

**University of Alberta**

Economic and Environmental Benefits from Growing Winter Wheat in the  
Prairie Provinces: a Bioeconomic Approach

by

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## **ABSTRACT**

Winter wheat fields provide upland nesting habitat for migrating birds. Duck nests built in winter wheat croplands experience lower probabilities of nest mortality due to farming practices compared to nests built in spring wheat croplands. Two dynamic optimization models are specified in order to measure economic (producer's profit) and environmental benefits (mallard population) derived from increases in winter wheat acreage in the Prairies. The first model, maximizes the farmer's revenue due to spring and winter wheat production, subject to mallard population dynamics. The second model uses a social planner point of view to maximize both the farmer's revenue obtained from wheat production, and social benefit associated with mallard population. The connection between duck population and winter wheat is specified using a logistic growth function where the intrinsic growth rate is a function of winter wheat acreage, and carrying capacity sets the maximum numbers of ducks in a specific area.

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## **Chapter 1: Introduction**

Winter wheat has gained popularity in the Prairie Provinces during the last decade because of the potential economic opportunities for farmers and the environmental benefits associated with its growth. The objective of this thesis is to develop mathematical models to calculate optimal acreages of winter wheat based on market opportunities and relationships with waterfowl populations. The profit/benefit maximization problems are specified in private and social contexts in order to compare results between models.

Wheat is the crop with the largest area seeded in Canada with approximately 36% of the total 2008 cropland (Statistics Canada 2008). Wheat is classified into three main categories: spring wheat, winter wheat, and durum wheat. Durum is the hardest type of wheat; it has high protein content and is mostly used for human consumption in the production of couscous and pasta. Spring wheat is also used mainly for human consumption in the production of flour for different types of bread and crackers; it can also be used for the production of noodles and some types of pasta. Winter wheat is used for human consumption in the production of low-to-medium protein requirement products and animal feed. Spring wheat is normally seeded in May and it takes about four months to complete its cycle to be harvested. Although durum wheat can be planted in spring or fall, almost all Canadian durum is seeded in the spring (USDA 2004). On the other hand, winter wheat is seeded in the fall; usually in second week of September and its harvest starts mid July, just a few weeks before spring wheat is harvested. The optimal seeding date and length of growth cycle may vary depending on the specific production area, weather conditions, seed and soil types, and other variables.

In 2008, 93.5% of the total acreage of spring and winter wheat in Canada was seeded in Alberta, Manitoba and Saskatchewan; 69.6% of this area was spring wheat, 22% was durum wheat, and 8.6% was winter wheat. Winter wheat still has a relatively small share over the total sown wheat fields. However, the seeded area of winter wheat has grown

over the last 20 years while the area seeded to durum wheat has remained relatively stable and spring wheat's area has decreased. These facts suggest the existence of incentives to farmers to grow less of the spring seeded crop and more of the fall seeded one. These incentives may be lower production costs, higher yields, better revenues for winter wheat, and reduced disturbance of wildlife, especially waterfowl population (Fowler 2002).

Agricultural practices have intensified considerably during the last five decades in Western Canada, which represents two thirds of the Prairie Pothole Region (PPR). The PPR of Western Canada is a region where approximately "eight million waterfowl and 20 million shorebirds use its wetlands and surrounding habitat to mate, nest, hatch and raise their young" (Ducks Unlimited Canada 2008). The PPR is considered the most important waterfowl breeding ground in North America. As a result, an important amount of natural breeding habitat for migratory waterfowl, including mallard ducks, has been converted into cropland (Klett et al. 1988; Bethke 1995; Greenwood 1995). Since the majority of crops grown in the Canadian Prairies are seeded in the spring and these imply the use of management practices such as tilling, the establishment of these crops contributes to low levels of waterfowl nesting success (Devries and Moats 2008). In addition, farmers consider waterfowl and other wildlife breeding near agricultural areas a nuisance because they interfere with the normal evolution of crop growth (Johansen and Skonhofs 2004; Skonhofs 1997; Clark et al. 1986). This situation intensifies the conflict between private human practices such as farming, and social concerns such as duck population protection.

On the other hand, areas seeded with winter wheat are a favorable upland nesting habitat for some species of migrating birds such as mallard ducks. Winter wheat fields contribute to higher rates of nest success of migrating waterfowl mainly because nests are kept undisturbed from farming practices (Thoroughgood 2008). Farmers who grow winter wheat as an alternative to spring wheat provide higher mallard nesting success by decreasing nest mortality associated with spring seeding management practices. Given that winter wheat fields provide a good breeding habitat to waterfowl, it is important to

adopt plans of action to encourage farmers to choose the fall seeded crop over those seeded in the spring.

Some of the programs available in the Canadian Prairies to increase winter wheat acreage either try to increase the profitability of winter wheat or decrease risks related to its production. Direct payments to farmers are an example of strategies designed to increase farmer's revenue. Ducks Unlimited Canada developed a program in some specific regions of the Prairie Provinces, where new growers of winter wheat receive a one time payment (of approximately CA\$10) per acre as an incentive to switch from spring to winter wheat. On the other hand, research focused on the development of new winter wheat varieties more resistant to cold, diseases and/or drought has been initiated in order to decrease farmers' production risk. Since "low temperature damage to the crown of the winter wheat plant during periods of cold is the main cause of winterkill on the Canadian prairies" (Fowler 2002, Chapter 12), and profit is one of the main drivers in the farmers seeding decision, two specific policies are going to be investigated in this thesis: improvements of winter wheat cold tolerance and direct payments to farmers.

Mallard ducks (*Anas platyrhynchos*) are one of the most harvested game birds breeding in the PPR (Cowardin, Gilmer and Shaiffer, 1985). Therefore they have high environmental value to society. The literature about mallards is extensive, for that reason there is an important amount of data available to do further research. Mallards represent an important component of the total waterfowl population in North America (Johnson, Sparling and Cowardin, 1987) therefore they can be used as a proxy for all ducks.

The mallard population in the PPR has experienced important oscillations in the last 55 years; however, numbers have evidenced a long-run decreasing trend. Between 1970's and mid 1990's mallard populations experienced low numbers reaching a minimum of approximately 5 million ducks in 1985 (Zimpfer et al. 2008). During this period research identified the main causes of low mallard numbers as loss of habitat due to agricultural expansion and intensification, and due to natural conditions such as climate change (Bethke and Nudds, 1995; Greenwood et al. 1995; Couinard et al. 2005). Measures such

as the North American Waterfowl Management Plan were undertaken to improve mallard and other waterfowl numbers to the same level they had in the mid 70's, approximately 7.5 million (U.S. Fish and Wildlife Service 1986; van Kooten 1993a). As a result, the duck population increased in the mid 90's, but dropped again in 2002. According to the results of the waterfowl breeding population survey developed by the US Fish and Wildlife Service, mallard numbers have not recovered since then.

Since an important component of waterfowl natural breeding habitat in the PPR is being used for agricultural practices, actions intended to increase waterfowl numbers should concentrate either on converting croplands into wildlife protected areas, or adopting environmentally friendly agricultural practices. Winter wheat production is an alternative that may provide economic benefits to farmers who grow it and environmental benefits to society because it supplies breeding habitat for various species of ducks. However, incentives to grow winter wheat need to be strong for farmers to switch from spring seeded crops to the fall seeded wheat. Cold tolerance is an important factor in winter wheat seeding decision. Cold tolerance improvements allow the plant to survive in areas where winters are colder or even longer. Therefore, regions where farmers have not traditionally grown winter wheat, because low temperatures would almost guarantee winterkill, can potentially be used to grow cold tolerant winter wheat varieties. Moreover, at lower levels of production risk due to winterkill, producers that have not included wheat in their crop rotation may be encouraged to do so; and wheat producers that have grown exclusively spring wheat in the past may be motivated to introduce winter wheat into their cropland.

The objectives of this thesis are first to mathematically model the relationship between farmers' decision to grow spring seeded or fall seeded wheat, and the size of mallard population under two scenarios: private and social. The private model is specified from a farmer's perspective where seeding decisions are only affected by production related factors such as price, cost and yield. The social model is specified from a social planner point of view where environmental and agricultural economic benefits are introduced as determinants of the farmer's seeding decision. The two models are specified in a dynamic

optimization framework using Hamiltonian functions to find optimal duck population numbers and optimal spring and winter wheat acreage that maximize private and social benefits. A second goal is to determine the policy that provides a bigger payback in terms of winter wheat acreage, either direct payments to farmers, or improvements in cold tolerance in this math model framework.

The farm level model's goal is to find the optimal amount of seeded area between spring and winter wheat that maximizes the farmer's total profit subject to a land constraint and a mallard population growth function linked to winter wheat acreage. The connection between the duck population and winter wheat is specified using a logistic growth function where the intrinsic growth rate is a function of winter wheat acreage, and the carrying capacity sets the maximum numbers of ducks a specific area can sustain. It is assumed that farmers do not assign any *value* to waterfowl and that duck population represents a cost to them. Costs arise because losses in crop yield are experienced in areas where ducks graze, trample and foul over swathed spring crops.

The social planner model's objective is to maximize producer's profit and society's benefit related to the mallard population, subject to a land constraint and a mallard population growth equation. The goal is to find the optimal seeding decision (acreage) of spring and winter subject to the available amount of land, revenue obtainable from each crop, and the duck population growth obtained from the improvements of nest survival rates influenced by the availability of winter wheat fields. Total benefit includes the income generated by the production of both, winter and spring wheat, direct payments from public or private sources seeking to stimulate wildlife protection, and social benefit generated by use, non-use and existence values of waterfowl (e.g. recreational services, hunting, bird watching, etc.). The main differences between the first and second models are that the first one focuses solely on the producer's benefit (production's profit) and that there is no explicit value for the duck population.

Optimum acreages for winter and spring wheat and mallard population are expected to be different between the two models. If economic and environmental benefits involved with

growing fall seeded wheat can overcome spring wheat profits, winter wheat acreage and duck optimum levels are expected to be higher in the social planner model. Results obtained from the two mathematical models are useful input to build policies that seek to encourage farmers to grow winter wheat as a strategy to increase mallard nesting success and therefore increase their population in the Canadian Prairies.

## **Chapter 2: Literature Review and Background**

The objective of this chapter is to give insights on wheat farming practices, the mallard population in the Prairie Provinces in Canada, and the way these may affect each other. Also, the methodologies that can be used to model interactions are reviewed in order to calculate optimal levels of Mallard ducks population versus spring and winter wheat acreage.

### ***2.1. Winter wheat***

Winter wheat is a variety of wheat seeded in the fall between the end of August and early September depending on location, and is harvested at the end of July or early August of the following year. Winter wheat's life cycle has 10 different stages (Fowler 2002, Chapter 10). The first stage is germination when the seed sprouts and begins to grow. The second is the seedling stage, in which the plant develops its first leaves. As winter wheat overwinters as a seedling, this is the stage in which winter wheat experiences higher levels of stress. "In order to cope with these stresses, winter wheat has evolved adaptive mechanisms which are temperature regulated and involve acclimation processes that can be reversed" (Fowler 2002, Chapter 12). This process allows the plant to survive the cold temperatures of winter season. This stage is very important because it is during this time plant survival is determined. The third stage is tillering, where the young plant develops its first branch. Stage 4 is where the lengthening of the stem occurs, followed by booting (stage 5), heading (stage 6) and flowering (stage 7). The eighth stage is the where the kernel formation begins; right after is the development stage (stage 9) where the kernel formation is completed. Finally, stage 10 is ripening, where seeds lose moisture and are ready to harvest. The complete growing process is depicted in figure 2.1.

Understanding the winter wheat acclimation process and the ways producers' management decisions can influence it, are very important to reduce the risk of winter kill. The most important management practices that help to increase probability of plant

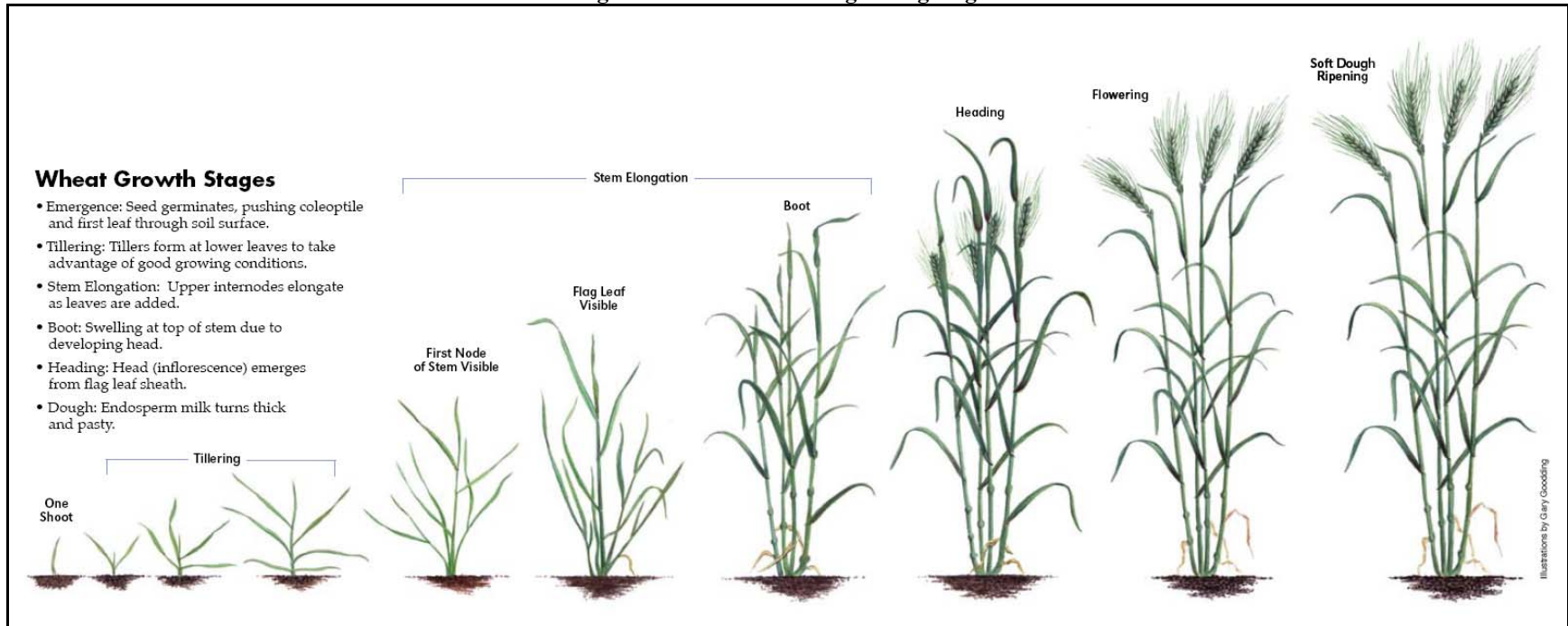
survival are: seeding date, seeding depth, and phosphorus and nitrogen fertilization (Fowler 2002, Chapter 7). Seeding date is important because it determines the amount of time at the right temperature that winter wheat has to acclimate. Weeks after seeding, cold acclimation of winter wheat begins when soil temperature falls under 9° in the fall, and is fully reached after 4 to 8 weeks under these temperatures. If during this period temperatures increase over 9°, cold hardiness (resistance) is gradually reduced and probabilities of survive winter temperatures decrease. Seeding date is very important because if seeded too early, winter wheat plants may experience excessive growth and are less resistant to cold temperatures; while if seeded too late plants will not be as vigorous and healthy to develop cold hardiness. Seeding depth determines how fast emergence occurs. If the seed is located too close to the surface, it may be overexposed to weather causing emergence not to occur; while, if seeded too deep, emergence is delayed resulting in weak plants that are more susceptible to damage from winter stress (Fowler 2002 , Chapter 12). Finally, utilization of the right amount of fertilizer gives plants the strength to optimally recover from winter damage.

There are other variables that also affect winter wheat cold tolerance and therefore winter survival. Examples are soil temperature and moisture and their relation with snow cover (Fowler 2002, Chapter12). A thick snow layer over the ground is usually required to prevent soil temperatures from falling below the minimum survival temperature (MST) for wheat. The MST varies depending on winter wheat variety, on the cold hardiness process, and management practices such as seeding date.

Winter wheat seeded area has grown in the Prairie Provinces in the last ten years (Figure 2.2). In 2008 Alberta's farmers seeded 300 thousand acres of winter wheat representing a 4.26% share of the total wheat cropland in that province. The winter wheat seeded area in Saskatchewan 2008 was two times the area in Alberta, representing a 4.49% share on its total wheat area. Winter wheat area in Manitoba was approximately 620 thousand acres on 2008, representing 19% of the province total area seeded with wheat. Possible reasons for this increase in winter wheat acreage are economic and environmental advantages involved with growing it.

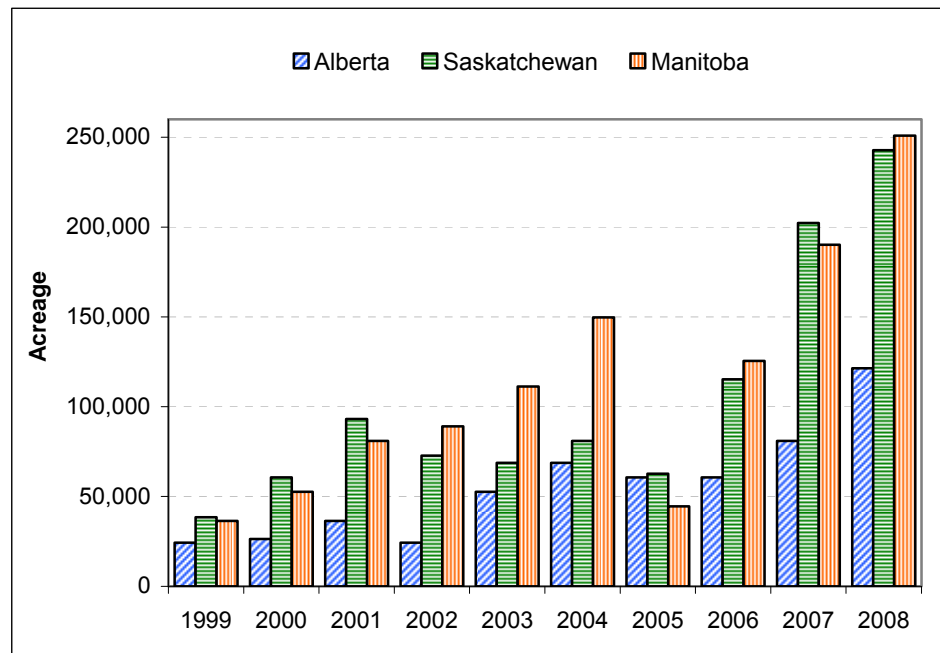


Figure 2. 1. Winter wheat growing stages



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**Figure 2. 2. Total winter wheat acreage in the Prairie Provinces (1999-2008)**

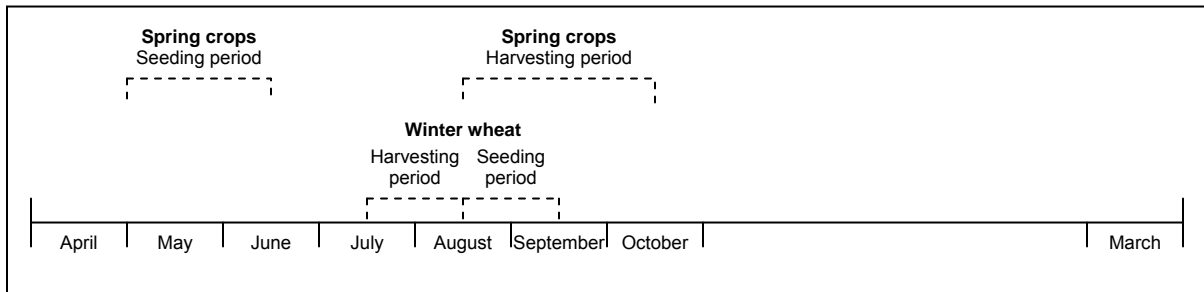


Data Source: Statistics Canada 2008

Although winter wheat area has grown in the Prairie Provinces, its proportion with respect to total wheat area remains relatively low. The majority of wheat producers prefer to grow spring wheat. The reason why farmers are still reluctant to adopt winter wheat varieties may be the challenges they have to deal with in the first year they grow winter wheat. Some of these challenges are:

1. Overlapping machinery use. Since vehicles and machinery are needed for harvesting spring crops and seeding winter wheat, time management may be an issue (Figure 2.3).
2. Labor overlapping is also a time management issue because labor is needed for both harvest of spring crops and seeding of winter crops. Farmers may have difficulties finding the extra labor needed for both activities.
3. Timing on the application of wild oat herbicide. The herbicide can only be applied when plant has already two or three leaves. If it is too cold at the time of application, the herbicide might lose effectiveness.
4. A special type of fertilizer like ESN (controlled release nitrogen fertilizer) is needed to grow winter wheat. If a normal fertilizer is used and it is applied at seeding time, it might not stay on the soil as long as needed (after winter).

**Figure 2. 3. Seeding and harvesting periods for winter wheat and spring crops**



Growing winter wheat has important economic and environmental implications that affect farmers and the rest of society. The most important economic implications are the farmers' profit. Winter wheat is characterized by having higher yields than spring wheat (including durum). Winter wheat grown in Alberta has averaged 8.39% higher yields compared to spring wheat over the last 20 years (Statistics Canada 2008). The same measure for Saskatchewan and Manitoba give values of 12.67% and 24.43% respectively. In addition, costs related to winter wheat production are lower than spring wheat production costs, particularly because of lower usage of chemical such as herbicides, insecticides or fungicides. On average, winter wheat production cost per acre can be from \$5 to \$15 lower than spring wheat cost per acre, depending on location (AAFRD 2008; MAFRI 2008; SAF 2008). Winter wheat higher yields and lower production costs are variables that potentially increase farmer's profit. However, winter wheat contains a lower protein content which translates into lower quality class and therefore a lower market price. On average, a bushel of winter wheat is \$0.62 less than the same quantity of spring wheat (CBW 2008). Still, the gain in profit due to higher yield, under average levels of winter kill (5%), and lower production costs may overcome the profit loss due to lower prices. Consequently, winter wheat net revenue per acre is potentially higher and therefore more economically profitable.

The environmental effects of growing fall seeded wheat are indirect, and are related to less wildlife disturbance in the spring, more efficient water utilization and less pollution caused by chemicals used for crop production (Fowler 2002). Since the environmental impact from growing winter wheat addressed in this thesis is related to mallard population, this section will be focused on wildlife disturbance.

Spring wheat is seeded in the spring, between early May and mid-June, at the same time that ducks who migrated through the Central Flyway<sup>1</sup> have already started their breeding season. At that point, thousands of female mallard and other duck species have built their nests all over the Prairie Pothole Region (PPR) area, including croplands. Since growing spring wheat and other spring seeded crops involve tillage practices to prepare the soil for seeding, a significant number of these nests are destroyed. On the other hand, winter wheat is seeded in the fall, between mid-August and mid-September. In fall the breeding season is over and migratory birds are preparing to fly south. In addition, winter wheat management practices do not involve tillage; therefore young ducklings' habitat is not disturbed. In conclusion, fall seeded crops have environmental advantages over spring seeded crops because its management practices do not disturb duck and other migrating birds breeding habitat less than spring crops.

Private or public organizations that are interested in protecting waterfowl and maintaining their habitat, may promote winter wheat adoption through different means. Ducks Unlimited Canada (DUC) uses three primary tools for encouraging the adoption of winter wheat: “a one time cash payment per acre of winter wheat to first time growers, extension of information through DUC agronomists and producer promoters, and media advertising of the production advantages of winter wheat relative to spring wheat” (Thoroughgood, 2008)<sup>2</sup>. It is believed that the DUC program explains in large part the growth of winter wheat acreage in the Prairie Provinces over the last decade. Although, DUC is apparently the only organization providing this type of payment to encourage farmers to grow fall seeded crops in Canada, the concept of using payments to encourage farmers to protect waterfowl habitat has been explored in the literature. Van Kooten (1993a), discussed how even though migratory birds have non-market value (this topic is discussed in Section 2.3.2), these values are not captured by land owners. Therefore it is important to encourage farmers to retain waterfowl habitats. The author also remarked on the importance of the design of the right type of payment or subsidy in order to

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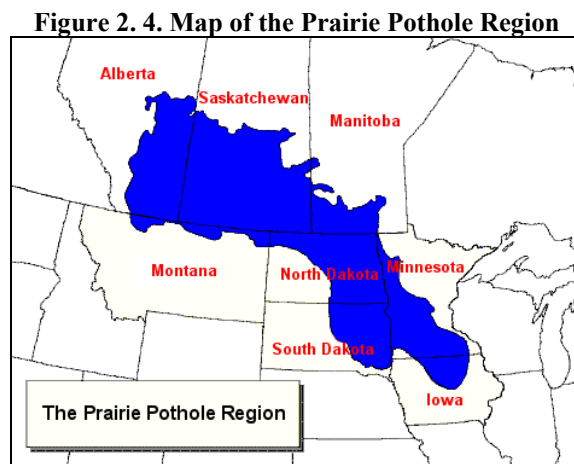
<sup>1</sup> The Central Flyway is a bird migration route that generally follows the Great Plains in the United States and Canada.

<sup>2</sup> Personal communication with Paul Thoroughgood. Regional Agrologist Western Region, Ducks Unlimited Canada.

effectively influence farmers' behavior and obtain the desired increase in waterfowl population.

## ***2.2. Mallard duck population***

The most important North American breeding grounds for ducks are the Prairie Pothole Region (PPR) of Alberta, Saskatchewan, Manitoba, Montana, North and South Dakota, and Minnesota (Cleary 1994). Over 80% of the PPR is located in Canada (Figure 2.4). Historically, this area probably produces more ducks than the rest of the continent combined (Cleary 1994). On average, 21.6 million ducks (about 51.1% of all estimated populated surveyed in the continent) used the PPR to breed between 1955 and 1985 (Greenwood et al 1995). However, an important section of the North American waterfowl breeding, migrating, and wintering areas are changing because of agricultural and land-clearing practices, northern prairie pothole drainage, and other projects that require land use reallocation. These activities have reduced the natural habitats of waterfowl and other birds affecting the total waterfowl population size.



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Mallards (*Anas platyrhynchos*) are dabbling ducks that feed by dabbling in shallow water and are often seen in the tipped-up position with their tail held vertically out of the water while looking for food beneath the surface. For that reason dabbling ducks are most often

found near water. Mallards can, however, fly long distances to and from favorite feeding grounds, which may include agricultural or upland sites (Cleary 1994). This species of duck is extremely adaptable, which means that they can survive both rural and urban environments. Primarily vegetarians, mallards feed on leaves, seeds, berries crop grains such as wheat, barley and corn, small worms and fish, insects fresh water snails and fish eggs (Goode, year not available).

Mallards select mates in the fall but do not breed until late March and April of the following year. Female and male search for an appropriate nesting site, usually one close to where the female was hatched. It is common that females come back year after year to the same nesting site even when habitat conditions have changed. Availability of upland areas suitable for nesting in regions traditionally used by waterfowl for that purpose, are important to assure duck population sustainability. Females may re-nest up to four times if her nest is destroyed, but will lay fewer eggs in each attempt due to lower energy reserves (Goode, year not available). The incubation period for a mallard nest is about 22 to 28 days. And each nest might have up to eight eggs. After breeding is over, usually between September and October, mallards migrate to central-south United States or even North of Mexico where water sources stay ice free (Goode, year not available).

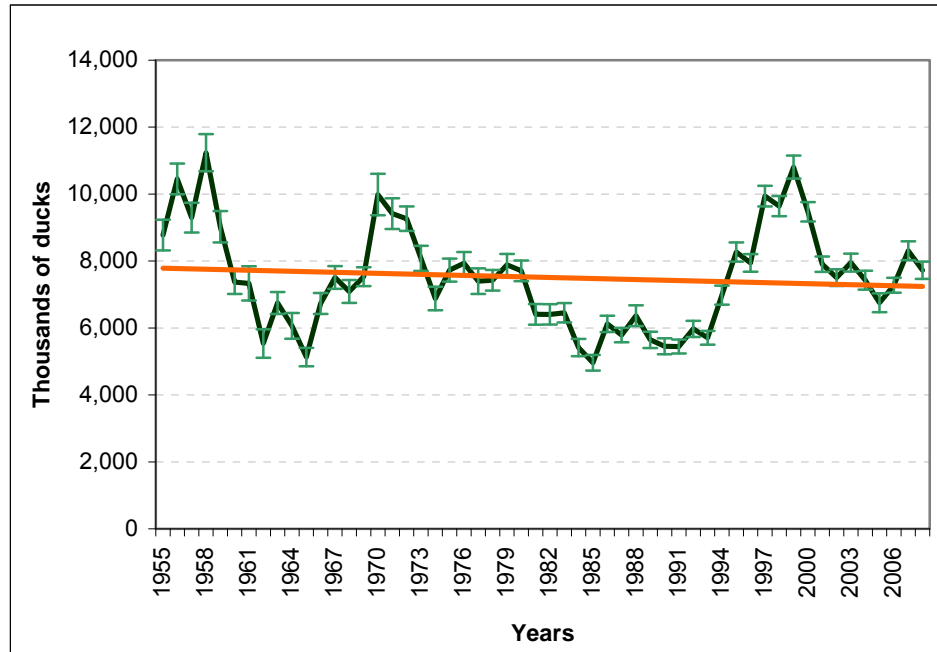
### **2.2.1. Change in waterfowl populations through time**

Waterfowl population numbers are characterized by cyclic behavior through time (Figure 2.5), experiencing low and high numbers at different periods<sup>3</sup> depending on habitat variability, weather conditions, and external shocks. However, several species of ducks, including mallards, have experienced a long-term decreasing trend in their population size. According to Bethke and Nudds (1995), the factors influencing the diminution in the numbers of breeding waterfowl in the Canadian parklands can be grouped in two main categories: habitat loss due to agricultural expansion and intensification, and habitat loss due to natural conditions such as climate change. Cowardin and Johnson (1979) argue that mallard survival, and therefore sustainability of their population, depends on a combination of hunting mortality, winter and breeding season survival.

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<sup>3</sup> Conclusion based on observation of data from Zimpfer et al. (2008)

**Figure 2. 5. Estimates of Mallard population size and confidence intervals (thousands) in the U.S. Fish and Wildlife Service's Traditional Survey Area (TSA)<sup>4</sup>**



Data source: Zimpfer et al. (2008)

Some authors have identified low nesting success due to changes in nesting habitat as the main cause of waterfowl population decrease in the PPR (Arnold et al. 2007). According to Klett et al. (1988:431), “Nest success rate in the PPR is a critical determinant of duck production and size of the fall flight.” The authors identified nest predation as the main cause of nest mortality, explaining 82% of unsuccessful mallard nests. In second place, agricultural practices such as tillage explained 7% of all destroyed nests. Greenwood et al. (1995) agreed that the main reason for the decline of mallards in the PPR was the low nesting success. Furthermore, the authors showed that estimated rates of nesting success (11%) were low compared to the levels necessary to assure population stability (15%). And, even though they agreed with the fact that predation was the main cause of destroyed and abandoned nests (78% of the cases), farming practices only explained 3% of the cases of nest mortality. In their case weather conditions and other causes had a higher impact on nest success. However, the authors found that the nest success was negatively correlated with the proportion of land cultivated annually.

<sup>4</sup> The Traditional Survey Area is the area where the U.S. Fish and Wildlife Service does the annual Waterfowl Breeding Population and Habitat Survey. The TSA's map is presented in Appendix 1.

In more recent research, factors other than nesting success were analyzed to explain mallard population dynamics (Hoekman 2002; Coluccy et al. 2008). In both studies, authors estimated the effect of vital rates of mallard's population<sup>5</sup> such as duckling survival, nest success, and survival of adult females during and outside breeding season, on population changes over time. Although both studies underlined the importance of knowing how mallard recruitment<sup>6</sup> is affected by different vital rates and how this knowledge helps to design better policies and obtain better outcomes, results differed between the two studies (Table 2.1). While Hoeakman (2002) identified nest success as the most important factor affecting mallard population, Coluccy et al. (2008) found that duckling survival and survival of breeding females were the variables affecting the most population dynamics. The possible explanation to such different findings is the area where each study took place. While Hoekmann's research involved the PPR of the United States and Canada, Coluccy et al.'s was done for mallards located in the Great Lakes States. It is apparent that landscape characteristics have a strong effect on Mallard vital rates and therefore their population dynamics.

**Table 2. 1. Estimated effects of vital rates of mallards over their population growth rates**

<i>Vital Rates</i>	<i>Coluccy et.al (2008)</i>	<i>Hoekman (2002)</i>
Nest Success	16%	43%
Duckling survival	32%	14%
Survival of adult females during breeding season	1.4%	19%
Survival of adult females outside the breeding season	36%	9%

Research by Sæther et al. (2008) supports the previous argument. The authors examined whether and how different geographical locations inside the PPR affect population dynamics of subgroups of ducks of the same species located in different areas. They found that mallard numbers varied significantly depending on location, and that the main

<sup>5</sup> Vital rate refers to different measures (rates) that describe birth and mortality of a population.

<sup>6</sup> Recruitment is the process of adding new individuals to a population. In the context of this thesis is a measure of biomass, which refers to ducklings that survive the breeding season.



variables affecting such variability were pond numbers, spring temperature, winter precipitation and latitude.

Pearse and Lester (2007) not only analyzed how vital rates affected mallard population but how the interaction between these factors contributed to changes in duck numbers. The authors argued that if covariation between factors is not accounted for when performing sensitivity and elasticity analyses, the importance of individual variables can be biased. They concluded that there was a strong correlation between estimates of nest and duckling survival of mallards living in south-central Saskatchewan. The possible reason for this correlation is that each life stage may have similar environmental factors that influenced the survival of nests and ducklings, including weather and predation community.

Devries (2008) measured relative nest abundance and nest survival among crop types (spring and fall seeded) and tested the influence of various landscape-scale covariates on these measures. The author observed that nests initiated in cropland, especially early nests, faced risks of destruction by seeding, tillage and spraying operations in addition to predation. He also found that apparent nest density is higher in winter wheat fields (0.39 nest/ha) compared to spring crops (0.03 nest/ha); and that the daily survival rate of nests was significantly higher in areas seeded with winter wheat (38%) compared to spring crops areas (12%). Finally, he concluded that “fall-seeded crops provide an opportunity for the provision of safe nesting habitat on private land and landscapes that attract high waterfowl populations but are predominantly cropland” (p. 1796)

Following the trend of study of mallard survival on croplands, Hoekman (2006) estimated female recruitment and population growth in Southern Ontario and New Brunswick during 1992 and 2000. According to the author’s results, breeding productivity of mallards in agricultural environments was sufficient to maintain populations, given annual survival typical to the region studied. Higher recruitments were attributed to higher female success resulting from nest survival and nesting effort. These

results suggest that mallard populations are very sensitive to nest survival, not only in eastern Canada but the whole PPR.

Some of the measures proposed in the literature to help improve duck population numbers are to restore habitat by restoring wetlands, retiring marginal agricultural lands from production (Bethke and Nudds 1995; Simpson 2006); conversion of important amounts of cropland into dense nesting cover (Arnold et al. 2007); promoting alternative agricultural practices to conserve soil and water while conserving wetlands and upland nesting habitat such as growing more fall-seeded crops as an alternative to those seeded in spring (Devries et al. 2008); predator control to protect eggs, ducklings and hens (Hoekman 2002; Brasher 2006); and the use of nest structures to increase nesting success (Chouinard et al. 2005).

### **2.2.2. Crop damage by waterfowl**

“Waterfowl and other birds such as mallards, pintails, geese and cranes probably have been feeding on farmer’s crops ever since cultivation began” (Hubbard 1991: 2). However, crop damage by waterfowl generally occurs in small areas and at certain times of the year (around late summer and early autumn harvest period). Normally, only a few farmers suffer damage, but when they experience it, damage can be substantial (Hubbard 1991). In addition, drainage of wetlands to create new cropland has intensified crop damage by waterfowl (Knittle and Porter 1988). In the Canadian Prairies the damage consists of direct consumption, trampling and fouling of swathed grain left in the field to dry (Clark et al. 1986). In fact, farmers who grow spring crops often leave swathed grains to dry on the field before these are combined, creating an opportunity for migrating birds to feed on the seeds. “The short growing season, possible early frost, uneven soil types, and topography sometimes prevent the even ripening needed for straight combining” (Williams-Whitmer, Brittingham-Brant and Casalena 1996). Damage to standing crops can also occur but is less common. This depredation occurs because swathed crops are an easy meal for waterfowl, especially at a time of year when millions of birds are preparing for a long journey to the south. Furthermore, young ducklings learn from their mothers to

use croplands as sources of food and keep doing so year after year. Wheat, barley, oats and millet are usually the most affected crops (Hubbard 1991).

Waterfowl grazing decreases crop yields, not only because of the grain eaten but because of wasted grain. Trampling compacts swathed crops extending the time period required for drying and makes swaths more susceptible to freezing to the ground (Hubbard 1991). Also, compacted swathed crops are difficult for combines to pick up the grain. In any case trampling translates into yield loss. Fouling involves fecal contamination of the grain and results in decreased crop quality and implies a lower market price of the crop. In order to maintain high prices swathed crops need to be cleaned before they are sold in the market. As a result, fouling affects farm profits either by decreasing crop price or increasing production costs.

Estimates of yield loss are difficult to quantify because these depend on different factors such as crop variety, grain moisture, weather conditions, etc. In the literature it is generally accepted that damage from trampling and fouling is around two or three times damage from direct feeding. Fariaizl (1981) estimated that 1,500 ducks ate 13 bushels (236 grams per duck a day) of durum wheat in two days and trampled and fouled an additional 39 bushels (707 grams per duck a day) for a total loss of 52 bushels (943.5 grams per duck a day). MacLennan (1973) estimated that one field-feeding duck could destroy a minimum of 660 grams (0.02 bu) of wheat per day. Sudgen (1979) calculated that an adult male mallard could consume 95-115 grams of 14% moisture-content grain daily. In a similar study, Jordan (1953) found that one wild mallard could consume between 73 and 82 grams of small grain a day. Other studies estimated losses at about 1.26 million bushels of wheat and barley (about 34,291 million grams) over a four year period in Alberta, and values of 35 million dollars a year of depredation loss in Saskatchewan (Hubbard, 1991).

Waterfowl crop damage can be potentially decreased if farmers grow less spring seeded crops and more winter crops. Winter grains are normally harvested and straight combined in July and August, long before migrating waterfowl arrive to cropland areas. Moreover,

“even though, winter young plants might be vulnerable to grazing and puddling damage by waterfowl in both the fall and spring, research has shown that light grazing of the winter rosette can actually increase stooling and grain yield” (Cleary 1994:133).

A number of conclusions can be drawn from the literature reviewed in this section. First, mallards are ducks whose survival depends significantly on available breeding habitat. Second, expansion and intensification of agricultural practices, wetland drainage, and other environmental disturbances arising from human activities have deteriorated mallard breeding habitat and decreased their numbers for decades. Third, the main management activity that could achieve a sustainable mallard population is breeding habitat restoration; this can be accomplished through conversion of cropland into more suitable breeding habitats, predator control, or implementation of environmentally sustainable agricultural practices such as non-tillage fall seeded crops.

### ***2.3. Bioeconomic models***

Economic modeling has been traditionally used to find solutions to problems related to production and demand of goods and services, and the benefits obtained from these economic activities. A new generation of economic models such as environmental, ecological and resource economic models has been used to tackle topics of utilization and management of resources. Results obtained with these models are a useful tool for policy design. Traditionally, biological models have been used to understand how different variables affect populations of animals, plants, people and other living beings. Bioeconomic models are the application of environmental and ecological economic methods to empirical biology in order to accomplish sustainable economic utilization of natural resources. Bioeconomic approaches have been widely used to model fisheries and other wildlife population dynamics, water utilization, forestry and farming activities (Grafton 2004).

Three steps need to be taken in order to build a robust bioeconomic model and be able to use its results to make decisions on management matters. First, is to define the objective

of the model in terms of some specific goal. Second, one must model the relationship between the goal and the parameters of the resource to be managed. Third, what results from the model are estimates of the parameters of the relationships among model elements (Cowardin and Johnson 1979).

The first step is to define the problem that is going to be modeled. Some examples of bioeconomic problems are, to maximize population numbers, maximize society's welfare related to environmental goods and services, or to minimize management costs of achieving a specific wildlife population goal. At the same time, the restrictions inherent to the problem need to be identified. Maximum amounts of available cropland if dealing with agricultural production, or a specific growth population dynamic if dealing with wildlife, are examples. In this step, functional forms of the different economic and biologic/ecologic expressions need to be specified.

The second step is to identify and specify the interaction between the variables included in the model. In bioeconomic models there is usually a tradeoff between economic activities and the environmental activities (e.g. hunting and wildlife population, harvesting of forest biomass and carbon emissions, or agricultural practices and water quality). The final step is to give values to the parameters used in the model. This usually involves information about environmental and resource values when dealing with society's welfare topics.

In order to give a complete review of the tools and concepts used in this thesis, this section is composed of three subsections. The first one deals with population dynamics and functional forms used in biological model to depict wildlife population evolution through time. The second one deals with measurements of environmental values. And finally, a review of the utilization of bioeconomic models in the literature is provided.

### **2.3.1. Population Dynamics: Logistic Growth Function**

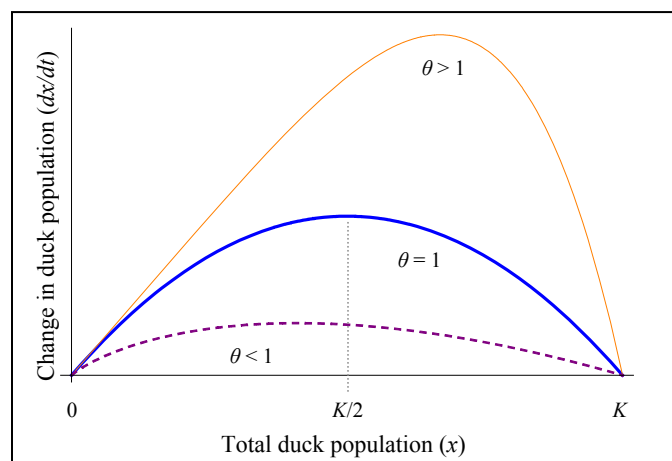
The logistic function is a population growth function widely used in the literature to model the intertemporal growth of a specific population (Gilpin and Ayala 1973;

Skonghoft 1998; van Kooten and Eiswerth 2007). It uses as parameters: the current population size ( $x$ ), the intrinsic growth rate ( $\gamma$ ) and the carrying capacity ( $K$ ) (Equation 2.1). The intrinsic growth rate is the rate at which a population is able to grow by its own natural conditions between successive time periods. The carrying capacity is defined as the population level of a specific species that an area can support given the food, habitat, water and other resources available within the ecosystem. In the context of mallard populations, the carrying capacity is the number of ducks that the habitat in a specific area can support without significant impacts on the population and the environment it self.

$$\frac{dx(t)}{dt} = \dot{x} = x(t)\gamma \left(1 - \frac{x(t)}{K}\right)^\theta \quad \text{with } \theta=1 \quad (2.1)$$

The logistic growth function describes the dynamic evolution of a population that is limited by the resources available in a specific area. Therefore, the population size ( $x$ ) changes depending on the capacity of the species to increase in numbers, and is limited by the competition for resources between members within the population. In other words, the population grows at a increasing rate until  $x$  equals  $K/2$ , the point at which population growth starts decreasing until  $x$  equals carrying capacity. Population dynamics ruled by a basic logistic growth function is pictured by the thick continuous line in figure 2.6. In this case, the function that relates population growth rate to population density is a parabola that intersects the x-axis at zero and  $K$  and is symmetric towards  $K/2$ .

**Figure 2. 6. An illustration of the logistic growth function**



Some variations to the logistic growth model have been used in the literature to introduce other variables affecting population dynamics such as density dependence, competition between species, and minimum viable populations. Anderson et al. (unpublished data) underlined how changes in the size of waterfowl, or any other population, are controlled both by density-independent and density-dependent factors. “Density-independent factors cause populations to increase or decrease irrespective of the species abundance. Density-dependence involves a negative relationship between abundance per unit of a limiting resource, and mortality and/or reproductive rates as a result of intra-specific competition for essential resources” (Anderson et al.: 34). To include a measure of density dependence into the Logistic function, a parameter ( $\theta \neq 1$ ) is added as an exponent to the expression  $(1 - x(t)/K)$ ; this function is called the theta logistic growth function. As explained by Gilpin and Ayala (1973), the addition of the parameter  $\theta$  removes the symmetry. For values of  $\theta > 1$ , the maximum rate of growth is obtained at some point greater than  $K/2$  (Figure 2.6). On the other hand, when  $\theta < 1$ , the maximum of the population growth rate is reached at some point less than  $K/2$  (Figure 2.6).

Examples of other variations of the Logistic growth function used in the literature are presented below. Gilpin and Ayala (1973) extended the original function to model the loss of carrying capacity due to competition between different species; Skonghoft (1998) used the original Logistic function, but added the loss of stock due to harvesting to estimate wildlife and livestock population numbers; Van Kooten and Eiswerth (2007) included the minimum viable population to the original Logistic model in order to obtain more realistic growth rates of wildlife populations.

The logistic growth function is a popular functional form to model population dynamics. It also is versatile, which allows researchers to modify its basic form to include other variables of interest such as density dependence, competition between species, harvest, habitat variations and levels of minimum viable populations.

### **2.3.2. Environmental Values and Welfare**

Environmental values are an essential input for bioeconomic models because these represent a “price” for goods and services supplied by an ecosystem. According to Grafton et al. (2004), measuring environmental values is important for six main reasons.

1. Environmental values are needed when doing cost-benefit analysis of development of public or private projects. Usually these projects involve non-market benefits or costs, which need to be measured in order to make responsible decisions about their implementation.
2. Environmental values are required in determining compensation of environmental damages.
3. Environmental values are indispensable to measure monetary values associated with changes in the standards of environmental goods and services. Information, not only on total levels of environmental capital, but on its variation is needed to balance marginal benefits and costs.
4. Environmental values are used in land use planning. By knowing all benefits and costs associated to different land uses, more efficient decisions can be made for the efficient management of public lands (Prins, Adamowicz and Phillips 1990).
5. Environmental and ecologic values are used in natural resource accounting. Measures of GDP usually do not take into account depreciation of environmental or natural resource stocks therefore do not represent sustainable economic growth.
6. To accomplish sustainable economic growth, information about environmental values is needed to impose sustainability constraints on the economic processes.

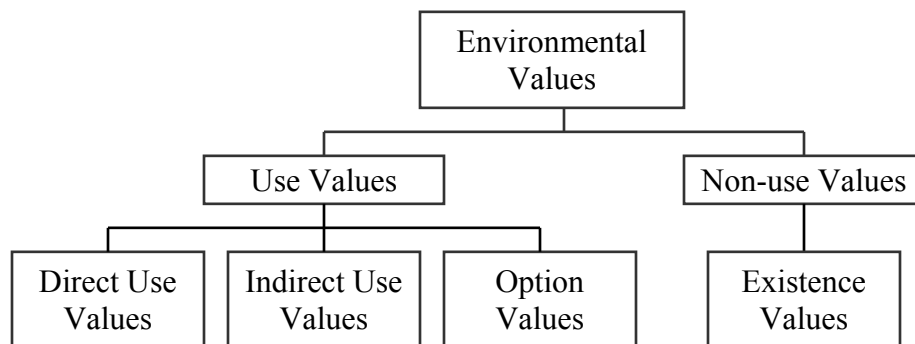
Environmental values can be classified into two main categories: use and non-use values (Figure 2.7). The former refers to benefits associated to the direct or indirect use of environmental goods and services, while the second one refers to the value associated with knowing that those goods and services are available even though, there is not an explicit intention of using them. Non-use values are usually defined as existence values; which means that people improve their welfare by the mere fact of knowing that environmental goods and services exist. These values are generally related to concepts of



social and environmental responsibility, altruistic behavior, or the desire of bequeathing to future generations a healthy ecosystem. Also, there is the concept of option values, which is traditionally linked to use-values (Pearce and Warford 1993) but can also be linked to non-use values. These represent the values of knowing that an environmental good or service is going to be available to use them in the future, even if present use is not intended.

The literature about environmental values includes a wide variety of economic activities, natural resources and management strategies. Prins, Adamowicz and Phillips (1990) review literature about the measurement of non-market values of products and services as wildlife, hunting, fishing and outdoor recreation activities given by forests. They concluded that: “non-market values are important as public land managers realize that wild-lands provide a large number of public goods and services in addition to producing traditional commodity resources” (p. 1).

**Figure 2. 7. Environmental values**



Source: Pearce and Warford (1993)

Olewiler (2004) illustrated the threats to Canadian natural capital due to the loss of natural areas to residential, commercial, industrial and agricultural use. She identified the costs of new land uses as loss of wetlands, forests, riparian areas and grasslands; soil erosion and increased sedimentation of rivers and streams; runoff of pesticides and nutrients from agricultural fertilizer and animal waste; and air pollution from the resulting economic activities. She also argued how important is the Government’s role to protect natural capital by providing information about environmental values, assisting on

decision making and funding to measure such values and to give incentives to land owners to conserve their land.

Tegtmeir and Duffy (2004) identified the most important external costs of agriculture in the United States as damage in natural resources such as soil and water, human health, wildlife and ecosystem biodiversity. In addition, the author estimated the monetary value of these costs to be between 5.7 and 16.9 US billion dollars per year.

Van Kooten and Schmitz (1992) compared the effectiveness of economic incentives versus moral suasion in designing programs to preserve waterfowl habitat in the Canadian Prairies. They found that farmers are willing to pay \$3.9 (5.24 in 2008 dollars) per acre a year to obtain permission to drain a 15-20 acre area for use in agricultural production; and that they were willing to accept \$26.8<sup>7</sup> (35.99 in 2008 dollars) compensation per acre a year not to drain an area of 30 to 40 acres in which they have the right to use for agricultural purposes.

In the line of literature estimating waterfowl values, Hammack and Brown (1974) used different model specifications of consumer surplus in order to estimate waterfowl use (hunting) values within the boundaries of the North American Pacific flyway. The authors calculated marginal hunting values in the range of US\$3.29/bird and US\$4.37/bird<sup>8</sup>.

Van Kooten (1993a) evaluated the social and economic values of the North American Waterfowl Management Plan. In order to do so, the author calculated costs and benefits associated with wetland conservation. Costs were defined as the expenditures attributed to promotion and securing leases, lease fees and costs of establishing and improving waterfowl habitat in the area reserved for that purpose. Benefits were calculated as the increase of waterfowl numbers in the control area, multiplied by their economic value. The author developed estimates of waterfowl economic values from the hunting value

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<sup>7</sup> These are nominal monetary values for the year 1993.

<sup>8</sup> These are nominal values for the year 1978. The corresponding values in 2008 Canadian dollars are \$15.31, and \$20.34, respectively.

perspective. The estimated values ranged from \$10 to \$25<sup>9</sup> per harvested bird. With this information, a target population size of waterfowl was determined so as to assure that preservation of wetlands on private agricultural areas was economically feasible.

As suggested by van Kooten (1993a), waterfowl are a source of welfare for society. Increases in their population levels enhance social welfare (Hammack and Brown, 1974; van Kooten & Schmitz, 1992). Social benefits provided by migratory birds come from various sources: consumptive values, non-consumptive values, and non-use values (van Kooten, 1993a). The benefits based on consumptive values such as hunting, are relatively easy to measure thanks to market prices associated with the activity (i.e. hunting license prices, equipment value, etc). However, this value can be measured using other methods such as travel and transfer costs methods (Hammack and Brown, 1974). Non-consumptive values of waterfowl are related to recreation activities such as viewing, in which case birds are an essential part of the activity, but are not harvested. The value of these non-consumptive activities are generally measured using travel costs people incur to be able to enjoy the activity (Grafton et al. 2004, Chapter 10). Finally, there is the non-use or existence value of waterfowl. In this case, people gain utility from the mere existence of the birds, even though they do not “use” them in any recreational activity (i.e. there is not a market). This value can be measured through stated preference methods (Grafton et al. 2004, Chapter 9), in which non-market or political behavior are measured (e.g. what is the willingness to pay to have a specific mallard population size).

Although, the measures of environmental values presented above are important in policy making, van Kooten and Eiswerth (2008) argue that simple measures of willingness to pay are not enough to design appropriate conservation policies, and that marginal values are required. For example, programs aimed to protect a wildlife species close to extinction cannot be designed the same way as programs that intend to protect species that are above minimum viable population level.

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<sup>9</sup> These are nominal values for the year 1993. The corresponding values in 2008 Canadian dollars are \$19.05 and \$34.10, respectively.

### **2.3.3. Modeling Bioeconomic Problems**

Bioeconomic models can be used to solve various types of problems such as minimizing costs while accomplishing a specific goal, or maximizing revenue or welfare based on the optimum allocation of resources or efforts towards a specific objective. These methods have been applied to solve economic problems in forestry, agriculture, fisheries, and natural resources management (e.g. wildlife, water). Literature dealing with these topics is reviewed in this section with the purpose of analyzing their objectives, methodologies and results.

Bioeconomic models can be grouped into two main approaches. Those that minimize the objective function subject to a set of restrictions, and those based on maximization. Papers using the first approach are presented as follows.

Yang et al. (2003) used a microeconomic approach to model farmers' decisions to retire land or continue crop production, and the environmental impacts of such decisions. The authors estimated and compared the differences in costs, amount of land retired from agriculture and patterns of retirement between two types of water sediment loading goals. In addition, they identified the characteristics of specific areas of land that should be retired to meet sediment loading costs in a cost-effective way.

Rashford and Adams (2007) built a bioeconomic model to determine a cost-effective conservation strategy for waterfowl population that depended on heterogeneity in landscape, management activities effectiveness, and existing correlations between activities. The objective was to minimize management costs subject to a biologic model that specified waterfowl production relationships based on landscape and management activities to. The authors found that predator control was the most difficult management activity to apply on a large scale, but that it was the most cost effective way to increase waterfowl population in the PPR.

Rashford, Dugger and Adams (2008) used an economic optimization model and a biological simulation model of breeding waterfowl to minimize management costs of

achieving specific management objectives in the PPR of the United States. The authors calculated the effectiveness of eight different management activities (cropland retirement, conservation tillage, delayed haying, planted cover, fenced planting cover, artificial nest structures, predation control, and semi permanent wetlands) and their respective costs. Next, they specified the problem as minimizing total cost of management (amount of each management activity applied times cost per acre), subject to mallard population response to management activities, population goal, and land constraints. This paper used an interesting approach because it measured the effect of each management activity independently, and the correlation between activities. This methodology allowed the identification of management activities that competed with each other and therefore should not be implemented simultaneously, and those activities that complemented each other.

Marshall, Homans and Haight (2005) built a simulation model in order to minimize management costs of achieving a minimum viable population goal for the Kirtland's Warbler. The model included forest dynamics, warbler population dynamics, logging yields and returns, and a component of uncertainty. Results showed that the warbler population dynamics depended on interactions with the habitat, habitat availability, and the tradeoff between economic incentives to harvest forest areas and conservation values. The results of the model included an optimal commercial logging rotation cycle and a probability of meeting the population goal.

Other approaches in this literature use bioeconomic models to find the maximum revenue, benefit or utility subject to a set of restrictions. Some examples are presented below.

Van Kooten (1993b) used a dynamic revenue maximization model to examine the effects of government support programs in the PPR for the conversion of wetland areas into agricultural lands. The author maximized a total revenue function, including the net benefits associated with agricultural production, value of wetlands, and costs of conversion; subject to the dynamics of land conversion, a land restriction, and a

restriction on the amount of marginal land that can be converted in each period. The author found that government grain support programs had contributed to wetland depletion in Western Canada, and that payments to farmers required to maintain waterfowl habitats were higher than they would be in the absence of agricultural subsidies. Finally, the author concluded that if waterfowl values were high enough, there would be incentives to restore the lost waterfowl habitats to agriculture.

Boman, Bostedt and Persson (2003) built a spatially differentiated dynamic bioeconomic model describing the management of a Swedish wolf population. The objective was to show the effectiveness of spatially differentiated policies. The authors argued that spatially differentiated analysis of conservation policies allowed them to identify and compare costs, benefits, and biological parameters. Their model maximized net benefits (environmental values minus economic costs) associated with wolf populations subject to the change in wolf numbers explained by prey abundance, migration between locations, and harvesting. The authors found that the size of wolf populations differed depending on location. In regions where costs associated with wolf populations were low the wolf numbers were close to carrying capacity. In areas where costs were high, wolf numbers were below carrying capacity. Therefore, policies needed to adjust to the needs of each region.

Johannesen (2007) used a bioeconomic model to evaluate the effects of increasing the size of protected areas for wildlife in regions where hunter-agrarian communities were located close to the protected areas' borders. The author used logistic growth functions to model the evolution of wildlife populations inside and outside protected areas, and Cobb-Douglas functions to model agricultural production and hunting practices. The objective was to find the optimal allocation of hunting effort and area of cropland that maximized revenue. In Johannesen's model, hunting effort did not depend on wildlife population size or its value; wildlife stocks were taken as exogenous variables.

Olaussen and Skonhoft (2005) used a bioeconomic model to analyze moose population dynamics, its economic value, and how these values vary depending on two different

management scenarios (hunting for meat or as a trophy). The first model maximized the hunting value based on meat value, subject to the population dynamics of both male and female moose. The second model maximized a land owner's profit obtained from meat value and income from hunting licenses, subject to moose population dynamics. This study found that females had a higher marginal value than males in both models. These results suggested that female moose value was higher because it included the value of young subpopulations.

Petersen et al. (2007) used a bioeconomic model to maximize the net revenue of individuals involved in aquaculture practices in Vietnam. The optimization problem was subject to the dynamic growth of the fish length, and the corresponding body weight related to each length. Based on the model results, the authors concluded that maximization of profit was not possible because of inefficiency in the input supplies, technology issues, and uncertainty on social planner policies.

Kundhlande, Adamowicz and Mapaire (2000) used an ecological-economic model to measure the value of environmental goods and services (carbon sequestration and water), from woodland areas in Zimbabwe. The authors measured how the ecosystem had changed due to disturbances generated by human activities and how these changes affected society's welfare. The authors found that the marginal value of water was very high in agriculture, carbon sequestration, firewood, and wild food economic activities suggesting high potential economic revenues in the application of water-conservation programs in the community. The public value of carbon sequestration was significant but did not overcome private benefits of clearing forested areas to use as cropland.

Watanabe, Adams and Wu (2006) evaluated four different environmental, biologic and economic models and compared their results over salmonid populations in various watersheds in the Pacific Northwest, in order to identify the best combination of management activities. Their objective was to increase fish population, acknowledging that both water temperatures and habitat conditions needed to be considered. The first, model minimized the costs of management activities aimed to decrease water temperature

to an ideal point for fish survival, subject to water temperature response rates, and the effect on habitat. The second model maximized the river's stream length in order to decrease water temperature subject to a budget constraint. In the third model the authors expected to find a specific water temperature target, given the water temperature needs of fish and varying budget constraints. The third model maximized fish numbers by controlling water temperature and other variables affecting fish populations, subject to a budget constraint. Their paper represents an example of how a problem can be specified in different setups, even if the objective is the same in each case.

Clearly bioeconomic models have been used in the literature to deal with a wide range of topics. Although, the objective of these models is to find the optimal allocation of resources that meet the maximum or minimum of the objective function, important insight about the issues related to the structure and state of the economic activities and communities under study can be deduced. Therefore, bioeconomic models can potentially give researchers information about the system in which economic, environmental and ecological problems take place.

Two papers from the reviewed literature were selected to be discussed in detail. These were selected because they dealt with economic and environmental problems similar to the one proposed in this thesis. Both modeled how agricultural producers maximize their profit subject to wildlife population evolution, the tradeoffs between agricultural practices and wildlife conservation, and welfare implications.

Skonhøft (1998), and Johannesen and Skonhøft (2004) used bioeconomic models to measure the benefits obtained by African communities from property rights over wildlife. These models dealt with the conflicts between natural parks agents, who seek to protect wildlife biodiversity, and rural communities that produce agricultural goods. The main issue was that wildlife was free to graze inside and outside protected areas, becoming a nuisance to farmers. In a scenario with no property rights over wildlife, agricultural communities face the costs of conservation, but do not benefit from conservation activities.



Skonhofs (1998) modeled three economic activities that generated income: livestock production for rural communities, hunting license revenue and tourism services for the park managers. Population dynamics for both livestock and wildlife were modeled using logistic growth functions. Competition for food between livestock and wildlife outside protected areas, and hunting practices were included in the logistic functions. The author specified a dynamic optimization model, where the present value of the sum of benefits of each activity was maximized, subject to the two population dynamic equations. The problem was specified in a continuous time over an infinite time horizon framework.

$$\text{Max} \int_0^{\infty} [py + V(x) + qh + W(z)] e^{-rt} dt \quad (2.2)$$

Subject to:

$$\dot{x} = \gamma_x x \left( 1 - \frac{x}{K_x} \right) - y \quad (2.3a)$$

$$\dot{z} = \gamma_z z \left( 1 - \frac{z}{K_z} \right) - \alpha z x - h \quad (2.3b)$$

From equation 2.2,  $p$  represents a constant price of a hunting license,  $y$  is the number of licenses sold, therefore  $py$  represents profit obtained by the park manager from selling hunting licenses.  $V(x)$  denotes profit of offering tourism services, such that  $V(0)=0$ ,  $V_x > 0$ , and  $V_{xx} < 0$ ;  $qh$  is the benefit from illegal hunting practiced by rural communities, where  $q$  denotes the marginal value of the offtake<sup>10</sup> and  $h$  is the number of animals harvested.  $W(z)$  represents the profit obtained from livestock production, such that  $W(0)=0$ ,  $W_z > 0$ , and  $W_{zz} \leq 0$ . And  $e^{-\delta t}$  represents the discount factor at discount rate  $r$ . Equation (2.2) represents social benefit/welfare, because it includes private and public benefits.

Equations (2.3a) and (2.3b) represent the population dynamic equations for wildlife and livestock, respectively. Variables  $x$  and  $y$  denote the biomass of the stock of each

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<sup>10</sup> Off-take refers to the amount of animals hunted.

population at a given point of time.  $\gamma_x$  and  $\gamma_z$  represented the intrinsic growth rate of each population, while  $K_x$  and  $K_z$  denote the carrying capacity of each population. Expression  $\alpha z x$  represents livestock loss because of competition for food with wildlife; and  $h$  denotes the number of animals hunted illegally.

This problem includes a trade-off between wildlife as a source of income to the park manager and livestock as income source for the rural communities. The solution to the problem yields optimal levels of biomass for each population at every period of time. Different optimal population levels are obtained when the original conditions of the problem vary. Skonhøft (1998) solved the problem under three different scenarios and compared the results in each case. The base case assumed that the social planner does not assign property rights over wildlife. In the second case the communities received a share of the harvesting profit. Finally rural communities received shares from both economic activities controlled by the park manager.

Johannesen and Skonhøft (2004) analyzed the migration process of wildlife in and out of the protected area, the nuisance they present to rural communities by feeding on crops or destroying agricultural products and the wildlife population dynamics. There are two economic activities the rural communities practice: growing crops and hunting. Hunting is encouraged as a measure to get rid of problem animals, for the meat, or just as sport that generates utility to the hunter. Depending on the specification of the property rights over wildlife, hunting can be legal or illegal. Two wildlife population growth functions are specified. In the case where the herd is outside the reserve area, population change depends on the hunting mortality only. And in the case where animals are inside the park area, the growth is given by a logistic function.

$$\text{Max} [p_H x_0 (1 - s) + p_A (A(N(x_0)) - D(x_0))] \quad (2.4)$$

Subject to: 
$$x_{0,t+1} = x_{T,t} + \gamma x_{T,t} \left(1 - \frac{x_{T,t}}{K}\right) \quad (2.5)$$

The expression  $p_H x_0(1-s)$ , from equation 2.4, represents the hunting benefit; with  $p_H$  being the marginal value (or price) of the offtake,  $x_0$  the herd size at the moment it leaves the park area and enters the agricultural area, and  $s$  the survival rate of the species.  $p_A(A(N(x_0))-D(x_0))$  represents the profit obtained from crop production.  $p_A$  denotes the price of the agricultural good,  $A(N(x_0))$  is the yield, as function of the cumulative effort in agriculture. This effort depends on the time assigned to agricultural practices and the time assigned to hunting.  $D(x_0)$  represent the loss in yield due to wildlife nuisance. Nuisance depends on the number of animals and their feed requirement per period of time. Equation 2.5 is a classic logistic function describing the wildlife population dynamics.

The question the authors wanted to answer is whether handing the property rights over to the local people result in higher wildlife abundance and more sustainable resource utilization (Johannesen and Skonhøft 2004). The model was solved under two different property right regimes: legal rights over wildlife and no legal rights. In the model without property rights,  $x_0$  is taken as an exogenous variable (i.e. the hunting decision does not take into account wildlife population in the future). In the model with property rights,  $x_0$  is endogenous and the state variable in the dynamic optimization problem (i.e. rural communities have incentives to protect wildlife and to hunt only an optimal amount of animal each period).

These two papers represent an example of how bio-economic models can be used to examine trade-offs between environmental values and agricultural profits. They also represent a multi-period problem and are therefore specified using dynamic optimization methodology (Chapter 3). Finally, the models are specified in two different setups: continuous and discrete. Both models represent interesting options for the Farm Level and Social Planner modeling frameworks proposed in this thesis.

## ***2.4. Conclusions***

Mallard and other waterfowl populations have a value to society. In spite of private and public institutional efforts on increasing waterfowl populations, mallard numbers are decreasing in the PPR of North America. The reasons for this decline have been mostly associated with the loss of breeding habitat due to expansion and intensification of agricultural practices. The main sources of low duck population growth in the Prairie Provinces in Canada have been linked to low nesting and duckling survival rates. Nest and duckling mortality are caused mainly by predators and agricultural practices used to grow spring seeded crops such as tillage. According to the literature, the most effective management activities to increase waterfowl populations are predator control and conversion of cropland to appropriate breeding habitat. However, reducing cropland area has significant negative effects in farm profits. In addition, mallard and other waterfowl represent a nuisance for most farmers because they graze, trample and foul swathed crops left in the fields to dry. Because of this, there are decreases in yields (because of grain eaten by ducks) and increases in cleaning costs for grain contaminated with duck waste.

As an alternative to decreasing agricultural areas, and to minimize costs associated with duck population in croplands, fall seeded crops such as winter wheat are proposed as substitutes for spring seeded crops. Winter wheat is seeded in the fall, therefore seeding practices do not interfere with Mallard breeding efforts. In addition, winter wheat harvest occurs earlier than spring crops. For that reason, the grain is not affected by duck predation. Winter wheat is also a convenient upland nesting habitat. It offers enough cover for female ducks to build their nests and be out of sight of predators.

Bioeconomic models are a useful tool to model producer or social planner problems. These types of models are usually used to find solutions to problems where there exists a trade-off between economic (e.g. maximize benefits or minimize costs) and environmental or ecological (e.g. wildlife conservation) objectives. Therefore their use is an suitable approach to implement in the mallard duck population-wheat production context.

## **Chapter 3: Methodology**

Bioeconomic models are generally based on optimization methods where a function is maximized (if dealing with benefits or utility), or minimized (if referring to costs) in order to allocate resources or efforts in the most efficient manner such that goals are met and restrictions hold. The objective of this chapter is to present the concept of optimization, its methods and the mathematical logic behind them. Also, it is discussed how optimization methodologies work, their purpose and utility, the classification of optimization techniques, and the similarities and differences between them.

### ***3.1. Optimization***

Economic optimization problems seek to find the optimal allocation of scarce resources to accomplish a specific objective. Consumer optimization problems usually maximize consumer utility by choosing the optimal levels of consumption of each good available in the market. This allocation is generally subject to a budget constraint. Then the problem becomes: how to maximize utility based on the consumption of goods that the consumer prefers, given a budget constraint. Producer optimization problems could be to maximize profits given a technology constraint, or minimize costs subject to a given level of production. All these examples use the same principle: find the maximum (or minimum) of an objective function subject to various constraints by choosing the optimal allocation of available resources.

Optimization problems can be specified in static and dynamic frameworks. Static optimization is used when dealing with economic problems that take place in a single period of time. Dynamic optimization is used when the economic problem has a duration of two or more periods, which means that the allocation of resources has to be made for more than one time interval. The number of periods for a dynamic approach may be fixed or infinite. In this case, the allocation in each period affects the allocation in all the other periods. Therefore the mathematical problem is more complex. Dynamic optimization

problems can be solved using two different frameworks: discrete or continuous time. A discrete framework implies the utilization of discrete steps or periods. A discrete time function can be defined as a set of discrete numbers over distinct time periods. A continuous time function can be defined in different ways; intuitively it is a function in which small changes in the input generates small changes in the output, where the time periods are instantaneous<sup>11</sup>.

## 3.2. Static optimization

### 3.2.1. Static optimization with equality constraints

Assume a maximization problem that needs to be solved for one period of time, and that the objective and the restriction functions are defined in terms of variables  $x$ ,  $y$  and  $z$ . Also, assume an equality restriction, which means that the restriction function must equal constant. The problem is specified as follows:

$$\begin{aligned} & \text{Maximize } f(x, y, z) \\ & \text{subject to } g(x, y, z) = c \end{aligned} \tag{3.1}$$

Mathematically, the solution of the problem is given by a point  $E^*$  where the functions  $f(x, y, z)$  and  $g(x, y, z)$  are tangent. This point is given by the coordinates  $f(x^*, y^*, z^*)$ , where the function  $f(x, y, z)$  is at its maximum. The isosurfaces<sup>12</sup> for the two functions  $f(\cdot)$  and  $g(\cdot)$  are tangent at point  $E^*$  only when their correspondent gradient vectors<sup>13</sup> have the same direction in the three dimensional space

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<sup>11</sup> Mathematically, the function  $f(\cdot)$  is continuous if when  $\lim_{n \rightarrow a} x_n = x_a$ , then  $\lim_{n \rightarrow a} f(x_n) = f(x_a)$

(Chiang 1984, p.147)

<sup>12</sup> An isosurface is a type of display that shows a three dimensional surface for a given value (i.e. the three dimensional representation of  $f(x, y, z) = a$ )

<sup>13</sup> The gradient vector of function  $f(x, y, z)$  is denoted  $\bar{\nabla}f$  and is defined as  $\bar{\nabla}f = (f_x, f_y, f_z)$ , where  $f_x = \partial f / \partial x$ ,  $f_y = \partial f / \partial y$ , and  $f_z = \partial f / \partial z$ . The direction of the gradient at a specific point  $(x_0, y_0, z_0)$  shows the direction in which function  $f(x, y, z)$  increases more quickly (Conrad and Clark 1987).

(i.e. geometric space where length, width, and depth are depicted). Formally, this means that  $\bar{\nabla}f(x, y, z) = \lambda \bar{\nabla}g(x, y, z)$  for some  $\lambda \neq 0$ . Then at point  $E^*$  the following holds:

$$\begin{cases} f_x - \lambda g_x = 0 \\ f_y - \lambda g_y = 0 \\ f_z - \lambda g_z = 0 \end{cases} \quad (3.2)$$

Equations (3.2) give the first approximation of an optimization problem using the Lagrangian method. The Lagrangian method is discussed in section 3.2.2.

If the problem did not have a restriction, the solution would be found by applying the first order conditions to the objective function (i.e.  $\partial f/\partial x = 0$ ,  $\partial f/\partial y = 0$ ,  $\partial f/\partial z = 0$ ), and solving the system of equations to find the optimum values for each of the variables. To assure that the solution is a maximum, the objective function must be concave (if dealing with a minimization problem, the procedure is the same but the objective function needs to be convex).

### 3.2.2. Lagrangian and Lagrange multipliers

The Lagrangian is an optimization technique that is characterized by the *Lagrangian multiplier* (e.g.  $\lambda$  from equation 3.2). The optimization problem (3.1) is solved using a Lagrangian by specifying the following function:

$$L = f(x, y, z) - \lambda [g(x, y, z) - c] \quad (3.3)$$

The first order conditions (FOC) for the Lagrangian, are the partial derivatives with respect to each variable, including the multiplier, set equal to zero (Equations 3.4). The FOC are the same expression found in 3.2 except for the additional equation given by the derivative with respect to  $\lambda$ .

$$\begin{aligned} \frac{\partial L}{\partial x} = \frac{\partial L}{\partial y} = \frac{\partial L}{\partial z} = 0 \\ \Rightarrow \frac{\partial f}{\partial x} - \lambda \frac{\partial g}{\partial x} = \frac{\partial f}{\partial y} - \lambda \frac{\partial g}{\partial y} = \frac{\partial f}{\partial z} - \lambda \frac{\partial g}{\partial z} = 0 \end{aligned} \quad (3.4a)$$

$$\frac{\partial L}{\partial \lambda} = -g(\cdot) + c = 0 \quad (3.4b)$$

The second order conditions (SOC) for the Lagrangian are given by the bordered Hessian matrix. The bordered Hessian is noted as  $\bar{H}$  and defined in equation 3.5. Intuitively,  $\bar{H}$  determines the curvature conditions on the objective function, determining if the solution found is a maximum or a minimum. The solution is a maximum if  $\bar{H}$  is negative definite, and a minimum if  $\bar{H}$  is positive definite.<sup>14</sup>

$$\bar{H} = \begin{bmatrix} 0 & g_x & g_y & g_z \\ g_x & L_{xx} & L_{xy} & L_{xz} \\ g_y & L_{yx} & L_{yy} & L_{yz} \\ g_z & L_{zx} & L_{zy} & L_{zz} \end{bmatrix} \quad (3.5)$$

Once the optimum values  $(x^*, y^*, z^*)$  are determined, the value for the Lagrangian multiplier  $\lambda$  is calculated. The parameter  $\lambda$  has both mathematical and economic interpretations. Mathematically, “ $\lambda$  equals the incremental change in value from an incremental change in the constraint parameter  $c$ ; in other words it represents the marginal value of relaxing the constraint” (Conrad & Clark 1987: 9). Economically,  $\lambda$  represents the shadow price of the variable restricted in  $g(x, y, z)$ . In other words, it represents the price for an additional unit of whatever  $g(x, y, z)$  is a function for.

<sup>14</sup>  $\bar{H}$  is negative definite if  $|\bar{H}_2| > 0, |\bar{H}_3| < 0, \dots, (-1)^n |\bar{H}_n| > 0$ .

$\bar{H}$  is positive definite if  $|\bar{H}_2| < 0, |\bar{H}_3| < 0, \dots, |\bar{H}_n| < 0$ .



### 3.2.3. Static optimization with inequality constraints

Assume problem 3.1. changes to the specification shown in equation 3.6.

$$\begin{aligned} & \text{Maximize } f(x, y, z) \\ & \text{subject to } g(x, y, z) \leq c \end{aligned} \quad (3.6)$$

In this case the maximum of the objective function  $f(x, y, z)$  may not be tangent to restriction  $g(x, y, z) = c$ . Actually, there are two possibilities for the optimum: first, when the restriction equality holds  $g(x, y, z) = c$  (corner solution), and second when the strict inequality holds  $g(x, y, z) < c$  (interior solution). These two cases can be combined to form a single condition known as *Kuhn Tucker condition* (Equations 3.7) which is the analog to the FOC in the equality constraint problem.

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial y} = \frac{\partial L}{\partial z} = 0 \quad (3.7a)$$

$$\lambda \begin{cases} = 0 & \text{if } g(x, y, z) < c \\ \geq 0 & \text{if } g(x, y, z) = c \end{cases} \quad (3.7b)$$

Generally, the condition for  $\lambda$  (equation 3.7b) is rewritten to its equivalent form specified in equation 3.8. The parameter  $\lambda$  has to be nonnegative. For the objective and restriction functions to be tangent on the optimum point  $E^*$ , the gradient vectors  $\vec{\nabla}f(\cdot)$  and  $\vec{\nabla}g(\cdot)$  must have the same direction from  $E^*$ . So, for  $\vec{\nabla}f(\cdot) = \lambda \vec{\nabla}g(\cdot)$  to hold,  $\lambda$  needs to be positive.  $\lambda$  may be equal to zero if at the tangent point with the restriction  $g(x, y, z)$ , the objective function  $f(x, y, z)$  is also in a local or global maximum. Then  $\vec{\nabla}f(\cdot) = 0$ , and for  $\vec{\nabla}f(\cdot) = \lambda \vec{\nabla}g(\cdot)$  to hold,  $\lambda = 0$  must be true.

$$\begin{aligned} \lambda [g(x, y, z) - c] &= 0 \\ \lambda &\geq 0 \end{aligned} \quad (3.8)$$

Thus far, methods used in discrete static optimization problems have been presented. However, many of the optimization problems in empirical work take into account several periods, and dynamic frameworks are needed.

### ***3.3. Dynamic optimization: an extension of the Lagrangian***

The objective of dynamic optimization problems is to find the optimal quantity for each variable included in the problem for every period of time within the whole planning period (discrete framework) or at each point of time in a specific time interval (continuous framework). To solve a dynamic optimization problem, the optimum values to all variables need to be found for each period or (point) of time. The complexity of this framework is that the optimum values in each period do not just depend on that specific period but on all periods before and after. “The solution of a dynamic optimization problem takes the form of an optimal time path for every choice variable, detailing the best of the variable today, tomorrow, and so forth, till the end of the planning period” (Chiang 1992: 3).

Dynamic optimization problems have two types of variables: state variables and control variables. Control variables, also known as choice variables, “determine the (expected) payoff in the current period and the (expected) state next period” (Woodward 2007: 3). According to Chiang (1992) control variables have two properties: first, they depend on the researcher choice, i.e. they are the variables researchers can control; second, they affect the state variable at every period of time. The state variable is easily identified in a dynamic problem because its evolution (intertemporal changes) is explicitly specified by the state equation, which is one of the restrictions specifying the problem.

A dynamic discrete-time problem is formulated in equation 3.9. The objective function  $f(\cdot)$ , which represents the function to be maximized, is defined in terms of the state variable  $x_t$  and the control variable  $y_t$ , and is evaluated for the periods  $0$  to  $T-1$ . The expression  $h(x_T)$  represents a *final function* indicating the value of alternative levels of the state variable at the last period ( $T$ ). In an economic context  $h(x_T)$  is interpreted as a

salvage value (e.g. value of a resource in the last period it is extracted). If the problem is specified for an infinite number of periods, the function  $h(x_T)$  disappears, and only  $f(\cdot)$  is maximized. The state equation is given by the difference equation  $x_{t+1} - x_t = g(\cdot)$ ; this defines the evolution of the state variable through the periods  $t = 1, 2, \dots, T-1$ . The initial condition for  $x$  is given by the value  $a$ .

$$\begin{aligned} & \text{Maximize}_{y_t} \sum_{t=0}^{T-1} f(x_t, y_t) + h(x_T) \\ & \text{subject to } x_{t+1} - x_t = g(x_t, y_t) \\ & \quad \quad \quad x_0 = a \end{aligned} \quad (3.9)$$

If the Lagrangian is used to solve this dynamic problem, the Lagrange function is specified as follows:

$$L = \sum_{t=0}^{T-1} [f(x_t, y_t) + \lambda_{t+1}(x_t - x_{t+1} + g(x_t, y_t))] + h(x_T) \quad (3.10)$$

Where  $\lambda_{t+1}$  is the Lagrange multiplier associated with  $x_{t+1}$ . In dynamic optimization problems,  $\lambda$  is also known as the costate variable. There is one  $\lambda$  for each  $x$  at every period of time. Given 3.10, the FOC are specified in equations 3.11<sup>15</sup>.

$$\frac{\partial L}{\partial y_t} = 0 \Rightarrow \frac{\partial f(\cdot)}{\partial y_t} + \lambda_{t+1} \frac{\partial g(\cdot)}{\partial y_t} = 0 \quad t = 0, \dots, T-1 \quad (3.11a)$$

$$\frac{\partial L}{\partial x_t} = 0 \Rightarrow - \left( \frac{\partial f(\cdot)}{\partial x_t} + \lambda_{t+1} \frac{\partial g(\cdot)}{\partial x_t} \right) = \lambda_{t+1} - \lambda_t \quad t = 0, \dots, T-1 \quad (3.11b)$$

$$\frac{\partial L}{\partial \lambda_{t+1}} = 0 \Rightarrow x_{t+1} - x_t = g(\cdot) \quad t = 0, \dots, T-1 \quad (3.11c)$$

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<sup>15</sup> The derivation of this equation 3.11b was done as follows: The derivation of the Lagrangian with respect to the state variable  $x_t$  equals zero, then we have:  $\frac{\partial L}{\partial x_t} = \frac{\partial f(\cdot)}{\partial x_t} + \lambda_{t+1} \left( 1 + \frac{\partial g(\cdot)}{\partial x_t} \right) - \lambda_t = 0$ . The term  $-\lambda_t$  appears when we go back to period  $t-1$  and we find a  $-x_t$ . Finally, rearranging the expression we end with equation 3.11b.

$$\frac{\partial L}{\partial x_T} = 0 \Rightarrow \frac{\partial h}{\partial x_T} = \lambda_T \quad (3.11d)$$

$$x_0 = a \quad (3.11e)$$

Equation 3.11a is composed of the marginal condition of the objective function and the influence of the control variable on the change of the state variable. Given the fact that the shadow price  $\lambda_{t+1}$  also appears in this expression, the right hand side can be interpreted as an intertemporal cost or benefit, depending on the way  $y_t$  affects the evolution of  $x$ . If an increase of  $y_t$  reduces the amount of the variable  $x_{t+1}$ , the expression  $\lambda_{t+1} \partial g(\cdot) / \partial y_t$  is usually called the *user cost*. If an increase of  $y_t$  increases  $x_{t+1}$ ,  $\lambda_{t+1} \partial g(\cdot) / \partial y_t$  could be interpreted as a benefit generated by  $y_t$ . Equation 3.11b is a difference equation which specifies how the multiplier must change with time to maintain the optimality. Equation 3.11c is also a difference equation specifying how the state variable changes optimally through time. Finally, 3.11d and 3.11e are boundary restrictions specifying the value of  $\lambda$  at the end of the planning period and the initial value of the variable  $x$  at the beginning of the planning period.

Condition 3.11d disappears if we deal with an infinite time horizon problem. In this case, it is assumed that there is a point in time where the “solution variables converge to a set of values and remain unchanged thereafter” (Conrad & Clark 1987: 16). This state where the solutions remain unchanged is called *steady state*. In the steady state  $\lambda_{t+1} - \lambda_t$  and  $x_{t+1} - x_t$  equal zero. The solution of the problem is based on a system of equations that have to be solved for the control, state and costate variables.

### 3.3.1. The Hamiltonian in a discrete framework

The Hamiltonian is an alternative approach to solve discrete (and continuous) dynamic optimization problems. It is closely related to the Lagrangian method. Using the problem specified in equation 3.9, the Hamiltonian is defined as:

$$H(x_t, y_t, \lambda_{t+1}) = f(x_t, y_t) + \lambda_{t+1} g(x_t, y_t) \quad (3.12)$$

Defining the Lagrangian in terms of the Hamiltonian:

$$L = \sum_{t=0}^{T-1} [H(x_t, y_t, \lambda_{t+1}) + \lambda_{t+1} (x_t - x_{t+1})] + h(x_T) \quad (3.13)$$

The FOC are:

$$\frac{\partial L}{\partial y_t} = 0 \Rightarrow \frac{\partial H(\cdot)}{\partial y_t} = 0 \quad t = 0, \dots, T-1 \quad (3.14a)$$

$$\frac{\partial L}{\partial x_t} = 0 \Rightarrow -\frac{\partial H(\cdot)}{\partial x_t} = \lambda_{t+1} - \lambda_t \quad t = 0, \dots, T-1 \quad (3.14b)$$

$$\frac{\partial L}{\partial \lambda_{t+1}} = 0 \Rightarrow \frac{\partial H(\cdot)}{\partial \lambda_{t+1}} = x_{t+1} - x_t \quad t = 0, \dots, T-1 \quad (3.14c)$$

$$\frac{\partial L}{\partial x_T} = 0 \Rightarrow \frac{\partial h(\cdot)}{\partial x_T} = \lambda_T \quad (3.14d)$$

$$x_0 = a \quad (3.14e)$$

This problem is solved by finding the optimal trajectory for the control variable  $y_t$ . Once  $y_t^*$  is found, using the first order conditions 3.14c and 3.14e, it is possible to find the optimal trajectory for the state variable  $x_t^*$ .

### 3.3.2. Dynamic optimization in a continuous time context

When working in a continuous time context the functions are assumed to be smooth and without jumps between points. In every unit of time (no matter how small) there is a value for each variable. This might not be realistic in some economic problems. However, continuous-time frameworks are widely used in economics because they offer powerful tools to solve problems. Rewriting the problem specified in equation 3.9 in a continuous time context gives:

$$\begin{aligned}
& \text{Maximize } \int_0^T f(x(t), y(t)) dt + h(x(T)) \\
& \text{subject to } \dot{x} = g(x(t), y(t)) \\
& \quad \quad \quad x(0) = a
\end{aligned} \tag{3.15}$$

Where  $\dot{x} = dx/dt$  (i.e. the change of variable  $x$  through time). Notice that there are four main technical differences between the discrete and the continuous frameworks. First, all variables are a function of time. Second, when working in continuous-time each variable is solved for every single point inside the  $0 \leq t \leq T$  interval. Third, instead of representing the dynamic of the model by adding the optimal values of each period, the integration over each point in the time interval is used; and finally, the evolution of the state variable is no longer specified by a difference equation but replaced by a differential equation.

Even though there are differences between the two frameworks, there are also similarities in the logic used to specify and solve the problem. To observe the similarities the Lagrangian for problem 3.15 is specified. The specification of the problem has the same form, except for the differences mentioned above. There is the integration of the objective function in terms of the control variable  $y(t)$  and the state variable  $x(t)$ , plus the costate variable  $\lambda(t)$  multiplying the state equation that represents the evolution of  $x(t)$  through time. Added to the integral is the value of  $h(x(T))$  (i.e. the value of  $x$  at the end of the time interval). As before, if the time interval is infinite, this last expression disappears.

$$L = \int_0^T [f(x(t), y(t)) + \lambda(t) (g(x(t), y(t)) - \dot{x})] dt + h(x(T)) \tag{3.16a}$$

By making some mathematical arrangements, the Lagrangian in equation 3.16a can be rewritten in terms of the changes in the costate variable instead changes in the state

variable. The integration by parts of the expression  $-\lambda(t)\dot{x}$ <sup>16</sup> and the reorganization of terms gives the Lagrangian as shown in equation 3.16b.

$$L = \int_0^T [f(x(t), y(t)) + \lambda(t) g(x(t), y(t)) + \dot{\lambda} x(t)] dt + h(x(T)) - [\lambda(T)x(T) - \lambda(0)x(0)] \quad (3.16b)$$

The Hamiltonian in a continuous time context is defined in equation 3.17. As before, the Hamiltonian is specified as the objective function plus the costate variable times the state equation. The difference with the discrete framework is that the variables are specified as functions of time.

$$H(x(t), y(t), \lambda(t)) = f(x(t), y(t)) + \lambda(t) g(x(t), y(t)) \quad (3.17)$$

Equation 3.18 represents the specification of the Lagrangian as a function of the Hamiltonian.

$$L = \int_0^T [H(x(t), y(t), \lambda(t)) + \dot{\lambda} x(t)] dt + h(x(T)) - [\lambda(T)x(T) - \lambda(0)x(0)] \quad (3.18)$$

The FOC are specified as:

$$\frac{\partial L}{\partial y(t)} = 0 \Rightarrow \frac{\partial H(\cdot)}{\partial y(t)} = 0 \quad (3.19a)$$

$$\frac{\partial L}{\partial x(t)} = 0 \Rightarrow -\frac{\partial H(\cdot)}{\partial x(t)} = \dot{\lambda} \quad (3.19b)$$

$$\frac{\partial L}{\partial \lambda(t)} = 0 \Rightarrow \frac{\partial H}{\partial \lambda(t)} = \dot{x} \quad (3.19c)$$

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<sup>16</sup>  $-\lambda(t)\dot{x}$  can be integrated by parts resulting in the expression  $\int_0^T \lambda(t)\dot{x} dt - [\lambda(T)x(T) - \lambda(0)x(0)]$ .

$$\frac{\partial L}{\partial x(T)} = 0 \Rightarrow \frac{\partial h(\cdot)}{\partial x(T)} = \lambda(T) \quad (3.19d)$$

$$x(0) = a \quad (3.19e)$$

Equations 3.19a, 3.19b, 3.19c and 3.19d together are known as the *maximum principle*. This principle states that the control variable  $y(t)$  must be maximized at every point in time; that the equation of motion for the costate variable  $\lambda(t)$  is given by equation 3.19b, that the equation of motion for the state variable  $x(t)$  is given by 3.19c; and finally that the transversality condition<sup>17</sup> 3.19d holds because  $\lambda(T) = 0$ .

### 3.3.3. Discounting

Solving problems that maximize benefits or returns in an intertemporal framework involves flows of money. Money does not have the same “value” in each period. This value is given by a *discount rate*. Money flows might have different values at different periods of time depending on the value of the discount rate. Additionally, the existence of a discount rate makes a difference in the present value of future flows of money. Without a discount rate, the present value of future flows of money is given by the sum of all flows; with a discount rate, the present value is given by the sum of future flows discounted in each period by a *discount factor* (Equation 3.20)

$$\text{Present Value} = \sum_{t=0}^T \frac{N_t}{(1+r)^t} \quad (3.20)$$

Discounting over a dynamic optimization model in a discrete time framework consists of including the discount factor in every period. The discount factor  $\tau$  is given by the expression on equation 3.21, where  $r$  is the periodic discount rate.

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<sup>17</sup> The transversality condition refers to the fact that the shadow price of the variable  $x$  should be driven to zero at the terminal time. The economic intuition behind this condition is that the state variable  $x$  will not have any economic value when it is not used anymore; therefore its shadow price is zero. (Chiang 1992, p. 209)



$$\tau = \frac{1}{(1+r)} \quad (3.21)$$

To discount an expression for more than one period, the discount factor for all the time periods is given by  $\tau^t = 1/(1+r)^t$ , where  $t$  is the number of periods. The discrete problem with discounting is presented in equation 3.22.

$$\begin{aligned} & \text{Maximize}_{y_t} \sum_{t=0}^{T-1} \tau^t f(x_t, y_t) + \tau^T h(x_T) \\ & \text{subject to } x_{t+1} - x_t = g(x_t, y_t) \\ & \quad \quad \quad x_0 = a \end{aligned} \quad (3.22)$$

Two discount factors are included in the problem. The first one,  $\tau^t$ , for the first  $T-1$  periods; and the second one,  $\tau^T$ , for the last period. The Lagrangian for this problem is specified as:

$$L = \sum_{t=0}^{T-1} \rho^t [f(x_t, y_t) + \tau \lambda_{t+1} (x_t - x_{t+1} + g(x_t, y_t))] + \tau^T h(x_T) \quad (3.23)$$

In the continuous-time framework, discounting is based on an instantaneous rate of discount  $\tilde{r}$ , defined as  $\tilde{r} = \ln(1+r)$ . The continuous discount factor is defined in equation 3.24.

$$e^{-\tilde{r}} = \frac{1}{(1+r)} \quad (3.24)$$

The continuous discount factor for several periods is given by  $e^{-\tilde{r}t} = 1/(1+r)^t$ . The continuous dynamic optimization problem with discounting is specified in equation 3.25.

$$\begin{aligned}
& \text{Maximize } \int_0^T f(x(t), y(t)) e^{-\tilde{r}t} dt + h(x(T)) e^{-\tilde{r}T} \\
& \text{subject to } \dot{x} = g(x(t), y(t)) \\
& \quad x(0) = a
\end{aligned} \tag{3.25}$$

The Lagrangian in the continuous set up with discounting is given by:

$$L = \int_0^T [f(x(t), y(t)) e^{-\tilde{r}t} + \lambda(t) (g(x(t), y(t)) - \dot{x})] dt + h(x(T)) e^{-\tilde{r}T} \tag{3.26}$$

### 3.3.4. Present Value and Current value Hamiltonian

Dynamic optimization problems with discounting can be solved using Hamiltonian functions. The difference with the traditional function presented in equation 3.17 is the introduction of the discount factor into the function. In this case, the optimization problem can be specified using two different approaches: a Present Value (PV) Hamiltonian, or a Current Value (CV) Hamiltonian. In the PV Hamiltonian cash inflows/outflows are discounted back to its present value (i.e. the value in the first period) and FOC's are derived using the same procedure presented in equations 3.19a to 3.10e. In this case the discount factor is treated as other set of parameters. Alternatively, the CV Hamiltonian “represents values from the perspective of period  $t$ ” (Conrad & Clark 1987: 33). The CV Hamiltonian is used to avoid the incorporation of the discount factor in the mathematical problem; yet, it involves a different specification of the Lagrange multiplier called current-value Lagrange multiplier ( $\mu_{t+1}$ ). The utilization of  $\mu_{t+1}$  implies a slightly different derivation of the FOC. The CV Hamiltonian required to solve the discrete-time problem described by equation 3.22 is specified as follows:

$$\begin{aligned}
\tilde{H}(x_t, y_t, \lambda_{t+1}) &= f(x_t, y_t) + \mu_{t+1} g(x_t, y_t) \\
& \text{where } \mu_{t+1} = \rho \lambda_{t+1}
\end{aligned} \tag{3.27}$$

The FOC are specified as:

$$\frac{\partial \tilde{H}(\cdot)}{\partial y_t} = 0 \quad t = 0, \dots, T-1 \quad (3.28a)$$

$$-\frac{\partial \tilde{H}(\cdot)}{\partial x_t} = \mu_{t+1} - \lambda_t \quad t = 0, \dots, T-1 \quad (3.28b)$$

$$\frac{\partial \tilde{H}(\cdot)}{\partial \mu_{t+1}} = x_{t+1} - x_t \quad t = 0, \dots, T-1 \quad (3.28c)$$

$$\frac{\partial h(\cdot)}{\partial x_T} = \lambda_T \quad (3.28d)$$

$$x_0 = a \quad (3.28e)$$

The corresponding specification of the CV Hamiltonian for the continuous optimization problem described in equation 3.25 is given by:

$$\begin{aligned} \tilde{H}(x(t), y(t), \lambda(t)) &= f(x(t), y(t)) + \mu(t) g(x(t), y(t)) \\ \text{where } \mu(t) &= \lambda(t) e^{\tilde{r}t} \end{aligned} \quad (3.29)$$

With the following FOC:

$$\frac{\partial \tilde{H}(\cdot)}{\partial y(t)} = 0 \quad (3.30a)$$

$$-\frac{\partial \tilde{H}(\cdot)}{\partial x(t)} = \dot{\mu} - \tilde{r}\mu(t) \quad (3.30b)$$

$$\frac{\partial H}{\partial \mu(t)} = \dot{x} \quad (3.30c)$$

$$\frac{\partial h(\cdot)}{\partial x(T)} = \lambda(T) \quad (3.30d)$$

$$x(0) = a \quad (3.30e)$$

### 3.3.5. Solution and Local Stability

After setting up the Hamiltonian (with an infinite time frame) and calculating the first order conditions, the result is a system of equations that needs to be solved in order to find the optimal solution of the problem. The solution is composed of the optimal values of the control, state and co-state variables associated with the equilibrium of the system. Reaching the equilibrium, also known in the literature as the steady state, implies zero growth of the variables involved in the system (i.e.  $\dot{x} = \dot{y} = \dot{\lambda} = 0$ ). This results because once an equilibrium is reached, the system does not offer any other options for the economic agent to be better off.

A graphic representation of the equilibrium in the  $x, y$  plane can be constructed based on the system of equations given by the FOC. After doing some algebraic manipulations, the expressions  $\dot{x} = F(x, y)$  and  $\dot{y} = G(x, y)$  can be specified. Since in steady state  $\dot{x} = 0$ , and  $\dot{y} = 0$ , each expression can be solved for  $x$  in terms of  $y$ . The point (or points) where the curves defined by these two new expressions intersect represents the steady state.

The local stability of the steady state can be determined based on the value of “the eigenvalues<sup>18</sup> of the *linearized* dynamical system evaluated at the steady state” (Conrad and Clark 1987: 45). In the case of a system composed by equations  $\dot{x} = F(x, y)$  and  $\dot{y} = G(x, y)$ , two eigenvalues ( $E_1$  and  $E_2$ ) exist. The signs of  $E_1$  and  $E_2$  determine the type of equilibrium the system has. Table 3.1 summarizes the stability properties of the system given by  $\dot{x}$  and  $\dot{y}$ . Stability is important because it ensures that the optimal solution can be reached at some point in time regardless of the location of starting point in the system.

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<sup>18</sup> Assume a linear system where:  $\dot{x} = ax + by$  and  $\dot{y} = cx + dy$ . The squared matrix  $A$  can be defined as  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ . The eigenvalues correspond to the values of the eigenvector  $r$ , such that  $Det(A - EI) = 0$  holds.

**Table 3. 1. Eigen values and their corresponding type of equilibrium**

<b>Eigenvalues</b>	<b>Type of equilibrium</b>	<b>Type of stability</b>
$E_1 < 0$ and $E_2 < 0$	Stable node	Wherever the starting point is, the system moves towards the equilibrium
$E_1 > 0$ and $E_2 > 0$	Unstable node	Wherever the initial point is, the system moves away from the equilibrium
$E_1 < 0$ , $E_2 > 0$ or vice versa	Saddle point	Depending on where the initial starting point is, the system moves towards or away from the equilibrium
$E_1$ and $E_2$ are complex with positive real part	Unstable spiral	Wherever the initial point is, the system spirals away from the equilibrium
$E_1$ and $E_2$ are complex with negative real part	Stable spiral	Wherever the starting point is, the system spirals towards the equilibrium
$E_1$ and $E_2$ are complex with real = 0	Vortex	Wherever the starting point is, the system ellipses around the origin.

### **3.4. Conclusions**

Optimization problems find the maximum or minimum of an objective function. In other words they involve finding the value that the variable (or variables) included in the objective function need to have at the highest or lowest point of such function. If the problem is unrestricted, the solution is obtained by finding the point or set of points in the objective function where the first derivative is zero. If the second derivative is negative (positive), the point is a maximum (minimum). If the problem is restricted, it is necessary to define a new function  $F(x)$  that includes the objective function and the restrictions; the restrictions are added to the objective function and multiplied by a new variable denominated the Lagrange multiplier. If the time frame of the problem is a single period, the problem is static and  $F(x)$  takes the form of a Lagrangian; if the problem is dynamic (i.e. needs to be solved for two or more time periods)  $F(x)$  is specified as a Hamiltonian. The Lagrangian and Hamiltonian are equivalent functions and have the same purpose in the optimization process. The difference between these two functions is that the former one is used for static problems, while the second is used for dynamic problems.

The Lagrangian derivatives taken with respect to each variable are equated to zero. The resulting system of equations is denominated First Order Conditions (FOC). The solution

of the FOC for all the variables (including the Lagrange multiplier), is the solution to the optimization problem.

Dynamic problems deal with three types of variables. Control variables, which can be modified by the researcher; state variables, which are affected by the control variables; and costate variables, which determine the way the state variable affects the solution. The costate variable is also known as the shadow price of the state variable (i.e. the state variable's value in the modeled system). Model dynamics is given by the introduction of a restriction specifying the change of the state variable through time.

Given the multi-period nature of dynamic optimization problems, a discount factor is usually used in the optimization process because monetary values are not *worth* the same in all time periods. Discount factors are used to obtain a solution in present value (i.e. convert the solutions of all periods into present monetary values). A Hamiltonian function including a discount factor is denominated Present Value (PV) Hamiltonian. However, the inclusion of a discount factor into the optimization problem may introduce complexity to the derivation of the FOC. In order to avoid this mathematical complexity, a Current Value (CV) Hamiltonian function, free of the discount factor, may be specified instead. "Current refers to the *undiscounted* nature of the new Hamiltonian" (Chiang 1992: 210). Even though, the PV and CV Hamiltonians have a slightly different set-up, their solutions are equivalent.

Dynamic optimization problems can be solved using two different setups: discrete and continuous. The two models proposed in this thesis have a discrete nature because the seeding decisions are taken on a yearly basis and because the increase in duck population is discontinuous. However, since the total mallard population is very large, then the new duck population is very small and then it can be considered as changing continuously (Shone, 2002). In addition, solving the models in continuous setup allow a more intuitive and interesting analysis. For instance, the solution of models specified in a discrete setup consists of numerical values of control, state and costate variables. On the other hand, the solution to models specified in a continuous setup provides equations defining the

optimum level of each variable, evolution paths for each variable, and their stability within the system. In general, a discrete analysis will only give numerical solutions, while a continuous set-up provides more information about the system generated by the problem.

## Chapter 4: Mathematical Models

Two mathematic models are developed to capture the effect of upland breeding habitats created by winter wheat crops for mallard populations in the Prairies, and the farmer's production decision on seeding winter or spring wheat. Prices, yields, costs and cropland interaction with duck population, were specified under two different scenarios: 1) From a farmer's perspective (private model) where the objective is to maximize the profit derived from wheat production. This model assumes that the mallard population represent a cost to farmers and that ducks do not have private economic value<sup>19</sup>. 2) From a social planner perspective (social model) where the objective is to maximize both farmer's profit and society's benefit associated with mallard populations. It is assumed that society assigns environmental values to mallards, and as a result there are incentives to develop policies that encourage farmers to substitute spring wheat with winter wheat. The two models are specified in continuous setups and using dynamic optimization methods. The purpose is to calculate the optimal acreage of spring and winter wheat, and optimum mallard population size that maximize private and social benefits.

### *4.1. Model Specification*

#### **4.1.1. Farm Level Problem**

In the farm level model, the farmer's objective is to maximize the net revenue associated with wheat production. Even though farmer's economic activities may include crops different from wheat, the model only deals with the section of the farmer's land that is used to grow spring and winter wheat. We assume that the producer divides his land in different sections of the same size and that crop rotation is done by seeding each section with a different crop every year. Therefore, the farmer's problem is to choose an optimal distribution of the land allocated to wheat production, between winter and spring wheat,

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<sup>19</sup> We note that this may not apply to all farmers in that some may derive utility from waterfowl (e.g. they could be hunters). However, we assume that most producers do not gain benefits from waterfowl.



that maximizes profit. The optimum acreage for each type of wheat is determined by the market price, production costs and yields.

The dynamic optimization problem of a farmer, who uses a fixed amount of land  $L$  to grow wheat in an area where cropland is used as breeding habitat by mallard duck populations every spring, is specified in equations 4.1 and 4.2.

$$\text{Max}_{y_w(t)} \int_0^{\infty} \{ [R_w - C_w(y_w)]y_w(t) + [R_s(y_s, x) - C_s(y_s)]y_s(t) - C_x(x) \} \cdot e^{-rt} dt \quad (4.1)$$

Subject to:

$$y_w(t) + y_s(t) = L \quad (4.2a)$$

$$\dot{x} = x(t) \cdot \gamma(y_w) \left[ 1 - \frac{x(t)}{K} \right] \quad (4.2b)$$

$$x(0) = a \quad (4.2c)$$

The objective function (i.e. function to be maximized) is composed of revenue per acre obtained from winter wheat ( $R_w$ ) and spring wheat ( $R_s$ ) production, cost per acre associated with wheat production ( $C_w$  and  $C_s$ , respectively), and costs associated with duck population different to yield loss ( $C_x$ ), such as crop contamination with bird droppings. Acreage of winter and spring wheat is denoted by  $y_w$  and  $y_s$ , respectively. The problem is specified in an infinite time horizon to simplify its mathematical solution. The discount term  $e^{-rt}$  is included as part of the intertemporal set-up specification.

While winter wheat profit per acre is independent of mallard duck population ( $\partial R_w / \partial x = 0$ ), spring wheat profit per acre is affected by duck population ( $\partial R_s(y_s, x) / \partial x < 0$ ). Spring crops seeded in areas where ducks nest and reside during nesting season may experience yields loss because ducks feed from the grain they find in croplands. Consequently, the duck population size affects farm profits generated from spring seeded crop production. In the private model, it is assumed that farmers do not

have incentives to “protect” the duck population or increase winter wheat acreage, other than via the net revenue obtained by winter wheat sales.

Equation 4.2a is a land restriction, where a fixed area of land ( $L$ ) is specified to grow both types of wheat. Restriction 4.2b represents the changes in mallard population ( $x$ ) through time. Equation 4.2c represents the size of the duck population at the beginning of the time horizon.

Functions and they associated parameters presented in equations 4.1 and 4.2 are justified in detail in section 4.2. Tables 1 and 2 define and summarize the functions, variables and parameters included in the Farm Level model.

#### **4.1.2. Social Planner Model**

The objective of the social planner model is to maximize the farmer’s profit associated with wheat production, and society’s benefit related with the use, non-use and existence values of mallard ducks. The solution to this problem will give optimal numbers of ducks and acreages of winter and spring wheat that maximize both the farmer’s and society’s benefit simultaneously. In this case optimal acreage levels will not only depend on wheat prices and yields, costs associated with wheat production and mallard population, but also on the value of ducks.

Since it is assumed that society values duck populations, there is a willingness to pay farmers to grow fall seeded wheat. These payments may have two sources: private organizations (e.g. Ducks Unlimited Canada) or public institutions (e.g. government). In this model, farmers receive a yearly payment  $\phi$  for each acre seeded with winter wheat. The payment can be made by a private or public organization. If the payment comes from a private source,  $\phi$  only affects the farmer’s revenue associated with winter wheat. However, if the payment comes from public funds,  $\phi$  positively affects the farmer’s profit and affects society’s benefit related with the duck population in a negative way.

**Table 4. 1. Definition of functions and parameters related with the control variables ( $y_w, y_s$ )**

<i>Winter wheat</i>	<i>Spring wheat</i>	<i>Definition</i>	<i>Type</i>
$y_w(t)$	$y_s(t)$	Wheat acreage	Control variable
$R_w$	$R_s(y_s, x)$	Wheat revenue (\$/acre)	Function
$p_w$	$p_s$	Price wheat (\$/bushel)	Parameter
$Y_w$	$Y_s$	Yield (bushel/acre)	Parameter
$\delta$	-	Winter kill rate (winter kill acreage/total acreage)	Parameter
-	$\kappa$	Loss of yield due to duck population in cropland (bushels/duck)	Parameter
$\phi$	-	Annual payment per acre of winter wheat	Parameter
$\varphi$	-	One time payment per acre of winter wheat	Parameter
$C_w(y_w)$	$C_s(y_s)$	Wheat production cost per acre	Function
$c_w$	$c_s$	Average production cost per acre	Parameter
$ac_w$	$ac_s$	Additional cost per acre related to different input uses	Parameter

**Table 4. 2. Definition of functions and parameters related with the state variable ( $x$ )**

<i>Variable</i>	<i>Definition</i>	<i>Type</i>
$x(t)$	Duck population	State variable
$\dot{x}(t)$	Duck population dynamics	Function
$\gamma(y_w)$	Duck population growth as a function of winter wheat acreage	Function
$\bar{\gamma}$	Intrinsic growth rate	Parameter
$\rho$	Gain in the duck population growth due to winter wheat acreage	Parameter
$K$	Carrying capacity of the mallard population	Parameter
$B(x)$	Social benefit of the mallard population	Function
$\alpha$	Value given by society to an additional unit of the mallard population	Parameter
$\beta$	Decreasing rate in which society's benefit increases with an additional duck	Parameter
$C_x(x)$	Costs involved with duck population on crop land not related to loss of yield	Function
$c_x$	Cost per duck not related to loss of yield.	Parameter
$r_p$	Private periodic discount rate	Parameter
$r_s$	Social periodic discount rate	Parameter

The mathematical models for the social planner problem when payments come from a private and public organization are specified in Equations 4.3a and 4.3b respectively. Restrictions of the social planner model are specified in Equations 4.4.

$$\text{Max}_{y_w(t)} \int_0^{\infty} \{ [R_w(y_w) - C_w(y_w)]y_w(t) + [R_s(y_s, x) - C_s(y_s)]y_s(t) - C_x(x) + B(x) \} \cdot e^{-rt} dt \quad (4.3a)$$

$$\text{Max}_{y_w(t)} \int_0^{\infty} \{ [R_w(y_w) - C_w(y_w)]y_w(t) + [R_s(y_s, x) - C_s(y_s)]y_s(t) - C_x(x) + B(x, y_w) \} \cdot e^{-rt} dt \quad (4.3b)$$

Subject to:

$$y_w(t) + y_s(t) = L \quad (4.4a)$$

$$\dot{x} = x(t) \cdot \gamma(y_w) \left[ 1 - \frac{x(t)}{K} \right] \quad (4.4b)$$

$$x(0) = a \quad (4.4c)$$

The differences between the Farm Level and Social Planner models are the addition of the annual payment  $\phi$  into the farmer's winter wheat revenue per acre (section 4.2.4), and the addition of the benefit function  $B(x)$  (or  $B(x, y_w)$  depending of the source of payment  $\phi$ ) in the objective function. The land, duck population dynamics and initial level restrictions are the same for each model.

## ***4.2. Model Justification: Functions and Parameters***

In this section each of the functions used in both the Farm Level (FL) and Social Planner (SP) models are described and justified.

### **4.2.1. Revenue functions for wheat production $R_w(y_w)$ and $R_s(y_s, x)$**

Revenue functions, for both winter and spring wheat were defined on a per acre basis. Revenue per acre was calculated by multiplying wheat prices (dollars per bushel) and

yield (bushels per acre). Total revenue was obtained by multiplying the revenue per acre by acreage. Prices for both types of wheat and winter wheat yields were taken as fixed. Winter wheat yield ( $Y_w$ ) depends on the winter kill rate, denoted by the parameter  $\delta$ . With lower values of  $\delta$ , the survival of the plants during winter is higher and yield level moves toward its maximum value. Spring wheat yield ( $Y_s$ ) depends on the wasted grain due to duck grazing defined in bushels per duck ( $\kappa$ ), and the size of mallard population. The higher the duck population feeding on the crop fields is, the greater the loss of yield due to grazing, and the lower is the yield. Although weather in other seasons can affect spring and winter wheat yields, we assume that this is fixed in the model. Yields for both winter and spring may change by adjusting parameters  $\delta$  and  $\kappa$ . However,  $Y_s$  is not only affected by parameter  $\kappa$ , but by both state and control variables. This means that winter wheat yield is fixed once a specific value of winter kill is assigned to  $\delta$ , while spring wheat yield is variable because it depends on the optimum values of the duck population and spring wheat acreage. Revenues per acre for winter and spring wheat are specified in equations in 4.5a and 4.5b, respectively.

$$R_w = p_w Y_w (1 - \delta) \quad (4.5a)$$

$$R_s(y_s, x) = p_s \left( Y_s - \kappa \frac{x(t)}{y_s(t)} \right) \quad (4.5b)$$

In the case of winter wheat, the more cold tolerant the seed used, the lower the winter kill rate (i.e.  $\delta$  approaches zero). In the case of spring wheat, the smaller is the damage made by ducks ( $\kappa$ ), and/or smaller the duck population ratio of duck population and spring wheat acreage ( $x/y_s$ ), the less is the yield loss.

#### 4.2.1.1. Loss of winter wheat yield ( $\delta$ )

In addition to management practices, there are other variables affecting winter survival that do not depend on the farmer (e.g. air temperature and snow cover<sup>20</sup>). As described in Chapter 2, all these variables together determine the probability of survival of winter wheat plants and are captured by this model in the parameter  $\delta$ . In other words,  $\delta$

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<sup>20</sup> Snow cover acts like a buffer helping soil temperature to be the least affected by air temperature changes.

represents the loss of yield due to winter kill. When  $\delta = 0$ , there is no risk of growing winter wheat due to winter kill.

#### 4.2.1.2. Loss of spring wheat yield ( $\kappa$ )

The coefficient  $\kappa$  represents the maximum loss of yield in the spring crop due to duck grazing on swathed crops during fall and is defined in a bushel/duck basis. This coefficient does not capture the loss in yield due to trampling, fouling (Cleary, 1994) or seed waste (Clark, Greenwood and Sugden 1986).

#### 4.2.2. Cost functions for wheat production $C_w(y_w)$ and $C_s(y_s)$

Production costs per acre for both spring and winter wheat are composed of two parts (Equations 4.6). The first part is interpreted as the average cost per acre ( $c_w$  and  $c_s$ ), including all necessary inputs (e.g. fertilizer, land cost, seeds, utilities); the second part is interpreted as the additional cost of seeding an extra acre that is not explained by the normal increase of input usage accounted in the average cost per acre ( $ac_w$  and  $ac_s$ ). Once cost per acre is multiplied by the total acreage, total production costs result in quadratic functions. The quadratic term adds concavity to the problem's objective function compensating for the linearity of the revenue functions.

$$C_w(y_w) = c_w + ac_w y_w(t) \quad (4.6a)$$

$$C_s(y_s) = c_s + ac_s y_s(t) \quad (4.6b)$$

There are various economic explanations to the addition of  $ac$ . The first is the loss in productivity when larger areas are seeded. Given the fact that prices and yields are fixed in the model, additional efforts might be needed to maintain revenues per acre (e.g. fertilizer usage or waterfowl damage prevention). The second explanation is the potential difference in land quality. It is intuitive to assume that farmers prefer to use their best sections of land first; the additional areas used to seed additional crop acres might not be as productive. At the same time, managing efforts increase with additional crop acres. This implies that managing efforts are not additive and therefore cannot be accounted in a per acre basis.

Values for  $ac_w$  and  $ac_s$  were specified using calibration techniques and are presented in Chapter 5.

#### **4.2.3. Cost of dealing with duck population different from yield loss $C_x(x)$**

With the presence of mallard ducks in agricultural areas, producers deal with another type of cost not associated with yield loss. This cost is usually related to damage on swathed crops caused by bird droppings. Wheat contaminated with wildlife excreta cannot be commercialized because it does not meet the Canadian Grain Commission standards. In this case, additional cleaning of the harvested crops is required. The additional cleaning cost depends on the severity of the contamination, and the severity of the damage depends on the number of birds residing in the crop production areas. In both the Farm Level and Social Planner models,  $C_x(x)$  is specified as a linear function of the duck population (Equation 4.7). The parameter  $c_x$  represents the cost associated with cleaning grain contaminated with bird excreta. The derivation of the value of  $c_x$  is discussed in the following chapter.

$$C_x(x) = c_x x(t) \quad (4.7)$$

#### **4.2.4. Payment per acre of winter wheat ( $\phi$ )**

As society benefits from Mallard population, the social planner model includes economic incentives to farmers to grow winter wheat as a measure to improve mallard breeding habitat. A direct payment per acre seeded with the winter wheat ( $\phi$ ) is added into the farmer's winter wheat revenue function to explore their effect in the farmer's seeding decision. The winter wheat revenue per are, including direct payment, is specified in Equation 4.8.

$$R_w(y_w) = p_w Y_w (1 - \delta) + \phi. \quad (4.8)$$

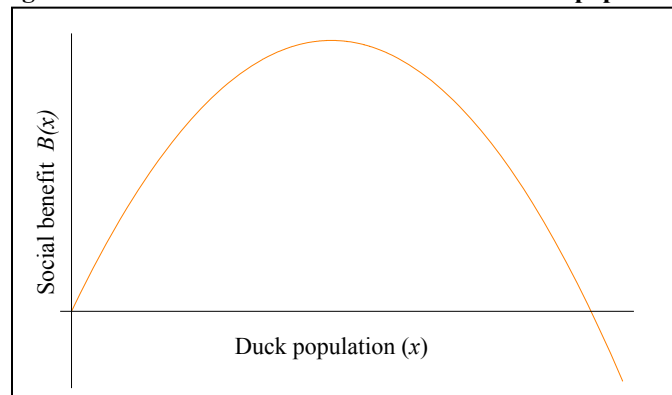
#### 4.2.5. Social benefit function of mallard population ( $B(x)$ )

Society's benefit associated with Mallard population was modeled as a function of duck population size and its associated use, non-use, and existence values. A quadratic functional form was used to specify such benefit (Equation 4.9)

$$B(x) = \alpha x(t) - \beta x(t)^2 \quad (4.9)$$

Parameter  $\alpha$  represents the amount by which utility increases with every additional duck, and  $\beta$  denotes the decreasing rate at which utility increases. This function has a concave shape (Figure 4.3) that represents how duck numbers positively affect the utility of people who value ducks. The utility increases at a decreasing rate, which means that every additional duck enhances utility but by a smaller amount than the duck before it. In extreme cases where the population reaches very high numbers, Mallards may become a nuisance to people in which case society's utility will start to decrease with each additional duck. If mallard population increases to a point where its management becomes very difficult (which is possible but improbable) the utility obtained by the presence of ducks could become negative (e.g. snow geese population in Canadian agricultural areas). It is intuitive to suggest that the duck population that maximizes society's utility is close to carrying capacity, and that extra numbers beyond that point will decrease utility due to higher duck mortality related to competition for resources.

Figure 4. 1. Social benefit as a function of Mallard population





Even though the mallard population improves social benefit, additional duck numbers may involve certain costs to society depending on how population growth is accomplished. Direct payments to farmers to encourage fall seeded crops acreage can be financed by private or public sources. Although, the provision of these types of payments to encourage farmers to grow winter wheat in Canada has been solely by private organizations such as DUC, the option of a public organization (e.g. government) as the source of the payment is also explored in this thesis. The source of the potential payments made by the government is assumed to be public funds (i.e. from taxes). When farmers receive these payments from public funds, the government budget, which could potentially be used for other public goods or services to benefit society, is decreased. Furthermore, the process of designing, planning and implementing the payment has a cost (in the model this transaction cost is denoted as  $\varepsilon$ ), that is also paid using public funds. Transaction costs, also called welfare cost, associated with environmental policies can be defined as “costs of information gathering, contracting, and controlling and/or enforcing established agreements” (Vatn 1998: 516). In the case where direct payments to farmers are supplied by the government, the payment value ( $\phi y_w$ ) and the transaction cost ( $\varepsilon$ ) must be subtracted from the objective function describing society’s benefit. In other words, when public funds are used as the payment’s source  $\phi$  and the transaction costs associated with  $\phi$  are indirectly paid by the society. Therefore, the benefit function specified in equation 4.9 becomes:

$$B(x, y_w) = \alpha x(t) - \beta x(t)^2 - \phi(1 + \varepsilon)y_w(t). \quad (4.10)$$

#### **4.2.6. Mallard population dynamics: the logistic growth function**

The basic logistic growth function presented in Chapter 2 (Equation 2.1) was modified to include the effect of upland nesting habitat obtained from winter wheat areas in the intrinsic growth rate of the mallard population. Waterfowl nest survival is higher in winter wheat croplands because areas seeded with winter crops are not disturbed in the spring nesting season. As a result, nesting habitat created in winter wheat areas decreases

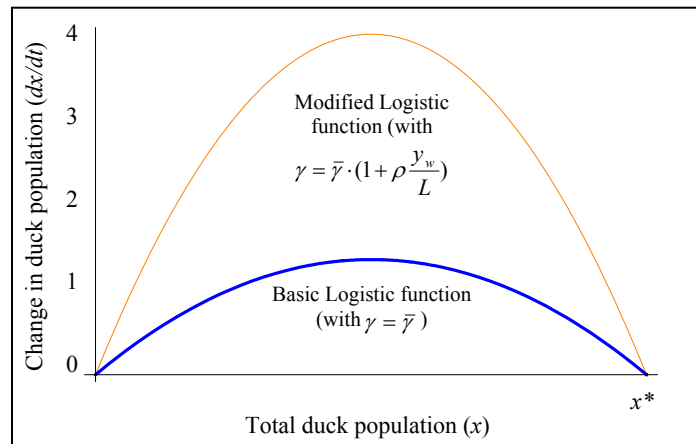
nest mortality rates and consequently increases population growth rates. Therefore,  $\gamma$  becomes a function of winter wheat acreage  $\gamma(y_w)$ . A linear functional form is used to model  $\gamma(y_w)$ . The intercept of the function is the intrinsic growth rate  $\bar{\gamma}$ , and the slope is given by the gain in  $\bar{\gamma}$ , determined by  $\rho$ , times the proportion of winter wheat acreage with respect to total wheat acreage. Parameter  $\rho$  represents the increases in growth rate ( $\gamma$ ) determined by winter wheat acreage. The function  $\gamma(y_w)$  is specified in equation 4.11, and the modified Logistic growth function is depicted in equation 4.12.

$$\gamma(y_w) = \bar{\gamma} \cdot \left( 1 + \rho \frac{y_w}{L} \right) \quad (4.11)$$

$$\dot{x} = x(t) \gamma(y_w) \left( 1 - \frac{x(t)}{K} \right) \quad (4.12)$$

These two equations imply that every additional acre seeded with winter wheat will increase mallard population growth rate by  $\rho$ . The result of an acreage increase of winter wheat in a habitat that can sustain  $x^*$  ducks is depicted in figure 4.2; the thick line represents the evolution of a mallard population if  $\gamma = \bar{\gamma}$ , while the thin line represents the evolution of the population where  $\gamma$  is a function of winter wheat acreage. The difference between the two parabolas indicates the gain in duck numbers due to the lower levels of nest mortality.

**Figure 4. 2. Basic and modified Logistic growth functions**



Since the modified logistic growth model assumes that areas where winter wheat is grown improve mallard breeding habitat, it could also be assumed that the new breeding habitat could potentially improve carrying capacity. However, there was no evidence in the literature about mallard ducks or any other North American waterfowl species population dynamics that supported this hypothesis. Waterfowl population carrying capacity is mainly explained by the availability of habitats such as lakes, ponds, rivers, potholes, woodland pools and surrounding uplands. In general, carrying capacity depends mostly on wetlands and associated vegetation that provides mallard's main food resources, vegetative cover that provides protection from predators, and isolation from other mallard pairs minimizing territory conflicts.

### ***4.3. Hamiltonian and Analytical Results***

In the following sections the Hamiltonian functions for the farm level model and the social planner model are presented and their corresponding first order conditions are derived.

#### **4.3.1. Farm Level Model**

After inserting revenue functions (Equation 4.5a and 4.5), production costs functions for spring and winter wheat (Equations 4.6a and 4.6b), and the duck cost function (Equation 4.7) into the objective function (Equation 4.1), the economic problem is to maximize the net revenue of wheat production subject to three restrictions: a land restriction, because the farmer has only a limited amount of land to grow wheat; a duck population growth function, which is positively affected by the acreage seeded with winter wheat; and the initial level of duck population. This set up is the start point to the optimization process.

At this point it is possible to specify the Hamiltonian as function of the state variable  $x(t)$ , the two control variables  $y_w(t)$  and  $y_s(t)$ , the costate variable  $\lambda(t)$ , and a land user cost  $\mu(t)$ . The costate variable  $\lambda(t)$ , also known as the shadow price, represents the implicit value of the a Mallard duck in the model. The variable  $\mu(t)$  can be interpreted as the opportunity

cost of using the land for agricultural purposes; in other words, it represents the *loss* of income of the land owner for not leasing the land. The Hamiltonian for the farm level model is specified as  $H_f(y_w(t), y_s(t), x(t), \lambda(t), \mu(t))$ , but will be denoted as  $H_f$  to simplify the notation.  $H_f$  is specified in equation 4.13.

$$H_f = \left\{ [p_w Y_w (1 - \delta) - c_w - a c_w y_w(t)] y_w(t) + \left[ p_s \left( Y_s - \kappa \frac{x(t)}{y_s(t)} \right) - c_s - a c_s y_s(t) \right] y_s(t) - c_x x(t) \right\} e^{-rt} \\ + \lambda(t) \left\{ x(t) \left[ \bar{\gamma} + \bar{\gamma} \rho \frac{y_w(t)}{L} \right] \left[ 1 - \left( \frac{x(t)}{K} \right) \right] \right\} + \mu(t) [L - y_w - y_s] \quad (4.13)$$

The software Mathematica 6 was used to derive the five FOC of the FL and SP models. The FOC are:

- $\partial H_f / \partial y_w = 0$  and  $\partial H_f / \partial y_s = 0$  for control variable winter and spring wheat acreage
- $-\partial H_f / \partial x = \dot{\lambda}$  for state variable duck population
- $\partial H_f / \partial \lambda = \dot{x}$  for the costate variable  $\lambda$
- $\partial H_f / \partial \mu = 0$  for the shadow price of land

The FOC with respect to winter wheat acreage is specified as:

$$\frac{\partial H_f}{\partial y_w(t)} = [p_w Y_w (1 - \delta)] e^{-rt} - [C_w(y_w) + C_w'(y_w) y_w(t)] e^{-rt} + \lambda(t) \left[ x(t) \gamma'(y_w) \left( 1 - \frac{x(t)}{K} \right) \right] - \mu(t) = 0 \quad (4.14)$$

Equation 4.14 was manipulated to obtain an equality between the marginal benefits (left side of the equation 4.15) and marginal costs (right side of the equation 4.15) associated with seeding an extra acre of winter wheat.

$$\underbrace{[p_w Y_w (1 - \delta)] e^{-rt}}_{\text{Farmer's revenue for every additional acre seeded with winter wheat}} + \underbrace{\lambda(t)x(t)\gamma'(y_w)\left(1 - \frac{x(t)}{K}\right)}_{\text{Implicit value/cost of the new mallard population influenced by winter wheat upland habitat}} = \underbrace{[C_w(y_w) + C_w'(y_w)y_w(t)] e^{-rt}}_{\text{Production cost of an additional acre seeded with winter wheat}} + \underbrace{\mu(t)}_{\text{Opportunity cost of land use}} \quad (4.15)$$

Marginal benefits are composed of two terms. First, the marginal revenue generated by the extra sales of winter wheat is represented by  $[p_w Y_w (1 - \delta)] e^{-rt}$ . Second, the value of the new duck population resulting from the reduction in nest mortality due to the upland habitat created by winter wheat cropland is  $\lambda(t)x(t)\gamma'(y_w)$ . This value can be interpreted as a cost because the model assumes that ducks do not have economic benefit to the farmer rather they represent a cost (i.e.  $\lambda(t)$  is expected to have a negative sign). The change in the mallard population is calibrated by the number of ducks that the habitat can support,  $[1 - (x(t)/K)]$ . Even though population growth increases due to less nest mortality, if there are not enough resources in the ecosystem for the new mallard population, mortality of ducklings and females might overcome the gain in nest success. Consequently, mallard population size is limited by the carrying capacity.

Marginal costs of additional acreage are composed of the average and additional costs associated with winter wheat production  $[C_w(y_w) + C_w'(y_w)y_w(t)] e^{-rt}$ , and the opportunity cost (or implicit cost) of using the land to grow crops  $\mu(t)$ .

The FOC with respect to spring wheat acreage is:

$$\frac{\partial H_f}{\partial y_s(t)} = \left\{ p_s Y_s - [C_s(y_s) + C_s'(y_s) \cdot y_s(t)] \right\} e^{-rt} - \mu(t) = 0 \quad (4.16)$$

At the maximum, the marginal revenue of growing an additional acre with spring wheat must equal its marginal costs (Equation 4.17). Revenues are determined by the income generated by the spring wheat produced with yield  $Y_s$  sold in the market at price  $p_s$ . The part of the revenue affected by the yield loss due to mallard damage to the spring crops is

not captured in this expression because it is a cost associated with duck population, not with the production process itself. The revenue generated by the production of spring wheat is represented by  $p_s Y_s e^{-rt}$ . The marginal cost, as in the winter wheat case, is given by the average and additional costs of seeding an extra acre  $[C_s(y_s) + C_s'(y_s) \cdot y_s(t)]e^{-rt}$ , and the opportunity cost of land  $\mu(t)$ .

$$\underbrace{p_s Y_s e^{-rt}}_{\substack{\text{Farmer's revenue} \\ \text{for every} \\ \text{additional acre} \\ \text{seeded with spring} \\ \text{wheat}}} = \underbrace{[C_s(y_s) + C_s'(y_s) \cdot y_s(t)]e^{-rt}}_{\substack{\text{Production cost of an} \\ \text{additional acre seeded with} \\ \text{spring wheat}}} + \underbrace{\mu(t)}_{\substack{\text{Opportunity} \\ \text{cost of land} \\ \text{use}}} \quad (4.17)$$

The first order condition with respect to duck population is given by:

$$-\frac{\partial H_f}{\partial x(t)} = [p_s \kappa + C_x'(x) - B'(x)]e^{-rt} - \lambda(t) \left[ \gamma(y_w) \left( 1 - \frac{2x(t)}{K} \right) \right] = \dot{\lambda} \quad (4.18)$$

By reorganizing equation 4.15, equation 4.19 can be derived. At the maximum, costs associated with the presence of ducks in croplands must equal the value of the mallard population. Additional duck numbers generate two costs to the farmer. First, the loss of yield of spring wheat due to grazing ducks:  $p_s \kappa e^{-rt}$ . Second, the cost associated with cleaning swathed crops contaminated with duck excreta,  $C_x'(t)$ . The duck population value is implicitly given by the shadow price  $\lambda(t)$  and its change through time ( $\dot{\lambda}$ ). The price per duck is represented by  $\lambda(t)\gamma(y_w)(1 - 2x(t)/K)$ ; given that the expression is a function of  $x(t)$ , the value of a mallard duck changes depending on the  $x(t) - K$  ratio; the closer the size of the population is to the carrying capacity, the lower is the value per duck.

$$\underbrace{p_s \kappa e^{-rt}}_{\substack{\text{Loss of yield} \\ \text{due to crop} \\ \text{depreciation by} \\ \text{ducks}}} + \underbrace{C_x'(x)}_{\substack{\text{Cost associated} \\ \text{with duck} \\ \text{population due} \\ \text{to crop} \\ \text{contamination}}} = \underbrace{\dot{\lambda}}_{\substack{\text{Change} \\ \text{of implicit} \\ \text{value per} \\ \text{duck}}} + \underbrace{\lambda(t)\gamma(y_w) \left( 1 - \frac{2x(t)}{K} \right)}_{\substack{\text{Implicit value/cost per duck} \\ \text{responding at changes of the} \\ \text{ratio of } x(t) \text{ and } K}} + \underbrace{B'(x)e^{-rt}}_{\substack{\text{Gain on} \\ \text{society's utility} \\ \text{due to duck} \\ \text{population}}} \quad (4.19)$$

Finally, the FOC with respect to the  $\lambda(t)$  states that the change in Mallard population must equal the equation of motion specified by the logistic growth model; and the FOC with respect to  $\mu(t)$  reaffirms the land restriction.

#### 4.3.2. Social Planner Model

The revenue expressions for winter wheat (Equation 4.8) and spring wheat production (Equation 4.5b), production costs (Equations 4.6a and 4.6b), duck population cost (Equation 4.7), and society's benefit due to mallard population (Equation 4.9) are substituted into the objective functions with payments from private source (Equation 4.3a) or public funds (Equation 4.3b). Using the three restrictions (Equations 4.4) the Hamiltonian for the SP model is specified as a function of the control, state and costate variables:  $H_{sp}(y_w(t), y_s(t), x(t), \lambda(t), \mu(t))$ .  $H_{sp}$  is maximized and solved for all five variables. The mathematical specifications of  $H_{sp}$ , when  $\phi$  comes from a private and public source are expressed in equations 4.19a and 4.19b, respectively.

$$H_{sp} = \left\{ [p_w Y_w (1 - \delta) + \phi - C_w(y_w)] y_w(t) + \left[ p_s \left( Y_s - \kappa \frac{x(t)}{y_s(t)} \right) - C_s(y_s) \right] y_s(t) - C_x(x) + B(x) \right\} e^{-rt} \\ + \lambda(t) \left[ x(t) \gamma(y_w) \left( 1 - \frac{x(t)}{K} \right) \right] + \mu(t) [L - y_w - y_s] \quad (4.20a)$$

$$H_{sp} = \left\{ [p_w Y_w (1 - \delta) + \phi - C_w(y_w)] y_w(t) + \left[ p_s \left( Y_s - \kappa \frac{x(t)}{y_s(t)} \right) - C_s(y_s) \right] y_s(t) - C_x(x) + B(x, y_w) \right\} e^{-rt} \\ + \lambda(t) \left[ x(t) \gamma(y_w) \left( 1 - \frac{x(t)}{K} \right) \right] + \mu(t) [L - y_w - y_s] \quad (4.20b)$$

The difference between expressions 4.20a and 4.20b is the specification of the benefit functions  $B(x)$  and  $B(x, y_w)$ . While the former depends uniquely on the duck population, the second is a function of both the duck population and total winter wheat acreage. Following the same procedure used in the Farm Level model, the FOC conditions were derived for  $H_{sp}$ . The results are expected to be similar to what was presented in section

4.3.1, except for the effects of the direct payments  $\phi$  over the farmer's winter wheat revenue and society's benefit.

The FOC with respect to winter wheat acreage are specified in equations 4.21a and 4.21b for  $\phi$  coming from private and public funds, respectively.

$$\begin{aligned} \frac{\partial H_{sp}}{\partial y_w(t)} &= [p_w Y_w (1 - \delta) + \phi] e^{-rt} - [C_w(y_w) + C_w'(y_w) y_w(t)] e^{-rt} \\ &+ \lambda(t) \left[ x(t) \gamma'(y_w) \left( 1 - \frac{x(t)}{K} \right) \right] - \mu(t) = 0 \end{aligned} \quad (4.21a)$$

$$\begin{aligned} \frac{\partial H_{sp}}{\partial y_w(t)} &= [p_w Y_w (1 - \delta) + \phi] e^{-rt} - [C_w(y_w) + C_w'(y_w) y_w(t)] e^{-rt} + B'(x, y_w)_{y_w} e^{-rt} \\ &+ \lambda(t) \left[ x(t) \gamma'(y_w) \left( 1 - \frac{x(t)}{K} \right) \right] - \mu(t) = 0 \end{aligned} \quad (4.21b)$$

Reorganizing terms in equation 4.21a, the FOC with respect to winter wheat acreage can be expressed as an equality between social marginal benefits and social marginal costs of an additional acre seeded with winter wheat (equation 4.22)<sup>21</sup>. This means that at the maximum, marginal benefits and marginal costs related to winter wheat production are the same. Marginal social benefits are composed of farmer's marginal revenue and the value of new duck population attributable to improvements in nest survival due to the availability of winter wheat fields. Marginal social costs include winter wheat production costs of an additional acre of winter wheat, changes in society's benefits and the opportunity costs of land.

$$\underbrace{[p_w Y_w (1 - \delta) + \phi] e^{-rt}}_{\text{Farmer's revenue for every additional acre seeded with winter wheat}} + \underbrace{\lambda(t) x(t) \gamma'(y_w) \left( 1 - \frac{x(t)}{K} \right)}_{\text{Implicit value of the new Mallard population influenced by winter wheat upland habitat}} = \underbrace{[C_w(y_w) + C_w'(y_w) y_w(t)] e^{-rt}}_{\text{Production cost of an additional acre seeded with winter wheat}} - \underbrace{B'(x, y_w)_{y_w} e^{-rt}}_{\text{Loss of society's benefit due to payments to farmers to promote winter wheat}} + \underbrace{\mu(t)}_{\text{Opportunity cost of land use}} \quad (4.22)$$

<sup>21</sup> Equality between marginal benefits and costs in the case of private  $\phi$  is omitted to simplify results discussion and is similar to equation 4.22.



There are some differences and similarities between the FL and SP marginal benefits. Farmer's marginal revenue due to winter wheat production (i.e. revenue per acre) is higher in the SP model than the FL model by  $\phi$ . However, the expression for the value of additional duck population is equivalent in both FL and SP models.

The main difference between FL and SP marginal costs is the inclusion of changes in society's benefit  $B'(x, y_w)_{y_w}$ .  $B'(x, y_w)_{y_w}$  represents the derivative of  $B(x, y_w)$  with respect to  $y_w$ . If  $\phi$  is paid to farmers by the government as an encouragement to seed larger areas with winter wheat, society's utility is decreased by  $B'(x, y_w)_{y_w} e^{-rt}$  with every additional acre of winter wheat. Total marginal costs associated with winter wheat production are composed of the costs to the farmer (i.e. production costs) and to society (benefit loss). Technically,  $B'(x, y_w)_{y_w} e^{-rt}$  is subtracted from wheat production costs, however as  $\partial B(x, y_w)/\partial y_w < 0$ , then the effect of winter wheat acreage on society's utility is actually added to  $[C_w(y_w) + C_w'(y_w)y_w(t)]e^{-rt}$  (equation 4.22).

On the other hand, if  $\phi$  is paid by a private organization, changes in winter wheat acreage do not directly affect society's benefit. For that reason, the expression  $B'(x, y_w)_{y_w} e^{-rt}$  would be eliminated from the right hand side of equation 4.22 and marginal costs associated with winter wheat production would be described by production costs and land opportunity cost:  $[C_w(y_w) + C_w'(y_w)y_w(t)]e^{-rt} + \mu(t)$ .

The results from FOC analysis with respect to spring wheat in the SP model are equivalent to results for the FL model because the payment  $\phi$  does not directly affect the seeding decision for the spring crop. Therefore equations 4.23 (FOC with respect to spring wheat acreage in the SP model) and 4.24 (equality between marginal social benefits and social costs) are identical to equations 4.16 and 4.17, respectively.

$$\frac{\partial H_{sp}}{\partial y_s(t)} = \left\{ p_s Y_s - [C_s(y_s) + C_s'(y_s) \cdot y_s(t)] \right\} e^{-rt} - \mu(t) = 0 \quad (4.23)$$

$$\underbrace{p_s Y_s e^{-rt}}_{\text{Farmer's revenue for every additional acre seeded with spring wheat}} = \underbrace{[C_s(y_s) + C_s'(y_s) \cdot y_s(t)]e^{-rt}}_{\text{Production cost of an additional acre seeded with spring wheat}} + \underbrace{\mu(t)}_{\text{Opportunity cost of land use}} \quad (4.24)$$

The FOC with respect to mallard duck population  $x(t)$  is expressed in equation 4.25. At the maximum, the costs associated with  $x(t)$  must equal its benefits. All the costs and benefits of ducks that come up in the social planner model with private  $\phi$  and public  $\phi$  are the same. There is only a different form for the society's benefit function; in the first case  $B(\cdot)$  is a function of  $x(t)$  only, while in the second case  $B(\cdot)$  is a function of both  $x(t)$  and  $y_w(t)$ . However, since what is evaluated in equations 4.25 and 4.26 are the effects of changes in  $x(t)$ ,  $B'(x)$  and  $B'(x, y_w)_x$  are equivalent. As a result, equation 4.25 and 4.26 represent exactly the same equality between costs and benefits generated by Mallard population.

$$-\frac{\partial H_{sp}}{\partial x(t)} = [p_s \kappa + C_x'(x) - B'(x, y_w)_x]e^{-rt} - \lambda(t) \left[ \gamma(y_w) \left( 1 - \frac{2x(t)}{K} \right) \right] = \dot{\lambda} \quad (4.25)$$

$$\underbrace{p_s \kappa e^{-rt}}_{\text{Loss of yield due to crop depredation by ducks}} + \underbrace{C_x'(x)e^{-rt}}_{\text{Cost associated with duck population due to crop contamination}} = \underbrace{\dot{\lambda}}_{\text{Change of implicit value per duck}} + \underbrace{\lambda(t) \left[ \gamma(y_w) \left( 1 - \frac{2x(t)}{K} \right) \right]}_{\text{Implicit value per duck responding at changes of the ratio of } x(t) \text{ and } K} + \underbrace{B'(x, y_w)_x e^{-rt}}_{\text{Gain on society's utility due to duck population}} \quad (4.26)$$

### 4.3.3. Mathematical results

The Hamiltonian functions specified in equations 4.13 and 4.20 were introduced in the software Mathematica, and the FOC's were calculated. Since in the steady state  $\dot{x}$  and  $\dot{\lambda}$  equal zero, all FOCs equaled to zero. The resulting system of equations was solved for  $y_w(t)$ ,  $y_s(t)$ ,  $x(t)$ ,  $\lambda(t)$ ,  $\mu(t)$ . The expressions derived for each variable represent the optimal levels of each variable.

Two sets of solutions were found from the FL model: one with the duck population equal to zero ( $x=0$ ), and another with the duck population equal to the carrying capacity ( $x=K$ ). The set of solutions with  $x=0$  is presented in the following equations.

$$x(t)^* = 0 \quad (4.27a)$$

$$y_s(t)^* = \frac{-c_s + c_w + 2Lac_w + p_s Y_s - p_w Y_w (1-\delta)}{2(ac_s + ac_w)} \quad (4.27b)$$

$$y_w(t)^* = \frac{c_s - c_w + 2Lac_s - p_s Y_s + p_w Y_w (1-\delta)}{2(ac_s + ac_w)} \quad (4.27c)$$

$$\lambda(t)^* = \frac{2e^{-rt}L(ac_s + ac_w)(c_x + p_s \kappa)}{\bar{\gamma}[2Lac_w + 2Lac_s + \rho(c_s - c_w + 2Lac_s - p_s Y_s + p_w Y_w (1-\delta))]} \quad (4.27d)$$

$$\mu(t)^* = -\frac{e^{-rt}(c_w ac_s + c_s ac_w + 2Lac_s ac_w - ac_w p_s Y_s - ac_s p_w Y_w (1-\delta))}{ac_s + ac_w} \quad (4.27e)$$

The expression for spring wheat acreage (Equation 4.27b) is defined in terms of land size and both spring and winter wheat revenues and costs coefficients. Spring wheat acreage is negatively related to its own costs coefficients ( $c_s$ ,  $ac_s$ ) and to winter wheat's revenue ( $p_w Y_w (1-\delta)$ ). It is positively related to land size, its own revenue ( $p_s Y_s$ ) and to winter wheat's cost coefficient  $ac_w$ . Also, it is inversely related to the additional cost coefficient for both spring and winter wheat production. All coefficients in equation 4.27b have the expected sign. Farmers will seed more spring wheat when its revenue increases (via prices or yields), and when production costs of winter wheat increase. Also, spring wheat acreage will decrease if its production cost increases and/or if revenue for winter wheat increases.

The expression for winter wheat acreage (Equation 4.27c) is also a function of land size, revenues and costs of both types of wheat, and is symmetric with equation 4.27b. The addition of both expressions equals the land size  $L$ .

The optimal value of the costate variable  $\lambda(t)$ , which represents the shadow price of duck population in the system, is a function of production costs and revenues of both types of

wheat, costs associated with duck population in croplands, average intrinsic growth rate ( $\bar{y}$ ), gain on  $\bar{y}$  due to winter wheat acreage ( $\rho$ ), and land size. Finally, the optimal use cost of land represented by  $\mu(t)$  is a function of winter and spring wheat revenues and production costs coefficients.

Results for  $x=K$ , have the same expressions for spring wheat acreage, winter wheat acreage and land shadow price. The difference with the first set of results, other than a positive mallard population, is that the shadow price for the duck population has the opposite sign of equation 4.27d.

Solution for the SP model with payments from a private source is composed of three sets of results:  $x=0$ ,  $x=K$  and  $x<0$ . Since a negative duck population is inconsistent with intuition, the latter set of results is dropped and only the first two are analyzed. The expressions for social optimal of control and co-state variables when  $x=0$  are presented in equations 4.28.

$$x(t)^* = 0 \quad (4.28a)$$

$$y_s(t)^* = \frac{-c_s + c_w + 2Lac_w + p_s Y_s - p_w Y_w (1 - \delta) - \phi}{2(ac_s + ac_w)} \quad (4.28b)$$

$$y_w(t)^* = \frac{c_s - c_w + 2Lac_s - p_s Y_s + p_w Y_w (1 - \delta) + \phi}{2(ac_s + ac_w)} \quad (4.28c)$$

$$\lambda(t)^* = \frac{2e^{-rt}L(ac_s + ac_w)(c_x - \alpha + p_s \kappa)}{\bar{y}[2Lac_w + 2Lac_s + \rho(c_s - c_w + 2Lac_s - p_s Y_s + p_w Y_w (1 - \delta) + \phi)]} \quad (4.28d)$$

$$\mu(t)^* = -\frac{e^{-rt}(ac_w c_s + ac_s c_w + 2ac_s ac_w L - ac_w p_s Y_s - ac_s p_w Y_w (1 - \delta) - ac_s \phi)}{ac_s + ac_w} \quad (4.28e)$$

Expression 4.28b differs from 4.27b in the subtraction of the payment  $\phi$  in the numerator. This means that direct payment as an incentive to grow winter wheat, discourages farmers to grow spring wheat; however, the effect the payment on the total

acreage depends on its magnitude. Likewise, equation 4.28c represents a higher winter wheat acreage compared with 4.27c, because of the addition of  $\phi$  in the numerator.

The expression of the optimal value of the co-state variable (Equation 4.28d) incorporates the negative of the coefficient  $\alpha$  in the numerator, and adds the farmers' payment  $\phi$  in the denominator. Consequently, both coefficients contribute to a smaller duck population shadow price compared to the FL model solution for zero mallard population. Finally, equations 4.27e and 4.28e differ in the fact that the latter adds the term  $ac_s \phi$  to the numerator. This means that direct payment contributes with a higher land-use value.

The second set of solutions of the SP model with  $\phi$  from a private source, (with  $x = K$ ) only differs from equations 4.28 on the expression for  $\lambda(t)^*$ . The difference is two twofold. First, the expression has the opposite sign; and second, the numerator also depends on the parameter  $\beta$  (Equation 4.29). In this case, coefficient  $\alpha$  contributes with a higher value of  $\lambda(t)^*$ , while  $\beta$  decreases it. This result is intuitive, since higher marginal values of ducks should increase the shadow price of Mallard population within the system.

$$\lambda(t)^* = -\frac{2e^{-rt}L(ac_x + ac_s)(c_x - \alpha + 2K\beta + p_s\kappa)}{\gamma[2ac_wL + 2ac_sL + \rho(c_s - c_w + 2Lac_s - p_sY_s + p_wY_w(1 - \delta) + \phi)]} \quad (4.29)$$

The solution for the SP model with payments from a public source also had three sets of solutions; for  $x=0$ ,  $x=K$  and  $x<0$ . Once again, the solution implying a negative duck population was not taken into account. The main difference between the results for two SP is the effect of the direct payment in each variable's solution. In the SP model with public payments,  $\phi$  is multiplied by  $\varepsilon$  in all the expressions. Since  $1 < \varepsilon < 0$ , then  $\varepsilon\phi < \phi$ ; the presence of a transaction cost associated with  $\phi$  decreases the effect of the payment in the variables of the system.

#### ***4.4. Conclusions***

The Farm Level (FL) model deals with the problem of a representative farmer in the Canadian Prairies who needs to make an optimal seeding decision regarding spring and winter wheat acreages, in order to maximize the profit obtained from wheat production. The solution of this model is a private optimum.

The Social Planner (SP) model deals with the problem of a social manager that needs to find the optimal acreages of spring and winter wheat, as well as the optimal mallard population in order to maximize both farmer's profit and society's benefit associated with mallard duck population. The solution of this model is a social optimum.

In the case of the SP model, two cases are analyzed. First, when direct payments to farmers to encourage improvements in winter wheat seeded area come from a private source; and second, when direct payments come from a public source. In the first case, society benefits from ducks without being affected by the amount paid to farmers to grow more winter wheat. In the second case, as public sources are used to pay farmers, society is indirectly making these payments and is also paying for the transaction cost that these imply. Therefore, society's benefit will increase with higher duck populations, but will decrease with every additional payment that needs to be made to encourage farmers to seed winter wheat.

The difference between the two models is that the SP deals with both the costs and benefits associated with mallard population and provides annual direct payments per acre to the farmers as an incentive to grow winter wheat. The FL assumes that the presence of the duck population represents a cost to farmers. For these two reasons it is expected that optimal mallard numbers and winter wheat acreage in the SP model are higher than the private optimal values.

The mathematical solution of the FL and SP models gave two feasible optimal duck population sizes, zero or a size equal to carrying capacity. This means that the solution of

the maximum of the private and social benefits has two local optimums; either when there is no duck population in crop areas, or when the population reaches the maximum level the habitat can support. The global maximum will be identified once all the parameter values are introduced into the mathematical solutions.

Optimal acreage levels of winter and spring wheat depend on land size, production cost coefficients, prices and yields of both types of wheat. In general winter wheat acreage will increase if its production costs decrease or if its revenues increase. Also, if spring wheat's production costs increase or revenues decrease, farmers would be encouraged to grow more winter wheat. In the SP model, the addition of the annual farm payment increases winter wheat acreage and therefore decreases spring wheat acreage.

## **Chapter 5: Data and Numerical Results**

In this chapter the data used in the model are presented and calibration methods are applied to the production cost and societal benefit functions to derive the corresponding coefficient values. Once all coefficients used in the models are justified, their values are replaced in the equations presented in section 4.3.3. in order to derive the numerical results of the FL and SP models. Sensitivity analyses are performed to evaluate the response of main results to changes in farm payments and winter wheat cold tolerance. Finally, the stability of the systems associated to the FL and SP models are examined.

### ***5.1. Data and Calibration of Parameters***

#### **5.1.1. Prices**

Wheat prices paid to farmers were calculated as the average of the values paid by the Canadian Wheat Board (CWB) per wheat variety, minus freight and elevation costs. Prices were assumed to equal CWB payments in store Vancouver or Saint Lawrence (i.e. prices paid at the ports of any of the two coasts), for the years 2007-08. Appendix 2 contains CWB payments to spring and winter wheat farmers used to calculate wheat price. Freight costs were assumed to equal the CWB deductions for wheat shipped from Calgary (AB), Regina (SK), and Winnipeg (MB) in the 2007-08 period. These data were obtained from the Alberta Agriculture and Rural Development (AARD) website. Appendix 3 contains information used to calculate freight costs. Elevation costs were obtained as an average of fees charged by the Canadian companies listed in the Canadian Grain Commission (CGC) website, for the year 2007. The list of companies and the corresponding fees are presented in Appendix 4.

Farm prices for spring and winter wheat in the Prairie Provinces were specified in a dollar per bushel (\$/bu) unit. Wheat price estimations are presented in table 5.1. Overall, spring wheat price was higher than winter wheat price in all three provinces. Alberta



registered the highest prices for both types of wheat, while Saskatchewan had the lowest for the crop year 2007-08.

**Table 5. 1. Estimated wheat prices in the Prairie Provinces 2008 (\$/bu)**

<i>Province</i>	<i>Winter wheat</i>	<i>Spring wheat</i>
Alberta	7.48	8.10
Saskatchewan	7.01	7.63
Manitoba	7.35	7.96

Data source: CWB (2008), AARD (2008), CGC (2008)

### 5.1.2. Yields

The source of yield data for spring and winter wheat is Statistics Canada (2008). Yields used in the models correspond to average yields by each province for the year 2007. The original data were in kilograms per hectare (kg/ha) units, and were converted into bushel per acre unit (bu/acre). Conversions were done based on 1 ha = 2.47 acres, and 1 metric ton = 36.74 bu of wheat, conversion units. Wheat yields are presented in table 5.2.

**Table 5. 2. Average wheat yields in the Prairie Provinces (bu/acre)**

<i>Province</i>	<i>Winter wheat</i>	<i>Spring wheat</i>
Alberta	52.60	47.30
Saskatchewan	41.00	36.10
Manitoba	67.90	45.50

Source: Statistics Canada (2008b)

### 5.1.3. Loss of winter wheat yield: winter kill

Based on expert opinion (Thoroughgood, 2008), the probability of winter kill in the Prairies is approximately 5%. Consequently, this is the value of  $\delta$  used in the base case in both FL and SP models.

### 5.1.4. Loss of spring wheat yield: duck depredation

Yield loss due to grazing ducks in croplands, was assumed to occur only on spring wheat, and caused only by Mallard population. Crop damage was calculated using information from Sugden (1979). It is assumed that spring wheat yield loss per duck equals 115 grams

a day, times the number of days ducks feed from the spring crop. Since the period when most crop depredation occurs is approximately 25 days long, this number of days was chosen to calculate yield loss per duck. Estimated spring wheat yield loss per duck for a crop-year is 0.1056 bu/duck.

### **5.1.5. Production costs**

#### *5.1.5.1. Data*

Data for production costs were obtained from estimates published yearly by Alberta Agriculture and Rural Development (AARD), Saskatchewan Agriculture and Food (SAF), and Manitoba Agriculture, Food, and Rural Initiatives (MAFRI). Only data corresponding to black and dark brown soils were used because these are the types of soil traditionally used in the Prairie Provinces to grow both spring and winter wheat. All cost units were specified on a per acre basis. Costs that were not comparable between provinces were excluded. The data were grouped into 10 categories. These are: seeding, fertilizer, chemicals, crop insurance, machinery and fuels, utilities and miscellaneous, land costs, depreciation, investment in capital, and labor.

Seeding costs refer to seed costs, treatment and cleaning. Fertilizer costs include nitrogen, phosphorous, potassium, and sulphur. Chemical costs are composed of herbicides, insecticides/fungicides and others. Machinery and fuel are based on the costs associated with machinery operation and repairs, and fuel utilization. Land cost refers to land rent or land investments and taxes, depending on the province. Depreciation (and investment) refers to all capital depreciation (investment), including machinery and buildings. Finally, labor refers to all costs associated with hired labor, custom work and management. Tables including data on production costs for spring and winter wheat grown in Alberta, Saskatchewan and Manitoba are presented in Appendix 5.

The initial aim was to use costs for five years (2004-2008) for each Province to build the average costs per acre for spring and winter wheat. However, there were no data available on winter wheat grown in Manitoba prior to 2008. Therefore, the cost data used in the model was information for the year 2008. Estimated costs for each type of wheat are presented in table 5.3.

**Table 5. 3. Estimated wheat production costs in the Prairie Provinces (\$/acre)**

<i>Province</i>	<i>Winter wheat</i>	<i>Spring wheat</i>
Alberta	219.82	234.30
Saskatchewan	182.39	186.70
Manitoba	221.58	214.58

Data source: AAFRD (2008), SAF (2008), MAFRI (2008)

### 1.5.1.2. Calibration of parameters

Cost functions used in Equations 4.6a for winter wheat and 4.6b for spring wheat are functions of the number of acres seeded with winter ( $y_w$ ) and spring wheat ( $y_s$ ), respectively, and parameters  $c_w$ ,  $ac_w$ ,  $c_s$ , and  $ac_s$ . In the case of winter wheat, it is necessary to estimate the values of  $c_w$  and  $ac_w$  using the available information of cost per acre ( $CPA$ ). Since there are two unknown values in the cost function 4.6a, it is necessary to specify a second equation containing these parameters in order to solve the system of equations for the two unknowns. The first equation of the system is the cost function equal to  $CPA_w$  (Equation 5.1a); this equation states that the average cost per acre seeded with winter wheat, should equal a constant cost denoted by  $c_w$  plus the additional value  $ac_w$  that multiplies the acreage. The second equation of the system states that at the maximum, the change of the farmer's profit related with of winter wheat production ( $\pi_w$ ), is zero (Equation 5.1b).

$$c_w + ac_w y_w(t) = CPA_w \quad (5.1a)$$

$$\frac{\partial \pi_w}{\partial y_w} = 0 \Rightarrow p_w Y_w (1 - \delta) - c_w - 2 ac_w y_w = 0 \quad (5.1b)$$

Solving the system of equations, expressions for  $c_w$  and  $ac_w$  are derived (Equations 5.2).

$$c_w = 2 CPA_w - p_w Y_w (1 - \delta) \quad (5.2a)$$

$$ac_w = \frac{p_w Y_w (1 - \delta) - CPA_w}{y_w} \quad (5.2b)$$

Following the same procedure but using costs and profit functions for spring wheat, expression for  $c_s$ , and  $ac_s$  are also derived (Equations 5.3).

$$c_s = 2 CPA_s - p_s Y_s \quad (5.3a)$$

$$ac_s = \frac{p_s Y_s - CPA_s}{y_s} \quad (5.3b)$$

These results present  $c_w$  and  $c_s$  as constant values (i.e. do not depend on the number of acres seeded); therefore, these two values can be interpreted as the fixed costs of growing winter and spring wheat, respectively. The values of  $ac_w$  and  $ac_s$  depend on the wheat acreage; with higher acreage, the additional value of growing wheat is lower. In equations 5.2b and 5.3b,  $y_w$  and  $y_s$  were assumed to be the model's maximum amount of land the farmer has available to grow wheat. This way,  $ac_w$  and  $ac_s$  take small values, which can be interpreted as a high productivity assumption.

#### 5.1.6. Other costs associated with duck population

Damaged grain due to trampling and fouling is approximately two times the amount of grain eaten by waterfowl (Hammond 1961; Clark, Greenwood and Sugden 1986). Therefore, wheat grain damaged by mallards in croplands located in the Prairie Provinces is approximately 0.211 bu/duck a year. Grain cleaning costs in each province were used as a proxy to the cost that farmers bear due to damaged grain by Mallards. Grain cleaning costs were obtained as an average of fees charged by the Canadian companies listed in the Canadian Grain Commission (CGC) website for the year 2007 (Table 5.4). The complete list of companies and their respective fees are found in Appendix 6.

**Table 5. 4. Average grain cleaning costs in the Prairie Provinces (2007)**

<i>Province</i>	<i>Cleaning costs (\$/bu)</i>
Alberta	0.394
Saskatchewan	0.408
Manitoba	0.375

Data source: CGC (2008)

The total cost per duck not associated with yield loss in one crop year was calculated as the amount of wasted and contaminated grain by a Mallard duck times cleaning costs. Estimations of these costs for each province are presented in Table 5.5.

**Table 5. 5. Estimated grain cleaning costs**

<i>Province</i>	<i>Duck population cost (\$/duck)</i>
Alberta	0.083
Saskatchewan	0.079
Manitoba	0.086

Data source: Sugden (1979), CGC (2008)

### **5.1.7. Social and private discount rates**

Discounting is an essential element of dynamic optimization. Consequently choosing an appropriate discount rate is imperative. Discount rates used in private and social contexts may differ because objectives are usually different. When dealing with environmental problems, private discount rates tend to be higher because economic agents are only concerned about profit generation and conservation is usually not a priority because “effects are often felt far in the future, sometimes across several generations” (Weitzman 1993: 200). In social contexts, utilization of natural resources is usually less intensive because “of lower time preference of returns, expectations of growing scarcity of resources in the future” (Pope and Parry 1989: 257). Social discount rates are usually between 3.5% and 5% (Moore et al. 2004; Evans and Sezer 2004; Weitzman 2001), while private discount rates may reach higher values. For the FL model, a discount rate of 8% was used; and for the SP model the discount rate was 4%.

### **5.1.8. Direct payments to winter wheat producers and transaction costs**

In order to specify the value of the direct payment to encourage farmers to grow winter wheat, the one time payment that Ducks Unlimited Canada currently makes to farmers is taken as a reference value. This payment is between \$8 and \$10 per acre, depending on the province. A one time payment of \$10 was used in the SP model. However, since the payment needs to be added to the farmer’s winter wheat revenue expression in every

period, the value of the annual payment is calculated using the Present Value of a perpetuity equation. Equation 5.4 expresses the present value of a perpetual annuity, where  $A$  represents the annual payment,  $r$  depicts the social annual discount rate, and the present value is given by the \$10 one time payment. Solving for  $A$ , the value of the annuity is found. With a discount rate of 4%, the annuity paid to farmers each crop year equals \$0.4/acre.

$$\text{One time payment} = \frac{A}{r} \quad (5.4)$$

Transaction costs associated with direct payments to farmers to encourage winter wheat production are specified as a proportion of the total amount provided to encourage winter wheat growth. Measures of welfare costs of taxes, which are equivalent to transaction costs defined in the SP model, have been calculated in the literature (Ballard, Shoven and Whalley 1985; Browning 1987). However, these measures depend on several factors such as policy design, the type of model used in the estimation (i.e. general-equilibrium or partial-equilibrium economic models), labor supply elasticity and the marginal tax rate<sup>22</sup>. The reserachers suggest that policy's transaction costs can take a wide range of values, ranging between 10% and 55% of the cost of the policy. Estimations for transaction costs of environmental policies of water irrigation for agriculture are close to 20% (Smith and Tsur 1997), while policies related to carbon sequestration experienced transaction costs of approximately 30% (McKitrick 1997). In the SP model, a transaction costs of 20% wass used, as the SP problem is closer to Smith and Tsur's (1997) analytical framework than McKitrick's (1997) policies.

### **5.1.9. Social benefit associated with duck population**

Calibration methods were used to estimate values for the coefficients  $\alpha$  and  $\beta$  in the total social benefit function associated with mallard population (Equation 4.9). Calibration for these two coefficients was done in a similar way to the production cost function approach (i.e. solving a system of two equations with two unknowns; Equations 5.5). The first

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<sup>22</sup> Welfare cost of taxation is calculated as a function of wage rate (because tax payment source is people's income), labor supply elasticity, and of course tax rate.

equation was specified as the average waterfowl value (\$/bird), while the second equation was the maximum principle of the benefit function. The average waterfowl value (AWV) was calculated based on the hunting values per bird estimated by Hammack and Brown (1974) and van Kooten (1993a), and assuming that preservation and non-use values of waterfowl are twice the hunting value<sup>23</sup>. AWV takes a value of \$61.88 per duck. Since equation 5.5b deals with the maximum social benefit associated to mallard population,  $x$  in equations 5.5 represents the maximum historic population. The maximum mallard population estimated by the U.S. Fish and Wildlife Service for the Traditional Survey Area (TSA) was 11.23 million birds in the year 1958. Given that the TSA is approximately 543.63 million acres (2.2 million km<sup>2</sup>), the population density was approximately 0.021 ducks/acre. Assuming that this population density also applied to the Prairie Provinces<sup>24</sup>,  $x$  in Equations 5.5 was around 62 ducks for the 3,000 acres land restriction specified in the FL and SP models.

$$\frac{B(x)}{x} \Rightarrow \alpha + \beta x = AWV \quad (5.5a)$$

$$\frac{\partial B(x)}{\partial x} \Rightarrow \alpha + 2\beta x = 0 \quad (5.5b)$$

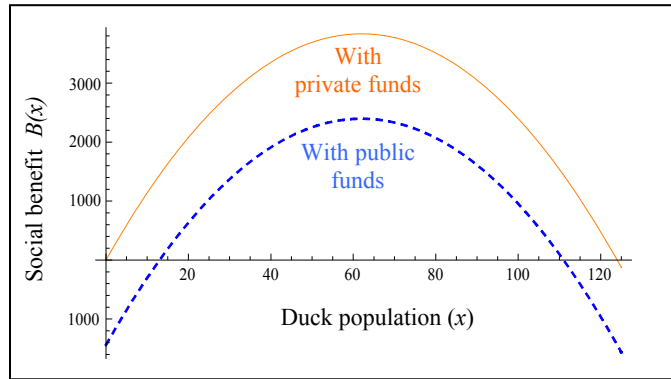
After solving the system of equations it was found that  $\alpha = 2AWV$  (\$123.75), and  $\beta = AWV/x$  (0.998). Figure 5.1 depicts social benefits when incentive payments to farmers who grow winter wheat are paid from private and public funds. As expected, society's benefit is lower when tax payments are used to persuade farmers to grow winter wheat (thick dashed line) compared to the case where farmers receive incentive payments from private sources (thin continuous line).

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<sup>23</sup> According to van Kooten (1993a), existence and non-use values of waterfowl can range from being the same to being for times hunting values (p. 773). Since, more specific measure of existence and non-use values were not found, these were assumed to be half of the maximum specified by van Kooten (1993a).

<sup>24</sup> This assumption is necessary since estimated numbers for Mallard population, carrying capacity and intrinsic growth rate, for only Alberta, Saskatchewan and Manitoba were not available in the literature reviewed.

**Figure 5. 1. Social benefit associated to Mallard population ( $\phi$  paid with private and public funds)**



### 5.1.10. Carrying Capacity and Intrinsic Growth Rates

Murray, Anderson, Steury (unpublished data) estimated values for carrying capacity and intrinsic growth rate for 10 duck species, including mallards, using a theta-logistic growth function and information from Waterfowl Breeding Population and Habitat Survey conducted by the U.S. Fish and Wildlife Service. Data used are from the TSA. The mallard's population intrinsic growth rate was specified as 0.332 and 0.149 for the periods 1955-1980 and 1981-2004, respectively. It was assumed the value of carrying capacity was uniform through all the surveyed area; therefore Alberta, Saskatchewan and Manitoba have the same coefficient value. The authors calculated carrying capacity as 7.77 million for the period 1955-1980 and 6.86 million for the period 1981-2004. In order to have a value per acre, it was assumed that carrying capacity was uniformly distributed through all the TSA; hence its value per acre was the ratio between the total number of ducks habitat could sustain and total area size. Carrying capacity in the 3,000 acre area, specified in the models, was calculated using estimated values for the latter period (i.e. 38 ducks).

### 5.1.11. Intrinsic growth rate as a function of winter wheat acreage

Mallard population growth rates,  $\gamma$  in this context, depend on various variables such as nest success, hen survival, and duckling survival. Hoekman (2002) argues that variations in nest success explain approximately 43% of the variations of population growth rate in



the PPR. Current rates of nest success in wheat fields are relatively low for Alberta and Saskatchewan, and slightly better for Manitoba (Table 5.6). Cowardin, Gilmer and Shaiffer (1985) argue that nest success of 15% is required to obtain a stable mallard population. Duck nest success in cropland areas may be improved if non-tillage crops are seeded instead of spring seeded crops. The parameter  $\rho$  in Equation 4.11 captures the increase in  $\gamma$  due to a higher availability of upland habitat free from agricultural practices that cause nest mortality.

**Table 5. 6. Weighted average of nest success in agricultural areas seeded with wheat (2008)**

	<i>Nest success rate</i>
Alberta	13.11%
Saskatchewan	13.17%
Manitoba	16.96%

Data source: Statistics Canada (2008); Hoekman (2002)

To specify the value of  $\rho$  various steps were taken. First, nest success and nest mortality rates were obtained from the literature for female mallards nesting in spring and fall seeded crops fields. Nest success was specified at 12% for spring crops, and between 18% and 38%<sup>25</sup> for fall seeded crops (Devries et al. 2008)<sup>26</sup>. Second, the main components in total nest mortality in spring wheat fields were identified as nest destruction or abandonment because of agricultural practices (6%), predation (78%), and other causes such as weather (4%) (Devries et al. 2008). Third, it was assumed that without the agricultural practices that cause nest mortality (i.e. tillage, and spraying), nest survival would increase by 6 percentage points, going from 12% to 18%. This 6 percentage points correspond to a 50% percentage increment in the survival rate. This percentage increment was multiplied by the value 0.43 to find the change in the intrinsic growth rate caused by the change in nesting success (21.5%). This is the value of  $\rho$ .

An alternative value of  $\rho$  was estimated based on the difference of nest survival between nests started on spring and winter wheat (38% - 12% = 26%); this value is significantly higher than the gain in nest success when damaging seeding practices are eliminated

<sup>25</sup> Nest success was 18% for fall rye, and 38% for winter wheat.

<sup>26</sup> Since, nest success rate + nest mortality rate = 1, nest mortality rates are 88% and 62% for nests started in spring and winter wheat, respectively.

(6%). This suggests that mallard utilization of winter wheat fields as nesting habitat, have more benefits than just the avoidance of soil disturbance in the seeding process during spring. Therefore the relative change in nest success when seeding winter instead of spring wheat would be up to 216.6%, and  $\rho$  would have a value of 93%.

### 5.1.12. Summary of coefficient values

Table 5.7 summarizes the coefficient values used in the FL and SP models. The intrinsic growth rate was assumed to be 14.9% a year. The carrying capacity for a crop land of 3,000 acres was assumed to be 37.88 ducks. The gain in intrinsic growth rate due to nesting habitat created by planting winter wheat in fields was specified at 21.5%. Winter kill probability was assumed to be 5%. The loss of spring wheat yield due to duck depredation was specified at 0.11 bu/duck. Society's welfare was assumed to increase by \$123.75 per duck at a decreasing rate of 0.998. The discount rates used for the FL and SP models were 8% and 4%, respectively. The annual payment to farmers was assumed to be \$0.4/acre (equivalent to a one time payment of \$10/acre). Finally, the transaction costs associated with such payments to farmers were assumed to be 20% of their total value.

**Table 5. 7. Summary of coefficient values different to prices and costs**

Parameter	Value	Description
$\bar{\gamma}$	14.9%	Intrinsic growth rate
$K$	37.88	Carrying capacity (# of ducks)
$\rho$	21.5%	Increase in $\bar{\gamma}$ due to winter wheat acreage
$\delta$	5%	Winter kill rate
$\kappa$	0.11	Loss in yields due to duck depredation (bu/duck)
$L$	3000	Cropland size (acres)
$\alpha$	\$123.75	Environmental value per duck (\$/duck)
$\beta$	0.998	Decreasing rate at which society benefit grows
$r_p$	8%	Private discount rate
$r_s$	4%	Social discount rate
$\phi$	\$10	One time farm payment per acre of winter wheat
$\phi_p$	\$0.8	Annual farm payment per acre of winter wheat (with $r_s$ )
$\phi_s$	\$0.4	Annual farm payment per acre of winter wheat (with $r_p$ )
$\varepsilon$	\$0.2	Transaction costs associated with $\phi$

## **5.2. Numerical Model Results**

After placing the coefficient values into the equations presented in section 4.3.3., numerical results for the optimal values of each variable and the corresponding profit and social benefit are calculated. Table 5.8 summarizes the results for the Prairie Provinces for each of the three models.

### **5.2.1. Farm Level results**

After comparing the two sets of results given by the Farm Level model, the group of equations implying  $x=0$  was chosen as the optimal solution for the three provinces. The reason is that the total benefit corresponding to the solution  $x=0$  is higher than the total benefit given by the case where  $x=K$ <sup>27</sup>. Since in the FL it is implicitly assumed that the mallard population represents only costs for farmers, the private optimum implying  $x=0$  is an intuitive result. Duck shadow prices for the optimum solution were found to be positive, which contradicts the assumption that mallards only represent a cost to farmers. The possible reasons why  $\lambda(t)^*$  may have a positive sign are: inconsistencies in the values used for coefficients, calibration problems, or instability of the model.

The optimal proportion of winter wheat acreage with respect to the total wheat area was found to be 58% for Alberta, 75.6% for Saskatchewan, and 74.4% for Manitoba. In all cases optimal levels of winter wheat acreage were significantly higher than the 2008 levels seeded in the each province (4.7%, 4.5%, and 19.1% respectively). It is noteworthy that optimal winter wheat acreage was found to be lower for Alberta than the other two provinces. A possible explanation for this result is that the difference between winter and spring wheat profit per acre is smaller for Alberta than for the other two provinces. Therefore the economic incentives of having higher winter wheat acreage are not as strong in Alberta as in Saskatchewan or Manitoba.

Annual farm profit obtained from the optimal distribution of spring and winter wheat resulted in \$489,998 for Alberta, \$287,752 for Saskatchewan and \$773,941 for Manitoba.

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<sup>27</sup> Calculations of the optimal values for each set of results can be found in Appendix 7.

**Table 5. 8. Numerical results of FL and SP models for Alberta, Saskatchewan and Manitoba**

<i>Model</i>	<i>Province</i>	$\lambda(t)^*$ (\$/ duck)	$\mu^*$ (\$/ acre)	$x^*$ (# ducks)	$y_w^*$ (# acres)	$y_s^*$ (# acres)	<i>Profit</i> (\$)	<i>B(x)</i> (\$)	<i>Total Benefit</i> (\$)
Farm Level	Alberta	5.60	105.29	0	1,737	1,263	489,998	-	489,998
	Saskatchewan	5.15	43.67	0	2,262	738	287,752	-	287,752
	Manitoba	5.33	121.28	0	2,233	767	773,941	-	773,941
Social Planner (private $\phi$ )	Alberta	270.59	105.46	38	1,740	1,260	489,962	3,256	493,218
	Saskatchewan	262.05	43.77	38	2,267	733	287,717	3,256	290,973
	Manitoba	262.42	121.39	38	2,235	765	773,906	3,256	777,162
Social Planner (public $\phi$ )	Alberta	281.70	105.26	38	1,737	1,263	489,963	2,422	492,385
	Saskatchewan	272.85	43.65	38	2,261	739	287,718	2,171	289,888
	Manitoba	273.17	121.26	38	2,232	768	773,906	2,184	776,091

Even though, Saskatchewan and Manitoba have similar acreage distribution, Manitoba evidenced almost three times the profit of Saskatchewan. This considerable difference is mainly driven by the high yields experienced by winter wheat seeded in Manitoba. On the other hand, the higher profit evidenced by Alberta (compared to Saskatchewan) is driven by higher wheat prices paid to Albertan farmers (Table 5.1).

While the mallard population had similar shadow prices in the three provinces (\$5.60, \$5.15, and \$5.33 for Alberta, Saskatchewan and Manitoba, respectively), land opportunity costs of land use differed significantly between them (\$105.3, \$43.7, \$121.3, respectively). The lack of significant differences in the values for  $\lambda^*$  suggests a similar importance of mallard population in each province. On the other hand, the contrasting values of  $\mu^*$  between provinces suggest that profit per acre obtained from the use of land for agricultural purposes directly affects the opportunity cost of land. Therefore, provinces where wheat production generate higher profits would have higher values of  $\mu^*$ .

### **5.2.2. Social Planner models**

The optimal set of results for both of the SP models (with public and private sources for  $\phi$ ) were the expression associated with  $x=K$ . A positive value for  $x^*$  is not a surprising result because in the social context the mallard population has value, even if it implies a cost for farmers. This result is the first difference between the social and private optimums.

Winter wheat acreage did not experience significant changes compared with the results from the FL model. Acreage in the SP model with private  $\phi$  increased less than 0.3% in every province, and decreased in the SP model with public  $\phi$  by less than 0.04%. This suggests that the winter wheat acreage is not greatly affected by the differences between the private and social contexts, but it is mainly determined by the common elements in both contexts (i.e. prices, yields and production costs).

Farmers' total profit did not experience significant changes either, except for a very minor decrease of approximately \$35 (which represents a change of -0.01%) in each province. Consequently, the loss of profit generated by the costs associated with duck population seems to be a minor problem for farmers in the SP models. However it is important to clarify that these costs are directly related with the number of ducks present in crop areas; the size of a group of ducks determines the cost of the damage.

In the SP model with private  $\phi$ , the social benefits generated by duck population equaled \$3,256 in each province. The benefits are the same in each location since the value  $x^*$  is the same and it was assumed that the value of mallards is the same in the three provinces. The value of  $B(x)^*$  also depends on the size of mallard population. However the benefit will reach its maximum at the point that the current duck population equals the highest historical population. In other words, society's benefit increases with the size of mallard population in each area as long as the ecosystem is able to sustain them.

In the SP model with public  $\phi$ , social benefits were lower compared to the SP model then with private  $\phi$ , with values of \$2,422 for Alberta, \$2,171 for Saskatchewan, and \$2,184 for Manitoba. The difference is due to the opportunity cost of using public funds to pay farmers to grow winter wheat instead of using it for other public goods or services, and also to the transaction costs associated with  $\phi$ . Since the optimal mallard population is the same in each province, the provinces with higher winter wheat acreage have lower  $B(x)$ . In the case that  $x^*$  grew with  $y_w$ ,  $B(x)$  would not necessary be lower in areas with high winter wheat acreage. However, this model assumes that winter wheat areas increase the growth of the population ( $\gamma$ ), but do not affect carrying capacity ( $K$ ). Therefore mallard numbers will be bounded by  $K$  regardless of the size of winter wheat acreage, as well as society's benefit due to  $x$ .

Total benefit, including private and social benefits, is higher for the SP models, especially in the case with private payments ( $\phi$ ). This suggests that the best scenario is where the

size of the mallard population is the maximum the ecosystem can sustain, and the incentives to grow winter wheat are given by a private organization.

Shadow prices for mallard ducks are significantly higher in the SP model than in the FL model. This is also an intuitive result because in a context where ducks generate benefits they will be considered more valuable. Once again, values for  $\lambda^*$  do not differ greatly between provinces. Finally, land opportunity costs remained nearly unchanged compared with the private context.

### **5.2.3. Sensitivity Analysis**

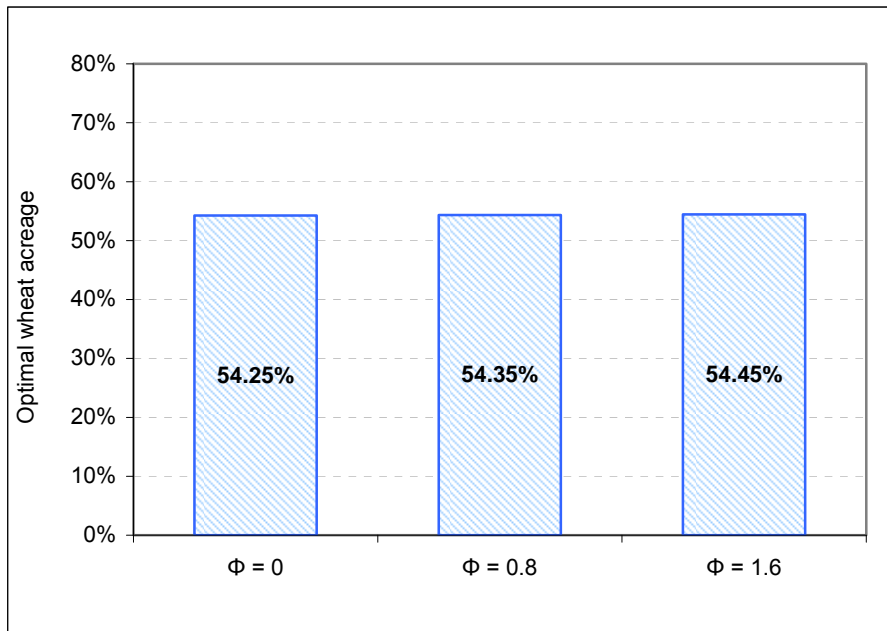
Changes in the optimal solutions for the FL and SP models as a response to changes in the values for the coefficients annual farm payments ( $\phi$ ) and winter kill rates ( $\delta$ ) were investigated. Improvements in cold tolerance and direct payments to increase winter wheat acreage are two of policies currently employed in the Prairie Provinces. By performing sensitivity analysis to evaluate how effective these policies are in the private and social contexts.

#### *5.2.3.1. Changes in annual farm payment ( $\phi$ )*

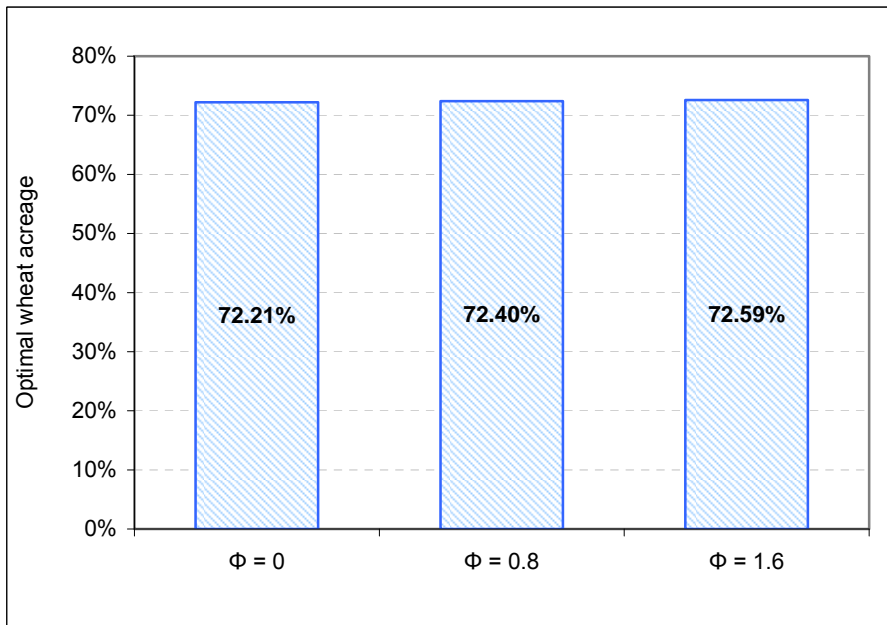
In the social planner model an annual payment ( $\phi$ ), equivalent to a one time payment ( $\phi$ ) of \$10/acre at the beginning of the time horizon, was paid. For the sensitivity analysis, the cases of no payment ( $\phi = 0$ ) and a payment two times the value used in the model ( $\phi = 1.6$ ) were considered. Results for the SP model with private  $\phi$  are presented in Figures 5.2, 5.3 and 5.4. Results for the SP model with public  $\phi$  are presented in Appendix 8.

Changes in winter wheat acreage were very small in the two SP models and for the three provinces. Results for Alberta (Figure 5.2) gave values of optimal acreage equal to 1,631 for  $\phi = 0.8$  (base case); acreage decreased to 1,627 with  $\phi = 0$ , and increased to 1,634 with  $\phi = 1.6$ . For Saskatchewan, winter wheat optimal acreage was 2,166, 2,172 and 2,178, for  $\phi = 0$ ,  $\phi = 0.8$  and  $\phi = 1.6$ , respectively (Figure 5.3). And for Manitoba these

**Figure 5. 2. Changes on winter wheat acreage due to changes in farmer's payments: Social Planner model with private  $\phi$  (Alberta)**

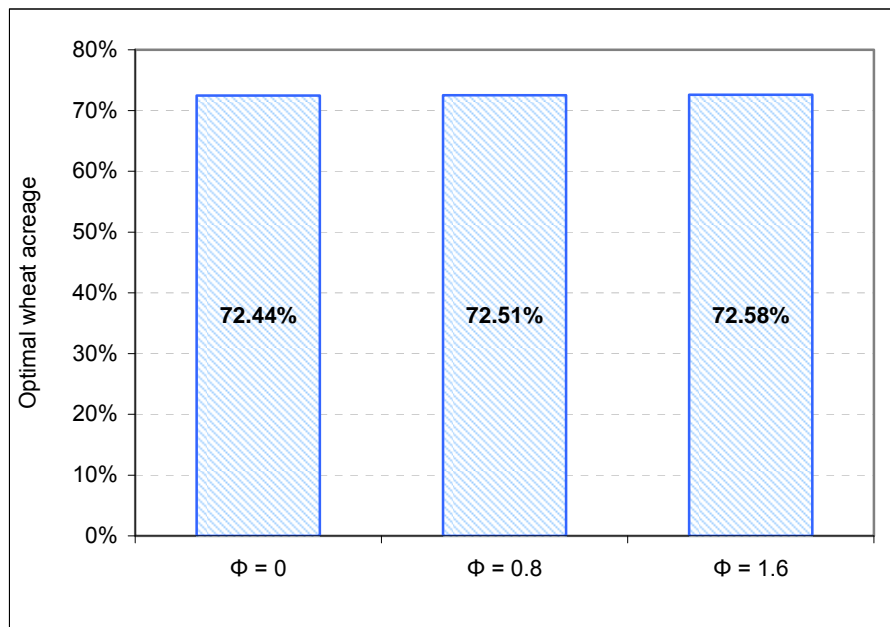


**Figure 5. 3. Changes on winter wheat acreage due to changes in farmer's payments: Social Planner model with private  $\phi$  (Saskatchewan)**





**Figure 5. 4. Changes on winter wheat acreage due to changes in farmer’s payments: Social Planner model with private  $\phi$  (Manitoba)**



values were 2,173, 2,175 and 2,177 acres, respectively (Figure 5.4). In all cases, acreage variations as a response to changes of \$0.8 in  $\phi$ , did not exceed the 6 acres (less than 0.27% of variation). This suggests that annual direct payments from private sources in the farmer’s seeding decision play a small role in seeding decisions.

Results of the sensitivity analysis for the SP model with public  $\phi$  were smaller than the results found for the SP model with private  $\phi$ . In this case, responses of winter wheat acreage to changes of \$0.8 in  $\phi$  were less than 2 acres in each province (Appendix 8). These results indicate that using public funds to make annual payments to farmers in order to encourage winter wheat seeding are less effective than in the case of using private funds. In addition, variations in the amount of payment  $\phi$  have strong implications on social benefit. Higher payments to farmers entail lower benefits to society.

Although different values of  $\phi$  did not generate large changes in winter wheat acreage in any of the provinces, responses differed slightly in each location. Saskatchewan has the

largest of the responses in both models, while Manitoba has the lowest. It seems that the difference of market conditions in each province may affect the effectiveness of the direct payment policy. In provinces such as Manitoba where the gross revenue per acre of winter wheat is much higher than the revenue obtained from spring wheat production, the policy might be less effective.

#### 5.2.3.2. *Changes in winter kill rate ( $\delta$ )*

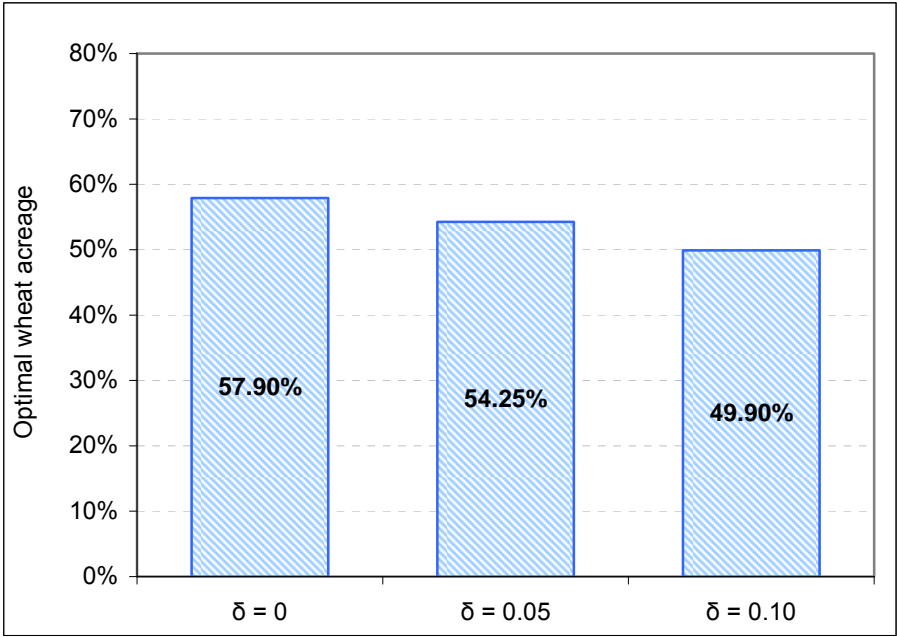
Sensitivity analyses were performed to the FL and SP models in order to evaluate the response of winter wheat acreage to changes in cold tolerance. Three values for  $\delta$  were considered: zero, 5% (base case) and 10%. Results for each province using FL and SL models are shown in Appendix 9.

The FL model for Alberta evidenced an increase of 6.73% in the winter wheat acreage in response to the diminution on the probability of winter kill from 5% to zero (Figure 5.5). The SP models with private and public  $\phi$  had similar results, with increments of 6.71% and 6.74%, respectively.

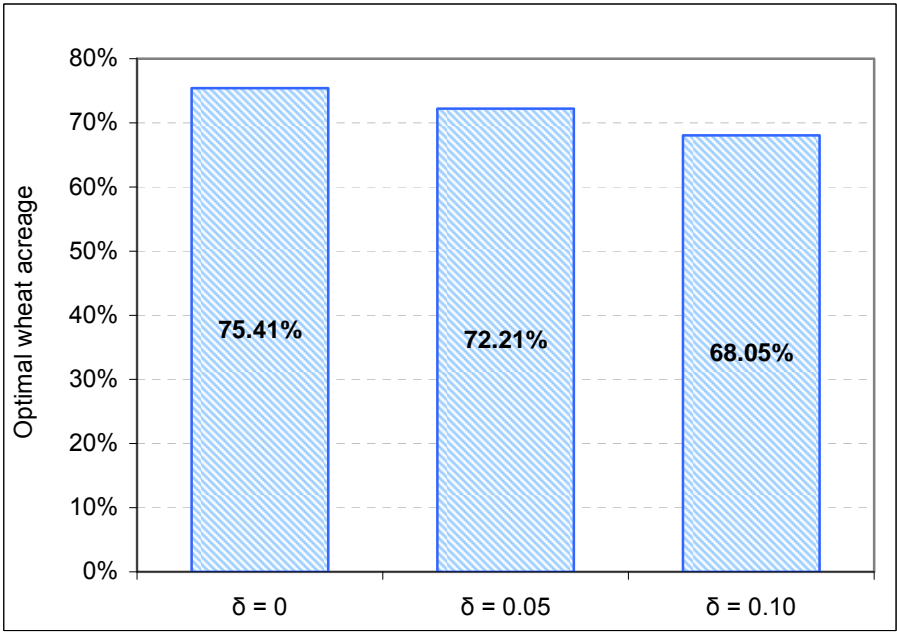
With an increase in  $\delta$  from 5% to 10%, optimal acreage of winter wheat in the FL model, dropped approximately 8% in Alberta, 5.7% for Saskatchewan and 3.2% for Manitoba. Results obtained with the SP models were very close to the FL model, with reductions in winter wheat acreage of approximately 8% in Alberta, 5.75% In Saskatchewan and 3.20% in Manitoba.

Responses in winter wheat acreage to changes in cold tolerance differ in each province. Alberta evidenced the highest response rate, while Manitoba evidenced the lowest. These dissimilarities between provinces are related to the corresponding differences between winter wheat net-revenue per acre and spring wheat net-revenue per acre. Results suggest that provinces with higher differences in profit per acre (Table 5.8) between fall seeded and spring seeded wheat have a lower response rate in winter wheat acreage to improvements in cold tolerance.

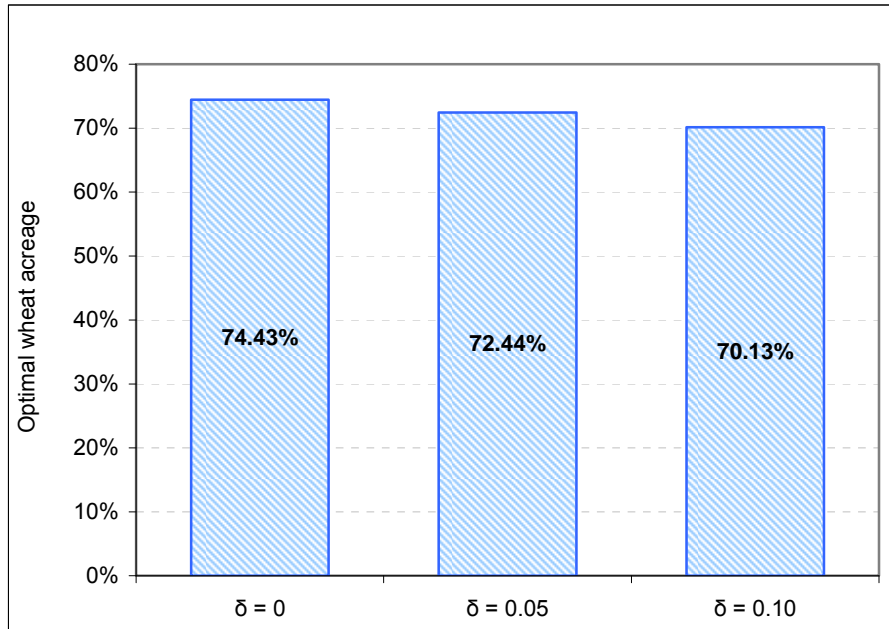
**Figure 5. 5. Changes of wheat acreage allocation due to changes cold tolerance: Farm Level model (Alberta)**



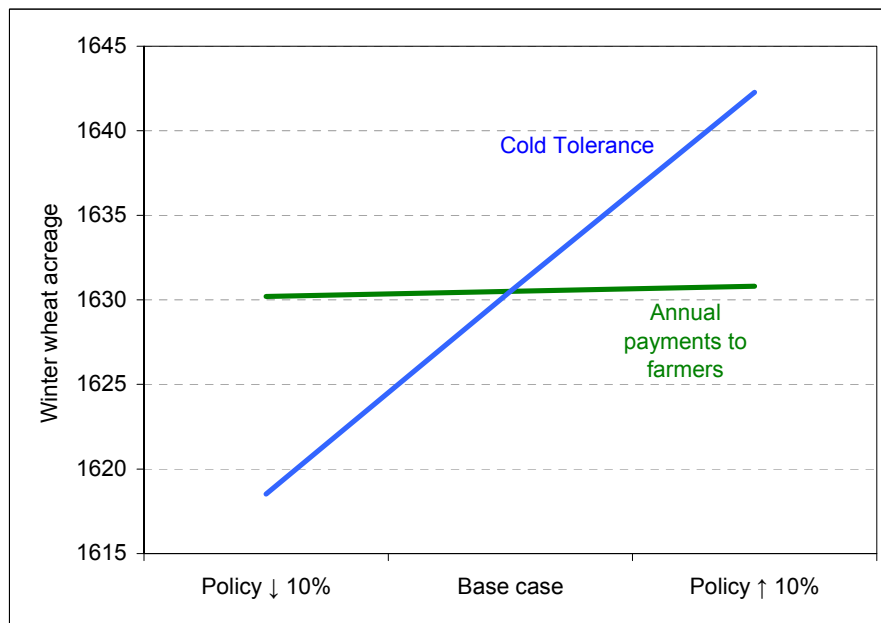
**Figure 5. 6. Changes of wheat acreage allocation due to changes cold tolerance: Farm Level model (Saskatchewan)**



**Figure 5. 7. Changes of wheat acreage allocation due to changes cold tolerance: Farm Level model (Manitoba)**



**Figure 5. 8. Response of winter wheat acreage to changes on annual payments to farmers and cold tolerance: Social planner model (with private  $\phi$ ) for Alberta**



In general, it was observed that improvements in winter wheat cold tolerance have a larger impact on winter wheat acreage than annual payments to farmers. Figure 5.8 illustrate how changes of 10% in the original level of cold tolerance have a higher impact on winter wheat area, compared to changes of 10% in the value per acre paid annually to farmers to encourage fall seeded crops. Apparently the farmers' seeding decision depends mostly on the level of production risk associated with winter wheat.

This analysis provides insight into the effect that changes in direct payments and winter wheat cold tolerance have over the farmer's seeding decision. However, the efficiency of the adoption of these potential policies still needs to be determined. Even though changes in cold tolerance have a stronger impact on the farmer's willing to grow fall seeded wheat, the cost of accomplishing a small change in  $\delta$  to meet a specific winter wheat acreage target may be much higher than the cost of increasing direct payments to farmers to meet that same target. The sensitivity analysis help to identify how the system reacts to changes in  $\phi$  and  $\delta$ , but does not give information on the relative cost of adopting one policy or the other.

#### **5.2.4. System Stability and Graphical Analysis**

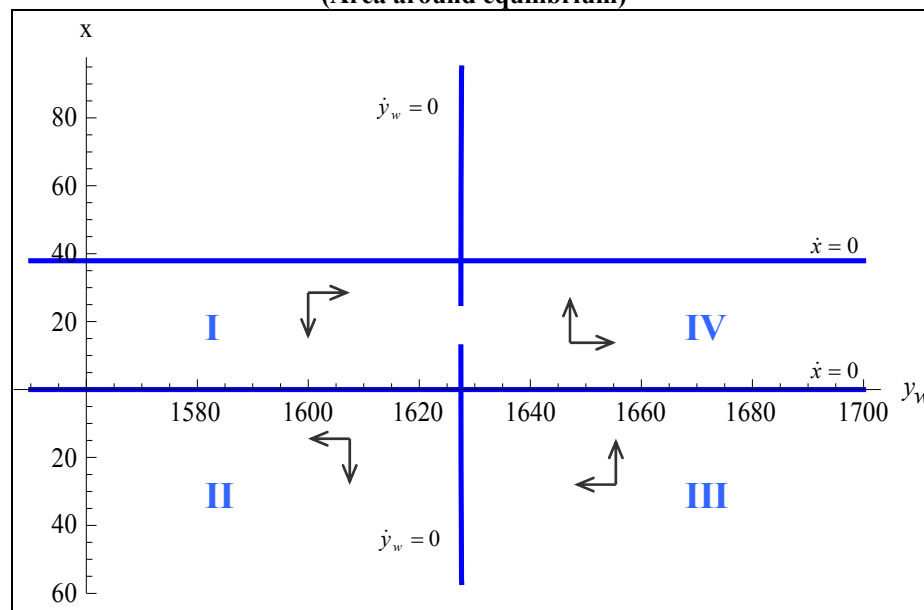
The next step is to understand how the system developed in each model behaves in order to evaluate if the equilibrium is reachable. In order to examine the stability of the equilibriums for the private and social models, phase diagrams are constructed and their stability properties are evaluated.

##### *5.2.4.1. Farm Level model*

Figure 5.9 depicts the phase diagram for the FL model for Alberta. The horizontal axis represents winter wheat acreage and the vertical axis represents duck population. Two equilibriums are observable in the  $(y_w, x)$  plane; these are  $(1,627.5, 0)$  and  $(1,627.5, 38)$ . Since, it was discussed in section 5.2.2 that the equilibrium that maximizes the farmer's profit is the one where duck population is 0, the stability of point  $(1,627.5, 0)$  is the one being analyzed in figure 5.9. Table 5.9 contains the eigenvalues ( $E_1$  and  $E_2$ ) of each of the models for each province; the values of  $E_1$  and  $E_2$  determine the stability of the equilibrium.

Figure 5.9 shows how the FL system divides the  $(y_w, x)$  plane in four quadrants. Each quadrant represents an area of starting points. The arrows in each quadrant depict how the forces of the system move the trajectory of the starting point towards or away the equilibrium. From figure 5.9, it is apparent that starting points located in quadrants I and III have a stable trajectory to the equilibrium, while starting points located in quadrants II and IV have trajectories that move away the equilibrium. Since quadrant III implies negative duck populations, this area is omitted from the analysis. Information in table 5.9 corroborates that the equilibrium is a saddle point, which means that there is a separatrix<sup>28</sup>; only along the separatrix the trajectories move toward the origin. The difference in the behaviors in each area and the separatrix are observable in the direction field depicted in Figure A.2 (Appendix 10). These results confirm that the private optimum given by the FL model is reachable only if the starting point is located along the separatrix located in quadrant I.

**Figure 5. 9. Phase Diagram: Farm level model for Alberta  
(Area around equilibrium)**



The FL models from Saskatchewan and Manitoba have the same behavior as Alberta (table 5.9). In the three cases the equilibrium is a saddle point and is only reachable if the

<sup>28</sup> Separatrix means the separation of the plane into two regions that have different behavior.

starting point is located over the separatrix in the quadrant I. Direction fields and phase diagrams for each province can be found in Appendix 10.

**Table 5.9. Stability properties for the FL and SP models' equilibriums: Eigenvalues**

<i>Model</i>	<i>Province</i>	<i>Equilibrium with <math>x^* = 0</math></i>		<i>Equilibrium with <math>x^* = K</math></i>	
		$E_1$	$E_2$	$E_1$	$E_2$
Farm Level	Alberta	0.1154	-0.1153	$-3.78E^{-05} + 0.11t$	$-3.78E^{-05} - 0.11t$
	Saskatchewan	0.1174	-0.1173	$-6.86E^{-05} + 0.12t$	$-6.86E^{-05} - 0.12t$
	Manitoba	0.1174	-0.1173	$-2.49E^{-05} + 0.12t$	$-2.49E^{-05} - 0.12t$
Social Planner (private $\phi$ )	Alberta	-0.0867	0.0768	$1.90E^{-03} + 0.08t$	$1.90E^{-03} - 0.08t$
	Saskatchewan	-0.0930	0.0741	$3.63E^{-03} + 0.08t$	$3.63E^{-03} - 0.08t$
	Manitoba	-0.0864	0.0797	$1.28E^{-03} + 0.08t$	$1.28E^{-03} - 0.08t$
Social Planner (private $\phi$ )	Alberta	-0.0867	0.0768	$1.90E^{-03} + 0.08t$	$1.90E^{-03} - 0.08t$
	Saskatchewan	-0.0930	0.0741	$3.63E^{-03} + 0.08t$	$3.63E^{-03} - 0.08t$
	Manitoba	-0.0864	0.0797	$1.28E^{-03} + 0.08t$	$1.28E^{-03} - 0.08t$

#### 5.2.4.2. Social Planner model

The phase diagram of the equilibrium for SP model (with private  $\phi$ ) for Alberta is depicted in figure 5.9. The system evidences two feasible equilibria in the  $(y_w, x)$  plane; these are  $(1,630.5, 0)$  and  $(1,630.5, 38)$ . As stated in section 5.2.2, the equilibrium that maximizes total social benefit (including farmer's profit and society's benefit) is the one with a positive duck population. The system is divided in quadrants I, II, III and IV. The arrows in each quadrant suggest a trajectory that moves around the equilibrium counterclockwise. The direction field depicted in figure A.4 (Appendix 10) supports this observation. To confirm if the trajectory is moving toward or away the equilibrium the Eigen values characterizing the system are calculated (Table 5.9). The values of  $E_1$  and  $E_2$  confirm that the equilibrium is an unstable spiral node, which means that the trajectory will spiral away the equilibrium. This means that unless the starting point is the optimum, the system will move away from the equilibrium.

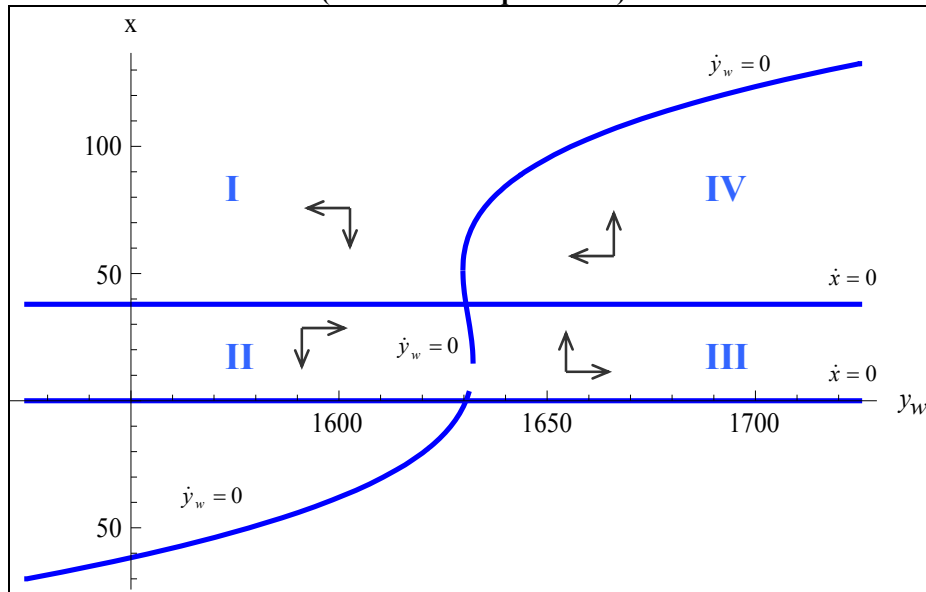
The system for the SP model with public  $\phi$  for Alberta has the same behavior described above (Figures 5.10 and A.6). All provinces evidenced the same behavior in the systems

derived from both SP models (Table 5.9). Direction fields and phase diagrams of the social planner models for Saskatchewan and Manitoba are available in Appendix 10.

### 5.2.4.3. Implications of lack of stability

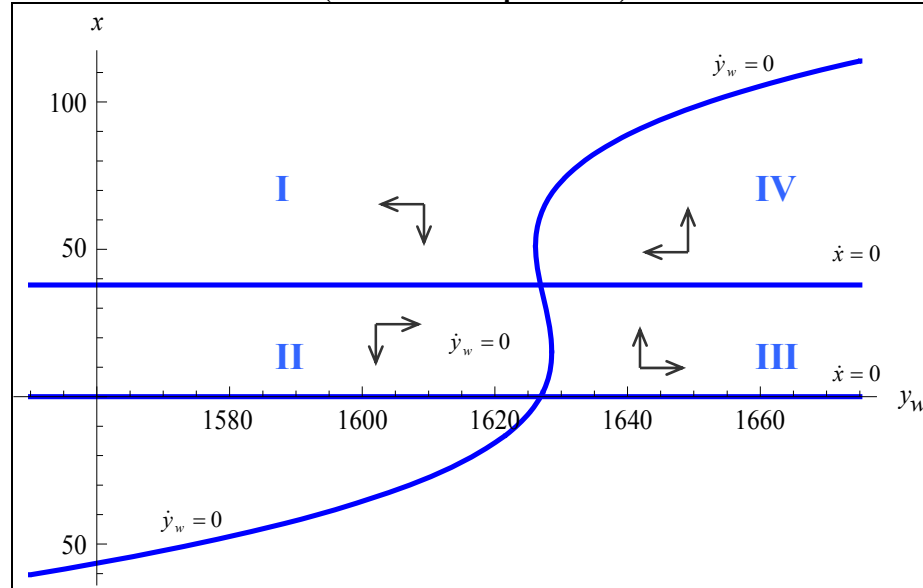
The lack of stability of the FL and SP models imply that duck population optimums are not reachable with the tools their systems provide. The conclusion for the FL model is that the optimal path is to choose an initial point over the separatrix, which is the only stable path of the system. However, the starting point may not be over this stable path and therefore divergent paths would be taken. Furthermore, if the starting point is over the stable path, errors in the estimation of the shadow price may drive the variables to divergent paths as well. There is no guarantee that the equilibrium would be reached.

**Figure 5. 10. Phase Diagram: Social Planner (with private  $\phi$ ) for Alberta  
(Area around equilibrium)**





**Figure 5. 11. Phase Diagram: Social Planner (with public  $\phi$ ) for Alberta  
(Area around equilibrium)**



For the SP model the conclusion is that unless the starting point is the equilibrium, the optimum will not be reached because the forces in the system will drive both variables away from it. If winter wheat acreage and mallard population start in quadrant I (with  $y_w$  being lower and  $x$  being higher than their optimums), acreage would decrease in order to decrease duck population, moving away of the equilibrium. If the starting point is in quadrant II (with both  $y_w$  and  $x$  being lower than their optimums), winter wheat acreage would increase to improve mallard population, however  $x$  will continue to decrease until the optimal acreage is reached. At this point forces will move variables to quadrant III, and then to quadrant IV, driving the  $y_w$  and  $x$  values further away of the equilibrium.

This suggests that the models do not include all variables affecting duck population and that the control variable (winter wheat acreage) does not influence the model enough to drive the system towards optimal mallard populations. Instability could also be related to the incorrect estimation of the shadow price (Shone, 2002).

### **5.5. Conclusions**

The optimal size of mallard duck population was found to be different in the FL and SP models. While the FL optimum implies a zero duck population, the SP optimum entails a population size equal to carrying capacity. This is an intuitive result since the FL model assumes that ducks only represent costs to farmers while in the SP model farmers are financially encouraged to generate breeding habitat appropriate for this type of ducks. In addition, society's benefit inclusion into the objective function in the SP model increases ducks' importance in the system.

Also, duck's shadow price entirely depends on the perspective in which the problem is evaluated. In the FL model each duck has a value of between CA\$5.2 and CA\$5.6, depending on the province. In the SP each duck has a value of approximately CA\$270. This evident difference in duck shadow prices suggests that mallard population is more economically valuable in a context where people's welfare is enhanced by the environmental services that duck populations provide. If the environmental value of ducks were low or inexistent, shadow prices from FL and SP models would not differ much.

Optimal winter wheat acreages given by the FL and SP models for Alberta, Saskatchewan and Manitoba are significantly higher than the current area evidenced in the Prairie Provinces. Results suggest that 52% of all wheat seeded in Alberta should be winter wheat in order to maximize wheat producer's total profit. The corresponding results for Saskatchewan and Manitoba are approximately 72%. The actual proportion of winter wheat area with respect to total wheat area in 2008 was 4.26%, 4.49%, and 19.08% for Alberta, Saskatchewan and Manitoba, respectively.

Even though the models provided intuitive numerical results and responded adequately to changes in different parameters, the system stability around the equilibriums seems to be problematic. In the FL model the equilibrium is only approachable for specific starting points where winter wheat acreage and duck population are lower than their optimal levels. In the SP models any starting point different to the optimum values will be driven away the equilibrium by the forces of the system.

## Chapter 6: Discussion and Conclusions

Mallard ducks and other waterfowl have environmental values that enhance people's welfare. For this reason there are incentives for society to protect duck populations. The Prairie Pothole Region (PPR) of North America has been considered the duck factory of the continent. Historically this area has produced more ducks than lands in the rest of the continent. However, the natural waterfowl habitat in the PPR has been severely disturbed in the last five decades by human action, mainly agricultural practices. Given the smaller and more challenging breeding habitat duck populations face, population numbers have decreased. Average mallard numbers in the period 1980-2008 decreased 10% compared with the average population during the 1955-1979 period.

A significant section of the new cropland area in the PPR of the Prairie Provinces is used to grow wheat. Approximately 91.4% of total wheat area is used to grow spring wheat, while the other 8.6% is used for winter wheat production. Production of crops that are seeded in the spring, such as spring wheat, involve herbicide spraying for wild oat and tillage practices at the beginning of the spring, when millions of female mallards have already set their nests in these areas. These agricultural practices explain approximately 7% of total mallard nests mortality. Conversely, winter wheat seeding practices do not disturb mallard ducks nesting habitat because it is seeded in the fall around spring crops harvesting period. Moreover, winter wheat fields provide an appropriate upland nesting habitat for waterfowl.

Two bioeconomic models were developed in order to model a farmer's decision between growing spring or winter wheat and its effect on mallard population growth from private and social perspectives. In order to link the biological and economic models, an extended logistic growth function where intrinsic growth rate is a function of winter wheat acreage was specified to represent the evolution of mallard population through time. Results of these models suggested that positive numbers of mallard ducks are only desirable in contexts where their environmental values improve people's welfare. In general, the

economic values of ducks depend on the environmental services they offer to society (e.g. recreation services).

Results also suggested that winter wheat acreage in the Prairie Provinces is currently lower than the optimal levels determined by the models. The difference between the model's optimum and actual winter wheat acreages is mainly explained by the way production risk is specified in the FL and SP models. The models assume that risk is fixed and equal to 5% winter kill (i.e. from every annual production, 5% is lost because of winter kill). However, the actual average rate of winter kill is approximately 5%, risk is not fixed and winter kill rates can be much higher (or lower) in a given year. Also, when winter wheat fields are affected by winter kill, all acreage and not just 5% is damaged. The risk perception is different. In the model the farmer knows that every year only 5% of winter crops are going to be lost. In a real scenario the farmer knows that the average probability of experiencing winter kill is 5%, but this risk can vary and if the crops are affected all production is going to be lost. The difference between the way the farmer from the model and the way an actual farmer perceives risk is the main driver of the differences between the model's and current winter wheat acreage in the Prairies.

Sensitivity analysis of the effects of changes in the two policies considered (annual direct payments to farmers and improvements in cold tolerance of winter wheat) over winter wheat acreage showed that the policy with the bigger payback in terms of increases in winter wheat acreage was improvements in cold tolerance. These results suggest that policies aimed to increase winter wheat acreage in the Prairie Provinces should focus efforts and resources in finding ways to decrease winter wheat production risk such as winter kill. Focusing policy on cold tolerance would appear to affect seeding decisions more than the annual financial incentives, and could thus have an additional benefit of increasing waterfowl numbers.

Even though the models provided intuitive results and provided useful insights they have some limitations. First, the equilibriums found in the solution of the model were partially stable or not stable at all. This questions the ability of the system to take us from a

realistic starting point (or even any starting point) to the equilibrium. The lack of stability of the equilibrium suggests that the way the model is specified does not have enough incentives or forces to take us from the current winter wheat acreage and duck population scenario to a private or social optimum.

Second, the specification of the extended logistic growth function (Equations 4.11 and 4.12) may be limiting. The assumption that winter wheat fields affect mallard population growth only through increments in the intrinsic growth rate may be unrealistic because improvements in breeding habitat may affect carrying capacity as well. The FL and SP models assume mallard population will only increase until a fixed level of carrying capacity, and that changes in carrying capacity are explained by variables external to the model (e.g. number of ponds or wetlands). Initially, the specification of the extended logistic growth model had both intrinsic growth rate and carrying capacity as functions of winter wheat acreage. However, there was no evidence in the literature that supported the assumption that improvements in breeding habitat could increase carrying capacity.

Third, the low impact that annual direct payments to farmers have on winter wheat acreage do not reflect the results of the Ducks Unlimited Canada (DUC) program as stated by their staff. The DUC program includes direct payments and provides extension information through DUC agronomists, and producer promoters. The SP models do not capture the effects that provision of information about the benefits associated with winter wheat production has over the farmer's seeding decision. In addition to the provision of economic incentives and information about winter wheat production and its benefits (e.g. high yields, lower herbicide use, water use efficiency, earlier availability than spring wheat), DUC also has programs to improve winter wheat acreage such as the development of new varieties (e.g. cold tolerant, improved grain quality), improvement of marketing options and agronomic practices.

In general, the FL and SP models could be improved by incorporating elements such as:

- Environmental benefits that are different than duck population such as lower herbicide use and efficiency in water utilization.

- Economic benefits different than higher yields such as earlier availability in the market compared to spring wheat and flexible marketing opportunities.
- A logistic growth function with carrying capacity as a function of wetland availability and weather conditions. This way the optimal mallard population given by the model will not be bounded by a fixed value.
- Since improvements of cold tolerance are accomplished via research usually funded by government funds, costs associated to research should be also be included in the social planner model.
- A variable winter kill rate in order to model in a more realistic way how production risk is perceived by the farmer.
- Yields as a function of cold tolerance and weather. Yields differ between years depending on variables such as temperature and precipitation. In order to obtain a most realistic outcome, yields should be able to vary within the model as a response to weather variables.

In conclusion, bioeconomic models can be a powerful tool for policy making. They provide useful information about optimum allocation of resources that potentially can be used to specify policy goals. However their utilization implies limitations in terms of the problem specification (e.g. functional forms, lack of available information, and need of assumptions to simplify the analysis) and complexity in their mathematical solution (e.g. derivable functions, system stability).

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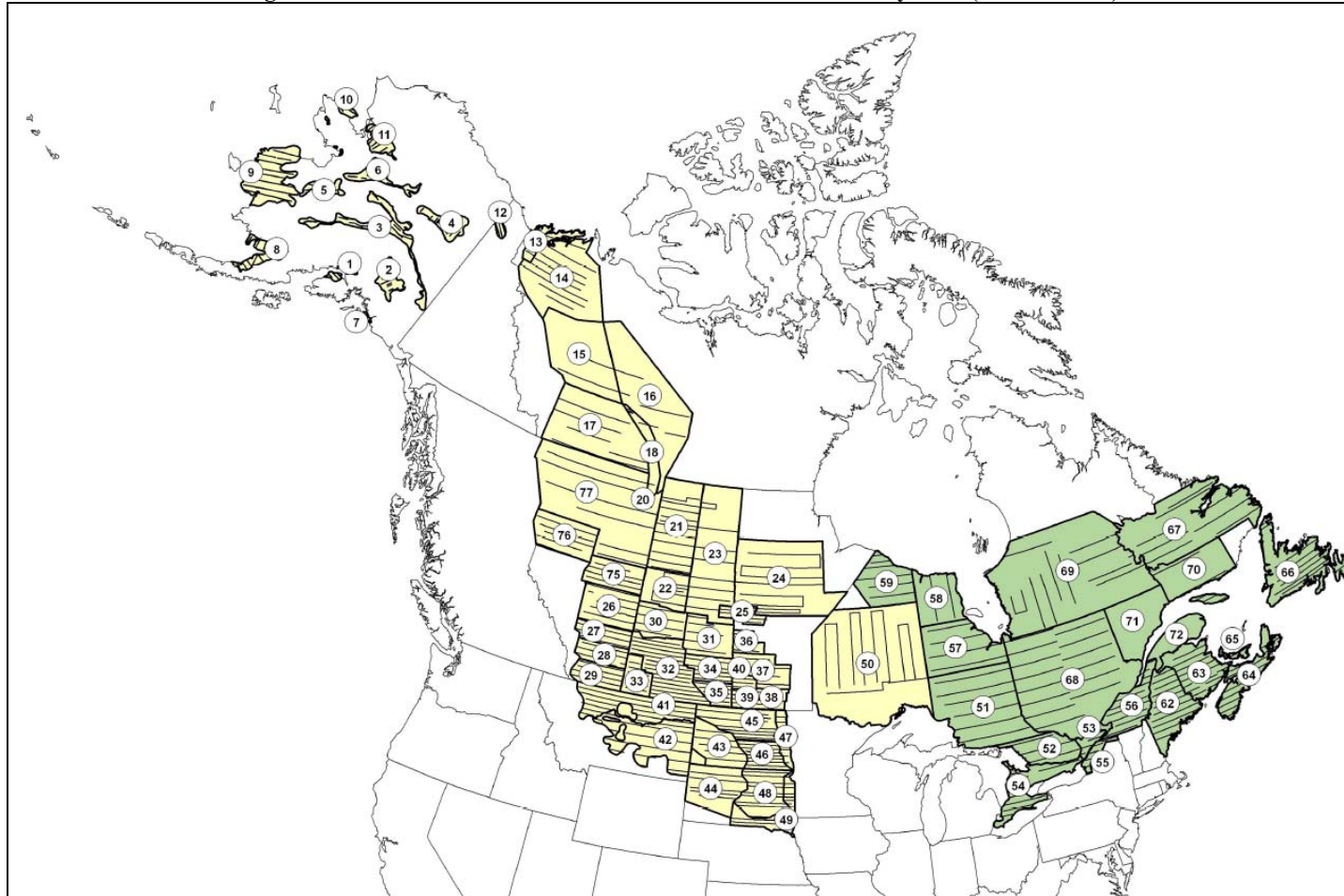
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## **APPENDICES**

*Appendix 1. Map of the U.S. Fish and Wildlife Service's traditional survey area of the Waterfowl Breeding Population and Habitat Survey*

**Figure A. 1. U.S. Fish and Wildlife Service's traditional survey area (areas 1 to 50)**



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*Appendix 2. Canadian Wheat Board Payments (2007-2008)*

**Table A. 1. Spring wheat prices by variety and grade (\$/bu) 2007-2008  
(In store Vancouver or St. Lawrence)**

<i>Variety</i>	<i>Grade</i>	<i>\$/bu</i>
Canada Western Red Spring	15.5	9.79
Canada Western Red Spring	15.4	9.78
Canada Western Red Spring	15.3	9.77
Canada Western Red Spring	15.2	9.75
Canada Western Red Spring	15.1	9.74
Canada Western Red Spring	15	9.73
Canada Western Red Spring	14.9	9.71
Canada Western Red Spring	14.8	9.7
Canada Western Red Spring	14.7	9.68
Canada Western Red Spring	14.6	9.67
Canada Western Red Spring	14.5	9.66
Canada Western Red Spring	14.4	9.64
Canada Western Red Spring	14.3	9.63
Canada Western Red Spring	14.2	9.62
Canada Western Red Spring	14.1	9.6
Canada Western Red Spring	14	9.59
Canada Western Red Spring	13.9	9.58
Canada Western Red Spring	13.8	9.56
Canada Western Red Spring	13.7	9.55
Canada Western Red Spring	13.6	9.53
Canada Western Red Spring	13.5	9.52
Canada Western Red Spring	13.4	9.51
Canada Western Red Spring	13.3	9.5
Canada Western Red Spring	13.2	9.49
Canada Western Red Spring	13.1	9.49
Canada Western Red Spring	13	9.48
Canada Western Red Spring	12.9	9.47
Canada Western Red Spring	12.8	9.47
Canada Western Red Spring	12.7	9.46
Canada Western Red Spring	12.6	9.45
Canada Western Red Spring	12.5	9.45
Canada Western Red Spring	12.4	9.44
Canada Western Red Spring	12.3	9.43
Canada Western Red Spring	12.2	9.43
Canada Western Red Spring	12.1	9.42
Canada Western Red Spring	12	9.41
Canada Western Red Spring	11.9	9.41
Canada Western Red Spring	11.8	9.4
Canada Western Red Spring	11.7	9.39
Canada Western Red Spring	11.6	9.39
Canada Western Red Spring	11.5	9.38
Canada Western Red Spring	11.4	9.37



Canada Western Red Spring	11.3	9.36
Canada Western Red Spring	11.2	9.36
Canada Western Red Spring	11.1	9.35
Canada Western Red Spring	11	9.34
Canada Western Red Spring		9.31
Canada Western Red Spring	15.5	9.63
Canada Western Red Spring	15.4	9.62
Canada Western Red Spring	15.3	9.6
Canada Western Red Spring	15.2	9.59
Canada Western Red Spring	15.1	9.58
Canada Western Red Spring	15	9.56
Canada Western Red Spring	14.9	9.55
Canada Western Red Spring	14.8	9.53
Canada Western Red Spring	14.7	9.52
Canada Western Red Spring	14.6	9.51
Canada Western Red Spring	14.5	9.49
Canada Western Red Spring	14.4	9.48
Canada Western Red Spring	14.3	9.47
Canada Western Red Spring	14.2	9.45
Canada Western Red Spring	14.1	9.44
Canada Western Red Spring	14	9.43
Canada Western Red Spring	13.9	9.41
Canada Western Red Spring	13.8	9.4
Canada Western Red Spring	13.7	9.39
Canada Western Red Spring	13.6	9.37
Canada Western Red Spring	13.5	9.36
Canada Western Red Spring	13.4	9.34
Canada Western Red Spring	13.3	9.34
Canada Western Red Spring	13.2	9.33
Canada Western Red Spring	13.1	9.32
Canada Western Red Spring	13	9.32
Canada Western Red Spring	12.9	9.31
Canada Western Red Spring	12.8	9.3
Canada Western Red Spring	12.7	9.3
Canada Western Red Spring	12.6	9.29
Canada Western Red Spring	12.5	9.28
Canada Western Red Spring	12.4	9.28
Canada Western Red Spring	12.3	9.27
Canada Western Red Spring	12.2	9.26
Canada Western Red Spring	12.1	9.26
Canada Western Red Spring	12	9.25
Canada Western Red Spring	11.9	9.24
Canada Western Red Spring	11.8	9.24
Canada Western Red Spring	11.7	9.23
Canada Western Red Spring	11.6	9.22
Canada Western Red Spring	11.5	9.22
Canada Western Red Spring	11.4	9.21
Canada Western Red Spring	11.3	9.2
Canada Western Red Spring	11.2	9.19

Canada Western Red Spring	11.1	9.19
Canada Western Red Spring	11	9.18
Canada Western Red Spring		9.15
Canada Western Red Spring	14.5	9.27
Canada Western Red Spring	14.4	9.25
Canada Western Red Spring	14.3	9.24
Canada Western Red Spring	14.2	9.23
Canada Western Red Spring	14.1	9.22
Canada Western Red Spring	14	9.21
Canada Western Red Spring	13.9	9.2
Canada Western Red Spring	13.8	9.19
Canada Western Red Spring	13.7	9.18
Canada Western Red Spring	13.6	9.17
Canada Western Red Spring	13.5	9.16
Canada Western Red Spring	13.4	9.15
Canada Western Red Spring	13.3	9.13
Canada Western Red Spring	13.2	9.12
Canada Western Red Spring	13.1	9.11
Canada Western Red Spring	13	9.1
Canada Western Red Spring	12.9	9.09
Canada Western Red Spring	12.8	9.09
Canada Western Red Spring	12.7	9.08
Canada Western Red Spring	12.6	9.07
Canada Western Red Spring	12.5	9.07
Canada Western Red Spring	12.4	9.06
Canada Western Red Spring	12.3	9.06
Canada Western Red Spring	12.2	9.05
Canada Western Red Spring	12.1	9.05
Canada Western Red Spring	12	9.04
Canada Western Red Spring		9
Canada Western Red Spring		8.62
Canada Western Hard White Spring	15.5	9.79
Canada Western Hard White Spring	15.4	9.78
Canada Western Hard White Spring	15.3	9.77
Canada Western Hard White Spring	15.2	9.75
Canada Western Hard White Spring	15.1	9.74
Canada Western Hard White Spring	15	9.73
Canada Western Hard White Spring	14.9	9.71
Canada Western Hard White Spring	14.8	9.7
Canada Western Hard White Spring	14.7	9.68
Canada Western Hard White Spring	14.6	9.67
Canada Western Hard White Spring	14.5	9.66
Canada Western Hard White Spring	14.4	9.64
Canada Western Hard White Spring	14.3	9.63
Canada Western Hard White Spring	14.2	9.62
Canada Western Hard White Spring	14.1	9.6
Canada Western Hard White Spring	14	9.59
Canada Western Hard White Spring	13.9	9.58
Canada Western Hard White Spring	13.8	9.56

Canada Western Hard White Spring	13.7	9.55
Canada Western Hard White Spring	13.6	9.53
Canada Western Hard White Spring	13.5	9.52
Canada Western Hard White Spring	13.4	9.51
Canada Western Hard White Spring	13.3	9.5
Canada Western Hard White Spring	13.2	9.49
Canada Western Hard White Spring	13.1	9.49
Canada Western Hard White Spring	13	9.48
Canada Western Hard White Spring	12.9	9.47
Canada Western Hard White Spring	12.8	9.47
Canada Western Hard White Spring	12.7	9.46
Canada Western Hard White Spring	12.6	9.45
Canada Western Hard White Spring	12.5	9.45
Canada Western Hard White Spring	12.4	9.44
Canada Western Hard White Spring	12.3	9.43
Canada Western Hard White Spring	12.2	9.43
Canada Western Hard White Spring	12.1	9.42
Canada Western Hard White Spring	12	9.41
Canada Western Hard White Spring	11.9	9.41
Canada Western Hard White Spring	11.8	9.4
Canada Western Hard White Spring	11.7	9.39
Canada Western Hard White Spring	11.6	9.39
Canada Western Hard White Spring	11.5	9.38
Canada Western Hard White Spring	11.4	9.37
Canada Western Hard White Spring	11.3	9.36
Canada Western Hard White Spring	11.2	9.36
Canada Western Hard White Spring	11.1	9.35
Canada Western Hard White Spring	11	9.34
Canada Western Hard White Spring		9.31
Canada Western Hard White Spring	15.5	9.63
Canada Western Hard White Spring	15.4	9.62
Canada Western Hard White Spring	15.3	9.6
Canada Western Hard White Spring	15.2	9.59
Canada Western Hard White Spring	15.1	9.58
Canada Western Hard White Spring	15	9.56
Canada Western Hard White Spring	14.9	9.55
Canada Western Hard White Spring	14.8	9.53
Canada Western Hard White Spring	14.7	9.52
Canada Western Hard White Spring	14.6	9.51
Canada Western Hard White Spring	14.5	9.49
Canada Western Hard White Spring	14.4	9.48
Canada Western Hard White Spring	14.3	9.47
Canada Western Hard White Spring	14.2	9.45
Canada Western Hard White Spring	14.1	9.44
Canada Western Hard White Spring	14	9.43
Canada Western Hard White Spring	13.9	9.41
Canada Western Hard White Spring	13.8	9.4
Canada Western Hard White Spring	13.7	9.39
Canada Western Hard White Spring	13.6	9.37

Canada Western Hard White Spring	13.5	9.36
Canada Western Hard White Spring	13.4	9.34
Canada Western Hard White Spring	13.3	9.34
Canada Western Hard White Spring	13.2	9.33
Canada Western Hard White Spring	13.1	9.32
Canada Western Hard White Spring	13	9.32
Canada Western Hard White Spring	12.9	9.31
Canada Western Hard White Spring	12.8	9.3
Canada Western Hard White Spring	12.7	9.3
Canada Western Hard White Spring	12.6	9.29
Canada Western Hard White Spring	12.5	9.28
Canada Western Hard White Spring	12.4	9.28
Canada Western Hard White Spring	12.3	9.27
Canada Western Hard White Spring	12.2	9.26
Canada Western Hard White Spring	12.1	9.26
Canada Western Hard White Spring	12	9.25
Canada Western Hard White Spring	11.9	9.24
Canada Western Hard White Spring	11.8	9.24
Canada Western Hard White Spring	11.7	9.23
Canada Western Hard White Spring	11.6	9.22
Canada Western Hard White Spring	11.5	9.22
Canada Western Hard White Spring	11.4	9.21
Canada Western Hard White Spring	11.3	9.2
Canada Western Hard White Spring	11.2	9.19
Canada Western Hard White Spring	11.1	9.19
Canada Western Hard White Spring	11	9.18
Canada Western Hard White Spring		9.15
Canada Western Hard White Spring	14.5	9.27
Canada Western Hard White Spring	14.4	9.25
Canada Western Hard White Spring	14.3	9.24
Canada Western Hard White Spring	14.2	9.23
Canada Western Hard White Spring	14.1	9.22
Canada Western Hard White Spring	14	9.21
Canada Western Hard White Spring	13.9	9.2
Canada Western Hard White Spring	13.8	9.19
Canada Western Hard White Spring	13.7	9.18
Canada Western Hard White Spring	13.6	9.17
Canada Western Hard White Spring	13.5	9.16
Canada Western Hard White Spring	13.4	9.15
Canada Western Hard White Spring	13.3	9.13
Canada Western Hard White Spring	13.2	9.12
Canada Western Hard White Spring	13.1	9.11
Canada Western Hard White Spring	13	9.1
Canada Western Hard White Spring	12.9	9.09
Canada Western Hard White Spring	12.8	9.09
Canada Western Hard White Spring	12.7	9.08
Canada Western Hard White Spring	12.6	9.07
Canada Western Hard White Spring	12.5	9.07
Canada Western Hard White Spring	12.4	9.06

Canada Western Hard White Spring	12.3	9.06
Canada Western Hard White Spring	12.2	9.05
Canada Western Hard White Spring	12.1	9.05
Canada Western Hard White Spring	12	9.04
Canada Western Hard White Spring		9
Canada Western Hard White Spring		8.62
Canada Prairie Spring Red		8.55
Canada Prairie Spring Red		8.39
Canada Prairie Spring White		8.55
Canada Prairie Spring White		8.39
Canada Western Extra Strong	12.5	9.16
Canada Western Extra Strong		9.13
Canada Western Extra Strong	12.5	9.03
Canada Western Extra Strong		9

Source: Canadian Wheat Board (2008)

**Table A. 2. Winter wheat prices by variety and grade (\$/bu)  
2007-2008**

<i>Winter wheat variety</i>	<i>Grade</i>	<i>\$/bu</i>
Canada Western Red Winter Select	14	8.94
Canada Western Red Winter Select	13.9	8.94
Canada Western Red Winter Select	13.8	8.93
Canada Western Red Winter Select	13.7	8.92
Canada Western Red Winter Select	13.6	8.92
Canada Western Red Winter Select	13.5	8.91
Canada Western Red Winter Select	13.4	8.9
Canada Western Red Winter Select	13.3	8.9
Canada Western Red Winter Select	13.2	8.89
Canada Western Red Winter Select	13.1	8.88
Canada Western Red Winter Select	13	8.88
Canada Western Red Winter Select	12.9	8.87
Canada Western Red Winter Select	12.8	8.86
Canada Western Red Winter Select	12.7	8.86
Canada Western Red Winter Select	12.6	8.85
Canada Western Red Winter Select	12.5	8.84
Canada Western Red Winter Select	12.4	8.84
Canada Western Red Winter Select	12.3	8.83
Canada Western Red Winter Select	12.2	8.82
Canada Western Red Winter Select	12.1	8.82
Canada Western Red Winter Select	12	8.81
Canada Western Red Winter Select	11.9	8.8
Canada Western Red Winter Select	11.8	8.79
Canada Western Red Winter Select	11.7	8.79
Canada Western Red Winter Select	11.6	8.78
Canada Western Red Winter Select	11.5	8.77
Canada Western Red Winter Select	11.4	8.77
Canada Western Red Winter Select	11.3	8.76
Canada Western Red Winter Select	11.2	8.75

Canada Western Red Winter Select	11.1	8.75
Canada Western Red Winter Select	11	8.74
Canada Western Red Winter	11.5	8.47
Canada Western Red Winter		8.41
Canada Western Red Winter Select	14	8.78
Canada Western Red Winter Select	13.9	8.77
Canada Western Red Winter Select	13.8	8.77
Canada Western Red Winter Select	13.7	8.76
Canada Western Red Winter Select	13.6	8.75
Canada Western Red Winter Select	13.5	8.75
Canada Western Red Winter Select	13.4	8.74
Canada Western Red Winter Select	13.3	8.73
Canada Western Red Winter Select	13.2	8.73
Canada Western Red Winter Select	13.1	8.72
Canada Western Red Winter Select	13	8.71
Canada Western Red Winter Select	12.9	8.71
Canada Western Red Winter Select	12.8	8.7
Canada Western Red Winter Select	12.7	8.69
Canada Western Red Winter Select	12.6	8.69
Canada Western Red Winter Select	12.5	8.68
Canada Western Red Winter Select	12.4	8.67
Canada Western Red Winter Select	12.3	8.67
Canada Western Red Winter Select	12.2	8.66
Canada Western Red Winter Select	12.1	8.65
Canada Western Red Winter Select	12	8.64
Canada Western Red Winter Select	11.9	8.64
Canada Western Red Winter Select	11.8	8.63
Canada Western Red Winter Select	11.7	8.62
Canada Western Red Winter Select	11.6	8.62
Canada Western Red Winter Select	11.5	8.61
Canada Western Red Winter Select	11.4	8.6
Canada Western Red Winter Select	11.3	8.6
Canada Western Red Winter Select	11.2	8.59
Canada Western Red Winter Select	11.1	8.58
Canada Western Red Winter Select	11	8.58
Canada Western Red Winter	11.5	8.3
Canada Western Red Winter		8.25

Source: Canadian Wheat Board (2008)

*Appendix 3. Freight costs for wheat shipped from the Prairie Provinces (2007-2008)*

**Table A. 3. Wheat freight costs from Prairie Provinces to main grain stores and CWB deductions (2007-08)**

<i>Province</i>	<i>Origin</i>	<i>Rail</i>	<i>Freight rate to Vancouver</i>		<i>Freight rate to Thunder Bay</i>		<i>CWB deductions (wheat)*</i>	
			<i>Value (\$/ton)</i>	<i>Value (\$/bu)</i>	<i>Value (\$/ton)</i>	<i>Value (\$/bu)</i>	<i>Value (\$/ton)</i>	<i>Value (\$/bu)</i>
AB	Calgary	CN	28.27	0.77	56.86	1.55	28.27	0.77
AB	Calgary	CP	25.57	0.70	47.68	1.30	25.57	0.70
MB	Winnipeg	CN	49.43	1.35	15.75	0.43	31.10	0.85
MB	Winnipeg	CP	53.56	1.46	22.00	0.60	34.35	0.93
SK	Regina	CN	45.68	1.24	38.84	1.06	45.68	1.24
SK	Regina	CP	46.99	1.28	35.67	0.97	43.97	1.20

Source: Alberta Agriculture and Rural Development (2008)

- \* The freight deductions producers pay when they deliver wheat is a combination of rail freight rates and the Freight Adjustment Factors (FAF)<sup>29</sup>. For wheat, farmers will be deducted the lesser of:
- a) the rail freight to Vancouver or
  - b) the rail freight to Thunder Bay plus the FAF

<sup>29</sup> Freight Adjustment Factors (FAF) were introduced in the 1995-96 crop year to account for a change in the eastern pooling basis point, from Thunder Bay to the Lower St. Lawrence, and for the location advantage of accorded shipments from delivery points near Churchill and markets in the United States. FAFs are established prior to the beginning of each crop year to reflect changes in sales opportunities, cropping patterns and Seaway freight rates.

*Appendix 4. Elevation costs for wheat (2007)*

**Table A. 4. Wheat elevation costs in the Prairie Provinces (2007)**

<i>Company</i>	<i>Province</i>	<i>Elevation cost (\$/bu)</i>	<i>Additional cost (\$/bu)</i>	<i>Total cost (\$/bu)</i>
Cargill Limited	MB	0.36	0.14	0.50
Cargill Limited	SK	0.36	0.00	0.36
Cargill Limited	AB	0.37	0.00	0.37
CMI Terminal Joint Venture	SK	0.39	0.14	0.53
Delmar Commodities	MB	0.33	0.13	0.46
Gardiner Dam Terminal	SK	0.37	0.14	0.51
Grain Solutions Inc.	AB	0.35	0.16	0.52
Great Northern Grain Terminal	AB	0.44	0.27	0.71
Great Sandhills Terminal Ltd.	SK	0.37	0.13	0.50
Lethbridge Inland Terminal Ltd.	AB	0.35	0.13	0.48
Louis Dreyfus	AB, SK, MB	0.37	0.15	0.52
Nature's Best Organics Inc.	SK	0.33	0.15	0.48
North East Terminal	SK	0.36	0.13	0.49
Parrish & Heimbecker	MB	0.37	0.15	0.52
Parrish & Heimbecker	SK	0.37	0.15	0.52
Parrish & Heimbecker	AB	0.38	0.15	0.53
Paterson Grain Div. of Pat. Global Foods Inc.	MB	0.37	0.14	0.51
Paterson Grain Div. of Pat. Global Foods Inc.	SK	0.37	0.14	0.51
Paterson Grain Div. of Pat. Global Foods Inc.	AB	0.37	0.14	0.51
Pioneer Grain	MB	0.37	0.14	0.51
Prairie West Terminal	SK	0.37	0.13	0.50
Providence Grain Group Inc.	AB	0.38	0.14	0.53
R.W. Organic Ltd.	SK	0.31	0.30	0.61
South West Terminal	SK	0.35	0.13	0.48
Tri Lake Agri Limited	MB	0.37	0.14	0.51
Vandaele Seeds Ltd.	MB	0.41	0.05	0.46
Viterra	AB, SK, MB	0.37	0.14	0.51
Westmor Terminals Inc.	AB	0.39	0.14	0.53
Westlock Terminals (NGC) Ltd.	AB	0.40	0.13	0.53

Source: Canadian Grain Commission (2008)

\* Additional cost refers to the tariffs charged by licensed primary elevators for removing dockage from various kinds of grain.



*Appendix 5. Production cost estimations for spring and winter wheat seeded in the Prairie Provinces (2004 – 2008)*

**Table A. 5. Cost per acre of spring and winter wheat seeded in Alberta (Dark-Brown soil)**

	<i>Spring Wheat</i>					<i>Winter Wheat</i>				
	<i>2004</i>	<i>2005</i>	<i>2006</i>	<i>2007</i>	<i>2008</i>	<i>2004</i>	<i>2005</i>	<i>2006</i>	<i>2007</i>	<i>2008</i>
Seeding	10.0	10.0	12.5	13.2	16.9	10.0	10.0	15.0	14.0	18.0
Fertilizer	31.0	31.3	28.0	44.0	58.0	31.0	31.3	28.0	49.0	64.0
Chemicals	24.5	24.5	24.5	29.8	29.8	7.5	7.5	12.0	9.8	9.8
Crop Insurance	8.0	8.0	11.0	9.1	15.0	8.0	8.0	12.0	9.9	13.4
Machinery and Fuels	15.2	17.1	17.5	20.0	22.7	15.2	17.1	17.5	20.0	22.7
Utilities and miscellaneous	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Land costs	30.0	30.0	35.0	40.0	40.0	30.0	30.0	35.0	40.0	40.0
Depreciation	22.0	22.0	23.0	25.0	25.0	22.0	22.0	23.0	25.0	25.0
Investment in capital	4.0	4.0	9.0	9.0	9.0	4.0	4.0	9.0	9.0	9.0
Labor	14.0	14.0	16.0	18.0	18.0	14.0	14.0	16.0	18.0	18.0
<b>Total costs</b>	<b>158.7</b>	<b>160.9</b>	<b>176.5</b>	<b>208.1</b>	<b>234.3</b>	<b>141.7</b>	<b>143.9</b>	<b>167.5</b>	<b>194.6</b>	<b>219.8</b>

Source of data: Alberta Agriculture, Food, and Rural Development

**Table A. 6. Cost per acre of spring and winter wheat seeded in Saskatchewan (Black soil)**

	<i>Spring Wheat</i>					<i>Winter wheat</i>
	<i>2004</i>	<i>2005</i>	<i>2006</i>	<i>2007</i>	<i>2008</i>	<i>2008</i>
Seeding	7.79	7.58	8.97	10.05	12.68	13.65
Fertilizer	28.80	30.60	36.00	32.70	45.30	53.55
Chemicals	20.03	24.38	24.44	23.46	25.58	14.65
Crop Insurance	5.19	4.59	4.52	4.72	5.24	5.89
Machinery and Fuels	18.50	17.76	17.96	14.80	19.06	19.06
Utilities and miscellaneous	4.72	4.93	5.18	5.30	5.41	5.41
Land costs	22.85	22.88	23.16	23.52	27.73	27.73
Depreciation	20.60	20.60	20.60	20.60	22.20	22.2
Investment in capital	11.766	10.656	10.878	12.65	14.75	14.75
Labor	6.25	6.25	7.00	7.75	8.75	5.5
<b>Total costs</b>	<b>146.49</b>	<b>150.22</b>	<b>158.71</b>	<b>155.55</b>	<b>186.70</b>	<b>182.39</b>

Source of data: Saskatchewan Agriculture and Food

**Table A. 7. Cost per acre of spring and winter wheat seeded in Manitoba (Black soil)**

	<i>Spring Wheat</i>					<i>Winter wheat</i>				
	<i>2004</i>	<i>2005</i>	<i>2006</i>	<i>2007</i>	<i>2008</i>	<i>2004</i>	<i>2005</i>	<i>2006</i>	<i>2007</i>	<i>2008</i>
Seeding	10.63	11.48	10.20	12.75	18.28	12	14	12	13.5	20
Fertilizer	32.34	33.41	40.37	31.36	45.03	54.7	55.55	59.85	49.38	65.8
Chemicals	31.00	31.00	30.20	34.25	30.75	23	23	23	23.5	18.5
Crop Insurance	5.92	5.52	5.44	5.73	5.97	6.12	5.53	5.53	5.98	7.23
Machinery and Fuels	21.50	21.50	23.25	24.50	25.00	17.5	17.5	18.9	20	20.5
Utilities and miscellaneous	7.50	7.50	7.50	7.50	7.50	7.5	7.5	7.5	7.5	7.5
Land costs	28.50	29.25	29.25	29.80	29.80	28.5	29.25	29.25	29.8	29.8
Depreciation	22.50	22.50	22.50	25.00	25.00	22.5	22.5	22.5	25.0	25.0
Investment in capital	9.0	9.0	9.0	10.0	10.0	9.0	9.0	9.0	10.0	10.0
Labor	15.00	17.25	17.25	17.25	17.25	15	17.25	17.25	17.25	17.25
<b>Total costs</b>	<b>183.9</b>	<b>188.4</b>	<b>195.0</b>	<b>198.1</b>	<b>214.6</b>	<b>195.8</b>	<b>201.1</b>	<b>204.8</b>	<b>201.9</b>	<b>221.6</b>

Source of data: Manitoba Agriculture, Food, and Rural Initiatives

*Appendix 6. Grain cleaning costs (2007)*

**Table A. 8. Wheat cleaning costs in the Prairie Provinces (2007)**

<i>Company</i>	<i>Province</i>	<i>Cleaning cost (\$/bu)</i>
Cargill Limited	MB	0.408
CMI Terminal Joint Venture (By Commercial Cleaners)	SK	0.463
Fillmore Seeds	SK	0.408
Gardiner Dam Terminal	SK	0.463
Great Northern Grain Terminals (By Commercial Cleaners)	AB	0.399
Great Northern Grain Terminals (By Grain Separators)	AB	0.348
Great Sandhills Terminal Ltd.	SK	0.435
Johnson Seeds, S.S.	AB	0.376
Lethbridge Inland Terminal Ltd.	AB	0.484
Louis Dreyfus	AB, SK, MB	0.528
Nature's Best Organics Inc.	SK	0.327
North East Terminal	SK	0.408
North West Terminal	SK	0.520
Parrish & Heimbecker	SK, MB	0.367
Parrish & Heimbecker	AB	0.408
Paterson Gr/Div of Paterson GlobalFoods Inc.	AB, SK, MB	0.354
Pioneer Grain	MB	0.252
Prairie West Terminal (By Commercial Cleaners)	SK	0.439
Prairie West Terminal (By Grain Separators)	SK	0.369
South West Terminal	SK	0.263
Tri Lake Agri. Ltd.	MB	0.252
Viterra	AB, SK, MB	0.463
Westmor Terminals Inc.	AB	0.381
Westlock Terminals (NGC) Ltd.	AB	0.402
Weyburn Inland Terminals	SK	0.313

Source: Canadian Grain Commission (2008)

*Appendix 7. Numerical results for the Farm Level and Social Planner models for Alberta, Saskatchewan and Manitoba*

**Alberta**

**Table A. 9. Numerical results for the FL and SP models for Mallard populations equal to zero/carrying capacity (Alberta)**

<i>Farm Level</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	<i>Revenue</i>	<i>B(x)</i>
$x = 0$	5.60	105.29	0	1,737	1,263	489,998	-
$x = K$	-5.60	105.29	37.88	1,737	1,263	489,963	-

<i>Social Planner (payment from private source)</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	<i>Revenue</i>	<i>B(x)</i>
$x = 0$	-732.87	105.46	0	1,740	1,260	489,998	0
$x = K$	270.59	105.46	37.88	1,740	1,260	489,962	3,256

<i>Social Planner (payment from public funds)</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	<i>Revenue</i>	<i>B(x)</i>
$x = 0$	-733.02	105.26	0	1,737	1,263	489,998	-834
$x = K$	281.70	105.26	37.88	1,737	1,263	489,963	2,422

**Saskatchewan**

**Table A. 10. Numerical results for the FL and SP models for Mallard populations equal to zero/carrying capacity (Saskatchewan)**

<i>Farm Level</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	<i>Revenue</i>	<i>B(x)</i>
$x = 0$	5.15	43.67	0	2,262	738	287,752	-
$x = K$	-5.15	43.67	37.88	2,262	738	287,718	-

<i>Social Planner (payment from private source)</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	<i>Revenue</i>	<i>B(x)</i>
$x = 0$	-681.49	43.77	0	2,267	733	287,750	0
$x = K$	262.05	43.77	37.88	2,267	733	287,717	3,256

<i>Social Planner (payment from public funds)</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	<i>Revenue</i>	<i>B(x)</i>
$x = 0$	-709.57	43.65	0.00	2,261	739	287,751	-1,085
$x = K$	272.85	43.65	37.88	2,261	739	287,718	2,171

## Manitoba

**Table A. 11. Numerical results for the FL and SP models for Mallard populations equal to zero/carrying capacity (Manitoba)**

<i>Farm Level</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	<i>Revenue</i>	<i>B(x)</i>
$X = 0$	5.33	121.28	0.00	2,233	767	773,941	-
$X = K$	-5.33	121.28	37.88	2,233	767	773,906	-

<i>Social Planner (payment from private source)</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	<i>Revenue</i>	<i>B(x)</i>
$X = 0$	-682.71	121.39	0	2,235	765	773,941	0
$X = K$	262.42	121.39	37.88	2,235	765	773,906	3,256

<i>Social Planner (payment from public funds)</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	<i>Revenue</i>	<i>B(x)</i>
$X = 0$	-710.67	121.26	0	2,232	768	773,941	-1,072
$X = K$	273.17	121.26	37.88	2,232	768	773,906	2,184

Appendix 8. Sensitivity analysis for the Social Planner models for Alberta, Saskatchewan and Manitoba: changes in direct payment to farmers ( $\phi$ )

**Alberta**

**Table A. 12. Effects of changes in  $\phi$  over numerical results for SP models (Alberta)**

<i>Social Planner (payment from private source)</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	$\pi$	$B(x)$
$\phi = 0$	272.55	98.65	38	1,627	1,373	448,195	3,256
$\phi = 0.8$	272.49	98.83	38	1,631	1,369	448,194	3,256
$\phi = 1.6$	272.44	99.02	38	1,634	1,366	448,192	3,256

<i>Social Planner (payment from public funds)</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	$\pi$	$B(x)$
$\phi = 0$	283.67	98.65	38	1,627	1,373	448,195	3,256
$\phi = 0.8$	283.68	98.61	38	1,627	1,373	448,195	2,475
$\phi = 1.6$	283.69	98.58	38	1,626	1,374	448,195	1,695

**Saskatchewan**

**Table A. 13. Effects of changes in  $\phi$  over numerical results for SP models (Saskatchewan)**

<i>Social Planner (payment from private source)</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	$\pi$	$B(x)$
$\phi = 0$	263.70	41.82	38	2,166	834	249,820	3,256
$\phi = 0.8$	263.60	41.93	38	2,172	828	249,819	3,256
$\phi = 1.6$	263.51	42.04	38	2,178	822	249,816	3,256

<i>Social Planner (payment from public funds)</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	$\pi$	$B(x)$
$\phi = 0$	274.46	41.82	38	2,166	834	249,820	3,256
$\phi = 0.8$	274.48	41.80	38	2,165	835	249,820	2,217
$\phi = 1.6$	274.50	41.78	38	2,164	836	249,820	1,179

## Manitoba

**Table A. 14. Effects of changes in  $\phi$  over numerical results for SP models (Manitoba)**

<i>Social Planner (payment from private source)</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	$\pi$	$B(x)$
$\phi = 0$	263.42	118.05	38	2,173	827	709,946	3,256
$\phi = 0.8$	263.39	118.16	38	2,175	825	709,946	3,256
$\phi = 1.6$	263.36	118.27	38	2,177	823	709,944	3,256

<i>Social Planner (payment from public funds)</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	$\pi$	$B(x)$
$\phi = 0$	274.17	118.05	38	2,173	827	709,946	3,256
$\phi = 0.8$	274.18	118.03	38	2,173	827	709,946	2,213
$\phi = 1.6$	274.19	118.01	38	2,173	827	709,946	1,170

*Appendix 9. Sensitivity analysis for the Farm Level and Social Planner models for Alberta, Saskatchewan and Manitoba: changes in winter wheat cold tolerance ( $\delta$ )*

**Alberta**

**Table A. 15. Effects of changes in  $\delta$  over numerical results for SP models (Alberta)**

<i>Farm Level</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	$\pi$	$B(x)$
$\delta = 0$	5.60	105.29	0	1,737	1,263	489,998	-
$\delta = 0.05$	5.64	98.65	0	1,627	1,373	448,230	-
$\delta = 0.10$	5.69	90.75	0	1,497	1,503	408,356	-

<i>Social Planner (payment from private source)</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	$\pi$	$B(x)$
$\delta = 0$	270.59	105.46	38	1,740	1,260	489,962	3,256
$\delta = 0.05$	272.49	98.83	38	1,631	1,369	448,194	3,256
$\delta = 0.10$	274.79	90.95	38	1,500	1,500	408,319	3,256

<i>Social Planner (payment from public funds)</i>							
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	$\pi$	$B(x)$
$\delta = 0$	281.70	105.26	38	1,737	1,263	489,963	2,422
$\delta = 0.05$	283.68	98.61	38	1,627	1,373	448,195	2,475
$\delta = 0.10$	286.07	90.71	38	1,496	1,504	408,320	2,538



## Saskatchewan

Table A. 16. Effects of changes in  $\delta$  over numerical results for SP models (Alberta)

	<i>Farm Level</i>						
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	$\pi$	$B(x)$
$\delta = 0$	5.15	43.67	0	2,262	738	287,752	-
$\delta = 0.05$	5.19	41.82	0	2,166	834	249,854	-
$\delta = 0.10$	5.23	39.41	0	2,041	959	212,788	-

	<i>Social Planner (payment from private source)</i>						
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	$\pi$	$B(x)$
$\delta = 0$	262.05	43.77	38	2,267	733	287,717	3,256
$\delta = 0.05$	263.60	41.93	38	2,172	828	249,819	3,256
$\delta = 0.10$	265.64	39.54	38	2,048	952	212,753	3,256

	<i>Social Planner (payment from public funds)</i>						
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	$\pi$	$B(x)$
$\delta = 0$	272.85	43.65	38	2,261	739	287,718	2,171
$\delta = 0.05$	274.48	41.80	38	2,165	835	249,820	2,217
$\delta = 0.10$	276.62	39.39	38	2,040	960	212,754	2,277

## Manitoba

Table A. 17. Effects of changes in  $\delta$  over numerical results for SP models (Manitoba)

	<i>Farm Level</i>						
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	$\pi$	$B(x)$
$\delta = 0$	5.33	121.28	0	2,233	767	773,941	-
$\delta = 0.05$	5.35	118.05	0	2,173	827	709,981	-
$\delta = 0.10$	5.37	114.28	0	2,104	896	646,836	-

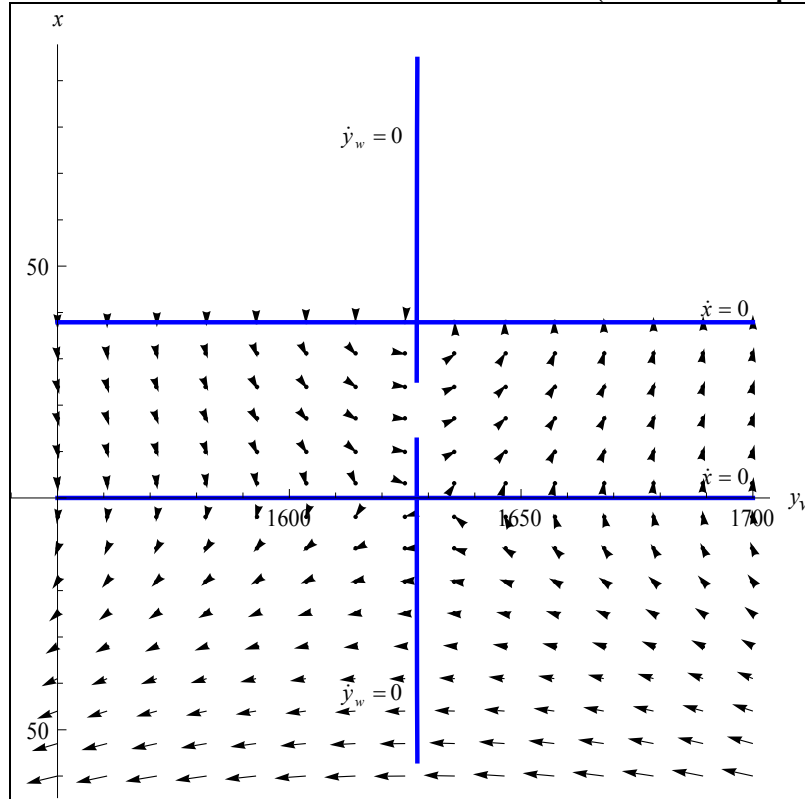
	<i>Social Planner (payment from private source)</i>						
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	$\pi$	$B(x)$
$\delta = 0$	262.42	121.39	38	2,235	765	773,906	3,256
$\delta = 0.05$	263.39	118.16	38	2,175	825	709,946	3,256
$\delta = 0.10$	264.53	114.39	38	2,106	894	646,801	3,256

	<i>Social Planner (payment from public funds)</i>						
	$\lambda$	$\mu$	$x$	$y_w$	$y_s$	$\pi$	$B(x)$
$\delta = 0$	273.17	121.26	38	2,232	768	773,906	2,184
$\delta = 0.05$	274.18	118.03	38	2,173	827	709,946	2,213
$\delta = 0.10$	275.37	114.25	38	2,103	897	646,801	2,246

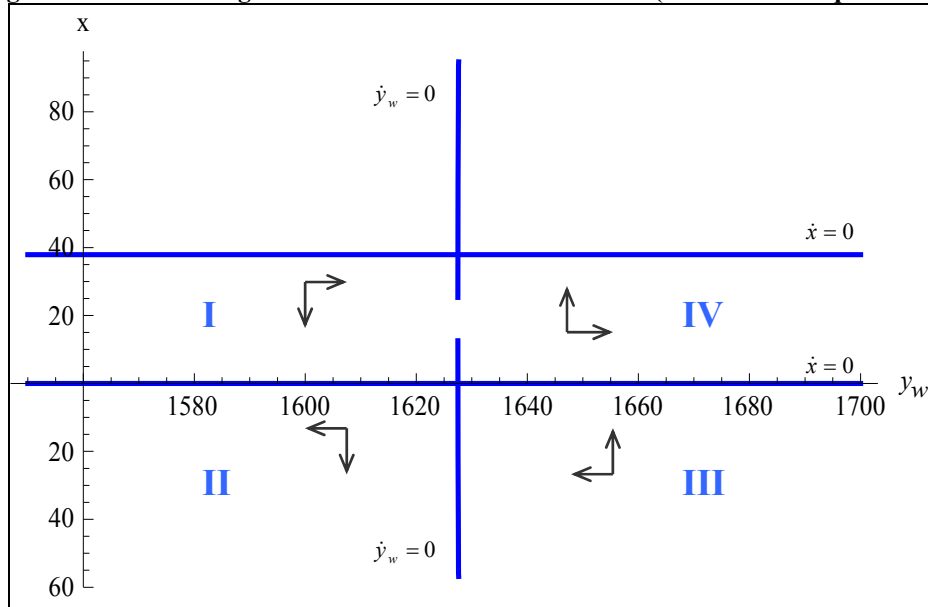
**ALBERTA**

**Alberta: Farm Level model**

**Figure A. 2. Direction Field: Farm level model for Alberta (Area around equilibrium)**

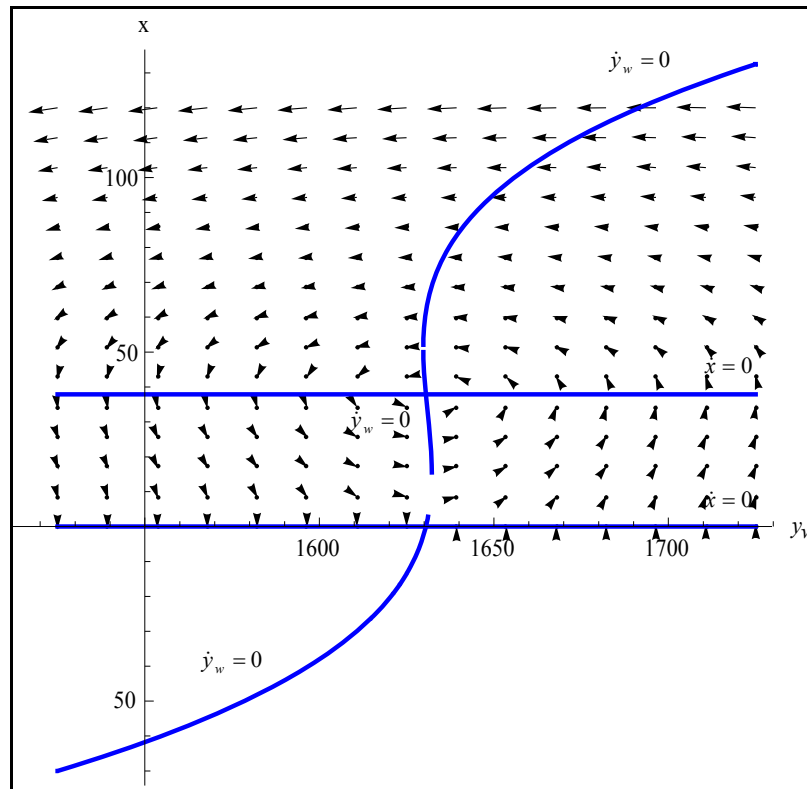


**Figure A. 3. Phase Diagram: Farm level model for Alberta (Area around equilibrium)**

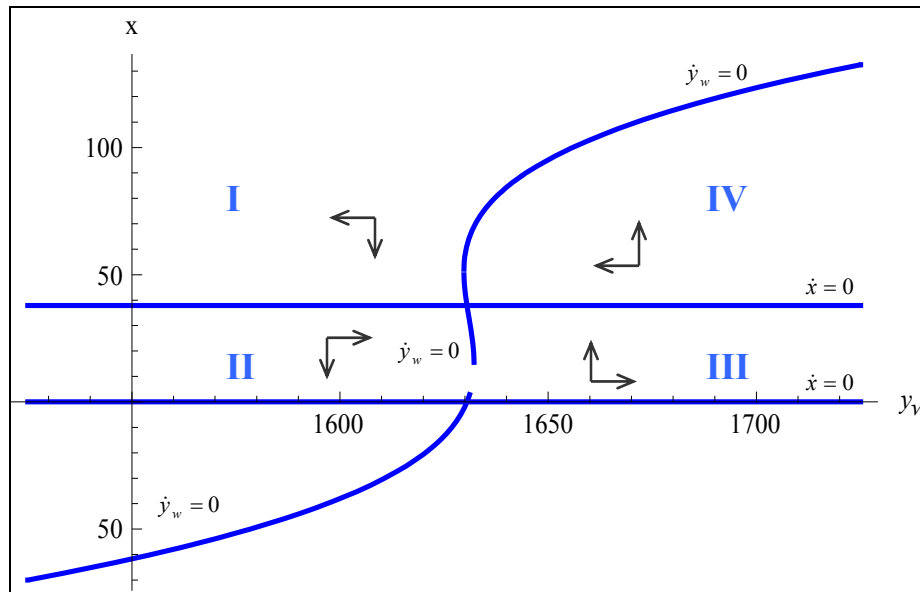


**Alberta: Social Planner model (private  $\phi$ )**

**Figure A. 4. Direction Field: Social Planner (with private  $\phi$ ) for Alberta (Area around equilibrium)**



**Figure A. 5. Phase Diagram: Social Planner (with private  $\phi$ ) for Alberta (Area around equilibrium)**



Alberta: Social Planner model (public  $\phi$ )

Figure A. 6. Direction Field: Social Planner (with public  $\phi$ ) for Alberta (Area around equilibrium)

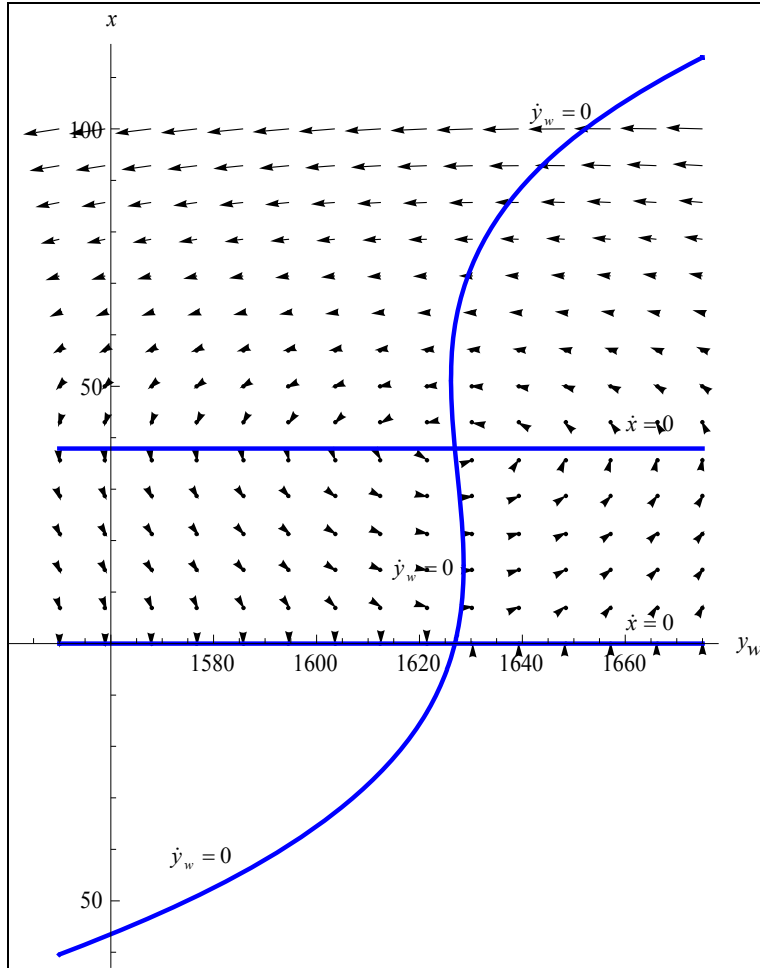
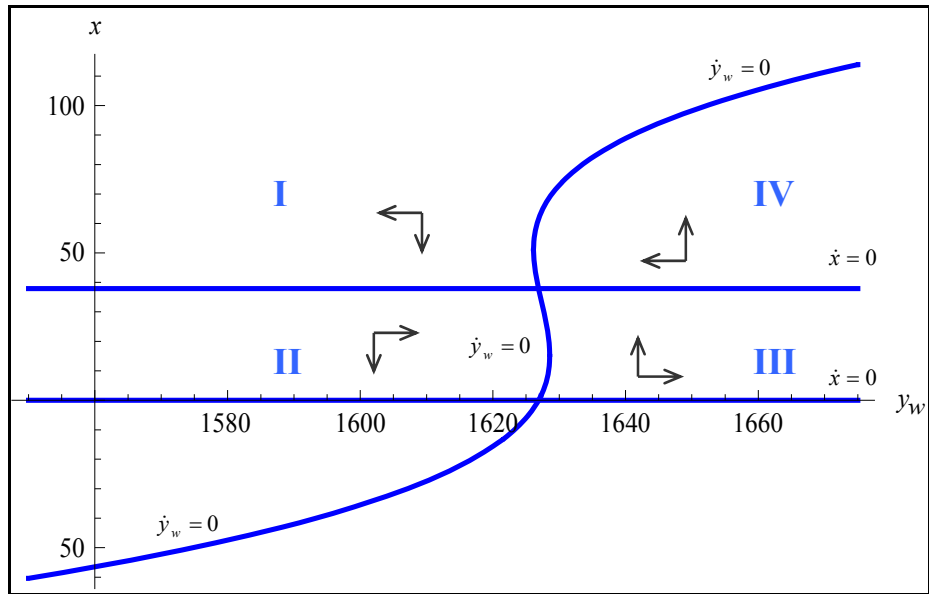


Figure A. 7. Phase Diagram: Social Planner (with public  $\phi$ ) for Alberta (Area around equilibrium)



# SASKATCHEWAN

## Saskatchewan: Farm level model

Figure A. 8. Direction Field: Farm Level model for Saskatchewan (Area around equilibrium)

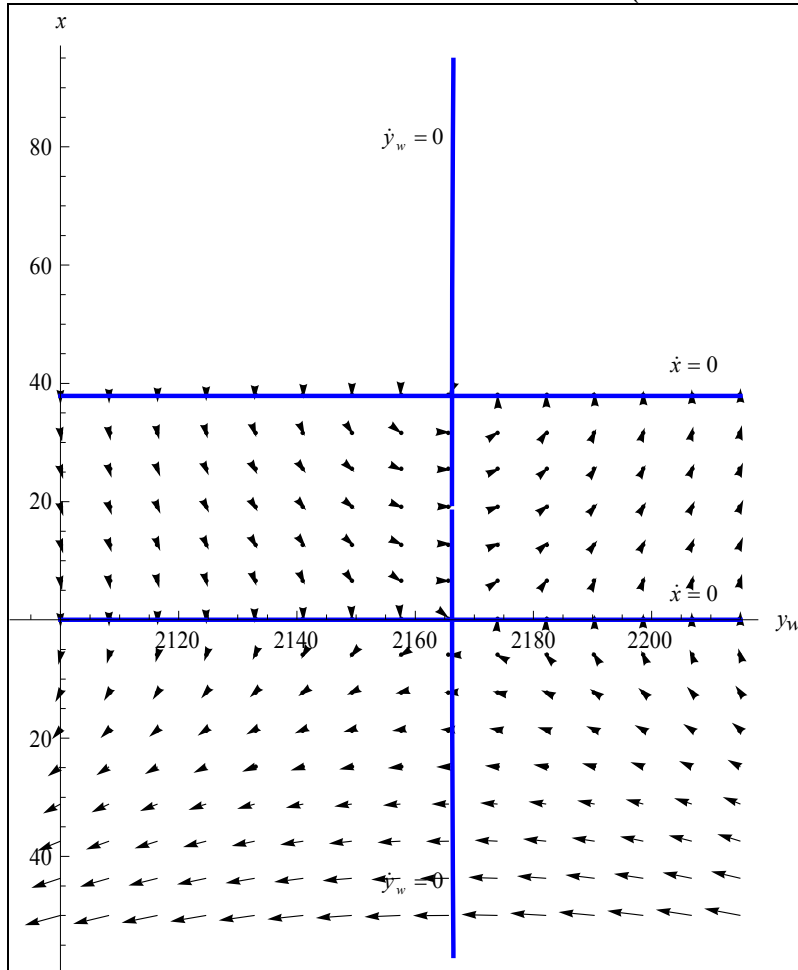
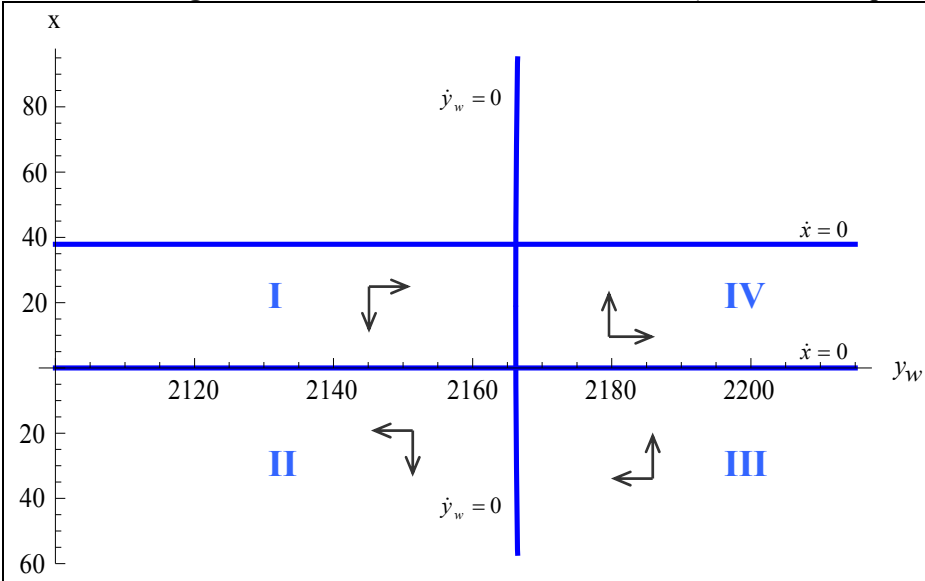


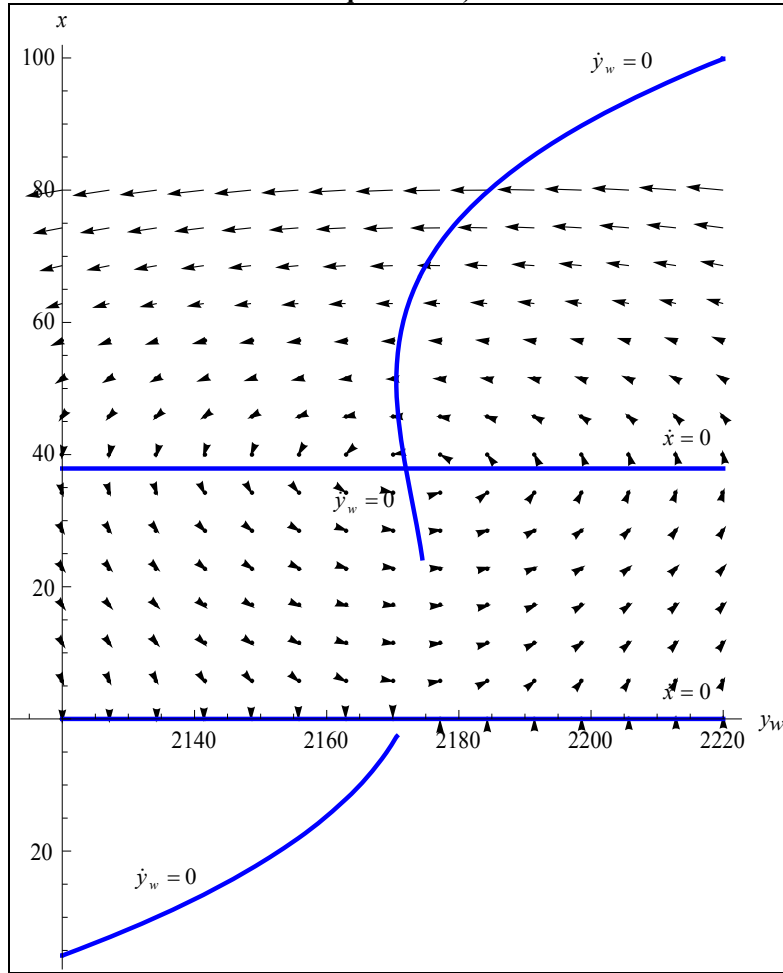
Figure A. 9. Phase Diagram: Farm Level model for Saskatchewan (Area around equilibrium)



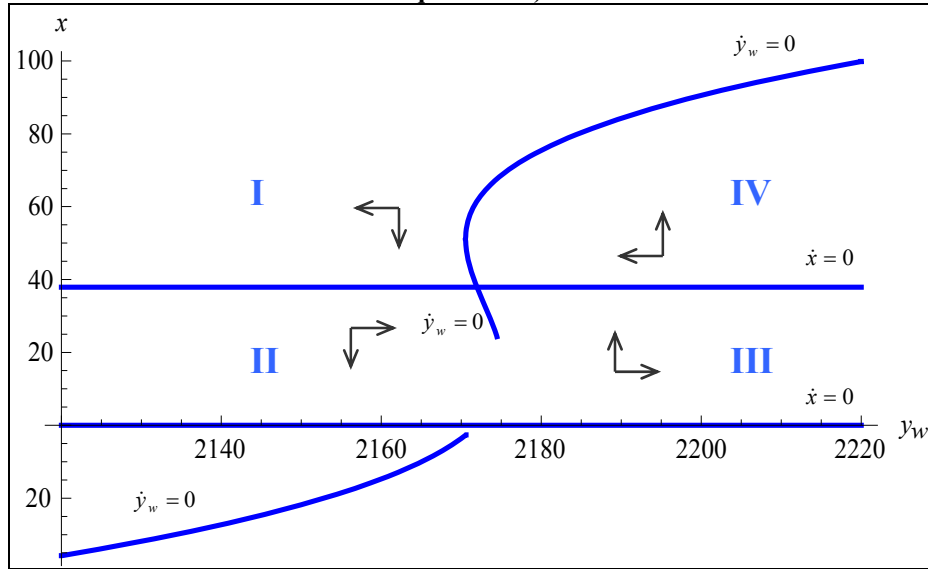


### Saskatchewan: Social planner model (private $\phi$ )

Figure A. 10. Direction Field: Social Planner (with private  $\phi$ ) for Saskatchewan (Area around equilibrium)

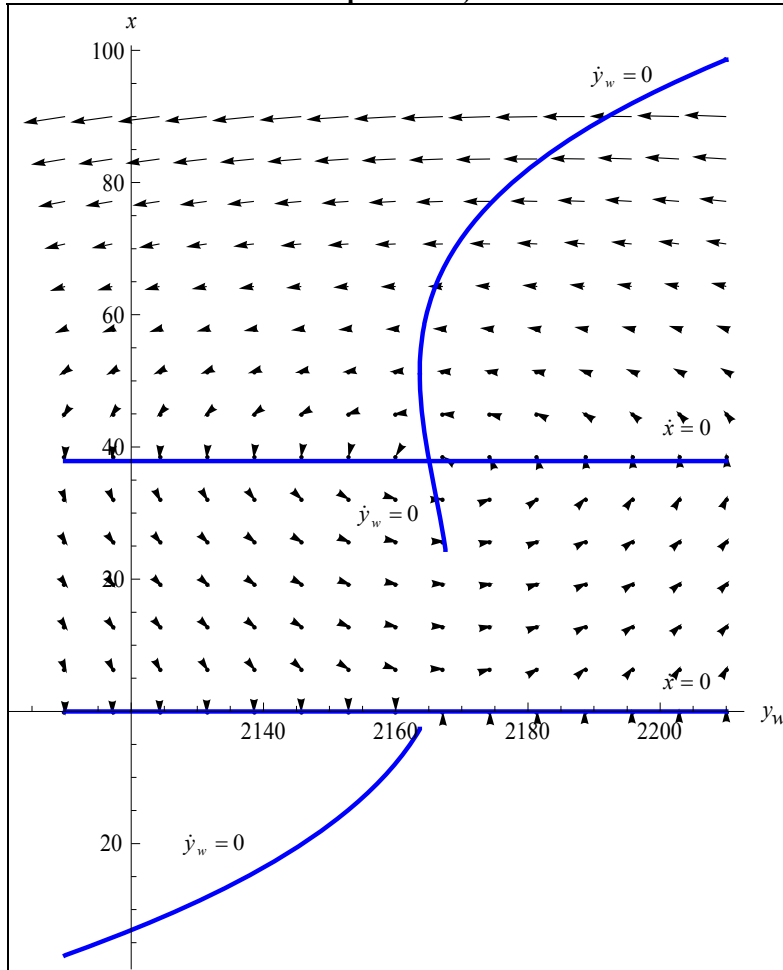


**Figure A. 11. Phase Diagram: Social Planner (with private  $\phi$ ) for Saskatchewan (Area around equilibrium)**

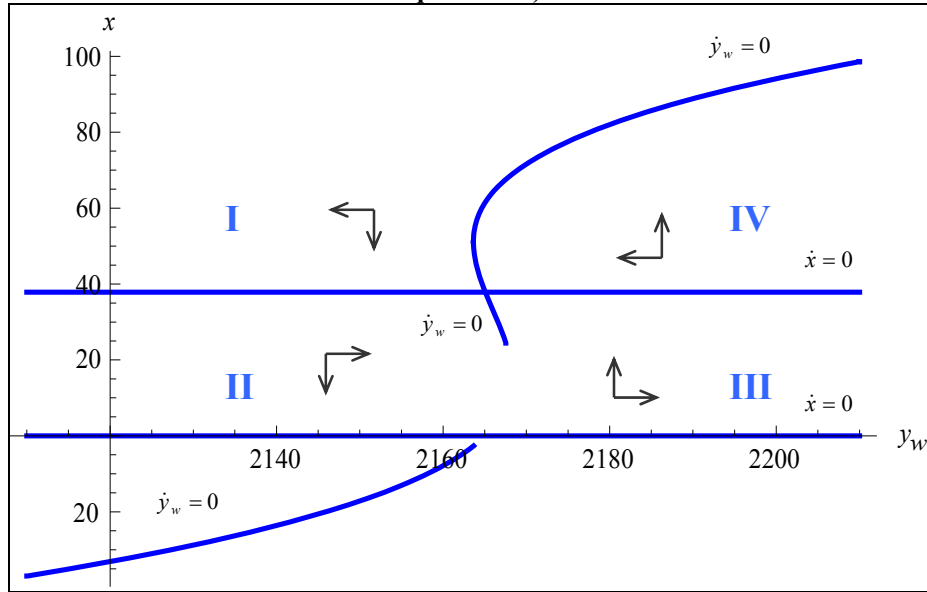


### Saskatchewan: Social planner model (public $\phi$ )

Figure A. 12. Direction Field: Social Planner (with public  $\phi$ ) for Saskatchewan (Area around equilibrium)



**Figure A. 13. Phase Diagram: Social Planner (with public  $\phi$ ) for Saskatchewan (Area around equilibrium)**



# MANITOBA

## Manitoba: Farm Level model

Figure A. 14. Direction Field: Farm level model for Manitoba (Area around equilibrium)

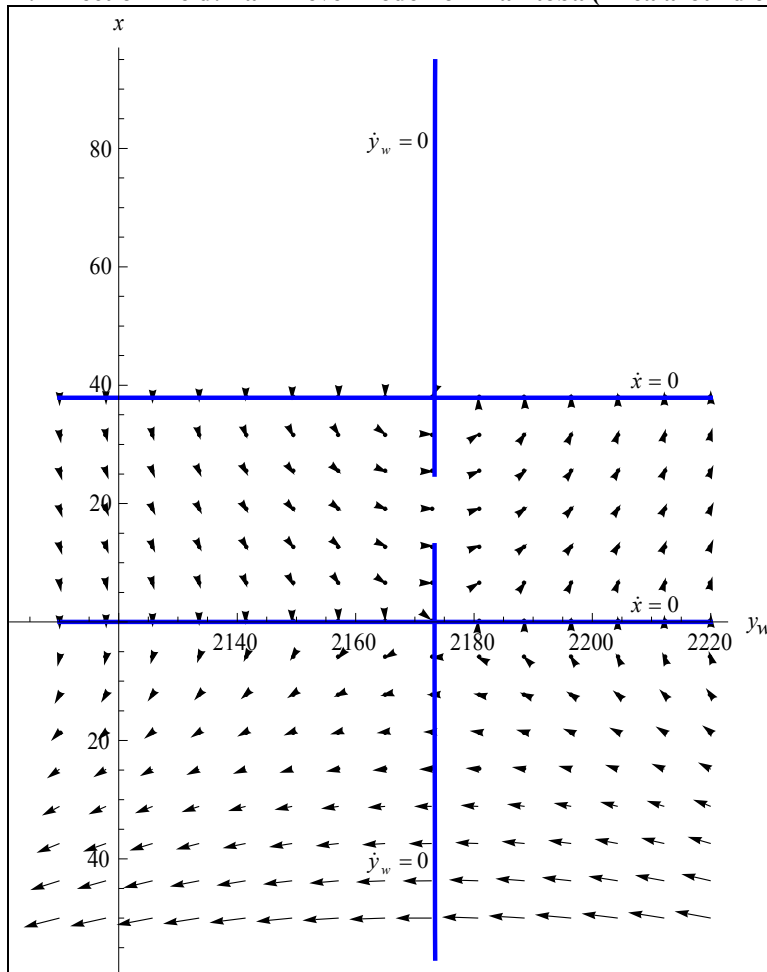
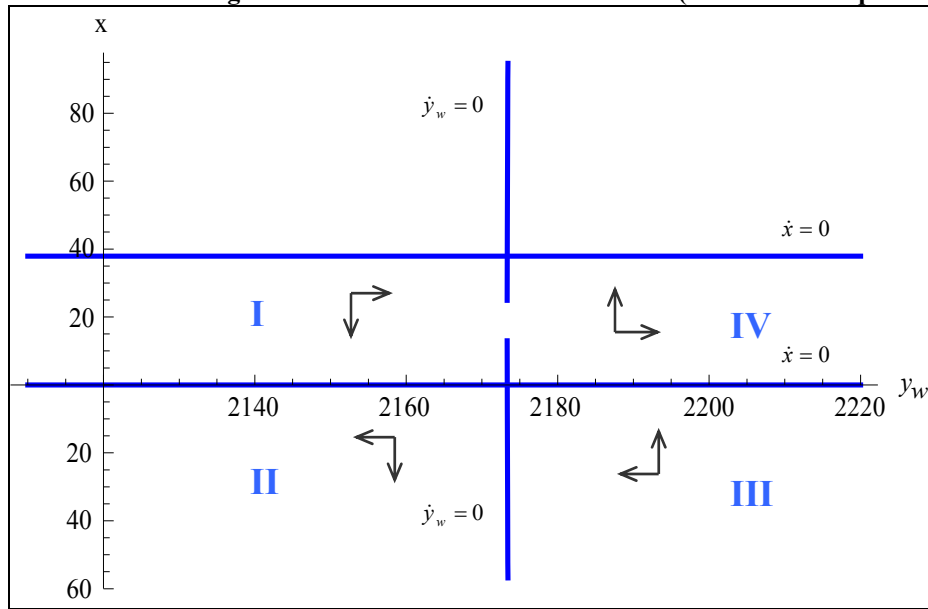


Figure A. 15. Phase Diagram: Farm Level model for Manitoba (Area around equilibrium)



**Manitoba: Social Planner model (private  $\phi$ )**

**Figure A. 16. Direction Field: Social Planner model (with private  $\phi$ ) for Manitoba (Area around equilibrium)**

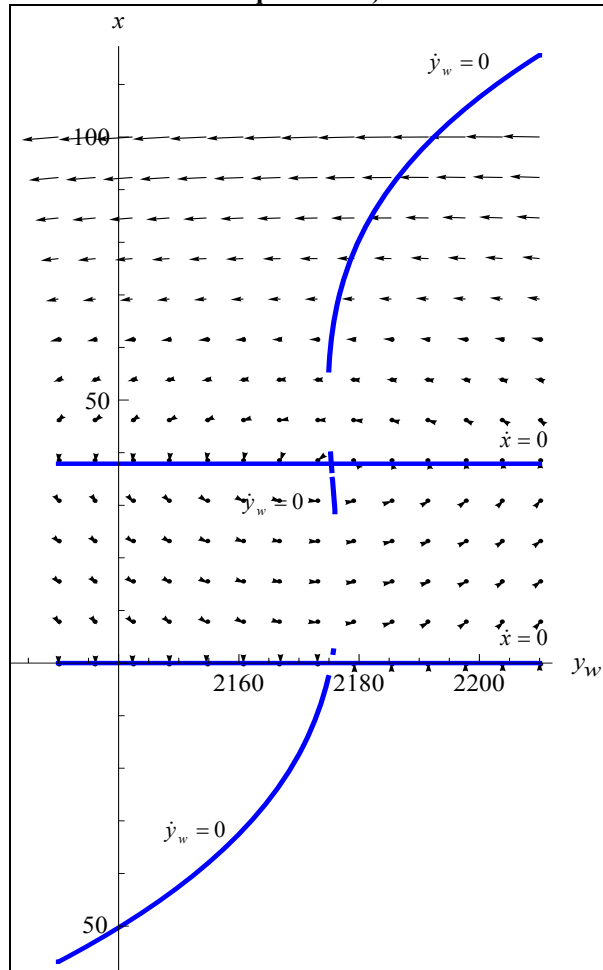
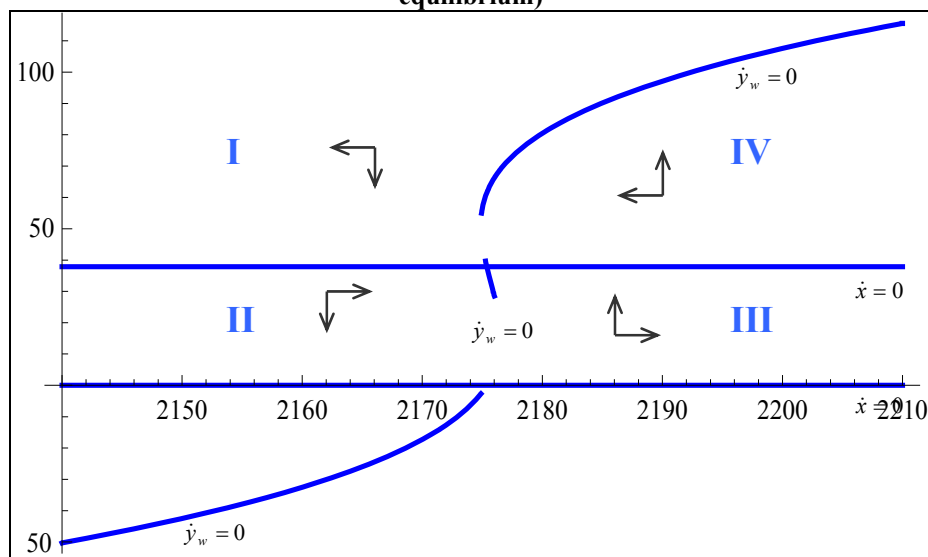


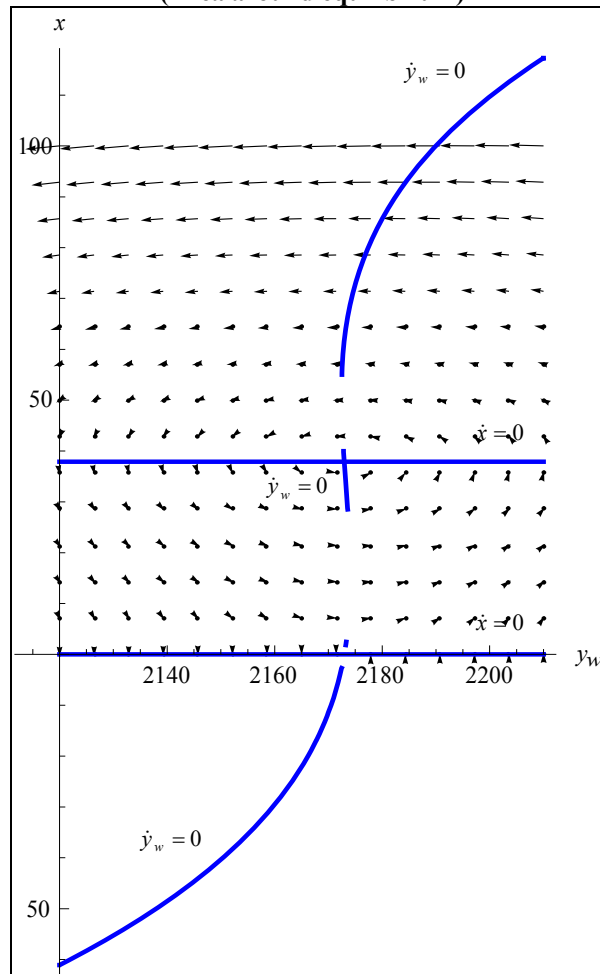
Figure A. 17. Phase Diagram: Social Planner model (with private  $\phi$ ) for Manitoba (Area around equilibrium)





# Manitoba: Social Planner model (public $\phi$ )

Figure A. 18. Direction Field: Social Planner model (with public  $\phi$ ) for Manitoba  
(Area around equilibrium)



**Figure A. 19. Phase Diagram: Social Planner model (with private  $\phi$ ) for Manitoba  
(Area around equilibrium)**

