

Definitions:

A *Field* is a set with two operations: $+$ and \times .

An Application is defined as a tuple: $A = \{name, O, R, m\}$

where O is a set of *Objects*, R is set of *Relations*, and m is a set of *methods* that apply to the Application as a whole.

For many purposes, an Application can be abbreviated: $A = \{O, R\}$

Closure:

$$a + b = c \in F$$

$$a \times b = c \in F$$

$$A_a + A_b = \{O_a, \} + \{O_b, \} = \{O_a \cup O_b, \}$$

$$A_a \times A_b = \{O_a, \} \times \{O_b, \} = \{O_a \cup O_b, O_a \leftrightarrow O_b\}$$

Associatively:

$$(a + b) + c = a + (b + c)$$

$$(a \times b) \times c = a \times (b \times c)$$

$$(A_a + A_b) + A_c = (\{O_a, \} + \{O_b, \}) + \{O_c, \} = \{(O_a \cup O_b), \} + \{O_c, \} = \{O_a \cup O_b \cup O_c, \}$$

$$(A_a \times A_b) \times A_c = \{O_a \cup O_b, O_a \leftrightarrow O_b\} \times \{O_c, \} = \{O_a \cup O_b \cup O_c, O_a \leftrightarrow O_b \leftrightarrow O_c\}$$

$$A_a \times (A_b \times A_c) = \{O_a, \} \times \{O_b \cup O_c, O_b \leftrightarrow O_c\} = \{O_a \cup O_b \cup O_c, O_a \leftrightarrow O_b \leftrightarrow O_c\}$$

\therefore

$$(A_a \times A_b) \times A_c = A_a \times (A_b \times A_c)$$

Commutative:

$$a + b = b + a$$

$$a \times b = b \times a$$

$$A_a + A_b = \{O_a \cup O_b, \} = \{O_b \cup O_a, \} = A_b + A_a$$

$$A_a \times A_b = \{O_a \cup O_b, O_a \leftrightarrow O_b\} = \{O_b \cup O_a, O_b \leftrightarrow O_a\} = A_b \times A_a$$

Identity:

$$a + 0 = a$$

$$a \times 1 = a$$

$$O = \{\{\phi\},\}$$

$$A_a + O = \{O_a \cup \phi,\} = \{O_a,\} = A_a$$

$$1 = \{\{\phi\},\{\phi\}\}$$

$$A_a \times 1 = \{O_a \cup \phi, O_a \leftrightarrow \phi\} = \{O_a,\} = A_a$$

Inverse:

$$a + (-a) = 0$$

$$a \times a^{-1} = 1$$

Distributive:

$$a \times (b + c) = (a \times b) + (a \times c)$$

$$A_a \times (A_b + A_c) = A_a \times \{O_b \cup O_c,\} = \{O_a \cup O_b \cup O_c, O_a \leftrightarrow O_b \leftrightarrow O_c\}$$

$$(A_a \times A_b) + (A_a \times A_c) = \{O_a \cup O_b, O_a \leftrightarrow O_b\} + \{O_a \cup O_c, O_a \leftrightarrow O_c\}$$

$$= \{O_a \cup O_b \cup O_c, \{O_a \leftrightarrow O_b, O_a \leftrightarrow O_c\}\} = \{O_a \cup O_b \cup O_c, O_a \leftrightarrow O_b \leftrightarrow O_c\}$$

\therefore

$$A_a \times (A_b + A_c) = (A_a \times A_b) + (A_a \times A_c)$$