Gain Analysis of Cooperative Broadcast in Two-Dimensional Wireless Ad Hoc Networks

by

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Abstract

Broadcast is a fundamental operation in wireless networks widely used to disseminate a message to every node in the network. Energy accumulation is an approach to reducing the power consumption of broadcast. In conventional approaches, a node can decode the message if the received power from a single transmission is above a threshold. In contrast, in the cooperative approach based on energy accumulation, a node can decode the message if the sum of the received powers from any set of transmissions exceeds the threshold. An important question is how much energy can be saved in broadcast if energy accumulation is employed. Since employing energy accumulation adds extra design complexities, answering this question can help in deciding whether or not it should be implemented. Previously, it was shown that this saving is limited in linear wireless networks, irrespective of the network size, and the location of the nodes in the network. In this work, however, we show that this saving can increase with the network size in two-dimensional networks. Also, despite the fact that both problems of cooperative and non-cooperative broadcast with minimum energy are NP-hard, we establish a bound on the maximum saving that can be obtained.

"There is a powerful driving force inside every human being that, once unleashed, can make any vision, dream, or desire a reality"

- - Anthony Robbins

Acknowledgements

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Contents

List of Figures

List of Abbreviations

List of commonly used abbreviations

2D	Two Dimensional
NP-hard	Non-deterministic Polynomial-time hard
MRC	Maximal Ratio Combining
AWGN	Additive White Gaussian Noise
IoT	Internet of Things
M2M	Machine-to-Machine

List of Symbols

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s	The source node
$\rho^{(c)}(.)$	Transmit power (cooperative broadcast)
$\rho^{(n)}(.)$	Transmit power (non-cooperative broadcast)
G_{tot}	Cooperation Gain – Definition 4
$\mathcal{R}(u)$	Closest node to u that transmits before u – (3.4)
D_u	Disk with radius $d_{u,\mathcal{R}(u)}$ centred at node u – (3.5)
$\mathcal{D}$	Set of all disks D_u, $u \in \mathscr{U} \setminus \{s\}$
$\mathcal{I}$	A subset of non-overlapping disks in $\mathcal{D}$
$f_u(.)$	Received power from node u capped at one – Definition 5
$\mathcal{F}(.)$	$\sum_{u \in \mathscr{U}} f_u(.)$

Chapter 1

Introduction

Today's application of wireless ad hoc networks ranges from Smart Grid [1, 2] and Internet of Things (IoT) [3, 4] to Machine-to-Machine (M2M) communications networks [5, 6] and smart environments [7, 8]. A fundamental communication operation widely used in these applications is broadcast in which a message is sent from a source node to all other nodes in the network. In a wireless ad hoc network, some nodes may be out of the transmission range of the source node. In this case, not only the source node but also a subset of other nodes that receive the message need to transmit the message to deliver the message to everyone.

Every time a message is broadcast, many transmissions occur in the network. Since each of these transmissions consumes power, and wireless devices (e.g., mobile devices) typically have limited energy supplies (they run on batteries), reducing the total power consumption of broadcast is of great interest.

To reduce the power consumption of broadcast, different approaches have been taken in the literature. These approaches are mainly based on transmission power control [9], and reduction in the number of redundant transmissions [10, 11].

A third approach, studied in this work, is energy accumulation, which is one of the main approaches in cooperative broadcast [12]. In cooperative broadcast with energy accumulation, which is simply referred to as cooperative broadcast, a node can decode the message if the sum of the received powers from any set of transmissions of the message is above the decoding threshold [13]. This generalizes the conventional non-cooperative broadcast algorithms in which a node can decode the message only if the received power from a single transmission of the message is above the decoding threshold. For example, suppose the decoding threshold is one, and a node u receives three transmitted signals with powers 0.5, 0.4, and 0.3. With energy accumulation, node u is able to decode since the sum of the received powers is 1.2, which is above the threshold. Without energy accumulation, however, decoding is not possible because, for each transmission, the received power at u is strictly less than one.

Energy accumulation can let transmitters reduce their transmission powers while still assuring other nodes will receive the message. This, as shown in [12] and [14] through numerical analysis, can reduce the power consumption of broadcast, hence saving energy. Theoretical analysis of the amount of this energy saving, which is the subject of this work, can shed more light on how much gain/saving can be achieved using energy accumulation.

If the minimum energy consumption of cooperative broadcast and non-cooperative broadcasts are P_c and P_n, respectively, then the cooperation gain (i.e., the energy saving) is defined as $\frac{P_n}{P_c}$. To evaluate the cooperation gain, therefore, we need to find values of P_n and P_c.

A major challenge in analyzing the cooperation gain is that the minimum-energy cooperative broadcast and the minimum energy non-cooperative broadcast are both NP-hard problems [15, 16]. To address this challenge, we first introduce a novel conversion method that transforms any cooperative broadcast algorithm into a non-cooperative broadcast algorithm. Then, we prove that the energy consumption of the converted non-cooperative algorithm is at most a $\mathcal{O}(\log n)$ factor higher than that of the cooperative algorithm, where n denotes the total number of nodes in the network. In other words, we show that if there is a cooperative broadcast algorithm with total power consumption of P_c, then there is a non-cooperative algorithm with total energy consumption of $P_n = P_c.\mathcal{O}(\log n)$. This implies that the cooperation gain is $\mathcal{O}(\log n)$.

In [17], it was proven that the cooperation gain is limited (constant) in linear wireless networks, irrespective of the number of nodes, and their locations in the network. In this work, we show that this limitation does not extend to two-dimensional (2D) networks. The main contributions of this work are two folds.

1. In 2D grid networks, we prove that the cooperation gain grows logarithmically with the number of nodes. Further, using numerical analysis, we show the same logarithmic growth in *random networks*. This establishes a *lower bound* of $\Omega(\log n)$ on the cooperation gain achievable in 2D networks.

2. Through a novel approach, we prove that the cooperation gain grows at most logarithmically with the number of nodes in any 2D network. This establishes an *upper bound* of $\mathcal{O}(\log n)$ on the maximum cooperation gain achievable in 2D networks. We remark that this upper bound is proven despite the fact that computing the minimum power consumption of both cooperative broadcast and non-cooperative broadcast is NP-hard [15, 16].

1.1 Related Work

Cooperative broadcast with energy accumulation can be performed at receivers utilizing maximal ratio combining (MRC) of orthogonal signals in time, frequency or code domain (see [18, 14, 13]). Existing cooperative broadcast algorithms using energy accumulation fall into two groups. The first group includes algorithms (e.g., [19, 18]) in which receiving nodes can combine signals from all previous transmissions to benefit from transmission diversity. These algorithms are called cooperative broadcast algorithms with memory. The other group includes "memoryless" cooperative broadcast algorithms such as the one proposed in [14]. In these algorithms, a node can only use transmissions in the present time slot to accumulate energy; Signals received from transmissions in previous time slots are discarded. The cooperative broadcast algorithms studied in this thesis are with memory.

The problem of cooperative broadcast with minimum energy can be broken into two sub-problems i) transmission scheduling, which determines the set of transmitters and the order of transmissions; ii) power allocation, in which the transmission powers are set. It was proven that, given a transmission scheduling, optimal power allocation can be computed in polynomial time, but finding an optimal scheduling that leads to a minimum power consumption is NP-hard [19, 12]. Minimizing the broadcast delay while keeping the power consumption at minimum was proven not only NP-hard but also $o(\log n)$ inapproximable [14].

The problem of finding non-cooperative broadcast with minimum energy was also proven to be NP-hard [16]. The best existing approximation algorithm to the problem was proposed by Caragiannis *et al.*[20]. In 2D wireless networks, their algorithm has a constant approximation ratio of 4.2 for Euclidean cost graphs, and a logarithmic approximation for non-Euclidean cost graphs.

Unlike 2D networks, in linear networks, the minimum energy of both cooperative and non-cooperative broadcast algorithms can be computed in polynomial time. For linear networks, the ratio of the two minimum power consumptions was proven to be constant with respect to the number of nodes in the network [17]. Our work, extends that study to 2D networks, where we show that the gain increases with the number of nodes.

1.2 Thesis Outline

This thesis is organized as follows. We provide the necessary background and fundamental concepts in Chapter 2. The core of our work is presented in Chapter 3 where we initially introduce the system model and definitions. Next, we analyze cooperation gain in 2D wireless networks and provide an upper bound for it. We take our analysis one step further and study the cooperation gain achievable in 2D grid networks. In Chapter 4, we verify our analytical results through numerical analysis. Finally, in Chapter 5, we conclude the thesis and introduce directions where our work can be extended.

Chapter 2

Background

2.1 Wireless Channels

2.1.1 Additive White Gaussian Noise Channel

Additive White Gaussian Noise (AWGN) is a basic and generally accepted model for thermal noise in communication channels. The main attributes of AWGN are

- The noise is additive since the received signal is a combination of the transmitted signal plus some noise where the noise is statistically independent of the signal.

- The noise is white, meaning that it has a constant power spectral density. Therefore, the autocorrelation of the noise, in the time domain, is zero for any time offset $\tau \neq 0$.

- The distribution of the noise samples is Gaussian.

A channel is called AWGN if its noise can be modeled as above.

2.1.2 Fading Channels

In wireless networks, the the signal sent from the transmitter experiences several paths with different path losses and phase shifts until received at the receiver's side. The transmission radios can be categorized into two general groups of i) (Line-Of-Sight) LOS, and ii) (Non-Line-of-Sight) NLOS.

A transmission is called line-of-sight (LOS) when there is a direct path between the transmitting node and the receiver, and the signal travels a free space without being impacted by any physical obstacles. On the other hand, a signal propagation is considered NLOS when the transmitted signal is partially obstructed by physical objects between transmitter and receiver like buildings, clouds, and etc. Signals traveling through a NLOS link experience phenomena such as reflection, diffraction, scattering, absorption and refraction. Figure 2.1 is an example of different paths between the transmitter and receiver.

The path loss associated with LOS transmission, referred to as large-scale fading, corresponds to attenuation of average received power. Let P_t be the transmit power. The power received under large-scale fading can be modeled using *Friis Free Space* equation as:

$$P_r = P_t G_t G_r (\frac{\lambda}{4\pi d})^{\alpha}, \tag{2.1}$$

where G_t and G_r are the transmit and receive antenna gains, respectively. λ is wavelength, d is the distance between transmitter and receiver, and α, the path loss exponent, usually has a value between 2 and 6. The path loss model can be simplified as

$$P_r = \beta \frac{P_t}{d^{\alpha}}, \tag{2.2}$$

where distance d is greater than Fraunhofer Distance (see Section 2.1.3), and parameter β depends on the frequency and other factors. We refer to this model as the simplified path-loss model. In Figure 2.2, the dashed line shows the average received power decreasing with distance.

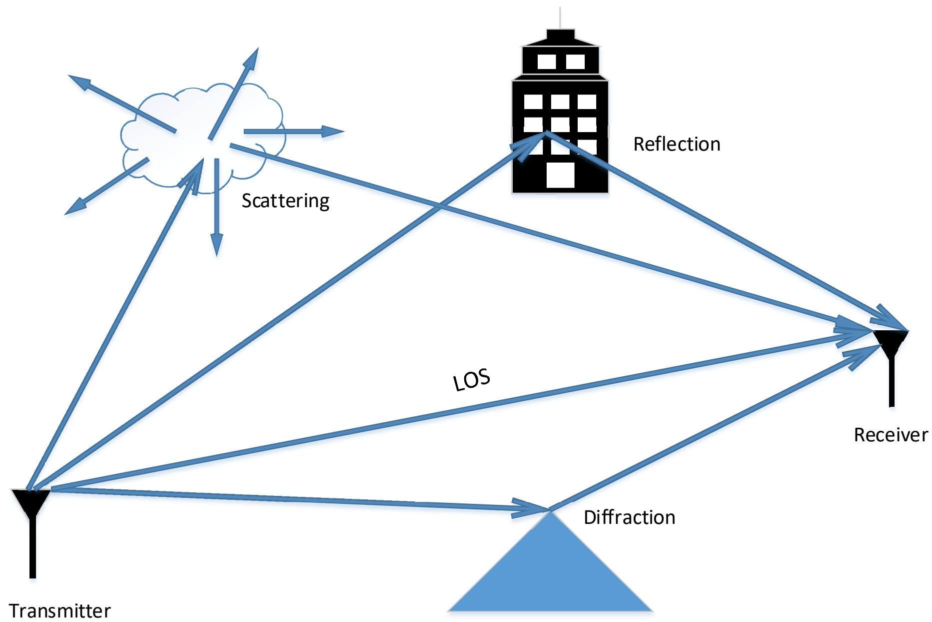

Figure 2.1: Common propagation mechanisms in wireless transmission

Signal transmission in an actual network is, however, not limited to LOS propagation. In other words, the received signal is composed of NLOS versions of the message (multi-path waves) as well as its LOS version. Traveling through diverse paths, multi-path waves arrive at slightly different times, and they can vary a lot in phase and amplitude. Therefore, their combination can result in rapid fluctuations in power and amplitude over short period of time or short travel distance. This characterization is called small-scale fading which is also referred to as fading. Figure 2.2 shows the received power under both large-scale and small-scale fading.

The randomness that is associated with fading requires employment of statistical models to study the behavior of the power and amplitude of the received

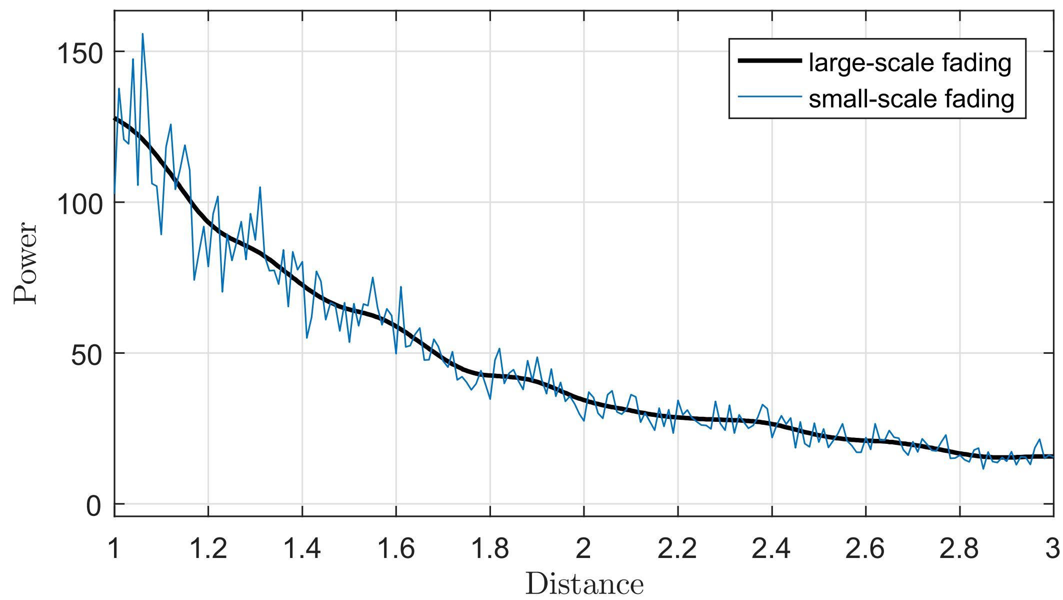

Figure 2.2: Received signal under fast and slow fading

signal. Some of the important models in the literature are

- Rayleigh fading model
- Ricean fading model
- Frequency selective fading models

Rayleigh fading model is used to model flat fading channels where there is no LOS between the transmitter and the receiver. Let $r(t)$ and $s(t)$ be received and transmitted signals, respectively. The discrete-time inputoutput, under Rayleigh fading model, is

$$r(t) = \alpha s(t) + n(t), \tag{2.3}$$

where path gain α is a complex zero-mean Gaussian random variable, and n_t the additive noise component which usually has Gaussian distribution.

Ricean fading model is another model used to analyze flat fading channels. This model is used when there exists dominant stationary component in addition to random multiple paths. The relationship between the baseband signals using this model is similar to 2.3. The main difference is that the path gain α is a complex Gaussian random variable with imaginary and real parts having non-zero mean.

Frequency Selective fading models. The inter-symbol interference of Frequency Selective fading channels can be modeled by the sum of delta functions that fade independently. Thereby, the discrete-time inputoutput relationship of such channels is

$$r(t) = \sum_{j=0}^{J-1} \alpha^j s(t-j) + n(t), \tag{2.4}$$

where α^js are independent path gains with complex Gaussian distributions.

2.1.3 Fraunhofer Distance

The Fraunhofer distance, or far-field distance, is given by:

$$d = \frac{2D^2}{\lambda}, \tag{2.5}$$

where D is the length of largest dimension of antenna, and λ is the wavelength of the radio wave. Fraunhofer distance is the key the differentiate between far-field and near-field in electromagnetic field. The simplified path-loss model is only valid when the distance between the transmitter and the receiver is equal or greater than the Fraunhofer distance.

2.2 Wireless Ad Hoc Networks

Contrary to infrastructure wireless networks where nodes collaborate using access points, wireless ad hoc networks (WANET) do not rely on central devices to manage network traffic. These networks are self-configuring meaning that the connections between nodes are made on the fly without going through the complex process of infrastructure setup and administration. In WANET, two nodes may be out of each other transmission ranges. Nevertheless, the two nodes can communicate with each other by first establishing a path of nodes between themselves, and then forwarding their messages through this path. The task of establishing paths/routes is done using routing algorithms, which rely on broadcast for route discovery.

Below are the two ad hoc categories that have been widely studied.

- Wireless Sensor Networks

  Wireless Sensor Networks (WSN) are WANETs where nodes are sensor-based devices that are spatially distributed in the environment. These networks are mostly employed for monitoring physical or environmental settings such as sound, pressure, climatic changes, and etc. Some applications of these networks include greenhouse monitoring, forest fire detection, traffic control, and data center monitoring. A commonly used communication operation in WSNs is broadcast using which the sink node sends its control messages to every sensor in the network.

  The pattern used for broadcasting the message could vary based on the application of the network. Figure 2.3 shows a wireless sensor network where the source (black node) broadcasts its message (an alert) to others.

- Vehicular Ad hoc Network

  WANETs can also be categorized with respect to the mobility of nodes. Mobile Ad hoc Network (MANET) is a continuously self-configuring WANET where nodes are able to move in the network. Nodes' mobility could lead some connections to break while new connections are formed as nodes get to each others proximity. Vehicular Ad Hoc Network (VANET) is a special class of MANETs, which can be used to improve road safety

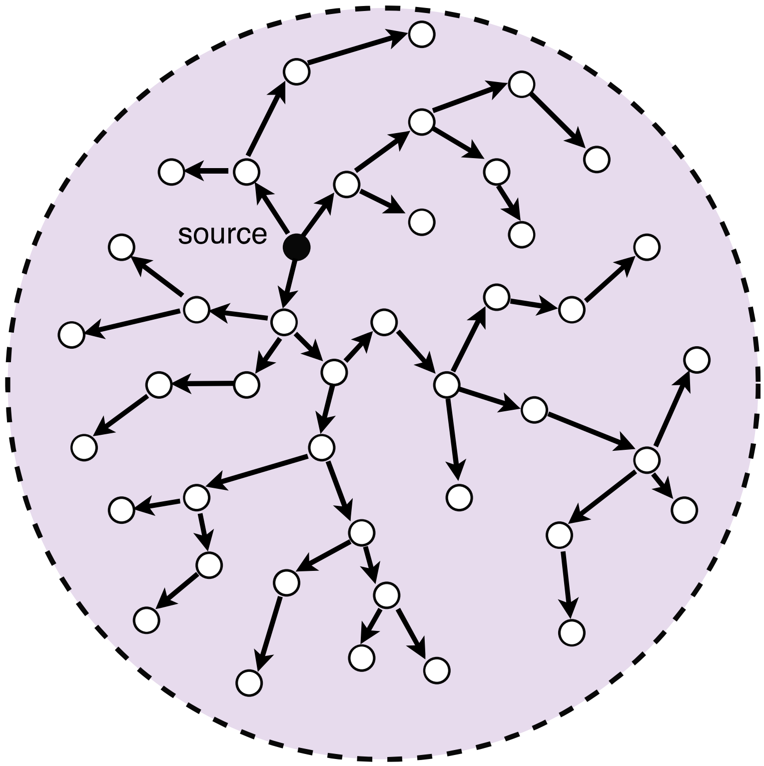

Figure 2.3: Broadcast in a sample wireless sensor network

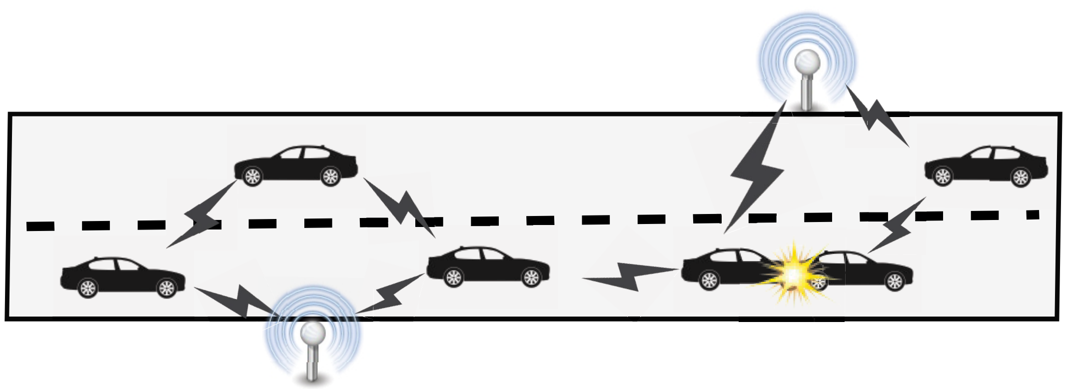

Figure 2.4: Application of Vehicular ad hoc networks in preventing additional damage

and provide travelers comfort. However, it differs from MANET in many aspects including challenges, and applications. The main communication architectures that are mostly employed in these networks are vehicle-to-vehicle and vehicle-to-roadside. Figure 2.4 shows a sample application of VANET in which an alert is locally broadcast in the network to inform nearby cars of an accident.

2.3 Broadcast

One-to-one and one-to-many are two main network communication primitives. In one-to-one communication, the transmitting node has only one target to send the message to, whereas in one-to-many transmission, the message is supposed to be received at multiple receivers. A well-studied class of one-to-many transmission is broadcast. It is a mechanism in wireless and ad hoc networks that is widely used to disseminate a message to every node in the network. Broadcast is done in either single-hop or multi-hop fashion. In single-hop broadcast, the transmitting node (source) adjusts its transmit power to deliver the message to the furthest node. This approach does not work when some nodes are out of the maximum transmission range of the source. When some nodes are out of the transmission range of the source, the broadcast is done in a multi-hop fashion. In multi-hop broadcast, in addition to the source, some other nodes transmit the message to propagate the message through the network. In this thesis, we study multi-hop broadcast, which we simply refer to as broadcast.

A major challenge in designing broadcast algorithms is to decide which nodes should transmit the message. A simple strategy is to have every node transmit the message. The broadcast algorithm based on this strategy is called flooding (see Figures 2.5-2.8). A major issue with flooding is that it can pose many redundant transmissions, hence wasting energy. For example, in the network shown in Figure 2.5, it can be simply verified that a broadcast can be accomplished using only five transmissions by nodes N_1, N_2, N_9, N_8, and N_6. In the same network, flooding uses nine transmissions, since in flooding every node must transmit the message. Figures 2.5-2.8 show the first few steps of the flooding algorithm.

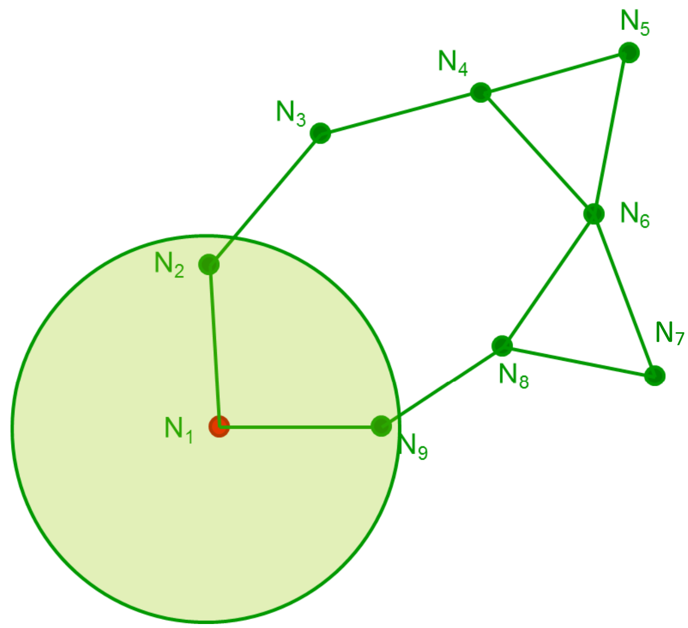

Figure 2.5: Step 1: The source node N_1 transmits the message. As the result, its neighbours N_2 and N_9 receive the message.

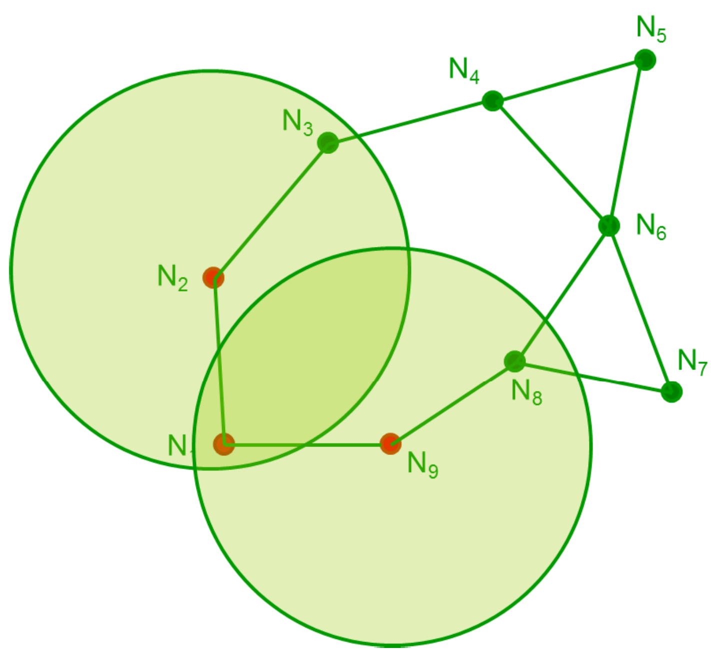

Figure 2.6: Step 2: Nodes N_2 and N_9 transmit the message. Therefore, N_3 and N_8 will receive the message.

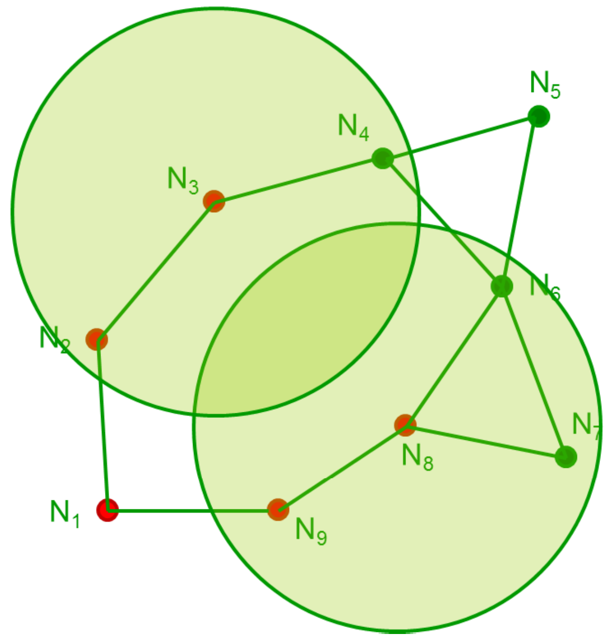

Figure 2.7: Step 3: Nodes N_8 and N_3 transmit the message. After this, every node except N_5 will have the message.

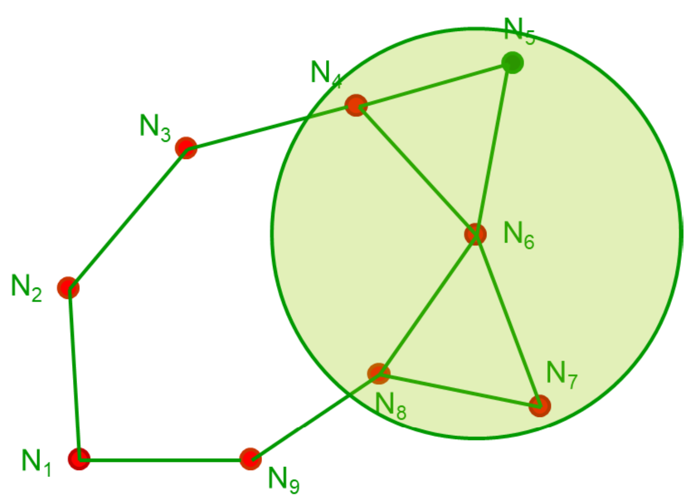

Figure 2.8: Step 4: N_6 transmits the message, thus N_5 will receive the message.

2.4 Energy Efficient Broadcast

Considering that wireless devices (e.g., mobile devices) typically have limited energy supplies (they run on batteries), reducing the energy consumption of broadcast is of great interest. Different approaches have been taken in the literature to reduce energy consumption of broadcast. These approaches are mainly based on transmission power control and reducing the number of redundant transmissions.

1. Transmission power control: Some wireless devices are able to reduce or increase their wireless transmission power. Algorithms based on transmission power control use this capability of devices and adjust their powers such that the sum of the powers is minimized while the network is still connected. See [9] for a survey on these algorithms.

2. Reducing the number of transmissions: Another approach to reduce the total power consumption of broadcast is to reduce the number of redundant transmissions. In general networks, it is not possible to eliminate all redundant transmissions due to the difficulty of the task (the problem of finding the minimum number of transmissions required in broadcast has been shown to be NP-hard [21]). Nevertheless, there are algorithms in the literature that perform well in reducing the number of redundant transmissions (e.g., [10, 11]).

2.4.1 Energy Accumulation

The two main accumulation mechanisms studied in communication systems are i) energy accumulation [12, 22], and ii) mutual-information accumulation [23, 24].

Using energy accumulation, a node can decode the message if the sum of the received powers from any set of transmissions of the message is above a threshold [13]. As an example to better understand energy accumulation, suppose that the decoding threshold is one, and three versions of the transmitted signal arrive at node u with powers 0.5, 0.4, and 0.3. With energy accumulation, node u is able to decode since the sum of the received powers is above the threshold of one. Without energy accumulation, however, decoding is not possible because, for each transmission, the received power at u is strictly less than one.

In practice, the power threshold is set higher that the noise floor. This is to enable out-of-range nodes to overhear the message. Using maximal-ration combining, these unreliable versions of the message, coming from various links, are combined together and the original message is recovered. MRC is often used in MIMO systems where signals arriving from various channels are multiplied by a weight factor that is proportional to the signal amplitude. The Key idea in MRC is to use linear coherent combining of signals to maximize the output SNR. Contrary to MIMO systems where multiple antennas are available to each receiver, in ad hoc networks, nodes are normally equipped with one antenna. This makes interference form concurrent transmissions an obstacle towards efficient implementation of MRC. To avoid interference, a common solution is to have the network operate in the wideband power limited regime with no co-channel interference.

2.5 Asymptotic Notations

Asymptotic notations are mathematical tools used to analyze growth rate of a function by identifying its behavior as the input size for the function increases. The three main notations used to represent time complexity of algorithms are:

1. $\mathcal{O}(.)$ notation, also referred to as **Big-O**, denotes an upper bound for the growth rate of a function. For the formal definition, let $f(n)$ and $g(n)$ be

two functions defined on some subset of the real numbers. We have

$$f(n) = \mathcal{O}(g(n)) \iff \forall n \geq n_0 : \quad |f(n)| \leq c|g(n)|,$$

where c and n_0 are positive real number. The example given in Figure 2.9 shows that $f(n) = \mathcal{O}(g(n))$. This implies that $f(n)$ does not grow faster than $g(n)$ as the input n increases. A very similar asymptotic notation is $o(.)$ (Small-o) which denotes the upper bound (that is not asymptotically tight) on the growth rate of runtime of an algorithm. It is formally defined as

$$f(n) = o(g(n)) \iff \forall_{n \geq n_0} : \quad |f(n)| < c|g(n)|.$$

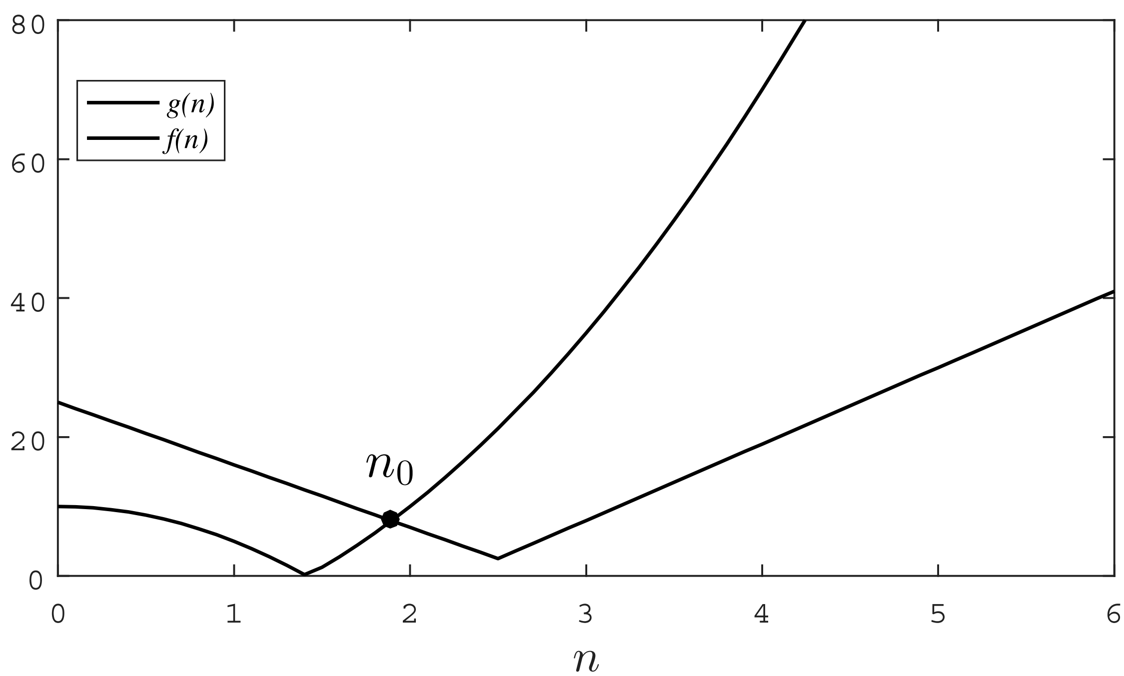

Figure 2.9: $f(n) = \mathcal{O}(g(n))$ as there exist a real number c such that for all numbers larger than n_0 we have $f(n) \leq cg(n)$.

2. $\Omega(.)$ notation is opposite to Big-O notation, and provides a lower bound

for the growth rate of a function. Formally speaking

$$f(n) = \Omega(g(n)) \iff \forall_{n \geq n_0} : \quad |f(n)| \geq c|g(n)|,$$

where $f(n)$ and $g(n)$ be two functions defined on some subset of the real numbers, and c and n_0 are positive real number. In Figure 2.9, we have $f(n) = \Omega(g(n))$ meaning that $g(n)$ grows faster than $f(n)$ as the input n increases.

3. $\Theta(.)$ notation is used to provide the asymptotically tight bound on the growth rate of an algorithm. Considering two functions $g(n)$ and $f(n)$, we have

$$f(n) = \Theta(g(n)) \iff f(n) = \mathcal{O}(g(n)) \;\&\; f(n) = \Omega(g(n)).$$

This implies that $f(n)$ grows as fast as $g(n)$ as the input n increases.

To have a better understanding of how these notation are used in the analysis of algorithms, consider the function $f(n) = 3n^3 + 7n - 21$. Towards finding a tight upper bound for this function, we should look at its summands $3n^3$, $7n$, -21 with respect to their growth rate. Among summands of $f(n)$, $3n^3$ has the fastest growing rate. Therefore, we have $f(x) = \mathcal{O}(n^3)$ since there are $n_0 = 2$ and $c = 4$ such that $\forall_{n \geq 4} : \quad 3n^3 + 7n - 21 \leq 4n^3$.

2.6 Problem Complexity

The theory of complexity classifies problems into four categories based on how complex they are to solve.

- P problem

  A problem is in class **P** (Polynomial Time) if there is an algorithm that can solve the problem in polynomial time (i.e. the running time of the algorithm is a polynomial function of the input size).

- NP problem

  There is another class of problem called **NP** (Non-deterministic Polynomial Time). Finding the solution to NP problems cannot necessarily be done in polynomial time. However, if a solution is found for them, it can be verified to be correct in polynomial time.

- NP-hard problem

  A problem is **NP-hard** if any NP problem can be reduced to it in polynomial time. Informally, NP-hard problems are at least as hard as the hardest problems in NP.

- NP-complete

  A problem which is both NP and NP-hard is referred to as **NP-complete**

2.7 Binary Relation

Consider two sets A and B. A *binary relation* $\mathcal{R}$ between A and B is a collection of ordered pairs (a, b) where $a \in A$ and $b \in B$. In other words, it is a subset G of $A \times B$. If $A = B$, it is called a "binary relation over A. The statement $(a, b) \in G$ is read "a is $\mathcal{R}$-related to b", and is denoted by $a\mathcal{R}b$.

Common binary relation categories are i) partially order, ii) total order, and iii) strict order.

1. *Partially Order*

   A binary relation $\mathcal{R}$ is a partial order over a set A if it has:

   - Reflexivity, meaning that $a\mathcal{R}a$ for all a in A.
   - Antisymmetry, meaning that if $a\mathcal{R}b$ and $b\mathcal{R}a$ we have $a = b$.
   - Transitivity, which means that $a\mathcal{R}b$ and $b\mathcal{R}c$ implies $a\mathcal{R}c$.

2. *Total Order*

   A binary relation $\mathcal{R}$ is a total order over a set A (i.e. $\mathcal{R}$ totally orders A) if the following properties hold.

   - Reflexivity.
   - Antisymmetry.
   - Transitivity.
   - Comparability, meaning that for any a, b in A, either $a\mathcal{R}b$ or $b\mathcal{R}a$.

3. *Strict Order*

   A binary relation $\mathcal{R}$ is a strict order over a set A if it has:

   - Irreflexivity, meaning that for any $a \in A$, $a\mathcal{R}a$ doesn't hold.
   - Asymmetry, meaning that if $a\mathcal{R}b$, we can have $b\mathcal{R}a$.
   - Transitivity.

Chapter 3

Analysis of Cooperation Gain in 2D Wireless Networks

In this section, we first introduce the system model and definitions that we used for the analysis of cooperation gain in 2D wireless networks. Then, we propose our novel conversion method that converts any given cooperative broadcast algorithm into a non-cooperative broadcast algorithm. Next, we leverage this *conversion method* to establish upper bounds on cooperation gain, G_{tot}. This is very challenging as the bound must be proven for every 2D network.

We consider 2D networks with path loss exponent $\alpha = 2$, and derive an upper bound on the cooperation gain by proving that $G_{tot} = \mathcal{O}(\log n)$. In [17], it is observed that the cooperation gain for $\alpha > 2$ is less than that for $\alpha = 2$. Therefore, the upper bound $G_{tot} = \mathcal{O}(\log n)$ is applicable to all 2D networks under different path loss exponents.

We take our analysis one step further and study the cooperation gain in 2D *grid* networks. We derive a lower bound of $G_{tot} = \Omega(\log n)$ for such networks, thereby showing that $G_{tot} = \Theta(\log n)$ when $\alpha = 2$. Without loss of generality, we assume $P_{th} = 1$ in the remaining of this work.

3.1 System Model and Definitions

We consider a static 2D wireless network with a set of n nodes $\mathscr{U}$, and a single source node $s \in \mathscr{U}$, which is to broadcast a single message to every other node in $\mathscr{U}$ in a multi-hop fashion. The channel between any pair of nodes u_i and u_j in the network is assumed to be additive white Gaussian noise (AWGN), and is described by a frequency non-selective link gain h_{u_i,u_j}. In addition, the distance between any pair of nodes in the network is considered to be larger than Fraunhofer distance. We adopt the simplified path-loss model used in [12], [19] and [25]. This model is commonly used for system design as it captures the essence of signal propagation without resorting to complicated statistical models [26]. In the simplified path-loss model, the link gain h_{u_i,u_j} is represented as $h_{u_i,u_j} = d_{u_i,u_j}^{-\alpha}$, where d_{u_i,u_j} is the distance between nodes u_i and u_j and α is the path loss exponent ($\alpha \geq 2$). In this work, we set $\alpha = 2$, and leave the case $\alpha > 2$ for future studies.

Note that multi-path fading and shadowing can affect the cooperation gain. Using numerical analysis and the Rayleigh fading model, it was shown that such effect is small in linear networks [17]. In our numerical analysis, we also observe a small improvement in cooperation gain in 2D networks when Rayleigh fading is used.

We divide time into slots, and assume that only one node transmits in a single time-slot. Furthermore, we assume that each node transmits at most once. These assumptions are not restrictive in studying the total energy consumption, because any broadcast algorithm can be converted into the above time-sloted model without affecting its total power consumption and delivery coverage (this may impact latency, which is not the concern of this work). For example, if a node u transmits, say twice at times t_1 and t_2 ($t_1 \leq t_2$) with powers P_1 and P_2, respectively, those transmissions can be merged into one transmission at time t_1 with power $P_1 + P_2$.

Based on the above model, a broadcast algorithm can be defined as follows:

Definition 1 (Broadcast Algorithm)**.** *We represent a broadcast algorithm by a tuple $(\mathscr{A}, <, \rho)$, where the binary relation $<$ is a strict total order on $\mathscr{U}$, and the transmit power function $\rho : \mathscr{U} \to \mathbb{R}_{\geq 0}$ is a function from set of nodes to real numbers $\mathbb{R}_{\geq 0}$ which returns the transmission power of each node. The set $\mathscr{A} \subseteq \mathscr{U}$ represents the set of nodes that transmit (i.e. set of nodes u for which $\rho(u) \neq 0$). The binary relation determines the order of transmission. In particular, we have*

- $u_i < u_j$ *if* $u_i, u_j \in \mathscr{A}$ *and* u_i *transmits before* u_j.
- $u_i < u_j$ *if* $u_i \in \mathscr{A}$ *and* $u_j \in \bar{\mathscr{A}} = \mathscr{U} \backslash \mathscr{A}$.
- $u_i = u_j$ *if* $u_i, u_j \in \bar{\mathscr{A}}$.

In the rest of this work, we represent a broadcast algorithm by $(\mathscr{A}, \rho)$ instead of $(\mathscr{A}, <, \rho)$, for convenience.

Assuming that the noise has unit power, we use maximal ratio combining (MRC) technique to verify that the original message has successfully been recovered at a node. In non-cooperative broadcast, the message is received and decoded if the received power at node u from a node v is above a decoding threshold. In cooperative broadcast, multiple copies of the message arrived from multiple transmitters are combined to decode the message. Therefore, in this case, the message is decodable at u if the sum of received powers at u is greater than the threshold.

Definition 1 of broadcast algorithm does not guarantee that the message is delivered to all nodes, as it does not put any condition on the transmission power ρ. In this work, we only study broadcast algorithms that guarantee full delivery. To achieve full delivery, extra conditions need to be imposed on transmit power function. The following two definitions enforce such conditions, and describe two general classes of broadcast algorithms with full delivery.

Definition 2 (Cooperative Broadcast Algorithm)**.** *A broadcast algorithm* $(\mathscr{A}, \rho)$ *is called a cooperative broadcast algorithm, and is denoted by* $(\mathscr{A}^{(c)}, \rho^{(c)})$*, if and only if*

$$\forall u_i \in \mathscr{U} \backslash \{s\} : \quad \sum_{u_j < u_i} \rho^{(c)}(u_j) h_{u_i, u_j} \geq P_{th}, \tag{3.1}$$

where P_{th} *is the decoding threshold. The inequality implies that the sum of received powers at every node (except the source) is not less than the threshold, hence every node receives the message.*

Definition 3 (Non-cooperative Broadcast Algorithm)**.** *A broadcast algorithm* $(\mathscr{A}, \rho)$ *is called a non-cooperative broadcast algorithm, and is denoted by* $(\mathscr{A}^{(n)}, \rho^{(n)})$*, if and only if:*

$$\forall u_i \in \mathscr{U} \backslash \{s\}, \exists u_j < u_i : \quad \rho^{(n)}(u_j) h_{u_i, u_j} \geq P_{th}. \tag{3.2}$$

Similar to (3.1), Inequality 3.2 ensures full delivery.

The cooperation gain G_{tot} is defined as follows:

Definition 4 (G_{tot})**.** *Let* $(\mathscr{A}_{\dagger}^{(c)}, \rho_{\dagger}^{(c)})$ *and* $(\mathscr{A}_{\dagger}^{(n)}, \rho_{\dagger}^{(n)})$ *be, respectively, optimal cooperative and non-cooperative broadcast algorithms with minimum power consumption. We define the cooperation gain* G_{tot} *as*

$$G_{tot} = \frac{\sum_{u_i \in \mathscr{U}} \rho_{\dagger}^{(n)}(u_i)}{\sum_{u_i \in \mathscr{U}} \rho_{\dagger}^{(c)}(u_i)}. \tag{3.3}$$

3.2 Conversion Method

In this section, we propose a carefully crafted conversion method that converts any cooperative broadcast algorithm $(\mathscr{A}^{(c)}, \rho^{(c)})$ into a non-cooperative broadcast algorithm $(\mathscr{A}^{(n)}, \rho^{(n)})$.

Consider any 2D network with a set of n nodes $\mathscr{U}$. Let $(\mathscr{A}^{(c)}, \rho^{(c)})$ be an arbitrary cooperative broadcast algorithm with power consumption $P_{tot}^{(c)}$. To better explain our conversion method, we will use the simple network shown in Figure 3.1 as an example where the transmitting nodes $\mathscr{A}^{(c)} = \{u_1, u_3, u_5, u_6\}$ are depicted by the asterisks, and the transmission order is $u_1 <^{(c)} u_3 <^{(c)} u_5 <^{(c)} u_6$ with u_1 being the source node. Since the set of nodes are fixed, the task of the conversion method is to 1) assign powers to the node, that is to determine the function $\rho^{(n)}$, and 2) determine the order of transmissions. Before explaining the power assignment and the order of transmissions, we need a few definitions.

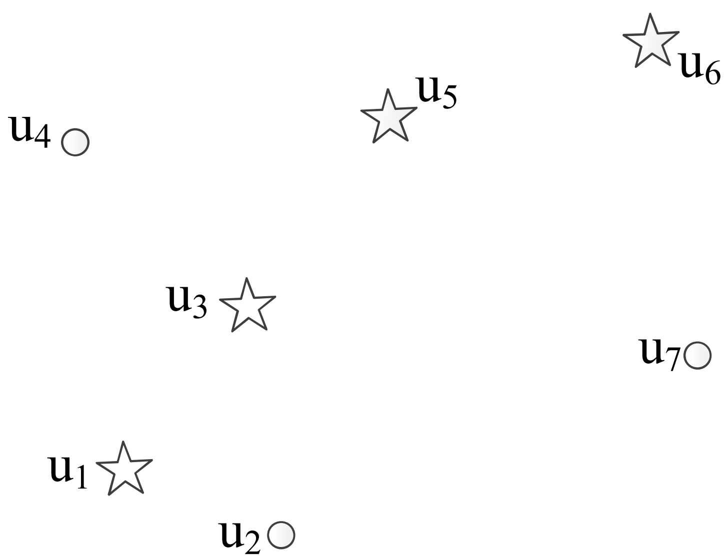

Figure 3.1: Sample network, asterisks are cooperative transmitting nodes

For any node $u \in \mathscr{U} \backslash \{s\}$, let $\mathcal{R}(u)$ be the closest node to u that transmits before u in the cooperative algorithm $(\mathscr{A}^{(c)}, \rho^{(c)})$. Formally

$$
\begin{aligned}
&\mathcal{R} : \mathscr{U} \backslash \{s\} \rightarrow \mathscr{A}^{(c)} \\
&\mathcal{R}(u) \in \operatorname*{argmin}_{v <^{(c)} u} d_{u,v}.
\end{aligned}
\tag{3.4}
$$

In our sample network, we have

$$\mathcal{R}(u_7) = u_6, \;\; \mathcal{R}(u_6) = u_5, \;\; \mathcal{R}(u_5) = u_3,$$
$$\mathcal{R}(u_4) = u_3, \;\; \mathcal{R}(u_3) = u_1, \;\; \mathcal{R}(u_2) = u_1.$$

Also, for any node $u \in \mathscr{U} \backslash \{s\}$ let

$$D_u = \{(x, y) | (x - x_u)^2 + (y - y_u)^2 \leq d^2_{u,\mathcal{R}(u)}\} \tag{3.5}$$

be the disk with radius $r_u = d_{u,\mathcal{R}(u)}$ centred at node u, where $d_{x,y}$ denotes the distance between two nodes x and y. Let

$$\mathcal{D} = \{D_u | u \in \mathscr{U} \backslash \{s\}\}.$$

In other words, $\mathcal{D}$ is the set of every disk centred at a node $u \in \mathscr{U} \backslash \{s\}$ with the closest node that transmits before u being located on the boundary of the disk. Figure 3.2 shows the disks $D_u \in \mathcal{D}$ for our sample network.

The power assignment is done in two steps:

1. Step 1: In this step, we find a subset $\mathcal{I}$ of non-overlapping disks in $\mathcal{D}$ through an iterative process. Initially, we set $\mathcal{I} = \{\}$. In each iteration, we find the largest disk in $\mathcal{D} \backslash \mathcal{I}$ that does not overlap with any disk in $\mathcal{I}$, and add that to $\mathcal{I}$. We stop when every disk in $\mathcal{D} \backslash \mathcal{I}$ overlaps with at least one disk in $\mathcal{I}$. Disks with thick boundary in Figure 3.3 demonstrate the set of non-overlapping disks $\mathcal{I}$ for our sample network.

2. Step 2: Let

   $$\mathcal{I} = \{D_{u_1}, D_{u_2}, ..., D_{u_{|\mathcal{I}|}}\},$$

   be the result of the first step. To every disk $D_{u_i} \in \mathcal{I}$, we assign a node w_i

   $$w_i = \min_{u \in \mathcal{S}_i}(u) \tag{3.6}$$

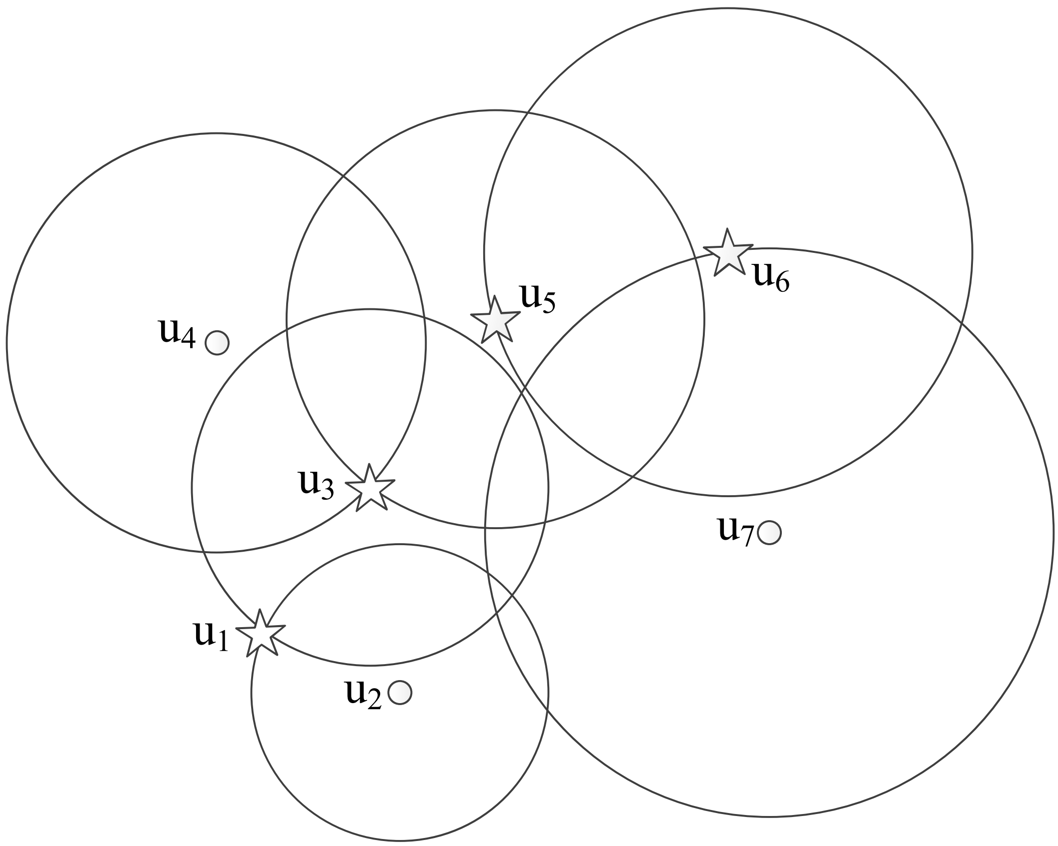

Figure 3.2: Disks $D_u \in \mathcal{D}$ in the sample network

where the minimization is with respect to $<^{(c)}$, and

$$\mathcal{S}_i = \{\mathcal{R}(v) | D_v \cap D_{u_i} \neq \emptyset, r_v \leq r_{u_i}, D_v \in \mathcal{D}\}. \tag{3.7}$$

This rather complex choice of node w_i and the set $\mathcal{S}_i$ are better understood in the proof of Theorem 1. Black filled asterisks in Figure 3.3 represent nodes w_i for disks $D_{u_i} \in \mathcal{I}$ in our sample network. Note that every node in $\mathcal{S}_i$ is a transmitting node because, by (3.4), $\mathcal{R}(v) \in \mathscr{A}^{(c)}$ for every node $v \neq s$. Equation 3.6 simply implies that w_i is the node in $\mathcal{S}_i$ that transmit before any other node in $\mathcal{S}_i$.

In our constructed non-cooperative broadcast algorithm $(\mathscr{A}^{(n)}, \rho^{(n)})$, only nodes w_i, $1 \leq i \leq |\mathcal{I}|$, are assigned non-zero transmission power accordingly to Algorithm 1.

Algorithm 1 Power Assignment

1: $\forall u \in \mathscr{U}$ set $\rho^{(n)}(u) \leftarrow 0$
2: **for** $i \leftarrow 1, |\mathcal{I}|$ **do**
3: $\quad \rho^{(n)}(w_i) \leftarrow \max\{\rho^{(n)}(w_i), (5r_{u_i})^\alpha\}$
4: **end for**

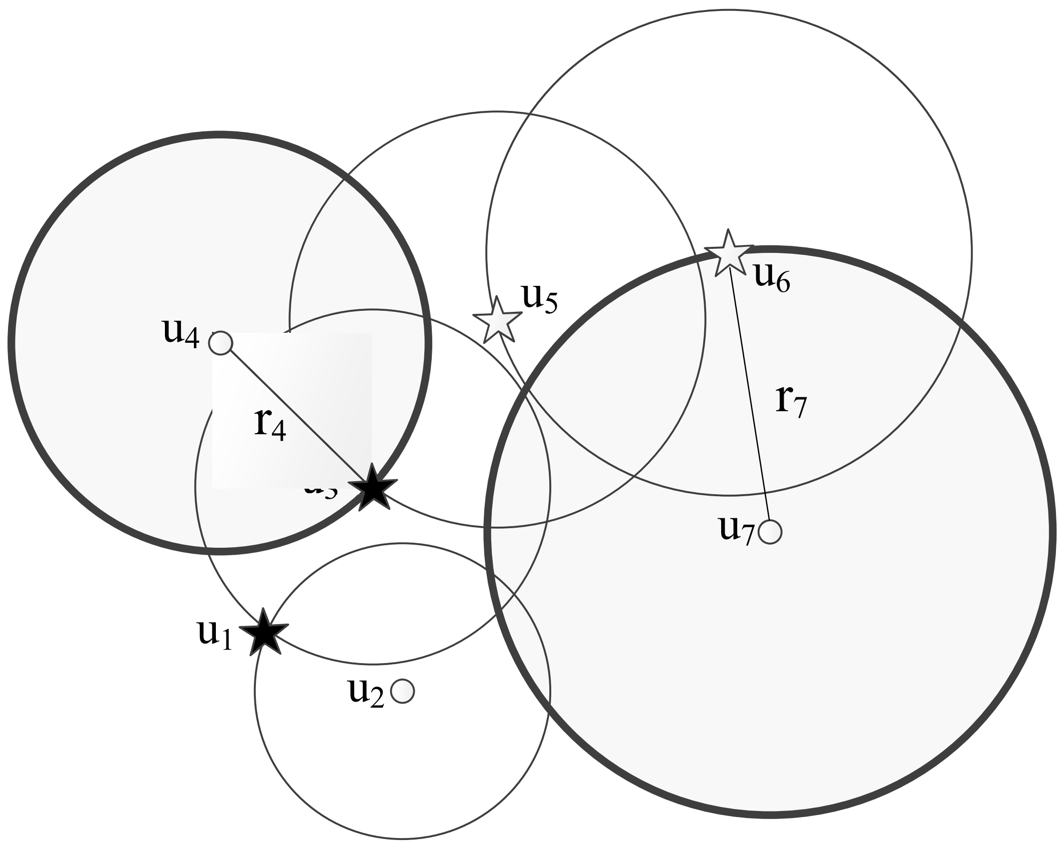

Figure 3.3: Finding transmitting nodes of non-cooperative broadcast algorithm

In the sample network, the transmitting set of our constructed non-cooperative algorithm is

$$\mathscr{A}^{(n)} = \{u_1, u_3\},$$

and the transmission powers are

$$\rho^{(n)}(u_1) = (5r_7)^\alpha, \qquad \rho^{(n)}(u_3) = (5r_4)^\alpha.$$

Note that the constructed non-cooperative algorithm is not necessarily efficient in terms of power consumption since it is designed to simplify the analysis of cooperation gain for 2D wireless networks.

Remark 1. By the description of Algorithm 1, we get

$$P_{tot}^{(n)} \leq \sum_{D_{u_i} \in \mathcal{I}} (5r_{u_i})^{\alpha}, \tag{3.8}$$

where $P_{tot}^{(n)}$ is the power consumption of non-cooperative algorithm.

Remark 2. It can be inferred from Algorithm 1 that $\mathscr{A}^{(n)} \subseteq \mathscr{A}^{(c)}$, since

$$\forall 1 \leq i \leq |\mathcal{I}| : \; w_i \in \mathscr{A}^{(c)}.$$

i.e, every node that transmits in the constructed non-cooperative algorithm, also transmits in the given cooperative algorithm. With the power assignment summarized in Algorithm 1, the only remaining task to fully define our constructed non-cooperative broadcast algorithm is to establish a transmission order among the transmitting nodes $\mathscr{A}^{(n)}$. To this end, in the constructed cooperative algorithm, we simply follow the same transmission order as in the non-cooperative algorithm, that is

$$\forall u, v \in \mathscr{A}^{(n)} : \; u <^{(c)} v \Rightarrow u <^{(n)} v, \tag{3.9}$$

or equivalently

$$\forall u \in \mathscr{A}^{(n)} : \; u <^{(c)} v \Rightarrow u <^{(n)} v. \tag{3.10}$$

Theorem 1. *The constructed broadcast algorithm* $(\mathscr{A}^{(n)}, \rho^{(n)})$ *is a non-cooperative broadcast algorithm. (i.e., the message is fully delivered to all the nodes in the network.)*

Proof. Towards showing a contradiction, we assume that there is at least one node that does not receive the message. Among nodes that have not received the message, let f be the smallest node with respect to the ordering $<^{(n)}$ (i.e. every node $u <^{(n)} f$ has successfully received the message). In the following, we will prove that there is a node v, $v < f$, which has received the message and transmitted with power $\rho^{(n)}(v) \geq d_{v,f}^2$. This implies that node f must have received the message from node v, a contradiction. We consider two cases. In the first case, we assume $D_f \in \mathcal{I}$; in the second case we assume $D_f \notin \mathcal{I}$.

1. Case 1: $D_f \in \mathcal{I}$:

   Since $D_f \in \mathcal{I}$, we assume that $f = u_i$, for some $i \in \{1, 2..., |\mathcal{I}|\}$. Figure 3.4 shows disk $D_{u_i} \in \mathcal{I}$ (with thick boundary) as well as all disks in $\mathcal{D}$ that are not bigger than D_{u_i} and intersect with D_{u_i}. For every disk D_u shown in Figure 3.4, node $\mathcal{R}(u)$ is represented by an asterisk. Therefore, the asterisks shown in Figure 3.4 are the set $\mathcal{S}_i$ defined in (3.7). Note that, w_i, defined in (3.6), is one of those asterisks. We have the following for w_i:

   (i) $\rho^{(n)}(w_i) \geq (5r_{u_i})^\alpha$, where r_{u_i} is the radius of the disk $D_{u_i} = D_f$:

   It is because, in the ith iteration of the power assignment algorithm (Algorithm 1), the power of node w_i (i.e. $\rho^{(n)}(w_i)$) is set to a number at least equal to $(5r_{u_i})^\alpha$. Also if $w_i = w_j$ for some integer $j > i$, $\rho^{(n)}(w_i)$ will not be reduced in iteration j.

   (ii) $d_{w_i,u_i} \leq 3r_{u_i}$:

   Let P be a point on the circumference of any disk shown in Figure 3.4. The distance between P and $f = u_i$ is at most $3r_{u_i}$ because the radius of every disk is at most equal to r_{u_i}.

   (iii) $w_i <^{(n)} u_i$:

   From (3.4), we get $\mathcal{R}(u_i) <^{(c)} u_i$, and by (3.6) we get $w_i <^{(c)} \mathcal{R}(u_i)$. Therefore, from (3.10), we have $w_i <^{(n)} u_i$, since $w_i \in \mathscr{A}^{(n)}$.

   (iv) w_i has received the message:

   This is simply by the assumption that every node $u <^{(n)} f$ has received the message and because $w_i <^{(n)} f$.

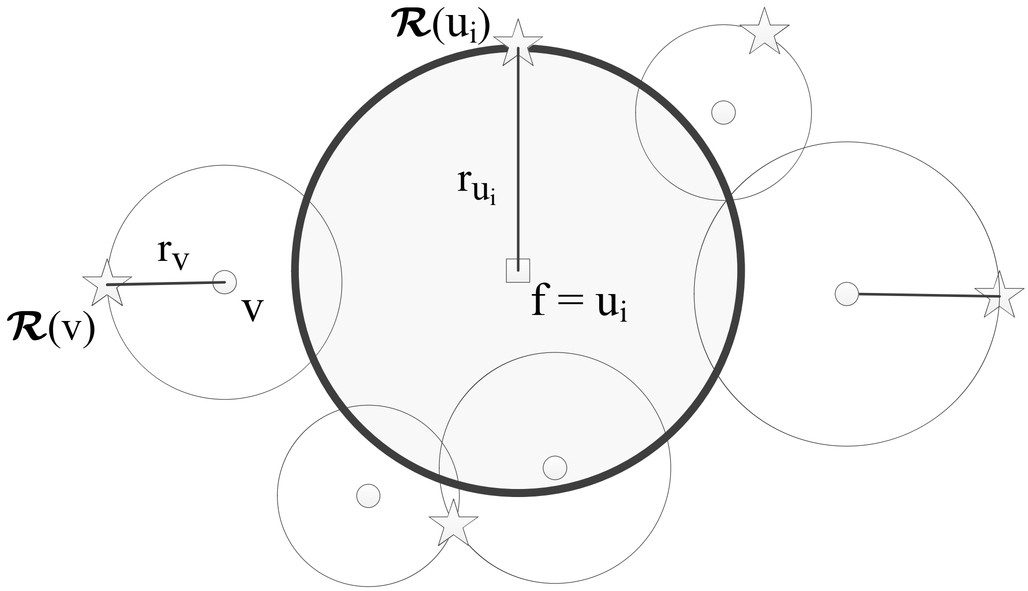

Figure 3.4: Disk $D_{u_i} = D_f$ (with bold border), and the disks in $\mathcal{D}$ that are not bigger than D_{u_i} and intersect with D_{u_i}. Asterisks represent the nodes in $\mathcal{S}_i$.

To sum up, there is a node $w_i <^{(n)} f$ that has received the message, and transmits with a power at least equal to $(5r_{u_i})^\alpha$, and is at most $3r_{u_i}$ away from f. Consequently f must have received the message from w_i.

2. Case 2: $D_f \notin \mathcal{I}$:

   Since $D_f \notin \mathcal{I}$, there must be at least a disk in $\mathcal{I}$ that intersects with D_f. Among such disks, let D_{u_i} be the largest one. The disk D_{u_i} must be at least as large as D_f as otherwise, D_f would have entered the set $\mathcal{I}$ instead of D_{u_i}. In Figure 3.5, the disk D_{u_i} is shown with thick boundary. The Figure also shows all the disks in $\mathcal{D}$ that intersect with D_{u_i} and are at most as large as D_{u_i}. Similar to Case 1, every node in $\mathcal{S}_i$ is represented by an asterisk, and w_i is one of the asterisks. Similar to Case 1, we have

   (i) $\rho^{(n)}(w_i) \geq (5r_{u_i})^\alpha$.

   (ii) $d_{w_i,u_i} \leq 5r_{u_i}$:

   This is because the radius of all disks is at most r_{u_i}. As illustrated in Figure 3.5, any point on the circumference of any disk is at most $5r_{u_i}$ away from node f.

(iii) $w_i <^{(n)} f$:

From (3.4), we get $\mathcal{R}(f) <^{(c)} f$, and (3.6) implies that $w_i <^{(c)} \mathcal{R}(f)$. Hence, by (3.10), we get that $w_i <^{(n)} f$.

(iv) w_i has received the message:

Again, this is by the assumption that every node $u <^{(n)} f$ has received the message, and because $w_i <^{(n)} f$.

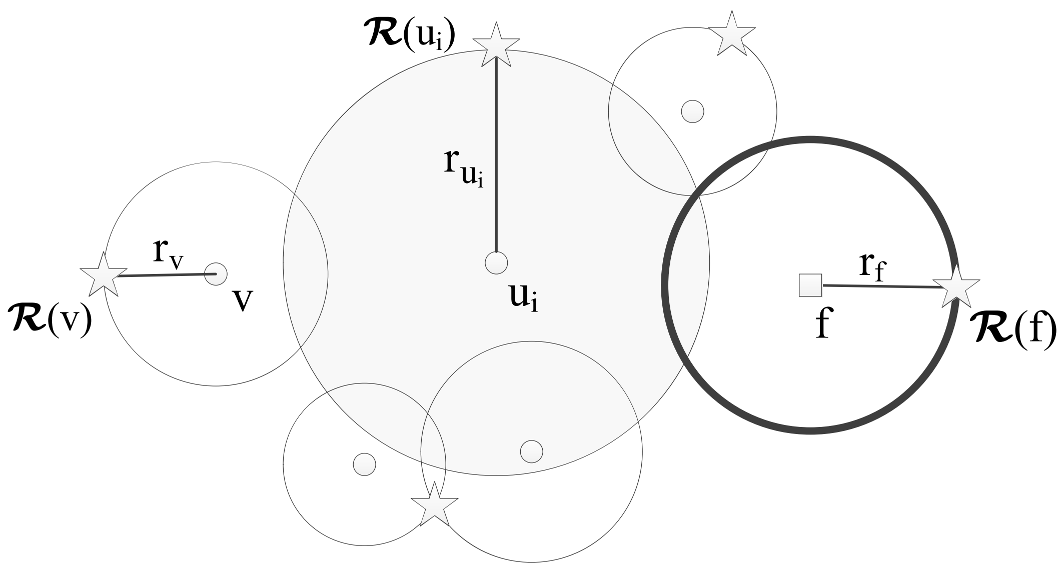

Figure 3.5: The shaded disk D_{u_i} is the biggest disk in $\mathcal{I}$ that intersects with D_f (the disk with bold border).

To sum up, node w_i has received the message, $w_i <^{(n)} f$. It transmits with a power at least equal to $(5r_{u_i})^\alpha$, and is at most $5r_{u_i}$ away from f. Consequently, as in Case 1, f must have received the message from w_i.

□

So far, we have shown how to construct a non-cooperative broadcast algorithm from any cooperative broadcast algorithm. Next, we analyze the total power consumption of the cooperative algorithm and its constructed non-cooperative algorithm to derive the upper bound for G_{tot}.

3.3 Upper Bound Analysis

In this section, we establish an upper bound on the cooperation gain. Note that signal power drops faster for larger path loss exponents, which results in a smaller cooperation gain. Therefore, to establish an upper bound, we set the path loss exponent to $\alpha = 2$.

In this section, we show that the total power consumption of the constructed non-cooperative algorithm is at most $\mathcal{O}(\log n)$ times that of the cooperative algorithm used for the conversion. This implies that the cooperation gain, G_{tot}, is in $\mathcal{O}(\log n)$ because any given cooperative algorithm can be converted to a non-cooperative algorithm with at most a factor of $\mathcal{O}(\log n)$ increase in the total transmission power. We start by some definitions and lemmas.

Let $(\mathscr{A}^{(c)}, \rho^{(c)})$ be a cooperative broadcast algorithm, and $(\mathscr{A}^{(n)}, \rho^{(n)})$ be the non-cooperative broadcast algorithm constructed from it.

Definition 5. *For every node $u \in \mathscr{A}^{(c)}$, we define*

$$\begin{aligned} & f_u : \mathbb{R}^2 \to (0, 1] \\ & f_u(p) = \begin{cases} 1 & d_{u,p} \leq \sqrt{\rho^{(c)}(u)} \\ \frac{\rho^{(c)}(u)}{d^2_{u,p}} & d_{u,p} > \sqrt{\rho^{(c)}(u)} \end{cases} \end{aligned} \tag{3.11}$$

where p is a point in the network, and $d_{u,p}$ is the distance between p and node u. The function $f_u(p)$ is simply the received power from node u at point p capped at one. Function $\mathcal{F}$, accordingly, is defined as

$$\begin{aligned} & \mathcal{F} : \mathbb{R}^2 \to \mathbb{R} \\ & \mathcal{F}(p) = \sum_{u \in \mathscr{U}} f_u(p). \end{aligned}$$

Lemma 2. *Let $|\mathscr{U}| = n$. We have*

$$\sum_{u \in \mathscr{A}^{(c)}} \frac{\rho^{(c)}(u)}{P^{(c)}_{tot}} \ln\left(\frac{P^{(c)}_{tot}}{\rho^{(c)}(u)}\right) \leq \ln(n)$$

Proof. Let

$$\alpha_u = \frac{\rho^{(c)}(u)}{P^{(c)}_{tot}}.$$

We have

$$\sum_{u \in \mathscr{A}^{(c)}} \alpha_u = 1.$$

Therefore, the sequence α_u can be seen as a probability distribution whose entropy is

$$\sum_{u \in \mathscr{A}^{(c)}} \alpha_u \ln(\frac{1}{\alpha_u}) \leq \ln(|\mathscr{A}^{(c)}|) \leq \ln(|\mathscr{U}|) = \ln(n)$$

□

Lemma 3. *For any real number R, $R \geq \sqrt{\rho^{(c)}(u)}$, we have*

$$\int_0^{2\pi} \int_0^R f_u(r,\theta) r \, \mathrm{d}r \, \mathrm{d}\theta = \pi\rho^{(c)}(u) + \pi\rho^{(c)}(u) \ln\left(\frac{R^2}{\rho^{(c)}(u)}\right),$$

where the function $f_u(r,\theta)$ is the function defined in (3.11) transferred into the polar coordinate system with the pole at node u.

Proof.

$$\begin{aligned}\int_0^{2\pi} \int_0^R f_u(r,\theta) r \, \mathrm{d}r \, \mathrm{d}\theta = & \int_0^{2\pi} \int_0^{\sqrt{\rho^{(c)}(u)}} f_u(r,\theta) r \, \mathrm{d}r \, \mathrm{d}\theta \, + \\ & \int_0^{2\pi} \int_{\sqrt{\rho^{(c)}(u)}}^R f_u(r,\theta) r \, \mathrm{d}r \, \mathrm{d}\theta \\ = & \int_0^{2\pi} \int_0^{\sqrt{\rho^{(c)}(u)}} r \, \mathrm{d}r \, \mathrm{d}\theta \, + \\ & \int_0^{2\pi} \int_{\sqrt{\rho^{(c)}(u)}}^R \frac{\rho^{(c)}(u)}{r^2} r \, \mathrm{d}r \, \mathrm{d}\theta \\ = & \pi\rho^{(c)}(u) + \pi\rho^{(c)}(u) \ln\left(\frac{R^2}{\rho^{(c)}(u)}\right).\end{aligned}$$

□

Let $U_{\mathcal{I}} = \bigcup_{D \in \mathcal{I}} D$, and $\Delta_{\mathcal{I}}$ be the area of $U_{\mathcal{I}}$.

Lemma 4. *We have*

$$\iint_{U_{\mathcal{I}}} f_u(r,\theta) r \,\mathrm{d}r \,\mathrm{d}\theta \leq \pi\rho^{(c)}(u) + \pi\rho^{(c)}(u) \ln(\frac{\Delta_{\mathcal{I}}}{\pi\rho^{(c)}(u)})$$

Proof. Let $D_{\mathcal{I}}$ be a disk with radius $R = \sqrt{\frac{\Delta_{\mathcal{I}}}{\pi}}$ centered at u. Let $U_{in} = U_{\mathcal{I}} \cap D_{\mathcal{I}}$, and $U_{out} = U_{\mathcal{I}} \backslash D_{\mathcal{I}}$, be the parts of $U_{\mathcal{I}}$ that respectively fall inside and outside of the disk $D_{\mathcal{I}}$. Let q be a point on the circumference of disk $D_{\mathcal{I}}$. For any point $p_{in} \in D_{\mathcal{I}}$ we have

$$f_u(p_{in}) \geq f_u(q),$$

since $d_{u,p_{in}} \leq d_{u,q}$, and the function $f_u(p)$ is non-increasing in terms of $d_{u,p}$. Similarly, we have

$$f_u(p_{out}) \leq f_u(q),$$

for any point $p_{out} \notin D_I$. Therefore, we have

$$\begin{aligned} \iint_{U_{out}} f_u(r,\theta) r \,\mathrm{d}r \,\mathrm{d}\theta &\leq \Delta_{U_{out}} \times f_u(q) \Delta_{D_{\mathcal{I}} \backslash U_{in}} \times f_u(q) \\ &\leq \iint_{D_{\mathcal{I}} \backslash U_{in}} f_u(r,\theta) r \,\mathrm{d}r \,\mathrm{d}\theta, \end{aligned} \tag{3.12}$$

where $\Delta_{U_{out}}$ and $\Delta_{D_{\mathcal{I}} \backslash U_{in}}$ are areas of U_{out} and $D_{\mathcal{I}} \backslash U_{in}$, respectively. Note that $\Delta_{U_{out}} = \Delta_{D_{\mathcal{I}} \backslash U_{in}}$. Consequently,

$$\begin{aligned} &\iint_{U_{\mathcal{I}}} f_u(r,\theta) r \,\mathrm{d}r \,\mathrm{d}\theta = \\ &\iint_{U_{in}} f_u(r,\theta) r \,\mathrm{d}r \,\mathrm{d}\theta + \iint_{U_{out}} f_u(r,\theta) r \,\mathrm{d}r \,\mathrm{d}\theta \leq \\ &\int_{U_{in}} f_u(r,\theta) r \,\mathrm{d}r \,\mathrm{d}\theta + \iint_{U_{\mathcal{I}} \backslash U_{in}} f_u(r,\theta) r \,\mathrm{d}r \,\mathrm{d}\theta = \\ &\iint_{D_{\mathcal{I}}} f_u(r,\theta) r \,\mathrm{d}r \,\mathrm{d}\theta \leq \pi\rho^{(c)}(u) + \pi\rho^{(c)}(u) \ln(\frac{\Delta_{\mathcal{I}}}{\pi\rho^{(c)}(u)}) \end{aligned}$$

where the last inequality is by Lemma 5.

□

Lemma 5. *For any point $p \in U_{\mathcal{I}}$, we have*

$$\mathcal{F}(p) \geq \frac{1}{4}$$

Proof. Let p be an arbitrary point in $U_{\mathcal{I}}$. Then, p must be inside one disk in $\mathcal{I}$. Let that disk be $D_v \in \mathcal{I}$. Since in $(\mathscr{A}^{(c)}, \rho^{(c)})$ node v receives the message, we must have

$$\sum_{u <^{(c)} v} f_u(v) \geq 1. \tag{3.13}$$

Note that for any node $u <^{(c)} v$, we have

$$d_{u,v} \geq r_v,$$

where r_v is the radius of disk D_v. Therefore, for any node $u <^{(c)} v$, we have

$$d_{u,p} \leq 2d_{u,v}. \tag{3.14}$$

Hence, by (3.13) and (3.14), we get

$$\begin{aligned} \mathcal{F}(p) \geq \sum_{u <^{(c)} v} f_u(p) &= \sum_{u <^{(c)} v} \left(f_u(v) \times \left(\frac{d_{u,v}}{d_{u,p}} \right)^2 \right) \\ &\geq \sum_{u <^{(c)} v} \left(f_u(v) \times \frac{1}{4} \right) \geq \frac{1}{4}. \end{aligned}$$

□

The following corollary directly follows from Lemma 5.

Corollary 6. *we have*

$$\iint_{U_{\mathcal{I}}} \mathcal{F}(p) \geq \frac{1}{4} \Delta_{\mathcal{I}}.$$

Let r_{u_i} denote the radius of disk $D_{u_i} \in \mathcal{I}$, $1 \leq i \leq |\mathcal{I}|$. We have

$$\Delta_{\mathcal{I}} = \pi \sum_{i=1}^{|\mathcal{I}|} r_{u_i}^2.$$

In Algorithm 1, each disk D_{u_i} , $1 \leq i \leq |\mathcal{I}|$, contributes a power of $25r_{u_i}^2$ at most once. Therefore,

$$P_{tot}^{(n)} \leq 25 \sum_{i=1}^{|\mathcal{I}|} r_{u_i}^2$$

Hence,

$$\Delta_{\mathcal{I}} \geq \frac{\pi}{25} P_{tot}^{(n)} \tag{3.15}$$

Theorem 7. *Let $(\mathscr{A}^{(c)}, \rho^{(c)})$ be a cooperative broadcast algorithm, and $(\mathscr{A}^{(n)}, \rho^{(n)})$ be the non-cooperative broadcast algorithm constructed from it. We have*

$$G_{tot} = \frac{\sum_{u_i \in \mathscr{U}} \rho^{(n)}(u_i)}{\sum_{u_i \in \mathscr{U}} \rho^{(c)}(u_i)} \in \mathcal{O}(\log n).$$

Proof. By Corollary 6, we have

$$\iint_{U_{\mathcal{I}}} \mathcal{F}(p) \geq \frac{1}{4} \Delta_{\mathcal{I}}.$$

To prove the theorem, we compute an upper bound on the same integration, and then compare the two bounds.

$$\begin{aligned}
\iint_{U_{\mathcal{I}}} \mathcal{F}(p) &= \iint_{U_{\mathcal{I}}} \sum_{u \in \mathscr{A}^{(c)}} f_u(p) \\
&= \sum_{u \in \mathscr{A}^{(c)}} \iint_{U_{\mathcal{I}}} f_u(p) \\
&\leq \sum_{u \in \mathscr{A}^{(c)}} \pi \rho^{(c)}(u) + \sum_{u \in \mathscr{A}^{(c)}} \pi \rho^{(c)}(u) \ln\left(\frac{\Delta_{\mathcal{I}}}{\pi \rho^{(c)}(u)}\right)
\end{aligned} \tag{3.16}$$

The second summation can be written as

$$\sum_{u\in\mathscr{A}^{(c)}} \pi\rho^{(c)}(u)\ln(\frac{\Delta_{\mathcal{I}}}{\pi\rho^{(c)}(u)}) =$$

$$\pi P_{tot}^{(c)} \sum_{u\in\mathscr{A}^{(c)}} \frac{\rho^{(c)}(u)}{P_{tot}^{(c)}} \ln\left(\frac{\pi P_{tot}^{(c)}}{\pi\rho^{(c)}(u)} \times \frac{\Delta_{\mathcal{I}}}{\pi P_{tot}^{(c)}}\right) =$$

$$\pi P_{tot}^{(c)} \sum_{u\in\mathscr{A}^{(c)}} \frac{\rho^{(c)}(u)}{P_{tot}^{(c)}} \ln\left(\frac{P_{tot}^{(c)}}{\rho^{(c)}(u)}\right) + \sum_{u\in\mathscr{A}^{(c)}} \pi\rho^{(c)}(u)\ln\left(\frac{\Delta_{\mathcal{I}}}{\pi P_{tot}^{(c)}}\right)$$

$$\leq \pi P_{tot}^{(c)} \sum_{u\in\mathscr{A}^{(c)}} \frac{\rho^{(c)}(u)}{P_{tot}^{(c)}} \ln\left(\frac{P_{tot}^{(c)}}{\rho^{(c)}(u)}\right) + \pi P_{tot}^{(c)} \ln\left(\frac{P_{tot}^{(n)}}{25P_{tot}^{(c)}}\right).$$

where the last inequality is by (3.15). Furthermore, by Lemma 2 we get

$$\begin{aligned}\pi P_{tot}^{(c)} \sum_{u\in\mathscr{A}^{(c)}} \frac{\rho^{(c)}(u)}{P_{tot}^{(c)}} \ln\left(\frac{P_{tot}^{(c)}}{\rho^{(c)}(u)}\right) + \pi P_{tot}^{(c)} \ln\left(\frac{P_{tot}^{(n)}}{25P_{tot}^{(c)}}\right) \leq \\ \pi P_{tot}^{(c)}\ln(n) + \pi P_{tot}^{(c)}\ln\left(\frac{P_{tot}^{(n)}}{25P_{tot}^{(c)}}\right).\end{aligned} \tag{3.17}$$

Using (3.17) in (3.16), we have

$$\begin{aligned}\iint_{U_{\mathcal{I}}} \mathcal{F}(p) &\leq \sum_{u\in\mathscr{A}^{(c)}} \pi\rho^{(c)}(u) + \sum_{u\in\mathscr{A}^{(c)}} \pi\rho^{(c)}(u)\ln(\frac{\Delta_{\mathcal{I}}}{\pi\rho^{(c)}(u)}) \\ &\leq \pi P_{tot}^{(c)} + \pi P_{tot}^{(c)}\ln(n) + \pi P_{tot}^{(c)}\ln\left(\frac{P_{tot}^{(n)}}{25P_{tot}^{(c)}}\right).\end{aligned} \tag{3.18}$$

By corollary 6 and (3.15), we get

$$\iint_{U_{\mathcal{I}}} \mathcal{F}(p) \geq \frac{1}{4}\Delta_{\mathcal{I}} \geq \frac{\pi}{100}P_{tot}^{(n)}. \tag{3.19}$$

From (3.18) and (3.19), we get

$$\pi P_{tot}^{(c)} + \pi P_{tot}^{(c)}\ln(n) + \pi P_{tot}^{(c)}\ln\left(\frac{P_{tot}^{(n)}}{25P_{tot}^{(c)}}\right) \geq \frac{\pi}{100}P_{tot}^{(n)} \tag{3.20}$$

Dividing both sides by $\pi P_{tot}^{(c)}$ yields

$$1 + \ln(n) + \ln(\frac{G_{tot}}{25}) \geq \frac{G_{tot}}{100} \tag{3.21}$$

Finally, assuming $n \geq 2$, it can be verified that (3.21) holds only when $G_{tot} \leq 127 \ln(n)$, which completes the proof. □

3.4 Cooperation Gain in Grid Networks

In the previous section, we proved that the cooperation gain grows *at most* logarithmically with the number of nodes n in 2-D networks. An interesting question is whether there is any 2-D network in which the cooperation gain increases exactly logarithmically with n. In this section, we prove that that this is the case for 2-D grid networks. Later, using numerical analysis, we show that this also holds for random networks.

Figure 3.6 shows the topology of a grid network with minimum node distance d, and $n = m^2$ nodes. The algorithm $(\mathscr{U}, \rho^{(n)}(u) = d^2)$ is a simple non-cooperative broadcast algorithm in which all nodes in $\mathscr{U}$ transmit with power d^2. The total power consumption of this algorithm is clearly $P_{tot}^{(n)} = nd^2$. There are non-cooperative broadcast algorithms with total power consumption less than nd^2. For example, if only nodes in every third row[1] (i.e., setting $L = 3$ in Figure 3.6) transmit, every node will receive the message and we get $P_{tot}^{(n)} \simeq \frac{m^2 d^2}{3} = \frac{nd^2}{3}$. The next proposition, however, shows that the total power consumption of any non-cooperative algorithm for the given grid network is $\Omega(nd^2)$. This implies that $(\mathscr{U}, \rho^{(n)}(u) = d^2)$ is asymptotically optimum.

Proposition 8. *For any non-cooperative broadcast algorithm over the 2D grid*

[1]To guarantee that every node receives the message, nodes in the top and bottom rows may need to transmit too.

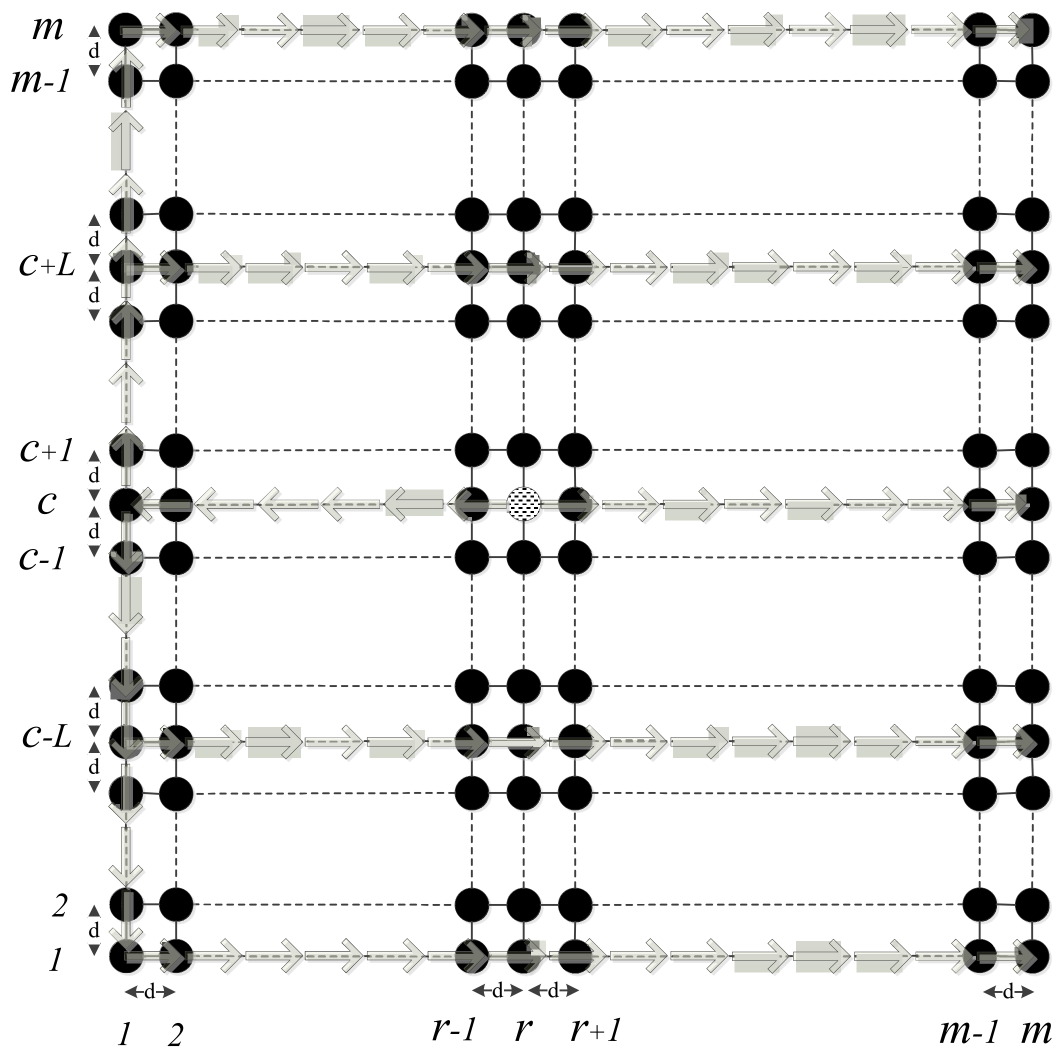

Figure 3.6: Node placement and broadcast strategy in a grid network.

network (Figure 3.6) with power consumption $P_{tot}^{(n)}$ *we have*

$$P_{tot}^{(n)} \in \Omega(nd^2).$$

Proof. Let $(\mathscr{A}^{(n)}, \rho^{(n)})$ be an arbitrary non-cooperative broadcast algorithm. If there is a node $u \in \mathscr{A}^{(n)}$ with $\rho^{(n)}(u) < d^2$, we can safely set $\rho^{(n)}(u) = 0$ (hence removing u from $\mathscr{A}^{(n)}$), since the transmission by u will not reach any other node in the grid network. For any node $u \in \mathscr{A}^{(n)}$, let $\mathcal{N}_u$ denote the set of nodes within the transmission range of u, that is

$$\mathcal{N}_u = \{v | d_{u,v} \leq \sqrt{\rho(u)}\},$$

where $d_{u,v}$ denotes the distance between nodes $u, v \in \mathscr{U}$. Note that all nodes in $\mathcal{N}_u$ are within the square with side length of $2\sqrt{\rho(u)}$ entered at u, because they are all inside the circle with radius $\sqrt{\rho(u)}$ centered at u. The number of nodes within a square with sides length of $2\sqrt{\rho(u)}$ is bounded by

$$|\mathcal{N}_u| \leq \left(2\left\lfloor\frac{\sqrt{\rho(u)}}{d}\right\rfloor + 1\right)^2.$$

Therefore

$$|\mathcal{N}_u| \leq 9\left(\frac{\rho(u)}{d^2}\right), \tag{3.22}$$

because

$$\begin{aligned}\left(2\left\lfloor\frac{\sqrt{\rho(u)}}{d}\right\rfloor + 1\right)^2 &\leq \left(2\frac{\sqrt{\rho(u)}}{d} + 1\right)^2 \\ &\leq \left(3\frac{\sqrt{\rho(u)}}{d}\right)^2 = 9\left(\frac{\rho(u)}{d^2}\right),\end{aligned}$$

where the last inequality holds since $\rho(u) \geq d^2$ (i.e. $\frac{\sqrt{\rho(u)}}{d} \geq 1$). Every node in the network, including the source, is within the transmission range of at least one transmitting node in $\mathscr{A}^{(n)}$. Thus, we must have

$$\sum_{\forall u \in \mathscr{A}^{(n)}} |\mathcal{N}_u| \geq n.$$

Hence by (3.22) we get

$$\sum_{\forall u \in \mathscr{A}^{(n)}} 9\left(\frac{\rho^{(n)}(u)}{d^2}\right) \geq n,$$

thus,

$$P_{tot}^{(n)} = \sum_{\forall u \in \mathscr{A}^{(n)}} \rho^{(n)}(u) \geq \frac{nd^2}{9}.$$

□

Next, we propose a simple cooperative broadcast algorithm $(\mathscr{A}^{(c)}, \rho^{(c)})$ with total power consumption $P_{tot}^{(c)} \in \mathcal{O}(\frac{nd^2}{\log n})$. Figure 3.6 illustrates this cooperative algorithm in which the transmission proceeds horizontally, from the source, in both directions. Reaching the left-most column, the broadcast continues vertically, in both directions. As depicted in Figure 3.6, nodes in every Lth row transmit the message, where L is an adjustable parameter of the algorithm. Every such row is called a *transmitting row*. The transmission power of each node $u \in \mathscr{A}^{(n)}$ is set to d^2. The following proposition shows that the algorithm illustrated in Figure 3.6 is a cooperative broadcast algorithm (i.e. every node will successfully decode the message) even when L is as large as $0.15\ln(n)$. This implies that total power consumption $P_{tot}^{(c)}$ can be as low as $\mathcal{O}(\frac{nd^2}{\log n})$.

Theorem 9. *The broadcast algorithm* $(\mathscr{A}^{(c)}, \rho^{(c)})$ *illustrated in Figure 3.6 is a cooperative broadcast algorithm (i.e the message is delivered to all nodes) for any* $L \leq 0.15\ln(n)$.

Proof. For any given node $u \in \bar{\mathscr{A}}^{(c)}$, there are at least $\lceil \frac{1}{2} \lfloor \frac{m}{L} \rfloor \rceil$ transmitting rows either below or above that node. Without loss of generality, we suppose that node u has at least $\lceil \frac{1}{2} \lfloor \frac{m}{L} \rfloor \rceil$ transmitting rows atop. Let $\mathcal{P}_r(u)$ be the sum of powers received at $u \in \bar{\mathscr{A}}^{(c)}$ from all nodes in all transmitting rows. Among nodes in a non-transmitting row, the one on the rightmost column has the least value of $\mathcal{P}_r(u)$. Hence, we assume that node u is in rightmost column. Let l denote the Euclidian distance between u and the closest upper transmitting row. The total power received at u from nodes in ith upper transmitting row, denoted as $\mathcal{P}_i(u)$, is

$$\begin{aligned}\mathcal{P}_i(u) &= \sum_{j=0}^{m-1} \frac{d^2}{((l+(i-1)L)d)^2 + (jd)^2} \\ &\geq \sum_{j=0}^{m-1} \frac{d^2}{(iLd)^2 + (jd)^2} \\ &= \sum_{j=0}^{m-1} \frac{1}{(iL)^2 + j^2}.\end{aligned} \tag{3.23}$$

Since the number of upper transmitting rows is at least $c = \lceil \frac{1}{2} \lfloor \frac{m}{L} \rfloor \rceil$, we get

$$\begin{aligned}\mathcal{P}_r(u) &\geq \sum_{i=1}^{c} \mathcal{P}_i(u) \\ &\geq \sum_{i=1}^{c} \sum_{j=0}^{m-1} \frac{1}{(iL)^2 + j^2},\end{aligned} \tag{3.24}$$

where the second inequality is by (3.23). We have $m \geq iL$, $1 \leq i \leq c$. Thus,

$$\sum_{i=1}^{c} \sum_{j=0}^{m-1} \frac{1}{(iL)^2 + j^2} \geq \sum_{i=1}^{c} \sum_{j=0}^{iL-1} \frac{1}{(iL)^2 + j^2} \geq \sum_{i=1}^{c} \frac{1}{2Li}.$$

Using the partial sum of harmonic series, we get

$$\begin{aligned}\sum_{i=1}^{c} \frac{1}{2Li} = \frac{1}{2L} \sum_{i=1}^{\lceil \frac{1}{2} \lfloor \frac{m}{L} \rfloor \rceil} \frac{1}{i} &\geq \frac{1}{2L} \ln \left\lceil \frac{1}{2} \lfloor \frac{m}{L} \rfloor \right\rceil \\ &\geq \frac{1}{2L} \ln \left(\frac{1}{2} (\frac{m}{L} - 1) \right).\end{aligned} \tag{3.25}$$

Thus, $\mathcal{P}_r(u) \geq 1$ (hence, u can decode the message) if

$$\frac{1}{2L} \ln((\frac{m}{2L} - \frac{1}{2})) \geq 1. \tag{3.26}$$

Finally, it can be verified that (3.26) holds for any $L \leq 0.3 \ln(m) = 0.15 \ln(n)$ and $n = m^2 \geq 4$, which completes the proof. $\square$

Considering the grid network (Figure 3.6), there are $\frac{m}{L}$ transmitting rows within each m nodes broadcast with power d^2. Therefore, we have $P_{tot}^{(c)} \in \mathcal{O}(\frac{nd^2}{\log n})$, hence, $G_{tot} \in \Omega(\log n)$.

Chapter 4

Numerical Analysis

In this section, using numerical analysis, we verify our analytical results by showing that, in grid networks, the cooperation gain increases logarithmically with n, the number of nodes in the network. Then, we implement the best existing cooperative and non-cooperative broadcast algorithms in the literature and compare their total power consumptions. We show that, in networks with nodes at random positions, the ratio of their total power consumption increases logarithmically with n both under small-sale and large-scale fading.

For grid networks, we simulate a slightly-modified version of the cooperative broadcast algorithm discussed in Section 3.4. In the modified version, not only the nodes in every Lth row, but also the nodes in the top and bottom rows transmit. For a given size of the grid network and a given position of the source node, we first search for the largest value of L that guarantees full delivery. Then, we use the maximum value of L in the cooperative algorithm, and calculate the total power consumption. As for the non-cooperative algorithm, we use the simple algorithm in which all the nodes transmit with power d^2, where d denotes the minimum distance between nodes in the grid. This simple non-cooperative algorithm was proven to be asymptotically optimal. In numerical analysis, we go up to a grid size of 50×50, with $n = 50 \times 50$ nodes. For a given grid size, we run the numerical analysis 100 times. In each run, the

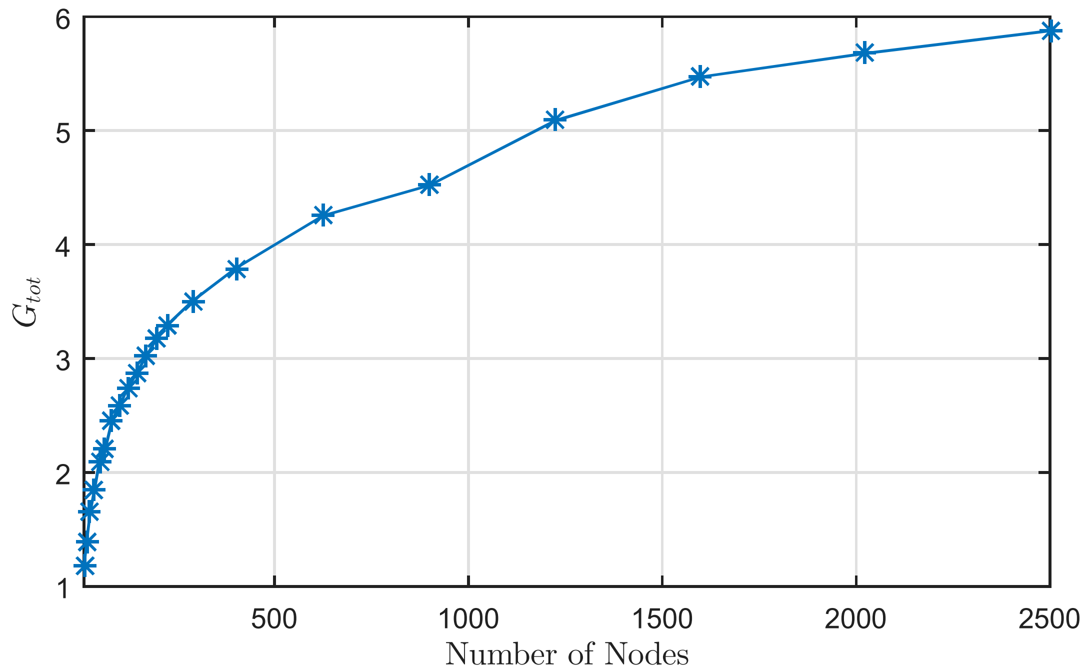

Figure 4.1: G_{tot} versus number of nodes for 2D grid network.

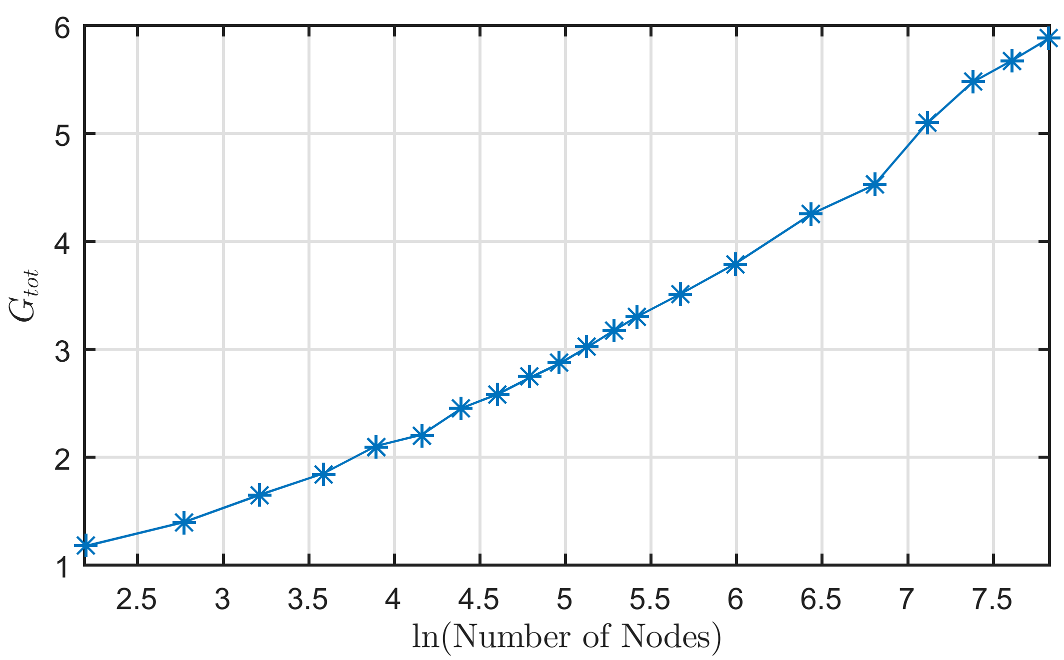

Figure 4.2: G_{tot} versus $\ln(n)$ for 2D grid networks.

source is selected uniformly at random from the nodes in the gird. Figure 4.2 shows the ratio of the total power consumption of the non-cooperative algorithm over that of the cooperative algorithm. Figure 4.1 shows the cooperation as the number of nodes n in the network increases. To show the logarithmic growth of the cooperation gain, in Figure 4.2, we plotted the gain versus $\ln n$.

To calculate the cooperation gain, we used the best existing algorithms in the literature. For non-cooperative broadcast, we implemented the proposed algorithm by Caragiannis et al. in [20]. This algorithm has the best approximation factor to the broadcast power consumption minimization problem, among the existing non-cooperative broadcast algorithms. As for the cooperative algorithm, we implemented the greedy filling algorithm proposed in [12]. In numerical analysis, a set of n nodes are placed uniformly at random in a disk. For each given number of nodes n, we execute the above two algorithms on 50 different node placements, Figure 4.3 and 4.4 show the results under both simplified path-loss model and Rayleigh fading, respectively, for different values of α.

We considered two other nodes distribution and performed *CA* and *greedy filling* algorithms. In one setting, the nodes had a Gaussian distribution with standard deviation of 0.5 around the center the disk. Figure 4.5 and 4.6 show the results under both simplified path-loss model and Rayleigh fading, respectively, for this nodes distribution.

Then, we considered a clustered structure for nodes distribution consisting of 5 cluster centers. Other nodes are located equally around these centers with a Gaussian distribution ($\sigma = 0.5$). Figure 4.7 and 4.8 show the results under both simplified path-loss model and Rayleigh fading, respectively, for this nodes distribution.

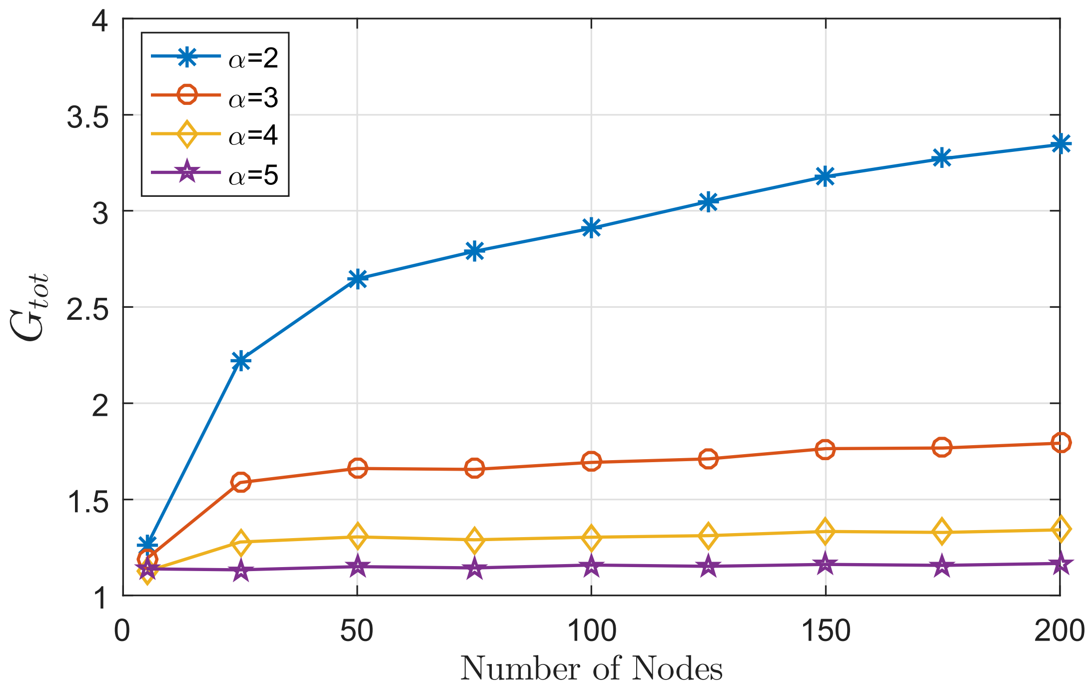

Figure 4.3: G_{tot} versus n under simplified path-loss model, nodes are uniformly distributed in the network

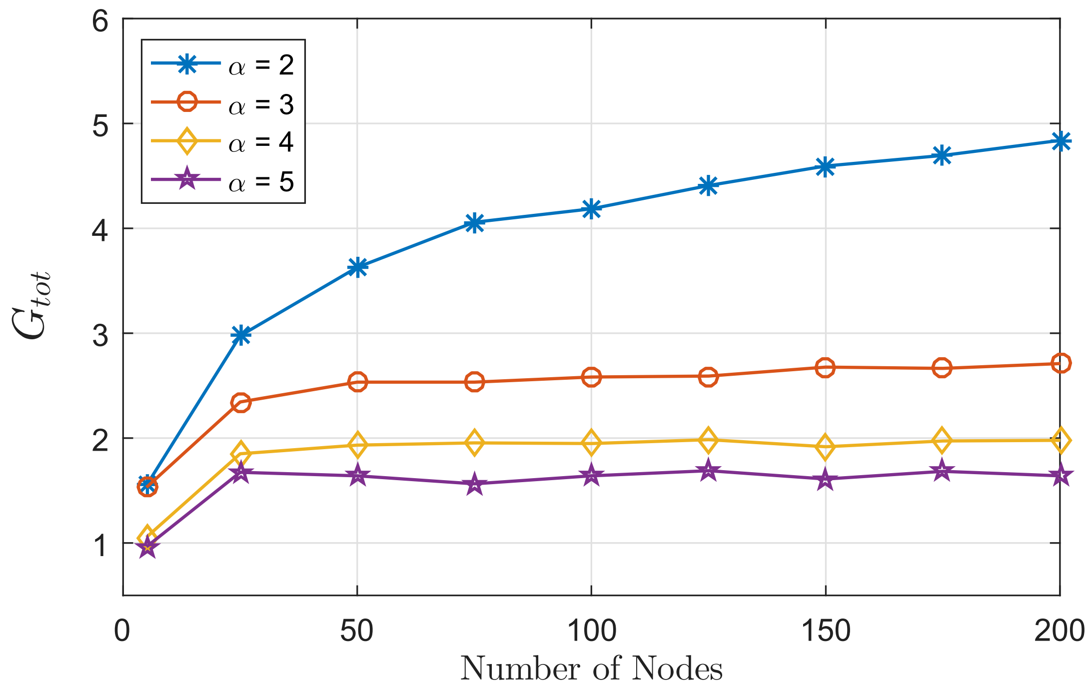

Figure 4.4: G_{tot} versus n under Rayleigh fading model, nodes are uniformly distributed in the network

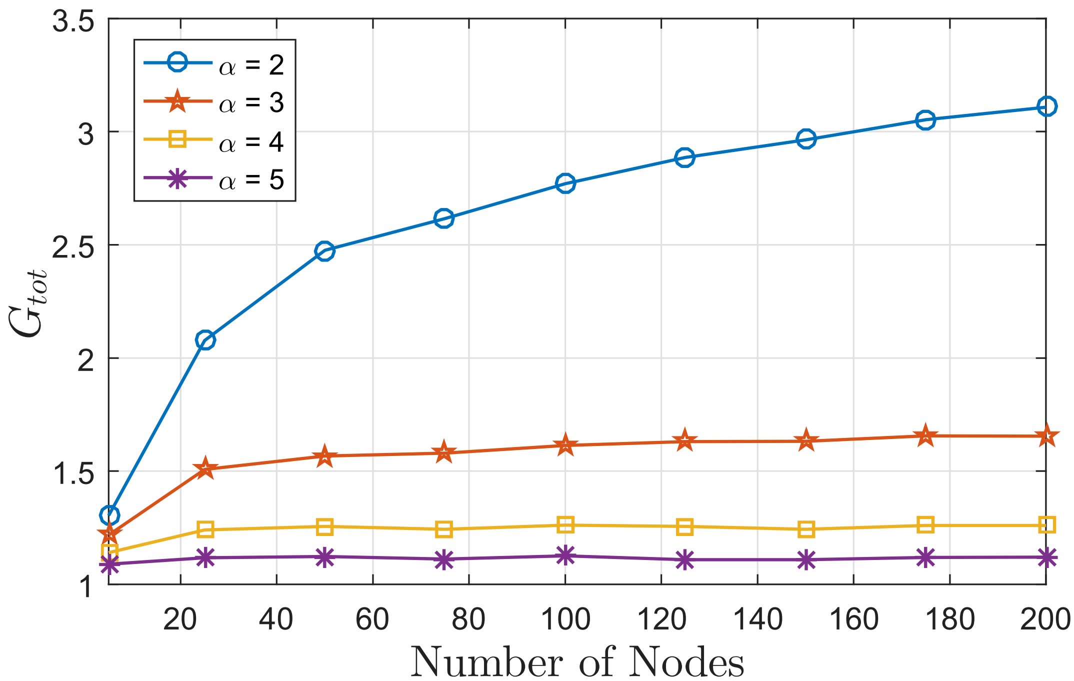

Figure 4.5: G_{tot} versus n under simplified path-loss model, nodes have Gaussian distribution around center

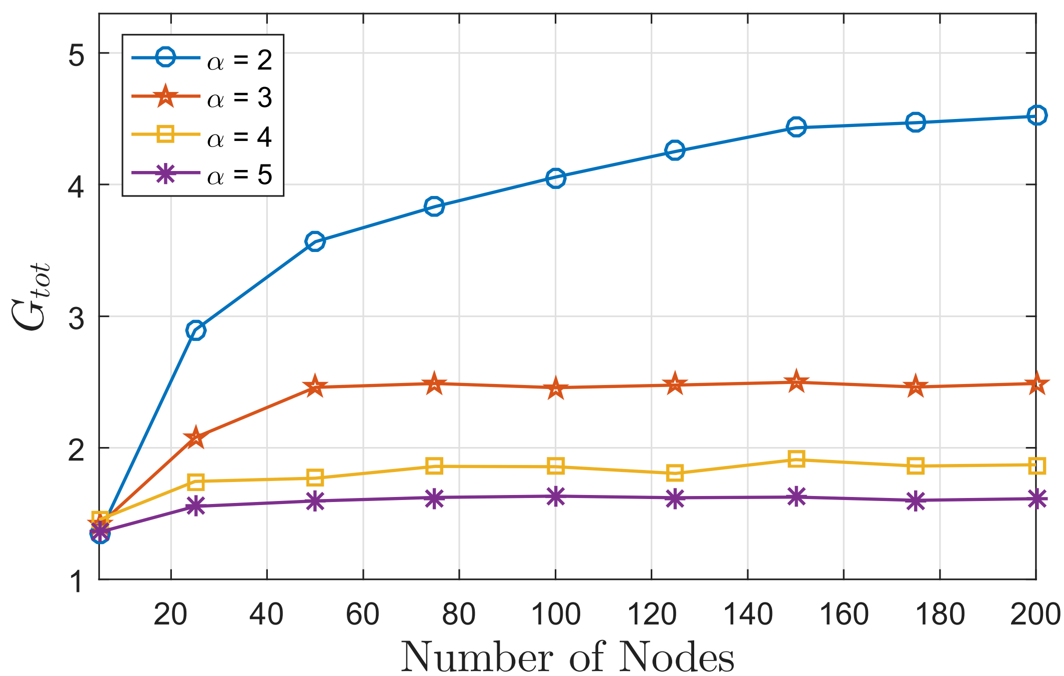

Figure 4.6: G_{tot} versus n under Rayleigh fading model, nodes have Gaussian distribution around center

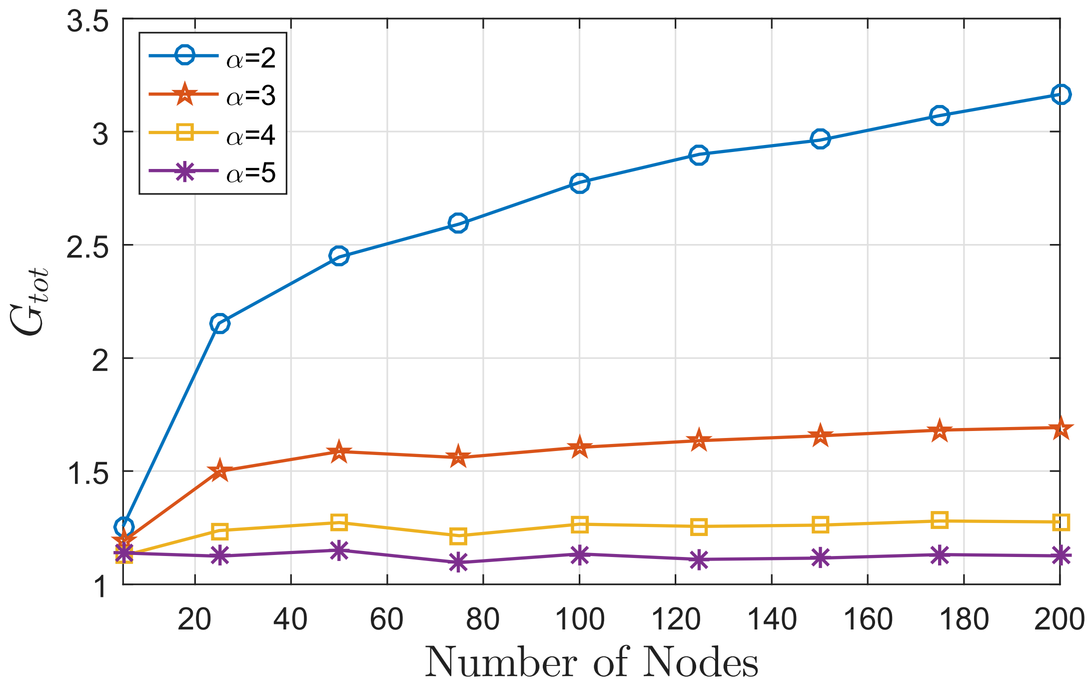

Figure 4.7: G_{tot} versus n under simplified path-loss model, nodes in a clustered structure

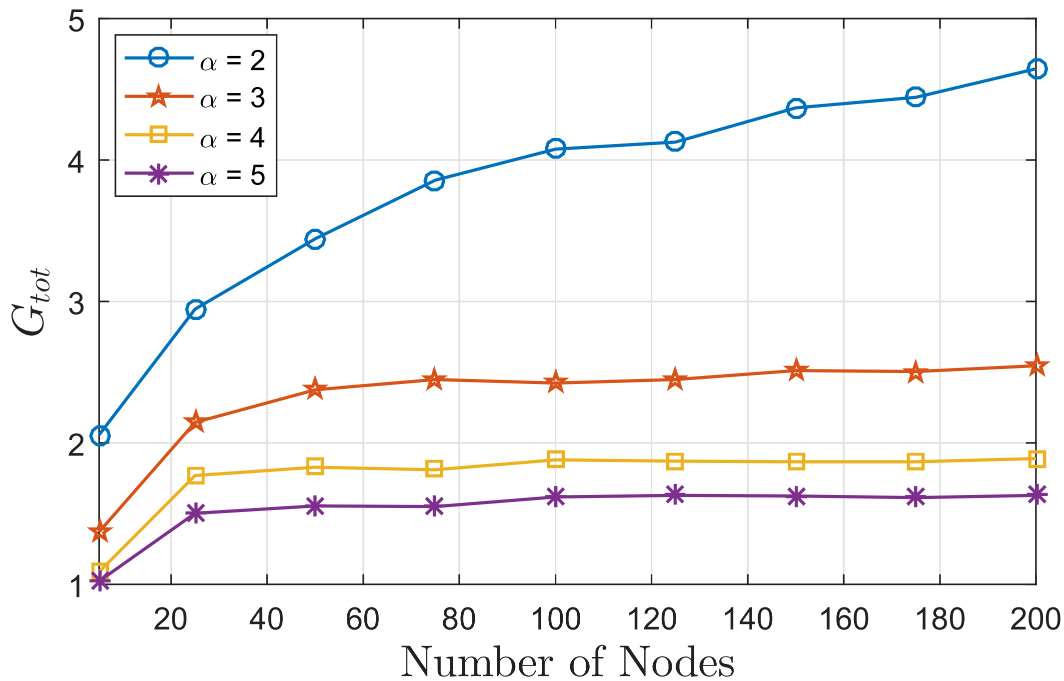

Figure 4.8: G_{tot} versus n under Rayleigh fading model, nodes in a clustered structure

Next, we evaluate our construction method explained in Section 3.3. This method converts any given cooperative broadcast algorithm with total power consumption of $P_{tot}^{(c)}$ to a non-cooperative broadcast algorithm with total power consumption $P_{tot}^{(n)} \leq 127 \ln(n)$ To verify this result, we considered networks with size up to $n = 400 \times 400$. We placed nodes uniformly at random in the network, and used the greedy filling algorithm proposed in [12] as the input to our construction method. For each value of n, we performed 1000 runs of numerical analysis. In every run, we verified that the constructed algorithm achieves full delivery. Figures 4.9 and 4.10 show the construction radio versus n and $\ln(n)$, respectively, where the construction ratio is defined as the ratio of the total power consumption of the constructed algorithm to that of the construction method input (in this case, the greedy filling algorithm). The maximum slope of the curve in Figure 4.10 is about five, much lower than the slope of 127 in the proven bound of $127 \ln(n)$ on the construction ratio (See proof of Theorem 7).

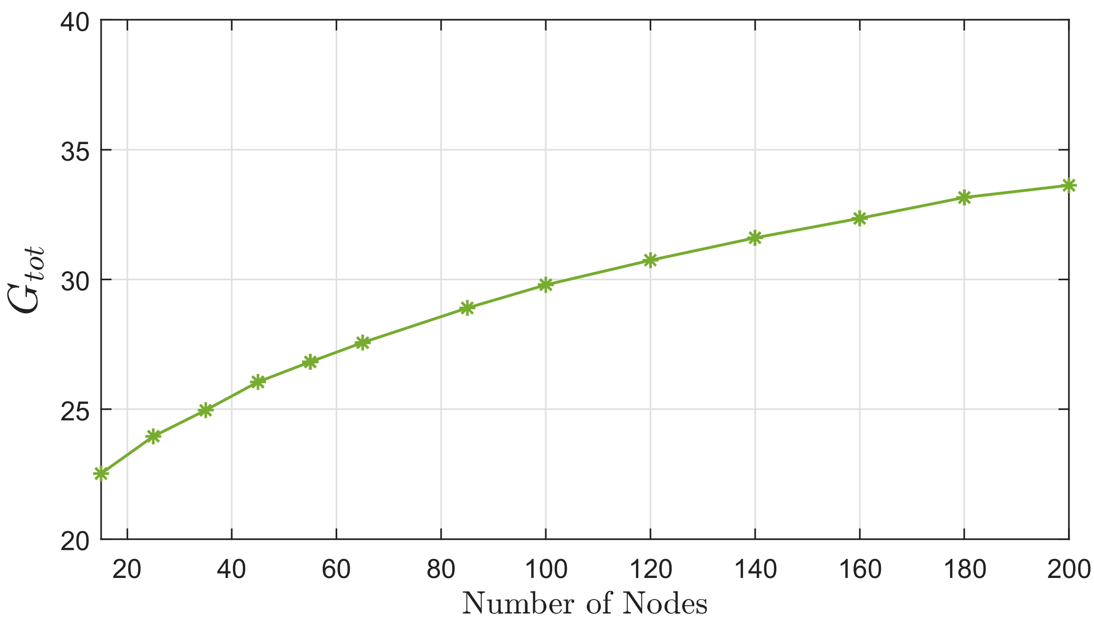

Figure 4.9: Construction ratio when the greedy algorithm is used as the input of the construction method.

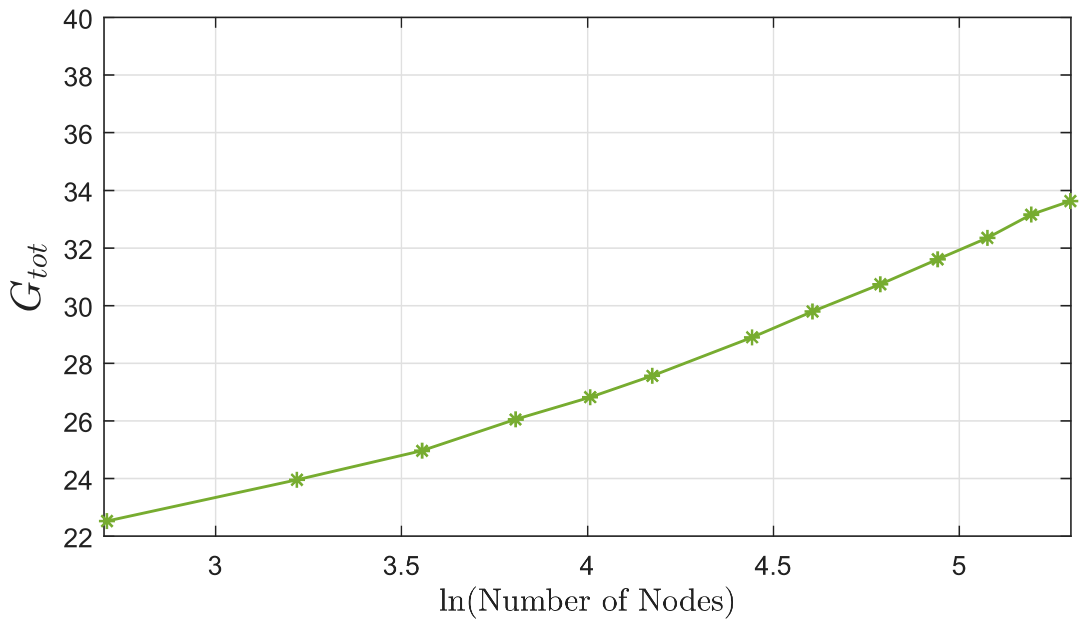

Figure 4.10: Construction ratio when the greedy algorithm is used as the input of the construction method.

Chapter 5

Conclusion & Future Works

The cooperation gain is a measure of how much energy can be saved when energy accumulation is used in wireless broadcast. In this work, we showed that, in 2D grid networks and random networks, the cooperation gain increases logarithmically with the number of nodes. This is in contrast to linear networks, where the gain was proven to be constant with respect to the number of nodes. Further, we proved that the gain is $\mathcal{O}(\log n)$ in any 2D networks with n nodes, that is, the cooperation gain grows at best logarithmically with n.

The above results can be extended in at least two ways. First is to account for the circuit power consumed in wireless transmissions. This is important in short-range transmissions, where the circuit power consumption is non-negligible. Second is to analyze the gain for larger path loss exponents.

Chapter 6

Bibliography

[1] Seth Blumsack and Alisha Fernandez. Ready or not, here comes the smart grid! *Energy*, 37(1):61–68, 2012.

[2] Patrick McDaniel and Stephen McLaughlin. Security and privacy challenges in the smart grid. *IEEE Security and Privacy*, 7(3):75–77, 2009.

[3] Friedemann Mattern and Christian Floerkemeier. From the internet of computers to the internet of things. In *From active data management to event-based systems and more*, pages 242–259. 2010.

[4] Luigi Atzori, Antonio Iera, and Giacomo Morabito. The internet of things: A survey. *Computer networks*, 54(15):2787–2805, 2010.

[5] Dusit Niyato, Lu Xiao, and Ping Wang. Machine-to-machine communications for home energy management system in smart grid. *IEEE Communications Magazine*, 49(4):53–59, 2011.

[6] Geng Wu, Shilpa Talwar, Kerstin Johnsson, Nageen Himayat, and Kevin D Johnson. M2m: From mobile to embedded internet. *IEEE Communications Magazine*, 49(4):36–43, 2011.

[7] Diane Cook and Sajal Das. *Smart environments: Technology, protocols and applications*, volume 43. John Wiley & Sons, 2004.

[8] Stefan Poslad. *Ubiquitous computing: smart devices, environments and interactions*. John Wiley & Sons, 2011.

[9] Azrina Abd Aziz, Y Ahmet Sekercioglu, Paul Fitzpatrick, and Milosh Ivanovich. A survey on distributed topology control techniques for extending the lifetime of battery powered wireless sensor networks. *IEEE communications surveys & tutorials*, 15(1):121–144, 2013.

[10] Majid Khabbazian, Ian F Blake, and Vijay K Bhargava. Local broadcast algorithms in wireless ad hoc networks: Reducing the number of transmissions. *IEEE Transactions on Mobile Computing*, 11(3):402–413, 2012.

[11] Milen Nikolov and Zygmunt J Haas. Towards optimal broadcast in wireless networks. *IEEE Transactions on Mobile Computing*, 14(7):1530–1544, 2015.

[12] Ivana Maric and Roy D Yates. Cooperative multihop broadcast for wireless networks. *IEEE Journal on Selected Areas in Communications*, 22(6): 1080–1088, 2004.

[13] Chenxi Qiu, Haiying Shen, and Lei Yu. Energy-efficient cooperative broadcast in fading wireless networks. In *IEEE INFOCOM*, pages 1114–1122, 2014.

[14] Marjan Baghaie and Bhaskar Krishnamachari. Delay constrained minimum energy broadcast in cooperative wireless networks. In *IEEE INFOCOM*, pages 864–872, 2011.

[15] Fulu Li and L Nikolaidis. On minimum-energy broadcasting in all-wireless networks. In *Local Computer Networks*, pages 193–202, 2001.

[16] Mario Čagalj, Jean-Pierre Hubaux, and Christian Enz. Minimum-energy broadcast in all-wireless networks: Np-completeness and distribution is-

sues. In *International conference on Mobile computing and networking*, pages 172–182, 2002.

[17] Zahra Mobini and Majid Khabbazian. Asymptotic gain analysis of cooperative broadcast in linear wireless networks. *IEEE Transactions on Wireless Communications*, 15(1):485–497, 2016.

[18] Birsen Sirkeci-Mergen and Anna Scaglione. On the power efficiency of cooperative broadcast in dense wireless networks. *IEEE journal on selected areas in communications*, 25(2):497–507, 2007.

[19] Yao-Win Hong and Anna Scaglione. Energy-efficient broadcasting with cooperative transmissions in wireless sensor networks. *IEEE Transactions on Wireless communications*, 5(10):2844–2855, 2006.

[20] Ioannis Caragiannis, Michele Flammini, and Luca Moscardelli. An exponential improvement on the mst heuristic for minimum energy broadcasting in ad hoc wireless networks. *IEEE/ACM Transactions on Networking (TON)*, 21(4):1322–1331, 2013.

[21] Brent N Clark, Charles J Colbourn, and David S Johnson. Unit disk graphs. *Discrete mathematics*, 86(1-3):165–177, 1990.

[22] Jiangzhuo Chen, Lujun Jia, Xin Liu, Guevara Noubir, and Ravi Sundaram. Minimum energy accumulative routing in wireless networks. In *INFOCOM 2005. 24th Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings IEEE*, volume 3, pages 1875–1886. IEEE, 2005.

[23] Andreas F Molisch, Neelesh B Mehta, Jonathan S Yedidia, and Jinyun Zhang. Wlc41-6: cooperative relay networks using fountain codes. In *Global Telecommunications Conference, 2006. GLOBECOM'06. IEEE*, pages 1–6. IEEE, 2006.

[24] Jeff Castura and Yongyi Mao. Rateless coding over fading channels. *IEEE communications letters*, 10(1):46–48, 2006.

[25] Ivana Maric and Roy D Yates. Cooperative multicast for maximum network lifetime. *IEEE Journal on Selected Areas in Communications*, 23(1):127–135, 2005.

[26] Andrea Goldsmith. *Wireless Communications*. Cambridge University Press, 2005.