product they perceived was a list of homework questions that was well-defined and bounded. Mrs. Finley drew together these two ideas – doing and time – as dominant discourse tools to mark the tasks students were required to complete. She contrasted time management and doing homework as “getting to the learning in the first place” with “picking key questions … writing in the margin about why” as ways to *learn* mathematics. Mrs. Finley recognized that students would need to move beyond these tasks to engage in meaningful mathematical learning.

## Characteristics of Tasks for Students

Tasks for students were given labels such as “homework” or “time management.” The uniformity in naming tasks arose from the systemically defined characteristic of tasks, that school as a normative structure has systematized these procedures without consideration of any particular student.

Mathematics teachers, as actors in the system, compel students to use the tasks through their authoritative stance, resulting in externally imposed ways to learn content. Mathematics students respond by *studenting*, a term proposed by Mason & McFeetors (2007) to distinguish behaviours that are "intending primarily to meet institutional expectations rather than to learn" (p. 304). Although the learner-participants desired to succeed in mathematics class and acknowledged that these tasks should support that success, they struggled to implement them to any effect. Ashley illustrates this in her first interactive writing when she wrote,

> One thing that I have been trying to improve this year to help me succeed at learning math is how do I study for math in the most effective way possible that will give me the best results. Even though I do practicing math and I work very hard lately it seems as if that is not enough, and the feedback I am receiving from my math teacher is maybe the way I am preparing for an assessment is incorrect.

In the entry, Ashley identifies working at improving her studying and also practicing. However, she also identifies these common tasks as ineffective in supporting her learning and a lack of further direction for her efforts.

The unquestioning implementation of tasks demonstrated the students' lack of intentions and personal investment in procedures espoused by school. Students perceived the tasks as being static and superficial. Teacher demands, especially for notes and homework, led students to use the task as prescribed even if they did not see it as supporting their learning. The students left the task unchanged and usually did not attempt to modify it, which demonstrated that the task was, for them, a static object. The task was perceived as static both in its implementation and in its lack of contribution to learning mathematics for understanding. The students' use of tasks contributed to a superficial form of learning (memorization of mathematical procedures), glossed over the challenges of learning, and viewed one way of learning as equally effective for all students.

Furthermore, the students were also positioned as static; static persons whose learning was also superficial through the implementation of tasks. Through the use of essentialized tasks, students did not have opportunities to be changed by deep engagement in a learning process and mathematics. They saw themselves as ineffective learners and prioritized the surface goal of high marks. In commenting on a middle school student in their study, Mcintyre Latta and Olafson (2006) describe how "the assigned tasks position her as one of the many nameless, faceless, voiceless students in the classroom" (p.86). In the same way, the students were positioned as generic persons in light of tasks, learners whose particular identities and personal ways of learning were not respected. Against this backdrop, the learner-participants and I began to explore the ways they were attempting to learn academic mathematics.

The positioning of the students with tasks brings forward the power differential in the relationship between students and teachers. The task is assigned by a teacher and it is up to the student whether to do it or not. Inherent in my use of the label "task" is how it is taken up by students. When I use the term "task" going forward, it is in recognition that such assignments are taken up in an instrumental manner. The student's relationship to his or her mathematics teacher is mediated by a task in such a way that the student completes the task as an act of submission within an authority structure (school and teachers), perhaps at best in a manner that trusts the teacher with the student's best interests.

The above description of students' tasks, while highlighting their static and superficial characteristics, is offered up as a portrayal of the work many students in the study were doing with the purpose of succeeding in mathematics class. The implementation of tasks by students does not preclude learning, nor is the description meant to exclude the possibility of learning through and with tasks. Learning, such as remembering through reinforcement of instruction, could have been taking place through tasks such as doing homework or studying. Learning, such as developing familiarity with mathematical terminology and symbols, could have been taking place through tasks such as copying notes from the board or working with peers. Learning, such as recall, could have been taking place through a task like reviewing. The learner-participants were performing to varying degrees of success on unit tests through their implementation of tasks.

Not only was some form of learning possible through the implementation of tasks, these were the experiences of the learner-participants in their mathematics class context. Interpreting this from the notion of the continuity of experience (Dewey, 1938/1997) acknowledges that the learner-participants' use of tasks was a place where they were building from for the activities and experiences within the research study. Learner-participants were familiar with how their mathematics classes as communities – both teachers and peers – used tasks to do mathematics. Bateson (1994) suggests that in communities "participation precedes learning" (p. 41). The learner-participants' participation in the tasks of mathematics class were important developmentally in the shaping of their own processes of learning mathematics and provided a starting point in the various conversational opportunities described in the previous chapter.

## Developing Approaches to Learning Mathematics

While many of the ways the learner-participants were doing work for mathematics class were characterized as tasks, some of the learner-participants had already been developing alternative, personal ways of learning mathematics. I will refer to these ways of learning mathematics as *learning strategies* in order to differentiate them from tasks. The phrase *learning strategies* also preserves the manner in which the learner-participants referred to their actions, maintaining their own naming of the ways they were going about learning mathematics.

My interpretation of the learner-participants' inquiry into their tasks and learning strategies resulted in four forms of engagement for developing mathematical learning processes. The four forms of engagement include: *becoming aware*, *incorporating suggestions*, *verbalizing possibilities*, and *(re)forming intentions*. These forms of engagement are constructed from and grounded in the learner-participants' data, and represent ways in which they shaped their learning of mathematics. Figure 5.1 represents the four forms of learner-participants' engagement in developing approaches to learning mathematics.

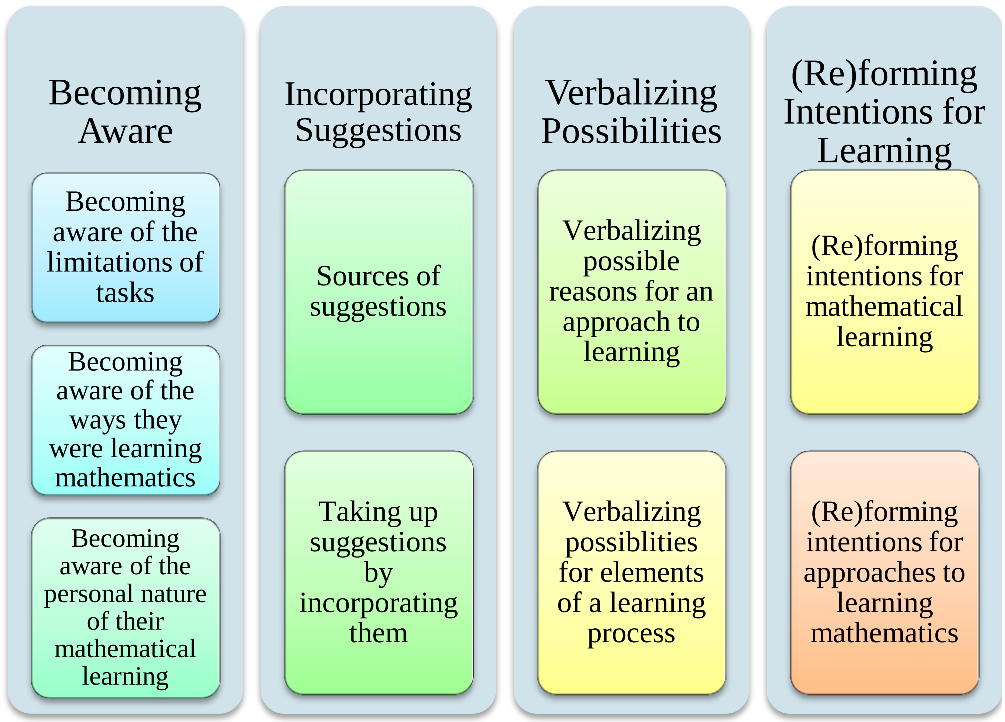



*Figure 5.1*. Four forms of engagement in developing processes for learning mathematics.

Taken together, the four forms of engagement provide an emerging picture of how the learner-participants engaged in inquiring into tasks and learning strategies as a process in which the learner-participants were active and interactive. Their activity was marked by a kind of tentativeness and uncertainty, both in wondering about and experimenting with various approaches. Their interactions highlighted the interpersonal dimension of learning with others. Being enacted simultaneously and in an integrated fashion, the forms of engagement represent ways in which learner-participants shaped how they were learning mathematics. Experiences within the Mathematical Learning Skills class and the procedures for data construction in the research project provided opportunities for the learner-participants to shape their ways of learning. In the following sections, I provide an explanation of each of the forms of engagement, use specific examples from learner-participants' statements whose data exemplify the engagement, and weave in extant literature to locate the work conceptually.

**Becoming Aware**

As learner-participants developed ways of learning mathematics they were simultaneously *becoming aware* that they engaged in particular and sometimes multiple ways of learning mathematics. Bateson (1994) points to the possibility that "there is even room for awareness of the process of learning" (p. 96) as she explores the notion of attending. My interactions with the learner-participants in the study demonstrate that it was possible for them to attend to the ways in which they were learning mathematics. As I invited leaner-participants to notice how they were learning mathematics through our conversations and work together, they were becoming aware of multiples ways of learning. Grace's exclamation, "Wow, that's a lot! … Oh, I thought I only had two or three ways to learn math, kind of thing. Just never really think about it" initiated my conceptualization of this form of engagement. In some cases, they were able to point with words to their growing awareness of their ways of learning. As I interpreted the experiences of the learner-participants, their engagement in the process of learning how to learn mathematics demonstrated that growth in awareness was important to shaping their learning.

After conceptualizing *becoming aware* as a form of learner-participants' engagement in developing learning processes, I sought out extant literature. Gattegno (1987) claims that "only awareness is educable" (p. 106). While I do not wish to debate the essentializing of learning in his claim, what he offers is the notion that attending to, rather than memorizing, opens up possibilities for students of learning in rich and meaningful ways.

Two particular works informed my thinking. Polanyi (1964/1969) delineates "two kinds of awarenesses—the subsidiary and the focal" (p. 140). Focal awareness refers to a state of noticing things (e.g., objects, ideas, emotions) that are at the centre. Subsidiary awareness refers to a state of the implicit noticing of things that are at the edge, to acknowledge a presence of the thing but not directly attend to it. Polanyi evoked a visual metaphor of space where a phenomenon could be drawn from peripheral view (subsidiary awareness) into focus.

Marton and Booth (1997) locate their development of awareness within a learning-based context. Their notion of multiple forms of awareness is finely nuanced:

> Our awareness has structure to it. At any instant certain things are to the fore—they are figural or thematized—whereas other things have receded to the ground—they are tacit or unthematized. But to stress the dichotomous nature of awareness—figure-ground, thematized-unthematized, explicit-implicit—would be to oversimplify. There are different degrees of how figural, thematized, or explicit things or aspects are in our awareness. … Gurwitsch (1964) made a distinction between the object of focal awareness, the *theme*, and those aspects of the experienced world that are related to the object in which it is embedded, the *thematic field*. … All that which is coexistent with the theme without being related to it by dint of the content or meaning, Gurwitsch called the *margin*. (p. 98)

The parallel of focal/thematized and subsidiary/marginal awareness is apparent, and yet the thematic field and the notion of the differing intensities of awareness provide a rich explication of awareness within learning. For the learner-participants, this meant that while learning processes might have been drawn into thematized awareness, the importance of the mathematical content still had a high degree of intensity within the thematic field.

The awareness that I take up in the remainder of this section is of the thematized or focal nature. And, rather than awareness as a static state, the learner-participants' form of engagement was in the *becoming* aware. The learner-participants' *becoming aware* was a growth in their awareness of the ways in which they were or could learn mathematics. Becoming aware occurred slowly, over time, and not in a singular moment. This dynamic nature is stressed in the use of the gerund – becoming – rather than in the static past tense – became – as related to the learner-participants' rising awareness. The learner-participants were becoming aware of the limitations of the tasks they were undertaking to learn mathematics, of the ways in which they were learning mathematics, and of the personal nature of their mathematical learning.

**Becoming Aware of the Limitations of Tasks**. Learner-participants were *becoming aware* of the limitations of their current approaches to learning mathematics, both the tasks they were given by mathematics teachers and some of their own learning strategies. Awareness of limitations provided the opportunity for learner-participants to identify a specific starting place to improve their approach to learning mathematics and to cast their gaze forward in imagining how to shape a learning process. As such, the tasks they were using were drawn from marginal awareness into a high degree in the thematic field. The context of being dissatisfied with a current approach opened up space for the learner-participants and me to be in conversation and actively work to further develop ways of learning mathematics.

Learner-participants were becoming aware of the limitations of tasks they were given by mathematics teachers. In addition to being told to copy out notes in class, students were often exhorted to read through their notes as a way to review new content daily. Teresa described this task as "a learning process 'cause it's already written … you've kind of went through it. Now it's just learning it and getting it." In her third interactive writing, Teresa explained, "It's one thing to understand by reading notes, but another to actually do the work. Also by doing homework, I can understand it and do it without looking at my notes, meaning I truly learned the material." She demonstrates a growing awareness that a passive approach to reading notes was not sufficient for her learning. Vanessa, in our first small group session clarifies the passive approach by explaining, "when I'm reading, I get bored and it's like, 'Oh, okay' and then I drift off." Vanessa illustrates not only an awareness that the task of reading notes was limiting, but also a nascent awareness of why the task was limiting for her.

Teresa was also becoming aware that the limitation of reading through notes stemmed from the insufficiency of the explanations in class notes. She mentioned, "if you look at notes and it doesn't really tell you, you can't really figure it out." Teresa went on to demonstrate the inadequacy of reading notes as a task when she identified a section within the notes by stating, "I don't really know what it's saying. So, I don't know if it's really important" as she skimmed her notes looking for ideas to record on her transition page. The growing awareness that reading through notes was inadequate in beginning a homework assignment provided an opening for trying a new approach to transitioning from copying notes to doing homework. The transition page was an opportunity for Teresa and Vanessa to record short explanations that were not in their notes as they made sense of the mathematical ideas. After our first attempt at designing a transition page, Teresa noted that "it made this chapter look a lot easier" and Vanessa added, "It's less intimidating, when you see your own writing." Against the backdrop of becoming aware of the limitation of reading notes, Teresa and Vanessa began to develop an alternate way of learning mathematics.

Learner-participants were also becoming aware of the limitations of some of their personal learning strategies. Kylee, who had previously created a system of cue cards for learning terms in biology, noticed their limitation for mathematics. She explained, "I realize how much of my time I waste making Q-cards [*sic*] before my test when I could instead be studying them." Through our conversations in class and through interactive writing, Kylee found an opening to refine her existing learning strategy of reviewing through the use of cue cards. The opening was created as Kylee was *becoming aware* that her learning strategy was not supporting her mathematical learning in the way she wanted.

The awareness of limitations of tasks and strategies drew learning into the thematic field of awareness as learner-participants noticed elements where the tasks and strategies could be enhanced to support their mathematical learning.

**Becoming Aware of the Ways They were Learning Mathematics**. Many of the tasks assigned by mathematics teachers were systematized to the point that the learner-participants were only marginally aware of the ways they were learning mathematics. Through conversational opportunities, learner-participants were becoming more aware of the ways in which they were learning mathematics. Near the beginning of the first interview with the learner-participants, I asked them to show me what they had been recently learning in mathematics class. As they each pointed to specific mathematics content, I followed up with questions like, "So, how did you learn that?" The prompt sponsored the learner-participants to shift their attention from the specific mathematics topic to the approaches to learning. The mathematical content was still within the thematic field as it remained important to the conversation, but it was no longer what was of central concern to our attending. The learner-participants pointed to the notes they copied out or the homework questions which they had been assigned.

There was something more interesting occurring in the shift in attention that brought into view the numerous ways students were going about learning mathematics. The learner-participants noticed that they were active in learning rather than simply perceiving the learning happening to them through the mathematics teachers' lectures in class. Nadia demonstrates this transition, when she explains that:

> listening in class and really focusing on what is being taught I would consider 50% of learning, then when you get home, the other 50% is in the

> time you put into doing your homework. I feel that these two ways of learning must be balanced.

When we discussed this interactive writing in the first interview, Nadia elaborated on her active stance of engaging in the learning:

> Nadia: I think engaging just means that you're actually, you're actually using your brain to kind of figure, figure things out that you didn't know before. And that you're in, that you're trying to get the answer and you're trying to understand it. … I think it's a lot of, patterns, kind of. And you just have to know where to start. 'Cause you can have like a, when you go into a test and you don't study, you can kind of see like, what you roughly have to do. But in order to actually do it fully and know each step, you have to--, you have to know the process from beginning to end. Yeah.
>
> Janelle: Can you say more about that idea of pattern?
>
> Nadia: Umm, patterns. Well, I think in like a, in one unit, let's say this one. They all kind of follow the same basic procedure. So once you find that pattern, it's easier to apply it to different questions.
>
> Janelle: Yeah. Is your teacher telling you that pattern or is that something that is part of the figuring out in homework?
>
> Nadia: I think that's part of the figuring out in homework. I mean, they can show you in the easier question, compared to a harder question, that they both kind of follow the same principle. But, when you're doing your homework, it's--, you see it, for yourself.
>
> Janelle: Yeah. Is that important for you to see it for yourself?
>
> Nadia: Yeah, for sure. 'Cause then I can use it in different situations on tests and further, stuff like that.

Whereas previous mathematics homework had not figured largely in Nadia's approach to learning mathematics, she had a growing awareness that her active stance of "figuring out" while completing homework questions contributed to her understanding of mathematical ideas and procedures. Nadia noticed that she was active in learning mathematics.

Homework figured prominently in the learner-participants' talk about learning in mathematics class, often viewed as a task to be completed. However, two learner-participants begin to offer a different perspective on homework in relation to learning mathematics. For both, their aim was to understand mathematical ideas and procedures. Laurel and Jocelyn, within our conversations, were able to point to homework as the way in which they were learning mathematics that made a difference to their progress in mathematics class. Consider Jocelyn's interactive writing:

> Balancing homework and asking questions has really helped me feel more confident in math. I feel like Learning Skills has really helped my math skills and study skills in general. The hardest part for me about studying is finding enough time to do homework and study so I think now I am going to work more on time management skills.

> *Jocelyn,*
>
> *You've helped me see a new perspective on improving time management. Often, students tell me they have to manage time first before improving math or study skills. I really appreciate your experience of making big improvements and that compels you to make sure you can squeeze it all in. I'm looking forward to hearing about moments when you realized your study skills improved this year.* *Janelle*

Jocelyn's statement about homework and the relation to time management does not seem to be a striking example on the surface. However, as I point out in my reply, Jocelyn's idea of first being aware that homework was making a difference to her and because of that awareness moving toward planning to ensure enough time to learn from homework stands in stark contrast to most of her peers. Jocelyn's peers would often spend lots of energy in managing their time, with little energy left to invest in learning from homework. In my observations during the Learning Skills class, Jocelyn was one of only a small handful of students who consistently used the class to learn from her homework as she worked with peers, sought help from teachers, and completed the assignments.

Like Jocelyn, Laurel renewed her commitment to homework for her grade 12 mathematics course. Laurel explained, "So then once I start working through it and there's examples, it makes sense to me, and I see the pattern and it -- then it starts making sense." Laurel was aware that homework was enabling her to make sense of the mathematics. At the same time, I observed and talked with many students in Mathematical Learning Skills class who were not finding that homework was making as big a difference for them as it was for Laurel. When I wondered aloud about this, Laurel responded by saying, "Yeah. Maybe they -- well, I don't know. Maybe they're not thinking while they're doing it. They're just kind of doing it." The challenge Laurel had in explaining the difference between thinking during homework and simply doing homework highlights the dynamic quality of and tentativeness of *becoming*, in that her awareness of being aware of her effective processes with homework was nascent.

*Becoming* is always in process, and indeterminacy within the growth of awareness can be expected. I am reminded of Bateson's (1994) writing:

> The lessons of school gain authority because they are layered onto earlier informal learning in the home, which is where we learn how and what to learn and how to transfer knowledge from one situation to another. These vital skills mostly remain outside of awareness (p. 42).

While Bateson's context and focus of informal learning are different from the context and focus of Laurel's learning to learn, what I see Laurel experiencing through the study are the earlier layers in learning to learn mathematics. Just as Laurel was first aware of her sense-making while doing homework, she was in the process of *becoming aware*. The notion of sense-making as a process was beginning to be drawn into her noticing so that it had the possibility of becoming part of her thematized awareness.

**Becoming Aware of the Personal Nature of Their Mathematical Learning**. Learner-participants were also *becoming aware* that their approaches to learning mathematics could fit with who they saw themselves as learners. Some learner-participants already had a sense of self as a particular learner and as they engaged in shaping learning processes, they began to notice how the learning process responded to them personally, as learners.

Shane was beginning to notice that a new way of learning mathematics fit with his view of himself as a learner. Shane saw himself as a conceptual learner. In informal conversations in Learning Skills class, Shane explained to me the various ways that he excelled academically in courses like English and Social Studies and yet continued to struggle in Pure Mathematics. Shane's second interactive writing points to a conceptual approach to learning in courses he succeeded in when he explained, "as for study methods for English and Social, when studying them I tend to review by reading the notes over and relating them to things I already know." Hiebert and Lefevre (1986) describes conceptual knowledge in mathematics in a similar way, where it "is characterized most clearly as knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information" (pp. 4-5). Shane continued to identify making content-based connections in the fourth interactive writing when he writes in response to the prompt "What was the exponents and logarithms unit about?":

> I've really never thought about what it "means". Perhaps if I did it would be easier to learn, kind of like knowing what the math I'm learning is in the grand scheme of things. Like, in biology, I've taken a huge interest in the brain since gaming (video gaming) is basicly [*sic*] neuroscience.

Shane juxtaposes his conceptually-based learning in biology by making connections to his personal interests with a lack of opportunity to view mathematics conceptually. The prompt asked what the content was *about* in the unit, yet Shane responds with what the content *means* and how the content might be seen as connected. Through Shane's opening two statements, the notion that mathematics content has the possibility of being viewed conceptually seems to be an idea he is noticing and considering for one of the first times. Perhaps this is one of the early moments in Shane's *becoming aware* of mathematics as a conceptual discipline.

Throughout the study, Shane and I returned often to the idea of conceptual learning. In the first interview, Shane directed the conversation toward the development of summary sheets. Summary sheets was an addition to Shane's approaches to learning mathematics.

> Janelle: How do you think that making the summary sheets is helping you learn the math?
>
> Shane: Helping me learn the math? I remember I was thinking about it to myself. It makes the math more than just numbers, 'cause I'm a very conceptual kind of person. … it becomes more like English or biology when it's concepts rather than just numbers.
>
> …

Janelle: Can you think of a certain concept in the transformations unit, now that you've been working on that a bit, that's kind of stuck out for you or as an example of getting at the concept?

Shane: Getting at the concept. Um, what was a good one? Hmm. When I was looking at the -- the stretches, I think. Well, when we were doing it in class, a stretch to me was just editing the coefficient of one of the variables so it changes the shape of the graph. But when you write it as a concept, you get to look at the formula and -- like, I played around with my calculator a bit. … Like, they say in the book you should, just to see how they work. Yeah. Mm-hmm.

Janelle: Neat. So you're using – that's in the textbook, then --

Shane: Yeah.

Janelle: -- that you're going back to and using as a resource.

Shane: Yeah. Like, I'm -- they -- they say in there the -- yeah. I think if you have a concept that you teach, I think you should be allowed to play around with it so you get what it does. Like, if you change this number, this happened. If you change this number, this happens.

Shane raised the idea of focusing on learning conceptually in direct relation to his new approach to learning mathematics by creating summary sheets. He also provided a specific mathematical example to support his understanding of learning mathematics conceptually. Shane's subsequent insight, "I would focus on learning how these numbers work and now I guess how the numbers work is a concept in itself, but I never thought of it that way," is more explicit in signaling a new awareness that his mathematical learning could fit his identity as a conceptual learner. In the second interview, Shane mentioned that, "I guess now the more I think about it, it was pretty obvious that I was a conceptual learner (laughter), because I like those big concepts." In bringing our conversation back to conceptually-based learning, Shane demonstrates that his growing awareness of how to learn mathematics conceptually fit well with how he saw himself as a learner and a thinker.

Through attending to the learner-participants' data and reading related literature, I was also becoming aware of learning I was engaged in as a researcher. Awareness was not one of the sensitizing concepts in chapter 2. However, I see it as an extension of the sensitizing concept of voice. The growth in the learner-participants' awareness was situated in a space where their voices could be heard and valued, even in its tentative state. Awareness is also related to the learner-participants' relationship with sources of knowledge. In the interactions through the study, the learner-participants were respected as sources of self-knowledge about themselves as learners and knowers. Within this context, opportunities arose for them to attend to their experiences of learning mathematics and to become aware of approaches that supported their mathematical learning. Becoming aware, and the different forms of awareness developed in extant literature, is a new way of viewing students' experiences of learning mathematics for me, and I anticipate that I will incorporate awareness as an additional sensitizing concept as I continue to engage in research around students' mathematical learning beyond this study.

## Incorporating Suggestions

As the learner-participants shaped ways of learning during the study, they were *incorporating suggestions* from their peers and from me. The movement from tasks and strategies toward learning processes necessitated an active stance of the learner-participants. As learner-participants incorporated suggestions they engaged by acting on the suggestion. The idea of "acting on" is useful in two of its senses. The learner-participants were acting on the recommendations of another person, trying out the approach to learning mathematics. They were also acting on the approach suggested – it was not viewed as a static prescription, like tasks, but something that called on the learner-participant to shape it as he or she used it. In order to explicate the difference between a teacher telling a task and a teacher or peer suggesting a way to improve a learning strategy, I explore the notion of suggestions and from where the suggestion originated. Then, I explain what it meant for the learner-participants to be engaged in incorporating, more so than trying or implementing, a suggestion.

**Sources of Suggestions**. Looking outside of the learner-participants themselves, there were at least two sources of suggestions for developing meaningful ways of learning mathematics. The first source was me, acting in the role of a mathematics teacher and researcher. In contrast to telling, as I described above in the assigning of tasks, I viewed my offers of ideas as suggestions. More importantly, the learner-participants also perceived what I offered as suggestions. Kylee, in our second interview, described my interactions with her as "you weren't telling me to do something or getting mad because I did that on a math test. You were just encouraging" as I offered alternatives and worked from learner-participants' current capabilities in learning mathematics.

Several features of suggestions shift learner-participants' perception of them from prescribed instructions to suggestions from a teacher. The first two interactive writings between Vanessa and I exemplify these suggestions. At the beginning of the project, Vanessa was struggling to implement the tasks prescribed by her teacher. As I interacted with her, my intent was to sponsor a shift from tasks toward approaches that would support mathematical learning. Consider the interactive writing entries:

Journal #1: Vanessa

1. I tried reading my math notes when I get home. But I find that when I try to do the homework and understand the notes, I've already forgotten how and what to do by the end of the day. To be honest, I sort of gave up on reading my notes because I never know how to actually start off on some problems, so I just give up and work on other subjects. I also tried getting help from my friends but sometimes I have a hard time focusing. I try to listen in class to improve, but I need to get easier worksheets and problems first, then get progressively harder ones. Also, when I listen in class, sometimes I cannot comprehend what the teacher is

saying. But overall, I've been trying to read my notes, got help from friends, and listened in class to improve in math.

2. I want to learn how to approach problems without freaking out, and know how to eliminate wrong answers on multiple choice questions. I also want to learn how to make notes that are organized and easy to comprehend and read. I have to work harder in class, listen more, get better notes from my math teacher and ask teachers for help more than my friends.

*Vanessa,*

*It sounds like you have been trying a lot of different strategies to succeed at learning math this year. It's a good idea to try multiple approaches to see which ones help the most. And with trying them out, you have also been able to identify places you would like to improve. I noticed taking notes in class came up a couple of times, along with being able to listen. I wonder if these are connected with not freaking out about starting to work on questions. Are you able to get all the notes from class? If so, I have a suggestion for you to consider: before starting a homework assignment, read through your notes and write down the important* <u>*ideas*</u> *and* <u>*formulas*</u> *from the lesson. It might help you figure out how to start questions and is a quick reference sheet. Let's think together this semester more about math notes.* *Janelle*

Journal #2: Vanessa

Yes, I do find I freak out when approaching questions. That's a great idea writing the important ideas and formulas down first. Sometimes, I am lazy that's why I am not able to get around that. This simester [*sic*] I am going to try to be more motivated and not give up so easily. I am going to try and boost my marks up and at least try to get them at a 70% especially in bio and chemistry. I don't want to accept the fact that I have to upgrade, because I don't want to give up write [*sic*] now. So, I'm just going to have to try my best and find some motivation to do better, with a positive attitude.

*Vanessa,*

*I wonder if writing down the important ideas and formulas will give you a chance to take a deep breath before trying a homework assignment (whether in math, bio, or chemistry). Perhaps this small change in approach to homework – and something new to try – will be more motivation? And when a question seems to stump you, you'll know you have a reference sheet to go back to in order to help you out. Over the next few weeks, I'm looking forward to hearing how this new strategy helps you be more successful.* *Janelle*

The suggestion I provided began to take shape as I listened to what Vanessa was telling me in the first entry. I used an interpretive stance to come to understand Vanessa's experiences in trying to learn mathematics. I related my interpretive listening in my reply by affirming the multiplicity of strategies, by highlighting a common theme of using notes, and by making use of some of Vanessa's words.

Unlike Elise who indicated that copying notes and listening in class simultaneously was difficult (see next section below), I understood Vanessa's difficulties to be located in what to do with the notes – not understanding when reading her notes later in the day and not being able to use the notes to begin a homework assignment. Suggestions begin, not with the offering of them, but with listening.

Building on interpretive listening, suggestions work from a student's current capabilities. By this I mean that I moved back to what I noticed a learner-participant could do and shaped my suggestion to move forward in the process of learning mathematics. Rather than fixing what might appear to be deficiency, working from a student's current capabilities aims to build on what is effective. For Vanessa, working from her current capabilities meant cutting through what Vanessa said she could not do and what was not working to interpreting that she was copying out the mathematics teacher's notes in class as a starting place. Learning can be seen as "an aim, and points to the future, in distinct contrast with causal explanations that point to events in the past" (Marton & Booth, 1997, p. 51). With the same intent for learning, my suggestion began with the notes Vanessa was taking in class to address seeing forward in the learning to process the notes so that she could both begin to understand them and begin her homework. The suggestion can also be identified as a small modification to existing tasks or learning strategies, rather than a complete change. Working from a student's current capability means offering suggestions that make incremental changes where students can imagine the possibility.

The manner in which a suggestion is offered is another feature of a suggestion. Rather than telling a learner-participant to use a certain approach, a suggestion is offered by wondering or inquiring about an approach. Notice the tentativeness of the phrases, "I wonder …", "a suggestion for you to consider", and "it might help …" in my reply to the first interactive writing and "I wonder if …" in my reply to the second interactive writing. Both of my replies end with an invitation for a conversation about the suggestion. A sense of wondering invites a student to respond, to try, to imagine. In other words, it opens up space for a learner-participant to imagine possibilities or even simply the possibility of the suggestion offered. While Vanessa did not necessarily move to verbalizing the possibility of writing down the important ideas or formulas from a lesson's notes, she did engage in reciprocity through her receptivity demonstrated when she responded by writing, "that's a great idea." I formed the Transition Small Group – of which Vanessa, Teresa, and Robyn were members – out of the suggestion I offered to Vanessa and her positive response.

The second source of suggestions was peers. The small group sessions provided many opportunities for me to attend to the ways in which the learner-participants offered suggestions to each other. In particular, the peer interactions of the Summary Sheets Small Group provide illuminating examples around Ashley's first use of summary sheets. In the first session, Danielle shared her approach to summary sheets with Ashley by explaining

Danielle: These are just all the laws [of logarithms], right? And then sometimes, if I'm doing a question on another page then I forget

I'll just look at it or I can take it off and put it beside the question so I know exactly what--

Ashley: Oh, that's smart!

Danielle: You know. And then--

Ashley: Oh, okay. I see.

Danielle: Umm, down here, is just examples, right? And then sometimes I have little notes over here. Like, what kind of questions are they about? Like, why is it a different kind of question? Or, whatever.

Danielle's suggestion was offered as a sharing of her experience of the learning process. In a study of post-secondary students' learning, Baxter Magolda (1992) noticed that a contextual knower "expected to learn from her classmates … Her peers' knowledge came from their experiences" (p. 176). The learner-participants were interacting in similar ways to contextual knowers as they were open to learning from each other's experiences. More assertive than the stance of tentative wondering I often used, describing a personal experience was a powerful yet non-prescriptive suggestion.

The learner-participants were also active in eliciting suggestions from their peers, rather than relying on themselves or a teacher as a source of knowledge about learning processes. In the second Summary Sheets Small Group session, the learner-participants were beginning to construct their summary sheet for the Transformations of Functions unit. As they began to decide what content to include, Ashley inquired into Danielle's sticky note organization:

Ashley: So what are, Danielle, what are you writing down on your sticky notes, the little ones?

Danielle: Umm, well, right now, it's just little-- what's it called? Remember how I did in the other one, where I like, put little things that I would need to remember throughout the chapter?

Ashley: Yeah.

Danielle: So, that's kind of what I'm doing now.

Ashley: That's what you're doing right now? Okay

Danielle: Yeah

Ashley: Oh, okay. I was and then you, and then you go from section to section after that?

Danielle: Mm, hmm

Ashley: Oh, okay!

Danielle: Well, I mean, this is section, this is the first lesson, right?

Ashley: Thank you

Similar to the first session, Danielle's suggestions were a relating of her experience in creating the summary sheet. However, Ashley prompted and continued to ask clarifying questions to elicit Danielle's steps to consider as a suggestion as Ashley made decisions about how to start her summary sheet. Later, in our first interview, Ashley explained the importance for her of "sitting down with other people and talking to them and asking them, you know, how are you doing this." Being able to ask her peers for suggestions was part of Ashley's development of new ways of learning mathematics.

Chickering and Reisser (1993) identify "finding the courage to ask for needed assistance" (p. 141) as a characteristic of being interdependent. In the above example, Ashley had elicited ideas for summary sheets from Danielle, especially because this was a new approach to learning for Ashley. As the Summary Sheets Small Group continued to meet, there was an emerging stance of interdependence among the learner-participants as they offered suggestions for the approach among themselves. In the third session when we were discussing the role of specific examples, Ashley explained that she placed

> the question on the sticky note and then all of the solutions and the work to that question underneath it. Because this is, if I look at this and if I say, like, if I have some notes here, and then I have an example, then I'll do the example and then if I really have to look, I'll flip the sticky over.

Later in the session, Chelsea affirmed Ashley's suggestion when she identified "I kind of like Ashley's information here and then you flip over and it's an example. I like that" as an approach she would like to try. As Ashley incorporated suggestions from earlier sessions, she shared her approach to summary sheets and Chelsea used this as a suggestion for further refinement in her process. Ashley exemplifies that learner-participants who experienced a new approach to learning felt confident in sharing their expertise with their peers.

Defining interdependence as "respecting the autonomy of others and looking for ways to give and take with an ever-expanding circle of friends" (p. 14), Chickering and Reisser's (1993) psychosocial framework focuses on the dynamics of the group. While each of the members of the Summary Sheet Small Group maintained a stance that "I guess I can include it like that. But I still kind of want to do it my own way!", as Shane remarked, there was a reciprocity in offering suggestions through personal experience with summary sheets. Baxter Magolda's (1992) explication of the epistemological stance of contextual knowers provides a more nuanced understanding of the interdependence among the learner-participants in this small group. She noted that, "contextual knowers conferred expert status on classmates when they had valid knowledge" (p. 175) and that "the credible opinions of others must be integrated into one's own view" (p. 188). In the case of the learner-participants, a peer was seen as having expertise through shaping an approach to learning mathematics. At the same time, the learner-participants did not end their reception of a suggestion by simply listening – they moved toward taking up the suggestion to improve their learning of mathematics.

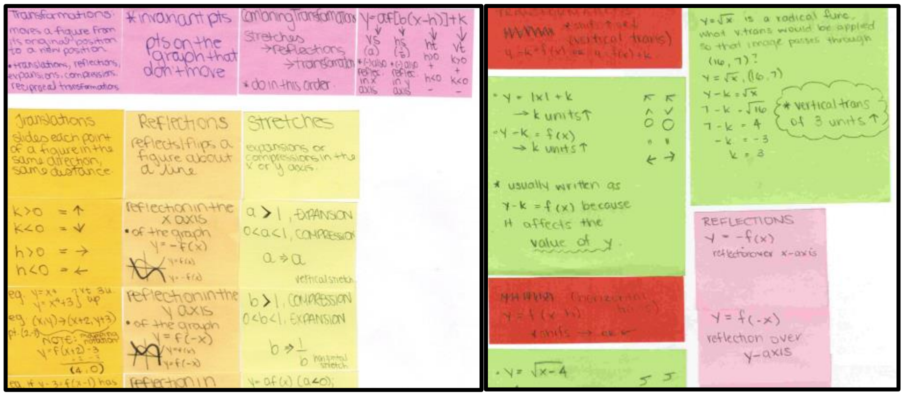
**Taking Up Suggestions by Incorporating Them**. What is important to the development of learning processes is the quality of the learner-participants' engagement with suggestions. As the suggestions were offered up in a tentative manner, the learner-participants did not often implement the suggestion exactly the way it was explained by following a step-by-step prescription. Rather, they took up the suggestion offered by *incorporating* the ideas. In other words, they modified the suggestion to fit how they saw themselves learning math and as a learner.

Elise exemplifies the notion of incorporating suggestions by actively modifying suggestions. In one Big Ideas Small Group session, the learner-participants had completed homework questions related to reflections of functions (in the *y*-axis, in the *x*-axis, and in the line $y = x$). After completing questions, I encouraged the learner-participants "to record some of the big ideas or skills from this homework assignment" that they had noticed in their worked solutions. In her recording "big ideas" at the end of a homework assignment, Elise asserted that "it really showed the function notation of it. And what we wrote down here broke down what it actually meant. So, I understand what to do when I have a question." My intention had been for the learner-participants to draw together the main concepts or skills from the lesson. Elise shows that she had adapted my suggestion of identifying one or two main concepts from a lesson and, instead, described seeing the component parts of a lesson as the process.

In the small group example above, Elise had modified the intention for the learning strategy as she incorporated it into her repertoire of ways of learning. In addition to modifying intentions, Elise incorporated suggestions by shaping the process she used within the strategy. After noticing Danielle's emergent learning process of making summary sheets, Elise began making summary sheets, too. In this case, Elise modified Danielle's approach of using sticky notes on summary sheets to make it her own. Elise explained:

> I kind of like the way [Danielle] does it, but I think it works better for me the way I do it. But she uses different sizes of sticky notes too, so that works really well. … So I'm doing that for chem and math, using the sticky notes, because there's – there's so much information in both subjects and so many examples, but I think there's – there's not many principles, and so I can just put those into little sticky notes and just put them all down and line them up. And then I can – I'll make note of what page in my SNAP book or in my math book to look for a certain example that's really big that I'm not gonna write all out, right? … I just noticed Danielle's just putting sticky notes. Like, I don't know how she's putting them on and what kind of way she's doing it. But I know she used sticky notes and just put them on sheets. So I was like, "Oh, well, that's a good idea." So I just did it my way…. I have to do it for me, because if I do it -- because it's the way I think, right? So she might think a different way than I do, and using different-sized sticky notes might stick in her mind more, but -- I use different colours, and I put them in certain ways certain patterns and stuff, on the sheet. So that matters more to me.

Elise pointed to two differences in the organization of the summary sheets. In contrast to Danielle's various sizes of sticky notes, Elise used all the same-sized sticky notes. Also, Elise searched for and made a reference to lengthy examples in her workbook, whereas Danielle only included short examples on her summary sheet. Figure 5.2 below contains excerpts from both learner-participants' summary sheets for the Transformations of Functions unit. Elise reiterated several times the reason for shaping a suggestion for a learning strategy, that the approach needed to fit who she was as a mathematical learner.




Elise's Summary Sheet Danielle's Summary Sheet

*Figure 5.2.* Transformations summary sheet for Elise and Danielle.

There were rare times where learner-participants did take up suggestions but did not end up incorporating the suggestion into their repertoire of learning strategies. Consider the following exchange between Elise and me in our first interview:

Elise: So he'll [the mathematics teacher] go through the questions, and he'll write everything out and explain how to do it. So at least he explains it, but for me I find that he talks really fast, and everybody else gets what he's saying, but I need somebody to go really slow, and he doesn't do that. So, we'll finish the lesson, and then we just have the rest of the time to do homework. And I'll go, "I don't even understand."

…

Janelle: And it's tough to get the steps down here and listen to what he's saying at the same time?

Elise: Yeah, exactly.

Janelle: I know I had one student in a research project a few years ago that told me the same thing, and what she did was -- she got together with a friend of hers, and all she would do is listen. She wouldn't pick up her pencil in class, and her friend would write everything down in class but not listen. And then they would get together after class, and as she was copying out what her friend had written down, she was, at the same time, explaining to her friend what the teacher had said.

Elise: Oh, that makes sense. That's good.

Janelle: I don't know if you have a friend in class and if you'd want to give that a try.

Elise: Yeah, that's really good, actually, 'cause then that way you're teaching yourself as you teach it to someone else.

In response to Elise's difficulty in taking notes and listening to the teacher at the same time, I offered an approach that had been effective for another high school mathematics student. Elise verbalized the possibility of the approach working for her and a willingness to try a new way of taking notes. Upon returning in the second interview, Elise reported that:

> I did what you had told me to do where I just sit and listen, and then I'll get the notes from somebody else, and that didn't work. I can't just sit there. I have to be writing while he's talking.

Elise's active stance helps tease out the difference between trying the approach and shaping the approach. In this example, contrasted to the interactions described above in the Big Ideas Small Group, Elise does not demonstrate a shaping of the approach, and this could explain the ineffectiveness she perceived in the alternative note-taking approach. In Elise's report on the unsuccessful use of the approach, she is thoughtful as she explains how the new way of taking notes did not match her sense of herself as a learner.

It was with risk to the learner-participants' sense of self and mathematics that they began to incorporate suggestions as part of the development of mathematical learning processes. For learner-participants like Teresa and Vanessa who retained a strong sense of the teacher as source of knowledge, while they engaged in trying the transition approach during our small group sessions, they failed to sustain it outside of our sessions together. *Incorporating suggestions* was not an easy process, as the learner-participants needed to begin to see themselves as capable of learning mathematics, a reorienting of their perspective of themselves as capable of developing ideas for their particular ways of learning mathematics. *Incorporating suggestions* was also demanding in the effort the learner-participants undertook. The suggestions were not prescriptions easily carried out and the learner-participants sometimes needed to try multiple times as they actively engaged in the process of incorporating. While the students listened to the ideas of others, they began to recognize that they themselves had ideas for their own learning worth building on. The learner-participants were becoming sources of knowledge about how they learned.

**Verbalizing Possibilities**

When learner-participants deliberated on how to modify ways of learning and put ideas into their own words, they were *verbalizing possibilities* for ways to learn mathematics. The verbalizing of possibilities builds on my theorizing about the emergence of voice for Consumer Mathematics students in my master's research. I wrote that being verbal "implies more than just putting words to thoughts and saying them out loud (whether it is written or oral). Rather being *verbal* means that the individual is pointing towards specific objects through the selective use of words" (McFeetors, 2003, p. 235). I drew on Freire's (2000) process of naming:

> To exist, humanly, is to *name* the world, to change it. Once named, the world in its turn reappears to the namers as a problem and requires of them a new *naming*. … it must not be a situation where some name on behalf of others. (pp. 88-89)

*Verbalizing possibilities*, as an act of naming, then is a human endeavour where the naming is dynamic and is located in an agentic person. In this way, the learner-participants were saying in their own words and for themselves and others, a reason for and a specific process within an approach to learning mathematics. In this section, I explore these two foci for verbalizing possibilities.

**Verbalizing Possible Reasons for an Approach to Learning**. Learner-participants were *verbalizing possibilities* when they put in their own words what they imagined they would get out of shaping an approach to learning as they incorporated suggestions that had been offered to them. With a possible result in view, the learner-participants could begin to articulate the reason for trying a suggestion. After I suggested to Grace to "try identifying groups of questions which are similar and make a note about what they are asking. It might help you look at <u>types</u> of questions, rather than individual questions," Grace responded in an interactive writing:

> I think by grouping questions at the end of the homework would help me since I would see the different examples. It'd especially help w/ exams [unit test] since I wouldn't get confused. Also, it'd help me w/ reading questions also I think I have the most problems with questions that have more than 5 steps. I'll def. [*sic*] try this strategy for my log exam. Because logs has a lot of variety in questions and it'll help me.
> Thanks for the suggestion ☺

The interactive writing represents Grace's first attempts to consider the suggestion of identifying types of questions in a homework assignment. Notice how Grace shifts from my use of "groups" as a noun to verbalize the possibility of a process-based approach, where she is actively "grouping" the homework questions. This demonstrates that Grace was naming, in her own words and for herself, a different approach to learning mathematics content. Grace moved quickly to verbalizing three possible benefits in the grouping approach, namely being able to identify differences among questions, to alleviate confusion when working on a unit test, and to interpret questions in order to solve them. Grace also verbalized a possible reason for a match between the new approach and the current content, "because logs has a lot of variety in questions." *Verbalizing possible reasons* for an approach to learning mathematics was a form of engagement which allowed me to notice how the learner-participants began forming intentions for particular learning processes.

I created the Big Ideas Small Group with Grace's verbalizing the possibility of grouping homework questions in mind. In our small group sessions, we did not explicitly discuss similar types of questions or groups of questions in the homework assignments the learner-participants used to generate their big idea record sheets. However, Grace explained a connection between grouping questions and the learning in the small group when she mentioned:

> I think grouping questions, I think it was journal two, it was like in the logs unit. So, like the logs unit was huge! There's so many things you can do with logs. So, I think by grouping the questions section by section, it was like easier for me to look at. It's like, logs are like the big theme, but then there's actually so many, it's almost like a tree branch. So there's a lot of things branching out, and sometimes it's hard to understand, see the whole tree. So, I just broke it up. … I break down like the types of

> questions. And then from that type of question, I break down how I'm supposed to do it. And then I can see the difference all around. So it's not so confusing.

The naming of the cognitive process of "breaking down" was important to both Elise and Grace. In both the small group sessions and the interview, I interpret Grace's naming of "breaking down" as pointing to the role of her voice in verbalizing the possibility of how she was making sense of mathematical ideas.

Interactive writing provided moments for the learner-participants to consider and reflect on a suggestion offered to them and to subsequently verbalize a possible benefit or reason for the suggestion through their writing. *Verbalizing the possibility* of the suggestion also occurred in spoken conversations. Returning to Elise's example of having difficulty taking notes and listening in class, she explained the possibility in the suggestion I offered, "that's really good, actually, 'cause then that way you're teaching yourself as you teach it to someone else." In her own words, she articulated the possible benefit of explaining notes to a friend as teaching both herself and her friend to support her mathematical learning. For the learner-participants, *verbalizing possible reasons* for incorporating or shaping an approach to learning was a way of naming the approach to learning for themselves and deliberating on why they might incorporate the process.

**Verbalizing Possibilities for Elements of a Learning Process**. When learner-participants explained an emerging idea related to a learning process to me or their peers, they were *verbalizing the possibility* of a way of going about learning mathematics. As they considered a new approach or shaped a learning strategy, learner-participants named elements within the development of a learning process. The learner-participants used their own words to make apparent to themselves and to me how they were actively engaging in shaping a personal approach to learning mathematics.

It was through listening that I was able to attend to a moment when Danielle verbalized an emerging idea of how she might initiate an evolution of her summary sheets approach. Early in the study, I recorded in my field notes:

> I gave the prompt for the second interactive writing to Danielle, Chelsea, and Grace to catch up on in class. … Danielle gave me her's later in class. When she brought up the writing and handed the paper to me, she explained how she would use different colours of sticky notes to represent different kinds of content such as definitions, formulas, and examples.

Although I was listening attentively to Danielle's verbal explanation of the use of sticky notes on her summary sheets, I assumed that she was telling me orally what she had written in the interactive writing. As I read her interactive writing later on, I was surprised to find no mention of sticky notes in her clarification to me of summary sheets (which she interchangeably referred to as cheat sheets):

> Yes, my cheat sheets are definitely used to create connections with the ideas of each chapter. … I do try and put ideas together and seeing how each relate to other concepts in the chapter. I kind of use the cue cards only when needed. I usually put really important notes on the cue cards, or concepts which I need to remember throughout the chapter. The

> cheat sheets on the other hand, I put examples and explanations on why I take certain steps in doing different questions. I haven't quite figured out this method yet and I honestly don't use it for <u>every</u> test, but I find that it helps a lot when I'm feeling really stressed about a test.

After noticing what Danielle wrote in the interactive writing itself, I understood her oral utterance to be her first attempt to articulate for herself and me the details of a new possibility for her summary sheets.

Through her independent work and our efforts together in a small group on this studying process, Danielle refined her explanations for the use of sticky notes as an organizational technique as she related in an interactive writing:

> I am handing in my logs and exponents review/cheat sheets. This chapters [*sic*] sheets were done quite thoroughly because of how much time I put into planning them. In the past my review sheets were often rushed and messy. Though they helped they seemed a little useless because it was so disorganized. With [the] method I developed, my ideas are organized and layed [*sic*] out in a way that really helps me understand.

More than just a proposal of using sticky notes, Danielle is able to articulate an intention, understanding the mathematical content, as a result of refining an element of her summary sheet learning process. This growing sophistication in her explanation aids in highlighting the nascent quality of her initial utterance when handing in the second interactive writing earlier in the study. Danielle also used the sticky notes to demonstrate how she connected mathematical ideas across a unit of content when she offered the suggestion to Ashley. The *verbalizing of possibilities* occurred both orally and in writing through my conversations with learner-participants and demonstrated that individuals, like Danielle, could be sources of knowledge for how they learn and perceived their voices as being valued in the learning context.

Danielle's enthusiasm for her emerging process of creating summary sheets was apparent as she also shared with Mrs. Finley the idea of using sticky notes. In our first conversation, Mrs. Finley explained that Danielle "couldn't wait for us to read it [interactive writing 2], she had to come and tell me! Well, because she had even for herself refined what she wrote in her journal. … So obviously she was thinking about it." Mrs. Finley highlights the important process of refining an approach to learning mathematics through the *verbalizing of possibilities* implicit in the development of a learning process. Danielle's enthusiasm to verbalize the emerging idea to both Mrs. Finley and me demonstrates that using her voice to give life to a possibility and refining her summary sheet approach as she verbalized the possibility mattered to Danielle.

In the movement from tasks to learning processes, *verbalizing possibilities* was a means of working out how learner-participants could shape an approach to learning in order for it to fit them as mathematical learners. Bruner (1986) writes about the difficult task of a writer in authoring a story so that it is at once both "'accessible' to readers, [but] at the same time … set forth with sufficient subjunctivity to allow them to be *rewritten* by the reader, rewritten so as to allow play for the reader's imagination" (p. 35). The indeterminacy of a learning process before it is formed – whether suggested by another or being created by a learner-participant – is rewritten by the learner-participant as he or she puts into his or her own words possible reasons for and ways to go about the approach. It was a way for the learner-participants to imagine how they might shape a learning strategy. It was a way for the learner-participants to commit to working on an approach to learning, they were authoring the process for themselves.

### (Re)Forming Intentions for Learning

Learner-participants were *(re)forming intentions* for particular ways of learning mathematics as they inquired into the systematized tasks assigned to them. When I use the notation "(re)forming" I am representing the idea that learner-participants were forming and forming again (reforming). The way in which the learner-participants actively engaged with intentions was to sometimes return to inspecting intentions they already had held and to re-form them. Other times, especially as they added on to their repertoire of ways of learning mathematics, they were forming for the first time an aim and reason for the incorporation of a new approach.

In identifying the sensitizing concept of intentionality in chapter 2, I described intentionality as an internal construct which gives meaning to actions. Intentions, then, are characterized as being both an aim and a process. As an aim, intentions are dynamic in pointing toward an end-in-view which is not specifically defined. As a process, intentions can be seen as informing an orientation which encapsulates how a student might go about acting intentionally to move toward an aim. In my master's research, I attended to how learners in Consumer Mathematics were intentional in their use of voice to affect themselves, their learning, and their relationship with their teacher.

But what arose in the data constructed from the learner-participants' experiences in this research project is not simply that they were acting with intentions – as in *being* intentional – but something much more finely nuanced. The learner-participants were *(re)forming* intentions, as the meaning I have made by interpreting the learner-participants' development of learning processes. What is important to understanding the learner-participants' experiences was the growth toward intentional learning – the forming of intentions for learning. In exploring this form of engagement, I explicate how learner-participants were developing learning processes through engaging in *(re)forming intentions* for mathematical learning and for particular approaches to learning mathematics.

**(Re)Forming Intentions for Mathematical Learning**. Although mathematics was not a compulsory course for grade 12 graduation, the learner-participants took the course for a range of reasons such as social pressure, unquestioned assumptions, and university entrance. In light of multiple reasons for taking a grade 12 mathematics course, getting high marks was a prominent goal and memorization-based approaches inherent in assigned tasks were purported by teachers and peers to reach a marks-based goal. However, it seems that even near the beginning of the research study the learner-participants were noticing limitations of the tasks and learning strategies they had been working on because their marks were not what they expected.

As the learner-participants became aware of the limitations of their current approaches, they began to express the intention of understanding the mathematical topics they were introduced to in mathematics class. The learner-participants were *(re)forming their intentions* for mathematical learning from memorizing toward making sense of mathematical ideas. Over the project, these intentions became progressively more well-formed as learner-participants explained to me the evolving intentions. Consider the following discussion contrasting understanding and memorizing among Ashley, Danielle, and Chelsea:

> Ashley: At the beginning of grade eleven, my biggest problem was, I was, I had to memorize in math. I don't know if that makes sense, but I had to memorize the math to understand it. But, I never really, I never really fully understood what was going on
>
> Danielle: Oh yeah. I get what you mean.
>
> Ashley: But, I, I thought to myself, "Okay, well if I memorize how to do each question and what, what the work of each question visually looked like, I'd be fine." But I found that every time I got to the test or the exam, I would sit there and be like, "Oh, my, goodness. I have no clue what I'm doing."
>
> Chelsea: I do that, too. … I always knew that it was, like, wrong, to like, just memorize the way to do things. But it was just the simplest way to try and pass
>
> Danielle: Yeah
>
> Chelsea: But it just hasn't worked. Yeah.

For these learner-participants, their unsatisfactory marks in grade 12 mathematics classes prompted a reforming of their intentions where they began to see the purpose of learning mathematics as understanding the mathematical ideas. Later on in the second interview, Chelsea explained her evolving intention for mathematics as, "I want to understand, like -- instead of just memorizing it." Chelsea engaged in developing learning processes by reforming her intention for mathematical learning.

Grace told me that grade 12 mathematics was "more of the process than the answer. And sometimes the answer and the process don't really look the same, especially with transformations. The process is the most important. It's not in the answer. It's a process." She explained how she refined her approaches to learning with the intention of developing understanding of mathematical processes. Students are sometimes encouraged by their mathematics teachers, as a task, to work with peers either daily on homework or to study for unit tests. In order to implement the teacher's instructions, Grace met regularly with a small group of students to work on homework. When I asked Grace if her group had improved in how they discussed homework, she responded:

> Yeah! From since September till now. September was like, "Oh, this is how you get the answer!" It's just the last two steps. It's how you got the answers. First we did transformations, yeah, it's like, "Oh, you just flip it over" and that's it. But now, we actually talk about what's actually happening with the question. 'Cause when you know the answer, it's like,

> "Oh, you know the answer, whatever." Right? But then, on the test, you're like, "'kay, I'm supposed to know how to get the answer."

Comparing answers when working together, as Grace indicates her study group did when they first started meeting, is consistent with an intention to finish the work and focus on a final answer. The students in the group initially did not attend to the process or understand the process of completing questions. Grace then points to a shift toward collaboration and learning together as they discussed the process within questions. She also provided a specific example of this focus:

> So, we pick a question and then we all try to do it. Like, just on ourselves. And then we stop and then we explain, it's like, "Oh. No. You're supposed to start with this." Or, "Oh, you have the wrong idea." 'Cause permutations and combinations, sometimes it's like, "Oh, which one is it?" So, we usually just work it out. And we discuss why you're doing it, why is it a combination and why is it a permutation. Sometimes, the wording is really confusing for homework. So, you know what you're doing. It's just you have the wrong mindset of it. So, that's what we talk about.

The students' talk in her study group shifted from "how you got the answers" to "we discuss why you're doing it" when completing homework questions. Grace was reforming her intentions for mathematical learning toward understanding processes and concepts. Over the year, the learner-participants were reforming intentions toward making sense of mathematical ideas; often the tasks they were told to complete by their teachers were not necessarily supporting their evolving intentions for mathematical learning.

It could be said that the learner-participants had intentions for mathematical learning prior to the forming of intentions for making sense of mathematical ideas. In this particular case, what I interpreted of the learner-participants' experiences of developing approaches to learning is that they were forming alternate intentions for their mathematical learning. And, it was in light of these evolving intentions that the learner-participants actively engaged in shaping processes that supported rich mathematical learning.

**(Re)Forming Intentions for Approaches to Learning Mathematics**. Often when the learner-participants expressed the reforming of their intentions for mathematical learning, it was done within the context of an approach to learning mathematics they were developing. As was demonstrated in the previous section, learner-participants were verbalizing possible reasons for shaping a learning process. The explanations the learner-participants gave helped me to see that they were also *forming intentions* for various approaches to learning mathematics. Within Grace's study group, she was forming the intention to learn, through collaboration, with her peers. She initially came to the group intending to compare answers to homework questions. As their work together evolved over the year, the group's discussions about mathematical processes enabled Grace to deepen her understanding through their interactions. Grace's *reforming of intentions* shaped her ways of learning mathematics.

Laurel’s reorientation in her approach to homework also enabled her shift from doing homework as a task to learning from homework as a learning process:

> I think ’cause I understand in class, but then once I get home, maybe I don’t understand something. So then once I start working through it and there’s examples, it makes sense to me, and I see the pattern and it -- then it starts making sense.
>
> …
>
> Everyone’s always said, “Do your homework. The home study is the biggest thing.” So I noticed that I never did any homework, and it kind of caught up to me, ’cause there was questions in your homework right on the exam, and you’re like, “Well, you could have known that before the exam, and that’s an easy question.” And so just going through your homework and just understanding it, that’s why I started doing it. And then I started noticing that I understood it all, and I didn’t need to know the question. I just needed to know how to do the question.
>
> …
>
> I think ’cause once I started doing my homework, I started asking more questions. I’m sort – I’m more confident. And now I’m starting to help people with their homework and then just explaining and teaching people. And so, the more I do it and the more I’m engulfed in it, just, I keep doing better. It’s not just the homework. It’s being able to teach it back to someone. That’s the better part of it.

Laurel’s intention at first was to complete the homework, in part because she was told to do so. Her intentions for homework evolved toward an expectation that she would be learning mathematics as she noticed patterns within types of questions in a homework assignment. She continued to reform her intentions for homework, so that through understanding she could also explain mathematical ideas to her friends and further refine her own understanding. Laurel was becoming aware that homework was a place in which she could learn mathematics for understanding. The learner-participants were forming the intention of learning with and through various approaches to learning mathematics, and this provided an opportunity for something as task-oriented as homework to be a site for learning mathematics.

In developing learning processes, the learner-participants were often *forming intentions* for particular approaches as they verbalized possibilities and incorporated suggestions. Rather than attempting to place the forming of intentions temporally in relation to the other forms of engagement in the development of learning processes, an interpretive understanding of the learner-participants’ experiences needs to recognize the complexity of pulling apart each of these forms of engagement. The *(re)forming of intentions* as a form of engagement demonstrates that learner-participants were thinking critically about the qualities of their mathematical learning and how their ways of learning would support their aims in learning.

## Processes for Learning Mathematics

As the learner-participants inquired into the tasks and learning strategies they had been using to make progress in mathematics class, they engaged in developing learning processes which supported their sense-making of mathematical ideas. This chapter opened with an explanation of how the learner-participants had been going about learning academic mathematics when the study began, through prescribed tasks and personal learning strategies. As the learner-participants took up opportunities to be in learning-based conversations, they started to develop other ways of learning mathematics. What resulted from their learning is what I conceptualize as "processes for learning mathematics." In this final section of the chapter, I explore the naming for learner-participants' ways of learning mathematics, describe examples of process for learning mathematics, and characterize these processes in comparison to doing tasks.

### Exploring "Processes for Learning" as a Naming for Ways of Learning Mathematics

In the above sections of the chapter, I have referred to the learner-participants developing ways of learning mathematics or mathematical learning processes or approaches to learning mathematics. As I was focused on explicating an interpretive understanding of the learn-participants' engagement in shaping learning strategies, I used the namings of what resulted indiscriminately.

At the outset of the research project and as the learner-participants and I were co-constructing data, I used the phrase "processes of learning mathematics" without inquiring into this naming. As I listened to the learner-participants and interpreted their experiences of learning to learn mathematics, I came to see that the phrase "processes of learning mathematics" did not necessarily reflect what the learner-participants had developed and what they had come to use in order to learn. I recognized the need to develop a more representative language to express the significance of these ways of learning.

I considered several word choices for the notion of "processes," not wanting to leave the naming uninterrogated. I purposefully used the labels of "tasks" and "strategies" earlier and needed to move beyond those labels. As I was using "ways" and "approaches" in the above sections of this chapter, I came to see that these two words were too broad to point at any particular experience or activity of the learner-participants in the study. The same was the case for "experiences" even though I found Chamberlin (2009) used it to clarify what was meant by "learning processes" in teacher professional development and "metacognitive experiences" has been used as an implementation of metacognitive knowledge concerned with the affective dimension of metacognitive knowledge and control (Efklides, Kiorpelidou, & Kiosseoglou, 2006; Mevarech & Fridkin, 2006; Vrugt & Oort, 2008). Upon a brief introduction to Wenger's (1998/2008) notion of practice, I felt that the phenomena he was pointing at through the use of the word "practice" was very similar to what I was noticing in the learner-participants' approaches to learning mathematics. Consider the description:

> Practice is, first and foremost, a process by which we can experience the world and our engagement with it as meaningful. … It does not address simply the mechanics of getting something done, individually or in groups; it is not a mechanical perspective. … *Practice is about meaning as an experience of everyday life*. (p. 31).

What resonates for me with my interpretation of the learner-participants' experiences is the active nature of practice that is malleable and aims at understanding. Embedded in Wenger's definition is the word "process" which also represents a strong connection to my understanding. However, for many mathematics students and teachers the idea of practice often carries negative connotations as thoughtless repetition of similar types of questions.

At the same time, I have been mindful of communicating the idea of learning processes with a broader audience. The words I select to represent my understanding of the phenomenon of learning processes, while important that it is meaningful to me, also needs to resonate with others as they consider the ideas. After exploring different possibilities, I decided to retain the use of "processes" to mark what the learner-participants were developing as they improved their approaches to learning mathematics. I perceive processes to be dynamic activity that have particular ways of being enacted and have intentions imbued in them, but at the same time are not step-by-step procedures.

There are occasions where "learning processes" is used in literature in mathematics education. As one example, De Corte (2007) uses the term to refer to what students are required to do to develop competence in the context of mathematics instruction, a way of building knowledge and acquiring skills. Rather than speaking directly to what characterizes learning processes, De Corte explores four aspects (constructive, self-regulated, situated, and collaborative) of learning which are involved in learning processes. Mathematics reforms have included mathematical processes viewed as "critical aspects of learning, doing and understanding mathematics (WNCP, 2008, p. 6) and "ways of acquiring and using content knowledge" (NCTM, 2000, p. 28). Viewing student learning through the lens of active participation in processes is a familiar perspective within mathematics education. Learning processes, like the examples developed by the learner-participants in this study, are also critical aspects of how students come to understand mathematical ideas. Whereas mathematical processes are integral to the development of mathematical cognition, learning processes are integral to the development of mathematical learning for students.

Continuing in my exploration, I also attended to the preposition and wondered if it needed to be modified. Instead of "processes of learning mathematics," which has somewhat a static connotation, "processes for learning mathematics" reflects more closely what students were developing. A similar distinction is made with the use of prepositions in assessment, where

> Assessments of learning are those assessments that happen after learning is supposed to have occurred to determine if it did. … Assessments for learning happen while learning is still underway. … provide students with feedback they can use to improve the quality of their work, and help

> students see and feel in control of their journey to success. … This is about getting better. (Stiggins, Arter, Chappuis, & Chappuis, 2004, p. 31)

The use of the preposition "for" highlights the dynamic nature of assessment and supports the improvement of students' learning. Correspondingly, "processes for learning mathematics" also is dynamic and occasions students' growth in approaches to learning mathematics. The processes, then, are used in order for students to learn mathematics and signify a formative approach, both in forming the processes and in forming the learner, akin to assessment feeding back into itself in assessment for learning. The "for" as a preposition invites a forward-looking orientation, both for the learner-participants who were developing processes for learning and for others who wish to take up this work to invite students to learn to learn mathematics and to understand their students' learning to learn. I use "processes for learning" and "learning processes" interchangeably.

## Examples of Processes for Learning Mathematics

Processes for learning mathematics are ways in which students make sense of mathematical content and are developed by students in response to the particularities of each one and to fulfil a specific purpose within the flow of a unit in mathematics class. There are five different processes for learning included in this section, used as specific illustrations of what I mean in saying that the learner-participants were developing learning processes. It is not intended to be an exhaustive list of the processes for learning mathematics I observed learner-participants develop during the study. Each of these processes supported learner-participants' sense-making of mathematical ideas. They illustrate how learning processes can be developed and used at different times within the sequencing of a unit of content. Each of the processes for learning represents the learner-participants' response to prescribed tasks as shaping a learning process that had fit with how they saw themselves learning. I have purposefully not included details beyond what is already contained in the dissertation because there is a risk of the processes for learning being reified – becoming tasks to be given to students in the future. The list provides an opportunity for the reader to pause and draw together the snapshots provided throughout the two preceding chapters.

***Designing a variety of types of notes*** was formed in contrast to the task of copying the teacher's notes in class. Learner-participants made, for themselves, written, word-based explanations of mathematical ideas. Margin notes were created concurrently with copying notes, where the learner-participants added their own or the teacher's explanation beside worked solutions copied from the board. Learner-participants sometimes added side notes to class notes while rereading notes after class, to construct meaning of symbolic steps. Learner-participants also crafted their own notes to accompany the viewing of online videos. Learner-participants were making sense of mathematical ideas both by designing their own notes in words they saw as their own and through the interaction of multiple forms of notes.

***Categorizing types of questions*** occurred as learner-participants engaged in pattern-noticing, attending to commonalities or differences either in the focus of questions or solution methods. Rather than seeing the task of "doing homework" as simply practicing questions to complete them, learner-participants critically viewed the questions posed within each lesson. It was through the process of categorizing types of questions that the learner-participants began to see the relatedness of mathematical ideas within a lesson and generalize mathematical procedures for themselves.

***Formulating verbal explanations*** consisted of putting words to mathematical ideas, and speaking them aloud to peers. Rather than "work with your friends" as a task, learner-participants were using such opportunities to say, often for the first time, how the steps they were using in finding a solution made sense to them. At the same time, learner-participants were aware of their audience and intended the explanation to be rendered sensible by their friends. Learner-participants were formulating verbal explanations while in the process of completing homework and also used understanding developed through homework to form the foundation of the verbal explanations they offered while assisting peers with difficulties.

***Identifying key mathematical ideas*** was comprised of breaking down elements of a lesson presented as a coherent whole, making note of the individual elements (whether skills, concepts, symbols), and then putting the elements back together. Learner-participants often worked from either their class notes or their completed homework assignments to identify key mathematical ideas. Content learner-participants identified as key ideas included concepts, formulas, and symbolic notation and were gathered together in organizing structures designed by the learner-participants. Both natural language and mathematical language were used to record key ideas.

***Creating summary sheets*** was a process learner-participants developed in order to learn through their studying for a unit test. In comparison to being told to "study" in an ambiguous sense, the learner-participants proceeded through each lesson in a unit and identified key mathematical ideas. In contrast to the previous learning process, learner-participants constructed more complex representations to illustrate their perception of the structure of content. Throughout the recording of key ideas and afterwards in using the summary sheets, the learner-participants were relating ideas and building connections of content across the unit.

In each of the above examples, the focus of the learner-participants' activity is represented by the use of gerunds. In other words, the process for learning mathematics was not "a variety of types of notes" but the *designing* that led to a variety of types of notes. Each of the gerunds was intentionally chosen to portray the generative element of the learning process. Additionally, engaging in the development of the learning processes often yielded a written record where learner-participants were producing an artifact to which they could return. For example, within the process of "creating summary sheets" the learner-participant would have a summary sheet in hand to demonstrate the result of the process and represent the learning that occurred. Sfard (2003) explains that "learning mathematics implies seeing structures on many different levels" (p. 360) and the learner-participants represented their personal development of structures through their written records.

The description of each of the examples is offered to represent the importance of the idea that the learner-participants were the ones who were shaping ways in which they were learning mathematics, in comparison to offering up specifics for the ways in which learner-participants learned mathematics. For the learner participants, the *development* of the approaches to learning mathematics seems to have had more impact on their learning than the approach itself. Additionally, learner-participants did not focus solely on one learning process, but were developing multiple approaches to learning mathematics. In fact, as the need arose they invoked the learning process that had the potential to support their learning at a particular moment.

**Characteristics of Processes for Learning Mathematics for Students**

The chapter opened by exploring tasks which students were told to do in order to learn mathematics. In contrast to tasks, processes for learning were dynamic and authentic processes created by the learner-participants. The processes for learning were dynamic because the learner-participants continued to shape them. The learner-participants also noticed how their peers were also continuing to modify processes for learning, especially within the context of the small group sessions. Additionally, the learner-participants' development of mathematical understanding was supported by the learning processes, suggesting the discipline of mathematics was seen by them as malleable and the content was of their own making.

The processes of learning were authentic because the learner-participants were aware of how the processes supported their learning. They were also authentic because the learner-participants had developed the processes and could describe that development. There was authenticity in their mathematical learning through learning processes because it enabled students to learn conceptually. Biggs (1988) refers to this authentic learning as a deep approach where students "become actively involved and can reflect upon what they are doing so that they may improve their approach" (p. 135).

Learner-participants, through the use of their processes for learning, were positioned as dynamic persons who developed a sense of authority in their approaches to learning mathematics. The learner-participants experienced growth as mathematical learners, a dynamic process, through their development and use of processes for learning. They were becoming aware of their shifts in approaches to learning and of themselves as learners. The learner-participants also came to have a nascent authorial stance as they saw themselves as personally developing their processes of learning. They saw themselves as authentic agents as they developed personal processes for learning.

Through my interpretation of the ways in which learner-participants were developing processes for learning mathematics, I have constructed an understanding of ways in which learner-participants were shaping personal ways of learning mathematics that fit their intentions for understanding mathematical ideas and how they saw themselves as learners. In the next chapter, I turn to examining processes of growth of the learner-participants' mathematical thinking which occurred alongside the development of ways of learning mathematics.

# Chapter 6
# Making for Themselves Mathematical Ideas

The learning processes the learner-participants shaped were aimed at developing mathematical understanding. One of the aspects of the learning processes in the previous chapter is that they enabled the learner-participants to engage in making sense of mathematical ideas. In other words, the learning processes provided opportunities for the learner-participants to interact with and generate mathematical content for themselves. Upon closer inspection of the learning processes, it became clear there were specific mechanisms the learner-participants used in order to learn mathematical content. This insight provides a more complex understanding of the learner-participants' learning within the study. Not only were they learning to learn mathematics, but they were simultaneously learning mathematical ideas. At the outset of the research project, I wondered in what ways the learner-participants' learning would be about increasing their mathematical competencies. Noticing how they were making for themselves mathematical ideas responds to my wondering through an understanding that the learner-participants were in fact learning mathematics so that it made sense, and that they did so through particular mechanisms which they developed and verbalized to me.

In this chapter, I take a third interpretive moment with the data. I increase the magnification from the previous chapter and zoom in further to examine within the learning processes the mechanisms by which the learner-participants were making sense of mathematical ideas. In this way, the chapter foregrounds the mathematical learning the learner-participants described and I noticed within the study. I see this as a necessary exploration because the aim for the learner-participants was to improve their mathematical learning and because the context of the study is in mathematics education. In the chapter, I explain the learner-participants' intentions for understanding mathematical ideas. My intention is to amplify the learner-participants' voices by using their naming of the ways in which they were making sense of mathematical ideas. I explore four mechanisms to illustrate what they identified as important to their emerging understanding.

## Learning Mathematical Ideas for Understanding

I established in the previous chapter that the learner-participants were (re)forming their intentions for their mathematical learning. They wanted to understand the procedures and concepts that were introduced to them in mathematics class each day. They recognized that a memorization-based approach to learning was not sufficient for them, and wanted to understand the mathematical skills they were required to perform on unit tests and course examinations. This understanding moves beyond performing the skills to knowing when to select which procedure and why the procedure leads to a correct solution. It was the intention of learning mathematical ideas for understanding that legitimatized the learner-participants' engagement in developing learning processes.

The word *understanding*, in mathematics educational literature, has been used in a variety of ways. Rather than presenting a survey of this literature, I selected a foundational piece within the mathematics education research community. Skemp (1976/2006) acknowledged that *understanding* is used to mean several things in relation to learning mathematics. He teased out a distinction between two primary ways *understanding* is used in relation to learning mathematics in school by attending to the way the word is already used by teachers and students. The first is labelled as "instrumental understanding," where students are able to use a mathematical rule or concept to arrive at a correct solution. The second is labelled as "relational understanding," where students not only know what to do to craft a solution to a mathematical question, but they know the reasons for why they carried out a particular procedure. In developing relational understanding, students connect mathematical ideas in order to flexibly solve mathematical problems.

While Cobb, Wood, Yackel, and McNeal (1992) cautioned against categorizing types of understanding as oversimplifying a complex activity, they also found Skemp's distinction effective in interpreting students' development of understanding in two different mathematics classroom cultures. Building on a relational sense of understanding, they noticed that, "students who participate in an inquiry mathematics tradition typically experience understanding when they can create and manipulate mathematical objects in ways that they can explain and, when necessary, justify" (p. 598). The authors highlight the notion that understanding means that students have made mathematical ideas for themselves. Through this generative process, students are then able to express reasons for the procedures and application of concepts to mathematical problems posed to them.

The interactions and conversations the learner-participants and I had during the development of their learning processes could be characterized as an inquiry approach to learning. Especially relevant to the mathematical learning of the learner-participants in this study is Cobb *et al.*'s (1992) recognition that a student who understands is one "who gives the symbols meaning in terms of his or her own mathematical ways of knowing" (p. 298). The mathematical work I observed was predominantly symbolic and was reflected in what was recorded as the result of shaping and using their personal learning processes. The records they constructed contained the meaning they made of a mathematical symbol system.

The learner-participants also used *understand* in multiple ways. Occasionally, a sense of instrumental understanding was expressed. For instance, Vanessa recognized that her approach to learning mathematics was to memorize each of the questions from examples in her notes. In admitting the short-comings of this approach, she stated, "If I were to understand, I'd know where to start and I'd know what words connect with what formulas." Here, Vanessa simply wanted to be able to identify key words in order to begin working on homework questions as she used an instrumental sense of understanding. She did not express an aim of developing relationships among mathematical ideas or exploring why particular formulas would help solve a problem. Vanessa's instrumental understanding provides a contrast to the richer sense in which other learner-participants used the word *understand*.

Most often, the learner-participants aimed at relational understanding that focused on why procedures they were required to learn worked to solve problems. Laurel demonstrates this in saying, “I think I’m more open to understanding why something happens and – I understand the reason stuff happens now. It’s not just – not just do it. I understand why it’s happening.” Laurel expresses a strong sense of relational understanding as she repeatedly mentions *why* a procedure would work in specific situations. She contrasts this with the limitations, for her mathematical learning, of an instrumental understanding approach.

The learner-participants also used other terms in relation to *understanding* to communicate the richness and complexity of relational understanding. They considered the *meaning* of mathematical ideas when they referred to developing an understanding of the content being presented in mathematics class. Consider Elise’s explanation:

> I just have that one principle or that statement or whatever on that sticky note and then I’ll go do a question. So I’ll understand what I’m supposed to do and what it means on the sticky note and then I have to apply that to a question. So once I can connect those two, then it’s like all these doors open up and I start to understand everything.

The learning process of creating summary sheets allowed Elise to move between the sticky notes that held singular mathematical ideas and practice questions, and back again. The *connecting* occurred for Elise as she moved between the mathematical idea and constructing a solution. Within the connecting she was making *meaning* of the ideas. Again, Elise uses the word understanding in a relational sense as she moved beyond understanding what to do and toward what the mathematical idea on the sticky note meant in relation to the questions she was answering. While making sense and understanding occurred within the connecting of an idea with a singular question, Elise could carry this understanding forward to the rest of the lesson and the unit.

At other times, learner-participants described their efforts to *make sense* of mathematics as another way to express coming to understand. For instance, Laurel explained:

> As soon as you figure out in your own way how to do it, it sticks. You’re not thinking, “Oh, what did Mrs. Finley say?” You’re thinking, “What did I do?” And it makes more sense when you’re teaching yourself. … when you go home, you process it in your own way, and then it makes sense, because you’re kind of teaching yourself.

Laurel shaped and used the learning process of categorizing types of questions when she was making sense during homework. She had an active stance, as she saw herself as the one who was engaged in sense-making. Her choice of the phrase “it sticks” could indicate that rather than being remembered, the mathematical ideas that made sense became a part of who Laurel was becoming, a mathematical thinker and learner. For Laurel and many of her peers, *making sense* of mathematical ideas is what occurred as they engaged in developing and using their learning processes.

In drawing together the words of the learner-participants and literature from within mathematics education, a provisional interpretation of what is meant by *understanding* in the context of this study can be explicated. *Understanding* implies that the learner-participants were active in the development of mathematical ideas for themselves. It is an act of making sense of mathematical ideas so that the ideas are personally meaningful to the learner-participants. They could explain with their own words the meaning of the mathematical ideas and could express connections among mathematical ideas. Understanding enabled the learner-participants to use the mathematical ideas for themselves. The remainder of the chapter will explore specific instances of the ways in which students were developing understanding through their personal learning processes.

## Mechanisms for Understanding Mathematical Ideas

The learner-participants had a broad repertoire of ways of making sense of mathematical ideas, which can be seen as component parts within the learning processes described at the end of the previous chapter. I interpret these ways of making sense as *mechanisms* for understanding mathematical content. Drawing out of this broad repertoire of ways of engaging is sense-making, I highlight four particular mechanisms for learning mathematics. The four mechanisms include: *breaking down*, *putting together*, *connecting*, and *writing down*. The four ways of coming to understand were selected because they were evident across multiple learning processes. The learner-participants identified the mechanisms as integral to their sense-making of mathematical ideas. I also made note of the mechanisms through *in vivo* codes, as they were named by the learner-participants. Finally, the mechanisms enabled the learner-participants to generate mathematical ideas for themselves. Figure 6.1 contains a diagram to portray the four mechanisms for making sense of mathematical ideas.

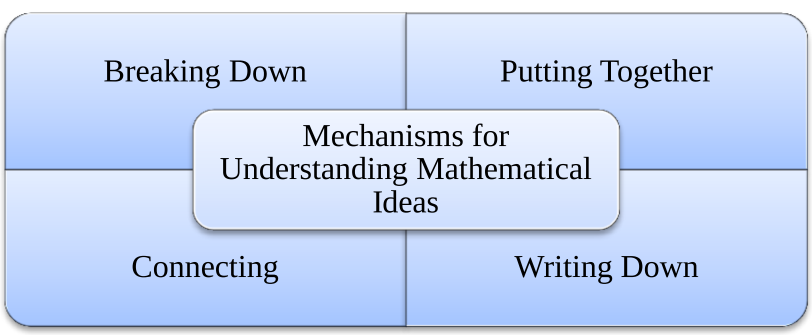



*Figure 6.1*. Four mechanisms for students developing understanding of mathematical ideas.

As I considered how the learner-participants were making sense of mathematical ideas through their learning processes, I came to see that the mechanisms for coming to understand were what enabled me to call "learning processes" just that. The mechanisms were the ways in which students were learning within the learning processes. In other words, the mechanisms are located within the learning processes. They are component elements of the learning processes. The mechanisms themselves are also processes, they are sense-making, meaning-constructing processes of mathematical content.

The learning processes explicated at the end of chapter 5 (including designing a variety of types of notes, categorizing types of questions, formulating verbal explanations, identifying key mathematical ideas, and creating summary sheets) and the four mechanisms for making sense of mathematical ideas explained below are interrelated. As will be illustrated below, individual mechanisms were used within multiple learning processes – the mechanisms were not unique to one particular learning process. At the same time, more than one mechanism was often employed within a singular learning process. One mechanism for understanding did not bear the whole of coming to understand.

I chose the term "mechanisms" to point to the ways in which (or processes through which) the learner-participants were making sense of mathematical ideas. The word choice also aims to clarify the different layers of the learner-participants' learning, rather than overuse of the term "processes" to point to multiple dynamic phenomena. Additionally, Liljedahl (2010) has used the notion of "mechanisms" to elucidate processes mathematics teachers undergo when they experience professional growth and change in their pedagogic practice. The term is used in a similar sense to what I mean by the term in this chapter when he points to teachers "beginning to make sense of why" (p. 416) in a parallel fashion to making sense of mathematical ideas. Liljedahl's dynamic meaning of "mechanisms" provides an example in extant literature of a similar use of the term.

In what follows, I provide an explanation of each of the four mechanisms for understanding mathematical ideas brought forward in the learner-participants' data. For each mechanism, the phrase is drawn from the learner-participants' words as they explained how they were learning mathematical ideas. Within the exploration of the mechanism, I make connections to a variety of learning processes as illustrative examples.

**Breaking Down**

Elise introduced the phrase *breaking down* in a session for the Big Ideas Small Group. A more thorough discussion of the origins of the naming of the mechanism can be found in chapter 3 ("Big Ideas Small Group" section) and chapter 5 ("Taking Up Suggestions by Incorporating Them" section). As a reminder, Elise regarded the way we had as a small group identified all the component parts of a single lesson, working from a completed homework assignment, as *breaking down* the lesson. In the particular small group session where Elise offered *breaking down* as a way through which she made sense, the learner-participants were trying to understand three types of reflections of functions. The mathematical content is depicted in Figure 6.2 with the first part of Nadia's big ideas record sheet.

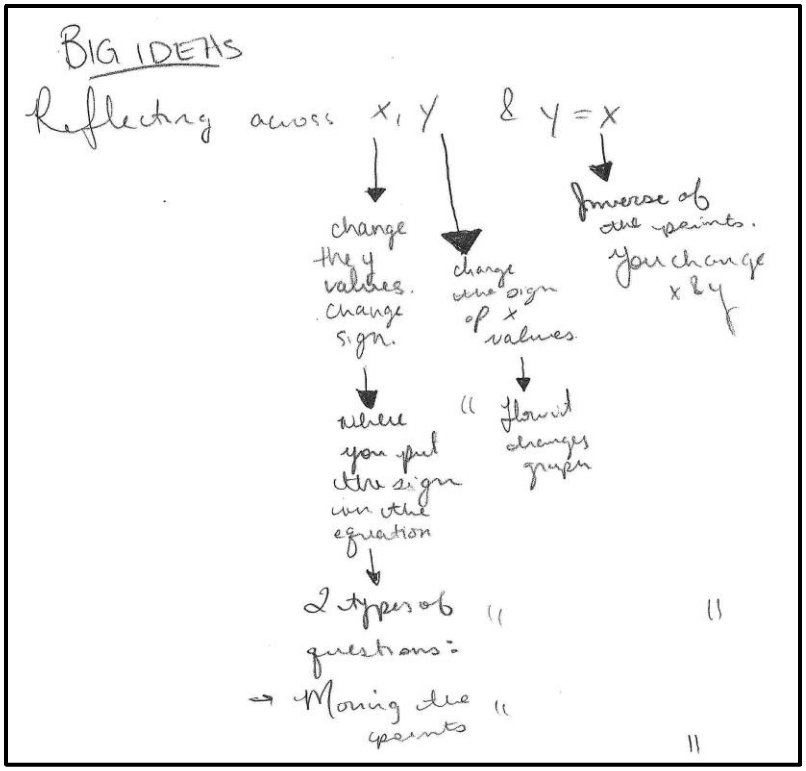



*Figure 6.2*. Excerpt from Nadia's big ideas record sheet.

At first glance, the sheet might appear similar to notes provided by a mathematics teacher. However, this sheet reflects the component parts of reflections of functions as Nadia broke down the mathematical content. As she was breaking down each of the types of reflections, she identified a singular type of reflection and recorded it on the sheet in a discrete fashion. The result of documenting her sense-making through *breaking down* is a sheet that contains all the component elements of the mathematical idea in one place. The mechanism of *breaking down* which generated ideas that made sense was what Nadia and her peers had done for themselves as they were making meaning of the symbolic system and acting on points on the function's graph. Breaking down as a way of learning mathematics held potency for the learner-participants, so much so that Grace incorporated and used the phrase to explain her mathematical learning in both interviews ("Verbalizing Possible Reasons for an Approach to Learning" section in chapter 5).

I wondered why the learner-participants perceived *breaking down* as such an important element of their processes for learning mathematics. What I came to understand is that they saw the mathematics teacher's presentation of a lesson as a whole in which the component parts were largely inaccessible. Vanessa pointed to the difficulty to access component parts when she commented on not understanding the notes after class or being able to use them to begin a homework assignment. The mechanism of *breaking down* provided the opportunity for the learner-participants to make sense of each of the mathematical elements within a lesson by generating the meaning of each element for themselves.

While Elise had generated the naming *breaking down* in the context of identifying key math ideas as a learning process, other learning processes included *breaking down* mathematical ideas from within a single lesson. The development toward creating a transition page which Teresa and Vanessa worked on during the small group sessions could be seen as another example of *breaking down*. For instance, in the first small group session Teresa and Vanessa were breaking down the component parts of sigma notation, $\sum_{i=1}^{n} a^i$. Teresa explained, “the top is how far you have to go. And then this, is the first one!” Her reference to “the top” is the *n* value in the expression, and “this” refers to the *i* value in the expression. As both learner-participants clarified the mathematical terminology afterwards, they recorded three different ways of referring to the lower and upper limits of a series written in sigma notation. Vanessa and Teresa were making meaning of the mathematical symbols and terms through their multiple ways of recording and explaining the component parts of sigma notation. Figure 6.3 contains the first half of Teresa’s transition page. Take note of her labeling of the lower and upper limits. As Elise and Grace were creating a record sheet of ideas within a lesson and as Teresa and Vanessa were creating a transition page, they were shaping the learning process of identifying key ideas.

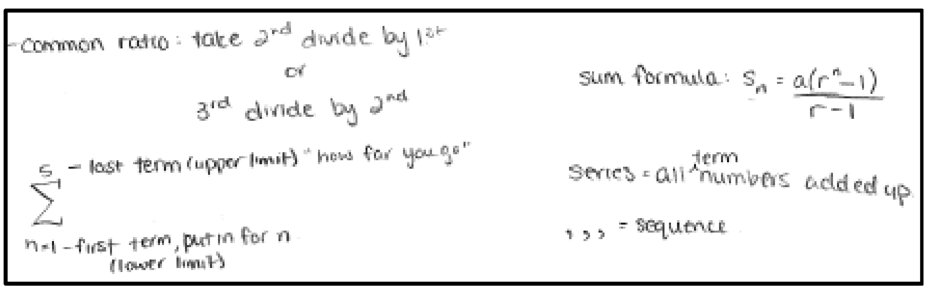



*Figure 6.3*. Excerpt from Teresa’s transition page on sigma notation.

A third example of *breaking down* occurred within the learning process of creating summary sheets. When Chelsea explained in her first interview how she used the creating of summary sheets to learn mathematics, she pointed out that she could “break up ideas … I don’t try to write too much on a sticky note … I write, kind of big ideas … I kind of separate the ideas.” Like Danielle and Elise, Chelsea would identify a singular idea to write about on a sticky note. Figure 6.4 contains two sticky notes, one focusing on common polynomial functions from the Transformations Summary Sheet and one focusing on reference and rotational angles from her Trigonometric Summary Sheet as examples of separate ideas.

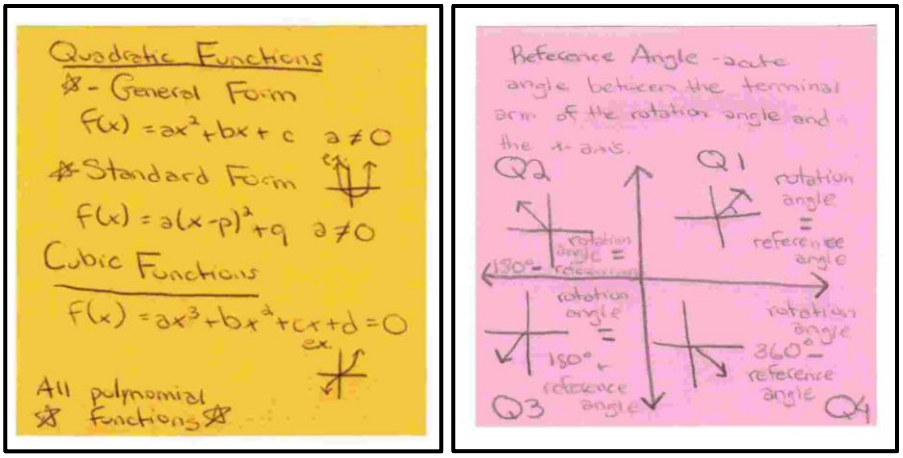



*Figure 6.4*. Two sticky notes from Chelsea's summary sheets.

In the use of sticky notes, the separating of ideas was both visual and tactile as Chelsea recorded discrete ideas on each sticky note for her summary sheet. By placing one mathematical idea on each sticky note, Chelsea could see one specific idea at a time. Danielle also used different colours of sticky notes to add a more substantial visual difference and moved the sticky notes around, as a tactile act, to connect ideas. It was in deciding what mathematical idea she would write on each sticky note that Chelsea was breaking up the component parts of each lesson.

It is intriguing to consider that Chelsea was not a member of the same small group that Elise and Grace were in, where the phrase *breaking down* was first noted in the data and became a common way to refer to making sense of mathematical ideas. So while the naming was significant within the Big Ideas Small Group, it was not isolated in its use as a way for the learner-participants to express how they were coming to understand mathematical ideas.

**Putting Together**

Not only was breaking down the mathematical ideas in a lesson important for the learner-participants, but the action that often followed was to reassemble the mathematical ideas into a coherent whole. The learner-participants named this mechanism for making sense of mathematical ideas as *putting together*. For example, Nadia explained, "if I have the main idea and then everything that could possibly be underneath that and I continue doing that for all the different information, then you can see that I really have everything put together." After *breaking down* the ideas from a lesson in the Big Ideas Small Group, Nadia used the phrase "put together" as representing what it meant to collect the fragments of mathematical ideas. The emphasis was *putting together* mathematical ideas within a discrete mathematics lesson, and in some ways reconstructing the cohesive whole that had been presented in mathematics class by the teacher. It is important to note that the scale of *putting together* existed at the level of a singular lesson within a unit. In the examples of lesson that learner-participants shared with me through notes and workbooks, a lesson usually focused on a single topic (e.g., Fundamental Counting Principle, reflections of functions, laws of logarithms).

When Chelsea was learning through creating summary sheets, she saw it as an opportunity where the mathematical ideas "come together … 'cause they look like separate ideas if they're – and then if they do combine, then, at the end, just do that." Even the prompt she posed to herself, "Oh, where can I put this?" indicates that she was considering how to put together the ideas in a lesson as she created a summary sheet. Additionally, Chelsea made it clear that in order to make sense of the grade 12 mathematics topics, "you have to put it all together." In this way, *breaking down* content to understand elements of a lesson was not an isolated mechanism for understanding, but was done in a way that allowed the learner-participants to combine the individual elements that were now sensible to them. Danielle's summary sheets represented *putting together* as she would place one or two lessons per side of a sheet of paper.

In the learning process of categorizing types of questions, Laurel noted that "once I starting working through it, and there's examples, it makes sense to me, and I see the pattern." Laurel saw patterns within a homework assignment, where she was putting together mathematical ideas or steps across questions. There were two types of patterns Laurel noticed when learning with homework, one where it was "not just doing the same questions over and over again. They all look the same, but there's different steps to them and different ways you have to solve them." The mathematical idea is similar across the questions, but there are different ways of going about using the idea. Another pattern was where "if you notice the steps are all the same, it's a different question, but it's the exact same step, then it's so easy to do the math and everything." In this case, the mathematical procedure was the same regardless of the question, which often was similar in structure but different in numerical coefficients. Laurel's process of noticing patterns in her homework is a different kind of example of *putting together,* where she is putting together similarities in questions through the description of a pattern.

Interestingly, the written records of many of the learning processes the learner-participants generated represented the action of *putting together* in their final form. While this is a visible record for an observer to notice and to think about, the learner-participants' use of *putting together* pointed to their activity of how mathematical ideas within a lesson was being enacted while in process, rather than pointing to the static document at the end. The mechanism of *putting together* highlights the notion that the learner-participants were generating mathematical ideas for themselves as a way of making sense of the ideas presented in their mathematics class. The whole of the mathematical ideas in the lesson were now of the learner-participant's making and having, rather than the teacher's providing.

**Connecting**

When learner-participants were *breaking down and putting together*, they were developing relationships among mathematical ideas within a lesson. The scale of the lesson was a single mathematical topic. The relationship-building, however, was not limited to connections within lessons as learner-participants named *connecting* as the mechanism for understanding that occurred when they made connections across a unit or multiple lessons. The scale of *connecting* was larger in comparison to *putting together*. Rather than making sense of the intricacies of a specific procedure within a lesson, learner-participants were able to identify relationships between previously learned ideas and new mathematical content. Nadia recognized connecting ideas across units was important for her sense-making because "when we start on the next chapter and it's a continuation on from the previous one and it kind of, it all builds off the previous chapters." She also recognized connecting ideas across grades when she pointed out the fundamental counting principle is connected to "tree diagrams, that's even in junior high … it kind of just builds off of that." Nadia was expressing an emergent understanding of fundamental counting principle in seeing why the structure of it worked in light of her previous knowledge of tree diagrams.

Vanessa extended Nadia's connection when Teresa introduced the factorial notation to her in a small group session and Vanessa exclaimed, "Oh, I get it! … It's like this, we don't want to draw, like a tree diagram so then you actually, oh okay. Yeah, yeah. It's like faster." Vanessa was in the moment of figuring out factorial notation and was developing a justification for the computation and notation. As she was building a connection, she was enacting the learning process of formulating verbal explanations. *Connecting* mathematical ideas across lessons enabled the learner-participants to generate relationships that rendered the mathematical ideas sensible. In this way, the mechanism of *connecting* afforded learning within learning processes like formulating verbal explanations.

The learning process of creating summary sheets was generally employed at the end of a unit and as a way to learn through studying for a unit test. The scale of a unit was often a collection of ten or more lessons. Creating summary sheets demonstrates how learner-participants were connecting the mathematical ideas within a unit in order for them to make sense. Danielle explained the process of *connecting*: "I don't really connect them to, till the end. … So right now, it's kind of just putting it all together. And then at the end, after I start looking over it, then I get it more." Notice the uses of both *connecting* and *putting together* to point to two different mechanisms. Danielle is putting together the ideas within a lesson that she had previously broken down. This putting together occurred as Danielle was creating her summary sheet. Afterwards, she was able to connect ideas across various lessons. The *connecting* mechanism provided the opportunity for Danielle to "get it more," or understand the mathematical ideas.

While the mathematical ideas were clearly and neatly recorded on Danielle's summary sheets, the connections among the ideas were not necessarily as visible. Figure 5.1 in the previous chapter contains an example. In her first interview, Danielle explained:

> All the ideas come one after the other and then you understand one thing and the next part. And then you're like, "Oh, okay. So this is why this happens in this section because of what I learned last lesson." … It's mostly in my head. I just make those connections by myself. And then --, usually I remember them, though.

Danielle maintained a linear structure in recording the mathematical ideas and procedures from the unit, moving from one lesson to the next in order. The connections she was making across the ideas were imperceptible, subject to difficulty in recording on her summary sheet. To her peers, during a small group session, she explained for her Exponents and Logarithms Summary Sheet that, “these are just all the [logarithm] laws, right? And then sometimes, if I’m doing a question on another page then I forget, I’ll just look at it or I can take it off and put it beside the question so I know exactly what to do.” To demonstrate the connections she had developed, she would move the sticky notes around, which was “always a constant reminder of why are we doing something. Or how you do it.” Demonstrating to others the connections that were made as part of her making sense of mathematical ideas was as equally challenging as the recording, but Danielle used the movement of sticky notes to show her action of *connecting*. Danielle repeatedly emphasized the justification, or the “why,” of a procedure when she was generating her own connections by moving sticky notes around on her summary sheets.

Ashley, in contrast to Danielle, chose to demonstrate her *connecting* directly on her summary sheets as she would “highlight or draw after in arrows how these ideas connect to the different parts of the chapter.” Figure 6.5 contains Ashley’s beginning efforts for the Exponents and Logarithms Summary Sheet, where the emergent connections are diagrammed. Visually, Ashley’s summary sheet was different from Danielle’s linear progression of content on her summary sheets. Ashley was *connecting* mathematical ideas not by assuming or taking on the teacher’s structuring of content, but by generating a relational structure of the content that made sense to her.

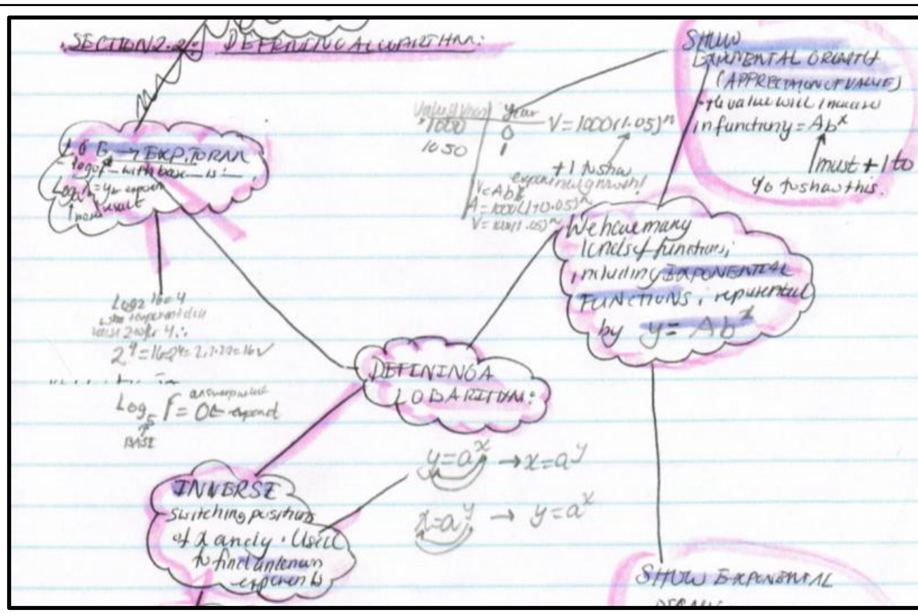



*Figure 6.5*. Excerpt from Ashley’s logarithmic and exponential functions summary sheet.

Connecting mathematical ideas to generate relational understanding also happened over time through engagement during a unit. Ashley provides an example of *connecting* as an important mechanism in designing a variety of types of notes and learning through homework. As a specific example, Ashley explained her understanding of logarithms in the fourth interactive writing:

> Initially when I began the unit it took me a while to really figure out the relationship between logs and exponents. Even when [my math teacher] pointed it out in class I couldn't visualize it in the beginning, but now I understand why we bring the exponent down, and solve for the unknown variable, which is the exponent, b/c [*sic*] logs are great for unknown exponents. So when I pieced that together the rest of my work for the questions came much easier. This is how I figured it out.

The idea of "piecing together" is left implicit in Ashley's interactive writing, however the writing does showcase Ashley's understanding of the relationship between logarithms and exponents. In the third small group session when the learner-participants were creating the summary sheet for the exponential and logarithmic functions unit, Ashley was more explicit about the mechanism of *connecting*:

> Each day you do a different section of the chapter. … But then when I have three or four sections of notes to work with … And I slowly, and I have time and I meticulously go through everything, that I'm "Oh, this connects to this. This is, applies to this." And then I can kind of see it better. … When you go back to maybe do homework, for some reason it kind of comes together at the end.

Ashley was intentionally returning to her notes to make connections, with an aim to "see it better," or understand. What she showed me in class was that she would create an additional set of notes as she carefully went through her class notes, engaging in the learning process of designing a variety of types of notes. Connecting mathematical ideas across several lessons in a unit, Ashley was developing a relational understanding of the interaction of logarithms and exponents. She was active in her engagement with notes and homework as she learned by developing connections. Her *connecting* shows that she was generating the relationship for herself, rather than memorizing what she had been told by her mathematics teacher. Ashley had a growing sense of authority over mathematical content through the process of *connecting*.

In addition to developing connections among mathematical ideas, some learner-participants were also connecting the mathematical ideas they were learning to experiences outside of mathematics class. Shane named this way of coming to understand mathematical ideas as "relating." Shane was one of the learner-participants who was becoming aware of how his mathematical learning could fit how he saw himself as a conceptual learner. Shane provided an example of "relating" with specific content of "pathways" (a particular class of problems based on Pascal's Triangle):

> Janelle: What does it mean for you to understand something in math class?
>
> Shane: To understand something?

Janelle: Yeah.
Shane: Um . . . I don’t know. To relate it to something that’s not completely classroom … like, pathways, relating it to a maze and thinking of it, how -- I don’t know. Have you seen the movie *Inception*?
Janelle: No. I’ve heard of it, though.
Shane: Okay. Well --
Janelle: You can tell me about it.
Shane: Okay. I remember, it just reminded me of this one -- like, when you’re designing a dream, you have to have, -- a designer, a little designer, and one of the tests that -- the character Leonardo Dicaprio was playing, he would -- he would ask the character to take two minutes to make a maze that would take five minutes to complete. And -- I don’t know -- that just made me thought of -- think of that, ’cause you’d have to take into consideration how many different ways you’d go into there.
…
Janelle: Now, these connections that you’re making outside of math class in order to be able to understand a math idea --
Shane: Mm-hmm.
Janelle: -- are they things outside of class that matter to you, and does it matter if you get to pick what that is?
Shane: Yeah, ’cause not everyone’s the same -- right? And it has to be suited to you. It’s more likely you’re going to follow something if it’s -- you think it’s your own idea rather than someone else telling you this is how you do it.

In this interview excerpt, Shane explains how he comes to understand mathematical ideas by “relating” them to his experiences outside of mathematics class. Within the combinatorics unit, the idea that Shane is relating pathways questions to a scene from a familiar movie provided the opportunity to make sense of a type of mathematical problem and be able to express this understanding to an audience. It is apparent in his closing statement in this interaction that he is generating ideas for himself as a mathematical learner.

Learner-participants were building several different types of connections with the learning processes they shaped. They were *connecting* mathematical ideas within units, for example, understanding the inverse quality of exponential and logarithmic functions. They were *connecting* mathematical ideas across multiple units, for example, understanding tree diagrams as the underlying structure for the fundamental counting principle. They were *connecting* mathematical ideas with experiences outside of mathematics class, for example, Pascal’s Triangle and creating mazes. Although an outside observer might see *putting together* and *connecting* as the same mechanism, for the learner-participants these mechanisms differed in their uses and in their results because of the difference in scale of the context of the relationships created. For each of these connecting activities, the learner-participants demonstrated that they were generating relationships among ideas for themselves. Similar to *breaking down* and *putting together,* the learner-participants' authority in making sense of mathematical ideas emerged through this mechanism.

**Writing Down**

Another mechanism for making meaning of mathematical ideas that the learner-participants named is that of *writing down*. While occasionally used in a casual sense, *writing down* carried with it the intensity and intentionality of understanding mathematical ideas. The learner-participants would commit to paper and commit to words only those mathematical ideas that they had rendered sensible. The choice of language is distinct from the task of scribing notes in class explained by Shane as being "like a drone -- copied down the notes" or by Grace as "you gotta take all the notes. Gotta take good notes." In fact, Grace continued on to explain that in addition to taking the teacher's notes that she would also "write down little notes for myself." She named these little notes as "side notes." Grace explained the side notes as, "I'm just writing it, and I'm looking at it. And I'm like, 'Am I writing down, … am I writing down correctly, like what's asking me?' … It's writing the numbers and then beside it why you did it." The explanation of the steps for a worked solution was the focus of what Grace was *writing down* beside the symbolic steps provided by the mathematics teacher. This provided an opportunity for Grace to design a form of notes that was an extension to what the teacher presented in class. *Writing down* was part of the learning process of designing a variety of types of notes.

Additionally, when Grace explained how she used videos from the Khan Academy website to learn mathematics independently outside of class, she related, "I've been going by the videos and just writing it down, making my own notes." Rather than the worked example portrayed, she was designing another form of notes that were her own – of her own making. More than making notes, Grace was *writing down* in order to make sense of mathematical ideas for herself. Interpreting Grace's data made me aware of the specific use of *writing down* as a mechanism through which the learner-participants were making sense of mathematical ideas for themselves.

Aware of the meaning with which Grace and others used the phrase *writing down*, I sought out more details about important elements of personal notes made by *writing down*. An aspect that was important to the learner-participants was that they used words they perceived as their own when *writing down*. When I asked Grace further about her side notes, she was emphatic that, "Oh, it's in my words!" Vanessa concurred that, "when you write something in your own words and own – like, how you'll get it, not how someone else will, you kind of know the material better." A possible insight as to why the learner-participants felt that putting mathematical ideas in their own words was important is that by doing so, they were active as opposed to simply enacting something they were told to do.

Grace and Vanessa clearly were active in generating their own way of explaining mathematical ideas when they were *writing down*. Moreover, Vanessa extends this notion toward making sense for herself when she explained that she found the transition sheets "helpful 'cause you write your own ideas and you're like, ‘Okay, I get this because’ – and sometimes you even make up little things.” Vanessa was implicitly referring to her idea of “how far you go” to point to an upper limit for sigma notation (see Figure 6.2 above to see Teresa’s recording of this idea). Within the mechanism of *writing down* the learner-participants were actively learning mathematical ideas through the use of words they saw as their own.

The use of the phrase *writing down* was used by other learner-participants in the context of learning processes other than designing a variety of types of notes. For example, Laurel explained that when she is helping a friend she would “try to write it down for them. … ’Cause sometimes math, they do the hardest way possible … but once you figure out the easier way, it’s kind of – you can cheat a little bit.” She used *writing down* in order to enhance the learning process of formulating verbal explanations. Laurel, in writing down steps as a solution for a homework question to show a friend, is committing to paper those mathematical procedures that she has already made sense of through her own learning.

When Danielle explained to Ashley how she was going about creating her summary sheet in our first small group session, she mentioned, “And so then I just wrote everything down individually. Make sure I understand them. That was pretty much it.” In this way, Danielle was saying that she would only commit to paper that which she understood. By creating a summary sheet, and the imbedded mechanism of *writing down*, the learner-participants were making a record they could look back at in preparation for tests and examinations.

Nadia used a slightly different phrase, “put down,” when she explained that on the big ideas record sheets “after we’ve learned the concept, we put it down into different sections, smaller sections to really get what’s going on.” Here, Nadia was using the label “putting down” as a mechanism within the learning process of identifying key mathematical ideas. Putting down, a similar process to *writing down*, was one way that the learner-participants were making sense of the mathematical ideas they were required to learn. Many of the processes for learning resulted in written records, and so *writing down* became a powerful idea for making mathematical ideas for themselves.

Often *writing down* was seen as a culminating activity by the learner-participants, as is evident in the above examples. Even within those examples, mathematical ideas were written down in a provisionary manner, marking the understanding the learner-participant had at that moment. Kylee’s learning process of making cue cards (introduced in chapter 4 in the “Preparation” section) provides an exemple of *writing down* as a dynamic activity. Kylee explained how, “instead of trying to concentrate on homework, I just wrote down, kind of like redoing the notes … the main important points and maybe two examples.” She named this process “making pages,” where she was *writing down* her emergent understanding of mathematical ideas. “Making pages” could be seen as one form of notes with the learning process of designing a variety of types of notes that Kylee was making to support her mathematical learning. Then, Kylee engaged in a further refining by *writing down* mathematical ideas on cue cards in order to, as she explained, “I’ll write down – write down what I understand after, and then I’m able to put them on the cue cards … to make sure I understand and then – it’s all definitely quite an active process." Kylee's *writing down* was dynamic as she continually refined the mathematical ideas she was recording as she made sense of them for herself. Through learning processes which included *writing down*, the learner-participants were coming to understand mathematical ideas.

## Mathematical Learning within Learning to Learn Mathematics

This chapter has explored one of the elements that made the learning processes hold potency for the students – they were able to make sense of mathematical ideas. The learner-participants expressed a desire to understand, in a connected way, the mathematical ideas they were required to learn in mathematics class. The mechanisms of *breaking down*, *putting together*, *connecting*, and *writing down* are four examples of the ways the learner-participants were actively making sense of mathematical ideas through their learning processes. What is clear from these examples is that the learner-participants were actively engaged in developing their own learning processes. Their level of engagement was significant and complex. At the same time that they were developing rich ways of learning mathematics, the learner-participants were also engaged in making sense of mathematical ideas. They were *learning* mathematics and how to learn mathematics in an interrelated and simultaneous process.

# Chapter 7
# Authoring Ways of Being In Mathematics

I open this chapter with a brief recapitulation of my building of the interpretive moments and a restatement of the outline of the interpretive moments from the previous three chapters. Each of the previous chapters contained an explanation of a unique interpretive moment with the data. For each of these chapters, my interpretation focused on a different facet of the learner-participants' learning to learn mathematics. While inquiring each time into the same phenomenon and the same set of data, I came to understand the complexity of the learning that was occurring through this multiplicity.

The aim of a study carried out with a constructivist grounded theory (CGT) framing is to engage in theorizing. Charmaz (2006) views theorizing as an act of developing "abstract interpretive understandings" (p. 9), using descriptive words like *indeterminacy*, *multiplicity*, *provisionality*, *contextualized*, *interpretive*, *connections*/*relationships*, and *abstracted*. Within these qualities of theorizing, I portray an account of my theorizing in this chapter, rather than an *object*ive theory. My intention is in synchrony with CGT in coming to understand in complex and interconnected ways the learner-participants' experiences of learning to learn mathematics – and that it remains an active process. The preceding chapters can be viewed as providing interpretive understandings, but what I seek to do in this chapter is to integrate these understandings together into an abstract interpretive understanding – culminating in the metaphor of authoring.

The separate interpretive moments could be described with a number of different metaphors, or in a variety of ways. I could have described it as a matter of perspective or standpoint. Yet, this approach calls on a movement from one static position to another where the researcher's sensitivities might be assumed to change. I could have described the different interpretive moves as peeling back layers. There is a kind of linearity in moving toward a centre, with an assumption that leads to the discovery (or creation) of a core essentialized truth. Additionally, layers often imply a hierarchy that was not necessarily present and the disjunction between something happening at one moment and another event at another moment. I chose to use an approach of magnification which takes up the movement of zooming in and out.

What a descriptive approach of magnification and the notion of zooming in and out provides for me is a way to characterize my theorizing as preserving the complexity of the learning in view and yet working with manageable elements to develop insightful, abstract understandings. What is intriguing about this explanatory space is that even within a particular magnification, I see myself as still experimenting with the focus as I move about within a particular magnification to draw into view data through which I can make meaning. The theorizing continues to be a dynamic act. The descriptive approach allows me to maintain a multiplicity of understandings, highlighting provisionality and the contextualization of understanding. In zooming out, the relating of the different understandings developed through altering magnification can be explained.

And so, with the idea of magnification and zooming in, I can draw together the previous three chapters and lead into the culminating interpretive moment in this chapter. I began the explanation of my theorizing, or interpretive endeavours, by zooming in slightly to make sense of the opportunities for conversations in which the learner-participants engaged (Chapter 4). Zooming into the learning that was occurring within the learning-based conversations, I increased the magnitude to draw into view the ways in which the learner-participants were developing learning processes (Chapter 5). Within this magnification, I noticed four forms of engagement in developing learning processes and also described some of the learning processes themselves. Increasing the magnification even more, I zoomed into the learning processes themselves to theorize about the mechanisms the learner-participants named in making sense of mathematical ideas (Chapter 6).

As I attended to these three interpretive moments, I wondered about how I could integrate them into an abstract interpretive understanding – or the theorizing that is the aim of a CGT study. It is in this chapter, now, I zoom out greatly to see how these interpretive moments fit together. In some ways, the fit is inherent because the learner-participants' experiences of learning to learn mathematics were happening all at once. There was no fragmentation of the learner-participants' learning through the inquiry, but a complex endeavor. Through the metaphor of authoring, I integrate the different interpretive moments to provide an abstract interpretive understanding in response to the research question.

In this chapter, I explicate my culminating understanding of the learner-participants' learning in this study through the use of the metaphor of authoring. As the learner-participants were learning to learn mathematics – developing processes for learning – they were authoring. I begin by exploring the metaphor space to find a starting point in the languaging to proceed through the chapter. Interpreting the experiences of the learner-participants in the study, I explore their learning situated within three sites in the main work of the chapter. Learner-participants were learning about and shaping personal ways of learning mathematics – they were authoring processes for learning mathematics. Learner-participants were learning mathematics – they were authoring mathematical ideas. Learner-participants were learning about who they were, (re)forming their identity – they were self-authoring as mathematical learners.

## The Metaphor Space of Authoring

Metaphors help us understand or make sense of complex phenomena. In this chapter I use the metaphor of *authoring* as an interpretive understanding of the learner-participants' learning in this study. In so doing, I respond to the research question. Within the field of mathematics education, the use of metaphors for understanding mathematics and for understanding mathematical learning is common. Mowat (2010) draws on the work of Lakoff and Johnson when she explains, "Metaphor, more than a mere figure of speech, is a central part of everyday thought." (p. 31) Her use of metaphoric networks enriches our understanding of the mathematical concept of exponentiation. Ernest (2003) demonstrates that exploring the use of an alternative (at that time) metaphor, conversation, can respond to problematic issues in theories of learning within mathematics education and that “much can be learned about theories in mathematics education and elsewhere by examining their underlying metaphors” (p. 1). Additionally, Pausigere and Graven (2014) demonstrate the potency of metaphor use to understand the nature of teacher learning. Pimm’s (1988) notion of metaphor resonates with me when he explains, “metaphor involves the seeing (and therefore the understanding) of one thing in terms of another; it is a conceptual rather than solely a linguistic phenomenon” (p. 30). As such, the metaphor of *authoring* provides the opportunity for me to further theorize as an act of coming to understand the learner-participants’ learning.

The metaphor of authoring, used as a way to understand the learner-participants’ learning in this study, draws together what were seemingly disparate parts of my data analysis. Polanyi (1964/1969) points to both the importance and struggle to theorize:

> William Whewell described how the merging of hitherto isolated observations into elements of a scientific theory changes their appearance.
>
> > To hit upon a right conception (he wrote) is a difficult step; and when this step is once made, the facts assume a different aspect from what they had before: that done, they are seen in a new point of view …
>
> We may say that a scientific discovery reduces our focal awareness of observations into a subsidiary awareness of them, by shifting our attention from them to their theoretical coherence. (p. 140)

While Polanyi was concerned with scientific theory, there is resonance for my act of theorizing.

Throughout the data analysis, I recognized the learner-participants were learning personal ways of learning mathematics, were learning particular mathematical ideas, and were learning about themselves as mathematical learners. As I pursued an interpretation that was an abstract explanation of a nuanced understanding of learning within these various sites, I wondered about the integration between the ways in which learner-participants were learning. It was in a moment when I began to think about the primacy of words that I heard echoes in my mind of a notion of self-authorship that I had read years earlier in Baxter Magolda (2001).

Returning to Baxter Magolda’s (2001) work, I found that it was my interpretive engagement with her text where I came to understand a dynamic process of authoring. The ideas in the book focused on “becoming the author of one’s life” (p. 119) and “self-authorship—the capacity to internally define … beliefs, identity, and relationships” (p. *xvi*). With this as a starting place, I imagined the possibility of authoring as a metaphor for learning. The metaphor extends my work in the emergence of voice and speaking with an emergent voice, where the saying of ideas around mathematical thinking and learning represented the ways in which students were learning. At the same time, authoring shifts to a different metaphor space that calls forth the use of words in a generative way. As

I began to search records of my earlier thinking, I found two instances of where authoring came up – one in my master's thesis as an extension of voice and one in a presentation proposal from this current research written half a year earlier. These two examples of foreshadowing demonstrate that, although I had not intentionally used the idea of authoring, my development of the metaphor of authoring as how learner-participants were generative in making sense of their learning was progressing.

Through enacting a pedagogic stance which invited the students to actively construct knowing and understanding of ways of learning and mathematical content in my own teaching practices, I have often wrestled with the metaphor space of "constructing." I saw that "constructing" provided a way to view students as actively shaping their mathematical knowing and developing relational understanding as they put ideas together to build a structure. However, there is rigidity in the metaphor of construction that points to inanimate edifices which dehumanizes the act of learning. Additionally, constructing directs attention to the content a student is coming to understand. The foregrounding of content development minimizes the recognition of the learner as a person who is being shaped by experiences of learning. It was my understanding of learners (re)forming their identity through learning that remained largely unaddressed by the metaphor of "constructing." Metaphors for learning are abundant, in part because "learning does not have an independent, reified, external existence—it is only that which we choose to call it, or, more accurately to conceptualize as learning" (Hodkinson, 2005, p. 110). An invitation exists to re-conceptualizing learning to recognize how the learner is becoming.

As I listened intently and intensely to the learner-participants in the study and as I sought to construct an abstract interpretation of their experiences in learning to learn mathematics, I began to shape my understanding of authoring and crafted a provisional statement. Authoring is a generative activity of making meaning of experiences and interactions which shapes self and the world. Engaging in the act of authoring implicates the author in self-making as he/she expresses understanding with a sense of authority through his/her voice to an audience. Authoring, then, can be used to understand learning in its complexity.

The etymology of author (as a verb) is "to be the author of an action; to originate, cause, occasion … to be the author of a statement; to state, declare, say" ("Oxford English Dictionary Online", 2013). Most often authoring is used in the context of words, the latter of the etymological foundations. However, authoring through action has been explored by Simmt (2000) where curriculum was "authored by the participants in action" (p. 16) through their doing, rather than the words written in a static document. This authoring took place through action and interaction.

Because the one who is authoring is central to the use of author as a verb, the origins of author are "the person who originates or gives existence to anything … one who sets forth written statements" ("Oxford English Dictionary Online", 2013). The etymological roots point to the generative activity of authoring, implicates the one who is engaged in authoring, and where the use of words is implied.

With this provisional understanding of authoring as a metaphor for learning, in the remainder of the chapter I return to the three sites of learning experiences by the learner-participants in the study. They were learning about processes for learning mathematics, they were learning mathematical ideas, and they were learning about themselves as mathematical learners. I use the three sites of learning to develop the metaphor of authoring as what it means to be actively engaged in learning mathematics.

## Authoring Processes for Learning Mathematics

The learner-participants' learning in the study can be understood as authoring processes for learning mathematics. In chapter 5, I represented my understanding of the learner-participants' engagement as developing processes for learning. They were developing processes for learning in four ways – becoming aware, incorporating suggestions, verbalizing possibilities, and (re)forming intentions for learning – by inquiring into the ways they had been learning mathematics. However, I was still left considering the question: what does the development of processes for learning mathematics mean?

The learner-participants were learning through their engagement in developing processes for learning. They were learning about how to go about learning mathematics personally, learning mathematical content, and learning more about themselves. In other words, the learner-participants were engaged in *authoring* processes for learning mathematics. Working within this familiar space of the ways in which learner-participants were developing processes for learning, in this section I develop authoring as a way to explain the interrelationships of the four forms of engagement. While the learner-participants were authoring processes that were unique to them and their contexts, the metaphor of authoring is abstracted from the context as a way to understand and offer to others a new way of attending to students' learning. Using my provisional statement of what it means to author, I perceive four key ideas of what authoring as a metaphor for learning might encompass: a generative activity, making meaning, implicating the author, and developing an emerging authority in relation to an audience. These four key ideas will be used to making meaning of the learner-participants' learning.

The learner-participants were *generative* as they were authoring processes for learning mathematics. They were generating, or making for themselves, approaches to learning that would support understanding of mathematical ideas. Borasi and Siegel (1990) assert that "to comprehend a text means to transform it in such a way as to produce understanding, that is, to duplicate the author's creative role and not simply the author's message" (p. 5). The learner-participants were authoring learning processes by incorporating suggestions. The shaping and adapting of a suggestion through incorporating was a creative act of an author. When Elise authored the process of creating summary sheets, she remarked on the creative act as supporting her in "figur[ing] out that there are ways that I can be creative, how I am, in order to learn something that I find so boring. … I can learn it in a way that I know works for me." As the learner-participants were authoring processes for learning, they were generating for themselves ways of making sense of mathematical content. In a similar fashion, Watson and Mason (2002) see acts of generativity as acts of learning where generating examples afforded students opportunities to make meaning of mathematical content and to see themselves as having authority. Watson and Mason's view of generativity supports the generative acts of authoring as what it means to learn.

The generative act of authoring also occurred as learner-participants were verbalizing possibilities of why they would engage in shaping learning processes and how to modify elements of the learning processes. Bakhtin (1986) explains that "an utterance … always creates something that never existed before, something absolutely new and unrepeatable" (pp. 119-120). As the learner-participants were putting to words – shaping utterances – the possibilities for a learning process they were authoring new ideas for learning. Additionally, the processes for learning themselves could be seen as utterances of the learning participants as they created unique ways of learning mathematics that were particular to the learner-participant.

The learner-participants were *making meaning* of their experiences of learning mathematics as they were authoring processes for learning. Meaning-making can be closely connected to authoring in that "in the making of meaning, we 'author' the world" (Holland, Skinner, Lachiocotte, & Cain, 1998, p. 170). For the learner-participants in the study, they were authoring their world of learning processes. These were not tasks to simply complete, which could be understood as a function of writing, but were processes imbued with meaning by an author – a distinction also made by Foucault (1984). The learner-participants were learning new approaches to learning mathematics, and were constructing meaning of the new approaches as they were developing. Sfard (2003) similarly notes that "the meaning of ideas is constructed anew each time anyone learns these ideas" (p. 357). What is striking in the learner-participants' data is that while they would occasionally explain how they needed to remember mathematical ideas, there is an absence of talk around remembering how to learn because the processes were meaningful to them and were a part of who they were becoming.

By incorporating suggestions, the learner-participants were immersed in the making meaning of their approaches as they actively worked on understanding mathematical content while authoring the processes for learning. However, incorporating suggestions also highlights the struggle of authoring – that it was not about following prescriptive rules but required the interrogation of learning processes. Coming to view critically tasks assigned by teachers, the learner-participants were also becoming aware of limitations within some elements of their learning processes as they continued to verbalize possibilities for the particular elements. There is an uncertainty as an author makes sense in order to generate, and the learner-participants demonstrate this through the tentative nature of becoming aware and the ongoing shaping of the various processes for learning mathematics. The meaning-making process of authoring interprets how the learner-participants were becoming aware of how they were learning mathematics. Laurel saw herself as actively making meaning of her experiences in authoring learning processes when she explained, "I think it's for you to figure out how to learn. It's not for [Mrs. Finley] to tell you, 'This is how you learn.' It's for her to explain different ways and for you to figure it out." The experience of figuring out speaks to the meaningfulness of the authoring of processes for learning mathematics.

The learner-participants were developing an emerging stance as *authors* of learning processes. An author, in this sense, is the one who is generating and making sense of processes for learning mathematics. The role of an author extends beyond the role of a writer (one who completes a written task), where the author is implicated in the process. There are two ways in which an author is implicated in the authoring of a piece. In the first way, who is doing the authoring matters because they put themselves into the authored piece. Polanyi (1958) acknowledges that "into every act of knowing there enters a passionate contribution of the person knowing what is being known" (p. viii). As the learner-participants were incorporating suggestions, the learning process they authored represented how they personally made sense of mathematical content. For instance, Ashley saw her web-based structure in summary sheets as representative of the connections she was building in order to understand the mathematical ideas. The learner-participants also expressed how they saw themselves as learners through the learning process they authored. For example, Shane saw his stance as a conceptual learner demonstrated in summary sheets. As learners of both learning and mathematics, the learner-participants infused who they were in their coming to understand how they could learn mathematics.

In the second way, the author is being shaped by the act of authoring, that to author is an educative experience (Dewey, 1938/1997). The generative, sense-making activity of authoring can be seen as an educative experience which impacts the learner. It was through the learner-participants' engagement in becoming aware of the personal ways of learning mathematics and as they (re)formed intentions for ways of learning mathematics that they had opportunities to see themselves as the ones who were generating learning processes. They were in the process of *becoming* capable learners of mathematics – authoring learning processes impacted who the learner-participants were becoming in relation to mathematics and to learning. In this way, the learner-participants were authoring themselves through their authorship of learning processes. I leave this idea only briefly stated because of its importance to the growth of the learner-participants in the study, and will return to the idea for an expanded manner in the section "Self-Authoring" below.

The learner-participants were developing a *sense of authority* in relation to their mathematical learning as they were authoring processes for learning. The sense of authority was growing out of their acts of authoring, in part meaning that their initial attempts at authoring processes for learning were tentative and that as they made sense through authoring there was consolidation toward authority. "Verbal and written expressions are echoes of the voice of authority" (Baxter Magolda, 1992, p. 273). Learner-participants, through verbalizing possibilities, demonstrated an emerging sense of authority as they expressed in what ways and for what effect they were authoring the particular learning processes. The authoritative stance was held with the recognition that the processes for learning were not static – they were never finished in their shaping – but that the learner-participants could be deliberate in their learning to learn mathematics. As the learner-participants deliberated, they were (re)forming their intentions. In contrast to incorporating suggestions where practical decisions were made by learner-participants in how to shape and use learning processes, (re)forming intentions occurred from a deep investment located within a sense of authority. Bakhtin (1981) notes, in relation to words, that "it becomes 'one's own' only when the speaker populates it with his own intentions … adapting it to his own semantic and expressive intention" (p. 293). The learner-participants saw each of the learning processes they had authored as their own because they had shaped it for themselves as a particular learner and imbued the processes for learning with their own intentions for the process and for mathematical learning.

The notion of authority carries with it the addressing of an audience, where authority can be understood in the relationship between the author and the audience. As the learner-participants were authoring processes for learning, not only were they attentive to the suggestions of others but they were also offering suggestions to each other. Offering suggestions arose from the growing sense of authority the learner-participants had through learning processes they saw as having authored. Danielle demonstrates this in her comment about using sticky notes in creating summary sheets as "the method I developed." Rather than relying on external sources of knowledge or authority, the learner-participants were developing an "internal authority" (Herbel-Eisenmann & Wagner, 2009, p. 154). The relationship in which authority is located is reciprocal, in that authority is demonstrated through authoring ideas while growth in the sense of authority occurs through this same meaning-making and generative activity. Povey, Burton, Angier, and Boylan (1999) develop the idea of "author/ity" which serves to highlight mathematical knowledge-making within a community where the author's (student's) voice is valued. In the case of the Summary Sheets Small Group leading a class session, the members of the small group shared their experience of authoring the learning process of creating of summary sheets from their sense of author/ity. Not only were their voices valued within the small group sessions as authority in relation to learning processes was shared, in coaching peers the members of the small group were seen as authors of a process for learning mathematics and were offering suggestions to their classmates in their first attempt. While learner-participants addressed others with a growing sense of authority in a variety of interactions, suggestions were not prescriptive in order to live out a shift in stance that perceived each student as having self-understanding in relation to learning mathematics.

Within the context of the learner-participants developing processes for learning mathematics, *authoring* provides an abstract interpretation of their learning. In my own authoring of this section, I came to see that the forms of engagement the learner-participants enacted in order to develop their learning process could be integrated. The integration of the ways in which the learner-participants were learning is a more faithful interpretation of the way the learner-participants were learning compared to presenting each way discretely in chapter

5. *Authoring* represents the richness and complexity of the learner-participants' learning.

## Authoring Mathematical Ideas

The learner-participants were learning more than how they learned during the study. Elise highlighted the importance of having mathematical content for a focus in developing learning processes when she explained, "I can't just have this [Mathematical Learning Skills] class. I have to have something to look back onto." The learner-participants' authoring of learning processes happened in the context of learning mathematical ideas. In the previous chapter, it was demonstrated that within the processes for learning, the learner-participants were making for themselves mathematical ideas so that they could understand. The learner-participants developed *breaking down*, *putting together*, *connecting*, and *writing down* as mechanisms for learning mathematical ideas for understanding. In other words, they were *authoring* mathematical ideas. If the metaphor of *authoring* is to be useful in understanding and explaining how students go about learning, then it is possible that it would also support an understanding of learning mathematical ideas. How might the metaphor of authoring be used to come to a more insightful understanding of their mathematical learning?

In some ways, I have already explored this idea in the above section as I teased out the important elements of my understanding of *authoring* within the context of processes for learning mathematics. The learner-participants, however, did not only hold the learning processes as their objects of learning, but also the mathematical content they were introduced to in their mathematics classes. Povey *et al.*'s (1999) definition of authoring in mathematics class is the creating of a mathematical narrative that aims at meaning-making of mathematical ideas. In the remainder of this section, I explore four key ideas in authoring – a generative activity, making meaning, implicating the author, and developing an emerging authority – for how the metaphor could support an understanding of the learner-participants' mathematical learning.

The learner-participants engaged in the *generative activity* of making for themselves mathematical ideas. While the mathematical ideas were presented in class by their teacher through representing worked examples, it was noted in the previous chapter that learner-participants viewed this as a whole conceptual object. This whole was an asserted fact, one that belonged to the teacher and the domain of mathematics to be given with the expectation that it would be taken. The learner-participants identified that their activity in response was to memorize concepts and procedures. Within the learning processes, though, learner-participants were *breaking down* the whole into the component mathematical ideas. Once the component elements were identified and each rendered sensible, the learner-participants would reassemble the mathematical ideas to reconstruct a cohesive whole. It is in this reassembling, through the mechanisms of *putting together* and *connecting* that the learner-participants were generating the mathematical ideas for themselves. Rather than passively receiving the mathematical content from their teachers, the learner-participants were transforming the ideas (Borasi & Siegel, 1990). Generating the skills and concepts through developing understanding within lessons and across an entire unit contributes to the conceptualization that the learner-participants were authoring mathematical ideas.

As the learner-participants were engaged in generating mathematical ideas for themselves, they saw these ideas as meaningful. They were *making meaning* of mathematical ideas through the four mechanisms of coming to understand. For many of the learner-participants, the reforming of their intentions for mathematical learning aimed for a relational understanding (Skemp, 1976/2006). Both Elise and Shane provided examples earlier in this document of remarking on the meaning of mathematical ideas as what it would look like to understand what they were required to learn. In addition to the generating of mathematical ideas, the learner-participants developed meaning of those ideas. They were authoring their world of mathematics, akin to Holland *et al.*'s (1997) notion of authoring the world.

Taken together, each of the four mechanism for understanding the learner-participants shaped and used afforded opportunities for meaning-making. The previous chapter contains examples within the descriptions of each of the mechanisms to illustrate specific mathematical ideas of which the learner-participants were making sense. At the same time, as the learner-participants began to tell me how they were making meaning of the mathematical ideas they were engaged in the interpretive act of giving meaning to the mechanisms. Through this reflection, I observed that the learner-participants had opportunity to engage in educative experiences (Dewey, 1938/1997): through activity they were making meaning of mathematical content that was presented in mathematics class and then consideration of the activity by putting to words their actions of authoring their mathematical world.

The learner-participants could be seen as emerging *authors* in relation to the mathematical ideas they were making for themselves. Similarly to being an author of learning processes, the learner-participants were developing as authors of mathematical ideas as they put of themselves into the authoring and were shaped by the act of authoring. Bakhtin (1986) notes the reflection of the author in that being authored when he wrote, "the *author* of the work—manifests his own individuality in his style … this imprint of individuality marking the work also creates special internal boundaries that distinguish this work from other works connected with it" (p. 75). As the learner-participants were *connecting* mathematical ideas, the connections they developed and expressed were personalized, or had the mark of their individuality. They were *putting together* and *connecting* in ways that made sense to them and in ways which were not prescribed by others. Danielle's dynamic *connecting* by moving sticky notes is a striking example of how the connections bore the learner-participants' individual perspectives as they were made in-the-moment. The learner-participants were aware that the relationships among ideas they developed needed to be meaningful for themselves. The representations of the mechanism of *putting together* and *connecting* were also unique to each learner-participant as they designed structures that characterized their personal connections.

Occasionally, I had a glimpse of how the learner-participants were beginning to see themselves as individuals who could understand mathematics. In this way, they were being shaped by the act of authoring. Povey *et al.* (1999) characterize an author as a learner who can "reflect upon what is being learned and how" (p. 232). In becoming aware of their mathematical learning, learner-participants were authors in this way. Grace recognized that not only were her approaches to learning mathematics improving, but her understanding of mathematical ideas. She reported, "You can tell how I'm improving. … I actually understand what I'm doing." Grace demonstrates confidence in her understanding of mathematical processes. Implicit in her response is an emerging stance as a person who could understand mathematical ideas, shaping herself as an author of mathematical ideas.

The becoming an author was manifest in the learner-participants' nascent *sense of authority* in relation to expressing understanding of mathematical content. This sense of authority differed in its intensity compared to authority in relation to learning processes. Their sense of mathematics as a discipline could be explanatory: that ideas in mathematics stand as an external body of knowledge, and while they have learned the required topics in a way that they have made the ideas for themselves (and in this way have enacted the role of author) the origin of those ideas lay outside themselves. There is a constraint of the prescribed mathematical content being viewed as an external authority. In contrast, Herbel-Eisenmann (2009) remarks that a

> kind of reliance on internal authority can help students learn mathematics with meaning. As Schoenfeld (1992) pointed out, however, the development of internal authority is rare in students who have 'little idea, much less confidence, that they can serve as arbiters of mathematical correctness, either individually or collectively' (p. 62). (p. 136)

Kylee provides an example of a *sense of authority* in her statement, "writing it down is definitely learning it in my own ways instead of just how it is in the workbook." She is making the mathematical content her *own*, rather than how the content is given in her class text. The learner-participants demonstrated early beginnings toward the development of internal authority.

Even within the tentativeness of a just emerging *sense of authority*, the learner-participants were expressing mathematical ideas that were their own to different audiences. For some, self was audience as they were making sense of mathematical ideas. The mechanism of *writing down* is the strongest example of describing mathematical skills and concepts through the processes for learning mathematics. Self was not the only audience for expressing mathematical understanding. For some of the learner-participants, they were able to demonstrate a growing sense of authority as they explained mathematical ideas to others. Grace demonstrated this earlier in *formulating verbal explanations* when she described her mathematical thinking to her study group. Laurel was also respected by her peers as an authority on the mathematical content they were learning as they sought her out to explain ideas with which they were struggling. Grace and

Laurel are examples of an emerging sense of authority in relation to mathematics and their peers.

The learner-participants were authoring mathematical ideas because they were learning mathematics in a meaningful way, expressing ideas that had become their own in their own way. The learner-participants were generating their personal understanding of the mathematical ideas, making sense of the procedures by connecting them to other mathematics content they understood and by seeking out responses to their queries about why procedures were effective in solving particular questions. The learner-participants' authority in relation to mathematical ideas was emerging as they expressed their understanding as it was forming to themselves and to others. I believe the metaphor of *authoring* holds potency for more than learning to learn in that it can provide a different perspective in understanding mathematical learning. Because the experiences of the learner-participants in this study primarily informs learning to learn mathematics, I anticipate further inquiry into and refinement of the notion of authoring mathematical ideas. There seemed to be a higher degree of tentativeness in authoring mathematical ideas compared to learning to learn. I wonder if this occurred, in part, in response to a prioritizing of the development of learning processes among the learner-participants and I compared to mathematical ideas and in response to my prompts which were not as often directed at authoring mathematical ideas.

## Self-Authoring

In the cases of learning to learn mathematics and learning mathematics, *authoring* is an effective metaphor through which to understand the complexities of the learning that took place for the learner-participants. But what makes the metaphor distinct, in comparison to a metaphor like constructing, is that the learner is noticed and attended to within the learning. The learner, as the one authoring, is not only authoring content but authoring himself or herself into the world. In other words, through experiences of learning, the learner is in the process of being and becoming. In this section, I concentrate on this identified uniqueness of the metaphor of *authoring* to attend to how the learner is engaged in a process of becoming.

At the outset of the study, I identified learning as an ontological orientation for me – that I see myself as a learner and I view the world from this perspective. Within learning as an ontological orientation, I had identified three perspectives, including: learning is living in the best way possible in that particular moment, learning is a way of being in relationship with others, and learning is a way of attending to others and the world to make sense of these things. While I wondered if the experiences of the learner-participants could afford the opportunity for me to explore the notion of learning as an ontological orientation further, I did not find this to necessarily be the case with them.

Rather, what was brought forward in my interpretation of the learner-participants' experiences of learning to learn mathematics is that I could explore an ontological approach to understanding learning. The notion of an ontological approach to learning is well documented in general educational literature (e.g., Packer, 1999; Wenger, 1998; Wortham, 2006) and mathematics education literature (e.g., Boaler & Greeno, 2000; Jansen, Herbel-Eisenmann, & Smith, 2012). I understand ontology to be the study of being and becoming, and an ontological approach to learning as understanding learning in light of students' being and becoming through learning experiences. What this means is that not only is intellectual development recognized in the process of learning, but that students' identity and how it is being shaped is also attended to in relation to learning. Learning is ontological because it shapes who learners are and how they see themselves in the world.

Within an ontological approach to learning, identity is a core concept. At the beginning of the study, I included (re)forming identity as a sensitizing concept. I conceptualized the notion of *identity* as an understanding or sense of self, with *(re)forming* emphasizing the dynamic nature of the continual shaping through experiences with others and the world. (Re)forming identity speaks to the changes in learners and to the shifts in how they see themselves in the world. In relation to this study, while the learner-participants were in the midst of developing processes for learning and mathematical ideas, they were themselves being changed by their learning. Jansen, Herbel-Eisenmann, and Smith (2012) also take on a dynamic perspective: "When students learn content, they not only develop knowledge of concepts, skills, and mathematical process, but they also develop beliefs about nature of the discipline and understandings about themselves as learners and doers of mathematics" (p. 289). The learner-participants in this study were not only being shaped, and they noticed their becoming as they understood who they were in different ways.

Using a*uthoring* as a metaphor of what it means to learn allowed me to notice the identity (re)formation of students while they are in the process of learning mathematics content. When we come to see learning through the metaphor of authoring, we add richness to what it means to learn beyond coming to know or knowledge-making (as is the case in Povey *et al.*'s (1999) use of authoring mathematical knowledge). The author, the learner who is in the process of authoring, is engaged in more than just authoring content. The author is implicated in a powerful way as he or she reflects within the learning experience and that reflection considers who he or she is and how he or she is becoming. In other words, the author is not only shaping that which is being authored, but is also shaping self in the process of authoring. It could be said that this is what it means to be in the process of *self-authoring*.

I have derived and come to use the term *self-authoring* from Baxter Magolda's (2001) development of the notion of self-authorship. As I mentioned earlier in the chapter, self-authorship emphasizes internal voice and students who were highly sophisticated in their learning were viewed as having "authored their own lives" (2004, p. 38). I understand this phase to be a strong consolidation of voice and certainty of self, and my learner-participants could be seen as emergent in the developmental trajectory. Additionally, Holland *et al.* (1998) point to, "in Bakhtin's account of 'self-authoring,' the 'I-for-myself' realizes itself explicitly in words and categories, naming the 'I-for-others' and the 'other-in-myself'" (p. 178). While Bakhtin and Holland *et al.* use the term *self-authoring* in the context of the production of humans within social and cultural situations, I locate and use the term within an educative context as a way to understanding the complexity of the learner-participants' learning and their sense of self for themselves.

I found limited examples in extant literature that the notion of *authoring* being used in relation to the being and becoming of individuals through learning situations. Graven (2012) connects the negotiating of identities and *authoring* by stating, "the re-authoring of identities is not only possible but could enable and give momentum to learning" (p. 130). Re-authoring can be used as a way to understand the important work of negotiating one's identity through learning. Nasir and Cooks (2009) suggest that, "identity is constructed as individuals both act with agency in authoring themselves and are acted upon by social others as they are positioned" (p. 41). Here, the authoring of self is seen as a component part of identity construction. For my learner-participants, the authoring of self was deeply personal and individual, and not as concerned with social positioning or identification. With the theoretical development of self-authoring as what it means to continually shape oneself and have a sense of that shaping – or the (re)forming of identity through learning – I turn back to some particular examples of the learner-participants experiences of self-authoring.

The learner-participants expressed self-authoring as individuals capable of doing mathematics, learning mathematics, and learning to learn mathematics. Some of these examples are already contained in chapter 5, where I explicated the learner-participants' development of processes for learning mathematics. Indeed, many of the examples of data throughout the dissertation could be re-viewed in relation to self-authoring. To explore these moments of self-authoring further, I begin with an excerpt from an interview with Laurel. I will use this excerpt as a primary example as I weave in the voices of several other learner-participants. In the second interview, Laurel and I had the following conversation:

Laurel: [In grade 11 math] I was just fed up, and I was like, "I can't learn math. I'm -- I don't understand it." … But this year with a more positive outlook with Mrs. Finley, it was -- I really wanted to kind of show that I could do math and learning how -- through Learning Skills, it was nice to figure out that I can -- do it.

Janelle: So do you see yourself as capable of learning math now?

Laurel: Mm-hmm. I come into every class just thinking, "I'm gonna understand it," not -- last year coming in -- being like, "Oh, I'm just going to block this out, 'cause it just doesn't make sense." And it's not a lot of -- I understand now it's not about just learning the ques- -- the way you do things. It's about learning how you learn, which is kind of the basis of high school, I feel. Not just figuring out this course is -- but figuring it out yourself for university. So, it felt so good coming into math and being like, "I know this, and I know how to do it." More confidence, I think.

Janelle: So did I hear you saying that that's sort of what high school is about, then, is learning how you learn?

Laurel: Yeah. My -- my dad has always said that to me, that it's not -- like, learning this stuff – it's important stuff as well, but it's about learning about how you learn in yourself. And I was always like, "Yeah. No, Dad. That's not it. (Laughter.) That's a terrible idea." But now it makes sense, because you don't really use this stuff in university. It's just learning how to process it all.

Janelle: Why does it make sense to you now?

Laurel: Because -- I don't know, actually. It just -- because I'm being more successful, and it's not that I just memorize something. It's that I've actually figured out what works best for me. It's not just this -- what I used to do is cram and memorize for an hour and then forget it all.

Laurel had authored herself in grade 11 as a person who did not understand mathematics. Through her experiences in grade 12 – both in mathematics class and in Mathematical Learning Skills class – she was authoring herself as capable of understanding mathematics, as a mathematical learner, and as a learner of learning.

Some of the learner-participants were self-authoring as capable of doing and understanding mathematics through the development of their learning processes. Laurel, in the above conversation, remarked, "it was nice to figure out that I can -- do it." She demonstrates that she saw herself as capable of doing mathematics. In this way, her self-authoring was different from her earlier experiences – she was authoring herself as a person who was able to do mathematics. In addition to making sense of mathematical ideas, Laurel was able to notice her success in making sense and express it to me.

The learner-participants' self-authoring of themselves as ones who can understand mathematical ideas often came through the use of their learning processes. Grace explained:

> I was trying to explain it to Vanessa, and I actually understood what I was explaining. … But I think when I was explaining it to Vanessa, I was like, 'Wow, I actually understand why' … So I think when you explain it to someone else it's like, it kind of shows how much you understand the concept.

Grace was self-authoring as a learner who understood mathematical ideas as she reflected on her use of the learning process, *formulating verbal explanations*. And for Danielle, creating summary sheets allowed her to notice herself as capable of understanding mathematics:

Janelle: Are you learning math by using summary sheets?

Danielle: Yeah! Most of the time I don't really understand the actual concept until I start doing the sheets.

In many instances, the learner-participants' statements of self-authoring were implicit. In other words, they did not express directly that they were "mathematical thinkers." Evidence of their self-authoring was enacted by them noticing that they were able to do mathematics. In this way, they were authoring themselves as mathematics-capable. Britzman (2003) points out a distinction in authorship: "whether we see ourselves as authors of, rather than as authored by, our experience" (p. 50). Laurel, Grace, and Danielle demonstrate that they were shaping their experiences of learning mathematics through their self-authoring as capable of understanding mathematical ideas.

The learner-participants were also self-authoring as mathematical learners. They saw their growing capabilities to *learn* mathematical ideas as a vital and dynamic process. Nasir and Cooks (2009) connect mathematical and learner identities in stating, "Boaler and Greeno … argue that the ability to do the math alone is not enough to support strong mathematical identities for students; rather, mathematical identities are tied to … seeing oneself as an effective mathematics learner in the classroom" (p. 44). This was the case for Laurel in her declarative statement, "I've figured out what works best for me" in her processes of learning mathematics. Her mathematics identity was related to her self-authoring as a mathematical learner.

Learner-participants also saw themselves as particular kinds of learners, and could fit how they saw themselves in the world. I explored this idea in chapter 5 (the section "Becoming Aware of the Personal Nature of Their Mathematical Learning"), using Shane as an example of a conceptual learner. Through creating summary sheets as a learning process, he was self-authoring as a mathematical learner:

> All I'd have to do is just take note of how I learned this, and it will be useful for future reference. Like in university, when I'm trying to learn something, I could just say, "This is what kind of learner I am, and I could do it this way, and this is how I learn more efficiently."

Shane is explicit in his identity statement of "what kind of learner I am" and makes apparent that how a person learns is informed by his or her identity as a particular learner. Elise also came to see herself as a mathematical learner in reflecting on the benefits of the Mathematical Learning Skills course:

> But this class has really helped me figure out that there are ways that I can be creative, how I am, in order to learn something I find so boring. … I can learn it in a way that I know works for me.

In addition to seeing Elise's statement as generating learning processes (see above), her explanation also demonstrates her self-authoring. Elise had a strong sense of self, expressed in this instance as being creative, and the learning process of creating summary sheets was an opportunity for her to author herself as a mathematical learner because she had space and support to figure out for herself an approach to learning that empowered her as a learner.

In interpreting the experiences of the learner-participants within the inquiry, I came to understand that most of them saw themselves as capable of *learning* mathematics through our conversations together. They could point to ways in which they had become better at learning mathematics, and that they had figured out for themselves how to learn. These were powerful moments of self-authoring that demonstrate growth. Graven (2012) captures the potency of this same phenomenon with teacher educators learning alongside mathematics teachers when she notices, “teacher educators explicitly author themselves as learners and ‘being a learner’ is thus given professional status” (p. 136).

While many of the learner-participants came to see themselves as mathematical learners, a few of them moved one step further and came to see themselves as learners who could figure out how they learned mathematics most effectively. When Laurel noted that high school is “about learning how you learn,” she was in the process of authoring herself as a learner who could develop for herself ways of learning. More than capable of learning mathematics, Laurel was able to continue increasing her mathematical learning capabilities because she came to see herself as a *learner*. Her identity as a learner empowered her as someone who was able to engage in a variety of learning situations – especially as she prepared to enter university. Danielle, in a similar fashion, saw herself as a person who was able to develop learning processes when she took on an agentic stance in relation to creating summary sheets. She referred to the learning process as the “method I developed.” (See the section “Verbalizing Possibilities for Elements of a Learning Process” in chapter 5 for a more detailed exploration of this data.) These examples demonstrate an emerging stance as a learner in the world. Perhaps what I was able to attend to and notice in their self-authoring was the nascent beginnings of a trajectory toward learning as an ontological orientation, as learners like Laurel and Danielle continued to engage in learning to learn and consolidated their sense of self as learners.

I am reminded that for many of the learner-participants, their self-authoring did not exist in a dramatic shift of how they saw themselves in mathematics class and in relation to learning mathematics. Often their statements of identity were expressed as the actions they were capable of enacting or had enacted. (Re)forming their identities as mathematical learners was a complex endeavor which would require the continuity of experience. Similarly, Toll (2012) recognizes that, “When one learns, one has a shift in identity. … shifts in identity occur in an evolutionary fashion, meaning they occur over time and in an organic manner in response to changing conditions” (p. 30). Over time and through learning-based conversations, the learner-participants were authoring themselves as they were in the process of learning how they learned mathematics for understanding. Through their self-authoring they were speaking themselves into the world as particular kinds of individuals. They were coming to see themselves as capable of doing and understanding mathematics, mathematical learners, and learners of learning in relation to others and to their beliefs about the nature of mathematics. In coming to understand the complexity of learner-participants’ learning within the study, I saw that they were *authoring themselves as mathematical learners*.

# Chapter 8
# Drawing the Inquiry to a Close

Among many reasons for engaging in dissertation research, one of the prominent aspects is the opportunity to engage in learning for myself. Rather than exploring my learning through the dissertation process as an exercise in the past tense – what I have learned – I have chosen to look forward in my learning as I draw this current inquiry to a close. The preceding chapters in the dissertation portray my learning, both the processes by which I learned and the results of what I have learned. The learning process built upon previous professional and scholarly learning, and will provide a foundation for further inquiry into students' learning in mathematics classes.

In this final chapter, I offer possible starting places for conversations and to guide future research building on the learning and theorizing in the dissertation. The first section positions the dissertation as a scholarly work in terms of its possible contributions to mathematics education. It begins with the interpretive understandings generated through data-based theorizing, and moves to suggesting the theorizing could be used as ways to see differently the learning of students in mathematics classrooms. The second section returns to methodological considerations and possibilities for engaging in further explorations within constructivist grounded theory (CGT) to continue to shape the methodology. The chapter closes with a brief personal reflection of my own learning.

## Contributing to Mathematics Education

In drawing this inquiry to a close, I am reminded that this process was not a solitary act of learning. As I have benefitted in building on the contributions of other scholars to the field of educational research, and mathematics education in particular, I offer a response to their work and join the ongoing conversation about improving students' opportunities to learn mathematics through my own theorizing in this dissertation. Within constructivist grounded theory, theorizing contributes to the scholarly community by "creating abstract interpretive understandings of the data" (Charmaz, 2006, p. 9). Theorizing is the act and the result of pausing and listening closely to learner-participants' experiences and authoring their meaning to communicate to others. Consistent with this conceptualization, I have sought to put in words abstractions which tentatively portray a nuanced understanding of the learner-participants' learning within the study.

I see my theorizing as occurring in multiple moments as represented in Chapters 4 through 7 in this dissertation. My inquiry into students' experiences of developing personal processes for learning high school mathematics led to a culminating exploration of the metaphor of *authoring*. Authoring, in relation to what it means to learn, is a generative activity in which students express emergent understandings of their experiences of learning as they are in the process of growth (Chapter 7). I view the developing of this abstract interpretive understanding to be an assembling of other moments of theorizing as I (re)searched within the data from multiple perspectives to make sense of the learner-participants experiences. From the perspective of mathematical learning, the learner-participants were shaping and enacting particular processes for making sense of mathematical content explicated through the four mechanisms for mathematical understanding (Chapter 6). The four mechanisms include: *breaking down, putting together, connecting,* and *writing down*.

From the perspective of learning to learn mathematics, the learner-participants enacted four forms of engagement in developing processes for learning mathematics (Chapter 5). The four forms of engagement include: *becoming aware*, *incorporating suggestions*, *verbalizing possibilities*, and *(re)forming intentions for learning*. Finally, from the perspective of a particular way of being with students, four features of learning-based conversations opened up space for the learner-participants to focus on their approaches to learning mathematics (Chapter 4). The four features include: *preparation*, *presence*, *mode*, and *pace*. Taking all these moments of theorizing together, the learner-participants, through engagement in learning-based conversations, were authoring their learning and themselves.

**Reflecting on My Theorizing**

In this section, I reflect on my theorizing to explore what it offers to the discipline of mathematics education as an endeavour which is simultaneously practical and scholarly. In order to assess theorizing through CGT, Charmaz (2006) provides for criteria: credibility, originality, resonance, and usefulness. She also delineates reflective questions for each of these criteria. Reading these questions interpretively, I respond to the main ideas encapsulated in the questions to reflect on my theorizing. Because readers are welcome to view again the examples developed in detail earlier in the dissertation, I provide only brief examples in this section.

*Credibility* (Charmaz, 2006, p. 182) refers to the quality of the use of data to support the theorizing. Three ideas in particular are important: the depth of the use of data as a subtle and thorough familiarity, the breadth of the incorporation of data through a wide range of data used, and the strength of the connections made between the data and the interpretive renderings. I have provided evidence in chapter 3 explicitly about the intimacy I developed with the data, both in its initial co-construction and through coding. This deep familiarity and understanding of a large amount of data is further demonstrated in the nuances I theorized about throughout Chapter 5 (for example, through the emergence of becoming aware and the initial verbalizing of possibilities for learning processes) and chapter 6 (for example, the learner-participants' particular use of *writing down* as an act of making sense).

In addressing breadth, within the examples shared in each of the Chapters 4 to 7, I draw on all the learner-participants, across multiple learning processes and data construction processes as a means to capture a broad range of learner-participants' learning. Strength in connection-building demonstrated by the metaphor of *authoring* in Chapter 7 draws together the full span of what learner-participants were attempting in developing their capacities for learning mathematics, understanding the mathematical content, and shaping their identity as mathematical learners. Each aspect of how the metaphor of *authoring* emerged as an interpretive device was connected back to examples from the data to strengthen the relationship of the metaphor as emergent theory.

*Originality* (Charmaz, 2006, p. 182) addresses directly the potential of the theorizing for mathematics education as a field of scholarship. In considering the contribution, originality rests on the innovation of new conceptual understandings that have arisen because of the theorizing and how it enters into the ongoing conversation about improving students' learning of mathematics. In my scholarly efforts in preparing for the research study I found claims for the importance of students improving their learning, but without suggestions for how students might accomplish the identified improvements. The four ways in which the learner-participants developed learning processes (the forms of engagement explicated in Chapter 5) are a new insight into understanding learning to learn within the mathematics education context. Additionally, in theorizing learning as authoring, I have challenged "constructing," one of the dominant metaphors in conceptualizations of the learning of mathematics, to encourage the use of a new metaphor which attends to a student's self-authoring as he or she experiences growth in becoming a mathematical learner.

*Resonance* (Charmaz, 2006, pp. 182-183) indicates the authenticity and insightfulness of the theorizing. In particular, consideration is given to the way in which the participants feel the interpretive work resembles and supports their understanding of their experiences in the inquiry. In the initial interpretive work I engaged in during the phase of constructing data, I provided the learner-participants access to that work through my replies to interactive writing and through the interview prompts, both of which contained my emerging understanding of the learner-participants' experiences in learning to learn mathematics. In Chapter 5, I provided details about the learner-participants' responses to this feedback, including their incorporation of elements of the feedback into their emerging perspectives of their ways of learning.

Resonance also depends on the attention given to the whole of the experiences in the inquiry, rather than a narrow or snapshot-like views of isolated events. Elements of the trajectory of experience were interpreted individually across all learner-participants as well as longitudinally, one learner-participant at a time. When the data informed more than the learning of mathematics content, and more than the learning of learning processes within mathematics, moving to their emerging understanding of their identity as mathematical learners, the interpretation moved to address the whole of the experiences. The metaphor of authoring (Chapter 7) – by its particular choice and design – unifies the learning of mathematics and the learning of processes for learning mathematics with the fullness of students' experiences in mathematics class as individuals who are in the process of becoming.

Based on these three criteria alone, I believe there is potency for others available in the artifacts of my theorizing – an abstract interpretive understanding

– of the phenomenon I set out to research. However, one criterion remains unexplored thus far, and I will do so in the next section.

**Offering Up My Theorizing for Others**

I also see the act of theorizing as not only an interpretive endeavour, but also as an invitation for others who read and interact with the theorizing in order to sponsor their thoughtfulness and growth. This is similar to Charmaz's (2006) fourth criterion for reflecting on a grounded theory: *usefulness*. Conceptually, usefulness refers to others being able to use particular ideas from the research in their own contexts, sponsors further inquiry, and improves how we live in the world. Similarly, Kieren (1997) proposes the notion of *theories for*, where sharing the theorizing is not a prescription for action but suggests for a teacher and/or inquirer "insights she can use in observing and listening differently to the mathematical actions and languaging of her students, and in entering into a different form of conversation with them" (p. 32). Just as the learner-participants in the research project benefitted from developing learning processes for themselves rather than being told what to do to learn mathematics, the idea of *theories for* has resonance for me in inviting others to improve the learning experiences of students in mathematics classrooms. I intend this invitation to be to mathematics teachers and to inquirers carrying out research with mathematics students.

But what does it mean to offer up *theories for* others? Is it simply a stating of my theorizing? Do I bear responsibility beyond the explications? Even though an offering up of theorizing might begin with the sharing and leaving space for others to interpret and to thoughtfully consider, I feel some responsibility to suggest trajectories along which to consider implications for learning with students in mathematics classrooms and further inquiry. In living within this tension, I choose to imagine the ways in which my theorizing could offer different ways of seeing and acting for mathematics teachers and inquirers. In my imagining, my intention is to invite others to consider possibilities. I imagine myself not only offering tentative suggestions to others, but also wrestling with seeing differently as I continue to learn alongside students about learning mathematics and thinking mathematically. As I imagine for myself how my inquiry through this dissertation is a springboard for further research, I am obliged to pose invitations for other inquirers in mathematics education to consider the meanings of this work for their own future efforts to improve the mathematical learning experiences of students in schools.

For mathematics teachers or those working in schools with mathematics students, particularly in high school as this was the context of this study, I want to first offer encouragement. The encouragement is that students are capable of, and often have desire of, being in conversations with others about what matters to them, and their learning in school is of great concern. Mathematics teachers can find willing conversation partners in their students. These students are also capable of developing ways of learning mathematics richly that fit their identity as mathematical learners. Interacting with students begins with listening to the students with a responsiveness that meets students where they are and envisioning together possibilities for improved learning. I imagine that teachers could incorporate or enliven their learning-based conversations with students by attending to the various and complex ways in which the conversations could take place in their classrooms. Even though a concern about the amount of time available to talk about learning in mathematics class may be raised, I imagine the opening up of small moments over time for students to inquire into and shape their processes of learning mathematics. I invite teachers to use learning-based conversations as a site for generative activity, as they come alongside students who are in the midst of developing their own approaches to learning mathematics. As teachers begin to consider their practice from the perspective of students and of the learning occurring in their classrooms, I hope they think first about how students can make sense of mathematical ideas within a lesson and then about learning across lessons in a unit (maybe in the context of didactic instruction). Students need opportunities to not only see the lesson as a coherent whole as the teacher presents it, but also to make sense of the elements in the lesson. They will then need opportunities and leadership as the unit proceeds to make connections across the various lessons.

The usefulness of theorizing also rests on its function to generate further inquiry. In this way, I offer my understanding of students' learning to learn within my study to other inquirers (whether mathematics teachers or mathematics educational researchers) as a place to begin further research. Rather than waiting until the end of high school to support students' development of learning processes, I would encourage further research to explore learning-based conversations and the development of learning processes with younger students in school. Perhaps trajectories of students' learning to learn could be outlined over time, looking across multiple years of school within a developmental approach to improving students' ways of learning mathematics. Further inquiries into the development of processes for learning mathematics could enlarge the repertoire of illustrative examples offered in this dissertation, deepening the range of the ways in which teachers could enable students to develop their personal learning processes. I invite inquirers to enrich our shared understanding of students' development of learning processes from the perspective of the students themselves, in order to explore the nuances of learning which makes a difference to students.

While writing this dissertation, I have been participating as an instructor in pre-service teacher education, imagining how this research might be extended into inquiries with pre-service teachers. Within mathematics education courses, I wonder about how opportunities could be afforded in which pre-service teachers begin to attend to their own learning. As pre-service teachers have these experiences of a learning-focused education, it would be worthwhile to explore how this focus affects their own processes of ongoing professional development. As well, I wonder in what ways noticing and attending to their own learning might provide foundational experiences to shape their ways of noticing and attending to children's mathematical learning in their own classrooms. Within teacher education, I invite further inquiries into how pre-service teachers might incorporate conversations about their learning into their emerging practice as mathematics teachers.

What I find most invigorating is the emergent metaphor space of authoring as a way to come to understand learning as it occurs in mathematics classrooms. Because the notion of authoring brings into view learners' deeply individual and personal involvement in their learning – the (re)forming of their identity as mathematical learners – I am hopeful that as a construct it could help to humanize the discipline of mathematics education. My contribution is in its emergent phase and I eagerly anticipate further use and development of the metaphor to provide a more nuanced understanding of learning which attends not only to the objects of learning, but to the learners themselves. As I see myself as an inquirer into students' experiences in mathematics education, I anticipate that each of the suggestions I have raised here are avenues along which I could further develop my own research program.

## Identifying Methodological Conversations within Constructivist Grounded Theory

Much of my own learning about conducting research using CGT is recorded in Chapter 3, as I wrestled with the processes that led to theorizing from data. Out of that learning, I have identified three methodological contributions. These methodological contributions are invitations for further exploration and development within CGT to strengthen the methodology.

In selecting CGT as a methodological framing, I searched out researchers who had published critiques or criticisms of the methodology to present a balanced view when describing the methodology earlier in the dissertation. I found no such critiques, perhaps because CGT is a relatively new methodological framing and because it responds to many of the criticisms expressed about many of the variations of grounded theory. I found CGT helpful in framing my inquiry and often found a sense of synchrony with Charmaz's methodological writing. It is with respect for the work Charmaz has done in the re-grounding of grounded theory to shape CGT and with no intention to raise broad-sweeping criticisms of the methodology that I offer possibilities for making CGT an even stronger research methodology in the social sciences. In this section, I identify three methodological contributions: regrounding coding and categorizing in postpositivist discourse, using interpersonal insights in analyzing data, and shifting toward a relational methodology for CGT. This section illuminates how my learning through the use of CGT can contribute to a methodological conversation.

### Regrounding Coding and Categorizing

As noted in Chapter 2, Charmaz (2000, 2005, 2006, 2009) has regrounded the philosophical orientation of grounded theory within American Pragmatism and placed it within a postpositivist tradition. Except for the methods-based caveat of providing guidelines as opposed to prescriptive steps for data analysis, the processes of working with data remained largely the same as Glaser and Strauss' (1967) original work. Without the regrounding of data analysis processes, the work remains within a structuralist tradition. Structuralism could be defined as "a method of analysis and a philosophical orientation which privileges structures, systems, or sets of relations over the specific phenomena which emerge in, are constituted by, and derive their identity from those structures and sets of relations" (Pinar, Reynolds, Slattery, & Taubman, 2008, pp. 452-453). In this way, the coding and categorizing in CGT are privileged after the construction of data. Rather than the researcher using the data to guide the analytic process, the procedures of coding and categorizing could dominate the researcher's thinking of the data and are taken as given. I am not asserting that CGT guidelines promote that the phenomena in the inquiry are entirely ignored, but that the procedures in working with the data become elevated and are of central concern to the researcher. My observation caused me to wonder whether the analytic processes had been interrogated in the same way that the philosophical orientation had been during the re-grounding of grounded theory.

Dey (2010) has already begun the reconceptualizing work of imagining a different orientation to coding and categorizing in grounded theory. His thinking provides an image of how coding and categorizing (and their respective products of codes and categories) can be seen differently, as being guided by the data beyond the stage of coding. His exploration includes strong reasons in moving away from a conventional sense of categorizing. The piece begins an important conversation. What I propose is an enriching of this emergent conversation, where there is an interrogation of the grounding of processes like coding and categorization in CGT. The inquiry could explore a re-grounding of the analytic processes of CGT from a post-structuralist perspective and then provide suggestions for researchers using CGT for processes consistent with a post-structuralist perspective. It seems that with a postpostivist turn in CGT, that a post-structural orientation toward the processes of data analysis would provide a cohesive methodological package.

In identifying the need to interrogate the grounding of processes like coding and categorizing in CGT, I only invite the possibility of such an inquiry. The intended focus of this current work in not methodological and so I recommend as a future project, and perhaps with more examples of studies using CGT with the reconceptualized coding and categorizing processes, a regrounding so that the theoretical provenance of poststructuralist thought and the empirical base are present. de Freitas and Nolan (2008) identified a burgeoning direction in mathematics education of a poststructuralist turn, "building on the strengths of structuralist programs … but employing these theoretical frameworks reflexively and without reliance on positivist epistemologies" (p. 3). It seems that a re-inspection of the processes of CGT is well situated to be carried out within mathematics education.

**Interpersonal Insights in Analyzing Data**

In the development of categories and codes, some grounded theorists (e.g., Charmaz, 2006; Dey, 2010) recognize two main sources of provenance in constructing categories. One source of provenance is the empirical data generated from the research study. Another source of provenance is the researcher's theoretical background, especially through sensitizing concepts. These two sources inform the researcher as he or she constructs conceptual categories which seek to explain the phenomenon under study. I believe there is another experience of the researcher which can be important in the development of categories in the data analysis phase of a grounded theory study. When a researcher is immersed in the research context and living alongside the participants over time, the experiences of the researcher within the study can also inform data analysis.

I found that my immersion in the research context was an experience that shaped my data analysis. I came to the data analysis phase in this study with a certain kind of intimacy already with the data. I had been present at the time of the co-construction of the data and had recollections that were a part of who I am because I experienced the Mathematical Learning Skills class and because I was implicated in the learner-participants' actions. The *co-construction* of the data is worth noting: that as a *participant*-inquirer I also contributed data in the form of replies within interactive journal writing and my observations which were recorded in field notes. There are limitations in making notes of observations and recording interviews, and yet I experienced those events in a fuller way than is recorded. Prototypical exemplars arose for me before I engaged in data analysis and I was able to situate them more robustly because of my sustained experiences in the research context. The interpersonal relationships that I formed with each of the learner-participants and the teacher-participant provided depth in my understanding of moments of data because the moments were situated within a larger framing.

It would also follow from a symbolic interactionist perspective – where meanings are negotiated or shaped in interaction, and that these negotiated meanings shape the individuals who are interacting – that the relationality of the data construction informs later analytic work. I also held in view the whole of the learner-participant, as a whole person and across all the experiences I had been present for, which guarded against the fragmentation that coding or categorizing could have forced. I have not located a report of similar experiences of a researcher in publications for CGT, perhaps in part because often interviews are singular events with each participant or are conducted by research assistants on behalf of a principal researcher. The forming of interpersonal relationships through immersion in the research context informs insights during data analysis that are richly nuanced – an *interpersonal provenance* to categorizing.

**Shifting CGT to a Relational Methodology**

If, as I suggest, an interpersonal provenance to categorizing can occur within a CGT study, it would mean that the research was carried out within a relational methodological framing. I understand a relational methodological framing as a researcher's orientation to a research project that is imbued in relationships with participants. In this way, the researcher seeks to forge relationships with participants through immersion in the research context and over sustained periods of time. The researcher's actions and inquiry come out of a concern for these relationships, which include processes such as data construction, analysis of data, and shaping of interpretive understanding of the participants' experiences during the study.

Building from related literature, I have selected three sources which address the notion of relational research. Loewenthal's (2007) uses the term *relational research* to point to "the use of the relationship between the researcher and the researched as a means of research" (p. 1), and examples in the book highlight the practical efficiencies of finding research participants through existing relationships within social sciences professions. Gunzenhauser (2006) explored both epistemological and ethical dimensions of relationships between an inquirer and participants in relational research and explicates five core elements:

> First, a relational ethic positions the researcher as one-caring, reflecting displacement and engrossment. Second, the researcher-researched relation is one of knowing subjects, who both contribute knowledge to the relation. Third, contact between knowing subjects calls not for objectivity but enhanced subjectivity, a move that integrates epistemology and ethics as moral epistemology, a basis for understanding the relation between the researcher and the researched as a difficult process of "knowing others and their concerns." Fourth, the relation is characterized by particularity; as such, the moral epistemology serves not as a foundation for research but suggests places to begin, such as particular moral commitments. Fifth, the knowledge claims produced through the researcher-researched relation come about by mutual critique. The interaction of the researcher and the researched leads to complex, tentative understandings that respect both as knowing subjects. (pp. 630-631)

Within narrative inquiry – which has been identified as a relational approach to research (e.g., Foster & Bochner, 2008) – Clandinin and Connelly (2000) address the practices of relational research when they describe their processes as, "a collaboration between researcher and participants, over time, in a place or series of places, and in social interaction with milieus" (p. 20). Drawing these three sources together, relational research projects can be viewed as inquiries carried out within relationships between the inquirer and participants, where coming to understand a complex social phenomenon of mutual concern is a collaborative endeavour valuing the varied perspectives. Relationality can be seen as the interpersonal dimension of the inquiry.

In light of this conceptualization of a relational research methodology, my study provides an example of how a CGT could be conceived of as a relational methodology. I was immersed in the research context, over an extended period of time, and sought to develop relationships with the participants. As I acted within a caring relation formed with each participant, and my notion of co-constructing data in the study, where data is constructed collaboratively through a growing relationship, positions both the learner-participants and me as knowing individuals, as Gunzenhauser (2006) described. I see this collapse of the duality of researcher-researched having fit with the nondualist ontology I explicated at the outset of this study. The relational stance of my research even extends to my initial coming to understand the data, as I sought out the learner-participants' understanding of our shared experiences through the interviews. This moves beyond the dichotomy of objectivity-subjectivity in the prioritizing of the relationality as informing emerging understandings. Most importantly, in considering the metaphor space of authoring as what it means to learn, I recognize that it was not only the learner-participants who were engaging in authoring, but I was authoring for myself alongside the learner-participants.

With my study as a singular example of how CGT could be recast as a relational methodology, I view my methodological contribution as extending an invitation for others using CGT to take on a relational orientation in carrying out a research project. I imagine the possibility of more rewarding data co-construction and more profound understanding of complex social phenomena through interpersonal insights. While I did locate one other example of a grounded theory study labeled as relational research, the relationality referred to using a professional practice relationship of midwifery to conduct discrete interviews with participants (Edie & Loewenthal, 2007), rather than a moral commitment to a relationship developed over time which respects all involved as knowing beings and informs interpretive actions. There are several possibilities for exploration. One possibility would be to simply add more studies as examples of a relational orientation to CGT for other researchers to consider and from which to learn. Another possibility would be to examine how Belenky *et al*.'s (1997) conception of "connected knowing" might inform how a researcher comes to understand through a relational orientation to CGT. Another possibility is to explore the tensions of a relational orientation, especially as data tends to be voluminous when the researcher is immersed in the setting over an extended period of time.

## Looking Forward

> To get outside of the imprisoning framework of assumptions learned within a single tradition, habits of attention and interpretation need to be stretched and pulled and folded back upon themselves, life lived along a Mobius strip. These are lessons too complex for a single encounter, achieved by garnering doubled and often contradictory visions rather than by replacing one set of ideas with another. When the strange becomes familiar, what was once obvious may become obscure. The goal is to build a complex structure in which both sets of ideas are intelligible, a double helix of tradition and personal growth. (Bateson, 1994, pp. 43-44)

I opened this final section with a quote from Mary Catherine Bateson because, for me, it represents what this learning journey has been about and causes me to look forward. In each of my research projects that I have shaped and learned from, I seek as a core endeavour to come to understand the learning of learners in mathematics classrooms. In so doing, I aim to attend to the becoming of learners through their encounters with others and with mathematics. I see these lessons as too complex to be noticed once and interpreted in just one way, but demands a cycling back to get a glimpse and to understand more richly the potency of learners' efforts in being and becoming. This inquiry, then, represents another moment for me as an inquirer in coming back to gain a different perspective of learning which enriches my growing sense of the profound nature of learning that forms who we are. Through the inquiry, I have reinvigorated my commitment to continuing this focus of inquiry and I am excited about future possibilities for research.

Alongside the learners in the study who were authoring their learning and themselves, I, too, have been authoring my learning and myself as a scholar and learner. As I close, writing the final words of this document that has represented an incredible opportunity to learn, I can assert that I was in a process of authoring. I have listened closely and have come to understand for myself nuances of learning within the context of mathematics. And within this, I have been authoring myself as an inquirer who thinks about learning in multiple and complex ways. I have increased my capacity to see these meanings as provisional and as invitations to continue to return and to wonder. I see myself as an author, one who is able to be generative when attending to the learning of others and self, one who is able to make sense of this learning in profound ways, one who is able to express to others through the use of words and ideas I have deliberated on with a sense of emerging authority. I see engaging in authoring as a continual process, one in which my experiences of authoring ideas and myself represented in this document provides a new starting place for future authoring. I have been engaged in the process of becoming, of self-authoring, knowing that who I speak myself into the world as will impact my future interactions and scholarship.

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# Appendix A
# Interactive Writing Prompts for Students

**Journal 1** (Feb 25, 2011)
What is one thing you have been trying to improve this year, to help you succeed at learning math?
What are do you want to learn through this course? List two specific things you will do to reach your learning goals.

**Journal 2** (Mar 7, 2011)
Response to my reply to Journal 1. (If they did not have it available and did not remember, they were to write what their goals or plans were for the week)

**Journal 3** (Mar 18, 2011)
On a scale of 1 to 10 (from least to most important), how do you rate homework as a tool for learning? Defend your rating in a paragraph. Use a specific example from your homework this week.

**Journal 4** (Mar 24, 2011)
What was the exponents and logarithms unit about? How did you figure that out?

**Journal 5** (April 7, 2011)
What evidence are you handing in to Mrs. Finley of working on a learning strategy? Describe the process of developing and using it.

**Journal 6** (May 5, 2011)
Write two report card comments for the "Math Learning Skills" course that communicates what you have been learning in the course. Provide an explanation of them.

**Journal 7** (May 25, 2011)
How will you prepare for the final math exam? Be detailed and explain how each will help you succeed.
Tell me about the summary sheets. What did you learn about creating a summary sheet?

# Appendix B
# Big Ideas Small Group Guides

## Big Ideas Small Group Session 1: Conversational Guide

Members: Jocelyn, Laurel, Nadia, Grace, Elise

**Introduction**
Let's get started. I'm going to take a few minutes to explain the purpose of the small group conversation and what we will be doing during the times when we meet. In the interactive writing and future interviews, we'll get a chance to look back on how you have been learning to learn math better. The small group conversation is meant to see that learning how to learn in action. I would like to learn about how learning strategies are developed and refined by listening to it in action.

Your group will begin by looking back at a completed homework assignment and come up with the big ideas or skills from the lesson. Here's how the group was formed: I thought hard about what we wrote about in the first two journals and what you wanted to work on; I chatted with Mrs. Finley; and I thought about where you were in your approach to learning math and goals. The five of you all indicated that while you had some pieces under control – like homework and studying – you wanted to get more out of it so you could understand the math.

In terms of the research process, I would like to you keep in mind that we've promised to keep what is said confidential – that it won't leave this group. I hope that means that you will feel safe to take risks, to say what you are thinking. What I will be learning from is the overall conversation – so don't feel like you need to say who you are each time you talk, just respond and offer ideas as they come to you.

**Part 1: Introductions**
That being said, I would like to go around the group so that we get to know each other a bit. Say your name. And then, say a couple of sentences about what you might already be doing with this strategy or why it caught your attention as something useful for you. I can begin, just to give you a couple minutes to think. I'm Janelle. In my last few years of teaching, I sometimes asked students to do this with their homework, but only checked to see it was complete. I'm looking forward to learning how this strategy can help you learn better, to understand.
Jocelyn Laurel Nadia Grace Elise

**Part 2: Strategy Work**
Thanks! Let's get down to working on the strategy – coming up with big ideas or skills after you have worked on a homework assignment. My hope is that we can figure out how this strategy will work best for each of you, and how it is a

strategy that helps you learn from doing your homework (not just doing a bunch of questions). I would like this to be a conversation among the five of you, and I'll prompt and help you out as needed.

So, let's jump in. Grab your homework assignment from graphing exponents and logs. You might not have the same assignments, but that might help us out!

1) Has anyone started? (Laurel, Nadia, Grace) What do you have? How did you do it? Were there questions you asked yourself?

2) What ideas came up while you were working on the assignment?

3) What skills did you find were repeated across questions?

4) Is this related to anything else you've done this year or last? What skills or ideas did you use that you learned before this unit? What was new in this lesson?

5) How many ways can you use to represent exponential and logarithmic functions?

6) Why does know how to graph the functions matter? What do you learn about the functions because you know how to graph them?

7) Are there things you noticed that came up that weren't in your notes?

8) What could you be asked to do on a test or exam related to this topic?

--have the outcomes handy from the program of studies

## Big Ideas Small Group Session 2: Conversational Guide

Members: Jocelyn, Laurel, Nadia, Grace, Elise

### Opening the Conversation

--catch up Nadia on the focus of the group, our work before spring break (involve a student)

1) In our last small group, we worked on exponential and logarithmic graphs. Did the topic come up on the unit test? Can you say a little more?

2) We tried the strategy once together before spring break. Have you tried it again? Is there a reason why or why not?

### Working on the Strategy

Let's get down to working on the strategy – coming up with big ideas or skills after you have worked on a homework assignment. Today we'll do this a little differently. Since you're at different places in your math classes, I've gone back to the beginning of the year – transformations – and selected a topic: reflections. What I'm going to ask you to do is work on the questions for the assignment for the next 15-20 minutes, and then we'll try out our strategy. I hope this gives you a more realistic look at what you might do on your own.

3) Tell me a little bit about how the homework went for this topic.

4) What ideas came up while you were working on the assignment?

5) What skills did you find were repeated across questions? [types of questions?]

6) How is this connected to what is on your page already? How can you show the connection?

7) Is this related to anything else you've done previous? What was new in this lesson?

8) Let's look back at the notes for this topic. How is what is on your page connected to your notes?

9) Are there things you noticed that came up that weren't in your notes? Are these important? Why?

10) Here's some sample diploma exam questions. With the person beside you, look through the pages and decide if any connect to the homework you did today. How did you know?

**About the Strategy**

11) Tell me what you see on your page.

12) Think about how we went through your homework assignment today. What was helpful in this process? What questions/comments helped you pick out important ideas?

13) Now that we've gone through this strategy again, how would you do it on your own for tomorrow's homework?

Next time: Bring a few examples of trying this strategy on your own!

## Big Ideas Small Group Session 3: Conversational Guide

Members: Jocelyn, Laurel, Nadia, Grace, Elise

**Working on the Strategy**

1) Tell me a little bit about how the homework went for this topic.

2) What were you thinking about as you finished up the assignment? What ideas came up while you were working on the assignment?

3) What skills did you find were repeated across questions? [types of questions?]

4) How is this connected to what is on your page already? How can you show the connection?

5) Is this related to anything else you've done previous? What was new in this lesson?

6) Let's look back at the notes for this topic. How is what is on your page connected to your notes? Are there things you noticed that came up that weren't in your notes? Are these important? Why?

**About the Strategy**

7) Tell me what you see on your page.

8) Think about how we went through your homework assignment today. What questions/comments helped you pick out important ideas? What mathematics did the strategy help you learn?

9) If you were to do this on your own tonight for your homework, what would you do? Would you change anything in the process? One time this week, I'd like you try this on your own. I'll ask you for a copy of it next week.

10) So help me out here. In the last small group you mentioned that this was a good strategy because: "It's a nice way to summarize it."; "It cleared up things that I was confused about"; "what we wrote down here broke down what it actually meant … So, I understand what to do when I have a question; "it's like almost like my notes, but simplified". To me that seems beneficial. Yet while there are these benefits, there's not a lot of excitement about it. Can you help me understand that?

11) If I was to tell math teachers about this strategy, what should I call it? Why should I encourage them to teach it to their math students?

12) If we met one more time as a small group, what learning strategy or process would be beneficial for you to work on and talk about with each other?

# Appendix C
# Summary Sheet Small Group Guides

### Summary Sheet Small Group Session 1: Conversational Guide

Members: Ashley, Shane, Chelsea, Kylee, Danielle

**Introduction**
Let's get started. I'm going to take a few minutes to explain the purpose of the small group conversation and what we will be doing during the times when we meet. In the interactive writing and future interviews, we'll get a chance to look back on how you have been learning to learn math better. The small group conversation is meant to see that learning how to learn in action. I would like to learn about how learning strategies are developed and refined by listening to it in action.

Your group will begin by looking back at a unit and create a summary to dig out big ideas/skills and make connections to prepare for a test. Here's how the group was formed: I thought hard about what we wrote about in the first two journals and what you wanted to work on; I chatted with Mrs. Finley; and I thought about where you were in your approach to learning math and goals. The five of you all indicated that you wanted to get better at studying for math – either adding a new strategy or refining something you've been trying.

In terms of the research process, I would like to you keep in mind that we've promised to keep what is said confidential – that it won't leave this group. I hope that means that you will feel safe to take risks, to say what you are thinking. What I will be learning from is the overall conversation – so don't feel like you need to say who you are each time you talk, just respond and offer ideas as they come to you.

**Part 1: Introductions**
That being said, I would like to go around the group so that we get to know each other a bit. Say your name. And then, say a couple of sentences about what you might already be doing with this strategy or why it caught your attention as something useful for you. I can begin, just to give you a couple minutes to think. I'm Janelle. In my last few years of teaching, I sometimes suggested students to do this for test prep and have had students in a research project mention it, but never explored it in depth. I'm looking forward to learning how this strategy can help you learn better and to prepare effectively for a test.
Ashley Shane Chelsea Kylee Danielle

**Part 2: Strategy Work**
Thanks! Let's get down to working on the strategy – developing a way to summarize the ideas and skills in a unit. My hope is that we can figure out how

this strategy will work best for each of you as a study tool, and how it is a strategy that helps you understand the content from the unit better. I would like this to be a conversation among the five of you, and I'll prompt and help you out as needed.

So, let's jump in. Grab your notes and assignments from exponents and logs and anything you've started! You might different resources, but that might help us out!

9) Has anyone started? (Danielle, Ashley, Kylee) What do you have? How did you do it? Were there questions you asked yourself (include/not include)?

10) What resources do you have to use? Will you use just one, or multiple resources?

11) What are the big topics in exponents and logs unit? How can you record it?

12) How can you show that the ideas or skills are connected?

13) How can you identify skills from previous units that you need to be able to do? Can you connect the ideas or skills to other units in the course?

14) What was new in this unit?

15) Are there things you noticed that came up that weren't in your notes?

16) Questions about today: What's your reaction to our work today? What's been most helpful? Is there anything that surprised you? What about the focus on seeing connections?

--have the outcomes handy from the program of studies

## Summary Sheet Small Group Session 2: Conversational Guide

Members: Ashley, Shane, Chelsea, Kylee, Danielle

### Opening the Conversation

In our last small group, we talked about the summary sheet/cue card strategy and shared some of the ways you had made a summary sheet for the exponents and logs unit. Today, we're going to give the strategy try as a group – moving back to the transformation unit.

1) Did anyone have anything they thought about afterwards? I was interested in the idea that sometimes you have to wait for a few lesson before starting to make the connections.

**Working on the Strategy**
Let's get down to working on the strategy – developing a way to connect the ideas and skills in the transformations unit.

2) Did anyone make a summary sheet for this unit in the fall? What do you have? How did you do it? Were there questions you asked yourself (include/not include)?

3) What resources do you have to use? Will you use just one, or multiple resources?

4) What are the big topics in the transformations unit? How can you record it? How did you decide?

5) Does the type of function matter? What kind of "base" functions did you work with?

6) How can you show that the ideas or skills are connected within the unit?

7) Can you connect the ideas or skills to other units in the course? Previous courses?

8) What was new in this unit?

**About the Strategy**
9) Tell me what you see on your page.

10) Think about how we went through the transformations unit today. What was helpful in this process? What questions/comments helped you pick out important ideas?

11) Now that we've gone through this strategy again for the transformations unit, how are you going to complete?

12) Is there anything that surprised you? What about the focus on seeing connections?

Next time: Please work on this a little more and let's get together next week to see how you completed it. We'll also take a look at sample diploma exam questions and how to connect them to your representation – you guys talked last time about being able to recognize the questions on an exam, without memorizing.

**Summary Sheet Small Group Session 3: Conversational Guide**

Members: Ashley, Shane, Chelsea, Kylee, Danielle

**Sharing the Transformation Summary Sheets**

1) Could each of you walk us through your summary sheet?
   --why did you decide to do that?
   --what do you like best about yours?

2) What do you like about one other person's that was different than yours? Do you have a question for someone else?

3) Are these sheets a list of content? Do they get at how the ideas are connected? (like Ashley mentioned last time)

4) One of the things I noticed in our work together last time, that our talk was about the ideas in the unit – we didn't use specific examples very often. Is there a place for specific examples on the summary sheets? Why or why not?

5) When I started by asking about the big ideas last time, it was tough to begin. A couple of you mentioned specifically that you usually begin lesson-by-lesson.

**Working on/with the Summary Sheets**

6) We could keep working on the summary sheets if they aren't done, to make more progress.

7) Let's get down to working on the strategy more – developing a way to connect the ideas and skills in the transformations unit. How might we represent that the ideas are connected? We don't have to go in the same order as the lessons.
   --where did these topics pop up again this year? Is it worth including here?

8) I brought sample diploma exam questions, from the PureMath30.com website. Let's split up some pages and match up a question with a place on your summary sheet. Work with a partner.

**About the Strategy Work Today**

9) What did you each get out of sharing the summary sheets today? About sharing yours? About listening to someone else?

10) What is your reaction to my questions about the connections – or about developing the connections? Is there anything that stood out for you?

Next time: Should we move on to the trig unit? What could you do to prepare for next time to jump straight into the work on the summary sheet together? One idea would be to skim through your notes/homework and then make a skeletal frame of the important ideas in the unit.

**Summary Sheet Small Group Session 5: Conversational Guide**

Members: Ashley, Shane, Chelsea, Kylee, Danielle

**Sharing the Transformation Summary Sheets**

1) Could each of you walk us through your summary sheet? Let us know what you were thinking about as you made it. [For the others, think of questions to ask.]
   --why did you decide to do that?
   --what are some of the important features?
   --what might you do different for the next one? Why?

2) We'll be leading the class tomorrow. Here's my image:
   --first: you'll each get to say a few things about your summary sheet for transformations. What would you like to say?
   --second: we'll divide up the class into small groups, and each of you will sit with another small group. What unit do you recommend for them to work on – older content (trans that you've done, trig that you're working on) or the most current (combinatorics or stats/prob)?

**Working on the Summary Sheets**

3) Would you like some more time to work on the trig sheets? Last time we left of with the huge amount of details at the beginning of the unit. Do you want to organize that or go on to the graphs of functions?

**About the Strategy Work Today**

4) What kinds of things were said today to help you decide what to put down on your summary sheet?

5) What makes this a good learning strategy for you? Why do you continue to use it? How is it helping you learn math? A couple small groups ago you guys mentioned that "for some reason it kind of comes together at the end" and "And then I get it! … That makes sense!" – how does this learning strategy support that?

6) We've talked and spent time making the summary sheets. Is that the end of learning with them, or will you use them in some way afterwards? What will you do with them?

# Appendix D
# Transition Small Group Guides

## Transition Small Group Session 1: Conversational Guide

Members: Teresa, Robyn, Vanessa

**Introduction**
Let's get started. I'm going to take a few minutes to explain the purpose of the small group conversation and what we will be doing during the times when we meet. In the interactive writing and future interviews, we'll get a chance to look back on how you have been learning to learn math better. The small group conversation is meant to see that learning how to learn in action. I would like to learn about how learning strategies are developed and refined by listening to it in action.

Your group will begin by looking back at a set of notes from class and see how we can make a transition from the notes to starting the homework assignment. Here's how the group was formed: I thought hard about what we wrote about in the first two journals and what you wanted to work on; I chatted with Mrs. Finley; and I thought about where you were in your approach to learning math and goals. Each of you indicated that it's challenging to start in on the homework and I suggested in various ways about looking at the big ideas to get to your goals of understanding.

In terms of the research process, I would like to you keep in mind that we've promised to keep what is said confidential – that it won't leave this group. I hope that means that you will feel safe to take risks, to say what you are thinking. What I will be learning from is the overall conversation – so don't feel like you need to say who you are each time you talk, just respond and offer ideas as they come to you.

**Part 1: Introductions**
That being said, I would like to go around the group so that we get to know each other a bit. Say your name. And then, say a couple of sentences about what you might already be doing with this strategy or why it caught your attention as something useful for you. I can begin, just to give you a couple minutes to think. I'm Janelle. I find that, as teachers, we often take for granted what students need to do when they move from the lessons in class into homework. So, I'm interested in learning from you how students can successfully make the transition … as we do this in slow motion!

Teresa Robyn Vanessa

**Part 2: Strategy Work**
Thanks! Let's get down to working on the strategy – transitioning from the notes in class to starting on homework. My hope is that we can figure out how this strategy will work best for each of you, and how it is a strategy that helps you learn from both notes and homework. I would like this to be a conversation among the three of you, and I'll prompt and help you out as needed.

So, let's jump in. Grab your notes from geometric series. You might not have the same notes, but that might help us out!

17) Has anyone started looking through the notes to see what is there? How did you do it? Were there questions you asked yourself?

18) Let's look at the notes together. What skills/formulas are in the notes that you might need? What concepts are in the notes that you might need?

19) What ideas came up while you were taking notes and listening to your teacher? Did you find something coming up repeatedly?

20) What skills or concepts did you use in the notes/examples that were from previous lessons?

21) What was new in this lesson? How are you going to look out for that in the homework assignment?

22) Why is the topic of geometric series and summation in this unit?

23) Now that we've gone through this strategy once, is there something you would like to do differently when you take notes in class next time?

24) What could you be asked to do on a test or exam related to this topic?

--have the outcomes handy from the program of studies

## Transition Small Group Session 2: Conversational Guide

Members: Teresa, Robyn, Vanessa

**Opening the Conversation**
--catch up Robyn on the focus of the group, our work before spring break (involve a student)

1) We tried the strategy once together before spring break. Have you tried it again? Is there a reason why or why not?

**Working on the Strategy**
Today we will work with the fundamental counting principle lesson.

2) One of the things you mentioned last time was the challenge of taking notes and listening to an explanation. Think back to the lesson – what ideas came up while you were taking notes and listening to your teacher? Did you write that down?

3) What skills/formulas are in the notes that you might need? What concepts are in the notes that you might need?

4) Do you want to write that down or not? Why did you make that decision?

5) What can you add in that's in your own words, rather than what the teacher said/wrote?

6) What skills or concepts did you use in the notes/examples that were from previous lessons?

7) What was new in this lesson? How are you going to look out for that in the homework assignment?

8) Let's look at the homework assignment. How are the questions connected to what you have written down on your page?

**About the Strategy**

9) Tell me what you see on your page.

10) When do you think you could use this strategy?

11) Think about how we went through your notes today. What was helpful in the process? What questions were asked that helped you pick out important ideas?

12) Now that we've gone through this strategy again, is there something you would like to do differently when you take notes in class next time?

Next time: Bring a few examples of trying this strategy on your own!

# Appendix E
# Prompts for One-on-One Interviews with Learner-Participants

The words/phrases in italics in the following interview guides were place holders where I added specific examples from the learner-participant's data.

### Interview 1: Conversation Guide

1. In math class, you're in the middle of combinatorics. What is the most recent topic you have done? What page are you on? How did you learn from this page?

2. Homework is a major component of math class. In journal 3, I asked you to rate it as a learning strategy. One of the students said, "I'd rate it a 4, only because it's not really engaging but repetitive and kind of boring."
    a. Can you comment on that, about homework not being engaging?
    b. The student went on to say, "I understand that by completing my homework I'll retain information for the exams and quizzes … but in the same sense, is retaining information really learning?" Can you comment on this part?
    c. What is the last homework assignment that you did? What did you learn from doing the homework?
    d. How can you tell when you've done will with the homework?

3. I am noticing that many students are talking about understanding for math class – that it is necessary and that they desire it.
    a. What does it mean for you to understand in math class?
    b. How do you decide if you understand the day's lesson or not? How does that compare to doing a question? Can you give a specific example?

4. I went through all the ways of learning math you have mentioned in journals or tried out in class/small groups. Here's the list – which ones are learning strategies? Can you pick a strategy that you rely on and tell me more about it? [*specific_key_example*]
    a. Where did the strategy come from? Why did you start using it? [*example_from_journal*]
    b. How have you improved the strategy? How important is it for you to improve the strategy? [*example _from_other_data*]
    c. Why do you use it now? [*example _from_other_data*]
    d. How does the strategy help you ***learn*** math?
    e. Do you think you can use the strategy in other courses? Why? What makes it transferable? What makes it specific to math? [not that interested]

Is there another strategy where you changed the way you use it? Can you tell me more about it?

5. In journal *number*, you wrote that you were “*select_quote*.” Can you tell me more about trying? [As a back up: In journal 1, you wrote that you were “trying to *select_quote*” with a particular learning strategy. Can you tell me more about that – how often, effort, think about it after?]

6. In the small group conversation we’ve been working on *topic_for_small_group*.
    a. In our last small group session, you said, “*select_quote*.” Can you tell me more? [ could do a second quote if necessary]
    b. I often ask whether you think something is important to write down. Let’s look at this example [a sheet from the small group in my file], can you take me through it? Why was that important to write down?
    c. Tell me about your thoughts on the small group sessions.

7. I asked you about combinatorics at the beginning of our conversation, but I didn’t ask:
    a. Are combinatorics worth learning?
    b. Are you getting better at learning math by learning combinatorics?

8. In the last two weeks, who have you talked with about how you learn math? Are you getting better at learning math? In what ways? [How has it helped you become a better math learner?] [*an_example_from_other_data _or_what_I_noticed*]

9. The “Mathematical Learning Skills” course is quite unique. It’s an optional course, on top of taking Pure Math 30. What is the course for? [*anything_from_journal1*]
    a. What brought you to the learning strategies class?
    b. What are you learning in the class? What about learning strategies? Who have you learned it from? [what has Mrs. F introduced, “pulling unit together”]
    c. Could you tell me about a day in the class last week, that is typical of other days. How did you decide what to do that day?
    d. How has the learning strategies class changed your approach to math class? How have you been doing in math class with the addition of this course, compared to last year/semester?

10. Do you have any questions for me?

Also attached to the interview guide was a list of the ways of ways the learner-participant was going about learning mathematics that was recorded in interactive journal writing, small group sessions, and field notes.

**Interview 2: Conversation Guide**

1. The math exam isn't that far away. In your last journal you mentioned "*prep_for_exam*".
    a. Can you tell me a bit more?
    b. How will studying for the final exam be different from last year or from the mid-term?
    c. Do you learn when you prepare for final exams? [reviewing/studying/learning]

2. I asked you to bring your best example of a learning strategy that you've use this year. Can you tell me about this?
    a. How does this help me see you as succeeding in math class?
    b. How does this help me see you as better prepared for future math courses than you were at the beginning of the year?
    c. I've been thinking about what you've been telling me about your math learning during this research project. I've noticed that *interpretive_comment*. How does this example help me understand your learning?
    d. Does this example demonstrate how you've been getting better at the process of learning math?

3. With finishing grade 12 soon, I know you've been looking toward *next_years_plans*.
    a. What will be your next experience with math after high school?
    b. Math instructors next year are often skilled mathematicians, rather than teachers, and are often less accessible. Are you ready to succeed even if that demands more of you? [Do you think you have a comprehensive set of processes/strategies to learn math? ]
    c. What have you learned about yourself as a learner that will impact your success next year?
    d. In the last interview I asked you if you were getting better at learning math. Do you think you've become more aware of how you learn math? What have you done to figure out how you learn?

4. Being involved in a research project is a new experience for you. Why did you get involved?
    a. How would you describe the purpose of the research to a friend/someone else?
    b. What have you done over the last four months that counts as being part of the research project?
    c. Has participating mattered to you?
    d. What do you think might be one of the significant things you've said or written to me during the research?

5. Any questions or other comments?

# Appendix F
# Prompts for Conversations with Mrs. Finley

## Conversation 1: Early Interactive Journal Writing & Forming Small Groups

1. General impressions of the interactive writing (#1 and 2)
   --any students stand out?
   --surprises?
   --specifically:
      --bit more about Grace
      --Robyn –what would be the best support? (spinning wheels)

2. Small group work
   --to work on evidence building
   --where do these fit: more about Elise; Chelsea
   --I have a preliminary set of groups

## Conversation 2: Mid-Study Discussion about Interactive Journal Writing

Journal 4 – what the exponents and log unit is about.

> Shane: I've never thought about what this unit actually "meant". I mean, we did learn that there are applications such as exponentials scales like the Rictor Scale and Decibles. I saw that there were things we learned in earlier units re-appear in this one. I've really never thought about what it "means". Perhaps if I did it would be easier to learn, kind of like knowing what the math I'm learning is in the grand scheme of things. Like, in biology, I've taken a huge interest in the brain since gaming (vide gaming) is basicly neuroscience.
>
> Jocelyn: Logs and exponents have to do with graphing growth and decay within a function. The growth and decay can be used to calculate many different things such as earthquake intensity, bacteria growth, pH of an acid or base or geometric sequences. I figured this out through listening to Mrs. Finley in class and reading through her notes.

Journal 5 – range of evidence of learning skills (spreadsheet)

In the last small group session where we look at the big ideas after homework, here's a couple comments from the students:

> Elise: Well, it really showed the function notation of it. And what we wrote down here broke down what it actually meant. So, I understand what to do when I have a question.

Laurel: Oh, mine was just strictly going through the notes and writing it down and then I would do questions. But this is kind of just more, straight forward, like what you're supposed to do. I think there is more information on my notes than was necessary.

Grace: it's like almost like my notes, but simplified. So it's not all the examples and stuff, so it's easier to understand.