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UNIVERSITY OF ALBERTA

BEVERAGE CONTAINER DESIGN AND MATERIAL RECOVERY FROM SOLID WASTE

BY

© EDWIN KWAN LAP TAM

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE

IN

ENVIRONMENTAL ENGINEERING

DEPARTMENT OF CIVIL ENGINEERING

Edmonton, Alberta

FALL 1994

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FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommended to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled BEVERAGE CONTAINER DESIGN AND MATERIAL RECOVERY FROM SOLID WASTE submitted by EDWIN KWAN LAP TAM in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE.

Dr. C.A. Zeiss
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Dr. W. Adamowicz

Date: October 4 1994

DEDICATION

This thesis is dedicated to my family: my mother, father, brother, grandmother, and to the memory of my grandaunt. For them, the completion of thesis is the end of one journey, and the beginning of another.

ABSTRACT

Many beverage containers are potentially recyclable after use but only a portion of these are regularly recycled. To recover this portion that would otherwise be wasted, this thesis investigated if an alternate beverage container design as compared to a current design would improve the recovery of the container material after shredding and screening in a materials recovery operation. A desirable design alternative would enable the shredded material to concentrate into some specific size fraction during screening, allowing for easier separation, as opposed to having the material scattered throughout. Four physical design characteristics were investigated to determine if and how they affect the shredding behaviour. These factors were size, shape (cylindrical, rectangular), geometry (open ended, enclosed), and the container material. The selected test containers were Tetra-Pak aseptic packages, glass jars, and expanded polystyrene (EPS) containers, primarily because they had unique testing characteristics. The effects of the size, shape, and geometry on Tetra-Pak and glass containers shredding behaviours were tested using a 2^3 factorial design. The effect of size on EPS containers was tested by straight comparisons. The effect of the container material on the shredding behaviour of all containers and the effect of the Tetra-Pak container orientation just prior to shredding were also tested by straight comparisons. A final test investigated if specific Tetra-Pak containers would separate out from a mixture of recyclable items due to a characteristic - small container size - that was observed to produce a better shredded material concentration. Except for this effect of size on Tetra-Paks, the size, shape, and geometry of a container had little or no effect on the shredding behaviour of the containers. The Tetra-Pak orientation also had no effect. The material type was found to have an effect but a relationship between the material and the behaviour was not determined. The implications of the finding that smaller Tetra-Paks produced greater concentrations of shredded material for potential recovery were analyzed. It is concluded that the design of a container for improved recovery is only one consideration in the design of an "environmentally friendly" product.

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1.0 PROBLEM STATEMENT

Many beverage containers are potentially recyclable after use but only a portion of these are regularly recycled. The remainder are often disposed. To recover this portion that would otherwise be wasted, one option is to construct "easier-to-recycle" beverage containers for mechanical materials recovery systems that employ shredding and screening operations. However, this option has received little attention in the past. Therefore, investigating how the current physical design characteristics of a beverage container affects its shredding behaviour and subsequent size distribution after screening may reveal if an alternate container design can improve the recovery of the container material. If a characteristic or even a combination of characteristics, when compared to other design configurations, can better *isolate* the material into a specific size fraction after shredding, the recovery of such material can be improved. The shredded material has been concentrated into a specific size range and thus can be more easily captured by screening than if the material was distributed over a wide range of sizes. For the purposes of this research, it is not important into what specific size fraction a material may be isolated, so long as a significant portion of the shredded material is separated into some size fraction.

1.1 RESEARCH OBJECTIVES

Based on the above problem, the overall objective is to determine if an alternative design for a beverage container as compared to a current design will improve the recovery potential of the beverage container material and if so, what is this design. To fulfill this main objective, this research must first satisfy the following objectives.

I. It is necessary to determine what factors or characteristics significantly affect the shredding and screening behaviour of a beverage container. These factors include the container size, shape, geometry, material, and preshredding orientation and will be discussed in greater detail later.
II. If any of the above factors are significant, it must then be determined what is the relationship between any such factor(s) and the subsequent shredding and screening behaviour. It is also necessary to determine if this relationship would benefit the recovery of the container material.

Furthermore, providing a relationship exists, the implications of any such relationship for that beverage container must also be examined.

1.2 BEVERAGE CONTAINER CONTEXT

Beverage containers regulated by deposit-refund laws are often returned for recycling while other containers are recovered through various recycling collection programs. The rest are usually disposed due to either the lack of recycling programs for these containers or the inadequacy of existing recycling programs to handle such containers. The scenario is especially relevant to containers that are intended as single-use only items. As a result, the beverage container material, which is potentially reusable, is wasted.

The circumstances surrounding these beverage containers can be illustrated using an example involving plastics. Many Canadian produced plastic containers now have the familiar triangular identification label on the bottoms. When the contents are spent, the container can be identified and appropriately sorted for recycling. Although this seems to be a highly workable solution, in reality, this plastics coding system has several flaws (Recycling Canada, January 1992). One of the biggest problems is the sheer number of plastics available on the market. Because mixed plastics are not suitable for many plastics recycling programs, post-consumer used plastics must be sorted. Depending on the size of the collection base, this can be a costly proposition. Furthermore, because the general public may not be familiar with the specific requirements that a plastics recycling program demands, plastics that are not currently recyclable can be mixed with the recyclable ones (Glenn, 1990). This example illustrates that a *recyclable commodity does not necessarily get recycled* (Reinfeld, 1992, p.59). Despite the efforts in identification, a community may be unable to afford an adequate plastics collection and sorting program that will meet the demands of the plastics recycler. This may result in no plastics collection or a mixed plastics collection that must still be sorted. It is important to realize that while the technology to recycle a specific item may exist, true recyclability means economically viable and widespread collection, processing, and marketing systems also exist (CCME, 1993). And of course, there are individuals who may not recycle (Ottman, 1994, p.33). Recyclable items may simply be disposed of as garbage.

In either of these scenarios, recyclable items either end up in a commingled stream of recyclable items or in the waste stream. Recovering these recyclable items would be mainly accomplished by either manual sorting or mechanical sorting, or a combination of both. However, the heterogenous nature of both streams can impose great technical demands on mechanized operations to separate out desired items (Glenn, 1991). Handsorting could also be employed but may be very labour intensive and expensive, especially if a large volume of recyclables are handled. In either case, sorting would most likely be improved if the recyclable item was easier to isolate from the other materials present in the stream. This would mainly involve changing some aspect of the item itself.

Little research has been performed to investigate how specific characteristics and design of an item may impact waste generation (Conn, 1988). As an example of how knowledge of this aspect can be beneficial, consider a rectangular shaped item made of a recyclable material. If, because of its rectangularity, such an item was found to separate itself from other materials in the materials recovery sorting process, this characteristic of the item would be desirable. It may be advantageous to thus design similar items in a comparable fashion to improve the materials recovery. This emerging cradle-to-grave attitude, known as product stewardship, is reflected in a statement by the Canadian Council of Ministers of the Environment concerning packaging, "All packaging will be designed, manufactured, used, and disposed of in such a way as to minimise its effect on the environment and to achieve maximum diversion from the disposal options through the application of the Three R's: Reduce, Reuse, Recycle." (CCME, 1990)

Beverage containers were chosen to be studied because many of them are disposable "packages", discarded once the contents are consumed. For example, in the past, there were greater quantities of beverage containers that were reused (refillable glass containers) in the

United States. However, the American consumer prefers convenience and hence the increase in nonrefillable beverage containers (Miller, May 1992). In the United States in 1986, containers and packaging made up an estimated 30.3% of the discards to municipal solid waste (MSW) *after* materials recovery was performed. (Franklin Associates Ltd., 1988) The U.S. EPA estimates that 33.8% of MSW in 1986 is packaging and containers (Erwin and Healy, 1990). Furthermore, beverage containers are diverse in nature. The configurations and materials of currently available beverage containers, both of which are key design parameters, vary considerably. Selecting beverage containers for this research therefore offers a broad range of research possibilities.

It is obviously not possible to research all possible combinations of beverage containers and their respective characteristics and materials recovery methods. Therefore, the research will be limited to investigating selected *non-returnable* beverage containers. The familiar returnable drink bottles (metal and glass) and aluminum cans that require a deposit generally have significant recovery rates. *Screening* was chosen as the sorting method because it is one of the common methods to separate out materials (Tchobanolglous, 1993, p.258). Shredded materials would thus be separated based on *differences in size*. *Size reduction* is often done by most materials recovery facilities and waste facilities to improve the handling of the recyclable or waste material. However, many forms of size reduction exist. *Slow-speed shredding* is a relatively new trend in waste processing that appears to be enjoying reasonable success according to industry personnel. The topics of choosing non-returnable beverage containers, size reduction, and slow-speed shredding will be discussed in greater detail in Chapter Two. Note that this thesis will be limited to the North American beverage container situation.

Because of these limitations, applying the research results to scenarios outside of those specified in this research may not be valid. Nevertheless, this research contributes to the practical and scientific aspects of solid waste management. The realization that a specific characteristic or characteristics would render a beverage container recyclable at the end of the product's life may justify redesigning the product to incorporate that characteristic. A current recyclable product may likewise be more recyclable if such a characteristic(s) was incorporated or enhanced if it already existed. Furthermore, this research will detail how some common beverage container characteristics can affect the breakage of selected beverage containers. The results will contribute to the field of *breakage theory for solid wastes*. This field appears to be not well-understood. For example, one of the breakage theories that was used extensively in the past to describe how solid wastes break has been shown to be generally invalid for many solid wastes (Vesilind et al., 1986).

1.3 RESEARCH OUTLINE

mainder of this thesis is divided into the following chapters.

Chapter Two explains the selection of the beverage containers chosen for testing, the selection of the four design characteristics, the current recovery potential of these containers, and the role of size reduction in materials recovery and separation. This chapter also examines several of the current theories used in describing and predicting the breakage of solid wastes. Based on the discussion of these preceding topics, the chapter concludes by developing the hypotheses between the various physical container characteristics and their effect on the recovery of the container material.

Chapter Three examines the experimental design developed and used to test the hypotheses. Several different procedures were devised because not all of the hypotheses could be tested by a single method due to the nature of the beverage containers and limitations on research resources. Following this, the actual procedures used to carry out the design are explained. Any procedural deviations or complications are also noted in this chapter.

Chapter Four presents the summaries of the various test data and the analyses performed to evaluate the validity of the hypotheses. However, to perform these analyses, several other aspects had to be examined, including the applicability of the data gathered and the selection of a suitable method to "measure" the data. The chapter concludes with an evaluation of the hypotheses proposed in Chapter Two.

Chapter Five discusses the implications of the findings from Chapter Four for one of the selected beverage containers used in this research.

Chapter Six concludes this thesis and recommends several possibilities for future research.

Appendix A contains the original data and initial treatments of the data.

Appendix B details the various calculations required for the data analyses in Chapter Four.

CHAPTER TWO

Recovery Potential of Beverage Containers

2.0 INTRODUCTION

A variety of topics must be examined to determine if an alternate beverage container design can improve the recyclability of the container. The following topics will be discussed in this chapter.

I. Why certain non-returnable beverage containers were selected for testing and the recovery potential of these containers.
II. What beverage container characteristics were selected for testing. These characteristics would constitute the design of a beverage container.
III. Material properties of the chosen beverage containers and how they may affect the size reduction of the containers.
IV. The role of size reduction in materials recovery. This section will focus on slow-speed shredding as the chosen method of size reduction.
V. Existing theories relating to the breakage of solid wastes and how they could possibly be useful to this research.
VI. The hypotheses, developed from the preceding topics, that relate the characteristics of a beverage container, its behaviour during shredding, and the subsequent recovery of the container material.

2.1 SELECTION OF CONTAINERS FOR EXPERIMENTATION

2.1.1 Returnable versus Non-returnable Beverage Containers

The beverage containers currently on the market can be divided into two categories: returnable and non-returnable. Returnable containers are subject to deposit-refund regulations while non-returnable containers are not. As an incentive to return the beverage container for reuse, the consumer pays a deposit amount at the point of purchase that can be later redeemed at a depot when returning the empty beverage container. Many of these returnable containers are thus recovered for reuse or recycling. For example, the Province of Alberta currently has moderate to high return rates on refundable containers. These rates are shown in Table 2.1.

Because the existing systems governing returnable beverage containers appear to be effective, focussing on examining the recoverability of *non-returnable* beverage containers would be of greater benefit to the field of solid waste management. While some of these containers can be recycled currently with existing technology, no monetary incentive exists to motivate consumers to return the spent containers. Furthermore, some of these containers, such as aseptic packages, are single use items only. In addition, this scenario is complicated by the fact that certain beverage containers, such as "styrofoam" cups, although made of a recyclable material, have few recycling systems or markets in place to process the used containers. As a result, the great majority of such containers are disposed. However, alternate container designs may improve these containers' recovery and thus allow existing recycling programs to accept such containers. If certain characteristics prove very effective in isolating the desired container material, it may even be possible to separate out such containers from a waste stream.

Aluminum Cans	**77.60%**
Bi-Metal Cans	**33.35%**
Glass Bottle Fund	
500mL or less	**56.81%**
501mL to 1L	**61.61%**
Over 1L	**42.83%**
Plastic (HDPE/PVC)	
500mL or less	**43.50%**
501mL to 1L	**39.81%**
Over 1L	**43.20%**
Plastic (PET)	
500mL or less	**64.58%**
501mL to 1L	**55.28%**
Over 1L	**87.01%**
Liquor and Wine	**82.47%***
Imported Beer Cans	**99.86%***
Imported Beer Bottles	**74.03%***

TABLE 2.1: Rates of Return as of October 13, 1992 for the Province of Alberta
(*) Sales figures from September 1992.
Source: Alberta Recycling Branch, Alberta Environment.

2.1.2 Selection of Specific Beverage Containers for Testing

Based on the previous discussion and the context of this research, the following criteria were used to determine which containers would be chosen for testing. The container should:

1. Be non-returnable.
2. Not be currently recycled through existing recycling programs and thus more likely to be disposed.
3. Be made of a material that is potentially recyclable using existing technology, otherwise there would be little present use in recovering the material.
4. Possess a characteristic(s) that is unique and may therefore affect how the container shreds and screens.

Point four is important because one of the objectives is to determine if a *particular* design characteristic(s) of the beverage container will improve the recoverability of that container. If all the chosen beverage containers were similar, it is unlikely that any differences in the recoverability of the shredded materials would result. Furthermore, unique characteristics from various beverage containers could be potentially quite different from one another. The greater the difference, the more likely the effect of a particular characteristic will be clearly observed.

After considering these criteria, three types of containers were selected: Tetra-Pak aseptic packages[1], expanded polystyrene (EPS) cups, and glass jars (not returnable glass bottles). All three containers either match all or most of the criteria.

Tetra-Paks are multi-layered aseptic packages used to hold beverages. They are currently non-returnable, not accepted in most recycling programs, are potentially recyclable, and are one of the few rectangular shaped containers on the market. The aseptic packaging material is highly flexible. The aseptic packages also come in two distinct sizes: 250 mL and 1000 mL. Furthermore, the 250 mL aseptic packages are entirely enclosed during use except for the small drinking straw hole.

EPS containers also fulfill the first three requirements, although some urban centres have started recycling programs for used polystyrene containers. The EPS containers chosen however are the familiar foam "coffee" cups. These are unique because one end is fabricated entirely open for drinking and they are always cylindrical. The vertical angle of the cup will be ignored for the purposes of this research. EPS is also a brittle material.

Glass jars are also non-returnable but are usually accepted by most recycling programs. Glass jars are also made of a brittle material. However, of the various non-returnable beverage containers materials, glass is generally the heaviest and densest of all materials. Furthermore, glass is commercially available [illegible] many configurations and sizes. Glass jars can thus be compared against the other two container types.

2.1.3 Selection of Physical Design Characteristics

The three selected containers - Tetra-Paks, EPS containers, and glass jars - each have unique characteristics. These unique characteristics of *size, shape, geometry (open mouth or not), and container material* can be generalized into categories. It is observed that almost all beverage containers can be described by the following categories.

1. ***Size***. Many beverage containers come in a variety of sizes. This is usually quantified by the volume of beverage the container holds.
2. ***Shape***. Most beverage containers are cylindrical. However, certain containers are square or rectangular, such as milk cartons or aseptic packages. There are very few containers that are inbetween these two shapes.
3. ***Geometry***. Beverage containers are either enclosed or open at one end. Prior to use, it can be expected that such containers are sold closed but after use, the container may be disposed as an open container. There are varying degrees of enclosure; some containers may be quite wide at one end but have narrow openings.
4. ***Material***. Each beverage container is obviously constructed of some material. Many are single material containers while other may be composite material containers. The materials can also range from the very flexible to the very brittle and can vary in density.

These categories are the various *physical design characteristics* that may affect the shredding and screening behaviour of the beverage containers. These design characteristics would therefore be the physical basis of any alternative container design. Furthermore, the chosen

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[1] Tetra-Pak is the company name for the largest North American manufacturer of aseptic packages. Throughout this thesis, the term Tetra-Pak will be used to refer to aseptic packages.

beverage containers represent the extremes of these categories. For example, testing a rectangular container against a cylindrical container would illustrate what difference, if any, shape has on the recovery of the container material.

2.2 RECOVERY POTENTIAL OF SELECTED CONTAINERS

It is worthwhile to examine the present materials recovery situations of the chosen containers: Tetra-Paks, EPS containers, and glass jars. Although they physically possess the extremes of the characteristics to be tested and are therefore ideally suited for this research, this section will evaluate their recovery and recyclability from a *practical viewpoint*. The current efforts to recycle these containers will be described and the quantities available of each container for recycling will be analyzed.

2.2.1 Tetra-Pak Aseptic Packages

Aseptic packages have been heralded as one of the most significant food science developments within the last half-century. These packages, consisting of multi-layers of paper, polyethylene, and aluminum foil, include the 1 litre box and the 250mL "drink box". The drink box offers convenience, excellent beverage preservation, and ease of transport due to its rectangular shape. In fact, composite material containers generally require less material for their production. However, numerous concerns have been raised about the disposability (i.e. lack of recycling options) of these juice boxes (CCME, 1990). Until recently, no service or market was available to recover the spent drink boxes. These were disposed of with other non-recoverable wastes. Consumers' concerns include the feeling of guilt - the boxes are generally thrown away, unlike returnable glass and aluminum beverage containers. The State of Maine in 1990 banned the aseptic box, classifying it as unrecyclable due to its multi-layered composition. As of mid-1993, aseptic packages have not been placed under deposit-refund legislation, although there has been some discussion about doing so. Thus, in order to develop and demonstrate a successful post-consumer recovery market for the aseptic packages, used drink boxes have been collected on a limited scale for the production of plastic lumber. Another venture is to recover the paper content from the packages for pulp. However, these markets for used drink boxes have still not fully developed.

As an example of the recovery situation regarding aseptic packages, a company collects aseptic packages (specifically Tetra-Paks) from schools in the metropolitan Edmonton area of Edmonton, Fort Saskatchewan, and St. Albert. These Tetra-Paks are drained, bailed, and delivered to a local plastic lumber manufacturing company. At of the time of this thesis research, this operation is the only one of its kind in Canada. Tetra Pak Inc. delivers 1070 tonnes of multi-layered Tetra Pak material to Alberta per year for the construction of the aseptic boxes. Sixty percent, or 642 tonnes, are formed into the 250mL drink boxes for consumption in Alberta. The cities of Edmonton, Fort Saskatchewan, and St. Albert have a combined population of 670 965 out of the total Alberta population of 2 545 553. A complete population count for the County of Strathcona was not obtained. Assuming a constant per capita consumption rate of Tetra Paks within Alberta and assuming that all drink boxes formed are actually bought and used, the three municipalities above would use 26.4% of the 642 tonnes, or 169 tonnes, of the total 250mL Tetra Paks consumed in Alberta on a yearly basis, pro-rated based on the population distribution.

Approximately 4.53 to 7.26 tonnes, of Tetra Paks[2] are collected per month. Assuming these are only 250mL drink boxes and that any seasonal fluctuations average to a reasonably constant rate, 54.4 to 87.1 tonnes of the Tetra Paks, or a linear average of 70.8 tonnes, are recovered annually from the 169 tonnes used by the three municipalities. The collection program through the schools translates into a recovery rate of approximately 32% to 52%, or an average 42%, of *all* 250mL drink boxes in these three municipalities.

Capturing *all* used Tetra Paks is unlikely to generate a significant increase in the mass of material recovered compared to the amounts actually landfilled. The Province of Alberta generated 1 830 800 tonnes of waste in 1987 (Stanley Associates Engineering Ltd., 1988). If all 1070 tonnes of Tetra Pak material sent to Alberta was thrown into the waste stream, it would constitute only 0.058% of the total waste generated. These calculations assume the present waste quantities are similar in magnitude to the value reported for 1987.

The preceding calculations show that the quantity available for collection, when compared to the total waste stream, is diminutive. For comparison, in 1988, newspapers alone made up 7.4% of the MSW by mass in the U.S. (Miller, June 1992). However, attempting to recover aseptic packages would appeal to environmentally concerned consumers. The future of aseptic package recycling will also depend on present and future developments for used aseptic packages (e.g. pulp recovery, more uses for plastic lumber).

2.2.2 Expandable Polystyrene Containers (Cups)

As with the Tetra-Paks, environmental concerns about polystyrene food service items have increased over the last several years. The efforts to recycle polystyrene appear to be still in the beginning stages. A recent venture to recycle polystyrene packaging by a major fast food retailer was discontinued in favour of paper packaging. Despite this apparent set back, polystyrene recycling efforts have started in major centres throughout North America, including a full-scale polystyrene recycling plant in metropolitan Toronto. This program and others typically use source-separated polystyrene items; the majority of other contaminants have already been removed.

The 1990 Canadian consumption of expandable polvstyrene beads used for disposable cups was 6% of 157 000 tonnes of total polystyrene, or 9420 tonnes. (Chemical Economics Handbook, 1991) These cups produced from EPS beads are the familiar foam cups. However, the two other major forms of polystyrene, straight or crystal polystyrene, and impact grade polystyrene, are also used to produce disposable cups. No percentages of cups produced from each polystyrene have been found in the literature, but in Western Canada, it is *roughly estimated* that 75% of the typical "coffee cups" are manufactured from expandable polystyrene beads. For North America, this percentage is roughly 65% (Barry Middleton, June 7, 1993). By applying the 65% ratio, the total number of cups made from all types of polystyrene would be [135% x 9420 tonnes] or 12 717 tonnes. Canadians disposed of approximately 30 million tonnes of garbage in 1989. (Ministry of Supply and Services, 1991) The total garbage mass includes all types of municipal wastes, including construction wastes. Assuming the 1989 Canadian amount of garbage was approximately the same for 1990, the disposable cups would constitute 0.042% of the total solid wastes. This is a very small portion of the total MSW.

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[2] Combibloc brand produced containers have been assumed to contribute an insignificant weight.

Locally, the amount of waste plastics in the City of Edmonton is 36580 tonnes per year (CMG, February 1991). The residential plastic waste is 19374 tonnes, and the commercial waste is 12915 tonnes. In the residential waste stream, polystyrene makes up approximately 7% to 12% of the total plastic present. Assuming a mean value of 9.5% and applying this average to *both* the residential and commercial plastic wastes, 3067 tonnes of polystyrene are disposed of each year. From above, 6% of all polystyrene consumed in Canada was used to produce expandable polystyrene foam cups. Since expandable polystyrene is estimated to make only 75% of all disposable cups in Western Canada, and assuming the 6% value is valid in Edmonton, the mass of disposable cups is [6% x 1.25 x 3067 tonnes] or 230 tonnes of polystyrene cups per year.

An alternative method to determine the amount of polystyrene cups is to examine data from the Blue Box program in Edmonton and estimated polystyrene amounts from institutions and industries (CMG, March 1991). The Blue Box program collects about 363 kg of polystyrene per day, *selected* institutions, such as hospitals and colleges, would collect about 324 kg per day, and *selected* polystyrene industries would make available 39 kg per day. A total of 265 tonnes of polystyrene per year, would be available. Again, using the 6% and 1.25 factors, the total mass of polystyrene cups is 19.8 tonnes per year. Because this second calculation is based on the Blue Box program and selected institutions, considerably more polystyrene is available than would actually be collected.

In 1986, Edmonton landfilled 655 900 tonnes of solid waste (including construction debris) (Stanley Associates Engineering Ltd., 1988). At that time, 230 tonnes of polystyrene cups would have only constituted 0.035% of the total solid wastes. This figure agrees roughly with the national values calculated previously. Considering that the total amount of plastics landfilled in Edmonton in 1987 was 153 000 tonnes, the amount of polystyrene cups appears insignificant in terms of waste tonnage.

An article by M.B. Hocking compared the merits of the polystyrene foam bead cup against those of the paper cup (Hocking, 1991). He presents a life-cycle analysis for both cups and points out several important considerations.

a. For cups of the same size (8 oz.), the polystyrene cup weighs one-quarter of the paper cup.
b. The fuel requirement for the manufacturing of both cups is approximately the same.
c. More petroleum feed stock is required for the production of the polystyrene cup. Similarly, the paper cup requires wood pulp feed stock for its production. In both cases, there are environmental considerations (damage?) due to the exploration and extraction of the raw material.
d. The polystyrene cup requires significantly less steam and electricity than the paper cup and also less cooling water requirements for its manufacture.
e. Paper cup manufacturing involves chlorine, chlorine dioxide, sulphur, and sulphur dioxide emissions. These emissions have been reduced in recent years due to improved technology and processes. Polystyrene cup manufacturing emits pentane, which contributes to volatile hydrocarbon levels. Sulphur dioxide emissions are slightly less.
f. For cold beverages, both cups perform adequately. However, hot (water) beverages significantly weaken the stiffness of a paper cup, reducing its usefulness. Polystyrene cups behave similarly regardless of the beverage temperature.
g. Because of its surface, the paper cup, once used, cannot be easily washed for reuse. The polystyrene cup, since its structure is not altered by beverages, can be reused.

h. Both cups are recyclable, but the properties in (g) allow polystyrene cups to be more easily recycled since it can be cleaned more readily. Furthermore, the paper cup requires an adhesive to bond the components. This can complicate the recycling operation. Conversely, the polystyrene cup is a single material.
i. Polystyrene is one of the plastics that most readily breaks down into its primary constiuent.
j. Should incineration be chosen as the recovery method, polystyrene cups produce almost double the recoverable heat of paper cups based on mass. However, because polystyrene cups are significantly less dense, more cups are required to make up the heat generation.
k. When landfilled, the paper cup can biodegrade under ideal anaerobic conditions. This will contribute to leachate and methane gas generation. The polystyrene cup, however, is not biodegradable.

The article does not endorse the use of the polystyrene cup, but as demonstrated, the polystyrene cup has some advantages. Based on these arguments, attempting to recover polystyrene cups (perhaps even all polystyrene food ware) is worthwhile because of their ease in cleaning and reprocessing for recycling, especially since there are circumstances, typically for sanitation and convenience, which warrant disposable food ware. Conversely, the article also states, "More extensive recycling of paper would thus effect a far more significant reduction of the waste stream volume than would an equivalent fraction of any or all of the other packaging materials in the municipal waste stream." (Hocking, 1991) This agrees with the earlier calculations of polystyrene cup waste quantities.

Despite the apparent recoverability of polystyrene cups, there are certain obstacles to polystyrene recycling in general. One of the main problems is the transportation of the collected polystyrene to the reprocessing operation. Hauling such light material is usually not economical, hence shredding, compacting, and/or baling may be necessary, especially for long transport distances. (Gruder-Adams, May 1991) Furthermore, colour separation of the post-consumer polystyrene and the destruction of pathogens from food ware are also issues. Most of the recovered polystyrene is mixed in colour, and as a result, becomes lower-grade, dark coloured objects intended for non-food applications (e.g. office desk stationary). (Clayre, June 10, 93.) Most polystyrene cups are white coloured and thus would not present a problem by themselves. However, to make the recycling of polystyrene economically worthwhile, the cups would undoubtedly have to be mixed with other polystyrene items. Unless these other items are also colour separated, the effort to single out polystyrene cups may be wasted. Furthermore, all polystyrene that is destined for recycling must be cleaned of contaminants, such as food and paper.

While the polystyrene cup offers many advantages, the polystyrene recovery situation is similar to that of the Tetra-Pak: low quantities of waste when compared to the overall waste stream and developing recycling markets.

2.2.3 Glass Jars

Glass jars are not necessarily beverage containers only but can also hold a variety of food stuffs and beverages. For this research, the glass jars that will be tested are the common type of containers that would be thrown out as waste or else routed to a recyclables collection system, such as a blue box program, instead of being sent to a bottle depot. As was shown in

Table 2.1, returnable glass bottles have reasonably high rates of recovery, ranging from 43% to 74%.

Unlike Tetra-Pak or even polystyrene, glass recycling has been in place for some time. Source reduction has even been practised through the use of refillable glass bottles. In 1988, 6.3% or 10.2 million tonnes of the U.S. municipal solid waste stream was glass (Miller, May 1992). This figure is at least one order of magnitude higher than those of EPS or Tetra-Pak. However, it should be noted that this is a weight comparison. It is possible that glass, being brittle, will more than likely break during landfilling and so reduce that volume it occupies.

Glass is a highly recyclable material and although recycling technologies have existed for some time, glass recycling still has several notable difficulties. Glass recycling can suffer from contamination. The mixing of various glass bottle colours (green, amber, flint) and other materials, such as ceramics or heat-resistant glass, can pose problems for reusing glass. Furthermore, glass containers generally have a low value per ton. Paying to have scrap glass shipped to a distant market could prove too expensive for a recycling/collection operation. Despite these setbacks, glass containers, including refillable ones, had a 31% recycling rate in 1991 in the U.S. (Miller, May 1992).

2.2.4 Summary of Selected Containers' Recovery Potentials

Eventhough the quantity of polystyrene and Tetra-Pak is minute compared to the entire municipal solid waste stream, disposing such containers means losing potentially recoverable resources while adding to the growing solid waste problem of landfill shortages. This is particularly true since Tetra-Paks and EPS containers are generally *single-use, disposable* items. As mentioned in Chapter One, the use of non-reusable beverage containers is increasing. And although not widespread, recycling markets for Tetra-Paks and EPS containers have started recently.

The three chosen containers possess suitable physical characteristics for testing. The case for including the aseptic package centres on several major points: it has only two dominant sizes, all packages possess the same rectangular shape, distinguishing it from the majority of other cylindrical beverage containers, it is one of the few composite material containers, and the packaging material is quite ductile. All these points make the aseptic package unique. Testing glass jars in this research is justified mainly because it offers a distinct physical contrast to the other containers. Futhermore, it is made of a common and established material in the recycling market. Lastly, all of the characteristics that can be found in almost all beverage containers can be tested with these three containers. EPS beverage containers are typically used as an "open mouth" container. Tetra-Paks are usually used as an "enclosed" container, are pliable, and are uniquely rectangular. Glass containers are brittle and the majority are cylindrical. These reasons, combined with the merits from the potential recovery discussion above, indicate that the three chosen beverage containers are suitable for testing in the context of this research.

2.3 MATERIAL PROPERTIES

It was discussed earlier that the beverage containers' materials could affect the performances of the beverage containers during materials recovery operations. Furthermore, given the many types of container materials available, the material that a container is constructed out of is one of its unique characteristics. Based on the criteria established earlier, unique characteristics should be tested. However, little information was available on the behaviour of the materials used in the selected beverage containers undergoing post-consumer recovery. Nevertheless, some general guidelines were applicable and would later prove helpful in the analysis of the gathered data. The properties are discussed with respect to size reduction, which will be explained in a later section.

2.3.1 Expandable Polystyrene

In general, polystyrene has a stress-strain curve typical of a *brittle material*. Fracture normally occurs before the yield point[3] of the material. Expandable polystyrene containers would thus be expected to crack prior to any significant deformation in the overall shape. (This can be demonstrated by squeezing a foam cup.) The mechanical properties, such as tensile or shear strengths, of foamed polystyrene are a linear function of the density of the material (Svec, 1990, p.1990). The mechanical properties are also affected by the material cell size and by the orientation of the container.

2.3.2 Aseptic Packages - Composite Materials

Very little information was available on the overall mechanical properties of the aseptic package. However, since three different materials are used in its construction, the properties of the individual materials give some idea of how the overall container would behave.

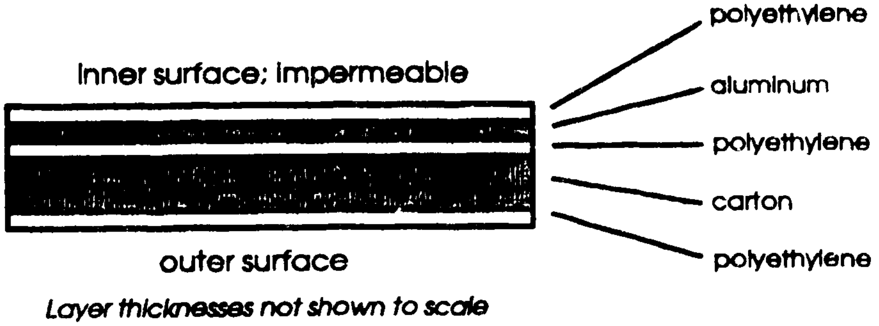

FIGURE 2.1: Cross-section of Aseptic Packaging Material

The carton is made of cardboard-like material, providing stiffness to the container. It is also foldable to allow sheets of aseptic packaging material to be formed into packages. Interestingly, one reference noted that larger cardboard sizes were not as strongly affected by shredding as were smaller sizes (Hasselriis, 1984, p.105). The inner and outer polyethylene sheets are of course flexible. The inner layer is primarily an impermeable layer while the outer layer helps seal the box and provides extra protection. The aluminum layer serves as a barrier against light and oxygen and is very thin and pliable. Because all three layers are very pliable, aseptic packages should exhibit ductile behaviour.

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[3] The yield point is the lowest point on a stress-strain curve at which strain increases without a further increase in stress.

2.3.3 Glass

The properties of any particular glass container are situation specific. For example, strength is not a material constant such as density or thermal expansion. Instead, the strength depends on the container size, preparation, load duration, and surrounding media (Zarzycki, 1991, p.679). The practical strength of glass is determined more by surface flaws than any other factor. Cracks, microcracks, and faults contribute to the discrepancy between the theoretical strength and the real strength, which can be several orders of magnitude lower.

Given the brittle nature of glass and the fact that glass containers found in the recyclable or MSW streams would have already been handled and exposed to external stresses, it is reasonable to expect glass containers to break readily if subjected to any significant impact.

2.4 SIZE REDUCTION

After selecting the beverage containers and the physical characteristics that will be tested, the next step is to determine how the beverage containers will be handled in the research. While it is unlikely that the research procedures would duplicate exactly what such beverage containers would undergo practically, it would be advantageous to model realistic conditions as much as possible.

The current trend for many materials recovery facilities is use a combination of both manual sorting and mechanical unit operations to separate materials (Tchobanoglous, 1993, p.254). Size reduction is one of these unit operations. Collected waste materials are mechanically reduced in size by shredding, grinding, or other similar means.

Size reduction produces a more uniform product that is generally smaller in size than the original product (Tchobanoglous, 1993, p.255). However, the case for uniformity is disputed by an extensive study conducted in 1977 (Trezek, 1977, p.1).

> Size distributions of shredded refuse typically span three to four orders of magnitude. This evidence contradicts the visual claims that state that after refuse is shredded, the size of the product is tends to be uniform. Actually, from the resource recovery point of view, it is fortunate that uniformity in size does not occur; as will be discussed, certain materials tend to fall into various size range bands, and as such, they can be separated.

Despite the contradiction, the last portion of this statement - that shredding results in better separation - actually supports the need for modelling size reduction in this research. Size reduction also performs the following important tasks to aid in the recovery of materials for recycling (Hasselriis, 1984, p.87).

i. Bags and containers are broken to release their contents.
ii. The tendency of materials to tangle is reduced.
iii. Additional removal operations, such as magnetic separaters or air-classifiers, are not as effective if large, bulky objects interfere with the removal process. Size reduction produces an easier-to-handle waste stream.

Based on these reasons, size reduction should be included when testing for the recovery of materials from beverage containers. Screening, which results in a size distribution of the size reduced material, would therefore be the means to "measure" the interaction between the size reduction process and the beverage container The next task was to select the appropriate size reduction method and thus equipment.

Until recently, hammermills were the most common devices to size reduce wastes (Vesilind et al., 1986). A hammermill consists a high speed rotating shaft with affixed stationary or swinging hammers. There may or may not be a grate to control the size of the output. Much of the research conducted in the 1970's to 1980's on how waste materials break involved testing with a hammermill. Consequently, during the initial stages of this research, a laboratory scale or pilot scale hammermill was sought to size reduce the chosen beverage containers. However, several of the major suppliers of size reduction equipment indicated that shear shredders, specifically slow-speed shredders, were more suited for materials recovery facilities (Glass, July 27, 93.) There appears to be a general consensus that the more pliable and fibrous materials encountered in a commingled recyclables stream would be more effectively size reduced by a shredder (Vesilind, Aug. 19, 93.; Whaley, Sept. 23, 93.) and that slow-speed shredders have fewer operating problems. High speed hammermills can suffer from rapid hammer wear and can emit significant quantities of dust and particles. Given these considerations, a slow-speed shredder was eventually rented to shred the beverage containers.

2.5 THEORIES RELATING TO THE BREAKAGE OF SOLID WASTES

Up to this point, what beverage containers should be tested, the characteristics that would be important, and the principle handling method of these containers have been established. The next important stage is to determine if some relationship exists between the physical design of an object and how it subsequently size reduces. Such a relationship (if it has already been proven) may therefore be able to predict how a beverage container will break. This would be useful in helping to design the experiment and would also provide some indication of the experimental results that could be expected.

Almost no information was found concerning the relationship between the shape or geometry of an object and the subsequent recovery of its material after size reduction. However, research has been conducted into how waste materials, in general, could be expected to break during size reduction, specifically from hammermilling. Several theories attempt to predict the size distribution of waste materials after breakage based on either the *size before and/or after the breakage*, the *material* itself, or a *combination of both*. Such theories would be important because they could provide a theoretical basis from which to evaluate and somehow incorporate the influence of the physical design of an object on its shredded material size distribution. The following discussion will focus on the Rosin-Rammler relationship and the II breakage theory.

2.5.1 Rosin-Rammler Relationship

The Rosin-Rammler relationship is widely used to predict the size distribution of size-reduced material. This relationship is a universal law for all powders, regardless of the material or method of size reduction (Vesilind, 1986) and was originally developed for broken coal.

The Rosin-Rammler relationship is written as:

$$Y = 1 - \exp[-x/x_o]^n \qquad [1]$$

In this relationship, x_0 is the characteristic particle size at which 63.21% by weight of the sample passes, x is the particle size, Y is the fraction by weight less than size x, and n is an empirical constant. Plotting the size distributions of broken materials on double logarithmic versus logarithmic paper generally produces straight lines. If this is done, the slope of the resulting size distribution curve is n, the empirical constant. To use this relationship to describe the size distribution for any broken material in any given situation, the constant n and the x_0 variable must first be experimentally determined. Low n values indicate the broken material is distributed over a wide size range whereas high n values indicate a more uniform size distribution over a narrow size range.

The Rosin-Rammler relationship was used initially to describe the various size distribution curves obtained in this research. The slopes, or n constants, were compared to determine if the factors of shape, size, or geometry influenced the shredding and subsequent size distribution of the beverage containers. However, the Rosin-Rammler relationship was later dropped in favour of a more direct method of describing the size distributions. This is discussed in Chapter Four.

2.5.2 Π Breakage Theory

One of the more prominent theories is the Π breakage theory. This theory was originally developed by B. Epstein in 1948 and further contributed to by S. Broadbent and T. Callcott. It was also originally developed for the breakage of coal and has been applied to the size reduction of municipal solid waste (Vesilind and Rimer, 1981; Trezek, 1977). The principle behind this theory can be summarized by the following statement (Vesilind et al. 1986, p.1111).

> ... some fraction of particles of a given size in the feed will be broken while others will not undergo breakage. The product in any specific size category will then be made up of particles from the unbroken fraction plus particles that arrive at this particular product size from the breakage of larger particles.

For each size of particles fed into the size reduction process, one function, the *breakage function*, describes the broken particles' size distribution, or more accurately, "... the fraction by weight of products that have a size less than *x* when particles of original size *y* are broken once" (Vesilind and Rimer, 1981, p.104). The breakage function used in this theory is written as

$$B_{xy} = \frac{1 - \exp(-x/y)}{1 - \exp(-1)} \qquad [2]$$

in which B_{xy} is the cumulative fraction by weight of the broken material smaller than any size x that resulted from the breakage of size y material, y is the original size of the material, and x is the size of material after breakage.

For each size of particles fed in at each stage, another function, the Π term, or *selection function*, describes what proportion of particles will be *selected* to break according to the breakage function. In its original development, this theory held the Π term as a constant. Later research into this theory (for example, refer to Vesilind and Rimer, 1981) revealed that this selection function can change because at different particle sizes, particles may break in different proportions. This situation can be accounted for by using a selection function term, *s*, for each separate size interval. However, this theory is not completely predictive because the Π (or s) term must first be determined for each material type tested. Once it has been calculated experimentally, it can then theoretically be used to predict how other items of the same material may break. The *product* for any given grade (size range) is made up of broken particles that were selected to break from previous grades and from within that same grade but did not fall through to the next grade. Because of this additional change through the grades and a possible nonconstant Π term, a series of product equations results. The Π breakage theory consequently lends itself to matrix calculations. For example, these equations can be written as:

$$p_1 = (b_{11}s_1f_1) + (1 - s_1)f_1$$
$$p_2 = (b_{21}s_1f_1 + b_{22}s_2f_2) + (1 - s_2)f_2$$
$$p_3 = (b_{31}s_1f_1 + b_{32}s_2f_2 + b_{33}s_3f_3) + (1 - s_3)f_3$$

Each successive equation represents the product in each decreasing grade *interval* (i.e. between any two grades). Again, the product at any grade is made up of broken particles that were selected to break from previous grades and from within that same grade but did not fall through to the next grade. The total product also contains the particles that have fallen to that grade interval but did not break. In this example, b_{21} represents the portion of the particles that end up between grades; the material breaks from grade 1 and falls to grade 2. b_{22} represents material that broke but did not fall to the next grade interval. The f terms are the portion of the material that fell to that grade interval but did not break. The selection function, s, governs how much material is selected to break. Conversely (1-s) indicates how much was selected to not break. The Π breakage theory is partially emprical in nature because for each material type, the selection terms must be *first* calculated from the above product equations using both the experimental data (actual product size distribution data) and the breakage function calculated b terms.

A study by Vesilind, Pas, and Simpson in 1986 found the Π breakage theory to be generally a poor predictor/descriptor of how waste materials may break down. Woodchips, paper (newspaper and cardboard), plastic (expandable polystyrene and polyurathane), aluminum, and glass were each tested separately. A pilot scale hammermill and test screens were used. The study compared the actual size distributions of the milled materials against the size distributions predicted by the breakage function and according to their results, only glass produced a curve that was similar for both cases. Their study suggests that the actual B_{xy} breakage function used in this theory may not be accurate for the other materials. This is plausible because glass, like coal, is a brittle material. The rest of the materials, wood, aluminum, plastic, have higher degrees of ductility. Their study also confirmed that the size reduction process is not adequately described by a constant Π term. The estimated s terms (selection function terms throughout) decreased with decreasing particle size, indicating that particles are more difficult to break as their size decreases. According to the results presented in their 1986 article, it appears that for all the materials tested with the exception of glass, the estimated selection functions vary between a low of 0.20 and a high of 1.00 for the *first two* upper particle size ranges for every material type, while the remaining lower size ranges vary between 0.0 and 0.28, with the zero value appearing very frequently. Glass had estimated selection function values that decreased continuously with each smaller particle size range.

An earlier study by Trezek (1977) and discussions by Vesilind and Rimer (1981) indicated that modifications to the breakage function may produce better results. For example, the breakage function was modified to the following form

$$B_{(x)} = \frac{1-\exp[-(x/x_o)^n]}{1-\exp(-1)} \qquad [3]$$

This is the modified Broadbent-Callcott equation. All terms are as defined previously except for n, which is a positive index varying between 0.845 and 1.0. According to Trezek (1977), this equation was found to produce good results in describing the second or third grinding of *municipal solid wastes* (as opposed to the hammermilling of separate components as studied by Vesilind et al. in 1986). Trezek also found that another form of the breakage function modelled after the Gaudin-Meloy distribution equation, produced good results in describing the primary grinding of solid waste. This equation can be written as

$$B_{(x,x_o)} = 1-(1-x/x_o)^r \qquad [4]$$

To test if the breakage of the various waste components, as performed by Vesilind et al. in 1986, could be better described, the modified Broadbent and Callcott equation [3] was applied to the original data in the study because the Vesilind et al. study shredded each refuse component twice. Thus, the product of the first shredding was used as the feed for the second shredding.

Styrofoam was selected as the component to test because expanded polystyrene will also be tested in this research. The data for the product (after second shredding) (Simpson, 1984), the values as predicted by the unmodified Broadbent-Callcott breakage function [2], and the values predicted using the modified Broadbent-Callcott equation [4] with n=0.845[4] were plotted in the following figure. In these calculations, *only the breakage function itself* and not the product equation was considered. As Vesilind et al. points out in the 1986 study, the selection function itself is highly dependent on the breakage function and thus the breakage function should first be determined to be adequate.

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[4] Because no other data existed, the value of 0.845 was arbitrarily used.

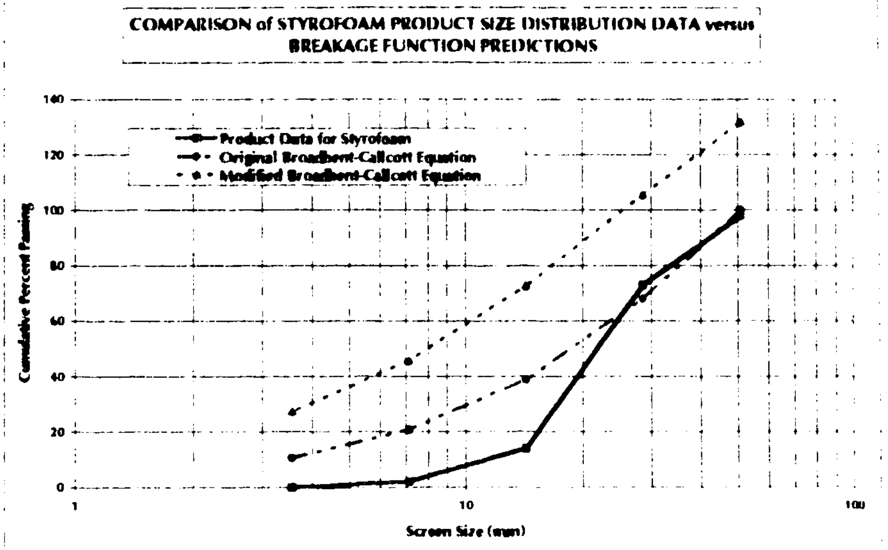


FIGURE 2.2: Comparison of Actual Data to Breakage Functions Descriptions
Source: Application of Π Breakage Theory to Refuse. Simpson, 1984.

From this plot, it appears that neither of the breakage function equations serve as a good descriptor of the actual distribution of styrofoam. The modified Broadbent-Callcott equation, using the n index equal to 0.845, also predicted distribution values greater than 100%, which is not physically possible. Figure 2.2 suggests that at some intermediate size range, the actual distribution of the product particles changes. Nonconstant selection terms may therefore account for this shift. From the Vesilind et al. 1986 study, the estimated selection functions for expanded polystyrene were:

Geometric Mean of Sieve (mm)	*Selection Function*	*Estimated Selection Function*
50.8	s_1	1.00
28.6	s_2	0.71
14.3	s_3	0.00
7.1	s_4	0.14

TABLE 2.2: Estimated Selection Functions for EPS
Source: Evaluation of Π Breakage Theory for Refuse Components. Vesilind et al. 1986.

These values will be modified and used in the product feed equations based on the *original work* by Simpson for the Vesilind et al. study to determine if the *product* predicted (which combines both the breakage and selection functions) using these newly estimated selection functions matches the actual particle size breakdown for EPS. Because the Vesilind et al. study concluded that the errors for each product equation were generally small, the error term will be ignored for these calculations. Note the following comparison will use the original Broadbent-Callcott equation [2].

Because the first two selection functions are significantly different from the last two, each pair of selection functions were averaged. Thus any calculations calling for s_1 or s_2 and s_3 or s_4 used the values of 0.855 and 0.07 respectively. Recalculating the original product equations and comparing them against the original product values gives the following table.

Geometric Mean of Sieve (mm)	*Actual Product*	*Calculated Product Using Selection Functions*
50.8	0.02	0.0543
28.6	0.25	0.210
14.31	0.59	0.501
7.14	0.12	0.139
3.57	0.02	0.0853

TABLE 2.3: Comparison of Actual and Calculated Products

From this table, using two different selection functions to represent the two main groupings of particles results in calculated products that are reasonably close to the actual products. However, this analysis appears partially flawed. For the separate refuse components analyzed in the 1986 Vesilind et al. study, the Broadbent-Callcott equations were found to inadequately describe the actual breakage pattern. Despite this failing, the breakage equations are needed in the product equations, along with empirical data, to calculate the selection function terms. The "best" selection function terms are then used to calculate the most descriptive product equation. In short, an inadequate parameter (the breakage function) is necessary to calculate the parameter (the selection function) that is necessary for predicting the size distribution. This "adjustment" does not affect the breakage function itself, but the product equation. The original problem concerning the adequacy of the breakage function does not appear to be addressed. One possibility to overcome this apparent deficiency is to incorporate the selection function concept within the breakage function itself.

Due to the inability of the various Broadbent-Callcott equations themselves to accurately describe material breakage, the unresolved problem of how to best modify the Π breakage theory, and the constraints on this thesis, this theory will not be used in this research.

2.6 HYPOTHESES

This chapter developed the reasons for selecting Tetra-Paks, EPS containers, and glass jars for testing. These containers possess suitable properties that make them unique and thus ideal for testing. At the same time, they also represent common attributes found in almost all other beverage containers. It was concluded that the physical characteristics of size, shape, geometry, and material are the important factors that may affect how a beverage container is shredded and consequently how well its material can be recovered. As stated in Chapter One, the purpose of testing these factors using these containers is to determine if a characteristic or even a combination of characteristics can better *isolate* the container material into a specific size fraction after shredding. This could lead to an alternative beverage container design that would subsequently improve the recovery of the container material.

The following hypotheses concerning the influence of the above factors on the shredding of the beverage containers are based upon interpreting the limited information available and the possible shredding scenarios.

1. ***Size - specifically, the volume of the container.***
   The smaller the size of the container, the smaller the pieces of material should be after shredding. This is hypothesized for two reasons: 1) smaller containers have less material to begin with and 2) smaller containers would probably be more easily caught in a shredder and torn apart. Conversely, larger containers may result in a greater size range of shredded materials, from large to small pieces. Smaller containers should therefore result in a more concentrated distribution of shredded material. Two common volumes will be tested, 250 mL versus 1000 mL, because all three container types are commercially available in these formats.
2. ***Shape - whether the container is rectangular or cylindrical.***
   Cylindrical containers may "roll" on top of the shredder blades for some brief period of time until a large enough mass of containers is accumulated to push the initial containers through. This may lead to incomplete shredding of the containers, affecting its subsequent size distribution. Conversely, rectangular shaped containers, such as aseptic packages, should allow for easier "grasping" of corners by the shredder blades. It is less likely that rectangular containers would roll on top of the blades and so any accumulated mass of containers above could more easily push the initial containers through, resulting in more complete shredding and a more concentrated size distribution.
3. ***Geometry - whether the container has an open mouth or is enclosed.***
   Containers that are commonly found "open-mouthed", such as EPS cups, should offer a greater area for blades to grasp onto and shred. Such containers should result in a greater degree of shredding and a more concentrated size distribution. Enclosed containers, however, should offer no such advantage. In addition, enclosed containers may offer greater structural integrity because all sides are "intact". Open-mouth containers are not reinforced on one side and thus may be less able to resist any shredding action.
4. ***Material used to construct the container.***
   The material the beverage container is constructed of should make a difference in how it is shredded. Although this research will not rigourously study the breakage mechanics of the containers, it is assumed that brittle materials will break into smaller pieces. Pliable or "soft" materials may not shred uniformly and may instead be "mangled", as opposed to being shredded, resulting in a poor size distribution. In this research, glass represents a very brittle material while Tetra-Paks represent a very ductile material.

CHAPTER THREE

Experimental Design and Procedure

3.0 INTRODUCTION TO THE EXPERIMENTAL DESIGN

The experimental design was devised to test the influence of the principal factors of size, shape, geometry, and material on the shredding behaviour of the chosen beverage containers: Tetra-Paks, expanded polystyrene (EPS) containers, and glass jars. The design and the results later gathered from the experiment should fulfill the design-orientated research objectives stated in Chapter One. They are briefly restated here.

I. Determine which factors significantly affect the shredding behaviour of the beverage container.
II. Determine the relationship between any such significant factor(s) and the subsequent shredding behaviour.

Satisfying these objectives will enable the research to later conclude if an alternate beverage container design is more advantageous compared to other, current designs and what the implications of this conclusion are for the beverage container.

The experimental design was divided into several major components. A *2^3 factorial design* was selected as the main experimental design to evaluate the effect(s) of the beverage container characteristics throughout this research. This design was applied to both Tetra-Pak and glass containers but not to expandable polystyrene containers. Instead, EPS containers were tested through simple comparisons. These tests are referred to as the ***preliminary tests*** and would evaluate the effect of size, shape, and geometry within the same container/material type.

The following tests were also planned.

I. ***Materials Test***. Due to the difficulty this factor would have created if it was tested along with the others in the factorial design, a separate test would be conducted to determine if the material the beverage container was constructed of influenced its shredding and screening behaviour.
II. ***Orientation Test***. It was suspected the beverage container orientation just prior to shredding would be important in affecting how it ultimately shredded. However, no prior information was found on this subject. This orientation factor is not considered unique to any particular beverage container and so was not included in the development of the hypotheses in Chapter Two. However, it may be an *operational* consideration for shredding materials. This test was devised to determine the signficance of the pre-shredding orientation on the shredding behaviour.
III. ***Commingled Recyclables with Beverage Containers Test***. If a particular characteristic(s) of a beverage container was found to improve its shredding and subsequent screening behaviour, an application of such a finding would be to determine if the characteristic(s) enabled the container to separate out from a mixture of several materials.This test would attempt to determine if certain beverage containers, because of their characteristics, were indeed better isolated from other recyclables after shredding.

Along with the EPS containers, the materials test, the orientation test, and the commingled recyclables test would be performed as simple comparisons between changing variables. However, because these last three tests are straightforward comparisons and could not be planned in detail without knowing the general outcome from the preliminary tests, these three tests are described in the later sections dealing with the experimental procedures.

This chapter will begin with a description of the factorial analysis. Afterwards, the various experimental procedures used during the testing will be described. In general, the actual experimental procedure was consistent throughout the research but any discrepancies or problems have been also noted.

3.1 FACTORIAL DESIGN AND ANALYSIS

This section will detail the factorial design and analysis devised for this research. The reference used is Statistics for Experimenters by Box, Hunter, and Hunter, 1978.

3.1.1 Overview of Factorial Design and Analysis for this Research

A 2^3 factorial experiment was conducted for both the Tetra-Pak and glass containers. Each involved testing three factors at two different levels. In this case, the three factors are size, shape, and geometry. The levels are, respectively, 250 mL and 1000 mL, cylindrical and rectangular, and open and enclosed. The characteristics of the chosen beverage containers therefore lend themselves ideally to a factorial analysis and vice versa. This situation requires a total of eight runs to test all combinations of factors and levels. Only size is a continuous variable; the other two could have varying degrees of "rectangularity" or "enclosure" but are for the most part a discrete variable (i.e. one or the other). The levels are further coded -1 for the lower level and +1 for the upper level.

Factors	*Level -1*	*Level +1*
Size	Small	Large
Shape	Cylindrical	Rectangular
Geometry	Open Mouth	Enclosed

TABLE 3.1: Factors and Levels for Factorial Design

The factorial design offers the ability to test the effects caused by each factor, called the *main effects*, in fewer runs than if one factor was held changed and the others were held constant. Furthermore, it is possible to determine if any interactions between variables produce an effect (*interaction effects*). The factorial design also produces a linear model that predicts the theoretical reponse given the various factors that have a significant effect. The factorial design for this experiment is shown in Table 3.2.

Each run represents a *container configuration* that would be shredded and screened to produce a data set, which is the *response variable or output*. A configuration could be tested several times, resulting in replicated data. It was discovered later that performing a consistent number of replicates throughout the experiment was not always possible. Chapter Four discusses what was eventually used as the response variable to characterize the resulting size distribution curves. For example, the size distribution curve for run six of the Tetra-Pak containers would come from shredding and screening large, rectangular, open mouth Tetra-Pak packages.

Run	*Size*	*Shape*	*Geometry*	*Description*
1	-	-	-	small, rectangular, enclosed
2	+	-	-	large, rectangular, enclosed
3	-	+	-	small, round, enclosed
4	+	+	-	large, round, enclosed
5	-	-	+	small, rectangular, open mouth
6	+	-	+	large, rectangular, open mouth
7	-	+	+	small, round, open mouth
8	+	+	+	large, round, open mouth

TABLE 3.2: Factorial Runs

Eventhough it was considered to be an important factor, the fourth characteristic, container material, was not included because it would have dramatically increased the complexity of the current 2^3 factorial design to a "3^4" format. There would have been four factors but only three levels for the material factor; EPS, Tetra-Pak, and glass. Finding a third level for the factors of shape and geometry would have involved establishing some acceptable definition for "half-rectangular" or "half-enclosed" respectively. It was ultimately decided that each container type should be tested for the first three factors independently and then tested against one another to determine the effect of different materials. In doing so, some information would be lost. Structuring the experiment in this manner does not evaluate any potential material interaction effect with one, two, or all of the other factors. This was deemed as an workable situation given the alternative of a difficult to manage, "incomplete" factorial design.

3.1.2 Significance of Effects

Once the main effects and interaction effects have been calculated, it is important to determine which effects, and hence factors, are actually significant in affecting the size distribution of the shredded beverage containers. This can be performed by constructing a *half-normal probability plot* of all the effects. The absolute value of each effect is plotted in increasing magnitude on a half-normal probability plot (Johnson and Leone, 1977, p.802). Effects that are insignificant are most likely the result of random variation and should thus be (approximately) normally distributed and lie on a reasonably straight line between these insignificant effects. Effects that are significant (i.e. that are the result of more than just random variation) should depart noticeably from the straight line.

Another significance determination method is to perform an *analysis of variance* (ANOVA) on the factorial analysis. In this case, the experimental conditions would be the "treatments" and the "blocks" would be the replications of each configuration for the ANOVA. Both of these techniques were used in Chapter Four to analyze the preliminary Tetra-Pak and glass tests to determine which effects were significant.

3.2 EXPERIMENTAL PROCEDURE

Each container configuration or experimental run described in the factorial design was subject to the same experimental procedure. In fact, almost all tests performed followed the same general procedure, although several did have modifications. However, prior to testing, Tetra-Pak containers, glass jars, and expanded polystyrene (EPS) cups and containers were acquired for testing. To fulfill the factorial design, cylindrical Tetra-Pak containers had to be hand constructed. Glass jars were generally available in all configurations required for the factorial design. Rectangular EPS containers suitable for testing could not be purchased or fabricated and thus this particular container configuration was not tested.

The beverage containers were shredded using a slow-speed shredder and screened using a mechanical screen. The total weights of each experimental container run were taken prior to shredding, after shredding, before and after air-drying (if applicable) to determine any losses due to shredding and screening or handling of the beverage containers, as well as evaporation of any fluid. The weight of each screen size was measured after screening to determine the final size distribution of the shredded beverage container material.

The general experimental method is outlined as follows.

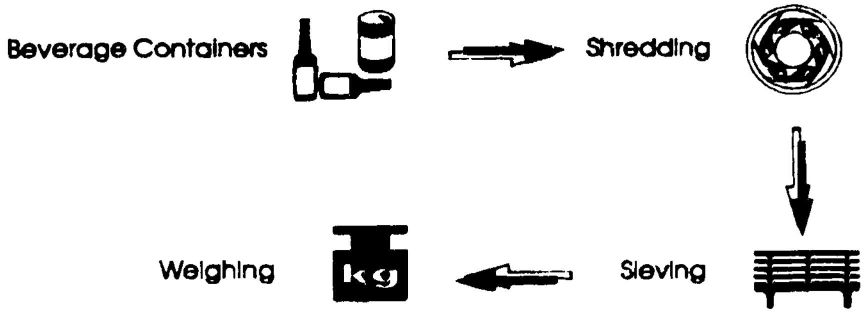


FIGURE 3.1: General Experimental Procedure

3.3 ACQUISITION OF BEVERAGE CONTAINERS

3.3.1 Tetra-Pak Containers

Commercial rectangular Tetra-Pak containers were available in both the 250 mL and 1000 mL sizes. The Tetra-Paks were used but the ones provided for testing were either whole or only slightly deformed. They were generally considered intact for testing purposes. The 250 mL Tetra-Paks often had the plastic drinking straws inside the package. Removing the straws would have damaged the container and so the straws were left inside. Aside from the punctured straw hole on the 250 mL Tetra-Paks and the cut-open spout on the 1000 mL Tetra-Paks, these containers matched the *rectangular, enclosed* configurations. The *rectangular, open mouth* configurations for both sizes were obtained by cutting the top surface (the "box lid") off a portion of the 250 mL and 1000 mL Tetra-Pak containers from The Recycle Zone.

Obtaining cylindrical configurations for the Tetra-Pak containers proved more difficult. No cylindrical containers were or are manufactured. However, using supplied aseptic package material, cylindrical Tetra-Pak containers were made by wrapping the aseptic packaging material around a cylindrical form. The dimensions of the cylindrical Tetra-Pak containers were made to closely resemble those of the rectangular Tetra-Paks and still have the same size. The *cylindrical, enclosed* configurations had both cylindrical ends. Drinking straws were included in the *250 mL, cylindrical, enclosed* configuration to imitate the straws found in the actual aseptic packages. Open configurations were simply made without one end. The hand made cylindrical containers also had to be joined in a similar method to the factory produced containers. Technical personnel at a Tetra-Pak packaging plant advised that using a hot melt glue gun to join the seams would most likely be the best and closest method for the hand made containers.

Hand construction of the cylindrical containers proved to be a very time consuming process. As a result, only one trial (i.e. no replicates) for the each of the four cylindrical configurations was planned. Conversely, the supplied "regular" rectangular Tetra-Paks were available in large quantities and several replicates were performed for these configurations.

Tetra-Pak Container	*Approximate Dimensions (mm)*
250 mL rectangular	63 length by 41 wide by 106 high
250 mL cylindrical	57 diameter by 98 high
1000 mL rectangular	95 length by 63 width by 167 high
1000 mL cylindrical	87 diameter by 168 high

TABLE 3.3: Dimensions of Tetra-Pak Containers

3.3.2 Glass Jars

The *250 mL cylindrical and rectangular* configurations, as well as the *1000 mL cylindrical* configurations glass jars were purchased. The cylindrical jars are *universal jars* and are similar to the "mayonnaise" type jars seen in food stores. The rectangular jars are similar to the square jam jars. Large rectangular or square jars are apparently no longer in Western Canada. Instead, mostly unused, second-hand 1litre square Mason brand canning jars were purchased. Although the corners are rounded, these jars have four distinct corners with relatively flat sides.

To satisfy the enclosed configurations, a portion of the glass jars were capped with metal twist or thread lids. Glass lids would have been preferred because then only one material is used throughout the preliminary glass tests. However, only metal or plastic lids were available. Using metal lids could possibly introduce another material variable into the research but this could not be avoided. Fortunately, as the analysis will later demonstrate, enclosing the glass jars did not make a significant difference. Enough glass jars were available to perform two trials for each configuration.

Glass Jar	*Approximate Dimensions (mm)*
250 mL rectangular	63 width square by 104 high
250 mL cylindrical	66 diameter by 101 high
1000 mL rectangular	84 width square by 195 high
1000 mL cylindrical	101 diameter by 170 high

TABLE 3.4: Dimensions of Glass Jars

3.3.3 Expanded Polystyrene Cups and Containers

EPS cups were available in the 250 mL size. EPS containers were available in the 1000 mL size. Although it is classified as a container, the 1000 mL size resembles a large cup. Both sizes are cylindrical, although there is a vertical slant. Two trials were performed for each configuration. Because no equivalent rectangular container was available, it was attempted to make rectangular EPS containers. One suggestion was to crease the EPS containers, creating four corners, and then heating them briefly to retain the square shape. This proved very difficult. Not only would the EPS crack after creasing, heating also produced observable material changes. This would alter the material properties before any testing was performed and possibly compromise the results. Rectangular EPS configurations were thus dropped from testing. Polystyrene lids were available to satisfy the *cylindrical, enclosed* configurations but these were not purchased. Because of restrictions on the available time and resources for testing, it was decided to test the EPS containers last. It was thought that the results from the Tetra-Pak and glass preliminary test would help later determine which tests on EPS should be performed.

EPS Container	*Approximate Dimensions (mm)*
250 mL cylindrical (SM9)	80 top diam., 50 bottom diam. by 100 high
1000 mL cylindrical (135)	119 top diam., 84 bottom diam. by 157 high

TABLE 3.5: Dimensions for EPS Containers

3.4 DESCRIPTION OF TESTING EQUIPMENT

All experimental runs were conducted in the shop area at the *Waste Management Incorporated West Edmonton Landfill and Recycle Facility*. A *RABCO 2032* slow-speed shredder was used to shred all beverage containers. Shredded material was screened using a *SELSBERG* Engineering mechanical test screen.

3.4.1 RABCO 2032 Slow-Speed Shredder

The RABCO 2032 shredder was the only small-scale shredder available for testing purposes. According to the supplier, this unit was designed specifically for shredding recyclables. The unit has a 20 inch by 32 inch intake opening. Fifteen fixed blades, approximately 0.5 inches or 13 mm wide, run along the back and front lengths of the actual shredding area. A horizontal shaft, rotating from back to front when viewed from above, has 16 staggered blades approximately 0.625 inches or 15mm wide. Each blade on the shaft actually consists of two separate blades spaced 180 degrees opposite from each other. The shaft rotates at approximately 15 rpm. The approximate clearance is 0.5 inches or 13 mm between any fixed blade surface and rotating blade surface. Directly overhead of the blade area is a bar equiped with 7 short stubs, rotating at approximately 8.5 rpm in a large circle, to push material down into the blade area. This circling bar, at its lowest point, will slightly overlap with the rotating blades. All beverage containers to be shredded were loaded into the shredder from the top.

The exact force of the shredder is not known but all operating conditions were held constant throughout, except where noted. The shredder is equipped with an air mechanism that allows difficult-to-shred or large objects to pass through unshredded to prevent damage to the shredder. This air mechanism was set at approximately 42 to 45 psi for all tests except for several tests of the commingled recyclables stream (set at 55 psi).

3.4.2 SELSBERG Test Screen

This test screen can handle a 10 kg sample of low density material and is designed specifically to analyze the size distribution of shredded waste. Nine screens are stacked on top of each other Shredded material is poured into the top (largest) screen and a lid is fastened down, securing all screens to the moving portion of the frame base. The material is then vertically and violently shaken to fully distribute material throughout the various screens. The mechanical motion sends the screens through a vertical amplitude of 50 mm. The screen operates at a minimum frequency of 200 rpm. The operating conditions for the test screen were held constant for all tests. The manufacturer recommends a screening time of two to three minutes. Except for glass container test runs, all runs were limited to a total of two minutes.

Each screen measured approximately 1 m by 1 m and was either 250 mm or 150 mm in depth. The test screen originally had the following screen sizes: 125 mm, 80 mm, 50 mm, 25 mm, 15 mm, 10 mm, 6 mm, 3 mm, and 0 mm (bottom tray). The bottom of each screen consisted of holes of the specified size diameter. Because of the nature of the material to be tested, the 3 mm screen was removed, leaving eight screens for the experimental runs. As shown in the data analysis, the smaller screen sizes collected insignificant amounts of material.

3.5 DETAILED EXPERIMENTAL PROCEDURE

Where possible, a 10 kg sample of beverage containers was measured for each experimental run (container configuration). Large cardboard boxes were weighed and used as containers during the weighing of the containers, for pouring the containers into the shredder hopper, and for collecting the shredded material afterwards. The scale was checked for accuracy against a known 2 kg mass prior to each day's experimental runs.

Depending on the actual volume of the 10 kg beverage container sample, all experimental runs attempted to shred the sample in one load. If the hopper was too small, the sample was broken into two smaller loads that were shred consecutively. The actual shredding continued until all containers had been fed through the blades. Depending on the container type, the shredder was then reversed (rotated backwards) and run forward for several cycles to free any shredded material caught within the blades. Shredded material that was too deeply embedded between the blades and could not be removed by the "reverse and run" procedure was left. Forcibly removing such material would tear the trapped material, altering its shredded size. However, this material would often dislodge and mingle with the next batch of shredded beverage containers, contaminating the following batch's results. Where possible, any visible contaminants were removed by hand after shredding. However, as shown by the data, the dislodged contaminants (loss due to shredding) usually amounted to only several hundred grams or less.

The shredded material was then poured into the top of the mechanical screen. The screen was operated for a maximum of two minutes. If necessary, the screening was divided into two 1 minute loads. Approximately half of the shredded batch was poured into the test screen, mechanically shaken for one minute, and then the remaining shredded material was poured into the test screen and the entire load was shaken for the final minute. It was observed early in the testing that low density materials such as the shredded Tetra-Pak material could potentially trap the smaller shredded pieces within the larger screen sizes by overpacking the upper screens, effectively reducing the amount of space available for agitating the pieces. This condition will be discussed in greater detail later.

After screening, each screen was individually weighed on the scale. The total weight of the screen, after subtracting the weight of the screen itself, determined the mass of shredded material in that size fraction. There was almost always a loss after screening due to handling and infrequently a mass gain due to contamination. *The total of all individual size fraction masses was used as the total mass when calculating the percentile size distribution.* When all size fractions had been weighed, a sample of that shredded batch was taken. The screens were then cleaned and restacked for the next experimental run.

Experimental runs were performed in no particular order. However, due to limitations on access to the facility and the equipment and supply availability, especially for the glass containers, several of the replicate runs had to be performed consecutively.

Due to the time constraints of hand fabricating the cylindrical Tetra-Pak containers, it was not possible to perform replicates for all desired Tetra-Pak container configurations. Several of the rectangular Tetra-Pak experimental runs also resulted in unusual and unexpected size distributions. If warranted, a *partial batch experimental* run of a container type was performed. This partial batch, subjected to the same experimental conditions as the other runs, ranged from 2 to 5 kg in mass.

3.5.1 Procedures Specific to Testing Tetra-Pak Batches

All Tetra-Pak batches were reversed and run four times after shredding to free any shredded material lodged between the shredder blades.

The rectangular aseptic packages usually had juice remaining in the boxes. This was especially true for the 250 mL size boxes. The 1000 mL containers were typically dry. To prevent excessive weighing discrepancies due to fluid loss during screening, the shredded 250 mL juice boxes were allowed to air-dry prior to screening. Because access to the facility was not available at all times, the time allotted for air-drying varied. It was observed, however, that time periods longer than "overnight drying" (approximately 18 hours) resulted in little difference in the amount of fluid evaporation.

The majority of the Tetra-Pak tests, especially for the 250 mL size boxes, were screened in two loads to avoid excessive entrapment of shredded pieces in the upper screens. It was noted that the 1000 mL size containers did not have the same degree of "packing" in the upper screens as the 250 mL size containers. In fact, for the *1000 mL, rectangular, open, Tetra-Pak* configuration, replicate one was screened in one load while replicate two was screened in two loads. Replicate one, surprisingly, had more material distributed to the lower screen sizes. Overpacking, however, did appear to be a concern for the *1000 mL, cylindrical, open, Tetra-Pak* configuration and a second partial batch was performed for this run. Because of these discrepancies, all later Tetra-Pak runs were screened in two loads. Earlier runs that produced unusual distributions were retested and screened in two loads to eliminate the possibility of error to overpacking. However, as will be discussed later, simply screening the shredded material in one load did not warrant eliminating the batch data from the entire data set.

3.5.2 Procedures Specific to Testing Glass Containers

It was generally not required to reverse and run the shredder to loosen any trapped glass pieces after shredding. If needed, the shredder was only reversed and run once.

Because of the brittle nature of glass, screening the shredded glass pieces was restricted to 15 seconds for the first glass test performed. The normal 2 minute mechanical screening would undoubtedly fracture the glass into even smaller pieces, resulting in a size distribution non-representative of the shredding process. Replicate one of the *1000 mL, cyclindrical, open, glass configuration* was screened for 15 seconds. However, even this short time period was found to result in excessive breakage. This data has since been disregarded. Instead, for all glass tests, the screens were shaken by hand in side-to-side rocking motion to distribute the glass throughout the screens and then subject to a 1 to 2 second mechanical screening period to ensure there were no pieces overlapping and thus trapping smaller pieces. This procedure appeared to minimize the amount of additional breakage while thoroughly screening the shredded glass.

3.5.3 Procedures Specific to Testing Expanded Polystyrene Cups

As with the Tetra-Pak containers, all of the expanded polystyrene (EPS) containers were reversed and run four times after shredding and subsequently screened in two loads.

Because of the very light density of the EPS cups, thousands of cups would have been required for a 10 kg sample. Instead, for each respective size, approximately the same number of cups as Tetra-Pak containers were tested. Nine hundred of the 250 mL size EPS cups and 300 of the 1000 mL EPS cup-containers were used.

3.6 CONTAINER MATERIALS COMPARISON

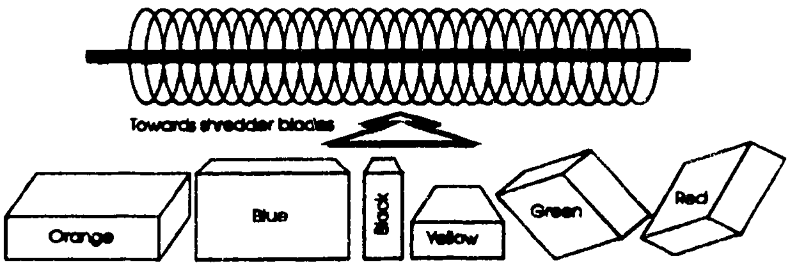
This test was devised to evaluate the difference in shredding behaviour due to the materials used for each beverage container. As discussed earlier, including this factor in the factorial analysis would have proved difficult to effectively manage. Instead, a separate test was conducted to compare one material against another.

Since glass and EPS are difficult to mold, Tetra-Pak containers were specifically constructed for this test, which were then shredded and screened as all other preliminary tests were conducted. The Tetra-Pak containers for the actual materials test were made by wrapping a Tetra-Pak sheet around a *1000 mL, rectangular, open, glass jar*. The resulting construct was *dimensionally similar* to the jar, except for the mouth end; the curves of the glass jar proved too difficult to duplicate. These containers were then shredded and screened in the same manner as all the other tests. Any difference between the shredding data of the Tetra-Pak containers and that of the *1000 mL, rectangular, open, glass jar* should then be due to material differences only, assuming small deviations in dimensional comparisons are insignificant. Chapter Four provides more details on this test.

3.7 EFFECT OF BEVERAGE CONTAINER ORIENTATION TEST

It was suspected that the orientation of a beverage container just as it was shredded would affect its subsequent shredding behaviour. The pre-shredding orientation of the beverage container is assumed to depend mainly on the random placing of the container after being poured into the shredder. The main tests on Tetra-Pak, glass, and EPS cups therefore used a large number of containers that were simply poured into the shredder hopper to duplicate this randomness in orientations and thus hopefully eliminate any orientation bias. However, a series of tests were conducted specifically to determine what effect the orientation would have on the shredded material. The orientation of the container is a procedural consideration that while not unique to a specific container in the random case, may still impact the outcome of this research. In this respect, the orientation of a container may be as important of a factor as the four physical characteristics. If a specific orientation was found to produce a higher degree of material isolation after shredding than the others, this may suggest aligning the beverage containers for improved materials recovery in a materials recovery operation.

The orientation test was performed three times. In each case, six 1000 mL rectangular, enclosed Tetra-Paks were used. Each was spraypainted a different colour so that the shredded material could be tracked and related to its original orientation prior to shredding. Originally, the six Tetra-Paks were to be randomly poured into the shredder and its arbitrary resting position considered its pre-shredding orientation. However, it was found that several of the boxes had the similar orientations. Other boxes would overlap each other and once the shredder started, would shift in orientation. As a result, the six containers were placed in specific orientations, as illustrated in Figure 3.2, along the fixed and rotating blades of the shredder.



Overhead View of Containers Placed In Shredder
(Set Three)

FIGURE 3.2: Pre-shredding Position of Tetra-Paks in Orientation Test

The pre-shredding positions are described in the following table. Note that the colour/position combination varied for the other two tests.

CONTAINER COLOUR	*PRE-SHREDDING POSITION*
Orange	*Side, Narrow.*
Red	*End, Corner (Spout Down)*
Black	*End, Narrow*
Green	*End, Corner (Spout Up)*
Blue	*Side, Broad*
Yellow	*End, Broad (Spout Up)*

TABLE 3.6: Pre-shredding Positions for Orientation Test

Shredding of the containers occurred almost immediately after the shredder was started. The shredded material was collected and pieces were regrouped according to colour. The types of breakage were described. All experimental procedures up to this point were videotaped and photographed. The shredded material was then mechanically screened. The quantities in each screen size were quite small and were therefore bagged and weighed using a laboratory analytical balance.

3.8 COMMINGLED RECYCLABLES AND BEVERAGE CONTAINER TEST

The objectives of this research were to determine if a particular characteristic(s) of a beverage container would improve its shredding and subsequent screening behaviour. If a characteristic(s) was found to have such an effect, an application of such a finding would be to determine if the characteristic(s) enabled the container material to separate out from a mixture of several materials after shredding. Such a mixture could be found in, for example, a recovery operation that collects a commingled stream of various recyclable items that must be later separated for processing. To test this, a series of four shredding and screening tests were performed which combined *250 mL, rectangular, enclosed Tetra-Paks* with a typical commingled recyclables mixture. 250 mL size Tetra-Paks were selected for the following reasons.

1. The preliminary tests results from the shredded and screened 250 mL size Tetra-Paks were observed to have a reasonably high degree of isolation compared to the 1000 mL Tetra-Paks, particularly into the 125 to 80 mm and 80 to 50 mm size ranges. This observation that size appeared to produce a higher degree of isolation indicated that size was most likely a characteristic that would improve the recovery of the beverage container material.
2. Large quantities of Tetra-Paks were easily available. This was especially important since the test had to be repeated several times.

3.8.1 Quantities of Commingled Recyclable Items for Testing

The breakdown and test materials for the commingled recyclables stream was provided by the *Edmonton Recycle Society (ERS)* which operates the curbside recycling program in north Edmonton.

Based on a 13 month average from 1993 to 1994, the percentages for material types received by the curbside program were calculated. Both the paper and metal categories were subdivided into additional categories based on the *sales* of these collected materials to recyclable materials markets. At the time, no further subdivisions based on collection amounts were available. However, the sales figures provide an approximate estimate of these breakdowns and correlate roughly with the total material type collected.

MATERIAL TYPE	*PERCENTAGE COLLECTED*
Paper and paper items	*82.2%*
Glass	*5.0%*
Metal (all types)	*9.3%*
Plastics (all types)	*3.5%*

TABLE 3.7: Percentage of Material Type Collected by Edmonton Recycling Society

PAPER	*As Percent of Paper Only*	*As Percent of Total*
Newspaper	*71.9%*	*59.1%*
Mixed Waste	*9.6%*	*7.9%*
Cardboard	*10.1%*	*8.3%*
Phone Books	*6.1%*	*5.0%*
Milk Cartons	*2.3%*	*1.9%*
METAL	*As Percent of Metal Only*	*As Percent of Total*
Aluminum	*3.5%*	*0.33%*
Steel	*96.5%*	*9.0%*

TABLE 3.8: Percentage Paper Type and Metal Type Collected

The test batches were limited to approximately 10 kg because of the large amount of newspaper present, which qualified as a loosely packed, light density material for using the test screen. Aluminum constitutes a very small percentage of the total metal. Consequently, no attempts were made to model the aluminum collected. Based on a 10 kg sample size, the approximate following quantities were weighed out for each test run.

For the purposes of testing, *mixed waste* was considered to be various types of bond paper, publications, glossy prints, and magazines. *Cardboard* also included boxboard, such as cereal boxes and packaging boxes. No phone books were actually tested in the experimental runs.

MATERIAL TYPE	*MASS (kg)*
Newspaper	*5.91*
Mixed Waste	*0.79*
Carboard	*0.83*
Phone Books	*0.50*
Milk Cartons	*0.19*
Glass	*0.50*
Metal	*0.93*
Plastic	*0.35*

TABLE 3.9: Test Quantities of Recyclable Materials

3.8.2 Quantities of Tetra-Pak to Test with Commingled Recyclables

Tetra-Paks are not currently collected by the ERS as part of their recycling program. However, from Chapter Two, approximately 169 000 kg of Tetra-Pak are used in the metropolitan Edmonton area annually. The *ERS* does not collect from all of metro-Edmonton, but for order-of-magnitude calculations, this discrepancy was ignored. From March 1993 to March 1994, the *ERS* collected 13 504 579 kg of recyclabe materials. If the above quantity of Tetra-Pak were included in the *ERS* collection quantities, it would constitute 1.2% of the total collected. This percentage of a 10 kg sample would be only 0.120 kg, a quantity too small to effectively test in the experimental runs. Instead, 1.0 kg of *250 mL, rectangular, enclosed Tetra-Paks* were used in each test run.

3.8.3 Commingled Recyclables Testing Procedure

Approximate quantities of each item were weighed out as listed in the previous table. Each 1.0 kg load of Tetra-Pak was spraypainted a different colour to allow for tracking of the Tetra-Pak after shredding and screening. All test materials, including the Tetra-Paks, were placed in a large box and handmixed. The test batch was then poured into the hopper and shredded. After shredding, the test batch was screened and each individual test screen weighed. The Tetra-Pak pieces were then separated from the other items and the test screens reweighed. These differences gave the size distribution of the Tetra-Pak material.

Due to some initial, unusual results, the shredder air mechanism was increased to 55 psi. This apparently made little difference in later test results.

3.9 SUMMARY OF EXPERIMENTAL DESIGN

The experimental procedure was designed to determine which physcial characteristics would affect the shredding behaviour of the selected beverage containers and in what manner.

The **preliminary tests** would evaluate the effects of size, shape, and geometry within the same container/material type. To avoid a difficult analysis, the **materials comparison test** would be perfomed to evaluate the effect of the container material separately from the other three factors.

It was suspected that the orientation of a beverage container just prior to shredding may affect how its shredding behaviour. To test this, an **orientation test** was designed to examine this possibility. Lastly, a **commingled recyclables/Tetra-Pak** container test was devised. This would evaluate if a particular characteristic that was found to favourably affect the shredding behaviour of a Tetra-Pak by concentrating its shredded material would similarly separate out the container material from a mixture of various recyclable materials.

CHAPTER FOUR
Experimental Results and Data Analysis

4.0 INTRODUCTION TO THE EXPERIMENTAL RESULTS AND DATA ANALYSIS

This chapter presents the experimental results of the various tests outlined in the experimental design. Analyzing these results will determine if any of the various factors are significant in affecting the shredding behaviour of the beverage containers and if so, what relationship exists between any such factors and the shredding behaviour. By satisfying these two objectives, the analysis will provide the basis for examining potential alternative beverage container designs.

Meeting these objectives principally involves testing the hypotheses. They are briefly restated here.

1. ***Size - specifically, the volume of the container.***
   The smaller the size of the container, the smaller the pieces of material should be after shredding, resulting in a more concentrated distribution of shredded material. This is hypothesized for two reasons: 1) smaller containers have less material to begin with and 2) smaller containers would probably be more easily caught in a shredder and torn apart. Conversely, larger containers may result in a greater size range of shredded materials, from large to small pieces.
2. ***Shape - whether the container is rectangular or cylindrical.***
   Cylindrical containers may "roll" on top of the shredder blades for some brief period of time until a large enough mass of containers is accumulated to push the initial containers through. This may lead to incomplete shredding of the containers, affecting its subsequent size distribution. Conversely, rectangular shaped containers, such as aseptic packages, should allow for easier "grasping" of corners by the shredder blades, resulting in more complete shredding and a more concentrated size distribution.
3. ***Geometry - whether the container has an open mouth or is enclosed.***
   Containers that are commonly found "open-mouthed", such as EPS cups, should offer a greater area for blades to grasp onto and shred. Such containers should result in a greater degree of shredding and a more concentrated size distribution. Enclosed containers, however, should offer no such advantage. In addition, enclosed containers may offer greater structural integrity because all sides are "intact". Open-mouth containers are not reinforced on one side and thus may be less able to resist any shredding action.
4. ***Material used to construct the container.***
   The material the beverage container is constructed of should make a difference in how it is shredded. It is presumed that brittle materials will break into smaller pieces. Pliable or "soft" materials may not shred uniformly and may instead be "mangled", as opposed to shredded, resulting in a poor size distribution.

The data obtained from the various experimental runs can be found in **Appendix A**. A detailed description of the calculations used throughout this chapter is located in **Appendix B**. The remainder of this chapter is divided into the following main sections.

I. The data were first scrutinized to determine if it was valid for further analyses. It was concluded that, with a few exceptions, most of the data was generally valid.

II. Several parameters were then tried to determine which parameter would best describe each *data set*. A data set is considered to be the set of values describing the size distribution of the shredded material from one trial (or replicate) of a container configuration or experimental run. A single parameter that would best characterize each data set was desirable for the factorial analysis and analysis of variance. The *modal interval density* was eventually used as the descriptive parameter. The modal interval densities were checked to determine if they were normally distributed to allow for further analysis.

III. The *preliminary tests* for Tetra-Pak, glass, and EPS were then examined. The analysis of these tests determined if the factors of size, shape, and geometry were significant. In general, contrary to the hypothesized outcomes, most of the various characteristics did not produce significant differences within a container material type, except for size.

IV. The *materials test* analyzed if the container material significantly affected the shredding behaviour. A definitive conclusion was not possible because of inadequate data. However, it is shown that material appears to play a critical role in defining the shredding behaviour of any beverage container.

V. The results of the *orientation tests* on the Tetra-Pak containers were analyzed. It was found that the pre-shredding container orientation appeared to have no effect.

VI. The *commingled recyclables/Tetra-Pak tests* revealed that the Tetra-Pak containers appeared to separate out similarly whether as a single material or mixed with other items.

4.1 VALIDITY OF COLLECTED DATA

4.1.1 Selection of Applicable Data Sets

Chapter Three discussed the potential problem with screening light density materials such as shredded Tetra-Pak in one load. During the early stages of experimentation, it was thought that screening in one load may contribute to over packing and result in poorly screened samples. However, it was generally observed that the prescribed 10 kg sample size often had a fair amount of vertical space, or "freeboard" for the shredded materials to bounce during vertical agitation in the top screen layer. Examining the *individual percentage of materials* contained in each screen for those screened in one load versus those screened in two loads did not reveal any conclusive trends. Furthermore, because so few replications could be performed, as many data sets as possible were retained. It is possible that over packing, if it is real event, may randomly occur. Discarding such data would therefore bias the analysis.

Only three data sets were finally deemed unsuitable. Data for replicate one and replicate four for the *250 mL, rectangular, enclosed, Tetra-Pak* configuration were discarded. Replicate one used too large a sample size - 13 kg - for the mechanical screen. Replicate four had the shredder operating at 55 psi, instead of the 42 psi used for all other preliminary tests. Replicate one data for *1000 mL, cylindrical, open, glass* was not used. The shredded glass was subject to screening for 15 seconds. This proved too long and resulted in additional breakage beyond that provided by the shrededer. Subsequent glass screenings were held to 1 to 2 seconds. Both of the discarded replicate one data for the Tetra-Pak and the glass were the very first tests performed for either container type.

Chapter Three stated that in several cases, due to either supply constraints or to check unusual distributions, *partial batches* were tested instead of the full 10 kg sample size. These usually ranged from 2 to 5 kg in size, depending on what was available. Including these data sets would violate the conditions of assumed normality and equal variances because the sample size is inconsistent throughout. It was generally observed that partial batches produced comparable, but slightly different, distributions to the full batches. However, one of the reasons for using a 10 kg sample was to duplicate the random orientations a container may be placed in prior to shredding. (Ten kilograms is also the recommended limit for light density material during screening, although lesser quantities can be used.) Even with the partial batches, pouring the containers into the shredder was observed to produce a good, if not the same, degree of orientation randomness. For these reasons of limited supply, reasonable random container orientation, and since only three of the sixteen Tetra-Pak preliminary tests were partial batches, the partial batches were assumed acceptable and included in the data analysis.

4.1.2 Hand-Built Containers

For the glass and EPS preliminary tests, container variances are not a concern since all containers were factory produced. These two container types could be expected to have equal variances throughout. However, in order to test the Tetra-Pak containers, half of the configurations had to built by hand. As discussed before, every effort was made to duplicate a factory produced Tetra-Pak container. Due to time constraints, it was not possible to devise a separate test to determine if hand-built containers were equivalent in performance to factory produced containers. However, it was possible to make some comparisons after the preliminary Tetra-Pak test analysis. This evaluation of the hand-built containers appears out of sequence. It was possible only after the analysis was completed under the assumption that the variances were equal throughout but because this section concerns the data validity, hand-built containers will be discussed here.

Because size was later determined to be the most significant factor, shape to be only marginally significant, and all other factors to be nonsignifcant, hand-built and factory built containers of the same size can be compared. A visual comparison of the *cumulative percent of material retained* is perhaps the easiest and most descriptive comparison. The screen size will be measured logarithmically, compressing the horizontal scale to allow for an easier comparison of the size distribution curves. Because the shape factor cannot be totally discounted, seeking similar *trends* in curvature to indicate similar performance is the objective, as opposed to having precisely the same curve. Comparing *modal interval densities* is not as preferable in this case since data from only one run was available for certain configurations. Furthermore, a statistical analysis, such as a t-test, that would incorporate these estimates would confound the effect shape (although such an effect may be very small) with any potential difference due to the construction technique.

The following charts present both the rectangular, factory produced containers and the cylindrical, hand-built containers. The trend of the distribution curve is similar between the hand-built and factory produced containers across the same sized containers. The difference in the trend of the distribution curve when going from the smaller to larger size is also reasonably consistent between the two construction techniques. Although not a statistical analysis, this visual comparison suggests that the hand-built containers were constructed to behave similarly to their factory produced counterparts.

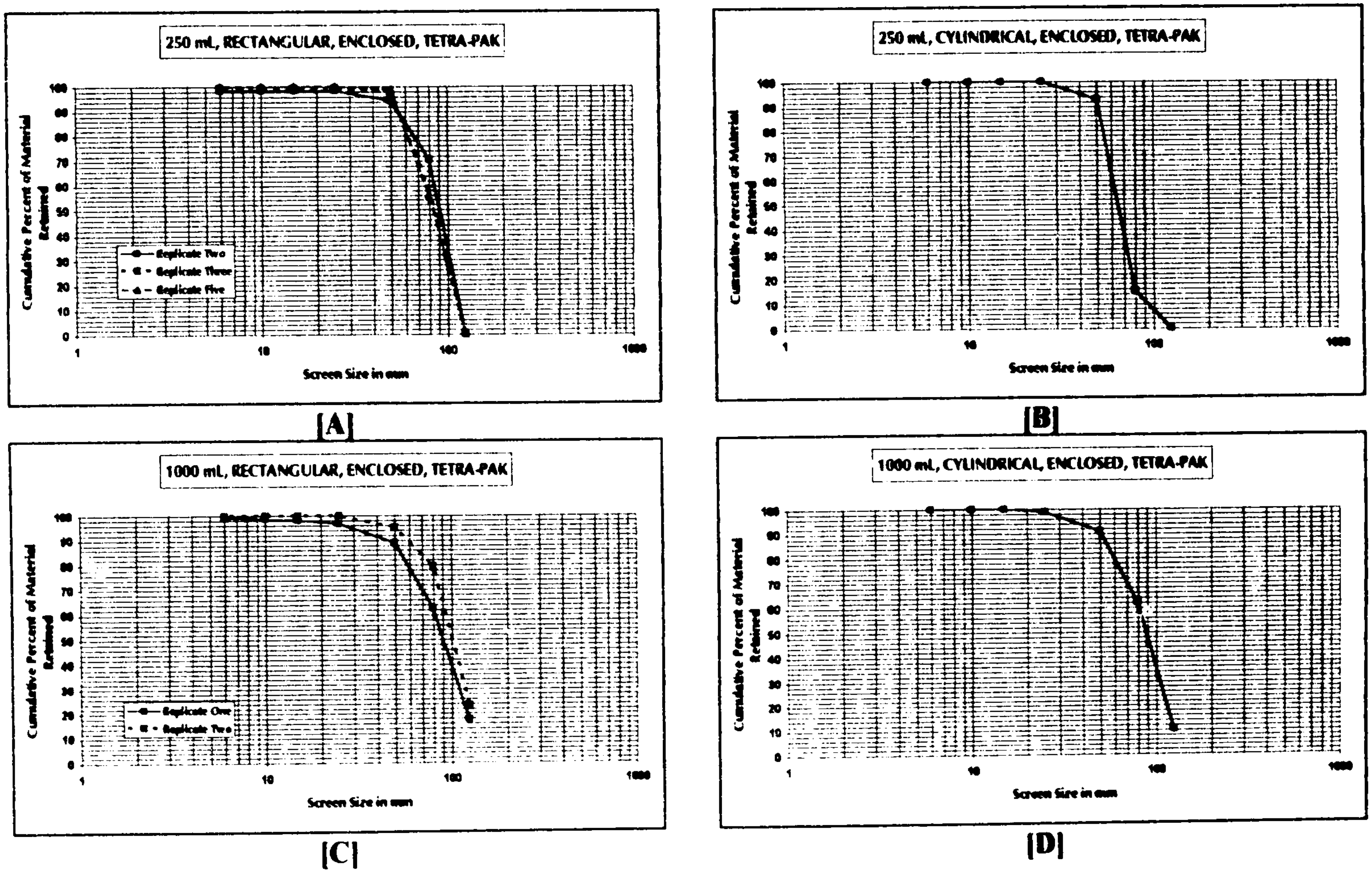


FIGURE 4.1: Comparison of Hand-Built versus Factory Built Tetra-Pak Containers

4.2 SELECTION OF THE RESPONSE CHARACTERIZATION PARAMETER

Having concluded that the data are initially acceptable for the intended analyses, the next step involved selecting a suitable means to represent the gathered data. Because the output data gathered for each batch would be in the form of a size distribution consisting of several values, a single parameter was sought that could represent the batch data. In the Tetra-Pak and glass preliminary tests, the factorial design evaluates the effect of a certain combination of factors, or container configuration, on how well the shredded container material is *separated and isolated.* Whatever single parameter was chosen, it must reflect if this configuration improved or worsened this isolation.

4.2.1 Rosin-Rammler Slope

Municipal solid waste (MSW) has been widely size characterized by the *Rosin-Rammler* method. This method involves plotting the natural log of the reciprocal of the percent size distribution retained against the log of the screen sizes. The resulting graph produces two useful parameters. The first is the *characteristic particle size*, or the size through which 63.21% of the sample passes, and the second is the *slope* of the distribution line (Hasselriis, 1984, p.102). The key parameter would be the slope. A steep or large slope would indicate that the shredded material had a narrow distribution and would consequently be better isolated among fewer screen sizes. Conversely, a flat or small slope would indicate that the shredded material had a much greater distribution throughout various screen sizes.

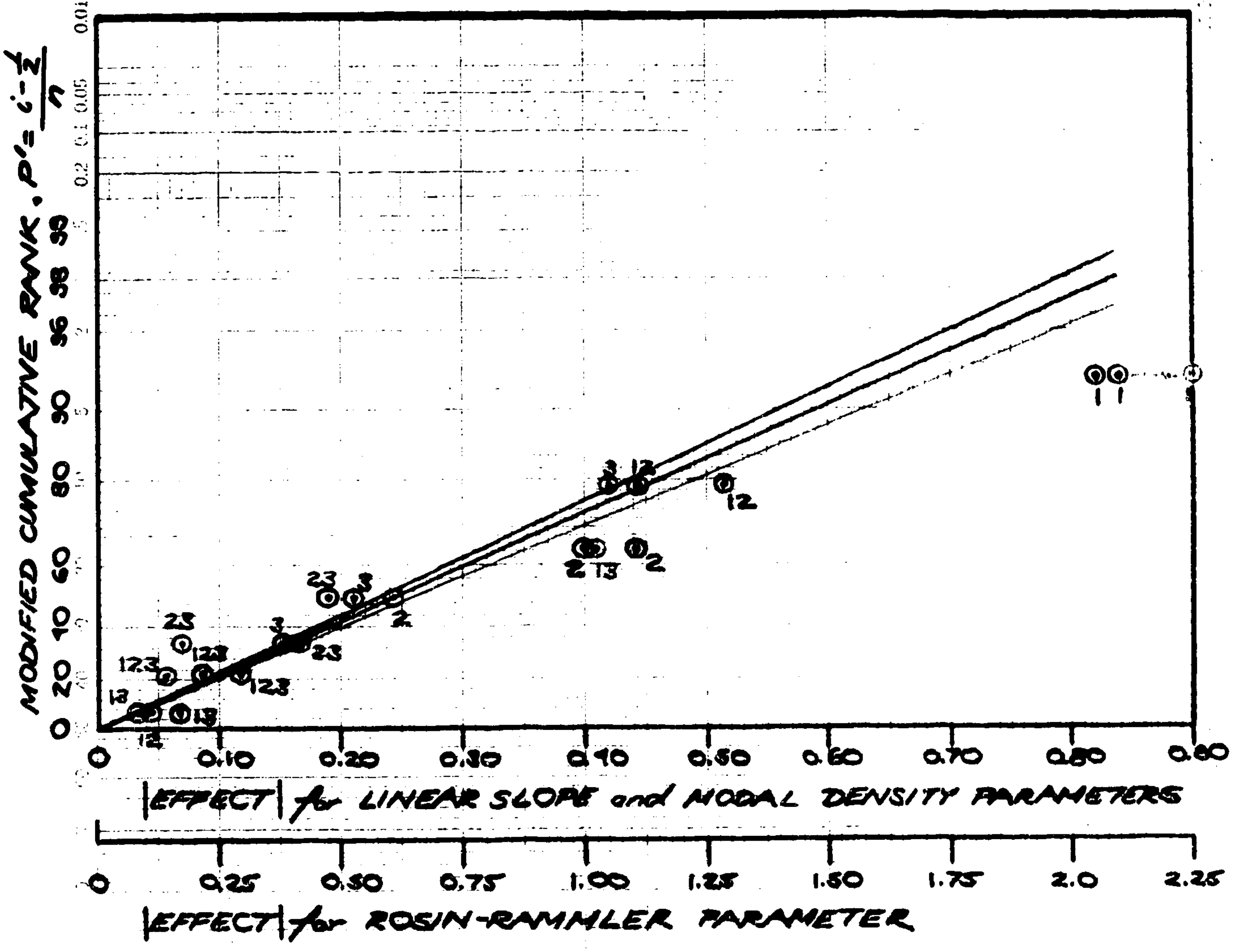
The Rosin-Rammler method was first applied to the Tetra-Pak *cumulative percentage retained* data. The data from each batch or configuration was transformed as described above and plotted. From each batch, the slope was determined. If replicates were made of a particular configuration, a mean slope and standard deviation from the replicates were determined.

After a slope or mean slope was calculated for each Tetra-Pak configuration, the slopes were used to complete the factorial analysis. Of all the factors, the ***size*** of the container was the most significant effect. The increase in container size from 250 mL to 1000 mL could be expected to decrease the slope of the curve by 2.351, with a standard error of 1.00. The ***geometry*** of the container, going from open to enclosed, improved the slope by 1.054, while the combination of both ***size and geometry*** combined, that is, going from open to enclosed and increasing the size, decreased the slope by 1.031. Again, the standard error for these is 1.00.

These two factors and interaction were determined to be significant using the *half-normal probability plot* (Johnson and Leone, 1977, p.802) shown in Figure 4.2. There were difficulties interpreting which data points should be included when drawing the best fit straight line in the half-normal probability plot. For example, the first three points could be considered as the defining points for a best fit line. However, a line that attempts to fit more of points in order to isolate the truly signficant effects was drawn. The resulting plot indicates that while the calculated effect of size on the isolation of the shredded containers confirmed that size appeared to make a difference during experimentation, the importance of geometry and the size/geometry interaction was unexpected. On the following half-normal plot, these latter effects were "somewhat close" to the straight line, but it was debatable if these effects could be

FIGURE 4.2

HALF-NORMAL PLOT for SIGNIFICANT TETRA-PAK EFFECTS



LEGEND:

⊙ ROSIN-RAMMLER
⊙ LINEAR SLOPE
⊙ MODAL DENSITY

Numbers indicate designated factor or interaction.

considered truly insignificant. In fact, it was observed during experimentation that the shape factor appeared to make a more significant difference than geometry.

These unexpected results could have been the result of using the Rosin-Rammler method. Although used in the characterization of municipal solid waste, the transformations involved - the reciprocal and natural log functions, and the log graph scale - could render the slope insensitive to the effects of the factors. Furthermore, the slopes were hand-estimated from the mid-section of each curve. Extreme portions of the curve of each graph often had little or no meaning because either none of the shredded material had been accumulated or else all the shredded material had been already accumulated in preceding screens. Estimating the slope from log-scaled axes is unlikely to be the most accurate method. These reasons, combined with the difficulties in drawing a confident straight line in the half-normal probability plot, led to using a second characterization parameter.

4.2.2 Linear Slope Method

Instead of using the logarithmic transformations required previously in the Rosin-Rammler approach to plot the data, the data were simply plotted as the cumulative percentage retained against the screen sizes. The slope of each curve, termed the linear slope since no transformations were required for the data or the scales, was calculated by hand. As with the Rosin-Rammler method, a mean slope and standard deviation was determined for replicates of the same configuration.

These slopes were then used in the factorial analysis. From the same half-normal probability plot (Figure 4.2), *size* was again determined to be the most significant factor. Increasing the container size from 250 mL to 1000 mL decreased the slope by 1.446 %/mm. The *shape* factor was also significant. Going from a cylindrical to rectangular container improved the slope by 0.442 %/mm. The standard error of both effects is 0.220. Estimating the best fit straight line in the half-normal probability plot was not as difficult.

Although the results of using the linear slope more closely match what was observed during experimentation and should be considerably more sensitive than the Rosin-Rammler slope, the linear slopes were still estimated. Furthermore, the slope provides an estimate of how narrowly or widely the shredded material is distributed but does not give a direct measure of the concentration or isolation of the shredded material.

4.2.3 Modal Density Parameter

Because the spacings of the screen sizes do not have consistent ranges, collecting 70 percent of the shredded material over a range of, for example, 45 mm (from the 125 mm screen to the 80 mm screen) is not equivalent to collecting 70 percent of the shredded material over a 30 mm range (from the 80 mm to the 50 mm screen). The latter case displays a higher concentration of material, implying better isolation.

To correct this, the *density of shredded material for each size interval* was calculated (Kennedy and Neville, 1986, p.18). This would measure the material by a percentage per mm size range in any particular size range, effectively allowing for "fair comparisons" between various size intervals and hence different batch data sets. The above examples would give densities of 1.556 %/mm and 2.333 %/mm. Density parameters were thus calculated for all tests.

For some tests, amounts were found in the 125 mm screen. As the top screen, there was no limit on how large an item could be in this screen. This open-endedness would prevent calculating an interval density. In such cases, the *largest dimension of the largest item* that could possibly be found in the size interval was used as the upper size limit for the 125 mm screen interval. For example, a 1000 mL rectangular, open, Tetra-Pak could conceivably be shredded and opened up into a single sheet (minus one end because it is an open configuration). Thus, the diagonal of this sheet would be the largest dimension. This assumes that no other information was available. In the case of the *1000 mL rectangular, enclosed, Tetra-Pak*, the longest dimension after shredding was known because these same containers were used in the orientation tests and additional length measurements had been made. However, the percentages collected in this top screen were usually small compared to the amounts collected through the middle and/or lower size ranges. Any inaccuracies in calculating the top interval density are assumed to not significantly affect the analysis.

From the various interval densities calculated for each batch, the *highest interval density, or mode*, was selected as the single numerical parameter to represent the batch results. Thus, the modal interval density described the maximum concentration or isolation of shredded material that could be expected from that particular container configuration. The modal interval density was finally chosen as the parameter to use throughout the analysis. It offers several advantages over the slope parameters that were calculated previously.

1. Using the modal interval density allows for unbiased comparisons between other intervals and other container configurations. Unusually large or small percentages are viewed in context to the screen interval they are found in.
2. It is not dependent on judging which data points are significant, as is the case when determining the slopes from charts.
3. The results of the factorial analysis are easier to understand. For example, the model may state that increasing the size of the container decreases the density interval by X %/mm. This directly relates the effects on the concentration or isolation of shredded material. The slope parameters, however, would only give a relative measurement of how narrow or broad is a size distribution.
4. Determining the modal interval density also determines the corresponding screen (size range). Knowing what screen will collect the most material could be important for practical applications.

4.3 NORMAL DISTRIBUTION OF MODAL INTERVAL DENSITIES

Although using the modal interval densities has several advantages, this use also requires a separate check be made to determine if the these modal densities are approximately normally distributed. If so, the further analyses which depend on the conditions of normality can be carried out.

Because the highest interval density from each size distribution has been selected to represent that particular size distribution, the remainder of the data from the distribution no longer impacts the analyses (although it may be important in materials recovery applications). Instead of determining if these original size distributions were normally distributed, it is now necessary to determine if the modal interval densities from all the data sets are themselves approximately normally distributed.

There are sixteen modal interval density values from the Tetra-Pak preliminary test data, fifteen values from the glass preliminary test data, and only four values from the EPS preliminary test data.

If the modal interval density values are normally distributed, they should form a straight line that passes through the centroidal point (mean value of the variate, 50 percent probability) when plotted on normal probability paper (Kennedy and Neville, 1986, p. 198). Furthermore, greater attention is given to the points in the centre of the plot, as opposed to those at the extremes. To construct these normal probability plots, the modal values for each container type were ranked in increasing magnitude and assigned to various percentiles. This is essentially the same technique used in constructing the half-normal probability plot and was necessary because the modal interval densities no longer depend on the original cumulative percentage frequency of shredded material (which would have served as the y-axis percentile in the original size distributions).

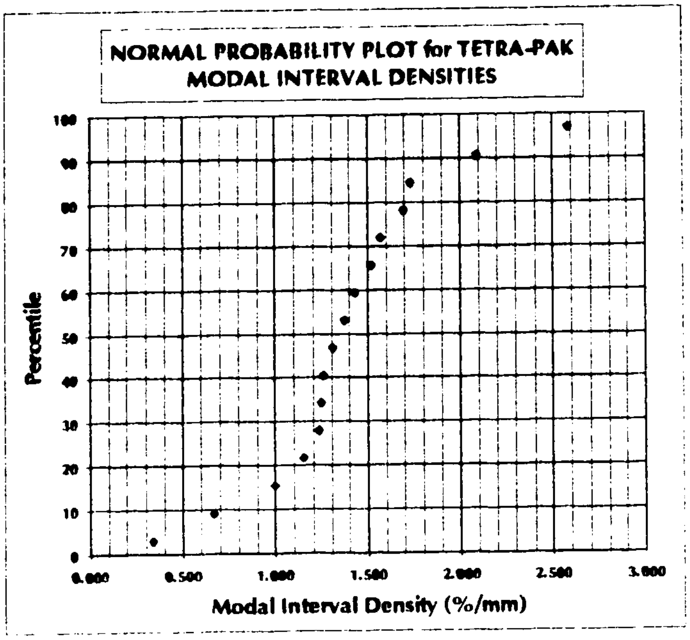


FIGURE 4.3: Normal Probability Plot for Tetra-Pak Modal Interval Densities

In Figure 4.3, the mean of the sixteen modal interval densities is at 1.389 %/mm. The best fit straight line appears to intersect the centroidal point of the plot, confirming that the Tetra-Pak modal interval densities do appear to be normally distributed.

The modal interval densities from the glass data also appear to be normally distributed, as shown in Figure 4.4. The mean value of the glass modal interval densities is 2.745 %/mm. The best fit straight line also appears to intersect the centroid of the plot. However, it should be noted that these values span a much smaller density range than the Tetra-Pak values.

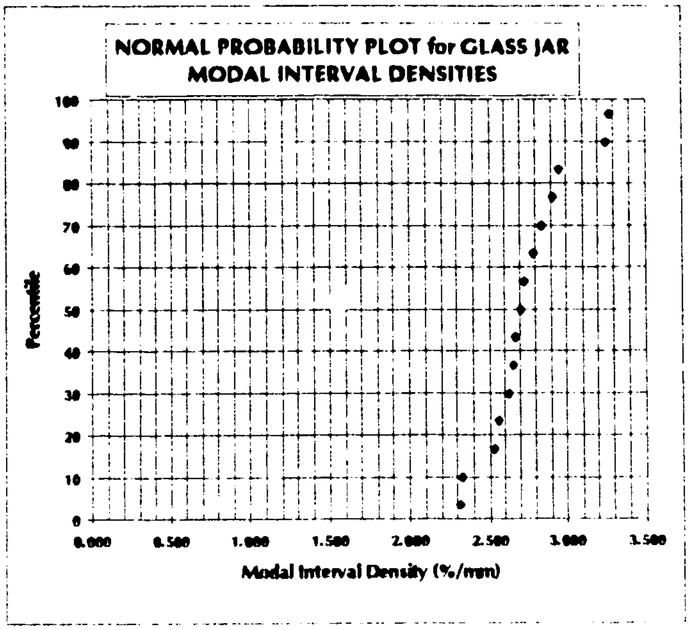


FIGURE 4.4: Normal Probability Plot for Glass Jar Modal Interval Densities

A normal probability plot was also constructed for the EPS modal interval densities. However, from the data, only four values were available. It is doubtful whether a best fit straight line could be drawn with any confidence. However, given the apparent normal distributions of the last two container types, it will be assumed that the EPS modal values are also normally distributed.

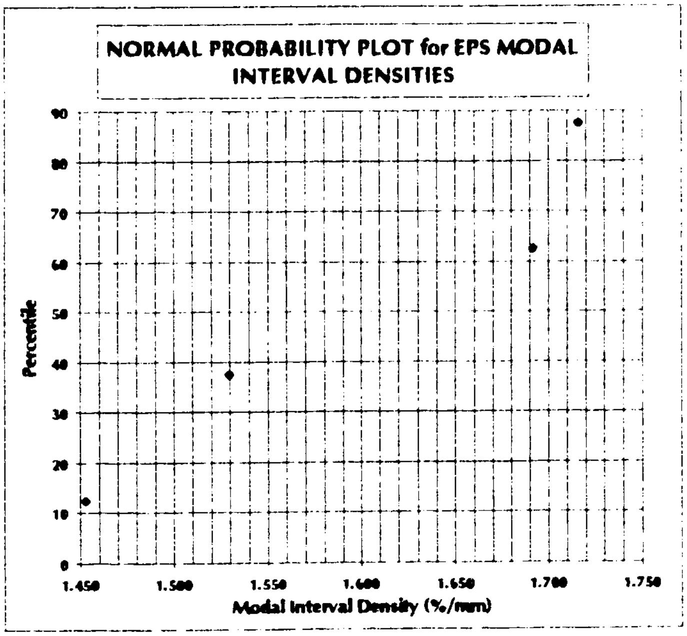


FIGURE 4.5: Normal Probability Plot for EPS Modal Interval Densities

4.4 RESULTS ON PRELIMINARY TETRA-PAK CONTAINER RUNS

4.4.1 Factorial Analysis for Tetra-Pak Containers

The modal interval densities for all container configurations were used in the factorial analysis for the Tetra-Pak containers. The factorial analysis, as described in Chapter Three, produced the following linear model.

$$\eta = 1.499 - 0.420x_1 - 0.200x_2 + 0.076x_3 + 0.222x_1x_2 - 0.018x_1x_3 - 0.095x_2x_3 + 0.059x_1x_2x_3 \qquad [5]$$

The standard error of each effect is ∓ 0.1957, except the for constant which is ∓ 0.0979. In the above equation, η represents the model outcome, the constant is the average value of all configurations (or runs), which is subsequently altered by adding or subtracting the x-terms. Recall that the actual effect of each factor, when going from the -1 to +1 level, is double the magnitude of the coefficient for that factor listed in the equation.

For those configurations with replicated runs, the mean of the highest interval densities were used in the factorial analysis. A standard deviation for that configuration was calculated. Available standard deviations from the various configurations were used to calculate the standard error of each effect. Runs which have no replication have no standard deviation and consequently no degrees of freedom. This is subsequently reflected in the final standard error of effects calculation. See Appendix B for the detailed calculations.

Again, using the half-normal plot (Figure 4.2) for Tetra-Pak, the significant effect was ***size***. The effect of ***shape*** was perhaps marginally significant. Increasing the size of the container from 250 mL to 1000 mL would decrease the interval density by (2 x -0.420) or 0.841 %/mm. The average modal density from all runs is the constant, or 1.499%/mm. This average is derived from the testing of four large and four small container configurations and thus represents a midpoint. Therefore, the 0.841 %/mm change represents a deviation of 0.420 %/mm to either size level from the midpoint. This would represent a midpoint-to-extreme change of approximately 28%.

Given this evaluation, the model appears as

$$\eta = 1.499 - 0.420x_1 - 0.200x_2 \qquad [6]$$

with the remaining terms eliminated because they were judged insignificant. Using this model, the theoretical or *predicted value* for each run was calculated. The actual modal interval densities used in each run (not the average used in the factorial) were compared against their respective predictive values to calculate the residuals. Analyzing these residuals should point out if the model is reasonably accurate (Kennedy and Neville, 1986, p.546) and if the necessary underlying assumptions, such as if the errors are normally distributed, are valid.

The following two residuals plots in Figure 4.6 were generated. The first is a scatter plot of the residuals versus the predicted modal interval densities. If the residuals appear unrelated to responses, the model is probably appropriate (Box et al., 1978, p.186). The plot indicates the residuals seem to cluster into two areas: above 0.88 and below 1.7. However, the second plot indicates that the residuals are approximately normally distributed, indicating that the errors are normally distributed and that the other effects were most likely due to "random noise" (Box et al., 1978, p.334).

Because Figure 4.6a indicates that the model may not be appropriate, one possibility is to transform the modal interval densities (or average modal density if present) of each run by using their logarithms. The logarithms may be more appropriate for the factorial analysis calculations and result in a better model. This transformation was performed and the results are shown in Figure 4.7.

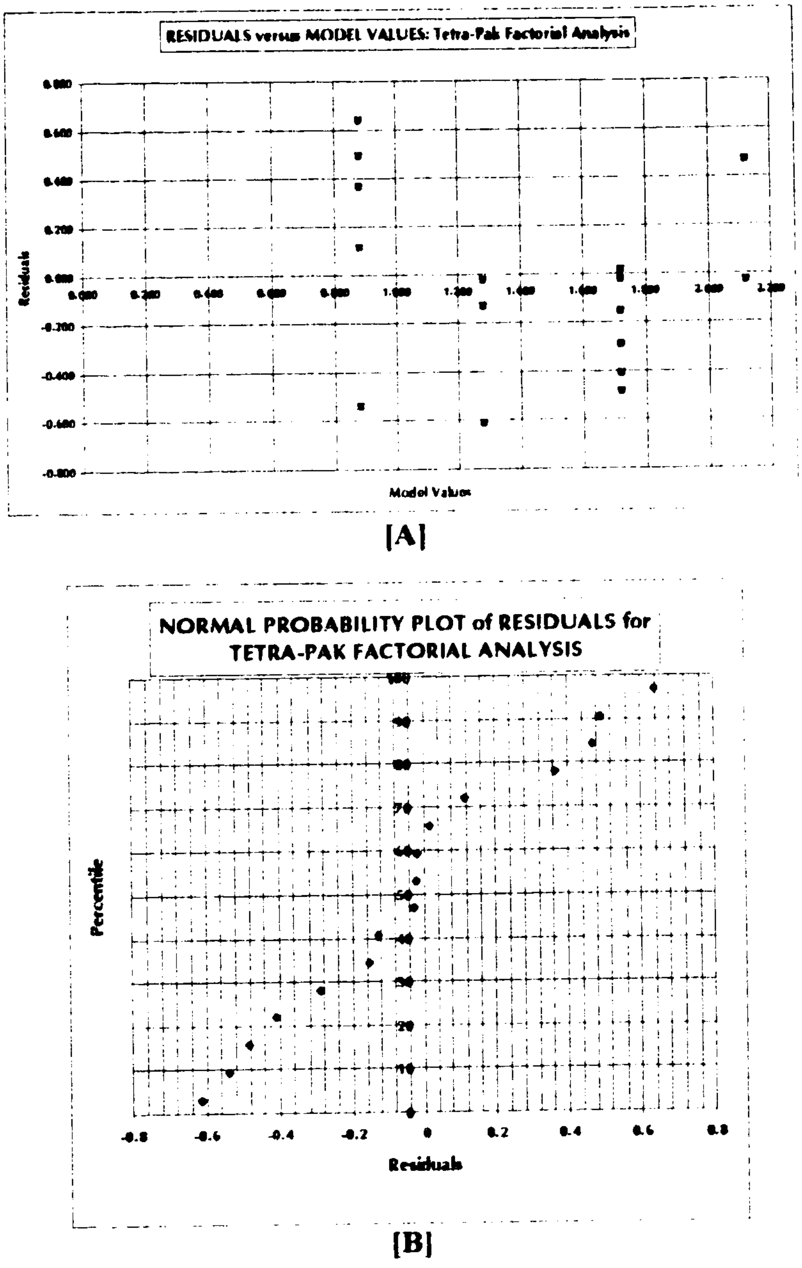
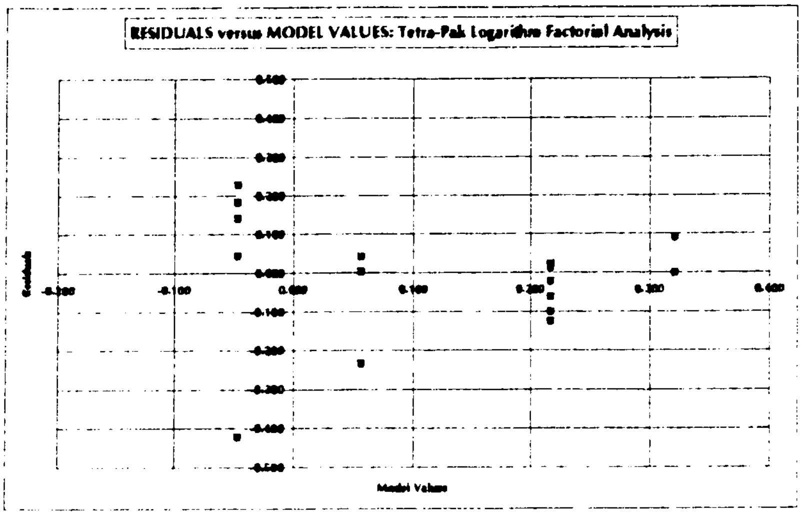
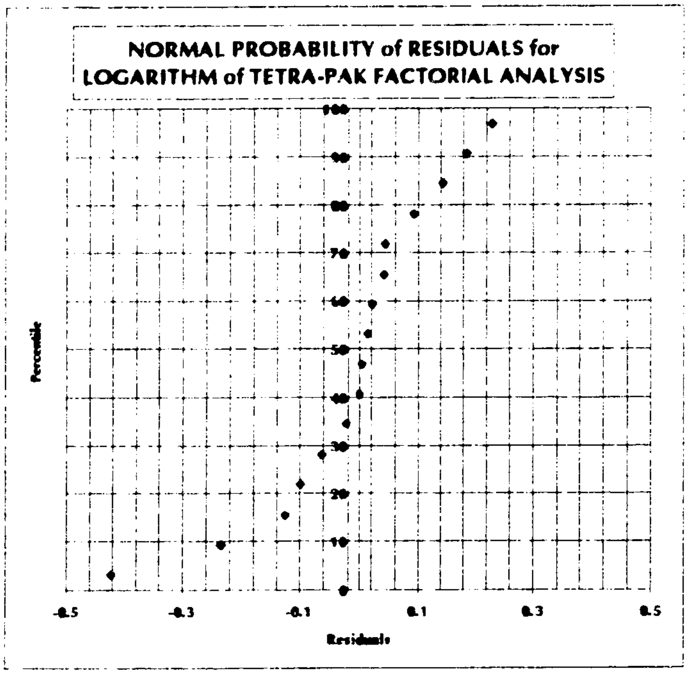


FIGURE 4.6: Residual Plots from Tetra-Pak Factorial Analysis

Although the vertical displacement of the data clustering in the scatter plot is reduced, the same data trend that was seen in Figure 4.6 is also present. Again, the normal plot in Figure 4.7 produces an approximate straight line, although the normal plot in Figure 4.6 prior to the logarithmic transformations appears to produce a better straight line. Because there is no conclusive evidence to suggest otherwise, the data will be deemed acceptable and the model generated as being appropriate.



[A]



[B]

FIGURE 4.7: Residual Plots from Tetra-Pak Analysis, Logarithms

4.4.2 Analysis of Variance for Tetra-Pak Containers

As a last check on the conclusions reached in the previous section, a two-way analysis of variance was performed on the Tetra-Pak data. For the purposes of the ANOVA, the configurations were considered the treatments while the replicates were considered as blocks. However, several of the Tetra-Pak configurations had three replicates, others had two replicates, while others had only one run. Since there were eight configurations, and the maximum number of usable replicates in any of the configurations was three, there were potentially twenty-four modal interval densities. Only sixteen modal interval densities were actually available from the data. To facilitate the ANOVA, the remaining eight modal interval densities were estimated using a procedure outlined in Hicks, 1982 and referred to by Kennedy and Neville, 1986. Refer to Appendix B. A missing value is estimated by determining what value will minimize the sum of the squares of the errors. This can be done for "... any reasonable number of missing values." (Hicks, 1982, p.75) No reference has been found as to what constitutes "reasonable". Afterwards the analysis is carried out as before, resulting in an approximate ANOVA with the degrees of freedom for the error term reduced by the number of estimated values. However, this ANOVA should be considered alongside the half-normal plot, and not as a stand-alone analysis.

The subsequent one-sided F-tests revealed that at both the 0.05 and 0.01[5] significance level, all factors and interactions, including the replications, were insignificant except for size. Size was significant at both levels. Shape was significant only at the 0.05 level but only produced a F_{calc}-to-F_{tab} ratio of 1.087, just slightly above 1.0. Shape was not significant at the 0.01 level. These findings confirm the interpretation of the half-normal plot.

4.4.3 Summary of Preliminary Tetra-Pak Tests

Based on the results of the factorial analysis and the ANOVA results, the size of a Tetra-Pak container is important in determining the isolation of shredded material into any given size interval while the other factors are not. The smaller size apparently results in a greater concentration of shredded material. This supports the earlier assertion in Chapter Two that larger cardboard sizes are not as affected as smaller cardboard sizes. Shape is only marginally important.

Source of Variation	*Deg. of Free., d.f.*	*SS*	*MS (SS/d.f.)*	*F_{calc} = MS/MSError*	*Sig. @ 0.05?*	*Sig. @ 0.01?*
Replicates	2	0.5026	0.251	1.549	*No*	*No*
Size	2	4.379	2.190	13.519	*Yes*	*Yes*
Shape	2	1.810	0.905	5.586	*Yes*	*No*
Size/Shape	4	1.459	0.365	2.253	*No*	*No*
Geometry	2	0.243	0.122	0.753	*No*	*No*
Size/Geom.	4	0.001	0.00025	0.00154	*No*	*No*
Shape/Geom	4	0.252	0.063	0.389	*No*	*No*
3-factor	6	0.066	0.011	0.0679	*No*	*No*
Error	6	0.972	0.162			

TABLE 4.1: ANOVA Results for Preliminary Tetra-Pak Container Tests

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[5] The commonly used 0.05 and 0.01 significance levels were accepted as the test levels in the absence of other information.

4.5 RESULTS OF PRELIMINARY GLASS CONTAINER RUNS

4.5.1 Factorial Analysis for Glass Containers

The results of the glass container runs were handled in the same manner for the factorial analysis as the Tetra-Pak data. Since almost all runs had two replicates, the mean of the modal interval densities was calculated along with the corresponding standard deviation. Only one run did not have replicated data. The factorial analysis produced the following model.

$$\eta = 2.719 - 0.151x_1 - 0.056x_2 + 0.067x_3 + 0.149x_1x_2 - 0.028x_1x_3 - 0.013x_2x_3 - 0.093x_1x_2x_3 \qquad [7]$$

The standard error of each effect is ∓0.0999, except for the constant, which is ∓0.05. The actual effect of factor is double the magnitude of the respective coefficient for that factor when going from the -1 to +1 level.

According to the half-normal plot in Figure 4.8 for these effects, the only significant effect is from the ***size*** factor. Increasing the glass container size from 250 mL to 1000 mL will decrease the modal interval density by 0.301 %/mm. From the half-normal plot, it is difficult to ascertain if the ***size/shape interaction*** factor is important. If so, the combination of increasing the size and switching from cylindrical to rectangular glass containers increases the modal interval density by 0.299 %/mm. Unlike the Tetra-Pak model however, in which the significant effect of size would produce a midpoint-to-extreme change of 28 %, the effect of size for glass would affect the constant by only 0.151 %/mm to either extreme size level. This is only a change of approximately 6%, suggesting that while size may be important relative to the other factors, it may not be that important on an absolute scale. Again, the residuals from the factorial analysis were calculated. However, these plots *assume that only the size factor is significant*. The plots of these residuals are shown in the Figure 4.9.

The clusterings in Figure 4.9A about the predicted values appear to show a distinctive clustering[6] but no distinctive pattern within a cluster. Figure 4.9B, however, produces a reasonably straight line with one exceptional point in the far left corner. This suggests that the model could be adequate and that the assumptions of normality are most likely valid. An additional check should be performed to confirm the earlier conclusions about the significance of size and the size/shape interaction. As with the Tetra-Pak analysis, an ANOVA will be performed on the glass analysis.

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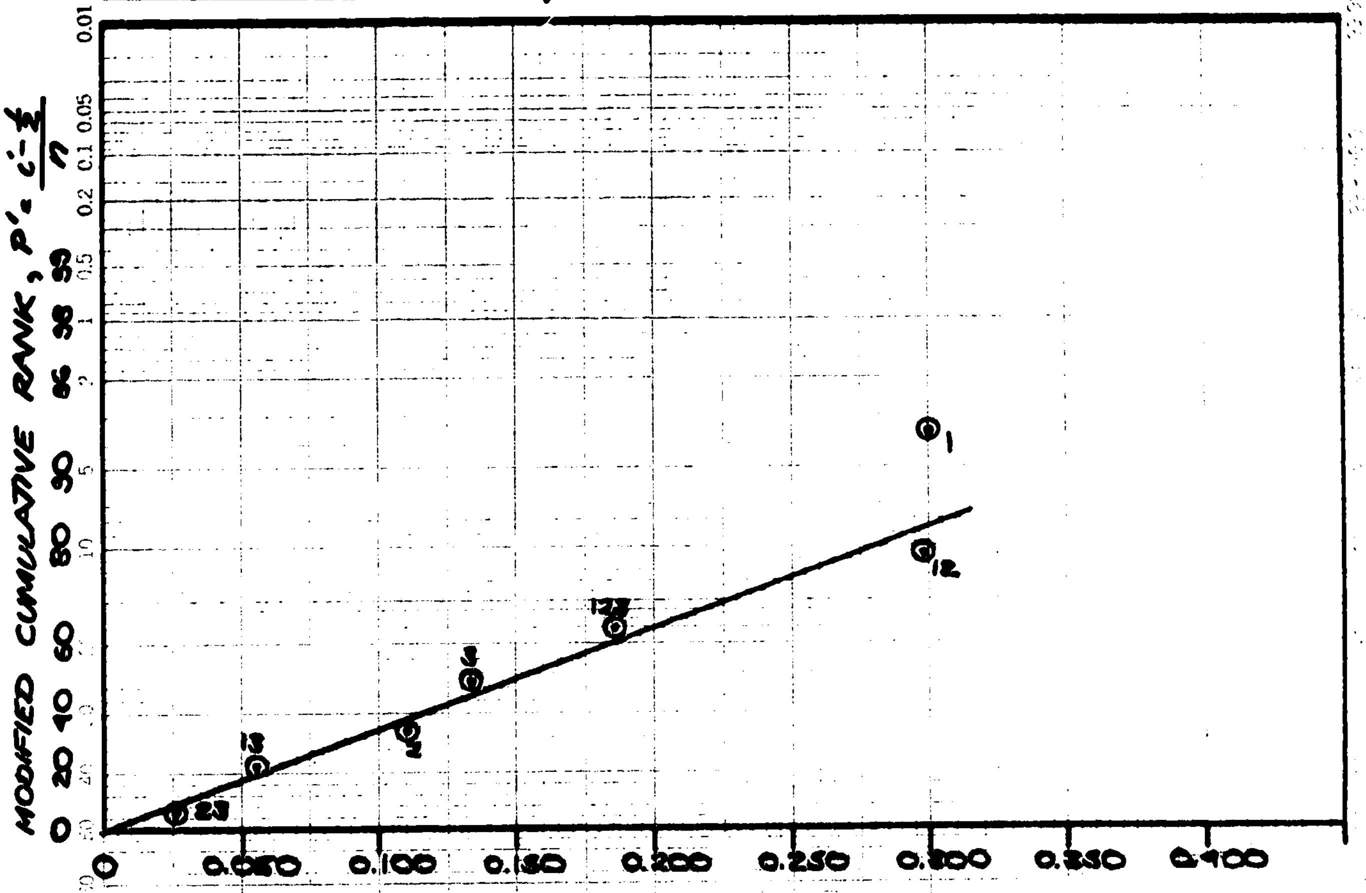
[6] It appears as if there is a distinctive clustering, but this is also due in part to the fact only one factor is considered significant. In the calculation of the model values, the model values alternate between one of two numbers (the constant ± one factor). Thus all residuals would be compared to only two numbers instead of across the entire horizontal axis.

FIGURE 4.8

HALF-NORMAL PLOT for SIGNIFICANT GLASS EFFECTS



|EFFECT| for GLASS using MODAL DENSITY PARAMETER

Numbers indicate designated factor or interaction.

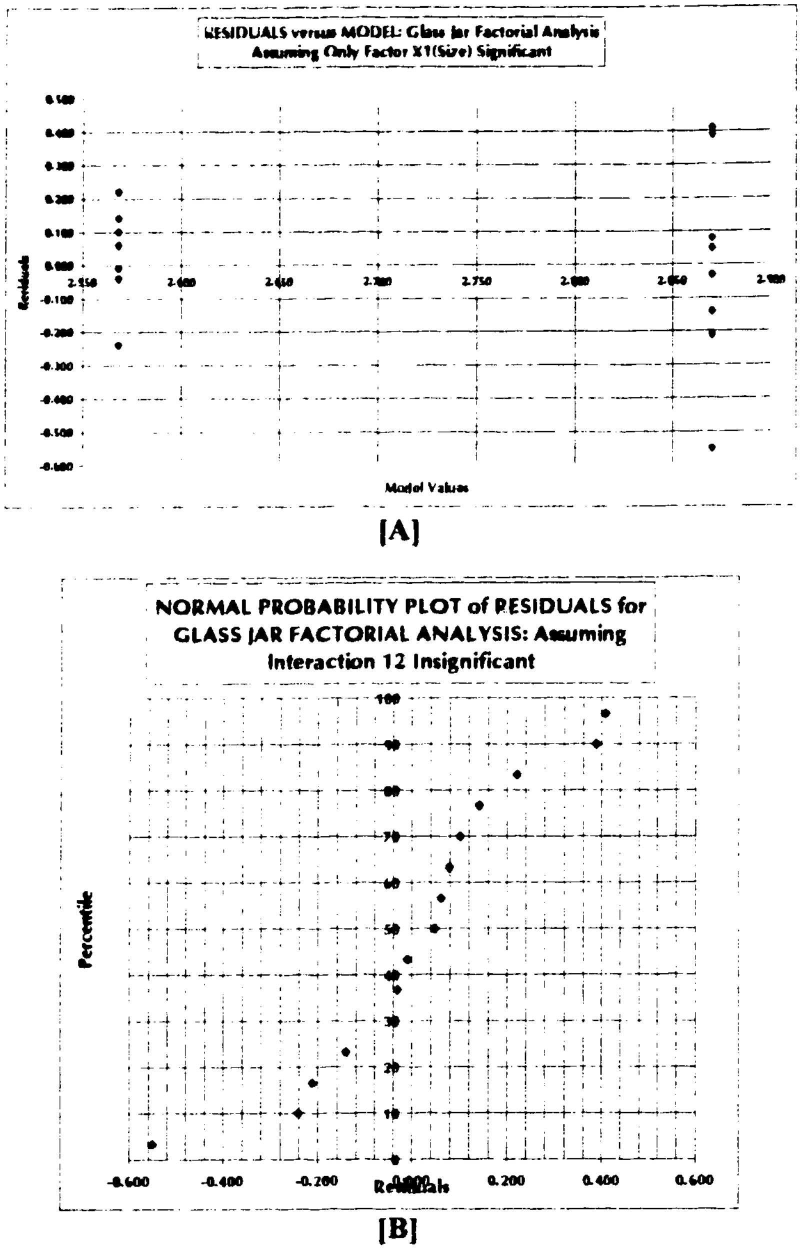


FIGURE 4.9: Residual Plots from Glass Jar Factorial Analysis

4.5.2 Analysis of Variance for Glass Containers

Because one of the glass container data sets had to be discarded, one configuration had only one run while all others had two replicates. The same technique used in the Tetra-Pak ANOVA was also used for glass to estimate the missing modal interval density.

Based on a one-sided F-test at the 0.05 and 0.01 level of significance, all factors were insignificant except for the size factor and size/shape interaction factor. Note that the sum of squares of error, SSE, has six degrees of freedom ((8 runs -1) x (2 replicates -1) - 1 estimation). The size and size/shape interaction factors were both significant at the 0.05 level but insignificant at the 0.01 level. This closely matches our interpretation from the factorial analysis and the half-normal plot that even if these two effects *appear* significant, the significance is most likely limited.

Source of Variation	*Deg. of Free., d.f.*	*SS*	*MS (SS/d.f.)*	*F_{calc} = MS/MSError*	*Sig? @ 0.05*	*Sig? @ 0.01*
Replicates	1	0.0346	0.0346	0.876	*No*	*No*
Size	1	0.334	0.334	8.456	*Yes*	*No*
Shape	1	0.061	0.061	1.544	*No*	*No*
Size/Shape	1	0.28	0.28	8.304	*Yes*	*No*
Geometry	1	0.059	0.059	1.494	*No*	*No*
Size/Geom.	1	0.019	0.019	0.481	*No*	*No*
Shape/Geom	1	0.001	0.001	0.0253	*No*	*No*
All Factors	1	0.121	0.121	3.063	*No*	*No*
Error	6	0.237	0.0395			

TABLE 4.2: ANOVA Results for Preliminary Glass Container Tests

4.5.3 Summary of the Preliminary Glass Tests

These results suggest that none of the tested factors, other than perhaps the material properties of glass itself, are important in determining the modal interval density for glass. The size factor and the interaction of size and shape appear to have little effect. Furthermore, since the geometry factor has apparently no effect, the presence or absence of *metal* lids appears to have not made any difference.

4.6 RESULTS OF EXPANDED POLYSTYRENE CONTAINER TESTS

Because no EPS containers were available in rectangular form and hand-making such containers proved too difficult, no factorial experiment was performed. Instead, of the available EPS containers, two configurations similar to those tested for Tetra-Pak and glass were chosen. Two replicates were run for each of *250 mL, cylindrical, open* and *1000 mL, cylindrical, open*. These tests would provide additional data as to how different materials performed and also test the effect of size, which earlier had been observed to be the most important factor for Tetra-Paks.

The F-test was performed to determine if there were any significant differences between the two variances because of the different sizes. F_{calc} is significantly lower than F_{tab}, indicating there is no difference, although the accuracy of this test suffers from having only one degree of freedom for both configurations. Next, a two-sample t-test was performed to evaluate if the means of the two EPS containers are different.

Size (mL)	***Mean Largest Interval Density (%/mm)***	***Standard Deviation, s***	***Variance, s^2***	***Degrees of Freedom, d.f.***
250	1.492	0.054	0.00292	1
1000	1.692	0.034	0.00116	1

F_{calc}	***Numerator d.f.***	***Denominator d.f.***	***Significance***	***$F_{tabulated}$***
2.526	1	1	0.025 (for 0.05)	647.8

TABLE 4.3: F-Test on Results for EPS Containers

Combined Variance, s_c^2	***Combined Std. Deviation, s_c***	***Standard Deviation, s_d***	***Sample n_1***	***Sample n_2***	***t_{calc}***
0.002038	0.04514	0.04514	2	2	4 ‘30

Total d.f.	***Significance***	***$t_{tabulated}$***	***Significant?***
2	0.05	4.303	*Yes*
2	0.01	9.925	*No*

TABLE 4.4: t-test on Results for EPS Containers

Based on this comparison, the means are different at the 0.05 significance level, but just barely so; the ratio of t_{calc}-to-t_{tab} is 1.02. At the 0.01 significance level, there is no real difference between the means. This suggests that size is not an important factor in the shredding behaviour of EPS containers. Visually examining the plot of the two container types in in the following figure also supports this reasoning. The curves are very close together, again strongly suggesting that the container size has little influence on the shredding. Note that the values plotted are the average of the two replicates of the *cumulative percent retained* data, as in a typical size distribution curve, and not the interval densities. Thus, for an EPS container, the size of the container does not significantly affect how it will shred.

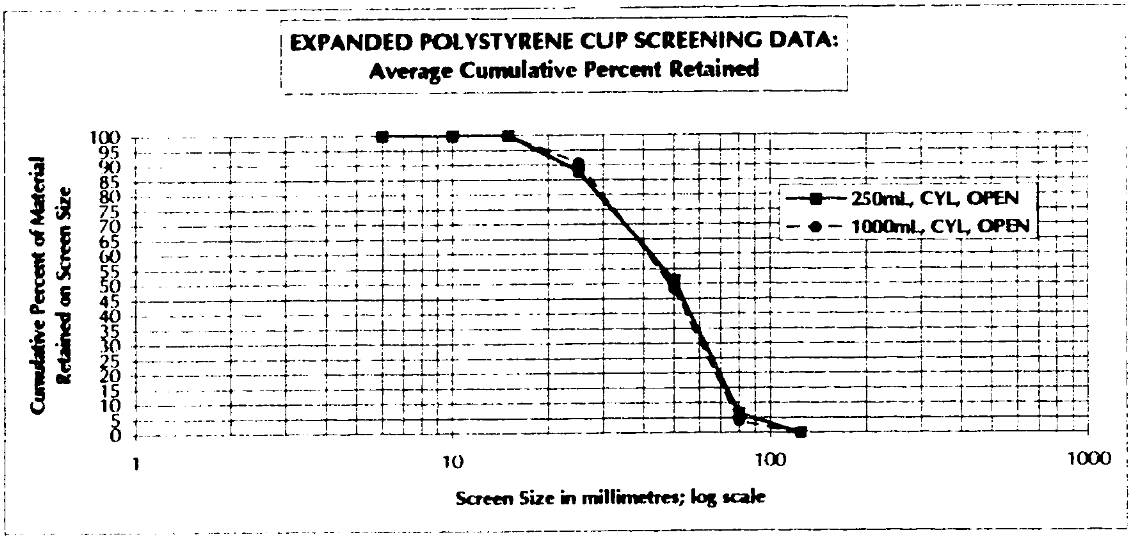


FIGURE 4.10: EPS Container Screening Data

4.7 RESULTS OF MATERIALS COMPARISON TEST

4.7.1 Selection of Test Data

The previous preliminary tests concluded that the physical characteristics of size, shape, and geometry have little effect on how a container shreds, as is the case with glass and EPS. Only the Tetra-Pak containers appeared to be affected by the size of the container. Given the general insignificance of these first three factors, and considering the earlier discussion about the material of the container in Chapter Two, the last physical characteristic, the material, may the signficant factor that affects the shredding behaviour especially since all three container/material types seem to produce different size distributions.

To evaluate the effect of the container material, special Tetra-Pak containers were hand-built by wrapping aseptic packaging material around a 1000 mL, rectangular, glass jar. Aside from small dimensional differences at the ends of the containers, the two different container types were similar except for the material. Any differences in shredding behaviour should therefore be due to the differences in the container material. Initially, it was thought that the existing data from the *1000 mL, rectangular, open, Tetra-Pak* configuration was not usable. Although volumetrically similar, these post-consumer Tetra-Paks have different dimensions. Moreover, by wrapping the aseptic material around a tested glass jar, it was possible to use the preliminary test data for this glass jar in materials test.

Approximately 5.0 kg of these specially constructed Tetra-Paks were shredded and screened. Its modal interval density is 1.196 %/mm. Only one separate test run was performed. Due to time constraints, no replicates could be performed. However, the other two *1000 mL, open, Tetra-Pak* configurations from the preliminary tests produced similar modal interval densities. Since it was determined that size is the significant factor, it should be possible to compare these three 1000 mL sized Tetra-Pak container results.

Tetra-Pak Configuration (Test)	*(Mean) Modal Interval Density (%/mm)*	*Standard Deviation*	*Number of Replicates*	*Standard Error of Mean*
Materials Test	1.196	N/A	1	N/A
1000 mL, rect, open	1.079	0.644	3	0.372
1000 mL, cyl, open	0.962	0.418	2	0.296

TABLE 4.5: Comparison of Similar Tetra-Pak Container Data

The modal interval density for the materials test configuration lies within the range of the two other configuration means, 1.079 %/mm ∓ 0.372 %/mm and 0.962 %/mm ∓ 0.296 %/mm. Although this comparison is not rigid test, it does signify that the modal interval density from this separate non-replicated Tetra-Pak materials test can be used with a reasonable degree of confidence.

As mentioned previously, no rectangular EPS containers could be purchased or constructed. For the purposes of comparing materials, the *1000 mL, cylindrical, open, EPS* configuration data will be used. Because shape was not a significant factor for the other two container/material types, it is unlikely that shape would be an important factor for EPS containers. Furthermore, the mechanical properties of polystyrene foam depend mainly on its density (Svec, 1990, p.190). However, it should be kept in mind that in these materials comparisons, the EPS containers tested have a cylindrical shape.

4.7.2 Calculation of Density of Container Material

A quantitative and objective parameter would be needed to identify the container material in order to accurately correlate a relationship between the shredding behaviour and the material, assuming such a relationship exists. It was originally thought that the *modulus of elasticity*[7] would best describe the material. However, depending on what orientation the container was in while being shredded, the modulus of elasticity would most likely vary. All possible orientations and their subsequent moduli would have to be evaluated in separate laboratory experiments. Furthermore, some of the cylindrical shapes, such as the sloped EPS containers, would complicate the testing process. Therefore, the modulus was not used as a material parameter.

Instead, the *density* of the container material was chosen to describe the material. Density is a unique parameter and is unlikely to affected by the orientation of the container. This also follows the description that EPS mechanical properties depend primarily on its density. The density of the three container types - the materials test Tetra-Pak, the glass jar, and the EPS container - were calculated by first weighing three or four containers of each type. Determining the volume of the containers proved more difficult than initially thought due to the thinness of the Tetra-Pak material, the curves of the glass jar, and the sloped surface of the EPS container. The volume was finally calculated by measuring the vertical displacement of each container when submerged in a water filled beaker. Thirteen millimetres of vertical displacement was equivalent to 250 mL of water; this refinement was necessary since the beaker gradations were inadequate. Note that the containers examined below are only those selected for this materials comparison test.

Material/ Container	*Mean Density (g/mL)*	*Standard Deviation*	*Standard Error of Mean*	*Modal Interval Density (%/mm)*
Tetra-Pak	0.749	0.067	0.034	1.196
Glass Jars	2.492	0.022	0.013	2.730
EPS	0.0544	0.0013	0.0007	1.692

TABLE 4.6: Materials Comparison Test Data

The modal interval density was plotted against the material density in Figure 4.11. While the points are located quite differently from one another, this plot does not reveal any trend between the modal interval density and the container material density among the three container material types. However, from observing the behaviour of the different container/material types during testing, it is apparent that the modal interval density appears to vary with the degree of ductility inherent in the material. This ironically suggests that the E modulus, however imperfect, may have been the preferable material characteriziation parameter. Figure 4.12 plots the modal interval density against a "description" of the ductility or brittleness of the container/material type.

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[7] The modulus of elasiticity is the slope of the stress-strain curve for a material.

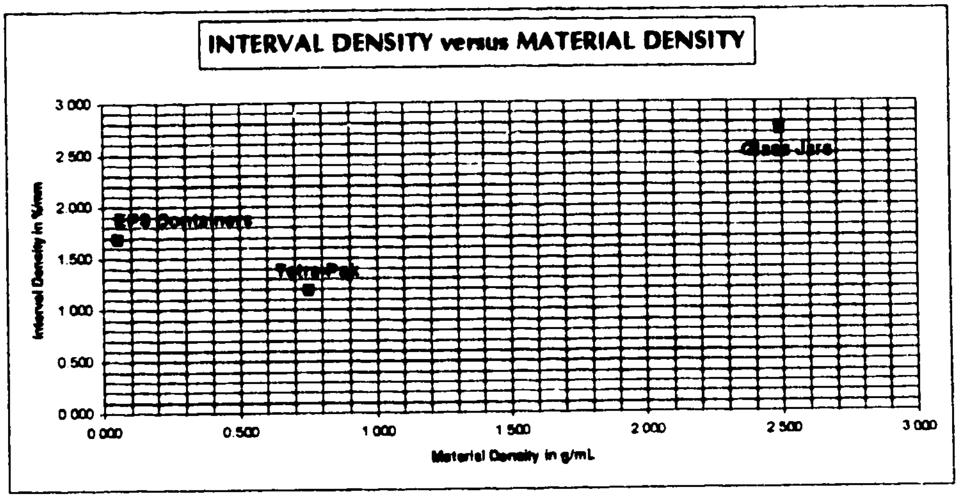


FIGURE 4.11: Modal Interval Density versus Container Material Density

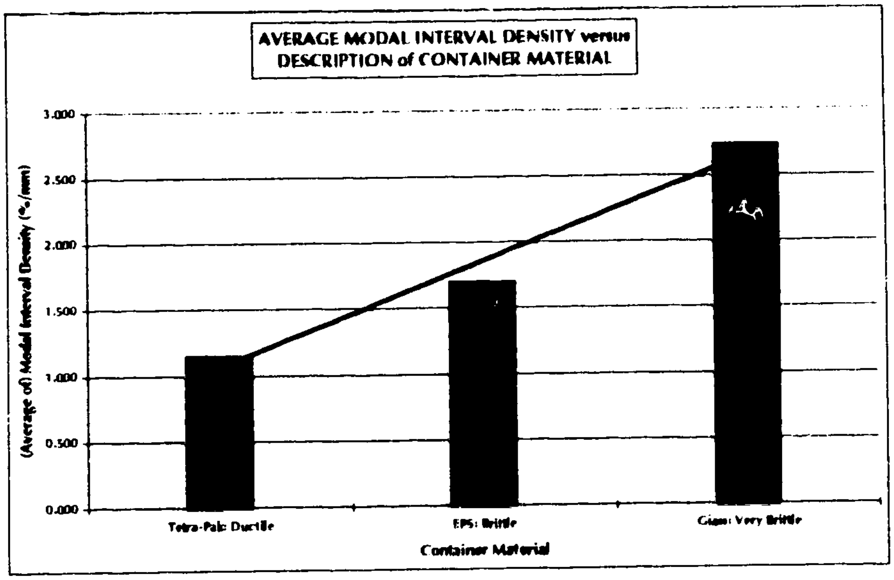


FIGURE 4.12: Modal Interval Density versus Description of Container/Material Type

It should be noted that in Figure 4.12, the Tetra-Pak modal interval density is actually composed of two different sets of values: 1) the Tetra-Pak material test data itself, and 2) the two modal interval densities from the 1000 mL, rectangular, enclosed, Tetra-Pak configuration. Since only size was considered truly signficant, shape marginally significant, and geometry insignificant for Tetra-Paks, including the preliminary test data was possible.

4.7.3 Summary of the Materials Test

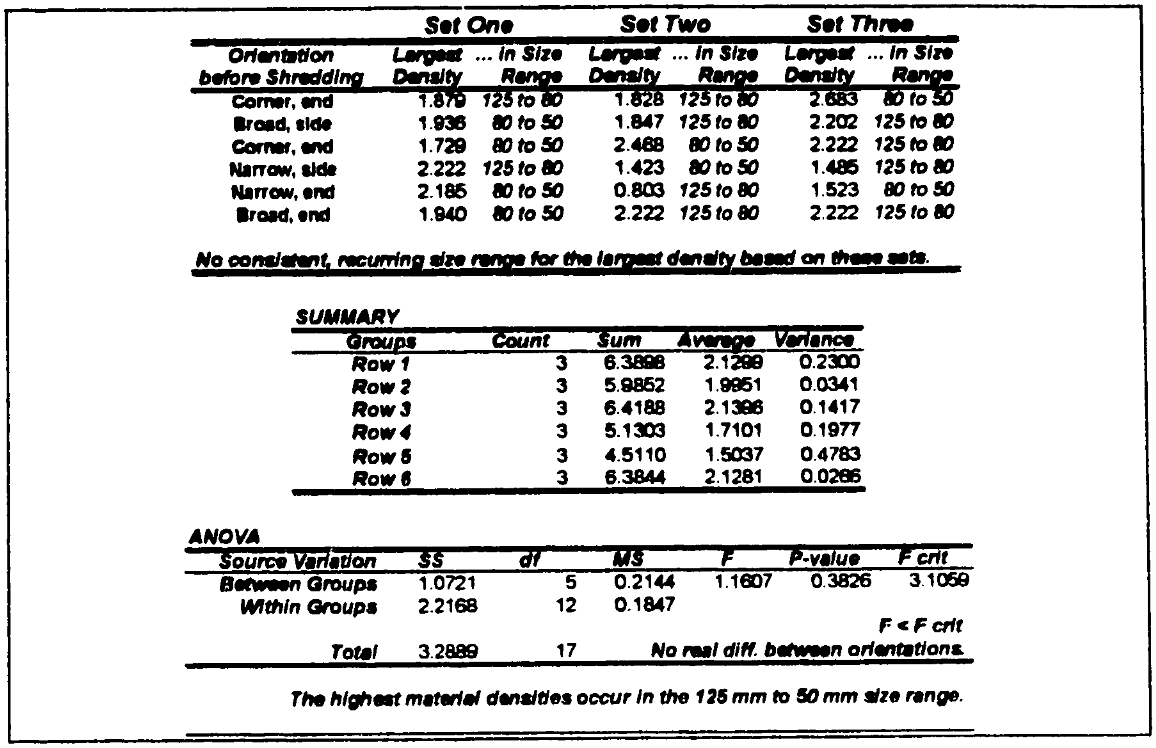
Comparing the shredding behaviour of one material against another confirms that the different container materials do produce different modal interval densities. It appears that the modal interval density increases as the brittleness of the material increases. More information than what is available from this research and a better material identification parameter are needed to reach a conclusive relationship.

4.8 RESULTS OF THE ORIENTATION TESTS

Since it was suspected that the orientation of the container prior to shredding could affect how the container was shredded, these tests were conducted to determine if specific orientations affected the shredding behaviour. The shredded containers were described in two manners: 1) visually describing the breaks in the Tetra-Paks and 2) screening the Tetra-Pak as was done for the preliminary tests. The screened data was again analyzed for the largest interval densities. As previously detailed, the largest dimension of the largest piece found in each test set was the upper range limit for the top 125 mm screen size for calculating the interval densities. However, all the greatest interval densities were found below the 125 mm screen.

The six orientations described in Chapter Three were tested in three different sets. The final results are listed in Figure 4.13. No one orientation produced the largest interval density in the same size range throughout the three test sets. The largest interval densities were spread over two screen size ranges, from 125 mm to 80 mm and 80 mm to 50 mm. The ANOVA calculates that the F_{calc} is less than F_{tab} at the 0.05 significance level, rejecting the alternative hypothesis that a real difference exists between the various orientations. To confirm the validity of this ANOVA, two plots of the residuals, or the differences between the average or "model" value and the actual values for each orientation, were plotted. Figure 4.14A shows no particular trend among the scatter plot of the residuals against the predicted values while Figure 4.14B demonstrates that the residuals, with the possible one exception of the far left value, are approximately normally distributed. These two plots indicate the ANOVA results were valid and that the assumptions necessary for ANOVA (e.g. normal distribution of the errors) are upheld. As a further confirmation, the logarithms of the largest interval densities found in the orientation tests were also calculated, analyzed, and the residuals similarly plotted. It was possible that the logarithms may have revealed additional information. This was not the case. Both residual plots generated from the logarithms were similar to those in Figure 4.14.

The visual descriptions of the major breaks were also examined. No consistent breakage trend could be found throughout all three sets for each orientation. Based on these data, the orientation of a Tetra-Pak container prior to shredding does not appear to affect its shredding behaviour. However, this assertion is not necessarily valid for other beverage containers. For example, Chapter Two discussed that the behaviour of glass was dependent on the specific circumstances of the situation surrounding the glass. This would include the container orientation. More research would need to be conducted to determine if this is true.



	Set One		Set Two		Set Three	
Orientation before Shredding	Largest Density	... In Size Range	Largest Density	... In Size Range	Largest Density	... In Size Range
Corner, end	1.879	125 to 80	1.828	125 to 80	2.683	80 to 50
Broad, side	1.936	80 to 50	1.847	125 to 80	2.202	125 to 80
Corner, end	1.729	80 to 50	2.468	80 to 50	2.222	125 to 80
Narrow, side	2.222	125 to 80	1.423	80 to 50	1.485	125 to 80
Narrow, end	2.185	80 to 50	0.803	125 to 80	1.523	80 to 50
Broad, end	1.940	80 to 50	2.222	125 to 80	2.222	125 to 80

No consistent, recurring size range for the largest density based on these sets.

SUMMARY

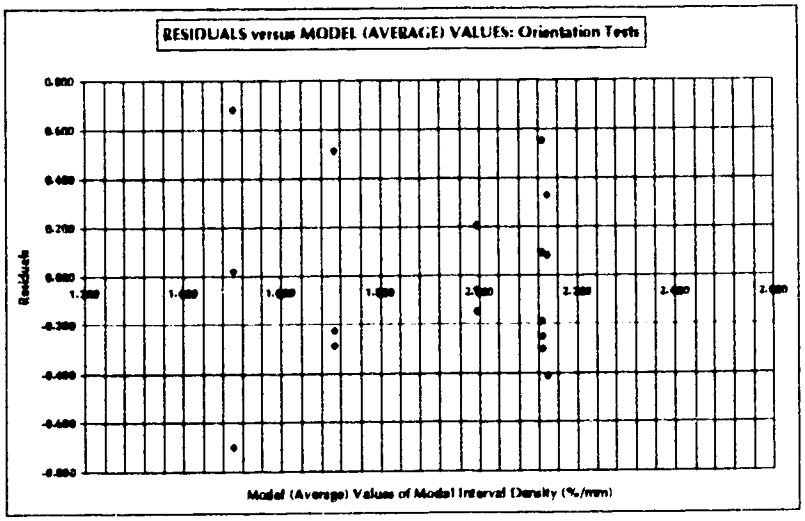
Groups	Count	Sum	Average	Variance
Row 1	3	6.3898	2.1299	0.2300
Row 2	3	5.9852	1.9951	0.0341
Row 3	3	6.4188	2.1396	0.1417
Row 4	3	5.1303	1.7101	0.1977
Row 5	3	4.5110	1.5037	0.4783
Row 6	3	6.3844	2.1281	0.0266

ANOVA

Source Variation	SS	df	MS	F	P-value	F crit
Between Groups	1.0721	5	0.2144	1.1607	0.3826	3.1059
Within Groups	2.2168	12	0.1847			
						F < F crit
Total	3.2889	17	No real diff. between orientations.			

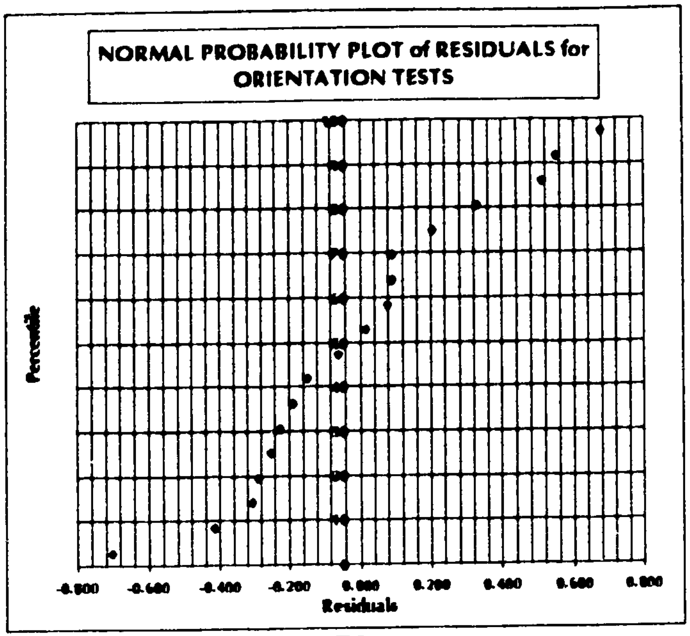
The highest material densities occur in the 125 mm to 50 mm size range.

FIGURE 4.13: Final Results and ANOVA for Orientation Tests



[A]

FIGURE 4.14: Residuals Plots for Orientation Test ANOVA



[B]

FIGURE 4.14: Residuals Plots for Orientation Test ANOVA

4.9 RESULTS OF COMMINGLED RECYCLABLES/TETRA-PAK TESTS

These tests would help demonstrate the application of the preliminary tests results to a materials recovery operation. If a certain characteristic was found to produce a higher degree of material isolation after shredding, this same characteristic may enable the container material to be more easily separated from other materials if the container was in a mixture of items. During the experimentation, the size of a beverage container appeared to make the most significant difference in the shredding behaviour for the Tetra-Pak containers. Thus, 250 mL, rectangular, enclosed Tetra-Paks were mixed with a typical commingled recyclables sample and tested as discussed in Chapter Three. However, the initial commingled tests displayed some unusual results that prompted a change in the experimental procedure.

The test one batch was shredded and then screened. After shredding, many of the Tetra-Paks appeared relatively undamaged, suffering little breakage and only some compression. This was again observed for the second test. Conversely, the Tetra-Paks from the preliminary tests were obviously shredded. After the second test, it was decided to increase the air valve pressure to 55 psi from 42 psi. Since the tests involved other items, such as large cans and cardboard boxes, the shredder might allow these bulkier items to pass through with minimal or no shredding to avoid damage to the shredder. Tetra-Paks may inadverdently pass through with these other items as a result. The supplier of the shredder suggested increasing the air valve pressure to allow fewer items to pass through unshredded.

Tests three and four were conducted with the shredder at 55 psi. Although a visual inspection of the shredded material showed that more of the Tetra-Paks were shredded than in test three, some were still intact. Test four had intact Tetra-Paks very similar to tests one and two. In fact, a soup can passed through literally untouched in test four. After all four tests were completed, the distribution of Tetra-Paks among the various screen sizes were very similar for tests one, three, and four. Changing the air valve psi appears to have made little, if any, difference. Only test two had some observable difference and this was prior to increasing the air valve to 55 psi. This can be seen in the following figure. However, there was no reason to discard the data from set two since all experimental conditions, other than the altered air valve pressure, were unchanged.

Again, the interval densities for the shredded Tetra-Pak were calculated to determine the highest interval density. To calculate the upper range for the 125 mm screen size range, the largest dimension was the diagonal of a 380 mm by 300 mm folded newspaper. In all four tests, such newspapers were often found relatively unshredded and so were usually the largest item with the largest dimension. The highest densities of Tetra-Pak occured in the 125 mm to 80 mm size range except for test four, which occurred in the 80 mm to 50 mm size range. Even then, the interval density for the 125 mm to 80 mm size range in test four was very close to the highest interval density. These interval densities versus the screen size ranges are plotted in Figure 4.15.

The largest Tetra-Pak interval densities from these commingled tests are 1.3072 %/mm, 0.7407 %/mm, 1.3072 %/mm, and 1.2500 %/mm, giving a mean interval density of 1.151 %/mm, with a standard deviation of 0.275 %/mm and a variance of 0.0756. The preliminary tests that used *250 mL, rectangular, enclosed Tetra-Paks* have a mean largest interval density of 1.436 %/mm with a standard deviation of 0.128 %/mm and a variance of 0.01638. According to the following two-sided F-test for a 0.05 significance level, F_{calc} is less than F_{tab}. This upholds the null hypothesis that the two sample variances estimate the same population variance. Next, a t-test was performed to determine if the mean of the preliminary tests and the mean of the commingled tests were truly different from one another.

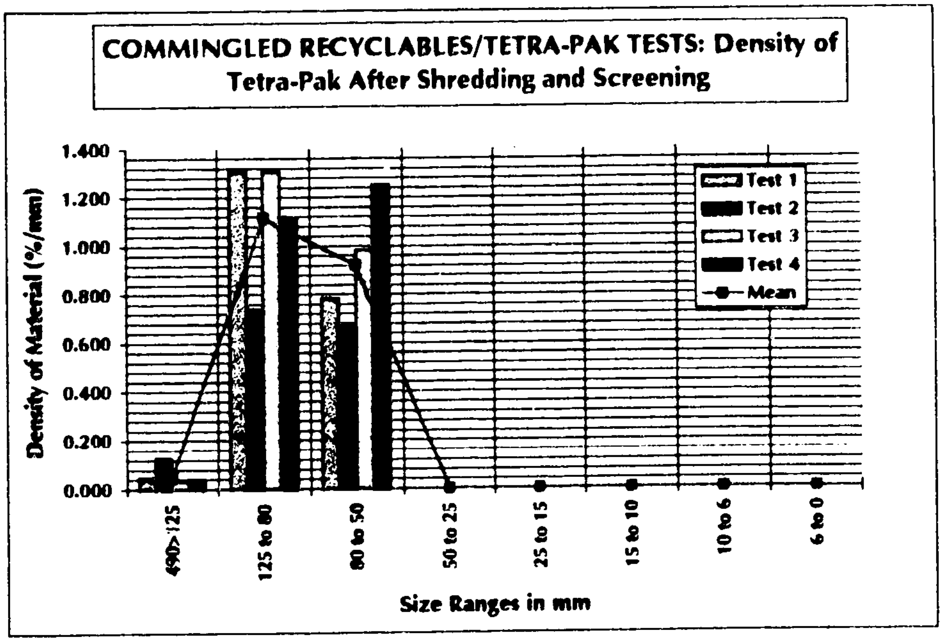


FIGURE 4.15: Tetra-Pak Interval Densities for Commingled Recyclables Test

Test	***Mean Interval Density (%/mm)***	***Variance, s2***	***Degrees of Freedom, d.f.***
Commingled	1.151	0.0756	3
Preliminary	1.436	0.01638	2

F_{calc}	***Numerator d.f.***	***Denominator d.f.***	***Significance***	$F_{tabulated}$
4.615	3	2	0.025 (for 0.05)	39.17

TABLE 4.7: F-test on Results of Commingled Recyclables/Tetra-Pak Tests

At both significance levels, t_{calc} is less than t_{tab}. The two means are not different from one another, implying that *whether the 250 mL rectangular, enclosed Tetra-Pak containers are by themselves or commingled with other recyclable items, the containers will produce similar modal interval densities*. However, this does not definitively conclude that Tetra-Paks do so *because* of the container characteristics. Furthermore, eventhough shredding Tetra-Paks either alone or commingled produced similar results, the results are not identical. In the containers-only situation the Tetra-Paks were actually shredded but in the commingled case a fair majority were relatively undamaged. This suggests another variable(s), possibly the interference from the other recyclables, in the commingled situation must be investigated.

Combined Variance, s_c^2	***Combined Std. Deviation, s_c***	***Standard Deviation, s_d***	***Sample n_1***	***Sample n_2***	t_{calc}
0.05191	0.2278	0.1740	4	3	1.6379

Total d.f.	***Significance***	$t_{tabulated}$	***Significant?***
5	0.05	2.571	*No*
5	0.01	4.032	*No*

TABLE 4.8: t-test on Results of Commingled Recyclables/Tetra-Pak Tests

4.11 CONCLUSIONS OF DATA ANALYSIS

The main objectives of this chapter were to determine if the size, shape, geometry, and material characterstics of the selected beverage containers - Tetra-Paks, glass jars, and EPS containers - would affect the shredding behaviour of that beverage container and if so, what such a relationship would be. If a characteristic or combination of characteristics better isolated the container material after shredding, the separability and subsequent recovery of the container material would most likely be improved. Based on the various test findings in this chapter, the following general statements with respect to these objectives can be made.

4.11.1 Preliminary Tests

These tested the effects of the size, shape, and geometry factors on the shredding behaviour of the beverage containers within the same container/material type. The factors were each tested at two levels: 250 mL and 1000 mL sizes, rectangular or cylindrical shapes, and whether the container was open ended or enclosed.

The preliminary tests on the Tetra-Paks indicate that the size of the aseptic package affects the isolation of the container material after shredding. The 250 mL sized Tetra-Paks produce significantly higher concentrations of shredded material than their 1000 mL sized counterparts. The shape of the Tetra-Pak is, at best, marginally significant in affecting how the container shreds. The geometry of the Tetra-Pak container appears to make no difference.

The shredding behaviour of the glass jars appear to be minimally affected by the size of the container and the interaction between the size and shape of the container. In general, the size, shape, and geometry of the beverage container appears to have no effect. This suggests that the material itself is the deciding factor in affecting the shredding behaviour.

Both the 250 mL and 1000 mL sized EPS containers shredded similarly. Size therefore does not appear to affect the shredding behaviour of these containers. The factors of shape and geometry could not be evaluated because suitable containers for testing these the influences of these characteristics could not be obtained. However, because the other two container types are minimally affected, if at all, by the factors of shape and geometry, it is quite possible that the EPS containers are also similarly unaffected. This presumption is also supported by observing that both glass and polystyrene are brittle materials. Some similarity between the performance of the two materials during shredding could be expected. As with the case for glass, the preliminary test for EPS suggests that the material itself may be the deciding factor.

4.11.2 Materials Comparison Test

Except for the role of size in the Tetra-Pak test, the general insignificance of the size, shape, and geometry factors suggests another factor significantly affects the shredding behaviour of the selected beverage containers. Based on the background information in Chapter Two and the materials test, the material the container is constructed of is most likely the significant factor. Each material was described using the density of the material. The materials comparison test demonstrated that all three different containers did indeed shred differently and produce different modal interval densities because each container was made of a different material. However, no observable trend could be seen between the modal interval densities and the container material densities. Instead, the modal interval densities were compared to a description of the ductility or brittleness of the container material. This comparison demonstrated that the greater the brittleness of the material, the greater the modal interval density. Further research is needed to quantify this relationship.

4.11.3 Orientation Test

The orientation of the beverage container just prior to shredding was originally suspected to affect how the container was shredded. While the orientation is not a physical characteristic of the container, it may be an important consideration for actual materials recovery operations. To test this, six different orientations using Tetra-Pak containers were used. However, the results indicate that the pre-shredding orientation of the container does not appear to make any significant difference in the shredding behaviour of the Tetra-Paks. Thus, at least to Tetra-Pak containers, the orientation is unimportant. However, the orientation of the beverage container may be significant to other beverage containers such as glass jars.

4.11.4 Commingled Recyclables/Tetra-Pak Test

250 mL, rectangular, enclosed Tetra-Paks were mixed with a variety of other recyclable items to evaluate if the small size, which was found to improve the concentration of shredded material in the preliminary tests, also resulted in the separation of the Tetra-Pak material from other items after shredding the entire mixture. The test results indicate that whether the Tetra-Paks are shredded alone, as in the preliminary test, or are commingled with a typical recyclables mixture, the shredding behaviour is similar. Therefore, given that:

i. the smaller Tetra-Paks produced a better concentration of Tetra-Pak material in the preliminary tests than the other Tetra-Pak configurations, and
ii. the highest concentration means of Tetra-Pak material from the commingled tests and the preliminary tests were not significantly different,

it is logical to conclude that smaller Tetra-Paks would indeed be more effectively recovered from a shredded and screened commingled recyclables stream. However, this cannot be concluded definitively for two reasons.

1. As mentioned previously, the Tetra-Paks in the commingled recyclables test were observed to be quite intact, whereas the preliminary test Tetra-Paks underwent considerably more shredding. This suggests another variable(s) should be investigated, such as the interference from other materials during the shredding and/or screening process.
2. An additional test is required which would compare the results from, for example, a commingled recyclables/large Tetra-Paks against those of the commingled recyclables/small Tetra-Paks[8]. Such a test would directly compare the effects of the different sizes within a mixed materials sample and evaluate if the smaller size does improve the materials recovery potential.

4.11.5 Hypotheses

Based on the test results and analyses, the second and third hypotheses postulated in Chapter Two are disproven. The shape and geometry of the container appears to be generally insignificant in affecting the shredding behaviour of the selected beverage containers. The first hypothesis proposed that smaller sized containers would result in a more concentrated size distribution after shredding than larger sized containers. This appears to be borne out in only one instance. 250 mL Tetra-Paks do produce a greater concentration of shredded material than 1000 mL Tetra-Paks. However, the size factor appears to have little, if any, effect on the performance of the glass and EPS containers. The last hypothesis asserted that the material a beverage container is made of will affect the shredding behaviour of the container. This was found to be true but a conclusive relationship between the material type and the subsequent shredding behaviour could not be determined.

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[8] Unfortunately, no more 1000 mL Tetra-Paks were left and insufficient quantities were available from the supplier at the conclusion of this experiment.

CHAPTER FIVE
Applications

5.0 APPLICATION OF THE TETRA-PAK FINDING

What are the implications for a particular product if it is discovered that a change in the physical design can improve the recovery of the product materials after consumer use? As an example, the Tetra-Pak package was the only container type that appeared to be affected by its physical design. The smaller, 250 mL size container produced a higher concentration of shredded material than the larger, 1000 mL size container. Does this finding imply that all Tetra-Pak containers should be *redesigned* to the smaller size?

To examine this question, a limited life-cycle analysis will be performed. This discussion is not intended to be a comprehensive examination of the entire product cycle of a Tetra-Pak aseptic. Also, the scenario that follows is hypothetical and does not necessarily reflect what would actually occur in a materials recovery operation. However, the following discussion does illustrate that the research findings should be interpreted with respect to other important variables. This proposed scenario incorporates the effect of physical design to maximize the concentration of desired material for materials recovery.

5.1 TETRA-PAK CONCLUSIONS FROM RESEARCH

The factorial analysis performed on the Tetra-Pak data concluded that the average modal interval density is 1.499 %/mm. The transition from a smaller to larger container (going from the 250 mL to 1000 mL size) would decrease the modal interval density by 0.841 %/mm. Because the average density of 1.499 %/mm is actually at the midpoint size between these two container sizes (recall that this is the average of *all* container runs, both small and large sizes), the modal interval density at the smaller size would be (1.499 + 0.420) or 1.919 %/mm. Similarly, the modal interval density at the larger size would be (1.499 - 0.420) or 1.079 %/mm. These last two densities represent the highest concentration of shredded and screened material predicted by the model for their respective sized materials. It is assumed that the other factors and interactions are insignificant and thus will be omitted from this comparison. To translate these densities into more useful quantities, it is necessary to find out over what range these densities would occur.

There were totally eight runs from all the Tetra-Pak data that used 250 mL sized containers. Four of these runs had their modal interval densities occur in the 45 mm range while the other four runs had theirs in the 30 mm range. The average range is therefore 37.5 mm. There were also eight runs using 1000 mL sized containers. Six of these runs had their modal interval densities occur in the 45 mm range; two runs had theirs in the 30 mm range. The adjusted average size range is [6(45) + 2(30)]/8 or 41.25 mm.

Assuming that the modal interval densities represent a realistic amount of material that could be captured, the predicted recovery amounts would be as shown in Table 5.1. Thus, for a given amount (mass) of 250 mL sized containers and using appropriate screens to separate out the highest concentration of shredded material, it would be possible to recover up to 72.0 % of the original amount. However, only 44.5% of the 1000 mL sized containers could be expected to be recovered. Therefore, it would appear to be more "environmentally friendly" in terms of material recovery to redesign all Tetra-Pak containers to the 250 mL size format.

Container Size (mL)	*Modal Interval Density (%/mm)*	*Size Range for Density (mm)*	*Percent Recovery of Material*
250	1.919	37.5	72.0
1000	1.079	41.25	44.5

TABLE 5.1: Predicted Recovery Percentages for Tetra-Pak

5.2 APPLICATION TO THE TETRA-PAK PACKAGE LIFE CYCLE

This section compares the current situation of having both 250 mL and 1000 mL Tetra-Pak containers against having only 250 mL Tetra-Pak containers. The life-cycle data used in the following calculations has been adapted from the April 1991 entitled ***Energy and Environmental Profiles in Canada of Tetra-Brik Aseptic Carton and Glass Bottle Packaging Systems***. All life-cycle quantities are for *every 1000 L of Tetra-Pak packaged.*

Based on the solid wastes generated from the current allocation between the 250 mL and 1000 mL sized containers, the following table calculates the theoretical amount of Tetra-Pak material recoverable.

Container Size (mL)	*Solid Waste (kg/1000 L)*	*Recovery (%)*	*Mass Recovered (kg)*
250	71.5	72.0	51.5
1000	49.4	44.5	22.0

TABLE 5.2: Theoretical Recovery Amounts - Current

A total of 73.5 kg out of an original 120.90 kg, or 60.8% would thus be theoretically recovered. Consider the case if only 250 mL Tetra-Pak containers were produced. The solid waste amount of 250 mL containers would be *double* the current amount to compensate for the absence of 1000 mL containers. The quantities given are for 1000 L worth of packaging, regardless of the container size. Thus, an extra 71.5 kg of 250 mL containers would be required to hold the same amount of beverage as 49.4 kg of 1000 mL containers. The theoretical recovery amount would be of course 72.0%. However, an extra [2(71.5 kg) - 120.90 kg] or 22.1 kg of solid waste has been generated as as well. While the recovery would have increased from 60.8% to 72.0%, or 11.2%, the solid waste generated would have increased by 18.3% from the original 120.90 kg.

The consequences of improved container design for recovery results in more waste. However, there are also the manufacturing and ancillary processes that must be considered. Several of these have been selected as examples. The following tables compare the current situation in which both 250 mL and 1000 mL sized containers are manufactured against the alternate situation of manufacturing only 250 mL sized containers. It is assumed that doubling the quantity of 250 mL containers also doubles whatever quantity is required for that particular process.

Container Size (mL)	*Total Material Consumption (kg)*
250	71.8
1000	51.4
Grand Total	123.2

Container Size (mL)	*Total Material Consumption (kg)*	*Original Total Consumption (kg)*	*Difference from Original Total*
250	71.8 x 2 = 143.6	123.2	16.5% increase

TABLE 5.3: Comparison of Materials Consumption

Container Size (mL)	*Water Consumption (m^3)*
250	6.414
1000	4.831
Grand Total	11.25

Container Size (mL)	*Water Consumption (m^3)*	*Original Water Consumption (m^3)*	*Difference from Original Total*
250	6.414 x 2 = 12.83	11.25	14% increase

TABLE 5.4: Comparison of Water Consumption

Container Size (mL)	*Energy Consumption (MJ)*
250	3567
1000	2355
Grand Total	5922

Container Size (mL)	*Energy Consumption (MJ)*	*Original Energy Consumption (MJ)*	*Difference from Original Total*
250	3567 x 2 = 7134	5922	20% increase

TABLE 5.5: Comparison of Energy Consumption

5.3 CONCLUSION TO THE TETRA-PAK APPLICATION

Redesigning all Tetra-Paks to the smaller 250 mL size improves the potential recovery of shredded Tetra-Pak by 12% over the current mix of both 250 mL and 1000 mL containers. However, producing 250 mL sized containers only would also increase the initial material consumption, water consumption, energy consumption, and solid waste generation by various amounts, ranging from 14% to 20%. Redesigning all Tetra-Pak containers to the smaller size does not appear justified. There are clearly other aspects, such as the manufacturing processes, that must considered.

CHAPTER SIX
Conclusions and Recommendations

6.0 THESIS SUMMARY

Many beverage containers are potentially recyclable after use but only a portion of these are regularly recycled. To recover these containers that would otherwise be disposed, this thesis investigated if an alternate beverage container design would improve the recovery of the container material in a materials recovery operation, or if the container was not currently recycled, actually allow a materials recovery operation to recapture this container material. It was assumed that the beverage container would be shredded and screened during the recovery process. A desirable alternate beverage container design would enable the shredded container material to be better isolated or concentrated into some specific size group as opposed to having the shredded material scattered throughout many size ranges.

Three types of beverage containers were eventually chosen for testing: Tetra-Pak aseptic packages, expanded polystyrene (EPS) cups, and glass jars (not returnable glass bottles). Examining these selected beverage containers revealed unique characteristics that could be generalized into the categories of *size, shape, geometry (open-ended or enclosed)*, and *container material*. These categories can be used to describe almost any beverage container and are the various *physical design characteristics* that may affect the shredding and screening behaviour of the beverage containers. These design characteristics would thus be the physical basis of any alternative container design. It was hypothesized that the smaller the size of the container, the smaller the pieces of material should be after shredding; that cylindrical containers would result in less complete shredding of the containers; that "open-mouthed" containers would result in a greater degree of shredding; and that brittle materials should break into smaller pieces while pliable materials may not shred uniformly.

With only one exception, it was concluded that in general, the factors of size, shape, and geometry were insignificant in affecting the shredding and screening behaviour of the chosen beverage containers. Smaller sized Tetra-Paks appeared to have higher concentrations of shredded material than their larger sized counterparts. However, based on the background information and the data results, the material the container is constructed of is most likely the significant factor. All three different containers did indeed shred differently and produce different shredding behaviours; brittle materials produced the greatest concentration of shredded material. Further research is needed to determine precisely what is this relationship.

However, the research does demonstrate that in *some* instances, an alternate design may lead to higher materials recovery. Specifically, smaller sized Tetra-Paks produced a higher concentration of shredded material. To illustrate the implications of such a conclusion, a limited life cycle analysis that incorporates the size-effect finding for the Tetra-Pak container was performed. It was discovered that designing a container for the *sole* purpose of maximizing its material recovery potential was not justifiable.

6.1 CONCLUSIONS

Based on the results of the analyses and the outcomes of the hypotheses, the following conclusions can be made in the context of this research with regards to improving the materials recovery from shredded and screened beverage containers by using an alternate beverage container design.

I. For beverage containers similar to the glass jars and expanded polystyrene containers used in this research, it is highly unlikely that redesigning the container in terms of size, shape, or geometry will improve the recovery of the container material after shredding.
II. For beverage containers similar to the Tetra-Pak aseptic packages, it is highly unlikely that redesigning the container in terms of shape or geometry will improve the recovery of the container material after shredding. However, the effect of having a smaller sized container does appear to increase the concentration of shredded material and could potentially lead to improved recovery of the container material. Redesigning Tetra-Pak packages to smaller sized formats to facilitate better recovery after consumer use is therefore a possible consideration.
III. Redesigning beverage containers by altering the material used may or may not improve the materials recovery. It was observed that the brittle materials, glass and EPS, produced higher concentrations of shredded material than the shredded Tetra-Pak. However, no definite conclusion can be drawn about the relationship between the type of container material and the subsequent recovery of that material based on the findings of this research. More work is needed to determine what this relationship is, assuming it exists.

Physically designing (or redesigning) an item for the sole purpose of materials recovery does not necessarily result in the most environmentally beneficial product. Instead, the potential to improve the materials recovery of the spent product is only one, although important, consideration of the design process. There are clearly other considerations, such as the use of raw materials and energy consumption, that should also be incorporated into the design process. As stated in Chapter One, the product should be, "... designed, manufactured, used, and disposed of in such as way as to minimise its effect on the environment..." (CCME, 1990).

6.2 RECOMMENDATIONS

At the outset of this thesis, it was thought that all four characteristics - size, shape, geometry, and material - would play an important role. The research suggests that the material a beverage container is constructed of is the most important factor while the others are insignificant in almost all instances, but the data gathered is insufficient to draw a definite or quantitive conclusion concerning the effect of the container material. Future research into this subject should be directed towards investigating how the shredding of beverage containers is related to its material composition. This would consequently entail including more materials, such as metals (e.g. juice cans) in the tests and conducting a more detailed investigation into how the container material would be best described. This thesis research eventually used qualitative descriptions such as brittle or flexible. Although suitable for the current purposes, a more meaningful and/or quantitive characteristic, such as the modulus of elasticity is most likely required for future studies.

More research into the validity of the Π breakage theory would also be beneficial. Although the theory in its present form appears inadequate for describing the breakage of solid wastes, its basic concepts have some merit. For example, it was demonstrated that the use of a nonconstant selection function, while not preferred in the original theory development, does appear to more realistically model the actual breakage. However, the theory suffers from an inadequate breakage function which is the "starting point" for Π breakage theory. Future research into this subject could be directed at improving the basic premise of the theory, such as incorporating the concept of selection directly into the breakage function rather than using the breakage function to derive the selection function.

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APPENDIX A
Collected Data

A1.0 INTRODUCTION TO COLLECTED DATA

This appendix contains the various size distribution data collected during the research. Also presented are the initial treatments, such as the calculations for the modal interval densities, performed on the data, and the analyses of variances of the orientation test data (which proved to be an uncomplicated matter). Lengthy calculations and analyses are found in **Appendix B**.

A1.1 TETRA-PAK PRELIMINARY TEST DATA

The data from the preliminary Tetra-Pak tests are shown as the percentage of material found in each screen interval, as well as the cumulative percentages. The shredded material interval densities, along with the computer calculated kurtosis and skewness values, are also presented.

SCREENING DATA SET
DATA ANALYSIS

Container Type Tested

Vol. (ml.) *1000*
Shape *rect*
Geometry *open*
Material *T-Pak*

Interval Size (mm)	*Date* February 25, 1994 *Replicate One* Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	*Date* May 11, 1994 *Replicate Two* Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	*Date* June 3, 1994 *Replicate Three* Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	*Averages* Percent Mat'l. in Interval	Cum. % Mat'l. in Interval
>125	76.6	76.6	6.6	6.6	0.0	0.0	27.7	27.7
125 to 80	7.3	83.9	68.5	75.1	61.8	61.8	45.9	73.6
80 to 50	10.2	94.1	19.9	95.0	38.2	100.0	22.8	96.4
50 to 25	3.9	98.0	5.0	100.0	0.0	100.0	3.0	99.3
25 to 15	1.0	99.0	0.0	100.0	0.0	100.0	0.3	99.7
15 to 10	0.5	99.5	0.0	100.0	0.0	100.0	0.2	99.8
10 to 6	0.0	99.5	0.0	100.0	0.0	100.0	0.0	99.8
6 to 0	0.5	100.0	0.0	100.0	0.0	100.0	0.2	100.0
Number of Screening Loads	1		2		1			
	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)
	-0.10	-0.25	-0.25	-0.15	-0.20	-0.20	-0.18	-0.20

Range (mm)	Density of Mat'l in Interval %/mm Rep. One	Density of Mat'l in Interval %/mm Rep. Two	Density of Mat'l in Interval %/mm Rep. Three
45	0.162	1.522	1.373
30	0.340	0.663	1.273
25	0.156	0.200	0.000
10	0.100	0.000	0.000
5	0.100	0.000	0.000
4	0.000	0.000	0.000
6	0.083	0.000	0.000

Largest Densities:
0.340 1.522 1.373
Average of Densities:
1.079
Standard Deviation:
0.644
Logs of Largest Densities:
-0.46852 0.10248 0.13778

EXCEL GENERATED DESCRIPTIVE STATISTICS

Using *NON-ZERO* Values of Density of Mat'l Distributio and Assuming Uni-modal Distribution

Statistical Parameter	Rep. One	Rep. Two	Rep. Three
Mean	0.1716	0.7952	1.3233
Standard Error	0.0441	0.3873	0.0500
Median	0.1560	0.6633	1.3233
Mode	0.1000	#N/A	#N/A
Standard Deviation	0.0987	0.6709	0.0707
Sample Variance	0.0097	0.4501	0.0050
Kurtosis	3.2903	#DIV/0!	#DIV/0!
Skewness	1.7487	0.8502	#DIV/0!
Range	0.2400	1.3222	0.1000
Minimum	0.1000	0.2000	1.2733
Maximum	0.3400	1.5222	1.3733
Sum	0.8582	2.3856	2.6467
Count	5.0000	3.0000	2.0000
Confidence Level(95.000%)	0.0865	0.7592	0.0980

Average of Logs:
-0.04942

Comments Replicate Three was a "partial-batch" of approx. 2 kg to confirm replicate one behaviour and was screened in one load.

SCREENING DATA SET
DATA ANALYSIS

Container Type Tested

Vol. (ml.) *1000*
Shape *cyl*
Geometry *open*
Material *T-Pak*

Interval Size (mm)	*Date* March 25, 1994 *Replicate One* Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	*Date* June 3, 1994 *Replicate Two* Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	*Averages* Percent Mat'l. in Interval	Cum. % Mat'l. in Interval
362>125	52.1	52.1	2.4	2.4	27.3	27.3
125 to 80	18.9	71.0	56.6	59.0	37.8	65.0
80 to 50	20.0	91.0	32.5	91.5	26.3	91.3
50 to 25	7.4	98.4	8.4	99.9	7.9	99.2
25 to 15	1.1	99.5	0.0	99.9	0.6	99.7
15 to 10	0.0	99.5	0.0	99.9	0.0	99.7
10 to 6	0.0	99.5	0.0	99.9	0.0	99.7
6 to 0	0.5	100.0	0.0	99.9	0.3	100.0
Number of Screening Loads	1		1			
	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)
	-0.10	-0.15	0.00	-0.25	-0.05	-0.20

Range (mm)	Density of Mat'l in Interval %/mm Rep. One	Density of Mat'l in Interval %/mm Rep. Two
237	0.220	0.010
45	0.420	1.258
30	0.667	1.083
25	0.296	0.336
10	0.110	0.000
5	0.000	0.000
4	0.000	0.000
6	0.083	0.000

Largest Densities:
0.667 1.258
Average of Densities:
0.962
Standard Deviation:
0.418
Logs of Largest Densities:
-0.17609 0.09960

EXCEL GENERATED DESCRIPTIVE STATISTICS

Using *NON-ZERO* Values of Density of Mat'l Distribution and Assuming Uni-modal Distribution

Statistical Parameter	Rep. One	Rep. Two
Mean	0.3425	0.6718
Standard Error	0.0955	0.2977
Median	0.2960	0.7097
Mode	#N/A	#N/A
Standard Deviation	0.2135	0.5953
Sample Variance	0.0456	0.3544
Kurtosis	0.5596	-4.1869
Skewness	0.8568	-0.1791
Range	0.5567	1.2477
Minimum	0.1100	0.0101
Maximum	0.6667	1.2578
Sum	1.7125	2.6872
Count	5.0000	4.0000
Confidence Level(95.000%)	0.1872	0.5834

Average of Logs:
-0.03824

Comments Replicate Two was a "half-batch" of approximately 4 kg and was thus screened in one load.
The "materials test" using approx. 1000ml. rect., open Tetra-Pak containers produced similar results to Replicate Two.

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SCREENING DATA SET
DATA ANALYSIS

Container Type Tested		Interval Size (mm)	Date February 25, 1994 Replicate One Percent Mat'l. in Interval	Replicate One Cum. % Mat'l. in Interval	Date April 1, 1994 Replicate Two Percent Mat'l. in Interval	Replicate Two Cum. % Mat'l. in Interval	Averages Percent Mat'l. in Interval	Averages Cum. % Mat'l. in Interval	Range (mm)	Density of Mat'l in Interval %/mm Rep. One	Density of Mat'l in Interval %/mm Rep. Two
		430>125	17.6	17.6	23.5	23.5	20.6	20.6	305	0.058	0.077
Vol. (mL)	*1000*	125 to 80	44.8	62.4	56.1	79.6	50.5	71.0	45	0.996	1.247
Shape	*rect*	80 to 50	26.7	89.1	15.8	95.4	21.3	92.3	30	0.890	0.527
Geometry	*encl*	50 to 25	7.6	96.7	4.6	100.0	6.1	98.4	25	0.304	0.184
Material	*T-Pak*	25 to 15	1.4	98.1	0.0	100.0	0.7	99.1	10	0.140	0.000
		15 to 10	0.5	98.6	0.0	100.0	0.3	99.3	5	0.100	0.000
		10 to 6	0.5	99.1	0.0	100.0	0.3	99.6	4	0.125	0.000
		6 to 0	1.0	100.1	0.0	100.0	0.5	100.1	6	0.167	0.000
		Number of Screening Loads	1		2						

Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)
0.00	0.30	0.00	-0.20	0.00	0.05

Largest Densities:
0.996 1.247
Average of Densities:
1.121
Standard Deviation:
0.178
Logs of Largest Densities:
-0.00193 0.09575

Comments

EXCEL GENERATED DESCRIPTIVE STATISTICS

Using *NON-ZERO* Values of Density of Mat'l Distribution and Assuming Uni-modal Distribution

Statistical Parameter	Rep. One	Rep. Two
Mean	0.3474	0.5086
Standard Error	0.1327	0.2641
Median	0.1533	0.3553
Mode	#N/A	#N/A
Standard Deviation	0.3754	0.5281
Sample Variance	0.1410	0.2789
Kurtosis	-0.0789	1.1797
Skewness	1.3202	1.3014
Range	0.9379	1.1696
Minimum	0.0577	0.0770
Maximum	0.9956	1.2467
Sum	2.77[illegible]	2.0344
Count	8.0000	4.0000
Confidence Level(95.000%)	0.2602	0.5175

Average of Logs:
0.04691

SCREENING DATA SET
DATA ANALYSIS

Container Type Tested		Interval Size (mm)	Date March 7, 1994 Replicate One Percent Mat'l. in Interval	Replicate One Cum. % Mat'l. in Interval	Range (mm)	Density of Mat'l in Interval %/mm
		419>125	10.1	10.1	294	0.034
Vol. (mL)	*1000*	125 to 80	51.8	61.9	45	1.151
Shape	*cyl*	80 to 50	29.1	91.0	30	0.970
Geometry	*encl*	50 to 25	8.0	99.0	25	0.320
Material	*T-Pak*	25 to 15	1.0	100.0	10	0.100
		15 to 10	0.0	100.0	5	0.000
		10 to 6	0.0	100.0	4	0.000
		6 to 0	0.0	100.0	6	0.000
		Number of Screening Loads	1			

Diff. Due to Shred (kg)	Diff. Due to Screen (kg)
0.00	-0.05

Largest Density:
1.151
Log:
0.06112

EXCEL GENERATED DESCRIPTIVE STATISTICS

Using *NON-ZERO* Values of Density of Mat'l Distribution and Assuming Uni-modal Distribution

Statistical Parameter	
Mean	0.5151
Standard Error	0.2294
Median	0.3200
Mode	#N/A
Standard Deviation	0.5131
Sample Variance	0.2632
Kurtosis	-2.7503
Skewness	0.4979
Range	1.1168
Minimum	0.0344
Maximum	1.1511
Sum	2.5755
Count	5.0000
Confidence Level(95.000%)	0.4497

Comments

SCREENING DATA SET
DATA ANALYSIS

			Date March 7, 1994 *Replicate One*		*Date* April 27, 1994 *Replicate Two*		*Date* June 6, 1994 *Replicate Three*			*Averages*		Density of	Density of	Density of
		Interval Size (mm)	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**		**Mat'l in Interval %/mm**	**Mat'l in Interval %/mm**	**Mat'l in Interval %/mm**
Container Type Tested											**Range (mm)**	**Rep. One**	**Rep. Two**	**Rep. Three**
		>125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
Vol. (mL)	***250***	**125 to 80**	78.2	78.2	55.6	55.6	45.9	45.9	59.9	59.9	**45**	1.738	1.236	1.020
Shape	***rect***	**80 to 50**	15.4	93.6	32.6	88.2	51.0	96.9	33.0	92.9	**30**	0.513	1.087	1.700
Geometry	***open***	**50 to 25**	5.3	98.9	9.6	97.8	3.1	100.0	6.0	98.9	**25**	0.212	0.384	0.124
Material	***T-Pak***	**25 to 15**	0.5	99.4	1.1	98.9	0.0	100.0	0.5	99.4	**10**	0.050	0.110	0.000
		15 to 10	0.0	99.4	0.0	98.9	0.0	100.0	0.0	99.4	**5**	0.000	0.000	0.000
		10 to 6	0.0	99.4	0.0	98.9	0.0	100.0	0.0	99.4	**4**	0.000	0.000	0.000
		6 to 0	0.5	99.9	1.1	100.0	0.0	100.0	0.5	100.0	**6**	0.083	0.183	0.000
		Number of Screening Loads	1		2		1							

Diff. Due to Shred (kg)	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**
-0.10	-0.10	-0.15	0.00	0.00	0.00	-0.13	-0.05

Largest Densities:
1.738 1.236 1.700
Average of Densities:
1.558
Standard Deviation:
0.200
Logs of Largest Densities:
0.23999 0.09186 0.23045
Average of Logs:
0.18744

EXCEL GENERATED DESCRIPTIVE STATISTICS

Using *NON-ZERO* Values of Density of Mat'l Distribution and Assuming Uni-modal Distribution

Statistical Parameter	**Rep. One**	**Rep. Two**	**Rep. Three**
Mean	0.6283	0.7041	0.9480
Standard Error	0.3821	0.2714	0.4564
Median	0.3627	0.7353	1.0200
Mode	#N/A	#N/A	#N/A
Standard Deviation	0.7642	0.5429	0.7905
Sample Variance	0.5840	0.2947	0.6248
Kurtosis	2.7003	-4.4343	#DIV/0!
Skewness	1.6414	-0.1511	-0.4065
Range	1.6878	1.1256	1.5760
Minimum	0.0500	0.1100	0.1240
Maximum	1.7378	1.2356	1.7000
Sum	2.5131	2.8162	2.8440
Count	4.0000	4.0000	3.0000
Confidence Level(95.000%)	0.7489	0.5320	0.8945

Comments No weight loss data due to shredding or screening available for replicate 3; not included in average losses.
Replicate three was a "half-batch" of approx. 4.5 kg to confirm behaviour of earlier replicates; screened in one load.

SCREENING DATA SET
DATA ANALYSIS

			Date May 25, 1994 *Replicate One*			Density of
		Interval Size (mm)	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**		**Mat'l in Interval %/mm**
Container Type Tested					**Range (mm)**	
		>125	0.0	0.0		
Vol. (mL)	***250***	**125 to 80**	30.0	30.0	**45**	0.667
Shape	***cyl***	**80 to 50**	62.8	92.8	**30**	2.093
Geometry	***open***	**50 to 25**	7.0	99.8	**25**	0.280
Material	***T-Pak***	**25 to 15**	0.0	99.8	**10**	0.000
		15 to 10	0.0	99.8	**5**	0.000
		10 to 6	0.0	99.8	**4**	0.000
		6 to 0	0.0	99.8	**6**	0.000
		Number of Screening Loads	2			

Diff. Due to Shred (kg)	**Diff. Due to Screen (kg)**
-0.15	-0.20

Largest Density:
2.093
Log:
0.32084

EXCEL GENERATED DESCRIPTIVE STATISTICS

Using *NON-ZERO* Values of Density of Mat'l Distribution and Assuming Uni-modal Distribution

Statistical Parameter	
Mean	1.0133
Standard Error	0.5514
Median	0.6667
Mode	#N/A
Standard Deviation	0.9551
Sample Variance	0.9122
Kurtosis	#DIV/0!
Skewness	1.4182
Range	1.8133
Minimum	0.2800
Maximum	2.0933
Sum	3.0400
Count	3.0000
Confidence Level(95.000%)	1.0808

Comments

SCREENING DATA SET
DATA ANALYSIS

Container Type Tested		Screen Sizes (mm)	Interval Size (mm)	Replicate Two Date April 1 & 8, 1994 Percent Mat'l. in Interval	Replicate Two Cum. % Mat'l. in Interval	Replicate Three Date May 31, 1994 Percent Mat'l. in Interval	Replicate Three Cum. % Mat'l. in Interval	Replicate Five Date June 7, 1994 Percent Mat'l. in Interval	Replicate Five Cum. % Mat'l. in Interval	Averages Percent Mat'l. in Interval	Averages Cum. % Mat'l. in Interval	Range (mm)	Density of Mat'l in Interval %/mm Rep. Two	Density of Mat'l in Interval %/mm Rep. Three	Density of Mat'l in Interval %/mm Rep. Five
		125	>125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
Vol. (mL)	***250***	80	125 to 80	70.5	70.5	59.0	59.0	56.0	[illegible]	61.8	61.8	45	1.567	1.311	1.244
Shape	***rect***	50	80 to 50	23.9	94.4	39.0	98.0	42.9	98.9	35.3	97.1	30	0.797	1.300	1.430
Geometry	***encl***	25	50 to 25	4.0	98.4	1.5	99.5	1.1	100.0	2.2	99.3	25	0.160	0.060	0.044
Material	***T-Pak***	15	25 to 15	0.0	98.4	0.0	99.5	0.0	100.0	0.0	99.3	10	0.000	0.000	0.000
		10	15 to 10	0.0	98.4	0.0	99.5	0.0	100.0	0.0	99.3	5	0.000	0.000	0.000
		6	10 to 6	0.0	98.4	0.0	99.5	0.0	100.0	0.0	99.3	4	0.000	0.000	0.000
		0	6 to 0	0.6	99.0	0.0	99.5	0.0	100.0	0.2	99.5	6	0.100	0.000	0.000
Number of Screening Loads				1		2		2							

Replicate Two Diff. Due to Shred (kg)	Replicate Two Diff. Due to Screen (kg)	Replicate Three Diff. Due to Shred (kg)	Replicate Three Diff. Due to Screen (kg)	Replicate Five Diff. Due to Shred (kg)	Replicate Five Diff. Due to Screen (kg)	Averages Diff. Due to Shred (kg)	Averages Diff. Due to Screen (kg)
-0.20	-0.10	-0.30	-0.25	-0.15	0.20	-0.22	-0.05

Largest Densities:
1.567 1.311 1.430
Average of Densities:
1.436
Standard Deviation:
0.128
Logs of Largest Densities:
0.19498 0.11764 0.15534
Average of Logs:
0.15598

Comments Replicate One was rejected because the mass screened (13.95 kg) exceeded the recommended limit for the mechanical screen (10 kg). Replicate Four was rejected because the shredder limit was altered to 55 psi. All other tests were performed at 42 psi.

EXCEL GENERATED DESCRIPTIVE STATISTICS
Using *NON-ZERO* Values of Density of Mat'l Distribution and Assuming Uni-modal Distribution

Statistical Parameter	Rep. Two	Rep. Three	Rep. Five
Mean	0.8411	0.8904	0.9061
Standard Error	0.4067	0.4152	0.4344
Median	0.7967	1.3000	1.2444
Mode	#N/A	#N/A	#N/A
Standard Deviation	0.7044	0.7191	0.7524
Sample Variance	0.4962	0.5172	0.5661
Kurtosis	#DIV/0!	#DIV/0!	#DIV/0!
Skewness	0.2828	-1.7316	-1.6143
Range	1.4067	1.2511	1.3860
Minimum	0.1600	0.0600	0.0440
Maximum	1.5667	1.3111	1.4300
Sum	2.5233	2.6711	2.7184
Count	3.0000	3.0000	3.0000
Confidence Level(95.000%)	0.7971	0.8138	0.8514

Because of the limited number of non-zero values, the low density values may distort skewness calculations. These low values reflect the tapering off range towards zero.

SCREENING DATA SET
DATA ANALYSIS

Container Type Tested		Interval Size (mm)	Replicate One Date May 13, 1994 Percent Mat'l. in Interval	Replicate One Cum. % Mat'l. in Interval	Range (mm)	Density of Mat'l in Interval %/mm
		>125	0.0	0.0		
Vol. (mL)	***250***	125 to 80	15.4	15.4	45	0.342
Shape	***cyl***	80 to 50	77.7	93.1	30	2.590
Geometry	***encl***	50 to 25	6.9	100.0	25	0.276
Material	***T-Pak***	25 to 15	0.0	100.0	10	0.000
		15 to 10	0.0	100.0	5	0.000
		10 to 6	0.0	100.0	4	0.000
		6 to 0	0.0	100.0	6	0.000
Number of Screening Loads			1			

Diff. Due to Shred (kg)	Diff. Due to Screen (kg)
-0.10	-0.20

Largest Density:
2.590
Log:
0.41330

EXCEL GENERATED DESCRIPTIVE STATISTICS
Using *NON-ZERO* Values of Density of Mat'l Distribution and Assuming Uni-modal Distribution

Statistical Parameter	
Mean	1.0694
Standard Error	0.7605
Median	0.3422
Mode	#N/A
Standard Deviation	1.3173
Sample Variance	1.7352
Kurtosis	#DIV/0!
Skewness	1.7271
Range	2.3140
Minimum	0.2760
Maximum	2.5900
Sum	3.2082
Count	3.0000
Confidence Level(95.000%)	1.4906

Because of the limited number of non-zero values, the low density values may distort skewness calculations. These low values reflect the tapering off range towards zero.

Comments

80

A1.2 GLASS JAR PRELIMINARY TEST DATA

The data from the preliminary glass jar tests are shown as the percentage of material found in each screen interval, as well as the cumulative percentages. The shredded material interval densities are also presented.

SCREENING DATA SET
DATA ANALYSIS

Container Type Tested		Size Range (mm)	Interval Size (mm)	*Date* May 11, 1994 *Replicate One* Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	*Date* May 13, 1994 *Replicate Two* Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	*Averages* Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	Interval	Density of Mat'l. in Interval %/mm Rep. One	Density of Mat'l. in Interval %/mm Rep. Two
			>125	0.0	0.0	0.0	0.0	0.0	0.0	>125	0.000	0.000
Vol. (mL)	***250***	45	125 to 80	0.0	0.0	0.0	0.0	0.0	0.0	125 to 80	0.000	0.000
Shape	***cyl***	30	80 to 50	8.2	8.2	7.4	7.4	7.8	7.8	80 to 50	0.273	0.247
Geometry	***encl***	25	50 to 25	39.5	47.7	34.7	42.1	37.1	44.9	50 to 25	1.580	1.388
Material	***glass***	10	25 to 15	29.2	76.9	32.6	74.7	30.9	75.8	25 to 15	2.920	3.260
		5	15 to 10	11.8	88.7	12.6	87.3	12.2	88.0	15 to 10	2.360	2.520
		4	10 to 6	6.2	94.9	6.8	94.1	6.5	94.5	10 to 6	1.550	1.700
		6	6 to 0	5.1	100.0	5.8	99.9	5.5	100.0	6 to 0	0.850	0.967

Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)
-0.05	-0.40	-0.05	-0.35	-0.05	-0.38

Largest Densities:
2.920 3.260
Average of Densities:
3.090
Standard Deviation:
0.240

Comments

SCREENING DATA SET
DATA ANALYSIS

Container Type Tested		Size Range (mm)	Interval Size (mm)	*Date* May 11, 1994 *Replicate One* Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	*Date* May 13, 1994 *Replicate Two* Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	*Averages* Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	Interval	Density of Mat'l. in Interval %/mm Rep. One	Density of Mat'l. in Interval %/mm Rep. Two
			>125	0.0	0.0	0.0	0.0	0.0	0.0	>125	0.000	0.000
Vol. (mL)	***1000***	45	125 to 80	0.5	0.5	2.0	2.0	1.3	1.3	125 to 80	0.011	0.044
Shape	***cyl***	30	80 to 50	13.5	14.0	10.6	12.6	12.1	13.3	80 to 50	0.450	0.353
Geometry	***encl***	25	50 to 25	41.1	55.1	44.9	57.5	43.0	56.3	50 to 25	1.644	1.796
Material	***glass***	10	25 to 15	27.1	82.2	25.3	82.8	26.2	82.5	25 to 15	2.710	2.530
		5	15 to 10	9.4	91.6	9.6	92.4	9.5	92.0	15 to 10	1.880	1.920
		4	10 to 6	4.7	96.3	4.0	96.4	4.4	96.4	10 to 6	1.175	1.000
		6	6 to 0	3.6	99.9	3.5	99.9	3.6	99.9	6 to 0	0.600	0.583

Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)
0.00	-0.30	0.00	-0.20	0.00	-0.25

Largest Densities:
2.710 2.530
Average of Densities:
2.620
Standard Deviation:
0.127

Comments For both replicates, observed some breakage from 80mm size fraction downwards after mechanical screening for approx. 1 second.

SCREENING DATA SET
DATA ANALYSIS

Date 15-Apr-94 (*Replicate One*)
Date 19-Apr-94 (*Replicate Two*)

Container Type Tested

Vol. (mL) ***250***
Shape ***rect***
Geometry ***open***
Material ***glass***

		Replicate One		*Replicate Two*		*Averages*	
Size Range (mm)	Interval Size (mm)	Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	Percent Mat'l. in Interval	Cum. % Mat'l. in Interval
	>125	0.0	0.0	0.0	0.0	0.0	0.0
45	125 to 80	0.0	0.0	0.0	0.0	0.0	0.0
30	80 to 50	6.2	6.2	4.5	4.5	5.4	5.4
25	50 to 25	52.1	58.3	48.2	52.7	50.2	55.5
10	25 to 15	23.2	81.5	26.6	79.3	24.9	80.4
5	15 to 10	9.8	91.3	11.6	90.9	10.7	91.1
4	10 to 6	4.6	95.9	5.0	95.9	4.8	95.9
6	6 to 0	4.1	100.0	4.0	99.9	4.1	100.0

Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)
-0.10	-0.20	0.00	-0.20	-0.05	-0.20

Interval	Density of Mat'l. in Interval %/mm Rep. One	Density of Mat'l. in Interval %/mm Rep. Two
>125	0.000	0.000
125 to 80	0.000	0.000
80 to 50	0.207	0.150
50 to 25	2.084	1.928
25 to 15	2.320	2.660
15 to 10	1.960	2.320
10 to 6	1.150	1.250
6 to 0	0.683	0.667

Largest Densities:
2.320 2.660
Average of Densities:
2.490
Standard Deviation:
0.240

Comments

SCREENING DATA SET
DATA ANALYSIS

Date 15-Apr-94 (*Replicate One*)
Date 19-Apr-94 (*Replicate Two*)

Container Type Tested

Vol. (mL) ***1000***
Shape ***rect***
Geometry ***open***
Material ***glass***

		Replicate One		*Replicate Two*		*Averages*	
Size Range (mm)	Interval Size (mm)	Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	Percent Mat'l. in Interval	Cum. % Mat'l. in Interval
	>125	0.0	0.0	0.0	0.0	0.0	0.0
45	125 to 80	0.0	0.0	2.1	2.1	1.1	1.1
30	80 to 50	5.4	5.4	5.2	7.3	5.3	6.4
25	50 to 25	48.0	53.4	48.2	55.5	48.1	54.5
10	25 to 15	27.9	81.3	26.7	82.2	27.3	81.8
5	15 to 10	9.8	91.1	9.9	92.1	9.9	91.6
4	10 to 6	4.9	96.0	4.2	96.3	4.6	96.2
6	6 to 0	3.9	99.9	3.7	100.0	3.8	100.0

Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)
-0.05	-0.10	0.00	0.35	-0.03	-0.23

Interval	Density of Mat'l. in Interval %/mm Rep. One	Density of Mat'l. in Interval %/mm Rep. Two
>125	0.000	0.000
125 to 80	0.000	0.047
80 to 50	0.180	0.173
50 to 25	1.920	1.928
25 to 15	2.790	2.670
15 to 10	1.960	1.980
10 to 6	1.225	1.050
6 to 0	0.650	0.617

Largest Densities:
2.790 2.670
Average of Densities:
2.730
Standard Deviation:
0.085

Comments

SCREENING DATA SET
DATA ANALYSIS

Date 15-Apr-94 (*Replicate One*)
Date 19-Apr-94 (*Replicate Two*)

Container Type Tested

Vol. (mL) ***250***
Shape ***rect***
Geometry ***encl***
Material ***glass***

		Replicate One		*Replicate Two*		*Averages*	
Size Range (mm)	Interval Size (mm)	Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	Percent Mat'l. in Interval	Cum. % Mat'l. in Interval	Percent Mat'l. in Interval	Cum. % Mat'l. in Interval
	>125	0.0	0.0	0.0	0.0	0.0	0.0
45	125 to 80	0.0	0.0	0.0	0.0	0.0	0.0
30	80 to 50	6.5	6.5	8.6	8.6	7.6	7.6
25	50 to 25	37.5	44.0	38.9	47.5	38.2	45.8
10	25 to 15	29.5	73.5	27.3	74.8	28.4	74.2
5	15 to 10	13.0	86.5	12.6	87.4	12.8	87.0
4	10 to 6	7.5	94.0	7.1	94.5	7.3	94.3
6	6 to 0	6.0	100.0	5.6	100.1	5.8	100.1

Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)	Diff. Due to Shred (kg)	Diff. Due to Screen (kg)
0.00	-0.10	-0.05	-0.15	-0.03	-0.13

Interval	Density of Mat'l. in Interval %/mm Rep. One	Density of Mat'l. in Interval %/mm Rep. Two
>125	0.000	0.000
125 to 80	0.000	0.000
80 to 50	0.217	0.287
50 to 25	1.500	1.556
25 to 15	2.950	2.730
15 to 10	2.600	2.520
10 to 6	1.875	1.775
6 to 0	1.000	0.933

Largest Densities:
2.950 2.730
Average of Densities:
2.840
Standard Deviation:
0.156

Comments

SCREENING DATA SET
DATA ANALYSIS

				Date March 21, 1994 ***Replicate One***		***Date*** 15-Apr-94 ***Replicate Two***		***Averages***			**Density of Mat'l. in Interval %/mm**	**ensity o Mat'l. in Interval %/mm**
Container Type Tested		**Size Range (mm)**	**Interval Size (mm)**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Interval**	**Rep. One**	**Rep. Two**
			>125	0.0	0.0	0.0	0.0	0.0	0.0	**>125**	0.000	0.000
Vol. (mL)	***250***	**45**	**125 to 80**	0.0	0.0	0.0	0.0	0.0	0.0	**125 to 80**	0.000	0.000
Shape	***cyl.***	**30**	**80 to 50**	6.1	6.1	4.0	4.0	5.1	5.1	**80 to 50**	0.203	0.133
Geometry	***open***	**25**	**50 to 25**	43.1	49.2	40.4	44.4	41.8	46.8	**50 to 25**	1.724	1.616
Material	***glass***	**10**	**25 to 15**	28.4	77.6	32.8	77.2	30.6	77.4	**25 to 15**	2.840	3.280
		5	**15 to 10**	12.2	89.8	11.6	88.8	11.9	89.3	**15 to 10**	2.440	2.320
		4	**10 to 6**	5.6	95.4	6.1	94.9	5.9	95.2	**10 to 6**	1.400	1.525
		6	**6 to 0**	4.6	100.0	5.1	100.0	4.9	100.0	**6 to 0**	0.767	0.850

Diff. Due to Shred (kg)	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**
0.05	-0.15	-0.10	-0.10	-0.03	-0.13

Largest Densities:
2.840 3.280
Average of Densities:
3.060
Standard Deviation:
0.311

Comments

SCREENING DATA SET
DATA ANALYSIS

				Date March 21, 1994 ***Replicate One***		***Date*** 1-Apr-94 ***Replicate Two***		***Averages***			**Density of Mat'l. in Interval %/mm**	**ensity of Mat'l. in Interval %/mm**
Container Type Tested		**Size Range (mm)**	**Interval Size (mm)**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Interval**	**Rep. One**	**Rep. Two**
			>125	0.0	0.0	0.0	0.0	0.0	0.0	**>125**	0.000	0.000
Vol. (mL)	***1000***	**45**	**125 to 80**	0.0	0.0	0.5	0.5	0.3	0.3	**125 to 80**	0.000	0.011
Shape	***cyl.***	**30**	**80 to 50**	3.0	3.0	8.5	9.0	5.8	6.0	**80 to 50**	0.100	0.283
Geometry	***open***	**25**	**50 to 25**	30.5	33.5	54.5	63.5	42.5	48.5	**50 to 25**	1.220	2.180
Material	***glass***	**10**	**25 to 15**	39.6	73.1	23.3	86.8	31.5	80.0	**25 to 15**	3.960	2.330
		5	**15 to 10**	14.2	87.3	7.4	94.2	10.8	90.8	**15 to 10**	2.840	1.480
		4	**10 to 6**	6.6	93.9	3.2	97.4	4.9	95.7	**10 to 6**	1.650	0.800
		6	**6 to 0**	6.1	100.0	2.6	100.0	4.4	100.0	**6 to 0**	1.017	0.433

Diff. Due to Shred (kg)	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**
0.00	-0.10	-0.05	-0.45	-0.03	-0.28

Largest Densities:
3.960 2.330
Average of Densities:
3.145
Standard Deviation:
1.153

Comments Replicate One was screened for 15 seconds. All other glass tests were screened for only 1 to 2 seconds. The violent mechanical screening actually caused additional glass breakage.
Replicate One data was not used.

SCREENING DATA SET
DATA ANALYSIS

				Date 15-Apr-94 ***Replicate One***		***Date*** 27-Apr-94 ***Replicate Two***		***Averages***			**Density of Mat'l. in Interval %/mm**	**ensity of Mat'l. in Interval %/mm**
Container Type Tested		**Size Range (mm)**	**Interval Size (mm)**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Interval**	**Rep. One**	**Rep. Two**
			>125	0.0	0.0	0.0	0.0	0.0	0.0	**>125**	0.000	0.000
Vol. (mL)	***1000***	**45**	**125 to 80**	1.0	1.0	5.3	5.3	3.2	3.2	**125 to 80**	0.022	0.118
Shape	***rect***	**30**	**80 to 50**	8.5	9.5	10.5	15.8	9.5	12.7	**80 to 50**	0.283	0.350
Geometry	***encl***	**25**	**50 to 25**	47.2	56.7	38.4	54.2	42.8	55.5	**50 to 25**	1.888	1.536
Material	***glass***	**10**	**25 to 15**	25.6	82.3	26.3	80.5	26.0	81.4	**25 to 15**	2.560	2.630
		5	**15 to 10**	9.0	91.3	10.0	90.5	9.5	90.9	**15 to 10**	1.800	2.000
		4	**10 to 6**	4.0	95.3	4.7	95.2	4.4	95.3	**10 to 6**	1.000	1.175
		6	**6 to 0**	4.5	99.8	4.7	99.9	4.6	99.9	**6 to 0**	0.750	0.783

Diff. Due to Shred (kg)	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**
-0.05	-0.25	0.05	0.25	0.00	0.00

Largest Densities:
2.560 2.630
Average of Densities:
2.595
Standard Deviation:
0.049

Comments Screened for approx. 1 second. Observed some minor breakage from 80mm size down.
Replicate Two had some "slightly different" 1000ml glass jars.
Many lids relatively undamaged from shredding operation.

A1.3 EXPANDED POLYSTYRENE PRELIMINARY TEST DATA

The data from the preliminary expanded polystyrene container tests are shown as the percentage of material found in each screen interval, as well as the cumulative percentages. The shredded material interval densities are also presented.

SCREENING DATA SET
DATA ANALYSIS

Container Type Tested

Vol. (mL)	***250***
Shape	***cyl.***
Geometry	***open***
Material	***EPS***

		Date June 8, 1994 ***Replicate One***		***Date*** June 8, 1994 ***Replicate Two***		***Averages***				
Screen Sizes (mm)	**Interval Size (mm)**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Range (mm)**	**Density of Mat'l. in Interval %/mm Rep. One**	**Density of Mat'l. in Interval %/mm Rep. Two**
125	**>125**	0.0	0.0	0.0	0.0	0.0	0.0		0.000	0.000
80	**125 to 80**	5.4	5.4	7.7	7.7	6.6	6.6	**45**	0.120	0.171
50	**80 to 50**	45.9	51.3	43.6	51.3	44.8	51.3	**30**	1.530	1.453
25	**50 to 25**	37.8	89.1	35.9	87.2	36.9	88.2	**25**	1.512	1.436
15	**25 to 15**	10.8	99.9	12.8	100.0	11.8	100.0	**10**	1.080	1.280
10	**15 to 10**	0.0	99.9	0.0	100.0	0.0	100.0	**5**	0.000	0.000
6	**10 to 6**	0.0	99.9	0.0	100.0	0.0	100.0	**4**	0.000	0.000
0	**6 to 0**	0.0	99.9	0.0	100.0	0.0	100.0	**6**	0.000	0.000

Diff. Due to Shred (kg)	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**
-0.15	-0.25	0.05	-0.25	-0.05	-0.25

Largest Densities:
1.530 1.453
Average of Densities:
1.492
Standard Deviation:
0.054

Comments EPS cups came in "stacks" and were individually pulled apart before shredding. However, during random placing of cups into box for pouring into shredder, some cups would randomly orient themselves and 2 to 5 cups would "self-stack".
Some stacked cups were pulled apart by hand, although most remained "as is".

SCREENING DATA SET
DATA ANALYSIS

Container Type Tested

Vol. (mL)	***1000***
Shape	***cyl.***
Geometry	***open***
Material	***EPS***

		Date June 8, 1994 ***Replicate One***		***Date*** June 8, 1994 ***Replicate Two***		***Averages***				
Screen Sizes (mm)	**Interval Size (mm)**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Range (mm)**	**Density of Mat'l. in Interval %/mm Rep. One**	**Density of Mat'l. in Interval %/mm Rep. Two**
125	**>125**	0.0	0.0	0.0	0.0	0.0	0.0		0.000	0.000
80	**125 to 80**	4.1	4.1	4.2	4.2	4.2	4.2	**45**	0.091	0.093
50	**80 to 50**	42.9	47.0	45.8	50.0	44.4	48.5	**30**	1.430	1.527
25	**50 to 25**	42.9	89.9	41.7	91.7	42.3	90.8	**25**	1.716	1.668
15	**25 to 15**	10.2	100.1	8.3	100.0	9.3	100.1	**10**	1.020	0.830
10	**15 to 10**	0.0	100.1	0.0	100.0	0.0	100.1	**5**	0.000	0.000
6	**10 to 6**	0.0	100.1	0.0	100.0	0.0	100.1	**4**	0.000	0.000
0	**6 to 0**	0.0	100.1	0.0	100.0	0.0	100.1	**6**	0.000	0.000

Diff. Due to Shred (kg)	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**
0.00	-0.30	-0.05	-0.35	-0.03	-0.33

Largest Densities:
1.716 1.668
Average of Densities:
1.692
Standard Deviation:
0.034

Comments Same "self-stacking" note as with 250mL, cyl, open, EPS cups, although not as prevalent.

A1.4 MATERIALS COMPARISON TEST DATA AND ANALYSIS

This section contains the data and calculations for determining the density of the actual container material. Also included are the size distribution data used to perform the materials comparison. The data from the preliminary glass and expanded polystyrene data has been repeated.

CONTAINER MATERIALS COMPARISON TESTS

DENSITY OF CONTAINER MATERIALS

Tetra-Pak Containers: Approx. 1000 mL, Rect., Open, Mat'l Test

Sample	*Mass (g)*	*Ht.(mm)*	*Wd. (mm)*	*Lg. (mm)*	*Ht. Water (mm)*	*Vol. (mL)*	*Glass Rod (mL)*	*Net Vol. (mL)*	*Density (g/mL)*
1	30.7345	180	83	82	2.3	44.23	0.12	44.12	0.697
2	29.9307	180	83	81	1.9	36.54	0.10	36.44	0.821
3	30.3576	180	83	82	2.0	38.46	0.10	38.36	0.791
4	30.3073	178	83	82	2.3	44.23	0.12	44.12	0.687
Mean Value	30.3325	179.5	83	81.75	2.13	40.87	0.11	40.76	**0.749**
Std. Dev.	0.3288	1	0	0.5	0.21	3.96	0.01	3.95	0.067
Std. Error of Mean	0.1644	0.5	0	0.25	0.10	1.98	0.01	1.98	0.034

13 mm of vertically displaced water equals 250 mL in a 4 L beaker.

Glass rod used to place containers in beaker has volume equal to water height x 8 mm diameter.

Glass Jars: 1000 mL, Rect., Open

Sample	*Mass (g)*	*Ht.(mm)*	*Wd. (mm)*	*Lg. (mm)*	*Ht. Water (mm)*	*Vol. (mL)*	*Glass Rod (mL)*	*Net Vol. (mL)*	*Density (g/mL)*
1	474.4				9.9	190.38		190.38	2.492
2	475				10.0	192.31		192.31	2.470
3	473.8				9.8	188.46		188.46	2.514
Mean Value	474.40				9.90	190.38		190.38	**2.492**
Std. Dev.	0.60				0.10	1.92		1.92	0.022
Std. Error of Mean	0.35				0.06	1.11		1.11	0.013

13 mm of vertically displaced water equals 250 mL in a 4 L beaker. No glass rod used.

EPS Containers: 1000 mL, Cyl., Open

Sample	*Mass (g)*	*Ht.(mm)*	*Wd. (mm)*	*Lg. (mm)*	*Ht. Water (mm)*	*Vol. (mL)*	*Glass Rod (mL)*	*Net Vol. (mL)*	*Density (g/mL)*
1	9.1506				8.9	171.15	2.76	168.39	0.0543
2	9.1504				8.6	165.38	2.76	162.62	0.0563
3	8.9685				8.8	169.23	2.76	166.47	0.0539
4	9.0472				9.0	173.08	2.76	170.31	0.0531
Mean Value	9.079175				8.83	169.71	2.76	166.95	**0.0544**
Std. Dev.	0.088404				0.17	3.28	0.00	3.28	0.0013
Std. Error of Mean	0.044202				0.09	1.64	0.00	1.64	0.0007

13 mm of vertically displaced water equals 250 mL in a 4 L beaker.

Glass rod used to place containers in beaker has volume equal to 55 mm x 8 mm diameter.

CONTAINER MATERIALS COMPARISON TESTS

COMPARISON OF SHREDDED MATERIAL INTERVAL DENSITY VERSUS CONTAINER MATERIAL DENSITY

Tetra-Pak Containers

SCREENING DATA SET — ***Date***

DATA ANALYSIS — May 31, 1994

Container Type Tested

			Replicate One			
		Interval Size (mm)	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Range (mm)**	**Density of Mat'l in Interval %/mm**
		394>125	6.6	6.6	**269**	0.025
Vol. (mL)	***1000***	**125 to 80**	53.8	60.4	**45**	1.196
Shape	***rect***	**80 to 50**	29.7	90.1	**30**	0.990
Geometry	***open***	**50 to 25**	9.9	100.0	**25**	0.396
Material	***T-Pak***	**25 to 15**	0.0	100.0	**10**	0.000
MATERIALS TEST RUN		**15 to 10**	0.0	100.0	5	0.000
		10 to 6	0.0	100.0	**4**	0.000
		6 to 0	0.0	100.0	**6**	0.000
		Number of Screening Loads	1			

Diff. Due to Shred (kg)	**Diff. Due to Screen (kg)**
-0.20	-0.25

Largest Density: **1.196**

EXCEL GENERATED DESCRIPTIVE STATISTICS

Using *NON-ZERO* Values of Density of Mat'l Distribution and Assuming Uni-modal Distribution

Statistical Parameter	
Mean	0.6515
Standard Error	0.2691
Median	0.6930
Mode	#N/A
Standard Deviation	0.5382
Sample Variance	0.2896
Kurtosis	-3.1886
Skewness	-0.2710
Range	1.1710
Minimum	0.0245
Maximum	1.1956
Sum	2.6061
Count	4.0000
Confidence Level(95.000%)	0.5274

DENSITY of CONTAINER MATERIAL (g/mL)

Mean Value	**0.749**
Std. Dev.	0.067
Std. Error of Mean	0.034

Comments This materials test was a "half-batch" run to confirm behaviour and was screened in one load.

Glass Jars

SCREENING DATA SET

DATA ANALYSIS

				Date 15-Apr-94		*Date* 19-Apr-94						
				Replicate One		***Replicate Two***		***Averages***				
		Size Range (mm)	**Interval Size (mm)**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Interval**	**Density of Mat'l. in Interval %/mm Rep. One**	**Density of Mat'l. in Interval %/mm Rep. Two**
Container Type Tested												
			>125	0.0	0.0	0.0	0.0	0.0	0.0	**>125**	0.000	0.000
Vol. (ml.)	***1000***	**45**	**125 to 80**	0.0	0.0	2.1	2.1	1.1	1.1	**125 to 80**	0.000	0.047
Shape	***rect***	**30**	**80 to 50**	5.4	5.4	5.2	7.3	5.3	6.4	**80 to 50**	0.180	0.173
Geometry	***open***	**25**	**50 to 25**	48.0	53.4	48.2	55.5	48.1	54.5	**50 to 25**	1.920	1.928
Material	***glass***	**10**	**25 to 15**	27.9	81.3	26.7	82.2	27.3	81.8	**25 to 15**	2.790	2.670
		5	**15 to 10**	9.8	91.1	9.9	92.1	9.9	91.6	**15 to 10**	1.960	1.960
		4	**10 to 6**	4.9	96.0	4.2	96.3	4.6	96.2	**10 to 6**	1.225	1.050
		6	**6 to 0**	3.9	99.9	3.7	100.0	3.8	100.0	**6 to 0**	0.650	0.617

Diff. Due to Shred (kg)	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**
-0.05	-0.10	0.00	-0.35	-0.03	-0.23

Largest Densities:
2.790 2.670
Average of Densities:
2.730
Standard Deviation:
0.085

DENSITY of CONTAINER MATERIAL (g/mL)

Mean Value	**2.492**
Std. Dev.	0.022
Std. Error of Mean	0.013

EPS Containers

SCREENING DATA SET

DATA ANALYSIS

				Date June 8, 1994 *Replicate One*		*Date* June 8, 1994 *Replicate Two*		*Averages*				
Container Type Tested		**Screen Sizes (mm)**	**Interval Size (mm)**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Percent Mat'l. in Interval**	**Cum. % Mat'l. in Interval**	**Range (mm)**	**Density of Mat'l. in Interval %/mm Rep. One**	**Density of Mat'l. in Interval %/mm Rep. Two**
		125	**>125**	0.0	0.0	0.0	0.0	0.0	0.0		0.000	0.000
Vol. (mL)	***1000***	**80**	**125 to 80**	4.1	4.1	4.2	4.2	4.2	4.2	**45**	0.091	0.093
Shape	***cyl.***	**50**	**80 to 50**	42.9	47.0	45.8	50.0	44.4	48.5	**30**	1.430	1.527
Geometry	***open***	**25**	**50 to 25**	42.9	89.9	41.7	91.7	42.3	90.8	**25**	1.716	1.668
Material	***EPS***	**15**	**25 to 15**	10.2	100.1	8.3	100.0	9.3	100.1	**10**	1.020	0.830
		10	**15 to 10**	0.0	100.1	0.0	100.0	0.0	100.1	**5**	0.000	0.000
		6	**10 to 6**	0.0	100.1	0.0	100.0	0.0	100.1	**4**	0.000	0.000
		0	**6 to 0**	0.0	100.1	0.0	100.0	0.0	100.1	**6**	0.000	0.000

Diff. Due to Shred (kg)	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**	**Diff. Due to Shred (kg)**	**Diff. Due to Screen (kg)**
0.00	-0.30	-0.05	-0.35	-0.03	-0.33

Largest Densities:
1.716 1.668
Average of Densities:
1.692
Standard Deviation:
0.034

DENSITY of CONTAINER MATERIAL (g/mL)

Mean Value	**0.0644**
Std. Dev.	0.0013
Std. Error of Mean	0.0007

Comments Same "self-stacking" note as with 250mL., cyl, open, EPS cups, although not as prevalent.

QUALITATIVE RELATIONSHIP BETWEEN MATERIAL TYPE AND MODAL INTERVAL DENSITY

While there does not appear to be a direct relationship between the material density and the modal interval density, there does appear to be a relationship between the ***relative ductility or brittleness*** of a material and the resulting modal interval density.

Since it was determined that geometry appears to make no difference, shape was marginally significant, and that the size factor was the most significant for the Tetra-Paks, include the 1000 mL, rectangular, enclosed, Tetra-Pak data with the data specifically designed for the materials test. This argument is also based on the previous discussion concerning accepting the materials test Tetra-Pak data.

Matl. Description	*Modal Interval Densities*			*Average*	*Std.Dev.*
Tetra-Pak: Ductile	1.196	0.996	1.247	1.146	0.133
EPS: Brittle		1.716	1.668	1.692	0.034
Glass: Very Brittle		2.790	2.670	2.730	0.085

Anova: Materials Test

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Row 1	3	3.439	1.146333	0.0176
Row 2	2	3.384	1.692	0.001152
Row 3	2	5.46	2.73	0.0072

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	3.020613	2	1.510306	138.7108	0.000202	6.944276
Within Groups	0.043553	4	0.010888			
Total	3.064165	6		*Groups differ significantly.*		

A1.5 ORIENTATION TESTS ON TETRA-PAKS DATA AND ANALYSIS

This section contains the data and calculations used to determine if the orientation of the Tetra-Pak container just prior to shredding affected the shredding behaviour of the containers.

ORIENTATION TESTS on TETRA-PAKS

SET ONE

Box Colour	*Before Mass (g)*	*Orientation before Shredding*	*Major Break After Shredding*	*Screened Fractions (g)* 125 mm	80 mm	50 mm	25 mm	*Total (g)*	*Mass Lost (g)*
Red	30.985	**Corner, end**	2 halves; 1 corner intact	0.000	25.628	2.526	2.150	30.303	-0.682
Green	33.236	**Broad, side**	Cross-cut; tore ends off	0.000	10.370	15.152	0.565	26.087	-7.149
Blue	32.133	**Corner, end**	Random, uniform pieces	0.000	15.356	16.544	0.000	31.900	-0.234
Yellow	33.232	**Narrow, side**	Cross-cut; tore 1 end	0.000	32.818	0.000	0.000	32.818	-0.414
Orange	32.512	**Narrow, end**	Random, uniform pieces	0.000	4.088	15.191	3.891	23.170	-9.342
Black	29.348	**Broad, end**	1 big piece; 1 long piece	0.000	7.365	16.953	4.812	29.131	-0.218

Largest piece was 430 mm long.

SET TWO

Box Colour	*Before Mass (g)*	*Orientation before Shredding*	*Major Break After Shredding*	*Screened Fractions (g)* 125 mm	80 mm	50 mm	25 mm	*Total (g)*	*Mass Lost (g)*
Green	32.849	**Corner, end**	Cross-cut at angle; 2 ends	0.000	26.759	5.779	0.000	32.538	-0.311
Orange	31.950	**Broad, side**	1 end; 2 corners	0.000	26.255	4.241	1.093	31.589	-0.361
Yellow	32.853	**Corner, end**	1 end; 1 cross piece; random	0.000	7.631	23.428	0.585	31.644	-1.209
Red	30.844	**Narrow, side**	1 long cross piece; random	0.000	17.181	12.797	0.000	29.978	-0.866
Blue	32.611	**Narrow, end**	2 big corner pieces; diagonal	14.418	11.580	6.059	0.000	32.057	-0.555
Black	32.072	**Broad, end**	1 end; 1 strip	0.000	31.908	0.000	0.000	31.908	-0.164

Largest piece was 315 mm long.

SET THREE

Box Colour	*Before Mass (g)*	*Orientation before Shredding*	*Major Break After Shredding*	*Screened Fractions (g)* 125 mm	80 mm	50 mm	25 mm	*Total (g)*	*Mass Lost (g)*
Red	32.037	**Corner, end**	Diag. cut, rand., unif. chunks	0.000	6.010	24.793	0.000	30.803	-1.234
Blue	32.059	**Broad, side**	Torn, long end; mid cross-cut	0.000	21.769	0.000	0.199	21.968	-10.091
Green	32.760	**Corner, end**	2 ends; big pieces, cross-cut	0.000	32.582	0.000	0.000	32.582	-0.178
Orange	30.286	**Narrow, side**	3 pieces;mid cross-cut, 2 ends	0.000	19.561	9.708	0.000	29.270	-1.016
Black	32.001	**Narrow, end**	Uniform pieces, 2 ends	0.000	14.196	11.941	0.000	26.138	-5.864
Yellow	32.693	**Broad, end**	Random, same size cuts	0.000	22.166	0.000	0.000	22.166	-10.527

Largest piece was 300 mm long.

ANALYSIS of ORIENTATION TESTS on TETRA-PAKS

SET ONE

Orientation	Screened Fractions (g)				Total	Percent in Screen Fractions				Density (%/mm) in Screen Ranges (mm)			
before Shredding	125 mm	80 mm	50 mm	25 mm	(g)	125 mm	80 mm	50 mm	25 mm	305	45	30	25
Corner, end	0.000	25.628	2.526	2.150	30.303	0.000	84.570	8.336	7.094	0.000	1.879	0.278	0.284
Broad, side	0.000	10.370	15.152	0.565	26.087	0.000	39.751	58.083	2.166	0.000	0.883	1.936	0.087
Corner, end	0.000	15.356	16.544	0.000	31.900	0.000	48.139	51.861	0.000	0.000	1.070	1.729	0.000
Narrow, side	0.000	32.818	0.000	0.000	32.818	0.000	100.000	0.000	0.000	0.000	2.222	0.000	0.000
Narrow, end	0.000	4.088	15.191	3.891	23.170	0.000	17.644	65.564	16.792	0.000	0.392	2.185	0.672
Broad, end	0.000	7.365	16.953	4.812	29.131	0.000	25.283	58.198	16.519	0.000	0.562	1.940	0.661

Largest piece was 430 mm long.

SET TWO

Orientation	Screened Fractions (g)				Total	Percent in Screen Fractions				Density (%/mm) in Screen Ranges (mm)			
before Shredding	125 mm	80 mm	50 mm	25 mm	(g)	125 mm	80 mm	50 mm	25 mm	190	45	30	25
Corner, end	0.000	26.759	5.779	0.000	32.538	0.000	82.238	17.762	0.000	0.000	1.828	0.592	0.000
Broad, side	0.000	26.255	4.241	1.093	31.589	0.000	83.115	13.425	3.460	0.000	1.847	0.447	0.138
Corner, end	0.000	7.631	23.428	0.585	31.644	0.000	24.115	74.035	1.850	0.000	0.536	2.468	0.074
Narrow, side	0.000	17.181	12.797	0.000	29.978	0.000	57.313	42.687	0.000	0.000	1.274	1.423	0.000
Narrow, end	14.418	11.580	6.059	0.000	32.057	44.978	36.122	18.900	0.000	0.237	0.803	0.630	0.000
Broad, end	0.000	31.908	0.000	0.000	31.908	0.000	100.000	0.000	0.000	0.000	2.222	0.000	0.000

Largest piece was 315 mm long.

SET THREE

Orientation	Screened Fractions (g)				Total	Percent in Screen Fractions				Density (%/mm) in Screen Ranges (mm)			
before Shredding	125 mm	80 mm	50 mm	25 mm	(g)	125 mm	80 mm	50 mm	25 mm	175	45	30	25
Corner, end	0.000	6.010	24.793	0.000	30.803	0.000	19.511	80.489	0.000	0.000	0.434	2.683	0.000
Broad, side	0.000	21.769	0.000	0.199	21.968	0.000	99.094	0.000	0.906	0.000	2.202	0.000	0.036
Corner, end	0.000	32.582	0.000	0.000	32.582	0.000	100.000	0.000	0.000	0.000	2.222	0.000	0.000
Narrow, side	0.000	19.561	9.708	0.000	29.270	0.000	66.832	33.168	0.000	0.000	1.485	1.106	0.000
Narrow, end	0.000	14.196	11.941	0.000	26.138	0.000	54.314	45.686	0.000	0.000	1.207	1.523	0.000
Broad, end	0.000	22.166	0.000	0.000	22.166	0.000	100.000	0.000	0.000	0.000	2.222	0.000	0.000

Largest piece was 300 mm long.

ANALYSIS of ORIENTATION TESTS on TETRA-PAKS

Treating as All Sets Combined

Orientation	*Mean Densities (%/mm)*				*Standard Deviations of Densities*				*Standard Error of the Means*			
before Shredding	***125 mm***	***80 mm***	***50 mm***	***25 mm***	***125 mm***	***80 mm***	***50 mm***	***25 mm***	***125 mm***	***80 mm***	***50 mm***	***25 mm***
Corner, end	0.000	1.380	1.184	0.095	0.000	0.820	1.307	0.164	0.000	0.474	0.755	0.095
Broad, side	0.000	1.644	0.795	0.087	0.000	0.682	1.014	0.051	0.000	0.394	0.585	0.029
Corner, end	0.000	1.276	1.399	0.025	0.000	0.862	1.267	0.043	0.000	0.498	0.731	0.025
Narrow, side	0.000	1.660	0.843	0.000	0.000	0.498	0.747	0.000	0.000	0.288	0.431	0.000
Narrow, end	0.079	0.801	1.446	0.224	0.137	0.407	0.781	0.388	0.079	0.235	0.451	0.224
Broad, end	0.000	1.669	0.647	0.220	0.000	0.959	1.120	0.381	0.000	0.553	0.647	0.220

Treating as Replicated Sets

	Set One		***Set Two***		***Set Three***	
Orientation before Shredding	***Largest Density***	***... in Size Range***	***Largest Density***	***... in Size Range***	***Largest Density***	***... in Size Range***
Corner, end	1.879	*125 to 80*	1.828	*125 to 80*	2.683	*80 to 50*
Broad, side	1.936	*80 to 50*	1.847	*125 to 80*	2.202	*125 to 80*
Corner, end	1.729	*80 to 50*	2.468	*80 to 50*	2.222	*125 to 80*
Narrow, side	2.222	*125 to 80*	1.423	*80 to 50*	1.485	*125 to 80*
Narrow, end	2.185	*80 to 50*	0.803	*125 to 80*	1.523	*80 to 50*
Broad, end	1.940	*80 to 50*	2.222	*125 to 80*	2.222	*125 to 80*

No consistent, recurring size range for the largest density based on these sets.

For ANOVA

Largest Density		
Set One	***Set Two***	***Set Three***
1.879	1.828	2.683
1.936	1.847	2.202
1.729	2.468	2.222
2.222	1.423	1.485
2.185	0.803	1.523
1.940	2.222	2.222

Anova: Single Factor with Sets as Replicates

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Row 1	3	6.3898	2.1299	0.2300
Row 2	3	5.9852	1.9951	0.0341
Row 3	3	6.4188	2.1396	0.1417
Row 4	3	5.1303	1.7101	0.1977
Row 5	3	4.5110	1.5037	0.4783
Row 6	3	6.3844	2.1281	0.0266

ANOVA

Source Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	1.0721	5	0.2144	1.1607	0.3826	3.1059
Within Groups	2.2168	12	0.1847			
Total	3.2889	17				

F < F crit

No real diff. between orientations.

The highest material densities occur in the 125 mm to 50 mm size range.

A1.6 COMMINGLED RECYCLABLES/TETRA-PAK TESTS DATA AND ANALYSIS

This section contains the data and calculations used to determine if 250 mL, rectangular, enclosed Tetra-Paks separated out from a mixture of commingled recyclables. It was observed during the preliminary tests that the smaller sized Tetra-Paks produced higher concentrations of shredded materials. This test would evaluate if the effect of size would similarly improve the separation of these containers in a commingled situation. It should be noted that in all four separate test trials it was observed a portion of the Tetra-Paks appeared undamaged and unshredded, suffering only minimal deformation.

COMMINGLED RECYCLABLES/TETRA-PAK TESTS

TEST ONE: May 30, 1994

Prior to Shredding and Screening

Recyclable Item	*Mass (kg)*	*Max. Dim. (mm)*	*Min. Dim. (mm)*	*Typ. Dim. (mm)*	*Comments*
Newspaper	5.85	380 x 300		345 x 305	Varying thickness.
Mixed Paper Waste	0.80		265 x 105	280 x 210	Fibrous, flat, glossies, bond.
Card/boxboard	0.80	460 x 180 x180	270 x 120 x 70		More rigid, some flattened.
Glass	0.55	Dia.100 x 155	Dia.60 x 73		Cylindrical, no lids.
Metal	1.00	Dia.154 x 176	Dia.75 x 112	Dia.84 x 115	Typical - "soup can" size.
Plastic	0.35	1100 x 250	180 x 180		Also rigid plastic containers.
Tetra-Pak	1.00				
Total Weight:	**10.35**				

Shredding

Jogged and run 2x after.
Shredder @ 42 psi.

Shred Loss (kg): **-0.10**

Screening

In two, 1-minute loads. Top layer "cleared" for 2nd load for adequate screening.

After Shredding and Screening; Tetra-Pak Categorization

Screen Size Range	*Range (mm)*	*Wt. w/Screen (kg)*	*Weight w/o T-Pak (kg)*	*Weight of T-Pak (kg)*	*Percent of T-Pak*	*Density of T-Pak (%/mm)*	*Comments*
490>125	***365***	25.95	25.80	0.15	17.65	0.0483	Mixed. Whole T-Paks.
125 to 80	***45***	21.00	20.50	0.50	58.82	1.3072	Mixed. Whole T-Paks.
80 to 50	***30***	20.20	N/A	0.20	23.53	0.7843	Mixed. Whole T-Paks.
50 to 25	***25***	16.50	0.00		0.00	0.0000	Mostly newsprint.
25 to 15	***10***	16.30	0.00		0.00	0.0000	Paper and glass.
15 to 10	***5***	16.10	0.00		0.00	0.0000	Paper and glass.
10 to 6	***4***	15.55	0.00		0.00	0.0000	Paper and glass.
6 to 0	***6***	19.95	0.00		0.00	0.0000	Paper and glass.

Total Weight of Tetra-Pak: **0.85**

Unusual that Tetra-Paks were found relatively undamaged from 125 to 50 mm screens
Newspaper can still be found in relatively whole pieces.
The largest pieces in the 125 mm size range were generally (nearly) unshredded newspaper.
The diagonal of 380 mm by 300 mm was taken as the largest dimesion for the size range.

TEST TWO: May 31, 1994

Prior to Shredding and Screening

Recyclable Item	*Mass* (kg)	*Max. Dim.* (mm)	*Min. Dim.* (mm)	*Typ. Dim.* (mm)	*Comments*
Newspaper	5.90	380 x 300		345 x 305	Varying thickness.
Mixed Paper Waste	0.75	570 x 350	170 x 87	280 x 210	Glossies, envelopes, flyers.
Card/boxboard	0.75	790 x 560	Dia.45 x 105	270 x 120 x 60	Boxes, cartons, packaging.
Glass	0.70	Dia.95 x 200	Dia.57 x x73	Dia.80 x 130	Glass jars.
Metal	0.90	Dia.155 x 170	Dia.65 x 36	Dia.75 x 109	Typical - "soup can" size.
Plastic	0.35	Dia.110 x 225	Dia.85 x 19		Containers, trays, lids, bags.
Tetra-Pak	1.00				
Total Weight:	**10.35**				

Shredding

Jogged and run 2x after.
Shredder jammed once.
Shredder @ 42 psi.

Shred Loss (kg): **-0.05**

Screening

In two, 1-minute loads. Mixed top layer for 2nd load for adequate screening.

After Shredding and Screening; Tetra-Pak Categorization

Screen Size Range	*Range* (mm)	*Wt. w/Screen* (kg)	*Weight w/o T-Pak* (kg)	*Weight of T-Pak* (kg)	*Percent of T-Pak*	*Density of T-Pak* (%/mm)	*Comments*
490>125	***365***	27.30	26.95	0.35	46.67	0.1279	
125 to 80	***45***	20.15	19.90	0.25	33.33	0.7407	
80 to 50	***30***	19.95	19.80	0.15	20.00	0.6667	
50 to 25	***25***	15.60	0.00		0.00	0.0000	Bits of paper and glass.
25 to 15	***10***	16.25	0.00		0.00	0.0000	Bits of paper and glass.
15 to 10	***5***	0.00	0.00		0.00	0.0000	Bits of paper and glass.
10 to 6	***4***	0.00	0.00		0.00	0.0000	Bits of paper and glass.
6 to 0	***6***	0.00	0.00		0.00	0.0000	Bits of paper and glass.

Total Weight of Tetra-Pak: **0.75**

Unusual that Tetra-Paks were found relatively undamaged from 125 to 50 mm screens

Newspaper can still be found in relatively whole pieces, esp. in 125 mm screen. Perhaps newsprint hindered screening?

TEST THREE: May 31 to June 1, 1994

Prior to Shredding and Screening

Recyclable Item	*Mass* (kg)	*Max. Dim.* (mm)	*Min. Dim.* (mm)	*Typ. Dim.* (mm)	Comments
Newspaper	5.95	380 x 300		345 x 305	Varying thickness.
Mixed Paper Waste	0.85	275 x 275	265 x 105	280 x 210	Glossies, publication, etc.
Card/boxboard	0.75	440 x 410	300 x 100 x 60		Compressed, partly flattened.
Glass	0.65	Dia.97 x 160	Dia.57 x 73		
Metal	1.00	Dia.155 x 177	Dia.60 x 95	Dia.75 x 112	Some lids still attached.
Plastic	0.35	250 x 150 x 150	90 x 90 x 50		Jugs, pots, tray, bags.
Tetra-Pak	1.00				
Total Weight:	**10.55**				

Shredding

Jogged and run 2x after.
Shredder @ 55 psi.

Shred Loss (kg): **0.05**

Screening

In two, 1-minute loads.
Top layer partly cleared
for 2nd load for
adequate screening.

After Shredding and Screening; Tetra-Pak Categorization

Screen Size Range	*Range* (mm)	*Wt. w/Screen* (kg)	*Weight w/o T-Pak* (kg)	*Weight of T-Pak* (kg)	*Percent of T-Pak*	*Density of T-Pak* (%/mm)	Comments
490>125	***365***	24.45	24.35	0.10	11.76	0.0322	Similar to Test One.
125 to 80	***45***	21.80	21.30	0.50	58.82	1.3072	Similar to Test One.
80 to 50	***30***	21.15	20.90	0.25	29.41	0.9804	Similar to Test One.
50 to 25	***25***	15.80	0.00		0.00	0.0000	Newsprint, glass.
25 to 15	***10***	16.35	0.00		0.00	0.0000	Newsprint, glass.
15 to 10	***5***	16.00	0.00		0.00	0.0000	Newsprint, glass.
10 to 6	***4***	15.50	0.00		0.00	0.0000	Newsprint, glass.
6 to 0	***6***	19.90	0.00		0.00	0.0000	Newsprint, glass.

Total Weight of Tetra-Pak: **0.85**

Not as many undamaged Tetra-Paks but still some relatively intact.
Newspaper shredded more due to increased psi but can still be found in large pieces.

TEST FOUR: June 2, 1994

Prior to Shredding and Screening

Recyclable Item	*Mass (kg)*	*Max. Dim. (mm)*	*Min. Dim. (mm)*	*Typ. Dim. (mm)*	*Comments*
Newspaper	5.85	380 x 300	310 x 170	345 x 305	Varying thickness.
Mixed Paper Waste	0.80			280 x 204	Magazines.
Card/boxboard	0.90	800 x 610	330 x 230 x 60		Some flattened and crushed.
Glass	0.70	133 x 90 x 70	Dia.57 x 73		
Metal	1.00	Dia.157 x 176	Dia.86 x 40	Dia.75 x 112	Typical is "soupcan" size.
Plastic	0.35	660 x 640	250 x 146 x 146		Jugs, bags.
Tetra-Pak	1.00				
Total Weight:	**10.60**				

Shredding

Jogged and run 2x after.
Jammed-jogged 5x.
Shredder @ 55 psi.

Shred Loss (kg): **-0.35**

Screening

In two, 1-minute loads.
Top layer partly cleared
for 2nd load for
adequate screening.

After Shredding and Screening; Tetra-Pak Categorization

Screen Size Range	*Range (mm)*	*Wt. w/Screen (kg)*	*Weight w/o T-Pak (kg)*	*Weight of T-Pak (kg)*	*Percent of T-Pak*	*Density of T-Pak (%/mm)*	*Comments*
490>125	***365***	25.45	25.40	0.10	12.50	0.0342	Similar to Test One.
125 to 80	***45***	20.70	20.30	0.40	50.00	1.1111	Similar to Test One.
80 to 50	***30***	20.80	20.50	0.30	37.50	1.2500	Similar to Test One.
50 to 25	***25***	15.80	0.00		0.00	0.0000	Newsprint, glass.
25 to 15	***10***	16.35	0.00		0.00	0.0000	Newsprint, glass.
15 to 10	***5***	16.10	0.00		0.00	0.0000	Newsprint, glass.
10 to 6	***4***	15.55	0.00		0.00	0.0000	Newsprint, glass.
6 to 0	***6***					0.0000	Newsprint, glass.

Total Weight of Tetra-Pak: **0.80**

Some Tetra-Pak still quite whole in 125, 80 mm screens. Soup can literally untouched in 80 mm screen!
Newspapers, magazines, and bags, some quite intact, dominate 125 mm fraction.

DATA ANAYLYSIS of COMMINGLED RECYCLABLES/TETRA-PAK TESTS

	Range	Tetra-Pak Interval Density(%/mm)						Std.Error
Screen Size Range	**(mm)**	**Test 1**	**Test 2**	**Test 3**	**Test 4**	**Mean**	**Std. Dev.**	**of Mean**
490>125	**365**	0.0483	0.1279	0.0322	0.0342	0.0607	0.0454	0.0227
125 to 80	**45**	1.3072	0.7407	1.3072	1.1111	1.1166	0.2671	0.1335
80 to 50	**30**	0.7843	0.6667	0.9804	1.2500	0.9203	0.2550	0.1275
50 to 25	**25**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
25 to 15	**10**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
15 to 10	**5**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10 to 6	**4**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6 to 0	**6**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

APPENDIX B
Calculations

B1.0 SAMPLE CALCULATIONS

This appendix details the calculations used throughout the thesis.

B1.1 BREAKAGE FUNCTION CALCULATIONS

The values used to plot Figure 2.2 in Chapter Two are as follows:

Geometric Screen Mean (mm)	*Average Product*	*Retained Cumulative*	*Passing Cum. (%)*	*Unmodified B-C*	*Modified B-C*
50.8	0.02	0.02	98	100	131.7
28.6	0.25	0.27	73	68.1	105.6
14.31	0.59	0.86	14	38.84	72.6
7.14	0.12	0.98	2	20.74	45.72
3.57	0.02	1	0	10.74	27.4

TABLE B1.1: Data for Plotting Various Breakage Functions

The screen sizes and average product are calculated from the original work conducted by Simpson (1984) in the Π breakage theory using styrofoam as one of the test components. The geometric mean is the average screen size between two successive screens. The average product is the mean fraction of material (from all trials) that is "retained" on the geometric screen mean after the secondary shred. The B-C abbreviation stands for the Broadbent-Callcott equation (both modified and unmodified). The following example illustrates how the breakage function value for a given interval is calculated using the modified Broadbent-Callcott equation. A similar calculation is performed for the unmodified Broadbent-Callcott equation.

$$B(14.31\text{mm},\ 25.5\ \text{mm}) = \frac{1-\exp[-(14.31/25.5)^{0.845}]}{1-\exp(-1)} = 0.726\ (\text{or } 72.6)$$

B1.2 ROSIN-RAMMLER CALCULATIONS

The Rosin-Rammler method used initially to characterize the curve was calculated as shown in the following example using the data from the *250 mL, rectangular, enclosed, Tetra-Pak* configuration. The objective was to determine the slope. A small slope would indicate a narrow distribution of shredded material, whereas a large small would indicate a wide distribution of shredded material.

SCREENING DATA SET
ROSIN-RAMMLER DATA ANALYSIS

						Date April 1 & 8, 1994 *Replicate Two*			*Date* May 31, 1994 *Replicate Three*			*Date* June 7, 1994 *Replicate Five*
		Screen Sizes (mm)	Interval Size (mm)	Percent Mat'l. in Interval	Cum.Wt. Fraction> (1-Y)	ln[1/(1-Y)]	Percent Mat'l. in Interval	Cum.Wt. Fraction> (1-Y)	ln[1/(1-Y)]	Percent Mat'l. in Interval	Cum.Wt. Fraction> (1-Y)	ln[1/(1-Y)]
Container Type Tested												
		125	>125	0.0	0.000	#DIV/0!	0.0	0.000	#DIV/0!	0.0	0.000	#DIV/0!
Vol. (mL)	*250*	80	125 to 80	70.5	0.705	0.350	59.0	0.590	0.528	56.0	0.560	0.580
Shape	*rect*	50	80 to 50	23.9	0.944	0.058	39.0	0.980	0.020	42.9	0.989	0.011
Geometry	*encl*	25	50 to 25	4.0	0.984	0.016	1.5	0.995	0.005	1.1	1.000	0.000
Material	*T-Pak*	15	25 to 15	0.0	0.984	0.016	0.0	0.995	0.005	0.0	1.000	0.000
		10	15 to 10	0.0	0.984	0.016	0.0	0.995	0.005	0.0	1.000	0.000
		6	10 to 6	0.0	0.984	0.016	0.0	0.995	0.005	0.0	1.000	0.000
		0	6 to 0	0.6	0.990	0.010	0.0	0.995	0.005	0.0	1.000	0.000
Number of Screening Loads				1			2			2		

FIGURE B1.1: Example of Rosin-Rammler Data for Tetra-Pak

As explained in Chapter Two, x_0 is the characteristic particle size at which 63.21% by weight of the sample passes. On this double logarithmic graph, x_0 is the screen size wherever the curve intersects the horizontal line at ln[1/(1-Y)] equals 1.0, in which Y is 0.6321. The $\log_{10}$ of this entire term equals zero and is thus useful for determining the slope. By extending the straight line portion of the curves, the x_0 values are: replicate two, 109 mm; replicate three, 88 mm; and replicate five, 86 mm. The slope of the line is then calculated by

$$slope,\ n = \frac{\log\{\ln[1/(1-Y)]\}}{\log x - \log x_o}$$

250 mL, RECTANGULAR, ENCLOSED, TETRA-PAK

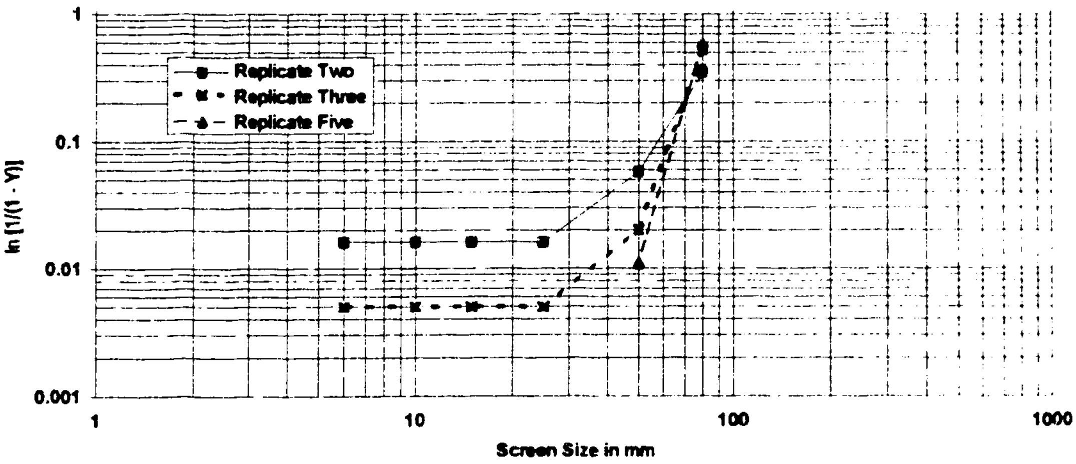

FIGURE B1.2: Example of Rosin-Rammler Plotted Data for Tetra-Pak

A second point was chosen to perform the slope calculation. For example, for replicate two, ln[1/(1-Y)] equals 0.0576 at 50 mm. The slope is therefore

$$n = \frac{\log(0.0576)}{\log(50) - \log(109)} = 3.66$$

Replicate	*ln[1/(1-Y)]...*	*... at x mm*	*x_o*	*Slope, n*
Two	0.0576	50	109	3.66
Three	0.0202	50	88	6.90
Five	0.0111	50	86	8.31

TABLE B1.2: Example of Slopes Determined by Rosin-Rammler Method

The average of these slopes is 6.29, with a standard deviation of 2.384, and a standard error of 1.377. All Tetra-Pak data was initially analyzed using this Rosin-Rammler method. A factorial analysis was later performed on these final slopes.

B1.3 LINEAR SLOPE CALCULATIONS

Because the Rosin-Rammler method produced results contrary to what was observed, it was thought the double logarithmic scale may prevent accurate estimation of the various data points. The next parameter used was the linear slope. This is essentially the slope from the straight line portion of the various cumulative size distribution curves. The axes scales are linear and not modified by exponential functions. Again, the example uses the data from the *250 mL, rectangular, enclosed, Tetra-Pak* configuration.

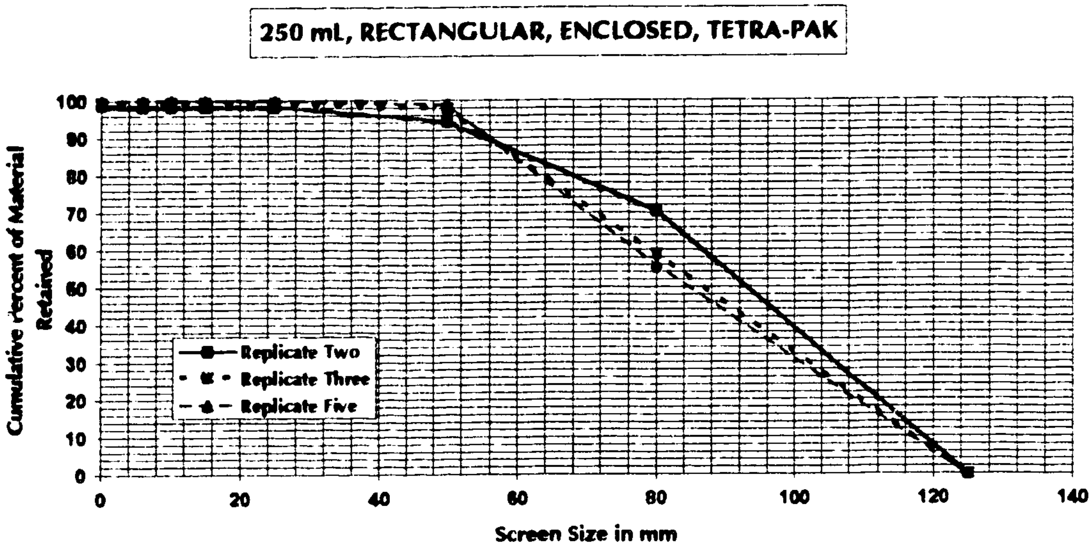

FIGURE B1.3: Example of Linear Slope Plotted Data for Tetra-Pak

The slopes of each curve were determined by calculating the rise over the run for the straight line portions. In this example, the slopes are

$$n_2 = \frac{58 - 0}{88 - 125} = -\ 1.568$$

$$n_3 = \frac{64 - 0}{76 - 125} = -\ 1.306$$

$$n_5 = \frac{52 - 0}{84 - 124} = -\ 1.30$$

$$n_{mean} = -\ 1.391$$

$$s_{deviation} = 0.153$$

These calculations were performed for all Tetra-Pak data. A factorial analysis was also performed on this data. However, because of the estimation still required and the advantage of using normalized material densities for each size range, the *modal interval density* was eventually adopted as the parameter to characterize each curve. The details of determining this parameter were outlined in Chapter Four.

B1.4 FACTORIAL ANALYSIS

The factorial analysis in the Tetra-Pak and glass preliminary tests were 2^3 designs in which three factors were each tested at an upper and lower level. Thus, each test had eight runs. This design simplifies calculating the effects.

In the following example, to determine the summations, each x-factor or x-interaction is multiplied by the average (or single if no replicates) Rosin-Rammler determined slope for that beverage container configuration. The columns are then summed and each sum divided by eight. The resulting terms form the polynomial model. However, with the exception of x_0, which is the constant and average of all runs, the rest of the effects must be multiplied by two to determine the full effect of that factor or interaction between the lower and upper levels. The standard error of each effect is determined by using the available standard deviations and corresponding degrees of freedom from the eight runs (Box et al., 1978, p.319). Note that the standard error of the constant is half of the standard error of the effects.

The same calculation method as outlined for the Rosin-Rammler method was used for the linear slope and modal interval density values.

FACTORIAL ANALYSIS

FOR TETRA-PAK USING ROSIN-RAMMLER SLOPE PARAMETER

		Volume	Shape	Geom	Interactions				R-R Slope	Std. Dev.	DegFree	Variance	
Run	Xo	X1	X2	X3	X1X2	X1X3	X2X3	X1X2X3	n	s	d.f.	s2	s2 x d.f.
1	1	-1	-1	-1	1	1	1	-1	4.923				
2	1	1	-1	-1	-1	-1	1	1	3.629	1.770	1	3.1329	3.1329
3	1	-1	1	-1	-1	1	-1	1	4.168	2.302	2	5.2992	10.5984
4	1	1	1	-1	1	-1	-1	-1	2.822	1.740	1	3.0276	3.0276
5	1	-1	-1	1	1	-1	-1	1	6.970				
6	1	1	-1	1	-1	1	-1	-1	3.340				
7	1	-1	1	1	-1	-1	1	-1	6.290	2.384	2	5.6835	11.3669
8	1	1	1	1	1	1	1	1	3.156	0.380	1	0.1444	0.1444

Summations

Xon	X1n	X2n	X3n	X1X2n	X1X3n	X2X3n	X1X2X3n
4.923	-4.923	-4.923	-4.923	4.923	4.923	4.923	-4.923
3.629	3.629	-3.629	-3.629	-3.629	-3.629	3.629	3.629
4.168	-4.168	4.168	-4.168	-4.168	4.168	-4.168	4.168
2.822	2.822	2.822	-2.822	2.822	-2.822	-2.822	-2.822
6.97	-6.97	-6.97	6.97	6.97	-6.97	-6.97	6.97
3.34	3.34	-3.34	3.34	-3.34	3.34	-3.34	-3.34
6.29	-6.29	6.29	6.29	-6.29	-6.29	6.29	-6.29
3.156	3.156	3.156	3.156	3.156	3.156	3.156	3.156
35.298	**-9.404**	**-2.426**	**4.214**	**0.444**	**-4.124**	**0.698**	**0.548**

Total s2 x d.f.	**28.2702**
Total d.f.	**7**
Pooled Estimate of Run Variance is (total S2 x d.f.)/(total d.f.)	**4.0386**
Total Runs, N	**16**
Variance of Each Effect is 4/N x pooled variance	**1.00965**
Standard Error of Effect is square root of variance	$\boxed{1.0048}$

Although 16 runs were made, only 7 degrees of freedom are possible since Run 4 had one immeasureable R-R slope.

Divide by 8

MODEL PARAMETERS

	Volume	Shape	Geom	Interactions			
Xo	X1	X2	X3	X1X2	X1X3	X2X3	X1X2X3
4.412	**-1.176**	**-0.303**	**0.527**	**0.055**	**-0.516**	**0.087**	**0.069**
EFFECTS							
	-2.351	**-0.607**	**1.054**	**0.111**	**-1.031**	**0.175**	**0.137**

FIGURE B1.4: Factorial Analysis for Tetra-Pak, Rosin-Rammler Method

FACTORIAL ANALYSIS

FOR TETRA-PAK USING LINEAR SLOPE PARAMETER

		Volume	Shape	Geom	Interactions				Linear Slope	Std. Dev.	DegFree	Variance	
Run	Xo	X1	X2	X3	X1X2	X1X3	X2X3	X1X2X3	n	s	d.f.	s2	s2 x d.f.
1	1	-1	-1	-1	1	1	1	-1	-2.070				
2	1	1	-1	-1	-1	-1	1	1	-0.888	0.512	1	0.2621	0.2621
3	1	-1	1	-1	-1	1	-1	1	-1.368	0.324	2	0.1050	0.2100
4	1	1	1	-1	1	-1	-1	-1	-1.040	0.703	2	0.4942	0.9884
5	1	-1	-1	1	1	-1	-1	1	-2.600				
6	1	1	-1	1	-1	1	-1	-1	-1.111				
7	1	-1	1	1	-1	-1	1	-1	-1.391	0.153	2	0.0234	0.0468
8	1	1	1	1	1	1	1	1	-1.101	0.211	1	0.0445	0.0445

Summations

Xon	X1n	X2n	X3n	X1X2n	X1X3n	X2X3n	X1X2X3n
-2.07	2.07	2.07	2.07	-2.07	-2.07	-2.07	2.07
-0.888	-0.888	0.888	0.888	0.888	0.888	-0.888	-0.888
-1.368	1.368	-1.368	1.368	1.368	-1.368	1.368	-1.368
-1.04	-1.04	-1.04	1.04	-1.04	1.04	1.04	1.04
-2.6	2.6	2.6	-2.6	-2.6	2.6	2.6	-2.6
-1.111	-1.111	1.111	-1.111	1.111	-1.111	1.111	1.111
-1.391	1.391	-1.391	-1.391	1.391	1.391	-1.391	1.391
-1.101	-1.101	-1.101	-1.101	-1.101	-1.101	-1.101	-1.101
-11.569	**3.289**	**1.769**	**-0.837**	**-2.053**	**0.269**	**0.669**	**-0.345**

Total s2 x d.f.	**1.5519**
Total d.f.	**8**
Pooled Estimate of Run Variance is (total S2 x d.f.)/(total d.f.)	**0.1940**
Total Runs, N	**16**
Variance of Each Effect is 4/N x pooled variance	**0.04850**
Standard Error of Effect is square root of variance	$\boxed{0.2202}$

Divide by 8

MODEL PARAMETERS

	Volume	Shape	Geom	Interactions			
Xo	X1	X2	X3	X1X2	X1X3	X2X3	X1X2X3
-1.446	**0.411**	**0.221**	**-0.105**	**-0.257**	**0.034**	**0.084**	**-0.043**
EFFECTS							
	0.822	**0.442**	**-0.209**	**-0.513**	**0.067**	**0.167**	**-0.086**

FIGURE B1.5: Factorial Analysis for Tetra-Pak, Linear Slope

Since the modal interval density appeared to be the parameter that best described the size distribution curves for the purposes of this experiment, glass data was not subjected to the Rosin-Rammler or linear slope methods. Only the modal interval density was used. Note that the same calculation techniques were used when the logarithms of the densities were later analyzed.

FACTORIAL ANALYSIS FOR TETRA-PAK

FOR TETRA-PAK USING MATERIAL DENSITY PARAMETER

Run	*Xo*	*Volume* *X1*	*Shape* *X2*	*Geom* *X3*	*Interactions* *X1X2*	*X1X3*	*X2X3*	*X1X2X3*	*Density %/mm*	*Std. Dev. s*	*Degree d.f.*	*Variance s2*	*s2 x d.f.*
1	1	-1	-1	-1	1	1	1	-1	2.093				
2	1	1	-1	-1	-1	-1	1	1	0.963	0.418	1	0.1747	0.1747
3	1	-1	1	-1	-1	1	-1	1	1.558	0.280	2	0.0784	0.1568
4	1	1	1	-1	1	-1	-1	-1	1.078	0.644	2	0.4147	0.8295
5	1	-1	-1	1	1	-1	-1	1	2.590				
6	1	1	-1	1	-1	1	-1	-1	1.151				
7	1	-1	1	1	-1	-1	1	-1	1.436	0.128	2	0.0164	0.0328
8	1	1	1	1	1	1	1	1	1.122	0.177	1	0.0313	0.0313

Summations

Xon	*X1n*	*X2n*	*X3n*	*X1X2n*	*X1X3n*	*X2X3n*	*X1X2X3n*
2.093	-2.093	-2.093	-2.093	2.093	2.093	2.093	-2.093
0.963	0.963	-0.963	-0.963	-0.963	-0.963	0.963	0.963
1.558	-1.558	1.558	-1.558	-1.558	1.558	-1.558	1.558
1.078	1.078	1.078	-1.078	1.078	-1.078	-1.078	-1.078
2.59	-2.59	-2.59	2.59	2.59	-2.59	-2.59	2.59
1.151	1.151	-1.151	1.151	-1.151	1.151	-1.151	-1.151
1.436	-1.436	1.436	1.436	-1.436	-1.436	1.436	-1.436
1.122	1.122	1.122	1.122	1.122	1.122	1.122	1.122
11.991	**-3.363**	**-1.603**	**0.607**	**1.775**	**-0.143**	**-0.763**	**0.475**

Total s2 x d.f.	1.2251
Total d.f.	8
Pooled Estimate of Run Variance is (total S2 x d.f.)/(total d.f.)	0.1531
Total Runs, N	16
Variance of Each Effect is 4/N x pooled variance	0.03828
Standard Error of Effect is square root of variance	$\boxed{0.1957}$

Divide by 8

MODEL PARAMETERS

	Xo	*Volume* *X1*	*Shape* *X2*	*Geom* *X3*	*Interactions* *X1X2*	*X1X3*	*X2X3*	*X1X2X3*
	1.499	**-0.420**	**-0.200**	**0.076**	**0.222**	**-0.018**	**-0.095**	**0.059**
EFFECTS		**-0.841**	**-0.401**	**0.152**	**0.444**	**-0.036**	**-0.191**	**0.119**

FIGURE B1.6: Factorial Analysis for Tetra-Pak, Modal Interval Density

FACTORIAL ANALYSIS
FOR GLASS USING MATERIAL DENSITY PARAMETER

Run	Xo	Volume X1	Shape X2	Geom X3	Interactions X1X2	X1X3	X2X3	X1X2X3	Density %/mm	Std. Dev. s	DegFree d.f.	Variance s2
1	1	-1	-1	-1	1	1	1	-1	3.060	0.311	1	0.0967
2	1	1	-1	-1	-1	-1	1	1	2.330			
3	1	-1	1	-1	-1	1	-1	1	2.490	0.240	1	0.0576
4	1	1	1	-1	1	-1	-1	-1	2.730	0.085	1	0.0072
5	1	-1	-1	1	1	-1	-1	1	3.090	0.240	1	0.0576
6	1	1	-1	1	-1	1	-1	-1	2.620	0.127	1	0.0161
7	1	-1	1	1	-1	-1	1	-1	2.840	0.156	1	0.0243
8	1	1	1	1	1	1	1	1	2.595	0.049	1	0.0024

Total s2 x d.f.
Total d.f.
Pooled Estimate of Run Variance
is (total S2 x d.f.)/(total d.f.)

Total Runs, N

Variance of Each Effect is
4/N x pooled variance

Standard Error of Effect is
square root of variance

Summations Xon	X1n	X2n	X3n	X1X2n	X1X3n	X2X3n	X1X2X3n
3.06	-3.06	-3.06	-3.06	3.06	3.06	3.06	-3.06
2.33	2.33	-2.33	-2.33	-2.33	-2.33	2.33	2.33
2.49	-2.49	2.49	-2.49	-2.49	2.49	-2.49	2.49
2.73	2.73	2.73	-2.73	2.73	-2.73	-2.73	-2.73
3.09	-3.09	-3.09	3.09	3.09	-3.09	-3.09	3.09
2.62	2.62	-2.62	2.62	-2.62	2.62	-2.62	-2.62
2.84	-2.84	2.84	2.84	-2.84	-2.84	2.84	-2.84
2.595	2.595	2.595	2.595	2.595	2.595	2.595	2.595
21.755	-1.205	-0.445	0.535	1.195	-0.225	-0.105	-0.745

Divide by 8
MODEL PARAMETERS

	Volume X1	Shape X2	Geom X3	Interactions X1X2	X1X3	X2X3	X1X2X3
Xo 2.719	-0.151	-0.056	0.067	0.149	-0.028	-0.013	-0.093
EFFECTS	-0.301	-0.111	0.134	0.299	-0.056	-0.026	-0.186

FIGURE B1.7: Factorial Analysis for Glass

B1.5 NORMAL DISTRIBUTION OF MODAL INTERVAL DENSITIES

The modal interval densities were analyzed to determine if they were normally distributed.

TREATMENT OF MODAL INTERVAL DENSITIES

TETRA-PAK

										Pi=100(i-0.5)/m, where m=16	
Container Type			**Modal Interval Densities**			**Average**	**Std.Dev.**	**All MID's.**	**Rank**	**Ranked**	**Pi (%)**
250	rect.	encl.	1.567	1.311	1.430	1.436	0.128	1.567	1	0.340	3.125
250	cyl.	encl.	2.590					2.590	2	0.667	9.375
250	rect.	open	1.738	1.236	1.700	1.558	0.280	1.738	3	0.996	15.625
250	cyl.	open	2.093					2.093	4	1.151	21.875
1000	rect.	encl.	0.996	1.247		1.121	0.178	0.996	5	1.236	28.125
1000	cyl.	encl.	1.151					1.151	6	1.247	34.375
1000	rect.	open	0.340	1.522	1.373	1.079	0.644	0.340	7	1.258	40.625
1000	cyl.	open	0.667	1.258		0.962	0.418	0.667	8	1.311	46.875
								1.311	9	1.373	53.125
	Mean	1.389			***Range***	2.250		1.236	10	1.430	59.375
	Standard Error	0.131			***Minimum***	0.340		1.247	11	1.522	65.625
	Median	1.342			***Maximum***	2.590		1.522	12	1.567	71.875
	Mode	#N/A			***Sum***	22.219		1.258	13	1.700	78.125
	Standard Deviation	0.523			***Count***	16		1.430	14	1.738	84.375
	Sample Variance	0.274			***Confidence Level(95.000%)***	0.256		1.700	15	2.093	90.625
	Kurtosis	1.425						1.373	16	2.590	96.875
	Skewness	0.293									

FIGURE B1.8: Normal Distribution Data for Tetra-Pak Modal Interval Densities

GLASS JARS

									Pi=100(i-0.5)/m, where m=15			
Container Type			**Modal Interval Densities**		**Average**	**Std.Dev.**	**All MID's.**	**Rank**	**Ranked**	**Pi (%)**	**LogRank**	**Pi (%)**
250	rect.	open	2.840	3.280	3.060	0.311	3.280	1	2.320	3.333	0.365	3.333
1000	cyl.	open		2.330			2.330	2	2.330	10.000	0.367	10.000
1000	rect.	encl.	2.560	2.630	2.595	0.049	2.630	3	2.530	16.667	0.403	16.667
250	rect.	open	2.320	2.660	2.490	0.240	2.660	4	2.560	23.333	0.408	23.333
1000	rect.	open	2.790	2.670	2.730	0.085	2.670	5	2.630	30.000	0.420	30.000
250	rect.	encl.	2.950	2.730	2.840	0.156	2.730	6	2.660	36.667	0.425	36.667
250	cyl	encl.	2.920	3.260	3.090	0.240	3.260	7	2.670	43.333	0.427	43.333
1000	cyl.	encl.	2.710	2.530	2.620	0.127	2.530	8	2.710	50.000	0.433	50.000
							2.840	9	2.730	56.667	0.436	56.667
	Mean	2.745		***Range***	0.96		2.560	10	2.790	63.333	0.446	63.333
	Standard Error	0.072		***Minimum***	2.32		2.320	11	2.840	70.000	0.453	70.000
	Median	2.710		***Maximum***	3.28		2.790	12	2.920	76.667	0.465	76.667
	Mode	#N/A		***Sum***	41.18		2.950	13	2.950	83.333	0.470	83.333
	Standard Deviation	0.280		***Count***	15		2.920	14	3.260	90.000	0.513	90.000
	Sample Variance	0.078		***Confidence Level(95.000%)***	0.141592		2.710	15	3.280	96.667	0.516	96.667
	Kurtosis	0.189										
	Skewness	0.513										

FIGURE B1.9: Normal Distribution Data for Glass Jar Modal Interval Densities

EPS CONTAINERS

									Pi=100(i-0.5)/m, where m=4			
Container Type			**Modal Interval Densities**		**Average**	**Std.Dev.**	**All MID's.**	**Rank**	**Ranked**	**Pi (%)**	**LogRank**	**Pi (%)**
250	cyl	open	1.530	1.453	1.492	0.054	1.453	1	1.453	12.5	0.162	12.5
1000	cyl	open	1.716	1.668	1.692	0.034	1.530	2	1.530	37.5	0.185	37.5
							1.692	3	1.692	62.5	0.228	62.5
							1.716	4	1.716	87.5	0.235	87.5

Mean	1.598	***Skewness***	-0.277
Standard Error	0.064	***Range***	0.263
Median	1.611	***Minimum***	1.453
Mode	#N/A	***Maximum***	1.716
Standard Deviation	0.127	***Sum***	6.391
Sample Variance	0.016	***Count***	4
Kurtosis	-4.097	***Confidence Level(95.000%)***	0.124485

FIGURE B1.10: Normal Distribution Data for EPS Modal Interval Densities

B1.6 RESIDUALS AND MODEL ADEQUACY

Examples of the various residuals and model adequacy checks performed throughout Chapter Four are presented in this section. Note that the same residuals calculations method was performed when the logarithms of the Tetra-Pak modal interval densities were analyzed.

NORMAL PROBABILITY PLOT

TO DETERMINE SIGNIFICANCE OF EFFECTS

Effects	Rank	Percentile	Factors
[illegible]	1	7.1	X1
[illegible]	2	21.4	X2
[illegible]	3	35.7	X1X3
0.119	4	50.0	X1X2X3
0.152	5	64.3	X3
0.191	6	78.6	X2X3
0.444	7	92.9	X1X2

From the half-normal probability plot and the normal probability plot of the effects, factors 1 (size) and 2 (shape) are significant. It is debatable whether interaction 12 is significant on the normal probability plot because there are so few data points for a straight line comparison. It will be considered insignificant.

DIAGNOSTIC CHECK: EVALUATE RESIDUALS

Model is therefore...

Xo	*X1*	*X2*
1.499	-0.420	-0.200

Run	*Xo*	*X1*	*X2*	*Model*
1	1.499	0.420	0.200	2.119
2	1.499	-0.420	0.200	1.279
3	1.499	0.420	-0.200	1.719
4	1.499	-0.420	-0.200	0.879
5	1.499	0.420	0.200	2.119
6	1.499	-0.420	0.200	1.279
7	1.499	0.420	-0.200	1.719
8	1.499	-0.420	-0.200	0.879

Pi=100(i-0.5)/n, where n=16

Run	*Actual*	*Model*	*Res.*	*Rank*	*i*	*Pi*
1	2.093	2.119	-0.026	-0.612	1	3.1
2	0.667	1.279	-0.612	-0.539	2	9.4
2	1.258	1.279	-0.021	-0.483	3	15.6
3	1.738	1.719	0.019	-0.408	4	21.9
3	1.236	1.719	-0.483	-0.289	5	28.1
3	1.700	1.719	-0.019	-0.152	6	34.4
4	0.340	0.879	-0.539	-0.128	7	40.6
4	1.522	0.879	0.643	-0.026	8	46.9
4	1.373	0.879	0.494	-0.021	9	53.1
5	2.590	2.119	0.471	-0.019	10	59.4
6	1.151	1.279	-0.128	0.019	11	65.6
7	1.567	1.719	-0.152	0.117	12	71.9
7	1.311	1.719	-0.408	0.368	13	78.1
7	1.430	1.719	-0.289	0.471	14	84.4
8	0.996	0.879	0.117	0.494	15	90.6
8	1.247	0.879	0.368	0.643	16	96.9

FIGURE B1.11: Residuals for Tetra-Pak Factorial Analysis

NORMAL PROBABILITY PLOT

TO DETERMINE SIGNIFICANCE OF EFFECTS

Effects	Rank	Percentile	Factors
-0.301	1	7.1	X1
-0.186	2	21.4	X1X2X3
-0.111	3	35.7	X2
-0.056	4	50.0	X1X3
-0.026	5	64.3	X2X3
0.134	6	78.6	X3
0.299	7	92.9	X1X2

It is much more difficult to ascertain which effects are significant from the normal probability plot. Instead, the half-normal probability plot will be used instead. From the half-normal plot, the significant effects are from factor 1 (size) and possibly from interaction 12 (size and shape), although this last one is difficult to precisely determine.

DIAGNOSTIC CHECK: EVALUATE RESIDUALS

Model is therefore (assuming interaction 12 significant)...

Xo	*X1*	*X1X2*
2.719	-0.151	0.149

Run	*Xo*	*X1*	*X2*	*Model*
1	2.719	0.151	-0.149	2.721
2	2.719	-0.151	-0.149	2.419
3	2.719	0.151	0.149	3.019
4	2.719	-0.151	0.149	2.717
5	2.719	0.151	-0.149	2.721
6	2.719	-0.151	-0.149	2.419
7	2.719	0.151	0.149	3.019
8	2.719	-0.151	0.149	2.717

Pi=100(i-0.5)/n, where n=15

Run	*Actual*	*Model*	*Res.*	*Rank*	*i*	*Pi*
1	2.840	2.721	0.119	-0.699	1	3.333
1	3.280	2.721	0.559	-0.359	2	10.000
2	2.330	2.419	-0.089	-0.289	3	16.667
3	2.320	3.019	-0.699	-0.157	4	23.333
3	2.660	3.019	-0.359	-0.089	5	30.000
4	2.790	2.717	0.073	-0.087	6	36.667
4	2.670	2.717	-0.047	-0.069	7	43.333
5	2.920	2.721	0.199	-0.047	8	50.000
5	3.260	2.721	0.539	0.073	9	56.667
6	2.710	2.419	0.291	[illegible]	10	63.333
6	2.530	2.419	0.111	[illegible]	11	70.000
7	2.950	3.019	-0.069	0.199	12	76.667
7	2.730	3.019	-0.289	0.291	13	83.333
8	2.560	2.717	-0.157	0.539	14	90.000
8	2.630	2.717	-0.087	0.559	15	96.667

FIGURE B1.12: Residuals for Glass Jar Factorial Analysis

	Largest Density				Residuals (Actual - Ave)			Pi=100(i-0.5)/n, where n=18				
								Rank			Corresponding	
	Set One	Set Two	Set Three	Average	Set One	Set Two	Set Three	Res.	i	Pi	Average	Res.
Corner, end	1.879	1.828	2.683	2.1299	-0.251	-0.302	0.553	-0.701	1	2.778	2.1299	-0.251
Broad, side	1.936	1.847	2.202	1.9951	-0.059	-0.148	0.207	-0.411	2	8.333	1.9951	-0.059
Corner, end	1.729	2.468	2.222	2.1396	-0.411	0.328	0.083	-0.302	3	13.889	2.1396	-0.411
Narrow, side	2.222	1.423	1.485	1.7101	0.512	-0.287	-0.225	-0.287	4	19.444	1.7101	0.512
Narrow, end	2.185	0.803	1.523	1.5037	0.682	-0.701	0.019	-0.251	5	25.000	1.5037	0.682
Broad, end	1.940	2.222	2.222	2.1281	-0.188	0.094	0.094	-0.225	6	30.556	2.1281	-0.188
								-0.188	7	36.111	2.1299	-0.302
								-0.148	8	41.667	1.9951	-0.148
								-0.059	9	47.222	2.1396	0.328
								0.019	10	52.778	1.7101	-0.287
								0.083	11	58.333	1.5037	-0.701
								0.094	12	63.889	2.1281	0.094
								0.094	13	69.444	2.1299	0.553
								0.207	14	75.000	1.9951	0.207
								0.328	15	80.556	2.1396	0.083
								0.512	16	86.111	1.7101	-0.225
								0.553	17	91.667	1.5037	0.019
								0.682	18	97.222	2.1281	0.094

FIGURE B1.13: Residuals for Orientation Tests

B1.7 HALF-NORMAL PROBABILITY PLOTS CALCUATIONS

To determine which effects are significant, the absolute values of the effects of the factors and factor-interactions (not including the constant) are plotted in increasing rank order on a modified probability paper. The results of the modal interval density factorial analysis illustrate the required procedure.

Rank, i	*Factor or Interaction*	*/Effect /*	*P={(i-0.5)/n}x100, where n = 7*
1	13	0.036	7.1%
2	123	0.118	21.4%
3	3	0.152	35.7%
4	23	0.190	50.0%
5	2	0.400	64.3%
6	12	0.444	78.6%
7	1	0.840	92.9%

TABLE B1.3: Half-Normal Probability Plot Procedure for Tetra-Pak

In this example, the notation for interaction 13 represents the interaction between factor one, size, and factor three, geometry. The percentiles are then plotted against the absolute values of the effects. Non-significant effects should lie approximately on a straight line between the origin and and these effects. Significant effects will depart noticeably from this line. This half-normal probability plot procedure was performed for the other two Tetra-Pak parameters (Rosin-Rammler and linear slope) and also for the glass factorial analysis results.

Rank, i	*Factor or Interaction*	*/Effect/*	*P={(i-0.5)/n}x100, where n = 7*
1	23	0.026	7.1%
2	13	0.056	21.4%
3	2	0.111	35.7%
4	3	0.134	50.0%
5	123	0.186	64.3%
6	12	0.299	78.6%
7	1	0.301	92.9%

TABLE B1.4: Half-Normal Probability Plot Procedure for Glass

B1.8 MISSING ANOVA CALCULATIONS

B1.8.1 Missing ANOVA Calculations for Glass

The technique used to estimate the missing value(s) to complete the analysis of variance for the Tetra-Pak and glass data is presented in Fundamental Concepts in the Design of Experiments by Charles R. Hicks, 1982, pages 73 to 76. The sample calculation will be based on the data collected for the shredding and screening of *glass* jars. Because only one value was missing from the glass data, this will be easier to illustrate than the Tetra-Pak analysis.

Level of Factor			*Modal Interval Density (%/mm)*			
A	*B*	*C*	*Replicate*	*Replicate*		
Vol.	*Shape*	*Geom.*	*One*	*Two*	*Total*	*Mean*
-1	-1	-1	2.840	3.280	6.120	3.060
+1	-1	-1	2.330	*y*	*y*+2.330	(*y*+2.330)/2
-1	+1	-1	2.320	2.660	4.980	2.490
+1	+1	-1	2.790	2.670	5.460	2.730
-1	-1	+1	2.920	3.260	6.180	3.090
+1	-1	+1	2.710	2.530	5.240	2.620
-1	+1	+1	2.950	2.730	5.680	2.840
+1	+1	+1	2.560	2.630	5.190	2.595

TABLE B1.56: ANOVA Calculation Table for Glass Data

Using conventional ANOVA analysis notation, in which SST is the total sum of squares, SSTr is the treatment sum of squares (runs in this example), SSBl is the block sum of squares (replicates in this example), and SSE is the error sum of squares, the calculations are as follows:

$$SST = SSTr + SSBl + SSE$$

$$C = \frac{T^2}{ab} = \frac{1}{8 \times 2}[6.12 + y + 2.33 + 4.98 + \ldots + 5.19]^2$$

$$C = 105.99 + 5.148y + 0.0625y^2$$

$$SST = \sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij}^2 - C = 2.84^2 + 2.33^2 + \ldots + y^2 + \ldots + 2.63^2 - C$$

$$SST = 114.15 + y^2 - C$$

$$SSTr = \frac{\sum_{i=1}^{a} T_{i.}^2}{b} - C = \frac{1}{2}\left[6.12^2 + (y + 2.33)^2 + \ldots + 5.19^2\right] - C$$

$$SSTr = 111.17 + 0.5y^2 + 2.33y - C$$

$$SSBl = \frac{\sum_{j=1}^{b} T_{.j}^2}{a} - C = \frac{1}{8}\left[(2.84 + 2.33 + \ldots + 2.56)^2 + (3.28 + y + \ldots + 2.63)^2\right] - C$$

$$SSBl = 106.16 + 4.94y + 0.125y^2 - C$$

$$SSE = SST - SSTr - SSBl$$

$$SSE = 114.15 + y^2 - C - \left[111.17 + 0.5y^2 + 2.33y - C\right]$$

$$-\left[106.16 + 4.94y + 0.125y^2 - C\right]$$

$$SSE = 2.81 + 0.4375y^2 - 2.122y$$

To estimate the missing value, minimize SSE. Therefore, the partial derivative of SSE with respect to y is equal to zero.

$$\frac{\partial}{\partial y} SSE = 0$$

$$\frac{\partial}{\partial y} SSE = 0.4375(2)y - 2.122 = 0$$

$$y = 2.425$$

The calculated missing y value is 2.425. This is then substituted back into the ANOVA equations for C, SST, SSTr, SSBl, and SSE to complete the analysis.

$$C = 118.84 \quad SSTr = 0.921 \quad SSBl = 0.0346 \quad SSE = 0.237 \quad SST = 1.19$$

The degrees of freedom for SSE must be modified because missing values were calculated.

$$v = (a-1)(b-1) - \text{number of estimated values}$$
$$v = (8-1)(2-1) - 1 = 7$$

To calculate the individual sum of squares for each run, a technique known as *Yate's Algorithm* was used. A table is constructed and the values are arrived at by successive addition or subtraction. The same number of calculation columns as factors are required. There are the eight values in the *totals* column as shown in the following tables. The first four values in column one are calculated by adding each successive pair in the totals column. Conversely, the last four values in column one are calculated by subtracting each successive pair in the totals column. This is repeated for the required number of columns. The SS (sum of squares) column is obtained by squaring the value in column three and dividing by (2 x 8 runs) or 16.

YATE'S ALGORITHM

For Glass Data Analyzed with Missing Anova Value Technique

Experimental Condition	Total	Column 1	Column 2	Column 3	I.D.	SS
1	6.12	10.88	21.32	43.61	*I*	118.865
a	4.76	10.44	22.29	-2.31	*A*	0.334
b	4.98	11.42	-0.88	-0.99	*B*	0.061
ab	5.46	10.87	-1.43	2.29	*AB*	0.328
c	6.18	-1.36	-0.44	0.97	*C*	0.059
ac	5.24	0.48	-0.55	-0.55	*AC*	0.019
bc	5.68	-0.94	1.84	-0.11	*BC*	0.001
abc	5.19	-0.49	0.45	-1.39	*ABC*	0.121
					Sum (not including SS of I)	0.922
					SS Replicate	0.0346
					SS Error	0.237
					SS Total	**1.193**

FIGURE B1.14: Yate's Algorithm for Glass Data

EXCEL ANOVA RESULTS

Calculated Missing Anova Value Assumed as Real Data Point; Check Hand Calculations

	Experimental Conditions							
	1	*a*	*b*	*ab*	*c*	*ac*	*bc*	*abc*
Rep. 1	2.840	2.330	2.320	2.790	2.920	2.710	2.950	2.560
Rep. 2	3.280	2.425	2.660	2.670	3.260	2.530	2.730	2.630

ANOVA: TWO-FACTOR WITHOUT REPLICATION; Treating Replicates as Separate Blocks

SUMMARY

	Count	*Sum*	*Average*	*Variance*
Rep. 1	8	21.42	2.6775	0.062221
Rep. 2	8	22.185	2.773125	0.102864
1	2	6.12	3.06	0.0968
a	2	4.755	2.3775	0.004513
b	2	4.98	2.49	0.0578
ab	2	5.46	2.73	0.0072
c	2	6.18	3.09	0.0578
ac	2	5.24	2.62	0.0162
bc	2	5.68	2.84	0.0242
abc	2	5.19	2.595	0.00245

ANOVA

Source	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Rows	0.036577	1	0.036577	1.111335	0.326814	5.59146
Columns	0.925211	7	0.132173	4.015918	0.043389	3.787051
Error	0.230386	7	0.032912			
Total	1.192173	15				

FIGURE B1.15: Computer ANOVA Analysis for Glass

The preceding ANOVA calculations were performed using built-in EXCEL spreadsheet functions. Note that these computer calculated values are not precisely the same as the hand calculated results for SST, SSTr, SSBl, and SSE because the calculated missing value is assumed to be an actual value. Instead, these computer values serve as a check on the hand calculations. The final analysis of variance results are shown in the following table.

Source of Variation	***Deg. of Free., d.f.***	***SS***	***MS (SS/d.f.)***	***F_{calc} = MS/MSError***	***Sig? @ 0.05***	***Sig? @ 0.01***
Replicates	1	0.0346	0.0346	0.876	*No*	*No*
Size	1	0.334	0.334	8.456	*Yes*	*No*
Shape	1	0.061	0.061	1.544	*No*	*No*
Size/Shape	1	0.28	0.28	8.304	*Yes*	*No*
Geometry	1	0.059	0.059	1.494	*No*	*No*
Size/Geom.	1	0.019	0.019	0.481	*No*	*No*
Shape/Geom	1	0.001	0.001	0.0253	*No*	*No*
All Factors	1	0.121	0.121	3.063	*No*	*No*
Error	6	0.237	0.0395			

Numerator d.f.	***Denominator d.f.***	***Significance Level***	***Tabulated F-value***
1	6	0.05	5.99
1	6	0.01	13.74

TABLE B1.6: Final ANOVA Results for Glass Tests

B1.8.2 Missing ANOVA Calculations for Tetra-Pak

The same technique was used to calculate the missing values for the Tetra-Pak ANOVA.

Level of Factor			***Modal Interval Density (%/mm)***				
A	***B***	***C***	***Rep.***	***Rep.***	***Rep.***		
Vol.	***Shape***	***Geom.***	***One***	***Two***	***Three***	***Mean***	***Total***
-1	-1	-1	2.093	y_1	y_2	$(2.093+y_1+y_2)/3$	$2.093+y_1+y_2$
+1	-1	-1	0.667	1.258	y_3	$(1.925+y_3)/3$	$1.925+y_3$
-1	+1	-1	1.738	1.236	1.700	1.558	4.674
+1	+1	-1	0.340	1.522	1.373	1.079	3.235
-1	-1	+1	2.590	y_4	y_5	$(2.590+y_4+y_5)/3$	$2.590+y_4+y_5$
+1	-1	+1	1.151	y_6	y_7	$(1.151+y_6+y_7)/3$	$1.151+y_6+y_7$
-1	+1	+1	1.567	1.311	1.430	1.436	4.308
+1	+1	+1	0.996	1.247	y_8	$(2.243+y_8)/3$	$2.243+y_8$

TABLE B1.7: ANOVA Calculation Data for Tetra-Pak

$$SST = SSTr + SSBl + SSE$$

$$C = \frac{T^2}{ab} = \frac{1}{(8)(3)}\left[2.093 + 1.925 + \ldots + 2.243 + y_1 + y_2 + \ldots + y_8\right]^2$$

$$C = \frac{1}{24}\left[22.219 + y_1 + y_2 + \ldots + y_8\right]^2$$

$$SST = \sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij}^2 - C = \left[(2.093)^2 + (0.667)^2 + \ldots + y_1^2 + \ldots + y_8^2\right] - C$$

$$SST = y_1^2 + y_2^2 \ldots + y_8^2 + 34.963 - C$$

$$SSTr = \frac{\sum_{i=1}^{a} T_i^2}{b} - C$$

$$SSTr = \frac{1}{3}\left[(2.093 + y_1 + y_2)^2 + (1.925 + y_3)^2 + \ldots + (4.308)^2 + (2.243 + y_8)^2\right] - C$$

$$SSBl = \frac{\sum_{j=1}^{b} T_j^2}{a} - C$$

$$SST = \frac{1}{3}\left[(2.093 + 0.667 + \ldots + 0.996)^2 + (y_1 + 1.258 + \ldots + 1.247)^2 + (y_2 + y_3 + \ldots + y_8)^2\right] - C$$

$$SSE = SST - SSTr - SSBl$$

$$SSE = y_1^2 + y_2^2 + y_3^2 + y_4^2 + y_5^2 + y_6^2 + y_7^2 + y_8^2 + 34.963$$
$$-\frac{1}{3}[(2.093 + y_1 + y_2)^2 + (1.925 + y_3)^2 + (2.590 + y_4 + y_5)^2 + (1.151 + y_6 + y_7)^2 + (2.243 + y_8)^2 + 50.87]$$
$$-\frac{1}{8}[124.14 + (y_1 + y_4 + y_6 + 6.574)^2 + (y_2 + y_3 + y_5 + y_7 + y_8 + 4.503)^2]$$
$$+\frac{1}{24}[22.219 + y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8]^2$$

Minimize SSE. Therefore, the partial derivative of SSE with respect to each *y* is equal to zero. For example,

$$\frac{\partial SSE}{\partial y_1} = 0$$

$$\frac{\partial SSE}{\partial y_1} = 1.1666y_1 - 0.5834y_2 + 0.08333y_3 - 0.1667y_4 + 0.08333y_5 - 0.1667y_6 + 0.08333y_7 + 0.08333y_8 - 1.1865$$

This can be repeated for the remaining seven derivatives, producing

$$0.669 = -0.5834y_1 + 1.1667y_2 - 0.1667y_3 + 0.08333y_4 - 0.1667y_5 + 0.08333y_6 - 0.1667y_7 - 0.1667y_8$$
$$0.5573 = 0.08333y_1 - 0.1667y_2 + 1.1667y_3 + 0.08333y_4 - 0.1667y_5 + 0.08333y_6 - 0.1667y_7 - 0.1667y_8$$
$$1.518 = -0.1667y_1 + 0.08333y_2 + 0.08333y_3 + 1.1667y_4 - 0.5834y_5 - 0.1667y_6 + 0.08333y_7 + 0.08333y_8$$
$$1.00 = 0.08333y_1 - 0.1667y_2 - 0.1667y_3 - 0.5834y_4 + 1.1667y_5 + 0.08333y_6 - 0.1667y_7 - 0.1667y_8$$
$$0.559 = -0.1667y_1 + 0.08333y_2 + 0.08333y_3 - 0.1667y_4 + 0.08333y_5 + 1.1667y_6 - 0.5834y_7 + 0.08333y_8$$
$$0.0413 = 0.08333y_1 - 0.1667y_2 - 0.1667y_3 + 0.08333y_4 - 0.1667y_5 - 0.5834y_6 + 1.1667y_7 - 0.1667y_8$$
$$0.769 = 0.08333y_1 - 0.1667y_2 - 0.1667y_3 + 0.08333y_4 - 0.1667y_5 + 0.08333y_6 - 0.1667y_7 + 1.1667y_8$$

The y values can be determined by solving this as a system of linear equations.

$$y_1 = 2.3455 \quad y_2 = 2.4343 \quad y_3 = 1.1776 \quad y_4 = 2.8424$$
$$y_5 = 2.9310 \quad y_6 = 1.4041 \quad y_7 = 1.4928 \quad y_8 = 1.3365$$

The various sum of squares can now be determined with these calculated values. Note that the degrees of freedom for SSE is correspondingly reduced by the number of calculated values.

$$C = 60.748 \quad SST = 9.685 \quad SSTr = 8.211 \quad SSBl = 0.5026 \quad SSE = 0.9716$$
$$v_{SSE} = (8-1)(3-1) - 8 = 6$$

Yate's algorithm was also used to determine the individual sum of squares for each run. Following this, the significance or nonsignificance of each factor or interaction was determined.

YATE'S ALGORITHM

For Tetra-Pak Data Analyzed with Missing Anova Value Technique

Experimental Condition	Total	Column 1	Column 2	Column 3	I.D.	SS
1	6.8728	9.98	17.88	38.18	*I*	60.752
a	3.1026	7.91	20.30	-10.25	*A*	4.379
b	4.674	12.41	-5.21	-6.59	*B*	1.810
ab	3.235	7.89	-5.04	5.92	*AB*	1.459
c	8.3634	-3.77	-2.07	2.42	*C*	0.243
ac	4.049	-1.44	-4.52	0.17	*AC*	0.001
bc	4.308	-4.31	2.33	-2.46	*BC*	0.252
abc	3.5795	-0.73	3.59	1.25	*ABC*	0.066
					Sum (not including SS of I)	8.210
					SS Replicate	0.5026
					SS Error	0.972
					SS Total	**9.684**

FIGURE B1.16: Yate's Algorithm for Tetra-Pak Data

Source of Variation	*Deg. of Free., d.f.*	*SS*	*MS (SS/d.f.)*	F_{calc} = *MS/MSError*	*Sig. @ 0.05?*	*Sig. @ 0.01?*
Replicates	2	0.5026	0.251	1.549	*No*	*No*
Size	2	4.379	2.190	13.519	*Yes*	*Yes*
Shape	2	1.810	0.905	5.586	*Yes*	*No*
Size/Shape	4	1.459	0.365	2.253	*No*	*No*
Geometry	2	0.243	0.122	0.753	*No*	*No*
Size/Geom.	4	0.001	0.00025	0.00154	*No*	*No*
Shape/Geom	4	0.252	0.063	0.389	*No*	*No*
3-factor	6	0.066	0.011	0.0679	*No*	*No*
Error	6	0.972	0.162			

Numerator d.f.	*Denominator d.f.*	*Significance Level*	*Tabulated F-value*
2	6	0.05	5.14
4	6	0.05	4.53
6	6	0.05	4.28

Numerator d.f.	*Denominator d.f.*	*Significance Level*	*Tabulated F-value*
2	6	0.01	10.92
4	6	0.01	9.15
6	6	0.01	8.47

TABLE B1.8: Final ANOVA Results for Tetra-Pak Tests