The Logic for Metaphysical Conceptions of Vagueness

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Abstract

Vagueness is a phenomenon whose manifestation occurs most clearly in linguistic contexts. And some scholars believe that the underlying cause of vagueness is to be traced to features of language. Such scholars typically look to formal techniques that are themselves embedded within language, such as supervaluation theory and semantic features of contexts of evaluation. However, when a theorist thinks that the ultimate cause of the linguistic vagueness is due to something other than language – for instance, due to a lack of knowledge or due to the world's being itself vague – then the formal techniques can no longer be restricted to those that look only at within-language phenomena. If, for example a theorist wonders whether the world itself might be vague, it is most natural to think of employing many-valued logics as the appropriate formal representation theory. I investigate whether the ontological presuppositions of metaphysical vagueness can accurately be represented by (finitely) many-valued logics, reaching a mixed bag of results.

Keywords: Vagueness, Many-valued Logic, Evans-argument.

Introduction

Even though people sometimes point to vague memories (e.g., of that very first date you had) or vague objects (like the cloud above me as I write, or the mist that covered St. Petersburg a few nights ago), it is in language where vagueness most clearly manifests itself, and where most theorists focus their attention. The reasons for this are not hard to fathom:

- The majority of our linguistic terms admit borderline cases;

- We are unable to resolve the application vs. non-application of many scalar predicates;

- We sometimes may not be able to determine what proposition (if any) is asserted when using certain vague terms.

But even if it is in the realm of language where we find vagueness manifested, there is still the question What is the "ultimate" cause of the vagueness? Is it perhaps a matter of lack of knowledge? Perhaps lack of knowledge of some relevant features of the world? Or perhaps lack of knowledge of the relevant context? Or is it instead that the precise language is correctly representing a vague reality? Or is it merely that language itself does not completely and precisely represent (the non-vague, precise) reality?

It is traditional to divide viewpoints concerning the ultimate cause of vagueness into three sorts: (a) Epistemological Vagueness, where vagueness is claimed to be due to a lack of knowledge – an inability to tell whether some statement is true or false, even though it might correctly represent reality or represent it incorrectly; (b) Linguistic Vagueness, where vagueness is claimed to be due to a shortcoming in the language itself – the language is not adequate to correctly or fully represent the detailed features of the world; and (c) Metaphysical (or Ontological) Vagueness, where vagueness is claimed to be inherent in reality – our language correctly represents reality, but these items are themselves vague. We will look briefly at each in turn, before we focus on the use (and motivation for the use) of many-valued logic.

Most accounts of vagueness, of all these different types, focus on *properties* that can manifest vagueness, particularly properties that characterize a "scale" – such as tallness, or being a heap, or intelligence, or .... Less time has been spent on the possibility of vague *objects*.[1] (Of course, some scholars think that one way to have a vague object is for it to manifest one of the vague properties in a vague manner.) In this paper we investigate the vague objects more closely than vague properties, although of necessity we talk also about vague properties.

1 Epistemological vagueness

The natural way to understand this viewpoint on vagueness is that the "world" is precise, determinate, definite, and so on, but our apprehension of these precise facts is limited in one way or another by our finite epistemological powers. In the "world"

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[1]Compare the differences in focus and detail of the papers [1] and [20].

there are objects that have precise boundaries, and all properties have sharp cut-off points (or maybe: all objects are in the clear extension or anti-extension of all the properties).

However, because we lack knowledge, vagueness is introduced: For example, old Prof. Worthington, the ancient don at Wembley College, only vaguely/indeterminately/fuzzily remembers who was present at his Doctoral Viva. He can *almost* remember someone with white or maybe blond hair. But he can't recall clearly whether it was the long-dead Coppleston or the equally dead Millingston.

That was a case of "individual vagueness" on Worthington's part. But there can be wider and wider cases of vagueness: *no one* quite remembers what the priest looked like at old Dr. Benoit's baptism. And maybe it is even more pervasive: a feature of the way the world has developed (all relevant people have died, and no one left any unambiguous memoirs) – for instance, did Galileo *actually* drop balls of different weights from on high? And did he also tether together different weighted balls in order to determine how fast the composite object fell? These are events that actually happened or didn't happen – totally and completely – in the actual world. But since there is now no evidence of any sort to decide which way the world actually went, we say it is vague whether Galileo dropped balls of different weights from a height. One might even go so far as to say that this is the category of "verifiable in principle but not actually verifiable".

And it could be more radical than this: For example, the Epistemological Vagueness position holds that in reality there is *in fact* a particular number of grains of sand that would make this pile of sand be a heap (say, m grains). However, we *can't know* that m grains of sand make a heap because all the evidence that we (or anyone) have available is the same for adding one grain of sand to an $(m-2)$-grains pile as it is for adding one grain of sand to an $(m-1)$-grains pile. (Since by hypothesis we can't discern a change when only one grain is added). Yet in the former case we don't know that a pile has become a heap (because by hypothesis it hasn't). So in the latter case we can't know either (even though it *has* become a heap).[2]

What would be an appropriate representational medium for this conception of vagueness? Well, since the view holds that

- *in the world* there is no indeterminacy... every factual sentence either is true or is false, every object is unique, distinct, and separate from all others, and

- vagueness comes from a lack of positive or negative knowledge of these facts, including lack of knowledge as to what proposition is being asserted,

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[2]The epistemic conception of vagueness is most famously championed by [21, 22, 29].

it seems to follow that some sort of epistemic logic is called for. Thus the epistemic interpretation really involves two logics: classical, two-valued logic for "the world" and the just-mentioned epistemic logic to accommodate the state of knowledge of people. Vagueness seems then just to be identified with conceptual indeterminacy on the part of a speaker. Such an epistemic logic would employ a modal operator that means "is vague", but of course, in this conception, being vague is interpreted as being epistemically indeterminate, and so something can be *non*-vague by being definitely (in the epistemic sense) false, as well as by being definitely true (again, in the epistemic). Thus, if something is vague (epistemically), then so is its negation, under this conception. Using ∇ to represent this indeterminacy (and $\triangle$ for determinacy), typical postulates of such a logic include, among others

$$\text{if } \vDash \varphi, \text{ then } \triangle\varphi$$

$$\triangle\varphi \leftrightarrow \triangle\neg\varphi$$

$$\triangle\varphi \leftrightarrow \neg\nabla\varphi$$

So the required modal logic couldn't be a Kripke-normal modal logic. In [12], I proposed a class of logics of epistemic vagueness (or epistemic indeterminacy): every statement is in fact either true or false (at a world), but when inside the epistemic vagueness operator, we are to evaluate what is going on at a certain class of related worlds. But as I mentioned, this class is not determined in a classical Kripke-manner, but rather in terms of "neighbourhood semantics".

2 Linguistic vagueness

Linguistic Vagueness posits the same ontology as Epistemological Vagueness, namely that the "world" is precise, determinate, definite, and so on. But it differs from the epistemological version by saying that our *description* of these precise facts is limited in one way or another, rather than our knowledge of the precise facts. It holds that in the "world" there are objects that have precise boundaries, and all properties have sharp cut-off points. Vagueness in this conception is a matter of a kind of mismatch between language and "the world" and not a matter of a mismatch between people's knowledge and "the world", as it is in the epistemological conception. (Of course, different versions of Linguistic Vagueness will have differing accounts of what specific parts of language exhibit the mismatch.)

One version of this mismatch might hold, for example, that when Allen says that George is tall, the name 'George' picks out some specific individual in the world (namely, George) who has some specific height such as 180 cm. But it might

hold that there is no such *primitive* property in the world as being tall, for only the specific heights count as primitive properties. In this view, either the property TALLNESS doesn't exist, or if it does, then at least it is not a "basic" property[3] but is instead defined, in one way or another, in terms of the more basic, specific properties and (perhaps) "contexts of use" (as in some of the "contextual theories of vagueness", [8, 16, 19]).

It is a shortcoming of our language, according to some (but not all) of the believers in Linguistic Vagueness, that it has developed with these sorts of predicate-terms. Some also hold it to be a shortcoming of our language that *the denotation relation* is not precise: the name 'Mt. Everest' does not unproblematically designate a specific region of the Earth; so when people use this linguistic term they are not accurately identifying what is the case "in the world". When a person says "This rock is a part of Mt. Everest", the imprecision of the denotation relation forces the sentence as a whole to be vague.[4]

Advocates of the explication of vagueness in terms of a linguistic mismatch have formed the largest group of philosophers, at least starting with Frege. Some were dismayed by the fact that natural language had vague predicates, and saw the ideal language as remedying this[5]:

> We have to throw aside concept-words that do not have a *Bedeutung*. These are...such as have vague boundaries. It must be determinate for every object whether it falls under a concept or not; a concept word which does not satisfy this condition on its *Bedeutung* is *bedeutungslos*. [7, p. 178]

Some others who also thought that vagueness was linguistic believed instead that it was a *good* thing in natural language:

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[3]I use 'basic' and "primitive' in an intuitive manner, allowing that the relevant theories will be obliged to provide a detailed analysis of these notions.

[4]This view of vagueness – although without the feeling that it is a shortcoming – is expressed in [10]:

> The only intelligible account of vagueness locates it in our thought and language. The reason it's vague where the outback begins is not that there's this thing, the outback with imprecise borders; rather, there are many things, with different borders, and nobody's been fool enough to try to enforce a choice of one of them as the official referent of the word 'outback'. (p. 212)

A similar view is expressed in [27].

[5]Actually, it is very difficult to find any theorist of vagueness – of whatever sort – who thinks that vagueness is a shortcoming in language as a whole. What is more problematic, they would say, is the use of some vague term or phrase in a context where more precision, accuracy, or definiteness is desired and is available for use but just not chosen.

> ...a vague belief has a much better chance of being true than a precise one, because there are more possible facts that would verify it. ...Precision diminishes the likelihood of truth." [18, p. 91]

> Vagueness is a natural consequence of the basic mechanism of word learning. The penumbral objects of a vague term are the objects whose similarity to ones for which the verbal response has been rewarded is relatively slight. ...Good purposes are often served by not tampering with vagueness. Vagueness is not incompatible with precision. [15, pp. 113–115]

> There are contexts in which we are much better off using a term that is vague in a certain respect than using terms that lack this kind of vagueness. One such context is diplomacy. [2, pp. 85–86]

(For example, "We will take strenuous measures to block unwanted aggression whenever and wherever it occurs" allows for a wide course of actions, whereas any non-vague statement would not allow such freedom.)

What would be an appropriate representational medium for this conception of vagueness? Well, since the view holds that

- *in the world* there is no indeterminacy...every factual sentence that uses only the *basic* predicates and the correct denotation relation either is true or is false, and

- vagueness comes from the use of non-basic predicates (and "ambiguously denoting" singular terms) where there is no relevantly determined method of stating how they are related to the basic predicates,

it seems to follow that some semantic technique is needed for displaying the various types of results that might hold between the non-basic predicates used in some linguistic expression and the basic predicates that describe "the world".

For example, one might decide that one class of non-basic predicates actually are abbreviations of some (ordered) range of the basic predicates, and that it is "context" that determines which part of this ordered range is relevant to evaluating the truth value of the expression. (Supervaluations and maybe some other semantic techniques, as introduced by [23, 24], and developed by [3, 28], are plausible candidates for this sort of evaluation, as are some of the theories that employ context, like [8, 16, 19]).

Unlike the Epistemic conception of vagueness in which every (declarative) sentence either *is* true or *is* false (but in some cases we may not know which, so that vagueness is a type of epistemic shortfall), in the Linguistic conception only *some*

sentences are true and only *some* are false. Among the ones that are true or are false are those composed with basic predicates (and no funny stuff with the denotation relation). Many of the sentences containing non-basic predicates will be given the value 'vague' (i.e., 'neither true nor false'). But not all of these latter type of sentence will be vague, as for example when the specific object of a predication *clearly* satisfies the predicate. For instance, when 200 cm. in height LeBron James is said to be tall, this is true despite the vagueness inherent in 'tall'.

And there can even be true (also false) sentences about the tallness of middle-height people... and similarly for other non-basic terms. For example, supervaluation theory allows that classical logical truths and contradictions are true/false. And perhaps different semantic techniques, such as contextual theories, could generate other examples.

3 Ontological vagueness

Ontological/Metaphysical/Realistic Vagueness locates vagueness "in the world". So, as opposed to being unclear as to whether a situation actually obtains or not (Epistemic Vagueness), and as opposed to being vaguely described by a language that contains non-basic predicates (Linguistic Vagueness), Realistic Vagueness claims that certain objects in the world just plain are vague. (The intent here, which I will in general follow, is to target physical objects with this characterization, although it might also apply to abstracta, events, relations, and so on.) Few writers have explained it, but [18], who is an advocate of Linguistic Vagueness, assures us that it used to be a common view: "...it is a case of the fallacy of verbalism – the fallacy that consists in mistaking the properties of words for the properties of things."

One might also point to fictional entities as neither having nor lacking certain properties: Hamlet neither has nor lacks a 5 mm wart on his left shoulder. Even though this example is from the realm of fiction, Realistic Vagueness might claim that for an vague actual object, there is some property which it neither has nor lacks.

As is well-known, [5] claims that all views advocating ontological vagueness must invoke the claim that, for certain names a and b, the sentence $a = b$ is neither definitely true nor definitely false. (That is, Ontological Vagueness predicts that there are vague objects in the world, and when we have vague objects, then whether they are or are not the same object can also be indeterminate, at least according to some advocates. This gives at least a sufficient condition for metaphysical vagueness.) [25, 26] also proposed that the crucial test would be a situation in which the question 'In talking about x and y, how many things are we talking about?' has the

features that 'none' is definitely a wrong answer; 'three', 'four', etc., are all definitely wrong answers; and neither 'one' nor 'two' is either definitely a right or definitely a wrong answer to it. I call this viewpoint about what is the underlying feature of Ontological Vagueness the Evans/van Inwagen criterion, or, when discussing specific argumentation that turns on exactly how this criterion is to be represented formally, the Evans assumption (or the van Inwagen assumption), and when considering the argumentation that makes use of the Evans assumption I call it Evans' argument and sometimes *an* Evans argument (to emphasize that, while it is not exactly Evans' argument, it is an argument inspired by Evans' argument).

What would be an appropriate representational medium for this conception of vagueness? Well, since the view holds that

- *in the world* there is indeterminacy... vague objects actually have the *real* property: being neither red nor not-red, for example, and

- for any object/property pair, either the object has the property (definitely), or the object lacks the property (definitely), or else it neither has nor lacks it – and this last fact is, in its own way, just as definite as the former two,

it seems that the appropriate representation of this conception will employ a many-valued logic. If a has property F, then Fa is true; if a lacks F, then Fa is false; so in the case where a neither has nor lacks F, Fa must take on some other truth value (counting 'neither True nor False' as a truth value).

Employing a modal logic would not accurately capture Realistic Vagueness, for a modal logic presumes that in each world, every sentence either *is* true or *is* false. Employing unusual semantic techniques also does not adequately capture Realistic Vagueness, for the Realist insists that *all* the properties under discussion *are* in fact "real" and "basic". Only a many-valued logic could capture the Realist's attitude toward vagueness.

And it is to many-valued logics that I now turn.

4   A 3-valued logic embodying vagueness

There are three values: intuitively, TRUE, FALSE, INDETERMINATE.[6] These are taken to describe *three different ways the actual world might relate to a sentence describing it*. That is, the portion of the actual world that is under discussion is *actually* one of: definitely the way being described, definitely not the way being described, or correctly described as indeterminate.

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[6]We turn later to logics with more than three values, when we discuss the possibility of describing "degrees of vagueness" as an account of "higher-order vagueness".

We would like our language to be able to express the facts that sentence φ is TRUE, FALSE or INDETERMINATE (calling these semantic values T, F, I). So let us invent sentence operators ("parametric operators") that do that: $\boldsymbol{D_t}, \boldsymbol{D_f}, \boldsymbol{V}$. They are ordinary, extensional logic operators, having the following truth tables.

φ	$\boldsymbol{D_t}\varphi$	$\boldsymbol{D_f}\varphi$	$\boldsymbol{V}\varphi$
T	T	F	F
F	F	T	F
I	F	F	T

We use standard 3-valued (Łukasiewicz) interpretations of negation, and, or. (And use the convention that the truth values are ordered: $T > I > F$).

φ	$\neg\varphi$
T	F
I	I
F	T

$$[\varphi \wedge \psi] = \min([\varphi], [\psi])$$
$$[\varphi \vee \psi] = \max([\varphi], [\psi])$$

I am going to steer clear of the intricacies involved in the interpretation of the conditional and biconditional, other than to advocate on behalf of these principles:

$$[\boldsymbol{D_t}\text{-AXIOM}]: \quad \vDash \boldsymbol{D_t}\varphi \to \varphi$$
$$[\boldsymbol{EQ}\text{-RULES}]: \text{If } \vDash (\varphi \leftrightarrow \psi), \text{then infer } \vDash (\boldsymbol{D_t}\varphi \leftrightarrow \boldsymbol{D_t}\psi)$$
$$\text{If } \vDash (\varphi \leftrightarrow \psi), \text{then infer } \vDash (\boldsymbol{D_f}\varphi \leftrightarrow \boldsymbol{D_f}\psi)$$
$$\text{If } \vDash (\varphi \leftrightarrow \psi), \text{then infer } \vDash (\boldsymbol{V}\varphi \leftrightarrow \boldsymbol{V}\psi)$$

Although neither $(\varphi \wedge \neg\varphi)$ nor $(\neg\boldsymbol{D_t}\varphi \wedge \neg\boldsymbol{D_f}\varphi)$ is a contradiction in a three-valued logic, contradictions *can* be described by insisting on the Uniqueness of Semantic Value in 3-valued logic[7].

[$\boldsymbol{USV_3}$]: Every sentence takes exactly one of the three values:

$$(\boldsymbol{D_t}\varphi \vee \boldsymbol{D_f}\varphi \vee \boldsymbol{V}\varphi) \wedge \neg(\boldsymbol{D_t}\varphi \wedge \boldsymbol{D_f}\varphi) \wedge \neg(\boldsymbol{D_t}\varphi \wedge \boldsymbol{V}\varphi) \wedge \neg(\boldsymbol{D_f}\varphi \wedge \boldsymbol{V}\varphi).$$

Lemma. *If the main operator of formula Φ is $\boldsymbol{D_t}, \boldsymbol{D_f},$ or $\boldsymbol{V}$, then $[\![\boldsymbol{V}\Phi]\!] = F$.*

Proof. If the main connective of Φ is one of the three parametric operators, then (as can be seen from their truth tables) the value of Φ is either T or F. But then $\boldsymbol{V}\Phi$ will be F. $\qquad\qquad\square$

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[7]Although in Graham Priest's logic *LP* [14], the third value is claimed to be *both T and F simultaneously*, and "not really" a different third value.

Corollary. *If all sentential parts of formula* Φ *are in the scope of any of* $\boldsymbol{D_t}$, $\boldsymbol{D_f}$, $\boldsymbol{V}$, *then* $[\![\boldsymbol{V}\Phi]\!] = F$, *or equivalently,* $[\![\neg\boldsymbol{V}\Phi]\!] = T$ *or equivalently,* $[\![\boldsymbol{D_f}\boldsymbol{V}\Phi]\!] = T$.

With regards to using a three-valued interpretation in the predicate logic (with identity), I do not give a full characterization, but only three principles:

[$\boldsymbol{V}$-$\forall$]: $\vDash \boldsymbol{V}[(\forall x)F(x)] \rightarrow \neg(\exists x)\boldsymbol{D_f}[F(x)]$

(i.e., if it is vague that everything is F, then there cannot be anything of which it is definitely FALSE that it is F)

[**ref**$_=$]: $\vDash \boldsymbol{D_t}[\alpha = \alpha]$

(i.e., self-identities are definitely TRUE).

[**LL**]: $\vDash a = b \leftrightarrow (\forall F)(Fa \leftrightarrow Fb)$

(Leibniz's Law, as this is usually called: Two things are identical if and only if they share all properties.[8])

Now, while some scholars might find some of these principles questionable (I mention Graham Priest in footnote 9 below), the holders of Ontological Vagueness have pretty much uniformly taken them on.

5   An argument against vague objects in this logic

The argument

An Evans argument proceeds by assuming the Evans/van Inwagen criterion of what the believers in Ontological Vagueness hold: that it can be vague whether there is one or two objects before a person; and it continues, using the principles mentioned above about many-valued logic. The following version is given in [13].

a.	$\boldsymbol{V}[a = b]$	the Evans assumption
b.	$a = b \leftrightarrow (\forall F)(Fa \leftrightarrow Fb)$	**LL**
c.	$\boldsymbol{V}[a = b] \leftrightarrow \boldsymbol{V}(\forall F)(Fa \leftrightarrow Fb)$	(b) and [**EQ**-RULE]
d.	$\boldsymbol{V}(\forall F)(Fa \leftrightarrow Fb)$	(a), (c), $\leftrightarrow$-elim
e.	$\neg(\exists F)\boldsymbol{D_f}(Fa \leftrightarrow Fb)$	(d) and [**V**-$\forall$]
f.	$(\forall F)[\boldsymbol{D_t}(Fa \leftrightarrow Fb) \vee \boldsymbol{V}(Fa \leftrightarrow Fb)]$	(e) and [**USV**$_3$]
g.	$\boldsymbol{D_t}[\boldsymbol{D_t}[a = a] \leftrightarrow \boldsymbol{D_t}[a = b]]\vee$	(f), instantiate $(\forall F)$ to
	$\quad\boldsymbol{V}[\boldsymbol{D_t}[a = a] \leftrightarrow \boldsymbol{D_t}[a = b]]$	$\lambda x\boldsymbol{D_t}[a = x]$ and λ-convert
h.	$\boldsymbol{D_t}[\boldsymbol{D_t}[a = a] \leftrightarrow \boldsymbol{D_t}[a = b]]$	(g), [Lemma], disjunctive syllogism
i.	$\boldsymbol{D_t}[a = a] \leftrightarrow \boldsymbol{D_t}[a = b]$	(h) and [$\boldsymbol{D_t}$-AXIOM]
j.	$\boldsymbol{D_t}[a = b]$	(i) and [**ref**$_=$], $\leftrightarrow$-elim
k.	$\neg\boldsymbol{V}[a = b]$	(j) and [**USV**$_3$]

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[8]As both [6,11] remark, Leibniz himself only took pains to argue for the right-to-left aspect of [LL], and that with a restriction on the types of properties that F designate. Presumably everybody finds the left-to-right direction of [LL] undeniable. As is often noted, there is a peculiarity with this verbalization of the formula, since the formula gives a condition for there being just one thing under consideration, not two, and says that any property this one thing has is a property it has.

Although not every pair of formulas of the form φ and $\neg\varphi$ contradict one another in a three-valued logic, (a) and (k) do really contradict each other. (a) is either T, F, or I (by [**USV₃**]); by the truth-table for **V** it cannot be I; so it is either T or F. But this argument shows that if (a) is T then it is F, but if (a) is F then it is T (à la (k) and the truth table for ¬). But [**USV₃**] claims that no formula can be both T and F.[9]

Comments & defense of the argument

Was there any "cheating" going on in this proof, or with the postulates? Is there something "funny" about the **V**-operator? Might one question the λ-abstract: is it a "real" property? Might one have concerns about one of the principles used: [D_t-AXIOM], [**EQ**-RULES], [**USV₃**], [**LL**], [**V**-∀], [**ref**₌]?

The argument I presented proceeds by λ-abstraction, using the predicate: 'being definitely TRUE of x that it is identical to a'. Does that predicate correspond to a *real* property? If not, then this is not a legitimate case of λ-abstraction, by the standards of Ontological Vagueness.

For the advocate of Metaphysical Vagueness, the answer must be 'yes, it is a genuine property'. For, it is a feature of this position that *in the world* there is vagueness, and its contrary, definiteness. These are *real, actual* properties that are designated by these predicates. And unless the advocates of this position want there to be some sort of "ineffability" when it comes to their postulated properties-in-the-world, they have to admit that such expressions *do* designate such properties. The language is entirely extensional – there is no "funny business" going on about 'opaque contexts' or rigid vs. non-rigid names or . . . . The λ-abstraction picks out what the believer in Ontological Vagueness must acknowledge is a legitimate property.

But let's look again at this presumed notion of vagueness and definiteness. It cannot be the modal notion of the epistemicists, since that characterizes one's epistemic states rather than reality. *That* kind of (in)definiteness does *not* characterize an item "in the world" but rather cognizer's *apprehension* of objects. Any indefiniteness operator of this variety will endorse a principle like $\nabla\varphi \leftrightarrow \nabla\neg\varphi$, as I mentioned above, and such a principle does not characterize the usual ontological vagueness theorists' view.[10]

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[9]This shows that if we were to interpret the middle value **V** as it is in Priest's [14] – as being *both* **T** *and* **F** – we would have to rephrase the interpretation of [**USV₃**]. And in fact, Priest (p.c.) says that he *denies* [**USV₃**], believing that there are but *two* truth values, but that some formulas can take both. Therefore, I propose this argument only against those who do not think that vagueness leads to true contradictions.

[10]Well, except perhaps for dialetheic views like that of Priest [14].

The view of Heck in [9] that $\triangledown$ should in fact obey this principle shows that his argumentation is not really directed against nor in favour of metaphysical vagueness, but rather at or in favour of some hybrid view of metaphysical and epistemic vagueness.

I say again: only the many-valued logic viewpoint accurately captures the ontological vagueness theorist's view.

So far as I am aware, no one has faulted the following principles that are used in the proof (well, so long as they are willing to allow a 3-valued logic in the first place, and as long as they see the extra values as being truly distinct from TRUE and FALSE, contrary to Priest's viewpoint expressed in footnote 9):

- **USV$_3$**: $\vDash (\boldsymbol{D_t}\varphi \vee \boldsymbol{D_f}\varphi \vee \boldsymbol{V}\varphi) \wedge \neg(\boldsymbol{D_t}\varphi \wedge \boldsymbol{D_f}\varphi) \wedge \neg(\boldsymbol{D_t}\varphi \wedge \boldsymbol{V}\varphi) \wedge \neg(\boldsymbol{D_f}\varphi \wedge \boldsymbol{V}\varphi)$;

- **D$_t$-AXIOM**: $\vDash \boldsymbol{D_t}\varphi \rightarrow \varphi$;

- **ref$_=$**: $\vDash \boldsymbol{D_t}[\alpha = \alpha]$;

- **V-$\forall$**: $\vDash \boldsymbol{V}[(\forall x)\Phi(x)] \rightarrow \neg(\exists x)\boldsymbol{D_f}[\Phi(x)]$.

In the case of those believers in ontological vagueness who hold there to be more "degrees of metaphysical vagueness" than just the three we have been assuming, a strictly analogous proof to the very same conclusion can be crafted, as discussed in [13]. One changes the [**USV**] axiom to accommodate the further truth values, and generalizes the [**V-$\forall$**] axiom for the extra truth values.

We will return to a discussion of the argument after a brief excursion into higher-order vagueness.

6   Higher-order vagueness

The topic of higher-order vagueness concerns the issue of whether it can be vague that something is vague (and even further iterations, such as being definite that it is vague that it is vague). For the believer in Ontological Vagueness, the first iteration amounts to wondering whether it can be vague that some aspect of reality is vague? It is not clear to me that a proponent of Vagueness-in-Reality will wish to accept this as a part of their doctrine concerning *Reality*. They might instead prefer to view it as a mixture of different types of vagueness: "We don't know whether it is true or false that such-and-so is metaphysically vague", and would thereby prefer some mixture of a many-valued logic with an epistemic logic of vagueness (like that of [12]) added on. Certainly, if they *do* wish to have metaphysical higher-order vagueness,

they wouldn't represent it by iterating the $\boldsymbol{V}$-operator (nor with iterated mixtures of any of the $\boldsymbol{V}$-, $\boldsymbol{D_t}$, and $\boldsymbol{D_f}$-operators). The previously-mentioned Lemma precludes this.

Instead they would increase the number of truth values... and with them, the number of truth-operators in the language. For this purpose, it is common in discussions of many-valued logic to take the truth-values to be integers, with $\mathbf{1}$ being "most true" and (for an n-valued logic) to make $\mathbf{n}$ be the "most false" value. And then it is common to introduce the so-called $\mathbf{J_i}$-operators [17]. Such an operator is a generalization of the ideas behind our $\boldsymbol{D_t}, \boldsymbol{D_f}$ and $\boldsymbol{V}$ operators – like our operators, the $\mathbf{J}$-operators have a formula as an argument, and are semantically valued as being "completely true" (that is, take the value $\mathbf{1}$) if the formula-argument takes the value indicated in the subscript of the $\mathbf{J}$-operator, and "completely false" (that is, take the value $\mathbf{n}$) otherwise. Semantically this is to say, for any value i of an n-valued logic ($1 \le i \le n$)

$$[\![\boldsymbol{J_i}(\varphi)]\!] = \mathbf{1} \text{ if } [\![\varphi]\!] = \mathbf{i}$$
$$= \mathbf{n} \text{ otherwise}$$

For example, a five-valued logic would have $\boldsymbol{J_1}, \boldsymbol{J_2}, \boldsymbol{J_3}, \boldsymbol{J_4}, \boldsymbol{J_5}$ as $\mathbf{J}$-operators), with truth tables:

φ	$\boldsymbol{J_1}\varphi$	$\boldsymbol{J_2}\varphi$	$\boldsymbol{J_3}\varphi$	$\boldsymbol{J_4}\varphi$	$\boldsymbol{J_5}\varphi$
1	1	5	5	5	5
2	5	1	5	5	5
3	5	5	1	5	5
4	5	5	5	1	5
5	5	5	5	5	1

and this account might say that $\boldsymbol{J_3}\varphi$ makes the claim that φ is completely vague, while $\boldsymbol{J_2}\varphi$ asserts that it is vague whether φ is completely vague or is true; and that $\boldsymbol{J_4}\varphi$ claims that it is vague whether φ is completely vague or false.

With suitable additions of the number of truth-values, this seems as plausible a way to represent higher-order metaphysical vagueness as it is in modal logic to represent higher order epistemological vagueness by the iteration of a modal operator $\triangledown\triangledown\Phi$. However: a version of the Argument can be made using *any* (finitely-) many valued logic (with suitable emendations to the various principles). I don't rehearse the proof of that fact here; details can be found in [13].

I think the ability to represent higher-order vagueness (of any finite number of iterations) shows that it is not that many-valued logics are incapable of giving some sense to higher-order vagueness, but that the argument in [13] demonstrates that there is some other, perhaps deeper, incoherency with Metaphysical Vagueness.

Although the ploy of increasing the number of truth values shows that many-valued logics are in fact capable of giving a plausible account of higher-order vagueness for any finite number of iterations, there remains still the issue just mentioned: no matter how (finitely) many truth-values our ontological vagueness proponent wishes to invoke as a way of handling higher-order vagueness for a finite number of truth values, there is a generalization of the Argument that can be turned against it. A question naturally arises then concerning the interaction of higher-order vagueness with the number of truth values. We've just seen that to have one iteration of higher-order vague, we would increase the number of truth values from three to five. If we had another iteration, and wanted all possible combinations to be represented, we would need many more. And if we thought it possible to have any level of iterated higher-order vagueness, then the conclusion would be that we need an infinite-valued logic to accommodate this. Infinite-valued logics come with their own share of unusual characteristics, such as that a quantified formula can be assigned true ($[\![J_i(\exists xFx)]\!]=1$) even without there being any object a in the domain such that $[\![J_i(Fa)]\!]=1$. I think that most vagueness-in-reality theorists either hold that higher-order vagueness of any sort is impossible (as various authors have claimed, even independently of whether they believed in metaphysical vagueness), or else that it is bounded by some finite number of iterations. (It is not clear how this latter possibility might be argued for by our ontological metaphysicians. Most arguments to this conclusion come from the point of view of it being *cognitively impossible* to have infinite iterations... and that's not very relevant to the ontological conception of vagueness.) Anyway, I'm not going to consider it further.

7   Returning to the argument

In [4], Cowles and White object to the statement of Leibniz's Law in the form given in [LL] above, namely
   $\vDash a = b \leftrightarrow (\forall F)(Fa \leftrightarrow Fb)$,
and prefer to see it as (what they call "Classical LL"):
   $\boldsymbol{D}_t[a = b] \leftrightarrow \boldsymbol{D}_t[\forall F(Fa \leftrightarrow Fb)]$

They also deny the full force of the [**EQ**-RULES]: They claim that just having
   $\vDash \varphi \leftrightarrow \psi$
does not justify
   $\vDash \boldsymbol{V}[\varphi] \leftrightarrow \boldsymbol{V}[\psi]$,
nor
   $\vDash \boldsymbol{D}_f[\varphi] \leftrightarrow \boldsymbol{D}_f[\psi]$.

As they show, their position has the effect of denying both:

- $\boldsymbol{V}[a = b] \leftrightarrow \boldsymbol{V}[\forall F(Fa \leftrightarrow Fb)]$

- $\boldsymbol{D}_f[\forall F(Fa \leftrightarrow Fb)] \rightarrow \boldsymbol{D}_f[a = b]$

(although it does validate $\boldsymbol{D}_f[a = b] \rightarrow \boldsymbol{D}_f[\forall F(Fa \leftrightarrow Fb)]$).

Just how plausible are these denials? Not very, it seems to me. Is it really plausible to claim that when we have a *logical* truth that two formulas are equivalent, we cannot conclude that one of them is vague just in case the other one is? Nor that one of them is definitely false just in case the other one is? How plausible is it to claim that even when it is *definitely* false that two objects share all properties, it might yet not be definitely false that these are the same object?

On the other hand, I should admit that because of the plausibility of the [**EQ-RULES**], as well as the other rules, I had originally – when I wrote [13] – thought that the Argument showed the complete *im*plausibility of the conception of Metaphysical Vagueness. However, I hadn't internalized these facts (or maybe I hadn't even noticed them):

1. Although the proof given was framed as showing that a contradiction followed from the assumption of $\boldsymbol{V}[a = b]$, *it equally is a proof of*

   $$\boldsymbol{V}[a = b] \rightarrow \neg\boldsymbol{V}[(\forall F(Fa \leftrightarrow Fb)]$$

   i.e., even if it is vague that $a = b$, it can't be vague that they share all the same properties.

2. And of course: $\boldsymbol{D}_t[a = b] \rightarrow \neg\boldsymbol{V}[(\forall F(Fa \leftrightarrow Fb)]$

   i.e., if it is definitely TRUE that $a = b$, then it isn't vague that they share all the same properties: intuitively, it is definitely TRUE that they *do* share all the same properties.

3. Furthermore, clearly: $\boldsymbol{D}_f[a = b] \rightarrow \neg\boldsymbol{V}[\forall F(Fa \leftrightarrow Fb)]$

   i.e., given that it is definitely FALSE that $a = b$, it can't be vague that they share all the same properties: intuitively, it has to be definitely TRUE that they differ on at least one property.

But [**USV**] asserts that one of the three cases must hold, so we can conclude

$$\vDash \neg\boldsymbol{V}[\forall F(Fa \leftrightarrow Fb)]$$

That is, it is *never* the case that it is vague that two(?) objects have all properties in common. (Or, that it is *never* vague that an object has all the properties it has).

In conclusion

I used to think that the original argumentation showed:

- The conception of Metaphysical Vagueness is committed to representing its doctrines with a many-valued logic.

- The conception was committed to various logical principles (listed above), as a consequence of its metaphysics.

- Part of Metaphysical Vagueness was a commitment to the Evans/van Inwagen criterion.

- The Argument showed that any many-valued logic which embodied those principles led to a contradiction.

- And I concluded that Metaphysical Vagueness – Vagueness in Reality – was an incoherent notion.

But given that the Argument proves $\vDash \neg\boldsymbol{V}[\forall F(Fa \leftrightarrow Fb)]$, (which by one of the [**EQ**-RULES] shows $\neg\boldsymbol{V}[a = b]$), perhaps we should instead follow a different route:

- Continue to hold to the requirement of a many-valued logic with the specified logical principles to describe the view, *BUT*

- Deny the background assumption given to us by Evans [5] and van Inwagen [25, 26] that Metaphysical Vagueness is committed to instances of $\boldsymbol{V}[a = b]$.

- And similarly deny the [25] version of Evans' assumption to the effect that one cannot count vague objects – because it is *never* TRUE or FALSE that such a thing is one object and it is *never* TRUE or FALSE that it is two objects.

So, by this line of thought the Argument does *not* show Metaphysical Vagueness to be incoherent, it shows instead that *the Evans/van Inwagen criterion of metaphysical vagueness is incorrect.* So believers in Vagueness-in-Reality should turn their attention to finding a different way of stating their basic metaphysical position, and not allow their opponents to define the field for them. Since I am not an advocate of metaphysical vagueness, I cannot therefore offer anything for them.

However, until they do that, Metaphysical Vagueness remains a "deeply dark and dank conception" that one should avoid.

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