THE UNIVERSITY OF ALBERTA

INDICATORS OF GROWTH WITHIN A LOGO[1] / MOTION GEOMETRY CURRICULUM ENVIRONMENT

BY

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled "Indicators Of Growth Within A Logo / Motion Geometry Curriculum Environment" submitted by Susan C. Ludwig in partial fulfillment of the requirements for the degree of Master of Education.

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ABSTRACT

A theory proposes specified levels of Logo use and relates them to the van Hiele levels of geometric thinking. This theory was the subject of the reseacher's investigation. For the study a teaching unit was developed which integrated, within a Logo turtle geometry environment, the van Hiele levels and phases of learning with the Alberta grade seven transformation geometry curriculum objectives. The major motion geometry topics covered were congruency, translations, rotations and reflections. A curriculum incorporating both the instructional focus of the van Hiele theory and the instructional levels of Logo (Kieren, 1984) was designed involving demonstrations and activities in Logo corresponding to the intents of the van Hiele phases of learning.

To explore student Logo and geometry behaviors within this environment, the researcher served as a teacher / observer throughout eleven biweekly eighty minute instructional sessions on the motion geometry unit. Two heterogeneous grade seven mathematics classes were involved in the project. The work and progress of ten students was closely monitored. Daily, and again at the end of the project, video tapes were reviewed and systematic summaries were recorded on observation grids and then analyzed.

To characterize the relationship found between Logo use and the van Hiele theory, the findings for each pair of students were presented for each topic in a threefold perspective - a descriptive summary of each group's work, a tabulation of the Logo activity observed and a commentary on the students' behavior related to the van Hiele theory and Logo use. Additionally, the growth of knowledge as observed through the changing student behaviors over the four topics was summarized and presented with statistical analysis.

Two post-instructional tests showing the students successfully achieved the prescribed curriculum objectives provided the study's validation.

Generally the students' Logo activities began in direct mode and relied on trial and error construction using the turtle as a "drawing tool". Students began to see the value of procedure writing, although the procedures were frequently used only to capture under a single name a previously established sequence of commands. Even when

students worked directly in procedure mode they continued to function at a visual level, adding and debugging one or two commands at a time.

At project initiation, students showed Basic Level thinking. They were able to correctly identify transformations, but not to explain their identification rationale. In the early stages they were capable of correctly constructing the figures, though seldom used any analysis of geometric shapes for correct angles or distances.

The students showed movement toward Level I thinking as they began to view the transformations as composed of discrete pieces. The original and image were seen as a group of commands and were eventually represented under a single name as a subprocedure. The visual identification of construction errors led in turn to recognition of properties, although students' definitions remained imprecise and primarily visual. The students' geometry behaviors, and their perceptions of geometric shapes, were observably reflected in the nature of their Logo activities.

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CHAPTER I

THE RESEARCH QUESTION

1.1 BACKGROUND TO THE STUDY

Logo is a computer language developed at and around the Artificial Intelligence Laboratory at M.I.T. in 1967. The Logo research group has involved many Feurzig, Abelson, DiSessa, Watt and Lawler to name a few but the key philosopher and spokesperson has been Seymour Papert. The Logo language is suitable for use by children and yet was developed as a powerful professional programming language that would contribute to the study of human intelligence and its development (Robertson, 1976).

Logo was first created on a large mainframe computer. The initial application was related to language using the list processing characteristics (Bitter and Watson, 1983). The first graphics project involved a peripheral device, a robot on wheels called a mechanical turtle, which traced its movements on large sheets of paper. Movements were controlled by a remote button box using commands described by diagrams depicting arrows pointing forward, back, right turn 90, penup and so on. Since then Logo has become a complex and sophisticated language, but still allows for easy initiation to its use. The "turtle" has become a two-dimensional, triangularly shaped cursor that leaves its tracks on the screen rather than on a sheet of paper. Programming the turtle to draw geometric figures on the screen is called "turtle geometry" and has become the best known and most widely used function of Logo.

The use of turtle geometry to develop an intuitive understanding of geometrical concepts has caught on quickly in many mathematics classrooms. Papert (1980) argues that the central theme in this intuitive development is the syntonic learning involved in programming the turtle. Planning the turtle's movements on the screen is often understood by students as they simulate the turtle's path by actually walking it out. This is often called "playing turtle" and is considered to be body syntonic. The geometric learning in this environment has been called ego syntonic in that programming the turtle's movements, in exploring concepts such as displacement, rotation and angle, congruence, variables and measurement, relates to the learner's previous knowledge and experience and can be useful in building a hierarchy of geometric knowledge. The different levels of complexity at which one can work in a Logo environment strengthens this syntonic nature.

Kieren (1984) has suggested different levels of Logo use and the instructional structure for Logo illustrated in Figure 1. The levels of programming outlined in the diagram involve a sequential progression from that of functioning in direct mode where single commands (Level 1) or simple lists of commands (Level 2) give immediate results. The third level of use occurs where initial programming involves lists of commands to define new terms that can then be used as primitives. The fourth and fifth levels of use involve more sophisticated programming. The ability to preplan, generalize and structure the programming into families of subprocedures, and the use of variables and recursion, are increasingly employed after level three.

The van Hiele theory (Wirszup, 1976; Hoffer, 1983) presents one way of looking at geometry stratified in levels which match the perception/thinking process of the students. The theory stratifies learning of plane geometry into the following sequential levels, stated briefly here and described later in detail.

Basic Level The objects are seen as wholes and are recognized by appearance alone.

Level I The properties of figures are perceived but remain unrelated.

Level II Definitions are meaningful and local logical relationships between properties and between figures are perceived and deduced.

Level III The relationships are deductible from an axiomatic system.

FIGURE 1 - LEVELS OF VAN HIELE AND LOGO USE

Debugging

Screen Debugging: (use of PENERASE, BACK, DRAW etc.) → Direct Mode

List Editing: (changing elements of lists to change corresponding actions using EDIT mode and particularly using editing keys to change parts of procedures.) → Naive Programming Mode

Logic Debugging: (changing procedures where there is no direct correspondence between a line of the program and a simple screen action.) → Planned Programming Mode

—— Strong Connections

---- Lesser Connections

Levels of Logo use

Direct Mode

Naive Programming Mode

Planned Programming Mode

Formal Programming Mode

Turtle Geometry

I. Logo is word oriented. A word drives an action.

II. Logo is list oriented. A list of words relates to a sequence of actions.

III. Logo is procedure oriented
- a. Procedures as named lists.
- b. Procedures with list patterns noted (eg.REPEAT used.)
- c. Bottom up superprocedures.
- d. Simple use of list commands.

IV. Logo is structure oriented.
- a. Procedures which are families of procedures (use of colon words of variables.)
- b. Procedures which call subprocedures.
  1) use of top down procedures.
  2) use of struc-charts.
- c. Procedures which call themselves: tail recursion.

V. Logo is list processing oriented and involves levelled logic.
- a. Procedures which use logical primitives
- b. Procedures which call themselves in the middle of a procedure (level change).
- c. Development of structured list processing procedures.

van Hiele Levels

Basic Level — Objects as wholes

Level I — Objects as bearers of properties

Level II — Objects as bearers of logical relationships

Level III — Deduction as a means of developing all of the geometry theory under consideration

Level IV — Modern standard of rigor exemplified by Hilbert's work. Development of theories without concrete interpretations.

This sequential development occurs through learning, thus the importance of the content and instructional methods are stressed. The process that leads to the next level of development has been broken down into five phases of learning: information, directed orientation, explanation, free orientation and integration (Wirszup, 1976).

The development of insight in their students was the focus of the van Hieles' description of levels and the five phases of learning. This theory has been suggested as a curriculum model for geometric learning (Wirszup, 1976; Coxford, 1978; Hoffer, 1980).

Kieren and Olson (1983) suggest a link between the levels of Logo programming in turtle geometry and the van Hiele theory of geometric development (Figure 1). Through the integration of these theories one can envision using Logo and its different levels of use to enhance a curriculum where the recognition, production and naming of figures as wholes, and a intuitive understanding of their properties, are studied through direct mode activities. An intuitive understanding is implicit in the writing of a list of commands to describe an object. From there an explicit understanding of the properties of those figures and then of the relationships between the figures and their properties become apparent in the naive programming mode and can be extended using more structured programming (Kieren and Olson, 1983). For example, writing a single procedure that will draw squares, rectangles and parallelograms requires an understanding of the relations between the properties of these figures.

1.2 PURPOSE OF THE STUDY

This study sought to explore and test the relationship between Logo use and the van Hiele theory of learning. A teaching unit integrating the principles of van Hiele geometry, described in Hoffer (1983) and the Grade 7 motion geometry objectives using Logo experiences as a vehicle was devised. It was felt that the nature of the Logo language with its graphic capabilities, together with the students' conjectures and active involvement in program definition could help develop, and then build on, the geometric intuition necessary for students to move from concrete experiences to more abstract reasoning. The following major topics in motion geometry were covered: translations, rotations, and reflections.

The following research questions further delineate the purpose of the study.

1. Can a set of experiences be developed in the Logo environment that incorporate the van Hiele levels and phases of learning?

2. What are the Logo and geometric behaviors of the students while working in such an environment?

3. Are levels of Logo use and the van Hiele levels of thinking observably reflected in the activities of the students?

4. Are these behaviors consistent with the theorized relationship linking levels of Logo use and the van Hiele theory?

5. Are there behavioral markers of transition between Logo levels? To what extent are such markers observable?

1.3 DELIMITATIONS

The researcher acted as a teacher/researcher in a classroom setting over an eleven week period. A team approach involving the classroom teacher and researcher allowed each to assume the role of advisors and facilitators to small groups of students as they progressed through the unit. The researcher's observation of, and discussion with, the students about their work arose through this role. Although all members of two classes were involved in the Logo/geometry curriculum the intense monitoring of students' progress necessary to gather the data to answer the questions above required limiting of the number of subjects to ten student volunteers.

As the intention of both the study and the unit was aimed at mathematical development, and not advancing the students' knowledge of Logo programming, Logo activities were limited to those programming concepts the students were previously familiar with: primitive commands which move the turtle, procedures, variables, simple subprocedures, tail-end recursion and disk operating commands.

1.4 DEFINITION OF TERMS

Bug - An error or omission in command use or in the logical progression of the program.

Debugging - The process of correcting errors or "bugs".

Ego-syntonic - Geometry derived from and matched to the natural idea development of the learner.

Levels

- van Hiele levels - The levels of geometric thought outlined by the van Hieles and presented by Wirszup (1976), Coxford (1978) and Hoffer (1981, 1983).
- Logo programming levels - The five levels of Logo programming outlined by Kieren (1984).

Logo - The name of a programming language available in several versions for use on a variety of different microcomputer models;

Geometric Intuition - Intuition is the immediate knowing or learning of a topic without the conscious use of reasoning. In mathematics this entails the use of imagery, informal use of language and thinking tools. Geometric intuition with regards to this study, involves the immediate recognition of images and the use of informal definitions and terms for identification. The intent is to build on this geometric intuition. The process requires experience of recognition and construction of geometric figures specified by the curriculum, and the use of terms and names in procedural titles. From there, the development of procedure definitions can be used as thinking tools in understanding classifications and the realization and analysis of the properties of the figures.

Phases of Learning - The sequence of instruction necessary for students to progress from one thought level to the next, outlined by the van Hieles, and presented by Hoffer (1983).

State Transparent - In drawing a closed shape the turtle ends in the same position and orientation as when it started.

Total Turtle Trip Theorem (Rule of 360^0) - In order to draw a state transparent closed shape, the turtle must be turned through a total angle which is an exact multiple of 360^o (Watt, 1983).

1.5 SIGNIFICANCE OF THE STUDY

Although the literature expounding upon the capabilities and suggested uses of Logo and turtle geometry is pervasive, documentation of the effect in developing geometry knowledge has yet to be revealed. This study seeks evidence as to whether such an environment structured according to the van Hiele stage theory can facilitate geometric development. The basis of the study is an instructional unit designed to provide appropriate computer-based learning experiences to help students build on their intuitive geometry understanding to study the basic motions of transformation geometry. Knowing the effects of such experiences may provide important implications for geometry instruction.

1.6 ORGANIZATION OF THE REPORT

The present chapter is an outline and preview of the study. Chapter II presents the theoretical and research background to both the van Hiele theory and the Logo language.

Chapter III describes the curriculum development and research design. Chapters IV presents the findings of each of the instructional topics in the motion geometry unit. The concluding chapter, Chapter V contains the summary, conclusions and implications of the research study.

CHAPTER II

REVIEW OF THE RELATED LITERATURE

The purpose of the study is to explore and test for relationships between the van Hiele theory of geometry learning and the use of the computer language, Logo in Turtle graphics mode. Chapter II presents background information and a framework for the study. A brief history of the learning theories related to geometric ideas will initiate the discussion. A detailed account of the van Hiele theory of geometric development and an examination of related research follows. The chapter concludes with a discussion of the philosophical roots of the Logo language and the research related to the use of this language with students.

2.1 HISTORICAL PERSPECTIVE

The theory of development of spatial and geometric ideas outlined by Piaget and his colleagues describes a child's progression from topological to projective and Euclidean concepts in the understanding of representational space. The important differences between the topological, projective and Euclidean stages is the way in which different figures are related to each other, with topological and projective concepts involving qualitative properties while Euclidean concepts involve quantitative properties (Smock, 1976). Through increased understanding, the ability to differentiate between the relevant and irrelevant attributes grows. Many studies have been done which generally confirm the sequence of these stages, although discrepancies have arisen regarding the age of attainment of the stages (Dodwell 1963, Lovell 1959, Bober 1973, Montangero 1978). Martin (1976) has shown that not all topological concepts develop before the projective and Euclidean. A number of studies (Perham 1978, Schulz 1978, Kucheman 1980, Thomas 1978) affirm that understanding progresses from qualitative to quantitative.

Although Piaget was not concerned with curricula and schools some general principles have been derived from his theory which may guide educational procedures (Ginsburg and Opper, 1979). It has been suggested that the child's progression through developmental stages may be affected by instruction, but this is a long process. Experience within the environment is the key. Physical and logico-mathematical experiences give children the ability to transpose physical actions to a plane of thought and reflect on them, so that they can come to appreciate the significance of their actions. The environment should provide children with activities of their own choosing at which they can work individually but with social interaction (Ginsburg and Opper, 1979). Such free problem solving activities would likely have an effect on the transition between stages. Bober (1973) and Kuper (1978) have shown such experiences do speed up the transition.

Skemp (1971) outlined similar ideas on reflective intelligence. Mathematics is learned at an intuitive level long before we can function at a reflective level. At the intuitive level the child is dependent on the new material being closely associated with his existing mathematical schemas and on concrete thinking from "which the learner can resynthesize the structures in his own mind" (Skemp, 1971 pp. 67).

2.2 THE VAN HIELE THEORY

A theory more specific to plane geometric concepts, curricula and teaching is that of the van Hieles. These two Dutch educators were interested in the development of insight in their students. To have insight, students must understand what they are doing, why they are doing it and when to do it. They can apply their knowledge in order to solve problems (Hoffer 1983). Through their respective doctoral dissertations in 1957 the van Hieles formulated a theory stratifying geometric learning into levels of thought. Their work did not become known in North America until introduced in English by Freudenthal (1973). Wirszup (1976) expanded on this introduction with a paper describing the following levels of geometric development:

> Basic Level: Figures are recognized by appearance alone and are seen in their totality. Students do not see the parts of figures, nor do they perceive the relationships among the components of the figure. They cannot compare figures with common properties with one another. For example, a square would be conceived as totally different from a rectangle.

> Level I: Students begin to analyze the components of figures and also establish relationships between individual figures. At

this level individual figures can be analyzed but figures and their properties are not explicitly interrelated, therefore a square is not a rectangle.

Level II: Students relate figures and their properties. A logical ordering of the properties of a figure and of classes of figures begins. Definitions become meaningful. Students can see that every square is a rectangle and can organize sequences of statements to justify their observations.

Level III: Students understand the significance of deduction as a means of constructing and developing all geometric theory. The transition to this level is assisted by the understanding of axioms, definitions and theorems. However they do not understand the need for rigor, nor the relationships between other deductive systems.

Level IV: Students analyze the various deductive systems with a high degree of rigor and are able to make abstract deductions. A person at this level develops a theory without making concrete interpretations.

Hoffer (1983) has labelled these levels as recognition, analysis, order, deduction and rigor. Dina van Hiele Geldof called the levels I through IV: the aspect of geometry, the essence of geometry, insight into the theory of geometry and scientific insight into geometry (Usiskin 1982).

It is inherent to this theory that progression through the levels occurs in sequence and one level must be mastered before moving on, as each level is "about" the previous level. The objects at level n are extensions of or "about", objects at level n-1, and thought which was intrinsic at the preceding level becomes extrinsic. An extension of this is that people reasoning on different levels cannot understand one another (van Hiele, 1980). In particular, a teacher communicating or posing tasks at level n, to a student reasoning at level x, x<n, can anticipate a communications breakdown or lack of understanding on the part of the student.

The van Hieles also proposed the following sequence of learning in five phases. Just as Skemp suggested they also think that learning should move from teacher directed instruction to student independence from the teacher. Generally each of the instructional phases is necessary for passage from one thought level to the next.

Phase 1 - Inquiry - The teacher engages the students in conversation about the objects studied. The teacher learns how the students interpret the words and gives them some understanding of the topic to be studied. Questions are raised and observations made using the vocabulary and objects of the study setting the stage for further study.

Phase 2 - Directed Orientation - The teacher carefully sequences activities for student exploration by which students begin to realize what direction the study is taking and they become familiar with the characteristic structures. Many activities in this phase are one step tasks that elicit specific responses.

Phase 3 - Expliciting - The students, building from previous experiences, with minimal prompting by the teacher, refine their use of the vocabulary and express their opinions about the inherent structures of the study. During this phase the students begin to form the system of relations of the study.

Phase 4 - Free Orientation - The students now encounter multi-step tasks or tasks that can be completed in different ways. They gain experience in finding their own way in resolving the tasks. By orienting themselves in the field of investigation, many of the relations between objects of the study become explicit to the students.

Phase 5 - Integration - The students now review the methods at their disposal and form an overview. The objects and relations are internalized into a new domain of thought. The teacher aids this process by providing global surveys of what the students already know, being careful not to present new or discordant ideas. At the close of the fifth phase, the new level of thought is attained.

(Hoffer, 1983 pp.208)

In summary, the major characteristics of the van Hiele levels of thought are that 1) the levels are sequential 2) each level has its own language, set of symbols and network of relations, 3) what is implicit at one level becomes explicit at the next, 4) explanations above a students level will not be understood, 5) progress from level to the next is more dependent on instruction than on age or maturation, and 6) progress from one level to the next involves five phases of instruction.

2.3 RESEARCH ON THE VAN HIELE THEORY

Soviet psychologists have adopted the van Hiele ideas. Many studies have been done illustrating the importance of the phases of learning and formed a basis for developing a new eight year program with a curriculum which insured a single continuous line of geometric development for the pupils. Pyshkalo describes the benefit, in that, by the end of grade 3 all pupils did reach the second level (van Hiele level I), surpassing the progress of seventh grade students using the traditional curriculum (Hoffer 1983). Kilpatrick and Wirszup (1969) edited a series of translated studies, "Soviet Studies in the Psychology of Learning and Teaching of Mathematics". The Soviet psychologists stressed the importance of the role of instruction in cognitive development and that the passage from one level to the next is not a spontaneous process with age. These studies using various modes of instruction illustrated how the learning or knowledge building process can be affected by instruction.

North American research began with Hoffer (1983) writing a secondary level geometry text for a one year course with the van Hiele levels in mind. His intention in using the text was to allow for investigations and activities in the first semester, preparing students to work in a deductive system (van Hiele Level III) in the second semester. A proof writing test was given at the end of the year to classes who had used the materials and those who had not. Participating teachers felt students in the experimental classes had learned more geometry especially under the topics of area, volume, and transformations, and could write original proofs as well as the control classes.

The Chicago Project (Hoffer 1983, Usiskin 1982) tested the van Hiele levels of students entering high school geometry courses and used this information to describe and predict the students' achievement in the course. Depending on the criteria used for the number of correct responses, 67 to 90 percent of the students could be assigned to a van Hiele level. Analysis of the data showed that these levels are a good predictor of achievement on year end standardized geometry content and proof writing tests. Using the students' van Hiele levels as a predictor, Usiskin then suggests that half of the students are enrolled in geometry courses where they have less than 50 percent chance of success.

This study confirms the use of the van Hiele theory to explain the difficulties many students have with formal geometry due to lack of foundational experience. The van Hiele level test developed within the project probed a few concepts to predict an overall van Hiele level. A discrepancy with the theory is found in that some students being able to answer questions at a higher level and yet incorrectly answering lower level questions.

Mayberry (1981) constructed a test based on seven common geometric concepts to assess van Hiele levels of eighty-four elementary preservice teachers. Her results showed that the students can be assigned a level but there is no consensus across the concepts thus people can be at different levels for various concepts. The inconsistency with the Chicago test results might be attributable to this lack of consensus.

The Oregon Project (Burger and Shaughnessy, 1986, Hoffer, 1983) investigated the extent to which the van Hiele levels serve as a model for assessing students' understanding of geometry. The subjects (45 students drawn from kindergarten to college math majors) responded to sequenced tasks on triangles and quadrilaterals in clinical interview sessions. The tasks included drawing shapes, identifying and defining shapes, sorting shapes, determining a mystery shape, establishing properties of parallelograms and comparing components of mathematical systems.

The interviews were analyzed to reveal descriptors of students' perceptions and comprehension levels and to assign the students to van Hiele levels of predominant reasoning. The interview data revealed a number of recurring behaviors labelled "Level Indicators". Students who seemed to be reasoning predominantly at a particular van Hiele level tended to exhibit some or all of these behaviors.

Basic Level

1. Use of imprecise properties (qualities) to compare drawings and to identify, characterize, and sort shapes.
2. References to visual prototypes to characterize shapes.
3. Inclusion of irrelevant attributes when identifying and describing shapes, such as orientation of the figure on the page.
4. Inability to conceive of an infinite variety of types of shapes.
5. Inconsistent sortings; that is, sortings by properties not shared by the sorted shapes.
6. Inability to use properties as necessary conditions to determine shape; for example, guessing the shape in the mystery shape task after far too few clues, as if the clues triggered a visual image.

Level I
1. Comparing shapes explicitly by means of properties of their components.
2. Prohibiting class inclusions among general types of shapes, such as quadrilaterals.
3. Sorting by single attributes, such as properties of sides, while neglecting angles, symmetry, and so forth.
4. Application of a litany of necessary properties instead of determining sufficient properties when identifying shapes explaining identifications, and deciding on a mystery shape.
5. Descriptions of types of shapes by explicit use of their properties, rather than by type names, even if known. For example, instead of rectangle, the shape would be referred to as a four sided figure with all right angles.
6. Explicit rejection of textbook definitions of shapes in favor of personal characterizations.
7. Treating geometry as physics when testing the validity of a proposition; for example, relying on a variety of drawings and making observations about them.
8. Explicit lack of understanding of mathematical proof.

Level II
1. Formation of complete definitions of types of shapes.
2. Ability to modify definitions and immediately accept and use definitions of new concepts.
3. Explicit references to definitions.
4. Ability to accept equivalent forms of definitions.
5. Acceptance of logical partial ordering among types of shapes, including class inclusions.
6. Ability to sort shapes according to a variety of mathematically precise attributes.
7. Explicit use of "if, then" statements.
8. Ability to form correct informal deductive arguments, implicitly using such logical forms as the chain rule (if p implies q and q implies r, then p implies r) and the law of detachment (modus ponens).
9. Confusion between the role of axiom and theorem.

Level III
1. Clarification of ambiguous questions and rephrasing of problem tasks into precise language.
2. Frequent conjecturing and attempts to verify conjectures deductively.
3. Reliance on proof as the final authority in deciding the truth of a mathematical proposition.
4. Understanding of the roles of the components in a mathematical discourse, such as axioms, definitions, theorems, proof.
4. Implicit acceptance of the postulates of Euclidean geometry.

(Burger and Shaughnessy, 1986 p 43)

This study confirmed van Hieles' original general description and sequence of the levels. However the discreteness of levels, particularly those of analysis and abstraction, was not confirmed. Consistent with Usiskin's (1982) observations the use of formal deduction among secondary or post secondary geometry students was nearly absent.

The Brooklyn Project (Fuys, Geddes, Tischler 1985, Hoffer 1983,) developed three modules of instruction; properties of polygons, angle measurement and area. The use of concrete objects in the modules offered definite instructional advantages and assisted in the identifying the students' thought processes. Some module activities were similar to those used in the Oregon Project. The instruction modules were used as research tools to obtain data in six hours of video taped clinical interviews with sixth and ninth grade students. The interviews of individual students working through the modules were analyzed using protocol forms.

Results of the clinical interviews indicate that the van Hiele model provides a reasonable structure for describing students' geometry learning. Video tape analysis provides insight and information on students' the levels of thinking, language difficulties, learning styles and thinking processes. Analysis of the interviews indicated the entry level for the sixth grade students was mainly at the Basic Level while ninth grade students entered at the Basic Level or Level I. Indicators similar to those described in the Oregon project arose with the students identifying and sorting shapes on an "it looks like" basis rather than by the use of properties. Initial student descriptions were imprecise.

The project also included an analysis of the geometry strands found in three widely used commercial text book series (K-8). The analysis involved the three levels of thinking and entailed four components: goals, expository section, exercises and tests, with the following results. Only in the texts for Grades 6-8 were even occasional sections or exercises found above the Basic Level and even then there were gaps between the expository sections requiring first or second levels of understanding and the corresponding Basic Level exercises involving mainly recognition or identification of figures. The texts examined included few exercises of an analytical or informal deductive nature (Levels I or II).

2.4 LOGO PHILOSOPHY OF EDUCATION

Although Logo is a computer programming language, Piaget's theories of development are central to it's design. Some of these ideas were expressed by Papert (1980):

> Children learn by interacting with their environment and thinking about what they have done.
>
> Children learn by building on past experiences with meaningful models.
>
> Children can direct their own learning.
>
> Children are builders of their own intellectual structures, they can learn without being taught.

Having made these ideas central in to the language, the M.I.T. research group developed a philosophy of education surrounding the use of this language. Logo was written to relate to child's world. Through Logo, children learn by doing and then reflect on what they have done. This idea is also reflected in the theories of Piaget, Skemp and also the van Hiele phases of learning. Papert advocates that writing a program in a high level language involves an expression of thought processes not normally formed. When a mistake or bug occurs, the child can retrace his own thoughts and understand his own mistakes (Robertson 1976). This rethinking helps children become clear, precise, logical thinkers. In "Teaching Children to be Mathematicians vs. Teaching about Mathematics", Papert (1971) argues that many skills are hard to teach only because they are hard to describe. Logo, he feels, is a catalyst for communication because, in learning within a descriptive system, concepts become clearer and more communicable, stimulating interaction between the teacher and students, and between students, when they collaborate on a project. Teachers themselves benefit as learning Logo helps them understand and express what they are teaching step by step.

It is inherent to this educational philosophy that children should work in micro worlds, task domains or problem spaces that are delimited learning environments where interesting things happen and powerful ideas are formed (Goldenberg, 1982). These powerful ideas should be "simple, general, useful and syntonic" (Lawler, 1982). This idea of learning being syntonic, that is relating ideas and being appropriate to the person's previous knowledge, is also evident in the van Hiele phases. Ideas can only gain power when they are shown by concrete models. The turtle is central to this philosophy of a concrete model, children can program it, telling the computer how to duplicate their own body movements and thus experience "mathematizing" (Papert, 1971) for themselves. Many of these microworlds can be built from a simple recursive procedure in which the input variables can be changed with interesting results.

2.5 RESEARCH ON LOGO

Much of the literature proposing uses of Logo and research studies involving Logo have been published in popular computer magazines. Most of the uses arose out of projects in elementary and junior high schools where teachers' initial reports were very positive. Teachers commented that students become enthusiastic, self-sufficient and often that students who do poorly in regular classroom situations excel in Logo projects thus increasing their motivation. It is also noted that teacher-student relationships often changed as the teacher is moved from a position of knowledge giver and authority figure, to one of advisor where they work with students on solutions.

The Brookline Logo Project (Watt 1982) involved fifty grade six students: case studies of sixteen of them documenting their different learning styles was included. The factors of planning, analysis and interpretation, assistance from the teacher plus the mathematical concepts encountered by the students, in their freely chosen projects, were all recorded. A brief summary of the students' learning with regard to the above factors was the final report; there were no explanations of how the data was recorded or analyzed nor was there any objective testing included. A conclusion is given that children created their own microworlds in which they worked on a limited but expandable set of concepts with which they were comfortable. All students experienced success with the Logo programming.

The Texas, Lamplighter Project was similar: where Logo was used in all grades for instructional purposes. The teachers' reports were positive, but the principal expressed concern about the time involved and available to teachers in developing appropriate curricula (Watt 1982).

Another project commencing at the same time was at Bank Street College School (Watt 1982). The students had extensive access to Logo, with the researchers focussing on students learning problem solving techniques and social interaction among students as they worked on Logo activities. Teachers were to help students as little as possible in order to foster discovery learning. The initial report (Jewson and Pea 1982) showed that children interact more when they are working with Logo activities. Other results of the project were disappointing (Green, 1985) showing no differences with a control group when the children were tested on planning skills. Papert (Green, 1985) questioned the results in view of the lack of learner support from teachers.

Logo has also been used with learning disabled students (Watt 1978) where success was reported through changes in attitudes toward learning, school and self concept. A report describing a breakthrough in communication with an autistic child (Weir and Emmanuel, 1976) through the use of Logo, concludes that the improvement was due to the child's interaction with the turtle and his being in control of that environment.

Two studies have been done with preservice teachers with low mathematics achievement and no computer training. Austin's (1976) focus was students' adaptation to the computer/mathematics environment and the transfer of the knowledge gained in the laboratory experiences to real world situations. The students developed many mathematics and programming capabilities. They profited more from their mistakes but were less willing to try new ideas than children who had used the same materials.

Duboulay's (1977) study centered on those preservice teachers who were weak in and disliked mathematics with the thought that the Logo experience would strengthen their understanding of mathematical concepts. Overall conclusions were that the Logo experience promoted self-awareness of the learning taking place and aided the subjects to identify the process of teaching mathematics, allowing insight into the difficulties children encounter in learning mathematics.

Using Logo in grade seven mathematics was a long term study at the Artificial Intelligence Laboratory at Edinburgh University (Howe and Ross, 1981). Eleven year old boys low in mathematical achievement and discouraged by their inability to learn mathematics, worked with Logo for two years. The first year involved programming concepts and in the second year procedures were used to illustrate mathematical concepts. All activities were based on structured worksheets.

Achievement test showed no significant differences between the experimental group and the control group, but the experimental group did show a greater improvement in their understanding of the concepts. The experimental group was also described by their teachers as able to argue more sensibly about mathematical ideas and explain their difficulties more clearly. The researchers suggest that improvements could be attributed to the increased time and the assistance these students received.

Hillel (1984) examined the mathematical and programming concepts acquired by 8 and 9 year old children who worked in a somewhat restricted Logo environment for twelve hours. It was hoped that the Logo could be used as a tool for learning some mathematics. Once the children had explored a command they were to use it in some goal directed activity. Children were encouraged to advance plan activities.

The childrens' work and their conversations with the investigators were recorded. Any work away from the computer was also kept. An observation grid was developed containing the following aspects identified as important: perceptual analysis, researcher intervention, error coping, and mathematical and procedural concepts. Detailed analysis of the students' work was done after the session and entered on the grid. All sixty sessions were summarized on the grid for each pair of children and then summarized for each of the five pairs. Therefore, the results obtained are indicative of the majority of children involved.

No dramatic conceptual changes occurred in the children by the end of the twelve sessions; traditional obstacles to the acquisition of mathematical concepts do not disappear. Analysis was unable to reveal definite conclusions about conceptual understanding, many concepts were just emerging although the students' achievement in dealing with a formal language and mathematically sophisticated concepts was remarkable. Children worked mostly in direct mode, writing procedures only to save their work.

Two subsequent studies (Hillel and Samurcay, 1985; Hillel, 1985) used similiar methodologies to probe childrens' understanding and conceptual difficulties with Logo variables. Four children (participants in the earlier study) were observed as they worked through structured activities.

Initially five recurring difficulties with variables were perceived: 1) The variable was identified but a general procedure was not defined. 2) A variable was declared but not used with in the procedure. 3) A fixed procedure was assigned inputs.
4) Assumptions that the variable can take on different values within the same procedure. 5) Confusion about what the variable name signifies. Later the children showed a greater fluency in the definition and use of general procedures including parameters other than length, thinking about two variables and the spontaneous definition of general procedures.

Some of the difficulties the children continued to experience related to their lack of understanding of the essence of procedure use and the relationship between procedures and objects they produce. Resolving inter-procedural relations and coming to terms with the intrinsic nature of the geometry continued to be areas of difficulties.

Hillel's studies provide a valuable contribution to Logo research with extension beyond qualitative analysis. The detailed analysis with respect to programming style and mathematical concepts provided a prototype methodology for the present study.

The Atlanta - Emory Logo Project (Olive, 1985) is an ongoing investigation into students' understanding of geometric relationships. A pilot study investigated the potential for using the Logo language to generate a geometric microworld of relational learning cycles for students, and to assess the effectiveness of the teaching methodology, sequence and content for generating relational learning.

Twenty ninth grade students were taught programming through Turtle graphics in a series of "guided discovery" learning episodes, of two hours per day / three days per week for a six week period. The instructional focus for geometric relationships was based on rationale of the van Hiele model of geometric thought. The teaching methodology and curricular ideas were based on a theory of relational learning cycles intended to help students achieve a higher level of abstraction in their mathematical thinking. Student interactions with Logo were saved using dribble file technology, allowing students computer work to be visually recreated.

While for most students the instructional sequence was too fast, those students able to keep pace shifted to a more abstract mode of functioning with the Logo and the relational understanding of many of the geometric concepts introduced.

Two key sequencing elements for generating relational learning cycles arose from this pilot study; an instructional sequence should progress through a Fixed Procedure, a Variable Procedure to a Generalized Procedure for a particular type of Logo object before introducing a more complex Logo object. Secondly each new Logo object is introduced only after the students have had the opportunity to reflect on what they have been doing with existing Logo objects and the relationships they have discovered using those objects.

2.6 SUMMARY

Current North American research based on the van Hiele model has sought information concerning the hierarchical nature of the levels and the placement of students within the structure (Burger and Shaughnessy, 1986; Mayberry 1981). Other research has investigated the effects of instruction on a student's predominant van Hiele level (Fuys, Geddes and Tischler, 1985). The geometric abilities of students as a function of van Hiele levels have also been measured (Usiskin, 1982). These studies have provided relevant conclusions; the levels are hierarchical in nature, but seem to be dynamic rather than static. Individual students can be difficult to assign to a van Hiele level as students can be on different levels for different concepts and students in transition are particularly difficult to classify reliably. Recent research on the van Hiele theory has allowed for the elaboration of the model providing indicators characterizing the levels operationally as well as by descriptions of the protocol analysis of the clinical interview sessions.

Most of the research to date on Logo has been extremely positive, however many of the articles, published as anecdotes in popular computer magazines, do not include evidence of rigorous standards or statistical evidence of the effects of Logo use. Hillel's project (1985) was the first to provide ideas and techniques and to challenge activities pertaining to the instruction in Logo use by the researchers. This study is also the first in providing a detailed analysis of the children's understanding of the mathematical and Logo concepts encountered. The definition of instructional strategies is an important outcome of the Atlanta - Emory project (Olive, 1985). The role of instruction in the use of Logo in classrooms and its integration into the mathematics curricula have become an important research questions.

The reviewed theory and research has influenced the current study in many ways. The propositions below indicate this influence and relate directly to the research questions of the study.

1. A teaching unit is used as a research tool in a similiar manner to that of the van Hiele research of the Brooklyn Project. (Fuys, Geddes, Tischler 1985, Hoffer, 1983,)
2. The van Hiele phases of learning (Hoffer, 1983) are the basis of the teaching unit.
3. Descriptions of behaviors at the various van Hiele levels Wirszup, 1976, Coxford, 1978, Hoffer, 1981, 1983) as well as the level indicators as outlined in the Oregon Project (Burger and Shaughnessy, 1986) provide reference for the analysis of students' perception and behavior in order to assign the students to a predominant level of reasoning.
4. Hillel's (1984, 1985) studies which examine the mathematical and programming concepts acquired by 8 and 9 year olds provides a prototype methodology for data collection and analysis observation grids.

CHAPTER III

METHODOLOGY

As the central purpose of this study was to explore and test the relationship between Logo use and the van Hiele theory of learning, a teaching unit was devised integrating the principles of van Hiele geometry and the Grade seven motion geometry objectives using Logo experiences. Curriculum development was the primary step in answering the research questions and was followed by observation data of the students' activities and behaviors throughout the teaching of the unit. This chapter presents a description of the curriculum development and validation, the research subjects and the collection and analysis of data.

3.1 CURRICULUM DEVELOPMENT

In the Alberta Junior High Mathematics Curriculum (Alberta Education, 1978) for grades seven and eight, the basic motions (translations, rotations, and reflections) of transformation geometry provide the vehicle for an investigation of the properties of two dimensional figures. Thus transformation geometry was chosen as the topic for an instructional unit integrating the van Hiele theory with a Logo environment. In preparing the teaching project unit, the motion geometry concepts outlined for grade seven in the Junior High Mathematics Curriculum Guide were identified and then classified according to van Hiele levels. To initiate the analysis, typical tasks related to the curriculum objectives was derived from student exercises presented in the prescribed and supplementary reference texts outlined in Alberta Junior High Mathematics Curriculum (Alberta Education, 1978).

Further analysis into levels involved the identification of basic skills necessary to complete these tasks. Hoffer (1981) extened the van Hiele model by identifying basic skills (visual, verbal, drawing, logical and applied) at each level of thought. The motion geometry tasks were assessed with respect to these basic skills and were classified accordingly the corresponding van Hiele level of thought. Figure 2 presents a summary of the task analysis.

Transformation geometry is a continuation of a topic introduced in

FIGURE 2 – ANALYSIS OF THE VAN HIELE LEVEL OF CURRICULUM TASKS

OBJECTIVES

- Creates and discusses simple repeated patterns in terms of translations, reflections and rotations.
- Constructs polygons using protractor, straight-edge, compass and straight-edge or Mira, as specified by the teacher.
- Constructs the image of a figure given a transformation or a combination of transformations.
- Given congruent figures on geopaper, names the transformations or combinations of transformations that move one figure onto to another.
- Represents a translation by a slide arrow, a reflection by a reflection line, and a rotation by a turn center and turn arrow.

EXAMPLES OF TYPICAL TASKS	SKILLS	VAN HIELE LEVEL OF THOUGHT
- Given a simple original and a representation of the motion draw the image. - Given the original and the image, represent the transformation by a turn arrow, slide arrow or mirror line. - Given the image and the representation of the motion, draw the original.	→ - Accurately draws the figure	BASIC LEVEL
- Determine which motion or combinations of motion was used to produce an image.	→ - Recognizes the image	BASIC LEVEL
- Construct a "Goes To" table for congruent polygons produced by the transformation.	→ - Understands conservation in various orientations	BASIC LEVEL
- Combine successive slides, rotations or reflections to produce an image and compare the results.	→ - Accurately draws the figure	BASIC LEVEL
- Identify and construct mirror symmetry.	→ - Identifies included information	LEVEL I
- Complete an invariance table using the properties of length, angle, congruence and parallelism for the slide, reflection and 1/2 turn.	→ - Distinguishes properties	LEVEL I
- Make wallpaper patterns by combining motions.	→ - Accurately draws the figure	→ BASIC LEVEL

the Alberta Elementary Mathematics Curriculum (Alberta Education,1982). In grade seven the experience continues with basic level tasks: constructing the correct images for these transformations, representing the transformations through appropriate notation, and identifying the transformations or combinations of them. Identification of the transformations may involve the properties of invariance in the transformations, a Level I task. The algebra of the composition of transformations is not included in the Alberta Grade Seven Geometry Curriculum and was not included in this study.

The lessons were developed using this assessment of the objectives. The instructional focus of the van Hiele theory - the phases of learning recommended to assist students in moving from one level to the next - was used as a model. As described in Chapter II, the five phases within each level are: Information, Guided Orientation, Explication, Free Orientation and Integration. To incorporate these phases into the motion geometry unit, suitable Logo activities were needed for each phase. Using the instructional levels of Logo (Kieren, 1985) the following van Hiele / Logo curriculum model emerged.

The Demonstration/Discussion phase identified by the van Hieles as Inquiry included many examples and non-examples demonstrated very effectively by the computer. This enhanced the observation, vocabulary and discussion necessary to give some understanding of the topic and to set the stage for further study.

In this model, the second phase was labelled Turtle Tracking. The students were given teacher designed procedures allowing for one step exploration into the new concepts. Some examples are: a procedure given which required the supply of inputs to complete the image, the identifying of a specific aspect in a given procedure, creation of a design or motion in direct mode, or completing or debugging given procedures to complete the motion.

Extensions were related to the third phase of learning. Here the intent of most of the activities was to have students design and debug procedures related to the topic. They were assisted in writing a simple list of commands using the knowledge gained in the previous phase.

The Free Orientation phase was titled Turtle Excursions. In this phase, students extended their procedures to include variables, explored recursive procedures related to the topic, or both.

The final Project phase was an attempt to integrate the knowledge gained about the topic in two ways. First, given some suggestions, the students wrote procedures requiring the use of the knowledge gained through at least three of the last four phases. At this stage students were to work with little assistance at this in finding their own solutions. Second, on demonstrating the various projects to the class, the students' explanations and discussions of the various approaches provided a survey of the insights gained on the topic. An example of the Translation Lesson Module is included in Appendix A. The framework of the curriculum model is summarized in Figure 3.

Figure 3 - DEVELOPMENT OF THE MOTION GEOMETRY UNIT THROUGH VAN HIELE PHASES

VAN HIELE PHASE	LOGO / MOTION GEOMETRY
INQUIRY	**DEMONSTRATION / DISCUSSION**
discussion, the teacher learns how the students interpret the words and gives some understanding of the topic to be studied, involving questions and vocabulary	demonstration of many examples and nonexamples involving observation, discussion and an introduction to the vocabulary
DIRECTED ORIENTATION	**TURTLE TRACKING**
the teacher carefully designs sequences of activities for student exploration so they become familiar with the characteristic structures	teacher designed procedures; creation of motion in direct mode; completion or debugging of given procedures
EXPLICITING	**EXTENSIONS**
building from previous experience with minimal prompting from the teacher; begin to form the system of relations of the study	design and debug procedures related to the to the topic; write simple lists of commands using knowledge gained in Turtle Tracking
FREE ORIENTATION	**TURTLE EXCURSIONS**
students encounter multi-step tasks, they gain experience finding their own way in resolving the tasks	extend procedures to include variables; explore recursive procedures related to the topic
INTEGRATION	**PROJECT**
students review methods at their disposal and form an overview; objects and relations are internalized into a new domain of thought.	given some suggestions, students will write procedures requiring the knowledge gained in the last 4 phases; students work with as little assistance as possible finding their own solutions; demonstration of projects to the class to explain and discuss the various approaches to give a survey.

Five modules were designed for use in the instructional unit; congruency, translations, rotations, reflections and combination of motions, however time constraints restricted the modules used by the students to the first four.

3.2 ENTRY INTO THE SCHOOL SETTING

In order to complete such a unit the students must be familiar and comfortable with Logo programming concepts which when included with the geometry unit, required an extensive commitment for all involved. Thus the project took place in an urban junior high school at which the researcher had previously taught mathematics for five years. The administration and the grade seven mathematics teacher offered their co-operation with the project. Two heterogeneous grade seven mathematics classes were involved in the project with twenty-two students in 7A and sixteen students in 7B. Although the classes were not intentionally academically streamed, students not involved in the French as a Second Language Option comprised the 7B class. Academic achievement for this group was consistently lower in all subject areas.

Prior to the commencement of the geometry unit, Logo programming concepts were introduced in one eighty minute session per week for twelve weeks. The researcher instructed the 7B class and the mathematics teacher (with no previous Logo programing experience), after observing and assisting with the 7B class, implemented a similar program with the 7A class. In this introduction to Logo, the students became comfortable with Logo primitives, writing their own procedures and, simple subprocedures as well as disk operating commands. They also explored teacher designed variable and tail-end recursive procedures and wrote simple variable procedures.

During the teaching of the motion geometry unit the instruction time for the two classes varied due to differences in the class timetables. The 7A's instruction time was limited to one eighty minute session per week for twelve weeks while the 7B class's instruction time was two eighty minute sessions per week for a ten week period. During the project the students worked on the Logo activities in pairs. Although all members of both classes were involved in the project, six students in the 7B class and four students in the 7A agreed to have their progress closely monitored. A sample letter for the student and parents is included in Appendix B. Eight students initially volunteered in the 7B class, but one pair withdrew prior to project completion.

The classes were divided into two groups for the Demonstration / Discussion phase, with the researcher and the teacher each working with one of the groups. The four remaining phases were completed with student pairs working at the computers. The teacher and researcher offered guidance and assistance to the students as they worked on the Turtle Tracking, Extensions and Turtle Excursion phases. The Project phases were completed with as little assistance as possible.

3.3 DATA COLLECTION

Although all the students in both classes were involved in the project only ten students were closely monitored. As they worked in pairs at the computers all of their work was recorded using video cassettes connected directed to the computer monitor. An external microphone connected directly to the VCR unit provided and audio recording of the conversations between the student pairs as well as the interactions with the teacher or researcher. Completed procedures were saved on floppy disk. Field notes were also kept of any activities away from the computer such as preplanning and any follow-up classroom discussions.

3.4 CURRICULUM VALIDATION

The curriculum was validated by reference to three questions.

a) Did the curriculum on the whole, and on a day to day activity basis, match the theoretical model?
b) Could the curriculum be successfully implemented by a teacher?
c) Did students achieve the geometry knowledge required by the Alberta curriculum in this environment?

The pre-design process established the face validity; the instructional modules appear to cover relevant content. An interactive design process used throughout the instruction also confirmed the validity. Between class sessions, review of the videotapes and completed projects, as well as discussions with the teacher, allowed for diagnosis, revision and extension of the instructional activities. The 7A class began the geometry unit approximately three weeks after the 7B class, allowing time for further revision.

Further validation was established through the successful use of Logo / van Hiele curriculum materials in the 7A class by the mathematics teacher although she was not previously experienced in the Logo use. The teacher and the researcher met regularly to discuss the progress, direction and problems with the unit.

Following the instructional phase of the teaching project, the validity of the curriculum materials was also checked through interviews with the ten participating students as well as two paper and pencil tests (Appendix C and D). Individual interviews involving paper and pencil tasks were used to ascertain each student's knowledge and understanding of the motion geometry topics studied. The students' written responses and audio tape recordings of the interviews were another source of data.

The students' knowledge of geometry was also assessed in a motion geometry unit test as well a school system year end examination. Objectives of the motion geometry curriculum were tested in a 35 item multiple choice test. The test was constructed in conference with the classroom teacher from a test item bank recommended for use in the school system. Individual and class results were recorded. The school system evaluation consisted of 100 multiple choice items, covering all grade seven mathematics topics. Of particular interest to this study were the students results on the 20 items comprising the geometry section of the examination. Individual student results for both classes were recorded as well class and system averages. The data from both the interviews and test results is reported and analyzed on page 87.

3.5 DATA ANALYSIS

Given a valid instructional experience in the Logo / Motion Geometry, it is useful to study the Logo and geometric behaviors of the students and the development of their mathematical ideas in this environment. This was done through the analysis of the video tape data which was an ongoing process throughout the teaching project. The following three stages of analysis were involved (Pothier, 1981).

Immersion Stage

The attempts that students made in completing tasks and the students' behaviors were monitored so that appropriate adjustments of tasks, assistance and responses on the part of the researcher could occur during the sessions. Field notes of important comments and questions were also recorded and analyzed at this time.

Reflection Stage

This stage occurred between the classroom sessions when review of the video tapes, completed projects and discussions with the teacher allowed for extension and revision of the activities, the interactive design process. The 7A class began the geometry unit approximately three weeks after the 7B class, allowing for further reflection and revision where necessary.

The reflection stage also allowed a thorough review of the students' successes and failures with the activities, so that adjustments could be made in the following sessions.

Documentation Stage

Following the collection of data, all sources were reviewed and tabulated. The video tapes were replayed many times as systematic summaries were written on observation grids to analyze the descriptive data on the students' behaviors, as they progressed through the activities.

In this phase the reliability of the observation summaries was affirmed through the review of selected episodes by a school system consultant experienced in junior high mathematics and computer education.

To illustrate the use of the observation grids, a representative episode of direct mode activity will be reproduced as Figure 4.

Turtle Tracking Phase

Group I
Topic Reflections
Activity # 16

Description of Task: Construct the following reflection. Begin at the mirror line to draw the original. Return to the starting point to draw the image.

Preplanning:

__ Discussion with partner
__ Sketching of expected outcome prior to input
__ planning in action (ongoing discussion as commands are input)
✓ paper pencil planning (preplanned command lists)

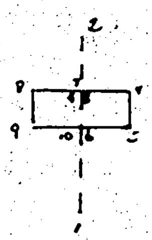
Unusual preplanning:

Difficulties with Understanding: Instructions, Geometry, Logo Programming

None

Activity Summary:

Diagram | Command List | Debugging



Repeat 4 [FD 20 PU FD 20 PD] → REPEAT 9 [FD 5 PU FD 5 PD]
PU BK 40
RT 90 FD 30 → BK 30 PD FD 30
RT 90 FD 20
RT 90 FD 30
RT 90 LT 90 → RT 90 PU FD 20
LT 90 FD 30 → BK 30 PD FD 30
LT 90 FD 20
LT 90 FD 30
LT 90 FD 20

Analysis used for determining angles or distances

Mentally calculated the distance to move back to the midpoint of the mirror line through the analysis of the repeat command

Mode of Operation: __ Trial and Error ✓ Input Preplanned Lists

Worked in direct mode

Assistance / Intervention required: * At the mirror line #(6) they planned to go straight to construct the image. They were reminded to go back to the starting point of the original.

Specific student behaviors: eg., Students' comments, properties mentioned or errors overlooked, work done as procedures.

Corrected their preplanned lists on an ongoing basis
Paul noticed the flaw in not using PU between #10 : 11

Task Completed

✓ Yes __ No ✓ Accurately completed **Time Involved** 6 min

Figure 4 - OBSERVATION GRID

EXPLANATION OF THE ENTRIES TO THE OBSERVATION GRID

Each separate activity was labelled an "episode". For example, in the Turtle Tracking phase of the reflection lesson the students were asked to construct four given examples of specified reflections in direct mode; each example constructed was labelled an episode.

A description of the task, as assigned to the students in the instructional materials, is included as a reference. Any preplanning done prior to computer work on the activity was recorded, this group had prepared a list of commands (hence paper and pencil planning).

Any difficulties in understanding the students encountered with the activity instructions, a lack of necessary geometry knowledge or a lack of fluency in the Logo language were recorded. This entry assisted in lesson revision. No such difficulties were evident in the reported activity.

A summary of the activity as executed is recorded in alternate forms. The preplanned list of commands were entered under command lists. The errors and omissions were noticed on input, and corrections were recorded as debugging. A diagram of the visual output was also included. The numbers indicate the order of production including any errors.

Most frequently the students guessed at the angle and distances thus any analysis or distance or angle measurement was recorded. Analysis generally involved the identification of an angle measurements through the recognition of a geometric shape or as in this case, the length of the mirror line was determined through an analysis of the repeat command

Preplanned lists were input in direct mode to indicate the predominant mode of operation. The researcher intervened to remind this group to make the original and image state transparent.

Students' comments and behaviors revealing definitions, mentioning of properties or indicating their level of thinking, useful in assessing the van Hiele level, were recorded. Hoffer's (1981) list of Basic Skills for each van Hiele level was used as a reference for the analysis of this entry. This group easily debugged their preplanned list on input thereby successfully integrated the two activities. Paul expressed concern about the flaw in leaving the pendown on the mirror line between 10 and 11. These entries show the students' ability to use appropriate names (mirror line, original, image), accurately draw figures and understand conservation in different orientations these indicate recognition or basic Level skills.

The episode was considered to be successfully completed if the finished figure closely resembled that given. Size differences between the assigned task and the screen construction were overlooked if the constructed original and image were congruent.

The analysis shown above was repeated for all recorded episodes for each pair of students. The items in the observations were summarized for each pair of students and then summarized over the five pairs. The episodes were then analyzed to reveal descriptors of students' perceptions and comprehension levels and to assign them to a van Hiele level of predominant reasoning using the level indicators outlined by Burger and Shaughnessy (1986) previously listed on page 13. Items recorded as checks on the grid were summarized statistically. Observations of the students' behaviors were also summarized but as brief descriptions for each group, displaying either trends in their behavior or key observations highlighting level of thought and or Logo use. The results are reported in the following chapter for each student pair over the four topics studied. Each summary chart contained the following categories of behavior:

- Started in Direct Mode
- Started as a Procedure
- Successfully Completed
- Episode abandoned before Completion
- Properties Mentioned or Discussed
- Trial and Error Construction
- Assistance Needed
- Debugging Done Visually Error by Error

Comparison charts were analyzed as a whole, by category and by intergroup comparison in order to understand the particular contributions of the Logo / motion geometry curriculum as experienced by these students.

3.6 SUMMARY

In accordance with the purposes of this study, the methodologies used were selected to test for the validity of the curriculum and then to allow a close study of effects on children. In correspondence with the former criterion:

- A detailed theoretical analysis of the van Hiele instructional propositions, of Logo in Turtle Geometry Mode and a parallel analysis of motion geometry was made. The resulting curriculum was matched against all three perspective.
- All students were given prior instruction which enabled them at a behavioral level to exhibit Logo techniques necessary and possible in this study.
- A teacher with no initial knowledge of Logo was given continuing instruction and was evaluated as very capable of carrying out the instruction in the study.
- The two classes were evaluated using a parallel for geometry test against the geometric achievement of the previous five years of grade seven students in this school. Further, they were evaluated against the achievement of all other grade seven Catholic school students in Edmonton on the geometry portion of the final exam.

The detailed instructional effects were studied using the following techniques:

- A selected group of 5 pairs of students had all of their Logo computer work transferred to video tape which tracked their in session verbal interactions as well.
- Video tapes were analyzed using the methodology of Pothier (1981).
- In the initial phase of each geometric topic and then after final completion of the instructions phase of the study students were interviewed with particular attention to their geometric ideas.
- Reliably coded behaviors were counted and categorized. These vectors of category scores were analyzed for indicators of growth and change, among categories, across time and among groups.

CHAPTER IV

PRESENTATION OF THE FINDINGS

It was the purpose of this research to explore and test the relationship between Logo use and the van Hiele theory of geometric learning. Observations of students as they progressed through the Logo / motion geometry curriculum developed for this study are the basis of the research findings presented in this chapter. Four topics, congruency, translations, rotations and reflections comprised the unit of study. An outline of the Logo / motion geometry activities for each topic is followed by observations of students' behaviors as they worked through the acitivities as well as by intepretations of the observation data.

The curriculum and observation data concludes with a summary of all groups Logo and geometry behaviors across topics for the procedure mode activities

Other included data are the students' achievement scores on the two postinstructional examinations and student responses to interview questions.

CONGRUENCY

4.1 INSTRUCTIONAL PHASES

A. DEMONSTRATIONS / DISCUSSIONS

Inquiry

The concept of congruency was introduced with students viewing demonstration procedures. Throughout the demonstration a teacher led discussion sought to elicit the students' understanding of the concept as well as to illustrate the following aspects of congruency.

a) Congruent figures were drawn in different orientations to illustrate congruent figures can be translations, rotations or reflection images of the original,
b) Similar figures of increasing size were used as nonexamples to emphasize the importance of equal size of congruent figures.
c) Two figures, differing only slightly, were used to emphasize

congruent figures must be the same shape.

d) Recursive procedures producing many different congruent shapes were used to initiate discussion and identification of congruent shapes.

B. DIRECT MODE ACTIVITIES

Turtle Tracking

Two activities introduced the concept of checking congruency, by tracing. Students were familiar with this as a paper and pencil strategy but had to be acquainted with using the turtle for this concept. Tracing could help reinforce the concept that congruent figures must be the same size and shape. This could be checked figurally by superimposing one figure on top of another and in a language sense by checking for identical command lists.

a) Students were given a picture (an arrow) on the screen, and the turtle was used to trace the shape. A record of the commands could be accessed by the text screen. When the tracing was complete, the list of commands was transferred to a procedure. By superimposing one image on top of the other, students could check for congruency.

b) The students viewed a rectangle on the screen and were asked to identify the correct command list to construct it. Congruency was checked by inputting the commands to see if one shape fit on the other.

C. PROCEDURE MODE ACTIVITIES

Congruent figures were used to create designs or patterns by repetition of the shape through the motions of translation, rotation or reflection. These motions were the topics of later study. The activities would help prepare students to realize the importance of congruency in these motions. Working with procedures not only would give students practice in identifying congruent figures, but also assist them in understanding how procedures include congruent shapes repeated many times.

Extensions

a) A given procedure for a regular hexagon constructed from six congruent equilateral triangles was input. Students were to design a composite-procedure to draw five congruent hexagons, either vertically or horizontally.

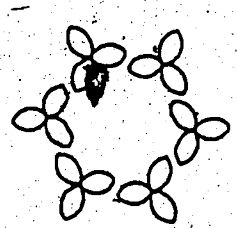
b) A procedure using the hexagon to create a snowflake pattern was given. When input this procedure was used to identify congruent shapes and served as a pattern for a composite procedure drawing many congruent shapes.

Project

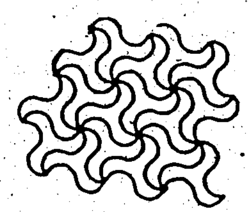
Two projects were included that involved repeated congruent shapes.

a) Two procedures were given as building blocks "ARC" and "TRILLIUM". Several trilliums were then to be joined as a garland (Figure 5).

b) Another two building block procedures "S" and "Propeller" were given. Interesting patterns could be fashioned by positioning several propellers (Figure 5).



(Burnett, 1982)



(Microquests, 1984)

Figure 5 - Congruency Projects

4.2 OBSERVATIONS OF THE SESSIONS

A. OBSERVATIONS - DEMONSTRATIONS / DISCUSSIONS

At the outset of the demonstration the students were familiar with the term congruency and were able to correctly identify congruent figures. Throughout the discussion individual students were asked to explain how they identified the congruent shapes or give definitions for the terms that arose in the discussion. The following descriptions were some of the students' responses.

TABLE 1 - STUDENT DESCRIPTIONS OF CONGRUENCY

In two of the four groups the students were not sure if congruent figures could be in different orientations.

The original is the "real" picture, while the image is the second picture.

Congruent figures must be the same shape and size. [This description was given in the latter part of the demonstration.]

During the Inquiry phase the students also discussed representation of the screen results and agreed that ten turtlesteps would represent the space between two dots on the dot paper.

B. OBSERVATIONS - DIRECT MODE ACTIVITIES

Group I

During the tracing students typically moved the turtle in small steps until the correct distance or angle was achieved. The two students in Group I used a distinctive strategy. After an erroneous trial in attempting to determine a 120^o angle, they always returned to the original heading and began afresh. All other groups used a trial and error strategy building from incorrect estimates rather than beginning a "drawing" again.

Group II

This group also used trial and error strategy. Only 90^o angles were recognized without trials. Although this group too worked in small steps, but before recording one command on paper their small steps were totalled mentally.

Group III

The simplest method, used by all groups to greater or lesser degrees, was recognizing geometric shapes, within the pictures, for which they knew the correct angles. This group in particular recognized the arrowhead as an equilateral triangle, recalled from previous introductory Logo activities. They used their knowledge of the repeat command and the 120^o angle in this new situation and eliminated the need for trial and error strategy.

For this topic, only Group III checked for repeated list patterns before they wrote their list as a procedure.

Group V

This group also recognized the triangle shape and had no need of trial and error strategy to determine the angle. Group V seemed particularly adept at building (by adding) the correct angles and distances to determine the correct measurement.

Summary of Congruency - Direct Mode

As can be seen in Table 2, in all seven episodes observed

Table 2 Topic - Congruency - Direct Mode Activities

Data Categories Displaying Number of Episodes	Student Groups I	II	III	V	Total
Total Episodes	2	1	2	2	7
Successfully Completed	2	1	2	2	7
Trial and Error Construction	2	1	2	2	7
Properties Mentioned or Discussed	0	0	1	1	2
Analysis used for determination of Distances or Angles	1	1	2	2	6

students used a trial and error strategy to discover the list of commands. Properties of congruent figures were mentioned only twice as the students worked. Students generally moved the turtle in small steps until the correct distance or angle was achieved, but without exception the recorded commands were written as summations of these small increments.

C. OBSERVATIONS - PROCEDURE MODE ACTIVITIES

Group I

Without preplanning, discussion or error, the students in Group I correctly wrote a procedure incorporating both the given procedure and the correct distances between congruent shapes to construct five vertical hexagons. They discussed, but did not complete, revisions to their procedure to eliminate screen wrap around.

They indicated uncertainty on planning their projects. Thus the location and orientation of the turtle after the execution of given procedures was determined with direct mode trial and error analysis. The necessary information on measurements of angles and distances were also checked. Accuracy of measurement was discussed as being important; in fact, the use of rulers and protractors was suggested, though not adopted. Following some preliminary exploration, a procedure based on the repeat command was conceived. Planning consisted of no more than ongoing discussion and visual results checks in immediate mode after the addition of every one or two commands.

After mastering one garland of flowers, they attempted to preplan a procedure to include two garlands. Still unsure of the new orientation and distances needed, they returned to the "planning in action" mode of operation.

Groups II and III

Groups II and III employed similar approaches to the extension activities. Both wrote their procedures without preplanning and checked the results visually step by step. The five hexagons were congruent but overlapping. Both groups used trial and error analysis to determine the accurate distance needed to move the turtle between each hexagon. After several attempts they were assisted, by way of leading questions, in looking for the necessary distance (the length of the side of the triangle, i.e. the apothem) within the hexagon procedure. They immediately made the connection and corrected their procedure. Group III proceeded to design and, with one trial, completed a procedure for a horizontal hexagon pattern.

Project work for Group II involved only the construction of a garland of flowers. They were unable to preplan without working first in direct mode. The garland was completely finished in direct mode before the list was transferred to a procedure. They searched for a repeatable pattern for the rotation and for the distance moved between flowers. After many unsuccessful attempts a reminder of the rule of 360^{0} proved useful to them in finding a solution.

Group IV

Group IV students work on the project was limited to direct mode. They never conceived of an overall pattern or plan that could draw these shapes repeatedly. Instead, commands were added one at a time and errors dealt with by clearing the screen and beginning again. At the close of the class period they had completed the propeller and had two flowers on the garland, but had failed to record many of their commands with the result that a procedure could not be defined. As they were aware of the role of congruency in these patterns, it was decided these projects would remain incomplete and that they would move to the next topic, translations.

Summary of Congruency - Procedures

This topic involved only two phases, extensions and projects

Table 3 Topic - Congruency - Procedures

Data Categories Displaying Number of Episodes	Student Groups				
	I	II	III	IV	Total
Total Episodes	4	2	2	2	10
Started in Direct Mode	1	2	0	2	5
Started as a Procedure	3	0	2	0	5
Successfully Completed	3	2	2	1	8
Episode Abandoned before Completion	1	0	0	1	2
Properties Mentioned or Discussed	1	0	1	0	2
Trial and Error Construction	1	2	2	2	7
Assistance Needed	0	1	1	1	3
Debugging Done Visually, Error by Error	1	2	2	2	7

emphasizing procedurally oriented work. Table 3 displays the data observed from the ten episodes of procedure activities on congruency.

Bearing in mind that the intent of these activities was to work in procedural mode, subject to each group's ability to cope, it is interesting to note that one half of the episodes were begun in direct mode. Another feature of this direct mode work is the number of episodes involving trial and error construction (7 out of 10) with no evidence of preplanning. Eight of the ten episodes were completed successfully (a figure demonstrating congruency appropriately).

4.3 COMMENTARY ON THE OBSERVATIONS OF THE CONGRUENCY TOPIC

The predominant behavior evident in all groups was a trial and error approach to construction. Functioning in direct mode is compatible with this strategy, but even when students wrote procedures they found ways to continue this approach. The only form of planning was ongoing discusion while inputting commands. The strategy expected was that students would proceed from direct mode to procedure when they had conceived an overall, workable, pattern and were able to preplan. Group I made strong attempts to operate in this manner but, on the whole found preplanning impossible at this stage. As a result procedures were built one command at a time with constant visual checking for the accuracy of each move. This inability to preplan is evident in the following observations:

- After each attempt at finding the correct angle, during the extension phase, Group I returned to the original heading .
- During the project activities, Group I's attempts at preplanning ended when they realized they were missing important information.
- At a procedural level, all groups worked only with small bits of information at one time and required frequent shifts between direct mode and the editor.
- When working at the project stage in direct mode, groups II, III and IV constantly cleared the screen and began from scratch.

INTERPRETATIONS OF THE VAN HIELE THEORY AND LEVELS OF LOGO USE

The foregoing observations can be interpreted within the following van Hiele descriptors;

Students at a basic level are capable of the reproduction of a figure without error (Coxford, 1978).

- As demonstrated in this topic, all groups were able to successfully complete most of the activities through trial and error strategy.

Students can only see the figures they are working with as wholes; they are not capable of seeing the relationships or parts (Coxford, 1978).

- Their clearing the screen after each error, inability to preplan and building through single commands clearly point to the students' method of coping with inability to isolate the shapes' components.

Along with this inability to see the parts of the figures comes an inability to analyze the geometric shapes on their own. They seem to see one shape entirely differently from another. (Wirszup, 1976).

- When directed students were able to seek out and identify the needed information through prompting. Left to their own devices, students simply used a guess and check system to determine unknown angles or distances.
- Frequently, even after the students had assistance in determining a measurement or a method of arriving at that measurement they could not independently apply that information to new situations. Each figure and situation seems to have been viewed independently.

TRANSLATIONS

4.4 INSTRUCTIONAL PHASES

A. DEMONSTRATIONS / DISCUSSIONS

Inquiry

A group demonstration was used to introduce the students to the concept of translations and generate class discussion. The discussion assisted in both illuminating the students' understanding of translations and highlighting key concepts of translations. The demonstration involved the following three approaches:

a) The components of a translation: the original, slide arrow and image were displayed and discussed.

b) Examples and nonexamples of translations were demonstrated. The nonexamples included rotations, reflections and incongruous original and image. Students were asked to identify examples of translations and discuss their reasons.

c) Students were asked to provide inputs involving variables for directions and distance to locate the image in specified positions on the screen. Although the teachers directed the demonstration procedures at the computer, students provided the required inputs after one or two examples.

B. DIRECT MODE ACTIVITIES

Turtle Tracking

This phase included the following three activities intended to prepare students to write their own translation procedures:

a) Students were to represent four given translation procedures on dot paper. To determine the size and distance, students could enter the edit mode and examine the procedures.

b) A regular pentagon procedure needing an input variable for size of the side was used by the students as the object to translate in direct mode. They were asked to produce three different translations and to record their commands. Using a procedure with a variable to draw the original and image was used to help students realize the two must be congruent.

c) Four incomplete or otherwise incorrect procedures as well as diagrams of intended outcomes were given for students to debug and complete. Typically, the original or image was missing, or the translation was in the wrong direction.

C. PROCEDURE MODE ACTIVITIES

Three phases of learning, Extensions, Turtle Excursions and Projects, emphasized work at the procedural level of translations. The activities within each of these phases were intended to allow students to explore the designing of a procedure to construct a translation. This exploration progressed from developing simple procedural lists for specified tasks to tackling self directed projects. It was the intent that student knowledge of the translation concept be expanded through this progression.

Extensions

The Extension phase of this topic involved two different activities.

a) Students were asked to develop procedures to reconstruct three given diagrams. The object of translation was a rectangle (Figure 7).

b) Students were to create their own design to use as the translation object. Then they were to define procedures to translate the design to three different screen locations.

Turtle Excursions

In the Turtle Excursion phase, students were asked to develop a general procedure capable of drawing any translation using variable inputs for the angle and of translation distance. In a teacher led lesson involving class discussion, the group defined a variable translation procedure.

Project

The topic of translations concluded with a project. Two procedures to draw circles (RCIRCLE drew a circle to the right and LCIRCLE drew a circle to the left) were given. Suggestions were given and students were to design procedures that involved translations. The most popular suggestion was to use a circle procedure to draw the following "slinky".

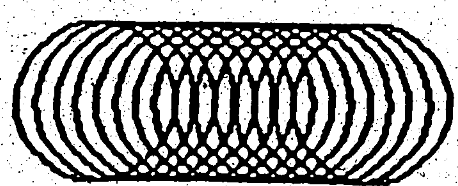


Figure 6 (Watt, 1984)

5 OBSERVATIONS OF THE SESSIONS

A. OBSERVATIONS - DEMONSTRATIONS / DISCUSSIONS

Discussion of the students' conception of a translation was encouraged throughout the Inquiry Phase. This discussion provided insights into the students' definitions for this motion. When asked how they had identified an example as a translation the most prevalent response was that "they just knew". However Table 4 below presents remarks expressed during the demonstration revealing students preliminary understanding of translations.

Table 4 - Representative Student Descriptions of Translations

Slides are like something is moved over.
Slides are like a bulldozer has moved the object over.
Slides always point in the same direction.
Slides always look the same, upright.

Although the terms <u>original</u> and <u>image</u> had been introduced in the previous topic, the students did not use them in any of their discussions.

During the final stage of the demonstrations the students were asked to predict the outcome of their inputs prior to viewing the output. Generally, they predicted distances much larger than the actual outcome. The students' choices of inputs were positive numbers that placed the image in the upper right quadrant of the screen. They had to be encouraged to use negative inputs to locate the image in other quadrants. Frequently a location was specified by the teacher and a student was asked to provide the appropriate inputs to slide the image to that position. Initially the students had difficulty providing the correct inputs, but they improved with practice.

B. OBSERVATIONS - DIRECT MODE ACTIVITIES

Group I

Group I students' discussion concluded the original and the translation image of the pentagon must be the same size, therefore the same value for the variable was necessary. This group was also able to produce an image at an oblique angle to the original, the only such occurrence. All other student examples were either vertical or horizontal.

In the third activity, involving the completion or correction of translation procedures, the entire command list was input immediately. Possible errors and missing information, were discussed and corrections made, line by line. Any uncertainty resulted in testing for accuracy in direct mode before editing the procedure.

Group II

At the outset, students in Group II quickly produced two examples but were unconcerned with either the size or the orientation of the original and the image. As they moved on to the next activity, they experienced difficulty. When asked to repeat this activity at the beginning of the following session, they were able to complete three examples accurately. In redoing the assignment, they readily discussed (when questioned) the importance of congruency of original to image, but still struggled to achieve the original heading before constructing the image. They eventually discovered that when they turned the turtle to produce the translation, they would have to "turn back that same amount".

Because of poor grounding in translations after their first attempt at Turtle Tracking, Group II students met with great difficulty debugging the given procedures. With assistance, they were able to complete three examples but not the fourth. Their problems completing the exercise seemed to lead to frustration which in turn, apparently escalated the number of typing errors. Unable to distinguish typing errors from incorrect or missing information they abandoned the activity. As previously mentioned the Turtle Tracking phase was repeated before they continued with the Extension phase.

Group III

Group III indicated hesitancy about overlapping the original and image, and as well their uncertainty as to the necessity of congruency. When asked to recall examples from the demonstration, they concluded that congruency was necessary and overlapping irrelevant. With this knowledge, they proceeded to complete three examples successfully.

Group III students also experienced some difficulty with the debugging activity. Like Group I, they input the entire command list and then studied the visual results. They could identify the errors but struggled in locating their position within the procedures. In the fourth example they were unable to use the visual results and switched to recreating the procedure one line at a time in direct mode.

Group IV

Initially, Group IV students experienced difficulty inputting a variable to execute the pentagon and six alternatives were attempted before success was achieved. In producing their first example they turned RIGHT 90 to produce a horizontal translation but neglected to return to the original heading before drawing the image. They described the results as "a sort of a twist" which they knew to be incorrect. Unsure of how to correct the "twist", they resolved their problem by producing all three translations in a vertical line. In the following session they were assisted in drawing a horizontal translation but were still unable to complete an oblique example without step by step guidance.

To complete the third activity of turtle tracking phase, Group IV students debugged the given procedures in direct mode by identifying the errors visually. Little attention was directed at the diagram labels of "O" for original and "I" for image and thus they often incorrectly focussed on the direction of the translation as the error and ignored other possibilities.

Summary of Translations - Direct Mode Activities

In the fourteen episodes observed, (see Table 5) all students used trial and error strategy to accurately complete eleven examples. The double entries for Group II indicate the repeated assignment.

Properties of congruency in the original and image were mentioned and successfully incorporated. No evidence of preplanning was observed.

Table 5 Topic - Translations - Direct Mode Activities

Data Categories Displaying Number of Episodes	Student Groups I	II	III	IV	Total
Total Episodes	3	2/3	3	3	14
Successfully Completed	3	0/3	3	2	11
Trial and Error Construction	3	2/3	3	2	13
Properties Mentioned or Discussed	1	0/1	1	0	3
Analysis used for determination of Distances or Angles	0	0/0	0	0	0

C. OBSERVATIONS - PROCEDURE MODE ACTIVITIES

Group I

This group approached both Extension activities systematically. A rectangle procedure was defined to act as both the original and image in the first activity and then used as a subprocedure in a composite translation procedure. Although they realized that the first example could be used as a building block for the second example, they did not use their procedure name but rather recopied their entire list of commands. Preplanning was not in evidence at this stage; plans arose through ongoing discussion. The third example, however, disclosed a significant change in strategy. Not only was it preplanned but it was composed of two distinct procedures, one constructing both the original and image and the other describing the translation. These were executed successively in direct mode.

For Extensions, activity B, Group I chose to translate an equilateral triangle. They completed horizontal and vertical examples easily. A oblique translation was attempted for the third example. The translation was initiated with a 45^{0} left turn; on completion they did not return to the original heading. The visual results drew the comment "that's not a slide it's a twist". Their solution has been described previously in Group IV's direct mode activities, the left 45^{0} was simply deleted and the translation became horizontal. Later discussion with this group generated a more appropriate solution to the problem.

While waiting for group instruction and discussion on the Turtle Excursions phase activities, Paul (Group I) was able to plan the following procedure independently.

```
TO SLIDE :A :D
TRIANGLE
RT :A FD :D
TRIANGLE
END
```

While observing a trial run, he quickly realized that once again the turn back to the original heading had been omitted.

During the Turtle Excursions phase, the 7A class looked at the idea of a variable procedure and together were able to develop a procedure through appropriate leading questions and explanations.

Many of the questions asked by the other students indicated a lack of understanding in the process or value of working with a variable procedure for a translation. On returning to their computer stations it was suggested they input and try out this variable procedure. Most students did not. They began the project activities which seemingly interested them more.

Paul was the exception. Through the instruction he was able to debug his procedure by adding a LEFT :A prior to drawing the image. At the computer he input and tried his procedure and was able to explain to a visiting teacher the importance he saw in using a variable procedure to draw any translation rather than defining new procedures for each different translation.

Group I project work began testing variations in circle size with input changes; only after they felt confident using the circle procedures were they able to plan. Their first two projects were preplanned and input without error. Project three, entitled the Olympic rings, required an oblique translation, a concept they had consistently had difficulty with. They were unable to preplan and began by writing long lists of commands. They jumped in and out of the procedure to continue with their trial and error planning. Unable to analyze the translation distances necessary from the information given on radius and diameter, they guessed at the distances repeatedly. A later discussion revealed them both to be aware of radius and diameter and when asked to use the measures, they could in fact calculate the distances. Although not in perfect position, they were the only group to complete an Olympic rings project.

Group II

Group II began their first Extensions activity by defining a rectangle procedure to act as the original and image, but their rectangle procedure was defined so that the turtle's beginning and ending position was in the upper left vertex. Previous rectangle procedures had always originated from the lower left vertex. They seemingly did not notice the difference and proceeded to plan their translation procedure. Understandably the results did not correspond with their plan. They would not consider changing the rectangle procedure, preferring instead to debug the translation procedure. The discrepancy between preplanned and the actual outcomes, arose in the following two examples as they never did reconcile the difference between the originating position of the rectangle and the length of translation between the rectangles.

Their original plan of using the rectangle procedure as a subprocedure to draw the entire motion was abandoned as they reverted to direct mode in their second example. At this point they began to write long horizontal strings of commands and repeatedly cleared the screen whenever a bug arose. This tended to be a time consuming process but when they did achieve a correct list of commands, a procedure was then defined.

Group III

Group III attempted to define a rectangle procedure to use as a subprocedure in their first Extension activity. Their planning consisted of discussion while defining the translation procedure. They too returned to direct mode when the visual results were unexpected and fraught with bugs. As in Group II's experience, this group wrote long horizontal strings of commands and cleared the screen repeatedly whenever a bug arose. When they finally acheived a correct list of commands they defined a procedure.

They realized that the translation procedure designed for the first example could also be used as a building block for the second example. An attempt was made to incorporate the procedure, not by name, but rather by retyping the entire command list.

When they attempted to add a fourth image to this example they lost sight of their planned use of the rectangle as a subprocedure and constructed the image one side at a time, ignoring even the very familiar repeat command.

To complete Extensions activity B, this group followed what was becoming for them an established mode of operation. The design they chose to translate was a regular hexagon. The turning angle to draw this shape was first sought through trial and error. After being reminded of the rule of 360° they easily arrived at the 60° angle. Preplanning was omitted as they entered direct mode and developed a list of commands that was later defined as a procedure. This group began their second example in procedure mode, but they quickly returned to direct mode when they became uncertain of the next move. When a successful list of commands had been developed, they were able to shorten this list using a repeat command. They then created a procedure, translating the original and creating five different images.

The two students in Group III worked on two projects in the same familiar routine. Pencil and paper planning led to direct mode testing from which they extracted a repeat command from their list and defined it as a procedure. As throughout this topic, commands continued to be entered in fairly long horizontal strings and visual debugging occurred one error at a time.

Group IV

As did the three other groups, Group IV also started the first Extension activity with the construction of a rectangle, but they chose to work in direct mode building a list one side at a time. This was subsequently altered to a repeat command and then to a procedure. This procedure was then used as a subprocedure within a composite translation procedure. This group also faced unexpected output and reacted with the quick return to direct mode operation. Using long strings of commands and repeated screen clearing to develop a correct list became their pattern of behavior. Group IV was unable to reach the point of actually defining a procedure during the first session. In the following class period, they were given guidance on designing a translation procedure. At this point they seemed to have an established notion of the pattern that would produce a translation and a procedure was preplanned. Initially the translation distance did not match the sides of the rectangle, but this was easily debugged.

Following the assistance given in activity 1, Group IV had no difficulty in preplanning the procedure to slide their own design. The procedure omitted the original but was easily debugged on viewing the ouput. This translation procedure effectively incorporated a subprocedure.

Summary of Translations - Procedure Mode Activities

Table 6 displays the results of forty episodes of procedure work on

Table 6 Topic - Translations - Procedures

Data Categories Displaying Number of Episodes	Student Groups I	II	III	IV	Total
Total Episodes	12	9	9	10	40
Started in Direct Mode	1	7	4	6	18
Started as a Procedure	10	3	5	4	22
Successfully Completed	11	8	9	10	38
Episode Abandoned before Completion	1	1	0	0	2
Properties Mentioned or Discussed	4	0	1	1	6
Trial and Error Construction	5	9	4	2	20
Assistance Needed	2	4	1	1	8
Debugging Done Visually, Error by Error	10	8	9	10	37

translations. In almost half of the episodes (18 of 40) the students chose to work first in direct mode. Even in those episodes attempted initially as procedures, the students either returned to working in direct mode or worked visually with procedures by adding and debugging one command at a time. This was the predominant strategy (37 of 40 episodes) and is apparently similar to working in direct mode. Any mention of properties was incidental and noncomprehensive, yet approximately seventy-five percent of these episodes were done accurately.

4.6 COMMENTARY ON THE OBSERVATIONS OF THE TRANSLATION TOPIC

INTERPRETATIONS ON THE VAN HIELE THEORY

BASIC LEVEL

It was in this topic that the students first encountered formal instruction on transformation geometry. Previous interpreted observations on the congruency lessons indicate students entered this topic functioning at a Basic level of understanding, seeing the objects only as wholes. This was verified again in the Inquiry Phase of this topic as students were able to do little more than identify translations and could give no explanation of their identification process. However, the limited definitions they gave included use of imprecise qualities and visual prototypes, both attributes identified as level indicators in Burger's interviews (1985 p 2).

For example,

Slides are like something is moved over.
Slides are like a bulldozer has moved the object over.

During the turtle tracking phase students' behavior again substantiated the following Basic Level indicators.

Students at a Basic Level are able to understand conservation of shape in different orientations (Hoffer 1983).

- Evidence of this arose with Groups I and IV when they immediately recognized and discussed the concept of congruent original and image and the necessity of identical inputs to the variable procedure.

Burger (1985) further describes students operating at the Basic Level as having an imprecise concept of the properties of the figure. They frequently ignore relevant attributes and concentrate on irrelevant ones.

- Group II gave no independent consideration of the relevance of congruency or orientation as they produced translations in immediate mode even though they had seen many examples displaying these characteristics.
- Group III needed assistance to overcome their misconception that overlapping figures could not form a translation and to focus on congruency as an important characteristic.
- Group IV ignored relevant information in the given diagrams when they debugged and completed procedures.

A student at the Basic Level can imitate correctly but has no view of his own activity until he has reached a new level (Wirszup, 1976).

- The observation summaries show the students accurately completed most of the assigned tasks predominantly through direct mode, trial and error initiatives. Little preplanning at this stage predicated a step by step drawing or copying mode of operation for these students.

The Transition To Level I

Instruction, discussion and assigned tasks in each phase of this topic exposed the classes to the components of translations and to the relationships between the components. Observation of the students' attempts to assimilate this knowledge as they progressed through the phases revealed the beginning of movement towards Level I thinking. A transition between Basic Level understanding and that of Level I begins when students experimentally establish properties of the figures through observation, measurement and model making (Wirszup, 1976). Informal analysis of the figures perceived is necessary to establish these properties. The following observations allow for further descriptions of this transition.

- Each group conceived a plan to use one procedure to represent both original and image within a composite procedure which included the motion.

The preplanning of composite procedures indicates the students were able to recognize both the component pieces of a translation and the congruent relationship of the original and image. It was also necessary that they understand the translation relationship between the original and image.

- This relationship was a source of difficulty for each of the groups. Their preplanned procedures defining the actual motion all involved bugs.
- All groups responded to bugs in their procedures with a return to direct mode functioning.
- Group II made an important discovery when they realized that had to "turn back the same amount" before constructing the image. This discovery was made in the Turtle Tracking phase and they consistently used this information in constructing their procedures to draw horizontal translations.

A return to direct mode allowed the students to continue with a trial and error approach; it was a return to functioning at the Basic level where the translation was visually recreated step by step. But, the return to direct mode after preplanning seemed to hinder the trial and error process as they attempted to incorporate some of their preplanned lists.

- All groups began to use long horizontal strings of commands. Students had difficulty distinguishing typing errors from incorrect or missing information. Visual debugging became a difficult task.
- Through this process Group III appears to have lost sight of their initial plan to use a single procedure to represent both the original and image. The final image was created by copying or drawing one side at a time.
- Each of the groups repeatedly encountered the same errors and only by returning to direct mode were they all able to produce vertical and horizontal translation procedures by the Project phase.
- Groups I, II and IV were able to preplan and execute correct procedures, while Group III continued to establish their list of commands in direct mode before defining a procedure.

- The plans of Groups I and IV to create an oblique translation were altered to a vertical translation when they encountered bugs they could not deal with.
- Group I had to struggle with the relationship between the components when constructing an oblique translation in every phase. They eventually were able to define procedures involving this relationship but required some guidance in the process.

Each level has its own language, its own set of symbols and its own network of relations uniting these symbols. If the students have not yet reached a new level of thought, instruction at the new level is not likely to be understood by the students. The students may accept the explanation of the teacher, but the subject will not be internalized (Wirszup, 1976). Observations from Turtle Excursions phase of this topic illustrate such a situation.

- During the Turtle Excursions phase the students were encouraged to define a variable procedure to be used to construct any translation. Most of the students were able to construct this procedure only when prompted every step on the way.
- Questions asked by some of the students revealed their lack of understanding of how a variable procedure worked.
- Most of the students were unwilling to even try the variable procedure at the computer, indicating a lack of understanding of the advantages of using such a procedure.
- Paul's grasp of this concept seems to indicate his comprehension at a higher level than the other students in the class. He was able to use the instruction to clarify his understanding of the concept and independently debug his own procedure successfully.
- Paul demonstrated an understanding of the usefulness of a general procedure and more, an ability to articulate his understanding.
- This jump in levels was limited to this occurrence. Perhaps a teaching opportunity was missed which could have extended these insights in the subsequent lessons. The predominance of Basic Level thinking throughout the class seems to have impeded Paul's potential progress.

ROTATIONS

4.7 INSTRUCTIONAL PHASES

A. DEMONSTRATION / DISCUSSIONS

Inquiry

Students were introduced to rotations through a group demonstration; a dialogue between the students and teachers was encouraged throughout the demonstration. This dialogue served to identify for the teachers the students' prior familiarity with, and understanding of, the topic and to highlight those concepts the students found confusing or unfamiliar.

Rotations were demonstrated in the following three ways:

a) The following components of rotations were displayed and discussed: original, image, rotation center, and rotation arrow (including size and direction).

b) Examples, and nonexamples, of rotations were displayed. Nonexamples included translation and reflection images as well as incongruous figures. During the identification process students were encouraged to discuss their reasons for their identifications.

c) To allow students to explore the rotation concept and its components, they were provided with rotation procedures requiring student inputs for rotation and distance from the rotation center.

B. DIRECT MODE ACTIVITIES

Turtle Tracking

Students were asked to create rotations in direct mode using a simple figure (a rectangle was suggested) to serve as the original and image. Students were asked to complete three different examples and record their commands. It was hoped that having students create lists of direct mode commands would prepare them to define rotation procedures in subsequent phases and would also allow for further exploration of the relationships between the components introduced in the inquiry phase.

. PROCEDURE MODE ACTIVITIES

The activities of the Extension, Turtle Excursions and Project phases of learning were primarily intended to provide experience in defining procedures to construct rotations. This experience would also allow for geometry development in the following areas: correctly identifying rotations, realizing command errors in such identification, and comparing and contrasting location and orientation of original and image, thus illuminating some basic properties of rotations. Through this process students would develop their own definition of rotations. A description of each of the activities is presented.

Extensions

a) Students were asked to write a procedure to draw a simple design (a flag, rectangle or triangle, for example) and then use this design as the original, in defining three different rotation procedures. The rotation center was to be included. The distance from the rotation center and the degree of rotation were to vary.

b) Students were to fill in the missing information on two incomplete procedures. Diagrams of the completed rotations were included to assist students in determining the missing information.

Turtle Excursions

a) Patterns were created using a rotation procedure. A subprocedure constructing a square served as the object of rotation. Students initially had to provide missing information for the given procedures and then explored the effects of altering the degrees of rotation and the number of images.

b) A tail-end recursive procedure was also explored as an alternative to constructing a rotation. Again students needed to supply missing information in the procedures provided for them and then explore changes to the degrees turned or the figure used as the object of rotation.

Project

Choosing from four projects, students were given recursive procedures to use as building blocks in a composite rotation procedure. The students were able to use the given recursive procedures, however their understanding of embedded recursion was simply knowing the variable would increase or decrease to a set limit at which point the turtle would return to the starting point. Here is an example of one procedure and the project (Figure 7) it was to be used in.

```
TO TRI :S
IF :S<5 THEN STOP
FD :S RT 118
TRI :S-2
LT 118 BK :S
END
```

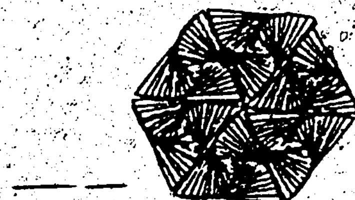


Figure 7 (Moore, 1984)

4.8 OBSERVATIONS OF THE SESSIONS

A. OBSERVATIONS - DEMONSTRATIONS / DISCUSSIONS

The students' initial understanding of rotations was uncovered through the student/teacher discussion prompted by the demonstration procedures. The students were unfamiliar with the center of rotation as well as with the concept of a rotation arrow representing the direction and amount of rotation.

Generally, the students could correctly identify the examples of rotations from nonexamples but were unable to explain their identification criteria. However, the following student remarks expressed during the demonstration illustrate their conception of rotations.

Table 7 Student Descriptions of Rotations

Turns are in a circle.
A turn center that is not away fom the original [and image] is not sensible.

In one of the demonstration procedures an original flag was constructed. Then the distance from the rotation center and the amount of rotation in degrees were asked for as inputs before the rotation image was completed.

Students quickly entered into a spirit of experimentation, providing inputs for the demonstration. The following examples described typical explorations.

The angle for a three-quarter turn was discovered through trial and error; one student immediately suggested that an input of LT -90 would give the same results.

Students began to experiment first with the full turn, then used inputs of 359^{0} and 361^{0} to see the effect. They discovered that some inputs are better than others for transfer to dot paper and the students tried many before representing the motion on dot paper. For example, a 90^o rotation was found easier to transfer than a 60^o rotation.

Two students felt that a turn center that was 1 or fewer turtlesteps from the original was possibly correct but not sensible.

OBSERVATIONS - DIRECT MODE ACTIVITIES

One student group was unable to complete the research project and the recorded observations of this group were omitted. The direct mode observations on rotations has been limited to three groups.

All three groups required guidance in completing the first example. Initial guidance consisted of asking students to discuss orally the steps involved and to plan a diagram on dot paper of the completed motion. This helped Group I but the students in Groups II and V were unable to position the image. They were encouraged to draw a rotation step by step in direct mode and further assisted through directed questions as illustrated in the following excerpt of conversation between the researcher and two students, Donna and Mary. The students' responses are capitalized.

> "What would you do first? DRAW THE ORIGINAL (Donna) [the original was then drawn on the screen]. What now? DRAW THE TURN CENTER (Donna). Do you want to draw the turn center on the original? NO WE WOULD HAVE TO MOVE BACK (Mary) [the commands BK 10 were then issued]. Do we want the line visible? NO, WE SHOULD HAVE USED A PU (Donna) [corrections were made]. Now we need? A TURN CENTER (Mary). [To draw the dot, first they tried to use the period key, then they tried typing in the word DOT]. Why doesn't that work? WE WOULD HAVE TO TELL THE COMPUTER HOW TO DO THAT FIRST (Mary). WE COULD MAKE A VERY SMALL CIRCLE (Donna). Yes that would work but there is an easier way. [A long pause]. MAYBE WE COULD MAKE JUST A TINY LITTLE LINE. YEAH, THAT WOULD WORK [they used the commands FD 1]. WE WILL HAVE TO HIDE THE TURTLE TO SEE IF THERE IS A DOT; YES THERE IS...! (Donna)"

Group I

After completing the first example with assistance, Group I students attempted to construct their second example as a procedure which they named "TO TURN", but demonstration procedures pre-existing in the computer memory included a procedure using the same name. Confused by this, they resorted to direct mode. Their second example was a replica of their first list of commands changing only the rotation from a RT 90 to a LT 90. They realized and discussed the similarity of the two examples, but decided that they had acceptable solutions to the assigned tasks.

Group II

Group II students' first construction used a LT 90 rotation, but the original and the image were unequal distances from the rotation center, something which they did not recognize as an error. They used this list of commands to produce a second example in which they used a rotation of RT -90 and were very pleased that they had created the same rotation. For their third example, they altered the amount of rotation to 180^0 in the preceeding list of commands. Viewing the results, they realized the original and image were not equidistant from the rotation center and that a change in distance did not appear correct to them. After discussion they concluded this was and error and were able to correct it.

Group V

After establishing a list of commands for a rotation in example 1 they easily produced two more examples changing the rotation in the first example and the distance from the rotation center in the second.

Summary of Rotations - Direct Mode

Once again a trial and error strategy was employed by all groups.

Table 8 Topic - Rotations - Direct Mode Activities

Data Categories Displaying Number of Episodes	Student Groups			
	I	II	V	Total
Total Episodes	2	3	3	8
Successfully Completed	2	1	3	6
Trial and Error Construction	2	3	3	8
Properties Mentioned or Discussed	0	1	0	1
Analysis used for determination of Distances or Angles	0	0	0	0
Use of Procedures	1 (attempt)	1	0	2

All three groups required guidance in completing the first example, then easily went on to complete two additional examples. Table 8 summarizes the students activities.

C. OBSERVATIONS - PROCEDURE MODE ACTIVITIES

Common Observations

During the first activity of the Extension phase all groups were able to preplan and to work in procedure mode. Each group defined three very similar procedures. Even though students worked at a procedural level, errors were still debugged one at a time after viewing each output. Groups I, II, and III erred in differently distancing for the original and image from the rotation center. Through this error these students came to realize and discuss the rotations' property of equidistance.

All groups worked on at least two of the four projects. Projects 1 and 2 were easily identified as turns by all groups although some exploration was necessary to arrive at the correct rotation angle was necessary.

All groups attempted project 3 (Figure 8). The procedure developed in project 1 was identified as the center to which points were to be added.

Project 1 Project 3

Rotations Projects - Figure 8

All groups experienced similar problems in moving the turtle the correct distance and angle to add the points. Although they all continue to work in procedure mode, they began adding one command and then visually checking its accuracy. No group was able to complete this project.

Group I

In project 1, a hexagon was constructed by rotating a recursive triangle procedure six times. Group I had just previously arrived at the correct angle of rotation to create a hexagon in the Turtle Excursions phase. In completing project 1, a 60^o rotation angle was again determined through trial and error. Group I approached this through an interesting sequence. They first chose a 20^o angle, which, being too small, caused the triangles to overlap. To remedy this they enlarged the recursive triangle, with a similar result. Suddenly they realized that size was not the important element and began trying different angles and eventually arrived at a rotation of 60^o.

Using the procedure developed in project 1 as a subprocedure in project 3 actually caused Group I some difficulty. Their first project used the input for the size of the recursive triangle. This input was also the distance required to move to the vertex of the hexagon in project 3 and could easily be determined by referring to the procedure for project 1. However, this group was unable to trace this information through subprocedures and switched instead to direct mode and trial and error strategies. Once they had confirmed that distance, they began to define their procedure. They reverted to the slow process of adding one move, checking its accuracy in direct mode and then repeating the process. They were convinced that there was a repeatable pattern that would add all the points to make a star, but after many unsuccessful attempts they resorted to placing each point individually.

Group II

Conflict arose in Group II during the Extension activities over the use of procedures as a subprocedures. Previously their strategy had been to recopy the command list, but now Mary realized and eventually proved to Donna, that only the procedural name was necessary within another procedure. They had successfully worked with this concept in prior Logo instruction.

They also showed confusion in matching rotations represented by fractions with the correct number of degrees. A 1/4 turn was thought to be 45^o and a 1/2 turn 90^o. This confusion became more evident when they worked on completing the procedures, the second Extension activity.

knew visually that a half turn was needed but their input of RIGHT 90 did not give the expected outcome. They fell to guessing the necessary 180 angle.

In the Turtle Excursions phase, only Group II independently and successfully used the rule of 360° to change the number of repeats and the angle to complete the pattern. The other groups, as they altered the existing numbers, failed to match the number of repeats with the rotation angle and thus the design either overlapped or did not complete a full rotation.

Group II though was unable to transfer their successful use of the rule of 360° into the Project phase. Even after using the 60° to produce a hexagon previously, this group found it necessary to rediscover the angle through trial and error.

Group III

Initially, Group III seemed unaware of their error in using unequal distances from the center of rotation to both the original and image in two examples using a 90° rotation. A 180° rotation seemed to make this discrepancy more apparent to them. When this error was discovered they did not debug the existing procedure but rather began again defining a new procedure. This discrepancy did not become sufficiently apparent to them to warrant debugging their first two examples.

Group V

Without preplanning, Group V was able to write three rotation procedures correctly using a triangle as the object of rotation during the Extension phase. Finishing far ahead of the class, they were challenged to include more than one rotation in their procedures. The result was a procedure that rotated six triangles constructing a hexagon. The number of repeats and the angle of rotation for this procedure was arrived at through trial and error. In defining the rotation procedure the entire repeat command constructing the hexagon was incorporated as the original and a rotation center exterior to the hexagon was established, but a single triangle was used for the image (also involving a repeat command). On viewing the output they expressed concern about somehow "losing a procedure" and were unable to determine why the hexagon had been omitted. Assistance was needed to resolve this problem.

During the Project phase this group completed projects 1 and 2. In both cases they wrote procedures on their first attempt, having perceived the correct angle of rotation on sight.

Summary of Rotations - Procedures

The observations of twenty six episodes of procedure activities are displayed in Table 9.

Table 9 Topic - Rotations - Procedures

Data Categories Displaying Number of Episodes	Student Groups				
	I	II	III	V	Total
Total Episodes	8	8	3	7	26
Started in Direct Mode	0	1	1	2	4
Started as a Procedure	8	7	2	5	22
Successfully Completed	6	5	2	6	19
Episode Abandoned before Completion	2	1	1	1	5
Properties Mentioned or Discussed	2	2	0	2	6
Trial and Error Construction	2	2	2	3	9
Assistance Needed	1	0	0	1	2
Debugging Done Visually, Error by Error	3	4	2	2	11

The significant number of episodes beginning as procedures (25 out of 26) evidences students' much improved ability to work at a procedural level by the time they have reach the later phases of this topic. Only nine of these involved trial and error planning, and students were able to complete sixteen of these episodes either easily without any planning or were able to preplan their procedures. The visual debugging (continuously jumping into direct mode to view results) was still evident although students completed more procedures correctly on first input.

4.9 COMMENTARY ON THE OBSERVATIONS OF THE ROTATIONS TOPIC

INTERPRETATIONS ON THE VAN HIELE THEORY AND LEVELS OF LOGO USE

Basic Level

Interpretation of the observations indicate the students entered the rotations topic at the Basic level of understanding. They viewed rotations as wholes and were able to correctly identify rotations. The following observations support this interpretation.

- The only means of identification suggested by the students ("Turns are in a circle") was impreoise and strictly visual (Burger, 1985).
- In the initial direct mode activity they were unable without prompting to separate the components to find an appropriate starting point.
- For the most part they were much more able to incorporate the correct angles and distances through visual recognition based on previous experience than was seen in earlier lessons.
- Group II confused the rotation angles and their corresponding fractions. On realizing the outputs were visually incorrect they immediately altered their angle. However, they seemed unable to recognize or reassess the source of the problem.
- A hexagon produced through the rotation of a triangle was used in both the Turtle Excursions and the Project phases. However the triangle used in the Project phase used a recursive procedure and fills the interior. To Groups I, II, and III one shape appears entirely different from the other and they are unable to view the similarity and tranfer the angle of rotation without reconfirmation through trial and error.

Conservation of shape in different orientations (Hoffer, 1983) seems to be a Basic Level concept mastered by these students.

- The subject of congruency between the original and image did not arise, but was evident in their work and seems to have been obvious to students at this point.

Transition to Level I

Wirzup (1976) described students at the Basic Level of understanding as one able to imitate correctly but having no view of their own activity until they have reached a new level. In the direct mode activities of this topic the students have begun to imitate, analyze and alter their own lists of commands.

- Every group altered their initial list of direct mode commands, changing either the rotation angle or the distance from the rotation center. They moved beyond using the turtle as a step by step drawing tool and showed insights involving the aspects of the language reflecting the changes they envision.

Their ability to distinguish the rotation components within the command lists was further extended in the procedural phases. Experience with these elements and the relationship between them established the basis of Level I thinking for these students.

- Attempts at designing a rotation procedure began as early as the Turtle Tracking phase for Group I.
- Once again all of the groups were able to preplan a composite procedures to represent a rotation generally involving a subprocedure to represent the original and image by the Project phase.
- The relationship between the object of rotation and the actual motion was a source of difficulty for all groups.
- Group II did not initially recognize that equal distancing of the orignal and image from the rotation center was a property to be considered. The property came to their attention visually when attempting an example involving a 180° rotation. Even though they recognized this property in this instance, they failed to relate the information to their previous, incorrect, examples. The other groups encountered similar experiences throughout the topic.
- Project 3 involved two distinct rotation patterns. All groups were easily able to complete the interior rotation pattern but were unable to perceive the exterior rotation relationship involving the object of rotation being moved away from the turn center. Although the students remained in procedure mode they

regressed to adding individual commands, visually checked in direct mode. They seemed unable to analyze in preplanning or reflecting on, the relationship between their building blocks and repeatedly guessed at the needed distances and angles.

The transtion to Level I understanding is also reflected in the students almost exclusive approach to activities at a procedural level by the Extension phase. As procedure work became more routine by the later phases of this topic, subprocedures began to enter the students' focus of attention as they began (not with out difficulty) to incorporate subprocedures in their rotations procedures. This progression in Logo use is illustrated in the following examples:

- Group II discussed and in order to settle the resulting disagreement, needed to verify the use of a subprocedure identified by name within another procedure.
- Locating the required information in a previously input subprocedure was not possible for Group I. Unable to locate the necessary input for distances they returned direct mode and guess work.
- To use a procedure as a subprocedure, Group V incorporated the whole list of commands rather than simply referring to the procedure name.

REFLECTIONS

4.10 INSTRUCTIONAL PHASES

A. DEMONSTRATION / DISCUSSION

Inquiry

The concept of reflections was introduced as students viewed and discussed demonstration procedures as small groups with the teachers. Both familiar and unfamiliar concepts, plus the students understanding of reflections could surface during these discussions.

The demonstration procedures illustrated reflection concepts the following ways:

a) Components of reflections (original, image and mirror line) were displayed and discussed;

b) The identification process for reflections was highligted by displaying of examples and nonexamples. A discussion by the students' of their reasoning was encouraged as they identified reflections;

c) Experience in working with the components and the relationship between the components was provided through reflection procedures requiring inputs for the angle of the mirror line and for the distance of the original and image from the mirror line.

B. DIRECT MODE ACTIVITIES

Turtle Tracking

The direct mode activities were meant to allow students to explore reflections independently so that they could arrive at their own command lists. Four examples of simple reflections were given as diagrams; students were asked to construct command lists to recreate these reflections. They were instructed to first draw the mirror line and then construct state transparent originals and images.

The students were asked to preplan their command lists before moving to the computer. Preplanning was encouraged to assist the students in differentiating between the command lists for the original and the image.

In a follow-up activity students were to choose two of their direct mode lists of commands and define them as procedures.

C. PROCEDURE MODE ACTIVITIES

The procedure activities were intended to provide the students with experiences promoting an understanding of the reflection motion as a composite of individual parts. Experience with the activities would allow students to further explore the identification, construction and differentiation of the original, image and mirror line as elements that must be combined in a particular way to create an accurate reflection procedure.

Examples of procedure activities implemented in the Extension, Turtle Excursions and Project phases are outlined below.

Extensions

What follows exemplifies the Extension activities. Students were given the following procedure constructing the design "the original".

```
TO ORIGINAL.DESIGN
FD 30 RT 120 FD 20 LT 120
FD 50 RT 120 FD 40 LT 120
FD 60 PU BK 60
RT 120 BK 40 LT 120 BK 50
RT 120 BK 20 LT 120 BK 30 PD
END
```

Constructing this:

Figure 9 (Moore, 1984)

and were instructed to create two procedures, one constructing the reflection image, the other combining the original and the image. These procedures were designed in the classroom setting prior to working in the computer room. After successfully completing these two procedures, they were to alter their combination procedure to include a mirror line. Three activities of this nature were given.

Turtle Excursions

Students were asked to define procedures constructing reflections with mirror lines orientated at 45^{o} angles rather than vertical or horizontal.

Project

The project phase included three examples. Students were given recursive procedures to use as building blocks in reconstructing the reflection motion. The students level of understanding of embedded recursive procedures was limited to knowing the variable increased or decreased to a set limit and the turtle then returned to the starting point. Then they were asked to complete procedures to draw the reflection images.

For example:

```
TO TRI :S                                    TO FLIP.TRI :S
IF :S<5                                      IF :S<5
FD :S            is changed to this          ____________
RT 118           and then completed:         ____________
TRI :S-2                                     FLIP.TRI :S-2
LT 118                                       ____________
BK :S                                        BK :S
END                                          END
```

(Moore, 1984)

After completing FLIP.TRI, these procedures were to be used as the original and image in a combination procedure that could also include a mirror line.

4.11 OBSERVATIONS OF THE SESSIONS

A. OBSERVATIONS - DEMONSTRATION / DISCUSSIONS

Insight into the students' existing conceptions of reflections were provided through discussion centered around the reflection demonstration procedures. Students could easily identify reflections and expressed the following comments when asked to explain how they could identify them.

Table 10 - Representative Student Descriptions of Reflections

Flips are in opposite directions
The mirror line is the place where the flip occurs.
Flips and reflections are not the same thing. A flip does not have a mirror line and a reflection does.

As is evident in their descriptions, the students used the term "flip" and seemed unfamiliar with the terms "reflection" and "mirror line". They seemed easily able to transfer their knowledge of the terms "original" and "image" from previous topics to the appropriate components within the reflections.

Not until this topic did a student suggest the use of a negative input prior to initiation by the teacher. This student also correctly predicted the negative input would position the mirror line in a reverse orientation to that which occured with a positive input. For example:

Original Image Original Image

Positive Input Negative Input

Figure 10

B. OBSERVATIONS OF THE SESSIONS - DIRECT MODE ACTIVITIES

Group I

The two students in Group I were able to use their preplanned lists. To complete their first example, some trial and error visual screen debugging was used to make corrections. In their second example, the preplanned lists were corrected prior to input using the knowledge just acquired. There was some experimentation to determine the correct angle of the object in relation to the mirror line. On completion of the original, they reversed their commands to arrive back at the starting position. No other students employed this technique, continuing instead to move forward, determining distances and angles through trial and error.

Group I totally preplanned the third example using subprocedures, labelling the procedure for the original "A", and the procedure for the image "B". The original and the image were input and debugged separately, then the mirror line was added. This was the only group that analyzed the repeat command and the distance they wanted to move back before beginning their original.

Asking students in Group I to preplan direct mode activities proved a valuable experience for them. For students in Groups II, III, IV and V, preplanning resulted in a totally different experience.

Groups II and III

Although all of the students did write a list of commands representing their thoughts on the correct path for the turtle, these two groups were unable to group these commands in any way. As a consequence, they wrote long, horizontal strings of commands, often covering two lines. Such lists frequently contained a single error. Then, the entire list would have to be input again and again, as is necessary in direct mode. It was also very difficult for the students to locate any geometric errors in these long strings. This premature pre-planning appeared to lead to frustration and confusion. Quickly responding to the frustration, students started from scratch, ignored their preplanned lists and reverted to the familiar trial and error strategy.

Group II students attempted to complete the original without returning to the starting position of the original. They found this a difficult task and and were assisted with making their command list state transparent. From a visually recognizable error, they noted, that the distances from the mirror line to both the original and the image must be equal.

In the 7B class, one student suggested it would be beneficial to write a procedure to draw a mirror line. Previously, they had input a repeat command in direct mode whenever they wished to use the mirror line. All students immediately felt it advantageous to develop such a procedure and enthusiastically set about planning and inputting commands for one. Group III sucessfully wrote, input and verified their mirror line procedure but on returning to their assignment, they regressed to the repeat command in direct mode rather than using their mirror line procedure.

Group IV

Group IV had similar difficulties caused by overly long strings of direct mode commands but they resolved these problems by switching to procedure mode, where they jumped from the editor after adding only one or two new commands, to immediate mode to check the accuracy of the resulting image. They in effect, employed the same trial and error strategy they had found useful in direct mode. Within the procedures the command lists continued to be long, horizontal, strings.

Group V

In the one episode observed Group V used the long horizontal strings of commands. They also attempted to use a procedure to draw the mirror line. In fact they went further, inputting procedures for both horizontal and vertical mirror lines named "M.L.1" and "M.L.2". However, they could never remember which name represented which procedure. After frequent bad guesses, each necessitating debugging, they returned to the repeat command in direct mode.

Summary of Reflections - Direct Mode

Table 11 displays a summary of the data on 11 direct mode episodes on reflections. In direct mode the most salient feature

Table 11 Topic - Reflections - Direct Mode Activities

Data Categories Displaying Number of Episodes	Student Groups I	II	III	IV	V	Total
Total Episodes	3	3	2	2	1	11
Successfully Completed	3	1	1	1	0	6
Trial and Error Construction	0	2	2	2	1	7
Properties Mentioned or Discussed	2	1	1	1	1	7
Analysis used for determination of Distances or Angles	2	0	1	0	0	3
Use of procedures	2	0	0	1	0	3

of this topic was the effect of preplanning on the various groups. Group I was able to use the preplanning effectively. They were able to see the reflection motion as a composite of building blocks that could be first be broken down and then put together or grouped as procedures and subprocedures. Although the other groups were involved in preplanning, they were unable to use their preplanned lists. In fact their preplanning seemed to hinder them; with command lists input as long horizontal strings, making debugging an onerous task. This is reflected in the limited number of episodes successfully completed.

C. OBSERVATIONS OF THE SESSIONS - PROCEDURES

Group I

Scheduling concerns limited the class time available and as a result Group I students completed only the three episodes of the Extension activities. During the Extension phase these students easily designed procedures for the images as a preplanned activity and discussed the opposite angle direction of originals and images. They also input all three as procedures before checking their results. Two types of errors were apparent; one was typing errors on input, while the other originated when the image procedure was designed. The students had neglected to reverse some of the turns. As soon as they realized this particular error they debugged every procedure, before checking the output, in the following manner. First checking the accuracy of the procedures they had preplanned, they then checked for errors in procedures in the editor. All three activities then worked without error. Planning was again implemented when they began to add the mirror line. First they ran only the original procedure for the second example to check on the turtle's final position. The designs were all state transparent, beginning and ending at the bottom right. They wrote a procedure drawing the mirror line and then added that procedural name to the combination procedure between the procedures for the original and image. On viewing the results they realized the mirror line procedure was not state transparent. They discussed the necessity for the original and image to be opposite one another and quickly debugged by adding BK 100 commands to the combination procedure. They ran into the same difficulty doing a second example and resolved it in a similar fashion. The problem was circumvented in their third example by changing the composite procedure in which the mirror line was drawn after the original and image.

Group II

Group II was also limited to the three Extension activities. Preplanning and advance writing of the procedures for the images and the composite procedure was possible. All procedures relating to the first activity were input, including a procedure for a mirror line. As had Group I, they too neglected to change the direction of some of the angles and committed some typing and copying errors. They knew debugging was necessary but had difficulty delving deeper than the superprocedure level, paticularly as the mirror line procedure placed between the original and image was not state transparent. Assistance in identifying and debugging one procedure at time was needed; they also removed the mirror line procedure. Debugging was done through direct mode viewing and corrections were made one at a time. When the procedure was corrected they reincluded the mirror line in the first example only, initially by direct mode and then as a procedure added to the superprocedure.

Group III

Group III was able to complete all phases of this topic. The preplanning requested in the Extension phase posed little difficulty to this group, although they too were careless in designing the flip procedure, compounding their failure to change all the directions of the angles by making many typing and copying errors. They were able to locate the errors in the individual procedures but spent a great deal of time jumping in and out of the edit mode to check and change every error. The mirror line was added first in direct mode. Later it appeared as a command line in the composite reflection procedures but only after all three examples were debugged. They still experienced difficulty in determining the distance the turtle should move back after they had drawn the dotted line to represent the mirror line and once again managed through trial and error guessing.

At the outset of the Turtle Excursions phase a class discussion evolved on how best to approach this activity. The class consensus was to develop separate procedu[illegible] each component and then to combine them in a superprocedure as [illegible] done in the previous phase. The class as a whole expressed con[illegible] about the mirror line being on a diagonal. It was suggested they first construct the figure horizontally. The diagonal orientation could be added later as a single command in the superprocedure. Despite the class discussion, Group [illegible] first began in direct mode. The original, mirror line and image were not kept separate. Instead the students wrote horizontal strings of commands too long to debug. They eventually decided to switch to procedures in which they were much more willing to work with an individual procedure for each component. They were able to debug each part before beginning the next. When all was working well they built a combination procedure.

During the Project phase this group had little difficulty adapting the procedure to a reflection image once they had familiarized themselves with calling a variable procedure. Initially when they input the reflection image procedure they neglected to change the name used in the recursive call. They needed assistance in locating and correcting this error, but apparently did not entirely understand this relationship as the error was repeated and again assistance was needed.

Group V

Although this group could easily define reflection procedures in the Extensions phase, they too made similar translation, typing and copying errors. After two attempts at debugging visually they recognized the potential problems and coped with the remainder of the errors at a procedural level checking their preplanning and input for errors or missing commands. Only when they had carefully checked all of the input procedures did they check their results visually. The mirror line was written as a procedure and added to the superprocedure after both the original and image were constructed thereby avoiding the need to make this procedure state transparent.

Group V began the Turtle Excursions activities by breaking the ultimate figure into components and then defining an individual procedure for each one. They were able to construct one part at a time by adding a few commands in the edit mode, viewing and debugging in direct mode and then repeating the process. After completing all three procedures they defined a superprocedure and were very pleased to find a 45^0 turn would then change this reflection from a horizontal position to a diagonal one (this last step was done with guidance).

During the Project phase this group was able to convert the original procedures to reflection images. Other than the difficulty encountered inputting the recursive procedure with the correct syntax, they were able to independently complete and debug their projects.

Summary of Reflections - Procedures

Table 12 displays the results arising from the 20 observations of the procedural work involved in the reflections

Table 12 Topic - Reflections - Procedures

Data Categories Displaying Number of Episodes	Student Groups				
	I	II	III	V	Total
Total Episodes	3	3	7	7	20
Started in Direct Mode	0	0	1	0	1
Started as a Procedure	3	3	6	7	19
Successfully Completed	3	3	6	7	19
Episode Abandoned before Completion	0	0	1	0	1
Properties Mentioned or Discussed	2	2	4	4	12
Trial and Error Construction	0	2	4	1	7
Assistance Needed	0	1	2	1	4
Debugging Done, Visually, Error by Error	0	1	4	1	6

topic. The students in the 7A class were able to complete the work only up to and including the Extension phase of this topic, thus the small number (3) of episodes observed involving the students in Group I and Group II. In this topic, there was a sharp increase in the number successfully completed episodes that began as procedures. There was also a significant decrease in the number of episodes involving trial and error strategies; preplanning appears to have been productive. As work progressed, the students were more talkative about properties inherent in the subject matter, concentrating on the difference between the procedure for the original and image as being a change in direction of the angles. The low number of episodes in which the students used visual debugging seem to be indicative of two things. They made fewer errors that needed to be corrected and were more willing to examine the entire list of commands within procedures for errors rather than jumping immediately into direct mode checking.

4.12 COMMENTARY ON THE OBSERVATIONS OF THE REFLECTIONS TOPIC

INTERPRETATIONS ON THE VAN HIELE THEORY AND LEVELS OF LOGO USE

Basic Level

Consistent with the results of the previous topics, student comments during the inquiry phase of this topic indicate a Basic Level of understanding of reflections. Observations revealing an unfamiliarity with terms, use of imprecise qualities for identification as well as some incorrect information substantiate this interpretation.

- Generally students could correctly identify reflections but their description were limited to "flips are in opposite directions".
- Reflections were identified only as "flips". Students seemed unfamiliar with the terms "reflection" and "mirror line" as well as with the representation of the mirror line as a dotted line.
- One student perceived flips and reflections to be different concepts.

Hoffer (1983) suggested students at a Basic Level of a topic understand conservation of shape and size in different orientations.

- During the topic, congruency of the original and image never emerged as an issue, although students illustrated, through their work, their assimilation of this property.

Additional indicators of Basic Level understanding suggest that the students perceived the figures they were working on only as wholes; they were not capable of seeing the relationship of parts (Coxford, 1978). A student at this level can also be described as able to imitate but having no view of his own activity until he has reached a new level of thought (Wirszup, 1976).

- Although all groups were encouraged to preplan their direct mode activities, four groups had no view of how to proceed. In preplanning they could not conceptualize the reflection motion as discrete pieces, thus command lists evolved as strings, and not grouped in any way. These preplanned lists were input as horizontal strings (somewhat like working in procedure mode where entire command lists are input at once).

- Debugging these lists was impossible. Students resorted to trial and error strategies attempting to copy the motion one step at a time. They continued to be hindered by entering long stings. The relationship between the Logo language and the geometry was impeded by this method of data input.
- Groups III and V demonstrated the inability to reflect on their own activity when they grasped at another students suggestion that they develop a procedure to represent the mirror line. They spent time designing and debugging procedures they could not use, unable to perceive how to effectively incorporate these procedures.

Transition to Level I

As the students move closer to a new level of understanding they must be allowed to experimentally establish the properties of the topic through observation, measurement and model making (Wirszup, 1976). The following comments illustrate the experimentation and model making observed as the students identified and developed relationships between the components of reflections.

- All groups were able to transfer existing knowledge of the original and image to appropriate components of reflections within the first phase of the the lesson.
- Preplanning the direct mode activities appears to have been beneficial for Group I. This group was able to segment a reflection into its components and then preplan command lists constructing the motion on the screen in their first attempt. They progressed rapidly to preplan and input separate procedural components correctly by their third example.
- By the extension phase all groups were able to use procedures and subprocedures to represent the components of the reflection motion.
- When creating an original procedure, Group I reversed their commands and backed out, making their figure state transparent. All other groups, also making state transparent originals, continued to move forward in an attempt to complete the circuit.

This difficult task, involving repetitious trial and error, can overshadow the intended conceptualization of reflections. Backing out of a procedure would relieve this problem and would be a beneficial strategy taught to all students.

- Students can lose all concept of reflections if their command list for the reflection object is not state transparent. This strategy too ought to be taught to all students.
- It appears that whenever the students struggled with the relationship between the components of a reflection, they returned to trial and error guessing. Along with this return came long horizontal strings of commands.

Students began to understand the relationship between the original, image and mirror line as is apparent in their approach to debugging. To debug preplanned procedures as a unit requires the students to conceptually break down their original model into its parts and make revisions. They then were able to reflect on their own activity.

- Throughout this topic, Groups I and V were able to debug preplanned lists and entire procedures rather than emphasizing the step by step visual debugging still evident in the work of Groups II and III. Groups I and V illustrated they could now alter language lists to reflect the changes they wish to include, without requiring visual verification of every move.
- All groups had difficulty editing and adding new ideas to the appropriate subprocedure; instead they had to add command lists to the driver procedure.

Also apparent in this topic and indicative of a transiton to Level I, the students began to establish a more precise concept of the properties involved in the figures.

- One property of reflections repeatedly discussed was the difference in angles between the original and image.
- As well, the importance of an image and original being equidistant from the mirror line was discussed. For most of the students, two or three examples were developed before they recognized this

inconsistency in their model. However, on perceiving this need they did not correct any of their previous examples.

4.13 SUMMARY OF PROCEDURE MODE ACTIVITIES

The students' Logo / geometry behaviors as they progressed through the teaching unit are summarized in Table 13. Throughout the four topics the students behaviorial changes have been subjected to statistical analysis. To characterize growth in this environment the performances of the five groups on the first topic, congruency were compared with their performances on the last topic, reflections. A multiple comparison of proportions based on the ideas of Tukey, developed by Glass and Maquire (1973) was used in Table 13. The work of of five groups has been amalgamated in this summary for statistical purposes due to the relatively small number of episodes per topic per group.

Table 13 Summary - Procedure Mode Activities - ALL GROUPS

Data Categories Displaying Number of Episodes	Topics			
	Congruency	Translations	Rotations	Reflections
Total Episodes	10	40	26	20
Started in Direct Mode	.50	.45	.14	.05
Started as a Procedure	.50	.55	.86	.95 ***
Successfully Completed	.80	.95	.74	.95
Episode Abandoned before Completion	.20	.05	.19	.05
Properties Mentioned or Discussed	.20	.15	.22	.60 **
Trial and Error Construction	.70	.50	.35	.35 *
Assistance Needed	.30	.20	.08	.20
Debugging Done Visually, Error by Error	.70	.93	.40	.30 **

Proportion Comparisons between Congruency and Reflections, Significant at 0.01***; 0.05**; and 0.10*.

Consistent throughout the topics was the high rate of successful completion of all the assigned activities. Behavior changes, however were evident in the students' approaches to the acitivities. In the procedure mode activities students were encouraged to preplan and then to work at a procedural level. The increase in procedural level work relates to the steady decline of trial and error construction; preplanning became a viable possibility. The number of episodes the students rely on visual debugging steadily declined as the students were more able to alter the language lists without constant visual checks. In the early sessions (congruency and translations) the students relied strongly on a trial and error approach to their geometry constructions, the turtle used mainly as a "drawing tool". Procedure writing consisted of building a command list through a trial and error approach with frequent shifts between direct and procedure mode. Activities throughout the rotation and reflection lessons showed a sharp increase in the use of procedures for construction. Tentative use of subprocedures began in these later topics.

The students' geometry behaviors indicate a Basic Level of thinking (seeing the figures as wholes) throughout the introduction to each topic. While able to identify the transformation correctly, they were unable to define or explain identification criteria.

As they progressed through the phases of each topic the students were capable of reproducing figures in Logo with a drawing or copying mode of operation, recreating the figures step by step. Frequently the screen was cleared and the work begun a new. Independent analysis of geometric figures for correct angles or distances was infrequent, and advice assisting with analysis or construction strategies was never applied in a new situation. In the latter phases of rotations and reflections, the students began to establish a pattern for figure construction based on the recognition of the components of the transformation and the motions between them. Often properties were recognized through visual identification of errors.

Variations in the students' progress and in their approaches to procedure mode activites are highlighted by a comparison of the performances of Group I and Group II. Group I showed progress beyond the other four groups in their approach to the activities, while Group

II is represe[illegible] of all other groups. In Table 14 Fisher z scores compare the performances of these two groups in all procedure mode activities for all topics.

Table 14 Comparison Summary - Procedure Mode Activities Group I and Group II

Categories	Group I	Group II	Fisher-z
Total Episodes	27	22	
Started as a Procedure	0.93	0.55	3.10**
Successfully Completed	0.85	0.82	0.28
Properties Mentioned or Discussed	0.33	0.18	1.19
Trial and Error Construction	0.3	0.68	2.65**
Assistance Needed	0.09	0.32	2.03*
Debugging Done Visually, Error by Error	0.52	0.68	1.13

Significant differences at the 0.005**; and 0.05*.

From a study of this analysis, indicators of growth in the use of Logo to represent geometric ideas are: an increase in the use of procedures as a thinking tool, reduced dependency on trial and error approaches to construction and a more explicit characterizations of properties. The visual aspect of geometry knowledge building remained prominent as the students composed and debugged procedures one or two commands at a time.

POST-INSTRUCTIONAL EXAMINATION RESULTS

The students' geometry knowledge was assessed in a motion geometry unit test and as well in a school system year end examination. Both sets of test results will be reported in this section of the chapter.

4.14 MOTION GEOMETRY UNIT TEST

Knowledge of the motion geometry objectives prescribed in the Alberta Grade Seven Curriculum were assessed in a 35 item paper and pencil test. The test was constructed by the researcher in conference with the classroom teacher from a test item bank recommended for use in the school system. Class averages as well as the individual scores of the participating students, will be presented in Table 15.

Table 15 - Motion Geometry Test Results

a) Class Averages

7A	76%
7B	68%

b) Participating Student's Scores   Total 35

Group I	Paul	27 (77%)
	David	22 (63%)
Group II	Donna	32 (91%)
	Mary	28 (80%)
Group III	Sheila	21 (60%)
	Tracey	22 (63%)
Group IV	Diane	25 (71%)
	Richard	23 (66%)
Group V	Gerald	20 (57%)
	Franco	25 (71%)

4.15 School System Evaluation

The school system examination consisted of 100 mutiple choice items, covering the following topics: number systems - 55 items, ratio and proportion - 8 items, measurement - 10 items, exponents - 3 items, geometry - 20 items, and graphing - 4 items. For the purposes of this study, the 20 items comprising the geometry section of the examination and the overall test scores are presented for both classes, the whole school system and the individual student particpants as well as scores on previous parallel system evaluations will be presented in Table 16.

Table 16 – School System Evaluation Results

		Overall Test	Geometry Section
a)	7A Class	71%	78%
	7B Class	54%	68%
	School Average	62%	73%
	School System	61%	68%
b)	School Average Previous Four Years		
		62%	68%
		62%	66%
		61%	66%
		63%	64%

c) Participating Students Scores

		Overall Test Total 100	Geometry Section Total 20	
Group I	Paul	78	17	(85%)
	David	71	16	(80%)
Group II	Donna	75	16	(80%)
	Mary	62	15	(75%)
Group III	Sheila	51	13	(65%)
	Tracey	55	11	(55%)
Group IV	Diane	55	11	(55%)
	Richard	50	14	(70%)
Group V	Gerald	53	12	(60%)
	Fred	68	16	(80%)

4.16 COMMENTARY ON THE EVALUATION RESULTS

The reported test results of both the motion geometry unit test and the year end school system evaluation were included as a validity check on the Logo / van Hiele curriculum developed. The achievement of the Grade 7 geometry objectives as set down in the Alberta Mathematics Curriculum was assessed through both of of these tests. Class averages on both assessments (72% and 73%) indicate successful achievement of the curriculum objectives. Individual results of the particpating students show a range of achievement from satisfactory to excellent. The students using this curriculum did learn geometry.

This provides a validation for the instructional system developed in this investigation. Since the instructional system is functioning well in the sense of achievement of curriculun objectives, the observations on the qualities of student performance can be associated with a successful instructional environment.

INTERVIEW RESULTS

The results of the postinstructional interviews will be presented. A brief outline of the interview structure is followed by a summary and by excerpts of the student interview responses. A commentary on these results concludes the chapter.

4.17 STRUCTURE OF THE INTERVIEWS

All participating students were interviewed subsequent to the instructional phases. Individual interviews involving paper and pencil tasks were used to ascertain each student's knowledge and understanding of the motion geometry topics studied (translations, rotations and reflections). With respect to the structure of the interview questions, students were asked to:

a) given an original, construct the image for a specified motion (translation, rotation or reflection). Students were then asked to label their construction, give the appropriate notation and then attempt to explain how they "knew" they had constructed a specified motion.

b) supply the correct notation for examples of each motion.

c) identify each specified motion from a series of examples and explain the identification process. Samples to choose from included translations, rotations, reflections and incongruous images.

d) to connect corresponding vertices on the original and image of a given, completed, example of each of the motions, and to revise or clarify their definitions if this activity provided them with additional information or insight into the motions.

4.18 RESPONSES

In many instances the students responded similarly to the interview questions. The following summarizes the common responses.

1. Students could correctly identify slides, rotations, and reflections.
2. Usually identification was done through a process of elimination according to visual discrimination.

3. Students were able to use the appropriate translation and rotation notation.
4. Four of the ten students confused the angles of rotation and the corresponding fraction notation.
5. Students were hesitant to identify as reflections those constructed on an oblique mirror line.
6. Reflection images constructed using a 45^{o} mirror line were difficult for five of the students to construct.

Individual student responses provide insight into the their understanding of each of the motions. Specific comments and explanations arising as the students attempted to explain or describe their process of identification or construction are included for each topic.

TRANSLATIONS

The following excerpts are the student's explanations as to how they were able to identify examples as translations. Four of the students were able to clarify their descriptions of translations later in the interviews [after connecting corresponding vertices] and these extensions to their definitions are also included.

Group I

Paul: "...because they are facing the same way and are exactly the same shape." After connecting the corresponding vertices on an original and its translations image he described the connecting lines as parallel, but responded with an emphatic "No" when asked if the connecting lines would be parallel in every translation. On viewing some examples of translations he changed his response to "...well, maybe they all are parallel."

David: Each identification was explained as a result of eliminating other motions. David too, after connecting the corresponding vertices on an original and its translations image described the connecting lines as parallel. He then described a translation as "the original and slide image should be congruent and parallel" and was sure that this definition would apply to every translation.

Group II

Donna: "...the flag is still upright. It moves to another spot. The angle dosen't change or anything, and they must be congruent."

Mary: Mary's identification seemed intuitive and visual; each identification was explained by eliminating the other motions.

Group III

Sheila: "... because I went straight across. It is exactly the same, except moved." After connecting the corresponding vertices on an original and its translations image, Sheila said, "...The slides are parallel." She thought all translations would be parallel and was asked verify her statement using previously constructed or identified examples. On encountering, an incorrect construction, Sheila noted the original and image were not parallel. Asked if that would change her idea about all slides being parallel, she responded, "No, that is still true, I made a mistake in drawing it." She was able to correct the error.

Tracey: "It's the same picture; it doesn't turn or anything. It is the same angle". She included a slide with a larger image, and when questioned indicated that the size of the staff length on the flag (the object of translation) was not important.

Group IV

Diane: Diane only defined translations as "the original and image are congruent". When asked to provide appropriate slide notation, she did not independently distinguish the original and image, thus frequently giving the incorrect direction within the notation.

Richard: "It is a slide because it is not a reflection or a turn."

Group V

Gerald: "...it's moved over and it does not change the position." Then, after connecting the corresponding vertices on an original and its translations image he was able to specify that "Slides are parallel and equal distance apart." He was quite sure these properties would apply to all translations.

Fred: "It is just moved over, it has to face the same way. It has to be congruent."

ROTATIONS

The student responses to the rotation interview questions follow. The responses include the students' attempts to explain their identification of rotations and how they "knew" that they had constructed a rotation. Some students were unable to explain their identification criteria but could label the rotation components appropriately.

Group I

Paul: Paul labelled the rotation components in his construction as the original, image and turn center. The rotation was identified as a 90^0 or 1/4 turn "...because the image is perpendicular to the original." He also noted that corresponding sides on an original and a 1/2 turn image were parallel.

David: The rotation components were identified as the original, image and turn center. David also noted that the turn center must be an equal distance from the original and image, and that in addition the original and the image must be the same size.

Group II

Donna: Donna indicated the original and image must be congruent and equal distances from the turn center. She also noted that the corresponding sides of an original and half turn image were parallel.

Mary: Rotation components were identified as the original, image and turn center. She constructed a 90^0 rotation which was identified as such because "it looks like a picture that goes around four times."

Group III

Sheila: Sheila correctly labelled the original and image but omitted the turn center. The 1/4 rotation was identified as 45^0 and the 1/2 rotation to be 90^0. She knew the 1/2 was incorrect but concluded "I don't know what to do."

Tracey: During the identification procedure, Tracey included incongruous rotation images (the staff on the flag is longer) and indicated (when questioned) that congruemcy did not matter. She was unable to consistently construct rotations correctly.

Group IV

Diane: Diane was able to accurately construct rotations, label the three components (original, image and turn center) and use the correct notation. Several attempts were needed to complete a 180^{o} rotation.

Richard: Richard identified rotations because "...they have the turn center and the clockwise rotation." Later he added the rotation image must be the same distance from the turn center as the original and must congruent to the orignal.

Group V

Gerald Identification was made, "...because they are different than slides," later adding "you can have 1/4, 1/2 and 3/4 turns and the image must be congruent and the same distance [as the original] from the turn center". He eliminated some turns during identification because they were too far away from the original.

Fred: Fred correctly labeled the components, identified rotations, and matched the angle of rotation and corresponding fraction. He realized he had incorrectly constructed a 180^{o} turn because it "looked " wrong and tried unsuccessfully to "fix" his error.

REFLECTIONS

Student responses to questions involving the reflection motion during the interviews follow. The excerpts are the reponses to the question "Can you explain how you know this is a reflection?"

Group I

Paul: "They must be congruent, the mirror line must be halfway between the original and image. Lines joining the matching points meet the mirror line at 90^{o}"

David: "Reflections are opposites facing each other, or are backwards. They must be equal distances from the mirror line."

David correctly identified and constucted reflections and was able to use his definition to correct an error he had made in construction (his construction was not "backwards").

Group II

Donna: "The mirror line cuts the original and image in half; you can have a diagonal mirror line but not often." She correctly identified reflections but constucted two of the reflection images incorrectly.

Mary: "The original and image cannot overlap but they can touch the mirror line. The original and image shoud be congruent and the same distance from the mirror line. The mirror line can go through a figure like a square." She correctly constructed and identified all reflections throughout the interview.

Group III

Sheila: "...[illegible] the mirror line things look backwards, the image must be congruent, and opposite. They must be equal distance from the mirror line." With the exception of an example involving a 45^{0} mirror line, Sheila correctly identified and constructed the examples of reflections.

Tracey: Tracey's only explanation for identification of reflections was, "It is a reflection because it is not a turn or a slide. The mirror line can be at different angles 90, 45, 180, 270 and 360." She correctly identified and constructed reflections except those examples using an oblique mirror line.

Group IV

Diane: "...The original and image must be congruent, the mirror line is a dotted line a[illegible]ngle between two objects."

Richard: He described th[illegible] congruent and an equal distance from the mirror line as [illegible] original.

Group V

Gerald: Gerald was unable to explain how he identified reflections, although he could correctly identify and construct all examples. He was hesitant to identify, as an example of a reflection, one constructed with an oblique mirror line, but encountered no difficulty in the construction of the image for such an example.

Fred: "...I fold it in my mind on the mirror line, the mirror line reflects and makes the other side backwards. The original and image have to be the same distance from the mirror line. Fred correctly constructed and identified all examples, but experienced difficulty constructing an image on an oblique mirror line.

4.19 COMMENTARY ON THE INTERVIEW RESULTS

The interview responses indicate the students completed questions based on the Grade 7 motion geometry curriculum successfully. In this, the students demonstrated their ability to: create and discuss simple repeated patterns in terms of translations, reflections or rotations, construct the image of a figure given a transformation, name the transformation or combination of transformations that moves one figure onto the other, and represent a transformation by the appropriate notation.

INTERPRETATIONS OF THE THE VAN HIELE THEORY

The students' explanations and comments as they completed these transformation interview activities show correlations to the experiences observed during the lessons; similar connections to the Van Hiele theory can be made. Throughout the lesson observations the students entered activities primarily at the Basic Level according to the van Hiele levels of thought. This was especially apparent in the definitions (Table 17) given during the inquiry phases of each of the topics.

Table 17 - Representative Student Descriptions during the Demonstration / Discussion Phases

CONGRUENT FIGURES:
Congruent figures must be the same shape and size.
The original is the "real" picture.
The image is the second picture.
In two groups the students were not sure if congruent figures could be in different orientations.

TRANSLATIONS:
Slides are like something is moved over.
Slides are like a bulldozer has move the object over.
Slides always point the same direction.
Slides always look the same upright.

ROTATIONS:
Turns ace in a circle.
A turn center that is not away form the original and image, is not sensible. (sic)

REFLECTIONS:
Flips are opposite directions.
The mirror line is the place where the flip occurs.
Flips and reflections are not the same thing. A flip does not have a mirror line and a reflection does.

However, as the instructional phases progressed there was evidence of the students' emerging awareness of properties involved in identifying and constructing the transformations studied, indicating movement towards Level I understanding. The interview responses demonstrate a similiar transition between these two levels of thought as illustrated in the following discussion.

Burger and Shaughnessy's investigations of the van Hiele thought levels have suggested that the comparisons of shapes explicitly by means of properties and their components are indicators of Level I understanding. Coxford (1978) also suggested that at Level I understanding properties serve as a means of recognition of figures. Unlike the definitions during the demonstration / discussion phase of each of the topics most of the students during the postinstructional interviews were able to give an explanation (or definition) for their identification of each transformation. Many of these explanations denote the students' awareness of the specific properties of the transformations.

- Congruency of the original and image in each transformation was recognized by all but one student. Most mentioned congruency as part of their defintions.
- The components of the transformation were easily identified by all students.
- Four of the students showed an awareness that a segment maps to a parallel segment in a translation.
- Sheila was able to use the idea of mapping parallel segments in translations to correct an error in construction, she had not initially perceived.

- That a point and its image are the same distance from the turn center was a property of rotations recognized by most of the students. This property also arose frequently during the instructional phases.
- Paul recognized a segment and its 1/4 rotation to be perpendicular to one another, while the 1/2 rotations were parallel.
- A parallel image in the 1/2 rotation was also perceived as an important condition by two other students.
- Most students recognized the mirror line as the centre of a reflection and as well that the reflection reverses the orientation. These properties also frequently arose in the instructional phase.

Despite the students' awareness of specific properties of the transformations, they also relied on many characteristics identified in Burger and Shaughnessy's interviews as Basic Level indicators. Students at a Basic Level of understanding use visual prototypes, imprecise qualities to compare drawings and identify and characterize figures. Throughout the interviews students, continued to rely on visual considerations based on the figures as wholes.

- "It's the same picture, just moved over."
- "It's just moved over, it has to face the same way."
- "It looks like a picture that goes around four times."
- Students frequently tried to correct their constructions when they "looked" wrong, even though they could not explain the error. Sheila knew intuitively her construction of a 1/2 turn was incorrect and expressed her frustration with "I just don't know what to do."
- In their identification of reflections students used expressions such as "I fold it in my mind", "it looks backward" and "its a reflection because it is not a slide or rotation" highlighting their visual consideration of the drawings.
- One student's idenification criteria included a description of the mirror line.

An inablility to conceive of an infinite variety of types of shapes is labelled as a Basic Level indicator (Burger, 1985). Student responses here also limited the acceptable examples.

- One student limited rotations to 1/4, 1/2 and 3/4 turns. Examples too far away from the center of rotation were also eliminated.
- Although Donna identified a reflection constructed on an oblique mirror line, this was thought to be rare occurence.
- Tracey limited the mirror line construction to 90, 45, 180, 270 and 360 degree angles.

Basic Level understanding is also illustrated by a focus on irrelevant attributes and an exclusion of the relevant.

- Tracey did not see congruency of the original and image as a determining attribute in motion geometry.
- Diane did not attend to labelling the original and image on the diagrams and thus made frequent errors.
- Mary disqualified examples of reflections that included overlapping of original and image.

CHAPTER V

SUMMARY, CONCLUSIONS AND IMPLICATIONS OF THE STUDY

This chapter presents a summary and the conclusions of the study, some of the implications for educators, and concludes with some recommendations for further research.

5.1 SUMMARY OF THE INVESTIGATION

The purpose of the study was to explore and test the relationship between Logo use and the van Hiele theory of geometry learning. This relationship was studied through a teaching unit integrating the van Hiele levels and phases of learning, and the grade seven motion geometry curriculum objectives within a turtle geometry environment. The investigation of the Logo / van Hiele relationship involved two related components.

The development of a curriculum model incorporating the van Hiele phases of learning into the instructional levels of Logo was integral to the investigation. This model became a basis for lessons on the motion geometry topics: congruency, translations, rotations and reflections. For each topic the following components were devised. An "Demonstration / Discussion" phase involved computer demonstrations and group discussion. In the second phase, "Turtle Tracking" the students used teacher designed procedures, to create a motion in direct mode or debugged the given procedures to complete the motion. The intention in the "Extensions" phase was for students to design and debug procedures related to the topic. They were assisted in writing procedures by lists of commands recorded in the previous phase. "Turtle Excursions" for students at higher Logo or van Hiele levels, was to extend students' procedures related to the topic. In the final "Project" phase students were given some initial activity suggestions and were encouraged to approach these activities with as little assistance as possible using the knowledge gained through the previous phases to write their own procedures.

The studies' second element involved the researcher serving as a teacher / observer throughout the eleven weeks of instruction on the motion geometry unit. Two heterogeneous grade seven mathematics classes were involved in the project. Although all members of both classes were involved in the project, the work and progress of ten students, six fromone class and four from the other, was closely monitored. The students worked in pairs at the computers. The work and conversations at the computer of the ten students were saved by a video tape recorder and an external microphone. Field notes of activities both at and away from the computer were also kept. This curriculum and instructional phase was validated as follows: a detailed full validation with reference to the van Heile theory, the mathematical concepts, and the curriculum was done; there was a daily and topical validation based on student responses; a teacher without Logo experience was trained and carried out this experiment with one of the two classes involved as subjects; the results obtained by the subjects, compared with those of four previous years of students and with those of an entire school system, indicated that students learned the geometry well.

Daily, and at the end of the project, the video tapes were reviewed and systematic summaries were written on observation grids and then analyzed. The students were also interviewed and tested at the end of the teaching unit.

To give as complete a picture as possible of the relationship found between Logo use and the van Hiele theory, the findings for each pair of students were presented for every topic in a three fold perspective: a descriptive summary of each group, a tabulation of their Logo activity and a commentary on those students' behaviors related to the theories. In addition, a summary of growth over the four topics was presented and analyzed.

This final chapter presents conclusions and implications of the study.

5.2 CONCLUSIONS

The exploratory study focussed on five research questions. The questions and the responses to them inferred from the research findings are presented.

1. Can a set of experiences be developed in a Logo environment incorporating the van Hiele levels and phases of learning?

A Logo / van Hiele curriculum model was developed and used throughout the instruction of motion geometry. This curriuculum and the associated student activites were found to have face validity with repect to the van Hiele theory. The effectiveness of that curriculum model was shown by its successful use in a class whose teacher was not experienced in Logo use, as well as by the classes successful achievement on a motion geometry unit test and on the school system geometry year end examination. This provides a validation of the curriuculum model developed in this investigation. Since the instructional environment was functioning well in the sense of achievement of curriculum objectives, then observations of student performance can be associated with a successful instructional system. Thus an effective teaching unit can be developed within a Logo environment which incorporates the van Hiele levels and phases of learning.

The responses for the following two questions will be grouped under the topics of Logo behaviors and geometry behaviors.

2. What are the Logo and geometry behaviors of the students working in this environment?

3. Are levels of Logo use and van Hiele levels of thinking observably reflected in the activities of the students.

a) The students' Logo behaviors

In each topic the students' activities began in direct mode. Students relied strongly on trial and error approach to the construction of the motions with the turtle used mainly as a "drawing tool". As the students progressed through the four topics, they began to see the value of procedure writing and the results show that they did indeed use procedures. However, most procedures took the form of capturing under a name a sequence of commands which drew something. Frequently procedures were initiated but the students either abandoned the procedure mode or worked at a visual level, in adding and debugging procedures one or two commands at a time; frequent shifts between the two modes resulted. Activities during rotations and reflections topic showed evidence of students moving to the naive programming mode. There was a more routine use of procedures and a tentative use of subprocedures. Only one student was able to independently define a general procedure to translate the original any variance of angle and distance.

b) Geometry behaviors

Through the precursory Demonstration / Discussion phase of the topics, the students showed evidence of a Basic Level of geometric understanding. They could correctly identify the transformations but were unable to explain their identification process.

As they progressed through the phases the students' behaviors continued to reflect Basic Level thinking. They were capable of reproducing figures using Logo without error and constructions were completed as a drawing or copying mode of operation, recreating the figures step by step. When errors were made students frequently cleared the screen and began again. Seldom did the analysis of geometric shapes for correct angles of distances arise independently. If assisted with this analysis, any information gained or style of attack suggested, was not applied to a new situation; it seemed each figure was viewed independently.

A movement toward Level I (property based) thinking began when the students were able to establish a pattern for figure construction, a recognition based on the components of the transformations and the motions between them. The activities began to alter as the perceptions of the motions changed from viewing them as wholes to viewing them in discrete pieces. The Logo constructions began to reflect this new perception. The original and image were seen as a group of commands and were eventually represented under a single name as a subprocedure. Correctly including the motion between the original and image within a planned procedure continued to be a struggle throughout the unit. A recognition and discussion of properties began to emerge, often beginning with visual identification through construction errors. The students' definitions given in final interviews remained imprecise and visual; however there was definite evidence of a beginning awareness of the properties of transformations.

4. Were the behaviors consistent with the theorized relationship linking the Levels of Logo use and the van Hiele theory?

The students' geometry behaviors and the way they perceived geometric shapes was reflected in the nature of the Logo activities. Initially, observing the figures as wholes restricted them to constructing these shapes in a drawing or copying mode; thus the constructions (although nearly all successfully completed) were done through trial and error constructions, adding and debugging one command at a time, whether in direct or procedure mode. There is little evidence of preplanning at this level. Only when students began to see the figures as comprised of components and the relationships between the components (movement to Level I thinking) were they able to write composed procedures including list patterns and, eventually, subprocedures. At this stage planning becomes effective. They have now moved into the Naive Programming Mode. Logo use seems to rely on how the students perceive the geometry, thus the link between Logo and van Hiele appears to be consistent with relationship theorized by Kieren and Olson at the Basic and Level I.

5. Are there behavioral markers of the transition between Logo levels? To what extent are such markers observable?

Students seem to be moving towards the naive programming mode when they see the shapes as composed of discrete pieces. This was evident when preplanned lists of commands began to be grouped. Input of the list was usually at a procedural level. Copying a list of commands for the original to later be used to represent the image is also an early marker of this transition. At a more advanced stage of this transition, students they would define procedures and then use the procedural name within subprocedures. At this stage they were able to debug their procedures; however, for most this was still accomplished with visual direct mode checking. Students at Level 1 had the ability to alter parts of a procedure itself to reflect the changes they envision, whereas those in transition were more likely to be able to relate errors to pieces of a list of direct mode commands.

Throughout the transition students struggled with the relationship between the components of the motions. They usually resorted to trial and error strategies in direct mode to resolve the problem. Students were either able to overcome their difficulties and move back to procedure mode or seemed to flounder between the two modes as command lists were discarded as students returned to visual construction. Preplanning early in the transitions phase seems counter-productive. Because the students still thought of drawing step-by-step, students would ignore preplanned work or perhaps worse, would enter long, horizontal strings of commands which led to frustrating, uncontrollable, errors. A premature instructional emphasis on preplanning seems inadvisable.

Logo because of characteristics of language in "use" allows us to see the transition in the student's geometric perceptions and likely sponsors level changes in student behavior.

5.3 IMPLICATIONS

Research to date on the van Hiele theory has shown many students in high school and university geometry courses struggling at levels of understanding beneath that at which the courses are being taught. All recommendations have pointed towards the provision of informal geometry experiences in the middle grades to build on students' geometric intuition. This study is a first step in confirming the relationship of Logo and the van Hiele levels, and using this link provides an important vehicle for curriculum development in informal geometry. The curriculum model developed in this study provides a basis on which to build curriculum in other areas of geometry instruction and possibly other mathematical areas. The data, both in terms of student geometric achievement and in terms of levelled student actions, are supportive of the van Heile instructional theory.

As stressed in the van Hiele theory, this study also shows the role of instruction to be vital. The decision for the topic of study, transformation geometry, was based on the scope and sequence of the provincial curriculum objectives and proved to be a providential decision. The nature of construction of transformations in Logo encourages a transition in Logo use from simple command lists developed through trial and error, to a more structured programming style based on building block construction. Programming which has a building block approach is more apt to reveal the geometric relationships involved. The foundation of the Logo / van Hiele instructional environment must foster this transition so that the innate geometric relationships can emerge.

Throughout the instructional activities the students frequently oscillated between the direct mode and the naive programming mode. The markers of the transition between these modes must be carefully observed by teachers so that appropriate assistance ensues. If students are encouraged to approach activities at levels they are not ready for, they may not be challenged students, but rather frustrated and discouraged.

5.4 RECOMMENDATIONS FOR FURTHER RESEARCH

The findings of this study suggest the need for further research in the following areas.

1. Observation - The present investigation was based on an introduction to the motion geometry topics of translations, rotations and reflections.

   Research Questions - Can this curriculum model be extended to include operations on the transformations and geometry of two dimensional figures? What Logo / van Hiele experiences would encourage growth in these areas? Can this curriculum model be used (at least for modelling purposes) in other areas, such as ratio/proportion and equation solving or more advance topics as vector space?

2. Observation - This curriculum was executed in a real classroom environment, but under special conditions.

   Research Question - In what ways and to what effect can this curriculum be implemented in a large number of classrooms?

3. Observation - The students had twelve / eighty minute weekly sessions introducing Logo programming to them prior to beginning the motion geometry unit.

   Research Question - Would the relationship between Logo and van Hiele levels change with a more extensive Logo background?

4. Observation - The findings reveal a relationship between the first Logo Levels of use and the van Hiele levels of thought?

   Research Question - Does this link continue to exist at Levels II, III, and IV?

5. Observation - Students' achievement scores on related examinations were generally higher than usual.

Research Question - What are the long term effects on academic achievement in a Logo / van Heile environment?

6. Observation - Students' geometric ideas seem to be shaped by, and related to, the Logo activities.

   Research Question - What are the mathematical opportunities and ideational consequences of work in a Logo / van Heile environment?

7. Observations - The curriculum here conformed to the geometric ideas expressed in the curriculum guide.

   Research Question - In what ways can Logo experiences be constructed which carefully reflect the form of mathematical ideas such as vector spaces, graphs, et cetera? What is the effect of using appropriately "formed" Logo representations on student mathematical ideas?

8. Observation - The effects of Logo use in this study were primarily cognitive. It was observed, however, that students maintained a high level of social interactions in this environment.

   Research Question - What are the affective consequences for students of working in such an environment?

9. Observation - Some teacher acts seemed to sponsor student growth in this environment, some to inhibit growth.

   Research Question - What is the nature of such positive and negative mathematical teaching acts?

V. CONCLUDING STATEMENT

The van Hieles were concerned with developing insight in their students. The goal of the present study was an exploration; through the integration of the van Hiele phases of learning within a Logo environment and the intense observation of students in this environment insights into the teaching and learning of transformation geometry were sought. The value of this research rest on the extent to which it allows for further insights into levels of learning.

BIBLIOGRAPHY

Alberta Education. Elementary Mathematics Curriculum Guide. Edmonton: Alberta Education, 1982.

Alberta Education. Junior High School Mathematics Curriculum Guide. Edmonton: Alberta Education, 1978.

Austin, H. Teaching Teachers Logo: The Lesley Experiement. M.I.T Logo Memo No. 23, 1976.

Backus, D.W., Chandler, C., Kieren, T.E. & Belcher B. It's Mathematics 7. Toronto: Gage, 1984.

Bearden, B. & Martin, K. The world of symmetry. Microquests. Grapevine, Texas: Martin-Bearden Inc. 1984, October.

Bitter, G. & Watson, N. Apple Logo Primer. Reston, Virginia: Reston Publishing, 1983.

Bober, W.C. Role of maturity and experince in junior high school geometry. Unpublished master's thesis, University of Alberta, 1973.

Burger, W. Geometry. Arithmetic Teacher. 1985, 6, 52-56.

Burger, W. The van Hiele Levels: A Framework for Describing Reasoning Processes in Geometry. Unpublished manuscript, Oregon State University, 1985.

Burger, W. and Shaughnessy, J.M., Characterizing the van Hiele levels of development in geometry. Journal for Research in Mathematics Education, 1986, 1, 31-48

Burger, W. and Shaughnessy, J.M., Spadework prior to deduction in geometry. Mathematics Teacher, 1985, 6, 419-427.

Burger, W., Hoffer, A., Mitchell, B. and Shaughnessy J.M. Goals for geometry - can we meet them? The Oregon Mathematics Teacher, November-December, 1981, 12-15.

Burnett, J.D. Logo an Introduction. Morris Plains, New Jersey: Creative Computing Press, 1982.

Carpenter, Thomas. Research in cognitive development. In Shumay, R.J. (Ed.), Research in Mathematics Education. Reston Virginia: National Council of Teachers of Mathematics, 1980.

Copeland, R. How Children Learn Mathematics. New York: Macmillan, 1984.

Coxford, A.F. Research directions in geometry. In Lesh, R. & Mierkiewicz (Eds.), Recent Research Concerning the Development of Spatial and Geometric Concepts. Columbus, Ohio: ERIC, 1978.

du Boulay, B. Relearning mathematics through Logo: Helping student teachers who don't understand mathematics. In Howe, J.A.M. & Ross, P.M. (Eds), Microcomputers in Secondary Education: Issues and Techniques, London: Kogan Page 1981, 69-81.

Fey, James T. (Ed.), Computing and Mathematics, The Impact on Secondary School Curricula. University of Maryland: National Council of Mathematics, 1984, 31-53.

Freudenthal, H. Mathematics as an Educational Task. Dordrecht: Reidel, 1973.

Fuys, D., Geddes, D. and Tischler, R. An Investigation of the van Hiele Model of Thinking in Geometry among Adolescents Final Report. National Science Foundation, 1985.

Geometry and Visualization. In, The Mathematics Resource Project. Palo Alto, California: Creative Publications, 1978.

Ginsburg, H. & Opper, S. Piaget's Theory of Intellectual Development. Englewoods Cliffs, New Jersey: Prentice Hall, 1979, 206-238.

Glass, G. & Maquire, T. Multiple Comparisons of Proportions, Division of Educational Research Services, Edmonton, Alberta: University of Alberta, 1973.

Groen. G. J. The theoretical ideas of Piaget and educational practice. In Suppes, P. (Ed.), Impact of Research on Education: Some Case Studies. Washington, D.C.: National Academy of Education, 1978.

Goldenberg, E.P. Logo - a cultural glossary. Byte, August, 1982, 210-228.

Harvey, B. Why Logo? Byte, August, 1982, 163-193.

Hillel, J. Mathematical and Programming Concepts Acquired by Children, Aged 8-9, in a Restricted Logo Environment. Montreal, Quebec: Concordia University, 1984.

Hillel, J. On Logo squares, triangles and houses. For the Learning of Mathematics, FLM Publishing Association: Montreal, Quebec, June, 1985.

Hillel, J. and Samurcay, R. Analysis of a Logo Environment for Learning the Concept of Procedures with Variables. Montreal, Quebec: Concordia University, 1985.

Hoffer, A. Geometry is more than proof. Mathematics Teacher. 1981, 5, 11-18.

Hoffer, A. Van Hiele - based research. In Lesh, R. and Landau M. (Eds.), Acquisition of Mathematics Concepts and Processes. New York: Academic Press, 1983.

Howe, J.A.M. & O'Shea, F. Teaching mathematics through Logo programming: an evaluation study. In Lewis, R. & Tagg, E.D. (Eds.), Computer Assisted Learing: Scope, Progress and Limits. New York: North Holland, 1980, 85-134.

Kieren, T.E. Logo in Education: What, How, Where, Why and Consequences. Alberta Education: Edmonton, Alberta, 1984.

Kieren, T., & Olson, A. Linking Logo and Learning Levels. Paper presented at the Northwest Computing Conference, Corvallis, OR, 1983.

Kilpatrick, J. & Wirszup, I. (Ed.), Soviet Studies in the Psychology of Learning and Teaching Mathematics. Chicago: University of Chicago, 1969.

Kuchemann, D.E. Childrens difficulties with single reflections and rotations. Maths in School, 1980, 12-13.

Kuper, M. The effects of model building on geometrical maturity. University of Alberta: Unpublished master's thesis, 1979

Lawler, R.W. Designing computer-based microworlds. Byte, August, 1982, August 138-160.

Libeskind, S., Billstein, R. & Lott, J.W. Logo Activities in Elementary Geometry. University of Montana: Missoula, Montana, 1983.

Martin, J.L. An analysis of some of Piaget's topological tasks from a mathematical point of view. Journal for Research in Mathematics Education, 1976, 7-24.

Mayberry, J. The van Hiele levels of geometric thought in undergraduate preservice teachers. Journal for Research in Mathematics Education, 1983, 14, 58-69.

Moore, M.L. Geometry Programs for Logo Discoveries, Explorations in Turtle Geometry. Palo Alto, California: Creative Publications, 1984.

Moore, M.L. Logo Discoveries, Explorations and Programming Activities for Beginners. Palo Alto, California: Creative Publications, 1984.

Olive, J. Logo programming and relational learning in a geometric microworld - implications for instruction. Damarin, S. and Shelton, M. (Eds.), In Proceedings of The 7th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Columbus, Ohio, 1985, 351-356.

Papert, S. Teaching Children to be Mathematicians vs. Teaching Children About Mathematics (Logo Memo No.4). Cambridge, MA: MIT Artificial Intelligience Laboratory, 1971.

Papert, S. Mindstorms: Children, Computers, and Powerful Ideas. New York: Basic Books, 1980.

Perham, F. An investigation into the effect of instruction on the acquisition of transformation geometry concepts in first grade children and subsequent transfer to general spatial ability. In Lesh, R.A. & Miekiewicz, D. (Eds.), Recent Research Concerning the Development of Spatial and Geometrical Concepts. Columbus: ERIC, 1978.

Pothier, Y. Partitioning: construction of rational number in young children. Unpublished doctoral dissertation, University of Alberta, 1981.

Robertson, Miranda. Artificial Intelligence in Education. Nature, August, 1976, 435-437.

Schulz, K. A. Variables influencing the difficulty of rigid transformations during the transition between the concrete and formal operations stages of cognitive development. In Lesh, R.A. & Miekiewicz, D. (Eds.), Recent Research Concerning the Development of Spatial and Geometric Concepts. Columbus, Ohio: ERIC, 1978.

Skemp, R. The Psychology of Learning Mathematics. Middlesex, England: Penquin Books, 1971.

Smock, C.D. Piaget's thinking about the development of space concepts and geometry in Martin, J.L. (Ed.), Space and Geometry: Papers from a Research Workshop. Columbus: ERIC, 1976.

Thomas, D. Students' understanding of selected transformational geometry concepts. In Lesh, R.A. & Mierkiewicz, D. (Eds.), Recent Research Concerning the Development of Spatial and Geometric Concepts. Columbus, Ohio: ERIC, 1978.

Usiskin, Z. Van Hiele Levels and Achievement in Secondary School Geometry. Final Report of the CDASSG Project. Chicago: University of Chicago, 1982.

van Hiele, P.M. Levels of Thinking, How to Meet Them, How to Avoid Them. Paper presented to a Special Interest Group of Research in Mathematics AERA, 1982.

Waite, M., Martin, D., Martin, J. 88 Apple Logo Programs. Indianapolis, Indiana: Howard W. Sams, 1984.

Watt, D. Final report of the Brookline Logo projects. Part III: Profiles of individual students' work. Columbus, Ohio: ERIC, 1979.

Watt, D. Learning with Logo. New York: McGraw-Hill, 1983.

Watt, D. Logo in the schools. Byte, August, 1982, 116-134.

Weir, S. & Emmanuel, R. Using Logo to Catalyze Communication in an autistic child. D.A.I. Research Report # 15: University of Edinburgh, 1976.

Wirszup, I. Breakthroughs in the psychology of learning and teaching geometry. In Martin, J.L. and Bradbard, D.A. (Eds), Space and Geometry: Papers from a Research Workshop. Columbus: ERIC, 1976

Zykova, V.I. The psychology of sixth-grade pupils mastery of geometry concepts. In Kilpatrick, J. and Wirszup, I. (Eds.), Soviet Studies in the Psychology of Learning and Teaching Mathematics. Chicago: University of Chicago, 1969.

APPENDIX A

TRANSLATION LESSON MODULE

TRANSLATIONS (SLIDES) - TEACHER PAGES

OBJECTIVES: IDENTIFY A TRANSLATION
CONSTRUCT AN ORIGINAL, SLIDE ARROW AND ITS IMAGE

MATERIALS: DEMONSTRATION DISK (DISK FILE SLIDES INCLUDES DEMONSTRATIONS 1 THROUGH 3)

SUGGESTED ACTIVITES:

TEACHER DIRECTED ACTIVITIES

1. Discuss with the students their understanding of a slide. Use DEMONSTRATION 1 produces translations randomly, showing the original, the image and the slide arrow. The original and its image are in different colours. Discuss the three parts of the translation, they can be shown separately. Show how a slide arrow is represented on dot paper. Discuss the congruency of the original and its image.

2. Students will predict the results of a slide when entering vertical and horizontal inputs to complete the slide.

STUDENT ACTIVITIES

1. Given slide procedures the students will represent each motion on dot paper
2. Students will complete translations in direct mode using a general polygon procedure
3. Four incomplete or incorrect translations are given to the students to complete
4. Students will create three translations on their own using a rectangle as the object. Diagrams of the specified outputs are given
5. Students are asked to create their own design as well as a translation procedure for the motion. Three examples are required
6. Students are asked to create a general procedure to construct translations. Distances moved should be inputs
7. Students are given three projects on translations to complete.

TURTLE TRACKING

Throughout Turtle Tracking you will be using procedures saved on your disk.

READ "SLIDES to load these procedures.

1. Use the procedures SLIDE1, SLIDE2, SLIDE3, and SLIDE4. The procedure will give you the slide notation. Draw the ORIGINAL, SLIDE-ARROW, and IMAGE to represent each procedure.

a

b

c

d

2. Use the following procedure to draw a pentagon.

```
TO PENTAGON :SIDE
REPEAT 5 [FD :SIDE RT 72]
END
```

Using this procedure draw a pentagon of any size. In direct mode complete three different slides of your pentqon. Keep a list of your commands.

EXAMPLE 1

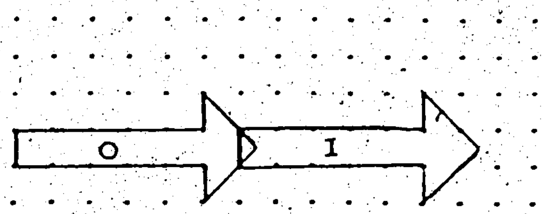
EXAMPLE 2

EXAMPLE 3

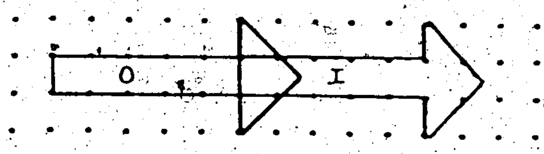
TURTLE TRACKING CONTINUED

3. The following four procedures are not complete. Diagrams of the expected results are shown. Try and determine the missing commands and then check your answer by teaching the turtle the procedure and check the results. You will be using the arrow procedure already loaded in the computer memory from exercise 1.

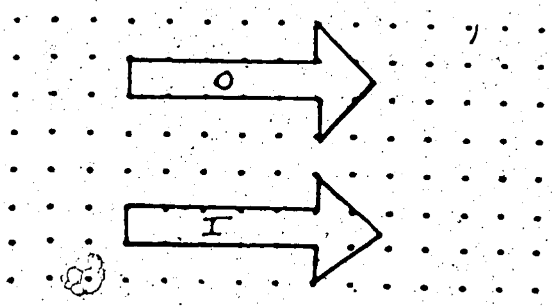
a)
```
TO SLIDE.A
ARROW
PU RT 90 FD 60 LT 90 PD
END
```



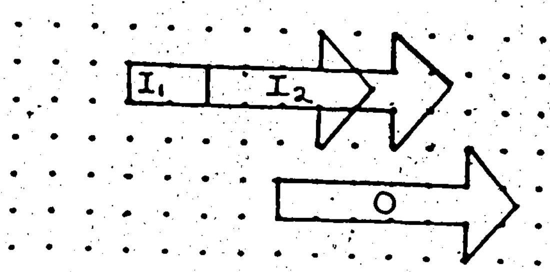
b)
```
TO SLIDE.B
ARROW
PU RT 90 FD 50 PD
ARROW
END
```



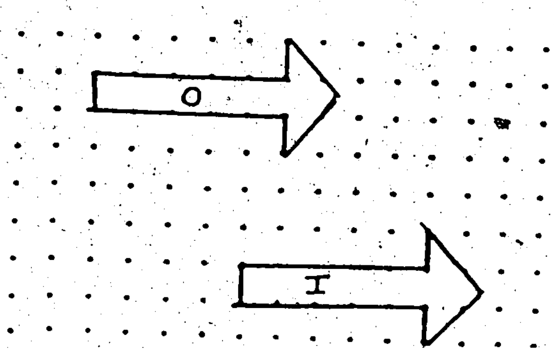
c)
```
TO SLIDE.C
PU BK 40 PD
ARROW
END
```



d)
```
TO SLIDE.D
PU LT 90 FD 40 RT 90 FD 30 PD
ARROW
PU RT 90 FD 20
ARROW
END
```



e)
```
TO SLIDE.E
ARROW
LT 45 BK 65 RT 45
END
```



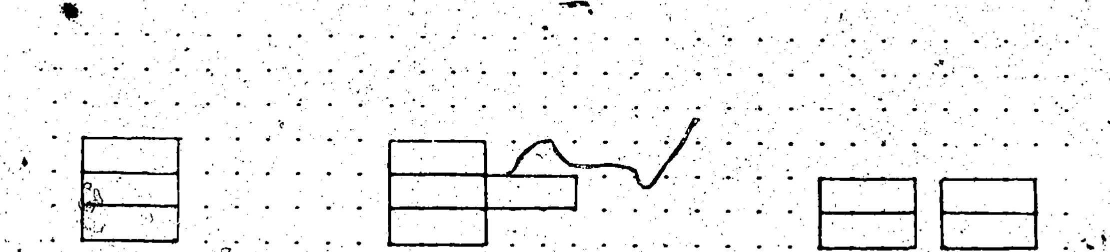
EXTENSIONS

1. You can use a rectangle as a original and then create procedures to slide your rectangle to create designs.

First write a procedure to draw a rectangle.

TO RECTANGLE

Using the idea of slides write procedures that will draw on the screen the following diagrams.



a) b) c)

Record your command lists.

TO SLIDE.REC1 TO SLIDE.REC2 TO SLIDE.REC3

EXTENSIONS CONTINUED

2. Write a procedure for a simple design of your choice.

TO DESIGN

Now write three different slide procedures to move your design. Record your commands and save your procedures on your disk.

a) TO SLIDE.DESIGN1

DIAGRAM

b) TO SLIDE.DESIGN2

DIAGRAM

c) TO SLIDE.DESIGN3

DIAGRAM

TURTLE EXCURSIONS

Try and create a general procedure for a slide using your design as the origijnal from the last extensioom activity. The distances and angles for sliding should be input. Sketch your design and record your procedure.

SKETCH

PROCEDURE

Save your procedure on your disk.

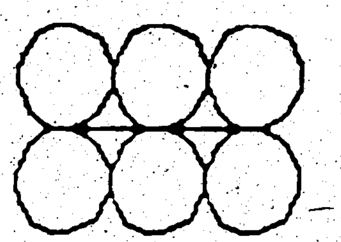
SLIDE PROJECTS

You will be using procedures saved on your disk.

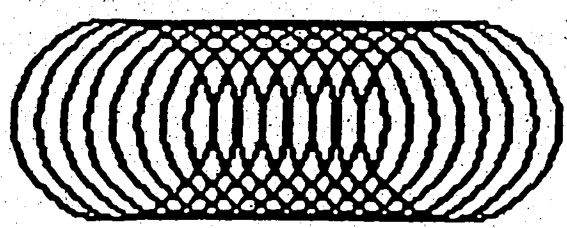
READ "SLIDES to load these procedures.

Use the two circle procedures. LCIRCLE will draw a circle to the left and RCIRCLE to the right. Either procedure needs one input, the radius of the circle (the distance from the center of the circle to the outside edge). Here are some designs you can make with circles. If you hide the turtle while drawing drawing the circles, they will be drawn faster. Use the idea of slides and try to write procedures to create these designs on the screen. Save your procedures on your disk.

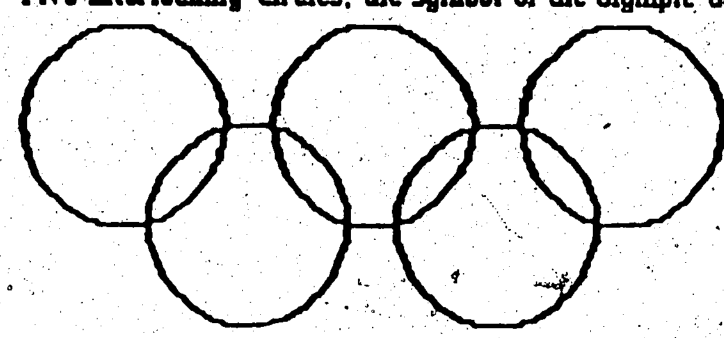
a)



b) A Slinky design.



c) Five interlocking circles, the symbol of the Olympic Games.



APPENDIX B

LETTER TO STUDENTS AND PARENTS

University of Alberta
Edmonton, Alberta
T6G 2G5
February 18, 1985

Dear Parents and Student:

Computers are a new and not very well understood tool in education. There is much to be learned about how to use them most effectively.

I have been teaching for ten years, the last five at .... School. For the 1984-85 school year I have taken a leave of absence in order to do graduate work at the University of Alberta in the use of computers in mathematics education.

You may aready know that this year I have been assisting .... with using computers in grade seven mathematics. .... and the school principal, ..., have graciously allowed me to include the Grade Seven students in my reasearch. My study involves using the computer as a tool to teach and learn geometry. We trust the result of my research will benefit all students and teachers.

Althouugh all students will work with .... and me in this new approach, I plan to work more closely with ten students as they follow the lessons. It will be necessary to tape record their comments and discussions with their partners and me and to record their work both at and away from the computer. I anticipate my study will take approximately two months. When the data is analyzed, the childrens' identities will be withheld. Students will be chosen to take part in the observation part of the study only if he/she and his/her parents consent. Please complete the attached form and have it returned to ....

Yours truly,

Susan Blair

Susan Blair

Attach.

Dear Miss Blair:

We are willing to have ____________________________ participate
Pupil's name
in the observation phase of your research project, as outlined in your your letter of February 18, 1985, should he/she be selected.

________________________ ________________________
Pupil's signature Parent's signature

OR

We are not willing to have ____________________________ participate
Pupil's name
in the observation phase of your research project.

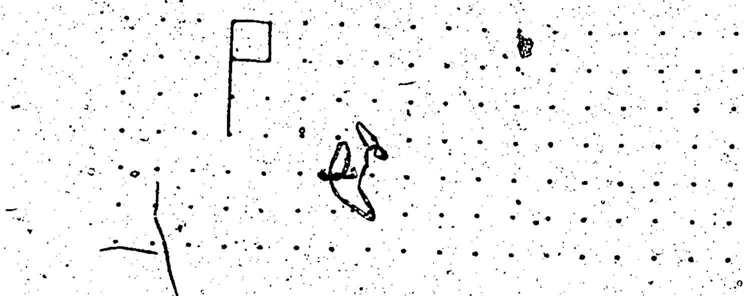
APPENDIX C

INTERVIEW QUESTIONS

TRANSFORMATION ACTIVITIES

ACTIVITY 1: TRANSLATIONS

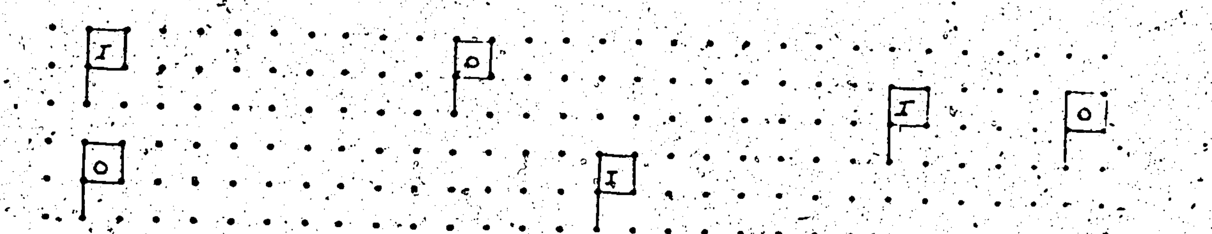
DRAW ANY SLIDE OF THE FLAG



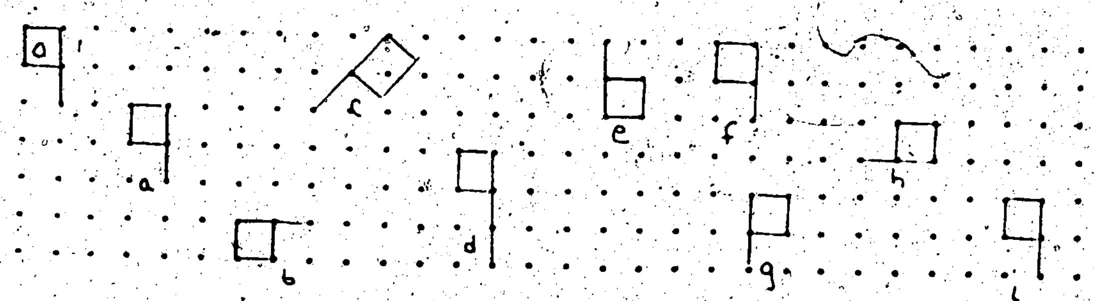
Script:

1. Can you explain what a slide is?
2. Draw another slide that is different in some way?
3. Can you tell me how you know these are slides?
4. How are your two slides different?

HERE ARE THREE FIGURES AND THEIR SLIDE IMAGES. GIVE THE SLIDE NOTATION FOR EACH.



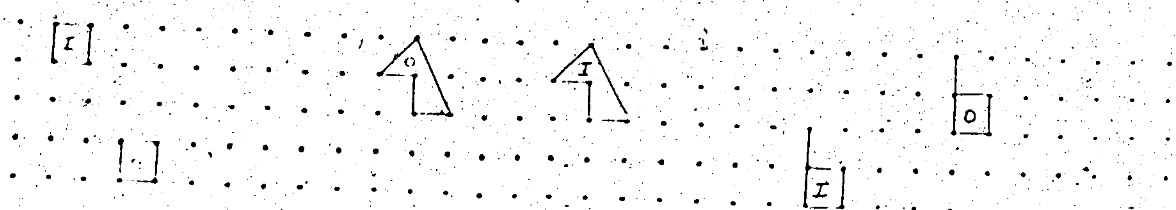
WHICH OF THE FOLLOWING ARE SLIDE IMAGES?



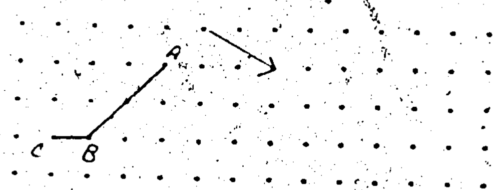
Script:

1. Have the student check those images they think are slides.
2. Using the examples the students have identified, ask them to explain how they can identfy examples that are slides and those that are not.

DRAW THE SLIDE ARROW THAT REPRESENTS THE FOLLOWING SLIDES OF THE ORIGINAL. DRAW YOUR ARROW AWAY FROM THE FIGURES.



DRAW THE SLIDE IMAGE OF ABC, LABEL IT DEF



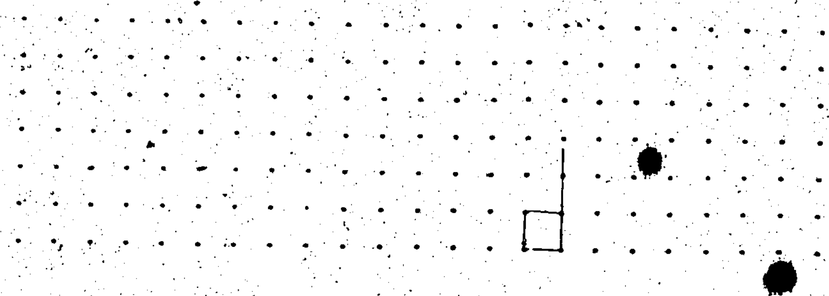
Script:

If the students complete the translation correctly then:

1. Ask the students to compare the lengths of segments AB and DE as well as BC and EF.
2. Can you explain what parallel lines are?
3. Are there parallel lines in this translation?
4. Draw segments joining AD, BE and CF. Can you notice anything about the lines you have drawn?
5. If the students recognize that these segments are parallel, ask them if that would that would be the case in all slides. Have them look at the previous examples of slides to confirm their response.

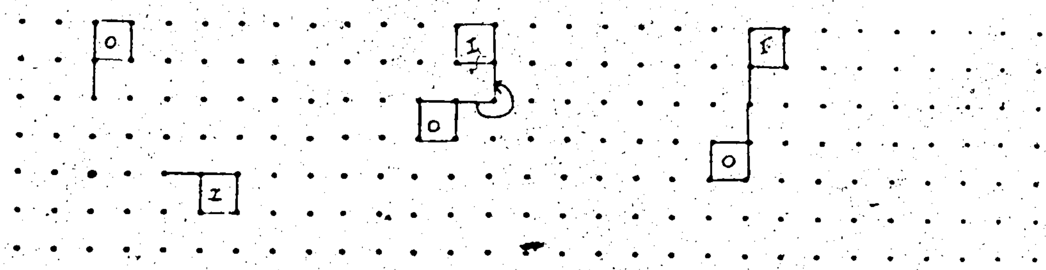
ACTIVITY 2: ROTATIONS

DRAW ANY ROTATION OF THE FLAG

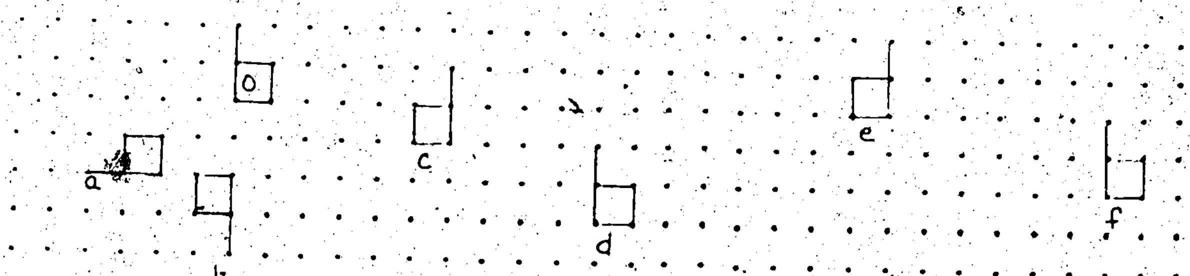


Script:
1. Can you explain what a turn is?
2. Draw another turn that is different in some way.
3. Can you tell me how you know these are turns?
4. How are your two turns different?
5. Name the parts of a turn.

HERE ARE THREE FIGURES AND THEIR TURN IMAGES. GIVE THE AMOUNT AND DIRECTION OF THE TURN FOR EACH.



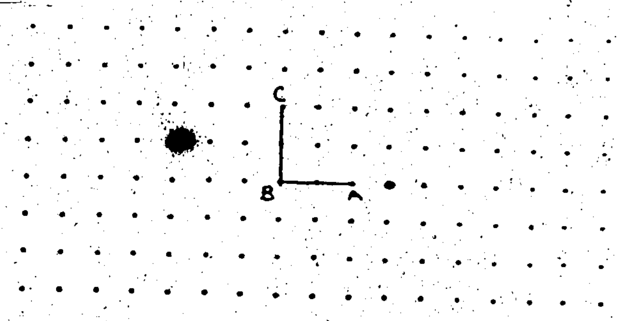
WHICH OF THE FOLLOWING ARE TURN IMAGES?



Script:
1. Have the student check those images they think are turns.

2. Using the examples the students have identified, ask them to explain how they can identify examples that are turns and those that are not.

DRAW THE HALF TURN IMAGE OF ABC, LABEL THE IMAGE DEF?



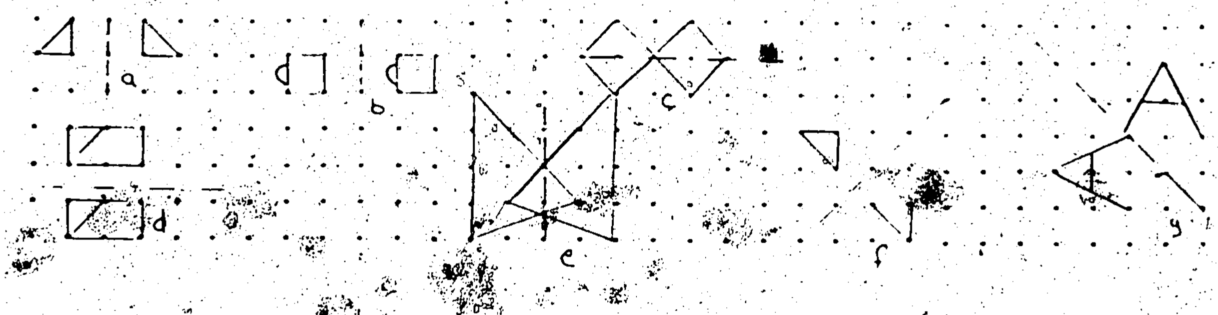
Script:

If the students can correctly complete this rotation then:

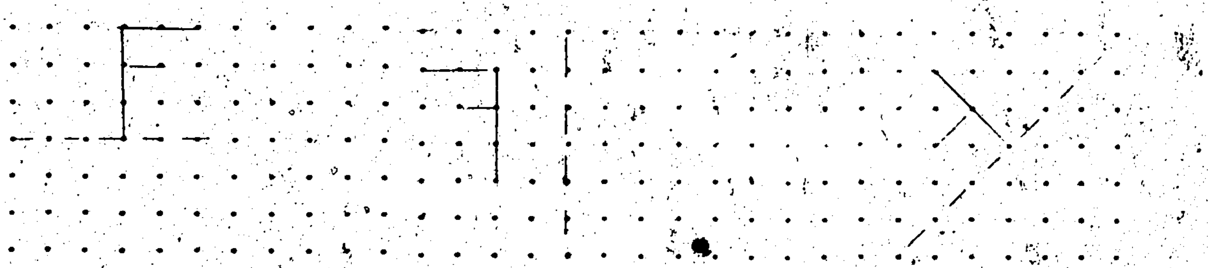
1. Ask the students to compare the lengths of segments AB and DE as well as BC and EF.
2. Are there parallel lines in this rotation?
3. Draw segments joining AD, BE and CF. Can you notice anything about the lines you have drawn?
4. If the students recognize that these lines intersect (better still, bisect) with the turn center, ask them if that would be the case in all turns half turns, and in all turns. Have them look at the previous examples of turns to confirm their response.

ACTIVITY 3: REFLECTIONS

TELL WHETHER THE FIRST FIGURE IS A REFLECTION OF THE SECOND. TRY AND EXPLAIN WHY OR WHY NOT.



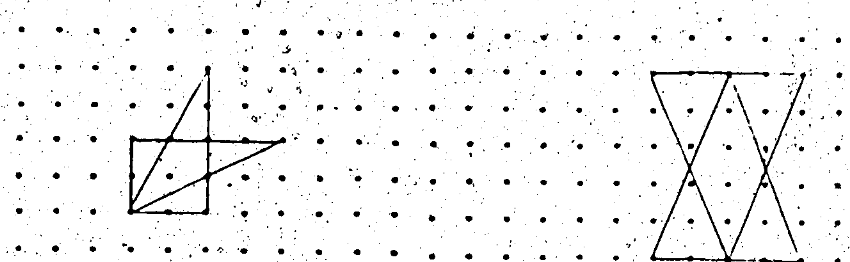
DRAW THE FOLLOWING REFLECTION IMAGES.



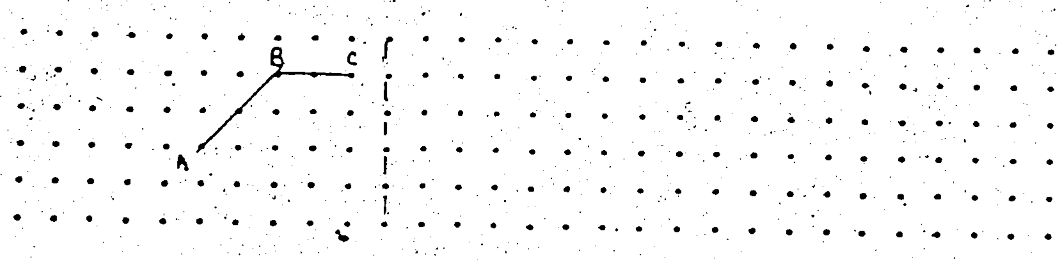
Script:

1. Can you explain what a reflection is?
2. How do you know these are reflections?

CAN YOU DRAW THE MIRROR LINE FOR THESE REFLECTIONS.



DRAW THE REFLECTION IMAGE OF ABC; LABEL THE IMAGE DEF?



Script:

If the students can correctly complete this reflection then:

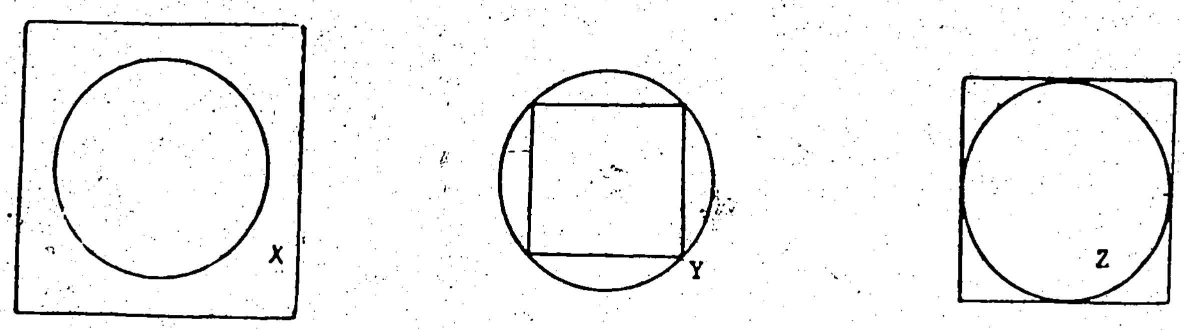
1. Ask the students to compare the lengths of segments AB and DE as well as BC and EF.
2. Are there parallel lines in this reflection?
3. Draw segments joining AD, BE and CF. Can you notice anything about the lines you have drawn?
4. If the students recognize that the mirror line is a perpendicular bisector of these lines, (they will explain it in their own vocabulary) and these segments are parallel, ask them if that would be the case in for all reflections. Have them look at the previous examples of reflections confirm their response. Ask them if there is anything else they notice about segments AD, BE and CF.

APPENDIX D

MOTION GEOMETRY UNIT TEST

. Which of the following circles are congruent?

A. X and Y
B. Y and Z
C. X and Z
D. X, Y and Z



. Which pair of figures is congruent?

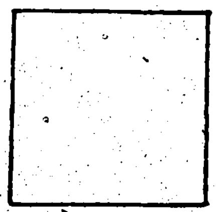


FIG. 1

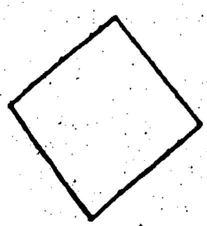


FIG. 2

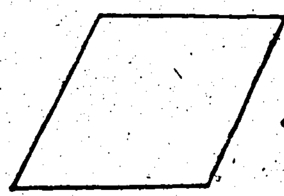


FIG. 3

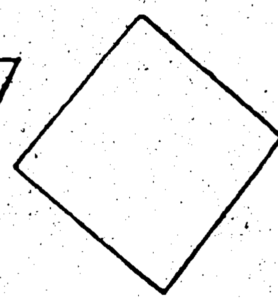


FIG. 4

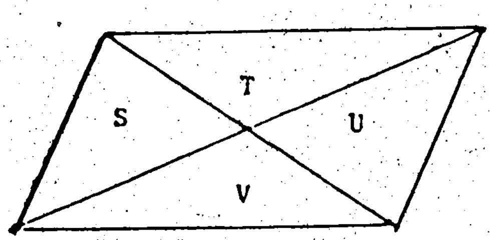
A. 1 and 2
B. 1 and 3
C. 1 and 4
D. 3 and 4

If a tracing of one circle fits exactly on the other, the circles are said to be

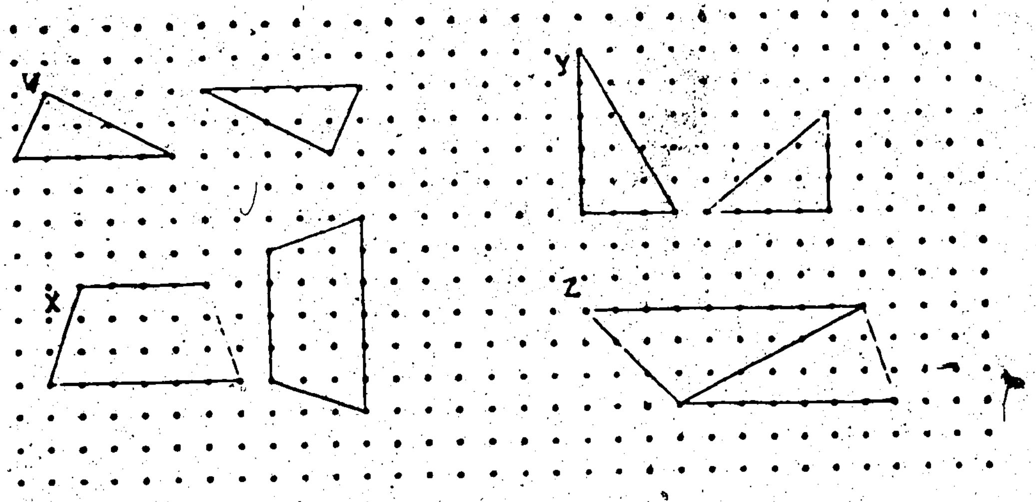
A. equal
B. the same
C. congruent
D. paired

. In the following parallelogram, which figures are congruent?

A. S and V are congruent
B. S and T are congruent,
U and V are congruent
C. S and U are congruent,
T and V are congruent
D. S, V, U and T are congruent



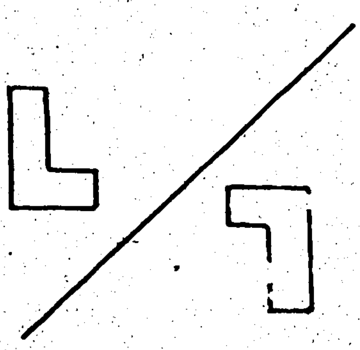
Which sets of polygons are congruent?



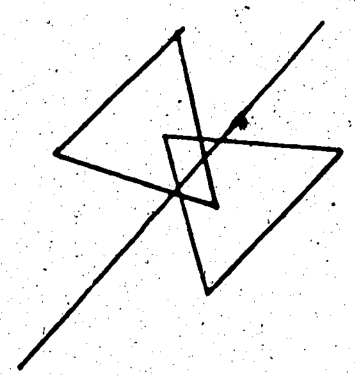
A. W and X
B. W and Z
C. W, X and Z
D. W, Y and Z

A pair of polygons produced by a flip is:

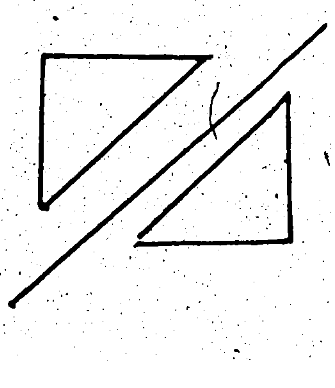
a.



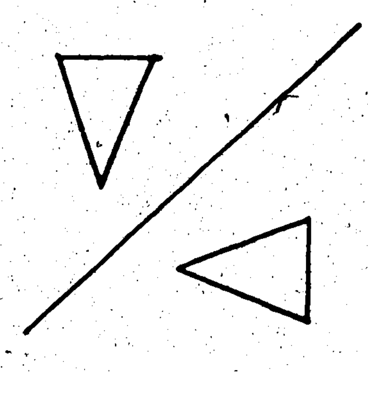
b.



c.

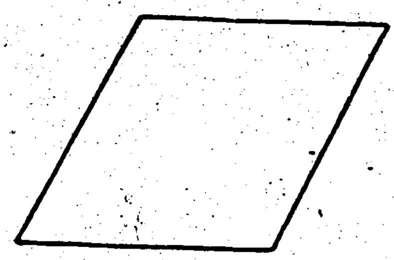


d.



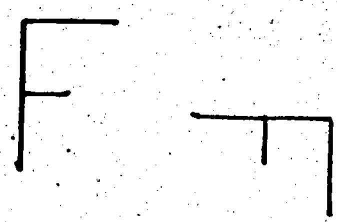
The number of "mirror symmetries" for the rhombus pictured to the right is

a. 0
b. 2
c. 4
d. infinite

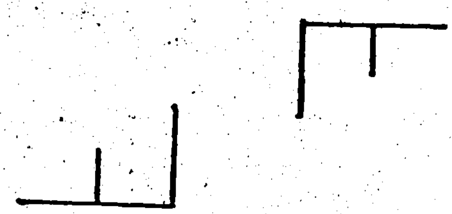


The pair of figures which was produced by a reflection is

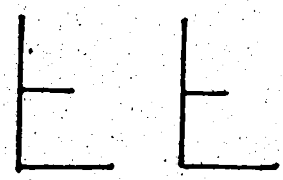
a.



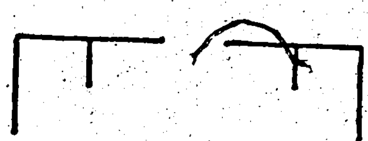
b.



c.

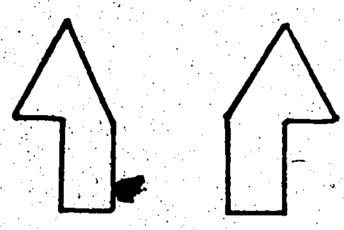


d.

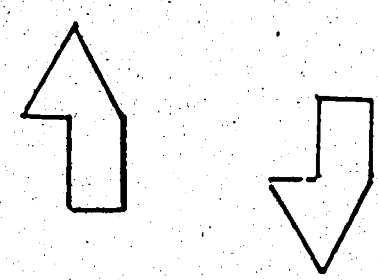


The pair of polygons produced by a slide is

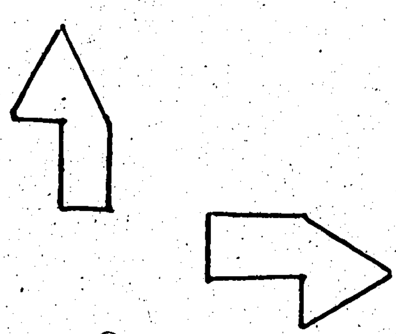
a.



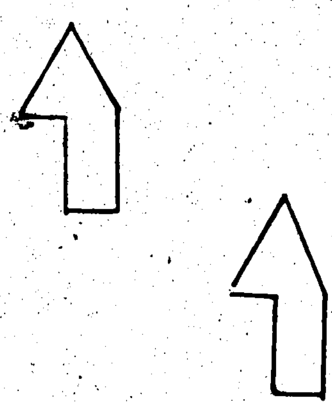
b.



c.

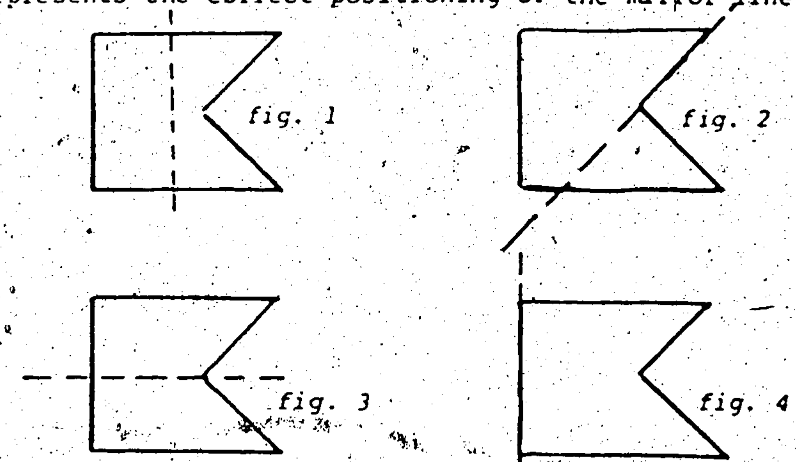


d.



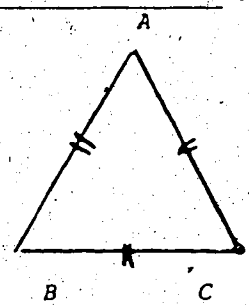
0. *Which figure represents the correct positioning of the mirror line?*

a. fig. 1
b. fig. 2
c. fig. 3
d. fig. 4



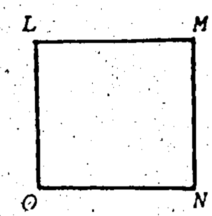
1. *Triangle ABC has ____________ axes of symmetry?*

a. 1
b. 2
c. 3
d. 4



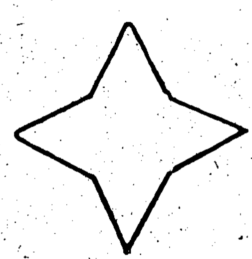
2. *Square LMNO has ____________ axes of symmetry?*

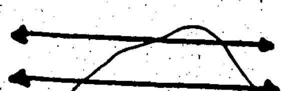
a. 2
b. 3
c. 4
d. 5



. *The star has ____________ axes of symmetry?*

a. 2
b. 4
c. 6
d. 8

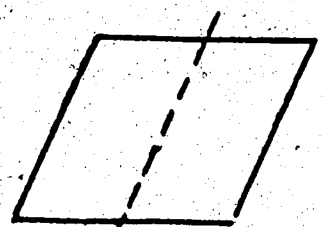


.  represents

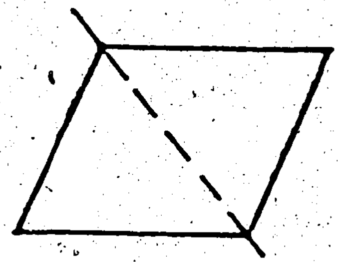
a. perpendicular lines
b. parallel rays
c. perpendicular segments
d. parallel lines

Which figure represents the correct mirror line?

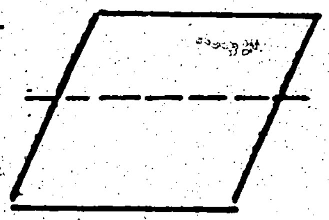
a.



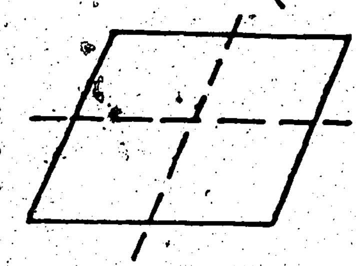
b.



c.

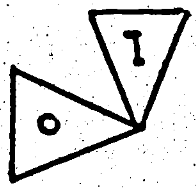


d.

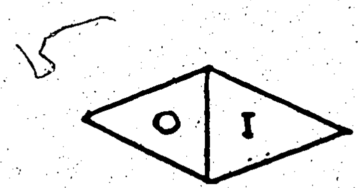


Which figure represents $\frac{1}{4}$ turn clockwise:

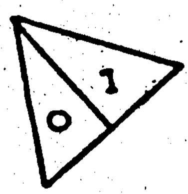
a.



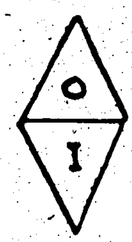
b.



c.



d.



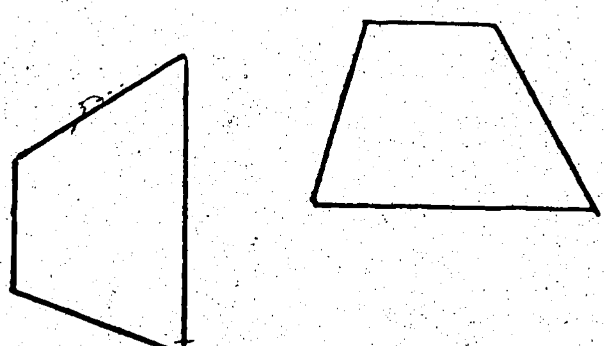
The pair of figures is produced by a turn. Which turn command would produce this?

a. 

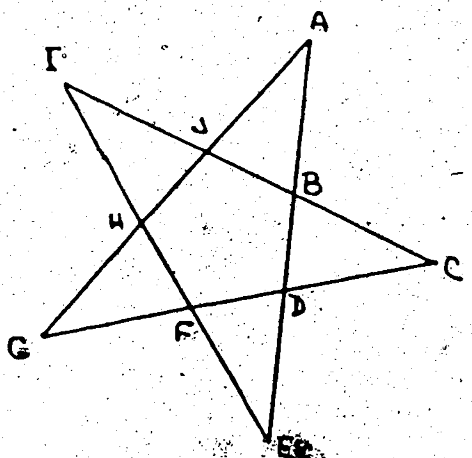
b. 

c. 

d. 



8.



Name a regular pentagon.

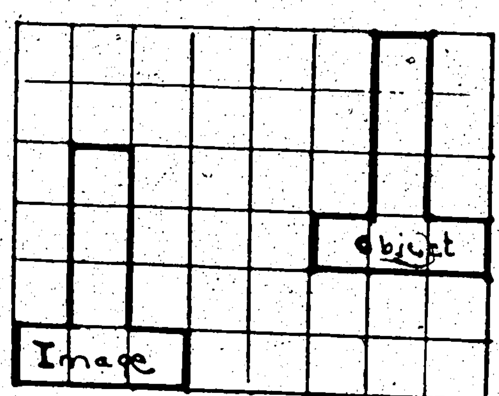
a) IHGDB

b) AGFE

c) JBDFH

d) BCD

9. Give the rule for the slide.



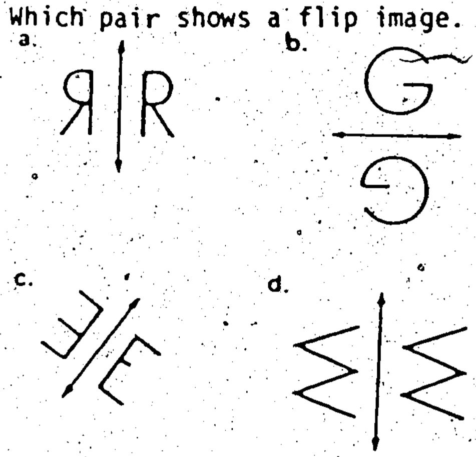
a) 5 L

b) 2 D

c) (5 L, 2D)

d) (2D, 5 L)

1. Which pair shows a flip image.



. Each angle of a square measures:

a. 45^{0}

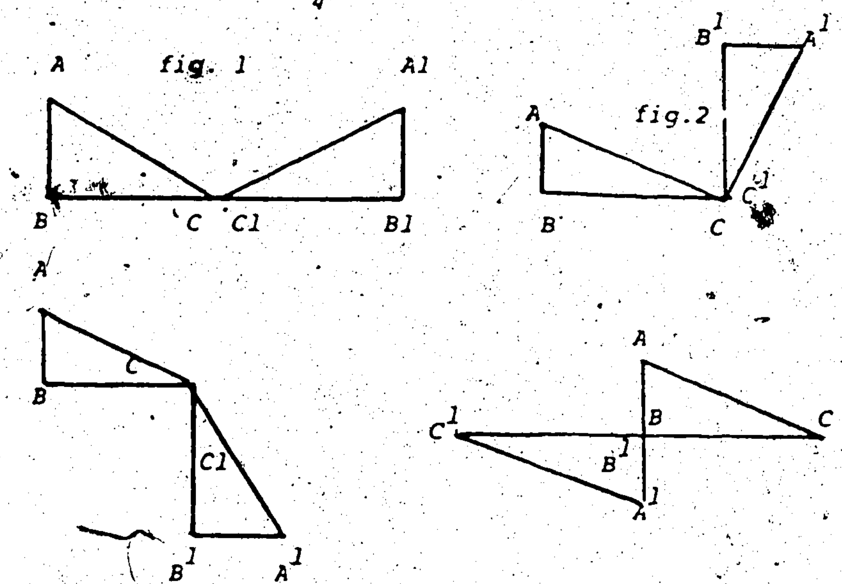
b. $22\frac{1}{2}^{\,0}$

c. 90^{0}

d. 180^{0}

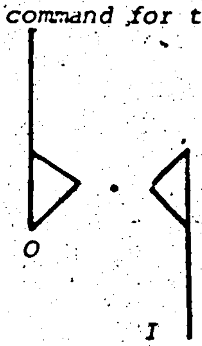
22. *The correct representation for a $\frac{1}{4}$ turn clockwise is*

a. *fig. 1*
b. *fig. 2*
c. *fig. 3*
d. *fig. 4*



23. *The correct turn command for the figure below is*

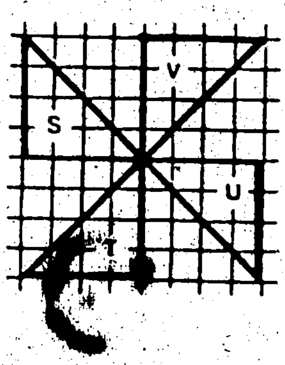
a. 
b. 
c. 
d. 



24. *A polygon with 4 turn symmetries is*

a. *square*
b. *parallelogram*
c. *triangle*
d. *rectangle*

25. Which motion maps S onto U.

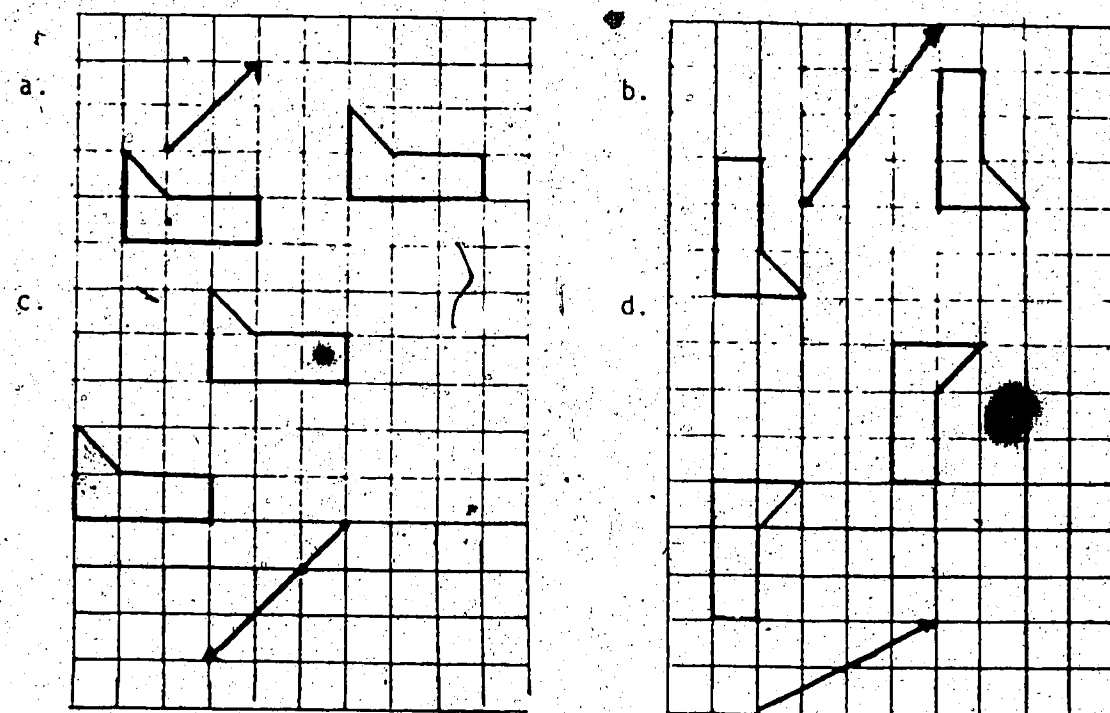


a) slide
b) flip
c) turn
d) enlargement

26. What slide should be added to (3R, 7U) to obtain a single slide of (5L, 2D)?

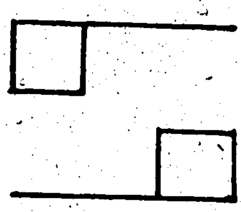
A. (2L, 5U)
B. (8R, 9U)
C. (2R, 5U)
D. (8L, 9D)

27. The pair of figures which was produced by the indicated slide arrow is:

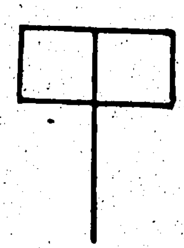


28. A pair of figures produced by a $\frac{1}{2}$ turn counter clockwise is:

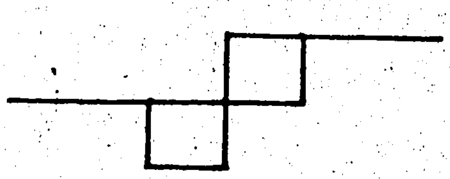
a.



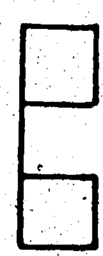
b.



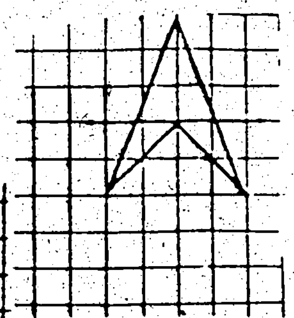
c.



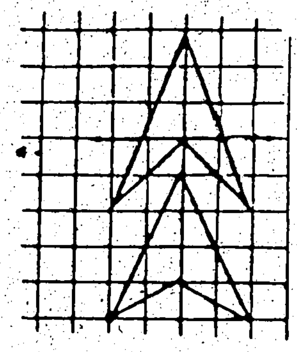
d.



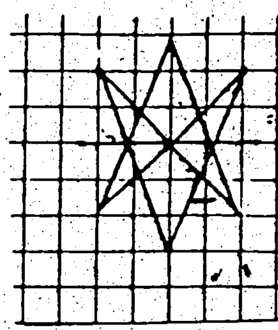
29. The flip image of the figure would be represented by:



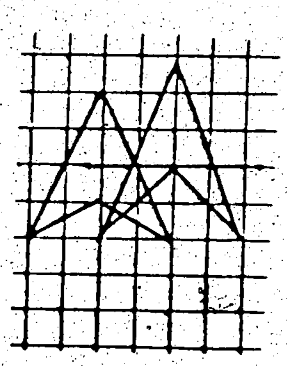
a)



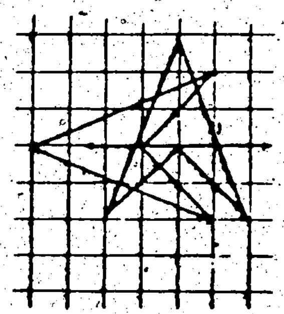
b)



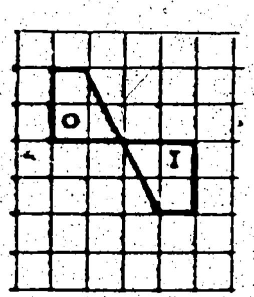
c)



d)



30. Name the turn angle that map the object unto its image.



a) 90^0

b) 180^0

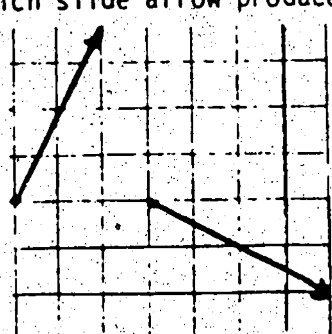
c) 270^0

d) 360^0

31. Which slide arrow produces the slide? (4R, 2U)

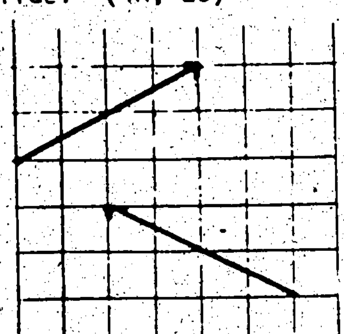
a.

b.



c.

d.



32. Which motion occurs in this activity: turning a door knob.

a) slide

b) flip

c) turn

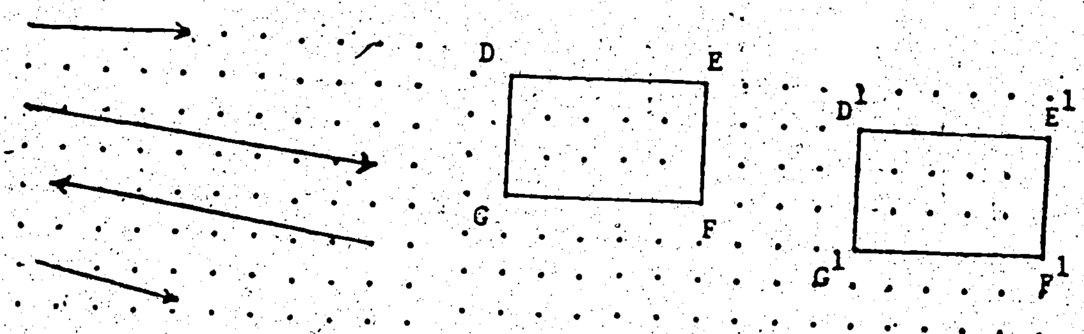
d) enlargement

33. (3L, 1U) is used to get the image. From this image a second image is obtained by (4L, 3U). The slide notation that would describe the second image from the original would be

A. (1L, 2U)
B. (7L, 4U)
C. (7L, 2U)
D. (1L, 4U)

34. Given Figure DEFG and its image $D^1E^1F^1G^1$, find the slide arrow.

A.
B.
C.
D.



35. If we make two successive reflections and the lines of reflection are perpendicular, we could obtain the same result by using a

A. slide
B. rotation
C. flip
D. mirror