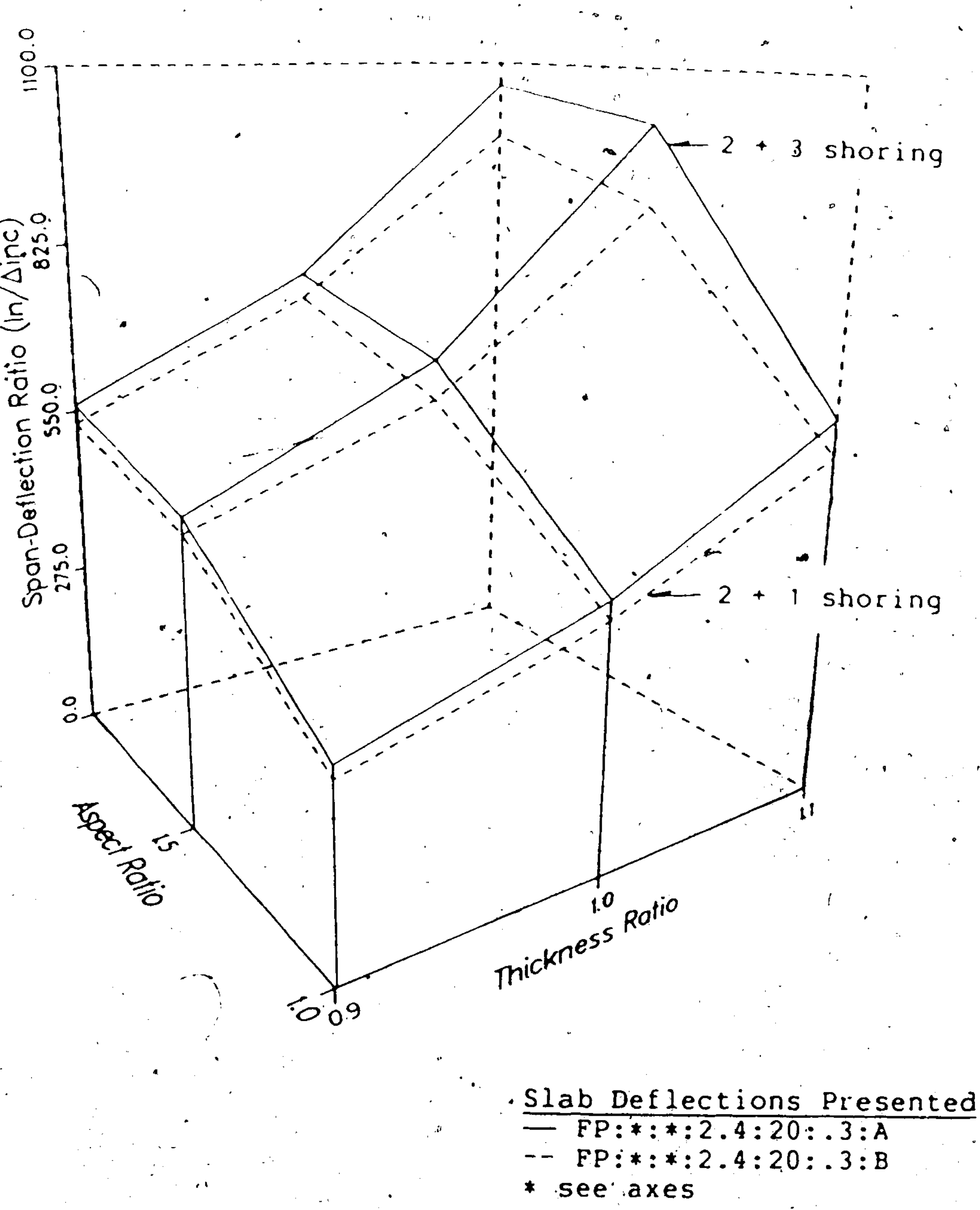
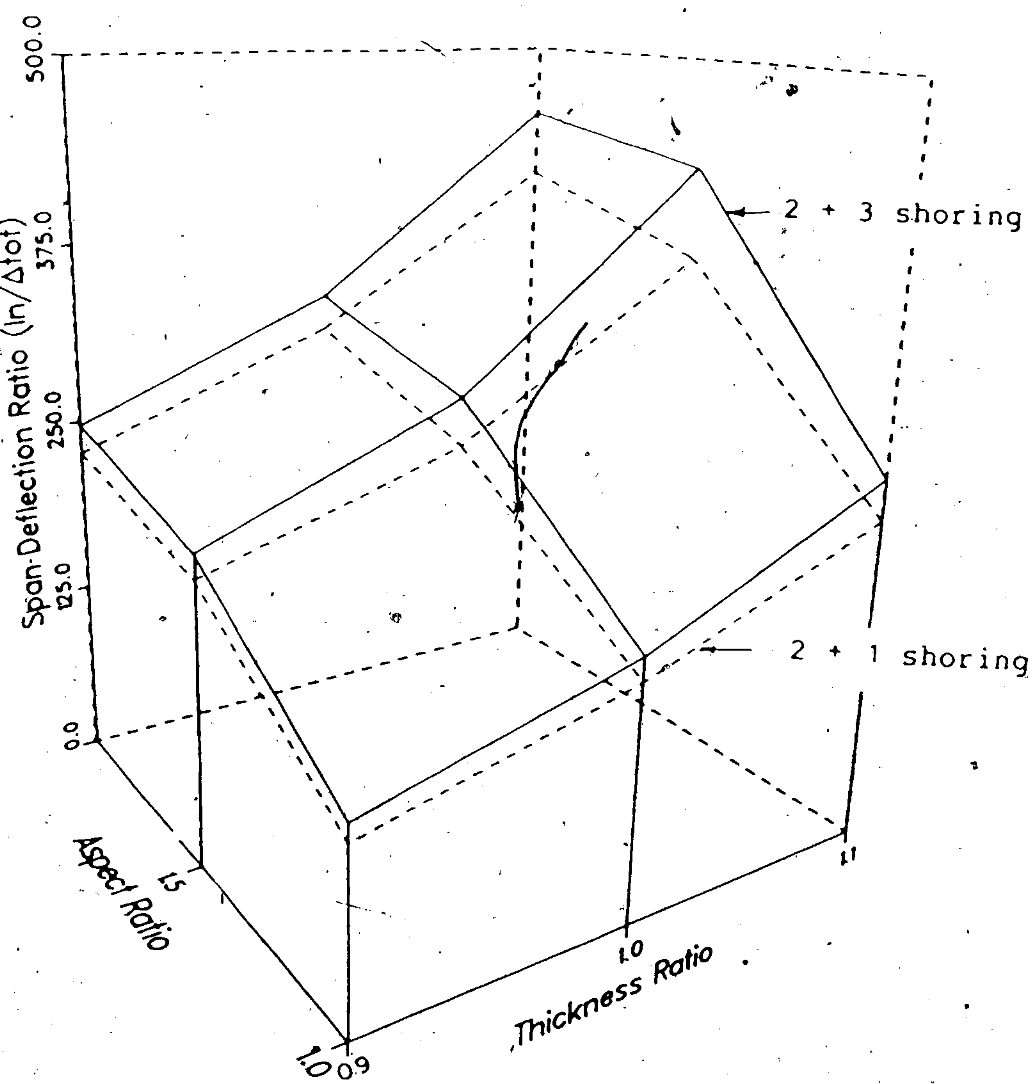


Figure 5.5 Effect of Different Shoring Systems on Calculated Incremental Deflections



Slab Deflections Presented
— FP:*:*:2.4:20:.3:A
-- FP:*:*:2.4:20:.3:B
* see axes

Figure 5.6 Effect of Different Shoring Systems on Calculated Total Deflections

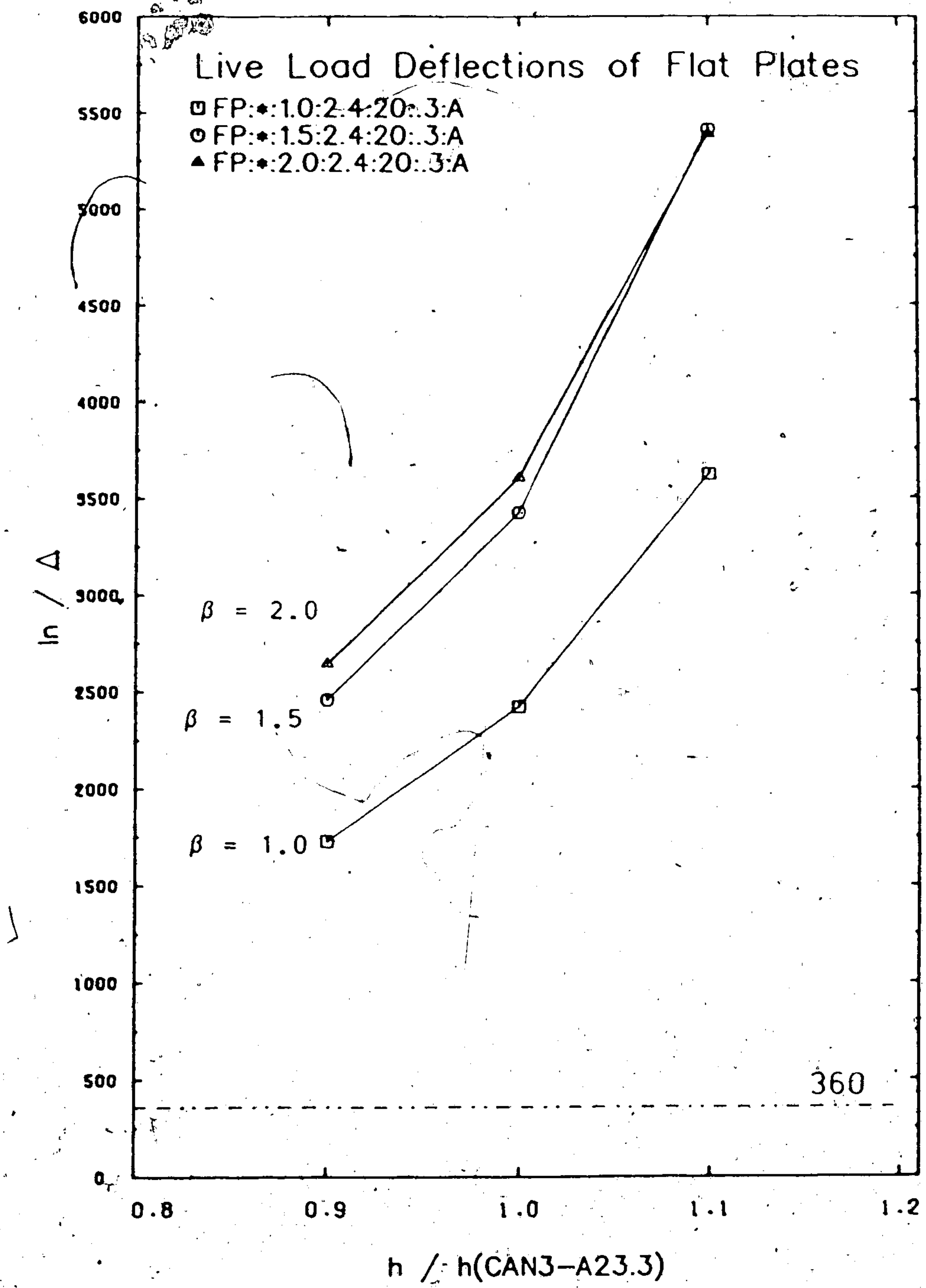
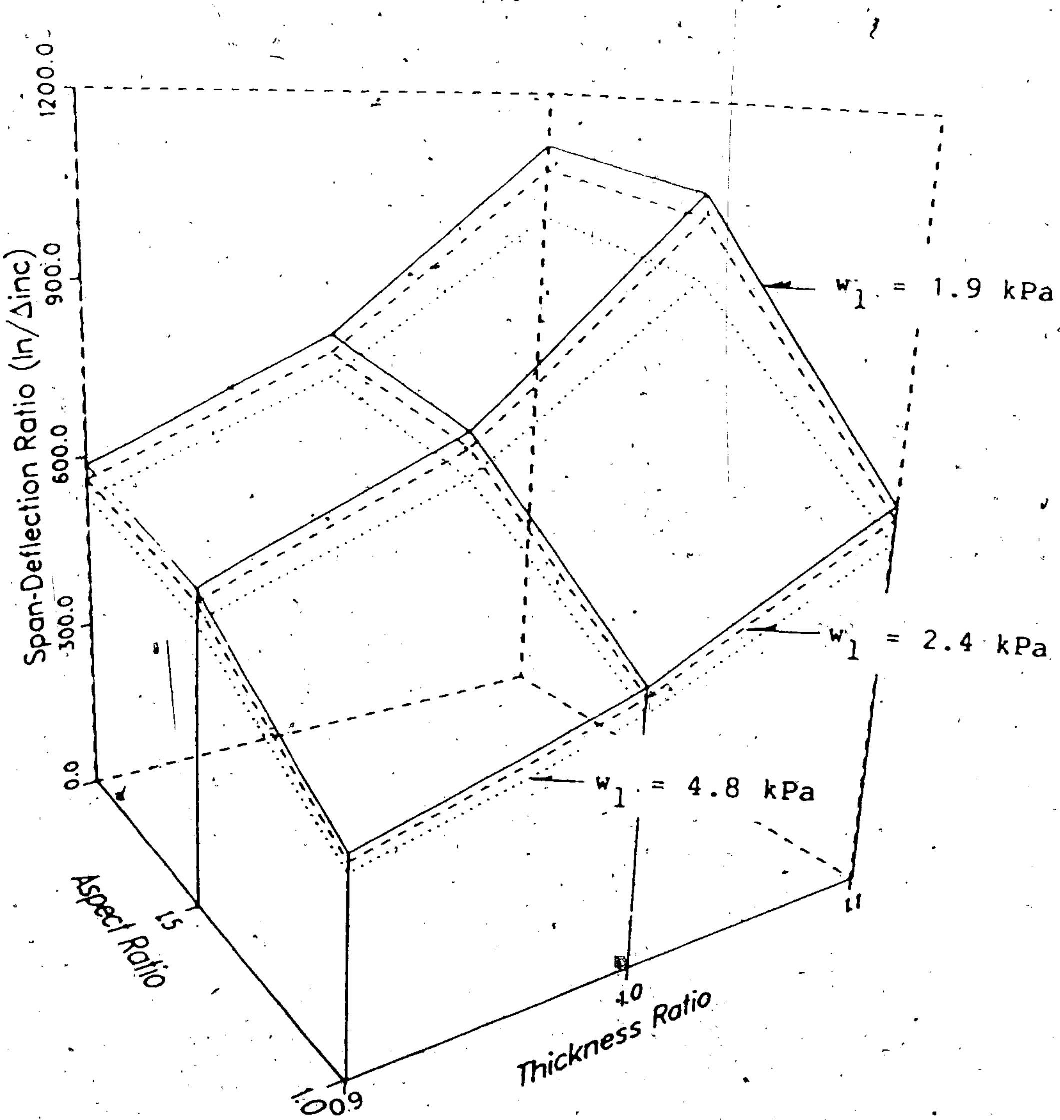


Figure 5.7 Calculated Live Load Deflections of Flat Plates



Slab Deflections Presented
— FP:*:*:1.9:20:.3:A
-- FP:*:*:2.4:20:.3:A
.. FP:*:*:4.8:20:.3:A
* see axes

Figure 5.8 Effect of Different Live Loads on Calculated Incremental Deflections

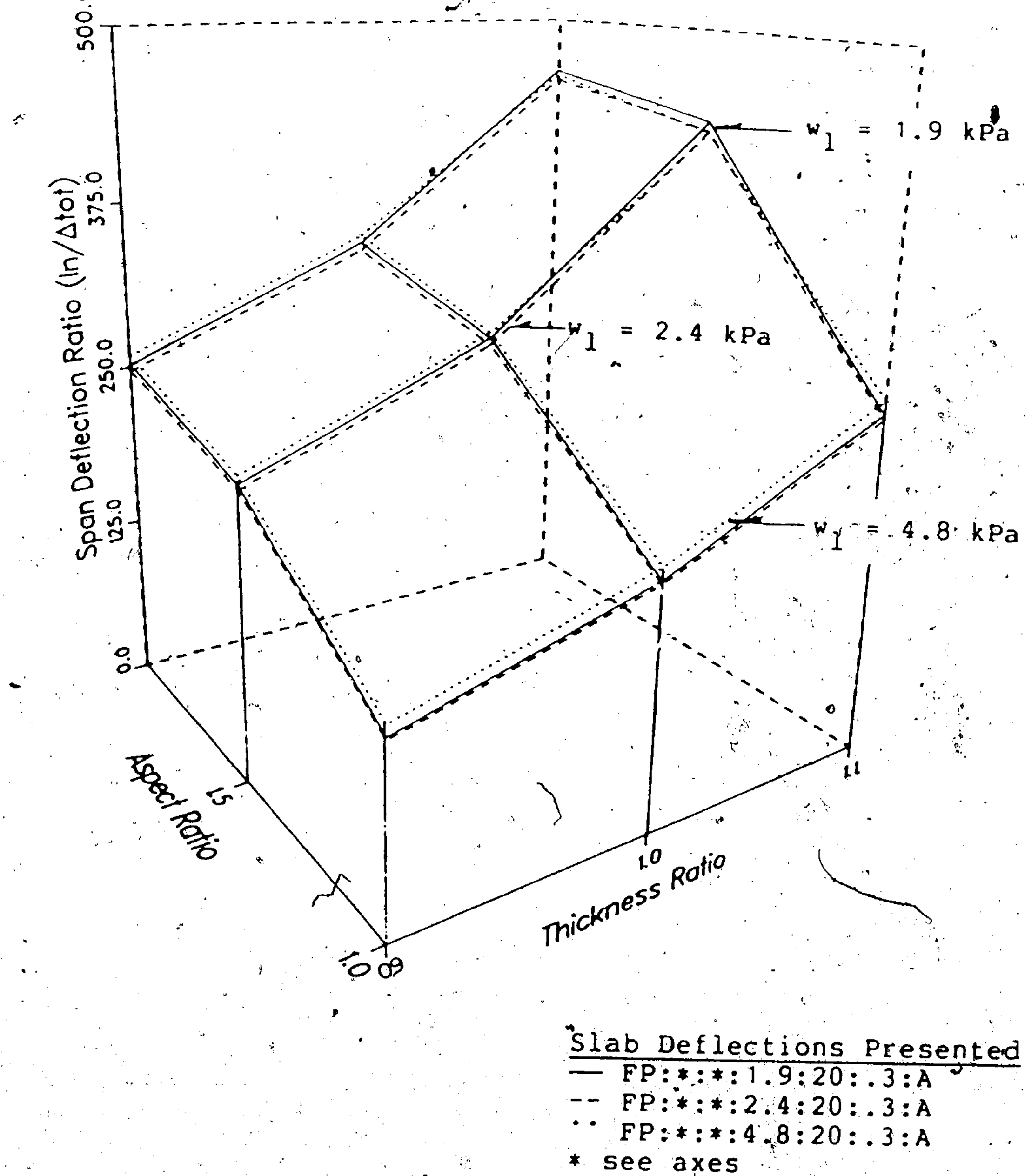


Figure 5.9 Effect of Different Live Loads on Calculated Total Deflections

slabs under office loading and twelve percent greater than slabs under apartment loading. The largest difference in calculated incremental deflections occurred with an aspect ratio (β) of 1.0 because of the use of the live load reduction factor with office and apartment loads.

Effects of different shoring systems and different live loads on incremental deflections for flat plates with β = 1.0 are compared in Figure 5.10. Changing from office to retail loading on slabs constructed using a 2 + 3 shoring system increases incremental deflections by six percent. Constructing an office loaded slab using 2 + 1 shoring instead of 2 + 3 shoring increased the deflection by eight percent. This greater effect of construction loads on incremental deflections is further emphasised when one considers that the retail loaded slab should fail by punching shear according to design calculations.

5.2.3 Geometric Properties

The location of the minimum span-deflection ratio is given in Table 5.1. For slabs with β = 1.0 the minimum span-deflection ratio occurs at midpanel, not along the column strip. For aspect ratios of 1.5 and 2.0, the minimum ratios were found along the column strips. The parameter study indicated that, for slabs using an effective modulus of rupture of $.30\sqrt{f'_c}$, the maximum deflection would occur in

Table 5.1 Location of Minimum Span-Deflection Ratios

Slab Parameters		Flat Slab with Drop Panels		Flat Plate		Flat Plate with Edge Beams	
β	$\frac{h}{h_m}$	f_e	f_r	f_e	f_r	f_e	f_r
1.0	0.9	C	C	C	A,C*	D	B
1.0	1.0	D	C	D	C	D	D
1.0	1.1	D	C	D	C	D	D
1.5	0.9	A	A	A	A	-	-
1.5	1.0	A	B	B	A	-	-
1.5	1.1	B	B	B	A	-	-
2.0	0.9	A	A	A	A	-	-
2.0	1.0	A	B	B	A	-	-
2.0	1.1	B	B	B	A	-	-

Notes:

A-exterior panel, column strip deflection

B-interior panel, column strip deflection

C-exterior panel, midpanel deflection

D-interior panel, midpanel deflection

* result from analysis using 2+1 shoring system

$f_e = .30\sqrt{f'_c}$

$f_r = .60\sqrt{f'_c}$

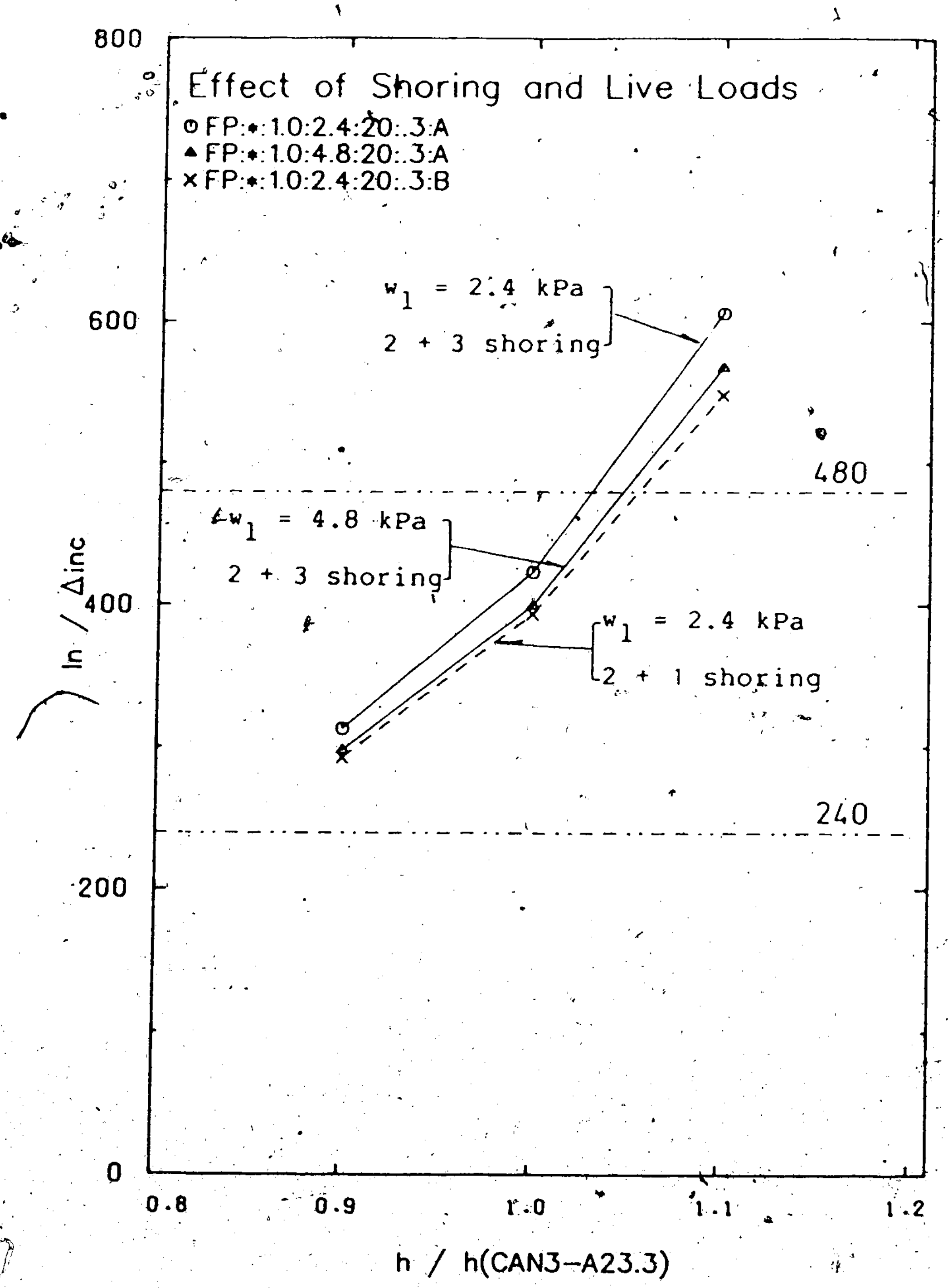


Figure 5.10 Comparison of the Effect of Different Live Loads or Shoring Systems on Calculated Incremental Deflections

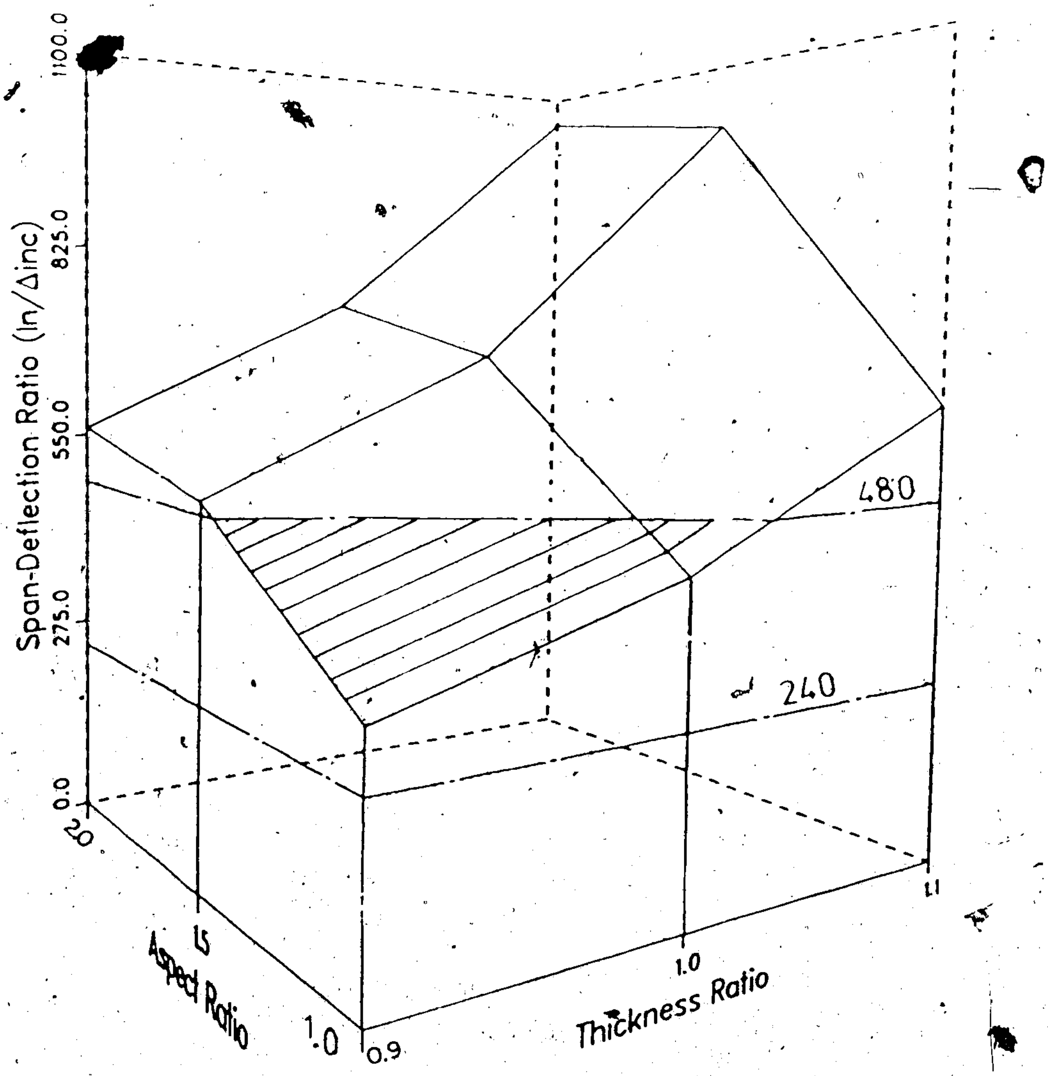
the exterior panel when the slab thickness was less than the code minimum, and was located in the interior panels with slabs meeting code requirements.

Figures 5.11 to 5.22 present the performance of the different types of slabs constructed using 2 + 3 shoring, 20 MPa concrete, under office loads with $f_e = .30\sqrt{f_c}$.

5.2.3.1 Flat Plates

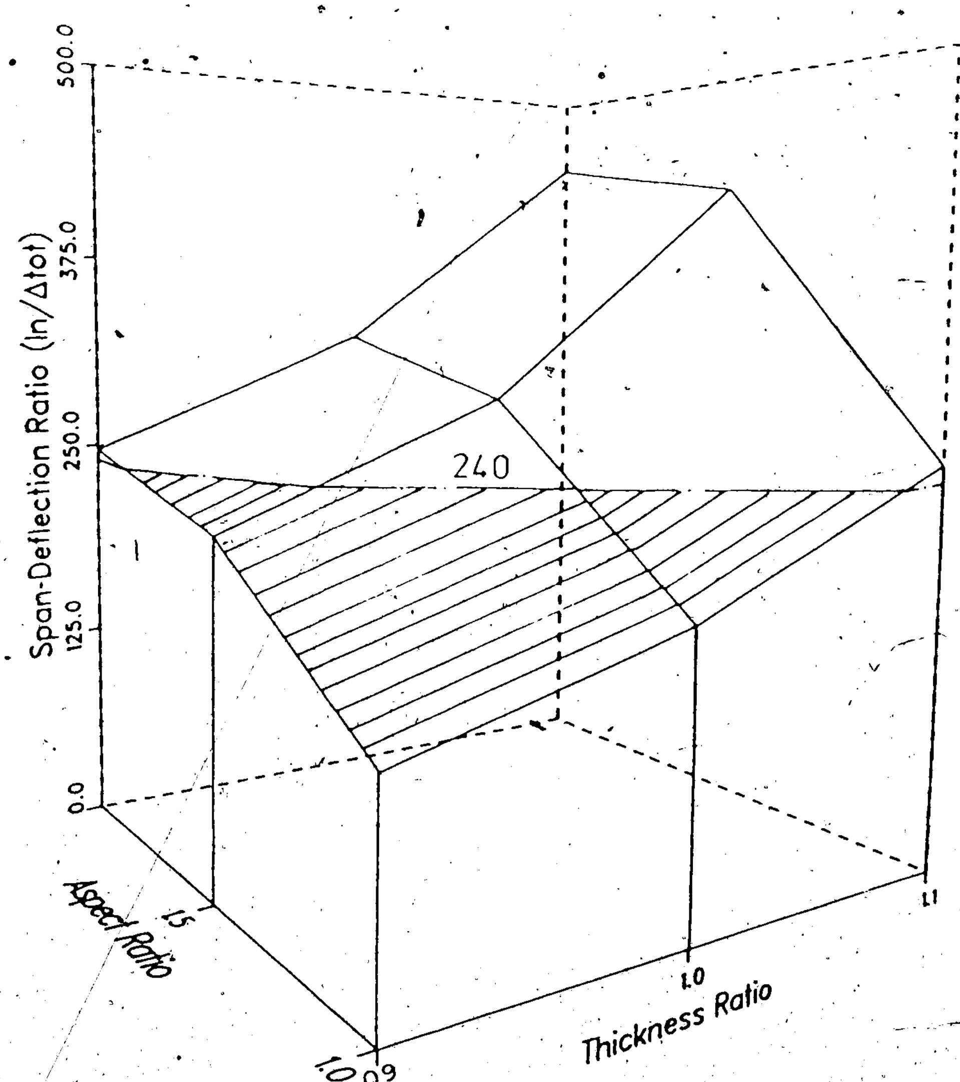
Figures 5.11 and 5.12 show the performance of flat plates simulated in the study when considering calculated incremental deflections, from two months to five years, and total deflections at five years. The shaded areas highlight the regions where the calculated deflections exceed the deflection limits indicated. Calculated incremental deflections shown in Figure 5.11 exceed the deflection limit of $\frac{\ell_n}{480}$ when the panel aspect ratio (β) is equal to 1.0 and the slab thickness is less than 1.03 times the code minimum thickness (h_m). This is also the case when the thickness is $.90h_m$ and β less than 1.5. Simulations indicated that h_m would satisfy the deflection limits for β greater than 1.1. In all cases $.90h_m$ would meet the deflection limit of $\frac{\ell_n}{240}$.

Figure 5.12 shows calculated total deflections of the flat plates only satisfy the deflection limit $\frac{\ell_n}{240}$ when the thickness is $1.10h_m$ if β = 1.0 and if, β = 2.0 when the thickness was $.90h_m$. Flat plates will meet the deflection limit for h_m if β is greater than 1.3.



Slab Deflections Presented
FP:*:*:2.4:20:.3:A
* see axes

Figure 5.11 Minimum Span-Calculated Incremental Deflection Ratios of Flat Plates Analysed



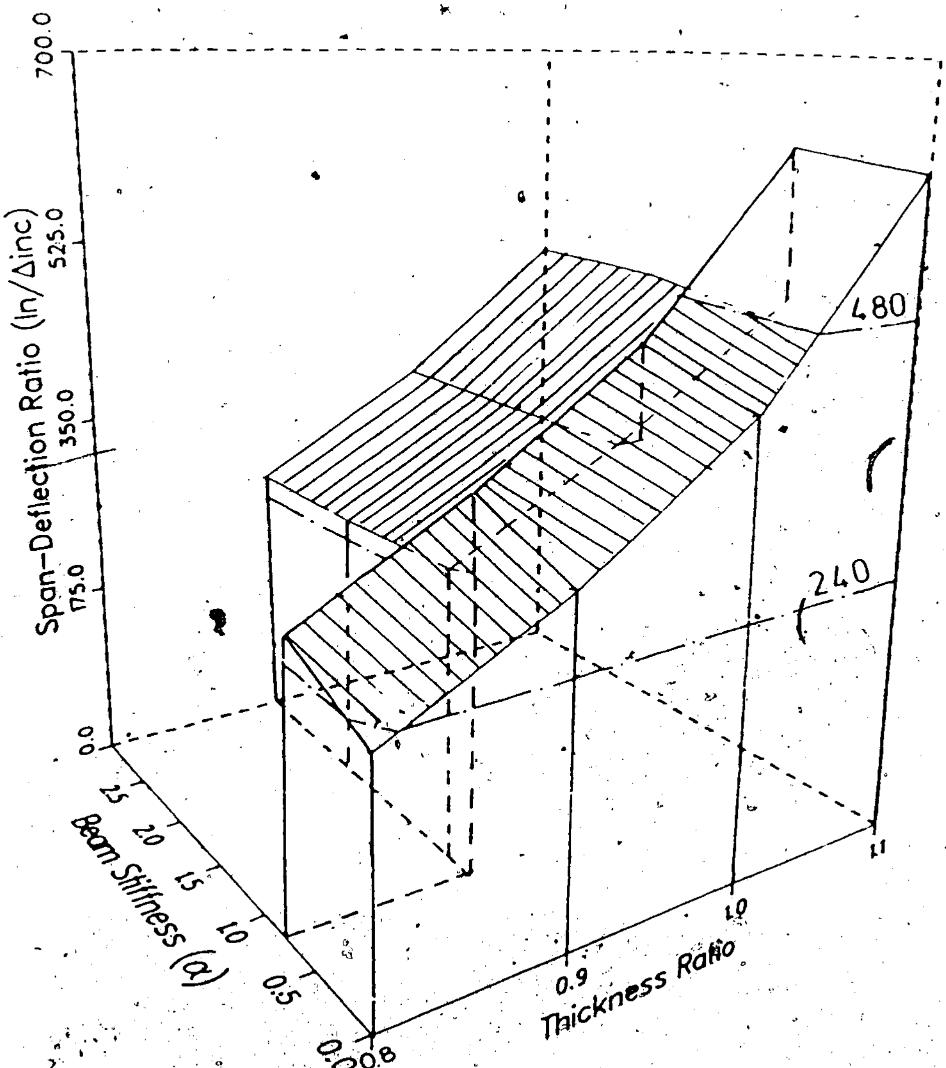
Slab Deflections Presented
FP:*:*:2.4:20:.3:A
* see axes

Figure 5.12 Minimum Span-Calculated Total Deflection Ratios of Flat Plates Analysed

5.2.3.2 Flat Plates with Edge Beams

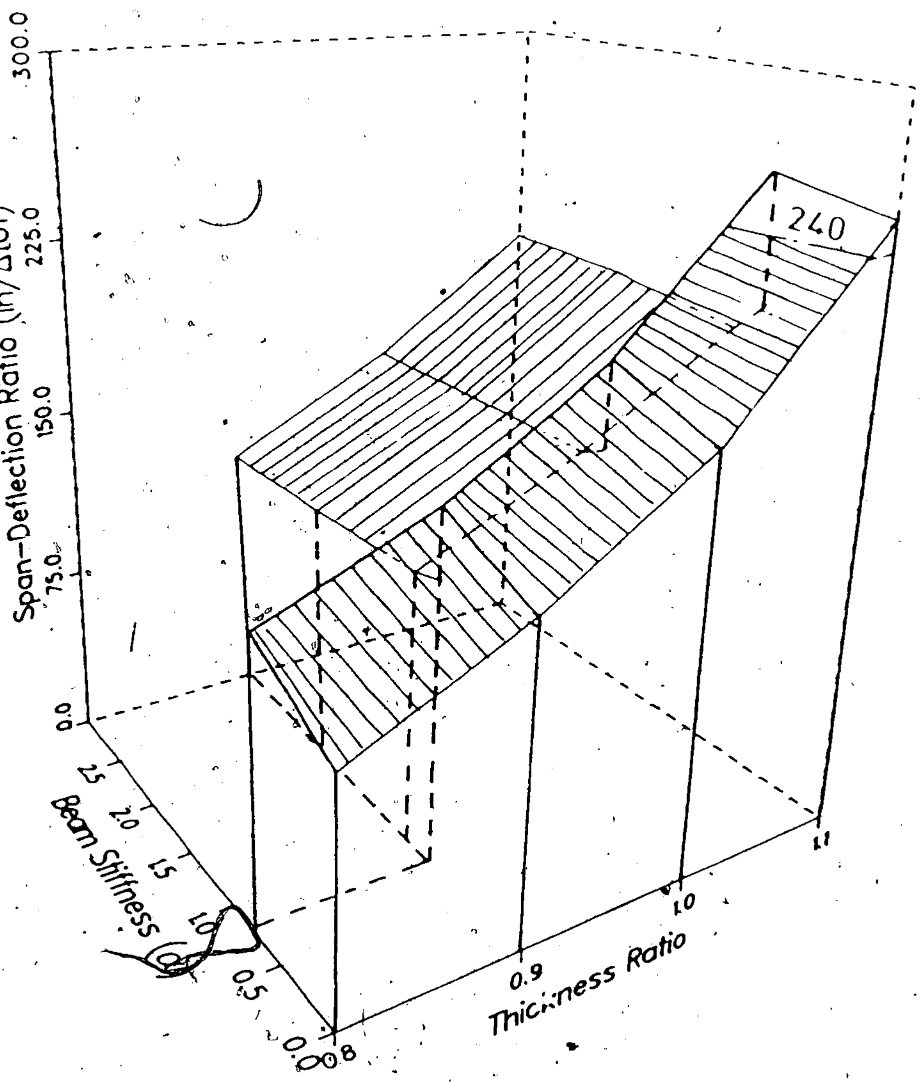
Figures 5.13 to 5.15 present calculated deflections of flat plates, β = 1.0, with edge beams. Calculated incremental deflections for slabs with edge beams with a beam slab stiffness ratio (α) greater than 0.8 exceeded the deflection limit of $\frac{\ell_n}{480}$ but not $\frac{\ell_n}{240}$. Calculated total deflections (Figure 5.14) also exceed the deflection limit when α is greater than 0.8. Figure 5.15 shows the effect of edge beams on calculated incremental deflections for slabs with various thickness. The minimum span-deflection ratio increased with stiffer beams. The greatest improvement, an increase of eleven percent, occurred when α increased from 0.0 to 0.8. The difference between calculated minimum ratios when α=2.0 and α=3.0 was less than point two percent and decreased slightly when the slab thickness was 145 or 160 mm. Although the inclusion of edge beams improve the serviceability of flat plates the improvement is less than predicted by the code.

Reasons for the minimal changes in minimum calculated span-deflection ratios with stiffer edge beams are explained in Figures 5.16 and 5.17. The figures show calculated deflections of slabs FP:160:1.0:2.4:20:.3:A and BP:160:0.8:2.4:20:.3:A under construction loads. The inclusion of edge beams (Figure 5.17) decreases the calculated deflections of the exterior panel, but increases the deflections of the



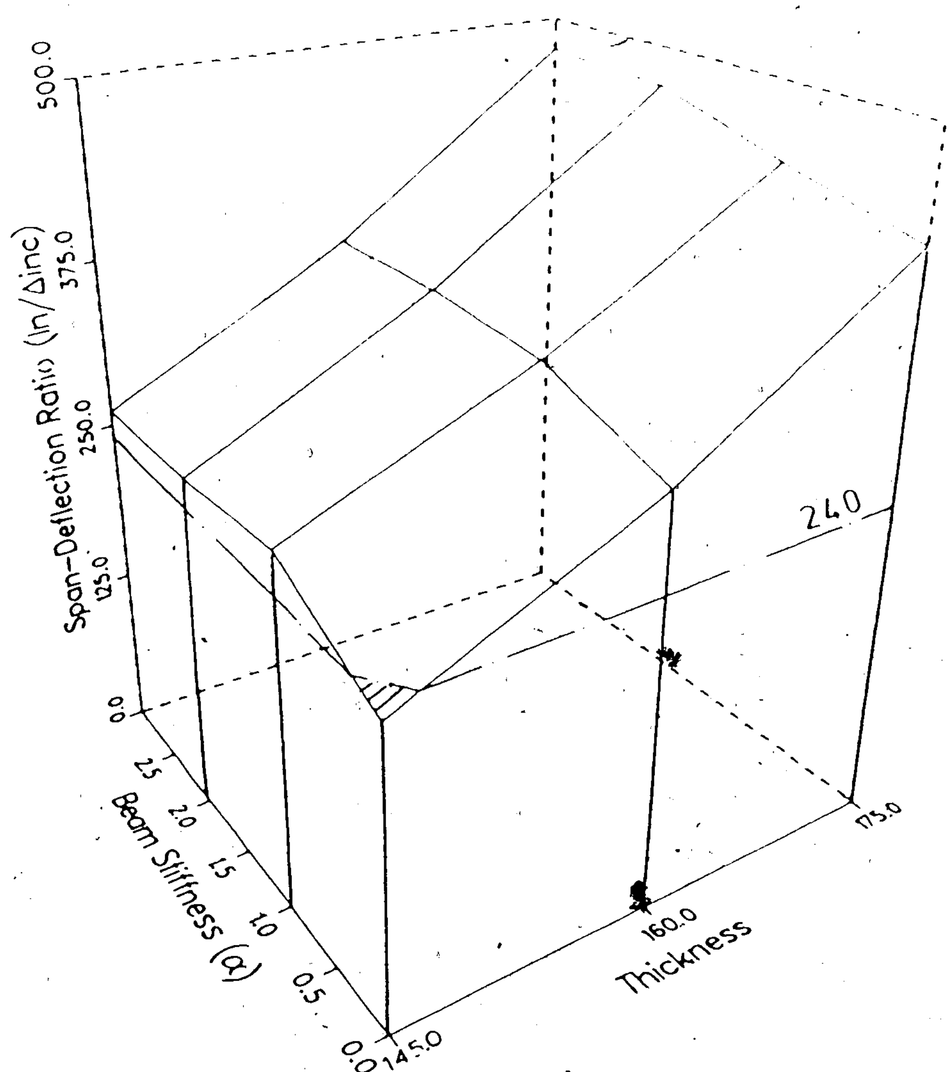
Slab Deflections Presented
BP:*:*:2.4:20:.3:A
* see axes

Figure 5.13 Minimum Span-Calculated Incremental Deflection Ratios of Flat Plates with Edge Beams Analysed:slab thickness ratio



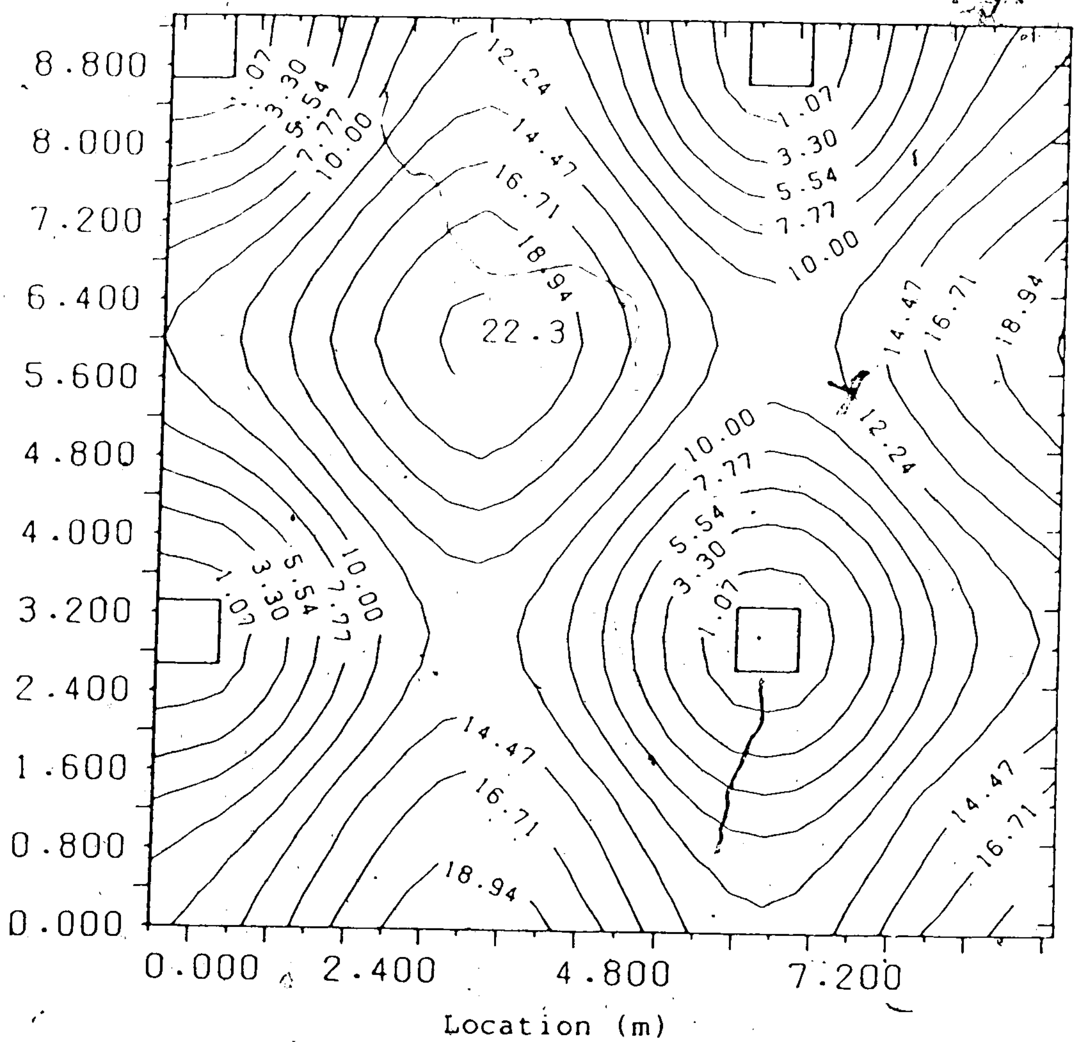
Slab Deflections Presented
BP:*:*:2.4:20:.3:A
* see axes

Figure 5.14 Minimum Span-Calculated Total Deflection Ratios of Flat Plats with Edge Beams Analysed



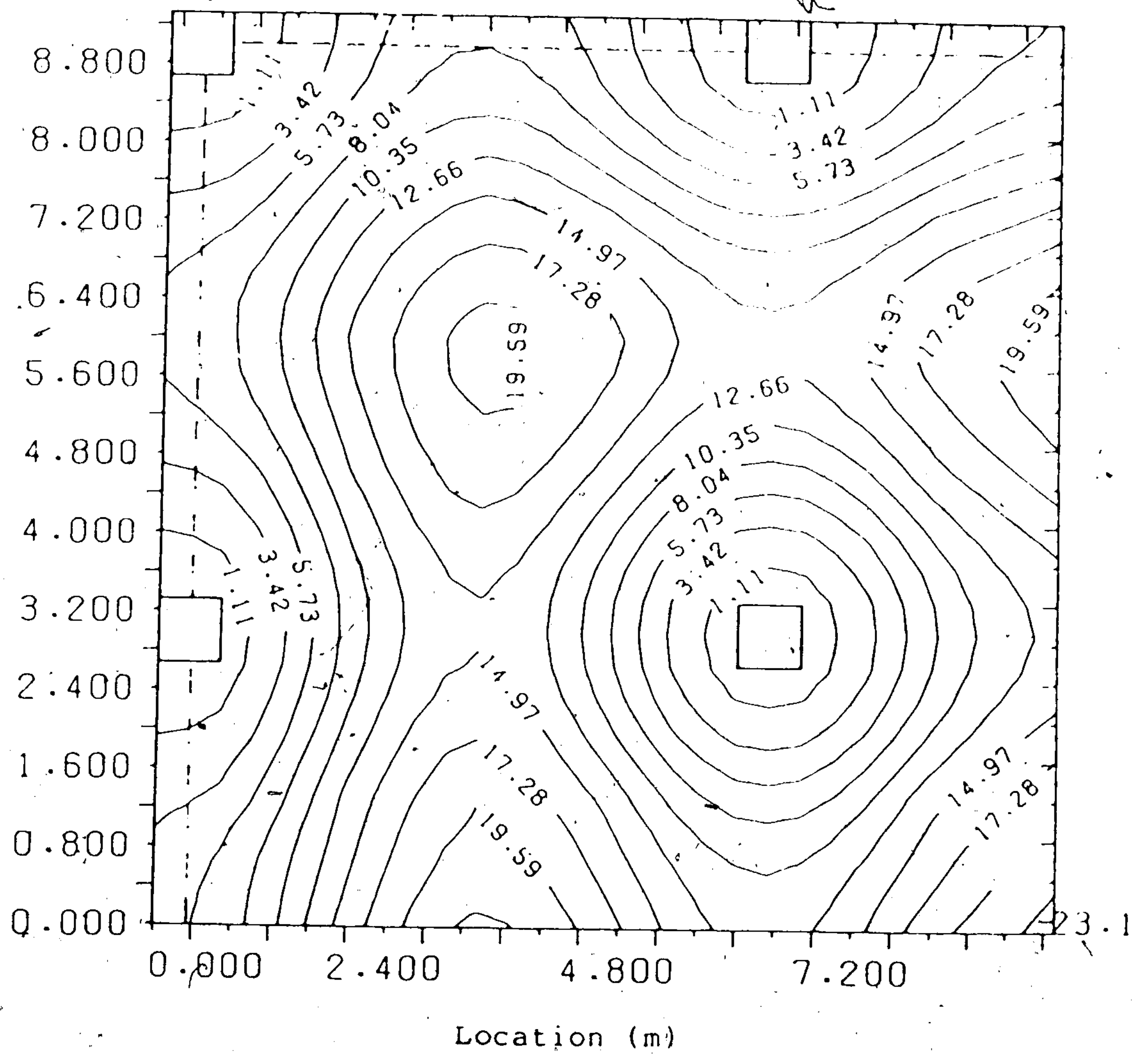
Slab Deflections Presented
BP:*:*:2.4:20:.3:A
* see axes

Figure 5.15 Minimum Span-Calculated Incremental Deflection Ratios of Flat Plates with Edge Beams Analysed:absolute slab thickness



Deflection Contours (mm)

Figure 5.16 Calculated Deflections of FP:145:1.0:2.4:20:.3:A under Construction Loads



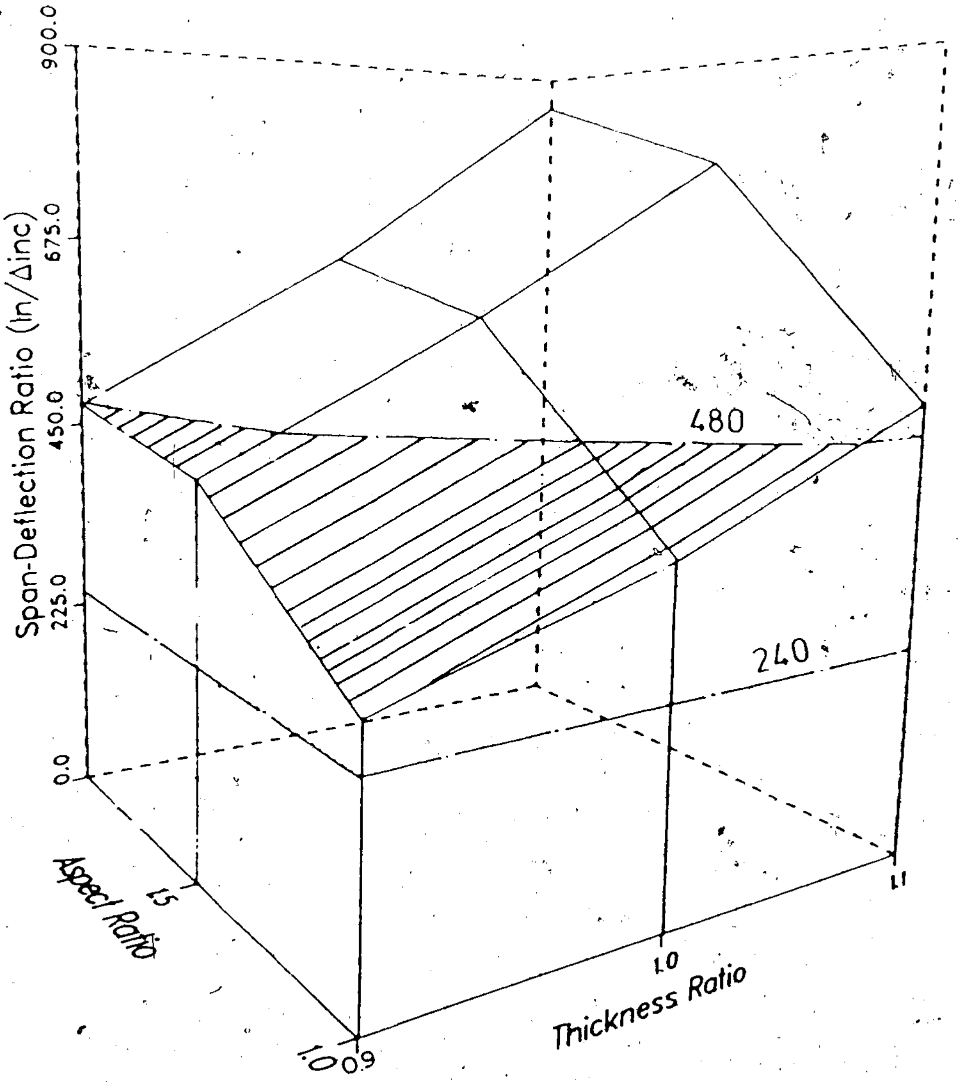
Deflection Contours (mm)

Figure 5.17 Calculated Deflections of BP:145:0.8:2.4:20:.3:A under Construction Loads interior panel. Therefore the inclusion of edge beams will decrease the calculated deflections of exterior panels but increase the deflections of interior panels. Since the interior and exterior deflections were of similar magnitude with no edge beams, the overall improvement in performance is not significant and in some cases poorer.

5.2.3.3 Flat Slabs with Drop Panels

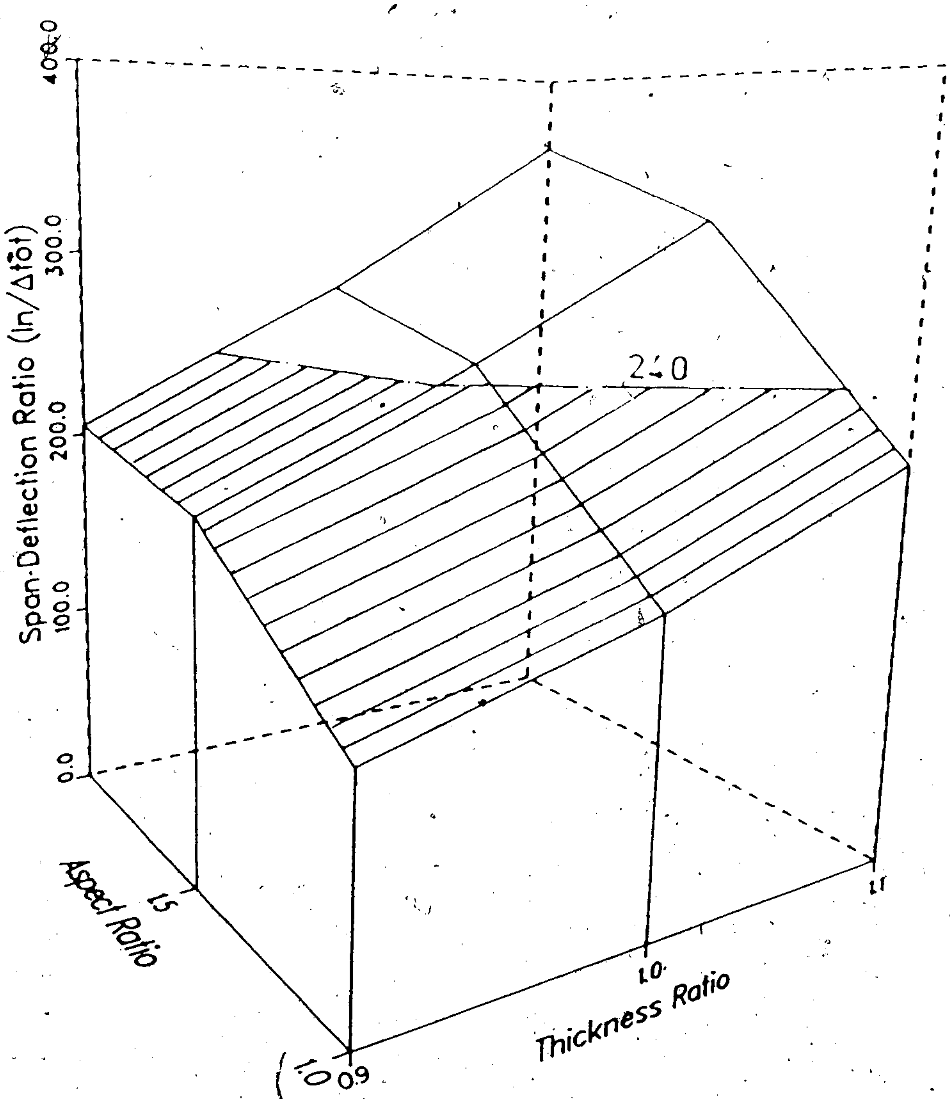
CAN3 A23.3 permits the slab thickness to be reduced by ten percent if drop panels are included in the slab configuration. Figure 5.18 illustrates the predicted performance of this type of slab. The results indicate that when h_m (including reduction for drop panels) is used, the slab would satisfy the incremental deflection limit $\frac{\ell_n}{480}$ when β is greater than 1.3. The figure also indicates that when β= 1.0 the slab must be eight percent thicker than code minimum to satisfy the deflection limit. When β= 2.0, $.90h_m$ would be satisfactory.

The performance of this type of slab when considering calculated total deflections is shown in Figure 5.19. The figure indicates three major points. They are; a) the slab must be thicker than $1.10h_m$ to satisfy the deflection limit when β = 1.0, b) $.95h_m$ would be satisfactory when β = 2.0, and c) the minimum code thickness will meet the deflection limit when β is greater than 1.5.



Slab Deflections Presented
DP:*:*:2.4:20:.3:A
* see axes

Figure 5.18 Minimum Span-Calculated Incremental Deflection Ratios of Flat Slabs with Drop Panels Analysed



Slab Deflections Presented
DP:*:*:2.4:20:.3:A
* see axes

Figure 5.19 Minimum Span-Calculated Total Deflection Ratios of Flat Slabs with Drop Panels Analysed

The study also investigated ways of decreasing the calculated deflections of slabs with drop panels. The investigation considered the effect of increasing the relative thickness of the drop panels. Figure 5.20 shows the effect on incremental deflections when $\beta = 1.0$. If the drop panel thickness was increased to 1.5h from 1.25h the calculated incremental deflections would satisfy $\frac{\ell_n}{480}$, and in the case of total deflections the drop panels would have to be 1.65h (Figure 5.21) to provide acceptable results.

5.3 Discussion

Results of the parameter study have identified two variables which have a significant effect on the slab's performance but are not considered in CAN3 A23.3 minimum thickness provisions for slabs without beams. CAN3 A23.3 does not account for either the concrete strength or the aspect ratio of the panel. Equations 5-1 and 5-2 could be used as modification factors with the present CAN3 A23.3 minimum thickness equations to provide more consistent deflection results.

$$h_{f'_c} = \left(\frac{20}{f'_c}\right)^{\frac{1}{4}} h_{20} \qquad (5\text{-}1)$$

$$h_\beta = \left(\frac{1}{\beta}\right)^{\frac{1}{3}} h_{1.0} \qquad (5\text{-}2)$$

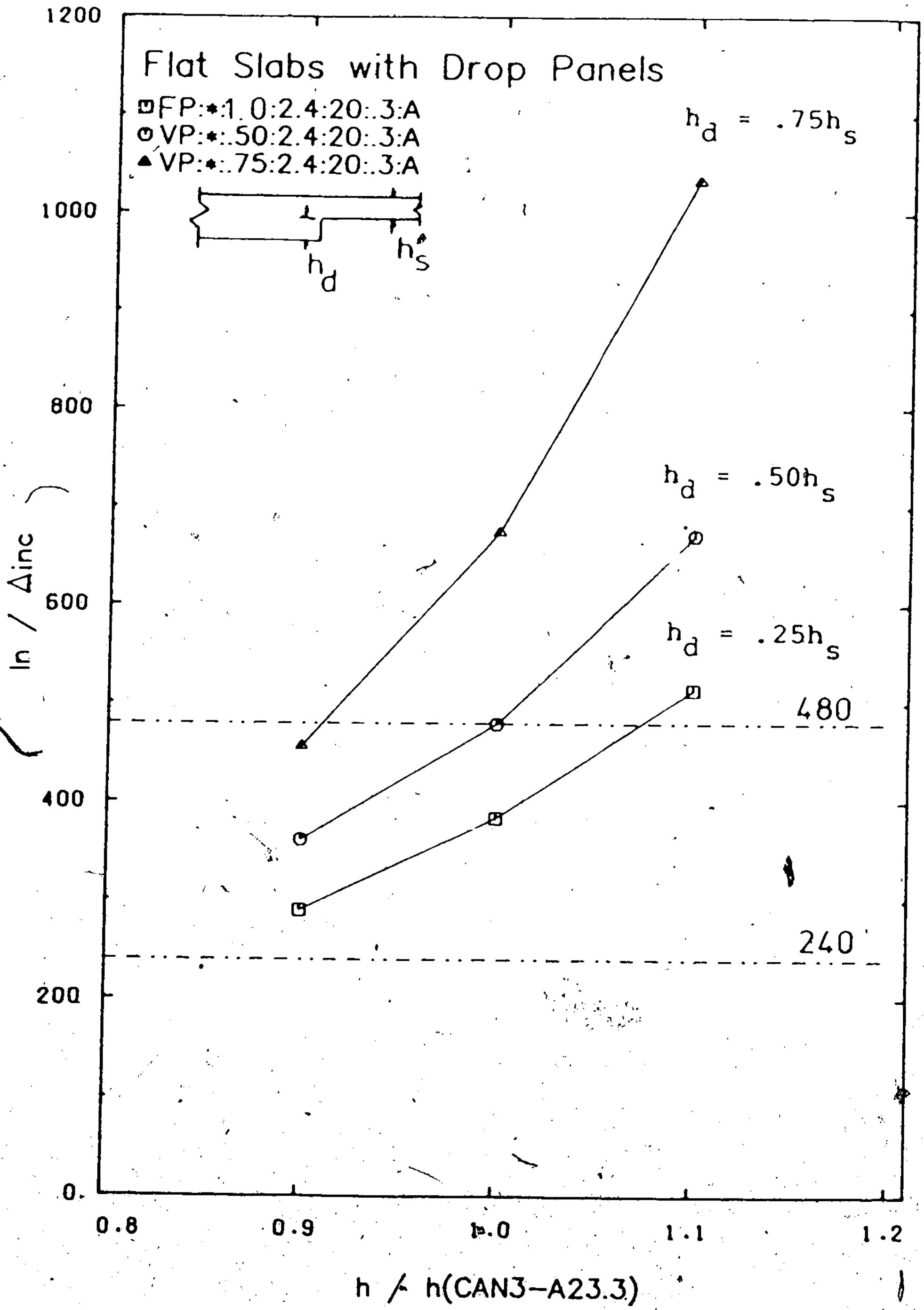


Figure 5.20 Minimum Span-Calculated Incremental Deflection Ratios of Flat Slabs with Various Drop Panel Thicknesses

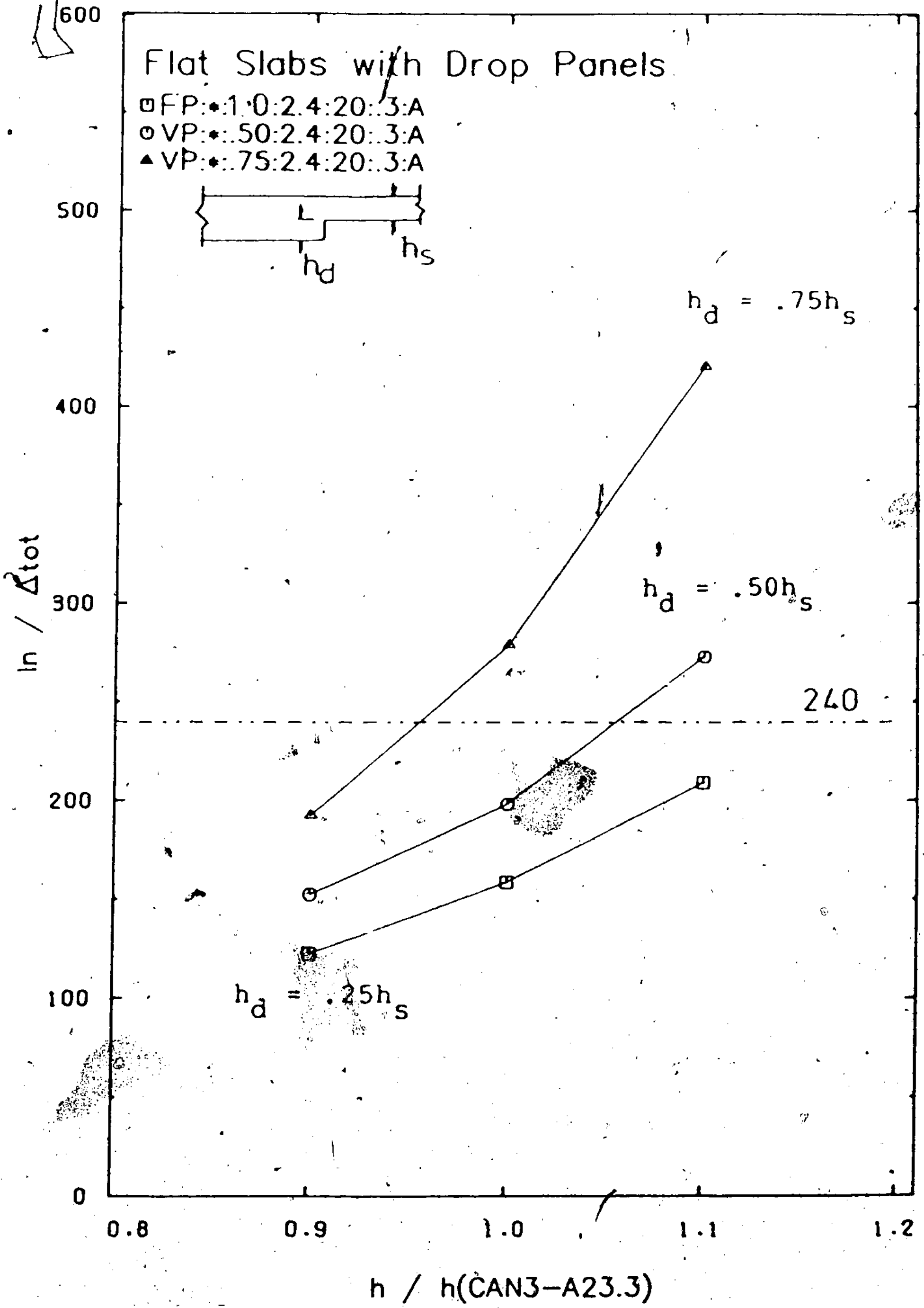


Figure 5.21 Minimum Span-Calculated Total Deflection Ratios of Flat Slabs with Various Drop Panel Thicknesses

Equations 5-1 and 5-2 were evaluated by comparing the calculated span-deflection ratios from the computer simulations of the slabs shown in Tables 5.2 and 5.3. Equation 5-1 predicts that slabs with 30 MPa concrete that are ten percent thinner than slabs with 20 MPa concrete should have the same calculated deflection. Table 5.2 compares calculated deflections of slabs, considered in the study, using 20 MPa concrete with those of slabs using 30 MPa that are ten percent thinner. The table indicates calculated deflections for these two groups slabs are within six percent of each other.

Equation 5-2 predicts that the calculated deflections of a slab with β = 1.0 and $1.10h_m$ and a slab with β = 2.0 and $.90h_m$ would be the same. Table 5.3 compares calculated deflections of such slabs considered in the study. The results indicate that Eq. 5-2 can be used for slabs governed by total deflections but not for incremental deflections. Although the difference in incremental deflections of the two slabs is eight percent, the difference would affect the slab thickness by only three percent.

All slabs considered in this study had constant thickness and equal long spans for all panels. When $f_e = .30\sqrt{f'_c}$ was used the calculated minimum span-deflection ratios for the exterior and interior panels are within five percent of each other.

Figure 5.22 presents calculated incremental deflections for interior panels of flat slabs with different

Table 5.2 Prediction of Effect of f_c on Slab Thickness

Slabs Compared	$\frac{\ell}{\Delta}$	$1.10h_m$	h_m	Percent
Concrete Strength f'_c		20 MPa	30 MPa	Difference
FP:*:1.0:2.4:#:.3:A	$\ell/\Delta tot$	252	256	-1.6
FP:*:1.0:2.4:#:.3:A	$\ell/\Delta inc$	606	604	0.3
FP:*:1.5:2.4:#:.3:A	$\ell/\Delta tot$	426	431	-1.2
FP:*:1.5:2.4:#:.3:A	$\ell/\Delta inc$	1011	1002	0.9
FP:*:2.0:2.4:#:.3:A	$\ell/\Delta tot$	449	441	1.8
FP:*:2.0:2.4:#:.3:A	$\ell/\Delta inc$	1058	1017	4.0
DP:*:1.0:2.4:#:.3:A	$\ell/\Delta tot$	209	206	1.5
DP:*:1.0:2.4:#:.3:A	$\ell/\Delta inc$	514	497	3.4
DP:*:1.5:2.4:#:.3:A	$\ell/\Delta tot$	320	320	0.0
DP:*:1.5:2.4:#:.3:A	$\ell/\Delta inc$	778	763	2.0
DP:*:2.0:2.4:#:.3:A	$\ell/\Delta tot$	358	344	4.1
DP:*:2.0:2.4:#:.3:A	$\ell/\Delta inc$	860	813	5.8

Notes:
concrete strength is specified in chart
* slab thicknesses are as follows
when f'_c = 20 MPa $1.10h_m$=195 (FP) and 210 (DP)
f'_c = 30 MPa h_m=175 (FP) and 190 (DP)
if the difference is negative the deflection is less with the thinner slab

Table 5.3 Prediction of Effect of β on Slab Thickness

Slabs Compared	$\frac{\ell}{\Delta}$	$1.10h_m$	$.90h_m$	Percent
Panel Aspect Ratio β		1.0	2.0	Difference
FP:*:#:2.4:20:.3:A	$\ell/\Delta tot$	252	252	0.0
FP:*:#:2.4:20:.3:A	$\ell/\Delta inc$	606	560	8.2
DP:*:#:2.4:20:.3:A	$\ell/\Delta tot$	209	205	2.0
DP:*:#:2.4:20:.3:A	$\ell/\Delta inc$	514	474	8.4

Notes:
aspect ratio is specified in chart
* slab thicknesses are as follows
when β= 1.0 $1.10h_m$=195 (FP) and 210 (DP)
β= 2.0 $0.90h_m$=160 (FP) and 175 (DP)
if the difference is negative the deflection is less with the thinner slab

thicknesses. For the minimum thicknesses specified by CAN3 A23.3, $\frac{\ell_n}{33.75}$ (flat plate) and $\frac{\ell_n}{37.1}$ (flat slab with drop panels), the incremental deflections exceed the most severe deflection limit of $\frac{\ell_n}{480}$ when β_o is less than 1.5. When the thicknesses are increased by ten percent to $\frac{\ell_n}{30.4}$ and $\frac{\ell_n}{33.75}$, respectively, deflections exceed the $\frac{\ell_n}{480}$ deflection limit only when β is less than 1.2.

Since the most severe deflection limit is only slightly exceeded when β is between 1.0 and 1.2, a minimum thickness of $\frac{\ell_n}{30.4}$ for flat plates and $\frac{\ell_n}{33.75}$ for flat slabs with drop panels may be used for limiting incremental deflections to $\frac{\ell_n}{480}$ for interior panels. These minimum thicknesses can be applied to exterior panels also, since the deflections are similar. For exterior panels of flat slabs with no edge beams these thicknesses are used as the minimum thicknesses under the present code provisions.

The use of the same thickness for the minimum thicknesses for both interior and exterior panels would simplify the code. This can be done by replacing CAN3 A23.3 Equation 9-9 (Eq. 2-1) by Eq. 5-3.

$$h = \frac{\ell_n}{30} \qquad \text{when } f_y = 400 \text{ MPa} \qquad (5\text{-}3)$$

Equation 5-3 is the same as the minimum thickness requirement used in ACI 318-63 except that the clear span (ℓ_n) is used instead of the centre to centre span (ℓ).

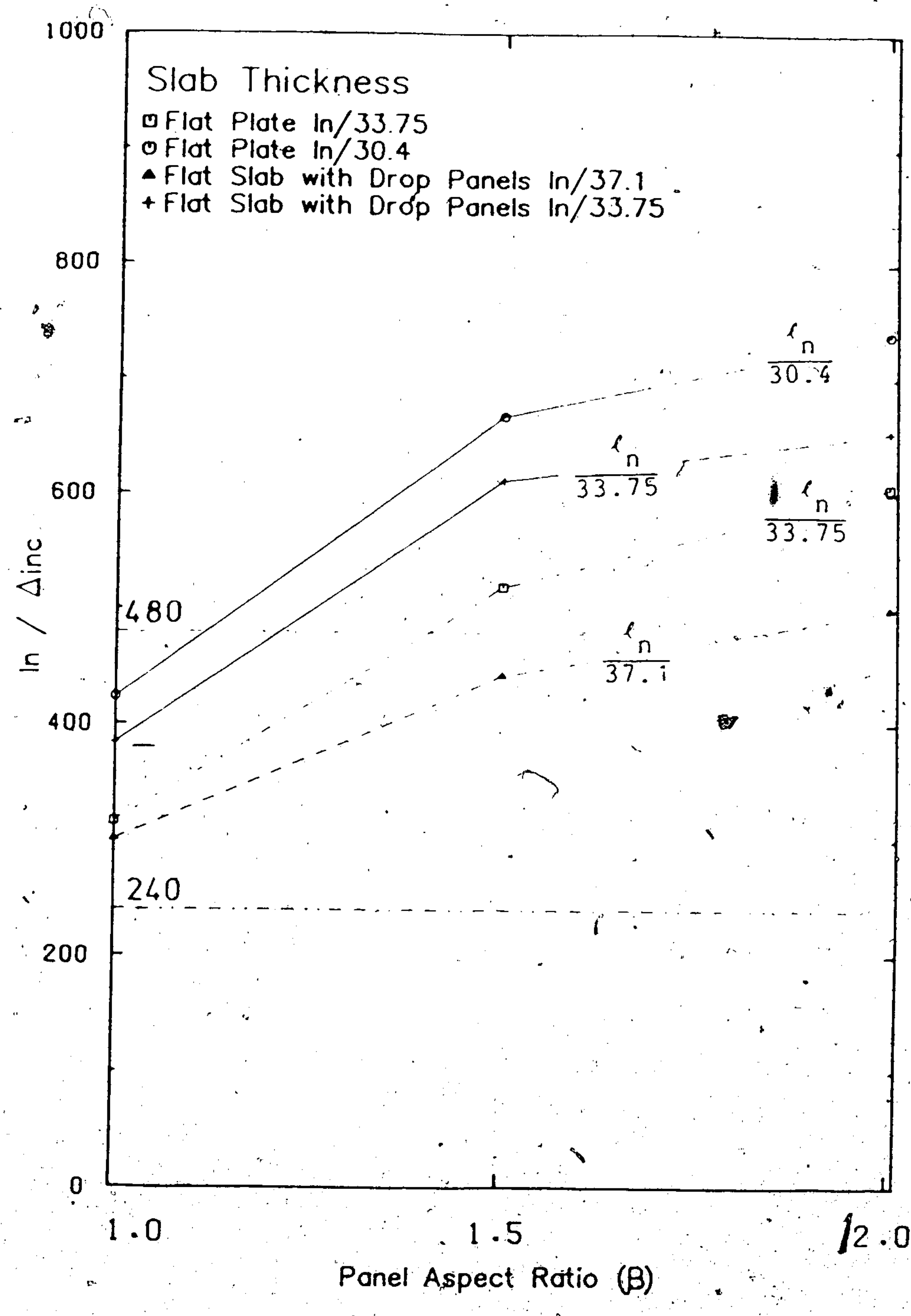


Figure 5.22 Comparison of Minimum Span-Calculated Incremental Deflection Ratios of Interior Panels of Different Thicknesses

5.4 Evaluation of Minimum Thickness Provisions

5.4.1 Live Load Deflections

All the slabs simulated in this thesis meet the deflection limit of $\frac{\ell_n}{360}$. The calculated deflections were at most one quarter of the limit.

5.4.2 Incremental Deflections

Flat plates investigated using $f_e = .60\sqrt{f'_c}$, or having a concrete strength of 30 MPa had calculated deflections less than the deflection limit of $\frac{\ell_n}{480}$. All flat plates met the limit of $\frac{\ell_n}{240}$. Flat plates with 20 MPa concrete and $f_e = .30\sqrt{f'_c}$ have calculated deflections less than $\frac{\ell_n}{480}$ when β is greater than 1.25 for all live loads. Thus the only area of concern for flat plates would be of slabs with 20 MPa concrete and β = 1.0 supporting partitions likely to be damaged by large deflections.

Flat plates with edge beams using $f_e = .60\sqrt{f'_c}$ have calculated deflections less than $\frac{\ell_n}{480}$. Slabs under apartment loads will satisfy $\frac{\ell_n}{480}$ when α is greater than 2.0 and concrete strength of 30 MPa. In all other cases the calculated deflections were between $\frac{\ell_n}{240}$ and $\frac{\ell_n}{480}$.

Deflections calculated for slabs with drop panels were less than $\frac{\ell_n}{480}$ when $f_e = .60\sqrt{f'_c}$. The calculated deflections with $f_e = .30\sqrt{f'_c}$ were smaller than the $\frac{\ell_n}{480}$ limit when β was greater than 1.3, but were between $\frac{\ell_n}{480}$ and $\frac{\ell_n}{240}$ with β less than 1.3 for slabs with 30 and 20 MPa concrete.

The incremental deflections calculated in this study were based on an early time of installation of partitions, one week after removal of shoring, and are a conservative estimate of the slabs' performance. Better results would be obtained if the installation of the partitions was delayed, and this time variable needs further investigation.

In summary, slabs studied in this thesis meeting the code minimum thickness satisfied the deflection limit to prevent damage to partitions not likely to be damaged by large deflections. However, with $f_e = .30\sqrt{f'_c}$ when $\beta = 1.0$, these slabs did not satisfy the limit to prevent damage to partitions likely to be damaged by large deflections.

5.4.3 Total Deflections

Flat plate calculated total deflections met the deflection limit $\frac{\ell_n}{240}$ except in the following case. The deflections would exceed the limit when $f_e = .30\sqrt{f'_c}$, f'_c = 20 MPa and β less than 1.3. When edge beams were included in the slab configuration for $\beta = 1.0$ the calculated deflections would exceed the deflection limit except when $f_e = .60\sqrt{f'_c}$.

The calculated deflections of slabs with drop panels did not exceed the deflection limit when $f_e = .60\sqrt{f'_c}$, or $f_e = .30\sqrt{f'_c}$ with f'_c = 20 MPa and β greater than 1.5. The slab's performance improved when f'_c = 30 MPa, but the deflections still exceed the limit when $\beta = 1.0$. There should be some concern about using slabs of minimum thickness with aspect ratios of less than 1.5.

5.5 Summary

The results of the parameter study were presented and evaluated considering live load, incremental, and total deflections. Results indicated that two variables not considered by CAN3 A23.3, concrete strength and panel aspect ratio of the slab, had a significant effect on calculated deflections for slabs without beams. Conclusions and recommendations of the study are presented in the next chapter.

6. Summary, Conclusions and Recommendations

6.1 Summary

CAN3 A23.3 allows two methods for controlling deflections of slabs. Slab deflections need not be checked if the slab thicknesses are larger than those specified in the code. If thinner slabs are used the deflections must be calculated to ensure that they do not exceed the code-specified limits. The minimum thickness provisions of the code are based on satisfactory performance of previously built slabs, but these thicknesses have never been checked to see what their calculated deflections are. This parametric study has investigated factors affecting deflections of Flat Plates and Flat Slabs to evaluate CAN3 minimum thickness provisions.

The study used a computer simulation to calculate deflections of different types of slabs. The simulation was done in two parts. The first part used a finite element program to simulate short-term deflections, which included cracking of the slab. An effective modulus of rupture of $f_e = .30\sqrt{f'_c}$ was used to account for the effect of shrinkage on the slab. The results of this program were modified in the second part of the simulation, using ACI 209 recommended equations to simulate the long-term effects of creep, shrinkage and the increasing strength of the concrete. The results of the simulation were evaluated using the code-specified limits with two additional criteria. The diagonal span of the panel was considered as well as column line span and a total deflection limit was also used.

The effects of using different concrete strengths, loads and thicknesses for a variety of rectangular slabs were investigated indicating areas of concern with the present provisions. The parametric study identified two parameters which have a significant effect on deflections but are not considered in the present provisions. These parameters are the panel aspect ratio and the concrete strength. In addition, the study shows the present provisions are inadequate for interior panels and indentifies one parameter (edge beam stiffness ratio α) used in the code that does not have as significant an effect as previously thought.

6.2 Conclusions

Based on the results of the parametric study the following conclusions are presented.

6.2.1 Material Properties

1. Calculated deflections of slabs analysed using $f_r=.60\sqrt{f'_c}$ are 40 percent of those of the same slabs using $f_e=.30\sqrt{f'_c}$
2. Calculated deflections of slabs analysed using f'_c= 30 MPa are 75 percent of those of the same slabs using f'_c= 20 MPa

3. Slabs having thicknesses related by Eq. 5-1 have similar calculated deflections.

$$h_{f'_c} = \left(\frac{20}{f'_c}\right)^{\frac{1}{4}} h_{20} \qquad (5\text{-}1)$$

6.2.2 Loading

1. Construction methodology has a greater influence on slab deflections than service live loads.
2. Different Live Loads have little effect on calculated deflections.

6.2.3 Geometric Properties

1. In most cases the minimum span-deflection ratio of the slab was found in the interior bay.
2. When the slab panel aspect ratio (β) equalled 1.0 the minimum span-deflection ratio was found along the diagonal of the panel.
3. Slabs having thicknesses related by Eq. 5-2 have similar calculated deflections.

$$h_{\beta} = \left(\frac{1}{\beta}\right)^{\frac{1}{3}} h_{1.0} \qquad (5\text{-}2)$$

4. Flat Slabs with drop panels giving a total thickness 1.5 times the slab thickness meet the $\frac{\ell_n}{480}$ incremental deflection limit.
5. Although Edge Beams improve the minimum span-deflection ratio of exterior panels, they do not significantly improve the overall minimum span-deflection ratio of a

slab. In square panel slabs with edge beams the minimum span-deflection ratio occurs in the interior panel.

6. Interior and exterior panels with thicknesses set using Eq. 5-3 have satisfactory calculated deflections.

$$h = \frac{\ell_n}{30} \qquad \text{when } f_y = 400 \text{ MPa} \qquad (5\text{-}3)$$

6.2.4 Code Deflection Limits

1. Calculated deflections meet the code deflection limits for all slabs analysed using the code value for $f_r = .60\sqrt{f'_c}$.
2. All slabs studied meet the deflection limit for live load deflections.
3. All slabs studied meet the incremental deflection limit of $\frac{\ell_n}{240}$.
4. The incremental deflection limit of $\frac{\ell_n}{480}$ was exceeded when $f_e = .30\sqrt{f'_c}$ and $\beta = 1.0$ by all the slabs which used the code minimum thickness.
5. The incremental deflection limit of $\frac{\ell_n}{480}$ was exceeded when $f_e = .30\sqrt{f'_c}$ and $\beta \leq 1.5$ by all interior panels using the code minimum thickness.
6. All flat plates with edge beams studied had calculated deflections that exceeded the limits for incremental deflections of $\frac{\ell_n}{480}$ using $f_e = .30\sqrt{f'_c}$.

6.2.5 Selected Total Deflection Limit

1. For the slabs studied the total deflection limit of $\frac{\ell_n}{240}$ was a more restrictive limitation than an incremental deflection limit of $\frac{\ell_n}{480}$.
2. All flat plates with edge beams studied had calculated deflections which exceeded the total deflection limit $(\frac{\ell_n}{240})$ using $f_e = .30\sqrt{f'_c}$.
3. Calculated total deflections exceeded the limit for slabs of code minimum thickness using $f_e = .30\sqrt{f'_c}$ when $\beta = 1.0$.

6.3 Recommendations for Revisions to CAN3 A23.3

Based on the results of this parametric study the following revisions to CAN3-A23.3 are presented for consideration.

1. Clause 9.5.3.3.1 - Eq. 9-9 should be revised to Eq. 5-3

$$h = \frac{\ell_n}{30} \qquad \text{when } f_y = 400 \text{ MPa} \qquad (5\text{-}3)$$

2. Clause 9.5.3.3.2. - note (a) should be revised to read:- The drop panels project at least one-half of the slab thickness below the slab
3. Clause 9.5.3.3.3. -*"At discontinuous edges, an edge beam shall be provided with a stiffness ratio, α, not less than 0.8, or the minimum thickness required by Eq. 9-9 shall be increased by at least ten percent in the panel with the discontinuous edge"*

-should be deleted

4. Table 9.2 -A total deflection limit should be included. The British and Australian Concrete Codes use a total deflection limit of $\frac{\ell}{250}$.
5. Table 9.2 -ℓ should be clearly defined as the span either along the column line or the diagonal of a panel in two-way slab systems.

6.4 Recommendations for Design Practice

1. Do not decrease the slab thickness if edge beams are used unless the interior spans are significantly shorter.
2. Use drop panel thicknesses of 1.5 times the slab thickness instead of 1.25.
3. Do not use less than the code minimum thickness for slabs supporting partitions likely to be damaged by large deflections, with an aspect ratio less than 1.5.
4. Thinner slabs than the code minimum thickness can be used for supporting partitions not likely to be damaged by large deflections, but there will be a noticeable total deflection.

6.5 Recommendations for Future Research

1. Investigation of the effects of the time of installation of partitions, and span length on slab deflections.

2. Investigation into the serviceability of slabs with beams and one-way slabs.
3. Review of present shrinkage and creep experimental data to produce more accurate time functions for these properties.
4. Development of a finite element program which will simulate the effect of normal forces as well as cracking on concrete slab deflections.
5. Investigation of the effect of slab cracking on the load ratios developed by Grundy and Kabalia.
6. Development of proper load factors and ϕ factors for design of concrete slabs under construction loads.
7. Experimental investigation to verify the appropriateness of using f_e in Branson's I_e equation for the calculation of deflections of concrete members with end restraints.

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Appendix A

Code Provisions for

Minimum Slab Thickness

A.1 Excerpts from CAN3 A23.3-M84

The numbering of sections, equations and tables are taken directly from A23.3-M84.

Notation

h overall thickness of member, mm

ℓ_n length of clear span, in the direction moments are being determined, measured face-to-face of supports, mm

f_y specified yield strength of non-prestressed reinforcement, MPa

α ratio of flexural stiffness of beam section to the flexural stiffness of a width of slab bounded laterally by the centreline of the adjacent panel, if any, on each side of the beam (see clause 13)

α_m average value of α for all beams on the edges of a panel

β ratio of clear spans in the long to short direction of a two-way construction

β_s ratio of length of continuous edges to total perimeter of a slab panel

9.5.3.1 - A slab thickness less than the minmimum thickness required by Clauses 9.5.3.2, and 9.5.3.3 may be used, if shown by computation that deflection will not exceed the limits stipulated in Table 9.2. Deflections shall be computed taking into account the size and shape of the panel, the conditions of support, and the nature of restraints at the panel edges. For deflection computations, the modulus of elasticity, E_c, for concrete shall be as specified in Clause 8.5.1. The effective moment of inertia shall be that given by Eq.(9-3); other values may be used if the computed deflection is in reasonable agreement with results of comprehensive tests. Additional long-time deflection shall be computed in accordance with Clause 9.5.2.5.

9.5.3.2-Two-way slabs with beams between all supports

9.5.3.2.1 - The minimum thickness of slabs or other two-way construction designed in accordance with provisions of Clause 13, and having a ratio of long to short span not exceeding 2 and having beams satisfying Clause 13.6.1.6 between all supports, shall be governed by Eq.(9-7), (9-8) and (9-9) and the other provisions of Clause 9.5.3.2.

$$h = \frac{\ell_n\ (800 + f_y/1.5)}{36,000 + 5000\beta(\alpha_m - .5(1 - \beta_s)(1+1/\beta)} \qquad (9-7)$$

but not less than

$$h = \frac{\ell_n\ (800 + f_y/1.5)}{36,000 + 5000\beta(1 + \beta_s)} \qquad (9-8)$$

but need not be greater than Eq. 9-9.

9.5.3.2.2. - For a panel with a discontinuous edge, the beam at the discontinuous edge shall have a stiffness ratio, α, not less than 0.80, or the minimum thickness required by Clause 9.5.3.2.1 for that panel shall be increased by at least ten percent.

$$h = \frac{\ell_n \,(800 + f_y/1.5)}{36,000} \qquad (9\text{-}9)$$

9.5.3.2.3 - The slab thickness shall not be less than:

(a) value of α_m greater than or equal to 2.0... 90mm

(b) value of α_m less than 2.0..................120mm

9.5.3.3 - Two-way slabs without beams between interior supports

9.5.3.3.1 - The minimum thickness of two-way slabs without beams or with beams only at discontinuous edges designed in accordance with Clause 13 and having a ratio of long to short span not exceeding 2, shall be governed by Eq. (9-9) and the other provisions of Clause 9.5.3.3.

9.5.3.3.2 - For slabs with drop panels, the slab thickness calculated from Eq. (9-9) may be reduced by ten percent provided:

(a) the drop panels project at least one quarter of the slab thickness below the slab,

(b) the drop panels extend beyond the support centre-lines, a distance in each direction not less than 1/6 the centre-to-centre span length in that direction.

9.5.3.3.3 - At discontinuous edges, an edge beam shall be provided with a stiffness ratio, α, not less than 0.80 , or the minimum thickness required by Eq. (9-9) shall be increased by at least ten percent in the panel with the discontinuous edge.

9.5.3.3.4 - The slab thickness shall not be less than

(a) panels with drop panels conforming to Clause 9.5.3.3.2....100mm

(b) all other panels.......120mm

Table 9.1

Thickness Below Which Deflections Must be Computed for Non Prestressed Beams or One-Way Slabs Not Supporting or Attached to Partitions or Other Construction Likely to be Damaged by Large Deflections

Minimum thickness ,(h)	Simply supported	One end continuous	Both Ends continuous	Cantilever
Solid one-way slabs	$\frac{\ell}{20}$	$\frac{\ell}{24}$	$\frac{\ell}{24}$	$\frac{\ell}{10}$
Beams or ribbed one-way slabs	$\frac{\ell}{16}$	$\frac{\ell}{18.5}$	$\frac{\ell}{21}$	$\frac{\ell}{8}$

Values given shall be used directly for members with normal density concrete (γ_c = 2400 kg/m^3) and Grade 400 reinforcement. For other conditions, the values shall be modified as follows:

(a) For structural low density concrete and structural semi-low density concrete, the values shall be multiplied by 1.65 - 0.0003c, but not less than 1.00, where c is the density in kg/m^3.

(b) For fy other than 400 MPa, the values shall be multiplied by $(0.4 + \frac{f_y}{670})$.

Table 9.2

Maximum Permissible Computed Deflections		
Type of member	Deflection to be considered	Deflection limitation
roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to specified live load L	$\frac{\ell}{180}$*
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to specified live load L	$\frac{\ell}{360}$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long time deflection due to all sustained loads and the immediate deflection due to any additional live load)	$\frac{\ell}{480}$t
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\frac{\ell}{240}$§

*-Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and considering long-time effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

t-Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.

§-Long-time deflections shall be determined in accordance with Clause 9.5.2.5 or 9.5.4.4 but may be reduced by amount of deflection calculated to occur on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

- But not greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.

A.2 Excerpts from CEB-FIP Model Code MC 78 for Concrete Structures

The numbering of sections, equations and tables are taken directly from CEB-FIP MC 78.

<u>16.2.4 Cases for which checking of the Deflections may be omitted</u>

A check on the deflections is not necessary where the spans or slenderness of slabs or beams do not exceed certain values and if there is no danger of damage to surface finishes owing to the deflections or the rotations at the supports.

<u>Commentary</u>

For information it may be stated that checking the deflections is not indispensable in the following cases:

(a) slabs or beams with spans not exceeding five meters,

(b) one or two way slabs for which slenderness ratio $\zeta \ell / h$ does not exceed 30,

(c) beams for which $\zeta \ell / h$ does not exceed 25.

Values for ζ may be taken from table 16.1. For floor slabs which support partitions the behaviour of which is affected by the deflections, the latter must be checked unless the slenderness ratio $\zeta \ell / h$ does not exceed $150/\zeta \ell$ (ℓ in m).

Table 16.1

Boundary Conditions	Value of ζ
Simply Supported	1.0
Exterior Span	0.8
Interior Span	0.6
Cantilever	2.4

A.3 Excerpts from BS 8110:1985

The numbering of sections, equations and tables are taken directly from BS 8110:1985.

Notation

b the effective width of a rectangular beam

d the effective depth

$A_{s.req}$ the area of tension reinforcement required at mid-span to resist the moment due to design ultimate loads (at support for a cantilever)

$A_{s.prov}$ the area of tension reinforcement provided at mid-span (at support for cantilever)

f_s the estimated design service stress in the tension reinforcement

f_y the characteristic strength of the reinforcement

β_b the ratio:

$$\frac{\text{moment at the section after redistribution}}{\text{moment at the section before redistribution}}$$

from the respective maximum moments diagram

3.2 Serviceability Limit States

3.2.1 Excessive deflections due to vertical loads

3.2.1.1 *Appearance* For structural members that are visible, the sag in a member will usually become noticeable if the deflection exceeds $\ell/250$, where ℓ is either the span or, in the case of a cantilever, its length.

This shortcoming can in many cases be at least partially overcome by providing an initial camber. If this is done, due attention should be paid to the effects on construction tolerances, particularly with regard to thicknesses of finishes.

This shortcoming is naturally not critical if the element is not visible.

3.2.1.2 *Damage to non-structural elements.* Unless partitions, cladding and finishes, have been specifically detailed to allow for the anticipated deflections, some damage can be expected if the deflection after the installation of such finishes and partitions exceeds the following values:

(a) L/500 or 20 mm, whichever is the lesser, for brittle materials

(b) L/350 or 20 mm, whichever is the lesser, for nonbrittle partitions or finishes;

where L is the span or, in the case of a cantilever, its length.

<u>3.5.7 Deflection of Solid Slabs</u>

Deflections may be calculated and compared with the serviceability requirements given in section three of BS 8110:Part2 but, in all normal cases, it will be sufficient to restrict the span/effective depth ratio. The appropriate ratio for a solid slab may be obtained from the Table 3.10, modified by Table 3.11.

Only the reinforcement at the centre of the span in the width of slab under consideration should be considered to influence deflection. The ratio for a two-way spanning slab should be based on the shorter span and its amount of reinforcement in that direction.

Table 3.10

Basic span/effective depth ratios for rectangular beams

Support Conditions	Ratio
Cantilever	7
Simply supported	20
Continuous	26

3.4.6.4 Long Spans

For spans exceeding 10m Table 3.10 should be used only if it is not necessary to limit the increase in deflection after the constfuction of partitions and finishes. Where limitation is necessary the values in Table 3.10 should be multiplied by 10/span except for cantilevers where the design should be justified by calculation.

3.4.6.5 Modification of span/depth ratios for tension reinforcement.

Deflection is influenced by the amount of tension reinforcement and its stress and therefore the span/effective depth ratios should be modified according to the area of reinforcement provided and its service stress at the centre of the span (or at the support of the case of a cantilever). Values of span/effective depth ratio obtained from Table 3.10 should therefore be multiplied by the appropriate factor obtained from Table 3.11.

Table 3.11

Modification factor for tension reinforcement									
Steel stress	$\frac{M_u}{bd^2}$								
	0.50	0.75	1.00	1.50	2.00	3.00	4.00	5.00	6.00
100	2.00	2.00	2.00	1.86	1.63	1.36	1.19	1.08	1.01
150	2.00	2.00	1.98	1.69	1.49	1.25	1.11	1.01	0.94
$(f_y = 250)$ 156	2.00	2.00	1.96	1.66	1.47	1.24	1.10	1.00	0.94
200	2.00	1.95	1.76	1.51	1.35	1.14	1.02	0.94	0.88
250	1.90	1.70	1.55	1.34	1.20	1.04	0.94	0.87	0.82
$(f_y = 460)$ 288	1.68	1.50	1.38	1.21	1.09	0.95	0.87	0.82	0.78
300	1.60	1.44	1.33	1.66	1.06	0.93	0.85	0.80	0.76

Note 1. The values in the table derive from the relationship:

$$\text{Modification factor} = 0.55 + \frac{(477 - f_s)}{120(0.9 + M_u/bd^2)} \leq 2.0$$

Where M_u is the design ultimate moment at the centre of the span or, for a cantilever, at the support.

Note 2. The design service stress in the tension reinforcement in a member may be estimated from the equation:

$$f_s = \frac{5\ f_y A_{s.req}}{8\ A_{s.prov}} \times \frac{1}{\beta_b}$$

Note 3. For a continuous beam, if the percentage of redistribution is not known but the design ultimate moment at mid-span is obviously the same as or greater than the elastic ultimate moment, the stress f_s, in this table may be taken as $\frac{5}{8} f_y$.

A.4 Excerpts from draft of AS 1480-85

The numbering of sections, equations and tables are taken directly from AS 1480-85.

10.3.4. Deemed to comply span-to-depth ratio for reinforced concrete slabs.

10.3.4.1 One-way slabs and two-way flat slabs.

Total or incremental deflections of a reinforced concrete one-way slab, or two-way flat slab subject to uniformly distributed loads and having multiple spans, shall be deemed to comply with the requirements of Clause 4.3.2, if the effective span to effective depth ratio is not greater than

$$L_e/d = k_3 k_4 \left(\frac{(\Delta/L_e).1000.E_c}{F_{d.ef}}\right)^{\frac{1}{3}}$$

where

Δ/L_e=the deflection limit selected in accordance with clause 4.3.2(a) and the deflection is taken on the centreline between the supports used to calculate L_e.

L_e= the effective span (the longer span for two-way flat slabs)

E_c = the modulus of elasticity of the concrete in MPa

$F_{d.ef}$ = the appropriate effective design load in kN/m^2 determined from Clause 9.5.3.3,

k_3 = 1.0 for a one-way slab

= 0.95 for a two-way flat slab without drop panels

= 1.05 for a two-way flat slab with drop panels, which extend at least L/6 in each direction on each side of support centreline and have an overall depth not less than

1.3 D where D is the slab thickness beyond the drops.

k_4 = a deflection constant which

(a) for simply supported slabs

= 1.6; or

(b) for continuous slabs, where the ratio of the longer to the shorter of any two adjacent spans does not exceed 1.2 and the, live load, q, is not greater than the dead load, g,

= 2.1 for an end span

= 2.6 for an interior span.

<u>10.3.4.2 Rectangular slabs supported on four sides.</u> Deflection of a reinforced concrete slab supported on four sides by walls or beams shall be deemed to comply with the deflection limited selected in accordance with Clause 4.3.2(a) if the ratio of the effective short span to the effective depth complies with the requirements of Clause 10.3.4.1 except that the value of k_3 shall be taken as 1.0, and the value of k_4, which in this clause takes into account edge support conditions, ratio of long to short side and torsional stiffness, shall be determined from Table 10.3.4.2.

<u>9.5.3.3 Long-term deflection of reinforced beams by multiplier method</u>

The long-term deflection of a normal-weight concrete reinforced beam may be calculated by using the effective second moment of area, I_{ef}, from Clause 9.5.3.1 and an effective design load, $F_{d.ef}$, given by

(a) $F_{d.ef} = (1.0 + k_{cs})G + (\Psi_s + k_{cs}\Psi_1)Q$ for total deflection;

or

(b) $F_{d.ef} = k_{cs}G + (\Psi_s + k_{cs}\Psi_l)Q$ for incremental deflection.

Where

$k_{cs} = 2 - 1.2 \frac{A_{sc}}{A_{st}}$ greater than or equal to 0.8, A_{sc} and, A_{st} being the reinforcement provided

(i) at midspan, for a simply supported or continuous beam;or

(ii) at the support, for a cantilever beam;

3.3 Load Combinations for Serviceability Limit States.

The design load for the serviceability limit states shall be taken as one of the following service load combinations:

a) Short-term load ......$G + P + \Psi_s Q + W_s$

b) Long-term load ....$G + P + \Psi_l Q$

Where P is the force in the prestressing steel and Ψ_s and Ψ_l are the short-term and long-term factors respectively. In the absence of more accurate data, the values of Ψ_s and Ψ_l may be taken from Table 3.3 and W_s may be taken as 0.5 W_{50}. In (a) and (b) above, the dead load G shall be taken as the weight of structural members plus superimposed dead loads including surfacing material, permanent services and permanent partitions. Liquid and earth pressure loads may be considered as superimposed dead loads.

Table 3.3

Term Factors for the Serviceability Limit States		
Live Load for-	Short-term factor, Ψ_s	Long-term factor, Ψ_l
(a) dwellings	0.7	0.4
(b) Offices	0.7	0.4
(c) Parking areas	0.7	0.4
(d) Retail Stores	0.7	0.4
(e) Storage areas	1.0	0.5 to 0.8
(f) Bridges 10m to 100m span	varies linearly from 0.7 to 0.5	0.0

4.3.2 Design of beams and slabs for excessive deflection.

The limit state of excessive deflection for beams and slabs shall be taken into account as follows:

(a) A deflection limit for the member shall be chosen appropriate to the structure and its intended use. In no case shall the values chosen exceed the relevant value specified in Table 4.3.2.

(b) The member shall be designed so that, under the design load for the serviceability limit state given in Clause 3.3, the deflections calculated in accordance with Sections 9 or 10, as appropriate, do not exceed the value determined in (a) above.

(c) For the purpose of calculating deflections at the serviceability limit state, it may be assumed that, provided allowance is made for concrete cracking, a concrete structure will act in a linear-elastic manner under service loads of short duration.

Table 4.3.2

Deflection Limitations			
1	2	3	4
Type of Member	Deflection to be considered	Deflection limitation Δ/L_e for spans	Deflection limitation Δ/L_e for cantilevers
Members generally	Total deflection	1/250 from as-cast position	1/100 from as-cast position
Members with a pre-camber not exceeding $L_e/300$	Total deflection less pre-camber	1/300 below horizontal	1/150 below horizontal
Members supporting masonry partitions	Incremental deflection, being that part of the total deflection which occurs after the addition of non-structural elements. It comprises that long-term deflection due to all sustained loads and the immediate deflection due to any additional live load	1/500 to 1/1000 depending on the provision made to minimize the effect of movement	1/250 to 1/500 depending on the provision made to minimize the effect of movement.
Bridge members	Live load (and impact) deflection	1/800	1/400

Note:

In two-way slab construction the deflection to which the above limits apply is the theoretical deflection of the line diagram representing the idealized frame defined in Clause 8.5.2.

Appendix B

Theoretical Background of Australian and British Code Provisions

B.1 Australian Span Effective Depth Equations

To understand the 1985 Australian code provisions one must review the equations and assumptions used in their derivation. Effective depth will be referred to as 'depth' in describing the derivations and equations used in the derivations are located in Table B.1.

The objective of this exercise is to develop a span to depth ratio that uses Branson's effective moment of inertia. This is done by considering a one-way member under uniform loading. The total deflection will be composed of two components, short term deflection and long term deflection (the additional deflection caused by shrinkage and creep). The mid-span deflection can be written, Eq. B-2, with different values for β depending on end conditions of the member. For Example $\beta = \frac{5}{384}$ for a simply supported beam.

Assuming the member will be under full live and dead loads, both immediate and long term deflection can be written using I_e shown in Eq. B-3. Further, writing I_e as a function of bd^3, Eq. B-4 is derived. Equation B-5 gives an allowable span to depth ratio and can be arrived at by algebraic manipulation. This equation provides a span to depth ratio in terms of the required total deflection.

The derivation of an equation to calculate span-depth ratio's for incremental deflections is very similar to that for total deflections. The deflection to be checked is the long term incremental deflection after the installation of non-structural elements. This is achieved by modifying Eq.

Table B.1 Derivation of Australian Span Depth Equations

$$\Delta_t = \Delta_e + \lambda \Delta_{e,sus} \qquad \text{(B-1)}$$

$$\Delta_e = \beta \frac{w l^4}{E_c I}$$

Total Deflection

$$\Delta_t = \beta \frac{w_t l^4}{E_c I_1} + \lambda \beta \frac{w_{sus} l^4}{E_c I_2} \qquad \text{(B-2)}$$

Let

$$I_e = I_1 = I_2$$

$$\Delta_t = \beta \frac{l^4}{E_c I_e} (w_t + \lambda w_{sus}) \qquad \text{(B-3)}$$

Let

$$I_e = \Lambda b d^3$$

$$\Delta_t = \beta \frac{l^4}{E_c \Lambda b d^3} (w_t + \lambda w_{sus}) \qquad \text{(B-4)}$$

$$\frac{l^3}{d^3} = \frac{\Delta_t \Lambda b E_c}{l \beta (w_t + \lambda w_{sus})}$$

$$\frac{l}{d} = \left(\frac{\Lambda}{\beta}\right)^{\frac{1}{3}} \left[\frac{\Delta_t}{l} \frac{b E_c}{(w_t + \lambda w_{sus})} \right]^{\frac{1}{3}} \qquad \text{(B-5)}$$

Incremental Deflection

$$\Delta_{inc} = \beta \frac{w_t l^4}{E_c I_e} + \beta \frac{w_{sus} l^4}{E_c I_e} - \beta \frac{w_{dead} l^4}{E_c I_e} \qquad \text{(B-6)}$$

$$\frac{l}{d} = \left| \frac{\Lambda}{\beta} \right|^{\frac{1}{3}} \left| \frac{\Delta_{inc}}{l} \frac{b E_c}{(w_t + \lambda w_{sus})} \right|^{\frac{1}{3}} \qquad \text{(B-7)}$$

Table B.2 Derivation of Australian Span Depth Equations: continued

$$\frac{l_e}{d} = k_1 k_2 \left| \frac{\Delta_{inc}}{l} \frac{bE_c}{(w_l + \Delta w_{sus})} \right|^{\frac{1}{3}} \quad \text{(B-8)}$$

Code Provisions

$$\frac{l_e}{d} = k_3 k_4 \left| \frac{\Delta_{inc}}{l} \frac{1000E_c}{k_{cs} g + k_{cs} \Psi_l q + \Psi_s q} \right|^{\frac{1}{3}} \quad \text{(B-9a)}$$

$$\frac{l_e}{d} = k_3 k_4 \left| \frac{\Delta_{tot}}{l} \frac{1000E_c}{(1 + k_{cs}) g + k_{cs} \Psi_l q + \Psi_s q} \right|^{\frac{1}{3}} \quad \text{(B-9b)}$$

Effective Moment of Inertia

$$I_e = \left(\frac{M_{cr}}{M_a} \right)^3 I_g + \left| 1 - \left(\frac{M_{cr}}{M_a} \right)^3 \right| I_{cr} \quad \text{(B-10)}$$

$$I_e = \left(\frac{M_{cr}}{M_a} \right)^3 \left| I_g - I_{cr} \right| + I_{cr} \quad \text{(B-11)}$$

$$M_a = \rho b d f_s j d \qquad M_{cr} = f_r \frac{bh^2}{6} \quad \text{(B-12)}$$

Let

$$f_s = .6 f_y \quad f_y = 410\ MPa. \quad j = .87 \quad d = 9h$$

$$I_{cr} = \frac{bd^3}{12} \left| 4k^3 + 12 n\rho (1 - k)^2 \right| \quad \text{(B-13)}$$

$$I_g = 1.36 \frac{bd^3}{12} \quad \text{(B-14)}$$

Substituting into I_e

$$I_e = \frac{bd^3}{12} \left| \left(\frac{0006 \sqrt{f_c'}}{\rho} \right)^3 \left(1.36 - \left| 4k^3 + 12 n\rho (1 - k)^2 \right| \right) + 4k^3 + 12 n\rho (1 - k)^2 \right| \quad \text{(B-15)}$$

$$\lambda = 1955 \sqrt{n\rho} \leq 111 \qquad n\rho \geq 045 \quad \text{(B-16a)}$$

$$\lambda = \frac{0019}{n\rho} \leq 067 \qquad n\rho \leq 045 \quad \text{(B-16b)}$$

B-2 as indicated in Eq. B-6, and will provide the span to depth equation shown in Eq. B-7.

Rangan[10] included a modification factor (k_2) to accommodate T & L beams in the equation and leading to the relationship shown in Eq. B-8. Gilbert[12] extended these equations to slab deflections by using k_2 as a slab modification factor to account for two-way action.

The code equations (Eq. B-9a & B-9b)[13] for slab depth take the same form as those for beams but the location factor (k_3), from $(\frac{\Lambda}{\beta})^{\frac{1}{3}}$ in Eq. B-7, is different due to the use of a different reinforcement ratio (ρ). The code provisions for the 1985 version of AS 1480 are shown in Appendix A. Comments on the assumptions used are listed below:

1. **Uniformly distributed loads are considered.** Although uniformly distributed loads are most commonly used in slab design it should be recognized that the ratios can not be directly applied to slabs designed for concentrated loads.
2. **Only midspan deflections are considered.** Midspan deflections are not always the maximum deflection but are very close to the maximum value. The effect on the result of the equation is approximately 1% to 2%.
3. **β was developed from elastic analysis.** The β variable used to calculate the midspan deflection used by Rangan was developed for elastic beams. Rangan's assumptions do not allow for redistribution of moments after cracking

in continuous beams or slabs.

A study was carried out to investigate the appropriateness of the magnitude of moments used to calculate k_2. The study included beams having three to six spans shown in Figure B.1 and it was found that much smaller values of β could be used, Table B.3.

4. **The moment of inertia used is calculated at only one level of loading (full dead load plus live load).**

A number of authors indicated that construction loads on slabs often exceed specified design loads and using I_e under full-service loading reflects the real situation for slabs. Therefore this is a reasonable assumption for most slabs.

5. **Slab deflections are modelled as a one dimensional problem.** This is the major assumption used by Gilbert in extending Rangan's equation and has no stated theoretical basis. An emperical slab factor (k_1) was developed from computer simulations to account for variables effecting the slab deflections. A theoretical proposal to explain this factor is given in Appendix C.

6. **The code provisions (span-depth equations) will encourage designers to use a similar amount of reinforcing steel in all their slab designs.** These provisions discourage variation in design of slabs in different areas of the country where cost factors are different. Since the code assumes that a certain amount of steel will be used in slab design and this is not the

Table B.3 Calculated Values for k_2 using Elastic and Redistibuted Moments

Number of Spans	Exterior Spans		Interior Spans			
3	1.24	1.24	2.92			
*	1.19	1.19	1.76			
4	1.27	1.27	1.91	1.91		
*	1.21	1.21	1.54	1.54		
5	1.27	1.27	2.11	1.63	2.11	
*	1.21	1.21	1.61	1.41	1.61	
6	1.27	1.27	2.11	1.66	1.66	2.11
*	1.21	1.21	1.61	1.44	1.44	1.61
#Rangan	1.30	1.30	1.50	1.50	1.50	1.50

Note:

* negative moments were reduced by 15 percent as permitted in CAN3 A23.3-M84 clause 8.4

values of k_2 used by Rangan

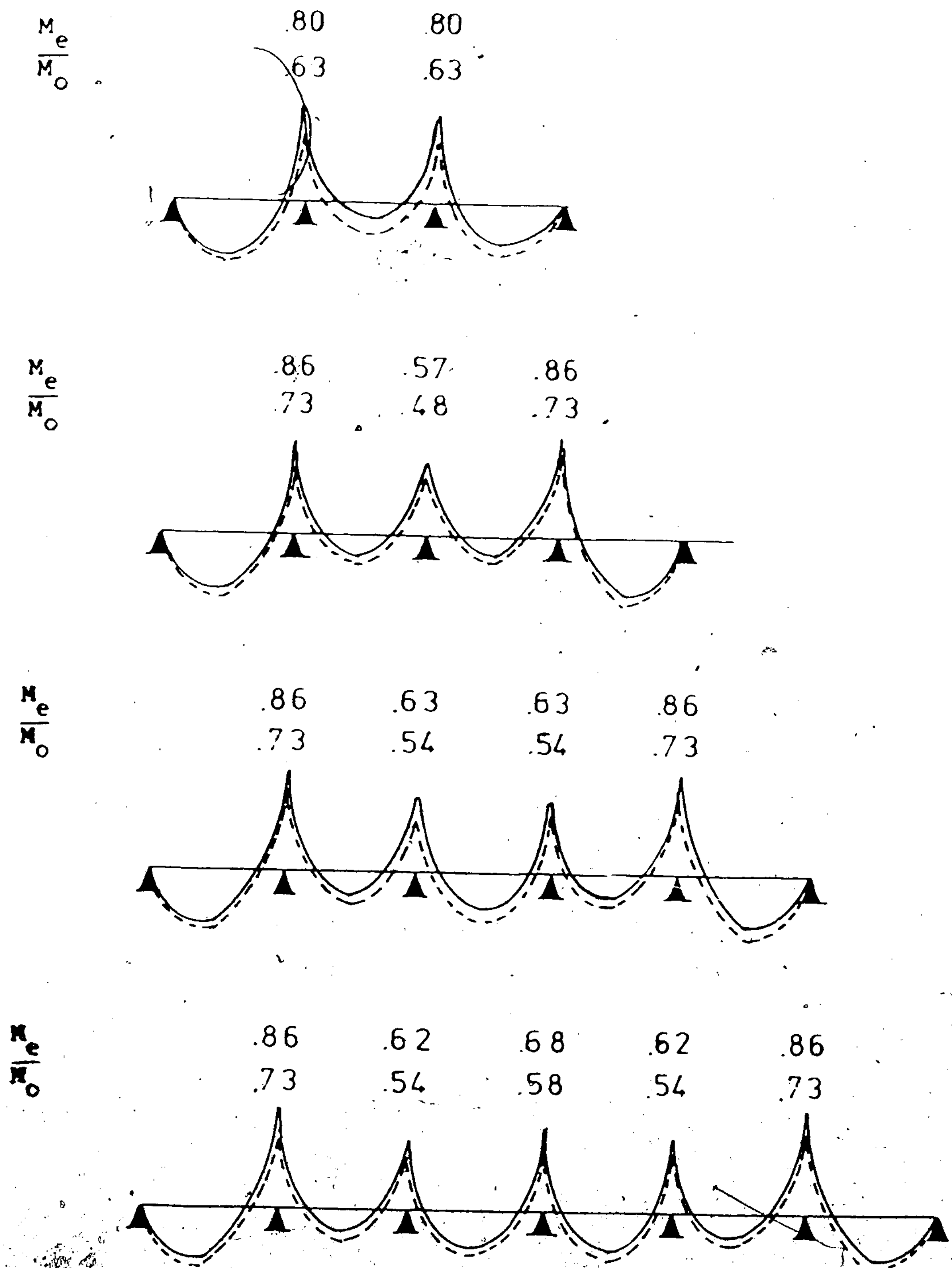


Figure B.1 Elastic Moments of Equal Spans same quantity required for moment resistance, different deflections than those used in the equation must be expected. Examples are shown in Figures B.2 to B.7 These figures indicate that the code value for $n\rho$ (.034) should be reconsidered especially for slabs under retail loading. This analysis also indicated that when only small deflections, ($\frac{\ell_e}{500}$), are allowed that only minimum reinforcement was required to satisfy the moment capacity.

B.2 Rangan's Approximation of Effective Moment of Inertia

The equations used in the approximation of the effective moment of inertia (I_e) are shown in Table B.2. In attempting to reduce I_e to a ratio of bd^3 from Branson's cubic equation, Eq. B-10, the original equation can be rewritten in the form of Eq. B-11 and from this one can see four areas that must be investigated: the cracking moment, service load moment, the gross moment of inertia and cracked moment of inertia. The cracking moment (M_{cr}) and the service load moment (M_a) must be considered at the same time. Looking first at the service load moment, Rangan made three assumptions to arrive at a solution,

1. the section considered will be only singly reinforced,
2. the stress in the steel will be 60% of the yield stress and only 410 MPa steel is used,
3. the elastic value j will be .87.

These assumptions allow M_a to be expressed as a function of

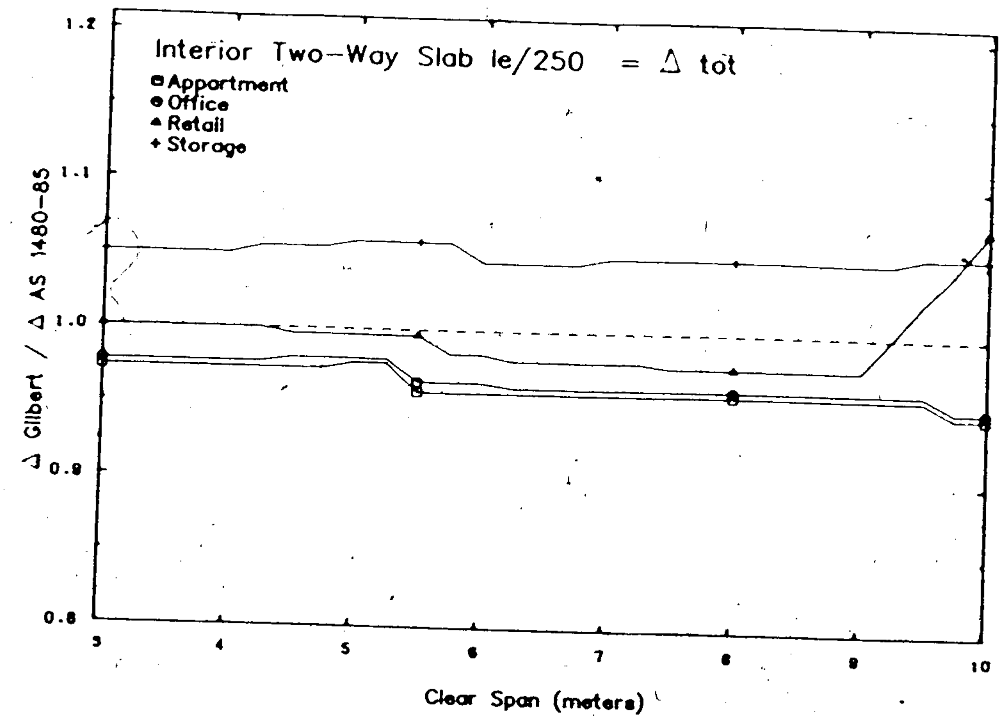


Figure B.2 Predicted Deflections for Different Occupancies

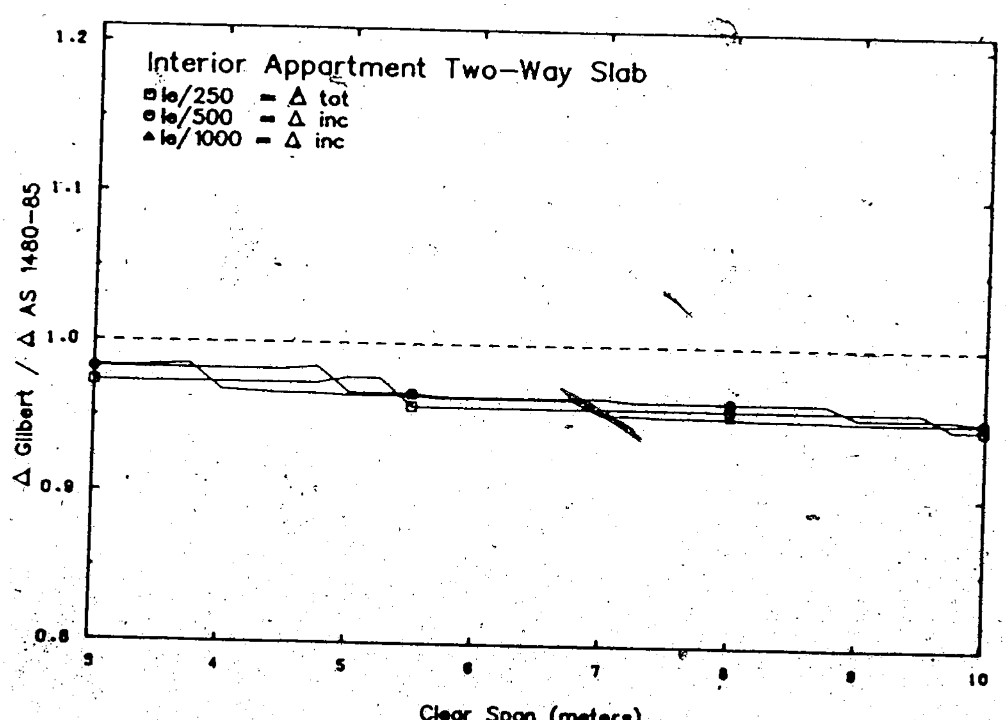


Figure B.3 Predicted Deflections for Different Deflection Criteria

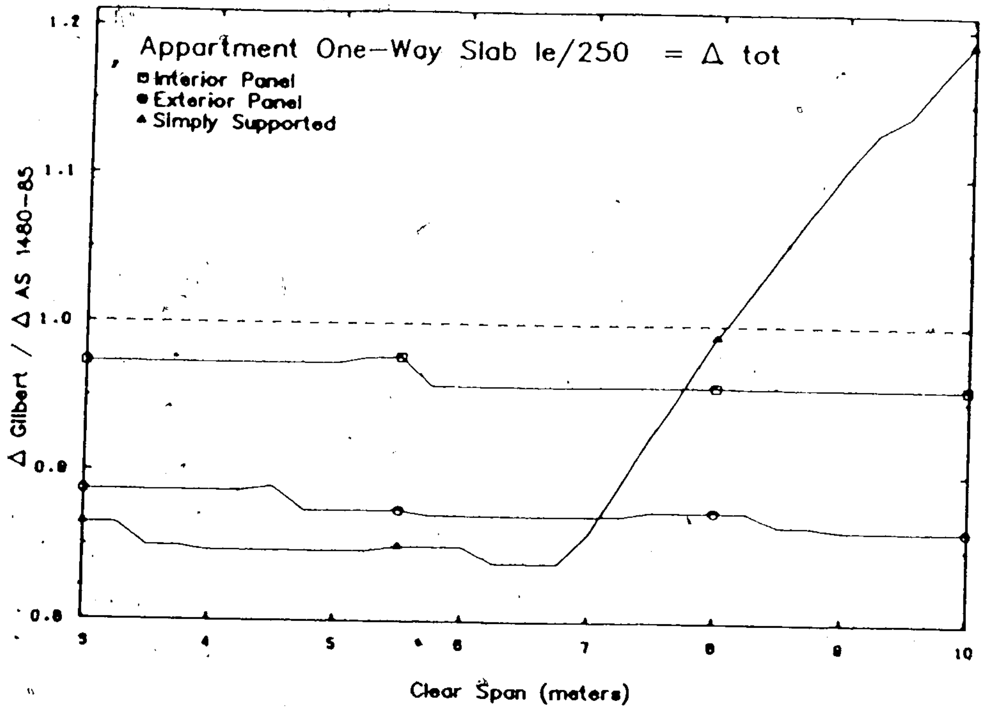


Figure B.4 Predicted Deflections for Different Locations: Apartment One-way Slabs

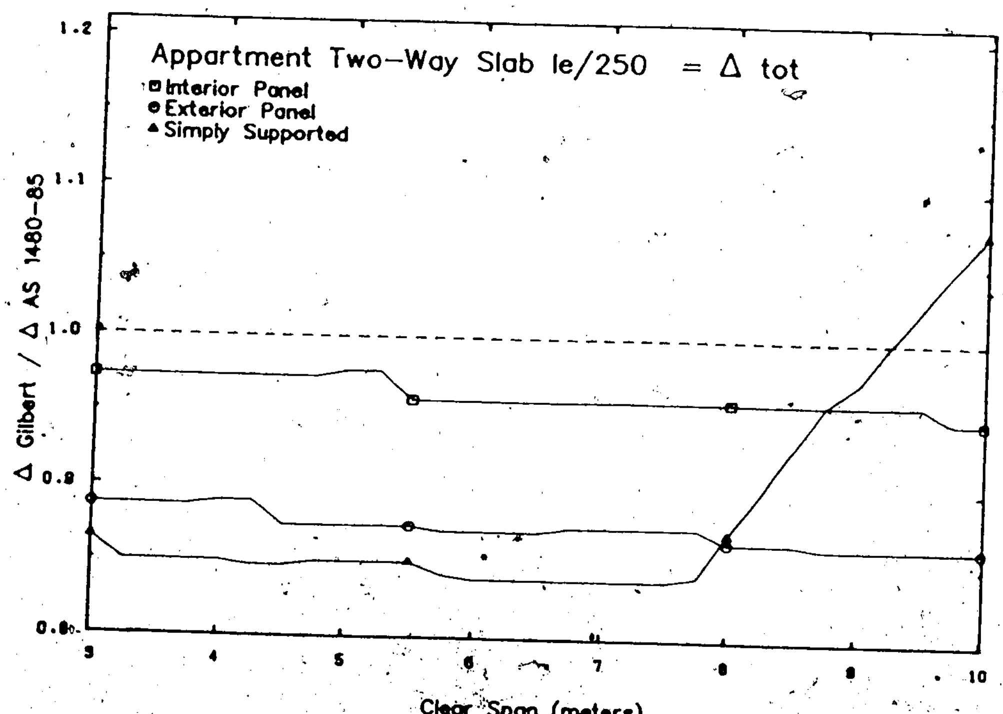


Figure B.5 Predicted Deflections for Different Locations: Apartment Two-way Slabs

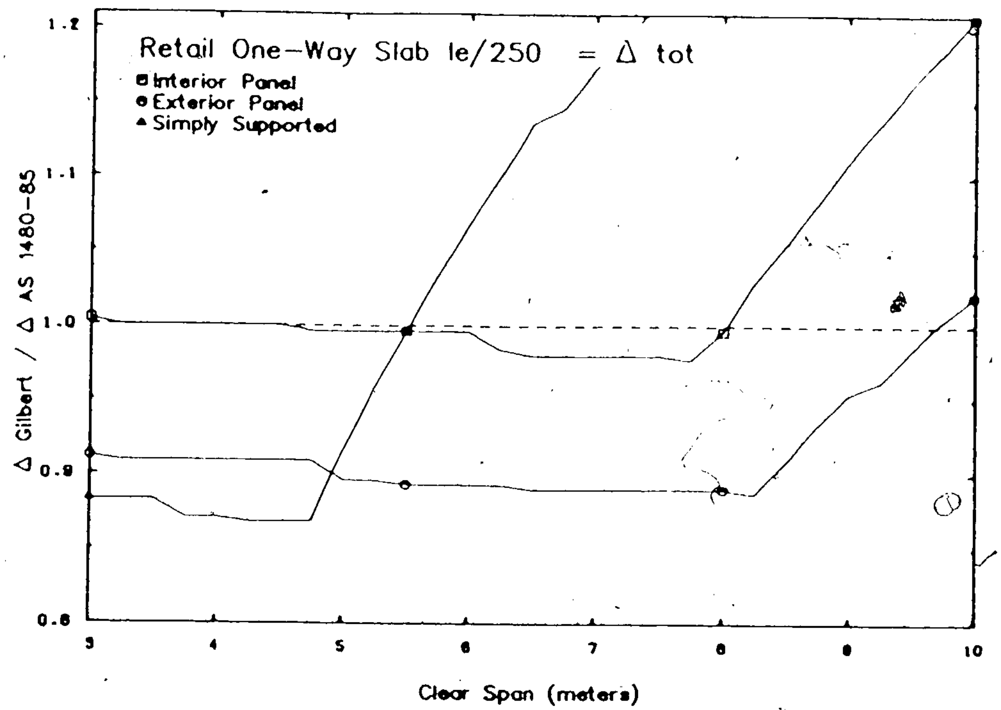


Figure B.6 Predicted Deflections for Different Locations: Retail One-way Slabs

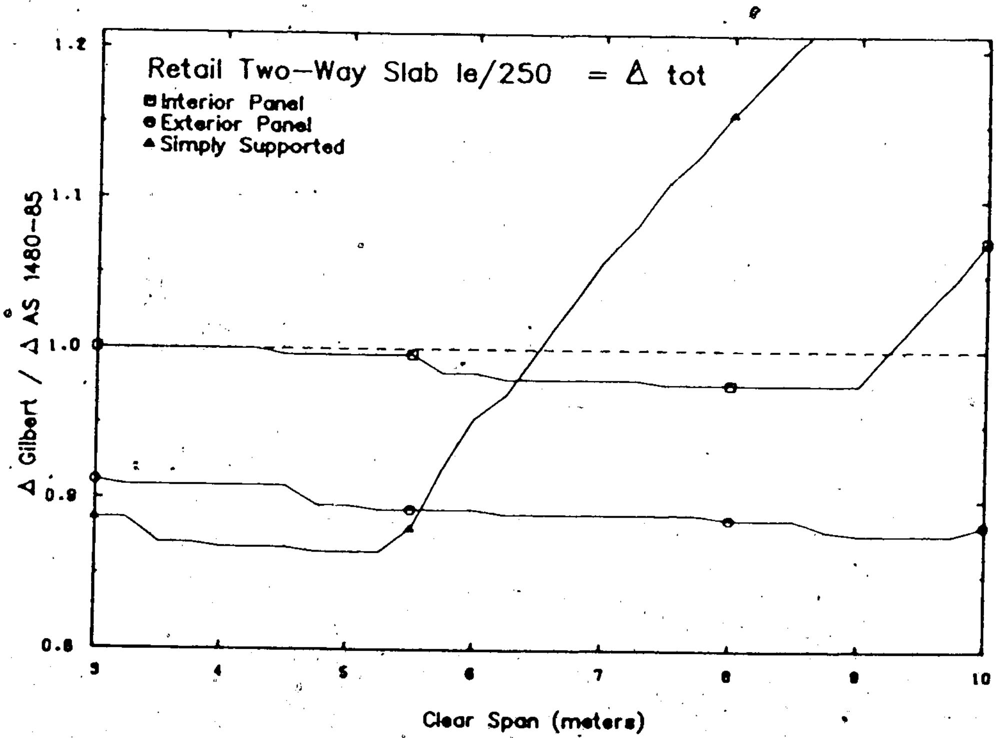


Figure B.7 Predicted Deflections for Different Locations: Retail Two-way Slabs

bd^2, Rangan assumed a depth to thickness ratio of .9 allowing the ratio of M_{cr} versus M_a to be considered as a function of ρ.

$$\frac{M_{cr}}{M_a} = 1.24 \frac{1}{0.9} \frac{b\ d^2}{\rho\ b\ d^2\ j\ f_s}\ f_r \qquad \text{(B-17)}$$

The cracked moment of inertia, since it is of a singly reinforced member, can be defined as shown in Eq. B-13 while the gross moment of inertia is defined in Eq. B-14 using a depth to thickness ratio of .9. These equations can then be combined to create Eq. B-15. This can be reduced to a function of steel area (ρ) and the concrete strength. Rangan developed Eq. B-16a & B-16b, which approximates Branson's I_e, under these assumptions. Recognizing the necessity of limiting tension stiffening Rangan limited I_e to $.60I_g$ for low values of ρ while restricting I_e to I_g for ρ greater than .7 percent

Although Rangan's effective depth formulas are insensitive to variation in I_e, since the formulas use the cubic root of I_e, a review of assumptions is still in order.

Rangan made five assumptions to arrive at a simplified equation for I_e:

1. **Only singly reinforced members are considered.** This is a conservative assumption that allows the effective moment of inertia to be reduced to a function of the reinforcing ratio (ρ).
2. **j is assumed to be .87 and f_s is assumed to be $.60f_y$.**

These two assumptions are being discussed together for two reasons: they are both functions of ρ, and are unnecessary. The $\frac{M_{cr}}{M_a}$ relationship can be derived without using these assumptions by equating M_a and the factored moment (M_f) to find the steel stress in terms of the load factors as illustrated below.

$$\frac{M_a}{M_f} = \frac{w_l + w_d}{\alpha_l w_l + \alpha_d w_d} \tag{B-18}$$

$$M_f = M_n \tag{B-19}$$

$$M_a = \frac{w_l + w_d}{\alpha_l w_l + \alpha_d w_d} M_n \tag{B-20}$$

All additional factors used in this derivation cancel out.

A regression analysis of I_e was done not using these two assumptions and new equations were found for Λ, Eq. B-16a and B-16b, in Table B.2. They are,

$$\Lambda = .0228 + .345n\rho \le .111 \quad \text{when } n\rho \ge .045 \tag{B-21}$$

$$\Lambda = \frac{.00018}{n\rho^{1.7}} \le .067 \quad \text{when } n\rho \le .045 \tag{B-22}$$

3. **An effective depth to thickness ratio ($\frac{d}{h}$) of .9 was used.** This is a realistic value for beams but not slabs and a range of .75 to .85 provides a better set of $\frac{d}{h}$ values. The equation for I_e can be modified to allow

different $\frac{d}{h}$'s and this is shown in Figure B.8.

4. **Only 410 MPa reinforcing steel is used.** This is a assumption restricts the applicablility of the equation to slabs using 410 MPa reinforcement. There has been a relationship found between $\frac{I_e}{I_g}$ versus $\frac{M_{cr}}{M_a}$ for different steel strengths that could be used.

5. **Limiting Tension Stiffening.** Tension stiffening can be described mathematically as the $(\frac{M_{cr}}{M_a})^3(I_g - I_{cr})$ part of Branson's equation (see Eq. B-11). For members with low reinforcement this part of the equation governs the value of I_e and will theoretically lead to an infinite stiffness for a member with no reinforcing because of the small value of M_a. Branson recommends that I_e be limited to the transformed or gross moment of inertia to prevent the use of unrealistic values. Rangan[10] and others[3] argue that this is an unreasonable value for slabs because of additional cracking caused by shrinkage restraint. Rangan recommends that I_e be no greater than $.60I_g$ for members with low reinforcement to provide better results. Gilbert[2], using this suggestion, found good correlation between his finite element analysis and field measurements. Scanlon and Murray[4], while agreeing on the need to reduce the tension stiffening, suggested a different method of reducing I_e. They proposed that an effective modulus of rupture (f_e), which takes into consideration the effects of shrinkage stresses to reduce f_r to a reasonable value, be used. Work done in

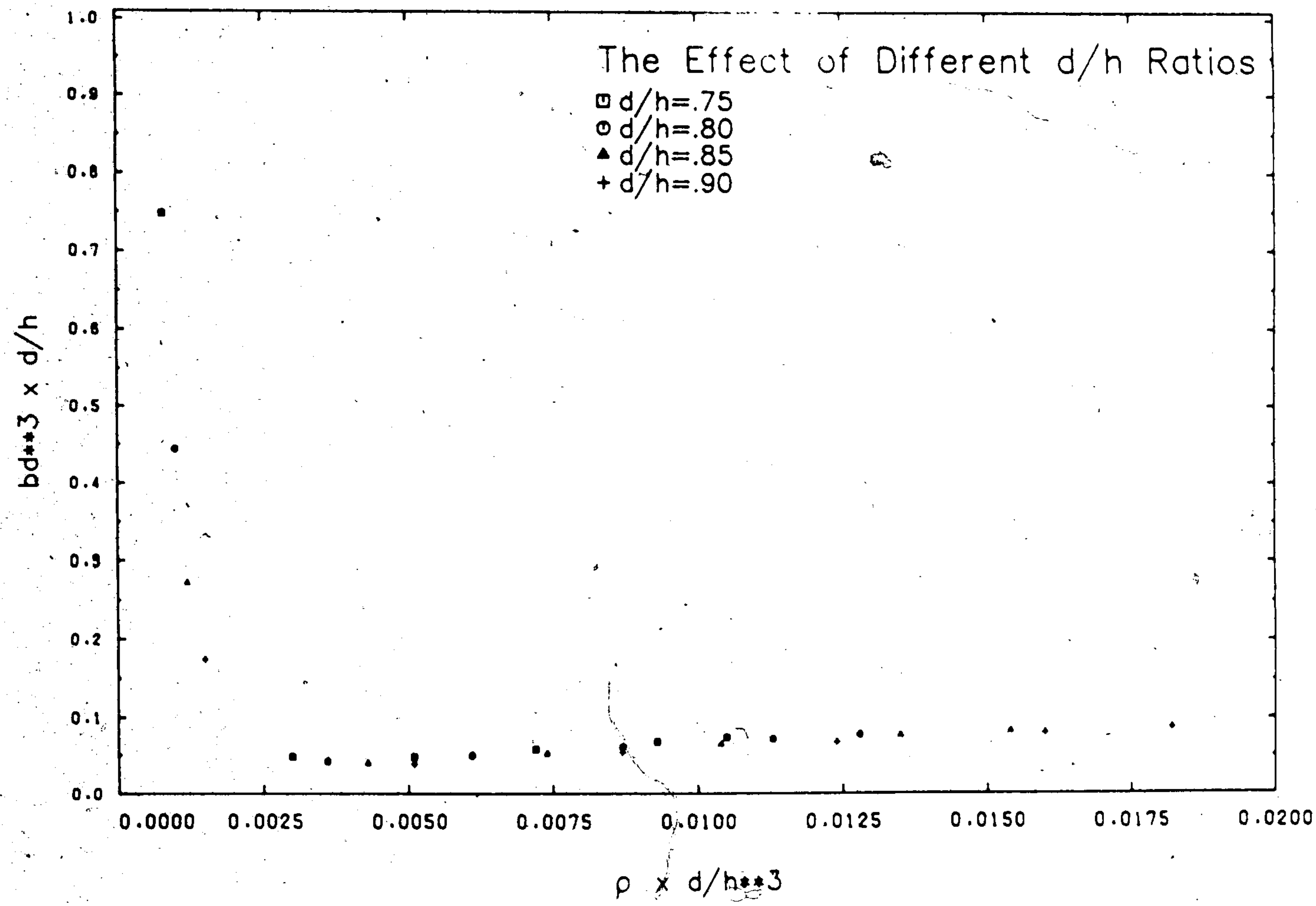


Figure B.8 Relationship Between $\frac{d}{h}$ and bd^3

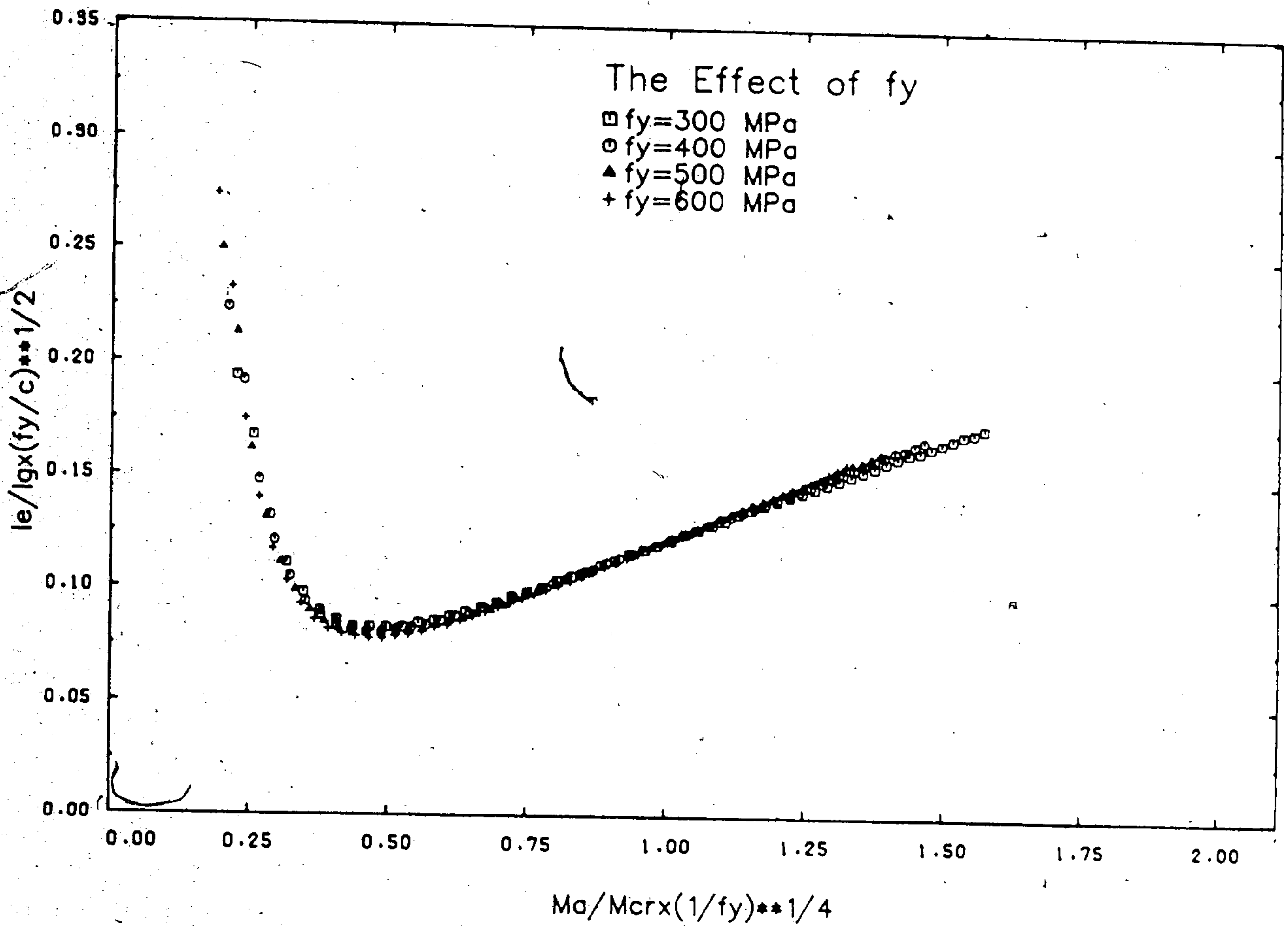


Figure B.9 Relationship Between f_y and I_e

this area[18] indicates that a good value would be $f_e = f_r/2$. Further support for this method arises out of experimental work done at the University of Alberta[11] in which one-way slabs were loaded without major shrinkage stresses being allowed to form. The experimenter found in all members with 20 MPa concrete, the cracking moment was at or above that predicted by the concrete code. All these members had a reinforcement ratio of less that .5%. Figures B.9 and B.10 show the differences in the three methods (Branson, Rangan and Scanlon) for moment curvature at minimum reinforcement and $\frac{I_e}{I_g}$ for 25 MPa concrete. In conclusion, assuming that Branson's I_e is accurate and that I_e should be limited to $.60I_g$, Figure B.11 shows the percentage difference of Rangan's approximations compared with Branson's I_e and the error this causes in the span-depth equations. The plot indicates that when $\frac{d}{h}$=.9 at $n\rho$=.034 the approximation causes the span-depth equation to be unconservative by three percent allowing slab deflections to be ten percent greater than predicted. Using $\frac{d}{h}$ = .85, which can be considered a appropriate value for slabs, the approximation is ten percent conservative. Therefore, although there are errors in both the approximation and the $\frac{d}{h}$ value they cancel out.

A general approximation of Branson's I_e equation and a modified slab effective depth equation has been developed and is described in Appendix C.

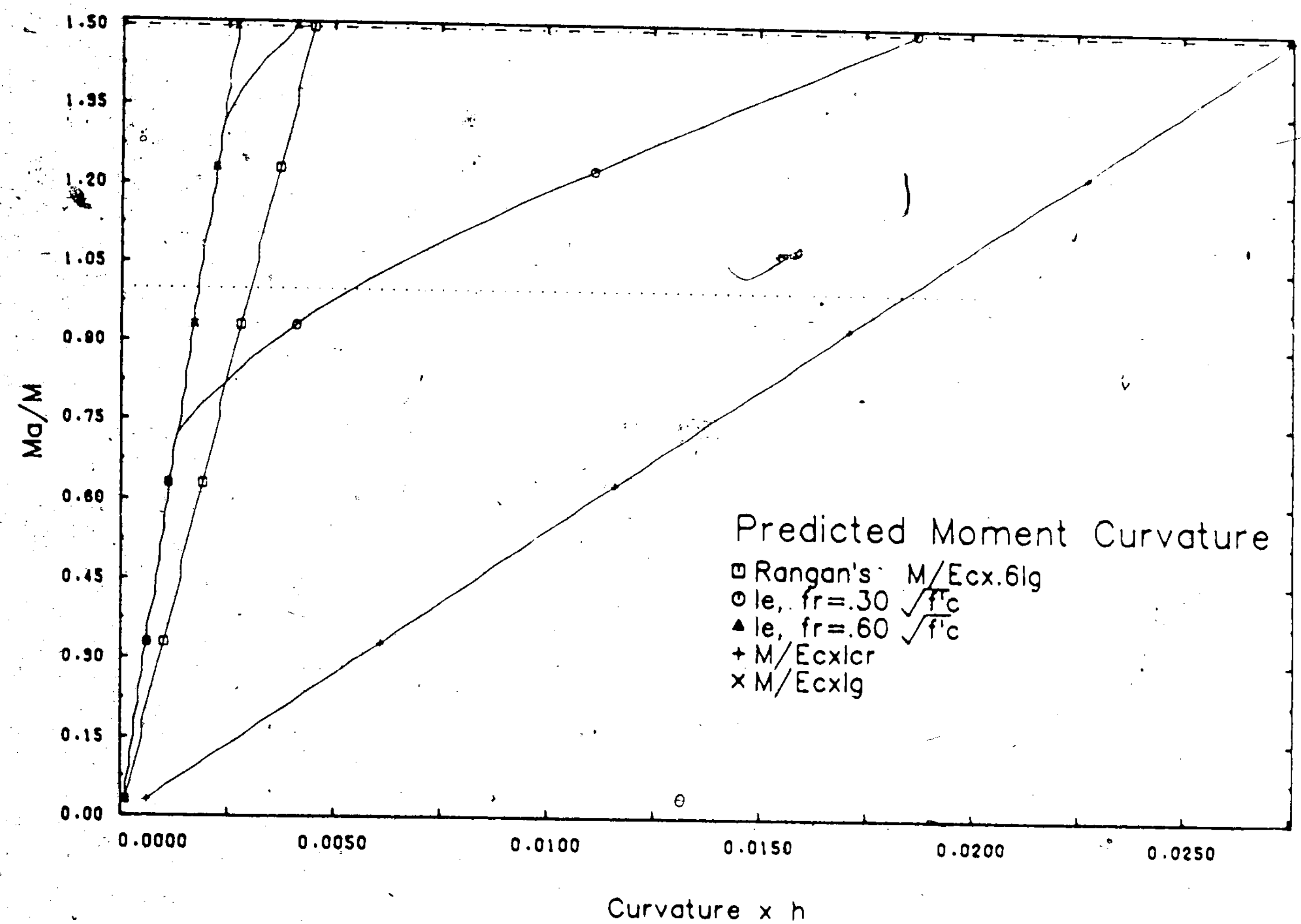


Figure B.10 Different Moment Curvature Relationships for Minimum Reinforcement

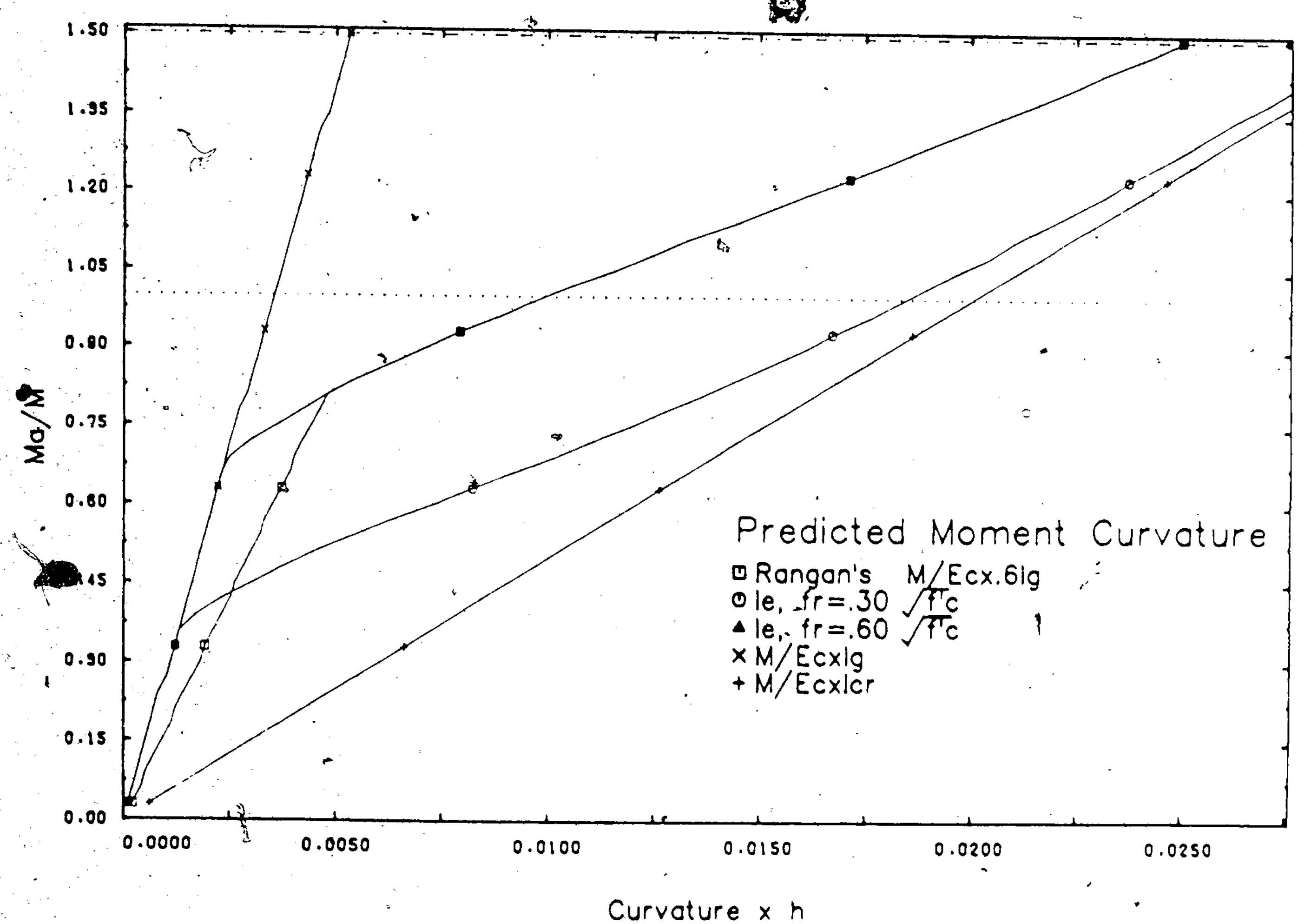


Figure B.11 Different Moment Curvature Relationships for Column Strip Reinforcement

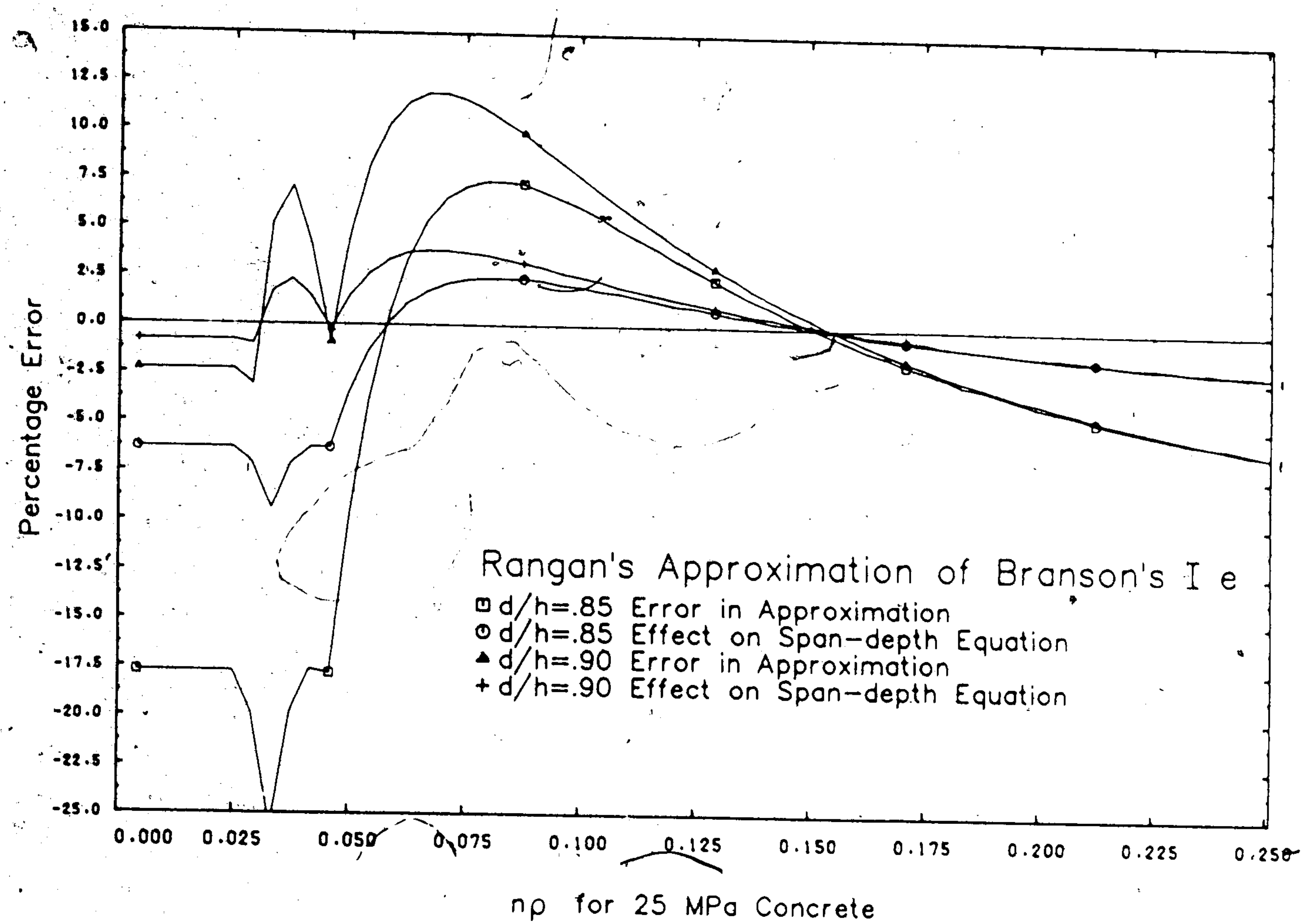


Figure B.12 Percentage Difference Between I_e Predicted by Rangan versus Branson

B.3 Assumptions Used in British Parameter Study

To complete the computer program used in the parametric study Beeby[7] made seven assumptions, they were:

1. Plane sections remain plane for calculation of strains,
2. Reinforcement has a modulus of elasticity of 200 GPa,
3. Concrete has a modulus of elasticity of $4.5\sqrt{f_u}$ GPa, which is approximately $5\sqrt{f_c'}$ GPa using cylinder concrete strengths for short term deflections, and $4.5 \frac{1}{1+\phi} \sqrt{f_u}$ for long term deflections. ϕ accounts for the effect of creep and ranges from 3.5 to 4.5,
4. Stresses in the concrete in tension have a triangular distribution with a tensile stress of 1.0 MPa at the level of the reinforcement,
5. The neutral axis was located based on the assumption that the concrete could not take tension,
6. The effective depth (d) of steel was .9 of the total thickness (h),
7. The service load would occur at two thirds of the factored load.

The first five assumptions are commonly used and their effects are well known; however, the last two assumptions require some discussion. The use of d as .90h is a conservative assumption when considering slabs. A better range for d in slabs would be .8 to .85h, i.e. d = 120 in a 150mm slab. The effect of using .9 instead of .85 would be about a five percent increase in the moment of inertia and therefore a five percent decrease in actual deflection.

However the effect on a span-effective depth ratio is less than two percent.

The assumption that the service load is two thirds of the factored load means that the dead load and live loads are equal. This is a reasonable assumption for slabs under retail loads but not for those with office loadings. The office loaded slabs would have their deflections underestimated by five to eight percent due to this assumption.

After the parameter study was completed, Beeby normalized his results to a standard beam so that a designer could use modifiers to account for different geometries, tensile steel stresses, compression reinforcement, loading and severe long term deflection conditions. This standard beam has a concrete strength of 30 MPa (cube strength) which is equivalent to 27 MPa by cylinder testing. This strength is higher than normally used in North America where slabs are usually designed with 20 to 25 MPa concrete. Equation B-23 was derived by the author to evaluate the effect of the use of such high concrete strength.

$$d_{fu} = \left(\frac{30}{fu}\right)^{\frac{1}{6}} d_{Beeby} \qquad \text{(B-23)}$$

Equation B-23 was arrived at by equating, using Eq. B-24, two slabs with the same span, loading and deflection but with different concrete strengths.

$$\Delta = \frac{w \ell^4}{EI} \qquad (B-24)$$

$$E_1 I_1 = E_2 I_2 \qquad (B-25)$$

but, $I = \Lambda b d^3$ and $E = 4.5\sqrt{f_u}$

therefore

$$\left(\frac{d_1}{d_2}\right)^3 = \left(\frac{f u_2}{f u_1}\right)^{\frac{1}{2}} \qquad (B-26)$$

which gives

$$\frac{d_1}{d_2} = \left(\frac{f u_2}{f u_1}\right)^{\frac{1}{6}} \qquad (B-27)$$

In the case of 20 MPa concrete Eq. B-23 indicates this could lead to a five percent underestimation of the required effective depth. This underestimation is rectified if Eq. B-23 is used with the standard ratios to account for different strengths of concrete.

Another problem that occurs with Beeby's basic span-effective depth ratios is that they do not match the results of the computer study, i.e., the basic ratio for a simply supported beam of 20 would have a deflection of $\frac{\ell}{185}$ instead of $\frac{\ell}{250}$. Beeby, justifiably, claims that the deflection would be less due to less creep and shrinkage, lower loading and increased stiffness due to finishes and end restraints. Therefore, it is reasonable to say that a ratio of 20 is adequate. Although Beeby's reasoning is sound, there will be errors induced by the approximations of the above effects.

A comparison between minimum thickness using Beeby's approach and the A23.3 minimum thickness is shown in Figures B.11 and B.12. In order to provide a better comparison, the modification factor Eq. B-23 was used. Figures B.13 and B.14 show the modified Beeby thickness as a fraction of the A23.3 minimum thickness for clear spans of three to ten meters. The thicknesses are almost equal for office loading in the longer clear spans and the larger thicknesses are due to an absolute minimum thickness of 125 mm was used for Beeby's slabs compared to 120 mm, flat plates, or 100 mm, drop panels, for the A23.3 values.

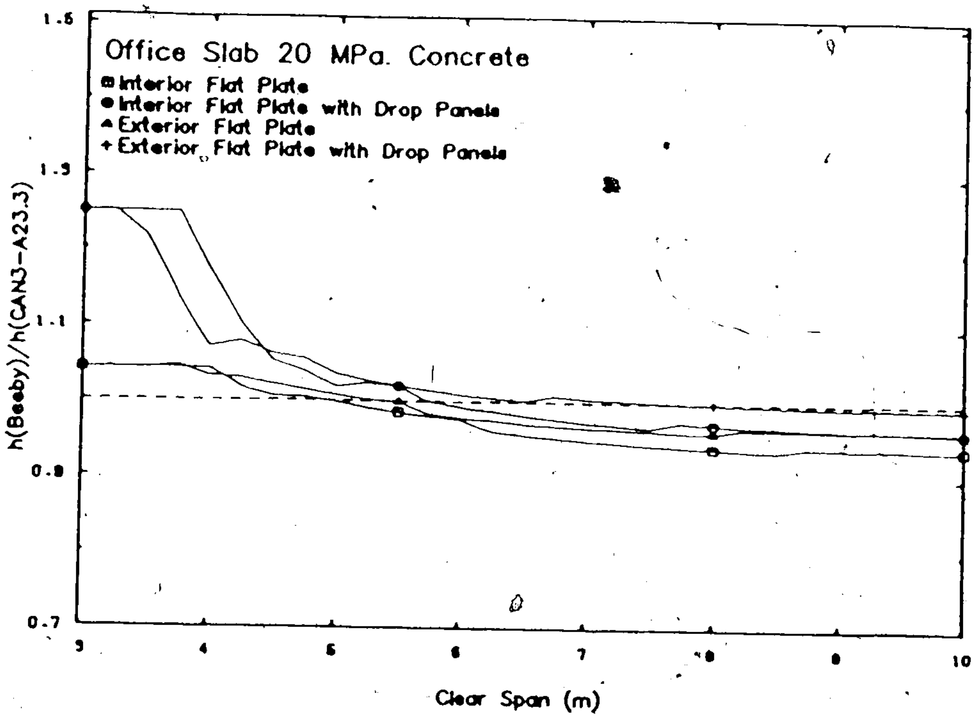


Figure B.13 Comparison of Beeby's and A23.3 Minimum Thickness under Office Loadings

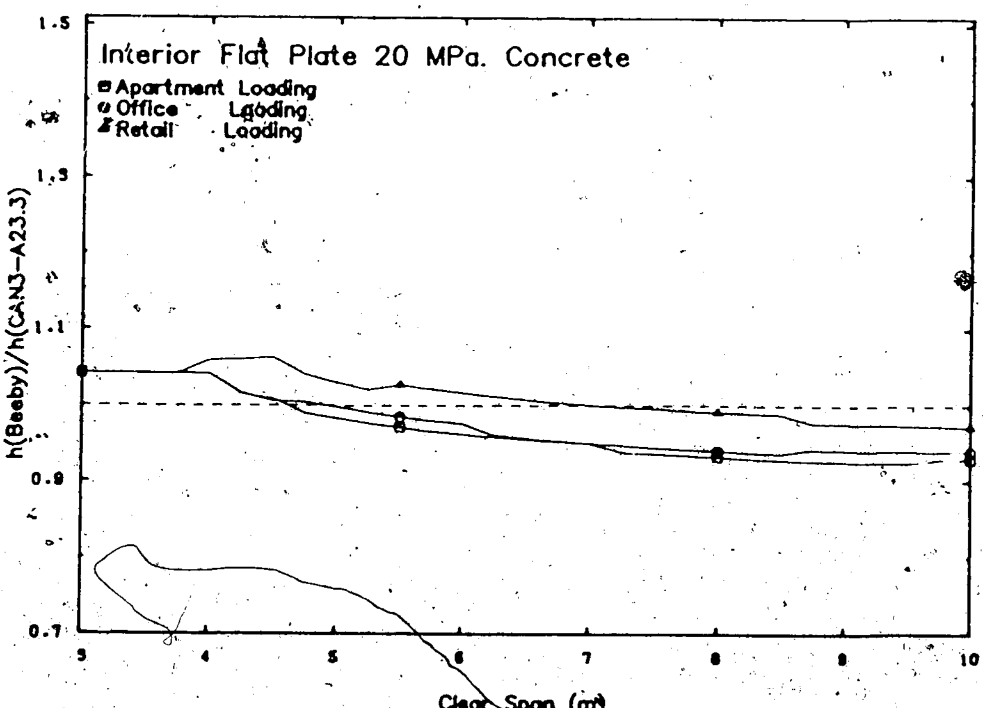


Figure B.14 Comparison of Beeby's and A23.3 Minimum Thicknesses under Different Loadings

Appendix C

Maximum Slab

Span-Thickness

Equations

C.1 General Equations

Theoretical Span-Thickness equations have been derived to investigate the factors influencing the setting of minimum thicknesses. An attempt has been made to develop all the factors used in the equation theoretically and not to depend on experimental results. The equations were developed using the same approach as Rangan[10] and Gilbert[12].

The final equations in Table C.1 (Eq. C-9 and C-120) differ from Gilbert's equations in two areas, I_e and the span. The equations use an I_e calculated using column strip reinforcement instead of an average of the middle and column strips, with a $\frac{d}{h}$ ($\frac{\text{effective depth}}{\text{total thickness}}$) value of .8 rather than .9. The use of column strip reinforcement only simplifies the estimation of I_e for use in the general equation. The value for $\frac{d}{h}$ of 0.9 is not appropriate for slabs. Consider a 125 mm slab, using clear cover of 20 mm and a #10 bar the $\frac{d}{h}$ value is 0.8. This value, 0.8, is more appropriate for slabs. The equations are written in terms of ℓ_n and h versus ℓ_e and d. The use of h instead of d simplifies the use of the equations without any loss of accuracy, and the choice of ℓ_n was felt to provide a realistic value for the span[38].

The equations were derived using four major assumptions. They are:

1. The midpanel deflection of a slab can be found using a crossing beam analogy.
2. The slab will have been subjected to full design loading before the deflections are calculated.

3. The stress distribution through a section of the slab can be modelled as linear for service loads and by the Whitney Stress Block at ultimate loads.
4. The moment distribution in the slab is the same as assumed in the Direct Design Method.

A derivation of a general Span-Thickness Equation for both incremental and total deflections for a concrete slab is presented over the next three pages. Multipliers used in Eq. C-8, C-9, C-10, and C-11 are described in detail later.

These equations have not been checked against the results of the parameter study and should not be used for design purposes.

Span Thickness Equations

$$\Delta_t = \Delta_e + \lambda \Delta_{e,sus} \qquad \text{(C-1)}$$

$$\Delta_e = \beta \frac{w \ell^4}{E I}$$

Total Deflection

$$\Delta_{tot} = \beta \frac{w_{tot} \ell^4}{E_c I_1} + \lambda \beta \frac{w_{sus} \ell^4}{E_c I_2} \qquad \text{(C-2)}$$

let $$I_e = I_1 = I_2$$

$$\Delta_{tot} = \beta \frac{\ell^4}{E_c I_e} (w_{tot} + \lambda w_{sus}) \qquad \text{(C-3)}$$

let $$I_e = \Lambda b h^3$$

$$\Delta_{tot} = \beta \frac{\ell^4}{E_c \Lambda b h^3} (w_{tot} + \lambda w_{sus}) \qquad \text{(C-4)}$$

$$\frac{\ell^3}{h^3} = \frac{\Delta_{tot} \Lambda b E_c}{\ell \beta (w_{tot} + \lambda w_{sus})}$$

$$\frac{\ell}{h} = \left(\frac{\Lambda}{\beta}\right)^{\frac{1}{3}} \left[\frac{\Delta_{tot}}{\ell} \frac{b E_c}{(w_t + \lambda w_{sus})}\right]^{\frac{1}{3}} \qquad \text{(C-5)}$$

Incremental Deflection

$$\Delta_{inc} = \beta \frac{w_{tot} \ell^4}{E_c I_e} + \beta \frac{w_{sus} \ell^4}{E_c I_e} - \beta \frac{w_{dead} \ell^4}{E_c I_e} \qquad \text{(C-6)}$$

$$\frac{\ell}{h} = \left[\frac{\Lambda}{\beta}\right]^{\frac{1}{3}} \left[\frac{\Delta_{inc}}{\ell} \frac{b E_c}{(w_l + \lambda w_{sus})}\right]^{\frac{1}{3}} \qquad \text{(C-7)}$$

If we let

$$\alpha = \frac{\Lambda}{\beta_{simp}}$$

the span-thickness equation can be written as

$$\frac{l}{h} = \left[\frac{\beta_{simp}}{\beta}\right]^{\frac{1}{3}} \left[\frac{\Delta_{inc}}{l} \frac{\alpha b E_c}{(w_l + \lambda w_{sus})}\right]^{\frac{1}{3}}$$

which can be rewritten as

$$\frac{l}{h} = \left(\frac{1}{(1.2\frac{M_m}{M_o} - 0.2)}\right)^{\frac{1}{3}} \left[\frac{\Delta_{inc}}{l} \frac{\alpha b E_c}{(w_l + \lambda w_{sus})}\right]^{\frac{1}{3}} \qquad \text{(C-8)}$$

If we are to use this Equation for two-way slabs, the equations must ensure that at no location is the deflection-span ratio exceeded. Therefore the equations should include a second factor to account for deflections at other locations.

$$\frac{l}{h} = \left(\frac{(1+\beta_{panel}^2)^{\frac{1}{2}}}{\left[1 + \left[.6666\left[\frac{1}{\beta_{panel}}\right]^3\right]\right]}\right)^{\frac{1}{3}} \left(\frac{1}{\Psi_{load}(1.2\frac{M_m}{M_o} - 0.2)}\right)^{\frac{1}{3}} \left[\frac{\Delta_{inc}}{l} \frac{\alpha b E_c}{(w_l + \lambda w_{sus})}\right]^{\frac{1}{3}} \qquad \text{(C-9)}$$

<u>Maximum Slab Span-thickness Equations</u>

$$\frac{l}{h} = k_1 k_2 \left[\frac{\Delta_{tot}}{l} \frac{\alpha b E_c}{(w_{tot} + \lambda w_{sus})}\right]^{\frac{1}{3}} \qquad \text{(C-10)}$$

$$\frac{l_n}{h} = k_1 k_2 \left[\frac{\Delta_{inc}}{l} \frac{\alpha b E_c}{(w_l + \lambda w_{sus})}\right]^{\frac{1}{3}} \qquad \text{(C-11)}$$

where

$$k_1 = \left(\frac{(1 + \beta_{panel}^2)^{-\frac{1}{2}}}{\left[1 + \left[.6666 \left[\frac{1}{\beta_{panel}} \right]^3 \right] \right]} \right)^{\frac{1}{3}} \quad \text{(C-39)}$$

$$k_2 = \left(\frac{\alpha}{\psi_{load} \left(1.2 \frac{M_m}{M_o} - 0.2 \right)} \right)^{\frac{1}{3}} \quad \text{(C-48)}$$

$$\alpha = \frac{384}{5} (d/h)^3 \left[\left(\frac{M_{cr}}{M_a} \right)^3 \left(\frac{1}{(d/h)^3} - \left[4k^3 + 12n\rho(1-k)^2 \right] \right) + 4k^3 + 12n\rho(1-k)^2 \right] \quad \text{(C-29)}$$

$$\frac{M_{cr}}{M_a} = \left[\frac{1}{(d/h)^2 6} \; \frac{f_r}{\phi_s f_y \rho \left[1 - .59 \frac{\phi_s f_y}{\phi_c f'_c} \rho \right]} \left(\frac{\left(\alpha_l \frac{w_l}{w_d} + \alpha_d \right)}{\left(\frac{w_l}{w_d} + 1 \right)} \right) \right] \quad \text{(C-25)}$$

$$\lambda = \frac{\xi}{1 + 50\rho'}$$

The accuracy of the span-thickness equations derived are dependent on assumptions used in calculating the Moment of Inertia (I_e), Slab Factor (k_1), Location Factor (k_2), and the long-term deflection multiplier (λ).

C.2 Long-term Multiplier

The λ used in last page is the CAN3 A23.3 equation but research has indicated that this equation underestimates the long-term deflection of concrete slabs. Table C.1 compares the values from the code with the recommended values of Graham and Scanlon[11] when $f_e = .30\sqrt{f'_c}$. The values were modified to account for time (Eq. C-13) by adjusting the creep ($\nu_c(t)$) and shrinkage ($\epsilon_{sh}(t)$) values in Eq. C-12 with the time functions recommened by ACI 209[20].

$$\lambda = 2.5 \times \frac{\nu_c(t)}{2.35} + .75 \times \frac{\epsilon_{sh}(t)}{760 \times 10^{-6}} \qquad \text{(C-12)}$$

$$\lambda = 2.5 \times \left(1 - \frac{t^{.6}}{10 + t^{.6}}\right) + .75 \times \left(1 - \frac{t}{35 + t}\right) \qquad \text{(C-13)}$$

when t is time of installation of the partitions in days after the placement of concrete.

The values proposed by Graham and Scanlon are larger than those used in the code, and multipliers of similar magnitude have been proposed. Use of Eq. C-12 is recommended for λ with the Span-Thickness Equations.

Table C.1 Comparison of Long-term Deflection Multipliers

Time of Installation of Partitions	CAN3-A23.3 Code Value λ	Recommended Value of λ
0	2.0	3.25
3	1.0	1.22
6	0.8	0.84
12	0.6	0.62

C.3 Effective Moment of Inertia

The Branson cubic equation[38] for the effective moment of inertia (I_e) can be reduced to a function of six variables as shown on the next two pages. The variables shown in Eq. C-5 are the reinforcement ratio (ρ), concrete strength (f'_c), steel yield strength (f_y), $\frac{d}{h}$, modulus of rupture (f_r) and the ratio of live to dead loads ($\frac{w_l}{w_d}$). k and j were not included in the variables since they are both functions of ρ.

A computer program was written based on Eq. C-5 to investigate the relationships between the six variables and I_e. I_e was found to be insensitive to change in $\frac{w_l}{w_d}$ (Figure C.1). The effect of variation in f'_c on I_e was minimized when f'_c was incorporated as a combined variable with ρ as $\frac{E_s}{E_c}\rho$ or $n\rho$ in calculating I_e (Figure C.2). Values of the last three variables were assumed to simplify the derivation, although in the cases of f_y and $\frac{d}{h}$ relationships can be found demonstrated in Figures C.3 and C.4. The reinforcing was assumed to be 400 MPa and $\frac{d}{h} = 0.8$ for reasons previously stated. The value of f_e was assumed to be $0.30\sqrt{f'_c}$ based on the result of studies by Tam and Scanlon[18] and Graham and Scanlon[19]. Branson's cubic equation can be reduced to Eqs. C-14 and C-15 using these relationships, which allows I_e to be calculated knowing only f'_c and ρ.

$\Lambda = .0083 + .2n\rho$ when $n\rho \geq .040$ (C-14)

$\Lambda = .047 - 1.03n\rho$ when $n\rho \leq .040$ $\Lambda > 0.016$ (C-15)

The approximations provide a close estimate of I_e.

Effective Moment of Inertia

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_g \qquad \text{(C-16)}$$

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \left[I_g - I_{cr}\right] + I_{cr} \qquad \text{(C-17)}$$

Consider M_a

$$M_a = \rho b d f_s j d \qquad M_f = \Phi_s f_y b d^2 \rho \left[1 - .59 \frac{\Phi_s f_y}{\Phi_c f_c} \rho\right] \qquad \text{(C-18)}$$

$$M_a = (w_l + w_d)\frac{l^2}{8} \qquad M_f = (\alpha_l w_l + \alpha_d w_d)\frac{l^2}{8} \qquad \text{(C-19)}$$

$$M_a = \frac{(w_l + w_d)}{(\alpha_l w_l + \alpha_d w_d)} M_f \qquad \text{(C-20)}$$

Also

$$M_a = \frac{(w_l + w_d)}{(\alpha_l w_l + \alpha_d w_d)} \Phi_s f_y b d^2 \rho \left[1 - .59 \frac{\Phi_s f_y}{\Phi_c f_c} \rho\right] \qquad \text{(C-21)}$$

$$j f_s = \frac{(w_l + w_d)}{(\alpha_l w_l + \alpha_d w_d)} \Phi_s f_y \left[1 - .59 \frac{\Phi_s f_y}{\Phi_c f_c} \rho\right] \qquad \text{(C-22)}$$

$$M_{cr} = \frac{bh^2}{6} f_r = \frac{bd^2}{(d/h)^2 6} f_r \qquad \text{(C-23)}$$

$$\frac{M_{cr}}{M_a} = \left[\frac{1}{(d/h)^2 6} \frac{f_r}{\Phi_s f_y \rho \left[1 - .59 \frac{\Phi_s f_y}{\Phi_c f_c} \rho\right]} \left(\frac{(\alpha_l w_l + \alpha_d w_d)}{(w_l + w_d)}\right)\right] \qquad \text{(C-24)}$$

$$\frac{M_{cr}}{M_a} \left| \frac{1}{(d/h)^2 6} \; \frac{f_r}{\Phi f_y \rho \left| 1 - 59 \frac{\Phi f_y}{\Phi f'_c} \rho \right|} \left(\frac{(\alpha_l \frac{w_l}{w_d} + \alpha_d)}{(\frac{w_l}{w_d} + 1)} \right) \right| \qquad \text{(C-25)}$$

or for ACI 318-83

$$\frac{M_{cr}}{M_a} = \left| \frac{1}{(d/h)^2 6} \; \frac{f_r}{\Phi f_y \rho \left| 1 - 59 \frac{f_y}{f'_c} \rho \right|} \left(\frac{(\alpha_l \frac{w_l}{w_d} + \alpha_d)}{(\frac{w_l}{w_d} + 1)} \right) \right|$$

$$I_{cr} = \frac{bd^3}{12} \left| 4k^3 + 12n\rho(1-k)^2 \right|$$

$$I_g = \frac{bd^3}{(d/h)^3 12} \qquad \text{(C-26)}$$

Substituting into I_e

$$I_e = \frac{bd^3}{12} \left| \left(\frac{M_{cr}}{M_a} \right)^3 \left(\frac{1}{(d/h)^3} - \left| 4k^3 + 12n\rho(1-k)^2 \right| \right) + 4k^3 + 12n\rho(1-k)^2 \right| \qquad \text{(C-27)}$$

or

$$I_e = (d/h)^3 \frac{bh^3}{12} \left| \left(\frac{M_{cr}}{M_a} \right)^3 \left(\frac{1}{(d/h)^3} - \left| 4k^3 + 12n\rho(1-k)^2 \right| \right) + 4k^3 + 12n\rho(1-k)^2 \right|$$

Giving us Λ

$$\Lambda = \left| \left(\frac{M_{cr}}{M_a} \right)^3 \left(\frac{1}{(d/h)^3} - \left| 4k^3 + 12n\rho(1-k)^2 \right| \right) + 4k^3 + 12n\rho(1-k)^2 \right| \qquad \text{(C-28)}$$

or

$$\Lambda = (d/h)^3 \left| \left(\frac{M_{cr}}{M_a} \right)^3 \left(\frac{1}{(d/h)^3} - \left| 4k^3 + 12n\rho(1-k)^2 \right| \right) + 4k^3 + 12n\rho(1-k)^2 \right|$$

If we assume $\frac{d}{h} = 0.8$, $f_r = 0.3\sqrt{f'_c}$, and using CAN3-A23.3-M84 we get

$\Lambda = .0083 + .2n\rho$ when $n\rho \geq .040$

$\Lambda = .047 - 1.03n\rho$ when $n\rho \leq .040$ but $\Lambda > .016$

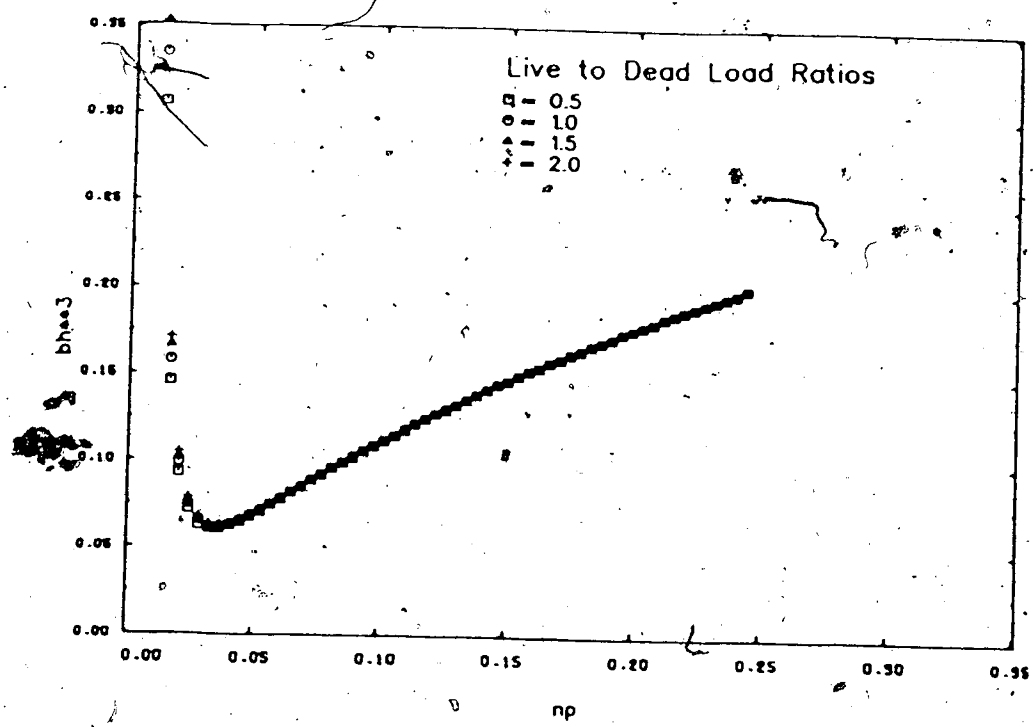


Figure C.1 Effect of Live to Dead load Ratios on I_e

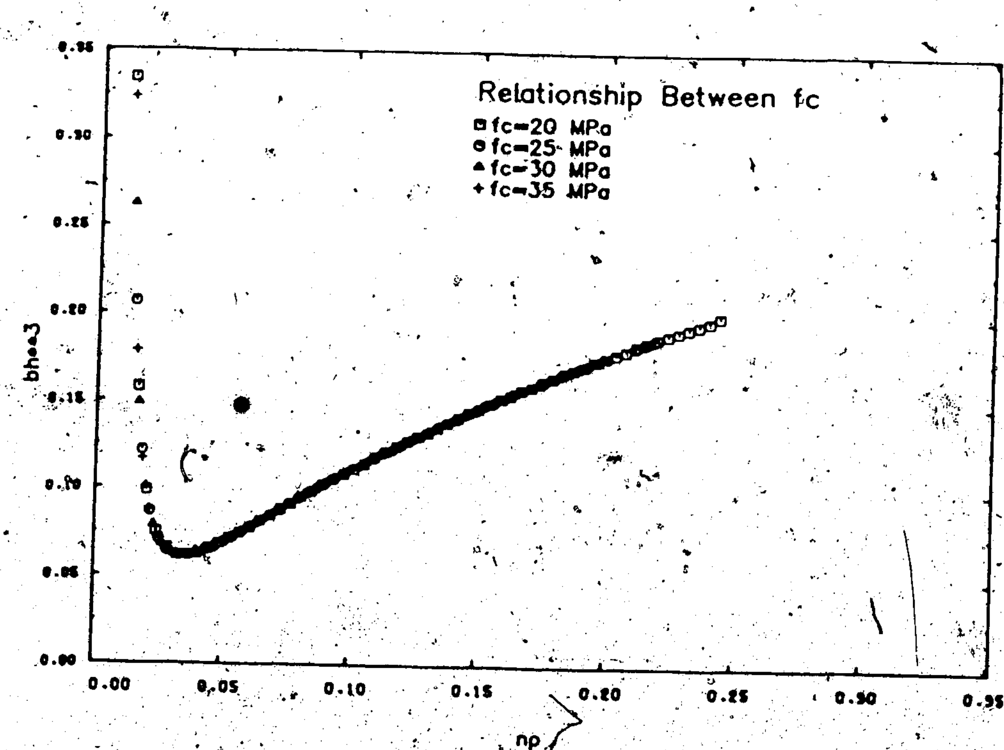


Figure C.2 Effect of Concrete Strength on I_e

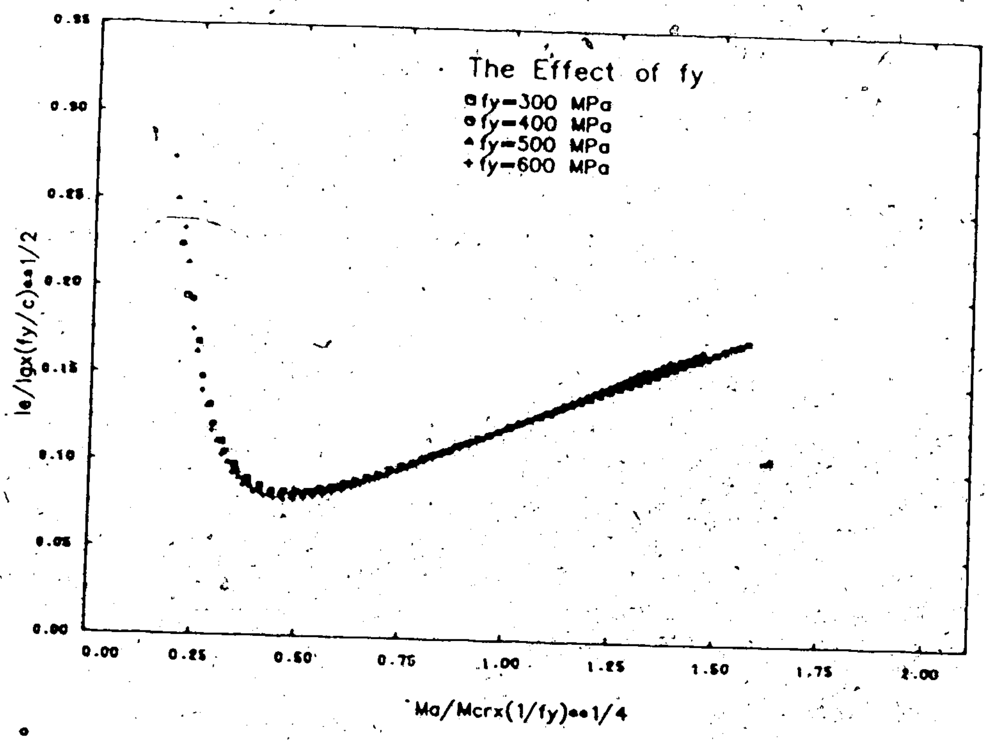


Figure C.3 Effect of Reinforcing Yield Strengths on I_e

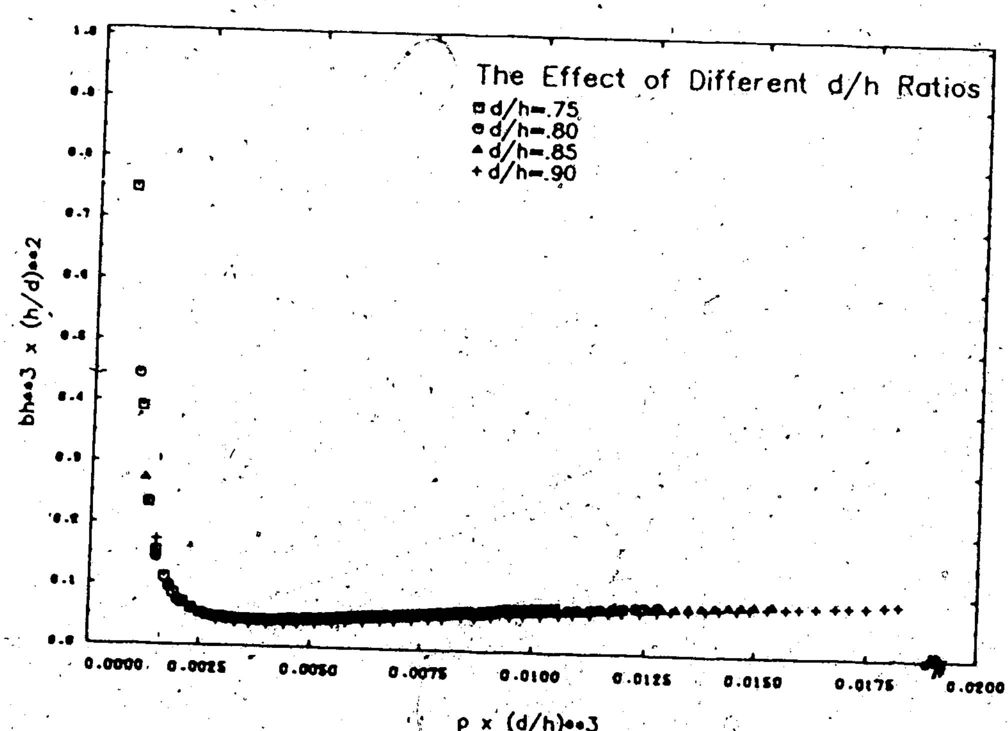


Figure C.4 Effect of $\frac{d}{h}$ ratios on I_e

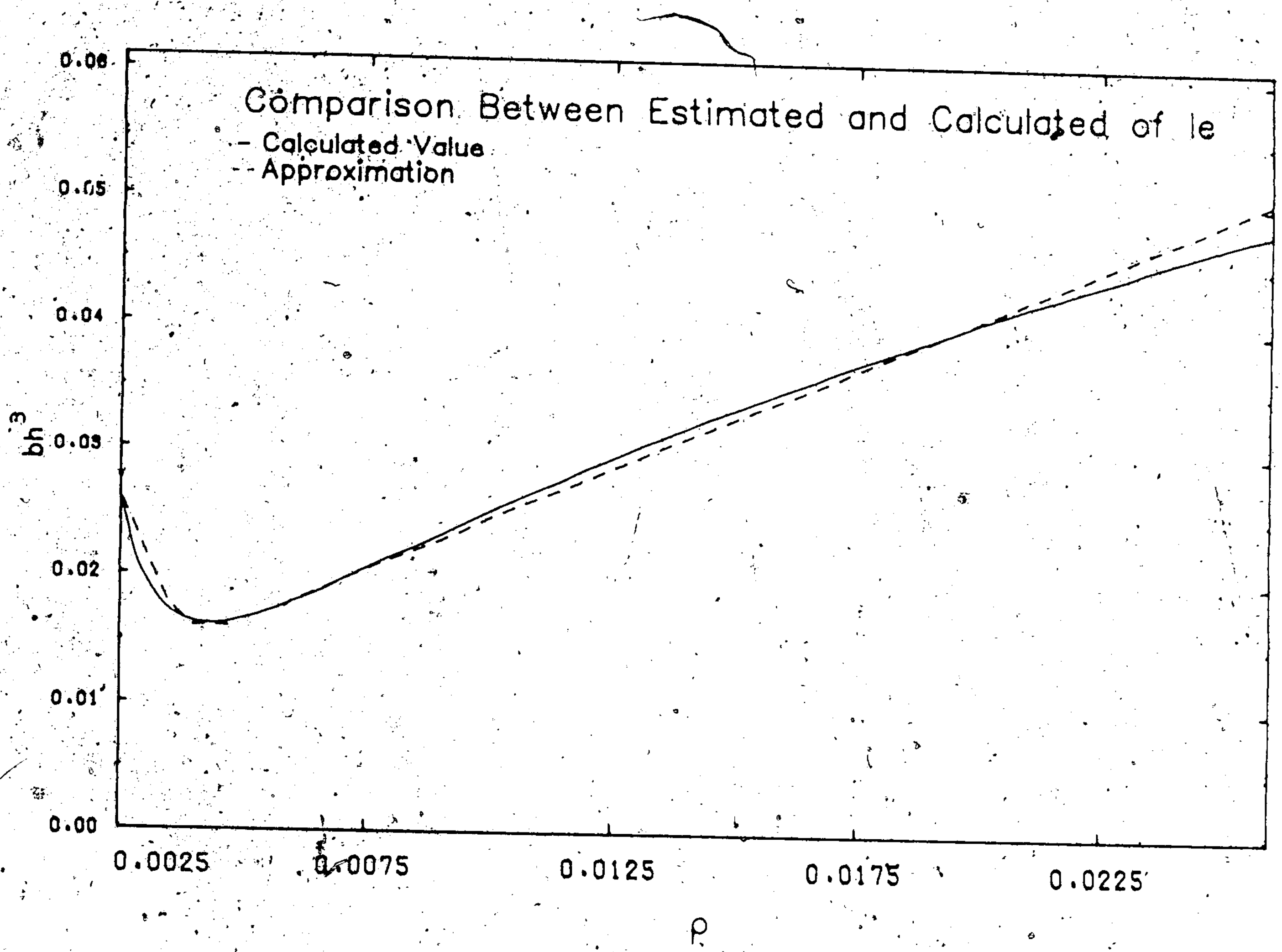


Figure C.5 Approximation of Branson's I_e using $f_e = 0.30\sqrt{f_c'}$

C.4 Slab Factor

The Slab Factor (k_1) derived on the following pages was completed by assuming the slab could be modelled as assumed in the Direct Design Method, and that the Moment of Inertia of the slab column strip was the same as the middle strip.

The crossing beam analogy was used in the derivation. It assumes that the slab middle and column strips can be modelled as wide beams with uniform loads, allowing the midpanel deflection to be calculated by summing the column and middle strip deflections. The magnitude of the uniform loads were calculated using the moment fields from the Direct Design Method. The use of the moment fields from the Direct Design Method should provide a realistic moment distribution, since most slabs are designed using the Direct Design or Equivalent Frame Methods' which have similar moment fields.

The last assumption used was that the middle and column strips would have the same stiffness. If one assumes that all the middle strips will have minimum reinforcement of .25 % and the column strip has typically .3 to .7 % reinforcing steel, Eqs. C-14 and C-15 shows that the column strip will not as stiff. The assumpιion of similar stiffnesses can thus be considered as conservative.

Slab Factor k_1

If we consider the slab column and middle strips as a number of crossing beams, the middle panel span-deflection ratio can be written in terms of both the column and middle strip span-deflection ratios.

Let

$$Minimum\ Span-Deflection\ ratio = \left|\frac{l}{\delta}\right|_{min}$$

If the column strip span is always longer than the middle strip the spans can be written as

$$l_{col} = l_1 \quad and \quad l_{mid} = \frac{l_1}{\beta_{panel}}$$

Also column and middle deflections can be written as

$$\Delta_{col} = \beta_{col}\frac{w_{col}}{E_c I_{col}} l_1^4 \quad and \quad \Delta_{mid} = \beta_{mid}\frac{w_{mid}}{E_c I_{mid}}\left|\frac{l_1}{\beta_{panel}}\right|^4 \qquad (C-30)$$

The middle strip deflection can then be written in terms of the column strip deflection

$$\Delta_{mid} = \frac{\Delta_{mid}}{\Delta_{col}}\Delta_{col} \quad which\ is \quad \Delta_{mid} = \left|\frac{\beta_{mid} I_{col} w_{mid}}{\beta_{col} I_{mid} w_{col}}\left|\frac{1}{\beta_{panel}}\right|^4\right|\Delta_{col}$$

and the midpanel deflection can be written in terms of the column strip deflection as well

$$\Delta_{panel} = \Delta_{col} + \Delta_{mid} \qquad \Delta_{panel} = \Delta_{col} + \left|\frac{\beta_{mid} I_{col} w_{mid}}{\beta_{col} I_{mid} w_{col}}\left|\frac{1}{\beta_{panel}}\right|^4\right|\Delta_{col} \qquad (C-31)$$

Thus span-deflection ratio for the diagonal span can be defined in terms of the column strip deflection and span.

$$\frac{l_{diag}}{\Delta_{panel}} = \frac{(1+\beta_{panel}^2)^{\frac{1}{2}}}{\left[1+\left|\frac{\beta_{mid} I_{col} w_{mid}}{\beta_{col} I_{mid} w_{col}}\left|\frac{1}{\beta_{panel}}\right|^4\right|\right]}\frac{l_{col}}{\Delta_{col}} \qquad (C-32)$$

If the slab is not to exceed the span-deflection criterion.

$$\frac{l_{col}}{\Delta_{col}} = \frac{\left[1 - \left[\frac{\beta_{mid} l_{col} w_{mid}}{\beta_{col} l_{mid} w_{col}}\right]\left[\frac{1}{\beta_{panel}}\right]^4\right]}{(1 - \beta^2_{panel})^{\frac{1}{2}}} \left[\frac{l}{\delta}\right]_{min} \geq \left[\frac{l}{\delta}\right]_{min} \qquad \text{(C-33)}$$

If we consider the slab moment distribution as assumed in the Direct Design Method, the load and deflection in the column strip for an interior span will be:

$$M_e = 75 \times 65 \frac{wl_1}{\beta_{panel}} \frac{l_1^2}{8}$$

$$M_m = 60 \times 35 \frac{wl_1}{\beta_{panel}} \frac{l_1^2}{8}$$

Therefore the total moment will be

$$M_o = (75 \times 65 + 60 \times 35) \frac{wl_1}{\beta_{panel}} \frac{l_1^2}{8}$$

This allows us to write load in the column strip as,

$$w_{col} = 6975 \frac{wl_1}{\beta_{panel}} \qquad \text{(C-34)}$$

and the deflection coefficient is (see k_2)

$$\beta_{col} = .1613 \beta_{simp} \qquad \text{(C-35)}$$

Similarly, this can be done in the middle strip

$$M'_e = 25 \times 65 wl_1 \frac{l_1^2}{8} \left[\frac{1}{\beta_{panel}}\right]^2$$

$$M_m = 40 \times 35 wl_1 \frac{l_1^2}{8} \left[\frac{1}{\beta_{panel}}\right]^2$$

Therefore the total moment will be,

$$M_o = (25 \times 65 + 40 \times 35) w l_1 \frac{l_1^2}{8} \left| \frac{1}{\beta_{panel}} \right|^2$$

the load in the middle strip,

$$w_{mid} = 3025\, w l_1 \qquad \text{(C-36)}$$

and the deflection coefficient is equal to

$$\beta_{mid} = 3554\, \beta_{simp} \qquad \text{(C-37)}$$

If we conservatively assume that I_{mid} is equal to I_{col}, the span-deflection criterion can be written in terms of the design load and panel aspect ratio.

$$\frac{l_{col}}{\Delta_{col}} = \frac{\left| 1 + \left| \dfrac{3025\, w l_1\, 3554\, \beta_{simp}}{6975 \dfrac{w l_1}{\beta_{panel}}\, 1613\, \beta_{simp}} \left| \frac{1}{\beta_{panel}} \right|^4 \right| \right|}{(1 + \beta_{panel}^2)^{\frac{1}{2}}} \left| \frac{l}{\delta} \right|_{min} \geq \left| \frac{l}{\delta} \right|_{min} \qquad \text{(C-38)}$$

which reduces to

$$\frac{l_{col}}{\Delta_{col}} = \frac{\left| 1 + \left| .6666 \left| \dfrac{1}{\beta_{panel}} \right|^3 \right| \right|}{(1 + \beta_{panel}^2)^{\frac{1}{2}}} \left| \frac{l}{\delta} \right|_{max} \geq \left| \frac{l}{\delta} \right|_{max}$$

This relationship is used in the span-thickness equation when using the deflection-span ratio and is included as k_1

$$k_1 = \left(\frac{(1 + \beta_{panel}^2)^{\frac{1}{2}}}{\left| 1 + \left| .6666 \left| \dfrac{1}{\beta_{panel}} \right|^3 \right| \right|} \right)^{\frac{1}{3}} \leq 1.0 \qquad \text{(C-39)}$$

C.5 Location Factor

This factor (k_2) is derived from an elastic beam deflection equation, as was done by Rangan. The derivation was extended to put the midspan deflection in terms of the positive midspan moment.

An additional factor, Ψ_{load}, has been included in k_2 to account for varying loads and material properties of the slab at different locations. The different loads were calculated using the Direct Design moment fields to estimate an equivalent unit load on the column strip, allowing the use of the design loads. For example if sixty percent of the design load is influencing the column strip, a factor of 1.2 (60 % of the load divided by 50 % of the width) would be used. A factor of 1.50 was used in the case of a simply supported two-way slab, and this was calculated using the Hillerborg strip method and assuming the load would disperse equally in both directions. The second part of calculating Ψ_{load} is evaluating any change in material properties. The only material property considered was shrinkage. It was assumed that different minimum values of I_e due to f_e should be used in different locations. This derivation assumed f_e would equal $.30\sqrt{f'_c}$ for interior spans, $.45\sqrt{f'_c}$ for exterior spans and simply supported slabs. All the variables discussed in this section are summarized in Table C.2 along with the recommended values for k_2.

Location Factor k_2

If we consider a simply supported beam with end moments, the midspan deflection can be written as

$$\Delta = \frac{5}{48}\frac{M_o l^2}{EI} - \frac{(M_l + M_r)}{16}\frac{l^2}{EI} \qquad \text{(C-40)}$$

Therefore

$$\Delta = \frac{l^2}{48EI}(5M_o - 3(M_l + M_r))$$

This can be rewritten as

$$\Delta = \frac{5l^2}{48EI}(M_o - .6(M_l + M_r)) \qquad \text{(C-41)}$$

which is the equation that Rangan[10] used in his Span-Effective Depth formulation. We can extend the deflection equation with the following relationship between the moments.

$$M_o = \frac{(M_l + M_r)}{2} + M_m \qquad \text{(C-42)}$$

$$(M_l + M_r) = 2(M_o - M_m)$$

This gives us

$$\Delta = \frac{5l^2}{48EI}(M_o - 1.2(M_o - M_m)) \qquad \text{(C-43)}$$

$$\Delta = \frac{5l^2 M_o}{48EI}(1.2\frac{M_m}{M_o} - 0.2) \qquad \text{(C-44)}$$

but

$$M_o = \frac{wl^2}{8}$$

Therefore

$$\Delta = \frac{5wl^4}{384EI}\left(1.2\frac{M_m}{M_o} - 0.2\right) \quad \text{(C-45)}$$

$$\Delta = \beta_{simp}\left(1.2\frac{M_m}{M_o} - 0.2\right)\frac{wl^4}{EI} \quad \text{(C-46)}$$

Therefore

$$\beta = \beta_{simp}\left(1.2\frac{M_m}{M_o} - 0.2\right) \quad \text{(C-47)}$$

This relationship is used in the Span-Thickness equation as k_2

$$k_2 = \left(\frac{1}{\Psi_{load}\left(1.2\frac{M_m}{M_o} - 0.2\right)}\right)^{\frac{1}{3}} \quad \text{(C-48)}$$

Table C.2 Values used in Location Factor

	Simply Supported	Exterior Span	Interior Span
Ψ_{load}			
$*f_e$	$\frac{1}{1.42}$	$\frac{1}{1.42}$	1.0
Load	1.5	1.43	1.40
Total Factor	1.06	1.01	1.40
$\frac{M_m}{M_o}$	1.00	0.45	0.30
$1.2 \frac{M_m}{M_o} - 0.2$	1.00	0.45	0.16
k_2	0.98	1.42	1.65
k_2 with a included	1.08	1.56	1.81

Notes:

1. The moments are calculated from the Direct Design Method

2. The a value included with k_2 is the minimum value $(.016 \times \frac{384}{5})$

* ratio of minimum value of I_e using different f_e values

Appendix D

Construction

Shoring Load Ratios

Grundy's and Kabaila's[24] method of analysis presented in 1960 uses construction load ratios (R). These ratios were calculated assuming one shoring sequence. The order of operations assumed was:

1. Erect shores and formwork on new level and cast slab.
2. Strip the formwork and shores on the lowest level of shoring and in the case where reshores are used , reshore that level.
3. When reshoring is used, remove the lowest level of reshores.

Repeat Step 1

The analysis calculated the load ratios by distributing the weight of the floor cast to the supporting floors using the stiffnesses of the slabs. In their analysis Grundy and Kabaila assumed the ground level to be rigid and therefore until the shoring is above the ground floor none of the slabs take any load. The loads are then allocated to the floors as follows;

1. Allocate the weight of the slab cast in step 1 to the supporting floors so each floor deflect the same amount.
2. Allocate the load in the lowest level of shores to the floors above so they have equal deflections when the shores are removed in step 2. The floors below these shores, when reshoring is used, will rebound upward and these floors will only be supporting their own weight.

Repeat the first calculation .

If the slab thicknesses are assumed equal the loads will be proportioned according to the floor's Modulus of Elasticity (E_c). In this thesis a seven day construction cycle was assumed. Table D.1 shows the calculated Modulus of Elasticity of the floors at different times compared to that of E_c at twenty eight days and the values in Table D.1 were used to calculate the load ratios.

Table D.2 provides the maximum load ratios for a series of shoring systems. The Table illustrates that the load ratios are history dependent with the ratios fluctuating on the lower floors and converging to a constant ratio at higher levels. In this thesis only the converged value of the load ratio was considered. In cases of a 2 + 1 and 2 + 3 shoring systems, these values will be valid for any floor above the sixth level (2 + 1) and the ninth level (2 + 3).

ACI Committee 347 recommends[33] Eq. D-1 be used to insure the slab's capacity.

$$w_{con} \leq \frac{w_{ult}}{F.S.} \times \frac{f_c(t)}{f'_c} \qquad \text{(D-1)}$$

where w_{con} =maximum load imposed on the slab during construction.

=1.1 x1.1 x R_{max} x w_{slab} + 2.4 or 3.6 kPa

$f_c(t)$ =strength of the concrete at the time when w_{con} occurs, t is the time in days

w_{slab} =weight of slab

w_{ult} =ultimate design load

F.S. =the factor of safety, 1.4 is recommended.

Table D.1 Modulus of Elasticity of Concrete at Different Ages

Age	$\frac{E_c(t)}{E_c(28)}$
7	0.84
14	0.94
21	0.98
28	1.00
35	1.02

Note:
The Age is the number of days since placement of concrete

Table D.2 Construction Shoring Load Ratios for Different Shoring Systems

Floor Level	Load Ratio			
	(2 + 0)	(2 + 1)	(2 + 2)	(2 + 3)
G	-	-	-	-
1	1.52	1.36	1.28	1.22
2	2.28	1.86	1.28	1.22
3	1.92	1.76	1.78	1.22
4	2.09	1.81	1.63	1.73
5	2.01	1.78	1.70	1.56
6	2.05	1.79	1.67	1.64
7	2.03	1.79	1.69	1.60
8	2.04	1.79	1.69	1.62
9	2.04	1.79	1.69	1.61
10	2.04	1.79	1.69	1.61
11	2.04	1.79	1.69	1.61
12	2.04	1.79	1.69	1.61

Notes:
1.) The shoring systems are designated by;
(No. of Shores + No. of Reshores)
2) The load ratios were calculated using a varying E_c

Appendix E

Parameter Study Results

The complete results of the parameter study are tabulated in Table E.1 on the next page. The table list the minimum calculated span deflection ratios for the 344 simulations evaluated. The table includes the slab designation, span deflection ratios for total, incremental, live load, and construction load deflections. Their locations are also shown. The parameters evaluated in each simulation are described by the slab designation. The parameters are described in the following manner.

AA:BBB:CCC:DDD:EE:FF:GG

where AA slab type

FP	Flat Plate
BP	Flat Plate with edge beams
DP	Flat Slab with drop panels
VP	Flat Slab with drop panels of various thicknesses

BBB slab thickness(mm); $.90h_m$, h_m, $1.10h_m$

FP	160,175,195
BP	145,160,175
DP	170,190,210
VP	170,190,210

CCC geometric properties

FP-aspect ratio; 1.0,1.5,2.0

DP-aspect ratio; 1.0,1.5,2.0
(thickness of drop panels below slab:0.25)

BP-stiffness ratio of edge beam and slab; 0.8, 2.0, 3.0 (aspect ratio equal to 1.0 in all cases)

VP-relative thickness of drop panel below slab; 0.50, 0.75 (aspect ratio equal to 1.0 in all cases)

DDD service live load (kPa); 1.9, 2.4, 4.8

EE concrete strength (MPa); 20, 30

FF effective modulus of rupture multiplier of f_e; 0.60, 0.30

GG construction load -A, load from 2+3 shoring system

-B, load from 2+1 shoring system

The construction span-deflection ratios in table E.1 are the results of the finite element program described in chapter 3 and is included to allow for further simulation of different load histories after the shoring loads are removed. The ratios bracketted in Table E.1 exceed the CAN3 A23.3 Deflection Limits and for total deflection $\frac{\ell_n}{240}$. Figure E.1 indicates the location of the minimum span-deflection ratios listed in the table.

Minimum Span-Deflection Ratios

Slab Designation	Loc.	ln / Δ l	ln / Δ inc	ln / Δ tot	ln / Δ con
FP:145:1.0:1.9:20:.3:A	10	1547	(239)	(103)	285
FP:145:1.0:1.9:20:.6:A	9	3263	504	(217)	600
FP:145:1.0:1.9:30:.6:A	9	2022	(312)	(134)	372
FP:145:1.0:1.9:30:.6:A	7	5127	792	341	943
FP:145:1.0:2.4:20:.3:A	10	1224	(230)	(101)	285
FP:145:1.0:2.4:20:.6:A	9	2583	484	(213)	600
FP:145:1.0:2.4:30:.6:A	9	1601	(300)	(132)	372
FP:145:1.0:2.4:30:.6:A	7	4059	761	335	943
FP:160:1.0:1.9:20:.3:A	9	2187	(325)	(137)	367
FP:160:1.0:1.9:20:.6:A	10	5053	751	317	847
FP:160:1.0:1.9:30:.3:A	12	2909	(433)	(183)	488
FP:160:1.0:1.9:30:.6:A	9	7499	1115	471	1257
FP:160:1.0:2.4:20:.3:A	9	1732	(313)	(135)	367
FP:160:1.0:2.4:20:.6:A	10	4000	723	312	847
FP:160:1.0:2.4:30:.3:A	12	2303	(416)	(180)	488
FP:160:1.0:2.4:30:.6:A	9	5937	1073	463	1257
FP:160:1.0:4.8:20:.3:A	10	852	(298)	(146)	439
FP:160:1.5:1.9:20:.3:A	10	3108	517	(221)	595
FP:160:1.5:1.9:20:.6:A	4	7928	1319	564	1518
FP:160:1.5:1.9:30:.3:A	3	4350	724	309	833
FP:160:1.5:1.9:30:.6:A	4	10603	1764	754	2030
FP:160:1.5:2.4:20:.3:A	10	2461	496	(217)	595
FP:160:1.5:2.4:20:.6:A	4	6276	1264	553	1518
FP:160:1.5:2.4:30:.3:A	3	3443	694	304	833
FP:160:1.5:2.4:30:.6:A	4	8394	1691	740	2030
FP:160:1.5:4.8:20:.3:A	10	1331	(466)	(227)	686
FP:160:1.5:4.8:20:.6:A	4	3013	1054	515	1552
FP:160:1.5:4.8:30:.3:A	3	1820	636	311	937
FP:160:1.5:4.8:30:.6:A	4	3975	1390	679	2048
FP:160:2.0:1.9:20:.3:A	10	3342	586	252	682
FP:160:2.0:1.9:20:.6:A	4	8462	1484	638	1726
FP:160:2.0:1.9:30:.3:A	3	4428	776	334	903
FP:160:2.0:2.4:20:.3:A	10	2646	560	247	682
FP:160:2.0:2.4:20:.6:A	4	6699	1418	625	1726
FP:160:2.0:2.4:30:.3:A	3	3506	742	327	903
FP:160:2.0:4.8:20:.3:A	10	1517	531	259	781
FP:160:2.0:4.8:20:.6:A	4	3436	1202	587	1770
FP:160:2.0:4.8:30:.3:A	10	1969	689	336	1014
FP:175:1.0:1.9:20:.3:A	10	3063	(439)	(182)	471
FP:175:1.0:1.9:20:.6:A	9	7591	1089	452	1168
FP:175:1.0:1.9:30:.3:A	9	4370	627	260	672
FP:175:1.0:1.9:30:.6:A	9	10323	1481	615	1588
FP:175:1.0:2.4:20:.3:A	10	2425	(424)	(180)	471

* see Figure E.1 for location of minimum span-deflection ratio

Minimum Span-Deflection Ratios continued

Slab Designation	Loc.	ln / Δ l	ln / Δ inc	ln / Δ tot	ln / Δ con
FP:175:1.0:2.4:20:.6:A	9	6010	1050	445	1168
FP:175:1.0:2.4:30:.3:A	9	3459	604	256	672
FP:175:1.0:2.4:30:.6:A	9	8172	1428	606	1588
FP:175:1.0:4.8:20:.3:A	10	1176	(400)	(192)	556
FP:175:1.5:1.9:20:.3:A	10	4328	695	292	761
FP:175:1.5:1.9:20:.6:A	4	10862	1745	733	1910
FP:175:1.5:1.9:30:.3:A	3	6502	1045	439	1143
FP:175:1.5:1.9:30:.6:A	4	14375	2310	970	2527
FP:175:1.5:2.4:20:.3:A	10	3426	667	287	761
FP:175:1.5:2.4:20:.6:A	4	8599	1675	720	1910
FP:175:1.5:2.4:30:.3:A	3	5147	1002	431	1143
FP:175:1.5:2.4:30:.6:A	4	11380	2216	953	2527
FP:175:1.5:4.8:20:.3:A	10	1818	618	296	860
FP:175:1.5:4.8:20:.6:A	4	4099	1395	668	1939
FP:175:1.5:4.8:30:.3:A	3	2658	904	433	1257
FP:175:1.5:4.8:30:.6:A	4	5384	1832	878	2547
FP:175:2.0:1.9:20:.3:A	10	4552	771	326	852
FP:175:2.0:1.9:20:.6:A	4	11744	1989	841	2199
FP:175:2.0:1.9:30:.3:A	4	6274	1063	449	1175
FP:175:2.0:2.4:20:.3:A	10	3604	738	320	852
FP:175:2.0:2.4:20:.6:A	4	9298	1904	825	2199
FP:175:2.0:2.4:30:.3:A	4	4967	1017	441	1175
FP:175:2.0:4.8:20:.3:A	10	2038	693	332	964
FP:175:2.0:4.8:20:.6:A	4	4706	1601	767	2226
FP:175:2.0:4.8:30:.3:A	10	2767	941	451	1309
FP:195:1.0:1.9:20:.3:A	10	4577	628	256	635
FP:195:1.0:1.9:20:.6:A	9	11359	1558	635	1576
FP:195:1.0:1.9:30:.3:A	9	6937	952	388	962
FP:195:1.0:1.9:30:.6:A	9	15219	2088	851	2111
FP:195:1.0:2.4:20:.3:A	10	3623	606	252	635
FP:195:1.0:2.4:20:.6:A	9	8993	1504	626	1576
FP:195:1.0:2.4:30:.3:A	9	5492	919	382	962
FP:195:1.0:2.4:30:.6:A	9	12048	2016	838	2111
FP:195:1.0:4.8:20:.3:A	10	1727	568	266	737
FP:195:1.5:1.9:20:.3:A	10	6837	1051	433	1084
FP:195:1.5:1.9:20:.6:A	4	15928	2449	1009	2525
FP:195:1.5:1.9:30:.3:A	4	10526	1619	667	1669
FP:195:1.5:1.9:30:.6:A	4	21035	3235	1333	3334
FP:195:1.5:2.4:20:.3:A	10	5413	1011	426	1084
FP:195:1.5:2.4:20:.6:A	4	12609	2354	993	2525
FP:195:1.5:2.4:30:.3:A	4	8333	1556	656	1669
FP:195:1.5:2.4:30:.6:A	4	16653	3109	1311	3334
FP:195:1.5:4.8:20:.3:A	10	2757	906	425	1176

* see Figure E.1 for location of minimum span-deflection ratio

Minimum Span-Deflection Ratios continued

Slab Designation	Loc.	ln / Δ 1	ln / Δ inc	ln / Δ tot	ln / Δ con
FP:195:1.5:4.8:20:.6:A	4	5967	1961	919	2545
FP:195:1.5:4.8:30:.3:A	4	4205	1382	648	1793
FP:195:1.5:4.8:30:.6:A	4	7831	2573	1206	3340
FP:195:2.0:1.9:20:.3:A	10	6803	1103	457	1148
FP:195:2.0:1.9:20:.6:A	4	17100	2773	1149	2887
FP:195:2.0:1.9:30:.3:A	4	11203	1817	753	1891
FP:195:2.0:2.4:20:.3:A	10	5385	1058	449	1148
FP:195:2.0:2.4:20:.6:A	4	13537	2660	1129	2887
FP:195:2.0:2.4:30:.3:A	4	8869	1743	740	1891
FP:195:2.0:4.8:20:.3:A	10	2944	967	453	1255
FP:195:2.0:4.8:20:.6:A	4	6806	2237	1048	2903
FP:195:2.0:4.8:30:.3:A	10	4714	1549	726	2010
BP:145:0.8:1.9:20:.3:A	10	1786	(276)	(119)	329
BP:145:0.8:1.9:20:.6:A	8	4030	622	268	741
BP:145:0.8:1.9:30:.3:A	12	2305	(356)	(153)	424
BP:145:0.8:1.9:30:.6:A	12	5802	896	385	1067
BP:145:0.8:2.4:20:.3:A	10	1414	(265)	(117)	329
BP:145:0.8:2.4:20:.6:A	8	3190	598	263	741
BP:145:0.8:2.4:30:.3:A	10	1825	(342)	(151)	424
BP:145:0.8:2.4:30:.6:A	12	4593	861	379	1067
BP:145:2.0:1.9:20:.3:A	10	1777	(274)	(118)	327
BP:145:2.0:1.9:20:.6:A	8	4080	630	271	751
BP:145:2.0:1.9:30:.3:A	12	2398	(370)	(159)	441
BP:145:2.0:1.9:30:.6:A	12	5832	901	387	1073
BP:145:2.0:2.4:20:.3:A	10	1407	(264)	(116)	327
BP:145:2.0:2.4:20:.6:A	8	3230	606	266	751
BP:145:2.0:2.4:30:.3:A	10	1898	(356)	(157)	441
BP:145:2.0:2.4:30:.6:A	12	4617	866	381	1073
BP:145:3.0:1.9:20:.3:A	10	1759	(272)	(117)	324
BP:145:3.0:1.9:20:.6:A	8	4091	632	272	753
BP:145:3.0:1.9:30:.3:A	12	2417	(373)	(161)	445
BP:145:3.0:1.9:30:.6:A	12	5831	900	387	1073
BP:145:3.0:2.4:20:.3:A	10	1392	(261)	(115)	324
BP:145:3.0:2.4:20:.6:A	8	3239	607	267	753
BP:145:3.0:2.4:30:.3:A	10	1914	(359)	(158)	445
BP:145:3.0:2.4:30:.6:A	12	4616	865	381	1073
BP:160:0.8:1.9:20:.3:A	10	2435	(362)	(153)	408
BP:160:0.8:1.9:20:.6:A	8	5794	862	364	971
BP:160:0.8:1.9:30:.3:A	12	3235	481	(203)	542
BP:160:0.8:1.9:30:.6:A	12	8077	1201	507	1353
BP:160:0.8:2.4:20:.3:A	10	1928	(349)	(150)	408
BP:160:0.8:2.4:20:.6:A	8	4587	829	358	971
BP:160:0.8:2.4:30:.3:A	12	2561	(463)	(200)	542

* see Figure E.1 for location of minimum span-deflection ratio

Minimum Span-Deflection Ratios continued

Slab Designation	Loc.	l_n / Δ_l	l_n / Δ_{inc}	l_n / Δ_{tot}	l_n / Δ_{con}
BP:160:0.8:2.4:30:.6:A	12	6394	1156	499	1353
BP:160:2.0:1.9:20:.3:A	10	2480	(369)	(156)	416
BP:160:2.0:1.9:20:.6:A	12	5850	870	367	980
BP:160:2.0:1.9:30:.3:A	12	3368	501	(211)	564
BP:160:2.0:1.9:30:.6:A	12	8014	1192	503	1343
BP:160:2.0:2.4:20:.3:A	10	1963	(355)	(153)	416
BP:160:2.0:2.4:20:.6:A	12	4631	837	361	980
BP:160:2.0:2.4:30:.3:A	10	2667	482	(208)	564
BP:160:2.0:2.4:30:.6:A	12	6344	1147	495	1343
BP:160:3.0:1.9:20:.3:A	10	2472	(368)	(155)	414
BP:160:3.0:1.9:20:.6:A	12	5837	868	366	978
BP:160:3.0:1.9:30:.3:A	12	3394	505	(213)	569
BP:160:3.0:1.9:30:.6:A	12	7994	1189	502	1339
BP:160:3.0:2.4:20:.3:A	10	1957	(354)	(153)	414
BP:160:3.0:2.4:20:.6:A	12	4621	835	360	978
BP:160:3.0:2.4:30:.3:A	10	2687	486	(210)	569
BP:160:3.0:2.4:30:.6:A	12	6328	1144	494	1339
BP:175:0.8:1.9:20:.3:A	10	3318	(476)	(198)	511
BP:175:0.8:1.9:20:.6:A	12	8217	1179	490	1264
BP:175:0.8:1.9:30:.3:A	12	4987	716	297	767
BP:175:0.8:1.9:30:.6:A	12	10935	1569	651	1682
BP:175:0.8:2.4:20:.3:A	10	2627	(459)	(195)	511
BP:175:0.8:2.4:20:.6:A	12	6505	1136	482	1264
BP:175:0.8:2.4:30:.3:A	12	3948	690	293	767
BP:175:0.8:2.4:30:.6:A	12	8657	1512	641	1682
BP:175:2.0:1.9:20:.3:A	10	3414	490	(203)	525
BP:175:2.0:1.9:20:.6:A	12	8187	1175	488	1260
BP:175:2.0:1.9:30:.3:A	12	5102	732	304	785
BP:175:2.0:1.9:30:.6:A	12	10841	1556	646	1668
BP:175:2.0:2.4:20:.3:A	10	2703	(472)	(200)	525
BP:175:2.0:2.4:20:.6:A	12	6481	1132	480	1260
BP:175:2.0:2.4:30:.3:A	10	4039	706	299	785
BP:175:2.0:2.4:30:.6:A	12	8583	1499	636	1668
BP:175:3.0:1.9:20:.3:A	10	3443	494	(205)	530
BP:175:3.0:1.9:20:.6:A	12	8185	1175	488	1259
BP:175:3.0:1.9:30:.3:A	12	5120	735	305	788
BP:175:3.0:1.9:30:.6:A	12	10814	1552	644	1664
BP:175:3.0:2.4:20:.3:A	10	2726	(476)	(202)	530
BP:175:3.0:2.4:20:.6:A	12	6480	1132	480	1259
BP:175:3.0:2.4:30:.3:A	10	4053	708	300	788
BP:175:3.0:2.4:30:.6:A	12	8561	1495	634	1664
DP:170:1.0:1.9:20:.3:A	10	2247	(301)	(125)	323
DP:170:1.0:1.9:20:.6:A	9	5155	690	286	741

* see Figure E.1 for location of minimum span-deflection ratio

Minimum Span-Deflection Ratios continued

Slab Designation	Loc.	ln/Δ 1	ln/Δ inc	ln/Δ tot	ln/Δ con
DP:170:1.0:1.9:30:.3:A	12	2757	(369)	(153)	397
DP:170:1.0:1.9:30:.6:A	9	7425	993	411	1068
DP:170:1.0:2.4:20:.3:A	10	1779	(290)	(123)	323
DP:170:1.0:2.4:20:.6:A	9	4081	666	282	741
DP:170:1.0:2.4:30:.3:A	12	2183	(356)	(151)	397
DP:170:1.0:2.4:30:.6:A	9	5878	960	406	1068
DP:170:1.5:1.9:20:.3:A	10	3053	(456)	(191)	499
DP:170:1.5:1.9:20:.6:A	3	6994	1044	437	1142
DP:170:1.5:1.9:30:.3:A	3	3882	579	243	634
DP:170:1.5:1.9:30:.6:A	3	9923	1481	620	1621
DP:170:1.5:2.4:20:.3:A	10	2417	(438)	(188)	499
DP:170:1.5:2.4:20:.6:A	3	5537	1004	430	1142
DP:170:1.5:2.4:30:.3:A	3	3073	558	(239)	634
DP:170:1.5:2.4:30:.6:A	3	7856	1425	610	1621
DP:170:1.5:4.8:20:.3:A	10	1170	(402)	(194)	569
DP:170:1.5:4.8:20:.6:A	3	2467	847	408	1200
DP:170:1.5:4.8:30:.3:A	3	1477	507	245	718
DP:170:1.5:4.8:30:.6:A	3	3398	1167	563	1652
DP:170:2.0:1.9:20:.3:A	10	3057	494	(209)	549
DP:170:2.0:1.9:20:.6:A	4	7251	1172	495	1301
DP:170:2.0:1.9:30:.3:A	3	3746	605	256	672
DP:170:2.0:1.9:30:.6:A	4	10410	1682	710	1868
DP:170:2.0:2.4:20:.3:A	10	2420	(474)	(205)	549
DP:170:2.0:2.4:20:.6:A	4	5740	1124	486	1301
DP:170:2.0:2.4:30:.3:A	3	2965	581	251	672
DP:170:2.0:2.4:30:.6:A	4	8241	1614	698	1868
DP:170:2.0:4.8:20:.3:A	10	1288	(442)	(213)	626
DP:170:2.0:4.8:20:.6:A	4	2769	951	458	1347
DP:170:2.0:4.8:30:.3:A	3	1568	538	260	762
DP:170:2.0:4.8:30:.6:A	4	3898	1338	645	1895
DP:190:1.0:1.9:20:.3:A	10	3107	(397)	(161)	402
DP:190:1.0:1.9:20:.6:A	9	7994	1021	415	1034
DP:190:1.0:1.9:30:.3:A	12	4019	513	(208)	520
DP:190:1.0:1.9:30:.6:A	9	11098	1417	576	1435
DP:190:1.0:2.4:20:.3:A	10	2460	(384)	(159)	402
DP:190:1.0:2.4:20:.6:A	9	6329	988	409	1034
DP:190:1.0:2.4:30:.3:A	12	3181	497	(206)	520
DP:190:1.0:2.4:30:.6:A	9	8786	1372	568	1435
DP:190:1.5:1.9:20:.3:A	10	4327	617	253	636
DP:190:1.5:1.9:20:.6:A	4	10746	1532	629	1578
DP:190:1.5:1.9:30:.3:A	4	5548	791	325	815
DP:190:1.5:1.9:30:.6:A	4	14943	2131	874	2195
DP:190:1.5:2.4:20:.3:A	10	3425	595	249	636

* see Figure E.1 for location of minimum span-deflection ratio

Minimum Span-Deflection Ratios continued

Slab Designation	Loc.	ln / Δ l	ln / Δ inc	ln / Δ tot	ln / Δ con
DP:190:1.5:2.4:20:.6:A	4	8507	1477	619	1578
DP:190:1.5:2.4:30:.3:A	4	4392	763	320	815
DP:190:1.5:2.4:30:.6:A	4	11830	2054	861	2195
DP:190:1.5:4.8:20:.3:A	3	1594	528	249	697
DP:190:1.5:4.8:20:.6:A	10	3719	1233	581	1626
DP:190:1.5:4.8:30:.3:A	4	2062	683	322	902
DP:190:1.5:4.8:30:.6:A	4	5067	1679	791	2216
DP:190:2.0:1.9:20:.3:A	10	4362	674	279	704
DP:190:2.0:1.9:20:.6:A	4	10954	1692	700	1768
DP:190:2.0:1.9:30:.3:A	4	5475	846	350	884
DP:190:2.0:1.9:30:.6:A	4	15560	2404	994	2511
DP:190:2.0:2.4:20:.3:A	10	3453	648	274	704
DP:190:2.0:2.4:20:.6:A	4	8672	1627	689	1768
DP:190:2.0:2.4:30:.3:A	4	4334	813	344	884
DP:190:2.0:2.4:30:.6:A	4	12318	2311	978	2511
DP:190:2.0:4.8:20:.3:A	10	1766	585	276	772
DP:190:2.0:4.8:20:.6:A	4	4145	1374	647	1812
DP:190:2.0:4.8:30:.3:A	4	2173	720	339	950
DP:190:2.0:4.8:30:.6:A	4	5774	1914	902	2525
DP:210:1.0:1.9:20:.3:A	10	4338	531	(212)	510
DP:210:1.0:1.9:20:.6:A	9	11513	1408	563	1352
DP:210:1.0:1.9:30:.3:A	12	6274	768	307	737
DP:210:1.0:1.9:30:.6:A	9	15833	1937	774	1860
DP:210:1.0:2.4:20:.3:A	10	3434	514	(209)	510
DP:210:1.0:2.4:20:.6:A	9	9114	1365	555	1352
DP:210:1.0:2.4:30:.3:A	12	4967	744	303	737
DP:210:1.0:2.4:30:.6:A	9	12534	1877	764	1860
DP:210:1.5:1.9:20:.3:A	10	5898	806	325	787
DP:210:1.5:1.9:20:.6:A	4	15337	2096	846	2046
DP:210:1.5:1.9:30:.3:A	4	8477	1159	467	1131
DP:210:1.5:1.9:30:.6:A	4	21373	2921	1178	2852
DP:210:1.5:2.4:20:.3:A	10	4669	778	321	787
DP:210:1.5:2.4:20:.6:A	4	12142	2024	833	2046
DP:210:1.5:2.4:30:.3:A	4	6711	1119	461	1131
DP:210:1.5:2.4:30:.6:A	4	16920	2820	1161	2852
DP:210:1.5:4.8:20:.3:A	10	2275	729	337	903
DP:210:1.5:4.8:20:.6:A	4	5284	1694	782	2099
DP:210:1.5:4.8:30:.3:A	4	3216	1031	476	1277
DP:210:1.5:4.8:30:.6:A	4	7231	2318	1070	2872
DP:210:2.0:1.9:20:.3:A	10	6029	893	363	884
DP:210:2.0:1.9:20:.6:A	4	16023	2374	965	2349
DP:210:2.0:1.9:30:.3:A	3	8780	1301	529	1287
DP:210:2.0:1.9:30:.6:A	4	22379	3316	1348	3281

* see Figure E.1 for location of minimum span-deflection ratio

Minimum Span-Deflection Ratios continued

Slab Designation	Loc.	ln / Δ 1	ln / Δ inc	ln / Δ tot	ln / Δ con
DP:210:2.0:2.4:20:.3:A	10	4773	860	358	884
DP:210:2.0:2.4:20:.6:A	4	12685	2286	950	2349
DP:210:2.0:2.4:30:.3:A	3	6951	1253	521	1287
DP:210:2.0:2.4:30:.6:A	4	17717	3192	1327	3281
DP:210:2.0:4.8:20:.3:A	10	2431	780	360	966
DP:210:2.0:4.8:20:.6:A	4	5988	1920	886	2378
DP:210:2.0:4.8:30:.3:A	3	3446	1105	510	1369
DP:210:2.0:4.8:30:.6:A	4	8275	2653	1225	3286
VP:170:.50:1.9:20:.3:A	12	2799	(374)	(155)	403
VP:170:.50:1.9:20:.6:A	10	7242	969	401	1042
VP:170:.50:1.9:30:.3:A	12	3634	486	(201)	523
VP:170:.50:2.4:20:.3:A	12	2216	(362)	(153)	403
VP:170:.50:2.4:20:.6:A	10	5733	936	395	1042
VP:170:.50:2.4:30:.3:A	12	2877	(470)	(198)	523
VP:170:.75:1.9:20:.3:A	12	3531	(472)	(196)	508
VP:170:.75:1.9:20:.6:A	10	9982	1335	553	1436
VP:170:.75:1.9:30:.3:A	12	5654	756	313	813
VP:170:.75:2.4:20:.3:A	12	2795	(456)	(193)	508
VP:170:.75:2.4:20:.6:A	10	7903	1290	545	1436
VP:170:.75:2.4:30:.3:A	12	4476	731	309	813
VP:190:.50:1.9:20:.3:A	12	3880	496	(201)	502
VP:190:.50:1.9:20:.6:A	10	10890	1391	565	1408
VP:190:.50:1.9:30:.3:A	12	5944	759	308	769
VP:190:.50:2.4:20:.3:A	12	3071	(479)	(199)	502
VP:190:.50:2.4:20:.6:A	10	8621	1346	557	1408
VP:190:.50:2.4:30:.3:A	12	4706	735	304	769
VP:190:.75:1.9:20:.3:A	12	5455	697	283	705
VP:190:.75:1.9:20:.6:A	10	14961	1911	776	1935
VP:190:.75:1.9:30:.3:A	12	9368	1197	486	1211
VP:190:.75:2.4:20:.3:A	12	4319	674	279	705
VP:190:.75:2.4:20:.6:A	10	11844	1849	766	1935
VP:190:.75:2.4:30:.3:A	12	7416	1158	479	1211
VP:210:.50:1.9:20:.3:A	12	5660	692	277	665
VP:210:.50:1.9:20:.6:A	10	15617	1911	763	1835
VP:210:.50:1.9:30:.3:A	12	9403	1150	459	1105
VP:210:.50:2.4:20:.3:A	12	4481	671	273	665
VP:210:.50:2.4:20:.6:A	10	12364	1851	753	1835
VP:210:.50:2.4:30:.3:A	12	7444	1115	454	1105
VP:210:.75:1.9:20:.3:A	12	8717	1066	426	1024
VP:210:.75:1.9:20:.6:A	10	20816	2547	1017	2445
VP:210:.75:1.9:30:.3:A	9	16231	1986	793	1907
VP:210:.75:2.4:20:.3:A	12	6901	1033	421	1024
VP:210:.75:2.4:20:.6:A	10	16480	2468	1004	2445

* see Figure E.1 for location of minimum span-deflection ratio

Minimum Span-Deflection Ratios continued

Slab Designation	Loc.	In / Δ l	In / Δ inc	In / Δ tot	In / Δ con
VP:210:.75:2.4:30:.3:A	9	12849	1924	783	1907
FP:160:1.0:1.9:20:.3:B	9	2044	(304)	(125)	317
FP:160:1.0:1.9:20:.6:B	10	4500	670	275	698
FP:160:1.0:1.9:30:.3:B	12	2646	(394)	(161)	410
FP:160:1.0:1.9:30:.6:B	9	7097	1056	433	1100
FP:160:1.0:2.4:20:.3:B	9	1618	(293)	(123)	317
FP:160:1.0:2.4:20:.6:B	10	3563	644	270	698
FP:160:1.0:2.4:30:.3:B	12	2095	(379)	(159)	410
FP:160:1.0:2.4:30:.6:B	9	5618	1016	426	1100
FP:160:1.5:1.9:20:.3:B	10	2941	490	(203)	521
FP:160:1.5:1.9:20:.6:B	4	7262	1209	502	1286
FP:160:1.5:2.4:20:.3:B	10	2328	(469)	(200)	521
FP:160:1.5:2.4:20:.6:B	4	5749	1159	493	1286
FP:160:2.0:1.9:20:.3:B	10	3153	553	(231)	595
FP:160:2.0:2.4:20:.3:B	10	2496	529	(227)	595
FP:175:1.0:1.9:20:.3:B	10	2860	(409)	(166)	408
FP:175:1.0:1.9:20:.6:B	7	6791	971	393	968
FP:175:1.0:1.9:30:.3:B	12	3851	550	(223)	549
FP:175:1.0:1.9:30:.6:B	9	9885	1413	572	1409
FP:175:1.0:2.4:20:.3:B	10	2264	(394)	(163)	408
FP:175:1.0:2.4:20:.6:B	7	5376	936	387	968
FP:175:1.0:2.4:30:.3:B	12	3049	530	(220)	549
FP:175:1.0:2.4:30:.6:B	9	7826	1362	564	1409
FP:175:1.5:1.9:20:.3:B	10	3932	629	258	641
FP:175:1.5:1.9:20:.6:B	4	10386	1662	682	1692
FP:175:1.5:2.4:20:.3:B	10	3113	604	254	641
FP:175:1.5:2.4:20:.6:B	4	8222	1595	670	1692
FP:175:2.0:1.9:20:.3:B	10	4310	727	300	748
FP:175:2.0:2.4:20:.3:B	10	3412	696	295	748
FP:195:1.0:1.9:20:.3:B	10	4179	568	(227)	538
FP:195:1.0:1.9:20:.6:B	9	10812	1469	587	1392
FP:195:1.0:1.9:30:.3:B	9	6186	841	336	797
FP:195:1.0:1.9:30:.6:B	9	14608	1985	793	1881
FP:195:1.0:2.4:20:.3:B	10	3308	548	(224)	538
FP:195:1.0:2.4:20:.6:B	9	8559	1419	579	1392
FP:195:1.0:2.4:30:.3:B	9	4897	812	331	797
FP:195:1.0:2.4:30:.6:B	9	11564	1917	782	1881
FP:195:1.5:1.9:20:.3:B	10	5976	911	368	879
FP:195:1.5:1.9:20:.6:B	4	15063	2295	928	2217
FP:195:1.5:2.4:20:.3:B	10	4731	876	362	879
FP:195:1.5:2.4:20:.6:B	4	11925	2207	913	2217
FP:195:2.0:1.9:20:.3:B	10	6260	1006	409	981
FP:195:2.0:2.4:20:.3:B	10	4956	965	402	981

* see Figure E.1 for location of minimum span-deflection ratio

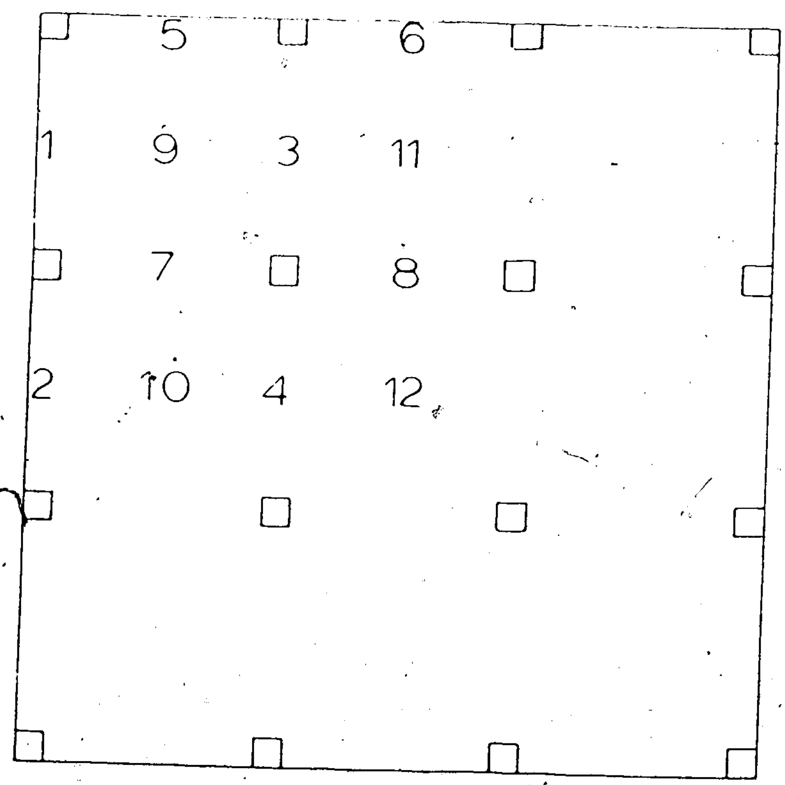


Figure E.1 Location of Minimum Span-Deflections

Figures E.2-E.4 illustrate the redistribution of the moment due to cracking in the slab under construction loading. The loads move away from the interior support as shown by the high negative moment in the elastic analysis, Figure E.2, to both the interior and exterior midspans (Figures E.3 and E.4). The redistribution is more pronounced when the Effective Modulus of Rupture is $.30\sqrt{f'_c}$, Figure E.3, than when $.60\sqrt{f'_c}$ is used. The moment fields shown in Figures E.3 and E.4 were found using the reduced stiffness of the slab due to cracking without any yielding of the reinforcing steel. These moment fields are different than that assumed by the Direct Design Method. But, no yielding of the reinforcing steel occurs at service loads and therefore, it can be concluded that for the slabs analysed that the moment field in the slab at service loads is different than at ultimate loads.

It is interesting to note that the slab around the exterior column does not crack as much as the interior column, shown in Figures E.5 and E.6, and the magnitude of the positive principal moments in the extior and interior panels are almost equal. This explains why in some cases the minimum span-deflection ratio is located in the interior panels.

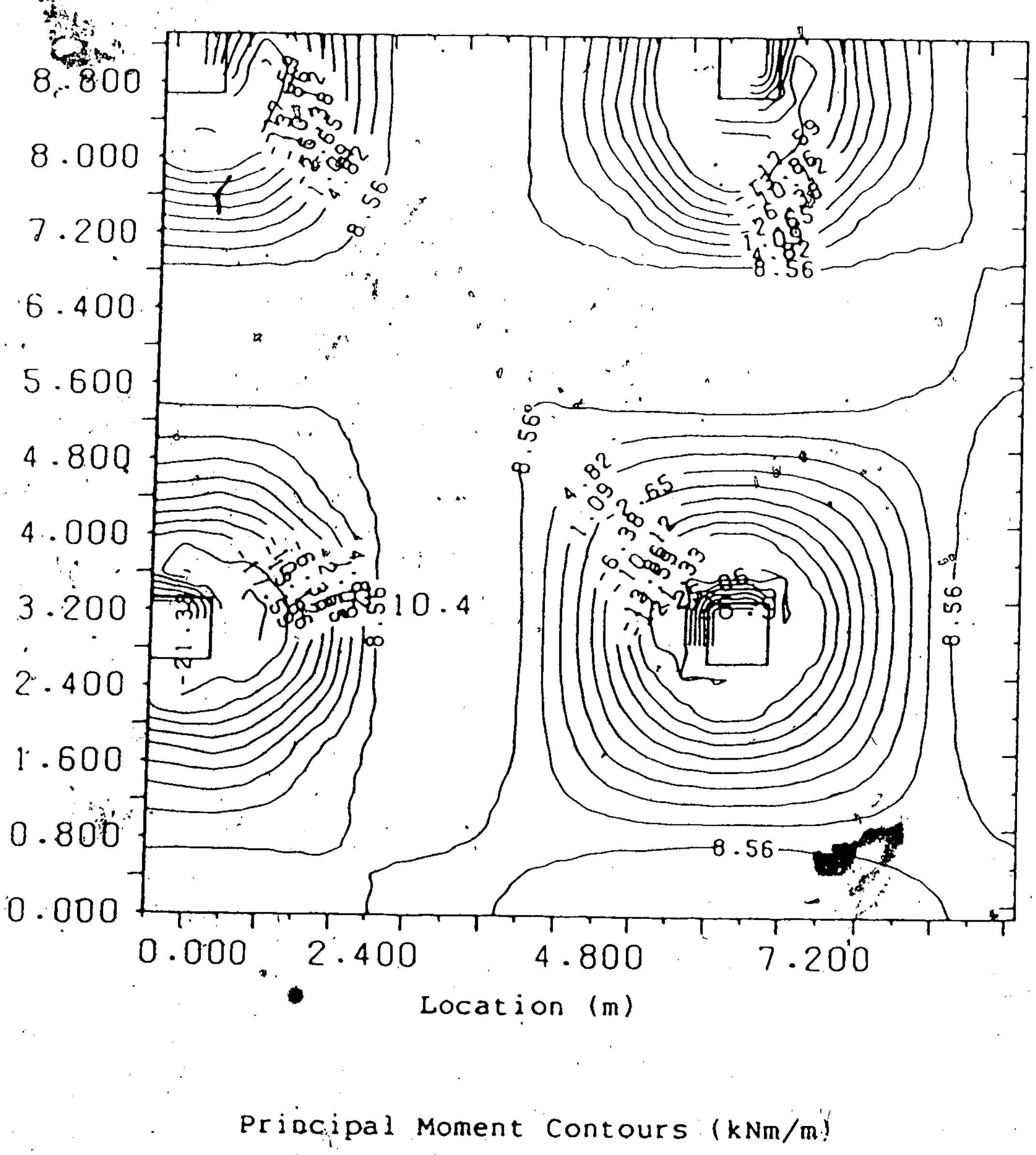
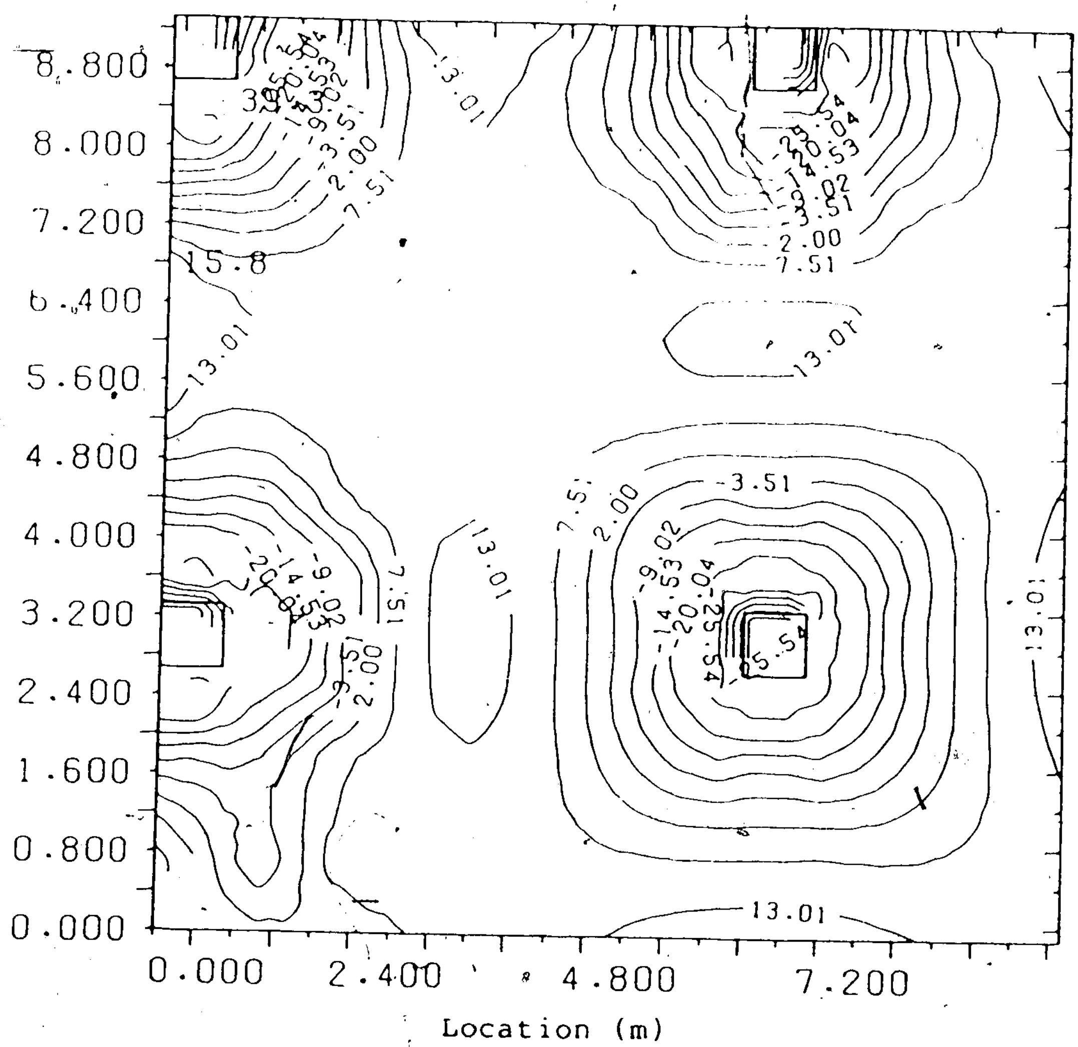
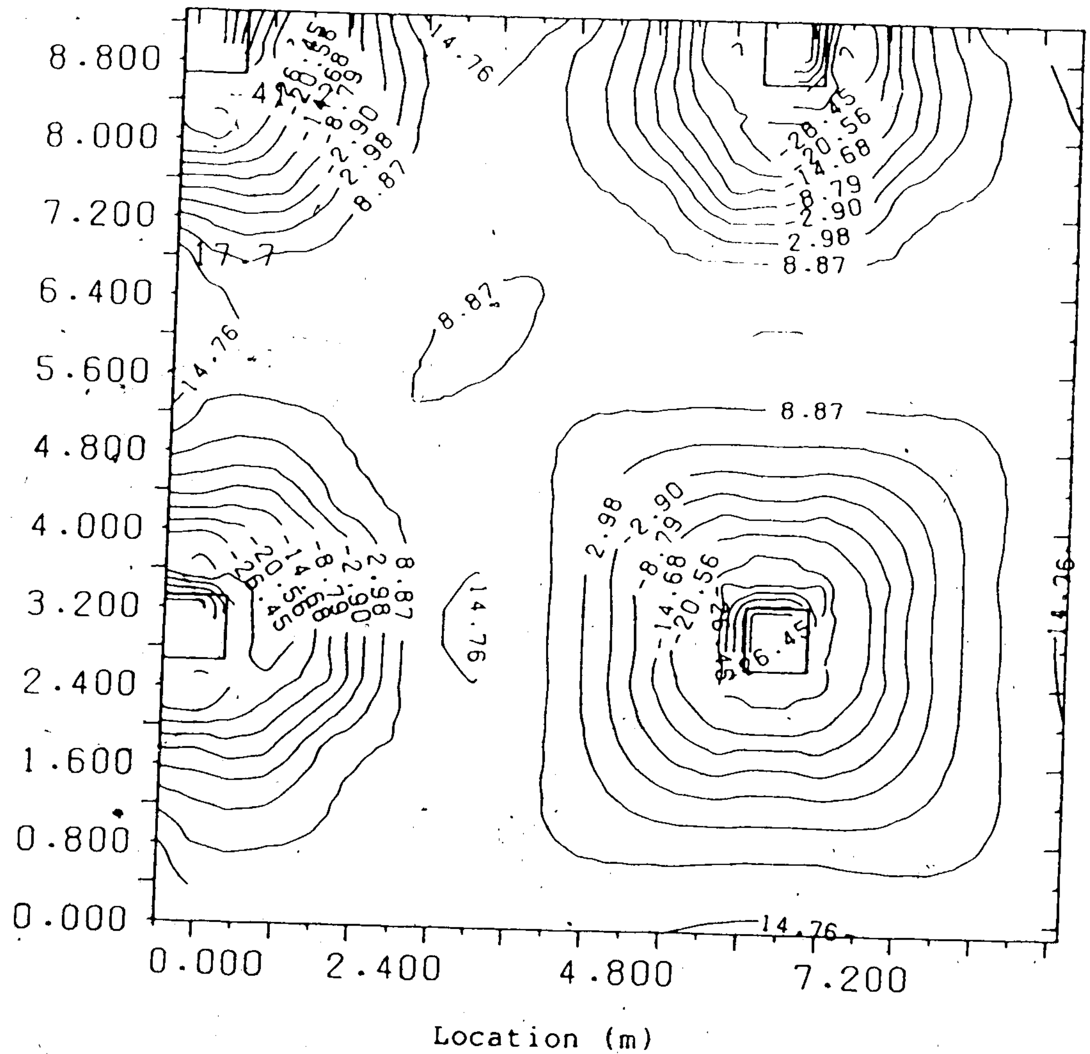


Figure E.2 Principal Moments of FP:160:1.0:2.4:20:.3:A under Construction Loads: Elastic Analysis



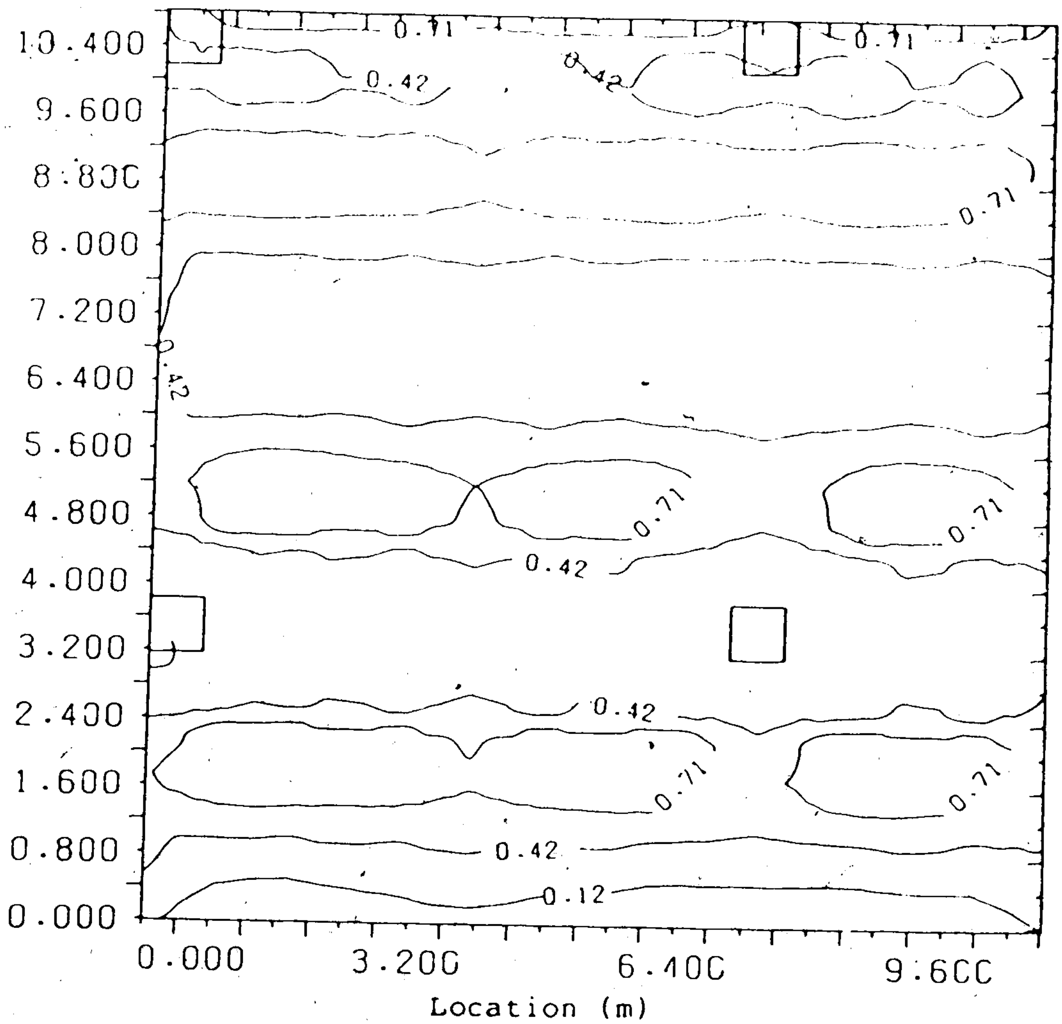
Principal Moment Contours (kNm/m)

Figure E.3 Principal Moments of FP:160:1.0:2.4:20:.3:A under Construction Loads: $f_e = .60\sqrt{f_c}$



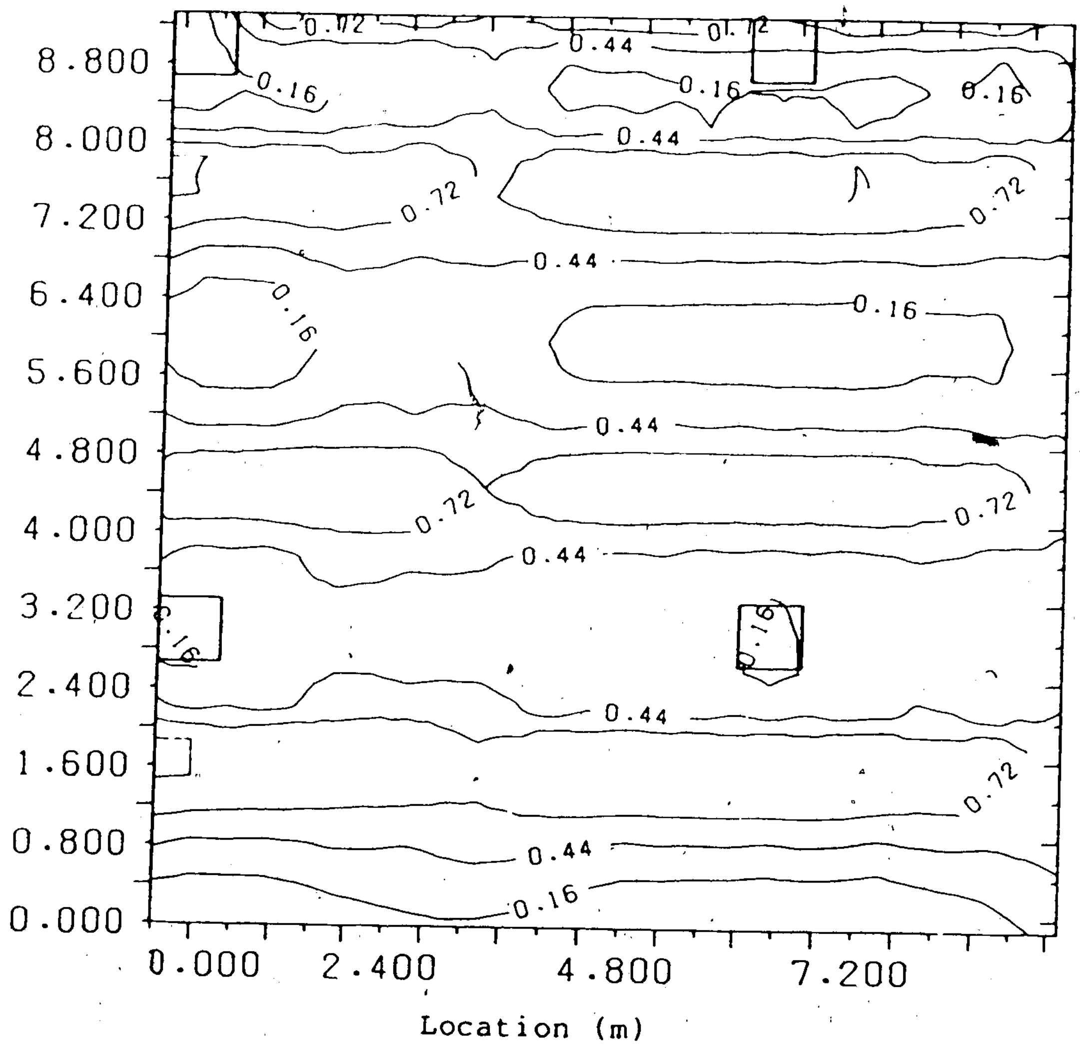
Principal Moment Contours (kNm/m)

Figure E.4 Principal Moments of FP:160:1.0:2.4:20:.3:A under Construction Loads: $f_e = .30\sqrt{f_c}$



Reduced Plate Siffness $(\frac{I_{ey}}{I_g})$

Figure E.5 Reduced Stiffness of DP:190:1.0:2.4:20:.6:A[a] under Construction Loads: $f_e = .60\sqrt{f_c}$



Reduced Plate Siffness ($\frac{I_{ey}}{I_g}$)

Figure E.6 Reduced Stiffness of FP:160:1.0:2.4:20:.3:A under Construction Loads: $f_e = .30\sqrt{f_c}$

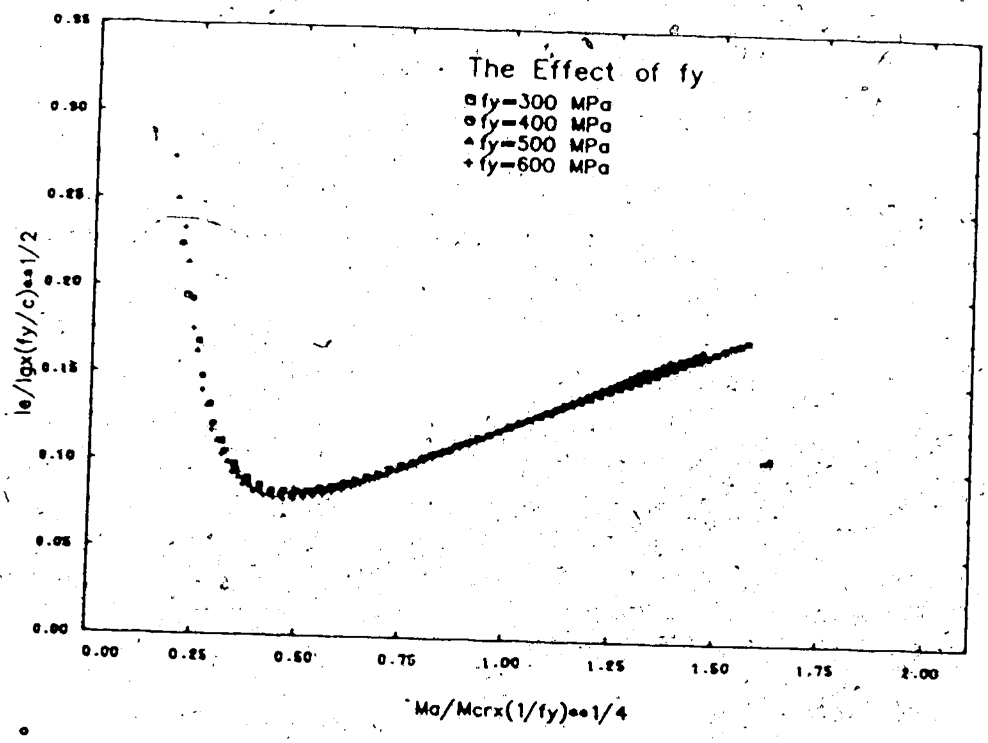


Figure C.3 Effect of Reinforcing Yield Strengths on I_e

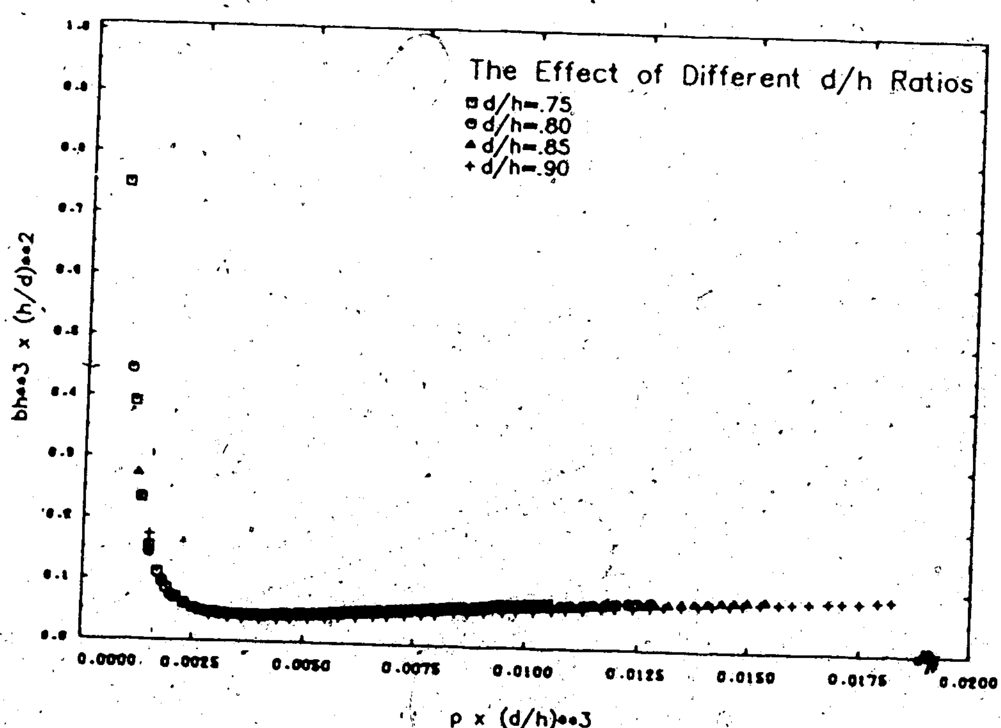


Figure C.4 Effect of $\frac{d}{h}$ ratios on I_e

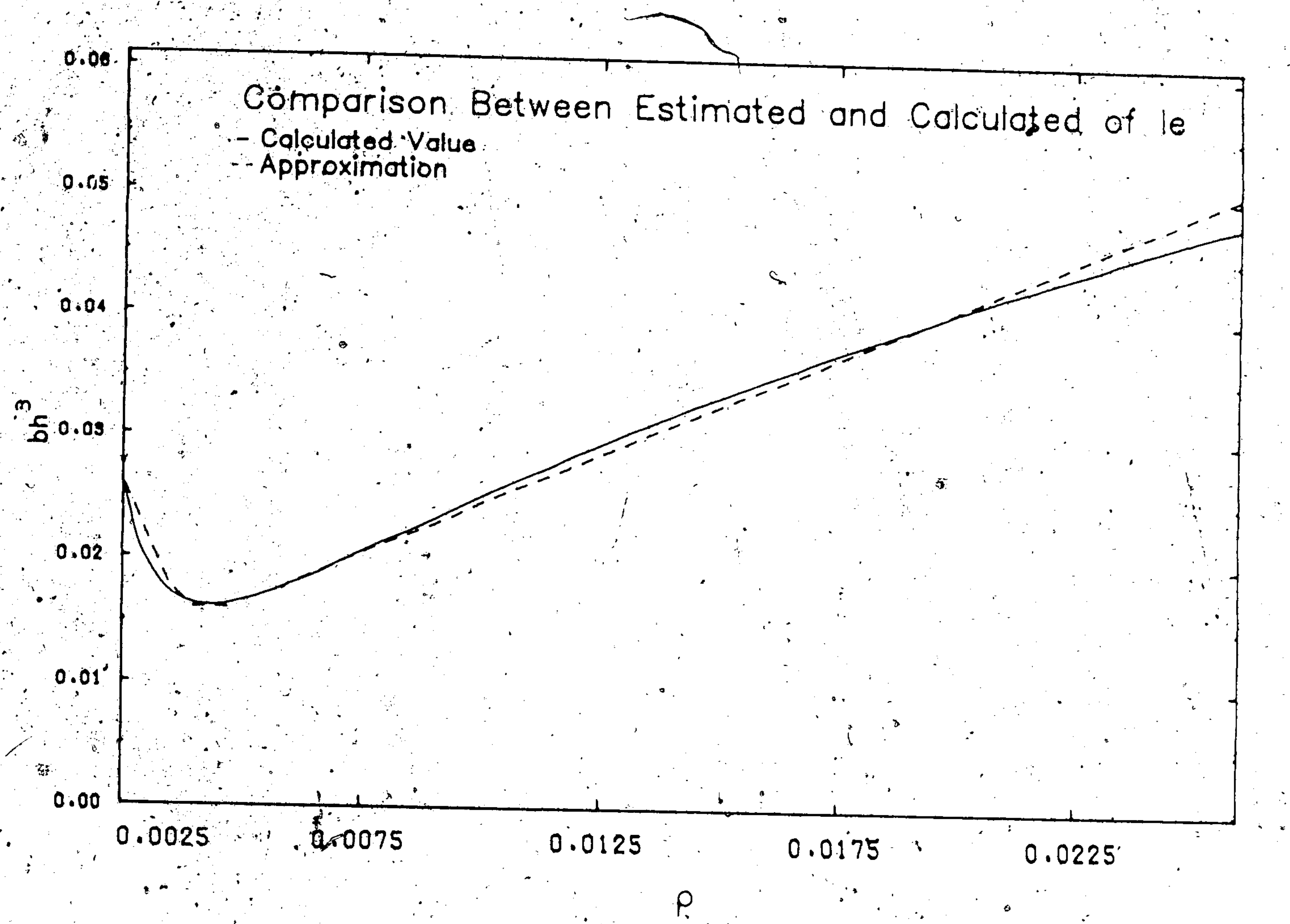


Figure C.5 Approximation of Branson's I_e using $f_e = 0.30\sqrt{f'_c}$

C.4 Slab Factor

The Slab Factor (k_1) derived on the following pages was completed by assuming the slab could be modelled as assumed in the Direct Design Method, and that the Moment of Inertia of the slab column strip was the same as the middle strip.

The crossing beam analogy was used in the derivation. It assumes that the slab middle and column strips can be modelled as wide beams with uniform loads, allowing the midpanel deflection to be calculated by summing the column and middle strip deflections. The magnitude of the uniform loads were calculated using the moment fields from the Direct Design Method. The use of the moment fields from the Direct Design Method should provide a realistic moment distribution, since most slabs are designed using the Direct Design or Equivalent Frame Methods' which have similar moment fields.

The last assumption used was that the middle and column strips would have the same stiffness. If one assumes that all the middle strips will have minimum reinforcement of .25 % and the column strip has typically .3 to .7 % reinforcing steel, Eqs. C-14 and C-15 shows that the column strip will not as stiff. The assumpuion of similar stiffnesses can thus be considered as conservative.

Slab Factor k_1

If we consider the slab column and middle strips as a number of crossing beams, the middle panel span-deflection ratio can be written in terms of both the column and middle strip span-deflection ratios.

Let

$$Minimum\ Span-Deflection\ ratio = \left|\frac{l}{\delta}\right|_{min}$$

If the column strip span is always longer than the middle strip the spans can be written as

$$l_{col} = l_1 \quad and \quad l_{mid} = \frac{l_1}{\beta_{panel}}$$

Also column and middle deflections can be written as

$$\Delta_{col} = \beta_{col}\frac{w_{col}}{E_c I_{col}} l_1^4 \quad and \quad \Delta_{mid} = \beta_{mid}\frac{w_{mid}}{E_c I_{mid}}\left[\frac{l_1}{\beta_{panel}}\right]^4 \tag{C-30}$$

The middle strip deflection can then be written in terms of the column strip deflection

$$\Delta_{mid} = \frac{\Delta_{mid}}{\Delta_{col}}\Delta_{col} \quad which\ is \quad \Delta_{mid} = \left[\frac{\beta_{mid} I_{col} w_{mid}}{\beta_{col} I_{mid} w_{col}}\left[\frac{1}{\beta_{panel}}\right]^4\right]\Delta_{col}$$

and the midpanel deflection can be written in terms of the column strip deflection as well

$$\Delta_{panel} = \Delta_{col} + \Delta_{mid} \qquad \Delta_{panel} = \Delta_{col} + \left[\frac{\beta_{mid} I_{col} w_{mid}}{\beta_{col} I_{mid} w_{col}}\left[\frac{1}{\beta_{panel}}\right]^4\right]\Delta_{col} \tag{C-31}$$

Thus span-deflection ratio for the diagonal span can be defined in terms of the column strip deflection and span.

$$\frac{l_{diag}}{\Delta_{panel}} = \frac{(1+\beta_{panel}^2)^{\frac{1}{2}}}{\left[1+\left[\frac{\beta_{mid} I_{col} w_{mid}}{\beta_{col} I_{mid} w_{col}}\left[\frac{1}{\beta_{panel}}\right]^4\right]\right]}\frac{l_{col}}{\Delta_{col}} \tag{C-32}$$

If the slab is not to exceed the span-deflection criterion

$$\frac{l_{col}}{\Delta_{col}} = \frac{\left[1 - \left[\frac{\beta_{mid} I_{col} w_{mid}}{\beta_{col} I_{mid} w_{col}}\right]\left[\frac{1}{\beta_{panel}}\right]^{4}\right]}{(1+\beta_{panel}^{2})^{\frac{1}{2}}}\left[\frac{l}{\delta}\right]_{min} \geq \left[\frac{l}{\delta}\right]_{min} \tag{C-33}$$

If we consider the slab moment distribution as assumed in the Direct Design Method, the load and deflection in the column strip for an interior span will be:

$$M_e = .75 \times .65 \frac{w l_1}{\beta_{panel}} \frac{l_1^2}{8}$$

$$M_m = .60 \times .35 \frac{w l_1}{\beta_{panel}} \frac{l_1^2}{8}$$

Therefore the total moment will be

$$M_o = (.75 \times .65 + .60 \times .35) \frac{w l_1}{\beta_{panel}} \frac{l_1^2}{8}$$

This allows us to write load in the column strip as,

$$w_{col} = .6975 \frac{w l_1}{\beta_{panel}} \tag{C-34}$$

and the deflection coefficient is (see k_2)

$$\beta_{col} = .1613\,\beta_{simp} \tag{C-35}$$

Similarly, this can be done in the middle strip

$$M_e' = .25 \times .65\, w l_1 \frac{l_1^2}{8}\left[\frac{1}{\beta_{panel}}\right]^{2}$$

$$M_m = .40 \times .35\, w l_1 \frac{l_1^2}{8}\left[\frac{1}{\beta_{panel}}\right]^{2}$$

Therefore the total moment will be,

$$M_o = (.25 \times .65 + .40 \times .35)\,wl_1 \frac{l_1}{8}\left[\frac{1}{\beta_{panel}}\right]$$

the load in the middle strip,

$$w_{mid} = .3025\, wl_1 \qquad \text{(C-36)}$$

and the deflection coefficient is equal to

$$\beta_{mid} = .3554\,\beta_{simp} \qquad \text{(C-37)}$$

If we conservatively assume that I_{mid} is equal to I_{col}, the span-deflection criterion can be written in terms of the design load and panel aspect ratio.

$$\frac{l_{col}}{\Delta_{col}} = \frac{\left[1 + \left[\dfrac{.3025\, wl_1\, .3554\,\beta_{simp}}{.6975\,\dfrac{wl_1}{\beta_{panel}}\,.1613\,\beta_{simp}}\left[\dfrac{1}{\beta_{panel}}\right]^4\right]\right]}{(1+\beta_{panel}^2)^{\frac{1}{2}}}\left[\frac{l}{\delta}\right]_{min} \geq \left[\frac{l}{\delta}\right]_{min} \qquad \text{(C-38)}$$

which reduces to

$$\frac{l_{col}}{\Delta_{col}} = \frac{\left[1 + \left[.6666\left[\dfrac{1}{\beta_{panel}}\right]^3\right]\right]}{(1+\beta_{panel}^2)^{\frac{1}{2}}}\left[\frac{l}{\delta}\right]_{max} \geq \left[\frac{l}{\delta}\right]_{max}$$

This relationship is used in the span-thickness equation when using the deflection-span ratio and is included as k_1

$$k_1 = \left(\frac{(1+\beta_{panel}^2)^{\frac{1}{2}}}{\left[1 + \left[.6666\left[\dfrac{1}{\beta_{panel}}\right]^3\right]\right]}\right)^{\frac{1}{3}} \leq 1.0 \qquad \text{(C-39)}$$

C.5 Location Factor

This factor (k_2) is derived from an elastic beam deflection equation, as was done by Rangan. The derivation was extended to put the midspan deflection in terms of the positive midspan moment.

An additional factor, Ψ_{load}, has been included in k_2 to account for varying loads and material properties of the slab at different locations. The different loads were calculated using the Direct Design moment fields to estimate an equivalent unit load on the column strip, allowing the use of the design loads. For example if sixty percent of the design load is influencing the column strip, a factor of 1.2 (60 % of the load divided by 50 % of the width) would be used. A factor of 1.50 was used in the case of a simply supported two-way slab, and this was calculated using the Hillerborg strip method and assuming the load would disperse equally in both directions. The second part of calculating Ψ_{load} is evaluating any change in material properties. The only material property considered was shrinkage. It was assumed that different minimum values of I_e due to f_e should be used in different locations. This derivation assumed f_e would equal $.30\sqrt{f'_c}$ for interior spans, $.45\sqrt{f'_c}$ for exterior spans and simply supported slabs. All the variables discussed in this section are summarized in Table C.2 along with the recommended values for k_2.

Location Factor k_2

If we consider a simply supported beam with end moments, the midspan deflection can be written as

$$\Delta = \frac{5}{48}\frac{M_o l^2}{EI} - \frac{(M_l + M_r)}{16}\frac{l^2}{EI} \qquad \text{(C-40)}$$

Therefore

$$\Delta = \frac{l^2}{48EI}(5M_o - 3(M_l + M_r))$$

This can be rewritten as

$$\Delta = \frac{5l^2}{48EI}(M_o - 6(M_l + M_r)) \qquad \text{(C-41)}$$

which is the equation that Rangan[10] used in his Span-Effective Depth formulation. We can extend the deflection equation with the following relationship between the moments.

$$M_o = \frac{(M_l + M_r)}{2} + M_m \qquad \text{(C-42)}$$

$$(M_l + M_r) = 2(M_o - M_m)$$

This gives us

$$\Delta = \frac{5l^2}{48EI}(M_o - 1.2(M_o - M_m)) \qquad \text{(C-43)}$$

$$\Delta = \frac{5l^2 M_o}{48EI}\left(1.2\frac{M_m}{M_o} - 0.2\right) \qquad \text{(C-44)}$$

but

$$M_o = \frac{wl^2}{8}$$

Therefore

$$\Delta = \frac{5wl^4}{384EI}(1.2\frac{M_m}{M_o} - 0.2) \qquad (C-45)$$

$$\Delta = \beta_{simp}(1.2\frac{M_m}{M_o} - 0.2)\frac{wl^4}{EI} \qquad (C-46)$$

Therefore

$$\beta = \beta_{simp}(1.2\frac{M_m}{M_o} - 0.2) \qquad (C-47)$$

This relationship is used in the Span-Thickness equation as k_2

$$k_2 = \left(\frac{1}{\Psi_{load}(1.2\frac{M_m}{M_o} - 0.2)}\right)^{\frac{1}{3}} \qquad (C-48)$$

Table C.2 Values used in Location Factor

	Simply Supported	Exterior Span	Interior Span
Ψ_{load}			
$*f_e$	$\frac{1}{1.42}$	$\frac{1}{1.42}$	1.0
Load	1.5	1.43	1.40
Total Factor	1.06	1.01	1.40
$\frac{M_m}{M_o}$	1.00	0.45	0.30
$1.2\,\frac{M_m}{M_o} - 0.2$	1.00	0.45	0.16
k_2	0.98	1.42	1.65
k_2 with a included	1.08	1.56	1.81

Notes:

1.The moments are calculated from the Direct Design Method

2.The a value included with k_2 is the minimum value $(.016 \times \frac{384}{5})$

* ratio of minimum value of I_e using different f_e values

Appendix D

Construction

Shoring Load Ratios

Grundy's and Kabaila's[2] method of analysis presented in 1960 uses construction load ratios (R). These ratios were calculated assuming one shoring sequence. The order of operations assumed was:

1. Erect shores and formwork on new level and cast slab.
2. Strip the formwork and shores on the lowest level of shoring and in the case where reshores are used , reshore that level.
3. When reshoring is used, remove the lowest level of reshores.

Repeat Step 1

The analysis calculated the load ratios by distributing the weight of the floor cast to the supporting floors using the stiffnesses of the slabs. In their analysis Grundy and Kabaila assumed the ground level to be rigid and therefore until the shoring is above the ground floor none of the slabs take any load. The loads are then allocated to the floors as follows;

1. Allocate the weight of the slab cast in step 1 to the supporting floors so each floor deflect the same amount.
2. Allocate the load in the lowest level of shores to the floors above so they have equal deflections when the shores are removed in step 2. The floors below these shores, when reshoring is used, will rebound upward and these floors will only be supporting their own weight.

Repeat the first calculation .

If the slab thicknesses are assumed equal the loads will be proportioned according to the floor's Modulus of Elasticity (E_c). In this thesis a seven day construction cycle was assumed. Table D.1 shows the calculated Modulus of Elasticity of the floors at different times compared to that of E_c at twenty eight days and the values in Table D.1 were used to calculate the load ratios.

Table D.2 provides the maximum load ratios for a series of shoring systems. The Table illustrates that the the load ratios are history dependent with the ratios fluctuating on the lower floors and converging to a constant ratio at higher levels. In this thesis only the converged value of the load ratio was considered. In cases of a 2 + 1 and 2 + 3 shoring systems, these values will be valid for any floor above the sixth level (2 + 1) and the ninth level (2 + 3).

ACI Committee 347 recommends[3] Eq. D-1 be used to insure the slab's capacity.

$$w_{con} \leq \frac{w_{ult}}{F.S.} \times \frac{f_c(t)}{f'_c} \qquad \text{(D-1)}$$

where w_{con} =maximum load imposed on the slab during construction.

=1.1 x1.1 x R_{max} x w_{slab} + 2.4 or 3.6 kPa

$f_c(t)$ =strength of the concrete at the time when w_{con} occurs, t is the time in days

w_{slab} =weight of slab

w_{ult} =ultimate design load

F.S. =the factor of safety, 1.4 is recommended.

Table D.1 Modulus of Elasticity of Concrete at Different Ages

Age	$\frac{E_c(t)}{E_c(28)}$
7	0.84
14	0.94
21	0.98
28	1.00
35	1.02

Note:
The Age is the number of days since placement of concrete

Table D.2 Construction Shoring Load Ratios for Different Shoring Systems

Floor Level	Load Ratio			
	(2 + 0)	(2 + 1)	(2 + 2)	(2 + 3)
G	-	-	-	-
1	1.52	1.36	1.28	1.22
2	2.28	1.86	1.28	1.22
3	1.92	1.76	1.78	1.22
4	2.09	1.81	1.63	1.73
5	2.01	1.78	1.70	1.56
6	2.05	1.79	1.67	1.64
7	2.03	1.79	1.69	1.60
8	2.04	1.79	1.69	1.62
9	2.04	1.79	1.69	1.61
10	2.04	1.79	1.69	1.61
11	2.04	1.79	1.69	1.61
12	2.04	1.79	1.69	1.61

Notes:
1) The shoring systems are designated by;
(No. of Shores + No. of Reshores)
2) The load ratios were calculated using a varying E_c

Appendix E

Parameter Study Results

The complete results of the parameter study are tabulated in Table E.1 on the next page. The table list the minimum calculated span deflection ratios for the 344 simulations evaluated. The table includes the slab designation, span deflection ratios for total, incremental, live load, and construction load deflections. Their locations are also shown. The parameters evaluated in each simulation are described by the slab designation. The parameters are described in the following manner.

AA:BBB:CCC:DDD:EE:FF:GG

where AA slab type

FP Flat Plate

BP Flat Plate with edge beams

DP Flat Slab with drop panels

VP Flat Slab with drop panels of various thicknesses

BBB slab thickness(mm); $.90h_m$, h_m, $1.10h_m$

FP	160,175,195
BP	145,160,175
DP	170,190,210
VP	170,190,210

CCC geometric properties

FP-aspect ratio; 1.0,1.5,2.0

DP-aspect ratio; 1.0,1.5,2.0 (thickness of drop panels below slab:0.25)

BP-stiffness ratio of edge beam and slab; 0.8, 2.0, 3.0 (aspect ratio equal to 1.0 in all cases)

VP-relative thickness of drop panel below slab; 0.50, 0.75 (aspect ratio equal to 1.0 in all cases)

DDD service live load (kPa); 1.9, 2.4, 4.8

EE concrete strength (MPa); 20, 30

FF effective modulus of rupture multiplier of f_e; 0.60, 0.30

GG construction load -A, load from 2+3 shoring system
-B, load from 2+1 shoring system

The construction span-deflection ratios in table E.1 are the results of the finite element program described in chapter 3 and is included to allow for further simulation of different load histories after the shoring loads are removed. The ratios bracketted in Table E.1 exceed the CAN3 A23.3 Deflection Limits and for total deflection $\frac{\ell_n}{240}$. Figure E.1 indicates the location of the minimum span-deflection ratios listed in the table.

Minimum Span-Deflection Ratios*

Slab Designation	Loc.	In-/Δ l	In/Δ inc	In/Δ tot	In/Δ con
FP:145:1.0:1.9:20:.3:A	10	1547	(239)	(103)	285
FP:145:1.0:1.9:20:.6:A	9	3263	504	(217)	600
FP:145:1.0:1.9:30:.6:A	9	2022	(312)	(134)	372
FP:145:1.0:1.9:30:.6:A	7	5127	792	341	943
FP:145:1.0:2.4:20:.3:A	10	1224	(230)	(101)	285
FP:145:1.0:2.4:20:.6:A	9	2583	484	(213)	600
FP:145:1.0:2.4:30:.6:A	9	1601	(300)	(132)	372
FP:145:1.0:2.4:30:.6:A	7	4059	761	335	943
FP:160:1.0:1.9:20:.3:A	9	2187	(325)	(137)	367
FP:160:1.0:1.9:20:.6:A	10	5053	751	317	847
FP:160:1.0:1.9:30:.3:A	12	2909	(433)	(183)	488
FP:160:1.0:1.9:30:.6:A	9	7499	1115	471	1257
FP:160:1.0:2.4:20:.3:A	9	1732	(313)	(135)	367
FP:160:1.0:2.4:20:.6:A	10	4000	723	312	847
FP:160:1.0:2.4:30:.3:A	12	2303	(416)	(180)	488
FP:160:1.0:2.4:30:.6:A	9	5937	1073	463	1257
FP:160:1.0:4.8:20:.3:A	10	852	(298)	(146)	439
FP:160:1.5:1.9:20:.3:A	10	3108	517	(221)	595
FP:160:1.5:1.9:20:.6:A	4	7928	1319	564	1518
FP:160:1.5:1.9:30:.3:A	3	4350	724	309	833
FP:160:1.5:1.9:30:.6:A	4	10603	1764	754	2030
FP:160:1.5:2.4:20:.3:A	10	2461	496	(217)	595
FP:160:1.5:2.4:20:.6:A	4	6276	1264	553	1518
FP:160:1.5:2.4:30:.3:A	3	3443	694	304	833
FP:160:1.5:2.4:30:.6:A	4	8394	1691	740	2030
FP:160:1.5:4.8:20:.3:A	10	1331	(466)	(227)	686
FP:160:1.5:4.8:20:.6:A	4	3013	1054	515	1552
FP:160:1.5:4.8:30:.3:A	3	1820	636	311	937
FP:160:1.5:4.8:30:.6:A	4	3975	1390	679	2048
FP:160:2.0:1.9:20:.3:A	10	3342	586	252	682
FP:160:2.0:1.9:20:.6:A	4	8462	1484	638	1726
FP:160:2.0:1.9:30:.3:A	3	4428	776	334	903
FP:160:2.0:2.4:20:.3:A	10	2646	560	247	682
FP:160:2.0:2.4:20:.6:A	4	6699	1418	625	1726
FP:160:2.0:2.4:30:.3:A	3	3506	742	327	903
FP:160:2.0:4.8:20:.3:A	10	1517	531	259	781
FP:160:2.0:4.8:20:.6:A	4	3436	1202	587	1770
FP:160:2.0:4.8:30:.3:A	10	1969	689	336	1014
FP:175:1.0:1.9:20:.3:A	10	3063	(439)	(182)	471
FP:175:1.0:1.9:20:.6:A	9	7591	1089	452	1168
FP:175:1.0:1.9:30:.3:A	9	4370	627	260	672
FP:175:1.0:1.9:30:.6:A	9	10323	1481	615	1588
FP:175:1.0:2.4:20:.3:A	10	2425	(424)	(180)	471

* see Figure E.1 for location of minimum span-deflection ratio

Minimum Span-Deflection Ratios continued

Slab Designation	Loc.	ln/Δ 1	ln/Δ inc	ln/Δ tot	ln/Δ con
FP:175:1.0:2.4:20:.6:A	9	6010	1050	445	1168
FP:175:1.0:2.4:30:.3:A	9	3459	604	256	672
FP:175:1.0:2.4:30:.6:A	9	8172	1428	606	1588
FP:175:1.0:4.8:20:.3:A	10	1176	(400)	(192)	556
FP:175:1.5:1.9:20:.3:A	10	4328	695	292	761
FP:175:1.5:1.9:20:.6:A	4	10862	1745	733	1910
FP:175:1.5:1.9:30:.3:A	3	6502	1045	439	1143
FP:175:1.5:1.9:30:.6:A	4	14375	2310	970	2527
FP:175:1.5:2.4:20:.3:A	10	3426	667	287	761
FP:175:1.5:2.4:20:.6:A	4	8599	1675	720	1910
FP:175:1.5:2.4:30:.3:A	3	5147	1002	431	1143
FP:175:1.5:2.4:30:.6:A	4	11380	2216	953	2527
FP:175:1.5:4.8:20:.3:A	10	1818	618	296	860
FP:175:1.5:4.8:20:.6:A	4	4099	1395	668	1939
FP:175:1.5:4.8:30:.3:A	3	2658	904	433	1257
FP:175:1.5:4.8:30:.6:A	4	5384	1832	878	25[illegible]
FP:175:2.0:1.9:20:.3:A	10	4552	771	326	852
FP:175:2.0:1.9:20:.6:A	4	11744	1989	841	2199
FP:175:2.0:1.9:30:.3:A	4	6274	1063	449	1175
FP:175:2.0:2.4:20:.3:A	10	3604	738	320	852
FP:175:2.0:2.4:20:.6:A	4	9298	1904	825	2199
FP:175:2.0:2.4:30:.3:A	4	4967	1017	441	1175
FP:175:2.0:4.8:20:.3:A	10	2038	693	332	964
FP:175:2.0:4.8:20:.6:A	4	4706	1601	767	2226
FP:175:2.0:4.8:30:.3:A	10	2767	941	451	1309
FP:195:1.0:1.9:20:.3:A	10	4577	628	256	635
FP:195:1.0:1.9:20:.6:A	9	11359	1558	635	1576
FP:195:1.0:1.9:30:.3:A	9	6937	952	388	962
FP:195:1.0:1.9:30:.6:A	9	15219	2088	851	2111
FP:195:1.0:2.4:20:.3:A	10	3623	606	252	635
FP:195:1.0:2.4:20:.6:A	9	8993	1504	626	1576
FP:195:1.0:2.4:30:.3:A	9	5492	919	382	962
FP:195:1.0:2.4:30:.6:A	9	12048	2016	838	2111
FP:195:1.0:4.8:20:.3:A	10	1727	568	266	737
FP:195:1.5:1.9:20:.3:A	10	6837	1051	433	1084
FP:195:1.5:1.9:20:.6:A	4	15928	2449	1009	2525
FP:195:1.5:1.9:30:.3:A	4	10526	1619	667	1669
FP:195:1.5:1.9:30:.6:A	4	21035	3235	1333	3334
FP:195:1.5:2.4:20:.3:A	10	5413	1011	426	1084
FP:195:1.5:2.4:20:.6:A	4	12609	2354	993	2525
FP:195:1.5:2.4:30:.3:A	4	8333	1556	656	1669
FP:195:1.5:2.4:30:.6:A	4	16653	3109	1311	3334
FP:195:1.5:4.8:20:.3:A	10	2757	906	425	1176

* see Figure E.1 for location of minimum span-deflection ratio

Minimum Span-Deflection Ratios continued

Slab Designation	Loc.	ln/Δ l	ln/Δ inc	ln/Δ tot	ln/Δ con
FP:195:1.5:4.8:20:.6:A	4	5967	1961	919	2545
FP:195:1.5:4.8:30:.3:A	4	4205	1382	648	1793
FP:195:1.5:4.8:30:.6:A	4	7831	2573	1206	3340
FP:195:2.0:1.9:20:.3:A	10	6803	1103	457	1148
FP:195:2.0:1.9:20:.6:A	4	17100	2773	1149	2887
FP:195:2.0:1.9:30:.3:A	4	11203	1817	753	1891
FP:195:2.0:2.4:20:.3:A	10	5385	1058	449	1148
FP:195:2.0:2.4:20:.6:A	4	13537	2660	1129	2887
FP:195:2.0:2.4:30:.3:A	4	8869	1743	740	1891
FP:195:2.0:4.8:20:.3:A	10	2944	967	453	1255
FP:195:2.0:4.8:20:.6:A	4	6806	2237	1048	2903
FP:195:2.0:4.8:30:.3:A	10	4714	1549	726	2010
BP:145:0.8:1.9:20:.3:A	10	1786	(276)	(119)	329
BP:145:0.8:1.9:20:.6:A	8	4030	622	268	741
BP:145:0.8:1.9:30:.3:A	12	2305	(356)	(153)	424
BP:145:0.8:1.9:30:.6:A	12	5802	896	385	1067
BP:145:0.8:2.4:20:.3:A	10	1414	(265)	(117)	329
BP:145:0.8:2.4:20:.6:A	8	3190	598	263	741
BP:145:0.8:2.4:30:.3:A	10	1825	(342)	(151)	424
BP:145:0.8:2.4:30:.6:A	12	4593	861	379	1067
BP:145:2.0:1.9:20:.3:A	10	1777	(274)	(118)	327
BP:145:2.0:1.9:20:.6:A	8	4080	630	271	751
BP:145:2.0:1.9:30:.3:A	12	2398	(370)	(159)	441
BP:145:2.0:1.9:30:.6:A	12	5832	901	387	1073
BP:145:2.0:2.4:20:.3:A	10	1407	(264)	(116)	327
BP:145:2.0:2.4:20:.6:A	8	3230	606	266	751
BP:145:2.0:2.4:30:.3:A	10	1898	(356)	(157)	441
BP:145:2.0:2.4:30:.6:A	12	4617	866	381	1073
BP:145:3.0:1.9:20:.3:A	10	1759	(272)	(117)	324
BP:145:3.0:1.9:20:.6:A	8	4091	632	272	753
BP:145:3.0:1.9:30:.3:A	12	2417	(373)	(161)	445
BP:145:3.0:1.9:30:.6:A	12	5831	900	387	1073
BP:145:3.0:2.4:20:.3:A	10	1392	(261)	(115)	324
BP:145:3.0:2.4:20:.6:A	8	3239	607	267	753
BP:145:3.0:2.4:30:.3:A	10	1914	(359)	(158)	445
BP:145:3.0:2.4:30:.6:A	12	4616	865	381	1073
BP:160:0.8:1.9:20:.3:A	10	2435	(362)	(153)	408
BP:160:0.8:1.9:20:.6:A	8	5794	862	364	971
BP:160:0.8:1.9:30:.3:A	12	3235	481	(203)	542
BP:160:0.8:1.9:30:.6:A	12	8077	1201	507	1353
BP:160:0.8:2.4:20:.3:A	10	1928	(349)	(150)	408
BP:160:0.8:2.4:20:.6:A	8	4587	829	358	971
BP:160:0.8:2.4:30:.3:A	12	2561	(463)	(200)	542

* see Figure E.1 for location of minimum span-deflection ratio

Minimum Span-Deflection Ratios continued

Slab Designation	Loc.	ln/Δ l	ln/Δ inc	ln/Δ tot	ln/Δ con
BP:160:0.8:2.4:30:.6:A	12	6394	1156	499	1353
BP:160:2.0:1.9:20:.3:A	10	2480	(369)	(156)	416
BP:160:2.0:1.9:20:.6:A	12	5850	870	367	980
BP:160:2.0:1.9:30:.3:A	12	3368	501	(211)	564
BP:160:2.0:1.9:30:.6:A	12	8014	1192	503	1343
BP:160:2.0:2.4:20:.3:A	10	1963	(355)	(153)	416
BP:160:2.0:2.4:20:.6:A	12	4631	837	361	980
BP:160:2.0:2.4:30:.3:A	10	2667	482	(208)	564
BP:160:2.0:2.4:30:.6:A	12	6344	1147	495	1343
BP:160:3.0:1.9:20:.3:A	10	2472	(368)	(155)	414
BP:160:3.0:1.9:20:.6:A	12	5837	868	366	978
BP:160:3.0:1.9:30:.3:A	12	3394	505	(213)	569
BP:160:3.0:1.9:30:.6:A	12	7994	1189	502	1339
BP:160:3.0:2.4:20:.3:A	10	1957	(354)	(153)	414
BP:160:3.0:2.4:20:.6:A	12	4621	835	360	978
BP:160:3.0:2.4:30:.3:A	10	2687	486	(210)	569
BP:160:3.0:2.4:30:.6:A	12	6328	1144	494	1339
BP:175:0.8:1.9:20:.3:A	10	3318	(476)	(198)	511
BP:175:0.8:1.9:20:.6:A	12	8217	1179	490	1264
BP:175:0.8:1.9:30:.3:A	12	4987	716	297	767
BP:175:0.8:1.9:30:.6:A	12	10935	1569	651	1682
BP:175:0.8:2.4:20:.3:A	10	2627	(459)	(195)	511
BP:175:0.8:2.4:20:.6:A	12	6505	1136	482	1264
BP:175:0.8:2.4:30:.3:A	12	3948	690	293	767
BP:175:0.8:2.4:30:.6:A	12	8657	1512	641	1682
BP:175:2.0:1.9:20:.3:A	10	3414	490	(203)	525
BP:175:2.0:1.9:20:.6:A	12	8187	1175	488	1260
BP:175:2.0:1.9:30:.3:A	12	5102	732	304	785
BP:175:2.0:1.9:30:.6:A	12	10841	1556	646	1668
BP:175:2.0:2.4:20:.3:A	10	2703	(472)	(200)	525
BP:175:2.0:2.4:20:.6:A	12	6481	1132	480	1260
BP:175:2.0:2.4:30:.3:A	10	4039	706	299	785
BP:175:2.0:2.4:30:.6:A	12	8583	1499	636	1668
BP:175:3.0:1.9:20:.3:A	10	3443	494	(205)	530
BP:175:3.0:1.9:20:.6:A	12	8185	1175	488	1259
BP:175:3.0:1.9:30:.3:A	12	5120	735	305	788
BP:175:3.0:1.9:30:.6:A	12	10814	1552	644	1664
BP:175:3.0:2.4:20:.3:A	10	2726	(476)	(202)	530
BP:175:3.0:2.4:20:.6:A	12	6480	1132	480	1259
BP:175:3.0:2.4:30:.3:A	10	4053	708	300	788
BP:175:3.0:2.4:30:.6:A	12	8561	1495	634	1664
DP:170:1.0:1.9:20:.3:A	10	2247	(301)	(125)	323
DP:170:1.0:1.9:20:.6:A	9	5155	690	286	741

* see Figure E.1 for location of minimum span-deflection ratio

Minimum Span-Deflection Ratios continued

Slab Designation	Loc.	ln / Δ 1	ln / Δ inc	ln / Δ tot	ln / Δ con
DP:170:1.0:1.9:30:.3:A	12	2757	(369)	(153)	397
DP:170:1.0:1.9:30:.6:A	9	7425	993	411	1068
DP:170:1.0:2.4:20:.3:A	10	1779	(290)	(123)	323
DP:170:1.0:2.4:20:.6:A	9	4081	666	282	741
DP:170:1.0:2.4:30:.3:A	12	2183	(356)	(151)	397
DP:170:1.0:2.4:30:.6:A	9	5878	960	406	1068
DP:170:1.5:1.9:20:.3:A	10	3053	(456)	(191)	499
DP:170:1.5:1.9:20:.6:A	3	6994	1044	437	1142
DP:170:1.5:1.9:30:.3:A	3	3882	579	243	634
DP:170:1.5:1.9:30:.6:A	3	9923	1481	620	1621
DP:170:1.5:2.4:20:.3:A	10	2417	(438)	(188)	499
DP:170:1.5:2.4:20:.6:A	3	5537	1004	430	1142
DP:170:1.5:2.4:30:.3:A	3	3073	558	(239)	634
DP:170:1.5:2.4:30:.6:A	3	7856	1425	610	1621
DP:170:1.5:4.8:20:.3:A	10	1170	(402)	(194)	569
DP:170:1.5:4.8:20:.6:A	3	2467	847	408	1200
DP:170:1.5:4.8:30:.3:A	3	1477	507	245	718
DP:170:1.5:4.8:30:.6:A	3	3398	1167	563	1652
DP:170:2.0:1.9:20:.3:A	10	3057	494	(209)	549
DP:170:2.0:1.9:20:.6:A	4	7251	1172	495	1301
DP:170:2.0:1.9:30:.3:A	3	3746	605	256	672
DP:170:2.0:1.9:30:.6:A	4	10410	1682	710	1868
DP:170:2.0:2.4:20:.3:A	10	2420	(474)	(205)	549
DP:170:2.0:2.4:20:.6:A	4	5740	1124	486	1301
DP:170:2.0:2.4:30:.3:A	3	2965	581	251	672
DP:170:2.0:2.4:30:.6:A	4	8241	1614	698	1868
DP:170:2.0:4.8:20:.3:A	10	1288	(442)	(213)	626
DP:170:2.0:4.8:20:.6:A	4	2769	951	458	1347
DP:170:2.0:4.8:30:.3:A	3	1568	538	260	762
DP:170:2.0:4.8:30:.6:A	4	3898	1338	645	1895
DP:190:1.0:1.9:20:.3:A	10	3107	(397)	(161)	402
DP:190:1.0:1.9:20:.6:A	9	7994	1021	415	1034
DP:190:1.0:1.9:30:.3:A	12	4019	513	(208)	520
DP:190:1.0:1.9:30:.6:A	9	11098	1417	576	1435
DP:190:1.0:2.4:20:.3:A	10	2460	(384)	(159)	402
DP:190:1.0:2.4:20:.6:A	9	6329	988	409	1034
DP:190:1.0:2.4:30:.3:A	12	3181	497	(206)	520
DP:190:1.0:2.4:30:.6:A	9	8786	1372	568	1435
DP:190:1.5:1.9:20:.3:A	10	4327	617	253	636
DP:190:1.5:1.9:20:.6:A	4	10746	1532	629	1578
DP:190:1.5:1.9:30:.3:A	4	5548	791	325	815
DP:190:1.5:1.9:30:.6:A	4	14943	2131	874	2195
DP:190:1.5:2.4:20:.3:A	10	3425	595	249	636

* see Figure E.1 for location of minimum span-deflection ratio

Minimum Span-Deflection Ratios continued

Slab Designation	Loc.	$\frac{ln}{\Delta\ l}$	$\frac{ln}{\Delta\ inc}$	$\frac{ln}{\Delta\ tot}$	$\frac{ln}{\Delta\ con}$
DP:190:1.5:2.4:20:.6:A	4	8507	1477	619	1578
DP:190:1.5:2.4:30:.3:A	4	4392	763	320	815
DP:190:1.5:2.4:30:.6:A	4	11830	2054	861	2195
DP:190:1.5:4.8:20:.3:A	3	1594	528	249	697
DP:190:1.5:4.8:20:.6:A	10	3719	1233	581	1626
DP:190:1.5:4.8:30:.3:A	4	2062	683	322	902
DP:190:1.5:4.8:30:.6:A	4	5067	1679	791	2216
DP:190:2.0:1.9:20:.3:A	10	4362	674	279	704
DP:190:2.0:1.9:20:.6:A	4	10954	1692	700	1768
DP:190:2.0:1.9:30:.3:A	4	5475	846	350	884
DP:190:2.0:1.9:30:.6:A	4	15560	2404	994	2511
DP:190:2.0:2.4:20:.3:A	10	3453	648	274	704
DP:190:2.0:2.4:20:.6:A	4	8672	1627	689	1768
DP:190:2.0:2.4:30:.3:A	4	4334	813	344	884
DP:190:2.0:2.4:30:.6:A	4	12318	2311	978	2511
DP:190:2.0:4.8:20:.3:A	10	1766	585	276	772
DP:190:2.0:4.8:20:.6:A	4	4145	1374	647	1812
DP:190:2.0:4.8:30:.3:A	4	2173	720	339	950
DP:190:2.0:4.8:30:.6:A	4	5774	1914	902	2525
DP:210:1.0:1.9:20:.3:A	10	4338	531	(212)	510
DP:210:1.0:1.9:20:.6:A	9	11513	1408	563	1352
DP:210:1.0:1.9:30:.3:A	12	6274	768	307	737
DP:210:1.0:1.9:30:.6:A	9	15833	1937	774	1860
DP:210:1.0:2.4:20:.3:A	10	3434	514	(209)	510
DP:210:1.0:2.4:20:.6:A	9	9114	1365	555	1352
DP:210:1.0:2.4:30:.3:A	12	4967	744	303	737
DP:210:1.0:2.4:30:.6:A	9	12534	1877	764	1860
DP:210:1.5:1.9:20:.3:A	10	5898	806	325	787
DP:210:1.5:1.9:20:.6:A	4	15337	2096	846	2046
DP:210:1.5:1.9:30:.3:A	4	8477	1159	467	1131
DP:210:1.5:1.9:30:.6:A	4	21373	2921	1178	2852
DP:210:1.5:2.4:20:.3:A	10	4669	778	321	787
DP:210:1.5:2.4:20:.6:A	4	12142	2024	833	2046
DP:210:1.5:2.4:30:.3:A	4	6711	1119	461	1131
DP:210:1.5:2.4:30:.6:A	4	16920	2820	1161	2852
DP:210:1.5:4.8:20:.3:A	10	2275	729	337	903
DP:210:1.5:4.8:20:.6:A	4	5284	1694	782	2099
DP:210:1.5:4.8:30:.3:A	4	3216	1031	476	1277
DP:210:1.5:4.8:30:.6:A	4	7231	2318	1070	2872
DP:210:2.0:1.9:20:.3:A	10	6029	893	363	884
DP:210:2.0:1.9:20:.6:A	4	16023	2374	965	2349
DP:210:2.0:1.9:30:.3:A	3	8780	1301	529	1287
DP:210:2.0:1.9:30:.6:A	4	22379	3316	1348	3281

* see Figure E.1 for location of minimum span-deflection ratio

Minimum Span-Deflection Ratios continued

Slab Designation	Loc.	ln / Δ l	ln / Δ inc	ln / Δ tot	ln / Δ con
DP:210:2.0:2.4:20:.3:A	10	4773	860	358	884
DP:210:2.0:2.4:20:.6:A	4	12685	2286	950	2349
DP:210:2.0:2.4:30:.3:A	3	6951	1253	521	1287
DP:210:2.0:2.4:30:.6:A	4	17717	3192	1327	3281
DP:210:2.0:4.8:20:.3:A	10	2431	780	360	966
DP:210:2.0:4.8:20:.6:A	4	5988	1920	886	2378
DP:210:2.0:4.8:30:.3:A	3	3446	1105	510	1369
DP:210:2.0:4.8:30:.6:A	4	8275	2653	1225	3286
VP:170:.50:1.9:20:.3:A	12	2799	(374)	(155)	403
VP:170:.50:1.9:20:.6:A	10	7242	969	401	1042
VP:170:.50:1.9:30:.3:A	12	3634	486	(201)	523
VP:170:.50:2.4:20:.3:A	12	2216	(362)	(153)	403
VP:170:.50:2.4:20:.6:A	10	5733	936	395	1042
VP:170:.50:2.4:30:.3:A	12	2877	(470)	(198)	523
VP:170:.75:1.9:20:.3:A	12	3531	(472)	(196)	508
VP:170:.75:1.9:20:.6:A	10	9982	1335	553	1436
VP:170:.75:1.9:30:.3:A	12	5654	756	313	813
VP:170:.75:2.4:20:.3:A	12	2795	(456)	(193)	508
VP:170:.75:2.4:20:.6:A	10	7903	1290	545	1436
VP:170:.75:2.4:30:.3:A	12	4476	731	309	813
VP:190:.50:1.9:20:.3:A	12	3880	496	(201)	502
VP:190:.50:1.9:20:.6:A	10	10890	1391	565	1408
VP:190:.50:1.9:30:.3:A	12	5944	759	308	769
VP:190:.50:2.4:20:.3:A	12	3071	(479)	(199)	502
VP:190:.50:2.4:20:.6:A	10	8621	1346	557	1408
VP:190:.50:2.4:30:.3:A	12	4706	735	304	769
VP:190:.75:1.9:20:.3:A	12	5455	697	283	705
VP:190:.75:1.9:20:.6:A	10	14961	1911	776	1935
VP:190:.75:1.9:30:.3:A	12	9368	1197	486	1211
VP:190:.75:2.4:20:.3:A	12	4319	674	279	705
VP:190:.75:2.4:20:.6:A	10	11844	1849	766	1935
VP:190:.75:2.4:30:.3:A	12	7416	1158	479	1211
VP:210:.50:1.9:20:.3:A	12	5660	692	277	665
VP:210:.50:1.9:20:.6:A	10	15617	1911	763	1835
VP:210:.50:1.9:30:.3:A	12	9403	1150	459	1105
VP:210:.50:2.4:20:.3:A	12	4481	671	273	665
VP:210:.50:2.4:20:.6:A	10	12364	1851	753	1835
VP:210:.50:2.4:30:.3:A	12	7444	1115	454	1105
VP:210:.75:1.9:20:.3:A	12	8717	1066	426	1024
VP:210:.75:1.9:20:.6:A	10	20816	2547	1017	2445
VP:210:.75:1.9:30:.3:A*	9	16231	1986	793	1907
VP:210:.75:2.4:20:.3:A	12	6901	1033	421	1024
VP:210:.75:2.4:20:.6:A	10	16480	2468	1004	2445

* see Figure E.1 for location of minimum span-deflection ratio

Minimum Span-Deflection Ratios continued

Slab Designation	Loc.	ln/Δ l	ln/Δ inc	ln/Δ tot	ln/Δ con
VP:210:.75:2.4:30:.3:A	9	12849	1924	783	1907
FP:160:1.0:1.9:20:.3:B	9	2044	(304)	(125)	317
FP:160:1.0:1.9:20:.6:B	10	4500	670	275	698
FP:160:1.0:1.9:30:.3:B	12	2646	(394)	(161)	410
FP:160:1.0:1.9:30:.6:B	9	7097	1056	433	1100
FP:160:1.0:2.4:20:.3:B	9	1618	(293)	(123)	317
FP:160:1.0:2.4:20:.6:B	10	3563	644	270	698
FP:160:1.0:2.4:30:.3:B	12	2095	(379)	(159)	410
FP:160:1.0:2.4:30:.6:B	9	5618	1016	426	1100
FP:160:1.5:1.9:20:.3:B	10	2941	490	(203)	521
FP:160:1.5:1.9:20:.6:B	4	7262	1209	502	1286
FP:160:1.5:2.4:20:.3:B	10	2328	(469)	(200)	521
FP:160:1.5:2.4:20:.6:B	4	5749	1159	493	1286
FP:160:2.0:1.9:20:.3:B	10	3153	553	(231)	595
FP:160:2.0:2.4:20:.3:B	10	2496	529	(227)	595
FP:175:1.0:1.9:20:.3:B	10	2860	(409)	(166)	408
FP:175:1.0:1.9:20:.6:B	7	6791	971	393	968
FP:175:1.0:1.9:30:.3:B	12	3851	550	(223)	549
FP:175:1.0:1.9:30:.6:B	9	9885	1413	572	1409
FP:175:1.0:2.4:20:.3:B	10	2264	(394)	(163)	408
FP:175:1.0:2.4:20:.6:B	7	5376	936	387	968
FP:175:1.0:2.4:30:.3:B	12	3049	530	(220)	549
FP:175:1.0:2.4:30:.6:B	9	7826	1362	564	1409
FP:175:1.5:1.9:20:.3:B	10	3932	629	258	641
FP:175:1.5:1.9:20:.6:B	4	10386	1662	682	1692
FP:175:1.5:2.4:20:.3:B	10	3113	604	254	641
FP:175:1.5:2.4:20:.6:B	4	8222	1595	670	1692
FP:175:2.0:1.9:20:.3:B	10	4310	727	300	748
FP:175:2.0:2.4:20:.3:B	10	3412	696	295	748
FP:195:1.0:1.9:20:.3:B	10	4179	568	(227)	538
FP:195:1.0:1.9:20:.6:B	9	10812	1469	587	1392
FP:195:1.0:1.9:30:.3:B	9	6186	841	336	797
FP:195:1.0:1.9:30:.6:B	9	14608	1985	793	1881
FP:195:1.0:2.4:20:.3:B	10	3308	548	(224)	538
FP:195:1.0:2.4:20:.6:B	9	8559	1419	579	1392
FP:195:1.0:2.4:30:.3:B	9	4897	812	331	797
FP:195:1.0:2.4:30:.6:B	9	11564	1917	782	1881
FP:195:1.5:1.9:20:.3:B	10	5976	911	368	879
FP:195:1.5:1.9:20:.6:B	4	15063	2295	928	2217
FP:195:1.5:2.4:20:.3:B	10	4731	876	362	879
FP:195:1.5:2.4:20:.6:B	4	11925	2207	913	2217
FP:195:2.0:1.9:20:.3:B	10	6260	1006	409	981
FP:195:2.0:2.4:20:.3:B	10	4956	965	402	981

* see Figure E.1 for location of minimum span-deflection ratio

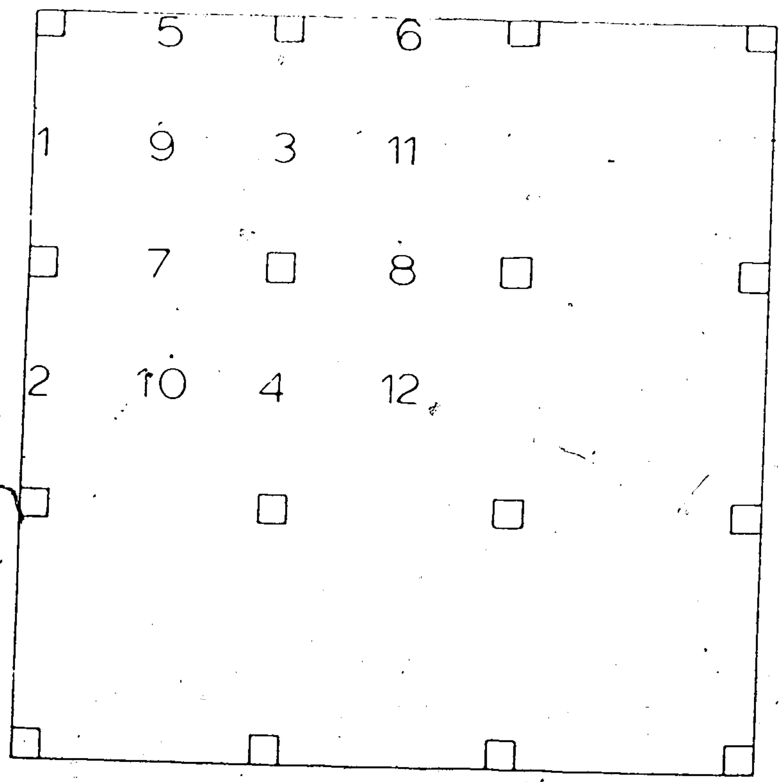


Figure E.1 Location of Minimum Span-Deflections

Figures E.2-E.4 illustrate the redistribution of the moment due to cracking in the slab under construction loading. The loads move away from the interior support as shown by the high negative moment in the elastic analysis, Figure E.2, to both the interior and exterior midspans (Figures E.3 and E.4). The redistribution is more pronounced when the Effective Modulus of Rupture is $.30\sqrt{f'_c}$, Figure E.3, than when $.60\sqrt{f'_c}$ is used. The moment fields shown in Figures E.3 and E.4 were found using the reduced stiffness of the slab due to cracking without any yielding of the reinforcing steel. These moment fields are different than that assumed by the Direct Design Method. But, no yielding of the reinforcing steel occurs at service loads and therefore, it can be concluded that for the slabs analysed that the moment field in the slab at service loads is different than at ultimate loads.

It is interesting to note that the slab around the exterior column does not crack as much as the interior column, shown in Figures E.5 and E.6, and the magnitude of the positive principal moments in the extior and interior panels are almost equal. This explains why in some cases the minimum span-deflection ratio is located in the interior panels.

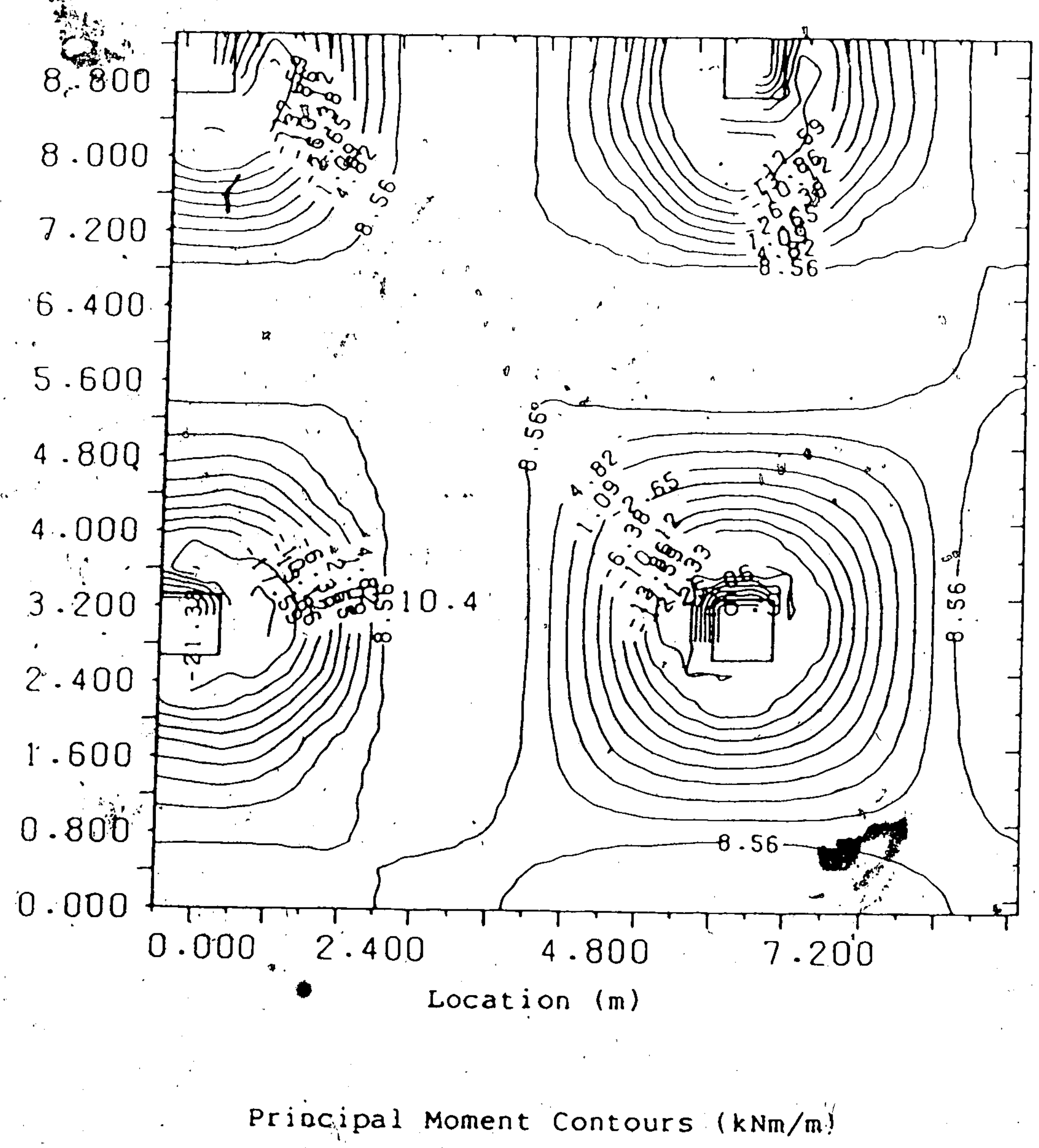
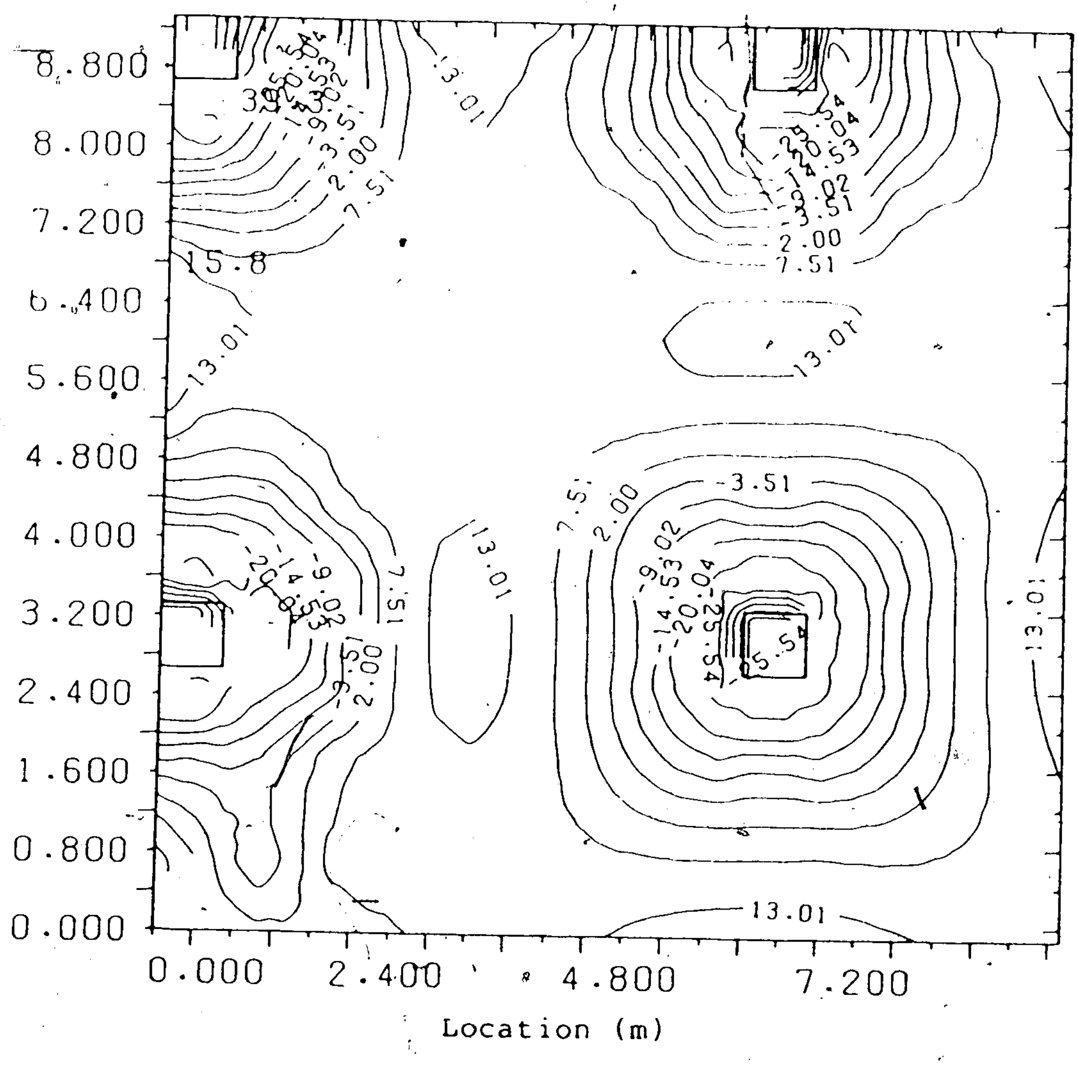
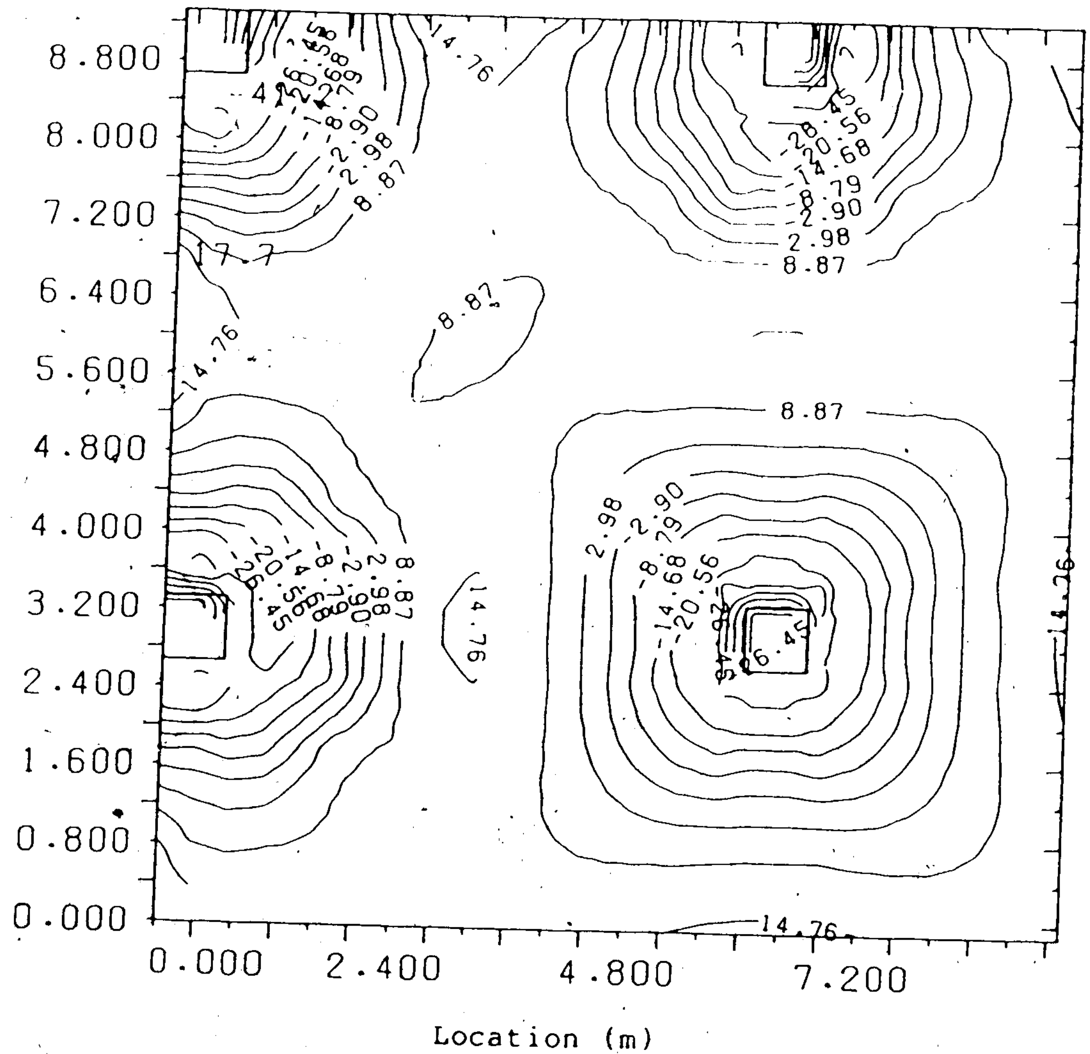


Figure E.2 Principal Moments of FP:160:1.0:2.4:20:.3:A under Construction Loads: Elastic Analysis



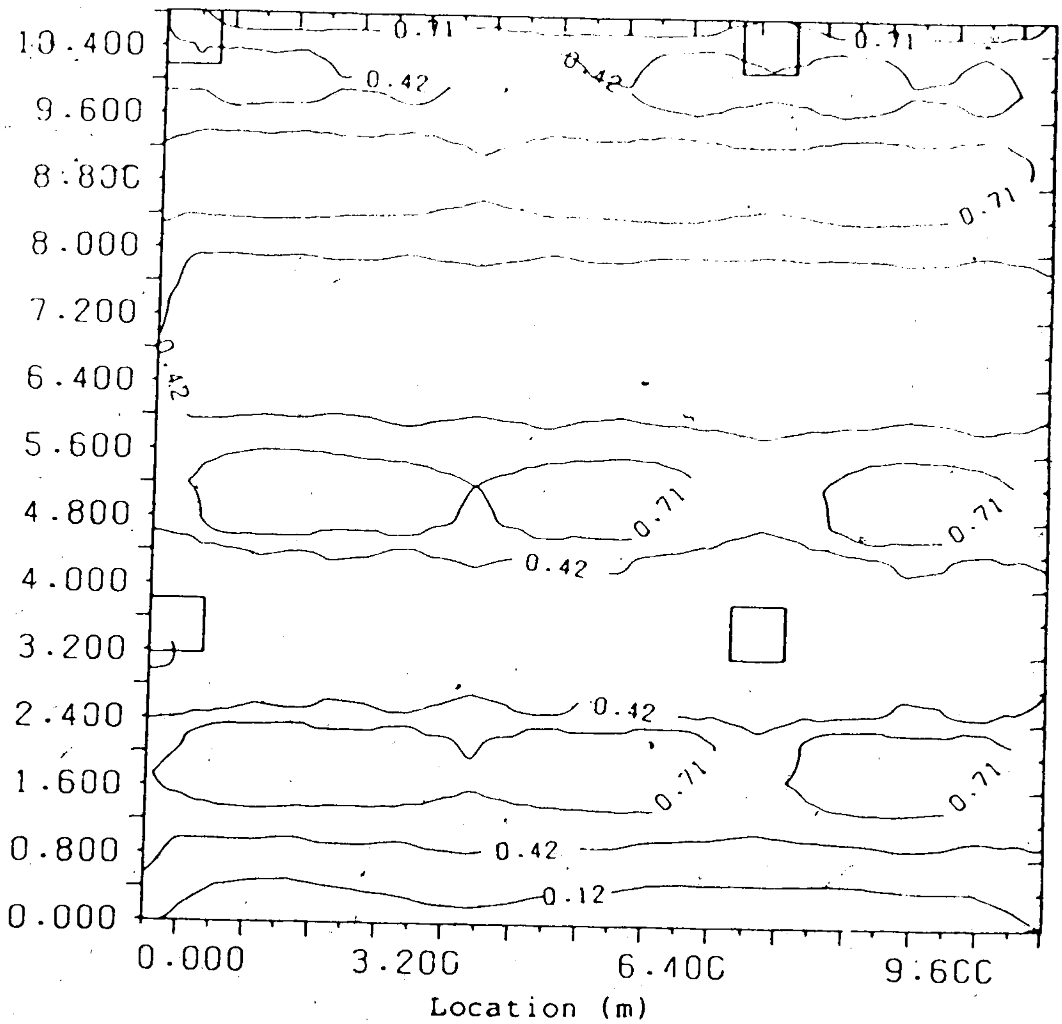
Principal Moment Contours (kNm/m)

Figure E.3 Principal Moments of FP:160:1.0:2.4:20:.3:A under Construction Loads: $f_e = .60\sqrt{f_c}$



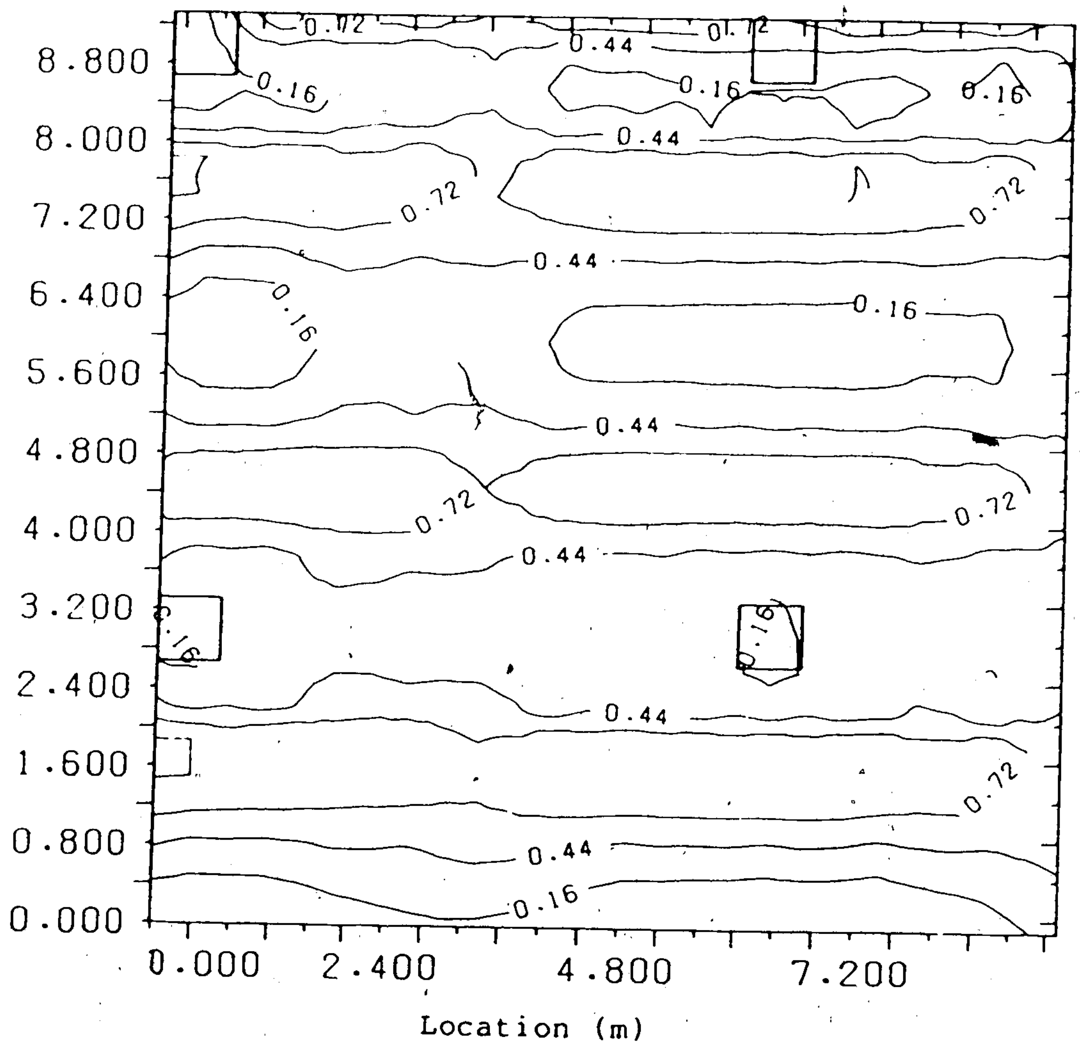
Principal Moment Contours (kNm/m)

Figure E.4 Principal Moments of FP:160:1.0:2.4:20:.3:A under Construction Loads: $f_e = .30\sqrt{f_c}$



Reduced Plate Siffness $(\frac{I_{ey}}{I_g})$

Figure E.5 Reduced Stiffness of DP:190:1.0:2.4:20:.6:A under Construction Loads: $f_e = .60\sqrt{f_c}$



Reduced Plate Siffness $(\frac{I_{ey}}{I_g})$

Figure E.6 Reduced Stiffness of FP:160:1.0:2.4:20:.3:A under Construction Loads: $f_e = .30\sqrt{f_c}$