UNIVERSITY OF ALBERTA

OPTION PRICING: THEORETICAL AND EMPIRICAL ISSUES

BY

DONALD A. CYR

A thesis sumbitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

IN

FINANCE

FACULTY OF BUSINESS

EDMONTON, ALBERTA

SPRING 1992

ISBN 0-315-73109-5

UNIVERSITY OF ALBERTA

RELEASE FORM

NAME OF AUTHOR: DONALD A. CYR

TITLE OF THESIS: OPTION PRICING: THEORETICAL AND EMPIRICAL ISSUES

DEGREE: DOCTOR OF PHILOSOPHY

YEAR THIS DEGREE GRANTED: 1992

9 Rutter Crescent
Saskatoon, Sask.
S7N 0W0

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled OPTION PRICING: THEORETICAL AND EMPIRICAL ISSUES submitted by DONALD A. CYR in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY IN FINANCE.

Dr. Giovanni Barone-Adesi

Dr. Prem Talwar

Dr. Randall Morck

Dr. Robin Lindsey

Dr. Paul Halpern

January 24, 1992

ABSTRACT

The three studies contained within this document represent the thesis requirement for the degree of Doctor of Finance at the University of Alberta. All three studies relate to the application of option pricing theory to valuation and empirical study.

The first study titled "An Application of Contingent Claims Analysis to Quasi-option Value", applies option pricing theory to the environmental resource issue of quasi-option value. An option pricing formula is developed for valuing the option implicitly associated with delaying irreversible development of a natural resource and a solution is provided for the critical asset ratio value at which development should take place.

Recently, the documenting of several calendar seasonalities in returns have cast doubt upon the hypothesis of market efficiency. A possible explanation for these anomalous high returns is the presence of a corresponding seasonality in risk. The second study in the series titled "A Test of Calendar Seasonalities in Stock Market Risk Implied from Options Markets" employs implied volatilities from options on Standard and Poor's 500 index futures and options on the Toronto Stock Exchange 300 and 35 indices to examine seasonality in market risk. In particular a test of calendar seasonalities in the time series of implied volatilities is performed utilizing multiple input intervention analysis. In general it is concluded that seasonalities in ex-ante market risk do not, as has been postulated, explain the observed return seasonalities.

The third study contributes to the growing literature on the effect of options and futures related trading on the underlying asset market at the time of expiration. "The Effect of Expiring Index Futures and Options on the Toronto Stock Exchange 35 Index" documents the presence of anomalous price changes in the level of the Toronto Stock Exchange 35 index, at the time of options and futures expiration. These observed prices changes occur despite settlement procedures designed to mitigate such effects.

Acknowledgement

I would like to acknowledge the help, advice, and time and effort of each of my committee members-- Giovanni Barone-Adesi, Prem Talwar, Randall Morck, Robin Lindsey and Paul Halpern. I would also like to thank Youngsoo Kim, Stephen Beveridge and Robert Korkie for their helpful comments.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I: INTRODUCTION

Most researchers of financial economics would currently suggest that the development of modern option pricing theory ranks as one of the landmarks of the field, alongside the advent of Markowitz's portfolio theory, the capital asset pricing model of Sharpe and Lintner, and the contribution of Modigliani and Miller to the question of capital structure and dividend policy. Since Black and Scholes' seminal article on option pricing in 1973, the number of researchers and consequent endeavors committed to the study of options and option pricing theory has grown at an immense rate. Mirroring this growth in research has been a similar growth in the extent and importance of markets for derivative securities in the economy. As well, the application of option pricing theory or more generally contingent claims analysis, has revolutionized the way in which problems deemed to be the realm of corporate finance are framed. Indeed the option pricing framework can be credited with the development in recent years, of a much needed internal consistency to the area of corporate finance.

In general, research in the area of option pricing or contingent claims analysis can be roughly classified into the following three categories:

1) The continued improvement in the theoretical pricing of options on various market traded assets including advances in the empirical testing of option pricing models, as well as related tests of market efficiency.

2) The growing area of empirical research into the impact of options markets on the markets for underlying assets and other derivative securities such as futures.

3) The application of the technology of option pricing to valuation problems in corporate finance and other financial and real markets.

The three research papers included in this document, which collectively represent the thesis requirement for the designation of Doctorate in Business Management at the University of Alberta, can be categorized as contributions to the above area of option pricing and contingent claims analysis. Indeed each paper can be clearly identified with one of the three aforementioned categories of options research.

Paper No. 1, entitled "An Application of Contingent Claims Analysis to Quasi-Option Value" exemplifies the research in category three above, of the application of option pricing theory to valuation and decision making problems involving real assets. This paper represents a contribution to the recent but growing set of literature which applies contingent claims analysis to the question of irreversible investment.

Paper No. 2, "A Test of Calendar Seasonalities in Stock Market

Risk Implied from Options Markets", characterizes some of the recent research involved in the first category. Here volatilities implied by an option pricing model are examined for calendar seasonalities, as a possible explanation for various return anomalies which have in recent years cast doubt on the general hypothesis of market efficiency.

Finally Paper No. 3, "Effect of Expiring Index Options and Futures on the Toronto Stock Exchange 35 Index", examines the impact of options and futures trading on the market for underlying instruments in the case of the Toronto Stock Exchange 35 index. The contribution of this paper to the second category above follows in the vein of recent research concerned with the short term impact of options and futures trading at the time of their expiration, upon that of the underlying asset.

CHAPTER II: AN APPLICATION OF CONTINGENT CLAIMS ANALYSIS TO QUASI-OPTION VALUE.

I. INTRODUCTION

The purpose of this paper is to apply the theory of contingent claims analysis (CCA), developed in the past two decades in the financial economics literature, to the resource and environmental economics concept of quasi-option value.

The concepts of option value and quasi-option value have been a source of considerable debate in the environmental economics literature, and although we will concentrate specifically on the problem of quasi-option value, confusion as to their exact relationship warrants the brief exposition of both concepts, presented in section II. In section III a general introduction to the theory of CCA and its recent applications to topics in economics and capital budgeting will be presented. In Section IV a contingent claims model of quasi-option value is developed by relating the concept to Margrabe's (1978) financial option to exchange one risky asset for another. Unlike the European option pricing model developed by Margrabe however, quasi-option is inherently American in nature, with the potential for early exercise. Given that a closed form solution for this class of problems has noc to date been identified, an approximate solution for the American option to exchange one risky asset for another is derived employing the quadratic approach of MacMillan (1986) and Barone-Adesi and Whaley (1987). The comparative statics of the model is then explored in terms of the explicit variables required for its solution.

It will be argued that the valuation formula specified by the model is clearly an advantage compared to current methodologies such as contingent valuation, involving survey techniques. In addition the valuation formula involves the specification of a decision rule in the form of a critical asset ratio value indicating the optimal time for the development of a natural environment. A summary of results is provided in Section V.

II. QUASI-OPTION AND OPTION VALUE

A major concern of the field of resource economics is the efficient allocation of natural resources and environments. If accomplished properly, cost-benefit analysis will correctly identify such an allocation and hence is undoubtedly one of the most important quantitative tools providing guidance to decision makers concerned with this endeavor. In many circumstances however the tool of cost-benefit analysis, or more generally the approach of discounted net benefits, is subject to severe limitations. It is the recognition of these shortcomings that gave rise to the environmental and resource economics concepts of quasi-option and option value.

These two concepts refer in a very broad sense to benefits that accrue to society and which are not accounted for in the traditional cost-benefit analysis framework. Although the purpose of this paper is to apply the theory of contingent claims analysis in valuing quasi-option value, the debates concerning quasi-option and the related concept of option value are highly intertwined and hence a brief review of both is in order. We will begin first by examining the literature concerning the relatively more basic concept of option value.

A. Willingness to Pay

In general cost-benefit analysis is a method of measuring welfare changes, when evaluating the impact of public policies, and hence it requires the identification of the total net benefits accruing to society from a particular decision concerning a good or asset(s). The measure of these benefits, that accrue from a good or asset, is theoretically the willingness of the consumer to pay for the good.

The accepted measure of this willingness to pay, in today's prices, is attained through measuring equivalent variation which uses status quo prices as the base case and asks what is the income change at current prices that would be equivalent to the proposed change. In practice, this is usually measured by expected consumers' surplus which is the difference between the maximum amount the consumer is willing to pay and the amount he or she actually pays. Expected consumers' surplus is, in many cases, an appropriate measure of the net benefits accruing to society from a particular asset with the standard procedure involving the estimation of current demand for the good and then projecting this estimate into the future.

It has long been recognized, however, beginning with Weisbrod (1964) and Krutilla (1967), that in some circumstances consumers' surplus is not an accurate measure of consumers' willingness to pay. In particular, significant benefits classed as "non-user" may be overlooked by this approach.

B. Non-User Benefits

In general, total benefits accruing to society include not only user benefits, which refer to benefits directly associated with the actual use of an asset, but also to a class of benefits termed non-user. Weisbrod (1964) and Krutilla (1967) first identified the concepts of non-user benefits when examining the decision as to the development, or alternative use of a natural environment.

These benefits, associated with the preservation of the natural environment or park, and which are not captured by consumers' surplus include: a) existence benefits; which represent the amount consumers are willing to pay for the knowledge that a natural environment is protected by wilderness designation, even though no recreational use is contemplated, b) bequest benefits; measured by the willingness to pay for the satisfaction derived from endowing future generations with the wilderness resource, and c) the benefits associated with the concept of option value, first identified by Weisbrod (1964).

Essentially Weisbrod argues that where consumers are uncertain about their future demand for a good, there may be value in retaining the option to consume the good in the future. Weisbrod developed this concept in response to Friedman (1962) who advocated the closing down of a national park if the commercial value of its lumber or minerals exceeded the willingness to pay, on the part of consumers, for the recreational benefits obtained from the park. Friedman argued that any park or wilderness area should be closed, and its resources allocated to other uses, when benefits to recreation users fall short of opportunity costs. Expected consumers' surplus in such circumstances would be calculated by estimating the current demand for the park. Weisbrod proposed that the willingness to pay on the part of recreational users of the park understates its value to society because of the existence of individuals who expect that they may possibly visit the park in the future and who would consequently be willing to pay for an option that would guarantee their future access. He argues that the amount consumers are willing to pay to preserve this option is a magnitude distinct from traditional consumers' surplus, but which should be included in any social benefit-cost analysis which involves the irreversible destruction of the park.

Since its inception, the concept of option value has been embroiled in considerable debate as to its precise definition and measurement. Bishop (1982) and Smith (1983) provide relatively recent syntheses of this debate as well as contributions of their own. In general the issues of concern have centered around 1) the appropriate measure and use of option value 2) the relationship of this measure to that of expected consumers' surplus, and 3) the nature of the uncertainty that results in option value's existence.

Much of this debate as to the precise definition and measurement of the concept of option value can be attributed to the rather informal manner by which Weisbrod developed his argument (Hanemann (1989)). Indeed Weisbrod used the term option value to refer to what is now in the prevailing terminology deemed "option price". This terminology, which distinguishes between option price and option value was first developed by Krutilla et al (1972). In terms of this framework, option price can be roughly defined as measuring both the value of retaining an option to consume a good and the expected value of actual consumption. Expected consumers' surplus alternatively measures only the latter. The difference between option price and expected consumers' surplus is what is termed option value (Plummer and Hartman (1986)).

C. Theoretical Interpretation and Sign of Option Value

The most dominant interpretation of the concept of option value is that of a form of risk premium. This interpretation was first developed by Cicchetti and Freeman (1971) and refined by, among others, Schmalensee (1972), Bohm (1975) and Graham (1981). Although some debate has centered around the appropriate definition of risk aversion (Smith (1983)) it is generally agreed that this risk premium arises due to the uncertainty as to the potential future value of the park if it is preserved.

Related to this interpretation of option value in a utility theoretic framework is the debate concerning the sign of option value. In general this discussion has centered around; 1) under what conditions or assumptions concerning preferences is the sign of option value positive (Freeman III (1984)), and 2) the resulting extent and direction of bias in using consumers' surplus as a measure of net benefits. In addition some authors have argued that the debate concerning the sign of option value is superfluous and that the most important consideration is whether option value or option price is the appropriate measure for researchers to attempt to value when considering alternative risky projects (Graham (1981), Smith (1987), Cory and Saliba (1987)).

D. Nature and Source of Uncertainty

A number of authors have examined the effect of the nature of the uncertainty concerning the future benefits of a park on the concept of option value (Plummer and Hartman (1986)). For example, demand uncertainty may arise due to 1) income uncertainty; having enough income to visit the park in the future, 2) quality uncertainty; uncertainty as to the quality of visits to the park, due perhaps to crowding, or weather, and 3) uncertainty concerning preferences. In general these analyses are concerned with the effect, if any, of the source of uncertainty on the sign of option value.

In addition to framing the concept in terms of the uncertainty of demand for the park faced by consumers, some authors have examined the nature of option value when uncertainty of supply is an issue (Bishop (1982), Freeman III (1985), Plummer (1986), and Segerson (1987)). In this framework the consumer is certain of demanding the services of an environmental asset in the future, but uncertain about its future availability. Bishop (1982), for example considers the scenario where a project under consideration involves the discharging of a known carcinogen into the Great Lakes water way. It is unknown whether this carcinogen will undergo a bioconcentration process but if so recreationally caught fish will be unfit for human consumption and the fishery will have to be closed indefinitely. Bishop argues that expected user benefits alone would be an underestimate of how much anglers would be willing to pay in order to avoid the risk to the fishery. He argues that anglers would be willing to pay an amount to retain the option to continue fishing in the Great Lakes.

In general the literature to date would suggest that the concept of option value exists under conditions of uncertainty as to either future demand and/or supply. (Walsh, Loomis and Gilliman (1984)).

E. Quasi-Option Value

Related to the concept of option value, and the issue upon which we will concentrate, is that of quasi-option value. First introduced by Arrow and Fisher (1974), and Henry (1974), quasi-option value is essentially defined as the positive benefit derived from delaying the development of a natural resource when there is uncertainty as to future benefits and costs, and development irreversibility. The concept of quasi-option value is usually couched in a time-sequenced framework where greater information about future benefits and costs is revealed in later periods.

Essentially the argument made by Arrow and Fisher is that the expected benefits of delaying irreversible development should include the benefit due to the retention of an option to not develop in the following period. This benefit, also classed as non-user in nature, is positive in value. In general, quasi-option value represents the adjustment that should be made to expected benefits to reflect the loss of options in situations where an irreversible decision is made and where the future will reveal additional relevant information (Plummer and Hartman (1986)). Given the significance of their work it is appropriate to present a brief review of Arrow and Fisher's initial example concerning quasi-option value.

F. The Arrow and Fisher Example

Arrow and Fisher (AF) focus on the decision as to how far, if at all, to proceed with the commercial development of an unspoiled natural area that yields benefits in its preserved state. The basic question asked is whether the introduction of uncertainty, as to the costs or benefits of a proposed development, has any effect on the formulated investment criterion beyond the replacement of known values with their expectations. They conclude that the expected benefits of an irreversible investment should be adjusted to reflect the loss of options it entails. Assuming risk neutrality in their model they find that quasi-option value is positive in sign and unlike in the theory of option value, does not depend upon an assumption of risk aversion.

Explicitly, AF consider the choice, at each moment in time, between preserving a virgin redwood forest for wilderness recreation, or opening it up to clear-cut logging. Although this transformation is technically reversible they argue that the length of time required for forest regeneration is so great that given a positive rate of time preference, it is effectively irreversible.

In their model the development of an area takes place over a two-period time horizon consisting of a first period followed by all future intervals compressed into a single second period. They assume no time discounting, for simplicity, and that development entails investment costs but preservation does not. The real difference between the alternatives of preservation or development, in their model, is that the former is assumed to be reversible and the latter not. Some amount of development is planned at the start of the first period, but the plan can be revised at the start of the second period, based upon information that has accumulated concerning benefits in the first period. Lastly, they assume that there are constant returns to any level of development or preservation, resulting in all benefits and costs specified as coefficients multiplied by the area developed.

The decision maker has expectations over the various benefits and costs in the two periods, and AF adopt a Bayesian information structure and a decision framework that allows deferral of the second period development decision until the beginning of period two. Hence, expected benefits in period two are conditional on what occurs in period one. If at the beginning of period two the expected net development benefits are greater (less) than expected preservation benefits, the remaining potential development will (will not) occur. What AF desired is a decision criterion for deciding on the amount of development to undertake in period one. In this framework information about the future consequences of development would arrive with time, independent of the development decision itself. AF show that when this possibility of acquiring information is recognized, there is a stronger case for postponing irreversible development than when no such possibility exists. They refer to the value associated with delaying development, as quasi-option.

G. Relationship of Quasi-Option and Option Value

The exact relationship between the concepts of quasi-option and option value has been frequently debated in the literature and, as of yet, is unresolved. Again part of the confusion is due to a disagreement concerning terminology. Unfortunately Henry (1974) who simultaneously developed the concept of quasi-option value along with Arrow and Fisher (1974), referred in his work to option value, what is generally now accepted as quasi-option value. This interchange of nomenclature, however, continues to appear in the literature. (Hanemann (1989))

Some authors (Hanemann (1989)) view the difference between these two concepts as being simply two broad interpretations of Weisbrod's (1964) original option value concept, whereas others interpret it as largely the difference between a "timeless", and "time-sequenced" modeling framework (Smith (1983)). Still others (Miller (1981)) have suggested that there is little in common between the two concepts.

Conrad (1980) remains the most significant article to attempt a reconciliation between option value and quasi-option value. He frames the question in terms of the theory of the value of information, and suggests that whereas option value can be viewed as the expected value of perfect information, quasi-option value is the expected value of information derived with the delay of development. A problem arises however, in that his model is set in a two period inter-generational framework which clouds the issue by introducing the existence of nonuser bequest benefits.

H. Sign of Quasi-Option Value and Dependent Learning

As in the case of option value, a substantial literature has been devoted to determining the sign of quasi-option value. The debate however has been significantly more one-sided, as it is generally agreed that quasi-option is non-negative in value.

The debate concerning sign has been intricately related to extensions of the traditional analysis of quasi-option value. In the traditional analysis, information concerning the future consequences of development arrives independently. Several authors however have relaxed this assumption and considered the case where information arrival is not independent of development, and the resulting effect on the size and sign of quasi-option value (Miller and Lad (1984), Freeman (1984), Fisher and Hanemann (1987), Crabbé (1987)). This assumption allows for the possibility of acquiring information concerning the consequences of development from carrying out an initial small amount, and is referred to as "dependent learning" (Fisher and Hanemann (1987)). In general, it has been determined (Fisher and Hanemann (1987)) that if the major source of uncertainty pertains to the benefit of preservation, then the case for preservation is further strengthened under dependent learning. Alternatively if the uncertainty concerns the benefits of development, then the case for development is enhanced.

Somewhat related to the studies of dependent learning is that of Viscusi (1988) who examines the case where later expansion of development is more costly than carrying out large scale development initially. Understandably this results in greater initial development in order to avoid "upward adjustment" costs.

III. CONTINGENT CLAIMS ANALYSIS AND ITS APPLICATIONS

A. Contingent Claims Analysis

Recently the Journal of Economic Perspectives included an article by Mark Rubinstein (1987), a major contributor to the theory of contingent claims analysis, which outlined some of the basic concepts of the area, and its growing contribution to the fields of finance and economics. This paper marks the growth of contingent claims analysis, which has its roots in the early work of Arrow (1964) but which for the most part began with the seminal work of Black and Scholes (1973).

Essentially contingent claims analysis (CCA) is a general methodology for determining the price of an asset whose payoff depends upon the prices of one or more other assets. The technique is fairly flexible and its applications wide-ranging. It has been used to determine the value of a number of complex financial instruments or securities, and has recently been extended to the valuation of real assets. The foundation of CCA lies in the recognition of various assets as combinations of simple option contracts, and although this technique is independent of the option pricing model used, it is generally associated with option pricing in the Black Scholes framework.

Although models of option pricing were developed early in the century, the significance of the Black and Scholes model is that it is conceived entirely in theory and has had an immediate and significant impact on financial markets. In addition the Black Scholes model is relatively easy to use and has been substantially validated by empirical research. In general, Black and Scholes derive a partial equilibrium model of option pricing that avoids the restrictive assumptions on individual risk preferences and market equilibrium price formation that have been the main drawbacks of previous models.

In its basic form the Black Scholes framework relies on the following assumptions:

A1) Markets are frictionless in that there are no transaction costs or differential taxes. Trading takes place continuously in time. Borrowing and lending are unrestricted and the borrowing rate equals the lending rate. Short sales are unrestricted with full use of proceeds.

A2) the riskless short term rate $r(t)$ is known over time.

A3) The dynamics for the underlying asset value, upon which a contingent claim depends, can be described by a diffusion type stochastic process.

Given these assumptions, Black and Scholes (1973) and Merton (1973) demonstrated that in terms of corporate securities a dynamic portfolio strategy can be derived that involves mixing positions in the firm (the underlying risky asset) with a risk-free asset to produce patterns of return that can exactly replicate the return to any corporate liability of the firm. This replicating portfolio is continuously adjusted (rebalanced) in response to changes in the value of the underlying asset and the passage of time. The continuous application of this replication argument results in a fundamental partial differential equation that must be satisfied by the prices of all of the firm's liabilities. The solution of this partial differential equation (PDE) subject to boundary conditions pertaining to the particular liability of interest results in an explicit valuation formula. The key factor in the Black Scholes framework is that the resulting PDE does not depend on the expected rate of return on the underlying asset, and hence does not require any assumptions about investors' risk preferences.

In the particular case of simple European options (options that can only be exercised at the maturity date of the option contract) as dealt with by Black and Scholes a closed form solution to the PDE subject to the appropriate boundary conditions exists. In other more complex cases including certain American options (where the option can be exercised at any time up to and including maturity) and options involving several underlying assets, a closed form solution may not exist. In such circumstances however, much work has been done in terms of approximating solutions through the use of such techniques as numerical analysis. These avenues of research have all been evolutionary in the development of the theory, and the basic structure of contingent claims analysis has remained intact. Even in complex cases when numerical solutions are not available, an options-based framework can offer significant payoffs in terms of qualitative understanding.

This basic analytical framework of Black Scholes has been extended in many ways and applied to many problems in finance and economics. Although many of these extensions and applications have involved the relaxation of the various basic assumptions the robustness of the theory has been encouraging.

B. Application to Valuation of Real Assets.

As we have noted, at first the theory of CCA and Black Scholes option pricing was applied with remarkable success to the valuation of financial options as well as other derivative financial securities. Recently however, significant extensions have been made in the area of applying contingent claims analysis to the valuation of real assets.[1] Examples of the diversity of these applications include Brennan and Schwartz (1985) on the valuation of a mine, Abowd and Manaster (1983) on valuing employment contracts, and Dothan and Williams' (1981) valuation of education as an option.

Applications in the area of capital budgeting have arose largely due to the shortcomings of traditional net present value analysis. These shortcomings are analogous to those associated with cost-benefit analysis. As with the tool of cost-benefit analysis, managers of corporations are dissatisfied with the current state of capital budgeting or net present value analysis because of its inability to recognize the additional value in projects resulting from "operating

---

[1]Mason and Merton (1983) and Rubinstein (1987) provide reviews of these applications.

flexibility". This "operating flexibility" associated with a project results from manangement's operating options or ability to revise a project at some future time, and the "strategic value" of a project resulting from its interdependencies with follow-up investments. (Trigeorgis (1988)).

Traditional NPV analysis makes implicit assumptions concerning an "expected scenario" of cash flows and presumes that decision makers irrevocably commit to a certain "operating strategy". Typically, the expected pattern of cash flows over a prespecified life is discounted at a risk-adjusted rate to arrive at the project's NPV. In the absence of managerial flexibility or real options, static NPV analysis would be correct. In the real world of uncertainty, however, the realization of cash flows will most likely differ from the decision maker's expectations. Over time new information arrives and uncertainty about future cash flows is gradually resolved; hence flexibility to revise the operating strategy originally anticipated has value. Recent work in the area of capital budgeting has centered around attempts to bridge this gap between traditional financial theory and strategic planning through the use of option valuation techniques or contingent claims analysis.

Essentially the decision maker's flexibility to adapt future actions to the future environment skews the probability distribution of NPV, relative to initial expectations, by improving the upside potential while limiting down side losses. As Trigeorgis (1988) indicates, this necessitates the use of an expanded NPV framework which includes a premium for the flexibility inherent in a project's operating options as well as the traditional static NPV of direct cash flows:

Expanded NPV = Static NPV + Option Premium.

Other approaches to overcome the problems of static NPV analysis such as simulation and decision tree analysis basically stumble on determining the appropriate discount rate with which to discount future cash flows (Ritchken and Rabinowitz (1988)). In a Black Scholes option pricing framework however the no-arbitrage hedge attainable under continuous trading allows a solution that is independent of risk attitudes and therefore may be conveniently obtained in a world of risk neutrality, circumventing the discount rate issue.

C. Reasonableness of Underlying Assumptions

All models abstract from the complexity of reality. The art of model building is to choose those abstractions which make the model tractable while capturing the essence of the real world environment in which it is to be applied. The basic Black Scholes model is certainly tractable, and in the case of financial assets which are traded on markets, the assumptions are fairly realistic. Understandably these assumptions are subject to criticism in the cases of real assets and financial assets not traded on public markets. This is certainly true in the application to quasi-option value postulated here. As with applications to corporate finance, this lack of public trading not only casts doubt on the assumption of the formation of a riskless arbitrage portfolio, but relates as well to the major drawbacks of CCA which have been the unobservability of the price of the underlying asset and the consequent problem of estimating the variance of the rate of return on the asset. In some cases however the argument can be made for the existence of an alternative traded asset whose return closely follows that of the unobservable asset and from which an estimate of return variance can be made.

Current methodologies for measuring option and quasi-option values however, referred to in the environmental literature as "contingent valuation", ask the consumer to assume markets exist for the benefit to be measured and then ask them to price the option value of interest. Consequently contingent claims analysis does not involve any more restrictive a set of assumptions than these current methodologies of practical valuation.

IV. A CCA MODEL OF QUASI-OPTION VALUE

A. Related Studies

A number of contributors to the literature of quasi-option value have recognized its potential relationship to the financial options literature but to date an explicit valuation employing CCA has not been attempted.

Crabbé (1985) in a discussion of the concepts of option value and quasi-option value relates the asymmetry in quasi-option value to that of financial options. This asymmetry exists, as Crabbê indicates, in the sense that only one tail of the probability distribution of the underlying variables affects the value. Although he recognizes the similarities in quasi-option value and financial options, he does not deal explicitly with CCA.

Hanemann (1989) makes a more accurate analysis of the relationship of quasi-option value and financial option theory. In a two period framework where development can take place in either period, he states that when development is not undertaken in the first period the result is the preservation of an option to develop in the future. However he incorrectly relates the value of a financial option to the total benefits of a decision not to invest in the first period.

Studies in a CCA framework which relate to a quasi-option application do exist, and are largely found in the emerging finance literature on irreversible investment. Recently Pindyck (1991) provides a concise review of this literature including a number of applications specifically in the contingent claims framework. Two such works include Majd and Pindyck (1987) and McDonald and Siegel (1986). Traditionally, discounted cash flow methods which treat the pattern of investment as fixed ignore the fact that the rate at which construction proceeds is flexible. The focus of Majd and Pindyck (1987) is consequently on a series of expenditures that must be made sequentially but which cannot exceed some maximum rate. They concentrate on the determination of optimal investment rules and maximum construction rates for a project, and model the problem as a series of compound options in which each unit of investment buys an option on the next unit.

As Majd and Pindyck note, this option is distinct and separate from the concept of quasi-option value. As opposed to the option a firm has in making sequential investments, quasi-option value relates to the option associated with a choice of projects that is foregone once the expenditure has been made. Similar to Majd and Pindyck however, we will address the opportunity cost associated with delaying the investment in a project, and will consequently develop an optimal investment rule or critical asset level at which investment will take place.

Closer in concept is the work of McDonald and Siegel (1986). This study addresses the optimal timing of investment in an irreversible project where, similar to our framework, the project and investment "cost" follow continuous time stochastic processes. In addition the case we will consider is similar to McDonald and Siegel's in that investment is lumpy in the sense of taking place instantaneously, as opposed to the sequential framework of Majd and Pindyck. Unlike the general equilibrium approach, however, used to develop the model in McDonald and Siegel, the model derived here is based upon a continuous hedging argument employed in option pricing theory. As well, we consider the case of finite-lived options as opposed to the infinitely lived option to invest that McDonald and Siegel consider. Consequently we are able to obtain a closed form approximate solution to the option valuation problem, whereas McDonald and Siegel do not.

The major drawback of the general equilibrium approach employed by McDonald and Siegel is that it requires a complete specification of an underlying model of capital equilibrium whose parameters are known. (See for example the framework developed by Cox, Ingersoll, and Ross (1985).) Hence a general limitation of this approach is that it is difficult to obtain refined estimates of the market model parameters.

Alternatively we adopt the approach of a continuous hedging argument described below, and which relies on the assumption of complete markets and continuous trading in assets. In particular this approach is based upon the formation of a risk free portfolio, requiring that one can trade in assets such as a mine or a national park.

Obviously this assumption is very strong and unrealistic, however the analysis still holds if we assume that there at least exists some other asset or portfolio of assets which is traded and whose price is perfectly correlated with the price of the mine or park. If the mine can be considered a simple function of a mineral or commodity price such as gold, this assumption is not unrealistic. In terms of the value of a national park, which depends upon society's consumption preferences we may perhaps assume that a portfolio of common stock of firms involved in the production or provision of recreational goods and services such as motor homes and camping equipment, may perhaps span the risk associated with a national park's value.

B. The Model

To be specific, let us consider the case of a natural environment or wilderness area which the government must decide whether to preserve as a national park or to allow development, say of a hydro-electric dam, mine, or other resource based project. Hence, the preservation and development decisions are mutually exclusive. Let us assume as well, that the project would be operated by the government and would yield benefits to society equivalent in monetary terms to those that would accrue to a private owner. In addition, for simplicity and in order to focus solely on the value of the option to delay investment, we will assume that investment in the construction of the facility takes place all at one time and that the net cash flows that arise from the facility begin immediately. Hence, as in Hanemann (1989) development is indivisible and irreversible.

Let M be equal to the present value at time t of the expected future net cash flows conditional on undertaking the project. This present value represents the appropriately discounted expected net cash flows, including initial investment, given all information available at time t. Hence M represents the market value of the completed project, which is the market value of a claim on the stream of net cash flows that arise from installing the facility at time t. We will make the assumption similar to Majd and Pindyck (1987) and McDonald and Siegel (1986) that the value M of the completed project is given exogenously and follows a particular stochastic process through time, that of geometric Brownian motion:

$$dM = (\alpha_M - D/M)\, Mdt + \sigma_M M dZ_M. \tag{1}$$

where dZ_M is the increment of a standard Wiener process. The central feature of this assumption is that M fluctuates stochastically over time, reflecting the arrival of new information concerning the future cash flows. The last term in equation (1) represents the unexpected component of changes in M. The parameter α_M is assumed constant and represents the expected rate of return on the project. This expected rate of return is the equilibrium rate established by the capital market.

The parameter D represents the opportunity cost of not investing in the project. Specifically, D represents the net cash flows from operating the completed facility, and is a deterministic function of the value of the project and time. These cash flows are net of any future investments required to maintain the productivity of the project and are not received unless the project is undertaken. They are analogous to the wages forgone by a student in school as in the Dothan and Williams (1981) option model of education.

A more elaborate model such as that of Brennan and Schwartz (1985) involving assumptions concerning the dynamics of output prices, operating costs, and the output rate of the project, could be developed. These assumptions would indeed outline a more complete model but would involve greatly increased complexity.

Recent works in the area of capital budgeting have focused on the various operating options available to managers once a project is undertaken. Equation (1) is not consistent with the existence of such options, as including their value will generally affect the dynamics of M. In particular M would not follow a lognormal process in their presence (Majd and Pindyck (1987)). Although it is possible to carry out the analysis with the presence of these implicit options assumed, this would again greatly increase the complexity of the model and would detract from our purpose of concentrating on the value of the option to delay the construction of the project.[2]

The above assumption for the stochastic process of the project value can be seen to be analogous to the usual assumption, in the case of financial option valuation concerning the stochastic process for

---

[2] Mason and Baldwin (1988) provide an example detailing how these operating options would affect valuation.

the value of a common stock which pays dividends. Indeed if the project was undertaken by a firm financed solely by equity where the project was its only asset and if cash flows from the project were paid out to shareholders as dividends, the value of the equity of the firm would follow the above process given in (1).

Now let us consider the benefits that accrue to society from preserving the park. In particular let P equal the present value at time t of the future benefits of preserving the park given all information available at time t. As in the case of the project these benefits are discounted at the appropriate discount rate as in the standard cost-benefit analysis framework. Future benefits associated with the preservation of the park are of course uncertain and hence the present value of the benefits is assumed to follow the stochastic process:

$$dP = \alpha_P P dt + \sigma_P P dZ_P \qquad (2)$$

where α_P represents the instantaneous expected growth rate of the present value of park benefits, and σ_P the instantaneous variance of the relative change in the present value of park benefits. Again, dZ_P is the increment of a standard Weiner process.

Analogous to the case of the mine value, this assumption rests upon the idea that the value of the park fluctuates stochastically over time reflecting the arrival of new information concerning the benefits of the park to society. This process through time is captured by equation (2) with the first term $\alpha_P Pdt$ representing the expected change in the park value over a unit of time, and the second term $\sigma_P PdZ_P$ representing the unexpected portion of the change in P. In addition let ρ_{MP} be the instantaneous correlation coefficient between dZ_P and dZ_M. Consequently $dZ_M dZ_P = \rho_{MP} dt$, where ρ_{MP} is assumed constant through time.

Given the above assumptions of stochastic processes for the underlying variables, we are implicitly considering the case where future information is independent of development. Hence we do not consider the possibility of dependent learning where a little development generates information about the future consequences of further development.

In this framework the problem the policy maker faces is simultaneously analogous to holding a call option on the present value of the project, where the exercise price is the present value of the park benefits and is stochastic, and a put option on the value of the park with a stochastic exercise price equal to the value of the project. Hence this option q(M,P,T) is a function of the project value, the value of the park benefits, and the time to maturity T of the option. As in Crabbé (1985) we can postulate a scenario where the option to develop exists for a finite length of time. In particular the example Crabbé considers is the diversion of the Kootenay River in

British Columbia for the purposes of hydroelectric power generation. The option to do so existed or was held by the Canadian government for a fixed period of time as set out by treaty agreement with the United States.

Given certain assumptions and the theory of contingent claims analysis we can value the option q. As has been indicated the derivation of the partial differential equation (PDE) which q must satisfy, relies crucially on the assumption of continuous trading of the underlying assets of the contingent claim in question. It is appropriate to again address the validity of this assumption as in the case outlined above, trading in financial markets is questionable particularly in terms of the present value of park benefits. The theory of CCA however, will still be valid as long as the capital market is sufficiently complete that the existence of the assets of concern does not change the opportunity set available to investors. For example, undertaking a large scale project that would change the future price of oil changes the opportunity set and consequently CCA cannot be employed as the project's value would be ambiguous. It would have one value given present prices and another value given prices that would prevail if the project were undertaken. Given the assumption of market completeness there always exists a portfolio of risky securities possessing returns perfectly correlated with the returns of the underlying assets.

In addition it should be noted that all capital budgeting procedures have as an objective the estimation of the value of a project which would be reflected in its price if it were traded in an efficient market. In this sense the CCA technique has the same limitation as any financial valuation method. If undertaking the project changes the opportunity set available to capital market investors then using either standard discounted cash flow techniques or contingent claim techniques will lead to an error in valuation. It should also be noted that projects with measurable impacts on prices and, therefore, the market opportunity set, are rare (Mason and Baldwin (1988)).

Employing conventional option pricing arguments it is possible to derive a partial differential equation (PDE) for the price of the option q. The derivation (given in Appendix (1)) is similar to that of previous derivations (Stulz (1982), Schwartz (1982)) for valuing contingent claims based on several underlying assets. The argument relies on no-arbitrage considerations and the use of Ito's Lemma. If the payout of an option can be perfectly replicated by a portfolio of traded securities, then by the law of one price the option price would be unambiguously determined by the current value of the replicating portfolio. Basically, a self-financing portfolio (one which does not require or produce cash payments until maturity) is constructed. Given that at maturity the value of this portfolio is precisely equal to that of the contingent claim, then to eliminate arbitrage profits the value of the portfolio must be equal to the option value at all points in time between now and the maturity date, if arbitrage profits are to be eliminated.

The PDE satisfied by q is:

$$(1/2)\sigma_M^2 M^2 q_{MM} + (1/2)\sigma_P^2 P^2 q_{PP} + \rho_{MP}\sigma_M\sigma_P MPq_{MP} + (rM-D)q_M + rPq_P - q_T - rq = 0 \quad (3)$$

and is appropriate for any contingent claim dependent on the same underlying assets of the project and the park. In general this PDE represents a restriction on the partial derivatives of the option value q and specifies the relationship of q to M, P, and T, that must hold given that the payoff structure of q can be replicated by a portfolio that involves investing in M, P, and the risk free asset.

Alternatively the above partial differential equation could be derived in a general equilibrium framework such as that of Cox, Ingersoll, and Ross (1985) with appropriate assumptions concerning preferences and technologies.Specifically, the assumption would have to be made that either consumers were risk neutral, or the expected growth rates of P and M were equivalent to the return on the risk free asset. In other words it would have to be that $\alpha_P = r$, and $(\alpha_M - D/M) = r$. Either of these assumptions would be extremely restrictive and surely not as palatable as the assumption of complete markets and continuous trading required for the derivation above.

C. European Option Value

Given the above partial differential equation the problem remains to solve for the value of the option, q, subject to certain terminal and boundary conditions dependent upon the nature of the option in question. These terminal and boundary conditions serve to give a unique representation to each contingent claim. For example both the European and American call option values are described by the PDE given by equation (3). For simplicity let us first consider the case of the European call option.

A European call option gives its owner the right, at the maturity date of the option and not before, to purchase a quantity of an asset at a prespecified price referred to as the exercise price. For example the holder of a European call option on a corporation's stock with an exercise price of $50.00 and a time to maturity of three months has the right, in three months time, to acquire a share in the corporation's stock, no matter what its value, at a price, or in exchange for, $50.00. This is analogous to the situation faced by the policy maker in that he holds a call option to invest in the project, and receive the net present value of its benefits in exchange for or at the cost of the present value of park benefits. Similar to the stock option holder forgoing the $50.00 in order to acquire a share in the corporation, the decision maker acquires the net present value of the project in exchange for the present value of the benefits associated with the park. The only difference being that the exercise price for the decision maker is stochastic in nature whereas it is a fixed value for the holder on the option on common stock. If we assume, for now, that he can exercise this right only at the time of maturity and not before, then the option he holds is European in nature.

Alternatively an American option is one that gives its holder the right to exercise the option at any time up to and including the final maturity date. An American option then, gives the holder the additional benefit of being able to exercise the option "early", if he so chooses. It can be shown (Merton (1973)) however, that in the case of an American call option on an asset that does not pay a "dividend", it is never optimal to exercise the option before the final maturity date and hence the American call can be valued as a European one.

If we assume that the option which the decision maker holds is European in nature, then the boundary condition at maturity that must be satisfied by the value of the option q is given by:

$$q(M,P,T = 0) = \max [M - P, 0] \quad (4)$$

This boundary condition specifies that at maturity (T = 0) the value of the option is equal to the maximum of the value of the project less the value of the park, or 0. This reflects the fact that exercising the option is a right and not an obligation. At maturity the decision maker will exercise the option and acquire the project for the value of the park only if the value of the project is greater than that of the park, at that time. Otherwise he will not exercise the option and its value is 0.

In general the PDE given by equation (3), subject to the boundary condition given by (4) will not have a closed form solution unless we assume that the cash flows received from the project are proportional to its value. In particular we will assume:

$$D = \delta M$$

where δM is the instantaneous rate of cash flow from operating the project and δ is assumed constant. This is not an unreasonable assumption as we would expect that the higher the cash flows from the project, the higher its market value. The partial differential equation, given this assumption is then:

$$(1/2)\sigma_M^2 M^2 q_{MM} + (1/2)\sigma_P^2 P^2 q_{PP} + \rho_{MP}\sigma_M\sigma_P MPq_{MP} + (r-\delta)Mq_M + rPq_P - q_T - rq = 0. \quad (5)$$

The solution of a PDE similar to that of equation (5), and subject to the boundary condition given by (4), for the value of the contingent claim q, has been developed by Margrabe (1978). He considered the problem of developing a valuation formula for the option to exchange one risky asset for another where there are no cash outflows or "dividends" from the assets. Among various problems he applies this solution to corporate share exchange offers, and the analysis of margin accounts. In the option we consider the decision maker can exchange the park for the project, however the project pays a "dividend", represented by its cash flows. Because of these cash flows it may be optimal for the option holder to exercise his option early in order to receive the cash flows from the underlying asset. We will consider for the moment that the option held by the decision maker is restricted to be European in nature. Consequently even though the project pays a "dividend" the decision maker cannot exercise the option until maturity. Analogous to Margrabe's solution, then, the value for q that satisfies (5) subject to the boundary condition (4) is given by:

$$q(M,P,T) = e^{-\delta T} MN(d_1) - PN(d_2) \text{ , where} \tag{6}$$

$$d_1 = \frac{\ln(M/P) + ((1/2)\sigma^2 - \delta)T}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T},$$

$N(\cdot)$ is the cumulative standard normal density function and

$$\sigma^2 = \sigma_M^2 + \sigma_P^2 - 2\rho_{MP}\sigma_M\sigma_P. \tag{7}$$

This solution differs from Margrabe's (1978) only by the term δ, as Margrabe does not consider the case of an opportunity cost associated with one of the assets, represented by a cash flow or dividend.

The term $e^{-\delta T} MN(d_1)$ represents, in a risk neutral world, the present value of the expected mine value M, at maturity, conditional on M being larger in value than P, times the probability that M will be greater than P. In other words, it represents the present value of the expected "benefit" of exercising the option in a risk neutral world, where the benefit derived is the receipt of the mine.

Alternatively the term $PN(d_2)$ represents, in a risk neutral world, the present value of the expected park value P, conditional on P being less than M at maturity, times the probability that M will be greater than P. In this sense $PN(d_2)$ represents the present value of the expected "cost" of exercising the option at maturity, in a risk neutral world, where this cost is due to the fact that the decison maker must give up the park if he exercises the option.

D. American Option Value

Given Margrabe's solution of the European option to exchange one asset for another, what is required is an extension of the analysis to an approximate solution that incorporates the possibility of early exercise.

As Merton (1973) has shown, in the case of a call option on an underlying asset that does not pay a cash "dividend", both the American and European option have the same value as it will not be optimal to exercise the option "early" relative to the final maturity date. This is due to the fact that no matter how much the value of the underlying asset exceeds the exercise price, there will always be a positive probability that the asset value will be higher in the future. Hence, the value of the option unexercised will be worth more than the proceeds obtained from exercising immediately. Consequently, the ability to exercise the option before maturity, associated with the American option, is worthless.

When the underlying asset pays a "dividend" as in the case considered here, there is an opportunity cost however, to not exercising the American option. This opportunity must be weighed against the expected benefit of not exercising. Given the assumption of a cash flow proportional to the asset value, then at any point in time there exists a value of the underlying asset for which immediate exercise would be optimal. Given this existence of a positive probability associated with early exercise the American option cannot be valued simply as a European option because the ability to exercise early will have value and hence the American option value will exceed that of the European by an "early exercise premium". The problem exists in determining the value of this early exercise premium.

As Margrabe indicates, the option to exchange one asset (the Park, P) for that of another (the project asset M) can also be viewed as a function of a call option on the ratio of the two assets with an exercise value of unity. This result is due to the fact that the option value, as proven by Merton (1973), is linearly homogeneous in the underlying asset value and the exercise price. Consequently the value of the call option C(M,P,T) that we wish to consider can be expressed as:

$$C[H,1,T] = (1/P)C(M,P,T) \tag{8}$$

$$\text{where } H = M/P \tag{9}$$

This result allows us to develop an approximate solution for the option C[H,1,T], which is considerably simpler than for the case of C(M,P,T) directly. The value of C(M,P,T) is then proportional to C[H,1,T] by a factor P.

Consider now the call option C[H,1,T] on the ratio, H, of the two assets M and P. In order to develop an approximate solution to the American option value for such an option we must first consider the partial differential equation that governs the option value through time. Given the general PDE in equation (5) for a contingent claim q(M,P,T), involving the two assets M and P, the PDE governing a contingent claim f(H,T) on the ratio of the two assets can be determined by substituting in equation (5) the relationship:

$$q(M,P,T) = Pf(H,T). \tag{10}$$

The resulting PDE as derived in Appendix 2 is given by:

$$(1/2)\sigma^2 H^2 f_{HH} - \delta H f_H - f_T = 0. \tag{11}$$

We will now present an analysis incorporating this PDE and based upon that of Barone-Adesi and Whaley (1987) and MacMillan (1986) for the development of an approximate solution to American option values.

The PDE represented by equation (11) governs the behavior of both the American and European value of an option on the ratio, H, of the two assets, M and P. Consequently, as BAW (1987) show, the value of the early exercise premium E, which represents the amount by which the American option value exceeds that of the European due to the additional benefit of being able to exercise early, must also be governed by equation (11) as:

$$C(H,T) = c(H,T) + E \tag{12}$$

where C(H,T) represents the American call option value and c(H,T) that of the European. As a result:

$$(1/2)\sigma^2 H^2 E_{HH} - \delta H E_H - E_T = 0. \tag{13}$$

For simplification let us first multiply (13) by $2/\sigma^2$ giving:

$$H^2 E_{HH} - BHE_H - JE_T = 0. \tag{14}$$

where $B = 2\delta/\sigma^2$ and $J = 2/\sigma^2$.

Let us now define the early exercise premium as $E(K,T) = K(T)g(H,K)$. Consequently $E_H = Kg_H$, $E_{HH} = Kg_{HH}$, and $E_T = K_T g + KK_T g_K$. Substituting these partial derivatives into (14) and factoring K yields:

$$H^2 g_{HH} - BHg_H - Jg[(K_T/K) + (K_T g_K/g)] = 0 \tag{15}$$

Choosing $K(T) = 1 - e^{-rT}$, substituting into (15) and simplifying results in

$$H^2 g_{HH} - BHg_H - [Jr(1 - K)/K]g - Jr(1 - K)g = 0 \tag{16}$$

As in the BAW (1987) and MacMillan (1986) derivations, the analysis performed up to this point has been exact. Consider now the last term in equation (16). As T approaches 0, K approaches 0 and so g_K approaches 0 and consequently the term $(1 - K)g_K$ approaches 0. Alternatively as T approaches infinity, K approaches 1, and again the term $(1 - K)g_K$ approaches 0. As an approximation then, to the partial differential equation (16), we can assume this last term to be equal to zero, resulting in the second order ordinary differential equation;

$$H^2 g_{HH} - BHg_H - [Jr(1 - K)/ K]g = 0 \tag{17}$$

This is a reasonable appproximation given that in most cases the option we are considering will have a reasonably long maturity, measured in years, relative to financial options having a typical maturity of less than one year. In addition tests of this approximation (BAW (1987)) in valuing these shorter maturity financial options have indicated insignificant error values.

As has been indicated by BAW and MacMillan, the general solution to differential equations of the type represented by (17) is given by:

$$g(H) = a_1 H^{y_1} + a_2 H^{y_2} \tag{18}$$

By substituting $g = aH^y$ into (17):

$$aH^y [y^2 - (1 + B)y - Jr(1 - K)/K] = 0 \tag{19}$$

where y_1 and y_2 are the roots of equation (19), given by:

$$y_1, y_2 = \frac{(1 + B) \pm \sqrt{(1 + B)^2 + 4Jr(1 - K)/ K}}{2}$$

Note that as $Jr(1-K)/ K > 0$, $y_1 > 0$ and $y_2 < 0$. Consequently if $a_2 \neq 0$, the function g(H) approaches ∞ as the asset price H approaches 0. As the early exercise premium of an American call becomes worthless when the underlying asset price approaches 0, this result is unacceptable. Consequently we will impose the constraint that $a_2 = 0$, and hence $f(H) = a_1 H^{y_1}$. As a result the approximate value of the American call option on H is, from (12), given by:

$$C(H,T) = c(H,T) + Ka_1 H^{y_1}. \tag{20}$$

What remains is the determination of the critical asset ratio H^* above which the option would be exercised, and the appropriate constraint on a_1.

Given that there exists, at any given point in time, a level of H, denoted by H^*, for which it would be optimal to exercise the option early then we know that at levels of H below H^*, the value of the option would be represented by equation (20), which is an increasing function of H, assuming a_1 is positive. At levels of H above H^* the value of the option would be equal to the exercisable proceeds of the option, H - 1, as the option would be exercised. Therefore at the exact value of H^* it must be that;

$$H^* - 1 = c(H^*,T) + Ka_1 H^{*y_1} \tag{21}$$

In addition, in order that the solution for the American call option value is not discontinuous in nature it must also hold that at H^* the slope of (20) must be equal to the slope of the line represented by the exercisable proceeds. Consequently we have, at H^*:

$$1 = e^{-\delta T} N(d_1(H^*)) + Ky_1 a_1 H^{*y_1 - 1} \tag{22}$$

Now from (22) we can solve for a_1:

$$a_1 = [\ 1 - e^{-\delta T} N(d_1(H^*))]/Ky_1 H^{*y_1 - 1} \tag{23}$$

and subsituting (23) into (21) results in the equation:

$$H^* - 1 = c(H^*,T) + [\ 1 - e^{-\delta T} N(d_1(H^*))]\ H^* / \ y_1 \tag{24}$$

from which H^* can be solved for iteratively. Efficient algorithms based on the Newton-Raphson technique have been developed for similar problems; see for example BAW (1987).

With H^* determined, and the value of a_1 provided by (23), the value of the American option is given by:

$$\begin{aligned} C(H,T) &= c(H,T) + [\ 1 - e^{-\delta T} N(d_1(H^*))]\ H^{*1-y_1}\ H^{y_1} / \ y_1 && \text{when } H < H^* \\ &= H - 1 && \text{when } H \geq H^* \end{aligned} \tag{25}$$

From equation (8) the value of the American option to exchange P for M is given by :

$$\begin{aligned} C(M,P,T) &= c(M,P,T) + A(H^* P)^{1-y_1} M^{y_1} && \text{when } H < H^* \\ &= M - P && \text{when } H \geq H^* \end{aligned} \tag{26}$$

where $A = [1 - e^{-\delta T} N(d_1(H^*))] / y_1$

and $c(M,P,T) = q(M,P,T)$.

Given this formulation of the decision to exchange the park for the project as being analogous to holding a contingent claim, the expression representing quasi-option value is simply the value of the above contingent claim, less the net benefits of exchanging the park for the project. Consequently the quasi-option value Q(M,P,T) is given by:

$$Q(M,P,T) = C(M,P,T) + P - M \quad \text{when } H < H^* \qquad (27)$$

$$= 0 \quad \text{when } H \geq H^*$$

As it can be shown that the value of an American option is greater than that of its exercisable proceeds for values of the underlying asset less than the critical value (see Merton 1973), it holds that:

$Q(M,P,T) \geq 0$

consistent with the utility theoretic framework findings, that quasi-option value is non negative.

E. Comparative Statics

quasi-option value, given by equation (27) is a function of several variables, in particular σ_P^2, σ_M^2, ρ_{MP}, M, P, r, δ, and T. Consequently it is insightful to examine the comparative statics of the model with respect to these variables. Results required for the derivation of these comparative statics are given in Appendix 3.

i. Comparative Statics with respect to σ_P^2, σ_M^2, and ρ_{MP}

Differentiation with respect to the variance parameters is given by:

$$\frac{\partial Q}{\partial x^2} = \frac{\partial Q}{\partial \sigma^2} \cdot \frac{\partial \sigma^2}{\partial x^2} \text{ where } x = \sigma_P^2, \sigma_M^2, \rho_{MP}.$$

and

$$\frac{\partial Q}{\partial \sigma^2} = \frac{\partial C}{\partial \sigma^2}.$$

It can be shown that the derivative of C with respect to σ^2 is given by:

$$\frac{\partial C}{\partial \sigma^2} = \frac{Me^{-\delta T}\sqrt{T}}{2\sigma} [\, N'(d_1) - (H/H^*)^{y_1 - 1} N'(d_1(H^*))]$$

$$+ (H/H^*)^{y_1 - 1} MA\, [\, \ln(H/H^*) + y_1^{-1}(1/2\sigma^2 - 1)\,]\, (\partial y_1/\partial \sigma^2) > 0, \quad \text{for } H < H^*$$

where

$$(\partial y_1/\partial \sigma^2) = -(1/\, 2\sigma^2)[\, B + (1/2\sqrt{\psi})(2(1+B)B + r(1-K)/\, K)] < 0,$$

$$\psi = (1 + B)^2 + JrK/\, (1-K)$$

and $N'(.)$ is the normal density function. As in the standard option result this derivative has a positive value for values of $H < H^*$. Since the derivative of σ^2 with respect to σ_P^2 is $1 - (\rho_{MP}\sigma_P/\, \sigma_M)$, this implies that quasi-option value is an increasing function of σ_P^2 if $\sigma_M > \sigma_P$.

Similarly the derivative of quasi-option value with respect to σ_M^2 is positive if $\sigma_P > \sigma_M$ since $\partial\sigma^2/\, \partial\sigma_M^2 = 1 - (\rho_{MP}\sigma_M/\, \sigma_P)$.

The sign of the derivative of quasi-option value with respect to the correlation coefficient ρ_{MP}, is, on the other hand negative, as the derivative of σ with respect to ρ_{MP} is given by $-2\sigma_M\sigma_P$. This indicates that quasi-option value decreases as the park and project value increase in correlation. This is understandable as we would expect an option to exchange one asset for another to be of less value if the two assets are highly correlated. Under such circumstances of highly correlated asset values when one asset is low in value so would be the other and hence the benefit derived from exchanging one for the other is relatively small.

This result has an interesting economic interpretation, however, specifically in the case of the option to exchange a park for a project. Consider the case where the project involves the clear cut logging of the park area for its timber resources, and assume that the value of the project increases as the value of timber increases. It can be argued that empirically an increase in scarcity of timber resources globally has corresponded with an increase in the scarcity of natural environments. Consequently, an interesting result is that quasi-option value is comparatively reduced under such circumstances, and the incentive for development increased.

ii. Comparative Statics with respect to M *and* P.

The derivative of quasi-option value with respect to the value of the project M is:

$$\frac{\partial Q}{\partial M} = \frac{\partial C}{\partial M} - 1 = e^{-\delta T}N(d_1) + [1 - e^{-\delta T}N(d_1(H^*))]\,(H/H^*)^{y_1 - 1} - 1 < 0,$$

for $H < H^*$.

The negative value of this derivative is explained by that fact that as M increases the critical exercise asset ratio H^* is approached and hence the value of the call option C(M,P,T) approaches the value of the exercisable proceeds M - P. Correspondingly Q(M,P,T) decreases, in value.

A similar explanation holds for the positive value of the partial derivative of the quasi-option value with respect to the value of the park P:

$$\frac{\partial Q}{\partial P} = \frac{\partial C}{\partial P} + 1 = 1 - N(d_2) + (M/P)A(1 - y_1)(H/H^*)^{y_1 - 1} > 0, \text{ for } H < H^*.$$

As P increases, H decreases, moving away from H^*, and hence an increase in quasi-option value.

iii. Comparative Statics with respect to T and r.

In terms of time to maturity T, and the risk free rate r, the derivatives of quasi-option value are ambiguous in sign. The derivative with respect to T is:

$$\frac{\partial C}{\partial T} = Me^{-\delta T}[\sigma N'(d_1)/\ 2\sqrt{T} - \delta N(d_1)]$$
$$+ (H/\ H^*)^{y_1 - 1} M\ \{\ A\ln(H/\ H^*)(\partial y_1/\ \partial T)\ - y_1^{-1}\delta e^{-\delta T}N'[d_1(H^*)]$$
$$- \delta e^{-\delta T}N[d_1(H^*)](y_1^{-1} - 1)\ \}\ \lesseqgtr 0\ ,\ \text{for } H < H^*,$$

where

$$(\partial y_1/\partial T) = -\ (.25JrK/\ K\sqrt{\psi})(1 + (rK/\ K)) < 0.$$

The derivative with respect to r is:

$$\frac{\partial C}{\partial r} = (H/H^*)^{y_1 - 1} M\ \{\ A\ln(H/\ H^*) + y_1^{-2}(1\ -\ e^{-\delta T}N[d_1(H^*)])(H^* - 1)\}(\partial y_1/\partial r)$$
$$\lesseqgtr 0,\ \text{for } H < H^*.$$

where

$$(\partial y_1/\ \partial r) = \frac{Ce^{-rT}}{4\ \sqrt{\psi}\ K^2}\ (1 - e^{-rT}\ - rT) \quad > 0$$

iv. Comparative Statics with respect to δ.

With respect to the rate of net cash flows δ derived from the project, the derivative of quasi-option value is again ambiguous in sign:

$$\frac{\partial C}{\partial \delta} = (H/H^*)^{y_1 - 1} M \left[\ln(H/H^*)(\partial y_1/\partial \delta) + Te^{-\delta T} N[d_1(H^*)](1 + y_1^{-1}) \right]$$
$$- TMe^{-\delta T} N[d_1] \lessgtr 0, \text{ for } H < H^*,$$

where

$$(\partial y_1/\partial \delta) = (1/\sigma^2)(1 + (1+B)/\sqrt{\psi}) > 0.$$

V. CONCLUSION

As in Hanemann (1989), we have shown that given that development is both irreversible and indivisible, a decision maker who ignores the retention of options associated with preservation can underestimate the benefits of preservation to some degree and tilts the balance in favor of development. This is due to the fact that irreversibility creates an asymmetry which has value. If the decision maker decides to preserve in the current period, the decision maker can always develop later if subsequent information warrants it; but if the decision to develop is made, it cannot be reversed regardless of what future information reveals. In the first case there is flexibility with respect to future decisions, and the information has some potential value; in the second case there is no flexibility and the information has no economic value. Thus, when future learning is taken into consideration, there is an extra component to the benefits associated with preservation. This extra component is termed quasi-option value and the purpose of this paper has been to develop an explicit valuation formula for quasi-option in a contingent claims framework.

Even in the presence of what may be objectionable assumptions concerning the trading of assets and the observability of the variance of their return, contingent claims analysis still provides a feasible alternative to the utility theoretic framework for quasi-option valuation. This is particularly true when we consider the accepted valuation technique of "contingent valuation", where surveys are carried out questioning individuals as to the price they would pay for certain "options" if markets for these contingent claims existed. Recent work (Boyle (1989)) indicates that such methods are subject to errors in that valuation by individuals is highly dependent on the description of the commodity in question.

Contingent claims analysis provides a robust theoretical model from which a relatively tractable explicit valuation equation can be obtained. In addition a decision rule of optimal investment in terms of a critical asset price ratio is specified by the CCA framework given that there is an opportunity cost in delaying development. Comparative static analysis reveals that quasi-option value can increase or decrease with an increase in uncertainty of either the benefits of development or preservation and is dependent on the relative uncertainty of the respective benefits. quasi-option value decreases however, the more positively correlated are the benefits of development and preservation.

Further work in this area would include the application of CCA to the concept of option value. In a sense option value, described as the option to visit the park in the future is distinct from the concept of quasi-option value defined as the option to exchange one risky asset for another. Although both concepts involve the benefits from the park as an underlying risky asset, for the purposes of valuing quasi-option, option value itself is a benefit associated with the park and is implicit in the value of P. In this sense option value has similarities to the operating options identified in applications of

CCA to capital budgeting. The application of CCA to the concept of option value and quasi-option value in a model of irreversible investment therefore holds promise in explaining the relationship between the two concepts.

In addition the valuation formula and critical exercise ratio developed are easily extendable to other investment decisions of an irreversible nature. Many such investment decisions exist in the arena of capital budgeting and the investment decision of firms. Any investment expenditure that could potentially be a sunk cost is essentially irreversible. Such expenditures would be investments in assets that are industry or firm specific such as marketing and advertising. In addition, strategic considerations such as the risk of entry into the market by other firms may result in a time limit or time to maturity for which the option to invest exists.

A direct application of the analysis presented here would be in the case of a firm currently involved in the production of a particular good and which is considering making an irreversible investment to its production line in order to produce an alternative good. Again the decision can be characterized as an American option to exchange one risky asset for another. An added complexity of course is that continuous cash flows or dividends emanate from both assets.

VI. REFERENCES

Abowd, J.M. and S. Manaster, 1983, "A General Model of Employment Contracting: An Application of Option Theory," University of Chicago Working Paper, 1983.

Arrow, K., 1964, "The Role of Securities in the Optimal Allocation of Risk Bearing," *Review of Economic Studies*, vol. 31, p.91-96.

Arrow, K.S. and A.C. Fisher, 1974, "Environmental Preservation, Uncertainty and Irreversibility ," *Quarterly Journal of Economics*, vol. 88, p.312-19.

Barone-Adesi, G. and R. Whaley, 1987, "Efficient Analytical Approximation of American Option Values," *Journal of Finance*, vol. 42, p.301-20.

Bishop, R.C., 1982, "Option Value: An Exposition and Extension," *Land Economics*, vol. 58. p.337-359.

Black, F. and M. Scholes, 1973, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, vol. 81, p.637-59.

Bohm, P., 1975, "Option Demand and Consumers' Surplus: Comment," *American Economic Review*, vol. 65, p.733-36.

Boyle, K.J., 1989, "Commodity Specification and the Framing of Contingent-Valuation Questions," *Land Economics*, vol. 65, p.77-63.

Brennan, M.J. and E.S. Schwartz, 1985, "Evaluating Natural Resource Investments," *Journal of Business*, vol. 58 p. 135-57.

Cicchetti, C.J. and A.M. Freeman, 1971, "Option Demand and Consumer Surplus; Further Comment," *Quarterly Journal of Economics*, vol. 85, p.528-39.

Conrad, 1980, "Quasi-Option Value and the Expected Value of Information," *Quarterly Journal of Economics*, vol. 94, p.813-20.

Cory, D.C. and B.C. Saliba, 1987, "Requiem for Option Value," *Land Economics*, vol. 63, p.1-10.

Cox, J.C., J.E. Ingersoll, and S.A. Ross, 1985, "A Theory of the Term Structure of Interest Rates," *Econometrica*, vol. 53, p.385-407.

Crabbé, P.J., 1985, "Option and Quasi-Option Values of Natural Resources," Paper Delivered at the Fifth World Congress of the Econometric Society, Boston, 1985.

Crabbé, P.J., 1987, "The Quasi-Option Value of Irreversible Investment: A Comment," *Journal of Environmental Economics and Management*, vol. 14, p.384-85.

Dothan, U. and J. Williams, 1981, "Education as an Option," *Journal of Business*, vol. 54 p. 117-39.

Fisher, A. and W.M. Hanemann, 1987, "Quasi-Option Value; Some Misconceptions Dispelled," *Journal of Environmental Economics and Management*, vol. 14, p.183-90.

Freeman III, A.M., 1984, "The Quasi-option Value of Irreversible Development," *Journal of Environmental Economics and Managment*, vol. 11, p.292-95.

Freeman, A.M., 1985, "Supply Uncertainty, Option Price, and Option Value," *Land Economics*, vol. 61 p.176-81.

Friedman, M., 1962, *Capitalism and Freedom*, Chicago: University of Chicago Press.

Graham, D.A., 1981, "A Cost-Benefit Analysis Under Uncertainty," *American Economic Review*, vol. 71, p.91-109.

Hanemann, W.M., 1989, "Information and the Concept of Option Value," *Journal of Environmental Economics and Management*, vol. 16 p.23-37.

Henry, C., 1974, "Option Value in the Economics of Irreplaceable Assets," *The Review of Economic Studies: Symposium on the Economics of Exhaustible Resources*, p.89-104.

Krutilla, J.V., 1967, "Conservation Reconsidered," *American Economic Review*, vol. 59, p.777-86.

Krutilla, J.V., C.J. Cicchetti, A.M. Freeman III, and C.S. Russell,1972, "Observations on the Economics of Irreplaceable Assets," in *Environmental Quality Analysis: Theory and Methods in the Social Sciences*, edited by J. Kneese and B.T. Bower, Baltimore; John Hopkins University.

MacMillan, L.W., 1986, "Analytical Approximation for the American Put Option," *Advances in Futures and Options Research*, vol. 1, p.119-39.

Majd, S. and R. Pindyck, 1987, " Time to Build, Option Value, and Investment Decision," *Journal of Financial Economics*, vol. 18, p.7-28.

Margrabe, W., 1978, "The Value of an Option to Exchange one Asset for Another," *Journal of Finance*, vol. 33, p.177-186.

Mason, S.P. and C.Y. Baldwin, 1988, "Evaluation of Government Subsidies to Large Scale Energy Projects: A Contingent Claims Approach," *Advances in Futures and Options Research*, vol. 3, p.169-181.

Mason, S.P. and R. Merton, 1985, "The Role of Contingent Claims Analysis in Corporate Finance," in Recent Advances in Corporate Finance, 1985, Irwin, editor: Altman, E.

McDonald, R. and D. Siegel, 1986, "The Value of Waiting to Invest," *Quarterly Journal of Economics*, vol. 101, p.707-27.

Merton, R., 1973, "The Theory of Rational Option Pricing". *Bell Journal of Economics and Management Science*, vol. 4, p.141-83.

Miller, J.R.. 1981, "On the Determinants of Quasi-Option Value," *Socio-Economic Planning Sciences*, vol.15, p.191-95.

Miller, J.R. and F. Lad, (1984), "Flexibility, Learning, and Irreversibility in Environmental Decisions; A Bayesian Approach," *Journal of Environmental Economics and Management*, vol. 11 p.161-72.

Pindyck, R.S., 1991, "Irreversibility, Uncertainty, and Investment," *Journal of Economic Literature*, vol 29, p.1110-48.

Plummer, M.L., 1986, "Supply Uncertainty, Option Price and Option Value; an Extension," *Land Economics*, vol.62, p.313-18.

Plummer, M.L. and R.C. Hartman, 1986, "Option Value: A General Approach," *Economic Inquiry*, vol. 24, p.455-71.

Ritchken, P. and G. Rabinowitz, 1988, "Capital Budgeting Using Contingent Claims Analysis: A Tutorial," *Advances in Futures and Options Research*. vol. 3 p.119-43.

Rubinstein, M., [illegible]7, "Derivative Assets Analysis," *Journal of Economic Perspectives*, vol. 1, p.73-93.

Schmalensee, R., 1972, "Option Demand and Consumer Surplus: Valuing Price Changes Under Uncertainty," *American Economic Review*, vol. 62, p.813-24.

Schwartz, E.S., 1982, "The Pricing of Commodity Linked Bonds," *The Journal of Finance*, vol. 37, p.525-39.

Segerson, K., 1987, "Supply Uncertainty and Option Value: Comment," *Land Economics*, vol. 63, p.406-8.

Smith, K.V., 1983, "Option Value: A Conceptual Overview," *Southern Economic Journal*, vol. 49, p.654-68.

Smith, K.V., 1987, " Uncertainty, Benefit-Cost Analysis, and the Treatment of Option Value," *Journal of Environmental Economics and Management*, vol. 14, p.283-92.

Stulz, R., 1982, "Options on the Minimum or the Maximum of Two Risky Assets; Analysis and Applications," *Journal of Financial Economics*, vol. 10, p.161-85.

Trigeorgis, L., 1988, "A conceptual Options Framework for Capital Budgeting," *Advances in Futures and Options Research*, vol. 3, p.145-67.

Viscusi, W.K., 1988, "Irreversible Environmental Investments with Uncertain Benefit Levels," *Journal of Environmental Economics and Management*, vol. 15, p.147-57.

Walsh, R.G., J.B. Loomis, and R.S. Gilman, 1984, "Valuing Option, Existence, and Bequest Demands for Wilderness," *Land Economics*, vol. 60, p.14-29.

Weisbrod, B.A., 1964, "Collective Consumption Services of Individual Consumption Goods," *Quarterly Journal of Economics*, vol. 78, p.471-77.

CHAPTER III. A TEST OF CALENDAR SEASONALITIES IN STOCK MARKET RISK IMPLIED FROM OPTIONS MARKETS.[1]

I. INTRODUCTION

Previous literature on stock returns documents the presence of significant seasonal patterns, with a majority of the focus on the finding of higher returns in January than in the remainder of the year. This January or, as first termed by Roll (1983), turn-of-the-year effect appears to be concentrated among smaller firms and strongest early in the month. Studies attesting to the existence of this effect include, among others, Rozeff and Kinney (1976), Keim (1983), Roll (1983), Chan (1986), Debondt and Thaler (1985), (1987), Ritter (1988) and Lakonishok and Smidt (1988).

Evidence in foreign markets is provided by Brown et al.(1983), Kato and Schalleim (1985), Tinic, Barone-Adesi, and West (1986) and Tinic and Barone-Adesi 1987), in the case of Australian, Japanese, and Canadian markets respectively.[2] As well, the presence of a January effect in the pricing of derivative securities is documented by Keim and Smirlock (1989) for stock index futures, and Dickinson and Peterson (1989) for options on common stocks.

Additional seasonalities in stock returns include 1) a monthly effect, documented by Ariel (1987) and Lakonishok and Smidt (1988), who find that returns are consistently higher in the early part of a month as opposed to the later part, and 2) a quarterly effect, as noted by Penman (1987) whose results show that returns are, on average, significantly higher early in calendar quarters two through four than in the remainder of the year.[3] As well, a number of authors have indicated the presence of a day-of-the-week effect with Monday returns generally lower than those of alternative weekdays.

Explanations of the existence of these seasonalities vary, and in general resulting tests have been inconclusive. They include, in the case of the January effect, tax-loss selling, insider trading, omitted risk factors and institutional constraints.[4] While no explanations

---

[1]A portion of this chapter, co-authored with G. Barone-Adesi, has been published in *Administrative Sciences Association of Canada 1991 Conference Proceedings*, *Finance Division*, Ed. G. Sick, under the title, "A Test of Calendar Seasonalities in the Implied Volatilities of Canadian Stock Indices".

[2]Jog (1988) presents a concise review of the evidence of stock return anomalies in the Canadian market. Higher returns in January have been noted in the TSE 300 as well as the Laval equally weighted and value weighted indices.

[3]Evidence of a monthly effect in the returns of the TSE 300 has been documented by Jog (1988).

[4]See Thaler (1987), Ritter (1988) and Haugen and Lakonishok (1988) for reviews of this literature.

have as yet been offered for the recently documented monthly effect, potential explanations of the quarterly effect in stock returns have centered on the arrival of earnings information throughout the year (Penmann (1987) and Peterson (1990)). Theories concerning the day-of-the-week effect have, alternatively, been expressed mainly in terms of insider trading and institutional factors.

An additional potential explanation of these seasonalities, explored by several authors, most notably in the case of the January effect, is the possible existence of seasonality in risk. Rozeff and Kinney (1976) found that the slope coefficients in the Fama and Macbeth (1973) study exhibited a significant January seasonality. Tinic and West (1984), while exploring Rozeff and Kinney's results more fully, found that the return to riskier stocks occurs exclusively in January, with these riskier stocks not earning higher returns in all other months. In addition, Smidt and Stewart (1984) found that the risk premium in January for an equally weighted index is significantly larger than for other months of the year.

Rogalski and Tinic's (1986) findings indicate that the daily return variances for 20 firm-size portfolios and an equally weighted index are significantly different across months. They found that January exhibits a significantly larger daily return variance than other months, in the case of small firm-size portfolios, and the third largest average daily return variance, following May and November, for the equally weighted index. In addition, tests of the equality of systematic and non-systematic risks across months indicated significant differences. Finally, Maloney and Rogalski (1989), in a study of volatilities implied from options on common stocks document an increase in the level of implied volatilities over year end periods.

The purpose of this paper is to test, simultaneously, for the existence of seasonality in total market risk corresponding to the January, monthly, and quarterly seasonalities observed in returns. Differing however, from the majority of past studies that have examined nonstationarity in risk employing the variance of historical returns, this study, like that of Maloney and Rogalski (1989) in the case of the January effect and Morse (1991) for day of the week effect, examines the expectations concerning risk using implied volatilities. Specifically, the presence of the observed return seasonalities, in implied volatilities derived from options on the S&P 500 index futures, is tested for, employing multiple-intervention time-series analysis. In addition, the test is extended to the Canadian market by examining implied volatilities from options on the TSE 35 and TSE 300 common stock indices.

Section II briefly outlines the option pricing models and data used to derive a time series of implied volatilities which proxy for expectations of total market risk. Section III describes the methodology of multiple-input intervention analysis, used to specifically test for the presence of January, monthly, and quarterly seasonalities in these expectations. Section IV presents the empirical results and implications for both the American and Canadian data, as well as suggesting some avenues of further research. Conclusions and a summary of the paper are provided in Section V.

II. OPTION PRICING MODELS AND DATA

A. Implied Volatilities from Options on Index Futures

A criticism common to the majority of studies to date, which have tested for the presence of seasonality of risk in the market, is the use of data which is ex-post in nature, in the sense of being based upon historical returns. The assumption made is that expectations on average are realized and that the sample size is large enough for these averages to be meaningful. In theory however, the riskiness of securities is an ex-ante measure and any ex-post measure is at best an unbiased proxy.

Latane and Rendleman (1976) were the first to indicate that the choice of an option pricing model leads to a measure of asset variance which is implied in the market prices of options. In this sense it is possible to derive an ex-ante measure of an underlying asset return variance which can be used to test hypotheses of risk expectations. In particular the appearance of options on stock index futures, in the past several years, provides instruments from which can be derived the implied expectations of the riskiness of market indices which proxy for the overall risk of the market return.

Consider the basic assumptions underlying the contingent claims approach to futures option valuation as first developed by Black (1975). The assumptions required to develop the standard partial differential equation describing the dynamics of the futures option price through time are:

A1) There exist no transaction costs in the options, futures, and bond markets.
A2) There exist no costless arbitrage opportunities.
A3) The short term riskless rate of interest is constant through time.
A4) The instantaneous futures price change relative can be described by the stochastic differential equation:

$$dF/F = \mu\, dt + \sigma\, dz$$

where μ is the expected instantaneous price change relative of the futures contract, σ is the instantaneous standard deviation, and z is a standard Wiener process.

It can be shown, (Whaley (1986)) given the assumption of a constant continuous riskless rate of interest r, and a constant (or nonconstant but deterministic function of time) continuous, proportional rate of receipt d for holding the underlying spot commodity, that assumption (A4) is consistent with the assumption that the price of the underlying spot commodity follows the stochastic differential equation:

$$dS/S = \alpha\, dt + \sigma\, dt.$$

Where α is the expected relative spot price change, $\mu = \alpha - (r - d)$ and σ is the instantaneous standard deviation equivalent to that for the futures price change. This can be shown by applying Ito's lemma to the cost-of-carry relationship:

$$S_t = F_t e^{-(r-d)T}.$$

where T is the time left to the maturity of the futures contract.

Consequently, implied volatilities derived from market prices of futures options give a measure of the volatility of the return on the spot itself while circumventing the difficulty involved in estimating the cost of carry, required in the case of options on the underlying commodity. All that is required is the choice of an option pricing model for pricing options on index futures.

B. Barone-Adesi and Whaley (1987) Option Pricing Model

For the purposes of this paper, implied volatilities are derived using the Barone-Adesi and Whaley (1987) (BAW) model for approximating the value of American options on futures.

Unlike the European option case, there is no known closed form solution to the partial differential equation, derived by Black, subject to the boundary conditions for an American call option on a futures contract. Models of American futures options have hence centered on approximation methods to solve for the option value. An alternative to the computationally expensive finite difference approximation methods of Ramaswamy and Sundaresan (1984), Brenner, Courtadon, and Subrahmanyam (1984) and the compound option method of Geske and Johnson (1984) adapted by Whaley (1984), is provided by the analytical approximation approach of BAW. This approach utilizes a quadratic approximation method first employed by MacMillan (1986) for the problem of valuing the American put option on common stock.

Black has shown that by forming a riskless hedge between the futures option and the underlying futures contract, the partial differential equation describing the movement of the futures option price V through time is:

$$1/2\ \sigma^2 F^2 V_{FF} - rV + V_t = 0.$$

The complexity involved in valuing an American call option on a futures contract versus that of the European call is due to the fact that there will always exist a value of the futures price above which it will be optimal to exercise the call early. Given this potential for early exercise, the partial differential equation must be solved subject to the boundary conditions:

$$C(F,T,X) = \max\,[0,\ F-X], \qquad T = 0,$$

$$C(F,T,X) = \max\,[C^*(F,T,X),\ F-X], \quad 0 < t < T$$

where $C^*(F,T,X)$ is the value of the call option with T periods left to maturity given that the option is not exercised at time t. The exercise price associated with the option is represented by X. BAW show that using the quadratic approximation approach, the value of the

American call option on a futures contract can be expressed as:

$$C(F,T,X) = c(F,T,X) + A_2\left(\frac{F}{F^*}\right)^{q_2} \qquad \text{for } F < F^*, \text{ and}$$

$$C(F,T,X) = F-X \qquad \text{for } F > F^*,$$

where

$c(F,T,X)$ = the value of the European call option.

$$A_2 = \left\{1-e^{-rT} N[d_1(F^*)]\right\} \frac{F^*}{q_2}$$

$$d_1(F^*) = \left[ln\left(\frac{F^*}{X}\right) + 0.5\ \sigma^2 T\right] \Big/ \sigma \sqrt{T}$$

$$q_2 = \left(1 + \sqrt{1+4K}\ \right) \Big/ 2 \text{ , and}$$

$$K = 2r \Big/ \left[\sigma^2 (1-e^{-rT})\right]$$

The critical futures price F^* above which the American futures call option will be exercised can be determined by solving iteratively for F^* from

$$F^* - X = c(F^*,T,X) + \left\{1-e^{-rT} N[d_1(F^*)]\right\} \frac{F^*}{q_2}$$

C. Implied Volatilities from Options on Index Futures

S&P 500 futures options contracts first began trading on the Chicago Mercantile Exchange in January of 1983, and are the most widely held of the index futures options. This makes them an obvious choice for the purpose of deriving implied volatilities which proxy for the riskiness of the market return. Consequently, weekly settlement prices for the S&P 500 futures contracts and weekly closing prices of call options on futures were obtained from the Wall Street Journal for the period June 1, 1984 to February 23, 1990, effectively giving over six and one half years of weekly data encompassing six turn-of-the-year periods. By excluding the 1983 data, any mispricing due to the infancy of the market was effectively avoided.

Weekly estimates of implied volatilities (ISDs) were calculated using a program developed at the University of Alberta. This program utilizes a Newton-Raphson technique to solve for the critical early exercise futures price F^* and then solves for the standard deviation of the futures price relative which equates the BAW model to the observed market price by way of an iterative procedure. The resulting series of weekly implied volatilities represented 299 observations.

As a proxy for the riskless rate of interest, the yield on the U.S. Treasury bill with a maturity most closely matching that of the options contract observed was utilized. These yields were computed using the average of the bid and ask discounts of the T-bills reported in *The Wall Street Journal*.

Weekly data was employed to derive a time series of implied volatilities because of the difficulties involved in using daily data. For the purposes of time series analysis, which underlies the testing methodology applied in this study, it is relatively important that the data represents a continuous series through time. The presence of market closures in either the futures and options, or treasury bill markets, results in missing observations for the purpose of a time series of implied volatilities.

Although there are a wide range of methods for dealing with missing data in time series (Dunsmuir (1981)), in general these procedures require the prediction of missing values from the available data set. For example, Abraham (1981) suggests that the missing observations be interpolated from an ARIMA (autoregressive integrated moving average) model fitted only on the observations occurring before the missing data period and the back cast of a model fitted on the observations occurring after. The relative frequency of missing data values in a series of daily ISD estimates, however, precludes the use of this methodology. In fact, the required resulting forecasts and back casts would be based on an average of only 14 observations if daily data were employed. ARIMA models identified on such short lengths of consecutive observations, particularly in the presence of any time series outliers, would be extremely variable and hence, result in questionable estimates for the missing values. The use of weekly closing data, although resulting in some loss of power in the test, circumvents this missing data problem.

The particular options employed to estimate the implied volatilities were call options closest to the money as these are the options for which trading is most frequent. At-the-money options are also more sensitive to changes in volatility than are in-the-money or out-of-the-money options (Cox and Rubinstein (1985)) and in this sense convey the most information about the expectations of the options market concerning future volatility.[5]

Theoretically, futures option pricing models require, as an input, the futures price at the instant the option is traded. Indeed, infrequent trading of the underlying asset can result in serious errors in implied volatilities based on closing data. The use of closing prices on at-the-money calls however, closely approximates the

---

[5]See also Harvey and Whaley (1991) for a concise proof that at-the-money options exhibit maximum sensitivity to volatility changes.

use of synchronous data as volume tends to be concentrated in these options. Any lack of synchronization between the closing futures and options prices is further minimized simply due to the extensive trading activity in both the futures and corresponding options markets for the S&P 500 index. In fact, three years after its introduction the S&P 500 futures contract was the second most active contract market in the U.S. futures industry and by 1987 the daily trading volume was comparable to the daily volume on the New York Stock Exchange (Gammill and Marsh (1988)). In addition, Whaley (1986), in a study of S&P 500 futures options contracts using transactions data for the year of 1983, observed that the average time between futures and futures options transactions was only 21 seconds, somewhat alleviating the concerns of non-synchronous trading, at the close of the day. This is true particularly when we consider the relative infancy of the market at that period in time. In addition, Harvey and Whaley (1991), in recent tests of various implied volatility estimation procedures, show that infrequent trading of the underlying asset is not an issue for options on indices, perhaps due to the fact that stocks in indices are traded relatively frequently at the end of the day.

Implied volatilities were calculated on options having the shortest times remaining to maturity to a minimum of three weeks. Consequently, options maturing in the month of observation were avoided due to the observed volatile options and futures price movements near expiration dates (Stoll and Whaley (1986) and (1987)).In general however, the use of relatively short term options allows for a better test of seasonality in risk as a temporary increase in volatility will have a greater effect on short maturity options than on long term ones. This is due to the fact that if implied volatility represents an average over the time to maturity of the instantaneous volatility, then a temporary increase in volatility causes a larger change in the average volatility to maturity the shorter is the time to maturity.

Although option contracts exist during much of the period in question, having maturity months differing from the regular quarterly maturity cycle, observations based on these irregular option contracts were avoided due to the nonsynchronous expiration date of the underlying futures contracts. In addition, utilizing options maturing only during the regular March June September and December cycle, along with the above time to maturity restrictions, results in a systematic pattern of time to maturity upon which the implied volatility estimates are based. Within a calendar year, this pattern of time of observation to time of maturity is shown below.

month of observation	option maturity month
January	March
February	March
March	June
April	June
May	June
June	September
July	September
August	September
September	December
October	December
November	December
December	March

In summary, the options employed had an average time to maturity of 63 days with a minimum and maximum of 21 and 112 days respectively.

Figure III-1 shows the plot of the resulting weekly ISD's calculated from the above options on S&P 500 index futures. It is important to note, that this series of ISD's does not represent a homogeneous time series of observations, as the nature of the maturity of the options from which the ISD's are calculated changes in the above noted systematic manner through time. Consequently, given the overlapping nature of the time periods that the weekly ISD observations pertain to, significant serial correlation is to be expected in the series. As evidence of this serial correlation, Table III-1 presents the results of times-series analysis carried out on the total series of data. As Table III-1 indicates the time series model identified for the data as a result of the standard Box-Jenkins (1970) analysis is an auto regressive model of order three (AR(3)). This model best describes the behavior of the ISD's through time.

As noted however, undoubtedly much of the observed autoregressive nature of the series is due to the overlapping periods to which the implied volatilities apply. Hence, an interpretation of this series in terms of the exact time process of the market's instantaneous expectations concerning risk cannot be made. The purpose of this analysis, however, is simply to test for and indicate the presence of autocorrelation in the ISD series, whatever its source and not to make conclusions as to the exact generating process of the market's expected volatility. As will be shown in section III the presence of this autocorrelation, however, must be taken into account when measuring the significance of any seasonality variable on the ISD estimates.

It is interesting to note however, that Schwert (1990) in a study of the persistence of volatility shocks in the market, estimates that volatilities of the S&P 500 index return derived from daily data are best described by an autoregressive model consisting of twenty-two lagged volatilities. This gives further evidence of the estimated three week autoregressive pattern of volatilities observed here.

In order to test for stationarity of the time series model that best describes the ISD series, Box-Jenkins analysis was also performed on subsections of the series corresponding approximately to one year lengths. The resulting models, for each of the subperiods examined, are also shown in Table 1. These models provide some indication of nonstationarity in terms of time series models, as they range from an AR(1) for the period June 6, 1986 to May 29, 1987 to an AR(3) for the period June 7, 1985 to May 30, 1986. Most likely, this nonstationarity in the process describing the behavior of the ISDs through time is due to significant time series outliers present in the data, in the form of temporary level shifts and trends. The implication is that any test of seasonality in the ISD's should perhaps be performed on the shorter subperiods of the series, and not simply on the series as a whole.

D. Implied Volatilities for the Canadian Market

In order to test for the presence of risk seasonalities in the Canadian stock market, options on indices themselves must be considered as options on index futures are not, at this time, in existence. Consequently, it is not possible to avoid the estimation of a dividend yield associated with the underlying index as with the case of options on index futures.

Two series of data exist which are of interest to us in deriving implied volatilities. The first of these is provided by options on the TSE 300 index which first began trading in 1984 and which were later discontinued in 1987. The second is provided by the relatively more successful options on the TSE 35 index. These options first began trading in 1987 and having proved to be more popular than those written on the TSE 300, continue trading today.

In order to test for seasonalities in the implied volatilities for options on the TSE 300, weekly option prices and closing levels of the index were obtained from the *Globe and Mail* for the period July, 20, 1984 to October 6, 1987, representing almost all of the published data available on these contracts. This resulted in a time series of 170 implied volatility estimates.

The last three weekly observations in October 1987, although available, were excluded from the analysis as these observations coincide with the observed crash in the U.S. and Canadian stock markets at that time. As options on the TSE 300 ceased trading after October 1987, the exclusion of these three observations does not significantly shorten the series, whereas their inclusion would only reduce the power of a test of seasonality due to the noise they entail. Figure III-2 shows the plot of the series of weekly ISDs implied by options on the TSE 300 and includes the last three observations in October 1987. The significance of these three "outliers" is apparent from the plot.

Again, only call options closest to the money, and having the shortest time to maturity to a minimum of three weeks, were employed to estimate implied volatilities. Consequently, these options had a minimum and maximum time to maturity of only 21 and 49 days respectively. The average time to maturity of only 33 days, associated with these options, is significantly shorter than that of the contracts utilized in the case of options on the S&P 500 index futures, basically because the calendar cycle of the options on the

TSE 300 is a monthly one. This calendar cycle of monthly maturity and the above time to maturity restrictions results in the following calendar pattern of observed options to month of maturity.

<u>month of observation</u>	<u>option maturity month</u>
January	February
February	March
March	April
April	May
May	June
June	July
July	August
August	September
September	October
October	November
November	December
December	January

As these options are American in nature, the model utilized to estimate implied volatilities was basically Merton's continuous dividend yield option pricing model, modified to take into account the possibility of early exercise. This modified continuous dividend yield model is provided by Barone-Adesi and Whaley (1987), and again is based upon the quadratic approximation.

Continuous dividend yields, required for the option pricing model, and applying to the time left to maturity for each option observed, were estimated from monthly dividend yield forecasts reported in the Toronto Stock Exchange monthly publication *The Review*. Although many studies use actual dividends to estimate a dividend yield under the assumption that expectations are fulfilled, the availability of *The Review* forecasts allows for the use of a true measure of expected dividends. Indeed traders in Canadian index futures and options regularly employ the TSE dividend yield forecasts in their pricing models. For each option observation, these monthly dividend yield forecasts were combined as weighted averages pertaining to the proportion of time each respective month represented out of the total time to maturity remaining for the option. This mitigates to some extent the criticism of Harvey and Whaley (1991) that the assumption of a constant dividend yield can induce large pricing errors which are transmitted to implied volatility estimations. This is true when the dividend yield is estimated over a long period of time and assumed constant over many implied volatility estimations. In the above procedure the dividend yield is assumed constant only over the life of the particular option, and is updated for each implied volatility observation. The riskless rate of interest was proxied for by the estimated continuous yearly return on 3-month Government of Canada Treasury bills, observed on a weekly basis.

Table III-2 contains the results of Box-Jenkins time series analysis performed on the resulting ISDs. As indicated, the most appropriate time series model, identified for the total series of TSE

300 ISD estimates, is an AR (2). Most likely, this autoregressive model of order two, as opposed to order three for the S&P 500 ISD data, is due to the shorter overlapping time periods that the individual TSE 300 ISD's pertain to. In addition, time series analysis was performed on the subperiods of July 20, 1984 to May 5, 1985, June 7, 1985 to May 30, 1986, and June 6, 1986 to October 16, 1987 indicating time series models of AR(1), AR(2), and AR(2) respectively. Again, as in the S&P 500 data, nonstationarity of the basic underlying time series process is indicated by these results.

Options on the TSE 35 first began trading in 1987 and essentially replaced the TSE 300 option contracts due to the unpopularity of the latter with institutional investors. The TSE 35 index was essentially designed to replicate the behavior of the TSE 300 and in fact exhibits a correlation of approximately 0.99 with the TSE 300.(Faseruk and Johnson (1989)). The smaller group of stocks involved in the 35 index proved however, to be more appealing to investors.

Weekly closing prices for options on the TSE 35 and the closing level of the index were compiled from the *Globe and Mail* for the period of July 10, 1987 to March 30, 1990, giving a time series of 143 weekly implied volatility estimates. As a proxy for the risk free rate, the observed yield on Government of Canada 3-month Treasury bills was utilized and again estimates of the continuous yield pertaining to the time to maturity of each option observed, were made from the monthly dividend yield forecasts available in the Toronto Stock Exchange *Review* publication. The same restrictions noted previously, as to moneyness and time to maturity of the options from which ISDs were estimated, applied as well to the options utilized in the case of the TSE 35. Again, the calendar cycle of maturity of these options is, as in the case of the TSE 300, a monthly one, resulting in an average time to maturity of 33 days and a systematic calendar relationship of month of observation to month of maturity as given above for the TSE 300 options.

Unlike options on the TSE 300 index, however, options on the TSE 35 are European in nature, hence the option pricing model used to estimate implied volatilities was Merton's continuous dividend yield model for European options.

Figure III-3 displays the plot of the resulting weekly TSE 35 ISD estimates and Table III-3 shows the results of time series analysis performed on the estimates for the total period of July 10, 1987 to March 30, 1990. As indicated, the total series is best described by an AR(2) time series model.Again, to test the stationarity of the model, Box-Jenkins analysis was also performed for the subperiods of July 10, 1987 to June 30, 1988, July 8, 1988 to June 30, 1989, and July 7, 1989 to March 30, 1990. The results, also reported in Table III-3, again indicate some nonstationarity with optimal ARIMA models of AR(2), AR(2), and AR(1) respectively.

III. METHODOLOGY OF MULTIPLE INPUT INTERVENTION ANALYSIS

A. Autocorrelation in ISD Estimates and Regression Residuals

Given the presence of significant autocorrelation in the ISD estimates, as indicated by the above time series analyses, tests of the effect of variables coded to capture various seasonalities in the data, based upon ordinary least squares regression are in general inappropriate.

These tests involving regression analysis are of course conditional upon the assumptions of the general linear model. In particular, the assumption of lack of serial correlation in the error terms is made. If this assumption does not hold, then the resulting ordinary least squares regression coefficients, while still unbiased, may be quite inefficient. Consequently, tests of the significance of explanatory variables using the t-distribution are no longer strictly applicable.

Significant autocorrelation in the dependent variable can often be the the cause of residual autocorrelation, due to the existence of a "missing variables" problem if the explanatory variables in the regression do not significantly explain the movement through time of the dependent variable. It is highly unlikely that explanatory variables representing January, monthly, and quarterly seasonalities will, by themselves, specify a linear model sufficiently complete to fully explain the serial correlation observed in the ISD estimates. Indeed, residuals from preliminary regressions of the ISD estimates on these seasonal variables exhibited significant serial correlation. Given the presence of significant autocorrelation in the ISD estimates, a more appropriate test of the hypothesis of seasonality of market risk can be carried out through the time series procedure of intervention analysis.

Intervention or impact analysis, as first developed by Box and Tiao (1975), is used to test the hypothesis that a postulated event causes a change in a variable that is measured as a time series. The primary advantage of this methodology, as noted by Larcker, Gordon, and Pinches (1980), is the ability to specifically test for a shift in a series, given the presence of significant autocorrelation.

B. Basic Methodology of Intervention Analysis

In intervention analysis, the ARIMA model describing the stochastic behavior of a time series process is comprised of what is referred to as the noise component of the model, and an intervention component. The intervention model for a time series variable y_t can therefore be expressed as:

$$y_t = f\ (I_t) + N_t$$

where N_t denotes the noise component, and $f\ (I_t)$ the intervention component. The intervention component describes the deterministic relationship between the event variable, I_t, and the time series variable, while the noise component is essentially the ARIMA process representing the stochastic portion of the time series variable not explained by its relationship to the event variable. In actuality, intervention analysis is transfer function modeling with a stochastic output series and a deterministic input variable.

The simplest forms of intervention components are those of the pulse and step functions where:

$$f\,(I_t) = \omega_o I_t$$

and

$$I_t = \begin{cases} 0 & \text{prior to an event} \\ 1 & \text{for all periods after the event} \end{cases}$$

in the case of the step function, and

$$I_t = \begin{cases} 0 & \text{for all periods not equal to the event time} \\ 1 & \text{for the event period} \end{cases}$$

in the case of the pulse function.

For the purposes of testing for seasonality in the ISD estimates however, more complex variables must be employed.

C. January, Monthly, and Quarterly Seasonal Intervention Variables

In usual circumstances, the modeling of an intervention variable to capture the effect of a change in level in a times series would involve a step function form. For example, in the case of a January seasonality a step function modeling of the event structure would imply an abrupt increase in the process level of the ISD estimates in December followed by an abrupt return in level in the month of January. Although appropriate if an instantaneous volatility were observable, this particular structure is questionable in the case of ISDs due to the overlapping time periods for which the weekly estimates apply.[6]

Merton (1973) has shown that if the instantaneous variance rate is nonconstant and can be expressed as a deterministic function of time, $\sigma^2(t)$, then the variance implied by a constant variance assumption model, $\sigma^2(T)$, can be more generally defined as the average variance rate per unit time from the valuation date to the time of the options expiration:

---

[6]Patell and Wolfson (1981) deal with a similar problem in a study of the effect of quarterly earnings announcements on implied volatilities.

$$\sigma^2(T) = T^{-1} \int_0^T \sigma^2(t)dt.$$

Continuing with our example, to measure the average variance corresponding solely to say the month of January, $\sigma^2(J)$, that is implied by options with times to maturity that include the January period, we would theoretically have to integrate implied variance over time where the bounds of integration would be the periods represented by the beginning J_b and end J_e of the month of January:

$$\sigma^2(J) = T_J^{-1} \int_{J_b}^{J_e} \sigma^2(t)dt.$$

This would, of course, necessitate the assumption of a functional form for the relationship of instantaneous variance to time.

Given that the assumed constant volatilities implied by options represent an average of the market's expectations of volatility, over the time to the options maturity, the expected volatility corresponding to the month of January receives a weighting proportional to its representation of the time left to maturity.

Consider now the time series of ISD estimates derived from options on S&P 500 index futures and their aforementioned calendar pattern of observation time to maturity month. Beginning in the month of December, as implied volatilities from options maturing in March are observed, the proportion of time left to maturity that the month of January represents is nonzero, and in fact increases with each consecutive weekly observation in December. The proportion representing January reaches a maximum with the last weekly observation in December, then decreases throughout the month of January to a value of zero with the first observation in February. Consequently, if the month of January is associated with a higher expected volatility than that of other months, the effect will be a gradual increase in the level of weekly implied volatility estimates throughout December, followed by a decrease throughout the month of January.

Hence, in order to test for the possibility of this impact structure, an intervention variable was coded for each ISD observation which effectively represents the proportion of the time left to maturity of the option observed that the month of January comprised. For all options where the month of January did not represent a portion of the time remaining to maturity, the variable was given a value of zero. The effect of this particular modeling of the turn-of-the-year event is to approximate the integration of the implied volatility over the January time period. Similarly, January effect intervention variables were coded for the time series of ISD estimates based on the options on the TSE 300 and TSE 35 indices.

In the same manner, variables can be coded to measure the effect of a monthly and quarterly seasonality in the ISD estimates given their overlapping nature. In the case of monthly seasonality an intervention variable was coded, for all three time series, as the proportion of time remaining to maturity represented by the last trading day and first eight trading days of any month. This is consistent with Ariel's (1987) findings of significantly higher returns in the first 9 trading days of a month, with a trading month extending from, and including the last trading day of the preceding calendar month up to but not including the last trading day of the current calendar month.

In the case of quarterly seasonality, intervention variables were coded, for all three time series as the proportion of time remaining to maturity represented by any part of the first two weeks of the months of March, June, and September. This is consistent with Penman's (1987) findings of higher returns associated with the first two weeks of the calendar quarters two through four.

D. Time to Maturity and October 87 Crash Intervention Variables

Along with the seasonality variables described above two other variables were identified as being appropriate to include in the intervention tests. These consisted of an intervention variable coded to correspond with the October 1987 stock market crash, and a variable representing the time to maturity of the options employed.

Preliminary observations of the time series of ISD estimates based on the S&P 500 index futures options and TSE 35 index options indicate a significant intervention in the ISD series coinciding with the OCT 1987 stock market crash. On October 19, 1987 the S&P 500 index exhibited the largest one-day drop in the history of major stock indices (Schwert 1990). As can be seen in Figures III-1 and III-3 the ISD estimates for these two time series exhibit a significant increase in level corresponding with this event, which then decays in effect over a period of approximately one year. The presence of such an obvious outlier in the time series of ISD estimates can result in significant identification errors in the process of specifying the model form for the noise component of the series.

The source of this aberrant observation is simply a non-repetitive exogenous intervention, the cause of which is irrelevant for our purposes. Several authors have however, studied the contaminating effects of such outliers on the sample autocorrelation function (SACF) and sample partial autocorrelation function (SPACF) of a series. Miller (1980) Chang and Tiao (1983) and Tsay and Tiao (1984) have shown that the the presence of such outliers can cause substantial biases in these statistics and hence jeopardize their use as identification tools. Depending upon their form, the existence of such outliers can also lead to erroneous transformations and differencing of the time series examined due to the apparent heteroskedasticity and nonstationarity they may instill.

The overall effect of this event, then, is to add noise to the ISD estimates for the purpose of testing for calendar seasonalities in instantaneous volatilities. In order to control for this additional noise in the ISD estimates due to the crash, an intervention variable was coded as a value of zero for all observations before the crash. The variable was given a maximum value of one, corresponding with the weekly ISD estimate that included the date of the crash, and then was coded to exponentially decay in value to approximately zero over a period of 52 observations. Although Schwert (1990) in a test of the effect of the crash on stock return volatility estimates that such volatility returned to normal levels by March of 1988, estimates of the coefficient of a dummy variable coded to decay over a 6-month period were insignificantly different from the one year decay variable employed.

An additional concern, given the constant variance option pricing models employed here to derive the ISD estimates, is the systematic biases that would be present in the ISDs if the assumption of constant variance is incorrect. Unfortunately, although a number of authors have examined option pricing models under the assumption of stochastic volatility, (for examples see Hull and White (1987) and Wiggins (1987)) these models have either been intractable or quite cumbersome.

Although Wiggins (1987) suggests that the use of at-the-money options would minimize any stochastic volatility biases, Hull and White (1987) have shown that if volatility is stochastic and uncorrelated with the security price then, relative to a stochastic volatility option pricing model, at-the-money options will have a tendency to be overpriced by a model assuming constant volatility. The effect of this is that if the assumption of constant variance over the life of the option is not true, then implied volatilities estimated from a constant variance model will be negatively correlated with time to maturity. Indeed this negative association between implied volatility and time to maturity has been observed by Rubinstein (1985) and Martin and French (1987) for options on common stock.

In order to control for the presence of any mispricing due to the assumption of constant volatility, an intervention variable was included in the analysis, coded simply as the fraction of a year that the time to maturity, associated with the option observed, represents.

E. Multiple-input Intervention Analysis

In order to simultaneously test the significance of the three seasonality, October 1987 crash, and time to maturity intervention variables, a multiple-input intervention or transfer function approach is required. This approach has been applied with success to similar problems such as the identification of time series models in the presence of calendar variation (Hillmer, Bell, and Tiao (1981) and Liu (1986)) and trading day effects (Hillmer (1982) and Liu(1986)). As in the present analysis involving several variables, calendar variation and trading day effect variables were included in these studies, in a transfer function approach as multiple deterministic (intervention) inputs.

These multiple-input intervention models are a subset of a larger class of multi-equation transfer function models that include vector ARIMA models (see Tiao (1985) for a brief review). As indicated by Liu (1987), they are natural extensions of "classical" regression models and the simultaneous equations systems used in econometrics. The primary difference is that the residuals for classical regression and econometrics models are assumed to be white noise while the transfer function models allow the noise to follow time series processes.

The major difficulty in multiple input transfer function models is the identification of the form of the ARIMA model for the noise component, in the presence of the deterministic inputs. In an intervention analysis of the form originally dealt with by Box and Tiao, the time series model of the noise component would be determined by fitting a model to the data preceding the postulated intervention period (see McCleary and Hay (1980)). The rationale behind this is that estimating the model over data where the intervention is present can result in serious identification problems, similar to those discussed in the case of outliers. This is not always possible however in the case of multiple intervention modeling as pre-event data may not be available. Alternatively, Lui and Hanssens (1982) have developed a methodology of identifying the noise component that is designed to handle this identification problem. For multiple-input transfer function models the first step in this identification process is to identify an ARIMA model for the output series alone, which in the present case, is the series of ISD estimates. Each series is then prewhitened or "filtered" by the AR/MA factors of the first common one among them and the stationary factors from their own univariate model. The impulse response weights are then estimated using least squares. The residuals from this estimation are then examined to determine whether a second common filter is required. If so each series is filtered again by this second common filter. The impulse response weights are again estimated. These weights are then applied to the original series and the residuals are then the estimated noise series from which a preliminary noise model is identified. The parameters of the full model including those of the preliminary noise model are then estimated simultaneously using non-linear least squares based on the Marquardt algorithm and backcasting as discussed by Box and Jenkins (1970). From there, insignificant parameters are dropped in the usual Box-Jenkins identification-estimation and validity approach. In the case of multiple input intervention models these filters are applied only to the output series.

IV. EMPIRICAL RESULTS

To test the significance of the seasonality, crash, and maturity variables, multiple-input intervention analysis using the Lui-Hannsen common filtering technique was performed on the ISD estimates for the three series of data. In addition the technique was performed on sub periods of each of the three series, corresponding to roughly one year lengths.

A. Results for S&P 500 ISDs

The results of the above multiple-input intervention analysis in the case of the ISD estimates derived from options on S&P 500 index futures are shown in Table III-4. For the total period of observation, and each sub period examined, this table gives the estimated intervention variable coefficients and corresponding t-statistics either just before excluding the respective variable in the modeling process, in the case of insignificance, or its value in a final estimated model. In addition the table shows the identified and estimated noise parameters included in the final model.

In terms of the January seasonality variable, denoted JAN, results of the analysis performed on the complete time series of S&P 500 ISDs indicate that this variable has a negative (-.0059) although insignificant effect on the ISD estimates. Tests carried out on sub periods of data indicate that only for the 86/87 and 89/90 periods is the January intervention variable significant. These results however are not consistent as the effect of the January variable is negative (-.0721) for the former sub period and positive (.2013) in the case of the latter. Overall three of the six sub periods of S&P 500 ISDs exhibited a negative coefficient for the January seasonal variable and three exhibited a positive coefficient.

In general, we would have to conclude that a January seasonality in risk is at most transitory over the total period of 1984 to 1990 and overall not significant. The analysis was also carried out employing an alternative turn-of-the-year variable which was coded to capture the proportion of time to maturity represented by the last trading day of the year, and the first four trading days of the consecutive year. The results however were similar.

Results in the case of a monthly seasonality denoted by the variable MON indicate that for the overall period there is not a significant monthly seasonality in risk as measured by S&P 500 implied volatilities. Sub period results indicate that for three of the six sub periods the monthly seasonality variable exhibited a positive coefficient and a negative coefficient for the other three sub periods. Only three of the six sub periods, 84/85, 86,/87, and 89/90 exhibited significant monthly seasonality with negative coefficients for the 84/85 and 86/87 sub periods and a positive coefficient for the 89/90 sub period. Again the results indicate weak evidence of a monthly seasonality pattern to risk, and at best an intermittent one.

Similarly the test for a quarterly seasonality in risk does not in general indicate significant results. The variable QUART, coded to capture this seasonality has a negative coefficient value of -.0271 in the case of the analysis on the complete series, however it is insignificant at a t-statistic level of -1.22. Sub period results indicate that this variable has a negative coefficient for four of the six sub periods and a positive one for the other two. Only two of the sub period results, 86/87 and 88/89 indicated a significant quarterly seasonality and in both cases the sign of the coefficient was negative. The implications are that contrary to the seasonality observed in the case of stock returns, when significant, instantaneous volatilities have a tendency to be lower in the beginning of quarters two through four, than in the rest of the year.

Although the intervention variable coded for the October 1987 market crash (denoted CRASH in table 1) is strongly significant, the time to maturity variable MAT is not. For the overall period the coefficient for the time to maturity is negative, but insignificant. In only one sub period, 88/89 was the time to maturity variable significant, with again, a negative coefficient value. It should be noted however that four of the six sub periods, including that of 88/89, exhibited negative coefficient values while two of the sub periods exhibited positive ones. This result of a predominantly negative coefficient value indicates that a stochastic variance model may be more appropriate for pricing options on S&P 500 index futures.

B. **TSE 300 ISD Results.**

Table III-5 contains the results of multiple intervention analysis carried out on the ISD's derived from options on the TSE 300 index. Again this table gives the estimated coefficient and corresponding t-statistic values for the January, monthly, and quarterly seasonality variables as well as the time to maturity variable. Table III-5 also provides the estimated parameters of the ARIMA process identified for the noise series portion of the intervention model. The table provides the results of a multiple intervention model identified and estimated for the total series, as well as the three sub periods of a) July 7, 1984 to May 31, 1985 (84/85), b) June 7, 1985 to May 30, 1986 (85/86) and c) June 6, 1986 to June 10, 1987 (86/87).

The results in the case of the January effect intervention variable give mixed evidence as to the significance of a January seasonality. In terms of the total series the coefficient of the January intervention variable is negative in value, but insignificant. For two of the three sub periods the January variable coefficient is positive in value whereas for the 86/87 period the value is negative. However, two of the three sub periods, 85/86 and 86/87 exhibit a significant January variable coefficient with a positive value associated with the 85/86 period and a negative value associated with the 86/87 period. Again the data indicates that a January seasonality in market risk is at most transitory.

In general the results for the TSE 300 ISD's indicate that a monthly seasonality in risk is nonexistent. The monthly seasonality variable is insignificant in the case of the total period data and exhibits significance in only one of the three sub periods, 85/86, analyzed. The coefficient value is negative in the case of the total series and as well has a negative value for two of the three sub periods, including that of 85/86. This would appear to indicate that even when a monthly seasonality is present in the instantaneous volatility of the Canadian stock market, risk is on average lower during the beginning of trading months.

Results in the case of a quarterly seasonality in TSE 300 ISD's are very similar to that of the monthly seasonality. The coefficient for the quarterly seasonality variable is insignificant in the case of the whole series and is significant only for the 85/86 sub period. Again the predominant sign of the coefficient including that of the 85/86 sub period is negative in value.

As the time period of this data does not include the October 1987 crash the only additional variable, aside from the seasonal ones, included in the multiple intervention analysis, was the time to maturity. It is interesting to note that the coefficient of this variable was negative in value and significant in the case of the analysis carried out on the series as a whole, as well as for the three sub periods. This provides fairly strong evidence that a stochastic variance model may be more appropriate for pricing options on Canadian stock indices.

C. TSE 35 ISD Results

Table III-6 provides the results of the multiple-intervention analysis carried out on the time series of ISD's derived from options on the TSE 35 index. Again this table provides the estimated coefficients and associated t-statistics for the seasonal, crash, and time to maturity variable included in the multiple-intervention analysis. Results are provided for the total series as a whole as well as for the sub periods of a) July 10, 1987 to June 30, 1988 (87/88), b) July 8, 1988 to June 30, 1989, (88/89) and c) July 7, 1989 to March 30, 1990 (89/90).

As in the case of the S&P 500 and TSE 300 above, results for the January seasonal variable indicate that such a seasonality in TSE 35 instantaneous volatilities is largely nonexistent. The estimated coefficient for the variable JAN is insignificant for the series a whole and is significant in only one of the three sub periods (87/88) examined. The sign of the coefficient in this sub period is positive and negative in the other two sub periods.

Although the estimated coefficient is predominantly negative in sign, with only one of the three sub periods (89/90) exhibiting a positive coefficient value, the monthly seasonal variable was not significant for the total period, nor for any of the sub periods. Similarly, the quarterly seasonality variable, having a predominantly negative estimated coefficient, is significant in only the 89/90 sub period of the series. The sign of this significant coefficient is negative in value implying that if a quarterly seasonality exist, volatility is lower during the first two weeks of quarters two through four.

The crash variable is predictably significant for the TSE 35 volatility series with a positive coefficient value. The time to maturity variable, however, exhibits somewhat different results than in the case of the S&P 500 and TSE 300 ISD series. The estimated coefficient for this variable is positive in value and predominantly significant with only the 89/90 sub period exhibiting an insignificant coefficient. This is contrary to Hull and White's (1987) predicted negative association between the ISD's derived from a constant volatility model and the time to maturity, for at-the-money options. Even in the presence of correlation, positive or negative, between volatility and the index level, Hull and White show that the assumption of constant volatility would still result in the overpricing of at-the-money options and hence a negative association between ISD and time to maturity. Consequently, their results cannot explain the positive association noted here. The need for further research into the pricing of options on the TSE 35 is indicated by these results.

D. Implications of Maloney and Rogalski's (1989) Study

Recently, Maloney and Rogalski (1989) in a similar study, examined implied volatilities from options on common stocks for evidence of January and monthly calendar seasonalities.[7] Contrary to the results presented here, they find evidence of a January seasonality in implied volatilities and hence a discussion of their methodology, results and the implications for the present study is appropriate.

i. Differences in Methodology

Several questions must be addressed in an attempt to reconcile the seemingly conflicting evidence of the Maloney and Rogalski study, and the results presented here for options on indices and index futures. First, let us examine the differences in testing methodology employed in these two studies.

The methodology Maloney and Rogalski use, to test for a turn-of-the-year seasonality in the regression residuals, is similar to that employed by Patell and Wolfson (1981) in examining the effect of earnings announcements on ISDs. Basically, this approach involves comparing the average implied volatility between different observation dates to determine whether or not a significant difference in average ISD exists. In the case of Maloney and Rogalski, they examine volatilities implied from call options on common stock for the calendar year end periods of 1973 to 1984 inclusive. In particular, they employed options on the common stock of 29 different firms, where

---

[7]The present study was well under way when Maloney and Rogalski's work was published.

the time to maturity of the options spanned year end periods.

Using all the available call options for each particular firm, they calculated implied volatilities based on the call option pricing model of Black (1976), for five different trading days around year end periods. Similar to the present study, they attempted to control for any biases in the data due to the choice of an option pricing model by regressing the calculated ISDs on a firm specific constant, the natural log of the stock price to exercise price ratio, and the time to maturity of the options employed. The intended effect of this regression was to "purge" the data of any time-to-maturity, or moneyness biases.

The Patell and Wolfson test was then conducted on the residuals from this regression by examining the mean residuals for the observation times of t = -30, -10, -2, +2, and + 10, where t is the number of trading days relative to a turn-of-the year. In summary, they found a significant average increase in ISDs between the trading days of -30 and -10, an insignificant decrease between trading days -10 and -2, and significant decreases between trading days -2 and +2, and between +2 and +10. In general this evidence gives some support of a January effect, with implied volatilities on common stocks tending to increase between mid-November and mid-December, and then subsequently declining during the turn of the year.

In terms of the specific methodology employed to derive a measure of implied volatility, the issues involved in the differences of approach between the Maloney and Rogalski study and the present case are of relatively little importance. Maloney and Rogalski effectively incorporate implied volatilities from all call prices available on the observation days of interest, regardless of moneyness, as opposed to the single observation of an at-the-money call option employed here. The issue however, concerning the optimal measure of implied volatility, in terms of a weighting of option contracts employed, is still relatively unresolved in the options literature. Various weighting schemes have been employed by different authors; however no direct comparison of these approaches has been made except in the case of Wilson and Fung (1991). Employing options on grain futures Wilson and Fung compared the implied volatilites estimated with four different weighting schemes:

1) The simple use of an at-the-money option
2) an equal weighted average of ISDs from options of all degrees of moneyness
3) the Chiras and Manaster (1978) weighted average where ISDs are weighted by the relative price elasticity of their option with respect to volatility
) the Chiras (1977) weighted average where weights are simply based on the price sensitivity of the option with respect to volatility.

They find that ISDs are not significantly different across the four weighting schemes.

In addition Brenner and Galai (1984), in a study of ISDs estimated from transaction data for options on common stock suggest that the average ISD during the day is a more reliable estimate of the risk of common stocks than estimates based on last daily observations.

They find though, that the difference between the daily average and that based on the last observation is minimized with well traded options. As well, they find that the existence of nonsymmetric deviations of ISDs based on the last daily observation from that based on the daily average implies that averaging ISDs across option classes does not avoid the problems associated with ISDs based on last trade prices. Given this evidence, and the relatively large trading volume involved in options on indices and index futures, it is unlikely there would be significant differences in the use of a single near-the-money option to calculate ISDs and the approach of averaging across multiple option classes employed by Maloney and Rogalski. If the difference in approach to ISD estimation is significant, it is again unlikely that it would result in any systematic differences in the ISD estimates which would have implications for a test of seasonality, other than a loss of power.

More importantly however is the fact that Maloney and Rogalski employ closing prices of options on common stocks to estimate volatility. As the options market closes 15 minutes after the stock market Harvey and Whaley (1991) document that ISDs estimated in this manner will exhibit negative serial correlation in their changes. The use of options on the S&P 500 index futures avoids this estimation problem as both the options and futures market close at the same time daily.

Both studies attempt to correct for any pricing biases though, due to the choice of an option pricing model. This is done by way of a two-stage "filtering" in the case of Maloney and Rogalski, and simultaneous estimation in the present study. More importantly however, is how potential serial correlation, present in the data is dealt with. By not addressing this issue in their methodology, Maloney and Rogalski face the risks associated with employing inefficient t-statistics to test their data.

Although a specific test for the presence of auto correlation in the mean residuals associated with varying trading days is not mentioned in their paper, the evidence of auto correlation in ISDs found in the present study and others (see for example Schwert (1990)) indicates that this could be a factor in their data. The question arises as to whether or not averaging across different time periods eliminates any serial correlation in the ISD residuals, given the averaging across years that their methodology employs. To explore this possibility, the following test was carried out on the ISD estimates derived from the options on the S&P 500 index futures.

Similar to Maloney and Rogalski, the weekly ISD's derived from options on S&P 500 index futures were first "purged" of any influences due to 1) the degree of moneyness, and 2) the time to maturity, by regressing the ISDs on the natural log of the ratio of futures price to exercise price, and the time to maturity as a fraction of a year. The results of this regression are given below:

Variable	Coefficient	t statistic
Constant	0.179	15.179
ln(F/ X)	0.418	0.841
Time to Maturity	-0.037	-0.583

R^2 = .003

F = .443
N = 312
Durbin Wat... statistic = .500

There are some interesting differences in the above regression results and those of the similar regression carried out by Maloney and Rogalski for options on common stocks, which should be noted. First, Maloney and Rogalski report an R^2 of .9162 in terms of their regression, indicating that along with a constant, the time to maturity and degree of moneyness explain the majority of the variance in the implied volatility estimates. Alternatively, the R^2 reported here for implied volatilities from options on index futures is only .003 indicating that very little of the variance in index futures ISDs is explained by the above variables. There may be several reasons for this difference. First, Maloney and Rogalski employ all classes of options to calculate implied volatilities, whereas the options employed for the S&P 500 index futures were those closest to-the-money. Hence, the degree of moneyness of the options employed to calculate index futures ISDs is to some extent already controlled for, possibly explaining the insignificance of this variable in the regression.

Second, the choice of an option pricing model may significantly alter the regression results. The option pricing model employed for the options on index futures takes in to account, more fully, the possibility of early exercise, than does the Black (1976) model used by Maloney and Rogalski. This factor could result in enhanced pricing biases showing up in the ISDs derived form options on common stocks which would be significantly explained by the time to maturity and degree of moneyness variables. Indeed Harvey and Whaley (1991) stress the danger of estimating implied volatilities from a European option pricing model when the true nature of the options are American.

It should also be noted, from the above regression results, that consistent with the previous multiple input intervention analysis, the relationship of ISD to time to maturity is a negative one, as proposed by Hull and White in the case of stochastic volatility. This is contrary to the results of Maloney and Rogalski who find that in the case of common stocks, and similar to the above findings for the options on Canadian stock indices, higher implied volatilities were associated with longer times to maturity. This again may simply be due to pricing biases associated with the particular option pricing model employed, or more importantly, may perhaps be due to fundamental differences in the return generating processes for common stocks and that for index futures. The need for further research is indicated by these results.

With a Durbin Watson statistic of .500 the presence of at least first order auto correlation in the residuals of the regression is indicated. Indeed, when Box-Jenkins analysis is carried out on the residuals it is found that the time series model that best describes the behavior of the residuals through time, is an AR(1) with a coefficient value of 0.4568 and a corresponding t statistic of 3.60 This indicates that even after averaging across years, significant auto correlation exists in the ISD's after "filtering" out any effects due to time to maturity and degree of moneyness.

ii. Implications of Contradictory Results

It is important to note that these results, concerning the possible contaminating effects of auto correlation do not cast doubt on Maloney and Rogalski's findings of a significant January seasonality. If auto correlation is present in the common stock ISDs and the regression residuals that they employ, then their test of the equality of mean ISDs across trading days spanning the turn-or-the-year is simply inefficient, but not biased. What these results do indicate however, is that a test specifically controlling for the time series properties of ISDs, such as that of multiple input intervention analysis employed in the present study, is a more appropriate one. In this sense the evidence provided here, conflicting with that of Maloney and Rogalski as to the significance of a turn-of-the-year seasonality in ISDs, is not simply due to a difference in testing methodology.

The conflict in evidence is however of particular interest, especially when we note that the 29 firms Maloney and Rogalski consider in their study, are all relatively large in size. This is understandable as very few relatively small firms have active options contracts trading on their common stock. In this sense however, the average ISDs employed by Maloney and Rogalski are not characteristic of the risk of the market as a whole.

The S&P 500 index however, although also comprised of mainly large firms, is intrinsically more indicative of the total market. Consequently, the presence of a significant ISD turn-of-the-year seasonality in Maloney and Rogalski's collection of large firms as opposed to that of the S&P 500 indicates that this seasonality may be predominantly a large firm effect.

This is especially interesting when we consider the evidence of some authors that the turn-of-the-year seasonality in returns can be largely attributed to small firms. Obviously, further research is required to reconcile the differences in the results presented here, and those of Maloney and Rogalski, and to test the above hypothesis. In particular, where possible, a detailed examination of the behavior of ISDs for common stocks of firms comprising the S&P 500, including any employed in the Maloney and Rogalski set, is warranted. It would also be worthwhile, as much as data permits, to examine the behavior of ISD's from options on stocks of relatively small firms, as well as that of the Value Line index which gives a proportionally greater weighting to small firm returns.

An alternative explanation to the conflict in results is that perhaps a January effect in market risk is transitory in nature and in particular is no longer present in the market in recent years. The Maloney and Rogalski study examines ISDs over the period of 1973 to 1984, while the present study, due to data limitations, only examines the period of 1984 to the present. Indeed Maberly and Maris (1991) have recently noted that while the January effect has not altogether been eliminated, arbitrage opportunities in the S&P 500, due to the effect have been substantially reduced since the advent of derivative security markets in 1982 and 1983. Further research to determine the extent of a turn-of-the-year seasonality in implied volatilities over different time periods is hence also required.

V. CONCLUSION

In general, given the above empirical results, we have to conclude that calendar seasonalities corresponding to those documented in returns, are not present in the expectations of total risk of the market, and hence do not provide an explanation for the corresponding return seasonalities. There are, however, several potential reasons as to why the above test may fail to capture such seasonalities in risk.

Recently, Stein (1990) in a study of the "term structure" of implied volatilities derived from index options finds that longer term maturity options tend to "overreact" to changes in implied volatilities of shorter maturity options. This is effectively a test of Merton's hypothesis that the implied volatility represents an average over time of the instantaneous volatility. If incorrect, as Stein's results appear to suggest, then seasonality intervention variables based simply upon a time to maturity weighting, as employed in this paper, would be inappropriate. In terms of an intervention variable designed to capture a January effect, for example, the weight corresponding to the January period would have to be greater than the proportion of time to maturity that January represents, for relatively longer term options. Consequently, the presence of this overreaction phenomena in the index and index futures options market would reduce the significance of a January intervention variable coded simply as the proportion of time to maturity that January represents.

In addition it should be noted that the implied volatilities derived for the study apply to stock indices. Obviously we make the assumption, when testing for seasonality in total market risk, that these indices are unbiased proxies of the true optimal portfolio consisting of all risky assets in proportion to their outstanding values. If this assumption does not hold and assets are priced according to the capital asset pricing model, then it is possible that the observed seasonal returns are due to nonstationarity in the systematic risk of the particular stock indices employed. If total risk does not increases with systematic risk, such nonstationarities would escape detection in the present testing methodology.

However, given the above caveats, it is also worth noting that a test of seasonality in ISD consists of a joint test of a) the option pricing model employed and b) the efficiency of the options market. Given the documented evidence of seasonality in risk variables based on historical returns, the lack of any seasonality in implied volatilities would suggest the rejection of this joint hypothesis. This would not be inconsistent with other findings such as that of Sheikh (1989) who observes that post-stock split increases in volatility of common stocks is not reflected in implied volatilities spanning the split announcement dates.

Aside from the above lack of significant seasonalities in the implied volatilities, this paper does, however, present a rigorous testing procedure based upon time series analysis which can be employed in any test of nonstationarity of ISDs given their usual overlapping nature and resulting autocorrelation. In addition, the empirical results presented here for the implied volatilities from options on S&P 500 index futures, indicating a lack of turn-of-the-year seasonality, are fairly suggestive, given previous evidence of such a seasonality in the ISDs derived from options on relatively large firm stocks. Further research in to the nature and extent of the turn-of-the-year seasonality in implied volatilities is indicated by these results.

Figure III-1

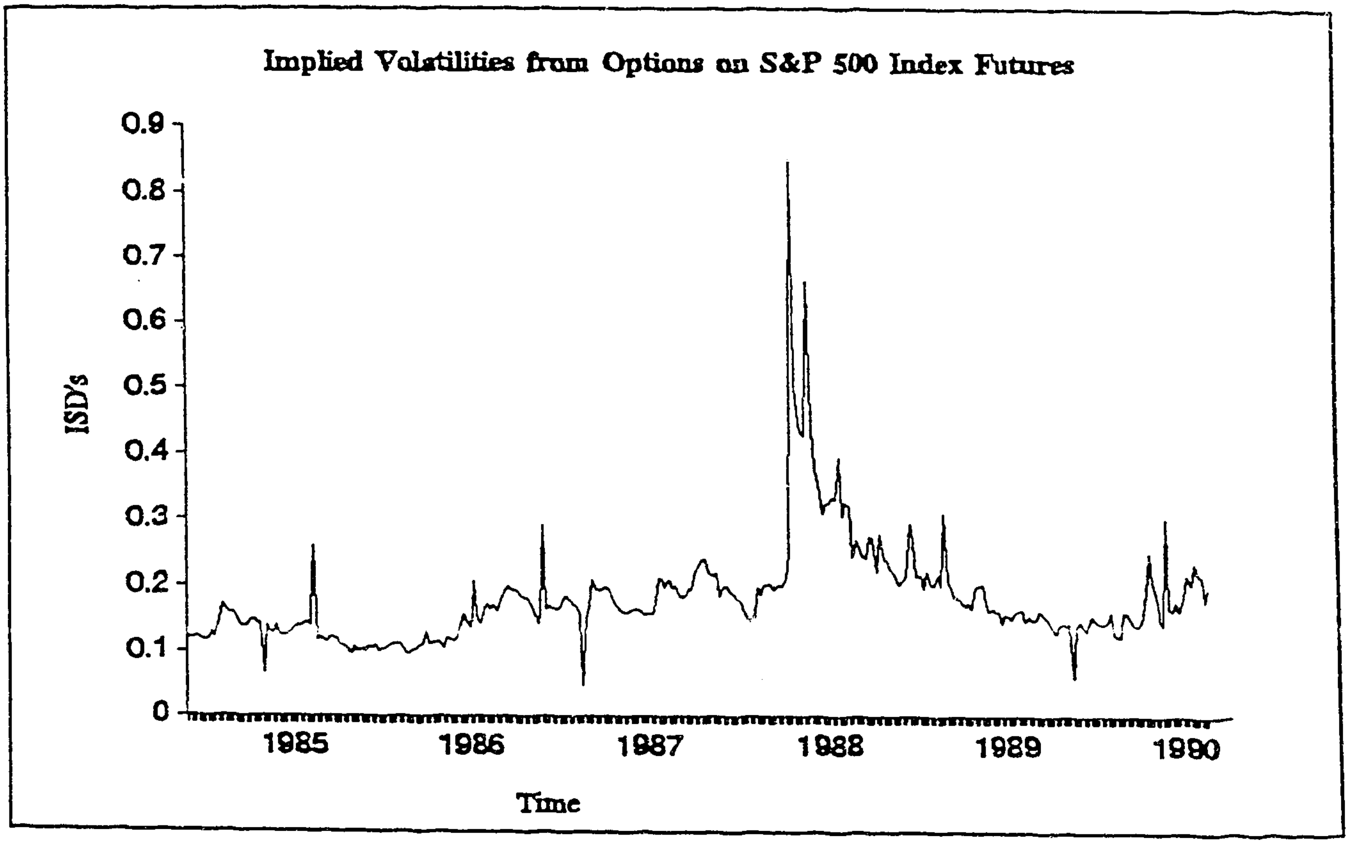
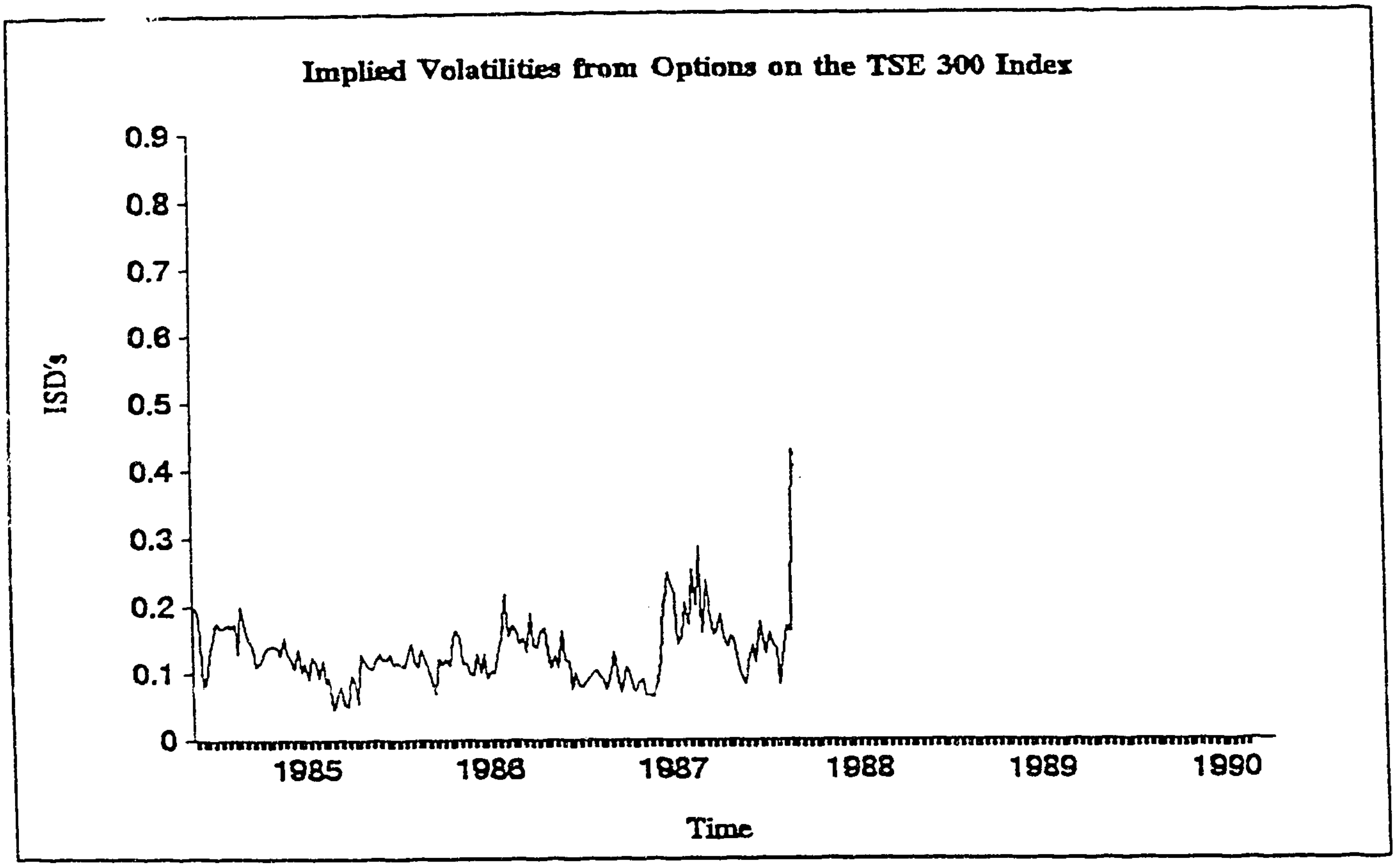
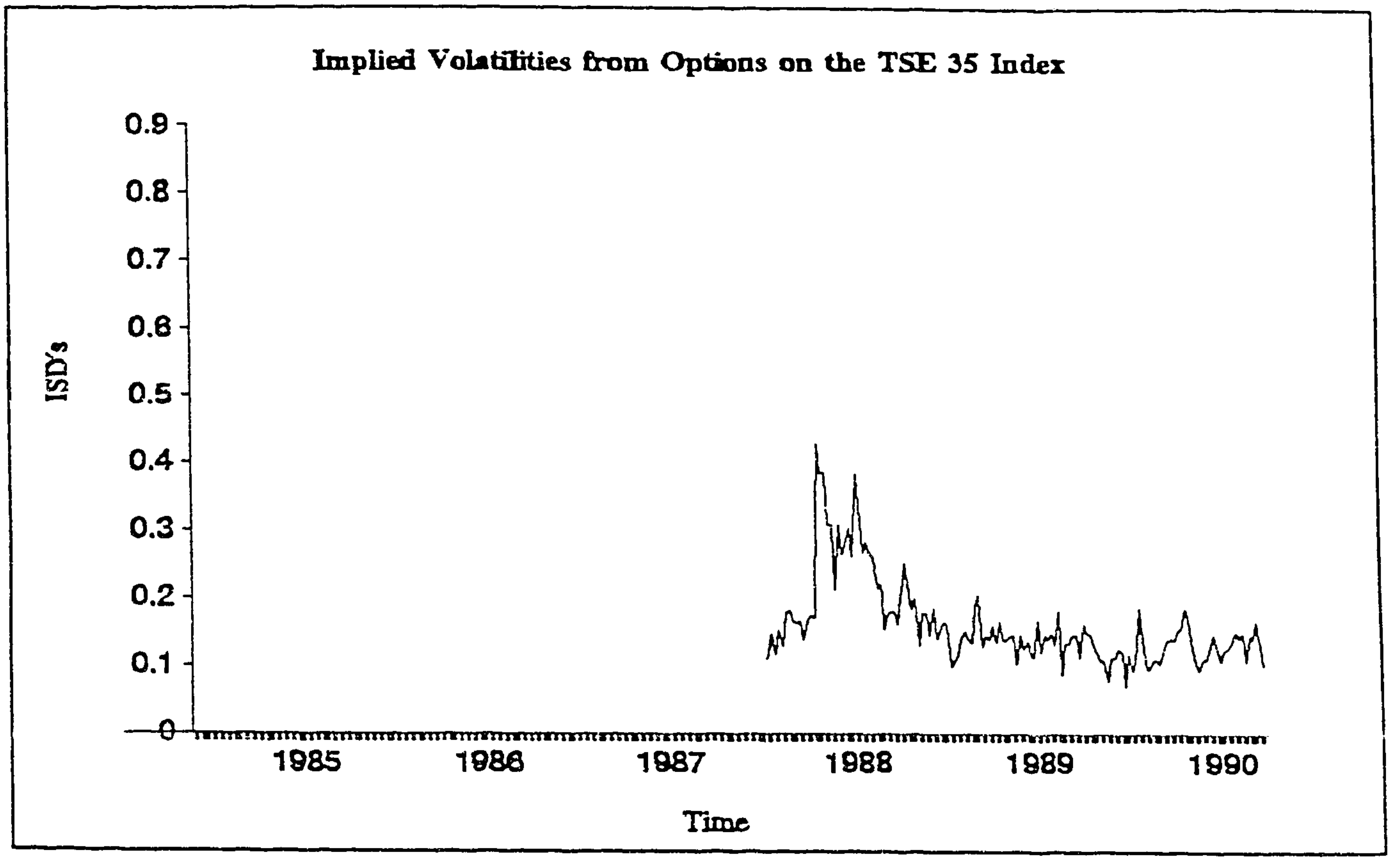

Implied Volatilities from Options on S&P 500 Index Futures
ISD's
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1985
1986
1987
1988
1989
1990
Time

Figure III-2



Implied Volatilities from Options on the TSE 300 Index
ISD's
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1985
1986
1987
1988
1989
1990
Time

Figure III-3



Implied Volatilities from Options on the TSE 35 Index
ISD's
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1985
1986
1987
1988
1989
1990
Time

Table III-1: Time-series Models Identified for Weekly Implied Volatilities Estimated from Options on S&P 500 Index Futures

Period	ARIMA Model	Mean	Parameters ϕ_1	ϕ_2	ϕ_3
Total Period					
01/06/84 - 30/02/90	(3,0,0)	.1309	.5161	.2549	.1301
		(2.51)	(8.86)	(3.98)	(2.23)
Subperiods					
01/06/84 - 31/05/85	(2,0,0)	.1299	.2406	.2372	
		(29.08)	(1.70)	(1.66)	
07/06/85 - 30/05/86	(3,0,0)	.2744	.3490		.6165
		(2.62)	(1.76)		(3.16)
06/06/86 - 29/05/87	(1,0,0)	.1815	.6460		
		(19.78)	(5.97)		
05/06/87 - 26/05/88	(2,0,0)	.2870	.4950	.2105	
		(5.74)	(3.44)	(1.46)	
03/06/88 - 25/05/89	(2,0,0)	.1725	.4588	.2286	
		(16.06)	(4.39)	(2.19)	

(Numbers in parentheses are t-statistics)

Table III-2: Time-series Models Identified for Weekly Implied Volatilities Estimated from Options on the TSE 300 Index

Period	ARIMA Model	Mean	Parameters ϕ_1	ϕ_2	ϕ_3
<u>Total Period</u>					
20/07/84 - 16/10/87	(2,0,0)	.1261	.5143	.2381	
		(13.60)	(6.81)	(3.17)	
<u>Subperiods</u>					
20/07/84 - 31/05/85	(1,0,0)	.1172	.6772		
		(9.08)	(6.40)		
07/06/85 - 30/05/86	(2,0,0)	.1264	.4280	.1934	
		(14.83)	(3.09)	(1.49)	
06/06/86 - 16/10/87	(2,0,0)	.1310	.5104	.2846	
		(6.44)	(4.36)	(2.43)	

(Numbers in parentheses are t-statistics)

Table III- 3: Time-series Models Identified for Weekly Implied Volatilities Estimated from Options on TSE 35 Index

Period	ARIMA Model	Mean	Parameters ϕ_1	ϕ_2	ϕ_3
Total Period					
10/07/87 - 30/03/90	(2,0,0)	.1611	.5748	.2814	
		(7.29)	(7.01)	(3.44)	
Subperiods					
10/07/87 - 30/06/88	(2,0,0)	.2216	.5515	.2306	
		(6.54)	(3.89)	(1.65)	
08/07/88 - 30/06/89	(2,0,0)	.1324		.2657	
		(27.55)		(1.83)	
07/07/89 - 30/03/90	(1,0,0)	.1253	.4627		
		(21.31)	(3.66)		

(Numbers in parentheses are t-statistics)

Table III-4: Estimated Multiple-input Intervention Model for Implied Volatility Estimates from Options on S&P 500 Index Futures.

Period	Intervention Inputs: JAN	MONTH	QUART	CRASH	MAT	Noise Model[a]: Constant	ϕ_1	ϕ_2	ϕ_3
Total Period									
01/06/84	-.0059	.0147	-.0271	.5631*	-.0621	.1711	.5374	.2582	
30/02/90	(- .14)	(.39)	(-1.22)	(16.32)	(-1.42)	(17.13)	(9.52)	(4.60)	
Subperiods									
01/06/84	.0027	-.0725*	.0192	NA	-.0729	.1586	.4199		
31/05/85	(.09)	(41.56)	(.45)		(-1.51)	(20.64)	(2.75)		
07/06/85	-.0210	.0027	.0093	NA	.0265	.5297	.3655		.6257
30/05/86	(- .51)	(.04)	(.34)		(.75)	(.21)	(1.92)		(3.30)
06/06/86	-.0721*	-.2891*	-.1132*	NA	.0814	.2703			
29/05/87	(-2.75)	(-6.99)	(-4.00)		(1.61)	(20.31)			
05/06/87	.1325	.0463	-.0812	.6615*	-.0423			b	
26/05/88	(1.14)	(.52)	(-.72)	(26.10)	(-1.40)				
03/06/88	-.0115	-.0021	-.1845*	NA	-.1963*	.2124	.8252		
25/05/89	(-.17)	(-.04)	(-12.34)		(-12.27)	(23.88)	(16.96)		
02/06/89	.2013*	.3198*	-.0646	NA	-.0895	.0825	.6478	c	
30/02/90	(2.73)	(4.48)	(-1.39)		(- .69)	(5.58)	(4.62)		

(Numbers in parentheses are t-statistics)

* significant at the 95% confidence level

[a] ϕ_i refers to an autoregressive parameter of order i, and θ_i refers to a moving average parameter of order i

[b] The series in this model has been differenced and the noise component consists of a moving average parameter θ_1 = .9690 (44.54)

[c] The noise model included a moving average parameter θ_7 = .9269 (52.49)

Table III-5: Estimated Multiple-input Intervention Model for Implied Volatility Estimates from Options on the TSE 300 Index.

Period	Intervention Inputs					Noise Model[a]			
	JAN	MONTH	QUART	CRASH	MAT	Constant	ϕ_1	ϕ_2	θ_7
Total Period									
20/07/84	-.0061	-.0427	-.0028	NA	-.1474*	.1410	.4929	.2537	-.2153
16/10/87	(- .28)	(-1.05)	(- .14)		(-2.19)	(11.56)	(6.51)	(3.35)	(-2.71)
Subperiods									
20/07/84	.0333	.0004	-.0206	NA	-.2473*	.1421	.7479		
31/05/85	(.90)	(.003)	(-.47)		(-1.92)	(7.33)	(7.51)		
07/06/85	.0677*	-.1409*	-.0566*	NA	-.6213*	.2304	.3465	.3819	-.9144
30/05/86	(4.36)	(-5.34)	(-4.62)		(-9.14)	(24.97)	(2.45)	(2.74)	(-7.88)
06/06/86	-.1320*	-.0034	.0308	NA	-.1466*	.1480		.6642	-.7056
16/10/87	(-3.67)	(- .13)	(1.81)		(-6.28)	(29.42)		(6.38)	(-7.64)

(Numbers in parentheses are t-statistics)

* significant at the 95% confidence level

[a] ϕ_i refers to an autoregressive parameter of order i, and θ_i refers to a moving average parameter of order i

Table III-6: Estimated Multiple-input Intervention Model for Implied Volatility Estimates from Options on the TSE 35 Index

	Intervention Inputs					Noise Model[a]			
Period	JAN	MONTH	QUART	CRASH	MAT	Constant	ϕ_1	ϕ_2	θ_3
Total Period									
10/07/87	.0195	-.0269	-.0073	.2633*	.2408*	.1290	.5123	.3195	
30/03/90	(.82)	(.57)	(- .31)	(9.42)	(3.07)	(8.09)	(6.30)	(3.92)	
Subperiod									
10/07/87	.1543*	-.0169	.0131	.2315*	.2937*	.1636	.2835		
30/06/88	(4.97)	(- .33)	(.47)	(4.78)	(4.26)	(10.11)	(1.66)		
08/07/88	-.0086	-.0659	-.0183	NA	.4567*	.0912		.2700	-.4777
30/06/89	(- .42)	(-1.14)	(-.83)		(4.74)	(8.75)		(1.87)	(-3.70)
07/07/89	-.0132	.0772	-.0630*	NA	.1249	.1304	.4788		
30/03/90	(- .51)	(1.07)	(-2.14)		(1.14)	(21.05)	(3.68)		

(Numbers in parentheses are t-statistics)

* significant at the 95% confidence level

[a] ϕ_i refers to an autoregressive parameter of order i, and θ_i refers to a moving average parameter of order i

VI. REFERENCES

Abraham, B., 1981, "Missing Observations in Time Series," *Communications in Statistics*, A-10, p.1643-1653.

Ariel, R., 1987, "A Monthly Effect in Stock Returns," *Journal of Financial Economics*, Vol.18, p.161-74.

Barone-Adesi, G. and R.E. Whaley, 1987, "Efficient Analytical Approximation of American Option Values," *Journal of Finance*, Vol.42, p.301-320.

Black, F., 1975, "Fact and Fantasy in the Use of Options," *Financial Analysts Journal*, Vol.31, p.36-41.

Black, F., 1976, "The Pricing of Commodity Contracts," *Journal of Financial Economics*, Vol.3, p.167-79.

Box, G.P., and G.M. Jenkins, 1970, *Time Series Analysis Forecasting and Control*, San Francisco; Holden Day.

Box, G.P., and G.C. Tiao, 1975, "Intervention Analysis with Applications to Economic and Environmental Problems," *Journal of the American Statistical Association*, Vol.70, p.70-79.

Brenner, M., G.R. Courtadon, and M. Subrahmanyam, 1985, "Option on the Spot and Options on the Futures," *Journal of Finance*, Vol.40, p.1303-17.

Brenner, M., and D. Galai, 1984, "On Measuring the Risk of Common Stocks Implied by Options Prices: A Note," *Journal of Financial and Quantitative Analysis*, Vol.19, p.403-412.

Brown, P., D. Keim, A. Kleidon, and T. Marsh, 1983, "Stock Return Seasonalities and the Tax-Loss-Selling Hypothesis: Analysis of the Arguments and Australian Evidence," *Journal of Financial Economics*, Vol.12, p.105-127.

Chan, K.C., 1986, "Can Tax-Loss Selling Explain the January Seasonal in Stock Returns?," *Journal of Finance*, Vol.41, p.1115-1128.

Chang, I., and G.C. Tiao, 1983," Estimation of Time Series Parameters in the Presence of Outliers," Technical Report 8, University of Chicago Statistics Research Center.

Chiras, D., 1977, "Predictive Characteristics of Standard Deviation of Stock Returns Deferred from Option Prices," Ph.D. dissertation, University of Florida.

Chiras, D., and S. Manaster, 1978. "The Information Content of Options Prices and a Test of Market Efficiency," *Journal of Financial Economics*, Vol.7, p.213-34.

Cox, J., and M. Rubinstein, 1985, *Options Markets*, Prentice-Hall Inc

Debondt, W.F.M., and R. Thaler, 1985, "Does the Stock Market Overreact?," *Journal of Finance*, Vol.40, p.793-805.

Debondt, W.F.M., and R. Thaler, 1987, "Further Evidence of Investor Overreaction and Stock Market Seasonality," *Journal of Finance*, Vol.42, p.557-581.

Dickinson A., and D.R. Peterson, 1989, "Seasonality in the Option Market," *The Financial Review*, Vol.24, p.529-540.

Dunsmuir, W., 1981, "Estimation for Stationary Time Series when Data are Irregularly Spaced or Missing," in *Applied Time Series Analysis II*, ed. D.F. Findley, New York; John Wiley, p.609-650.

Fama, E.F., and J.D. MacBeth, 1973. "Risk Return and Equilibrium: Some Empirical Tests," *Journal of Political Economy*, Vol.71, p.607-36.

Faseruk, A., and K. Johnson, 1989, "The TSE 35 Index Option Contract: Pricing, Boundary Conditions and Put Call Parity," in *Proceedings of the 1989 Asac Conference: Finance Division*. Administrative Sciences Association of Canada.

Gammill, J.F., and T.A. Marsh, 1988, "Trading Activity and Price Behavior in the Stock and Stock Index Futures Markets in October 1987," *Journal of Economic Perspectives*, Vol.2 p.25-44.

Geske, R., and H.E. Johnson, 1984, "The American Put Valued Analytically," *Journal of Finance*, Vol.39, p.1511-24.

Harvey, C.R., and R.E. Whaley, 1991, "S&P 100 Index Options Volatility," *Journal of Finance*, Vol.46, p.1551-1561.

Haugen, R.A., and J. Lakonishok, 1988, *The Incredible January Effect*, Dow Jones-Irwin, Homewood, Ill.

Hillmer, S.C., 1982, "Forecasting Time Series with Trading Day Variation," *Journal of Forecasting* Vol.1, p.385-95.

Hillmer, S.C., W.R. Bell, and G.C. Tiao, 1981, "Modeling Considerations in the Seasonal Adjustment of economic Time Series," *Journal of the American Statistical Association*, Vol.76, p.22-33.

Hull J., and A. White, 1987, "The Pricing of Options on Assets with Stochastic Volatilities," *The Journal of Finance*, Vol.42, p.281-300.

Jog, V., 1988, "Stock Pricing Anomalies: Canadian Experience," *Canadian Investment Review*, Vol.1, p.55-62.

Kato, K., and J.S. Schallheim, 1985, "Seasonal and Size Anomalies in the Japanese Stock Market," *Journal of Financial and Quantitative Analysis*, Vol.20, p.243-260.

Keim, D.B., 1983, "Size Related Anomalies and Stock Return Seasonality," *Journal of Financial Economics*, Vol.12, p.13-22.

Keim, D B., and M. Smirlock, 1989, "Pricing Patterns in Stock Index Futures," in *Handbook of Stock Index Futures and Options*, eds. F.J Fabozzi and F.M. Kipnis, Dow Jones-Irwin.

Lakonishok, J., and S. Smidt, 1988, "Are Seasonal Anomalies Real? A Ninety-Year Perspective," *The Review of Financial Studies*, Vol.1, p.403-425.

Larcker, D.F., G.A. Gordon and G.E. Pinches, 1980, "Testing for Market Efficiency: A Comparison of the Cumulative Average Residual Methodology and Intervention Analysis," *Journal of Financial and Quantitative Analysis*, Vol.15, p.267-87.

Latane, H.A., and R.J. Rendleman Jr., 1976, "Standard Deviations of Stock Price Ratios Implied in Option Prices," *Journal of Finance*, Vol.31, p.369-382.

Liu, L., 1986, "Identification of Time Series in the Presence of Calendar Variation," *International Journal of Forecasting*, Vol.2, p.357-372.

Liu, L., 1987, "Sales Forecasting Using Multi-Equation Transfer Function Models," *Journal of Forecasting*, Vol.6, p.223-228.

Lui, L., and D.M. Hanssens, 1982, "Identification of Multiple-input Transfer Function Models," *Communications in Statistics*, A-11, p.297-314.

Maberly, E.D., and B.A. Maris, 1991, "The January Effect, Arbitrage Opportunities, and Derivative Securities: Has Anything Changed?" *The Journal of Futures Markets*, Vol.11, p.253-57.

MacMillan, L.W., 1986, "Analytical Approximation for the American Put Option," *Advances in Futures and Options Research*, Vol.1, p.119-39.

Maloney, K.J., and R.J. Rogalski, 1989, "Call-Option Pricing and the Turn of the Year," *Journal of Business*, Vol.62, p.539-52.

Martin, D. W., and D.W. French, 1987, "The Characteristics of Interest Rates and Stock Variances Implied in Option Prices," *Journal of Economics and Business*, Vol.15, p.279-288.

McCleary, R.. and R. A. Hay, Jr., 1980. *Applied Time Series Analysis for the Social Sciences*, Beverly Hills CA, SAGE publications.

Merton, R., 1973, "The Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science*, Vol.4, p.141-83.

Miller, R.B., 1980, "Comments on Robust Estimation on Autoregressive Models by Martin," in *Directions in Time Series*, eds. D.R. Brillinger and G.C. Tiao, Hayward, CA: Institute of Mathematical Statistics p.255-262.

Morse, J.N., 1991, "An Intrweek Seasonality in the Implied Volatilities of Individual and Index Options," *The Financial Review*, Vol.26, p.319-341.

Patell, J.M., and M.A. Wolfson, 1981, "The Ex Ante and Ex Post Price Effects of Quarterly Earnings Announcements Reflected in Option and Stock Prices," *Journal of Accounting Research*, Vol.19, p.434-457.

Penman, S., 1987, "The Distribution of Earnings News Over Time and Seasonalities in Aggregate Stock Returns," *Journal of Financial Economics*, Vol.18, p.199-228.

Peterson, D.R., 1990, "Stock Return Seasonalities and Earnings Information," *Journal of Financial and Quantitative Analysis*, Vol.25, p.187-202.

Ramaswamy. K., and S.M. Sundaresan. 1985, "The Valuation of Options on Futures Contracts," *Journal of Finance*, Vol.40, p.1319-40.

Ritter, J.R. 1988, "The Buying and Selling Behavior of Individual Investors at the Turn of the Year," *Journal of Finance*, Vol.43 p.701-717.

Ritter, J.R., and N. Chopra, 1988, "Risk, Return, and January," Unpublished University of Michigan working paper.

Rogalski, R.J., and S.M. Tinic, 1986, "The January Size Effect: Anomaly or Risk Mismeasurement?," *Financial Analysts Journal*, Vol.43 p.63-70.

Roll, R. 1983, "Vas Ist Das? The Turn of the Year Effect and the Return Premia of Small Firms," *Journal of Portfolio Management*, Vol.9, p.18-28.

Rozeff, M., and W. Kinney, 1976, "Capital Market Seasonality: The Case of Stock Returns," *Journal of Financial Economics*, Vol.3, p.379-402.

Rubinstein, M., 1985, "Nonparametric Tests of Alternative Option Pricing Models Using All Reported Trades and Quotes on the 30 most Active CBOE Option Classes From August 23, 1976 Through August 31, 1978," *Journal of Finance*, Vol.40, p.455-480.

Schwert, G., 1990, "Stock Volatility and the Crash of '87," *The Review of Financial Studies*, Vol.3, p.77-102.

Sheikh, A.M., 1989, "Stock Splits, Volatility Increases, and Implied Volatilities," *The Journal of Finance*, Vol.24, p.1361-1372.

Smidt, S., and Stewart, S.D., 1984, "Risk Premium Seasonality," School of Management Working Paper, Cornell University.

Stein, J., 1989, "Overreactions in the Options Market," *Journal of Finance*, Vol.44, p.1011-24.

Stoll, H.R., and R.E. Whaley, 1986, "Expiration Day Effects of Index Options and Futures," *Monograph Series in Finance and Economics*, New York University, Monograph 1986-3.

Stoll, H.R., and R.E. Whaley, 1987, "Program Trading and the Expiration Day Effects of Index Options and Futures," *Financial Analysts Journal*, Vol.43, p.16-28.

Thaler, R., 1987, "Anomalies: The January Effect," *Journal of Economic Perspectives*, Summer 1987, p.197-201.

Tinic, S., G. Barone-Adesi and R. West, 1986, "Seasonality in Canadian Stock Prices: A Test of the Tax-Loss Selling Hypothesis," *Journal of Financial and Quantitative Analysis*, Vol.22. p.51-64.

Tinic, S., and G. Barone-Adesi, 1987, "Stock Return Seasonality and the Tests of Asset Pricing Models: Canadian Evidence" in *Stock Market Regularities*, ed. E. Dimson, Cambridge University Press.

Tinic, S., and R.R. West, 1984, "Risk and Return: January vs. the Rest of the Year," *Journal of Financial Economics* Vol.13, p.561-74.

Tiao, G.C., 1985," Autoregressive Moving Average Models, Intervention Problems and Outlier Detection in Time Series," in *Handbook of Statistics*,Vol.5, edited by E.J. Hannan, P.R. Krishnaiah, and M.M. Rao, Elsevier Science Publishers.

Tsay, R.S., and G.C. Tiao, 1984, "Consistent Estimates of Autoregressive Parameters and Extended Sample Autocorrelation Functions for Stationary and Non-Stationary ARMA models," *Journal of the American Statistical Association*, Vol.79, p.84-96.

Whaley, R.E., 1984, "On Valuing American Futures Options," Working Paper number 4, Institute for Financial Research, University of Alberta.

Whaley, R.E., 1986, "Valuation of American Futures Options, Theory and Empirical Tests," *Journal of Finance*, Vol.41, p.127-150.

Wiggins, J., 1987, "Option Values Under Stochastic Volatility," *Journal of Financial Economics*, Vol.19, p.351-372.

Wilson, W.W., and H. Fung, 1991, "Put-Call Parity and Arbitrage Bounds for Options on Grain Futures," *Journal of Agricultural Economics*, Vol.42, p.127-50.

CHAPTER IV. EFFECT OF EXPIRING INDEX OPTIONS AND FUTURES ON THE TORONTO STOCK EXCHANGE 35 INDEX

I. INTRODUCTION

Recently, several authors have examined the effects of expiring options and futures on the price of underlying assets. With few exceptions these studies have concentrated on American markets with an emphasis on common stocks and stock indices.

In the present paper we examine these effects with respect to the relatively new Toronto Stock Exchange 35 index (TSE 35). Options and futures on the TSE 35 began trading in 1987 and with extensive growth have developed to be the predominant markets for index arbitrage activities in Canada. Consequently, the probability of price effects due to expiring options and futures is high.[1] This is tempered somewhat by the fact that although the TSE 35 based contracts have effectively revolutionized derivative security trading in Canada, there still appears to be some reluctance in their use on the part of institutional investors (Gagnon (1990)). As well, settlement procedures for the TSE 35 differ from those pertaining to American markets studied in earlier papers, and were specifically designed to minimize expiration day effects.

In addition U.S. indices such as the S&P 500 have differing cycles of expiration with regards to futures and options contracts. As a result previous authors studying the expiration effects of these contracts have been able to determine the relative importance of options versus futures in the arbitrage related effects. Options and futures on Canadian indices however, expire on the same monthly calendar cycle and hence it is unclear from existing studies which derivative security predominates in the determination of expiration effects.

Consequently, the goals of this study are two-fold. The first is to determine the extent of price effects in the TSE 35 market in general, particularly in light of settlement procedures designed to mitigate such effects. Second is the identification of determinants which lead to expiration day effects on the Canadian market.

In addition, the possibility that Canadian expiration effects are related to U.S markets is explored, as well as the impact of trading in Toronto 35 Index Participation Units (TIPs). The introduction of these units of a portfolio replicating the TSE 35 is believed to have led to an increase in index arbitrage related trading in Canada (Gagnon (1990)) and hence a potential increase in expiration day effects.

---

[1] The impetus for this study was Sopher (1991), who noted significant expiration day effects in the TSE 35 in an exploratory study of the period of Sept. 1989 to August 1990. The exploratory study was performed under the guidance of this author as requirement for Mr. Sopher's undergraduate honors degree.

Section II will briefly review some of the previous literature concerning expiration day effects in U.S. and Canadian markets. Section III will describe the data and tests carried out to explore for the presence of these effects in the TSE 35, and to identify the relative importance of various determinants of the effects. Section IV concludes and suggests the potential of future research endeavors.

II. PREVIOUS LITERATURE ON THE EFFECTS OF EXPIRING INDEX OPTIONS AND FUTURES.

Although several studies (Klemkowsky (1978), Officer and Trennepohl (1981), and Klein (1986)) have noted the effect of expiring options on underlying common stocks, Stoll and Whaley (1986 and 1987) were the first to document significant price effects in stock indices, on the expiry dates of index options and futures.

Upon expiration, index options and futures specify cash settlement rather than delivery of the underlying portfolio of stocks. If the number of open positions to be unwound in the cash market is large, abnormally high trading volumes in the stocks comprising the index are expected, along with corresponding price effects.

Stoll and Whaley coined the phrase "triple witching hour" in reference to the last hour of trading on quarterly expiration days on which index futures, index options, and individual stock options expire simultaneously, and in which significant volume and price effects appear to exist. In general, they found that index volume is nearly doubled in the last hour of expiration Friday trading versus that of non-expiration day Fridays.

As well, price effects on the expiration Friday and in particular in the last hour of trading were noted. Although the effect of unwinding of arbitrage positions could have a negative or positive effect on stock prices, depending on net positions, the price effects recorded by Stoll and Whaley took the form of an average lower return during these periods with a tendency to reverse in the first half hour of the next day trading. In general, they found that the effects were mainly associated with the expiration of index futures contracts as only minor volume and price effects were noted on days when option contracts expired solely.

Although the relative size of expiration day price effects is small, their presence has attracted much attention from the various securities regulatory bodies. In June of 1987 the Chicago Mercantile Exchange, the New York Stock Exchange, and the New York Futures Exchange changed the settlement of the S&P 500 and NYSE index futures and options contracts from the close of trading on the expiry day to the opening price, with trading ending on the Thursday before the Friday expiry. This change was made in consideration of the documented expiration day effects noted above.

Although several alternative settlement procedures were suggested by the SEC in the debate concerning expiration day effects it was believed that settlement at the open was optimal largely because it would give the specialist time to attempt to match any large imbalances in orders due to the unwinding of arbitrage positions.[2]

Recently Stoll and Whaley (1991) have reexamined the expiration day issue in light of these settlement changes, and have found that

---

[2]See Kling (1987) and Stoll (1988) for interesting discussions of alternative settlement procedures.

expiration day effects, although mitigated somewhat, still exist. They document a decline in price effects at the close of Fridays and an increase in opening price effects after the June 1987 change in settlement procedures. Although the resulting price effects at the open are somewhat smaller than they were when settlement occurred at the close, Stoll and Whaley suggest this is due to expiration day trading being split between open and close, as futures and options on the Major Market Index, the S&P 100 and the Value Line Index continue to settle at the Friday close.

The concern of regulators in Canada has also been noted with the design of options and futures on the TSE 35 also based on a Friday opening settlement. Likewise administrators of the Toronto Stock Exchange have noted the possible presence of excess volatility and price effects due to expiring TSE 35 options and futures. (See Dancer (1991))

A. Canadian Evidence of Expiration Day Effects on Stock Indices

Although expiration day effects in U.S. markets, as noted by Stoll and Whaley, have been known for some time now, there is a sparsity of documentation of the effects in alternative markets although derivative securities markets in other countries have existed for a reasonable length of time. One notable exception is that of Chamberlain, Cheung, and Kwan (1989) who examined expiration day effects on the TSE 300 index.

Options and futures on the TSE 300 began trading in January 1984 and were later discontinued in favor of the contracts on the TSE 35. Volume activity w[illegible] great in the TSE 300 derivative securities markets relative [illegible] of the TSE 35, and it has been suggested that the American natu[illegible] the TSE 300 options contracts and large portfolio of stocks required for arbitrage activities were not popular with institutional investors.

Options and futures on the TSE 300 had a monthly cycle of maturity and like the original index options and futures contracts on the U.S indices the TSE 300 contracts settled at the close of the third Friday of the expiry month. One advantage of the monthly maturity cycle, for studies of expiration day effects, is the availability of a larger number of expiry day observations in a given period, as opposed to the U.S. markets where expirations are largely quarterly in nature.

Chamberlain, Cheung, and Kwan examined time stamped Friday and Monday data for the TSE 300 for the roughly one and one half year period of November 1985 to May 1987.[3] In general, they did not find significant differences in the average level or standard deviation of volume between expiration and non-expiration Fridays. They did however find significantly higher returns and return volatility in the last half hour of trading on expiration Fridays versus that of

---

[3]Unfortunately the time stamped data on the TSE 300, required for their study, was not available until the 1985 period.

non-expirations.

One possible explanation which the authors suggest for the higher expiration day returns is that of a "spill-over" from the expiration day effect observed in the American markets. In a test of this hypothesis they removed the expiration Fridays that correspond to the U.S. quarterly expirations from their data set and found that the remaining expiration days were no longer significantly different from non-expiration Friday returns. Volatility however, remained significantly different.

III. DATA AND EMPIRICAL RESULTS

A. Data

Daily open, high, low, and close prices as well as trading volume for the TSE 35 for the period of May 1987, when calculation of the index commenced, until June of 1991, was provided for this study by the Toronto Stock Exchange.

As noted previously, although options and futures on the TSE 35 only commenced trading in May of 1987 their calendar cycle of maturity is a monthly one as opposed to the quarterly maturities of some of the American index derivative securities. As a result, the TSE 35 data supplies three times the number of expiration day observations as does the American data for a given period. The original Stoll and Whaley (1986) study involved only 7 expiration days in which both futures and options expired and an additional 17 days of option-only expirations. Although Chamberlain, Cheung, and Kwan also dealt with monthly expiring contracts the time period of their study provided only 31 options and futures expiration dates. Alternatively this study involves a total of 50 expiration day observations.

B. Empirical Tests

Tests of expiration day effects have for the most part concentrated on several issues: 1) Whether there is a difference in volume of trading due to the expiry of options and futures. 2) The presence of price effects in the sense of significant differences in average return and volatility for expiration versus non-expiration periods. 3) The extent of reversal of any price effects, and finally 4) the predictability and determinants of the effects. In this section we will examine these issue in turn for the TSE 35.

Throughout the analysis we will also concentrate on the stationarity of the effects through time by examining the data in one year periods to determine whether expiration effects are concentrated in the early or later portion of the period of study. Due to the lack of volume in the fledgling Canadian derivative securities markets it is conceivable the the effects may not be present early in the period of study. Arguments against finding the effect later in the period are based largely on the assumption of efficient markets. Indeed informal conversations with Toronto Stock Exchange officials indicate that they believe a number of traders attempt to profit from the effects in the Canadian market.

i. Expiration-Day Volume Effects

Arbitrage positions involving TSE 35 derivative securities would typically be unwound at or near the close of trading on expiration Thursdays. Hence, the most appropriate data of study would be volume during the last half hour of trading on the expiration Thursdays, as well as the trades at the opening price on the expiration Friday mornings. Unfortunately this data was not available. As an alternative, and with some loss of power, we will examine daily volume statistics for Thursdays and Fridays.

Table IV-1 lists the average and standard deviation of the daily volume figures for expiry Thursdays and Fridays versus non-expiry Thursdays and Fridays. Results are reported for separate years and for the period in total. As well, the t-statistic for the hypothesis of equality of means and the F-statistic for equality of variances between expiration and non-expiration days are reported.

Although average volumes appear to be higher on expiration Thursdays, the increase is not significant at reasonable levels either for the period as a whole, or for individual years. The hypothesis of equal variance of volume cannot however be accepted, for the overall period, with a significant increase in volume volatility on expiration days. This result is largely consistent with the data on a yearly basis although years later in the period show no significant difference between expiration and non-expiration days.

There does however appear to be an overall significant increase in daily volume for expiration Fridays versus non-expiration days. These results vary however when volume is examined across one year periods with the majority of the years, excluding 1990, indicating no significant difference in volume average. These results are in general contrary to the findings of Chamberlain, Cheung, and Kwan (1989) of no significant volume effects due to expiration in the case of the TSE 300.

Results in terms of the variance of volume are mixed however with the first three years (1987-89) indicating significant differences in variances although no significant difference for the overall period.

ii.Expiration-Day Price Effects

The examination of expiration day price effects in previous studies, where settlement of options and futures was based upon the closing level of the index on expiration Fridays, centered on the Friday returns with particular focus on the returns corresponding to the last half hour of expiration day trading. In the case of options and futures on the TSE 35, where trading stops on the close of Thursday and settlement is based on the Friday open, it is appropriate to examine the returns for both expiration Thursdays and Fridays as some unwinding of positions may be carried out late in the day on the expiration Thursdays. Consequently, Table IV-2 lists the average and standard deviation of continuous returns for expiration and non-expiration Thursdays, Fridays, and the returns over the period of Thursday close to Friday open.

As indicated in the table, although it would appear that lower daily returns are associated with expiration Thursdays the effect is not significant either for the overall period or for individual years. As well, although the volatility of returns appears to be, on average, higher on expiration Thursdays the effect is again not significant except in the 1991 period. Further study may be indicated however as the F-statistic for equality of variances is reasonably high in some cases.

The results for daily returns corresponding to Fridays would appear to indicate higher returns overall on expiration days although none of the results are significant except for the 1988 period. Indeed, in some years the average daily Friday return appears to be lower on expiration days. Oddly enough, the volatility of daily returns appears to be significantly lower on expiration Fridays than those of non-expiration days, with the effect concentrated mainly in the 1988 and 1989 period. Again this may indicate the need for additional study into changes in volatility due to expiration of options and futures.

It is reasonable to assume that given the settlement procedures for the TSE 35 options and futures, any expiration day price effects would tend to be concentrated at the time of the close on the Thursday, to the open on the Friday and any subsequent reversals to take place throughout the day on Friday. Consequently, Table IV-2 also provides the results for returns extending from the close on Thursday to the Friday open. It would appear that when returns are measured from the Thursday close to Friday open, that indeed significant price effects are indicated with higher returns associated with expiration for the whole period of study as well as for the individual years of 1988 and 1990. As well, a significant increase in volatility of the close-to-open returns is indicated for the overall period, concentrated in the years of 1988, 1989, and 1991.

iii. Estimates of Intraday Variance

As noted above, the results for daily return volatility for expiration Thursdays and Fridays would appear to indicate the need for further study. Although no significant difference in return volatility for expiration Thursdays was indicated, the sparsity of data suggests alternative measures.

Parkinson (1980) and Garman and Klass (1980), have noted that the classical approach to the estimation of volatility, based on close-to-close prices does not consider other readily available information that may improve the estimator efficiency, such as the opening value and daily maximum and minimum prices. To take advantage of this readily available daily price data, several estimators have been developed by these authors based upon daily open, high and low prices, which improve on the efficiency of the standard close-to-close price based estimators. Intuitively these estimators are superior due to the incorporation of the range of prices observed over the entire day. We will examine two of these estimators for significant differences in expiration day price variances. Both estimators assume that prices follow Brownian motion with zero drift over both trading and non-trading periods.

The first such estimator examined is that of Parkinson (1980), modified by Garman and Klass (1980) to take into account the availability of open and close prices, and is given by:

$$VAR1_t = \frac{0.17\,(\ln O_t - \ln C_{t-1})^2}{f} + \frac{(1-0.17)(\ln H_t - \ln L_t)^2}{(1-f)\,4\,\ln 2}$$

where:

O_t = opening price for day t

C_{t-1} = closing price for day t-1

H_t = daily high for day t

L_t = daily low for day t

f = fraction ot the day during which trading does not take place.

Garman and Klass show that if trading is continuous, this estimator has an estimation variance 6.2 times lower than that of the usual estimator based on closing prices. As a result, for a given level of accuracy in the standard return variance estimator, about 84% less data is required for this extreme value measure.

The second estimator examined is that of Garman and Klass (1980) which also incorporates opening prices as well as the minimum and maximum values during the day. This estimator has an estimation variance 8.4 times lower than the classical price variance estimator, and is given by:

$$VAR2_t = \frac{0.12\,(\ln O_t - \ln C_{t-1})^2}{f} + (1-0.12)\left[\frac{\hat{\sigma}^2}{(1-f)}\right]$$

$$\hat{\sigma}^2 = \{0.511(u - d)^2 - 0.019[c(u+d)-2ud] - 0.383c^2\}$$

where $u = \ln H_t - \ln O_t$

$d = \ln L_t - \ln O_t$

$c = \ln C_t - \ln O_t$

Investigations into the actual performance of these extreme value estimators has been relatively sparse. Marsh and Rosenfeld (1986) in a simulation study compare the above estimators to the standard close-to-close estimator under the assumption of discrete trading and find that discrete trading can induce a downward bias and reduce their efficiency. Garman and Klass (1980) recognized this potential for downward bias when transactions are infrequent (less than 500 transactions on a daily basis). In general however, this bias is not of concern in the present application given the volume of trading in the stocks of the TSE 35.

Beckers (1983), in a test of the ability of the extreme-value estimators to predict future volatilities finds that despite the theoretical biases and efficiency losses associated with discrete trading, these estimators outperformed the close-to-close measure.

Recently however, Wiggins (1991) in a thorough test of the empirical performance of these estimators, finds that recording errors in high or low prices can result in poor performance on the part of the extreme-value estimators. When outliers are controlled for in the data, however the extreme value measures clearly outperform the close-to-close estimator. Fortunately the potential for outliers due to erroneous recordings of high or low prices in individual stocks is substantially reduced in the case of the indices as the individual values make up only a portion of the index level.

Table IV-3 and Table IV-4 provide mean values and standard deviations of these daily estimators for expiration and non-expiration Thursdays and Fridays. As well, results of the nonparametric Mann-Whitney-Wilcoxon test for equality of distribution between expiration and non-expiration days are given. Although the sign of the z-score associated with the Mann-Whitney-Wilcoxon test of equality of distribution may at times appear to be at odds with the average values reported in the table, it must be noted that the Mann-Whitney-Wilcoxon test is a rank order test, and does not involve the standard average value calculation.

From these tables the higher efficiency of the Garman and Klass estimator over the Parkinson estimator is apparent with a higher number of significant differences in daily price variance indicated by VAR2 than with VAR1.

Although differences in terms of the year by year data are indicated, the basic results of the Garman and Klass variance analysis are consistent with the findings of the standard variance estimator in the case of Thursdays. Thursday expiration days do not appear to have significantly different daily return variances from non-expiration Thursdays except for the years 1988 and 1989 which give conflicting results.

Contrary to the results of the standard variance estimator however, the Garman and Klass estimator indicates that expiration Fridays have significantly higher daily price variances than do non-expiration Fridays. Although this result may be largely due to the inclusion of Friday opening prices in the Garman and Klass estimator, the standard Parkinson estimator (not shown here) which does not incorporate opening prices also indicated a higher rank average in the case of expiration days. The z score is however not significant at reasonable levels.

iv. Spill Over Effect of U.S. Markets

In their study of expiration day effects on the TSE 300 index, Chamberlain, Cheung, and Kwan (1989) explore the hypothesis that the expiration day price effects noted are due to a "spill over" from the U.S markets where expiration dates for the U.S. index futures, options, and futures options occur on a quarterly basis. When they removed the observations from their sample which corresponded to the quarterly U.S expirations they found that the remaining Friday expiration returns for the TSE 300 were no longer significantly higher than non-expiration Fridays.

In a similar process the expiration Friday observations which corresponded with expirations in the months of March, June, September, and December, were separated from the non-quarterly expiration observations, for the TSE 35. Table IV-5 reports the mean and standard deviations for non-expiration Fridays, non-quarterly expiration Fridays, and quarterly expiration Fridays close-to-close and close-to-open returns and daily volume. As well t-statistics for equality of means and F-statistics for equality of variances are reported.

Table IV-5 indicates that although volumes on quarterly expiration Fridays appear to be on average higher than non-quarterly expiration days, the difference is not significant. While a significant difference in the close-to-close returns between quarterly and non-quarterly expiration days is not indicated it is interesting to note that the average close-to-open return is significantly lower on quarterly expirations than on non-quarterly expirations. A test of equality of means and variances indicates that quarterly expiration close-to-open returns are not significantly different from non-expiration days while those of non-quarterly expirations are significantly higher.

These results indicate that contrary to the findings of Chamberlain, Cheung, and Kwan (1989) the expiration day effects noted in Canadian markets are not simply a spill over effect due to the expiration of U.S derivative index securities. In fact, expiration day effects appear to be reduced on the quarterly expirations when Canadian and U.S contracts expire simultaneously. A more detailed analysis of the impact of U.S markets would have to involve an examination of whether quarterly expirations are on average associated with net long arbitrage positions outstanding at the time of expiration.

v. Price Reversals

To a large extent the literature on expiration day price effects has concentrated on the extent of price "reversal" that takes place after settlement of the underlying futures and options contracts. The basic hypothesis is that if unwinding of arbitrage positions at expiration drives stock prices temporarily out of equilibrium, then prices should rebound in the opposite direction after expiration.

Traditionally the study of these price reversals has concentrated on the returns based upon the half hour after expiration. There is however no evidence to support the proposition that specialists will necessarily unwind positions within 30 minutes of the open of the market. This point has been made by Stoll and Whaley (1991) and indeed they find no qualitative difference in the results of their study where reversals were measured based upon the return corresponding to the Friday open to close. As a result we will measure price reversal as:

$$REV1_t = \begin{cases} R_t \text{ if } R_{t-1} < 0 \\ -R_t \text{ if } R_{t-1} \geq 0 \end{cases}$$

where $REV1_t$ = reversal for expiration day t and

$$R_t = \ln(C_t/O_t)$$

$$R_{t-1} = \ln(O_t/C_{t-1})$$

With the above formulation the index reversal measure is negative in value when stock price movement after expiration continues in the same direction as before, and positive if the index reverses in direction between Friday open and close.

Table IV-6 reports the average and standard deviation as well as the relevant t and F statistics for quarterly expiration, non-quarterly expiration and non-expiration Friday price reversals. As indicated, returns tend to reverse on expiration days, with on average a positive value for quarterly and non-quarterly price reversals whereas prices tend not to reverse on non-expiration days.

Tests of equality of means indicate that average price reversals on non-quarterly expirations are significantly different from non-expiration Fridays whereas quarterly expiration price reversals are not. No significant differences in the volatility of price reversals are indicated.

In order to control for the possibility that the measure of reversal above may be biased in terms of the size of return calculated from the open to the close of the expiration day, table IV-6 also presents the results for the measure of reversal as:

$$REV2_t = \begin{cases} R_{t-1} \text{ if } R_t < 0 \\ -R_{t-1} \text{ if } R_t \geq 0 \end{cases}$$

This measure is alternatively based upon the Thursday close to Friday open return and assumes that the price movements are exactly reversed in the cases of reversal. No qualitative difference in results is indicated for reversals measured as REV2.

vi. Effects of the Introduction of TIPS trading.

A recent innovation of the Toronto Stock Exchange has been the introduction of Toronto Index Participation Units or TIPs. A TIP is essentially a unit of a trust operated by the Toronto Stock Exchange which holds a basket of the stocks of the TSE 35. These units are redeemable and pay dividends on a quarterly basis, equal to the aggregate dividends paid by the composite stocks during the quarter. TIPS first became listed on the exchange in March 1990.

The advantage of the units is to enable investors to participate easily in the 35 basket of stocks and facilitates hedging and arbitrage activities involving the TSE 35 options and futures contracts. TIPS prices are quoted on the Toronto Stock Exchange and literature from the Exchange concerning TIPS indicates that the unit price may, at times, differ from the underlying TSE 35 for several reasons, including simple supply and demand.

An interesting question arises as to whether or not the introduction of TIPS has altered expiration day price effects in any way. Gagnon (1990) suggests that the introduction of TIPS would lead to an increase in stock index arbitrage activities particularly in trading strategies designed to arbitrage between the index futures and spot. Since previous literature attributes much of the expiration day effects to the unwinding of futures arbitrage positions the introduction of TIPS may be associated with an increase in such activities and subsequent expiration effects.

Alternatively arbitrage trading involving TIPS may result in the migration of expiration day effects to the price of the units themselves, with a resultant "wedge" driven between the unit price and that of the index, at expiration.

In order to test for the possibility that TIPS trading has altered expiration day effects, the expiration day Friday close-to-open returns were subdivided into the period before and after the introduction of TIPS. Table IV-7 provides the mean and standard deviation of the expiration day Friday volumes, close-to-open returns, and reversals for the periods previous and subsequent to TIPS trading. As well, the average and standard deviation of daily volumes, close-to-open returns and reversals in proportion to the corresponding values for the second Friday of the month are reported, to control somewhat for the growth in volume, or differences in returns over time.

The results of t and F tests are also provided in Table IV-7 and in general indicate no significant difference in mean levels between the two periods in volume, close-to-open return, or reversal measured in absolute terms or as a relative measure. Consequently it would appear that the introduction of TIPS has not led to a significant increase in arbitrage activities or alternatively any additional expiration day effects due to increased volumes. It is possible however that expiration day effects are occurring in the value of the TIPs units themselves. A detailed exploration of the latter hypothesis involves examining the deviations of the price of TIPS units from that of the TSE 35, and is the subject of future study.

vii. Determinants of Expiration Day Effects

In general, the accepted explanation of expiration day effects is that of arbitrage program trading. Program traders essentially take opposite positions in the cash stock, and the stock index futures and options markets to earn profits on perceived mispricings. These positions are then closed out, or unwound at the time of futures or options expiration in order to earn arbitrage profits. This is accomplished with market orders placed at the exact moment that the derivative security expires. It is argued that this congestion or build up of market orders gives rise to the potentially anomalous price effects in the underlying securities.

An interesting question in the case of expiration day effects on the TSE 35, is the extent to which these effects can be attributed to the unwinding of trading strategies in index futures or in index options. Previous literature documents that a large part of expiration price effects are due to the unwinding of futures positions and not that of options. Stoll and Whaley (1986) in their seminal study of expiration day effects on the U.S indices find price effects associated with futures contracts expiration but little if any with the expiration of options contracts. In related studies, Stoll and Whaley (1987, and 1990) find that the individual stocks comprising the S&P 500 do not exhibit price effects upon options expiration.

Similar studies of the expiration effects of options on individual stocks include Officer and Trennepohl (1981), Ferris, Chance, and Wolf (1989) and Pettengill (1989) as cited in Hancock (1991). Their results suggest that price effects due to options expiration tend to be at best minor in magnitude. Studies involving other underlying securities include Bhattacharya (1987) who finds no evidence of option expiration price effects on treasury-bond futures and Hancock (1991) who finds no significant price effects in the case of S&P 500 futures due to the expiration of the corresponding index futures options.

To a large degree these findings simply document the fact that arbitrage trading may less frequently involve options than futures contracts. Indeed Stoll and Whaley (1986) document that conversations with market participants suggest that index options arbitrage activities are less frequent and of lesser magnitude than arbitrage in index futures. This may be due in part to the early exercise feature of the U.S. index options, which are American in nature. This early exercise feature creates uncertainty for arbitrageurs in terms of the life expectancy of an option contract.

Alternatively the options on the TSE 35 are European in nature and hence we would expect that index options arbitrage may play a relatively higher role in the Canadian market than the U.S. Unfortunately as the futures and options on the TSE 35 expire simultaneously on the third Friday of every month, it is not possible to examine expiration days where only futures or options expire.

Previous studies which have concentrated mainly on the predictability of expiration day price effects of course make assumptions concerning the underlying determinants. Understandably, given the above results the majority of the focus has been on arbitrage trading in the futures market, and resulting measures to predict expiration effects. The general approach has been to identify some measure of the net positions of arbitrageurs prior to contract expiration days. The most common measure examined has been the number of days for which futures contracts are mispriced relative to the theoretical net cost of carry model.[4]

In general, the net overpricing measure is based on the assumption that a contract which has been overpriced more days than it has been underpriced will be associated with an accumulation of net long cash positions by arbitrageurs. These long cash position accumulations must be sold at the expiration day unwinding and thus net overpricing will predict a stock price drop at expiration. Conversely net daily underpricing of the futures would predict program traders to have accumulated a net short position in stocks which must be covered by stock purchases at expiration. Thus net underpricing would predict a stock price rise at expiration.

In general, it is recognized that this measure of potential congestion is an approximation in that it assumes known future dividends, no pricing of the marked-to-market risk inherent in futures prices, and no transaction costs. (See Merrick (1989) for detail concerning these costs.)

Merrick (1989) also argues that a measure based simply on "net days over or under priced" does not recognize the possibility of early and delayed (contract rollovers) unwindings. Rollovers involve replacing the initial futures position with a similar position in the subsequent expiring futures contract while keeping the cash side of the initial arbitrage position intact. This may occur if at expiration day the next maturing contract is mispriced as well.

Early unwindings may occur if prior to expiration a mispricing of the futures contract opposite to that leading to the initial position, occurs. In such circumstances it may be profitable to unwind and earn a larger return than that of holding until maturity.

Merrick's work raises the question as to whether arbitrage activity is at the crux of the expiration day effects. In a study of 16 expiration days, Merrick determines a net arbitrageur position, prior to expiration day, based upon some simple unwinding and rollover rules incorporating transaction costs, through out the trading life of the expiring contract. Although this measure of net arbitrageur position appears to perform significantly better in predicting the sign of expiration day effects for the last half of trading, than the net days overpriced measure, only 7 of the 16 contracts involved a prediction of expiration day unwindings.

By using daily data his test is subject to the criticism that it ignores intraday arbitrage activities (as is the net days overprice measure), however his study does cast doubt on the role of arbitrage activities in expiration day effects, and the efficiency of the market.

In the present study we explore the potential of alternative measures of potential congestion associated with the unwinding of

---

[4]See for example Stoll and Whaley (1986) and Chamberlain, Cheung, and Kwan (1989).

arbitrage trading in the futures and in options contracts, with the main purpose of determining the relative importance of the two with respect to expiration day effects in the TSE 35. The variables we will utilize will be based upon open interest levels in the futures and options contracts at the time of expiration.

If open interest is not large on expiration days, no potential for large stock market volumes arising from the unwindings of futures and options positions would exist. As Stoll and Whaley note however the implication of open interest for stock market trading is relatively unclear.

In terms of the futures market, it must first be noted that not all the open interest in existence at the time of expiration will result in stock market trading. A portion of the open interest may be rolled over into the next maturing contract as noted by the results of Merrick (1989) above. Secondly, it is possible that if both long and short arbitrage opportunities arose during the life of the contract and were held to expiration, then the net unwinding of these positions may not imply an imbalance in trading in the stock market. Nevertheless, a large open interest on expiration day could portend a large unwinding of arbitrage positions in the stock market.

The ambiguity in terms of the relationship of options open interest at expiration and the subsequent effects of arbitrage position unwinding is greater still. As in the case of futures, options positions may be rolled over into more distant maturities, and arbitrage positions taken over the time to maturity may net out. In addition Stoll and Whaley state that some holders of options may have long term positions in the underlying stock which they have no intention of changing. Institutional option writers may for example sell stock options to earn additional income but may have no intention of selling the underlying shares.

The measures we will explore in identifying the determinants of expiration day effects on the TSE 35 make the following simplified assumptions. First we will assume that a majority or substantial amount of trading in the TSE 35 options and futures markets is done by institutional investors involved in hedging and arbitrage trading strategies. Previous studies suggest that this is not an unreasonable assumption. Stoll and Whaley (1986) quote the Commodity Futures Trading Commission report of "Commitments of Traders" which indicates that 50% to 60% of the open interest in S&P 500 index futures is held by institutional investor or broker-dealer firms.

Somewhat related observations are given by Mandron (1988b) who finds that the Canadian stock option market is dominated by institutions and market makers. It should be noted however that Mandron (1988a) also finds substantial arbitrage opportunities in the market prices of options which would lead us to believe that little arbitrage trading takes place. Mandron's study though, involves a relatively early period (1977-1980) of Canadian option trading and therefore her observations may not be relevant today.

Secondly, we will make the broad assumption that during the life of options and futures contracts, opportunities arise leading to the formation of arbitrage portfolios which result in a net increase in the level of open interest in futures and options contracts. The overall level of open interest in these contracts, on the Thursday before expirations would hence provide a measure of the extent of positions to be unwound and the subsequent expiration day effects. As well, the change in the level of open interest in the next expiring contract would provide some measure of the number of strategies which are rolled over at expiration and hence would not add to expiration day effects.

The daily level of Thursday and Friday open interest for all contracts expiring in the current month and those expiring in the subsequent month reported in the *Globe and Mail*, was collected, corresponding to the period of study.

In general, we would expect that volumes and price effects on expiration days would be a function of several open interest variables. A positive association would be expected between expiration day effect measures and 1) the level of open interest in the expiring call options on the Thursday before expiration, 2) the level of open interest in the expiring put options on the Thursday before expiration and 3) the level of open interest in the expiring futures contracts on the Thursday before expiration.

As well, due to the possibility of rollovers we would expect to find, given our assumptions above, a negative relationship between the expiration day effect measures and the variables defined as the change in open interest between the expiry Thursday and Fridays for the next month expiring 4) call, 5) put, and 6) futures contracts. The basic premise here is that if a large amount of the open interest in expiring contracts is rolled over, indicated by an increase in open interest in the next expiring contracts, then the expiration day effects will be lessened.

Tables IV-8 and IV-9 provide the results of multiple regressions of various expiration effect measures on the open interest measures denoted above. Table IV-8 presents the results in the case of expiration periods whereas Table IV-9 provides the corresponding regression results for non-expiration periods.

When expiration Friday volumes are regressed upon the open interest variables we find some predicted results and some not foreseen. As predicted, and consistent with the findings of previous studies the level of open interest in the expiring futures contracts on the Thursday before expiry is a significant and positive determinant of the level of volume in the TSE 35. Although the sign of the coefficient for the change in open interest in the next expiring futures contract has the predicted negative value, the t-statistic indicates that the variable is not a significant determinant.

Interestingly the open interest in expiring call options immediately before expiration appears to have a negative association with the level of expiration day cash volume. As well, the coefficient for the change in open interest in subsequent expiring calls, although insignificant, is contrary to that predicted, as it is positive in sign. Oddly enough the similar regression for non-expiration Fridays indicates the same if not stronger, negative relationship between the level of open interest in the current month call options and the subsequent day volume in the TSE 35. As well, the change in open interest for the following month expiring contracts has a significant and positive relationship with the cash volume level. Oddly enough we will see that the previous day level of open interest in the current month expiring call options will have a significant and negative association with the following Friday daily return in the case of non-expiration days.

Although the results are not reported here, an F-test for equality of regressions for expiration and non-expiration periods was performed and indicated that the two regressions were significantly different. As well univariate regressions showed no change in coefficient sign for significant independent variables, in either the expiration or non-expiration case.

Measures of put open interest were not significant in the case of expiration periods but were so in the case of non-expiration. It appears that non-expiration Friday volumes are significantly and positively related to the preceding day level of current month put open interest, and the change in open interest in following month expiring contracts.

We can conclude from these results that in general Friday cash volumes are impacted by the preceding day levels of option open interest. This relationship is obscured however in the case of expirations when the open interest outstanding in expiring futures has a significant impact on the expiration day cash volumes. This indicates that futures arbitrage activities appear to play the dominant role in expiration day volume effects.

In addition, although not shown here, when quarterly expirations were removed from the expiration data set the significance of the previous Thursday levels of call options and futures open interest increased. An F-test for equality of regressions however indicates no significant difference between the regressions for non-quarterly expirations and those of quarterly expirations.

The price effect variables examined in terms of identifying expiration day effect determinants were the daily returns on Fridays, both the returns and absolute value of returns from the Thursday close to the Friday open, and the two measures of reversal previously examined. Again table IV-8 provides the results of multiple regressions of these variables on open interest measures in the case of expiration periods and table IV-9 for non-expirations.

Similar to the results for volume, we see from table IV-9 that the previous day level of open interest in the current month expiring options contracts has some significance in terms of returns. For daily returns the Thursday level of call open interest appears to have a significant positive association with the next day return. As well the change in open interest in the next expiring month call option contracts has a negative impact on the day's return.

Although the results for the regression of daily returns on the open interest variables appear to be different for expiration days as given in table IV-8, an F-test of equality of regressions indicates no significant difference. This is consistent somewhat with our findings that the daily return on expiration Fridays was not significantly higher than non-expiration days. Given no significant difference in regressions, the expiration and non-expiration data were pooled and the multiple regression results (not reported) indicate a significant positive relationship between daily Friday returns and the preceding day level of call open interest as well as the change in open interest for next month expiring futures contracts. The preceding day level of open interest in the expiring futures contracts was significantly and negatively associated with the next day return as was the change in call option open interest for contracts expiring in the following month.

These results are interesting in terms of their implications for market efficiency particularly if the above relationship of daily returns to preceding day open interest holds for other days of the week as well. Preliminary results (Sopher (1991)) for the 89-90 period indicate that this may be the case in terms of the preceding day level of call open interest.[5]

Recently, a number of studies have documented lead-lag relationships between various markets, particularly the options and spot, in terms of volumes and prices. Most of these studies concentrate however on intraday data with lead and lag times measured in minutes.[6]

Anthony (1988), however, documents that volume in call options markets appears to lead trading in the underlying shares with a one day lag. To the extent that open interest proxies for volume, we may be observing, in the Canadian market, evidence of such a lead-lag relationship between the options market and the underlying stocks comprising the index. Similarly Chance (1989) provides some evidence that the ratio of put option volume to call option volume has some predictive power in terms of market direction in the case of the S&P 100 index. Although few studies have involved open interest, Schachter (1988) in a study of 95 firms and their corresponding stock options provides evidence that call option open interest is significantly lower than normal for several days prior to earnings announcement dates. The need for further study into the interrelationship of options and cash markets in Canada is evidenced by these results.

In terms of Thursday close to Friday open returns, an F-test for equality of regressions for expiration and non-expiration periods indicates no significant difference. A resulting pooled multiple regression of the data indicates (although not reported here) a significant negative association of close-to-open returns with the preceding day level of futures open interest, and a positive association with the change in open interest for the following month expiring futures contracts.

---

[5]In the data set examined by Sopher (1991) which included other days of the week a regression of daily returns on previous day level of call open interest showed a positive and significant relationship.

[6]Examples of such studies include Anthony (1988), Stephan and Whaley (1990), and Chan, Chan, and Karolyi (1990).

Although the average close-to-open return is significantly higher on expiration days than that of non-expiration, the level of open interest in futures or options has no fundamental relationship to actual return, in the sense that the preceding day levels do not indicate net potential long or short positions to be unwound in the cash market. Only an expiration day price effect, either positive or negative, is potentially indicated.

Consequently tables IV-8 and IV-9 present multiple regression results for the absolute value of close-to-open returns in the case of expiration and non-expiration periods respectively. The two regressions would appear to give varying results and indeed an F-test of equality of regressions indicates that they are significantly different. While the non-expiration data appears to indicate significant relationships between absolute returns and 1) open interest levels in the current expiring calls, and 2) the changes in open interest levels for following month expiring calls, this is not the case for the expiration period data. None of the open interest variables appear to be significant in the case of expiration close-to-open absolute returns. The implications for expiration day effects are similar to those of Merrick's (1989) study in that it casts doubt on the hypothesis that the effects are due mainly to arbitrage trading strategies.

The most accepted measure of expiration day price effects is that of price reversal. Table IV-8 and IV-9 again present the multiple regression results for expiration and non-expiration data for the two measures of price reversal previously explored.

The results for REV1 which employs the Friday open to close return as the magnitude of reversal indicate that the expiration and non-expiration regressions are significantly different. In both cases the changes in open interest of the next expiring call and put option contracts are significant with the difference being the sign of the coefficient of change in put open interest. In the case of non-expiration periods, reversals are negatively associated with the put open interest change, whereas the association is a positive one in the case of expirations. The association between the change in call open interest and REV1 is negative in both cases. At best these results indicate some role of put option arbitrage on expiration day price effects.

Contrary to the results for REV1, those for REV2 which employs the Thursday close to Friday open return as the magnitude of price reversal, indicate no significant difference in regressions. When pooled, the data implies a significant and negative association between price reversal and the change in the open interest of following month expiring calls, and a positive association with the change in the level of futures open interest.

In general the results are relatively perplexing in the case of expiration day price effects and at most levels of open interest as proxies for arbitrage unwinding congestion indicate that futures and perhaps put option arbitrage are the relatively more important determinants of the effects.

Our overall conclusion however, when combined with the results for expiration day volume, would be that futures arbitrage activities are most likely the relatively more important determinant in expiration day effects on the TSE 35 when compared to options arbitrage. If we accept the measure of open interest as a proxy for potential congestion however, it appears that the explanatory power of arbitrage activities in general is relatively low. The expiration day volume regression had an R^2 of 37.4% whereas for the REV1 regression the R^2 was only 19.25%

IV. CONCLUSIONS

Although the possibility of expiration effects have been noted by Toronto Stock Exchange officials the literature has to date not documented their existence in the Toronto Stock Exchange index of 35 stocks. The main purpose of this paper has been to examine the behavior of the index at the monthly expiration of index options and futures for the presence of these effects, particularly in light of contract settlement procedures specifically designed to mitigate them.

In general we find that although there appears to be no significant increase in stock trading volumes on the TSE 35 on the Thursdays immediately preceding settlement there is some indication of higher variance of volume on these days particularly in the earlier years of the index history. Significantly higher volumes are noted however on expiration Fridays, possibly due to arbitrage portfolio unwindings involving cash market trades occurring at the Friday opening price.

Although there appears to be no significant price effects occurring in average daily returns on expiration Thursdays and Fridays, volatility of returns appears to differ on expiration versus non-expiration Fridays. The classical variance estimator would indicate that expiration Friday returns are significantly less volatile than non-expiration days whereas other more efficient estimators of daily price volatility incorporating opening, high and low prices would appear to indicate a higher volatility of return on expiration Fridays. Further study is indicated by these results, perhaps incorporating volatilities implied from option prices such as in the case of Day and Lewis (1988).

More importantly, results show that returns measured from Thursday close to Friday open are on average higher, and are associated with significantly higher volatility, on expiration days. This would tend to suggest that on average, a net short spot position involved in arbitrage trading of derivative securities, is outstanding at the time of expiration.

The evidence of significant price effects in the TSE 35 is further substantiated by the examination of price reversal measures, indicating significant reversals in price level during the remainder of the day on expiration Fridays, after settlement of options and futures contracts at the opening price.

In general we can conclude there are significant price effects on the open of the TSE 35 stocks which obviously transmit to the index calculation, on the expiry days of index options and futures. These expiration day price effects appear to occur even early in the life of the TSE 35, in contradiction to the relatively low volumes in the index options and futures at that time. No significant changes in the effects appear to be associated with the introduction of TIPS trading, and contrary to previous Canadian market evidence, the effects do not appear to be due to a spill over from the U.S. market. Although not significant there is some indication that quarterly expirations corresponding to those of the U.S. are associated with a mitigation of expiration day effects in the TSE 35.

In addition, we have attempted to identify the most important determinants of expiration day effects, in terms of futures or options arbitrage trading activities for the TSE 35. In order to do this we have incorporated open interest variables at the time of expiration in an attempt to proxy for the potential congestion associated with the arbitrage portfolio unwindings. It appears as if, consistent with previous studies, the main determinant of expiration day effects is that of index futures trading. However, the explanatory power of all measures of potential congestion employed is fairly low, and casts some doubt on the to date, accepted role of arbitrage trading in derivative securities as the main determinant of expiration day effects. In examining the relative importance of these derivative securities we note some evidence that open interest in index call options leads daily returns in the index regardless of expiration date. The need for further research is indicated by this finding.

Further research into expiration effects in the Canadian index market would most likely follow the lines of Stoll and Whaley (1987 and 1990) in the examination of the behavior of the individual stocks comprising the TSE 35 on expiration days. In addition, examination of the pricing of the relatively new TIPS units in relation to the index level itself may prove to be fruitful in identifying whether arbitrage traders are beginning to incorporate this security in their portfolios with a subsequent migration of expiration day effects to the TIPS unit price.

Obviously the adoption of Friday opening settlement for the TSE 35 derivative contracts has not disseminated the congestion in orders due to arbitrage strategy unwindings. Stoll and Whaley (1991) refer to several suggestions made by the Securities Exchange Commission in the U.S, in 1986, when addressing the expiration day effect issue. Perhaps some of these suggestions as to trading procedures should be reexamined by the Toronto Stock Exchange and Trans Canada Options. Among the proposed solutions was the disclosure of market-to-close orders prior to close. This would be similar to the procedure of disclosing non-informational based sunshine trading in order to mitigate sudden price swings. In addition, the suggestion of a trading halt before the close, was made, with the objective of giving the market time to respond to order imbalances. In fact, the NYSE as noted by Stoll and Whaley has adopted special closing procedures for expiration days which involve the requirement that all market-on-close orders must be received one and a half hours before closing. Extreme order imbalances (in excess of 50,000 shares) are then disseminated to off-floor market participants.

Table IV-1. TSE 35 Volume on Expiration and Non-Expiration Thursdays and Fridays[1]

	1987	1988	1989	1990	1991	TOTAL
Thursdays						
EXP: Average	6035631	8493472	7981703	7084801	10044158	7846377
Std. Dev.	1101288	8839888	1607667	2978889	4142592	4849355
Obs.	7	12	12	13	6	50
N-EXP: Average	6312352	6090780	7763012	6711669	7835558	6887063
Std. Dev.	3027309	1727990	3617577	2720748	3413779	2961125
Obs.	25	40	40	39	19	163
t-statistic[2]	-.89	.94	.20	.42	1.32	1.33
F-statistic[3]	7.56*	26.17*	4.94*	1.20	1.47	2.68*
Fridays						
EXP: Average	7429752	6991960	7821788	8250062	8919209	7801821
Std. Dev.	2915707	3493885	1635317	2586540	3775976	2807480
Obs.	7	12	12	12	6	49
N-EXP: Average	5529058	5977821	7456415	5952930	7504634	6448238
Std. Dev.	1687742	2256107	2466613	2244495	2263892	2345066
Obs.	23	38	39	39	18	157
t-statistic[2]	1.64	.94	.59	2.99*	1.12	3.36*
F-statistic[3]	2.98*	2.39*	2.28*	1.32	2.78	1.43

[1]EXP and N-EXP refer to expiration period and non-expiration period Thursdays and Fridays respectively.

[2]Standard t-test for equality of mean values between expiry and non-expiry days. Where the equality of variances between expiry and non-expiry days could not be accepted the t-statistic reported has been adjusted for unequal variances.

[3]Standard F-test for equality of variances between expiry and non-expiry days.

*Indicates significance at the 5% level.

Table IV-2. TSE 35 Thursday and Friday Close to Close Returns and Thursday close to Friday Open Returns for Expiration and Non-Expiration Periods.[1]

	1987	1988	1989	1990	1991	TOTAL
Thursdays:cl-c						
EXP:Average	-0.00414	-0.00165	-0.00133	-0.00165	-0.00119	-0.00158
Std. Dev.	0.01023	0.00763	0.00642	0.00843	0.01004	0.00802
Obs.	7	12	12	13	6	50
N-EXP:Average	0.00005	-0.00119	0.00173	-0.00169	0.00212	-0.00002
Std. Dev.	0.01435	0.00605	0.00533	0.00624	0.00504	0.00778
Obs.	25	40	40	39	19	163
t-statistic[2]	-0.69	-0.21	-1.01	0.02	-0.78	-1.23
F-statistic[3]	1.97	1.59	1.45	1.82	3.97	1.06
Fridays: cl-cl						
EXP:Average	0.00139	0.00528	-0.00015	-0.00092	-0.00252	0.00092
Std. Dev.	0.01237	0.00359	0.00496	0.00638	0.00622	0.00696
Obs.	7	12	12	12	6	49
N-EXP:Average	0.00209	-0.00015	0.00090	-0.00155	0.00049	0.00016
Std. Dev.	0.01242	0.01100	0.00941	0.00673	0.00491	0.00933
Obs.	23	38	39	39	18	157
t-statistic[2]	-0.13	2.63*	-0.50	0.29	-1.22	.61
F-statistic[3]	.99	9.41*	3.60*	1.11	1.60	1.80*
Fridays: cl-op						
EXP:Average	-0.00159	0.00482	0.00201	0.00529	0.00013	0.00276
Std. Dev.	0.00596	0.00733	0.00559	0.00412	0.00751	0.00633
Obs.	7	12	12	12	6	49
N-EXP:Average	0.00098	-0.00095	-0.00004	-0.00041	-0.00060	-0.00027
Std. Dev.	0.00887	0.00309	0.00312	0.00570	0.00196	0.00494
Obs.	23	38	39	39	18	157
t-statistic[2]	-0.71	2.65*	1.21	3.21*	.23	3.06*
F-statistic[3]	2.21	5.63*	3.22*	1.91	14.65*	1.64*

[1]EXP and N-EXP refer to expiration period and non-expiration period Thursdays and Fridays respectively.Cl-cl refers to returns based on the previous day close to the present day close. Cl-op refers to returns based on the Thursday close to Friday open.

[2]Standard t-test for equality of mean values between expiry and non-expiry days. Where the equality of variances between expiry and non-expiry days could not be accepted the t-statistic reported has been adjusted for unequal variances.

[3]Standard F-test for equality of variances between expiry and non-expiry days.

*Indicates significance at the 5% level.

Table IV-3. Results of Parkinson (VAR 1) and Garman-Klass (VAR 2) Daily Return Variance Estimation for Expiration and Non-Expiration Thursdays.[1]

Thursdays	1987	1988	1989	1990	1991	TOTAL
VAR 1						
EXP: Average	0.00021	0.00013	0.00011	0.00012	0.00013	0.00013
Std. Dev.	0.00021	0.00015	0.00007	0.00013	0.00013	0.00014
Obs.						
N-EXP: Average	0.00062	0.00016	0.00008	0.00011	0.00009	0.00019
Std. Dev.	0.00115	0.00018	0.00008	0.00012	0.00008	0.00054
Obs.						
[2]M-W-W Test (z)	.11	-0.50	1.74	.26	-0.13	.79
VAR 2						
EXP: Average	0.00009	0.00001	0.00004	0.00002	0.00001	0.00003
Std. Dev.	0.00017	0.00007	0.00003	0.00002	0.00001	0.00007
Obs.						
N-EXP: Average	0.00023	0.00004	0.00001	0.00001	0.00003	0.00006
Std. Dev.	0.00045	0.00012	0.00003	0.00003	0.00011	0.00022
Obs.						
[2]M-W-W Test (z)	-.52	-2.02*	2.63*	1.74	.45	.71

[1]EXP and N-EXP refer to expiration period and non-expiration period Thursdays and Fridays respectively.

[2]Mann-Whitney-Wilcoxon nonparametric test for equality of distribution.

*Indicates significance at the 5% level.

Table IV-4. Results of Parkinson (VAR 1) and Garman-Klass (VAR 2) Daily Return Variance Estimation for Expiration and Non-Expiration Fridays.[1]

Fridays	1987	1988	1989	1990	1991	TOTAL
VAR 1						
EXP: Average	0.00034	0.00015	0.00011	0.00016	0.00012	0.00017
Std. Dev.	0.00037	0.00021	0.00008	0.00023	0.00005	0.00022
Obs.						
N-EXP: Average	0.00033	0.00022	0.00014	0.00013	0.00009	0.00018
Std. Dev.	0.00064	0.00056	0.00036	0.00011	0.00006	0.00042
Obs.						
[2]M-W-W Test (z)	.42	.48	1.31	-0.13	1.27	1.32
VAR 2						
EXP: Average	0.00005	0.00014	0.00008	0.00019	0.00010	0.00012
Std. Dev.	0.00008	0.00033	0.00009	0.00028	0.00012	0.00022
Obs.						
N-EXP: Average	0.00006	0.00007	0.00002	0.00012	0.00003	0.00006
Std. Dev.	0.00011	0.00029	0.00007	0.00058	0.00004	0.00033
Obs.						
[2]M-W-W Test (z)	-.47	1.04	2.91*	3.46*	1.67	3.94*

[1]EXP and N-EXP refer to expiration period and non-expiration period Thursdays and Fridays respectively.

[2]Mann-Whitney-Wilcoxon nonparametric test for equality of distribution.

*Indicates significance at the 5% level.

Table IV-5. Non-Expiration, Quarterly Expiration, and Non-Quarterly Expiration Close to Close and Close to Open Friday Returns, and Volumes for the 1987-1991 Period.[1]

		Cl-Cl	Cl-Op	Volume
Fridays				
N-EXP:	Average	0.00016	-0.00265	6448240
	Std. Dev	0.00934	0.00494	2345066
	Obs.	157	157	157
Q-EXP:	Average	0.00090	0.00086	8292345
	Std. Dev	0.00716	0.00534	3287634
	Obs.	17	17	17
NQ-EXP:	Average	0.00093	0.00377	7541230
	Std. Dev	0.00697	0.00666	2534492
	Obs.	32	32	32
Q Vs. N				
t-statistic[2]		.39	.88	2.25*
F-statistic[3]		1.70*	1.17	1.97*
NQ Vs. N				
t-statistic[2]		.53	3.25*	2.37*
F-statistic[3]		1.79*	1.81*	1.16
NQ Vs. Q				
t-statistic[2]		-0.01	-1.55	0.88
F-statistic[3]		1.05	1.55	1.68

[1] N-EXP, Q-EXP, and NQ-EXP refer to non-expiration periods, quarterly expiration periods and non-quarterly expiration periods respectively.Cl-cl refers to returns based on the previous day close to the present day close, and Cl-op refers to returns based on the Thursday close to Friday open.

[2] Standard t-test for equality of mean values between expiry and non-expiry days. Where the equality of variances between expiry and non-expiry days could not be accepted the t-statistic reported has been adjusted for unequal variances.

[3] Standard F-test for equality of variances between expiry and non-expiry days.

*Indicates significance at the 5% level.

Table IV-6. Price Reversals for Non-Expiration Fridays, Quarterly Expiration Fridays, and Non-Quarterly Expiration Fridays.[1]

		REV1[2]	REV2[3]
Fridays			
N-EXP:	Average	-0.00012	0.00028
	Std. Dev	0.00819	0.00493
	Obs.	157	157
Q-EXP:	Average	0.00155	0.00142
	Std. Dev	0.00676	0.00521
	Obs.	17	17
NQ-EXP:	Average	0.00389	0.00427
	Std. Dev	0.00760	0.00634
	Obs.	32	32
Q Vs. N			
t-statistic[3]		0.81	0.90
F-statistic[4]		1.46	0.89
NQ Vs. N			
t-statistic[3]		2.55*	3.96*
F-statistic[4]		1.16	0.61
Q Vs. NQ			
t-statistic[3]		-1.06	-1.59
F-statistic[4]		1.26	1.48

[1] N-EXP, Q-EXP, and NQ-EXP refer to non-expiration periods, quarterly expiration periods and non-quarterly expiration periods respectively.Cl-cl refers to returns based on the previous day close to the present day close, and Cl-op refers to returns based on the Thursday close to Friday open.

[2]REV1 represents reversal calculations employing return magnitudes over the period of Friday open to Friday close. REV2 represents reversal calculations employing return magnitudes over the period of Thursday close to Friday open.

[3]Standard t-test for equality of mean values between expiry and non-expiry days. Where the equality of variances between expiry and non-expiry days could not be accepted the t-statistic reported has been adjusted for unequal variances.

[4]Standard F-test for equality of variances between expiry and non-expiry days.

*Indicates significance at the 5% level.

Table IV-7. Results for Period Previous and Subsequent to TIPS Trading.[1]

		Volume	Cl-Op	REV1	V_F/V_{F-1}	C_F/C_{F-1}	R_F/R_{F-1}
Fridays							
NT-EXP.	Average	7482820	0.00250	0.00240	1.13797	-2.61851	-1.46648
	Std. Dev	2644526	0.00652	.00753	0.54869	18.56850	8.16264
	Obs.	33	33	33	33	33	33
T-EXP:	Average	8459761	0.00330	0.00449	1.42697	-4.13617	0.31897
	Std. Dev	3101145	0.00611	0.00693	0.59196	18.89524	26.15162
	Obs.	16	16	16	16	16	16
Q Vs. N							
t-statistic[1]		1.15	.41	0.93	1.69	-0.27	0.27
F-statistic[2]		1.38	0.88	0.85	1.16	1.04	10.26*

[1] NT-EXP and T-EXP refer to expiration periods prior and subsequent to TIPS trading respectively. Cl-op refers to returns based on the Thursday close to Friday open. V_F/V_{F-1}, C_F/C_{F-1}, and R_F/R_{F-1}, refer to Friday expiration volume, close to open return, and reversal as ratios of second Friday of the month values.

[2]REV1 represents reversal calculations employing return magnitudes over the period of Friday open to Friday close.

[3]Standard t-test for equality of mean values between expiry and non-expiry days. Where the equality of variances between expiry and non-expiry days could not be accepted the t-statistic reported has been adjusted for unequal variances.

[4]Standard F-test for equality of variances between expiry and non-expiry days.

*Indicates significance at the 5% level.

Table IV-8. Expiration Fridays Regression Results for Various Expiration Effect Variables Regressed upon Potential Expiration Day Determinants.[1]

Independent Variables[3]	Dependent Variables[2]					
	Volume	Cl-Cl	Cl-Op	|Cl-Op|	REV1	REV2
Th-CCOI	-908.82 (-1.925)*	8.991E-07 (0.725)	-1.015E-06 (-0.856)	-7.246E-07 (-0.814)	-5.587E-07 (-0.399)	-7.732E-07 (-0.698)
Th-CPOI	30.44 (0.123)	-3.446E-07 (-0.531)	-2.071E-09 (-0.003)	-6.660E-07 (-1.432)	-6.347E-07 (-0.867)	-7.930E-07 (-1.370)
Th-CFOI	4333.24 (3.902)*	-6.053E-06 (-2.074)*	-1.810E-06 (-0.649)	2.615E-06 (1.248)	2.032E-06 (0.617)	4.038E-06 (1.550)
Ch-NCOI	907.44 (1.198)	-5.500E-06 (-2.761)*	-1.514E-06 (-0.796)	-1.841E-06 (-1.289)	-5.213E-06 (-2.319)*	-3.053E-06 (-1.717)*
Ch-NPOI	924.37 (1.025)	1.769E-06 (0.746)	8.362E-07 (0.369)	2.304E-06 (1.355)	4.619E-06 (1.727)*	2.883E-06 (1.363)
Ch-NFOI	-645.74 (-0.282)	1.366E-05 (2.272)*	1.605E-05 (2.794)*	6.482E-06 (1.502)	7.783E-06 (1.146)	9.830E-06 (1.831)*
Constant	6740557 (6.757)*	1.976E-03 (0.754)	3.161E-03 (1.262)	5.030E-03 (2.673)*	3.301E-03 (1.115)	2.644E-03 (1.130)

[1]Values in the table represent coefficient estimates for a multiple regression of each dependent variable on the six independent variables identified. Values in parentheses represent t-statistics for the respective coefficient estimates.

[2]Cl-cl refers to returns based on the Thursday day close to the Friday close, and cl-op refers to returns based on the Thursday close to Friday open. |cl-op| represents the absolute value of the Thursday close to Friday open return. REV1 represents reversal calculations employing return magnitudes over the period of Friday open to Friday close. REV2 represents reversal calculations employing return magnitudes over the period of Thursday close to Friday open.

[3]TH-CCOI, TH-CPOI and TH-CFOI represent the level of open interest in the current month expiring call and put options and futures contracts, respectively, at the close of trading on Thursday. CH-NCOI, CH-NPOI, and CH-NFOI represent the change in level of open interest between Thursday to Friday of the call and put options and futures contracts respectively, which expire in the following month.

*Indicates significance at the 5% level.

Table IV-9. Non-Expiration Fridays Regression Results for Various Expiration Effect Variables Regressed upon Potential Expiration Day Determinants.

Independent Variables	Dependent Variables					
	Volume	Cl-Cl Rtn	Cl-Op Rtn	ABS: Cl-Op Rtn	REV1	REV2
Th-CCOI	-394.61 (-2.192)*	1.731E-06 (2.363)*	4.924E-07 (1.242)	6.769E-07 (2.153)*	3.477E-07 (0.543)	-4.003E-07 (-1.020)
Th-CPOI	340.98 (2.072)*	-5.771E-07 (-0.861)	-4.232E-07 (-1.167)	-4.042E-07 (-1.406)	4.057E-07 (0.693)	2.026E-07 (0.564)
Th-CFOI	797.85 (1.456)	-2.547E-06 (-1.142)	-2.114E-06 (-1.752)*	-9.947E-07 (-1.039)	-8.720E-07 (-0.447)	2.720E-07 (0.228)
Ch-NCOI	673.51 (2.453)*	-5.069E-06 (-2.674)*	-1.151E-06 (-1 122)	1.626E-06 (1.999)*	-4.404E-06 (-2.658)*	-2.584E-06 (-2.545)*
Ch-NPOI	1030.60 (2.453)*	-1.408E-06 (-0.823)	-1.728E-07 (-0.187)	-1.409E-06 (-1.919)*	-3.342E-06 (-2.235)*	-4.841E-07 (-0.530)
Ch-NFOI	351.35 (0.115)	7.429E-06 (0.598)	-1.054E-07 (-0.016)	-4.041E-06 (-0.758)	-8.301E-06 (-0.765)	-1.221E-06 (-0.184)
Constant	5775209. (11.250)*	-7.223E-04 (-0.346)	1.076E-03 (0.951)	3.004E-03 (4.037)	-1.122E-03 (-0.614)	8.025E-04 (0.717)

[1]Values in the table represent coefficient estimates for a multiple regression of each dependent variable on the six independent variables identified. Values in parentheses represent t-statistics for the respective coefficient estimates.

[2]Cl-cl refers to returns based on the Thursday day close to the Friday close, and cl-op refers to returns based on the Thursday close to Friday open. |cl-op| represents the absolute value of the Thursday close to Friday open return. REV1 represents reversal calculations employing return magnitudes over the period of Friday open to Friday close. REV2 represents reversal calculations employing return magnitudes over the period of Thursday close to Friday open.

[3]TH-CCOI, TH-CPOI and TH-CFOI represent the level of open interest in the current month expiring call and put options and futures contracts, respectively, at the close of trading on Thursday. CH-NCOI, CH-NPOI, and CH-NFOI represent the change in level of open interest between Thursday to Friday of the call and put options and futures contracts respectively, which expire in the following month.

*Indicates significance at the 5% level.

V. REFERENCES

Anthony, J.H., 1988, "The Interrelation of Stock and Options Market Trading-volume Data," *The Journal of Finance*, Vol.43, p.949-964.

Beckers, S., 1983, "Variances of Security Price Returns Based on High, Low, and Closing Prices," *Journal of Business*, Vol.56, p.97-112.

Bhattacharya, A.K., 1987, "Option Expirations and Treasury Bond Futures Prices," *Journal of Futures Markets*, Vol.7, p.49-64.

Chamberlain, T.W., C.S. Cheung, and C.Y. Kwan, 1989, "Expiration-Day Effects of Index Futures and Options: Some Canadian Evidence," *Financial Analyst Journal*, Vol.45, p.67-71.

Chan, K., K.C. Chan, and G. A. Karolyi, 1990, Intraday Volatility in the Stock Index and Stock Index Futures Markets," *The Review of Financial Studies*, Vol.4, p.657-684.

Chance, D.M., 1989, "Option Volume and Stock Market Performance," *Journal of Portfolio Management*, Vol.16, p.45-51.

Dancer, M., 1991, "Program Trading - Relieving the Misconceptions," *Open Interest*, January/February.

Day, T. and C. Lewis., 1988, "The Behavior of the Volatility Implicit in the Prices of Stock Index Options," *Journal of Financial Economics*, Vol.22, p.103-122.

Ferris, S.P., D.M. Chance, and G.A. Wolfe, 1989, "A Transactional Data Study of Stock Returns and Trading Activity During Options Expiration Periods," Working Paper, Virginia Polytechnic Institute.

Garman, M.B., and M.J. Klass. 1980, "On the Estimation of Security Price Volatilities from Historical Data," *Journal of Business*, Vol.53, p.67-78.

Gagnon, L., 1990, "Exchange-Traded Financial Derivatives In Canada: Finally Off the Launching Pad," *Canadian Investment Review*, Vol.3, p.63-69.

Hancock, G.D., 1991, "Futures Option Expirations and Volatility in the Stock Index Futures Market," *The Journal of Futures Markets*, Vol.11, p.319-330.

Klein, L.S., 1986, "The Effect of Option Expiration on Security Pricing," Working Paper, Florida State University.

Klemkowsky, R.C., 1978, "The Impact of Options Expirations on Stock Prices," *Journal of Financial and Quantitative Analysis*, Vol.13, p.507-517.

Kling, A., 1987, "How the Stock Market Can Learn to Live with Index Futures and Options," *Financial Analysts Journal*, Vol.43, p.33-39.

Mandron, A., 1988a, "Some Empirical Evidence about Canadian Stock Options Part I: Valuation," *Canadian Journal of Administrative Sciences*, Vol.5 p.1-13.

Mandron, A., 1988b, "Some Empirical Evidence about Canadian Stock Options Part II: Market Structure," *Canadian Journal of Administrative Sciences*, Vol.5 p.14-21.

Marsh, T., and E. Rosenfeld, 1986, "Non-trading, Market Making and Estimates of Stock Price Volatility, *Journal of Financial Economics*, Vol.15 p.359-72.

Merrick, J. J., 1989, "Early Unwindings and Rollovers of Stock Index Futures Arbitrage Programs: Analysis and Implications for Predicting Expiration Day Effects." *The Journal of Futures Markets*, Vol.9, p.101-111.

Officer, D.T., and G. Trennepohl, 1981, "Price Behavior of Corporate Equities Near Option Expiration Dates," *Financial Management*, Vol.10, p.75-80.

Parkinson, M., 1980, "The Extreme Value Method for Estimating the Variance of the Rate of Return," *Journal of Business*, Vol.53, p.61-65.

Pettengill, G.N., 1989, "The Effect of Option Expiration on Underlying Securities: A Case of Transitory Inefficiency?" Working Paper, Emporia State University.

Schachter, B., 1988, "Open Interest in Stock Options Around Quarterly Earnings Announcements," *Journal of Accounting Research*, Vol.26, p.353-372.

Sopher, M., 1991, "Expiration Day Effects on the TSE 35 Index," Student Honors Paper, University of Saskatchewan.

Stephan, J.A., and R.E. Whaley, 1990, "Intraday Price Change and Trading Volume Relations in the Stock and Stock Option Markets," *The Journal of Finance*, Vol.45, p.191-220.

Stoll, H.R., 1988, "Index Futures, Program Trading and Stock Market Procedures," *Journal of Futures Markets*, Vol.8, p.391-412.

Stoll, H.R., and R.E. Whaley, 1986, "Expiration Day Effects of Index Options and Futures" *Monograph Series in Finance and Economics*, 1986-3. Salomon Brothers Center for the Study of Financial Institutions.

Stoll, H.R., and R.E. Whaley, 1987, "Program Trading and Expiration Day Effects," *Financial Analysts Journal*, Vol.43, p.16-28.

Stoll, H.R., and R.E. Whaley, 1990, "Program Trading and Individual Stock Returns: Ingredients of the Triple Witching Brew," *Journal of Business*, Vol.63, p.165-192.

Stoll, H.R., and R.E. Whaley, 1991, "Expiration Day Effects: What has Changed?" *Financial Analysts Journal*, Vol.47, p.26-36.

Wiggins, J.B., 1991, "Empirical Tests of the Bias and Efficiency of the Extreme-Value Variance Estimator for Common Stocks," *Journal of Business*, Vol.64, p.417-432.

CHAPTER V: CONCLUSION

This Chapter will briefly summarize the three studies which as noted in Chapter I, collectively make up the thesis requirement for the Doctorate of Finance at the University of Alberta. All three endeavors fall within the realm of the relatively recent sub field of financial economics, commonly referred to as option pricing theory.

Chapter II is a study on the application of recent theoretical developments in the pricing of financial options to a problem of irreversible investment within the environmental economics field, commonly referred to as Quasi-option value. This paper outlines how option pricing theory in financial economics can be utilized to solve for the value of delaying the conversion of some natural environment to some alternative commercial use. Although not the first to apply this technology to the problem of irreversible investment in general, it is the first attempt to address the specific issue of quasi-option value. An added advantage of the particular methodology employed and the resulting valuation formula is the specification of an optimal decision rule for when to invest.

In an world where choices of preservation or development are increasing in importance, this paper provides a timely and tractable approach for optimal choices. In addition the framework has the potential to be easily extended to decisions facing the firm regarding optimal investment timing and the valuation of flexibility inherent in certain projects.

The second study in the series, presented in Chapter III, examines volatilities implied from options on S&P 500 index futures and TSE 300 and 35 index options, for the presence of January, monthly and quarterly seasonalities. The testing methodology employed is that of multiple-input, time series intervention analysis which provides a rigorous test of nonstationarity in implied volatilities given the presence of serial correlation. Contrary to previous studies, results indicate the absence of any of the above seasonalities, in ex-ante market risk. Combined with those of previous authors however the results are suggestive of the possibility of a large firm effect in terms of a January seasonality in implied volatilities.

Finally Chapter IV presented the results of a study into the effects on the Toronto Stock Exchange 35 index, of expiring index options and futures. In spite of settlement procedures design to mitigate such effects it is shown that significant price and volume effects occur on the third friday of every month, when settlement of the derivative securities takes place. In addition new measures of potential unwinding of arbitrage positions, and consequent expiration day effects, are proposed.

Appendix II-1:
Derivation of Partial Differential Equation for a Generic Contingent Claim F on the Value of Two Assets M and P Having Constant Proportional Dividend Yields

Given the stochastic differential equations describing the movement through time of the values of the assets M and P:

$$dM = (\alpha_M - D_M/M)Mdt + \sigma_M M dZ_M \quad (A.1)$$

$$dP = (\alpha_P - D_P/P)Pdt + \sigma_P P dZ_P \quad (A.2)$$

where $dz_M dz_P = \rho_{MP} dt$ (A.3)

consider the value of the generic contingent claim F(M,P,T). Applying Ito's lemma to F we have:

$$dF = F_M dM + F_P dP - F_T dt + (1/2)\left(F_{MM} dM^2 + F_{PP} dP^2 + 2F_{MP} dMdP \right). \quad (A.4)$$

Substituting in the expressions for dM and dP and simplifying gives:

$$dF/F = \alpha_F(M,P,T)\ dt + \sigma_1(M,P,T)\ dz_M + \sigma_2(M,P,T)\ dz_P \quad (A.5)$$

where

$$\alpha_F = \frac{F_M(\alpha_M M - D_M) + F_P(\alpha_P P - D_P) - F_T + (1/2)(F_{MM} M^2 \sigma_M{}^2 + F_{PP} P^2 \sigma_P{}^2 + 2F_{MP} MP \rho_{MP} \sigma_M \sigma_P)}{F}$$

(A.6)

and

$$\sigma_1(M,P,T) = F_M \sigma_M M / F, \qquad \sigma_2(M,P,T) = F_P \sigma_P P / F. \quad (A.7)$$

Consider now forming a portfolio Y by investing the amounts X_M in the asset M, X_P in the value of the asset P, and X_F in the contingent claim F. The instantaneous total return on this portfolio, dY, will then be given by:

$$dY = X_M \frac{dM + D_M dt}{M} + X_P \frac{dP + D_P dt}{P} + X_F \frac{dF}{F}. \quad (A.8)$$

Substituting A(1), A(2), and A(5) into A(8) and simplifying gives

$$dY = (X_M\alpha_M + X_P\alpha_P + X_F\alpha_F)dt + (X_M\sigma_M + X_F\sigma_1)dZ_M + (X_P\sigma_P + X_F\sigma_2)dZ_P \quad (A.9)$$

Now by choosing X_M, X_P, and X_F such that:

$$(X_M\sigma_M + X_F\sigma_1) = 0 \text{ and } (X_P\sigma_P + X_F\sigma_2) = 0 \qquad (A.10)$$

then the portfolio return dY is riskless. Assuming no possibility of arbitrage profits the return on the portfolio must be equal to the return on the risk free asset and hence we have:

$$X_M(\alpha_M - r) + X_P(\alpha_P - r) + X_q(\alpha_q - r) = 0. \qquad (A.11)$$

Note also that from A(10) we have

$$X_M = - X_F\sigma_1/\sigma_M \quad \text{and} \quad X_P = - X_F\sigma_2/\sigma_P. \qquad (A.12)$$

Hence substituting A(12) into A(11) and simplifying we have

$$-\frac{\sigma_1(\alpha_M - r)}{\sigma_M} - \frac{\sigma_2(\alpha_P - r)}{\sigma_P} + \alpha_F - r = 0 \qquad (A.13)$$

and substituting A(7) into A(13) results in:

$$-\frac{F_M M}{F}(\alpha_M - r) - \frac{F_P P}{F}(\alpha_P - r) + \alpha_F - r = 0 \qquad (A.14)$$

Consequently substituting A(6) into A(14) and simplifying results in the partial differential equation:

$$(1/2)\sigma_M^2 M^2 F_{MM} + (1/2)\sigma_P^2 P^2 F_{PP} + \rho_{MP}\sigma_M\sigma_P MPF_{MP} + (rM-D_M)F_M + (rP-D_P)F_P - F_T - rF = 0$$

(A.15)

Now assuming constant proportional dividend yields:

$D_M = \delta_M M$ and $D_P = \delta_P P$

and substituting into (A.15) we have:

$$(1/2)\sigma_M^2 M^2 F_{MM} + (1/2)\sigma_P^2 P^2 F_{PP} + \rho_{MP}\sigma_M\sigma_P MPF_{MP} + (r-\delta_M)MF_M + (r-\delta_P)PF_P - F_T - rF = 0$$

(A.16)

Appendix II-2

Partial Differential Equation Governing the Option f on the Ratio of Two Risky AssetsH.

STarting with the partial differential equation for the option F to exchange P for M, as developed in Appendix 1 and given by equation (A.16):

$$(1/2)\sigma_M^2 M^2 F_{MM} + (1/2)\sigma_P^2 P^2 F_{PP} + \rho_{MP}\sigma_M\sigma_P MPF_{MP} + (r-\delta_M)MF_M + (r-\delta_P)PF_P - F_T - rF = 0$$

(A.16)

Now given that:

$$F(M,P,T) = f(H,1,T)P \text{ where } H = M/P \qquad (A.17)$$

this implies the partial derivatives:

$F_M = f_H$, $F_{MM} = f_{HH}P^{-1}$, $F_{MP} = -f_{HH}MP^{-2}$, $F_P = -f_H MP^{-1} + f$, $F_{PP} = f_{HH}M^2P^{-3}$, and $F_T = Pf_T$.

Substituting these partial derivatives into (A.16) and simplifying results in:

$$(1/2)\sigma_M{}^2M^2P^{-1}f_{HH} + (1/2)\sigma_P{}^2M^2P^{-1}f_{HH} - \rho_{MP}\sigma_M\sigma_P M^2P^{-1}f_{HH} - \delta Mf_H - Pf_T - rPf = 0$$

(A.18)

where $\delta = \delta_M - \delta_P$.

Factoring out P and substitutiing in for H = M/P:

$$(1/2)\sigma^2H^2f_{HH} - \delta Hf_H - f_T - rf = 0 \qquad (A19)$$

which is equation (17) in the text.

Appendix II-3

Comparative Statics for Q(M,P,T)

A) Comparative Statics of Q(M,P,T) with respect to σ^2:

Differentiating C(M,P,T) with repect to σ^2 gives:

$$\frac{\partial Q}{\partial \sigma^2} = \frac{Me^{-\delta T}\sqrt{T}}{2\sigma} + A(H^*P)^{1-y_1}M^{y_1}\ln(M/H^*P)\frac{\partial y_1}{\partial \sigma^2} + (1-y_1)AP^{1-y_1}M^{y_1}H^{*-y_1}\frac{\partial H^*}{\partial \sigma^2}$$

$$+(H^*P)^{1-y_1}M^{y_1}\frac{\partial A}{\partial \sigma^2}$$

where it can be shown that

$$\frac{\partial A}{\partial \sigma^2} = (e^{-\delta T}N[d_1(H^*)] - 1)y_1^{-2}\frac{\partial y_1}{\partial \sigma^2} + \frac{e^{-\delta T}N'[d_1(H^*)]\, d_2(H^*)}{2\sigma^2}$$

$$- \frac{e^{-\delta T}N'[d_1(H^*)]y_1^{-1}}{\sigma\sqrt{T}\,H^*}\frac{\partial H^*}{\partial \sigma^2}, \quad \text{and}$$

$$\frac{\partial H^*}{\partial \sigma^2} = H^*e^{-\delta T}N'[d_1(H^*)]\,(2\sigma)^{-1}[\sqrt{T} + y_1^{-1}d_2(H^*)\sigma^{-1}]\,\phi^{-1}$$

$$+ (e^{-\delta T}N[d_1(H^*)] - 1)y_1^{-2}H^*(2\sigma)^{-1}\phi^{-1}\frac{\partial y_1}{\partial \sigma^2}, \quad \text{where}$$

$$\phi = (1 - e^{-\delta T}N[d_1(H^*)])(1-y_1) + \frac{e^{-\delta T}y_1^{-1}N[d_1(H^*)]}{\sigma\sqrt{T}}$$

Consequently, substituting in for $\frac{\partial A}{\partial \sigma^2}$ and $\frac{\partial H^*}{\partial \sigma^2}$ we have that:

$$\frac{\partial Q}{\partial \sigma^2} = \frac{Me^{-\delta T}\sqrt{T}}{2\sigma} [\ N'(d_1) - (H/H^*)^{y_1 - 1} N'[d_1(H^*)\ ]$$
$$+ (H/H^*)^{y_1 - 1} MA\ [\ \ln(H/H^*) + y_1^{-1}(1/2\sigma^2 - 1)\ ]\ (\partial y_1/\partial \sigma^2) \quad > 0$$

where

$(\partial y_1/\partial \sigma^2) = -(1/\ 2\sigma^2)[\ B + (1/2\sqrt{\psi})(2(1+B)B + r(1-K)/\ K)]$ and

$\psi = (1 + B)^2 + JrK/\ (1-K)$

B) Comparative Statics of Q(M,P,T) with respect to M and P.

Differentiating Q(M,P,T) with respect to M:

$$\frac{\partial Q}{\partial M} = e^{-\delta T}N(d_1) + Ay_1(H/H^*)^{y_1 - 1} + (1-y_1)AP(H/H^*)^{y_1}\frac{\partial H^*}{\partial M} + M(H/H^*)^{y_1 - 1}\frac{\partial A}{\partial M}$$

where $A = (1 - e^{-\delta T}N[d_1(H^*)])y_1^{-1}$

However it can be shown that:

$\frac{\partial H^*}{\partial M} = 0$ and consequently $\frac{\partial A}{\partial M} = 0$ as well.

As as result we have that:

$$\frac{\partial Q}{\partial M} = e^{-\delta T}N(d_1) + [1 - e^{-\delta T}N(d_1(H^*))]\ (H/H^*)^{y_1 - 1} \quad > 0$$

Similarly the derivative of Q(M,P,T) with respect to P is given by:

$$\frac{\partial Q}{\partial P} = -N(d_2) + (M/P)A(1-y_1)(H/H^*)^{y_1 - 1} \quad < 0$$

as it can be shown that $\frac{\partial H^*}{\partial P} = 0$, and consequently $\frac{\partial A}{\partial M} = 0$ as well.

C) Comparative statics of Q(M,P,T) with respect to T:

Differentiating Q(M,P,T) with respect to T:

$$\frac{\partial Q}{\partial T} = M(H/H^*)^{y_1-1} \frac{\partial A}{\partial T} + (1-y_1)AP(H/H^*)^{y_1} \frac{\partial H^*}{\partial T} + AM(H/H^*)^{y_1-1} \ln(H/H^*) \frac{\partial y_1}{\partial T}$$

where it can be shown that:

$$\frac{\partial A}{\partial T} = (e^{-\delta T}N[d_1(H^*)] - 1)y_1^{-2} \frac{\partial y_1}{\partial T} - e^{-\delta T}N'[d_1(H^*)]y_1^{-1}d_1(H^{*-1})(2T)^{-1}$$

$$- e^{-\delta T}N'[d_1(H^*)]y_1^{-1}(H^*\sigma T)^{-1} \frac{\partial H^*}{\partial T} \text{, and}$$

$$\frac{\partial H^*}{\partial T} = \Big[\delta H^* e^{-\delta T}N[d_1(H^*)](y_1^{-1}-1) + H^* e^{-\delta T}N'[d_1(H^*)]y_1^{-1}(\delta - d_1(H^{*-1})(2T)^{-1})$$

$$+ H^*(e^{-\delta T}N[d_1(H^*)] - 1)y_1^{-2} \frac{\partial y_1}{\partial T} \Big] \phi^{-1} \text{, where as before}$$

$$\phi = (1 - e^{-\delta T}N[d_1(H^*)])(1-y_1) + \frac{e^{-\delta T}y_1^{-1}N[d_1(H^*)]}{\sigma\sqrt{T}}$$

Substituting in for these results and simplifying gives:

$$\frac{\partial Q}{\partial T} = Me^{-\delta T}[\sigma N'(d_1)/ 2\sqrt{T} - \delta N(d_1)]$$

$$+ (H/ H^*)^{y_1-1} M \{ A\ln(H/ H^*)(\partial y_1/ \partial T) - y_1^{-1}\delta e^{-\delta T}N'[d_1(H^*)]$$

$$-\delta e^{-\delta T}N[d_1(H^*)](y_1^{-1}-1) \} \qquad > 0$$

where

$$(\partial y_1/\partial T) = - (.25JrK/ K\sqrt{\psi})(1 + (rK/ K))$$

D) Comparative statics of Q(M,P,T) with respect to r:

Differentiating Q(M,P,T) with respect to r:

$$\frac{\partial Q}{\partial r} = M(H/H^{*})^{y_1 - 1}\frac{\partial A}{\partial r} + AM(H/H^{*})^{y_1 - 1}\ln(H/H^{*}) \frac{\partial y_1}{\partial r} + (1-y_1)AP(H/H^{*})^{y_1}\frac{\partial H^{*}}{\partial r}$$

where,

$$\frac{\partial A}{\partial r} = - e^{-\delta T}N'[d_1(H^{*})]y_1^{-1}(H^{*}\sigma \sqrt{T})^{-1} \frac{\partial H^{*}}{\partial r}, \text{ and}$$

$$\frac{\partial H^{*}}{\partial r} = H^{*}(e^{-\delta T}N[d_1(H^{*})] - 1)y_1^{-2} \phi^{-1} \frac{\partial y_1}{\partial r} \quad \text{with } \phi \text{ defined as above.}$$

Now substituting in these results and simplifying gives:

$$\frac{\partial Q}{\partial r} = (H/H^{*})^{y_1 - 1} M \{ A\ln(H/ H^{*})+y_1^{-2}(1 - e^{-\delta T}N[d_1(H^{*})])(H^{*}-1)\}(\partial y_1/\partial r)$$

where

$$(\partial y_1/ \partial r) = \frac{Ce^{-rT}}{4 \sqrt{\psi} K^2} [(1-e^{-rT} -rT)$$

E) Comparative Statics of Q(M,P,T) with respect to δ:

Differentiating Q(M,P,T) with respect to δ gives:

$$\frac{\partial Q}{\partial \delta} = -TMe^{-\delta T}N(d_1) + M(H/H^{*})^{y_1 - 1} \frac{\partial A}{\partial \delta} + AM(H/H^{*})^{y_1 - 1}\ln(H/H^{*}) \frac{\partial y_1}{\partial \delta}$$

$$+ (1-y_1)AP(H/H^{*})^{y_1} \frac{\partial H^{*}}{\partial \delta}$$

where it can be shown that,

$$\frac{\partial A}{\partial \delta} = (e^{-\delta T}N[d_1(H^*)] - 1)y_1^{-2} \frac{\partial y_1}{\partial \delta} - e^{-\delta T}N'[d_1(H^*)]y_1^{-1}(H^*\sigma \sqrt{T})^{-1} \frac{\partial H^*}{\partial \delta}$$

$$+ Te^{-\delta T}N[d_1(H^*)]y^{-1} + e^{-\delta T}N'[d (H^*)]y^{-1}\sqrt{T}\,\sigma^{-1} \quad \text{and,}$$

$$\frac{\partial H^*}{\partial \delta} = \Big[H^*(e^{-\delta T}N[d_1(H^*)] - 1)y_1^{-2} \frac{\partial y_1}{\partial \delta} - H^*Te^{-\delta T}N[d_1(H^*)]$$

$$+ H^* e^{-\delta T}N'[d (H^*)]y^{-1}\sqrt{T}\,\sigma^{-1} \Big] \phi^{-1}$$

with ϕ defined as above.

Substituting in these results and simplifying gives:

$$\frac{\partial Q}{\partial \delta} =(H/ H^*)^{y_1 -1} M \,[\, \ln(H/ H^*)(\partial y_1/ \partial \delta)+Te^{-\delta T}N[d_1(H^*)](1+y_1^{-1})]$$

$$- TMe^{-\delta T}N[d_1]$$

where

$$(\partial y_1/ \partial \delta) = (1/ \sigma^2)(1 + (1+B)/ \sqrt{\psi}) \,.$$