University of Alberta

Selective maintenance for systems under imperfect maintenance policy

by

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Abstract

Due to the advent of complex engineering systems in the last few decades, reliability and maintenance issues are gaining more attention than ever. Maintenance is important to keep a system running and ensure that it performs its functions satisfactorily. Often, there are limited resources to complete the maintenance of a system. Such a limitation may arise due to limited budget, time, or repairman availability, etc. Under these circumstances, it is required to optimally allocate the available resources in a way that selected components within a system and maintenance actions performed on the selected components assure satisfactory performance of the system after maintenance. This maintenance policy is called selective maintenance.

At the time of maintenance, it is not always necessary that a system is replaced. It may undergo other maintenance options instead, such as minimal repair (when a failed system is as bad as old after maintenance) or imperfect maintenance (better than minimal repair but not as good as a new component). This Ph.D. research studies the selective maintenance modeling for systems when imperfect maintenance is also possible in addition to replacement and minimal repair. Models are developed in this thesis to reflect the effect of different maintenance actions on system reliability. In juxtaposition with these models, selective maintenance policy is developed for systems.

A maintenance policy is influenced by several factors such as the age of the system, failure modes in the system, maintenance history, and the performance levels of the system. At first in this thesis, the effect of the system age and maintenance budget on the maintenance decision is modeled and selective maintenance is performed. A single mission selective maintenance model is developed for a system that can exist in any of the two possible states: working and failed (also called binary system), and all failures within the system are maintainable.

Maintenance may or may not affect different failure modes in a system, and accordingly the system can be maintainable and non-maintainable with respect to these failure modes. Therefore, the presence of these two types of failures in a binary system is also studied and a single mission selective maintenance problem is solved.

If maintenance is required more than once in a given planning horizon, then the single mission selective maintenance is no longer adequate. In these conditions, the time to perform maintenance is important in order to keep a system reliable throughout the planning horizon. Hence, maintenance scheduling is required for a system along with the selective maintenance decision during each of the maintenance breaks. Thus, the selective maintenance scheduling problem is solved for a binary system in this thesis.

Conventionally, it is assumed that a system has binary states. But a system may also have more than two performance states. For such a multistate system, the binary selective maintenance model is not applicable. Therefore, a comprehensive model is developed in this thesis for selective maintenance of a multistate system.

The proposed selective maintenance models are applied to different examples. The results demonstrate the effectiveness and advantages of the proposed models.

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Contents

List of Figures

List of Tables

Abbreviations

ABAO As Bad As Old

AGAN As Good As New

CR Component Replacement

DE Differential Evolution

DN Do Nothing

EA Evolutionary Algorithm

IM Imperfect Maintenance

IR Imperfect Repair

MR Minimal Repair

MRL Mean Residual Life

MSS Multistate System

PM Preventive Maintenance

pdf Probability Density Function

SM Selective Maintenance

Chapter 1

Introduction

Recent developments in science and technology have ensured that complex operations can be performed by modern and more powerful engineering systems. The advancements in technology and operation bring sophistication in systems and their components. This means that the reliability issues need greater attention and management. Engineering systems and components are designed to perform predefined objectives over a defined duration. Reliability measures the ability of a system to perform its intended functions under stated conditions for a specified period of time. It is important to keep a system reliable throughout its intended period of use. Failure to do so may lead to system failures as well as cause severe damage to the environment and society.

The following examples demonstrate the damage caused by systems' failures. The Northeast blackout of 2003 is an example that caused a widespread power outage throughout parts of the Northeastern and Midwestern United States and the Canadian province of Ontario. More than 55 million people were without power for 1-2 days. The power outage had wide range impact on the communication network, emergency services, water treatment, supply and distribution, food distribution, banking services, traffic services, and government services. According to a report by the Government of Canada [1], the blackout reduced Ontario's gross domestic product (GDP) by 1.4%, which in turn reduced the national GDP by 0.7%. It cost Ontario's economy between $1 billion and $2 billion. A similar blackout took place in India in 2012, affecting 620 million people, about 9% of the world population [2]. Most recently,

Figure 1.1: Texas fertilizer blast[3]

a blast in a Texas fertilizer plant led to loss of lives and major property. It damaged the environment with toxic fumes (Fig.1.1).

In all stages of modern engineering processes, including design, manufacturing, operation, and services, reliability should be given due importance [4, 5, 6]. To describe system reliability, it is necessary to specify *the state* and *the configuration* of the system and its *failure process* [7]. Thus, to determine a system failure and reliability, the system definition should be clearly outlined.

1.1 System definition

A system consists of a group of components to perform one or more specified operational functions. The state of a system helps in defining the conditions of system failure. Traditionally, systems or components are considered to be in only two possible states: either working or failed. Such systems or components are said to have binary states and are called binary system or binary components, respectively. As shown in Fig.1.2, a system has a working and a failed state.

In the binary assumption, a system is assumed to perform its desired function satisfactorily until it fails. The random failure from the working to the failed state is assumed to follow a certain probability density function (pdf), to be explained in Section 1.2. A comprehensive discussion about the binary system reliability modelling can be found in Kuo and Zuo [7].

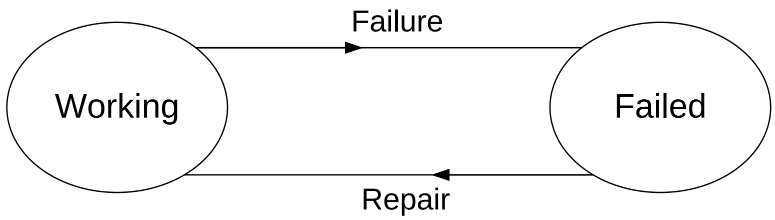

Figure 1.2: A binary system

For some systems or components the binary assumption does not actually reflect the possible states that each of them may experience. They can perform their tasks with more than two distinguished levels of efficiency, known as "performance rates." A system that has more than two performance rates varying from perfect operation to complete failure is called a multi-state system (MSS). The MSS has been investigated by many researchers to evaluate its reliability. A detailed review of the multistate reliability modelling can be found in [8, 9].

MSSs can be divided into two categories as: (1) continuous-state MSS [10] and (2) discrete-state MSS [11]. In the continuous-state MSS, the system is said to have continuous performance levels. The system is considered failed when it degrades beyond a predefined threshold. The problem in the implementation of continuous-state MSS is its mathematical complexity [12]. Owing to this challenge the discrete-state MSS is popular for reliability modelling of an MSS. In this thesis, we focus on a discrete-state MSS only.

As shown in Fig.1.3, a discrete-state MSS has total $v + 1$ states ($v \geq 1$) where v is an integer. State 0 is the complete failure state, state v is the best performance state for an MSS, and states 1 to $v - 1$ are the intermediate states. When the system is brand new, it is in the best state v. However, as time progresses, it degrades to the lower performance states. Maintenance is then needed to improve the performance of the system and bring it to a higher performance state. From this point on, we will refer a discrete-state MSS as an MSS only without using the term "discrete state."

There are many practical applications of an MSS such as the power supply system, wireless communication system, pumping system, and coal trans-

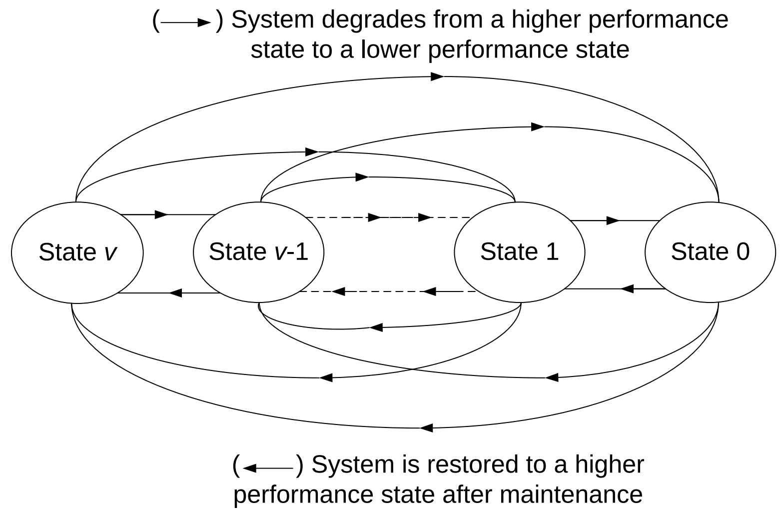

Figure 1.3: A multistate system

portation system [9, 13, 14]. A power supply system consists of generating and transmitting facilities where each generating unit can function at different capacity levels. The generating units are a complex assembly of many parts. Due to the failure or degradation of different parts, the unit may continue to operate but with a reduced capacity. In a wireless communication system there are different transmission stations. The state of each station may be defined by the number of subsequent stations covered in its range, which depends on the number of the amplifiers and conditions of the signal propagation (weather, solar activity, etc.). A pumping system relies on the capacities of the pumps in the system. In a coal transportation system, the coal supplied to the boiler depends on the condition of en-route elements, for example, feeder, conveyor.

The system configuration describes how the system is connected and its rules of operation. For example, if n components (where $n > 1$) in a system are connected in a series (Fig.1.4) then for this series configuration, the system performance may be defined as the minimum of the performance of the components. For example, in a manufacturing unit, three types of machines are arranged in a series with the production capacities of 60 units/hour, 50 units/hour, and 70 units/hour, respectively. For this machine line, the maximum achievable production capacity is 50 units/hour only because the second

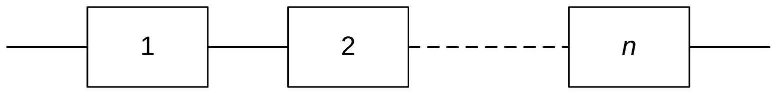

Figure 1.4: A system with n components in a series configuration

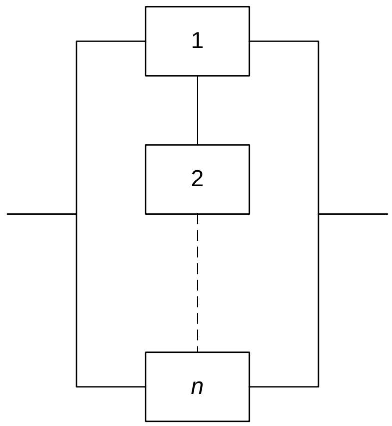

Figure 1.5: A system with n components in a parallel configuration

machine is the bottleneck and it cannot produce more than 50 units/hour.

Similarly, for a parallel configuration given in Fig.1.5, the system performance may be defined as the sum of the performances of the components. For instance, a city water pump station has two water pumps connected in a parallel arrangement, and each of them can pump water at a rate of 500 liters/hour. For this pump station, the total capacity would be 1000 liters/hour, which is the sum of the capacity of individual pumps.

A system may also have several subsystems connected in series where each subsystem has components connected in parallel. This arrangement is called a series-parallel configuration. In this configuration, first the performance of each subsystem is calculated in a parallel arrangement, which is the sum of performances of individual components in that subsystem. Since all subsystems in the system are in a series configuration, the system performance is the minimum of the performances of the subsystems.

Most of the engineering systems, such as pumps, machines, generators, deteriorate with age and usage. This system degradation behavior needs proper attention; otherwise the system may fail unexpectedly, leading to significant failure and subsequent repair costs. Therefore, it is important to assess the health of a system and capture its failure process.

1.2 Assessing system health and the failure process

In order to keep a system performing satisfactorily and decrease the chances of unexpected failures, it is important to understand the degradation behavior of the system. Based on this understanding, proper maintenance actions can be taken to keep the system reliable. The failure process defines the probability law that governs the failure mechanism of the system [7]. How to assess a system's health and understand its failure process are important aspects in defining the reliability and maintenance policy for that system.

A very common approach to reflect the system degradation is to use a hazard rate [15, 16, 17]. The hazard rate provides an estimate of how prone a system is to failure. Hazard rate $h(t)$, also referred in literature as failure rate, is defined as the probability that a component will fail in the next unit of time given that it has survived up to the point of time t:

$$h(t) = \lim_{\triangle t \to 0} \frac{1}{\triangle t} \frac{\Pr(t < T \leq t + \triangle t)}{\Pr(T > t)} = \frac{f(t)}{R(t)}, \tag{1.1}$$

where $f(t)$ is the pdf of the lifetime of the component and $R(t)$ is the reliability function of the component. Recently, condition–based health indicators like vibration signals, acoustic signals, and lube oil signals have also been used to assess the health of a system. A detailed survey of the condition monitoring can be found in [18]. In this thesis we have not performed signal collection or signal analysis for condition assessment and hence these condition–based indicators are not studied here.

The lifetime of a component is a random variable of interest in reliability analysis. It is continuous and can only take nonnegative values. Hence, continuous distributions are mainly used in the reliability analysis [19]. The best fit continuous distribution of failure observations of a system is used to define its hazard rate. A goodness–of–fitness test can be done to check how well a distribution fits a set of failure observations. A detailed discussion about the goodness–of–fitness tests can be found in [20, 21]. Two widely used failure distributions in reliability analysis are the exponential distribution and the Weibull distribution.

1. **Exponential distribution**: A random variable T has an exponential distribution if its pdf can be expressed in the following form:

$$f(t) = \lambda e^{-\lambda t}, \quad t \geq 0, \tag{1.2}$$

   where $\lambda > 0$ is the parameter of the distribution. The exponential random variable has the following reliability function and hazard rate function, respectively:

$$R(t) = e^{-\lambda t}, \quad t \geq 0, \tag{1.3}$$

$$h(t) = \frac{f(t)}{R(t)} = \lambda, \quad t \geq 0. \tag{1.4}$$

   From equation (1.4), it is observed that the hazard rate function of the exponential distribution is a constant. This means that a device with age t is as good as a new device with age 0. This is called the memoryless property of the exponential distribution. Due to mathematical simplicity, the exponential distribution is the most widely used distribution in reliability analysis [17]. Many products exhibit a roughly constant failure rate during their useful life period, as shown in the bathtub curve (Fig.1.6).

   A typical bathtub curve has three intervals. The first interval, which is usually short, shows the decreasing hazard rate function. This is also referred to as the early-failure period. The failures in this period occur mainly due to manufacturing defects. In the second interval, the hazard rate function is roughly constant. This interval covers most of the useful life of a system. The failures in this interval are caused by chance events like accidents, overloading. The third interval is often called the increasing hazard rate or wear-out period. The failures in this period are due to wear-out, aging, or serious deterioration. It should be noted that the shapes of bathtub curves for different system may be dramatically

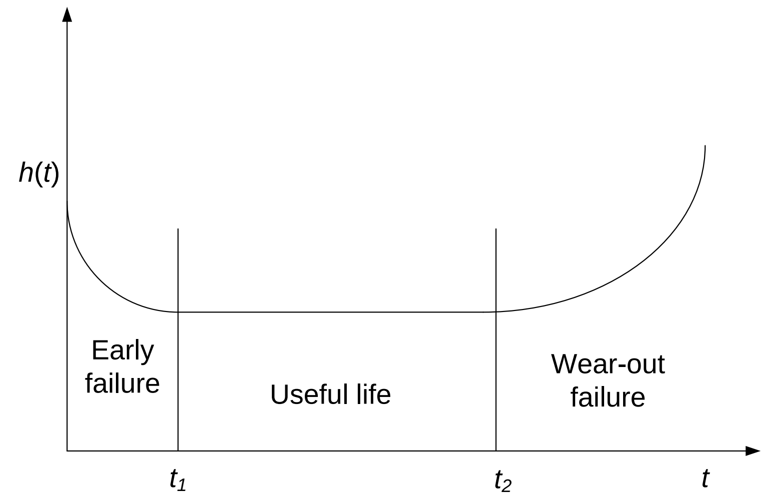

Figure 1.6: Bathtub curve of hazard rate function

different [7]. More discussions about the bathtub curves can be found in [22, 23].

2. **Weibull distribution**: A random variable T has the Weibull distribution if its pdf can be given by:

$$f(t) = \frac{\beta t^{\beta-1}}{\alpha^{\beta}} e^{-(t/\alpha)^{\beta}}, \quad t \geq 0, \tag{1.5}$$

where $\beta > 0$ is the shape parameter and $\alpha > 0$ is the scale parameter of the distribution. For a Weibull random variable, the reliability function and the hazard rate function are given by:

$$R(t) = e^{-(t/\alpha)^{\beta}}, \quad t \geq 0, \tag{1.6}$$

$$h(t) = \frac{f(t)}{R(t)} = \frac{\beta t^{\beta-1}}{\alpha^{\beta}}, \quad t \geq 0. \tag{1.7}$$

When $0 < \beta < 1$, $h(t)$ is a decreasing function. When $\beta = 1$, $h(t) = \frac{\beta}{\alpha}$ is constant, and the Weibull distribution reduces to the exponential distribution. When $\beta > 1$, $h(t)$ is an increasing function. Thus the Weibull distribution is a very flexible distribution in reliability analysis. It can be used to model all three regions of the bathtub curve (Fig.1.6).

An engineering system, such as pumps, machines, or generators, works under different stress conditions such as temperature, pressure, or load. With information about the states, configuration, and failure process of a system, its reliability can be determined. However, proper maintenance is also needed to keep an engineering system reliable. Reliability issues, maintenance problems and useful solutions are gaining more attention than ever. Maintenance strategies are developed with a focus on system reliability. In order to develop a maintenance strategy, it is important to define maintenance decisions and their effect on the system.

1.3 System maintenance

As a system is used, gradual deterioration occurs. It is important to keep a system in working order so that the required demand is met for a defined period of time. Maintenance can be defined as all activities necessary to keep a system in working order. System performance or reliability can be improved by adopting some appropriate maintenance policies and performing appropriate maintenance actions.

1.3.1 Maintenance actions

Maintenance can dramatically impact a system's overall performance and useful life [24]. Accordingly, researchers and practitioners are trying to improve system maintenance practices. If timely maintenance is not performed, a system may fail, leading to a significant cost associated with both the failure and the subsequent corrective actions.

Corrective maintenance is performed after a system failure, and the system is able to perform desired functions only after repair. Corrective maintenance is usually expensive; hence, it is important to prevent failures.

Preventive maintenance (PM) is performed at pre-specified intervals or as per some criteria such that the system reliability is increased and failures may be prevented. The failure and corrective maintenance costs may be saved using PM. PM has a number of advantages that make it worth utilizing [25, 26, 27, 28]:

1. It extends the life expectancy of the systems, thereby eliminating premature replacements.

2. Scheduled maintenance can reduce unexpected equipment downtime and the number of major repairs.

3. The maintenance crew is used more economically, and the cost of overtime is decreased because everyone is working according to a schedule, not according to breakdowns and repairs.

4. Improved safety and quality conditions.

Maintenance brings a system to a state where its functions can be performed satisfactorily. Hence, depending on the requirements, different levels of maintenance may be performed. One maintenance decision could be to replace the system with a new one. Another decision could be to restore a failed system to a state similar to just before failure. Also, a system state can be improved to a better state than it was just before maintenance, without replacing it. Hence, maintenance can be categorized as follows:

1. **Perfect maintenance or replacement**: Replacement of a system, whether it is working or failed, restores it to an as–good–as–new (AGAN) condition. Upon perfect maintenance, the health state of a component is the same as that of a new component.

2. **Minimal repair**: If the health state of a component after repair is the same as it was just before it failed, the repair action is called minimal repair. The system operating state is as bad as old (ABAO) after minimal repair. Changing the headlight of a truck could be an example of minimal repair because it does not change the overall failure intensity of the truck.

3. **Imperfect maintenance**: Traditionally, it is assumed that maintenance brings a system back to as good as new (AGAN) or as bad as old (ABAO) condition. However, maintenance can restore a system to

a state somewhere between AGAN and ABAO conditions. Such maintenance actions are called imperfect maintenance. Replacing some parts of a system can be one example. It makes the system better than ABAO but not AGAN.

Depending on the health state of a system before maintenance and the reliability requirement after maintenance, maintenance actions can be selected. However, it is not necessarily true that maintenance performed on a system will affect all types of failures. For example, crack and spalling in a bearing can not be changed by a maintenance action while the lack of lubrication can be taken care by maintenance. Thus, two types of failure modes may exist in a system: maintainable and non-maintainable failure modes.

1.3.2 Maintainable and non-maintainable failure modes

The failure of a system may take place due to more than one reason. A system may fail in one or another type of failure modes. One popular definition of failure mode states that "all technical items are designed to fulfill one or more functions; a failure mode is thus defined as non-fulfillment of one of these functions [29]." According to another definition, "a system is called to be failed when it is unable to fulfil a function to a standard of performance which is acceptable to the user, and all the events which are reasonably likely to cause each failed state are known as failure modes [30]." For example, spalling of a bearing (Fig.1.7) may lead to the bearing failure. Similarly insufficient lubrication may also cause damage to the bearing, leading to its failure. Hence, spalling and insufficient lubrication are two different types of failure modes in a bearing system.

Conventionally, all random failures of the system are treated equally and used to define the system's pdf and reliability function. However, some failure modes in the system can be maintained and, with respect to these failure modes, the system can once again be restored to AGAN condition. Other failure modes cannot be maintained without replacement; in these cases, the system remains in ABAO condition with respect to these failure modes. How-

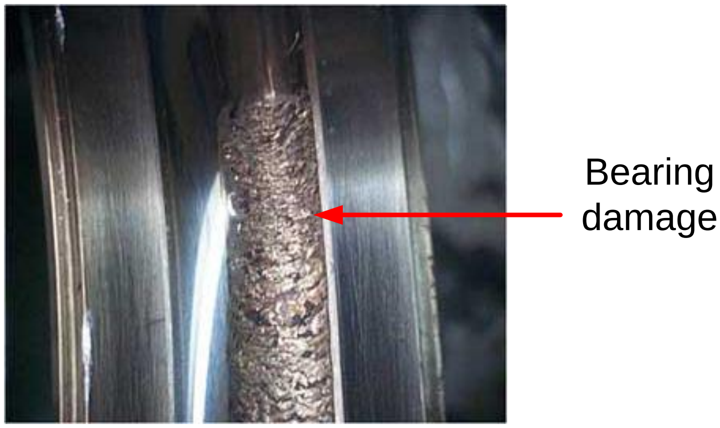

Figure 1.7: The spalling damage to a bearing race [31]

ever after replacement, a system is restored to AGAN condition with respect to both types of failure modes. These failure modes are called the maintainable failure modes and the non-maintainable failure modes, respectively [32]. For example, in a belt conveyor system, the damage to the belt is a maintainable failure mode because the belt can be easily replaced with a new one. However, the overall structural damage due to wear and fatigue is non-maintainable.

The definition that a failure is maintainable or not depends on the system boundary definition as well. For instance, if the focus is only on a bearing system, then spalling damage is a non-maintainable failure mode, because the only feasible maintenance action is to replace the bearing. However, if a belt conveyor system is considered, the failure mode due to the bearing would be called maintainable with respect to the conveyor. This is because there are several bearings attached to the idler that runs the system's conveyor belt. After replacing the bearing, the failure modes, with respect to that bearing, are restored to the AGAN condition.

In a system, usually some components exhibit maintainable failure modes and other components exhibit non-maintainable failure modes. In the conveyor belt system, the overall structural deterioration can be categorized as the non-maintainable failure. As the structure ages due to wear and corrosion, its vibration increases, which may cause faster degradation to the idler bearings mounted on the structure. Here, a bearing can be replaced easily (AGAN), but the structural fatigue wear is not recovered by maintenance. The hazard rate due to the maintainable failure modes is thus affected by the maintenance actions performed; however, the hazard rate due to the non-maintainable failure modes is unaffected by the maintenance actions performed on the system. The interaction between the two failure modes can be used to find the interaction between the components in the system experiencing these failure modes [33]. Additionally, a study of such interaction helps in understanding the effect of different maintenance options on the system and their effect on the overall system reliability.

Different maintenance options provide alternatives and flexibility to the maintenance crew. At the same time, the decision making becomes complicated with more options, especially when limited resources are in hand. For a system with multiple components, the maintenance department needs to decide the component(s) to be selected for maintenance as well as the maintenance level. This leads to the problem of selective maintenance optimization. This thesis focuses on the selective maintenance problem and explores maintenance actions and their effects on selective maintenance decisions.

1.4 Selective maintenance

The decision about maintenance actions on a system depends on the system requirements after maintenance and the resources available for maintenance. A wrong maintenance decision could lead to the extra use of resources or the system being unable to meet its operational requirements. When there are several components in a system and each component has several maintenance options available at the time of maintenance, it is even more critical to decide what components to select and what maintenance to perform on those components.

In many industrial environments, a system is required to perform a sequence of operations (or missions) with a finite break between two successive missions. These breaks provide an opportunity to perform maintenance on the component(s) of the system. However, it may be impossible to perform all desirable maintenance activities before the next mission has to start due to limited resources such as time, cost, and repairman availability. In such a case, a subset of maintenance activities is chosen so that the subsequent mission is successfully completed despite limited resources. This maintenance policy is called Selective Maintenance [5, 34]. Such maintenance may be required for manufacturing systems, vehicles, military equipment, power generating units, and coal transportation systems, etc. In these applications, during a break between two successive missions, a decision is made about allocating available resources.

A manufacturing system may work for a week or two and maintenance can be performed on a weekend. For instance, the manufacturing process of a connecting rod in an automobile engine workshop may consist of several NC (numerically controlled) machines in the machine line performing different operations [35]. At a certain maintenance time point, e.g., Sunday, some maintenance actions can be performed during the maintenance break, e.g., minimal repair, PM actions (PM of the drive system, lubrication system, etc.) or machine overhaul. After maintenance, the system is expected to work satisfactorily during the next mission until the next scheduled maintenance break. Similarly, a coal transportation system consists of several components such as feeders and conveyors. The system may need to work for weeks before a break. During this break, it is important to maintain the system so that it can perform reliably during the next mission [13]. In each of the above cases, a subset of components and feasible maintenance actions are to be selected to meet the system requirements during the next mission. Since each of the available maintenance options consumes some maintenance resources (e.g., time and cost), resources should be optimally allocated as well.

Maintenance of one component in a system may influence the performance of the whole system. Therefore, all components should be accounted for simultaneously and their maintenance should be prioritized while performing selective maintenance. To achieve this, it is necessary to determine a component's degradation and the effect of maintenance decisions on its health. Based on the effect of maintenance on the components' health, it is possible to determine what set of components and what level of maintenance will ensure the desirable post-maintenance reliability for the system.

A selective maintenance policy defines the selected components and maintenance actions to be performed on these selected components in a system. Proper consideration of the system states, the failure process, different failure modes and their relationship, is necessary. Decision-making becomes more complicated due to such considerations. If a system and its components are defined to exist in binary states, then depending on the number of working/-failed components and their arrangement, the system is determined to be in either a working or a failed state. In either case, the number of working components selected for imperfect maintenance or replacement and the number of failed components selected for minimal repair or imperfect repair, or replacement, are determined. A system is made up of many components. In general, improving the health of some components is likely to cause bigger improvement in the overall system reliability than improving the health of other components. It is therefore important to find a way to measure the effect of resource allocation on a component's health improvement and how does it affect the system reliability. This issue will be explained thoroughly in Chapter 2.

Failure modes in a system can be classified as maintainable and non-maintainable failure modes as explained in Section 1.3.2. The presence of these two types of failure modes and their interaction may affect the system reliability and also the maintenance decision, which is aimed at improving the system reliability after maintenance. A selective maintenance decision, where all of the components in the system and their degradation influence the resource allocation, calls for incorporating the effect of the two types of failure modes and their interaction. This issue is elaborately discussed in Chapter 3.

For a selective maintenance decision, the planning horizon should be defined first. A maintenance planning horizon can be assumed to be an infinite planning horizon or a finite planning horizon. The majority of the works in maintenance scheduling assume an infinite horizon and consider a single component system [36, 37, 38, 39]. This assumption facilitates the mathematical analysis; it is often possible to derive an analytical solution for such problems.

As the number of components in a system increases, different maintenance actions on the components and their combined effect on the overall system make the maintenance problem hard to solve over an infinite horizon. Further, when a maintenance decision is required under limited resources for a multi-component system (that is selective maintenance), the infinite horizon problem becomes even harder.

The finite planning horizon problem is usually solved for systems having demand information available only for a given time in the future. Examples of such systems include power generation system [40], transportation and material handling systems [41], or manufacturing system [35], etc. When demand information is available only for a given time, maintenance decisions are focused on the time period. A connecting rod manufacturing unit may need to produce 50,000 units within the next year, and a coal transportation system may have to carry a given amount of coal per hour to the boiler for power generation in a winter season. Based on the updated information at the end of the current planning horizon, maintenance decision can be made for the next planning horizon [15, 42, 43, 44, 45]. If limited resources are available to perform maintenance on such a system, selective maintenance is needed to find the suitable maintenance decision. Therefore, it is important to lay down the maintenance plan for a multi-component system in a finite planning horizon . In this thesis, we focus on the selective maintenance problem in a finite planning horizon only. From now on, unless mentioned specifically, a planning horizon refers to a finite planning horizon.

When a finite planning horizon consists of only one mission, the decision about maintenance is required only once at the beginning of the planning horizon. However, many times, it is necessary to divide a planning horizon into multiple missions because performing maintenance only once is not good enough to meet the reliability requirement for the entire planning horizon. In these cases, the decision–making becomes twofold. Firstly, the number of maintenance breaks within the given planning horizon (scheduling) is decided; and secondly the maintenance decisions during the maintenance breaks are determined such that the minimum reliability requirement is achieved during every mission.

In a finite planning horizon, the number of maintenance breaks can be chosen at equal or unequal intervals. When the system is maintained at integer multiples of some fixed interval like every two weeks, every month or every year, it is called periodic maintenance [15, 16, 19, 42, 46]; otherwise it is called non-periodic (or sequential) maintenance. The sequential maintenance is mostly performed on a single component system in an infinite horizon, based on the assumption that as the system ages it needs more frequent maintenance [36, 37, 39]. In the selective maintenance of a multi-component system, simultaneous consideration of all components is required. Considering one component at a time is not beneficial in selective maintenance. Therefore, to solve the finite horizon problem, we limit ourselves to periodic scheduling only.

Scheduling maintenance activities plays a very important role in a system's successful, economical, and reliable operation. It represents a major task in medium– and long–range planning [47]. Detailed reviews about maintenance scheduling can be found in [15, 42, 45]. Too frequent maintenance breaks will increase the maintenance budget, whereas too few maintenance breaks will increase the number of faults and outages. Hence, when to perform maintenance and the maintenance decisions during each of the breaks in a planning horizon are key variables in maintenance scheduling decisions (Fig.1.8). Such decisions should ensure that the total cost incurred during the planning horizon is the minimum.

Few studies investigate the selective maintenance scheduling of a multi-component system. Even fewer studies have included the imperfect maintenance model in solving the selective maintenance scheduling problem [41]. No study has investigated the effect of a maintenance budget and the system's age on the selective maintenance schedule. In this thesis, our focus is on the finite horizon selective maintenance scheduling problem for multi-component systems under imperfect maintenance. To establish a maintenance plan under limited resources, it is important to determine the maintenance priority and a schedule for the components. This issue is addressed in detail in Chapter 4.

If a system and its components exist in multiple states, then the state of

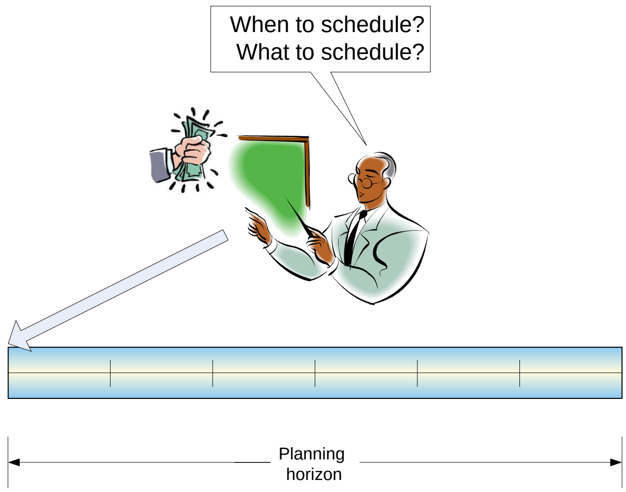

Figure 1.8: Selective maintenance scheduling

the system at any point in time depends on the combination of its components' states. Along with this dependence, the failure process from a higher to lower state for each component in the system and the effect of maintenance on a component's state are important in determining the selective maintenance policy for an MSS. Depending on the state of the components after maintenance, system state after maintenance and system reliability during the next mission are determined. Further discussion about the selective maintenance of an MSS is provided in Chapter 5.

System reliability can be improved by adopting some appropriate maintenance policies. To clearly define the maintenance policy, it is necessary to consider the system's configuration, failure process, state, and the effect of possible maintenance actions. This research will address the selective maintenance problem for a system with a focus on each of the above aspects. The detailed research scope and objectives of this thesis are described next.

1.5 Research scope and objectives

The objectives of this thesis are to derive selective maintenance models for systems under imperfect maintenance, taking into consideration the effects of the failure modes, maintenance scheduling in a finite planning horizon, and system and components states in the decision making. We aim to investigate the factors influencing the selective maintenance decision and develop models that reflect the effect of maintenance decisions on system reliability. To address the problem of selective maintenance for systems under imperfect maintenance with different configurations, failure processes, finite planning horizon, and states, we have divided this research work into four stages as shown in Fig.1.9.

In the first stage, a single mission selective maintenance problem is solved for binary systems under imperfect maintenance. The model is focussed on a binary system with binary components only. There aren't enough studies that reflect the effect of aging and maintenance budget on a system's health. We have developed a model, which considers the age of a system and the maintenance budget used, to determine the improvement factors associated with imperfect maintenance. Usually, the more is the used maintenance budget, the better improvement in the health of the system is. Changes in the improvement factors due to maintenance are thus related to the maintenance budget used. A system's response to maintenance may also change with its age, in the sense that a new system responds to maintenance actions better than a relatively older system [13, 48]. If a system is old, it is more expensive to achieve some improvement in its health as compared to a relatively new component.

The aim at this stage is to develop a model to incorporate the effect of a system's age and the maintenance budget on its overall health after maintenance. This model is incorporated in the selective maintenance decision–making of a binary system when imperfect maintenance is possible on the system. In this first stage, we have assumed that all failure modes are maintainable and a maintenance decision is required for the next single mission only. This is described in Chapter 2.

We have used the model developed in the first stage as the foundation and improved it by introducing more realistic scenarios, namely the two types of failure modes (as discussed in Section 1.3.2), the selective maintenance scheduling (as discussed in Section 1.4) and the MSSs (as explained in Section 1.1), respectively. Aforementioned aspects of selective maintenance modelling are studied individually, at later stages.

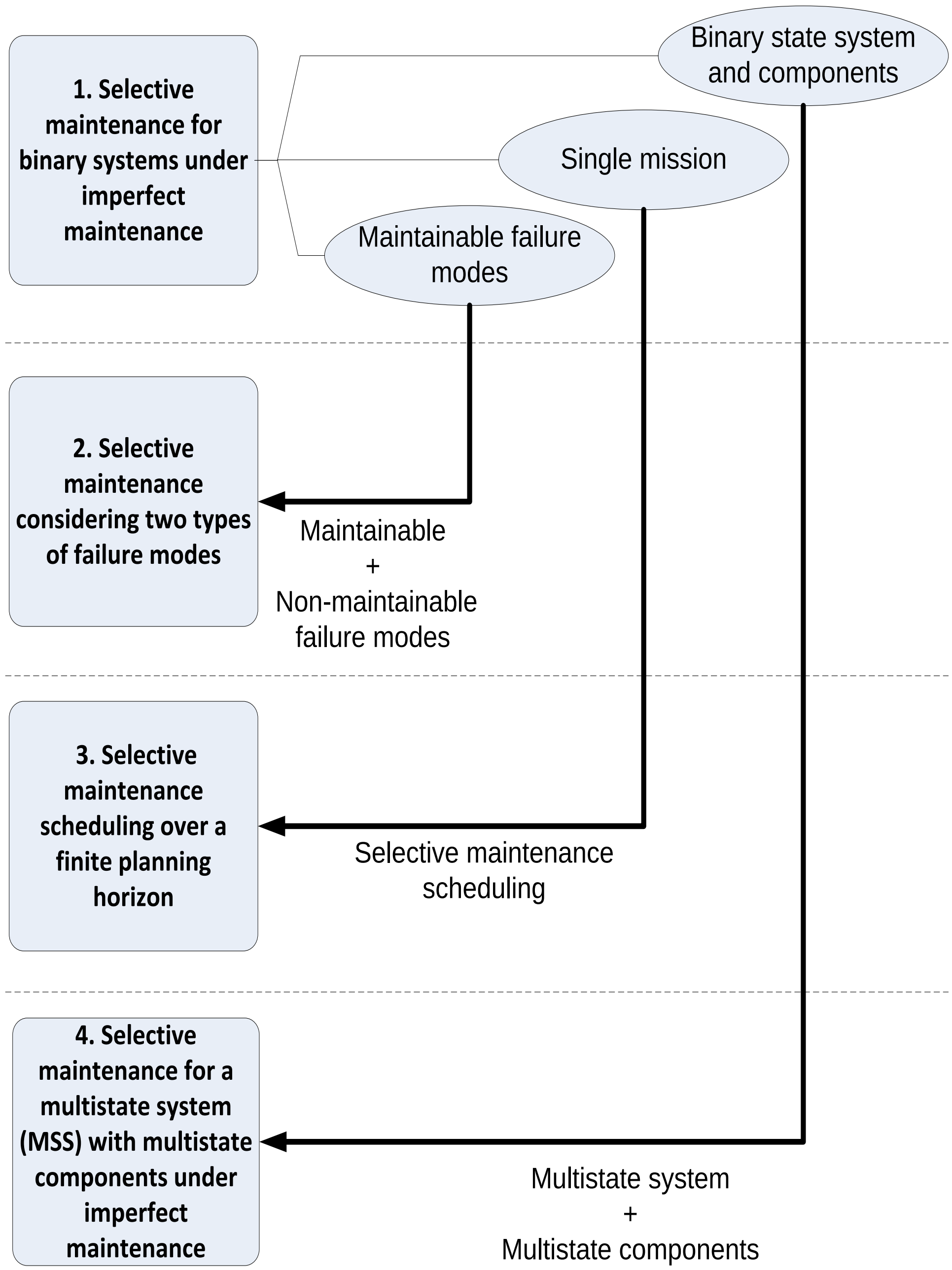

Figure 1.9: Thesis outline

In the second stage, we have solved a single mission selective maintenance problem for a binary system when two types of failure modes are present. We have used the first stage model; however, we have considered the presence of the maintainable and non-maintainable failure modes and defined their relationship. As shown in Fig.1.9, the second stage studies the relationship between the two types of failure modes and how their presence affects the selective maintenance decision.

If maintainable and non-maintainable failure modes co-exist in a system, it is possible to establish a relationship between the hazard rate due to the maintainable failure modes and the hazard rate due to the non-maintainable failure modes [49]. The purpose of establishing such a relationship is to consider the effect of the system degradation due to the non-maintainable failures on the system degradation due to the maintainable failures. As the system becomes older, the non-maintainable component's condition deteriorates, which may affect the hazard rate of a coupled maintainable component. Therefore, when a selective maintenance decision is made for a system in the presence of the two types of failure modes, their effect on the system reliability and the maintenance decision should be given due consideration. Chapter 3 contains a detailed explanation about the relationship defined between the two types of failure modes and selective maintenance with the two types of failure modes.

In the third stage of the research, the selective maintenance scheduling problem for a finite planning horizon is solved. We have used the first stage model and developed it for the finite planning horizon scheduling problem. In this process, some additional formulations are provided to incorporate the effect of imperfect maintenance over the successive missions. The optimum number of maintenance breaks and the maintenance decisions during each break is determined. As given in Fig.1.9, we have taken the binary system model used in the first stage and developed it to solve the selective maintenance scheduling problem. A detailed description about this stage is provided in Chapter 4.

As explained in Section 1.1, some systems may perform with more than two levels of efficiency. This kind of system is an MSS. The conventional binary model cannot be applied for the selective maintenance of an MSS. It is required to consider the relationship between the states of the components and the systems status. In the fourth stage (Fig.1.9), a complete set of formulations are provided for the modeling of a single mission selective maintenance of an MSS with multi-state components. A demonstration is provided as to how the components performance states affect the system states and how to solve the problem for selective maintenance of an MSS with multi-state components. In this final stage of the research, a step by step model is proposed to perform the selective maintenance decision–making for an MSS. Details about the selective maintenance modelling for an MSS are given in Chapter 5.

The definitions and models developed in this thesis– that is, (i) the relationship of the improvement factors with the age and maintenance budget, (ii) the relationship between the two types of failure modes, (iii) improvement factors for successive missions, (iv) modeling maintenance of an MSS with multi-state components– have the potential to be used not only in the selective maintenance domain but also can be utilized and extended in the other domains of reliability and maintenance as well, such as the joint redundancy (adding redundant component(s) to improve system reliability) and imperfect maintenance strategy, warranty and maintenance decisions. The background and implementation of the aforementioned stages will be elaborated upon in the upcoming chapters.

1.6 Thesis outline

The guidelines from the Faculty of Graduate Studies and Research (FGSR) at the University of Alberta have been followed to prepare this paper based thesis. This thesis consists of six chapters.

After the introduction in Chapter 1, Chapter 2 explains the selective maintenance modelling for binary systems with binary components. It represents the stage one of this research as given in Fig.1.9. A single mission problem is solved with the maintainable failure modes in the system. The effect of imperfect maintenance is modeled using the age-cost-based improvement factors.

The major contributions of this chapter have been published in the journal *Reliability Engineering and System Safety* [5], the conference proceedings of the *International Conference on Quality, Reliability, Risk, Maintenance, and Safety Engineering* [50], and published as a book chapter in *Reliability Modeling with Applications (Essays in Honor of Professor Toshio Nakagawa on His 70^{th} Birthday)* [51].

Chapter 3 uses the model derived in Chapter 2 and introduces a relationship between maintainable and non-maintainable failure modes in the system. The presence of these two failure modes and their effect on the selective maintenance decision is studied in this chapter as shown in stage two of Fig.1.9. Versions of this chapter have been accepted for publication in the *International Journal of Strategic Engineering Asset Management* [52] and published in the conference proceedings 19^{th} *ISSAT Conference on Reliability and Quality in Design* [53].

Chapter 4 describes the selective maintenance scheduling problem along with the decision-making during successive maintenance breaks in a multi-mission finite planning horizon. In this chapter, selective maintenance is performed during consecutive maintenance breaks such that a minimum desired reliability level is maintained during all missions. The imperfect maintenance model used in Chapter 2 is developed to address the scheduling and multiple mission maintenance decision-making problem (Fig.1.9). The model is improved in Chapter 4 to incorporate the effect of successive imperfect maintenance on a component. The number of missions and maintenance decisions for each mission is determined. Results of this chapter have been submitted for publication in the *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability* [54] and published in the conference proceedings of the *Reliability and Maintainability Symposium 2013* [27].

Chapter 5 solves the selective maintenance problem for an MSS with multi-state components. The idea developed in Chapter 2, about the selective maintenance modeling of the binary system, is extended to formulate the model for an MSS with multi-state components. This model explains, in steps, how to determine the effect of maintenance on the components' and the system's performances in a multi-state condition. This is the fourth and final stage of the research as given in Fig.1.9. Versions of this chapter have been published in the journal *IIE Transactions* [11] and published as a book chapter in *Reliability Modeling with Applications (Essays in Honor of Professor Toshio Nakagawa on His* 70^{th} *Birthday)*[51].

Chapter 6 draws the conclusion with important observations and introduces the possible directions for future work based on the outcomes of the research.

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Chapter 2

Selective Maintenance for Binary Systems under Imperfect Maintenance

This chapter is devoted to a single mission selective maintenance modeling of binary systems under imperfect maintenance. As explained in Section 1.3.1, several maintenance options are available for a system when it arrives at a maintenance depot after a mission. These options include minimal repair, replacement or imperfect maintenance of components within a system. In this first stage of this research, a relationship is developed among the maintenance budget used, the age of a component, and the change in its hazard rate due to maintenance. After developing this relationship, a maintenance model is used to perform selective maintenance of a binary system with multiple binary components. All failure modes in a component in the system are assumed to be maintainable, as defined in Section 1.3.2. Therefore, in this chapter, the hazard rate of the components in the system represents the maintainable hazard rate only.

An introduction to selective maintenance of binary systems is provided in Section 2.1. Section 2.2 explains the maintenance cost and time associated with the maintenance options. Section 2.3 introduces the relationship among the hazard rate improvement, cost used during maintenance and the component's age. In this section an imperfect maintenance model is also explained. Section 2.4 presents the system reliability estimation and selective maintenance modelling followed by the solution methodology in Section 2.5. Results and discussion are given in Section 2.6. A summary is provided in Section 2.7. Preliminary work related to this chapter has been published in the conference proceedings [1]. The fully developed model and results have been published as a journal paper [2]. An extended discussion about this work has been published as a book chapter [3].[1] This chapter is mostly based on the journal paper [2].

2.1 Introduction

An engineering system is required to operate satisfactorily throughout its service, to meet required demand. Maintenance can be defined as all activities necessary to keep a system in working order. Such activities may include inspection, lubrication, adjustment, repair, and replacement. As discussed in Section 1.4, a system may have to perform a sequence of operations (or missions) with a break between any two successive missions. These breaks provide an opportunity to perform maintenance on the system. However, it may not be feasible to perform all possible maintenance activities during the break due to limited maintenance resources such as time, budget, and repairman availability. Thus, a selective maintenance policy is adopted in which a subset of maintenance actions is chosen so that the subsequent mission is successfully completed.

Many types of equipment or systems perform a sequence of missions such that breaks between the missions offer the best opportunity to perform maintenance. Such systems may include manufacturing equipments, military vehicles, and power generation units. Manufacturing equipment may work during the week and be maintained during weekends; similarly, power generation units

[1] Versions of this chapter have been published in "M.Pandey, M.J. Zuo, R. Moghaddass, and M.K. Tiwari., Selective Maintenance for Binary Systems under Imperfect Repair. *Reliability Engineering and System Safety*, 113(1):42–51, 2013," "M. Pandey, M.J. Zuo, and R. Moghaddass, Selective Maintenance for Binary Systems Using Age-Based Imperfect Repair Model, *Proceedings of 2012 International Conference on QR2MSE*, pages 385-389," and "M. Pandey, Y. Liu and M.J. Zuo, Book Chapter, *Selective Maintenance for Complex Systems Considering Imperfect Maintenance Efficiency*, pages 17-49. World Scientific (Singapore). DOI:10.1142/9789814571944_0002."

may work for the whole week and maintenance can be performed early Sunday morning. Military equipment may be maintained between operations. In the above cases, a subset of feasible maintenance actions is required to be selected to meet the system requirement during the next mission. For instance, an automobile engine workshop that manufactures connecting rods may consist of several NC (numerically controlled) machines in the machine line performing different operations [4]. At the end of an operating week, each machine may be either working or failed. After inspection, there could be some possible maintenance actions to be performed during the weekend, e.g., no repair, minimal repair, different preventive maintenance actions or machine overhaul. After maintenance, the system should work with maximum reliability during the next week till the next scheduled maintenance break. Each of the maintenance options consumes some maintenance resources; therefore, optimal allocation of the resources, such as cost and time, is needed.

The selective maintenance problem was introduced by Rice et al. [5]. Rice looked at a system with a series-parallel configuration, constant component failure rates (the exponential distribution), and only one type of maintenance action (replacement of failed component). The model presented in [5] was extended by Cassady et al. [6] in the sense that cost was included as an additional resource constraint. By selecting as an objective either reliability, cost or time, and the remaining two as constraints, three different selective maintenance models were developed in [6]. Further, Cassady et al. [7] included age as a factor in reliability determination, and assumed a case where components' lifetimes follow the Weibull distribution. They proposed that maintenance action on a failed/working component could be minimal repair of the failed component or replacement of the failed component or replacement of the functioning component (preventive maintenance). Their study was limited to time as the only resource constraint. Later, Schneider and Cassady [8] considered multiple systems simultaneously and termed those a fleet. They adapted the model used in [5] and solved the selective maintenance problem for a fleet (consisting of multiple systems together) performance.

The enumeration methods were presented in Rajagopalan and Cassady [9] to improve the selective maintenance optimization. This work was aimed at reducing the CPU time for optimizing the selective maintenance. This approach was based on the assumption that all components in a subsystem were similar and only replacement was possible for a failed component. However, when components in a subsystem are non-identical, the number of maintenance options increases, or the time required for maintenance varies from one component to another, the heuristic becomes inefficient. It was found in Lust et al. [10] that for a system with a large number of components, the enumeration method was no longer useful as the problem became combinatorial in nature. They proposed a heuristic to generate an initial solution and used it as input to the branch and bound procedure and Tabu search. They found that the Tabu search provided an optimal or close to the optimal solution quickly as compared to the branch and bound method. In this work, for the first time, the Tabu search was used to solve the selective maintenance problem, and it was found to be useful. Some other works on selective maintenance include Iyoob et al. [11] and Maillart et al. [12]. Iyoob et al. [11] focused on the resource allocation for subsequent missions under selective maintenance, whereas Maillart et al. [12] considered selective maintenance for a series-parallel arrangement. Maillart et al. assumed that all components within a subsystem were identical, and their lifetime followed the exponential distribution.

In most of the works, either time or cost was considered as the available resource. However, usually maintenance personnel are limited both in time and cost. All the above works were focused on replacement and/or minimal repair of components only. However, the system can be maintained somewhere between as good as new and as bad as old, which is called imperfect maintenance. Imperfect maintenance was considered for components in a system by Liu and Huang [13]. They assumed that the system's age is affected by a maintenance action. However, the hazard rate of a system can also change due to maintenance [14]; hence it is more realistic to assume both the age reduction and the hazard adjustment (hybrid model) for imperfect maintenance [15]. Based on the above shortcomings, a hybrid imperfect maintenance model is considered in this thesis to solve the selective maintenance problem for binary systems. In Lin et al. [15], only a fixed imperfect maintenance option is available on a component during all maintenance breaks until it reaches the end of its useful life. Replacement is performed only at the end of this useful life. Lin et al. did not consider a component's age or maintenance budget while determining the age reduction or the hazard adjustment factors for the imperfect maintenance model. In this chapter, we focus on a single mission maintenance decision problem for a multi-component system. We determine not only the components to be selected during the given maintenance break but also the maintenance actions to be performed on the selected components. The term selective maintenance means that we do not have a single fixed maintenance action for a component during a maintenance break; rather, we need to choose any of the available maintenance alternatives which are minimal repair, replacement, and several imperfect maintenance options for a selected component.

In the imperfect maintenance optimization literature [14, 15], imperfect maintenance refers to any fixed maintenance action performed on a component which improves its condition to somewhere between between bad as old (minimal repair) and good as new (replacement). In the proposed work, we select any of the available maintenance actions to be performed on the components. These actions include do-nothing, minimal repair, replacement, or a certain level of imperfect maintenance. The selected maintenance action for a component depends on the maintenance objective, which is to maximize system reliability. To consider the effect of a component's age, a new formulation for the characteristic constant is proposed to determine whether a component is relatively young or old. This characteristic constant is then used in the formulation of the imperfect maintenance improvement factors. We have also developed an equation which relates the age and the maintenance budget to the imperfect maintenance factors. Thus, new cost-age-based age reduction and hazard adjustment factors are defined in this chapter. Sometimes, a maintenance manager is flexible in terms of time but constrained by budget, or vice versa. Hence, the effect of the variation of resources on the selective maintenance planning is also studied. To solve the above problem, the following assumptions are considered in this chapter:

1. The system consists of multiple, repairable components.

2. The components as well the system are in the binary state, i.e., the system is either working or failed.

3. After replacement, the component is as good as new (AGAN) and if minimal repair is performed it is as bad as old (ABAO). Maintenance is also possible such that the component health may lie between as good as new and as bad as old; i.e., maintenance can be modeled by imperfect repair.

4. Limited resources (budget and time) are available and the amount of resources required for maintenance activities are known and fixed.

2.2 Maintenance cost and time

In this chapter, a series-parallel system is considered where $s\,(\gamma = 1, 2, ..., s)$ independent subsystems are connected in a series. Each subsystem γ has n_γ $(\gamma' = 1, 2, ..., n_\gamma)$ components connected in parallel. A component in the system is denoted by i and there is a total of $n\,(i = 1, 2, ..., n)$ components in the system. Each component, subsystem, and system can be in one of two possible states: working or failed. During a maintenance break, different maintenance actions are possible for a component. It is reasonable to assume that a component's health can be improved by taking some maintenance actions during the maintenance interval, (e.g., oiling/cleaning, repairing/replacing some parts of a component or replacing the whole component). Corresponding to the available maintenance options, some discrete levels of maintenance (l_i) can be assigned to a component i. With all these maintenance options, let's assume that N_i maintenance levels, $l_i \in \{1, 2, 3, ..., N_i\}$, are available for component i. Here, $l_i = 1$ denotes the "do nothing" case when no maintenance is performed on component i, and l_i=N_i shows a replacement of component i. For each component i in the system, the available maintenance options (N_i) may be different. Related to these alternatives, cost and time estimation is provided next.

2.2.1 Maintenance cost

Depending on the system reliability requirement, a component may or may not be selected for maintenance. When it is not selected ($l_i = 1$), the corresponding maintenance cost is zero. However, if component i is selected for maintenance ($l_i > 1$), it consumes some of the maintenance budget. The expression for maintenance cost for component i can be given as:

$$C_i\left(l_i\right) = c_{i,l_i}^{fix} + c_{i,l_i}, \tag{2.1}$$

where c_{i,l_i}^{fix} is the fixed cost and c_{i,l_i} is the variable cost of maintenance for component i. The values of these costs depend on the level of maintenance l_i. For $l_i = 1$, $c_{i,l_i}^{fix} = 0$, $c_{i,l_i} = 0$, and for l_i=N_i, $c_{i,l_i} = C_i^R$, where C_i^R is the replacement cost of component i. Fixed cost is incurred when a component is selected for maintenance, no matter what is the level of maintenance. This cost is related to the cleaning, dusting, assembling, set-up, etc. If component i is in the working state at the time of maintenance, then $2 \le l_i < N_i$ denotes intermediate maintenance actions. An intermediate maintenance action for a working component is defined as an action between the no maintenance option ($l_i = 1$), and the replacement option ($l_i = N_i$). Each of the discrete intermediate maintenance levels has an associated cost $\left(c_{i,l_i}\right)$. If component i is in the failed state at the time of maintenance, $l_i = 2$ denotes minimal repair of the component, that is, $c_{i,l_i} = C_i^{MR}$, where C_i^{MR} is the cost of minimal repair for a failed component i. In this case, $3 \le l_i < N_i$ denotes intermediate repair actions. An intermediate repair action for a failed component is defined as the repair action between the minimal repair option ($l_i = 2$) and the replacement option ($l_i = N_i$). These options may be regarded as the improvement once minimal repair of the failed component is done; that is, the failed component is put into the working order first by a minimal repair, and then an additional maintenance action is performed to further improve the component health condition. With the help of the decision variable l_i, the cost and time incurred in maintenance of any component i in the system can be estimated. Thus the total maintenance cost for the whole system can be determined as:

$$C = \sum_{i=1}^{n} C_i \left(l_i \right). \tag{2.2}$$

In equation (2.2), only selected components cost will be added to which $l_i > 1$ because when $l_i = 1$, $C_i \left(l_i \right) = 0$.

2.2.2 Maintenance time

Similar to the maintenance cost, time to perform maintenance ($T_i \left(l_i \right)$) on a component (i) can be estimated as follows:

$$T_i \left(l_i \right) = t_{i,l_i}^{fix} + t_{i,l_i}, \tag{2.3}$$

where t_{i,l_i}^{fix} is the fixed time needed if component i is selected for maintenance. Time t_{i,l_i} is the variable time associated with maintenance of component i, which depends on the maintenance option $l_i \in \{1, 2, 3, ..., N_i\}$ selected for the component. If $l_i = 1$, we get $T_i \left(l_i \right) = 0$, and for l_i=N_i, $t_{i,l_i} = T_i^R$, where T_i^R is the time to replace component i. If a working component (i) is selected for intermediate maintenance ($2 \leq l_i < N_i$), then there is a maintenance time (t_{i,l_i}) associated with each of the options. If a failed component (i) is selected, then $t_{i,l_i} = T_i^{MR}$ for $l_i = 2$, where T_i^{MR} is the time to perform minimal repair of the failed component (i). For $3 \leq l_i < N_i$, intermediate repair actions are done on the failed component. Hence, for a decision variable l_i, related maintenance time for a component (i) can be estimated and the total maintenance time for the whole system can be determined as:

$$T = \sum_{i=1}^{n} T_i \left(l_i \right). \tag{2.4}$$

It is evident from equations (2.2) and (2.4) that for a particular decision variable for the maintenance level l_i, the corresponding cost and time involved in system maintenance can be determined. Here selective maintenance is required to be performed under a limited budget and time. When some cost is used for maintenance of a component, its health is likely to improve. However, to determine the improvement, it is important to find the maintenance effect on a component's age and its hazard rate. In the next section, an imperfect maintenance/repair model is provided to represent the changes in the effective age and hazard rate of a component due to maintenance. The factors influencing the model will also be discussed in detail.

2.3 Imperfect maintenance/repair model and maintenance options

2.3.1 Imperfect maintenance/repair model

Two preventive maintenance models were proposed by Nakagawa [14], where adjustment/improvement factors were considered in the hazard rate and effective age for a preventive maintenance (PM) policy. The time elapsed since a system was first operational, is called the calender age; and the time for which the system is in use is called the actual usage age. However, the age of a system that has to be accounted for in the assessment of its likelihood to fail is not the calendar age or usage age, but a fictitious time (effective age) accounting for the effect of maintenance undergone by this system [16]. Since the calender age and actual usage age are always increasing for a system, the effective age phenomenon is used to assess a system's health.

If maintenance actions have been correctly performed, the effective age is usually less than the calendar age and actual usage age. It reflects the effect of the aging of a system with time and the rejuvenation after the different maintenance interventions made on the system. For instance, assume that a new system is put into operation today and there is a scheduled maintenance intervention after two months. At the time of maintenance, the calender age of the system would be two months, but after maintenance, it may no longer perform as a two-month-old system. It is likely that due to maintenance it will perform better, for example, like a similar one-month-old system. Thus, after maintenance, even though the system will have a calender age of two months, it is said to have an effective age of one month only.

After maintenance, the useful life of a system may increase and its condition may improve. The effective age may indicate the effect of different maintenance actions on the system's age. Since the effective age can reflect a system's current (fictitious) age, the hazard rate given in equation (1.1) in Section 1.2 can be denoted as a function of the effective age to reflect the degradation behavior of the system and effect of maintenance. Thus, maintenance of a system can be characterized by the change in its effective age and the hazard rate. A higher value of hazard rate indicates that the system has a higher probability of failure in the next time unit as compared to a lower hazard rate. When performing selective maintenance, the consequence of a decision on a system's effective age and hazard rate is a deciding factor in what components should be selected and what level of maintenance to perform, to maximize the system's reliability after maintenance [2, 13]. At the same time, care should be taken to ensure that these maintenance actions are performed using available resources.

The first PM model proposed in Nakagawa [14] is a hazard rate adjustment model. In this model, the hazard rate in the next PM interval becomes $ah(x)$ where $h(x)$ is the hazard rate in the previous interval. The adjustment factor is $a \geq 1$ and $x \geq 0$ represents the time elapsed from the previous PM time. The second model is the age reduction model; according to which, if the effective age of a component is t right before the PM, it reduces to bt right after PM, where $0 \leq b \leq 1$ is the improvement factor for the effective age. The hazard rate adjustment model assumes that the hazard rate right after a PM reduces to zero and increases more quickly as compared to the previous interval before PM. The age reduction model assumes that maintenance reduces the effective age, and right after maintenance the effective age may be greater than zero. The hazard rate remains a function of the effective age. In a more general case, maintenance may not only reduce the effective age but also increase the hazard rate [15]. If the hazard rate function at time $t \in \{0, t_1\}$ is $h_0(t)$, PM at a maintenance break $[t_1, t_2]$ will change the hazard rate to $h_1(t)$ for $t \in \{t_2, t_3\}$. If the effective age of a component (i) before maintenance is B_i then the combined hybrid model, which includes the effect of the hazard adjustment and age reduction, can be written as:

$$h_1(t_2 + x) = ah_0(b \times B_i + x), \tag{2.5}$$

where, $a \geq 1$ and $0 \leq b \leq 1$, and $x \in \{0, t_3 - t_2\}$. When $a = 1$, the above model is the same as that for the age reduction model and for $b = 0$, it is the same as the hazard adjustment model. For selective maintenance, depending on the different maintenance alternatives (l_i), different values of improvement factors are obtained. This is explained in the next section.

2.3.2 Maintenance alternatives

Whenever a system comes in after a mission, a maintenance decision is to be made for each component. The component can be in either working or failed condition after a mission. Depending on the next mission requirement, the following maintenance/repair options are possible for working/failed components:

Maintenance Option#1: Do nothing ($l_i = 1$)

Maintenance Option#2: Perform imperfect maintenance/repair ($l_i > 1$)

Further, if option#2 is selected, the decision is required for age reduction as well as hazard rate adjustment for the next mission (see Fig.2.1). During the maintenance interval $[t_1, t_2]$, maintenance action performed on a component may change its effective age at the beginning of the next mission, as well as the slope of the hazard rate during the next mission.

This decision-making will be done for each component in the system such that available cost and time are used optimally during the maintenance break and, simultaneously, system reliability for the next mission is maximized. However, for the imperfect repair model, we need to determine the improvement factors as given next:

2.3.3 Age reduction factor

In general, there is a correlation between maintenance quality and the portion of the budget allotted to maintenance. As reported in Lie and Chun [17], the maintenance cost used and the age of the component are two important factors

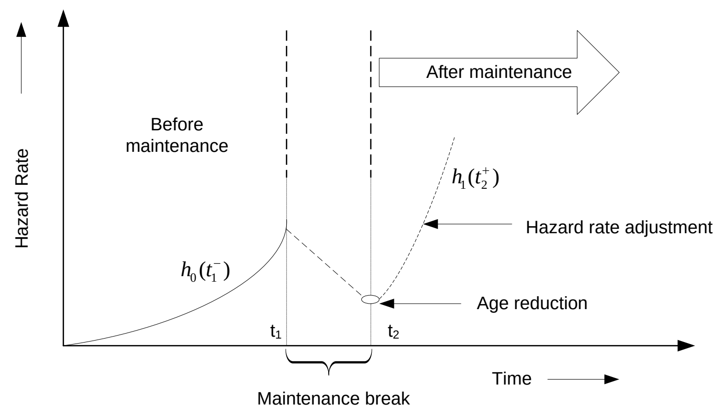

Figure 2.1: Hybrid imperfect preventive maintenance model

for determining a component's age reduction factor (b). Maintenance cost is related to the improvement done (reduction achieved after maintenance) in a component's effective age [13, 17]. A relationship is provided in [13, 17] for the age-reduction factor:

$$b\left(l_i\right) = 1 - \left(\frac{C_i\left(l_i\right)}{C_i^R}\right)^m, \tag{2.6}$$

where $m \geq 0$ is a characteristic constant that determines the exact relationship between maintenance cost and age reduction, $C_i\left(l_i\right)$ is the PM cost of the component which depends on the level of maintenance l_i, and C_i^R is the replacement cost of the component i. However, the minimal repair cost is not considered in the maintenance cost in formulation (2.6). If the minimal repair cost is included in the maintenance cost, it may give a smaller age reduction value than actually experienced. Since minimal repair does not contribute to the age reduction and only brings a failed component back to the as bad as old (ABAO) condition, its cost should not influence the determination of the age reduction factor either. Therefore, the above formulation is redefined in this chapter. Let Y_i be the state of a component i before maintenance, and $Y_i = 0$ denotes that the component is in the failed state and $Y_i = 1$ denotes that the component is in the working state. Then, the age reduction factor is redefined as follows:

$$b\left(l_i\right)=\begin{cases}1-\left(\frac{C_i\left(l_i\right)-C_i^{MR}}{C_i^R}\right)^m, & \text{for } Y_i=0, \quad 2\leq l_i<N_i,\\ 1-\left(\frac{C_i\left(l_i\right)}{C_i^R}\right)^m, & otherwise.\end{cases} \tag{2.7}$$

In equation (2.7), minimal repair cost does not influence the age reduction factor in the case of $Y_i = 0$. Here, for the options $2 \leq l_i < N_i$, minimal repair is included in the maintenance model without influencing the determination of the age reduction factor. If the minimal repair option $l_i = 2$, is selected as a maintenance action for a failed component, then $C_i\left(l_i\right) - C_i^{MR} = 0$, hence $b\left(l_i\right) = 1$. This shows that there is no change in the age of the component when minimal repair is performed. If any of the imperfect repair options, $3 \leq l_i < N_i$ are selected for a failed component, then out of the total maintenance cost, minimal repair cost is used to bring back the component to the ABAO condition and minimal repair does not contribute to any reduction in the component's effective age. The additional cost incurred in the maintenance option $C_i\left(l_i\right) - C_i^{MR}$, will determine the level of age reduction $b\left(l_i\right)$ for a component i.

2.3.4 Characteristic constant

As discussed in Lie and Chun [17] and Liu and Huang [13], a smaller value of m is related to a younger component, whereas, the m value increases as the component ages. However, there is no method or formulation available to determine m. In the present chapter, a formulation of m is proposed which reflects that a component is relatively younger or older. When a component ages, its effective age increases and the remaining useful life (RUL) diminishes. Let T_f be the time to failure of the component and suppose that the component has survived up to the effective age B_i; then the conditional random variable $T_f - B_i$ (defined when $T_f > B_i$, that is, the remaining time to failure), denotes the component's remaining useful life. A method to calculate (estimate) the expected residual life, often called the mean residual life (MRL), is given as [18]:

$$\text{MRL} = E\left(T_f - B_i | T_f > B_i\right) = \frac{\int_{B_i}^{\infty} R\left(x\right)dx}{R\left(B_i\right)} \tag{2.8}$$

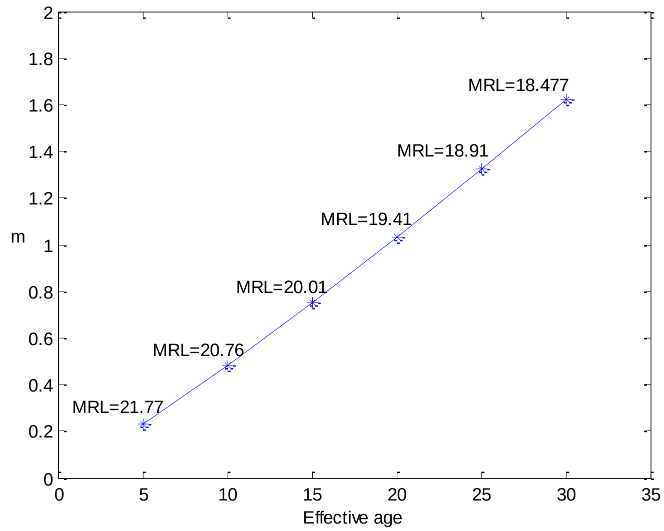

Figure 2.2: Variation of parameter m with component's effective age

This chapter defines the characteristic constant of a component, based on its effective age and the MRL. If a component's effective age is less than its MRL, it is assumed to be relatively younger. However, when the MRL of the component is less than its effective age , it is said that the component is relatively old. A new formulation is proposed in this chapter, according to the above definitions, to calculate the m value for a component as given in equation (2.9). According to this formulation, if a component is relatively young (when $B_i <$ MRL); then $m < 1$. Similarly, for relatively older components (when $B_i >$ MRL); $m > 1$.

$$m\left(B_i\right) = \frac{B_i}{\mathrm{MRL}} = \frac{B_i}{\left(\frac{\int_{B_i}^{\infty} R(x)dx}{R(B_i)}\right)} = \frac{B_i \times R\left(B_i\right)}{\int_{B_i}^{\infty} R\left(x\right)dx} \tag{2.9}$$

For example, assuming that a component follows the Weibull distribution with the scale parameter $\alpha = 25$ and the shape parameter $\beta = 1.2$, m is determined at different effective ages (Fig.2.2). It can be seen in Fig.2.2 that as the effective age of the component increases and the MRL decreases, or in other words as the component becomes older, its m value increases. When $B_i <$ MRL, $m < 1$, while $m > 1$ for $B_i >$ MRL.

With the new definition of $m\left(B_i\right)$, the age reduction factor can be rewritten

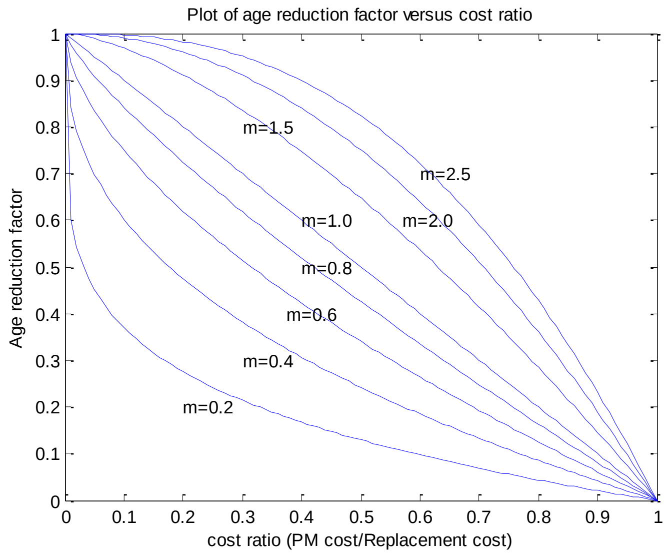

Figure 2.3: Age reduction factor versus cost-ratio for different values of m

as:

$$b\left(B_i, l_i\right) = \begin{cases} 1 - \left(\frac{\left(C_i(l_i) - C_i^{MR}\right)}{C_i^R}\right)^{m(B_i)}, & \text{for } Y_i = 0, \quad 2 \leq l_i < N_i \\ 1 - \left(\frac{C_i(l_i)}{C_i^R}\right)^{m(B_i)}, & otherwise \end{cases} \tag{2.10}$$

It is obvious that as per the new formulation, the age reduction factor depends on the imperfect maintenance/repair level (l_i) as well as the effective age (B_i) of the component. For different values of m, a variation of the age reduction factor $b\left(B_i\right)$ with the cost-ratio is shown in Fig.2.3.

Fig.2.3 depicts that the maximum age reduction is possible when a component is replaced. In the case of replacement, $C_i\left(l_i\right) = C_i^R$ and $b\left(B_i, l_i\right) = 0$. When the cost-ratio is less than 1 – that is the selected maintenance option is other than replacement – the age reduction factor is greater than 0. It is also evident from Fig.2.3 that for a relatively older component (a component with a higher m value), higher maintenance cost (cost-ratio) is required to achieve an age reduction that a relatively younger component can achieve with a smaller maintenance cost (cost-ratio). For example, when a component is young and it has a m value of 0.4, a cost-ratio of about 0.2 is enough to achieve an age reduction of about 0.5. However, when the component becomes older and its m value reaches 1, it needs 0.5 cost-ratio to achieve the same age reduction of

0.5. Our finding is similar to what is discussed in [13, 17].

By defining the characteristic constant as a function of the effective age (B_i) and the age reduction factor as a function of the cost-ratio, the formula (2.10) establishes a way to relate the component's age and maintenance cost with the improvement in its effective age due to maintenance. However, in addition to the effective age, the slope of the hazard rate of a component may also get affected by maintenance. Therefore, the effect of maintenance on the hazard adjustment is also determined in this chapter.

2.3.5 Cost-based hazard adjustment factor

Whenever maintenance is performed on a component, the slope of the hazard rate may also change with the effective age. A higher slope of the hazard rate denotes that the chances of a component failing in the next time unit are higher than when there is a smaller slope. In the literature, it is assumed that the change in the slope of the hazard rate is constant during a particular maintenance break. Constant hazard adjustment factor for a component is considered in [14, 15, 19, 20] and [21]. In addition to the time of a particular maintenance break, the hazard rate after maintenance may also be affected by the used maintenance budget. If the used budget is small, little improvement in component health is expected. The component's hazard rate increment after maintenance will be higher if there were a smaller budget used in maintenance. Thus, the hazard rate adjustment factor after maintenance depends on the amount of resources used. Also, as the component ages, it needs more and more resource to improve its health. There is a need to relate the hazard adjustment factor with the amount of resource used and the effective age of the component.

Following the above discussion, a new hazard adjustment factor is proposed in this chapter for one life cycle of a component. This proposed factor depends on the component's age and the resources used in its maintenance. The characteristic constant m is used in the formulation of the hazard adjustment factor to incorporate the effect of the effective age, while the cost-ratio is used to incorporate the effect of the maintenance budget. The new hazard adjustment factor is defined in equation(2.11).

$$a\left(B_i,l_i\right)=\begin{cases}\dfrac{p}{\left((p-1)+\left(\frac{\left(C_i(l_i)-C_i^{MR}\right)}{C_i^R}\right)^{\frac{1}{m(B_i)}}\right)} \\ \qquad\qquad \textit{for } Y_i=0,\ \ 3\leq l_i<N_i, \\ \qquad 1, \\ \qquad\qquad \textit{for } l_i=1 \text{ and for } Y_i=0,\ \ l_i=2, \\ \dfrac{p}{\left((p-1)+\left(\frac{C_i(l_i)}{C_i^R}\right)^{\frac{1}{m(B_i)}}\right)} \\ \qquad\qquad \textit{otherwise}, \end{cases} \tag{2.11}$$

In equation (2.11), p is calculated based on the maximum allowable hazard increment for a component, that is, it is related to the upper limit of the hazard adjustment factor that a component can achieve after a maintenance break. The smaller is the value of p, the larger is the maximum allowable hazard adjustment and vice-versa. The maximum hazard adjustment for a component could be estimated through the historical maintenance data about the system [17] and, accordingly, the p value can be selected. For instance, if the maximum allowable hazard adjustment for a component is found to be 1.2, then the value of p is selected such that $\frac{p}{(p-1)}=1.2$. This gives a value of $p=6$. Now this p value can be used in equation (2.11). When there is no maintenance ($l_i=1$), or the minimal repair action is performed on a failed component, there is no change in the hazard rate. Similar to our discussions for the age reduction factor in Section 2.3.3, for any imperfect repair action $3\leq l_i<N_i$ on a component in the failed state $Y_i=0$, minimal repair does not affect the hazard adjustment factor.

For the same cost-ratio, as the component ages, that is, the m value increases, the hazard adjustment factor of the component also increases. At a fixed value of the characteristic constant, that is a fixed effective age, the hazard adjustment factor varies with the amount of maintenance budget used. At the time of selective maintenance, the component's effective age is known; the only decision variable is the level of imperfect maintenance/repair (l_i). PM cost $C_i\left(l_i\right)$ of a component can be determined as given in Section 2.2. With the cost-ratio (PM cost/Replacement cost) in hand at a particular imperfect maintenance level l_i, the corresponding hazard adjustment factor can be found

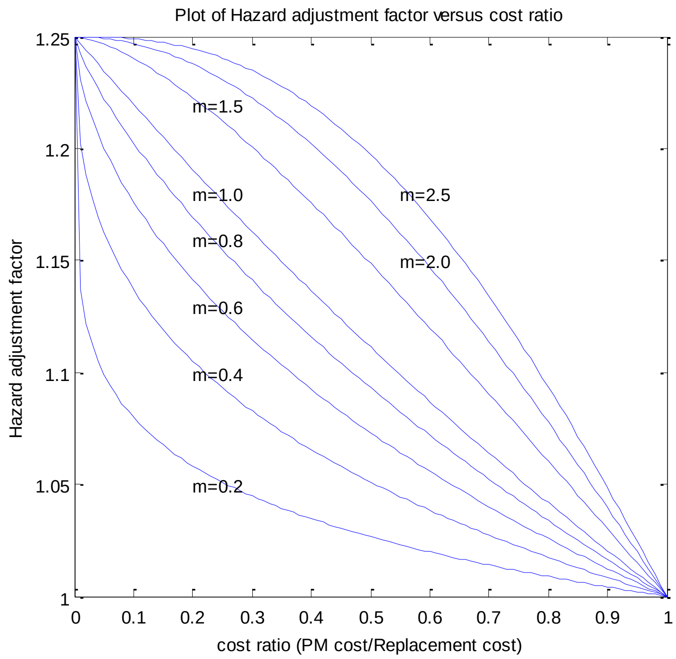

Figure 2.4: Hazard adjustment factor versus cost-ratio for different values of m (for p=5)

using the proposed formulation (2.11). Fig.2.4 shows the variations of the hazard adjustment factor with respect to the cost-ratio. The maximum allowable value of hazard adjustment factor is selected to be 1.25 (for p=5).

As shown in Fig.2.4, when the cost-ratio is small, i.e., a smaller budget is used for maintenance of a component, the hazard rate will increase faster after maintenance (higher value of hazard adjustment factor) and vice-versa. It also shows that for a fixed hazard adjustment factor, the amount of budget required increases as the component ages (i.e., m increases).

With known m and p values, plots 2.3 and 2.4 can be useful for maintenance managers to determine the amount of budget to invest in order to achieve the desired age reduction and hazard adjustment. The age reduction and hazard adjustment factors are required to be calculated in the case of intermediate maintenance actions only. In the case of no maintenance or minimal repair, there is no change in the effective age or the hazard rate of a component, and after replacement a new life cycle of the component starts. Based on the maintenance decision, the age reduction and the hazard adjustment factors will determine the hazard rate after maintenance, which in turn will determine the system reliability during the next mission. The next section provides information about evaluating system reliability, and the relationship between system reliability and different maintenance options is shown.

2.4 Probability of mission completion and selective maintenance modeling

2.4.1 Functioning probability of a component, subsystem and system

Let's assume that the system has arrived for maintenance after a previous mission and is required to perform the next mission, of length L. For each component i, the status at the end of the previous mission is denoted by Y_i;

$$Y_i = \begin{cases} 1, & \text{if component } i \text{ is working at the end of the previous mission }, \\ 0, & \text{otherwise.} \end{cases}$$

When a system comes to maintenance after the previous mission, the state Y_i and effective age B_i are known. For every component, a maintenance action is selected. Depending on the maintenance performed (i.e., the decision variable selected l_i $(1 \leq l_i \leq N_i)$), the component state may change after maintenance. The status of the component at the beginning of the next mission is given by X_i;

$$X_i = \begin{cases} 1, & \text{if component } i \text{ is working at the beginning of the next mission }, \\ 0, & \text{otherwise.} \end{cases}$$

Let p_{i,l_i} be the probability that a component i, after undergoing maintenance option l_i, finishes its next mission successfully. This probability depends on the effective age of the component at the beginning of the next mission and shows the reliability of the component for a given mission duration. If the length of the next mission is L, and t_2 is the beginning of the next mission, the component's hazard rate during the next mission $(h_{i,1,l_i}(t_2 + x))$ can be obtained from equation (2.5):

$$h_{i,1,l_i}(t_2 + x) = a(B_i.l_i) \times h_{i,0}(b(B_i, l_i) \times B_i + x), 0 \leq x \leq L. \tag{2.12}$$

In $\left(h_{i,1,l_i}\left(t_2+x\right)\right)$, subscript "1" refers that this is the first maintenance break for the component. The cumulative hazard rate of component i for the next mission can be defined as:

$$H_{i,l_i}\left(x\right) = \int_0^L h_{i,1,l_i}\left(t_2+x\right)\, dx. \tag{2.13}$$

The probability of this component successfully completing the next mission is:

$$p_{i,l_i} = \exp\left(-H_{i,l_i}\left(x\right)\right). \tag{2.14}$$

Thus, the reliability of component i can be defined as:

$$R_{i,l_i} = p_{i,l_i} \times X_i. \tag{2.15}$$

Hence, subsystem reliability where components within a subsystem are connected in a parallel arrangement can be defined as:

$$R_i\left(\boldsymbol{l}\right) = 1 - \prod_{\gamma'=1}^{n_\gamma}\left(1 - R_{i,l_i}\left(\gamma,\gamma'\right)\right), \tag{2.16}$$

where $\boldsymbol{l} = \{l_1, ..., l_i, ..., l_n\}$ is a vector comprising the maintenance decision variable l_i for all components in the system and $R_{i,l_i}\left(\gamma,\gamma'\right)$ is the reliability of component i during the next mission as given in equation (2.15). This component i is also the γ'th component in the subsystem γ. Similarly, for the whole system where the subsystems are connected in a series, the system reliability for the next mission can be given as:

$$R\left(\boldsymbol{l}\right) = \prod_{\gamma=1}^{s} R_i\left(\boldsymbol{l}\right) = \prod_{\gamma=1}^{s}\left(1 - \prod_{\gamma'=1}^{n_\gamma}\left(1 - R_{i,l_i}\left(\gamma,\gamma'\right)\right)\right). \tag{2.17}$$

The probability to finishing the next mission can be recursively determined for each component using its initial state, effective age at the beginning of the next mission, and mission duration. Thus, the reliability for the system can be determined using equation (2.17).

2.4.2 Selective maintenance modeling

If a system comes to maintenance after a mission with a known state Y_i, effective age B_i, and lifetime distribution parameters for all components, due to limited resources, only a subset of maintenance action can be performed. Thus the selective maintenance model is intended to:

1. identify the components (i) to be selected and determine maintenance action (l_i) on the selected components,

2. find the budget ($C_i(l_i)$) to be invested in each of the selected components,

3. find the amount of time ($T_i(l_i)$) to be invested in each of the selected components,

4. maximize the system reliability ($R(\boldsymbol{l})$) during the next mission.

The associated integer decision variable is l_i depending on which time ($T_i(l_i)$) and cost ($C_i(l_i)$) involved in maintenance are determined for each component. Also, system reliability ($R(\boldsymbol{l})$) is determined for any subset of maintenance actions following equation (2.17). Let the budget constraint on the total maintenance cost during the maintenance break be given by C_0 and available maintenance duration be T_0. The non-linear formulation to maximize the probability of successfully completing the next mission is developed as:

Objective:

$$Max \quad R(\boldsymbol{l}) = \prod_{\gamma=1}^{s} \left(1 - \prod_{\gamma'=1}^{n_\gamma} \left(1 - R_{i,l_i}(\gamma, \gamma')\right)\right), \tag{2.18}$$

Subject to:

$$\sum_{i=1}^{n} C_i(l_i) \leq C_0, \tag{2.19}$$

$$\sum_{i=1}^{n} T_i(l_i) \leq T_0, \tag{2.20}$$

$$R_{i,l_i} = p_{i,l_i} \times X_i, \tag{2.21}$$

$$V_i = \begin{cases} 1, & \text{if } l_i > 1, \\ 0, & \text{otherwise}, \end{cases} \tag{2.22}$$

$$X_i = \begin{cases} Y_i + V_i \text{ , if } Y_i = 0, \\ Y_i, \text{ otherwise,} \end{cases} \tag{2.23}$$

$$1 \leq l_i \leq N_i. \tag{2.24}$$

In this formulation, constraints (2.19) and (2.20) exhibit the limited available resources to perform maintenance. A component's reliability during the next mission is determined using equation (2.21), and constraints (2.22) and (2.23) set the component state at the beginning of the next mission depending on the state at the end of the previous mission and the maintenance action performed. Constraint (2.24) shows the available maintenance options for a component.

2.5 Solution methodology

Selective maintenance optimization for binary systems under imperfect repair is a nonlinear programming problem as presented in equations (2.18)-(2.24). Due to their ease of use and adaptability to the problem, evolutionary algorithms (like genetic algorithm (GA), differential evolution (DE), etc.) are widely used in maintenance optimization [10, 13, 22]. In this thesis, DE [23] is used to solve the selective maintenance problem. It is to be noted here that any other evolutionary algorithm can also be used to solve the problem. However, comparison of solution approaches is beyond the scope of this chapter and not discussed here.

To apply an algorithm to the problem, solution representation is an important procedure. Each solution string in the population has n elements. For each component i, the maintenance level $l_i \in \{1, ..., N_i\}$ is to be determined. Thus the possible maintenance alternative for the whole system is given by a string $\boldsymbol{l} = \{l_1, ..., l_i, ..., l_n\}$. For each solution point, the maintenance budget and time can be determined. For example, let us consider a system with three subsystems connected in a series, where each subsystem has two components connected in parallel (thus six components in total). Assume that for the components, $Y_i = \{1, 0, 1, 0, 0, 1\}$, $c^{fix}_{i,l_i} = \$2$ (in '000), and $t^{fix}_{i,l_i} = 2$ hrs, $C^{MR}_i = \$4$(in '000) , $T^{MR}_i = 4$ hrs, C^R_i=\$10 (in '000) and

T_i^R=10 hrs, respectively. Let the total possible number of actions (N_i) be six for each working components and seven for each failed component. For intermediate maintenance/repair actions, $c_{i,l_i} = \{\$5, \$6, \$7, \$8\}$ (in '000) and $t_{i,l_i} = \{5 \text{ hr}, 6 \text{ hr}, 7 \text{ hr}, 8 \text{ hr}\}$ ($2 \leq l_i < N_i$ in the case of working components, and $3 \leq l_i < N_i$ for components in the failed state). Then, for a specific solution string generated during the optimization process, say, $l = \{1, 3, 6, 5, 2, 2\}$, the following cost and time are observed. Component (1, 1) does not undergo any change. Component (1, 2), which was failed before maintenance, is in working condition now and its maintenance cost is $c_{i,l_i}^{fix} + c_{i,l_i}$ =\$2+\$5=\$7 (in '000). Similarly, maintenance time for component (1, 2) is seven hours. Component (2, 1) was in the working state at the time of maintenance but now it is replaced; hence, the total used cost is $c_{i,l_i}^{fix} + c_{i,l_i} = c_{i,l_i}^{fix} + C_i^R$=\$2+\$10=\$12 (in '000). Likewise, time to perform maintenance for component (2, 1) is 12 hrs. In an analogous way, cost and time for other components can be calculated.

With the level of imperfect maintenance/repair as the decision variable, the cost and the time to perform maintenance can be determined as shown above. The effective age and the hazard rate at the beginning of the next mission can be calculated by using imperfect maintenance/repair model discussed in Section 2.3. With the above information, system reliability can be evaluated as discussed in Section 2.4.

2.6 Results and discussion

2.6.1 Illustrative example

To demonstrate the advantages of the proposed model, an illustrative case is taken from Cassady et al. [6]. Their model is a special case of the proposed imperfect maintenance/repair model. If we restrict our model to minimal repair and replacement as the only possible maintenance actions, and consider time as the only constraint, it will be the same as [6]. In this example, a series parallel system is considered which consists of two subsystems connected in a series. Each subsystem has two components connected in parallel. This system is shown in Fig.2.5.

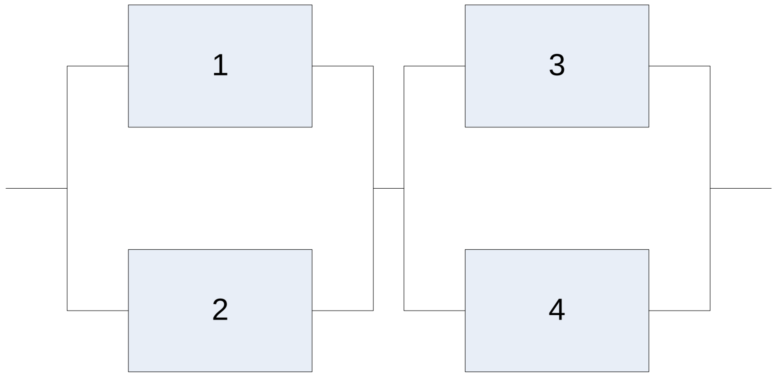

Figure 2.5: A series-parallel system

It is assumed that there are four intermediate maintenance actions possible for each component other than replacement or minimal repair, thus N_i=6 and N_i=7 for all working and failed components, respectively. In this example, time and cost are further divided based on Y_i, whether a component was working ($Y_i = 1$) or failed ($Y_i = 0$) at the time of maintenance. For $Y_i = 1$, $T_i^R = T_i^{WR}$ and $C_i^R = C_i^{WR}$ and for $Y_i = 0$, $T_i^R = T_i^{FR}$ and $C_i^R = C_i^{FR}$, respectively. Here, T_i^{WR} and C_i^{WR} are the time and cost of replacement when $Y_i = 1$ and T_i^{FR} and C_i^{FR} are the time and cost of replacement when $Y_i = 0$. It is assumed here that for intermediate maintenance actions, associated time and cost varies linearly as: $t_{i,l_i} = (l_i - 1) \times \Delta t_i^W$ and $c_{i,l_i} = (l_i - 1) \times \Delta c_i^W$ for $Y_i = 1$ and $t_{i,l_i} = T_i^{MR} + (l_i - 2) \times \Delta t_i^F$ and $c_{i,l_i} = C_i^{MR} + (l_i - 2) \times \Delta c_i^F$ for $Y_i = 0$. Here Δt_i^W, Δc_i^W, Δt_i^F, and Δc_i^F indicate the time and cost required to increase the intermediate maintenance level by unity for working and failed components, respectively. It is assumed that p=8 for each component in the system as shown in equation (2.11). The system parameters, time required for various maintenance actions and costs associated with maintenance of the components are given in Table 2.1.

The next mission length is $L = 8$ time units. Maintenance is performed within a given time window and available budget such that the maximum system reliability is achieved. Since fixed maintenance cost was not considered in the original problem, it is assumed here that t_{i,l_i}^{fix} and c_{i,l_i}^{fix} is zero for all components.

To solve the problem and compare the results, the example is analyzed and selective maintenance decisions and associated cost and time are investigated.

Table 2.1: System parameters, maintenance time and cost

i	α_i	β_i	Y_i	B_i	T_i^{MR}	T_i^{WR}	Δt_i^W	T_i^{FR}	Δt_i^F	C_i^{MR}	C_i^{WR}	Δc_i^W	C_i^{FR}	Δc_i^F
1	15	1.5	1	15	3	5	0.25	1	0.25	6	12	2	12	1
2	15	1.5	1	20	3	5	0.25	1	0.25	5	12	1.75	12	1
3	20	3	0	8	2	4	0.2	2	0.2	5	14	1.5	14	2
4	20	3	1	15	2	4	0.2	2	0.2	6	15	1.6	15	1.5

Effect of resource limitations and its sensitivity to the maintenance decision, role of characteristic constant on component selection, and effects of different imperfect maintenance/repair models for system reliability evaluation are discussed in detail. In the following discussion, IM shows the intermediate maintenance, WR is the replacement of a working component, MR is the minimal repair, FR denotes the replacement of a failed component, and DN is do nothing as the maintenance action. A detailed discussion is given in the following section.

2.6.2 Selective maintenance decision with time limit only

At first, we find the optimal solution when only time is limited (16 units), which was also assumed in the original problem. Results obtained by the proposed model is the same as those obtained by Cassady et al. [6]. The maximum reliability achieved is 0.8925. In this case all components are replaced. The age of all components at the beginning of the next mission becomes zero. Thus our model is verified by this result. Since time was only considered as an available resource by Cassady et al. [6], we first compared the proposed hybrid model with only time as a constraint. It is assumed that available time is T_o=9 units without any limitation on the available budget. The results are shown in Table 2.2.

Table 2.2 shows that when only replacement and minimal repair are considered, the maximum achievable system reliability is 0.7753. However, after incorporating imperfect maintenance/repair, the maximum achievable system reliability increases to 0.7969, an increase of more than 2%. In the first case, components 2 and 3 are selected for replacement, but two units of time remain unused because it is not possible to replace any of the remaining components within this unused time. In the second case, since intermediate maintenance action is possible due to imperfect maintenance/repair, components 1 and 4 are also selected in addition to the replacement of components 2 and 3. Thus, out of the remaining 2 units, 1.8 units of time is used for imperfect maintenance/repair and system reliability is further improved. Though all four of the components are in working condition in both cases, there is a difference in

Table 2.2: Selective maintenance decision and comparison when only replacement/minimal repair are used and when imperfect repair/maintenance is included with only time as a constraint.(T_o =9 units)

Comp(i, j)	With imperfect repair (Proposed model)				Replacement and minimal repair only			
	l_i*	T_i*	X_i*	A_i*	l_i	T_i	X_i	A_i
1	IM*	1	1	7.8071	DN*	0	1	15
2	WR*	5	1	0	WR	5	1	0
3	FR*	2	1	0	FR	2	1	0
4	IM	0.8	1	12.8936	DN	0	1	15
		$\sum$ =8.8				$\sum$ =7		
$R(l)$	0.7969				0.7753			

* T_i=Time spent on (i), X_i=state of (i) after maintenance, $1 \leq l_i \leq 6$ for $Y_i = 1$, $1 \leq l_i \leq 7$ for $Y_i = 0$, A_i= effective age after maintenance, IM= imperfect maintenance, WR=replacement of a working component, FR=replacement of a failed component, DN= do nothing.

the effective age of the components at the beginning of the next mission. In the first case, the effective age of components 1 and 4 remains unchanged to 15 units each. But in the second case, the effective age of these components reduces to 7.8071 and 12.8936 time units, respectively.

2.6.3 Selective maintenance decision with both time and cost limits

Now, additional cost limitation (C_o) is introduced in the above problem and its effect on the maintenance decision is analyzed. It is assumed that C_o=25 units and T_o= 9 units. The results for this case are shown in Table 2.3. It can be seen that with the additional cost constraint, the maintenance decision changes. For the case of only replacement and minimal repair as available maintenance options, the budget is sufficient to replace only component 2 and perform a minimal repair to component 3. With these actions, the system reliability is 0.6140 only during the next mission. When imperfect maintenance/repair is considered, the system reliability for the next mission increases to 0.7293. With minimal repair and replacement as the maintenance options, only 7 time units and 17 cost units are used and remaining time and budget are unused. However, when imperfect maintenance/repair is considered, a total 7.8 time units and all of the available 25 cost units are consumed.

It is obvious that incorporating imperfect maintenance/repair as maintenance options makes it possible to increase the system reliability by more than 11%, which is a large difference. Hence, it is important to include the imperfect maintenance/repair as an action for selective maintenance. It provides flexibility to use available resources in an optimal manner such that the system reliability is maximized. Depending on the available resources, the number of components selected and allocation of resources to these components has also changed. As can be seen in Tables 2.2 and 2.3, for a hybrid imperfect model with limitations on time only, all four components are selected for maintenance. Of those four, two are replaced and two undergo intermediate maintenance. However, with cost as an additional constraint, two components are selected and only one of those two is replaced. Hence, the allocation of

Table 2.3: Selective maintenance decision and comparison when only replacement/minimal repair is used and when imperfect repair/maintenance is included with both time and cost constraints (T_o =9 units, C_o=25 units)

Comp(i)	With imperfect repair (Proposed model)					Replacement and minimal repair only				
	l_i^*	T_i^*	C_i^*	X_i^*	A_i^*	l_i	T_i	C_i	X_i	A_i
1	DN*	0	0	1	15	DN	0	0	1	15
2	WR*	5	12	1	0	WR	5	12	1	0
3	IM*	2.8	13	1	2.7466	MR	2	5	1	20
4	DN	0	0	1	15	DN	0	0	1	15
		$\sum$ =7.8	$\sum$ =25				$\sum$ =7	$\sum$ =17		
$R(l)$	0.7293					0.6140				

* T_i=Time spent on (i), C_i=cost spent on (i), X_i=state of (i) after maintenance, $1 \leq l_i \leq 6$ for $Y_i = 1$, $1 \leq l_i \leq 7$ for $Y_i = 0$, A_i= effective age after maintenance, DN= do nothing, WR=replacement of a working component, IM= imperfect maintenance, MR= minimal repair.

resource is also critical while making decisions about selective maintenance. It also verifies that simultaneous consideration of all components is required for selective maintenance in the sense that the optimal allocation of resources is possible. If one component is considered at a time then optimal allocation would not be possible.

2.6.4 Effect of characteristic constant on the maintenance decision

Another noticeable observation is about the components' selection for maintenance. It is found that components 2 and 3 are selected to be repaired in all cases. The characteristic constant m values for all four components – 1, 2, 3, and 4 – are 1.813, 2.66, 0.752, and 2.30, respectively. Component 2 has the maximum and component 3 has the minimum m value. The m value shows that component 2 is relatively older; investing resources into this component for maintenance (other than replacement) will not result in considerable improvement in reliability. Hence, replacing component 2 facilitates better system reliability. Because component 3 is relatively younger, resource investment will result in incrementally better system reliability. Hence, component 3 is a suitable candidate upon which to perform minimal or intermediate maintenance actions within available (and limited) resources.

2.6.5 Sensitivity of maintenance resources

There could be instances when the maintenance crew is limited in terms of one resource but has flexibility on others. For example, a maintenance crew can have a fixed time limit, but might have flexibility with the budget. Similarly, a maintenance crew might have a tight budget but have flexibility in terms of time to perform maintenance tasks. In such conditions, it is important to find the effect of varying resources on the final system reliability so that an optimal allocation of resources is possible. Thus sensitivity of the selective maintenance decision with respect to the resource limitation is required to be investigated. To find the effect of the variation of time and cost limits, see the plot in Fig.2.6.

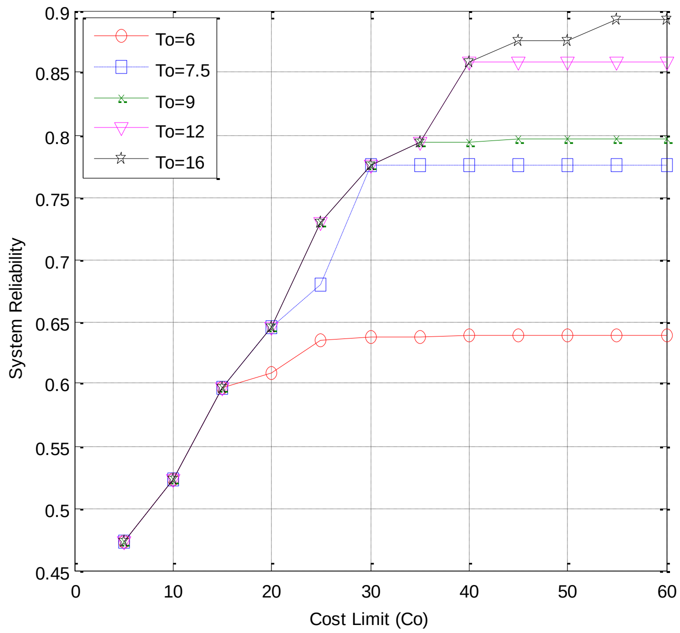

Figure 2.6: Sensitivity of system reliability with resource variation

Fig.2.6 shows the variation of the cost with the system reliability for different time limits (6, 7.5, 9, 12, and 16 time units). Such a plot is helpful in the budget and time estimation to achieve a certain reliability limit. For instance, 63.54% system reliability is achievable with 6 time units and an investment of 25 cost units. However, if the available time is 9 units, 73% system reliability can be achieved by investing 25 cost units only.

It can also be observed from Fig.2.6 that if one resource is constrained and the other resource is increased to achieve higher reliability, there is a limit after which no increase in system reliability is possible. For example, with T_o=12 time units, the maximum achievable reliability is 0.8589, which is achieved with a cost consumption of 38 units. A further increase in the maintenance budget is useless as there is no time available to consume that extra cost. Hence, no further increase in the the system reliability is possible. A similar observation can be found if the cost limit is kept constant and the time limit varies. As shown for C_o=30 units, an increase in T_o from 7.5 to 16 units does not improve system reliability further from 0.7753. Hence, a sensitivity analysis helps in deciding how to allocate resources optimally and perform maintenance/repair of components so as to achieve maximum system reliability. Such an analysis is helpful in deciding whether to use extra resources, especially if doing so would not improve system performance considerably. For example, for T_o=9, 12, and 16 units, an increase in cost limit C_o from 30 to 35 units leads to an increase in the system reliability by less than 2%. Hence, a maintenance manager can decide whether it is worth spending extra time or cost to make a minimal change in the system reliability.

2.6.6 System reliability and imperfect maintenance models

Since the proposed imperfect maintenance/repair model is a hybrid model which includes both the age reduction and hazard adjustment, it can be generalized to any of these two models as discussed in Section 2.3. With T_o =9 units and C_o=25 units as constraints, the age reduction and hazard adjustment models are also compared with the proposed hybrid model. It is found that for the age reduction and hazard adjustment models, the next mission system reliability is 0.7324 and 0.88, respectively. The system reliability is higher for the individual imperfect models as compared to the hybrid model (0.7293). This is because in the age reduction model, there is no hazard rate increment after maintenance (i.e., there is lesser probability of failure), and hence a higher reliability is achieved. For only hazard adjustment model, just after maintenance, the hazard rate starts with zero during the next mission (for all cases except minimal repair and no maintenance). Hence, a higher system reliability is achieved. In the hybrid model, both the age reduction of the components as well as the hazard rate increment are conceived, which is more realistic. Due to the combined effect of these two factors, system reliability is less. A similar observation was found in Lin et al. [24] that for a hybrid imperfect repair model more frequent PM is needed than an age reduction or hazard adjustment model because in a case with a hybrid imperfect repair, there is lower reliability. Our results are in line with the above outcome.

From the above discussions, it can be concluded that selective maintenance under imperfect maintenance/repair provides better reliability than selective maintenance with minimal repair and/or replacement only. It is also observed that a relatively younger component responds better to the resource allocated than an older component. However, allocation of resources depends on the state of the components as well as the overall system performance. Also, it is advantageous for a maintenance manager to be aware of the sensitivity of the system performance with respect to the resource limitations. In a flexible resource environment, it is suggested to determine the impact that variations in resources have on the system performance before any maintenance decision is made.

2.7 Summary

In this chapter, a single mission selective maintenance problem for binary systems under imperfect maintenance/repair is addressed. A more generalized hybrid imperfect maintenance model is used to formulate the components' improvement after maintenance/repair. This model includes both the age reduction and the hazard adjustment factors. A formulation for the characteristic constant m is also proposed, which determines whether a component is relatively younger or older. Based on the probability of successfully completing a mission, a selective maintenance model is formulated. This problem is solved and comparisons are provided between the proposed model and earlier methods where imperfect maintenance quality was not considered. Incorporating imperfect maintenance/repair action into selective maintenance yields better system output. Only the maintainable hazard rate is studied in this chapter; that is, the hazard rate considered in this chapter is affected by maintenance actions. Both the maintainable and non-maintainable hazard rates for a single mission selective maintenance problem will be studied in Chapter 3. When more than one mission is desired in a planning horizon, it is necessary to schedule selective maintenance. Simulating the dynamic probability of successfully completing the mission for multiple subsequent missions will be explored in Chapter 4. A system may also have multiple states. The selective maintenance problem for a multistate system will be solved in Chapter 5.

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Chapter 3

Selective Maintenance Considering Two Types of Failure Modes

Selective maintenance under imperfect maintenance is presented in Chapter 2, where the effect of cost and age on the imperfect maintenance improvement factors are modeled. It is assumed in Chapter 2 that only maintainable hazard rate is present in the system and it is affected by the maintenance actions. However, as mentioned in Section 1.3.2, the hazard rate due to non-maintainable failures is not affected by maintenance. Therefore, it is important to study the result of selective maintenance when both the maintainable and non-maintainable hazard rates define the failure behavior of components in the system. In this chapter a model is developed to incorporate the presence of the two types of failure modes and their effect on the selective maintenance decision making.

This chapter is organized as follows. The introduction is provided in Section 3.1, followed by preventive maintenance (PM) models in Section 3.2. In Section 3.2, a relationship between the two types of hazard rates and how they change the imperfect maintenance model are defined. System reliability evaluation and selective maintenance modeling is provided in Section 3.3. The solution methodology is presented in Section 3.4. Results and discussions are given in Section 3.5. Summary of the chapter is given in Section 3.6. Preliminary work related to this chapter is published in the conference proceedings

[1]. Detailed model and results have been accepted for publication as a journal paper [2].[1] This chapter largely covers the work presented in the journal paper [2].

3.1 Introduction

All equipment and systems deteriorate over time and need maintenance to improve their reliability and availability. If a system is not able to perform its intended function(s), it is said to have failed. As discussed in Section 1.3.2, all technical items are designed to fulfill one or more functions; a failure mode is thus defined as non-fulfillment of one of these functions [3]. Thus, if the corresponding function is unavailable, this is described as "failure with respect to a given failure mode of the system [4]." PM of a machine may include lubrication, tightening screws, cleaning, and shaft alignment. But such PM activities can only influence the failure modes that are affected by the working conditions associated with PMs. PM activities can reduce the hazard rate of such failure modes. These failure modes are called maintainable failure modes. However, the hazard rate of the non-maintainable failure modes cannot be changed by PM activities. These failure modes are related to the inherent design of a system, such as a crack in a shaft or gear.

It is possible to maintain or replace some components of a system in order to prevent the failure modes relevant to these components. However, failures associated with the fatigue or wear of the system over time are usually not maintainable. One type of failure mode may be affected by another type of failure mode in a system [5]. It is suggested in Yang et al. [6] that maintenance decisions and maintenance resource allocations may be affected by the type and frequency of these failure modes. The interaction between different failure modes and their effect on the system design, performance [7, 8], and

[1] Versions of this chapter have been accepted for publication in in "M.Pandey, and M.J. Zuo, Selective Maintenance Considering Two Types of Failure Modes. *International Journal of Strategic Engineering Asset Management*, accepted in July 2013", and published in "M. Pandey, M.J. Zuo, and D.D. Cuong, Selective maintenance for a multi-component system with two types of failure modes under age-based imperfect maintenance, *Proceedings of* 19^{th} *ISSAT conference on Reliability and Quality in Design*, pages 439-443, 5-7 August 2013."

maintenance decision making [9], are critical. In this chapter, the interaction between the two types of failure modes is studied to find their effect on the selective maintenance decision making.

Several examples have been reported in the literature in which different failures within a system are coupled. In Zequeira and Brenguer [4], an example of the electric truck motor is provided, which is a complex series system. In this system, damage in the armature winding (a non-maintainable failure) may increase the system's temperature, which may cause the lubrication to burn more quickly, resulting in inadequate lubrication (a maintainable failure). It is also mentioned in [4] that the interdependence of failures can be used to study their effects on the overall system reliability. Another example would be a belt conveyor system, in which the wear and tear of the conveyor structure (a non-maintainable failure) may increase the vibration level that may then accelerate the idler bearing failure, which is a maintainable failure for the conveyor system. It may also result in the slipping of the conveyor belt, which is a maintainable failure mode. Similarly, the coating of the pulley wears off over a period of time, which is a non-maintainable type of damage. It may lead to increased friction and may cause the belt joints to fail, which is a maintainable type of failure.

As mentioned in Section 1.3.2, whether a failure mode is maintainable or not, depends on the system boundary definition as well. A bearing crack failure is non-maintainable type of failure if the bearing itself is considered as a system. However, when bearing is one of the several components in a system and can be replaced along with other components, bearing crack should be considered as a maintainable failure mode with respect to that system.

In Lin et al. [10], the concept of maintainable and non-maintainable failure modes is used. They scheduled maintenance activities for a system with these two types of failure modes. They assumed that the maintainable and non-maintainable failure modes were independent and there was no interaction between the components in the system that were experiencing the non-maintainable and/or maintainable failures. However, an interaction between components experiencing the two types of failures may exist that characterizes the system degradation behavior. It is mentioned in [4] that the interaction between components can be used to characterize the system degradation. This interaction may include vibration or high temperature. They studied the maintainable and non-maintainable failure modes and suggested that there may exist a relationship between the two failure modes and can be stated in terms of the hazard rates. They suggested that the hazard rate due to the maintainable failure modes is related to the hazard rate due to the non-maintainable failure modes. Such a relation allows one to take into account the possible interactions between the failure modes when the failures are coupled. They proposed a relation between the hazard rates due to the maintainable and non-maintainable failure modes by using a coupling function. However, determination of this function was difficult.

Another relationship between the hazard rates due to the maintainable and non-maintainable failure modes is proposed in Castro [11]. It is shown in [11] that the hazard rate due to the maintainable failure modes depends on the cumulative effect of the hazard rate due to the non-maintainable failure modes. It is suggested in Lin et al. [10] that the hazard rate due to non-maintainable failure modes depends on the effective age of the component. However, [11] suggested that the hazard rate due to non-maintainable failure modes depends on the calendar age (excluding downtime) of the component up to the last maintenance. Both of the above assumptions have their shortcomings. The effective age of a component is affected by a maintenance action. Therefore, the hazard rate due to the non-maintainable failure modes will be lower for an effective age-based model as given in [10] as compared to a calendar age-based model as presented in [11]. The latter is more logical because non-maintainable failure modes are not affected by a maintenance action, and therefore the associated hazard rate is a continuously increasing function. If this hazard rate depends on the effective age, which may decrease after maintenance, it will no longer be a continuously increasing function. Further, in [11], the hazard rate of the maintainable failure modes during a mission was assumed to be dependent on the hazard rate of the non-maintainable failure modes up to the last maintenance only. It did not consider the interaction between these two hazard rate functions during the current mission. Therefore, the hazard rate due to the maintainable failure modes during the current mission evaluated by [11] is inaccurate. Recently, Chen et al. [12] applied the model presented in [11] and assumed that the cumulative hazard rate due to non-maintainable failure modes during the period immediately preceding the current calendar time affected the hazard rate due to maintainable failure modes. This consideration was also not logical. It is more reasonable to assume that non-maintainable failure modes up to the current calendar time affect the hazard rate due to maintainable failure modes. In this chapter, we have proposed a model in which the hazard rate due to the maintainable failure modes depends on the instantaneous hazard rate due to the non-maintainable failure modes.

Maintenance is critical to a system's performance and its reliability. Maintenance strategy aims to determine a trade-off between profits and the maintenance budget [13]. PM actions may increase the lifetime of a piece of equipment and decrease its breakdown frequency. As given in Section 1.3.1, traditionally, it is assumed that the maintenance of a system can improve its condition to as good as new (AGAN, also called replacement) or as bad as old (ABAO also called minimal repair). However, this assumption is not always realized in practice. For example, if only a few components are replaced, the whole system can be considered to be in between AGAN and ABAO conditions [14]. Such a maintenance policy is called imperfect maintenance. The system's effective age or hazard rate function can be adjusted to model the effect of a maintenance action [15, 16]. The improvement factors in the hazard rate and the effective age were introduced by Nakagawa [17] to consider the effect of imperfect maintenance. Later, Lin et al. [18] proposed that a maintenance action can simultaneously affect the effective age and the hazard rate of a system. They introduced a hybrid imperfect PM model to represent the effect of the maintenance in which the effective age of a system is reduced by a factor and the hazard rate due to maintainable failure modes increases by a factor after maintenance. It is assumed in [11] that after maintenance, the system is restored to as good as new condition with respect to the maintainable failure modes. In Chapter 2, we have developed the cost and effective age-based imperfect maintenance factors for the age reduction and hazard adjustment. Based on the model presented in Chapter 2, we have modified the imperfect maintenance factors for age reduction and hazard adjustment considering the two types of failure modes. This new hybrid imperfect maintenance model has been used in this chapter, and selective maintenance decision has been made for a series-parallel system.

Selective maintenance is required when it is not possible to perform all feasible maintenance actions due to limited resources. In modern industries and military applications, a system is often required to perform successive missions with a break between them. Because each of the available maintenance options consumes some maintenance resources like time and cost, an optimal allocation of resources is required. Selective maintenance was introduced by Rice et al. [19] for a series-parallel system with identical components and limited maintenance time. They assumed a constant hazard rate and that replacement was the only possible maintenance action. Later, Cassady et al. [20] considered both the cost and time as constraints and developed a maintenance optimization model for series-parallel structures. In Cassady et al. [21], it was assumed that a component's lifetime followed the Weibull distribution. Minimal repair and replacement were considered as possible maintenance actions. Although they considered the Weibull distribution, their study was limited in the sense that time was the only resource constraint. In Schneider and Cassady [22], selective maintenance for multiple systems, termed as a fleet, was performed.

Lust et al. [23] found that for a system with a large number of components, the optimization problem became combinatorial in nature, and the enumeration method was not useful. They found that Tabu search was useful in solving selective maintenance optimization problems. In Iyoob et al. [24], a resource allocation problem was solved for the subsequent missions under selective maintenance. Liu and Huang [25] assumed that only the effective age of a system was affected by maintenance actions. They did not consider the change in the hazard rate due to maintenance. None of the previous works in selective maintenance considered the presence of the two types of failure modes in a system. In all of the previous works on selective maintenance, it was assumed that all failure modes in a system are maintainable. As explained earlier, some failure modes may not be maintainable. For example, gear crack, bearing failure, fatigue, and overall wear of equipment cannot be maintained. However, conveyor belt joint failure, conveyor belt slippage, and lubrication loss are maintainable types of failures.

In this chapter, we first define a relationship between maintainable and non-maintainable types of failure modes. The maintainable and non-maintainable failure modes for a component are related in a manner that is similar to one described by [11]; however, we have assumed that imperfect maintenance is also possible for a component with respect to maintainable failure modes. We further propose that the hazard rate due to maintainable failure modes is related to the hazard rate due to non-maintainable failure modes up to the current calendar age, and not just up to the previous maintenance break as proposed in the model presented in [11]. We have developed an imperfect maintenance model with the two types of failure modes. Based on the model presented in Chapter 2, a new characteristic constant is defined in this chapter, and a system reliability equation is derived using the characteristic constant. It is followed by a formulation for the selective maintenance model with the maintainable and non-maintainable failure modes under hybrid imperfect maintenance. The aim is to determine the effect of the two types of failure modes on selective maintenance decision-making.

A multi-component series–parallel system is studied in this chapter. It has several subsystems connected in a series arrangement. A subsystem consists of components connected in a parallel way. Main contributions of this chapter can be summarized as: (i) the development of a model that relates maintainable and non-maintainable hazard rates considering the current age of the system, (ii) the development of a hybrid imperfect maintenance model with the two types of failure modes, and (iii) the formulation of a selective maintenance model considering the two types of failures and determination of its effect on maintenance decisions. An expression is developed for the cost and age-based age reduction and hazard adjustment factors in the hybrid imperfect maintenance model when the two types of failure modes are present in the system. To solve this problem, it is assumed that: (i) both the components and the system are in two possible states: working or failed, (ii) a limited amount of time and budget are available to perform maintenance, (iii) components states are known by inspection as soon as the components come in for maintenance at a maintenance depot, (iv) the failures of the components are independent of each other in the system, and (vi) the hazard rate of maintainable failure modes is related to the hazard rate of non-maintainable failure modes.

3.2 Models for preventive maintenance

A multi-component series parallel system is studied in this chapter. Each component can undergo a range of possible maintenance actions, which are minimal repair, imperfect maintenance/repair, or replacement. We assume that there are s ($\gamma = 1, 2, ..., s$) subsystems and each subsystem γ has $n_\gamma(\gamma' = 1, 2, ..., n_\gamma)$ components. There are n components in the whole system, that is, $\sum_{\gamma=1}^{s} n_\gamma = n(i = 1, 2, ..., n)$. The state of component i before maintenance is represented by Y_i. If a component is working then $Y_i = 1$, otherwise $Y_i = 0$. After maintenance, the state of component i is denoted by X_i. If a component is working after maintenance then $X_i = 1$, otherwise $X_i = 0$. Several discreet levels of maintenance are available for a component i. These options are denoted by l_i, $l_i \in \{1, 2, ..., N_i\}$. Here, $l_i = 1$ is the minimal level of maintenance, and $l_i = N_i$ is the best possible maintenance for a component that is replacement. Thus, for the whole system, we have a total of $N = \sum_{i=1}^{n} N_i$ PM actions available. These options for the system are called $\boldsymbol{l}_{system}$. When $Y_i = 1$, $1 \leq l_i \leq N_i - 1$ denotes imperfect maintenance options. When $Y_i = 0$, $l_i = 1$ denotes minimal repair of the failed component i. It represents the component being put into ABAO condition after repair. Other options $2 \leq l_i \leq N_i - 1$ are related to imperfect repair.

Depending on the available resources, a different number of components can be selected for maintenance. The selected number of components is denoted by $k(k \leq n)$, and the corresponding set of selected components is denoted as $\boldsymbol{i}_{selected} = \{i_1, i_2, ..., i_k\}$. Only one maintenance action can be performed from available $\{1, 2, ..., N_i\}$ options for each selected component. We denote these selected maintenance actions as $\boldsymbol{l}_{selected} = \{l_{i_1}, l_{i_2}, ..., l_{i_k}\}$; obviously, $\boldsymbol{l}_{selected} \subset \boldsymbol{l}_{system}$. According to the selected maintenance actions during a maintenance break, the hazard rate of selected components and the total cost and time of maintenance for the system vary.

3.2.1 Maintainable and non-maintainable failure modes

The maintainable and non-maintainable failure modes have associated hazard rates for a component i given by $h_{m,i}$ and $h_{n,i}$, respectively. They are called the maintainable and non-maintainable hazard rates. It is assumed in this chapter that failures due to the maintainable failure modes follow the Weibull distribution with the shape and scale parameters $\beta_{m,i}$ and $\alpha_{m,i}$, whereas failures due to the non-maintainable failure modes follow the Weibull distribution with the parameters $\beta_{n,i}$ and $\alpha_{n,i}$. The following relation between the hazard rates due to the maintainable and non-maintainable failure modes is proposed in [11]:

$$h_{m,i}(t) = h_{0,i}(t - t_1)\,\mu^{H_{n,i}(t_1)}, \tag{3.1}$$

where $h_{0,i}$ is the maintainable hazard rate when component i is AGAN, t_1 is the chronological time of the last maintenance (maintenance was assumed to be instantaneous in [11]), t is the chronological time such that $t > t_1$, and $H_{n,i}(t_1)$ is the cumulative hazard rate for the non-maintainable failure modes at time t_1. The maintainable hazard rate in equation (3.1) consists two parts. The first part $h_{0,i}(t - t_1)$ represents the effect of the maintainable failure modes while the second part $\mu^{H_{n,i}(t)}$ represents the effect of the non-maintainable failure modes, where μ is a constant.

In the proposed work, we have made two major changes in the model presented in [11]. The first major change is to consider the effect of the non-maintainable hazard rate up to the current time t rather than considering it up to the last maintenance break t_1. It is stated in [11] that the hazard rate due to the maintainable failure modes was affected by the non-maintainable hazard rate up to the time of previous maintenance break only. The cumulative hazard rate of the non-maintainable failure modes during the current mission also affects $h_{m,i}(t)$. Therefore, rather than using $H_{n,i}(t_1)$, as given in equation (3.1), we use $H_{n,i}(t)$. This change is important because considering the hazard rate only up to the previous maintenance break will give an inaccurate result related to the current time-point. This change will ensure that the effect of the cumulative non-maintainable hazard rate up to the current time is incorporated.

The second major change is to consider the effect of imperfect maintenance. It is assumed in [11] that the components are in AGAN condition with respect to the maintainable failure modes after maintenance. However, it is possible that components are in between AGAN and ABAO conditions with respect to the maintainable failure modes after maintenance. For instance, not all bearings in a belt conveyor system are replaced during a maintenance break. Hence, a hybrid imperfect maintenance model with the age reduction and the hazard adjustment factors is used to replace $h_{0,i}(t - t_1)$ in equation (3.1). This model is explained later in Section 3.2.3. Another minor change that we have incorporated concerns the value of the constant μ. Originally, Castro [11] defined $\mu > 1$; however, this would not cover the case in which the two types of failure modes are not related at all. Therefore, we have extended the definition and have redefined the constant as $\mu \geq 1$. When $\mu = 1$, the hazard rates due to the maintainable and non-maintainable failure modes are independent. When $\mu > 1$, there exists a relationship between these two hazard rates. The higher the value of μ is, the stronger the relationship is. Also, maintenance was considered instantaneous in [11], which is not realistic. We have assumed maintenance duration in the proposed model as given in Section 3.2.3.

The non-maintainable hazard rate of a component is an increasing function of time. However, if a component is replaced during a maintenance break, then the component becomes AGAN with respect to the non-maintainable failure modes as well. If the non-maintainable hazard rate, just before maintenance, is denoted by $g_{n,i}$, then after maintenance it becomes:

$$h_{n,i}(t) = \begin{cases} h_{n,i}^{0}(t - t_1), & \text{if } l_i = N_i, \\ g_{n,i}(t), & \text{otherwise,} \end{cases} \tag{3.2}$$

where $t - t_1 > 0$ and $h_{n,i}^{0}$ is the non-maintainable hazard rate for a new component and $g_{n,i}(t)$ is the non-maintainable hazard rate before maintenance. The cumulative hazard rate for the non-maintainable failure modes up to the current time t after the maintenance can be calculated as:

$$H_{n,i}(t) = \begin{cases} \int_{t_1}^{t} h_{n,i}(t)\,dt, & \text{if } l_i = N_i, \\ \\ \int_{0}^{t} h_{n,i}(t)\,dt, & \text{otherwise.} \end{cases} \tag{3.3}$$

We have explained the hazard rates and the cumulative hazard rate definitions in the imperfect maintenance and the selective maintenance models for the system in later sections.

3.2.2 Cost and time of maintenance

Upon arrival of the system for maintenance, the inspection determines the state Y_i and the effective age B_i for each component. Depending on the next mission requirement, a component may or may not be selected for maintenance. If selected, the allocated maintenance cost and time depends on maintenance option l_i. The maintenance cost for a component i is defined as:

$$C_i(l_i) = c_i^{fix} + c_{i,l_i}, \tag{3.4}$$

where c_i^{fix} is the fixed cost of maintenance and c_{i,l_i} is the variable cost of maintenance. The fixed maintenance cost is related to general maintenance actions (dusting, oiling, and so on) and setup cost. Variable cost is the cost associated with the selected maintenance option l_i. For a component, c_{i,l_i} for $l_i = N_i$ equals to the replacement cost denoted by C_i^R. When $Y_i = 0$, c_{i,l_i} for $l_i = 1$ denotes the minimal repair cost C_i^{MR}, and c_{i,l_i} for $2 \leq l_i \leq N_i - 1$ denotes the imperfect repair cost. When $Y_i = 1$, c_{i,l_i} for $1 \leq l_i \leq N_i - 1$ denotes the imperfect maintenance cost. The total maintenance cost for the whole system can be given by:

$$C = \sum_{i=i_1}^{i_k} C_i(l_i). \tag{3.5}$$

Similar to the maintenance cost, the maintenance time for a component can be given as:

$$T_i\left(l_i\right) = t_i^{fix} + t_{i,l_i}, \tag{3.6}$$

where t_i^{fix} is the fixed maintenance time and t_{i,l_i} is the variable time of maintenance that depends on the selected maintenance option l_i. For a component, when $l_i = N_i$, $t_{i,l_i} = T_i^R$, where T_i^R is the time required to replace the component, when $Y_i = 0$, t_{i,l_i} for $l_i = 1$ denotes the time to perform a minimal repair T_i^{MR}, and t_{i,l_i} for $2 \leq l_i \leq N_i - 1$ denotes the imperfect repair duration, and when $Y_i = 1$, t_{i,l_i} for $1 \leq l_i \leq N_i - 1$ denotes the time associated with the imperfect maintenance. The total maintenance time for the whole system can be given by:

$$T = \sum_{i=i_1}^{i_k} T_i\left(l_i\right). \tag{3.7}$$

It should be noted that $i(i = 1, 2, ..., n)$ denotes the components in the system. When i is used as a subscript with a variable/parameter, it shows that the variable/parameter is associated with the component i. In this chapter, c_{i,l_i} represents the variable cost of maintenance related to the maintenance option l_i selected for the component i. The value of c_{i,l_i} would be different for different l_i values even for the same component i. It should also be noted that depending on $l_i \in \{1, 2, ..., N_i\}$, which may vary from one component to another, the cost, time, and hazard rate vary. Similarly, t_{i,l_i} shows the variable time of maintenance for the selected component i and maintenance option l_i, and $h'_{m,i}\left(t, l_i\right)$ denotes the hazard rate of component i (for the maintainable failure modes) after a maintenance action l_i is performed during the maintenance break. The selective maintenance decision is not only about selecting the components for maintenance, but also selecting the maintenance option from all of the available options for that particular component.

Thus, if the maintenance decision for each of the k selected components is known, the total cost and time for maintenance can be calculated. If a component is not selected for maintenance, there is no cost or time involved. Depending on the level of maintenance, the hazard rate of the maintainable failure modes of a component changes. However, there is no effect of maintenance (other than replacement) on the hazard rate of non-maintainable failure modes.

3.2.3 Imperfect maintenance/repair model

Castro [11] assumed that after a maintenance break, a component is AGAN with respect to maintainable failure modes. However, imperfect maintenance of a component is also possible with respect to the maintainable failure modes. Therefore, it is important to consider the effect of imperfect maintenance in the modeling of maintainable failure modes (e.g., when only few components are maintainaed/replaced in a system). We have addressed this issue, and a hybrid imperfect maintenance model is used for this purpose. The hazard rate function $h_m(t)$ at time t reflects the condition of a component with respect to maintainable failure modes that depend on its operating history including operating conditions, failure and repairs, and PM actions. In a hybrid imperfect PM model, (i) the hazard rate after the PM becomes $ah_m(x)$ where $a \geq 1$ is the hazard adjustment factor and $x \geq 0$ represents the time elapsed from the previous PM; (ii) if the effective age of a component is t' immediately before a PM, it reduces to bt' immediately after the PM, where $b \leq 1$ is the age reduction factor. Given a certain maintainable hazard rate function $h_m^0(t)$ for $t \in \{0, t_1\}$, the PM activity during the maintenance interval $[t_1, t_2]$ changes the hazard rate to $h_m^{'}(t)$ for $t \geq t_2$ (Fig.3.1). The hybrid imperfect PM model can be given as:

$$h_m^{'}(t_2 + x) = ah_m^0(bt_1 + x), \tag{3.8}$$

where $a \geq 1$, $b \leq 1$ and $x \geq 0$.

In this chapter, the effective age of a component i before maintenance is given by B_i. If the hazard adjustment factor and age reduction factor for component i are represented by a_i and b_i, respectively, and the maintainable hazard rate before maintenance is $g_{m,i}$, then the first part of the maintainable hazard rate (in equation (3.1)) after maintenance $h_{0,i}(t - t_1)$ is replaced by a

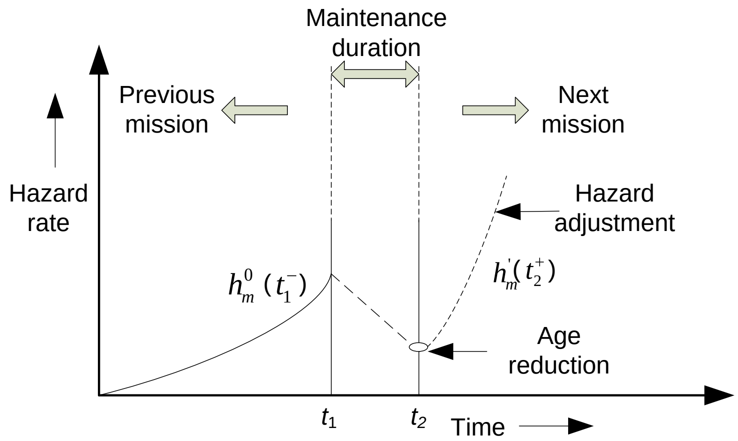

Figure 3.1: Hybrid imperfect maintenance model (effect of maintenance on the maintainable hazard rate)

hybrid imperfect maintenance based hazard rate $h'_{m,i}(t, l_i)$ as follows:

$$h'_{m,i}(t, l_i) = \begin{cases} h^0_{m,i}(t - t_2), & \text{if } l_i \in \boldsymbol{l}_{selected}, l_i = N_i, \\ a_i g_{m,i}(b_i.B_i + (t - t_2)), & \text{if } l_i \in \boldsymbol{l}_{selected}, l_i \neq N_i, \\ g_{m,i}(B_i + (t - t_2)), & \text{otherwise}, \end{cases} \tag{3.9}$$

where $t - t_2 > 0$, $a_i \geq 1$, and $b_i \leq 1$. The first part of equation (3.9) shows that after replacement the hazard rate of a component is AGAN. The second part of equation (3.9) shows that upon imperfect maintenance/repair of a component, the age reduction and the hazard adjustment factors are used. The third part of equation (3.9) shows that if a component is not selected for maintenance, its maintainable hazard rate after maintenance remains the same as before maintenance.

The maintainable hazard rate function in the next mission depends on the hazard rate at the end of the previous mission, PM activities performed on the component during the maintenance break, and non-maintainable hazard rate function. The hazard adjustment factor a_i and age reduction factor b_i depend on the amount of the maintenance budget used (which depends on the maintenance level l_i) as well as on the effective age of a component at the beginning of maintenance B_i, as discussed in detail in Section 2.3.2 in Chapter 2. If a component is relatively young, a smaller budget can attain improvement in its condition; but for the same improvement, more budget is expected when the component becomes old. Hence, to incorporate the effect of the amount of budget used and the effective age of the component, a cost-based age reduction factor is used as given in Chapter 2:

$$b_i\left(l_i, B_i\right) = \begin{cases} 1 - \left(\frac{C_i(l_i) - C_i^{MR}}{C_i^R}\right)^{m(B_i)}, \text{for } Y_i = 0, \quad 1 \leq l_i < N_i, \\ 1 - \left(\frac{C_i(l_i)}{C_i^R}\right)^{m(B_i)}, \quad \text{otherwise.} \end{cases} \tag{3.10}$$

Equation (3.10) shows that when a component is in the failed state before maintenance and an imperfect repair action is performed on the component, the minimal repair cost is not used in age reduction because minimal repair brings the component back to the ABAO condition. In all other cases, the entire maintenance budget is used in improving the condition of a component. Here, $m\left(B_i\right)$ is a characteristic constant that denotes the relative age of a component. As explained in Section 2.3.2, it is defined as:

$$m\left(B_i\right) = \frac{B_i}{\text{MRL}} = \frac{B_i}{\left(\frac{\int_{B_i}^{\infty} R_i(x)dx}{R(B_i)}\right)} = \frac{B_i \times R\left(B_i\right)}{\int_{B_i}^{\infty} R_i\left(x\right)dx}. \tag{3.11}$$

Since we have two types of failure modes present in a component, we need to redefine $R_i\left(x\right)$ as:

$$R_i\left(x\right) = \exp\left(-\left(\mu^{(x/\alpha_{n,i})^{\beta_{n,i}}}\left(x/\alpha_{m,i}\right)^{\beta_{m,i}} + \left(x/\alpha_{n,i}\right)^{\beta_{n,i}}\right)\right). \tag{3.12}$$

The characteristic constant $m\left(B_i\right)$ is defined as the ratio of the effective age B_i of the component just before maintenance and its mean residual life (MRL). If $m\left(B_i\right) > 1$, it is said to be a relatively old component; otherwise, it is relatively young. To check the variation of $m\left(B_i\right)$ for a component, we have considered an example with $\mu = 1.02$, the Weibull scale and shape parameters $\alpha_{m,i} = 300$ and $\beta_{m,i} = 1.5$ for the failure distribution of the maintainable failure modes, and $\alpha_{n,i} = 100$, $\beta_{n,i} = 1.3$ for the non-maintainable failure modes, respectively. When the effective age of the component is 50, 100, 150, 200, and 250 units, respectively, values of $m\left(B_i\right)$ are shown in Fig.3.2. Fig.3.2 shows that as a component becomes older and its effective age B_i increases, its m value also increases. For $m\left(B_i\right) = 1$, the effective age of a component is equal to the MRL of the component. In Fig.3.2, for $B_i = 50$, 100, and 150

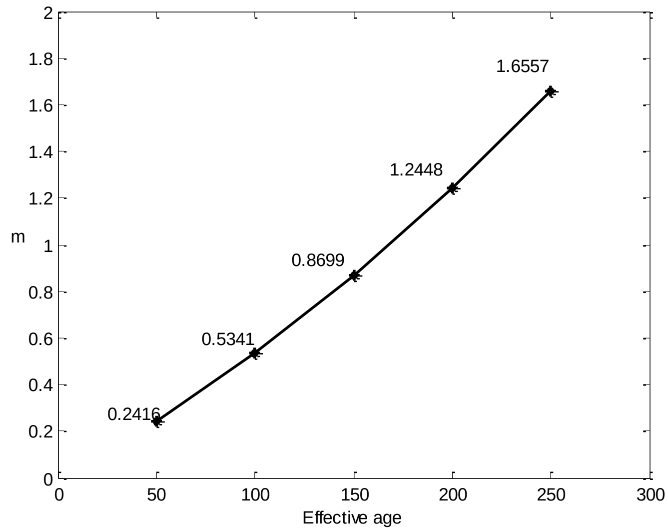

Figure 3.2: Variation of characteristic constant m with the effective age

units, $m(B_i)$ is less than 1. Thus, at these effective age values, we say that the component is relatively young. For $B_i = 200$ and 250 units, when $m(B_i)$ is greater than 1, the component is relatively old.

Similar to the age reduction factor, the hazard adjustment factor for maintainable failure modes is also defined in the way that it depends on the used maintenance budget and effective age of the component. As given in Section 2.3.5:

$$a(l_i, B_i) = \frac{p}{(p-1) + \left(\frac{C_i(l_i)}{C_i^R}\right)^{\frac{1}{m(B_i)}}}, \tag{3.13}$$

where $p > 1$ depends on the maximum hazard adjustment factor that a component can achieve after a maintenance break. The value of this maximum hazard adjustment factor could be estimated through the historical maintenance data about the component [26], and accordingly the p value can be decided as explained in Section 2.3.5. The larger the maximum hazard adjustment factor is, the smaller the value of p is.

In equation (3.9), $h'_{m,i}(t, l_i)$ gives the first part of the maintainable hazard rate in equation (3.1) after a maintenance break, while $\mu^{H_{n,i}(t)}$ is the second part of the maintainable hazard rate in equation (3.1). When these two parts are put together in equation (3.1), we obtain the new final expression for the maintainable hazard rate after maintenance:

$$h_{m,i}(t,l_i) = h'_{m,i}(t,l_i)\,\mu^{H_{n,i}(t)},$$
$$= \begin{cases} h^0_{m,i}(t-t_2)\,\mu^{H_{n,i}(t)}, & \text{if } l_i \in \boldsymbol{l}_{selected}, l_i = N_i, \\ a_i g_{m,i}(b_i.B_i + (t-t_2))\,\mu^{H_{n,i}(t)}, & \text{if } l_i \in \boldsymbol{l}_{selected}, l_i \neq N_i, \\ g_{m,i}(B_i + (t-t_2))\,\mu^{H_{n,i}(t)}, & \text{otherwise.} \end{cases} \quad (3.14)$$

Once a maintenance decision is made and the corresponding cost and time values are determined, we can then determine the achievable system reliability during the next mission. Using equations (3.2) and (3.14), the cumulative hazard rate for each component can be determined. This cumulative hazard rate can be used to evaluate system reliability as explained in the next section.

3.3 Mission reliability evaluation and selective maintenance modeling

3.3.1 Mission reliability evaluation

The cumulative hazard rate of a component i for the next mission of length L can be defined as:

$$H_{i,l_i}(L) = \int_{t_2}^{t_2+L} \left[h_{m,i}(t,l_i) + h_{n,i}(t)\right] dt. \quad (3.15)$$

The probability of this component successfully completing the next mission is

$$p_{i,l_i} = \exp\left(-H_{i,l_i}(L)\right). \quad (3.16)$$

Thus, the reliability of component i can be defined as:

$$R_{i,l_i} = p_{i,l_i} \times X_i, \quad (3.17)$$

where X_i represents the state of component i after maintenance. Its value is 1 when the component is working; otherwise, it is 0. Hence, system reliability for the next mission where components within a subsystem are connected in parallel and the subsystems are connected in series can be expressed as:

$$R(\boldsymbol{l}_{selected}) = \prod_{\gamma=1}^{s}\left(1 - \prod_{\gamma'=1}^{n_\gamma}\left(1 - R_{i,l_i}(\gamma,\gamma')\right)\right), \quad (3.18)$$

where $\boldsymbol{l}_{selected} = \{l_{i_1}, ..., l_{i_k}\}$ is an array of k maintenance decisions corresponding to the selected components $\boldsymbol{i}_{selected} = \{i_1, ..., i_k\}$ during the maintenance break and $R_{i,l_i}(\gamma, \gamma')$ is the reliability of the ith component in the system, which is also the γ'th component in the γth subsystem. The probability of completing the next mission successfully can be recursively determined for each component using its initial state, calendar age, effective age at the beginning of the mission, and the mission duration. Thus, the reliability for the whole system can be determined using equation (3.18).

3.3.2 Selective maintenance modeling

Our goal is to determine the components for maintenance and decide the level of maintenance to be performed on these components. We need to find $\boldsymbol{i}_{selected} = \{i_1, i_2, ..., i_k\}$ and $\boldsymbol{l}_{selected} = \{l_{i_1}, ..., l_{i_k}\}$ for the system. The budget C and maintenance time T used on the selected components during the maintenance break is decided. These decisions ensure that the maximum system reliability is achieved during the next mission using available resources. Let the budget constraint on the total maintenance cost during the maintenance break be C_0 and available maintenance duration be T_0. The non-linear formulation to maximize the next mission reliability is given as:
Objective:

$$Max \quad R(\boldsymbol{l}_{selected}) = \prod_{\gamma=1}^{s} \left(1 - \prod_{\gamma'=1}^{n_\gamma} (1 - R_{i,l_i}(\gamma, \gamma'))\right), \tag{3.19}$$

Subject to:

$$\sum_{i=i_1}^{i_k} C_i(l_i) \leq C_0, \tag{3.20}$$

$$\sum_{i=i_1}^{i_k} T_i(l_i) \leq T_0, \tag{3.21}$$

$$R_{i,l_i} = p_{i,l_i} \times X_i, \tag{3.22}$$

$$V_i = \begin{cases} 1, & \text{if } l_i \in \boldsymbol{l}_{selected}, \\ 0, & \text{otherwise}, \end{cases} \tag{3.23}$$

$$X_i = \begin{cases} V_i \text{ , if } Y_i = 0, \\ 1, \quad \text{otherwise}, \end{cases} \tag{3.24}$$

$$1 \leq l_i \leq N_i. \tag{3.25}$$

In this formulation, constraints (3.20) and (3.21) set the limitations of the available budget and time to perform maintenance. Equation (3.22) gives the value of a component reliability; constraints (3.23) and (3.24) set the components' states at the beginning of the next mission depending on their states at the end of the previous mission and the maintenance actions performed. If a component is selected for maintenance, in other words, if $l_i \in \boldsymbol{l}_{selected}$, V_i is 1; otherwise it is zero. Now, from equation (3.24), if $Y_i = 0$, X_i is equal to V_i, i.e., $X_i = 1$ if $l_i \in \boldsymbol{l}_{selected}$; otherwise $X_i = 0$. In all other cases, when a component was working before maintenance, it will remain in a working state after maintenance break whether or not it is selected for maintenance. State X_i affects the system reliability evaluation as given in equation (3.17). Constraint (3.25) shows the possible maintenance levels for a component i.

3.4 Solution methodology

To solve the non-linear optimization problem of selective maintenance, we have used an evolutionary algorithm. Evolutionary algorithms, for example, genetic algorithm, differential evolution, and so forth, are easy to use and adaptable to the problem [23, 27, 28]. Differential evolution (DE) [29] is used in this chapter to solve the problem. DE starts with a population and this population evolves to find the optimal/near optimal solution. A population is a set of solution strings. A solution string is represented by $\boldsymbol{S}$ where $\boldsymbol{S} = \{s_1, s_2, ..., s_{K_{\max}}\}$. Here $K_{\max}$ is the number of elements in a solution string. The value of $K_{\max}$ is chosen to be greater than or equal to the total possible number of maintenance actions, N, for the whole system. Each element of the string $\{s_1, s_2, ..., s_{K_{\max}}\}$ is one of the N PM actions available for the system. For each element position, maintenance action is randomly generated from the available N options for the whole system. Thus, PM actions related to one component may appear more than once in the solution string. We will therefore consider only the first appearing PM action for a component.

For a system, the number of PM actions that satisfies the constraints given in equations (3.20) – (3.25) may vary from one solution string to another depending on the set of maintenance actions (l_i) in that solution string. This is because different maintenance actions consume different amounts of resources; and they affect system reliability differently. It is needed to assign a number to represent this useful number of PM actions in a solution string. We denote this position in the solution string by P. Only useful parts of the solution string $\{s_1, s_2, ..., s_P\}$ will define the PM plan, while part of solution string $\{s_{P+1}, s_{P+2}, ..., s_{K_{\max}}\}$ will not contribute to the final PM solution. The elements of the string from s_{P+1} to $s_{K_{\max}}$ do not affect the objective function or the constraints, but they may affect the offspring by participating in the DE steps.

The following procedure is used to determine the objective function and constraint values of an arbitrary integer solution string $\boldsymbol{S} = \{s_1, s_2, ..., s_{K_{\max}}\}$.

1. Define $K_{\max}$. Set total cost $C = 0$, total time $T = 0$, system reliability $R = 1$.

2. Define sets $\boldsymbol{i}_{selected} = \phi$ and $\boldsymbol{l}_{selected} = \phi$.

3. For $K = 1$, the first element s_1 corresponds to the maintenance action l_i for a component i. This component i is the first component to be selected; hence, we assign a number $k = 1$ such that $i_{k=1} = i$ and $\boldsymbol{i}_{selected} = \bigcup i_k$, $\boldsymbol{l}_{selected} = \bigcup l_{i_k}$. Here k denotes the order of the selected components for maintenance. Corresponding to the decision $\boldsymbol{l}_{selected}$, update the total cost C using equation (3.5), total maintenance time T using equation (3.7), and system reliability R using equation (3.18).

4. If $C < C_0$, $T < T_0$, $K < K_{\max}$, and $k < n$, set $K = K + 1$, go to step 6.

5. If $C \geq C_0$ or $T \geq T_0$ or $K > K_{\max}$ or number of elements in $\boldsymbol{i}_{selected} = n$ (total number of components), go to step 9.

6. For K, s_K is an element of $\boldsymbol{l}_{system}$, which represents a maintenance action l_i for component i, check if $i \in \boldsymbol{i}_{selected}$, i.e., check whether a component is already selected for maintenance in the solution string.

7. If step 6 is true, then $K = K + 1$; go back to step 6. Otherwise go to step 8.

8. For K and maintenance action l_i given by element s_K, update $k = k+1$, $i_k = i$, $\boldsymbol{i}_{selected} = \bigcup i_k$ and $\boldsymbol{l}_{selected} = \bigcup l_{i_k}$. Update the total cost C, total maintenance time T, and the system reliability R. Go to step 4.

9. Stop. Reliability R is the final value for the given solution string. Finally, $\boldsymbol{l}_{selected} = \bigcup l_{i_k}$ gives the maintenance option selected for the system and $\boldsymbol{i}_{selected} = \bigcup i_k$ gives the components selected corresponding to $\boldsymbol{l}_{selected}$. If $K < K_{\max}$, then $P = K$.

For example, two components are connected in a series, and for each component three maintenance options are available; hence, $N_i = 3$ for both components and $N = \sum_{i=1}^{2} N_i = 6$. In this example, we define $K_{\max} = N = 6$. If a randomly generated solution string is given as $\boldsymbol{S} = \{2, 3, 4, 6, 5, 4\}$, then for $K = 1$, s_1 is the second element of $\boldsymbol{l}_{system}$, which denotes second option $l_i = 2$ for the component $i = 1$. For this selection, we have $k = 1$ and the first selected component is $i_1 = 1$. Thus, $\boldsymbol{i}_{selected} = \{i_1\} = \{1\}$, and $\boldsymbol{l}_{selected} = \{2\}$. We assume that the cost and time calculated for this decision is within constraint limits. Now for $K = 2$, $l_i = 3$. However, corresponding to this maintenance decision the component $i = 1$ is already selected; hence, we do not consider s_2 in the PM solution. We move to the next $K = K + 1 = 2 + 1 = 3$. Next element $s_3 = 4$ denotes the 4^{th} maintenance action in $\boldsymbol{l}_{system}$. This corresponds to $l_i = 1$ for component $i = 2$. Because $i = 2 \notin \boldsymbol{i}_{selected}$, we update $k = 1 + 1 = 2$, and the second selected component becomes $i_2 = 2$. Thus $\boldsymbol{i}_{selected} = \{i_1, i_2\} = \{1, 2\}$ and $\boldsymbol{l}_{selected} = \{2, 4\}$. We calculate the cost, time, and system reliability corresponding to this maintenance decision $\boldsymbol{l}_{selected}$. Equations (3.5), (3.7), and (3.18) are used to calculate the maintenance cost, time and system reliability for any $\boldsymbol{l}_{selected}$. No other solution can be selected,

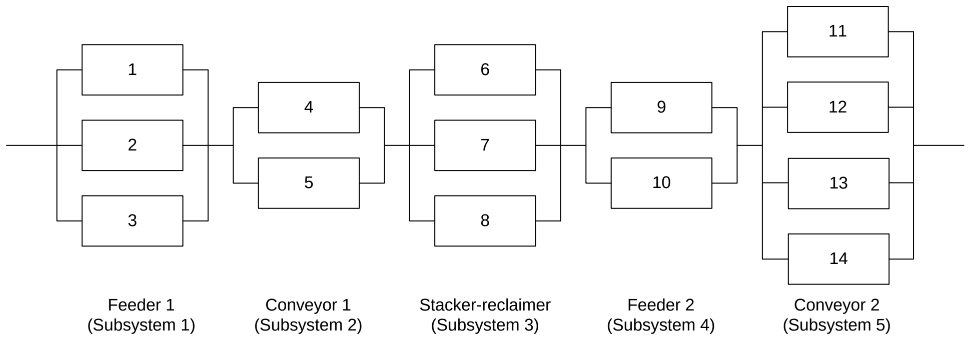

Figure 3.3: Block diagram of a coal transportation system [25]

as maintenance decisions corresponding to both components in the system are already realized. Since we already selected one maintenance action for each of the components, we can not perform any other maintenance action.

3.5 Results and discussion

To demonstrate the applicability of the above model, we have used an example of a coal transportation system in a power generation system that was also employed by [25]. This coal transportation system is used to supply coal to a boiler in a power station. It includes five basic subsystems, as shown in Fig.3.3.

Feeder 1 transfers coal from a bin to conveyor 1. Conveyor 1 transports the coal from feeder 1 to the stacker reclaimer, which lifts the coal up to burner level. Feeder 2 then loads conveyer 2, which transfers the coal to the boiler's burner feeding system. Each of the subsystem consists of a different number of components and each component has different parameters, as shown in Table 3.1. In Table 3.1, i=component, Y_i=state of component i before maintenance, B_i= effective age of component i before maintenance, $\beta_{m,i}$, $\alpha_{m,i}$=shape and scale factors for the Weibull distribution of the maintainable failure modes, $\beta_{n,i}$, $\alpha_{n,i}$=shape and scale factors for the Weibull distribution of the non-maintainable failure modes, c_i^{fix}, t_i^{fix}= fixed maintenance cost and time, l_i= maintenance options for component i, $\boldsymbol{l}_{system}$= maintenance options for the system, and c_{i,l_i}, t_{i,l_i}= cost and time associated with option l_i. There are two possible imperfect repair/maintenance levels for each component. For a working component (i.e., $Y_i = 1$), $l_i \in \{1, 2, 3\}$. Here, $l_i = 1,\ 2$ denote two imperfect maintenance levels and $l_i = 3$ denotes replacement of the component. For a failed component ($Y_i = 0$), minimal repair is also an option. In this case, $l_i \in \{1, 2, 3, 4\}$ are the possible maintenance levels. Here, $l_i = 1$ represents the minimal repair, $l_i = 2,\ 3$ are the imperfect repair levels, and $l_i = 4$ is the replacement of the component.

Table 3.1: System parameters, maintenance time and cost

i	Y_i	B_i	$\beta_{m,i}$	$\alpha_{m,i}$	$\beta_{n,i}$	$\alpha_{n,i}$	c_i^{fix}	t_i^{fix}	l_i	$\boldsymbol{l}_{system}$	c_{i,l_i}	t_{i,l_i}
1	1	120	1.5	300	1.5	900	3	0.25	1	1	8	0.15
									2	2	20	0.5
									3	3	40	1
2	1	120	2.4	300	2.0	900	4	0.25	1	4	6	0.15
									2	5	20	0.40
									3	6	35	0.75
3	1	120	1.6	250	1.5	900	3	0.25	1	7	8	0.15
									2	8	16	0.50
									3	9	40	1
4	0	85	2.6	400	2.0	1000	5	0.30	1	10	6	0.20
									2	11	15	0.30
									3	12	30	0.60
									4	13	45	1.00
5	1	120	1.8	400	1.8	900	2	0.30	1	14	9	0.25
									2	15	24	0.50
									3	16	45	1
6	1	120	2.4	375	1.6	900	3	0.15	1	17	9	0.25
									2	18	25	0.50
									3	19	38	1
7	1	120	2.5	400	1.8	900	6	0.30	1	20	10	0.15
									2	21	30	0.40
									3	22	40	0.75
8	1	120	2.0	375	1.2	900	5	0.10	1	23	15	0.25
									2	24	25	0.45
									3	25	42	0.90
9	1	120	1.2	400	1.2	850	3	0.40	1	26	10	0.20
									2	27	26	0.40
									3	28	45	1

Continued on Next Page...

Table 3.1 – Continued

i	Y_i	B_i	$\beta_{m,i}$	$\alpha_{m,i}$	$\beta_{n,i}$	$\alpha_{n,i}$	c_i^{fix}	t_i^{fix}	l_i	$\boldsymbol{l}_{system}$	c_{i,l_i}	t_{i,l_i}
10	0	100	1.4	400	1.4	850	6	0.20	1	29	6	0.20
									2	30	18	0.40
									3	31	26	0.75
									4	32	42	1.25
11	1	120	2.8	450	1.5	900	7	0.15	1	33	9	0.25
									2	34	21	0.50
									3	35	36	1
12	1	120	1.5	450	1.6	900	4	0.25	1	36	8	0.15
									2	37	20	0.40
									3	38	38	1
13	1	120	2.4	425	1.5	1000	6	0.35	1	39	10	0.20
									2	40	24	0.40
									3	41	40	0.80
14	0	100	2.2	400	1.2	900	3	0.35	1	42	9	0.12
									2	43	16	0.25
									3	44	28	0.50
									4	45	38	1.10

It is assumed that the system has stopped for its first maintenance break after 120 days of mission. The next mission duration is 90 days. In this example, 1 cost unit is \$1000 and 1 time unit is 1 day. We assume that $p = 20$ in equation (3.13); and similar to Castro [11] we have considered $\mu = 1.02$ in equation (3.14) for all components. The effect of imperfect maintenance/repair on selective maintenance decision making, resource constraints and their sensitivity, and maintainable and non-maintainable failure modes and their relationship are analyzed in detail. Point wise discussions are provided in the next section. In the following discussion, IM is imperfect maintenance, IR is imperfect repair, MR is minimal repair, and CR is component replacement.

Table 3.2: Only cost as a constraint ($C_0 = 400$ units)

Maintenance Decision, $\boldsymbol{l}_{selected}$	6, 9, 13, 16, 19, 22, 28, 32, 33, 43
Maintenance action	CR*, CR, CR, CR, CR, CR, IM*, IR*
Components selected, $\boldsymbol{i}_{selected}$	2, 3, 4, 5, 6, 7, 9, 10, 11, 14
Total time consumed, T	10.90 units
Total cost consumed, C	397
System reliability, R	96.04%

* CR= Component replacement, IM=imperfect maintenance, IR=imperfect repair.

3.5.1 Effect of resource constraints

We determine the effect of the resource constraints on selective maintenance decision-making and analyze resource sensitivity. At first, we consider $C_0 =$ 400 units as the only resource constraint. The results are shown in Table 3.2.

It can be seen from Table 3.2 that when cost is the only constraint, 10 components are selected during the maintenance break. Components 2, 3, 4, 5, 6, 7, 9, and 10 undergo replacement while imperfect maintenance is performed on component 11. Imperfect repair is performed on failed component 14. No maintenance is performed on the other 4 components, components 1, 8, 12, and 13. In this case, out of the available 400 cost units, 397 units are utilized and 10.90 time units are used in maintenance. When we introduce time as an additional constraint and keep $T_0 = 7$ units, the maintenance decision is changed. The results are given in Table 3.3.

With time as an additional constraint, only 250 units out of a total budget of 400 units and 6.80 units of time are used. The remaining 150 cost units are unused because there is no time available to perform maintenance/repair of any component that can further improve system reliability. When both constraints are considered, only 6 components are selected due to the time limitation, and no maintenance/repair action is performed on the remaining 8 components. It is evident that with an increase in the number of constraints on resources, the maintenance decision changes. This also demonstrates that

Table 3.3: Both cost and time as constraints ($C_0 = 400$ units, $T_0 = 7$ units)

Maintenance Decision, $\boldsymbol{l}_{selected}$	6, 13, 22, 28, 32, 43
Maintenance action	CR*, CR, CR, CR, CR, IR*
Components selected, $\boldsymbol{i}_{selected}$	2, 4, 7, 9, 10, 14
Total time consumed, T	6.80 units
Total cost consumed,C	250 units
System reliability, R	95.09%

* CR= Component replacement, IR=imperfect repair.

maintenance decisions are sensitive to resource variation. It is thus important for a maintenance manager to perform a sensitivity analysis before making a final maintenance decision.

To perform a sensitivity analysis, we have varied C_0 as 50, 100, 200, 300, and 400 units whereas T_0 as 3, 5, 7, and 9 units. The variation is shown in Fig.3.4. When $C_0 = 50$ units, the maximum achievable system reliability is 0.8772 for all values of T_0. For this budget, the selected maintenance actions are imperfect maintenance of components 6 and 8, which need 42 cost units. Although only 0.95 time unit is required for this maintenance, no further maintenance is possible because no budget is available for any other maintenance that can improve system reliability. Similarly, for $C_0 = 100$ units, the maximum achievable system reliability is 0.9113. This reliability can be achieved when $T_0 = 3$. Any further increase in time is not useful. Thus for each of the given budgets, there is a limit on time beyond which no further increase in the system reliability is possible.

Similar observations can be found for the time limit as well. For $T_0 = 3$ units, system reliability is 0.8772 for $C_0 = 50$ units, 0.9113 for $C_0 = 100$ units, and 0.9134 for $C_0 = 200$ units. However, any further increase in C_0 will not increase system reliability because there is no time available to perform other maintenance options. This demonstrates that a maintenance manager needs to be clear about the relative consumption of resources for each of the possible maintenance actions so that an optimal allocation of resources can

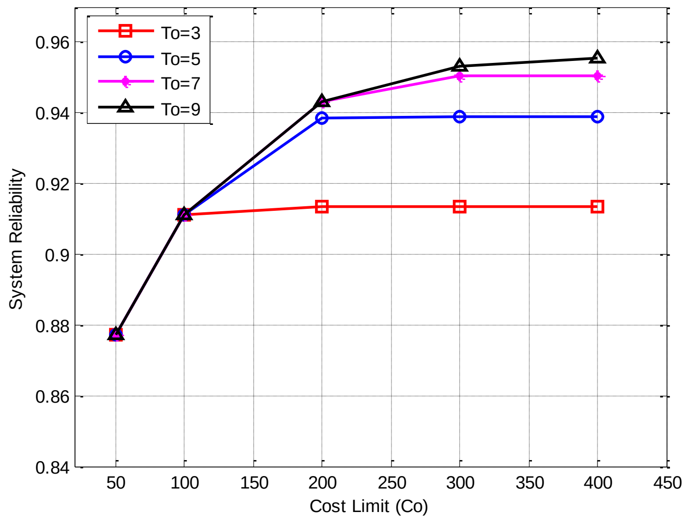

Figure 3.4: Sensitivity analysis of available budget and maintenance time

be performed. It can also be seen that for T_0= 9 units, an increase in the maintenance budget from 300 to 400 units leads to an increase in system reliability of less than 1%. Hence, the maintenance manager can decide whether it is beneficial to assign extra resources for the minimal increase in system reliability.

3.5.2 Comparing replacment/minimal repair and imperfect maintenance/repair as maintenance options

When only replacement and minimal repair are considered as possible maintenance actions, only 17 options are available for the systems, which are 14 replacement options and 3 minimal repair options for the failed components 4, 10, and 14. Let us assume that 500 units of budget are available for maintenance, and only 13 days are available to perform maintenance. The maintenance crew has to determine the components and the maintenance actions so that maximum system reliability is achieved during the next mission. The results are shown in Table 3.4.

Table 3.4 depicts that 11 components (component# 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 14) are selected for maintenance. Minimal repair is performed on

Table 3.4: Only replacement and minimal repair are possible ($C_0 = 500$ units, $T_0 = 13$ units)

Maintenance decision, $\boldsymbol{l}_{selected}$	3, 6, 9, 13, 16, 19, 22, 25, 28, 32, 42
Maintenance option	CR*, CR, CR, CR, CR, CR, CR,CR, CR, CR, MR*
Components selected, $\boldsymbol{i}_{selected}$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14
Total time consumed, T	12.62 units
Total cost consumed, C	311units
System reliability, R	92.68%

* CR= Component replacement, MR=minimal repair.

component 14 while all other selected components are replaced. The maximum achievable system reliability is 92.68%. It is interesting to note that out of 500 units of budget, only 311 units are used. The remaining 189 units of budget are unused because there is no time available to perform additional maintenance. This verifies that it is critical for a maintenance manager to determine the effect of maintenance resources on the final maintenance decision and accordingly decide about resource requirements. More time is required to better utilize the remaining budget. No maintenance is done on components 11, 12, and 13.

When imperfect maintenance/repair is introduced in addition to replacement and minimal repair, higher system reliability is achieved (Table 3.5). Table 3.5 shows that when imperfect maintenance is introduced in addition to replacement, system reliability is 96.26%, an increase of about 4%. This illustrates that the use of imperfect maintenance/repair is beneficial compared to replacement only. Higher system reliability is possible because the introduction of imperfect maintenance/repair enables using most of the available resources, which may not be possible if only replacement is performed. Another reason for the improvement in reliability due to imperfect maintenance is that it allows an option where more components can be imperfectly maintained.

3.5.3 Effect of the relationship between maintainable and non-maintainable failure modes

In this chapter, the hazard rate due to the maintainable failure modes is related to the hazard rate due to the non-maintainable failure modes. We have also investigated how the relationship between these two failure modes affects the final maintenance decision. We can rework the example with $C_0 = 400$ units and $T_0 = 7$ units; however, this time we assume that there is no relation between maintainable and non-maintainable failure modes (the maintainable and non-maintainable hazard rates are independent), i.e., $\mu = 1$. The results are shown in Table 3.6. Tables 3.3 and Table 3.6 suggest that the maintenance decisions remain the same for both cases. To check the sensitivity of the selective maintenance decision with respect to the constant μ value, we changed

Table 3.5: Imperfect maintenance/repair, replacement and minimal repair are possible ($C_0 = 500$ units, $T_0 = 13$ units)

Maintenance decision, $\boldsymbol{l}_{selected}$	3, 6, 9, 13, 16, 19, 22, 23, 28, 32, 34, 44
Maintenance option	CR*, CR, CR, CR, CR, CR, CR, CR, MR*, CR, CR, IR*
Components selected, $\boldsymbol{i}_{selected}$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14
Total time consumed, T	13.00 units
Total cost consumed, C	484 units
System reliability, R	96.26%

* CR= Component replacement, MR=minimal repair, IR=imperfect repair.

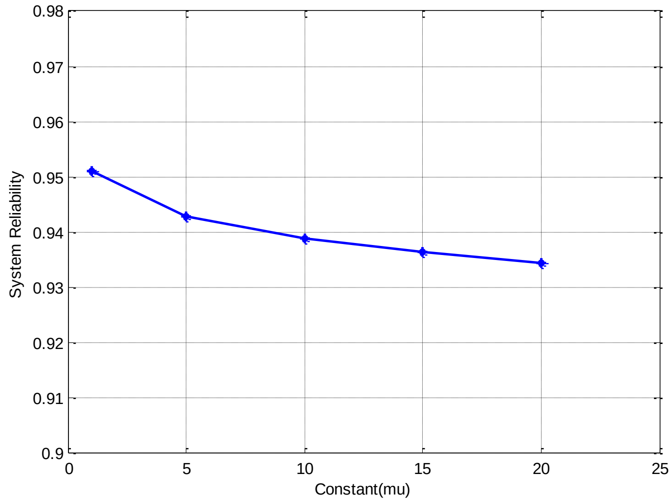

Figure 3.5: System reliability versus the constant μ

the value of μ to 1, 5, 10, 15, and 20, where $C_0 = 400$ units and $T_0 = 7$ units. A larger value of μ denotes a stronger relationship between the two types of hazard rates. The results are shown in Fig.3.5.

Fig.3.5 depicts that for the given example, system reliability is not very sensitive to the constant μ value in equation 3.14. There is a change of only about 2% in system reliability when μ changes from 1 to 20. It is because the value of the cumulative hazard rate due to non-maintainable failure modes is small in this example (much less than 1); therefore, even a larger increase in the constant does not result in too much variation in the system reliability. However, if we change the value of $\alpha_{n,i}$ for all components to 500 in Table 3.1, for $\mu = 5$, system reliability reduces to 0.8843. This indicates that when a component is old or its non-maintainable hazard rate is high, the value of the constant μ becomes important. It is therefore important to evaluate the value of the constant carefully for such a system. Another interesting finding is that for all of the above cases, the maintenance decision remained the same: the replacement of components 2, 4, 7, 9, 10 and IR of component 14. This was because, for the given cost and time constraints, maximum system reliability was achieved for the above decision for each of the μ values.

We have improved the model presented in [11] by changing the value of

Table 3.6: No relationship between the failure modes ($C_0 = 400$ units, $T_0 = 7$ units)

Maintenance Decision, $\boldsymbol{l}_{selected}$	6, 13, 22, 28, 32, 43
Maintenance action	CR*, CR, CR, CR, CR, IR*
Components selected, $\boldsymbol{l}_{selected}$	2, 4, 7, 9, 10, 14
Total time consumed, T	6.80 units
Total cost consumed, C	250 units
System reliability, R	95.10%

* CR= Component replacement, IR=imperfect repair.

$H_{n,i}(t_1)$ to $H_{n,i}(t)$ in calculating the maintainable hazard rate as given in equation (3.14). Because t_1 is the time of the last maintenance in Castro [11], Castro's model does not consider the current condition of the component in determining the present maintainable hazard rate. We have considered the instantaneous time point t in order to find the effect of the non-maintainable hazard rate on the maintainable hazard rate. Consideration of instantaneous time t in the non-maintainable hazard rate for calculating the instantaneous maintainable hazard rate during the next mission will give more accurate results compared to the case when a fixed value (the time of the last maintenance t_1) is used. This will also affect system reliability. When we use $H_{n,i}(t_1)$ for $C_0 = 400$ units and $T_0 = 7$ units with the mission duration of 90 time units and the constant μ value of 5, the best possible maintenance decision, the replacement of the components 2, 4, 7, 9, and 10 and IR of component 14, generates a system reliability of 0.9453. However, this system reliability value reduces to 0.9429 when $H_{n,i}(t)$ is used. Since the cumulative hazard rate up to the previous maintenance break is smaller, it gives a higher estimate of the system reliability than the actual reliability in the current time. Hence, it may affect maintenance decisions where reliability is important and the decision is based on system reliability evaluation.

It can be seen from the above discussion that considering the imperfect maintenance/repair is advantageous over only minimal repair/replacement as maintenance options. The former allows maximum utilization of available resources. Also, selective maintenance decisions depend on the number of resource constraints. We have found it to be advantageous to perform a sensitivity analysis so that resource allocation can be done wisely in order to attain desired reliability. The dependency between the maintainable and non-maintainable failure modes may also affect maintenance decisions. Hence, it is useful to find an appropriate value of the constant μ. We have also found that considering the effect of the non-maintainable hazard rate only up to the previous maintenance break may be misleading and gives an optimistic reliability value that is higher than the actual system reliability. This may lead to failure of the system while the maintenance crew believes that the system is more reliable than it actually is.

3.6 Summary

A single mission selective maintenance policy has been established in this chapter, where a series-parallel system experiences two types of failure modes: maintainable and non-maintainable. There are hazard rates corresponding to both types of failure modes. A formulation is proposed to relate these hazard rates when imperfect maintenance/repair of the components is possible. A hybrid imperfect maintenance model has been used to reflect the effect of age reduction and hazard adjustment. Cost and age based imperfect maintenance/repair factors have been derived. The changes are incorporated into the imperfect maintenance model in order to consider both types of the hazard rates and their relationship. The effect of the non-maintainable hazard rate up to the current time is modeled rather than considering it up to the previous maintenance break only. The latter may mislead maintenance decision because it gives a reliability value higher than the actual system reliability.

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Chapter 4

Selective Maintenance Scheduling over a Finite Planning Horizon

A single mission selective maintenance under imperfect maintenance is presented in Chapter 2. It is assumed in Chapter 2 that only one mission is performed in a planning horizon; hence, maintenance is needed only once at the beginning of the mission. However as mentioned in Section 1.4, it is possible that a system may need more than one maintenance breaks in a finite planning horizon because performing maintenance only once may not make the system reliable enough for the entire planning horizon. In such a condition, the number of maintenance breaks and the maintenance decisions during each of the maintenance breaks are determined. Therefore, a finite planning horizon selective maintenance scheduling problem is solved in this chapter. In this chapter, the number of periodic maintenance breaks within a finite planning horizon is found out in the manner that maintenance actions during each of the maintenance breaks ensure a minimum desired reliability limit during every mission. Also, these maintenance actions are performed within limited available time. Based on the single mission imperfect maintenance model introduced in Chapter 2, a model is developed in this chapter to find the imperfect maintenance improvement factors when maintenance is performed on a component in successive maintenance breaks.

This chapter is organized as follows: introduction to selective maintenance scheduling is given in Section 4.1. Maintenance options and system reliability evaluation are provided in Section 4.2. Cost and time involved in maintenance are discussed in Section 4.3. Problem formulation is presented in Section 4.4. Results and related discussion are given in Section 4.5. A summary is provided in Section 4.6. Preliminary work related to this chapter is published in the conference proceedings [1]. Fully developed model and results related to this chapter are submitted for publication in [2].[1] This chapter is mostly based on the work presented in the paper [2].

4.1 Introduction

All equipment and systems tend to deteriorate with age and usage. Preventive maintenance (PM) is often performed on a repairable system to improve the overall system reliability and availability. To establish a maintenance strategy for a repairable system, it is required to find the maintenance priority of the components within available resources. PM scheduling plays a very important role in the successful, economical, and reliable operation of systems. If maintenance actions are performed rarely, it can cause a large number of faults and outages; if performed too often, it may lead to a considerable increase in the maintenance cost. The time to perform maintenance (maintenance schedule) and maintenance actions during maintenance breaks are key decision variables for any PM policy. For many systems such as a semiconductor manufacturing system [3], a power plant [4], transportation and material handling systems [5], since the demand information is usually available for a known time horizon only, maintenance is scheduled for a finite horizon. This chapter presents a mathematical model for planning and scheduling maintenance activities for a repairable and maintainable system with multiple components, each of them deteriorates over discrete number of periods.

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Due to constrained resources, not all possible maintenance activities can be performed on a system. Optimal allocation of maintenance resources and selection of a subset of maintenance activities that fulfill the system requirement during a given planning horizon, are needed. For a multi-component system, maintenance models are concerned with the optimal maintenance policy for components with the stochastic failure process. The number of PM options available for a system depends on the PM options available for each component within the system. Obviously, failure or maintenance of any of the components will have an impact on the system performance. Considering this effect, along with selecting maintenance options for each component during maintenance breaks are major challenges in optimizing maintenance activities for a multi-component system.

Scheduling PM for a multi-component system was explored by some researchers in the past. In Dekker et al. [6], a heuristic was proposed to replace components within a finite planning horizon. Their heuristic becomes inefficient if the number of components increase. Wildeman et al. [7] grouped maintenance activities of the components in a multi-component system based on the optimal periodic maintenance interval of individual component. In their approach, only replacement and minimal repair were considered and, the maintenance duration was assumed to be negligible. Yao et al. [8] performed a limited study to optimize preventive maintenance scheduling in the semiconductor manufacturing operations. They did not consider the effect of maintenance actions on the components or system. Tsai et al. [9] optimized the periodic preventive maintenance schedule for the finite service life of a mechanical series system. They defined a formula to calculate the discarded life and used it as a replacement criterion. However, in their approach, the system can be discarded even if it has some useful life left. Tsai et al. [10] used improvement factor for a PM action and maximized system availability during each interval. They calculated the optimum periodic maintenance interval for each component in a series system and considered the minimum interval for a component as the system maintenance interval. However, this assumption may not be true in every case. Depending on the failure rates and repair costs of other components, the minimum interval for a component could be too frequent for the overall system and may increase the maintenance cost. Hence, the complete system should be considered simultaneously for the optimum schedule. Also, there were no limitations on available maintenance resources in the above works.

Bris et al. [11] minimized the maintenance cost for a fixed planning horizon under the availability constraint for a series parallel system. In their model, it was assumed that components were replaced at the time of maintenance. No other maintenance option was possible in their model. They considered each of the components separately and assumed that the hazard rate of the components was constant throughout their life, that is, they followed the exponential distribution. Later, Samrout et al. [12], and Wang and Lin [13] used the same model and same assumptions as [11] and only changed the solution approach. However, simultaneous consideration of all components is required to schedule maintenance in a multi-component system. Considering one component at a time may increase the downtime cost considerably. All of the above works consider that maintenance is instantaneous, which may not be true, especially in a finite horizon planning. Therefore, maintenance duration should be considered in the maintenance modeling.

Laggoune et al. [14] considered the periodic maintenance of all components simultaneously at predefined intervals. However, they assumed only replacement of the components. In reality, in addition to the replacement, minimal repair and imperfect maintenance are also possible. Recently, Moghaddam and Usher [15] minimized the cost of maintenance and failure under reliability constraint; and reliability was maximized under cost constraint for a series system within a given time horizon. Further, cost and reliability functions are combined and solved as a multi-objective problem in [5]. It is assumed in the above works that the number of maintenance breaks is fixed. Such a fixed number of breaks may not be the optimal for the given time horizon. Maintenance should be performed at an optimal frequency. Also they assumed that the maintenance time was negligible. However, usually maintenance does take some time and limited time is available to the maintenance crew. They also defined the system reliability for the whole planning horizon as the multiplication of the reliability for individual mission. This definition may cause uneven performance from one mission to another. For example, if a system reliability limit is defined as 90% for the entire planning horizon comprising two missions, then for the first mission a system reliability of 99% and for the second mission a system reliability of 91% make the system reliability for the entire planning horizon greater than 90%. However, it may lead to a maintenance decision where system performance varies considerably from the one mission (99% reliability) to another (91% reliability). Therefore, it is better to use individual mission reliability limit as a constraint to decide maintenance actions to be performed on the system for a consistent performance throughout the planning horizon. Moghaddam and Usher [5, 15] assumed that the effect of maintenance on the age of a component was same whether the component was new or old. However, a component's response to maintenance may be affected by its effective age and the maintenance resources consumed, as discussed in Chapter 2. Moghaddam and Usher [5, 15] did not consider the cost of maintenance downtime either. Vu et al. [16] proposed a similar model as [7] but their work was limited to replacement only and without consideration of maintenance duration.

It is found that cost alone is included in the above studies. However, maintenance duration is also critical especially in the finite time duration scheduling problems. Also, maintenance of a component not only changes its effective age but also it may change the slope of the hazard rate. Therefore, it is important to consider the hybrid imperfect repair model with both the age reduction and hazard adjustment. Rather than considering the entire planning horizon reliability, individual mission reliability should be considered in the maintenance decision making to achieve consistent system performance. Keeping the system's performance in perspective, a selective maintenance decision is required during each maintenance break regarding the components to be selected for maintenance and the maintenance actions to be performed on the selected components. Selective maintenance was proposed by Rice et al. [17]. He considered components replacement as the only maintenance option. Many other works on selective maintenance focused on the replacement as the only maintenance option [18, 19, 20, 21, 22]. In Chapter 2, we have discussed in detail about the selective maintenance under imperfect maintenance for a single break. A formulation is proposed in Chapter 2 to relate the component's age, maintenance budget used, and the imperfect maintenance improvement factors. We have taken the formulation from Chapter 2 for the improvement factors and extended them in this chapter in the context of successive maintenances in a multi-period finite horizon scheduling.

To thoroughly address the finite horizon selective maintenance scheduling problem in this chapter, we have found the optimum number of periodic maintenance breaks within the given finite horizon. Also, we have determined the selective maintenance decision during each of the maintenance breaks. The highlights of the contribution of this chapter are to: (i) include the effect of imperfect maintenance during consecutive missions in the selective maintenance scheduling and consider the hybrid imperfect maintenance model, (ii) include the effect of age and maintenance budget in defining the improvement factors in the hybrid imperfect maintenance model for scheduling, (iii) consider the maintenance duration along with the maintenance cost and system reliability, (iv) include the shutdown cost in the model, and (v) find the optimum number of maintenance breaks and perform selective maintenance decision during each of the maintenance breaks. To address the aforementioned problems, we have used the following assumptions in this chapter:

1. The system and the components within are in a binary state, that is, they are either working or failed.

2. The system consists of multiple, repairable components.

3. After replacement, a component is as good as new (AGAN). When minimal repair is performed it becomes as bad as old (ABAO). Maintenance is also possible such that the component health may lie between AGAN and ABAO, that is, maintenance can be modeled by imperfect maintenance.

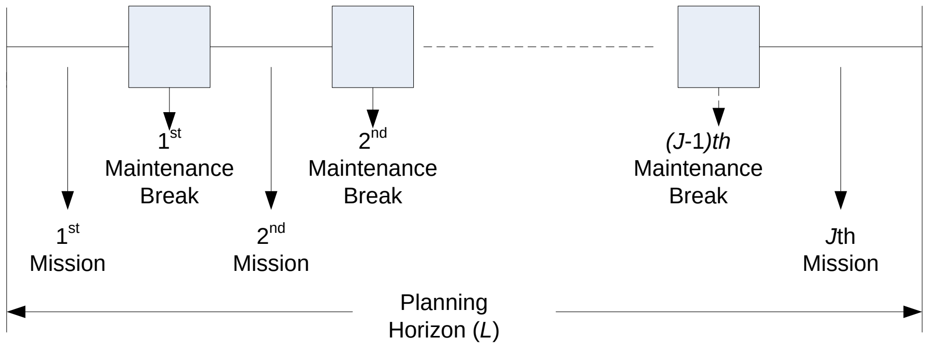

Figure 4.1: Maintenance breaks and missions in a finite planning horizon

4. Limited resources (cost and time) are available and the amount of resources required for maintenance activities are known.

5. Components and subsystems within the system are statistically independent.

6. Minimal repair is performed as soon as a component fails during a mission.

4.2 Maintenance model and system reliability evaluation

A series-parallel system is studied in this chapter that consists of s subsystems connected in a series and each subsystem γ $(\gamma = 1, ..., s)$ has n_γ $(\gamma' = 1, ..., n_\gamma)$ components connected in a parallel arrangement. There are in total $n = \sum_{\gamma=1}^{s} n_\gamma (i = 1, ..., n)$ possibly non-identical components in the system. We assume that the lifetime of each component follows a Weibull distribution with perhaps different parameter values. A schedule is to be established over the finite planning horizon $[0,\ L]$. All components are new at the beginning of the planning horizon (Fig.4.1).

The planning horizon $[0,\ L]$ is divided into J discrete equal intervals denoted as L_j, $(j = 1, ..., J)$. Each interval consists of one mission and one maintenance break at the end of each mission (except the last mission for the given planning horizon). The length of the jth mission and the jth maintenance duration are denoted by O_j and M_j, respectively.

Maintenance options for a system are denoted by $\boldsymbol{l}_{system}$. It has total N possible maintenance options available. If the total maintenance options available for the component i is denoted by N_i, then $\sum_{i=1}^{n} N_i = N$. Maintenance options for component i are denoted by $l_i \in \{1,\ 2,\ ..., N_i\}$. Thus we can represent the maintenance options available for the system ($\boldsymbol{l}_{system}$) as the combination of available maintenance options for all components, that is, $\boldsymbol{l}_{system} = \{l_1, l_2, ..., l_i, ..., l_n\}$. These options are related to imperfect maintenance, and replacement. Here, $1 \leq l_i \leq N_i - 1$ represents several imperfect maintenance options and $l_i = N_i$ represents replacement. Depending on the available resources and mission requirement, the different number of components can be selected for maintenance during each break. The selected number of components during the jth maintenance break is denoted by $k_j (k_j \leq n)$ and the corresponding set of selected components is denoted as $\boldsymbol{i}'_j = \{i_{1_j}, i_{2_j}, ..., i_{k_j}\}$. Only one maintenance action can be performed from the available $\{1, 2, ..., N_i\}$ options for each selected component during a maintenance break. We denote the selected maintenance actions during the jth break as $\boldsymbol{l}'_j = \{l_{i_{1_j}}, l_{i_{2_j}}, ..., l_{i_{k_j}}\}$. For the whole planning horizon, the complete set of the selected components is denoted by $\boldsymbol{i}_{selected} = \{\boldsymbol{i}'_1, \boldsymbol{i}'_2, ..., \boldsymbol{i}'_{J-1}\}$. Similarly, the corresponding maintenance decision is denoted by $\boldsymbol{l}_{selected} = \{\boldsymbol{l}'_1, \boldsymbol{l}'_2, ..., \boldsymbol{l}'_{J-1}\}$. It is assumed that the total number of components selected for the entire planning horizon is k, that is, the number of elements in $\boldsymbol{i}_{selected}$ and $\boldsymbol{l}_{selected}$ is k. According to the selected maintenance actions during the maintenance break(s), the total time and cost of maintenance and system reliability can vary. Also, depending on the level of maintenance, the imperfect maintenance improvement factors change.

4.2.1 Imperfect maintenance model

Whenever a component is replaced, it is in AGAN condition; however, imperfect maintenance brings it to somewhere in between ABAO and AGAN condition. Two preventive maintenance models were proposed by Nakagawa [23] wherein the hazard rate and effective age of a component were affected by PM. The first model is called the hazard rate adjustment model. In this

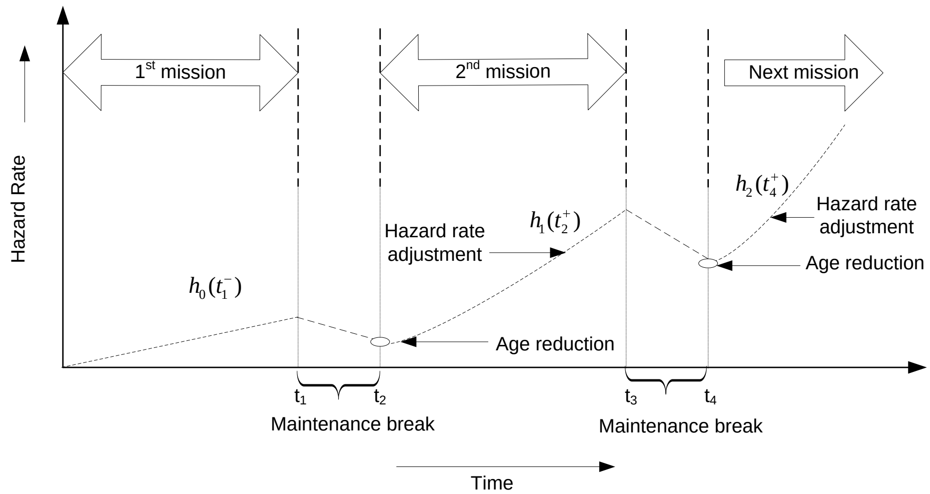

Figure 4.2: Hybrid imperfect maintenance model for successive missions

model, the hazard rate in the next PM interval becomes $ah(x)$ where $h(x)$ is the hazard rate in the previous PM interval. The hazard adjustment factor is $a \geq 1$ and $x \geq 0$ represents the time elapsed from the previous PM. In the second model, also known as the age reduction model, if the effective age of a component is t right before the PM then it reduces to bt right after PM, where $0 \leq b \leq 1$ is the improvement factor in the effective age. PM not only reduces the effective age but may also increase the hazard rate. The hazard rate function of a component in the next mission depends on the hazard rate at the end of the previous mission and the PM action performed on the component. The hybrid imperfect PM model can be given as [24]:

$$h_1(t_2 + x) = ah_0(bt_1 + x), \tag{4.1}$$

where $a \geq 1$, $b \leq 1$, and $x \geq 0$. During the maintenance interval $[t_1, t_2]$, maintenance action is performed on the component, which may change its effective age at the beginning of the next mission as well as the slope of the hazard rate during the next mission (Fig.4.2). As given in Fig.4.2, the age reduction and hazard adjustment are experienced when imperfect maintenance is performed during the first maintenance break. During the second mission, component's hazard rate will change following equation (4.1). If imperfect maintenance is performed again during the second maintenance break $[t_3, t_4]$, then the cumulative effect of the age reduction and hazard adjustment is used to find the hazard rate, in the third mission, after the second maintenance break.

Using the hybrid model [24, 25], the hazard rate function for a component i for $x \geq 0$ after the jth PM and in the $(j+1)$th mission can be expressed as:

$$h_{i,j+1}\left(t_{j+1}+x\right) = A_{i,j} h_0 \left(b_{i,j} B_{i,j} + x\right). \tag{4.2}$$

Here t_{j+1} is the chronological time at the beginning of the mission $j+1$, $A_{i,j} = \prod_{j'=1}^{j} a_{i,j'}$, represents the cumulative effect of the hazard adjustment on the hazard rate, and $B_{i,j}$ is the effective age just before the jth PM. We have the hazard adjustment factors $(a_{i,1}, a_{i,2}, ..., a_{i,j}) \geq 1$ and the age reduction factors $(b_{i,1}, b_{i,2}, ..., b_{i,j}) \leq 1$ for component i from the 1st to jth PM, respectively. The effective age of the component i right after the jth PM becomes $b_{i,j} B_{i,j}$.

As explained in Section 2.3, the improvement in the health of a component depends on the amount of resources used and the relative age of the component; and it is reasonable to assume that the age reduction and hazard adjustment factors depend on PM action (l_i) and the effective age of the component $(B_{i,j})$. A hybrid imperfect maintenance model, similar to the model provided in Section 2.3, is used in this chapter that considers the relative age of a component and maintenance budget used. The age reduction factor for a component i for the jth PM is calculated as:

$$b_{i,j}\left(B_{i,j}, l_i\right) = 1 - \left(\frac{c_{i,j,PM}\left(l_i\right)}{C_i^R}\right)^{m\left(B_{i,j}\right)}, \tag{4.3}$$

where $c_{i,j,PM}$ is the cost of maintenance for component i during maintenance break j, which depends on maintenance action l_i, and C_i^R is the replacement cost for ith component, which is equal to $c_{i,j,PM}\left(l_i = N_i\right)$, that is, the cost of maintenance when the maintenance decision is $l_i = N_i$. In the formulation for the age reduction factor, $m\left(B_{i,j}\right)$ is the characteristic constant, which shows the relative age of the component. It is defined as the ratio of the effective age of the component and its Mean Residual life (MRL) [26] at the current effective age. Based on the above definition, characteristic constant (m) becomes:

$$m\left(B_{i,j}\right)=\frac{B_{i,j}}{\mathrm{MRL}_{i,j}}=\frac{B_{i,j}}{\left(\frac{\int_{B_{i,j}}^{\infty}R_{i,j}(x)dx}{R(B_{i,j})}\right)}. \tag{4.4}$$

Here $R\left(B_{i,j}\right)$ is the reliability of component i at the effective age $B_{i,j}$ and $R_{i,j}\left(x\right)$ is the reliability function of component i for $t > B_{i,j}$.

In Chapter 2, the formulation of $m\left(B_{i,j}\right)$ was limited to a single mission and a new component only which has not undergone any maintenance yet. However, when multiple missions are required to be considered in a planning horizon, a component may experience several imperfect maintenance actions during these breaks. In such a situation, the hazard rate and reliability function $R_{i,j}\left(x\right)$ of a component changes. It is then required to find a formulation of the characteristic constant $m\left(B_{i,j}\right)$ that can be used in the subsequent intervals in the maintenance scheduling problem. To derive this expression for $m\left(B_{i,j}\right)$, we have assumed that component i follows the Weibull distribution with the scale and shape parameters α_i and β_i, respectively. In the expression for $m\left(B_{i,j}\right)$, with the known effective age and current reliability function, $B_{i,j}$ and $R\left(B_{i,j}\right)$ can easily be calculated. However, a formulation is needed to determine the reliability function $R_{i,j}\left(x\right)$. If we consider the $(j-1)$th maintenance break, then during the jth mission, $\int_{B_{i,j}}^{\infty}R_{i,j}\left(x\right)dx$ is given as:

$$\int_{B_{i,j}}^{\infty}R_{i,j}\left(x\right)dx=\exp\left(\frac{A_{i,j-1}}{\alpha_i^{\beta_i}}\left(b_{j-1}B_{j-1}\right)^{\beta_i}\right)\times$$
$$\int_{B_{i,j}}^{\infty}\exp\left(-\frac{A_{i,j-1}}{\alpha_i^{\beta_i}}\left(b_{j-1}B_{j-1}+x\right)^{\beta_i}\right)dx. \tag{4.5}$$

Proof: $L.H.S.=\int_{B_{i,j}}^{\infty}R_{i,j}\left(x\right)dx=\int_{B_{i,j}}^{\infty}\exp\left(-H_{i,j}\left(x\right)\right)dx$,

where $H_{i,j}\left(x\right)$ is the cumulative hazard rate function. From equation (4.2),

$$H_{i,j}\left(x\right)=\int_0^x h_{i,j}\left(t\right)dt=\int_0^x A_{i,j-1}h_0\left(b_{i,j-1}B_{i,j-1}+t\right)dt$$
$$=\int_0^x A_{i,j-1}\frac{\beta_i}{\alpha_i^{\beta_i}}\left(b_{i,j-1}B_{i,j-1}+t\right)^{\beta_i-1}dt=\frac{A_{i,j-1}\beta_i}{\alpha_i^{\beta_i}}\left[\frac{\left(b_{i,j-1}B_{i,j-1}+t\right)^{\beta_i}}{\beta_i}\right]_0^x$$

$\because\left(\int\left(ax+b\right)^n dx=\frac{(ax+b)^{n+1}}{a(n+1)}\right)$. Therefore,

$$H_{i,j}\left(x\right)=\frac{A_{i,j-1}}{\alpha_i^{\beta_i}}\left[\left(b_{i,j-1}B_{i,j-1}+t\right)^{\beta_i}\right]_0^x=\frac{A_{i,j-1}}{\alpha_i^{\beta_i}}\left[\left(b_{i,j-1}B_{i,j-1}+x\right)^{\beta_i}-\left(b_{i,j-1}B_{i,j-1}\right)^{\beta}\right]$$

Using this value of $H_{i,j}(x)$, we get,

$$\int_{B_{i,j}}^{\infty} R_{i,j}(x)\,dx = \int_{B_{i,j}}^{\infty} \exp\left(-\frac{A_{i,j-1}}{\alpha_i^{\beta_i}}\left[(b_{i,j-1}B_{i,j-1}+x)^{\beta_i} - (b_{i,j-1}B_{i,j-1})^{\beta_i}\right]\right)dx =$$

$$\int_{B_{i,j}}^{\infty} \exp\left(\frac{A_{i,j-1}}{\alpha_i^{\beta_i}}(b_{i,j-1}B_{i,j-1})^{\beta_i}\right) \times \exp\left(-\frac{A_{i,j-1}}{\alpha_i^{\beta_i}}(b_{i,j-1}B_{i,j-1}+x)^{\beta_i}\right)dx =$$

$$\int_{B_{i,j}}^{\infty} \exp\left(\frac{A_{i,j-1}}{\alpha_i^{\beta_i}}(b_{i,j-1}B_{i,j-1})^{\beta_i} - \frac{A_{i,j}}{\alpha_i^{\beta_i}}(b_{i,j-1}B_{i,j-1}+x)^{\beta_i}\right)dx =$$

$$\exp\left(\tfrac{A_{i,j-1}}{\alpha_i^{\beta_i}}(b_{i,j-1}B_{i,j-1})^{\beta_i}\right) \times \int_{B_{i,j}}^{\infty} \exp\left(-\tfrac{A_{i,j-1}}{\alpha_i^{\beta_i}}(b_{i,j-1}B_{i,j-1}+x)^{\beta_i}\right)dx =$$

$R.H.S.$

Putting the value of $\int_{B_{i,j}}^{\infty} R_{i,j}(x)\,dx$ from equation (4.5) in equation (4.4), we get,

$$m(B_{i,j}) = \frac{B_{i,j} \times R(B_{i,j})}{\exp\left(\tfrac{A_{i,j-1}}{\alpha_i^{\beta_i}}(b_{j-1}B_{j-1})^{\beta_i}\right) \times \int_{B_{i,j}}^{\infty} \exp\left(-\tfrac{A_{i,j-1}}{\alpha_i^{\beta_i}}(b_{j-1}B_{j-1}+x)^{\beta_i}\right)dx}. \tag{4.6}$$

According to the characteristic constant definition; a component is relatively young then $m<1$ and relatively old when $m>1$. Similar to the age reduction factor, the hazard adjustment factor is also defined such that it depends on the relative age of the component and ratio of the PM cost with the replacement cost. It is calculated as shown in Section 2.3.5:

$$a_{i,j}(B_{i,j}, l_i) = \frac{p}{\left((p-1) + \left(\frac{c_{i,j,PM}(l_i)}{C_i^R}\right)^{\frac{1}{m(B_{i,j})}}\right)},\ p > 1, \tag{4.7}$$

where $p > 1$ depends on the maximum hazard adjustment factor that a component can achieve after a maintenance break, as given in Section 2.3.5. The value of this maximum hazard adjustment factor can be estimated through the historical maintenance data about the component [27].

4.2.2 System reliability evaluation

Depending on the set of maintenance decision $\boldsymbol{l}_j'$ selected for a system, the hazard rate of the components after maintenance is determined. If a component is selected for maintenance during the $(j-1)$th break, its hazard rate after maintenance, that is, the hazard rate during the jth mission will change; otherwise it will remain the same. Hence, a set of definitions is provided in this chapter for the hazard rate after maintenance, which depends on the maintenance decision during a maintenance break, as given below:

$$h_{i,j,l_i}(t_j + x) = \begin{cases} h_{i,0}(x), & \text{if } l_i \in \boldsymbol{l}'_{j-1}, l_i = N_i, \\ a_{i,j-1} h_{i,j-1}(b_{i,j-1}.B_{i,j-1} + x), & \text{if } l_i \in \boldsymbol{l}'_{j-1}, l_i \neq N_i, \\ h_{i,j-1}(B_{i,j-1} + x), & \text{otherwise.} \end{cases} \quad (4.8)$$

The first part of equation (4.8) provides that if a component is replaced during a maintenance break, its hazard rate is the same as a new component after maintenance, which is $h_{i,0}$. The second part gives the hazard rate when imperfect maintenance is performed on the component. The third part shows that when a component is not selected for maintenance, its hazard rate remains the same as it was before the maintenance break . For a component i during the jth mission, its reliability is defined as:

$$R_{i,j}(l_i, J) = \exp(-H_{i,j}(x, l_i, J)) = \exp\left(\int_0^{O_j} h_{i,j,l_i}(t_j + x)\, dx\right). \quad (4.9)$$

Here $H_{i,j}(x, l_i, J)$ is the cumulative hazard rate during the jth mission. Thus, the system reliability for the jth mission for a series-parallel system can be given as:

$$R_j(\boldsymbol{l}'_{j-1}, J) = \prod_{\gamma=1}^{s}\left(1 - \prod_{i=1}^{n_\gamma}(1 - R_{i,j}(l_i, J))\right). \quad (4.10)$$

Since the first mission starts at time "0," there is no maintenance action before the first mission. System reliability for the first mission varies depending on the mission duration only. During maintenance breaks, limited time is available to perform maintenance and the maintenance decision should be completed within available time such that the total cost is minimized.

4.3 Maintenance cost and time

Our aim is to develop a selective maintenance scheduling model and minimize the total cost, which includes the cost of maintenance during maintenance breaks and the cost of failure during missions, for the entire planning horizon. The following costs are considered in this model:

4.3.1 Failure cost

For the future periods of a system operation, costs due to the unplanned component failures must account for. The failure cost has been widely used in the maintenance scheduling [5, 14, 15, 24]. At the start of the planning horizon $j = 1$, we cannot predict exactly when these failures will take place. However, we can predict that as the hazard rate increases, we are at the risk of experiencing a higher number of failures hence the higher cost associated with failures. Similarly, lower hazard rate should induce a lower cost of failure. To incorporate this, we denote the cost of unit failure for a component i as $c_{i,f}$ (in units of dollar/failure event), which allows us to calculate the total failure cost using expected number of failures $H_{i,j}(x, l_i, J)$ as follows:

$$c_{i,j,f}(l_i, J) = c_{i,f} \times H_{i,j}(x, l_i, J) = c_{i,f} \times \int_0^{O_j} h_{i,j,l_i}(t_j + x)\, dx. \tag{4.11}$$

This gives us the cost of failure for a component i during a mission j.

4.3.2 Maintenance cost

If a component is selected during a maintenance break, it will experience some maintenance cost; otherwise the maintenance cost would be zero. The cost of maintenance for component i during the jth maintenance break can be calculated as given in Section 2.2:

$$c_{i,j}(l_i) = \begin{cases} c_i^{fix} + c_{i,PM}(l_i), \text{ if } l_i \in \boldsymbol{l}'_j \\ 0 \end{cases}, \tag{4.12}$$

where c_i^{fix} is the fixed maintenance cost for component i and $c_{i,PM}(l_i)$ is the variable PM cost associated with the maintenance option l_i. The fixed cost is related to the set up cost, oiling, cleaning, etc., which are to be performed on a component if maintenance is required on it. The variable cost is known for each maintenance option l_i for component i. If no maintenance is performed then the maintenance cost is zero. The total cost of maintenance and failure for the system for the entire planning horizon can be calculated as:

$$c_1(\boldsymbol{l}_{selected}, J) = \sum_{j=1}^{J}\sum_{i=1}^{n} c_{i,j,f}(l_i, J) + \sum_{j=1}^{J-1}\sum_{i=i_{1_j}}^{i_{k_j}} c_{i,j}(l_i) \tag{4.13}$$

The first part of $c_1(\boldsymbol{l}_{selected}, J)$ shows that the failure cost of all components is considered for all missions, while the second part shows that maintenance cost is considered only for the selected i_{k_j} components during the jth maintenance break. There is no maintenance break at the end of the Jth mission.
In available literature for the finite horizon scheduling, the cost associated with the shutdown of the system and making it ready for the inspection, is not considered. This cost will be in addition to the production loss during the maintenance breaks. For a fixed total duration of the maintenance breaks, the production loss may remain the same, but with an increase in the number of maintenance breaks, the shutdown cost of the system will increase proportionally. We have included this cost in our model and termed it as the planned shutdown cost.

4.3.3 Planned shutdown cost

Whenever a system is required to undergo maintenance, shutdown of the whole system is done. After shutdown, the system has to be prepared for the inspection personnel. These actions involve costs and it is important to consider them while scheduling maintenance activities because with an increase in the number of maintenance breaks, the planned shutdown cost will also increase. It is assumed that every time the system is shut down for maintenance, a cost c_s is experienced. For all maintenance breaks, the total cost of planned shutdown 'C_s' is given as:

$$C_s = (J-1) \times c_s \tag{4.14}$$

The total cost for the entire planning horizon is estimated as:

$$C(\boldsymbol{l}_{selected}, J) = \sum_{j=1}^{J} \sum_{i=1}^{n} c_{i,j,f}(l_i, J) + \sum_{j=1}^{J-1} \sum_{i=i_{1_j}}^{i_{k_j}} c_{i,j}(l_i) + C_s \tag{4.15}$$

4.3.4 Maintenance time

Similar to the cost, time to perform maintenance on component i during the jth break can be calculated as:

$$t_{i,j}(l_i) = \begin{cases} t_i^{fix} + t_{i,PM}(l_i), \text{ if } l_i \in \boldsymbol{l}'_j \\ 0 \end{cases}, \quad (4.16)$$

where $t_{l_i}^{fix}$ is the fixed maintenance time and $t_{i,PM}(l_i)$ is the time to perform PM associated with the maintenance option l_i. Time $t_{i,PM}(l_i = N_i)$ is the time taken to replace the component i. For maintenance options $1 \leq l_i \leq N_i - 1$, $t_{i,PM}(l_i)$ represents the time required for imperfect maintenance. The maintenance time taken for the selected components during the jth maintenance break can be calculated as:

$$T(\boldsymbol{l'}_j) = \sum_{i=i_{1_j}}^{i_{k_j}} t_{i,j}(l_i). \quad (4.17)$$

We aim to find the set of maintenance actions for the jth maintenance break $\boldsymbol{l'}_j$ such that maintenance time $T(\boldsymbol{l'}_j)$ remains within the available time limit. It is to be noted here that maintenance cost and time depend on the maintenance decision.

4.4 Selective maintenance model and preventive maintenance scheduling formulation

It is assumed that the system is new with all new components. It has to achieve a minimum reliability level of R_0 during each of the missions. Due to limited time (break) between missions, it may not be possible that all PM actions are performed. The total available maintenance downtime for the entire horizon is assumed to be known and denoted as M. Equal maintenance time is assumed to be available during each interval and given as $M_j = \frac{M}{J-1}$ for all j. Our aim is to find the optimal number of intervals (J) within a planning horizon, the decision vector $\boldsymbol{l}'_j = \{l_{i_{1_j}}, l_{i_{2_j}}, ..., l_{i_{k_j}}\}$, and corresponding component vector $\boldsymbol{i}'_j = \{i_{1_j}, i_{2_j}, ..., i_{k_j}\}$ for each maintenance break j. Therefore, for the entire planning horizon, our objective is to find the decision vector $\boldsymbol{l}_{selected} = \{\boldsymbol{l}'_1, \boldsymbol{l}'_2, ..., \boldsymbol{l}'_{J-1}\}$ and corresponding component vector $\boldsymbol{i}_{selected} = \{\boldsymbol{i}'_1, \boldsymbol{i}'_2, ..., \boldsymbol{i}'_{J-1}\}$. The maintenance decision vector $\boldsymbol{l}_{selected}$ depends on the number of intervals, available maintenance time, and the desired reliability limits. The nonlinear formulation to minimize the total cost for scheduling maintenance in a planning horizon is given as:

Objective:

$$Min \quad C\left(\boldsymbol{l}_{selected}, J\right) = \sum_{j=1}^{J}\sum_{i=1}^{n} c_{i,j,f}\left(l_i, J\right) + \sum_{j=1}^{J-1}\sum_{i=i_{1_j}}^{i_{k_j}} c_{i,j}\left(l_i\right) + C_s, \tag{4.18}$$

Subject to:

$$\sum_{i=i_{1_j}}^{i_{k_j}} t_{i,j}\left(l_i\right) \leq M_j, \qquad 1 \leq j \leq J-1, \tag{4.19}$$

$$\prod_{\gamma=1}^{s}\left(1 - \prod_{i=1}^{n_\gamma}\left(1 - R_{i,j}\left(l_i, J\right)\right)\right) \geq R_0, \qquad 1 \leq j \leq J, \tag{4.20}$$

$$J = 2, 3, ..., \tag{4.21}$$

$$O_j = \left(t_{j+1} - t_j\right) - M_j, \qquad 1 \leq j \leq J-1, \tag{4.22}$$

$$O_J = L - t_J, \tag{4.23}$$

$$t_1 = 0, \tag{4.24}$$

$$1 \leq l_i \leq N_i. \tag{4.25}$$

In the above formulation, equations (4.19) and (4.20) show the limited time available for maintenance break and desired reliability limit, respectively. Equation (4.21) denotes that number of intervals can be any positive integer greater than one. When there is only one mission in the planning horizon, then no maintenance action is required as all components are new at the beginning of the planning horizon. Equation (4.22) gives the value of the jth mission duration for $1 \leq j \leq J-1$, while equation (4.23) provides the mission duration for the last mission. Equation (4.24) shows that the starting point of the planning horizon is time "0." Equation (4.25) provides maintenance options for component i.

Since the time to perform periodic maintenance is not known, it is required to find the optimal number of intervals for the given planning horizon. Too frequent PM may increase the maintenance and shutdown costs while too seldom PM may increase the failure cost. Therefore, in the selective maintenance model given in equations (4.19)-(4.25), J is also a decision variable in addition to the maintenance decision vector $\boldsymbol{l}_{selected} = \{\boldsymbol{l}'_1, \boldsymbol{l}'_2, ..., \boldsymbol{l}'_{J-1}\}$. In order to find the number of maintenance breaks and maintenance decisions, we start with $J = 2$ (note that for $J = 1$ all components are new and no maintenance decision is needed) and solve the problem. Afterward, we increase the number of intervals by unity ($J = J + 1$) and run the algorithm again to find the solution. It is repeated until an interval value is reached for which the total cost (objective function value) is the minimum. If we increase or decrease the number of intervals from this optimal interval value, the total cost will increase in either case. Hence, the scheduling problem is solved for one J value at a time. Evolutionary algorithms (like Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Differential Evolution (DE), and Simulated Annealing (SA)) are widely used in maintenance optimization [9, 14, 15, 16]. In this chapter, DE [28] is used to solve the selective maintenance problem. Details about the differential evolution algorithm can be found in [29, 30, 31].

In the selective maintenance scheduling model developed in this chapter, we have incorporated the effect of imperfect maintenance with both the age reduction and hazard adjustment. Also, we have improvised the definition of the characteristic constant so that the age-cost based improvement factors can be used, in the selective maintenance scheduling problem, when imperfect maintenance is performed on a component during successive maintenance breaks. It is the first time that the age-cost based improvement factors are used in the selective maintenance scheduling. Further, we have included the maintenance duration as an additional constraint in our model, which was not considered in the previous literature. We have also introduced the planned shutdown cost in our model to consider the effect of the number of shutdowns in the maintenance scheduling.

4.5 Results and discussion

For a planning horizon, maintenance scheduling was mostly performed in literature for the problems where replacement was considered as the only maintenance actions. Recently, the age reduction factor based imperfect maintenance model was considered in Moghaddam and Usher [5, 15], but they did not include the effect of the component age and maintenance budget. Also, they did not consider maintenance time in their model. To verify our model, we solved Moghaddam and Usher [5, 15] problem for 98% system reliability limit, 5 components and 6 missions. We found the total cost of $4584.60 for the problem. The solutions provided in their work for this problem lies in the range of $4503.80-$4768.97, for different solution approaches (branch and bound method and evolutionary algorithms). Our solution lies in their given range, thus verifies our model. To further evaluate the proposed methodology we have considered an example of the coal transportation system from Liu and Huang [32]. This coal transportation system is used to supply coal to a boiler in a power station. It includes five basic subsystems, as shown in Fig.4.3. Feeder 1 (subsystem 1) transfers coal from a bin to conveyor 1. Conveyor 1 (subsystem 2) transports coal from feeder 1 to the stacker reclaimer (subsystem 3) that lifts the coal up to the burner level. Feeder 2 (subsystem 4) then loads conveyer 2 (subsystem 5), which transfers the coal to the boiler's burner feeding system. The value of parameters for each component, e.g., parameters of the Weibull life distribution, maintenance options, related costs and times are shown in Table 4.1. We have assumed that $p = 20$ in equation (4.7) for all components in the system. In the following discussions, 1 cost unit = $1000 and 1 time unit = 1 day.

System demand is known for a given planning horizon of 378 days out of which 18 days are available as maintenance duration. The system is expected

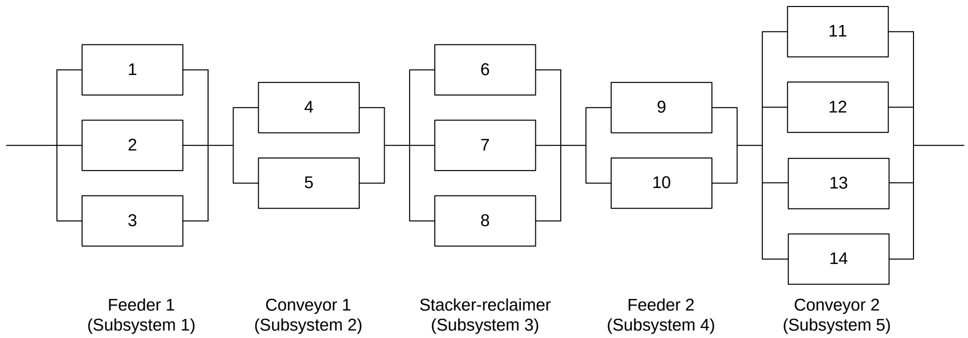

Figure 4.3: Block diagram of a coal transportation system [33]

to maintain a minimum reliability level of 96% during each mission. The shutdown cost (c_s) is 80 cost units. To present the results in details, point-wise discussions are provided as follows.

Table 4.1: System parameters, maintenance time and cost

i	β_i	α_i	$c_{i,f}$	c_i^{fix}	t_i^{fix}	l_i	$\boldsymbol{l}_{system}$	$c_{i,PM}(l_i)$	$t_{i,PM}(l_i)$
1	1.5	300	25	3	0.25	1	1	8	0.15
						2	2	20	0.5
						3	3	40	1
2	2.4	300	32	4	0.25	1	4	6	0.15
						2	5	20	0.40
						3	6	35	0.75
3	1.6	250	35	3	0.25	1	7	8	0.15
						2	8	16	0.50
						3	9	40	1
4	2.6	400	35	5	0.30	1	10	10	0.25
						2	11	25	0.50
						3	12	45	1.00
5	1.8	400	34	2	0.30	1	13	9	0.25
						2	14	24	0.50
						3	15	45	1
6	2.4	375	20	3	0.15	1	16	9	0.25
						2	17	25	0.50
						3	18	38	1
7	2.5	400	30	6	0.30	1	19	10	0.15
						2	20	30	0.40
						3	21	40	0.75

Continued on Next Page…

Table 4.1 – Continued

i	β_i	α_i	$c_{i,f}$	c_i^{fix}	t_i^{fix}	l_i	$\boldsymbol{l}_{system}$	$c_{i,PM}(l_i)$	$t_{i,PM}(l_i)$
8	2.0	375	35	5	0.10	1	22	15	0.25
						2	23	25	0.45
						3	24	42	0.90
9	1.2	400	28	3	0.40	1	25	10	0.20
						2	26	26	0.40
						3	27	45	1
10	1.4	400	35	6	0.20	1	28	8	0.25
						2	29	20	0.50
						3	30	42	1.25
11	2.8	450	32	7	0.15	1	31	9	0.25
						2	32	21	0.50
						3	33	36	1
12	1.5	450	35	4	0.25	1	34	8	0.15
						2	35	20	0.40
						3	36	38	1
13	2.4	425	36	6	0.35	1	37	10	0.20
						2	38	24	0.40
						3	39	40	0.80
14	2.2	400	38	3	0.35	1	40	10	0.20
						2	41	20	0.45
						3	42	38	1.10

4.5.1 Optimal number of intervals

It is desired by a maintenance manager to schedule the maintenance activities within a given planning horizon. Even if a pre-specified maintenance schedule is available, it may be needed to check whether this pre-imposed maintenance scheduling is the right choice or not. Therefore, the optimal number of maintenance breaks is determined for a given planning horizon that can minimize the total cost. A plot of the total cost versus the number of maintenance breaks is given in Fig.4.4.

In Fig.4.4, when the number of intervals is less than four, the length of

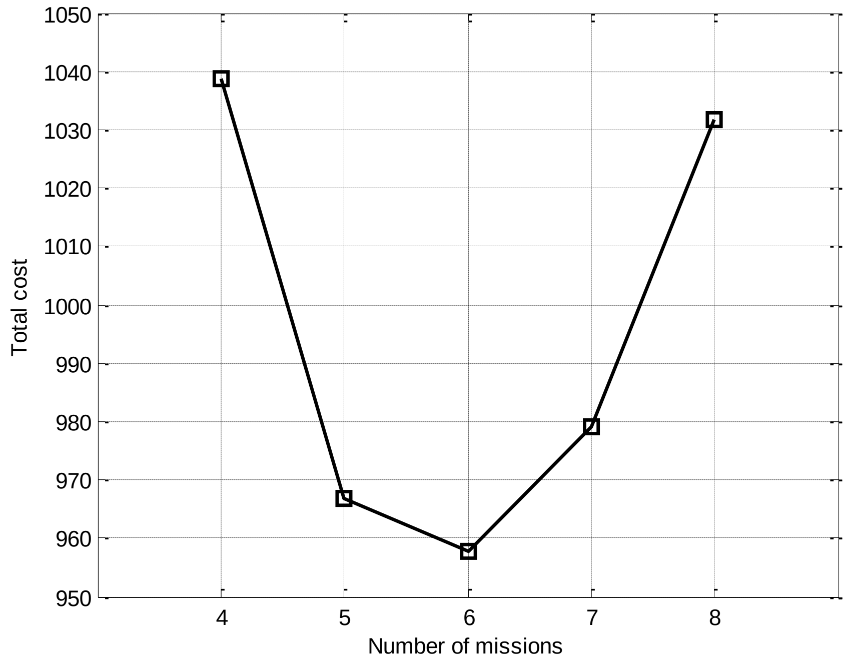

Figure 4.4: Finding optimal number of maintenance breaks within given planning horizon

the first mission is too long and the system reliability is less than 96%. For example, when the number of intervals is three, the first mission reliability is 95.37%, which is less than the desired reliability of 96%. Hence, the number of missions cannot be three or less. When the number of periods is four, the total cost of 1038.79 units is experienced. The results for four missions are shown in Table 4.2. When four missions and three periodic maintenance breaks are scheduled in the given planning horizon, the duration of each maintenance break is 18/3=6 days. Hence, all four missions have a mission length of 360/4=90 days. The final maintenance schedule obtained for this example is given in Table 4.2.

The best maintenance set up for the entire planning horizon is $\boldsymbol{l}_{selected} = \{5,$ 7, 10, 25, 29, 6, 12, 18, 30, 6, 10, 15, 21, 27, 29$\}$, which incurs a total cost of 1038.79 units. As shown in Table 4.2, components 2, 3, 4, 9 and 10 are required to be selected during the first maintenance break. All these components undergo imperfect maintenance during the first break. During the

Table 4.2: Maintenance scheduling decision for four missions ($R_0 = 96\%,\ M_j = 6$ units)

	j=1	j=2	j=3	j=4
Maintenance Decision	5, 7, 10, 25, 29	6, 12, 18, 30	6, 10, 15, 21, 27, 29	
Maintenance Action	IM*, IM, IM, IM, IM	CR*, CR, CR, CR	CR, IM, CR, CR, CR, IM	
Components Selected	2, 3, 4, 9, 10	2, 4, 6, 10	2, 4, 5, 7, 9, 10	
Total Maintenance Time	2.9 units	4.9 units	6.0 units	
Mission Reliability	0.9793	0.9603	0.9622	0.9610
Total Cost	1038.79 units			

*IM=Imperfect maintenance, CR=component replacement.

second maintenance break components 2, 4, 6 and 10 are all selected for replacement. During the third maintenance break components 2, 4, 5, 7, 9 and 10 are selected from which components 2, 5, 7, 9 are replaced while components 4 and 10 undergo imperfect maintenance. During the first mission, the system reliability is 97.93%, which depends on the mission duration only as all components are new at the beginning of the planning horizon. During the second, third, and fourth missions, the system reliability achieved are 96.03%, 96.22% and 96.03%, respectively, which are higher than the desired system reliability of 96%. Out of the available 6 units of maintenance time for each break, 2.9 time units are used during the first maintenance break, 4.9 units of time are used during the second maintenance break while a total of 6 units of time are used during the third maintenance break.

When the number of missions is increased to five, that is, the number of maintenance breaks is four, the total cost decreases to 966.87 units. For five missions, maintenance decisions are shown in Table 4.3. The maintenance decision for $J = 5$ is $\boldsymbol{l}_{selected} = \{9,\ 12,\ 16,\ 6,\ 30,\ 12,\ 21,\ 29\}$. These decisions ensure a system reliability of 98.86%, 96.65%, 96.01%, 96.14% and 96.26% during the 1st, 2nd, 3rd, 4th, and 5th missions, respectively. No maintenance is required during the first maintenance break. During the second maintenance break, components 3 and 4 are replaced while component 6 undergoes imperfect maintenance. During the third maintenance break, components 2 and 10 are replaced. For the fourth maintenance break, components 4 and 7 are selected for replacement while component 10 undergoes imperfect maintenance. From the available 4.50 units of time for each break, 2.95 time units, 2.45 time units, and 3.05 time units are used in the second, third, and fourth breaks, respectively.

When the number of missions is increased to $J = 6$, the minimum total cost of 957.89 units is obtained. The maintenance decision, associated cost, time, and system reliability are given in Table 4.4. Table 4.4 depicts that during the first and second maintenance breaks, no maintenance action is required on the components, as the system will maintain the reliability of 99.30% during the first mission, 98.11% during the second mission, and 96.43% during the

Table 4.3: Maintenance scheduling decision for five missions ($R_0 = 96\%,\ M_j = 4.50$ units)

	j=1	j=2	j=3	j=4	j=5
Maintenance Decision		9, 12, 16	6, 30	12, 21, 29	
Maintenance Action	NM*	CR*, CR, IM*	CR, CR	CR, CR, IM	
Components Selected		3, 4, 6	2, 10	4, 7, 10	
Total Maintenance Time		2.95 units	2.45 units	3.05 units	
Mission Reliability	0.9886	0.9665	0.9601	0.9614	0.9626
Total Cost	966.87 units				

*NM=No maintenance, IM=imperfect maintenance, CR=component replacement.

third mission. During the third maintenance break components 2 and 4 are selected for maintenance. Both components undergo replacement so that a system reliability of 97.22% is achieved during the fourth mission. During the fourth maintenance break only component 6 is selected for replacement to ensure a reliability of 96.07% during the fifth mission. During the fifth and the final maintenance break, components 2 and 4 are selected; out of those, component 2 is replaced while component 4 is imperfectly maintained. These actions give a system reliability value of 96.08% during the last mission.

The total cost is higher when the number of intervals (or maintenance breaks) is low. It reaches the minimum value for $J = 6$ and increases again for $J > 6$. When the number of intervals is increased to seven, the total cost increases to 979 units; and for eight intervals, it further increases to 1031 units. It is obvious that for the given problem, the total minimum cost is obtained when the number of intervals is six. Hence, the optimum number of intervals is six. All missions have more than 96% percent of system reliability and the total maintenance downtime is within the limit of 18 time units.

4.5.2 Maintenance duration and selective maintenance scheduling

In the previous works, maintenance was considered to be instantaneous. However, maintenance does take some time and it is important to include this time in the modeling and find its effect on the maintenance decision, system reliability, and the total maintenance cost. To find how much time is needed to achieve a desired reliability limit, we have relaxed the time limitation for the four mission problem given in Table 4.2. When the reliability limit is 96%, a maintenance break of 6 units is good enough to achieve the desired reliability limit during each mission. It shows that the total maintenance duration of 18 units (for three breaks) within the entire planning horizon should be sufficient. When the reliability limit is 95%, the longest maintenance break is of 4.33 time units. Thus, a total maintenance duration of 13 units is good for the planning horizon. Similarly, for 94% reliability limit, the desired maintenance duration for each break is 4 units, that is, the total maintenance duration of

Table 4.4: Maintenance scheduling decision for six missions ($R_0 = 96\%,\ M_j = 3.60$ units)

	j=1	j=2	j=3	j=4	j=5	j=6
Maintenance Decision			6, 12	18	6, 11	
Maintenance Action	NM*	NM	CR* , CR	CR	CR, IM*	
Components Selected			2, 4	6	2, 4	
Total Maintenance Time			2.30 units	1.15 units	1.80 units	
Mission Reliability	0.9930	0.9811	0.9643	0.9722	0.9607	0.9608
Total Cost	957.89 units					

*NM=No maintenance, IM=imperfect maintenance, CR=component replacement

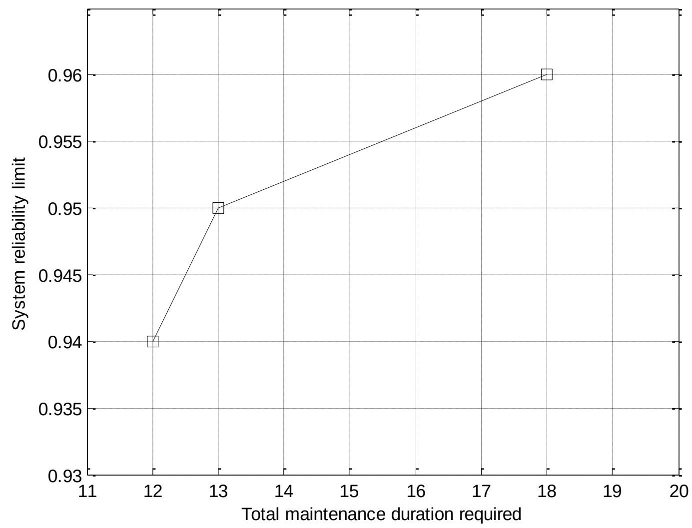

Figure 4.5: Maintenance duration needed to achieve the desired system reliability

12 units is needed for the entire planning horizon. If less time is available, fewer maintenance actions can be performed and it is likely that lower system reliability is achieved. It is clear that for different levels of desired system reliability, the required maintenance time varies. In other words, depending on the available maintenance duration, different levels of system reliability can be achieved. This is demonstrated in Fig.4.5, which provides a plot of the system reliability limit and the total maintenance duration required to achieve the desired reliability after maintenance.

Fig.4.5 shows that system reliability after maintenance depends on the total maintenance duration. When the required system reliability is increased from 94% to 95%, an increment in the total maintenance duration by one unit, from 12 units to 13 units, is sufficient to achieve the desired reliability. However, when the desired reliability is increased from 95% to 96%, the required maintenance duration increases from 13 to 18 units (a five unit increment). Thus, whether the desired reliability limit is achievable or not, depends on the available maintenance duration as well. This justifies our argument that neglecting time, especially in a finite horizon scheduling, is not appropriate because the achievable system reliability after maintenance is sensitive to the available maintenance duration. It is, therefore, important for a maintenance manager to check the variation of the maintenance duration with system reliability so that he(she) can determine whether a desired reliability limit is achievable or not within the given maintenance duration. The proposed model to consider the maintenance duration in the selective maintenance scheduling is important and more realistic in the finite horizon maintenance decision making.

From the above discussions, we can see that it is advantageous to consider the known planning horizon information in the maintenance decision modeling. An optimum schedule enables that the total cost is minimized for the given planning horizon. The maintenance duration should be included in the scheduling decision making rather than assuming instantaneous maintenance actions. Also, maintenance duration should be given due consideration by a maintenance manager while defining the desired reliability limit for a finite planning horizon.

4.6 Summary

A selective maintenance scheduling model is developed in this chapter for a given finite planning horizon where imperfect maintenance based PM modeling is proposed. During the jth PM, cumulative effect of the previous age reductions and hazard adjustments to a component is considered. The characteristic constant is defined for this purpose and a formulation is provided to find its value for each component, in the subsequent missions, considering the maintenance history. The total cost of maintenance and failure during the entire planning horizon is then determined such that maintenance between the successive missions are carried out within available time and the desired minimum system reliability is maintained during each mission. We have computed the optimal selective maintenance schedule and found the optimal number of missions and maintenance break, which minimizes the total cost.

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Chapter 5

Selective Maintenance Modeling for a Multistate System with Multistate Components under Imperfect Maintenance

In previous chapters, the selective maintenance modelings are done for systems in binary state only. However, as given in Section 1.1, a system may have more than two performance rates, that is, multiple states. For such a multistate system, a binary selective maintenance model is not applicable. Thus, there is a need to develop a selective maintenance model for a multistate system with multistate components. In this chapter, a thorough description is provided and step by step modeling is done for this purpose. A single mission selective maintenance problem for an MSS with multistate components under imperfect maintenance is solved in this chapter.

After the introduction in Section 5.1, system details and maintenance modeling are provided in Section 5.2. Evaluation of component state probabilities and system reliability are described in Section 5.3. Selective maintenance modeling and solution methodology are presented in Section 5.4. An example is illustrated and results are enumerated in Section 5.5. Concluding remarks are provided in Section 5.6. Results presented in this chapter are published in the journal paper [1] and the book chapter [2].[1]. This chapter is based on the

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journal paper [1].

5.1 Introduction

In an engineering environment, systems are required to perform specific objectives over a specified period of time. In many cases, a system is required to perform a sequence of operations (or missions) with a finite time break between two successive missions. These breaks provide an opportunity to perform maintenance on the component(s) of the system. However, it may be impossible to perform all desirable maintenance activities before the start of the next mission due to limited maintenance resources. In such cases, a subset of maintenance activities is chosen to ensure successful completion of the subsequent mission. This maintenance policy is called selective maintenance.

Cassady et al. [3] solved the selective maintenance problem for repair of failed components. They considered series-parallel systems and assumed that the states of the components, as well as the system were binary. The system and components that may be in two possible states – either working or failed – are said to have binary states and are called binary systems and binary components, respectively. The binary system reliability was maximized in [3], and cost and time were considered as available resources. Further, Cassady et al. [4] included age as a factor and considered that the components' lifetimes follow the Weibull distribution.

Lust et al. [5] established that Tabu search was useful in solving the selective maintenance problems. Selective maintenance for a series-parallel arrangement in a manufacturing system is applied in Zhu et al. [6]. They minimized maintenance cost during the maintenance break and production loss during the next mission when limited time was available for maintenance. Their study focused on a binary system with minimal repair, replacement, and a fixed maintenance level as maintenance options for components.

All of the above works focused on the binary systems. However, some sys-

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Y. Liu and M.J. Zuo, Book Chapter, *Selective Maintenance for Complex Systems Considering Imperfect Maintenance Efficiency*, pages 17-49. World Scientific (Singapore). DOI:10.1142/9789814571944_0002."

tems can perform their tasks with various discrete levels of efficiency known as "performance rates," varying from perfect operation to complete failure. Such a system is defined as a multistate system (MSS). Only a few researchers have addressed the problem of selective maintenance in a multistate system. Chen et al. [7] proposed a preliminary work on the selective maintenance optimization for a multistate series-parallel system. Their study did not show the desired maintenance on a component or component's state after maintenance, nor did it provide information about the system configurations and their relationship to the system reliability. Another work on selective maintenance for an MSS was done by Liu and Huang [8]. In their study, individual components within an MSS could have only two possible states, either working or failed; however, the system could have multiple states.

If the MSS is considered, components can also exhibit multiple performance levels. In this chapter, a selective maintenance problem is formulated for a MSS with multistate components. Based on the state of the components before maintenance, system demand during the next mission, and available resources, the desired components' states after maintenance is determined. Hence, maintenance actions and resources required for each component are found in this chapter. Multiple resource considerations in selective maintenance modeling of an MSS and their effect on maintenance decisions are also investigated. To solve the above problem, the following assumptions are used in this chapter:

1. The system consists of multiple, repairable components.
2. The components, as well as the system may be in multiple states; that is, both the components and the system have several discrete performance levels.
3. Replacement brings the component back to the best possible state.
4. Maintenance is possible only during a maintenance break; no repair/maintenance can be performed during a mission (the system and the components only degrade during operation). Maintenance may bring a component to a better state.
5. At the end of a mission, the current component/system states are observable.

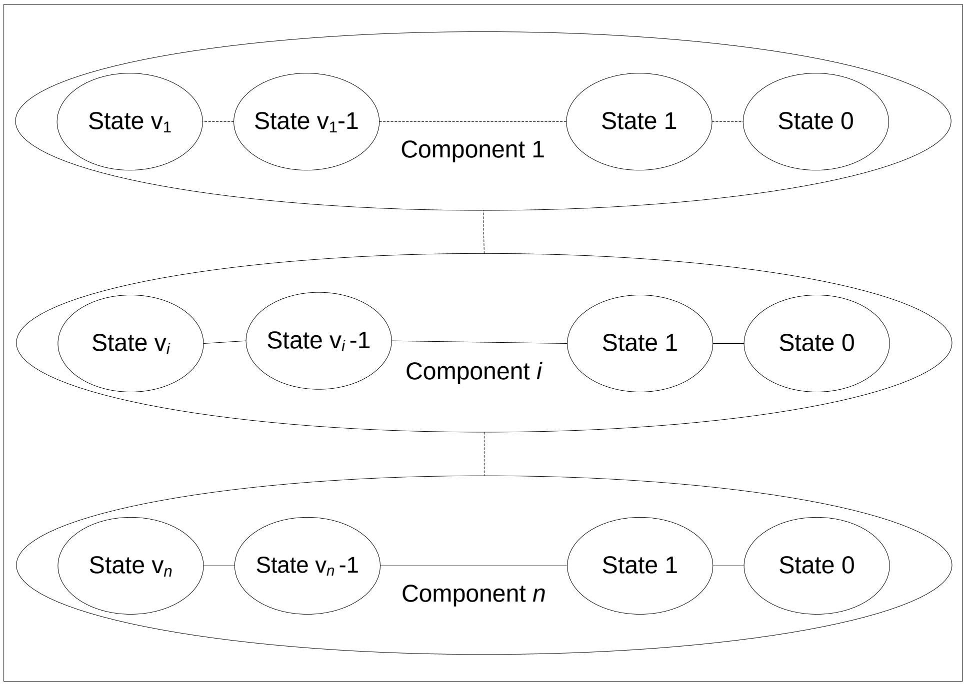


Figure 5.1: Different multistate components in an MSS

6. System degradation is modeled using a homogeneous Markov model, that is, the transition time between the component states follows the exponential distribution.
7. Limited resources (budget and time) are available, and the amount of resources required for maintenance activities is known and fixed.

5.2 System description and maintenance modeling

5.2.1 System description

All technical systems are designed to perform their intended tasks in a given environment. Some systems can perform their tasks with various distinguished levels of efficiency, usually referred to as performance rates [9, 10]. In an MSS with multistate components, each component in the system can have discrete performance rates (also called states) as shown in Fig.5.1.

In the given multi-state series-parallel system, the states of the components are statistically independent, that is, the sojourn time of a component in a state (that is the time spent by a component in a state) is independent of the sojourn time of another component in its own state. If the state of component i is denoted by j_{c_i}, then "$j_{c_i} = 0$" is the complete failure state and "$j_{c_i} = v_i$" is the best possible state. Corresponding to each state is an associated performance rate, as given in equation (5.1):

$$g_i(t) = \{g_{i,0}, g_{i,1}, ..., g_{i,v_i}\}, \qquad i = 1, 2, ..., n\ . \tag{5.1}$$

In this chapter $g_i(t)$ is used to denote the performance rate of a component i at any instant $(t \geq 0)$. It is a discrete random variable and can have any value from $g_{i,0}$ to g_{i,v_i}. The MSS performance rate is a random variable that depends on the components' performance rates:

$$G(t) = \Psi(g_1(t), ..., g_n(t))\,. \tag{5.2}$$

Depending on the possible combination of components' performance rates at any instant t, the system performance rate can have any discrete performance value from the set $G(t) = \{G_0, G_1, ..., G_S\}$, where $G_0, G_1, ...,$ and G_S are the performance rates that the system can have at any given time.

Based on the components performances, first the subsystems performances and then the complete system performance can be estimated. In the case of a group of n_S components connected in parallel, the performance rate of the subsystem is the sum of the performances of the components as given in equation (5.3). For a series arrangement, subsystem performance equals to the minimum of the performances of n_S components (equation 5.4).

$$g^{subsystem}(t) = \sum_{i=1}^{n_S} g_i(t)\,. \tag{5.3}$$

$$g^{subsystem}(t) = \min_{1 \leq i \leq n_S} (g_i(t))\,. \tag{5.4}$$

At any instant t, the performance level $G(t)$ of a series-parallel system can be evaluated as:

$$\begin{aligned} &g_\gamma^{subsystem}(t) = \textstyle\sum_{i=1}^{n_\gamma} g_i(t)\,, \quad \gamma = 1, 2, ..., s, \\ &G(t) = \min_{1 \leq \gamma \leq s} \left(g_\gamma^{subsystem}(t)\right). \end{aligned} \tag{5.5}$$

Nourelfath et al. [11] and Shrestha et al. [12] modeled the relationship between the performances of the system and its components. At any instant t, the system performance can be described completely if components' performance levels are known. System degradation takes place over its working time duration [13]. During a maintenance break, maintenance action on a component is determined on the basis of the current states of the components, how the action will affect the system performance rate during the next mission after maintenance, and how much resources are available for maintenance [14]. Resources are required to perform maintenance, and for different maintenance options, resource consumption varies. In the following section, a model is presented to describe the maintenance options,associated costs and required time.

5.2.2 Maintenance options

Whenever a system comes in for maintenance after a mission, a component may be in any of the possible states j_{c_i}, $(0 \leq j_{c_i} \leq v_i)$. In this chapter, the decision variable x_i denotes the state to which a component i is maintained during the maintenance break using available maintenance options (ζ_i). Depending on the current state of component i, the following maintenance actions (ζ_i) are feasible:

1. **Do nothing (DN):** No action is done. Leave the component as is. In this condition, decision variable $x_i = y_i$, that is, component state before maintenance y_i is the same as the component state after maintenance x_i.

2. **Component Replacement (CR):** A new component is installed in place of the old component. After replacement, the component state becomes $x_i = v_i$.

3. **Imperfect repair/maintenance (IR/IM):** Whenever repair/maintenance is done to improve the current degraded state of a component from complete failure $(y_i = 0)$/non-failure $(0 < y_i < v_i - 1)$ state to a higher improved state (other than v_i), it is defined as imperfect repair/maintenance. In this case, $y_i < x_i < v_i$. Imperfect repair (IR) implies that

the component was in the failed state (state "0") when it came to the maintenance depot. Now, if some repair action (but not replacement) is performed on it such that its performance level is improved, then it is called imperfect repair. On the other hand, imperfect maintenance (IM) refers to the maintenance action performed on a non-failed component such that its performance is further improved without replacement. Corresponding to each maintenance option is an associated required resource as discussed in the next section.

5.2.3 Maintenance cost

For maintenance of a multistate system comprising multistate components, the current state of components is assumed to be known when they come in for maintenance. Based on the current performance level of a component before maintenance and the decision variable x_i $(y_i \leq x_i \leq v_i)$, maintenance on a component can be categorized as DN, CR, or IR/IM. During a maintenance break, the maintenance cost for a component i can be formulated as follows:

$$C_{i,x_i} = c_{i,x_i}^{fix} + c_{i,x_i}. \tag{5.6}$$

The fixed term c_{i,x_i}^{fix} represents the cost of maintenance irrespective how extensive the maintenance is. It may cover actions such as cleaning, oiling, dusting, disassembling, and assembling of the component. The variable term c_{i,x_i} reflects the extent of the maintenance. The variable cost will depend on the improvement in the component state from the current state y_i to a higher state x_i . If no maintenance is performed on the component, then there is no cost incurred during the maintenance break for that component. Hence, for $x_i = y_i$ $c_{i,x_i}^{fix} = 0$, $c_{i,x_i} = 0$, and $C_{i,x_i} = 0$. In all other maintenance options, some fixed cost c_{i,x_i}^{fix} will be experienced. If a component i is replaced during a maintenance break, i.e., $x_i = v_i$, then $c_{i,x_i} = C_i^R$, where C_i^R is the replacement cost for component i. In the case of IR/IM options, where component i state after maintenance is $y_i < x_i < v_i$, there will be a cost associated with each of the possible maintenance options. In the following section, possible variations of cost with imperfect repair/maintenance options is discussed.

5.2.4 Variations of cost for imperfect repair/maintenance options

For an MSS, imperfect maintenance/repair options bridge the maintenance gap between the "do nothing" case and the replacement of the component. Generally, an increase in the used maintenance budget results in a better system performance. Thus, we have assumed that a higher performance level of a component (higher state) can be reached only through a higher maintenance cost. Imperfect repair/maintenance brings the component from the current state to a higher performance state (other than the best possible state, as the best performance state can be achieved only through replacement). For example, if there are several component performance states available to which a component can be repaired/maintained during a maintenance break, then selecting the better imperfect maintenance option can achieve a higher component performance level. At the same time, the cost of maintenance will also go up.

With the above discussions of the fixed and the variable maintenance costs for all components, the total maintenance cost for the whole system can be determined as:

$$C = \sum_{i=1}^{n} C_{i,x_i}. \tag{5.7}$$

It is obvious from equation (5.7) that with decision variable x_i for each component i, the maintenance cost for individual components and thus for the whole system can be determined.

5.2.5 Maintenance time

Similar to the maintenance cost, the time to perform maintenance for a component i can be expressed as:

$$T_{i,x_i} = t_{i,x_i}^{fix} + t_{i,x_i}. \tag{5.8}$$

In the case of no maintenance decision on a component i, i.e., $x_i = y_i$, both t_{i,x_i}^{fix} and t_{i,x_i} are equal to zero. For $x_i = v_i$, $t_{i,x_i} = T_i^R$, where T_i^R is the time to replace the component i. In the case of IR/IM, there is an associated time

t_{i,x_i} related to each option x_i. Based on the decision variable x_i, corresponding time spent maintaining each component can be determined using a method similar to determining the cost. Hence, maintenance time for the whole system is determined as:

$$T = \sum_{i=1}^{n} T_{i,x_i}. \tag{5.9}$$

Both the cost function and time function depend on the maintenance performed on a component. Decision about a particular maintenance action for a component depends on the available resources and the action's effect on the system performance. System reliability during the next mission is used as an indicator of the component's performance, which in turn depends on the components' state probabilities.

5.3 Component state probability and system reliability evaluation

An MSS's behavior is characterized by its evolution in the space of states [15]. An MSS state is said to be acceptable, if it is higher than some minimum desired performance level. When the system enters into an unacceptable state, it is said to enter into failure. An MSS's reliability can be defined as its ability to remain in the acceptable states during the whole mission [9]. Maintenance is performed during a mission break such that the probability of the system to stay in the acceptable states during the next mission increases. Hence, reliability of the system during the next mission can be used to find the effect of different maintenance actions on the system performance.

5.3.1 Component state probability evaluation

Once a system arrives for maintenance, different maintenance actions are possible for components depending on their current states revealed after inspection. We assume that the next mission under consideration dictates the given degradation behavior during the mission. During the next mission duration (L), no maintenance of any component is possible. However, components degrade during mission time; hence, it is important to evaluate the state probabilities for

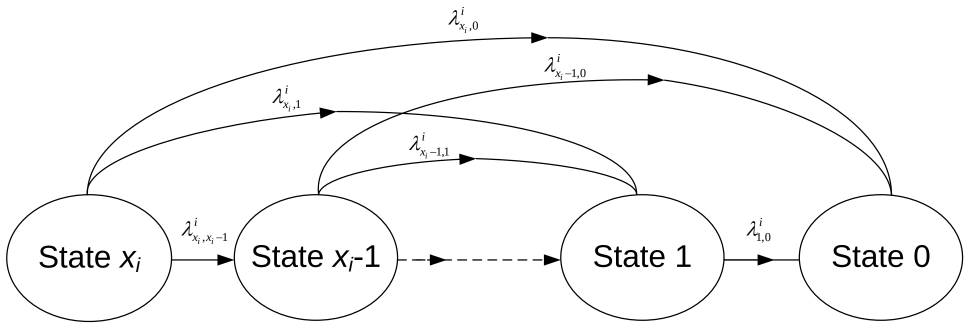


Figure 5.2: Component degradation during a mission

all components at the end of a mission. Once components' states probabilities are known, the probabilities associated with acceptable system states can be determined. It is assumed that for a component in the system, the transition time from one state to another follows the exponential distribution. Details about the transition rate estimation can be found in Welton and Ades [16], Gupta et al. [17], and Moghaddass and Zuo [18]. In the present chapter, constant transition rates between any two states, e.g., $\lambda_{1,2}^{i}$ between states 1 and 2 of component i, are assumed to be known for all components. The continuous time Markov chain of component i for the next mission is given in Fig.5.2.

To find the state probabilities $p_{i,j_{c_i}}(t,x_i)$, a system of differential equations (also called the Chapman-Kolmogorov equation) is solved [9]. For the state-space diagram shown in Fig.5.2, the set of differential equations can be written as follows:

$$\begin{aligned}
&p_{i,x_i}^{'}(t,x_i) = -p_{i,x_i}(t,x_i) \times \textstyle\sum_{j_{c_i}=0}^{x_i-1} \lambda_{x_i,j_{c_i}}^{i},\\
&p_{i,k_i}^{'}(t,x_i) = \textstyle\sum_{j_{c_i}=k_i+1}^{x_i} [\lambda_{j_{c_i},k_i}^{i} \times p_{i,j_{c_i}}(t,x_i)] - p_{i,k_i}(t,x_i) \times \textstyle\sum_{j_{c_i}=0}^{k_i-1} \lambda_{k_i,j_{c_i}}^{i},\\
&k_i = 1,2,3,...,x_i-1,\\
&p_{i,0}^{'}(t,x_i) = \textstyle\sum_{j_{c_i}=1}^{x_i} [\lambda_{j_{c_i},0}^{i} \times p_{i,j_{c_i}}(t,x_i)].
\end{aligned} \tag{5.10}$$

After solving the above equations, the state probabilities $p_{i,j_{c_i}}(t,x_i)$, $0 \leq j_{c_i} \leq x_i$ for all components can be determined. The probability distribution associated with the different states of a component i at any instant t can be represented by the following set:

$$p_i(t,x_i) = \{p_{i,0}(t,x_i), p_{i,1}(t,x_i), ..., p_{i,v_i}(t,x_i)\}, \tag{5.11}$$

where $p_{i,v_i}(t)$ is the probability that for component i, $g_i(t) = g_{i,v_i}$. The state probabilities satisfy the condition $\sum_{j_{c_i}=0}^{v_i} p_{i,j_{c_i}}(t,x_i) = 1$, because at any instant t, the component can be in any one of the states and all states of the component form the sample space consisting of mutually exclusive events. For the system, the instantaneous probabilities associated with different states can be represented by the following set:

$$P(t,\boldsymbol{X}) = \{P_0(t,\boldsymbol{X}), P_1(t,\boldsymbol{X}), ..., P_S(t,\boldsymbol{X})\}. \tag{5.12}$$

The system state probabilities also satisfy the condition $\sum_{J_S=0}^{S} P_{J_S}(t,\boldsymbol{X}) = 1$. Here $\boldsymbol{X} = \{x_1, x_2, ..., x_n\}$ is the decision vector comprising components' states after maintenance. Based on the components' probabilities, the system state probabilities and the probability of the system successfully completing a mission are calculated using universal generating function (UGF) [19]. UGF is easy to use and assess the MSS reliability [20, 21].

5.3.2 Universal generating function (UGF)

UGF represents the probability mass function of a discrete random variable via a polynomial form [22, 23]. For a multistate component, the performance rate distribution at any time instant t can be given by:

$$u_i(z,t,x_i) = \sum_{j_{c_i}=0}^{v_i} p_{i,j_{c_i}}(t,x_i).z^{g_{i,j_{c_i}}}. \tag{5.13}$$

In order to find the UGF of an arbitrary series-parallel system, it is necessary to apply a composition operator Φ, as given in equation (5.14). The final expression in equation (5.14) covers all possible mutually exclusive combinations and the associated probabilities corresponding to the values of the function $\Phi\left(g_{1,j_{c_1}}, g_{2,j_{c_2}}, ..., g_{n,j_{cn}}\right)$, which are determined by the structure of the system and the performance rate combination property discussed in Section 5.2.1.

$$\begin{aligned} &U_{sys}(z,t,\boldsymbol{X}) = \Phi\left[u_1(z,t,x_1), u_2(z,t,x_2), ..., u_n(z,t,x_n)\right] = \\ &\Phi\left[\sum_{j_{c_1}=0}^{v_1} p_{1,j_{c_1}}(t,x_1).z^{g_{1,j_{c_1}}}, \sum_{j_{c_2}=0}^{v_2} p_{2,j_{c_2}}(t,x_2).z^{g_{2,j_{c_2}}}, ..., \sum_{j_{cn}=0}^{v_n} p_{n,j_{cn}}(t,x_n).z^{g_{n,j_{cn}}}\right] = \\ &\sum_{j_{c_1}=0}^{v_1}\sum_{j_{c_2}=0}^{v_2}...\sum_{j_{cn}=0}^{v_n}\left[\prod_{i=1}^{n} p_{i,j_{c_i}}(t,x_i)\, z^{\Phi\left(g_{1,j_{c_1}}, g_{2,j_{c_2}}, ..., g_{n,j_{cn}}\right)}\right]. \end{aligned} \tag{5.14}$$

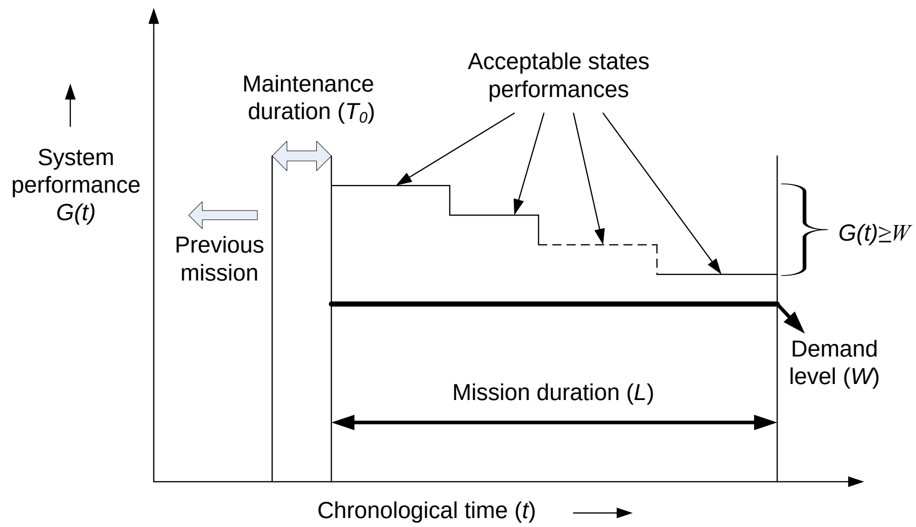


Figure 5.3: System performance during a mission and demand level

The composition operator $\Phi = \Phi_{PAR} = sum\left[g_1\left(t\right), ..., g_{n_s}\left(t\right)\right]$, in the case of parallel arrangement; and $\Phi = \Phi_{SER} = min\left[g_1\left(t\right), ..., g_{n_s}\left(t\right)\right]$, in the case of series arrangement of components. For the MSS, the system states and associated probabilities during the next mission can be represented as:

$$U_{sys}\left(z, t, \boldsymbol{X}\right) = \sum_{J_S=0}^{S} P_J\left(t, \boldsymbol{X}\right).z^{G_{J_S}}. \tag{5.15}$$

With the system state distribution and the associated probabilities in hand, system reliability can be obtained, as explained in the next section.

5.3.3 System reliability evaluation

During a mission, both the system and its components are non-repairable. Thus, the states of the system and its components are monotone non-increasing. Because an MSS degrades with time, it is significant that the MSS performance does not fall below the demand level (W) at any time t during the mission, i.e., $G\left(t\right) \geq W$. Fig.5.3 illustrates that a demand level W is required to meet during a mission of duration L.

System reliability is the probability that the system is in an acceptable state $(G\left(L\right) \geq W)$ at the end of the upcoming mission (t=L). Hence, the reliability of the MSS can be calculated as:

$$Prob\left(G\left(L\right) \geq W\right) = \varphi\left(\sum_{J_S=0}^{S} P_{J_S}\left(L, \boldsymbol{X}\right).z^{G_{J_S}-W}\right), \tag{5.16}$$

where φ is the distributive operator defined by the following equation:

$$\varphi\left(P\left(L, \boldsymbol{X}\right).z^{\sigma-W}\right) = \begin{cases} P\left(L, \boldsymbol{X}\right), \text{ if } \sigma \geq W \\ 0 \ \text{, if } \sigma < \text{W} \end{cases}, \tag{5.17}$$

Hence, depending on the mission demand W and the set of maintenance decision variables $\boldsymbol{X}$, the system reliability at the end of mission duration L can be expressed as:

$$R_S\left(W, L, \boldsymbol{X}\right) = \varphi\left(\sum_{J_S=0}^{S} P_{J_S}\left(L, \boldsymbol{X}\right).z^{G_{J_S}-W}\right) = \sum_{J_S=0}^{S}\left(\varphi\left(P_{J_S}\left(L, \boldsymbol{X}\right).z^{G_{J_S}-W}\right)\right). \tag{5.18}$$

With the use of distributive operator φ, equation (5.18) can also be written as:

$$R_S\left(W, L, \boldsymbol{X}\right) = \sum_{G_{J_S}(t) \geq W} P_{J_S}\left(L, \boldsymbol{X}\right). \tag{5.19}$$

Using equation (5.19), system reliability can be estimated for the next mission.

5.4 Selective maintenance modeling and solution methodology

5.4.1 Selective maintenance modeling

Upon the arrival of the system for maintenance, although several maintenance alternatives are available, only a subset of maintenance actions can be performed due to limited resources. Thus, the selective maintenance model is aimed at: (i) identifying the components i to be selected and determine the state of the component x_i after maintenance on the selected components; finding the set of components on which no maintenance will be performed, (ii) finding the budget C_{i,x_i} to be invested in each of the selected components, (iii) determining the amount of time T_{i,x_i} to be invested in each of the selected components, and (iv) making a decision such that system reliability $R_S\left(W, L, \boldsymbol{X}\right)$ during the next mission is maximized.

The associated integer decision variable is x_i and the decision vector comprising decision variables for all components in the system is $\boldsymbol{X}$. The budget constraint on the total maintenance cost during the maintenance break is given by C_0, and the available maintenance time duration is limited to T_0. The nonlinear formulation to maximize the probability of successfully completing the next mission is expressed as:

Objective:

$$Max \qquad R_S\left(W, L, \boldsymbol{X}\right) = \sum_{G_{J_S}(t) \geq W} P_{J_S}\left(L, \boldsymbol{X}\right), \tag{5.20}$$

Subject to:

$$\sum_{i=1}^{n} C_{i,x_i} \leq C_0, \tag{5.21}$$

$$\sum_{i=1}^{n} T_{i,x_i} \leq T_0, \tag{5.22}$$

$$y_i \leq x_i \leq v_i. \tag{5.23}$$

In this formulation, constraints (5.21) and (5.22) show the limited resources available to perform maintenance; constraint (5.23) sets the component state at the beginning of the next mission depending on the state at the end of the previous mission and the maximum possible performance level for a component. This chapter focuses on a single mission selective maintenance optimization problem. The model for this one mission optimization is given in equations (5.20)–(5.23), wherein the expressions of total cost (C), total time (T), and system reliability (R_S) are given in equations (5.7), (5.9), and (5.19), respectively. This is a typical constrained nonlinear optimization problem involving integer variables only. An evolutionary algorithm has been used in this chapter to solve such optimization problems, as explained in Section 5.4.2.

5.4.2 Solution methodology

Selective maintenance optimization for multistate systems under imperfect maintenance/repair is a nonlinear programming problem, as presented in (5.20)–(5.23). Evolutionary algorithms (like genetic algorithm (GA), and differential evolution (DE)) are commonly used in the maintenance optimization [8, 22, 24, 25]. In this paper, DE [26, 27] is used to solve the selective maintenance problem. More details about the differential evolution algorithm can be found in [28, 29].

To apply an algorithm to the problem, solution representation is an important procedure that should be defined clearly. All possible maintenance alternatives for the whole system are given by a string $\boldsymbol{X} = \{x_1, x_2, ..., x_n\}$. For each solution point, the maintenance budget, time, and system reliability can be determined. For instance, let us consider an example where two components are connected in parallel and then connected with a third component in a series. Each of the components is assumed to have three states {0, 1, 2}. When the system arrives for maintenance, its component states are given by $\boldsymbol{Y} = \{y_1, y_2, y_3\} = \{1, 0, 1\}$. The system has to fulfill a constant demand of W=30 units during the next mission. Let's assume that a solution string generated to denote the components states after maintenance is $\boldsymbol{X} = \{x_1, x_2, x_3\} = \{1, 2, 2\}$. Since the state of component 1 does not change during maintenance break, $c^{fix}_{i,x_i} = \$0$, c_{i,x_i} =\$0 and $t^{fix}_{i,x_i} = 0$ unit, t_{i,x_i} =0 unit. For both components 2 and 3, replacement is done during the maintenance break. It is assumed that fixed maintenance cost $c^{fix}_{i,x_i} = \$1$, replacement cost $c_{i,x_i} = C^R_i = \$2$, fixed maintenance time $t^{fix}_{i,x_i} = 1$ unit and replacement time t_{i,x_i} =$T^R_i = 2$ units, respectively for both components 2 and 3. The assumed capacity of each state of the components, the state of each component before and after maintenance, and the state probability distribution at the end of the next mission for all components are given in Table 5.1.

Component 1 is in state 1 at the beginning of the mission. Since no maintenance is possible during a mission, the probability for component 1 to be in higher state 2 is 0 at the end of the mission. Components 2 and 3 start the mission with their best possible states, that is, state 2. Hence, at the end of the mission, both components could be in any of the three possible states. This is shown by their respective probability distribution. The maintenance cost and time for component 1 are $C_{i,x_i} = c^{fix}_{i,x_i} + c_{i,x_i} = \$0$ and $T_{i,x_i} = t^{fix}_{i,x_i} + t_{i,x_i} = 0$ unit, respectively. For both components 2 and 3, main-

Table 5.1: Information about the multistate components

	Subsystem 1						Subsystem 2		
	Component1			Component2			Component3		
Possible States	0	1	2	0	1	2	0	1	2
Capacities of states	0	20	30	0	25	50	0	30	60
State before maintenance	1			0			1		
State after maintenance	1			2			2		
State probabilities at the end of the mission	0.3	0.7	0	0.1	0.4	0.5	0.2	0.4	0.4

tenance cost and time are $C_{i,x_i} = c_{i,x_i}^{fix} + c_{i,x_i} = 1 + 2 = \$3$ and $T_{i,x_i} = t_{i,x_i}^{fix} + t_{i,x_i} = 1 + 2 = 3$ units, respectively. For these maintenance actions, system reliability is evaluated using UGF. The UGF of a subsystem's components is defined according to equation (5.13) as: $u_1(z, L, 1) = 0.3z^0 + 0.7z^{20} + 0\ z^{30}$, $u_2(z, L, 2) = 0.1z^0 + 0.4z^{25} + 0.5z^{50}$, and $u_3(z, L, 2) = 0.2z^0 + 0.4z^{30} + 0.4z^{60}$. The system UGF is then obtained according to equation (5.14):

$U_{sys} = \Phi_{SER}(\Phi_{PAR}(u_1, u_2), u_3)$. Therefore,

$U_{sys} = \Phi_{SER}(\Phi_{PAR}((0.3z^0 + 0.7z^{20} + 0\ \mathrm{z}^{30}), (0.1z^0 + 0.4z^{25} + 0.5z^{50})),$
$(0.2z^0 + 0.4z^{30} + 0.4z^{60})) =$
$\Phi_{SER}(\Phi_{PAR}(0.03z^0 + 0.07z^{20} + 0.12z^{25} + 0.28z^{45} + 0.15z^{50} + 0.35z^{70}),$
$(0.2z^0 + 0.4z^{30} + 0.4z^{60})) =$
$0.224z^0 + 0.056z^{20} + 0.096z^{25} + 0.312z^{30} + 0.112z^{45} + 0.06z^{50} + 0.14z^{60}$.

To calculate the Prob($G(L) \geq W$), that is, for the total capacity of the system to be not less than the required demand level W=30 units, the φ operator is applied, as given in equation (5.18). Hence,

Prob($G(L) \geq W$)=

$$\varphi\begin{pmatrix} 0.224z^{0-30} + 0.056z^{20-30} + 0.096z^{25-30} + 0.312z^{30-30} \\ + 0.112z^{45-30} + 0.06z^{50-30} + 0.14z^{60-30} \end{pmatrix}$$

$\therefore R_S = 0.312 + 0.112 + 0.06 + 0.14 = 0.624 = 62.4\%$.

Similarly, maintenance cost, time, and associated system reliability for other possible maintenance alternatives can be evaluated. A self-explanatory flowchart of solution methodology is provided in Fig.5.4. In the next section, an illustrative example is provided and the results are discussed to explain the benefits of the proposed model for selective maintenance.

5.5 Results and discussion

To demonstrate the advantages of the proposed selective maintenance model, an illustrative example is presented in this section. It is followed by detailed results and related discussions. System performance based on the performance rates of individual components is established and the benefits of imperfect maintenance in selective maintenance decision-making is provided.

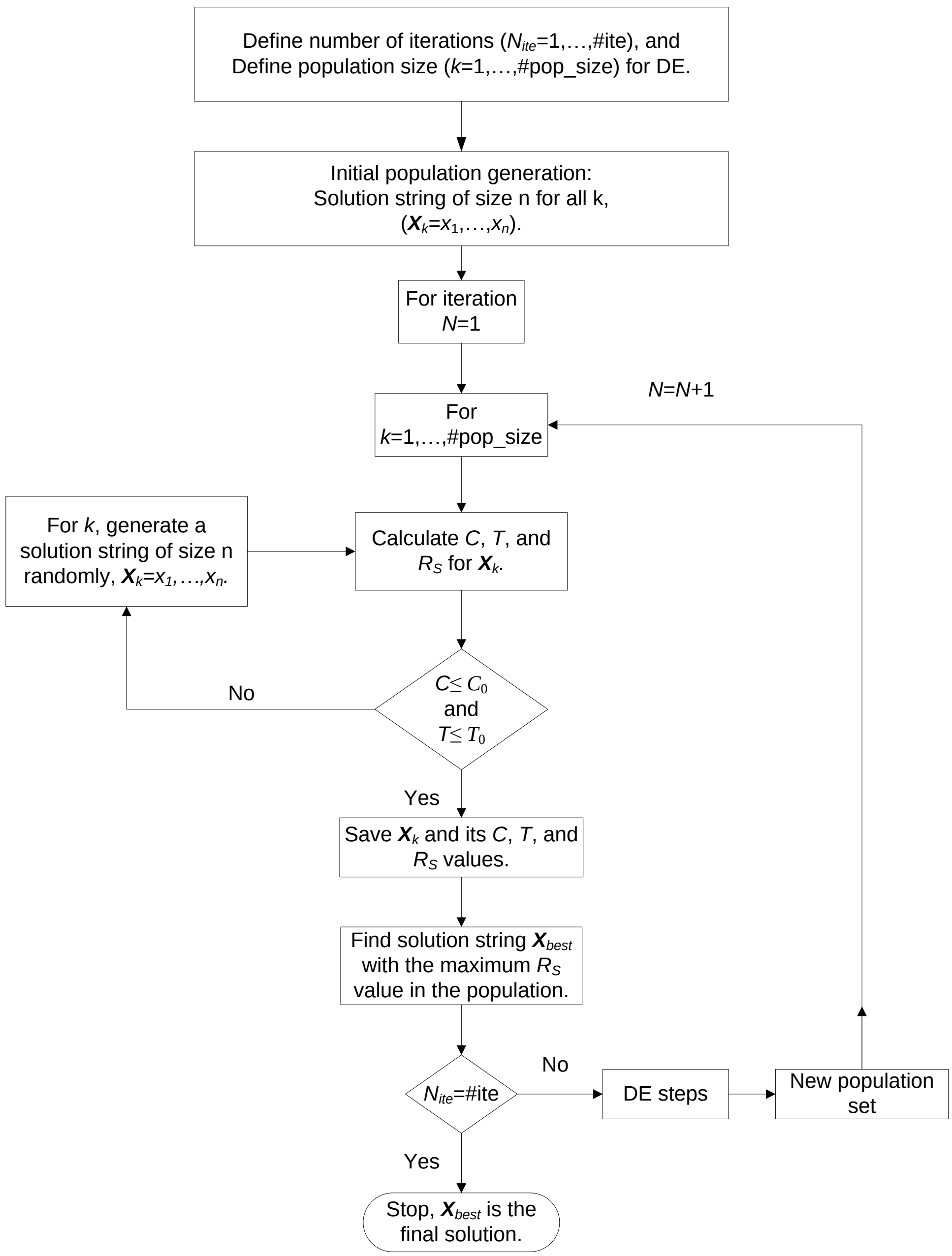


Figure 5.4: Flowchart of the solution methodology

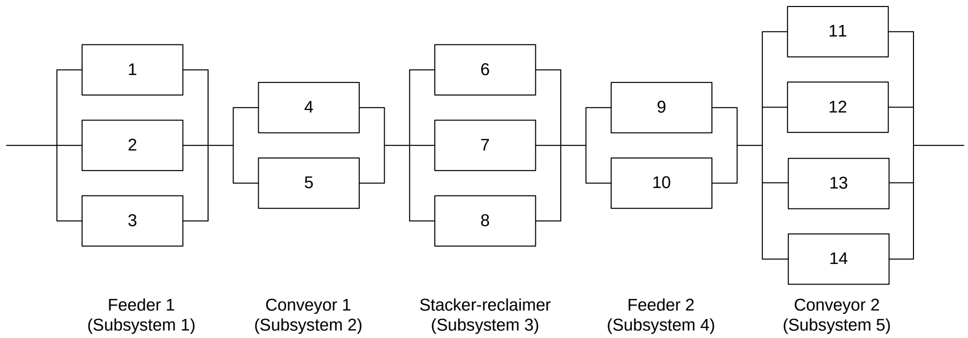


Figure 5.5: Block diagram of a coal transportation system

5.5.1 Illustrative example

A multistate coal transportation system with multistate components is studied as an example, which is taken from Liu and Huang [30]. This coal transportation system is used to supply coal to a boiler in a power station. It includes five basic subsystems, as shown in Fig.5.5. Feeder 1 transfers coal from the bin to conveyor 1. Conveyor 1 transports the coal from feeder 1 to the stacker reclaimer that lifts the coal up to the burner level. Feeder 2 then loads conveyer 2, which transfers the coal to the boiler's burner feeding system. Each of the subsystems consists of a different number of components, and each component has different performance rates and associated load capacities, as shown in Table 5.2.

A component stays in its current state for a random amount of time (following exponential distribution) and then makes a transition to a lower state. The process is characterized by transition intensities. The transition intensity for each component is given in Table 5.3. It is assumed in this example that for imperfect maintenance/repair actions, the associated cost and time vary linearly as: $c_{i,x_i} = \left(\frac{g_{i,x_i} - g_{i,y_i}}{g_{i,v_i}}\right) \times C_i^R$ and, $t_{i,x_i} = \left(\frac{g_{i,x_i} - g_{i,y_i}}{g_{i,v_i}}\right) \times T_i^R$. The components' states before maintenance and their maintenance cost and time parameters are provided in Table 5.4.

In Table 5.4, the values of y_i are known constants, c^i_{fix}, t^i_{fix} values are the fixed cost and time of maintenance for a component. If no maintenance is performed on a component, then $c^{fix}_{i,x_i} = 0$ and $t^{fix}_{i,x_i} = 0$; otherwise $c^{fix}_{i,x_i} = c^i_{fix}$ and $t^{fix}_{i,x_i} = t^i_{fix}$. The cost and time parameters are used if a component is selected

Table 5.2: Capacities of each component (tons/day) [30]

Component	State	0	1	2	3	4
Subsystem 1	C1	0	40	60	80	-
	C2	0	50	80	100	-
	C3	0	20	60	80	-
Subsystem 2	C4	0	70	120	-	-
	C5	0	90	130	-	-
Subsystem 3	C 6	0	40	80	100	-
	C7	0	30	60	80	-
	C8	0	30	70	90	-
Subsystem 4	C9	0	30	50	80	-
	C10	0	40	80	120	-
Subsystem 5	C11	0	10	40	60	80
	C12	0	25	50	70	90
	C13	0	25	45	75	95
	C14	0	25	65	80	100

Table 5.3: State transition intensities (per year) [30]

Comp(i)	$\lambda^i_{1,0}$	$\lambda^i_{2,0}$	$\lambda^i_{2,1}$	$\lambda^i_{3,0}$	$\lambda^i_{3,1}$	$\lambda^i_{3,2}$	$\lambda^i_{4,0}$	$\lambda^i_{4,1}$	$\lambda^i_{4,2}$	$\lambda^i_{4,3}$
1	0.5	0.2	0.3	0.25	0.2	0.2	-	-	-	-
2	0.3	0.2	0.3	0.15	0.3	0.2	-	-	-	-
3	0.2	0.4	0.3	0.2	0.4	0.3	-	-	-	-
4	0.5	0.3	0.2	-	-	-	-	-	-	-
5	0.2	0.2	0.2	-	-	-	-	-	-	-
6	0.4	0.2	0.25	0.3	0.4	0.3	-	-	-	-
7	0.3	0.15	0.12	0.3	0.2	0.4	-	-	-	-
8	0.2	0.1	0.18	0.15	0.3	0.5	-	-	-	-
9	0.4	0.2	0.1	0.2	0.2	0.4	-	-	-	-
10	0.3	0.2	0.15	0.2	0.2	0.3	-	-	-	-
11	0.5	0.08	0.2	0.2	0.3	0.4	0.2	0.12	0.2	0.2
12	0.2	0.3	0.2	0.2	0.2	0.25	0.15	0.2	0.15	0.25
13	0.3	0.2	0.15	0.15	0.3	0.3	0.2	0.12	0.3	0.3
14	0.2	0.2	0.3	0.25	0.2	0.4	0.115	0.15	0.25	0.2

Table 5.4: Maintenance cost and time for components (costs in thousands of dollars and time in days)

Comp(i)	1	2	3	4	5	6	7	8	9	10	11	12	13	14
y_i	1	1	1	1	2	2	3	2	2	3	3	2	1	2
c^i_{fix}	1.2	1.0	1.1	1.1	1.2	0.75	1.15	0.8	1.4	1.0	0.8	1.1	1.3	0.6
t^i_{fix}	0.25	0.25	0.25	0.30	0.30	0.15	0.30	0.1	0.40	0.20	0.15	0.25	0.35	0.35
C_i^R	20	15	20	14	20	10	15	12	20	15	10	15	18	12
T_i^R	2	1.5	2	1.25	2	1	1.5	1.2	2	1.5	1	1.5	1.75	1.25

for maintenance, and they do not depend on either the state of the component entering or exiting the maintenance depot. However, the total maintenance cost and time for a component depend on the state of the component exiting the maintenance depot. For example, if for component 1, $y_i = 1$; then, $g_{i,y_i} = 40$ units. Let's assume that $x_i = 2$, hence $g_{i,x_i} = 60$ units. For component 1, the maximum capacity is $g_{i,v_i} = 80$ units. Table 5.4 gives that C_i^R is 20 units and T_i^R is 2 units for component 1. Hence we get $c_{i,x_i} = 5$ units and $t_{i,x_i} = 0.5$ units, respectively. Now, using equations (5.6) and (5.8) and c_{fix}^i, t_{fix}^i values from Table 5.4, we get $C_{i,x_i} = 1.2 + 5 = 6.2$ units and $T_{i,x_i} = 0.25 + 0.5 = 0.75$ units, respectively. Thus, with the given information, maintenance cost and time for all components can be calculated for any maintenance decision. Let's assume that the next mission length is L= 0.5 years and the system demand is W=50 tons/day. It is necessary to perform maintenance within the available budget and/or given time window such that the maximum system reliability is achieved.

To solve the problem and compare the results, the example is analyzed in detail and selective maintenance decisions and associated cost and time are calculated. In the following discussion, 1 cost unit=$1000 and 1 time unit = 1 day.

5.5.2 Selective maintenance decision with cost constraint only

At first, only a cost limit of 100 units is assumed (C_0=$100,000). Table 5.5 shows the results. When CR is the only possible maintenance action, the maximum achievable system reliability is 0.9308. At the beginning of maintenance duration, components #1, 2, 3, 4, and 13 were in the failed state, that is, state 0. Of these components, components #1, 2, 4, and 13 are selected for replacement, while component #3 is left in state "0." The remaining components #5, 6, 7, 8, 9, 10, 11, 12, and 14 were at some of the intermediate states when the system arrived for maintenance. Of these, only component # 9 is selected for replacement. The cost incurred for the complete selective maintenance strategy is 93 units, and the time taken is 10.05 units.

Table 5.5: Selective maintenance decision with only cost as constraint (costs in thousands of dollars and time in days)

	Comp i	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	v_i	3	3	3	2	2	3	3	3	3	3	4	4	4	4
	y_i	0	0	0	0	1	1	2	1	1	2	2	1	0	1
	x_i	3	3	0	2	1	1	2	1	3	2	2	1	4	1
	Actions	CR*	CR	DN*	CR	DN	DN	DN	DN	CR	DN	DN	DN	CR	DN
I*	C_{i,x_i^*}	21.20	16	0	15.10	0	0	0	0	21.40	0	0	0	19.30	0
	T_{i,x_i^*}	2.25	1.75	0	1.55	0	0	0	0	2.40	0	0	0	2.10	0
		R_s=0.9309, C=93 units, T=10.05 units													
	x_i	2	2	3	2	1	2	2	2	2	2	2	1	2	2
	Actions	IR*	IR	CR	CR	DN	IM*	DN	IM	IM	DN	DN	DN	IR	IM
II*	C_{i,x_i}	16.20	13	21.10	15.10	0	4.75	0	6.13	6.40	0	0	0	9.83	5.40
	T_{i,x_i}	1.75	1.45	2.25	1.55	0	0.55	0	0.63	0.90	0	0	0	1.18	0.55
		R_s=0.9634, C=97.91 units, T=10.81 units													

*I= Only replacement as option, II= imperfect maintenance included with replacement as option. C_{i,x_i} =the budget used in the maintenance of component i, T_{i,x_i} = the time used in the maintenance of component i, DN= do nothing, IM= imperfect maintenance, IR= imperfect repair, CR= component replacement.

When IR/IM is also considered as a maintenance option along with the CR, a system reliability of 0.9634 is achieved, an increase of more than 3%. It is because with only CR as maintenance option, the budget consumed is only 93 units. Of the total 100 units available, 7 units remain unused. On the other hand, when IM/IR is included as maintenance options, the total cost incurred is 97.91 units. This shows that almost the entire available budget is utilized in order to achieve a higher reliability during the next mission. Also, only components #3 and 4 are selected to be replaced in this case, while components #1, 2, and 13 undergo IR and components #6, 8, 9, and 14 are selected for IM actions.

5.5.3 Selective maintenance decision with both time and cost constraints

Often, both cost and time impose restrictions on the available maintenance options. In addition to the cost limit of 100 units, a time limitation of 10 units is included here. The results when both time and cost are involved as constraints are presented in Table 5.6. Table 5.6 shows that when only replacement is considered as an option, the maximum achievable system reliability is 0.91774. The total maintenance cost and maintenance time are 86.30 units and 9.25 units, respectively. However, in the case of IR/IM, the maximum achievable reliability increases to 0.9613, which is about 5% higher than the case of replacement-only actions. In IR/IM case, the cost and time used during maintenance are 87.50 units and 9.76 units, respectively. It is clear that resources are better utilized if IR/IM is included as options for selective maintenance rather than choosing replacement only. If only replacement is a maintenance option, then components #1, 2, 4, 9, and 14 are selected. For IR/IM case, only component #4 is replaced. Components #1, 2, 3, and 13 undergo IR, while components #6, 8, and 9 have IM.

It is also evident from Table 5.5 and Table 5.6 that an optimal allocation of resources is critical to achieve the maintenance goal. With an increase in the number of resource restrictions, it becomes even more vital to determine the resource allocation. In Table 5.5, when only the available budget is limited, the maximum achievable reliability for replacement and imperfect maintenance options is 0.9308 and 0.9634 with consumption of 10.05 and 10.81 time units, respectively. However, when maintenance time is also considered and restricted to T_0=10 units, this will change the maintenance decision, that is, components to be selected, as well as required maintenance to be performed on the selected components. In the case of the imperfect maintenance option without time constraint, component #3 is replaced and #14 undergoes IM. However, when time constraint is added, #3 undergoes IM while no action is done on #14.

5.5.4 Sensitivity of maintenance resources

Sometimes, a maintenance crew has flexibility of resources, i.e., a limit on one resource but flexibility on another. In such conditions, it is critical to find the consequence of resource variation on the final system reliability so that optimal resource allocation can be performed. Hence, sensitivity of selective maintenance decisions with respect to the resource limitation becomes important and needs to be investigated. The effect of variation of time and cost limits on system reliability is shown in Fig.5.6, which illustrates the variation of cost with reliability for different time limits (6, 8, 10, and 16 units).

Such a plot (Fig.5.6) is helpful in the budget and time allocation to achieve a certain reliability limit. For instance, 93.82% system reliability is achievable with 8 time units and investment of 60 cost units. However, if 95% system reliability is desired within 8 units of maintenance duration, then a maintenance budget of 80 cost units will be required.

It can also be observed from Fig.5.6 that if one resource is constrained and the other resource continues to increase, then there is a limit after which no increase in the system reliability is possible. For example, with T_o=6 units, the maximum achievable system reliability is 91.89%, which is achieved with the cost consumption of 80 units. Further increase in the maintenance budget is useless, as there is no time available to consume that extra cost. Hence, no further increase in the system reliability is possible. A similar observation can be found if the cost limit is kept constant and the time limit varies. As shown for C_o=80 unit, increasing T_o to more than 10 units does not improve

Table 5.6: Selective maintenance decision with both replacement and imperfect maintenance/repair as options (costs in thousands of dollars and time in days)

	Comp i	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	v_i	3	3	3	2	2	3	3	3	3	3	4	4	4	4
	y_i	0	0	0	0	1	1	2	1	1	2	2	1	0	1
I*	x_i	3	3	0	2	1	1	2	1	3	2	2	1	0	4
	Actions	CR*	CR	DN*	CR	DN	DN	DN	DN	CR	DN	DN	DN	DN	CR
	C_{i,x_i^*}	21.20	16	0	15.10	0	0	0	0	21.40	0	0	0	0	12.60
	T_{i,x_i^*}	2.25	1.75	0	1.55	0	0	0	0	2.40	0	0	0	0	1.30
		R_s=0.91774, C=86.3 units, T=9.25 units													
II*	x_i	2	2	2	2	1	2	2	2	2	2	2	1	2	1
	Actions	IR*	IR	IR	CR	DN	IM*	DN	IM	IM	DN	DN	DN	IR	DN
	C_{i,x_i}	16.20	13	16.10	15.10	0	4.75	0	6.13	6.40	0	0	0	9.83	0
	T_{i,x_i}	1.75	1.45	1.75	1.55	0	0.55	0	0.63	0.90	0	0	0	1.18	0
		R_s=0.9613, C=87.5096 units, T=9.7623 units													

*I= Only replacement as option, II= imperfect maintenance included with replacement as option. C_{i,x_i} =the budget used in the maintenance of component i, T_{i,x_i} = the time used in the maintenance of component i, DN= do nothing, IM= imperfect maintenance, IR= imperfect repair, CR= component replacement.

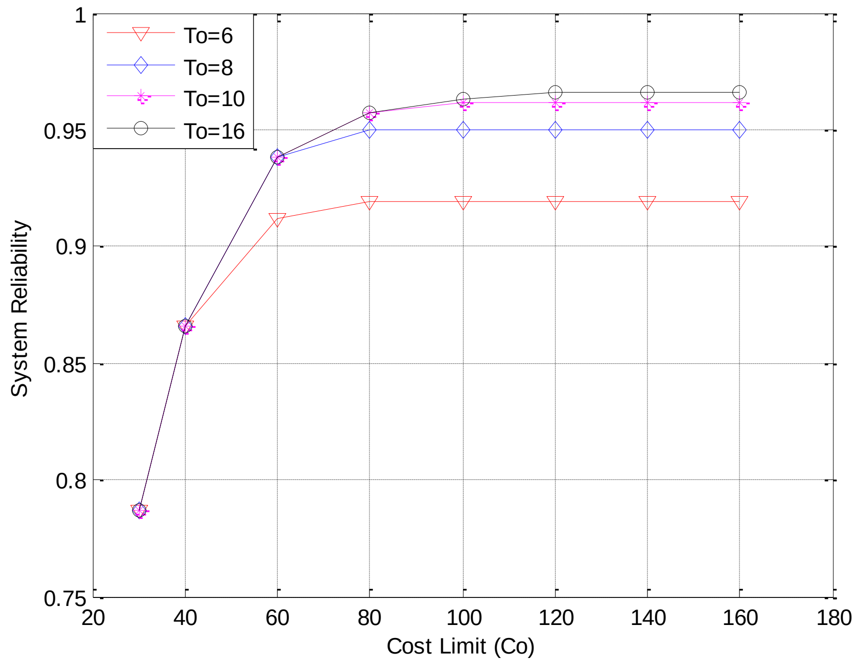


Figure 5.6: Sensitivity of system reliability with resource variation

the system reliability beyond 95.75%.

Such an analysis is helpful in determining whether using extra resources would improve system performance considerably. As can be seen for T_o=10 and 16 units, an increase in the budget limit C_o from 80 to 100 units leads to a less than 1% increase in system reliability. Hence, the maintenance manager can decide whether it is worth spending extra time or cost for a minimal change in the system reliability.

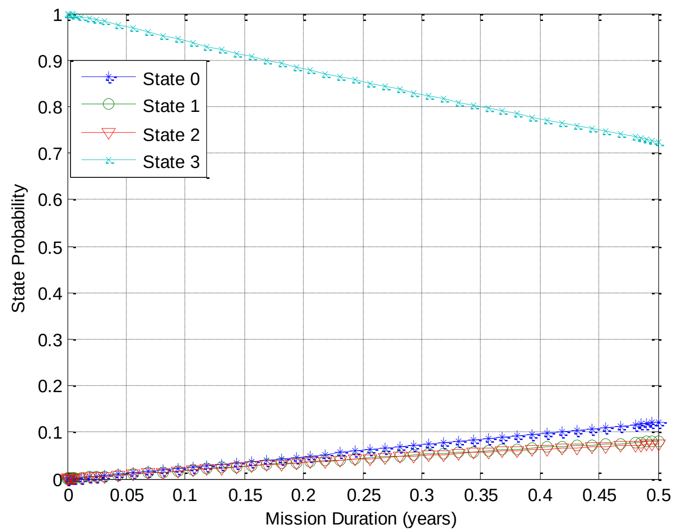
5.5.5 Component state probability variation during mission

After maintenance, each component is set at a distinct performance level, that is, its performance level is known and fixed right after maintenance. In other words, the probability that the component i is in state x_i at time t=0 is 1, i.e., $p_{i,x_i}(0, x_i) = 1$. But as the mission time progresses, depending on the transition rates from one state to another, component's states probabilities change. State probability for state x_i decreases from 1 while state probability of other states $j_{c_i} < x_i$ increases over mission duration. The state probability for states $j_{c_i} > x_i$ remain zero because the system is assumed to be non-repairable during a mission. Hence, once in a lower performance state at the beginning of the mission, a component cannot be brought to a higher performance state during the mission. To illustrate this, variation of state probabilities for components #1 and #5 are given in Fig.5.7, with x_i=3 and 1 (the replacement-only case in Table 5.5), respectively.

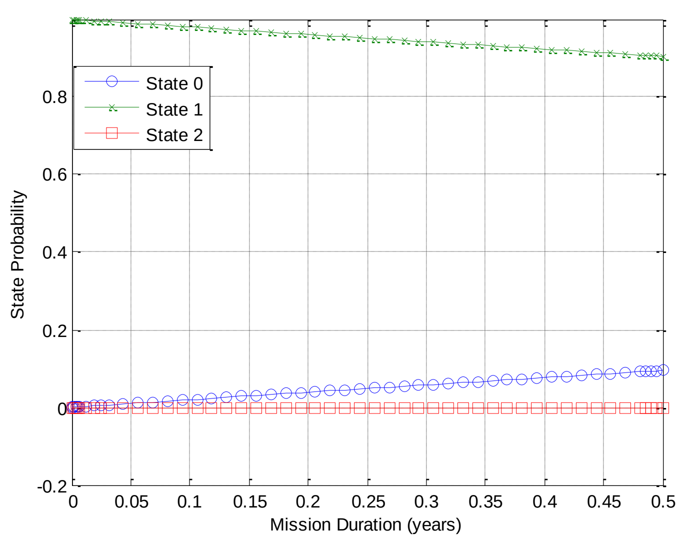
State x_i= 3 is the best possible state for component #1; as the mission progresses, the probability in state 3 decreases while for the probabilities in states 2, 1, and 0 increase. For component #5, state x_i=1 is an intermediate state. Hence, the probability of a lower state "0" increases while that of state 1 decreases over the mission duration. Because state 2 cannot be reached at any time during the mission, its probability value is always zero in the case of component #5. The state probabilities of all other components follow a similar variation over the mission duration. It can also be seen in Fig.5.7 that at any instant t during the mission duration, the sum of the state probabilities is 1, i.e. $\sum_{j_{c_i}=0}^{\nu_i} p_{i,j_{c_i}}(t, x_i) = 1$.

The example considered in this paper has 14 components and each has up to five states. We have solved differential equations given in equation (5.10) and the optimization problem using MATLAB. In a computer with an AMD Opteron 2Ghz processor and 3 GB RAM, it took around 3150 seconds to solve the problem, as described in Section 5.5.3 with both the time and cost as constraints, and get the final solution. In practice, a system may have a large number of components. However, the number of states of each component is usually not that high. Most reported works on multistate systems considered 3 to 6 state levels [20, 31]. Nevertheless, even if the number of states for each component is much larger (e.g., in the tens), today's computing packages are capable of handling the differential equations and the optimization processes. However, if the number of components is very large (e.g., in the hundreds), then the solution methodology used in this paper may not be suitable. Thus, it is a challenge to apply the proposed model to a very large system.

It can be concluded from the results that the selective maintenance under



(a)



(b)

Figure 5.7: Variation of state probabilities (a) Component #1 (b) Component #5

imperfect maintenance/repair provides better mission reliability. It is also observed that with a change in the number of resources, not only the selected components but also the maintenance actions on selected components change. The allocation of resources depends on the state of the components and the overall system performance. Further, it is useful for the maintenance manager to get information about the sensitivity of maintenance resources and the system performance. In the case of a flexible resource environment, investigating the variation of the system performance with the resources helps in the maintenance decision-making.

5.6 Summary

In this chapter, the selective maintenance problem is addressed for multistate systems subject to degradation during a mission. It is assumed that the components within the system may also be in multistate. A generalized maintenance model is developed where the relationship between the level of maintenance and consumed resources (cost and time) is established. The reliability evaluation for multistate system is performed for different maintenance options that depend on the available maintenance resources. An illustrative example is used and comparisons are provided between the replacement-based and imperfect maintenance/repair–based selective maintenance policies. Incorporating imperfect maintenance/repair yields better system performance.

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Chapter 6

Conclusion and Future Work

Maintenance is of prime importance for the reliable operation of engineering systems. Maintenance of a system may include minimal repair, replacement or imperfect maintenance. This thesis studies the selective maintenance modeling for systems, with special attention paid to the effect of imperfect maintenance. To thoroughly study the selective maintenance, this thesis research is divided into four stages. At the first stage, a single mission selective maintenance model is developed for a binary system with only maintainable failures. The model developed in the first stage is then extended into three parallel directions, which are the next three stages in this research. The three tasks associated with the next three stages are: to incorporate the maintainable and non-maintainable failure modes in the selective maintenance modeling, to solve the selective maintenance scheduling problem comprising successive multiple missions in a finite planning horizon, and to develop a selective maintenance model for a multistate system with multistate components. This chapter summarizes my contributions to selective maintenance modeling, describes some problems that remain to be addressed, and suggests directions for future work.

6.1 Summary and conclusion

6.1.1 Selective maintenance for binary systems under imperfect maintenance

Most of the engineering systems work under different load and stress conditions, and they tend to deteriorate over time. As a system ages, its response to the resources used during maintenance may get affected. A newer component is easy to maintain, whereas a relatively older component may need more resources in restoring it to a better condition. A formulation is proposed relating the maintenance quality with the effective age and the maintenance cost. It is found that for the same investment in a relatively younger component as compared to an older component, better improvement can be achieved. An expression for the characteristic constant "m" is also proposed, which defines whether a component is relatively young or old. A hybrid imperfect maintenance model is used to consider the effect of maintenance/repair on the component's health. It includes both the age reduction and hazard adjustment factors. This assumption is more realistic and more general. Based on the next mission reliability, the selective maintenance model is formulated. Comparisons between the proposed model and earlier models show that incorporating imperfect maintenance/repair into selective maintenance yields better system output.

6.1.2 Selective maintenance considering maintainable and non-maintainable failure modes

It is possible that not all failure modes in a system are affected by maintenance. Some failure modes in a system may be maintainable while others may be non-maintainable. Corresponding to these failure modes are associated hazard rates. A formulation is proposed in this thesis to relate these hazard rates when imperfect maintenance of a component is possible. Cost and age based imperfect maintenance factors are derived when the two types of failure modes are present in the system. Also, changes are incorporated into the imperfect maintenance model to incorporate both types of hazard rates and their relationship.

The relationship between the maintainable and non-maintainable failure modes is found to be important, which may affect the selective maintenance decision. The stronger the relationship is, the smaller achievable system reliability is for the next mission. It is found that the cumulative effect of the non-maintainable hazard rate should be considered up to the current time point, and not only up to the previous maintenance break as suggested in previous literature. Otherwise, a reliability value higher than the actual system reliability is obtained, which may mislead the maintenance crew about the actual system condition.

Furthermore, a comparison is provided when only cost or both cost and time are used as the available resources. It is found that more resource constraints change the selected components as well as the maintenance decision on a selected component. A resource sensitivity analysis has also been performed to help in deciding whether it is useful to invest extra resources to increase the system reliability.

6.1.3 Selective maintenance scheduling over a finite planning horizon

To complete a mission in the given planning horizon, maintenance is required to keep a system reliable. It is possible that maintenance performed only once at the beginning of the mission is not sufficient. It is then needed to schedule maintenance breaks in the given planning horizon. A selective maintenance scheduling model is developed in this thesis for a given finite planning horizon where imperfect maintenance based PM modeling is proposed. During a PM, the cumulative effect of the previous age reductions and hazard adjustments to a component is considered. The characteristic constant formulation is modified for this purpose. An expression is derived to find its value for each component considering the previous maintenance history.

Furthermore, it is found that assuming maintenance duration as negligible is not advisable because it affects the selective maintenance decision in a finite planning horizon. Maintenance duration is included in the proposed selective maintenance scheduling model. The total cost of maintenance and failure during the entire planning horizon is minimized such that maintenance between the successive missions are carried out within available time, and the desired minimum system reliability is maintained during each mission. An example is solved to find the optimal number of missions and maintenance breaks. It is found that the determination of the optimal schedule for a known planning horizon is beneficial and it can minimize the total cost.

6.1.4 Selective maintenance of a multistate system with multistate components under imperfect maintenance

Components in a multistate system can also have multiple states. Therefore, a selective maintenance model is developed for a multistate system with multistate components. An imperfect maintenance based selective maintenance model is developed, which depends on the resources utilized in improving the performance level of a component during the maintenance break. The reliability evaluation of a multistate system is performed. Based on the probability of successfully completing the mission, components are selected for maintenance and their performance level after maintenance is decided. An illustrative example is presented for this problem. Comparisons are provided between the replacement-based and imperfect maintenance/repair–based selective maintenance policies. Incorporating imperfect maintenance/repair yields better system performance.

6.2 Future work

Although the structure of this thesis is defined in the sense that important challenges and limitations of the current models in selective maintenance are covered; there are still some problems that need to be further addressed. Also, the proposed models have some new challenges, which need to be further described.

1. It is assumed in the selective maintenance scheduling that minimal repair is performed on a component if it fails during a mission. In some applications where it is not easy to perform repair of failed components during a mission, the effect of probable failure of a component on PM schedule can be studied in more explicit way. Furthermore, maintainable and non-maintainable failure modes can be included in the scheduling problem.

2. In the selective maintenance modeling for an MSS, the transition time between component states are assumed to follow the exponential distribution. In a more general case, other distributions, such as the Weibull distribution can be used.

3. It is assumed throughout this thesis that the components within the system are independent. However, it is possible that components within a system are dependent and failure of one component affects the failure of another component in the system. In such a case, the selection of components and maintenance decision pose a challenge that need to be addressed.

Additionally, a trade-off between the maintenance budget, time and mission reliability, as well as other resources (e.g., multiple repairmen) need to be solved. Multi-objective optimization approaches may be utilized to address this problem. In this thesis, research is not done on the solution methodology to solve the selective maintenance problems. Different solution approaches can be used and results can be compared for the proposed models. The differential evolution is used to solve the problems in this thesis. The proposed models can also be solved with the deterministic approaches like branch and bound method and results can be compared. Also, the computational complexity of the problem increases with increase in the number of components. If the number of components within a system are too many, say in hundreds, then it is a challenge to solve the problem.

Appendix A

In this thesis, the differential evolution (DE)[1] is used to solve the non-linear optimization problem of selective maintenance. Here, the steps involved in the differential evolution is presented. Like other evolutionary algorithms, DE is a population-based and stochastic global optimizer. DE starts with a population of size NP wherein each member of the population is a n–dimensional vector representing a candidate solution. Each individual can be represented as: $\mathbf{\Theta}_i = \theta_i(1), \theta_i(2), ..., \theta_i(n), i = 1, 2, ..., NP$. Starting from a randomly initialized population $\mathbf{POP} = \mathbf{\Theta}_1, \mathbf{\Theta}_2, ..., \mathbf{\Theta}_{NP}$ in the feasible solution domain, the DE algorithm employs mutation and crossover operators to generate new candidates (offsprings). Then one-to-one selection scheme is applied to determine whether the offspring or the parent survives in the next generation. The above process is repeated until a predefined termination criterion is reached.

In the DE algorithm, a mutant vector, $\mathbf{\Upsilon}_i = \gamma_i(1), \gamma_i(2), ..., \gamma_i(n), i = 1, 2, ..., NP$ is generated using a mutation operator. This mutation strategy can be described as follows:

$$\mathbf{\Upsilon}_i = \mathbf{\Theta}_{best} + F \times (\mathbf{\Theta}_{r1} - \mathbf{\Theta}_{r2}), \qquad \text{(A.1)}$$

where $\mathbf{\Theta}_{best}$ is the best individual in the current parent population, $\mathbf{\Theta}_{r1}$ and $\mathbf{\Theta}_{r2}$ are two individuals randomly selected from the current parent population such that $r1 \neq r2 \neq i \in \{1, 2, ..., NP\}$, and $F > 0$ is a mutation scale factor for scaling the differential variation between the two individuals. Following mutation, a crossover operation is performed to increase the potential diversity

[1] J. Brest, S. Greiner, B. Bokovi, M. Mernik, and V. Zumer. Self-adapting control parameters in differential evolution: A comparative study on numerical benchmark problems. IEEE Transactions on Evolutionary Computation, 10(6):646-657, 2006.

of the population. A trial vector $\mathbf{\Omega}_i = \omega_i(1), \omega_i(2), ..., \omega_i(n), i = 1, 2, ..., NP$ is generated by considering mutant vectors and its corresponding parent individual as follows:

$$\omega_i(j) = \begin{cases} \gamma_i(j), & if \quad r_j \leq CR \quad or \quad j = n_j, \\ \theta_i(j), & otherwise, \end{cases} j = 1, 2, ..., n, \tag{A.2}$$

where n_j is an index randomly chosen from the set $1, 2, ..., n$ to ensure that at least one dimension of the trial individual $\mathbf{\Omega}_i$ differs from its counterpart $\mathbf{\Theta}_i$ in the current generation, CR is the crossover probability in the range $[0, 1]$, and $r_j \in \{0, 1\}$ is a uniformly generated random number.

The selection scheme is based on the survival of the fittest between the trial vector $\mathbf{\Omega}_i$ and its parent counterpart $\mathbf{\Theta}_i$. For a maximization problem, it can be given as follows:

$$\mathbf{\Theta}_i = \begin{cases} \mathbf{\Omega}_i, & if \quad f(\mathbf{\Omega}_i) \geq f(\mathbf{\Theta}_i), \\ \mathbf{\Theta}_i, & otherwise, \end{cases} \tag{A.3}$$

where $f(\mathbf{\Omega}_i)$ and $f(\mathbf{\Theta}_i)$ are the objective function values of $\mathbf{\Omega}_i$ and $\mathbf{\Theta}_i$, respectively. If the problem is minimization type, then rather than checking for the inequality $f(\mathbf{\Omega}_i) \geq f(\mathbf{\Theta}_i)$ in equationA.3, $f(\mathbf{\Omega}_i) \leq f(\mathbf{\Theta}_i)$ is checked.

This new set of population again undergoes the DE steps of mutation, crossover and selection, and the process is repeated until the stopping (termination) criterion is reached.