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ASSESSMENT OF CONCRETE STRENGTH IN EXISTING STRUCTURES

by

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Abstract

A simple method is presented for the determination of an equivalent specified strength of concrete, using a small number of core tests, which can be substituted directly for the specified strength, f'_c, in conventional design equations to assess the safety of an existing structure. The method is developed from investigations concerning the interpretation of core strength data and the strength of concrete in structures.

Various aspects of the testing procedure which affect the compressive strength of concrete cores are quantified, using weighted linear and nonlinear regression analyses of core test data from conventional concretes investigated by others and new data from high performance concretes investigated in this study. Strength correction factors which depend on the length to diameter ratio, test moisture condition, and diameter of the core are obtained. The factors for length to diameter ratio are only slightly different from those recommended in Canadian and American published standards. These standards currently recommend short periods of drying or soaking before the specimen is tested which create a moisture gradient between the surface and the interior of the specimen that artificially biases the test result. Damage to the cut surface of the core causes cores with small diameters to fail at lower apparent strengths than cores with larger diameters obtained from the same element.

A statistical description of the strength of concrete in structures is developed using standard cylinder data from 108 concrete mixes produced in Alberta between 1988 and 1993, and using core and standard cylinder data reported by others. The average in situ compressive strength of 28 day old conventional concretes in columns is 1.3 f'_c, with a coefficient of variation of 18.6%. If the effects of within-member, between-member, and batch-to-batch strength variation are accounted for, there is roughly a 13.5% probability that the concrete strength in a 28 day old structure will be less than f'_c. It is probable that recalibration of load and resistance factors using this statistical description of concrete will yield greater factored concrete strengths than are used in current design practice.

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The following symbols are used:

a	depth of equivalent rectangular stress block, or age of concrete at time of core test;
A	area of cross section of core specimen;
A_I	area of interior (undamaged) region of core specimen;
A_O	area of outer (damaged) region of core specimen;
A_r	area of cross section of core specimen with reference diameter;
A_s	area of steel reinforcement;
b	width of concrete compression zone;
c	weight of cement per unit volume of concrete;
C_1, C_2	empirical constants in Bolotin's size effect equations;
d	diameter of concrete core specimen, or effective depth of reinforcement;
$E[\cdot]$	expected value of a derived random quantity;
f	failure stress;
F_{age}	factor to account for effect of concrete age on concrete strength;
fa^C	weight of Class C fly ash per unit volume of concrete;
fa^F	weight of Class F fly ash per unit volume of concrete;
f_c	strength of a concrete core specimen (mean value = $\bar{f}_c$, predicted value = $\hat{f}_c$);
f^*_c	strength of a concrete core specimen, modified to account for spatial variation of *in situ* strength (mean value = $\bar{f}^*_c$, predicted value = $\hat{f}^*_c$);
$\bar{f}_{c,ad}$	average strength of as-drilled concrete core specimens (predicted value = $\hat{f}_{c,ad}$);
$\bar{f}_{c,dry}$	average strength of air dried concrete core specimens (predicted value = $\hat{f}_{c,dry}$);

$f_{c,is}$	*in situ* concrete strength (mean value = $\bar{f}_{c,is}$, predicted value = $\hat{f}_{c,is}$, characteristic value = $f_{c,is,k}$, 90% limit on (one-sided) confidence interval of mean value = $(\bar{f}_{c,is})_{90}$);
f_{cn}	normalized core strength (predicted value = $\hat{f}_{cn}$);
$f_{c,NS}$	strength of cores with $1 < l/d < 2$ (mean value = $\bar{f}_{c,NS}$);
$f_{c,S}$	strength of cores with $l/d = 2$ (mean value = $\bar{f}_{c,S}$, predicted mean value = $\hat{\bar{f}}_{c,S}$);
$\bar{f}_{c,wet}$	average strength of soaked concrete core specimens;
f'_c	specified strength of concrete (equivalent specifed strength = $f'_{c,eq}$);
f_{cu}	cube strength (characteristic value = $f_{cu,k}$);
F_{cure}	factor which accounts for effect of curing on *in situ* strength;
f_{cyl}	result of a single standard concrete strength test, the average strength of 2 or 3 standard cylinders cast from one batch of concrete (mean value = $\bar{f}_{cyl}$, characteristic value $f_{cyl,k}$);
$\bar{f}_d$	average strength of cores with constant diameter d;
F_d	strength correction factor for core damage due to drilling;
F_{dia}	strength correction factor for core diameter;
F_{HV}	factor to account for effect of direction of loading on concrete strength;
f_I	failure stress of interior (undamaged) region of core specimen;
$F_{l/d}$	strength correction factor for core length to diameter ratio;
F_{mc}	strength correction factor for core moisture condition;
f_n	nominal strength of concrete in an element;
f_r	failure stress of a core specimen with reference diameter;
F_r	strength correction factor for core containing reinforcing steel;
f_{sm}	stress in the reinforcement at the maximum applied moment;
F_{sust}	factor to account for effect of rate of loading on concrete strength;
F_x, F_y, F_z	strength correction factors for variation of *in situ* strength in x, y and z directions;

F_1	ratio of average cylinder strength to specified strength (predicted value = $\hat{F}_1$);
F_2	ratio of average *in situ* strength to average cylinder strength (predicted value = $\hat{F}_2$);
$\bar{f}_4$	average strength of cores with 4 inch (100 mm) diameter;
g_1, g_2	functions reflecting the variation of core strengths due to systematic testing factors and systematic within-member spatial variation;
h	height of element;
H_0	the null hypothesis in a statistical test;
k	ratio of failure stress of interior region of core to outer region of core;
k_k	ratio of characteristic *in situ* strength to specified cube strength;
k_1	a constant used in Equation (9.17);
k_2	a constant used in Equation (9.18);
K	a constant defined in Equation (5.11);
K_1	a constant defined in Equation (7.5);
l	length of concrete core specimen;
l	length of element;
M	variable for moisture content in regression model;
M_{ad}	average moisture content of core immediately after drilling;
M_t	average moisture content of core at time of test;
M_{max}	maximum moment due to externally applied loads;
$\sqrt{MS_E}$	standard error from least squares analysis;
n	number of replicate observations of a quantity;
n_{col}	number of columns represented in a set of core data;
N_{low}	number of strength tests less than the specified strength;
N_m	number of different mixes studied in an investigation;

p	probability of the null hypothesis being true, or number of parameters estimated using regression analysis;
P	failure load of core specimen;
r_i	width of a range which bounds half of the set of observations (minimum value = r_{min});
R_d	ratio of average strength of cores with diameter d to average strength of cores with 4 inch (100 mm) diameter;
R_i	single test to predicted ratio;
R^2	coefficient of determination in regression analysis;
s	standard deviation (predicted value = $\hat{s}$);
s_B	sample standard deviation due to between-member strength variation;
$s_{c,is}$	sample standard deviation of *in situ* concrete strength;
sf	weight of silica fume per unit volume of concrete;
sl	weight of slag per unit volume of concrete;
s_M	sample standard deviation due to within-member strength variation;
s_o	robust estimate of standard deviation;
t	thickness of damaged outer region of core, or thickness of element;
T	air temperature;
T_{max}	Maximum temperature within element, or at the centre of gravity of the core during hydration;
T_1	outlier test statistic;
t_1, t_2	transition points in spline regression model;
UPV	ultrasonic pulse velocity;
u, v	transformed coordinate system;
V	coefficient of variation;
V	volume of core specimen;

V_{adj}	coefficient of variation of core strengths after adjustment to account for spatial variability of *in situ* strength;
V_B	coefficient of variation of due to between-member strength variation;
V_{bb}	coefficient of variation of cylinder strength due to batch-to-batch variation;
V_{core}	coefficient of variation of strength of replicate core specimens;
$V_{c,is}$	coefficient of variation of *in situ* strength;
V_{cyl}	coefficient of variation of cylinder strengths;
V_d	coefficient of variation attributable to core drilling damage;
V_{dia}	coefficient of variation of strength correction factor F_{dia};
V_F	coefficient of variation of strength correction factor;
V_{fcNS}	coefficient of variation of the strength of cores with $1<l/d<2$;
$V_{f\hat{c}NS}$	coefficient of variation of the predicted strength of cores with $1<l/d<2$;
V_{fcS}	coefficient of variation of the strength of cores with $l/d = 2$ (predicted value $= \hat{V}_{fcS}$);
$V_{f\hat{c}S}$	coefficient of variation of the predicted strength of cores with $l/d = 2$;
V_{fI}	coefficient of variation of strength of interior region of core;
$V_{l/d}$	coefficient of variation of strength correction factor $F_{l/d}$;
V_{mc}	coefficient of variation of strength correction factor F_{mc};
V_{model}	coefficient of variation representing the accuracy of a prediction model;
V_M	coefficient of variation of due to within-member strength variation;
V_r	coefficient of variation of strength correction F_r;
V_r	volume of core specimen with reference diameter;
V_s	coefficient of variation due to systematic spatial variation of *in situ* concrete strength;
V_{wb}	coefficient of variation of cylinder strength due to within-batch variation;
V_{wt}	coefficient of variation of cylinder strength due to within-test variation;
V_{WS}	coefficient of variation of *in situ* strength within a structure;

$V_{t/p}$ coefficient of variation of test to predicted ratios;

V_{Xi}, V_{Yi} coefficients of variation of X_i, Y_i;

V_{Za} component of core strength coefficient of variation attributed to core age;

Var [•] variance of a derived random quantity;

W weight given to an observation for regression analysis;

w_0 inverse of variance of a new observation;

x, y, z coordinates of centre of gravity of core;

x', y', z' normalized coordinates of centre of gravity of core;

$\bar{x}$, $\bar{y}$, $\bar{z}$ coordinates of center of gravity of all cores from one element;

X regressor variable (mean value = $\bar{X}$, minimum value = X_{min}, single recorded value = X_i);

Y response variable (mean value = $\bar{Y}$, single observed value = Y_i);

$\hat{y}_0$ predicted mean value of response variable;

$\hat{y}_N$ predicted value of new observation;

z' normalized distance from centre of core to base of column;

Z_a indicator variable for age of core;

Z_{air} indicator variable for air entrainment;

Z_b indicator variable for bridge construction;

Z_{bo} indicator variable for manner of core removal;

Z_{bot} indicator variable for cores from bottom of Bloem's 8 inch thick slab;

Z_c indicator variable for cores with epoxy coated sides;

Z_{cap} indicator variable type of capping material;

Z_{cen} indicator variable for platen eccentricity during core test;

Z_{cu} indicator variable for curing quality;

Z_{dia} indicator variable for specimen diameter;

Z_h indicator variable for high elements;

Z_{HV}	indicator variable for orientation of the core;
Z_i	general indicator variable;
$Z_{l/d}$	indicator variable for specimen length to diameter ratio;
Z_{mc}	indicator variable for moisture condition of the specimen;
Z_p	indicator variable for precast construction;
Z_s	variable indicating if coefficient of variation of core strength may be inflated by *in situ* strength variation;
$Z_{s/d}$	indicator variable for specimen location along length of the core;
Z_{seal}	indicator variable for cores sealed in plastic bags before testing;
Z_{seg}	indicator variable for likelihood of mix segregating;
Z_{sup}	indicator variable for supplementary cementitious materials;
Z_t	indicator variable for thin elements;
Z_{top}	indicator variable for cores from top of Bloem's 8 inch thick slab;
Z_w	indicator variable for wall elements;
α	empirical constant in Bolotin's size effect equations;
$\beta,\beta_0 ..\beta_{11}$	unknown parameters estimated by regression analysis (predicted values = $\hat{\beta}, \hat{\beta}_0 \ldots \hat{\beta}_{11}$);
$\Gamma(\bullet)$	Gamma function;
γ_n	normalized density of a core specimen (predicted value = $\hat{\gamma}_n$);
ε	normally distributed error in regression analysis;
ζ	standard deviation of transformed lognormal population;
η	mean of transformed lognormal population;
$\theta,\theta_0 ..\theta_3$	unknown parameters estimated by regression analysis (predicted values = $\hat{\theta}, \hat{\theta}_0 \ldots \hat{\theta}_4$);
μ	mean value;
σ^2	variance (predicted value = $\hat{\sigma}^2$);

σ_{bb} standard deviation of cylinder strengths attributable to batch-to-batch variation;

σ_I standard deviation of strength within a member after systematic strength variation accounted for (predicted value = $\hat{\sigma}_I$);

σ_M standard deviation of strength within a member if systematic strength variation is not accounted for (predicted value = $\hat{\sigma}_M$);

σ_S standard deviation attributable to systematic strength variation (predicted value = $\hat{\sigma}_S$);

σ_T total standard deviation of the strength of standard cylinders;

σ_{wt} standard deviation of cylinder strengths attributable within-test variation; and

$\Phi(\cdot)$ standard normal cumulative distribution function.

Chapter 1: Introduction

1.1 Assessment of Concrete Strength in Existing Structures

Many existing concrete buildings and bridges resist loads which are considerably larger than those envisaged at the time of original design and construction. If the safety of these structures is a matter of concern, detailed evaluation is warranted so that resources are not wasted on unnecessary rehabilitation (Buckland and Bartlett, 1992). As part of the evaluation process, it is common to obtain samples from the structure for testing to assess the quality of material (e.g. CSA, 1990). The primary technique for assessing the compressive strength of concrete is the core test. The research presented in this thesis is focussed on the assessment of concrete strength in an existing structure by interpretation of core strength test data.

A major goal of this investigation is to address the distinction between the specified concrete strength in resistance equations used for design, f'_c, and the strength disclosed by the core test results. This is necessary if core test data are to be analysed in a manner which conforms to the intent of Clause 12.14 of *Supplement No. 1 - 1990 "Existing Bridge Evaluation"* (CSA, 1990) to CSA Standard *CAN/CSA-S6-88 "Design of Highway Bridges"* (CSA, 1988), which states:

> "Material test results should not be used directly in the resistance equations; however, testing may be used to establish a strength equivalent to the specified strength upon which resistance equations are based."

Unfortunately, the Clause gives no practical guidance on how to accomplish this from concrete core test data.

In the past, researchers have suggested that the conversion of core strengths into equivalent standard specimen strengths is fraught with difficulties. In his landmark paper on core testing, Malhotra (1977, p. 168) observed that:

> "more often than not, the drilling and testing of cores can create more problems than they solve if an attempt is made to relate the strength of cores to the strength of control cylinders".

Similarly, Bloem (1965, p. 687) concluded that "they [core tests] cannot be translated to terms of standard cylinder strength with any degree of confidence", and Neville (1981, p. 568) states that "dogmatic conversions of core strengths into notional cylinder and cube strengths must not be made". The intent of Clause 12.14 therefore seems quite unrealistic and impractical in light of these concerns.

It is instructive to review the reasons given for these definitive statements. Malhotra (1977, p. 165) cites "a large number of factors which can affect the compressive strength of cores, and unless allowance is made for the effect of these factors, the data from various sources present a dismal picture". He then enumerates many of these factors, and cites various investigations which provide contradictory evidence about the effects of these factors on the core test result. Bloem (1965) and Neville (1981) offer similar reasons for their concerns about conversions of core strength to cylinder strength. Therefore, the various factors which affect the strength of concrete cores must be quantified so that core strengths may be interpreted correctly. Additional research is necessary to determine whether the influence of these factors is the same for modern high performance concretes with strengths greater than 55 MPa as it is for normal concretes.

The dimensions of a structure are initially chosen using a codified procedure which essentially has been proved by a history of successful past practice. With the development of Limit States Design, simple probabilistic descriptions of random variables representing loadings and resistances have been developed (MacGregor, 1976). These are used for calibration, allowing satisfactory past experience to be extrapolated to new structural forms, materials and construction methods. When evaluating the safety of existing structures, it is necessary to maintain consistency with these simple models so that a consistent measure of safety is obtained.

For example, the concrete strength value used in conventional resistance equations is a deterministic quantity which is specified by the designer. The actual strength of concrete in the structure is a random variable which is affected by many factors including the age of the concrete, the quality of compaction and curing provided, and the size and type of the element (Malhotra, 1977). To evaluate the safety of an existing structure in terms that are consistent with those used for design, it is therefore necessary to relate the actual *in situ* strength to the *in situ* strength that might have been expected from the specified strength used for design, instead of the actual strength originally specified.

Thus it is necessary to develop a statistical description of the concrete strength in structures. It is possible to adopt the description developed by Mirza et al. (1979) for the calibration of the load and resistance factors in Canadian Standard CAN3-A23.3-M84 "Design of Concrete Structures for Buildings" (CSA, 1984) and other standards (Mirza and MacGregor, 1982). However, it may be more appropriate to develop a new statistical description of the concrete strength which reflects data that have become available recently, including data for modern high performance concretes. Although a new description will not necessarily correspond to the nominal safety levels used in past code calibration, it will reflect the actual safety levels associated with the use of current load and resistance factors.

1.2 Description of this Study

The following three topics are investigated in this study:

1. How are core test data to be interpreted? Specifically, how is the strength of a core affected by various testing practices which are conventionally accepted? Do the effects of these testing factors extend to cores from high performance concretes?

2. What is the statistical description of the strength of concrete in structures? What is the expected value of the *in situ* strength, and how does it vary with the concrete age, the quality of curing provided, the size and type of element, the concrete strength and other factors? How accurately can the average *in situ* strength be predicted for a known specified strength? How much does the strength of concrete vary within a structure?

3. Using results from the investigation of the first two topics, what equivalent specified strength based on core test data should be used for the evaluation of existing structures?

An outline of the material presented in this thesis is shown in Figure 1.1.

Chapters 2 through 6 consider the interpretation of core test data. The experimental procedures and preliminary analyses of cores from high performance concrete beams are presented in Chapter 2. These data are combined with data from tests of conventional concretes reported in other published and unpublished sources. In Chapters 3, 4 and 5, regression analyses of these combined sets are used to determine core strength correction factors which account for the effects of the length to diameter ratio, the moisture condition and the diameter of the core, respectively, on the strength. A procedure for converting a core test result to the equivalent *in situ* strength based on these factors is presented in Chapter 6.

The procedure developed in Chapter 6 is used to investigate the strength of concrete in structures in Chapters 7 and 8. In Chapter 7, a relationship between the *in situ* strength and the specified strength of concrete is developed. In Chapter 8, the variation of the *in situ* strength in structures due to systematic and random factors is investigated.

Finally, in Chapter 9, a procedure for determining an equivalent specified strength from core test data is presented. The procedure is partially based on the method for converting core test results to equivalent *in situ* strengths in Chapter 6, and incorporates the statistical descriptions of the strength of concrete in structures developed in Chapters 7 and 8.

1.3 Thesis Format

This thesis is prepared in accordance with the regulations for a Paper Format Thesis as set out by the Faculty of Graduate Studies and Research at the University of Alberta (FGSR, 1992). Each chapter, except for the first and last chapters, is presented in a paper format without an abstract but with its own bibliography. Tables and figures for each chapter are grouped at the end of each chapter before the bibliography. The nomenclature is consistent throughout the thesis, and is listed in the prefatory pages. References to chapters which have been submitted for publication, or are intended to be submitted, take the form "(Bartlett and MacGregor, 1993, [Chapter 6])", which refers to the paper by Bartlett and MacGregor that appears as Chapter 6 of this thesis.

A considerable quantity of material, typically details of calculation methods and tables of experimental data, is not provided in the main chapters but is instead presented in appendices which follow the last chapter. Each appendix is identified by a number, which indicates which chapter it pertains to, and a letter. For example, Appendix 5D is the fourth of six appendices containing material that relates to Chapter 5. References cited in an appendix are included in the list at the end of the associated chapter.

Both Systeme International (S. I.) and United States Customary units of measurement are used in this thesis. Generally the units adopted for a particular set of data are the same as those used by the original investigators to report the data. This approach facilitates checking to detect and rectify transcription errors. Generally data from Canadian sources, including the data from the laboratory investigation conducted at the University of Alberta reported in Chapter 2, are in S. I. units, while data from American sources are generally in inch-pound units. Equivalent S. I. values are given in parentheses to aid the reader who may be unfamiliar with inches and pounds. However the provision of S. I. equivalents is not exhaustive, since the various data are usually analysed to obtain strength correction factors and ratios which are dimensionless.

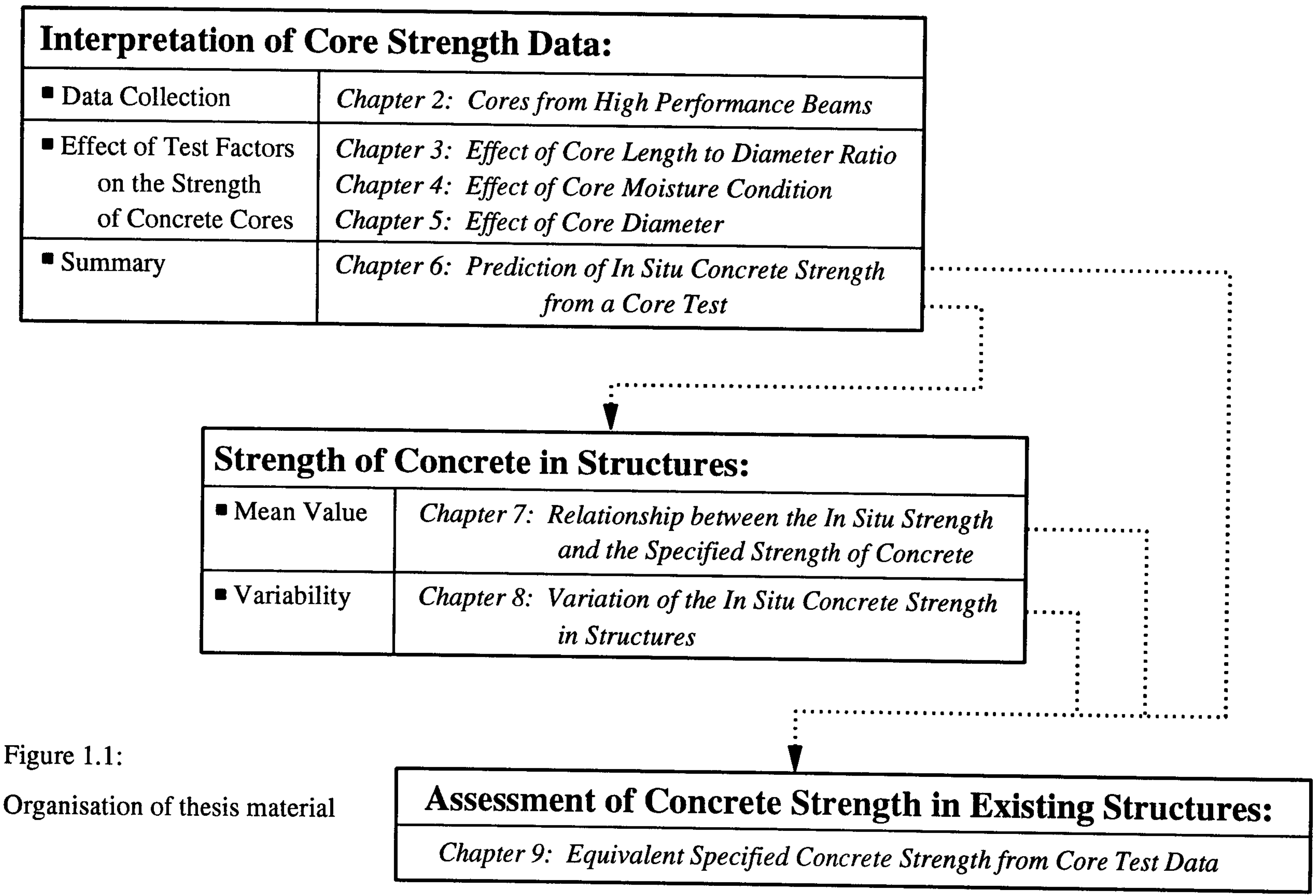

Figure 1.1:

Organisation of thesis material

References:

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Chapter 2: Cores from High Performance Concrete Beams[1]

2.1 Introduction

The compressive strength of concrete cores is affected by the sampling, moisture conditioning, and testing procedures adopted. Investigations by others have identified the sensitivity of the measured strength to the ratio of core length to diameter (*l*/d) (Meininger et al., 1977), the core moisture condition (Bloem, 1965), the core diameter (Bungey, 1979), and the orientation of the core axis with respect to the direction of concrete placement (Sanga and Dhir, 1976). These past investigations have typically been limited to concretes with standard cylinder strengths less than 7000 psi (50 MPa). There are little published data to substantiate extrapolating these core strength correction factors from conventional concretes to high strength concretes.

Moreover, the distinction between the strength of standard cores and the actual *in situ* concrete strength has not been fully investigated. Some researchers (e.g. Mak et al., 1990) have assumed that the core strength equals exactly the *in situ* strength. Others (McIntyre and Scanlon, 1990, Bartlett and MacGregor, 1994c [Chapter 5]) have suggested that the strength of cores is less than the true *in situ* strength because the surface of the core may be damaged while the core is being drilled.

As part of the ongoing research programme concerning high performance concrete at the University of Alberta, a series of large reinforced concrete beams were tested to flexural failure. Later, cores were obtained from these specimens to investigate the correlation between the standard core strength and the beam strength, and to assess the effect of various core sizes and core treatments on the core strength. In this paper the experimental procedures are described, the data are reported, the relationship between the core strength and the *in situ* concrete strength as assessed from the flexural beam tests is investigated, and the effects of core orientation and microcracking on the measured core strengths are analysed. The analysis of data pertaining to the effect of the length to diameter ratio, diameter, and moisture condition of the core from this and other investigations is reported elsewhere (Bartlett and MacGregor, 1994a-c [Chapters 3-5]).

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1 A version of this chapter has been accepted for publication in the *American Concrete Institute Materials Journal*

2.2 Research Significance

The experimental data obtained in this investigation indicate the effect of variations of core size and test moisture condition on the measured strength of cores from elements made of high performance concrete. This research also addresses a basic evaluation problem: given a core strength, what can be inferred about the structural capacity of the member from which the core was obtained? It is shown that the *in situ* concrete strength, and hence the capacity of the member, can be accurately assessed from the core strength if effects due to damage sustained during drilling and the moisture condition of the core are accounted for.

2.3 Testing Program

In a separate investigation by Alca, four medium sized beams and four large beams were cast using ordinary portland cement concretes. The cylinder strength and other information for each beam is shown in Table 2.1; further details concerning the concrete mixes and casting procedures are reported by Alca (1993).

The beams are designated using two letters and one number. The first letter is M for medium sized beams (235 by 450 by 5860 mm) or L for large beams (335 by 650 by 8380 mm). The second letter is L for beams cast from 50 MPa concrete with a maximum aggregate size of 19 mm or H for beams cast from 90 MPa concrete with a maximum aggregate size of 14 mm. The number indicates whether the beam is the first or second cast. In Table 2.2, cores are designated using the beam designation plus the letter N or S to indicate whether the core came from the North or the South end of the beam.

The beams were supported simply near their ends and loaded with two equal point loads located approximately at the third points to produce a region of constant moment through the middle third of the span. In all cases, flexural failures occurred which were initiated by yielding of the reinforcing steel. Typically a single beam test lasted for 10 to 12 hours. Further details of the tests are reported by Alca (1993).

Cores from all medium sized beams and the two low strength large beams were obtained from zones near the support reactions, which were not damaged during Alca's test. To avoid drilling through reinforcing steel, the top and bottom ends of the stirrups in the medium sized beams were exposed by chipping away the concrete before the cores were drilled. Unreinforced concrete blocks measuring 335 x 680 x 860 mm were cast at the same time as Beams LH1 and LH2 for coring later since these beams had insufficient space between the stirrups to accommodate 100 mm diameter cores.

Typically all specimens from one end of one beam or from one block were drilled during an eight hour period. It took between 6 and 15 minutes to drill a 235 mm long core, using a watercooled diamond bit operating at 450 rpm. Each core was sawn into two or three test specimens of the desired nominal length. The specimen from the surface near the drill was designated "s", the specimen from the surface furthest from the drill was designated "d", and if a specimen was obtained from the middle of the core, it was designated "m". Before testing, the ends of the specimens were lapped.

The data for the cores from the medium sized beams, shown in Table 2.2, were used to investigate the effect of the moisture condition and the core orientation on the measured strength. A total of 145 specimens with diameters of 50 mm and nominal lengths of 100 mm were obtained. They were soaked in water or dried in air in accordance with the provisions of ASTM C42-90 (ASTM, 1992), although the average soaking period of 22 days is considerably more than the minimum 40 hour soaking period specified. As part of a separate study addressing the effects of moisture condition on core strength (Bartlett and MacGregor, 1994b [Chapter 4]), the side surfaces of a few cores were painted with an epoxy waterproofing coating before being soaked or left to dry. The beams were cast upside down, so the direction of casting (or concrete placement) was vertical. All vertical cores were drilled from the top of the beam to the depth of the main reinforcing steel, parallel to the direction of casting, and broken out. All horizontal cores were drilled through the thickness of the beams, perpendicular to the direction of casting, half from one side and half from the other.

The data for cores from the large beams, shown in Table 2.3, were used to investigate the effect of the core diameter, length to diameter ratio, and moisture condition on the core strength. Altogether 46 specimens of 50 mm diameter and 96 specimens of 100 mm diameter were obtained. After drilling, the cores were left in the laboratory air for about 15 minutes to allow cooling water to evaporate. They were then stored in sealed plastic garbage bags. During sawing, the specimens were again superficially wetted. After sawing each specimen was left in laboratory air until its surface was visibly dry before being returned to its plastic storage bag. All specimens from Beams LL1 and LL2 were left in plastic bags until they were tested. About one third of the 100 mm diameter cores from blocks LH1 and LH2 were left in plastic bags until they were tested. The remainder were either soaked in lime water or left to dry in laboratory air in accordance with the provisions of ASTM C42-90.

Detailed data concerning the location, orientation, length and compressive strength of each core are presented in Appendices 2A and 2B for cores from the medium sized beams and large beams respectively.

The specimens were tested in a MTS Rock Mechanics testing machine under stroke control. Load and stroke were recorded. The stroke rate was doubled for the 200 mm long specimens so that the strain rates would be consistent for all specimens. The corresponding stress rates, determined for an intermediate loading range with a linear load-deflection response, varied between 0.10 and 0.26 MPa/sec. Although the lower rates lie just outside the range of 0.14 to 0.34 MPa/sec permitted in ASTM C39-86 (ASTM Committee C-9, 1992), the effect of these slight variations on the measured strengths is probably small (Mirza et al., 1979). Typically all specimens from one end of one beam or from one block were tested within an eight hour period.

The diameter of the hemisphere on the suspended bearing block of the testing machine was 100 mm, twice that of the 50 mm diameter test specimens. To achieve a satisfactory seating with uniform contact, this bearing block was vigorously struck with a rubber tipped mallet as the initial load was applied to the specimen.

2.4 Data Analysis

Data from each end of each beam, or from each block, were analysed separately. Typically the analysis included reviewing the data for outliers, conducting regression analysis on the cleansed data, and adjusting the data to compensate for the effect of spatial variation of the *in situ* concrete strength.

The strength of each 50 mm diameter core was modified to account for the actual l/d ratio of the core after its ends were ground to give the equivalent strength of a core with l/d = 2. The modification was based on

$$F_{l/d} = 1 - 0.20\left(2 - \frac{l}{d}\right)^2 \tag{2.1}$$

where $F_{l/d}$ is the modification factor and the constant value of 0.20 is as reported by Bartlett and MacGregor (1994c) [Chapter 5] for 50 mm diameter cores. The effect of this adjustment is slight; the shortest 50 mm diameter core had a length of 82 mm, for which the strength reduction is less than 3%. The actual lengths of the 100 mm diameter cores were very close to the nominal values of 100 and 200 mm and therefore did not require modification.

2.4.1 Outlier Identification

To identify possible outlying strength values, the core strength data were sorted into sets with common *l*/d, treatment, diameter, orientation and concrete mix. Typically each set contained results from 8 or 9 cores of 50 mm diameter or from 5 to 8 cores of 100 mm diameter. Criteria given in ASTM E178-80 (ASTM, 1989) were used to assess whether the extreme values within each set were probable outliers. Also, all specimens were visually inspected and the load-deflection response of each specimen was checked for irregularities.

The 13 core strengths shown in Table 2.4 were deemed to be low outliers and were not included in subsequent analyses. No high outliers were identified. The p(2s) values shown are the probabilities that the observed strength value would occur by chance according to the two tailed test criteria in ASTM E178-80. The critical value for this test is $p(2s) < 0.05$, indicating that the suspect value has less than a 5% chance of occurring by chance and so is probably an outlier. By this criterion, only the strengths of core 86d from Beam LL1 and perhaps core 134d from Block LH2 would be considered to be probable outliers.

In several cases, the load-deflection response could be used to identify extreme low strength values as outliers. This technique was useful because conventional statistical tests for outliers have little power when applied to sets of 5 or 6 replicate observations. For example, the curves shown in Figure 2.1 are all for 100 by 100 mm cores from Beam LL2 which were given identical moisture conditioning treatments. Six of the eight cores in this set had load-deflection curves similar to that shown for core 94d. Cores 99s and 93s, which come from opposite ends of the region sampled, were considerably weaker than the rest. Clearly their load-deflection curves are atypical; core 99s had an unusually long maximum load plateau and core 93s had a sudden, premature failure. Both these observed responses may be caused by imperfect contact between the core and the loading platens, perhaps due to poor grinding of the ends of the specimen. The corresponding low values can therefore be removed from the data set even though, as shown in Table 2.4, they don't qualify as outliers according to the criteria in ASTM E178-80.

For these data, the identification of outlying values using statistical methods is insensitive to systematic variation of the *in situ* strength. While such variation may make the extreme strength values seem more extreme, it also inflates the standard deviation and so does not affect the ASTM E178-80 tests which compare the observed extreme values to the observed standard deviation. The choice of outliers does not affect the regression analysis described in the following section because, for 11 of the 13 outliers identified in Table 2.4, the observed low strengths can be explained by credible physical reasons.

2.4.2 Regression Analysis

Multiple linear regression analysis was performed on the cleansed data from each beam end or block. The general regression model used accommodates possible spatial variation of the *in situ* strength as well as the effects of the various core test factors:

$$f_c = \beta_0 + g\ (\text{spatial variation}) + g\ (\text{test factors}) + \varepsilon \qquad (2.2)$$

where f_c is the observed core strength, β_0 is a parameter estimated by regression analysis, and ε is the error term. The value of β_0 represents the strength of a hypothetical air dried horizontal core located at the centroid of all cores from the region sampled, which has dimensions 50 by 100 mm if it comes from one of the medium sized beams or 100 by 200 mm if it comes from one of the large beams.

The effect of spatial variation of the *in situ* strength was modelled as:

$$g\ (\text{spatial variation}) = \beta_1 x' + \beta_2 x'^2 + \beta_3 y' + \beta_4 z' \qquad (2.3).$$

In this equation, $\beta_1 .. \beta_4$ are parameters estimated by regression analysis and

$$x' = x/\bar{x}, \qquad y' = y/\bar{y} \qquad \text{and} \qquad z' = z/\bar{z} \qquad (2.4)$$

where x, y and z are the coordinates of the centre of gravity of the core in mm, z being through the beam thickness, and $\bar{x}$, $\bar{y}$, and $\bar{z}$ are the coordinates of the centre of gravity of the set of all cores.

The effect of the various core test factors was modelled as:

$$g\ (\text{test factors}) = \beta_5 Z_{s/d} + \beta_6 Z_{l/d} + \beta_7 Z_{HV} + \beta_8 Z_{mc} + \beta_9 Z_c + \beta_{10} Z_{dia} + \beta_{11} Z_{seal} \qquad (2.5).$$

In this equation $\beta_5..\beta_{11}$ are the parameters estimated by regression analysis for the various indicator variables. Indicator variable $Z_{s/d}$ equals 0 if the test specimen came from the half of the core near the drill or 1 otherwise. Similarly $Z_{l/d}$ equals 1 for cores with nominal l/d = 1 or 0 for cores with nominal l/d = 2; Z_{HV} equals 1 for cores with their axis parallel to the direction of casting or 0 otherwise; Z_{mc} equals 1 for cores soaked in water before testing or 0 otherwise; Z_{seal} equals 1 for cores stored in plastic bags until testing or 0 otherwise; Z_c equals 1 for cores with epoxy coated sides or 0 otherwise; and Z_{dia} equals 1 for 100 mm diameter cores or 0 otherwise. If any indicator variable was irrelevant for the data set under consideration, it was not included in the analysis.

The analyses generally followed a backwards elimination procedure, yielding the fitted equations shown in Table 2.5. All parameter estimates shown are significantly different from zero at the 10% probability level. In two cases, indicated by the "no regression" entries, no parameter estimates are significantly different from zero. In these cases the standard error of regression, $\sqrt{MS_E}$, is the sample standard deviation.

2.4.3 Spatial Variation of *In Situ* Strength

The parameter estimates shown in Table 2.5 indicate that spatial variation of the *in situ* concrete strength is a significant factor in 7 of the 12 elements cored. The scatter of strengths within sets of cores with common l/d, treatment, diameter and drilling direction is inflated by the spatial variation. The average strength values are not appreciably affected because, within each set, the core locations are distributed fairly uniformly throughout the sampling area.

The standard deviations and coefficients of variation that may be attributed to spatial variation are shown in Table 2.6 for the seven elements where the spatial variation is a significant factor. The average standard deviation is 2.5 MPa, and the average coefficient of variation is 4.0%.

Modified core strengths, f^*_c, were determined from the raw core strengths f_c to account for the effect of any *in situ* strength variation using the equation:

$$f^*_c = f_c - \hat{\beta}_1(x' - \overline{x'}) - \hat{\beta}_2(x'^2 - \overline{x'^2}) - \hat{\beta}_3(y' - \overline{y'}) - \hat{\beta}_4(z' - \overline{z'}) \quad (2.6).$$

The modifications addressed only significant variations of the *in situ* strength. If the estimate of a parameter, $\hat{\beta}_1 .. \hat{\beta}_4$, was not found to be significantly different from zero at the 10% probability level, then a zero value was assigned to that parameter in Equation (2.6). The effect of these adjustments is generally to reduce the standard deviation of the strength results for a set of common cores without changing the average value appreciably.

The sample statistics for the various sets of core strengths, after adjustment to account for significant spatial variation, are given in Tables 2.2 and 2.3. The coefficients of variation range from 3.2% to 11.7% for cores from the medium sized beams and from 1.6% to 7.0% for cores from the large beams and blocks. The coefficients of variation shown for the medium sized beams tend to be larger than those shown for the large beams because the cores from the medium sized beams were all 50 mm in diameter and have slightly more variable strengths that the 100 mm diameter cores from the large beams.

2.5 Test Factors Affecting Core Strengths

Some of the test factors which affect the core strength are considered in this section. The effects due to the core length to diameter ratio, the core diameter and the core moisture condition are addressed elsewhere (Bartlett and MacGregor, 1994 a-c [Chapters 3-5]).

2.5.1 Damage due to Microcracking

Although Beam MH2 was cast from one load of ready mix concrete, the average strength of cores from the North end is consistently larger than the average strength of cores from the South end. The difference is statistically significant both for the sets of horizontally drilled cores (the probability of a difference of at least this magnitude, p, is about 0.03) and the vertically drilled cores ($p < 0.01$). The average strength difference is 6.6 MPa, which represents a measured strength difference of 11% between the North and South ends.

The reason for this difference may be the presence of microcracks at the South end of the beam. A 30 pound chipping hammer was used to remove cover concrete from Beam MH2 only. At the North end, only the bottom ends of the stirrups were exposed. At the South end, a much larger quantity of concrete was removed so that test specimens from both layers of longitudinal steel could be obtained as part of Alca's investigation. The extent of the damage is recorded in Figures 2.2 (a) and (b), which clearly confirm that the South end was considerably more damaged than the North end.

2.5.2 Test Specimen Location Along Core

Meininger et al. (1977) drilled horizontal cores through the thickness of six walls and sawed each core to obtain two test specimens. Analysis of their data (Meininger et al., 1977, Bartlett and MacGregor, 1994a [Chapter 3]) indicated that typically the strengths of specimens obtained from one end of the core were consistently larger than the strengths of specimens obtained from the other end. The particulars of the original experimental procedure made it impossible to determine whether this difference was due to a strength variation through the thickness of the elements cored, or instead due to some spurious effect associated with the drilling process.

In the present investigation, it is possible to distinguish between through the thickness spatial variation and effects due to the drilling process because approximately half of the horizontal cores were drilled from each side of each medium sized beam. Effects due to through thickness variation of the *in situ* strength are accounted for in the analysis by variable z' in Equation (2.3). Similarly effects attributable to the drilling process are accounted for by variable $Z_{s/d}$ in Equation (2.5). The regression equations shown for the medium sized beams in Table 2.5 indicate the significance of these factors. The variation of strength through the thickness of the element is significant at the 10% level in the medium sized beams in 3 of 8 cases. The difference between the strengths of specimens obtained from the part of the core nearest the drill and those obtained from the part farthest from the drill is not significant at the 10% level in any of the 8 cases.

2.5.3 Core Orientation

Some investigators have reported that the strength of vertical cores, drilled parallel to the direction of concrete placement, is 8 to 12% larger than that of horizontal cores, drilled perpendicular to the direction of concrete placement (Sanga and Dhir, 1976, Takahata et al., 1991). The difference has been attributed to moisture forming under aggregate particles, which causes a stress raiser when loaded in the transverse direction (Johnston, 1973). Others have reported no significant difference between the strengths of vertical and horizontal cores (Bloem, 1965).

Ratios of the average strength of vertical cores to the average strength of horizontal cores from the present investigation, and estimated 95% confidence limits on these values, are shown in Figure 2.3. The observed ratios for cores from the north end of Beam ML1 and from both ends of Beam MH2 are not significantly different from 1.0. While a significant strength loss may be attributed to microcracking at the South end of Beam MH2, this factor does not seem to affect the relative strengths of horizontal and vertical cores.

The average strength of vertical cores obtained from Beam ML2 is significantly larger than the average strength of the horizontal cores. As indicated in Table 2.1, Beam ML2 was cast from a ready mix supplier's "standard 50 MPa" product, which, unlike the mixes used to cast Beams ML1 and MH2, included fly ash and a vinsol resin air entraining admixture. It is therefore possible that the significant difference observed for the cores from ML2 may be attributed to the use of either fly ash or the air entraining admixture.

2.5.4 Moisture Condition of Small Diameter Cores

The strength of cores from mature concrete is increased if the cores are left to dry in air before testing. Conversely, the strength is reduced if the cores are soaked in water

(Bloem, 1965). An analysis of data for 100 mm diameter cores from both the present and other investigations indicates that the strength of air dried cores is on average 13% greater than that of soaked cores (Bartlett and MacGregor, 1994b [Chapter 4]).

Data from the present investigation suggest that the strength loss caused by soaking the cores may be relatively larger for small diameter cores, as indicated by the relative strength ratios and their estimated 95% confidence limits shown in Figure 2.4. The average of the four strength ratios for the cores from the medium sized beams soaked 22 days is 0.769. This is significantly less ($p < .01$) than the value of 0.874 reported by Bartlett and MacGregor (1994b) [Chapter 4] for 100 mm diameter cores soaked for 2 days. This difference seems to be due to two factors:

(a) The soaking treatment causes greater strength loss for 50 mm diameter cores than for 100 diameter cores. For cores soaked for two days, the average observed ratios for the 100 x 50 mm diameter cores from blocks LH1 and LH2 is 0.839. This is significantly smaller ($p < .01$) than the value for soaked 200 x 100 mm diameter cores from the blocks, 0.875.

(b) For 50 mm diameter cores, the strength loss is larger for longer soaking periods. This is illustrated by the data for 50 mm diameter cores from both ends of Beam MH1 and from blocks LH1 and LH2, which were all cast using a common mix design. The average of the ratios observed for cores soaked 22 days, from both ends of Beam MH1, is 0.776. This is significantly smaller ($p = 0.01$) than the value for 100 x 50 mm cores from blocks LH1 and LH2 which were soaked for only 2 days, 0.839.

2.6 Core Strength as a Measure of *In Situ* Strength in Beams

The relationship between core strengths and the structural capacity of the corresponding member was investigated using data from the four large beams only, since the 100 x 200 mm cores obtained from these beams need no correction for diameter or l/d ratio. Also, during the large beam testing program, concrete strength changes over time were monitored by testing one set of cylinders on the day of the beam test and another set after all the cores were tested.

Strengths are shown in Figure 2.5 for the concrete used for Beams LL1 and LL2, and in Figures 2.6(a) and 2.6(b) for the concrete used for Beams LH1 and LH2 respectively. The mean value and approximate 95% confidence limits on the mean value are shown for the various cylinder, core, and *in situ* beam strengths. The methods used to calculate these mean values and dispersions are briefly summarised below.

Values shown for the cylinder strengths are based on the sample averages and sample standard deviations from the cylinder tests. The 95% confidence limits on the mean values have been determined using a Student's t distribution. No account has been taken of the moisture condition of the cylinders; generally, they were removed from a limewater bath and left exposed to laboratory air for periods varying between 2 days and 7 days before testing.

The *in situ* strength of concrete is usually larger than the 100 mm diameter core strength by a factor of 1.06 (Bartlett and MacGregor, 1994c [Chapter 5]), due to damage sustained during drilling. Therefore, the sample averages and sample standard deviations of the measured core strengths have been multiplied by 1.06 to give the mean values and standard deviations of the core strengths shown in Figures 2.5 and 2.6.

The strength values shown for the beams have been estimated by assuming an equivalent rectangular distribution of concrete stress at the ultimate moment, in accordance with conventional practice (CSA, 1984, ACI Committee 318, 1989). Moment equilibrium requires that

$$M_{max} = A_s f_{sm} \left(d - \frac{a}{2} \right) \tag{2.7}$$

where M_{max} is the maximum moment at the section due to externally applied loads including the beam self weight, A_s is the area of the steel tension reinforcement, f_{sm} is the stress in the reinforcement corresponding to the maximum moment, and d is the effective depth of the reinforcement from the extreme compression fibre. The depth of the equivalent rectangular stress block, a, is obtained from horizontal force equilibrium

$$A_s f_{sm} = 0.85\, F_{sust} F_{HV} F_{age}\, f_{c,is}\, ab \tag{2.8}$$

where $f_{c,is}$ is the equivalent *in situ* strength of the beam and b is the width of the compression zone. The factor F_{sust} accounts for a strength differences attributable to the relative rates of loading. Although the cylinders and cores were loaded rapidly and the beam was loaded slowly, it is assumed that $F_{sust} = 1$ because this effect is believed to be included in the empirically determined constant, 0.85, in the description of the rectangular stress block. The factor F_{HV} accounts for strength differences attributable to concrete being loaded in a direction perpendicular to or parallel to the direction of placement. It is assumed that $F_{HV} = 1$ because, as indicated in Figure 2.3, there is no evidence of this effect being significant in concrete proportioned to Alca's mix design. The factor F_{age} accounts for strength differences attributable to the greater age of the cores at testing. It is assumed that $F_{age} = 1$ since, as shown in Figures 2.5 and 2.6, the cylinders tested after the cores were tested are not appreciably stronger than the cylinders tested on the day that the beams were tested. Using Equation (2.7) to eliminate a from Equation (2.8), the *in situ* strength of the beams is therefore

$$f_{c,is} = A_s f_{sm} \left\{ 2(0.85) b \left(d - \frac{M_{max}}{A_s f_{sm}} \right) \right\}^{-1} \quad (2.9).$$

The estimated 95% confidence limits shown for the *in situ* beam strengths in Figures 2.5 and 2.6 reflect the uncertainty of the "known" quantities on the right hand side of Equation (2.9). Estimates of these measurement errors are shown in Table 2.7. The standard deviation of $f_{c,is}$ was determined by approximating Equation (2.9) by a first order Taylor series expansion about the mean values and assuming that errors from all sources have normal probability distributions.

The uncertainties in the measured values of b, d, and M_{max} are small. Formwork tolerances are responsible for the variability of b and d; it is interesting to note that the standard deviations of these quantities increase with each reuse of the forms. The standard deviation of M_{max} was estimated by considering the effects of formwork tolerances and the variability of concrete density on the moment due to the beam self weight and the effects of calibration error, data acquisition "noise", and shear span variability on the recorded applied moment.

The most significant source of error is due to the uncertainty of $A_s f_{sm}$. Average strains in the reinforcing steel were determined using Alca's measurements of the extreme fibre concrete strain and the average curvature. For the bars in Beams LL1, LL2, and LH1, load-strain curves were determined from tests of 23 coupons provided by the supplier. For the bars in LH2, load-strain curves were determined from coupons retrieved from each bar in the beam after the flexural test. The force in the reinforcing steel was estimated from the load-strain curves for the estimated strains, taking account of the very slow test loading rate.

The effect of modelling errors associated with the equivalent rectangular stress block was not considered. The ratios of the test ultimate moment to the corresponding nominal value determined using the equivalent rectangular stress block reported by Alca (1993) have a mean value of 1.007 and a coefficient of variation of 0.015. As a check, the maximum concrete stress was calculated for a parabolic stress strain relationship acting over the full depth of the compression zone and satisfying horizontal equilibrium at first yield of the steel and moment equilibrium at the maximum moment. The calculated maximum concrete stresses were within 2% of 0.85 $f_{c,is}$ for Beams LL1, LH1 and LH2. The discrepancy was about 20% for Beam LL2, but subsequent investigation indicated that the result from the check calculation is very sensitive to the depth of the compression zone.

Inspection of Figures 2.5 and 2.6 suggests that the *in situ* beam strength, as calculated using the procedure just described, approximately equals the strength of the sealed cores as adjusted to account for damage sustained during drilling. This trend is confirmed by the ratios of the *in situ* beam strengths to the various core and cylinder strengths shown in Table 2.8. The average ratios of both the *in situ* beam strength to cylinder strength and the *in situ* beam strength to sealed core strength are close to 1.0. However, the coefficient of variation of the ratio based on cylinders is more than 2.5 times as large that based on sealed cores.

The values shown in Table 2.8 also indicate the consistency of the relative strengths for various core moisture treatments. The ratios based on both the air dried core strengths and the soaked core strengths are consistent, since the values shown for Beams LH1 and LH2 are similar. The ratios of the *in situ* beam strength to the sealed core strength shown are not as consistent, possibly due to the difficulty of achieving a controlled sealed condition. Therefore, while the sealed core strengths give the least biased estimate of the *in situ* strength, it may be more effective to test soaked or air dried core strengths and correct the strengths to account for the bias caused by the soaking or air drying treatments. However, the sensitivity of the correction factor for soaked cores to the core diameter and the duration of the soaking period, as shown in Figure 2.4, should be considered.

2.7 Conclusions

A total of 287 concrete core specimens were obtained from large beams cast in a laboratory using ordinary portland cement concrete with cylinder strengths ranging from 45 to 90 MPa. All were tested in compression when the concrete was between 49 and 128 days old. The core specimen sizes were 50 x 100 mm, 100 x 100 mm, and 100 x 200 mm. Before testing, some cores were left to dry in laboratory air for 7 days and some were soaked in water for at least 2 days, as recommended in ASTM C42-90. Others were left in air for a brief period to allow excess water to evaporate and then were stored in sealed plastic bags. Thirteen strengths were deemed to be low outliers and were removed from the data set.

From the analysis of these data, the following conclusions are drawn:

1. Statistical tests given in ASTM E178-80 to identify outlying values are not powerful for these data. It is generally more effective to examine the load-deflection curves for irregularities which might suggest imperfect test conditions to explain observed low strengths.

2. Even though the volume of the regions sampled is less than 0.2 m^3, significant spatial variation of the *in situ* strength is present in 7 of 12 cases. In the cases where spatial variation is significant, the component of the coefficients of variation of the strength which can be attributed to spatial variation is about 4%.

3. The strength of cores with their axes perpendicular to the direction of casting does not differ significantly from the strength of cores from the same element with their axes parallel to the direction of casting in 3 of 4 cases. For the only concrete mix containing fly ash and air entraining admixtures, the cores with axes parallel to the direction of casting are 14% stronger than the cores with axes perpendicular to the direction of casting.

4. Consistent through thickness strength differences can not be attributed to any effect associated with the drilling process, but can be attributed to variation of the *in situ* concrete strength through the thickness of the beam.

5. The relative strength loss due to soaking 50 x 100 mm cores for 2 days before testing is more severe than soaking 100 x 200 mm cores for the same time period. Extending the soaking period causes a further strength loss in the 50 x 100 mm cores.

6. The least biased estimate of the *in situ* beam strength is found by increasing the strength of "sealed" cores, which are left in air only until any excess water absorbed during drilling evaporates and then sealed to prevent further moisture change, by 6% to account for damage sustained during drilling of the core. However, a more accurate estimate of the *in situ* beam strength may be obtained by correcting the strengths from tests of air dried or soaked cores to account for the both core damage and moisture condition factors.

Table 2.1: Details of Beams Cast by Alca (1993)

Beam Mark	Concrete Source	cylinder tests age (days)	cylinder tests $\bar{f}_{cyl}$ (MPa)	Notes
ML1	Lab. mixer, 5 batches	56	52.7	Beam roughly handled before coring. No admixtures.
ML2	Ready mix supplier	38	54.1	Mix design by ready mix supplier, included fly ash, superplasticiser, and air entraining admixtures.
MH1	Lab. mixer, 5 batches	56	90.3	Mix included superplasticiser, no air entrainment.
MH2	Ready mix supplier	73	73.4	Stirrups exposed using 30 pound chipping hammer. Mix included superplasticiser, no air entrainment.
LL1	Lab. mixer, 12 batches	52	54.2	No admixtures.
LL2	Lab. mixer, 12 batches	44	43.8	No admixtures.
LH1	Lab. mixer, 13 batches	46	90.3	Mix included superplasticiser, no air entrainment.
LH2	Lab. mixer, 13 batches	47	87.7	Mix included superplasticiser, no air entrainment.

Table 2.2: Strength of Cores from Medium Sized Beams

Beam Mark	End	Age at Drilling (days)	Age at Testing (days)	Core Treatment	Orientation	n	$\bar{f}_c^*$ (MPa)	s (MPa)	V (%)
ML1	N	106	125	19 days in water	H	8	36.6	4.3	11.7
				19 days in water	V	8	35.2	2.3	6.5
ML1	S	106	128	14 days in air, then:					
				8 days in air	H	7[a]	58.3	2.4	4.1
				8 days in water	H	8	42.8	2.1	4.9
				8 days in air after coating surface	H	6	56.9	1.8	3.2
ML2	N	100	118	18 days in water	H	8	51.4	4.9	9.5
				18 days in water	V	8	58.4	4.8	8.2
ML2	S	100	125	13 days in air, then:					
				12 days in air	H	8	69.6	3.0	4.3
				12 days in water	H	8	54.9	2.0	3.6
				12 days in water after coating surface	H	5[a]	61.5	3.3	5.4
MH1	N	93	115	13 days in air, then:					
				9 days in air	H	8	97.7	6.7	6.9
				9 days in water	H	7[a]	76.6	4.9	6.4
				9 days in air after coating surface	H	4[a]	93.9	3.6	3.8
MH1	S	93	115	13 days in air, then:					
				9 days in air	H	7[a]	97.9	8.3	8.5
				9 days in water	H	8	75.2	4.2	5.6
				9 days in water after coating surface	H	5[a]	88.1	6.4	7.3
MH2	N	80	98	29 days in water	H	6[b]	59.7	4.7	7.9
				29 days in water	V	7[a]	61.6	2.7	4.4
MH2	S	80	94	29 days in water	H	6[b]	53.8	4.6	8.6
				29 days in water	V	7[a]	54.4	4.2	7.7

Note: Orientation key: H = core drilled through beam thickness, perpendicular to casting direction; V = core drilled through beam depth, parallel to casting direction. [a] - one low outlier removed from data, [b] - two low outliers removed from data.

Table 2.3: Strength of Cores from Large Beams

Beam	Age at Drilling (days)	Age at Testing (days)	d (mm)	l/d	Core Treatment	n	$\bar{f}_c^*$ (MPa)	s (MPa)	V (%)
LL1	65	71	50	2	sealed in plastic	7[a]	46.0	2.6	5.7
			100	2	bags for 6 days	7[a]	44.0	2.9	6.6
			100	1		8	48.8	2.6	5.3
LL2	51	55	50	2	sealed in plastic	7	41.2	2.7	6.6
			100	2	bags for 4 days	8	45.7	1.8	3.9
			100	1		6[b]	48.2	3.2	6.6
LH1	44	55	50	2	sealed in plastic	7	93.8	6.6	7.0
			100	2	bags for 4 days, then	5	94.3	3.2	3.4
			100	1	dried 7 days in air	5	100.7	4.0	4.0
			50	2	sealed in plastic	6[a]	78.0	3.2	4.1
			100	2	bags for 9 days, then	5	82.1	3.3	4.0
			100	1	soaked 2 days in water	5	87.6	3.3	3.8
			100	2	sealed in plastic	6	89.0	4.8	5.4
			100	1	bags for 11 days	6	99.3	3.8	3.8
LH2	42	49	50	2	dried for	8	92.3	5.0	5.4
			100	2	7 days in air	5	89.9	3.5	3.9
			100	1		5	101.4	2.9	2.9
			50	2	sealed in plastic	8[a]	78.0	3.8	4.9
			100	2	bags for 5 days, then	5	79.0	1.3	1.6
			100	1	soaked 2 days in water	5	83.8	1.4	1.7
			100	2	sealed in plastic	5[a]	81.8	5.0	6.1
			100	1	bags for 7 days	6	90.1	2.5	2.8

Note: The axis of all cores drilled was perpendicular to the direction of casting. [a] - one low outlier removed from data, [b] - two low outliers removed from data.

Table 2.4: Core Strength Outliers

Beam	End	Outlier	Criteria
ML1	North	20*	Large groove in top of core For $n = 5$, $p\ (2s) \approx 0.05$
ML1	South	2d	Core cracked through its base For $n = 8$, $p\ (2s) \approx 0.10$
ML2	South	26d	Top loading platen loose
MH1	North	46s	Core cracked through its base For $n = 8$, $p\ (2s) > 0.20$
MH1	North	52s	Core accidentally given wrong treatment
MH1	South	58d	Core accidentally given wrong treatment
MH1	South	64d	Crack occurred during Alca's test (before drilling core)
LL1	North	86d	Very low strength For $n = 8$, $p\ (2s) < 0.002$
LL1	North	89s	Irregular load-deflection curve For $n = 8$, $p\ (2s) < 0.05$
LL2	North	93s	Irregular load-deflection curve For $n = 8$, $p\ (2s) > 0.20$
LL2	North	99s	Irregular load-deflection curve For $n = 8$, $p\ (2s) > 0.20$
LH2	(Block)	134d	Low strength (poor end seating ?) For $n = 9$, $p\ (2s) \approx 0.03$
LH2	(Block)	138d	Load-deflection curve has two peaks For $n = 6$, $p\ (2s) > 0.20$

Table 2.5: Results of Regression Analyses

Beam	End	Regression Equation		R^2	$\sqrt{MS_E}$ (MPa)
ML1	North	$\hat{f}_c = 35.9$	(no regression)	0	3.4
ML1	South	$\hat{f}_c = 59.5 - 14.9\, Z_{mc} - 1.8\, z'$		0.93	2.2
ML2	North	$\hat{f}_c = 46.4 + 6.9\, Z_{HV} + 5.0\, z'$		0.39	5.0
ML2	South	$\hat{f}_c = 96.1 - 14.8\, Z_{mc} + 6.5\, Z_c - 55.0x' + 26.5\, x'^2$		0.89	2.6
MH1	North	$\hat{f}_c = 96.4 - 19.8\, Z_{mc}$		0.76	5.6
MH1	South	$\hat{f}_c = 70.5 - 22.8\, Z_{mc} + 12.9\, Z_c + 27.4\, y'$		0.78	6.2
MH2	North	$\hat{f}_c = 60.7$	(no regression)	0	3.7
MH2	South	$\hat{f}_c = 47.3 + 6.8z'$		0.25	4.4
LL1	North	$\hat{f}_c = 45.0 + 2.0\, Z_{dia} + 5.7\, Z_{l/d}$		0.53	2.4
LL2	North	$\hat{f}_c = 49.5 - 4.6\, Z_{dia} + 2.4\, Z_{l/d} - 8.2\, x' + 4.5\, y'$		0.48	2.8
LH1	--	$\hat{f}_c = 93.6 + 8.2\, Z_{l/d} - 13.8\, Z_{mc} - 3.5\, Z_{seal}$		0.76	4.3
LH2	--	$\hat{f}_c = 92.0 + 7.9\, Z_{l/d} - 14.3\, Z_{mc} - 9.9\, Z_{seal} + 2.5\, x' - 4.2\, y' + 1.9\, z'$		0.81	3.8

Table 2.6: Effect of Spatial Variation of *In Situ* Strength

Beam Mark	End	Standard Deviation (MPa)	Coefficient of Variation (%)
ML1	South	0.9	1.7
ML2	North	2.3	4.2
ML2	South	2.0	3.2
MH1	South	4.8	5.5
MH2	South	2.5	4.6
LL2	North	2.8	6.3
LH2	--	2.4	2.7

Table 2.7: Statistics for Calculation of *In Situ* Beam Strength

Quantity		Units	Beam LL1	Beam LL2	Beam LH1	Beam LH2
b	Avg.	mm	337.0	337.1	338.2	338.5
	s	mm	0.6	0.7	1.3	1.7
d	Avg.	mm	516.2	515.1	516.6	517.0
	s	mm	0.1	0.4	0.9	1.0
$A_s f_{sm}$	Avg.	kN	1712	1637	3407	3459
	s	kN	33.5	15.5	33.0	7.9
M_{max}	Avg.	kN.m	772.9	747.7	1548.7	1567.6
	s	kN.m	0.9	0.5	0.6	0.8
$f_{c,is}$	Avg.	MPa	46.2	48.9	95.5	94.2
	s	MPa	5.4	3.2	6.0	2.0

Table 2.8: Ratios of *In Situ* Beam Strength to Cylinder and Core Strengths

		$f_{c,is} / 1.06 f^*_c$		
	$f_{c,is} / \bar{f}_{cyl}$	air dried	sealed	soaked
Beam LL1	0.85	--	0.99	--
Beam LL2	1.12	--	1.01	--
Beam LH1	1.06	0.95	1.01	1.10
Beam LH2	1.07	0.99	1.09	1.13
Average	1.025	0.971	1.025	1.111
V	0.114	--	0.041	--

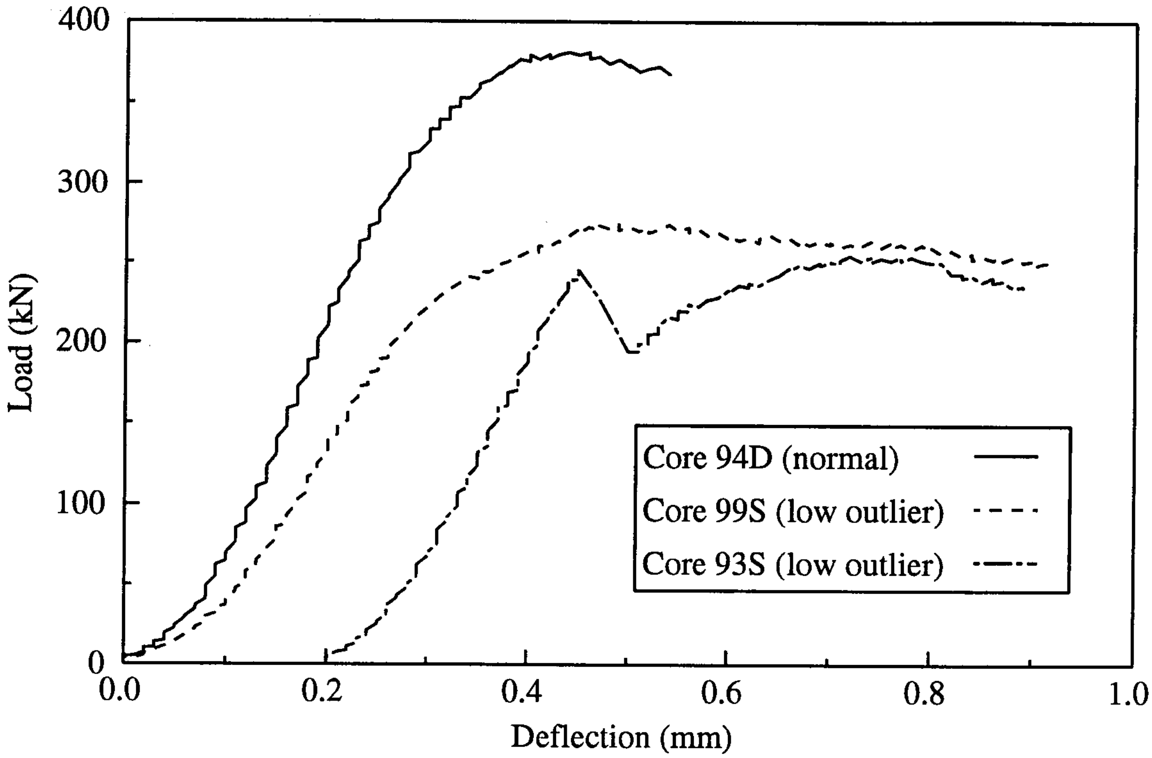
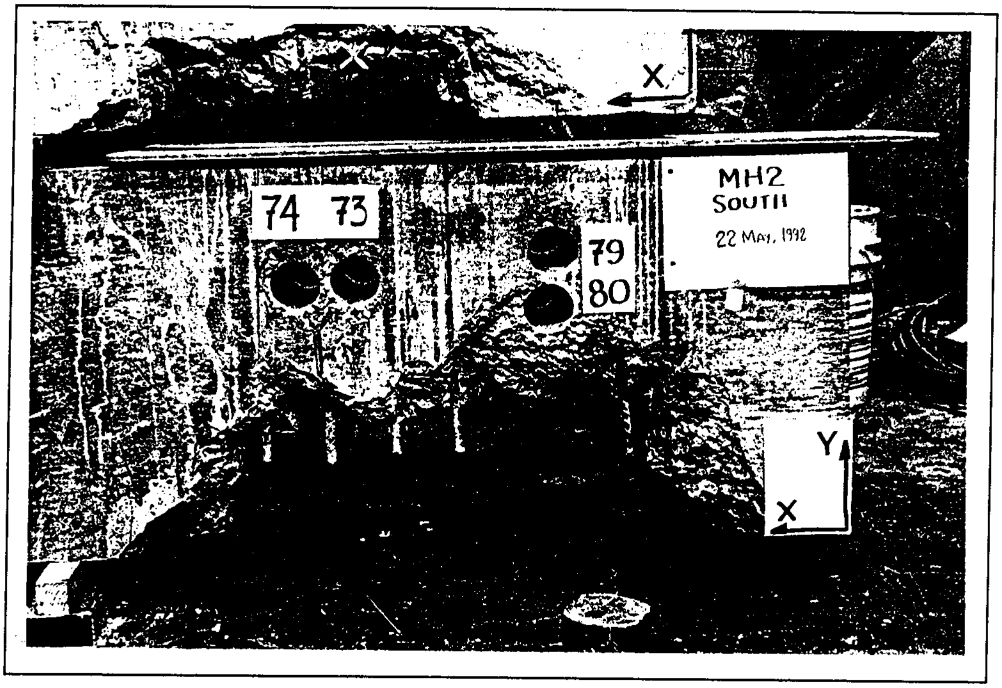
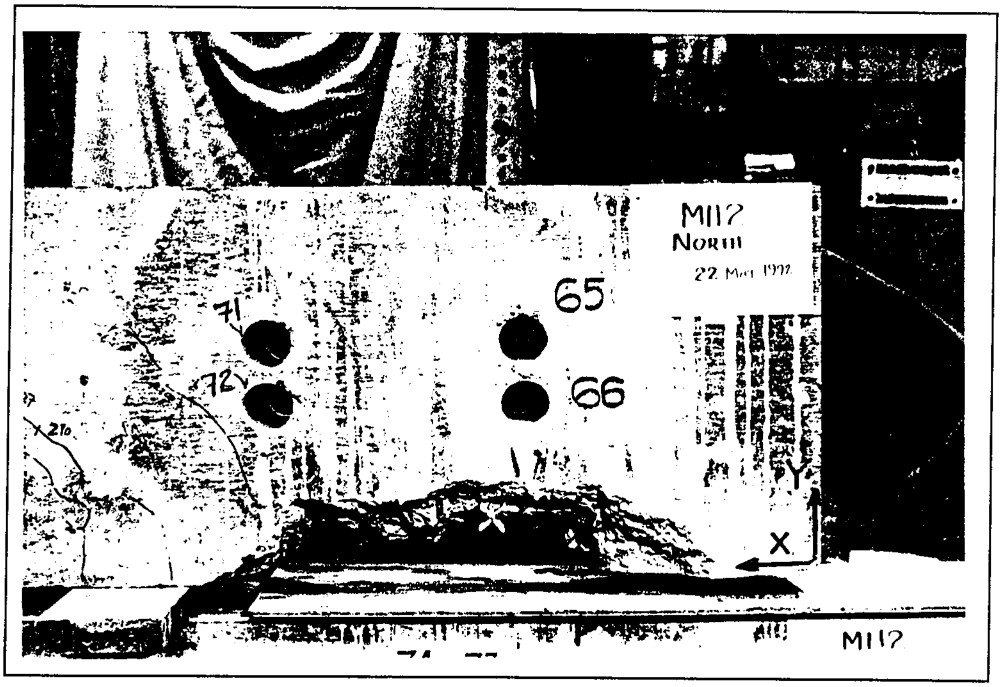


Figure 2.1: Load-deflection curves for 100 x 100 mm cores



(a) South End



(b) North End

Figure 2.2: Damage at ends of Beam MH2

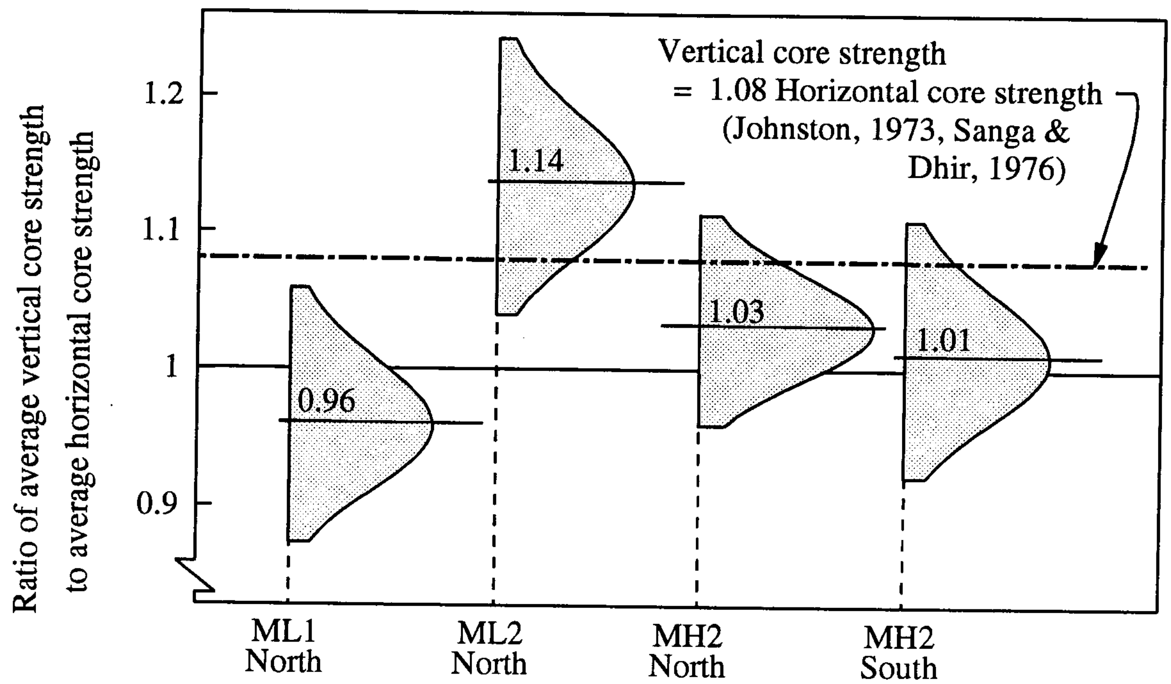


Figure 2.3: Approximate 95% confidence limits on ratio of average vertical core strength to average horizontal core strength

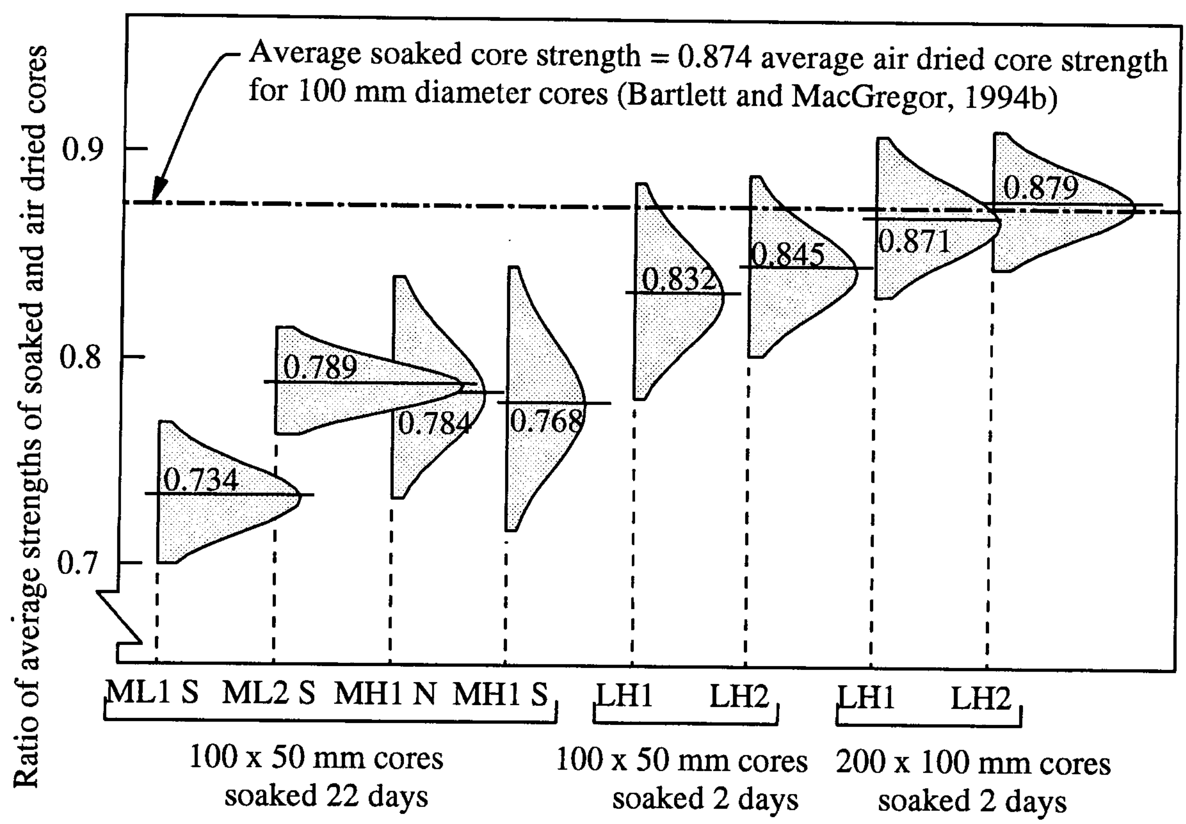


Figure 2.4: Effect of moisture condition on the strength of small diameter cores

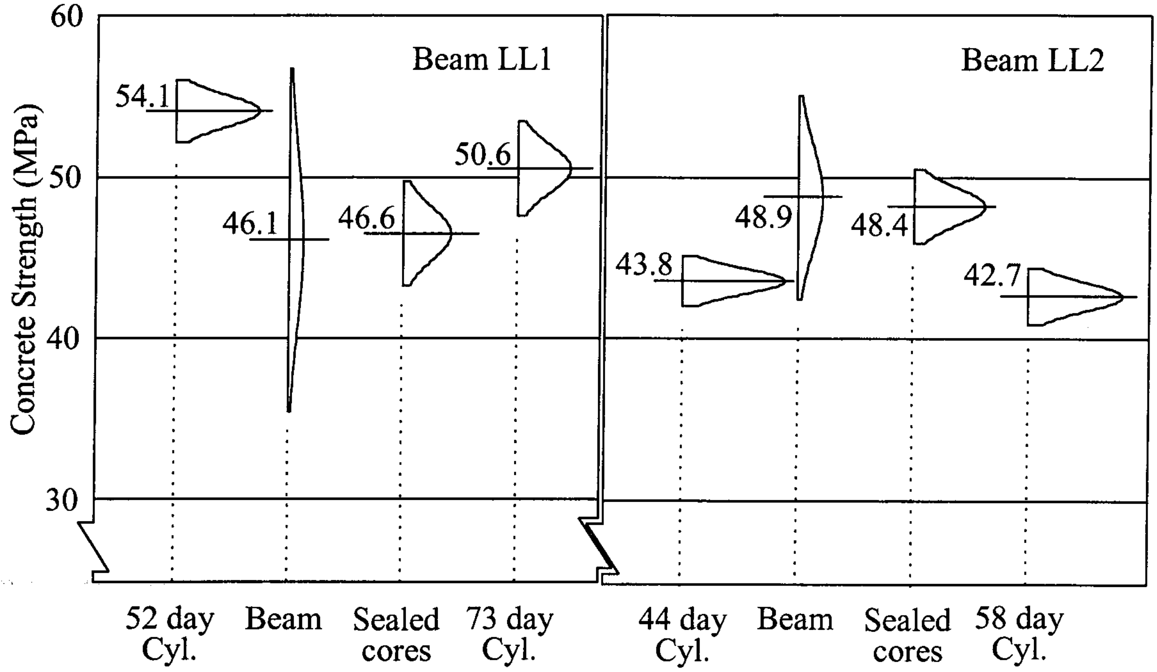


Figure 2.5: 95% confidence limits on mean concrete strengths, Beams LL1 and LL2

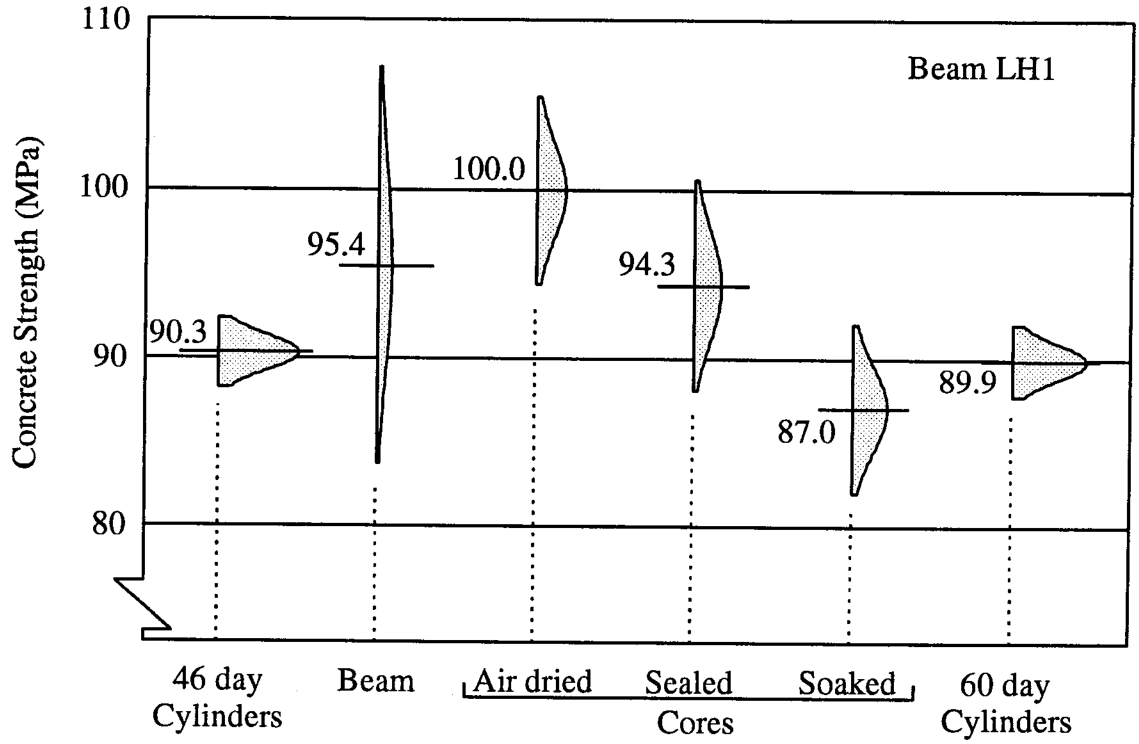


Figure 2.6 (a): 95% confidence limits on mean concrete strengths, Beam LH1

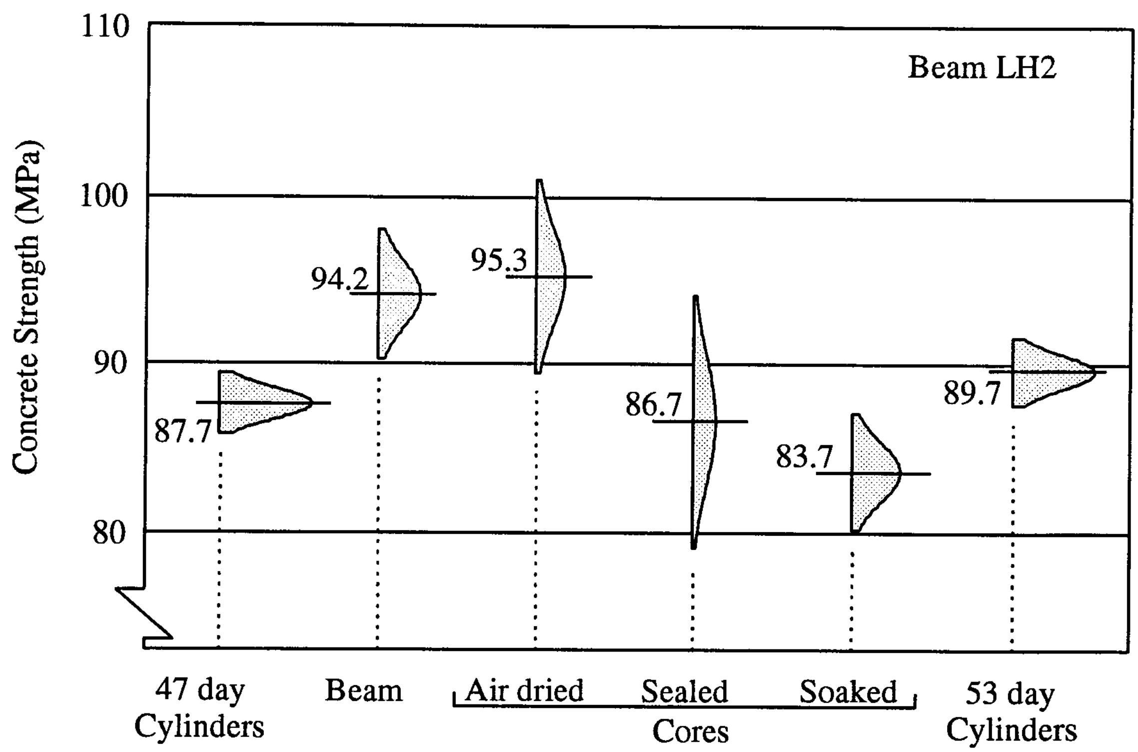


Figure 2.6 (b): 95% confidence limits on mean concrete strengths, Beam LH2

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Chapter 3: Effect of Core Length to Diameter Ratio on Concrete Core Strengths[1]

3.1 Introduction

The length to diameter ratio of concrete compression test specimens has long been recognised as a factor which influences the failure load (e.g. ASTM, 1927). Short specimens fail at greater loads because the steel loading platens of the testing machine restrain lateral expansion throughout the specimen more effectively (Newman and Lachance, 1964, Ottosen, 1984). The effect of end restraint is conventionally assumed to be negligible for a standard specimen with a length to diameter ratio, *l*/d, of 2 (ASTM, 1990).

In practice it may be advantageous to test short specimens instead of standard specimens since they take less time to drill, are easier to handle, and leave a smaller void to be patched. Moreover, ASTM C42-90 recommends trimming off those portions of a core which contain reinforcing steel as long as the final *l*/d ratio is at least one (ASTM, 1990). In this case, it is conventional to determine the strength of an equivalent standard specimen by reducing the actual failure stress of the short specimen by a correction factor. Five different sets of correction factors recommended by ASTM since 1927 are shown in Table 3.1. The current values are similar to those published sixty five years ago.

Past investigations have typically been limited to concretes with standard cylinder strengths less than 50 MPa. The values given in ASTM C42-90 are somewhat qualified by a note indicating that "correction factors depend on various conditions such as strength and elastic moduli". There is some indication that, as the concrete strength increases, smaller corrections (or correction factors closer to 1.0) are required (e.g. Kesler, 1959, Munday and Dhir, 1984). However, there are few published data to substantiate extrapolating current strength correction factors to high performance concretes.

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1 A version of this chapter has been accepted for publication in the *American Concrete Institute Materials Journal*

3.2 Research Significance

The main objectives of the present study are to determine length to diameter correction factors for concrete cores with strengths up to 100 MPa and to investigate the precision of these factors. Correction factor values are proposed which account for both the moisture condition and the strength of the core test specimen. The proposed factors are shown to be reasonably accurate for cores with l/d between 1 and 2. It is also shown that the variability of the strength of cores with l/d = 1 is not appreciably greater than that of standard cores.

3.3 Selection of Form of Model

The selection of a form of mathematical model to represent the l/d strength correction factor, $F_{l/d}$, was based on a review of correction factors proposed by others. Values recommended in various editions of ASTM C42 have already been identified. The Concrete Society (1976) suggested:

$$F_{l/d} = \frac{2}{1.5 + d/l} \tag{3.1}$$

Based on a "semi-rational" approach, Chung (1979) recommended:

$$F_{l/d} = \frac{1}{1 + 0.8(1 - 0.5(l/d))^2} \tag{3.2}$$

After a subsequent experimental program involving very short moulded cylinder specimens, Chung (1989) proposed changing Equation (3.2) slightly to:

$$F_{l/d} = \frac{1}{1 + 0.8(d/l)(1 - 0.5(l/d))^2} \tag{3.3}$$

This brief review indicated that the various proposed relationships for correction factors are all nonlinear. The recommended values for a core with length equal to diameter range from 0.80 (Concrete Society, 1976) to 0.91 (ASTM, 1968).

The selected model has the form

$$F_{l/d} = 1 - \theta\left(2 - \frac{l}{d}\right)^{\beta_1} \tag{3.4}$$

where the constants θ and β_1 are determined by regression. The family of curves defined by this model give correction factors of $(1-\theta)$ at $l/d = 1$ and 1.0 at $l/d = 2$. The value of β_1 determines how quickly the correction factor approaches 1 for intermediate length to diameter ratios. Values of θ and β_1 can be chosen which allow Equation (3.4) to closely simulate the various sets of correction factor values found in the literature, although obviously no unique values of θ and β_1 can closely simulate them all.

The selected form cannot accommodate correction factors larger than one. Theoretically this should not pose a problem since the effects of end confinement should always cause stronger strengths for shorter specimens. However the large scatter of test values sometimes causes correction factors which are larger than one, especially for cores with l/d ratios greater than 1.5. Hence a quadratic form, which gives $F_{l/d} = 1$ at $l/d = 2$ and allows for larger correction factors at smaller length to diameter ratios, was also considered:

$$F_{l/d} = 1 + \theta_1\left[2 - \frac{l}{d}\right] + \theta_2\left[4 - \left(\frac{l}{d}\right)^2\right] \tag{3.5}$$

Preliminary analyses using both models were carried out for a data set which included some correction factors greater than one. Severe multicollinearity (e.g. Montgomery and Peck, 1982) was present in the quadratic model due to the strong correlation between the predictor variables $(2-l/d)$ and $\{4-(l/d)^2\}$. This caused the variance of the parameter estimates of θ_1 and θ_2 to be inflated by a factor of about 300, and so the parameter estimates were not significantly different from zero. The mean square errors for both models were similar, suggesting little difference in accuracy between them. The quadratic model was abandoned since it did not give a better fit and has undesirable consequences due to the observed multicollinearity.

3.4 Data Sources

The data used in the present investigation are from three sources. The majority are from a series of experiments conducted in 1975 at the laboratories of the North Carolina Department of Transportation (NCDoT) and the joint laboratories of the National Sand and Gravel Association (NSGA) / National Ready Mixed Concrete Association (NRMCA). The research, which led to the *l*/d correction factor values currently recommended in ASTM C42 (ASTM, 1990), was originally published by Meininger, Wagner, and Hall (1977). The raw data have been made available for the present study through the kind cooperation of Meininger. To extend the scope of the present investigation to higher strength concretes, additional specimens were obtained from two beams and two blocks cast at the University of Alberta (Bartlett and MacGregor, 1994 [Chapter 2]).

The test specimens were drilled from ten different elements. The dimensions and coring locations of three walls investigated by the NCDoT, identified in this study as Elements 1, 2, and 3, are shown in Figure 3.1 (a). Three walls investigated by the NSGA/NRMCA, shown in Figure 3.1 (b), are identified in this study as Elements 4, 5, and 6. The remaining elements, two large beams of moderate strength concrete and two high strength concrete blocks, are shown in Figures 3.1 (c) and (d) and are identified as Elements 7 through 10.

These particular data are used in the present study for several reasons. A primary objective of each investigation was to study the *l*/d effect. In each case, the experimental procedures are sufficiently well documented to confirm that variable factors were closely controlled. Each data set comprises large numbers of replicate observations, which allow relatively precise average values to be calculated. Each set is complete, so that outlier tests can be carried out on each individual strength value. Finally the precise location of each core in the element is known, so that any effect caused by variation of the *in situ* strength throughout the element can be quantified.

3.4.1 Experimental Procedures

The various experimental procedures are thoroughly documented elsewhere (Meininger et al., 1977, Alca, 1993 and Bartlett and MacGregor, 1994 [Chapter 2]). Therefore, only a brief summary of the similarities and differences in experimental procedure are presented here.

All elements were cast using ordinary portland cement concretes. The types of aggregate varied: Elements 1-3 had quartz fine aggregate and crushed gravel coarse aggregate; Elements 4-6 had limestone fine and coarse aggregate; and Elements 7-10 had quartzite fine aggregate and a blended quartzite and hard sandstone coarse aggregate. Generally the air contents of the concretes were low, except for Elements 2 and 3 which had air contents in excess of 5%. The concrete in Element 6 contained a water reducing admixture, and that used for Elements 9 and 10 contained a superplasticiser.

Each element was laid on its side and cores were drilled vertically through the 12 or 13 inch thickness, perpendicular to the direction of casting. The age of the concrete at the time of drilling varied between 28 and 95 days. All cores were 4 inches in diameter. Each 12 or 13 inch long core was sawn to give two test specimens of 4 and 8, 5 and 7, or 6 and 6 inch lengths.

The cores were subjected to various moisture conditioning treatments between drilling and testing. Some were left in the laboratory air for 7 days before testing, as is now permitted in ASTM C42-90. Others were soaked in limewater for two days or for 7 days, as has been specified in ASTM standard methods since 1927. A few cores from Elements 7-10 were stored in plastic bags between drilling and testing.

Different procedures were carried out to prepare the ends of the specimen for testing. All cores from Elements 1-3 were capped with a sulfur capping compound. All cores from Elements 7-10 were lapped. The air dried cores from Elements 4-6 were all capped with a high strength gypsum plaster compound, whereas in each group of soaked cores with common length to diameter ratio, four specimens had lapped ends, four were capped with sulphur, four were capped with neat cement paste and four were capped with the gypsum plaster compound.

The soaked core data from Elements 4-6 were examined to see whether the different end preparations corresponded to significant differences of the average strength. For each wall, replicate strength observations for each *l*/d category were analysed using Tukey's pairwise comparisons (Larsen and Marx, 1986). Generally, observed differences were found not to be statistically significant at the 5% level. This confirms the results of a different analysis by Meininger et al. (1977), who concluded that end preparation procedures generally had no significant effect on the observed average *l*/d correction factor.

3.5 Preliminary Data Review

Before carrying out a general analysis of all the data to determine *l*/d correction factors, data from each element were analysed separately to identify outlying values and to quantify any effect caused by spatial variation of the *in situ* strength throughout the element.

3.5.1 Identification of Outliers

Imperfect drilling, handling, capping or testing procedures may damage the core specimen and so cause low strength observations. The complete set of data, containing 779 values, was reviewed and 21 low strengths were identified as probable outliers and removed.

In 6 cases, observations made by the original investigators provided credible explanations for the observed low strengths. This simplifies the problem of outlier identification considerably. In one instance, the observed low strength could be attributed to excessive cap thickness, since the recorded difference between the uncapped and capped specimen lengths is about 7/8 inch. Upon inspection after testing, two cores were found to contain foreign material. The load-deformation responses of three other cores were highly irregular and consistent with imperfect lapping of the specimen ends, causing in one case a premature splitting failure and, in the other two instances, premature localised crushing failures.

It is difficult to identify reliably outliers solely on the basis of the sample statistics, without credible physical reasons to corroborate the hypothesis that an observed extreme value is spurious. In light of this difficulty, the other 15 outliers were identified using two independent methods, those recommended in ASTM E178-80 "Standard Practice for Dealing with Outlying Observations" (ASTM, 1989) and other criteria suggested by Rousseeuw and Leroy (1987). Both are more powerful when used on large data sets. For example, for the case of only 3 replicate observations there is virtually no difference between ASTM E178 criteria for rejection at the 20% and the 0.02% levels of significance. Fortunately the NCDoT/NRMCA data in particular includes large numbers of replicate observations. Sample calculations based on the two methods for outlier identification calculations are presented in Appendix 3A.

3.5.2 Spatial Variation of the *In Situ* Strength

Details of the analysis used to quantify the variation of the *in situ* strength throughout each element are summarised below for Elements 1 through 6. The details of a similar procedure used for Elements 7 through 10 may be found elsewhere (Bartlett and MacGregor 1994 [Chapter 2]).

Separate regression analyses of the wet and dry core strength data for each wall were carried out using the model

$$f_n = f_c \theta_0 F_{l/d} F_x F_y F_z + \varepsilon \quad (3.6).$$

In this equation, the dependent variable f_n represents the nominal wall strength and is constant for each wall. Initially this value need not be known precisely since it is effectively adjusted by the dimensionless parameter θ_0 which is determined by regression. Independent variable f_c represents the observed strength of each individual core. It is modified by the *l*/d strength correction factor $F_{l/d}$, and factors F_x, F_y, and F_z which respectively account for possible spatial variation of the *in situ* strength over the length, height, and thickness of the wall. Linear gradients of *in situ* strength are assumed, leading to

$$F_x = 1 + \theta_1 x \quad (3.7),$$

$$F_y = 1 + \theta_2 y \quad (3.8)$$

and

$$F_z = 1 + \theta_3 z \tag{3.9}$$

where x, y, and z give the original location of the centre of the core within the wall according to the coordinate systems shown in Figures 3.1 (a) and 3.1 (b). The variable ε in Equation (3.6) represents the normally distributed random error component of the dependent variable f_n.

Parameter estimates, determined using the NLIN (nonlinear) regression procedures in the SAS software package (SAS Institute, 1990), are shown in Table 3.2. All values shown are significantly different from zero at the 90% confidence level unless indicated otherwise.

Small but significant variation of core strength with core location exists in all walls. The variation of the *in situ* strength along the length of walls 1-3 is indicated by the partial regression plots shown in Figure 3.2. The estimates of the location parameters shown in Table 3.2 for the wet core strengths are quite consistent with the corresponding estimates for the dry core strengths, especially for walls 1, 2, and 3. Since the wet and dry core analyses are based on mutually exclusive sets of data, this consistency represents a reassuring check of the estimated variation of the *in situ* strength. The estimates of θ_2 for wet and dry cores from walls 4-6 are less consistent, though this might be expected since the dry cores tend to come from the upper area of the drilling region and the wet cores tend to come from the lower area, as shown in Figure 3.1 (b).

The values shown in Table 3.2 also indicate the effect of the moisture conditioning treatment on both the core strength and the *l*/d strength correction factor. In all cases, the value of $\hat{\theta}_0$ for wet cores is larger than for dry cores, confirming the conventional wisdom that cores tested dry are stronger than cores of the same material tested wet. The magnitude of $\hat{\theta}$ tends to be lower for cores tested wet than for cores tested dry. Large differences of $\hat{\beta}_1$ for wet and dry cores are evident, though the effect of these differences on actual correction factor values is less dramatic.

The observed core strengths were adjusted to account for the effects of spatial variation of the *in situ* strength using the equation

$$f_c^* = f_c \frac{F_x F_y F_z}{\overline{F}_x \overline{F}_y \overline{F}_z} \tag{3.10}$$

where f_c is the raw core strength and f^*_c is the modified core strength. Spatial strength variation factors F_x, F_y, and F_z were determined by substituting the preliminary parameter estimates, shown in Table 3.2, into Equations (3.7), (3.8) and (3.9). If a parameter estimate was not significantly different from zero, it was assumed equal to zero and so did not contribute to the modification of the strength. Normalizing factors $\overline{F}_x$, $\overline{F}_y$ and $\overline{F}_z$ are the average values of the corresponding strength variation factors for the set of wet or dry cores from a particular element.

Sample statistics for the modified data, after discarding the 21 low strength values identified as outliers, are shown in Table 3.3. The average values observed for each group of replicate specimens are insensitive to the effects of spatial variation because, within each category of replicate observations, the specimens were generally distributed uniformly throughout the element cored. The observed dispersions of each set of replicate observations are inflated somewhat by the spatial variation of the *in situ* strength.

3.6 Regression Analysis for Strength Correction Factors

The model used to determine the strength correction factors was

$$\overline{f}_{c,S} = F_{l/d}\overline{f}_{c,NS} + \varepsilon \tag{3.11}$$

where $\overline{f}_{c,S}$ is the average strength of standard cores with $l/d = 2$, $\overline{f}_{c,NS}$ is the average strength of companion cores with l/d ratios not equal to 2, and $F_{l/d}$ is the strength correction factor given by Equation (3.4). The parameter θ in Equation (3.4) was modified to accommodate possible variation of the strength correction factor with both the moisture condition and the strength of the test specimens:

$$\theta = \beta_2 + \beta_3 Z_{mc} + \beta_4 \overline{f}_{c,NS} \tag{3.12}$$

In this equation, variable Z_{mc} indicates the moisture condition of the cores at the time of testing and equals 0 for air dried cores, 1 for soaked cores, or 0.5 for sealed cores.

A weighted regression analysis was carried out since these data violate two assumptions necessary for conventional regression analyses. The independent variable $\bar{f}_{c,NS}$ is random with an error of the same order as that observed in the dependent variable, $\bar{f}_{c,S}$. Also, both $\bar{f}_{c,NS}$ and $\bar{f}_{c,S}$ do not appear to have constant variance, violating the necessary condition of homoscedasticity. The data in Table 3.3 indicate that, for the cores from Elements 7-10, the sample standard deviations are relatively high and the numbers of replicate specimens are relatively low, so that the variance of the sample means for these data will be relatively large. To accommodate these violations, the analysis was carried out by weighting each observation by a factor inversely proportional to the variance of the total error. The calculation of the weighting factors is described in detail in Appendix 3B.

The analysis was performed using the NLIN (nonlinear) regression procedures in the SAS software package (SAS Institute, 1990). The square root of the residual mean square error was 1.51. The parameter estimates, standard errors, and p-values calculated using the exact algorithms proposed by Lackritz (1984) are shown in Table 3.4. For practical purposes, the value of $\hat{\beta}_1$ may be rounded to 2 without significant loss of accuracy, yielding:

$$\hat{f}_{c,S} = \left[1 + \left(-0.144 + 0.027 Z_{mc} + 0.0030 \frac{\bar{f}_{c,NS}}{1000}\right)\left(2 - \frac{l}{d}\right)^2\right]\bar{f}_{c,NS} \qquad (3.13).$$

The data indicate that both the core moisture condition and the core strength have a small but significant effect on the strength correction factor. For strengths reported in MegaPascals (MPa), the equivalent relationship is

$$\hat{f}_{c,S} = \left[1 + (-0.144 + 0.027 Z_{mc} + 0.00044 \bar{f}_{c,NS})\left(2 - \frac{l}{d}\right)^2\right]\bar{f}_{c,NS} \qquad (3.13a).$$

The variation of the observed and predicted correction factors with the l/d ratio is shown for air dried cores in Figure 3.3 (a) and for soaked cores in Figure 3.3 (b). Since the observed correction factors are a ratio of two random variables, both the mean observed correction factors and approximate 95% confidence limits on the mean values are shown. The predicted values according to Equation (3.13) are shown as a range, with the larger value corresponding to $\bar{f}_{c,NS}$ = 14,000 psi (96.5 MPa) and the smaller value corresponding to $\bar{f}_{c,NS}$ = 2000 psi (13.8 MPa). Also shown on the figures are the predicted correction factors according to ASTM C42-68, ASTM C42-77, Chung (1989) and the Concrete Society (1976).

Figures 3.3 (a) and 3.3 (b) indicate that, *for these data*, the strength correction factors proposed by the Concrete Society (1976) consistently overestimate the amount of correction required and so are conservative. The current ASTM C42-77 values closely simulate the observed values for air dried cores, whereas the previous set of values from ASTM C42-68 seem to more closely simulate the observed values for soaked cores. It should come as no surprise that the values predicted using Equation (3.13) are close to the ASTM C42-77 values, since they are largely based on the same data set. These observations are corroborated by the ratios of observed to predicted correction factors shown in Table 3.5.

3.7 Validation of the Strength Correction Factors

The most effective method of validating a regression model with respect to its prediction performance is to collect fresh data and compare the model predictions against it (Montgomery and Peck, 1982). Data collected by Bloem (1965) were selected for this purpose, since these experiments were carefully controlled and well documented. Strength results for individual specimens were again made available through the kind cooperation of Meininger.

Bloem's experiments were primarily designed to investigate the relationship between standard cylinder strengths, field cylinder strengths, and *in situ* concrete strengths as represented by core specimens. The experiments were not specifically designed to investigate the effect of core length to diameter ratio on core strength, but cores with nominal *l*/d ratios of 1 and 2 were obtained and tested.

The data used to validate the predictive power of Equation (3.13) include cores from the Series 183 A slabs and the Series 183 C columns. Only the data for well-cured slabs and columns were considered, so that differences of concrete quality over the length of the core specimen were minimised. All cores were 4 inches (100 mm) in diameter. Since the slab core strength averages are based on only 3 replicate observations and the column core strength averages are based on only 4 replicate observations, outliers could not be identified with any confidence.

The observed average non-standard core strengths, predicted standard core strengths, and observed average standard core strengths are shown in Table 3.6. The values in the right hand column are the ratios of the observed average standard core strengths to the predicted average standard core strengths. Although the individual entries vary somewhat, the overall average of these values is very close to 1.0, indicating that the predictive power of Equation (3.13) for these data is quite good.

3.8 Precision of Core Strengths and Strength Correction Factors

While ASTM C42-90 states that the single operator coefficient of variation on core compressive strengths has been found to be 3.2%, it does not say whether this refers only to standard cores or to all cores with *l*/d ratios between 1 and 2. The data shown in Table 3.3 can be used to investigate this, since all replicate observations were obtained by a single operator on a single testing machine.

The coefficients of variation for sets of replicate cores with various *l*/d ratios are shown in Table 3.7. Both a simple averaging procedure and a simple regression method were used to obtain the coefficients of variation. In the regression method, the sample standard deviation of each set of replicate strengths is regressed on the corresponding sample average using a linear model with no constant. The slope of the line represents the ratio of standard deviation to mean value and so is a measure of the coefficient of variation. The result differs from simply taking the average because more weight is given to observations with large mean strength values.

If spatial variation of the *in situ* strength is not accounted for, the average coefficients of variation range from 4.8 to 5.1 percent. These values corroborate the 5% value reported by Meininger et al. (1977). Accounting for spatial variation of the *in situ* strength reduces the coefficients of variation; the range is between 3.6 and 4.5 percent. The coefficients of variation determined by the regression method range from 2.7 to 4.0 percent if the spatial variation of the *in situ* strength is accounted for.

The values shown in Table 3.7 indicate clearly that the single operator coefficient of variation may be assumed to be independent of the core l/d ratio. The values also suggest that the single operator coefficient of variation reported in ASTM C42-90 may be slightly optimistic.

The accuracy of the prediction equation is indicated in Figure 3.4, where the residual standard strength errors are shown plotted against the predicted standard strengths. The residual strength errors are the difference between the observed average strength of the standard cores and the corresponding values predicted using Equation (3.13). A negative residual error indicates that the predicted value exceeds the observed value and so is unconservative. The region where the unconservative error exceeds 5% of the predicted value is also shown on Figure 3.4; only 3 of the 50 points shown lie in or near this region.

The precision of the l/d strength correction factors themselves can also be approximated using the standard errors of the parameter estimates shown in Table 3.4. From Equation (3.5), for $\beta_1 = 2$, the l/d correction factor has variance

$$\mathrm{Var}[F_{l/d}] = \mathrm{Var}[\hat{\theta}]\left\{\left(2-\frac{l}{d}\right)^2\right\}^2 \qquad (3.14)$$

and coefficient of variation

$$V_{l/d} = \frac{\sqrt{\mathrm{Var}[\hat{\theta}]}\quad (2-l/d)^2}{1-\hat{\theta}(2-l/d)^2} \qquad (3.15).$$

From Equation (3.12), assuming variables Z_{mc} and $\bar{f}_{c,NS}$ to be deterministic and assuming the other parameters to be statistically independent, the variance of $\hat{\theta}$ is

$$\mathrm{Var}[\hat{\theta}] = \mathrm{Var}[\hat{\beta}_2] + Z_{mc}^2\mathrm{Var}[\hat{\beta}_3] + \left(\frac{\bar{f}_{c,NS}}{1000}\right)^2\mathrm{Var}[\hat{\beta}_4] \qquad (3.16).$$

Hence for $Z_{mc} = 0.5$ and $\bar{f}_{c,NS} = 4000$ psi (28 MPa) one obtains approximately

$$V_{l/d} \cong 0.020\left(2 - \frac{l}{d}\right)^2 \qquad (3.17).$$

(The rationale used to derive Equation (3.16) has two minor flaws. First, the parameter estimates $\hat{\beta}_2$, $\hat{\beta}_3$, and $\hat{\beta}_4$ are not statistically independent as assumed, but correlated. Accounting for their correlation reduces Var[$\hat{\theta}$] and $V_{l/d}$. Second, the standard errors of the parameter estimates shown in Table 3.4 underestimate the true values because the square root of the residual mean square error exceeds 1.0. Addressing this deficiency increases Var[$\hat{\theta}$ and $V_{l/d}$. Because these two effects counteract each other, Equation (3.17) remains reasonably accurate. A more thorough analysis of the errors of the strength correction factors is given in Bartlett and MacGregor (1993) [Chapter 6].)

This value of $V_{l/d}$ is an estimate of the coefficient of variation of the strength correction factor itself. The coefficient of variation of the predicted equivalent standard strength, $V_{f\hat{c}S}$, can be calculated from the coefficient of variation of the observed strengths of cores with l/d less than 2, V_{fcNS}, using the equation

$$V_{f\hat{c}S} = \sqrt{V_{l/d}^2 + V_{fcNS}^2} \qquad (3.18).$$

It should be noted that this represents the coefficient of variation of the predicted standard strength and reflects the fact that the conversion factor is not known with absolute certainty. The effect of $V_{l/d}$ is negligible if the non-standard cores have l/d = 1.75 and is more significant if the non-standard cores have with l/d = 1. If, for some reason, it was necessary to invert Equation (3.13) to predict the equivalent strength of cores with l/d less than 2, $V_{fc\hat{N}S}$, from the strengths of standard cores with l/d = 2, the effect of the uncertain prediction factor would again be to increase the coefficient of variation of the predicted values

$$V_{fc\hat{N}S} = \sqrt{V_{l/d}^2 + V_{fcS}^2} \qquad (3.19).$$

3.9 Summary and Conclusions

Weighted nonlinear regression analysis has been performed to determine the concrete core compressive strength correction factor which accounts for the length to diameter ratio effect. In accordance with current North American practice, the correction factor is applied to the strength of a core with *l*/d ratio between 1 and 2 to give the equivalent strength of a standard core with *l*/d = 2. Data representing tests of 758 cores with diameters of 100 mm have been analysed. The cores were drilled from ten elements cast of ordinary portland cement concrete with strengths between 14 and 96 MPa which were at least 28 days old.

From the analysis of these data, the following conclusions are drawn:

1. The required strength correction is significantly reduced for high strength concretes. As the concrete strength increases, correction factors closer to 1.0 are appropriate.

2. The required strength correction is slightly but significantly reduced if the cores are soaked before testing, instead of being left to dry in laboratory air for 7 days.

3. The best fit to the data is given by Equation (3.13). The current set of strength correction factors in ASTM C42-90 (ASTM, 1990) fit the data for air dried cores well. The previous set of factors (ASTM, 1967) fit the data for soaked cores slightly better than the current values do.

4. The observed single operator coefficient of variation on cores is about 5%. If the effect of spatial variation of the *in situ* concrete strength throughout the element is accounted for, the observed coefficient of variation drops to about 4%. Both values are larger than that given in the precision statement in ASTM C42-90.

5. There is no difference between the observed coefficient of variation of strengths of standard cores with *l*/d = 2, and strengths of cores with *l*/d = 1.

6. The prediction error associated with predicting standard core strengths using Equation (3.13) can be assumed to have negligible bias and a coefficient of variation of roughly $0.02\ (2-l/d)^2$.

Table 3.1: *l*/d Strength Correction Factors Recommended by ASTM Standard C42

Edition of ASTM C42	Specimen length to diameter ratio, *l*/d			
	1.0	1.25	1.5	1.75
1927	0.85	0.94	0.95	0.98
1949	.85	.94	.96	.98
1961	.89	.94	.96	.98
1968	.91	.94	.97	.99
1977 to present	.87	.93	.96	.98

Table 3.2: Parameter Estimates, Separate Analyses of Data from Elements 1-6

Element No.		*l*/d parameters $\hat{\theta}$	$\hat{\beta}_1$	location parameters $\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_{0\,dry}/\hat{\theta}_{0\,wet}$
1	wet	.124	3.17	-.0025	-.016	-.0045	
	dry	.139	2.10	-.0044	-.011	-.0047	.884
2	wet	.073	0.80	-.0048	-.014	.0107	
	dry	.109	2.19	-.0033	-.018	.0077	.867
3	wet	.102	3.94	-.0045	NS	-.0030	
	dry	.141	1.31	-.0033	NS	-.0015	.844
4	wet	.081	3.19	.0053	NS	.0037	
	dry	.172	5.31	NS	-.027	.0065	.884
5	wet	.141	1.92	-.0076	NS	NS	
	dry	.161	2.11	-.0115	NS	NS	.885
6	wet	.145	2.13	-.0052	NS	-.0092	
	dry	.139	1.60	-.0039	NS	-.0105	.903

Note: NS - parameter not significantly different from zero at the 90% confidence level.

Table 3.3: Core Strength Data for *l*/d Analysis

Element	Z_{mc}	*l*/d	n	$\bar{f}^*_c$ (psi)	s (psi)	V (%)
1	1	1.01	14	4640	256	5.53
1	1	1.26	11[a]	4296	167	3.89
1	1	1.51	11[a]	4194	145	3.46
1	1	1.75	12	4115	189	4.58
1	1	1.99	9[a]	4059	179	4.41
1	0	1.01	13[a]	5381	195	3.63
1	0	1.26	12	5006	113	2.26
1	0	1.51	12	4840	133	2.75
1	0	1.75	9[c]	4631	115	2.49
1	0	2.00	10	4726	217	4.59
2	1	1.00	12	2174	155	7.12
2	1	1.25	10	2166	156	7.18
2	1	1.50	14	2141	119	5.56
2	1	1.75	14	2050	68	3.30
2	1	2.00	8	2042	144	7.04
2	0	1.00	14	2623	179	6.83
2	0	1.25	10	2526	140	5.56
2	0	1.50	14	2381	149	6.25
2	0	1.74	15	2381	139	5.83
2	0	1.99	9	2335	106	4.54
3	1	1.01	19[a]	2965	139	4.68
3	1	1.26	9[a]	2749	119	4.33
3	1	1.52	7[a]	2740	38	1.38
3	1	1.75	8	2663	74	2.78
3	1	2.00	8[a]	2686	85	3.15
3	0	1.00	25	3574	187	5.24
3	0	1.26	9	3364	163	4.85
3	0	1.51	11	3241	165	5.09
3	0	1.76	7[b]	3160	31	1.00
3	0	2.00	10	3085	169	5.49
4	1	1.02	16	3369	189	5.61
4	1	1.52	16	3124	89	2.85
4	1	1.99	16	3128	147	4.71
4	0	1.04	6	4022	176	4.39
4	0	1.29	6	3560	85	2.38
4	0	1.52	6	3382	123	3.65
4	0	1.78	6	3472	160	4.61
4	0	1.99	6	3495	110	3.15
5	1	1.03	16	5734	196	3.42
5	1	1.49	16	5152	272	5.29
5	1	1.98	16	4945	234	4.73
5	0	1.04	6	6362	265	4.16
5	0	1.29	6	5859	198	3.38
5	0	1.52	6	5680	273	4.81
5	0	1.75	6	5451	200	3.67
5	0	2.01	5[a]	5481	145	2.64
6	1	1.03	16	7919	325	4.10
6	1	1.50	16	7152	228	3.18
6	1	1.95	14[b]	6831	247	3.62
6	0	1.04	6	8465	302	3.57
6	0	1.28	5[a]	8260	83	1.01
6	0	1.50	6	7707	245	3.18
6	0	1.72	6	7526	288	3.83
6	0	1.97	6	7418	212	2.85
7	0.5	1.00	8	7083	379	5.35
7	0.5	2.04	7[a]	6375	425	6.66
8	0.5	1.01	6[b]	6984	460	6.59
8	0.5	2.00	8	6624	267	4.04
9	0	0.99	5	14603	585	4.00
9	0	2.03	5	13670	457	3.34
9	0.5	1.00	6	14409	552	3.83
9	0.5	2.01	6	12908	701	5.43
9	1	0.99	5	12705	474	3.73
9	1	2.03	5	11906	434	3.65
10	0	0.98	5	14701	418	2.84
10	0	2.00	5	13033	509	3.91
10	0.5	0.99	6	13067	357	2.74
10	0.5	1.99	5[a]	11862	739	6.23
10	1	0.99	5	12148	209	1.72
10	1	1.99	5	11454	182	1.59

Note: [a] - one, [b] - two, or [c] - three low outliers removed from data.
Z_{mc} = 0 for air dried cores, 0.5 for sealed cores, or 1 for soaked cores.

Table 3.4: Parameter Estimates, Standard Errors, and p-Values

Parameter	β_1	β_2	β_3	β_4
Estimate	2.120	-0.144	0.027	0.0030[a]
Standard Error	0.329	0.015	0.012	0.0015[a]
p-Value	0.0014 for $H_0{:}\beta_1 = 1$	$2\ (10^{-7})$ for $H_0{:}\beta_2 = 0$	0.029 for $H_0{:}\beta_3 = 0$	0.051 for $H_0{:}\beta_4 = 0$

Note: [a] - Parameter estimate and standard error have units [1/ksi].

Table 3.5: Ratios of Observed to Predicted Strength Correction Factors

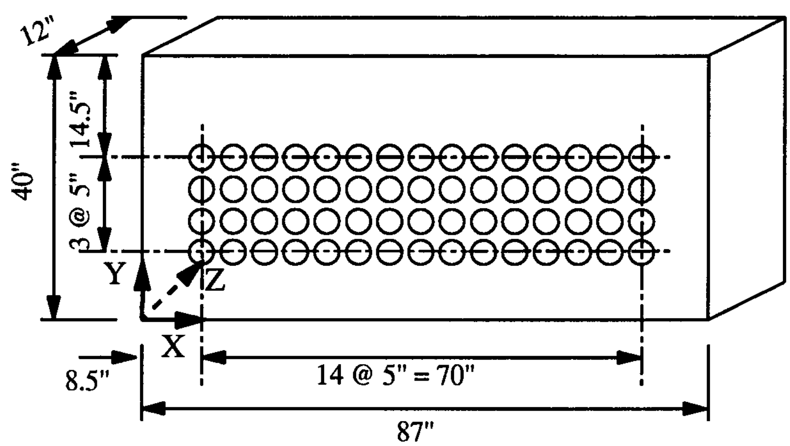
Method	l/d = 1.00		l/d = 1.25		l/d = 1.50		l/d = 1.75		Overall	
	Avg.	V (%)	Avg.	V (%)	Avg.	V (%)	Avg.	V (%)	Avg.	V (%)
Eq. (3.13)	1.00	3.2	1.02	2.8	1.00	2.5	1.00	1.5	1.00	2.6
Chung (89)	1.07	3.9	1.02	2.8	1.00	2.3	1.00	1.5	1.04	4.4
Chung (79)	1.08	3.7	1.04	2.8	1.02	2.3	1.01	1.4	1.04	3.9
Conc. Soc.	1.12	3.8	1.08	2.8	1.05	2.3	1.03	1.4	1.08	4.4
ASTM C42-68	0.99	3.4	1.00	2.8	1.00	2.4	1.01	1.5	1.00	2.9
ASTM C42-77	1.03	3.4	1.01	2.8	1.02	2.4	1.02	1.5	1.02	2.9

Table 3.6: Verification Data

Source	Test Moisture Condition	Z_{mc}	Avg. l/d	$\bar{f}_{c,NS}$	$\hat{f}_{c,S}$	$\bar{f}_{c,S}$	$\bar{f}_{c,S}/\hat{f}_{c,S}$
Series 183 A, 4" cores from top of slab	As Drilled	0.5	1.07	5280	4754	4800	1.01
	Soak 48 Hours	1	1.06	4750	4316	4450	1.03
	Soak 28 Days	1	1.06	5080	4621	4500	0.97
	Air Dry 7 Days	0	1.07	6010	5352	5440	1.02
Series 183 A, 4" cores from bottom of slab	As Drilled	0.5	1.16	5353	4919	4800	0.98
	Soak 48 Hours	1	1.09	5143	4708	4450	0.95
	Soak 28 Days	1	1.10	5183	4755	4500	0.95
	Air Dry 7 Days	0	1.06	6443	5730	5440	0.95
Series 183 C Column cores	Soak 48 Hours	1	1.09	3730	3402	3553	1.04
	Soak 28 Days	1	1.12	3801	3489	3542	1.02
	Air Dry 7 Days	0	1.11	4556	4084	4238	1.04
					Average		0.995
					Standard Deviation		0.038

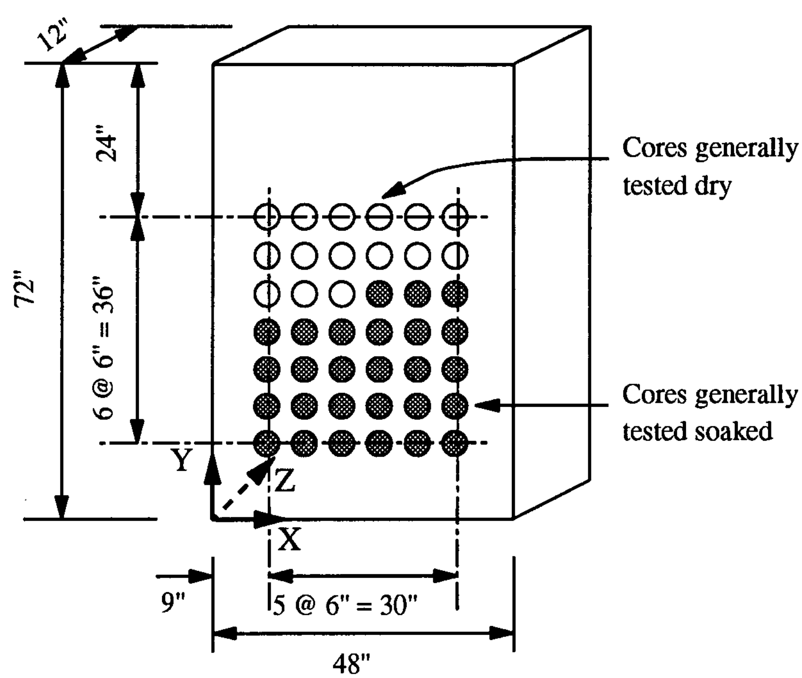
Table 3.7: Coefficients of Variation

Core length to diameter ratio	2.0	1.75	1.5	1.25	1.0
Number of Sets of Replicate Observations	20	9	12	9	20
Average CoV: (%) (spatial variation not accounted for)	4.92	5.11	4.81	4.80	5.04
Average CoV: (%) (after accounting for spatial variation)	4.29	3.56	3.95	3.87	4.45
CoV estimated by regression: (%) (after accounting for spatial variation)	4.07	3.60	3.70	2.65	3.60



(a) Elements 1-3 (North Carolina DoT walls)

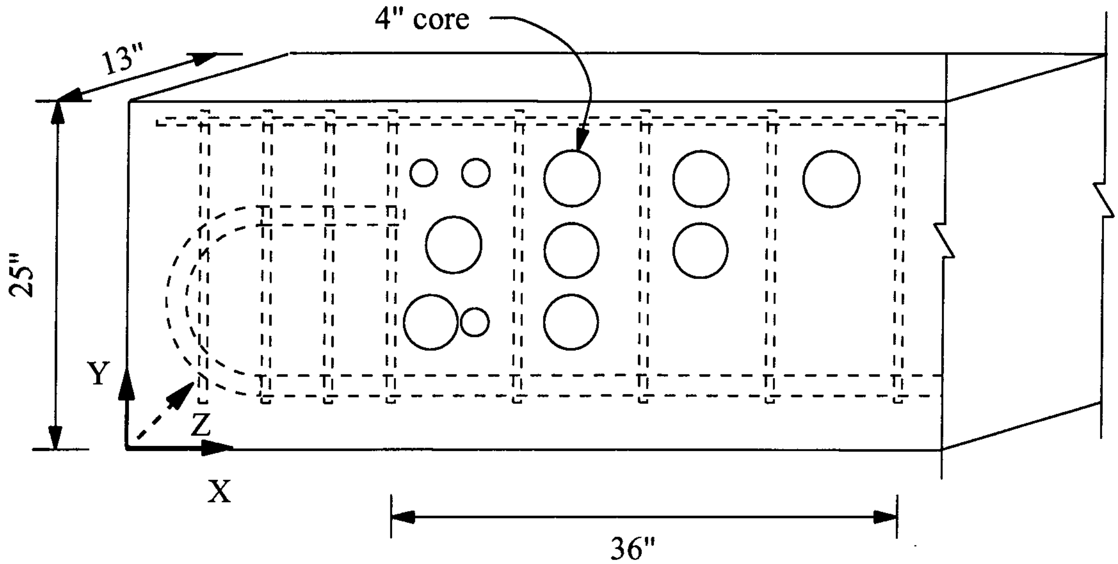
(Note: wall size is 2210 x 1020 x 300 mm)



(b) Elements 4-6 (NSGA/NRMCA walls)

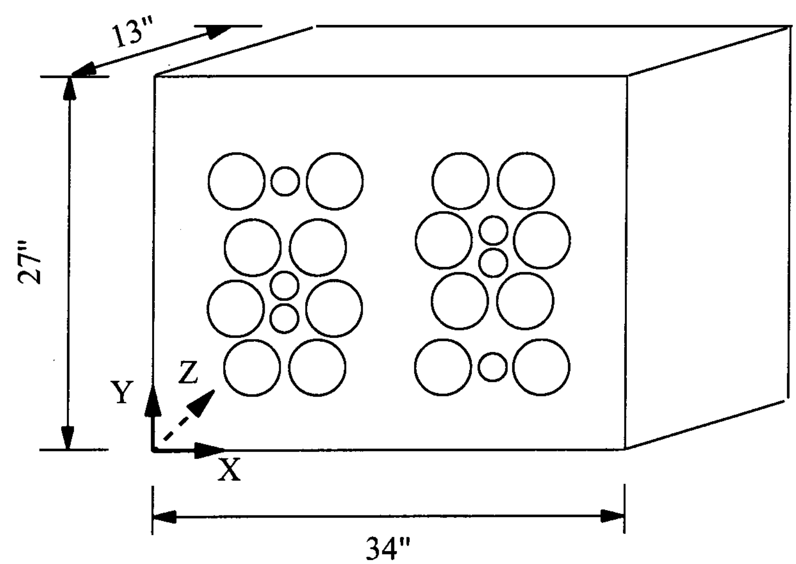
(Note: Wall size is 1830 x 1220 x 300 mm)

Figure 3.1: Element dimensions and core locations



(c) Elements 7 and 8 (University of Alberta beams)

(Note: Region cored is 910 x 640 x 330 mm)



(d) Elements 9 and 10 (University of Alberta blocks)

(Note: Block size is 860 x 690 x 330 mm)

Figure 3.1 (concluded): Element dimensions and core locations

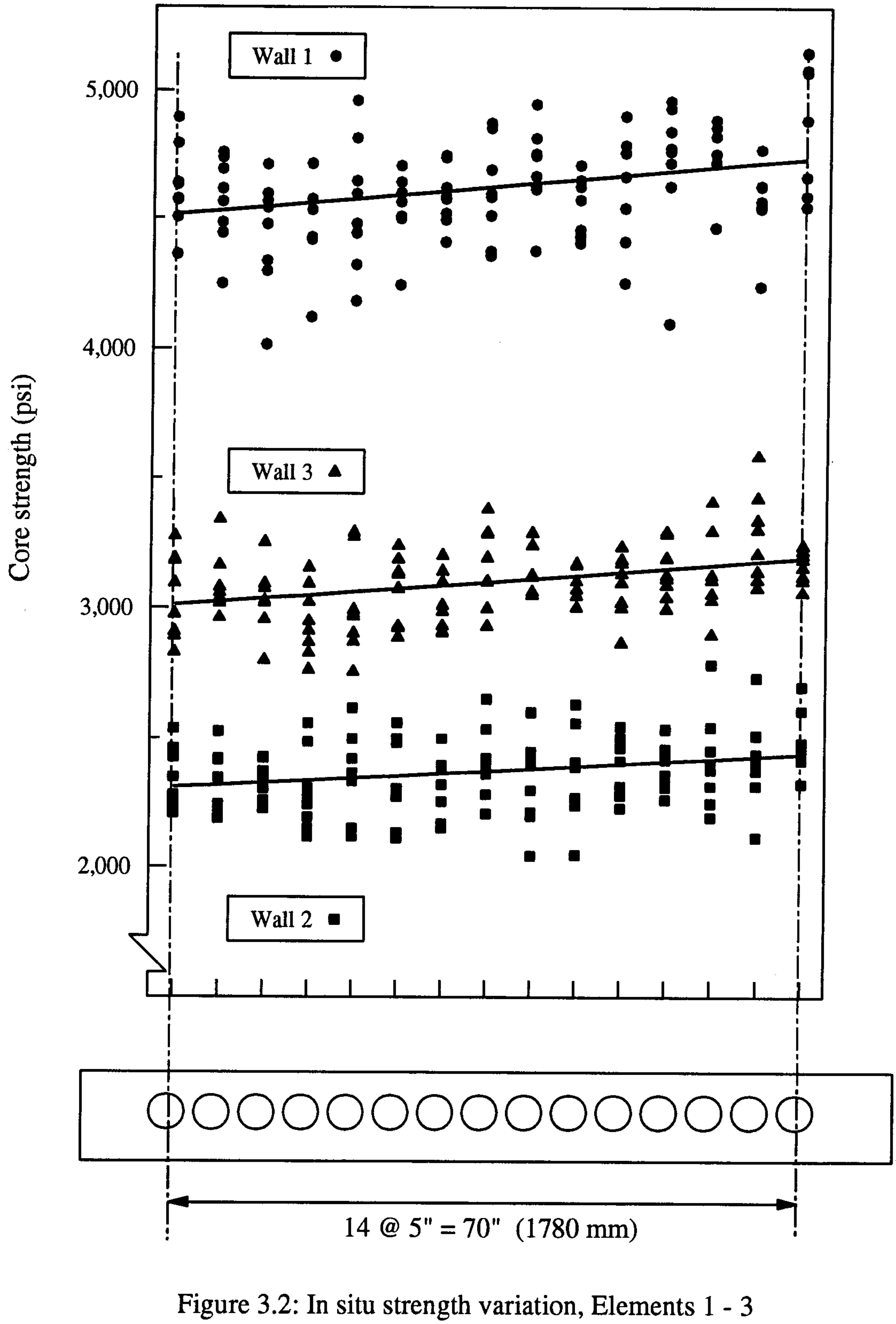


Figure 3.2: In situ strength variation, Elements 1 - 3

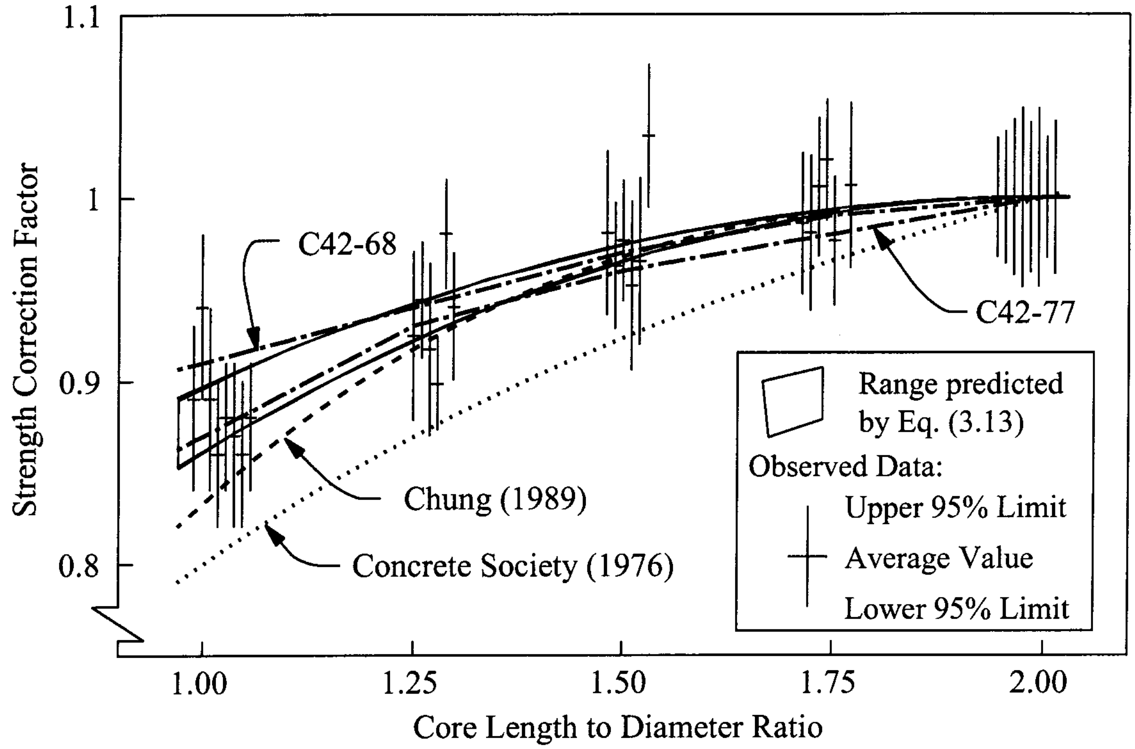


Figure 3.3 (a): Observed and predicted correction factors for dry cores

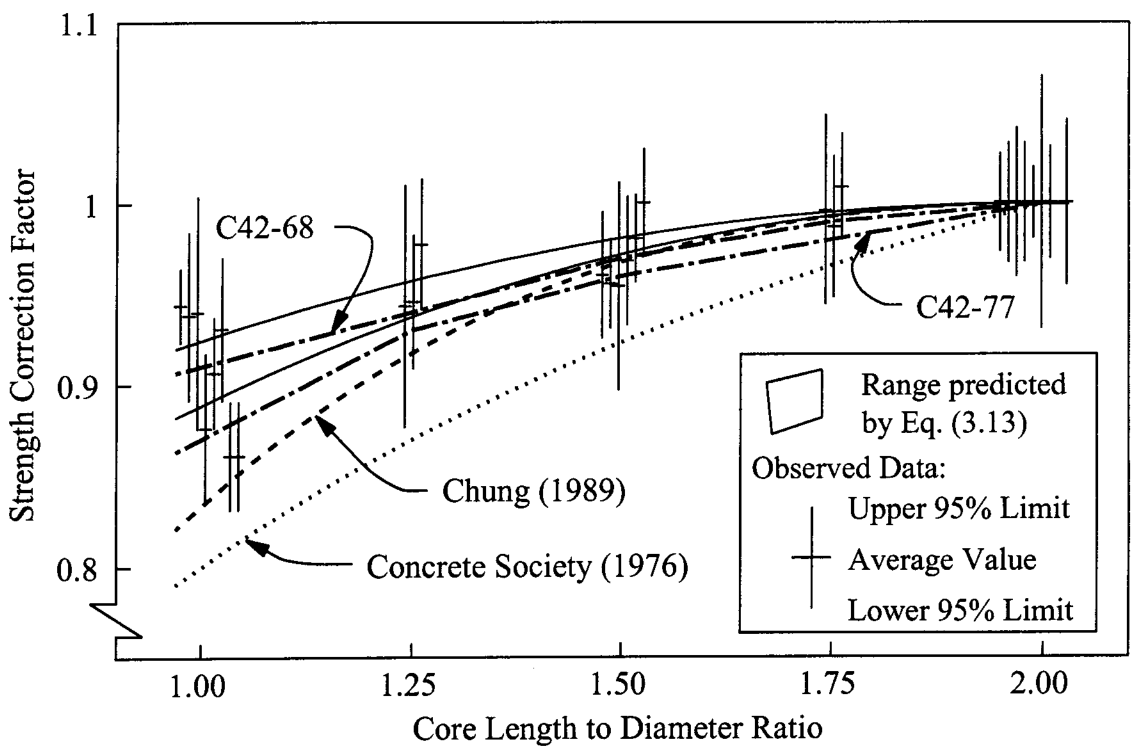


Figure 3.3 (b): Observed and predicted correction factors for wet cores

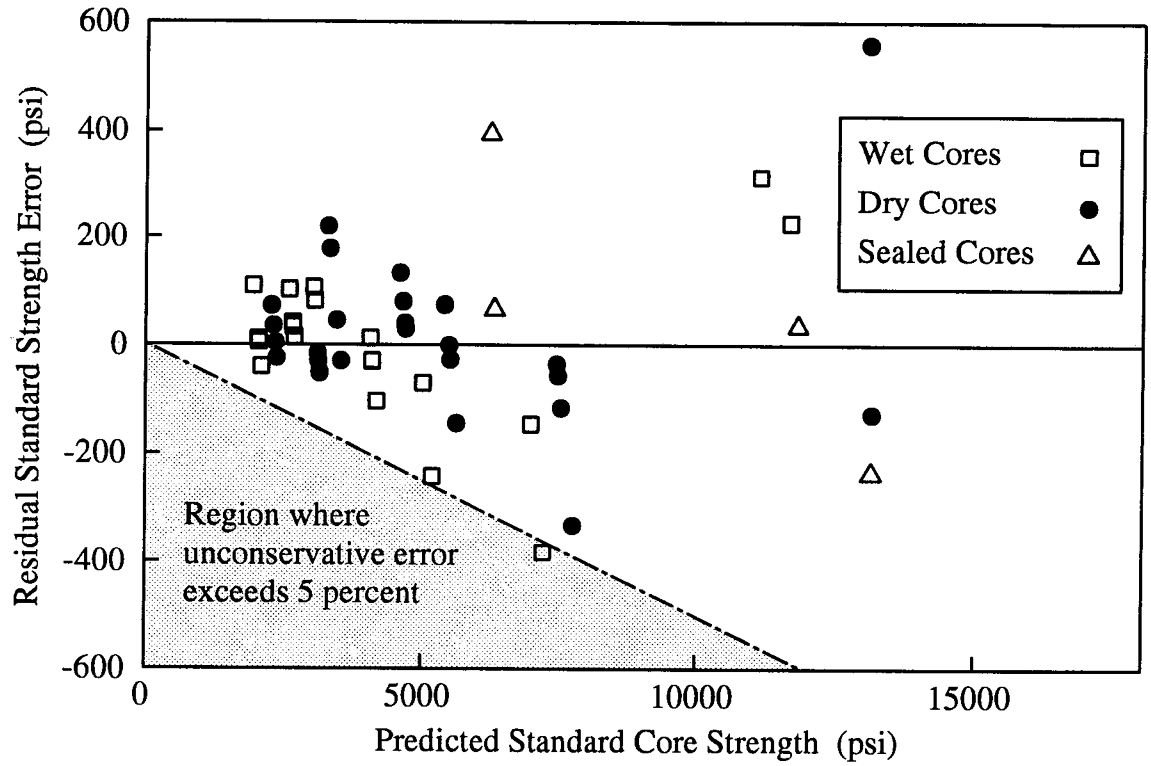


Figure 3.4: Residuals versus predicted standard core strength

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Chapter 4: Effect of Moisture Condition on Concrete Core Strengths[1]

4.1 Introduction

The practice of soaking concrete cores in water for at least 40 hours before testing has evolved from the 1927 version of ASTM C42 ("Standard Methods of Securing Specimens of Hardened Concrete from the Structure"):

> "In order that the tests be made under uniform conditions as to moisture content, the test specimens shall be completely submerged in water for 48 hours and the compression test shall be made immediately thereafter."

Studies by Bloem (1968) and Meininger et al. (1977) indicate that cores tested after air drying are consistently 10 to 20% stronger than cores tested after soaking in water. Based on these studies, the provisions of the ACI Building Code, ACI 318-89 (ACI Committee 318, 1989), require cores to be immersed in water for at least 40 hours before testing if the concrete in the structure will be more than superficially wet. Otherwise the cores shall be dried in air at 60-80 degrees Fahrenheit and relative humidity less than 60% for 7 days before testing. These provisions have also been adopted in the current version of ASTM C42 (ASTM, 1992).

Similarly, a report on concrete core testing by the Concrete Society (1976) recommends that cores be capped and then soaked for at least two days before testing. If the concrete in the structure is wet, the equivalent actual cube strength is taken to be 1.5 times the crushing strength of a core with length to diameter ratio equal to one. If the concrete in the structure is dry, the equivalent actual cube strength is taken to be 1.65 times the core crushing strength.

Both ACI 318-89 and the Concrete Society (1976) therefore allow the evaluator of an existing structure to take account of the beneficial effect of dry in-service moisture conditions on the apparent concrete strength. This is a departure from conventional

1 A version of this chapter has been accepted for publication in the *American Concrete Institute Materials Journal*

design practice, since any effect of the *in situ* moisture condition on the concrete strength is neglected when proportioning the structure. Moreover, it is tacitly assumed in both publications that the *in situ* moisture conditions can be accurately reproduced by conditioning the specimen in air or in water for relatively short periods of time.

4.2 Research Significance

The main objectives of the present study are to determine the magnitude of the differences between the strengths of soaked and air dried concrete cores, and to identify reasons for these differences. It is shown that both of these moisture conditioning treatments, which are recommended in ACI 318 and ASTM C42, cause moisture gradients across the cross section of the test specimen which appreciably affect its strength. To avoid these artificial factors which affect the test outcome, the testing of cores in their as-drilled condition is investigated.

4.3 Literature Survey

The strength loss caused by soaking concrete compression test specimens in water has been attributed to the absorption of water by the gel pores. Soaking the specimens in benzine or paraffin, which are not absorbed by the cement gel, has no effect on strength (Neville, 1981). The same phenomenon is likely responsible for the strength gain which occurs after leaving specimens to dry in air. However, in either case, it is useful to distinguish between uniform and non-uniform moisture changes throughout the volume of the specimen, and their respective influences on the specimen strength.

4.3.1 Effect of Uniform Moisture Change on Strength

Water absorbed into the gel pores is believed to cause distension within the test specimen (e.g. Neville, 1981, Popovics, 1986, Wittmann, 1973). As the applied load increases, a hydrostatic excess pressure develops since the absorbed water is prevented from being squeezed out of the specimen. This pressure must be resisted by the solid matrix along the sides of the specimen, thus causing a transverse bursting effect. Distension is eliminated by drying the specimen.

Bessey and Dilnot (1949) state that, according to the distension hypothesis, the moisture effect for autoclaved specimens should be much less than that for specimens steam cured at atmospheric pressure. Their experimental data indicate that the relative strength of soaked autoclaved specimens is only slightly less than that of their other specimens. They therefore conclude that the strength loss due to soaking should not be entirely attributed to distension.

It has been suggested (Feldman and Sereda, 1970) that the presence of adsorbed water on the gel particles provides an environment where the Si-O bonds break more readily to form Si-OH HO-Si bonds. When the concentration of water molecules is sufficient to maintain the delivery of moisture to a spreading crack, no further decrease in strength will occur. This is disputed by Glucklich and Korin (1975), who question whether enough water can be continually present at the crack tip to maintain the necessary aggressive environment.

A third explanation (Wittmann, 1973, Glucklich and Korin, 1975) is based on Griffiths fracture theory. When water is absorbed into the gel, forcing the gel surfaces further apart, Van der Walls forces between gel particles are reduced. These adhesive forces are proportional to the specific surface energy; the critical stress for a given crack size is therefore reduced when the intermolecular distances are increased. Calculations following this approach have reproduced trends observed experimentally (Glucklich and Korin, 1975, Wittmann, 1973).

Data pertaining to the relationship between compressive strength and moisture content have been reported to substantiate the various hypotheses. Observations by Bessey and Dilnot (1949), who tested 3 inch (75 mm) cubes of an aerated cement/sand mortar, are shown in Figure 4.1. The strength is constant for high moisture contents and becomes increasingly larger for lower moisture contents, which agrees with Feldman and Sereda's stress corrosion cracking explanation. A similar form of the relationship between compressive strength and moisture content for very small paste and mortar specimens has been reported by Feldman and Sereda (1970), Wittmann (1973), Glucklich and Korin (1976), and Pihlajavaara (1974). In all cases, it may reasonably be assumed that the variation of the moisture content throughout the specimen volume is negligible, given the small sizes of specimen tested or the long period of moisture conditioning applied.

4.3.2 Effect of Non-uniform Moisture Changes on Strength

Recently, de Larrard and Bostvironnois (1991) have investigated the effect of air drying 150 mm diameter moulded cylinder specimens by measuring the variation of moisture content across the cross section using gammadensimetry. In Figure 4.2, their data from specimens made of an ordinary portland cement concrete with a strength of roughly 40 MPa are reproduced. The figure shows the moisture change from the condition at demoulding for a cross section through the axis of the cylinder. After a 27 day long drying period, only the material within about one inch of the surface has been affected, while, after 4 years, the moisture loss is relatively constant over the full cross section. The two 90 day old specimens have similar moisture losses at their centres, but the specimen demoulded at one day has lost considerably more moisture than the specimen moulded at 28 days. If the specimens were left in the air for only 7 days, according to the provisions of ASTM C42-90 (1992), the moisture gradients between the surface and the centre of the specimen would be even more severe than those shown in Figure 4.2.

Similarly, rough calculations based on concrete permeability constants reported by Neville (1981) indicate that, after 2 days soaking, water is unlikely to penetrate more than perhaps 3 mm from the surface of the core. For a 4 inch diameter core, which is the smallest permitted by ASTM C42-90 (1992), the annular region affected is only about 12% of the overall area of the cross section.

Thus the length of the moisture conditioning periods specified for cores in ASTM C42-90 (1992) or ACI 318-89 (1989) are too short to allow a uniform change of moisture to occur throughout the entire volume of the specimen. It is interesting to note that, while the original rationale for soaking cores was to achieve "uniform conditions as to moisture content" (ASTM, 1927), the effect of the recommended short soaking period is to cause a moisture gradient between the surface and the interior of the core which is hardly uniform.

The effect of a moisture gradient on the specimen has been considered by Popovics (1986). Soaking causes swelling at the surface of the core where the moisture content is increased. Air drying causes the surface layer to shrink. These volume changes are restrained by the material at the centre of the core, which does not gain or lose moisture. The restraint creates a set of residual strains and corresponding self-equilibriated residual stresses.

The unrestrained swelling of the top and bottom ends of a soaked core is shown in Figure 4.3 (a). Restraint of the swelling causes small compressive stresses in the soaked area and slight tensile stresses in the adjacent unsoaked region. These stresses are unlikely to significantly reduce the failure load, because this region is confined by the loading platen and so the failure is initiated elsewhere (e.g. Ottosen, 1984).

The swelling of the surface layer in a circumferential direction is shown in Figure 4.3 (b). Restraint of this swelling causes compression stresses in the soaked outer region and slight tension stresses throughout the unsoaked inner region. The confinement of the inner region by the outer region is slightly reduced, which causes a very slight reduction of the capacity of the inner region.

The swelling of the surface layer in a direction parallel to the axis of the core is shown in Figure 4.3 (c). Restraint of this swelling causes compression stresses in the soaked surface layer and small tension stresses in the unsoaked central region. The stresses in the skin are largest at the core surface and decrease rapidly.

If the core is air dried, the surface layer shrinks and the sense of the deformations and residual stresses are reversed from the soaked condition shown in Figure 4.3. Circumferential surface shrinkage may cause cracks or microcracks parallel to the axis of the core which would relieve the residual stresses. Surface shrinkage parallel to the axis of the core has been identified as the reason for direct tension tests of air dried cores giving lower strengths than splitting tests (Raphael, 1984), since microcracks may occur.

Therefore, the effect of conventional core moisture conditioning treatments on the moisture content of the interior of the core is negligible. The capacity of the inner region may be slightly changed by residual circumferential strains in the surface layer. Residual longitudinal strains in the surface layer cause residual longitudinal stresses in the inner region. These two effects counteract each other, whether the core has been dried in air or soaked in water.

Conversely, the moisture content and residual stresses near the surface of the core are greatly affected by brief soaking or air drying treatments. Soaking reduces the strength of the concrete and causes swelling which in turn causes residual compressive stresses in this region. Given the relatively small area affected by soaking, it is likely that these two factors in combination are responsible for the significant overall strength loss. These factors also combine to increase the surface layer strength, and overall strength, of cores dried in air.

De Larrard and Bostvironnois (1991) observe a significant loss of strength for very high strength silica fume concrete specimens left in air for extended time periods. In contrast to the behaviour described above, they attribute the strength loss to the moisture gradients caused by drying. In their silica fume concrete specimens, the moisture losses after 4 years air drying produce moisture gradients which are more severe than that shown in Figure 4.2 for the 90 day old specimens demoulded at one day. However the moisture losses extend much further into the specimen than might be expected for cores after a seven day drying period. The corresponding residual tension stresses at the surface of the silica fume concrete specimens dried for 4 years would therefore be smaller than those for cores dried for 7 days, and the internal residual compressive stresses would be comparatively large. The capacity of the interior region to resist applied compression loads therefore may be reduced to a much greater extent. It is also possible that, since de Larrard and Bostvironnois apparently do not consider the consequences of circumferential shrinkage in their analysis, their conclusion may be flawed.

4.4 Analysis of Bloem's (1965) Data

As part of the research for the paper "Concrete Strength Measurement: Cores versus Cylinders" (Bloem, 1965), changes to the average moisture content of cores were monitored from the initial *in situ* condition to the final test condition. The variation of core strength with core moisture content can be examined by analysing these data. Mimeographed tabulations showing results for individual specimens were retrieved from the files of the National Aggregate Association / National Ready Mixed Concrete Association and kindly made available for use in the present study by Meininger.

The original investigation considered the effects of excellent and average curing on the strength of normal and lightweight concrete slabs and columns. The present investigation considers only the data for two well cured, normal weight concrete slabs. One 4 inch (100 mm) thick slab, one 8 (200 mm) inch slab, and a number of 6 x 12 inch (150 x 300 mm) cylinders were cast from one batch of ready mixed concrete. The concrete had a standard cylinder strength of 5130 psi (35.4 MPa), an air content less than 2%, and a maximum aggregate size of about 1 inch (25 mm). The slabs were left in the forms, and a curing membrane was sprayed on the top surface promptly after pouring. Next they were covered with damp burlap and polyethylene for 7 days, and then they were left in the laboratory for seven weeks with only the top surface exposed. At the age of 91 days, 4 inch diameter cores were drilled vertically through the slab thickness using a water cooled diamond bit. Half of the cores from the 8 inch slab were each sawn into two 4 inch long test specimens. Four categories of cores resulted: 8" cores, 4" cores from the top of the 8" slab, 4" cores from the bottom of the 8" slab, and 4" cores from the 4" slab. Twelve core specimens were obtained for each category.

Each set of twelve cores was given four different treatments before testing. Three cores were tested two to three hours after drilling, three were tested after being soaked for 48 hours, three were tested after drying in laboratory air for 7 days, and three were tested after 28 days soaking. All cores were capped with a sulphur capping compound before testing.

The mimeographed tabulations reveal that the original investigators kept a running record of the moisture content of each core between the time of drilling and the time of testing. Each core was weighed immediately after drilling, immediately before and after capping, and immediately before and after testing. Core damage during testing was limited by stopping the test after the load had dropped off 5 to 10%. The sulphur caps were carefully removed to permit oven drying of each core at 225 to 230°F (107-110°C) to constant weight. Average moisture contents at various stages of treatment were calculated, expressed as percentages of the weight of the core after oven drying.

4.4.1 Effect of Treatment on Average Moisture Content

The average changes of the average moisture content due to different durations of soaking and air drying treatments are shown in Figure 4.4. Two curves are shown, since the initial moisture conditions were different for the 4 and 8 inch slabs. The thinner slab had a larger ratio of surface area to volume, and therefore dried out more during the 7 week long period of storage. The initial *in situ* moisture content values shown are the averages of two push-out cylinders from each slab. These cylinders were cast in metal sleeves in the slab and received identical curing as the surrounding concrete which was cored.

The average core moisture content immediately after coring is larger than the *in situ* moisture content due to absorption of cooling water by the concrete during the drilling process. The change of moisture content due to soaking or air drying treatments is initially very rapid. Almost two thirds of the total moisture gain after 28 days soaking occurred within the first 24 hours. Similarly, about a quarter of the total moisture loss after 7 days air drying took place in the first 3 hours.

4.4.2 Effect of Moisture Gain on Strength

The 48 core strength observations consist of three replicate values in each of 16 categories. Each group of three replicate strength observations was checked for outliers, using the procedures of ASTM E178-80 (ASTM, 1989). No outliers were identified, although the ASTM E178 procedures are not very powerful when applied to only three replicate observations.

The data were adjusted slightly to account for the age of the specimens at the time of testing, which ranged from 91 days to 119 days. An equivalent 91 day strength was calculated for cores which were more than 91 days old. The adjustment factor for 119 day old cores was assumed equal to the ratio of the average strength of three 6 by 12 inch cylinders tested at 91 days age to that of three companion cylinders tested at 119 days. These cylinders were cast separately from the slabs but were cured in a manner which simulated the curing of the test slabs. The adjustment factor for cores which were between 91 and 119 days old was assumed to vary linearly with time from 1.0 at 91 days to 0.959 at 119 days. The raw strength data, age adjustment factors, adjusted core strengths and moisture content information for all cores are shown in Appendix 4A.

Preliminary scatter plots suggested a second order polynomial fit for the relationship between the core strength and the average moisture content in the core at the time of testing. The relationship between the core strength and the change in core moisture content between drilling and testing is also well simulated by a second order polynomial. Preliminary analyses of variance indicated a significant difference between the strengths of the cores from the top and the bottom of the 8 inch slab. Also the cores from the 4 inch slab were appreciably stronger than cores from the 8 inch slab.

In light of these findings, a model was selected with the form

$$f_c^* = (\beta_0 + \beta_1 M + \beta_2 M^2)(1 + \beta_3 Z_{top})(1 + \beta_4 Z_{bot})(1 + \beta_5 Z_t) + \varepsilon \qquad (4.1).$$

In this model f^*_c is the strength of the core as modified to account for the different ages of the cores at testing, β_0 .. β_5 are the various parameters estimated by regression analysis, and ε represents the random error component of f^*_c which is assumed to be normally distributed with constant variance. Indicator variable Z_{top} equals 1 for the 4 inch cores from the top of the 8 inch slab or 0 otherwise, so the factor $(1 + \beta_3 Z_{top})$ reflects any difference in strength between these cores and the rest. Similarly, Z_{bot} equals 1 for cores from the bottom of the 8 inch slab or 0 otherwise and Z_t equal 1 for cores from the 4 inch slab or 0 otherwise.

Analyses were first carried out setting the variable M in Equation (4.1) to the average moisture content in the core at the time of testing, M_t, expressed as a percentage of the oven dried core weight. Nonlinear regression analysis using the SAS software package (SAS Institute, 1990) gave the fitted model

$$\hat{f}_c^* = (16200 - 4160M_t + 366M_t^2)(1 + 0.099Z_{top})$$
$$(1 + 0.164Z_{bot})(1 + 0.164Z_t) \qquad (4.2)$$

with all parameter estimates significantly different from zero at the 95% confidence level. The standard error of regression is 240 psi, and the coefficient of determination, R^2 is 0.87.

The analyses were repeated with the variable M in Equation (4.1) equal to the change in moisture content in the core between drilling and testing, M_t - M_{ad}, again expressed as a percentage of the oven dried core weight. The corresponding fitted model is

$$\hat{f}_c^* = [4660 - 607(M_t - M_{ad}) + 273(M_t - M_{ad})^2]\,(1 + 0.088Z_{top})\,(1 + 0.150Z_{bot})\,(1 + 0.230Z_t) \qquad (4.3)$$

with all parameter estimates significant. The standard error of regression in this case is 190 psi and R^2 is 0.92. Equation (4.3) fits the data reasonably well, as indicated by the partial regression plot shown in Figure 4.5.

Comparison of the standard errors of regression and the coefficients of determination suggests that the model based on the average moisture content, Equation (4.2), does not seem to reflect the strength change as well as the model based on the moisture gain (or loss), Equation (4.3). This preliminary conclusion is corroborated by the graphs of the residual errors versus the various moisture conditioning treatments shown in Figure 4.6. There is a clear pattern to the residuals from the model based on average moisture content, since the residuals for the air dried cores are typically greater than zero and the residuals for the as-drilled cores are typically less than zero. There is no corresponding pattern to the residuals from the model based on the moisture gain.

The effect of moisture treatment on core strength is therefore related to the moisture change of the specimen before testing instead of the average moisture content of the specimen at the time of testing. Since the region affected is a small proportion of the overall specimen volume, the moisture conditioning effect on the specimen strength represents essentially an artificial testing bias. The implicit assumption in ACI 318-89 and in the provisions recommended by the Concrete Society (1976), that the moisture conditioning effect observed on small core specimens can be extrapolated to concrete in large structures, seems unrealistic.

An attempt was made to quantify the dimensions of the inner region and the surface layer, to estimate the moisture differential, and to determine the relationship between the strength and the moisture content of the surface layer using regression analysis. The attempt failed because the available data were insufficient for the task. In particular, both the surface moisture content and surface layer strength were extremely sensitive to the unknown surface layer thickness.

4.5 Analysis of Cores from Alca's Beam MH1

To further investigate the effect of moisture condition on core strengths, a number of 50 mm diameter cores were obtained from beam MH1 tested by Alca at the University of Alberta (Alca, 1993). The beam was cast from a superplasticised ordinary portland cement concrete which had negligible entrained air. The cores were drilled through the thickness of the beam, perpendicular to the direction of concrete placement, when the beam was 93 days old. The cores were left in laboratory air for 9 days while they were sawn to nominal 100 mm lengths and their ends were ground. At that time, the side surfaces of some of the cores were painted with a waterproof epoxy coating. The ends of these cores were not coated. Then half the cores were soaked for 9 days in water, while the other half were left in laboratory air. Complete details may be found elsewhere concerning the mix design and casting procedures (Alca, 1993) and the core drilling, conditioning and testing procedures (Bartlett and MacGregor, 1994a [Chapter 2]). Four low outlying strength observations were identified using the criteria given in ASTM E178 (ASTM, 1980). The strengths of the remaining 39 cores are summarised in Table 4.1.

The differences of the average strengths of cores from North and South ends of the beam are not statistically significant, whether the cores were air dried ($p = 0.96$) or soaked ($p = 0.62$). (It is conventional (e. g. Larsen and Marx, 1986) to assume that a difference between two observed mean values is statistically "significant" if, given the variability of the mean values, the probability of observing the difference by chance is less than 5%). Direct comparisons of the strengths of cores from both ends can therefore be made. The difference between the average strengths of the uncoated soaked and uncoated air dried cores is 21.7 MPa. If the sides of the cores are coated, the strength difference between soaked and air dried cores is only 5.7 MPa, about one quarter of the value for uncoated cores. Since this small difference is statistically significant ($p = 0.048$), some of the observed strength change must be due to either a moisture gradient or to a reduction in capacity in the regions at the ends of the cores. The remainder, almost three quarters of the observed total strength change, is probably caused by moisture gradient effects along the sides of the core.

The average strengths of the uncoated air dried cores are slightly greater than the average strengths of the coated air dried cores, although the difference is not statistically significant ($p = 0.30$). Given that both sets of cores were left in air for 9 days before the coating was applied, one would perhaps expect only a slight difference anyway. The observed difference may indicate that the coating causes a slight loss of strength. It also may indicate that the uncoated cores continued to gain strength because their sides were exposed to laboratory air for the extra 9 days. The latter explanation is consistent with the continued loss of moisture of specimens exposed to laboratory air for prolonged periods as shown in Figure 4.2.

4.6 Strength Correction Factors for Core Moisture Condition

Regression analyses were carried out to determine the relationships between the strength of air dried, soaked, and as-drilled cores.

4.6.1 Air Dried Core Strengths versus Soaked Core Strengths

The relationship between air dried and soaked core strengths was examined using data reported by Meininger et al. (1977), Bloem (1965), and Bartlett and MacGregor (1994a) [Chapter 2]. The moisture conditioning treatments used in all cases, either storage in air at 50 to 70°F (10 to 21°C) at 40 to 60% relative humidity for 7 days, or storage in lime saturated water for at least 40 hours, are in accordance with ASTM C42-90 (ASTM, 1990). Altogether, these data represent tests of 617 four inch (100 mm) diameter cores obtained from 10 separate elements which were cast using ordinary portland cement concretes with standard cylinder strengths ranging from 2200 to 13400 psi (15 to 92 MPa).

The numbers of specimens, average strengths and corresponding standard deviations are shown in Table 4.2 for soaked and air dried specimens with common length to diameter ratios. The strength of Bloem's (1965) cores have been modified to account for minor variations in their age at the time of testing as described previously. All strengths for cores with nominal length to diameter ratios less than 2 have been transformed to equivalent strengths of standard cores with l/d ratios of 2 using the correction factors proposed by Bartlett and MacGregor (1994b) [Chapter 3].

The model selected has the form

$$\bar{f}_{c,dry} = \beta_6 + \beta_7\left(\frac{l}{d}\right) + \beta_8\bar{f}_{c,wet} + \varepsilon \qquad (4.4)$$

where $\bar{f}_{c,dry}$ and $\bar{f}_{c,wet}$ are the average strengths of the air dried and soaked cores respectively, β_6 .. β_8 are unknown parameters determined by regression analysis, and ε is the error. A weighted regression analysis was carried out since, for these data, the variance of the dependent variable $\bar{f}_{c,dry}$ may not be constant for all observations and also the independent variable $\bar{f}_{c,wet}$ includes a significant measurement error. The weights for each observation were determined following the procedure given by Bartlett and MacGregor (1994b) [Chapter 3].

Analysis using the linear regression procedures of the SAS software package (SAS Institute, 1990) indicated that only the parameter estimate of β_8 is significant at the 5% level. The fitted model is

$$\hat{\bar{f}}_{c,dry} = 1.144\bar{f}_{c,wet} \tag{4.5}$$

with R^2 equal to 0.998 and the standard error of regression equal to 2.31. The high R^2 value indicates that the wet and dry average core strengths are strongly correlated. However, the standard error of regression is also rather high, indicating that the fit of Equation (4.5) to the data is actually not very good. This is also suggested by Figure 4.7, which shows considerable scatter of the observed average values about the predicted value of 1.144.

4.6.2 As-drilled Core Strengths versus Soaked Core Strengths

The relationship between as-drilled core strengths and soaked core strengths was examined using data from 115 core tests reported by Bloem (1965, 1968) and Bartlett and MacGregor (1994a) [Chapter 2]. The as-drilled strengths are for cores subjected to slightly different test conditions. In Bloem's 1965 investigation, cores were obtained using a water cooled drill and tested after about 3 hours drying in laboratory air, whereas, in his 1968 investigation, cores were obtained using an air cooled bit and tested the same day. The cores investigated by Bartlett and MacGregor were left in air for about 15 minutes until the drilling water had evaporated from the sides, and then were stored in plastic bags until the test. Thus it is reasonable to assume that the moisture gradient in the as-drilled cores from these three investigations is negligible. In all cases, the soaked cores were stored in lime saturated water for at least 40 hours.

The numbers of specimens, average strengths and corresponding standard deviations are shown in Table 4.3 for the as-drilled and soaked specimens. The values shown have again been adjusted slightly to account for differing ages of the cores at the time of testing, and have been converted to equivalent strengths of standard cores with *l*/d of 2. Three sets of cores were obtained by Bloem from each element in his 1968 investigation, at ages of 28 days, 91 days and 364 days. Only the 28 day and 91 day data is shown in Table 4.3, since the strengths of the 364 day old soaked cores are in all cases considerably less than the corresponding 91 day strengths.

Analysis was carried out on a model similar to Equation (4.4) using the linear regression procedures of the SAS software package (SAS Institute, 1990). The best fit was obtained with a constant term in the regression

$$\hat{f}_{c,ad} = 275 + 1.044\bar{f}_{c,wet} \tag{4.6}$$

where $\hat{f}_{c,ad}$ is the predicted average strength of the as-drilled cores. Both parameters in Equation (4.6) are significant, R^2 is equal to 0.994 and the standard error of regression is equal to 1.32. If the constant term is neglected, the fit is

$$\hat{f}_{c,ad} = 1.090\bar{f}_{c,wet} \tag{4.7}$$

with R^2 equal to 0.998 and the standard error of regression equal to 1.50. Comparing the R^2 values alone erroneously indicates that Equation (4.7) is a better fit than Equation (4.6), because R^2 is defined differently for models with and without a constant. In fact, although Equation (4.6) is a slightly better fit than Equation (4.7), both are reasonably good. The scatter of the observed averages around the value predicted from Equation (4.7) is shown in Figure 4.7.

4.6.3 Air Dried Core Strengths versus As-drilled Core Strengths

The available data are insufficient to determine precisely the relationship between the strength of cores tested as-drilled and those left to dry in air for 7 days. The ratio of the average air dried core strength to the average as-drilled core strength for the cores from elements 7 through 10 has an average of 1.09 and a standard deviation of 0.05. This ratio can also be estimated by dividing Equation (4.5) by Equation (4.7), which gives 1.05. Thus, on average, air dried cores are probably 5 to 9% stronger than as-drilled cores.

4.6.4 Accuracy of Strength Conversion Factors

In Figure 4.8, the residual errors obtained from Equations (4.5) and (4.7) are plotted against the corresponding predicted value for the data shown in Tables 4.2 and 4.3. Lines where the residual error equals +/-5% of the predicted value are also shown. Three of the 14 average as-drilled core strengths and 9 of the 32 average air dried core strengths fall outside these limits.

Typically, residual errors for the strengths of cores drilled from a common source are clustered together. For example, the four large positive residuals for $\bar{f}_{c,dry} \cong 5500$ psi are all from elements 7 and 8, which were cored by Bloem. Similarly, the three large negative residuals for $\bar{f}_{c,dry} \cong 6000$ psi are all from element 5 and the three large negative residuals for $\bar{f}_{c,dry} \cong 8000$ psi are all from element 6. This suggests that the general relationships given by Equations (4.5) and (4.7) may not be accurate for cores from a specific element. If accurate conversion factors are required it may be necessary to obtain cores from the element of interest and, after carrying out the moisture conditioning treatments, establish the specific factors directly.

4.7 Discussion

In his landmark paper, Bloem (1965) concludes that "for the conditions employed in this research, which simulated interior structural concrete, it appeared that cores dried for 7 days to eliminate water absorbed during drilling provided the most accurate measure of strength in place". This conclusion is based on the observation that the average strength of three replicate cores drilled and tested dry was consistently slightly greater than the average strength of three replicate cores drilled wet and tested after 7 days air drying for four slabs cast of lightweight concrete.

To determine the true *in situ* strength, it is clearly desirable to eliminate any moisture gradient effect caused by absorption of the drilling water. However it seems probable that a 7 day drying period is excessively long for this purpose. Figure 4.4 shows that the cores from Bloem's 8 inch normal weight concrete slab achieved their *in* *situ* moisture content after about 3 hours drying and the cores from the 4 inch slab achieved their *in situ* moisture content after about 24 hours drying. The observations of de Larrard and Bostvironnois shown in Figure 4.2 indicate that the extent of the moisture gradient, and therefore the strength of the specimen, is affected by the length of the drying periods. The analysis presented in the previous section, although based on a small quantity of data, indicate that the effect of 7 days air drying is to cause a moisture gradient which artificially increases the strength of the specimen by about 5% above the true *in situ* strength.

In the same paper, Bloem also observes that "the standard treatment - soaking for 48 hours - is probably more indicative of the in-place strength of wet concrete." The literature clearly indicates that concrete strength is reduced if the moisture content is increased uniformly throughout the specimen volume. It is also true that soaking the specimen causes a moisture gradient which reduces the test strength obtained. However the first factor reflects a true influence of the *in situ* concrete strength whereas the second factor is a more artificial effect which biases the test outcome. If it is necessary to evaluate the strength of wet concrete, it may be more appropriate to obtain cores by drilling with a water cooled bit, let the cores sit a few minutes in air to allow excess water to evaporate and then test the cores in that condition.

4.8 Summary and Conclusions

In accordance with the provisions of ASTM C42-90 and ACI 318-89, it is current practice to either dry concrete core specimens in air for 7 days or soak them in lime saturated water for at least 40 hours before they are tested. In this paper, the effect of moisture condition on the strengths of mature cores obtained from well cured elements is investigated by reviewing available literature and performing regression analyses of data from tests of 727 core specimens. The elements were all cast using ordinary portland cement concrete with standard strengths between 15 and 92 MPa.

From the literature review and the data analyses, the following observations and conclusions appear warranted:

1. The compressive strength of a concrete specimen is decreased if its moisture content is uniformly increased throughout its volume. Conversely, the strength is increased if the moisture content is uniformly decreased.

2. The compressive strength is also considerably affected if a moisture gradient is created between the exterior and the interior of the specimen. Soaking the specimen causes swelling at the surface. The interior of the specimen does not undergo any change in moisture content and so restrains the swelling at the surface. This restraint causes a set of self equilibriated residual stresses which in turn cause a reduction in the compressive strength of the specimen. Drying the specimen causes shrinkage at the surface and increases the compressive strength.

3. Analysis of Bloem's (1965) data indicates that the strength of cores is affected by the change in moisture content between drilling and testing instead of the total moisture content at the time of testing. The observed change in strength should therefore be attributed mostly to the moisture gradient effect.

4. The lengths of both the drying and soaking periods specified in ASTM C42-90 and ACI 318-89 are too short to cause a uniform change of moisture content throughout the volume of the core. The effect of these treatments is to create a moisture gradient which artificially biases the test result.

5. The most accurate estimate of the *in situ* concrete strength is obtained from a specimen which has no gradient of moisture content through its volume. Such a specimen may be obtained using an air cooled drill, or by letting the excess water evaporate from cores which have been obtained using a water cooled drill. The seven days air drying treatment permitted by ASTM C42-90 and ACI 318-89 seems excessively long for letting the excess cooling water evaporate.

6. The strength of cores left to dry in air for 7 days is on average 14% larger than the strength of cores soaked at least 40 hours before testing. The strength of cores with a negligible moisture gradient is on average 9% larger than the strength of soaked cores.

These general average values are constant for concretes with strengths ranging from 15 to 92 MPa. However, the strength ratios for any particular mix may differ appreciably from these general average values.

Table 4.1: Strength of Cores from Alca's Beam MH1

Treatment	Beam End	n	$\bar{f}_c$ (MPa)	s (MPa)
Air Dried	South	7[a]	96.7	8.2
	North	8	96.5	6.6
	combined	15	96.5	7.2
Coated and Air Dried	North	4[a]	92.7	3.6
Coated and Soaked	South	5[a]	86.9	4.2
Soaked	South	8	74.2	6.3
	North	7[a]	75.6	4.8
	combined	15	74.9	5.5

Note: [a] - one low strength outlier removed from data.

Table 4.2: Strength Data for Air Dried and Soaked Cores

Reference	Element No.	Air dried core data				Soaked core data			
		l/d	n	$\bar{f}_{c,dry}$ (psi)	s (psi)	l/d	n	$\bar{f}_{c,wet}$ (psi)	s (psi)
Meininger et al. (1977)	1	1.00	13[a]	4699	170	1.01	14	4167	230
	1	1.26	12	4648	105	1.26	11[a]	4049	158
	1	1.51	12	4687	129	1.51	11[a]	4090	141
	1	1.75	9[c]	4594	114	1.75	12	4088	187
	1	2.00	10	4726	217	1.99	9[a]	4059	179
	2	1.00	14	2264	155	1.00	12	1934	138
	2	1.25	10	2331	130	1.25	10	2031	146
	2	1.50	14	2300	144	1.50	14	2082	116
	2	1.74	15	2360	138	1.75	14	2036	67
	2	1.99	9	2335	106	2.00	8	2042	144
	3	1.00	25	3102	163	1.01	19[a]	2653	124
	3	1.26	9	3116	151	1.26	9[a]	2585	112
	3	1.51	11	3137	160	1.52	7[a]	2671	37
	3	1.76	7[b]	3135	31	1.75	8	2645	73
	3	2.00	10	3085	169	2.00	8[a]	2686	85
	4	1.04	6	3527	155	1.02	16	3025	170
	4	1.52	6	3277	120	1.52	16	3048	87
	4	1.99	6	3495	110	1.99	16	3128	147
	5	1.04	6	5630	234	1.03	16	5190	178
	5	1.52	6	5510	265	1.49	16	5016	265
	5	2.01	5[a]	5481	145	1.98	16	4945	234
	6	1.04	6	7540	269	1.03	16	7218	296
	6	1.50	6	7476	238	1.50	16	6979	222
	6	1.97	6	7417	212	1.95	14[b]	6829	247
Bloem (1965)	7	2.11	3	5381	21	2.13	6	4375	197
	7	1.07	3	5299	130	1.06	6	4370	166
	7	1.06	3	5674	140	1.09	6	4620	191
	8	1.09	3	5701	122	1.09	6	4790	151
Bartlett & MacGregor (1994a) [Chapter 2]	9	0.99	5	13122	525	0.99	5	11691	436
	9	2.03	5	13670	457	2.03	5	11906	434
	10	0.98	5	13173	374	0.99	5	11150	192
	10	2.00	5	13033	509	1.99	5	11454	182

Note: [a] - one, [b] - two, or [c] - three low strength outliers removed from data

Table 4.3: Strength Data for As-drilled and Soaked Cores

Reference	Element No.	As-drilled core data l/d	n	$\bar{f}_{c,ad}$ (psi)	s (psi)	Soaked core data l/d	n	$\bar{f}_{c,wet}$ (psi)	s (psi)
Bloem (1965)	7	2.17	3	4803	85	2.13	6	4375	197
	7	1.08	3	4771	215	1.06	6	4370	166
	7	1.11	3	4864	339	1.09	6	4620	191
	8	1.11	3	5536	58	1.09	6	4790	151
Bartlett & MacGregor (1994a) [Chapter 2]	9	2.01	6	12908	701	2.03	5	11906	434
	9	1.00	6	13155	504	0.99	5	11691	436
	10	1.99	5[a]	11862	739	1.99	5	11454	182
	10	0.99	6	11838	324	0.99	5	11150	192
Bloem (1968)	11	1.50	3	4193	239[b]	1.50	3	3543	252[c]
	11	1.50	3	4339	247[b]	1.50	3	4022	286[c]
	12	1.50	3	3094	176[b]	1.50	3	2958	210[c]
	12	1.50	3	3667	209[b]	1.50	3	3280	233[c]
	13	1.50	3	3317	189[b]	1.50	3	2978	211[c]
	13	1.50	3	3444	196[b]	1.50	3	3095	220[c]

Note: [a] - one low strength outlier removed from data
[b] - calculated using avg. coefficient of variation of 5.7% (Bloem, 1968)
[c] - calculated using avg. coefficient of variation of 7.1% (Bloem, 1968)

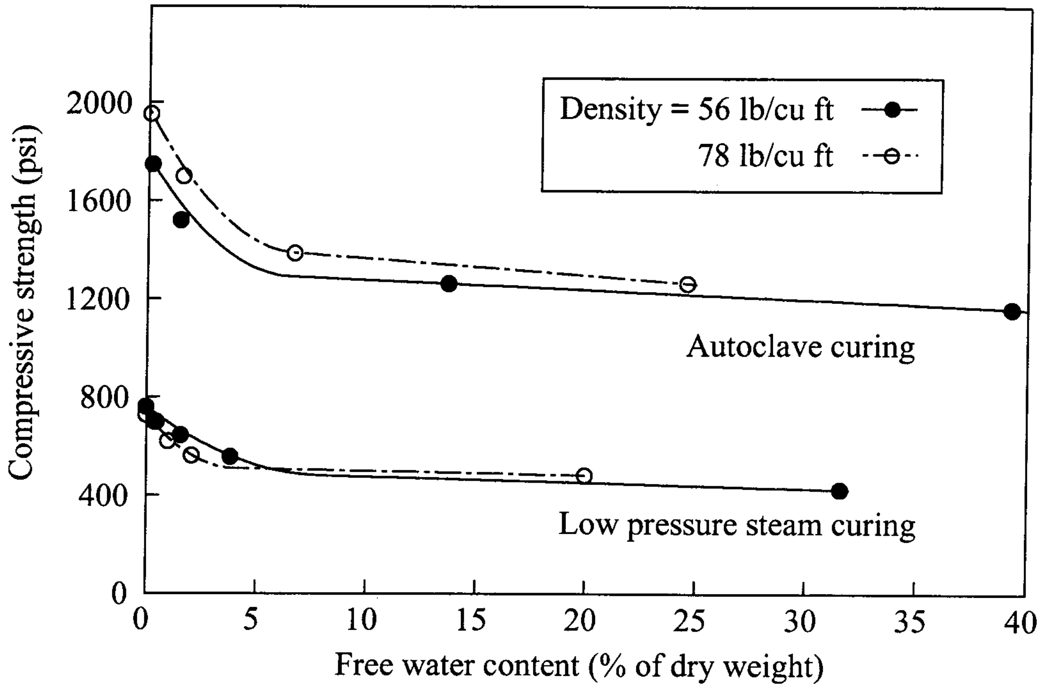

Figure 4.1: Strength versus moisture content for aerated mortars (from Bessey and Dilnot, 1949)

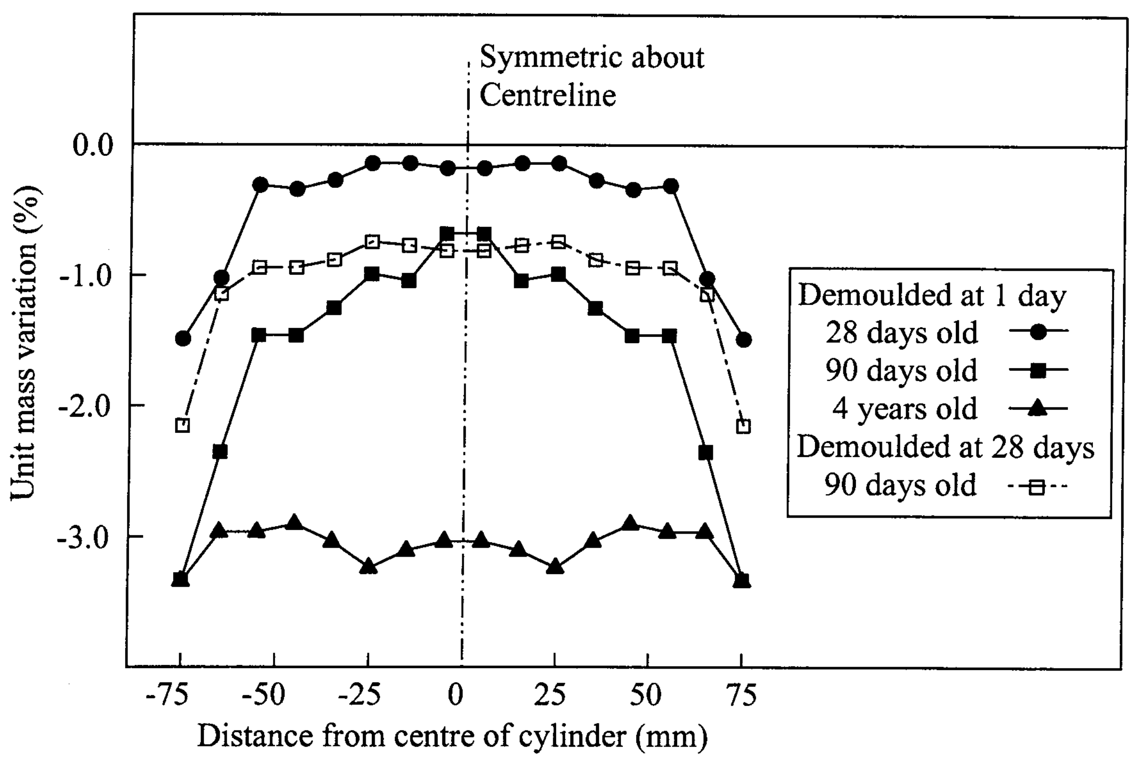

Figure 4.2: Unit mass variation across width of air dried cylinders (from de Larrard and Bostvironnois, 1991)

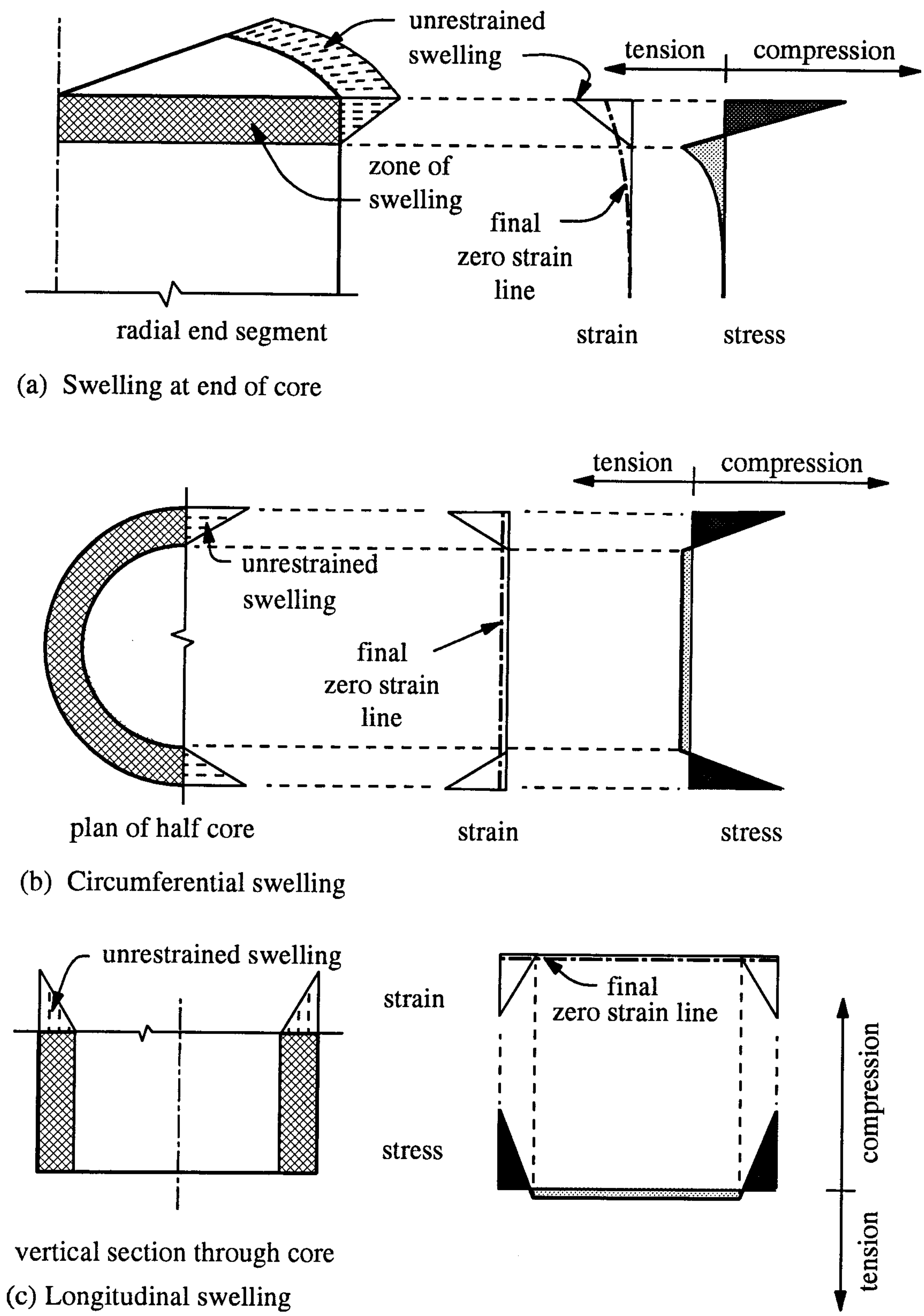

Figure 4.3: Swelling due to soaking and associated residual stresses

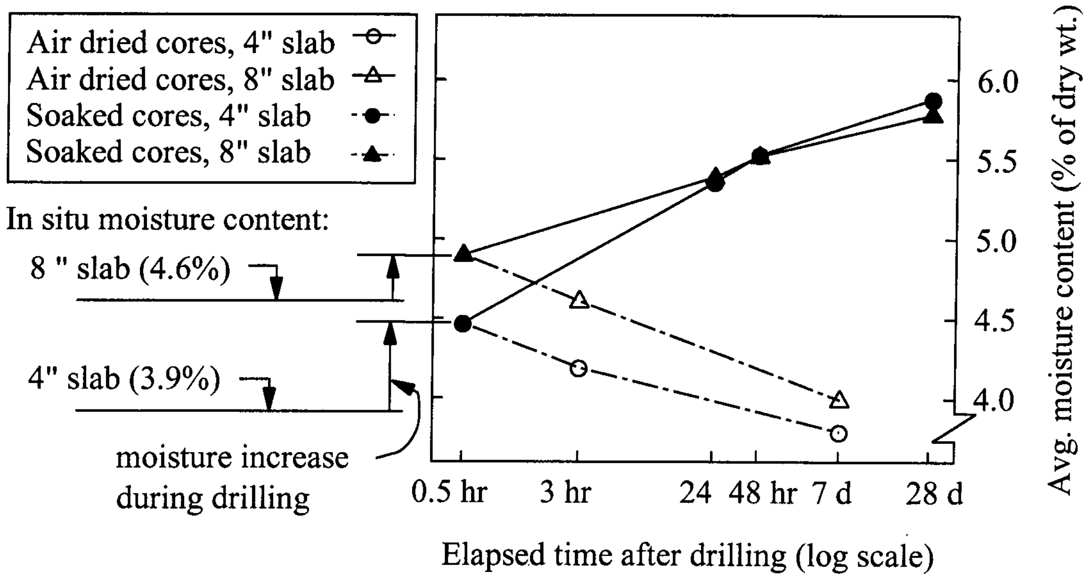

Figure 4.4: Average moisture content change for core moisture treatments

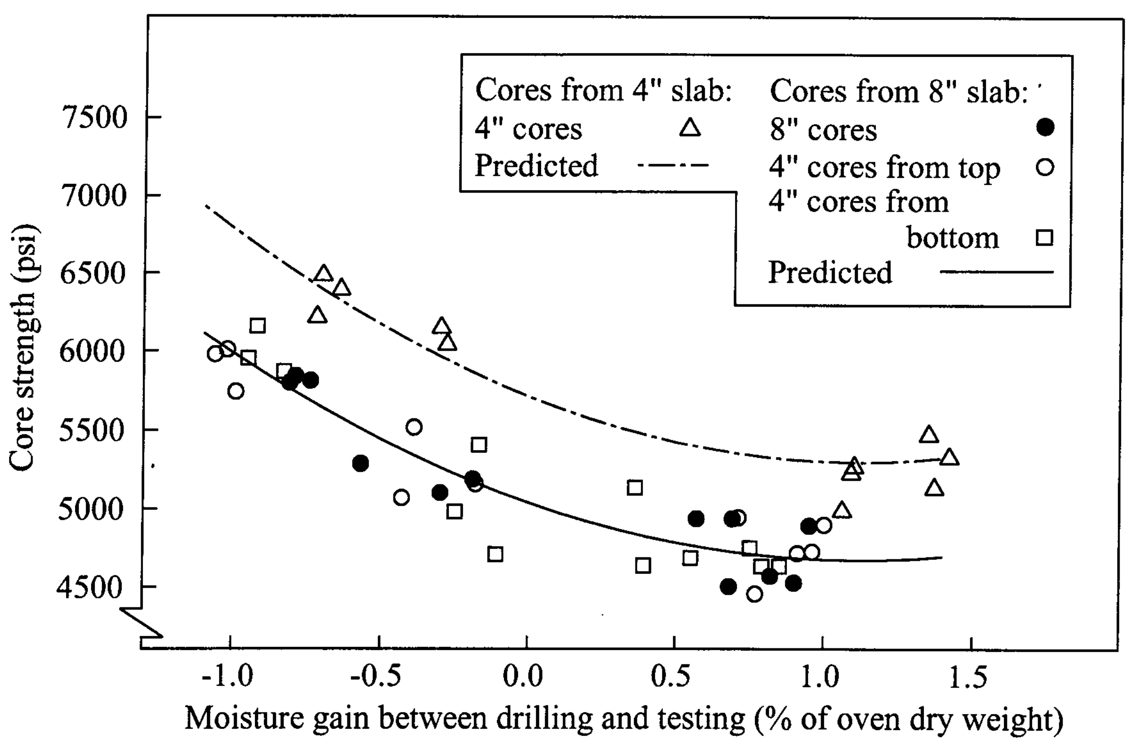

Figure 4.5: Partial regression plot of core strength versus moisture gain

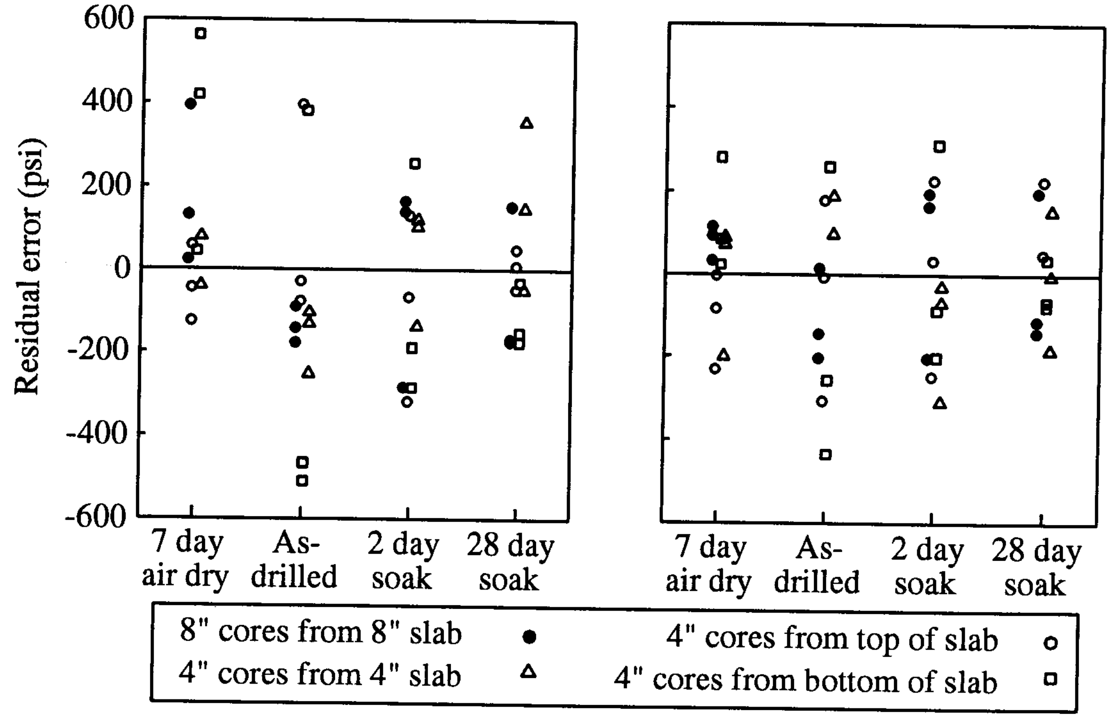

Figure 4.6: Core strength residuals versus moisture conditioning treatments

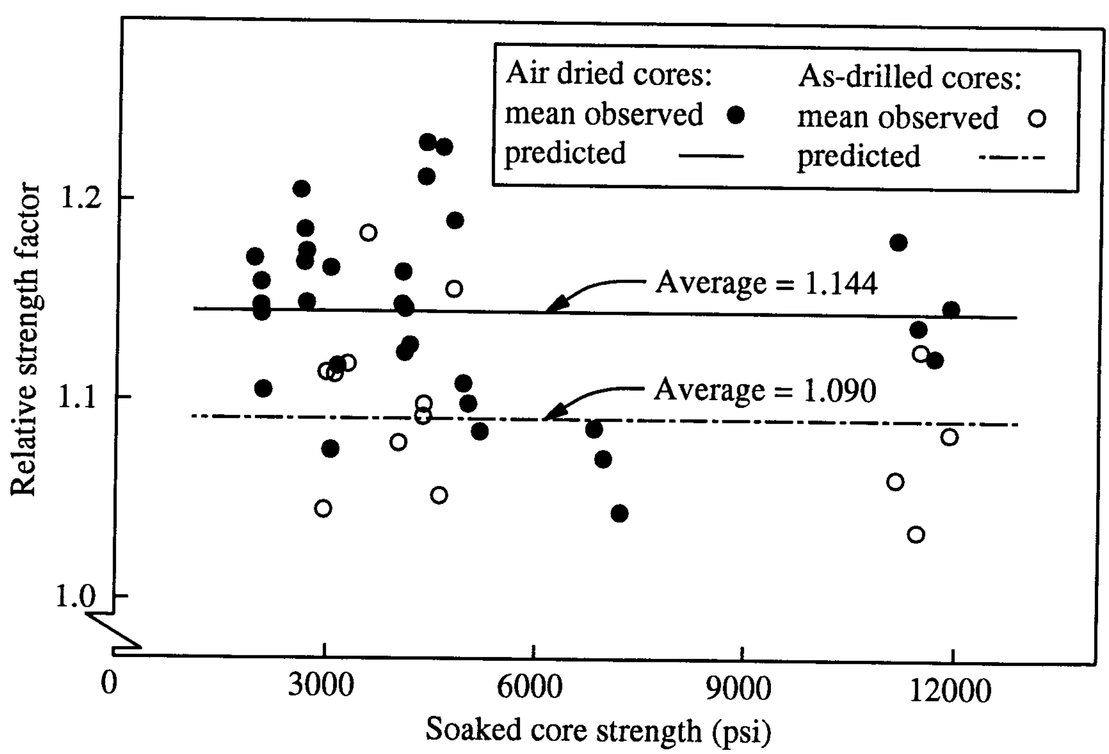

Figure 4.7: Strength of air dried and as-drilled cores relative to soaked cores

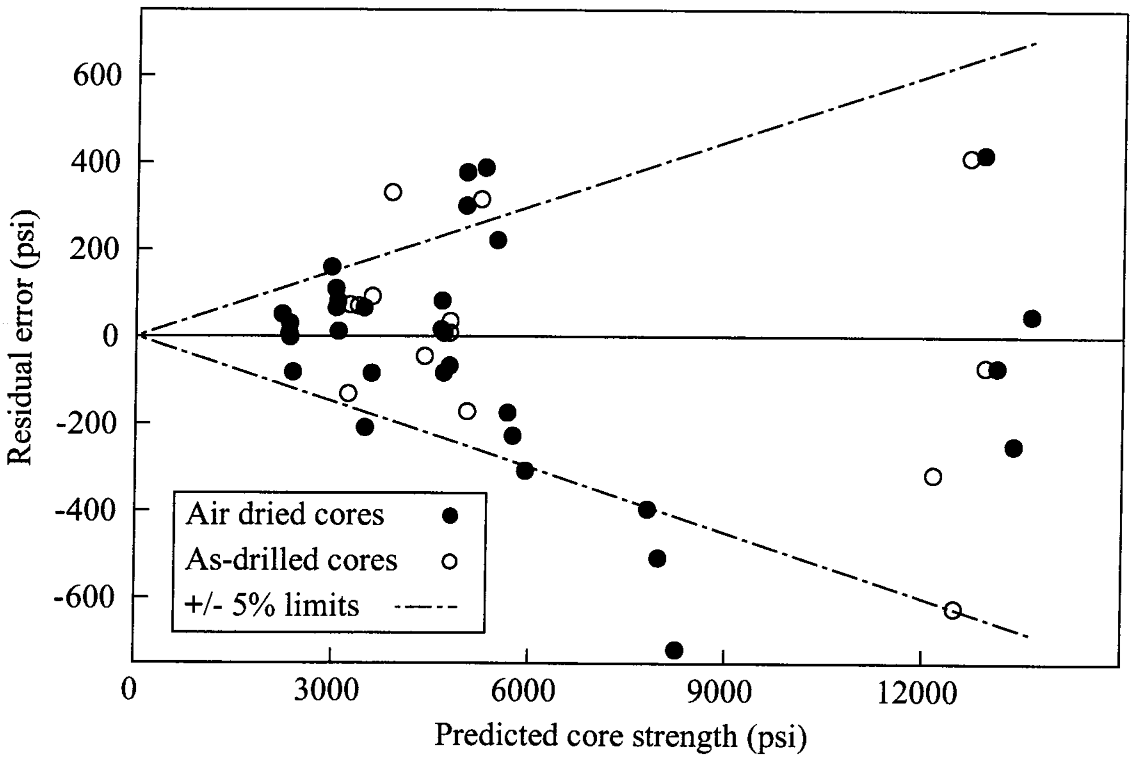

Figure 4.8: Residuals versus predicted core strengths

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Chapter 5: Effect of Core Diameter on Concrete Core Strengths[1]

5.1 Introduction

Small diameter cores are often used to assess concrete strength because they are easier to drill, handle and store than larger cores. Possible conflicts with reinforcing steel are minimised, and a smaller hole is left for subsequent repair. It is usually feasible to obtain a relatively large number of small cores, and so sample a large area of the structure. Finally, small diameter cores may be the only alternative if standard core specimens are desired; for example, if specimens with length to diameter ratios of two are to be obtained from thin elements. For these reasons, the Concrete Society published an addendum to its report on core testing (Concrete Society, 1976) in 1987 which permits the use of 2 and 3 inch (50 and 75 mm) diameter cores.

The most common criticism of small diameter cores is that they are unreliable. There are conflicting reports on the effect of core diameter on core strength; some investigators found no effect (Meininger, 1968, Lewis, 1976), others found the strength of small cores to be much less than that of larger cores (Campbell and Tobin, 1967). Small cores are also more susceptible to damage during drilling, handling and storing. These shortcomings are likely the reasons why the provisions of ASTM C42-90 "Standard Test Method for Obtaining and Testing Drilled Cores and Sawed Beams of Concrete" (ASTM, 1992) limit the minimum diameter of concrete cores to 4 inches (100 mm).

In this paper, the reasons for a diameter effect suggested by others are summarised. Core strength data collected by others to investigate the diameter effect are analysed. Then the various explanations of the diameter effect are reviewed to see whether they are corroborated by the core strength data.

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1 A version of this chapter has been accepted for publication in the *American Concrete Institute Materials Journal*

5.2 Research significance

While several theories have been developed relating compressive strength to the size of the concrete test specimen, the applicability of these theories to drilled core specimens has not been investigated. The research summarised in this paper indicates that the compressive strength of small cores is less than that of large cores, probably due to damage to the cut surface of the core specimen. The research also indicates that the variability of small core specimens strengths is sensitive to the spatial variability of the *in situ* concrete strength.

5.3 Literature review

Four general explanations of the diameter effect found in current literature are summarised in this section.

5.3.1 Systematic Bias Caused by Testing Procedures

Hansen et al. (1962) suggest that bending deformations of the loading platen may be more severe for large specimens. These deformations cause the load to be concentrated near the center of the specimen instead of being uniformly distributed over the entire cross section. This causes a splitting effect which reduces the strength of the specimen considerably (Newman and Lachance, 1965). Recently, apparent strength loss due to platen flexibility was noted during tests of high strength concrete specimens (Lessard and Aitcin, 1992). However the current criteria for bearing block dimensions given in ASTM C39-86 "Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens" (ASTM, 1990) preclude the possibility of large bending deformations. These criteria were included in a tentative revision to ASTM C39 in 1956, and were adopted in 1959. An observed diameter effect cannot be attributed to platen flexibility if the provisions of ASTM C39-86 are met.

The axial stiffness of cylindrical specimens increases in proportion to the specimen diameter for specimens with constant length to diameter ratios. The stiffness of the testing machine relative to that of the specimen is therefore larger for smaller specimens. This may cause a greater ultimate strength for small specimens, since a relatively stiff machine will tend to release energy at a rate that the specimen can tolerate (Neville, 1981).

5.3.2 Theoretical Size Effect Laws

If core specimens with identical height to diameter ratios are considered, the diameter effect is identical to a general volume or size effect. In 1939, Weibull obtained a significant size effect by applying probabilistic principles to the behaviour of ideal brittle materials (Madsen et al., 1986). According to this "weakest link" theory, specimens with large volumes are more likely to contain a defect and so fail at lower applied stress.

Weibull's original derivation assumed that the minimum strength of the material, as would be obtained by testing an infinitely large specimen, is zero (Madsen et al., 1986). The theory has since been extended to ideal brittle materials with minimum strengths greater than zero. Bolotin (1969) obtained the following expressions for the expected value and variance of the strength of specimens of different volumes:

$$E[f] = E[f_r] \cdot \left\{ C_1 + C_2 \left(\frac{V_r}{V} \right)^{\frac{1}{\alpha}} \right\} \tag{5.1}$$

$$Var[f] = \left\{ C_2 \left(\frac{V_r}{V} \right)^{\frac{1}{\alpha}} \phi(\alpha) \right\}^2 \tag{5.2}$$

where f is the strength of a specimen with volume V, f_r is the strength of the reference specimen with volume V_r, and E[•] and Var[•] refer to the expected value and variance of a quantity. Positive constants C_1, C_2 and α are selected to best fit the data, and

$$\phi(\alpha) = \sqrt{\frac{\Gamma\left(1+\frac{2}{\alpha}\right)}{\Gamma^2\left(1+\frac{1}{\alpha}\right)} - 1} \tag{5.3}$$

where $\Gamma(\cdot)$ is the gamma function.

Tucker (1945) rejected the weakest link theory because his data from tests of moulded cylinders did not show an appreciable size effect. Instead he proposed a "summation strength" theory, with the strength of the whole specimen equal to the sum of the strengths of the individual parts instead of the least strength of the individual parts. This departure requires the material to behave in an ideal ductile manner. His expressions for the expected value and variance of the compressive strength are

$$E[f] \;=\; E[f_r] \qquad (5.4)$$

$$Var[f] \;=\; \left(\frac{A_r}{A}\right) \; Var[f_r] \qquad (5.5)$$

where f is the strength of a specimen with area A and f_r is the strength of the reference specimen with area A_r.

Both the weakest link and summation strength theories neglect spatial correlation of the material strength. If the strengths of unit areas of the cross section are perfectly correlated, then the variance is constant for specimens with different sizes.

5.3.3 Effect of Damage to Core Surface

Cores are susceptible to surface damage such as microcracking caused by drilling. Drilling also cuts through aggregate particles, which may pop out during testing since they are not wholly contained within the concrete matrix (McIntyre and Scanlon, 1990). If the affected zone has a constant thickness irrespective of the core diameter, the damaged area of the cross section is proportionally much larger for a small core than for a large core.

The failure load of the core can be expressed as the sum of the loads resisted by the damaged and undamaged regions:

$$P \;=\; f_I A_I \;+\; k f_I A_O \qquad (5.6)$$

where P is the failure load, f_I is the strength of the undamaged inner region with area A_I and k f_I is the strength of the damaged outer region with area A_O. If the damaged region is assumed to have uniform thickness t which less than 10% of the diameter d, then a linear approximation of the expression for A_O is

$$A_O \;\approx\; 3.75\left(\frac{t}{d}\right)A \qquad (5.7)$$

where A is the total area of the cross section. Hence the stress at failure, f_c, is

$$f_c = f_I\left\{1 - 3.75\left(\frac{t}{d}\right)(1-k)\right\} \tag{5.8}$$

Assuming f_I, t, and k to be independent random variables, the expected value of f_c is

$$E[f_c] = E[f_I]\left\{1 - \left(\frac{3.75}{d}\right)E[t]\,(1 - E[k])\right\} \tag{5.9}$$

and the variance of f_c is

$$\begin{aligned}Var[f_c] = {} & Var[f_I]\left\{1 + \left(\frac{3.75}{d}\right)^2 E[t]^2 (1 + E[k]^2)\right\} \\ & + \left(\frac{3.75}{d}\right)^2 E[f_I]^2 \{(1 + E[k]^2)Var[t] + E[t]^2\, Var[k]\}\end{aligned} \tag{5.10}$$

The strength of the undamaged inner region is not necessarily constant but may vary as described by a theoretical size effect law. Results from Equations (5.1) to (5.5) could therefore be substituted for the values of $E[f_I]$ and $Var[f_I]$ in Equations (5.9) and (5.10).

5.3.4 Variation of *In Situ* Concrete Strength

Small cores are more likely to reflect any inhomogeneity of the *in situ* concrete strength (Lewis, 1976). In practice, *in situ* strength variations are unavoidable. For example, the concrete at the top of a slab is usually weaker than that on the bottom surface (Concrete Society, 1976) due to bleeding and other effects. The apparent strength of small cores as compared to large cores may be influenced by such strength gradients, depending on the sampling procedures adopted.

5.3.5 Literature Summary

Testing procedures, including the use of a single testing machine to test cores of different diameters, may cause small specimens to have greater apparent strength than large specimens.

Small cores should have greater average strengths than large cores with the same length to diameter ratio according to conventional Weibull theory. According to Tucker's summation strength theory, the average strength is independent of core size. Both theories predict that the strength of small cores is more variable than that of large cores. Both theories may be flawed because they neglect spatial correlation of concrete strength.

Damage to the cut surface of the core may affect a large proportion of the area of small cores. Such damage would cause small cores to be weaker and more variable than large cores.

In some cases, strength differences attributed to core size may instead be caused by a non-uniform distribution of strength within the element being cored.

5.4 Analysis of Meininger's (1968) Data

To investigate the variation of core strength with core diameter, data collected by Meininger (1968) were reanalysed. A wall, a slab, and a number of standard test cylinders were cast from one batch of concrete with a 28 day standard cylinder strength of 3110 psi (21.4 MPa). The wall and slab were cured under wet burlap for 90 days before coring. Cores with diameters of 1.9, 4.0 and 5.9 inches (48, 102, and 150 mm) were drilled using a water cooled diamond bit. Most cores were drilled right through, but six 4 inch cores were drilled approximately 10 inches into the wall and broken out. The cores were sawn to give length to diameter ratios of approximately two. A 16 inch long core therefore provided four 1.9 inch diameter specimens, two 4.0 inch diameter specimens, or one 5.9 inch specimen from either the near side or the far side of the element as drilled. All specimens were soaked in water before capping and testing, half for 40 hours, the remainder for 28 days.

The data for the wall and the slab were analysed separately using the NLIN procedures of the SAS regression analysis software package (The SAS Institute, 1990). The models had the form

$$f_c = \beta_0 (1+\beta_1 x+\beta_2 x^2)(1+\beta_3 y)(1+\beta_4 z)(1+\beta_5 d)(1+\beta_6 Z_a)(1+\beta_7 Z_{bo})+\varepsilon \qquad (5.11).$$

In this equation, f_c is the core breaking stress and $\beta_0..\beta_7$ are the various parameters estimated using regression analysis. Regressors x, y and z are the coordinates of the centre of the core, and d is the core diameter. Indicator variable Z_a equals 0 for cores tested after 40 hours soaking at about 91 days age, or 1 for cores tested after 28 days soaking at about 121 days age. Indicator variable Z_{bo} equals 1 if the core was broken out or 0 otherwise and so is neglected in the slab model. The random component of the breaking stress, assumed normally distributed with zero mean and constant variance, is represented by ε.

Parameter estimates for the slab and wall data are shown in Table 5.1. All parameter estimates differ significantly from zero at the 95% confidence level.

The regression analyses were repeated after removing the factor for core diameter, $(1 + \beta_5 d)$, from the model. The resulting parameter estimates, also shown in Table 5.1, show little sensitivity to the removal of this variable. To isolate the diameter effect, the parameter estimates from the analyses with the diameter excluded were used to adjust the core strengths to account for the effects of the other regressor variables.

The average values, standard deviations, and coefficients of variation for the adjusted core strengths are shown in Table 5.2. The average strength of cores from the wall decreases as the core diameter increases. The trend for the slab is different; the average strength of cores increases slightly as the core diameter increases. The standard deviation of the strengths of the 4 inch slab cores is significantly less than that of both the 1.9 and 5.9 inch cores ($p < 0.01$). No other differences of standard deviation are statistically significant (in all cases, $p > 0.20$). In particular, while there is a trend for the strengths of the 1.9 inch cores to be more variable than the strengths of the 5.9 inch cores, the differences of standard deviation are not significant.

5.4.1 Conformance with Weakest Link and Damage Explanations

The adjusted core strength data shown in Table 5.2 were checked for conformance with the weakest link and damage explanations. Clearly the slab data supports the damage explanation and does not conform to the weakest link theory since the average core strengths increase ($p = .16$) with increasing core diameter. However the average values and standard deviations of the wall core strengths are in general agreement with the weakest link theory, since they both increase with decreasing core diameters.

Constants C_1, C_2, and α in Equations (5.1) and (5.2) were determined for the average wall core strength and standard deviation values shown in Table 5.2. Ideally, the best fit should be obtained by some type of regression analysis, since the six available equations representing the expected value and variance of each size of core are inconsistent. However the numerical complexity of Equation (5.3) complicates any regression analysis considerably and so the constants were determined using different subsets of three of the available equations. Several different subsets were considered.

One subset comprises the sample mean and variance of specimens with the reference volume and the sample variance of specimens with a different volume. The solution may not be precise, since it uses two sample variances which may not be accurately estimated from small numbers of tests, but is obtained without iteration using the procedure described in Appendix 5A. Taking a 5.9 by 11.8 inch core as the reference volume, and fitting to the variance of the 1.9 inch diameter cores yields:

$$E[f_c] = E[f_r] \cdot \left\{0.715 + 0.285 \left(\frac{V_r}{V}\right)^{\frac{1}{8}}\right\} \qquad (5.1a).$$

This equation predicts the mean strength of the 1.9 inch diameter wall cores to be 9.3 percent greater than the observed value.

Another subset contains the sample mean strength values for the specimens of each diameter. This should give a more accurate solution, since sample averages are more precise than sample variances. The solution, obtained by iteration, is:

$$E[f_c] = E[f_r] \cdot \left\{0.9955 + 0.0045 \left(\frac{V_r}{V}\right)^{\frac{1}{1.33}}\right\} \qquad (5.1b).$$

This equation indicates that the strength of the 5.9 inch diameter reference cores is the minimum strength, since the product 0.9955 $E[f_r]$ approximately equals $E[f_r]$. The associated predicted variances are only 0.65%, 3.15% and 45% of the observed values for the 5.9, 4.0 and 1.9 inch diameter cores respectively. The value of α obtained, 1.33, is lower than might be expected (Bolotin, 1969, Madsen et al., 1986).

Hence even though the sample statistics for the wall core strengths shown in Table 5.2 seem to conform to the weakest link theory, the data do not fit the theory well. It may be said either that the change in average strength is too small given the variances or that the variances are too large given the change in average strength. These exceptions are consistent with the effects which would be caused by damage to the cut surface of the core indicated in Equations (5.9) and (5.10).

5.4.2 Effect of *In Situ* Strength Variation

If the core strengths are not adjusted for the *in situ* variation of concrete strength, the coefficients of variation are as shown in Table 5.2 in the column headed V_M. The coefficients of variation due to the *in situ* spatial variation of concrete strength, V_s, are also shown. These values have been determined using the equation

$$V_s^2 = V_M^2 - V_{adj}^2 \tag{5.12}$$

where V_{adj} is the coefficient of variation of the strengths as adjusted for core location. The large total variability of the strength of the 1.9 inch cores from the slab is caused by the large *in situ* strength gradient between the top and bottom surfaces. For the 5.9 inch slab cores, V_s is negligible since the distance between the centroids of cores taken from the top and the bottom of the slab is small. Also, the strength correction factors in the x and y directions cause the variation of the strengths of the 5.9 inch diameter cores, V_{adj}, to increase slightly.

The variation of strength along the length of the wall is the major component of the overall strength variation of this element. Similarly, the through thickness variation dominates the overall strength variation of the slab. Partial regression plots for these variables are shown in Figures 5.1 and 5.2 respectively. It is interesting to note that the wall and the slab were each cast from a single batch of concrete.

The values of V_s suggest that, for the wall cores, the coefficient of variation of strength due to core location is relatively constant for cores of all sizes. This is logically consistent with the small variation of strength through the wall thickness shown and the uniform scattering of core drilling locations over the height and length of the wall.

5.5 Analysis of Diameter Effect using Models based on Core Damage

In the previous section, it was shown that Meininger's data do not conform to the weakest link theory, probably because of the effects of damage to the cut surface of the core. In this section, average strength and variability data for cores from various sources are analysed using models which are based on the core damage.

5.5.1 Effect of Core Diameter on Mean Core Strength

The effect of core diameter on mean core strength was studied by regression analysis of published and unpublished average core strength data from nine sources. The primary goal of each investigation was to obtain and test cores. The details of each investigation is summarised in Table 5.3. The full data set is presented in Appendix 5E. The data from all sources represent tests of approximately 860 core specimens made from 36 different concrete mixes with standard cylinder strengths between 1440 and 13400 psi (9.9 and 92 MPa). The cores were drilled either parallel to or perpendicular to the direction of concrete placement. All have length to diameter ratios equal to 2. The specimens were soaked or left to dry in laboratory air and subsequently were tested when the concrete was between one and three months old.

The diameter effect was isolated by dividing the average strength of cores of each diameter by the average strength of companion cores with diameters of 4 inches:

$$R_d = \bar{f}_d / \bar{f}_4 \qquad (5.13).$$

This normalizing procedure was intended to eliminate the effect of other factors which affect the core strength, including the drilling direction, length to diameter ratio, treatment before testing, age at testing, and, obviously, the quality of the parent concrete. Although necessary, this normalizing procedure complicates the analysis since the response variable R_d is not normally distributed and has a relatively large variance.

A preliminary analysis of the ratios of average strengths of two inch and four inch diameter cores was carried out. Two outliers in the data set were identified using "Least Median of Square" techniques (Rousseeuw and Leroy, 1987) and removed. Regression analyses were performed to determine whether the ratio of average strengths was affected by the concrete strength, the maximum aggregate size, the direction of drilling, and the age of the concrete at the time of testing. None of these factors were significant at the 15% level.

A histogram of the ratios of average strengths of six inch and four inch cores is shown in Figure 5.3. The population represented by these data is clearly bimodal, with one peak at 0.98 and the other between 1.22 and 1.30. The high ratios of average strengths are all from one published source (Campbell and Tobin, 1967) and one unpublished study (Confidential Communication, 1990). Campbell and Tobin drilled cores when the concrete was two to three weeks old and returned the specimens to their respective holes in each slab until they were removed and soaked prior to testing; it is possible, though improbable, that these unconventional storage conditions may be a factor. In the other investigation, only two replicate 3.75 inch cores and two replicate 5.75 inch cores were obtained for each mix. Although the precision of the associated average strength values is adversely affected, this does not seem to be sufficient to explain the observed large ratios of average strength values. Although no reasons for these apparently anomalous data were found, the data clearly differ from that obtained by others ($p < 0.0001$) and therefore were neglected in the present investigation.

The scatter plot shown in Figure 5.4 suggests that the normalized average strength increases as the core diameter increases. Accordingly, the form of the model adopted was developed from Equation (5.9):

$$E[f_c] = E[f_i]\left\{1 - \frac{\beta}{d}\right\} \tag{5.9a}$$

where β, the unknown parameter to be determined by regression analysis, equals 3.75 E[t] (1-E[k]).

As described in Appendix 5B, a model was derived from Equation (5.9a) which has the normalized average core strength, R_d, as the regressor variable. In this derivation it is assumed that the mean strength of the undamaged inner region of the core is independent of the core diameter. A nonlinear weighted regression analysis using the NLIN procedure of the SAS software package (the SAS Institute, 1990) gives parameter estimates which imply that:

$$E[f_c] = E[f_I]\left\{1 - \frac{0.2265}{d}\right\} \qquad (5.9b).$$

The analysis was repeated to see whether β could be expressed as a linear combination of the other regressor variables. Only the treatment of the core between drilling and testing was found to be significant:

$$E[f_c] = E[f_I]\left\{1 - 0.308\frac{(1 - Z_{mc})}{d}\right\} \qquad (5.9c)$$

where $Z_{mc} = 0$ for cores dried for 7 days in air before testing or 1 for cores soaked for at least 40 hours in water before testing. This implies that the diameter effect is only significant for air dried cores. However this finding may not be accurate since the data set does not include many soaked cores. Moreover, in the only case where both soaked and air dried cores from a single mix were tested, the trend was for the soaked cores to have a greater diameter effect than the air dried cores (Bartlett and MacGregor, 1994a) [Chapter 2]. The result suggested by Equation (5.9c) therefore seems spurious.

The strength ratios predicted using Equation (5.9b) are superimposed on the scatter plot of normalized average core strength versus core diameter in Figure 5.4. The strength of a 2 inch diameter core with l/d = 2 is predicted to be 94% of the strength of a 4 inch diameter core or 92% of the strength of a 6 inch diameter core.

If it is assumed that the strength of the damaged outer layer of the core is negligible, it can be shown from Equations (5.9) and (5.9b) that the thickness of the outer layer is roughly 0.06 inches or 1.5 millimetres.

The strength of a concrete core is related in Equation (5.9b) to the strength of the undamaged interior region of the core. Perhaps the best estimate of $E[f_I]$ would be obtained from a "field cured" concrete cylinder which is cast from the same mix and subjected to the same temperature and moisture conditions as the element from which the core was obtained. For this case, Equation (5.9b) predicts the average strengths of four and six inch diameter cores with l/d = 2 to be 94% and 96% of the strengths of companion field cured cylinders respectively. These values are reasonably consistent with those reported by others. McIntyre and Scanlon (1990) found core strengths to average 92% of corresponding field cylinder strengths; Murphy (1977) reported this ratio to be 94%.

5.5.2 Effect of Core Diameter on the Precision of the Core Strength

The effect of core diameter on the precision of core strength measurements was assessed by regression analysis of published data from seven sources. These data represent tests of 780 cores from 23 concrete mixes with standard cylinder strengths varying from 1450 to 13400 psi (10 to 92 MPa). The complete data set is presented in Appendix 5F. The scope of the various investigations are summarised in Table 5.4. To limit the uncertainty caused by small sample sizes, each sample statistic in the data set is calculated from at least 5 replicate core strength tests.

The precision of each strength measurement was assumed to be best represented by the sample coefficient of variation, even though this ratio of two random variables is slightly less precise than the sample standard deviation. The raw sample standard deviation values require modification to compensate for factors other than the core diameter such as the moisture condition of the core before testing. In contrast, the sample coefficient of variation needs no adjustment for these factors since both the sample mean and standard deviation values are affected to the same degree.

The variation of the measured strengths is inflated by the variation of the *in situ* concrete strength, as demonstrated by the values shown in Table 5.2. The effects of *in situ* strength variation in Meininger's (1968) data and in Bartlett and MacGregor's (1994a) [Chapter 2] data were identified and the data were adjusted to remove the influence of these factors. Similarly the data reported by Meininger et al. (1977) as analysed by Bartlett and MacGregor (1994b) [Chapter 3] were also modified. Data from other sources could not be adjusted since the magnitude of the *in situ* strength variation could not be quantified.

An indicator variable was assigned to the data from each source to indicate whether the effect of spatial variation of *in situ* strengths was likely to be present or not. The small beams cored by Munday and Dhir (1984) are likely of uniform strength. In contrast, the coring locations in the slabs investigated by Lewis (1976) and Mather and Tynes (1961) were defined using a Latin Square layout; any *in situ* strength variation across the plan dimensions of the slabs would therefore affect the observed core strengths. The slab and wall elements cored by Ramirez and Barcena (1979) were of sufficient size that the effect of spatial variation of the *in situ* strengths is likely also present in their data.

The slab core entries in Table 5.2 also demonstrate that the apparent variability of core strengths is inflated by variation of the *in situ* strength through the thickness of the element being cored. The published experimental procedures indicate that only the data reported by Ramirez and Barcena might be affected by this factor. Preliminary analyses indicated that the variability of the strengths of all 2 inch (50 mm) diameter cores reported by Ramirez and Barcena was especially large. Since this is consistent with strength variation through the thickness of the elements which were cored, these data were removed from the analyses.

The model for regression analysis was developed from the equations for expected value and variance considering core damage, Equations (5.9) and (5.10). As described in Appendix 5D, the form of the model evolved to:

$$V_{fcS}^2 = \{V_{cyl}^2 + V_s^2 + V_{Za}^2\}\,K + \frac{3.75^2\,\mathrm{Var}[t]}{(d-0.2265)^2} + \varepsilon \qquad (5.14)$$

where V_{fcS} is the observed coefficient of variation and V_{cyl}, V_s, and V_{Za} represent the components of the coefficient of variation associated with the standard cylinder strength, the *in situ* spatial strength variation, and the age of the concrete at the time of testing. The variable K is defined as

$$K = \left\{1 + \left(\frac{0.2265}{d}\right)^2\right\} / \left\{1 - \frac{0.2265}{d}\right\}^2 \qquad (5.11).$$

Regression analysis using the NLIN procedure of the SAS software package (the SAS Institute, 1990) yielded:

$$\hat{V}_{fcS}^2 = \left\{\left(\frac{77}{\bar{f}_{cyl}}\right)^2 + (0.048\,Z_s)^2 + (0.027\,Z_a)^2\right\} K + \frac{0.00692}{(d-0.2265)^2} \qquad (5.14a)$$

where $\hat{V}^2_{fcS}$ is the predicted value of the square of the coefficient of variation. All parameter estimates are significant at the 5% level. In this equation, $\bar{f}_{cyl}$ is the standard 28 day old cylinder strength, Z_s is an indicator variable equal to 1 if the observed V_{fcS} is likely to be inflated by *in situ* strength variation or 0 otherwise, and Z_a is a variable ranging from 0 for cores tested when the concrete was 4 to 5 weeks old to 1 for cores tested when the concrete was 11 to 15 weeks old.

The observed and predicted coefficients of variation are plotted in Figure 5.5. Five of the six neglected data points are shown; the sixth, with V_{fcS} equals 32%, lies well off the graph. Clearly the observed coefficients of variation for these neglected points are considerably larger than the predicted values.

The component of the overall coefficient of variation associated with the spatial variability of the *in situ* strength is estimated to be 4.8%. This value is relatively close to the values shown in Table 5.2 for the cores from the wall, which range from 4.6 to 5.8%.

For cores with diameters larger than 6 inches, the coefficient of variation of the core strength depends mostly on the variability of the undamaged interior region and therefore on V_{cyl}, V_s, and V_{Za}. For cores with 2 inch diameters, the coefficient of variation of the core strength depends mostly on the variance of the thickness of the damaged region. Since this variance seems to be constant, its effect on the overall core strength uncertainty is much more pronounced for cores with small diameters.

5.6 Precision of Strength Correction Factors for Core Diameter Effect

Equation (5.9b) was used to predict the average strength of 4 inch diameter cores from the average strength of corresponding 2 and 6 inch diameter cores for the core data summarised in Table 5.3. In Figure 5.6, the differences between the observed average strength and the predicted value are shown. With the exception of the neglected data, the strengths predicted using the average strengths of 6 inch diameter cores generally fall within the lines indicating the limit where the residual error exceeds 5% of the predicted value. Many of the strengths predicted using the 2 inch core data do not; although Equation (5.9b) simulates the overall average relationship reasonably accurately, it has quite poor predictive power for specific cases. If correction factors are required to convert 2 inch core strengths to 4 inch core strengths (or vice versa), it may therefore be appropriate to obtain a few cores of each size from the structure in question and calculate the correction factor directly.

5.7 Influence of Length to Diameter Ratio on Core Diameter Effect

Data reported by Yip and Tam (1988) were analysed to investigate whether the diameter effect is constant for cores with different length to diameter ratios. The data are average values for 2 and 4 inch diameter cores trimmed to *l*/d ratios of 1, 1.5 and 2 from 12 different concrete mixes. A total of 576 cores were tested.

Histograms of the observed ratios of the average 2 inch diameter core strength to average 4 inch diameter core strength, R_2, are shown in Figure 5.7. A simple linear relationship

$$R_2 = 1.091 - 0.102\left(\frac{l}{d}\right) \tag{5.16}$$

fits the average values quite well, suggesting that the diameter effect may be negligible for short cores and more significant for cores with *l*/d = 2.

Equation (5.16) implies that the effect of length to diameter ratio on core strength is more significant for two inch diameter cores than for four inch diameter cores. Bartlett and MacGregor (1994b) [Chapter 3] report that the strength of a four inch diameter core with *l*/d is about 12% larger than that of a standard core with *l*/d = 2. For two inch diameter cores, Equation (5.16) predicts this difference to be about 20%, which is in general accordance with results reported by others (e.g. Bungey, 1979).

5.8 Discussion

It has been suggested (Hansen et al., 1962, Lessard and Aitcin, 1992, Neville, 1981) that a systematic bias in the testing procedure can cause large concrete cylinders to fail at lower loads than small cylinders. In Figure 5.4, the normalized average strength data indicate a trend in the opposite sense. Hence, even if systematic bias associated with the test procedure is a factor, its effect is overwhelmed by a counteracting effect.

Theoretical size effect laws have been obtained from the application of probabilistic principles to the behaviour of brittle materials (Bolotin, 1969, Madsen et al., 1986) which infer that the strength of small cores should be larger and more variable than that of large cores. Again, the trend in Figure 5.4 indicates that this effect, if present, is overpowered by a counteracting effect. This observation is reinforced by the analysis of Meininger's (1968) wall core data; although those data superficially conform to the weakest link theory, the observed size effect is too small given the observed variance of the strengths.

Concrete cores differ from molded specimens because their surfaces may be damaged during removal from the hardened concrete. The extent of the damaged region may depend on the characteristics of the aggregate, although the data are insufficient to investigate this adequately. A simple model which accounts for this damage indicates that small cores should be weaker and more variable than large cores. The analyses in Section 5.5 suggest that this model can explain the observed relationships between the core diameter and both the average strength and the variance. The damage effect on the mean core strength is opposite to those which would be inferred by the weakest link theory or by systematic bias caused by testing procedures.

The variation of the *in situ* strength of the element being cored is more likely to influence the strength and variability of small concrete cores than large cores (Lewis, 1976). This is confirmed by the data in Table 5.2 and by the significance of the regressor variable for *in situ* strength variation in Equation (5.14a). The analysis of Meininger's (1968) slab data indicates that the variation of strength through the thickness of a concrete slab may have a particularly strong effect on the strength and variability of small cores.

5.9 Summary and Conclusions

The effect of core diameter on the compressive strength of concrete core specimens has been investigated by reviewing literature and by analysing data from previous studies. The experimental data represent tests of 1080 core specimens obtained from elements of moderate size with standard cylinder strengths varying from 10 to 92 MPa.

From the literature review and the data analysis, the following observations and conclusions appear warranted:

1. The effect of damage to the surface layer in reducing the strength of small cores counteracts and overwhelms any effect that might be caused by systematic bias due to testing procedures or that might be explained by the weakest link theory.

2. The predicted variation of core compressive strength with core diameter is given in Equation (5.9b). The predicted average strength of a 2 inch (50 mm) diameter core is 94% of the predicted average strength of a 4 inch (100 mm) diameter core and 92% of the predicted average strength of a 6 inch (150 mm) diameter core.

3. Ratios of the average strength of 2 inch and 4 inch diameter cores have considerable scatter. If correction factors are required for converting 2 inch core strengths to 4 inch core strengths (or vice versa), they should be determined directly using a few cores of each size obtained from the structure in question.

4. The core length to diameter ratio effect is greater for 2 inch diameter cores than for 4 inch diameter cores. If a factor of 0.88 is used for 4 inch diameter cores to reduce the strength of a core with $l/d = 1$ to the equivalent strength of a core with $l/d = 2$, then a factor of about 0.80 should be used to account for this effect for 2 inch diameter cores.

5. The variability of measured strengths of small diameter cores is particularly sensitive to being inflated by the variability of the *in situ* strength across the dimensions of the element being cored. Slabs are particularly susceptible to through thickness strength variation; the strength in one slab studied increased by 17% in the 16 inch (400 mm) distance from top to bottom.

6. For cores with large diameters, the coefficient of variation of the core strength depends mostly on the variability of the undamaged interior region. For cores with small diameters, the coefficient of variation of the core strength depends mostly on the variance of the thickness of the damaged region.

7. If the effects of through-the-thickness strength variation are avoided, the systematic *in situ* strength variability of slabs and walls of moderate size cast from one batch of concrete may be considered as a component of the overall coefficient of variation with a magnitude of 5%.

Table 5.1: Parameter Estimates for Slab and Wall Cored by Meininger (1968)

Regressor	Parameter	Unit	Parameter Estimates, Slab		Parameter Estimates, Wall	
			$(1+\beta_5 d)$ in	$(1+\beta_5 d)$ out	$(1+\beta_5 d)$ in	$(1+\beta_5 d)$ out
	β_0	psi	2060	2100	2420	2290
x	β_1	1/inch	-0.00499	-0.00516	-0.00771	-0.00775
x^2	β_2	1/in.^{2}	0.000141	0.000145	0.000125	0.000105
y	β_3	1/inch	0.00287	0.00286	0.00580	0.00589
z	β_4	1/inch	0.0149	0.0148	-0.00434	-0.00452
d	β_5	1/inch	0.00513	--	-0.0140	--
Z_a	β_6	--	0.0657	0.0667	0.0609	0.0631
Z_{bo}	β_7	--	--	--	-0.0628	-0.0745
$\sqrt{MS_E}$		psi	107.	108.	131.	141.

Table 5.2: Strengths and Spatial Variability of Meininger's Cores

Element	d (inch)	n	$\bar{f}_d$ (psi)	s (psi)	V_{adj} (%)	V_M (%)	V_s (%)
Slab	1.9	28	2090	101	4.85	9.14	7.74
	4.0	12	2100	37	1.75	6.78	6.55
	5.9	12	2130	89	4.15	3.69	0
Wall	1.9	33	2350	144	6.13	7.67	4.61
	4.0	18	2240	103	4.57	6.86	5.12
	5.9	12	2230	94	4.24	7.14	5.75

Note: All strength values have been adjusted to account for age at testing, strength loss due to breaking out. The values of $\bar{f}_d$, s, and V_{adj} include additional adjustments which account for the spatial variation of the *in situ* concrete strength.

Table 5.3: Summary of Investigations Providing Mean Core Strength Data

Reference	n_m	$\bar{f}_{cyl}$ (psi)	d. (in)	n	age (days)	orien-tation[f]
Bartlett & MacGregor (1994a)	4	6380 to 13380	2 4	43 35	46 to 52	//
Bhargava (1969)	4	3440 to 6500	4 6	8 8	28	//
Campbell & Tobin (1967)	2	4510 to 6300	4 6	16[d] 16[d]	28, 84	//
Lewis (1976)	4	3260 to 6500	4 6	48 24	30	//
Meininger (1968)	1	3110	1.9 4 5.9	61 30 24	91	//, T
Munday & Dhir (1984)	1	5000[e]	2[b] 4	6 6	28	T
Ramirez & Barcena (1979)	3	1440 to 2760	2[b] 4 6	96[e] 96[e] 96[e]	90	//, T
Yip & Tam (1988)	12	2330[a] to 7280[a]	2 4	120 72	28, 84	//, T
Confidential[c] Communication (1990)	5	3020 to 4320	2.25 3.75 5.75	37[d] 10[d] 10[d]	28	//
Total	36			862		

Notes: [a] assumed 80% of published cube strength
[b] ASTM C39-86 (ASTM, 1990) maximum aggregate size limit exceeded
[c] data provided on the condition that its source not be identified
[d] data not included in final analysis
[e] estimated
[f] key: // - core axis parallel to casting direction, or
T - core axis perpendicular to casting direction.

Table 5.4: Summary of Investigations Providing Data about the Precision of Core Strengths

Reference	n_m	$\bar{f}_{cyl}$ (psi)	d (in)	n	age (days)	orientation[d]
Bartlett & MacGregor (1994a)	4	6380 to 13380	2 4	43 35	46 to 52	//
Lewis (1976)	4	3260 to 6500	4 6	48 24	30	//
Mather & Tynes (1961)	4	1870 to 4490	6[b] 8[b] 10[b]	32 32 32	28	//
Meininger (1968)	1	3110	1.9 4 5.9	61 30 24	91	//, T
Meininger et al. (1977)	6	2270 to 7100	4	117	35 to 100	T
Munday & Dhir (1984)	1	5000[c]	2[b] 3 4	6 6 6	28	T
Ramirez & Barcena (1979)	3	1440 to 2760	2[ab] 4 6	96[ac] 96[c] 96[c]	90	//, T
Total	23			784		

Notes: [a] data not included in final analysis
[b] ASTM C39-86 (ASTM, 1990) maximum aggregate size limit exceeded
[c] estimated
[d] key: // - core axis parallel to casting direction, or
T - core axis perpendicular to casting direction.

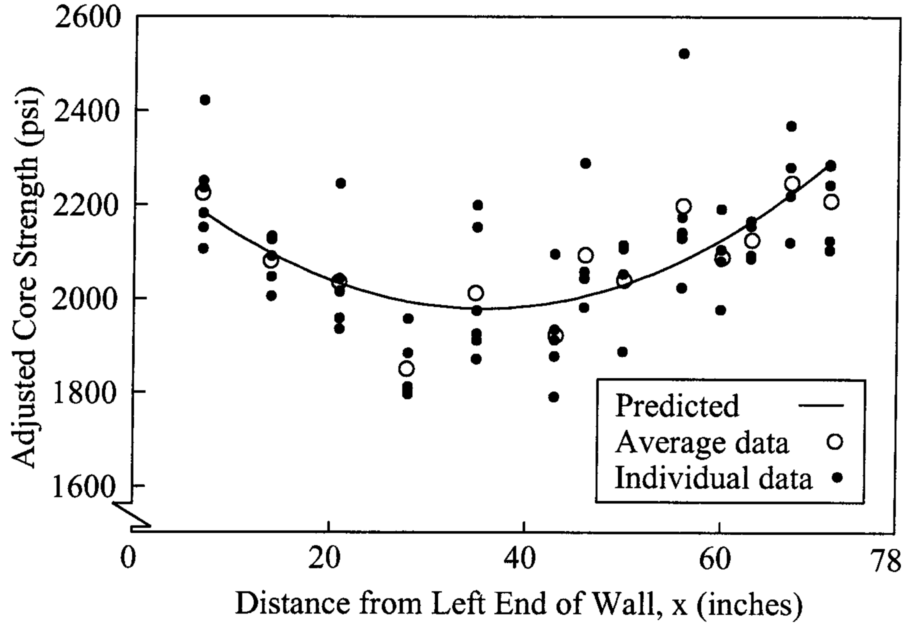

Figure 5.1: Variation of core strength along length of wall
(Data from Meininger, 1968)

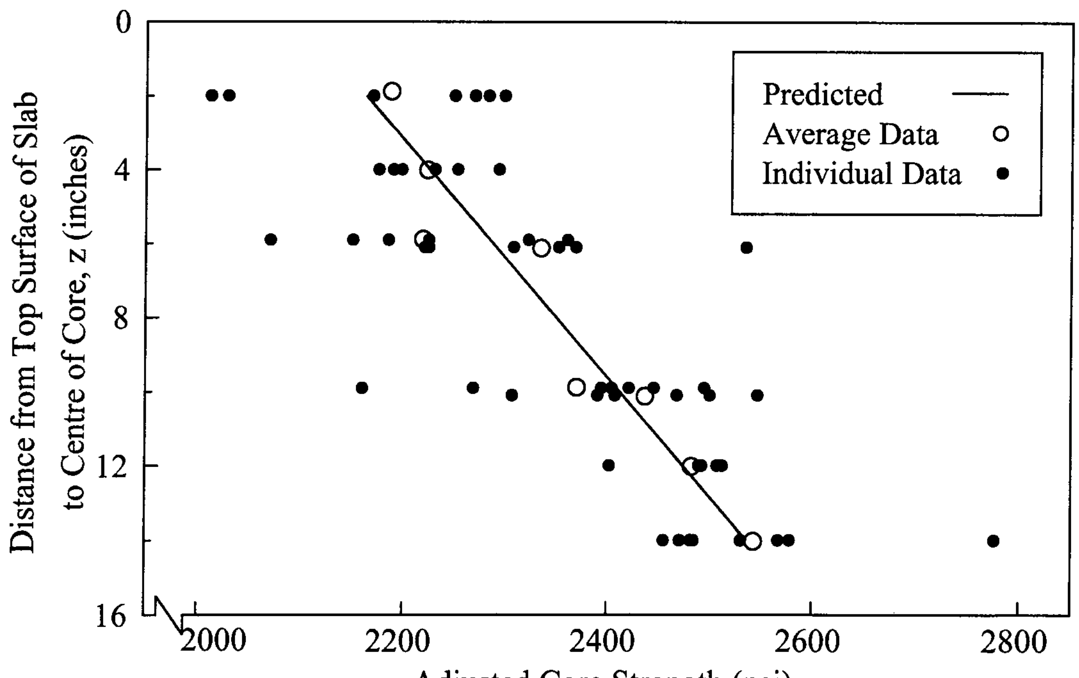

Figure 5.2: Core strength variation through slab depth
(Data from Meininger, 1968)

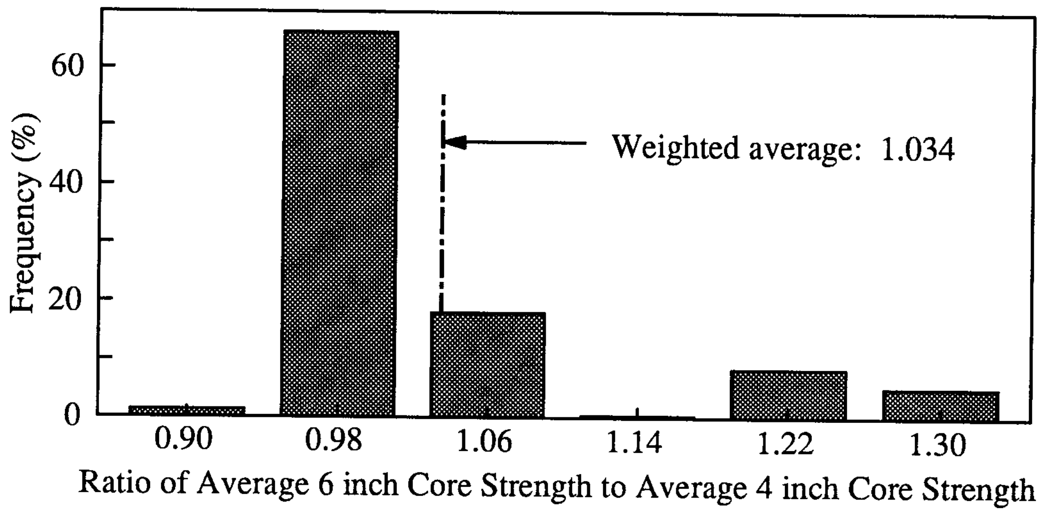

Figure 5.3: Histogram of ratios of average 6 inch core strengths to average 4 inch core strengths

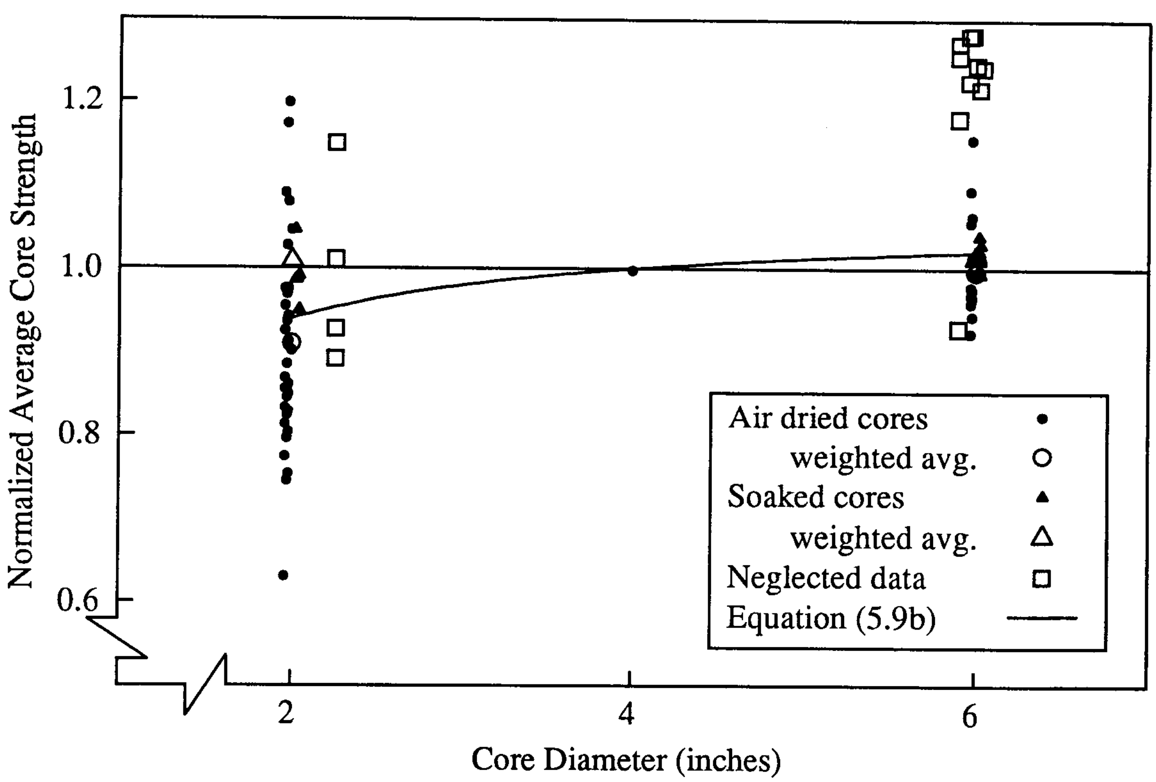

Figure 5.4: Normalized average core strength versus core diameter

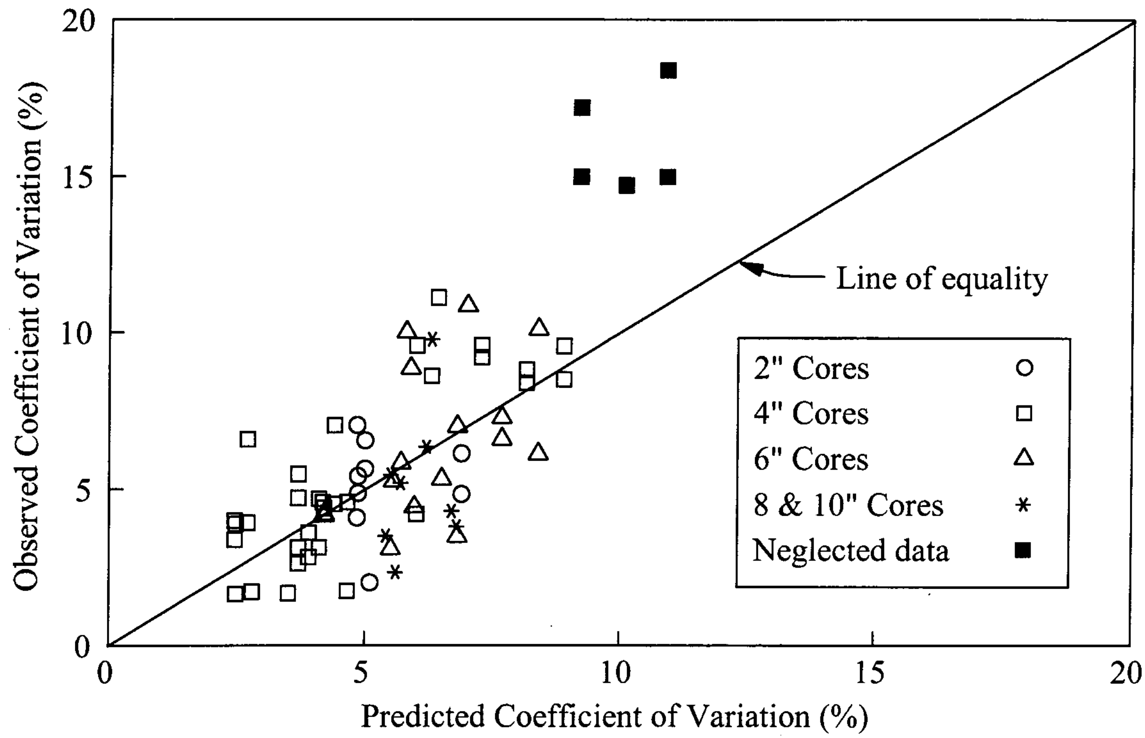

Figure 5.5: Observed versus predicted coefficients of variation

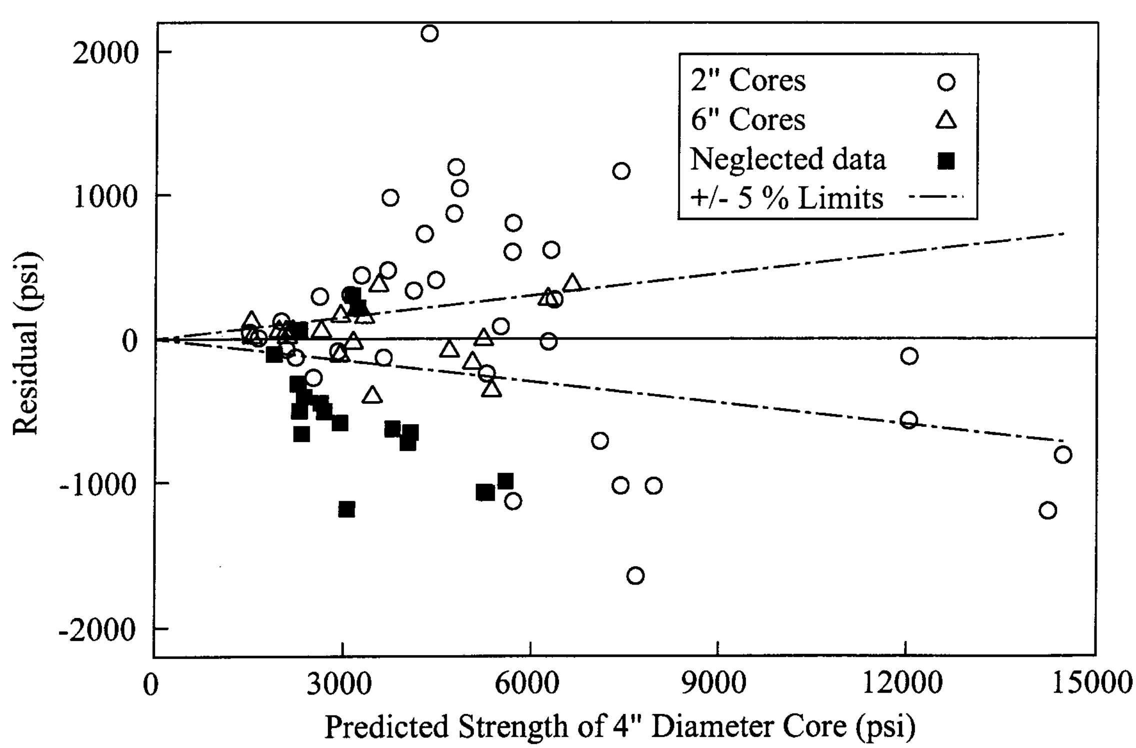

Figure 5.6: Residuals versus predicted strengths of 4" diameter cores

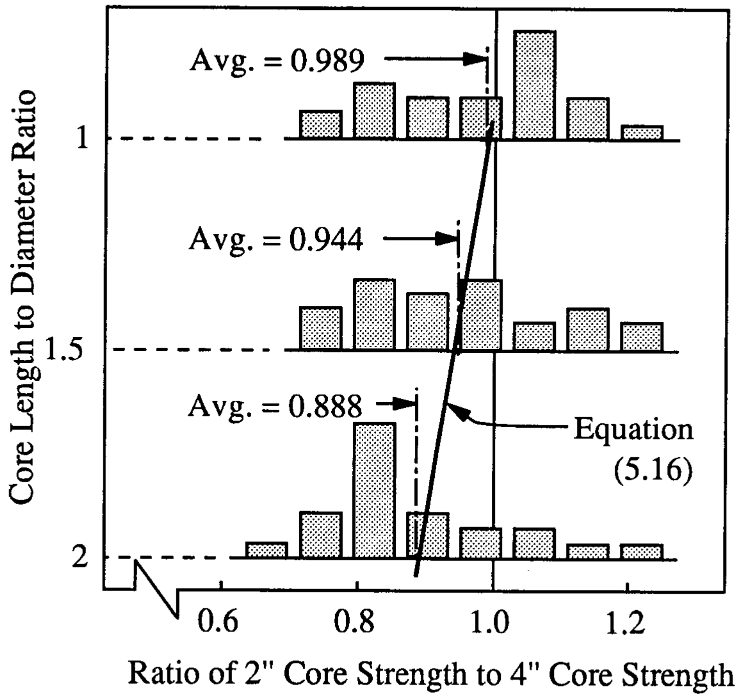

Figure 5.7: Diameter effect for cores with different length to diameter ratios

References:

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Chapter 6: Prediction of *In Situ* Concrete Strength from a Core Test[1]

6.1 Introduction

In this paper, the dependence of the strength of a concrete core specimen on the *in situ* strength of the volume from which the core was obtained is investigated. The relationship is illustrated schematically in Figure 6.1. It is assumed that the *in situ* strength within the volume from which the core was obtained has some constant but unknown value.

The core test result may be biased if the test specimen differs from a "standard" test specimen which is 4 inches (100 mm) in diameter by 8 inches (200 mm) long and does not contain any pieces of reinforcing bar. The effect of the bias is modelled as

$$f_{c,S} = F_{l/d} F_{dia} F_r f_{c,NS} \tag{6.1}$$

where $f_{c,S}$ and $f_{c,NS}$ are the strengths of the "standard" and "non-standard" specimens respectively. The factors $F_{l/d}$, F_{dia}, and F_r account for the effect of the length to diameter ratio, the diameter, and the presence of reinforcing bar pieces on the strength of the non-standard core.

The *in situ* strength may differ from the standard core strength because the condition of the core differs from that of the *in situ* concrete. This is modelled as

$$f_{c,is} = F_{mc} F_d f_{c,S} \tag{6.2}$$

where $f_{c,is}$ is the *in situ* concrete strength and factors F_{mc} and F_d account for the effect of the test moisture condition and damage sustained during drilling respectively on the strength of the standard core specimen.

Approximate values for some or all of the factors in Equations (6.1) and (6.2) are given in ASTM C42-90 (ASTM, 1992), the report of the Concrete Society (1976) or

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1 A version of this chapter was presented at the American Concrete Institute Fall Convention in Minneapolis, Minnesota, on 11 November 1993, and has been submitted for publication in a Special Publication of the American Concrete Institute.

elsewhere (e.g. Bartlett and MacGregor, 1994b-d [Chapters 3-5]). In some instances, specific values can be derived using strength data for cores obtained from the mix, component, or structure of interest.

Even if the actual *in situ* strength of the concrete at the location from which the core is obtained is uniform, the *in situ* strength value predicted from the core strength is uncertain due to two sources of error. The core strength itself includes a measurement error which, since the test is destructive, cannot be directly evaluated. The various strength correction factors in Equations (6.1) and (6.2) are the other source of uncertainty.

6.2 Research Significance

It is common to predict the *in situ* strength of concrete in a structural element from the strength of drilled cores. In this paper, the relationship between the *in situ* strength and the core strength is presented and the precision of the predicted *in situ* strength is investigated.

6.3 Precision of Core Strength Measurements

The precision of the measured core strength may be affected by normal variations in the rate of loading, the alignment of the specimen in the testing machine, the quality of the caps or other end preparations, and the deviation of the actual cross section from its assumed area. The error due to these factors can only be determined by tests of replicate specimens, and therefore is observed in conjunction with any variability of the quality of the material tested. The extent of damage sustained while the core is being drilled also affects each core strength to an uncertain extent. The relationship between these various factors can be modelled as

$$V_{core}^2 = V_{wt}^2 + V_{wb}^2 + V_s^2 + V_d^2 \qquad (6.3)$$

where V_{core} is the coefficient of variation of the core strengths observed from tests of replicate specimens and V_{wt} is the coefficient of variation due to normal variations of the test procedure. The effect of the variability of the quality of material tested is represented by two variables: V_{wb} is the coefficient of variation due to the inherent randomness of the material strength and V_s is the coefficient of variation due to systematic variation of the *in situ* strength throughout the volume of the element which inflates the coefficient of variation of the core tests. The coefficient of variation which reflects the uncertain extent of drilling damage is represented by V_d.

Various reported values of V_{core} are shown in Table 6.1. The higher values shown may be inflated by the effect of systematic spatial variation of the *in situ* strength, which should be allotted to the V_s term in Equation (6.3). If these effects are negligible or have been accounted for, a value of 4% seems appropriate. Investigations by others have indicated the insensitivity of V_{core} to other factors including the length to diameter ratio of the core (Bartlett and MacGregor 1994b [Chapter 3]) and the moisture condition of the core at the time of testing (Bloem, 1965). The variability of small diameter cores is not significantly larger than that of large diameter cores, but can be appreciably inflated by spatial variation of the *in situ* strength (Bartlett and MacGregor 1994d [Chapter 5]).

The coefficient of variation of the strength of replicate core specimens can also be calculated from Equation (6.3). It is reasonable to assume that the effects of normal variations of the specimen cross section area, end preparation, alignment of the specimen in the testing machine and test loading rates are roughly the same for cylinders and cores. The inherent randomness of the material quality is also the same, since any systematic *in situ* strength variation is included separately as the V_s term in Equation (6.3). Therefore, by inspecting the reported values of coefficients of variation for cylinders shown in Table 6.2, the coefficient of variation due to the combined effects of V_{wt} and V_{wb} is found to be roughly 3.0%. In the next section, it is suggested that the coefficient of variation due to drilling damage, V_d, is roughly 2.5%. If the systematic *in situ* strength variation, V_s, is assumed negligible, then the value of V_{core} calculated from Equation (6.3) is 3.9%. This corroborates the value of 4% obtained by inspection of Table 6.1.

6.4 Precision of Strength Correction Factors

The various core strength correction factors have typically been determined (Bartlett and MacGregor, 1994b-d [Chapters 3-5]) using weighted regression analyses of data from many sources. Conventional regression analysis solutions give sound and rigorously correct values for the precision of the strength correction factors if the underlying model is correct. However, as shown in Appendix 6A, they give erroneously small estimates of the error if, as in the present case, the underlying models are imperfect.

The precision of the strength correction factors may be estimated from techniques developed to assess model error for the calibration of Limit States Design (similar to LRFD) criteria. Ravindra and Galambos (1978) proposed a professional factor to account for the precision and bias of element resistance models, based on the ratios of test results to the corresponding predicted values. Mirza and MacGregor (1982) recognised that this simple ratio of test to predicted values includes any error inherent in the test procedure itself, and so overestimates the true model error. They therefore determine the coefficient of variation of the model, V_{model}, from

$$V_{model} = \sqrt{V_{t/p}^2 - V_{wt}^2 - V_{wb}^2} \tag{6.4}$$

where $V_{t/p}$ is the coefficient of variation obtained by direct calculation from the test to predicted ratios, and V_{wt} and V_{wb} are coefficients of variation reflecting the influence of errors in the measurement of loads, definition of failure, reported dimensions, and within batch strength variation on the recorded test result.

In the present investigation, the core strength correction factors have been calculated using the average strengths of replicate specimens from various sources. The variance of the average value varies, often considerably, for data from each source. Hence the inherent test error is not constant as implied by Equation (6.4) and so must be evaluated more rigorously.

The various core strength correction factors have the common mathematical form

$$\hat{Y}_i = F X_i \tag{6.5}$$

where X_i is a raw core strength, F is the strength correction factor, and $\hat{Y}_i$ is the corresponding predicted equivalent strength. The test to predicted ratio

$$R_i = \frac{Y_i}{F X_i} \tag{6.6}$$

is uncertain due to errors in the test value Y_i, the correction factor F and the raw strength X_i. The effect of these errors, which are assumed to be statistically independent, can be evaluated from a first order expansion of Equation (6.6) about the mean values $\overline{Y}_i$, $\overline{F}$, and $\overline{X}_i$

$$R_i \approx \frac{\overline{Y}_i}{\overline{F}\,\overline{X}_i} + \frac{1}{\overline{F}\,\overline{X}_i}(Y_i - \overline{Y}_i) - \frac{\overline{Y}_i}{\overline{F}^2\,\overline{X}_i}(F - \overline{F}) - \frac{\overline{Y}_i}{\overline{F}\,\overline{X}_i^2}(X_i - \overline{X}_i) \qquad (6.7).$$

The expected value of R_i is

$$E[R_i] = \frac{\overline{Y}_i}{\overline{F}\,\overline{X}_i} \qquad (6.8)$$

and the variance of R_i is

$$Var[R_i] = (E[R_i])^2 \left(\frac{Var[Y_i]}{\overline{Y}_i^2} + \frac{Var[F]}{\overline{F}^2} + \frac{Var[X_i]}{\overline{X}_i^2} \right) \qquad (6.9).$$

Recognising that $E[R_i] \approx 1$, Equation (6.9) can be simplified to:

$$Var[R_i] = V_{Yi}^2 + V_F^2 + V_{Xi}^2 \qquad (6.10)$$

where V_{Yi} and V_{Xi} are the known coefficients of variation of X_i and Y_i, and V_F is the unknown coefficient of variation of the strength correction factor. If V_{Yi} and V_{Xi} are constant for all observations, then V_F could be determined by rearranging Equation (6.10) to a form similar to that of Equation (6.4). Since in the present case they are not, another method must be found to evaluate V_F. In a weighted regression analysis, the variance factor, equal to the square of the standard error, equals one if the weighting is appropriate (Peterson and Peterson, 1988). Therefore, the coefficient of variation of the strength correction factor may be determined using the following iterative procedure:

(a) guess a value of V_F.

(b) calculate the variance of the test to predicted ratio for each observation from Equation (6.10).

(c) calculate the weight for each test to predicted ratio value as the inverse of the variance calculated from step (b).

(d) calculate the average and variance factor for the weighted test to predicted ratios. If the variance factor equals 1, the value of V_F assumed in step (a) equals the correct coefficient of variation of the strength correction factor. If the variance factor exceeds 1, it is necessary to repeat steps (a) through (d) with a larger estimate of V_F.

To illustrate the method, the precision of strength correction factors for core diameter is calculated from analysis of the test to predicted ratios. The specific model is

$$\hat{Y} = F_{dia} X \tag{6.5a}$$

where $\hat{Y}$ is the predicted strength of a 4 inch diameter core, and X is the strength of a 2 or 6 inch diameter core. The strength correction factor for non-standard core diameters, F_{dia}, is

$$F_{dia} = \frac{1 - \beta/4}{1 - \beta/d} \tag{6.11}$$

where d is the core diameter in inches and β is an unknown parameter. Weighted regression analysis gives $\hat{\beta}$, the estimate of β, as 0.2265 inches (Bartlett and MacGregor, 1994d [Chapter 5]), which corresponds to F_{dia} values of 1.064 for 2 inch diameter cores and 0.980 for 6 inch diameter cores.

To determine the precision of F_{dia} from the test to predicted ratios, a simple weighted regression analysis of a model without an intercept was carried out using the SAS software package (the SAS Institute, 1990). For the 2 inch diameter core data, the variance factors obtained were 0.958, 0.992 and 1.020 for three different analyses with assumed V_F values of 0.115, 0.1175 and 0.120 respectively. The correct coefficient of variation for the strength correction factor, V_F, is therefore about 0.118. Similarly, for the 6 inch diameter core data, $V_F = 0.018$.

Alternatively, the precision of F_{dia} may be estimated from the variance of the parameter estimate, Var [$\hat{\beta}$], using the first order approximation

$$\mathrm{Var}[F_{dia}] = \left(\frac{\partial F_{dia}}{\partial \beta}\right)^2 \mathrm{Var}[\hat{\beta}] \tag{6.12}$$

where, from Equation (6.11),

$$\frac{\partial F_{dia}}{\partial \beta} = \left(\frac{1}{d} - \frac{1}{4}\right)\left(1 - \frac{\beta}{d}\right)^{-2} \tag{6.13}.$$

The original weighted regression analysis (Bartlett and MacGregor, 1994d [Chapter 5]) gives Var [$\hat{\beta}$] = (0.046 inches)^2. Hence for 2 inch diameter cores, $\partial F_{dia} / \partial \beta = 0.318$ from Equation (13), Var [F_{dia}] = $(0.0146)^2$ from Equation (6.12), and so $V_F = 0.0146/1.064 = 0.0138$. Similarly, for 6 inch diameter cores, $V_F = 0.0039$. These values are considerably smaller than the corresponding values determined by analysis of the test to predicted ratios. They considerably underestimate the true error in F_{dia} because the model used to obtain Var [$\hat{\beta}$] is imperfect.

The residual errors of the predicted equivalent strengths of 4 inch diameter cores are shown on Figure 6.2. Limits where the residual error exceeds 5 and 20 percent of the predicted strength are shown by lines on the Figure. The residuals of the strengths predicted from 2 inch diameter cores tend to lie within the +/- 20% limits, and the residuals of the strengths predicted from the 6 inch diameter cores tend to lie within the +/- 5% limits. These ranges are reasonably consistent with the V_F values obtained from the test to predicted ratios, and are considerably wider than would be suggested by the V_F values obtained using the variance of the parameter estimates from the original regression analyses.

6.4.1 Conversion from Non-standard Core Strength to Standard Core Strength

The magnitude and precision of correction factors for converting the strength of non-standard cores to the equivalent strength of standard cores are shown in Table 6.3. The mean values of the correction factors for the core length to diameter ratio and the core diameter are from previous investigations (Bartlett and MacGregor, 1994b, 1994d [Chapter 3, Chapter 5]). The mean value of the correction factor for cores containing reinforcing steel has been obtained from data for 54 moulded cylinder specimens containing one or two pieces of reinforcing bar tested by Gaynor (1965). The coefficients of variation of the correction factors have been determined from ratios of the observed test strengths to the corresponding predicted values.

The correction factors shown in Table 6.3 are not necessarily accurate for cores which are not standard due to more than one factor. For example, the diameter effect for cores with l/d = 1 is negligible (Bartlett and MacGregor, 1994c [Chapter 5]), which implies that the length to diameter effect for cores with 2 inch diameters is greater than indicated by the factors in Table 6.3. Similarly, a recent investigation by Loo et al. (1989) indicates that the effect of pieces of reinforcing bar in the core is negligible for cores with l/d = 1.

6.4.2 Conversion from Standard Core Strength to *In Situ* Strength

The magnitude and precision of strength correction factors for converting standard core strengths to equivalent *in situ* strengths are shown in Table 6.4.

The values which account for the moisture condition of the core at the time of testing are based on a previous investigation (Bartlett and MacGregor, 1994c [Chapter 3]) which indicated that the air drying and soaking treatments recommended in ASTM C42-90 (ASTM, 1992) cause a moisture gradient within the test specimen which affects its strength. The average strength of air dried cores was found to be 1.14 times larger than the average strength of soaked cores. The coefficient of variation of this factor, calculated from test to predicted ratios, is 3.5%. Similarly the coefficient of variation of the factor for the difference in strengths between sealed and soaked cores was estimated from a small quantity of data to be 2.0%. The estimated coefficients of variation shown in Table 6.4 lie between these extremes.

The strength correction factor which accounts for damage sustained during drilling is

$$F_d = \left(1 - \frac{0.2265}{d}\right)^{-1} \tag{6.14}$$

where d is the diameter of the core in inches (Bartlett and MacGregor, 1994d [Chapter 5]). The effect predicted by this equation has been corroborated by other experimental data. Bloem (1968) tested 4 inch diameter cores drilled from slabs and 4 inch diameter push out cylinders, which were cast in metal sleeves in the slab. For the specimens obtained from well cured slabs and tested when the concrete age was 28 or 91 days, the average push out cylinder strength was 1.074 times the average core strength, which corroborates the value of 1.06 predicted using Equation (6.14). Similarly, the investigation by Campbell and Tobin (1967) included tests of 84 day old 6 inch diameter cores from slabs and 6 inch diameter cylinders made from normal weight concrete. Both the cores and the cylinders were stored in the slabs for 9 weeks before testing. The average cylinder strength was 1.045 times the average core strength, which again is in close agreement with the value of 1.039 predicted using Equation (6.14).

The estimate of the coefficient of variation of the strength correction factor for core damage shown in Table 6.4 is based on several considerations. Calculation using the variance of the parameter estimates from the original regression analysis give V_d = 1.2%, which is lower than the true coefficient of variation for the reasons described in Appendix 6A. The coefficient of variation of the ratios of the average strengths of cores and push out cylinders from Bloem's (1968) well cured slab is 3.9%. This value includes the effect of inherent testing and measurement errors and therefore is larger than the true value of V_d. The estimated value of 2.5% shown in Table 6.4 lies between these limits and, as already shown, reasonably reflects the relationship between V_{core} and V_{wt} according to Equation (6.3).

The combined effect of the strength correction factors for core moisture condition and core damage is also corroborated by strength data for cores from high performance concrete beams (Bartlett and MacGregor, 1994a [Chapter 2]). It was found that the least biased estimate of the *in situ* concrete strength corresponded to the strength of 4 inch diameter sealed cores, which have no moisture gradient due to soaking or air drying, multiplied by a factor of 1.06 to account for core damage. This finding is consistent with the average values of F_{mc} and F_d shown in Table 6.4.

6.5 Accuracy of Predicted *In Situ* Concrete Strength

The predicted *in situ* concrete strength, $\hat{f}_{c,is}$, as determined from the strength of non-standard core specimens, may be determined by substituting Equation (6.1) into Equation (6.2):

$$\hat{f}_{c,is} = F_{mc} F_d F_{l/d} F_{dia} F_r f_{c,NS} \tag{6.15}$$

where the values of the various strength correction are as shown in Tables 6.3 and 6.4. Alternatively, unique values of $F_{l/d}$, F_{dia} or F_r could be derived using strength data for cores obtained from the mix, component, or structure of interest.

It is assumed that, although the strength variation throughout the component may be appreciable, the variability of the strength within the small volume from which the core is obtained is negligible. The coefficient of variation of the predicted *in situ* strength therefore depends only on the inherent testing error and the uncertainty of the strength correction factors. Assuming these errors to be statistically independent

$$V_{c,is}^2 = V_{mc}^2 + V_d^2 + V_{l/d}^2 + V_{dia}^2 + V_r^2 + V_{wt}^2 \qquad (6.16)$$

where $V_{c,is}$ is the coefficient of variation of the predicted *in situ* strength. The values of V_{mc}, V_d, $V_{l/d}$, V_{dia} and V_r, which are the coefficients of variation of the strength correction factors for core moisture condition, drilling damage, length to diameter ratio, diameter and the presence of reinforcing bars respectively, are shown in Tables 6.3 and 6.4. It is appropriate to assume $V_{wt} = 3\%$ since the coefficient of variation due to drilling damage appears separately in Equation (6.16).

Obviously if the predicted *in situ* strength from Equation (6.15) is unaffected by a particular strength correction factor, then the corresponding coefficient of variation in Equation (6.16) is zero. For example, for standard cores $V_{l/d}$, V_{dia} and V_r do not contribute to $V_{c,is}$.

Substitution of the various coefficients of variation in Tables 6.3 and 6.4 into Equation (6.16) indicates that the coefficient of variation of the predicted *in situ* strength from a test of a 4 or 6 inch diameter core is between 4 and 5.5%. This implies that roughly 2/3 of the time the true *in situ* strength will be within 94.5 to 105.5% of the value predicted from a test of a single 4 or 6 inch diameter core. If the *in situ* strength is predicted from a test of a 2 inch diameter core, the coefficient of variation of the predicted *in situ* strength is 12.6%, and 2/3 of the time, the true *in situ* strength will be within 87.5 to 112.5% of the predicted value. This relatively large uncertainty is not due to the variability of the strengths of 2 inch diameter cores themselves, but instead reflects the large uncertainty in the factor relating the strengths of 2 and 4 inch diameter cores.

6.6 Summary

A two step method for converting a core test result to the *in situ* strength of the volume from which the core was obtained is presented. The equivalent strength of a standard core is first determined by accounting for the effects of the core length to diameter ratio, the core diameter, and the presence of reinforcing bars in the core. This is done by multiplying the core test result by the mean value of each of the applicable strength correction factors in Table 6.3. The equivalent *in situ* strength, which accounts for differences between the condition of the standard core and that of the *in situ* concrete, is then calculated by multiplying the equivalent strength of the standard core by the mean value of the applicable correction factors given in Table 6.4.

The accuracy of the *in situ* strength predicted in this manner is investigated. The precision of the predicted value is affected by the inherent error of the core strength measurement itself, and by the uncertainty of the various strength correction factors. The accuracy of the strength correction factors is determined by a weighted regression analysis of ratios of observed to predicted values which accounts for non-uniform variances of the dependent and independent variables.

The coefficient of variation of the *in situ* strength predicted from a test of a 4 or 6 inch (100 or 150 mm) diameter core is between 4 and 5.5%. If the *in situ* strength is predicted from a test of a 2 inch (50 mm) diameter core, the coefficient of variation of the predicted *in situ* strength is roughly 12.5%. These error estimates do not account for possible variation of *in situ* strength throughout the volume of the element being cored.

Table 6.1: Coefficients of Variation for Strengths of Replicate Cores

Source	V_{core} (%)
The Concrete Society (1976)	6.0
ASTM C42-90 (1992), for single operator	3.2
Meininger et al. (1977)	5.0
Bloem (1968)	5.3-7.1
Bloem (1965)	3.4
Bartlett and MacGregor (1994b), including V_s	4.8-5.1
- after removing effect of V_s	3.6-4.5

Table 6.2: Within Test Coefficients of Variation of Cylinder Strengths

Source	V_{wt} (%)
Alca (1993) (high strength concretes)	2.4-4.2
ACI Committee 214-77 (1976) (very good control)	2.0-3.0
Bloem (1968)	1.5
Bloem (1965)	2.2
Halstead (1969) (cube tests)	3.0
Malhotra[a]	3.0
MKM Consultants (1987)	2.7

Note: [a] - private communication, 1991

Table 6.3: Magnitude and Precision of Strength Correction Factors[a] for Non-standard Cores

Factor		Mean Value	V (%)
$F_{l/d}$: length to diameter ratio[b]		$1-\{0.144-0.027Z_{mc} -3(10^{-6})f_{c,NS}\}\left(2-\frac{l}{d}\right)^2$	$2.5\left(2-\frac{l}{d}\right)^2$
F_{dia}: core diameter -	2" cores	1.06	11.8
	6" cores	0.98	1.8
F_r: reinforcing steel present -	one bar	1.08	2.8
	two bars	1.13	2.8

Note: [a] - to obtain the strength of an equivalent "standard" core, multiply the strength of the non-standard core by the appropriate strength correction factor.

[b] - $f_{c,NS}$ is in psi. For $f_{c,NS}$ in MPa, the constant is -4.3 (10^{-4}).

Table 6.4: Magnitude and Precision of Factors[a] for Predicting *In Situ* Strengths from Standard Core Strengths

Factor		Mean Value	V (%)
F_{mc}: core moisture content -	soaked	1.09	2.5[e]
	air dried	0.96	2.5[e]
F_d: damage due to drilling		1.06	2.5[e]

Note: [a] - to obtain the equivalent *in situ* concrete strength, multiply the standard core strength by the appropriate factor(s).
[e] - estimated.

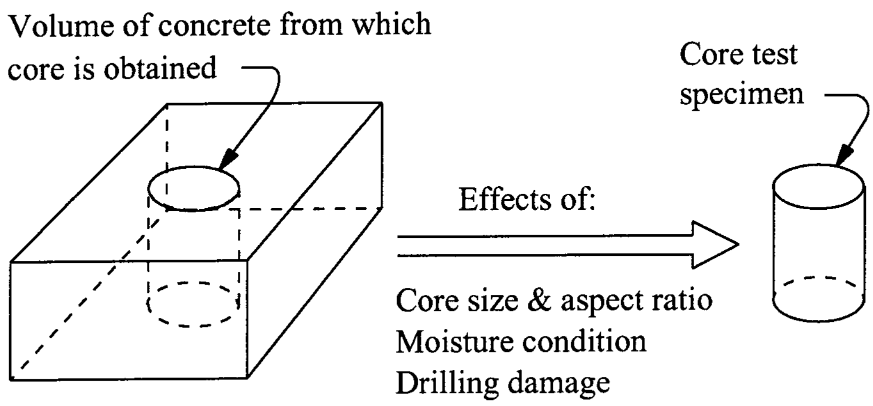

Figure 6.1: Relationship between in situ strength and core strength

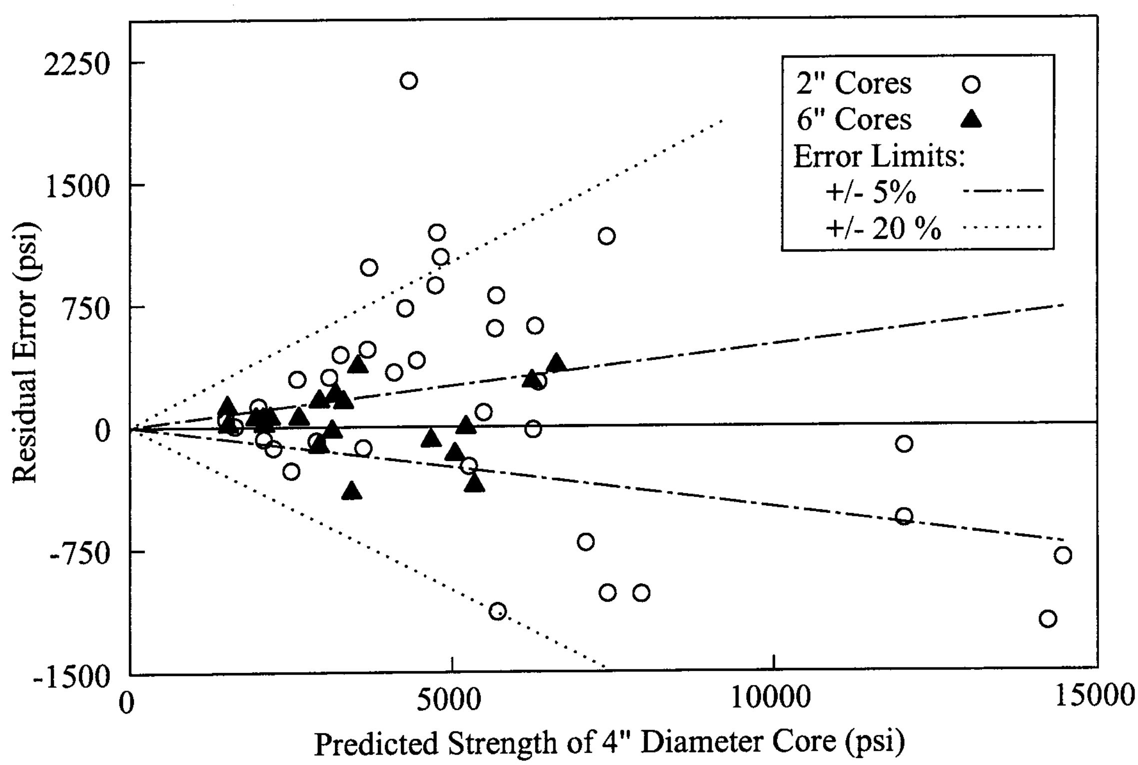

Figure 6.2: Residual errors of strength correction factors for core diameter

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Chapter 7: The Relationship between the *In Situ* Strength and the Specified Strength of Concrete

7.1 Introduction

Murphy (1979) has commented that "the engineer is often able to make little use of any information on the strength of the *in situ* concrete because this does not figure in any of the normal design processes and is, therefore, unfamiliar to him." The objective of this paper is to address this deficiency, so that *in situ* strength information can be rationally accounted for in the assessment of the safety of an existing structure. Specifically, the relationship between the average *in situ* strength and the specified strength, f'_c, which appears throughout the normal design processes and is familiar to engineers, is sought.

The nature of the relationship between the specified strength of concrete and the corresponding compressive strength in an element or structure consists of two contributing factors, as shown in Figure 7.1. The quality of material produced for a particular specified strength is checked using 28 day old cylinders which are molded, cured, capped and tested in accordance with the provisions of Canadian Standard CAN/CSA-A23.2-M90 (CSA, 1990b). Factor F_1 relates the average strength of these standard cylinders, $\bar{f}_{cyl}$, to the specified strength, f'_c:

$$\bar{f}_{cyl} = F_1 f'_c \qquad (7.1).$$

Typically the average standard cylinder strength exceeds the specified strength, since the concrete supplier usually overdesigns the mix to obtain some assurance that the 28 day strength of the product will be satisfactory. The actual quality of the concrete in the structure may differ from that represented by the cylinders because the age, consolidation or curing of the *in situ* material may not be well represented by the standard test specimens. Factor F_2 is therefore necessary to relate the average *in situ* strength of the concrete, $\bar{f}_{c,is}$, to the average cylinder strength:

$$\bar{f}_{c,is} = F_2 \bar{f}_{cyl} \qquad (7.2).$$

The value of F_2 may also depend on the on the size and type of member. The value for modern high performance concretes incorporating supplementary cementitious materials may differ from that for conventional concretes.

Probability density functions for the cylinder strength and the *in situ* strength for a given specified strength, as shown in Figure 7.1, are therefore different. The cylinder strength distribution depends on the magnitude and variability of factor F_1 only. The *in situ* strength distribution depends on the magnitude and variability of the product of factors F_1 and F_2. In this paper, statistical parameters for F_1 and F_2 are obtained by analysis of data so that the probability distribution for the average *in situ* strength can be determined.

7.2 Research Significance

In order to derive load and resistance factors so that new structures will achieve acceptable levels of safety, it is necessary to define the relationship between the specified concrete strength and the *in situ* strength in probabilistic terms. For conventional concretes, the relationship developed by MacGregor (1976) has been used for code calibration purposes (Mirza and MacGregor, 1982). If existing structures are to be evaluated, it is necessary to invert the relationship so that an equivalent specified strength may be obtained from test data which will result in reliability levels which are consistent with those currently accepted for new construction.

Based on the analysis of a large quantity of data, expressions are obtained for the mean value and coefficient of variation of the ratio of the average *in situ* strength to the specified strength. These expressions reflect the current quality of concrete produced in Alberta and account for the concrete age and the type of element. The relationship between the average *in situ* strength and the average cylinder strength of modern high strength concretes containing supplementary cementitious materials is also investigated.

7.3 Ratio, F_1, Between the 28 day Cylinder Strength and the Specified Strength

Guidelines for determining target strengths to be used for proportioning concrete mixes are given in CAN/CSA-A23.1-M90 "Construction Materials and Methods of Concrete Construction" (CSA, 1990a). The guidelines are based on the assumption that cylinder tests are normally distributed. To achieve a probability of at least 99% that the average of 3 consecutive tests exceeds the specified strength, it is recommended that

$$\bar{f}_{cyl} = f'_c + 1.4s \qquad (7.3)$$

where s is the standard deviation estimated from at least 30 consecutive strength tests of concrete which is similar to that to be produced. To achieve a probability of at least 99% that no individual test falls more than 3.5 MPa below the specified strength, it is further recommended that

$$\bar{f}_{cyl} = f'_c + 2.4s - 3.5 \qquad (7.4).$$

If concrete is proportioned to satisfy the criterion given by Equation (7.3), which governs if the standard deviation is less than 3.5 MPa, then the average cylinder strength will be 1.4 standard deviations greater than the specified strength, and so the probability of an individual test falling below f'_c will be 8.1%. If the standard deviation exceeds 3.5 MPa, Equation (7.4) governs and the corresponding probability that an individual test falls below f'_c will be less than 8.1%.

The criteria for determining the required average compressive strength used as the basis for selection of mix proportions in ACI 318-89 "Building Code Requirements for Reinforced Concrete" (ACI, 1989) are quite similar to those given by Equations (7.3) and (7.4), and yield slightly larger probabilities that individual tests will fall below f'_c. However, a more significant difference is that the criteria given in ACI 318-89 are compulsory whereas those given in A23.1-M90 are not.

Those who provided strength test data for this study also offered some insight into the current state-of-practice of proportioning concrete mixes. Durability concerns cause minimum cement content or maximum water-cement ratios to be specified which often require the producer to supply a stronger material than would normally be provided. Air entrained concretes are usually proportioned to have satisfactory strength at the maximum specified air content, and so are overstrength if the air content is less. It is considered prudent by at least one producer to proportion mixes containing fly ash to be just satisfactory if the fly ash does not "kick in" (that is, contribute to the strength). Requirements for strengths at 28 days may be superceded by criteria which are due to the construction schedule, such as early strengths required to meet the floor construction cycle of a high rise building. Reduced payment schedules for understrength concrete may cause the producer to supply material of a better quality than usual, depending on the financial risk. If the material supplied at the beginning of a job is considerably better than specified, the producer may request permission to reduce the quality but, in practice, only if the entire project is proceeding smoothly. This anecdotal evidence, although not in a form which can be readily corroborated using analytical statistics, indicates that the process of setting the target strength of a particular mix may be more complex than implied by Equations (7.3) and (7.4).

7.3.1 Standard Cylinder Test Data

Results of 3576 tests of 8662 standard cylinders from 108 concrete mixes were investigated to determine the relationship between the standard cylinder test strength and the specified strength. Each test is the average of two or three cylinder breaks. The data represent four categories of concrete produced between 1988 and 1993: general ready mix concrete which contains or does not contain entrained air; and concretes with entrained air supplied for precast or cast-in-place bridge construction. A summary of the data for all 108 mixes is presented in Appendix 7A. The breakdown of the data between the various specified strengths and categories is shown in Table 7.1. The mean values and standard deviations shown are based on at least 14 strength tests for each mix. Although the data can be weighted to reflect the relative inaccuracy of the mean and standard deviation values obtained from the smaller samples, this refinement was considered unwarranted. In cases where a long record of data for a standard product was available, several sets of data have been extracted and are considered as different mixes.

The strength test data for ready mix concretes were obtained from three local producers who together batch over 300 000 cubic metres per year. Almost all mixes contain fly ash. All strength testing was by independent CSA-certified testing laboratories. Most of the 29 air entrained concrete mixes and 26 mixes without air entrainment had specified strengths of 25 or 30 MPa.

The data for concretes used in bridge construction were obtained from the local provincial highways ministry, Alberta Transportation and Utilities. All of the mixes had entrained air, and none contained fly ash. The strength tests, each the average of three 28 day cylinders, were conducted by independent CSA-certified testing laboratories. Although concretes for cast-in-place and precast construction are produced to meet different specifications, both specifications include a reduced payment schedule for batches which do not achieve the specified strength at 28 days. For cast-in-place concretes, the target strength used to proportion the mix is specified to be 15% greater than the specified strength. The precast concretes were probably proportioned to produce sufficient strength at 18 hours to allow transfer of prestress. Although the precast components received steam curing, the cylinders used to monitor the 28 day strengths did not. Generally the precast concretes were produced in urban centres whereas the cast-in-place concretes were supplied by rural producers. Altogether, data were obtained from 30 cast-in-place projects using normal weight concrete and 23 projects using semi-lightweight precast concrete.

7.3.2 Analysis of the Average Strength of Standard Specimens

Three variables were used to investigate the relationship between the average strength of standard test specimens and the specified strength: the absolute overdesign, the overdesign factor, and the Studentized overdesign ratio. The absolute overdesign is the difference between the average strength and the specified strength, $(\bar{f}_{cyl} - f'_c)$, and has units of stress. The overdesign factor is the ratio of the average to specified strengths, $(\bar{f}_{cyl}/f'_c)$. The Studentized overdesign ratio is the absolute overdesign divided by the standard deviation of the mix, $(\bar{f}_{cyl} - f'_c)/s$. Both the overdesign factor and the Studentized overdesign ratio are dimensionless.

Initially the data were reviewed to see whether guidelines given in A23.1-M90 (CSA, 1990a) are an accurate reflection of the current state-of-practice. If the absolute overdesign is isolated on the left hand side of Equations (7.3) and (7.4), then the variation with the standard deviation is bilinear as shown in Figure 7.2. Clearly the actual data for bridge and general ready mix concretes, also shown in Figure 7.2, are not clustered around this relationship. The recommendations in A23.1 are met by 23 of the 26 general ready mix concretes without entrained air, 19 of the 29 general ready mix concretes with entrained air, 20 of the 30 mixes for cast-in-place bridge construction, and all of the 23 mixes used for precast bridge construction. The majority of the mixes exceed the recommended target strengths considerably. Thus while some producers may base preliminary mix designs on the non-compulsory guidelines given in A23.1, the mix design criteria do not accurately represent the degree of overstrength of concretes recently produced in the Edmonton area.

To investigate the relationship between the average strength of standard test specimens and the specified strength, scatter plots of the absolute overdesign, the overdesign factor, and the Studentized overdesign ratio versus the specified strength, the standard deviation and the coefficient of variation were prepared and reviewed. As shown in Figure 7.3, a slight relationship is apparent between the Studentized overdesign ratio and the inverse of the coefficient of variation:

$$\frac{(\bar{f}_{cyl} - f'_c)}{s} = \frac{K_1}{V} \qquad (7.5)$$

where K_1, a constant, equals 0.177. Since the coefficient of variation is equal to the standard deviation of the test strengths divided by the mean, Equation (7.5) is mathematically equivalent to a relationship of the form

$$\bar{f}_{cyl} = \frac{f'_c}{(1 - K_1)} \qquad (7.6).$$

This in turn implies that the overdesign factor, $\bar{f}_{cyl}/f'_c$, is constant and equal to approximately 1.22. Hence subsequent analyses considered only the overdesign factor, which was designated F_1.

Regression analysis of the overdesign factor on the specified strength yielded the fitted equations shown in Table 7.2. All parameter estimates are statistically different from zero at the 95% level. The predicted values of the strength ratio, $\hat{F}_1$, are close to the value of 1.22 noted above. The overdesign factor for general ready mix concretes with entrained air reduces as the specified strength varies from 25 to 35 MPa, and increases for higher specified strengths. As producers gain more experience with high strength concretes, it is probable that the corresponding target strengths will be reduced. Concretes for cast-in-place bridge construction generally have larger overdesign factors than general air entrained mixes of the same specified strength. However the trends indicate that rural producers may have difficulty supplying concretes with specified strengths greater than 30 MPa for cast-in-place bridge construction.

The low values of the coefficient of determination, R^2, for all of the analyses summarised in Table 7.2 suggest that the dependence of the overdesign factor on the specified strength is not strong. The data within each category of concrete may therefore be assumed to come from a homogeneous population. Cumulative distributions for each data set indicated little difference between the cast-in-place bridge concretes, the air entrained ready mix concretes, and the ready mix concretes without entrained air. These data were pooled, and the cumulative distribution of the sample was plotted on normal probability paper as shown in Figure 7.4. Clearly the fit of a normal distribution using the method of moments is good, and a slightly better fit to the lower half of the sample can be obtained. As also shown in Figure 7.4, a normal distribution fit to all the precast concrete data is not particularly accurate, but a good fit can be obtained for the lower tail. The parameters for the distributions fit to the lower tails are shown in Table 7.3. Also shown in Table 7.3 are parameters for distributions fit to the lower tails of the natural logarithm of the overdesign factor. The sample sizes are too small to identify whether it is more accurate to use a lognormal distribution or a normal distribution.

7.3.3 Analysis of Batch-to-batch Variation

Since each set of strength test cylinders is made from concrete from a single batch, the standard deviation of the test strengths of concrete produced using a particular mix design is a measure of the batch-to-batch variation of the mix. Regression analysis of the relationship between the standard deviations and the average test strengths yielded the fitted equations shown in Table 7.4, with all parameter estimates significantly different from zero at the 95% level. Generally the batch-to-batch variation for ready mix concretes which do not contain entrained air is less than that for the general ready mix concretes with entrained air. The batch-to-batch variation of mixes used for cast-in-place bridge construction, which all contain entrained air, is also less than that for general air entrained ready mix concretes. The batch-to-batch variation of concretes used for precast bridge construction is the least of all, even though all of these concretes contain entrained air.

Generally the standard deviation increases as the average strength increases, which suggests the possibility of expressing the batch-to-batch variability as a constant coefficient of variation. Analysis gave constant coefficients of variation of 10.2% and 8.9% respectively for ready mix concretes with and without entrained air, and 7.9% and 5.6% respectively for concretes used for cast-in-place and precast bridge construction. The accuracy of the predicted values of the batch-to-batch variation is similar to that obtained from the analysis of standard deviations shown in Table 7.4.

Data from six of the least overdesigned cast-in-place concrete mixes and two of the least overdesigned precast concrete mixes were reviewed to determine the nature of the batch-to-batch variation. Cumulative distributions for the data from each mix tended to plot as reasonably straight lines on normal probability paper. It is therefore reasonable to assume that the batch-to-batch variation follows a normal distribution.

7.3.4 Check of Fitted Relationships

The fitted relationships were used to predict the probability that an individual strength test would fall below the specified strength. The distribution of the cylinder test strengths was assumed normal, with the expected value of an individual test equal to the expected value of the average test strength:

$$E[f_{cyl}/f'_c] \quad = \quad E[\bar{f}_{cyl}/f'_c] \qquad (7.7).$$

The variance of an individual test is equal to the sum of the variance of the average values and the variance due to batch-to-batch variation, s_{bb}^2, within a mix:

$$Var[f_{cyl}/f'_c] \quad = \quad Var[\bar{f}_{cyl}/f'_c] \quad + \quad \left(\frac{1}{f'_c}\right)^2 s_{bb}^2 \qquad (7.8).$$

Assuming the batch-to-batch variation to have a constant coefficient of variation, V_{bb},

$$\left(\frac{1}{f'_c}\right)^2 \; s_{bb}^2 \quad = \quad \left(\frac{V_{bb}\,\bar{f}_{cyl}}{f'_c}\right)^2 \quad \approx \quad V_{bb}^2\,(E[\bar{f}_{cyl}/f'_c])^2 \qquad (7.9)$$

so the variance of an individual test result is, by substitution of Equation (7.9) into

Equation (7.8):

$$Var[f_{cyl}/f'_c] = Var[\bar{f}_{cyl}/f'_c] + V^2_{bb}(E[\bar{f}_{cyl}/f'_c])^2 \qquad (7.10).$$

The probability of a strength test result falling below f'_c was determined using values of $E[\bar{f}_{cyl}/f'_c]$ and $Var[\bar{f}_{cyl}/f'_c]$ shown in Table 7.3 for cast-in-place and precast concretes, and values of coefficients of variation as given in the previous section. For all of the ready mix cast-in-place concretes, the estimated probability of a low test result was 7.4%, which is slightly larger than the observed probability of 5.8%. For the precast concretes, the estimated probability was 1.6%, which is again slightly larger than the observed value of 0.9%. Thus the fitted relationships give realistic estimates of the probability of low test results, and are slightly conservative.

7.3.5 Summary of F_1 Values

The magnitude and variation of the ratio, F_1, of the average standard cylinder strength to the specified strength are indicated by the cylinder test data analysed. For cast-in-place concretes, F_1 can be represented by a normal distribution with a mean value of 1.25 and a standard deviation of 0.131, or by a lognormal distribution with a bias coefficient of 1.27 and a coefficient of variation of 0.14. For precast concretes, F_1 can be represented by a normal distribution with a mean value of 1.19 and a standard deviation of 0.058. The equations recommended for the design of concrete mixes in CAN/CSA-A23.1-M90 (CSA, 1990a) do not accurately reflect the actual margin of overstrength observed. The batch-to-batch variation is normally distributed with a constant coefficient of variation of roughly 10% for cast-in-place concretes or 6% for precast concretes.

7.4 Ratio, F_2, Between Average *In Situ* Strength and Average Cylinder Strength

A large data set was abstracted from published and unpublished investigations concerning the strength of cores and cylinders from common batches of concrete. The cylinders all received standard moist curing, and were tested at 28 days. In two instances the standard cylinder strength was estimated as 80% of the reported strength of moist cured cubes tested at 28 days. The cores were drilled when the concrete was at least 14 days old, and tested when the concrete was at least 28 days old. The core diameters were typically 100 or 150 mm and always at least 75 mm. The core length to diameter ratios varied between 1 and 2. In all cases, the moisture condition of the core at the time of testing was reported. Data which do not meet these general criteria were not included.

The locations from which the cores were drilled within the element are not known for all cases. Cores were obtained from the middle of most columns, walls, and large blocks, where the strength is reasonably uniform (Bartlett and MacGregor, 1994 [Chapter 8]). Strengths of cores from the top and bottom of the short column specimens tested by Haque et al. (1990) were averaged to estimate the strength in the middle. The cores from most of the slabs were drilled through the slab thickness; many were not trimmed before testing.

The *in situ* strengths were calculated from the actual core strengths following procedures given by Bartlett and MacGregor (1993) [Chapter 6]. The reported core strengths were modified to account for the effects of the core moisture condition and damage due to drilling after first being converted, if necessary, to the equivalent strength of a standard core 100 mm in diameter by 200 mm long. Using these strength correction factors, the *in situ* strength is 1.155 times the strength of a standard core which has been soaked for at least 40 hours before testing, or 1.018 times the strength of a standard core which has been left to dry in laboratory air for at least 7 days before testing.

The quality of curing given to the various elements varied widely and the actual descriptions of the curing provided are often incomplete. Data from a few investigations where the curing conditions were clearly described and carefully controlled were used to assess the effect of curing on the *in situ* strengths. Cores from the elements given good curing in these investigations were also included in the larger data set from the remaining investigations where either the curing conditions were clearly described but not controlled, or where the curing conditions were not clearly described.

The data were divided into two groups on the basis of the concrete strength: conventional concretes, which have 28 day cylinder strengths less than 55 MPa, and high performance concretes, which have higher strengths. Most of the conventional and high performance concretes were made with Type 10 cement. The 19 investigations which provided data for conventional concretes are summarised in Table 7.5, and represent tests of approximately 1080 cores from 108 mixes at concrete ages of between 28 days and 2 years. The 5 investigations which provided data for high performance concretes are summarised in Table 7.6, and represent tests of 771 cores from 22 mixes. Complete tabular listings of the data for conventional concretes and high performance concretes are given in Appendixes 7B and 7C respectively.

7.4.1 F_2 Values for Conventional Concretes

Preliminary analyses suggested that the F_2 values for columns and walls were consistently larger than those for shallow beams and slabs. This was accounted for using an indicator variable, Z_h, which was set equal to 1 for elements at least 450 mm high or 0 otherwise. The relative strength of cores from slabs may be reduced because the cores contain weak concrete from near the top of the element (Bartlett and MacGregor, 1994 [Chapter 8]). The difference may also be due to the greater sensitivity of the *in situ* strength of slabs to poor curing practices, as discussed in the next section. Preliminary analysis also indicated that, as the age of the concrete increased, the *in situ* strength increased relative to the standard 28 day cylinder strength. The strength gain was assumed to increase linearly with the natural logarithm of the age of the concrete.

Subsequent simple regression analyses eventually yielded:

$$\hat{F}_2 = 0.948 + 0.084\, Z_h + 0.100\, ln\left(\frac{a}{28}\right)\left[1 + 0.90\left(\frac{fa^F}{c}\right)\right] \qquad (7.11)$$

where $\hat{F}_2$ is the predicted F_2 value, a is the concrete age in days, and fa^F and c are the weights of Class F fly ash and cement respectively, per cubic metre. None of the mixes analysed contained Class C fly ash, silica fume, or slag. Although this fit has an R^2 of only 0.53, all parameters are significant at the 95% level and the standard error is 0.15. The average *in situ* strength of 28 day old concrete is therefore about 95% of the standard 28 day cylinder strength for shallow beams and slabs or 103% of the average 28 day cylinder strength for higher elements. The average *in situ* strength is greater at later ages, especially if the mix contains significant quantities of Class F fly ash.

It has been reported (Neville, 1981, Naik, 1990) that the ratio of *in situ* strength to standard test specimen strength decreases as the concrete strength increases. The residual errors of Equation (7.11) are plotted against the standard cylinder strength in

Figure 7.5 and do not indicate any such dependence. This corroborates the finding by McIntyre and Scanlon (1990) that the ratio of core strengths to companion field cured cylinder strengths does not depend on the concrete strength.

It may be advantageous for some applications to assume that the distribution of F_2 is lognormal. Nonlinear regression analysis (The SAS Institute, 1990) based on this assumption yielded:

$$ln\{\hat{F}_2\} = ln\left\{0.936 + 0.085\, Z_h + 0.097\, ln\left(\frac{a}{28}\right)\left[1 + 0.88\left(\frac{fa^F}{c}\right)\right]\right\} \qquad (7.11a)$$

with a standard error of 0.139. Analysis of the residuals of Equation (7.11a) did not indicate significant correlation with any other variable, including the standard cylinder strength. As shown in Figure 7.6, the residuals of Equation (7.11a) plot on normal probability paper as a reasonably straight line. This indicates that F_2 can indeed be approximated by a lognormal distribution.

There was no clear relationship between the F_2 value, or the residual error of the predicted F_2 value, and the quality of curing provided for these data, possibly because the "poor curing" conditions were generally not rigorously controlled. In many cases the poorly cured specimens were stored outdoors and hence the quality of curing depended on the local seasonal weather conditions. The scatter of F_2 values in these cases tended to be large. Thus the effects of various curing conditions are represented in the analysis by the mean values from Equations (7.11) and (7.11a) and by standard error values that are greater than might be obtained for uniform curing conditions.

7.4.2 Effect of Curing on *In Situ* Strength

The effect of curing on the *in situ* strength can be investigated by examining results for identical elements cast from one batch of concrete which have been subjected to different curing conditions. The results from investigations which adopted this strategy to address the effect of curing elements in air are summarised in Table 7.7. The well cured slabs investigated by Bloem (1965, 1968) were initially sprayed with a curing compound and then covered with moist burlap for one or two weeks; the remainder were left uncovered and had their bottom surfaces exposed to laboratory air 3 days after casting. The well cured column investigated by Bloem (1965) was covered in moist burlap for a week and left in the form until the cores were drilled, while the companion column was stripped after 1 day and left exposed to laboratory air. The well cured cores from the slab cast by Szypula and Grossman (1990) were drilled when the concrete was 2 days old and stored in a moist room; the remainder were obtained, just before testing, after the slab had been stored in a laboratory with its with top and bottom surfaces exposed. It should be noted that although this slab was loaded to simulate early construction loadings, in the present investigation only cores drilled from areas which were not subjected to large bending moments are considered. The wall specimens investigated by Gaynor (1970) were stripped after one day and either stored in a moist room or left with the side and top surfaces exposed in laboratory air. The beams tested by Meynink and Samarin (1979) were stripped after one day and either dry cured, wet cured, or wet cured for 7 days and dry cured thereafter. The complete data set for these investigations is presented in Appendix 7B.

The provisions of CAN/CSA-A23.1-M90 (CSA, 1990a) specify minimum levels of curing and protection to be provided to freshly deposited concrete. Essentially, temperature and moisture conditions at the surfaces of the concrete are to be controlled for a basic period of 3 days, or until the concrete has attained 35% of the specified 28 day strength. For concretes exposed to aggressive environments in service, the curing period is extended to 7 days, or until the concrete has attained 70% of the specified 28 day strength. Additional provisions address extreme temperature conditions and massive elements. For the investigations listed in Table 7.7, the well cured elements received better curing than required by CAN/CSA-A23.1-M90, and the poorly cured elements received worse curing than required.

Each value of F_{cure} shown in Table 7.7 is the ratio of the *in situ* strength of the poorly cured element to the *in situ* strength of the companion element which received good curing. The values shown do not vary appreciably with the age of the element. The effect of poor curing is apparently less severe for elements where the unformed top surface is a small fraction of the total surface area, such as columns and walls, than for elements where the unformed top surface is a large portion of the total surface area, such as slabs and beams. This may also be a partial cause of the difference in the average F_2 values noted between shallow and tall elements which is represented by the Z_h term in Equations (7.11) and (7.11a). The data for the poorly cured slabs may reflect particularly severe conditions because both top and bottom surfaces were exposed to air. For slabs and beams, the average value of F_{cure} is 0.77 and the standard deviation is 0.07. For walls and columns, the average value of F_{cure} is 0.90 and the standard deviation is 0.05. There are insufficient data to establish the type of distribution of F_{cure}.

These findings are consistent with *in situ* strength data reported by Petersons (1964) for columns which received extremely good curing. The columns, which were 300 by 300 by 3000 mm high, were cast upright and then immersed completely in a temperature controlled water bath one day after casting. Half of Petersons' mixes were intentionally designed to be liable to segregate. When the concretes were 28 days old, the *in situ* strengths of the mixes not liable to segregate averaged 127% of the strength of companion horizontally cast cylinders cured in the same water bath. For the mixes which were liable to segregate, the average value was 116%. These values both considerably exceed the value of 1.03 predicted for 28 day old columns using Equation (7.11a). The relative *in situ* strengths of Petersons' columns are also larger than those of the well cured columns investigated by Bloem (1968) which were not as well cured.

The effect of early exposure to cold weather on the *in situ* strength is indicated by the investigations summarised in Table 7.8. The slabs investigated by Malhotra (1973) were stored in a protective enclosure for 3 days at 23°C before exposure, unprotected, to winter conditions in Ottawa. The slabs and columns investigated by Berwanger and Malhotra (1974) were stored for 3 days at 10°C before being exposed to similar conditions. The relative strength of Malhotra's specimens at 28 days is larger, which, given the warmer initial curing temperatures applied, is consistent with conventional maturity concepts (Neville, 1981). The *in situ* strength at approximately 4 months, expressed as a percentage of the *in situ* 28 day strength, is reasonably consistent between both investigations. The *in situ* strengths are initially quite low but, after 4 months for the specimens investigated by Malhotra, or after 1 year for the specimens investigated by Berwanger and Malhotra, are within the range predicted by Equation (7.11a) for elements not subjected to extreme cold temperatures.

7.4.3 F_2 Values for High Performance Concretes

The data for high performance concretes were split into two separate groups for analysis because, as indicated in Table 7.6, the maximum heat sustained during hydration was recorded by all investigators except Read, Carette and Malhotra (1991). Preliminary analyses of both data sets indicated that the F_2 values for mixes containing blast furnace slag, silica fume, Class C fly ash or Class F fly ash were consistently lower than those for mixes which contained only ordinary portland cement. Therefore, indicator variable Z_{sup} was introduced which was set equal to 1 for mixes containing supplementary cementitious materials or 0 for mixes containing only ordinary portland cement. The *in situ* strength was assumed to increase linearly with the natural logarithm of the age of the concrete, with one exception as noted below. Typically all elements investigated were of sufficient size that the curing conditions at the exposed surfaces did not affect the strength of the interior concrete greatly. Indicator variable Z_h was not included in the analyses since it was equal to one for all elements investigated.

Simple regression analysis of the data reported by Read, Carette and Malhotra (1991) yielded:

$$\hat{F}_2 = 1.007 - 0.156\, Z_{sup} + 0.066 Z_t + 0.117\left(\frac{fa^F}{c}\right)$$
$$+0.122\, ln\left(\frac{a}{28}\right)\left[1 - 4.52\left(\frac{sf}{c}\right) + 2.86\left(\frac{fa^F}{c}\right)\right] - 0.052\left\{\left(\frac{fa^F}{c}\right) ln\left(\frac{a}{28}\right)\right\}^2 \qquad (7.12)$$

where indicator variable Z_t = 1 for data from the thin walls, or 0 for data from the thick walls or columns, and (sf/c) is the ratio of the weight of silica fume to cement. All the parameter estimates are significant at the 95% level, R^2 = 0.96, and the standard error is 0.057. The *in situ* strengths of the thin walls are consistently stronger than those of the thick walls and columns. The last term of Equation (7.12) indicates that the rate of strength gain for the only fly ash mix investigated by Read et al. gradually decreased with time.

Simple regression analysis of the data reported in the other 4 investigations yielded:

$$\hat{F}_2 = 0.993 - 0.125\, Z_{sup} - 0.0055(T_{max} - 63.8)$$
$$+0.133\, ln\left(\frac{a}{28}\right)\left[1 - 2.52\left(\frac{sf}{c}\right) - 0.82\left(\frac{fa^C}{c}\right)\right] \qquad (7.13)$$

where (fa^C/c) is the weight of class C fly ash divided by the weight of cement and T_{max}, the maximum temperature recorded during hydration, averaged 63.8°C for all the elements considered. All parameter estimates are significant at the 95% level, $R^2 = 0.89$, and the standard error is 0.063. The reported F_2 values reduce as T_{max} increases, as indicated by the partial regression plot shown in Figure 7.7. The trend is consistent with the observation by Neville (1981, p. 318) that a higher temperature during placing and setting may cause a rapid initial hydration and so form products of a poorer physical structure. High internal temperatures may also cause thermal gradients which may damage concrete near the surface of the element (Mak et al., 1990). Cores from near the exposed surfaces of the columns investigated by Mak et al. (1990), which have been appreciably weakened by the effect of thermal gradients, have not been included in the present study.

The parameter estimates given by Equations (7.12) and (7.13) for the two analyses are quite similar. If the average maximum temperature during hydration is assumed to be equal for the elements from both data sets, then the predicted F_2 values at 28 days range from 0.993 to 1.003 for mixes that to not contain supplementary cementitious materials and from 0.851 to 0.868 for mixes which do. The effect of the maximum temperature sustained during hydration in Equation (7.13) is consistent with the larger *in situ* strengths for thin wall elements predicted in Equation (7.12). The variation of strength with time is quite consistent for the mixes containing ordinary portland cement, slag, or silica fume. The only discrepancy is that the initial and long term F_2 values for the single mix investigated by Read et al. (1991) which contained Class F fly ash are appreciably greater than those for the other two Class F fly ash mixes investigated by Burg and Ost (1992). This discrepancy may be due to differences between the Class F fly ashes used in each investigation.

The F_2 values for high performance concretes containing only ordinary portland cement are similar to those for conventional concretes predicted using Equation (7.11) or (7.11a), both at 28 days and at later ages. The F_2 values for high performance concretes containing supplementary cementitious materials are 0.13 to 0.16 lower than those for conventional concretes. For some concretes containing Class F fly ash, the long term strength gain more than compensates for this difference. This effect may also be counteracted to some extent if the supplementary cementitious materials also reduce

T_{max}. The relative strength gain of silica fume concretes after 28 days is about half that of conventional concretes. The variability of the F_2 values for the high performance concretes is appreciably less than the variability of F_2 for conventional concretes.

7.4.4 Check of Predicted F_2 Values

The accuracy of the F_2 values predicted using Equations (7.11a) and (7.13) was checked using *in situ* strength data from real structures reported by the United States Bureau of Reclamation (1966), Price (1951), Wagner (1963), Henzel and Grube (1968) and Keiller (1985). These data were not used in the original analysis because the strengths reported are of cores and cylinders from the same structure but not necessarily the same batch of concrete. It is also possible that the long term strengths reported in the investigations published before 1970 are appropriate for older cements which were coarsely ground, but may not represent modern cements which are more finely ground. The complete data set is presented in Appendix 7B.

The F_2 values for these data show considerable scatter, as indicated in Figure 7.8. The best fit line to the data is:

$$\hat{F}_2 = 1.115 + 0.106\, ln\left(\frac{a}{28}\right) \tag{7.14}$$

with both parameters significant at the 95% level, $R^2 = 0.2$ and a standard error of 0.20. The predicted values, both for tall elements according to Equation (7.11a) and for thick elements containing only ordinary portland cement according to Equation (7.12), are superimposed on Figure 7.8 and slightly underestimate the average F_2 value. Equations (7.11a) and (7.13) are therefore reasonably accurate and slightly conservative for these data.

7.4.5 Summary of F_2 Values

The magnitude and variation of the ratio, F_2, of the average *in situ* strength in an element to the average standard cylinder strength have been determined for elements cast from conventional concretes and high performance concretes containing supplementary cementitious materials. For elements cast from conventional concretes, the average F_2 value is 0.95 for 28 day old slabs or shallow beams or 1.03 for walls or columns. The *in situ* strength of slabs may be weaker because the cores investigated contain weak concrete from the top of the element, or because the strength of slabs is particularly sensitive to poor curing practices. The average *in situ* strengths increase by about 25% between 28 days and one year. For these data, the F_2 value is independent of the concrete strength.

For elements cast using high performance concretes, the F_2 value decreases as the maximum temperature sustained during hydration increases. If the concrete contains silica fume, slag, or fly ash, the F_2 value is about 0.15 smaller than that of concretes which contain only ordinary portland cement. The average *in situ* strength of all elements continued to increase after 28 days. The rate of strength gain of silica fume concretes after 28 days was about half that of concretes containing ordinary portland cement.

7.5 Relationship between Average *In Situ* Strength and Specified Strength

The relationship between the average strength of concrete in structures and the specified strength is obtained from the substitution of Equation (7.1) into Equation (7.2):

$$\bar{f}_{c,is} = F_1 F_2 f'_c \qquad (7.15).$$

For concretes used in cast-in-place construction, F_1 can be represented by a lognormal distribution with the parameters shown in Table 3, with a mean value of 1.27 and a coefficient of variation of 0.122. The mean value of F_2 may be approximated by the predicted value, $\hat{F}_2$, as obtained from Equation (7.11a). For conventional concretes which do not contain fly ash, F_2 has a mean value of

$$E[F_2] = \hat{F}_2 = 0.945 + 0.085\, Z_h + 0.098\, ln\left(\frac{a}{28}\right) \qquad (7.16)$$

and a coefficient of variation of 0.140. The coefficients in Equation (7.16) are slightly different from those in Equation (7.11a) because the mean value of a variable which has a lognormal distribution is not simply the inverse logarithm of the expected value of the logarithms of the variable.

Assuming F_1 and F_2 to be statistically independent, the mean value of their product has a lognormal distribution with a mean value of

$$E[F_1 F_2] = 1.205 + 0.108\, Z_h + 0.125\, ln\left(\frac{a}{28}\right) \tag{7.17}$$

and a coefficient of variation of 0.186. Thus the average *in situ* strength of conventional concretes at 28 days is about 1.20 times the specified strength for shallow elements, or about 1.31 times the specified strength for tall elements. The 95% confidence limits for the ratio of the average *in situ* strength at 28 days to the specified strength are 0.82 to 1.70 for shallow elements or 0.90 to 1.85 for tall elements. When the concrete is one year old, the ratio of the average *in situ* strength to the specified strength averages 1.33 or 1.44 for shallow or tall elements respectively, with a coefficient of variation of 0.186.

Additional research is necessary to determine the relationship between the mean *in situ* strength and the specified strength for precast elements and for modern high performance concretes which incorporate supplementary cementitious materials. Specifically, the F_2 relationship for precast elements which are steam cured and the F_1 relationship for concretes incorporating supplementary cementitious materials are not well represented by the data analysed in this study. For precast units which are not steam cured, the ratio of the mean *in situ* strength to specified strength has a lognormal distribution with a mean value roughly 6% less than that given by Equation (7.17) and a coefficient of variation of 0.15.

The mean *in situ* strength for a given specified strength from Equation (7.17) is greater than that proposed previously by MacGregor (1976). However the variability of the mean *in situ* strength determined in the present investigation is also greater than the overall variability of the *in situ* strength assumed by Mirza and MacGregor (1982) for code calibration purposes. If, in the present investigation, the overall variability of the *in situ* strength were determined, it would be greater than the variability of the mean *in situ* strength because the effects of batch-to-batch variation and systematic spatial strength variation within the element are not fully accounted for. Thus the previous calibrations based on a relatively low *in situ* strength with a relatively low variability may give reliability levels which are consistent with those that would be obtained from the results of the present study.

7.6 Summary and Conclusions

The relationship between the average *in situ* strength of concrete in a structure and the corresponding specified compressive strength may be represented as the product of two independent factors, F_1 and F_2. Factor F_1 represents the ratio of the average strength of standard cylinder specimens to the specified strength, f'_c. To have some assurance that the material provided will meet specifications, the concrete producer typically ensures that F_1 exceeds 1.0. Factor F_2 is the ratio of the average *in situ* strength to the average strength of 28 day old standard cylinder specimens, and depends on the age and height of the element and on the quality of curing provided.

To investigate the degree of overstrength provided by the concrete producer, 3756 strength tests from 108 concrete mixes supplied from 1988 to 1993 in Alberta were obtained. The data represent mixes with and without entrained air used for cast-in-place and precast construction. The following conclusions apply to these data:

1. For cast-in-place concretes, F_1 is normally distributed with a mean value of 1.25 and a standard deviation of 0.131. For precast concretes, F_1 is normally distributed with a mean value of 1.19 and a standard deviation of 0.058. For either type of construction, F_1 may be represented by a lognormal distribution without significant loss of accuracy.

2. Equations which give suggested target strengths for the design of concrete mixes in CAN/CSA-A23.1-M90 "Construction Materials and Methods of Concrete Construction" (CSA, 1990a) do not accurately reflect the actual margin of overstrength observed.

3. The batch-to-batch variation has a constant coefficient of variation of roughly 10% for cast-in-place concretes or 6% for precast concretes.

To investigate the ratio, F_2, between the average *in situ* strength and the strength of standard cylinders tested at 28 days, core and cylinder data from 108 conventional concrete mixes and 22 high performance concrete mixes incorporating supplementary cementitious materials were obtained from various published and unpublished sources.

The following conclusions apply to for the data from conventional concretes:

4. At 28 days, the average value of F_2 is 0.95 for elements less than 450 mm high or 1.03 for elements at least 450 mm high. The average *in situ* strength increases by about 25% between 28 days and 1 year. The ratio may be represented by a lognormal distribution with a coefficient of variation of 14%.

5. Thin slabs given insufficient moisture during curing have *in situ* strengths that average 77% of the strength of identical slabs given moist curing. For columns or walls, the corresponding value is 90%.

6. The value of F_2 is independent of the concrete strength.

7. The average *in situ* compressive strength at 28 days of conventional concretes for cast-in-place construction is about 1.2 times the specified strength, f'_c, for shallow elements or 1.3 times the specified strength for tall elements. The ratio of the average *in situ* strength to the specified strength has a coefficient of variation of 18.6%

The following conclusions apply to the data for high performance concretes:

8. If the concretes contain silica fume, slag, or fly ash, the average F_2 value is about 0.15 smaller than that for concretes which do not contain supplementary cementitious materials.

9. The F_2 value decreases as the maximum temperature which occurs within the element during hydration increases.

10. Concretes containing silica fume gain *in situ* strength after 28 days at about half the rate of concretes containing only ordinary portland cement.

Table 7.1: Categorization of Cylinder Test Data

f'_c (MPa)	General Ready Mix: Air entr.	General Ready Mix: No air	Bridge Construction: c-i-p	Bridge Construction: precast	Total
25	18 (10)	18 (7)	15 (3)	0	51
30	4 (3)	5 (1)	13 (4)	0	22
32	1 (1)	1 (1)	0	0	2
35	4 (3)	1	2 (2)	20 (5)	27
40	0	1	0	3	4
45	2	0	0	0	2
Total	29 (17)	26 (9)	30 (9)	23 (5)	108

Note: Numbers in parentheses indicate number of mixes with at least one test below f'_c

Table 7.2: Regression Analysis of Overdesign Ratio on Specified Strength

	Fitted Equation	R^2	$\sqrt{MS_E}$	$\hat{F}_1$ for f'_c of 25	30	35	40	45
Ready mix -air entrained	$\bar{f}_{cyl}/f'_c = 2.559 - 0.08f'_c + 0.0009\,f'^{2}_c$	.28	0.079	1.25	1.16	1.13	1.17	1.26
Ready mix -no air	$\bar{f}_{cyl}/f'_c = 1.247$	.00	0.081	1.25	1.25	1.25	1.25	-
C-i-p bridge construction	$\bar{f}_{cyl}/f'_c = 2.048 - 0.03f'_c$	.37	0.124	1.32	1.18	1.02	-	-
Precast bridge construction	$\bar{f}_{cyl}/f'_c = 1.209$	.00	0.096	-	-	1.21	1.21	-

Table 7.3: Overdesign Ratio Distribution Parameters

	Random Variable	Average	Standard Deviation
All ready mix concrete	$\bar{f}_{cyl}/f'_c$	1.250	0.131
	$ln\ (\bar{f}_{cyl}/f'_c)$	0.234	0.122
Precast concrete	$\bar{f}_{cyl}/f'_c$	1.190	0.058
	$ln\ (\bar{f}_{cyl}/f'_c)$	0.180	0.053

Table 7.4: Regression Analysis of Standard Deviation on Average Test Strength

	Fitted Equation	R^2	$\sqrt{MS_E}$	$\hat{s}$ (MPa) for $\bar{f}_{cyl}$ of 30	35	40	45	50	55
Ready mix -air entrained	$s = -11.6 + 0.75\bar{f}_{cyl} - 0.0086\bar{f}^2_{cyl}$	.39	0.70	3.07	4.01	4.52	4.60	4.25	3.47
Ready mix -no air	$s = 0.376 + 0.081\bar{f}_{cyl}$	.20	0.87	2.80	3.21	3.62	4.02	4.42	-
C-i-p bridge construction	$s = 2.80$	.00	0.80	2.80	2.80	2.80	2.80	-	-
Precast bridge construction	$s = -0.75 + 0.074\bar{f}_{cyl}$	.26	0.55	-	-	2.21	2.58	2.95	3.32

Table 7.5: Summary of *In Situ* Strength Data Investigated

Reference	Year	N_m [a]	Range of $\bar{f}_{cyl}$ (MPa)	Test age (d)	Element l x t x h (m x m x mm)	Element Curing Details
Mather & Tynes	1961	4	13-31	28	1.5x1.5x500	
Bloem	1965	1 1	35 29	95 95	?x?x200, 100 0.7x.2x3050	7 days moist, then in forms until test
Bloem	1968	3	25-38	28, 91 & 365	2.7x1.5x150	14 days moist, then in lab air
Meininger	1968	1	21	93	2.0x0.4x1220 1.1x1.1x400	continuous moist
Gaynor	1970	2	26-42	91	0.6x.15x1220	continuous moist
Lewis	1976	4	22-45	30, 93	3.0x3.0x350	initially covered
Meininger, Wagner & Hall	1977	3 3	16-32 19-45	35-91 28, 60 -102	2.1x0.3x1070 1.2x0.3x1830	7 days moist, lab air continuous moist
Meynink & Samarin	1979	2	26-42	28	?x0.15x150	7 days moist, lab air or continuous moist
Malhotra,Carette	1980	7	18-51	28, 91	0.6x.6x305	continuous moist
Yip & Tam	1988	12	16-50[e]	28	0.3x.2x300 0.3x.2x150	continuous moist, or exposed to lab air
Akers	1990	2	35-37	28	"pads"	
Keiller	1984	21	16-47[e]	28, 365	1.2x0.3x600	after 4 days, left exposed outdoors
Szypula,Grossman	1990	1	26	28, 56	9.7x2.4x240	continuous moist
Haque et al.	1988	21	13-42	182	0.3x.3x450	after 1 day, left exposed outdoors
Confidential[b]	1991	5	21-29	28	?x?x140,290	outdoor exposure
ACI 214	1992	3	34-51	56	?x?x250	
Bickley & Read	1992	1 1	37 38	28, 91	3.0x.3x2000 3.0x3.0x300	outdoor exposure
Haque et al.	1991	8	33-52	28, 91 & 365	0.4x.4x150	after 1 day, left exposed outdoors
Langley et al.	1992	3	22-45	42, 365 & 730	3.1x3.1x3050	2 wks at 12°C, then outside

Note: [a] - N_m = number of mixes
[b] - data provided on the condition that its source not be revealed
[e] - estimated as 0.8 x cube strength

Table 7.6: Summary of High Performance Concrete Data Investigated

Ref.	Mix	$\bar{f}_{cyl}$ (MPa)	c $\left(\frac{kg}{m^3}\right)$	sf /c	sl /c	fa^C /c	fa^F /c	T_{max} (°C)	Test Ages (days)	Elements Tested
Read	1	79.9	315	0.111	0.43	0	0	nr[a]	28, 91,	walls: [b]
et al.	2	71.6	317	0	0.53	0	0	nr	182, 365,	0.5x2.5x1.5m,
(1991)	3	81.6	449	0.087	0	0	0	nr	546, 730,	0.25x3.x1.5m,
	4	90.6	427	0.138	0	0	0	nr	and 912	and column
	5	54.7	150	0	0	0	1.33	nr		2x1.2x1.35m
	6	70.1	485	0	0	0	0	nr		
Yuan	1	80.4	359	0	0	0.42	0	88.9	28, 56,	1.83m cubes
et al.	2	83.4	358	0	0	0.42	0	95	180 &	
(1991)									365	
Mak	1	58.2	380	0	0	0	0	44.6	28, 90,	columns: [c]
et al.	2	67.0	500	0	0	0	0	69.9	and 450	0.8x0.8x1.2m
(1990)	3	92.4	550	0	0	0	0	64.4		0.4x0.4x1.2m
(1993)	4	92.2	480	0	0.25	0	0	61.8		
	5	91.2	460	0.087	0	0	0	66.4		
	6	112.9	500	0.090	0	0	0	51.8		
Burg &	1	78.6	464	0	0	0	0	77.8	91	1.22m cubes
Ost	2	88.6	475	0.051	0	0	0.12	77.8	and 430	
(1992)	3	91.9	487	0.097	0	0	0	70.4		
	4	119.0	564	0.158	0	0	0	72.5		
	5	107.0	475	0.156	0	0	0.22	66.6		
	6	73.1	327	0.083	0	0	0.27	57.6		
Miao	1	90.4	435	0.080	0	0	0	68.0	91	1x1x2m
et al.	2	108.2	500	0.080	0	0	0	63.0		columns
(1993)										

Notes: [a] - not recorded
[b] - no thin wall cast from Mix 5
[c] - no small column cast from Mix 1

Table 7.7: Relative Strength Factors for Elements Cured in Air

Reference	Mix	$\bar{f}_{cyl}$ (MPa)	Element Type	F_{cure} for ages (days) of: 28	56	91	365
Bloem (1965)	1	35.4	100 mm slab	--	--	0.70	--
			200 mm slab	--	--	0.82	--
Bloem (1968)	1	37.9	150 mm slabs	0.84	--	0.76	0.81
	2	25.7		0.73	--	0.64	0.62
	3	28.6		0.84	--	0.85	0.82
Szypula (1990)	1	26.5	240 mm slab	0.74	0.76	--	--
Meynink & Samarin (1979)	1	26.0	150 mm beams	0.77	--	--	--
	2	41.5		0.79	--	--	--
Overall, Beams & Slabs			Avg.	0.78	0.76	0.76	0.75
			std. dev.	(.05)		(.08)	(.11)
Bloem (1965)	2	29.4	column	--	--	0.96	--
Gaynor (1970)	1	26.2	walls	--	--	0.87	--
	2	41.9		--	--	0.88	--
Overall, Columns & Walls			Avg.			0.90	
			std. dev.			(.05)	

Table 7.8: F_2 Factors for Elements Cured in Cold Temperatures

Reference	Mix	$\bar{f}_{cyl}$ (MPa)	F_2 for ages (days) of: 28	59	93	116	121	355
Malhotra (1973)	1	28.8	0.76	0.86	0.82	.92	--	--
Berwanger & Malhotra (1974)	1	17.4	0.48	--	--	--	0.60	0.98
	2	31.7	0.61	--	--	--	0.68	1.01
	3	35.7	0.62	--	--	--	0.70	1.12
	4	37.3	0.66	--	--	--	0.77	1.18
	5	42.0	0.66	--	--	--	0.76	1.08
Overall Average			0.63				0.70	1.07

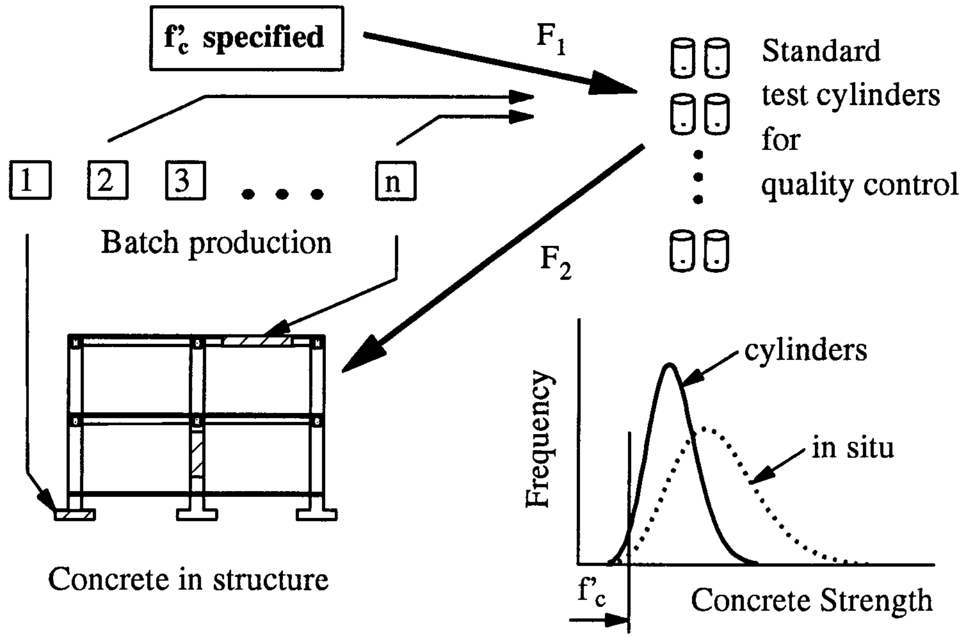

Figure 7.1: Nature of relationship between specified and in situ strengths

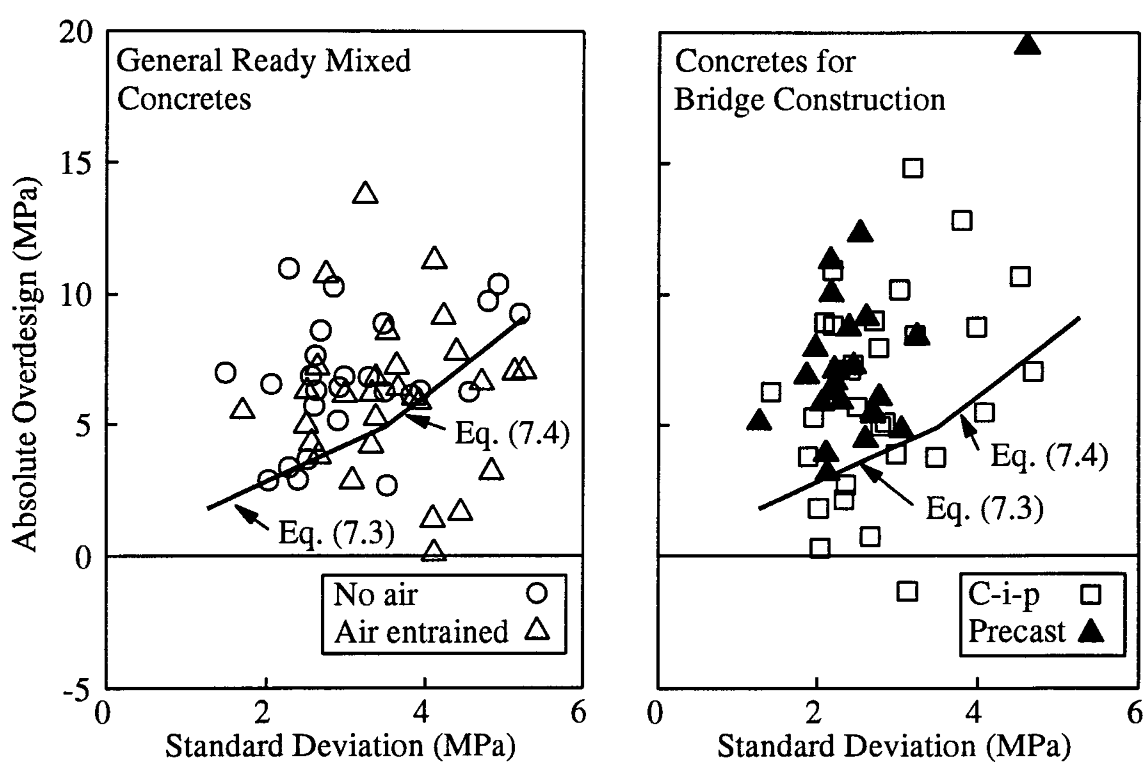

Figure 7.2: Relationship between mix designs and non-mandatory guidelines given in CAN/CSA-A23.1-M90

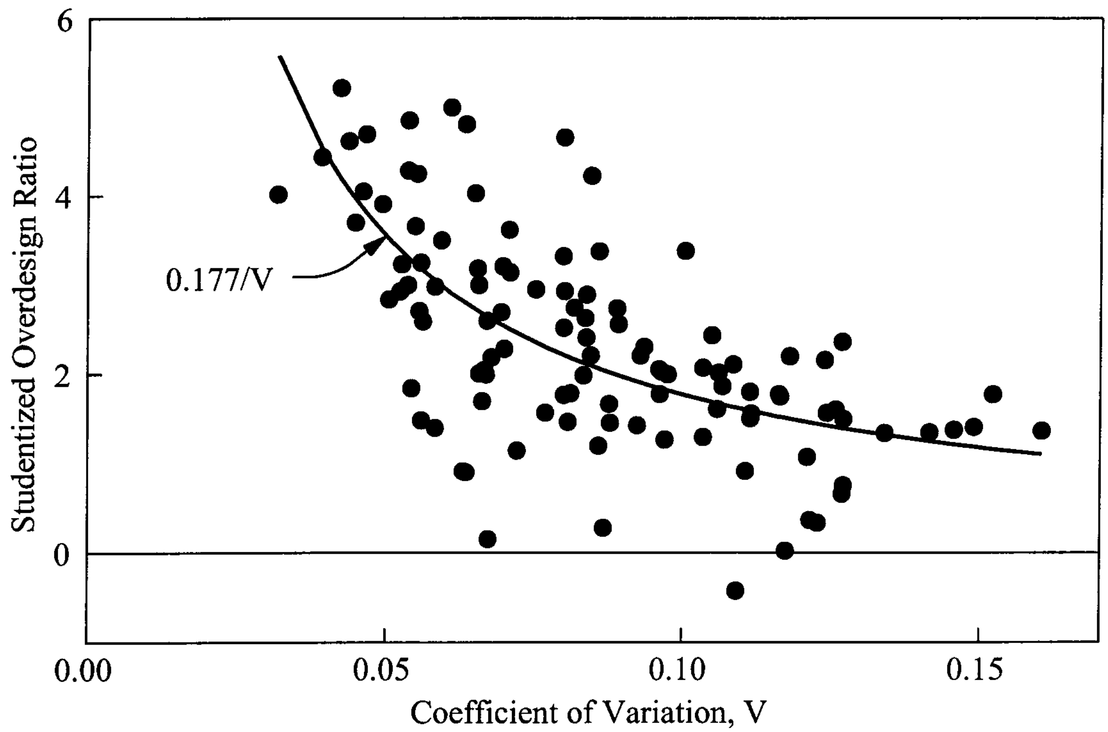

Figure 7.3: Studentized overdesign ratio versus coefficient of variation

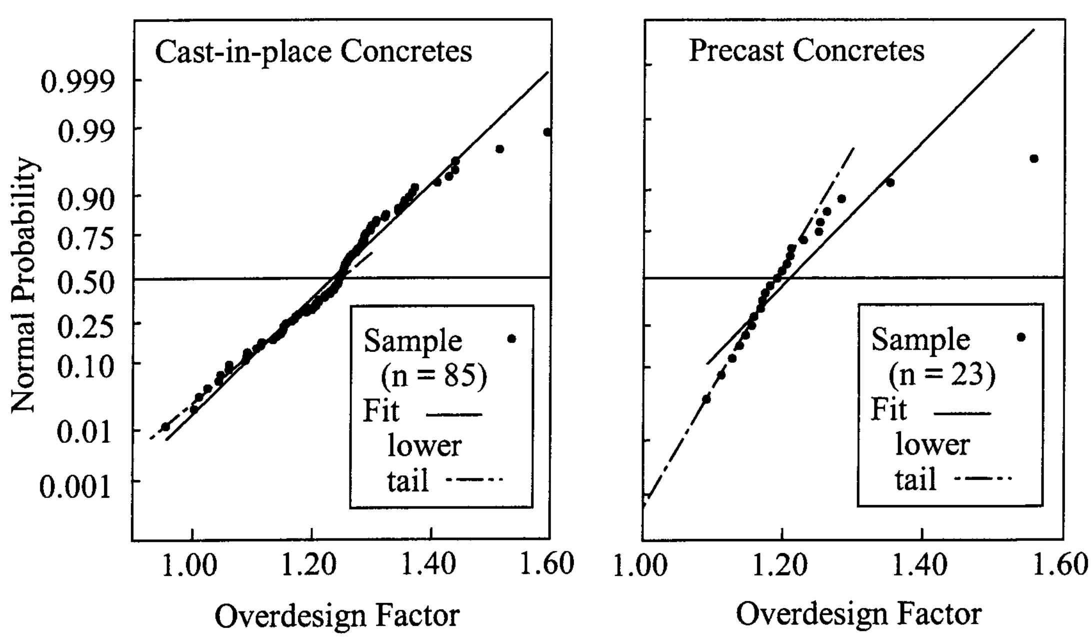

Figure 7.4: Sample cumulative distribution functions for overdesign factors

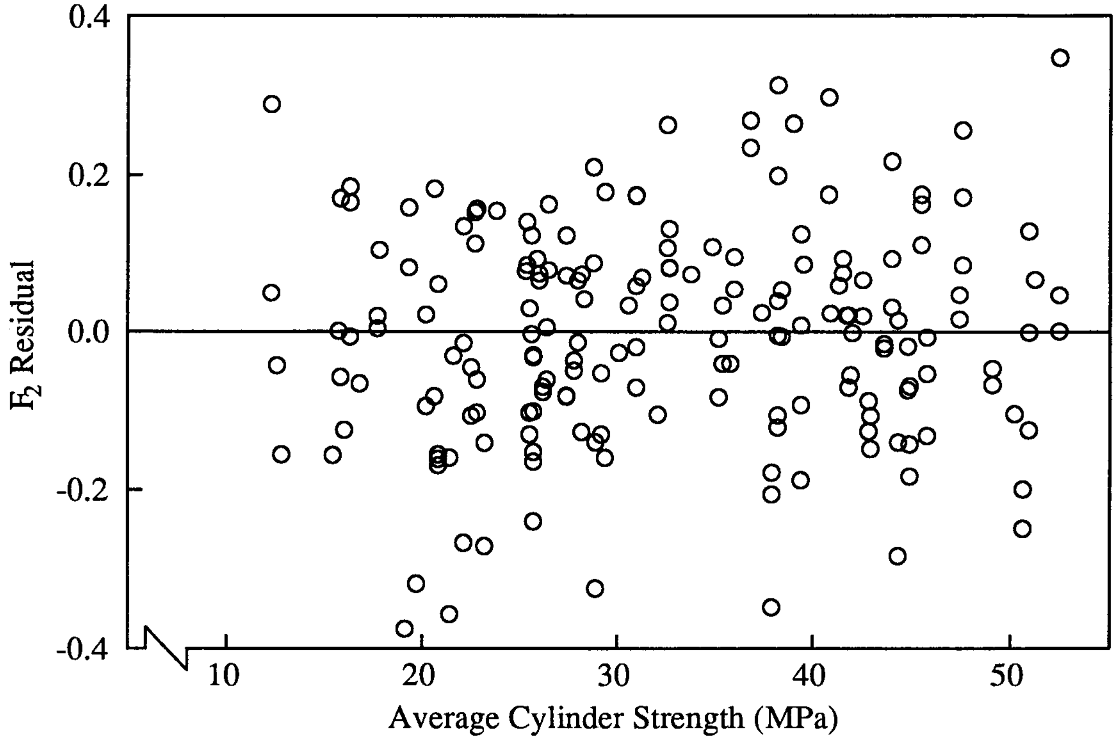

Figure 7.5: F_2 residuals versus average cylinder strength

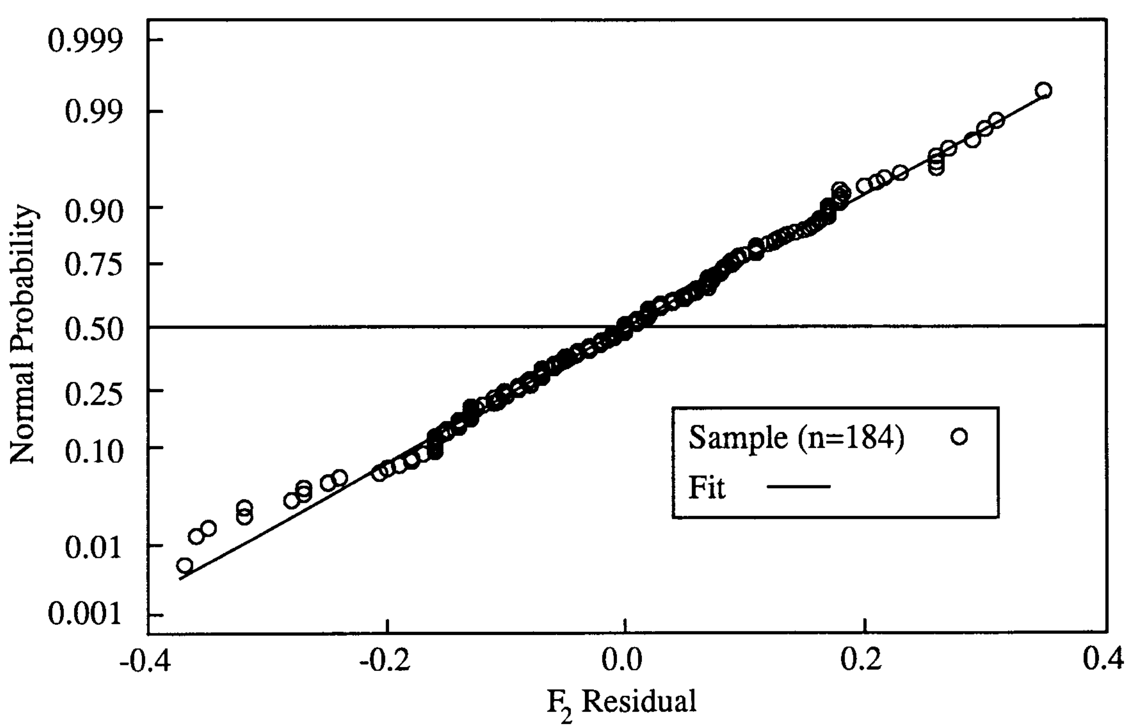

Figure 7.6: CDF for F_2 residuals from lognormal model

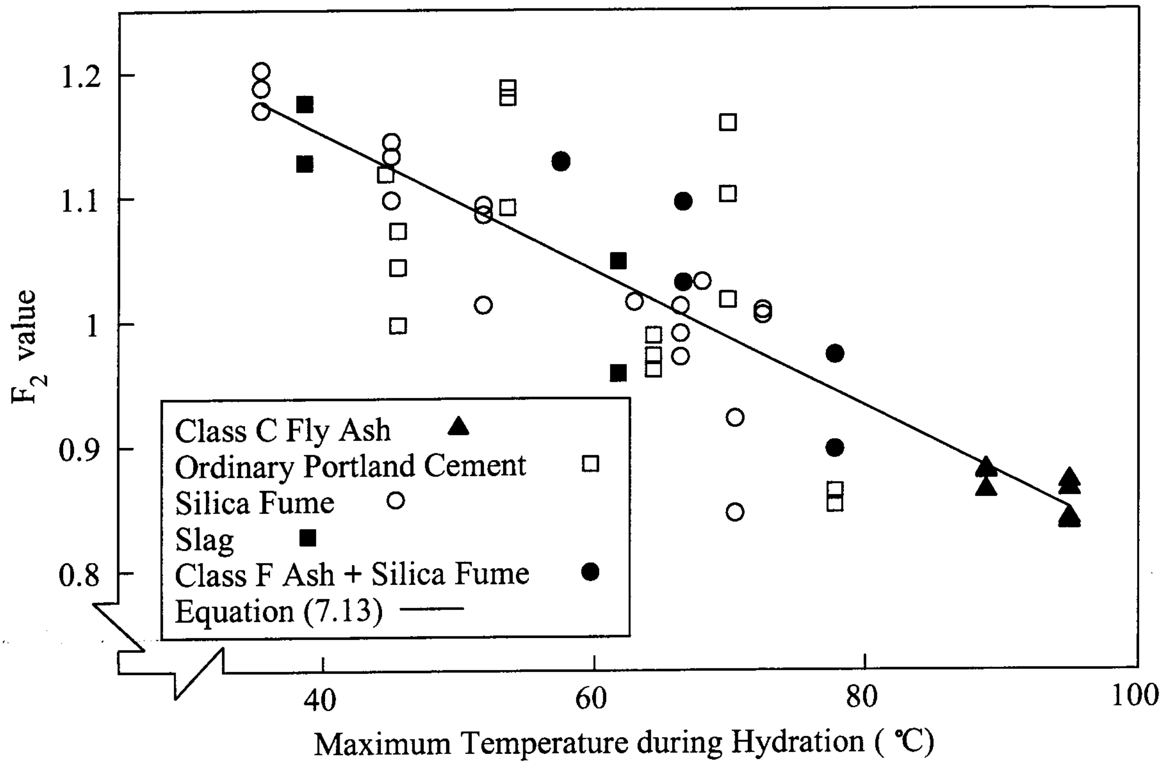

Figure 7.7: Partial regression plot of F_2 on maximum hydration temperature

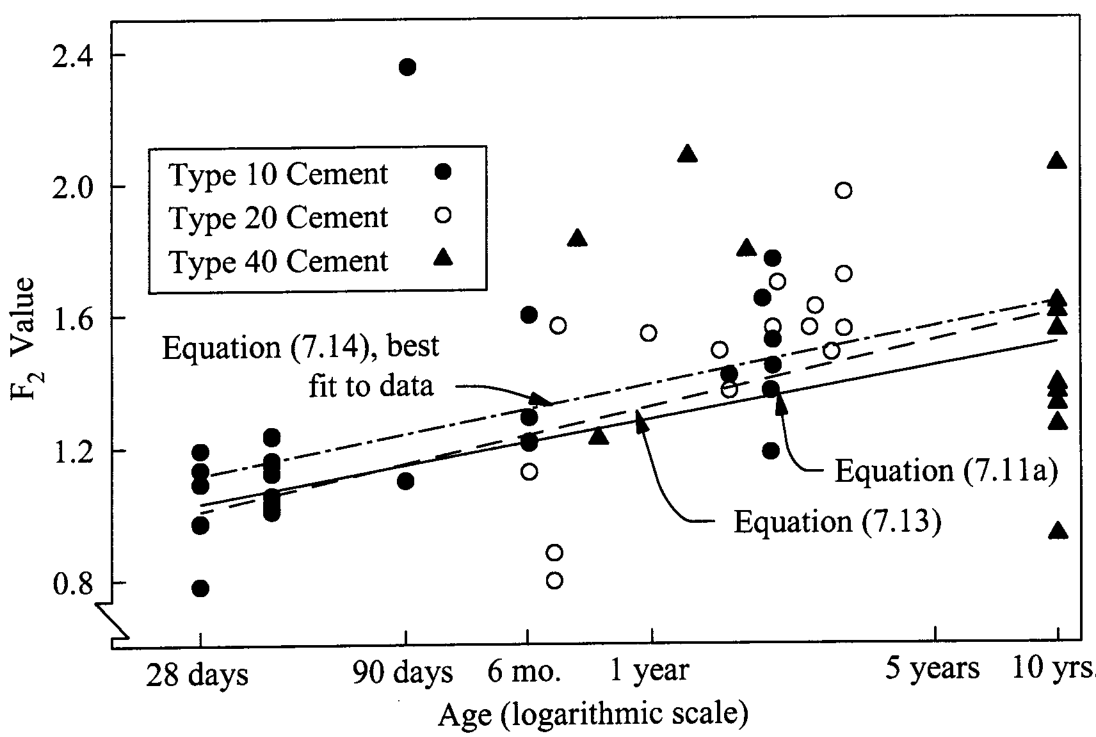

Figure 7.8: Comparison of fitted equations with historic in situ strength data

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Chapter 8: Variation of *In Situ* Concrete Strength in Structures

8.1 Introduction

To assess the safety of a concrete structure it is necessary to determine expressions for the average value and variability of the *in situ* concrete strength. In a companion paper (Bartlett and MacGregor, 1994d [Chapter 7]), relationships between the average *in situ* compressive strength and the specified compressive strength are developed which depend on the age of the concrete and the height of the member. In this paper, the variability of the *in situ* concrete strength is investigated.

To a concrete producer, the variability of concrete strength is represented by the within-batch variation and the batch-to-batch variation of standard cylinder specimens. Within-batch variation is essentially due to the inherent randomness of the strength of different parts of a single batch of concrete. If the standard cylinder test specimens are carefully cast, cured, and tested, then the within-batch variation may be a large component of the within-test variation measured on cylinders cast from a single batch. Batch-to-batch variation can be attributed to factors that may vary between batches, such as the amounts, characteristics and properties of the ingredients and the details of the batching and mixing procedures. According to ACI 214-77 "Recommended Practice for Evaluation of Strength Test Results of Concrete" (ACI Committee 214, 1977), the overall variation of standard test cylinder strengths is related to the batch-to-batch and the within-test variation by the expression:

$$\sigma_T^2 = \sigma_{wt}^2 + \sigma_{bb}^2 \tag{8.1}$$

where σ_T is the overall standard deviation and σ_{wt} and σ_{bb} are the standard deviations due to within-test variation and batch-to-batch variation respectively. ACI 214-77 does not give expected values of the batch-to-batch variation, but it does give ranges of values for the overall standard deviation and the within-test coefficient of variation, V_{wt}. From Equation (8.1), the batch-to-batch variation may therefore be calculated as

$$\sigma_{bb} = \sqrt{\sigma_T^2 - (V_{wt}\bar{f}_{cyl})^2} \tag{8.2}$$

where $\bar{f}_{cyl}$ is the average cylinder strength. In Figure 8.1, the within-test and batch-to-batch standard deviations are shown for cylinders from laboratory trial batches and general construction batches. The values shown for fair and very good degrees of control are calculated from Equation (8.2) using values of σ_T and V_{wt} which are in the middle of the ranges given in ACI 214-77. Clearly, the values in ACI 214-77 imply that most of the variability of the strengths of standard cylinders used for general construction testing is batch-to-batch variation.

However, the variation of the *in situ* compressive strength in structures is not completely described by the variation of standard cylinder strengths. As noted in a companion paper (Bartlett and MacGregor, 1994d [Chapter 7]), methods used to place and consolidate the wet concrete and the temperature, moisture conditions and duration of the curing period also affect the *in situ* strength. Three additional sources of variability must therefore be considered: systematic within-member strength variation, strength variation between different members of a common type, such as columns, and strength variation between different types of members. Systematic within-member variation may occur if the consolidation, water content or curing conditions vary in a consistent manner within the member. Systematic between-member strength variations may also be due to different curing conditions. For example, ambient temperatures for columns on different floors of a building which are cast at different times of the year may vary. The strength variation between different types of members is not considered here because expressions are presented in a companion paper (Bartlett and MacGregor, 1994d [Chapter 7]) which address the average differences between the *in situ* strength of short and tall elements.

When assessing the *in situ* strength variation of concrete in structures, it is conventional to report within-member strength variation and between-member strength variation (eg. Tso and Zelman, 1968, Davis, 1976). The within-member strength variation includes within-batch strength variation, systematic spatial strength variation, and, if the member is cast from more than one concrete batch, batch-to-batch variation. The between-member strength variation may also be inflated by batch-to-batch variation if each member is cast from one batch (or part of one batch). Thus the number of batches used to cast each member must be accounted for so that the effects of within-batch variation, batch-to-batch variation, systematic within-member variation and between-member variation may be uncoupled and individually quantified.

In this paper, a method is presented for assessing the systematic within-member strength variation in members cast from a single batch of concrete. The method is used to analyse data from elements cast in laboratories and elements cast in the field. The effects of batch-to-batch variation on the within-member and between-member variation of field cast elements is also investigated.

8.2 Research Significance

An understanding of the variation of the *in situ* strength of concrete in structures is necessary for the use of probabilistic methods of design and evaluation. This knowledge also facilitates the design of sampling plans which are efficient and unbiased for assessing the strength in an existing structure.

The research also indicates that the strength of concrete can vary significantly throughout a structural member - even a member cast in a laboratory from one batch of concrete.

8.3 Assessment of Systematic Within-member Strength Variation by Regression Analysis

A technique based on regression analysis can be used to isolate the systematic variation of the *in situ* concrete strength in a member if large quantities of data are available. The member must be cast from one batch of concrete. Spatial coordinates corresponding to each strength measurement must be known, and the locations sampled must be distributed uniformly throughout the volume of the member. In this section, the method is described for the case where the *in situ* strength is based on core test data.

The systematic variation of the *in situ* concrete strength is that part of the overall strength variability which is not random but varies systematically throughout the volume of the element. The component of variance which may be attributed to systematic spatial strength variation, σ^2_S, is isolated by taking the difference of two variances:

$$\sigma_S^2 = \sigma_M^2 - \sigma_I^2 \tag{8.3}$$

where σ^2_M is the total within-member variance of the measured *in situ* strength and σ^2_I is the variance of the strength which has been reduced by using regression analysis techniques to account for significant effects due to systematic spatial strength variation.

The analysis is based on the general regression model

$$f_c = \beta_0 + g_1(\beta_i, Z_i) + g_2(\theta_j, x, y, z) + \varepsilon \qquad (8.4)$$

where f_c is the observed core strength, β_0 is an unknown constant determined by regression analysis, and ε is the model error. The function $g_1(\beta_i, Z_i)$ accounts for the effects of various testing factors, Z_i, that affect the measured strength, such as the core moisture condition, diameter, and length to diameter (l/d) ratio. The function $g_2(\theta_j, x, y, z)$ accounts for variation of the *in situ* strength in the x, y, and z directions. The forms of g_1 and g_2, and the values of parameters β_0, β_i and θ_j are generally not known *a priori*.

For the data from each element, the forms of g_1 and g_2 are determined which minimise the model error and give estimates of the parameters which differ significantly from zero. Several iterations of regression analysis may be required to complete this process. (In accordance with the conventional notation used for regression analyses, a hat (ˆ) is used to distinguish between the estimate of an unknown parameter and the unknown parameter itself. Thus $\hat{\beta}_i$ and $\hat{\theta}_j$ are the estimates, as obtained by regression analysis, of unknown parameters β_i and θ_j.) The final fit, which best simulates the actual core strength data, is used to determine predicted core strength values

$$\hat{f}_c = \hat{\beta}_0 + g_1(\hat{\beta}_i, Z_i) + g_2(\hat{\theta}_j, x, y, z) \qquad (8.5).$$

Since g_2 is included in Equations (8.4) and (8.5), any significant effects due to spatial variation are accounted for. Hence $\hat{\sigma}^2_I$, the estimate of σ^2_I, may be determined from the squares of the residual errors

$$\hat{\sigma}^2_I = \frac{\Sigma(f_c - \hat{f}_c)^2}{(n - p)} \qquad (8.6)$$

where n is the number of core strength measurements and p is the total number of parameter estimates, $\hat{\beta}_0$, $\hat{\beta}_i$ and $\hat{\theta}_j$, obtained by regression analysis.

To estimate σ^2_M, the variance of the measured *in situ* strength neglecting spatial variation, the predicted strengths $\hat{f}^*_c$ are determined from

$$\hat{f}^*_c = \hat{\beta}_0 + g_1(\hat{\beta}_i, Z_i) + g_2(\hat{\theta}_j, \bar{x}, \bar{y}, \bar{z}) \tag{8.5a}$$

where the various parameter estimates $\hat{\beta}_i$ and $\hat{\theta}_j$ are identical to those in Equation (8.5) and $\bar{x}$, $\bar{y}$ and $\bar{z}$ are the average of the x, y, and z values for all locations. This essentially assumes that the location corresponding to each strength is unknown and must therefore be represented by its best estimate, which is simply its average value. The value of σ^2_M is estimated from the sum of the squared residual errors

$$\hat{\sigma}^2_M = \frac{\Sigma(f_c - \hat{f}^*_c)^2}{(n - p^*)} \tag{8.6a}$$

where p^* is the number of non-zero values of $\hat{\beta}_0$ and $\hat{\beta}_i$.

If there is no systematic spatial variation of the *in situ* concrete strength, the parameter estimates $\hat{\theta}_j$ are not significantly different from zero and therefore are not included in the model. In this case, the final forms of Equations (8.5) and (8.5a) are identical, $\hat{\sigma}^2_M = \hat{\sigma}^2_I$, and, from Equation (8.4), $\sigma^2_S = 0$.

If a consistent testing procedure has been followed for all data, the function $g_1(\beta_i, Z_i)$ will not be included in the model. In this case, $\hat{f}^*_c$ and $\hat{\sigma}^2_M$ are simply the mean and variance of the measured strengths. However, if the final fit includes estimates of β_i and θ_j which differ significantly from zero, $\hat{f}^*_c$ and $\hat{\sigma}^2_M$ are not necessarily the same as the values that would be obtained by regressing the strengths on the function $f_1(\beta_i, Z_i)$.

8.3.1 Example - Series 183E Column Investigated by Bloem

To illustrate the method, the variation of concrete strength over the height of the Series 183E column tested by Bloem will be investigated. Cores from this column were originally obtained to investigate the effects of drilling direction, capping material, and alignment in the testing machine on the core strength (Bloem, 1965). Mimeographed tabulations of the individual core strength data were kindly made available for the present investigation by Meininger of the National Aggregate Association / National Ready Mixed Concrete Association.

A single column, 12 inches (300 mm) by 24 inches (600 mm) in plan by 7 feet (2130 mm) high, was cast and cured under burlap kept moist with slowly dripping water for seven days. It was then sawn into seven 12 inch long blocks, which were stored in a moist room. When the concrete was 28 days old, 4 inch (100 mm) diameter by 12 inch long cores were drilled from each block, in both horizontal and vertical directions. Each core was sawn to give a 4 inch long specimen and an 8 inch long specimen. After seven days storage in the moist room, the specimens were capped with either lumite cement or sulphur and either carefully centred in the testing machine or placed 1/4 inch (6 mm) off centre when tested. The length, strength, location, orientation, capping material and testing condition for each of the 112 cores tested are shown in Appendix 8A.

The form of $g_1(\beta_i, Z_i)$ was chosen to account for the effects of the drilling orientation, capping material, length to diameter ratio and test eccentricity variables:

$$g_1(\beta_i, Z_i) = \beta_1 Z_{HV} + \beta_2 Z_{cap} + \beta_3 Z_{l/d} + \beta_4 Z_{cen} \quad (8.7)$$

where $Z_{HV} = 0$ for cores with axes perpendicular to the direction of casting or 1 for cores with axes parallel to the direction of casting, $Z_{cap} = 0$ for cores with lumite caps or 1 for cores with sulphur caps, $Z_{l/d} = 0$ for cores with $l/d = 2$ or 1 for cores with $l/d = 1$ and $Z_{cen} = 0$ for cores centred in the test machine or 1 for cores offset 1/4 inch from the centre of the platens.

The model for $g_2(\theta_j, x, y, z)$ was selected, after reviewing scatter plots, to be of the form

$$g_2(\theta_j, x, y, z) = \theta_1 z' + \theta_2[z'^2 - (z' - 0.25)_+^2] + \theta_3(z' - 0.75)_+^2 \quad (8.8)$$

where z' is the distance from the bottom of the column to the core location, divided by the height of the column. Equation (8.8) is a spline model which accommodates parabolic strength variations over the bottom quarter and top quarter of the column length, and a linear variation over the middle half of the column length. The values of $g_2(\theta_j, x, y, z)$ and the first derivative of $g_2(\theta_j, x, y, z)$ for the parabolic and linear relationships are forced to be identical at the transition points z' = 0.25 and z' = 0.75. There are therefore no gaps or kinks in the relationship at these points. These continuity restrictions are enforced by the expressions $(z'-.25)_+$ and $(z'-.75)_+$, where the subscript "+" indicates that if the expression within the parentheses yields a negative result, a value of zero is assumed.

Linear regression analysis of the full model yields

$$\hat{f}_c = 4000 - 91Z_{HV} - 195Z_{cap} + 604Z_{l/d} - 65Z_{cen} - 3519z'$$

$$+6875[z'^2 - (z' - 0.25)_+^2] - 16210(z' - 0.75)_+^2 \qquad (8.9)$$

with all parameter estimates significantly different from zero at the 95% level and R^2 equal to 0.86. The variation of the observed strengths and the predicted values along the height of the column are shown in Figure 8.2. To isolate the strength variation over the column height, the data shown in Figure 8.2 have been adjusted, using the parameter estimates from Equation (8.9), to equivalent strengths for vertical cores with l/d = 2 which have lumite caps and are carefully centred under the loading platens.

The fit has a standard error of 179 psi (1.23 MPa) which, since the effects of spatial variation are included in Equation (8.9), corresponds to $\hat{\sigma}_I$ in Equation (8.6). If the individual values of spatial variables z', $[z'^2 - (z'-.25)_+^2]$, and $(z'-.75)_+^2$ are replaced by their average values, the estimate of the standard deviation of the measured *in situ* strength neglecting spatial variation, $\hat{\sigma}_M$, is 307 psi (2.11 MPa). Thus, from Equation (8.3), the component of the standard deviation which may be attributed to systematic spatial variation of the *in situ* strength is 249 psi (1.71 MPa).

8.3.2 Limitations of the Method

A necessary condition for accurate assessment of the systematic *in situ* strength variation using regression analysis is that the full model, Equation (8.4), must be correct. In particular, this implies that the selected form of $g_2(\theta_j, x, y, z)$ must be correct.

It is always possible to fit an equation with n unknown constants to n points exactly. Thus an artificially "good" fit with an apparently large R^2 may be achieved if $g_2(\theta_j, x, y, z)$ includes too many of the higher order terms in x, y and z. This causes more of the strength variability to be attributed to spatial variation than is really the case, since the $\hat{\sigma}_I^2$ term in Equation (8.3) is underestimated. The overspecification of $g_2(\theta_j, x, y, z)$ is prevented to some extent if any of the parameter estimates $\hat{\theta}_j$ which are not significantly different from zero are removed from the model.

Similarly, if too few parameters are included in $g_2(\theta_j, x, y, z)$ the strength variability attributed to spatial variation will be erroneously underestimated. For example, if a linear variation of strength over the full height of Bloem's Series 183E column is assumed, the fitted full model is:

$$\hat{f}_c = 3792 - 99Z_{HV} - 195Z_{cap} + 631Z_{l/d} - 41Z_{cen} - 670z' \qquad (8.10)$$

with all parameters significantly different from zero and R^2 equals 0.76. The parameter estimates corresponding to variables Z_{cap}, Z_{cen}, Z_{HV} and $Z_{l/d}$ are similar to those in Equation (8.9), so the values of $\hat{\sigma}_M^2$ from both models are similar. However, as indicated by the partial regression plot shown in Figure 8.2, the assumed linear strength variation from Equation (8.10) does not fit the data as well as the parabolic-linear-parabolic spline fit from Equation (8.9). The standard error associated with Equation (8.10) of 228 psi yields σ_S = 198 psi from Equation (8.3), which is less than the value of 229 psi obtained previously using the standard error associated with Equation (8.9).

A second necessary condition for using the method is that the data points must be uniformly distributed through the volume of the element. Referring again to Figure 8.2, it is clear that the effect of spatial variation would be underestimated if a majority of the cores were obtained from the middle half of the column. If a majority of the cores were obtained from the top and bottom quarters of the column, the effect of spatial variation would be overestimated.

8.4 Within-member Strength Variation in Laboratory Cast Elements

An investigation of systematic within-member strength variation in 69 elements cast in various laboratories has been carried out using the regression analysis technique. Between 6 and 122 cores were obtained from each element to assess the *in situ* strength. A distinction has been made between data from columns, which may have appreciable systematic strength variation over their height, and other elements, such as beams, blocks, slabs or walls, which may have appreciable systematic strength variation along their length or width. In nearly all cases, the set of cores obtained from each element was drilled from a region cast from a single batch of concrete.

8.4.1 Columns

The *in situ* strength of concrete in a column often varies systematically over its height, with the strength at the bottom larger than the strength at the top. Petersons (1964) observed lower *in situ* strengths in the top 600 mm of his column specimens but noted no correlation with the density of the concrete and so concluded that only a small part of the strength loss should be attributed to segregation. Conversely, others have attributed the strength variation to the effects of segregation and bleeding which cause a greater concentration of the lighter components near the upper surface of the concrete (Concrete Society, 1976). In the present investigation, the nature and extent of the strength variation over the column length is investigated, using core strength data from 43 vertically cast columns investigated by Petersons (1964), Bloem (1965) and Tso and Zelman (1968).

Petersons' specimens, 300 mm square by 3000 mm high, were removed from the forms 24 hours after casting and then totally immersed in a shallow water bath for 25 days. The columns were then cut into 300 mm segments and the ends of the segments were ground smooth. Cores with diameters of 150 mm were drilled in a direction parallel to the original column axis and tested when the concrete was 28 days old.

Bloem's two Series 183C columns were 8 inches (200 mm) by 26 inches (660 mm) by 10 feet (3050 mm) high. One was cured 7 days under moist burlap and was then left in the forms until coring; the other was simply left exposed in the laboratory after the concrete was one day old. When the concrete age was 91 days, 4 inch diameter cores were drilled through the 8 inch dimension, capped, soaked in water for two days, and tested. The details of Bloem's other column specimen, Series 183E, have already been presented above.

The twelve columns cored by Tso and Zelman were 12 inches (300 mm) square by 10 feet (3050 mm) tall. After casting, they were protected by wet burlap for three days and then left exposed in the laboratory. When the concrete was older than 28 days, the columns were cut into 12 inch segments and 6 inch diameter cores were drilled parallel to the column axis. The age, capping procedure, and moisture condition of the cores at the time of testing were not reported.

Core strength data from each individual column were analysed using a model with the general form of Equation (8.8):

$$g_2(\theta_j, z') = \theta_1 z' + \theta_2\left[z'^2 - (z' - t_1)_+^2\right] + \theta_3(z' - t_2)_+^2 \tag{8.11}$$

where z' is the distance from the bottom of the column to the centre of gravity of core divided by the column height. This model accommodates possible quadratic variation of the strength at the ends of the column and linear variation in the middle. The strength and its rate of change are consistent at the transition points t_1 and t_2 which were selected after reviewing scatter plots. It cannot be determined whether the strength variation over the column height correlates better with the absolute distance or the relative distance from the base of the column because the columns investigated were of similar heights.

The dependence of the *in situ* strength on the normalized distance from the bottom of the column was found to be statistically significant in 32 of the 43 columns investigated. The fitted relationships shown in Figure 8.3 indicate the wide range of variation encountered. A few columns, such as Petersons' specimen E3, had strengths which increased linearly from the top of the member to the bottom. Others had a sharp reduction in strength at the top as exhibited by Petersons' specimen E1, a sharp increase in strength at the bottom, as exhibited by Tso and Zelman's specimen 32, or both.

Data from 29 columns, each of which provided at least 8 test cores, were used to estimate the component of the standard deviation attributable to systematic variation of the *in situ* strength, $\hat{\sigma}_S$. Data from the other 14 columns were not included in the analysis since fewer than 8 cores were obtained from each column and so the quantity of data was insufficient to estimate $\hat{\sigma}_S$ accurately. Significant spatial variation of the *in situ* strength was noted in 26 of the 29 columns as indicated by the non-zero values of $\hat{\sigma}_S$ shown in Table 8.1.

To investigate the overall average *in situ* strength variation, data were normalized by dividing each core strength by the average strength of the cores from the middle of the column ($0.3 \leq z' \leq 0.7$). The normalized data were pooled, after reducing the quantity of data from Bloem's columns to be roughly equivalent to that from each of the other columns. The model which best fit the pooled data had no strength variation through the middle half of the column length and quadratic strength variation in the upper and lower ends

$$\hat{f}_{cn} = 1.138 + 2.138[z'^2 - 0.5z' - (z' - 0.25)_+^2] - 3.611(z' - 0.75)_+^2 \qquad (8.12)$$

where $\hat{f}_{cn}$ is the normalized core strength. This fit has R^2 equal to 0.35 and a standard error of 0.061, which reflect the considerable scatter of the normalized core strength values. The fit given by Equation (8.12) is fairly robust, since similar parameter estimates were obtained from analyses of subsets of the data from which the results of one or more of the published investigations were neglected.

The variation of *in situ* strength predicted by Equation (8.12) is shown in Figure 8.4. The observed mean normalized core strengths and the 95% confidence limits on these sample mean values are also shown. The scatter of the individual values of the normalized strengths is much larger than that implied by the confidence intervals for the mean normalized strengths shown in Figure 8.4. The correspondence between the sample averages and the mean values predicted using Equation (8.12) indicates that the fit is quite good, except perhaps at z' equal to 0.4 and 0.8. Given the wide variety of curves shown in Figure 8.3 which simulate the strength variation in individual columns, it must be recognised that Equation (8.12) represents the variation in an average column only and may not be a good representation for any individual column. However, on average, the strength is constant throughout the middle half of the column and ranges from 86% of the midheight value at z' = 0.95 to 109% of the midheight value at z' = 0.05. Greater deviations from the midheight values are predicted by Equation (8.12) at the ends of the columns, but these predicted values require extrapolation outside the range of data used to determine Equation (8.12) and so may be inaccurate.

Core density data from Petersons' columns and Bloem's Series 183C columns were also analysed. Scatter plots of strength versus density for cores from individual columns indicated slight positive correlation in some cases and no correlation in other cases. To investigate the overall average density variation, the data were normalized by dividing each core density by the average density of cores from the middle of the column ($0.3 \le z' \le 0.7$). The variation of normalized density over the column height is shown in

Figure 8.5. The linear fit to these data is

$$\hat{\gamma}_n = 1.0027 - 0.00629z' \tag{8.13}$$

with both parameters significant, R^2 equals 0.1, and a standard error of 0.0051. The predicted normalized density, $\hat{\gamma}_n$, decreases slightly from the bottom to the top of the column, which corroborates the notion that strength and density are proportional. However, the constant of proportionality corresponding to a 13% change in strength for a 0.5% change in density between z' = 0.1 and z' = 0.9 seems unrealistically large. This confirms Petersons' view that the change in density is not likely the dominant reason for the change in strength.

8.4.2 Elements Subjected to Variable Temperatures during Hydration

It is shown elsewhere (Bartlett and MacGregor, 1994d [Chapter 7]) that as the maximum temperature sustained during hydration increases, the *in situ* concrete strength decreases. In large elements, because the maximum temperature sustained during hydration varies throughout the volume of the element (Cook et al., 1992), there may be a systematic variation of the *in situ* strength. To investigate this effect, the strengths of 4 inch diameter cores from six 4 foot (1220 mm) cubes reported by Burg and Ost (1992) are analysed.

The experimental procedures are reported in detail by Burg and Ost (1992) and need only be briefly summarised here. Each cube was cast from a single batch of superplasticized high strength concrete. The top and bottom surfaces of the cubes were insulated by layers of closed cell polystyrene, and thermocouples were placed along a horizontal plane through the middle of each block to monitor the temperature as hydration progressed. Horizontally drilled core specimens were obtained after 3 months and again after one year. A plan view of a typical block, indicating the thermocouple locations and core locations, is shown in Figure 8.6 (a). Raw data including the strength and location of each core were made available for the present study through the kind cooperation of Burg.

The core strengths were analysed using two separate regression models. Both models included a regressor to indicate the age of the cores, Z_a, which was set equal to 0 for the 3 month old specimens or 1 for the year old specimens. Both models also included a regressor, y, representing the vertical distance from the middle of the block to the axis of the core. One model considered the plan coordinates, x and z, of the centre of gravity of the core as shown in Figure 8.6 (a). The other accounted for the peak temperature, T_{max}, that was reached at the centre of the core during hydration. The method for estimating the temperatures at the core locations from the temperatures recorded by the thermocouples is given in Appendix 8B.

Results of the analyses, with only the statistically significant variables included in the equations, are shown in Table 8.2. Blocks 2 and 3 do not have a statistically significant correlation between the core strength and either the core plan coordinates or the peak temperature due to hydration. However the peak temperature is a significant factor for the other four blocks, and the distance between the drilling surface and the centre of gravity of the core, z, is a significant factor in three blocks. For the models where the peak temperature is significant and therefore is included, the distance from the drilling surface to the centre of gravity of the core is not a significant factor, and vice versa. The standard errors for the two fitted models are similar for each block, suggesting that the precision of the models are equivalent. Thus the systematic spatial variation may be partially attributed either to the distance between the core centre of gravity and the drilling surface, or to the peak temperature.

A partial regression plot of core strength versus peak temperature reached during hydration for the cores from Block 1 is shown in Figure 8.6 (b). For blocks 1, 4, 5, and 6, which have significant correlation between the core strength and the temperature, the strength loss averages roughly 3.5% for every 10°F increase. This average strength change is similar to the value of -2.8% per 10°F obtained from an analysis of the *in situ* strength of cores from 28 elements cast for four separate investigations (Bartlett and MacGregor, 1994d [Chapter 7]). The two analyses, which both include the data reported by Burg and Ost, differ slightly. In the present investigation, the strength loss is correlated to the maximum temperature at the centre of each core. In the other analysis, the strength loss is correlated to the maximum temperature as measured at the centre of the element. Since the results of the two analyses are similar, it is reasonable to conclude that systematic spatial variation of the *in situ* strength of concrete may be attributed to variation of the maximum temperature which occurs during hydration within the volume of the element.

8.4.3 Other Elements

Core strength data from 26 laboratory cast beams, blocks, and walls were analysed to determine the systematic *in situ* strength variation within each element. The strength data included 2 and 4 inch diameter cores from beams cast from high performance concrete by Alca (1993), 4 inch diameter cores from walls cast by Meininger et al. (1977), and 2, 4, and 6 inch diameter cores from a wall and a slab cast by Meininger (1968). Models used to isolate the *in situ* strength variation in these elements are documented elsewhere (Bartlett and MacGregor, 1994 a-c [Chapters 2, 3 and 5]). The data set also included the cores from the blocks cast by Burg and Ost (1992) which were analysed as described in the previous section. In almost all cases, cores were not obtained from regions adjacent to the top surface of each element but were uniformly distributed through a region located midway between the top and bottom surfaces.

The standard deviation due to systematic *in situ* strength variation in the 26 elements is shown in Table 8.3. Systematic spatial strength variation is statistically significant in 20 of the 26 elements. In each case where the strength varied significantly over the depth of the element, the concrete near the top of the element as placed was weaker than the concrete near the bottom.

8.4.4 Summary of Systematic Within-member Strength Variation in Laboratory Elements

Multiple linear regression analysis methods were used to investigate the variation of the values of $\hat{\sigma}_S$ obtained for laboratory cast elements with the other variables shown in Table 8.3. The standard deviation due to systematic *in situ* strength variation is not significantly dependent on the volume of the element or the age of the core, but is strongly correlated with the average concrete strength. It may therefore be more appropriate to represent the spatial strength variation by a coefficient of variation instead of a standard deviation. The average coefficient of variation due to *in situ* strength variation, V_s, is 4.7% for the 29 columns shown in Table 8.1, and 3.0% for the 26 other elements shown in Table 8.3. If only the elements with statistically significant spatial variation are considered, the average values of V_s are 5.2% for the columns and 3.9% for the other elements.

The systematic *in situ* strength variability in the columns is due to the difference between the strength at the top or bottom of the column and the strength at midheight, as suggested by Equation (8.12). The values of $\hat{\sigma}_S$ shown in Table 8.3 for the other elements do not include this variability, since cores were generally not obtained from near the top or bottom surface of each element. Since it is probable that the *in situ* strength is less near the top surface of any element (Concrete Society, 1976), the true values of V_s for these elements are probably larger than shown in Table 8.3 and may be conservatively estimated as the square root of the sum of the squares of the average V_s values for columns and the V_s values shown in Table 8.3. This estimated value ranges from $V_s = 5.6\%$ if all the data are considered, to $V_s = 6.5\%$ if only data from those columns and other elements with statistically significant spatial variation are considered.

8.5 Within-member Strength Variation in Structures

In this section, systematic variation of the *in situ* strength of concrete in structures is assessed to see whether it differs appreciably from that observed in elements cast in the laboratory. Typically the available data are nondestructive testing results, such as ultrasonic pulse velocities or measured pull-off loads, instead of core strengths. Uncertainty in the conversion from these nondestructive test values to the actual *in situ* strengths causes any inferences about the *in situ* strength variation in these data to be less certain.

8.5.1 Members Cast from One Batch: Building Columns

In Table 8.4, the average ratios of the strength of concrete at the top or bottom of a column to the corresponding strength at the midheight are shown for data from 4 published investigations. Tso and Zelman (1968) and Drysdale (1973) reported ultrasonic pulse velocity measurements in buildings of various ages which have been converted to equivalent compressive strengths using the factors derived in Appendix 8C. Murray and Long (1987) reported strength ratios based on pull-off tests of columns in a parking structure under construction. Davis (1976) reported strength ratios based on ultrasonic pulse velocity measurements for columns which were 28 and 91 days old. Generally each member in each investigation was sufficiently small to have been cast from a single batch of concrete.

Ranges of the normalized strength values predicted using Equation (8.12) for regions at the top and bottom of the column are also shown in Table 8.4. The bases for comparing these predicted values directly with the observed values are imperfect, because the definitions of "top" and "bottom" may be slightly inconsistent in the various field studies and because uncertainty is introduced when the nondestructive test measurements are converted to equivalent strengths. However, the field data summarised in Table 8.4 indicate that the average strength variation over the height of individual columns in structures is similar to that for columns cast in the laboratory.

Approximate values of the average overall standard deviations of the strengths within individual field columns are shown in Table 8.5. The values combine the effects of inherent randomness and any systematic variation of the *in situ* strength within a column, and are therefore analogous to the σ_M values shown in Table 8.1. For a direct comparison to be made, it is necessary to convert the reported σ_M values to equivalent standard deviations of core strengths in psi. For the Tso and Zelman data, the correction factor is 73.8 psi (core) per 100 feet/second (UPV reading) as derived in Appendix 8C. For the other data, a conversion factor of 110 psi (core) per MPa (cube) has been assumed to account for the different units of stress and the differences in strength between a drilled core and a cast cube. The equivalent within-member standard deviations shown in Table 8.5 range from 200 to 560 psi. This is within the range of 125 to 580 psi for the various columns shown in Table 8.1.

This brief study suggests that the characteristics of the laboratory cast columns which were analysed in the previous section are reasonably representative of columns cast in the field. The data shown in Tables 8.4 and 8.5 indicate that the average strength variation over the column height, and the overall within-member strength variation, are reasonably consistent for the laboratory cast and field cast columns.

8.5.2 Members Cast from More than One Batch: Bridge Girders

The *in situ* strength variation of the concrete in the girders of the Barons Bridge can be assessed using ultrasonic pulse velocity measurements reported by Mikhailovsky and Scanlon (1985). The three-span bridge, constructed in 1950, consists of a cast-in-place slab on 5 cast-in-place girders with spans of 40, 60 and 40 feet. The girders are continuous over the two interior supports. They project 44 inches below the slab soffit at the interior supports and 14 inches below the slab soffit at the middle of the centre span. The concrete was batched on site using a concrete mixer with a capacity of one sack that likely produced about 7 cubic feet of concrete per batch. The girders were cast individually from one end of the bridge to the other, each girder pour lasting for a day or two. A pour break is located at the slab soffit.

In the mid 1980's, the bridge was taken out of service. At that time, Mikhailovsky and Scanlon laid out a 14 by 14 inch square grid on the webs of the 5 girders in the centre span and measured ultrasonic pulse velocities at 92 points through the web of each girder. The number of measurements along a vertical grid line varied from one throughout the middle third of the span to three in the haunched regions at the supports. Cores were also obtained. Factors for converting the ultrasonic pulse velocities to equivalent *in situ* core strengths are derived in Appendix 8C.

To assess the *in situ* strength variation in the girders, a model was chosen which allows for quadratic variation of the strength along the girder length and independent linear variation over the girder depth at the North and South ends of the span. The form of the model is

$$f_c = \beta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 y_N + \theta_4 y_S + \varepsilon \qquad (8.14)$$

where f_c is the equivalent core strength estimated from the UPV measurements, x is the coordinate in feet of the vertical gridline from the span centreline and y_N and y_S are the distances in feet from the slab soffit to the horizontal gridline at the North and South ends of the span respectively. Unknown parameters β_0 and θ_1 to θ_4 are estimated by regression analysis, and ε is the model error.

Significant systematic *in situ* strength variation is present in each girder, as indicated by the fitted equations shown in Table 8.6. The general trend is for the strength at midspan to be less than the strength at the ends. The difference between the predicted strength values at these locations ranges from 300 psi (2 MPa) in Girder B to 940 psi (6.6 MPa) in Girder D. The average coefficient of variation due to within-girder spatial variation is 4.3%, which is close to the average value of 3.9% for the 20 elements in Table 8.3 which have significant spatial strength variation.

The average within-member coefficient of variation of the strengths for the girders is 9.9%, which is larger than the value of 6.9% noted for the laboratory cast elements shown in Table 8.3. Batch-to-batch variation is the reason for this difference. The elements listed in Table 8.3 were each cast from one batch and the within-member strength variation is not affected by batch-to-batch variation. Roughly 24 batches were required for each girder, so the within-member strength variation of the girders is considerably inflated by batch-to-batch variation.

8.6 Between-member Strength Variation in Structures

As noted above, the relationship between the between-member variability of the *in situ* strength in structures and the batch-to-batch variability of the concrete strength depends on the number of batches required to cast each member. To demonstrate this relationship further, extreme cases where the batch-to-batch variability inflates either the within-member variation or the between-member variation can be investigated.

The data shown in Table 8.6 reflect the case where a single member is cast from many batches and therefore the batch-to-batch variability inflates the within-member variation. It can be further demonstrated that batch-to-batch variability does not increase the between-member variation. From Table 8.6, the average within-member standard deviation of the strength is 495 psi. This value includes an average standard deviation due to systematic variation of 215 psi. By rearranging Equation (8.3), the average standard deviation of the "inherently random" part of the strength variation, σ_I, is found to be 446 psi. If it is assumed that this is mostly batch-to-batch variation, the standard deviation of the mean strength of a girder is roughly $446/\sqrt{24} = 91$ psi, since roughly 24 batches are required for each girder. This value is quite close to the sample standard deviation of the 5 mean girder strengths shown in Table 8.6, which is 114 psi. The slight difference may be due to systematic between-member variation, since the trend of the average strengths is highest for the inside girder and lowest for the outer girders. Clearly however the between-member variation for this case is not greatly affected by batch-to-batch variation.

The data shown in Table 8.7 reflect the case where each single member is cast from one batch and therefore the batch-to-batch variability inflates the between-member variation. The data are from ultrasonic pulse velocity measurements taken at 5 different elevations over the height of various building columns reported by Tso and Zelman (1968). The units of the various sample standard deviations shown are pulse velocity in hundreds of feet per second, but since the standard deviations are fairly small relative to the sample mean values, it can be assumed that they are proportional to the standard deviations in strength units. Since five measurements are taken per column, the standard deviation of the mean strengths should roughly equal the within-member standard deviation, s_M divided by $\sqrt{5}$. The between-member variation, s_B, which is the observed standard deviation of the mean column strengths, exceeds $s_M/\sqrt{5}$ considerably due to the effect of batch-to-batch variation on the between-member strength.

8.7 Summary and Conclusions

The variation of *in situ* strength in a structure is due to within-batch variation, batch-to-batch variation, systematic within-member strength variation and systematic between-member strength variation. Batch-to-batch variation is often particularly significant, and may either inflate the within-member variation if each member is cast from many batches, or inflate the between-member variation if each member is cast from a single batch. In order to evaluate accurately the overall variation of the *in situ* strength in existing structures, it is therefore necessary to know the number of batches represented by the selected sampling locations.

Multiple regression analysis techniques have been used to assess the variation of the *in situ* strength of concrete in both laboratory specimens and structures cast from one batch of concrete. The investigation of laboratory cast elements considered 585 cores from 43 columns with average strengths ranging from 2150 to 5250 psi (15 to 36 MPa) and 1062 cores from 26 blocks, beams, slabs, and walls with average strengths ranging from 2300 to 17400 psi (16 to 120 MPa). The investigation of structures considered ultrasonic pulse velocity and pull off test results from columns in 29 buildings and 5 bridge girders.

The following conclusions appear warranted:

1. Significant *in situ* strength variation along the length was detected in 32 of 43 columns. Typically the top region was 3 to 14% weaker than the region in the middle and the bottom region was 3 to 9% stronger than the region in the middle. A general

relationship for the average strength variation through the column is given by Equation (8.12), but does not reflect the unique relationships describing the variation in individual columns.

2. On average, densities near the top of a column are slightly but significantly lower than those near the bottom. The range of the density variation observed seems insufficient to be the primary cause of the strength variation.

3. In 4 of 6 high strength concrete blocks investigated, core strengths were significantly negatively correlated to the peak temperature sustained during hydration. For the 4 blocks with significant correlation, the strength loss averaged roughly 3.5% for every 10°F (6.3% per 10°C) increase in peak temperature at the core location.

4. Significant systematic *in situ* strength variation was detected in 20 of 26 beams, blocks, slabs, and walls. In each case where the strength varied significantly over the depth of the element, the concrete near the top of the element as cast was weaker than the concrete near the bottom.

5. The coefficient of variation associated with the systematic spatial variation of the *in situ* concrete strength is on average about 5% for columns or 6% for other elements including slabs, walls, beams and blocks.

6. No evidence was found, from the brief investigation of building columns and bridge girders, to suggest that Conclusions 1 and 5 are invalid for elements in real structures.

Table 8.1: Systematic Within-member Strength Variation in Laboratory Cast Columns

Reference	Element	n	$\bar{f}_c$ (psi)	Vol. (ft^3)	age (days)	h (ft)	$\hat{\sigma}_M$ (psi)	V_M (%)	$\hat{\sigma}_S$ (psi)	V_S (%)
Tso and Zelman (1968)	Col. 31	17	3920	10.0	> 28	10.0	453	11.6	416	10.6
	32	17	3700	10.0	> 28	10.0	244	6.6	207	5.6
	33	16	3750	10.0	> 28	10.0	219	5.8	150	4.0
	34	17	3610	10.0	> 28	10.0	204	5.7	137	3.8
	Col. 41	16	4540	10.0	> 28	10.0	224	4.9	186	4.1
	42	17	4270	10.0	> 28	10.0	133	3.1	68	1.6
	43	17	4530	10.0	> 28	10.0	177	3.9	154	3.4
	44	17	4450	10.0	> 28	10.0	193	4.3	126	2.8
	Col. 51	17	4750	10.0	> 28	10.0	165	3.5	115	2.4
	52	18	4560	10.0	> 28	10.0	242	5.3	172	3.8
	53	18	4760	10.0	> 28	10.0	181	3.8	154	3.2
	54	18	4906	10.0	> 28	10.0	235	4.8	168	3.4
Bloem (1965)	Ser. 183C	25	3510	14.4	93	10.0	372	10.6	242	6.9
	Ser. 183C	25	3260	14.4	93	10.0	274	8.4	205	6.3
	Ser. 183E	122	3700	14.0	35	7.0	307	8.3	250	6.8
Petersons (1964)	Col. A1	8	2150	10.6	28	9.9	124	5.8	0	0
	A2	9	3340	10.6	28	9.9	154	4.6	90	2.7
	A3	9	2800	10.6	28	9.9	132	4.7	124	4.4
	A4	9	2930	10.6	28	9.9	159	5.4	0	0
	Col. B3	9	2400	10.6	28	9.9	181	7.5	179	7.5
	B4	8	2400	10.6	28	9.9	121	5.0	82	3.4
	Col. C1	9	4410	10.6	28	9.9	222	5.0	0	0
	C2	9	4420	10.6	28	9.9	280	6.3	259	5.9
	C3	9	5230	10.6	28	9.9	491	9.4	440	8.4
	C4	9	5650	10.6	28	9.9	456	8.1	452	8.0
	Col. D1	9	4600	10.6	28	9.9	286	6.2	242	5.3
	D2	9	4700	10.6	28	9.9	581	12.4	483	10.3
	D3	9	5000	10.6	28	9.9	283	5.7	242	4.8
	D4	8	5030	10.6	28	9.9	310	6.2	290	5.8
Total	29	500					Avg.	6.3		4.7

Table 8.2: Within-member Strength Variation in Blocks Cast by Burg and Ost

N°	Model with Z_a, y, z	$\hat{\sigma}_I$ (psi)	Model with Z_a, y, T_{max}	$\hat{\sigma}_I$ (psi)
1	$\hat{f}_c = 11650\ (1+0.176\ Z_a)\ (1-0.0043\ y)\ (1-0.0024\ z)$	367	$\hat{f}_c = 23290\ (1+0.183\ Z_a)\ (1-0.0030\ y)\ (1-0.0031\ T_{max})$	369
2	$\hat{f}_c = 12130\ (1+0.109\ Z_a)$	341	$\hat{f}_c = 12130\ (1+0.109\ Z_a)$	341
3	$\hat{f}_c = 11700\ (1+0.086\ Z_a)\ (1-0.0014\ y)$	422	$\hat{f}_c = 11700\ (1+0.086\ Z_a)\ (1-0.0014\ y)$	422
4	$\hat{f}_c = 17220\ (1+0.122\ Z_a)\ (1-0.0035\ y)\ (1-0.0037\ z)$	679	$\hat{f}_c = 40000\ (1+0.126\ Z_a)\ (1-0.0038\ T_{max})$	583
5	$\hat{f}_c = 16560\ (1+0.060\ Z_a)\ (1-0.0025\ y)(1-0.0004\ y^2)$	575	$\hat{f}_c = 32700\ (1+0.065\ Z_a)\ (1-0.0002\ y^2)\ (1-0.0034\ T_{max})$	534
6	$\hat{f}_c = 12140\ (1+0.133\ Z_a)\ (1-0.0035\ z)$	533	$\hat{f}_c = 24600\ (1+0.140\ Z_a)\ (1+0.0014\ y)\ (1-0.0041\ T_{max})$	496

Table 8.3: Systematic Within-member Strength Variation in Laboratory Cast Shallow Elements

Reference	Element	n	$\bar{f}_c$ (psi)	Vol. (ft^3)	Age (days)	h (ft)	$\hat{\sigma}_M$ (psi)	V_M (%)	$\hat{\sigma}_S$ (psi)	V_S (%)
Bartlett & MacGregor (1993a)	ML1 N	16	5210	0.9	125	1.5	492	9.4	0	0
	S	21	7511	0.6	128	1.5	397	5.3	131	1.7
	ML2 N	16	7960	1.3	118	1.5	806	10.1	335	4.2
	S	21	8990	0.7	125	1.5	475	5.3	290	3.2
	MH1 N	19	14000	0.6	115	1.5	811	5.8	0	0
	S	20	12600	0.6	115	1.5	1230	9.8	695	5.5
	MH2 N	13	8800	0.8	98	1.5	539	6.1	0	0
	S	13	7840	0.8	94	1.5	730	9.3	360	4.6
	LL1 S	22	6525	2.5	71	2.2	362	5.5	0	0
	LL2 S	21	6500	2.5	55	2.2	570	8.8	405	6.2
	LH1	45	13570	3.4	55	2.3	622	4.6	0	0
	LH2	47	12600	3.4	49	2.3	645	5.1	350	2.8
Meininger et al. (1977)	wall N1	113	4600	9.8	91	3.5	235	5.1	128	2.8
	wall N2	120	2290	9.8	35	3.5	171	7.5	95	4.1
	wall N3	113	3110	9.8	35	3.5	165	5.3	74	2.4
	wall J1	78	3360	9.4	53	6.0	188	5.6	126	3.8
	wall J2	77	5470	9.4	74	6.0	250	4.6	130	2.4
	wall J3	75	7540	9.4	91	6.0	346	4.6	252	3.3
Meininger (1968)	Wall	63	2310	11.6	92/118	4.0	188	8.1	125	5.4
	Slab	52	2500	9.3	92/118	1.3	201	8.0	169	6.8
Burg & Ost (1992)	Mix 1	17	12329	64.0	91/365	4.0	1352	11.0	729	5.9
	Mix 2	18	12850	64.0	91/365	4.0	754	5.9	0	0
	Mix 3	17	12188	64.0	91/365	4.0	699	5.7	248	2.0
	Mix 4	12	17398	64.0	91/365	4.0	1504	8.6	791	4.5
	Mix 5	16	16372	64.0	91/365	4.0	829	5.1	527	3.2
	Mix 6	17	12257	64.0	91/365	4.0	1204	9.8	455	3.7
Total	26	1062					Average	6.9		3.0

Table 8.4: Average Spatial Strength Variation in Field Cast Columns

Reference	Normalized Strength: Top of Column	Normalized Strength: Bottom of Column	Scope of Investigation
Tso & Zelman (1968)	0.919	1.028	10 buildings
Drysdale (1973)	0.940	1.050	14 buildings
Murray & Long (1987)	0.93	1.05	1 building
Davis (1976)	0.91	not reported	4 buildings
Equation (8.12): $0.05 < z' < 0.15$ $0.85 < z' < 0.95$	 0.968 - 0.860	 1.090 - 1.026	29 columns cast in laboratories

Table 8.5: Average Within-member Strength Variation in Field Cast Columns

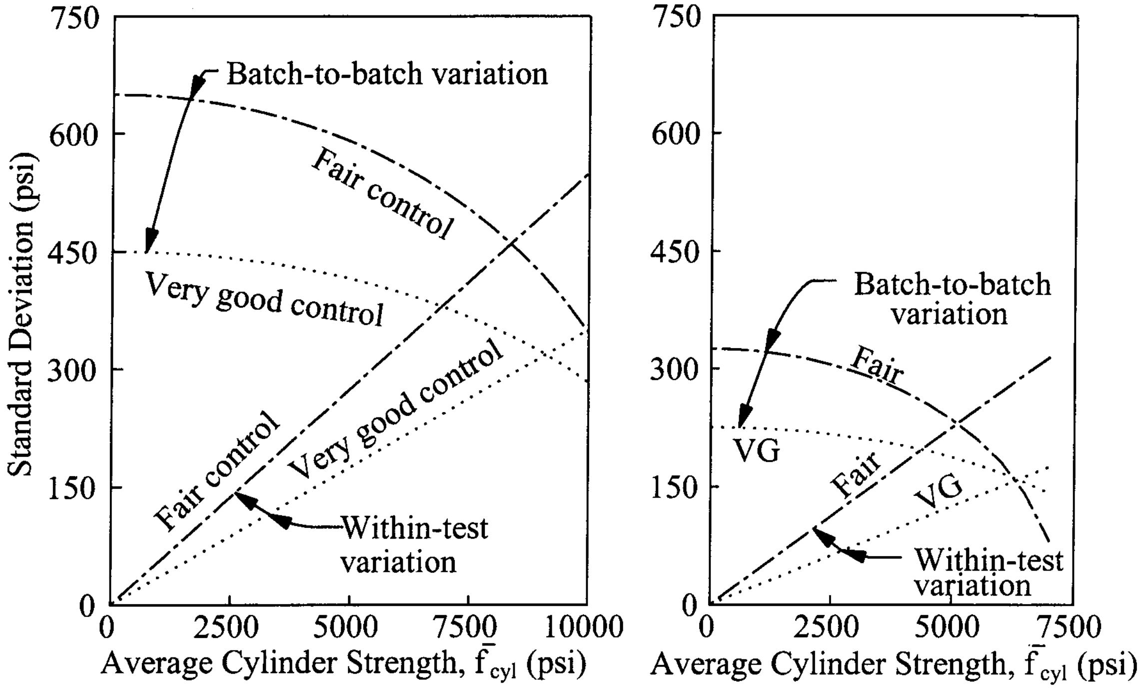
Reference	Site	Reported σ_M: Value	Reported σ_M: Unit	Conversion Factor	Equivalent σ_M (psi, cores)
Tso & Zelman (1968)	3	3.1	100 ft per second (UPV)	73.8 psi core per 100 ft per second	230
	4	7.6			560
	5	3.4			250
	6	4.0			290
	7	4.4			330
	8	4.3			320
	9	3.4			250
	10	2.8			200
Davis (1976)	A	2.7	MPa, cube strength	110 psi core per MPa cube	290
	B	2.8			310
	D	3.5			380
	E	3.9			430
Murray & Long (1987)	-	4.8	MPa, cube	110 psi core per MPa cube	530

Table 8.6: Systematic *In Situ* Strength Variation in Barons Bridge Girders

Girder	n	Fitted Equation	$\bar{f}_c$ (psi)	$\hat{\sigma}_M$ (psi)	V_M (%)	$\hat{\sigma}_S$ (psi)	V_S (%)
A	87	$\hat{f}_c = 4740 + 0.78x^2 - 250y_S$	4940	515	10.4	213	4.3
B	89	$\hat{f}_c = 4940 + 0.34x^2 - 290y_S$	4970	458	9.2	165	3.3
C	92	$\hat{f}_c = 5060 + 15x + 0.69x^2 - 470y_S$	5160	604	11.7	258	5.0
D	91	$\hat{f}_c = 4610 + 11x + 1.04x^2 - 160y_S$	4920	494	10.0	293	6.0
E	89	$\hat{f}_c = 4670 + 0.40x^2 + 125y_S$	4850	407	8.4	148	3.1
		Average	4970	495	9.9	215	4.3

Table 8.7: Within-member and Between-member Variation of Building Columns

Building	Dir'n	n_{col}	Average (100 ft /sec)	s_M (100 ft /sec)	$s_M/\sqrt{5}$ (100 ft /sec)	s_B (100 ft /sec)
3	E/W	21	150.5	3.14	1.40	6.94
3	N/S	13	151.4	3.12	1.40	4.93
4	E/W	64	156.1	7.57	3.39	5.90
5	E/W	25	156.7	3.41	1.52	3.21
6	E/W	14	173.7	4.01	1.79	9.09
7	E/W	17	161.0	4.67	2.09	8.38
7	N/S	16	156.0	3.96	1.77	9.55
8	E/W	19	159.5	4.30	1.92	7.70
9	E/W	42	170.2	3.46	1.55	7.18
9	N/S	25	167.5	3.40	1.52	5.78
10	E/W	39	167.0	2.40	1.07	4.23
10	N/S	16	168.2	3.10	1.39	3.03



(a) General construction testing (b) Laboratory trial batches

Figure 8.1: Within-test variation and between-batch variation for 28 day standard cylinders according to ACI 214-77

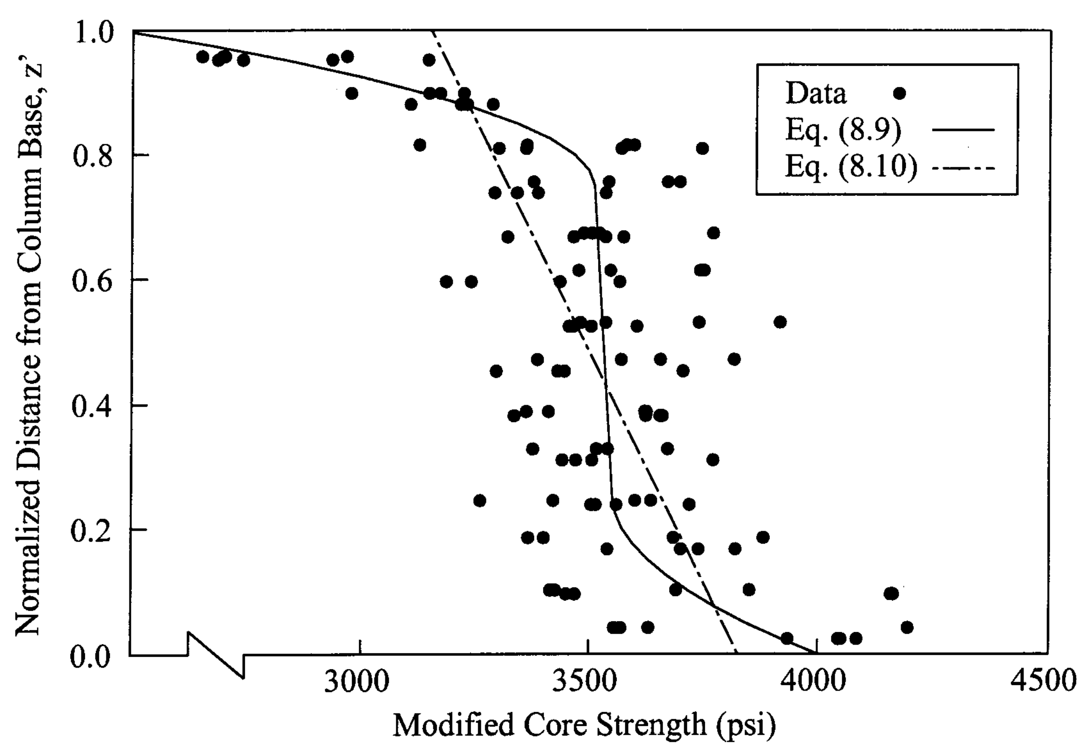


Figure 8.2: Partial regression plot of core strength versus height for Bloem Series 183E Column

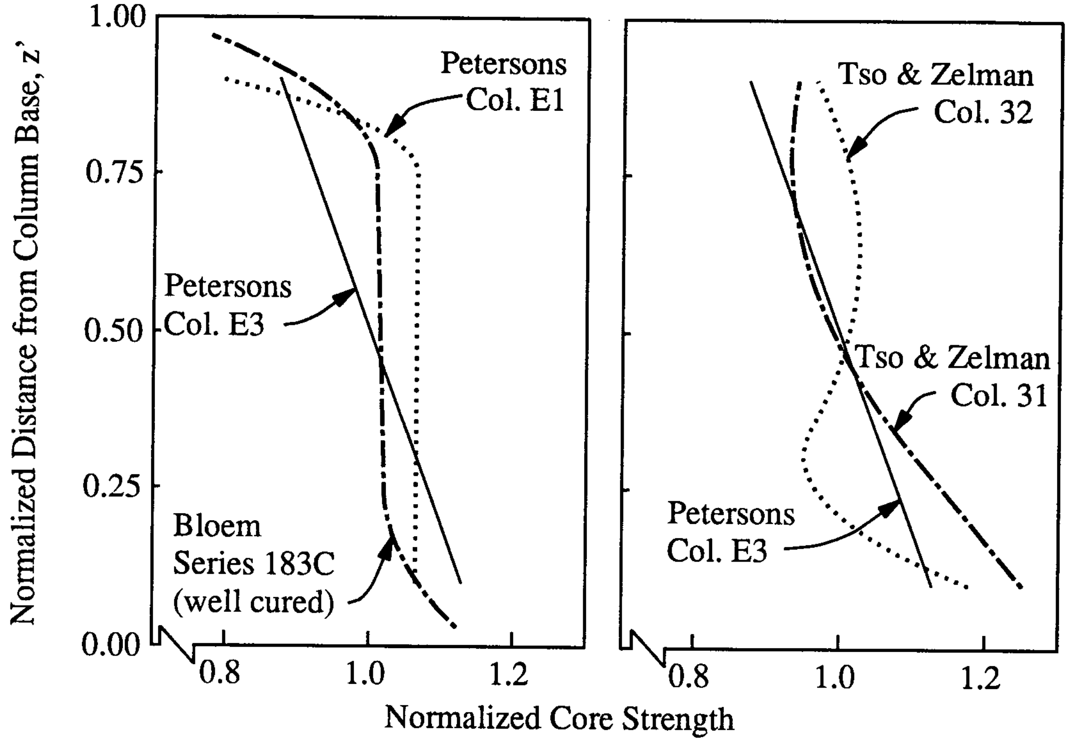


Figure 8.3: Variation of strength over height of individual columns

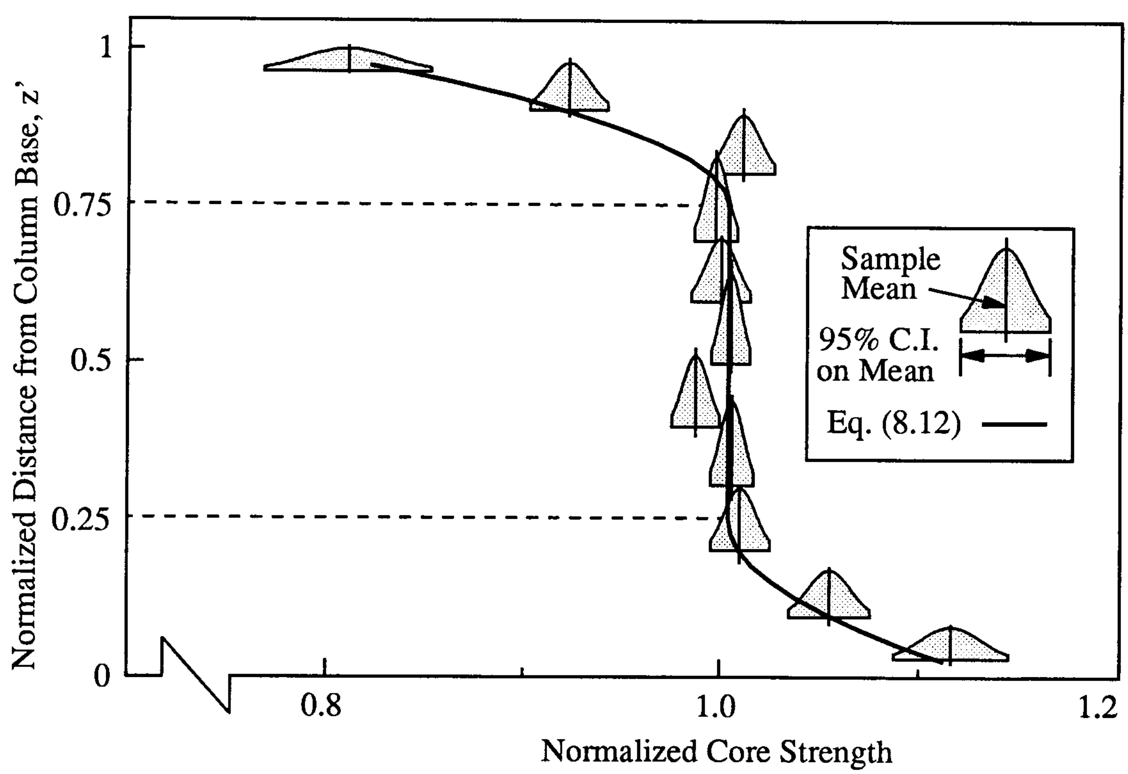


Figure 8.4: Variation of normalized core strength over column height for all columns

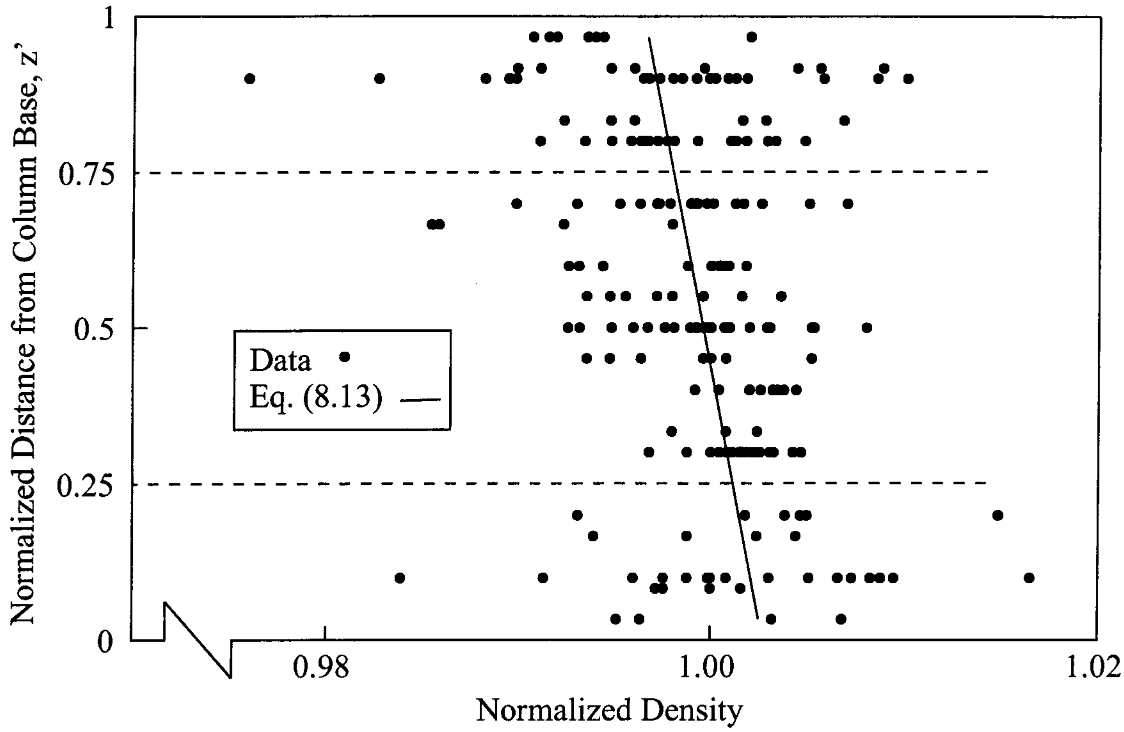


Figure 8.5: Normalized density versus normalized distance from column base

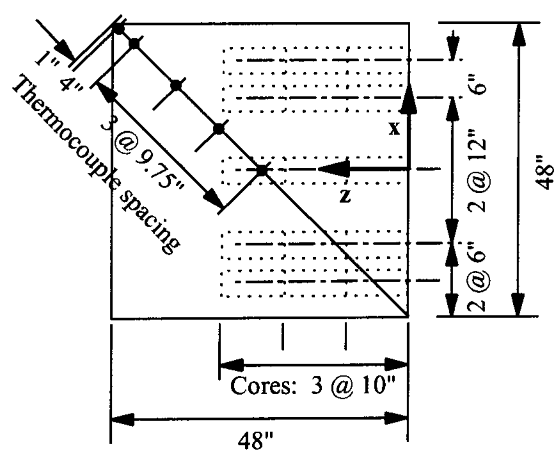


Figure 8.6 (a): Plan view of thermocouple and core locations in blocks cast by Burg & Ost

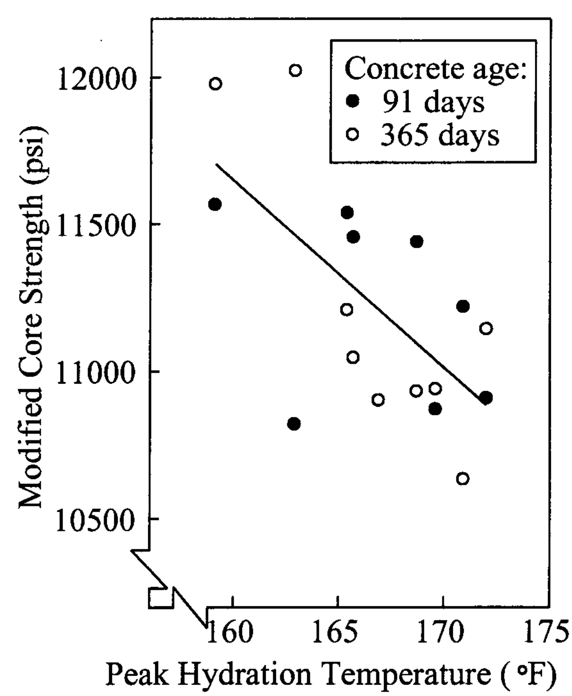


Figure 8.6 (b): Partial regression plot of core strength versus peak hydration temperature for Burg & Ost Block 1

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Chapter 9: Equivalent Specified Concrete Strength from Core Test Data

9.1 Introduction

Core specimens are obtained from an existing structure to determine various physical and chemical characteristics of the concrete, including the compressive strength. Standard procedures for drilling, conditioning and testing the cores are readily available, such as those specified in CAN/CSA-A23.2-M90 "Methods of Test for Concrete" (CSA, 1990b). If the cores have been obtained because the strength of standard 28 day old cylinder specimens is less than specified, criteria such as those given in CAN/CSA-A23.1-M90 "Concrete Materials and Methods of Concrete Construction" (CSA, 1990a) can be used to determine whether the concrete is acceptable or not. However, more general criteria for using core strength data to assess the safety of an existing structure are not available.

The purpose of this paper is to present a simple procedure for determining a value of concrete strength to be used for structural evaluation from core strength data. In practice, the number of cores obtained for testing is usually limited by budgetary constraints, and, in some cases, by concern about unduly weakening the structure. It is convenient for the resulting strength value to be equivalent in definition to the specified strength of concrete, f'_c, which is the basis for design, appears throughout the normal design process, and is therefore familiar. Thus a procedure for determining an equivalent f'_c value from a small sample of core strength data is sought.

9.2 Research Significance

Simple criteria are presented for determining an equivalent value of f'_c based on a small quantity of core test data. The equivalent f'_c value is consistent with the assumptions used to derive resistance factors for structural design, and therefore can be directly substituted into conventional resistance equations. In addition, practical guidelines are presented concerning the scope of the core drilling program, the sample size, and the detection of spurious low strength test values which should be discarded from the data set.

The research also indicates that the *in situ* strength of concrete assumed by others for the calibration of structural codes may be less than, and less variable than, the true value. For resistances which are sensitive to the concrete strength, the resistance factors in current use may be unduly conservative.

9.3 Target Safety Level and Method of Safety Assessment

In this section, aspects of structural safety associated with the design of new structures are reviewed in the context of the evaluation of existing structures. Issues concerning the target safety level, the method used to calculate safety, and the nature of the uncertainty of the safety of new and existing structures are addressed.

The target safety level for the assessment of existing structures need not be more stringent than that used for the design of new structures. An analysis presented in CSA-S408 "Guidelines for the Development of Limit State Design" (CSA, 1981) concludes that life safety criteria govern over economic criteria when setting target reliability levels at ultimate limit states. Life safety criteria, which measure the consequences of failure in terms of the number of lives lost, are identical for new and existing structures. Target safety levels for design and evaluation must therefore be identical if life safety criteria govern.

It is possible that the analysis given in CSA-S408 may not apply to the case of evaluation because the marginal cost of safety is greater for repairs to existing structures than for the design of new structures (e.g. Buckland and Bartlett, 1992). However for the present investigation this possibility is neglected and, perhaps conservatively, the target reliability levels for design and evaluation are assumed to be identical.

A unique probability of failure may exist for a specific component in a specific structure subjected to a particular loading regime. However in practice it is some estimate of this probability of failure, and not the true value itself, which is used to assess safety. Thus, for example, design codes are calibrated using reliability indices (CSA, 1981) which correspond to notional probabilities of failure because the true reliability levels are too difficult to determine. A complete calibration to determine load and resistance factors for evaluation using core strength data is outside the scope of this paper. Instead, it is assumed that the safety levels associated with modern design practice are adequate, although unknown. It is therefore only necessary to determine nominal values, in this case an equivalent value of f'_c, which are consistent with those assumed in design to attain a safety level which, although unknown, is also adequate.

Safety may be considered as an attribute of our state of knowledge of a structure (Madsen et al., 1986). During the design phase of a new structure, the safety assessment is intended to ensure that a random realisation of the structure is unlikely to fail during its design life. When the structure is put into service, many random factors have been realised, such as the material quality, workmanship, and at least some of the loading history. The probability of failure therefore changes from the original target value envisaged by the designer and, if the various realisations are known and accounted for, is less uncertain. From the time of design to the time of evaluation, the assessment of safety therefore evolves from considering the uncertainty associated with random factors which have not been realised to considering the uncertainty associated with random factors which have been realised but have not been precisely quantified.

9.4 Statistical Description of the Concrete Strength Variable in Conventional Resistance Equations

In this section, a statistical description of the concrete strength variable used in resistance equations, which is consistent with the information obtained from core test results, is developed. The various aspects which are disclosed by the core data are enumerated, and statistical descriptions of these aspects are provided which are consistent with those used as a basis for the design of new structures.

Provisions are given in a companion paper (Bartlett and MacGregor, 1993 [Chapter 6]) for converting core test data into the equivalent *in situ* strength of concrete, expressed in terms of the strength of a cylindrical concrete specimen with a length to diameter ratio of 2. Effects due to the length to diameter ratio, diameter, and moisture condition of the core, and to the effect of damage sustained during drilling are accounted for using strength correction factors. The equivalent *in situ* strength therefore reflects the proportions and qualities of materials in the concrete mix, the effects of batching, delivery, placing, consolidation and curing procedures, and the development of concrete strength up to the time of the test. These factors are all unknown at the time of design.

The compressive strength of concrete at failure is affected by many other factors, including the rate of loading, the size of the specimen, and the presence of biaxial or triaxial stress states. These factors are individually addressed in the equations used to calculate the resistance and the associated resistance factors. For example, in the statistical description of concrete strength reported by Mirza et al. (1979), the effect of loading rate was accounted for by an equivalent strength appropriate for a member loaded to failure in one hour. The effect of specimen size on the lower tail of the strength distribution was investigated, found to be slight, and neglected. Biaxial and triaxial stress states were not addressed by Mirza et al. but are included in the behavioural models where their effects are significant. The effects of dimensional variations and inaccuracies created by the use of simplified behavioural models were also accounted for. Because these other factors are individually included in the models used to calibrate load and resistance factors, it may be concluded that the concrete strength used in resistance equations is essentially analogous to the equivalent *in situ* strength, as defined in the previous paragraph.

For the calibration of the resistance factors in Canadian Standard CAN3-A23.3-M84 "Design of Concrete Structures for Buildings" (CSA, 1984), it was assumed that the strength of concrete in a structure does not increase after the concrete is 28 days old (Mirza and MacGregor, 1982). The conservatism associated with this assumption was countered to some degree by neglecting possible long-term deterioration of the concrete and the reinforcing steel. The extent of deterioration of an existing structure may be established by inspection and the effect of any deterioration observed may be accounted for in the safety assessment. If this practice is followed, then it is appropriate to base the equivalent specified strength on the actual strengths observed instead of some reduced equivalent 28 day strengths.

9.4.1 The Mean Value of the *In Situ* Strength

In resistance equations, the concrete strength is expressed in terms of the specified compressive strength at 28 days, f'_c. Associated with this deterministic value is the *in situ* concrete strength, as defined above, which is random. Various relationships for the expected value of the *in situ* strength for a given specified strength have been proposed.

The design provisions in CAN3-A23.3-M84 "Design of Concrete Structures for Buildings" (CSA, 1984) are based on the following relationships for the mean *in situ* strength suggested by MacGregor (1976):

$$\bar{f}_{c,is} = 0.675f'_c + 7.6 \tag{9.1}$$

but

$$\bar{f}_{c,is} \leq 1.15f'_c \tag{9.2}$$

where $\bar{f}_{c,is}$ is the mean *in situ* strength, in MPa, at the rate of loading specified for standard cylinder tests (e.g. CSA, 1990b). Equation (9.1) was derived for the most commonly produced grades of concrete, which have specified strengths greater than 16 MPa.

According to Thorenfeldt (1986), the provisions of the Norwegian Code for concrete structures, NS 3473 (NCBS, 1990) are based on

$$f_{c,is,k} = f_{cu,k}\, k_k \tag{9.3}$$

where $f_{c,is,k}$ is the 5% fractile of the *in situ* strength and $f_{cu,k}$ is the 5% fractile of the cube strength, which is specified by the designer. The factor k_k is obtained from

$$k_k = 0.775 + \frac{3.1}{f_{cyl,k}} \tag{9.4}$$

where $f_{cyl,k}$, the 5% characteristic value of standard cylinder specimens with l/d = 2, is in MPa. Thus the characteristic value of the *in situ* strength is

$$f_{c,is,k} = f_{cu,k}\left[0.775 + \frac{3.1}{f_{cyl,k}}\right] \tag{9.5}.$$

Using the relationship from NS 3473 (NCBS, 1990) for concretes with $f_{cyl,k} < 44$ MPa that

$$f_{cyl,k} = 0.8f_{cu,k} \tag{9.6},$$

Equation (9.5) may be simply expressed as

$$f_{c,is,k} = 0.775 f_{cyl,k} + 3.1 \qquad (9.7).$$

To compare this relationship with Equation (9.1), it is necessary to replace the 5% characteristic values $f_{c,is,k}$ and $f_{cyl,k}$ with the mean *in situ* strength and the specified strengths respectively. Analysis of standard cylinder test data from concrete batches produced in Alberta between 1988 and 1993 for cast-in-place construction indicates that the probability of a cylinder test falling below the specified strength is 5.8% (Bartlett and MacGregor, 1994a [Chapter 7]). Thus, the 5% fractile of the cylinder strength, $f_{cyl,k}$, may be considered roughly equivalent to the specified strength f'_c. If the *in situ* strengths are assumed to be normally distributed, the relationship between the 5% fractile and the mean value is

$$f_{c,is,k} = \bar{f}_{c,is} \quad [1 - 1.645 V_{c,is}] \qquad (9.8)$$

where $V_{c,is}$, the coefficient of variation of the *in situ* strength, typically lies between 0.1 and 0.2. If $V_{c,is} = 0.1$, then, from Equation (9.8), $f_{c,is,k} = 0.836\ \bar{f}_{c,is}$ and, from Equation (9.7),

$$\bar{f}_{c,is} = 0.74 f'_c + 3.7 \qquad (9.9a).$$

Similarly, if $V_{c,is} = 0.2$, $f_{c,is,k} = 0.671\ \bar{f}_{c,is}$, and,

$$\bar{f}_{c,is} = 0.92 f'_c + 4.6 \qquad (9.9b).$$

According to Thorenfeldt (1986), the relationship between the *in situ* strength and the specified strength represented by the factor k_k was not directly determined from analysis of *in situ* strength data. Instead, a relationship between the design strength and the specified strength was selected on the merit of its computational convenience, and the expression for k_k shown in Equation (9.4) was obtained by back calculation.

In a companion paper (Bartlett and MacGregor, 1994a [Chapter 7]), a relationship between the average *in situ* strength and the specified strength is determined from the analysis of a large quantity of published and unpublished data. The average *in situ* strength at 28 days, as corrected for the effect of the core moisture condition and damage sustained during drilling, approximately equals the average cylinder strength. The average cylinder strength is usually greater than the specified strength, because the concrete producer desires assurance that the product will meet specifications. The relationship obtained by regression analysis between the average *in situ* strength at 28 days and the specified strength is

$$\bar{f}_{c,is} = [1.205 + 0.108\, Z_h] f'_c \tag{9.10}$$

where Z_h is an indicator variable equal to 0 for elements less than 450 mm high or 1 otherwise.

The various relationships between the average *in situ* strength and the specified strength, as represented by Equations (9.1), (9.9a), (9.9b) and (9.10) are shown in Figure 9.1. Also shown is the *in situ* strength of 28 day old columns and walls in a multi-storey car park investigated by Murray and Long (1987) using a break-off tester. Murray and Long's data were excluded from the set analysed to obtain Equation (9.10) because cores were not tested. However the data do corroborate the values predicted using Equation (9.10), which are considerably greater than the values predicted by Equations (9.1) and (9.9).

9.4.2 Variability of the *In Situ* Strength

The relationships proposed to represent the variability of the *in situ* strength proposed by Mirza et al. (1979), which were the basis of the design provisions of CAN3-A23.3-M84 "Design of Concrete Structures for Buildings" (CSA, 1984), depend on the degree of control of the concreting operation. For specified strengths less than 28 MPa, the coefficient of variation of the strengths of standard cylinder test specimens was assumed equal to 10, 15 or 20 percent for excellent, average, or poor control respectively. For strengths greater than 28 MPa, the standard deviation of the strength of standard cylinders was assumed equal to 2.8, 4.2 or 5.6 MPa for excellent, average, or poor control. The coefficient of variation of the *in situ* strength, $V_{c,is}$, was determined from

$$V_{c,is} = \sqrt{V_{cyl}^2 + V_{WS}^2 - V_{wt}^2} \tag{9.11}$$

where V_{cyl} is the coefficient of variation of the cylinder strengths, V_{WS} is the coefficient of variation due to within-structure variation, and V_{wt} is the coefficient of variation due to within-test variation. Since V_{cyl} includes V_{wt}, which is an effect due to testing and should not be included in $V_{c,is}$, the effect of V_{wt} is subtracted. Mirza et al. reported V_{WS} = 10% and V_{wt} = 4%, which allowed Equation (9.11) to be simplified to

$$V_{c,is} = \sqrt{V_{cyl}^2 + 0.0084} \quad (9.11a).$$

Alternatively, the variability of the *in situ* strength of concrete can be split into two categories which represent the uncertainty associated with the mean *in situ* strength and the variability of the strength throughout the structure for a given mean *in situ* strength. In a companion paper (Bartlett and MacGregor, 1994a [Chapter 7]), the mean *in situ* strength at 28 days is shown to have a lognormal distribution with a mean value given by Equation (9.10) and a coefficient of variation of 18.6%. This distribution is based on analysis of *in situ* strength data from single members cast from a single batch of concrete, and so does not include batch-to-batch variation or systematic between-member variation. Also, the distribution is also not greatly affected by systematic within-member variation or uncertainties attributable to testing factors since these do not greatly increase the variability of the mean *in situ* strength.

The variation of the strength throughout the structure for a given mean *in situ* strength is composed of the inherent within-batch variability of concrete, systematic strength variation within a member, systematic between-member variation if the structure contains more than one member, and batch-to-batch variation if the structure contains more than one concrete batch. The inherent within-batch variability of concrete may be represented by a coefficient of variation, V_{wb}, of about 3% (Bartlett and MacGregor, 1993 [Chapter 6]). The coefficient of variation due to systematic within-member strength variation, V_M, is roughly 5% for columns or 6% for other elements including slabs, walls, and large blocks (Bartlett and MacGregor, 1994b [Chapter 8]). The coefficient of variation due to systematic between-member variation, V_B, is not known but probably does not exceed 5% if the effects of batch-to-batch variation are accounted for separately. The coefficient of variation due to batch-to-batch variation, V_{bb}, of concretes produced in Alberta from 1988 to 1993 is roughly 10% for cast-in-place construction or 6% for precast construction (Bartlett and MacGregor, 1994a [Chapter 7]). The variation of the strength throughout the structure for a given mean *in situ* strength therefore depends on the number of members, the number of batches and the type of construction as shown in Table 9.1.

Thus the overall coefficient of variation of the *in situ* strength of concrete, for a cast-in-place structure composed of many members and cast from many batches, is

$$V_{c,is} = \sqrt{0.186^2 + 0.130^2} = 0.227 \qquad (9.12).$$

This value pertains to the concrete strength in a structure that has not yet been constructed, that is, before any of the random factors which affect the material quality have been realised. It represents considerably uncertainty, since it implies that the crude 95% confidence limits on the unrealised *in situ* strength in a structure are 55% and 145% of the value obtained from Equation (9.10). The value is larger than that suggested by Mirza et al. (1979) for any degree of control.

In the development of resistance factors for design using conventional concretes, the *in situ* concrete strength should be represented by Equation (9.10) and by Equation (9.12) evaluated using the appropriate values from Table 9.1. As noted elsewhere (Bartlett and MacGregor, 1994a [Chapter 7]), it is uneconomic and often impossible to produce high performance concretes with the same the difference between the average strength of standard cylinders and the specified strength as has been observed for conventional concretes. Thus, for modern high performance concretes it may be unsafe to use Equation (9.10) to represent the mean *in situ* strength.

9.4.3 Characteristic Value of the *In Situ* Strength Corresponding to f'_c

The characteristic value of the *in situ* strength of 28 day old concrete represented by the specified strength, f'_c, in conventional resistance equations may now be determined. It is assumed that the distribution of the estimated *in situ* strength is lognormal, since the distribution of the estimated average *in situ* strength is lognormal and the coefficient of variation of the predicted mean *in situ* strength dominates the overall coefficient of variation given by Equation (9.12). The desired probability is $\Phi(-\eta/\zeta)$ where $\Phi(\cdot)$ is the standard normal cumulative distribution function and η and ζ are the mean value and the standard deviation of the natural logarithms of the *in situ* strengths. The value of η corresponding to the mean value from Equation (9.10) is

$$\eta = ln\left(\frac{1.205 + 0.108Z_h}{\sqrt{1 + V_{c,is}^2}}\right) \qquad (9.13)$$

where $V_{c,is} = 0.227$ from Equation (9.12). The value of ζ corresponding to the standard deviation is

$$\zeta = \sqrt{ln(1 + V_{c,is}^2)} \qquad (9.14).$$

Thus for tall members $Z_h = 1$, $\eta = 0.247$, $\zeta = 0.224$, which infers that the probability that the *in situ* strength is less than the specified strength is $\Phi(-1.10) = 0.135$. For shallow members, $Z_h = 0$, $\eta = 0.161$, $\zeta = 0.224$, which infers that the probability that the *in situ* strength is less than f'_c is $\Phi(-0.72) = 0.235$. The resistance of tall elements including columns and walls is typically more sensitive to the compressive strength of concrete than the resistance of shallow elements including slabs and beams. Therefore it will be assumed that the fractile of the *in situ* strength represented by f'_c is the 13.5% characteristic value, that is, 13.5% of the *in situ* strengths at 28 days will be less than f'_c.

It is interesting to note that a much larger characteristic value of the *in situ* strength is associated with the statistical description of concrete reported by Mirza et al. (1979). Assuming the *in situ* strength to be normally distributed with a mean value given by Equation (9.1) and a coefficient of variation given by Equation (9.11a), the fractile of the *in situ* strength represented by f'_c ranges from the 40% characteristic value for 20 MPa concretes produced under poor control to the 98% characteristic value for 50 MPa concretes produced under excellent control. It is therefore probable that a recalibration based on the statistical description of concrete given by Equations (9.10) and (9.12) would yield higher factored concrete strengths for design than are presently in use.

9.5 Equivalent Specified Strength Value from *In Situ* Strength Data

In the preceding section, it was shown that f'_c is equivalent to the 13.5% characteristic value of the *in situ* strength. When evaluating an existing structure, it is necessary to define a equivalent f'_c, equal to the 13.5% characteristic value of the *in situ* strength, for use in resistance equations. In this section, two methods are described for determining the 13.5% characteristic value from the mean *in situ* strength disclosed by core tests. One method uses a tolerance limit approach which conforms to the definition of the material strength recommended in CSA Special Publication S408-1981 "Guidelines for the Development of Limit States Design". In the other method, a lower bound estimate of the mean *in situ* strength is obtained by analysis of core data, and then the corresponding 13.5% characteristic value is determined using a generic statistical description of the strength variation throughout the structure. The rationale for adopting the second procedure instead of the first is presented.

It will be assumed throughout that the *in situ* strength of concrete in a structure has a normal distribution. There is a general consensus that concrete strengths have either a normal or a lognormal distribution (e.g. Mirza et al., 1979). It is convenient to adopt the normal distribution because this permits the use of many other distributions which have been derived on the assumption of normality (although these tools could also be used with a lognormal distribution by working with the natural logarithms of observed strength values). Given that the objective is to determine the 13.5% characteristic value of the *in situ* strength, it is clear from Figure 9.2 that it is slightly conservative to assume a normal distribution instead of a lognormal distribution. The degree of conservatism introduced is very slight, since the difference in the 13.5% characteristic value is less than 1.1% for random variables with coefficients of variation less than 20%, as shown in Figure 9.2(b). The difference would be greater if a smaller characteristic value were sought.

This assumption may seem inconsistent with the assumption in the previous section that the estimated *in situ* strength associated with a given specified strength has a lognormal distribution. In fact, there is no inconsistency. In the previous section, the uncertainty of the *in situ* strength was dominated by the uncertainty of the mean *in situ* strength given the specified strength, which varies from structure to structure and may be simulated using a lognormal distribution. In this section, the variability of the *in situ* strength within the single structure being evaluated is considered, and is assumed to have a normal distribution.

In Clause 5.1 of CSA Special Publication S408-1981 "Guidelines for the Development of Limit States Design", it is recommended that the specified value of a material strength should "correspond to a probability level of 1 to 10 percent with a 75% confidence interval for statistical sampling". A method which conforms to the intent of this recommendation is to determine the one-sided 75% tolerance limit on the 13.5% characteristic value from the sample mean and standard deviation. Factors for this tolerance limit are based on a non-central t distribution and may be determined using the approximate calculation given by Madsen et al. (1986) or by interpolation from the values tabulated by Natrella (1963). In practice, the tolerance limit for a small sample of data is considerably less than the sample mean value. This is largely because the variability of a population is difficult to estimate using a small number of samples, as indicated by the one-sided confidence limits on the standard deviation shown in Figure 9.3. For example, the 70% and 80% one-sided confidence interval on the standard deviation as estimated from 10 tests are respectively 1.17 and 1.27 times the sample standard deviation.

It was shown in the previous section that, at the time of design, the variability of the average *in situ* strength dominates the overall variability of the *in situ* strength. It therefore seems appropriate to use core strength data to estimate the average *in situ* concrete strength, and then to assume that the additional variation throughout the structure is accurately represented by the generic values shown in Table 9.1. This method is developed for the case where the lower limit of the 90% (one-sided) confidence interval on the mean *in situ* strength is desired. The choice of the 90% confidence interval is arbitrary, but is probably conservative in light of the 75% confidence interval recommended in CSA Special Publication S408-1981 (CSA, 1981). The lower limit of the 90% (one-sided) confidence interval on the mean *in situ* strength, $(\bar{f}_{c,is})_{90}$, is obtained from

$$(\bar{f}_{c,is})_{90} = \bar{f}_{c,is} - 1.282\sqrt{\frac{(k_1\, s_{c,is})^2}{n} + [\bar{f}_{c,is}]^2 (V_{l/d}^2 + V_{dia}^2 + V_r^2 + V_{mc}^2 + V_d^2)} \qquad (9.15)$$

where $\bar{f}_{c,is}$ and $s_{c,is}$ are the sample mean and sample standard deviation of the equivalent *in situ* strengths. The first term under the square root represents the effect of the sample size, n, on the uncertainty of the mean value. Parameter k_1 is simply the ratio of the 90th percentile of the Student t distribution with (n-1) degrees of freedom (e.g. Natrella, 1963) to the 90th percentile of the standard normal distribution, 1.282. Values of k_1 are shown in Table 9.3. The second term under the square root represents the effect of the uncertainty of strength correction factors used to estimate the equivalent *in situ* strengths. The variables $V_{l/d}$, V_{dia}, V_r, V_{mc}, and V_d are the coefficients of variation associated with strength correction factors $F_{l/d}$, F_{dia}, F_r, F_{mc}, and F_d respectively. Values of these coefficients of variation are shown in Table 9.2.

The equivalent specified strength, $f'_{c,eq}$, is obtained from

$$f'_{c,eq} = k_2 \quad (\bar{f}_{c,is})_{90} \tag{9.16}$$

where values of k_2 are shown in Table 9.4. These values are simply (1-1.10 V_{WS}), where V_{WS}, the coefficient of variation due to variation of the *in situ* strength throughout the structure, depends on the number of members, the number of batches and the type of construction as shown in Table 9.1. The rounded values of k_2 shown in the right hand column of Table 9.4 are sufficiently accurate for use in practice.

In practice, the method based on Equations (9.15) and (9.16) gives a larger value of the equivalent specified strength than would be obtained using the non-central t distribution in accordance with CSA S408. Use of Equations (9.15) and (9.16) is justified for two reasons. Firstly, it is generally recognised that core tests tend to overestimate the true variability of the compressive strength of concrete (e.g. MKM Consultants, 1987). This is partly due to damage sustained by the core specimen during drilling (Bartlett and MacGregor, 1994e [Chapter 5]), which may vary between specimens obtained from a common element. However, it is also almost certainly due to other testing factors associated with the transportation, storage, moisture conditioning, capping and testing of the cores. It is therefore desirable to remove the effect of these factors from the sample variability to obtain the true variability of the *in situ* strength but, since they are very hard to quantify, it is impractical to do so. In the proposed method, the variability of the observed core strengths is reflected only in the interval estimate of the mean in situ strength in Equation (9.15). Secondly, the level of sophistication of a procedure which is consistent with the recommendation in CSA-S408-1981 (CSA, 1981), although similar to that which would be carried out for code calibration, is significantly higher than that associated with present design, specification, and acceptance practices.

9.6 Procedure for Obtaining and Analysing Core Test Data

The suggested procedure for estimating the equivalent specified strength of concrete in an existing structure using core strength test data is as follows:

1. *Plan the scope of the testing program.* Ensure that the members and the locations within each member that will be sampled are consistent with the eventual intended use of the strength information. The optimal number of core specimens varies depending on the cost and potential hazards associated with obtaining core specimens and the desired accuracy of the strength estimate. As a practical minimum, 6 specimens from each grade of concrete to be evaluated should be tested.

2. *Obtain and test cores.* Record the location in the structure from which each core is obtained. Use standard methods to prepare the specimens and carry out the tests. Recognise the strong likelihood that spurious low strength values will occur, and record information which may later be useful to identify reasons for these values, such as the load-deflection curve during the core test and the nature of the failure.

3. *Convert the core strengths to equivalent in situ strengths*, using factors proposed by Bartlett and MacGregor (1993) [Chapter 6].

4. *Check for low outliers in the set of equivalent in situ strengths.* The methods in ASTM E178-80 (ASTM, 1993b) are not very powerful for less than 8 replicate specimens, and the procedure given by Rousseeuw and Leroy (1987) is not powerful for less than 6 replicate specimens. If an outlier is detected using a statistical test, try to ascertain a physical explanation for the aberrant value.

5. *Calculate the equivalent specified strength from the in situ strength data.* This requires calculation of the sample mean and sample standard deviation of the *in situ* strengths, after any outliers have been removed. The limit of the one-sided (lower) 90% confidence interval on the mean value is then obtained from Equation (9.15), accounting for the sample size and the uncertainty introduced by the strength correction factors. Finally, the equivalent specified strength is determined from Equation (9.16) using the appropriate factor from Table 9.4.

In the following subsections, aspects of the first three steps are presented. Use of the proposed procedure is illustrated by an example calculation.

9.6.1 Scope of Core Testing Programme

The scope of the core testing programme should be consistent with the intended use of the information obtained. Two distinct objectives which are commonly sought are an overall estimate of the strength for general evaluation purposes and an estimate of the strength of specific elements located in a specific part of the structure which have been previously identified as potentially substandard. If an overall strength value is required, cores should be drilled at random locations throughout the structure. If the strength in a specific component or group of components is sought, then the cores should be obtained from these specific components only.

The locations from which the cores are obtained should be selected to limit bias in the sample. For example, the sample may be biased if it is obtained from a single batch of concrete in a structure cast from many batches. Bias may also be introduced if a disproportionate number of specimens are obtained from part of a member which has systematic within-member strength variation. In companion papers, systematic within-member strength variation due to maximum temperatures sustained during hydration (Bartlett and MacGregor, 1994a [Chapter 7]) and due to consolidation and curing factors which may vary systematically throughout the volume of the element (Bartlett and MacGregor, 1994b [Chapter 8]) are investigated. In practice, it is usually not difficult to limit sample bias if the overall objectives of the investigation are kept in mind.

The process of choosing a sample size requires an evaluation of the associated costs and benefits. As the sample size increases, greater costs are incurred, and, perhaps more significantly, the risk of unduly weakening the structure also increases. On the other hand, the accuracy of the result improves for larger sample size and, as elaborated in detail later, the likelihood of successfully detecting a spurious low value in the data set also improves. Therefore, the optimum sample size may often be unique for a particular project.

However, it is possible to develop guidelines for a minimum number of cores. In Figure 9.4 (a), the effect of the number of specimens on the lower limit of the confidence level on the mean value is shown for a population with V = 15%. It is clear that, irrespective of the confidence level selected, the rate at which the lower confidence limit approaches the sample mean diminishes rapidly as the number of cores increases. For example the 90% (one-sided) confidence interval on the mean value is 91% of the sample mean for a sample of 6 cores and increases only slightly to 92.5% of the sample mean for a sample of 8 cores. A similar conclusion may be drawn from Figure 9.4 (b), which shows the change in the sample mean, $(\bar{f}_c - \bar{f}_c^*)$, caused by a single undetected low outlier. For a sample of only one core, the error of the sample mean caused by the undetected outlier is 100% of the deviation between the spurious value and the true mean value. For a sample of 6 cores, the effect on the sample mean is only 17% of the outlier deviation. Inspection of Figures 9.4 (a) and (b) indicates that use of only 2 or 3 specimens may lead to large errors, and also that the benefits diminish rapidly if more than 5 or 6 specimens are selected. Thus a minimum sample size of 5 or 6 cores from each grade of concrete to be evaluated seems reasonable. In Section 9.6.3, it is shown that a sample size of at least 6 is desirable if it is necessary to detect outliers.

9.6.2 Conversion to Equivalent *In Situ* Strengths

The procedure for converting the core strengths into equivalent *in situ* strengths is described in detail by Bartlett and MacGregor (1993) [Chapter 6]. The conversion equation is

$$f_{c,is} = F_{l/d}\, F_{dia}\, F_r\, F_{mc}\, F_d\, f_c \qquad (9.17)$$

where $f_{c,is}$ is the equivalent *in situ* strength and f_c is the core strength. Strength correction factors $F_{l/d}$ and F_{dia} account for the effect of the length to diameter ratio and diameter of the core, and F_r accounts for the effect of any reinforcing bars oriented at right angles to the core axis. These factors equal 1.0 if the core is 100 mm in diameter, has a length to diameter ratio of 2, and does not contain reinforcing bars. Strength correction factors F_{mc} and F_d account for the core moisture condition and the strength loss due to damage sustained during drilling.

Mean values for the various strength correction factors are shown in Table 9.2. It follows that a 100 mm diameter core with l/d = 2 which does not contain any reinforcing bars and has been soaked before testing according to the provisions of ASTM C42-90 (ASTM, 1993a) has $f_{c,is} = 1.0x1.0x1.0x1.09x1.06\ f_c = 1.15\ f_c$.

Coefficients of variation for the various strength correction factors, which have been determined by weighted regression analysis of ratios of observed test to predicted values, are also shown in Table 9.2. These coefficients of variation reflect the uncertainty associated with the use of the various strength correction factors and, for cores from a single structure, remain constant irrespective of the number of specimens obtained. They therefore must be included in the calculation of the lower limit of the confidence interval on the mean value.

9.6.3 Detection of Outliers

An outlying observation or "outlier" is one which appears to deviate markedly from the other observations in a data set. There is always a strong possibility that imperfect drilling, handling, capping or testing procedures may result in the reported strength of a core being less than its true value. High outliers, due to mistakes which occur while recording or transcribing the data, are also possible. Obviously an accurate strength estimate may only be obtained if all true outliers, that is, those which are not simply extreme manifestations of the random variable, are detected and purged from the data set.

The most reliable test for detecting outliers is the existence of a plausible physical explanation for the high or low value. For example, inspection of the load-deflection response of each specimen may confirm that a difference exists between one specimen and the rest, and may indicate possible causes for the difference (Bartlett and MacGregor, 1994c [Chapter 2]). Other methods for detecting outliers based on statistical criteria are given in ASTM E178-80 (ASTM, 1993b) and Rousseeuw and Leroy (1987). The details of these methods are illustrated later in the calculation example.

The methods in ASTM E178-80 "Standard Practice for Dealing with Outlying Observations" (ASTM, 1993b) have a common general form: "the doubtful value is included in the calculation of the numerical value of a sample criterion (or statistic), which is then compared with a critical value based on the theory of random sampling to determine whether the doubtful observation is to be retained or rejected". The critical value is chosen based on the risk of erroneously rejecting a good observation, which is selected by the user of the test. It is recommended that a low significance level such as 1% be used, which would be adopted by a user willing to accept the risk of erroneously rejecting a valid observation once every hundred times the test is performed. It is further recommended that the use of significance levels in excess of 5% should not be common practice.

In Figure 9.5, the critical values of the test statistic $(\overline{X} - X_{min})/s$ from ASTM E178-80 are shown for various levels of significance and various sample sizes, where $\overline{X}$ and s are the sample mean and sample standard deviation, and X_{min} is the lowest observed value. The doubtful value is an outlier at the level of significance where the calculated value of this statistic equals the critical value shown in the figure. The maximum possible values of this statistic, which depend on the sample size, n, according to

$$\left\{ \frac{(\overline{X} - X_{min})}{s} \right\}_{max} = \frac{(n-1)}{\sqrt{n}} \tag{9.18}$$

are also shown. Figure 9.5 gives a good indication of the power of the test, which is the probability of erroneously retaining an observation which is in fact an outlier, for the various sample sizes. For n = 3, the maximum value of the statistic is virtually indistinguishable from the critical values at the 5 and 10% significance levels. It is therefore highly probable that all extreme observations would be retained and thus the test has virtually no power for samples of 3 values. If a 1% level of significance is adopted, then the test has little power for samples of 6 and perhaps only becomes useful for samples of 8 or more. If a 5% level of significance is adopted, which is the maximum recommended in ASTM E178-80, then the detection test has negligible power for samples of 4 or less and marginal power for samples of 6 or more. There is therefore little benefit from applying the tests in ASTM E178-80 to samples of fewer than 6 observations.

The method proposed by Rousseeuw and Leroy (1987) differs fundamentally from that in ASTM E178-80 because the doubtful value is not included in the calculations which define the characteristics of the sample upon which the detection test is based. Instead, the median of the data is estimated as the middle of the smallest interval that contains half of the observations, and the dispersion is determined from the width of this interval. Potential outliers are then tested using these robust estimates of the median and scale. Although Rousseeuw and Leroy have included a factor in their procedure which addresses the effects of small sample sizes, their method will not generally be particularly effective if applied to fewer than 6 observations.

Thus, statistical procedures for outlier detection are not particularly effective for small numbers of observations. It is usually desirable to be able to ascertain the reason for any aberrant values. It is therefore essential that the core locations and observations noted during drilling and testing the cores be accurately recorded. It may sometimes be helpful to return to the structure and verify the accuracy of the low observed strengths by drilling additional cores or by using nondestructive test methods.

9.6.4 Calculation Example

As part of an investigation of the use of nondestructive test methods to evaluate existing bridges, 32 cores were obtained from the centre span of Barons Bridge in southern Alberta (Mikhailovsky and Scanlon, 1985). The bridge consists of a cast-in-place slab on 5 parallel, haunched, cast-in-place girders with spans of 12.2, 18.3 and 12.2 metres which are continuous over the two interior supports. The concrete strength specified at the time of construction in 1950 was 3000 psi (20.7 MPa). In this calculation example, an equivalent specified strength of the concrete is sought for the general evaluation of the girders.

9.6.4.1 Scope of the Core Testing Programme

The strength calculation will be based on the results from 6 core tests, which is the minimum number suggested in Section 9.5.1. A subset of 6 test results was selected randomly from the full set of results from 32 cores reported by Mikhailovsky and Scanlon. As shown in Table 9.5, the locations from which the 6 cores were obtained are reasonably well distributed between the north and south ends of the centre span, and among the 5 girders. Batch-to-batch variation is probably represented in these data because, according to the original construction records, the concrete was mixed on site using a mixer with a capacity of roughly only 0.2 m^3 (7 ft^3) (Ramsay, Private Communication, 1993). No cores were obtained from the middle third of the span, but since the bending and shear capacities are more sensitive to the concrete strength near the support than at midspan, this is not a serious concern. Thus, given the overall strength value desired, the 6 core strengths shown in Table 9.5 should represent a reasonably unbiased sample.

9.6.4.2 Conversion to Equivalent In Situ Strengths

Mikhailovsky and Scanlon have reported the particulars of the core testing in sufficient detail to allow a reasonably accurate conversion to equivalent *in situ* strengths to be made (Mikhailovsky and Scanlon, 1985, p. 50). The cores, all 100 mm in diameter, were tested after being left to dry in air. Most had length to diameter ratios of 2, and the strengths of cores with *l*/d less than 2 are reported as equivalent strengths of cores with *l*/d = 2. The factors used for this correction are identical to those in ASTM C42-90 (ASTM, 1993a) and are quite similar to the strength correction factors recommended by Bartlett and MacGregor (1994d [Chapter 3]) for cores dried before testing. The specimens were loaded at rates which fall within the ranges recommended in CAN/CSA-A23.2-M90 "Methods of Test for Concrete" (CSA, 1990b).

Thus, to obtain the equivalent *in situ* strengths, account need be taken only of the core moisture condition at the time of the test and the effect of damage due to drilling. From the values shown in Table 9.2, $f_{c,is} = 0.96x1.06\ f_c = 1.02\ f_c$. Using this relationship, the equivalent *in situ* strength values for all six cores shown in Table 9.5 are calculated.

9.6.4.3 Detection of Outliers

The strength of core number 2, from the North end of Girder C, is appreciably lower than the rest and might therefore be considered doubtful. It will be examined using the outlier detection criteria in ASTM E178-80 (ASTM, 1993b) and in Rousseeuw and Leroy (1987).

The statistic for the ASTM E178-80 test is the ratio of the difference between the sample mean and the suspect value divided by the sample standard deviation. Thus, using the sample statistics for all 6 equivalent *in situ* strengths shown in Table 9.5, the test statistic is $(\bar{f}_{c,is} - f_{c,is,min})/s_{c,is}$ = (33.48 - 20.69)/7.55 = 1.69. From Table 1 of ASTM

E178-80, the critical value at the 5% (one-sided) significance level is 1.82 for a sample of 6 observations. The observed statistic is less than the largest critical value recommended by ASTM E178-80 for general purpose use and therefore the low value should not be rejected. It can also be noted that the observed value is close to the critical value of 1.73 at the 10% (one-sided) significance level. Thus if the low value is rejected, and criteria consistent with this decision are used in the future to detect outliers, then a valid observation will be erroneously rejected about once every 10 times.

Outlier identification according to Rousseeuw and Leroy is carried out in three steps. First, a robust estimate of median of the distribution is obtained as the middle of the smallest range of strengths which include $\left(\frac{n}{2}+1\right)$ of the observations. A convenient approach is to rank the data from largest to least and calculate the range widths which bound half of the observations plus one.

The 3 highest values are, in order:	39.62	38.71	38.00	MPa, and
the 3 lowest values are, in order:	35.90	27.99	20.69	
	-------	-------	-------	
so the range widths are:	3.72	10.72	17.31	MPa.

The least range width is 3.72 MPa, so the estimate of the median strength is (39.62+35.90)/2 = 37.76 MPa. The scale is estimated as

$$s_o = 1.48\left(1+\frac{5}{n-1}\right)\left(\frac{r_{min}}{2}\right) \tag{9.19}$$

where s_o is the robust estimate of the scale and r_{min} is the least range width which bounds half the observations. The various numerical values which appear in Equation (9.19) are constants recommended by Rousseeuw and Leroy. With n = 6:

$$s_o = 1.48\left(1+\frac{5}{6-1}\right)\left(\frac{3.72}{2}\right) = 5.52 \quad \text{MPa} \tag{9.19a}$$

The test for detecting outliers suggested by Rousseeuw and Leroy is simply to divide the difference between the robust median value and the low value by the robust scale estimate. If the result is larger than 2.5, the observation is an outlier. In this example, for the low observation, (37.76-20.69)/5.52 = 3.10, and since this value exceeds 2.5, the low observation should be discarded.

Thus the low observation is found to be an outlier according to the criteria of Rousseeuw and Leroy but not according to the criteria in ASTM E178-80. In practice, this dilemma might be resolved by reviewing observations made concerning the drilling and testing procedures and discovering an explanation for the low value. For the purposes of this example, the low value will be rejected. The 5 remaining observations have a mean value of 36.0 MPa and a standard deviation of 4.7 MPa.

It is interesting to note that, even considering all the data reported by Mikhailovsky and Scanlon, it is debatable whether this low observation should be retained or rejected. On one hand, two test specimens were sawn from the core obtained at this location; the strength of the other was 21.07 MPa. On the other hand, these *two* values are outliers in the set of 32 observations at the 5% (one-sided) significance level according to ASTM E178-80 criteria, and also according to the procedure in Rousseeuw and Leroy. Furthermore, ultrasonic pulse velocity measurements recorded by Mikhailovsky and Scanlon before coring do not indicate low strengths in this region. Therefore these two values are probably outliers.

9.6.4.4 Calculation of Equivalent Specified Strength

The calculation of the equivalent *in situ* strength requires simple application of Equations (9.17) and (9.18). There are 5 values in the reduced data set, so, from Table 9.3, the value of k_1 is 1.196. The lower limit of the 90% (one-sided) confidence interval on the mean *in situ* strength, $(\bar{f}_{c,is})_{90}$, is

$$(\bar{f}_{c,is})_{90} = 36.0 - 1.282\sqrt{\frac{(1.196x4.7)^2}{5} + 36.0^2(0^2 + 0^2 + 0^2 + 0.025^2 + 0.025^2)}$$

$$= 32.4 \quad \text{MPa} \qquad (9.15a).$$

It is assumed in this calculation that only V_{mc} and V_d contribute to the uncertainty associated with the strength correction factors. Ideally, account should also be taken of the error associated with the use of the length to diameter strength correction factors, although, as shown in Table 9.2, this error is small for *l*/d > 1.5.

This structure consists of several members cast from many batches of concrete. Thus, from Table 9.4, the k_2 value is 0.85, and the equivalent specified strength is

$$f'_{c,eq} = 0.85(32.4) = 27.6 \text{ MPa} \quad (9.16a).$$

This value is 33% larger than the originally specified value.

9.7 Summary

A simple procedure is presented for the determination of an equivalent specified concrete strength from a small number of core test results for the assessment of the safety of an existing structure. Analysis of core test data allows the evaluator to roughly quantify the realisations of various random variables that affect the concrete strength which, for a specific structure, are unknown at the time of design. The procedure yields a concrete strength value which is consistent with the statistical description of the concrete strength at the time of initial design and so can be substituted directly for f'_c in conventional resistance equations.

Analysis of data reported by Bartlett and MacGregor (1994a, 1994b) [Chapters 7, 8] indicates that there is approximately a 13.5% probability that the *in situ* strength of concrete in a structure which is 28 days old will be less than the strength specified originally. This is a much smaller probability than that associated with the statistical description of concrete strength used to calibrate the load and resistance factors which are currently used for design of new structures in Canada. In the present investigation, it is assumed that a safety level which is equivalent to that provided by current design criteria, and is therefore sufficient, will be achieved for evaluation using a nominal strength value equal to the 13.5% characteristic value in conventional resistance equations with conventional resistance factors. However, it is also probable that recalibration of the design load and resistance factors based on the statistical description of the concrete strength given by Equations (9.10) and (9.12) would yield greater factored concrete strengths than are currently used in practice.

The procedure for calculating an equivalent specified strength is carried out in two steps. A lower bound estimate of the mean *in situ* strength is determined by analysis of the core strength data. The 13.5% characteristic value associated with this lower bound estimate of the mean strength is then calculated based on a generic description of the variation of *in situ* strengths within a particular structure. The variability of the observed core strengths, which is generally greater than the variability of the *in situ* strength due to various testing factors which are difficult to quantify, is reflected only in the interval estimate of the mean *in situ* strength.

Guidelines are also presented for planning the core testing programme, and for detecting outliers in the data set using criteria from ASTM E178-80 (ASTM, 1993b) and from Rousseeuw and Leroy (1987). The recommended minimum sample size is 6 cores.

Table 9.1: Coefficient of Variation due to *In Situ* Strength Variation Throughout Structure

Structure Composed of	One Member	Many Members
One Batch of Concrete	$\sqrt{V_{wb}^2+V_M^2}$ = 0.067	$\sqrt{V_{wb}^2+V_M^2+V_B^2}$ = 0.084
Many Batches of Concrete	$\sqrt{V_{wb}^2+V_M^2+V_{bb}^2}$	$\sqrt{V_{wb}^2+V_M^2+V_B^2+V_{bb}^2}$
Cast-in-place	= 0.120	= 0.130
Precast	0.090	0.103

Table 9.2: Magnitude and Precision of Strength Correction Factors for Converting Core Strengths into Equivalent *In Situ* Strengths[a]

Factor	Mean Value	V (%)
$F_{l/d}$: l/d ratio[b]		
soaked[c]	$1-\{0.117-4.3(10^{-4})f_c\}\left(2-\frac{l}{d}\right)^2$	$2.5\left(2-\frac{l}{d}\right)^2$
air dried[c]	$1-\{0.144-4.3(10^{-4})f_c\}\left(2-\frac{l}{d}\right)^2$	$2.5\left(2-\frac{l}{d}\right)^2$
F_{dia}: core diameter		
50 mm	1.06	11.8
100 mm	1.00	0.0
150 mm	0.98	1.8
F_r: rebar present		
none	1.00	0.0
one bar	1.08	2.8
two bars	1.13	2.8
F_{mc}: core moisture content		
soaked[c]	1.09	2.5
air dried[c]	0.96	2.5
F_d: damage due to drilling	1.06	2.5

Note: [a] - to obtain the equivalent *in situ* concrete strength, multiply the standard core strength by the appropriate factor(s).
[b] - f_c is in MPa. For f_c in psi, the constant is -3 (10^{-6}).
[c] - standard treatment specified in ASTM C42-90 (ASTM, 1993a)

Table 9.3: k_1 Factors for Use in Equation (9.17)

Number of Cores	k_1	Number of Cores	k_1
2	2.401	9	1.090
3	1.471	10	1.079
4	1.278	12	1.063
5	1.196	16	1.046
6	1.151	20	1.036
7	1.123	25	1.028
8	1.104	30	1.023

Table 9.4: k_2 Factors for Use in Equation (9.18)

Structure Composed of	One Member	Many Members	Rounded Values
One Batch of Concrete	0.93	0.91	0.90
Many Batches of Concrete			
Cast-in-place	0.87	0.86	0.85
Precast	0.90	0.89	0.90

Table 9.5: Strength of Cores from Barons Bridge

Core No.	Girder	End	f_c (MPa)	$f_{c,is}$ (MPa)	Note
1	D	North	35.28	35.90	
2	C	North	20.33	20.69	(doubtful value)
3	A	North	38.04	38.71	
4	E	South	27.51	27.99	
5	C	South	38.93	39.62	
6	A	South	37.34	38.00	
			$\bar{f}_{c,is}$	33.48	
			$s_{c,is}$	7.55	

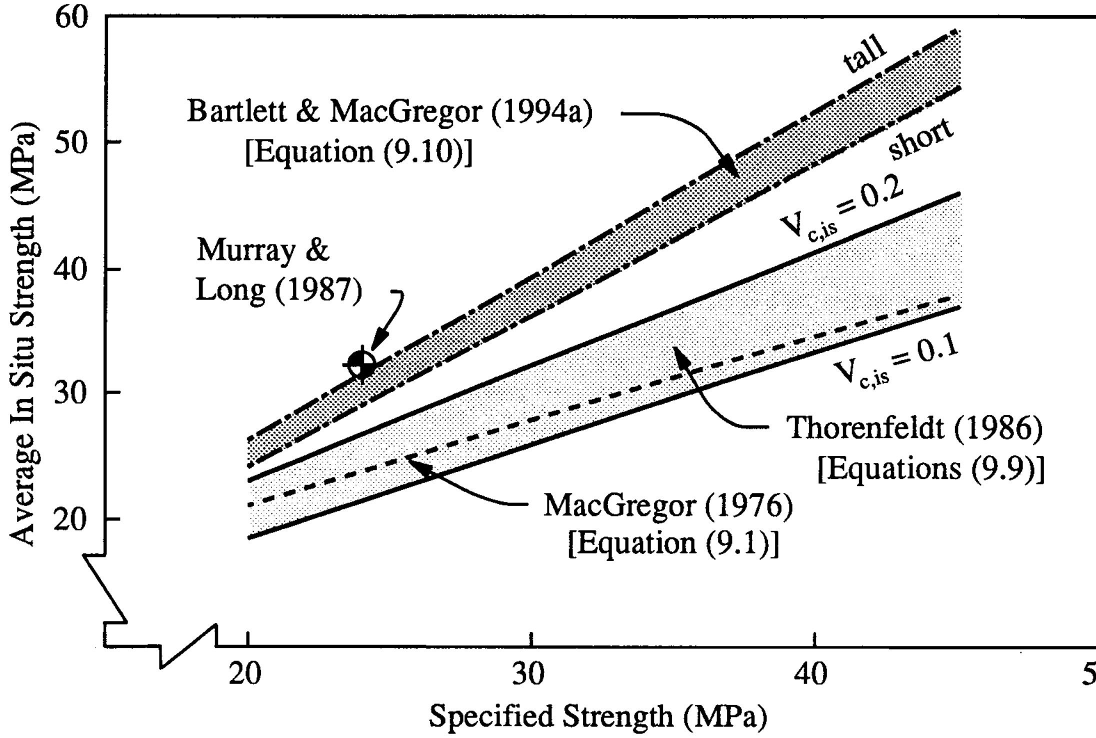
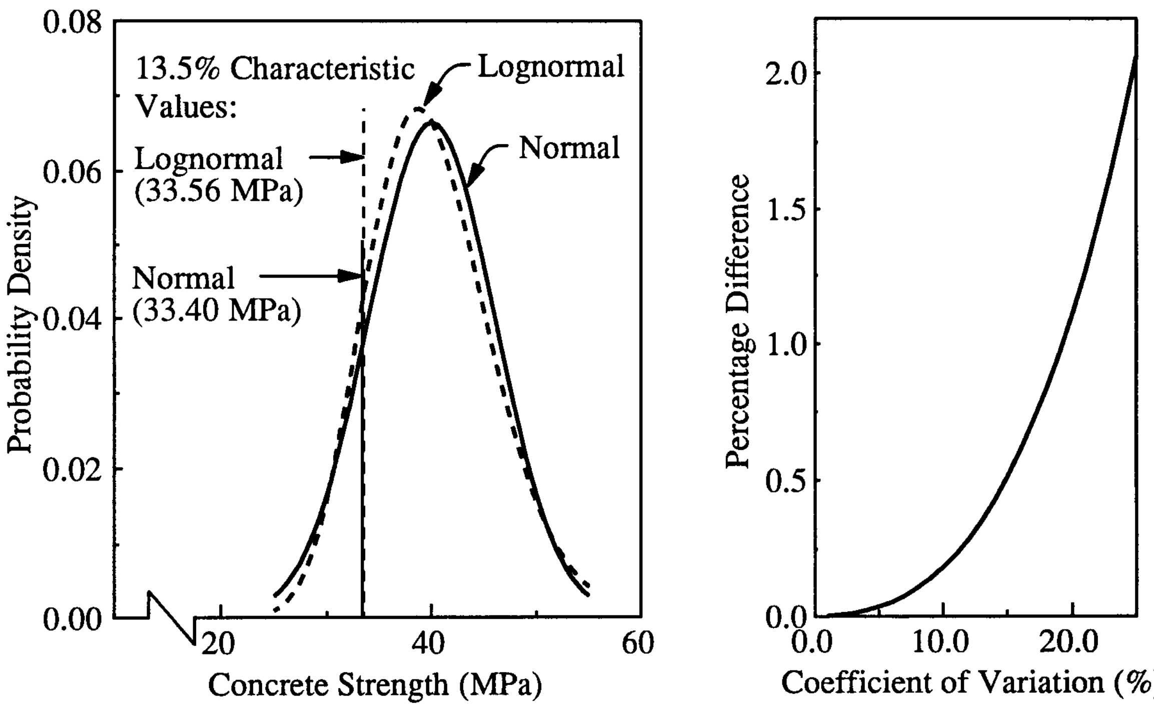


Figure 9.1: Average in situ strength versus specified strength



(a) Distributions with mean = 40 MPa and V = 15%

(b) Percentage difference in the 13.5% characteristic value

Figure 9.2: Comparison of normal and lognormal distributions

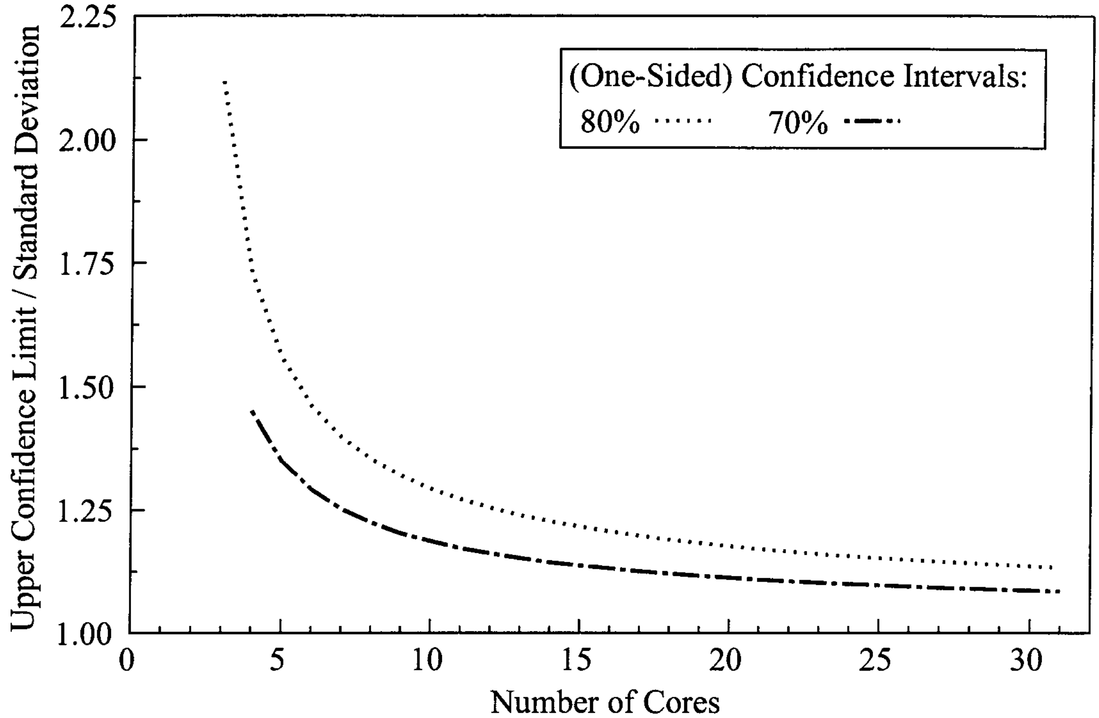
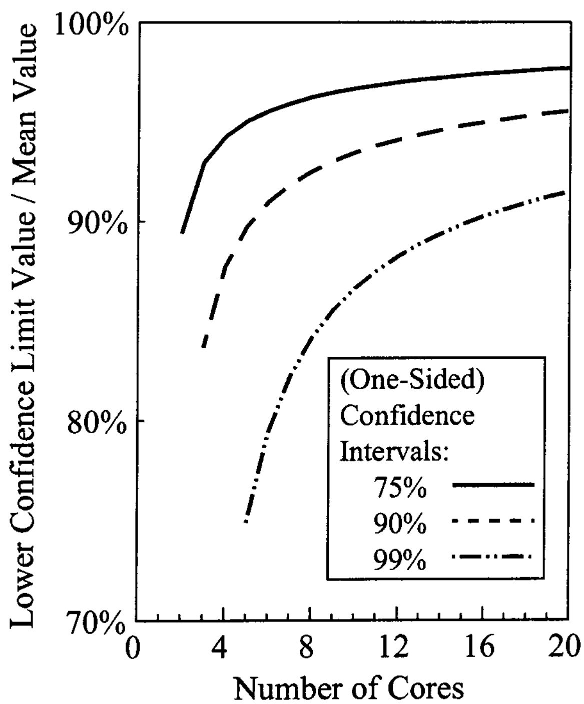
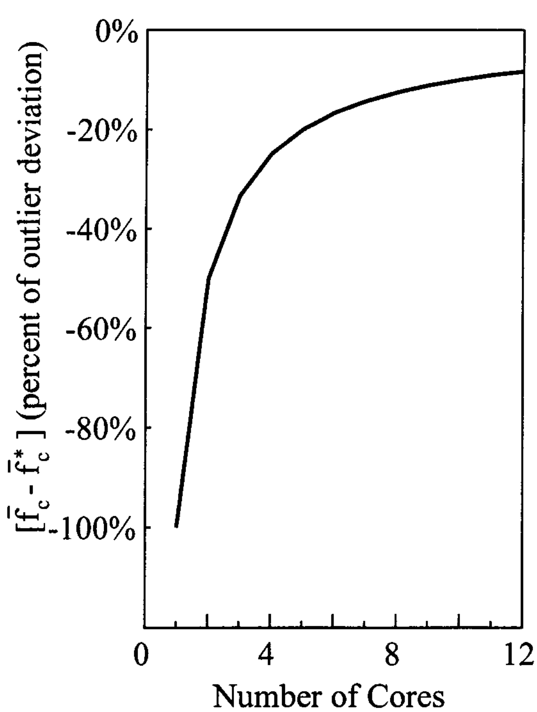


Figure 9.3: One-sided confidence intervals on the standard deviation



(a) Effect on lower confidence limit of mean value for V = 0.15



(b) Effect of undetected low outlier on sample mean value

Figure 9.4: Effect of number of core specimens

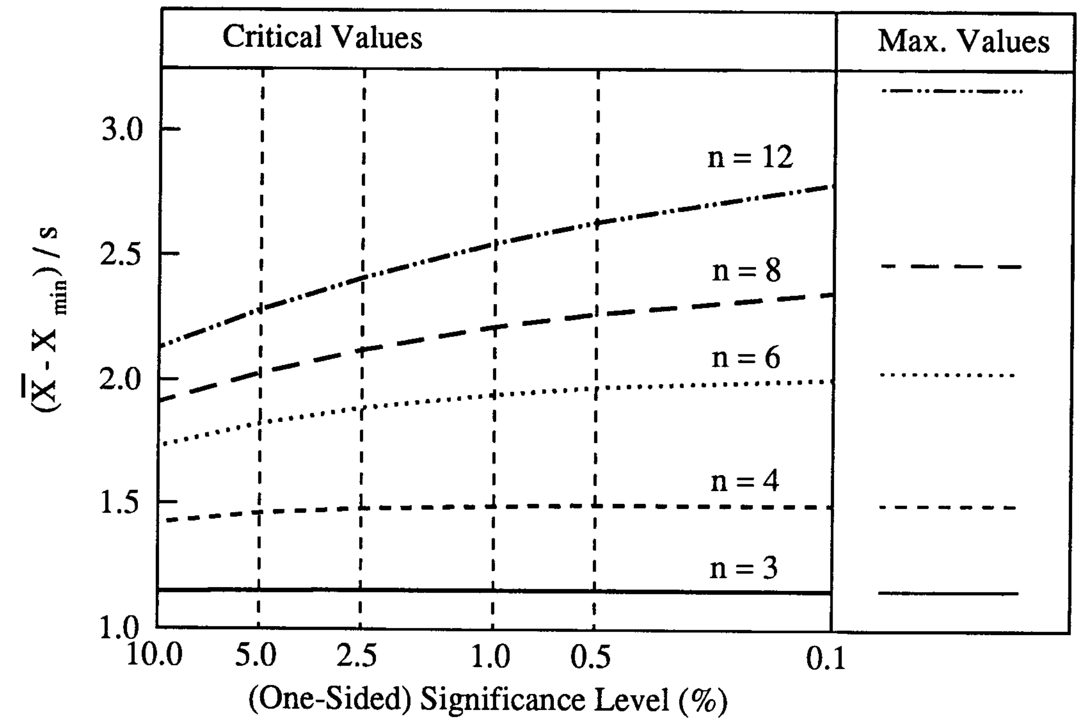


Figure 9.5: Power of ASTM E178-80 detection test for single outlier

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Chapter 10: Conclusions

A simple method is presented for the determination of the compressive strength of concrete using the results of tests of a small number of core specimens from an existing structure. The statistical definition of the value obtained is consistent with the statistical description of the *in situ* strength associated with the originally specified strength, f'_c. The value obtained is therefore equivalent to the specified strength and can be substituted directly for f'_c in conventional resistance equations, incorporating conventional resistance factors, to assess the safety of an existing structure.

A rational basis for the proposed method is developed from a comprehensive investigation of the factors affecting core test strengths and a detailed assessment of the strength of concrete in structures. These associated studies are unique in their use of sophisticated statistical analysis techniques to investigate large sets of data obtained from a number of independent sources. In this chapter, the various findings of these studies are briefly restated and suggested topics for future work are identified.

10.1 Interpretation of Core Strength Data

In Chapters 2 through 6, the various aspects of the testing procedure which affect the compressive strength of concrete cores are quantified. Strength correction factors which address these aspects are generally obtained by weighted linear and nonlinear regression analysis methods. The data used for these analyses are core test results for concretes with conventional strengths investigated by others and core test results for high performance concretes investigated as part of the present study.

An experimental investigation involving 12 data sets comprising a total of 287 concrete core specimens obtained from large beams cast by Alca (1993) using ordinary portland cement concretes with strengths between 45 and 90 MPa is described in Chapter 2. Statistical tests in ASTM E178-80 (ASTM, 1993b) are not powerful for identifying outliers in these data. Significant systematic spatial variation of the *in situ* strength is present in 7 of the 12 data sets, even though the volume of the regions sampled is less than 0.2 m^3. Detailed analyses of the data are presented in Chapters 3, 4, and 5 to assess whether the strength correction factors for the length to diameter ratio, moisture condition, and diameter of a core are valid for high performance concretes.

The effect of the ratio of the length to diameter of the specimen on the compressive strength of concrete cores is investigated in Chapter 3. Regression analyses are performed on data from tests of 758 core specimens, all 100 mm in diameter, obtained from 10 different elements cast from ordinary portland cement concretes with strengths between 14 and 96 MPa. Strength correction factors are presented for converting the strength of a core with an *l*/d ratio between 1 and 2 into the strength of an equivalent specimen with an *l*/d ratio of 2. The required strength correction is slightly but significantly reduced for high strength concretes. The required strength correction is also slightly but significantly reduced if the cores are soaked before testing instead of being left for 7 days exposed to air in a laboratory. The proposed strength correction factors for the length to diameter ratio of the core are only slightly different from those currently recommended in ASTM C42-90 (ASTM, 1993a) or CAN/CSA-A23.2-M90 (CSA, 1990b).

The effect of the moisture condition of the specimen on the compressive strength of concrete cores is investigated in Chapter 4. Regression analyses are carried out using data from tests of 727 core specimens obtained from well cured, mature elements cast using ordinary portland cement concretes with strengths between 15 and 92 MPa. The compressive strength is influenced by changes in the moisture content which occur uniformly throughout the volume of the specimen, and by moisture gradients between the surface and the interior of the specimen. The standard air drying and soaking periods specified for core specimens in ASTM C42-90 (ASTM, 1993a) and CAN/CSA-A23.2-M90 (CSA, 1990b) are too short to allow a uniform change of moisture content throughout the entire volume of the specimen. Instead, the effect of these treatments is to create a moisture gradient which artificially biases the test result. The most accurate estimate of the concrete strength is obtained from a test specimen which has no moisture gradient. The strength of cores dried 7 days in laboratory air before testing is on average 14% larger than the strength of cores soaked 2 days in water before testing. The strength of cores with no moisture gradient is on average 9% larger than the strength of soaked cores. These general average values are independent of the concrete strength, but may vary appreciably from the actual value for a particular concrete mix.

The effect of the diameter of the specimen on the compressive strength of concrete cores is investigated in Chapter 5. Regression analyses are performed on data from tests of 1080 core specimens obtained from elements of moderate size with concrete strengths varying from 10 to 92 MPa. It is shown that the effect of damage to the cut surface of the core counteracts and overwhelms any effect which might be inferred by the weakest link theory for the strength of brittle materials, or which might be attributed to systematic bias caused by testing procedures. The large variability commonly attributed to small diameter specimens is shown to be caused in many cases by the variability of the strength within the element being cored. The analyses indicate that the average strength of a core with a diameter of 50 mm is 94% of the average strength of a core with a diameter of 100 mm, and 92% of the average strength of a core with a diameter of 150 mm. These overall average values do not reflect the considerable scatter of the data, especially in the relationship between the strengths of cores with diameters of 50 and 100 mm, and should be used with caution. The data also indicate that the effect of the length to diameter ratio of the specimen on the compressive strength is more pronounced for cores with diameters of 50 mm than for cores with diameters of 100 mm.

Based on the findings presented in Chapters 3, 4, and 5, a two step method for converting a core test result to the *in situ* strength of the corresponding volume of concrete is presented in Chapter 6. A core test strength is first corrected to the strength of a standard core, which has a diameter of 100mm, a length of 200 mm and does not contain reinforcing bars, and then is converted to the equivalent *in situ* strength. Strength correction factors are required for these conversions which address the effects of the length to diameter ratio, moisture condition, and diameter of the core, the presence of reinforcing bars, and the damage sustained during drilling. Values of these factors are presented, and their accuracy is determined using a weighted regression analysis of ratios of test to predicted values which accounts for non-uniform variances of the dependent and independent variables. The precision of the estimated *in situ* strength is affected by the accuracy of the strength correction factors and by the inherent error of the core strength measurement itself. The coefficient of variation of the *in situ* strength predicted from a test of a core with a diameter of 100 or 150 mm is between 4 and 5.5%. The coefficient of variation of the *in situ* strength predicted from a test of a core with a diameter of 50 mm is 12.5%. These error estimates do not account for possible variation of the strength throughout the volume of the element from which the core is obtained.

The strength correction factors which address the effects of moisture condition and damage due to drilling presented in Chapter 6 are corroborated by the observed strengths of the cores from high performance concrete beams reported in Chapter 2. The *in situ* strength of the concrete corresponding to the maximum flexural resistance of the beams was determined by back calculation from information reported by Alca (1993). The strength of sealed core specimens, which are believed to have negligible moisture gradient, when increased by 6% to account for damage sustained during drilling as suggested in Chapter 6, corresponds well to the *in situ* beam strengths. Accurate estimates of the *in situ* beam strengths are also obtained by correcting the strengths of soaked or air dried cores using the strength conversion factors for moisture condition and damage due drilling presented in Chapter 6.

10.2 The Strength of Concrete in Structures

In Chapters 7 and 8, a statistical description of the strength of concrete in structures is developed. This description is necessary for the strength of concrete obtained from core testing to be interpreted in its proper context when assessing the safety of existing structures. The data used for this investigation are core test and ultrasonic pulse velocity test results obtained by others from elements cast in the laboratory, and from elements in structures. The strength correction factors presented in Chapter 6 are used to convert the core test data to equivalent *in situ* strengths for analysis in Chapters 7 and 8.

In Chapter 7, the relationship between the average strength of concrete in a structure is represented by independent factors F_1 and F_2. Factor F_1 is the ratio of the average strength of standard cylinder specimens tested at 28 days to the specified concrete compressive strength, f'_c. Factor F_2 is the ratio of the average *in situ* strength to the average strength of standard 28 day old cylinders cast from the same batch of concrete.

Concrete mixes are usually designed to have 28 day cylinder strengths in excess of the specified strength. Analysis of 3756 standard strength tests of 2 or 3 cylinders from 108 concrete mixes produced in Alberta between 1988 and 1993 indicates that F_1 may be represented by either a normal distribution or a lognormal distribution. For ready mix concretes used in cast-in-place construction, F_1 has a mean value of 1.25 and a standard deviation of 0.131. For concretes used in precast construction, F_1 has a mean value of 1.19 and a standard deviation of 0.058. Equations in CAN/CSA-A23.1-M90 (CSA, 1990a) which give suggested target average strengths for the design of concrete mixes do not accurately reflect the actual margin of overstrength observed.

The ratio, F_2, of the average *in situ* strength to the strength of 28 day old standard cylinder specimens is investigated using data from 108 conventional concrete mixes and 22 high performance concrete mixes obtained from various published and unpublished sources. When the concrete is 28 days old, the average value of F_2 is 0.95 for elements less than 450 mm high or 1.03 for elements greater than 450 mm high. The value of F_2 is independent of concrete strength but is smaller for concretes containing supplementary cementitious materials than for concretes with only ordinary portland cement. For conventional concretes, F_2 may be represented by a lognormal distribution with a coefficient of variation of 14%.

Combining the effects of factors F_1 and F_2, the average *in situ* compressive strength of 28 day old conventional concretes used for cast-in-place construction is 1.2 f'_c for shallow elements or 1.3 f'_c for tall elements. The ratio of the average *in situ* strength to the specified strength has a lognormal distribution with a coefficient of variation of 18.6%.

In Chapter 8, multiple regression analysis techniques are used to assess the systematic variation of the strength of concretes in laboratory specimens cast from one batch of concrete, and in structures. The investigation of laboratory cast elements includes 585 cores from 43 columns with average strengths ranging from 15 to 36 MPa and 1062 cores from 26 beams, blocks, slabs and walls with average strengths ranging from 16 to 120 MPa. The investigation of structures considers ultrasonic pulse velocity and pull-off test results from columns in 29 buildings and from girders in one bridge.

Significant systematic strength variation along the height of the column is detected in 32 of the 43 laboratory cast columns. Typically the top region was 3 to 14% weaker than the region in the middle and the bottom region was 3 to 9% stronger than the region in the middle. The slight difference between the density at the top and at the bottom of the column, although statistically significant, seems insufficient to be the primary cause of the strength variation. Significant systematic variation of the *in situ* strength is detected in 20 of the 26 beams, blocks, slabs and walls cast in various laboratories. In each instance where the strength varied over the depth of the element, the concrete near the top of the element as poured was weaker than the concrete near the bottom. No evidence to contradict these findings emerged from the investigation of the data from the building columns and the bridge girders.

10.3 Assessment of Concrete Strength in Existing Structures

In Chapter 9, a simple procedure is presented for the determination of an equivalent specified concrete strength from a small number of core tests for the assessment of the safety of an existing structure. Guidelines for planning the core testing programme and for detecting outliers in the data set are presented. The recommended minimum sample size is 6 cores from each grade of concrete being evaluated. Analysis of core test data allows the evaluator to roughly quantify the realisations of various random variables that affect the concrete strength which, for a specific structure, are initially unknown. The procedure yields a concrete strength value which is consistent with the statistical description of the concrete strength at the time of initial design and so can be substituted directly for f'_c in conventional resistance equations.

The findings of Chapters 7 and 8 indicate that there is roughly a 13.5% probability that the *in situ* strength of concrete in a structure which is 28 days old will be less than the originally specified strength. This is a much smaller probability than that associated with the statistical description of concrete strength used to calibrate the load and resistance factors which are currently used for design of new structures in Canada. In the present investigation, it is assumed that a safety level which is equivalent to that provided by current design criteria, and is therefore sufficient, will be achieved for evaluation using a nominal concrete strength value equal to the 13.5% characteristic value with current resistance equations and resistance factors. However, it is also probable that recalibration of the design load and resistance factors using the statistical description of concrete presented in Chapter 9 would yield greater factored concrete strengths than are used in current design practice.

The procedure for calculating the equivalent specified strength is carried out in several steps. Core strengths are converted to equivalent *in situ* strengths using the strength correction factors presented in Chapter 6, and checked for outliers. A lower bound estimate of the mean *in situ* strength is determined from these data. The 13.5% characteristic value associated with this lower bound estimate of the mean strength is then calculated based on the generic description of the variation of the *in situ* strengths within a structure reported in Chapters 7 and 8. The variability of the observed core strength, which is generally greater than the variability of the *in situ* strength due to various testing factors which are difficult to quantify, is reflected only in the interval estimate of the mean *in situ* strength.

10.4: Suggestions for Future Work

As already noted, the statistical description for the *in situ* strength of concrete developed in this study is different from that used to calibrate the load and resistance factors which are currently used for the design of new structures. A recalibration using the statistical description of concrete strength presented in Chapter 9 is warranted. It is likely that the resulting factored concrete compressive resistance will be 10 to 20% greater than that used in current design practice.

Additional research is necessary to determine the relationship between the mean *in situ* strength of precast elements and the average strength of standard 28 day old cylinders. For precast units which are not steam cured, the relationship for F_2 presented in Chapter 7 is valid. However, most precast units are steam cured so that a high early strength is obtained to maintain a rapid cycle of production. The effect of steam curing on the *in situ* strength at 28 days, and at later ages, is not represented by the relationships presented in Chapter 7 and should be investigated.

Additional research is also necessary to determine the relationship between the standard cylinder strength and the specified strength of high performance concretes incorporating supplementary cementitious materials. It is likely that the relationship for F_1 presented in Chapter 7 does not apply to high performance concretes. For conventional concretes with strengths less than 40 MPa, the cost to the concrete producer of supplying a material which is on average 20 to 25% stronger than specified is relatively small. If the specified strength is 100 MPa, the cost of providing this margin of overdesign is relatively great, and may in practice be impossible to achieve. If the relationship for F_1 presented in Chapter 7 is used to calibrate resistance factors for high performance concretes, unconservative results are likely.

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Appendix 2A: Strength Data for Cores from Medium Sized Beams

This Appendix gives strength and location data for all cores obtained from Alca's medium sized beams. The specimen from the surface near the drill is designated s and the specimen from the surface furthest from the drill is designated d. The core diameter is 50 mm in all cases. Coordinates x, y, and z locate the centre of gravity of the core according to the coordinate system shown in Figures 2A.1 (a) and (b). The raw core strength $f_{c,NS}$ after adjustment for core lengths l less than 100 mm is shown in the column headed "f_c". The strength after further adjustment as necessary to compensate for any statistically significant spatial variation of the *in situ* strength, summarised in Table 2.5, is shown in the column headed "f^*_c".

Table 2A.1: Cores from South End of Beam ML1

No.	x (mm)	y (mm)	z (mm)	Z_{mc}	Z_c	l (mm)	$f_{c,NS}$ (MPa)	f_c (MPa)	f^*_c (MPa)	Note
1 s	320	185	65	0	0	98	55.8	55.8	55.0	
1 d	320	185	170	1	0	100	40.3	40.3	41.1	
2 s	320	285	65	1	0	100	44.8	44.8	44.0	
2 d	320	285	170	0	0	96	49.5	49.5		low outlier
3 s	390	185	170	0	1	98	57.3	57.3	58.1	
3 d	390	185	70	0	0	99	60.3	60.3	59.5	
4 s	390	285	170	1	0	99	44.5	44.5	45.3	
4 d	390	285	65	0	1	98	55.8	55.8	55.0	
5 s	460	190	170	0	1	101	54.6	54.6	55.4	
5 d	460	190	60	1	0	99	46.9	46.8	46.0	
6 s	460	290	150	0	0	96	56.1	56.0	56.6	
6 d	460	290								no core
7 s	555	185	65	0	1	97	56.4	56.4	55.6	
7 d	555	185								no core
8 s	555	285	70	0	0	100	57.9	57.9	57.2	
8 d	555	285	175	1	0	97	40.9	40.9	41.8	
9 s	620	215	60	1	0	102	40.9	40.9	40.0	
9 d	620	215	165	0	0	100	61.2	61.2	61.9	
10 s	620	295	65	0	1	100	60.3	60.3	59.5	
10 d	620	295	170	0	0	100	57.0	57.0	57.8	
11 s	710	255	175	1	0	100	41.2	41.2	42.1	
11 d	710	255	70	0	0	100	60.9	60.9	60.1	
12 s	710	325	170	1	0	100	40.9	40.9	41.7	
12 d	710	325	55	0	1	96	58.8	58.8	57.7	
Avg.	509	249	117							

Note: Cores were drilled when the concrete was 106 days old, and left in the laboratory air for 14 days. At this time the sides of some cores, shown in the table by indicator variable $Z_c = 1$, were coated with an epoxy resin. Then some of the cores, shown in the table by indicator variable $Z_{mc} = 1$, were soaked in water while the remainder were left in laboratory air. All cores were tested when the concrete was 128 days old.

Table 2A.2: Cores from North End of Beam ML1

No.	x (mm)	y (mm)	z (mm)	Z_{HV}	l (mm)	$f_{c,NS}$ (MPa)	f_c (MPa)	f^*_c (MPa)	Note
13 s	415	295	75	1	89	40.0	39.6	39.6	
13 d	415	190	75	1	101	32.2	32.2	32.2	
14 s	420	295	150	1	102	26.9	26.9	26.9	
14 d	420	160	150	1	95	40.0	39.9	39.9	
15 s	330	285	165	0	99	38.8	38.8	38.8	
15 d	330	285	60	0	100	38.2	38.2	38.2	
16 s	330	185	165	0	100	39.4	39.4	39.4	
16 d	330	185	60	0	98	34.9	34.9	34.9	
17 s	550	285	65	0	99	37.9	37.9	37.9	
17 d	550	285	170	0	99	36.1	36.1	36.1	
18 s	550	185	70	0	101	34.6	34.6	34.6	
18 d	550	185	175	0	97	33.1	33.1	33.1	
19 s	710	295	60	1	100	35.5	35.5	35.5	
19 d	710	190	60	1	100	34.0	34.0	34.0	
20 s	710	365	155	1	98	37.6	37.6	37.6	
20 d	710	260	155	1	99	35.5	35.5	35.5	
20 *	710	158	155	1	95	26.3	26.2		low outlier
Avg.	502	246	113						

Note: Cores were drilled when the concrete was 106 days old. They were stored in water until the time of testing, when the concrete age was 125 days. In the table above, indicator variable $Z_{HV} = 1$ for cores with their axis parallel to the direction of casting, or 0 otherwise.

Table 2A.3: Cores from South End of Beam ML2

No.	x (mm)	y (mm)	z (mm)	Z_{mc}	Z_c	l (mm)	$f_{c,NS}$ (MPa)	f_c (MPa)	f^*_c (MPa)	Note
21 s	680	285	60	1	0	99	55.5	55.5	55.8	
21 d	680	285	164	0	0	100	74.0	74.0	74.3	
22 s	680	185	65	1	1	103	63.3	63.3	63.5	
22 d	680	185	170	1	0	97	55.8	55.8	56.0	
23 s	580	285	70	0	0	99	69.5	69.5	71.6	
23 d	580	285	175	1	1	101	56.7	56.7	58.8	
24 s	580	185	65	1	0	98	54.6	54.6	56.7	
24 d	580	185	170	0	0	100	68.9	68.9	71.0	
25 s	520	310	180	1	1	93	55.2	55.0	57.2	
25 d	520	310	82	1	0	95	50.1	50.0	52.2	
26 s	520	240	165	0	0	102	65.9	65.9	68.2	
26 d	520	240	60	1	1	91	46.9	46.5		low outlier
27 s	420	285	70	1	1	92	64.5	64.1	64.8	
27 d	420	285								no core
28 s	420	185	55	1	0	91	54.3	54.0	54.7	
28 d	420	185	144	0	0	86	71.0	69.9	70.6	
29 s	360	250	175	1	0	98	57.3	57.3	56.1	
29 d	360	250	75	0	0	91	68.0	67.6	66.4	
30 s	350	190	150	1	1	99	64.8	64.8	63.2	
30 d	350	190								no core
31 s	335	310	170	0	0	101	71.3	71.3	69.1	
31 d	335	310	65	1	0	98	58.5	58.5	56.2	
32 s	280	185	170	1	0	99	56.4	56.4	51.5	
32 d	280	185	65	0	0	99	70.1	70.1	65.2	
Avg.	484	242	119							

Note: Cores were drilled when the concrete was 100 days old, and left in the laboratory air for 14 days. At this time the sides of some cores, shown in the table by indicator variable $Z_c = 1$, were coated with an epoxy resin. Then some of the cores, shown in the table by indicator variable $Z_{mc} = 1$, were soaked in water while the remainder were left in laboratory air. All cores were tested when the concrete was 125 days old.

Table 2A.4: Cores from North End of Beam ML2

No.	x (mm)	y (mm)	z (mm)	Z_{HV}	l (mm)	$f_{c,NS}$ (MPa)	f_c (MPa)	f^*_c (MPa)	Note
33 s	275	360	60						no core
33 d	275	195	60	1	98	49.2	49.2	51.4	
34 s	425	305	112	1	95	65.7	65.5	65.3	
34 d	425	175	112	1	100	56.1	56.1	55.9	
34*	425	395	112	1	97	54.9	54.9	54.7	extra core
35 s	360	290	175	0	99	46.0	46.0	42.8	
35 d	360	290	70	0	102	53.7	53.7	55.5	
36 s	365	185	165	0	101	55.2	55.2	52.5	
36 d	365	185	60	0	101	54.6	54.6	56.8	
37 s	535	315	55	1	99	60.6	60.6	63.0	
37 d	535	210	55	1	103	53.4	53.4	55.9	
38 s	525	355	135	1	100	63.9	63.9	62.6	
38 d	525	170	135	1	93	59.7	59.4	58.2	
39 s	720	185	65	0	91	47.7	47.4	49.4	
39 d	720	185	165	0	97	58.2	58.1	55.5	
40 s	720	285	70	0	100	44.5	44.5	46.2	
40 d	720	285	175	0	100	55.8	55.8	52.7	
Avg.	500	251	108						

Note: Cores were drilled when the concrete was 100 days old. They were stored in water until the time of testing, when the concrete age was 118 days. In the table above, indicator variable $Z_{HV} = 1$ for cores with their axis parallel to the direction of casting, or 0 otherwise.

Table 2A.5: Cores from North End of Beam MH1

No.	x (mm)	y (mm)	z (mm)	Z_{mc}	Z_c	l (mm)	$f_{c,NS}$ (MPa)	f_c (MPa)	f^*_c (MPa)	Note
41 s	350	335								no core
41 d	350	335								no core
42 s	360	275	165	0	0	97	97.9	97.8	97.8	
42 d	360	275	60	1	0	101	78.8	78.8	78.8	
43 s	355	210	165	1	0	99	82.7	82.7	82.7	
43 d	355	210	60	0	0	100	88.6	88.6	88.6	
44 s	425	335	65	0	0	98	90.1	90.1	90.1	
44 d	425	335	170	0	1	98	89.8	89.8	89.8	
45 s	420	275	70	1	0	99	80.0	80.0	80.0	
45 d	420	275	175	0	1	98	97.3	97.3	97.3	
46 s	420	210	65	1	0	97	62.1	62.0		low outlier
46 d	420	210	170	0	0	99	104.1	104.1	104.1	
47 s	580	310	165	1	0	99	73.7	73.7	73.7	
47 d	580	310	60	0	0	100	96.4	96.4	96.4	
48 s	580	245	165	0	0	97	97.9	97.8	97.8	
48 d	580	245	60	1	0	101	74.0	74.0	74.0	
49 s	665	310	165	0	0	100	109.2	109.2	109.2	
49 d	665	310	60	1	0	100	78.8	78.8	78.8	
50 s	660	240	70	0	0	99	97.6	97.6	97.6	
50 d	660	240	175	0	1	93	97.0	96.6	96.6	
51 s	750	300	65	0	1	101	91.9	91.9	91.9	
51 d	750	300	170	1	0	97	68.3	68.3	68.3	
52 s	745	235	65	1	1	99	71.6	71.6		wrong treatment
52 d	745	235	170							no core
Avg.	532	269	114							

Note: Cores were drilled when the concrete was 93 days old, and left in the laboratory air for 14 days. At this time the sides of some cores, shown in the table by indicator variable $Z_c = 1$, were coated with an epoxy resin. Then some of the cores, shown in the table by indicator variable $Z_{mc} = 1$, were soaked in water while the remainder were left in laboratory air. All cores were tested when the concrete was 115 days old.

Table 2A.6: Cores from South End of Beam MH1

No.	x (mm)	y (mm)	z (mm)	Z_{mc}	Z_c	l (mm)	$f_{c,NS}$ (MPa)	f_c (MPa)	f^*_c (MPa)	Note
53 s	280	370								no core
53 d	280	370								no core
54 s	330	285	165	0	0	100	109.8	109.8	108.6	
54 d	330	285	60	1	0	99	83.3	83.3	82.0	
55 s	330	228	155	1	0	89	78.2	77.4	81.9	
55 d	330	228	60	0	0	89	101.8	100.8	105.3	
56 s	382	345	70	1	0	100	75.5	75.5	68.2	
56 d	382	345	175	1	1	98	90.1	90.1	82.8	
57 s	400	275	55	1	1	90	88.6	87.9	87.7	
57 d	400	275	148	0	0	83	106.2	103.8	103.6	
58 s	400	210	82	0	0	89	85.0	84.2	90.5	
58 d	400	210	178	0	1	88	85.9	85.0		wrong treatment
59 s	490	330	165	1	1	100	99.7	99.7	93.9	
59 d	490	330	60	1	0	98	82.1	82.0	76.3	
60 s	485	265	170	0	0	95	85.3	85.2	86.0	
60 d	485	265	65	1	0	102	78.2	78.2	79.0	
61 s	640	305	175	1	1	89	89.8	89.0	85.7	
61 d	640	305	82	0	0	86	103.0	101.3	98.1	
62 s	640	220	70	1	0	85	68.0	66.8	72.1	
62 d	640	220	175	1	1	96	85.0	84.9	90.2	
63 s	725	308	65	0	0	99	97.0	97.0	93.4	
63 d	725	308	170	1	0	99	80.9	80.9	77.3	
64 s	725	230	60	1	0	102	60.0	60.0	64.3	
64 d	725	230	165	0	0	97	86.8	86.8		precracked
Avg.	498	278	111							

Note: Cores were drilled when the concrete was 93 days old, and left in the laboratory air for 14 days. At this time the sides of some cores, shown in the table by indicator variable $Z_c = 1$, were coated with an epoxy resin. Then some of the cores, shown in the table by indicator variable $Z_{mc} = 1$, were soaked in water while the remainder were left in laboratory air. All cores were tested when the concrete was 115 days old.

Table 2A.7: Cores from North End of Beam MH2

No.	x (mm)	y (mm)	z (mm)	Z_{HV}	l (mm)	$f_{c,NS}$ (MPa)	f_c (MPa)	f^*_c (MPa)	Note
65 s	350	295	165	0	100	51.9	51.9	51.9	
65 d	350	295	60	0	97	56.7	56.7	56.7	
66 s	350	220	165	0	98	64.5	64.5	64.5	
66 d	350	220	60	0	99	62.4	62.4	62.4	
67 s	420	355	85	1	99	59.7	59.7	59.7	
67 d	420	250	85	1	99	58.8	58.8	58.8	
68 s	420	395	145	1	99	62.4	62.4	62.4	
68 d	420	290	145	1	98	66.2	66.2	66.2	
69 s	505	355	85	1	101	60.9	60.9	60.9	
69 d	505	250	85	1	99	59.4	59.4	59.4	
70 s	515	355	145						no core
70 d	515	225	145	1	98	63.6	63.6	63.6	
71 s	655	290	70	0	98	63.3	63.3	63.3	
71 d	655	290	150						no core
72 s	655	212	70	0	99	59.7	59.7	59.7	
72 d	655	212	175						no core
Avg.	455	281	105						

Note: Cores were drilled when the concrete was 80 days old. They were stored in water until the time of testing, when the concrete age was 98 days. In the table above, indicator variable Z_{HV} = 1 for cores with their axis parallel to the direction of casting, or 0 otherwise.

Table 2A.8: Cores from South End of Beam MH2

No.	x (mm)	y (mm)	z (mm)	Z_{HV}	l (mm)	$f_{c,NS}$ (MPa)	f_c (MPa)	f^*_c (MPa)	Note
73 s	587	330	160	0	100	54.0	54.0	50.4	
73 d	587	330	60	0	100	55.8	55.8	58.7	
74 s	658	330	140	0	84	51.0	50.0	47.7	
74 d	658	330	53	0	82	49.5	48.3	51.6	
75 s	507	391	85	1	84	52.2	51.2	52.4	
75 d	507	295	85	1	99	54.9	54.9	56.2	
76 s	507	375	145						no core
76 d	507	215	145	1	96	60.6	60.5	57.9	
77 s	418	395	85	1	99	47.7	47.7	49.0	
77 d	418	245	85	1	100	48.0	48.0	49.3	
78 s	418	405	145	1	96	62.4	62.3	59.6	
78 d	418	265	145	1	101	58.8	58.8	56.1	
79 s	348	360	90	0	97	54.9	54.9	55.8	
79 d	348	360							no core
80 s	355	295	80	0	100	57.0	57.0	58.6	
80 d	355	295							no core
Avg.	491	322	104						

Note: Cores were drilled when the concrete was 80 days old. They were stored in water until the time of testing, when the concrete age was 94 days. In the table above, indicator variable Z_{HV} = 1 for cores with their axis parallel to the direction of casting, or 0 otherwise.

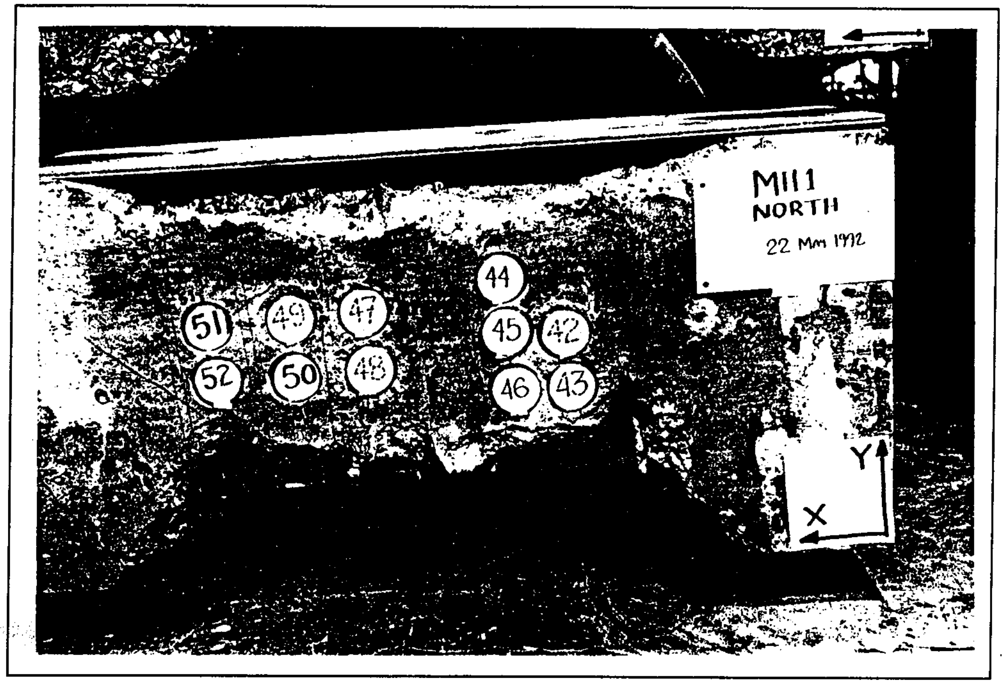

Figure 2A.1 (a): Core layout, North end of Beam MH1

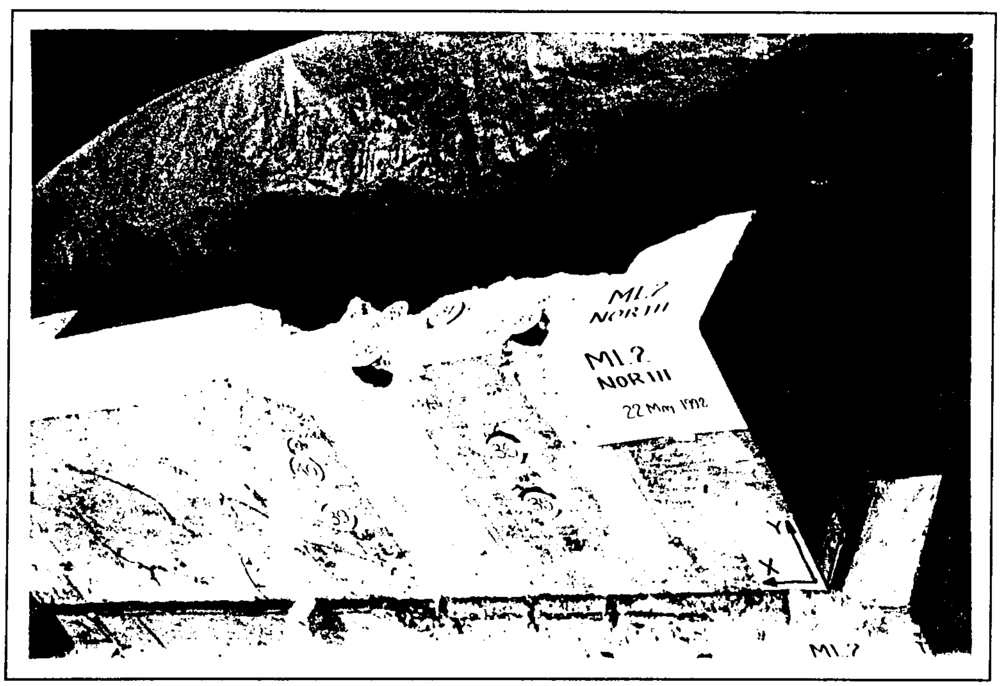

Figure 2A.1 (b): Core layout, North end of Beam ML2

Appendix 2B: Strength Data for Cores from Large Beams

This appendix gives strength, size, location and treatment data for cores obtained from Alca's large Beams LL1 and LL2 and from blocks cast at the same time as Beams LH1 and LH2. The specimen from the surface near the drill is designated s and the specimen from the surface furthest from the drill is designated d. Specimen obtained from the middle of the core are designated m. Coordinates x, y, and z locate the centre of gravity of the core according to the coordinate systems shown in Figures 2B.1 and 2B.2. The diameter d and length *l* were measured just before each core was tested. The raw core strength is shown in the column headed "f_c". The strength after adjustment to compensate for any statistically significant spatial variation of the *in situ* strength, as summarised in Table 2.5, is shown in the column headed "f^*_c".

Table 2B.1: Cores from North End of Beam LL1

No.		d (mm)	x (mm)	y (mm)	z (mm)	*l* (mm)	f_c (MPa)	f^*_c (MPa)	Note
81	s	100	590	370	130	206	45.1	45.1	
81	d	100	590	370	285	101	43.3	43.3	
82	s	100	550	230	75	98	47.7	47.7	
82	d	100	550	230	235	205	42.4	42.4	
83	s	100	805	480	50	99	49.2	49.2	
83	d	100	805	480	210	200	39.8	39.8	
84	s	100	805	360	120	204	47.7	47.7	
84	d	100	805	360	280	103	49.4	49.4	
85	s	100	805	230	100	203	41.5	41.5	
85	d	100	805	230	260	101	49.9	49.9	
86	s	100	1040	490	75	102	49.7	49.7	
86	d	100	1040	490	215	206	19.8		low outlier
87	s	100	1040	360	75	101	48.8	48.8	
87	d	100	1040	360	215	203	47.2	47.2	
88	s	100	1275	480	100	204	43.9	43.9	
88	d	100	1275	480	260	97	52.6	52.6	
89	s	50	535	500	62	98	34.0		low outlier
89	m	50	535	500	163	97	43.3	43.3	
89	d	50	535	500	262	96	46.3	46.3	
90	s	50	605	500	62	96	47.4	47.4	
90	m	50	605	500	163	97	46.9	46.9	
90	d	50	605	500	265	96	50.4	50.4	
91	s	50	630	230	90	99	44.2	44.2	
91	m	50	630	230					no core
91	d	50	630	230	255	98	43.6	43.6	
Avg.			771	394	167				

<u>Note:</u> Cores were drilled when the concrete was 65 days old, and tested when the concrete was 71 days old. During the period between drilling and testing, all cores were stored in sealed plastic bags.

Table 2B.2: Cores from North End of Beam LL2

No.		d (mm)	x (mm)	y (mm)	z (mm)	l (mm)	f_c (MPa)	f^*_c (MPa)	Note
92	s	100	590	355	125	200	48.9	47.5	
92	d	100	590	355	280	106	49.2	47.7	
93	s	100	530	230	75	100	32.4		low outlier
93	d	100	530	230	230	205	45.3	44.6	
94	s	100	805	475	105	200	47.0	46.5	
94	d	100	805	475	260	99	48.5	48.0	
95	s	100	805	360	55	98	48.7	49.5	
95	d	100	805	360	210	179	47.8	48.2	
96	s	100	805	245	125	202	44.5	46.7	
96	d	100	805	245	275	102	42.6	44.7	
97	s	100	1040	475	55	100	51.6	53.6	
97	d	100	1040	475	205	205	41.3	43.3	
98	s	100	1040	360	110	204	40.0	43.3	
98	d	100	1040	360	260	101	42.2	45.5	
99	s	100	1275	475	55	100	34.9		low outlier
99	d	100	1275	475	210	202	40.9	45.4	
100	s	50	535	500	50	100	40.6	36.9	
100	m	50	535	500	155	100	43.9	40.1	
100	d	50	535	500	280	100	42.4	38.6	
101	s	50	605	500	80	100	46.0	43.0	
101	m	50	605	500	180	100	47.1	44.2	
101	d	50	605	500					no core
102	s	50	630	230	65	100	41.2	41.6	
102	m	50	630	230					no core
102	d	50	630	230	265	100	43.3	43.7	
Avg.			771	399	163				

Note: Cores were drilled when the concrete was 51 days old, and tested when the concrete was 55 days old. During the period between drilling and testing, all cores were stored in sealed plastic bags.

Table 2B.3: Cores from Block Cast with Beam LH1

No.		d (mm)	x (mm)	y (mm)	z (mm)	l (mm)	Z_{mc}	Z_{seal}	f_c (MPa)	f^*_c (MPa)	Note
103	s	100	150	485	105	200	0	1	90.2	90.2	
103	d	100	150	485	260	103	0	1	103.7	103.7	
104	s	100	330	485	70	99	0	0	106.4	106.4	
104	d	100	330	485	230	205	0	0	94.2	94.2	
105	s	100	185	375	55	99	0	0	98.6	98.6	
105	d	100	185	375	210	203	0	0	93.0	93.0	
106	s	100	295	375	105	204	1	0	83.6	83.6	
106	d	100	295	375	260	100	1	0	88.8	88.8	
107	s	100	150	265	100	203	1	0	77.0	77.0	
107	d	100	150	265	255	97	1	0	89.8	89.8	
108	s	100	330	265	85	99	0	1	93.9	93.9	
108	d	100	330	265	240	203	0	1	93.4	93.4	
109	s	100	185	155	50	103	0	1	100.0	100.0	
109	d	100	185	155	220	198	0	1	95.4	95.4	
110	s	100	295	155	130	202	0	0	96.3	96.3	
110	d	100	295	155	290	99	0	0	102.5	102.5	
111	s	50	240	485	50	100	1	0	73.1	73.1	
111	m	50	240	485	155	101	1	0	78.8	78.8	
111	d	50	240	485							no core
112	s	50	240	300	50	102	0	0	93.1	93.1	
112	m	50	240	300	150	97	0	0	101.2	101.2	
112	d	50	240	300							no core
113	s	50	240	240	50	101	1	0	81.5	81.5	
113	m	50	240	240	150	98	1	0	75.2	75.2	
113	d	50	240	240							no core
114	s	100	565	485	125	206	1	0	84.6	84.6	
114	d	100	565	485	280	101	1	0	86.8	86.8	
115	s	100	675	485	75	99	0	1	102.9	102.9	
115	d	100	675	485	230	206	0	1	84.8	84.8	
116	s	100	530	375	55	98	0	1	96.0	96.0	
116	d	100	530	375	210	201	0	1	83.3	83.3	
117	s	100	710	375	105	202	0	0	98.0	98.0	
117	d	100	710	375	255	101	0	0	100.2	100.2	
118	s	100	565	265	105	202	0	0	89.7	89.7	
118	d	100	565	265	260	99	0	0	95.7	95.7	
119	s	100	675	265	52	100	1	0	82.3	82.3	
119	d	100	675	265	205	200	1	0	83.3	83.3	
120	s	100	530	155	55	99	1	0	90.3	90.3	
120	d	100	530	155	205	203	1	0	82.1	82.1	
121	s	100	710	155	105	200	0	1	86.9	86.9	
121	d	100	710	155	255	99	0	1	99.5	99.5	

Table 2B.3 (concluded): Cores from Block Cast with Beam LH1

No.		d (mm)	x (mm)	y (mm)	z (mm)	l (mm)	Z_{mc}	Z_{seal}	f_c (MPa)	f^*_c (MPa)	Note
122	s	50	620	410	60	102	0	0	99.1	99.1	
122	m	50	620	410							no core
122	d	50	620	410	165	100	0	0	92.5	92.5	
123	s	50	620	350	60	100	1	0	80.6	80.6	
123	m	50	620	350	165	96	1	0	78.8	78.8	
123	d	50	620	350							no core
124	s	50	620	155	60	101	0	0	83.6	83.6	
124	m	50	620	155	160	96	0	0	87.7	87.7	
124	d	50	620	155	265	98	0	0	99.7	99.7	
Avg.			434	317	151						

Note: Cores were drilled when the concrete was 44 days old, and stored in sealed plastic bags. Some cores, indicated in the table by indicator variables $Z_{mc} = Z_{seal} = 0$, were exposed to laboratory air for 7 days before testing. Some, indicated in the table by indicator variable $Z_{mc} = 1$, were soaked for 48 hours before testing. The remainder, indicated in the table by indicator variable $Z_{seal} = 1$, were left in the plastic bags until testing. The concrete was 55 days old on the day of the test.

Table 2B.4: Cores from Block Cast with Beam LH2

No.		d (mm)	x (mm)	y (mm)	z (mm)	l (mm)	Z_{mc}	Z_{seal}	$f_{c,NS}$ (MPa)	f_c (MPa)	f^*_c (MPa)	Note
125	s	100	150	485	105	201	0	1	80.4	80.4	84.6	
125	d	100	150	485	255	97	0	1	89.4	89.4	91.7	
126	s	100	330	485	105	195	0	0	86.4	86.4	89.6	
125	s	100	150	485	105	201	0	1	80.4	80.4	84.6	
125	d	100	150	485	255	97	0	1	89.4	89.4	91.7	
126	s	100	330	485	105	195	0	0	86.4	86.4	89.6	
126	d	100	330	485	255	96	0	0	105.0	105.0	106.4	
127	s	100	185	375	55	103	0	0	96.9	96.9	100.1	
127	d	100	185	375	210	202	0	0	90.5	90.5	91.8	
128	s	100	295	375	50	97	1	0	78.9	78.9	81.6	
128	d	100	295	375	230	192	1	0	76.8	76.8	77.3	
129	s	100	150	265	105	199	1	0	79.1	79.1	80.4	
129	d	100	150	265	255	98	1	0	85.2	85.2	84.7	
130	s	100	330	265	50	100	0	1	89.7	89.7	90.7	
130	d	100	330	265	205	201	0	1	73.6	73.6	72.7	
131	s	100	185	155	55	100	0	1	90.6	90.6	91.0	
131	d	100	185	155	205	201	0	1	85.0	85.0	83.5	

Table 2B.4 (concluded): Cores from Block Cast with Beam LH2

No.		d (mm)	x (mm)	y (mm)	z (mm)	l (mm)	Z_{mc}	Z_{seal}	$f_{c,NS}$ (MPa)	f_c (MPa)	f^*_c (MPa)	Note
132	s	100	295	155	105	200	0	0	92.1	92.1	91.3	
132	d	100	295	155	255	98	0	0	102.7	102.7	100.0	
133	s	50	240	485	45	96	0	0	83.3	83.2	87.6	
133	m	50	240	485	150	94	0	0	97.0	96.7	99.9	
133	d	50	240	485	270	98	0	0	85.9	85.9	87.6	
134	s	50	240	300	45	96	1	0	68.6	68.5	70.6	
134	m	50	240	300	165	90	1	0	79.1	78.4	79.0	
134	d	50	240	300	240	89	1	0	52.2	51.7		outlier
135	s	50	240	240	65	98	0	0	93.4	93.4	94.4	
135	m	50	240	240	165	99	0	0	95.8	95.8	95.6	
135	d	50	240	240	290	99	0	0	91.9	91.9	90.2	
136	s	100	565	485	105	206	1	0	77.2	77.2	79.2	
136	d	100	565	485	255	100	1	0	84.1	84.1	84.2	
137	s	100	675	485	55	97	0	1	84.5	84.5	86.6	
137	d	100	675	485	205	197	0	1	84.2	84.2	84.4	
138	s	100	530	375	55	98	0	1	91.5	91.5	92.9	
138	d	100	530	375	205	199	0	1	71.1	71.1		outlier
139	s	100	710	375	100	200	0	0	92.8	92.8	92.7	
139	d	100	710	375	255	97	0	0	103.1	103.1	101.1	
140	s	100	565	265	105	202	0	0	84.8	84.8	83.9	
140	d	100	565	265	255	96	0	0	101.9	101.9	99.2	
141	s	100	675	265	55	98	1	0	86.0	86.0	85.2	
141	d	100	675	265	205	196	1	0	82.4	82.4	79.8	
142	s	100	530	155	55	102	1	0	84.5	84.5	83.1	
142	d	100	530	155	205	200	1	0	81.5	81.5	78.1	
143	s	100	710	155	100	197	0	1	86.6	86.6	83.6	
143	d	100	710	155	255	99	0	1	92.5	92.5	87.6	
144	s	50	620	410	50	96	1	0	74.6	74.5	75.9	
144	m	50	620	410	145	93	1	0	77.6	77.3	77.5	
144	d	50	620	410	245	91	1	0	79.1	78.6	77.6	
145	s	50	620	350	80	99	0	0	96.4	96.4	96.7	
145	m	50	620	350								no core
145	d	50	620	350	270	98	0	0	88.6	88.6	86.5	
146	s	50	620	155	50	100	1	0	85.6	85.6	83.8	
146	m	50	620	155	145	96	1	0	81.8	81.7	78.6	
146	d	50	620	155	245	98	1	0	85.0	85.0	80.7	
Avg.			428	320	153							

Note: Cores were drilled when the concrete was 42 days old, and stored in sealed plastic bags. Some cores, indicated in the table by indicator variables $Z_{mc} = Z_{seal} = 0$, were exposed to laboratory air for 7 days before testing. Some, indicated in the table by indicator variable $Z_{mc} = 1$, were soaked for 48 hours before testing. The remainder, indicated in the table by indicator variable $Z_{seal} = 1$, were left in the plastic bags until testing. The concrete was 59 days old on the day of the test.

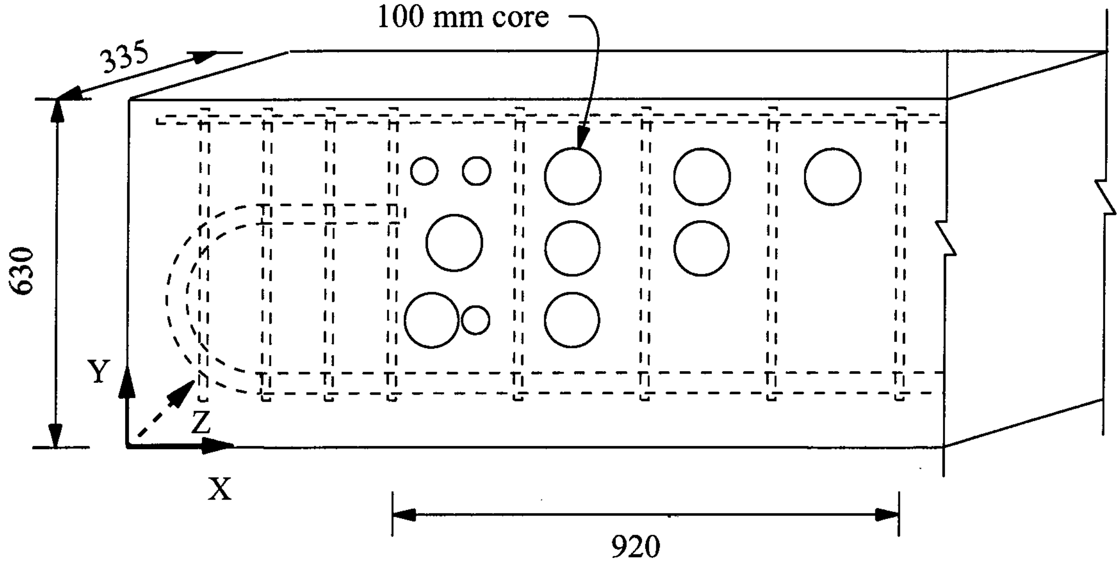

Figure 2B.1 (a): Coring locations, Beams LL1 and LL2

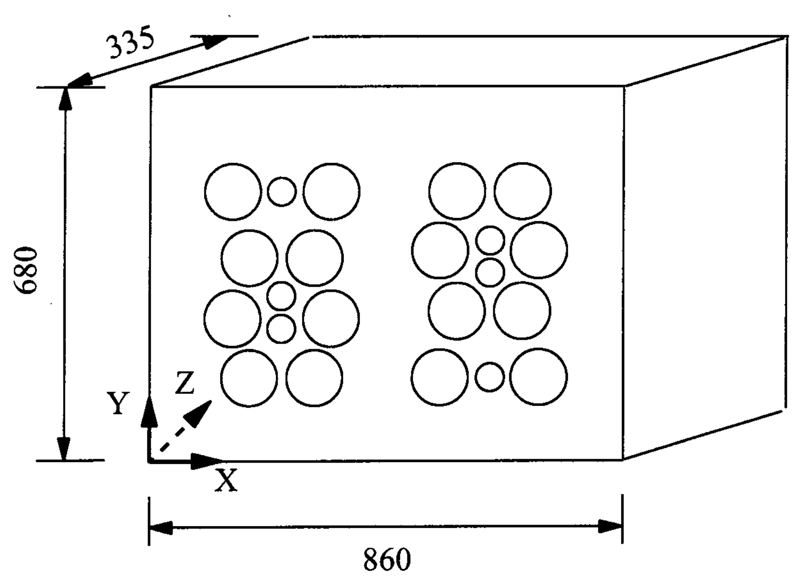

Figure 2B.1 (b): Coring locations, Blocks LH1 and LH2

Appendix 3A: Sample Outlier Identification Calculation

Given: Strength data for Element 1 soaked cores with l/d = 1.25.

Top cores: 3947, 4011, 4090, 4146, 4424, and 4615 psi.
Bottom cores: 3501, 4273, 4377, 4472, 4496, and 4600 psi.

Required: is the low bottom core strength of 3501 psi an outlier per ASTM E178-80 (1980) and Rousseeuw and Leroy (1987) criteria?

Solution:

a) Bottom cores have $\bar{f}_c = 4287$ psi, s = 400.5 psi. Hence the calculated $T_1 = (\bar{f}_c - 3501)/s = 1.961$. From Table 1 of ASTM E178-80, the T value (two sided test) for n = 6 observations is about 1.96 at the 1.5% significance level. An observation this far from the mean occurs by chance once per 66.6 times; since this is unlikely, the observation is probably an outlier.

b) Combining the top and bottom core strength values gives $\bar{f}_c = 4246$ psi, s = 324.5 psi (Note that analysis of variance indicates that the difference between top and bottom average values is not significant at the 5 % level, suggesting that it is not logically inconsistent to consider the combined sample). For the low bottom core strength $T_1 = 2.295$, which for n = 12 observations is significant at about the 10% level according to Table 1 of ASTM E178. This significance level would suggest to many analysts that the observation is not an outlier, since an observation lying this far from the mean occurs by chance 10 % of the time.

c) Outlier identification according to Rousseeuw and Leroy follows three steps:

1. Estimate the strength as the middle of the smallest range of strengths which include half the observations plus one. This can be easily calculated by ranking the data and calculating range widths which bound $\left(\frac{n}{2}+1\right)$ data values.

That is, the range bounding	3501	and 4377	is 876	psi, similarly
that bounding	3947	" 4424	" 477	"
" "	4011	" 4472	" 461	"
" "	4090	" 4496	" 406	"
" "	4146	" 4600	" 454	", and
" "	4273	" 4615	" 342	"

The least range width is 342 psi, so the strength estimate, θ, is (4273+4615)/2 = 4444 psi. This is an estimate of the median strength, and is not sensitive to the presence of outliers.

2. Estimate the scale as

$$s_0 = 1.4826\left(1+\frac{5}{n-1}\right)\left(\frac{r_{min}}{2}\right) \quad (3A.1)$$

where r_{min} is the least range width as obtained from step 1. So, with n = 12:

$$s_0 = 1.4826\left(1+\frac{5}{12-1}\right)\left(\frac{342}{2}\right) = 369\text{psi} \quad (3A.1a).$$

3. Calculate $(\theta\text{-}3501)/s_o = 2.56$. Since this value is greater than 2.5, the low observation is considered to be an outlier.

Thus, the low observation is found to be an outlier by procedures (a) and (c) but not by (b). On this basis, it was taken to be an outlier for this study, and has not been included in the data summarised in Table 3.3.

Appendix 3B: Weights for Regression Analysis

Consider the case of a regression analysis model where both the dependent and the independent variables are subject to error. The model has the form

$$Y + \varepsilon_Y = \beta_0 + \beta_1(X + \varepsilon_X) \quad (3B.1).$$

where ε_Y represents the error in dependent variable Y, ε_X represents the error in independent variable X, and parameters β_0 and β_1 are to be estimated by regression. Assume ε_Y is normally distributed with zero mean and variance σ_Y^2 and ε_X is normally distributed with zero mean and variance σ_X^2. The model may be rewritten as

$$Y = \beta_0 + \beta_1 X + (\beta_1\varepsilon_X + \varepsilon_Y) \quad (3B.1a)$$

This is a more conventional form for regression analysis, since the substitution $\varepsilon = \beta_1\varepsilon_X + \varepsilon_Y$ yields

$$Y = \beta_0 + \beta_1 X + \varepsilon \quad (3B.1b).$$

The error term, ε, has expected value

$$E[\varepsilon] = E[\beta_1 \varepsilon_X + \varepsilon_Y] = \beta_1 0 + 0 = 0 \qquad (3B.2)$$

and variance

$$Var[\varepsilon] = Var[\beta_1 \varepsilon_X + \varepsilon_Y] = \beta_1^2 \sigma_X^2 + \sigma_Y^2 \qquad (3B.3).$$

Hence, to achieve a best linear unbiased estimate, each observation should be weighted by a factor proportional to the inverse of the variance of the error term

$$W = \frac{1}{\beta_1^2 \sigma_X^2 + \sigma_Y^2} \qquad (3B.4).$$

If the variances of the error terms ε_X and ε_Y are not constant, but vary from observation to observation, then the weight given to the i^{th} observation is

$$W_i = \frac{1}{\beta_1^2 \sigma_{Xi}^2 + \sigma_{Yi}^2} \qquad (3B.5)$$

where σ_{Xi}^2 is the variance of the i^{th} observation of X and σ_{Yi}^2 is the variance of the i^{th} observation of Y.

The regression analysis for the unknown parameters in Equation (3.11) is simply a special case of Equation (3B.5), where the variances of ε_X and ε_Y are the sample variances of the observed mean values $\overline{X}$ and $\overline{Y}$ respectively. The desired weights are found using Equation (3B.5) with

$$\sigma_{Xi}^2 = \frac{s_{Xi}^2}{n_{Xi}} \qquad (3B.6)$$

and

$$\sigma_{Yi}^2 = \frac{s_{Yi}^2}{n_{Yi}} \qquad (3B.7).$$

In Equations (3B.6) and (3B.7), s_{Xi} and s_{Yi} are the sample standard deviations and n_{Xi} and n_{Yi} are the number of replicate observations used to determine the average values $\overline{X}_i$ and $\overline{Y}_i$. Since the estimate of parameter β_1 is not known *a priori* the analysis is repeated using the parameter estimated from the previous iteration until convergence is reached.

Appendix 4A: Core Data from Bloem (1965)

The table in this Appendix presents data for each of the cores drilled from two well cured slabs investigated by Bloem (1965). The data are reproduced from mimeographed tabulations prepared by Bloem, which were made available for the present study by Meininger of the National Aggregate Association / National Ready Mixed Concrete Association in Silver Spring, Maryland.

All cores had 4 inch diameters and were drilled through the slab thickness. Cores from the 8 inch thick slab were either tested as 8 inch long specimens or were sawn in half to give one 4 inch long specimen from the top of the slab and one 4 inch long specimen from the bottom of the slab. Four moisture conditioning treatments were investigated: "as-drilled" cores were tested after 2 to 3 hours exposure to laboratory air; "48 hour soak" and "28 day soak" cores were tested after being immersed in lime water for 2 days or 28 days respectively; and "7 day air dry" cores were tested after 7 days exposure to laboratory air. Further details are reported by Bloem (1965).

In the table, the core strength used for analysis, f^*_c, is the product of the measured core strength f_c and the age factor. The age factor equals 1.0 for 91 day old specimens and is less than 1.0 for older specimens. Other columns give the core length to diameter ratio, l/d, the average core moisture content, M_t, and the difference between the moisture content at the time of testing and the time of drilling, M_t-M_{ad}.

Table 4A.1: Strength and Moisture Content Data for Cores Tested by Bloem (1965)

	Treatment	f_c (psi)	l/d	Age (days)	Age Factor	f^*_c (psi)	M_t (pct)	$M_t - M_{ad}$ (pct)
8" long	As-	4720	2.19	91	1.00	4720	4.46	-0.30
cores	drilled	4800	2.19	91	1.00	4800	4.41	-0.19
from		4890	2.13	91	1.00	4890	4.37	-0.57
8" slab	48	4580	2.13	93	0.997	4567	5.24	0.69
	hour	4180	2.14	93	0.997	4168	5.16	0.68
	soak	4580	2.12	93	0.997	4567	5.32	0.57
	28	4410	2.16	119	0.959	4229	6.03	0.82
	day	4370	2.14	119	0.959	4191	5.87	0.90
	soak	4720	2.10	119	0.959	4526	5.91	0.95
	7	5430	2.12	98	0.990	5374	4.03	-0.74
	day	5420	2.10	98	0.990	5364	4.13	-0.81
	air dry	5460	2.11	98	0.990	5404	4.34	-0.79

Table 4A.1: Strength and Moisture Content Data for Cores Tested by Bloem
(concluded)

	Treatment	f_c (psi)	l/d	Age (days)	Age Factor	f^*_c (psi)	M_t (pct)	$M_t - M_{ad}$ (pct)
4" long	As-	5190	1.07	91	1.00	5190	4.64	-0.18
cores	drilled	5550	1.10	91	1.00	5550	4.72	-0.39
from		5100	1.08	91	1.00	5100	4.69	-0.43
top of	48	4760	1.06	93	0.997	4746	5.41	0.91
8" slab	hour	4990	1.06	93	0.997	4975	5.29	0.71
	soak	4500	1.07	93	0.997	4487	5.44	0.77
	28	4960	1.06	119	0.959	4757	5.91	0.96
	day	5140	1.06	119	0.959	4929	6.26	1.00
	soak	5140	1.06	119	0.959	4929	6.17	1.00
	7	6110	1.07	98	0.990	6047	3.82	-1.02
	day	6080	1.07	98	0.990	6018	3.97	-1.06
	air dry	5840	1.07	98	0.990	5780	4.07	-0.99
4" long	As-	5010	1.16	91	1.00	5010	4.65	-0.11
cores	drilled	5750	1.08	91	1.00	5750	4.84	-0.17
from		5300	1.08	91	1.00	5300	4.40	-0.25
bottom of	48	5480	1.09	93	0.997	5464	5.11	0.36
8" slab	hour	5000	1.11	93	0.997	4985	5.18	0.55
	soak	4950	1.09	93	0.997	4935	5.08	0.39
	28	5140	1.10	119	0.959	4929	5.87	0.79
	day	5270	1.07	119	0.959	5054	5.87	0.75
	soak	5140	1.06	119	0.959	4929	5.98	0.85
	7	6310	1.06	98	0.990	6245	4.05	-0.83
	day	6400	1.06	98	0.990	6334	4.27	-0.95
	air dry	6620	1.06	98	0.990	6552	4.21	-0.92
4" long	As-	6040	1.12	91	1.00	6040	4.09	-0.28
cores	drilled	6150	1.09	91	1.00	6150	3.91	-0.30
from		6040	1.12	91	1.00	6040	4.07	-0.28
4" slab	48	5280	1.08	93	0.997	5265	5.26	1.10
	hour	5000	1.11	93	0.997	4985	5.32	1.06
	soak	5240	1.11	93	0.997	5225	5.33	1.09
	28	5350	1.07	119	0.959	5131	6.19	1.37
	day	5550	1.07	119	0.959	5322	6.18	1.42
	soak	5700	1.08	119	0.959	5466	6.00	1.35
	7	6280	1.09	98	0.990	6216	4.01	-0.72
	day	6460	1.08	98	0.990	6394	3.89	-0.64
	air dry	6550	1.09	98	0.990	6483	3.91	-0.70

Appendix 5A: Fitting Bolotin's Size Effect Equations to Observed Data

A method is presented for determining positive constants C_1, C_2 and α for use in Equations (5.1), (5.2), and (5.3) given statistics for the observed strengths of two groups of specimens with differing sizes. Assume that the standard specimens, each with volume V_r, have sample mean $\bar{f}_r$ and sample variance s_r^2. Similarly the other specimens, each with volume V_1, have $\bar{f}_1$ and s_1^2.

A closed form solution is obtained by exactly satisfying the conditions that $E[f_c] = \bar{f}_r$ and $Var[f_c] = s_r^2$ at $V = V_r$, and $Var[f_c] = s_1^2$ at $V = V_1$. From Equation (5.1), the first condition yields:

$$C_1 + C_2 = 1 \qquad (5A.1).$$

From Equation (5.2), the second condition yields:

$$s_r = C_2 \; \phi(\alpha) \qquad (5A.2)$$

and the third condition yields:

$$s_1 = C_2 \left(\frac{V_r}{V_1}\right)^{\frac{1}{\alpha}} \phi(\alpha) \qquad (5A.3).$$

Dividing Equation (5A.3) by Equation (5A.2) and isolating α:

$$\alpha = ln\left(\frac{V_r}{V_1}\right) - ln\left(\frac{s_r}{s_1}\right) \qquad (5A.4).$$

With α known, $\phi(\alpha)$ may be obtained from Equation (5.3), C_2 from Equation (5A.2) and C_1 from Equation (5A.1). A check on the accuracy of the fit may be obtained by comparing the observed $\bar{f}_1$ with the value predicted using Equation (5.1).

Appendix 5B: Regression Analysis Model for Mean Core Strength

The normalized average core strength is the ratio of the average strength of cores with diameter d to that of cores with 4 inch diameter. From Equation (5.9), this is:

$$R_d = E[f_I]\left\{1-\frac{\beta}{d}\right\} / \left\{E[f_I]\left(1-\frac{\beta}{4}\right)\right\} \qquad \text{(5B.1).}$$

The term $E[f_I]$ appears in both the numerator and the denominator of this equation. If it is assumed, in accordance with the summation strength theory, that the mean strength of the undamaged inner region of the core is independent of the core diameter, then this equation may be simplified to:

$$R_d = \left\{1-\frac{\beta}{d}\right\} / \left\{1-\frac{\beta}{4}\right\} + \varepsilon \qquad \text{(5B.2)}$$

where the error term, ε, accounts for the uncertainty of the mean strengths of the interior regions of cores with diameter d and those with 4 inch diameters.

The unknown parameter β may be estimated using a restricted linear regression analysis (SAS Institute, 1986). The form is:

$$R_d = \beta_8 + \beta_9\left(\frac{1}{d}\right) + \varepsilon \qquad \text{(5B.3)}$$

with the restriction:

$$\beta_8 + 0.25\,\beta_9 - 1 = 0 \qquad \text{(5B.4)}$$

to force the line of best fit through $R_d = 1$ at $d = 4$ inches. Alternatively, the unknown parameter β may be estimated directly by nonlinear regression analysis of Equation (5B.2).

In the present investigation, a weighted nonlinear regression analysis was performed and the parameter estimates were checked using a weighted restricted linear regression analysis. In both analyses, the weight W_i applied to the i^{th} observation was:

$$W_i = \frac{n_{di}\, n_{4i}}{n_{4i} + n_{di}} \qquad (5B.5)$$

where n_{di} and n_{4i} are the numbers of cores with diameters of d and 4 inches respectively used to calculate the ratio of average strengths. As described in Appendix 5C, these weights crudely approximate the inverse of the variance of each ratio of average strengths. While they may be too crude to give the best linear unbiased estimate, they are likely to give estimates which are better than would be obtained using unweighted analysis.

Appendix 5C: Weighted Least Squares Analysis when the Response Variable is the Ratio of the Means of Two Random Variables

Consider a problem to be solved by least squares analysis where the response variable Z is the ratio of the means of random variables X and Y:

$$z_i = \frac{\bar{x}_i}{\bar{y}_i} \qquad (5C.1).$$

According to the central limit theorem, the distribution of $\bar{X}$ is normal with mean μ_X and variance σ_X^2/n_X, where μ_X and σ_X^2 are the mean and variance of X and n_X is the number of observations used to obtain $\bar{X}$. Similarly $\bar{Y}$ is normally distributed with mean μ_Y and variance σ_Y^2/n_Y. The distribution of Z is not normal, which violates a basic assumption of regression analysis. This can be mitigated by working with the natural logarithm of Z:

$$ln(z_i) = ln(\bar{x}_i) - ln(\bar{y}_i) \qquad (5C.2).$$

The natural logarithm of Z is normally distributed only if the natural logarithms of $\bar{X}$ and $\bar{Y}$ are also normally distributed, which violates the central limit theorem. However the difference between normal and lognormal distributions with the same expected value and variance is small in some cases. Stone and Reeve (1986) indicate that, for practical purposes, the difference may be neglected when the expected value is considerably greater than zero and the coefficient of variation is small.

If the distributions of $\overline{X}$ and $\overline{Y}$ are approximately lognormal, then *ln* Z has expected value

$$\mu_{\ln Z} = \mu_{\ln \overline{X}} - \mu_{\ln \overline{Y}} \tag{5C.3}$$

and variance

$$\sigma^2_{\ln Z} = \sigma^2_{\ln \overline{X}} + \sigma^2_{\ln \overline{Y}} \tag{5C.4}$$.

The expected value and variance of the distribution of *ln* $\overline{X}$ are obtained from the corresponding parameters of the distribution of X using the transformations (Ang and Tang, 1975, p. 104):

$$\mu_{\ln \overline{X}} = ln\left(\frac{\mu_X}{\sqrt{V^2_{\overline{X}} + 1}}\right) \tag{5C.5}$$

and

$$\sigma^2_{\ln \overline{X}} = ln\left(V^2_{\overline{X}} + 1\right) \tag{5C.6}$$

where

$$V_{\overline{X}} = \frac{\sigma_{\overline{X}}}{\mu_X \sqrt{n_X}} \tag{5C.7}$$.

The expected value and variance of *ln* $\overline{Y}$ may be obtained from identical transformations.

Conventional regression analysis also requires homoscedasticity; that is, the variance of the response variable must be constant throughout its range. This condition is satisfied if σ^2_X, σ^2_Y, n_X and n_Y are constant for all observations. In this case the best linear unbiased estimate of the parameters is obtained by assigning equal weight to each observation (Montgomery and Peck, 1982, p. 16). However in practice these values, particularly n_X and n_Y, may not be constant for all observations. In these circumstances, the best linear unbiased estimate of the parameters is obtained by weighting each observation by a factor which is proportional to the inverse of the variance of the response variable (Montgomery and Peck, 1982, p. 362).

To obtain the desired weights, Equation (5C.4) is revised using the transformations given in Equations (5C.6):

$$\sigma^2_{\ln Z} = ln\left(V^2_{\bar{X}} + 1\right) + ln\left(V^2_{\bar{Y}} + 1\right) \qquad (5C.8).$$

This can be written as

$$\sigma^2_{\ln Z} = V^2_{\bar{X}} + V^2_{\bar{Y}} + \text{error} \qquad (5C.9)$$

where the error is in the order of 2% for $V_{\bar{X}}$ and $V_{\bar{Y}}$ less than 0.2 and can be neglected.

Substitution of Equation (5C.7) into (5C.9) yields:

$$\begin{aligned} \sigma^2_{\ln Z} &= \frac{\sigma^2_X}{\mu^2_X n_X} + \frac{\sigma^2_Y}{\mu^2_Y n_Y} \\ &= \left(\frac{\sigma_X}{\mu_X}\right)^2 \left\{ n_Y + n_X \left(\frac{\mu_X \sigma_Y}{\mu_Y \sigma_X}\right)^2 \right\} \frac{1}{n_X n_Y} \qquad (5C.10). \end{aligned}$$

The desired weights are proportional to the inverse of $\sigma^2_{\ln Z}$. For the case where σ^2_Y and σ^2_X are constant for all observations, the weight for the i^{th} observation is:

$$W_i = \frac{n_{Xi}\, n_{Yi}}{n_{Yi} + n_{Xi}\, k^2\, (\mu_{Xi}/\mu_{Yi})^2} \qquad (5C.11)$$

where $k = \sigma_Y / \sigma_X$.

It can be seen from Equation (5C.11) that the weights depend on the ratio of the mean value of X to the mean value of Y for each observation. This ratio can be initially estimated as the observed z_i value, although this procedure makes the weights sensitive to the scatter of Z. It is therefore preferable to base preliminary values of the ratio (μ_{Xi}/μ_{Yi}) on the predicted values of Z obtained from an ordinary, unweighted, least squares analysis. If necessary, the results of the first weighted analysis can be used to improve the weights for a subsequent weighted analysis.

For the case where the sample variances are unknown (or approximately equal) and $(\mu_{Xi}/\mu_{Yi}) \cong 1$, the following simple expression for the weights is obtained from Equation (5C.11):

$$W_i = \frac{n_{Xi}\, n_{Yi}}{n_{Yi} + n_{Xi}} \qquad (5C.12).$$

Appendix 5D: Regression Analysis Model for Coefficient of Variation of Core Strength

The model for regression analysis was developed from the equations for expected value and variance considering core damage, Equations (5.9) and (5.10). Equation (5.10) was simplified by neglecting the terms $E[k]^2$ and $E[t]^2\, Var[k]$, to give:

$$Var[f_c] = Var[f_I]\left\{1 + \left(\frac{3.75}{d}\right)^2 E[t]^2\right\} + \left(\frac{3.75}{d}\right)^2 E[f_I]^2 Var[t] \qquad (5D.1).$$

To obtain a form containing coefficients of variation, both sides are divided by the square of $E[f_c]$ as defined in Equation (5.9) assuming E [k] is negligible. This gives:

$$V_{fcS}^2 = V_{fI}^2 K + \frac{3.75^2\, Var[t]}{(d - 3.75\, E[t])^2} \qquad (5D.2)$$

where V_{fcS} is the coefficient of variation of the overall core strength, V_{fI} is the coefficient of variation of the strength of the undamaged interior region, and

$$K = \left\{1 + \left(\frac{3.75\, E[t]}{d}\right)^2\right\} / \left\{1 - \frac{3.75\, E[t]}{d}\right\}^2 \qquad (5D.3).$$

These equations were further simplified using the analysis results for mean core strengths by Equation (5.9b), yielding:

$$V_{fcS}^2 \;=\; V_{fl}^2 K + \frac{3.75^2 \, Var[t]}{(d-0.2265)^2} \tag{5D.2a}$$

and

$$K \;=\; \left\{1+\left(\frac{0.2265}{d}\right)^2\right\} / \left\{1-\frac{0.2265}{d}\right\}^2 \tag{5D.3a}$$

The coefficient of variation of the strength of the undamaged interior of the core, V_{fl}, may itself depend on the regressor variables. Preliminary analyses indicated that the concrete cylinder strength, the spatial variation of the *in situ* strength, and the age of the concrete at the time of testing significantly influence V_{fl}. The maximum aggregate size and the core treatment are not significant factors. Thus the form of the model evolved to:

$$V_{fcS}^2 \;=\; \{V_{cyl}^2 + V_s^2 + V_{Za}^2\} \, K + \frac{3.75^2 \, Var[t]}{(d-0.2265)^2} + \varepsilon \tag{5D.4}$$

where V_{cyl}, V_s, and V_{Za} represent the components of the coefficient of variation associated with the standard cylinder strength, the *in situ* spatial strength variation, and the age of the concrete at the time of testing.

The component of the coefficient of variation affected by the concrete cylinder strength was initially assumed to be constant for $\bar{f}_{cyl}$ less than 28 MPa and consistent with constant standard deviation otherwise, in accordance with the findings of Mirza et al. (1979). However the residuals of preliminary analyses using this model indicated that a better fit would be obtained if the standard deviation were assumed constant for the entire range of $\bar{f}_{cyl}$. Thus the expression adopted for V_{cyl} was:

$$V_{cyl} \;=\; \beta_{10}\left(\frac{1}{\bar{f}_{cyl}}\right) \tag{5D.5}$$

where β_{10}, an unknown constant determined by regression analysis, represents the standard deviation which is constant for all concrete strengths.

The component of the coefficient of variation affected by the *in situ* spatial strength variation was assumed to have the form:

$$V_s = \beta_{11} Z_s \tag{5D.6}$$

where Z_s is an indicator variable equal to 1 if the observed V_{fcS} is likely to be inflated by spatial strength variation or 0 otherwise. Similarly, the component of the coefficient of variation affected by the age of the concrete was assumed to have the form:

$$V_{Za} = \beta_{12} Z_a \tag{5D.7}$$

where Z_a equals zero for cores tested when the concrete was 4 to 5 weeks old or 1 for cores tested when the concrete was 11 to 15 weeks old.

The response variable in Equation (5D.4), $V_{fcS}{}^2$, is small but always greater than zero. Hence the conventional regression analysis assumption of normally distributed random error ε is somewhat dubious. The model was revised to:

$$\mathit{ln}\,[V_{fcS}^2] = \mathit{ln}\left[\{V_{cyl}^2 + V_s^2 + V_{Za}^2\}\,K + \frac{3.75^2\,\mathrm{Var}[t]}{(d-0.2624)^2}\right] + \varepsilon \tag{5D.8}$$

to resolve this problem.

Appendix 5E: Ratios of Average Strengths of 2, 4 and 6 Inch Diameter Cores

Data reported by others concerning the average strength of 2 and 6 inch diameter cores, expressed as a fraction of the average strength of 4 inch diameter cores from the same batch of concrete, are shown in Table 5E.1. In the column headings, d is the core diameter in inches and d_r is the diameter of the reference cores, typically 4 inches. Indicator variable Z_r is equal to 1 for results from two specific studies which appeared to differ significantly from the rest, or 0 otherwise. Indicator variable Z_a is equal to 1 for cores tested when the concrete was 91 days old, or 0 for cores tested when the concrete was 28 days old. Variables n_d and n_4 represent the numbers of specimens used to determine the average strengths of the cores with diameter d and the cores with 4 inch nominal diameter respectively; these values are used to determine the weights for regression analysis, W_i. The 28 day standard cylinder strength, $\bar{f}_{cyl}$, for the data reported by Yip and Tam (1988) has been estimated as 80 % of the published cube strength. Indicator variable Z_{mc} is equal to 0 for cores left in laboratory air before testing, or 1 for cores soaked in water for at least 40 hours before testing. Indicator variable Z_{HV} is equal to 0 if the axis of the core is perpendicular to the direction of casting, or 1 if the axis is parallel to the direction of casting. The maximum aggregate size for each mix is a_m, in inches. R_d is the ratio of the average strength of cores with diameter d to the average strength of cores from the same mix with nominal 4 inch diameter.

Table 5E.1: Ratios of Strengths of 2, 4 and 6 Inch Diameter Cores

Reference	d (")	d_r (")	Z_r	Z_a	n_d	n_4	$\bar{f}_{cyl}$ (psi)	Z_{mc}	Z_{HV}	a_m	R_d	W_i
Bartlett & MacGregor (1994a)	2	4	0	.7	7	7	8120	.5	0	0.55	1.04	3.5
	2	4	0	.4	7	8	6380	.5	0	0.55	0.90	3.7
	2	4	0	.4	6	5	13380	1	0	0.55	0.95	2.7
	2	4	0	.4	7	5	13380	0	0	0.55	0.99	2.9
	2	4	0	.3	8	5	13080	1	0	0.55	0.99	3.1
	2	4	0	.3	8	5	13080	0	0	0.55	1.03	3.1
Bhargava (1969)	6.0	4.0	0	0	1	1	6530	0	1	0.75	0.98	0.5
	6.0	4.0	0	0	1	1	3443	0	1	0.75	0.97	0.5
	6.0	4.0	0	0	1	1	3642	0	1	0.75	0.92	0.5
	6.0	4.0	0	0	1	1	6530	0	1	0.75	1.06	0.5
	6.0	4.0	0	0	1	1	6587	0	1	0.75	1.15	0.5
	6.0	4.0	0	0	1	1	3443	0	1	0.75	1.09	0.5
	6.0	4.0	0	0	1	1	6587	0	1	0.75	0.98	0.5
	6.0	4.0	0	0	1	1	3642	0	1	0.75	0.97	0.5
Campbell & Tobin (1967)	6.0	4.0	1	0	4	4	5868	1	1	0.75	1.28	2.0
	6.0	4.0	1	1	4	4	6227	1	1	0.75	1.24	2.0
	6.0	4.0	1	1	4	4	4828	1	1	0.75	1.25	2.0
	6.0	4.0	1	0	4	4	4514	1	1	0.75	1.22	2.0
Confidential Communication (1990)	2.25	3.75	1	0	8	2	4230	0	1	0.5	0.89	1.6
	5.75	3.75	1	0	2	2	4230	0	1	0.5	0.93	1.0
	2.25	3.75	1	0	15	2	4190	0	1	0.5	1.15	1.8
	5.75	3.75	1	0	2	2	4190	0	1	0.5	1.26	1.0
	2.25	3.75	1	0	6	2	3693	0	1	0.5	0.93	1.5
	5.75	3.75	1	0	2	2	3693	0	1	0.5	1.27	1.0
	2.25	3.75	1	0	4	2	3360	0	1	0.5	1.15	1.3
	5.75	3.75	1	0	2	2	3360	0	1	0.5	1.18	1.0
	2.25	3.75	1	0	4	2	3020	0	1	0.5	1.01	1.3
	5.75	3.75	1	0	2	2	3020	0	1	0.5	1.30	1.0
Lewis (1976)	6.0	4.0	0	0	12	6	6500	1	1	0.75	1.02	4.0
	6.0	4.0	0	0	12	6	6220	1	1	0.75	1.04	4.0
	6.0	4.0	0	0	12	6	3830	1	1	0.75	1.03	4.0
	6.0	4.0	0	0	12	6	3260	1	1	0.75	1.00	4.0
Meininger (1968)	1.9	4.0	0	1	28	12	3110	1	1	0.75	0.99	8.4
	1.9	4.0	0	1	33	18	3110	1	0	0.75	1.05	11.6
	5.9	4.0	0	1	18	12	3110	1	0	0.75	0.99	7.2
	5.9	4.0	0	1	12	12	3110	1	1	0.75	1.01	6.0

Table 5E.1: Ratios of Strengths of 2, 4 and 6 Inch Diameter Cores (concluded)

Reference	d (")	d_r (")	Z_r	Z_a	n_d	n_4	$\bar{f}_{cyl}$ (psi)	Z_{mc}	Z_{HV}	a_m	R_d	W_i
Munday & Dhir (1984)	2.0	4.0	0	0	6	6	5000	1	0	0.75	0.99	3.0
Ramirez & Barcena (1979)	2.0	4.0	0	0	16	16	1445	0	1	1	0.94	8.0
	2.0	4.0	0	0	16	16	1445	0	0	1	0.91	8.0
	6.0	4.0	0	0	16	16	1445	0	1	1	0.94	8.0
	6.0	4.0	0	0	16	16	1445	0	0	1	1.01	8.0
	2.0	4.0	0	0	16	16	1791	0	1	1	0.97	8.0
	2.0	4.0	0	0	16	16	1791	0	0	1	0.89	8.0
	6.0	4.0	0	0	16	16	1791	0	1	1	0.99	8.0
	6.0	4.0	0	0	16	16	1791	0	0	1	1.00	8.0
	2.0	4.0	0	0	16	16	2760	0	1	1	0.97	8.0
	2.0	4.0	0	0	16	16	2760	0	0	1	0.86	8.0
	6.0	4.0	0	0	16	16	2760	0	1	1	1.06	8.0
	6.0	4.0	0	0	16	16	2760	0	0	1	0.96	8.0
Yip & Tam (1988)	2.0	4.0	0	0	5	3	5917	0	1	0.75	0.82	1.87
	2.0	4.0	0	1	5	3	5917	0	1	0.75	1.09	1.87
	2.0	4.0	0	0	5	3	5917	0	0	0.75	0.63	1.87
	2.0	4.0	0	1	5	3	5917	0	0	0.75	0.94	1.87
	2.0	4.0	0	0	5	3	2982	0	0	0.75	0.85	1.87
	2.0	4.0	0	1	5	3	2982	0	0	0.75	0.98	1.87
	2.0	4.0	0	0	5	3	3458	0	1	0.75	0.83	1.87
	2.0	4.0	0	1	5	3	3458	0	1	0.75	0.83	1.87
	2.0	4.0	0	0	5	3	5650	0	1	0.75	0.75	1.87
	2.0	4.0	0	1	5	3	5650	0	1	0.75	0.85	1.87
	2.0	4.0	0	0	5	3	4270	0	0	0.75	0.74	1.87
	2.0	4.0	0	1	5	3	4270	0	0	0.75	0.87	1.87
	2.0	4.0	0	0	5	3	5186	0	0	0.75	1.17	1.87
	2.0	4.0	0	1	5	3	5186	0	0	0.75	0.79	1.87
	2.0	4.0	0	0	5	3	5732	0	0	0.75	0.86	1.87
	2.0	4.0	0	1	5	3	5732	0	0	0.75	0.77	1.87
	2.0	4.0	0	0	5	3	6045	0	1	0.75	0.80	1.87
	2.0	4.0	0	1	5	3	6045	0	1	0.75	1.08	1.87
	2.0	4.0	0	0	5	3	7275	0	1	0.75	1.20	1.87
	2.0	4.0	0	1	5	3	7275	0	1	0.75	0.81	1.87
	2.0	4.0	0	0	5	3	2332	0	1	0.75	1.31	1.87
	2.0	4.0	0	1	5	3	2332	0	1	0.75	1.53	1.87
	2.0	4.0	0	0	5	3	5929	0	0	0.75	0.93	1.87
	2.0	4.0	0	1	5	3	5929	0	0	0.75	0.86	1.87

Appendix 5F: Data used for Study of Coefficient of Variation for Cores with Different Diameters

Data obtained by others concerning the coefficient of variation for cores of different diameters are shown in Table 5F.1. In the column headings, d is the core diameter in inches. "Age" is the age of the concrete in days when the core was tested, and $\bar{f}_{cyl}$ is the standard 28 day cylinder strength. The number of cores of each diameter from each mix is n. The average value, standard deviation and coefficient of variation of the strength of replicate core specimens are shown in columns headed $\bar{f}_c$, s, and V. The indicator variable Z_{mc} is equal to 1 for cores soaked in water before testing, or 0 for cores left in laboratory air. Indicator variable Z_{HV} is equal to 0 if the axis of the core is perpendicular to the direction of casting, or 1 if the axis is parallel to the casting direction. The maximum aggregate size, in inches, is shown as a_m. Indicator variable Z_s is equal to 1 if the reported coefficient of variation is likely to be inflated by the *in situ* variability of the strength, or 0 if the reported coefficient of variation is based on data which have been adjusted to account for *in situ* strength variability.

Table 5F.1: Strength Variability Data for Cores of Different Diameters

Source	d (in)	age (d)	n	$\bar{f}_{cyl}$ (psi)	$\bar{f}_c$ (psi)	s (psi)	V (%)	Z_{mc}	Z_{HV}	a_m (in)	Z_s
Bartlett	2	71	7	8120	6671	377	5.70	0.5	0	0.55	0
& MacGregor	4	71	7	8120	6381	421	6.60	0.5	0	0.55	0
(1994a)	2	55	7	6380	5975	392	6.60	0.5	0	0.55	0
	4	55	8	6380	6628	261	3.90	0.5	0	0.55	0
	2	55	6	13380	13604	957	7.00	0	0	0.55	0
	4	55	5	13380	13656	464	3.40	0	0	0.55	0
	2	55	7	13380	11312	464	4.10	1	0	0.55	0
	4	55	5	13380	11907	478	4.00	1	0	0.55	0
	2	49	8	13080	13387	725	5.40	0	0	0.55	0
	4	49	5	13080	13040	508	3.90	0	0	0.55	0
	2	49	8	13080	11310	551	4.90	1	0	0.55	0
	4	49	5	13080	11458	189	1.60	1	0	0.55	0
Lewis	4.0	30	12	6500	5214	220	4.22	1	1	1.0	1
(1976)	6.0	30	6	6500	5325	165	3.10	1	1	1.0	1
	4.0	30	12	3830	3111	268	8.61	1	1	1.0	1
	6.0	30	6	3830	3200	320	10.00	1	1	1.0	1
	4.0	30	12	3260	2683	298	11.11	1	1	1.0	1
	6.0	30	6	3260	2683	119	4.44	1	1	1.0	1
	4.0	30	12	6220	4590	440	9.59	1	1	1.0	1
	6.0	30	6	6220	4769	251	5.26	1	1	1.0	1

Table 5F.1: Strength Variability Data for Cores of Different Diameter (continued)

Source	d (in)	age (d)	n	$\bar{f}_{cyl}$ (psi)	$\bar{f}_c$ (psi)	s (psi)	V (%)	Z_{mc}	Z_{HV}	a_m (in)	Z_s
Mather & Tynes (1961)	6.0	28	8	2330	2081	111	5.32	1	1	6.0	1
	8.0	28	8	2330	2134	209	9.77	1	1	6.0	1
	10.0	28	8	2330	2128	135	6.35	1	1	6.0	1
	6.0	28	8	4490	3968	232	5.84	1	1	6.0	1
	8.0	28	8	4490	3960	216	5.46	1	1	6.0	1
	10.0	28	8	4490	3876	136	3.51	1	1	6.0	1
	6.0	28	8	1870	1613	175	10.85	1	1	3.0	1
	8.0	28	8	1870	1601	61	3.81	1	1	3.0	1
	10.0	28	8	1870	1636	71	4.31	1	1	3.0	1
	6.0	28	8	3510	3265	289	8.85	1	1	3.0	1
	8.0	28	8	3510	3313	172	5.20	1	1	3.0	1
	10.0	28	8	3510	3166	74	2.35	1	1	3.0	1
Meininger Wagner & Hall (1977)	4.0	91	10	4560	4541	208	4.59	0	0	1.0	0
	4.0	91	9	4560	3900	172	4.41	1	0	1.0	0
	4.0	35	9	2270	2325	105	4.54	0	0	1.0	0
	4.0	35	8	2270	2033	143	7.04	1	0	1.0	0
	4.0	35	10	2930	2962	163	5.49	0	0	1.0	0
	4.0	35	8	2930	2579	81	3.15	1	0	1.0	0
	4.0	58	6	2800	3752	118	3.15	0	0	1.0	0
	4.0	58	16	2800	3173	149	4.71	1	0	1.0	0
	4.0	79	5	5100	5214	138	2.64	0	0	1.0	0
	4.0	79	16	5100	4564	216	4.73	1	0	1.0	0
	4.0	100	6	7100	7038	201	2.85	0	0	1.0	0
	4.0	100	14	7100	6330	229	3.62	1	0	1.0	0
Meininger (1968)	1.9	91	28	3110	2086	101	4.85	1	1	0.75	0
	4.0	91	12	3110	2104	37	1.75	1	1	0.75	0
	5.9	91	12	3110	2133	89	4.16	1	1	0.75	0
	1.9	91	33	3110	2345	144	6.14	1	0	0.75	0
	4.0	91	18	3110	2241	103	4.60	1	0	0.75	0
	5.9	91	12	3110	2227	94	4.22	1	0	0.75	0
Munday & Dhir (1984)	2.0	28	6	5000	4955	100	2.02	1	0	0.75	0
	3.0	28	6	5000	5052	85	1.68	1	0	0.75	0
	4.0	28	6	5000	5027	87	1.73	1	0	0.75	0

Table 5F.1: Strength Variability Data for Cores of Different Diameter (concluded)

Source	d (in)	age (d)	n	$\bar{f}_{cyl}$ (psi)	$\bar{f}_c$ (psi)	s (psi)	V (%)	Z_{mc}	Z_{HV}	a_m (in)	Z_s
Ramirez	2.0	90	16	1445	1539	283	18.39	1	1	1.0	1
& Barcena	4.0	90	16	1445	1642	157	9.56	1	1	1.0	1
(1979)	6.0	90	16	1445	1550	95	6.13	1	1	1.0	1
	2.0	90	16	1445	1409	211	14.98	1	0	1.0	1
	4.0	90	16	1445	1544	131	8.50	1	0	1.0	1
	6.0	90	16	1445	1564	158	10.10	1	0	1.0	1
	2.0	90	16	1791	1963	289	14.72	1	1	1.0	1
	4.0	90	16	1791	2014	169	8.39	1	1	1.0	1
	6.0	90	16	1791	2000	132	6.60	1	1	1.0	1
	2.0	90	16	1791	1882	602	31.99	1	0	1.0	1
	4.0	90	16	1791	2125	187	8.80	1	0	1.0	1
	6.0	90	16	1791	2116	154	7.28	1	0	1.0	1
	2.0	90	16	2760	2729	409	14.99	1	1	1.0	1
	4.0	90	16	2760	2816	259	9.20	1	1	1.0	1
	6.0	90	16	2760	2992	105	3.50	1	1	1.0	1
	2.0	90	16	2760	2908	500	17.20	1	0	1.0	1
	4.0	90	16	2760	3398	326	9.59	1	0	1.0	1
	6.0	90	16	2760	3258	228	7.00	1	0	1.0	1

Appendix 6A: Precision of Parameters Estimated by Weighted Regression Analyses

The various core strength correction factors have been determined using weighted regression analyses of data from many sources, following the procedures described elsewhere (Bartlett and MacGregor, 1994b, d [Chapters 3, 5]). To introduce the notation, consider a simple linear model

$$\{Y\} = [X]\{\beta\} + \{\varepsilon\} \qquad (6A.1)$$

where $\{Y\}$ is the vector of n observed dependent variables, $[X]$ is the matrix of the corresponding independent variables with dimensions n by p, $\{\beta\}$ is the vector of p unknown parameters and $\{\varepsilon\}$ is the vector of the errors associated with the observations. For a weighted analysis, using a diagonal n by n matrix of weights, $[W]$, the vector of parameter estimates $\{\hat{\beta}\}$ is obtained from

$$\{\hat{\beta}\} = ([X]^T [W] [X])^{-1} [X]^T [W] \{Y\} \qquad (6A.2).$$

The best linear unbiased estimate of the unknown parameters is obtained when the weight applied to each observation is equal to the inverse of its variance (Montgomery and Peck, 1982). If the weighting is appropriate, the estimate of the variance factor (or mean square error) $\hat{\sigma}^2$ should not be statistically different from one (Peterson and Peterson, 1988). In other words, if the mathematical model is correct and the various errors are normally distributed, then all the uncertainty is attributable to the variance of the observations.

The precision of the prediction model is given by the variance of the predicted mean value of y

$$\mathrm{Var}(\hat{y}_0) = \hat{\sigma}^2 < X_0 > ([X]^T [W] [X])^{-1} \{X_0\} \qquad (6A.3)$$

where $\{X_0\}$ is the vector of independent variables used to predict the mean value of the dependent variable, $\hat{y}_0$. Recent reports concerning the assessment of *in situ* concrete strength have highlighted the importance of considering errors in both the dependent and independent variables when determining the error of the correlation curve (ACI

Committee 228, 1988). This can be accomplished using Equation (6A.3) if the weights are determined following the procedure described in Bartlett and MacGregor (1994b [Chapter 3]).

In the present investigation, weights are typically based on the actual variances of the average values from tests of replicate specimens. The corresponding variance factors range from 2 to 6 which implies that the models are imperfect, probably because they do not include various unknown factors. Although this conclusion has troublesome statistical implications, it does reflect the inherent difficulties in testing cores and interpreting the results (Malhotra, 1977).

To investigate the effect of variance factors greater than 1, consider the uncertainty of a predicted value after the effect of errors in the dependent and independent variables is removed. The variance of the predicted value of a new observation, calculated using the algorithm in the SAS software package (the SAS Institute, 1990) for weighted regression analysis, is

$$\mathrm{Var}(\hat{y}_N) = \hat{\sigma}^2 \left(\frac{1}{w_0} + \langle X_0 \rangle ([X]^T [W] [X])^{-1} \{X_0\} \right) \qquad (6A.4).$$

This equation indicates that the variance of a predicted new observation, $\hat{y}_N$, is simply the sum of the variance of the predicted mean value from Equation (6A.3) and the variance of the new observation, $1/w_0$. If the variance of the new observation is removed, one obtains

$$\mathrm{Var}(\hat{y}_0)^* = (\hat{\sigma}^2 - 1)\frac{1}{w_0} + \hat{\sigma}^2 \langle X_0 \rangle ([X]^T [W] [X])^{-1} \{X_0\} \qquad (6A.5)$$

which is equivalent to Equation (6A.3) only if the variance factor $\hat{\sigma}^2$ equals 1. If the variance factor is greater than 1, Equation (6A.5) indicates that the variance of the predicted mean value from Equation (6A.3) is always less than the true uncertainty of the strength correction factors. Therefore conventional regression analysis solutions give sound and rigorously correct values for the precision of the strength correction factors if the underlying model is correct. Unfortunately they give erroneously small estimates of the error if, as in the present case, the underlying model is imperfect.

Appendix 7A: Summary of Standard Cylinder Strength Data

The tables in this Appendix summarise 3576 strength tests from 108 concrete mixes. In the table headings, n is the number of tests (each test is the average of 2 or 3 cylinder breaks), f'_c is the specified strength and $\bar{f}_{cyl}$ and s are the average value and standard deviation of the test strengths. The number of tests falling below the specified strength is N_{low}. Indicator variable Z_b is 1 if the concrete was produced for bridge construction or 0 otherwise, Z_p is 1 for precast concrete or 0 for cast-in-place concrete, and Z_{air} is 1 for air entrained concrete or 0 otherwise.

Table 7A.1: Concrete for Cast-in-place Bridge Construction

n	f'_c (MPa)	$\bar{f}_{cyl}$ (MPa)	s (MPa)	N_{low}	Z_b	Z_p	Z_{air}
20	25	37.84	3.80	0	1	0	1
19	25	39.85	3.19	0	1	0	1
19	25	28.80	1.89	0	1	0	1
32	25	35.71	4.53	0	1	0	1
17	25	35.96	2.19	0	1	0	1
32	25	33.86	2.20	0	1	0	1
15	25	35.22	3.03	0	1	0	1
16	25	32.16	2.28	0	1	0	1
23	25	32.15	2.42	0	1	0	1
28	25	33.01	2.77	0	1	0	1
35	25	34.03	2.72	0	1	0	1
65	25	33.80	3.99	0	1	0	1
22	25	30.51	4.09	2	1	0	1
15	25	28.88	2.99	1	1	0	1
35	25	28.76	3.48	1	1	0	1
36	30	35.71	2.50	0	1	0	1
27	30	37.35	2.45	0	1	0	1
24	30	35.31	1.96	0	1	0	1
17	30	30.32	2.04	7	1	0	1
14	30	36.31	1.42	0	1	0	1
26	30	35.11	2.85	0	1	0	1
14	30	31.83	2.02	0	1	0	1
16	30	34.97	2.80	0	1	0	1
35	30	38.48	3.22	0	1	0	1
77	30	32.72	2.36	7	1	0	1
29	30	38.96	2.09	0	1	0	1
15	30	30.76	2.67	4	1	0	1
14	30	28.68	3.13	9	1	0	1
17	35	37.15	2.34	3	1	0	1
17	35	42.08	4.69	1	1	0	1

Table 7A.2: General Ready Mix Concretes

n	f'_c (MPa)	$\bar{f}_{cyl}$ (MPa)	s (MPa)	N_{low}	Z_b	Z_p	Z_{air}
41	25	30.28	3.38	1	0	0	1
41	25	32.24	2.64	0	0	0	1
41	25	33.59	3.53	0	0	0	1
41	25	31.81	3.38	2	0	0	1
41	25	30.90	3.93	4	0	0	1
55	25	31.65	4.72	4	0	0	1
41	25	27.68	3.52	10	0	0	0
41	25	31.34	3.94	2	0	0	0
41	25	31.84	3.29	0	0	0	0
41	25	31.88	2.98	0	0	0	0
41	25	31.57	2.06	0	0	0	0
40	25	34.27	5.21	0	0	0	0
41	30	37.11	5.26	3	0	0	1
41	30	37.82	4.40	2	0	0	1
41	30	38.91	3.47	0	0	0	0
41	30	40.30	2.85	0	0	0	0
41	30	38.63	2.69	0	0	0	0
52	35	35.10	4.12	25	0	0	1
66	35	46.29	4.12	0	0	0	1
41	35	45.40	4.93	0	0	0	0
36	40	49.77	4.80	0	0	0	0
41	45	55.74	2.75	0	0	0	1
41	45	58.75	3.24	0	0	0	1
41	25	32.06	5.14	1	0	0	1
41	25	31.22	3.33	1	0	0	1
67	25	28.81	2.66	4	0	0	1
49	25	31.29	3.49	2	0	0	0
40	25	27.89	2.40	6	0	0	0
25	25	31.17	2.99	0	0	0	1
20	25	31.30	4.56	1	0	0	0
18	25	31.46	2.92	0	0	0	0
18	25	30.16	2.90	0	0	0	0
14	25	31.31	2.51	0	0	0	1
26	30	36.91	2.56	0	0	0	0
15	30	37.27	3.64	0	0	0	1
21	25	29.98	2.50	0	0	0	1
31	32	33.39	4.10	12	0	0	1
55	32	34.85	2.03	6	0	0	0
34	30	36.17	3.83	2	0	0	0
41	30	34.21	3.32	4	0	0	1
30	25	28.67	2.52	2	0	0	0
27	25	31.35	2.63	0	0	0	0
28	25	28.36	2.29	2	0	0	0
31	25	35.97	2.28	0	0	0	0
30	25	30.74	2.60	0	0	0	0

Table 7A.2 (concluded): General Ready Mix Concretes

n	f'_c (MPa)	$\bar{f}_{cyl}$ (MPa)	s (MPa)	N_{low}	Z_b	Z_p	Z_{air}
32	25	32.67	2.62	0	0	0	0
30	25	32.01	1.49	0	0	0	0
36	25	30.53	1.70	0	0	0	1
48	25	27.85	3.08	6	0	0	1
40	25	31.43	3.66	1	0	0	1
34	25	29.30	2.57	2	0	0	1
32	25	34.17	4.24	0	0	0	1
24	25	31.08	3.86	0	0	0	1
39	35	36.64	4.45	12	0	0	1
32	35	38.20	4.85	8	0	0	1

Table 7A.3: Concretes for Precast Bridge Construction

n	f'_c (MPa)	$\bar{f}_{cyl}$ (MPa)	s (MPa)	N_{low}	Z_b	Z_p	Z_{air}
30	40	48.45	3.25	0	1	1	1
31	40	50.08	2.18	0	1	1	1
33	35	38.90	2.11	1	1	1	1
49	35	38.17	2.13	2	1	1	1
36	35	42.17	2.22	0	1	1	1
58	35	42.33	2.46	0	1	1	1
18	35	41.32	2.16	0	1	1	1
32	35	40.95	2.30	0	1	1	1
36	35	47.33	2.54	0	1	1	1
31	35	41.93	1.87	0	1	1	1
36	35	43.78	2.40	0	1	1	1
36	35	41.09	2.78	1	1	1	1
36	35	39.82	3.06	2	1	1	1
36	35	43.00	1.98	0	1	1	1
37	35	44.16	2.62	0	1	1	1
24	35	40.87	2.06	0	1	1	1
18	35	40.12	1.27	0	1	1	1
21	40	51.31	2.17	0	1	1	1
36	35	40.53	2.70	0	1	1	1
37	35	40.41	2.70	0	1	1	1
36	35	41.70	2.23	0	1	1	1
36	35	39.44	2.61	2	1	1	1
27	35	54.48	4.61	0	1	1	1

Appendix 7B: *In Situ* Strength Data for Conventional Concretes

Values of F_2 shown in Table 7B.1 represent the ratio between the average *in situ* strength and the average strength of standard test cylinders, $\bar{f}_{cyl}$. The length, thickness and height of the specimens are shown as l, t, and h respectively; an entry of -1 in these columns indicates that the actual dimension is unknown. Indicator variable Z_h equals 1 if the height of the member exceeds 450 mm, or 0 otherwise. The degree of curing of each element is crudely indicated by Z_{cu}, which is equal to 0 for no moist curing or 1 for initial moist curing for at least 7 days. The ratio of the weight of Class F fly ash to cement is shown in the column headed fa^F/c.

Table 7B.1: *In Situ* Strength Data for Conventional Concretes

Reference	Mix	$\bar{f}_{cyl}$ (MPa)	age (d)	l (mm)	t (mm)	h (mm)	Z_h	Z_{cu}	fa^F /c	F_2
Mather	1	16.3	28	1520	1520	500	1	1	0	1.014
& Tynes	2	31.0	28	1520	1520	500	1	1	0	1.001
(1961)	3	12.6	28	1520	1520	500	1	1	0	0.978
	4	25.5	28	1520	1520	500	1	1	0	1.052
Bloem	1	35.4	95	-1	-1	200	0	1	0	1.013
(1965)	1	35.4	95	-1	-1	100	0	1	0	1.091
	2	29.4	95	660	200	3050	1	1	0	0.972
Bloem	1	37.9	28	2700	1500	150	0	1	0	0.784
(1968)	1	37.9	91	2700	1500	150	0	1	0	0.855
	1	37.9	365	2700	1500	150	0	1	0	0.838
	2	25.7	28	2700	1500	150	0	1	0	0.906
	2	25.7	91	2700	1500	150	0	1	0	1.020
	2	25.7	365	2700	1500	150	0	1	0	1.006
	3	25.7	28	2700	1500	150	0	1	0	0.846
	3	25.7	91	2700	1500	150	0	1	0	0.903
	3	25.7	365	2700	1500	150	0	1	0	0.933
Meininger	1	21.4	93	1980	410	1220	1	1	0	0.797
(1968)	1	21.4	93	1070	1070	400	0	1	0	0.898
Gaynor	1	26.2	91	585	150	1220	1	1	0	1.051
(1970)	2	41.9	91	585	150	1220	1	1	0	1.074
Lewis	1	44.8	30	3050	3050	350	0	0.5	0	0.925
(1976)	1	44.8	93	3050	3050	350	0	0.5	0	0.978
	2	26.4	30	3050	3050	350	0	1	0	0.948
	2	26.4	93	3050	3050	350	0	1	0	0.991
	3	22.5	30	3050	3050	350	0	0.5	0	0.901
	3	22.5	93	3050	3050	350	0	0.5	0	0.946
	4	42.9	30	3050	3050	350	0	0.5	0	0.847
	4	42.9	93	3050	3050	350	0	0.5	0	0.908

Table 7B.1: *In Situ* Strength Data for Conventional Concretes (continued)

Reference	Mix	$\bar{f}_{cyl}$ (MPa)	age (d)	l (mm)	t (mm)	h (mm)	Z_h	Z_{cu}	fa^F /c	F_2
Meininger, Wagner & Hall (1977)	1	32.1	91	2130	300	1070	1	1	0	1.022
	2	15.7	35	2130	300	1070	1	1	0	1.043
	3	20.2	35	2130	300	1070	1	1	0	1.066
	4	19.3	28	1220	300	1830	1	1	0	1.109
	4	19.3	60	1220	300	1830	1	1	0	1.285
	5	35.2	28	1220	300	1830	1	1	0	0.940
	5	35.2	81	1220	300	1830	1	1	0	1.114
	6	49.1	28	1220	300	1830	1	1	0	0.954
	6	49.1	102	1220	300	1830	1	1	0	1.094
Meynink & Samarin (1979)	1	26.0	28	-1	150	150	0	1	0	1.000
	1	26.0	28	-1	150	150	0	1	0	1.009
	2	41.5	28	-1	150	150	0	1	0	1.030
	2	41.5	28	-1	150	150	0	1	0	1.010
Malhotra & Carette (1980)	1	17.7	28	610	610	305	0	1	0	0.940
	1	17.7	91	610	610	305	0	1	0	1.072
	2	22.8	28	610	610	305	0	1	0	0.881
	2	22.8	91	610	610	305	0	1	0	0.948
	3	31.0	28	610	610	305	0	1	0	0.872
	3	31.0	91	610	610	305	0	1	0	1.115
	4	31.0	28	610	610	305	0	1	0	1.115
	4	31.0	91	610	610	305	0	1	0	1.251
	5	38.4	28	610	610	305	0	1	0	0.988
	5	38.4	91	610	610	305	0	1	0	1.043
	6	41.8	28	610	610	305	0	1	0	0.872
	6	41.8	91	610	610	305	0	1	0	1.073
	7	50.6	28	610	610	305	0	1	0	0.767
	7	50.6	91	610	610	305	0	1	0	0.819
Yip & Tam (1988)	1	23.8	28	300	200	300	0	0	0	1.093
	2	29.4	28	300	200	150	0	0	0	1.120
	3	39.0	28	300	200	300	0	1	0	1.219
	4	40.8	28	300	200	150	0	1	0	1.260
	5	40.8	28	300	200	300	0	0	0	1.117
	6	35.8	28	300	200	150	0	0	0	0.899
	7	41.7	28	300	200	300	0	1	0	0.956
	8	39.5	28	300	200	150	0	1	0	1.021
	9	50.2	28	300	200	300	0	0	0	0.844
	10	40.9	28	300	200	150	0	0	0	0.958
	11	16.0	28	300	200	300	0	1	0	0.826
	12	20.6	28	300	200	150	0	1	0	1.124
Akers (1990)	1	34.9	28	-1	-1	300	0	1	0	1.045
	2	37.4	28	-1	-1	300	0	1	0	0.959

Table 7B.1: *In Situ* Strength Data for Conventional Concretes (continued)

Reference	Mix	$\bar{f}_{cyl}$ (MPa)	age (d)	l (mm)	t (mm)	h (mm)	Z_h	Z_{cu}	fa^F /c	F_2
Keiller (1984)	1	36.0	28	1200	300	600	1	0.5	0	1.079
	1	36.0	365	1200	300	600	1	0.5	0	1.400
	2	38.2	28	1200	300	600	1	0.5	0	0.904
	2	38.2	365	1200	300	600	1	0.5	0	1.264
	3	27.4	28	1200	300	600	1	0.5	0	0.940
	3	27.4	365	1200	300	600	1	0.5	0	1.367
	4	43.6	28	1200	300	600	1	0.5	0	1.004
	4	43.6	365	1200	300	600	1	0.5	0	1.244
	5	39.4	28	1200	300	600	1	0.5	0	0.930
	5	39.4	365	1200	300	600	1	0.5	0	1.053
	6	27.8	28	1200	300	600	1	0.5	0	0.971
	6	27.8	365	1200	300	600	1	0.5	0	1.224
	7	25.4	28	1200	300	600	1	0.5	0	1.177
	7	25.4	365	1200	300	600	1	0.5	0	1.385
	8	20.8	28	1200	300	600	1	0.5	0	0.869
	8	20.8	365	1200	300	600	1	0.5	0	1.351
	9	38.2	28	1200	300	600	1	0.5	0	0.918
	9	38.2	365	1200	300	600	1	0.5	0	1.321
	10	42.8	28	1200	300	600	1	0.5	0	0.900
	10	42.8	365	1200	300	600	1	0.5	0	1.163
	11	28.2	28	1200	300	600	1	0.5	0	0.899
	11	28.2	365	1200	300	600	1	0.5	0	1.368
	12	15.8	28	1200	300	600	1	0.5	0	0.963
	12	15.8	365	1200	300	600	1	0.5	0	1.507
	13	28.0	28	1200	300	600	1	0.5	0	1.007
	13	28.0	365	1200	300	600	1	0.5	0	1.358
	14	42.5	28	1200	300	600	1	0.5	0	1.092
	14	42.5	365	1200	300	600	1	0.5	0	1.296
	15	22.7	28	1200	300	600	1	0.5	0	1.191
	15	22.7	365	1200	300	600	1	0.5	0	1.423
	16	47.4	28	1200	300	600	1	0.5	0	1.071
	16	47.4	365	1200	300	600	1	0.5	0	1.291
	17	12.3	28	1200	300	600	1	0.5	0	1.073
	17	12.3	365	1200	300	600	1	0.5	0	1.694
	18	27.4	28	1200	300	600	1	0.5	1	0.941
	18	27.4	365	1200	300	600	1	0.5	1	1.686
	19	39.4	28	1200	300	600	1	0.5	1	1.029
	19	39.4	365	1200	300	600	1	0.5	1	1.689
	20	16.3	28	1200	300	600	1	0.5	0	1.228
	20	16.3	365	1200	300	600	1	0.5	0	1.500
	21	28.8	28	1200	300	600	1	0.5	0	1.259
	21	28.8	365	1200	300	600	1	0.5	0	1.388
Szypula & Grossman (1990)	1	26.5	28	9700	2400	240	0	1	0	1.014
	1	26.5	56	9700	2400	240	0	1	0	1.181

Table 7B.1: *In Situ* Strength Data for Conventional Concretes (continued)

Reference	Mix	$\bar{f}_{cyl}$ (MPa)	age (d)	l (mm)	t (mm)	h (mm)	Z_h	Z_{cu}	fa^F /c	F_2
Haque et al. (1988)	1	22.8	182	300	300	450	1	0	0	1.408
	2	17.8	182	300	300	450	1	0	0.25	1.382
	3	16.8	182	300	300	450	1	0	0.5	1.200
	4	12.8	182	300	300	450	1	0	1	1.166
	5	21.6	182	300	300	450	1	0	0.25	1.204
	6	20.2	182	300	300	450	1	0	0.5	1.167
	7	15.4	182	300	300	450	1	0	1	1.165
	8	30.1	182	300	300	450	1	0	0	1.171
	9	26.2	182	300	300	450	1	0	0.25	1.158
	10	20.6	182	300	300	450	1	0	0.5	1.181
	11	19.1	182	300	300	450	1	0	1	0.937
	12	25.6	182	300	300	450	1	0	0.25	1.237
	13	25.9	182	300	300	450	1	0	0.5	1.409
	14	19.7	182	300	300	450	1	0	1	0.991
	15	42.0	182	300	300	450	1	0	0	1.200
	16	32.7	182	300	300	450	1	0	0.25	1.290
	17	28.3	182	300	300	450	1	0	0.5	1.337
	18	25.3	182	300	300	450	1	0	1	1.472
	19	31.3	182	300	300	450	1	0	0.25	1.332
	20	30.6	182	300	300	450	1	0	0.5	1.327
	21	25.6	182	300	300	450	1	0	1	1.544
Confidential Communication (1991)	1	29.2	28	1140	460	140	0	0.5	0	0.888
	1	29.2	28	1830	730	290	0	0.5	0	0.822
	2	28.9	28	1140	460	140	0	0.5	0	0.814
	2	28.9	28	1830	730	290	0	0.5	0	0.678
	3	25.5	28	1830	730	290	0	0.5	0	0.822
	3	25.5	28	1140	460	140	0	0.5	0	0.845
	4	23.2	28	1140	460	140	0	0.5	0	0.813
	4	23.2	28	1830	730	290	0	0.5	0	0.714
	5	20.8	28	1140	460	140	0	0.5	0	0.791
	5	20.8	28	1830	730	290	0	0.5	0	0.802
ACI 214 (1992)	1	33.8	56	-1	-1	250	0	1	0	1.081
	2	41.3	56	-1	-1	250	0	1	0	1.066
	3	51.2	56	-1	-1	250	0	1	0	1.074
Bickley & Read (1992)	1	36.8	28	3000	300	2000	1	1	0	1.290
	1	36.8	91	3000	300	2000	1	1	0	1.485
	2	38.2	28	3000	3000	300	0	1	0	1.143
	2	38.2	91	3000	3000	300	0	1	0	1.437

Table 7B.1: *In Situ* Strength Data for Conventional Concretes (concluded)

Reference	Mix	$\bar{f}_{cyl}$ (MPa)	age (d)	l (mm)	t (mm)	h (mm)	Z_h	Z_{cu}	fa^F /c	F_2
Haque	1	32.7	28	375	375	150	0	0	0	1.016
et al.	1	32.7	91	375	375	150	0	0	0	1.200
(1991)	1	32.7	182	375	375	150	0	0	0	1.215
	2	52.4	28	375	375	150	0	0	0	1.326
	2	52.4	91	375	375	150	0	0	0	1.050
	2	52.4	182	375	375	150	0	0	0	1.172
	3	32.6	28	375	375	150	0	0	0.43	0.947
	3	32.6	91	375	375	150	0	0	0.43	1.218
	3	32.6	182	375	375	150	0	0	0.43	1.544
	4	44.3	28	375	375	150	0	0	0.43	0.814
	4	44.3	91	375	375	150	0	0	0.43	1.109
	4	44.3	182	375	375	150	0	0	0.43	0.894
	5	50.9	28	375	375	150	0	0	0.39	0.935
	5	50.9	91	375	375	150	0	0	0.39	1.241
	5	50.9	182	375	375	150	0	0	0.39	1.042
	6	45.5	28	375	375	150	0	0	0.43	1.116
	6	45.5	91	375	375	150	0	0	0.43	1.287
	6	45.5	182	375	375	150	0	0	0.43	1.326
	7	45.8	28	375	375	150	0	0	0	0.821
	7	45.8	91	375	375	150	0	0	0	0.996
	7	45.8	182	375	375	150	0	0	0	1.109
	8	47.6	28	375	375	150	0	0	0	1.112
	8	47.6	91	375	375	150	0	0	0	1.357
	8	47.6	182	375	375	150	0	0	0	1.218
Langley	1	44.9	42	3050	3050	3050	1	1	0	0.883
et al.	1	44.9	365	3050	3050	3050	1	1	0	1.102
(1992)	1	44.9	730	3050	3050	3050	1	1	0	1.248
	2	44.0	42	3050	3050	3050	1	1	1.22	1.369
	2	44.0	365	3050	3050	3050	1	1	1.22	1.585
	2	44.0	730	3050	3050	3050	1	1	1.22	1.843
	3	22.1	42	3050	3050	3050	1	1	1.25	0.845
	3	22.1	365	3050	3050	3050	1	1	1.25	1.768
	3	22.1	730	3050	3050	3050	1	1	1.25	1.660

Values of F_2 shown in Table 7B.2 are based on data reported by Petersons (1964), who cast 3000 mm high columns with 300 x 300 mm cross sections. The columns were cast in the upright position, and, one day later, immersed in a temperature-controlled water bath until the cores were drilled. In the table, indicator variable Z_{seg} equals 1 for the mixes which were intentionally proportioned to be susceptable to segregation, or 0 otherwise.

Table 7B.2: *In Situ* Strength Data for Petersons' Columns

Mix	$\bar{f}_{cyl}$ (MPa)	age (d)	F_2	Z_{seg}
A1	13.2	28	1.298	0
A2	20.3	28	1.296	0
A3	17.6	28	1.250	0
A4	16.5	28	1.367	0
C1	27.7	28	1.247	0
C2	31.0	28	1.277	0
C3	31.8	28	1.298	0
C4	32.7	28	1.388	0
C5	34.0	28	1.115	0
C6	33.7	28	1.241	0
C7	34.4	28	1.007	0
C8	34.5	28	1.223	0
C9	34.1	28	1.279	0
E1	43.6	28	1.377	0
E2	43.8	28	1.387	0
E3	45.8	28	1.312	0
E4	46.4	28	1.270	0
B1	19.5	28	1.188	1
B2	15.1	28	1.067	1
B3	16.1	28	1.183	1
B4	16.7	28	1.119	1
D1	30.8	28	1.179	1
D2	32.4	28	1.213	1
D3	32.7	28	1.223	1
D4	32.0	28	1.242	1
D5	33.5	28	1.127	1
F1	48.1	28	1.172	1
F2	46.6	28	1.153	1
F3	45.6	28	1.111	1
F4	44.8	28	1.289	1

Table 7B.3 (a): *In Situ* Strength Data for Poorly Cured Beams and Slabs

Reference	Mix	$\bar{f}_{cyl}$ (MPa)	age (d)	Element size (m x m x mm)	F_2 (poor cure)	F_2 (good cure)
Bloem (1965)	1	35.4	95	?x?x100	0.759	1.091
	1	35.4	95	?x?x200	0.832	1.013
Bloem (1968)	1	37.9	28	2.7x1.5x150	0.658	0.784
	1	37.9	91	2.7x1.5x150	0.648	0.855
	1	37.9	365	2.7x1.5x150	0.681	0.838
	2	25.7	28	2.7x1.5x150	0.661	0.906
	2	25.7	91	2.7x1.5x150	0.655	1.020
	2	25.7	365	2.7x1.5x150	0.621	1.006
	3	28.6	28	2.7x1.5x150	0.712	0.846
	3	28.6	91	2.7x1.5x150	0.770	0.903
	3	28.6	365	2.7x1.5x150	0.761	0.933
Meynink & Samarin (1979)	1	26	28	?x.15x150	0.780	1.000
	1	26	28	?x.15x150	0.780	1.009
	2	41.5	28	?x.15x150	0.801	1.010
	2	41.5	28	?x.15x150	0.801	1.030
Szypula & Grossman (1990)	1	26.5	28	9.7x2.4x240	0.752	1.014
	1	26.5	56	9.7x2.4x240	0.896	1.181

Table 7B.3 (b): *In Situ* Strength Data for Poorly Cured Walls and Columns

Reference	Mix	$\bar{f}_{cyl}$ (MPa)	age (d)	Element size (m x m x mm)	F_2 (poor cure)	F_2 (good cure)
Bloem (1965)	2	29.4	95	0.7x0.2x3050	0.933	0.972
Gaynor (1970)	1	26.2	91	0.6x.15x1220	0.917	1.051
	2	41.9	91	0.6x.15x1220	0.946	1.074

Table 7B.4: *In Situ* Strength Data for Elements Cured in Cold Temperatures

Reference	Mix	$\bar{f}_{cyl}$ (MPa)	age (d)	F_2	Element Size (m x m x mm)	Curing Conditions
Malhotra (1973)	1	28.8	28	0.756	0.6x.6x205	3 days at 23 °C then exposed to Ottawa winter conditions
	1	28.8	59	0.856	0.6x.6x205	
	1	28.8	93	0.820	0.6x.6x205	
	1	28.8	116	0.921	0.6x.6x205	
Berwanger & Malhotra (1974)	1	17.4	28	0.527	0.6x0.6x1500	3 days at 10° C then exposed to Ottawa winter conditions
	1	17.4	121	0.640	0.6x0.6x1500	
	1	17.4	355	1.080	0.6x0.6x1500	
	1	17.4	28	0.452	1.2x0.9x200	
	1	17.4	121	0.573	1.2x0.9x200	
	1	17.4	355	0.927	1.2x0.9x200	
	2	31.7	28	0.677	0.6x0.6x1500	
	2	31.7	121	0.664	0.6x0.6x1500	
	2	31.7	355	1.083	0.6x0.6x1500	
	2	31.7	28	0.577	1.2x0.9x200	
	2	31.7	121	0.692	1.2x0.9x200	
	2	31.7	355	0.966	1.2x0.9x200	
	3	35.7	28	0.665	0.6x0.6x1500	
	3	35.7	121	0.691	0.6x0.6x1500	
	3	35.7	355	1.200	0.6x0.6x1500	
	3	35.7	28	0.597	1.2x0.9x200	
	3	35.7	121	0.707	1.2x0.9x200	
	3	35.7	355	1.085	1.2x0.9x200	
	4	37.3	28	0.663	0.6x0.6x1500	
	4	37.3	121	0.682	0.6x0.6x1500	
	4	37.3	355	1.095	0.6x0.6x1500	
	4	37.3	28	0.654	1.2x0.9x200	
	4	37.3	121	0.817	1.2x0.9x200	
	4	37.3	355	1.217	1.2x0.9x200	
	5	42.0	28	0.635	0.6x0.6x1500	
	5	42.0	121	0.725	0.6x0.6x1500	
	5	42.0	355	1.080	0.6x0.6x1500	
	5	42.0	28	0.673	1.2x0.9x200	
	5	42.0	121	0.772	1.2x0.9x200	

Table 7B.5: Historical *In Situ* Strength Data

Reference	$\bar{f}_{cyl}$ (MPa)	cem. type	age (d)	F_2
United States Bureau of Reclamation (1966)	25.9	IV	3650	1.322
	21.9	IV	3650	1.629
	16.3	IV	3650	1.600
	11.4	IV	3650	2.050
	34.1	IV	3650	1.382
	34.1	IV	3650	0.926
	24.8	IV	3650	1.258
	24.6	IV	3650	1.358
	21.4	IV	3650	1.544
	25.9	II	182	1.123
	16.7	II	215	1.563
	25.4	I	690	1.642
	25.8	II	360	1.538
	27.2	II	540	1.486
	27.4	II	730	1.554
	27.8	II	900	1.554
	24.9	II	1095	1.713
	25.3	II	1095	1.968
	22.8	II	1095	1.550
	26.3	II	182	1.215
Price (1951)	34.9	I	720	1.179
	41.0	II	210	0.875
	25.2	II	1020	1.480
	31.0	IV	270	1.223
	32.0	IV	630	1.789
	21.9	IV	450	2.079
	18.2	IV	240	1.823
	24.0	I	91	2.355
	25.1	I	570	1.413
	31.4	I	720	1.366

Reference	$\bar{f}_{cyl}$ (MPa)	cem. type	age (d)	F_2
Price (1951)	29.9	I	182	1.596
	26.3	II	930	1.618
	27.6	II	750	1.690
	41.9	II	210	0.791
	25.1	II	570	1.366
Wagner (63)	44.8	I	28	0.778
Henzel (68)	22.0	?	90	1.098
Keiller (1984)	33.2	I	42	1.005
	33.2	I	42	1.020
	40.0	I	42	1.054
	41.6	I	42	1.120
	36.0	I	28	1.090
	27.2	I	182	1.287
	35.2	I	28	0.968
	36.0	I	28	1.191
	32.8	I	182	1.209
	39.2	I	42	1.017
	45.2	I	42	1.031
	33.6	I	28	0.968
	19.0	I	28	0.971
	20.8	I	28	1.132
	48.2	I	730	1.518
	47.8	I	730	1.763
	45.8	I	730	1.440
	33.6	I	42	1.236
	29.2	I	42	1.231
	35.6	I	42	1.152
	34.8	I	42	1.161
	33.6	I	42	1.122
	39.4	I	42	1.147

Appendix 7C: *In Situ* Strength Data for High Performance Concretes

Values of F_2 shown in these tables represent the ratio between the average *in situ* strength and the average strength of standard test cylinders, $\bar{f}_{cyl}$. If the concrete contains supplementary cementitious materials, indicator variable Z_{sup} is 1 and the ratios of the weights of silica fume, fly ash, and slag to the weight of cement are as shown in the columns headed sf/c, fa/c and sl/c respectively. Indicator variable Z_t equals 1 if the least dimension of the member is 400 mm or less, or 0 otherwise. Indicator variable Z_w equals 1 for walls or 0 otherwise. The ambient temperature at the time of casting is shown in the column headed T, and the maximum temperature recorded during hydration in the column headed T_{max}.

Table 7C.1: Data from Study by Read, Carette and Malhotra (1991)

Mix	Age (days)	F_2	$\bar{f}_{cyl}$ (MPa)	Z_{sup}	sf /c	fa^F /c	sl /c	Z_t	Z_w	T (°C)
1	28	0.918	79.9	1	0.11	0	0.43	0	1	32
1	91	0.984	79.9	1	0.11	0	0.43	0	1	32
1	182	0.972	79.9	1	0.11	0	0.43	0	1	32
1	365	1.057	79.9	1	0.11	0	0.43	0	1	32
1	546	1.031	79.9	1	0.11	0	0.43	0	1	32
1	730	1.122	79.9	1	0.11	0	0.43	0	1	32
1	912	1.057	79.9	1	0.11	0	0.43	0	1	32
1	28	0.970	79.9	1	0.11	0	0.43	1	1	32
1	91	1.105	79.9	1	0.11	0	0.43	1	1	32
1	182	1.071	79.9	1	0.11	0	0.43	1	1	32
1	365	1.084	79.9	1	0.11	0	0.43	1	1	32
1	546	1.068	79.9	1	0.11	0	0.43	1	1	32
1	730	1.175	79.9	1	0.11	0	0.43	1	1	32
1	912	1.100	79.9	1	0.11	0	0.43	1	1	32
1	28	0.921	79.9	1	0.11	0	0.43	0	0	32
1	91	0.939	79.9	1	0.11	0	0.43	0	0	32
1	182	0.923	79.9	1	0.11	0	0.43	0	0	32
1	365	1.041	79.9	1	0.11	0	0.43	0	0	32
1	546	1.016	79.9	1	0.11	0	0.43	0	0	32
1	730	1.043	79.9	1	0.11	0	0.43	0	0	32
1	912	1.085	79.9	1	0.11	0	0.43	0	0	32
2	28	0.959	71.6	1	0	0	0.53	0	1	30
2	91	1.020	71.6	1	0	0	0.53	0	1	30
2	182	1.039	71.6	1	0	0	0.53	0	1	30
2	365	1.127	71.6	1	0	0	0.53	0	1	30
2	546	1.137	71.6	1	0	0	0.53	0	1	30
2	730	1.326	71.6	1	0	0	0.53	0	1	30
2	912	1.279	71.6	1	0	0	0.53	0	1	30
2	28	1.016	71.6	1	0	0	0.53	1	1	30
2	91	1.048	71.6	1	0	0	0.53	1	1	30
2	182	1.041	71.6	1	0	0	0.53	1	1	30

Table 7C.1: Data from Study by Read et al., 1991 (continued)

Mix	Age (days)	F_2	$\bar{f}_{cyl}$ (MPa)	Z_{sup}	sf /c	fa^F /c	sl /c	Z_t	Z_w	T (°C)
2	365	1.255	71.6	1	0	0	0.53	1	1	30
2	546	1.270	71.6	1	0	0	0.53	1	1	30
2	730	1.393	71.6	1	0	0	0.53	1	1	30
2	912	1.338	71.6	1	0	0	0.53	1	1	30
2	28	0.939	71.6	1	0	0	0.53	0	0	30
2	91	0.937	71.6	1	0	0	0.53	0	0	30
2	182	1.027	71.6	1	0	0	0.53	0	0	30
2	365	1.128	71.6	1	0	0	0.53	0	0	30
2	546	1.170	71.6	1	0	0	0.53	0	0	30
2	730	1.254	71.6	1	0	0	0.53	0	0	30
2	912	1.242	71.6	1	0	0	0.53	0	0	30
3	28	0.917	81.6	1	0.09	0	0	0	1	35
3	91	0.907	81.6	1	0.09	0	0	0	1	35
3	182	0.992	81.6	1	0.09	0	0	0	1	35
3	365	1.042	81.6	1	0.09	0	0	0	1	35
3	546	1.050	81.6	1	0.09	0	0	0	1	35
3	730	1.025	81.6	1	0.09	0	0	0	1	35
3	912	1.152	81.6	1	0.09	0	0	0	1	35
3	28	0.939	81.6	1	0.09	0	0	1	1	35
3	91	0.991	81.6	1	0.09	0	0	1	1	35
3	182	1.042	81.6	1	0.09	0	0	1	1	35
3	365	1.077	81.6	1	0.09	0	0	1	1	35
3	546	1.138	81.6	1	0.09	0	0	1	1	35
3	730	1.064	81.6	1	0.09	0	0	1	1	35
3	912	1.135	81.6	1	0.09	0	0	1	1	35
3	28	0.868	81.6	1	0.09	0	0	0	0	35
3	91	0.898	81.6	1	0.09	0	0	0	0	35
3	182	0.928	81.6	1	0.09	0	0	0	0	35
3	365	1.029	81.6	1	0.09	0	0	0	0	35
3	576	1.043	81.6	1	0.09	0	0	0	0	35
3	730	1.024	81.6	1	0.09	0	0	0	0	35
3	912	1.095	81.6	1	0.09	0	0	0	0	35
4	28	0.875	90.6	1	0.14	0	0	0	1	23
4	91	0.852	90.6	1	0.14	0	0	0	1	23
4	182	0.948	90.6	1	0.14	0	0	0	1	23
4	378	1.024	90.6	1	0.14	0	0	0	1	23
4	546	1.032	90.6	1	0.14	0	0	0	1	23
4	730	1.104	90.6	1	0.14	0	0	0	1	23
4	912	1.134	90.6	1	0.14	0	0	0	1	23
4	28	0.888	90.6	1	0.14	0	0	1	1	23
4	91	0.837	90.6	1	0.14	0	0	1	1	23
4	182	0.992	90.6	1	0.14	0	0	1	1	23
4	365	1.031	90.6	1	0.14	0	0	1	1	23
4	546	1.031	90.6	1	0.14	0	0	1	1	23

Table 7C.1: Data from Study by Read et al., 1991 (concluded)

Mix	Age (days)	F_2	$\bar{f}_{cyl}$ (MPa)	Z_{sup}	sf /c	fa^F /c	sl /c	Z_t	Z_w	T (°C)
4	730	1.039	90.6	1	0.14	0	0	1	1	23
4	912	1.059	90.6	1	0.14	0	0	1	1	23
4	28	0.887	90.6	1	0.14	0	0	0	0	23
4	91	0.853	90.6	1	0.14	0	0	0	0	23
4	182	0.921	90.6	1	0.14	0	0	0	0	23
4	365	0.945	90.6	1	0.14	0	0	0	0	23
4	546	0.962	90.6	1	0.14	0	0	0	0	23
4	730	0.998	90.6	1	0.14	0	0	0	0	23
4	912	1.066	90.6	1	0.14	0	0	0	0	23
5	28	1.010	54.7	1	0	1.33	0	0	1	2
5	91	1.696	54.7	1	0	1.33	0	0	1	2
5	182	1.785	54.7	1	0	1.33	0	0	1	2
5	365	1.903	54.7	1	0	1.33	0	0	1	2
5	546	1.971	54.7	1	0	1.33	0	0	1	2
5	730	2.023	54.7	1	0	1.33	0	0	1	2
5	912	1.996	54.7	1	0	1.33	0	0	1	2
5	28	0.967	54.7	1	0	1.33	0	0	1	2
5	91	1.599	54.7	1	0	1.33	0	0	1	2
5	182	1.686	54.7	1	0	1.33	0	0	1	2
5	365	1.853	54.7	1	0	1.33	0	0	1	2
5	546	1.948	54.7	1	0	1.33	0	0	1	2
5	730	1.936	54.7	1	0	1.33	0	0	1	2
5	912	1.876	54.7	1	0	1.33	0	0	1	2
6	28	0.865	70.1	0	0	0	0	0	1	8
6	91	1.152	70.1	0	0	0	0	0	1	8
6	182	1.213	70.1	0	0	0	0	0	1	8
6	365	1.308	70.1	0	0	0	0	0	1	8
6	546	1.427	70.1	0	0	0	0	0	1	8
6	730	1.390	70.1	0	0	0	0	0	1	8
6	912	1.492	70.1	0	0	0	0	0	1	8
6	28	0.972	70.1	0	0	0	0	1	1	8
6	91	1.296	70.1	0	0	0	0	1	1	8
6	182	1.311	70.1	0	0	0	0	1	1	8
6	365	1.485	70.1	0	0	0	0	1	1	8
6	546	1.512	70.1	0	0	0	0	1	1	8
6	730	1.467	70.1	0	0	0	0	1	1	8
6	912	1.559	70.1	0	0	0	0	1	1	8
6	28	0.906	70.1	0	0	0	0	0	0	8
6	91	1.143	70.1	0	0	0	0	0	0	8
6	182	1.124	70.1	0	0	0	0	0	0	8
6	365	1.334	70.1	0	0	0	0	0	0	8
6	546	1.432	70.1	0	0	0	0	0	0	8
6	730	1.444	70.1	0	0	0	0	0	0	8
6	912	1.444	70.1	0	0	0	0	0	0	8

Table 7C.2: Data from Other Studies

Ref.	Mix	Age (days)	F_2	$\bar{f}_{cyl}$ (MPa)	Z_{sup}	sf /c	fa^C /c	fa^F /c	sl /c	T_{max} (°C)
Yuan et al. (1991)	1	28	0.731	80.4	1	0	0.42	0	0	88.9
	1	56	0.775	80.4	1	0	0.42	0	0	88.9
	1	180	0.893	80.4	1	0	0.42	0	0	88.9
	1	365	0.954	80.4	1	0	0.42	0	0	88.9
	2	28	0.715	83.4	1	0	0.42	0	0	95.0
	2	56	0.749	83.4	1	0	0.42	0	0	95.0
	2	180	0.854	83.4	1	0	0.42	0	0	95.0
	2	365	0.945	83.4	1	0	0.42	0	0	95.0
Mak et al. (1990), (1993)	1	28	1.094	58.2	0	0	0	0	0	44.6
	2	28	0.993	67.0	0	0	0	0	0	69.9
	2	90	1.233	67.0	0	0	0	0	0	69.9
	2	450	1.505	67.0	0	0	0	0	0	69.9
	2	28	1.067	67.0	0	0	0	0	0	53.6
	2	90	1.319	67.0	0	0	0	0	0	53.6
	2	450	1.526	67.0	0	0	0	0	0	53.6
	3	28	0.937	92.4	0	0	0	0	0	64.4
	3	90	1.104	92.4	0	0	0	0	0	64.4
	3	450	1.335	92.4	0	0	0	0	0	64.4
	3	28	0.973	92.4	0	0	0	0	0	45.5
	3	90	1.175	92.4	0	0	0	0	0	45.5
	3	450	1.419	92.4	0	0	0	0	0	45.5
	4	28	0.899	92.2	1	0	0	0	0.25	61.8
	4	90	0.965	92.2	1	0	0	0	0.25	61.8
	4	28	1.026	92.2	1	0	0	0	0.25	38.6
	4	90	1.134	92.2	1	0	0	0	0.25	38.6
	5	28	0.841	91.2	1	0.09	0	0	0	66.4
	5	90	0.985	91.2	1	0.09	0	0	0	66.4
	5	450	1.112	91.2	1	0.09	0	0	0	66.4
	5	28	0.995	91.2	1	0.09	0	0	0	45.0
	5	90	1.105	91.2	1	0.09	0	0	0	45.0
	5	450	1.238	91.2	1	0.09	0	0	0	45.0
	6	28	0.944	112.9	1	0.09	0	0	0	51.8
	6	90	1.057	112.9	1	0.09	0	0	0	51.8
	6	450	1.151	112.9	1	0.09	0	0	0	51.8
	6	28	1.053	112.9	1	0.09	0	0	0	35.4
	6	90	1.141	112.9	1	0.09	0	0	0	35.4
	6	450	1.325	112.9	1	0.09	0	0	0	35.4
Miao et al. (1993)	1	91	1.008	90.4	1	0.08	0	0	0	68.0
	2	91	0.992	108.2	1	0.08	0	0	0	63.0

Table 7C.2: Data from Other Studies (concluded)

Ref.	Mix	Age (days)	F_2	$\bar{f}_{cyl}$ (MPa)	Z_{sup}	sf /c	fa^C /c	fa^F /c	sl /c	T_{max} (°C)
Burg & Ost	1	91	0.996	78.6	0	0	0	0	0	77.8
(1992)	1	430	1.193	78.6	0	0	0	0	0	77.8
	2	91	0.961	88.5	1	0.05	0	0.12	0	77.8
	2	430	1.066	88.5	1	0.05	0	0.12	0	77.8
	3	91	0.892	91.9	1	0.10	0	0	0	70.4
	3	430	0.973	91.9	1	0.10	0	0	0	70.4
	4	91	0.951	119.0	1	0.16	0	0	0	72.5
	4	430	1.080	119.0	1	0.16	0	0	0	72.5
	5	91	1.042	107.0	1	0.16	0	0.22	0	66.6
	5	430	1.104	107.0	1	0.16	0	0.22	0	66.6
	6	91	1.104	73.1	1	0.08	0	0.27	0	57.6
	6	395	1.258	73.1	1	0.08	0	0.27	0	57.6

Appendix 8A: Strength of Cores from Bloem's Series 183E Column

The table in this Appendix presents data for each of the cores drilled from the Series 183E column investigated by Bloem (1965). The data are from mimeographed tabulations prepared by Bloem, which were made available for the present study by Meininger of the National Aggregate Association / National Ready Mixed Concrete Association in Silver Spring, Maryland.

In Table 8A.1, (h-z) is the distance between the centre of gravity of the core and the top of the column, from which the normalized distance from the base of the column, z', is calculated. Indicator variable $Z_{cap} = 0$ if the core had lumite caps or 1 if the core had sulphur caps, $Z_{cen} = 0$ if the core was centred under the loading platens or 1 if it was placed eccentrically, $Z_{HV} = 0$ if the core axis was horizontal or 1 if the core axis was vertical, parallel to the column axis, and $Z_{l/d} = 0$ for cores with $l/d = 2$ or 1 for cores with $l/d = 1$. The core strength is given in the column headed f_c.

Table 8A.1: Data from Bloem's Series 183E Column

(h-z) (inches from top)	Z_{cap}	Z_{cen}	Z_{HV}	$Z_{l/d}$	f_c (psi)	z'
3.5	0	0	0	0	2695	0.958
15.5	0	0	0	0	3360	0.815
27.5	0	0	0	0	3505	0.673
39.5	0	0	0	0	3480	0.530
51.5	0	0	0	0	3625	0.387
63.5	0	0	0	0	3260	0.244
75.5	0	0	0	0	3690	0.101
4	0	0	1	0	2775	0.952
16	0	0	1	0	3450	0.810
28	0	0	1	0	3665	0.667
40	0	0	1	0	3695	0.524
52	0	0	1	0	3745	0.381
64	0	0	1	0	3810	0.238
76	0	0	1	0	3560	0.095
8.5	0	1	0	0	2910	0.899
20.5	0	1	0	0	3475	0.756
32.5	0	1	0	0	3410	0.613
44.5	0	1	0	0	3505	0.470
56.5	0	1	0	0	3450	0.327
68.5	0	1	0	0	3335	0.185
80.5	0	1	0	0	3505	0.042
4	0	1	1	0	2765	0.952
16	0	1	1	0	3325	0.810
28	0	1	1	0	3560	0.667
40	0	1	1	0	3480	0.524
52	0	1	1	0	3360	0.381
64	0	1	1	0	3585	0.238
76	0	1	1	0	3475	0.095

Table 8A.1: Data from Bloem's Series 183E Column (continued)

(h-z) (inches from top)	Z_{cap}	Z_{cen}	Z_{HV}	$Z_{l/d}$	f_c (psi)	z'
3.5	1	0	0	0	2455	0.958
15.5	1	0	0	0	3385	0.815
27.5	1	0	0	0	3575	0.673
39.5	1	0	0	0	3545	0.530
51.5	1	0	0	0	3215	0.387
63.5	1	0	0	0	3440	0.244
75.5	1	0	0	0	3220	0.101
4	1	0	1	0	2830	0.952
16	1	0	1	0	3465	0.810
28	1	0	1	0	3215	0.667
40	1	0	1	0	3400	0.524
52	1	0	1	0	3520	0.381
64	1	0	1	0	3410	0.238
76	1	0	1	0	4055	0.095
8.5	1	1	0	0	2910	0.899
20.5	1	1	0	0	3410	0.756
32.5	1	1	0	0	3490	0.613
44.5	1	1	0	0	3395	0.470
56.5	1	1	0	0	3280	0.327
68.5	1	1	0	0	3425	0.185
80.5	1	1	0	0	3295	0.042
4	1	1	1	0	2975	0.952
16	1	1	1	0	3575	0.810
28	1	1	1	0	3295	0.667
40	1	1	1	0	3295	0.524
52	1	1	1	0	3490	0.381
64	1	1	1	0	3335	0.238
76	1	1	1	0	3995	0.095
3.5	0	0	0	1	3570	0.958
15.5	0	0	0	1	4200	0.815
27.5	0	0	0	1	4090	0.673
39.5	0	0	0	1	4520	0.530
51.5	0	0	0	1	4225	0.387
63.5	0	0	0	1	4025	0.244
75.5	0	0	0	1	4455	0.101
10	0	0	1	1	3800	0.881
22	0	0	1	1	3985	0.738
34	0	0	1	1	3880	0.595
46	0	0	1	1	4400	0.452
58	0	0	1	1	4465	0.310
70	0	0	1	1	4515	0.167
82	0	0	1	1	4745	0.024

Table 8A.1: Data from Bloem's Series 183E Column (concluded)

(h-z) (inches from top)	Z_{cap}	Z_{cen}	Z_{HV}	$Z_{l/d}$	f_c (psi)	z'
8.5	0	1	0	1	3760	0.899
20.5	0	1	0	1	4235	0.756
32.5	0	1	0	1	4280	0.613
44.5	0	1	0	1	4355	0.470
56.5	0	1	0	1	4210	0.327
68.5	0	1	0	1	3905	0.185
80.5	0	1	0	1	4735	0.042
10	0	1	1	1	3915	0.881
22	0	1	1	1	4015	0.738
34	0	1	1	1	4195	0.595
46	0	1	1	1	4075	0.452
58	0	1	1	1	4135	0.310
70	0	1	1	1	4330	0.167
82	0	1	1	1	4675	0.024
3.5	1	0	0	1	3110	0.958
15.5	1	0	0	1	3535	0.815
27.5	1	0	0	1	3930	0.673
39.5	1	0	0	1	3945	0.530
51.5	1	0	0	1	3770	0.387
63.5	1	0	0	1	4010	0.244
75.5	1	0	0	1	3835	0.101
10	1	0	1	1	3730	0.881
22	1	0	1	1	3840	0.738
34	1	0	1	1	3935	0.595
46	1	0	1	1	3930	0.452
58	1	0	1	1	3970	0.310
70	1	0	1	1	4240	0.167
82	1	0	1	1	4585	0.024
8.5	1	1	0	1	3490	0.899
20.5	1	1	0	1	3720	0.756
32.5	1	1	0	1	3890	0.613
44.5	1	1	0	1	3730	0.470
56.5	1	1	0	1	3720	0.327
68.5	1	1	0	1	4225	0.185
80.5	1	1	0	1	3975	0.042
10	1	1	1	1	3650	0.881
22	1	1	1	1	3970	0.738
34	1	1	1	1	3675	0.595
46	1	1	1	1	3730	0.452
58	1	1	1	1	3875	0.310
70	1	1	1	1	3975	0.167
82	1	1	1	1	4370	0.024

Appendix 8B: Estimation of Peak Temperature Sustained by Burg & Ost's Cores During Hydration

In this Appendix, a method is presented to extrapolate the peak temperatures recorded during hydration of Burg and Ost's specimens from the thermocouple locations to the core locations. It is assumed that no heat was lost from the top and bottom surfaces of each block, and therefore that the variation of temperature in the plan dimensions of the block is identical for all horizontal cross sections. Since the blocks were insulated by closed cell polystyrene insulation on their top and bottom surfaces, and since the cores were not drilled close to these surfaces, this assumption is probably valid.

In an investigation by Cook et al. (1992), 2-dimensional transient thermal finite element analyses were conducted to investigate thermal gradients in plain concrete columns 3.3 feet square by 6.6 feet high. The simulated distributions were fitted to temperatures measured in three real specimens cast using 35, 90 and 120 MPa concrete mixes. For all three mixes, the variation of temperature over a horizontal cross section through the midheight of the column was as indicated by the nested isotherms shown in Figure 8B.1 (a). It will be assumed that the shape of the isotherms in Burg and Ost's blocks is the same as that reported by Cook et al.

Since the temperature distribution along the diagonal of Burg and Ost's blocks is known from the thermocouple readings, the problem is simply to follow the isotherm from the centre of gravity of a given core to find where it crosses the diagonal. This is easily achieved if the geometry of the isotherm, shown in Figure 8B.1 (b), can be approximated. Since the isotherm has 4 planes of symmetry, it may be simulated by a simple function across a 45 ° arc with the boundary conditions

at $u = 0$, $\quad v = v_o \qquad$ (8B.1)

and $\quad \frac{dv}{du} = 0 \qquad$ (8B.2)

and at $u = u_o$, $\quad \frac{dv}{du} = -1 \qquad$ (8B.3).

If a parabolic form is chosen

$$v = a + bu + cu^2 \qquad (8B.4),$$

then the equation which satisfies the boundary conditions is

$$v = v_o - \frac{1}{2u_o} u^2 \qquad \text{(8B.5).}$$

By scaling information from the diagrams presented by Cook et al., the ratio u_o/v_o was found to be independent of the concrete strength and consistently equal to 0.643. Thus, the geometry of the isotherm is given by

$$v = v_o - \frac{1}{1.287\ v_o} u^2 \qquad \text{(8B.5a).}$$

Therefore, given some value of (u_1,v_1) which locates the centre of gravity of a particular core, the distance v_o to a point on the diagonal with the same temperature from the centre of the block is, by substitution into Equation (8B.5a):

$$v_o = 0.5\left(v_1 + \sqrt{v_1^2 + 3.108u_1^2}\right) \qquad \text{(8B.6).}$$

As an example, consider the core located on the vertical centreline of block 1 with its centre of gravity 5 inches in from the drilling face. As shown in Figure 8B.2 (a), the centre of gravity of the core has coordinates $u_1 = v_1 = 13.43$ inches, hence from Equation (8B.6), $v_o = 20.33$ inches. In Figure 8B.2 (b), the maximum temperatures recorded by the thermocouples along the diagonal about 24 hours after the block was cast are shown. By interpolation, $v_o = 20.33$ inches corresponds to a peak temperature of 165.7 ° F., which is therefore the estimated peak temperature of the core. As a matter of interest, the estimated peak temperatures of all of the cores from Block 1 range from 159.1 to 172.0°F.

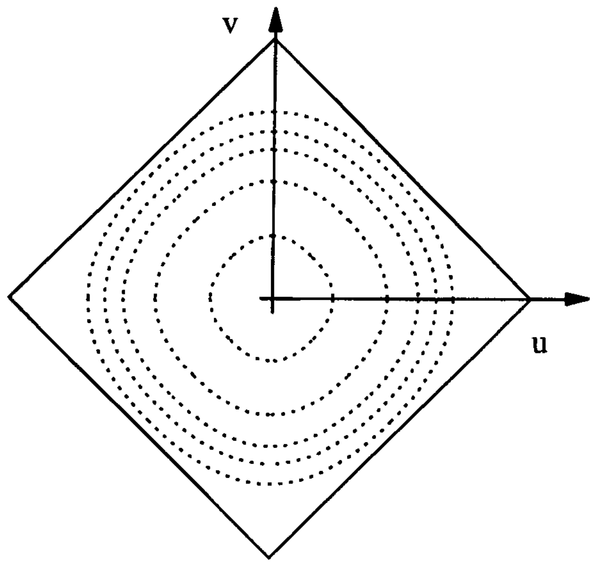

Figure 8B.1 (a): Plan view of block showing isotherms

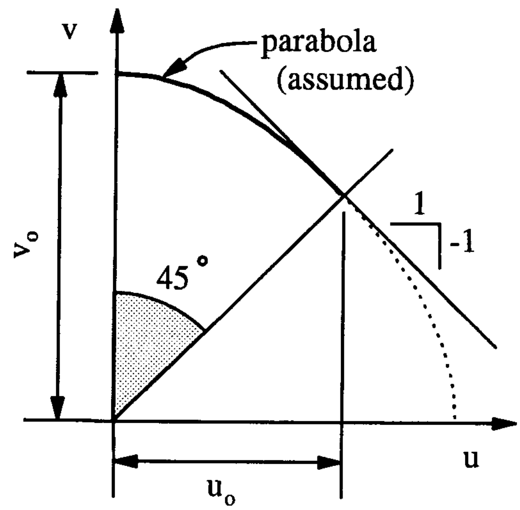

Figure 8B.1 (b): Geometry of isotherm

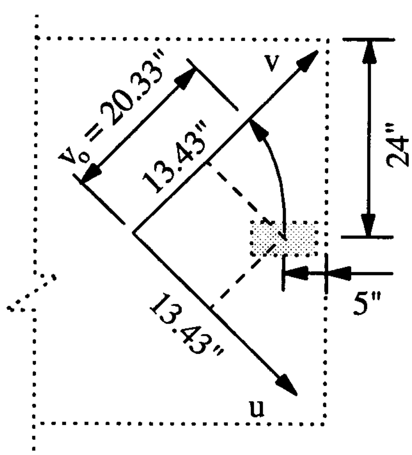

Figure 8B.2 (a): Example v_o calculation

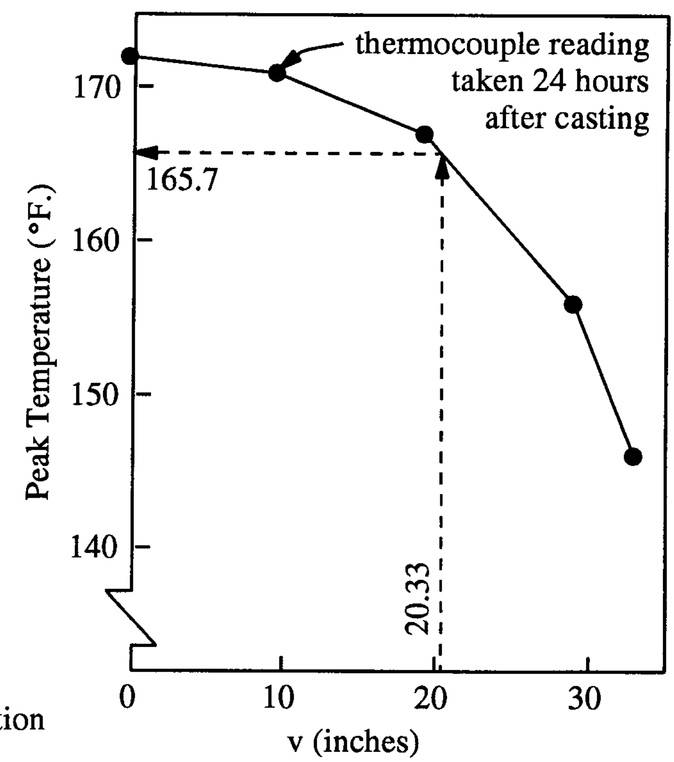

Figure 8B.2 (b): Interpolation between thermocouple readings along v axis, Block 1

Appendix 8C: Correlation between Ultrasonic Pulse Velocity and Core Strength

In this Appendix, factors are derived for converting ultrasonic pulse velocity (UPV) values measured by Tso & Zelman (1968), Drysdale (1973), and Mikhailovsky and Scanlon (1985) to equivalent core strengths.

Tso and Zelman (1968) measured ultrasonic pulse velocities on 12 columns cast in a laboratory using 3 batches of concrete with nominal strengths of 3000, 4000 and 5000 psi. The sonic tests were conducted through both the width and the thickness of the columns at 9 levels spaced 12 inches apart over the 10 foot height of the column. Subsequently the columns were cut into 12 inch long blocks and 6 inch diameter cores were drilled through the blocks, parallel to the original column axis. The investigators have reported all individual column UPV readings and the corresponding core strengths.

Each single point shown in Figure 8C.1 represents the average strength of the two cores obtained from each block and the average of the two corresponding UPV measurements. Although Tso and Zelman (1968) have suggested a quartic, and Mikhailovsky and Scanlon (1985) have suggested an exponential relationship between the ultrasonic pulse velocity and the concrete strength, a simple linear relationship was found to fit the data reasonably well. Simple linear regression analysis yielded:

$$\hat{f}_c = -7980 + 0.738 \quad \text{UPV} \tag{8C.1}$$

where $\hat{f}_c$ is the predicted core strength in pounds per square inch and UPV is the ultrasonic pulse velocity in feet per second. The fit has $R^2 = 0.63$ and a standard error of 295 psi.

The relationship given by Equation (8C.1) has been used to convert the ultrasonic pulse velocities measured on columns in buildings by Tso & Zelman (1968) and Drysdale (1973) to equivalent core strengths. This relationship is the best available for this purpose for these data because the same instrument and procedure was used to measure UPV values in the laboratory and in the field. However, since the calibration curve is not necessarily based on concrete mixes which are similar to those used in the buildings investigated, the predicted core strengths may not accurately represent the true *in situ* strengths.

Mikhailovsky and Scanlon (1985) measured ultrasonic pulse velocities through the webs of 5 girders on a bridge constructed in 1950. Subsequently cores were drilled though the webs at grid locations which corresponded to a few of the UPV measurements. Additional ultrasonic pulse velocity measurements were taken through the length of each core after its ends had been sawn square. The individual girder UPV readings, core UPV readings and core compressive strengths are reported.

The relationship between the core strengths and UPV measurements is shown in Figure 8C.2. The best fit of the core strengths to the UPV values measured on the girders is given by:

$$\hat{f}_c = 329 \quad e^{2.12 \times 10^{-4}(UPV)} \tag{8C.2}$$

with $\hat{f}_c$ in psi and UPV in feet per second. The best fit of the core strengths to the UPV values measured on the core is given by

$$\hat{f}_c = 25.7 \quad e^{3.89 \times 10^{-4}(UPV)} \qquad (8C.3).$$

The standard errors are roughly 840 psi and 550 psi for Equations (8C.2) and (8C.3) respectively. For a given core strength, the UPV measured through the girder is considerably less than the UPV measured through the core specimen with sawn ends. Since the assessment of the systematic *in situ* strength variation of the girder in the present investigation is based on the UPV measurements through the girder web, the equivalent core strengths for Mikhailovsky and Scanlon's data were determined using Equation (8C.2).

It is interesting to note that, for a given ultrasonic pulse velocity value, the core strength predicted using Equation (8C.1) is considerably smaller than that predicted using either Equation (8C.2) or Equation (8C.3). While the cores from both investigations had similar ranges of strength, the UPV values by Tso and Zelman are consistently much smaller than those measured by Mikhailovsky and Scanlon.

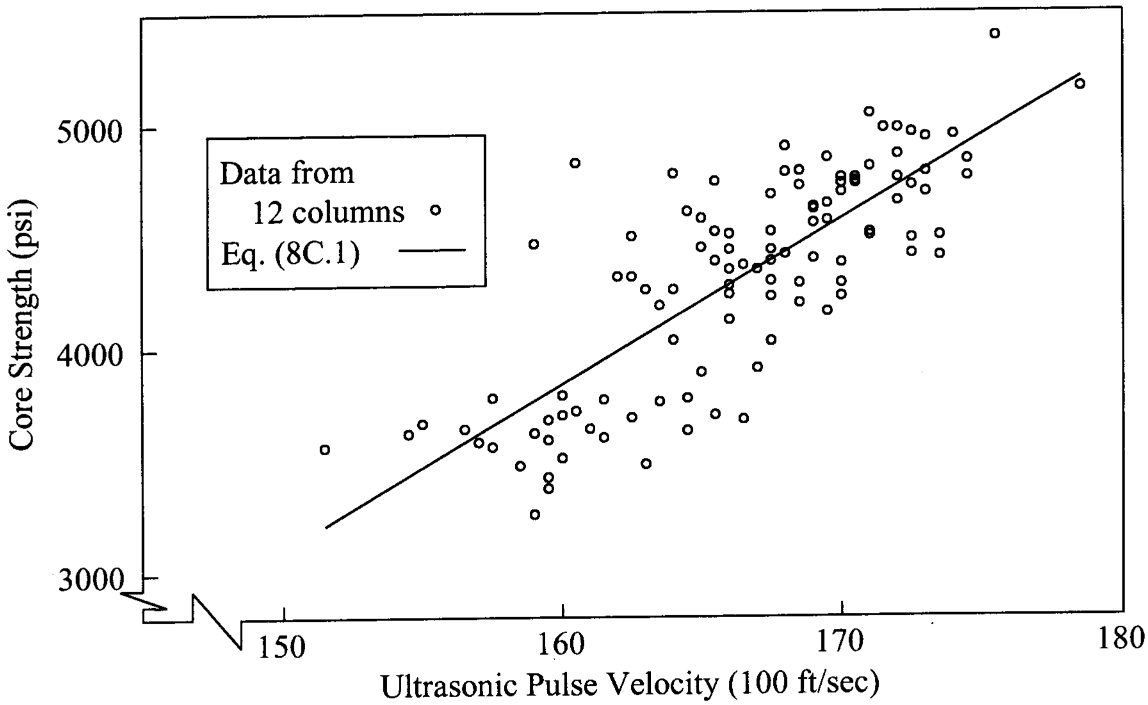

Figure 8C.1: Correlation between core strength and ultrasonic pulse velocity for Tso and Zelman (1965) data

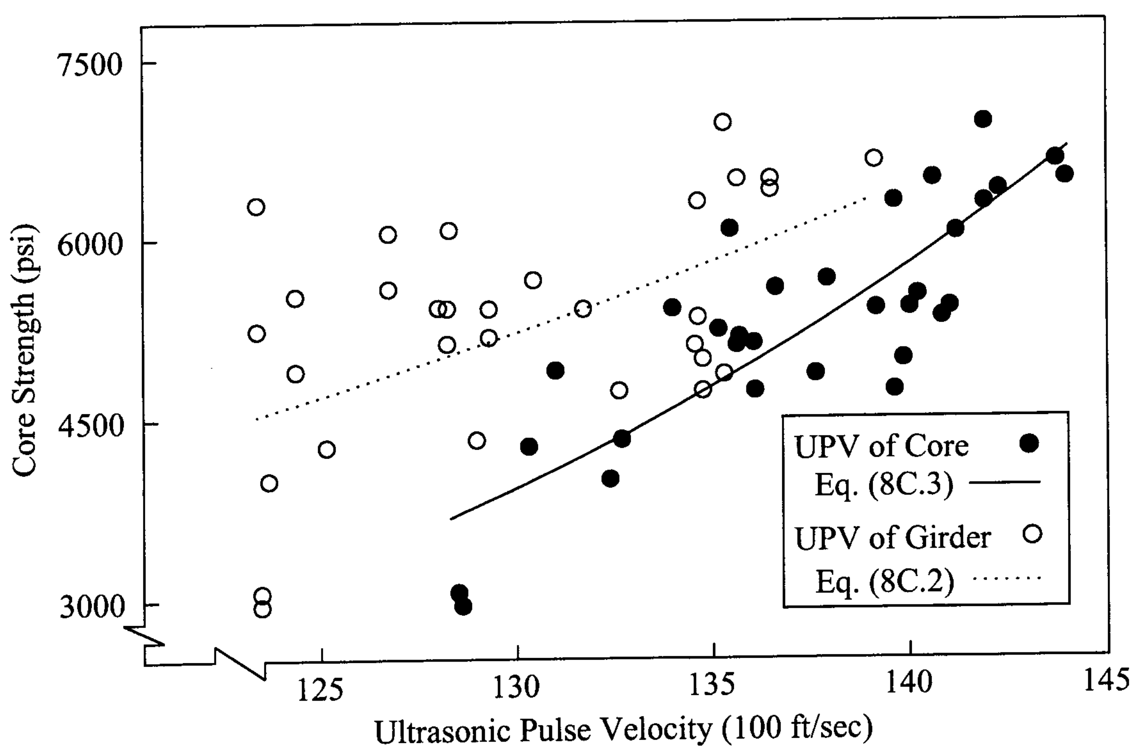

Figure 8C.2: Correlation between core strength and ultrasonic pulse velocity for Mikhailovsky and Scanlon (1985) data