Optimal Traffic Scheduling for Infrastructure-to-Infrastructure Communications in the Internet of Vehicles

by

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A thesis submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

in

Communications

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Abstract

There have been intensive research efforts on optimizing performance in wireless networks by adjusting certain parameters in the system. Most of the works are for decision making at individual moments, which are independent of each other. However, in many cases, a sequence of decisions need to be made in wireless networks, and the decisions are not independent, referred to as sequential decision making. The target of sequential decision making is to maximize the system efficiency in the long run.

In this research, we aim to design optimal sequential decision making process for infrastructure-to-infrastructure communications in the Internet of Vehicles, in which a roadside unit without the Internet connection needs to ask for the help of passing-by vehicles to forward its traffic to a roadside unit with the Internet connection. Upon a vehicle arrival, the source roadside unit needs to decide whether or not to forward its traffic to the vehicle. With the objective to minimize the rate of cost related to energy and delay, the sequential decision making process is modeled as optimal stopping problems.

Three cases related to the delay cost functions are considered: a) hard delay bound; b) soft delay bound; and c) multi-step soft delay bound. Threshold-based optimal stopping rules are derived for each of the above three cases.

Acknowledgements

First and foremost, I would like to express my immense gratitude for the support and guidance from my supervisor and academic model, Dr. Hai Jiang. I want to thank you from the bottom of my heart for continuously challenging me to think critically and reach my highest potential. This journey has been really long and could not been finished without your unwavering support.

I would like to acknowledge the members of my defense committee, Dr. Chintha Tellambura, Dr. Majid Khabbazian, Dr. Xihua Wang and Dr. Lian Zhao, for their time and input. Their suggestion, questions, and comments have largely improved the quality and the presentation of my works.

I was lucky to have amazing friends and office mates including Dr. Jie Gao, Dr. Rongfei Fan, Dr. Zhongjiang Yan, Dr. Zhou Zhang, Qiang Guo and many others throughout this journey. Becoming friends with them is wonderful memory of lifetime.

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Glossary of Terms

Acronyms	**Definition**
CDF	Cumulative Distribution Function
DSRC	Dedicated Short Range Communication
IoT	Internet of Things
IoV	Internet of Vehicles
ITS	Intelligent Transportation System
I2I	Infrastructure to Infrastructure
OSP	Optimal Stopping Problem
PDF	Probability Density Function
RSU	Roadside Unit
VANET	Vehicular Ad Hoc Networks
V2V	Vehicle to Vehicle
V2I	Vehicle to Infrastructure

Chapter 1

Introduction

1.1 Internet of Things

Electronic devices are getting more compact, faster, cheaper and more ubiquitous than ever before. Everyday consumer products such as cellphones, security monitors, thermostats, home appliances, garage door openers and much more are connected among themselves and to the Internet. Those data-spouting devices are embedded with sensors that gather, store and analyze data and the data can be uploaded to the Internet for further processing. As anything from home thermostats to heart rate monitors to security cameras on the street that can be connected are connecting to the Internet, we move towards the "Internet of Things" (IoT). IoT is already changing how we live and work [1, 2, 3, 4, 5, 6, 7].

An application of IoT is smart home, in which the appliances can be monitored and controlled effectively. Many technical companies have ventured into residential industry by developing IoT applications [8]. Google Nest learning thermostat has been adopted by more and more home owners to reduce utility bills. The nest learns your pattern of temperature setting overtime, programs itself, and can be controlled from your phone to reduce energy consumption. Apple also releases HomeKit, a platform on which smart-home app developers could develop software to increase energy efficiency and bring convenience to dwellers.

1.2 Internet of Vehicles

Vehicles and communication are indispensable parts of most people's daily life. The number of vehicles currently in use in the world is estimated to be approximately one billion [9] and will be two billion by 2035 [10]. A vehicular ad hoc network (VANET) [11] is proposed to support communication between people and vehicles using technologies such as dedicated short range communication (DSRC, also known as IEEE 802.11p), directional medium access control, and vehicular cooperative media access control [12]. VANET has the potential to make a safe, intelligent, and convenient transportation system [13, 14, 15, 16, 17, 18, 19, 20]. However, VANET does not attract much commercial interests [21] because of the issues related to pure ad-hoc networking [22], unreliable Internet access [23], to be upgraded personal devices [24] and unavailable cloud computing capabilities[25, 26].

The Internet of Vehicles (IoV) [27, 28, 29] is a new concept coming from the integration of the IoT and VANET. Many applications of IoV have been investigated [30, 31]. One major application area is Intelligent Transport System (ITS) which could improve traffic safety and efficiency[32, 28, 33, 34, 35]. The system of improving traffic safety is referred as the collision prevention system [36, 37, 38]. The system continuously monitors the vehicles and traffic conditions. When it detects any emergency situation such as traffic jam, accidents and bad road conditions, it immediately broadcasts warning messages to other vehicles. This will make daily commute safer. Global traffic efficiency [39, 40] can be improved by enabling connected vehicles to automatically send real-time traffic data to a traffic control center which coordinates the whole network of vehicles.

Another application of IoV is to serve smart cities [41, 42, 43, 44, 45]. For example, vehicles can serve as carrier to carry sensing data from locations in a city to a data center [46]. In work [47], vehicular sensor networks are used to monitor metropolitan air qualities and communicate the collected data to the public. In work [48], efficient waste collection is achieved in the context of smart cities, where high capacity waste trucks are mobile depots and waste bins are optimally placed for access.

1.3 Research motivation and thesis contributions

The increasing adoption of IoT leads to massive deployment of small cost-efficient sensors. Those sensors make is possible to perform environmental monitoring and information collection in a large-scale area, and thus, are essential for future smart cities. In order to minimize the deployment cost, those sensors are usually equipped with a limited energy source such as battery or solar energy. A primary concern in IoT is efficient power management to make sure those sensors will not run out of power soon and so that the network lifetime could be improved. In this research we aim to find power efficient data transmission strategies for road side units to facilitate the establishment of smart cities.

Many applications of IoV have been proposed base on three forms of vehicular communications. In vehicle-to-vehicle (V2V) communications [49, 50], vehicles can share with neighboring vehicles information such as its moving direction, its velocity, and its activities (e.g., braking, lane change, etc.). In vehicle-to-infrastructure (V2I) communications, roadside units (RSU) can serve as access points to provide Internet services to passing vehicles. In infrastructure-to-infrastructure (I2I) communications, vehicles can forward traffic from one RSU to another RSU if the two RSUs do not have direct connection between them

[51, 52].

It is in general assumed that all the RSUs are connected to a backbone network via wired links. However, in some cases, some RSUs may not be connected to the backbone network. For example, in remote areas, it is costly to connect all RSUs to the backbone network. Those *remote RSUs* (i.e, RSUs without backbone connection) need to send their data traffic to *central RSUs* (i.e., RSUs with backbone connection), and then the central RSUs forward the data traffic to the backbone network. A cost-effective method to achieve I2I communications is to use passing-by vehicles, which can carry messages from the remote RSUs and forward them to central RSUs on their path.

For such vehicle-aided I2I communications, the energy consumption of the remote RSUs is an issue. This is because those RSUs are usually deployed in remote areas, and thus, they do not have constant power supply. So the remote RSUs are often equipped with batteries, and the batteries can get recharged or renewed after a relatively long time (for example, a few months) [53]. Energy efficient data forwarding in VANETs has been investigated recently in the literature [54, 55, 56, 57, 58]. The work in [54] targets at energy consumption minimization for an RSU. A scheduling scheme is provided, which favors passing-by vehicles with higher velocity and/or shorter distance to the RSU. The work in [55] considers delivery of packets from a source to a destination by using relaying service of other nodes. A delay bound is set for each packet. If a packet cannot be delivered to the destination within the delay bound, the packet will be discarded. To maximize the packet delivery probability under an energy consumption constraint, it is shown that the threshold dynamic policy is optimal. The works in [56] and [57] take into account the energy for node discovery process as well as energy for information transmission, for two-hop routing and epidemic routing, respectively. Transmission policy is well designed such that packet delivery probability is maximized. The work in [58] proposes that the RSU-to-vehicle scheduling can be combined with vehicle-to-vehicle forwarding, which can largely lower the energy cost of the RSU.

Although the vehicle-aided I2I communications can tolerate a certain level of delay, timely delivery is still preferred [59, 60, 61]. In general, the total delay of a data packet at a source RSU consists of two components: the queuing delay at the source RSU, and the transit delay (i.e., the time needed by a helping vehicle to travel to the destination RSU to deliver its carried data traffic). A tradeoff exists between the queuing delay and the transit delay. To minimize the transit delay, the source RSU should wait for fast vehicles, which may result in a larger queuing delay. On the other hand, to minimize the queuing delay, the source RSU should pick up the first passing-by vehicle, which may lead to larger transit delay. The work in [62] considers finite-size traffic case and infinite-size traffic case. For the former case, the source file at the source RSU has a number of packets, and the time duration needed to deliver all packets is minimized by using a Markov decision process. For the latter case, the source file has an infinite number of packets, and the average delay of a packet is minimized by a Markov decision process. Both queuing delay and transit delay are considered and well balanced. The work in [63] assigns each passing-by vehicle a pick-up probability, which favors faster vehicles. For a vehicle, its assigned probability is the probability that its arrival moment at the destination is earlier than its next vehicle‘s expected arrival moment at the destination.

The work in [64] is the first research in the literature that considers both energy consumption and queuing delay. However, it does not consider the transit delay.

To fill this research gap, in this thesis, we investigate optimal traffic scheduling in vehicle-aided I2I communications, taking into account energy consumption, queueing delay, as well as transit delay.

The contributions of the thesis work are summarized as follows.

- In Chapter 3, a *hard delay bound* is used, which means that if an information unit at the source RSU cannot be delivered to the destination RSU within the delay bound, the information unit is considered useless and thus, is dropped at the source RSU. Cost is assigned to energy consumption as well as packet dropping (due to delay bound violation). An optimal scheduling scheme is formulated, which minimizes the rate of cost (i.e., the average cost per unit of time). To solve the problem, we first find the optimal rule when the waiting time at the source RSU is below a specific value. Then based on the result and with some mathematical manipulations, we find the optimal rule when the waiting time at the source RSU is above the specific value. The derived optimal rule can be implemented with negligible complexity.

- In Chapter 4 we consider a *soft delay bound*. In specific, it is desired that any information unit is delivered within the delay bound. However, when an information unit cannot be delivered within the delay bound, the information unit is considered to be partially useful and is still delivered, and a cost is charged for the delay bound violation.[1] When it is impossible to deliver on time the buffered information at the source RSU, then the source

[1] The rationale behind using soft delay bound in vehicle-aided I2I communications is as follows. One typical application of the source RSU is to serve as a gateway for a wireless sensor network that monitors the environments (fire detection, animal tracking, etc.) in remote areas. The sensing data of the wireless sensor network are sent to the source RSU and are subsequently delivered by the source RSU to a destination RSU with backbone connection. Then the destination RSU sends the data to the data center of the wireless sensor network. It is preferred that the sensing data are delivered within a delay bound. If the sensing data are beyond the delay bound, they still have some value (for example, for later historical studies), and will still be delivered. A similar soft delay bound model was used in [65, 66].

RSU is forced to select the next vehicle arrival to help forward its buffered information, referred to as *forced stop*. An optimal stopping problem is formulated. Due to the forced stop, methods used to solve traditional optimal stopping problems, including the method used in Chapter 3, do not work. To address this challenge, we develop a completely new method to solve the formulated problem. We first eliminate a set of non-optimal stopping rules. After finding some special features of the remaining rules, we theoretically derive optimal rule for the formulated problem. Interestingly, the optimal rule has a conditional pure-threshold structure, i.e., before a forced stop, the source RSU is optimal to transmit to a passing-by vehicle if the queuing delay is more than a threshold, conditioned on that the sum of the queuing delay and the transit delay is below the delay bound. We also theoretically derive the value of the threshold. The conditional pure-threshold structure makes the derived optimal rule easy to implement in a real network.

- In Chapter 5, we consider a more general case when the delay cost function is a multi-step case and the probability distribution function of the vehicle speed can be any distribution. We transform the originally formulated problem into multiple sub-problems. Then we solve the sub-problems sequentially, from the last sub-problem to the first sub-problem. The derived optimal rule has a multiple-threshold structure.

Chapter 2

Optimal Stopping Problem

In this research, we model the problem of optimal traffic scheduling between two roadside units in a vehicular network as a discrete time optimal stopping rule problem. This chapter gives an introduction to the optimal stopping problem and an example is given for readers to better understand optimal stopping problems. The monotone optimal stopping problem is explained at the end of this chapter.

2.1 Definition of optimal stopping problem

Optimal stopping problem is a stochastic optimization tool that is very powerful in dealing with sequential decision making and sequential analysis in statistics. The origin of optimal stopping problem is due to Wald's sequential analysis [67]. In sequential analysis, the number of observations is not fixed in advance but depends on the results of the observations. The natural question is "when is it optimal to stop" so as to get maximal reward (expected) or minimal cost (expected).

In an optimal stopping problem [68, 69], we observe a sequence of random variables $X_1, X_2, \ldots$. After observing n random variables $X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n$, we need to decide whether we should stop. If we choose to stop, we get the reward $y_n(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n)$. If we choose to continue, we will observe the realization of X_{n+1}. We define y_0 to be the reward if we do not take any observations and $y_\infty(X_1 = x_1, X_2 = x_2, \ldots)$ to be the reward if we never stop observing. The objective is to choose a moment to stop such that expected reward (or cost) is maximized (or minimized). A strategy of how to choose the stopping time is called a stopping rule.

The stopping rule $\boldsymbol{\psi}$ is a sequence of decision functions

$$\boldsymbol{\psi} = \{\psi_0, \psi_1(x_1), \psi_2(x_1, x_2), \ldots\}$$

where $\psi_n(x_1, x_2, \ldots, x_n)$ is 1 or 0. If we have observed the first n random variables as $X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n$, we need to decide whether we should stop at the nth observation. If $\psi_n(x_1, x_2, \ldots, x_n) = 1$, we stop at the nth observation. Otherwise, we continue to observe the $(n+1)$th random variable X_{n+1}. We also define ψ_0 to represent whether we should make any observations at all, i.e. if $\psi_0 = 1$ we will make no observations and if $\psi_0 = 0$ we will observe the first random variable X_1.

With the above sequences of decision functions and observations of random variables $(X_1, X_2, \ldots, X_n)$, we define the stopping time N to be the time when the stopping occurs. If $\psi_n(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) = 1$, then we have $N = n$ after we have observed $x_1, x_2, \ldots, x_n$ and the reward is $y_n(x_1, x_2, \ldots, x_n)$. If $N = \infty$, then stopping will never occur.

Our objective is to choose a stopping rule $\boldsymbol{\psi}$ in order to maximize the expected reward $V(\boldsymbol{\psi})$:

$$\begin{aligned} V(\boldsymbol{\psi}) =& \mathbb{E}\left[\left[y_N(X_1, X_2, \ldots, X_N)\right]\right] \\ =& \mathbb{E}\left[\sum_{i=0}^{\infty} \psi_i(X_1, X_2, \ldots, X_i) y_i(X_1, \ldots, X_i)\right]. \end{aligned}$$

2.2 An example

A die rolling game example is given in [70] to illustrate the concept of stopping problem. In the die rolling, if the player gets a 6, the player is forced to stop the game and earns nothing. If the player gets $\ell = 1, 2, ..., 5$, the player has two choices: 1) to roll again (with no reward for the current roll), or 2) to stop the game and get reward of ℓ dollars. The die rolling game continues until the player is forced to stop (i.e., at a roll of 6) or chooses to stop (i.e., at a roll of 1, 2, ..., or 5). The question is: at each roll, what is the optimal decision strategy of the player such that his/her expected reward can be maximized? As shown in [70], at a roll of ℓ, the optimal strategy is

- If ℓ is 1 or 2, the player should roll again;
- If ℓ is 3, the player may select to roll again or to stop, as the two choices have the same expected reward;
- If ℓ is 4 or 5, the player should stop and earn ℓ dollars;
- If ℓ is 6, the player is forced to stop and earns 0 dollar.

2.3 Monotone optimal stopping rule problem

The monotone stopping rule problem is first introduced in [71]. Let $(\Omega, \mathscr{F}, \mathbb{P})$ be a probability space with points $\omega \in \Omega$ and $\left(y_n(X_1, X_2, \ldots, X_n), \mathscr{F}_n\right), n = 1, 2, \ldots,$ be an integrable stochastic process: $y_n(X_1, X_2, \ldots, X_n)$ is a sequence of random variables, $\mathscr{F}_n$ is a sequence of σ-algebras with $\mathscr{F}_n \subset \mathscr{F}_{n+1} \subset \mathscr{F}$, $y_n(X_1, X_2, \ldots, X_n)$ is measurable with respect to $\mathscr{F}_n$ and $\mathbb{E}\left[y_n(X_1, X_2, \ldots, X_n)\right]$ exists with $-\infty \leq \mathbb{E}\left[y_n(X_1, X_2, \ldots, X_n)\right] \leq \infty$.

Definition 1. *Let*

$$B_n = \left\{ y_n(X_1, X_2, \ldots, X_n) \leq \mathbb{E}\left[y_{n+1}(X_1, X_2, \ldots, X_n, X_{n+1}) | \mathscr{F}_n \right] \right\}. \tag{2.1}$$

An optimal stopping problem is monotone *if*

$$B_0 \subset B_1 \subset B_2 \subset \ldots \quad a.s. \tag{2.2}$$

An important property of the monotone stopping rule problem is that the one-step look-ahead stopping rule is optimal. The one-step look-ahead stopping rule is also known as myopic stopping rule and it is defined as:

$$N_1 = \min \left\{ n \geq 0 : y_n(X_1, X_2, \ldots, X_n) \geq \mathbb{E}\left[y_{n+1}(X_1, X_2, \ldots, X_n, X_{n+1}) | \mathscr{F}_n \right] \right\}.$$

It asks for stopping when the return for stopping is not less than the expected return for stopping at the next observation. When it asks for skipping at nth observation, it is optimal to continue regardless whether the optimal stopping problem is monotone or not. When it asks for stopping at nth observation, it is optimal to stop if the optimal stopping problem is monotone.

The set B_n in the definition of the monotone stopping rule problem is the set on which the one-step look-ahead rule decides to stop at the nth observation. The condition $B_n \subset B_{n+1}$ means that if the one-step look-ahead rule decides to stop at the nth observation, then it will also decide to stop at the $(n+1)$th observation, regardless of the value of X_{n+1} a.s. Similarly, $B_n \subset B_{n+1} \subset B_{n+2} \subset \ldots$ means that if the one-step look-ahead rule decides to stop at the nth observation, then it will decide to stop at all subsequent observations a.s.

The optimal stopping problem in Section 2.2 is a monotone problem and the one-step look-ahead rule is optimal, i.e. to continue when $k = 1, 2$, and otherwise to stop.

Chapter 3

I2I Traffic Scheduling with a Hard Delay Bound[1]

Traffic scheduling problems in vehicular delay tolerant networks (VDTNs) have been attracting increasing research interests in the past years. One type of VDTN is installed in less-populated remote areas, which includes a number of roadside units (RSUs), and only a limited number of RSUs have connection to backbone networks [72]. The isolated RSUs (i.e., the RSUs without backbone network connection) are deployed to serve as gateways for sensor networks (for example, sensor networks for monitoring environment or wildlife [73, 74, 75, 76]) in less-populated remote areas. Since it may be costly to set up direct communication connections from the isolated RSUs to backbone networks, passing-by vehicles may provide a solution: passing-by vehicles can help forward traffic (e.g., sensed data) from isolated RSUs to RSUs that have connection to backbone networks [77]. When a vehicle arrives at a source RSU that is isolated, the source RSU may send its traffic to the vehicle, and then the vehicle stores the traffic in its local buffer and forwards the traffic to a destination RSU with backbone network connection when the vehicle arrives at the destination RSU [78]. The destination

[1] A version of this chapter has been published in IEEE Trans.Veh. Technol., 64: 1079-1094 (2015).

RSU then forwards (through backbone networks) the traffic to a data center that processes the data.

For such RSU-to-RSU communication in a VDTN, in general, delay (the duration from the moment that the traffic arrives at the source RSU until the moment that the traffic is delivered to the destination RSU) can be tolerated to a certain level. However, it is still desirable that the traffic is delivered before it becomes expired [59, 60, 61]. In other words, there is a delay bound associated with each traffic unit. In RSU-to-RSU communication, the delay of the traffic is composed of two parts: Queueing delay at the source RSU (from the moment that the traffic arrives until the moment that the traffic is sent to a passing-by vehicle) and transit delay (from the moment that the passing-by vehicle receives the traffic until the moment that the vehicle arrives at the destination RSU and forwards the traffic). There is a tradeoff between these two delay components. If each passing-by vehicle is used to forward traffic, then we can keep the smallest queueing delay; however, low-speed vehicles may make the transit delay very large. On the other hand, if only high-speed vehicles are selected to help, then we can keep small transit delay; however, the queueing delay might be out of control since it might take a long time for the next high-speed vehicle to arrive. Therefore, the source RSU should strike a balance between queueing delay and transit delay. In [62], the traffic to be sent is either a finite-size file or an infinite-size file. For the first case, the finite-size file is partitioned into segments, and the tool of Markov decision process is used so as to minimize the time needed by the destination RSU to receive all segments of the file. For the second case, there are an infinite number of segments in an infinite-size file, and thus, the tool of Markov decision process is used to achieve the minimum average delay of a segment. It can be seen that both queueing delay and transit delay are considered in [62]. In [63], when the source RSU is waiting for vehicles, its incoming traffic is aggregated into a bundle. The probability that a passing-by vehicle arrives at the destination later than the next vehicle does is calculated, and upon arrival of a vehicle, the source RSU transmits to the vehicle with that probability.

For the isolated RSUs in a VDTN, it is also likely that they are powered by batteries or renewable energy (for example, solar power). Therefore, energy consumption of the RSUs is another important performance measure in VDTNs [54, 55, 56, 57]. In [54], the objective is to design a scheduling algorithm that achieves minimum energy consumption of the RSUs while satisfying passing-by vehicles' communication requirement. It is shown that vehicles that are closer to the RSU and with higher speeds should be picked up. In [55], each packet has a delay bound deadline, and packets that have not been received at the destination by the deadline will be discarded at the source node. The source node probabilistically sends its traffic to a vehicle, and the optimal probability to send traffic (i.e., the probability that achieves the highest successful delivery probability under a constraint of energy consumption) is obtained. In [56] and [57], in addition to energy used for data transmission, energy used by the source RSU to detect a passing-by vehicle is also taken into account. By distributing available energy for vehicle detection and data transmission, the RSU maximizes the probability of successful traffic delivery prior to a delay bound, for two-hop routing case in [56] and epidemic routing case in [57].

Both queueing delay and energy consumption are taken into account in [64]. Cost is charged 1) for consumed energy to send information, and 2) when there is information loss due to delay bound violation. An optimal scheduling strategy, in which the rate of cost (i.e., the average cost per unit time) is minimized, is derived, and is shown to have a pure-threshold structure, that is, a passing-by vehicle should be picked up if the queueing delay exceeds a threshold whose value can be numerically calculated off-line.

The delay considered in [64] is the queueing delay at the source RSU. However, in a real application, we are more interested in the total delay including the queueing delay and the transit delay. The transit delay depends on the speeds of the passing-by vehicles. As aforementioned, there is a tradeoff between queueing delay and transit delay. So the source RSU's scheduling decision (i.e., upon a vehicle arrival, whether to stop at this vehicle and transmit, or continue to wait for other vehicles) should depend on both the queueing delay and the speed of the passing-by vehicle, to be addressed in this work. Another issue of the work in [64] is that in a forced stop (i.e., when the delay of any traffic unit in the queue of the source RSU is more than the delay bound, the RSU is forced to use the coming vehicle), a fixed penalty is charged regardless of the amount of traffic units whose delay is more than the delay bound. This may not be practical, since it is more reasonable to set up the penalty proportional to the amount of traffic units whose delay is more than the delay bound. This issue is to be addressed in this work. In specific, in this work, costs are assigned for both energy consumption and traffic loss: An amount of cost is associated with each consumed energy unit, and an amount of cost is charged if a traffic unit cannot be delivered (by the selected vehicle) before its delay bound and thus is discarded. We derive an optimal strategy for the source RSU to select passing-by vehicles to help deliver its traffic.

The remainder of this chapter is organized as follows. Section 3.1 presents the system model and problem formulation. Section 3.2 derives an optimal strategy. Section 3.3 shows performance evaluation of the derived optimal strategy. Section 3.4 investigates the effect of wireless transmission errors, followed by concluding remarks in Section 3.5. Appendices include proofs of the theorems. A list of notations used is given in Table 3.1.

3.1 System model and problem formulation

We consider a source RSU, which does not have backbone connection, and a destination RSU with backbone connection. The distance from the source RSU to the destination RSU is D (meters). Traffic arrives at the source RSU at a constant rate r (bits/second) (for example, when the source RSU is used as a gateway of underlying wireless sensor networks, the collected traffic by the source RSU is likely to be with almost constant rate). Each traffic unit should be sent to the destination RSU by a maximal delay of K (seconds) from the moment when the traffic unit arrives at the source RSU. No direct connection is assumed between the two RSUs. Therefore, passing-by vehicles are selected by the source RSU to forward its accumulated traffic to the destination RSU. At the source RSU, the traffic is kept buffered until a passing-by vehicle is selected, at which moment those accumulated traffic units whose delay (including the queueing delay and the transit delay, i.e., the time needed by the vehicle to arrive at the destination RSU) is less than K are sent to the selected vehicle, and other traffic units are discarded by the source RSU. Subsequently the vehicle forwards the traffic to the destination RSU when it arrives at the destination RSU. Assume that wireless transmission between RSUs and vehicles is error free (effects of wireless transmission errors will be investigated in Section 3.4).

Assume that the starting point of the process of observation is $T_0 = 0$ (second). At the source RSU, the arrival instant of the nth vehicle (called vehicle n) is T_n (second). Then $X_n = T_n - T_{n-1}$ ($n = 1, 2, ...$) is the vehicle inter-arrival duration. Since some empirical measurement [79] has shown that the vehicle

Symbol	Meaning
A_n	Amount of discarded traffic if vehicle n is selected
a, b	The smallest, largest transit delay
b^*	Convergence point of sequence $\{b_i\}_{i=1,2,...}$ $(b_1 = \phi(b); b_i = \phi(b_{i-1}), i \geq 2)$
B	Cost of discarding one traffic unit
C	Cost of energy used to transmit one traffic unit
D	Distance between source RSU and destination RSU
$F_G(g)$	Cumulative distribution function of transit delay
$F_V(v)$	Cumulative distribution function of vehicle speed
$f_G(g)$	Probability density function of transit delay
$\mathscr{F}_n$	Information of $T_1, T_2, ..., T_n$ and $G_1, G_2, ..., G_n$
G_n	The transit delay of vehicle n
K	Delay bound
$N^\dagger(\lambda)$	Optimal stopping rule of Problem (3.7)
P	Transmit power level
R	Data transmission rate from source RSU to a vehicle
r	Incoming traffic rate at the source RSU
T_n	The arrival time of vehicle n
V_n	Speed of vehicle n
$V(\lambda)$	Minimal cost of Problem (3.7)
$v_{\max}$	Maximum vehicle speed
$v_{\min}$	Minimum vehicle speed
W	Cost of energy per Joule
X_n	Duration between arrivals of vehicles $n-1$ and n
Y_n	Total cost of using vehicle n (given in (3.2))
$Z_n(\lambda)$	Cost function for Problem (3.7)
μ	Average vehicle inter-arrival duration
κ	Overhead duration for communication
ω	Outcome of $Y_1, Y_2, Y_3, ...$

Table 3.1: Used Notations in Chapter 3

inter-arrival durations at a roadside point follow independent and identically distributed (i.i.d.) exponential distributions, similar to [62, 63, 80, 81, 82, 83], we assume X_n's are independent and follow an exponential distribution with parameter μ, i.e., $\mathbb{E}[X_n] = \mu$ (seconds). Here $\mathbb{E}[\cdot]$ means expectation. In other words, vehicle arrivals at the source RSU follow a Poisson process with vehicle arrival rate being $1/\mu$. Similar to [62, 63, 82], we assume that, upon a vehicle arrival, the source RSU can detect the speed of the vehicle and the vehicle's speed does not change from the source RSU to the destination RSU. Denote V_n as the speed of the nth vehicle. Suppose the vehicle speeds are i.i.d. random variables with cumulative distribution function (CDF) being $F_V(v)$ for $v \in [v_{\min}, v_{\max}]$, where $v_{\min}$ (m/s) and $v_{\max}$ (m/s) are the minimum and maximum speeds, respectively. The transit delay of vehicle n is $G_n = D/V_n$ (seconds). So G_n's ($n = 1, 2, ...$) are i.i.d. random variables with CDF $F_G(g) = 1 - F_V\left(D/g\right)$ for $g \in [a, b]$, where $a \triangleq D/v_{\max}$ (seconds) and $b \triangleq D/v_{\min}$ (seconds) are the smallest and the largest transit delay, respectively.

Upon arrival of a vehicle, the source RSU has two choices: To skip the vehicle and continue to wait for future vehicles (referred to as *continue* in the sequel), or to stop waiting (referred to as *stop* in the sequel) and send its accumulated traffic (that can meet the delay bound requirement) to the vehicle by a constant rate R (bits/second) and a transmit power level P (Watt). Consider that vehicle n is selected to help (which implies that the source RSU does not stop at vehicles $1, 2, ..., n-1$). The amount of accumulated traffic (from moment 0 to moment T_n) is rT_n. Let A_n denote the amount of traffic units that cannot meet their delay requirement (i.e., when queueing delay plus transit delay is more than the delay bound K)[2] and thus are discarded. A_n is given as

$$A_n = \begin{cases} rT_n & \text{if} \quad K < G_n \\ r(T_n + G_n - K) & \text{if} \quad G_n \le K < T_n + G_n \\ 0 & \text{if} \quad K \ge T_n + G_n \end{cases} \tag{3.1}$$

for the following reason. When $K < G_n$, even the transit delay G_n alone is more than the delay bound K, so all accumulated traffic with amount rT_n will be discarded. When $K \ge T_n + G_n$, it means that even the oldest traffic unit in the source RSU's buffer can meet the delay bound (for the oldest traffic unit, its queueing delay is T_n, and its transit delay is G_n). So no traffic will be discarded. When $G_n \le K < T_n + G_n$, considering that vehicle n needs G_n duration to arrive at the destination RSU, only traffic accumulated in the past $(K - G_n)$ duration can meet delay bound requirement. So the amount of discarded traffic is $rT_n - r(K - G_n) = r(T_n + G_n - K)$.[3]

Similar to the weighted cost structure in [84, 65, 85], if vehicle n is selected, we have cost values associated with the energy consumption and traffic loss.

- Energy: The total energy consumption for the data transmission from the source RSU to the vehicle is $P(rT_n - A_n)/R$ (Joule). By letting W (unit of cost per Joule) denote the cost weight of energy consumption, the cost of the energy consumption of transmitting data is given by $WP(rT_n - A_n)/R$. For the data transmission, there is also communication overhead to setup the transmission (for example, the information exchanges of request-to-send [RTS] and clear-to-send [CTS]) and to acknowledge the transmission.

[2]The total delay is the summation of the queueing delay and the transit delay G_n. The transmission duration from the source RSU to the vehicle is not considered here, since the transmission duration is actually included in the transit delay.

[3]Considering that the delay bound K of a VDTN is usually not more than a few thousand seconds (e.g., 7200 seconds as used in [75]) and that the data coming rate r at the source RSU is usually not more than a few hundred bps [75, 76], it is reasonable to assume that all the traffic that is not discarded, with amount given as $rT_n - A_n$, can be transmitted from the source RSU to the passing-by vehicle within their contact time.

Assume the overhead also uses power level P, and the total duration of the transmission of the overhead is κ (seconds). Then the energy cost for communication overhead is $WP\kappa$. So the total cost for energy consumption is $WP(rT_n - A_n)/R + WP\kappa = C(rT_n - A_n) + WP\kappa$, where $C \triangleq WP/R$ is the energy cost of sending one traffic unit.

- Traffic loss: When some traffic units are discarded, a penalty of B (unit of cost) is charged for each discarded traffic unit. So the total cost for traffic loss is BA_n.

The total cost of using vehicle n, denoted Y_n, is thus given as[4]

$$Y_n = WP\kappa + C(rT_n - A_n) + BA_n. \tag{3.2}$$

Y_n's ($n = 1, 2, ...$) are random variables.

For the stopping problem, consider that we observe random variables $Y_1, Y_2, \ldots$, and get one realization of them as $y_1, y_2, \ldots$, corresponding to the costs of using vehicle $1, 2, \ldots$, respectively. In probability theory, a such realization, denoted as ω, is referred to as *one outcome*. And the sample space, denoted as Ω, is the set of all outcomes. An event is defined as a subset of Ω, which is assigned a probability.

For example, consider that we have observed the value of the first random variable Y_1 as y_1. This fact can be defined as an event $\mathcal{E}_1 = \{\omega : Y_1 = y_1\}$. In other words, the event is the set of outcomes that have the first observed value as y_1. We can write $Y_1(\omega) = y_1$ for $\omega \in \mathcal{E}_1$; and $Y_1(\omega) \neq y_1$ for $\omega \in \Omega \backslash \mathcal{E}_1$.

[4]If the source RSU selects vehicle n to help, it is optimal to transmit all $rT_n - A_n$ traffic to vehicle n, for the following reason. Assume an amount x of the $rT_n - A_n$ traffic is not transmitted to vehicle n. If the amount x traffic is transmitted to vehicle n, the extra cost is given as Cx. However, for any future vehicle to carry the amount x traffic, the cost is at least Cx, considering that all or part of the x amount of traffic may become expired when waiting for future vehicles.

After knowing $Y_1 = y_1$ or the event $\mathcal{E}_1$ happens, if the source RSU decides to stop at vehicle 1, the final cost of the stopping problem is y_1; otherwise, the source RSU continues to observe $Y_2, Y_3, \ldots$, which are unobserved random variables conditioned on the event $\mathcal{E}_1$. What we aim to achieve is to find the right moment to stop so as to minimize the cost.

Consider a *stopping rule* such that for each outcome $\omega \in \Omega$, the moment to stop is denoted $N(\omega)$ where $N(\omega) \in \{1, 2, \ldots\}$ is the index of the vehicle that the source RSU stops at. $N(\omega)$ is also used to denote the stopping rule. We can think of a stopping rule as a function in the domain Ω, which maps each $\omega \in \Omega$ to one positive number. So $N(\omega)$ (a stopping rule) is a random variable and is often written in short form as N. A simple example of stopping rules is $N = 1$ ($N(\omega) = 1$) for $\omega \in \Omega$, i.e., always stop at the first vehicle. To solve an optimal stopping problem is to find an optimal stopping rule that maps each outcome to a stopping moment to minimize the cost.

In this thesis, we consider stopping problems that are repeated in time. For the moment when the source RSU completes information exchange with a selected vehicle, this moment is considered as the end of the old stopping problem and also the beginning of a new stopping problem. If the source RSU stops at vehicle $N(\omega)$, whose cost is $Y_{N(\omega)}(\omega)$ and stopping time is $T_{N(\omega)}(\omega)$, then starting from the moment $T_{N(\omega)}(\omega)$, a new stopping problem begins with starting point denoted $T_0 = 0$ and the next arrival vehicle called vehicle 1. For such repeated stopping problems with stopping rule $N(\omega)$, denote the independent outcomes as $\omega_1, \omega_2, \ldots$, etc. Then we have i.i.d. stopping vehicle indices $\{N(\omega_1), N(\omega_2), \ldots, N(\omega_n), \ldots\}$, i.i.d. stopping moments $\{T_{N(\omega_1)}(\omega_1), T_{N(\omega_2)}(\omega_2), \ldots, T_{N(\omega_n)}(\omega_n), \ldots\}$, and i.i.d. costs $\{Y_{N(\omega_1)}(\omega_1), Y_{N(\omega_2)}(\omega_2), \ldots, Y_{N(\omega_n)}(\omega_n), \ldots\}$. The total cost is $\sum_i Y_{N(\omega_i)}(\omega_i)$ and the total amount of traffic that arrives at the source RSU is $\sum_i rT_{N(\omega_i)}(\omega_i)$.

So the rate of cost (average cost per traffic unit) is $\sum_i Y_{N(\omega_i)}(\omega_i)/\sum_i rT_{N(\omega_i)}(\omega_i)$, which converges to $\mathbb{E}\left[Y_{N(w)}(w)\right]/\mathbb{E}\left[rT_{N(w)}(w)\right]$, or in short form $\mathbb{E}[Y_N]/\mathbb{E}[rT_N]$, by the law of large numbers [86]. Here $\mathbb{E}[\cdot]$ means expectation. Therefore, our objective is to find a stopping rule N to minimize the rate of cost $\mathbb{E}[Y_N]/\mathbb{E}[rT_N]$. The optimal rate of cost is given as

$$\begin{aligned} Q^* &\triangleq \inf_{N\geq 1} \frac{\mathbb{E}[Y_N]}{\mathbb{E}[rT_N]} \\ &= \inf_{N\geq 1} \frac{WP\kappa + C\mathbb{E}[rT_N - A_N] + B\mathbb{E}[A_N]}{\mathbb{E}[rT_N]} \\ &= \inf_{N\geq 1} \frac{WP\kappa + C\mathbb{E}[rT_N] + (B-C)\mathbb{E}[A_N]}{\mathbb{E}[rT_N]} \end{aligned} \tag{3.3}$$

where $\{N \geq 1\}$ is the set of stopping rules that observe at least one vehicle.[5]

3.2 An optimal stopping rule

We make three comments here.

- We assume $a \leq K$, because otherwise the minimum transit delay a alone is more than the delay bound K, and thus, no vehicle can meet the delay requirement of any traffic unit.

- If $b > K$, which means the maximum transit delay b is greater than the delay bound K, then those vehicles whose transit delay are greater than K should be skipped, since they cannot meet the delay bound requirement of any traffic unit. Recall that the vehicle arrivals follow a Poisson process with rate $1/\mu$. Upon a vehicle arrival, it is with probability $1 - F_G(K)$ that the vehicle is skipped. Thus, the arrivals of considered (not skipped)

[5] In the expression of the optimal rate of cost in (3.3), we use "inf" instead of "min" because for a general stopping problem, it is possible that no optimal stopping rule N exists such that $\frac{\mathbb{E}[Y_N]}{\mathbb{E}[rT_N]} = Q^*$. Nevertheless, in Section 3.2 we will show that in our Problem (3.3), there exists an optimal stopping rule that has a rate of cost being Q^*.

vehicles follow a Poisson process with rate $F_G(K)/\mu$ [87]. Then the original stopping problem is equivalent to the case when vehicle arrivals follow a Poisson process with rate $F_G(K)/\mu$, the transit delay of a vehicle is within range $[a, K]$, and the CDF of transit delay is

$$\begin{cases} 0 & \text{if} \quad g < a \\ F_G(g)/F_G(K) & \text{if} \quad a \le g \le K \\ 1 & \text{if} \quad g > K. \end{cases}$$

Therefore, without loss of generality, we assume $K \ge b$ in the sequel. And thus, from (3.1), A_n now takes the form of

$$A_n = r(T_n + G_n - K)^+ \tag{3.4}$$

in which $(x)^+ = \max(x, 0)$.

- We should have

$$B > C + \frac{WP\kappa}{r(K-a)}. \tag{3.5}$$

The reason is as follows. First we have $B > C$ (this is because, if $B \le C$, it means the cost of discarding a traffic unit is not more than the energy cost of sending a traffic unit, then all traffic units should be discarded). Next we use proof by contradiction. Suppose $B \le C + WP\kappa/\big(r(K-a)\big)$, which means $WP\kappa - r(B-C)(K-a) \ge 0$. Since the maximal amount of traffic that a vehicle can carry is $r(K-a)$,[6] then for any amount of traffic $x \in (0, r(K-a)]$ carried by a selected vehicle, we have $WP\kappa - (B-C)x \ge WP\kappa - r(B-C)(K-a) \ge 0$, which leads to $WP\kappa + Cx \ge Bx$. This means that $WP\kappa + Cx$, the cost of energy consumption in sending the carried traffic, is not less than Bx, the cost of discarding the carried traffic.

[6]Considering that the transit delay of any vehicle is not less than a, a vehicle can take at most the traffic accumulated in the past $K-a$ duration. So the maximal amount of traffic that a vehicle can carry is $r(K-a)$.

Thus, the source RSU should discard all traffic. Therefore, we should not have $B \leq C + WP\kappa/\big(r(K-a)\big)$.

3.2.1 Transformation of Problem (3.3)

We consider a transformation of Problem (3.3). For $\lambda > 0$, define a cost function

$$\begin{aligned} Z_n(\lambda) &\triangleq Y_n - \lambda r T_n \\ &= WP\kappa + C(rT_n - A_n) + BA_n - \lambda r T_n \\ &= WP\kappa + r\,(C-\lambda)\,T_n + (B-C)\,A_n \end{aligned} \tag{3.6}$$

in which the physical meaning of λ is rate of cost.

Based on the cost function $Z_n(\lambda)$, we formulate a new stopping problem as

$$\begin{aligned} V(\lambda) &\triangleq \inf_{N\geq 1} \mathbb{E}\left[Z_N(\lambda)\right] \\ &= \inf_{N\geq 1} \mathbb{E}\left[WP\kappa + r\,(C-\lambda)\,T_N + (B-C)\,A_N\right]. \end{aligned} \tag{3.7}$$

Theorem 1. *For Problem (3.7) with $\lambda \in (0, B]$, there exists an optimal stopping rule, denoted $N^\dagger(\lambda)$, such that $\mathbb{E}\left[Z_{N^\dagger(\lambda)}(\lambda)\right] = V(\lambda)$.*

Proof. See Appendix 3.6.1. □

Theorem 2. *If i) there exists a λ^* such that $V(\lambda^*) = \inf_{N\geq 1}\mathbb{E}\left[Z_N(\lambda^*)\right] = 0$; and ii) for Problem (3.7) with λ^*, there exists an optimal stopping rule $N^\dagger(\lambda^*)$ such that $\mathbb{E}\left[Z_{N^\dagger(\lambda^*)}(\lambda^*)\right] = V(\lambda^*) = 0$, then $N^\dagger(\lambda^*)$ is an optimal stopping rule for Problem (3.3) and the optimal rate of cost (i.e., the optimal objective function of Problem (3.3)) is λ^*.*

Proof. See Appendix 3.6.2. □

The next theorem shows that the first condition in Theorem 2 is satisfied for Problem (3.7).

Theorem 3. *There exists a* $\lambda^* \in (C, B]$ *such that* $V(\lambda^*) = 0$.

Proof. See Appendix 3.6.3. □

From Theorem 1, it can be concluded that the second condition in Theorem 2 is also satisfied. Therefore, an optimal stopping rule for Problem (3.3) is in the form of $N^\dagger(\lambda^*)$. So if $N^\dagger(\lambda)$ and $V(\lambda)$ can be obtained for any $\lambda \in (C, B]$, then the value of λ^* can be obtained numerically such that $V(\lambda^*) = 0$, and thus $N^\dagger(\lambda^*)$ is an optimal stopping rule of Problem (3.3). Therefore, next we focus on derivation of $N^\dagger(\lambda)$ and $V(\lambda)$ for $\lambda \in (C, B]$ in Problem (3.7). For Problem (3.7) with a specific λ, we first derive an optimal strategy for vehicles arriving after moment $K - a$ in Section 3.2.2, and based on the result, we derive an optimal stopping rule for vehicles arriving before moment $K - a$ in Section 3.2.3. Then as a summary of Sections 3.2.2 and Section 3.2.3, in Section 3.2.4 we give an overall optimal stopping rule, i.e., $N^\dagger(\lambda)$, for Problem (3.7) with a specific λ, as well as its optimal cost $V(\lambda)$. In Section 3.2.5, we find λ^* such that $V(\lambda^*) = 0$, and thus, $N^\dagger(\lambda^*)$ is an optimal stopping rule of Problem (3.3).

3.2.2 Optimal stopping rule for Problem (3.7) (with a specific λ) when $T_n \geq K - a$

First we consider that the source RSU does not stop before moment $K - a$, i.e., we consider Problem (3.7) when $T_n \geq K - a$. For presentation simplicity, in Section 3.2.2, when we say "Problem (3.7)", it means "Problem (3.7) when $T_n \geq K - a$".

Here we introduce the notion of *myopic stopping rule*. The myopic stopping rule is the rule that calls for stopping at vehicle n if the cost of vehicle n [7] is not greater than the expected cost of vehicle $n+1$. For Problem (3.7), its myopic rule is to stop at $\min\left\{n \geq 1 : Z_n(\lambda) \leq \mathbb{E}\left[Z_{n+1}(\lambda) | \mathscr{F}_n\right]\right\}$. Here $\mathscr{F}_n$ means information

[7] Cost of a vehicle is the cost of stopping at the vehicle.

of $T_1, T_2, ..., T_n$ and $G_1, G_2, ..., G_n$, which is available at the source RSU when it decides whether or not to stop at vehicle $n+1$. Note that, upon arrival of vehicle n, if the myopic rule calls for continuation[8] (i.e., $Z_n(\lambda) > \mathbb{E}\left[Z_{n+1}(\lambda)|\mathscr{F}_n\right]$, and thus, the myopic rule decides not to stop at vehicle n), it is also optimal for Problem (3.7) to continue since $Z_n(\lambda) > \mathbb{E}\left[Z_{n+1}(\lambda)|\mathscr{F}_n\right]$ means that the expected cost of vehicle $n+1$ is smaller than the cost of vehicle n and thus, continuing to observe vehicle $n+1$ is optimal. On the other hand, if the myopic rule calls for stopping at a vehicle, generally it may not be optimal for Problem (3.7) to stop at the vehicle. However, if a stopping problem is *monotone*, under some mild conditions, if the myopic rule calls for stopping at a vehicle, it is also optimal for the stopping problem to stop at the vehicle (in other words, the myopic rule is an optimal stopping rule). Therefore, next we introduce the concept of *monotone problem*, and when Problem (3.7) is not monotone, we transform it to a monotone problem, for which we prove that its myopic rule is optimal.

Definition 2. *Let $\mathcal{B}_n$ denote the event $\left\{\omega : Z_n(\lambda) \leq \mathbb{E}\left[Z_{n+1}(\lambda)|\mathscr{F}_n\right]\right\}$. Problem (3.7) is monotone if $\mathcal{B}_1 \subseteq \mathcal{B}_2 \subseteq \mathcal{B}_3 \subseteq \ldots$ almost surely (a.s.) [68].*

$\mathcal{B}_n$ is the set of outcomes that the myopic rule calls for stopping at the vehicle n.[9] $\mathcal{B}_n \subseteq \mathcal{B}_{n+1} \subseteq \mathcal{B}_{n+2} \subseteq ...$ means that: If for one specific outcome ω the myopic rule calls for stopping at vehicle n, then the myopic rule will also call for stopping at vehicle $n+1$ (for whatever realization of T_{n+1} and G_{n+1}); and in general, the myopic rule will call for stopping at any future vehicle for whatever realization of $\{T_{n+1}, T_{n+2}, ...\}$ and $\{G_{n+1}, G_{n+2}, ...\}$ (a.s.).

[8] When we say a stopping rule calls for continuation (which means the source RSU continues to observe other vehicles) or stopping at vehicle n, it implies that the source RSU does not stop at vehicles $1, 2, ..., n-1$.

[9] For presentation simplicity, here vehicle n means the nth vehicle arriving after moment $K-a$ (recalling that in Section 3.2.2 we only consider vehicles arriving after the moment $K-a$).

Next we will try to transform Problem (3.7) into a monotone problem (if it is not a monotone problem), derive the myopic rule for the monotone problem and prove that the myopic rule is optimal for Problem (3.7).

Recall that in this subsection we consider $T_n \geq K - a$, which means $T_n + G_n \geq K$ since $G_n \geq a$. Then from (3.4) we have $A_n = r(T_n + G_n - K)$. Since $T_{n+1} > T_n \geq K - a$ and $G_{n+1} \geq a$, we also have $A_{n+1} = r(T_{n+1} + G_{n+1} - K)$. $Z_n(\lambda)$ in (3.6) can be rewritten as

$$\begin{aligned} Z_n(\lambda) =& WP\kappa + r(C-\lambda)T_n + r(B-C)(T_n + G_n - K) \\ =& WP\kappa + r(B-\lambda)T_n + r(B-C)(G_n - K). \end{aligned} \tag{3.8}$$

From (3.8), the expectation of $Z_{n+1}(\lambda)$ conditioned on that the source RSU has observed the first n vehicles but has not stopped at them is

$$\begin{aligned} & \mathbb{E}\left[Z_{n+1}(\lambda)|\mathscr{F}_n\right] \\ =& WP\kappa + r(B-\lambda)\mathbb{E}\left[T_{n+1}|\mathscr{F}_n\right] \\ & + r(B-C)\mathbb{E}\left[G_{n+1} - K|\mathscr{F}_n\right] \\ =& WP\kappa + r(B-\lambda)\mathbb{E}\left[T_n + X_{n+1}|\mathscr{F}_n\right] \\ & + r(B-C)(\mathbb{E}\left[G_{n+1}|\mathscr{F}_n\right] - K) \\ =& WP\kappa + r(B-\lambda)(T_n + \mu) + r(B-C)\left(\mathbb{E}\left[G_{n+1}\right] - K\right) \end{aligned} \tag{3.9}$$

where the last equality comes from the following fact. $\mathscr{F}_n$ does not carry information regarding X_{n+1} and G_{n+1}. In other words, if we know $\mathscr{F}_n$, we know the values of $\{X_k, k = 1, \ldots, n\}$ and $\{G_k, k = 1, \ldots, n\}$, but not X_{n+1} or G_{n+1}. Thus, we have $\mathbb{E}\left[T_n|\mathscr{F}_n\right] = T_n$, $\mathbb{E}\left[X_{n+1}|\mathscr{F}_n\right] = \mathbb{E}\left[X_{n+1}\right] = \mu$, and $\mathbb{E}\left[G_{n+1}|\mathscr{F}_n\right] = \mathbb{E}\left[G_{n+1}\right]$.

From (3.8) and (3.9), the difference between the cost of vehicle n and the expected cost of vehicle $n+1$ is

$$\begin{aligned}
&\mathbb{E}\left[Z_{n+1}(\lambda)|\mathscr{F}_n\right]-Z_n(\lambda)\\
=&r(B-\lambda)(T_n+\mu)+r(B-C)\left(\mathbb{E}\left[G_{n+1}\right]-K\right)\\
&-r(B-\lambda)T_n-r(B-C)(G_n-K)\\
=&r(B-\lambda)\mu+r(B-C)\left(\mathbb{E}\left[G_{n+1}\right]-G_n\right)\\
=&r(B-C)\left(\frac{(B-\lambda)\mu}{B-C}+\int_a^b xf_G(x)\,\mathrm{d}x-G_n\right)
\end{aligned}\tag{3.10}$$

where $f_G(x)$ is the probability density function of the transit delay of the vehicles. We define a function

$$\phi(g)\triangleq\frac{1}{F_G(g)}\left(\frac{(B-\lambda)\mu}{B-C}+\int_a^g xf_G(x)\,\mathrm{d}x\right)\quad a\le g\le b,\tag{3.11}$$

and define $b_1\triangleq\phi(b)$.[10] Since $F_G(b)=1$, we can re-write (3.10) as

$$\mathbb{E}\left[Z_{n+1}(\lambda)|\mathscr{F}_n\right]-Z_n(\lambda)=r(B-C)(b_1-G_n).\tag{3.12}$$

We consider two scenarios: $b_1\ge b$ and $b_1<b$. For each scenario, we get a monotone problem, obtain its myopic rule, and prove that the myopic rule is optimal for Problem (3.7), as follows.

Scenario with $b_1\ge b$

Since b_1 is not less than the largest transit delay b, it is not less than G_n. So for any vehicle n, expression (3.12) is always nonnegative. Thus, for any n, ignoring vehicle n and stopping at vehicle $n+1$ is expected to involve more cost than that of stopping at vehicle n. Therefore, the myopic rule for Problem (3.7) will require the source RSU to stop at the first vehicle (arriving after $K-a$), given as

$$N_{K-a}^{\mathrm{m}}(\lambda)=\min\{n:T_n\ge K-a\}\tag{3.13}$$

[10]Note that b_1 is a function of λ. For presentation simplicity, we do not show it in form of $b_1(\lambda)$. The subsequent $b_2,b_3,\ldots$ and b^* are treated similarly.

in which superscript 'm' means 'myopic' and subscript '$K-a$' means we start observation from moment $K-a$. And Problem (3.7) is a monotone problem for reason as follows. From (3.12), we always have $Z_n(\lambda) \le \mathbb{E}\left[Z_{n+1}(\lambda)|\mathscr{F}_n\right]$ for any n. Recalling that $\mathcal{B}_n$ denotes the event $\left\{\omega : Z_n(\lambda) \le \mathbb{E}\left[Z_{n+1}(\lambda)|\mathscr{F}_n\right]\right\}$, we have $\mathcal{B}_1 = \mathcal{B}_2 = \mathcal{B}_3 = ...$, and thus, from Definition 1, Problem (3.7) is a monotone problem.

Theorem 4. *When $T_n \ge K-a$ and $b_1 \ge b$, the myopic rule (3.13) is an optimal stopping rule for Problem (3.7) with $\lambda \in (0, B]$.*

Proof. See Appendix 3.6.4. □

So when $T_n \ge K-a$, the expected optimal stopping time is

$$\mathbb{E}\left[T_{N^{\mathrm{m}}_{K-a}(\lambda)}\right] = K - a + \mu \tag{3.14}$$

due to the fact that: Since the Poisson arrival process (of vehicle arrivals) is memoryless, starting from moment $K-a$, the expected waiting time for the next vehicle is μ (the average vehicle inter-arrival time).

Denote the optimal cost of Problem (3.7) when we start observation from moment $K-a$ as $V_{K-a}(\lambda)$. Then we have

$$\begin{aligned} V_{K-a}(\lambda) =& \mathbb{E}\left[Z_{N^{\mathrm{m}}_{K-a}(\lambda)}(\lambda)\right] \\ =& WP\kappa + r(B-\lambda)\mathbb{E}\left[T_{N^{\mathrm{m}}_{K-a}(\lambda)}\right] \\ & + r(B-C)(\mathbb{E}\left[G_n\right] - K) \\ =& WP\kappa + r(C-\lambda)K - r(B-\lambda)(a-\mu) \\ & + r(B-C)\int_a^b x f_G(x)\,\mathrm{d}x \end{aligned} \tag{3.15}$$

where the second equality comes from (3.8) and the third equality comes from (3.14).

Scenario with $b_1 < b$

We consider a sequence of values $\{b_i\}_{i=1,2,\dots}$, in which $b_i = \phi(b_{i-1})$ for $i \geq 2$ (recall that $b_1 = \phi(b)$). The following theorem will be useful in subsequent investigation.

Theorem 5. *When $b_1 < b$, the sequence $\{b_i\}_{i=1,2,\dots}$ has the following properties: 1) $b_i < b_{i-1}$ for $i \geq 2$; 2) $\{b_i\}_{i=1,2,\dots}$ converges to a value denoted b^*; 3) b^* is the unique root of $\phi(g) = g$ for $g \in (a, b]$; 4) for $g \in (a, b]$, $g = b^*$ minimizes $\phi(g)$.*

Proof. See Appendix 3.6.5. □

Next we show that when $b_1 < b$, Problem (3.7) is not a monotone problem, and then transform Problem (3.7) into a monotone problem. Upon arrival of vehicle n, if $G_n \leq b_1$, then from (3.12) we have $\mathbb{E}\left[Z_{n+1}(\lambda)|\mathscr{F}_n\right] - Z_n(\lambda) \geq 0$, and the myopic rule will call for stopping at vehicle n; otherwise, the myopic rule will ask for continuation. This means the decision for any vehicle depends on the transit delay of the vehicle. So Problem (3.7) is not a monotone problem when $b_1 < b$. To transform Problem (3.7) into a monotone problem, we have the following iterations.

Iteration 1: By observing (3.12), we notice that $\mathbb{E}\left[Z_{n+1}(\lambda)|\mathscr{F}_n\right] - Z_n(\lambda)$ depends only on G_n. Upon arrival of vehicle n, if $G_n > b_1$, then for whatever value of T_n, the myopic rule always asks for continuation and thus it is optimal for Problem (3.7) to skip this vehicle and continue to wait for other vehicles (recalling that if myopic rule calls for continuation, it is optimal for the source RSU to continue). So we can ignore those slow vehicles with transit delay larger than b_1, and only consider those vehicles whose transit delay is smaller than or equal to b_1. After ignoring those slow vehicles, the arrival rate of *considered* vehicles is $F_G(b_1)/\mu$. We use superscript '[1]' to denote the case when vehicles with transmit delay more than b_1 are not considered. The transit delay of the nth considered vehicle, denoted $G_n^{[1]}$, is in the range of $[a, b_1]$ with CDF $F_{G^{[1]}}(g) = F_G(g)/F_G(b_1)$ for $g \in [a, b_1]$. And similar to (3.10), we have

$$\begin{aligned}
&\mathbb{E}\left[Z_{n+1}^{[1]}(\lambda)|\mathscr{F}_n^{[1]}\right] - Z_n^{[1]}(\lambda)\\
=&r(B-\lambda)\mu/F_G(b_1) + r(B-C)\left(\mathbb{E}\left[G_{n+1}^{[1]}\right] - G_n^{[1]}\right)\\
=&r(B-\lambda)\mu/F_G(b_1) + r(B-C)\left(\frac{\int_a^{b_1} x f_G(x)\,\mathrm{d}x}{F_G(b_1)} - G_n^{[1]}\right)\\
=&r(B-C)\left(\frac{1}{F_G(b_1)}\left(\frac{(B-\lambda)\mu}{B-C} + \int_a^{b_1} x f_G(x)\,\mathrm{d}x\right) - G_n^{[1]}\right)\\
=&r(B-C)(\phi(b_1) - G_n^{[1]})\\
=&r(B-C)(b_2 - G_n^{[1]})
\end{aligned} \tag{3.16}$$

where the last two equalities come from the definition of $\phi(\cdot)$ in (3.11) and $b_2 = \phi(b_1)$, respectively.

From Theorem 5, we have $b_2 < b_1$.

Iteration 2: Upon arrival of the nth considered vehicle, if $G_n^{[1]} > b_2$, then $\mathbb{E}\left[Z_{n+1}^{[1]}(\lambda)|\mathscr{F}_n^{[1]}\right] - Z_n^{[1]}(\lambda) < 0$, and thus, it is optimal to continue since continuing to stop at vehicle $n+1$ expects to incur less cost than that of stopping at vehicle n. So we can ignore those slow vehicles with transit delay $G_n^{[1]} > b_2$, and only consider the remaining vehicles. After ignoring those slow vehicles, the arrival rate of considered vehicles is $F_G(b_2)/\mu$. And the transit delay of the nth considered vehicle, denoted $G_n^{[2]}$, is in the range of $[a, b_2]$ with CDF $F_{G^{[2]}}(g) = F_G(g)/F_G(b_2)$ for $g \in [a, b_2]$. Here superscript '[2]' means that those vehicles with transmit delay more than b_2 are not considered. And similar to (3.16), we have

$$\mathbb{E}\left[Z_{n+1}^{[2]}(\lambda)|\mathscr{F}_n^{[2]}\right] - Z_n^{[2]}(\lambda) = r(B-C)(b_3 - G_n^{[2]}).$$

Similarly, we can have Iterations 3, 4,...,etc. And in Iteration l ($l = 1, 2, ...$), vehicles with transit delay more than b_l should not be considered. Since sequence

$\{b_i\}_{i=1,2,...}$ converges to b^* (from Theorem 5), it can be concluded that, as an overall result after all iterations, vehicles with transit delay more than b^* should not be considered. The arrival rate of considered vehicles is $F_G(b^*)/\mu$. And the transit delay of the nth considered vehicle, denoted $G_n^{[o]}$, is in the range of $[a, b^*]$ with CDF $F_{G^{[o]}}(g) = F_G(g)/F_G(b^*)$. Here superscript '$[o]$' means the overall effect of the iterations (i.e., vehicles with transit delay more than b^* are not considered).

With only those considered vehicles, Problem (3.7) is a monotone problem. This is because for any considered vehicle $n = 1, 2, ...$, we have $G_n^{[o]} \leq b^*$, and similar to (3.16), we have $\mathbb{E}\left[Z_{n+1}^{[o]}(\lambda)|\mathscr{F}_n^{[o]}\right] - Z_n^{[o]}(\lambda) = r(B-C)(b^* - G_n^{[o]}) \geq 0$. Similar to the scenario with $b_1 \geq b$, Problem (3.7) with only those considered vehicles is monotone.

For Problem (3.7) with only those considered vehicles, since $\mathbb{E}\left[Z_{n+1}^{[o]}(\lambda)|\mathscr{F}_n^{[o]}\right] - Z_n^{[o]}(\lambda) \geq 0$ for any n, its myopic rule will call for stopping at the first considered vehicle after moment $K-a$ (recalling that we consider $T_n \geq K-a$ in this subsection). The myopic rule is expressed as

$$N_{K-a}^{\mathrm{m}}(\lambda) = \min\left\{n : T_n^{[o]} \geq K-a\right\}. \tag{3.17}$$

Theorem 6. *When $T_n \geq K-a$ and $b_1 < b$, the myopic rule (3.17) is an optimal stopping rule for Problem (3.7) when only vehicles with transmit delay less than b^* are considered.*

The proof is similar to the proof of Theorem 4, and is omitted here.

As aforementioned, for Problem (3.7) when the RSU does not stop before time $K-a$, it is optimal to ignore those vehicles with transit delay larger than b^* because for such a vehicle, the expected cost of stopping at the next vehicle is less. So stopping rule (3.17) is optimal for Problem (3.7). The optimal stopping rule can be rewritten as

$$N_{K-a}^{\mathrm{m}}(\lambda) = \min\left\{n : T_n \geq K-a, G_n \leq b^*\right\} \tag{3.18}$$

which means from moment $K-a$, the source RSU should stop at the first vehicle whose transit delay is not more than b^*. Similar to (3.14), the expected optimal stopping time is

$$\mathbb{E}\left[T_{N_{K-a}^{\mathrm{m}}(\lambda)}\right] = K - a + \frac{\mu}{F_G\left(b^*\right)} \tag{3.19}$$

since the arrival rate of vehicles whose transit delay is not more than b^* is $F_G(b^*)/\mu$.

Upon an optimal stopping, the expectation of transit delay (which is the average transmit delay of a vehicle conditioned on that its transit delay is not more than b^*) is

$$\mathbb{E}\left[G_{N_{K-a}^{\mathrm{m}}(\lambda)}\right] = \frac{\int_a^{b^*} g f_G(g)\,\mathrm{d}g}{F_G(b^*)}. \tag{3.20}$$

Recall that $V_{K-a}(\lambda)$ denotes the optimal cost of Problem (3.7) when we start observation from moment $K-a$. Thus, we have

$$
\begin{aligned}
& V_{K-a}(\lambda) \\
=& \mathbb{E}\left[Z_{N^{\mathrm{m}}_{K-a}(\lambda)}(\lambda)\right] \\
=& WP\kappa + r(B-\lambda)\mathbb{E}\left[T_{N^{\mathrm{m}}_{K-a}(\lambda)}\right] \\
& + r(B-C)\left(\mathbb{E}\left[G_{N^{\mathrm{m}}_{K-a}(\lambda)}\right] - K\right) \\
=& WP\kappa + r(B-\lambda)\left(K - a + \frac{\mu}{F_G\left(b^*\right)}\right) \\
& + r(B-C)\left(\frac{\int_a^{b^*} g f_G(g)\,\mathrm{d}g}{F_G(b^*)} - K\right) \\
=& WP\kappa + r(C-\lambda)K - r(B-\lambda)a \\
& + r(B-C)\frac{1}{F_G(b^*)}\left(\frac{(B-\lambda)\mu}{B-C} + \int_a^{b^*} x f_G(x)\,\mathrm{d}x\right) \\
=& WP\kappa + r(C-\lambda)K - r(B-\lambda)a + r(B-C)\phi(b^*) \\
=& WP\kappa + r(C-\lambda)K - r(B-\lambda)a + r(B-C)b^*
\end{aligned} \tag{3.21}
$$

in which the second equality comes from (3.8), the third equality comes from (3.19) and (3.20), and the last two equalities come from the definition of $\phi(\cdot)$ in (3.11) and $\phi(b^*) = b^*$ (from Theorem 5), respectively.

Remark: To get the optimal stopping rule, the value of b^*, which is the converging point of sequence $\{b_i\}_{i=1,2,\ldots}$, should be obtained. From Theorem 5, the value of b^* can be obtained by any method to find the unique root of $\phi(g) = g$ for $g \in (a, b]$ or by any method to find the minimum of $\phi(g)$ for $g \in (a, b]$.

3.2.3 Optimal stopping rule for Problem (3.7) (with a specific λ) when $0 \leq T_n < K - a$

Since we have obtained the optimal stopping rule for $T_n \geq K - a$ in Section 3.2.2, next we only need to consider vehicles arriving between $[0, K-a)$. Denote $V_t(\lambda)$ as the optimal cost of Problem (3.7) when we start observation from moment t

(not including moment t). So the optimal cost of Problem (3.7) is $V(\lambda) = V_0(\lambda)$. As a boundary condition, when $t = K - a$, $V_t(\lambda)$ is given in (3.15) when $b_1 \geq b$ or in (3.21) when $b_1 < b$. Next we will derive $V_t(\lambda)$ for $t \in [0, K - a)$ using this boundary condition.

Suppose a vehicle, say vehicle n, arrives at moment T_n, and if it is selected, the cost is $Z_n(\lambda)$. The source RSU needs to decide whether it is optimal to stop at this vehicle. The optimality equation of dynamic programming [68] states that, if $Z_n(\lambda) < V_{T_n}(\lambda)$ (i.e., the cost of current vehicle is less than the optimal cost of ignoring the current vehicle and continuing to wait for future vehicles), then it is optimal to stop at vehicle n; otherwise, it is optimal to continue observation of future vehicles.

For moment t, consider an interval $(t - \Delta t, t)$ in which $\Delta t > 0$ is sufficiently small. Since the vehicle arrival process is a Poisson process with average arrival rate $1/\mu$, within interval $(t - \Delta t, t)$, the probability that one vehicle arrives is $\Delta t/\mu$, the probability that no vehicle arrives is $(1 - \Delta t/\mu)$, and the probability that two or more vehicles arrive is $o(\Delta t)$ (higher order of Δt). Next we give an expression for $V_{t-\Delta t}(\lambda)$.

- If there is no vehicle arriving during the interval $(t - \Delta t, t)$: Then the source RSU has to wait for vehicles arriving after moment t for chance of transmission. For this case, the optimal costs starting from t and $(t - \Delta t)$ are the same, i.e., $V_{t-\Delta t}(\lambda) = V_t(\lambda)$.

- If there is one vehicle, say vehicle n, arriving at $T_n = t - \Delta t'$ where $0 < \Delta t' < \Delta t$: From (3.6) and (3.4), we have $Z_n(\lambda) = WP\kappa + r\,(C - \lambda)\,(t - \Delta t') + r\,(B - C)\,(t - \Delta t' + G_n - K)^+$.

When

$$Z_n(\lambda) = WP\kappa + r\,(C-\lambda)\,(t-\Delta t') \\ + r\,(B-C)\,(t-\Delta t' + G_n - K)^+ \le V_{t-\Delta t'}(\lambda), \quad (3.22)$$

then it is optimal to stop at vehicle n and there is no need to continue the observation after t. Define a function[11]

$$\rho(t,\lambda) \triangleq \max\big\{g \in [a,b] : WP\kappa + r\,(C-\lambda)\,t + \\ r\,(B-C)\,(t+g-K)^+ \le V_t(\lambda)\big\}.$$

Then (3.22) is equivalent to $G_n \le \rho(t-\Delta t', \lambda)$, which happens with probability $F_G(\rho(t-\Delta t', \lambda))$. When (3.22) holds, it is optimal to stop at vehicle n, and we have

$$\begin{aligned} &V_{t-\Delta t}(\lambda) \\ =&\mathbb{E}\left[Z_n(\lambda)\right] \\ =&WP\kappa + r\,(C-\lambda)\,(t-\Delta t') \\ &+ r(B-C)\frac{\int_a^{\rho(t-\Delta t',\lambda)}(t-\Delta t' + x - K)^+ f_G(x)\,\mathrm{d}x}{F_G(\rho(t-\Delta t',\lambda))}. \end{aligned}$$

If (3.22) does not hold (with probability $1 - F_G(\rho(t-\Delta t', \lambda))$), it is optimal to skip vehicle n and continue to wait for other vehicles that come after moment t. Therefore, we have $V_{t-\Delta t}(\lambda) = V_t(\lambda)$.

[11]Note that if $WP\kappa + r\,(C-\lambda)\,t + r\,(B-C)\,(t+g-K)^+ \le V_t(\lambda)$ never holds for $g \in [a,b]$, then $\rho(t,\lambda) = a$.

As a summary, we have

$$
\begin{aligned}
& V_{t-\Delta t}(\lambda) \\
=& (1-\Delta t/\mu)V_t(\lambda)+(\Delta t/\mu)\left(1-F_G(\rho(t-\Delta t',\lambda))\right)V_t(\lambda) \\
& +(\Delta t/\mu)F_G(\rho(t-\Delta t',\lambda))\Bigg(WP\kappa+r\,(C-\lambda)\,(t-\Delta t') \\
& +r\,(B-C)\,\frac{\int_a^{\rho(t-\Delta t',\lambda)}(t-\Delta t'+x-K)^+f_G(x)\,\mathrm{d}x}{F_G(\rho(t-\Delta t',\lambda))}\Bigg) \\
& +o(\Delta t) \\
=& V_t(\lambda)+(\Delta t/\mu)F_G(\rho(t-\Delta t',\lambda)) \\
& \times\Bigg(-V_t(\lambda)+WP\kappa+r\,(C-\lambda)\,(t-\Delta t') \\
& +r\,(B-C)\,\frac{\int_a^{\rho(t-\Delta t',\lambda)}(t-\Delta t'+x-K)^+f_G(x)\,\mathrm{d}x}{F_G(\rho(t-\Delta t',\lambda))}\Bigg) \\
& +o(\Delta t).
\end{aligned}
$$

After some algebraic operations, we have

$$
\begin{aligned}
& \mu\frac{V_t(\lambda)-V_{t-\Delta t}(\lambda)}{\Delta t} \\
=& F_G(\rho(t-\Delta t',\lambda))\Bigg(V_t(\lambda)-WP\kappa-r\,(C-\lambda)\,(t-\Delta t') \\
& -r\,(B-C)\,\frac{\int_a^{\rho(t-\Delta t',\lambda)}(t-\Delta t'+x-K)^+f_G(x)\,\mathrm{d}x}{F_G(\rho(t-\Delta t',\lambda))}\Bigg) \\
& +o(\Delta t)/\Delta t.
\end{aligned}
$$

Letting Δt approach zero, it follows

$$
\begin{aligned}
\mu\frac{\partial V_t(\lambda)}{\partial t} =& F_G(\rho(t,\lambda))\Bigg(V_t(\lambda)-WP\kappa-r\,(C-\lambda)\,t \\
& -r\,(B-C)\,\frac{\int_a^{\rho(t,\lambda)}(t+x-K)^+f_G(x)\,\mathrm{d}x}{F_G(\rho(t,\lambda))}\Bigg). \qquad (3.23)
\end{aligned}
$$

With the aforementioned boundary condition, equation (3.23) can be solved numerically to get $V_t(\lambda)$ for $t\in[0,K-a)$.

$$N^{\dagger}(\lambda) \tag{3.24}$$
$$= \begin{cases} \min\{n : WP\kappa + r\,(C-\lambda)\,T_n + \\ \qquad r\,(B-C)\,(T_n+G_n-K)^+ \le V_{T_n}(\lambda)\} & \text{if } T_n < K-a \\ \min n \text{ if } b_1 \ge b \text{ or } \min\{n : G_n \le b^*\} \text{ if } b_1 < b & \text{if } T_n \ge K-a. \end{cases}$$

Therefore, for Problem (3.7), upon arrival of vehicle n at moment T_n, if $T_n < K-a$, an optimal stopping rule works as follows. The source RSU first calculates (from (3.6)) the cost of vehicle n given as $Z_n(\lambda) = WP\kappa + r(C-\lambda)T_n + r(B-C)(T_n+G_n-K)^+$. If $Z_n(\lambda)$ is less than $V_{T_n}(\lambda)$, then the source RSU stops at vehicle n; otherwise, the source RSU continues to observe future vehicles. The optimal cost of Problem (3.7) is $V(\lambda) = V_0(\lambda)$.

3.2.4 Overall optimal stopping rule for Problem (3.7) (with a specific λ)

As a summary of Sections 3.2.2 and 3.2.3, for Problem (3.7), the optimal cost is $V(\lambda) = V_0(\lambda)$, and an optimal stopping rule works as shown in (3.24) on top of the next page. In other words, when the waiting time is less than $K-a$, then the source RSU stops at the first vehicle such that $Z_n(\lambda) = WP\kappa + r(C-\lambda)T_n + r(B-C)(T_n+G_n-K)^+ < V_{T_n}(\lambda)$; when the waiting time is more than $K-a$, then the source RSU stops at the next vehicle if $b_1 \ge b$ or stops at the next vehicle with transmit delay less than b^* if $b_1 < b$.

3.2.5 Optimal stopping rule for Problem (3.3)

From Theorem 3, there exists $\lambda^* \in (C, B]$ such that $V(\lambda^*) = 0$. From proof of Theorem 3, $V(\lambda)$ is continuous and decreasing for $\lambda \in [C, B]$. So λ^* can be found by a bisection search. Then from Theorem 2, an optimal stopping rule of Problem (3.3) is $N^{\dagger}(\lambda^*)$ in the form of (3.24), and the optimal rate of cost (i.e., the optimal objective function of Problem (3.3)) is λ^*.

To implement the optimal stopping rule $N^\dagger(\lambda^*)$, the value of λ^*, the values of $V_t(\lambda^*)$ for $t \in [0, K-a)$, and values of b, b_1 and b^* can be calculated off-line in advance and stored at the source RSU when the source RSU is setup. Then it can be seen that the computational complexity of the optimal stopping rule $N^\dagger(\lambda^*)$ in the form of (3.24) is fairly low. Thus, upon a vehicle arrival, the source RSU is able to quickly make a decision, and the energy consumption in the computation is negligible.

Next we continue to calculate the expected optimal stopping time of Problem (3.3) given as $\mathbb{E}\left[T_{N^\dagger(\lambda^*)}\right]$, the expected energy consumption per unit time, and the expected traffic loss rate.

Define function $\alpha(t) = \mathbb{E}\left[T_{N^\dagger(\lambda^*)} \,\middle|\, T_{N^\dagger(\lambda^*)} \geq t\right]$, which is the expected optimal stopping time if we know that the stopping time is not before time t. Recall that when we start observation from moment $K-a$, the expected optimal stopping time is $\mathbb{E}\left[T_{N^{\mathrm{m}}_{K-a}(\lambda^*)}\right] = K - a + \mu$ given in (3.14) when $b_1 \geq b$ or $\mathbb{E}\left[T_{N^{\mathrm{m}}_{K-a}(\lambda^*)}\right] = K - a + \mu/F_G(b^*)$ given in (3.19) when $b_1 < b$. If we extend the definition of b^* to the scenario with $b_1 \geq b$ such that $b^* = b$ when $b_1 \geq b$ holds, then we have a uniform expression of expected optimal stopping time after moment $K-a$ for both $b_1 \geq b$ and $b_1 < b$ as: $\mathbb{E}\left[T_{N^{\mathrm{m}}_{K-a}(\lambda^*)}\right] = K - a + \mu/F_G(b^*)$. In other words, a boundary condition of $\alpha(t)$ is

$$\alpha(K-a) = \mathbb{E}\left[T_{N^{\mathrm{m}}_{K-a}(\lambda^*)}\right] = K - a + \frac{\mu}{F_G(b^*)}. \tag{3.25}$$

Following the same method of deriving (3.23), we can derive the following equation

$$\mu \frac{\mathrm{d}\alpha(t)}{\mathrm{d}t} = \alpha(t) - F_G(\rho(t, \lambda^*))t. \tag{3.26}$$

Based on (3.25) and (3.26), $\alpha(t)$ can be numerically calculated for $t \in [0, K-a)$. And the expected optimal stopping time of Problem (3.3) is $\mathbb{E}\left[T_{N^\dagger(\lambda^*)}\right] = \alpha(0)$.

When the optimal stopping rule $N^\dagger(\lambda^*)$ is used, the objective function of Problem (3.3) is given as

$$\begin{aligned}
&\frac{WP\kappa + C\mathbb{E}\left[rT_{N^\dagger(\lambda^*)}\right] + (B-C)\mathbb{E}\left[A_{N^\dagger(\lambda^*)}\right]}{\mathbb{E}\left[rT_{N^\dagger(\lambda^*)}\right]} \\
&= \frac{WP\kappa + Cr\alpha(0) + (B-C)\mathbb{E}\left[A_{N^\dagger(\lambda^*)}\right]}{r\alpha(0)}.
\end{aligned}$$

Since an optimal stopping rule of Problem (3.3) should attain the optimal rate of cost (also the optimal objective function of Problem (3.3)) given as λ^* (from Theorem 2), we have

$$\frac{WP\kappa + Cr\alpha(0) + (B-C)\mathbb{E}\left[A_{N^\dagger(\lambda^*)}\right]}{r\alpha(0)} = \lambda^*$$

which leads to

$$\mathbb{E}\left[A_{N^\dagger(\lambda^*)}\right] = \frac{r\alpha(0)(\lambda^* - C) - WP\kappa}{B-C}. \tag{3.27}$$

Therefore, the traffic loss rate of the optimal stopping rule $N^\dagger(\lambda^*)$ is the ratio of the expected discarded traffic amount upon a stop to the expected total traffic amount accumulated before a stop, given as

$$\frac{\mathbb{E}\left[A_{N^\dagger(\lambda^*)}\right]}{r\mathbb{E}\left[T_{N^\dagger(\lambda^*)}\right]} = \frac{r\alpha(0)(\lambda^* - C) - WP\kappa}{r\alpha(0)(B-C)}.$$

And the energy consumption per time unit is

$$\frac{P\kappa + P\frac{r\mathbb{E}\left[T_{N^\dagger(\lambda^*)}\right] - \mathbb{E}\left[A_{N^\dagger(\lambda^*)}\right]}{R}}{\mathbb{E}\left[T_{N^\dagger(\lambda^*)}\right]} = \frac{P\kappa + P\frac{r\alpha(0) - \mathbb{E}\left[A_{N^\dagger(\lambda^*)}\right]}{R}}{\alpha(0)}$$

in which the numerator is the expected energy consumption when the source RSU stops, the denominator is the expected stopping time, and $\mathbb{E}\left[A_{N^\dagger(\lambda^*)}\right]$ is given in (3.27).

3.3 Performance evaluation

In this section, we evaluate the performance of the derived stopping strategy, and compare with Matlab simulations. If a vehicle is selected to help, four-way handshake of RTS-CTS-DATA-ACK is used. The data transmission rate is $R =$ 11 Mbps, and transmission power is $P = 15.5$ dBm $= 35.5$ mW. The overhead duration $\kappa = 938.91$ μs consists of the following components: RTS duration (ratio of RTS size to basic rate, plus preamble time), CTS duration (ratio of CTS size to basic rate, plus preamble time), ACK duration (ratio of ACK size to R, plus preamble time), as well as overhead for data transmission (MAC header time, given as ratio of MAC header size to R, and preamble time), in which basic rate is 2 Mbps, preamble time is 192 μs, and RTS, CTS, ACK, and MAC header have sizes of 20 bytes, 14 bytes, 14 bytes, and 34 bytes, respectively. The traffic coming rate is $r = 5$ bps at the source RSU. The energy cost is $W = 1$ unit of cost per μJoule. The delay bound K is usually application dependent (e.g., a delay bound of 120 minutes is adopted in [75]; the delay bound in [88] is 5–20 minutes). In our simulation, each traffic unit is expected to be delivered to the destination RSU within delay bound $K = 1800$ seconds. If a traffic unit is discarded at the source RSU, a cost of $B = 0.5$ per bit is charged. The distance of the source RSU to the destination RSU is $D = 10,000$ meters. At the source RSU, vehicle inter-arrival time follows an exponential distribution with parameter $\mu = 400$ seconds. The speeds of those vehicles are truncated Gaussian random variables [12] which have mean $\bar{v} = 25$ m/s and variance $\sigma^2 = 9$ [89]. The minimum and maximum speeds are $v_{\min} = 18$ m/s and $v_{\max} = 32$ m/s, respectively. Thus, speeds of vehicles are

[12] Here "truncated Gaussian random variable" means the probability density function of a Gaussian random variable is truncated with a minimum value and a maximal value.

i.i.d. random variables with CDF:

$$F_V(v) = \begin{cases} 0 & \text{if} \quad v < v_{\min} \\ \frac{\int_{v_{\min}}^{v} e^{-(x-\bar{v})^2/2\sigma^2}\,\mathrm{d}x}{\int_{v_{\min}}^{v_{\max}} e^{-(x-\bar{v})^2/2\sigma^2}\,\mathrm{d}x} & \text{if} \quad v_{\min} \le v \le v_{\max} \\ 1 & \text{otherwise.} \end{cases}$$

The transit delay of vehicle n is $G_n = D/V_n$, where V_n is the speed of vehicle n. And G_n's are i.i.d. random variables with CDF:

$$\begin{aligned} F_G(g) &= 1 - F_V\left(\frac{D}{g}\right) \\ &= \begin{cases} 0 & \text{if} \quad g < a \\ \frac{\int_{D/g}^{v_{\max}} e^{-(x-\bar{v})^2/2\sigma^2}\,\mathrm{d}x}{\int_{v_{\min}}^{v_{\max}} e^{-(x-\bar{v})^2/2\sigma^2}\,\mathrm{d}x} & \text{if} \quad a \le g \le b \\ 1 & \text{if } g > b \end{cases} \end{aligned}$$

where $a = D/v_{\max} = 312.5$ seconds and $b = D/v_{\min} = 555.6$ seconds are the smallest and the largest transit delay, respectively.

3.3.1 Optimal stopping rule (3.24) for Problem (3.7)

We first evaluate our optimal stopping rule (3.24) for Problem (3.7). For $\lambda \in [0.0032, 0.05]$,[13] Fig. 3.1 shows the numerically calculated values of $V(\lambda) = V_0(\lambda)$ (optimal cost of Problem (3.7)) based on our derivation in Section 3.2.3. Matlab simulations[14] are also carried out to get the cost of Problem (3.7) by using the stopping rule $N^\dagger(\lambda)$ given in (3.24), and the simulation results are also shown in Fig. 3.1. It can be seen that numerical and simulation results match well. As indicated in proof of Theorem 3, $V(\lambda)$ is a continuous and decreasing function of $\lambda \in [C, B]$.

[13]In this example, $C = WP/R = 0.0032$. Since $\lambda^* \in (C, B]$ (from Theorem 3), the minimum value of λ in Fig.3.1 is 0.0032.

[14]When simulations are carried out to evaluate the proposed stopping rule or the subsequently presented heuristic stopping rule, the stopping rule is applied for each vehicle arrival to decide whether or not to stop. And once the source RSU stops and forwards its traffic to the selected vehicle, it keeps observing subsequent vehicles for its next stopping decisions. In other words, in the simulations, repeated stopping problems are dealt with. And simulation statistics are averaged over 10^6 stops.

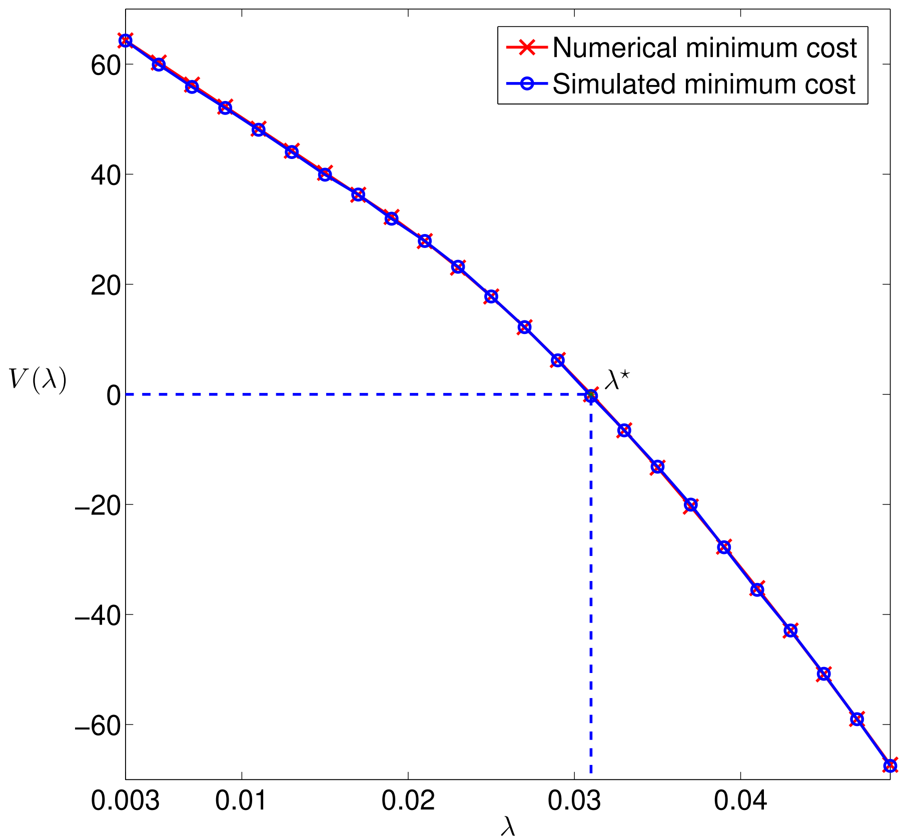

Figure 3.1: $V(\lambda)$ for $C \leq \lambda \leq B$ for Problem (3.7).

3.3.2 Optimal stopping rule of Problem (3.3)

Now we focus on Problem (3.3). From Fig. 3.1, we have $\lambda^* = 0.031$ since $V(\lambda^*) = 0$. And from our analysis in Section 3.2.5, an optimal stopping rule of Problem (3.3) is as follows, with optimal rate of cost being $\lambda^* = 0.031$.

- When $T_n < K - a = 1487.5$ seconds, by (3.24) with $\lambda = \lambda^* = 0.031$, an optimal stopping rule of Problem (3.3) is

$$
\begin{aligned}
&N^\dagger(0.031) \\
&= \min\{n : WP\kappa + r(C - 0.031)T_n \\
&\qquad\qquad + r(B - C)A_n \leq V_{T_n}(0.031)\} \\
&= \min\{n : 33.33 - 0.1389T_n + 2.48(T_n + G_n - 1800)^+ \\
&\qquad\qquad \leq V_{T_n}(0.031)\}.
\end{aligned}
$$

- When $T_n \geq K - a = 1487.5$ seconds and $\lambda = \lambda^*$, since $b_1 = \phi(b) = \phi(555.6) = 805.39 > b = 555.6$, by (3.24) with $\lambda = \lambda^* = 0.031$, an optimal stopping rule of Problem (3.3) is

$$\min\{n : T_n \geq K - a = 1487.5\}.$$

Next we vary the value of B from 0.2 to 1. For Problem (3.3) with each value of B, we numerically calculate λ^* (which is the numerically obtained optimal rate of cost for Problem (3.3), and can be found, as aforementioned in Section 3.2.5, by a bisection search such that $V(\lambda^*) = 0$) and obtain the optimal stopping rule $N^\dagger(\lambda^*)$ in the form of (3.24) with $\lambda = \lambda^*$. We also run computer simulations for the objective function of Problem (3.3) with stopping rule being $N^\dagger(\lambda^*)$ and get the simulated rate of cost. Both the numerically calculated optimal rate of cost (i.e., λ^*) and the simulated rate of cost for different B values are shown in Fig. 3.2. We can see that the numerical and simulation results match well.

Fig. 3.2 also shows the simulated rate of cost using a heuristic stopping rule for different values of B. The heuristic stopping rule is the simple myopic stopping rule: Upon a vehicle arrival (say vehicle n), if its rate of cost is less than the rate of cost of the next vehicle, vehicle $n+1$, by assuming that vehicle $n+1$ has an average inter-arrival time (i.e., $X_{n+1} = \mu$) and has an average transit delay

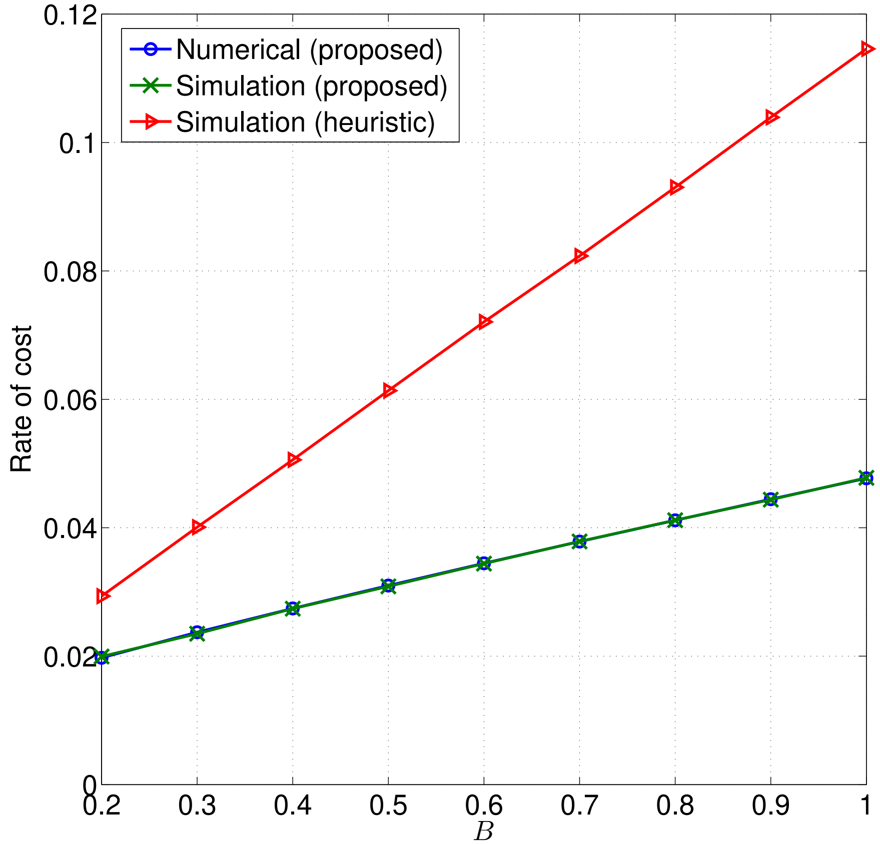

Figure 3.2: The rate of cost of the heuristic rule (simulation results) and the proposed optimal stopping rule (numerical and simulation results) for Problem (3.3).

(i.e.,$G_{n+1} = \int_a^b g f_G(g)\,\mathrm{d}g$), then the source RSU stops at vehicle n; Otherwise, the source RSU skips vehicle n and waits for other vehicles. In other words, the heuristic stopping rule is shown in (3.28) on top of next page. From Fig. 3.2, it can be seen that the proposed stopping rule has a much lower rate of cost than the heuristic rule.

Figs. 3.3-3.5 shows the average stopping time, average percentage of traffic loss, and average energy consumption per time unit, respectively, for both the proposed stopping rule (analytical results from Section 3.2.5 and simulation re-

$$\min\left\{n : \frac{WP\kappa + rCT_n + r(B-C)(T_n+G_n-K)^+}{rT_n} \right. \tag{3.28}$$
$$\left. \leq \frac{WP\kappa + rC(T_n+\mu) + r(B-C)(T_n+\mu+\int_a^b g f_G(g)\,\mathrm{d}g - K)^+}{r(T_n+\mu)}\right\}.$$

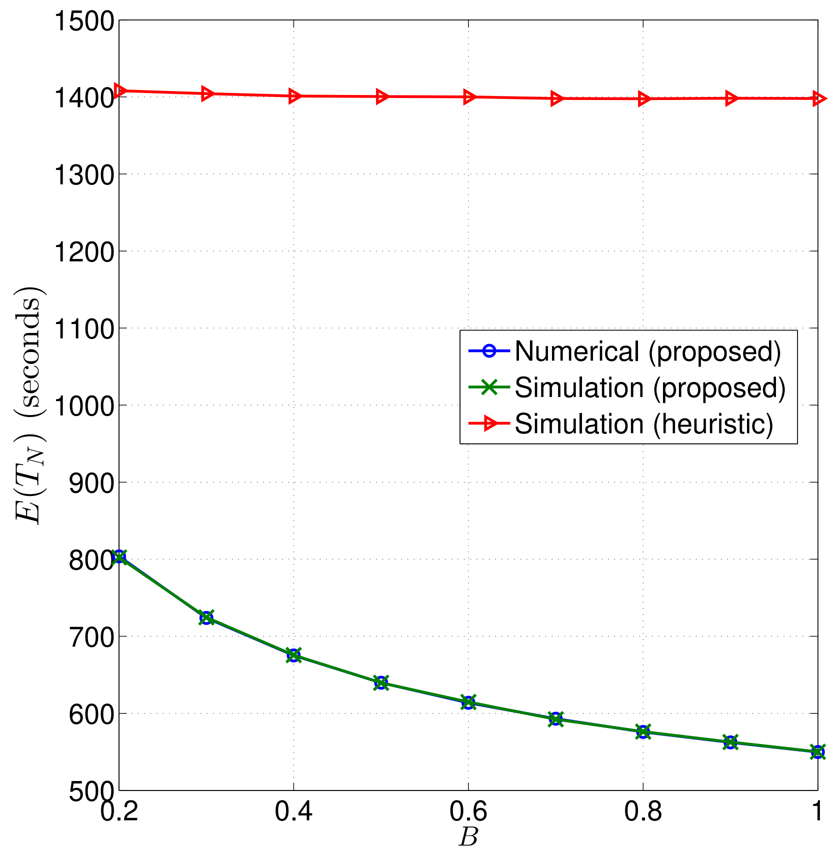

Figure 3.3: Average stopping time of the heuristic rule (simulation results) and the proposed optimal stopping rule (numerical and simulation results) for Problem (3.3).

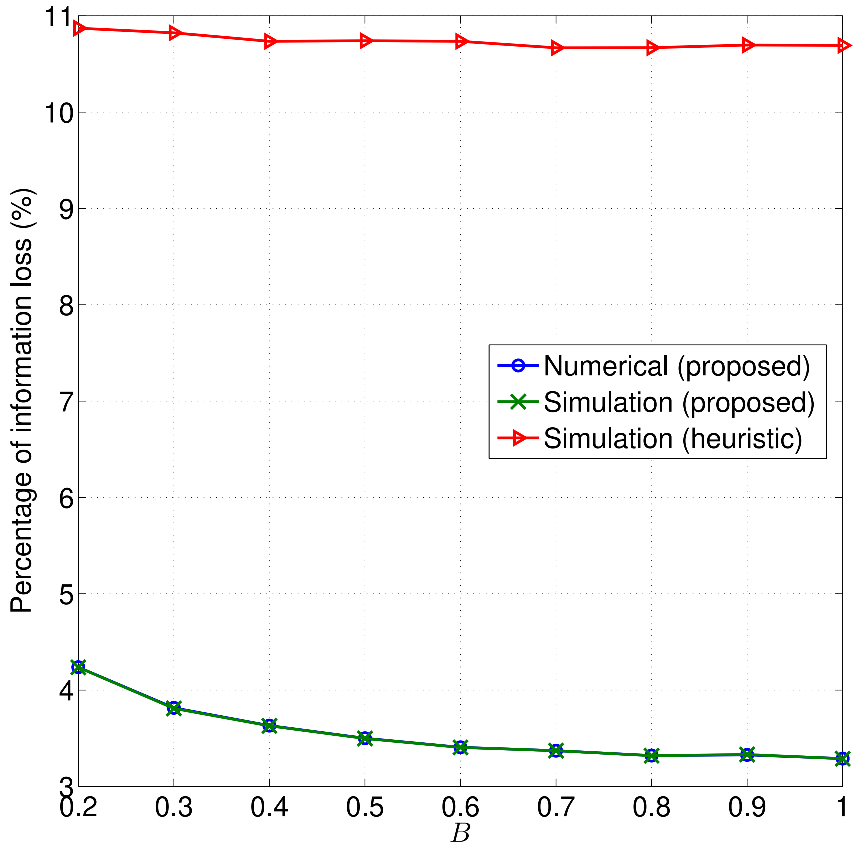

Figure 3.4: The percentage of traffic loss of the heuristic rule (simulation results) and the proposed optimal stopping rule (numerical and simulation results) for Problem (3.3).

sults) and the heuristic rule (simulation results). When penalty B increases, the proposed stopping rule is more conservative to decide on continuation, and therefore, the average stopping time decreases in Fig. 3.3 and the percentage of traffic loss decreases in Fig. 3.4. Since more percentage of traffic is delivered when B increases, more energy is consumed, as shown in Fig. 3.5. On the other hand, for the heuristic stopping rule, increase of B only slightly decreases the average stopping time. This can be roughly explained as follows.

- When $T_n \leq K - \mu - \int_a^b g f_G(g)\,\mathrm{d}g = 994.8$ second, we have $(T_n + G_n -$

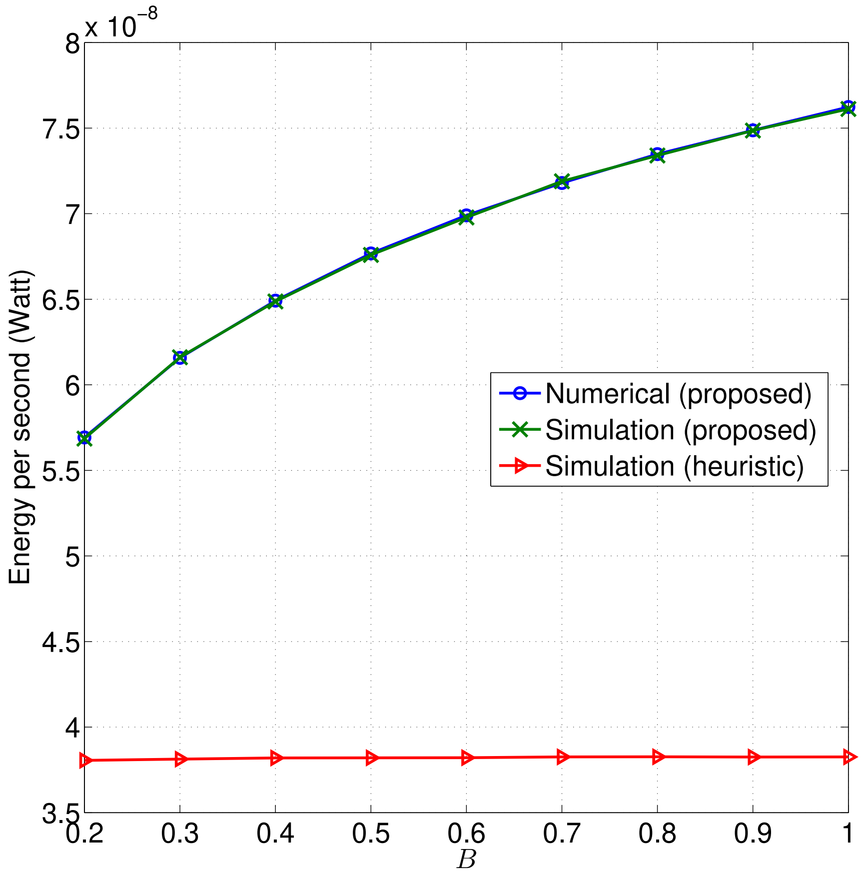

Figure 3.5: The energy consumption per second of the heuristic rule (simulation results) and the proposed optimal stopping rule (numerical and simulation results) for Problem (3.3).

$K)^+ \le (T_n + b - K)^+ \le (994.8 + b - K)^+ = 0$. Since $(T_n + G_n - K)^+ = \max(T_n + G_n - K, 0) \ge 0$, we have $(T_n + G_n - K)^+ = 0$. Further, we have $(T_n + \mu + \int_a^b g f_G(g)\,\mathrm{d}g - K)^+ = 0$. Therefore, the inequality in (3.28) becomes

$$\frac{WP\kappa + rCT_n}{rT_n} \le \frac{WP\kappa + rC(T_n + \mu)}{r(T_n + \mu)}$$

which apparently never holds. Therefore, when $T_n \le 994.8$ second, the heuristic stopping rule never decides on stopping.

- When $T_n \ge K - a = 1487.5$ second, we have $(T_n + G_n - K)^+ = T_n + G_n - K$,

$$\begin{aligned}&\frac{WP\kappa + rCT_n + r(B-C)(T_n+G_n-K)}{rT_n}\\ \le&\frac{WP\kappa + rC(T_n+\mu) + r(B-C)(T_n+\mu+\int_a^b gf_G(g)\,\mathrm{d}g - K)}{r(T_n+\mu)}.\end{aligned} \tag{3.29}$$

---

and $(T_n+\mu+\int_a^b gf_G(g)\,\mathrm{d}g-K)^+ = T_n+\mu+\int_a^b gf_G(g)\,\mathrm{d}g-K$, and therefore, the inequality in (3.28) becomes (3.29) on top of this page. Recall that B varies from 0.2 to 1 in Figs. 3.3-3.5. If B takes the minimum value 0.2, we have $C = 0.0032 \ll B-C$, $WP\kappa = 33.33 \ll r(B-C)(K-a) \le r(B-C)T_n$. So in (3.29), we can approximately omit $WP\kappa+rCT_n$ and $WP\kappa+rC(T_n+\mu)$ from the numerator on both sides, respectively, and get

$$\frac{T_n+G_n-K}{T_n} \le \frac{T_n+\mu+\int_a^b gf_G(g)\,\mathrm{d}g-K}{T_n+\mu}$$

in which B does not exist. Therefore, when $T_n \ge K-a = 1487.5$ second, the value of B almost does not affect the stopping time.

- When $T_n \in (994.8 \text{ second}, 1487.5 \text{ second})$, the heuristic stopping rule is more conservative when the penalty B increases.

Overall, for the heuristic stopping rule, the value of B only affects the stopping decision for vehicle n when $T_n \in$ (994.8 second, 1487.5 second). Therefore, when B increases, the average stopping time of the heuristic rule only slightly decreases in Fig. 3.3. This also explains that the percentage of traffic loss slightly decreases in Fig. 3.4, and that the average energy consumption per second slightly increases in Fig. 3.5.

3.4 Effect of wireless transmission errors

Now we briefly investigate the case when there are wireless transmission errors (e.g., due to wireless link outage or collisions). In specific, if a transmission error happens with the RTS, CTS, DATA, or ACK exchange between the source RSU and a vehicle, retransmission(s) will be needed, which consume more energy. And if the RTS-CTS-DATA-ACK handshake cannot be completed before the vehicle leaves the communication coverage area of the source RSU, the DATA will remain in the source RSU's buffer, and wait for future vehicles.

First we analyze the effect of wireless transmission errors on our optimal stopping rule. When the source RSU makes a decision on whether or not to stop, it does not know how much energy will be consumed if it decides to stop, since possible retransmissions may enlarge the energy consumption. Therefore, the source RSU needs to assess the *expected* energy consumption when it decides whether or not to stop. Denote the probabilities that an RTS, CTS, DATA, and ACK message is successfully received as p_1, p_2, p_3, and p_4, respectively. Then the information exchange between the source RSU and the vehicle follows a Markov chain with five states: RTS, CTS, DATA, ACK and SUCCESS, as shown in Fig. 3.6, in which SUCCESS means that the handshake is successfully completed. In the figure, $\mathrm{e_{RTS}}$, $\mathrm{e_{CTS}}$, $\mathrm{e_{DATA}}$, and $\mathrm{e_{ACK}}$ are the energy consumption for transmission of an RTS, CTS, DATA, and ACK message, respectively.

Denote $\overline{\mathrm{E}}_{\mathrm{RTS}}$, $\overline{\mathrm{E}}_{\mathrm{CTS}}$, $\overline{\mathrm{E}}_{\mathrm{DATA}}$, and $\overline{\mathrm{E}}_{\mathrm{ACK}}$ as the expected energy consumption to reach SUCCESS state if we begin with RTS, CTS, DATA and ACK state,

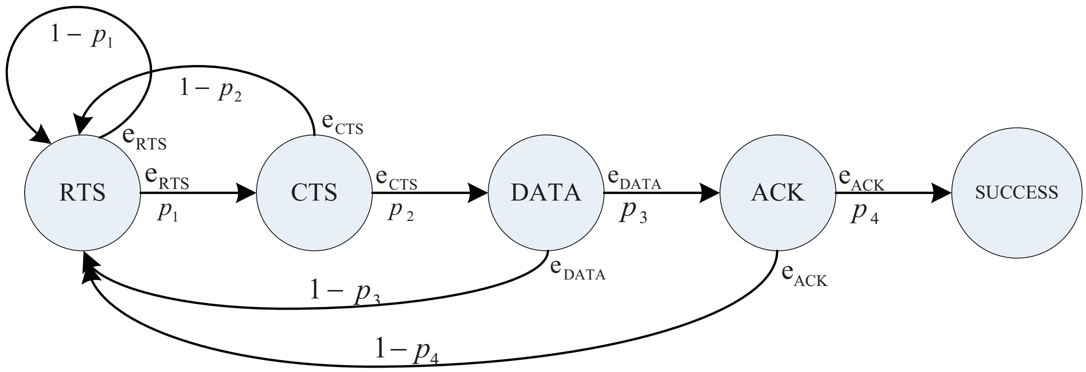

Figure 3.6: Information exchange between the source RSU and a vehicle.

respectively. We have the following equations:

$$\begin{aligned}
\overline{\mathrm{E}}_{\mathrm{ACK}} &= p_4 \mathrm{e}_{\mathrm{ACK}} + (1-p_4)\left(\mathrm{e}_{\mathrm{ACK}} + \overline{\mathrm{E}}_{\mathrm{RTS}}\right)\\
\overline{\mathrm{E}}_{\mathrm{DATA}} &= p_3\left(\mathrm{e}_{\mathrm{DATA}} + \overline{\mathrm{E}}_{\mathrm{ACK}}\right) + (1-p_3)\left(\mathrm{e}_{\mathrm{DATA}} + \overline{\mathrm{E}}_{\mathrm{RTS}}\right)\\
\overline{\mathrm{E}}_{\mathrm{CTS}} &= p_2\left(\mathrm{e}_{\mathrm{CTS}} + \overline{\mathrm{E}}_{\mathrm{DATA}}\right) + (1-p_2)\left(\mathrm{e}_{\mathrm{CTS}} + \overline{\mathrm{E}}_{\mathrm{RTS}}\right)\\
\overline{\mathrm{E}}_{\mathrm{RTS}} &= p_1\left(\mathrm{e}_{\mathrm{RTS}} + \overline{\mathrm{E}}_{\mathrm{CTS}}\right) + (1-p_1)\left(\mathrm{e}_{\mathrm{RTS}} + \overline{\mathrm{E}}_{\mathrm{RTS}}\right).
\end{aligned}$$

By solving the above equations, we have the expected energy consumption if the source RSU stops, as follows:

$$\overline{\mathrm{E}}_{\mathrm{RTS}} = \frac{\mathrm{e}_{\mathrm{RTS}} + p_1 \mathrm{e}_{\mathrm{CTS}} + p_1 p_2 \mathrm{e}_{\mathrm{DATA}} + p_1 p_2 p_3 \mathrm{e}_{\mathrm{ACK}}}{p_1 p_2 p_3 p_4}.$$

The energy consumed to send data (i.e., $\mathrm{e}_{\mathrm{DATA}}$) contains two parts: energy used to send MAC header and preamble: $\mathrm{e}_{\mathrm{MAC+preamble}}$, and energy used to send actual data: $\mathrm{e}_{\mathrm{PAYLOAD}}$ (which is equal to $P(rT_n - A_n)/R$). Since W is cost weight for energy consumption, to successfully transmit traffic from the source RSU to a vehicle, the expected cost of energy consumption is

$$\frac{WP\frac{rT_n - A_n}{R}}{p_3 p_4} + W\frac{\mathrm{e}_{\mathrm{RTS}} + p_1 \mathrm{e}_{\mathrm{CTS}} + p_1 p_2 \mathrm{e}_{\mathrm{MAC+preamble}} + p_1 p_2 p_3 \mathrm{e}_{\mathrm{ACK}}}{p_1 p_2 p_3 p_4}.$$

After a comparison with the cost expression of energy consumption given in Section II, it can be seen that: our analysis and solution in Section II and III

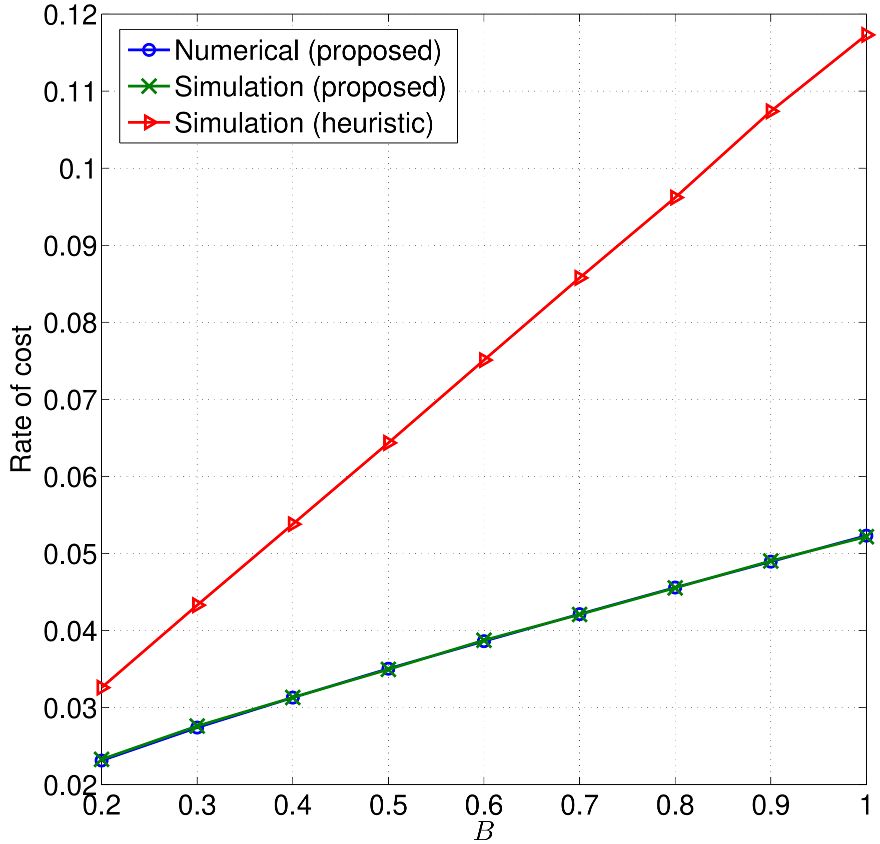

Figure 3.7: The rate of cost of the heuristic rule (simulation results) and the proposed optimal stopping rule (numerical and simulation results) for Problem (3.3) when there are transmission errors ($p_1 = p_2 = p_3 = p_4 = 0.9$).

are still valid, if we replace parameter C with $C/(p_3p_4)$ and replace parameter κ with

$$\frac{\mathrm{e_{RTS}} + p_1\mathrm{e_{CTS}} + p_1p_2\mathrm{e_{MAC+preamble}} + p_1p_2p_3\mathrm{e_{ACK}}}{Pp_1p_2p_3p_4}.$$

With these replacements, similar to Fig. 2, we get the numerical result and simulation result of our derived optimal stopping rule, as well as the simulation result of the heuristic rule, as shown in Fig. 3.7. In the numerical and simulation results, each message (RTS, CTS, DATA, or ACK) is successfully received with a probability 0.9, which means $p_1 = p_2 = p_3 = p_4 = 0.9$. In the simulation for our derived optimal stopping rule and the heuristic rule, if the RTS-CTS-DATA-ACK handshake cannot be completed before the vehicle is outside the communication coverage of the source RSU[15] (the coverage radius is set to be 300 meters in the simulation), the RSU keeps the traffic in its buffer and waits for future vehicle. It can be seen that the numerical and simulation results for our derived stopping rule match with each other. Compared to Fig. 2, wireless transmission errors lead to larger rate of cost in our derived stopping rule and the heuristic rule.

3.5 Conclusion

In this work, we have studied the traffic scheduling in vehicular delay tolerant networks by taking into account the queueing delay, the transmit delay, and the energy consumption. By setting the objective to minimize the rate of cost, we theoretically derive an optimal stopping rule of the source RSU.

3.6 Appendix

3.6.1 Proof of Theorem 1

According to [68], if the following two conditions are met:

$$1.\, \mathbb{E}\left[\inf_n Z_n(\lambda)\right] > -\infty \tag{3.30}$$

$$2.\, \lim_{n\to\infty} Z_n(\lambda) \geq Z_\infty(\lambda) \quad \text{almost surely (a.s.)}, \tag{3.31}$$

then there exits an optimal stopping rule $N^\dagger(\lambda)$ such that $\mathbb{E}\left[Z_{N^\dagger(\lambda)}(\lambda)\right] = V(\lambda)$, where $V(\lambda) = \inf_{N\geq 1} \mathbb{E}\left[Z_N(\lambda)\right]$.

From (3.4), we have

$$A_n \geq \begin{cases} 0 & \text{if} \quad T_n < K - a \\ r(T_n + G_n - K) & \text{if} \quad T_n \geq K - a. \end{cases} \tag{3.32}$$

[15] Note that this rarely happens, since the available communication time of the source RSU and the vehicle is large enough if the transmission successful probability is not extremely low.

For $\lambda \leq B$, we have

$$
\begin{aligned}
& Z_n(\lambda) \\
& = WP\kappa + r\,(C-\lambda)\,T_n + (B-C)A_n \\
& \geq WP\kappa + r\,(C-B)\,T_n + (B-C)A_n \\
& \overset{\text{(e1)}}{\geq} \begin{cases} WP\kappa + r\,(C-B)\,T_n & \text{if} \quad T_n < K-a \\ WP\kappa + r(B-C)(G_n-K) & \text{if} \quad T_n \geq K-a \end{cases} \\
& \overset{\text{(e2)}}{\geq} \begin{cases} WP\kappa + r\,(C-B)\,(K-a) & \text{if} \quad T_n < K-a \\ WP\kappa + r(B-C)(a-K) & \text{if} \quad T_n \geq K-a \end{cases} \\
& = WP\kappa + r(B-C)(a-K) > -\infty
\end{aligned}
$$

where inequality (e1) follows from (3.32), and inequality (e2) follows from $B > C$ and $G_n \geq a$. Thus, the first condition in (3.30) is established.

When $n \to \infty$, we have $T_n \to \infty$ a.s., and thus from (3.4), we have $A_n = r(T_n + G_n - K)$, and

$$
\begin{aligned}
& \lim_{n\to\infty} Z_n(\lambda) \\
& \qquad = \lim_{n\to\infty} \left\{ WP\kappa + r\,(C-\lambda)\,T_n + (B-C)r(T_n+G_n-K) \right\} \\
& \qquad\qquad = \lim_{n\to\infty} \left\{ WP\kappa + r\,(B-\lambda)\,T_n + (B-C)r(G_n-K) \right\}.
\end{aligned}
$$

Define $Z_\infty(\lambda) = \infty$. Then we have $\lim_{n\to\infty} Z_n(\lambda) = Z_\infty(\lambda)$ a.s., and the second condition in (3.31) is satisfied.

3.6.2 Proof of Theorem 2

According to the condition i), we have $\mathbb{E}\left[Y_N - \lambda^* r T_N\right] \geq 0$ for any stopping rule N, which leads to

$$
\frac{\mathbb{E}\left[Y_N\right]}{\mathbb{E}\left[rT_N\right]} \geq \lambda^*. \tag{3.33}
$$

From condition ii), we have $\mathbb{E}\left[Z_{N^\dagger(\lambda^*)}(\lambda^*)\right] = V(\lambda^*) = 0$, which means $\mathbb{E}\left[Y_{N^\dagger(\lambda^*)} - \lambda^* r T_{N^\dagger(\lambda^*)}\right] = 0$. This leads to

$$
\frac{\mathbb{E}\left[Y_{N^\dagger(\lambda^*)}\right]}{\mathbb{E}\left[rT_{N^\dagger(\lambda^*)}\right]} = \lambda^*. \tag{3.34}
$$

From (3.33) and (3.34), $N^\dagger(\lambda^*)$ is an optimal stopping rule for

$$\inf_{N\geq 1} \frac{\mathbb{E}\left[Y_N\right]}{\mathbb{E}\left[rT_N\right]}$$

which is Problem (3.3), and the optimal rate of cost is

$$\frac{\mathbb{E}\left[Y_{N^\dagger(\lambda^*)}\right]}{\mathbb{E}\left[rT_{N^\dagger(\lambda^*)}\right]} = \lambda^*.$$

3.6.3 Proof of Theorem 3

Based on the definition of $Z_n(\lambda)$ in (3.6), when $\lambda = C$, for any n we have $Z_n(\lambda)\Big|_{\lambda=C} \geq WP\kappa > 0$. Hence, based on the definition of $V(\lambda)$ in (3.7), we have $V(C) > 0$.

From (3.5), we have

$$K - a > \frac{WP\kappa}{r(B-C)}.$$

Define

$$\delta \triangleq K - a - \frac{WP\kappa}{r(B-C)} > 0.$$

When $\lambda = B$, from (3.6) we have $Z_n(B) = WP\kappa - (B-C)(rT_n - A_n)$. From (3.7), we have

$$V(B) = \inf_{N\geq 1} \mathbb{E}\left[Z_N(B)\right] \leq \inf_{N\geq 1, T_N > K, G_N < a+\delta} \mathbb{E}\left[Z_N(B)\right] \tag{3.35}$$

because $\{N : N \geq 1, T_N > K, G_N < a + \delta\} \subset \{N : N \geq 1\}$.

When $T_N > K, G_N < a + \delta$, from (3.4) we have $A_N = r(T_N + G_N - K)$, and further, we have

$$\begin{aligned} Z_N(B) &= WP\kappa - (B-C)(rT_N - A_N) \\ &= WP\kappa - (B-C)r(K - G_N) \\ &< WP\kappa - (B-C)r(K - (a+\delta)) \\ &= 0 \end{aligned} \tag{3.36}$$

where the last equality comes from the definition of δ.

From (3.35) and (3.36), we have $V(B) \leq 0$.

Next we show that $V(\lambda)$ is continuous in $[C, B]$. From Theorem 1, it is shown that for $\lambda \leq B$, there exists an optimal stopping rule denoted $N^{\dagger}(\lambda)$ for Problem (3.7). As to be shown in (3.14) and (3.19) in Section 3.2.2 (which show that the average optimal stopping time of Problem (3.7) is finite), we have $\mathbb{E}\left[T_{N^{\dagger}(\lambda)}\right] < \infty$.

Let $C \leq \lambda_1 < \lambda_2 \leq B$. Then

$$\begin{aligned} V(\lambda_1) &= \mathbb{E}\left[Y_{N^{\dagger}(\lambda_1)}\right] - \lambda_1 \mathbb{E}\left[rT_{N^{\dagger}(\lambda_1)}\right] \\ &> \mathbb{E}\left[Y_{N^{\dagger}(\lambda_1)}\right] - \lambda_2 \mathbb{E}\left[rT_{N^{\dagger}(\lambda_1)}\right] \\ &\geq V(\lambda_2) \end{aligned} \tag{3.37}$$

where the first equality comes from the optimality of stopping rule $N^{\dagger}(\lambda_1)$ for Problem (3.7) with $\lambda = \lambda_1$, the first inequality comes from $\lambda_1 < \lambda_2$, and the second inequality comes from the fact that $V(\lambda_2)$ is the minimum cost of Problem (3.7) with $\lambda = \lambda_2$.

From (3.37), it can be seen that $V(\lambda)$ is decreasing in $\lambda \in [C, B]$.

If $\lambda_2 - \lambda_1 < \epsilon/(r\mathbb{E}\left[T_{N^{\dagger}(\lambda_2)}\right])$ where ϵ is a very small positive value (recalling that $\mathbb{E}\left[T_{N^{\dagger}(\lambda)}\right]$ is finite for any $\lambda \in (0, B]$), we have

$$\begin{aligned} &|V(\lambda_2) - V(\lambda_1)| \\ &= V(\lambda_1) - V(\lambda_2) \\ &= V(\lambda_1) - \left(\mathbb{E}\left[Y_{N^{\dagger}(\lambda_2)}\right] - (\lambda_1 + (\lambda_2 - \lambda_1))\mathbb{E}\left[rT_{N^{\dagger}(\lambda_2)}\right]\right) \\ &= V(\lambda_1) - \left(\mathbb{E}\left[Y_{N^{\dagger}(\lambda_2)}\right] - \lambda_1 \mathbb{E}\left[rT_{N^{\dagger}(\lambda_2)}\right]\right) \\ &\qquad + (\lambda_2 - \lambda_1)\mathbb{E}\left[rT_{N^{\dagger}(\lambda_2)}\right] \\ &\leq (\lambda_2 - \lambda_1)\mathbb{E}\left[rT_{N^{\dagger}(\lambda_2)}\right] < \epsilon \end{aligned} \tag{3.38}$$

where the second equality comes from the optimality of $N^{\dagger}(\lambda_2)$ for Problem (3.7) with $\lambda = \lambda_2$, and the first inequality comes from the fact that $V(\lambda_1)$ is the minimum cost of Problem (3.7) with $\lambda = \lambda_1$.

From (3.38), it can be concluded that $V(\lambda)$ is continuous for $\lambda \in [C, B]$. Since $V(C) > 0$ and $V(B) \leq 0$ and $V(\lambda)$ is decreasing in $\lambda \in [C, B]$, according to Intermediate Value Theorem [90], there exists a unique $\lambda^* \in (C, B]$ such that $V(\lambda^*) = 0$.

3.6.4 Proof of Theorem 4

According to Theorem 2 and Corollary 2 in Chapter 5 of [68], if Problem (3.7) when $T_n \geq K - a$ is monotone, then its myopic rule (3.13) is optimal if the following conditions are satisfied:

- i) Z_n can be written as $Z_n = u_n + w_n$, where $\mathbb{E}\left[\sup_n |u_n|\right] < \infty$ and w_n is nonnegative and nondecreasing a.s.;
- ii) $\lim_{n\to\infty} Z_n = Z_\infty$ a.s..

We copy (3.8) here:

$$Z_n(\lambda) = WP\kappa + r(B - \lambda)T_n + r(B - C)(G_n - K).$$

Define $u_n = WP\kappa + r(B - C)(G_n - K)$ and $w_n = r(B - \lambda)T_n$. Since $\lambda \in (0, B]$, Condition i) is satisfied.

Define $Z_\infty(\lambda) = \infty$. Then

$$\begin{aligned} &\lim_{n\to\infty} Z_n(\lambda) \\ &= \lim_{n\to\infty} \big(WP\kappa + r(B - \lambda)T_n + r(B - C)(G_n - K)\big) \\ &= Z_\infty(\lambda) \text{ a.s.} \end{aligned} \tag{3.39}$$

which means Condition ii) is satisfied.

3.6.5 Proof of Theorem 5

Take the first-order derivative of $\phi(g)$ given in (3.11), we have

$$\begin{aligned}\frac{\mathrm{d}\phi(g)}{\mathrm{d}g} &= \frac{f_G(g)}{F_G(g)}\left(g - \frac{(B-\lambda)\mu}{(B-C)F_G(g)} - \frac{\int_a^g x f_G(x)\,\mathrm{d}x}{F_G(g)}\right) \\ &= \frac{f_G(g)}{F_G(g)}\big(g-\phi(g)\big) \\ &= \frac{f_G(g)}{(F_G(g))^2}\big(g-\phi(g)\big)F_G(g) \\ &= \frac{f_G(g)}{(F_G(g))^2}\Psi(g) \qquad (3.40)\end{aligned}$$

in which $\Psi(g) \triangleq \big(g-\phi(g)\big)F_G(g)$. Then $\Psi(b) = (b-\phi(b))F_G(b) = b-\phi(b) = b-b_1 > 0$. Replacing $\phi(g)$ with (3.11), $\Psi(g)$ can be rewritten as

$$\begin{aligned}\Psi(g) &= gF_G(g) - \frac{(B-\lambda)\mu}{B-C} - \int_a^g x f_G(x)\,\mathrm{d}x \\ &= \int_a^g (g-x) f_G(x)\,\mathrm{d}x - \frac{(B-\lambda)\mu}{B-C}\end{aligned}$$

from which it can be seen that $\Psi(g)$ is an increasing function of $g \in (a, b]$, and

$$\lim_{g>a,\ g\to a} \Psi(g) = -\frac{(B-\lambda)\mu}{B-C} < 0.$$

Recalling that $\Psi(b) > 0$, for $g \in (a, b]$ there is a unique root of $\Psi(g) = 0$. Denote the root as b^*. In other words, $\Psi(b^*) = (b^* - \phi(b^*))F_G(b^*) = 0$, which leads to $\phi(b^*) = b^*$. This means, for $g \in (a, b]$, curve $y = \phi(g)$ and curve $y = g$ have a unique common point at $g = b^*$, which proves Part 3) of Theorem 5. Since $\Psi(g)$ is an increasing function of $g \in (a, b]$ and $\lim_{g>a,\ g\to a} \Psi(g) < 0 < \Psi(b)$, we have

- When $g \in (a, b^*)$, $\Psi(g) < 0$. Then from (3.40) we have $\frac{\mathrm{d}\phi(g)}{\mathrm{d}g} < 0$, which means $\phi(g)$ is a decreasing function of $g \in (a, b^*)$;

- When $g \in (b^*, b]$, $\Psi(g) > 0$. Then from (3.40) we have $\frac{\mathrm{d}\phi(g)}{\mathrm{d}g} > 0$, which means $\phi(g)$ is an increasing function of $g \in (b^*, b]$.

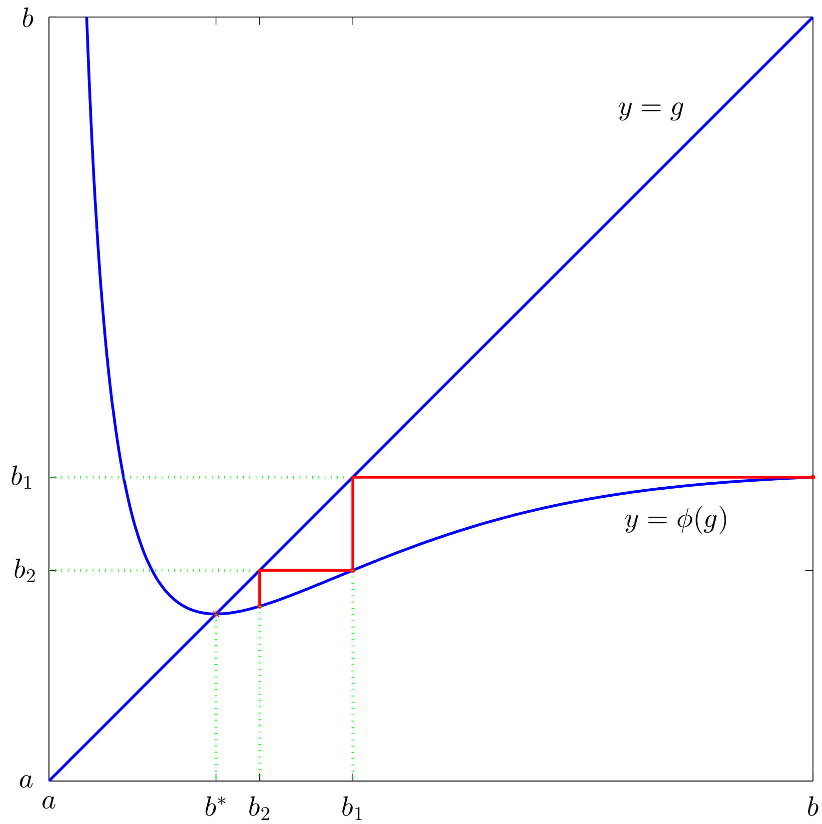

Figure 3.8: Curves $y = \phi(g)$ and $y = g$, and the procedure of obtaining $b \to b_1 \to b_2 \to \ldots$.

These also mean that for $g \in (a, b]$, $g = b^*$ minimizes $\phi(g)$, which proves Part 4) of Theorem 5.

Fig. 3.8 shows the curve $y = \phi(g)$ and curve $y = g$. The red curve in Fig. 3.8 shows how to get $b_1 = \phi(b)$ from b, get $b_2 = \phi(b_1)$ from b_1, ..., etc. It can be seen that the procedure of obtaining $b \to b_1 \to b_2 \to \ldots$ is actually the procedure of finding the unique common point of the curve $y = \phi(g)$ and curve $y = g$. Therefore, it can be concluded that sequence $\{b_i\}_{i=1,2,\ldots}$ converges to b^*, and $b > b_1 > b_2 > \ldots$, which proves Parts 1) and 2) of Theorem 5.

Chapter 4

I2I Traffic Scheduling with a Soft Delay Bound[1]

The contributions of this chapter are summarized as follows. 1) Based on the soft delay bound model, we formulate an optimal stopping problem. In the formulated optimal stopping problem, a concept of forced stop (the definition of forced stop is given in Section 4.1) is introduced. Due to the forced stop, the methods used in the literature to solve traditional optimal stopping problems, including the method used in Chapter 3, do not work here, and a completely new method is required. By characterizing the impact of forced stop, we develop a method to find optimal solution for the formulated problem. 2) We theoretically prove that it is optimal for the source RSU to take a *conditional pure-threshold strategy*. In specific, before a forced stop, the source RSU should transmit to a passing-by vehicle if the queuing delay is above a threshold[2], conditioned on that the total delay (queuing delay plus transit delay of the vehicle) is not more than the delay bound. The conditional pure-threshold structure can largely facilitate implementation of the strategy in a VANET. As a comparison, optimal solution in 3 does not have such a conditional pure-threshold feature, and thus, more

[1]A version of this chapter has been published in IEEE Trans. Intelligent Transportation Systems, 19: 839-853 (2018).

[2]In the sequel, "threshold" means threshold for queuing delay at the source RSU.

computation is needed to make a decision in optimal solution of 3. 3) We provide a method that quickly calculates the threshold off-line. Table 4.1 summarizes important notations used in this chapter.

The following sections are organized as follows. Section 4.1 describes the considered system and formulates the research problem. Section 4.2 derives an optimal strategy of the problem, and proves that the optimal strategy has a conditional pure-threshold structure. Section 4.3 provides an efficient method to obtain the threshold. Section 4.4 evaluates our derived strategy. Section 4.5 concludes this chapter. Appendix 4.6 includes proofs of the theorems in this chapter.

4.1 System model and problem formulation

The system model is similar to the one in Chapter 3, except that a soft delay bound is used here. A source RSU (S-RSU) has constant data traffic arrival rate r. The S-RSU is a remote RSU, and needs to send its data traffic to a destination RSU (D-RSU, which is a central RSU) by using the help of passing-by vehicles. The distance between the S-RSU and D-RSU is d. If a vehicle is selected to help, it takes all buffered data traffic at the S-RSU, and when the vehicle arrives at the D-RSU, it passes all the carried data traffic to the D-RSU .

At the S-RSU, denote the arrival moment of the nth ($n = 1, 2, ...$) vehicle as T_n. Without loss of generality, we set $T_0 = 0$. Similar to [62, 63, 91], the arrival process of vehicles at the S-RSU is a Poisson process with parameter μ, which means that the vehicle inter-arrival durations, denoted as $X_n = T_n - T_{n-1}$ ($n = 1, 2, ...$), are independent exponentially-distributed random variables with mean $1/\mu$. Similar to [92], the speed of the vehicles are independent random variables that are uniformly distributed between $v_{\min}$ (the minimal speed) and

Symbol	Meaning
a, b	Smallest, largest possible transit delay
$\mathcal{C}$	The set of all possible stopping strategies
d	Distance from S-RSU to D-RSU
D	Soft delay bound
$F_S(x)$	Cumulative distribution function of transit delay, given in (4.1)
$\mathscr{F}_{n_r}$	Information of arrival moments and transit delay of vehicles before Vehicle n_r
$N^\dagger(\lambda)$	Optimal stopping strategy of Problem (4.5)
P	Transmission power of RTS, CTS, DATA, and ACK
R	Transmission rate of DATA packets
r	Data traffic arrival rate at the S-RSU
S_n	The transit delay of the nth vehicle
T_n	The arrival moment of the nth vehicle
U_n	Total cost of using the nth vehicle (given in (4.2))
$V(\lambda)$	Optimal objective function of Problem (4.5)
$v_{\min}$	Minimal speed of vehicles
$v_{\max}$	Maximal speed of vehicles
X_n	Time interval from arrival of vehicle $n-1$ to arrival of vehicle n
$Z_n(\lambda)$	Cost function for Problem (4.5)
$1/\mu$	Average duration between two vehicle arrivals
κ	Communication overhead duration between S-RSU and selected vehicle
ω	Cost weight for energy consumption
β	Cost of violating soft delay bound

Table 4.1: Used Notations in Chapter 4

$v_{\max}$ (the maximal speed). As the transit delay is the duration for a vehicle to travel from the S-RSU to the D-RSU, it can be seen that the transit delay has cumulative distribution function (CDF) given as

$$F_S(x) = \begin{cases} 0 & \text{if} \quad x < a; \\ \frac{b(x-a)}{x(b-a)} & \text{if} \quad a \le x \le b; \\ 1 & \text{if} \quad x > b. \end{cases} \tag{4.1}$$

Here $a = d/v_{\max}$ is the smallest possible transmit delay, while $b = d/v_{\min}$ is the largest possible transit delay.

When a vehicle (say the nth vehicle) arrives, the system state is defined as the arrived vehicles' arrival moments (i.e., $T_1, T_2, ..., T_n$) and transit delay (denoted as $S_1, S_2, ..., S_n$), and the S-RSU needs to make a decision between two options, as follows.

- Option *wait*: the nth vehicle is skipped, and the S-RSU waits for later vehicles.

- Option *stop*: the S-RSU stops waiting, and passes its buffered traffic to the nth vehicle by using a four-way handshake RTS-CTS-DATA-ACK. Here RTS means request-to-send, CTS means clear-to-send, DATA carries the data traffic, and ACK means acknowledgement. Denote P as the transmission power of RTS, CTS, DATA, and ACK. Denote κ as the duration of all communication overhead (including RTS, CTS, ACK, as well as the medium access control [MAC] header of the DATA packet). Denote the transmission rate of a DATA packet as R. Similar to [84, 65, 85], we adopt a weighted cost structure for energy consumption and delay as follows. For energy consumption, we assign cost weight ω (unit of cost per Joule). Then the energy consumption cost if the S-RSU stops at the nth vehicle is expressed as $\omega P \left(T_n r/R + \kappa\right)$, in which $T_n r$ is the amount of buffered data traffic at the S-RSU. A soft delay bound D is set for each information unit in the data traffic. If one or multiple information units of the data traffic have a total delay (queuing delay plus transit delay[3]) larger than D, we say that a soft delay bound violation happens, and a fixed charge of β is set (the reason for a fixed charge for one or multiple information units with delay bound violation is given in Section 4.1.1). Therefore, delay cost if the S-RSU stops at the nth vehicle is expressed as $\beta \mathbb{1}_{[T_n+S_n>D]}$, where $\mathbb{1}_{[\cdot]}$ is an indicator function (which is equal to 1 if the event indicated in $\{\cdot\}$ happens, and equal to 0 otherwise), and S_n is transit delay of the nth vehicle. Overall, the total cost if the S-RSU stops at the nth vehicle is given as

$$U_n = \omega P \kappa + \frac{\omega r P T_n}{R} + \beta \mathbb{1}_{[T_n+S_n>D]}. \tag{4.2}$$

Recall that for any vehicle, the transit delay is always not less than a (the smallest possible transit delay). Thus, if the queuing delay at the S-RSU is more than $(D-a)$, the total delay (queuing delay plus transit delay) will be always more than the delay bound D. Therefore, when its queuing delay is more than $(D-a)$, the S-RSU is required to stop when the next vehicle arrives, referred to as a *forced stop*.[4] The index of the vehicle that is the first arrival after moment $(D-a)$ is denoted as $C \triangleq \min\{n : T_n > D-a\}$. Thus, if the queuing delay at the S-RSU is more than $(D-a)$, it will be forced to stop at the Cth vehicle, and the moment of the forced stop is denoted as T_C. The inter-arrival duration between Vehicle C and its previous vehicle is denoted as X_C.

[3]Note that when the S-RSU and a vehicle are exchanging information, the vehicle is also moving. Thus, the duration of transmissions between the S-RSU and the vehicle is included in the transit delay.

[4]Reference [66] uses a concept similar to forced stop. In [66], when the target soft delay bound for a service is crossed, the system must provide the service when the next chance appears.

Denote N as index of the vehicle upon arrival of which the S-RSU stops[5]. We also use N to denote the corresponding stopping strategy. We target at the S-RSU's optimal stopping strategy with minimal rate of cost (i.e., minimal cost per unit time). Define $Y_n \triangleq \omega P\kappa + \beta\mathbb{1}_{[T_n+S_n>D]}$, and $\mathcal{C} \triangleq \{N : 1 \le N \le C\}$ is the set of all possible stopping strategies (i.e., due to the forced stop concept, we exclude stopping rules which will stop at a vehicle arriving after the Cth vehicle). Similar to Chapter 3, to achieve our target, equivalently we should find

$$\begin{aligned} N^* &\triangleq \arg\inf_{N\in\mathcal{C}} \frac{\mathbb{E}[U_N]}{\mathbb{E}[T_N]} \\ &= \arg\inf_{N\in\mathcal{C}} \frac{\mathbb{E}[Y_N] + \omega r P \mathbb{E}[T_N]/R}{\mathbb{E}[T_N]} \\ &= \arg\inf_{N\in\mathcal{C}} \frac{\mathbb{E}[Y_N]}{\mathbb{E}[T_N]}, \end{aligned} \tag{4.3}$$

where $\mathbb{E}[\cdot]$ means expectation.[6] As the vehicle inter-arrival durations are exponentially distributed (which means that the vehicle inter-arrival durations are "memoryless"), we have $\mathbb{E}[T_C] = D - a + \mathbb{E}[X_C] = D - a + 1/\mu < \infty$. Thus, $\mathbb{E}[T_N] < \infty$ for all $N \in \mathcal{C}$.

Remarks: For the formulated problem, there is a tradeoff between wait and stop. If the S-RSU waits less time and picks up a vehicle, the selected vehicle can have a larger chance to deliver all data traffic before delay bound. But it is not energy efficient since the S-RSU needs to have information exchanges with more vehicles in a long term, thus consuming more energy. If the S-RSU waits longer time, it is energy efficient as the S-RSU needs to have information

[5] Note that after the S-RSU stops at a vehicle and transmits its data traffic, we denote this stop moment as $T_0 = 0$, and call the next arrived vehicle as Vehicle 1 again. In other words, the formulated problem is repeated after a stop.

[6] Here we minimize $\mathbb{E}[U_N]/\mathbb{E}[T_N]$ due to the following reason. Since the optimal stopping problem is repeated after a stop, we denote the stopping time in K stops as $T_{N_1}, T_{N_2}, ..., T_{N_K}$ (which are independent and identically distributed), and the corresponding cost in the K stops as $U_{N_1}, U_{N_2}, ...U_{N_K}$ (which are independent and identically distributed), respectively. Then the average cost per unit time is given as $(U_{N_1} + U_{N_2} + ... + U_{N_K})/(T_{N_1} + T_{N_2} + ...T_{N_K})$, which converges to $\mathbb{E}[U_N]/\mathbb{E}[T_N]$ by the law of large numbers [68].

exchanges with fewer vehicles in a long term, but the chance for delay bound violation is also higher. The major challenge in solving the problem is due to the forced stop, which makes the problem different from a traditional optimal stopping problem. A traditional optimal stopping problem does not have forced stop, and thus, methods used to solve traditional optimal stopping problems, including the method used in Chapter 3, cannot be used here. In the sequel, we will develop a completely new method to solve our optimal stopping problem with forced stop.

4.1.1 Reason to have a fixed charge for one or multiple information units with delay bound violation

As an example, we use the typical application of the S-RSU: serve as gateway for a wireless sensor network. So the data traffic at the S-RSU actually carries information of a number of "events" in the wireless sensor network, and each event is corresponding to a number of information units in the data traffic at the S-RSU.

We expect that only a very small portion of the data traffic will have delay bound violation (i.e., cannot be delivered before the delay bound). This is because, if a large portion of data traffic has delay bound violation, this means that the system is not effective to deliver the buffered data traffic at the S-RSU, and thus, a new system is needed (for example, by using cellular communications or satellite communications).

Therefore, when the S-RSU stops at a vehicle, it is very likely that the information units that have delay bound violation belong to the same event. For the same event, a single information unit with delay bound violation and multiple information units with delay bound violation have the same effect on processing of the event: both will make processing of the event at the receiver side delayed. Thus, when there is delay bound violation, we do not make the penalty charge proportional to the number of information units with delay bound violation. Rather, we use a fixed penalty charge for one or multiple information units with delay bound violation.

In addition, if the data traffic of the wireless sensor network is encrypted, an encryption segment consists of a number of information units. At the receiver side, decryption of an encryption segment can be done only after all information units in the segment are received. Then for an encryption segment, a single information unit with delay bound violation and multiple information units with delay bound violation both will make the decryption process at the receiver side delayed. Thus, it is reasonable to charge a fixed penalty when one or multiple information units have delay bound violation.

4.2 An optimal stopping strategy

We have four steps in the following four subsections to derive an optimal stopping strategy for Problem (4.3).

4.2.1 Transformation of the original problem

Define

$$Z_n(\lambda) = Y_n - \lambda T_n = \omega P \kappa + \beta \mathbb{1}_{[T_n + S_n > D]} - \lambda T_n, \ \lambda > 0. \tag{4.4}$$

Here λ can be viewed as rate of cost.

We will first transform Problem (4.3) into a stopping problem that minimizes $\mathbb{E}\left[Z_N(\lambda)\right]$ [68], i.e.,

$$N^{\dagger}(\lambda) = \arg \inf_{N \in \mathcal{C}} \mathbb{E}\left[Z_N(\lambda)\right]. \tag{4.5}$$

Theorem 7. *If i) for any particular $\lambda > 0$, Problem (4.5) has an optimal stopping strategy, denoted as $N^{\dagger}(\lambda)$, and ii) there exists a λ^* such that $\mathbb{E}\left[Z_{N^{\dagger}(\lambda^*)}(\lambda^*)\right] = 0$, then an optimal stopping strategy of Problem (4.3) is in the form of $N^{\dagger}(\lambda^*)$.*

Proof. See Appendix 4.6.1. □

In the subsequent two steps in Sections 4.2.2 and 4.2.3, we derive $N^{\dagger}(\lambda)$ for Problem (4.5). Then in the last step in Section 4.2.4, we prove that there exists λ^* satisfying the above condition ii).

4.2.2 Elimination of a set of non-optimal stopping strategies

In this step, we show that, for Problem (4.5), a set of stopping strategies are non-optimal, and thus, can be removed from our consideration.

For any stopping strategy $N \in \mathcal{C}$, define an event: $\{T_N \leq D - a, T_N + S_N > D\}$. This event means that when the S-RSU stops, it is not a forced stop, and the total delay (queuing delay plus transit delay) is more than the delay bound.

We first consider the following set of stopping strategies:

$$\mathcal{B} = \left\{N \in \mathcal{C} : \mathbb{P}\{T_N \leq D - a, T_N + S_N > D\} > 0\right\},$$

in which $\mathbb{P}\{\cdot\}$ means probability of an event. Next, we show that stopping strategies in $\mathcal{B}$ are strictly non-optimal for Problem (4.5). We can use proof by contradiction. Assume $N \in \mathcal{B}$ is optimal for Problem (4.5). Then based on N, we can construct a new stopping strategy, denoted as N'. The only difference of N' from N is that: if N advises a stopping such that $T_N \leq D - a, T_N + S_N > D$, then N' advises that the S-RSU waits until a forced stop. It can be easily shown that $\mathbb{E}\left[Z_N(\lambda)\right] > \mathbb{E}\left[Z_{N'}(\lambda)\right]$, which contradicts the assumption that strategy N is optimal.

Hence, we need only to search for an optimal stopping strategy within the following collection of stopping strategies:

$$\mathcal{N} = \mathcal{C} \backslash \mathcal{B} = \{N \in \mathcal{C} : T_N + S_N \leq D \text{ if } T_N \leq D - a\}.$$

In other words, upon a vehicle arrival, if the total delay (queuing delay plus transit delay of the vehicle) is above D, and the queuing delay is less than $D - a$ (i.e., it is before the forced stop, which also means that it is still possible for the S-RSU to pick up a later vehicle that can make the total delay bounded by D), then the S-RSU should continue to wait for the next vehicle. Or equivalently, the S-RSU should skip the vehicles which arrive at the S-RSU before moment $(D-a)$ and violate the delay bound. And we re-index the not skipped vehicles as $n_r = 1, 2, ...$ So we have $1 \leq n_r \leq C_r$, where $C_r \triangleq \min\{n_r : T_{n_r} > D - a\}$ means the forced stop.[7] Denote $X_{n_r} = T_{n_r} - T_{n_r - 1}$ ($n_r = 1, 2, ...$). Let N_r denote the corresponding stopping time (the new index of the vehicle upon arrival of which the S-RSU stops) and stopping strategy.

Then we have the following new stopping problem

$$N_r^\dagger(\lambda) = \arg \inf_{N_r \in \mathcal{N}} \mathbb{E}\Big[Z_{N_r}(\lambda) = \omega P \kappa + \beta \mathbb{1}_{[T_{N_r} + S_{N_r} > D]} - \lambda T_{N_r}\Big]. \tag{4.6}$$

4.2.3 Optimal stopping strategy for Problem (4.6) and Problem (4.5)

Consider Problem (4.6). The concept of myopic stopping strategy is given first. Upon a vehicle arrival, the myopic stopping strategy advises the S-RSU to stop if the cost of stopping at the vehicle is not more than the expected cost of skipping the vehicle and stopping at the next vehicle.

[7] Note that C_r (in the re-indexed system with some vehicles skipped) and C (in the initial system) both mean the forced stop, corresponding to the first vehicle arrival after moment $D - a$.

We use $\mathcal{A}_{n_r}$ to denote the event $\{Z_{n_r}(\lambda) \leq \mathbb{E}\left[Z_{n_r+1}(\lambda)|\mathscr{F}_{n_r}\right]\}$, in which $\mathscr{F}_{n_r}$ is the information up to time T_{n_r}. Here $\mathscr{F}_{n_r}$ includes the arrival moments and transit delay of all previous vehicles before Vehicle n_r. So $\mathcal{A}_{n_r}$ means that the myopic strategy advises the S-RSU to stop at Vehicle n_r. We have the following definition for a *monotone problem*.

Definition 3. *Problem (4.6) is monotone if* $\mathcal{A}_1 \subset \mathcal{A}_2 \subset \mathcal{A}_3 \subset \dots$ *almost surely (a.s.) [68].*

In this definition, $\mathcal{A}_{n_r} \subset \mathcal{A}_{n_r+1} \subset \mathcal{A}_{n_r+2} \subset \dots$ means that if the myopic strategy advises the S-RSU to stop at Vehicle n_r, then it will also advise the S-RSU to stop at any future vehicle[8] no matter what the realization of $(T_{n_r+1}, T_{n_r+2}, ...)$ will be (a.s.).

Now, we proceed to show that Problem (4.6) is a monotone problem. Since at moment T_{C_r} the S-RSU is forced to stop, we need only to consider $n_r < C_r$, i.e., $T_{n_r} = t \in [0, D-a]$. Then, we have

$$\begin{aligned}
Z_{n_r}(\lambda) \quad & = \omega P\kappa - \lambda T_{n_r}, \\
\mathbb{E}\left[Z_{n_r+1}(\lambda)|\mathscr{F}_{n_r}\right] \quad & = \mathbb{E}\left[Z_{n_r+1}(\lambda)|T_{n_r} = t\right] \\
& \overset{\text{(i)}}{=} \omega P\kappa + \beta\mathbb{P}\{n_r + 1 = C_r|T_{n_r} = t\} \\
& \quad - \lambda T_{n_r} - \lambda\mathbb{E}\left[X_{n_r+1}|T_{n_r} = t\right],
\end{aligned}$$

in which equality (i) uses the following two equations:

$$\mathbb{E}\left[\mathbb{1}_{\left[T_{n_r+1}+S_{n_r+1}>D\right]}|T_{n_r}=t\right] = \mathbb{P}\{n_r + 1 = C_r|T_{n_r}=t\},$$

$$T_{n_r+1} = T_{n_r} + X_{n_r+1}.$$

For $0 \leq t \leq D-a$, we define :

$$\begin{aligned}
m(t,\lambda) & \overset{\triangle}{=} \mathbb{E}\left[Z_{n_r+1}(\lambda)|\mathscr{F}_{n_r}\right] - Z_{n_r}(\lambda) \\
& = \beta\mathbb{P}\{n_r + 1 = C_r|T_{n_r} = t\} - \lambda\mathbb{E}\left[X_{n_r+1}|T_{n_r} = t\right].
\end{aligned}$$

[8] When we say that the myopic strategy advises the S-RSU to stop at a future vehicle, it is assumed that the S-RSU does not stop at vehicles before that future vehicle.

Theorem 8. *$m(t, \lambda)$ is continuous in $t \in [0, D-a]$. And if for some $t^* \in [0, D-a)$, we have $m(t^*, \lambda) \geq 0$, then $m(t, \lambda)$ is a strictly increasing function in $t \in [t^*, D-a]$.*

Proof. See Appendix 4.6.2. □

For Problem (4.6), if the myopic strategy advises the S-RSU to stop at Vehicle n_r, based on the definition of $m(t, \lambda)$, we have $m(T_{n_r}, \lambda) \geq 0$. Then from Theorem 8, we have $m(T_{n_r+1}, \lambda) > 0$, $m(T_{n_r+2}, \lambda) > 0, \ldots$, for $T_{n_r+1} < D-a, T_{n_r+2} < D-a, \ldots$. In other words, the myopic strategy also advises the S-RSU to stop at any vehicle after Vehicle n_r. Thus, Problem (4.6) is a monotone problem.

In general, the myopic strategy of Problem (4.6) advises the S-RSU to stop at the earliest possible vehicle such that the cost of stopping at the vehicle is not more than the expected cost of skipping the vehicle and stopping at the next vehicle. Thus, the myopic strategy for Problem (4.6) can be expressed as

$$N_r^m(\lambda) = \min \Big\{ \min \left\{ n_r : m(T_{n_r}, \lambda) \geq 0 \right\}, C_r \Big\}, \tag{4.7}$$

in which the superscript m stands for "myopic".

Theorem 9. *The myopic stopping strategy (4.7) is optimal for Problem (4.6).*

Proof. See Appendix 4.6.4. □

Considering the skipped vehicles when we transform Problem (4.5) to Problem (4.6), from Theorem 8 an optimal stopping strategy for Problem (4.5) is

$$N^\dagger(\lambda) = \min \Big\{ \min \left\{ n : T_n \geq T_{\text{th}}(\lambda), T_n + S_n \leq D \right\}, C \Big\}, \tag{4.8}$$

in which $T_{\text{th}}(\lambda)$ is given as

$$T_{\text{th}}(\lambda) = \begin{cases} t^* & \text{if} \quad \exists t^* \in [0, D-a], \; m(t^*, \lambda) = 0; \\ \infty & \text{if} \quad m(D-a, \lambda) < 0. \end{cases} \tag{4.9}$$

4.2.4 Optimal stopping strategy for Problem (4.3)

Recall that $N^{\dagger}(\lambda)$ denotes optimal strategy of Problem (4.5). For Problem (4.5), let $V(\lambda)$ denote the optimal objective function, i.e.,

$$V(\lambda)=\inf_{N\in\mathcal{C}}\left(\mathbb{E}\left[Y_N\right]-\lambda\mathbb{E}\left[T_N\right]\right)=\mathbb{E}\left[Y_{N^{\dagger}(\lambda)}\right]-\lambda\mathbb{E}\left[T_{N^{\dagger}(\lambda)}\right].$$

Theorem 10. *$V(\lambda)$ is strictly decreasing and continuous in $\lambda > 0$.*

Proof. See Appendix 4.6.5. □

If $\lambda \to 0$,

$$\lim_{\lambda\to 0} V(\lambda) = \lim_{\lambda\to 0} \mathbb{E}\left[Y_{N^{\dagger}(\lambda)}\right] \geq \omega P \kappa. \tag{4.10}$$

On the other hand, if $\lambda > \mu(\omega P\kappa + \beta)$, we have

$$\begin{aligned} V(\lambda) &\overset{\text{(ii)}}{=} \omega P\kappa + \beta\mathbb{P}\{T_{N^{\dagger}(\lambda)} + S_{N^{\dagger}(\lambda)} > D\} - \lambda\mathbb{E}\left[T_{N^{\dagger}(\lambda)}\right] \\ &\leq \omega P\kappa + \beta - \lambda\mathbb{E}\left[T_1\right] = \omega P\kappa + \beta - \frac{\lambda}{\mu} < 0, \end{aligned} \tag{4.11}$$

in which equality (ii) uses

$$\mathbb{E}\left[\mathbb{1}_{\left[T_{N^{\dagger}(\lambda)}+S_{N^{\dagger}(\lambda)}>D\right]}\right] = \mathbb{P}\{T_{N^{\dagger}(\lambda)} + S_{N^{\dagger}(\lambda)} > D\}.$$

From Theorem 10 and inequalities (4.10) and (4.11), it can be concluded that there exists one and only one $\lambda^* > 0$ such that $V(\lambda^*) = 0$. In other words, condition ii) of Theorem 7 is satisfied. Thus, according to Theorem 7, an optimal stopping strategy to the Problem (4.3) is

$$N^{\dagger}(\lambda^*) = \min\left\{\min\left\{n : T_n \geq T^*, T_n + S_n \leq D\right\}, C\right\},$$

in which $T^* = T_{\text{th}}(\lambda^*)$. It can be seen that, before a forced stop, the S-RSU is optimal to transmit to a passing-by vehicle if the queuing delay is more than the threshold T^*, conditioned on that the sum of the queuing delay and the transit delay is bounded by D. In other words, the optimal stopping strategy has a conditional pure-threshold structure.

4.3 Derivation of the optimal threshold T^*

To derive the optimal threshold T^*, it is intuitive to firstly obtain the threshold $T_{\text{th}}(\lambda)$ based on (4.9) for each $\lambda(>0)$ value (in which the solution of nonlinear equation $m(t^*, \lambda) = 0$ needs to be calculated numerically), secondly obtain the optimal objective function $V(\lambda)$ of Problem (4.5) for each λ value, and thirdly find λ^* such that $V(\lambda^*) = 0$. Although $T^* = T_{\text{th}}(\lambda^*)$ can be numerically calculated based on this intuitive method, the computational complexity is high. So next we propose to derive T^* from another perspective, to obtain T^* directly.

For Problem (4.3), for $t \in [0, D-a]$, we consider the following stopping strategies:

$$N(t) = \min\Big\{ \min\left\{n : T_n \geq t, T_n + S_n \leq D\right\}, C\Big\}$$

with corresponding objective function denoted as

$$k(t) = \frac{\omega P\kappa + \beta\mathbb{P}\{N(t) = C\}}{\mathbb{E}\left[T_{N(t)}\right]}. \tag{4.12}$$

Then T^* should be the value of t that minimizes $k(t)$.

Theorem 11. *$k(t)$ is continuous in $t \in [0, D-a]$.*

Proof. See Appendix 4.6.6. □

To minimize $k(t)$, we may investigate its derivative expressed as

$$\frac{\mathrm{d}k(t)}{\mathrm{d}t} = l(t)/\Big(\mathbb{E}\left[T_{N(t)}\right]\Big)^2,$$

where

$$\begin{aligned} l(t) = {} & \beta\,\frac{\mathrm{d}\mathbb{P}\{N(t) = C\}}{\mathrm{d}t}\,\mathbb{E}\left[T_{N(t)}\right] \\ & - \frac{\mathrm{d}\mathbb{E}\left[T_{N(t)}\right]}{\mathrm{d}t}\Big(\omega P\kappa + \beta\mathbb{P}\{N(t) = C\}\Big). \end{aligned} \tag{4.13}$$

We have the following theorem.

Theorem 12. *The function $l(t)$ is continuous in the interval $[0, D-a]$ with $l(0) < 0$ and $l(D-a) = 0$. Moreover, if there is a $t^{\ddagger} \in [0, D-a)$ such that $l(t^{\ddagger}) \geq 0$, then $l(t) > 0$ for $t \in (t^{\ddagger}, D-a)$.*

Proof. See Appendix 4.6.7. □

Theorem 12 implies that there is at most one root for $l(t) = 0$, $t \in [0, D-a)$. And if there is a root, denoted as $t^{\S}$, then $l(t) < 0$ in $t \in [0, t^{\S})$ (which means $k(t)$ is strictly decreasing in $t \in [0, t^{\S})$) and $l(t) > 0$ in $t \in (t^{\S}, D-a)$ (which means $k(t)$ is strictly increasing in $t \in (t^{\S}, D-a)$), and thus, the optimal threshold T^* should be $T^* = t^{\S}$.

Based on these conclusions, we can derive T^*, as follows. We consider the following three cases.

Case 1) If $l(D-b) \geq 0$:

Since $l(0) < 0$, and $l(t)$ is continuous in the interval $[0, D-a]$, we can see that $l(t) = 0$ ($t \in [0, D-a)$) has a unique root in $[0, D-b]$. From (4.40) in Appendix 4.6.7, the optimal threshold T^* is the root of

$$\begin{aligned}
&\mu\beta t e^{-\mu(D-t)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}} - \omega P\kappa \Big(1 + \mu e^{-\mu(D-b-t)} \\
&\times \big(g(D-b) - D + b - \frac{1}{\mu})\Big) = 0. \qquad (4.14)
\end{aligned}$$

The method of bisection search can be used to find the root. The corresponding minimum rate of cost is

$$\begin{aligned}
k(T^*) &= \frac{\omega P\kappa + \beta\mathbb{P}\{N(T^*) = C\}}{\mathbb{E}\left[T_{N(T^*)}\right]} \\
&\overset{\text{(iii)}}{=} \frac{\omega P\kappa + \beta e^{-\mu(D-T^*)}\left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}}}{\frac{1}{\mu} + T^* - \Big(D - b + \frac{1}{\mu} - g(D-b)\Big) e^{-\mu(D-b-T^*)}}, \qquad (4.15)
\end{aligned}$$

in which equality (iii) comes from (4.34) and (4.35) in Appendix 4.6.6, and function $g(\cdot)$ is defined in Appendix 4.6.2 and derived in Appendix 4.6.3.

Case 2): If $l(D-b) < 0$ and $l(t)|_{t=(D-a)^-} > 0$ (which is equivalent to $\big(t - g(t) + [\beta/(\omega P\kappa)]h(t)t\big)|_{t=D-a} > 0$ from (4.41) in Appendix 4.6.7, where function $h(\cdot)$ is defined in Appendix 4.6.2 and derived in Appendix 4.6.3):[9]

We can see that $l(t) = 0$ ($t \in [0, D-a)$) has a unique root in $(D-b, D-a)$. From (4.41), the optimal threshold T^* is the root of $t - g(t) + \beta h(t)t/(\omega P\kappa) = 0$. The method of bisection search can be used to find the root. The corresponding minimum rate of cost is

$$k(T^*) = \frac{\omega P\kappa + \beta\mathbb{P}\{N(T^*) = C\}}{\mathbb{E}\left[T_{N(T^*)}\right]} \overset{\text{(iv)}}{=} \frac{\omega P\kappa + \beta h(T^*)}{g(T^*)}, \tag{4.16}$$

in which equality (iv) comes from (4.36) and (4.37) in Appendix 4.6.6.

Case 3): If $l(D-b) < 0$ and $l(t)|_{t=(D-a)^-} \leq 0$ (which is equivalent to $\big(t - g(t) + [\beta/(\omega P\kappa)]h(t)t\big)|_{t=D-a} \leq 0$ from (4.41)):

From Theorem 12, we have $l(t) < 0$ in $[0, D-a)$. Thus, $k(t)$ is a strictly decreasing function in $[0, D-a)$. Since we want to minimize $k(t)$, the optimal threshold should be $T^* = D-a$. In other words, it is optimal to wait for a forced stop. The corresponding minimum rate of cost is

$$k(T^*) = \frac{\omega P\kappa + \beta\mathbb{P}\{N(D-a) = C\}}{\mathbb{E}\left[T_{N(D-a)}\right]} \overset{\text{(v)}}{=} \frac{\omega P\kappa + \beta}{g(D-a)}, \tag{4.17}$$

where equality (v) comes from $\mathbb{P}\{N(D-a) = C\} = 1$ and $\mathbb{E}\left[T_{N(D-a)}\right] = g(D-a)$ (which is from (4.37) in Appendix 4.6.6).

4.4 Performance evaluation

We use Matlab simulation to evaluate our derived stopping strategy. The distance of the S-RSU and D-RSU is $d = 10,000$ m. The S-RSU has a data arrival rate of $r = 5$ bits/second. The soft delay bound is set to be $D = 1,800$ seconds.

[9] Here x^- means a value that is smaller than x but with infinitely small difference.

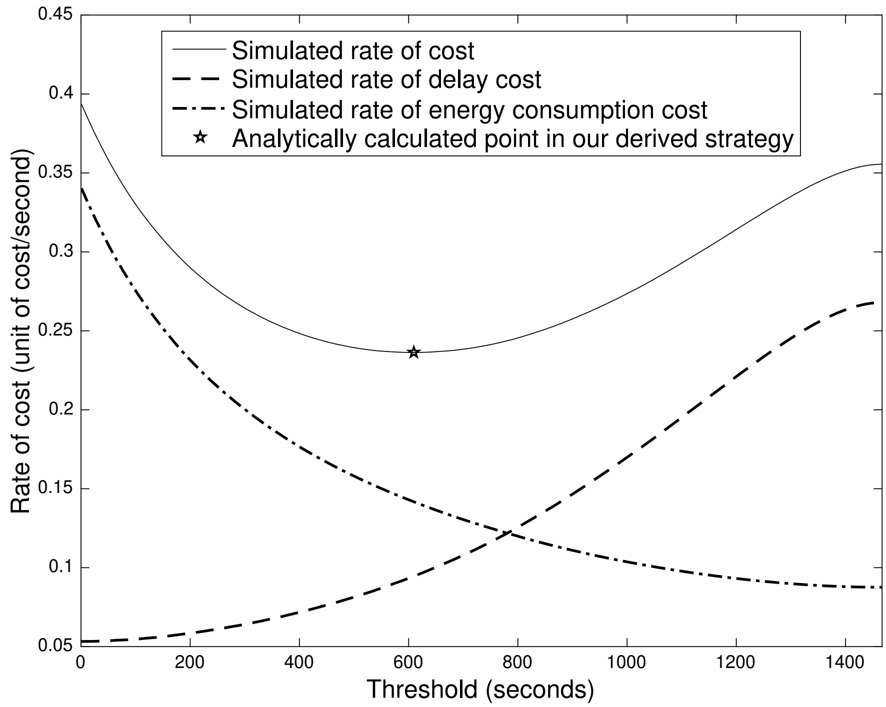

Figure 4.1: Rate of cost in conditional pure-threshold strategies with different thresholds.

Vehicle arrival process at the S-RSU is a Poisson process with parameter μ. If the S-RSU decides to stop at a vehicle, the communication overhead duration is $\kappa = 938.91$ μs.[10] The transmission rate of DATA packets is $R = 11$ Mbps. The transmission power of RTS, CTS, DATA, and ACK is $P = 15.5$ dBm= 35.5 mW. The cost weight for energy consumption is $\omega = 1$ unit of cost per μJoule. We collect simulation statistics over $100,000$ simulation runs.

We first demonstrate that our stopping strategy is optimal. For this purpose, we compare our strategy with other conditional pure-threshold strategies. Here a conditional pure-threshold strategy with threshold η works as follows: before

[10]The overhead 938.91 μs is calculated based on IEEE 802.11 Standard, which includes the following: RTS preamble (192 μs), ratio of RTS size (20 bytes) to RTS transmission rate (2Mb/s), CTS preamble (192 μs), ratio of CTS size (14 bytes) to CTS transmission rate (2Mb/s), DATA preamble (192 μs), ratio of DATA MAC header size (34 bytes) to DATA transmission rate (11 Mb/s), ACK preamble (192 μs), and ratio of ACK size (14 bytes) to ACK transmission rate (11Mb/s).

forced stop, the S-RSU selects the first vehicle (say the nth vehicle) such that $T_n > \eta$ and $T_n + S_n \leq D$; if the S-RSU cannot find such a vehicle, then the forced stop is decided on. We set $\beta = 500$, $\mu = 1/400$ vehicles/second, $v_{\min} = 10$ m/second, and $v_{\max} = 30$ m/second. In conditional pure-threshold strategies with threshold η varying from 0 to $(D-a)$, Fig. 4.1 shows the simulation results of the rate of cost, as well as the simulation results of the rate of energy consumption cost and rate of delay cost [11]. It can be seen that, when the threshold increases, the chance to have delay bound violation is larger, and thus, the delay cost is higher. On the other hand, a larger threshold means that the S-RSU has information exchanges with fewer vehicles in a long term, and thus, the energy consumption cost is lower. Fig. 4.1 also shows the analytically calculated threshold T^* and the corresponding analytically calculated rate of cost (i.e., $k(T^*)$ in (4.15)-(4.17) plus $\omega rP/R$, the difference of $\mathbb{E}[Y_N]/\mathbb{E}[T_N]$ from $\mathbb{E}[U_N]/\mathbb{E}[T_N]$ as shown in (4.3)) in our derived strategy. It is clearly shown that our derived strategy strikes an optimal balance between energy consumption cost and delay cost, and achieves the minimal rate of total cost.

We then vary the penalty cost β from 10 to $10,000$. And for each β value, we exhaustively search the simulated rate of cost in conditional pure-threshold strategies with threshold η varying from 0 to $(D-a)$. Fig. 4.2 shows the searched optimal threshold that achieves the (simulated) minimal rate of cost for each β value, and Fig. 4.3 shows the corresponding (simulated) minimal rate of cost for each β value. As a comparison, for each β value, Fig. 4.2 and Fig. 4.3 also show

[11] In a conditional pure-threshold strategy (including our derived strategy), a delay bound violation happens only at a forced stop. At a forced stop, the average amount of information units with delay bound violation is expressed as $r(1/\mu + \mathbb{E}[S] - a)$, in which $\mathbb{E}[S]$ is average transit delay of a vehicle. Therefore, if the rate of delay cost is expressed as R_d, the amount of information units per unit time with delay bound violation can be expressed as $(R_d/\beta)r(1/\mu + \mathbb{E}[S] - a)$, i.e., proportional to R_d. Thus, the rate of delay cost in our cost function actually can represent the performance of positive system throughput (defined as the amount of information units per unit time that can be delivered to the D-RSU within delay bound).

the analytically calculated threshold T^* in our derived strategy and corresponding analytically calculated rate of cost, respectively. It can be seen that the analytical results and exhaustively searched optimal simulation results match well. When β is small, the S-RSU waits until it is forced to stop, i.e., the optimal threshold is $(D-a) = 1,467$ seconds. This is because the penalty cost β is dominated by the benefit from delivering more traffic in a transmission. In fact, from Case 3) when deriving T^* in Section 4.3, we know that if

$$\big(t - g(t) + [\beta/(\omega P\kappa)]h(t)t\big)|_{t=D-a} \leq 0,$$

which means

$$\beta \leq \frac{\omega P\kappa \left(g(D-a) - D + a\right)}{(D-a)h(D-a)} = 36.4,$$

then the optimal threshold is $(D-a) = 1,467$ seconds. When the value of β increases, the penalty cost begins to dominate, and the optimal threshold value begins to decrease. As an extreme case, the optimal threshold becomes 0 when $\beta \to \infty$, which means that the cost of delay bound violation is too high to afford, and thus, the S-RSU should transmit to the first vehicle that meets the delay bound requirement (i.e., the sum of queuing delay and transit delay is not more than D).

Fig. 4.3 also shows the comparison of our derived stopping strategy with the following heuristic strategy: when the S-RSU's waiting time is less than $D-b$, it is impossible for any vehicle to violate the delay bound requirement (i.e., $T_n + S_n > D$), and thus, the S-RSU does not stop; when the S-RSU's waiting time is more than $D-b$, it is possible that a vehicle would violate the delay bound requirement, and thus, the S-RSU transmits to the next coming vehicle that satisfies the delay bound requirement. So the heuristic strategy is actually a conditional pure-threshold strategy with threshold being $(D-b)$. The

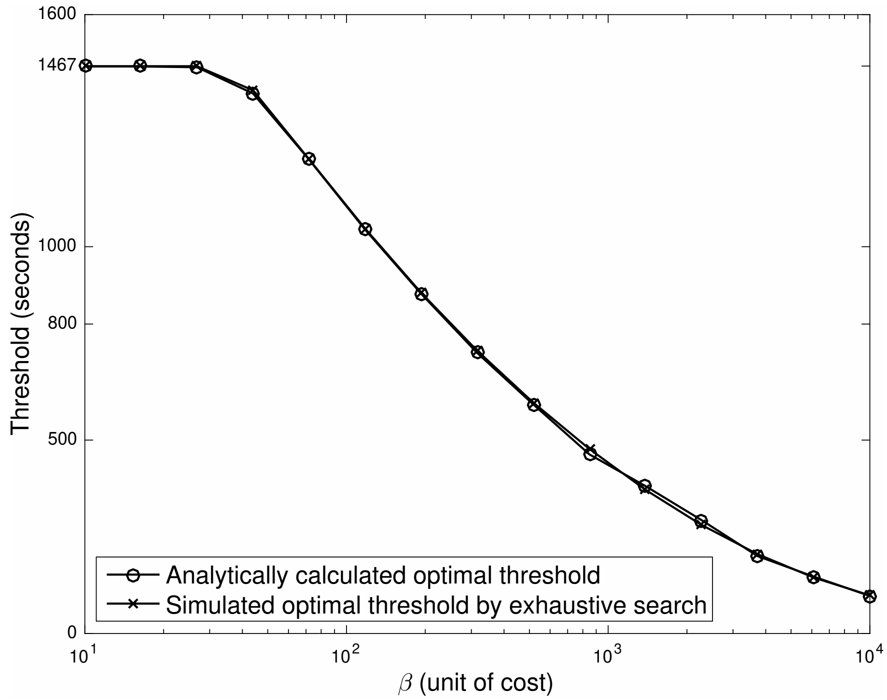

Figure 4.2: The optimal threshold in conditional pure-threshold strategies.

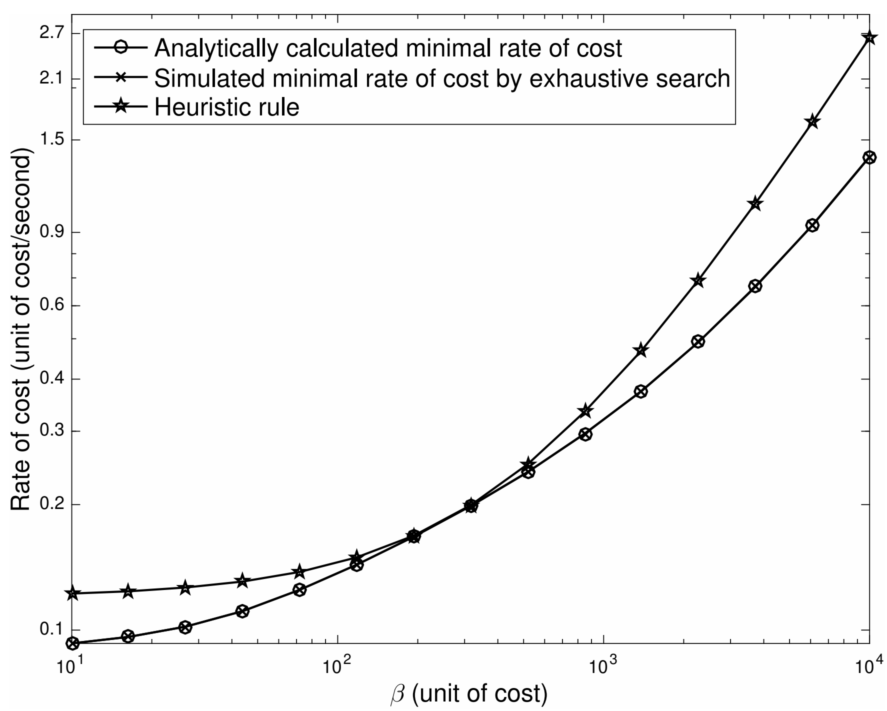

Figure 4.3: The rate of cost in conditional pure-threshold strategies.

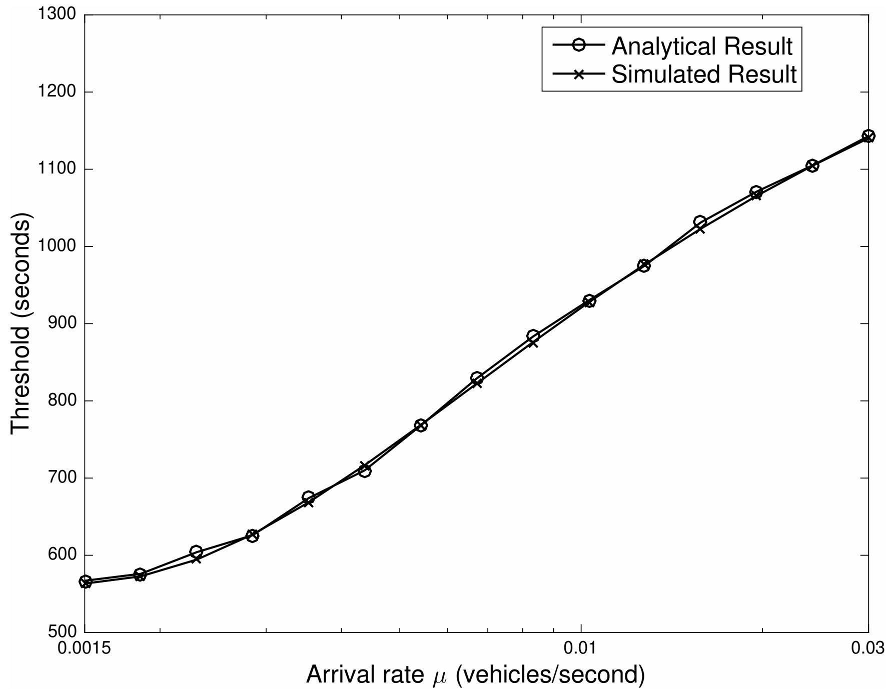

Figure 4.4: The optimal threshold value for different μ.

simulated rate of cost in the heuristic strategy for different β values is shown in Fig. 4.3. It is clear that the heuristic strategy is not optimal in general.

We continue to show how different arrival rates of vehicles at the S-RSU affect the optimal threshold and the rate of cost in the derived stopping strategy. We vary the arrival rate μ of vehicles from 0.0015 to 0.03 vehicles/second. For $\beta = 500$ and different μ, the analytically calculated and simulated (by exhaustive search) optimal thresholds are shown in Fig. 4.4, and analytical calculated and simulated minimal rates of cost are shown in Fig. 4.5. It can be seen that, when μ increases, the optimal threshold increases, and the minimal rate of cost decreases. This is because, when it is expected that vehicles arrive more frequently, the S-RSU can hold the traffic in its buffer for a longer time, and thus, each transmission can deliver more traffic, which leads to a smaller rate of cost.

Next we show how the value of the soft delay bound D affects the optimal

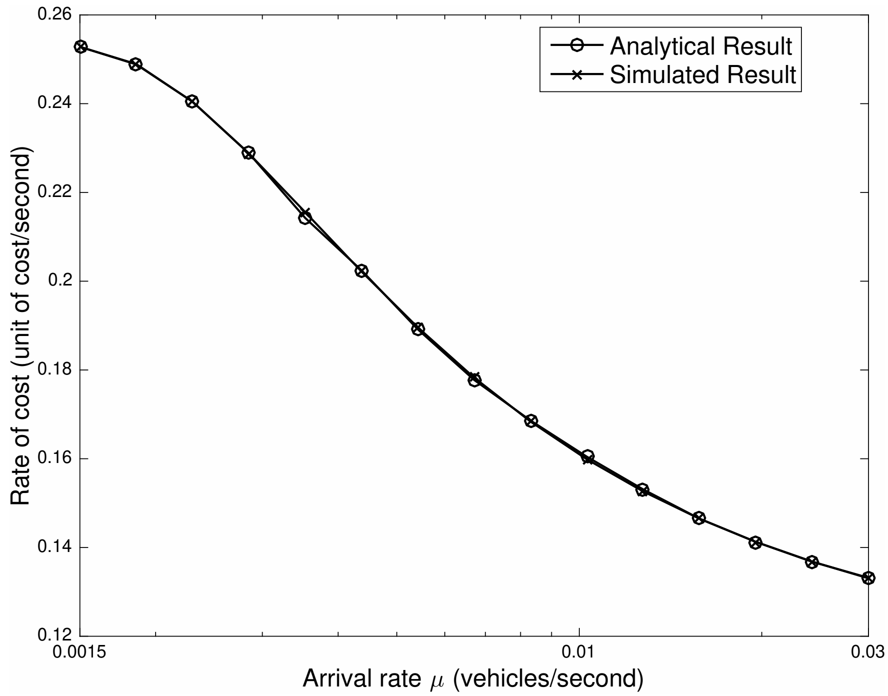

Figure 4.5: The minimal rate of cost for different μ.

threshold and the rate of cost in the derived stopping strategy. For $\beta = 500$ and $\mu = 1/400$ vehicles/second, we vary D from 1,000 seconds to 40,000 seconds. The analytically calculated and simulated (by exhaustive search) optimal thresholds are shown in Fig. 4.6, and analytical calculated and simulated minimal rates of cost are shown in Fig. 4.7. It can be seen that, with a larger delay bound D, the S-RSU can wait more time before stop, and thus, the optimal threshold increases, and the minimal rate of cost decreases (as the S-RSU has information exchanges with fewer vehicles in a long term, thus decreasing energy consumption).

4.5 Conclusion

This chapter studies vehicle-aided communications from a remote RSU to a central RSU. Costs are assigned to energy consumption as well as possible violation of a soft delay bound. We theoretically prove that an optimal stopping strategy

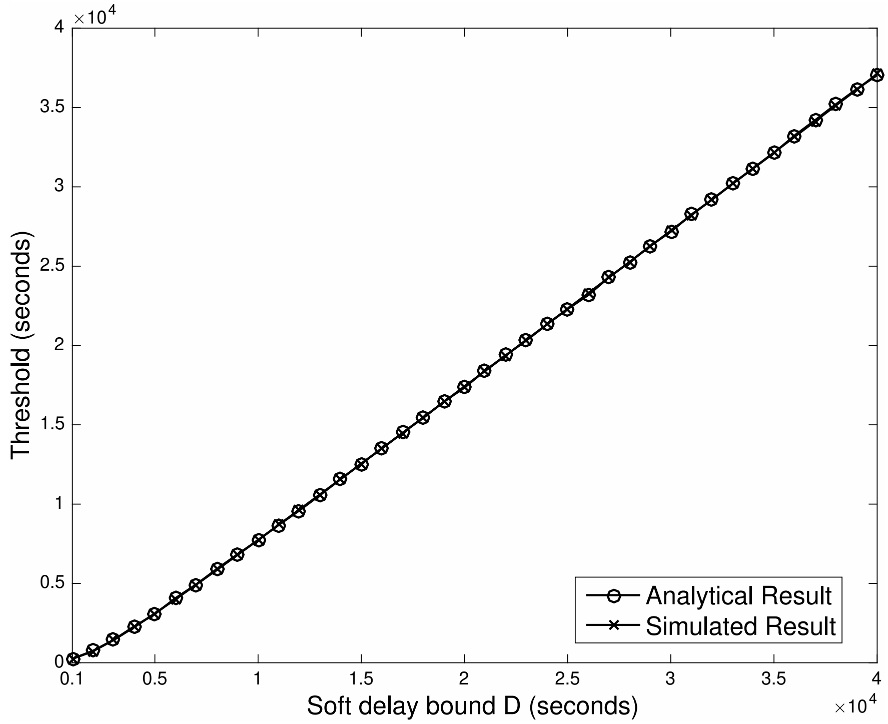

Figure 4.6: The optimal threshold value for different D.

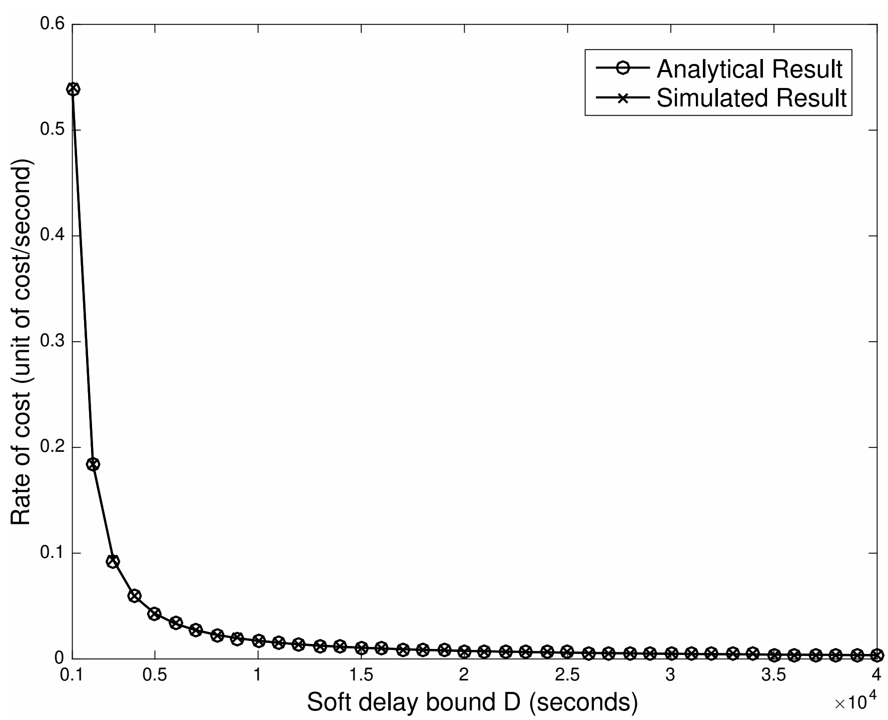

Figure 4.7: The minimal rate of cost for different D.

has a conditional pure-threshold structure, and the threshold can be calculated offline quickly by our provided method. Upon arrival of a passing-by vehicle, if the vehicle can meet the delay bound requirement, the S-RSU only needs to compare the arrival moment of the vehicle with the threshold to make its decision. Thus, the derived stopping strategy can be implemented in a VANET easily with very low complexity.

4.6 Appendix

4.6.1 Proof of Theorem 7

Considering Problem (4.5) with λ^*, from i) we know that $N^\dagger(\lambda^*)$ is optimal stopping strategy. In other words, for any stopping strategy $N \in \mathcal{C}$, we have $\mathbb{E}\left[Z_N(\lambda^*)\right] = \mathbb{E}\left[Y_N - \lambda^* T_N\right] \geq \mathbb{E}\left[Z_{N^\dagger(\lambda^*)}(\lambda^*)\right] = 0$, in which the last equality comes from ii). Based on this, we have $\mathbb{E}\left[Y_N\right]/\mathbb{E}\left[T_N\right] \geq \lambda^*$.

Further, from (4.4), we have $\mathbb{E}\left[Z_{N^\dagger(\lambda^*)}(\lambda^*)\right] = \mathbb{E}\left[Y_{N^\dagger(\lambda^*)} - \lambda^* T_{N^\dagger(\lambda^*)}\right]$. As $\mathbb{E}\left[Z_{N^\dagger(\lambda^*)}(\lambda^*)\right] = 0$ which is from ii), we have $\mathbb{E}\left[Y_{N^\dagger(\lambda^*)}\right]/\mathbb{E}\left[T_{N^\dagger(\lambda^*)}\right] = \lambda^*$. Together with $\mathbb{E}\left[Y_N\right]/\mathbb{E}\left[T_N\right] \geq \lambda^*$, we can see that among all stopping strategies in $\mathcal{C}$, $N^\dagger(\lambda^*)$ minimizes $\mathbb{E}\left[Y_N\right]/\mathbb{E}\left[T_N\right]$, and thus, is an optimal stopping strategy of Problem (4.3), with the optimal value of the objective function of Problem (4.3) being λ^*.

4.6.2 Proof of Theorem 8

Since the expression of $m(t, \lambda)$ includes the following two terms: $\mathbb{P}\{n_r + 1 = C_r | T_{n_r} = t\}$ and $\mathbb{E}\left[X_{n_r+1} | T_{n_r} = t\right]$ ($t \in [0, D - a]$), we need to calculate the two terms. We consider the following two cases.

- When $0 \le t < (D-b)$: We have

$$
\begin{aligned}
&\mathbb{P}\{n_r+1=C_r|T_{n_r}=t\}\\
\overset{\text{(vi)}}{=}&\mathbb{P}\{n_r+1=C_r|T_{n_r+1}\ge D-b, T_{n_r}=t\}\\
&\times \mathbb{P}\{T_{n_r+1}\ge D-b|T_{n_r}=t\}\\
\overset{\text{(vii)}}{=}&h(D-b)e^{-\mu(D-b-t)}\\
=&\left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}}e^{-\mu b}e^{-\mu(D-b-t)}\\
=&e^{-\mu(D-t)}\left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}}, \qquad (4.18)
\end{aligned}
$$

in which equality (vi) uses Total Probability Theorem and the fact $\mathbb{P}\{n_r+1=C_r|T_{n_r+1}<D-b, T_{n_r}=t\}=0$, and equality (vii) uses $\mathbb{P}\{n_r+1=C_r|T_{n_r+1}\ge D-b, T_{n_r}=t\}=\mathbb{P}\{n_r+1=C_r|T_{n_r+1}\ge D-b\}$ and $\mathbb{P}\{T_{n_r+1}\ge D-b|T_{n_r}=t\}=e^{-\mu(D-b-t)}$ (recalling that vehicle inter-arrival durations are exponentially distributed with parameter μ). Here $h(\tau)\triangleq\mathbb{P}\{n_r+1=C_r|T_{n_r+1}\ge\tau\},\ D-b\le\tau\le D-a$, is derived in Appendix 4.6.3.

$$
\begin{aligned}
&\mathbb{E}\left[X_{n_r+1}|T_{n_r}=t\right]\\
\overset{\text{(viii)}}{=}&\mathbb{E}\left[X_{n_r+1}|T_{n_r+1}<D-b, T_{n_r}=t\right]\\
&\times \mathbb{P}\{T_{n_r+1}<D-b|T_{n_r}=t\}\\
&+\mathbb{E}\left[X_{n_r+1}|T_{n_r+1}\ge D-b, T_{n_r}=t\right]\\
&\times \mathbb{P}\{T_{n_r+1}\ge D-b|T_{n_r}=t\}\\
\overset{\text{(ix)}}{=}&\int_0^{D-b-t}\mu x e^{-\mu x}\,\mathrm{d}x+\big(g(D-b)-t\big)\,e^{-\mu(D-b-t)}\\
=&\frac{1}{\mu}-\left(D-b+\frac{1}{\mu}-g(D-b)\right)e^{-\mu(D-b-t)}, \qquad (4.19)
\end{aligned}
$$

in which equality (viii) uses Total Probability Theorem, equality (ix) uses the fact that the vehicle inter-arrival durations are exponentially distributed with parameter μ, and $g(\tau) \triangleq \mathbb{E}\left[T_{n_r+1}|T_{n_r+1} \geq \tau\right],\ D-b \leq \tau \leq D-a$, is derived in Appendix 4.6.3.

- When $(D-b) \leq t \leq (D-a)$: We have

$$\begin{aligned}&\mathbb{P}\{n_r+1 = C_r|T_{n_r} = t\}\\ &= h(t) = \left(\frac{D-t}{a}\right)^{\frac{\mu ab}{b-a}} e^{-\frac{\mu b(D-a-t)}{b-a}}\end{aligned}$$

  and $\mathbb{E}\left[X_{n_r+1}|T_{n_r} = t\right] = g(t) - t$.

Then $m(t,\lambda)$ can be expressed as

$$\begin{aligned}&m(t,\lambda)\\ &= \begin{cases} \beta e^{-\mu(D-t)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}} - \lambda\Big(\frac{1}{\mu} - \big(D-b+\frac{1}{\mu} - g(D-b)\big) \\ \qquad \times e^{-\mu(D-b-t)}\Big), \text{ if } 0 \leq t < D-b; \\ \beta h(t) - \lambda\left(g(t) - t\right), \quad \text{if } D-b \leq t \leq D-a. \end{cases}\end{aligned} \tag{4.20}$$

It can be verified from (4.20) that $m(t,\lambda)$ is continuous for $t \in [0, D-a]$.

Suppose $\lambda > 0$ and there is a $t^* \in [0, D-a)$ satisfying $m(t^*,\lambda) \geq 0$.

If $t^* \in [0, D-b)$, from (4.20), we have

$$\begin{aligned}m(t^*,\lambda) =& \beta e^{-\mu(D-t^*)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}} - \lambda\Big(\frac{1}{\mu} - \big(D-b\\ &+ \frac{1}{\mu} - g(D-b)\big)e^{-\mu(D-b-t^*)}\Big) \geq 0.\end{aligned}$$

Then, we have

$$\left.\frac{\partial m(t,\lambda)}{\partial t}\right|_{t=t^*} = \mu m(t^*,\lambda) + \lambda > 0. \tag{4.21}$$

If $t^* \in [D-b, D-a)$, from (4.20), we have

$$m(t^*,\lambda) = \beta h(t^*) - \lambda\big(g(t^*) - t^*\big) \geq 0.$$

Then, we have

$$\begin{aligned}
\frac{\partial m(t,\lambda)}{\partial t}\bigg|_{t=t^*} &= \left(\beta\,\frac{\mathrm{d}h(t)}{\mathrm{d}t} - \lambda\,\frac{\mathrm{d}g(t)}{\mathrm{d}t} + \lambda\right)\bigg|_{t=t^*} \\
&\overset{\text{(x)}}{=} \beta\mu F_S(D-t^*)h(t^*) - \lambda\mu F_S(D-t^*) \\
&\quad \times \big(g(t^*)-t^*\big) + \lambda \\
&= \mu F_S(D-t^*)\Big(\beta h(t^*) - \lambda\big(g(t^*)-t^*\big)\Big) + \lambda \\
&= \mu F_S(D-t^*)m(t^*,\lambda) + \lambda > 0,
\end{aligned} \tag{4.22}$$

where equality (x) uses the following two equations:

$$\begin{aligned}
\frac{\mathrm{d}h(t)}{\mathrm{d}t} &= \mu F_S(D-t)h(t), \\
\frac{\mathrm{d}g(t)}{\mathrm{d}t} &= \mu F_S(D-t)\big(g(t)-t\big),
\end{aligned}$$

which are from Appendix 4.6.3.

Thus, if for some $t^* \in [0, D-a)$, we have $m(t^*,\lambda) \geq 0$, then from (4.21) and (4.22) we have

$$\frac{\partial m(t,\lambda)}{\partial t} > 0 \quad \text{for } t \in [t^*, D-a),$$

which means $m(t,\lambda)$ is strictly increasing in $t \in [t^*, D-a]$.

4.6.3 Derivation of $h(\tau)$ and $g(\tau)$

We consider the following stopping strategy:

$$N(\tau) = \min\Big\{\min\{n : T_n \geq \tau, T_n + S_n \leq D\}, C\Big\}$$

for $D-b \leq \tau \leq D-a$.

According to the definitions of $h(\tau)$ and $g(\tau)$ in Appendix 4.6.2, we have

$$h(\tau) = \mathbb{P}\{N(\tau) = C\}$$

$$g(\tau) = \mathbb{E}\left[T_{N(\tau)}\right]$$

with the following boundary conditions:

$$h(D-a)=1$$
$$g(D-a)=D-a+\frac{1}{\mu}.$$

We first derive $h(\tau)$. For $D-b<\tau\leq D-a$, consider a sufficiently small $\Delta\tau$ such that $\tau-\Delta\tau\geq D-b$. Recall that vehicles arrive at the S-RSU following a Poisson process with parameter μ. Thus, within duration $(\tau-\Delta\tau,\tau)$, the probabilities of no vehicle arrival, one vehicle arrival, and two or more vehicle arrivals are expressed as $(1-\mu\Delta\tau)$, $\mu\Delta\tau$, and $o(\Delta\tau)$ (higher order of $\Delta\tau$), respectively. Consider stopping strategy $N(\tau-\Delta\tau)$. If no vehicle arrives within duration $(\tau-\Delta\tau,\tau)$, then the S-RSU should continue to wait for the next vehicle that comes after moment τ. If one vehicle, say the nth vehicle, arrives within duration $(\tau-\Delta\tau,\tau)$, and $T_n+S_n\leq D$, then the S-RSU stops and there is no need to wait after τ. If one vehicle, say the nth vehicle, arrives within duration $(\tau-\Delta\tau,\tau)$, and $T_n+S_n>D$, then the RSU should skip this vehicle and continue to wait for the next vehicle that comes after moment τ. As a summary, we have

$$\begin{aligned}h(\tau-\Delta\tau)=&(1-\mu\Delta\tau)h(\tau)+\mu\Delta\tau\left(1-F_S(D-\tau)\right)h(\tau)\\&+o(\Delta\tau),\end{aligned}$$

which leads to

$$\frac{h(\tau)-h(\tau-\Delta\tau)}{\Delta\tau}=\mu F_S(D-\tau)h(\tau)-\frac{o(\Delta\tau)}{\Delta\tau}.$$

Letting Δt approach zero, we have

$$\frac{\mathrm{d}h(\tau)}{\mathrm{d}\tau}=\mu F_S(D-\tau)h(\tau). \tag{4.23}$$

Using the initial condition $h(D-a)=1$, we obtain

$$h(\tau)=e^{\mu\int_{D-a}^{\tau}F_S(D-x)\,\mathrm{d}x}=\left(\frac{D-\tau}{a}\right)^{\frac{\mu ab}{b-a}}e^{-\frac{\mu b(D-\tau-a)}{b-a}} \tag{4.24}$$

for $D-b \leq \tau \leq D-a$.

Since we have

$$\frac{\mathrm{d}h(\tau)}{\mathrm{d}\tau} = \mu F_S(D-\tau)h(\tau) > 0 \quad \text{for } \tau \in [D-b, D-a)$$

and

$$\frac{\mathrm{d}h(\tau)}{\mathrm{d}\tau}|_{\tau=D-a} = \mu F_S(a)h(D-a) = 0 \text{ (as } F_S(a)=0\text{)},$$

it can be concluded that $h(\tau)$ is strictly increasing in $[D-b, D-a]$.

Similar to the derivation of $h(\tau)$, for $g(\tau)$ we have

$$\begin{aligned} g(\tau-\Delta\tau) = (1-\mu\Delta\tau)g(\tau) &+ \mu\Delta\tau F_S(D-\tau)\tau \\ &+ \mu\Delta\tau(1-F_S(D-\tau))g(\tau) + o(\Delta\tau), \end{aligned}$$

which leads to

$$\frac{g(\tau)-g(\tau-\Delta\tau)}{\Delta\tau} = \mu F_S(D-\tau)\left(g(\tau)-\tau\right) - \frac{o(\Delta\tau)}{\Delta\tau}.$$

Letting $\Delta\tau$ approach zero, we have

$$\frac{\mathrm{d}g(\tau)}{\mathrm{d}\tau} = \mu F_S(D-\tau)\left(g(\tau)-\tau\right). \tag{4.25}$$

Using the initial condition $g(D-a) = D-a+1/\mu$, we obtain

$$\begin{aligned} g(\tau) = D - h(\tau)\Big[a - \frac{1}{\mu} + \frac{\mu b}{b-a}\left(\frac{a}{e}\right)^{\frac{\mu a b}{b-a}} \\ \times \int_a^{D-\tau}(z-a)z^{-\frac{\mu a b}{b-a}}e^{\frac{\mu b z}{b-a}}\,\mathrm{d}z\Big] \end{aligned} \tag{4.26}$$

for $D-b \leq \tau \leq D-a$, where $h(\tau)$ is given in (4.24).

4.6.4 Proof of Theorem 9

Since the S-RSU is forced to stop when $n_r = C_r$, for presentation simplicity we can set $T_{n_r} = T_{C_r}$ for $n_r > C_r$. Then the objective function of Problem (4.6) can be written as $Z_{n_r}(\lambda) = \omega P\kappa + \beta \mathbb{1}_{[n_r = C_r]} - \lambda T_{n_r}$. We have

$$
\begin{aligned}
\mathbb{E}\left[\inf{}_{n_r}(Z_{n_r}(\lambda))\right] &\geq \omega P\kappa + \mathbb{E}\left[\inf{}_{n_r}(-\lambda T_{n_r})\right] \\
&= \omega P\kappa - \mathbb{E}\left[\sup{}_{n_r}\lambda T_{n_r}\right] \\
&\geq \omega P\kappa - \lambda \mathbb{E}\left[T_{C_r}\right] \\
&= \omega P\kappa - \lambda\left(D - a + \frac{1}{\mu}\right) \\
&> -\infty. \qquad (4.27)
\end{aligned}
$$

According to Wald's Equation, $\mathbb{E}[T_{C_r}] = \mathbb{E}[C]/\mu$. Thus, $\mathbb{E}[C_r] \leq \mathbb{E}[C] < \infty$ (since $\mathbb{E}[T_{C_r}] < \infty$), which leads to $\mathbb{1}_{[C_r<\infty]} = 1$ a.s.. Again, since the RSU is required to stop when $n_r = C_r$, the stopping strategies that we consider have the property $N_r(\lambda) \leq C_r$ and thus

$$
\mathbb{E}\left[N_r(\lambda)\right] < \infty \quad a.s.. \qquad (4.28)
$$

According to Theorem 3.1 in [68], when the two inequalities (4.27) and (4.28) hold, there exists an optimal stopping strategy $N_r^{\dagger}(\lambda)$ for Problem (4.6), and the optimal (minimal) objective function of the problem is denoted as $V^*(\lambda) = \mathbb{E}\left[Z_{N_r^{\dagger}(\lambda)}\right]$.

To prove the optimality of the myopic stopping strategy $N_r^m(\lambda)$ in (4.7) for Problem (4.6), it suffices if we can show that the optimal objective function of Problem (4.6) is not less than the objective function of the myopic stopping strategy $N_r^m(\lambda)$, as follows.

For Problem (4.6), if the S-RSU is forced to stop when the vehicle index n_r is more than J, then we call this problem bounded at J. Recall that Problem (4.6) is monotone problem. Thus, Problem (4.6) bounded at J is a finite horizon monotone problem, and thus, according to Theorem 5.1 in [68], the corresponding myopic strategy for Problem (4.6) bounded at J, given as $N_r^{m,(J)}(\lambda) =$

$\min\Big\{\min\left\{n_r : m(T_{n_r},\lambda)\geq 0\right\}, C_r, J\Big\}$, is optimal, with the achieved objective function denoted as $V^{(J)}(\lambda)=\mathbb{E}\left[Z_{N_r^{m,(J)}(\lambda)}(\lambda)\right]$.

Based on $N_r^{\dagger}(\lambda)$, we define a new stopping strategy as $N_r^{[J]}(\lambda)=\min\{N_r^{\dagger}(\lambda), J\}$, for $J\geq 1$, and denote the corresponding objective function as $V^{[J]}(\lambda)$. Then we have $V^{[\infty]}(\lambda)=V^*(\lambda)$. Since stopping strategy $N_r^{m,(J)}(\lambda)$ is optimal for Problem (4.6) bounded at J, and $N_r^{[J]}(\lambda)$ is a stopping strategy for Problem (4.6) bounded at J, we have $V^{[J]}(\lambda)\geq V^{(J)}(\lambda)$. Then

$$\begin{aligned}
0\leq & V^{(J)}(\lambda)-V^*(\lambda)\leq \mathbb{E}\left[Z_{N_r^{[J]}(\lambda)}(\lambda)\right]-\mathbb{E}\left[Z_{N_r^{\dagger}(\lambda)}(\lambda)\right]\\
=&\mathbb{E}\left[\mathbb{1}_{\left[N_r^{\dagger}(\lambda)>J\right]}\left(Z_{N_r^{[J]}(\lambda)}(\lambda)-Z_{N_r^{\dagger}(\lambda)}(\lambda)\right)\right]\\
=&\mathbb{E}\left[\mathbb{1}_{\left[N_r^{\dagger}(\lambda)>J\right]}\left(Z_{N_r^{[J]}(\lambda)=J}(\lambda)-Z_{N_r^{\dagger}(\lambda)}(\lambda)\right)\right]\\
=&\mathbb{E}\Bigg[\mathbb{1}_{\left[N_r^{\dagger}(\lambda)>J\right]}\Bigg(\beta\Big(\mathbb{1}_{[T_J+S_J>D]}-\mathbb{1}_{\left[T_{N_r^{\dagger}(\lambda)}+S_{N_r^{\dagger}(\lambda)}>D\right]}\Big)\\
&\quad+\lambda(T_{N_r^{\dagger}(\lambda)}-T_J)\Bigg)\Bigg]\\
\leq&\mathbb{E}\left[\mathbb{1}_{\left[N_r^{\dagger}(\lambda)>J\right]}\left(2\beta+\lambda T_{N_r^{\dagger}(\lambda)}\right)\right]\\
\leq&\mathbb{E}\left[\mathbb{1}_{\left[N_r^{\dagger}(\lambda)>J\right]}\left(2\beta+\lambda T_{C_r}\right)\right].
\end{aligned}\tag{4.29}$$

Since $\mathbb{P}\{N_r^{\dagger}(\lambda)>J\}\to 0$ as $J\to\infty$, then we have $\mathbb{E}\left[\mathbb{1}_{\left[N_r^{\dagger}(\lambda)>J\right]}\left(2\beta+\lambda T_{C_r}\right)\right]\to 0$ as $J\to\infty$. Then from (4.29) we have

$$\begin{aligned}
V^*(\lambda)=\lim_{J\to\infty}V^{(J)}(\lambda)&=\lim_{J\to\infty}\mathbb{E}\left[Z_{N_r^{m,(J)}(\lambda)}(\lambda)\right]\\
&=\liminf_{J\to\infty}\mathbb{E}\left[Z_{N_r^{m,(J)}(\lambda)}(\lambda)\right]\\
&\overset{\text{(xi)}}{\geq}\mathbb{E}\left[\liminf_{J\to\infty}Z_{N_r^{m,(J)}(\lambda)}(\lambda)\right]\\
&\overset{\text{(xii)}}{=}\mathbb{E}\left[Z_{N_r^{m}(\lambda)}(\lambda)\right],
\end{aligned}\tag{4.30}$$

in which inequality (xi) follows from (4.27) by applying Fatou's lemma [86], and equality (xii) follows from the fact that $N_r^{m,(J)}(\lambda)$ is an increasing sequence of stopping strategies converging to $N_r^m(\lambda)$. Because of (4.28), $N_r^m(\lambda)$ is a fixed integer from some J on a.s.. Thus, we have $\liminf_{J\to\infty} Z_{N_r^{m,(J)}(\lambda)}(\lambda) = Z_{N_r^m(\lambda)}(\lambda)$ a.s..

Inequality (4.30) means that the achieved objective function in the optimal stopping strategy for Problem (4.6) is not less than the achieved objective function of the myopic stopping strategy $N_r^m(\lambda)$. So the myopic stopping strategy is optimal for Problem (4.6).

4.6.5 Proof of Theorem 10

Consider positive λ_2 and λ_1 satisfying $\lambda_2 > \lambda_1$. We have

$$\begin{aligned} V(\lambda_1) &= \mathbb{E}\left[Y_{N^\dagger(\lambda_1)}\right] - \lambda_1 \mathbb{E}\left[T_{N^\dagger(\lambda_1)}\right] \\ &> \mathbb{E}\left[Y_{N^\dagger(\lambda_1)}\right] - \lambda_2 \mathbb{E}\left[T_{N^\dagger(\lambda_1)}\right], \end{aligned} \tag{4.31}$$

in which the inequality is because $\lambda_1 < \lambda_2$.

For Problem (4.5) with parameter λ_2, its optimal strategy is denoted as $N^\dagger(\lambda_2)$. In other words, $N^\dagger(\lambda_2)$ minimizes the objective function of Problem (4.5) with parameter λ_2. Thus, we have

$$\begin{aligned} V(\lambda_2) &= \mathbb{E}\left[Y_{N^\dagger(\lambda_2)}\right] - \lambda_2 \mathbb{E}\left[T_{N^\dagger(\lambda_2)}\right] \\ &< \mathbb{E}\left[Y_{N^\dagger(\lambda_1)}\right] - \lambda_2 \mathbb{E}\left[T_{N^\dagger(\lambda_1)}\right]. \end{aligned} \tag{4.32}$$

Combining (4.31) and (4.32), we have $V(\lambda_1) > V(\lambda_2)$. Thus, $V(\lambda)$ is strictly decreasing in $\lambda > 0$.

Next we prove that $V(\lambda)$ is uniformly continuous, i.e., for any $\epsilon > 0$, there exists a δ such that for any λ_1 and λ_2 satisfying $|\lambda_2 - \lambda_1| < \delta$, we have $|V(\lambda_2) - V(\lambda_1)| < \epsilon$. We set $\delta = \epsilon / \mathbb{E}[T_C]$. Then for any λ_1 and λ_2 satisfying $0 < \lambda_1 <$

$\lambda_2 < \lambda_1 + \delta$, we have

$$
\begin{aligned}
&|V(\lambda_2) - V(\lambda_1)| \\
&= V(\lambda_1) - V(\lambda_2) \\
&< V(\lambda_1) - V(\lambda_1 + \delta) \\
&= V(\lambda_1) - \left(\mathbb{E}\left[Y_{N^\dagger(\lambda_1+\delta)}\right] - (\lambda_1 + \delta)\mathbb{E}\left[T_{N^\dagger(\lambda_1+\delta)}\right]\right) \\
&= V(\lambda_1) - \left(\mathbb{E}\left[Y_{N^\dagger(\lambda_1+\delta)}\right] - \lambda_1\mathbb{E}\left[T_{N^\dagger(\lambda_1+\delta)}\right]\right) \\
&\quad + \delta\mathbb{E}\left[T_{N^\dagger(\lambda_1+\delta)}\right] \\
&\overset{\text{(xiii)}}{\leq} \delta\mathbb{E}\left[T_{N^\dagger(\lambda_1+\delta)}\right] \overset{\text{(xiv)}}{\leq} \delta\mathbb{E}\left[T_C\right] = \epsilon,
\end{aligned}
$$

in which inequality (xiii) is because $V(\lambda_1)$ is the minimal objective function of Problem (4.5) with parameter λ_1, and inequality (xiv) is because $N^\dagger(\lambda_1+\delta) \leq C$.

Since a uniformly continuous function is also continuous [90], it can be concluded that $V(\lambda)$ is continuous in $\lambda > 0$.

4.6.6 Proof of Theorem 11

When $0 \leq t < (D - b)$, similar to (4.18), we have

$$
\begin{aligned}
\mathbb{P}\{N(t) = C\} &= e^{-\mu(D-b-t)}h(D - b) &(4.33)\\
&= e^{-\mu(D-t)}\left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}}, &(4.34)
\end{aligned}
$$

in which $h(\tau)$ is derived in Appendix 4.6.3. Similar to (4.19), we have

$$
\mathbb{E}\left[T_{N(t)}\right] = \frac{1}{\mu} + t - \left(D - b + \frac{1}{\mu} - g(D - b)\right)e^{-\mu(D-b-t)}. \tag{4.35}
$$

So $\mathbb{P}\{N(t) = C\}$ and $\mathbb{E}\left[T_{N(t)}\right]$ are both continuous in $t \in [0, D - b)$, and thus, from (4.12), $k(t)$ is continuous in $t \in [0, D - b)$.

When $(D - b) \leq t \leq (D - a)$, we have

$$
\mathbb{P}\{N(t) = C\} = h(t) = \left(\frac{D-t}{a}\right)^{\frac{\mu ab}{b-a}} e^{-\frac{\mu b(D-a-t)}{b-a}}, \tag{4.36}
$$

$$
\mathbb{E}\left[T_{N(t)}\right] = g(t), \tag{4.37}
$$

where $h(\tau)$ and $g(\tau)$ are derived in Appendix 4.6.3. So $\mathbb{P}\{N(t) = C\}$ and $\mathbb{E}\left[T_{N(t)}\right]$ are both continuous in $t \in [D-b, D-a]$, and thus, $k(t)$ is continuous in $t \in [D-b, D-a]$.

Moreover, around $t = D - b$ we have

$$\begin{aligned}\lim_{t<D-b,\ t\to(D-b)} \mathbb{P}\{N(t) = C\} &\overset{\text{(xv)}}{=} h(D-b) \\ &\overset{\text{(xvi)}}{=} \mathbb{P}\{N(D-b) = C\},\end{aligned}$$

in which equalities (xv) and (xvi) are from (4.33) and (4.36), respectively, and

$$\lim_{t<D-b,\ t\to(D-b)} \mathbb{E}\left[T_{N(t)}\right] = g(D-b) = \mathbb{E}\left[T_{N(D-b)}\right],$$

in which the two equalities are from (4.35) and (4.37), respectively. So $\mathbb{P}\{N(t) = C\}$ and $\mathbb{E}\left[T_{N(t)}\right]$ are both continuous at $t = D - b$, and thus, $k(t)$ is continuous at $t = D - b$.

Overall, $k(t)$ is continuous in $t \in [0, D-a]$.

4.6.7 Proof of Theorem 12

When $0 \le t < (D - b)$, from (4.34) we have

$$\frac{\mathrm{d}\mathbb{P}\{N(t) = C\}}{\mathrm{d}t} = \mu e^{-\mu(D-t)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}}. \tag{4.38}$$

From (4.35), we have

$$\frac{\mathrm{d}\mathbb{E}\left[T_{N(t)}\right]}{\mathrm{d}t} = 1 + \mu e^{-\mu(D-b-t)} \left(g(D-b) - D + b - \frac{1}{\mu}\right). \tag{4.39}$$

Then from (4.13), (4.34), (4.35), (4.38), and (4.39), we have expression of $l(t)$ in (4.40) on top of next page.

From (4.40) we know that $l(t)$ is continuous in $[0, D-b)$. Taking the first-order

$$
\begin{aligned}
l(t) =&\mu\beta e^{-\mu(D-t)}\left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}}\left(\frac{1}{\mu}+t-\left(D-b+\frac{1}{\mu}-g(D-b)\right)e^{-\mu(D-b-t)}\right)\\
&-\left\{1+\mu e^{-\mu(D-b-t)}\left(g(D-b)-D+b-\frac{1}{\mu}\right)\right\}\left(\omega P\kappa+\beta e^{-\mu(D-t)}\left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}}\right)\\
=&\mu\beta te^{-\mu(D-t)}\left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}}-\omega P\kappa\left(1+\mu e^{-\mu(D-b-t)}\left(g(D-b)-D+b-\frac{1}{\mu}\right)\right).
\end{aligned}
\tag{4.40}
$$

---

derivative of $l(t)$, we have

$$
\begin{aligned}
\frac{\mathrm{d}l(t)}{\mathrm{d}t} =&\mu\beta e^{-\mu(D-t)}\left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}}+\mu^2\beta te^{-\mu(D-t)}\left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}}\\
&-\omega P\kappa\mu^2e^{-\mu(D-b-t)}\left(g(D-b)-D+b-\frac{1}{\mu}\right)\\
=&\mu\Bigg\{\mu\beta te^{-\mu(D-t)}\left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}}-\omega P\kappa\mu e^{-\mu(D-b-t)}\\
&\quad\times\left(g(D-b)-D+b-\frac{1}{\mu}\right)-\omega P\kappa\Bigg\}\\
&+\mu\omega P\kappa+\mu\beta e^{-\mu(D-t)}\left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}}\\
=&\mu l(t)+\mu\omega P\kappa+\mu\beta e^{-\mu(D-t)}\left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}}>\mu l(t).
\end{aligned}
$$

Thus, if there is a $t^{\ddagger}\in[0,D-b)$ satisfying $l(t^{\ddagger})\geq 0$, then we have $\frac{\mathrm{d}l(t)}{\mathrm{d}t}>0$ for $t\in[t^{\ddagger},D-b)$, and subsequently we have $l(t)>0$ for $t\in(t^{\ddagger},D-b)$.

When $(D-b)\leq t\leq(D-a)$, from (4.36), (4.37), (4.23), and (4.25), we have

$$
\begin{aligned}
\frac{\mathrm{d}\mathbb{P}\{N(t)=C\}}{\mathrm{d}t}&=\frac{\mathrm{d}h(t)}{\mathrm{d}t}=\mu F_S(D-t)h(t),\\
\frac{\mathrm{d}\mathbb{E}\left[T_{N(t)}\right]}{\mathrm{d}t}&=\frac{\mathrm{d}g(t)}{\mathrm{d}t}=\mu F_S(D-t)\left(g(t)-t\right).
\end{aligned}
$$

Then from (4.13), we have

$$\begin{aligned}
l(t) =&\mu\beta F_S(D-t)h(t)g(t) \\
&- \mu F_S(D-t)\left(g(t)-t\right)\left(\omega P\kappa + \beta h(t)\right) \\
=&\mu F_S(D-t)\Big(t\left(\omega P\kappa+\beta h(t)\right) - g(t)\omega P\kappa\Big) \\
=&\omega P\kappa\mu F_S(D-t)\left(t - g(t) + \frac{\beta h(t)t}{\omega P\kappa}\right) \\
=&\omega P\kappa\mu F_S(D-t)p(t),
\end{aligned} \tag{4.41}$$

where $p(t) \stackrel{\triangle}{=} t-g(t)+\frac{\beta}{\omega P\kappa}h(t)t$. From (4.41), it can be seen that $l(t)$ is continuous in $t \in [D-b, D-a]$.

When $t \in [D-b, D-a]$, according to (4.25), we have

$$t - g(t) = \frac{-\dfrac{\mathrm{d}g(t)}{\mathrm{d}t}}{\mu F_S(D-t)}.$$

Then, we have

$$\begin{aligned}
p(t) &= \frac{-\dfrac{\mathrm{d}g(t)}{\mathrm{d}t}}{\mu F_S(D-t)} + \frac{\beta}{\omega P\kappa}h(t)t \\
&= \frac{\dfrac{\mathrm{d}h(t)}{\mathrm{d}t}\left(a-\frac{1}{\mu}+\frac{\mu b}{b-a}\left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}}\int_a^{D-t}(z-a)z^{-\frac{\mu ab}{b-a}}e^{\frac{\mu bz}{b-a}}\,\mathrm{d}z\right)}{\mu F_S(D-t)} \\
&\quad - \frac{h(t)\left(\frac{\mu b}{b-a}\left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}}(D-t-a)(D-t)^{-\frac{\mu ab}{b-a}}e^{\frac{\mu b(D-t)}{b-a}}\right)}{\mu F_S(D-t)} \\
&\quad + \frac{\beta}{\omega P\kappa}h(t)t \\
&= h(t)q(t),
\end{aligned} \tag{4.42}$$

where the second equality is from (4.26), the last equality is from (4.23) and (4.1), and $q(t)$ is defined as

$$\begin{aligned}
q(t) =& a - \frac{1}{\mu} + \frac{\mu b}{b-a}\left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}} \int_a^{D-t} (z-a) z^{-\frac{\mu ab}{b-a}} e^{\frac{\mu bz}{b-a}} \, \mathrm{d}z \\
& - \left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}} (D-t)^{1-\frac{\mu ab}{b-a}} e^{\frac{\mu b(D-t)}{b-a}} + \frac{\beta t}{\omega P \kappa}.
\end{aligned}$$

Taking the first-order derivative of $q(t)$, we have

$$\begin{aligned}
\frac{\mathrm{d}q(t)}{\mathrm{d}t} =& - \frac{\mu b}{b-a}\left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}} (D-t-a)(D-t)^{-\frac{\mu ab}{b-a}} e^{\frac{\mu b(D-t)}{b-a}} \\
& + \left(1 - \frac{\mu ab}{b-a}\right)\left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}} (D-t)^{-\frac{\mu ab}{b-a}} e^{\frac{\mu b(D-t)}{b-a}} \\
& + \frac{\mu b}{b-a}\left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}} (D-t)^{1-\frac{\mu ab}{b-a}} e^{\frac{\mu b(D-t)}{b-a}} + \frac{\beta}{\omega P \kappa} \\
=& \left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}} (D-t)^{-\frac{\mu ab}{b-a}} e^{\frac{\mu b(D-t)}{b-a}} + \frac{\beta}{\omega P \kappa} > 0.
\end{aligned}$$

As shown in Appendix 4.6.3, $h(t) > 0$ and $\frac{\mathrm{d}h(t)}{\mathrm{d}t} > 0$ in $t \in [D-b, D-a)$. So if there is a $t^{\ddagger} \in [D-b, D-a)$ such that $l(t^{\ddagger}) \geq 0$, then from (4.41) we have $p(t^{\ddagger}) \geq 0$, and further from (4.42) we have $q(t^{\ddagger}) \geq 0$. Together with the fact that $\frac{\mathrm{d}q(t)}{\mathrm{d}t} > 0, h(t) > 0, \frac{\mathrm{d}h(t)}{\mathrm{d}t} > 0$ for $t \in [D-b, D-a)$, we have that $p(t) = h(t)q(t)$ is strictly increasing and always positive for $t \in (t^{\ddagger}, D-a)$. Since $F_S(D-t) > 0$ for $t \in [D-b, D-a)$, from (4.41) it can be concluded that $l(t) > 0$ for $t \in (t^{\ddagger}, D-a)$.

Since $l(t)$ is continuous in the intervals of $[0, D-b)$ and $[D-b, D-a]$, and

$$\begin{aligned}
& \lim_{t<D-b,\ t\to(D-b)} l(t) \\
&= \mu\beta(D-b)e^{-\mu b}\left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}} + \omega P\kappa\mu\left(D-b-g(D-b)\right) \\
&= \omega P\kappa\mu\left(D-b-g(D-b) + \frac{\beta h(D-b)(D-b)}{\omega P\kappa}\right) \\
&= \lim_{t>D-b,\ t\to(D-b)} l(t)
\end{aligned}$$

(in which the first equality is from (4.40), the second equality is from (4.24), and the last equality is from (4.41) and $F_S(b) = 1$), it can be concluded that $l(t)$ is continuous in $[0, D - a]$.

From (4.40), we have

$$l(0) = -\omega P \kappa \left(1 + \mu e^{-\mu(D-b)} \left(g(D-b) - D + b - \frac{1}{\mu} \right) \right).$$

According to the definition of $g(t)$ in Appendix 4.6.2,

$$g(D-b) = \mathbb{E}\left[T_{n_r+1} | T_{n_r+1} \geq D - b \right] \geq D - b + 1/\mu,$$

which means $l(0) < 0$. From (4.41), it can be seen that $l(D - a) = 0$ due to the fact that $F_S(a) = 0$.

Chapter 5

I2I Traffic Scheduling with a Multi-Step Soft Delay Bound

In Chapter 4, we consider a soft delay bound. If the total delay is less than the delay bound, no cost is charged for delay. On the other hand, if we try to encourage sooner delivery, we may charge some cost if the total delay is less than but close to the delay bound. Thus, now we extend the delay cost function used in Chapter 4 to be a multi-step function. In addition, now we consider that the vehicle speeds can follow any distribution. Table 5.1 compares existing data traffic forwarding methods in VANETs, the methods used in Chapter 3 and Chapter 4 and the methods proposed in this chapter. The major contributions of this chapter are summarized as follows. 1) Based on the multi-step soft delay bound model, we formulate an optimal stopping problem. To solve the formulated optimal stopping problem, we decompose the original problem into a sequence of sub-OSPs. 2) We obtain an threshold based optimal stopping rule for each sub-OSP. 3) We obtain an optimal stopping problem for the original optimal stopping problem by combining the optimal stopping rules for the sequence of sub-OSPs. Table 5.2 summarizes those notations used in this chapter but not in Chapter 4.

Work	**Research Problem, Methodology, and Result**

[54]	Problem: Minimize energy consumption of an RSU that processes requests from vehicles. Methodology: Optimization formulation and approximation. Result: A scheduler based on vehicles' locations and velocities.
[55]	Problem: Maximize the multi-hop packet delivery probability from a source to a destination subject to an energy consumption constraint. Methodology: Continuous-time Markov framework. Result: The threshold dynamic policy is optimal.
[56]	Problem: Maximize packet delivery probability in two-hop routing subject to constraints on energy consumption and relay activation rate, considering energy in information transmission and node discovery. Methodology: Fluid approximation, optimal control theory. Result: Optimal two-dimensional threshold policy in closed form for transmission and activation.
[57]	Problem: Maximize message delivery probability in epidemic routing subject to total energy consumption constraint, considering energy in information transmission and node discovery. Methodology: Continuous-time Markov framework. Result: Optimal beaconing control solution.
[58]	Problem: in RSU-to-vehicle communications, minimize RSU energy consumption by using V2V forwarding. Methodology: Integer linear programming. Result: Greedy scheduling algorithms with low complexity.
	Problem: For I2I communications using vehicles as relays, minimize the delay for delivering all packets in a finite-size file, or the average packet delay for a file with infinite packets.

[62]	Methodology: Markov decision process. Result: Optimal scheduling algorithms and a low-complexity sub-optimal algorithm.
[63]	Problem: For I2I communications using vehicles as relays, minimize the sum of queuing delay and transit delay. Methodology: Queuing analysis. Result: Probabilistic scheduling scheme.
[64]	Problem: For I2I communications using vehicles as relays, minimize the rate of weighted cost of energy consumption and queuing delay. Methodology: Traditional optimal stopping theory. Result: Optimal pure-threshold strategy.
Chapter 3	Problem: For I2I communications (with hard delay bound) using vehicles as relays, minimize the rate of weighted cost of energy consumption, queuing delay, and transit delay. Methodology: Traditional optimal stopping theory. Result: Optimal strategy without threshold structure.
Chapter 4	Problem: For I2I communications (with soft delay bound) using vehicles as relays, minimize the rate of weighted cost of energy consumption, queuing delay, and transit delay. Methodology: New method to solve an optimal stopping problem with forced stop. Result: Optimal strategy with conditional pure-threshold structure.
Chapter 5	Problem: For I2I communications (with multi-step soft delay bound and the speed of vehicles being any distribution) using vehicles as relays, minimize the rate of weighted cost of energy consumption, queuing delay, and transit delay. Methodology: New method to solve multi-sub optimal stopping problems.

	Result: Optimal strategy with multi-thresholds structure.

Table 5.1: Comparison of Traffic Forwarding Methods in VANETs

Symbol	**Meaning**
$B_1, B_2, \ldots$	Costs of violating multi-step soft delay bounds
$c(t)$	Multi-step delay cost function
$f_G(x)$	Probability density function of transit delay
$F_G(x)$	Cumulative distribution function of transit delay
G_n	The transit delay of the nth vehicle
$K_1, K_2, \ldots$	Multi-step soft delay bounds
Q	Stages of the multi-step delay cost function
$\mathcal{N}$	The set of candidate stopping rules for Problem (5.2)
$N(\lambda)$	An optimal stopping rule for Problem (5.3)
$N_1(t, \lambda)$	1-stage look-ahead stopping rule starting from time t
N_Q	The first vehicle arriving after $(K_Q - a)$
$Th_1(\lambda), Th_2(\lambda), \ldots$	Optimal thresholds for the first sub-OSP, the second sub-OSP, ...
$T_1(\lambda), T_2(\lambda), \ldots$	Expected stopping time for the first sub-OSP, the second sub-OSP, ...
$V(\lambda)$	Optimal objective function of Problem (5.3)
$V(\lambda, t)$	Optimal objective function of Problem (5.3) if starting observation from time t
V_n	The speed of the nth vehicle
$Y_1(\lambda), Y_2(\lambda), \ldots$	Expected minimum costs for the first sub-OSP, the second sub-OSP, ...

Table 5.2: Used Notations in Chapter 5

The following sections are organized as follows. Section 5.1 gives the system model and research problem. Section 5.2 gives an optimal rule for the research problem, and multiple-threshold structure of the optimal rule is also proved.

Section 5.3 evaluates our derived strategy. Section 5.4 concludes this chapter. Appendix 5.5 includes proofs of the theorems in this chapter.

5.1 System model and problem formulation

There is a data stream of constant rate r arriving at the source RSU. The source RSU wishes to send these data to the destination RSU with minimal expected cost per bit. The distance between the source RSU and the destination RSU is D meters. Upon the arrival of a vehicle, the source RSU decides whether it should call for transmission. If the source RSU skips the current vehicle, it will continue to buffer incoming traffic and wait for the next vehicle. If it chooses the nth vehicle, whose arrival time is T_n, as the carrier of the accumulated data, then the amount of transmitted data is rT_n.

The total cost to transmit those data to the destination RSU includes the cost of energy consumption and the delay penalty. The source RSU transmits data at a constant rate of R with transmission power P. To start the process of data transmission, there needs a handshake process between the source (destination) RSU and the vehicle. Assume the transmission power of the overhead is also P, and the duration of the process of handshake transmission is κ. Then the cost of the total energy consumption for transmission of the data and the overhead is $\omega P \left(T_n r/R + \kappa\right)$, where ω (unit: unit of cost per Joule) is the cost weight of energy consumption. In addition, it is desirable that large queueing delays are avoided even in delay tolerant networks. When a vehicle arrives at the destination, a penalty is charged depending on the total delay comprised of the queueing delay and the transit delay. Similar to [65], the RSU is forced to transmit its accumulated data to a vehicle when the total delay cannot be less than a value K_Q. We consider the following multi-step delay cost function in this chapter:

$$c(t) = \sum_{i=1}^{Q} B_i \mathbb{1}_{\left[K_i \le t < K_{i+1}\right]}, \tag{5.1}$$

where $Q \geq 2$, $0 = B_1 < B_2 < \cdots < B_Q$ and $0 = K_1 < K_2 < \cdots < K_Q$.

Let $\{X_n\}, n \geq 1$ denote the waiting time for the nth vehicle after observing the $(n-1)$th vehicle. The arrival time of the nth vehicular is $T_n = \sum_{i=1}^{n} X_i$. The speed of the nth vehicle passing by the RSU is a random variable. We do not need to assume a specific distribute of the speed random variable as in the previous two chapters since the optimal stopping rule derived in this chapter applies to any distribution. However, for convenience of explanation, we still assume the sequence $\{V_1, V_2, \dots\}$ is a sequence of i.i.d. uniformly distributed random variables:

$$V_n \in [v_{\min}, v_{\max}],$$

where $v_{\min}$ is the minimum speed and $v_{\max}$ is the maximum speed. We assume $\{V_n\}$ to be an independent identically distributed sequence. The transit time of the nth vehicle from the source to the destination is

$$G_n = \frac{D}{V_n}.$$

It follows that $\{G_n\}$ is also an independent identically distributed sequence. The probability density function and cumulative distribution function of G_n are:

$$f_G(x) = \begin{cases} \frac{ab}{x^2(b-a)} & \text{if} \quad a \le x \le b \\ 0 & \text{otherwise} \end{cases} \qquad F_G(x) = \begin{cases} 0 & \text{if} \quad x < a \\ \frac{b(x-a)}{x(b-a)} & \text{if} \quad a \le x \le b \\ 1 & \text{otherwise} \end{cases},$$

where $a = \frac{D}{v_{\max}}$ is the smallest transit time, and $b = \frac{D}{v_{\min}}$ is the largest transit time.

Our objective is to derive one optimal stopping rule that minimizes the rate of cost:

$$\begin{aligned} N^* &= \arg\inf_{N\in\mathcal{N}} \frac{\omega P T_N r/R + \omega P\kappa + c(T_N + G_N)}{T_N} \\ &= \arg\inf_{N\in\mathcal{N}} \frac{\omega P\kappa + c(T_N + G_N)}{T_N} \\ &= \arg\inf_{N\in\mathcal{N}} \frac{Y_N}{T_N} \end{aligned} \tag{5.2}$$

where $Y_N = \omega P\kappa + c(T_N + G_N)$, $\mathcal{N} = \{N : 1 \le N \le N_Q\}$ is the set of candidate stopping rules and $N_Q = \min\{n : T_n \ge K_Q - a\}$ is a random variable. The random variable N_Q denotes the first vehicle arriving after $(K_Q - a)$.

5.2 An optimal stopping rule

The following six subsections describe the steps to get an optimal stopping rule for Problem (5.2).

5.2.1 Transformation of the original problem

Like the problems in the previous two chapters, we first transform the original problem to the following problem:

$$N(\lambda) = \arg\inf_{N\in\mathcal{N}} \mathbb{E}\left[Y_N - \lambda T_N\right] \tag{5.3}$$

$$V(\lambda) = \inf_{N\in\mathcal{N}} \mathbb{E}\left[Y_N - \lambda T_N\right] = \mathbb{E}\left[Y_{N(\lambda)} - \lambda T_{N(\lambda)}\right]. \tag{5.4}$$

We use the backward induction to decompose this optimal stopping problem (OSP) into a sequence of Q sub-OSPs. And the final optimal stopping rule $N(\lambda)$ for a specific λ is the composition of the Q optimal stopping rules for the decomposed Q sub-OSPs. Then we update λ to find the λ^* such that

$$V(\lambda^*) = 0. \tag{5.5}$$

And $N(\lambda^*)$ is one optimal stopping rule for the original problem (5.2).

Theorem 13. *$V(\lambda)$ is decreasing and concave in $\lambda \geq 0$.*

Proof. See Appendix 5.5.1. $\square$

According to Theorem 13, we can use Newton's method to find the root to the equation $V(\lambda) = 0$, which converges quadratically.

5.2.2 $T_n + G_n \geq K_Q$

We first solve the OSP (5.3) for vehicles whose total delay is not less than K_Q. It can be proved the same way as in Chapter [Add ref link] that it is optimal to skip those vehicles such that: $T_n < K_Q - a$ and $T_n + G_N \geq K_Q$.

Recall that $(K_Q - a)$ is the forced-stop threshold, which means the first vehicle arriving at or after $(K_Q - a)$ should be chosen as the vehicle to carry accumulated data to the destination. Thus for all vehicles whose total delay is $T_n + G_n \geq K_Q$ we have the optimal stopping rule $N_Q = \min\{n : T_n \geq K_Q - a\}$:

1. If $T_n < K_Q - a$, the RSU should not stop at this vehicle.
2. If $T_n \geq K_Q - a$, the RSU should stop at this vehicle.

This stopping rule is threshold based and we denote this threshold as

$$Th_1(\lambda) = K_Q - a. \tag{5.6}$$

The expected cost and stopping time for this first sub-OSP are

$$Y_1(\lambda) = \mathbb{E}\left[Y_{N_Q(\lambda)}\right] = \omega P \kappa + B_Q \tag{5.7}$$

$$T_1(\lambda) = \mathbb{E}\left[T_{N_Q(\lambda)}\right] = K_Q - a + \mathbb{E}\left[X_{N_Q}\right] = K_Q - a + 1/\mu \tag{5.8}$$

Denote the optimal value for the first transformed OSP as:

$$V(Th_1(\lambda), \lambda) = Y_1(\lambda) - \lambda T_1(\lambda).$$

$V(t, \lambda)$ is the optimal value could be achieved if we start observation from time t.

With the optimal stopping rule for vehicles whose total delay is $T_n+G_n \geq K_Q$ determined, we continue to find the optimal stopping rule for vehicles whose total delay is $K_{Q-1} \leq T_n + G_n < K_Q$.

5.2.3 $K_{Q-1} \leq T_n + G_n < K_Q$

In the preceding sub-OSP, the optimal stopping rule for those vehicles with total delay $T_n+G_n \geq K_Q$ is derived. In this sub-OSP, we can ignore those vehicles and focus on those vehicles whose total delay are $K_{Q-1} \leq T_n+G_n < K_Q$ Focusing here means we only consider what decision to make when we observe those vehicles whose delay are $K_{Q-1} \leq T_n+G_n < K_Q$ and not those whose delays are $T_n+G_n < K_{Q-1}$.

We will prove later in this chapter that this sub-OSP is a monotone problem, which means there exists a threshold $Th_2(\lambda)$ such that for those vehicles $K_{Q-1} \leq T_n + G_n < K_Q$:

1. If $T_n \geq Th_2(\lambda)$, it is optimal to stop.
2. If $T_n < Th_2(\lambda)$, it is optimal to continue.

Before continuing, we have the following two facts:

- It is optimal to stop at those vehicles such that $T_n \geq Th_2(\lambda)$ and $T_n+G_n < K_{Q-1}$ because:

$$\omega P\kappa + B_1 - \lambda T_n < \omega P\kappa + B_2 - \lambda T_n < \cdots <$$
$$\omega P\kappa + B_{Q-2} - \lambda T_n < \omega P\kappa + B_{Q-1} - \lambda T_n \leq V(T_n, \lambda).$$

  and $\omega P\kappa + B_{Q-1} - \lambda T_n \leq V(T_n, \lambda)$ is from the optimality of this stopping rule.

- $Th_2(\lambda) \leq Th_1(\lambda)$. This is obvious since $Th_1(\lambda) = K_Q - a$.

Based on the above information, for all vehicles arriving after $Th_2(\lambda)$ we have the optimal stopping rule:

1) If $Th_2(\lambda) \leq T_n < Th_1(\lambda)$ and $T_n + G_n < K_Q$, it is optimal to stop;

2) If $Th_2(\lambda) \leq T_n < Th_1(\lambda)$ and $T_n + G_n \geq K_Q$, it is optimal to continue;

3) If $T_n \geq Th_1(\lambda)$, use the optimal stopping rule for the previous sub-OSP.

Since this problem is a monotone problem, the optimal threshold is the root $t \in [0, Th_1(\lambda))$ to the following equation:

$$\mathbb{E}\left[Y_{N_1(t,\lambda)} - \lambda T_{N_1(t,\lambda)}\right] - (\omega P \kappa + B_{Q-1} - \lambda t) = 0, \tag{5.9}$$

where $N_1(t, \lambda)$ is the 1-stage look-ahead stopping rule starting from time t.

To compute $\mathbb{E}\left[Y_{N_1(t,\lambda)} - \lambda T_{N_1(t,\lambda)}\right]$, we should use the optimal stopping rule for vehicles arriving after $Th_1(\lambda)$. Consider that we are at some instant in time after $Th_2(\lambda)$ but before $Th_1(\lambda)$, we will wait for vehicles such that $T_n + G_n \leq K_{Q-1}$ to stop at. If we wait past $Th_1(\lambda)$, then from this point, we should use the optimal threshold rule for the first sub-OSP to decide whether to stop. This means for this second sub-OSP, we only need to consider the set of stopping rules whose decision after $Th_1(\lambda)$ is the same with the stopping rule for the first sub-OSP. And we can treat the first vehicle arriving after $Th_1(\lambda)$ as the first vehicle of the previous sub-OSP since the vehicle arrival process is a Poisson process and the exponential distribution is memoryless. Then, we have

$$
\begin{aligned}
&\mathbb{E}\left[Y_{N_1(t,\lambda)}\right] \\
=&(1-\alpha(Th_1(\lambda)-t, K_2-t)) * (\omega P\kappa + B_1) \\
&+ (\alpha(Th_1(\lambda)-t, K_2-t) - \alpha(Th_1(\lambda)-t, K_3-t)) * (\omega P\kappa + B_2) \\
&+ (\alpha(Th_1(\lambda)-t, K_3-t) - \alpha(Th_1(\lambda)-t, K_4-t)) * (\omega P\kappa + B_3) \\
&+ \dots \\
&+ (\alpha(Th_1(\lambda)-t, K_{Q-1}-t) - \alpha(Th_1(\lambda)-t, K_Q-t)) * (\omega P\kappa + B_{Q-1}) \\
&+ \alpha(Th_1(\lambda)-t, K_Q-t) * Y_1(\lambda) \\
&\mathbb{E}\left[T_{N_1(t,\lambda)}\right] \\
=&\alpha(Th_1(\lambda)-t, K_Q-t) * T_1(\lambda) + \beta(Th_1(\lambda)-t, K_Q-t) + t \\
&- \alpha(Th_1(\lambda)-t, K_Q-t) * (Th_1(\lambda) + 1/\mu),
\end{aligned}
$$

where α and β are defined in the Appendix. $\alpha(x, y)$ is the probability if we start observation from time $t = 0$, we cannot find a vehicle whose arrival time is $T_n < x$ and total delay is $T_n + G_n < y$. If there is not such a vehicle arrived before x, we stop at the first vehicle arriving at or after x. $\beta(x, y)$ is the expected stopping time of the above stopping problem.

Here we explain the terms in the expression of $\mathbb{E}\left[Y_{N_1(t,\lambda)}\right]$. In the first term

$$(1-\alpha(Th_1(\lambda)-t, K_2-t)) * (\omega P\kappa + B_1),$$

$(1-\alpha(Th_1(\lambda)-t, K_2-t))$ is the probability we observe a vehicle arriving after t and before $Th_1(\lambda)$ and whose total delay is less than K_2.

In the second term

$$(\alpha(Th_1(\lambda)-t, K_2-t) - \alpha(Th_1(\lambda)-t, K_3-t)) * (\omega P\kappa + B_2),$$

$(\alpha(Th_1(\lambda)-t, K_2-t) - \alpha(Th_1(\lambda)-t, K_3-t))$ is the probability we observe a vehicle arriving after t and before $Th_1(\lambda)$ and whose total delay is between $K_2 \leq t + G_n < K_3$.

The last term $\alpha(Th_1(\lambda) - t, K_Q - t) * Y_1(\lambda)$ is the expected cost if we do not observe a vehicle whose arrival time is after t and before $Th_1(\lambda)$, and whose total delay is less than K_Q, so that we use the optimal stopping rule for the previous sub-OSP.

Here we explain the terms in the expression of $\mathbb{E}\left[T_{N_1(t,\lambda)}\right]$. The term

$$\beta(Th_1(\lambda) - t, K_Q - t) + t$$

is the expected stopping time if we take into account of the definition of $\beta(x, y)$. However, recall that we are considering those optimal stopping rules such that it makes the same decision as the optimal stopping rule for the previous sub-OSP if we are observing those vehicle arriving after $Th_1(\lambda)$. Thus, we remove $\alpha(Th_1(\lambda) - t, K_Q - t) * (Th_1(\lambda) + 1/\mu)$ from $\beta(Th_1(\lambda) - t, K_Q - t) + t$ and then add the actual expected stopping time after $Th_1(\lambda)$ which is $\alpha(Th_1(\lambda) - t, K_Q - t) * T_1(\lambda)$.

We can find the root for equation (5.9) numerically. And the root is the value of $Th_2(\lambda)$. In the same way we can define:

$$Y_2(\lambda) = \mathbb{E}\left[Y_{N_1(t,\lambda)}\right]|_{t=Th_2(\lambda)}$$
$$T_2(\lambda) = \mathbb{E}\left[T_{N_1(t,\lambda)}\right]|_{t=Th_2(\lambda)}.$$

5.2.4 $K_{Q-2} \leq T_n + G_n < K_{Q-1}$

Now we find an optimal stopping rule for all vehicles whose total delay $T_n + G_n \geq K_{Q-1}$. We continue to solve the third sub-OSP. The third problem considers those vehicles such that $T_n < Th_2(\lambda)$ and $K_{Q-2} \leq T_n + G_n < K_{Q-1}$. Again, we try to find the root to the following equation:

$$\mathbb{E}\left[Y_{N_1(t,\lambda)} - \lambda T_{N_1(t,\lambda)}\right] - (\omega P\kappa + B_{Q-2} - \lambda t) = 0. \tag{5.10}$$

To compute $\mathbb{E}\left[Y_{N_1(t,\lambda)} - \lambda T_{N_1(t,\lambda)}\right]$, we should use the $Y_2(\lambda)$ and $T_2(\lambda)$ for vehicle arriving after $Th_2(\lambda)$.

$$\begin{aligned}
&\mathbb{E}\left[Y_{N_1(t,\lambda)}\right] \\
=&(1-\alpha(Th_2(\lambda)-t, K_2-t)) * (\omega P\kappa + B_1) \\
&+ (\alpha(Th_2(\lambda)-t, K_2-t) - \alpha(Th_2(\lambda)-t, K_3-t)) * (\omega P\kappa + B_2) \\
&+ (\alpha(Th_2(\lambda)-t, K_3-t) - \alpha(Th_2(\lambda)-t, K_4-t)) * (\omega P\kappa + B_3) \\
&+ \ldots \\
&+ (\alpha(Th_2(\lambda)-t, K_{Q-2}-t) - \alpha(Th_2(\lambda)-t, K_{Q-1}-t)) * (\omega P\kappa + B_{Q-2}) \\
&+ \alpha(Th_2(\lambda)-t, K_{Q-1}-t) * Y_2(\lambda) \\
&\mathbb{E}\left[T_{N_1(t,\lambda)}\right] \\
=&\alpha(Th_2(\lambda)-t, K_{Q-1}-t) * T_2(\lambda) + \beta(Th_2(\lambda)-t, K_{Q-1}-t) + t \\
&- \alpha(Th_2(\lambda)-t, K_{Q-1}-t) * (Th_2(\lambda) + 1/\mu)
\end{aligned}$$

We can find the root for equation (5.10) numerically. And the root is the value of $Th_3(\lambda)$. Again, we define:

$$Y_3(\lambda) = \mathbb{E}\left[Y_{N_1(t,\lambda)}\right]|_{t=Th_3(\lambda)}$$
$$T_3(\lambda) = \mathbb{E}\left[T_{N_1(t,\lambda)}\right]|_{t=Th_3(\lambda)}.$$

Before giving the optimal stopping problem for this sub-OSP, we need to show $Th_3(\lambda) \leq Th_2(\lambda)$. Suppose $Th_3(\lambda) > Th_2(\lambda)$. Then for a vehicle arriving between $Th_2(\lambda) \leq T_n < Th_3(\lambda)$, if $K_{Q-2} \leq T_n + G_n < K_{Q-1}$, then the optimal stopping rule for the previous sub-OSP asks for stopping and the optimal stopping rule for this sub-OSP asks for continuing. This is a contradiction.

Then all vehicles arriving after or at $Th_3(\lambda)$ have the optimal stopping rule:

1) If $Th_3(\lambda) \leq T_n < Th_2(\lambda)$ and $T_n + G_n < K_{Q-1}$, it is optimal to stop;

2) If $Th_3(\lambda) \leq T_n < Th_2(\lambda)$ and $T_n + G_n \geq K_{Q-1}$, it is optimal to continue;

3) If $T_n \geq Th_2(\lambda)$, use the optimal stopping rule in previous sub-OSP.

5.2.5 Final composite optimal stopping rule

In the same way we can find all Q thresholds: $Th_1(\lambda) \geq Th_2(\lambda) \geq \cdots \geq Th_Q(\lambda)$. The Q thresholds for a specific λ is the optimal stopping rules for the transformed problem. We can use Newton's method to find the λ^*. Denote the Q thresholds for λ^* as $Th_1(\lambda^*) \geq Th_2(\lambda^*) \geq \cdots \geq Th_Q(\lambda^*)$. This set of thresholds are the optimal thresholds for the original problem. When a vehicle arrives, we use its total delay to choose the threshold to use. For example, a vehicle arrives at time T_n and its transit delay is G_n.

1. If $T_n + G_n \geq K_Q$, use the threshold $Th_1(\lambda^*) = K_Q - a$.

2. If $K_i \leq T_n + G_n < K_{i+1}, i \leq Q-1$, then we use threshold $Th_{Q+1-i}(\lambda^*)$ to decide whether to stop. If $T_n \geq Th_{Q+1-i}(\lambda^*)$, then stop at this vehicle. Otherwise, continue to wait for the next vehicle until forced stop.

5.2.6 Proof of the monotone property of sub-OSP

In this section, we prove the monotone property of each sub-OSP. Suppose there is a t^* such that $K_{Q-i} \leq t^* + G_n < K_{Q-i+1}$:

$$
\begin{aligned}
0 =& \mathbb{E}\left[Y_{N_1(t^*,\lambda)}\right] - \lambda\mathbb{E}\left[T_{N_1(t^*,\lambda)}\right] - (\omega P\kappa + B_{Q-i} - \lambda t^*) \\
=& -\alpha(Th_i(\lambda) - t^*, K_2 - t^*) * (\omega P\kappa + B_1) \\
&+ (\alpha(Th_i(\lambda) - t^*, K_2 - t^*) - \alpha(Th_i(\lambda) - t^*, K_3 - t^*)) * (\omega P\kappa + B_2) \\
&+ \dots \\
&+ (\alpha(Th_i(\lambda) - t^*, K_{Q-i} - t^*) - \alpha(Th_i(\lambda) - t^*, K_{Q-i+1} - t^*)) * (\omega P\kappa + B_{Q-i}) \\
&+ \alpha(Th_i(\lambda) - t^*, K_{Q-i+1} - t^*) * Y_{i+1}(\lambda) \\
&- \lambda * (\alpha(Th_i(\lambda) - t^*, K_{Q-1} - t^*) * T_{i+1}(\lambda) + \beta(Th_i(\lambda) - t^*, K_{Q-1} - t^*) \\
&+ \alpha(Th_i(\lambda) - t^*, K_{Q-1} - t^*) * (Th_i(\lambda) + 1/\mu)) - (B_{Q-i} - B_1)
\end{aligned}
$$

Then we prove that for $t \geq t^*$,

$$
\mathbb{E}\left[Y_{N_1(t,\lambda)}\right] - \lambda\mathbb{E}\left[T_{N_1(t,\lambda)}\right] - (\omega P\kappa + B_{Q-i} - \lambda t) \geq 0.
$$

Define $m(t,\lambda) = \mathbb{E}\left[Y_{N_1(t,\lambda)}\right] - \lambda\mathbb{E}\left[T_{N_1(t,\lambda)}\right] - (\omega P\kappa + B_{Q-i} - \lambda t)$, and we are to prove $m(t,\lambda) \geq 0$ for $t \geq t^*$. According to the definition $\gamma(t,x,y)$ and $\eta(t,x,y)$ in the Appendix:

$$
\begin{aligned}
\frac{\mathrm{d}\gamma(t,x,y)}{\mathrm{d}t} &= \mu F_G(y-t)\gamma(t,x,y) \\
\frac{\mathrm{d}\eta(t,x,y)}{\mathrm{d}t} &= \mu F_G(y-t)\left(\eta(t,x,y) - t\right).
\end{aligned}
$$

we have:

$$
\frac{\mathrm{d}m(t,\lambda)}{\mathrm{d}t} = \mu F_G(Th_i(\lambda) - t)\left(m(t,\lambda) + B_{Q-i} - B_1 + \lambda t\right) \geq 0.
$$

Thus, $m(t,\lambda)$ is a non-decreasing function for $t \geq t^*$ when $m(t^*) = 0$.

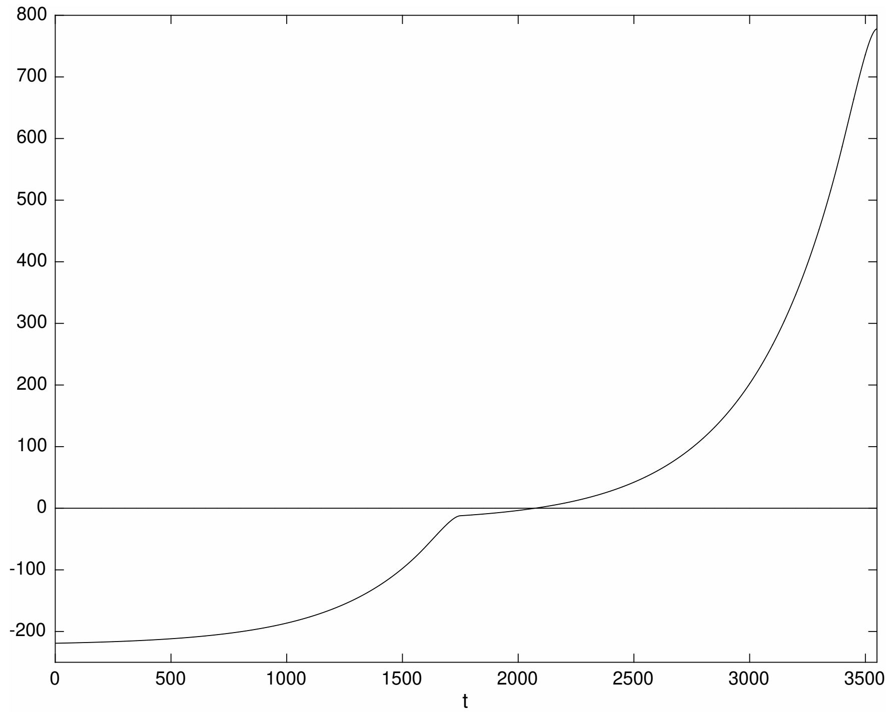

Figure 5.1: $m(t, \lambda_1)$ for $K_2 \leq t + G_n < K_3$.

5.3 Performance evaluation

This section we simulate for the case when $Q = 3$. The parameters used are mostly the same with the ones in the previous two chapters. In the simulation, we set $B_1 = 0$, $B_2 = 200$, $B_3 = 1000$, $K_1 = 0$, $K_2 = 1800$ and $K_3 = 3600$. During the iteration process of λ, it converges quickly to the optimal λ^*: The initial value of λ is set to $\lambda_0 = 0.1089178581$. The other λ's are $\lambda_1 = 0.0550637979$, $\lambda_2 = 0.0456309952$, $\lambda_3 = 0.0452523308$, and $\lambda_4 = 0.0452516678$. And $\lambda^* = \lambda_4$ is the optimal value of rate of cost. The corresponding optimal thresholds for λ^* are $Th_1 = 3550$, $Th_2 = 1993.12$ and $Th_3 = 736.58$.

For a specific $\lambda = \lambda_1 = 0.0456309952$, the optimal thresholds are $Th_1 = 3550$, $Th_2 = 2071.83$ and $Th_3 = 821.42$. The graphs of $m(t, \lambda_1)$ are Fig. 5.1 and Fig. 5.2. It is evident from these two figures that the monotone property is satisfied.

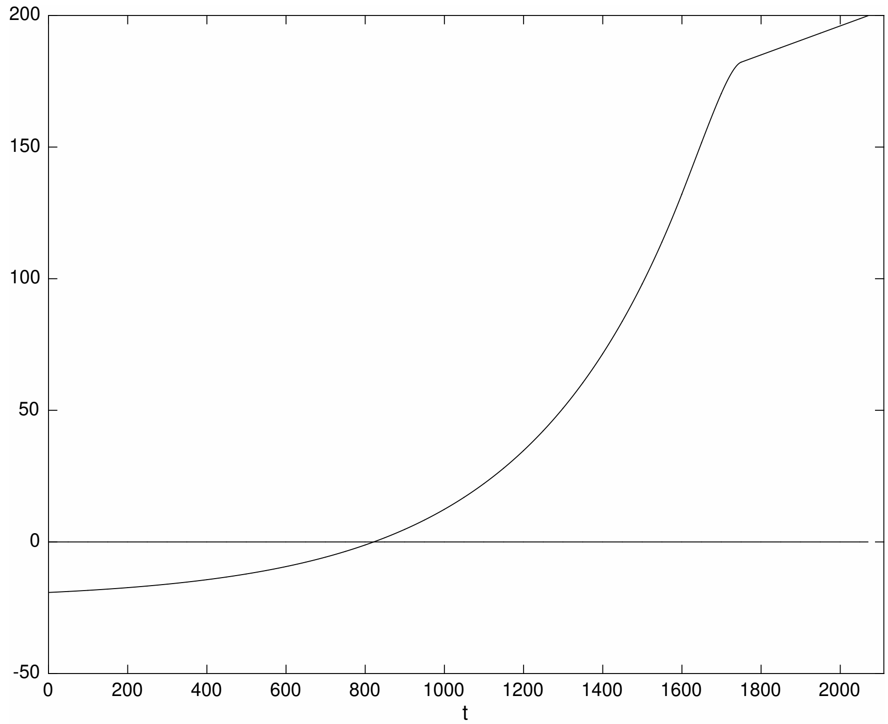

Figure 5.2: $m(t, \lambda_1)$ for $K_1 \leq t + G_n < K_2$.

Fig. 5.3 is the rate of cost for all possible combinations of $Th_3 \leq Th_2$ for $\lambda^* = 0.0452516678$. Using brute force search, the minimum rate of cost is found at the point ($Th_2 = 1993.12, Th_3 = 736.58$) which matches exactly with the theoretical results.

Fig. 5.4 and Fig. 5.5 show the optimal thresholds and minimal rate of cost, respectively, when μ (vehicle arrival rate) changes. Higher μ results in larger thresholds and smaller rate of cost. This is intuitive as higher μ means more choices of vehicles to select.

5.4 Conclusion

In this chapter, we consider multi-step soft delay bound. Accordingly, the optimal strategy has a multi-threshold structure. When a vehicle arrives at the source

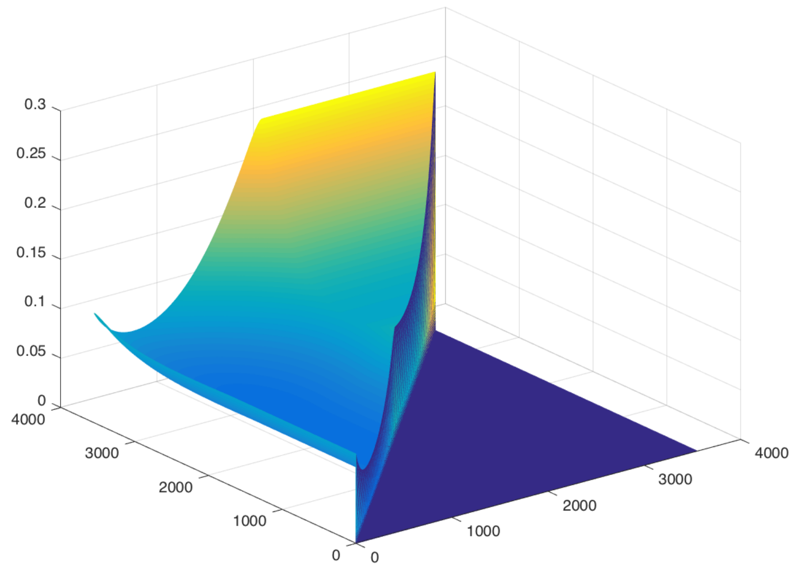

Figure 5.3: The rate of cost for all possible combinations of Th_2 and Th_3.

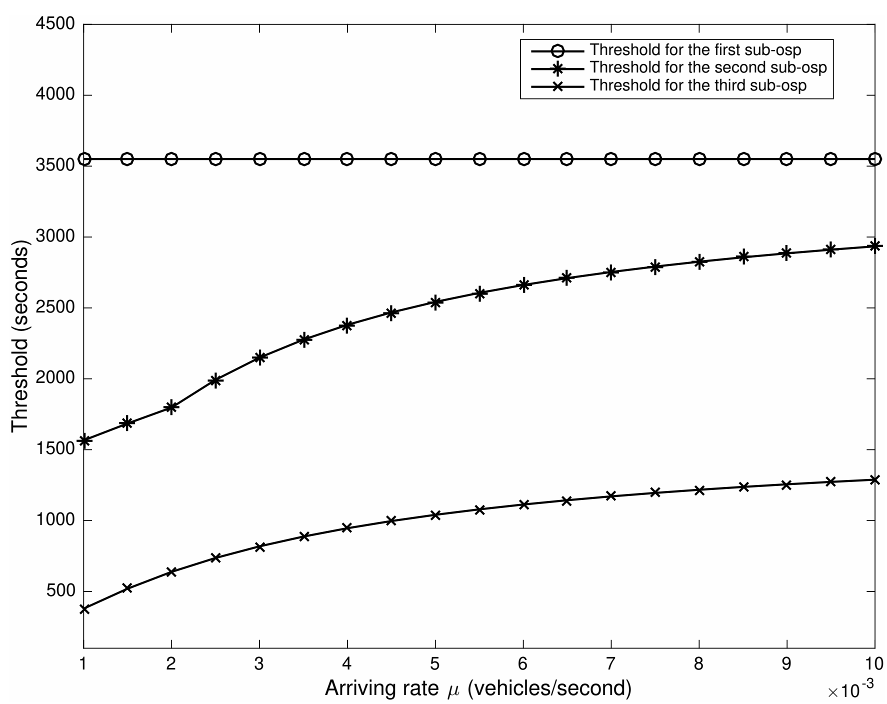

Figure 5.4: The optimal thresholds for different μ.

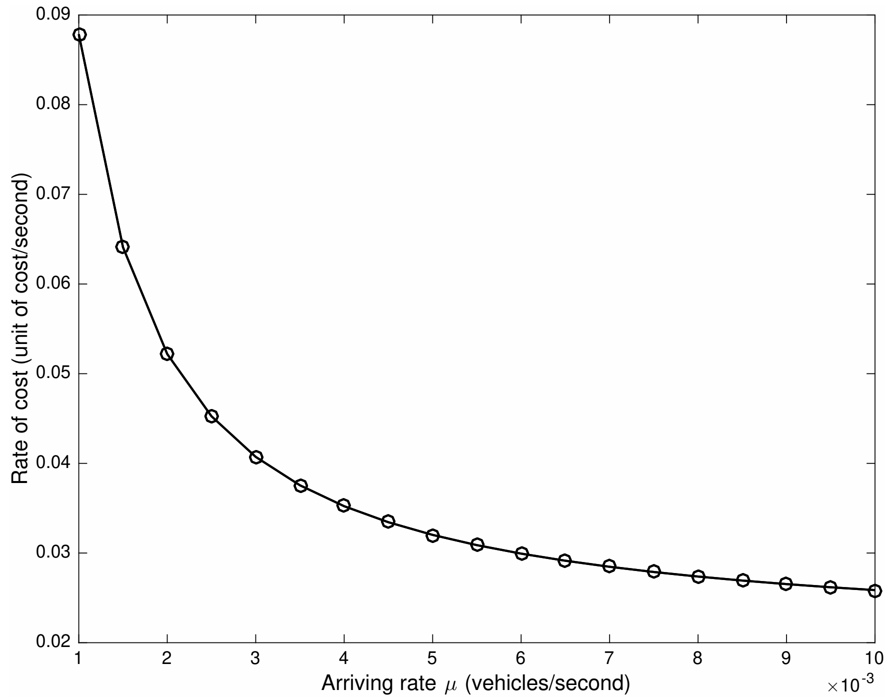

Figure 5.5: The minimal rate of cost for different μ.

RSU, after comparing the vehicle arriving time with one threshold, an optimal decision (continue or stop) can be made.

5.5 Appendix

5.5.1 Proof of Theorem 13

Similar to the proof of Theorem 10, here $V(\lambda)$ is decreasing with λ.

Let $0 < \zeta < 1$, $\lambda_1 > \lambda_2 > 0$, and $\lambda = \zeta\lambda_1 + (1-\zeta)\lambda_2$. Then

$$\begin{aligned} V(\lambda) =& \mathbb{E}\left[Y_{N(\lambda)}\right] - [\zeta\lambda_1 + (1-\zeta)\lambda_2]\mathbb{E}\left[T_{N(\lambda)}\right] \\ =& \zeta\left(\mathbb{E}\left[Y_{N(\lambda)}\right] - \lambda_1\mathbb{E}\left[T_{N(\lambda)}\right]\right) + (1-\zeta)\left(\mathbb{E}\left[Y_{N(\lambda)}\right] - \lambda_2\mathbb{E}\left[T_{N(\lambda)}\right]\right) \\ \geq& \zeta V(\lambda_1) + (1-\zeta)V(\lambda_2), \end{aligned}$$

so $V(\lambda)$ is concave in $\lambda > 0$.

5.5.2 Derivation of $\alpha(x, y)$

We consider an optimal stopping problem starting from time 0 with two parameters $x \geq 0$ and $y \geq 0$, and the stopping rules are:

$$N_\alpha = \min\{\min\{n : T_n + G_n < y\}, C_\alpha\}, \tag{5.11}$$

where $C_\alpha = \min\{n : T_n \geq x\}$. The stopping rule (5.11) means that when the RSU has observed the first vehicle such that $T_n < x$ and $T_n + G_n < y$, it stops to transmit. If there is no vehicle satisfying $T_n + G_n < y$ for $n < C_\alpha$, the RSU is forced to transmit using the C_αth vehicle.

We define a function

$$\alpha(x, y) = \mathbb{P}\left(N_\alpha = C_\alpha\right), \tag{5.12}$$

which is the probability that the RSU transmits using the C_αth vehicle.

If $x = 0$, then $C_\alpha = 1$. Thus, $\alpha(x, y) = 1$. In the following we assume $x > 0$.

We first consider the case $a + x \leq y \leq b$ and define

$$\gamma(t, x, y) = \mathbb{P}(N(t) = C_\alpha) \quad 0 \leq t \leq x,$$

where $N(t) = \min\{\min\{n : T_n \geq t, T_n + G_n < y\}, C_\alpha\}$. The stopping rule $N(t)$ means that the RSU stops at the first car that arrived after time t and $T_n + G_n < y$. If for all $n < C_\alpha$, either $T_n < t$ or $T_n + G_n \geq y$, then the RSU is forced to stop at C_α. When $t = x$,

$$N(x) = \min\{\min\{n : T_n \geq x, T_n + G_n < y\}, C_\alpha\} = C_\alpha.$$

In accordance with the above description, we have the following initial condition for $\gamma(t)$:

$$\gamma(x, x, y) = \mathbb{P}(N(x) = C_\alpha) = 1. \tag{5.13}$$

Notice that $\gamma(t, x, y)$ is the probability that the RSU starts observing the arrival process from time t, and is forced to transmit using the C_αth vehicle. Now, suppose $t < x$ and the RSU starts observation from time $t + \Delta t$, where $\Delta t > 0$ is sufficiently small such that $t + \Delta t < x$. Since the arrival process is a Poisson process, the probability that a vehicle arrived during the interval $(t, t+\Delta t)$ is $\mu\Delta t$, the probability that no vehicle arrived $1 - \mu\Delta t$, and the probability that more than one vehicle arrived $o(\Delta t)$, where $\lim_{\Delta t \downarrow 0} o(\Delta t)/\Delta t = 0$. If there is no vehicle arrived during the interval $(t, t + \Delta t)$, then the RSU has to wait cars arriving after time $t + \Delta t$ for the chance of transmission. Hence, the probabilities that the RSU is forced to transmit using C_αth car when it starts observing from time t and $t + \Delta t$ respectively, are the same. On the other hand, if there is a vehicle arrived during the interval $(t, t + \Delta t)$, and that vehicle satisfies $T_n + G_n < y$, then the RSU transmits using that vehicle. Under this situation, there is no need for the RSU to *restart* the observation from time $t + \Delta t$. Moreover, if there is a vehicle arrived during the interval $(t, t + \Delta t)$, but the total delay of the vehicle $T_n + G_n \geq y$, then RSU shall continue to wait for the next car that comes after $t + \Delta t$. Thus, under this scenario, the probabilities that the RSU is forced to transmit when it starts observing from time t and $t + \Delta t$ respectively, are the same. With the above description, we establish the following equation:

$$\begin{aligned}\gamma(t, x, y) =&(1 - \mu\Delta t)\gamma(t + \Delta t, x, y)\\ &+ \mu\Delta t[1 - F_G(y - t + \Delta t')]\gamma(t + \Delta t, x, y) + o(\Delta t),\end{aligned}$$

where $0 \leq \Delta t' \leq \Delta t$. Note that $t + \Delta t'$ is the arrival instant of the first vehicle after t. After some algebraic operations, we have

$$\frac{\gamma(t + \Delta t, , x, y) - \gamma(t, x, y)}{\Delta t} = \mu F_G(y - t + \Delta t')\gamma(t + \Delta t, x, y) + \frac{o(\Delta t)}{\Delta t}.$$

Letting Δt approach zero from above, we have

$$\frac{\mathrm{d}\gamma(t,x,y)}{\mathrm{d}t} = \mu F_G(y-t)\gamma(t,x,y).$$

Using the initial condition $\gamma(x,x,y) = 1$, for $0 \le t \le x$, we obtain

$$\gamma(t,x,y) = e^{\mu \int_x^t F_G(y-v)\,\mathrm{d}v}. \tag{5.14}$$

Since $N_\alpha = N(0)$,

$$\alpha(x,y) = \gamma(0,x,y) \quad a + x \le y \le b. \tag{5.15}$$

When $a < y \le b$ and $y \le x + a$, $\alpha(x,y) = \alpha(y-a,y)$.

When $y \le a$, $\alpha(x,y) = 1$.

When $y > b$ and $a \le y - x < b$, $\alpha(x,y) = e^{-\mu(y-b)}\alpha(x-y+b,b)$.

When $y > b$ and $y - x \ge b$, $\alpha(x,y) = e^{-\mu x}$.

When $y > b$ and $y - x < a$, $\alpha(x,y) = e^{-\mu(y-b)}\alpha(b-a,b)$.

When the distribution of $F_G(y)$ is the uniform distribution, we have

$$\begin{aligned}\gamma(t,x,y) = e^{\mu \int_x^t F_G(y-v)\,\mathrm{d}v} &= \left(\frac{y-t}{y-x}\right)^{\frac{\mu ab}{b-a}} e^{\frac{\mu b(t-x)}{b-a}} \\ \alpha(x,y) = \gamma(0,x,y) &= \left(\frac{y}{y-x}\right)^{\frac{\mu ab}{b-a}} e^{-\frac{\mu bx}{b-a}} \quad a + x \le y \le b.\end{aligned}$$

5.5.3 Derivation of $\beta(x,y)$

We still consider the optimal stopping rule (5.11) in Appendix 5.5.2, but focus on another property of the stopping rule, i.e.

$$\beta(x,y) = \mathbb{E}\left[T_{N_\alpha}\right], \tag{5.16}$$

which is the expected stopping time of the stopping rule N_α. For $x = 0$, it is readily to know that $\beta(0,y) = 1/\mu$ for $y \ge 0$. So we assume $x > 0$ in the following.

Define

$$\eta(t,x,y) = \mathbb{E}\left[T_{N(t)}\right] \quad 0 \le t \le x,$$

which is the expected stopping time if the RSU starts observation from time t. We have the following boundary condition for $\eta(t,x,y)$:

$$\eta(x,x,y) = \mathbb{E}\left[T_{N(x)}\right] = \mathbb{E}\left[T_{C_\alpha}\right] = x + \frac{1}{\mu}.$$

We first assume $a + x \le y \le b$. Similar to the derivation of $\gamma(t,x,y)$, for $\eta(t,x,y)$ we have

$$\begin{aligned}\eta(t,x,y) =& (1-\Delta t\mu)\eta(t+\Delta t,x,y) + \Delta t\mu F_G(y-t-\Delta t')(t+\Delta t') \\ &+ \Delta t\mu(1-F_G(y-t-\Delta t'))\eta(t+\Delta t,x,y) \\ =& \Delta t\mu F_G(y-t-\Delta t')(t+\Delta t') \\ &+ \left(1-\Delta t\mu F_G(y-t-\Delta t')\right)\eta(t+\Delta t,x,y) + o(\Delta t),\end{aligned}$$

where $0 \le \Delta t' \le \Delta t$. After some algebraic operations, we have

$$\frac{\mathrm{d}\eta(t,x,y)}{\mathrm{d}t} = \mu F_G(y-t)\left(\eta(t,x,y) - t\right). \tag{5.17}$$

With boundary condition $\eta(x,x,y) = x + \frac{1}{\mu}$, we obtain

$$\eta(t,x,y) = \gamma(t,x,y)\left(x + 1/\mu - \phi(x,t)\right), \tag{5.18}$$

where $\gamma(t,x,y)$ is defined in Equation (5.14), and

$$\phi(l,u) = \int_l^u zF_G(y-z)\gamma(z,x,y)\,\mathrm{d}z.$$

Since $N_\alpha = N(0)$, then

$$\beta(x,y) = \eta(0) = \alpha(x,y)(x + 1/\mu - \phi(x,0)). \tag{5.19}$$

When $a < y \le b$ and $y \le x + a$, $\beta(x,y) = \beta(y-a,y) + \alpha(y-a,y)(x-(y-a))$.

When $y \le a$, $\beta(x, y) = 1/\mu$.

When $y > b$ and $a \le y - x < b$,

$$\begin{aligned}\beta(x, y) &= \int_0^{y-b} v\mu e^{-\mu v}\,\mathrm{d}v + e^{-\mu(y-b)}\beta(x - y + b, b) \\ &= \frac{1}{\mu} + e^{-\mu(y-b)}\left(\beta(x - (y - b), b) - (y - b) - \frac{1}{\mu}\right).\end{aligned}$$

When $y > b$ and $y - x \ge b$, $\beta(x, y) = 1/\mu$.

When $y > b$ and $y - x < a$,

$$\begin{aligned}\beta(x, y) = &\int_0^{y-b} v\mu e^{-\mu v}\,\mathrm{d}v \\ &+ e^{-\mu(y-b)}\big(\beta(b - a, b) + \alpha(b - a, b)(x - (y - a))\big) \\ =&\frac{1}{\mu} + e^{-\mu(y-b)}\big(\beta(b - a, b) \\ &+\alpha(b - a, b)(x - (y - a)) - (y - b) - \frac{1}{\mu}\bigg).\end{aligned}$$

Chapter 6

Conclusions and Future Research

6.1 Conclusions

In this research, we have investigated how to make efficient data dissemination in IoT via vehicles. We have studied scheduling delay-tolerant data from a battery or solar powered RSU to the Internet by considering both the queueing delay and the transmit delay of the collected data, and the energy consumption. We aim to achieve the efficient data transmission from the RSU to the Internet by minimizing the total costs including the delay cost and the energy cost. The data transmission efficiency is measured by the rate of cost, i.e. the average of cost per unit time at the RSU.

Chapter 3 studies the case when the delay bound is a hard delay bound. If a bit cannot reach the destination RSU by the predefined deadline, this bit is discarded at the source RSU and a delay penalty is charged for this bit. The above problem has been formulated as a traditional optimal stopping problem. By deriving the optimal rules for the cases when the waiting time is above and below a specific value, we find an overall optimal stopping rule.

Chapter 4 considers the case when the delay bound is a soft delay bound. If the data buffered at the source RSU cannot reach the destination RSU by a predefined deadline, those data still get carried to the destination RSU but a delay penalty is charged. The problem cannot be solved using traditional optimal stopping methods. To solve this problem, we iteratively eliminate a set of non-optimal stopping rules and obtain a sequence of optimal stopping problems. And we prove that this sequence of optimal stopping problems will converge and the optimal stopping rule for the converged optimal stopping problem is an optimal stopping rule for the original problem.

Chapter 5 focuses on the case when the delay bound is a multi-step soft delay bound. This chapter is a more general form of the problem discussed in Chapter 4. Also, the probability distribution of the vehicle arrival process has been extended to any distribution instead of the uniform distribution assumed by the previous two chapters. A sequence of optimal stopping problems are solved to get the final composite optimal stopping rules for the original problem. The derived optimal stopping rule has a multi-thresholds structure and is easy to implement.

6.2 Further extensions and future research

In this thesis, it is assumed that all vehicles passing by the source RSU will also pass by the destination RSU. Actually our research can be extended to the case when some arrival vehicles do not pass by the destination RSU. Recall that vehicle arrivals at the source RSU follow a Poisson process with rate $1/\mu$ (μ is the average inter-arrival time). Let β denote the percentage of the passing-by vehicles at the source RSU that will also pass by the destination RSU. Then the problem is equivalent to the case that vehicles arrive with rate $\beta(1/\mu)$ (i.e., average inter-arrival time is μ/β) and all vehicles will pass by the destination RSU.

We also assume there is only one destination RSU. However, our research can be extended to the case when the source RSU can send its traffic to any of several destination RSUs (that have backbone connection). As an example, consider a source RSU that has two paths, with destination RSU #1 and destination RSU #2, respectively. The transit delay of the two paths have CDF $F_{G_1}(g)$ and $F_{G_2}(g)$, respectively. For the vehicles arriving at the source RSU, denote α and $1-\alpha$ as the percentage of vehicles passing by destination RSU #1 and #2, respectively. Then, the problem is equivalent to the case that only a single destination RSU exists and the transit delay of vehicles has the following effective CDF: $F_G(g) = \alpha F_{G_1}(g) + (1-\alpha)F_{G_2}(g)$.

From the above discussion it can be seen that if a vehicle does not pass by any destination RSU, then the source RSU (called source RSU #1) cannot ask the vehicle to help. However, if the vehicle will pass by another source RSU (called source RSU #2 that does not have backbone connection either) closer to a destination RSU, then it may be beneficial for source RSU #1 to forward its traffic to the vehicle, expecting that the vehicle will deliver the traffic to source RSU #2 and source RSU #2 will ask its passing-by vehicles to help deliver the traffic to a destination RSU. Further, a vehicle, who carries the traffic of a source RSU, may pass the traffic to another vehicle later, if it takes the latter vehicle less time to arrive at an RSU with backbone network connection. The cooperative traffic delivery will be considered as a future research topic.

In this thesis, we use optimal stopping theory to deal with infrastructure-to-infrastructure traffic forwarding. Actually optimal stopping theory might also be a powerful mathematical tool to investigate decision making in vehicle-to-vehicle information sharing, traffic forwarding, and communications, which could be another future research topic.

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