University of Alberta
Department of Civil Engineering

Structural Engineering Report No. 41

Plastic Design of Steel Frame-Shear Wall Structures

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August, 1972

ABSTRACT

This report presents a plastic design method for steel frame-shear wall structures. The design is based on the ultimate capacity of the structure and the method includes the effect of the formation of plastic hinges in the structure and the PΔ effect.

The frame portion of the structure is first designed plastically as a braced frame. The wall is then proportioned for strength considering the contribution of the frame in resisting combined loads. A simple direct design method is presented for the class of structures which do not reach their ultimate capacities once the shear wall develops a plastic hinge. For structures which do reach their ultimate capacities once the shear wall develops a plastic hinge, the design is based on an analysis in which the wall remains elastic.

Following this preliminary design the wall proportions are checked and possibly modified to resist the effect of full gravity loads alone, acting on the structure and to ensure that the working load deflections are satisfactory.

The adequacy of the design method is verified by a rigorous computer analysis of typical steel frame-shear wall structures designed as described above.

TABLE OF CONTENTS

Table of Contents (continued)

Table of Contents (continued)

LIST OF SYMBOLS

d	distance from the centroid, to the face of the shear wall
E	modulus of elasticity
h	story height
H	lateral load
I	moment of inertia
K	shear-sway rotation stiffness of a column
K_b	bracing stiffness
L	length of girder
L.F.	load factor
m	number of column stacks which offer resistance to sway
M	moment
M_{AD}, M_{BE}	girder end moments
M_b	moment at bottom end of a column
M_f, M_R	resisting moment of frame
$M_{CB}, M_{CD}, M_{R_{CB}}, M_{R_{CD}}$	moments at centroid of shear wall
M_h	overturning moment due to applied lateral forces
M_p, M_{pc}	plastic moment capacity of a girder, column
$M_{P\Delta}$	overturning moment due to $P\Delta$ effect
M_t	moment at top end of a column
N	number of stories
P	column axial load
P_c	load defined by equation 3.6
P_G	total gravity load applied to each floor
P_w	concentrated vertical load applied to the shear wall
P_u	ultimate column axial load

List of Symbols (continued)

V	resisting shear of a column
w	uniformly distributed vertical load
w_g	defined by equation 4.5
x	location of girder plastic hinge
Δ	sway deflection
λ	load factor on lateral loads
γ	defined by equation 4.2
β	factor to account for differing story heights
θ_C, θ_B	joint rotations
ρ	story sway rotation
ρ_f	sway rotation at which structure is on the verge of instability
ρ_u	sway rotation at which a column develops its ultimate resisting shear

Chapter I

INTRODUCTION

Modern structures frequently consist of frames coupled with shear walls. A plan view of a typical structure is shown in Figure 1.1. This structure consists of seven parallel steel bents, coupled with a reinforced concrete core through the floor system. If the floor diaphrams are rigid the structure may be idealized as shown in Figure 1.2. In this figure the bent containing the shear wall is linked by inextensible bars to a lumped bent, which is proportioned to represent the action of the framed bents in the structure(15).

The stiff shear wall dominates the behavior of the structure under lateral loads so that common design practice has been to assume that the walls resist all the lateral load(5). In tall structures, however, significant interaction may occur between the wall and the frame due to the incompatibility of the cantilever deformation of the wall and the portal deformation of the frame(15). This interaction, if neglected, may result in an unsafe or an uneconomical design.

The frame portion of the structure shown in plan in Figure 1.1 has been designed plastically as a braced frame(2,3). The behavior of this structure under combined loads is illustrated in Figure 1.3 which plots the load factor (L.F.) on lateral loads, λ, versus the roof sway, Δ. Gravity loads (L.F. = 1.3) have been applied initially, then held constant as the lateral loads are increased monotonically. In this figure, the dotted, dashed, and solid curves represent the results of a first order elastic analysis, a first order elastic-plastic analysis, and a second order elastic-plastic analysis, respectively(1,2).

The first order elastic analysis overestimates the stiffness of the structure and indicates that the formation of the first plastic hinge in the structure occurs on the initial application of gravity loads. Thus, according to elastic design philosophy the usable capacity of the structure corresponds to the origin of the λ-Δ plot. This in no way relates to the actual ultimate capacity, which may be utilized.

The first order elastic-plastic analysis indicates the gradual deterioration in stiffness due to the formation of plastic hinges in the structure. The λ-Δ response becomes horizontal at "b" which represents the formation of a simple plastic failure mechanism. The mechanism load grossly overestimates the actual failure load (which is reached before a mechanism forms) because the analysis neglects the second order ($P\Delta$) effect.

The second order elastic-plastic analysis accurately predicts the actual response of the structure[2,7]. The point "a" represents the formation of the first plastic hinge in the shear wall, and at "c" the ultimate capacity of the structure is reached. The increase in capacity between "a" and "c" is due to the plastic redistribution of forces between the wall and the frame.

From the above discussion it is apparent that a rational design procedure for frame-shear wall structures, based on the ultimate capacity, must include the effect of plastic hinging in the structure and the $P\Delta$ effect. The purpose of this thesis is to develop such a design procedure for steel frame-shear wall structures.

Plastic design methods are available for braced and unbraced multi-story steel frames[2]. The methods consist of dividing the frame into characteristic units which are designed as if their action were independent of the behavior of the rest of the frame. For braced frames the strength and stiffness of bracing required to resist lateral loads and PΔ forces are directly determined for each characteristic unit. This approach has little applicability to frame-shear wall structures, thus, direct methods for the plastic design of such structures are not presently available.

In Chapter II of this thesis, the literature pertinent to the development of a plastic design method for steel frame-shear wall structures is reviewed. The stability of frame-shear wall structures is investigated in detail in Chapter III, and a criterion is developed to determine the load stage at which the structure reaches its ultimate load.

In Chapter IV an analytical model for the steel frame in a frame-shear wall structure is developed. An analytical model for the entire frame-shear wall structure is presented in Chapter V. A plastic design procedure based on this model is also presented in Chapter V. Finally, the design procedure is verified by second order elastic-plastic analyses of typical steel frame-shear wall structures in Chapter VI.

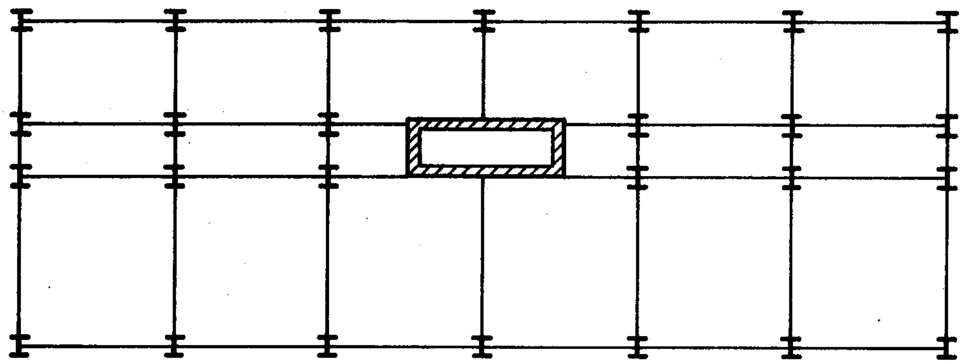


FIGURE 1.1 PLAN OF TYPICAL FRAME-SHEAR WALL STRUCTURE

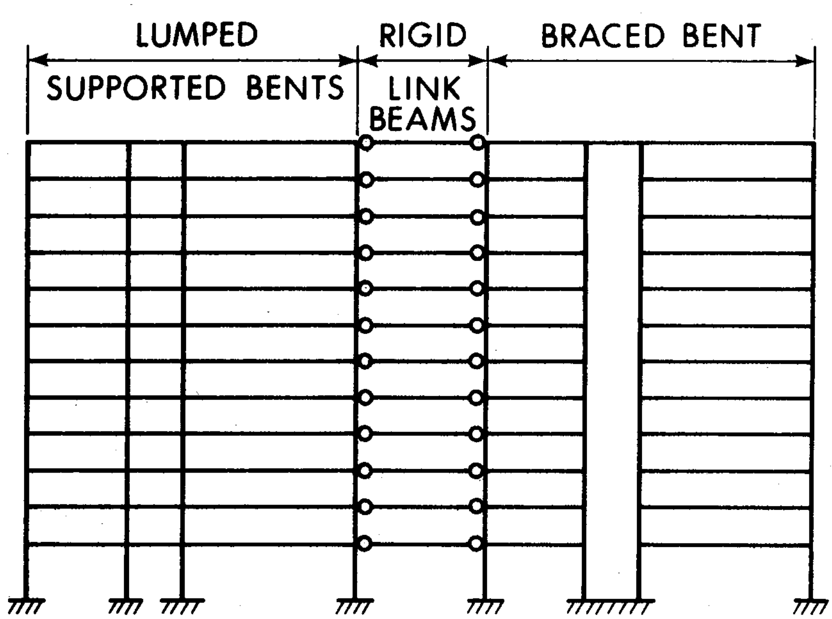


FIGURE 1.2 IDEALIZATION OF STRUCTURE

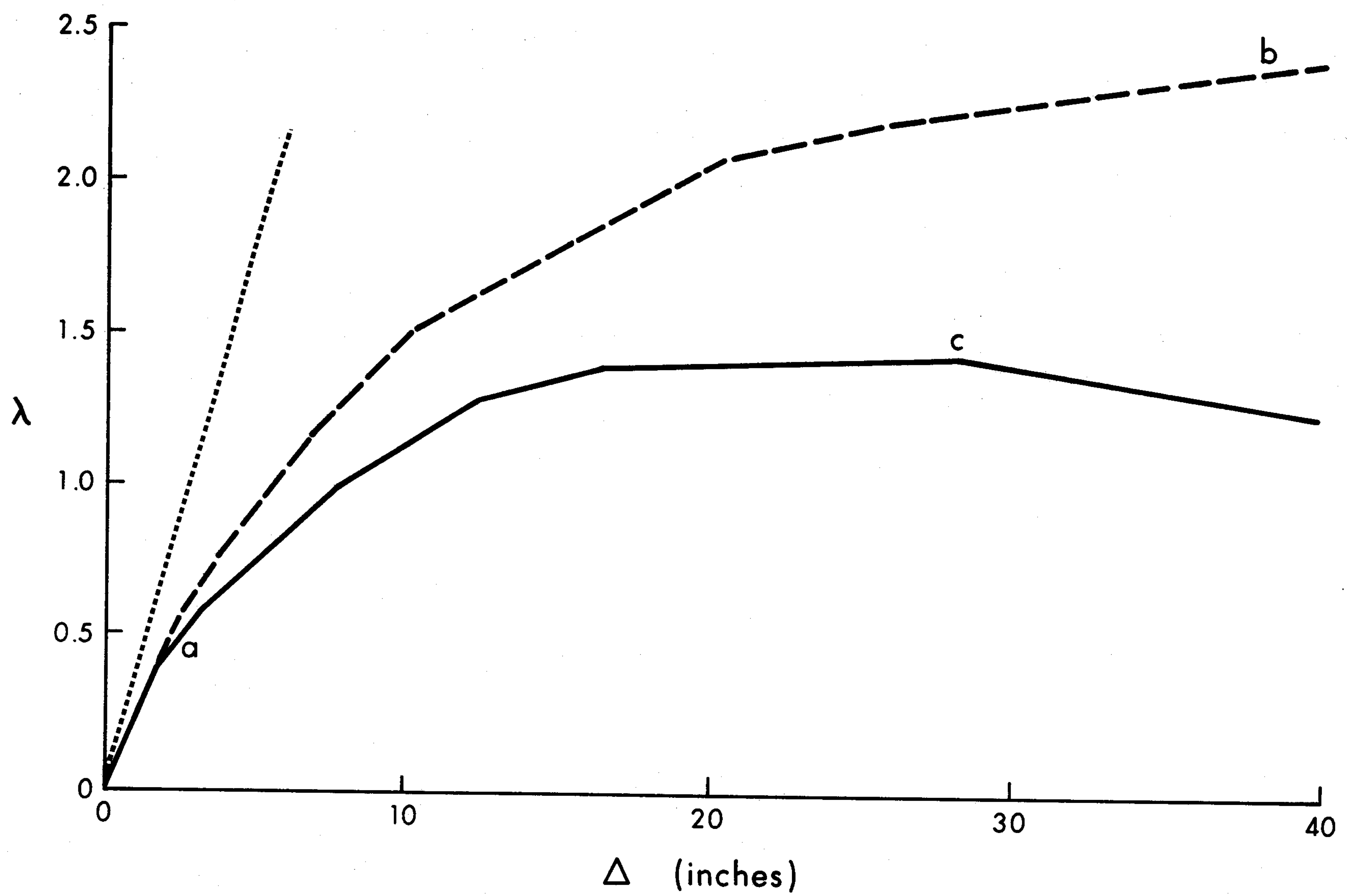


FIGURE 1.3 LOAD - SWAY RESPONSE

CHAPTER II

DEVELOPMENT OF THE DESIGN METHOD

2.1 Introduction

In this chapter the literature pertinent to the development of a plastic design method for steel frame-shear wall structures is reviewed. In the design method the member sizes in the framed portion of the structure are assumed to be selected to resist full gravity loads (L.F. = 1.7); the sway forces are assumed to be resisted by the bracing system. The beams are designed to develop a three hinge mechanism, and the columns are then proportioned to have sufficient capacity to resist the bending moments transmitted from adjacent girders in addition to the axial thrust(2,3).

The stiffness of the shear wall in a frame-shear wall structure is normally determined by architectural requirements. The wall must therefore be designed for strength so that the structure can resist the factored loads, and satisfy the serviceability requirements.

2.2 Design for Combined Loads

Numerous analytical models for frame-shear wall structures have been developed(13,14,15,16,17,18). The structure is reduced to a simplified planar model and then analyzed to determine the distribution of internal moments and forces.

A typical analytical model is that presented by Gould(16). Gould idealized the complete structure as a cantilever wall supported on an elastic foundation as shown in Figure 2.1. In this figure, the translational springs represent the lateral action of the frame in each story, and the rotational springs represent the restraining action of the girders framing directly into the wall. The rigid bars represent the floor system. The spring stiffnesses were derived from a model of the frame in which the sway rotations in successive stories were assumed equal, and points of inflection were assumed to occur at mid-height in the columns and mid-span in the beams. Gould then subjected the analytical model to a first order elastic analysis. In Chapter I it has been shown that this analysis is inadequate for a design based on the ultimate capacity.

Adams and MacGregor[6] have presented a second order elastic-plastic analysis of the analytical model shown in Figure 2.2. The members in the model[8] are designed to have lateral stiffnesses and strengths equivalent to those of the actual structure. The analysis is capable of including the actual moment-curvature relationship of the wall. To use the model the following design procedure was suggested:

1. Design the frame portion as a braced frame[2,3] under full gravity loads (L.F. = 1.7).

2. Analyze the above model under combined loads and adjust the strengths and stiffnesses of the wall. Iterate the analysis until the required load factor (L.F. = 1.3) is reached and the working load deflections are satisfactory.

Second order elastic-plastic analyses may be performed on more accurate models of the structure[9,10]. For example, the analysis reported in reference (10) is capable of tracing the complete load deflection history of planar structures, including the descending branch of the load deflection curve. The moment-curvature response of a section is assumed to be elastic perfectly-plastic. The effect of axial load on the column plastic moment capacity, stiffness and deformation is considered. Hinge reversal in the girders is treated; and the effect of column and wall finite width may be included.

Both the analyses described in (6) and (10) require complex computer programs. In addition, a design approach using these analyses alone, must be iterative.

The simple direct approach to design, developed in this thesis, is based on an estimate of the deflected shape of the structure at the ultimate state. The PΔ effect and the resistance of the frame portion may then be evaluated, and the wall proportioned to resist the remaining lateral forces. To obtain an estimate of the deflected shape, a stability criterion is required to determine the load stage at which the structure reaches its ultimate capacity.

A stability criterion similar to that suggested by Rosenbleuth (12) is developed in Chapter III. Rosenbleuth investigated the stability of a single story of a frame by comparing the stiffness of the frame with the PΔ effect in that story. The PΔ effect was represented as a "negative stiffness" which subtracts from the story stiffness. Rosenbleuth's approach is illustrated in Figure 2.3 which plots the moment, M, versus the story sway, Δ. The solid curve in this figure represents the resisting moment of the frame defined as:

$$M_R = \sum_{i=1}^{m} (M_t + M_b)_i \tag{2.1}$$

where M_t and M_b are the moments at the top end and the lower end, respectively, of the i^{th} column. The dashed curve represents the PΔ moment in the story defined as:

$$M_{P\Delta} = \sum_{i=1}^{m} P_i \, \Delta \tag{2.2}$$

where P_i is the axial load in the i^{th} column. The broken curve in this figure represents the total moment defined as:

$$M = M_R - M_{P\Delta} \tag{2.3}$$

At point "a" on the broken curve the stiffness of the frame equals the slope or "negative stiffness" of the PΔ curve, and the structure, having effectively "zero stiffness", reaches its ultimate capacity.

For a certain class of structures, the ultimate capacity is reached once a plastic hinge develops in the shear wall. An analysis based on full moment redistribution is not suitable for these structures. Instead a second order elastic-plastic analysis in which the wall remains elastic is necessary in order to determine the ultimate forces to be resisted by the wall. In Chapter V an analytical model, similar to Gould's, is presented for the design of such structures. In this model the action of the frame portion is based on the analytical model developed in Chapter IV and includes the effect of the formation of plastic hinges.

2.3 Design for Vertical Loads Alone

The behavior of structures under gravity loads alone, is indicated in Figure 2.4. This figure plots the vertical load, w, versus a characteristic lateral deflection, Δ. For perfectly symmetrical structures with no initial imperfections, the load may be increased without lateral deformation until the critical value, w_{cr}, is reached. At this point the structure lurches into a sidesway mode. If the structure is unsymmetrical, however, the load deflection relationship is shown by the solid curve in Figure 2.4. As the load increases the lateral deflections also increase and become very large near w_{cr}.

Under full gravity loads alone (L.F. = 1.7) sufficient bracing must be present in the structure to resist the vertical loads in a swayed position(2). Bracing systems designed to resist gravity loads, alone, are normally designed for stiffness, only, as though the structure were perfectly symmetrical(2,6). The design approach is to apply the full gravity loads and to subject the structure to a small lateral disturbance. If the bracing system is adequately stiff then the load is less than the critical load and the structure will return to the undeformed position.

Goldberg(19,20) investigated this approach and concluded that to ensure that the elastic critical load for sidesway buckling of a single story was above that for non-sway buckling, the bracing system must be sufficiently stiff to resist the PΔ shear in that story. Figure 2.5 shows the single story frame braced by a spring of stiffness, K_b. Equilibrium of the PΔ shear with the bracing force yields equation 2.4, from which the bracing stiffness may be calculated.

$$K_b \, \Delta \geq \frac{\Delta}{h} \sum_{i=1}^{m} P_i \tag{2.4}$$

In equation 2.4, Δ is the story sway disturbance, h is the story height and P_i is the non-sway buckling load for the i^{th} column.

The bracing system in a plastically designed frame is designed to resist the PΔ effect without assistance from the frame(2). The bracing stiffness required to resist the PΔ effect is determined by an approach similar to Goldberg's. Each story is given a sway disturbance, and equilibrium conditions dictate the required bracing stiffness for a story as:

$$K_b \geq \frac{1}{h} \sum_{i=1}^{m} P_{ui} \qquad (2.5)$$

In equation (2.5), P_{ui} is the ultimate load (L.F. = 1.7) on the i^{th} column in the story.

The above design approach neglects the strength required by the bracing system for real structures which are generally unsymmetrical. The bracing system for real structures must provide strength to resist the PΔ forces corresponding to the deformed position of the structure under full gravity loads.

In frame-shear wall structures the wall stiffness is normally determined by architectural requirements. The design approach taken in this thesis is to analyze the structure in a deflected position corresponding to point "a" in the solid curve of Figure 2.4, to determine the forces to be resisted by the bracing system. (For symmetrical structures, initial imperfections must be assumed so that the structure deflects laterally under gravity loads alone).

2.4 Summary

Plastic design methods are available for the design of the frame members in steel frame-shear wall structures. To proportion the wall under the combined loading case (L.F. = 1.3), an iterative approach using a complex computer program is required. Also, the forces to be resisted by the wall corresponding to the deformed position of the structure under full gravity loads (L.F. = 1.7) are not accounted for.

It is the purpose of this thesis to develop a design method for a simple, direct calculation of the ultimate forces to be resisted by the shear wall under combined loads. A rational approach to the design under full gravity loads, alone, is also presented.

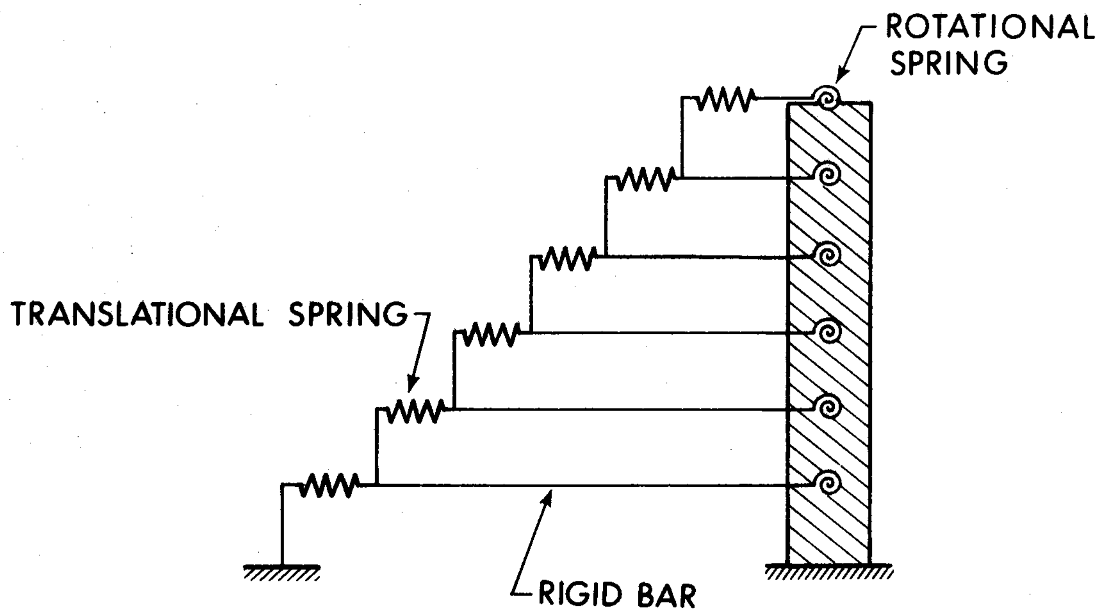


FIGURE 2.1 GOULD'S MODEL

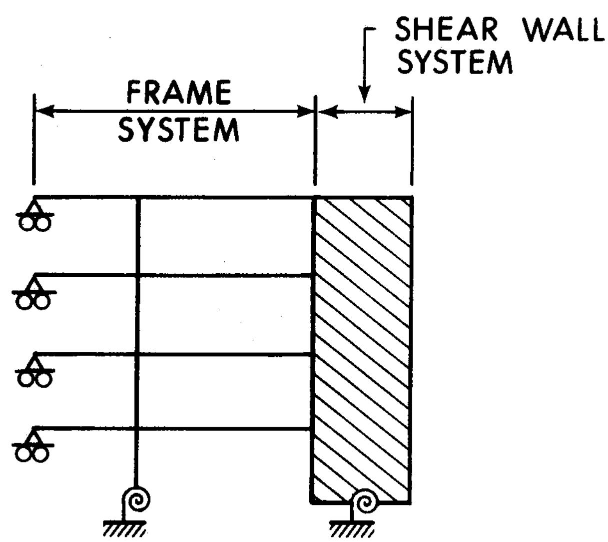


FIGURE 2.2 ADAMS AND MACGREGOR'S MODEL

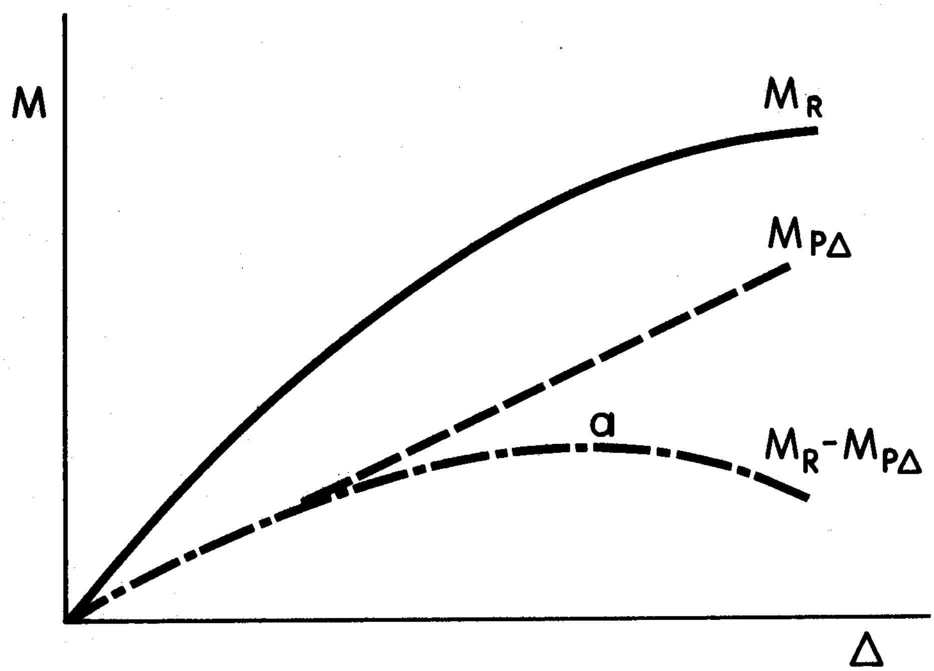


FIGURE 2.3 ROSENBLEUTH'S STABILITY CRITERION

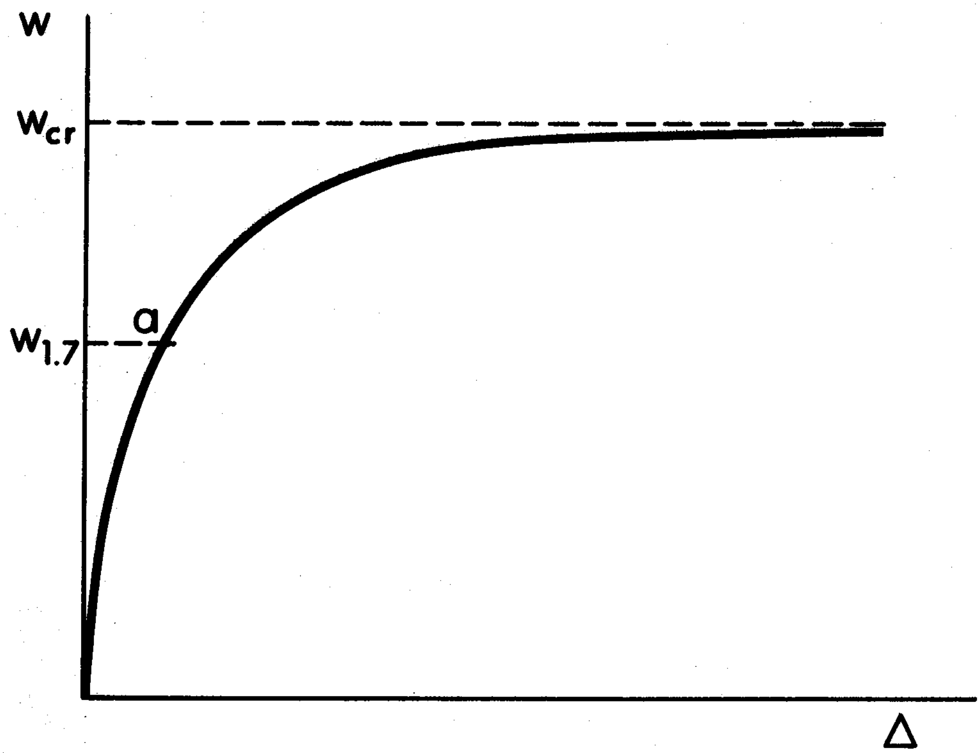


FIGURE 2.4 LOAD DISPLACEMENT RESPONSE

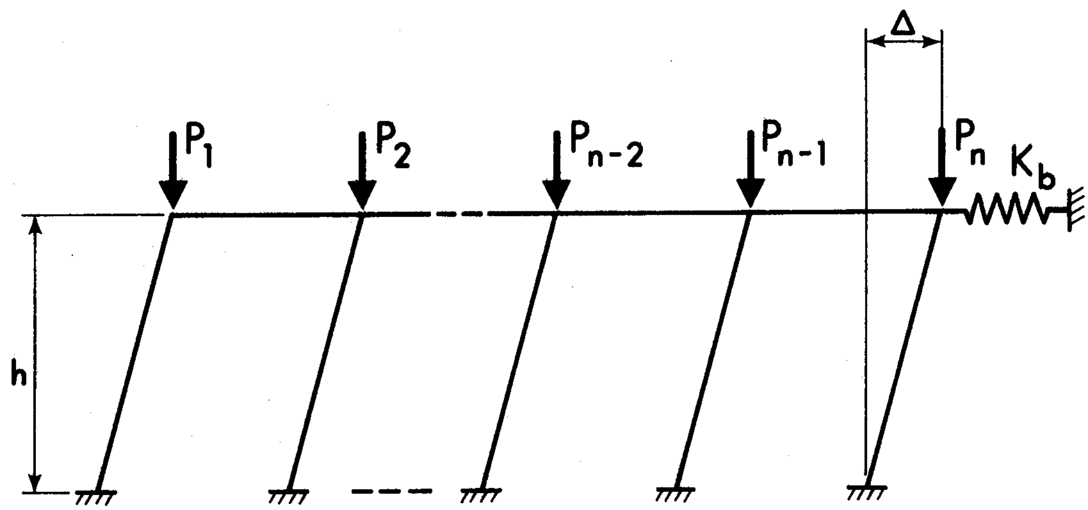


FIGURE 2.5 BRACED FRAME

CHAPTER III

STABILITY OF STEEL FRAME-SHEAR WALL STRUCTURES

3.1 Introduction

The previous chapters have emphasized the importance of providing sufficient lateral stiffness to resist the second order (PΔ) effects in addition to the applied lateral forces in tall buildings. The deterioration of stiffness due to the formation of plastic hinges in the structure has also been discussed. Both the PΔ effect and plastic hinging must be included in any rational design procedure based on the ultimate capacity of a structure.

In this chapter the behavior of steel frame-shear wall structures under combined loading is discussed. The behavior under gravity loads, alone, is discussed in Chapter V. Attention is focused on the determination of the loading stage at which the structure reaches its ultimate capacity.

3.2 Idealization of the Building

Figure 3.1(a) shows the plan view of a steel framed building. The frame portion has been designed plastically as a braced frame by the methods described in References (2) and (3). The frame is braced by a shear wall in the form of a central core. It is assumed that the structure is symmetrical and is symmetrically loaded; and that the floor system is infinitely rigid in its own plane. The building thus translates under the action of the applied loads but does not twist. Thus the structure may be treated as a series of plane frames as illustrated in Figure 3.1(b) where the bents are linked by rigid bars which enforce equal floor displacements.

Figure 3.2 shows a simpler representation of a plane frame-wall system. The frame is a lumped frame, linked by rigid bars to a wall at each floor level[15]. The properties of this structure are indicated in Table 3.1. The plastic moment capacities of the girders, M_p, have been reduced so that;

$$M_p \equiv \frac{1.7\ wL^2}{16} \tag{3.1}$$

where; w is the uniformly distributed design load on the girders (L.F. = 1.0), and L is the girder span. The factored loads (L.F. = 1.3) are indicated in Figure 3.2. The concentrated vertical loads, P_w, applied to the wall, were adjusted as described below, to study the PΔ effect.

3.3 Behavior of the Frame-Shear Wall

The structure shown in Figure 3.2 was analyzed by the rigorous second order elastic-plastic analysis described in Reference (10). Vertical loads were applied initially and held constant, while lateral loads were increased to define the response of the structure. Figure 3.3 plots the load factor, λ, defined as the ratio of lateral loads to the design values, versus the roof sway, Δ; for P_w = 0, 458 Kips, and 2000 Kips. The inserts to this figure show the hinging patterns. The number above each hinge indicates the load stage on the λ-Δ curve at which the hinge formed.

The responses in Figure 3.3 are similar, until a plastic hinge develops at the base of the shear wall at load stage #5. At this stage there is a drastic reduction in the stiffness of the structure. As the structure continues to sway, further resistance to the PΔ forces and applied lateral loads, is offered by the frame portion alone. For P_w = 0.0 the frame is able to resist additional applied loads as well as the $P\Delta$ forces, so that the capacity of the structure continues to increase. At load stage #7 the frame develops mechanisms, simultaneously, in each story, there is a drastic reduction in the stiffness of the structure, and beyond this stage the capacity of the structure is reduced. If the magnitude of the lateral loads at load stage #7 is maintained, then the structure becomes unstable at this stage. In this thesis it is assumed that lateral loads are maintained in this manner so that the attainment of the ultimate capacity represents instability of the structure.

For P_w = 458 Kips, the frame is just able to resist the $P\Delta$ forces, alone, so the load carrying capacity remains constant as the structure continues to sway. The structure again reaches its ultimate capacity (instability) at load stage #7, as the stiffness is drastically reduced, when the frame develops mechanisms, simultaneously, in each story.

As shown, for P_w = 2000 Kips, the frame has insufficient stiffness to resist the $P\Delta$ forces once a plastic hinge has formed in the wall so the maximum capacity of the structure is reached at load stage #5.

3.4 Significance of Behavior

For relatively low gravity loads, as shown for P_w = 0, and P_w = 458 Kips in Figure 3.3, the load carrying capacity of the structure increased or remained constant despite the drastic reduction in stiffness which occurred when the wall developed a plastic hinge. The capacity of the structure was reduced only after the stiffness deteriorated to the point where increases in the $P\Delta$ effect could not be resisted.

This occurred when both the wall and the frame had developed their ultimate capacities. The plastic design of such structures should, therefore, utilize moment redistribution in the structure beyond the stage where the wall, alone, reaches its ultimate capacity; up to the stage where the capacity of the entire structure is attained.

As shown, for P_W = 2000 Kips in Figure 3.3, the structure, subjected to relatively high gravity loads, reached its ultimate capacity as soon as the wall developed a plastic hinge. The plastic design of such structures should therefore be based on an analysis in which the wall remains elastic.

Before plastic design procedures can be applied to frame-shear wall structures, a criterion must be developed to determine the stage at which the structure becomes unstable. The criterion must only involve a comparison between the $P\Delta$ effect and the frame resistance, since the structure is likely to become unstable only after the wall has yielded.

3.5 Development of Stability Criterion

The stability of a frame under combined loading is commonly examined by studying the stability of each story in turn[2]. The stability of a single story is determined by comparing the shear resistance of the structure, with the $P\Delta$ shear in that story.

Figure 3.4 shows the forces acting on the columns and shear wall in a particular story. The resisting shears developed by the columns and the shear wall are plotted against the story sway rotation, ρ, in Figure 3.5. The total resisting shear (wall plus frame) is also shown by the dashed curve. To determine the resistance to applied shear, the $P\Delta$ shear, indicated as a negative quantity in Figure 3.5, is subtracted from the total resisting shear. At points A and B on the wall response curve, plastic hinges develop at A and B, respectively, in the wall.

Assuming that the structure is not unstable in the elastic range, the stability of the story must be tested at point A, where the stiffness is reduced by the first wall hinge. However, this test would require a comparison of the PΔ effect, indicated by the slope of the PΔ curve of Figure 3.5, with the total resisting shear, represented by the slope of the dashed curve in Figure 3.5. The total resisting shear consists of not just the frame resistance, but also the resistance of the partially yielded wall; a resistance which is difficult to obtain (Section 5.2). The suggested approach, therefore, is to examine the stability in each story by comparing the moment resistance of the structure with the overturning moments due to the PΔ effect.

Figure 3.6 plots the shear due to the PΔ effect and the shear resisted by the frame; for the structure shown in Figure 3.7 from the roof down to level "n", which is "n" stories from the roof. This structure is a series of multi-bay frames linked to a shear wall. The effects of the beams framing directly into the wall, shown in Figure 3.1(b), are neglected. The overturning moment due to the PΔ effect at this location, $M_{P\Delta}$, is:

$$M_{P\Delta} = \sum_{j=1}^{n} h_j [\sum_{k=1}^{j} P_k \rho_k] \qquad (3.2)$$

where: h_j is the height of the "j"th story.

P_k is the vertical load applied to the floor system of story "k".

ρ_k is the sway rotation of story "k".

The moment resisted by the frame, M_f, is:

$$M_f = \sum_{j=1}^{n} h_j [\sum_{i=1}^{m} V_{ji}] \qquad (3.3)$$

where: V_{ji} is the shear resisted by the column in the "j"th story of the "i"th bay.

and "m" is the number of bays in the frame.

The moment due to the applied lateral forces, M_h, is:

$$M_h = \sum_{j=1}^{n} H_j \left[\sum_{k=j}^{n} h_k \right] \qquad (3.4)$$

where: H_j is the concentrated lateral load applied at the "j"th level.

and h_k is the height of the "k"th story.

Figure 3.8 plots $M_{P\Delta}$, M_f, M_h and the moment resisted by the wall, M_w, against the story sway rotation, ρ. The total moment resistance of the structure ($M_f + M_w$) at level "n" is also indicated by the dashed curve. This figure is based on the structure shown in Figure 3.7, assuming that the deflected shape above level "n" is rectilinear, after the wall develops a plastic hinge (Section 5.3.1). The frame response is based on the model developed in Chapter IV. The points B, C, and D, on the M_f curve indicate the stages where bay types B, C and D, respectively, in Figure 3.7, fail. It was assumed that each bay of the frame portion of Figure 3.7 fails by developing mechanisms, simultaneously in every story; and that the wall develops a plastic hinge prior to failure of any bay of the frame (Section 5.5). It was also assumed that the wall has sufficient stiffness, that elastic instability does not occur.

The stiffness of the structure is significantly reduced, first, at load stage "A" in Figure 3.8, when the wall develops a plastic hinge. At this stage the stiffness of the structure is given by the slope of the $M_f + M_w$ curve, which is:

$$\frac{d}{d\rho}(M_f + M_w) = \frac{dM_f}{d\rho} \tag{3.5}$$

The stability is tested by comparing this resistance with the $P\Delta$ effect given by the slope of the $M_{P\Delta}$ curve, which is:

$$\frac{dM_{P\Delta}}{d\rho}$$

If the term:

$$\frac{dM_f}{d\rho} - \frac{dM_{P\Delta}}{d\rho}$$

is negative, the structure is unstable at "A". The total resistance of the structure is also reduced significantly at load stages B, C, and D in Figure 3.8, due to the deterioration of the frame stiffness when entire bay types fail. The stability criterion, which is an evaluation of the term:

$$\frac{dM_f}{d\rho} - \frac{dM_{P\Delta}}{d\rho}$$

must, therefore, be applied at stages B, C and D, to determine at which stage the structure becomes unstable.

If the wall develops a plastic hinge at its base the most severe stability conditions will occur in the bottom story of the structure. Therefore, the stability criterion, above, need not be applied to every story; only to the bottom story.

A simple formulation of the above stability criterion has been derived in Appendix B, for a multi-bay frame supported by a shear wall (Figure 3.7). The formulation simulates an evaluation of the term:

$$\frac{dM_f}{d\rho} - \frac{dM_{P\Delta}}{d\rho}$$

in the bottom story. The average total gravity load applied to each story is compared with P_c, given as:

$$P_c = \frac{12E}{(N+1)h^2} \sum_{i=1}^{m} \left[\frac{I_i}{\frac{2L_i}{h} + \frac{1}{2}} \right] \quad (3.6)$$

where: E is the modulus of elasticity for the frame.

N is the number of stories.

h is the story height (assumed constant).

I_i is the moment of inertia of the girders in bay i (all girders in bay i are assumed to be the same).

L_i is the span of bay i.

and m is the number of bays which offer resistance to sway at the load stage considered.

If the average total gravity load is greater than P_c at a certain load stage, then the structure will become unstable. If the average total gravity load, is equal to P_c, then the capacity will remain constant as the structure sways to the next load stage, at which there is a significant reduction in the stiffness. The response will thus be similar to that between load stages 6 and 7 for P_w = 458 Kips in Figure 3.3.

Equation 3.6 should be calculated at load stage "A" of Figure 3.8, where the stiffness of the structure is first significantly reduced. The summation process at this stage should be made over all the bays of the frame, with the exception of the bays with link beams, which are not considered to resist sway (Chapter IV). If the structure is stable at stage "A", equation 3.6 should then be evaluated at load stage "B", where the stiffness is again significantly reduced. In the summation process at this stage, therefore, the type "B" bays are neglected. If the structure is stable at "B", equation 3.6 should be calculated at "C", etc., to determine the load stage at which the structure becomes unstable.

To demonstrate the practical significance of the stability criterion discussed above, equation 3.6 was expressed graphically in Figure 3.9, for a multi-bay frame (shown in the insert) with a constant bay width, L, and a constant story height of 10 ft.; linked to a shear wall. This figure indicates the number of stories at which the structure will just be stable, once the wall develops a plastic hinge; in terms of the ratio of the girder moment of inertia, I, to the span, L; for two magnitudes of the uniformly distributed load, w, and three values of the girder span. If a particular structure lies above the appropriate curve then the structure will become unstable once the wall yields. If the point lies on, or below the curve then the structure will become unstable once both the wall yields and the frame develops mechanisms.

3.6 Summary

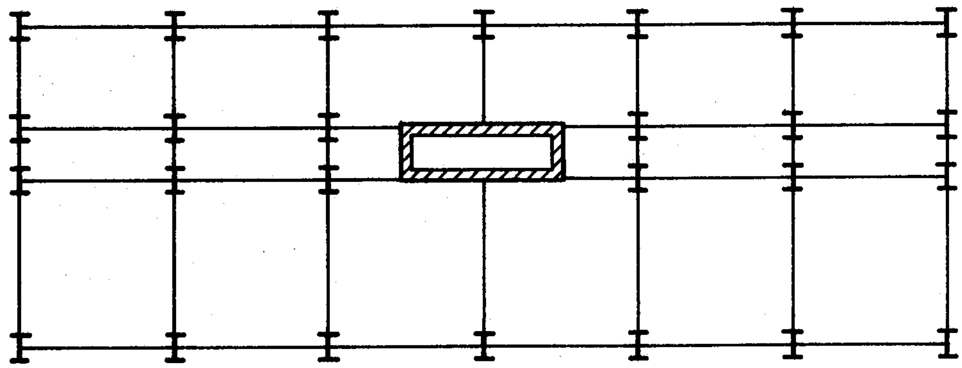
Plastic design procedures for frame-shear wall structures should utilize moment redistribution in a structure up to the stage at which the structure becomes unstable. This stage depends on the relative magnitudes of the gravity loads and the stiffness. The stiffness, in turn, is reduced by plastic hinging in the structure.

In this chapter a criterion has been developed (equation 3.6) to determine at which load stage the structure becomes unstable, once the wall yields. This criterion is necessary before plastic design procedures can be established.

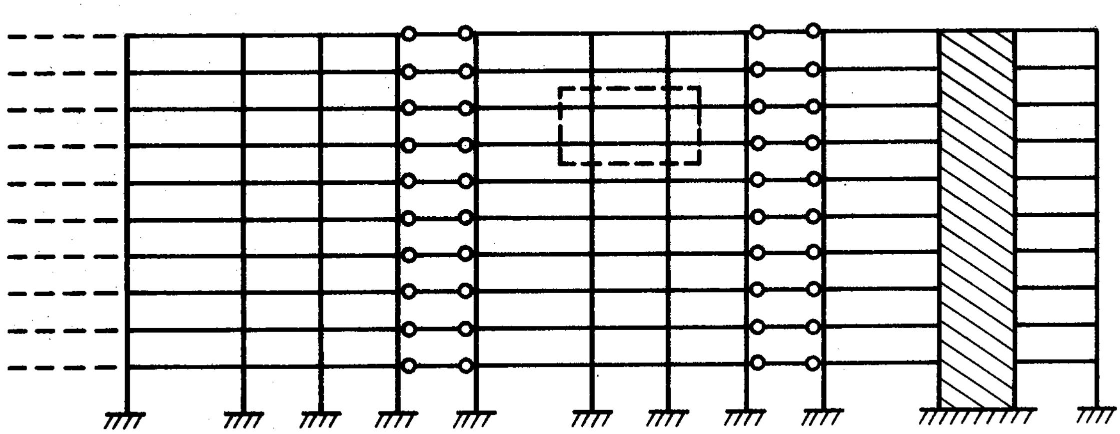
STORY	COLUMN
1	5 - 14 WF 30
2	5 - 14 WF 30
3	5 - 14 WF 30
4	5 - 14 WF 30
5	5 - 14 WF 43
6	5 - 14 WF 43
7	5 - 14 WF 43
BEAMS ALL 5 - 14 WF 30	
WALL EI = 2.5×10^{10} Kip-ins.^{2} WALL M_p = 25,000 Kip-ft.	

TABLE 3.1

PROPERTIES OF STRUCTURE OF FIGURE 3.2



(a) PLAN



(b) IDEALIZATION

FIGURE 3.1 STEEL FRAME-SHEAR WALL STRUCTURE

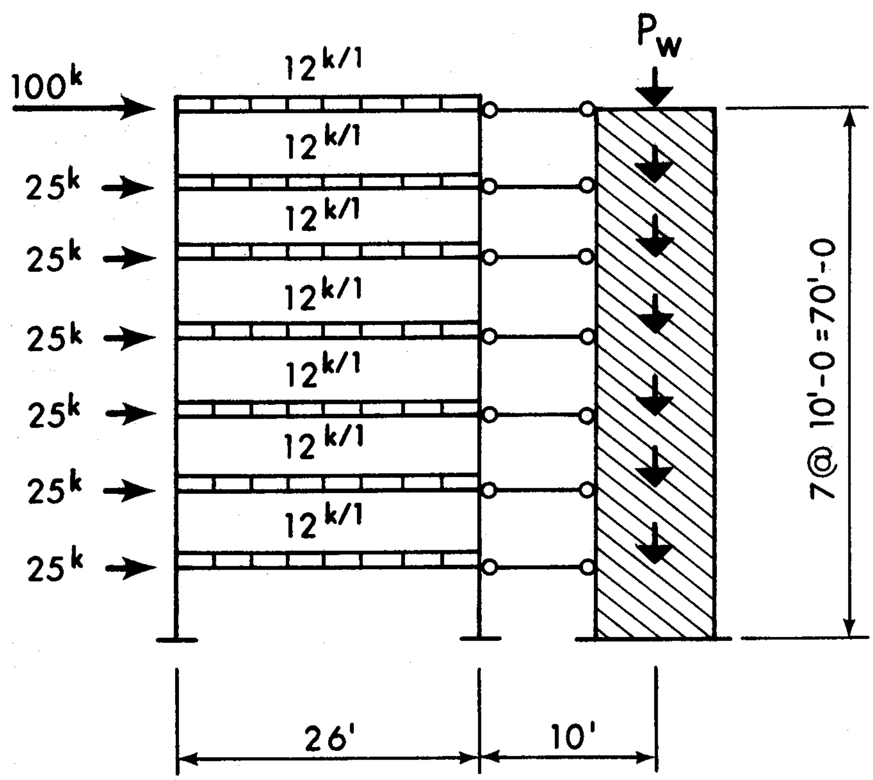


FIGURE 3.2 PLANE FRAME-WALL SYSTEM

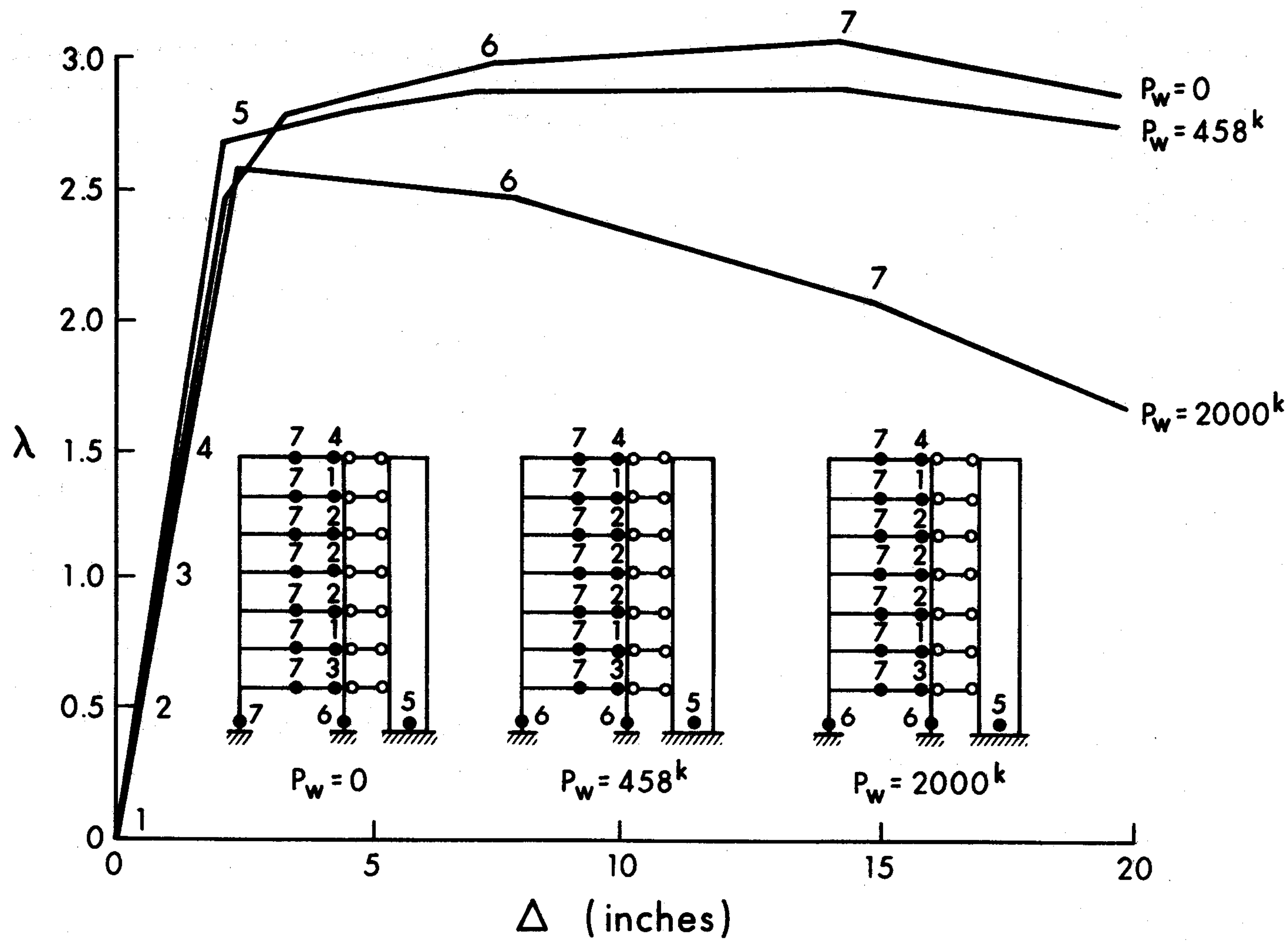


FIGURE 3.3 BEHAVIOR OF FRAME- SHEARWALL STRUCTURE

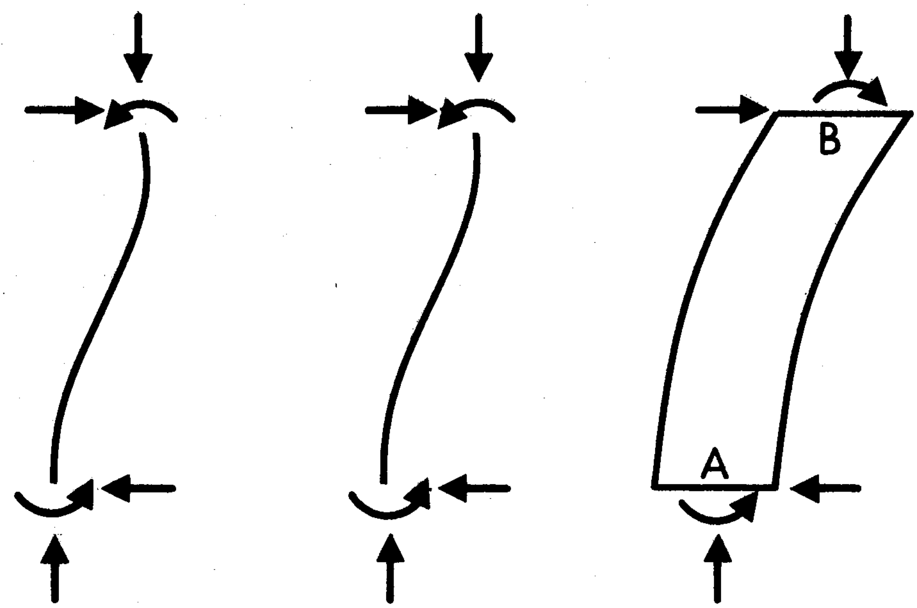


FIGURE 3.4 FORCES ACTING ON COLUMNS AND SHEAR WALL

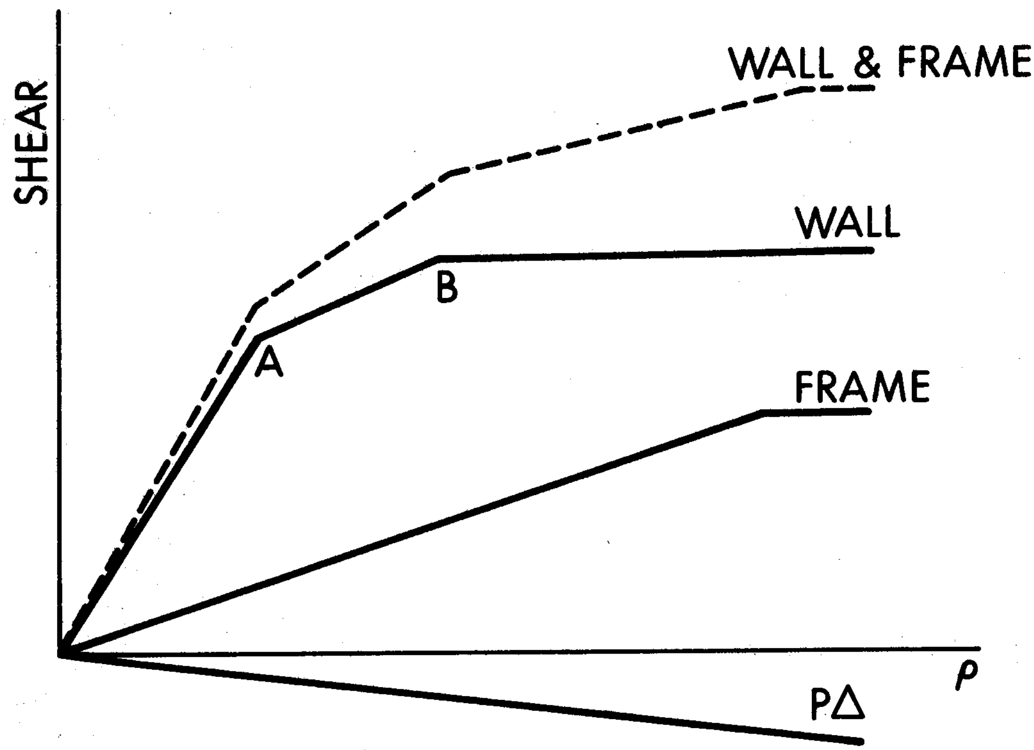


FIGURE 3.5 SHEAR-SWAY RESPONSE FRAME-SHEAR WALL

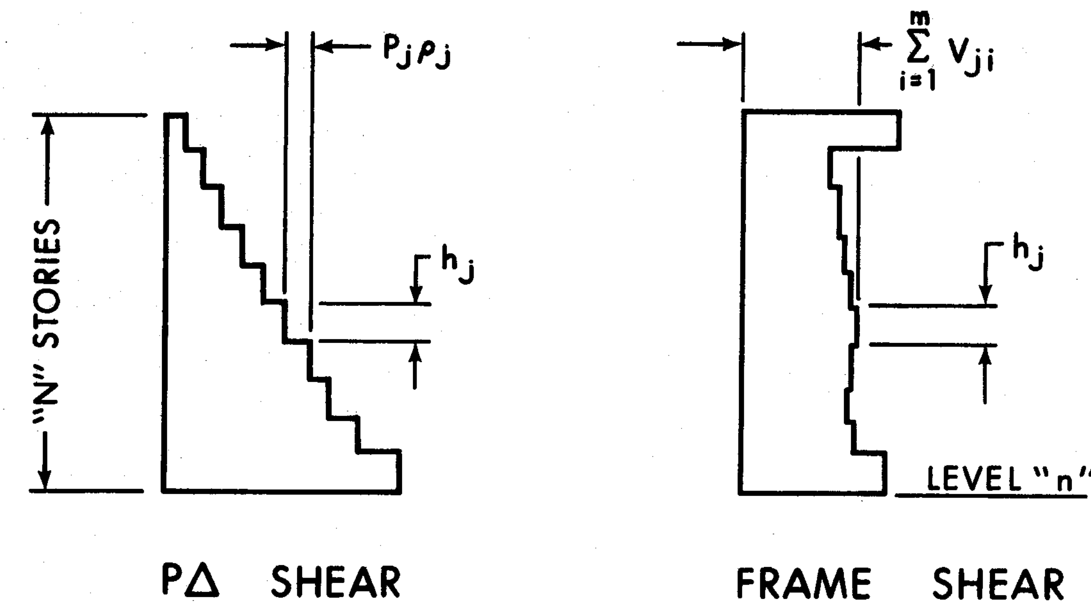


FIGURE 3.6 SHEAR DIAGRAMS

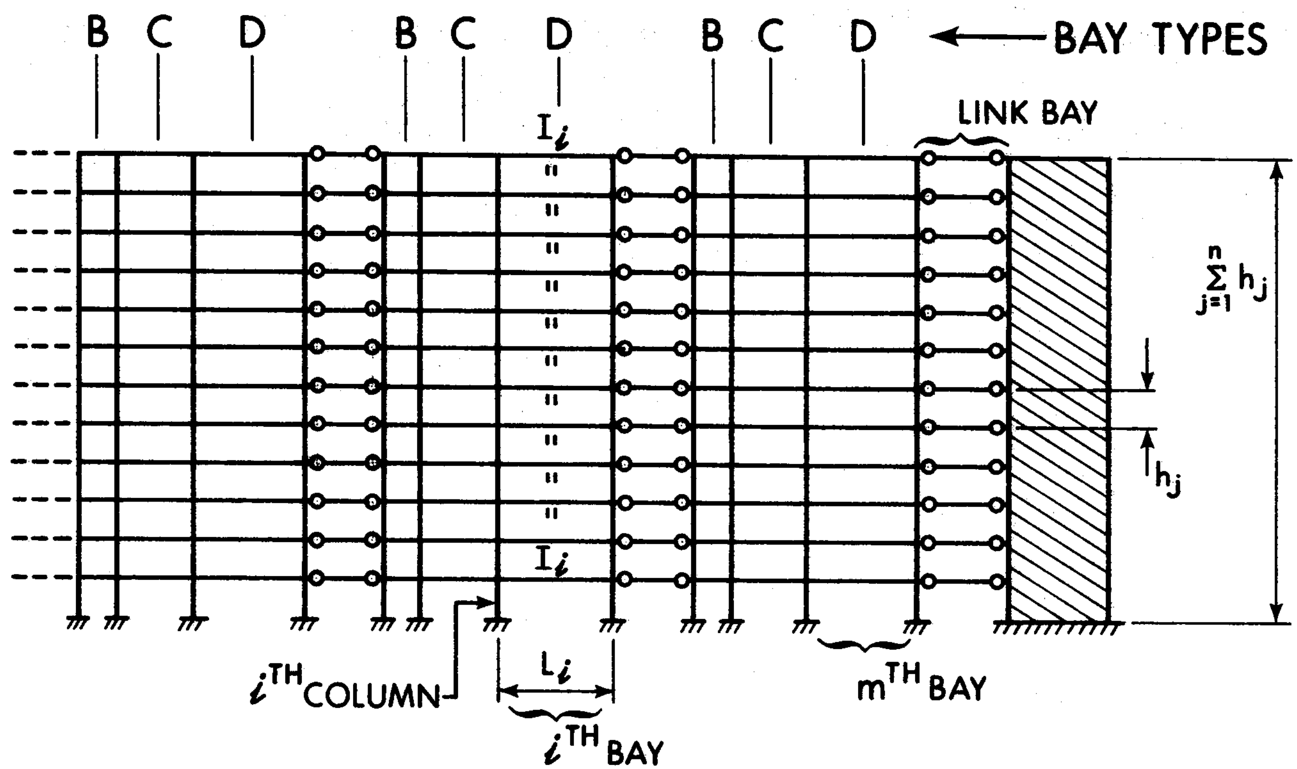


FIGURE 3.7 FRAME-SHEAR WALL STRUCTURE

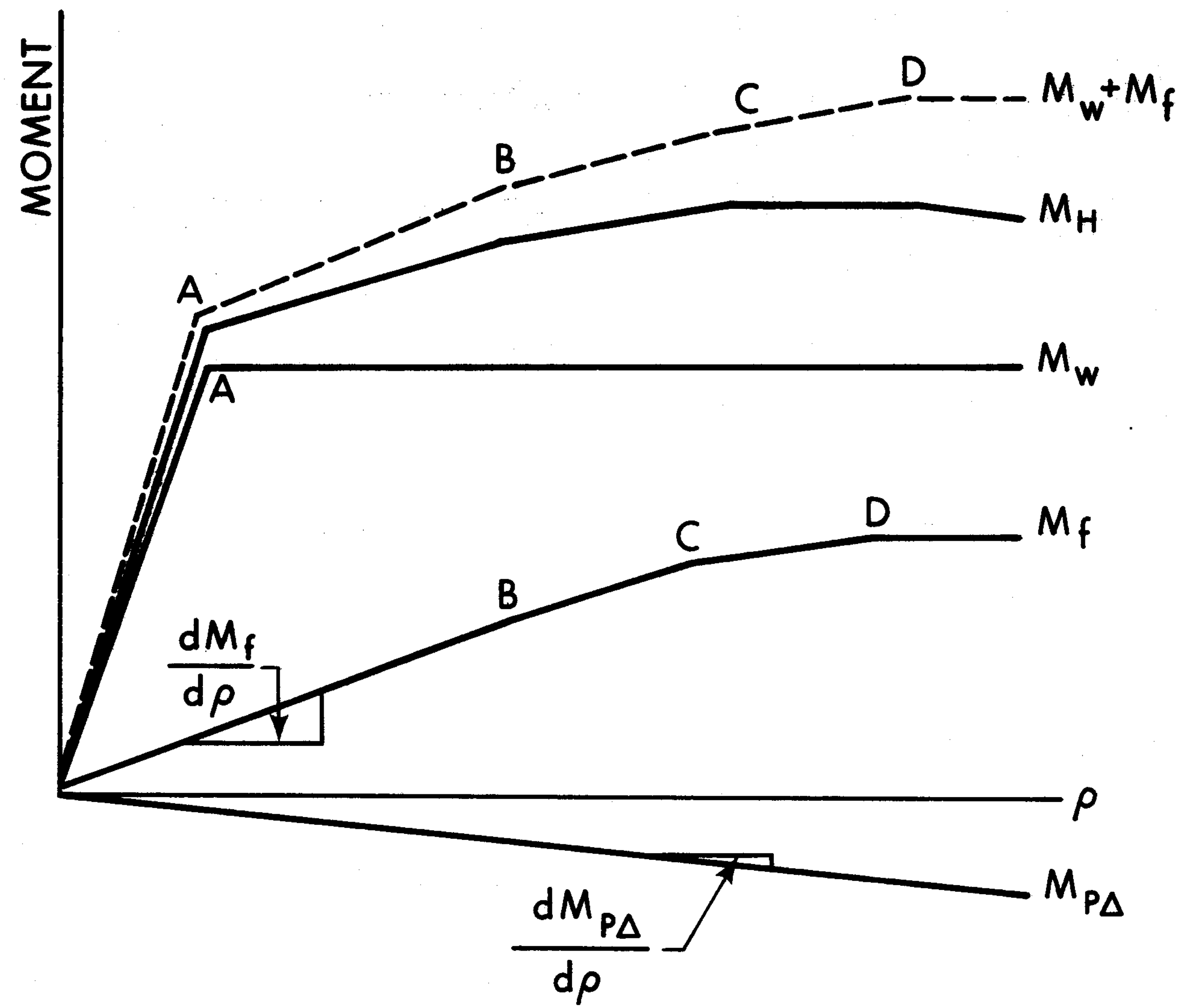


FIGURE 3.8 MOMENT DIAGRAMS

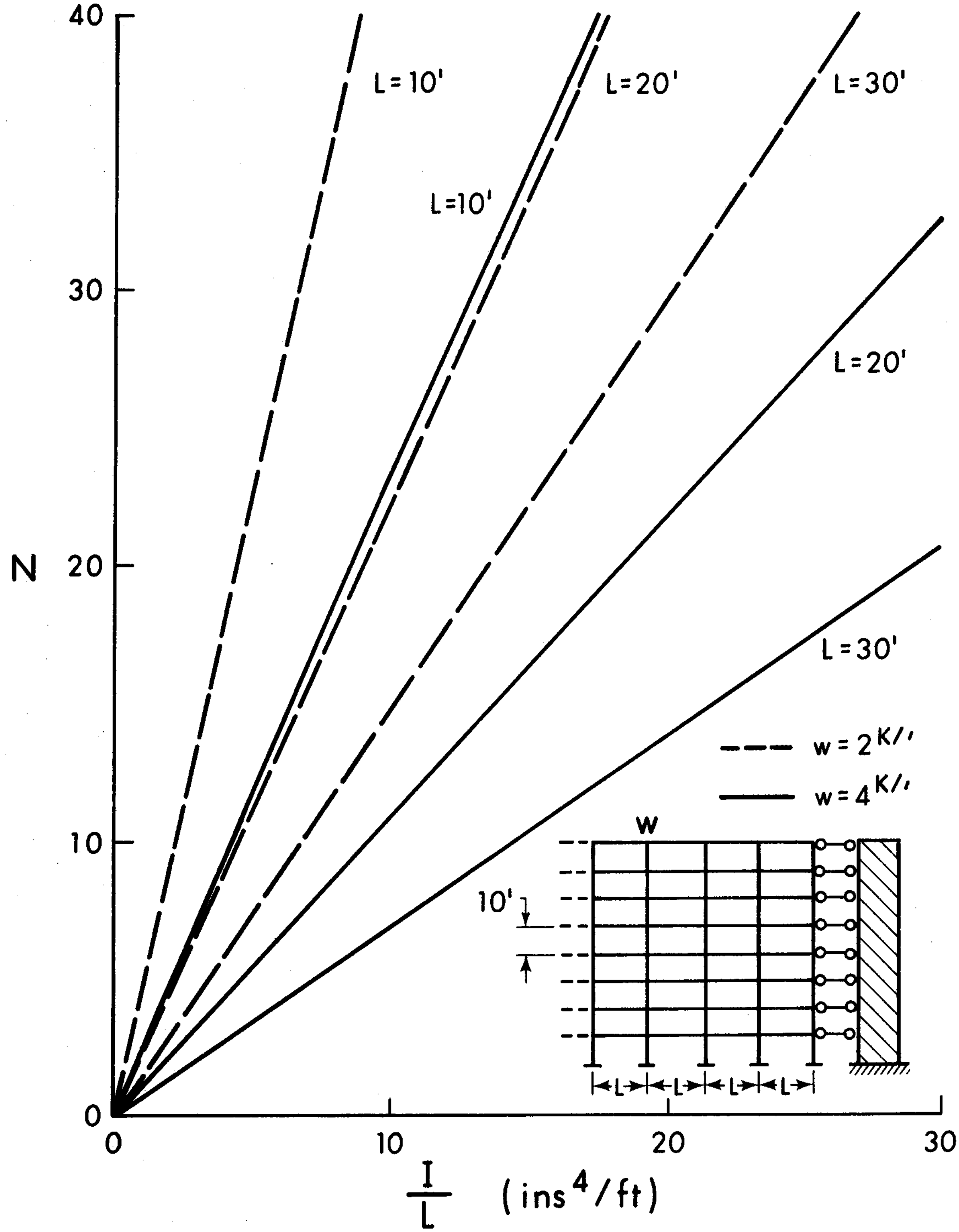


FIGURE 3.9 STABILITY CRITERION

CHAPTER IV

SINGLE STORY MODEL OF THE STEEL FRAME

4.1 Introduction

In this chapter an analytical model for one story of the frame portion of a steel frame-shear wall structure is developed. The model consists of a single column with its restraining members, thus, the response of each column in a given story can be obtained independently of the rest of the frame. The responses of all columns are then superimposed so that the complete frame response for the story is obtained. The process is repeated for each story to determine the response of the entire frame.

4.2 Selection of the Analytical Model

Figure 3.1(a) shows the plan view of a steel framed building. The framed portion has been designed plastically as a braced frame[(2,3)]. The building is idealized as a series of planar bents shown in Figure 3.1(b), and described in Section 3.2.

In Figure 3.1(b) a typical interior portion of one bay is enclosed by dashed lines. In Figure 4.1(a) this same portion is shown subjected to the factored gravity loads. (L.F. = 1.30 for the combined loading case.) Under this loading condition it is assumed that plastic hinges form at both ends of the girders[(2)]. As lateral loads are applied to the structure, the leeward girder hinge rotates plastically and the windward hinge reverses and behaves elastically[(2)].

This portion of the frame may be represented as shown in Figure 4.1(b). Here the enjoining structure, at the location of a leeward girder hinge, is replaced by a concentrated load equal to the girder vertical reaction and a moment equal to the plastic moment capacity of the hinged girder.

Points of inflection are assumed to occur at mid-height in the columns adjacent to the story under consideration. The resulting analytical model for the response of an individual column, isolated in Figure 4.1(c), is less sensitive than if the location of points of inflection were assumed in the story under question[2]. In Figure 4.1(c) points of inflection are located at h_1 and h_3 above and below the story in question, where the story height is denoted by h_2. If the story heights are equal then $h_1 = h_3 = h_2/2$. In this model it is assumed that the girders are designed for the same load and have the same moment of inertia, I. The moments of inertia of the columns are different and are denoted by I_1, I_2, and I_3.

In the analysis of the model it is assumed that the member response is elastic-perfectly plastic[1,2]. The plastic moment capacity of a girder is denoted by M_p and the plastic moment capacity of a column, M_{pc}, is reduced by the presence of axial load. It is further assumed that local and lateral torsional buckling do not influence the member response, however, the possibility of premature lateral torsional buckling is checked for the columns selected.

The influence of axial load on the stiffness and axial deformation of individual members is not considered in the model. The secondary story shears produced by gravity loads acting through their sway displacements (the $P\Delta$ effect) is included in the analysis of the entire structure by treating the $P\Delta$ shears as additional applied story shears[2].

An assumption peculiar to the model is that the sway displacements in adjacent stories are approximately equal. The validity of this assumption for the frame shearwall structure is tested in Chapter VI.

4.3 Initial Response of the Analytical Model

In determining the initial response of the model it is assumed that gravity loads, factored by 1.3, are first applied and remain constant as the structure is deformed by lateral loads. The model is analyzed under lateral loads only while maintaining the hinging effects of gravity loads. The relationship between the sway rotation, ρ, of the story under consideration and the resisting shear, V, developed by the column is derived in Appendix A as:

$$V = \frac{6EI_2}{h_2^2} \left\{ \frac{\frac{2I}{L}}{\gamma + \frac{I}{L} + \frac{2I_2}{h_2}} \right\} \rho \tag{4.1}$$

where:

$$\gamma = \frac{1}{2} \left\{ \frac{I_1}{h_1} + \frac{I_3}{h_3} \right\} \tag{4.2}$$

In equation 4.1, E denotes the modulus of elasticity and L the girder span. Equation 4.1 predicts a linear relationship between V and ρ that is valid until an additional plastic hinge forms in the model.

4.4 Ultimate Capacity of the Analytical Model

The capacity of the analytical model is reached when column BC (Figure 4.1(c)) can resist no additional shear. This occurs when the moments in the column at B and C cannot increase. The moment at B cannot increase if a plastic hinge forms at B in the column or a second hinge forms in the girder BE. The ultimate capacity of the model will be reached when one of these hinging patterns occurs at both ends of the column BC.

Figure 4.2 shows, schematically, the four possible mechanisms consistent with the assumed model behavior. If the failure hinge at one end of the column occurs in the column then it is probable that the failure hinge will occur in the column at the other end; since adjacent stories (having similar stiffness distributions and shears) will have similar force distributions. A similar conclusion is valid for the beam hinge pattern. Consequently, only two possible mechanisms are valid: those shown in Figure 4.2(a) and (d). The lower of the loads corresponding to each of these two mechanisms is the ultimate capacity of the analytical model.

The story shear corresponding to mechanism (a) is:

$$V = \frac{2M_{Pc}}{h_2} \tag{4.3}$$

The story shear corresponding to mechanism (d) depends on the location of the beam hinge and is derived in Appendix A as:

$$V = \frac{1.3\ wL^2}{h_2} \left\{ 0.578 \sqrt{\frac{w_g}{w}} - \frac{1}{2} \right\} \tag{4.4}$$

In equation 3.4, w is the design load for the girder and w_g is defined in equation 4.5 as:

$$w_g = \frac{16}{1.7} \frac{M_P}{L^2} \tag{4.5}$$

4.5 Single Story Response

The response of the analytical model is elastic perfectly plastic. The initial response is predicted by equation 4.1 and the ultimate capacity by equation 4.3 or 4.4. With the response curve determined for each column in a story, the response of the entire story is obtained by superimposing the response curves of the individual columns. The leeward column in a story is assumed not to develop a resisting shear.

This process is illustrated for the story slice shown in Figure 4.3. The story slice has been taken from a plastically designed braced frame[2]. The girders are subject to a uniformly distributed factored load of 1.9 Kip/ft. (L.F. = 1.3). The member properties of the slice are given in Table 4.1. With constant story height, then $h_1 = h_3 = 0.5\ h_2$. The response of each column, predicted by the analytical model, is summarized in Table 4.2 and shown by the dashed curves in Figure 4.4. In this figure the column responses are superimposed to give the entire story response indicated by the solid curve.

The response of the analytical model adjacent to the shear wall, Figure 3.1(b), is influenced by the finite width of the wall. The rotation of the wall in each story causes premature hinging in the girders framing into the wall, and an apparent increase in stiffness of the model. This effect is ignored in the present model[15].

The response of the model representing a slice from a typical story, is not suitable for either the top or bottom story because of the special boundary conditions in these stories. A model for the response of the top story is derived in Appendix A. The response of the bottom story is assumed to be the same as the response of a typical story. This assumption is valid because the response of the bottom story of the frame portion has little effect on the behavior of the entire frame-shear wall structure (Appendix B, Equation B.3).

4.6 Discussion of Model Response

The response of the story slice shown in Figure 4.3 has been analyzed by the Lehigh Subassemblage method[4] and by the rigorous elastic plastic analysis presented in Reference (10). The comparisons are shown in Figure 4.5. The solid, dashed and dotted lines represent the predictions of the analytical model, the rigorous analysis, and the subassemblage method respectively. The responses of the story, predicted by all three analyses are very similar; the analytical model being the most conservative.

The insert to Figure 4.5 shows the hinging pattern in the story as predicted by the analytical model and the rigorous analysis. The numbers indicate the story shear corresponding to the formation of the plastic hinge. The numbers in brackets represent the prediction of the analytical model.

The ability of the analytical model to represent significant changes in the moment of inertias of the columns has been investigated. In the story slice of Figure 4.3 the moment of inertias of columns 2-3 were maintained as in Table 4.1. However, the upper column segments 3-4 and the lower column segments 1-2, had moments of inertias reduced and increased, respectively, by 20% of those of columns 2-3. A rigorous analysis of this slice resulted in the dashed curve of Figure 4.6. This curve corresponds closely to the results obtained from the analytical model indicated by the solid curve.

Comparison of the dashed curves of Figures 4.6 and 4.5 implies that the story response is insensitive to the moments of inertias of the columns. If it is assumed that the frame is regular so that in equation 4.1:

$$\gamma = \frac{2I_2}{h_2} \tag{4.6}$$

then

$$V = \frac{6E}{h_2} \frac{1}{\frac{2L}{I} + \frac{h_2}{2I_2}} \rho \tag{4.7}$$

Equation 4.7 shows that the influence of the column moment of inertia on the initial response of the model is small. Thus, if the frame is regular, the term h_2/I_2 may be averaged over a number of stories and the model will produce a reasonable estimate of the response in each story.

The sensitivity of the model to the location of column inflection points was studied by changing h_1 and h_3 (Figure 4.4) by 40% of their assumed values of 0.5 h_2. The effect of these changes is shown in Figure 4.7. It is apparent that the analytical model is insensitive to the location of column inflection points.

A peculiar assumption of the analytical model is that plastic hinges form at both ends of the girders during the initial application of gravity loads. The effect of a violation of this assumption has been studied by increasing the plastic moment capacities of the girders, in Figure 4.3 by 20%. Figure 4.8 shows the comparison between the rigorous analysis and the analytical model. The model predicts the ultimate capacity of the story, however, the initial stiffness is underestimated by the model. This results because the girders exert a rotational restraint on the columns at their leeward ends until plastic hinges develop at the leeward ends of the girders.

4.7 Summary

In this chapter an analytical model for the response of the steel frame in a frame-shear wall structure has been developed. In the model it is assumed that the member response is elastic perfectly plastic. In the following chapter the application of this model to the design of steel frame-shear wall structures will be developed.

MODULUS OF ELASTICITY $E = 29{,}000$ KSI				
YIELD STRESS $\sigma_y = 40$ KSI				
MEMBERS	M_p (K-FT)	I (INS^4)	r_x (INS)	A (INS^2)
Girders AB	20	15	-	-
BC	80	147	-	-
CD	180	280	-	-
Columns A	30	20	2.1	5.0
B	100	200	5.4	5.0
C	200	300	5.2	10.6
D	200	300	5.2	10.6

TABLE 4.1

PROPERTIES OF STORY SLICE

COLUMN	V(KIPS) (EQN. 4.1)	V(KIPS) (EQN. 4.4)	ULTIMATE SWAY ρ
A	76.3 ρ	1.87	0.0245
B	406 ρ	7.5	0.0185
C	523 ρ	16.9	0.0323
D	0	0	0

TABLE 4.2

RESPONSE OF COLUMNS OF STORY SLICE

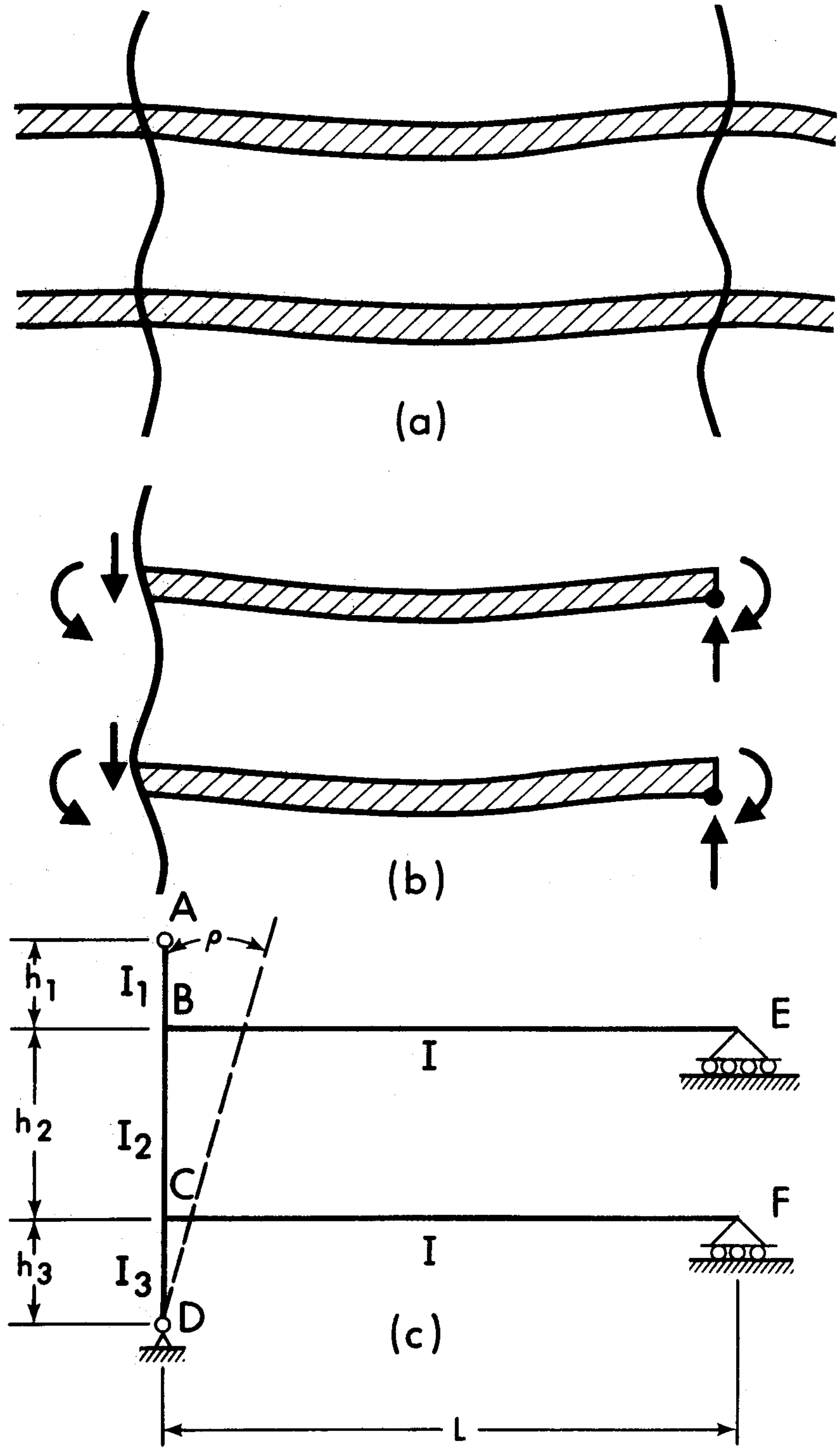


FIGURE 4.1 DEVELOPMENT OF ANALYTICAL MODEL

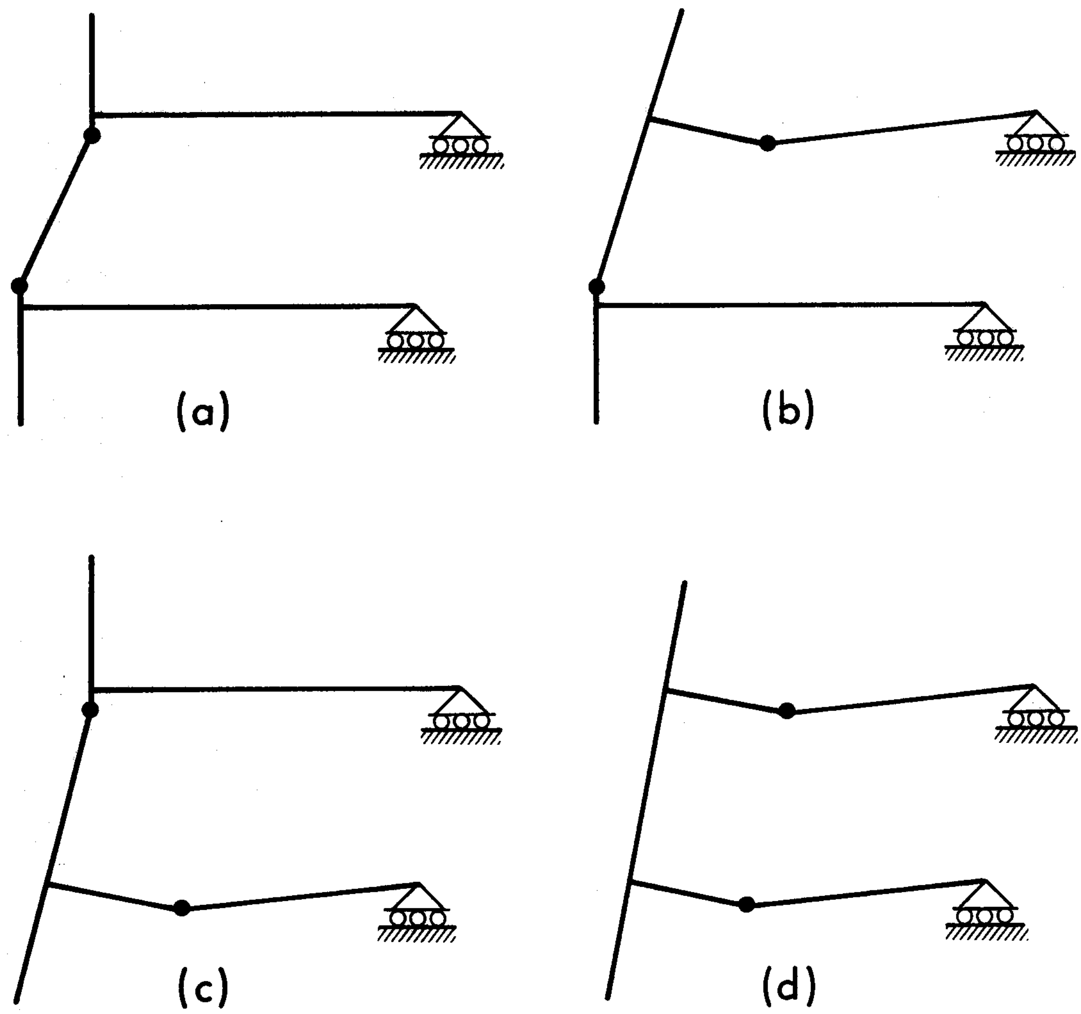


FIGURE 4.2 FAILURE MECHANISMS FOR ANALYTICAL MODEL

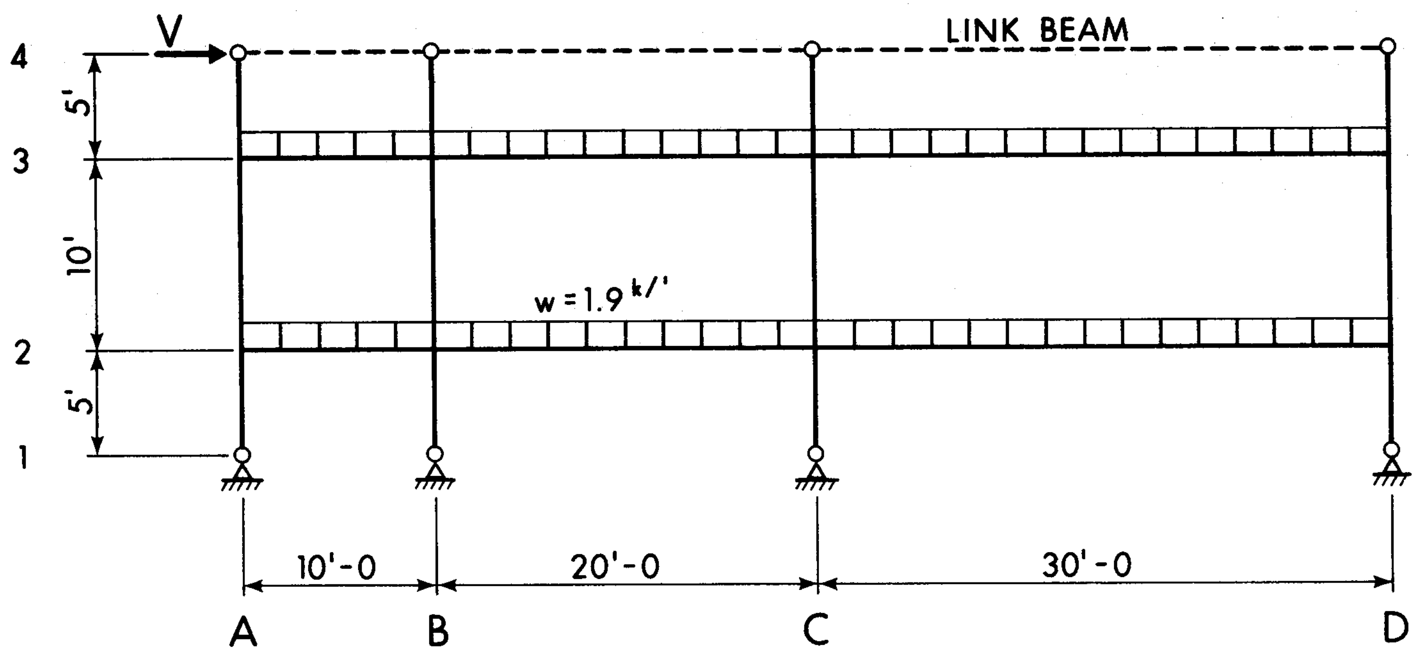


FIGURE 4.3 STORY SLICE OF STEEL FRAME

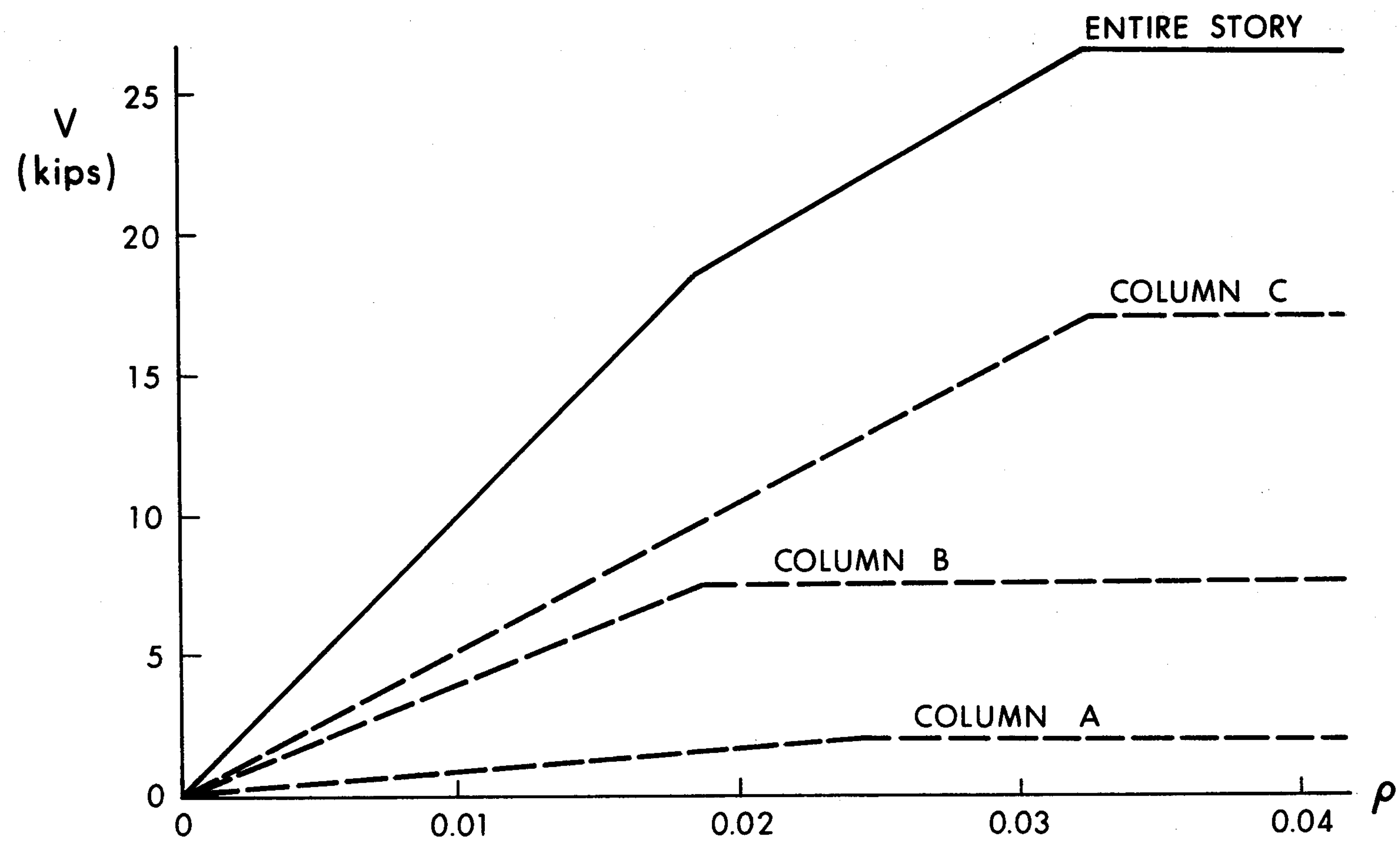


FIGURE 4.4 RESISTANCE OF A STORY SLICE

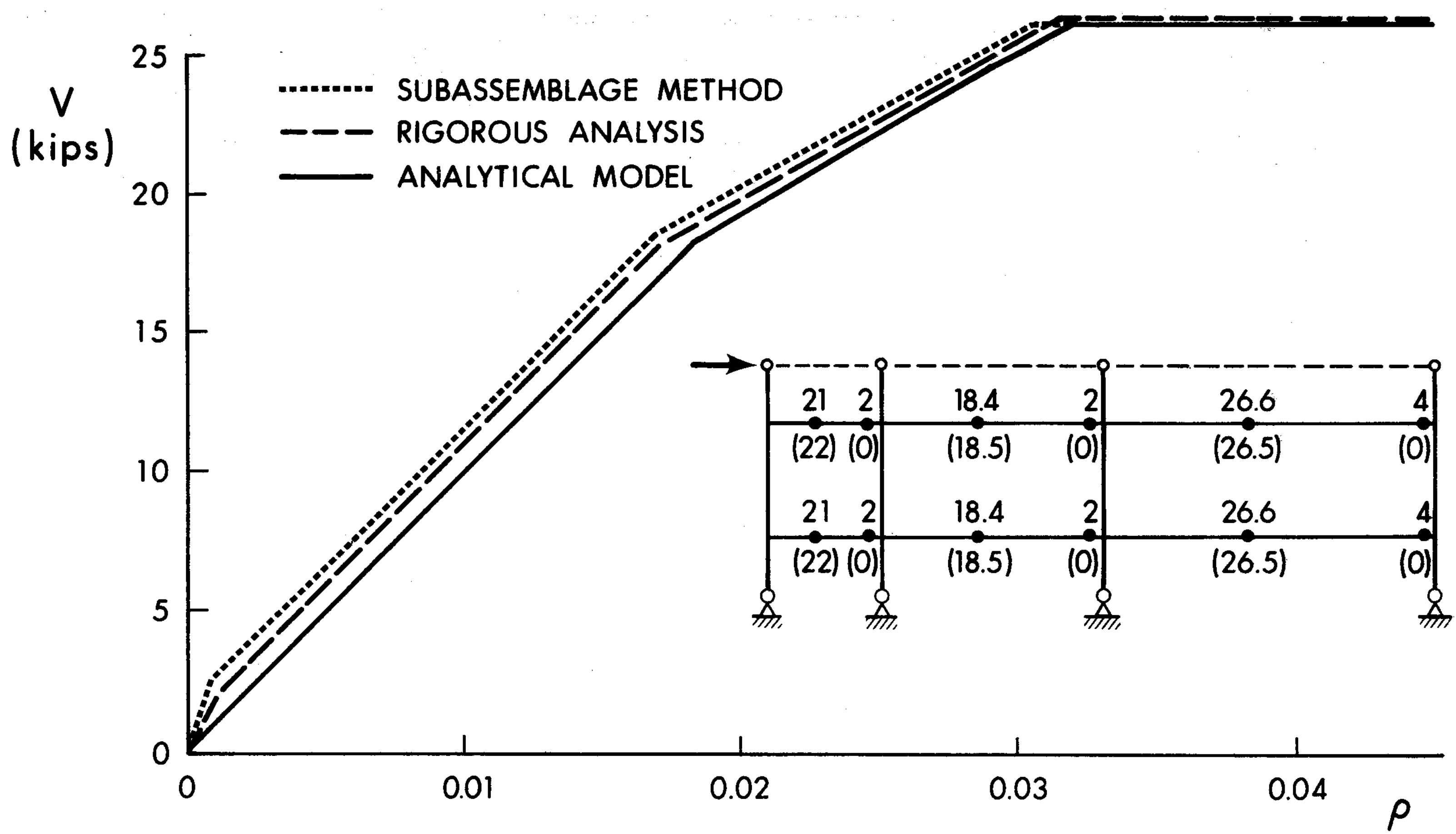


FIGURE 4.5 COMPARISON OF ANALYTICAL MODEL WITH SUBASSEMBLAGE METHOD AND RIGOROUS ANALYSIS

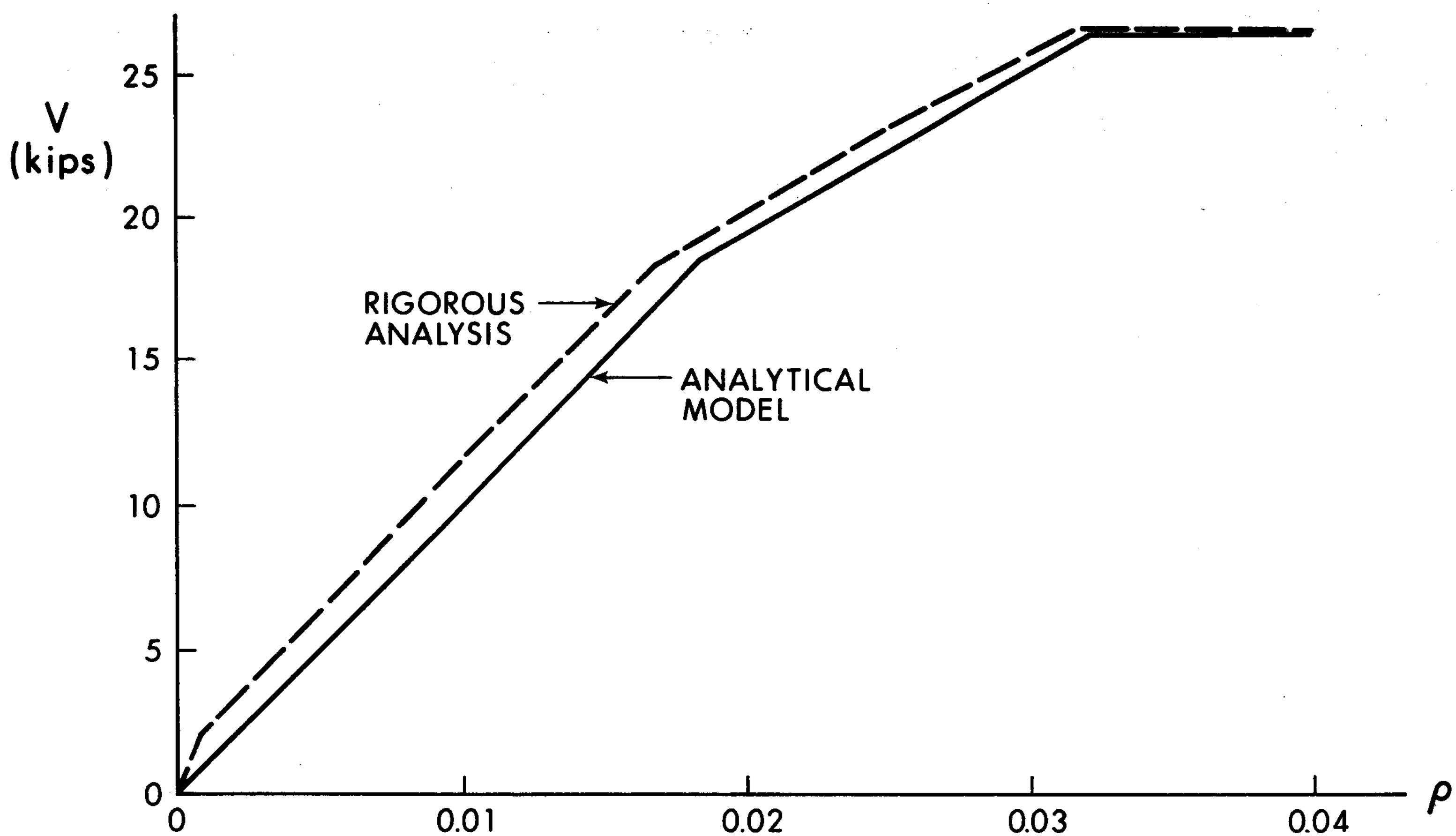


FIGURE 4.6 EFFECT OF COLUMN STIFFNESS ON RESPONSE

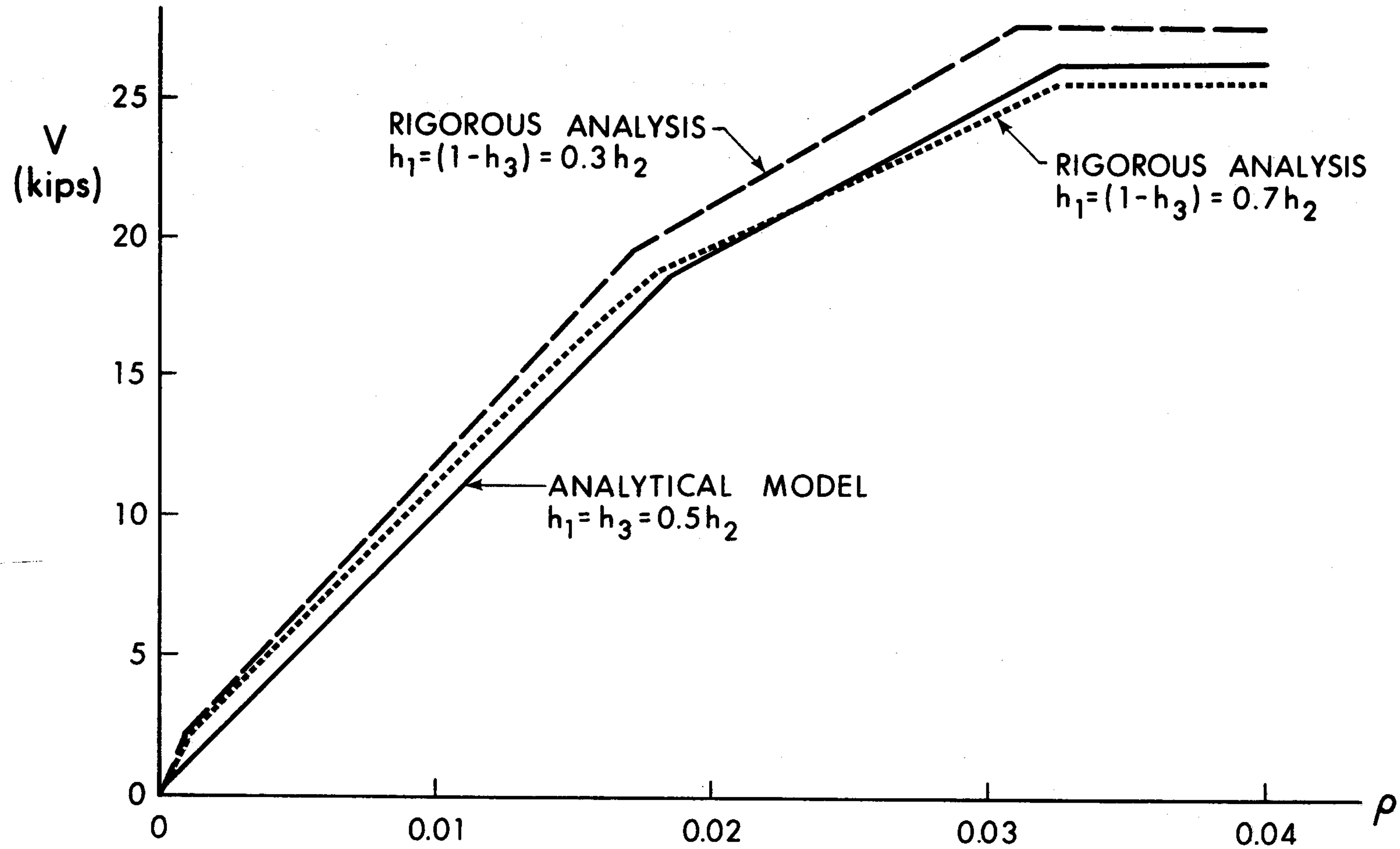


FIGURE 4.7 EFFECT OF LOCATION OF POINTS OF INFLECTION ON RESPONSE

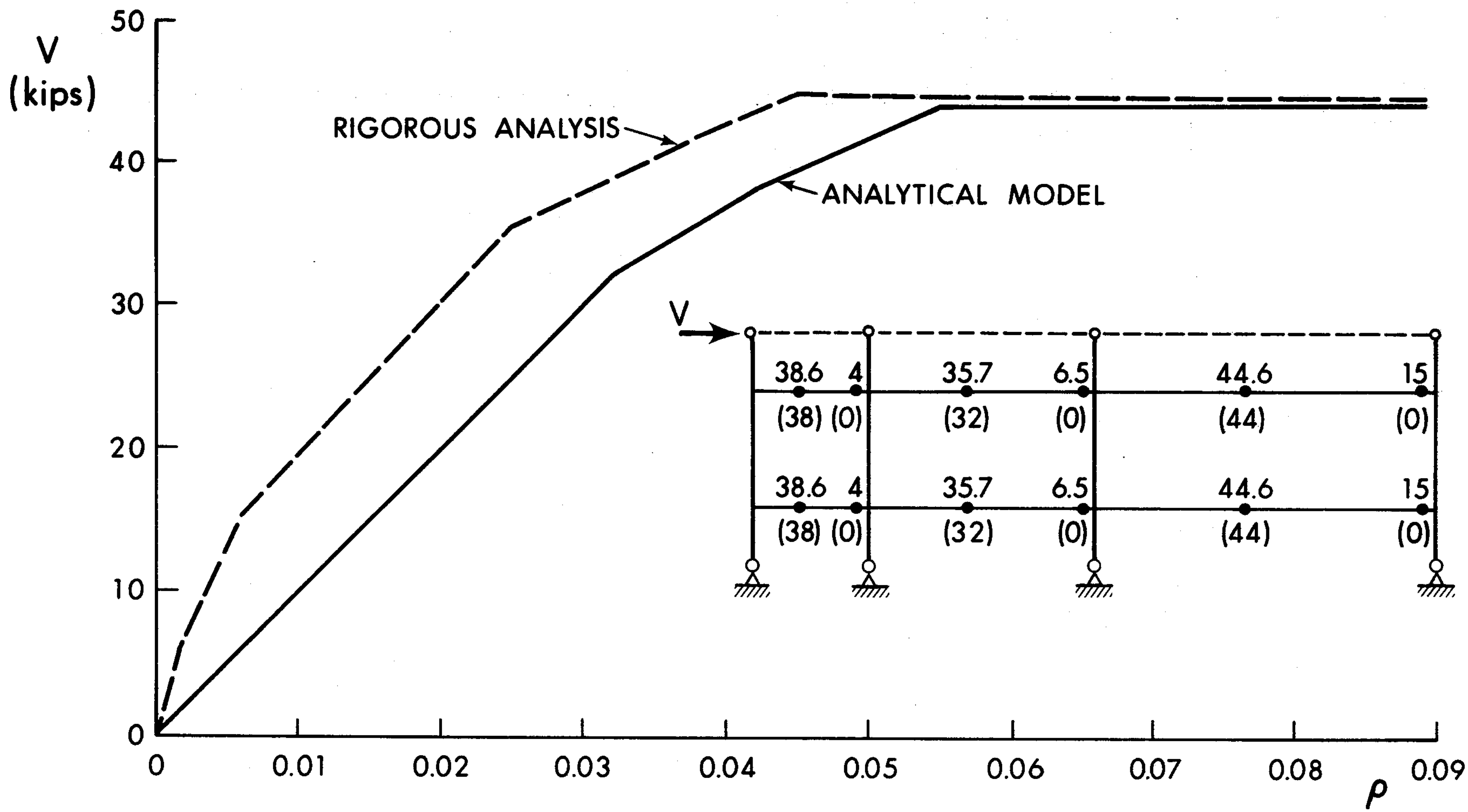


FIGURE 4.8 EFFECT OF INCREASING GIRDER M_p'S BY 20%

CHAPTER V

PLASTIC DESIGN METHOD FOR STEEL FRAME-SHEAR WALL STRUCTURES

5.1 Introduction

In this chapter a plastic design method for steel frame-shear wall structures is presented. The method is based on an analytical model for the complete frame shear wall structure, described in Section 5.2; and accounts for the formation of plastic hinges in the structure and the second order (PΔ) effects.

It is assumed that the frame portion of the structure (Figure 3.1(b)), has been designed plastically as a braced frame. It is also assumed that the dimensions of the shear wall are determined by functional or architectural requirements so that the stiffness of the wall is known. The aim of this chapter, therefore, is to determine the minimum strength required by the wall so that the entire structure has the capacity to resist the factored design loads. A check is made to ensure that the wall has adequate stiffness to prevent overall buckling of the structure under gravity loads and to control working load deflections within specified limits.

5.2 Analytical Model for the Frame-Shear Wall Structure

The behavior of the frame portion of a steel frame-shear wall structure alone has been approximated in Chapter IV by isolating a two story slice. A comparable slice from the frame-shear wall structure is shown in Figure 5.1. The columns have been isolated at their points of inflection and the restraints at the ends of the wall segment are represented by rotational springs. The action of the rotational springs, however, can only be determined by a second order, elastic-plastic analysis of the entire structure; therefore, the slicing technique is unsuited for the analysis of a combined frame-wall structure.

The analytical model selected is shown in Figure 5.2. The shear-wall is treated as a cantilever column; restrained at each floor level by translational and rotational springs, which represent the action of the frame; and restrained at the base by a rotational spring which represents the action of the foundation. The response of the translational spring in a particular story is that obtained for the steel frame portion, developed in Chapter IV. The rotational restraint in each story is due to the action of the beams framing directly into the wall. The response of a typical rotational spring is derived in Appendix C. The rotational restraint of the foundation is assumed to be linearly elastic.

Lateral loads, H_j, are applied as concentrated loads at each floor level. To evaluate the PΔ effect, concentrated loads, P_j, equal to the total gravity load on each floor, are assigned to the analytical model at each floor level. It is assumed that gravity loads are applied to the structure, initially; then held constant while the lateral loads are incremented, until the ultimate capacity of the structure is reached.

5.3 The Proposed Plastic Design Method

5.3.1 Assumed Failure Sequence

The design method assumes that the sequence of failure of each story is as follows: first, a plastic hinge develops in the shear wall so that the shear wall above the story rotates under the restraint of the frame portion, alone. If the structure is stable once the wall hinges, it is assumed that the wall rotates until the usable portion of the frame capacity is developed and the ultimate capacity of the structure is reached. It is assumed that the deflected shape of the structure, above the story in question, is rectilinear in the ultimate state. The effect of treating the above failure sequence in every story, may be obtained by assuming that plastic hinges occur in the wall in every story, and that the deflected shape of the entire structure is rectilinear in the ultimate state.

5.3.2 Steps in the Design Procedure

The proposed plastic design method (Appendix D) consists of the following steps:

1. Determine whether the structure becomes unstable once the wall develops a plastic hinge. The criterion described in Chapter 3 (equation 3.6) may be used. If the structure is unstable go to step 6. If not, continue with steps 2 through 5.

2. Model the structure according to section 5.2 and Figure 5.2. Calculate the response of the translational and rotational springs of Figure 5.2.

3. Determine the stage at which the structure becomes unstable. Select a sway rotation, ρ_f, for the entire structure, at which the structure will be on the verge of instability.

4. Rotate the entire structure through a sway rotation, ρ_f. Assuming a rectilinear deflected shape, calculate the shear resisted by the frame portion, and the $P\Delta$ shear. Calculate the net shear in the wall by subtracting the frame shear from the applied shear and the $P\Delta$ shear.

5. Calculate the moments, shears and axial loads which the wall must resist in each story, and proportion the wall accordingly.

6. If the structure becomes unstable once the wall yields;

the shear wall should be proportioned for strength by a second order elastic-plastic analysis of the structure, in which the wall remains elastic. A computer program to perform such an analysis is presented in Appendix E. The program analyzes the model shown in Figure 5.2; and is of interest because the important subroutines, FRAME and WALL, are small enough to be programmed in a small office computer[28].

5.4 Secondary Design Considerations

5.4.1 Stability Under Gravity Loads Alone

Under full gravity loads, alone, (L.F. = 1.7) it is assumed that beam mechanisms have formed in all girders of the frame. Therefore, the shear wall alone must provide lateral resistance to the $P\Delta$ effect.

The wall must therefore be designed by performing a second order analysis and proportioning the wall to resist the corresponding forces. For perfectly symmetrical structures, initial imperfections must be assumed in order to determine design forces for the wall (section 2.3). The computer program presented in Appendix E performs the analysis for any non-prismatic wall with arbitrary loading.

5.4.2 Serviceability Requirements

At working loads (L.F. = 1.0) serviceability requires that the story sway rotation be less than a specified index, usually 0.002[2]. Although plastic hinging in the frame may be acceptable, provided beam deflections are tolerable[2], it is assumed in this thesis that no plastic hinges should form in the shear wall at working loads. The minimum wall strength and stiffness thus required may be determined by the analysis suggested in step 6 of section 5.3.2. Since the entire structure will probably be elastic at working loads, the initial response of the translational springs of Figure 5.2 should be increased by a factor of four for interior columns and two for exterior columns in the above analysis to account for the fact that interior columns are restrained by two girders and exterior columns, by one girder which are twice as stiff as the girders under combined loads (L.F. = 1.3).

5.4.3 Flexible Shear Wall

In Section 5.3.1 it was assumed that the shear wall develops a plastic hinge before the frame portion develops mechanisms, corresponding to the attainment of the ultimate capacity. If the serviceability requirements of Section 5.4.2 are met then this assumption is most likely valid. The exception occurs if the wall is much more flexible than the frame. In such cases the structure may be unable to develop the expected ultimate capacity, since the entire PΔ effect at the ultimate load is not accounted for. The PΔ effect assumed in design is based on the rectilinear deflected shape of the structure corresponding to the development of the usable capacity of the frame. However, if the wall is still elastic at this stage, then the extra deformation required to develop a hinge in the wall represents the PΔ effect not accounted for in the design of the wall.

To check for such a situation, the following procedure is suggested:

Assume that the frame portion has developed its usable capacity corresponding to a sway rotation, ρ_f, in every story. Calculate the net lateral forces on the wall (applied lateral forces minus the frame resistance) and analyze the wall with the computer program in Appendix E. If the sway rotation of the wall in each story is less than ρ_f; then the entire PΔ effect at the ultimate state has been satisfactorily accounted for. If not, the wall should be redesigned for the moments and shears given by the computer program.

5.4.4 Rotational Capacity of the Steel Frame

The proposed plastic design method assumes that the steel frame is capable of resisting its share of the ultimate load. However, under combined loads (L.F. = 1.3), the axial loads in the leeward columns of a bent may be greater than those designed for under gravity loads, alone (L.F. = 1.7). The rotational capacities of the leeward columns may be terminated by lateral torsional buckling, before the ultimate state of the structure is reached. Studies by the writer have shown that the leeward columns of a bent carry a higher axial load under combined loads than designed for under gravity loads alone if the girders are eight percent stronger or more than required to support a beam mechanism (L.F. = 1.7). Thus, the possibility of lateral torsional buckling must be checked if the girders are so overdesigned.

5.4.5 Inelastic Rotation of the Shear Wall

It has been assumed in Section 5.3.2 that the wall can supply the inelastic rotations necessary to develop the resisting shear in the frame, corresponding to a sway rotation, ρ_f, in each story above the story of failure. The maximum ρ_f to be expected is in the order of 0.02 radians (Appendix D). This figure represents a very conservative upper bound to the inelastic rotational capacity required in the wall, since a significant portion of this rotation, in each story, is supplied by rotation of the wall foundation, elastic rotation of the wall, and inelastic rotation of stories below the portion of the wall considered.

At the University of Alberta, tests[24] of reinforced concrete walls with proportions, reinforcing and moment-shear-thrust ratios typical of practical shear walls, obtained inelastic rotations in the order of 0.03 radians. Other investigations have likewise confirmed such ductility of reinforced concrete members[21,22,23]. So it would appear that the ductility requirements of the design method will be satisfied.

5.5 Summary

A plastic design method for steel frame-shear wall structures has been presented. The method utilizes plastic redistribution between the wall and the frame and accounts for the $P\Delta$ effect in proportioning the shear wall. Examples illustrating the design steps and the secondary design considerations are presented in Appendix D.

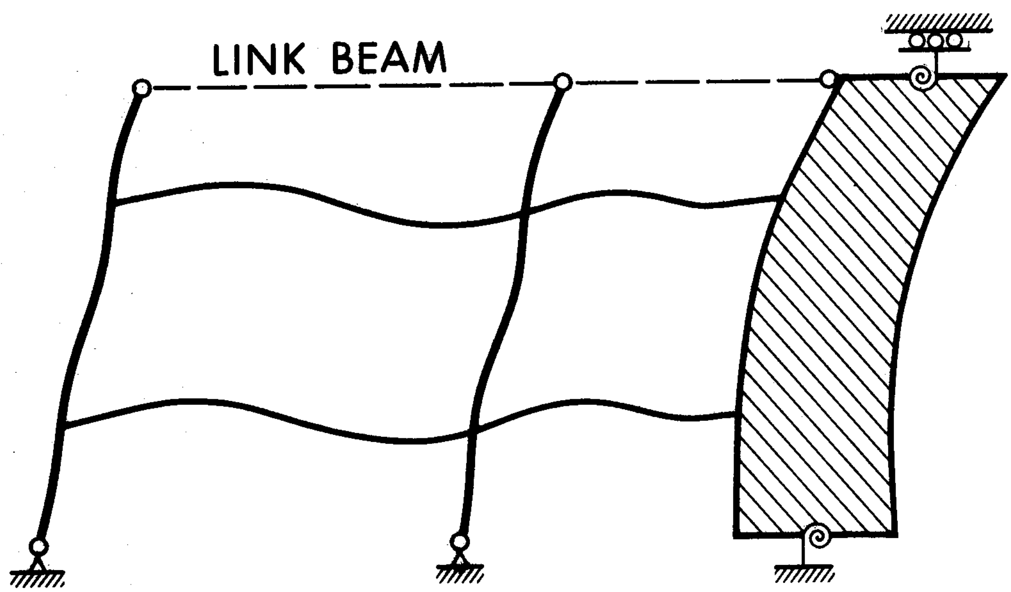


FIGURE 5.1 SLICE OF A FRAME-SHEAR WALL STRUCTURE

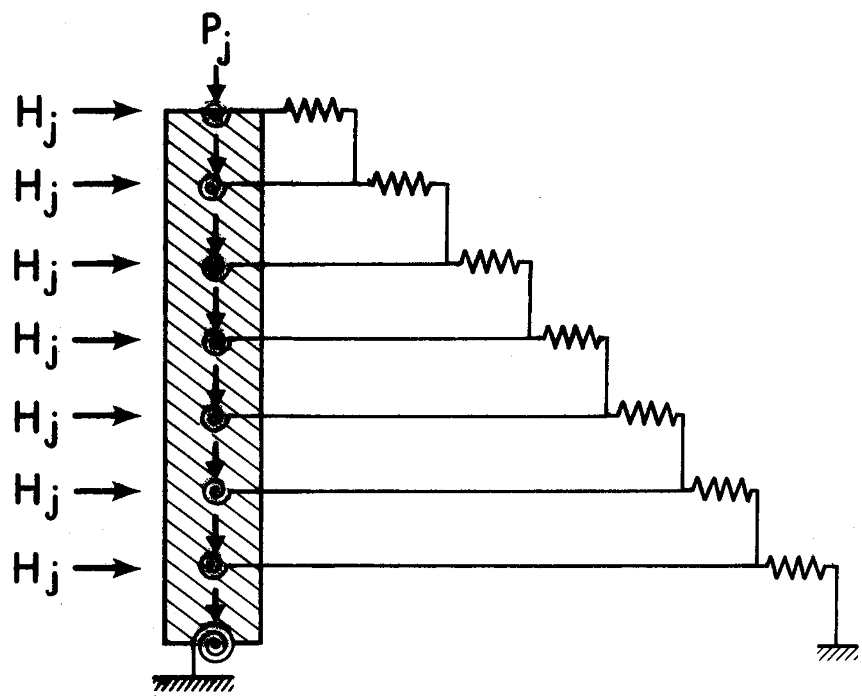


FIGURE 5.2 ANALYTICAL MODEL FOR THE FRAME-SHEAR WALL STRUCTURE

CHAPTER VI

VERIFICATION OF THE DESIGN METHOD

6.1 Introduction

In this chapter the adequacy of the design method presented in Chapter V is tested. Two steel frame-shear wall structures, one twelve story and one eighteen story, were designed and then analyzed by the rigorous second order elastic-plastic analysis described in reference 10. The structures were selected to illustrate designs using different degrees of plastic redistribution between the wall and the frame. The twelve story structure was designed (Appendix D) to utilize complete plastic redistribution, whereas the 18 story structure could efficiently use only partial redistribution. The behavior of each structure is compared with the behavior assumed in the design method.

6.2 The Design Examples

The plan view typical of both example structures is shown in Figure 6.1. The structures consist of seven parallel steel bents. Bent A-A incorporates a reinforced concrete core. The design loads (L.F. = 1.0) are 100 P.S.F. vertical load on each floor (including the roof), and 20 P.S.F. lateral load. The modulus of elasticity of steel and concrete are assumed to be 30,000 K.S.I. and 3,000 K.S.I., respectively. The yield stress of the steel is assumed to be 36 K.S.I.

Figures 6.2 and 6.3 show the idealization of the building of Figure 6.1, for the twelve story and the eighteen story structure, respectively. The bent A-A, containing the shear wall, is linked to a lumped bent[15] which represents the action of the remaining bents of the structure. The frame portion of each structure is designed plastically as a braced structure (no live load reduction). The members used for the columns are shown in Figures 6.2 and 6.3, and those used for the girders are given in Table 6.1, except that the moments of inertia, plastic section moduli and areas of the members used for the lumped bents are five times the values for the members indicated for these bents. The assumed dimensions of the reinforced concrete core are shown in Figure 6.1. The moment of inertia of the section was reduced by a factor of 2.5[11] to give an effective moment of inertia of 350 ft^4.

The numbering scheme used for floor levels and column stacks is indicated in Figure 6.2. A girder, designated as girder j,i, has its left end framing into column stack i at floor level j. Column j,i, has its lower end framing into floor level j in column stack i. Story j is the story between floor levels j and j+1; similarly, bay i is the bay between column stacks i and i+1.

6.3 The Twelve Story Structure

The shear wall in the twelve story structure shown in Figure 6.2 has been designed in Appendix D. Full plastic redistribution of forces between the wall and the frame was utilized and the wall was proportioned to deliver the moment capacities listed in column eight of Table D.3; the serviceability requirements of Section 5.4.2 were ignored. The structure was analyzed by a second order elastic-plastic analysis[10]; gravity loads (L.F. = 1.3) were applied initially and held constant, then lateral loads increased until the structure reached its ultimate capacity.

Figure 6.4 plots the load factor on lateral loads, λ, versus the roof sway, for this structure. The numbers on the curve represent load stages. Figure 6.5 shows the hinging pattern in the structure. The solid circles represent locations of plastic hinges. The numbers above each hinge indicate the load stage at which the hinge formed. The numbers are keyed to Figure 6.4. Hinges without numbers formed between load stages one and two. The open circles represent hinges which formed and subsequently reversed. Figure 6.6 shows the deflected shape of the structure at each load stage. The dashed line in this figure represents the assumed deflected shape at the ultimate state.

The shear wall develops plastic hinges in stories 7 and 8 at an early stage of loading. Beyond this stage the stiffness of the structure gradually deteriorates as the wall and the frame plastify. The structure above stories 7 and 8 deflects rectilinearly as assumed in Section 5.3.1. At load stage 10 there is a marked decrease in stiffness as most of the stories of bay 1 develop mechanisms. Between load stage 10 and 11 the frame portion is just able to resist the $P\Delta$ effect so the load-deflection curve of Figure 6.4 is almost horizontal. At load stage 11, corresponding to $\lambda = 1.42$, the remaining bays of the frame have developed mechanisms and the load capacity is reduced. The sway rotation of the top eight stories corresponds closely to the assumed sway rotation at the ultimate load. The maximum inelastic rotation of the wall was 0.0045, well within the capacity of practical reinforced concrete sections[24].

The analytical model for the steel frame has been shown, in Chapter IV, to adequately predict the response of a two story slice, assuming that the sway rotations in two successive stories are equal. The ability of the model to predict the response of each story of the steel frame in an actual frame-shear wall structure will be illustrated by comparing the assumed and actual resisting shears developed.

Figures 6.7 to 6.10 plot the total resisting shears developed by columns 1, 2, 3 and 4, against the story sway rotations, for stories 1, 5, 7 and 11, respectively. The solid curves indicate the responses predicted by the second order elastic-plastic analysis. The dashed curves indicate the response predicted by the analytical model of the steel frame developed in Chapter IV. Figures 6.7 and 6.8 show the typically good agreement for the top six stories. Figure 6.10 shows the typical agreement for the bottom four stories. For stories 7 and 8 the agreement was poor. Figure 6.9 shows the poorest agreement. For stories 7 and 8 the assumption of equal sway rotations in successive stories is not realized (Figure 6.6). The broken curve shown in Figure 6.9 indicates the response when the effect of different story sways is included in the analytical model (Appendix A). The poor agreement is adequately explained by this effect. Neglect of this factor in the design procedure is conservative and has little effect on the overall stability of the structure.

The restraining effect of the girders framing directly into the wall was predicted accurately by the model presented in Appendix C. Figures 6.11 and 6.12 show the response of the windward girder and the leeward girder, respectively, for story 5. The agreement between the model and the rigorous analysis, shown in these figures, was typical.

The twelve story structure was analyzed with the shear wall in its final design form. The secondary design considerations of stability under vertical loads alone (L.F. = 1.7) and serviceability requirements were included. The serviceability requirements selected were that under working loads no plastic hinging should occur in the wall, and that the sway rotation in each story should not exceed 0.002. The meet these requirements the effective moment of inertia of the shear wall was increased to 700 ft^4, and the moment capacities of the wall in stories 7 and 8 were increased by 130% and 20%, respectively. The final design moments for the wall are given in column (11) of Table D.3.

Figure 6.13 shows that the serviceability requirements were met by the design method. The solid curve represents the sway rotation of each story under working loads. The dashed line represents the deflection index of 0.002. The insert to this figure shows the plastic hinges which formed in the structure under working loads.

Figures 6.14, 6.15 and 6.16 show the plot of load factor on lateral loads, λ, versus the roof sway; the hinging pattern in the structure, and the deflected shape of the structure at each load stage, respectively. The shear wall develops the first plastic hinge between load stages 3 and 4. At this point there is a significant decrease in stiffness. The stiffness further deteriorates as the frame and the wall plastify, until the structure becomes unstable at load stage 12, corresponding to $\lambda = 1.52$. As in the previous analysis, the actual behavior corresponds well with the behavior assumed in the design procedure.

Figure 6.17 shows the inelastic rotations which occurred at each plastic hinge in the wall up to load stage 12. Such inelastic rotations are well within the capacity of reinforced concrete sections[24].

The economy of the design method is illustrated in Figure 6.18. The stepped dotted curve indicates the final design moments in each story for the wall. The dashed curve shows the absolute bending moment in the wall at load stage 12. Comparison of these two curves shows that there has been no overdesign in any story of the wall. For example, the design moment in the bottom story of the wall is 21,300 Kip-ft. and the maximum moment in this story at the ultimate load was 21,300 Kip-ft. The stepped solid curve in Figure 6.18, shows the design moments in the wall given by a second order elastic-plastic analysis in which the wall remained elastic. The average design moment in each story, given by this analysis is 9,140 Kip-ft. The average design moment, by the present design method, is only 6,930 Kip-ft., by comparison.

6.4 Eighteen Story Structure

The shear wall in the eighteen story structure shown in Figure 6.3 has been designed in Appendix D. The stability criteria of Chapter III indicated that an efficient design for this structure should utilize partial plastic redistribution of forces between the wall and the frame. The design of the wall is summarized in Table D.4. The sway rotation at the ultimate state, ρ_f, selected for the design, was 0.015; this meant that the structure would become unstable once bays 1 and 5, of the frame, only developed mechanisms. Since the minimum required moments of column (8) of Table D.4 were the final design moments (except for story 6 where the capacity had to be increased by 4%) the structure was analyzed in the final design form only. The serviceability analysis indicated that the effective moment of inertia of the wall should be at least 21,000 ft^4; this value was used in this analysis.

Figures 6.19, 6.20 and 6.21 show the plot of load factor on lateral loads, λ, versus the roof sway; the hinging pattern in the structure; and the deflected shape of the structure at each load stage, respectively. Again, the behavior of the structure corresponded to that assumed in the design procedure. Plastic hinges formed in the shear wall early in the loading history. The stiffness of the structure gradually deteriorated until the structure reached its ultimate capacity at load stage 10, corresponding to λ = 1.375. At load stage 10, mechanisms formed in bays 1 and 5 only as predicted.

Figure 6.22 shows the inelastic rotations in the wall at load stage 10. Again, the ductility requirements for the wall are within the capacity of practical reinforced concrete sections[24].

Figure 6.23 illustrates the economy of the design method. The economy of the present plastic design method over the elastic design is not as great as shown in Figure 6.18 for the 12 story structure, because only a partial redistribution of forces between the wall and the frame could be utilized for the 18 story structure.

6.5 Summary

Two steel frame-shear wall structures have been designed by the plastic design procedure outlined in Chapter V. Rigorous second order elastic-plastic analyses of the resulting structures proved the validity of the major design assumptions and showed that the method produces a satisfactory design.

GIRDER SPAN (FT.)	MEMBER
10	6 I 12.5
20	14 B 26
28	14 WF 43
30	18 B 40

TABLE 6.1

GIRDERS USED IN EXAMPLE STRUCTURES

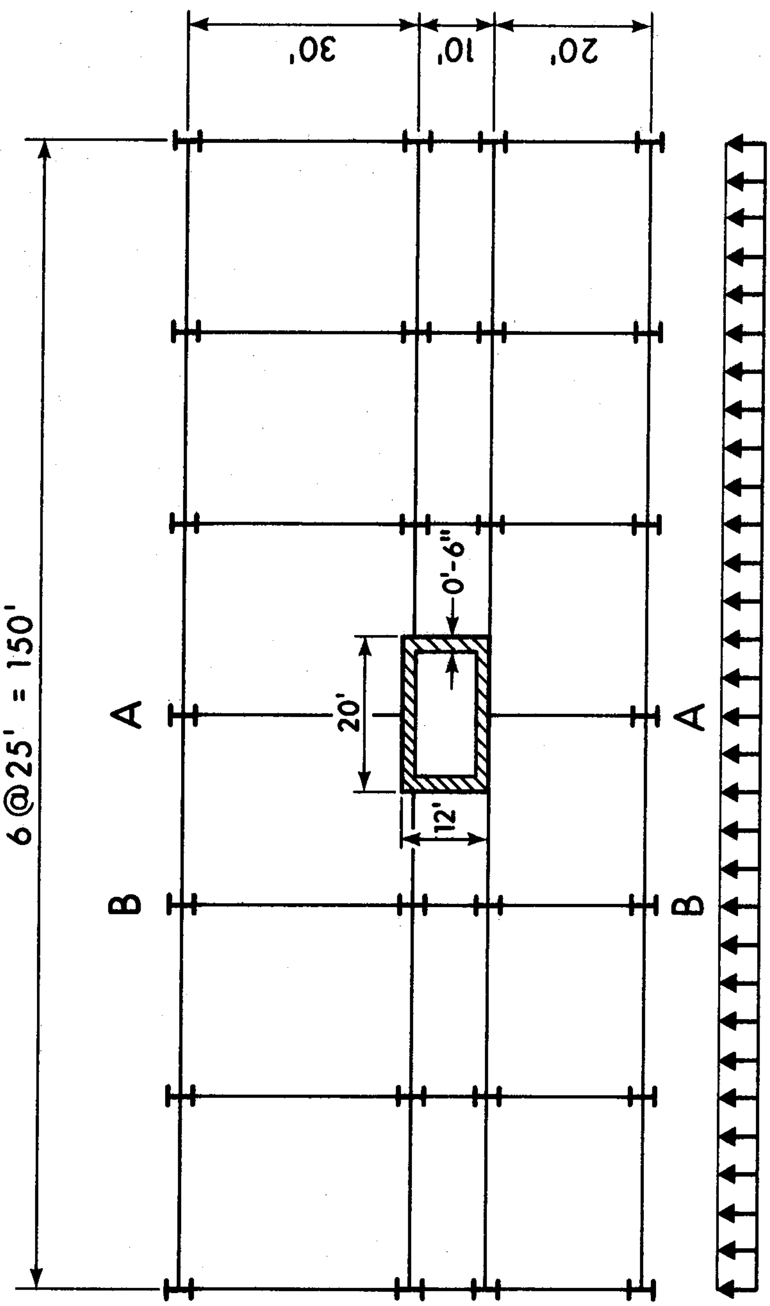


FIGURE 6.1 PLAN OF EXAMPLE STRUCTURES

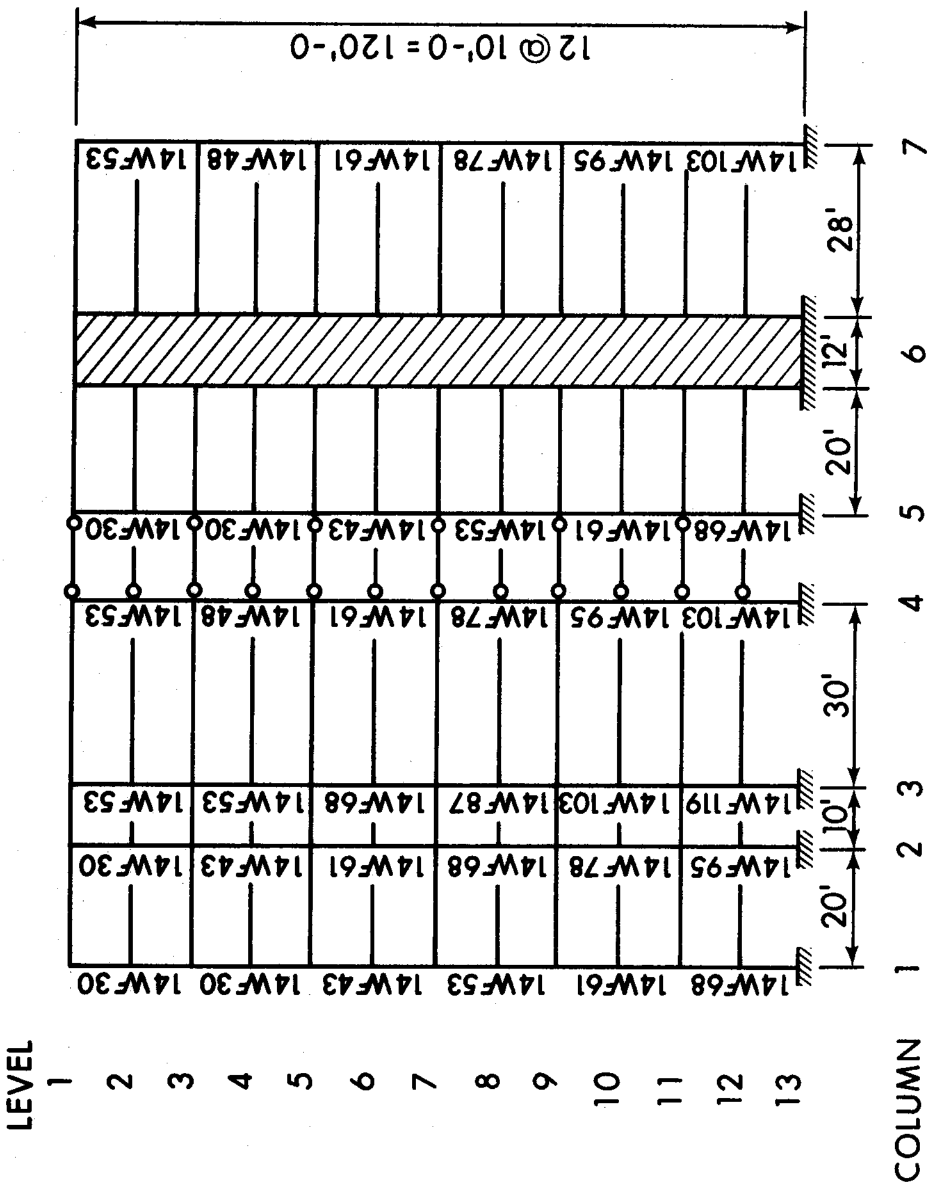


FIGURE 6.2 12 STORY STRUCTURE

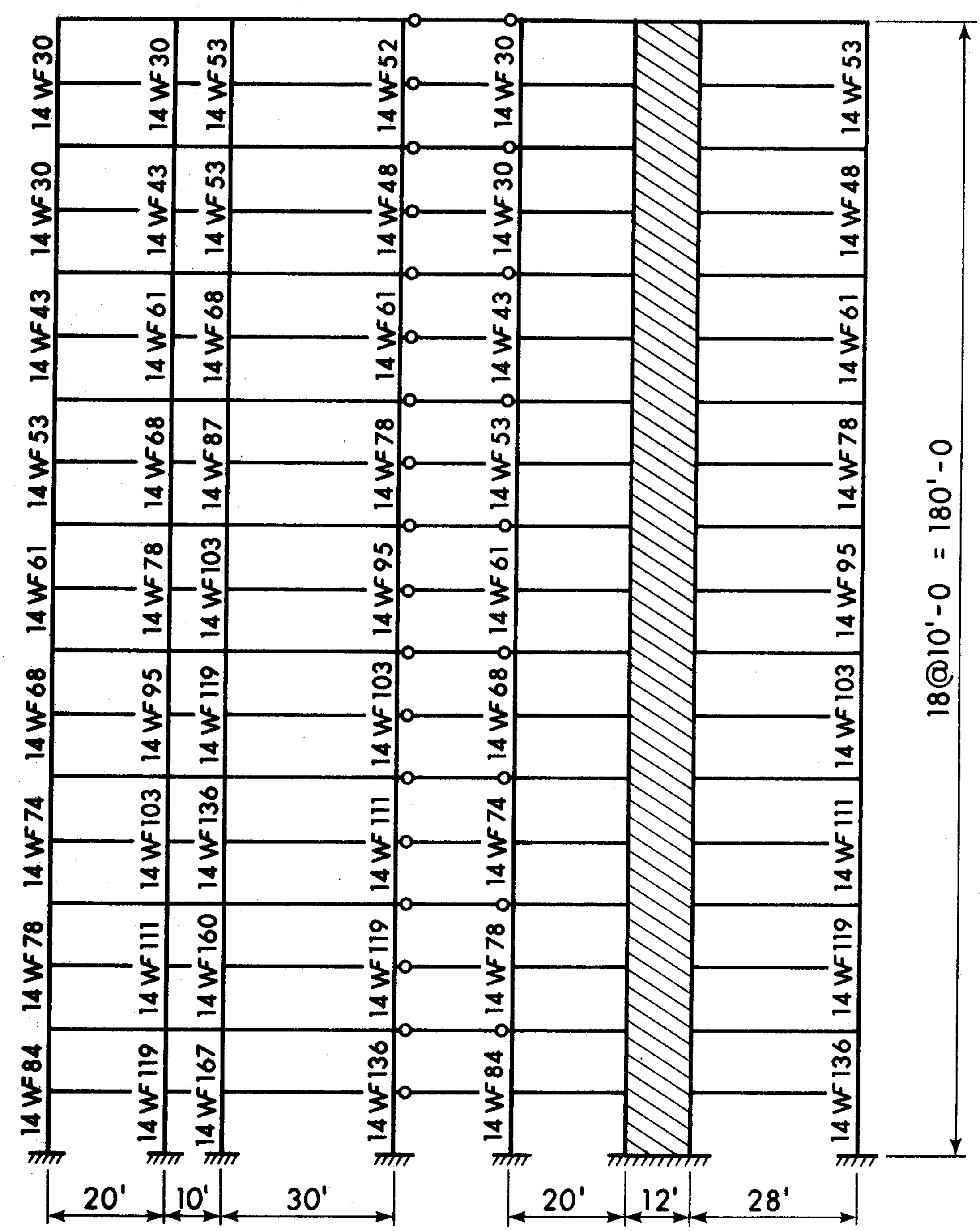


FIGURE 6.3 18 STORY STRUCTURE

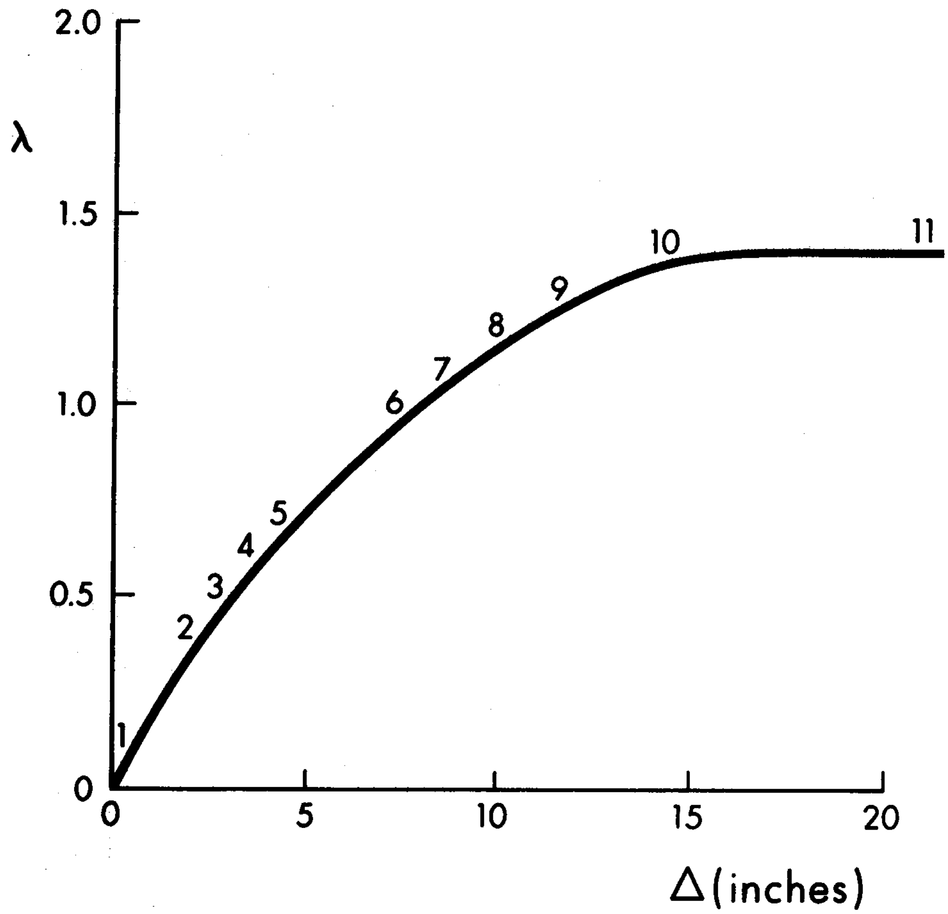


FIGURE 6.4 LOAD DISPLACEMENT RESPONSE 12 STORY STRUCTURE

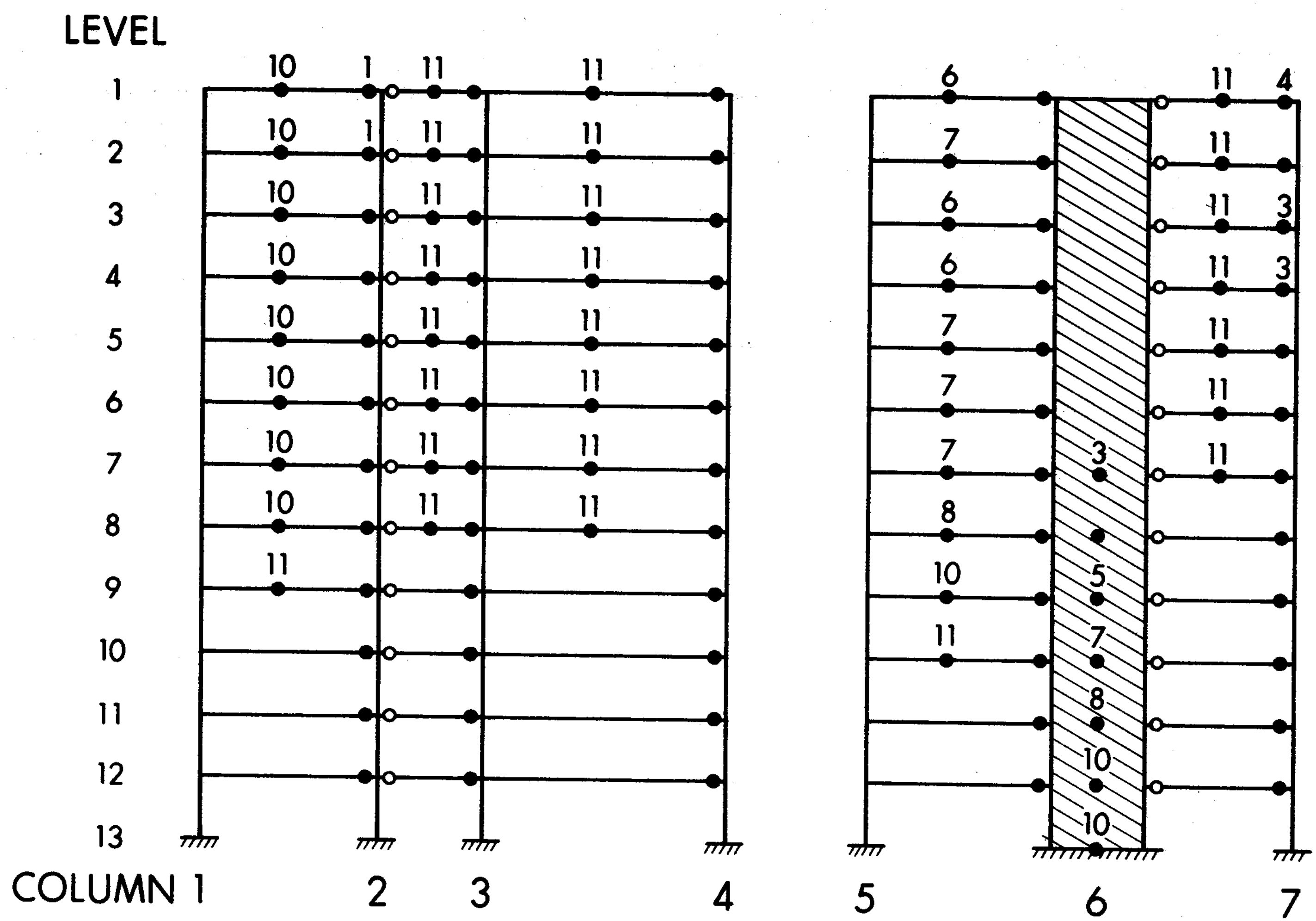


FIGURE 6.5 HINGING PATTERN 12 STORY STRUCTURE

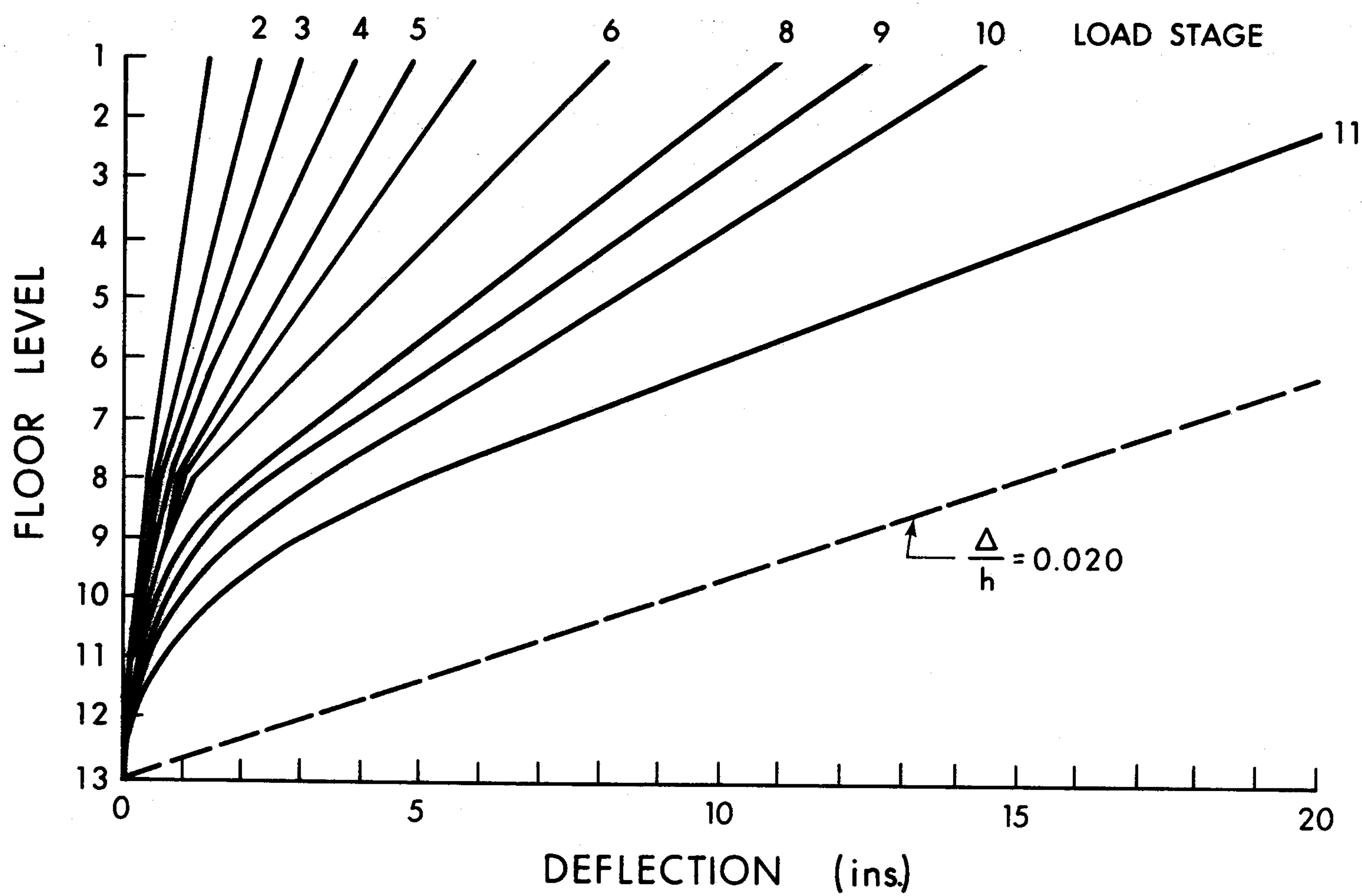


FIGURE 6.6 DEFLECTED SHAPE 12 STORY STRUCTURE

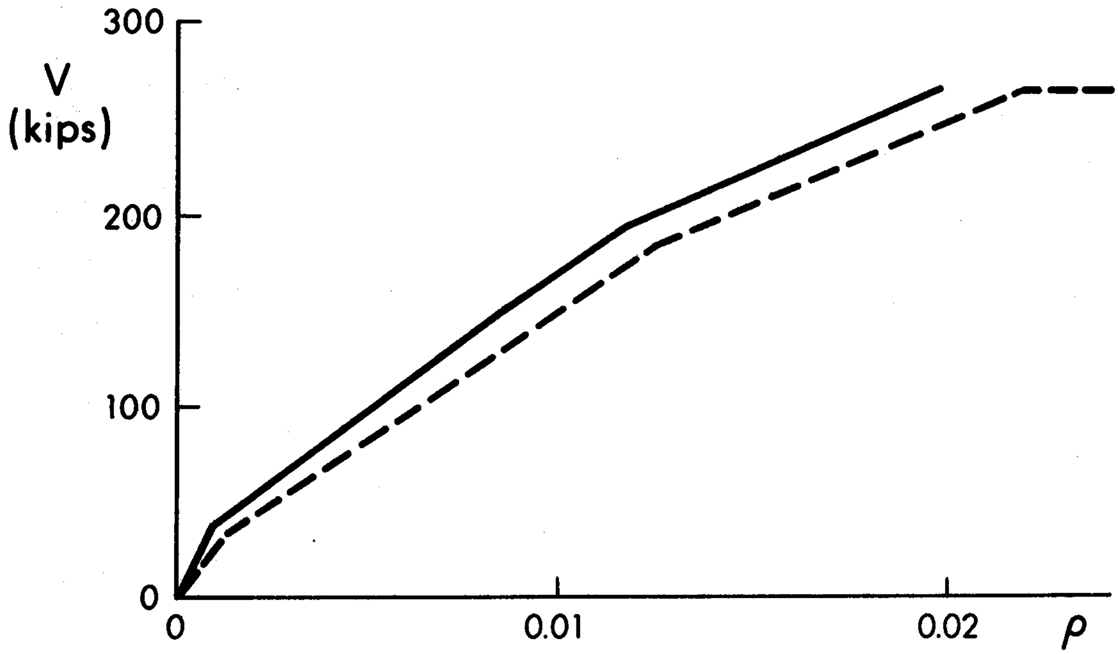


FIGURE 6.7 RESISTING SHEARS STORY 1

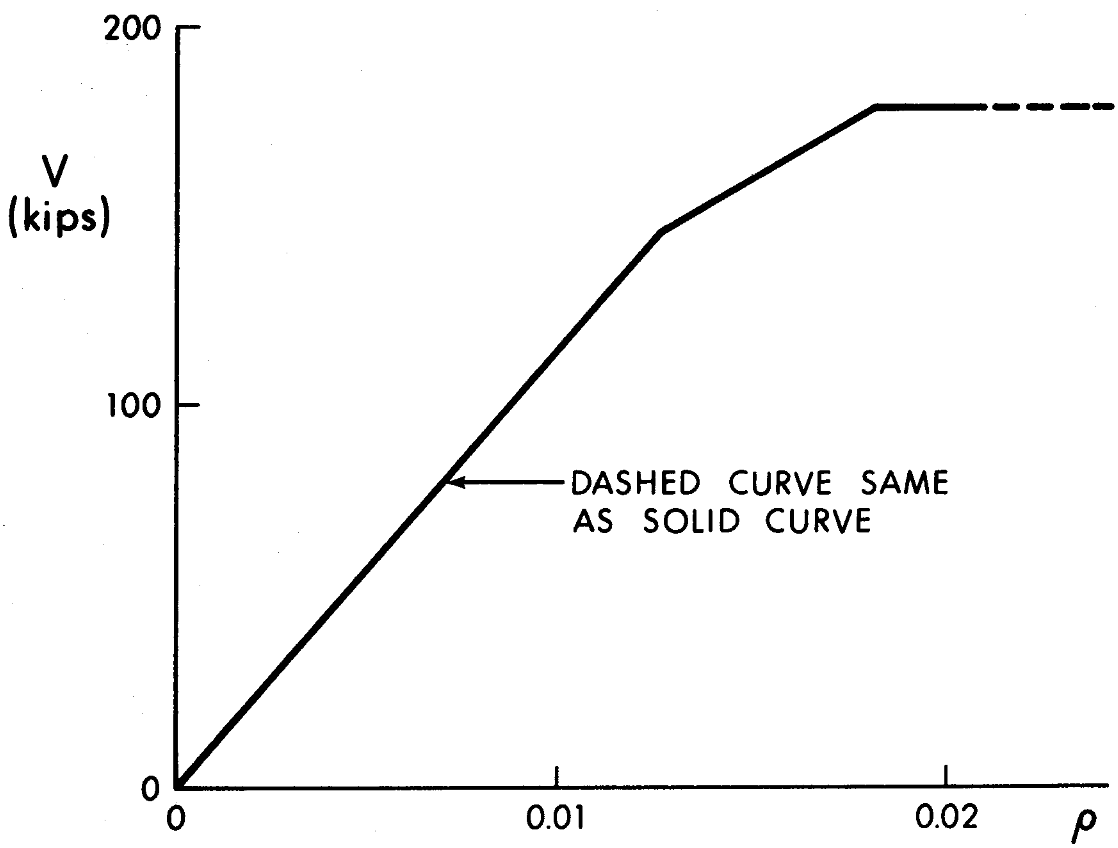


FIGURE 6.8 RESISTING SHEARS STORY 5

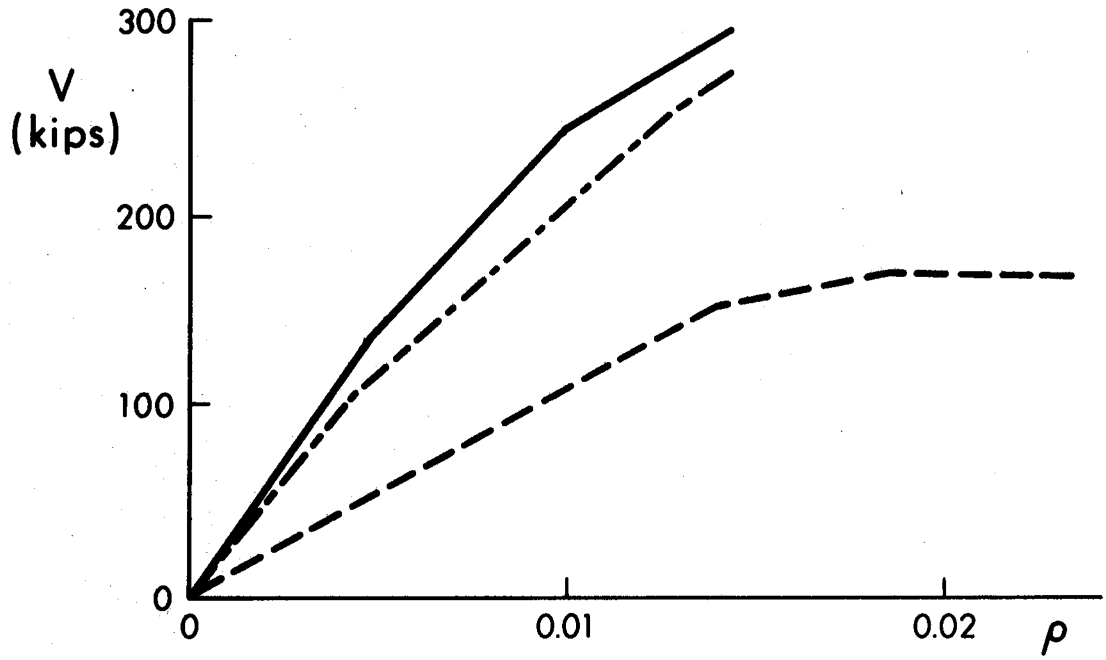


FIGURE 6.9 RESISTING SHEARS STORY 7

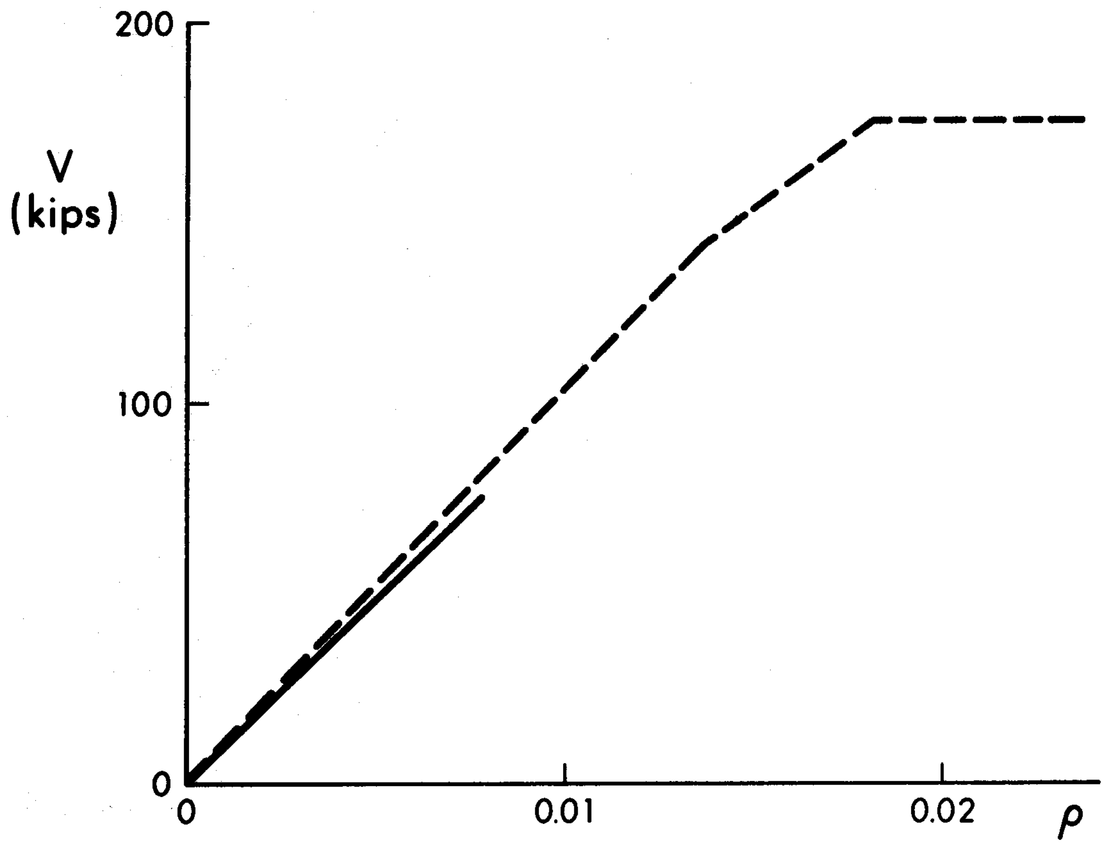


FIGURE 6.10 RESISTING SHEARS STORY 11

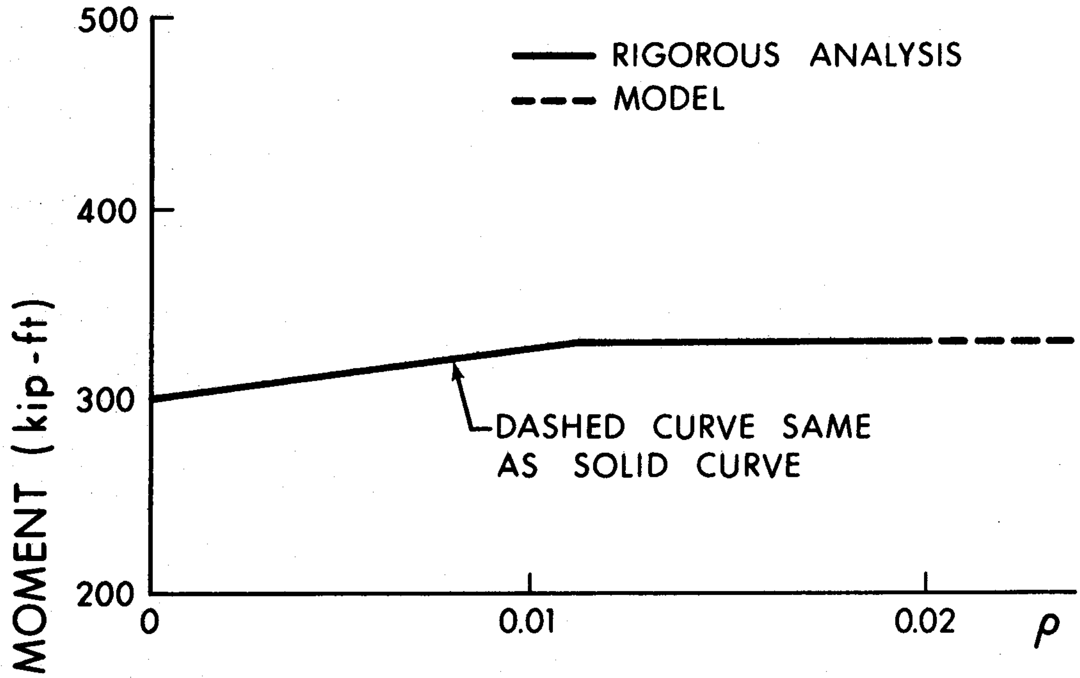


FIGURE 6.11 WINDWARD GIRDER

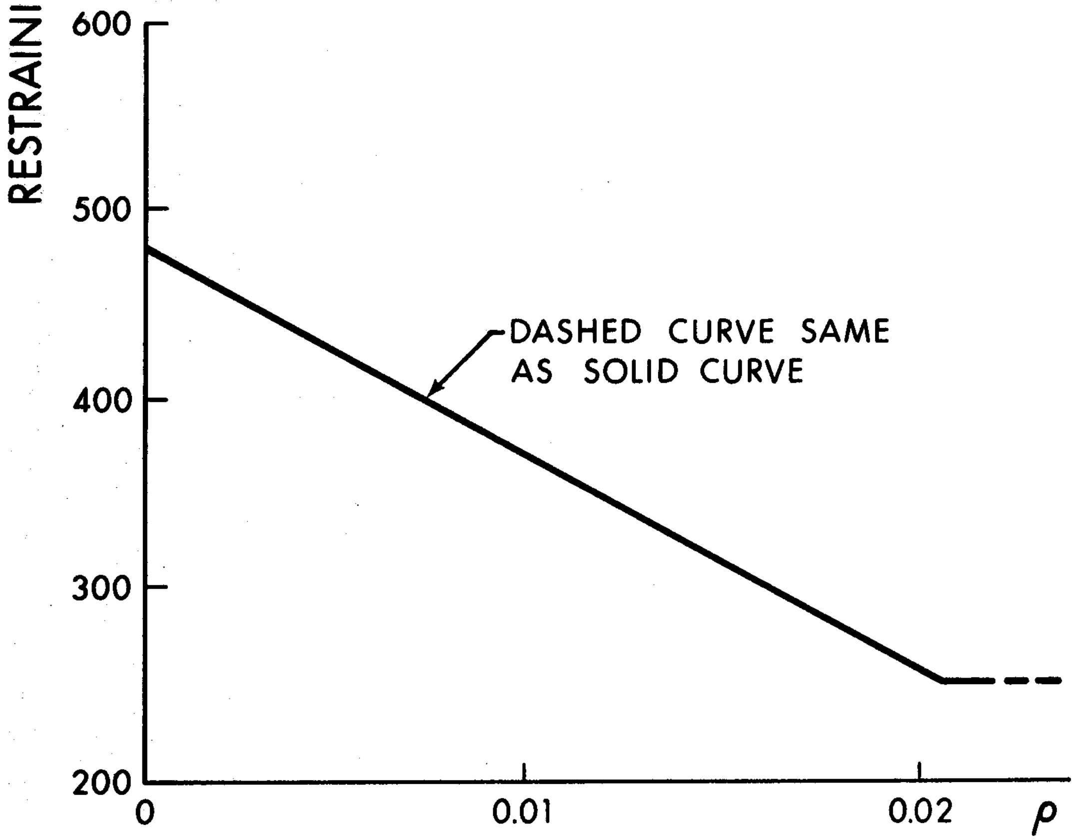


FIGURE 6.12 LEEWARD GIRDER

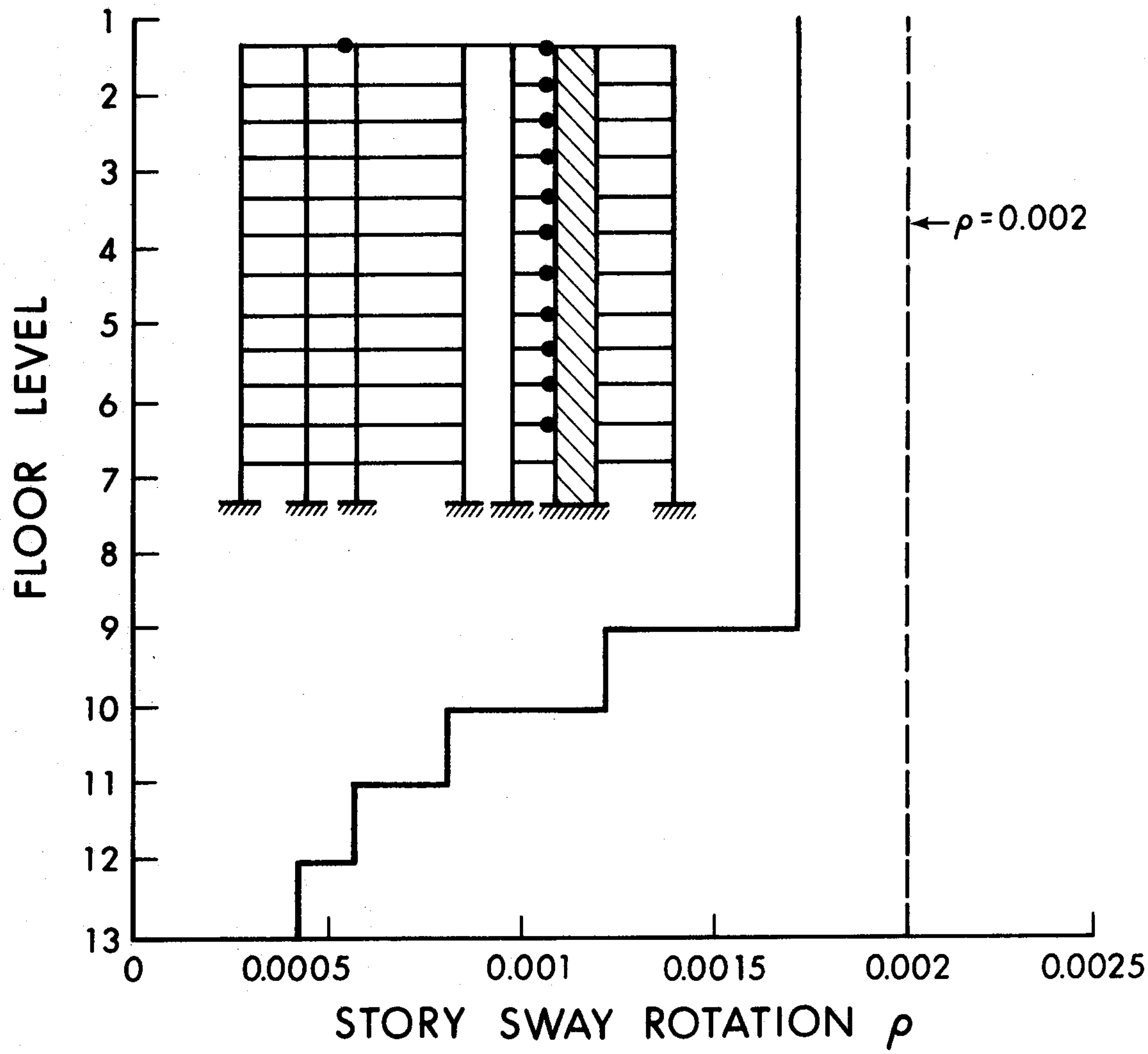


FIGURE 6.13 WORKING LOAD STORY SWAY ROTATIONS

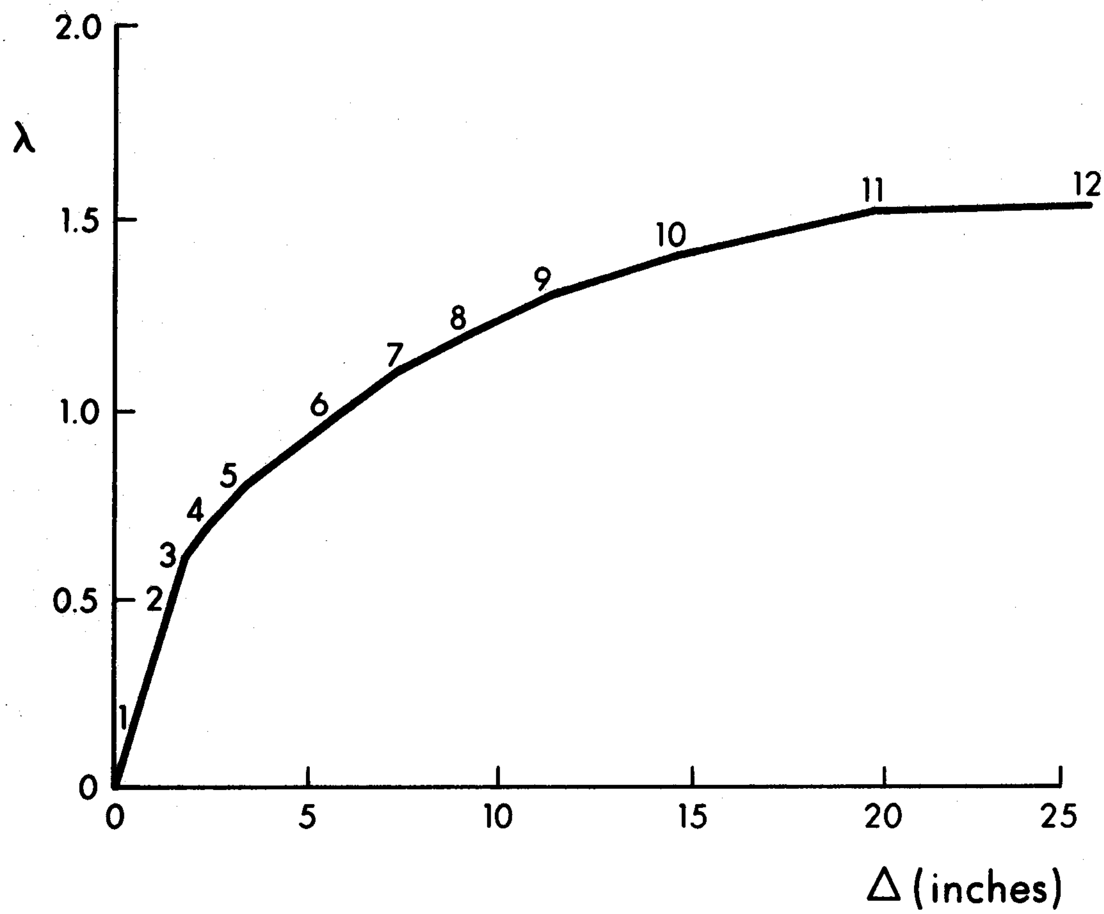


FIGURE 6.14 LOAD DEFLECTION RESPONSE 12 STORY STRUCTURE

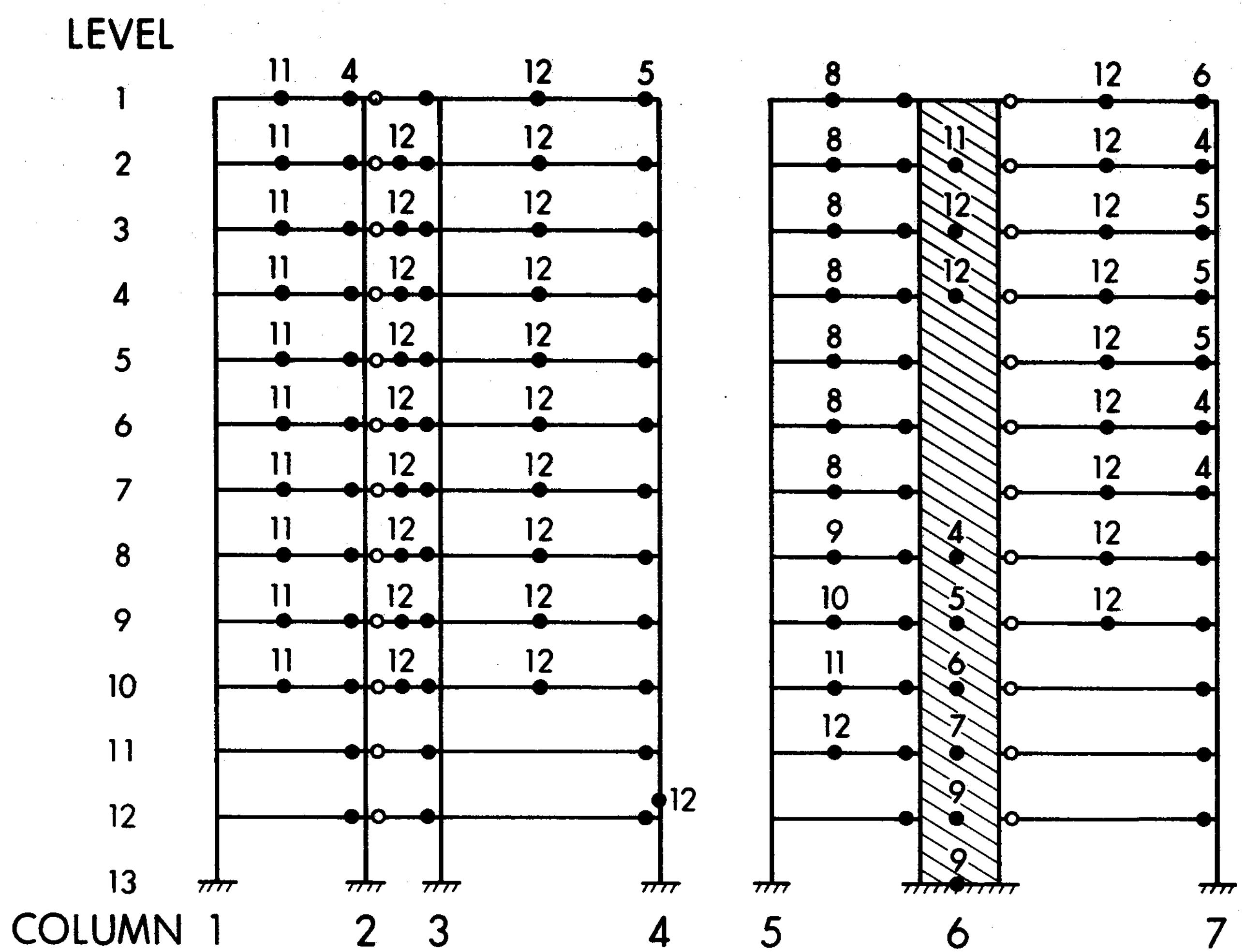


FIGURE 6.15 HINGING PATTERN 12 STORY STRUCTURE

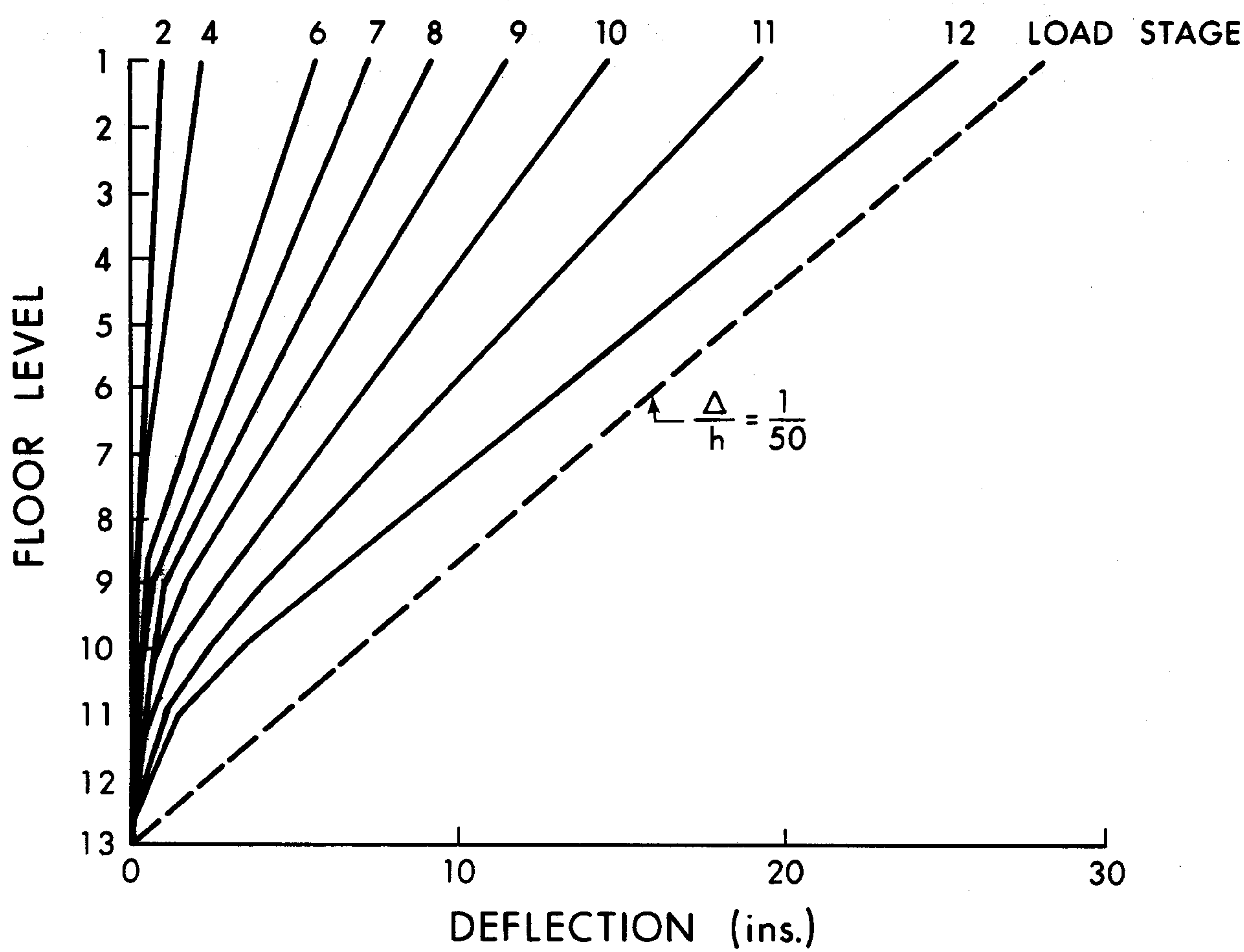


FIGURE 6.16 DEFLECTED SHAPE 12 STORY STRUCTURE

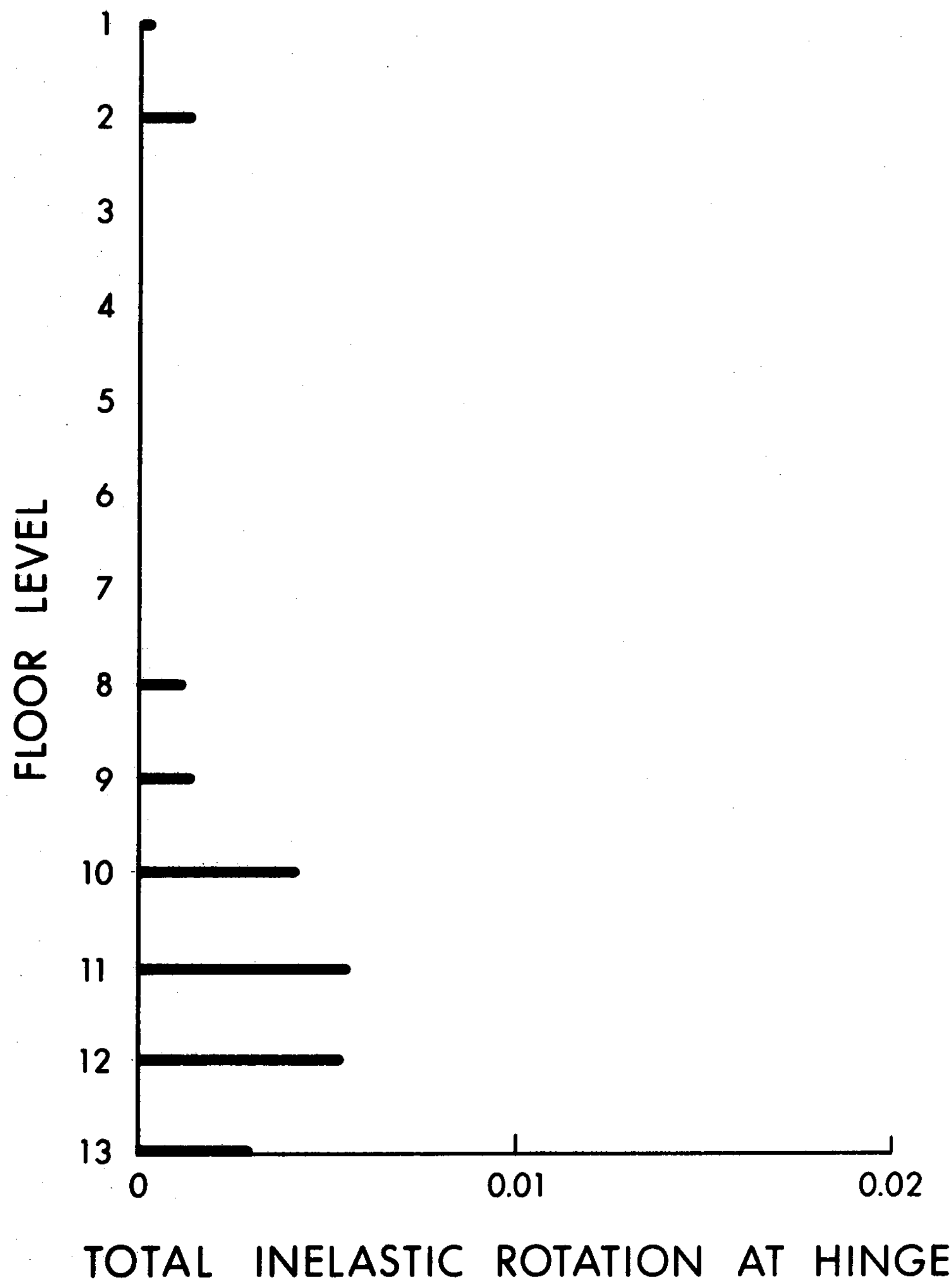


FIGURE 6.17 INELASTIC ROTATION IN WALL

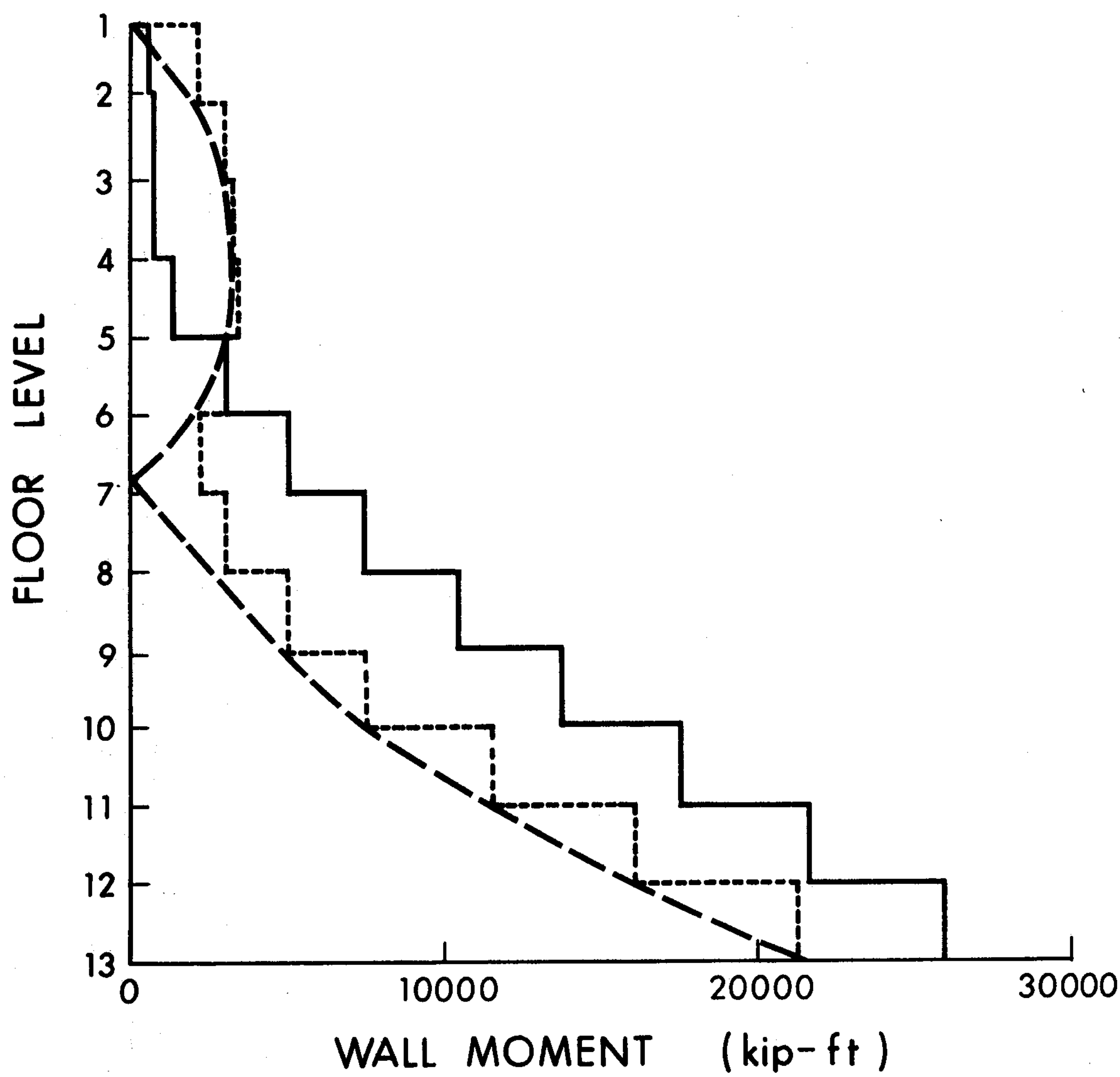


FIGURE 6.18 DESIGN AND ACTUAL ULTIMATE MOMENTS IN WALL

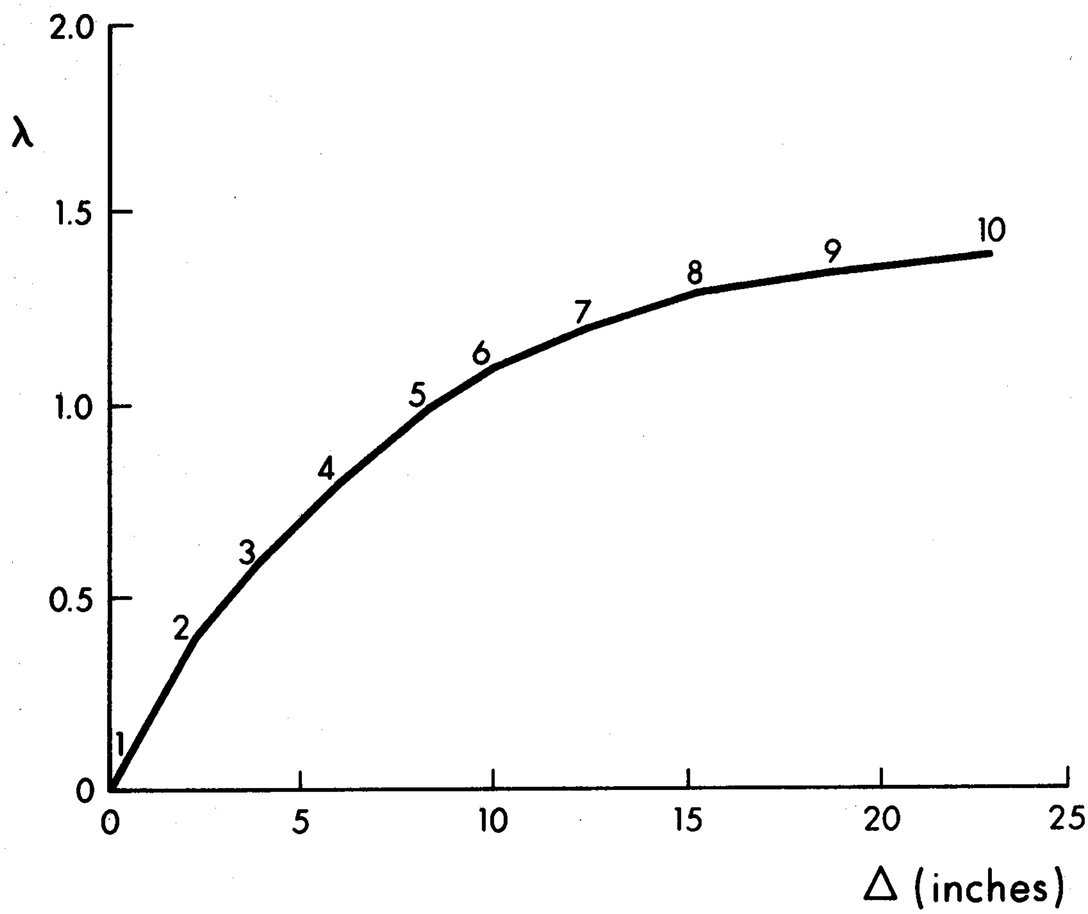


FIGURE 6.19 LOAD DEFLECTION RESPONSE 18 STORY STRUCTURE

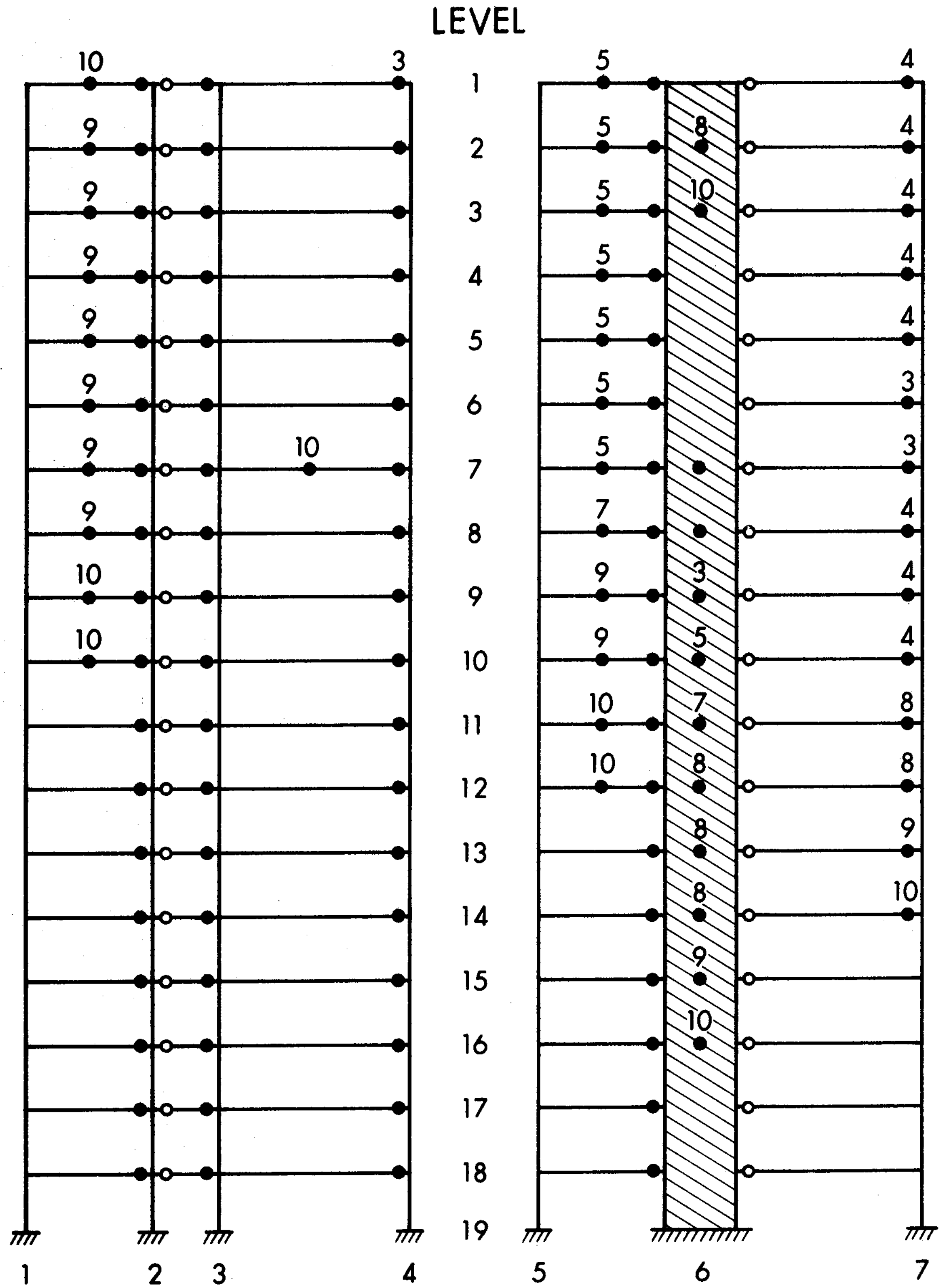


FIGURE 6.20 HINGING PATTERN 18 STORY STRUCTURE

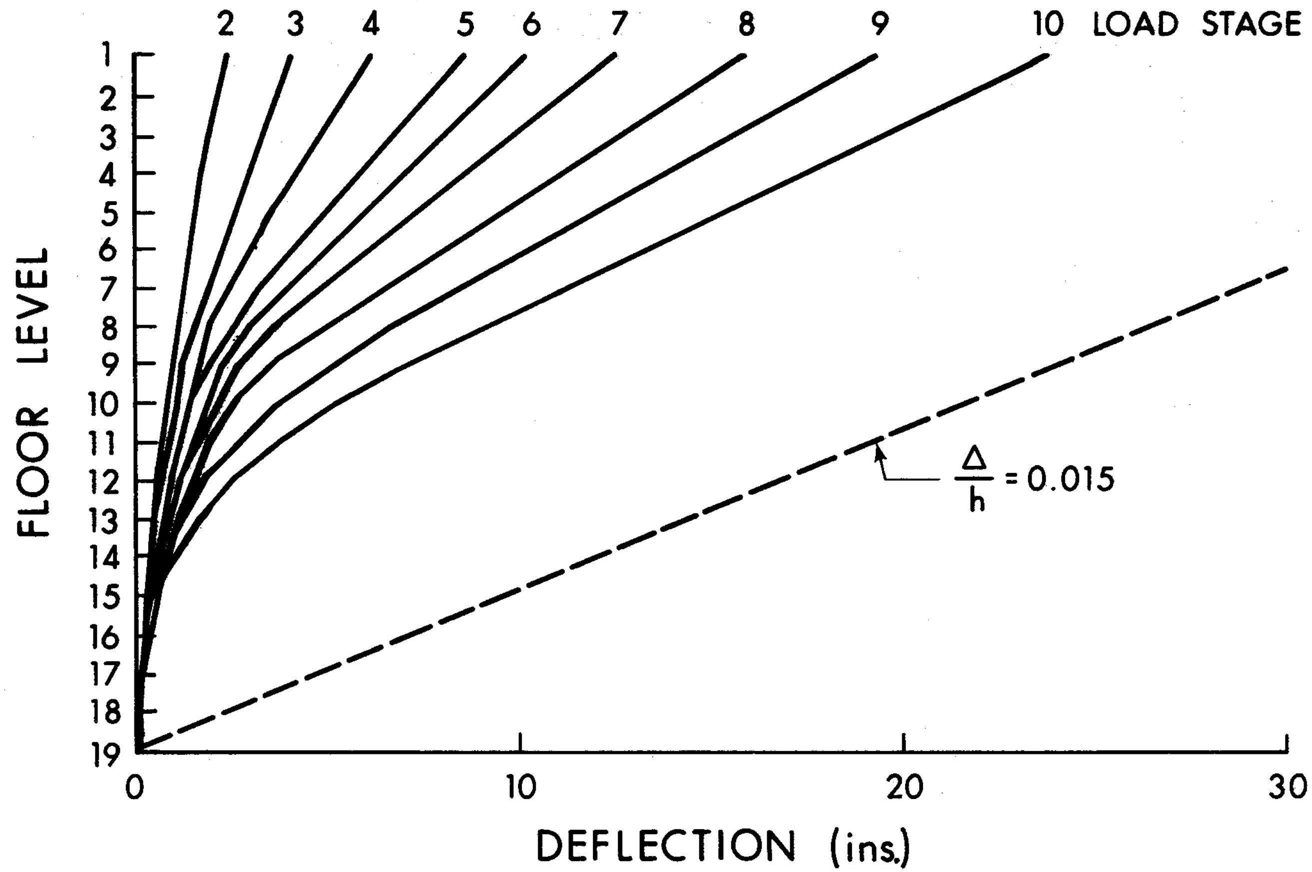


FIGURE 6.21 DEFLECTED SHAPE 18 STORY STRUCTURE

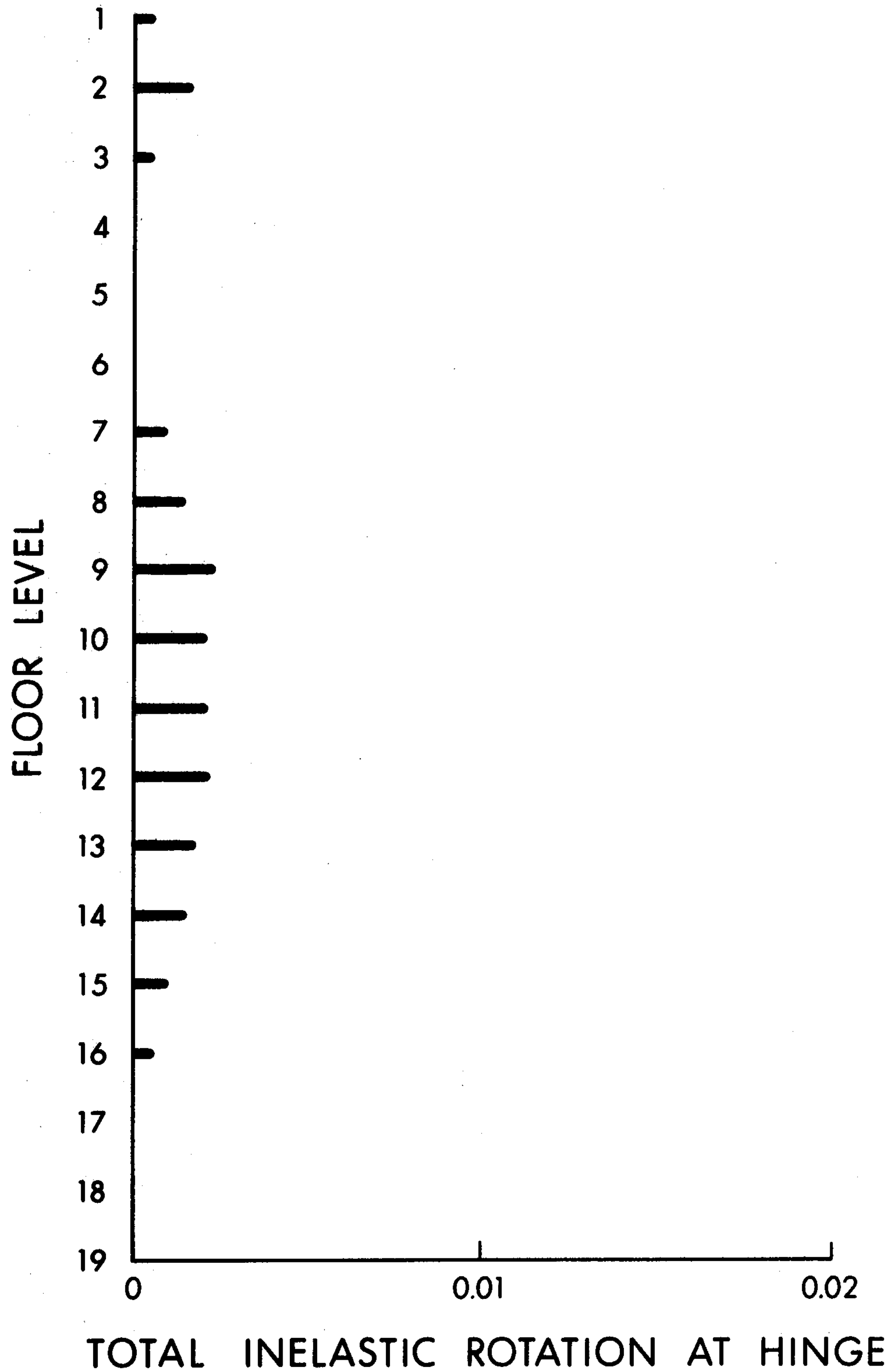


FIGURE 6.22 INELASTIC ROTATION IN WALL

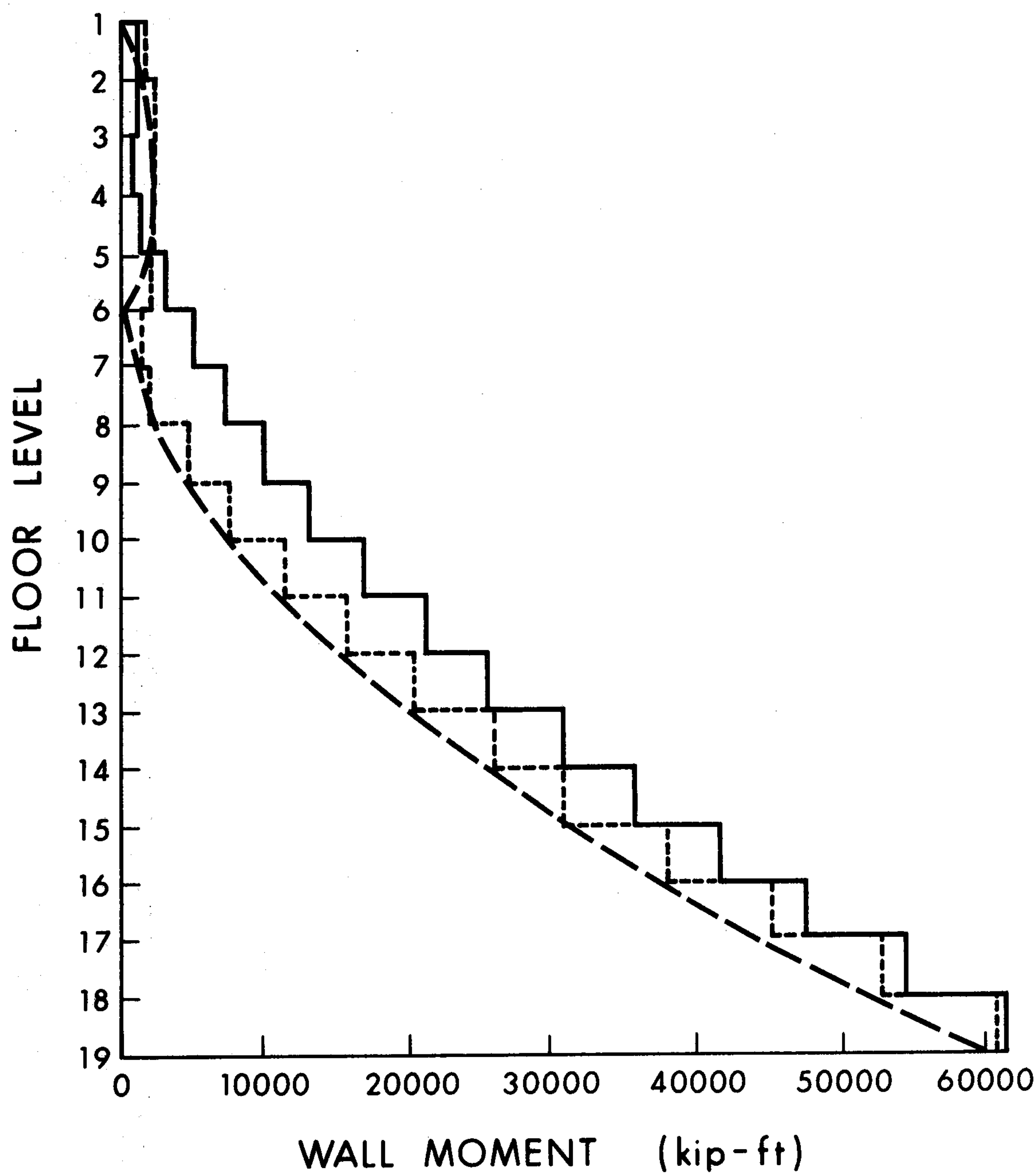


FIGURE 6.23 DESIGN AND ACTUAL ULTIMATE MOMENTS IN WALL

CHAPTER VII

SUMMARY

In this thesis a plastic design method for steel frame-shear wall structures has been presented. The design is based on the ultimate capacity of the structure and the method includes the effect of the formation of plastic hinges in the structure and the PΔ effect.

The frame portion of the structure is first designed plastically as a braced frame. The wall is then proportioned for strength considering the contribution of the frame in resisting combined loads (L.F. = 1.3). For the class of structures which do not reach their ultimate capacities once the shear wall develops a plastic hinge, a simple direct design is possible. The approach is based on an estimation of the deflected shape of the structure at the ultimate load. From this, the PΔ effect and the resistance of the frame may be calculated and the wall proportioned to resist the net lateral and vertical loads.

For structures which do reach their ultimate capacities once the shear wall develops a plastic hinge, the design is based on an analysis in which the wall remains elastic. A computer program which performs a second order elastic-plastic analysis on an approximate model of the frame-shear wall structure has been presented for the design of this type of structure.

Following this preliminary design the wall proportions are checked and possibly modified to resist the effect of full gravity loads (L.F. = 1.7) alone, acting on the structure and to ensure that the working load deflections (L.F. = 1.0) are satisfactory.

The validity of the assumptions and the adequacy of the design methods presented in this thesis have been verified by a rigorous computer analysis of a twelve story and an eighteen story steel frame-shear wall structure designed as described above.

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ACKNOWLEDGEMENTS

This study forms part of an investigation of the behavior of braced multi-story structures, currently in progress at the Department of Civil Engineering, University of Alberta, sponsored by the Canadian Steel Industries Construction Council. A scholarship awarded by the Alberta Regional Committee of the Canadian Institute of Steel Construction to John Bryson, for the 1971-72 academic year, provided personal financial assistance and is gratefully acknowledged.

The authors wish to express thanks to J.G. MacGregor, and J.S. Kennedy for their helpful suggestions.

The manuscript was typed by Mrs. Betty Boon, whose cooperation is appreciated.

APPENDIX A

RESPONSE OF THE ANALYTICAL MODEL FOR THE STEEL FRAME

APPENDIX A

RESPONSE OF THE ANALYTICAL MODEL FOR THE STEEL FRAME

In this appendix equations are derived to relate the sway rotation of a story to the lateral shear resistance of a single column of that story.

A.1 Analytical Model for a Typical Story

It is assumed that the structure under consideration is regular, so that the sway rotations in adjacent stories are equal; that is, $\rho_1 = \rho_2 = \rho_3 = \rho$, as shown in Figure A.1.

The structure is subjected to combined loading (L.F. = 1.3). Gravity loads are applied first and under these loads, plastic hinges form at either end of both girders. Lateral loads are then applied. During the sway motion of the structure the plastic hinges at the windward ends of the girders BE and CF reverse and provide an elastic rotational restraint to the columns at B and C, respectively, in Figure A.1. The plastic hinges at the leeward ends will continue to rotate plastically during the sway motion, providing no restraint to the columns at E and F.

To obtain the initial response of the model, the slope-deflection equations are written for each member, and moment equilibrium equations are written for each joint to eliminate the joint rotations as unknowns. In this process the following equation is obtained:

$$\left(\frac{3I_3}{h_3} + \frac{3I}{L} + \frac{6I_2}{h_2}\right)\theta_C + \left(\frac{3I_1}{h_1} + \frac{3I}{L} + \frac{6I_2}{h_2}\right)\theta_B = \left(\frac{12I_2}{h_2} + \frac{3I_1}{h_1} + \frac{3I_3}{h_3}\right)\rho \qquad (A.1)$$

The coefficients of θ_B and θ_C are assumed to be equal, and to be:

$$[\frac{1}{2}(\frac{3I_3}{h_3} + \frac{3I_1}{h_1}) + \frac{3I}{L} + \frac{6I_2}{h_2}] \qquad (A.2)$$

Solving for $(\theta_B + \theta_C)$, the shear equilibrium equation for column BC may be written. This results in a relationship between the sway rotation, ρ, and the resisting shear, V, developed by the column:

$$V = \frac{6EI_2}{h_2^2}\{\frac{2I/L}{\gamma + I/L + 2I_2/h_2}\}\rho \qquad (A.3)$$

where;

$$\gamma = \frac{1}{2}[\frac{I_1}{h_1} + \frac{I_3}{h_3}] \qquad (A.4)$$

In Chapter VI a model including the effect of different sway rotations in successive stories is required. So repeating the above procedure with $\rho_1 \neq \rho_2 \neq \rho_3$ gives the relationship for the resisting shear of column BC in terms of the sway rotations ρ_1, ρ_2 and ρ_3 as:

$$V = \frac{6EI_2}{h_2^2}\left[\frac{\frac{I_3}{h_3}(\rho_2-\rho_3) + \frac{I_1}{h_1}(\rho_1-\rho_2) + \frac{2I}{L}\rho_2}{\gamma + \frac{I}{L} + \frac{2I_2}{h_2}}\right] \qquad (A.5)$$

Equation (A.3) is valid as long as additional plastic hinges do not form in the model. As the structure sways, however, the plastic moment capacity, M_{pc}, of column BC may be reached at B and C, simultaneously, or the plastic moment capacities of the girders, M_p, may be reached at a distance from the leeward column, X, (Figure A.1), simultaneously. Either of these mechanisms limits the capacity of the column to offer shear resistance to additional sway.

The shear corresponding to a column mechanism is:

$$V = \frac{2M_{pc}}{h_2} \tag{A.6}$$

To evaluate the shear corresponding to a beam mechanism, consider a portion of the analytical model; beam BE and segments of the upper and lower columns isolated at their points of inflection, shown in Figure A.2. The points of inflection are assumed to be located at mid-height in the columns. The factor, β, in Figure A.2, accounts for differing story heights. For constant story height, $\beta = 1.0$.

Under the initial application of gravity loads alone, plastic hinges form at either end of the girder and the reaction at E in Figure A.2 is:

$$\frac{1.3wL}{2}$$

Under the superimposed lateral shear, V, corresponding to the formation of a plastic hinge at X in the beam, Figure A.2, the reaction at E is:

$$\frac{1.3wL}{2} + \frac{\beta h_2 V}{L}$$

The moment at the plastic hinge location is given by:

$$M_x = \frac{1.3w_g L^2}{12} = \frac{1.3wL}{2} \chi + \frac{\beta h_2 V}{L} \chi - \frac{1.3w_g L^2}{12} - 1.3 \frac{w \chi^2}{2} \tag{A.7}$$

In equation (A.7) w is the girder design load and w_g is defined by equation (1.5).

Solving equation (A.7) results in:

$$\chi = \left(\frac{L}{2} + \frac{\beta h_2 V}{1.3wL}\right) \pm \frac{1}{2} \sqrt{\left(L + \frac{2\beta h_2 V}{1.3wL}\right)^2 - \frac{4}{3} \frac{w_g}{w} L^2} \tag{A.8}$$

But χ must be located at a maximum in M_x. So differentiating M_x with respect to x gives:

$$\chi = \frac{L}{2} + \frac{\beta h_2 V}{1.3wL} \qquad (A.9)$$

Equations (A.8) and (A.9) yield an expression for V, corresponding to a beam mechanism:

$$V = \frac{1.3wL^2}{\beta h_2} [0.578\sqrt{\frac{w_g}{w}} - \frac{1}{2}] \qquad (A.10)$$

For the case; $w_g = w$:

$$V = \frac{0.078(1.3wL^2)}{\beta h_2} \qquad (A.11)$$

A.2 Analytical Model for the Top Story

The analytical model modified to suit the top story is shown in Figure A.3. The moments of inertia of the roof girder and the columns are assumed to be the same and are denoted by I_c. The floor girder is designed for a different load than is the roof girder and has a moment of inertia, I. It is assumed that the story height is the same in the top two stories, and that the point of inflection in the column adjacent to the top story occurs at mid-height. The story sways in the top two stories are assumed to be equal; that is; $\rho_1 = \rho_2 = \rho$.

The initial response of the model is obtained by writing the slope-deflection equations for each member, and moment equilibrium equations for each joint to eliminate the joint rotations as unknowns. The shear equilibrium equation for column AB then gives the shear sway response of column AB:

$$V = \frac{6EI_c}{h^2} \left\{ \frac{\frac{12}{L} + \frac{I}{I_c}\frac{1}{L} + 13.5\,\frac{h}{L^2}\,\frac{I}{I_c}}{\frac{18}{h} + \frac{15}{L} + \frac{8}{L}\,\frac{I}{I_c}} \right\} \rho \tag{A.12}$$

To evaluate the resisting shear corresponding to a beam mechanism, additional plastic hinges are assumed to have formed in girders AD and BE. The shear in column AB corresponding to this mechanism is:

$$V = \frac{M_{AD} + \frac{M_{BE}}{2}}{h} \tag{A.13}$$

In equation (A.13), M_{AD} and M_{BE} are the girder end moments and are given by equation (A.14), where V is defined by equation (A.10) for each girder:

$$M = \beta h V \tag{A.14}$$

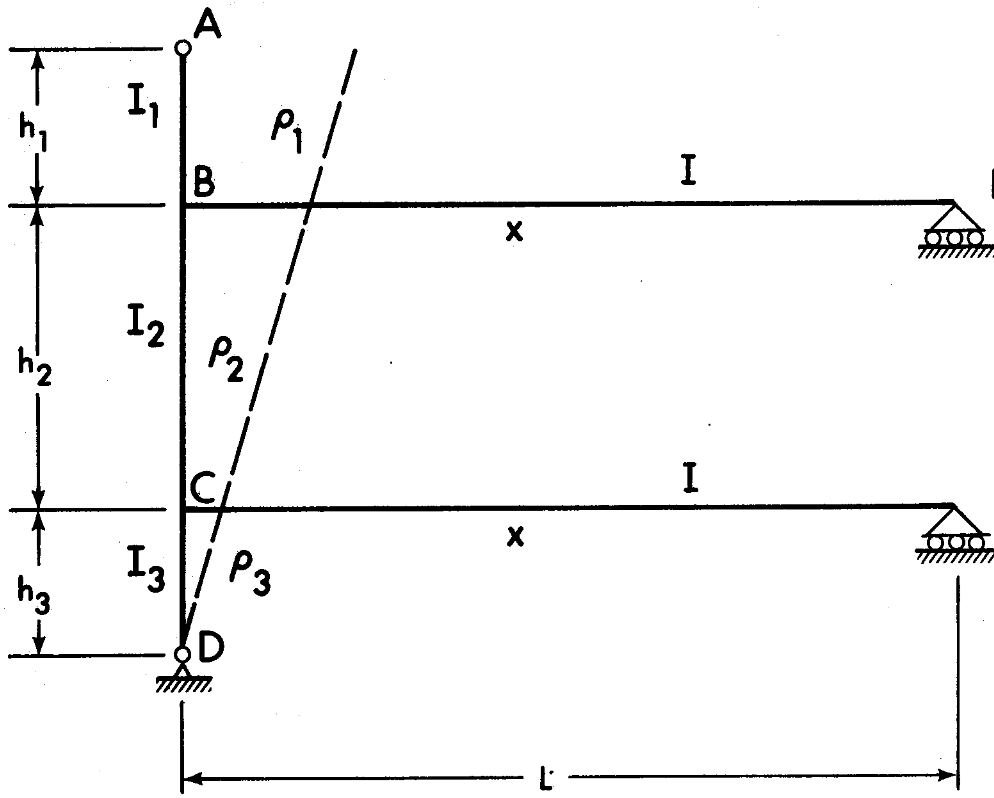


FIGURE A.1 ANALYTICAL MODEL FOR A TYPICAL STORY

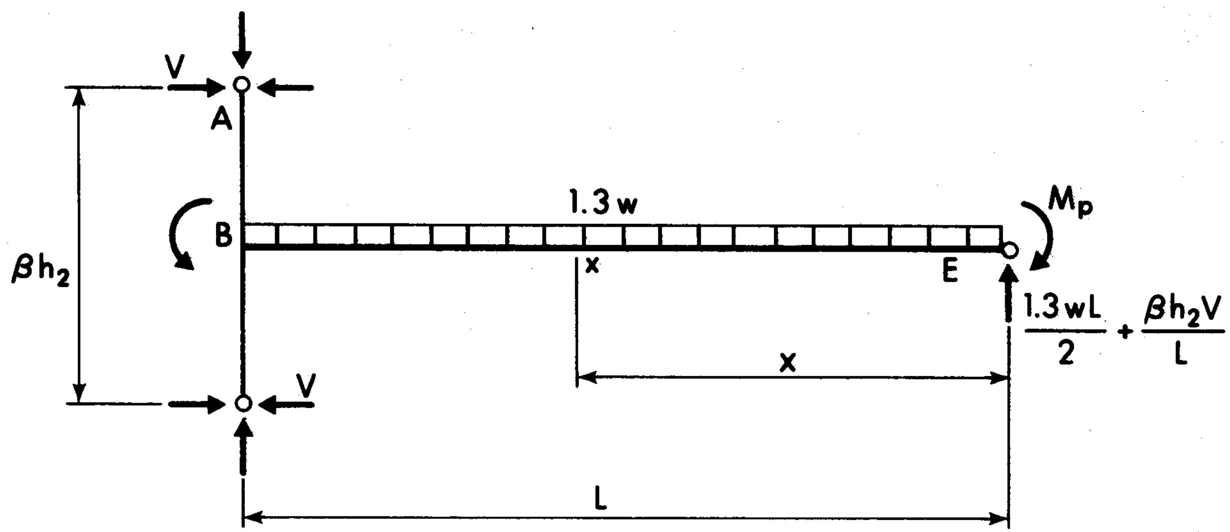


FIGURE A.2 PORTION OF THE ANALYTICAL MODEL UNDER COMBINED LOADS

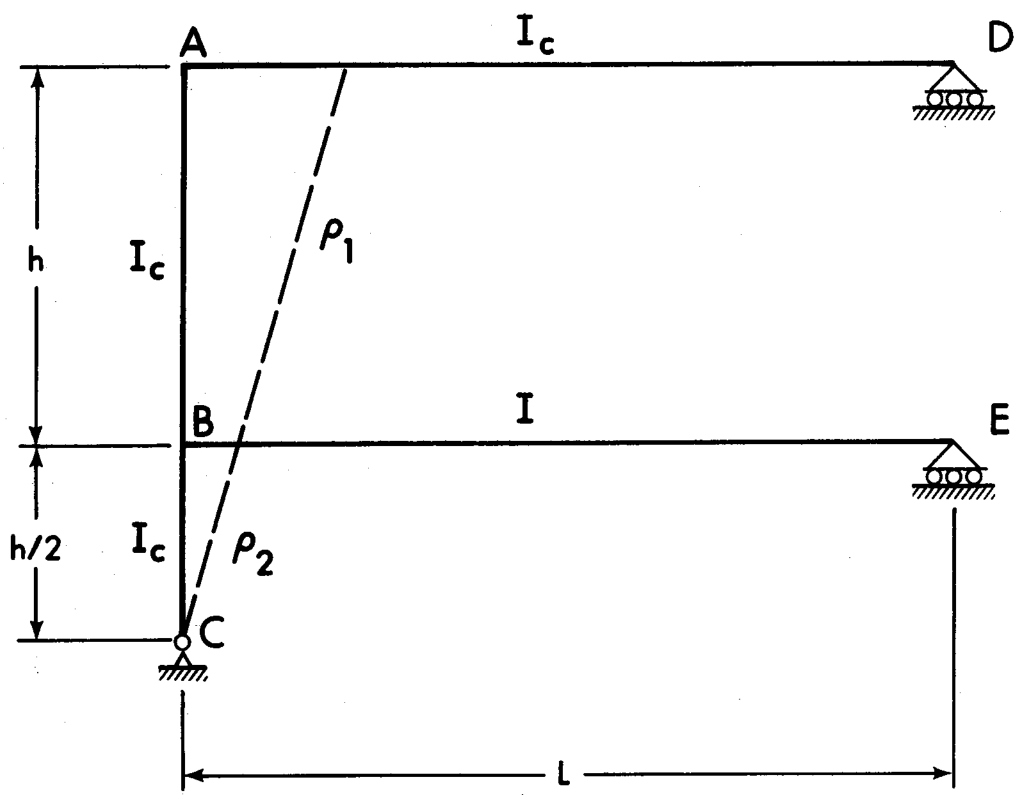


FIGURE A.3 ANALYTICAL MODEL FOR THE TOP STORY

APPENDIX B

STABILITY CRITERION FOR FRAME-SHEAR WALL STRUCTURES

APPENDIX B

STABILITY CRITERION FOR FRAME-SHEAR WALL STRUCTURES

In this appendix a simple formulation of the stability criterion developed in Section 3.5 of Chapter III is derived. The stability of a frame-shear wall structure at a load stage, is indicated by the term:

$$\frac{dM_f}{d\rho} - \frac{dM_{P\Delta}}{d\rho}$$

where $M_{P\Delta}$ and M_f are defined in section 3.5 of Chapter III, and ρ is the sway rotation of the story under consideration. If this term is negative, once the wall has yielded, then the structure is unstable.

The structure is first idealized as a series of linked bents, as shown in Figure 3.8. The structure has N stories, each of height h. The frame portion has M bays, which offer resistance to sway forces at a load stage. A bay containing link beams, offers no resistance to sway (Chapter IV). Each girder in the i^{th} bay carries a uniformly distributed load w_i, has a moment of inertia I_i, and spans L_i. The total gravity load on each floor, P_c, is assumed to be constant. It is assumed that the frame is regular enough so that the moment of inertia of each column in the i^{th} bay can be replaced by the average column moment of inertia, I_{c_i} (Section 4.6, Equation 4.7).

It is assumed that the deflected shape of the structure, once the wall has developed a plastic hinge, is rectilinear and that the sway rotation in each story is ρ. Thus the stability test need only be applied in the bottom story. It is also assumed that the response of each column (including those in the top and bottom stories) is given by equation (A.3) of Appendix A. The shear in the frame in each story, $\sum_{i=1}^{m} V_{ji}$, is thus constant; the shear diagram for the frame is shown in Figure B.1(a). The incremental PΔ shear in each story, $P_c\rho$, is also constant; the PΔ shear diagram is shown in Figure B.1(b).

The resisting moment developed at the bottom story of the structure due to the frame is:

$$M_f = \sum_{j=1}^{N} \sum_{i=1}^{m} V_{ji}\, h_j = Nh \sum_{i=1}^{m} V_{ji} \qquad \text{(B.1)}$$

Substituting equation (A.3) and conservatively assuming that $I_{c_i} = I_i$; and that the modulus of elasticity, E, is constant, gives:

$$M_f = \frac{6NE}{h}\rho \sum_{i=1}^{m} \left\{ \frac{I_i}{\frac{2L_i}{h} + \frac{1}{2}} \right\} \qquad \text{(B.2)}$$

Therefore:

$$\frac{dM_f}{d\rho} = \frac{6NE}{h} \sum_{i=1}^{m} \left\{ \frac{I_i}{\frac{2L_i}{h} + \frac{1}{2}} \right\} \qquad \text{(B.3)}$$

The moment in the bottom story, due to the PΔ effect is:

$$M_{P\Delta} = \sum_{j=1}^{N} [jP_c\, \rho h_j] = hP_c \frac{N(N+1)}{2} \rho \qquad \text{(B.4)}$$

Therefore:

$$\frac{dM_{P\Delta}}{d\rho} = hP_c \frac{N(N+1)}{2} \qquad \text{(B.5)}$$

Once the wall has developed a plastic hinge, the structure will just be stable at a load stage if:

$$\frac{dM_f}{d\rho} = \frac{dM_{P\Delta}}{d\rho} \tag{B.6}$$

Therefore, equating (B.3) and (B.5) gives:

$$P_c = \frac{12E}{(N+1)h^2} \sum_{i=1}^{m} \left[\frac{I_i}{\frac{2L_i}{h} + \frac{1}{2}} \right] \tag{B.7}$$

If the total gravity load applied to each story is greater than P_c, at a load stage after the wall has yielded, then the structure will be unstable.

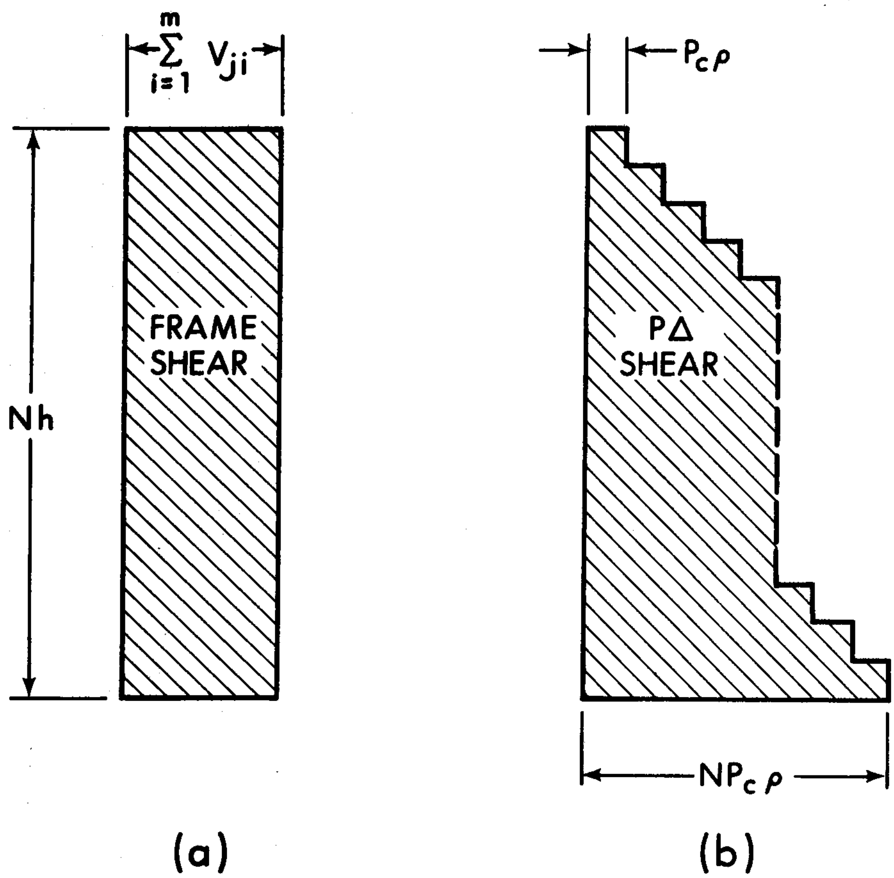


FIGURE B.1 FRAME SHEAR AND PΔ SHEAR

APPENDIX C

BEAMS FRAMING DIRECTLY INTO THE SHEAR WALL

APPENDIX C

BEAMS FRAMING DIRECTLY INTO THE SHEAR WALL

In this appendix the response of a typical rotational spring, which represents the action of the beams framing directly into the wall, shown in Figure 5.2, is derived. As the structure sways laterally, the rotation of the wall causes the beams framing into the wall to undergo significant vertical displacements at one end (Figures C.1(a) and C.1(b)). Moments and shears, in addition to those produced by the sway motion alone, are induced in the girders[15].

On the initial application of gravity loads, Figure C.1(a); the restraining moment at the centroid of the wall due to girders AB and DE, respectively, are:

$$M_{CB} = M_{P_{AB}} + \frac{1.3\ w\ L_{AB} d_{BC}}{2} \tag{C.1}$$

$$M_{CD} = -\ M_{P_{DE}} - \frac{1.3w\ L_{DE} d_{DE}}{2} \tag{C.2}$$

where: $M_{P_{AB}}$ and $M_{P_{DE}}$ are the plastic moment capacities of the girders.

L_{AB} and L_{DE} are the girder clear spans.

d_{BC} and d_{CD} are the depths of the wall from the centroid to each face.

and, w is the design uniform load (L.F. = 1.0) for the girders.

As the structure sways, the vertical reaction of girder AB at B, increases, as the moment at A increases. Assuming that the columns are much stiffer than the girders, then the rotation of girder AB at A will be equal to the story sway rotation, ρ. Therefore, the increase in moment A, due to both the sway and the vertical deflection at B, is:

$$\Delta M_{AB} = \frac{3EI_{AB}}{L_{AB}} \{1 + \frac{d_{BC}}{L_{AB}}\} p \tag{C.3}$$

Translating this into terms of restraining moment; the total restraining moment at the centroid of the wall is:

$$M_{R_{CB}} = M_{CB} + \frac{3EI_{AB}}{L_{AB}} \frac{d_{BC}}{L_{AB}} \{1 + \frac{d_{BC}}{L_{AB}}\} p \tag{C.4}$$

Assuming that the failure mechanism is a beam mechanism; then this response is terminated at the sway rotation:

$$\rho = \frac{0.94\ M_{P_{AB}}}{\frac{3EI_{AB}}{L_{AB}} \{1 + \frac{d_{BC}}{L_{AB}}\}} \tag{C.5}$$

At this point:

$$M_{R_{CB}} = M_{CB} + 0.94\ M_{P_{AB}} \frac{d_{BC}}{L_{AB}} \tag{C.6}$$

In equations (C.5) and (C.6) the term $0.94\ M_{P_{AB}}$ is the change in moment which occurs at A in Figure C.1 as the structure sways until a second plastic hinge forms in girder AB.

A similar analysis is valid for girder DE. The restraining moment at the centroid of the wall, due to girder DE is:

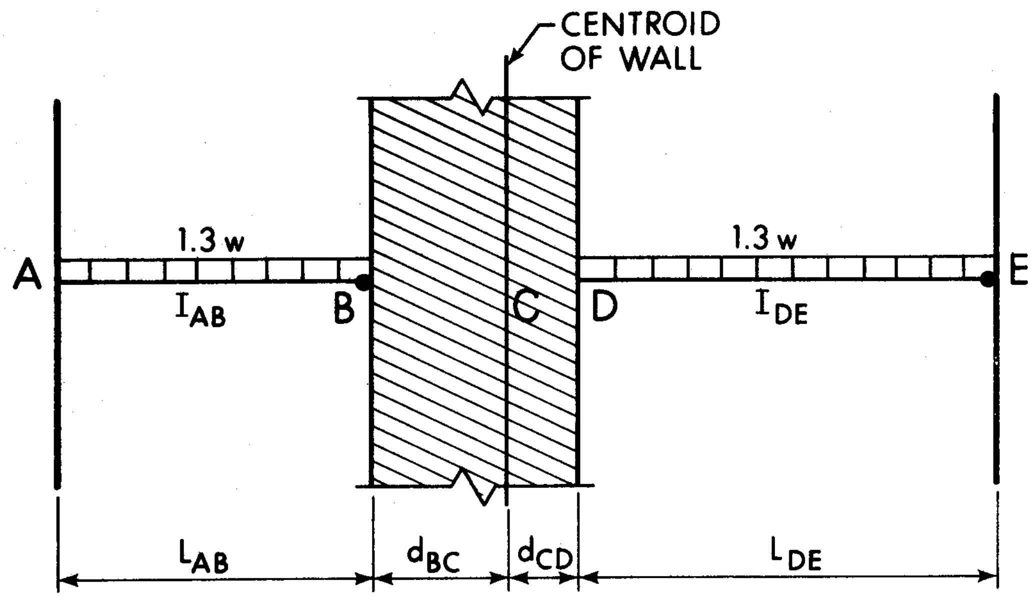
$$M_{R_{CD}} = M_{CD} + \frac{3EI_{DE}}{L_{DE}} \{1 + \frac{d_{CD}}{L_{DE}}\}^2 \rho \tag{C.7}$$

This response is terminated at the sway rotation:

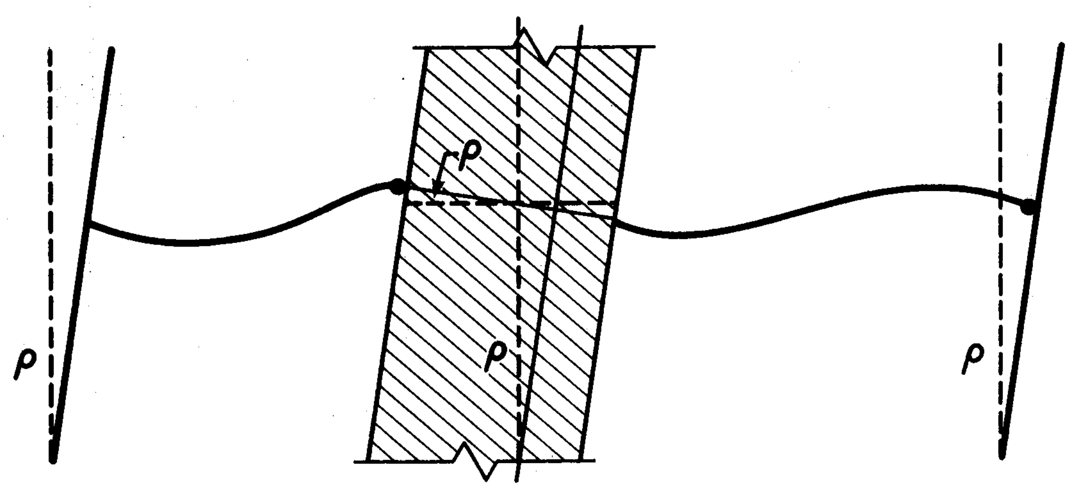
$$\rho = \frac{0.94\ P_{M_{DE}}}{\frac{3EI_{DE}}{L_{DE}} \{1 + \frac{d_{CD}}{L_{DE}}\}} \tag{C.8}$$

At this point, the girder has developed a second hinge, and the restraining moment on the wall is:

$$M_{R_{CD}} = M_{CD} + M_{P_{DE}} \{1 + \frac{d_{CD}}{L_{DE}}\} \tag{C.9}$$



(a)



(b)

FIGURE C.1 BEAMS FRAMING DIRECTLY INTO THE SHEAR WALL

APPENDIX D

DESIGN EXAMPLES

APPENDIX D

DESIGN EXAMPLES

In this appendix, two design examples based on the design method described in Chapter V are presented. The examples concern structures which become unstable following some degree of plastic moment redistribution between the wall and the frame. The examples therefore, illustrate the procedure described in section 5.3.2 of Chapter V.

The design examples are described in Chapter VI and illustrated in Figures 6.2 and 6.3, for examples one and two, respectively.

Example 1 - 12 Story Structure

The steps in the design procedure for proportioning the wall according to section 5.3.2 are:

STEP 1 Determine if the structure remains stable once the wall develops a plastic hinge.

$$P_c = \frac{12\ E}{(N+1)h^2} \sum_{i=1}^{m} \frac{I_i}{\frac{2L_i}{h} + 1/2} \qquad (3.6)$$

Column (1) of table D.1 gives the term:

$$\frac{I}{\frac{2L}{h} + 1/2}$$

for each of the columns of Figure 6.2 which offer resistance to sway once the wall develops a plastic hinge.

$$P_c = \frac{12 \times 30{,}000}{(12 + 1) \times 120 \times 120} \times 837.5 = 1610^{\text{KIPS}}$$

Total gravity load on each floor level is:

$$P_G = 1040 \text{ KIPS}$$

Since $P_G < P_c$, structure remains stable.

<u>STEP 2</u> The response of each column of the frame, which offers resistance to sway, is given in table D.2. For each column three values, K, V and ρ_u are tabulated which define the column response. K (KIPS) is the shear-sway rotation response for the column and is defined by equation A.12 for the columns in story 1, and by equation A.3 for the columns in the remaining stories. V(KIPS) is the ultimate shear capacity of the column, defined by equation A.5 or A.13 for the columns in story 1, and by equation A.5 or A.10 for the columns in the remaining stories. ρ_u (radians) is the story sway rotation at which the column develops its ultimate shear capacity and is defined as:

$$\rho_u = \frac{V}{K} \tag{D.1}$$

The response of the beams framing into the wall (Appendix C) are:

$$M_{R_{CB}} = 301 + 2970\,\rho \tag{C.4}$$

$$\rho = 0.010 \text{ radians} \tag{C.5}$$

$$M_{R_{CD}} = -481 + 14200\,\rho \tag{C.7}$$

$$\rho = 0.0158 \text{ radians} \tag{C.8}$$

The values ρ indicate the sway rotation at which a second hinge forms in the beams.

<u>STEP 3</u> Stage at which the Structure Becomes Unstable

From table D.2, columns 1 and 5 develop their capacities at $\rho \simeq 0.015$ radians. All columns have developed their capacities at $\rho \simeq 0.002$ radians. The stability of the structure must therefore be tested at $\rho \simeq 0.015$ where columns 1 and 5 offer no further resistance to sway. Referring to column (2) of table D.1 equation (3.6) gives:

$$P_c = 990^K < P_G .$$

Since $P_c \simeq P_G$ and equation (3.6) is conservative it is assumed that the structure is stable at this stage. ρ_f is therefore selected as 0.020 radians, since all columns develop their capacities at this sway rotation.

<u>STEPS 4 & 5</u> Steps 4 and 5 are tabulated in table D.3. In this table column (2) gives the total shear resistance of the frame portion in each story at $\rho_f = 0.020$ radians. Column (3) gives the applied shear due to the lateral load. Column (4) gives the shear due to the $P\Delta$ effect at $\rho_f = 0.020$ radians. This shear is imaginary and is used only to calculate the moments in the wall. Column (5) gives the net shear in the wall and is obtained by subtracting column (2) from the sum of columns (3) and (4) in each story. Column (6) gives the maximum moment in the wall in each story. Column (7) indicates the net restraining moment due to the girders framing into the wall, at $\rho = 0.020$ radians. Column (7) is added to column (6) to give column (8). This column gives the minimum moment required in each story of the wall to ensure that the structure reaches a L.F. = 1.3 under combined loads.

SECONDARY DESIGN CONSIDERATIONS

The requirements of Section 5.4 were checked. For serviceability, the analysis recommended in Section 5.4.3 was used. This indicated that the moment of inertia of the wall should be at least 700 ft^4 and that the wall should have the capacity given in column (9) of table D.3.

The analysis of Section 5.4.1 was used to check the stability of the structure under vertical loads alone (L.F. = 1.7). Column (10) gives the minimum moments required by this analysis.

The final design moments for the wall are given in column (11).

EXAMPLE 2

The 18 story structure of Figure 6.3 was designed in this example. The procedure was the same as in example 1. The major calculations are given in table D.4; the columns in this table correspond to those of table D.3.

For this structure ρ_f was selected as 0.015 radians. This meant that the design was based on the stage at which only half the columns in the frame have developed their capacities.

To satisfy the serviceability requirements the effective moment of inertia of the wall was adjusted to be 2100 ft^4.

TABLE D.1

COLUMN	(1)	(2)
1	27.0	0
2	43.5	43.5
3	47.0	47.0
4	54.0	0
$\sum$ =	837.5	513.5

TABLE D.1

STABILITY TERMS FOR COLUMNS OF FIGURE 6.2

	COLUMN 1			COLUMN 2			COLUMN 3			COLUMN 5		
STORY	K	V	ρ_u	K	V	ρ_u	K	V	ρ_u	K	V	ρ_u
1	5840	76.2	0.013	7500	19.0	0.0025	7920	171.5	0.0218	1168	15.24	0.013
2	3437	50.8	0.015	600	12.7	0.021	5815	114.3	0.020	687	10.16	0.015
3	3437	"	0.015	930	"	0.017	5815	"	0.020	687	"	0.015
4	3100	"	0.016	600	"	0.021	5400	"	0.021	620	"	0.016
5	3836	"	0.013	930	"	0.017	6815	"	0.018	767	"	0.013
6	3340	"	0.015	650	"	0.019	6510	"	0.021	668	"	0.015
7	3720	"	0.013	695	"	0.018	6485	"	0.018	756	"	0.013
8	3445	"	0.015	650	"	0.020	5760	"	0.021	689	"	0.015
9	3765	"	0.014	695	"	0.018	6355	"	0.018	753	"	0.014
10	3521	"	0.014	685	"	0.020	5845	"	0.020	703	"	0.014
11	3750	"	0.014	710	"	0.018	6365	"	0.018	749	"	0.014
12	3750	"	0.014	710	"	0.018	6365	"	0.018	749	"	0.014

TABLE D.2

RESPONSES OF COLUMNS OF FIGURE 6.2

(1) STORY	(2) V	(3) V_H	(4) $V_{P\Delta}$	(5) V_{WALL}	(6) M	(7) M_B	(8) M_{MIN}	(9) $M_{W.L.}$	(10) M_{STAB}	(11) M_{DESIGN}
1	264.4	19.5	22.6	-222.3	-2223	- 75.8	-2,300	- 774	416	2,300
2	188.0	58.5	45.2	- 84.3	-3066	-151.6	-3,218	- 877	638	3,218
3	188.0	97.5	67.8	- 22.7	-3293	-227.4	-3,520	- 877	867	3,520
4	188.0	136.5	90.4	38.9	-3293	-303.2	-3,596	- 678	1102	3,596
5	188.0	175.5	113.0	100.5	-2904	-379.0	-3,283	641	1343	3,283
6	188.0	214.5	135.6	162.1	-1899	-454.8	-2,353	1781	1588	2,353
7	188.0	253.5	158.2	223.7	1959	-530.6	1,428	3269	1835	3,259
8	188.0	292.5	180.8	285.3	4812	-606.4	4,206	5093	2081	5,093
9	188.0	331.5	203.4	346.9	8281	-682.2	7,600	7303	2323	7,600
10	188.0	370.5	226.0	408.5	12366	-758.0	11,608	9915	2560	11,608
11	188.0	409.5	248.6	470.1	17067	-838.8	16,233	12954	2785	16,233
12	188.0	448.5	271.2	531.7	22384	-909.6	21,474	16383	2793	21,474

TABLE D.3

SUMMARY OF DESIGN CALCULATIONS FOR SHEARWALL OF 12 STORY STRUCTURE

STORY	V	V_H	$V_{P\Delta}$	V_{WALL}	M	M_B	M_{MIN}	$M_{W.L.}$	M_{STAB}	M_{DESIGN}
1	213	19.5	17.	-177.	-1770	- 75.8	1846	-1066	415	1,846
2	156	58.5	33.9	- 63.6	-2406	-151.6	2558	-1362	631	2,558
3	158	97.5	50.9	- 9.6	-2502	-227.4	2730	-1362	854	2,730
4	152	136.5	67.9	52.4	-2502	-303.2	2805	-1347	1083	2,805
5	166	175.5	84.8	93.6	-1980	-379	2360	-1020	1317	2,360
6	152	214.5	101.8	163.9	-1042	-454.8	1497	576	1556	1,556
7	166	253.5	118.8	206.3	2,660	-530.6	2130	1854	1800	2,130
8	157	292.5	135.7	271.7	5,380	-606.4	4771	3457	2046	4,771
9	166	331.5	152.7	318.2	8,560	-682.2	7877	5392	2295	7,877
10	157	370.5	169.6	383.6	12,400	-758.0	11637	7667	2545	11,637
11	166	409.5	186.6	430.1	16,700	-833.8	15863	10290	2796	15,863
12	162	448.5	203.6	490.5	21,600	-909.6	20700	13269	3046	20,700
13	165	487.5	220.5	542.6	27,030	-985.4	26042	16612	3293	26,042
14	162	526.5	237.5	602.4	33,051	-1061.2	31990	20331	3537	31,990
15	165	565.5	254.5	654.6	39,600	-1137	38460	24435	3774	38,460
16	162	604.5	271.4	714.3	46,740	-1212.8	45530	28933	4005	45,530
17	166	643.5	288.4	766.4	54,404	-1288.6	53115	33837	4226	53,115
18	166	682.5	305.4	822	62,624	-1364.4	61040	39032	4232	61,040

TABLE D.4

SUMMARY OF DESIGN CALCULATIONS FOR SHEARWALL OF 18 STORY STRUCTURE

APPENDIX E

COMPUTER PROGRAM

APPENDIX E
COMPUTER PROGRAM

E.1 Description

In this Appendix the computer program described in step 6 of section 5.3.2 is presented. The program performs a second order elastic-plastic analysis of the analytical model, shown in Figure 5.2, in which the wall remains elastic. The analytical model represents a steel frame-shear wall structure; the translational and rotational springs (Figure 5.2) represent the action of the frame portion of the structure, and their responses are defined in Appendices A and C, respectively.

The analysis is similar to the iterative procedure described in Reference 15, and proceeds as follows:

1. Assume an initial deflected shape of the structure corresponding to a sway rotation of 0.003 radians in each story.

2. Compute the resistance of the translational and the rotational springs.

3. Calculate the net lateral forces acting on the wall (applied lateral forces minus frame resistance).

4. Calculate the deflected shape of the wall. This step includes the PΔ effect and consists of successive numerical integrations of the wall until convergence of the calculated deflected shape is obtained.

5. Recalculate the resistance of the translational and rotational springs.

6. Repeat steps 3 through 5 until compatibility between the wall and the frame; and equilibrium of applied, frame, and wall shears,

are satisfied.

Figures E.1 to E.4 present flow diagrams of the main program and the subroutines. Subroutine READIN reads in all the data necessary to describe the problem. This includes the size of the structure, the loads, and the properties of the translational and rotational springs. The data cards required are given in Section E.4. Subroutine FRAME calculates the resistance of the translational and rotational springs; and also, the net shear in the wall. Subroutine WALL performs the numerical integration of the wall to calculate the deflected shape.

A listing of the program is given in Section E.5 and typical output is given in Section E.6. The nomenclature for the program is defined in Section E.3.

E.2 Accuracy of the Analysis

The twelve and eighteen story structures described in Chapter VI and shown in Figures 6.2 and 6.3, respectively, were analyzed by the computer program described herein and by the program described in Reference 10. The load factor on gravity and lateral loads was 1.3. Figures E.5 and E.6 show the deflected shape and the bending moments in the wall, respectively, for the structure of Figure 6.2. The solid curve indicates results given by the analysis of Reference 10 and the dashed curve indicates the results of the present analysis. Figures E.7 and E.8 again compare the deflected shape and the bending moments in the wall, respectively; calculated by both methods, for the 18 story structure of Figure 6.3.

The better agreement indicated for the taller structure may be explained by the fact that the assumption of equal successive story sway rotations is valid for a greater proportion of stories in the 18 story structure than the 12 story structure. Generalizing, it may be concluded that the accuracy of the present analysis will increase with taller structures.

E.3 Nomenclature for Computer Program

BM(J)	Benging moment at level "j" in wall (K-FT).
CM(J)	Restraining moment due to beams framing into the wall (K-FT).
DEFLN(N,J)	Deflection of wall at level "j".
DELM(J)	Change in wall moment (K-FT).
DELV(J)	Change in wall shear (K).
E	Modulus of elasticity of wall (KSI).
ERTIA(J)	Moment of inertia of wall in story "j" (FT^4).
F(J)	First order wall shear (K).
FLAT(J)	Applied lateral force at level j (K).
FF(J)	Applied shear in story j (K).
FRAM(J)	Resisting shear of frame in story j (K).
FULT(J,I)	Capacity of i^{th} column in the j^{th} story of the frame (K). FULT=V where V is defined by equation A.13 (Appendix A) for the top story and by equation A.10 for the remaining stories.
FRAVM(J)	Restraining moment due to beams framing into the wall under gravity load only (K-FT). GRAVM = M_{CB} + M_{CD} where M_{CB} and M_{CD} are defined by equations (C.1) and (C.2), respectively, in Appendix C.

H(J)	Story height (FT).
HF(J,I)	Resisting shear of i^{th} column in the j^{th} story (KIPS). HF(J,I) = STIFF(J,I) x RO(J).
K	Number of stories.
L	= K + 1
N	Indicator of the number of cycles of iteration on the wall.
NCOL	Number of columns in the frame.
NR	Indicator of the number of iterations between wall and frame.
NSTRY	Number of stories.
P(J)	Vertical load applied at level j (K).
RO(J)	Story sway rotation.
ROF(J,I)	Story sway rotation which develops the capacity of the i^{th} column in the j^{th} story. ROM = $\frac{\rho_A + \rho_B}{2}$ where ρ_A and ρ_B are defined by equations (C.5) and (C.8), respectively, in Appendix C.
ROM(J)	Story sway rotation which develops the capacity of the beams framing into the wall.
ROT(J)	Rotation of wall at level j.
SOILK	Base spring constant (K-FT).
STIFF(J,I)	Initial response of the i^{th} column in the j^{th} story (K). STIFF = $\frac{V}{\rho}$ where V is defined by equation A.12 (Appendix A) for columns in the top story and by equation A.3 for columns in the remaining stories.
STIFM(J)	Rotational stiffness of beams framing directly into the wall (K-FT).

$$STIFM = \frac{M_{R_{CB}} + M_{R_{CD}} - GRAVM}{ROM}$$ where $M_{R_{CB}}$ and $M_{R_{CD}}$ are defined by equations (C.4) and (C.7), respectively, in Appendix C.

T(NR,J) Wall deflection after the NR^{th} iteration between the wall and the frame.

TP(J) Total vertical load at level j (K).

ULTM(J) Capacity of the beams framing into the wall (K-FT).

$ULTM = M_{R_{CB}} + M_{R_{CD}}$ where $M_{R_{CB}}$ and $M_{R_{CD}}$ are defined by equations (C.6) and (C.9), respectively, in Appendix C.

V(J) Second order shear in the wall (K).

E.4 Data Cards

The data cards are read in the following order:

1. Identification card (reproduces first 40 characters at top of output).
2. Number of columns in frame, number of stories, E, and the base spring constant.
   (2I5, F10.2, E11.4)
3. Story height, applied lateral force, applied vertical load, moment of inertia of wall, and response of beams framing into the wall.
   H, FLAT, P, ERTA, GRAVM, STIFM, ULTM, ROM (one card for each story; for definitions of GRAVM, STIFM, ULTM, and ROM see section E.3).
   (7F10.2, 1F10.9)
4. Column Initial Response, (one card for each story - read columns across each story in turn).
   STIFF(J,I) (See section E.3).
   (8F10.0)

5. Column Ultimate Shear Capacity, (one card for each story - read columns across each story in turn).

   FULT(J,I) (see Section E.3)

   (8F10.0)

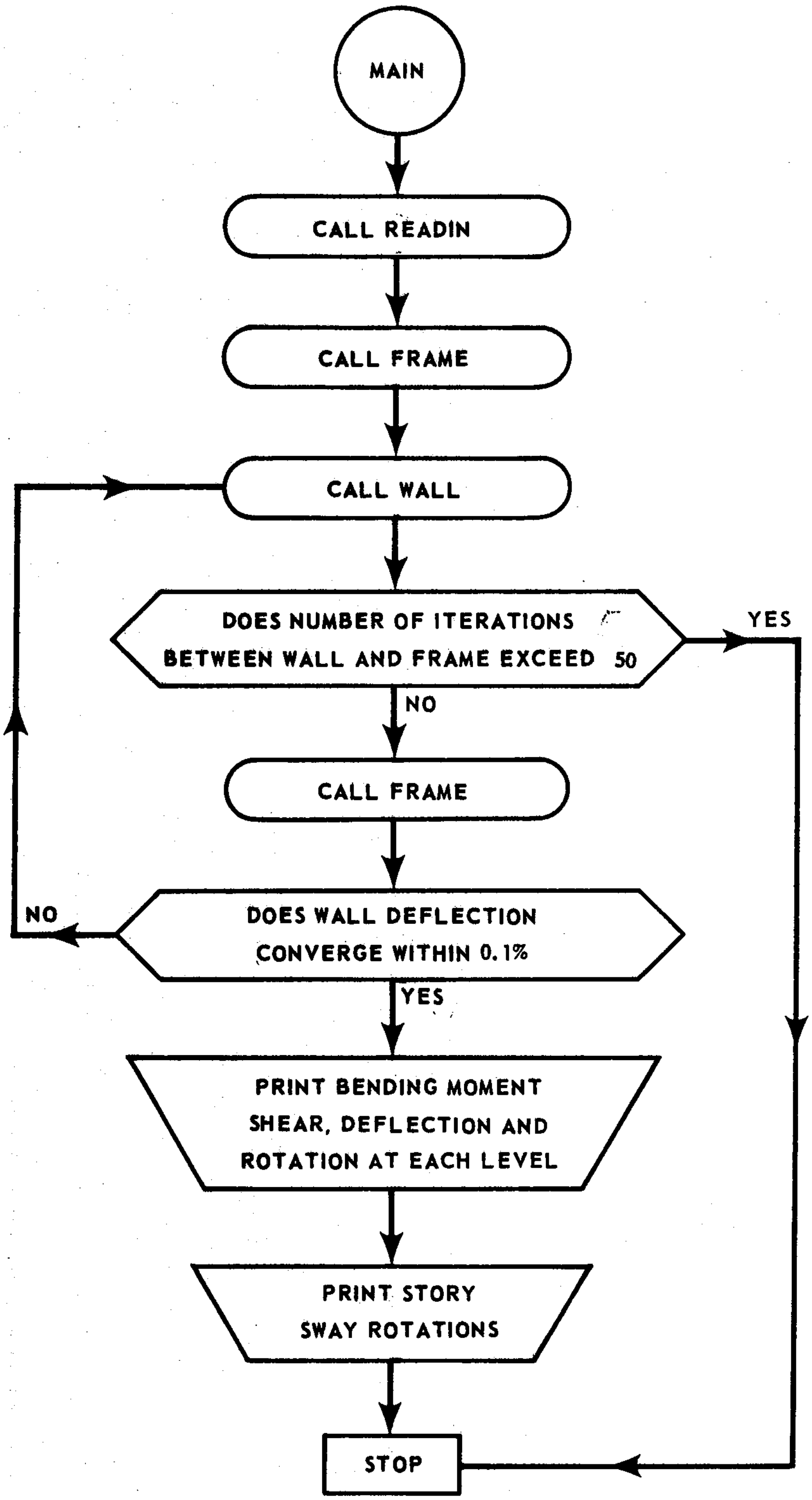


FIGURE E.1 FLOW DIAGRAM FOR THE COMPUTER PROGRAM

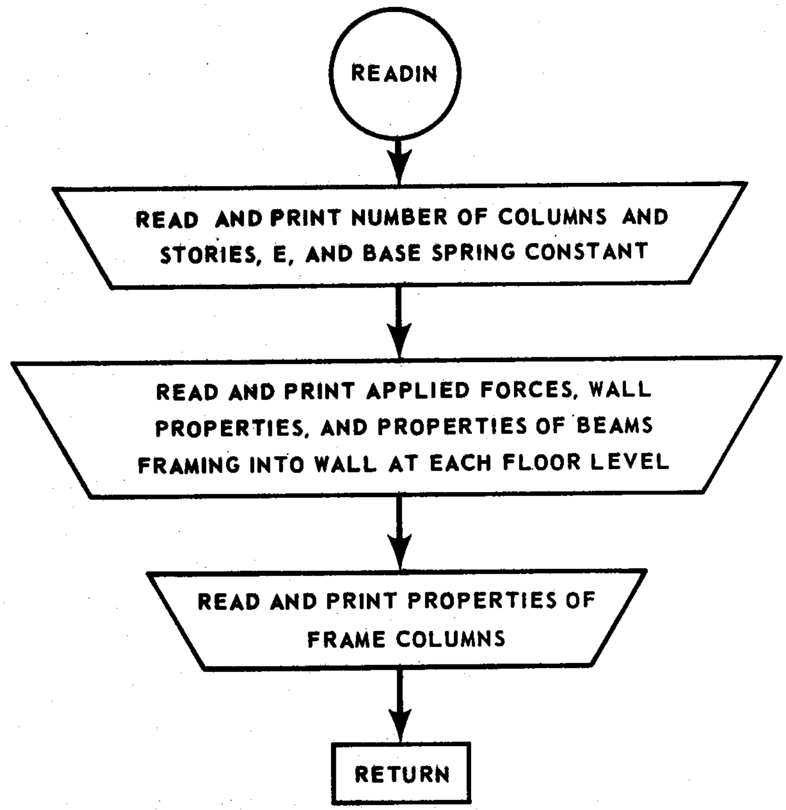


FIGURE E.2 FLOW DIAGRAM FOR SUBROUTINE 'READIN'

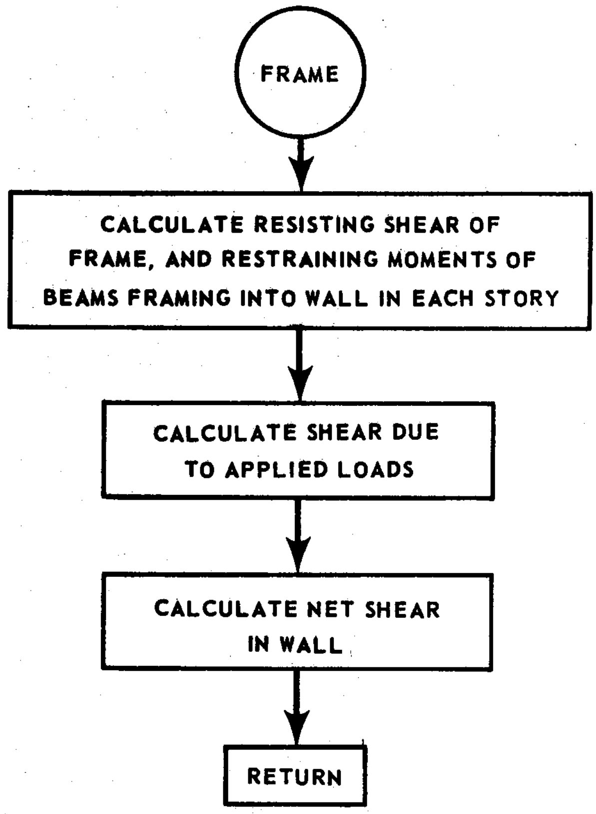


FIGURE E.3 FLOW DIAGRAM FOR SUBROUTINE 'FRAME'

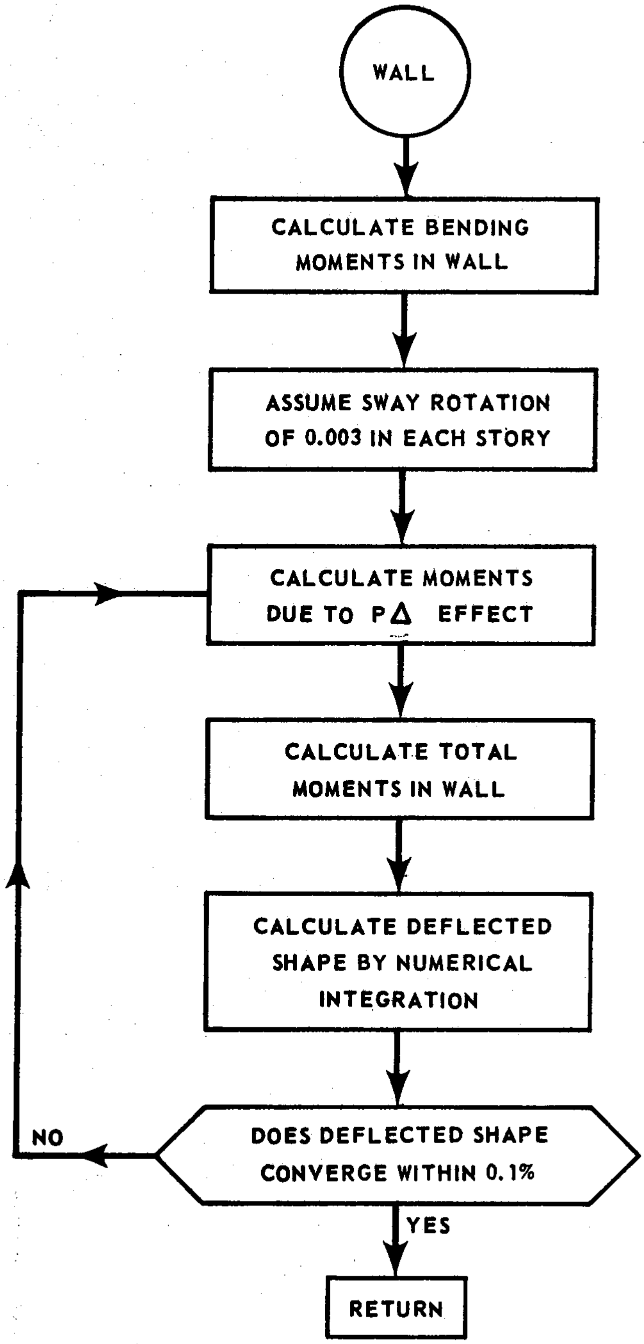


FIGURE E.4 FLOW DIAGRAM FOR SUBROUTINE 'WALL'

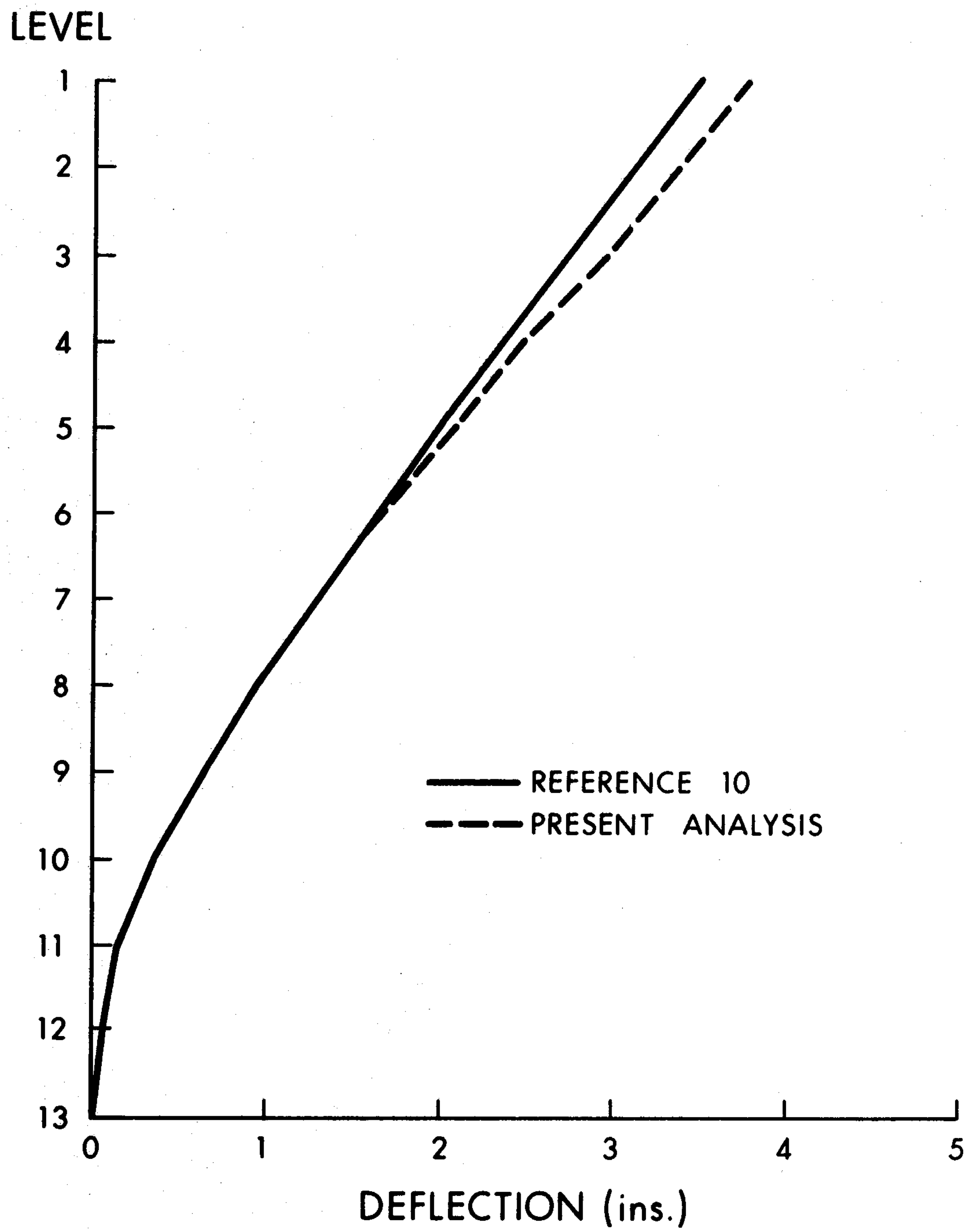


FIGURE E.5 DEFLECTED SHAPE 12 STORY STRUCTURE

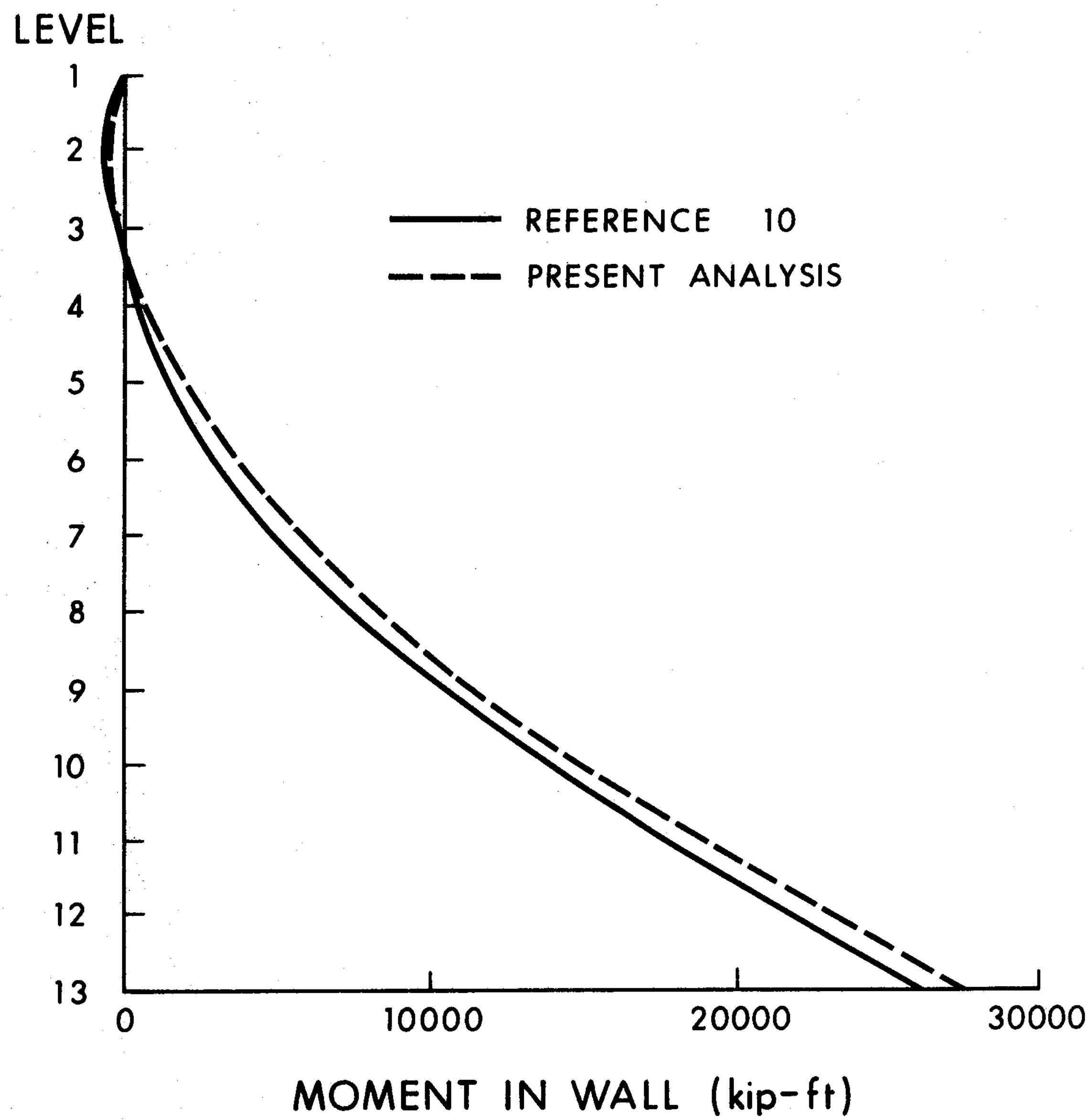


FIGURE E.6 BENDING MOMENT IN WALL 12 STORY STRUCTURE

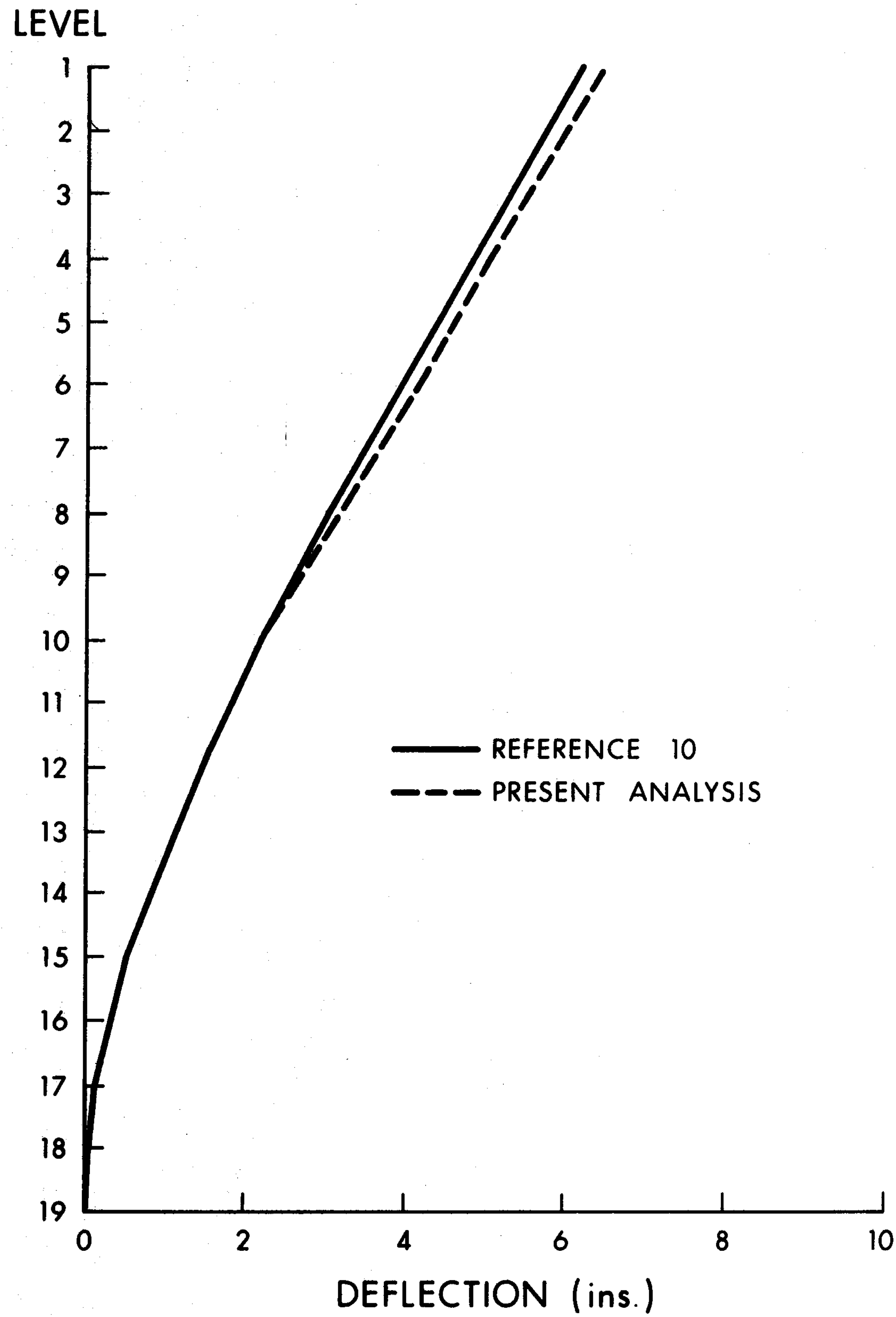


FIGURE E.7 DEFLECTED SHAPE 18 STORY STRUCTURE

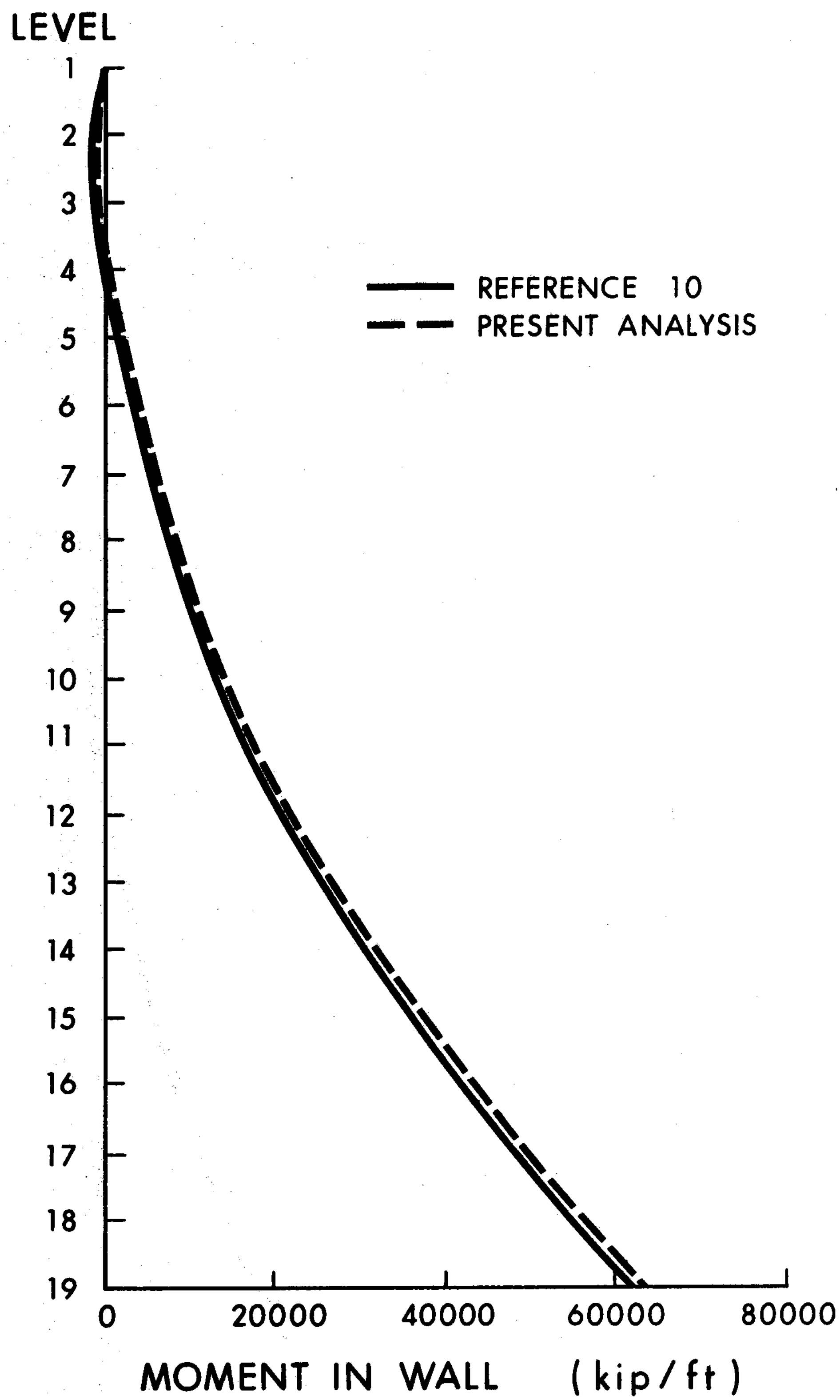


FIGURE E.8 BENDING MOMENT IN WALL 18 STORY STRUCTURE

```
      COMMON E,SOILK,NCOL,NSTRY,NN,NR,N,FLAT(40),P(40),ERTIA(40),H(40),C
     1M(40),GRAVM(40),STIFM(40),ROM(40),ULTM(40),STIFF(40,8),FULT(40,8),
     2ROF(40,8),RO(40),F(40),BM(40),ROT(40),DEFLN(50,40),T(50,40)
      CALL READIN
      DO 1 J=1,NSTRY
      T(1,J)=0.0
1     RO(J)=0.003
      NR=1
      CALL FRAME
3     NR=NR+1
      CALL WALL
      IF(NR.EQ.51) GOTO 4
      CALL FRAME
      DO 2 J=1,NSTRY
      TEST=ABS((T(NR-1,J)-T(NR,J))/T(NR,J))
      IF(TEST.GT.0.001) GOTO 3
2     CONTINUE
      WRITE(6,1000) NR
      WRITE(6,2074)
      L=NSTRY+1
      DO 18 J=1,L
      DEFLN(N,J)=12.*DEFLN(N,J)
18    WRITE(6,2300) J,BM(J),F(J),DEFLN(N,J),ROT(J)
      WRITE(6,4000)
      DO 19 J=1,NSTRY
19    WRITE(6,4001) J,RO(J)
      GOTO 6
4     WRITE(6,5)
6     STOP
5     FORMAT(1H0,'NUMBER OF PERMISSABLE CYCLES BETWEEN FRAME AND WALL EX
     1CEEDED')
1000  FORMAT(1H1,'NUMBER OF ITERATIONS BETWEEN FRAME AND WALL = ',I3)
2074  FORMAT(1H0,'LEVEL BENDING MOMENT        SHEAR     DEFLECTION     ROTAT
     1ION ')
2300  FORMAT(' ',I4,F16.2,F12.2,4X,E11.4,4X,E11.4)
4000  FORMAT(1H0,'STORY  SWAY ROTATION')
4001  FORMAT(' ',I3,4X,E11.4)
      END
```

```
C
C
      SUBROUTINE READIN
C
      COMMON E,SOILK,NCOL,NSTRY,NN,NR,N,FLAT(40),P(40),ERTIA(40),H(40),C
     1M(40),GRAVM(40),STIFM(40),ROM(40),ULTM(40),STIFF(40,8),FULT(40,8),
     2ROF(40,8),RO(40),F(40),BM(40),ROT(40),DEFLN(50,40),T(50,40)
      REAL*8 IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
      WRITE(6,2000)
       READ(5,1060) IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
      WRITE(6,1061) IDENT1,IDENT2,IDENT3,IDENT4,IDENT5
      WRITE(6,2002)
      READ(5,1000) NCOL,NSTRY,E,SOILK
      WRITE(6,2003) NCOL,NSTRY
      WRITE(6,2004) E,SOILK
      E=E*144.
      WRITE(6,2005)
      DO 1 M=1,NSTRY
         READ(5,1234) H(M),FLAT(M),P(M),ERTIA(M),GRAVM(M),STIFM(M),ULTM(
     1M),ROM(M)
1     WRITE(6,1001) M,H(M),FLAT(M),P(M),ERTIA(M),GRAVM(M),STIFM(M),ULTM(
     1M),ROM(M)
      WRITE(6,1002)
      WRITE(6,1003)
      DO 2 J=1,NSTRY
      READ(5,1007) (STIFF(J,I),I=1,NCOL)
2     WRITE(6,1009) J,(STIFF(J,I),I=1,NCOL)
      WRITE(6,1005)
      WRITE(6,1003)
      DO 3 J=1,NSTRY
      READ(5,1007) (FULT(J,I),I=1,NCOL)
      WRITE(6,1009) J,(FULT(J,I),I=1,NCOL)
      DO 3 I=1,NCOL
3     ROF(J,I)=FULT(J,I)/STIFF(J,I)
      RETURN
2000  FORMAT(1H1,'********************************************')
1060  FORMAT(5A8)
1061  FORMAT(1H0,5A8)
2002  FORMAT(1H0,'********************************************')
1000  FORMAT(2I5,F10.2,E11.4)
2003  FORMAT(1H0,I5,' COLUMNS',I5,' STORIES')
2004  FORMAT(1H0,'E = ',F10.2,5X,'  BASE SPRING CONSTANT = ',E11.4)
2005  FORMAT(1H0,'STORY  HT.     FLAT        P         ERTIA        GRAVM
     1STIFM      ULTM        ROM    ')
1001  FORMAT(I3,2X,F5.2,6F10.2,F10.4)
1234  FORMAT(7F10.2,1F10.9)
1002  FORMAT(1H0,'COLUMN  INITIAL  RESPONSE')
1003  FORMAT(1H0,'STORY  COLUMN =         1            2            3
     1      4            5            6            7            8       ')
1007  FORMAT(8F10.0)
1005  FORMAT(1H0,' COLUMN  ULTIMATE  CAPACITY')
1009  FORMAT(I5,10X,8F12.2)
      END
```

```
C
C
       SUBROUTINE FRAME
C
       COMMON E,SOILK,NCOL,NSTRY,NN,NR,N,FLAT(40),P(40),ERTIA(40),H(40),C
      1M(40),GRAVM(40),STIFM(40),ROM(40),ULTM(40),STIFF(40,8),FULT(40,8),
      2ROF(40,8),RO(40),F(40),BM(40),ROT(40),DEFLN(50,40),T(50,40)
       DIMENSION FRAM(40),HF(40,8),FF(40)
       DO 1 J=1,NSTRY
       FRAM(J)=0.0
       DO 1 I=1,NCOL
       IF(RO(J).GT.ROF(J,I)) GOTO 2
       HF(J,I)=STIFF(J,I)*RO(J)
       GOTO 1
2      HF(J,I)=FULT(J,I)
1      FRAM(J)=FRAM(J)+HF(J,I)
        DO 3 J=1,NSTRY
       IF(RO(J).GT.ROM(J)) GOTO 4
       CM(J)=GRAVM(J)+STIFM(J)*RO(J)
       GOTO 5
4       CM(J)=ULTM(J)
5      FF(J)=0.0
       DO 6 L=1,J
6      FF(J)=FF(J)+FLAT(L)
       F(J)=FF(J)-FRAM(J)
3      CONTINUE
       RETURN
       END
```

```
C
C
      SUBROUTINE WALL
C
      COMMON E,SOILK,NCOL,NSTRY,NN,NR,N,FLAT(40),P(40),ERTIA(40),H(40),C
     1M(40),GRAVM(40),STIFM(40),ROM(40),ULTM(40),STIFF(40,8),FULT(40,8),
     2ROF(40,8),RO(40),F(40),BM(40),ROT(40),DEFLN(50,40),T(50,40)
      DIMENSION DELM(40),DELV(40),V(40),TP(40)
      N=1
      K=NSTRY
      L=K+1
      V(1)=F(1)
      BM(1)=-CM(1)
      TP(1)=P(1)
      DELM(1)=0.0
      DEFLN(1,L)=0.0
       DO 2 J=2,K
      V(J)=F(J)
      TP(J)=TP(J-1)+P(J)
2      BM(J)=BM(J-1)+V(J-1)*H(J-1)-CM(J)
      V(L)=F(K)
       BM(L)=BM(K)+V(K)*H(K)
      DO 8 J=1,K
8     DEFLN(1,K-J+1)=DEFLN(1,K-J+2)+H(K-J+1)*0.003
      DELV(1)=P(1)*0.003
      DO 9 J=2,K
      DELM(J)=DELM(J-1)+TP(J-1)*H(J-1)*0.003
9     DELV(J)=TP(J)*0.003
      DELM(L)=DELM(K)+TP(K)*H(K)*0.003
      DELV(L)=DELV(K)
100   N=N+1
      IF(N.EQ.50) GOTO 101
      DO 16 J=1,L
      BM(J)=BM(J)+DELM(J)
16    V(J)=V(J)+DELV(J)
      ROT(L)=BM(L)/SOILK
      DEFLN(N,L)=0.0
      DO 12 J=1,K
      ROT(K-J+1)=ROT(K-J+2)+H(K-J+1)*(2.*BM(K-J+1)+V(K-J+1)*H(K-J+1))/(2
     1.*E*ERTIA(K-J+1))
12    DEFLN(N,K-J+1)=DEFLN(N,K-J+2)+ROT(K-J+2)*H(K-J+1)+(BM(K-J+1)*H(K-J
     1+1)**2/2.+V(K-J+1)*H(K-J+1)**3/3.)/(E*ERTIA(K-J+1))
      DELV(1)=P(1)*(DEFLN(N,1)-DEFLN(N-1,1)-DEFLN(N,2)+DEFLN(N-1,2))/H(1
     1)
      DO 15 J=2,K
      DELM(J)=DELM(J-1)+TP(J-1)*(DEFLN(N,J-1)-DEFLN(N-1,J-1)-DEFLN(N,J)+
     1DEFLN(N-1,J))
15    DELV(J)=TP(J)*(DEFLN(N,J)-DEFLN(N-1,J)-DEFLN(N,J+1)+DEFLN(N-1,J+1)
     1)/H(J)
      DELM(L)=DELM(K)+TP(K)*(DEFLN(N,K)-DEFLN(N-1,K))
      DELV(L)=DELV(K)
       DO 17 J=1,K
      TEST=ABS((DEFLN(N,J)-DEFLN(N-1,J))/DEFLN(N,J))
      IF (TEST.GT.0.001) GOTO 100
17    CONTINUE
      DO 5 J=1,K
5     T(NR,J)=DEFLN(N,J)
      DO 1 J=1,K
1      RO(J)=(T(NR,J)-T(NR,J+1)+T(NR-1,J)-T(NR-1,J+1))/(2.*H(J))
      GOTO 26
101   WRITE(6,1000)
      WRITE(6,1001)
      STOP
26    RETURN
1000  FORMAT(1H0,'NUMERICAL INTEGRATION ON WALL FAILS TO CONVERGE AFTER
     1 50 CYCLES')
1001  FORMAT('STRUCTURE IS UNSTABLE')
      END
```

****

18 STORY FRAME SHEARWALL STRUCTURE

****

4 COLUMNS 18 STORIES

E = 4500.00 BASE SPRING CONSTANT = 0.5000E 20

STORY	HT.	FLAT	P	ERTIA	GRAVM	STIFM	ULTM	ROM
1	10.00	15.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
2	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
3	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
4	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
5	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
6	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
7	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
8	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
9	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
10	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
11	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
12	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
13	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
14	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
15	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
16	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
17	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100
18	10.00	30.00	1130.00	350.00	-160.00	24000.00	76.00	0.0100

COLUMN INITIAL RESPONSE

STORY	COLUMN = 1	2	3	4	5
1	5840.00	7500.00	7920.00	1168.00	
2	3500.00	650.00	5800.00	700.00	
3	3500.00	650.00	5800.00	700.00	
4	3500.00	650.00	5800.00	700.00	
5	3500.00	650.00	5800.00	700.00	
6	3500.00	650.00	5800.00	700.00	
7	3500.00	650.00	5800.00	700.00	
8	3500.00	650.00	5800.00	700.00	
9	3500.00	650.00	5800.00	700.00	
10	3500.00	650.00	5800.00	700.00	
11	3500.00	650.00	5800.00	700.00	
12	3500.00	650.00	5800.00	700.00	
13	3500.00	650.00	5800.00	700.00	
14	3500.00	650.00	5800.00	700.00	
15	3500.00	650.00	5800.00	700.00	
16	3500.00	650.00	5800.00	700.00	
17	3500.00	650.00	5800.00	700.00	
18	3500.00	650.00	5800.00	700.00	

COLUMN ULTIMATE CAPACITY

STORY	COLUMN = 1	2	3	4	5
1	76.20	19.00	154.00	15.30	
2	50.80	12.70	114.00	10.20	
3	50.80	12.70	114.00	10.20	
4	50.80	12.70	114.00	10.20	
5	50.80	12.70	114.00	10.20	
6	50.80	12.70	114.00	10.20	
7	50.80	12.70	114.00	10.20	
8	50.80	12.70	114.00	10.20	
9	50.80	12.70	114.00	10.20	
10	50.80	12.70	114.00	10.20	
11	50.80	12.70	114.00	10.20	
12	50.80	12.70	114.00	10.20	
13	50.80	12.70	114.00	10.20	
14	50.80	12.70	114.00	10.20	
15	50.80	12.70	114.00	10.20	
16	50.80	12.70	114.00	10.20	
17	50.80	12.70	114.00	10.20	
18	50.80	12.70	114.00	10.20	

NUMBER OF ITERATIONS BETWEEN FRAME AND WALL = 25

LEVEL	BENDING MOMENT	SHEAR	DEFLECTION	ROTATION
1	-67.73	-146.38	0.1696E 02	0.9529E-02
2	-1485.62	-57.18	0.1582E 02	0.9561E-02
3	-1906.56	-28.03	0.1467E 02	0.9635E-02
4	-1926.93	1.11	0.1351E 02	0.9717E-02
5	-1544.63	30.44	0.1234E 02	0.9792E-02
6	-756.18	60.13	0.1116E 02	0.9841E-02
7	442.41	90.39	0.9976E 01	0.9847E-02
8	2054.92	121.40	0.8797E 01	0.9790E-02
9	4084.05	153.37	0.7629E 01	0.9653E-02
10	6530.61	186.48	0.6484E 01	0.9418E-02
11	9392.75	220.93	0.5373E 01	0.9066E-02
12	12665.04	256.91	0.4313E 01	0.8579E-02
13	16337.69	294.62	0.3321E 01	0.7939E-02
14	20395.73	334.22	0.2415E 01	0.7129E-02
15	24818.26	375.89	0.1617E 01	0.6133E-02
16	29577.66	419.78	0.9510E 00	0.4935E-02
17	34638.98	466.04	0.4415E 00	0.3521E-02
18	39959.34	514.78	0.1151E 00	0.1880E-02
19	45302.48	-0.00	0.0	0.9060E-15

STORY	SWAY ROTATION
1	0.9538E-02
2	0.9594E-02
3	0.9674E-02
4	0.9755E-02
5	0.9818E-02
6	0.9847E-02
7	0.9823E-02
8	0.9728E-02
9	0.9543E-02
10	0.9251E-02
11	0.8833E-02
12	0.8271E-02
13	0.7548E-02
14	0.6646E-02
15	0.5550E-02
16	0.4246E-02
17	0.2719E-02
18	0.9594E-03