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Title of Thesis — Titre de la thèse

Macro-Economic Model With Government Budget Restraint in an Open Economy

University — Université

University of Alberta

Degree for which thesis was presented — Grade pour lequel cette thèse fut présentée

M.A.

Year this degree conferred — Année d'obtention de ce grade

1982

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April 22/82

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THE UNIVERSITY OF ALBERTA

MACRO-ECONOMIC MODEL WITH GOVERNEMNT BUDGET RESTRAINT IN AN OPEN ECONOMY

by

© MASAYOSHI MATSUSHITA

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF ARTS

DEPARTMENT OF ECONOMICS

EDMONTON, ALBERTA

SPRING 1982

THE UNIVERSITY OF ALBERTA

RELEASE FORM

NAME OF AUTHOR MASAYOSHI MATSUSHITA

TITLE OF THESIS MACRO-ECONOMIC MODEL WITH GOVERNEMNT BUDGET RESTRAINT IN AN OPEN ECONOMY

DEGREE FOR WHICH THESIS WAS PRESENTED MASTER OF ARTS

YEAR THIS DEGREE GRANTED SPRING 1982

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ABSTRACT

When a government increases its expenditures, there must be a finance source to cope with the increased expenditures. The government has some means (policy variables) of financing the increased expenditures, such as printing new money, issueing new government bonds or increasing tax rate.

If a finance source of the government budget is recognized in a system, we can observe the effect of each different finance source in spite of the fact that an exogenous increase in government expenditures itself is a fiscal policy. Then we can form an equation of a government budget restraint.

The aim of this thesis is to recognize the government budget restraint in an open economy. The variables of wealth effects and interest payments are included in the system. A sterilization coefficient is taken into account for the pegged exchange rate system.

First, a simple concept of the government budget restraint is employed for analysis. That is, an exogenous increase in a government policy is simply equated to an endogenous increase in a finance source, ignoring the effects caused by a change in the income level.

Second, the effect of an endogenous change in the income level is explicitly recognized in the government budget restraint. Then, in order to satisfy the government budget restraint the second endogenous policy variable must

be adjusted according to the fluctuation of the income level, under each policy specification. Thus far, the price level is assumed to be rigid.

Third, the flexible price level and full-employment income level are assumed in the system along with the government budget restraint.

The method of analysis is a comparative statics and the IS-LM-BP paradigm. The multipliers of endogenous variabvles are observed for each government policy in the short run. In an open economy two systems can be employed. One is a floating exchange rate system where the balance of payments is assumed to be always equilibrated by the fluctuation of an exchange rate. The other is a pegged exchange rate system where the exchange rate is fixed and the balance of payments fluctuates. The effects of government policies are analyzed for both systems.

The results obtained from the case of the first government budget restraint and the case of the second government budget restraint are similar under the floating exchange rate system. Only one signifgicant difference can be observed on the income multipliers in a fiscal policy financed by goverbment bonds. Under the pegged exchange rate system, the first government budget restraint does not recognize the balance of payments, but the fluctuation of the balance of payments is assumed to be absorbed in a money market. The balance of payment variable is recognized in the second government budget restraint. In this case the fluctuation of the balance of payments is assumed to be adjusted by an endogenous policy variable. Then each multiplier of endogenous variables shows a significant difference from the case of the first government budget restraint.

The assumption of the flexible price level does not alter the effects of government policies on private endogenous variables significantly. However, since the income level is assumed to be rigid, the adjustment of the marginal tax rate is different from the case of the variable income level.

ACKNOWLEDGEMENTS

I would like to express sincere gratitude to Professor T. Tsushima who was a firm but always patient guide in my pursuit of knowledge. Without his outstanding knowledge and assistence, this thesis would never have been written.

I would also like to thank Professor T. Powrie for his sincere kindness and continuous encouragement which kept me working on this thesis.

I am grateful to Professor R. Mirus making himself available on very short notice and for his invaluable comments.

My best wishes and thanks to all of my colleagues for their friendship and encouragement. In particular, thanks go to Jerry Ochitwa, Michel Bailey, Susan Johnson, Jim Tiessen and Karen Richter who corrected my faulty English and proof-read many drafts.

I am endebted to my wife Elizabeth and parents-in-law Dr. and Mrs. Egger for their great generosity, understanding and support.

Table of Contents

LIST OF TABLES

page

LIST OF FIGURES

PAGE

LIST OF SYMBOLS AND ABBREVIATIONS

B = nominal interest payments

$\dot{B}$ = a change in the number of government bonds

C = net current account

E = real domestic private expenditure function

F = imports

g = real government purchases of domestic and foreign goods and services

i = nominal domestic interest rate

K = real capital inflow function

L = domestic demand for money function

M = domestic nominal stock of base money

$\dot{M}$ = a change in the stock of base money

M^* = total nominal stock of base money

$\dot{M}^*$ = a change in the total stock of base money

p = domestic price level

p^* = foreign price level

R = real foreign reserves

$\dot{R}$ = the balance of payments

r = exchange rate

s = sterilization coefficient

t = real tax collections

u = marginal tax rate

V = nominal autonomous tax flow less transfer payments

v = real autonomous tax flow less transfer payments

w = real private wealth

X = real net export function

X_e = exports

y = real domestic product and income

z = real domestic disposable income

ϵ = elasticity

Δ = Jacobian determinant

BFGE = bond financed government expenditures

ERC = exchange rate change policy

FERS = the floating exchange rate system

MFGE = money financed government expenditures

OMP = open market purchases

PERS = the pegged exchange rate system

I. LITERATURE REVIEW

A. Introduction

In recent years a government budget restraint has been given important recognition in macroeconomic theory. The concept of the government budget restraint was first introduced in Ritter(1955-56).[1] Christ(1967,1968) made further progress in the theoretical analysis of the government budget restraint.[2] Blinder-Solow(1973) also made a contribution in the analysis of government policies by using the government budget restraint.[3] Their paper involved the stability of a system in the controversy between the monetarists and the neo-Keynesians.[4] Other relevant papers have been published since and the controversy of the stability condition continues.

In the literature on the government budget restraint, however, most of the progress has been accomplished in the sphere of a closed economy. An open economy is a new field

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[1]L.S. Ritter, "Some Monetary Aspects of Multiplier Theory and Fiscal Policy," Review of Economic Studies XXIII(2), no.61 (1955-56): pp.126-31.
[2]C.F. Christ, "A Short-Run Aggregate-Demand Model of the Interdependence and Effects of Monetary and Fiscal Policies with Keynesian and Classical Interest Elasticities," American Economic Review (May 1967): pp.434-443; "A Simple Macroeconomic Model with a Government Budget Restraint," Journal of Political Economy (January 1968): pp.53-67.
[3]Alan S. Blinder and Roberto M. Solow, "Does Fiscal Policy Matter?" Journal of Public Economics (November 1973): pp.319-37.
[4]Tobin named those who have an "eclectic nonmonetarist view", "Neo-Keynesians", vs. monetarists.
J. Tobin, "Friedman's Theoretical Framework," Journal of Political Economy (September/October 1972): p.852.

of study since only a few relevant papers have been published, relating to the role of the government budget restraint. Our purpose is, therefore, to focus on this field of study and to attempt to make further progress in this thesis.

In this chapter, first the literature on the government budget restraint is reviewed in a closed economy. Then, the development of the traditional theory in an open economy is reviewed. Finally, the literature on the government budget restraint in an open economy is explored.

B. Review

1. Closed Economy on the Government Budget Restraint

The presence of a government budget restraint in a system has, in fact, been recognized by many different economists since the 1950's. The first article concerning the government budget restraint is found in Ritter(1955-56). In this paper the necessity of defining the government budget restraint in the system was vaguely implied.

After Ritter, or during the same period, many papers implied or considered the necessity of the government budget restraint in the system.[1] But the essential analysis of the

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[1]See C.F. Christ, "Some Dynamic Theory of Macroeconomic Policy Effects on Income and Prices under the Government Budget Restraint," Journal of Monetary Economics 4 (January 1978): p.45.

government budget restraint was done in two subsequent papers by C.F. Christ.

Christ(1967,1968) emphasized that the extreme choice between a monetary and a fiscal policy is misleading, because both policies are interdependent due t the presence of the government budget restraint. The government budget restraint has been defined in many different ways by different economists. The essence is that the government authorities cannot determine all policy variables exogenously. They are always constrained by one or more endogenous policy variables since the total government spending must be equated to the total of financing from all sources. In other words, given N policy variables available to apply to the nation's economy, the government authorities can at most predetermine N-1 policy. The Nth policy is left as endogenously determined.

One of the significant discoveries in the literature of the government budget restraint is that a long-run income multiplier with respect to government purchases is the reciprocal of the tax rate 1/u, assuming the tax rate u is held constant. Ritter(1955-56) recognized that only when a budget is balanced and the tax rate is fixed high enough can an increase in government purchases be offset by an induced tax revenue. Thus, if the multiplier effect of government purchases is less than the reciprocal of the tax rate 1/u, a deficit (surplus) will result when government purchases are increased (decreased). In the case of the budget deficit (surplus), government authorities must increase (decrease) the money stock by printing (destroying) money in order to balance the budget.

The recognition of the government budget restraint in Ritter(1955-56) is strictly a static concept. But Christ(1968) explains that when the equation of the government budget restraint is included in a system, it becomes a dynamic system since a change in money stock over time is involved. For example, if one unit of government purchases is increased, the government authorities must increase one unit of money stock once and for all to finance the budget deficit in the first period. Assuming that the government purchases are held constant from the first period on, government authorities still encounter the budget deficit. However, an increase in money stock will be less than one unit in the second period, because an increase in the income level, due to the multiplier effect of an increase in government purchases, creates an extra induced-tax revenue, given a constant tax rate. It continues until the deficit is completely financed by the induced-tax revenue (i.e. $dg=udy$ where dg is a change in government purchases and dy a change in income).

The economy at this stage goes back to a new equilibrium steady state. An increase in income stops at this stage and the multiplier of income is equal to the reciprocal of the tax rate $1/u$ with respect to government purchases at the constant tax rate. But Christ's illustration of 1/u is not a special case as Ritter described. It is the long-run multiplier from one steady state equilibrium to another. When an economy is at an equilibrium, government budget must be balanced (i.e. g=t where t is tax collections). Therefore, when the budget is unbalanced, the system is on the process of going toward another equilibrium state.

We have discussed how the inclusion of the government budget restraint in the model creates a dynamic model moving from one equilibrium state to the other, as long as the budget is unbalanced. This is the case when the money stock is endogenously determined. From the early days of macroeconomic field, efficacy of a monetary policy vs. a fiscal policy has been debated.

From the empirical study in Anderson-Jordan(1968), they concluded that a change in government spending financed by bond issue or tax collections would not only offset the total expansionary effect of the increased government spending, but also have a negative effect on income. [4] This empirical result was attacked on a theoretical ground by Tobin(1972). Tobin claimed that if their empirical result was correct, then the perfect interest inelastic demand for money would have to prevail in a system. Most empirical studies, however, had already proven that the demand for

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[4] L.C. Andersen and J.L. Jordan, "Monetary and Fiscal Actions: A Test of Their Relative Importance in Economic Stabilization," Federal Reserve Bank of St. Louis Review.51 (november 1968): pp.11-24.

money is in fact elastic with respect to interest rate.

Then monetarists argued that the perfect interest inelastic demand for money is irrelevant to this issue. The important issue, they argue, is to recognize an effect of budget deficit created by government borrowing.

Milton Friedman established his monetarist view as to the effectiveness of a monetary policy vs. the ineffectiveness of a fiscal policy in a series of subsequent papers published from 1970 to 1972. [7] His point is that an increase in government purchases particularly financed by bond issue has little expansionary effect on income in the long run. As long as the government has a budget deficit, an expansionary fiscal effect on income made by government purchases has to be diminished. Eventually the expansionary fiscal effect is completely crowded out when the government budget is balanced.

Friedman's view was once again counterattacked by the neo-Keynesians. At this stage recognition of the government budget restraint is involved with the monetarist and neo-Keynesian debate. Specifically, Blinder-Solow(1973) presented a systematic treatment of the effects of a fiscal policy with the recognition of a wealth effect and the government budget restraint in a simple Keynesian model.

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[7]M. Friedman, "A Theoretical Framework for Monetary Analysis," Journal of Political Economy (March/April 1970): pp.193-238; "A Monetary Theory of Nominal Income," Journal of Political Economy (March/April 1971): pp.323-37; and "Comments on the Critics," Journal of Political Economy (September/October 1972): pp.906-50

Their arguments were against the monetarist view of the ineffective fiscal policy.*

The key issue raised by Blinder-Solow(1973) was a consideration of the stability system. Their claim was that if Friedman's view was verified (that is, the assumption of increased government spending financed by bond issue is contractionary), then the system would be unstable. Under the fixed price level Blinder-Solow(1973) derived the following conclusions.

First, if the crowding out effect on the money market exceeds the positive wealth effect on the commodity market, then bond finance is contractionary. In this case the monetarist view is correct, but the system would be unstable.

Second, if an increased government spending followed by bond finance is expansionary, (that is, the net wealth effect increases the income level), then the extreme monetarist view is wrong. But even in this case where it is less expansionary than the increased government spending financed by money, the system will turn out to be unstable.

* $W=K+M/P+B/iP$

where K is capital stock, M/P real money stock and B/iP real value of outstanding government bonds. This wealth equation above specified by Christ(1967) and Blider-Solow(1973) enables the variable of government bonds to be included in a system.

We must note that both in Christ(1967) and Blinder-Solow(1973) B is specified as the number of government bonds, and each bond is a perpetuity paying $1.00 per year. Hence, B becomes the value of interest payments to the private sector, and the number of bonds divided by interest rate B/i is equal to the market value of the stock of bonds.

Finally, only if the increased government spending followed by bond finance is more expansionary than that by money finance will the system be stable.

The verification of the stable system under bond finance can be derived from the effect of interest payments to the private sector on the income level. In the case of bond finance, budget deficit is larger than in the case of money finance due to the existence of future interest payments. Therefore, it would require higher private expenditures to obtain sufficient induced-tax revenue to finance higher budget deficit. Thus, if the system is stable, the bond finance must have a higher expansionary effect on income than the money finance in the long run.

Following Blinder-Solow(1973) a number of papers have been published related to this controversy. ' In terms of recent literature of the government budget restraint, the focus was on a stability condition in the system of an economy. As the representative of these works we shall review Christ(1978,1979). The model in Christ(1978) is sophisticated, including a price adjustment equation and a

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' Karl Brunner and Allan H. Meltzer, "Money, Debt, and Economic Activity," Journal of Political Economy (September/October 1972): pp.951-77 and "An Aggregative Theory for a Closed Economy," in Monetarism, ed. Jerome L. Stein, (Amsterdam: North Holland Publishig Co., 1976): pp.69-103; J. Tobin and W.H. Buiter, "Long-Run Effects of Fiscal and Monetary Policy on Aggregate Demand," in Monetarism, ed. Jerome L. Stein: pp.273-309; C.F. Christ, "Some Dynamic Theory of Macroeconomic Model with a Government Budget Restraint," Journal of Monetary Economics 4 (January 1978): pp.45-70; and C.F. Christ, "On Fiscal and Monetary Policies and the Government Budget restraint," American Economic Review (September 1979): pp526-538.

concept of steady state growth rate in the system.

Now we focus on the stability condition of the system for each endogenous policy variable. The conclusion in Christ(1978) is that the increased government spending accompanied by bond finance is unlikely to be stable while the rest of endogenous policy variables (such as government purchases, autonomous nominal taxes and high powered money) lead the system to stable equilibrium. Using the same model in Christ(1978), Christ(1979) investigates the stability condition once again by varying the definition of government purchases, and finds that stability is dependent on how the variable of interest payments is incorporated in the government budget restraint equation.

Christ(1978) specified government purchases simply as the government expenditure variable. The tax equation and the government budget restraint are defined as follows:

I-1 $t=V/P-B/P+uB/P+uy$ where V/P is real autonomous taxes.

I-2 $g=t+\dot{M}/P+\dot{B}/iP$

Substituting the tax equation into the government budget restraint we have :

I-3 $g-V/P+(1-u)B/P-uy=\dot{M}/P+\dot{B}/iP$

In the case of bond finance, the high-powered money stock is kept constant (i.e. dM/P=0). I-3 shows that the deficit will show up due to an increase in interest payments net of tax, (1-u)B/P>0, assuming that an increase in government purchases dg is offset by induced-tax revenue udy in the long run. Hence, the system will not converge to the equilibrium position.

Two other specifications of a government expenditure variable can be defined as follows :

I-4 $g'=g+(1-u)B/P$

I-5 $g''=g+B/P$

With the same tax equation in I-1, the government budget restraint will be as follows for I-4 and I-5 respectively :

I-6 $g'-uy-V/P=\dot{M}/P+\dot{B}/iP$

I-7 $g''-uy-V/P-uB/P=\dot{M}/P+\dot{B}/iP$

First let us analyze from I-4 and I-6. I-4 gives government purchases plus government interest net of tax as the government expenditure variable. Substituting I-4 into I-3 we have I-6. Obviously it is a simpler government budget restraint than I-3 since the government interest net of tax is cancelled out by the tax equation. Although the term B/p disappears from the government budget restraint, the deficit will not change if government bonds are issued. Hence, the system will not reach an equilibrium in this case either.

Next, a government expenditure variable is specified in I-5; that is, government purchases plus debt interest gross of tax. Similarly substituting I-5 into I-3 the government budget restraint becomes I-7. In this case the stability of the system is quite hopeful when bonds are issued to finance a deficit. Because (-uB/P) is revenue, the deficit shrinks and eventually the system converges to an equilibrium position.

The difference between this system and the previous two systems lies in debt interest B/P in the government expenditure variable. When the exogenous variable g" is increased, and bonds are issued to finance a deficit, an increase in debt interest is counteracted by an equal decrease in government purchases.

By using the government expenditure variable g", Christ(1979) checked the stability conditions with the two models different from the previous model of Christ(1978). One is the model in Blinder-Solow(1973), and the other is the model of Tobin-Buiter(section 6)(1976). The model of Blinder-Solow(1973) with g" showed the definite stability when either bonds or money is endogenous. The original model of Tobin-Buiter(1976) showed the definite instability when bonds are endogenous, but with g'' the stability becomes uncertain. Hence, the stability is not impossible with bond finance, as long as a government makes a proper consideration for forthcoming increased interest payments.

2. Open Economy

The first half of the section reviews the development of the literature on monetary and fiscal policies in an open economy established mainly in the 1960's. Then the development of the analysis in a macro model recognizing a government budget restraint in an open economy is reviewed.

In a system with two goals (full employment and balance of payment equilibrium), and two instruments (the terms of trade and the interest rate), Mundell(1960) illustrated that the pegged exchange rate system would be more effective when capital mobility was high, while the floating exchange rate system would be more effective when capital mobility was low.[10] The reason for high effectiveness in the floating exchange rate system under a low capital mobility economy was explained by the fact that the instrumental variable (i.e. the interest rate) would have a direct effect on the financial market. On the other hand, the exchange rate would have a direct effect on the commodity market. Mundell(1960) stated,

> ....a system works best if variables respond to the markets on which they exert the most direct influence.[11]

This argument was referred to as the Principle of Effective Market Classification (hereafter PEMC).

In Mundell(1960) PEMC was recognized through capital mobility and the instrumental variables such as the exchange rate and the interest rate, but the government policies were not explicitly recognized. In Mundell(1962), however, monetary and fiscal policy were used as instruments to equilibrate the system.[12] Given the assumptions of the

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[10] R.A. Mundell, "The Monetary Dynamics of International Adjustment Under Fixed and Flexible Exchange Rates," Quarterly Journal of Economics (May 1960): pp.227-57.

[11] Ibid., p.250

[12] R.A. Mundell, "The Appropriate Use of Monetary and Fiscal Policy for Internal and External Stability," IMF Staff Papers IX, no.1 (March 1962).

pegged exchange rate system and no impositions of trade controls, Mundell's conclusion on PEMC was that monetary policy should be reserved for the purpose of the external stability and fiscal policy for the internal stability under the pegged exchange rate system.

Mundell(1963) expanded his theory of the pegged exchange rate system to the floating exchange rate system.[13] Mundell(1963) was different from the previous two papers in that it did not adhere to PEMC. However, it illustrated the effects of a monetary policy and a fiscal policy under both the floating exchange rate system and the pegged exchange rate system systematically. Mundell(1963) assumed perfect capital mobility. This means the perfect substitutability of all securities, domestic or foreign monetary policy was assumed to take the form of open market purchases of securities and the fiscal policy of bond financed government expenditures.

Under the floating exchange rate system Mundell(1963) concluded that the monetary policy should have a strong effect on income level and employment, given the assumption of the rigid price level. The reason for the strong effectiveness of the monetary policy is as follows. Open market purchases of domestic securities result in downward pressure on the interest rate, but the interest rate is

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[13]R.A. Mundell, "Capital Mobility and Stabilization Policy under Fixed and Flexible Exchange Rates," Canadian Journal of Economics and Political Science (November 1963): pp.475-485.

controlled by foreign interest rates under perfect capital mobility. This implies outflow of capital. Instead of the interest rate change, a depreciation of the exchange rate is incurred due to the balance of payment deficit. Finally, due to the exchange rate depreciation, the balance of trade goes up. The income level and employment level increase until the system reaches a new equilibrium level.

On the other hand, a fiscal policy has no effect on income level and employment under the floating exchange rate system. An increase in government expenditures raises the income level and has upward pressure on the interest rate. This in turn causes the appreciation of the exchange rate due to an inflow of capital. The exchange rate appreciation causes the income level to fall, which offsets exactly the increased income level. Therefore, employment cannot change under the floating exchange rate system when the fiscal policy is applied.

Under the pegged exchange rate system, Mundell(1963) concluded that a fiscal policy would have a strong effect on income level and employment. This was illustrated as follows. An increase in government expenditures raises the income level, employment level and the demand for money. Upward pressure of the interest rate, in turn, causes an inflow of capital and improves the balance of payments. The increase in the balance of payments fulfills an increase in the demand for money. Consequently the rise of the income and employment that originated in an increase in government expenditures remains at a new equilibrium.

The monetary policy, on the other hand, under the pegged exchange rate system was concluded to have no effect on income and employment. Open market purchases cause an outflow of capital which deteriorates the balance of payments. Since an outflow of capital decreases domestic money supply, income and employment decrease until the foreign exchange deficit equals the increased amount of money supply through open market purchases. Mundell's contribution was to illustrate the effectiveness of the monetary and the fiscal policies under the condition of perfect capital mobility, only.

Johnson(1966) proved Mundell's theory correct, but improved it by using the multipliers derived from the simple model of an open economy.[14] Johnson's contribution lies in the recognition of two different types of capital mobility. One as was the main theme of Mundell(1963), is mobility in response to a change in the interest rate. The other is mobility in response to a change in the income level. That is, the determination of international capital mobility depends not only on a change in the interest rate, but also on a change in the income level, hence a change in profitability of investment. The raised level of income was assumed to increase a net inflow of capital, because an

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[14] H.G. Johnson, "Some Aspects of the Theory of Economic Policy in a World of Capital Mobility," Essays in Honour of Marco Fanno ed. Gabiotti, Antonio Milani, Padova, 1966: pp.345-59

increase in the income and employment level resulting from the fiscal and the monetary policy would attract foreign investors to purchase domestic securities.

Another contribution of Johnson(1966) lies in the analysis of a non-perfect capital mobility system. Mundell's perfect capital mobility assumption made a clear distinction between the effects of the monetary and the fiscal policies for each condition of floating and the pegged exchange rate systems. However, Johnson's generalization of international capital mobility approached closer to a more realistic state of economy.

Although Johnson(1966) progressed in the theory of an open economy through the tool of mathematical analysis, its simple model had some drawbacks. In particular, sterilization policy was ignored in the pegged exchange rate system. Johnson assumed the rigid price level, no liquidity trap and no real balance effect in the system.

It was Takayama(1969) who constructed a generalized equilibrium model by integrating the contributions of Mundell(1963) and Johnson(1966), and eradicating their deficiencies.[18]

3. Government Budget Restraint on Open Economy

The government budget restraint on open macro model is still a new field of study. We shall review several papers

[18] Akira Takayama, "The Effects of Fiscal and Monetary Policies under Flexible and Fixed Exchange Rates," Canadian Journal of Economics (May 1969): pp.190-209.

that have treated this field.

The fundamental role of the government budget restraint in an open economy was well explained in Yu(1980) in the context of comparative statics under the pegged exchange rate system.[14] The model specified in Yu(1980) was the extension of the model in Johnson(1966). The model includes the simplest specification of the government budget restraint and the wealth effect. Two fiscal policies were chosen to be analyzed: an increase in government expenditures financed either by money stock (hereafter called money financed government expenditures) or by bond finance (hereafter called bond financed government expenditures). Only the balance of payment multiplier was derived for each policy. The effect of the government budget restraint and the wealth effect on the balance of payments in contrast to the traditional theory in the open economy, were analyzed.

Turnovsky(1977) made an extensive study of an open macro model with the government budget restraint.[15] The model in chapter 9 (Turnovsky(1977) pp.195-216) includes a sterilization coefficient in the money market. It defines a more detailed balance of payment equation (in that the variables of exports and imports are separated) than a net

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[14]Eden S.H. Yu, "Government Financing Constraint, Wealth Effects and External Balances," Southern Journal of Economics (October 1980): pp.303-316.
[15]Stephen J. Turnovsky, Macroeconomic Analysis and Stabilization Policy Cambridge: Cambridge University Press, 1977.

export equilibrium used in Johnson(1966) and Yu(1980).[18] The capital account equation defined in Turnovsky(1977)(chapter 9) is similar to the traditional specification (Johnson(1966), Takayama(1969) and Yu(1980)) where net capital inflow, k is the function of income, y and domestic interest rate, r (i.e. k=k(y,r)). capital consisting of the net change in domestic bonds purchased by foreigners, A/i, less the net change in foreign bonds purchased by domestic

Turnovsky's focus was rather on the specification of the balance of payment equation than on recognition of the government budget restraint. In this sense, Turnovsky did not explain explicitly the role of the government budget restraint in his analysis.

Some other papers which recognize the government budget restraint in the open economy can be found in the literature, such as Scarth(1975) and Allen(1977).[19] The government budget restraint specified in Scarth(1975) is complex in that it includes the variable of the balance of payments and the government debt from the private sector. The results of multipliers both under the floating exchange

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[18] The definition of the current account in Johnson(1966) and Yu(1980) was simple in that the exports were assumed to be kept constant. Therefore, the fluctuation of the imports was assumed to determine the current account. The clear distinction of exports and imports variables, as in Turnovsky(1977)(chapter 9 pp.201-2), made it possible to draw the Marshall-Lerner condition explicitly.

[19] W.M. Scarth, "Fiscal Policy and the Government Budget Constraint under Alternative Exchange-Rate Systems," Oxford Economic Papers 27 (March 1975): pp.10-20;

Polly R. Allen, "Financing Budget Deficits: The Effects on Income in Closed and Open Economies," European Economic Review 10 (1977): pp.345-373.

rete and pegged exchange rate systems were shown in a simulation context.

Allen(1977) demonstrates an extensive study on the government budget restraint with a complex model. It starts in the context of a closed economy and then goes on to an open economy. The model recognizes the wealth effect and the financing budget deficit to a full extent. The focus of this paper is the stability conditon of a system and the steady state condition for government policies.

C. Summary

Christ(1968) analysed the effects of government policies by recognizing the government budget restraint in the macro model. This paper demonstrated theoretically that domestic monetary and fiscal policies are interrelated if the government budget restraint is to be recognized in a system. It also demonstrated that the recognition of the induced tax revenue is an important factor to equilibrate the system.

Blinder-Solow(1973) demonstrated stability conditions of the system for money financed government expenditures and bond financed government expenditures, including an interest payment variable in the government budget restraint. They concluded that bond financed government expenditures are more expansionary than money financed government expenditures in the long run, because an increase in the interest payments due to new government bond issues has a multiplier effect on the private expenditures.

The traditional theory on an open economy was reviewed based on Mundell(1960, 1962 and 1963) and Johnson(1966). Mundell showed the Principle of Effective Market Classification. He concluded that the monetary policy is effective on income and employment under the floating exchange rate system and that the fiscal policy is effective under the pegged exchange rate system.

Regarding an open economy on the government budget restraint, although some papers related in this field have been published, we find that some more improvement must be made. Yu(1980) analysed the balance of payment multipliers for money financed government expenditures and bond financed government expenditures under the assumption of a complete sterilization policy, but did not solve the model in the floating exchange rate system. Turnovsky(1977) and Yu(1980) did not incorporate the interest payment variable, nor the variable of induced tax revenue. Scarth(1975) had all the variables in the model, but the multipliers were not explicitly shown as he resorted to the simulation method. All these papers were analyzed in the context of comparative statics.

Since the paper by Blinder-Solow(1973) appeared, the stability conditions of the system have become a main theme in many papers on the government budget restraint. In the closed economy, Christ(1978,1979) demonstrated that the specification of the government expenditures if they include or exclude interest payments is the significant factor to hold the stability conditions. In the open economy, Allen(1977) and Turnovsky(1977)(chapter 11) showed the stability conditions. Thus, the stability condition of the system has been debated extensively both in the closed and the open economy.

The approach in this thesis will be restricted to a short run period, and strictly to the context of comparative statics in the open economy rather than the dynamic system of analysis. Stability question in a system is not analyzed. We shall observe the short run impact of each government policy on each endogenous variable under both the floating exchange rate system and the pegged exchange rate system. Those variables which were not taken into account either in Turnovsky(1977) (chapter 9) or Yu(1980) will be incorporated into our model; that is, the inclusion of a sterilization coefficient under the pegged exchange rate system, and the interest payment variable on the government budget restraint. In particular, the recognition of induced tax revenue on the government budget restraint due to the increased income level will be a main concern of this thesis.

II. IMPLICIT RECOGNITION OF THE GOVERNMENT BUDGET RESTRAINT IN AN OPEN ECONOMY

A. Introduction

If a government budget restraint is recognized in the system of an economy, government authorities cannot simply vary a single policy variable. A change in the policy variable must be adjusted by a change in other policy variable(s) in order to satisfy the government budget restraint. Therefore, the effects of the change in the policy variable depend on how other policy variables are adjusted in the system. This implies that the traditional analysis of monetary and fiscal policies without concept of a government budget restraint is a special application of government policies.

In this chapter we shall build a simple macro model of an open economy, in which a government budget restraint is included, and observe the short run effects of government policies in the system associated with the traditional IS-LM framework. The three domestic government policies will be chosen for analysis. The first two policies are a change in government expenditures with either money finance or government bond finance. The former policy will be called hereafter money financed government expenditures and the latter, bond financed government expenditures. Another policy to be analyzed is a traditional monetary policy, namely open market purchases (i.e. a monetary expansion and a debt contraction).

Since the model is built for an open economy, each policy will be analyzed under the floating exchange rate system and the pegged exchange rate system. In addition to the three domestic policies, the effect of an external policy, namely exchange rate change policy, will be observed under the pegged exchange rate system. A sterilization coefficient will be incorporated in the system.[19] Throughout this thesis, the small country assumption is retained.

The effects on three endogenous variables will be analyzed for each policy in each system. That is, the income level, the interest rate, and the exchange rate for the floating exchange rate system and the balance of payments for the pegged exchange rate system. The price level will be assumed rigid in this chapter so that all variables are expressed in real terms. Special attention will be paid to the analysis of the exchange rate multiplier for the floating exchange rate system and the balance of payments multiplier for the pegged exchange rate system by assuming different slopes of the balance of payment curve (hereafter the BP curve). Under the pegged exchange rate system, the analysis will be divided in two stages. In the first stage holding the sterilization coefficient constant, the multipliers will display the effects of domestic

[19] Johnson(1966) and Yu(1980) assumed a complete sterilization policy. The introduction of the sterilization policy is referred to Takayama(1969) and Turnovsky(1977)(chapter 9).

policies on the endogenous variables. But the internal and external balances are shown to be disequilibrated by the effects of domestic policies under the pegged exchange rate system. Therefore, in the second stage, assuming non-sterilization policy, we shall observe how the system can attain a new equilibrium in the context of the IS-LM framework.

Following Mundell(1963), the effect of each policy will also be analyzed for the case of perfect capital mobility both for the floating exchange rate system and the pegged exchange rate system.

A stability condition is not examined, but it is assumed to be stable.

B. The model

The model consists of seven equations as follows:

II-1 $y=E(z,i,w)+X(z,r,p)+g$

II-2 $z=y+B/p-t$

II-3 $t=u(y+B/p)+v$

II-4 $M/p+R/p=L(y,i,w)$

II-5 $w=M/p+B/ip$

II-6 $\dot{R}/p=X(z,r,p)+K(y,i)$

II-7 $g+B/p+\dot{R}/p=\dot{M}/p+\dot{B}/ip+\dot{R}/p+t$

The endogenous variables are defined as follows:

y=Real domestic product and income;

z=Real domestic disposable income;

t= Real tax collections;

w=Real private wealth;

i=Nominal domestic interest rate;

p=Domestic price level;

R=Nominal foreign reserves;

$\dot{R}$=The balance of payments (a change in foreign reserves);

r=Exchange rate, the price of foreign currency in terms of the domestic currency;

The policy variables are defined as follows:

g=Real government purchases of domestic and foreign goods and services;

B=Nominal interest payments or total number of bonds issued by the government since each bond is assumed to bear a perpetuity paying $1 per year;

B=A change in the number of government bonds;

M=Domestic nominal stock of base money, bearing no interest;

$\dot{M}$=A change in the stock of base money;

u=Marginal tax rate where $0<u<1$;

v=Real autonomous tax flow less transfer payments which is assumed to be negative.

The functional notations of the system are as follows:

E=Real domestic private expenditure function;

L=Domestic demand for money function;

X=Real net export function;

K=Real capital inflow function.

From the traditional macroeconomic theory, the signs of the partial derivatives of II-1, II-4 and II-7 are as follows:

$\partial E/\partial z = E_1 > 0$, $\partial E/\partial i = E_2 < 0$, $\partial E/\partial w = E_3 > 0$,

$\partial X/\partial z = X_1 < 0$, $\partial X/\partial r = X_2 > 0$,

$\partial L/\partial y = L_1 > 0$, $\partial L/\partial i = L_2 < 0$, $\partial L/\partial w = L_3 > 0$,

$\partial K/\partial y = K_1 > 0$ and $\partial K/\partial i = K_2 > 0$.

In addition to the assumptions above, the following restrictions will be specified to assure a stable equilibrium:

$0 < E_2 < 1$, $0 < L_2 < 1$ and $0 < E_1 + X_1 < 1$.[21]

II-1 states that real domestic income is the sum of private expenditures, net exports and government expenditures. Domestic private expenditures are the function of disposal income, the interest rate and private wealth. Net exports are the function of disposable income and the exchange rate.

II-2 states that real domestic disposable income is the sum of real income plus real interest receipts minus tax payments to the government.

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[21] This inequality is derived by a plausible assumption that the increased magnitude of domestic private expenditures due to an increment of income is larger than the magnitude of net exports decreased due to the same increment of income.

II-3 states that the real tax equation is the marginal tax rate multiplied by real household income and interest receipts from holding government bonds plus real autonomous tax flow less transfer payments.

II-4 is the equilibrium condition for money market in an open economy. It states that the domestic stock of money plus the foreign reserves are equal to the real demand for money. Real demand for money is a function of real income, the domestic nominal interest rate and the real value of private wealth. We should note that the conventional meaning of the stock of money does not appear in this model. The stock of money here is assumed to be the base money stock or the high-powered money stock. Hence, it is controlled by the government as one of the policy variables. The base money bears no interest. The banks and the non-banking private sectors are assumed to be consolidated into a sole private sector so that banks' liabilities and assets of non-banking sector cancel out each other. The rest of money holdings in the private sector is currency in circulation plus deposits at the Central Bank by chartered banks which are equivalent to the base money stock or the high-powered money stock. The treasury and the Central Bank are assumed to be consolidated in a sole government sector.

II-5 states that real private wealth is the sum of real base money stock and real government bonds. Patinkin(1965) showed that the amount of the money stock does matter in an economy due to the existence of the real balance effect in the short run.[22] The wealth effect is virtually the same as real balance effect. It is at work only in the short run. It is controversial, however, whether government bonds should be counted into private wealth or not. One argument is that government bond holdings cannot be counted as wealth, because the bonds are the future tax liabilities to the private sector if people maintain a perfect foresight for the future. We assume, however, that people have an interest payment illusion on government bonds without being cognizant of the future tax liabilities. Then government bonds are included as a net asset to the private sector. By including government bonds explicitly in the private wealth equation in this manner, we can implicitly recognize the market for government bonds in the system by virtue of Walras law.

Besides the inclusion of government bonds and base money stock in the private wealth equation, we must take physical capital stock into account. But this can be abstracted from the private wealth equation by assuming that the physical capital stock is kept constant in the short run.

II-6 is the balance of payment equilibrium condition. The balance of payments is the net flow of foreign exchange reserves from one period to another. The balance of payments consists of two major accounts, namely the current

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[22] D. Patinkin, Money, Inflation and Prices. 2nd ed., New York: Harper & Row, 1965.

account and the capital account. The current account is the foreign exchange receipts from the sales of domestically produced goods and services abroad and also the foreign exchange payments from the purchases of foreign produced goods and services. The net receipts of these foreign exchange transactions appear as the net exports in the domestic income equation II-1. The net exports are assumed to be the function of real disposable income and the exchange rate. The exchange rate r is expressed as the price of foreign currency in terms of the domestic currency (i.e. $r=p/p*$ where $p*$ is a foreign price level). Hence, an increase in the exchange rate implies the devaluation of the domestic currency. If we express exports and imports explicitly in the current account, it will be written as follows:

II-8 $$C=X_e(r)-rF(z,r,p),$$

where C=the net current account,

X_e=exports,

C and X_e are expressed in terms of the domestic currency.

F=imports expressed in terms of the foreign currency.

The following restrictions are typically assumed in macro theory:

$$\partial X_e/\partial r>0,\ \partial F/\partial z>0,\ \partial F/\partial r<0 \text{ and } \partial F/\partial p>0.$$

Exports are determined by the economic activities in the rest of the world and the exchange rate. Imports are the function of the real disposable income, the exchange rate and the domestic price level. For consistency of terms in II-8, the imports expressed in the foreign currency must be converted into the domestic currency by multiplying the exchange rate. From II-8 we may observe the effects of the exchange devaluation (i.e. $dr>0$) on the balance of payments. Taking a partial derivative in II-8 with respect to the exchange rate, we have:

II-9 $\quad \partial C/\partial r = \partial X_0/\partial r - r \cdot \partial F(z,r,p)/\partial r - F(z,r,p).$

The elasticity of demand for exports ϵ_1 and imports ϵ_2 with respect to exchange rate are defined as follows:

II-10 $\quad \epsilon_1 = \partial X_0/\partial r \cdot r/X_0 \; >0$ and $\epsilon_2 = -\partial F/\partial r \cdot r/F \; \geq 0.$

Now we can express II-9 in the elasticity form, given the assumption that the balance of payments is at equilibrium prior to the exchange rate depreciation(i.e. $X_0(r) = rF(z,r,p)$ or $X_0/r = F$).

II-9(a) $\quad \partial C/\partial r = F \cdot (\epsilon_1 + \epsilon_2 - 1)$

This result holds true only for an equilibrium. From II-9(a) it can be seen that an improvement in the balance of payments will be accomplished if the condition that the sum of the elasticities of exports and imports exceeds one (i.e. $\epsilon_1 + \epsilon_2 > 1$) holds. Thus, the well-known Marshall-Lerner condition is assumed to be satisfied.

For the sake of simplification we may integrate exports and imports into one term net exports, without loss of generality; that is:

II-11 $\quad X(z,r,p) = X_0(r) - rF(z,r,p).$

Given the assumptions $\partial X_e/\partial r>0$ and $\partial F/\partial r<0$, the exchange rate change works in the same direction; that is, the devaluation of the exchange will drive exports up and imports down. As a result, the net exports will rise (i.e. $X_2>0$). The effects of disposable income on X will drive the net exports down (i.e.$X_1<0$). This is because an increase in disposable income will raise imports. However, since a change in disposable income has no impact on exports, the increased imports will drive down the foreign exchange receipts. As a result, net exports will decrease.

The capital account is the flow of funds between the sales of domestic assets to foreign investors and the purchases of foreign assets by domestic investors. Hence, the capital account is defined as the net inflow of foreign exchange. It is the function of the real income and the nominal domestic interest rate (i.e. $K=K(y,i)$). This is the traditional specification of the capital account. Although there have been criticisms of this specification, it will be adopted in our model, because the main objective of this study lies in the recognition of a government budget restraint in the system. Both increases in domestic income and in domestic interest rate are assumed to increase the net capital inflow (i.e. $K_1>0$ and $K_2>0$). K_1, which was introduced in Johnson(1966), means that an increase in income tends to raise investment and hence attract foreign capital. If capital mobility is not perfect, the fluctuation of the domestic interest rate will affect international capital flows as is specified. Under the perfect capital mobility assumption, however, the domestic interest rate is determined by foreign interest rates.

Thus, with the current account function and the capital account function we have the balance of payment equation in II-6. Under the floating exchange rate system the exchange rate will fluctuate endogenously to equilibrate the balance of payments. Hence, we set $dR=0$ and $dr\neq0$. Under the pegged exchange rate system the balance of payments will fluctuate endogenously to satisfy the equation. Hence, we set $dR\neq0$ and $dr=0$.

II-7 is the government budget restraint. The government budget restraint is similar to the one specified by Blinder-Solow(1973) and Christ(1978). It was pointed out by Blinder-Solow(1973) that interest payments on government bonds are the significant factor in determining a stability condition in the system. We must note that since B is the interest payments for bonds paying out $1 coupon, it will be the nominal market value of the stock of bonds when B is divided by interest rate (i.e. B/i).

In order to extend the government budget restraint for an open economy the variable of the balance of payments must be included in the equation. However, the receipts from the balance of payments in the public sector are assumed automatically to become part of the money supply in the private sector. Hence, the balance of payments in the government budget restraint is totally cancelled out and it is counted as a part of money supply in II-4. Substituting II-3 into II-7 and cancelling out R/p in II-7, we have:

II-7(a) $g+B/p=\dot{M}/p+\dot{B}/ip+u(y+B)+v$

The policy variables in II-7(a) are g, $\dot{M}$, $\dot{B}$, u and v. By virtue of the government budget restraint, one of policy variables is to be determined endogenously and the rest exogenously.

Price level is assumed to be fixed in this chapter (i.e. dp=0) so that a change in price level does not affect the real domestic income level.

Given the assumptions above and the seven equations, this system can be reduced to the following four equation system.

II-12 $y=E[y+B-u(y+B)-v,\ i,\ M+B/i]+X[y+B-u(y+B)-v,\ r]+g$

II-13 $M+R=L(y,\ i,\ M+B/i)$

II-14 $\dot{R}=X[y+B-u(y+B)-v,\ r]+K(y,\ i)$

II-15 $g+B=\dot{M}+\dot{B}/i+u(y+B)+v$

To be consistent in the context of comparative statics, we set $d\dot{M}=dM$, $d\dot{B}/i=dB/i$ and $d\dot{R}=dR$, assuming that the economy is initially at a steady state equilibrium. Then, totally differentiating II-15 we have:

II-16 $dg+dB=dM+dB/i+(y+B)du+udB+dv+udy-(\dot{B}/i^2)di$.

where $\dot{B}/i^2=0$ at the initial steady state equilibrium.

Now we must note that udy cannot be determined by the public sector, but is dependent on the economic activity of the private sector. udy is an induced amount of tax revenue due to a change in the income level. If this variable is explicitly recognized in the government budget restraint equation, the analysis will be complex. Hence, we shall avoid this complexity in the present chapter by making the assumption below. The full recognition of the government budget restraint is left as the task of the following chapter. Here we assume that the value udy is always offset by a change in autonomous tax flow less transfer payments (i.e. udy=dv where v is negative). Thus, we have the following government budget restraint equation:

II-16(a) $dg=dM+dB/i+(y+B)du-(1-u)dB.$

From II-16(a) we find four policy variables (i.e. dg, dM, dB and du). Given these policy variables the following particular government policies are specified.

(1) Money financed government expenditures

An increase in government expenditures dg is a fiscal policy with recognition of money finance dM as an endogenous policy variable. The rest of the policy variables are assumed to be constant. Hence, the government budget restraint will be specified as: $dg=dM$, $dB=du=0$.

(2) Bond financed government expenditures

This is a policy defined in traditional macroeconomics as a fiscal policy; but a finance source is not usually recognized. Hence, we incorporate this variable explicitly in the sysyem. The government budget restraint will be specified as:

dg=dB/i, dM=du=0.

(3) Open market purchases

A monetary policy is usually defined open market purchases in the traditional macroeconomics. The government increases the money supply for the same amount as the purchases of bonds from the private sector. We recognize the contraction of bonds explicitly in the system. Thus, the government budget restraint will be specified as:

dM=-dB/i, dg=du=0.

C. The Floating Exchange Rate System with Implicit Recognition of the Government Budget Restraint

With the assumptions of the floating exchange rate system (i.e. dR=0 and dr≠0), we have the following matrix form of the endogenous variables, dy, di and dr. It is derived by totally differentiating the behavioral equations II-12, II-13 and II-14.

II-17 $$\begin{bmatrix} 1-(1-u)(E_1+X_1) & E_2B/i^2-E_2 & -X_2 \\ -L_1 & L_2B/i^2-L_2 & 0 \\ (1-u)X_1+K_1 & K_2 & X_2 \end{bmatrix} \begin{bmatrix} dy \\ di \\ dr \end{bmatrix} =$$

$$\begin{bmatrix} [(1-u)(E_1+X_1)]dB+E_2dB/i+E_2dM-(E_1+X_1)(y+B)du+dg \\ L_2dB/i-(1-L_2)dM \\ -(1-u)X_1dB+X_1(y+B)du \end{bmatrix}$$

The Jacobian determinant of the system is:

$|J|=X_2\{[1-(1-u)E_1+K_1](L_2B/i^2-L_2)+L_1(E_2B/i^2-E_2+K_2)\}$

$=X_2\Delta_1>0$, where Δ_1 represents the terms inside curly brackets.

Since all of the terms are positive by assumption, the Jacobian determinant is positive.

In Keynesian macro theory, II-12 represents an IS curve, II-13 a LM curve and II-14 a balance of payment curve (BP curve). Differentiating II-12, II-13 and II-14, setting all other variables equal to zero except dy and di, and solving for di/dy, we find the slope of the three curves as follows.

IS curve:

II-18 $\quad di/dy=[1-(1-u)(E_1+X_1)]/(E_2B/i^2-E_2) <0$

LM curve:

II-19 $\quad di/dy=L_1/(L_2B/i^2-L_2) >0$

BP curve:

II-20 $\quad di/dy=-\{(1-u)X_1+K_1]/K_2 >0$ if $(1-u)X_1>K_1$

<0 if $(1-u)X_1<K_1$

The IS curve in II-18 states that if private expenditures are highly elastic with respect to a change in the interest

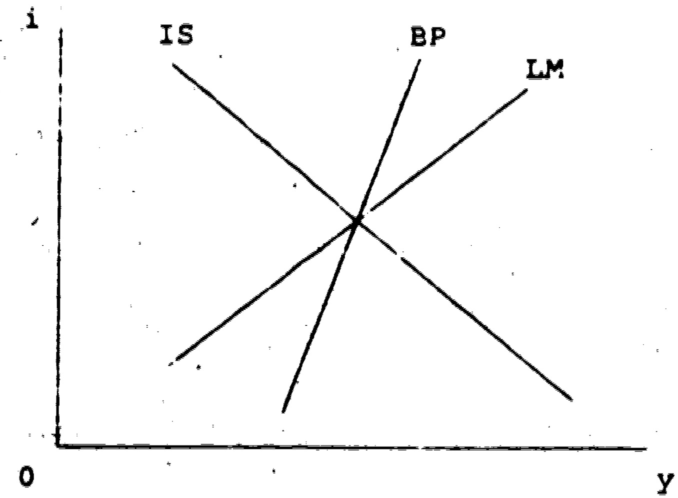

Figure II-1

The Initial Internal and External Equilibrium

rate, the slope of the IS curve will be flat. Otherwise the IS curve will be steep. The same is true for the LM curve in II-19. If demand for money is highly elastic with respect to a change in the interest rate, the positive slope of the LM curve will be flat, otherwise it will be steep. The slope of the BP curve in II-20 cannot be determined because of two conflicting forces. These are the changes in net exports $(1-u)X_1<0$, and capital inflows $K_1>0$ that result with a change in the income level. If the negative effect of net exports is stronger than the capital inflow effect (i.e. $|(1-u)X_1|>K_1$), the BP curve will have a positive slope. If it is the opposite (i.e. $K_1>|(1-u)X_1|$), then the BP curve will have a negative slope. Since these forces were illustrated in detail in Johnson(1966), we shall basically assume $|(1-u)X_1|>K_1$; that is, the negative effect on net exports is stronger than the effect on capital inflow with respect to a change in the income level. In this case the BP curve has a positive slope. It does so under the condition that capital mobility with respect to a change in the interest rate is positive, $K_1>0$. According to Mundell(1963), however, as K_1 approaches infinity the BP curve becomes completely flat, since the slope of the BP curve in II-20 approaches to zero regardless of magnitude of the numerator.

Thus, we can draw the three curves in the traditional IS-LM diagram in Figure II-1. We shall focus our attention, in particular, on the movement of the BP curve under the floating exchange rate system and the pegged exchange rate system.

1. Money Financed Government Expenditures

When an increase in the government expenditures is financed solely by an increase in money supply, we have the government budget restraint as dg=dM. Then we have the matrix form from II-17 as:

$$\text{II-17(i)} \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} dy \\ di \\ dr \end{bmatrix} = \begin{bmatrix} (1+E_2)dM \\ -(1-L_2)dM \\ 0 \end{bmatrix}$$

where a_{11}..........a_{33} are the same as in II-17. By solving for endogenous variables, dy, di and dr under this policy specification we obtain:

II-17(a)

$$dy/dM = 1/\Delta_1 \{(1+E_2)(L_2B/i^2-L_2)+(1-L_2)[(E_2B/i^2-E_2)+K_2]\} \quad >0$$

$$\text{II-17(b)} \quad di/dM = 1/\Delta_1 \{(1+E_2)L_1-(1-L_2)[1-(1-u)E_1+K_1]\}$$

$$\text{II-17(c)} \quad dr/dM = -1/X_2\Delta_1 [(1+E_2)\{L_1K_2+[(1-u)X_1+K_1](L_2B/i^2-L_2)\} -(1-L_2)\{[1-(1-u)(E_1+X_1)]K_2 -[(1-u)X_1+K_1](E_2B/i^2-E_2)\}]$$

As both the fiscal effect of an increase in government expenditures and the monetary effect of money finance have a positive effect on the income level, the sign of the income multiplier in II-17(a) is unambiguously positive. But the sign of the interest rate multiplier in II-17(b) is uncertain. This is due to the conflict between the fiscal and the monetary effects on the interest rate. As in the ordinary Keynesian IS-LM diagram our system displays that a fiscal effect shifts the IS curve rightward (i.e. an increase in income). This raises the interest rate while a monetary effect shifts the LM curve rightward (i.e. an increase in income). This lowers the interest rate unless economy is in a liquidity trap. [23] Hence, whether the interest rate goes up or down in II-17(b) depends on the slope and the magnitude of shifts of the IS-LM curves.

In the case of money financed government expenditures illustrated here, the role of the wealth effect is insignificant. The positive wealth effect in the commodity market (i.e. $E_3(L_3B/i^2-L_2)>0$) means a higher level of the private expenditures at any given income level. Hence, the wealth effect raises the IS curve further to the right in addition to the effect of the increased government expenditures. [24] This wealth effect can be interpreted similar to the real balance effect introduced in Patinkin(1965). The presence of the wealth effect in the money market affects the leftward shift of the LM curve. It is permissible to assume that the presence of the wealth effect in the money market weakens the magnitude of the

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[23] From II-12 the income and the interest rate multipliers with respect to an increase in the government expenditures for a given LM curve will be shown as:

(1) $dy/dg=1/[1-(1-u)(E_1+X_1)] >0$

(2) $di/dg=1/(E_3B/i^2-E_2)>0$

From II-13 the income and the interest rate multipliers with respect to an increase in the money supply for a given IS will be shown as:

(3) $dy/dM=1/L_1>0$

(4) $di/dM=-1/(L_3B/i^2-L_2)<0$

[24] The rightward shift of the wealth effect on the IS curve under money financed government expenditures can be inferred from II-12 as follows:

(1) $dy/dM=E_3/[(1-u)(E_1+X_1)] >0$

(2) $di/dM=E_3/(E_3B/i^2-E_2) >0$

rightward shift of the LM curve since $0<(1-L_2)<1$ in II-17(a). This concept of the wealth effect in the money market can be interpreted as an increase in demand for money as a result of the increased wealth experienced. Hence, the rightward shift of the LM curve is crowded out to some extent.

The balance of payment multiplier shows complex interactions of economic forces. With the assumptions of $(1-u)X_1+K_1<0$ where $X_1<0$ and $K_1>0$, we find the two forces inside brackets in the first term are in conflict with each other. But from II-19 and II-20, these two terms can be inferred to represent the LM and the BP curves. Suppose $L_1K_2<[(1-u)X_1+K_1](L_2B/i^2-L_2)$; that is, the BP curve is steeper than the LM curve. This is represented by BP_1 in Figure II-2. Then the sign of the exchange rate multiplier in II-17(c) is unambiguously positive. That is, the exchange rate will depreciate with an increase in money financed government expenditures. In this case BP_1 shifts rightward to the domestic equilibrium at A, under the floating exchange rate system in Figure II-2.

If the LM curve is steeper than the BP curve, the sign of the exchange rate multiplier will be uncertain. However, although the LM curve can be steeper than the BP curve namely, $L_1K_2>[(1-u)X_1+K_1](L_2B/i^2-L_2)$, as long as the positive value of the second term exceeds the negative value of L_1K_2, the exchange rate will depreciate. This is represented on BP_2 in Figure II-2. Only when the BP curve

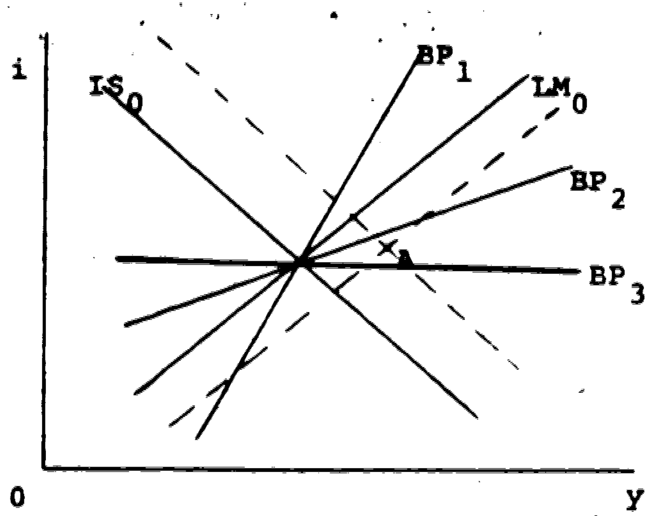

Figure II-2
The Effect of MFGE, the Assumption of the Positive BP Curve; the Floating Exchange Rate System

is so flat that the magnitude of (1+E,)L,K, exceeds the value of all other terms will the sign of II-17(c) be negative; and thus the exchange rate appreciates (i.e. BP, in Figure II-2). But this will occur only if the interest rate goes up. Unless the condition of (1+E,)L,>(1-L,)[1-(1-u)E,+K,] in II-17(b) holds, there is no possibility for dr/dM to become negative in II-17(c).

Observing the second term in II-17(c), and II-18 and II-20, we note that these two coefficients represent the slopes of the IS and the BP curves. We now assume a negatively sloped BP curve (i.e. (1-u)X,+K,>0) by breaking the original assumption. Then we can analyze the exchange rate multiplier associated with the IS curve. If the BP curve is steeper than the IS curve (i.e. BP, in Figure II-3), the magnitude of the second coefficient is larger than that of the first in the second term of II-17(c). In this case the exchange rate will appreciate (i.e. dr/dM<0).

Even if the IS curve is steeper than the BP curve (i.e. BP_2 in Figure II-3) the exchange rate will appreciate as long as the negative magnitude of the first term in II-17(c) exceeds the positive magnitude of the second term. Only when the second positive term exceeds that of the first term will the exchange rate depreciate. Unlike the case of BP_2 in Figure II-2, this will occur only if the interest rate goes down. Hence, BP_2 shifts down to the domestic equilibrium at B in Figure II-3.

Let us consider the situation of perfect international capital mobility. K_2, the partial derivative of capital with respect to the interest rate, is the term we must focus on now. Following Mundell(1963) we assume that K_2 goes to infinity for each multiplier from II-17(a) to II-17(c). Thus, we have the following:

II-17(a)' $dy/dM=(1-L_2)/L_1>0$

II-17(b)' $di/dM=0$

II-17(c)' $dr/dM=\{(1-L_2)[1-(1-u)(E_1+X_1)]-(1+E_2)L_1\}/L_1$

Mundell(1963) showed that monetary policy is effective on income and employment under the floating exchange rate system with perfect capital mobility. It induces a capital outflow without altering the interest rate. This brings about an export surplus as a consequence of the depreciation of the exchange rate. On the other hand, fiscal policy is

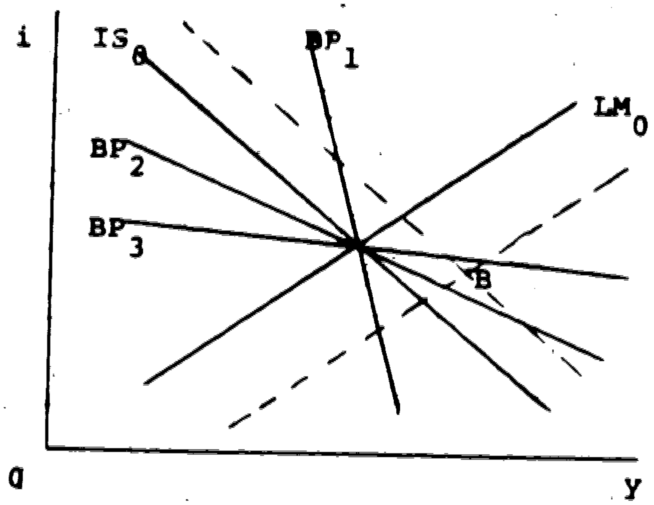

Figure II-3
The Effect of MPGE, the Assumption of the Negative BP Curve

not effective on income and employment under the floating exchange rate system with perfect capital mobility. The exchange rate appreciates through the effect of capital inflow when the government expenditures increase. Since the appreciation of the exchange rate has a negative effect on income, it completely offsets the positive multiplier effect on income of the original increase in government expenditures.

We can observe from II-17(a)', that the effect of fiscal policy in which the finance source (i.e. in this case, base money stock) is explicitly recognized, is an elaboration of the traditional theory. It is obvious that the multiplier in II-17(b)' is zero since i becomes, in fact, an exogenous variable under perfect capital mobility assumption.

The sign of the exchange rate multiplier in II-17(c)' is uncertain. The exchange rate appreciates by the fiscal effect while it depreciates by the monetary effect. We must note that the BP curve becomes a horizontal line since the domestic interest rate is dominated by foreign interest rates.

2. Bond Financed Government Expenditures

When an increase in the government expenditures is financed solely by an increase in government bonds, we have the government budget restraint as dg=dB/i. Then the matrix form II-17 can be expressed as follows:

$$\text{II-17(ii)} \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} dy \\ di \\ dr \end{bmatrix} = \begin{bmatrix} [(1-u)(E_1+X_1)i+E_3+1]dB/i \\ L_3 dB/i \\ -(1-u)X_1 idB/i \end{bmatrix}$$

where a_{11}..........a_{33} are the same as II-17.

By solving for endogenous variables, dy, di and dr, we obtain:

II-17(d) $dy/(dB/i)$

$$=1/\Delta_1 \{[(1-u)E_1 i+1](L_3 B/i^2-L_1)-E_3 L_2-L_3(K_3-E_2)\} >0$$

II-17(e) $di/(dB/i)$

$$=1/\Delta_1 \{[(1-u)E_1 i+E_3+1]L_1+[1-(1-u)E_1+K_1]L_3\} >0$$

II-17(f) $dr/(dB/i)=-1/X_2\Delta_1[$

$$[(1-u)(E_1+X_1)i+E_2+1]\{K_2L_1+[(1-u)X_1+K_1](L_2B/i^2-L_2)\}$$

$$+L_2\{[1-(1-u)(E_1+X_1)]K_2-[(1-u)X_1+K_1](E_2B/i^2-E_2)\}$$

$$+(1-u)X_1i\{[1-(1-u)(E_1+X_1)](L_2B/i^2-L_2)+(E_2B/i^2-E_2)L_1\}]$$

In the case of money financed government expenditures the wealth effect did not give a significant influence on determining the sign of the multipliers. However, in the case of bond financed government expenditures, since the effect of a change in government bonds can be observed only in the wealth equation in II-5, we must focus on this variable.

The income multiplier in II-17(d) consists of three different elements. The first term is the effect of an increase in government expenditures which raises the IS curve from IS_0 to IS_1 in Figure II-4. The second term (i.e. $-E_2L_1>0$ as $L_1<0$) in II-17(d) is the wealth effect in the commodity market which also raises the IS curve from IS_1 to IS_2. As in money financed government expenditures, it is clear that the increment of government expenditures coupled with the wealth effect shifts the IS curve rightward. This raises the income level assuming the positively sloping LM curve. The third term (i.e. $-L_2(K_2-E_2)<0$ as $L_2>0$, $K_2>0$ and $E_2<0$) in II-17(d) is the wealth effect in the money market. The presence of the wealth effect in the money market can be inferred in the following manner. Given a constant money supply (i.e. $dM=0$), the increment of government bonds will cause an excess demand for money at any level of income and interest rate. This wealth effect is difficult to illustrate in the two dimentional IS-LM diagram. In order to have an equilibrium in the money market, the increment of the wealth effect may be inferred for the income level to go down and for the interest rate to go up. Hence, the "desired" level of the money market equilibrium is indicated at A in Figure II-4. [25] [26]

It should be pointed out that these last two terms of the wealth effects in II-17(d) namely, $-E_3L_2-L_3(K_2-E_3)$, are the point of contention in Blinder-Solow(1973). The argument is focused on the magnitude of the wealth effects. If the wealth effect in the commodity market is higher than the wealth effect in the money market, namely $|E_3L_2|>|L_3(K_2-E_3)|$, the increment of bond finance will have a positive net impact on the income level. Then the system can be stable. On the other hand, if $|E_3L_2|<|L_3(K_2-E_3)|$, then the monetarists view which claims that a fiscal policy

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[25]The lower income level and the higher interest rate associated with the increment of government bonds can be shown from the money market equilibrium in II-13 as follows:
(1) $dy/(dB/i)=-L_3/L_1<0$
(2) $di/(dB/i)=-L_3/L_2>0$.

[26]As an alternative pure fiscal policy, we can observe the effect of the tax financed government expenditures in the system. Hence, we have the government budget restraint as: $dg=(y+B)du$ in II-17. In this policy the increment of the income level due to the increased government expenditures is crowded out by the decreased private expenditures due to the increase in tax rate. These movements are observed only on the IS curve, but the shift of the LM curve is not involved. The crowding out effect does not cause the IS curve to shift all the way back to the initial equilibrium and as a result, the effects on the income level and the interest rate are positive.

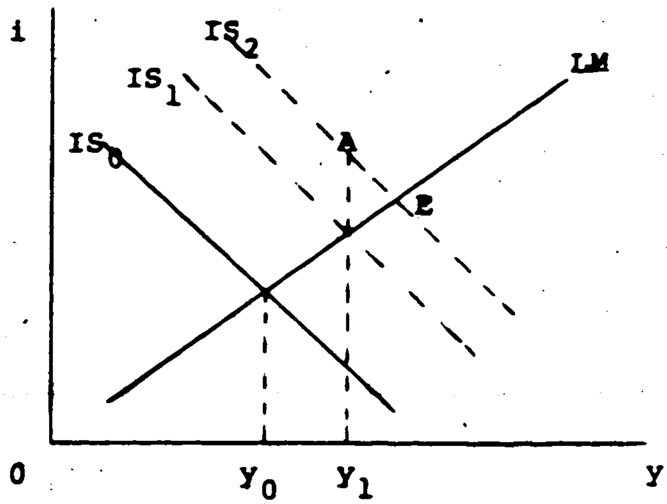

Figure II-4
The Effect of BFGE; Recognition of
of the Wealth Effects

is contractionary will be correct. But this implies that the system is unstable. We can observe this in Figure II-4. If the desired level of the LM curve settles the left hand side of A where $|E_2L_2|=|L_2(K_2-E_2)|$, the system will be unstable. This is because the negative wealth effect reduces the impact of the increased government expenditures on the income level period by period. Eventually the increase in the income level will be completely offset. On the other hand, the right hand side of A is the only "possible" region for a stable system.

However, Blinder-Solow(1973) state that even when government bond issue has an expansionary effect on the income level, it is necessary but not sufficient condition for the system to be stable. Only when the magnitude of the income multiplier for bond financed government expenditures is larger than that for money financed government expenditures in the long run, is the sufficient condition for the system to be stable satisfied.

> Provided that the net impact of the wealth effects is expansionary, the 'second round' increase in income will be greater under bond financing than under money financing, and this will continue to be true in subsequent rounds. The basic intuition is that under bond financing any given budgetary gap is harder to close because every increase in the number of bonds outstanding requires more expenditure on debt service. It therefore takes a greater rise in income to induce tax receipts sufficient to close the budgetary gap.[27]

Observing the first term of the numerator in II-17(d) for a bond financed government expenditures, namely, $[(1-u)E_1 i+1](L_2 B/i^2-L_1)$, we find that the magnitude of interest payments to the private sector raises the IS curve further to the right. This effect of interest payments on private expenditures is the factor which gives a chance of a stable system under bond financed government expenditures.

The sign of the interest rate multiplier in II-17(e) is clearly positive. The balance of payment multiplier in II-17(f) shows a complex result due to the involvememt of interest payments in the system. First, for simplicity, we consider the exchange rate multiplier excluding the terms of the interest payments.

II-17(ff) $dr/(dB/i)$

$$=-1/X_2\Delta_1\ [(1+E_2)\{L_1K_2+[(1-u)X_1+K_1](L_2B/i^2-L_2)$$

$$+L_2\{[1-(1-u)(E_1+X_1)]K_2-[(1-u)X_1+K_1](E_2B/i^2-E_2)\}]$$

As was the case in money financed government expenditures,

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[27] A.S. Blinder and R.M. Solow, "Does Fiscal Policy Matter?" Journal of Public Economicss (November 1973): p.327

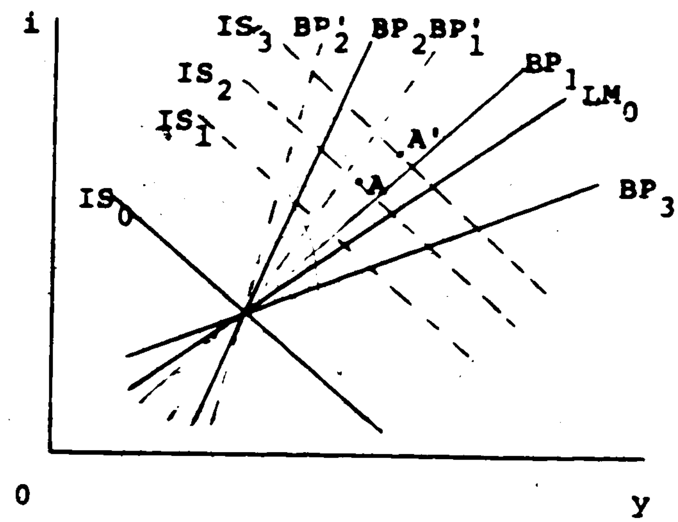

Figure II-5
The Effect of BFGE, the Assumption of the Positive BP Curve; Recognition of Interest Payments

the two forces of the first term conflict with each other under the assumption of $(1-u)X_1+K_1<0$. However, even if $L_1K_3<[(1-u)X_1+K_1](L_3B/i^2-L_2)$; that is, the BP is steeper than the LM (i.e. BP_1 in Figure II-5), we cannot conclude that the exchange rate will depreciate, because the sign of the second term is now negative. Only when the positive magnitude of the first term exceeds the negative magnitude of second term in II-17(ff) will the exchange rate depreciate (i.e. BP_2 in Figure II-5). However, if the LM curve is steeper than the BP curve (i.e. BP_3 in Figure II-5), the case of high capital mobility, then the exchange rate in II-17(ff) appreciates (i.e. $dr/(dB/i)<0$).

Under the assumption of $(1-u)X_1+K_1>0$, we can now observe the second term in II-17(ff) since the sign of the first term is clearly negative. The two elements of the second term can be inferred to represent the slopes of the IS and the BP curves respectively.

If the BP curve is flatter than the IS curve (i.e. BP_1 in Figure II-6), the sign of dr/(dB/i) is unambiguously negative in II-17(ff). But even if the BP curve is extremely steep as BP_2 in Figure II-6, there is no possibility for the exchange rate to depreciate since the internal equilibrium A always lies on the right hand side of the initial equilibrium C in the short run. Thus, regardless of the magnitude of the second term it can never exceed the magnitude of the first term in II-17(ff). Hence, the exchange rate appreciates unambiguously when $(1-u)X_1+K_1>0$.

Going back to the original exchange rate multiplier in II-17(f), we note that the effect of the interest payments must be enhanced in the analysis. As we learned earlier, the effect of interest payments shifts the IS curve further to the right. This is shown on IS_2 in Figure II-5. The interest payments to the private sector affects the slope of the BP curve. A higher income level due to the interest payments increases imports, thus decreasing net exports for any given interest rate. This is displayed in the third term in II-17(f). The positive third term may mean a steeper BP curve than the BP curve not including the interest payments. This is shown by BP_1' and BP_2' for BP_1 and BP_2 which were assumed to be drawn from II-17(ff). The steeper BP curves imply a wider range for exchange rate depreciation, or a smaller range for exchange rate appreciation. In the case of BP_1 the exchange rate

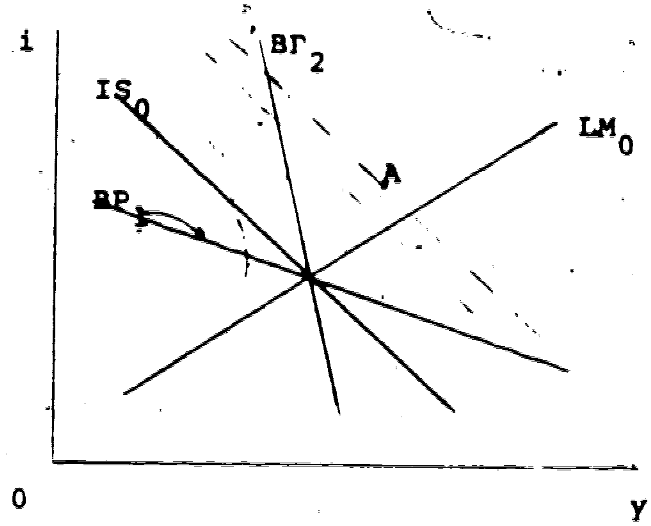

Figure II-6
The Effect of BFGE, the Assumption of the Negative BP Curve

appreciates at the internal equilibrium A in Figure II-5. It will depreciate the exchange rate at the internal equilibrium A' if the interest payments are included in the system.

We now consider the perfect capital mobility case. Taking K_a to be infinite for each multiplier from II-17(d) to (f), we have the following:

II-17(d)' $dy/(dB/i)=-L_2/L_1<0$

II-17(e)' $di/(dB/i)=0$

II-17(f)' $dr/(dB/i)$

$=-\{[(1-u)(E_1+X_1)i+E_2+1]L_1+[1-(1-u)(E_1+X_1)]L_2\}/X_2L_1$

<0

With the recognition of the bond finance, the income multiplier results in a negative sign due to the crowding out effect in the money market in II-17(d)'.

In II-17(e)' since the perfect capital mobility system paralyses the domestic interest rate movement, the multiplier is zero; that is, i is a given exogenous variable.

In II-17(f)' both the terms of the IS and the LM curves work in the same direction for the exchange rate. Thus the exchange rate will appreciate unambiguously. We must note that there is no effect of interest payments on the sign of the exchange rate multiplier since the interest rate does not change.

In money financed government expenditures we obtained the result that fiscal policy could be effective. However, with the bond finance, fiscal policy has a negative effect on the income level rather than the nil effect, as in traditional theory. The result of the exchange rate multiplier is consistent with the traditional theory under the floating exchange rate system.

3. Open Market Purchases

Open market operations are applied in an economy in order to regulate the reserve position of the banking system or to control the interest rate indirectly. The government purchases bonds from the private sector by printing new money, for the purpose of lowering the interest rate. This is open market purchases. If the government wishes to raise the interest rate indirectly, government bonds will be issued, and in turn money supply will be decreased. This is open market sales. In both cases the government budget restraint is the same, except for the signs of the variables; that is, $dM=-dB/i$ for open market purchases and $dB/i=-dM$ for open market sales. The elements of multipliers will be exactly the same for the both policies.[18]

Now we shall analyze the effects of open market purchases on the endogenous variables as a representative of open market operations. The system of the matrix system is expressed as:

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[18]Open market sales and bond financed government expenditures are similar policies in that government bonds are increased for the both policies. In the case of bond financed government expenditures, however, the amount of money stock purchased from the private sector was assumed to return automatically to the private sector in the form of increased government expenditures. Hence, the shift of the LM curve was captured as a "desired" level of money market equilibrium. In the case of open market sales, government bond sales to the private sector decreases the money supply.

$$\text{II-17(iii)} \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} dy \\ di \\ dr \end{bmatrix} = \begin{bmatrix} -[(1-u)(E_1+X_1)]idM \\ -dM \\ (1-u)X_2 idM \end{bmatrix}$$

where $a_{11} \ldots\ldots\ldots a_{33}$ are the same as II-17.

Solving for endogenous variables dy, di and dr, we obtain the following:

II-17(g) $dy/dM = 1/\Delta_1 \{-(1-u)E_2 i(L_3 B/i^2 - L_2) + [K_2 + (E_3 B/i^2 - E_2)]\}$

II-17(h) $di/dM = -1/\Delta_1 \{[1-(1-u)E_1 + K_1] + (1-u)E_1 iL_1\} < 0$

II-17(j) dr/dM

$$= 1/X_3\Delta_1 \; [(1-u)i\{[X_1 + (E_1+X_1)K_1](L_3 B/i^2 - L_2)$$
$$+ [(E_1+X_1)K_2 + (E_3 B/i^2 - E_2)X_1]L_1\}$$
$$+ [1-(1-u)(E_1+X_1)]K_2 - [(1-u)X_1 + K_1](E_3 B/i^2 - E_2)]$$

The sign of the income multiplier in II-17(g) is uncertain due to the conflicting effects of an increase in money supply and a decrease in private expenditures which are caused by a decrease in interest payments. The first term in II-17(g) displays the leftward shift of the IS curve resulting from the decreased interest payments to the private sector (i.e. IS_0 to IS_1 in Figure II-7). The income level will, hence, go down unless the LM curve is vertical. The increase in money supply shifts the LM curve rightward. This will raise the income level unless the IS curve is vertical (i.e. LM_0 to LM_1 in Figure II-7). This can be seen in the second term in II-17(g). We note that the magnitude of the LM shift is one in lieu of $(1-L_3)$ due to the cancellation of the wealth effect on the money market. It is plausible to assume here that the positive effect on the income level caused by an increase in money supply is larger than the negative effect on the income level caused by a decrease in interest payments to the private sector (i.e. $(K_2+E_3B/i^2-E_2)>(1-u)E_1i(L_3B/i^2-L_2)$). Hence, the income level increases from y_0 to y_1 in Figure II-7.

The sign of the interest rate multiplier in II-17(h) is unambiguously negative, since both an increase in money supply and a decrease in private expenditures lower the interest rate from i_0 to i_1 in Figure II-7.

The exchange rate multiplier consists of two main elements; the effect of less interest payments and the effect of an increase in money supply. The effect of private expenditures the first term inside the curly bracket in II-17(j) has two conflicting forces. This is because a decrease in the income level raises net exports while a fall in the interest rate causes capital outflow. The effect of the increased money supply in the second term works in the same direction for capital flow and net exports. Because an increase in the income level lowers net exports and the fall of the interest rate attracts capital outflow, the exchange

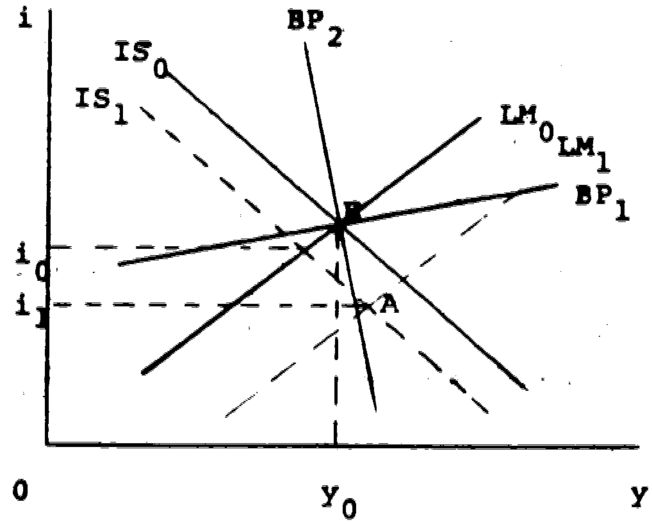

Figure II-7
The Effect of OMP;
The Positive and Negative BP Curves

rate depreciates in the second term. Assuming as before that the effect of the increased money supply has dominant power over less interest payments, we may conclude that the exchange rate will depreciate by open market purchases under the floating exchange rate system in II-17(j).

This can be observed in the context of the IS-LM framework in Figure II-7. Now that an internal equilibrium A lies under the initial equilibrium E in Figure II-7, regardless of the slope of the BP curve , there is no possible chance for BP_1 to lie on the right hand side of the internal equilibrium A. That is, the exchange rate will depreciate unambiguously under the assumption of the positive BP curve.

The exchange rate depreciates clearly in open market purchases under the assumptions of the higher income level (i.e. $dy/dM>0$) and the positively sloping BP curve(i.e. $(1-u)X_1+K_1<0$). Let us observe the behaviour of the exchange rate when each of these assumptions is changed. We assume first that the income level is decreased by open market purchases. In this situation the slope of the IS curve is so steep that the shift of the LM curve is paralysed and/or the impact of the substantial decrease in interest payments on the private expenditures exceeds the impact of the increased money supply on the income level. Under this condition (i.e. $dy/dM<0$) the exchange rate will appreciate only if the elasticity of net exports with respect to the income level is so high that the steep BP curve lies on the right hand side of the internal equilibrium. This can be seen from the exchange rate multiplier in II-17(j). The negtative magnitude in the first term exceeds the magnitude of the positive terms.

Next we assume that the BP curve is negatively sloping (i.e. $(1-u)X_1+K_1>0$) holding $dy/dM>0$. Then it is possible for the exchange rate to appreciate if the negative magnitude in the first and second terms exceeds the positive magnitude of the remaining terms. This means, in the context of the IS-LM framework, that the BP curve (BP_1) is steeper than the IS curve (IS_0) by the amount the BP curve lies on the left hand side of the internal equilibrium A in Figure II-7.

If we assume a decreased income level and a negatively sloping BP curve at the same time (i.e. $dy/dM<0$ and $(1-u)X_1+K_1>0$), there will be no possibility for the exchange rate to appreciate. In this case the internal equilibrium A lies on the left hand side of y_0 and the BP curve cannot lie on the left hand side of A regardless of the slope of the BP curve.

Now we consider the case of perfect capital mobility. Taking K_2 to be infinite for II-17(g), (h) and (j) we obtain the following:

II-17(g)' $dy/dM = i/L_1 \quad >0$

II-17(h)' $di/dM = 0$

II-17(j)' dr/dM

$$= \{(1-u)(E_1+X_1)iL_1 + [1-(1-u)(E_1+X_1)]\}/X_2L_1 \quad >0$$

Here the traditional theory concerning the effectiveness of monetary policy on the income level can be observed in the situation of perfect capital mobility under the floating exchange rate system. The interest rate is controlled by the foreign interest rate so that the interest rate multiplier is zero. The exchange rate depreciates unambiguously resulting from the capital outflow.

D. The Pegged Exchange Rate System with Implicit Recognition of the Government Budget Restraint

Under the pegged exchange rate system the balance of payments becomes an endogenous variable, and the exchange rate an exogenous policy variable. As can be seen in the money market equilibrium in II-4, the acumulation or decumulation of foreign reserves through changes in the balance of payments over a period of time will be a part of the domestic money sypply. However, in an actual economy the magnitude of the balance of payment deficit or surplus over a period of time is not transformed automatically into the domestic currency since a domestic economy may be disturbed by the fluctuation of the external balance.

Here a sterilization policy for the balance of payments must be introduced under the pegged exchange rate system. We assume that the government may insulate a new balance of payment deficit or surplus caused by a domestic policy such that the money market equilibrium can be controlled. Denoting a sterilization coefficient s where $1 \geq s \geq 0$, the non-sterilized magnitude of the balance of payments can be defined as sR. If s=0, it indicates a complete sterilization policy. This means that the amount of the balance of payment deficit or surplus does not affect the domestic money supply. On the other hand, if s=1, a non-sterilization policy prevails. That is, the whole balance of payments enters into the domestic money supply. Hence, including the sterilization coefficient s the money market equilibrium can be rewritten as:

II-4(a) $M/p+sR=L(y,i,w)$[19]

As was stated before, although the balance of payments goes through the government, we simply assume that the magnitude of non-sterilized balance of payments autmatically becomes a part of the domestic money supply. Hence, the balance of payment variable disappears from the government budget restraint equation.[20]

Under the pegged exchange rate system we set $dR \neq 0$ and $dr=0$ from the reduced form in II-12, II-13 and II-14. With implicit recognition of the government budget restraint the matrix form is expressed with respect to dy, di and dR as follows:

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[19] I a part of the balance of payments is included in the domestic money supply, it should be incorporated into the wealth effect equation in II-5. But as the current and the capital accounts were assumed to have no wealth effect in the balance of payment equilibrium for the sake of simplicity, we also assume that the non-sterilized balance of payments has no wealth effect in II-5, without loss of generality.

[20] This simplification can be done according to the specification that M/p is the domestic money supply excluding the quantity of the non-sterilized foreign reserves. If we assume, however, that M'/p is specified as a total amount of money supply including the non-sterilized foreign reserves; that is:

(1) $M'/p=M/p+sR/p$,

then II-4(a) will simply become:

(2) $M'/p=L(y,i,w)$.

But now $s\dot{R}/p$ must be incorporated in the government budget restraint equation from II-7 as:

(3) $g+B/p+s\dot{R}/p=\dot{M}'/p+\dot{B}/ip+t$,

where $\dot{M}'$ is defined a change in the total amount of money supply in the country including the non-sterilized balance of payments.

We must note here that the money market equilibrium is a stock concept while the government budget restraint equation is a flow concept. This implies that the external balance in the money market is foreign reserves, but in the government budget restraint it is the balance of payments.

II-21
$$\begin{bmatrix} 1-(1-u)(E_1+X_1) & E_2B/i^2-E_2 & 0 \\ L_1 & -(L_2B/i^2-L_2) & -s \\ (1-u)X_1+K_1 & K_2 & -1 \end{bmatrix}\begin{bmatrix} dy \\ di \\ dR \end{bmatrix} =$$

$$\begin{bmatrix} [(1-u)(E_1+X_1)]dB+E_2dB/i+E_2dM-(E_1+X_1)(y+B)du+dg+X_2dr \\ L_2dB/i-(1-L_2)dM \\ -(1-u)X_1dB+X_1(y+B)du-X_2dr \end{bmatrix}$$

The Jacobian determinant of the system is:

$$|J|=[1-(1-u)(E_1+X_1)][(L_2B/i^2-L_2)+sK_2]$$
$$+\{L_1-s[(1-u)X_1+K_1]\}(E_2B/i^2-E_2)$$
$$=\Delta_2 \quad >0$$

We assume as before that resulting from the expansion of income a net decrease in the current account outweighs an increase in the capital account (i.e. $(1-u)X_1+K_1<0$ where $(1-u)X_1<0$ and $K_1>0$). Then the sign of the Jacobian determinant Δ_2 is positive.

We must note that the level of the sterilization coefficient s affects the determination of the income level and the interest rate. We assume here, however, that s is a constant term from the moment the system leaves an initial equilibrium until it arrives at a new internal equilibrium.

We shall call this process a first stage. The multipliers derived through the method of comparative statics will display the results of the first stage. This is the point where an internal equilibrium and an external equilibrium will be separated from the initial equilibrium.

Next we assume that initiating from the internal and external disequilibrium of the first stage, the system will adjust toward an internal and external simultaneous equilibrium through the self-liquidating process built into the system. We shall call this process a second stage. The analysis of the second stage will be done diagramatically in the IS-LM framework.

As for the selection of government policies, the same three government policies (as in the floating exchange rate system) will be analyzed. And as an extention of a study, since the exchange rate can be controlled by the government under the pegged exchange rate system, exchange rate change policy will be examined as well.

1. Money Financed Government Expenditures

The government budget restraint is $dg=dM$ in money financed government expenditures so that the system of the matrix form will be expressed as:

$$\text{II-21(i)}\quad \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} dy \\ di \\ dR \end{bmatrix} = \begin{bmatrix} (1+E_2)dM \\ -(1-L_3)dM \\ 0 \end{bmatrix}$$

where a_{11}.........a_{33} are the same as II-21.

Solving for the endogenous variables dy, di and dR, we obtain the following multipliers:

II-21(a) $dy/dM=1/\Delta_3$

$$\{(1+E_3)[(L_3B/i^2-L_2)+sK_2]+(1-L_3)(E_3B/i^2-E_3)\} \quad >0$$

II-21(b) $di/dM=1/\Delta_3$

$$[(1+E_3)\{L_1-s[(1-u)X_1+K_1]\}-(1-L_3)[1-(1-u)(E_1+X_1)]]$$

II-21(c) $dr/dM=1/\Delta_3[(1+E_3)\{L_1K_2+[(1-u)X_1+K_1](L_3B/i^2-L_3)]$

$$-(1-L_3)\{[1-(1-u)(E_1+X_1)]K_2-[(1-u)X_1+K_1](E_3B/i^2-E_3)\}]$$

The sign of the income multiplier is unambiguously positive in II-21(a). It consists of three different elements. These are the fiscal effect, the monetary effect and the effect of capital inflow, all of which raise the income level.

The sign of the interest rate multiplier is uncertain due to the conflicting forces; the fiscal effect which raises the interest rate, and the monetary effect which lowers it in II-21(b).

The sign of the balance of payments multiplier is also uncertain. The first term in II-21(c) consists of the two conflicting forces in the bracket. That is, the higher capital inflow creates the balance of payments surplus. On the otherhand the lower net exports cause the balance of payment deficit given the assumption $(1-u)X_1+K_1<0$. The second term in II-21(c) shows the negative sign. This is because the monetary effect lowers the interest rate which creates the capital outflow and raises the income level which lowers the net exports. Both effects cause the balance of payment deficit.

These terms of the balance of payment multiplier in II-21(c) can be illustrated in the context of the IS-LM framework in Figure II-8. We know that the BP curve is positively sloping under $(1-u)X_1+K_1<0$. We first suppose that the BP curve is steeper than the LM curve (i.e. BP_1 in Figure II-8). In this case dR/dM in II-21(c) will be unambiguously negative. That is, the balance of payments will deteriorate since the domestic equilibrium point corresponding to the intersection A of IS_1 and LM_1 in Figure II-8 lies the right hand side of BP_1. This implies that the income level is so high that a decrease in net exports

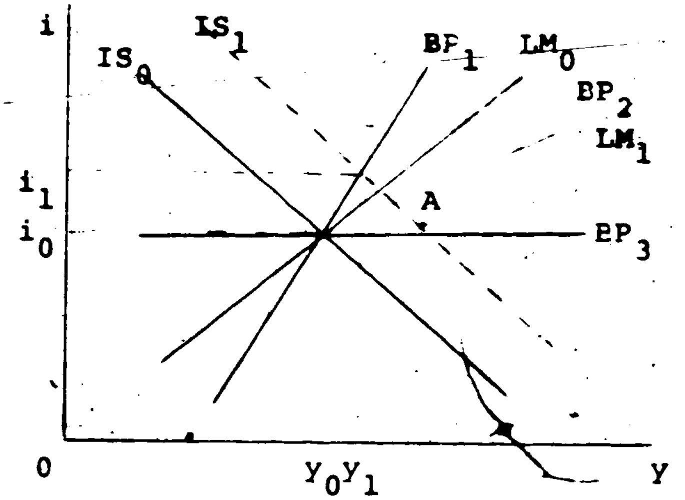

Figure II-8
The Effect of MFGE; the Pegged Exchange Rate System

exceeds the capital inflow.

We next suppose that the LM curve is steeper than the BP curve (i.e. BP_1 in Figure II-8). Although BP_1 lies the right hand side of LM_0 beyond y_0, so far as the internal equilibrium point A lies the right hand side of the BP curve, the balance of payment deficit will not be altered. This is because a decrease in net exports still exceeds an increase in the capital inflow. Only when a BP curve lies the right hand side of the internal equilibrium A beyond y_0 (i.e. BP_3 in Figure II-8), will the balance of payments show a surplus.

So far in the first stage the analysis of the balance of payment multiplier in money financed government expenditures under the pegged exchange rate system is similar to that of the the floating exchange rate system. However, the process of the simultaneous equilibrium in the internal and the external systems under the pegged exchange rate system is different from that under the floating exchange rate system. We learned that under the floating exchange rate system, the exchange rate was determined by the internal equilibrium so that a BP curve simply moved toward the internal equilibrium point. In the case of the pegged exchange rate system, since the exchange rate has no self-liquidating power, the adjustment of the external equilibrium with the internal equilibrium must be dependent on the fluctuation of the money supply. This means that the direct movement of a BP curve to the internal equilibrium will not occur under the pegged exchange rate system unless the exchange rate change policy is taken, but the magnitude of the balance of payments goes into the domestic money market and affects the internal equilibrium. Therefore, the simultaneous internal and external equilibrium will be achieved only through the money market under the pegged exchange rate system.

Let us observe in the second stage how the external equilibrium and the internal equilibrium, BP, and A respectively in Figure II-9 will eventually adjust into the simultaneous equilibrium. We now assume that there is no sterilized magnitude of the balance of payments so that all the value of the balance of payments goes into the money market (i.e. s=1). Since the income level increases and the interest rate is uncertain in money financed government expenditures the balance of payments will become deficit; unless the capital mobility is extremely high. After the

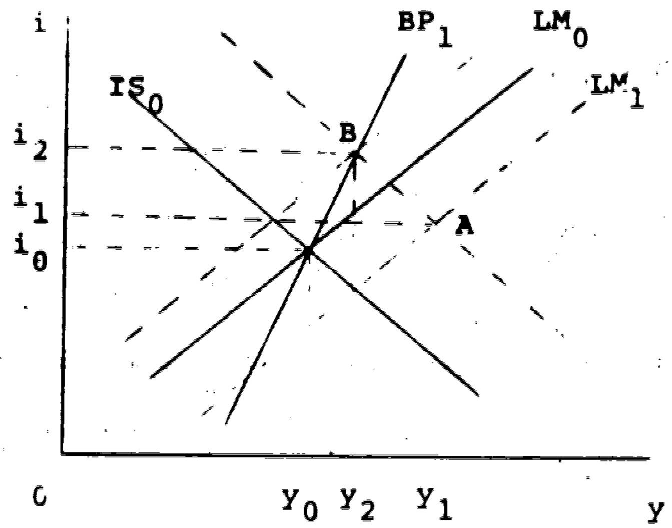

Figure II-9
The Internal and External Equilibrium in MFGE; the Steeper BP Curve than the LM Curve

adjustment of money financed government expenditures in the system, the internal equilibrium settles at A while the external disequilibrium BP_1 is left alone in Figure II-9. With the situation of the balance of payment deficit the quantity of money supply is reduced. As a result, there will be an excess demand for money in the money market. This situation will shift the LM curve backwards. The LM curve will be shifted along the IS curve (IS_0) until it reaches the external equilibrium curve on BP_1. Then the simultaneous equilibrium will be finally reached at B in Figure II-9. It should be noted that the effect of a decrease in net exports with respect to an increase in the income level is so large that the reduction of money supply exceeds the increased amount of money stock. Consequently, the LM curve (LM_1) lies on the left hand side of the original money market equilibrium (LM_0) in Figure II-9. At B the income level (y_2) is higher than the original income level (y_0), but it is not as high as y_1. The interest rate (i_2) becomes the highest at B since an increase in money supply under money financed government expenditures is completely paralysed.

Next let us observe the simultaneous equilibrium process in the case of BP_2 (i.e. the steeper LM curve than the BP curve) in Figure II-10. Though BP_2 is flatter than LM_0 in Figure II-10, the external disequilibrium is the same as BP_1 (i.e. the balance of payment deficit) since BP_2 lies the left hand side of A. The simultaneous equilibrium process is the same as the case of BP_1. As a result, the income level goes up from y_0 to y_2, but not as high as y_1 and the interest rate i_2 reaches the highest level. But now the reduction of money supply is not so high that the quantity of money supply (LM_2) remains higher than the original level (LM_0) in Figure II-10.

This concludes that the higher the elasticity of the reduction of net exports with respect to the increment of the income level, the lower the effect on the income level will be in the case of money financed government expenditures. As an extreme case, if the BP curve is the vertical line, there will be no increase in the income level. On the other hand, the higher capital mobility is the higher the level of income is in money financed government expenditures. We must note that the self-liquidating process of the simultaneous equilibrium can be attained only when s=1. If we assume s=0, the total

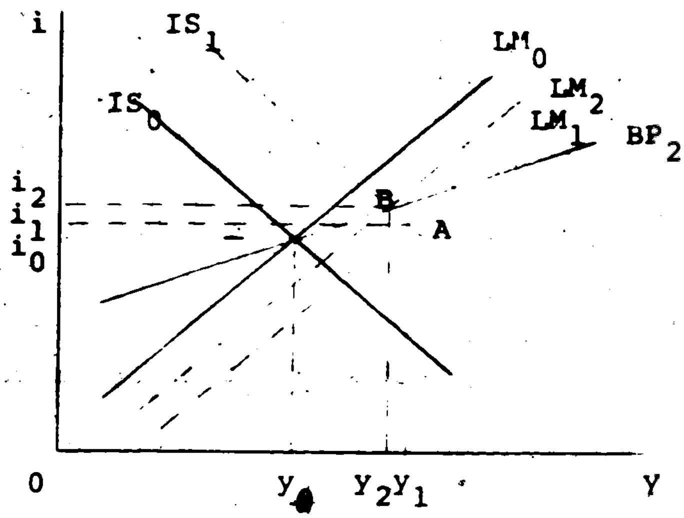

Figure II-10
The Internal and External Equilibrium in MFGE; the Steeper LM Curve than the BP Curve

balance of payments will be insulated from the domestic economy. Hence, both the capital mobility (i.e. K_2 and K_1) and the net exports (i.e. X_1) have no effect on the income and the interest rate multipliers in II-21(a) and II-21(b). This implies that the IS curve shift from IS_0 to IS_1 in Figure II-10 is smaller than IS_1 level which causes the lower income and the lower interest rate. But since the external balance is completely insulated from the internal equilibrium, A in Figure II-9 and Figure II-10 will not move. Hence, the system can keep the high level of the income level but the balance of payment deficit remains intact and accumulates hereafter.

Let us consider the case of perfect capital mobility. Taking K_2 to be infinite for each multiplier from II-21(a) to II-21(c), we have the following:

II-21(a)' $dy/dM=(1+E_2)/[1-(1-u)(E_1+X_1)] > 0$

II-21(b)' $di/dM=0$

II-21(c)' dR/dM

$$=\{(1+E_2)L_1-[1-(1-u)(E_1+X_1)](1-L_2)\}/s[1-(1-u)(E_1+X_1)]$$

The income level clearly goes up. The sign of the balance of payment multiplier is uncertain. If there is a pressure such that the interest rate rises, the balance of payments will improve. Otherwise it will deteriorate. With the assumption of non-sterilization policy, the internal and external equilibrium settles on the external curve BP, in Figure II-8. Although the adjustment of a simultaneous equilibrium is made by the shift of the LM curve, as we see in Figure II-8, the internal equilibrium and the external equilibrium are close to each other. This implies that the simultaneous equilibrium is attained by a marginal fluctuation of the money supply and the increment of the income level can be maximized in money financed government expenditures under perfect capital mobility.

2. **Bond Financed Government Expenditures**

The government budget restraint in bond financed government expenditures is $dg=dB/i$ so that the system of the matrix form will be expressed as:

$$
\text{II-21(ii)} \quad \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} dy \\ di \\ dR \end{bmatrix} = \begin{bmatrix} [(1-u)(E_1+X_1)i+E_2+1]dB/i \\ L_3 dB/i \\ -(1-u)X_1 i dB/i \end{bmatrix}
$$

where a_{11}.........a_{33} are the same as II-21.

Solving for endogenous variables dy, di and dR, we obtain the following multipliers:

II-21(d)

$$
dy/(dB/i)=1/\Delta_3[[(1-u)(E_1+X_1)i+1](L_2B/i^2-L_2)-E_2L_2L_3E_2 +s\{[(1-u)(E_1+X_1)i+E_2+1]K_2 +(1-u)X_2i(E_2B/i^2-E_2)\}]
$$

II-21(e)

$$
di/(dB/i)=1/\Delta_3[[1-(1-u)(E_1+X_1)][L_3-s(1-u)X_2i] +[(1-u)(E_1+X_1)i+E_2+1]\{L_1-s[(1-u)X_1+K_1]\}] \quad >0
$$

II-21(f)

$$
dR/(dB/i)=1/\Delta_3[[(1-u)(E_1+X_1)i+E_2+1]\{K_2L_1+[(1-u)X_1+K_1](L_2B/i^2-L_2)\} +L_3\{[1-(1-u)(E_1+X_1)]K_2-[(1-u)X_1+K_1](E_2B/i^2-E_2)\} +(1-u)X_2i\{[1-(1-u)(E_1+X_1)](L_2B/i^2-L_2)+(E_2B/i^2-E_2)L_1\}]
$$

The sign of the income multiplier is most likely positive since the IS curve shift is fairly large compared to the negative wealth effect on the money market as was introduced in Blinder-Solow(1973). But there are two elements which bring the income level down in II-21(d). The first element is the wealth effect in the money market due to an increase in government bonds. As we analyzed under the floating exchange rate system, this wealth effect (L_1E_2) affects a stability condition of the system. If $|E_1L_2|>|L_1E_2|$, this means that the bond finance has an expansionary effect so that the system can be stable. Another element is related with a sterilization coefficient s. Given the assumption s>0, capital inflow increases the income level (sK_1). While an increase in net exports due to an increase in interest paymemts affects the income level to go down ($s(1-u)X_1i$) in II-21(d).

The sign of the interest rate multiplier is unambiguously positive since all the terms work in the positive direction in II-21(e).

Since the numerator of the balance of payment multiplier in II-21(f) is the same as that of the exchange rate multiplier in II-17(f) in spite of the opposite signs, the analysis of the multiplier is similar to what we had in II-17(f). But as was the case of money financed government expenditures, the way the system approaches a simultaneous equilibrium under the pegged exchange rate system differs from that under the floating exchange rate system. Let us briefly analyze the balance of payment multiplier in the context of the IS-LM framework in the first stage. If the BP curve is flatter than the LM curve (i.e. BP_1 in Figure II-11), the balance of payments will clearly improve since

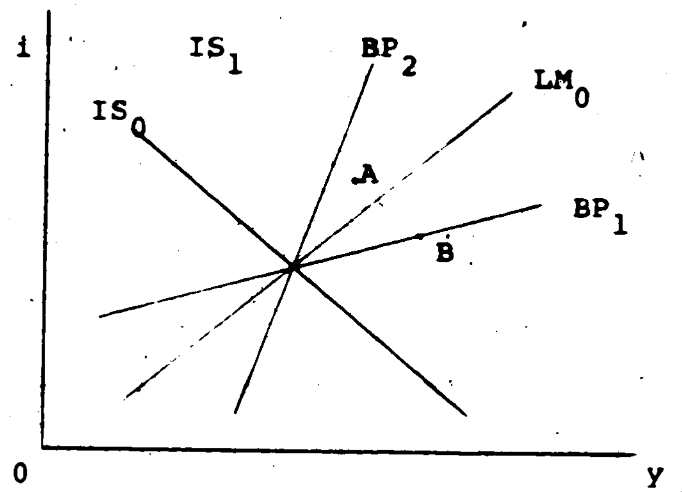

Figure II-11
The Effect of BFGE; the Pegged Exchange Rate System

capital inflow exceeds a decrease in net exports.[31] The balance of payments will improve if the curve lies on the right hand side of the internal desired equilibrium point A. Only when the BP curve lies the left hand side of A like BP_2, the balance of payments will deteriorate.

Starting from the initial internal and external equilibrium, we now recognize that the internal and the external equilibria are seprarated after the application of bond financed government expenditures in the system. Our second analysis starts from this stage. It is concerned with how the simultaneous internal and external equilibrium will be attained through the self-liquidating process under

---

[31]We learned that the third term in the exchange rate multiplier in II-17(f) which is identical with that in II-21(f) affects somewhat the slope of the BP curve and the magnitude of the IS curve compared with the multiplier without including interest payments. Here we assume that the slope of the BP curve and the magnitude of rightward shift on the IS curve in Figure II-11 have already taken the third term into account.

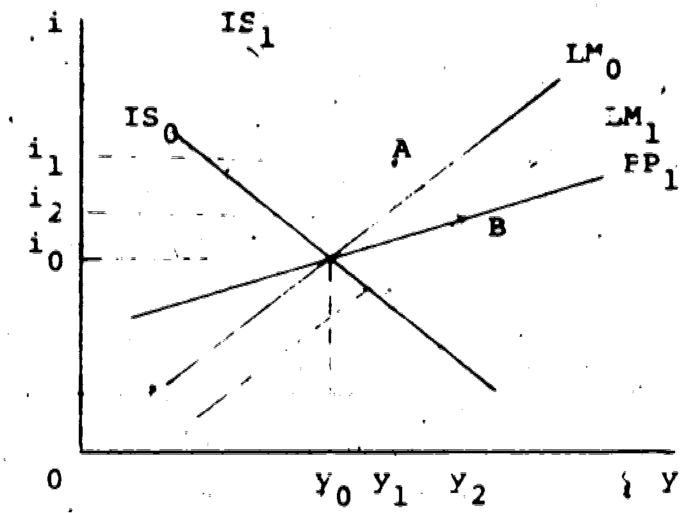

Figure II-12
The Internal and External Equilibrium in BFGE; the Steeper LM Curve than the BP Curve

the pegged exchange rate system.

Under a non-sterilization policy, in the case of bond financed government expenditures under the floating exchange rate system the wealth effects on the IS and the LM curves were important to recognize. But under the pegged exchange rate system as the LM curve shifts in lieu of the BP curve, the magnitude of the wealth effects will be assumed not to be a significant factor to determine the multipliers.

First, we shall consider the case that the BP curve is flatter than the LM curve (i.e. BP_1 in FiureII-12). With bond financed government expenditures, the internal equilibrium exists at A where the higher income level (y_1) and the higher interest rate (i_1) than the original equilibrium (y_0 and i_0). Since the internal equilibrium lies on the left hand side of the external equilibrium BP_1, foreign capital is attracted and as a result the balance of payments improves. Then since the balance of payment surplus goes into the money market so that the LM curve (LM_1) will shift rightward until it reaches the BP curve. The simultaneous equilibrium level is settled at B in Figure II-12. The income level (y_2) is higher than the internal equilibrium level (y_1) at A and the interest rate (i_2) goes down from the internal equilibrium, but higher than the initial equilibrium level (i_0).

Next let us consider the case where the BP curve lies on the left hand side of the internal equilibrium in Figure II-13. If net exports are highly elastic with respect to a change in the income level, the BP curve will be steep as BP_2. At the internal equilibrium point A the income level (y_1) is higher than the original level (y_0) in Figure II-13. This high income level decreases net exports more than the amount of an increase in capital inflow. As a result, the balance of payments deteriorates and a decrease in money supply shifts the LM curve leftwards until it reaches the external equilibrium at B. At the new simultaneous equilibrium B the income level is lower than the internal equilibrium A (i.e. y_1 to y_2), and the interest rate is even higher than the internal equilibrium (i.e. i_1 to i_2) in Figure II-13.

Finally let us consider the case of perfect capital mobility. Taking K_2 to be infinite for each multiplier we have the following:

II-21(d)' $dy/(dB/i)=[(1-u)(E_1+X_1)i+E_2+1]/[1-(1-u)(E_1+X_1)]$ >0

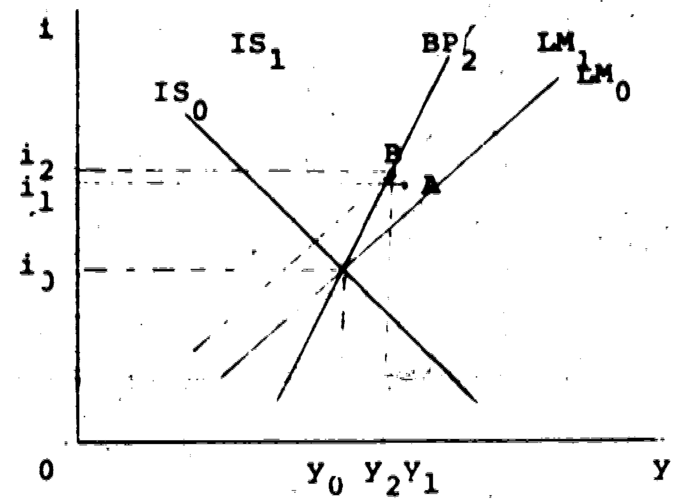

Figure II-13

The Internal and External Equilibrium in BFGE; the Steeper BP Curve than the LM Curve

II-21(e)' $di/(dB/i)=0$

II-21(f)' $dR/(dB/i)$

$=\{[(1-u)(E,+X,)i+E,+1]L,+L,[1-(1-u)(E,+X,)]\}/$

$s[1-(1-u)(E,+X,)] > 0$

Let us go back to Figure II-11 and assume that BP_1 is a horizontal line. This indicates the situation of perfect capital mobility. The increment of government bonds to finance the increased government expenditres causes excess demand for money. The desired internal equilibrium point at A lies on the left hand side of the original LM curve (LM_0) in Figure II-11. However, the balance of payment surplus shifts the LM curve all the way down to the external equilibrium level on BP_1. As a result, the simultaneous equilibrium settles on B where the income level is higher than the intertnal equilibrium at A and there is no change in the interest rate from the initial equilibrium. Thus, whether the government applies money financed government expenditures or bond financed government expenditures, the income level settles in a similar manner. In fact, ignoring the interest payments term $(1-u)(E_1+X_1)i$ in II-21(d)', we realize that the income multiplier in bond financed government expenditures is identical to that in money financed government expenditures in II-21(a)' when capital mobility is perfect. The sign of the balance of payments multiplier in II-21(f)' is positive. Unlike the case of money financed government expenditures in II-21(c)' the LM curve shift is solely done by the balance of payment surplus.

3. Open Market Purchases

The government budget restraint in open market purchases is $dM=-dB/i$ so that the system of the matrix form will be expressed as:

$$\text{II-17(iii)}\quad \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} dy \\ di \\ dR \end{bmatrix} = \begin{bmatrix} -[(1-u)(E_1+X_1)]idM \\ -dM \\ (1-u)X_1 idM \end{bmatrix}$$

where $a_{11}, \ldots, a_{33}$ are the same as II-21.

Solving for endogenous variables dy, di and dR, we obtain the following multipliers:

$$\text{II-21(g)}\quad dy/dM = 1/\Delta_3\{[1-s(1-u)X_1 i](E_2B/i^2-E_2) - (1-u)(E_1+X_1)i[(L_2B/i^2-L_2)+sK_2]\}$$

$$\text{II-21(h)}\quad di/dM = -1/\Delta_3[[1-(1-u)(E_1+X_1)][1-s(1-u)X_1 i] + (1-u)(E_1+X_1)i\{L_1-s[(1-u)X_1+K_1]\}] < 0$$

$$\text{II-21(j)}\quad dR/dM$$

$$= -1/\Delta_3[\{[1-(1-u)(E_1+X_1)]K_2-[(1-u)X_1+K_1(E_2B/i^2-E_2)\} + (1-u)i\{[X_1+(E_1+X_1)K_1](L_2B/i^2-L_2) + [(E_1+X_1)K_2+(E_2B/i^2-E_2)X_1]L_1\}]$$

We assumed in open market purchases under the floating exchange rate system that the effect on the income level caused by an increase in money supply is larger than the effect on the income level caused by a decrease in interest payments to the private sector. That is, the magnitude of the first term is larger than the magnitude of the second term in II-21(g). If we take this assumption here as well, the sign of the income level multiplier will be positive.

The sign of the interest rate multiplier is clearly negative since both a decrease in private expenditures and an increase in money supply have a downward pressure on the interest rate in II-21(h).

A decrease in net exports caused by an increased money supply together with a decrease in capital inflow causes the balance of payments to deteriorate in the first term in II-21(j). The effect of a decrease in private expenditures is uncertain on the balance of payments in the second term. But since we assumed the dominant effect of a change in money supply over the effect of a change in interest payments, it may be concluded that the balance of payments will deteriorate in open market purchases in II-21(j).

Now that the internal and the external equilibria are separated in the system, we must investigate next how the self-liquidating force will return to a simultaneous internal and external equilibrium in the context of a IS-LM framework.

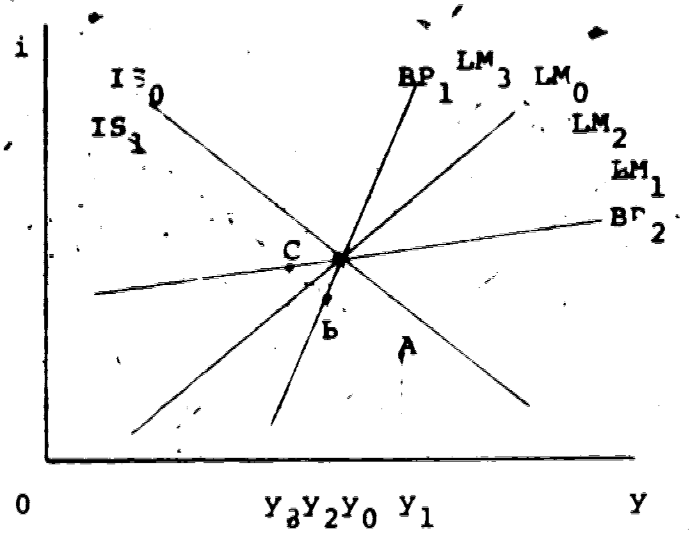

Figure II-14
The Internal and External Equilibrium in OMP

First we assume that the positively sloping BP curve is steeper than the LM curve. Assuming a non-sterilization policy, all the magnitude of the balance of payment deficit goes into the money market. Since a decrease in money supply causes an excess demand for money, the LM curve (LM_1) shifts leftward until it attains the intersection of the IS curve and the BP curve (IS_1 and BP_1) at B in Figure II-14. As a result, the income level is substantially decreased from the internal equilibrium A (y_1 to y_2), the lower level of the initial equilibrium ($y_0 > y_2$). The leftward shift of the LM curve (from LM_1 to LM_2) does not go back all the way to the original LM curve (LM_0), however.

Next we assume that the BP curve is flatter than the LM curve (BP_2). The high elastic BP curve with respect to a change in the interest rate leads the LM curve even further to the left hand side of the original LM curve (from LM_1 to LM_3) until it reaches the intersection of IS_1 and BP_2 at C in Figure II-14. As a result, the income level (y_3) is even lower than the case of the steep BP curve (y_2), but the interest rate at C is higher than that at B.

Let us consider the case of perfect capital mobility in open market purchases. Taking K_2 to be infinite for each multiplier from II-21(g) to II-21(j), we have the following:

II-22(g)' $dy/dM=-(1-u)(E_1+X_1)i/[1-(1-u)(E_1+X_1)] \quad <0$

II-21(h)' $di/dM=0$

II-21(j)' $dR/dM=-1/s-(1-u)(E_1+X_1)i/s[1-(1-u)(E_1+X_1)] \quad <0$

In the traditional theory, a monetary policy is ineffective on the income level. This occurs because the downward pressure of the interest rate due to an increase in money supply leads to the outflow of capital which is exactly the same amount of an increase in money supply. But when interest payments to the private sector are taken into account, the monetary policy has even an adverse effect on the income level. This happens because the fall of the interest rate due to an increased money supply and a decrease in private expenditures cause capital outflow.

4. Exchange Rate Change Policy

As we have learned so far, under the pegged exchange rate system there is no movement in an external balance schedule. Thus an external balance disequilibrium is adjusted through a change in money supply. But this way of equilibrating the system causes an internal disturbance. A sterilization policy can be used as one of the techniques to avoid the internal disturbance when the internal and external disequilibria occur. However, this does not solve the external disequilibrium at all. To solve the external disequilibrium without disturbing the internal equilibrium an exchange rate change policy can be applied in the system. Assuming an initial internal and external equilibrium as usual we shall observe the effects of a devaluation in the exchange rate (i.e. dr>0) on the three endogenous variables in the system. The system of the matrix form will be expressed as:

$$\text{II-21(iv)} \quad \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} dy \\ di \\ dR \end{bmatrix} = \begin{bmatrix} X_3 dr \\ 0 \\ -X_3 dr \end{bmatrix}$$

where a_{11}.........a_{33} are same as II-21.

Solving for the endogenous variables dy, di and dR, we obtain the following multipliers:

$$\text{II-21(k)} \quad dy/dr = 1/\Delta_2 X_2\{(L_1 B/i^2 - L_2) + s[(E_2 B/i^2 - E_2) + K_2]\} \quad > 0$$

$$\text{II-21(l)} \quad di/dr = 1/\Delta_2 X_2\{L_1 - s[1-(1-u)X_1 + K_1]\}$$

$$\text{II-21(m)} \quad dR/dr = 1/\Delta_2 X_2\{[1-(1-u)E_1 + K_1](L_2 B/i^2 - L_2) + L_1[(E_2 B/i^2 - E_2) + K_2]\} \quad > 0$$

The sterilization coefficient is first assumed to be kept constant and non-zero so that some part of the balance of payments affects the domestic money supply; and the Marshall-Lerner condition is satisfied (i.e. $X_2>0$). Given these conditions, the devaluation of the exchange rate raises the income level and the balance of payments while the interest rate is uncertain.

Let us observe these results in the IS-LM framework. The devaluation of the exchange rate shifts the BP curve rightward assuming $(X_1+K_1)<0$ from BP_1 to BP_2 in Figure II-15. The devaluation also shifts the IS and the LM curves rightward and form the internal equilibrium at A in Figure II-15. If a complete sterilization is applied, there will be no shift in the LM curve. As a result, we conclude the higher the magnitude of non-sterilized balance of payment surplus is, the higher will be an increase in the income level and the lower the interest rate.

Now that the devaluation has caused the internal and the external disequilibria, we must proceed to the second stage where the simultaneous equilibrium system is attained.

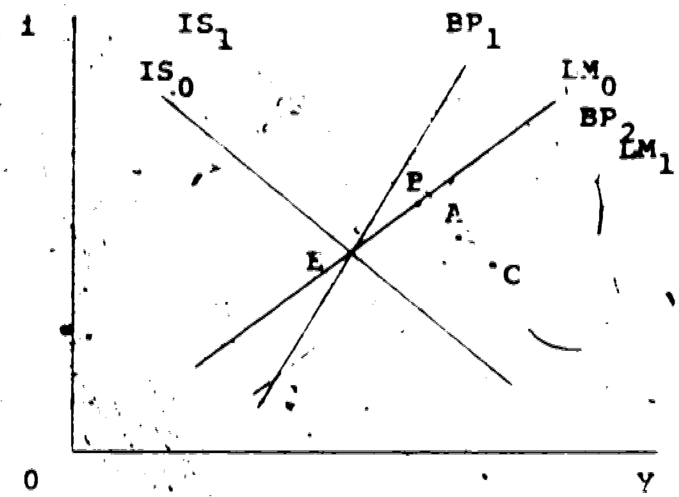

Figure II-15
The Internal and External Equilibrium in ERC

Here we assume that the government takes the non-sterilization policy. Since the internal equilibrium A lies the left hand side of the external equilibrium BP_1, the balance of payment surplus feeds into the domestic money supply and shifts the LM curve rightward. The movement of the LM curve (LM_1) continues along the IS curve until it reaches C where the external balance and the internal equilibrium intersects. Thus, the internal and external simultaneous equilibrium is attained at C in Figure II-15.

Let us consider the case of perfect capital mobility. Taking K_1 to be infinite for each multiplier we have the followings:

II-21(k)' $dy/dr=X_2/[1-(1-u)(E_1+X_1)] > 0$

II-21(l)' $di/dr=0$

II-21(m)' $dR/dr=X_2L_1/s[1-(1-u)(E_1+X_1)] > 0$

Assuming s≠0 the devaluation of the exchange rate raises the income level. The balance of payments improves due to a higher inflow of capital.

E. Summary

Table II-1 Government Policies: Summary of Effects

	FERS				PERS		
Endog. Var. \ Policy	MFGE	BFGE	OMP	MFGE	BFGE	OMP	ERC
y n-pcm[1]	+[3]	+	+	+	+	+	+
y pcm[2]	+	-[4]	+	+	+	-	+
i n-pcm	±[5]	+	-	±	+	-	±
i pcm	0[6]	0	0	0	0	0	0
r for FERS n-pcm	+?[7]	±	+?	-?[8]	±	-?	+
R for PERS pcm	±	-	+	±	+	-	+

1. Non-perfect capital mobility;
2. Perfect capital mobility;
3. Definitely positive;
4. Definitely negative;
5. Uncertain;
6. No change;
7. Most likely positive;
8. Most likely negative.

Under the condition of perfect capital mobililty, the income level decreases in bond financed government expenditures[4] for the floating exchange rate system and in open market purchases for the pegged exchange rate system while the rest of the policies raise the income level.

A fluctuation in the interest rate is determined by two forces, a monetary effect which lowers the interest rate and a fiscal effect which raises it. Thus, the sign of the interest rate multiplier is ambiguous in money financed government expenditures, goes up in bond financed government expenditures and goes down in open market purchases under the condition of non-perfect capital mobility.

The multipliers of the external balance are determined by the movements of the income level and the interest rate. A rise of the income level causes net exports to fall. Hence, the exchange rate depreciates or the balance of payments deteriorates. Here we note that the increased income level affects a capital flow as is expressed by $K_1>0$. But we assumed $X_1+K_1<0$. A rise of the interest rate causes capital of inflow. Hence, the exchange rate appreciates or the balance of payments improves. Since all the domestic policies have a positive impact on the income level, the current account always deteriorates. This implies that it is not possible for the exchange rate or the balance of payments to appreciate or improve unambiguously through the effect of a domestic policy. The results of the multipliers on the external balance, therefore, are either uncertain, to depreciate or to deteriorate. These differences are determined by the movements of the interest rate. If the interest rate increases, then the sign of multipliers is uncertain. If the interest rate decreases, then depreciation or deterioration results. In money financed government expenditures, since there is a minor change in the interest rate, the external balance is most likely to deteriorate. In bond financed government expenditures, the interest rate goes up clearly so that the sign of the external balance multipliers is uncertain. In open market purchases, there is a minor improvement of the external balance due to the decreased private expenditures. However, since the interest rate goes down clearly, the external balance is most likely to deteriorate. The devaluation of the exchange rate is the only policy which improves the external balance unambiguously.

If the assumption $X_1+K_1<0$ is altered into $X_1+K_1>0$, then all the results change. This assumption implies that the increased income level causes the external balance to improve due to the dominant force of capital inflow over the negative net exports.

These results are derived under the condition of non-perfect capital mobility. If the capital mobility is perfect, then the fluctuation of the income level no longer affects the determination of the external balance. The only factor to influence the external balance is the pressure on the interest rate. If a domestic policy has a force to raise the interest rate, then the external balance improves. Otherwise, it deteriorates.

Thus far, there is little difference in the analysis between the floating exchange rate system and the pegged exchange rate system. A significant difference in these systems can be seen, however, in the process of moving towards a new equilibrium level. Under the floating exchange rate system, the exchange rate always moves along with the internal equilibrium. On the other hand, the pegged exchange rate system has no variable to follow the internal equilibrium, The only way of reaching a new external and internal equilibrium is through the adjustment in the money market. That is, the amount of deficit or surplus can be adjusted by a change in the money supply. Thus, the movement of the LM curve leads the system to the new equilibrium.

III. EXPLICIT RECOGNITION OF THE GOVERNMENT BUDGET RESTTRAINT IN AN OPEN ECONOMY

A. Introduction

In the preceding chapter the endogenous variable (i.e. the income level y) in the government budget restraint was assumed to be offset by a change in autonomous tax flow (i.e. udy=dv). This has been a common assumption in many papers which recognize the government budget restraint in the system in order to simplify the analysis of government policy effects. Under this assumption the government loses one degree of freedom to choose a policy variable, because an endogenous variable is equated to an exogenous variable. However, if this assumption is relaxed, the government loses one more degree of freedom so that another policy variable must be adjusted endogenously to satisfy the government budget restraint. In the present chapter we shall analyze the effect of each government policy by incorporating the concept of the fully recognized government budget restraint into the system.

As in chapter II, under the floating exchange rate system the effects of the three government policies (namely, money financed government expenditures, bond financed government expenditures and open market purchases) and under the pegged exchange rate system the four government policies (namely, exchange rate change policy in addition to the three policies above) will be analyzed. Special attention in this chapter will be given to the analysis of the endogenous policy variable multiplier.

In chapter II the fluctuation of the balance of payments was simply assumed to be adjusted in the money market. But now that the government budget equilibrium is explicitly considered, the variable of the balance of payments is also incorporated into the government budget restraint under the pegged exchange rate system.

B. The Model

The model consists of the seven equations as follows:

III-1 $y=E(z,i,w)+X(z,r,p)+g$

III-2 $z=y+B/p-t$

III-3 $t=u(y+B/p)+v$

III-4 $M^*/p=L(y,i,w)$

III-5 $w=M^*/p+B/ip$

III-6 $\dot{R}=X(z,r,p)+K(y,i)$

III-7 $g+B/p+\dot{R}/p=\dot{M}^*/p+\dot{B}/ip+t$

$E_1>0,\ E_2<0,\ E_3>0,\ L_1>0,\ L_2<0,\ 0<L_3<1,$

$X_p<0,\ X_2>0,\ K_1>0,\ K_2>0$ and $0<E_1+X_1<1.$

All notations and the signs of the partial derivatives in the model above are the same as the model in the previous chapter, exept M^* in lieu of M. However, now that the assumption (i.e. $udy=dv$) is removed, the government budget restraint will be considered as one of the equations; that is, the matrix form will be expressed by a four-by-four in size.

Another important change in this model lies in the definition of money supply.[illegible] M^* is defined as total base money supply (i.e. $M^*=M+R$ where M is domestic base money supply and R foreign reserves). In the preceding chapter, the money supply was defined as the domestic money supply excluding foreign reserves. Therefore, the variable of foreign reserves was placed separately in the money market equilibrium. Since the receipts from the balance of payments in the public sector were assumed to become a part of money supply automatically, the balance of payment variable did not exist in the government budget restraint. Now that M^* is assumed to include the foreign reserves, the variable of the balance of payments remains on the left hand side of the government budget restraint in III-7.[illegible]

As in chapter II, we have the following reduced forms of a system. The price level is again in the present chapter assumed to be fixed so that we set p=1.

III-8 $\quad y=E[y+B-u(y+B)-v,\ i,\ M^*+B/i]+X[y+B-u(y+B)-v,\ r]+g$

III-9 $\quad M^*=L(y,\ i,\ M^*+B/i)$

III-10 $\quad \dot{R}=X[y+B-u(y+B)-v,\ r]+K(y,\ i)$

III-11 $\quad g+B+\dot{R}=\dot{M}^*+\dot{B}/i+u(y+B)+v$

III-8, III-9 and III-10 display IS, LM and BP curves respectively. The slope of each curve is also the same as

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[illegible]This concept was already introduced in chapter II.
[illegible]The inclusion of the balance of payment variable in the government budget restraint was also introduced by W.M. Scarth, "Fiscal Policy and the Government Budget Constraint under Alternative Exchange-Rate Systems," Oxford Economic Papers 27 (March 1975): 10-20.

in chapter II.

IS curve:

III-12 $di/dy=-[1-(1-u)(E_1+X_1)](E_2B/i^2-E_2)$ <0

LM curve:

III-13 $di/dy=L_1/(L_2B/i^2-L_2)$ >0

BP curve:

III-14 $di/dy=-[(1-u)X_1+K_1]/K_2$ >0 where $(1-u)X_1+K_1<0$

Let us now observe the new concept of the government budget restraint in III-11. Transforming the interest payments of government bonds B to the right hand side, we have:

III-11(a) $g+\dot{R}=\dot{M}^{*}+\dot{B}/i+uy-(1-u)B-v$

Taking total differentials in III-11(a) we have:

III-11(b) $dg+dR=dM^{*}+dB/i-(\dot{B}/i^2)di+udy-(1-u)dB+(y+B)du+dv$

In the context of comparative statics the system is, at the intial stage, assumed to be at a steady state equilibrium so that we realize $\dot{B}/i^2$ to be zero. Then we find five policy variables (i.e. dg, dM^{*}, dB, du and dv) and two endogenous variables (i.e. dR and dy) in III-11(b). Out of the five policy variables, since du and dv have a similar path, we assume dv=0 hereafter. Hence, we have the four policy variables. We must, however, note that this system will lose one degree of freedom to determine exogenous policy variables. We shall choose the marginal tax rate u as a representative endogenous policy variable. This policy variable multiplier will be analysed in addition to income, interest rate and exchange rate (under the floating exchange rate system) or the balance of payment (under the pegged exchange rate system) multipliers.

The approach of the present chapter can be considered a mere extension of chapter II in which we analyzed the basic effects of the three different government policies on the three endogenous variables, with implicit recognition of the government budget restraint.

In this chapter, however, we have two endognous policy variables: one generated by the government budget restraint equation (i.e. du), and another by each policy specification such as dM* for money financed government expenditures and dB for bond financed government expenditures. For money financed government expenditures, we get the following government budget restraint:

III-11(c) $dg=dM^{*}+udy+(y+B)du$ by setting $dB=0$ and $dR=0$.[34]

As in chapter II, an exogenous increase in government expenditures dg is financed by an endogenous increase in base money dM* so that $dg=dM^{*}$. The analysis of the preceding chapter terminated at this stage. But in chapter III we must recognize the effect of a change in the income level on the government budget restraint. If there is any fluctuation in the income level through money financed government expenditures, the government will encounter the different value of tax collections (i.e. an induced tax revenue). In order to satisfy the government budget

[34]The description of the government budget restraint here is the case of floating exchange rate system for simplification so that $dR=0$.

restraint equation, we shall assume that a change in an induced tax revenue is adjusted by an endogenous change in the marginal tax rate u.

For the case of bond financed government expenditures we have the following government budget restraint equation:

III-11(d) $dg=dB/i-(1-u)dB+udy+(y+B)du$ by setting $dM^{*}=0$.

This policy specification requires $dg=dB/i$. Therefore, the marginal tax rate must be adjusted not only by the fluctuation of the income level, but also by an increase in interest payments.

For open market purchases the government budget restraint equation will be as follows:

III-11(e) $0=dM^{*}+dB/i-(1-u)dB+udy+(y+B)du$ by setting $dg=0$.

The policy specification is $dM=-dB/i$. The marginal tax rate again will be adjusted by the fluctuation of the income level and net interest payments.

C. Floating Exchange Rate System with Explicit Recognition of the Government Budget Restraint

As before under the floating exchange rate system, we shall set $dR=0$ and $dr\neq 0$ in III-9 and III-10. The system of the matrix form will be expressed with respect to dy, di, dr and dU where $dU=(y+B)du$ as follows:

$$\text{III-16}\quad \begin{bmatrix} 1-(1-u)(E_1+X_1) & E_2B/i^2-E_2 & -X_2 & E_1+X_1 \\ -L_1 & L_2B/i^2-L_2 & 0 & 0 \\ (1-u)X_1+K_1 & K_2 & X_2 & -X_1 \\ u & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dy \\ di \\ dr \\ dU \end{bmatrix} =$$

$$\begin{bmatrix} (1-u)(E_1+X_1)dB+E_2dB/i+E_2dM^s+dg \\ L_2dB/i-(1-L_2)dM^s \\ -(1-u)X_1dB \\ (1-u)dB-dB/i-dM^s+dg \end{bmatrix}$$

The Jacobian determinant of the system is:

$$|J|=X_2\{(1-E_1+K_1)(L_2B/i^2-L_2)+L_1[K_2+(E_2B/i^2-E_2)]\}$$

$$=X_2\Delta_2 \quad >0$$

The Jacobian determinant Δ_2 is similar to Δ_1 obtained in chapter II. Only one different coefficient can be observed in Δ_2 from Δ_1. It is related to the marginal tax rate u. When the government budget restraint is explicitly recognized, u in the first term disappears from Δ_1 and as a result, it makes the lower magnitude of the term in Δ_2 (i.e. $1-(1-u)E_1>1-E_1$).

1. Money Financed Government Expenditures

Since the exogenous increase in the government expenditures is fully financed by an increase in the base money stock at the initial equilibrium point, the variable of the base money stock is equated to the variable of the government expenditures (i.e., $dM^s = dg$) on the right hand side of the matrix in III-16. The system of the matrix form is expressed as:

III-16(i)

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & 0 & 0 & a_{44} \end{bmatrix} \begin{bmatrix} dy \\ di \\ dr \\ dU \end{bmatrix} = \begin{bmatrix} (1+E_2)dM^s \\ -(1-L_2)dM^s \\ 0 \\ 0 \end{bmatrix}$$

where a_{11}.........a_{44} are the same as III-16.

Solving for endogenous variables, dy, di, dr and dU, we obtain the following multipliers:

III-16(a) $dy/dg = i/\Delta_2\{(1+E_2)(L_2B/i^2 - L_1) + (1-L_2)[K_2 + (E_2B/i^2 - E_1)]\} \quad > 0$

III-16(b) $di/dg = 1/\Delta_2[(1+E_2)L_1 - (1-L_2)(1-E_1+K_1)]$

III-16(c) dr/dg

$= -1/X_2\Delta_2$ [([illegible] $K_2L_1 + (X_1+K_1)(L_2B/i^2 - L_2)]$

$$-(1-L_2)\{[1-(E_1+X_1)]K_2-(X_1+K_1)(E_2B/i^2-E_2)\}]$$

III-16(d) dU/dg

$$=-1/\Delta_2[u\{(1+E_2)(L_2B/i^2-L_2)+(1-L_2)[K_2+(E_2B/i^2-E_2)]\}] \quad <0$$

The sign of the income multiplier is positive in III-16(a). We find that the numerator in III-16(a) is identical to that in II-17(a). This implies that the magnitude of the income multiplier with explicit recognition of government budget restraint is larger than that with implicit case in chapter II as $\Delta_2<\Delta_1$. This result is enhanced by observing the endogenous policy variable multiplier in III-16(d). The sign of III-16(d) is unambiguously negative. This implies that the marginal tax rate goes down to offset the government budget surplus which is due to an increase in the induced tax revenue.

The sign of the interest rate multiplier is uncertain, as is the usual case of money financed government expenditures, since the fiscal and the monetary effects work in opposite directions. The determining factors of the sign are the magnitude of shift and the slope of the IS and the LM curves. The only difference in III-16(b) compared to II-17(b) can be found in the second term. With explicit recognition of the government budget restraint the magnitude of the second term in III-16(b) becomes less than that in II-17(b) since $1-E_1+K_1<1-(1-u)E_1+K_1$. This implies that if the sign of the interest multiplier is positive, the magnitude of the multiplier in III-16(b) is higher than that in II-17(b).

Regarding the exchange rate multiplier in III-16(c), we again have a similar multiplier to that obtained in II-17(c). We should note that if we replace (X_1+K_1) with $[(1-u)X_1+K_1]$ in III-16(c), then the multiplier will become identical to II-17(c). We concluded with II-17(c) that the necessary and sufficient condition for the exchange rate depreciation is that the BP curve must be positively sloping and lie above the LM curve. This conclusion is not essentially altered in III-16(c).

2. Bond Financed Government Expenditures

Similar to money financed government expenditures, a change in the government expenditures is equated to a change in the value of government bonds (i.e. $dg=dB/i$) on the right hand side of the matrix in III-16. The system of the matrix form is expressed as:

$$\text{III-16(ii)} \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & 0 & 0 & a_{44} \end{bmatrix} \begin{bmatrix} dy \\ di \\ dr \\ dU \end{bmatrix} = \begin{bmatrix} [(1-u)(E_1+X_1)i + E_2+1]dB/i \\ L_2 dB/i \\ -(1-u)X_1 i dB/i \\ (1-u)i dB/i \end{bmatrix}$$

where a_{11}.........a_{44} are the same as III-16.

Solving for endogenous variables dy, di, dr and dU, we obtain the following multipliers:

III-16(e) $dy/(dB/i) = 1/\Delta_2\{(L_2B/i^2 - L_2) - E_2L_2 - L_2(K_2 - E_2)\}$ >0

III-16(f) $di/(dB/i) = 1/\Delta_2[L_2(1 - E_1K_1) + (1 + E_2)L_1]$ >0

III-16(g) $dr/(dB/i)$

$$= -1/X_2\Delta_2$$

$$[(1+E_2)[(K_2 - X_1)L_1 + (X_1 + K_1)(L_2B/i^2 - L_2)$$

$$-L_2\{(X_1 + K_1)(E_2B/i^2 - E_2) - [1 - (E_1 + X_1)]K_2\}]$$

III-16(h) $dU/(dB/i)$

$$= 1/\Delta_2\ [\ -u\{(1+E_2)(L_2B/i^2 - L_2) - L_2[K_2 + (E_2B/i^2 - E_2)]\}$$

$$+(1-u)i\{(1 - E_1 + K_1)(L_2B/i^2 - L_2) + L_1[K_2 + (E_2B/i^2 - E_2)]\}]$$

The income multiplier in III-16(e) consists of three different elements. The first two terms display the rightward shift of the IS curve including the effect of the increase in government expenditures and the wealth effect in the commodity market. The third term is the wealth effect in the money market. As was discussed in chapter II, the wealth effect in the money market is the factor which may cause the system to become contractionary under bond financed government expenditures. If $|E_2L_2| > |L_2(K_2 - E_2)|$, then the bond finance will have a net impact on the income level so that the system can be expansionary, but if it is the opposite case, the bond finance will become contractionary.

In contrast to the income multiplier in II-17(d) obtained in chapter II, the income multiplier in III-16(e) consists of somewhat different elements. First, we find that the magnitude of the IS curve shift (i.e. $(L_2B/i^2-L_1)-E_1L_2$) in III-16(e) is smaller than that obtained in II-17(d) (i.e. $[(1-u)E_1i+1](L_2B/i^2-L_1)-E_1L_2$). The term, $(1-u)E_1i$, which represents an increase in the private expenditures due to an increase in interest payments in II-17(d), does not appear in III-16(c). Second, the magnitude of Δ_3 is less than that of Δ_1. As a result, it is not possible to conclude which magnitude of multiplier is larger.

In order to clarify these two differences, we now turn to the endogenous policy variable multiplier in III-16(h). The multiplier in III-16(h) consists of two terms. The first term indicates the effect of a change in the marginal tax rate through the income level, and the second term the effect of interest payments to the private sector. The sign of the first term is negative while the sign of the second term is positive. Hence, we cannot conclude whether the marginal tax rate goes up or down.

In referring to the income multiplier in III-16(e), we can illustrate the effects of these two terms in III-16(h) in the following manner. In chapter II the interest payments to the private sector were displayed simply by increasing the private expenditures. But here the government must finance the increased interest payments in the form of raising the tax revenue. Hence, the private expenditures cannot increase as much as in II-17(d). The increased interest payments must be totally financed by the increased tax revenue and/or increased marginal tax rate. This positive induced tax revenue can be observed in the fact that the denominator Δ_3 has a smaller value than Δ_1. As a result, there is a trade-off between the effect of the increased income level and the effect of interest payments. If the positive induced tax revenue exceeds the increased interest payments; the marginal tax rate must go down, and otherwise it must be raised.

Thus, when the government budget equilibrium is required, the increased intererst payments do not affect the private expenditures.

Now we assume that government authorities take into account of the exact value of interest payments to the private sector. Then the government issues new bonds whose value is to finance not only the increased government expenditures, but also the interest payments of the increased government bonds. Under this policy specification the income and the endogenous policy variable multipliers are as follows: [33]

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[33] From the specification above we have the second government budget restraint as dg+(1-u)dB=dB/i. (see I-4 in chapter I) Then the right hand side of the fourth row in III-16(ii) completely disappears. Since dg+(1-u)dB is the actual increase in government expenditures, we equate dg+(1-u)dB to dg on the right hand side of the first row in III-16(ii). Then the right hand side of III-16 becomes as follows:

[2]

III-16(ee)

$$dy/dB/i = 1/\Delta_2\{[1+(1-u)E_1 i](L_2 B/i^2 - L_2) - E_2 L_2 - L_2(K_2 - E_2)\}$$

III-16(hh) $$dU/dB/i = 1/\Delta_2[-u\{[(1-u)E_1 i + E_2 + 1](L_2 B/i^2 - L_2) - L_2[K_2 + (E_2 B/i^2 - E_2)]\}$$

Now we find the effect of the interest payments in III-16(ee). Hence, the magnitude of shift on the IS curve is higher than the case in III-16(e).

The endogenous policy variable multiplier in III-16(hh) consists only of the effect of the induced tax revenue. However, there is no effect of the interest payments on the determination of the endogenous policy variable. Thus, we conclude that in order to have higher private expenditures caused by the increased interest payments, the government should issue that amount of new bonds which finance not only the increased government expenditures, but also would be increased interest payments.

As before, the interest rate multiplier in III-16(f) shows the definite positive sign. The sign of the exchange rate multiplier is uncertain in III-16(g). We must note that there are not the coefficients of tax rate and interest

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[2] (cont'd) $$\begin{bmatrix} [(1-u)(E_1+X_1)i+E_2+1]dB/i \\ L_3 dB/i \\ -(1-u)X_1 i dB/i \\ 0 \end{bmatrix}$$

payments in these multipliers due to recognition of endogenous policy variable.

3. **Open Market Purchases**

The system of the matrix form is expressed as:

$$\text{III-16(iii)}\quad \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & 0 & 0 & a_{44} \end{bmatrix} \begin{bmatrix} dy \\ di \\ dr \\ dU \end{bmatrix} = \begin{bmatrix} -(1-u)(E_1+X_1)idM^{\circ} \\ -dM^{\circ} \\ (1-u)X_1idM^{\circ} \\ -(1-u)idM^{\circ} \end{bmatrix}$$

where a_{11}..........a_{44} are the same as III-16. Solving for endogenous variables dy, di, dr and dU, we obtain the following multipliers:

$$\text{III-16(j)}\quad dy/dM^{\circ}=1/\Delta_3[K_2+(E_3B/i^2-E_2)] \quad >0$$

$$\text{III-16(k)}\quad di/dM^{\circ}=-1/\Delta_3(1-E_1+K_1) \quad <0$$

$$\text{III-16(l)}\quad dr/dM^{\circ}=1/X_2\Delta_3\{[1-(E_1+X_1)]K_2-(X_1+K_1)(E_3B/i^2-E_2)\}>0$$

$$\text{III-16(m)}\quad dU/dM^{\circ}=1/\Delta_3[\ -u[K_2+(E_3B/i^2-E_2)]$$
$$-(1-u)\{(1-E_1+K_1)(L_3B/i^2-L_2)+L_1[K_2+(E_3B/i^2-E_2)]\}] \quad <0$$

Let us first observe the endogenous policy variable multiplier in III-16(m). The endogenous policy variable multiplier consists of two elements. The first term in III-16(m) displays the induced tax revenue. Since the income level clearly increases, as will be explained shortly, the marginal tax rate goes down. The second term displays the effect of interest payments to the private sector on the endogenous policy variable. Now that government bonds are purchased from the private sector, the government does not have to pay as much in interest payments as they would have had to before. his causes a government budget surplus so that the marginal tax rate goes down. As a result, the endogenous policy variable goes down unambiguously in open market purchases.

The sign of the income level multiplier is unambiguously positive in III-16(j). In contrast to the income multiplier in II-17(g) we must note that there is no effect of the decreased interest payments in the private sector when the government budget restraint is explicitly recognized, as under the previous two policies. The reason the private expenditures do not decrease despite the fall of the interest payments to the private sector in III-16(j) can be illustrated as before. Since government bonds are purchased from the private sector by increasing base money stock in open market purchases, the government will have budget surplus in terms of a decrease in interest payments to the private sector. But by recognizing the endogenous policy variable, the amount of the surplus in government budget goes back to the private sector in the form of the tax rate reduction. As a result, there will be no effect of the decrease in interest payments on private expenditures. This means that there is no shift in the IS curve, but only the LM curve shifts rightward in open market purchases.

The sign of the interest rate is negative, as is expected.

The exchange rate multiplier, because of some cancellations of the terms, consists of only two terms. These two terms show an unambiguous positive sign in III-16(1). Similar to the income multiplier, the interest payment coefficients disappear from the exchange rate multiplier in III-16(1). This is because the fall of the marginal tax rate due to the surplus for interest payments in the governemnt sector keeps the private expenditures constant.

D. Pegged Exchange Rate System with Explicit Recognition of the Government Budget Restraint

Under the pegged exchange rate system the variable of the balance of payments is incorporated in the government budget equation, while dr is kept constant. Thus, the matrix form of the system is a four-by-four in size with respect to the income level, the interest rate, the balance of payments and the endogenous policy variable (i.e. the marginal tax rate). Totally differentiating the equations of III-8 to III-11 we have the following matrix form:

$$\text{III-17}\quad \begin{bmatrix} 1-(1-u)(E_1+X_1) & E_2B/i^2-E_2 & 0 & E_1+X_1 \\ L_1 & -(L_2B/i^2-L_2) & 0 & 0 \\ (1-u)X_1+K_1 & K_2 & -J & -X_1 \\ u & 0 & -s & 1 \end{bmatrix} \begin{bmatrix} dy \\ di \\ dR \\ dU \end{bmatrix} =$$

$$\begin{bmatrix} (1-u)(E_1+X_1)dB+E_2dB/i+E_2dM^*+dg+X_2dr \\ -L_2dB/i+L_2B/i^2-L_2dM^* \\ -(1-u)X_1dB-X_2dr \\ (1-u)dB-dB/i-dM^*+dg \end{bmatrix}$$

where $dU=(y+B)du$

As before, two endogenous policy variables will be chosen to equilibrate the government budget restraint for each policy. One variable is the marginal tax rate which is adjusted according to changes in the endogenous variable dy and the interest payments dB in the government budget restraint. However, we must note that under the pegged exchange rate system there exists another endogenous variable in the government budget restraint. That is the balance of payments dR. The fluctuation of the balance of payments is assumed to be adjusted also by the marginal tax rate.[34] Observing the variable of the balance of payments on the left hand side of the government budget restraint, we must note that the balance of payment surplus raises the marginal tax rate. On the other hand, the balance of payment deficit lowers the marginal tax rate to equilibrate the government budget restraint.

Another endogenous policy variable is determined by each policy specification. Since we assume that the balance of payments does not affect the quantity of money supply, dM^* is equated to dg for money financed government expenditures and to $-dB/i$ for open market purchases.

$$|J|=\{1-(E_1+X_1)+s[X_1+(E_1+X_1)K_1]\}(L_2B/i^2-L_3)$$
$$+L_1(1+sX_1)(E_2B/i^2-E_3)+sL_1K_2(E_1+X_1)$$
$$=\Delta^*_4$$

The Jacobian determinant Δ_2 in chapter II showed the clear positive sign with the assumption $(1-u)X_1+K_1<0$. But the Jacobian determinant Δ_4 does not guarantee a positive sign. To assure the positive Jacobian determinant Δ_4, some

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[34] This assumption implies that the total money supply M^* is not affected by the fluctuation of the balance of payments, but M^* is kept constant. If the balance of payments was assumed to change into a part of the money supply, the results would be identical to what we obtained in chapter II.

assumptions must be made. Observing the second term in Δ_4, we are not certain if $(1+sX_1)>0$ as $X_1<0$. However, we know that a sterilization coefficient s can be determined by the government exogenously, and s was simply assumed to be kept constant in chapter II. Thus, if the government determines this coefficient in such a way that the negative value of sX_1 does not exceed 1, then $(1+sX_1)$ can be assured to be positive. Thus, the government faces a narrower range in choosing s rather than $0\leq s\leq 1$. If $(1+sX_1)>0$, then $1-(E_1+X_1)+s(E_1+X_1)K_1>sX_1$ is also assured. Therefore, Δ_4 is positive.

1. Money Financed Government Expenditures

The system of the matrix form will be expressed as:

$$\text{III-17(i)}\quad \begin{bmatrix} a_{11} & a_{12} & 0 & a_{14} \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & 0 & -s & a_{44} \end{bmatrix} \begin{bmatrix} dy \\ di \\ dR \\ dU \end{bmatrix} = \begin{bmatrix} (1+E_2)dM^{\circ} \\ (1-L_3)dM^{\circ} \\ 0 \\ 0 \end{bmatrix}$$

where $a_{11}.........a_{44}$ are the same as III-16.

Solving for endognenous variables, dy, di, dR and dU we obtain the following multipliers:

III-17(a) $dy/dM^{\circ}=1/\Delta_4\{(1+E_2)(1+sX_1)(L_2B/i^2-L_2)$
$+(1-L_3)[(1+sX_1)(E_2B/i^2-E_2)+s(E_1+X_1)K_2]\}\quad >0$

III-17(b) $di/dM^{\circ}=1/\Delta_4[(1+E_2)(1+sX_1)L_1$
$-(1-L_3)\{1-(E_1+X_1)+s[X_1+K_1(E_1+X_1)]\}]$

III-17(c) $dR/dM^{\circ}=1/X_2\Delta_4[(1+E_2)[L_1K_2+(X_1+K_1)(L_2B/i^2-L_2)]$
$-(1-L_3)\{[1-(E_1+X_1)]K_2-(X_1+K_1)(E_2B/i^2-E_2)\}]$

III-17(d)

$dU/dM^{\circ}=1/\Delta_4[-u[(1+E_2)(L_2B/i^2-L_2)+(1-L_3)(E_2B/i^2-E_2)]$
$+s[(1+E_2)\{L_1K_2+[(1-u)X_1+K_1](L_2B/i^2-L_2)\}$
$+(1-L_3)\{[(1-u)X_1+K_1](E_2B/i^2-E_2)-[1-(1-u)(E_1+X_1)]K_2\}]]$

Let us observe the income multiplier in III-17(a). In contrast to the income multiplier obtained in II-21(a) we find in it not only the coefficient of capital flow K_2, but also the coefficients of net exports X_1. To assure a positive Jacobian determinant we assumed that the government determines the value of s such that $1+sX_1>0$ is satisfied. Holding this assumption hereafter, we have the positive income multiplier. These effects of sX_1 and sK_2 on the income multiplier can be clarified by observing the endogenous polcy variable multiplier in III-17(d). The second term in III-17(d) displays an effect of the balance of payments on the endogenous policy variable. The only coefficient which indicates the balance of payment surplus, namely the inflow of capital L_1K_2, raises the marginal tax rate. The rest of the coefficients which indicate the balance of payments deficit lower the marginal tax rate.

The analysis of the first term in III-17(d) is the same as the case in floating exchange rate system. An increase in the income level induces a tax revenue so that the marginal tax rate is lowered.

In referring to the income multiplier again, we can now elucidate the meaning of the coefficients sX_1's. By the effect of the balance of payment deficit the government lowers the marginal tax rate. The lowered tax rate raises the private expenditures. Then these increased private expenditures increase imports. As a result, the increased imports affect the income level so that it goes down. Thus, the amount of the increased income level due to the fall of the marginal tax rate is replaced by the increased imports sX_1 in III-17(a). Obviously, these effects are of simultaneous nature.

The effect of the capital flow is a similar case. The capital outflow due to a fall of the interest rate lowers the marginal tax rate. But the amount of the tax reduction is totally replaced by a capital inflow which increases the income level (i.e. $s(E_1+X_1)K_2>0$) in III-17(a).

The sign of the interest rate multiplier in III-17(b) is uncertain as in the usual case of money financed government expenditures.

The analysis of the balance of payment multiplier is analogous to that in chapter II. If a balance of payment deficit results in III-17(c), the sign of the endogenous policy variable multiplier will be clearly negative. If a balance of payment surplus results, it will be uncertain.

2. Bond Financed Government Expenditures

The system of the matrix form will be expressed as:

$$\text{III-17(ii)}\quad \begin{bmatrix} a_{11} & a_{12} & 0 & a_{14} \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & 0 & -s & a_{44} \end{bmatrix} \begin{bmatrix} dy \\ di \\ dR \\ dU \end{bmatrix} = \begin{bmatrix} [(1-u)(E_1+X_1)i+E_3+1]dB/i \\ -L_3dB/i \\ -(1-u)X_1idB/i \\ (1-u)idB/i \end{bmatrix}$$

where $a_{11}.........a_{44}$ are the same as III-16.

Solving for the endogenous variables dy, di, dR and dU, we obtain the following multipliers:

$$\text{III-17(e)}\quad dy/(dB/i)=1/\Delta_4\{(1+sX_1)[(L_3B/i^2-L_2)-E_3L_2+L_3E_2 -sL_3K_2(E_1+X_1)\}$$

$$\text{III-17(f)}\quad di/(dB/i)=1/\Delta_4[\{[1-(E_1+X_1)]+s[X_1+K_1(E_1+X_1)]\}L_3 +(1+E_3)(1+sX_1)L_1]$$

$$\text{III-17(g)}\quad dR/(dB/i)=1/\Delta_4[(1+E_3)[(E_1+X_1)(L_3B/i^2-L_2)+K_2L_1] +\{[1-(E_1+X_1)]K_2-(E_1+X_1)(E_3B/i^2-E_2)\}L_3]$$

III-17(h)

$$du/(dB/i)=1/\Delta_4[-u[(1+E_3)(L_3B/i^2-L_2)-L_3(E_3B/i^2-E_2)] +s[(1+E_3)\{[(1-u)X_1+K_1](L_3B/i^2-L_2)+K_2L_1\} +L_3\{[1-(1-u)(E_1+X_1)]K_2-[(1-u)X_1+K_1](E_3B/i^2-E_2)\}]$$

$$+(1-u)i[\{[1-(\underline{E}_1+\underline{X}_1)]+s[\underline{X}_1(\underline{E}_1+\underline{X}_1)K_1]\}(L_2B/i^2-\underline{L}_2)$$
$$+[(1+s\underline{X}_1)(\underline{E}_2B/i^2-\underline{E}_2)+s(\underline{E}_1+\underline{X}_1)K_2]L_1]]$$

Let us first observe the endogenous policy variable multiplier in III-17(h). The analysis of the first and the third term is similar to that which we previously had in bond financed government expenditures under the floating exchange rate system. If the income level goes up, there will be a positive value of the induced tax revenue which lowers the marginal tax rate. This is displayed in the first term. The third term is the effect of interest payments to the private sector. An explanation of this was that, since the government must raise the marginal tax rate to finance the increased interest payments, these interest payments do not affect the private expenditures.

In the balance of payment multiplier in III-17(g), the two coefficients of the capital flow lead the balance of payments to a surplus due to the rise of the interst rate. However, there is a conflict in the trade balance due to the income rise from the increased government expenditues and the negative wealth effect on the income level. Therefore, although the possibility of the balance of payment surplus is higher than in the case of money financed government expenditures, the sign of the balance of payment multiplier is still uncertain.

The effects of the balance of payments on the endogenous policy variable multiplier can be seen in the second term in III-17(h). The capital inflow raises the marginal tax rate whereas the fall of net exports due to the increased income level lowers the marginal tax rate. On the other hand, the rise of net exports due to the crowding-out effect in the money market raises the marginal tax rate. The movement of the marginal tax rate due to these effects of the balance of payments can be seen in the income level in III-17(e). The rise of the marginal tax rate due to the inflow of capital decreases private expenditures so that the income level is crowded out (i.e. $-sL_2K_2(E_1+X_1)<0$). The fall of the marginal tax rate due to the current account deficit increases private expenditures. But these increased private expenditures are replaced by an increase in imports. As a result, the income level is crowded out as well (i.e. $sX_1[(L_2B/i^2-L_1)-E_2L_1]<0$). On the other hand, the rise of the marginal tax rate due to the crowding-out effect in the money market increases net exports so that the income level goes up (i.e. $sX_1L_2E_2>0$ where $E_2<0$).

Thus, due to the effects of the endogenous policy variable, the sign of the income multiplier cannot be positive unless the crowding-out effect of capital flow in the second term is assumed not to exceed the positive first term.

The sign of the interest rate multiplier is positive as $(1+sX_1)>0$ by assumption for first and second terms in III-17(f).

The sign of the balance of payment multiplier is uncertain. We must note again that due to the recognition of the endogenous policy variable multiplier, the balance of payment multiplier is simple without the coefficients of the marginal tax rate and interest payments. If the balance of payment surplus results in bond financed government expenditures, the government will have a double burden to finance the deficits. If an increase in the induced tax revenue exceeds the magnitude of the balance of payment surplus and the increased interest payments, the marginal tax rate will go down. Otherwise, it will go up.

3. Open Market Purchases

The system of the matrix form is expressed as:

$$\text{III-17(iii)} \quad \begin{bmatrix} a_{11} & a_{12} & 0 & a_{14} \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & 0 & -s & a_{44} \end{bmatrix} \begin{bmatrix} dy \\ di \\ dR \\ dU \end{bmatrix} = \begin{bmatrix} -(1-u)(E_1+X_1)idM^* \\ dM^* \\ (1-u)X_1 idM^* \\ -(1-u)idM^* \end{bmatrix}$$

where a_{11}..........a_{44} are the same as III-16.

Solving for the endogenous variables dy, di, dR and dU, we obtain the following multipliers:

$$\text{III-17(j)} \quad dy/dM' = 1/\Delta_4[(1+sX_1)(E_2B/i^2-E_2)+sK_2(E_1+X_1)] \quad >0$$

$$\text{III-17(k)} \quad di/dM' = -1/\Delta_4\{1-(E_1+X_1)+s[X_1+(E_1+X_1)K_1]\} \quad <0$$

$$\text{III-17(l)} \quad dR/dM' = -1/X_2\Delta_4\{[1-(E_1+X_1)]K_2-(X_1+K_1)(E_2B/i^2-E_2)\} \quad <0$$

$$\begin{aligned}\text{III-17(m)} \quad dU/dM' = 1/\Delta_4[&-u(E_2B/i^2-E_2)\\ &-s[[1-(1-u)E_1+X_1]K_2-[(1-u)X_1+K_1(E_2B/i^2-E_2)]\\ &-(1-u)i[\{1+s[X_1+(E_1+X_1)K_1]\}(L_2B/i^2-L_2)\\ &+[(1+sX_1)(E_2B/i^2-E_2)+(E_1+X_1)K_2]L_1] \quad <0\end{aligned}$$

By now we are familiar with the coefficients of multipliers. First, let us observe the balance of payment multiplier in III-17(l). The sign of the multiplier is negative. As we learned in the effects of open market purchases in floating exchange rate system, there is only the rightward shift of the LM curve in the case of explicitly recognized government budget restraint. Thus, a fall of the interest rate and an increase in the income level cause the balance of payment deficit.

The effect of the balance of payments on the endogenous policy variable multiplier can be found in the second term in III-17(m) which shows the unambiguous negative sign. The third term in III-17(m) is the effect of interest payments on the endogenous policy variable. The sign of this term is also negative. This is plausible, because open market purchases is a bond contraction in return for a money expansion so that interest payments by the government are decreased. The negative first term in III-17(m) is the effect of the induced tax revenue on the endogenous policy variable. Thus in open market purchases all the terms in the endogenous policy variable multiplier work in the negative direction and hence the marginal tax rate will unambiguously go down.

Then let us observe the income multiplier in III-17(j). As was the case under the floating exchange rate system, there is no reduction of the income level for a portion of interest payments as well, despite the fact that the private sector receives less interest payments due to the bond contraction. This is because the government lowers the marginal tax rate to match the decreased amount of interest payments so that the private sector keeps its expenditures constant. The balance of payment deficit, which causes the marginal tax rate to go down, affects on the income level in two different ways. One is the fall of the marginal tax rate due to the outflow of capital. The lowered marginal tax rate causes an increase in the private expenditures and hence the income level goes up. The other is also the fall of the marginal tax rate due to the decreased net exports. But this lowered marginal tax rate consequently decreases net exports and hence the income level is crowded out.

The interest rate goes down in III-17(k). There is some positive force due to the crowding-out effect on the income level. But as we assumed $(1+sX_1)>0$, the sign of the interest rate is negative.

4. Exchange Rate Change Policy

Assuming that a sterilization coefficient is constant and non-zero, and that the Marshall-Lerner condition is satisfied (i.e. $X_2>0$), the effects of a devaluation in the exchange rate (i.e. $dr>0$) will be observed on the four endogenous variables in the system. The system of the matrix form will be expressed as:

$$\text{III-17(iv)} \quad \begin{bmatrix} a_{11} & a_{12} & 0 & a_{14} \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & 0 & -s & a_{44} \end{bmatrix} \begin{bmatrix} dy \\ di \\ dR \\ dU \end{bmatrix} = \begin{bmatrix} X_2 dr \\ 0 \\ -X_2 dr \\ 0 \end{bmatrix}$$

where a_{11}..........a_{44} are the same as III-17.

Solving for the endogenous variables dy, di, dR and dU, we obtain the following multipliers:

III-17(n) $dy/dr=1/\Delta_4 \; X_2(1-sE_1)(L_1B/i^2-L_2) \quad >0$

III-17(o) $di/dr=1/\Delta_4 \; X_2(1-sE_1)L_1 \quad >0$

III-17(p) $dR/dr=1/\Delta_4\ X_2\{[(1-E_1)+K_1](L_2B/i^2-L_2)+[K_2+(E_2B/i^2-E_2)]L_1\}\ >0$

III-17(q) $dU/dr=1/\Delta_4[-u(L_2B/i^2-L_2)+s\{[1-(1-u)E_1+K_1](L_2B/i^2-L_2)+[K_2+(E_2B/i^2-E_2)]L_1\}$

The income level and the interest rate increase unambiguously because of the exchange rate devaluation. The balance of payments improves. The increased income level causes a positive induced tax revenue which lowers the marginal tax rate. The balance of payment surplus causes the marginal tax rate to go up. Thus, the effect of the higher marginal tax rate due to the balance of payment surplus is displayed as a crowding-out effect on private expenditures in the income multiplier in III-17(m) and in the interest rate multiplier in III-17(o) (i.e. $-sE_1<0$).

E. Summary

Table III-1 Government Policies:

Effects on the Endogenous Policy Variable

	FERS			PERS			
Policy / Endog. Var.	MFGE	BFGE	OMP	MFGE	BFGE	OMP	ERC
Marginal Tax Rate	-	±	-	±	±	-	±

In this chapter the induced tax revenue is explicitly incorporated in the government budget restraint. This implies that another endogenous policy variable other than one in policy specification must be recognized to satisfy the budget equilibrium. The marginal tax rate serves this role. With this concept of the explicitly recognized government budget restraint, we find some different results from those obtained in chapter II; in particular, with respect to interest payments to the private sector.

Nevertheless, the signs of the three endogenous variables (the income level, the interest rate and the exchange rate or the balance of payments) do not show a significant difference with those in chapter II except in magnitudes. It is obvious that the magnitudes of such are an empirical question.

Therefore, we must focus the effects of a change in the marginal tax rate on the multipliers. Under the floating exchange rate system and in money financed government expenditures, only the induced tax revenue affects the marginal tax rate. Since the income level goes up clearly, the marginal tax rate goes down. As a result, the magnitude of the income level multiplier in this concept is larger than that in chapter II.

However, in bond financed government expenditures, the increased interest payments affect the determination of the marginal tax rate in addition to the induced tax revenue. Under the policy specification (i.e. dg=dB/i), which implies that the increased government expenditures are financed by the increased government bonds, the government encounters the budget deficit, because an increased interest payments must be taken account for the newly issued bonds. In chapter II we observed the effect of interest payments on the income level. As Blinder-Solow(1973) concluded, the increased interest payments to the private sector induces even higher private expenditures than the case of money financed government expenditures. If we recognize the government budget restraint explicitly as in this chapter, however, the coefficient of the multiplier expansion by the increased interest payments disappears from the income multiplier. It is obvious that if the government budget restraint is to be satisfied, the amount of interest payments must be financed by revenue. Then, the marginal tax rate must be raised to offset the budget deficit. As a result, the increased interest payments cannot raise the private expenditures. Thus, with the concept of fully recognized government budget restraint, the conclusion of Blinder-Solow(1973) cannot be supported. However, as Christ(1979) introduced, if we assume that government bonds are issued in order to finance not only the increased government expenditures, but also concomitant interest payments (i.e. $dg+(1-u)dB=dB/i$), the effect of interest payments can be seen on the income level as in chapter II. Therefore, *given the condition above*, we may conclude that bond finance can be more expansionary than money finance.

The multipliers obtained in open market purchases can be explained similarly to the case of bond financed government expenditures. In chapter II the decreased interest payments due to bond contraction affected the private expenditures. Hence, the income level went down. But with the concept of the explicit government budget restraint, the government lowers the marginal tax rate due to lower interest payments. This implies that the private expenditures are not affected by the decreased interest payments. As a result, the government receives the induced tax revenue and spends less interest payments, the marginal tax rate goes down unambiguously in open market purchases.

Under the pegged exchange rate system, the balance of payment variable is enhanced to be adjusted by the marginal tax rate in addition to the induced tax revenue and interest payments. As a result, the uncertain signs of the balance of payment multipliers in money financed government expenditures and bond financed government expenditures make the endogenous policy variable multiplier also uncertain. In open market purchases, however, the deterioration of the balance of payments lowers the marginal tax rate. The positive induced tax revenue, the lower interest payments and the balance of payment deficit, all of which work in the same direction on the endogenous policy variable lower the marginal tax rate.

With regard to the effects of the devaluation of the exchange rate, the balance of payments improves clearly. The income level goes up, because net exports go up. But the balance of payment surplus and the increment of the income level are in conflict in determining the marginal tax rate. Hence, the sign of the endogenous policy variable is uncertain.

IV. EXPLICIT RECOGNITION OF THE GOVERNMENT BUDGET RESTRAINT AND A FLEXIBLE PRICE LEVEL IN AN OPEN ECONOMY

A. Introduction

The purpose of this chapter is to observe the effects of policies on the price level and three other endogenous variables with explicit recognition of the government budget restraint under the floating exchange rate system and the pegged exchange rate system. Up to now we assumed the price level to be rigid (i.e. $dp=0$). In this chapter, however, we assume a condition of full-employment of labour so that there is no change in the income level, but instead the price level is assumed to be flexible (i.e. $dy=0$ and $dp\neq0$). This is the traditional aspect of the classical case. [37]

B. The Model

We shall have the following reduced forms of the system from the original model.

IV-1 $$y=E[y+B/p-u(y+B/p)-v,\ i,\ M^{\bullet}/p+B/ip]$$
$$+X[y+B/p-u(y+B/p)-v,\ r,\ p]+g$$

IV-2 $$M^{\bullet}/p=L(y,\ i,\ M^{\bullet}/p+B/ip)$$

IV-3 $$\dot{R}/p=X[y+B/p-u(y+B/p)-v,\ r,\ p]+K(y,\ i)$$

IV-4 $$g+B/p+\dot{R}/p=\dot{M}^{\bullet}/p+\dot{B}/p+u(y+B/p)+v$$

The sign of partial derivatives for each variable was

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[37] The flexible price level with recognition of a government budget restraint can be referred to Christ(1969,1978) and Brunner-Meltzer(1972,1976).

assumed as follows:

$E_1>0$, $E_2<0$, $0<E_3<1$, $X_1<0$, $X_2>0$, $\partial X/\partial p=X_3<0$, [15] $K_2>0$, $L_2<0$, $0<L_3<1$ and $0<E_1+X_1<1$.

Since the government budget restraint is explicitly recognized in the system, the system of the matrix form will be a four-by-four in size from IV-1 to IV-4. Similar to the preceding chapter, a marginal tax rate will be chosen as an endogenous policy variable and analyzed accordingly. [16] The government policies will be the same as those in the preceding chapters; that is, money financed government expenditures, bond financed government expenditures and open market purchases for the floating exchange rate system, and exchange rate change policy in addition to the three policies above for the pegged exchange rate system. As in chapter III, the autonomous tax revenue v will be assumed constant (i.e. dv=0).

Now let us observe the slope of each curve from IV-1 to IV-3 which can be derived by totally diferentiating each equation, holding the rest of variables constant but dp and di, and then solving for $di/(dp/p^2)$. Each slope of the curves is as follows:

is curve:

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[15] The sign of net exports with respect to a rise of the price level is negative (i.e. $X_3<0$). This is derived from a plausible assumption that the Marshall-Lerner condition is satisfied and a rise of the domestic price level increases imports while it decreases exports.

[16] As in chapter III, an endogneous policy variable will be set $dU=(y+B/p)du$ so that the total tax revenue is the actual endogenous policy variable.

IV-5 $\quad di/dp=-[(1-u)B(E_1+X_1)+WE_2-X_2]/p^2(E_2B/i^2p-E_3) \quad <0$

where $W=M^*+B/i$ (i.e. nominal wealth effect)

lm curve:

IV-6 $\quad di/dp=(M^*-WL_2)/p^2(L_2B/i^2p-L_3) \quad >0$

where (M^*-WL_2) is assumed positive.

bp curve:

IV-7 $\quad di/dp=-[X_2-\dot{R}-(1-u)BX_1]/p^2K_2 \quad >0$

where $|X_2-\dot{R}|$ is assumed larger than $|(1-u)BX_1|$.

Let us observe the government budget restraint more in detail, given the assumptions of the fixed income level and the flexible price level. Totally diferentiating IV-4, and setting dy=0, dR=0 and dv=0, we have the following:

IV-4(a) $\quad dg+[\dot{W}-\dot{R}-(1-u)B]dp/p^2+(\dot{B}/i^2p)di+dR/p$

$=dM^*/p+dB/ip-(1-u)dB/p+(y+B)du$

where $\dot{W}=\dot{M}^*+\dot{B}/i$

In chapter III, $\dot{B}/i^2$ was set at zero due to the condition of the initial steady state equilibrium. Similarly $\dot{W}$ and $\dot{R}$ can be set $\dot{W}=\dot{R}=0$. Thus we have the following government budget restraint:

IV-4(b) $\quad dg-(1-u)Bdp/p^2+dR/p=dM^*/p+dB/ip-(1-u)dB/p+(y+B)du$

We must note that in IV-4(b) a change in the price level affects only the real value of interest payments in the government budget restraint. If the price level goes up, the real value of interest payments will go down. Then the government will gain a budget surplus so that the marginal tax rate is lowered to offset the surplus. This implies that an increase in the price level is, in fact, an indirect tax collection for the government sector.[44]

C. Floating Exchange Rate System with Explicit Recognition of the Government Budget Restraint and a Flexible Price Level

As is usual, the balance of payments is assumed to be equilibrated through the fluctuation of the exchange rate (i.e. $dR=0$ and $dr\neq0$). Given the assumptions, totally differentiating IV-1 to IV-4, and expressing them in the matrix form with respect to dp, di, dr and dU where $dU=(y+B)du$, we have the following:

$$\text{IV-8} \quad \begin{bmatrix} (1-u)B(E_1+X_1)+WE_3-X_3 & E_3B/i^2p-E_2 & -X_2 & E_1+X_1 \\ -(M^s-WL_3) & L_3B/i^2p-L_2 & 0 & 0 \\ X_3-R-(1-u)BX_1 & K_2 & X_2 & -X_1 \\ (1-u)B & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dp/p \\ di \\ dr \\ dU \end{bmatrix} =$$

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[44] Under the condition of the flexible price level if money supply and government bonds are assumed to grow at a steady state growth level every period, the government budget restraint equilibrium will be affected by these variables as well. This is because the real values of money supply and government bonds are different from an initial equilibrium to a next equilibrium due to a change in the price level and the interest rate.

$$\begin{bmatrix} (1-u)(E_1+X_1)dB/p+E_2 dB/ip+E_3 dM^{\circ}/p+dg \\ L_2 dB/ip-(1-L_2)dM^{\circ}/p \\ -(1-u)X_1 dB/p \\ (1-u)dB-dB/ip-dM^{\circ}/p+dg \end{bmatrix}$$

where $W=M^{\circ}+B/i$

The Jacobian determinant of the system is:

$$|J|=X_2\{WE_3(L_3B/i^2p-L_2)$$

$$+(M^{\circ}-WL_3)[K_2+(E_3B/i^2p-E_2)]\}$$

$$=X_2\Delta_3 \quad >0$$

where Δ_3 represents the inside brackets { }. For a small magnitude of L_3, $(M^{\circ}-WL_3)$ may be assumed positive; that is, demand elasticity of money is less than unity and then the Jacobian determinant Δ_3 is positive.

1. **Money Financed Government Expenditures**

The system of the matrix form is expressed as:

$$\text{IV-8(i)}\quad \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & 0 & 0 & b_{44} \end{bmatrix} \begin{bmatrix} dp/p \\ di \\ dr \\ dU \end{bmatrix} = \begin{bmatrix} (1+E_2)dM^e/p \\ -(1-L_2)dM^e/p \\ 0 \\ 0 \end{bmatrix}$$

where b_{11}.........b_{44} are the same as IV-8.

Solving for endognenous variables we obtain the following multipliers:

IV-8(a) dp/dM^e

$$=p/\Delta_2\ \{(1+E_2)(L_2B/i^2p-L_2)+(1-L_2)[K_2+(E_2B/i^2p-E_2)]\}\quad >0,$$

IV-8(b) $$di/dM^e=1/p\Delta_2\{(1+E_2)(M^e-WL_2)-(1-L_2)WE_2\}$$

IV-8(c) $$dr/dM^e=-1/pX_2\Delta_2\ [(1+E_2)(M^e-WL_2)K_2 +(1-L_2)[(X_2-\dot{R})(E_2B/i^2p-E_2)-(WE_2-X_2)K_2]$$

IV-8(d) $$dU/dM^e=1/p\Delta_2\ [-(1-u)B\{(1+E_2)(L_2B/i^2p-L_2) +(1-L_2)[K_2+(E_2B/i^2p-E_2)]\}]\quad <0$$

The sign of the price multiplier is positive in IV-8(a). The sign of the interest rate multiplier is, as is usual in money financed government expenditures, uncertain.[41]

The exchange rate is determined by two elements. One is a change in the price level and the other is a change in the interest rate. The first term in IV-8(c) displays the inflow of capital caused by the increased government expenditures. The sign of the first term is negative so that the exchange rate appreciates. The second term displays the effect of the increased base money. An increase in the price level decreases net exports, and a fall of the interest rate causes the outflow of capital, both of which cause the exchange rate to depreciate. Hence, the sign of the exchange rate multiplier is uncertain in IV-8(c).

The endogenous policy variable multiplier consists only of one term in IV-8(d). Going back to the government budget restraint equation in IV-4(b), we find that the endogenous policy variable in the case of money financed government expenditures is adjusted only by the effect of a change in the price level. Thus, the increment of the price level lowers the real value of the interest payments to the private sector, so that the government can afford to lower the marginal tax rate in order to satisfy the government budget restraint equilibrium.

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[41]We must note here that as in the previous chapters, the wealth effects affect the magnitude shift of the curves. On the is curve the wealth effects work positively (i.e. $E_3 > 0$) whereas on the lm curve the wealth effect works negatively (i.e. $-L_3 < 0$).

Here let us recall the effect of the endogenous policy variable observed in chapter III. Under the assumption of the variable income level, the induced tax revenue was the main value motivating the marginal tax rate to change. The effect on the income level of the movement of the marginal tax rate on the income level could be seen in the denominator of the matrix in contrast to the denominator obtained in chapter II. Thus, let us derive a Jacobian determinant of an implicit government budget restraint case with the assumptions of a flexible price level and a fixed income level. From IV-8 we can take a three-by-three matrix with respect to the endogenous variables of the price level, the interest rate and the exchange rate; then we have a following Jacobian determinant:

$$|J| = X_2\{[(1-u)BE_1 + WE_2](L_1B/i^2p - L_2)$$
$$+ (M^o - WL_2)[K_2 + (E_1B/i^2p - E_2)]\}$$
$$= X_2\Delta_8' \quad > 0$$

The difference in Δ_8' from Δ_8 exists on the extra term $(1-u)BE_1 > 0$ in Δ_8', which raises the magnitude of the determinant in contrast to that in Δ_8. This implies that the magnitude of the price multiplier in the case of the explicit government budget restraint is higher than in the case of the implicit government budget restraint if both of the numerators are identical. Thus, as with the effect of the induced tax revenue in chapter III, we can differentiate between an explicit government budget restraint and an implicit government budget restraint as above with respect to the effect of a price level change.

2. Bond Financed Government Expenditures

The system of the matrix form is expressed as:

$$\text{IV-8(ii)} \quad \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & 0 & 0 & b_{44} \end{bmatrix} \begin{bmatrix} dp/p \\ di \\ dr \\ dU \end{bmatrix} = \begin{bmatrix} [(1-u)(E_1+X_1)i + E_2+1]dB/ip \\ L_2 dB/ip \\ -(1-u)X_1 idB/ip \\ (1-u)idB/ip \end{bmatrix}$$

where b_{11}.........b_{44} are the same as IV-8.

Solving for endogenous variables we obtain the following multipliers:'

IV-8(e) $dp/(dB/i) = p/\Delta_2\{(L_1B/i^2p-L_1)-E_2L_1+L_1(E_2-K_2)$ >0

IV-89(f) $di/(dB/i) = 1/p\Delta_2[(M^{\circ}-WL_1)+E_2M^{\circ}]$ >0

IV-8(g) $dr/(dB/i)$

$$=1/pX_2\Delta_2 \ \{-(1+E_2)[(M^{\circ}-WL_1)K_2+(X_2-\mathring{R})(L_1B/i^2p-L_2)]$$
$$+L_1[(X_2-\mathring{R})(E_2B/i^2p-E_2)-(WE_2-X_2)K_2]\}$$

IV-8(h) $dU/(dB/i) = 1/p\Delta_2[-(1-u)B\{(1+E_2)(L_1B/i^2p-L_1)$

$$-L_1[K_2+(E_2B/i^2p-E_2)]\}$$

$$+(1-u)i\{WE_2(L_2B/i^2p-L_2)$$

$$+(M^o-WL_2)[K_2+(E_2B/i^2p-E_2)\}]$$

The price multiplier in IV-8(e) consists of the same three elements we had in the income multipliers in the preceding chapters; that is, the effect of the increased government expenditures and the wealth effects both in the commodity and the money markets. The price level goes up unambiguously with the assumption that the negative wealth effect in the money market does not exceed the rest of the terms.

The sign of the exchange rate is uncertain in IV-8(g). The second term displays the effect of the money market equilibrium. Both the crowding-out effect, and the interest rate increase due to the increased government bonds in th money market, cause the exchange rate to appreciate. But there are forces in conflict in the first term which display the effect of the increased government expenditures on the exchange rate. Although the interest rate rise leads the exchange rate to appreciate, the increased price level causes a depreciation.

The sign of the endogenous policy variable is uncertain in IV-8(h). The first term is the effect of the changed value of interest payments which is due to the increment of the price level on the marginal tax rate. Thus, the endogenous policy variable multiplier implies that the government gains a surplus. This is because the real value of interest payments goes down, when the price level goes up.

3. Open Market Purchases

The system of the matrix form is expressed as:

$$\text{IV-8(iii)} \quad \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & 0 & 0 & b_{44} \end{bmatrix} \begin{bmatrix} dp/p^2 \\ di \\ dr \\ dU \end{bmatrix} = \begin{bmatrix} -(1-u)(E_1+X_1)idM^*/p \\ -dM^*/p \\ (1-u)X_1 idM^*/p \\ -(1-u)idM^*/p \end{bmatrix}$$

where b_{11}.........b_{44} are the same as IV-8.

Solving for endogenous variables we obtain the following multipliers:

$$\text{IV-8(j)} \quad dp/dM^* = p/\Delta_3 [K_2+(E_2B/i^2p-E_2)] \quad >0$$

$$\text{IV-8(k)} \quad di/dM^* = 1/p\Delta_3 (WE_2-X_2) \quad <0$$

$$\text{IV-8(l)} \quad dr/dM^* = -1/p\Delta_3 \; [(X_2-\dot{R})(E_2B/i^2p-E_2)-(WE_2-X_2)K_2] \quad >0$$

$$\text{IV-8(m)} \quad dU/dM^* = -1/p\Delta_3 [(1-u)B[K_2+(E_2B/i^2p-E_2)] + (1-u)i\{(WE_2-X_2)(L_2B/i^2p-L_2) + (M^*-WL_2)[K_2+(E_2B/i^2p-E_2)]\}] \quad <0$$

When the government budget restraint is recognized explicitly, the multipliers of endogenous variables tend to be simpler due to the endogenous adjustment of the policy variable. In particular, the sign of the multipliers in open market purchases is clear, since the cancellation of a number of coefficients can be made. The sign of the price level and the exchange rate multipliers is positive, while the sign of the interest rate and the endogenous policy variable multipliers is negative. Both the increased price level and the lower interest rate cause the exchange rate to depreciate in IV-8(1). Since the contraction of government bonds as well as the rise of the price level leads the government budget to surplus, the marginal tax rate must be lowered to satisfy the government budget restraint. The rise of the price level lowers the real value of the finance sources for government spending, while the higher value of government bonds due to the fall of the interest rate and the lower interest payments due to the purchase of government bonds from the private sector, give the government budget surpluses.

D. Pegged Exchange Rate System with Explicit Recognition of the Government Budget Restraint and a Flexible Price Level

With the assumptions of the pegged exchange rate system (i.e. $sdR \neq 0$ and $dr = 0$ where the sterilization coefficient is in the range of $0 \le s \le 1$), the explicit recognition of the government budget restraint, and the flexible price level (i.e. $dy=0$ and $dp \ne 0$), we have the following system of the matrix form with respect to dp, di, dr and dU where $dU=(y+B)du$:

$$\text{IV-9} \quad \begin{bmatrix} (1-u)B(E_1+X_1)+WE_2-X_2 & E_2B/i^2p-E_2 & 0 & E_1+X_1 \\ (M^{\circ}-WL_2) & -(L_2B/i^2p-L_2) & 0 & 0 \\ X_2-R-(1-u)BX_1 & K_2 & -1 & -X_1 \\ (1-u)B & 0 & -s & 1 \end{bmatrix} \begin{bmatrix} dp/p^2 \\ di \\ dR \\ dU \end{bmatrix} =$$

$$\begin{bmatrix} (1-u)(E_1+X_1)dB/p+E_2dB/ip+E_2dM^{\circ}/p+dg+X_2dr \\ -L_2dB/ip+(1-L_2)dM^{\circ}/p \\ -(1-u)X_1dB/p-X_2dr \\ (1-u)dB-dB/ip-dM^{\circ}/p+dg \end{bmatrix}$$

where $W=M^{\circ}+B/i$

The Jacobian derminant of the system is:

$$|J|=[WE_2-X_2(1-sE_1)+sWE_2X_1](L_2B/i^2p-L_2)$$
$$+(M^{\circ}-WL_2)[(1-sX_1)(E_2B/i^2p-E_2)-s(E_1+X_1)K_2]$$

$=\Delta_4 \quad >0$

Although there are some negative coefficients, all of which are related to a sterilized magnitude of the balance of payments, we assume as in chapter III, that the government determines the value of s exogenously in such a way that the sign of the Jacobian determinant is assured to be positive.

1. Money Financed Government Expenditures

The system of the matrix form is expressed as:

$$\text{IV-9(i)} \quad \begin{bmatrix} b_{11} & b_{12} & 0 & b_{14} \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & 0 & -s & b_{44} \end{bmatrix} \begin{bmatrix} dp/p \\ di \\ dR \\ dU \end{bmatrix} = \begin{bmatrix} (1+E_2)dM^*/p \\ (1-L_2)dM^*/p \\ 0 \\ 0 \end{bmatrix}$$

where $b_{11}.........b_{44}$ are the same as IV-9.

Solving for endogenous variables we obtain the following multipliers:

$$\text{IV-9(a)} \quad dp/dM^* = p/\Delta_5\{(1+E_2)(1+sX_1)(L_1B/i^2p-L_2) + (1-L_2)[(1+sX_1)(E_1B/i^2p-E_2)+s(E_1+X_1)K_2]\} \quad >0$$

$$\text{IV-9(b)} \quad di/dM^* = 1/p\Delta_4\{(1+E_2)(1+sX_1)(M^*-WL_2) - (1-L_2)[(1+sX_1)WE_2-(\dot{i}-sE_1)(X_2-\dot{R})]\}$$

$$\text{IV-9(c)} \quad dR/dM^{\circ} = 1/p\Delta \cdot [(1+E_2)(M^{\circ}-WL_2)K_2 + (1-L_2)[(X_2-\dot{R})(E_2B/i^2p-E_2)-(WE_2-X_2)K_2]$$

$$\text{IV-9(d)} \quad dU/dM^{\circ} = 1/p\Delta \cdot [-(1-u)B[(1+E_2)(L_2B/i^2p-L_2) + (1-L_2)(E_2B/i^2p-E_2)] + s[(1+E_2)\{[X_2-\dot{R}-(1-u)BX_1](L_2B/i^2p-L_2)+(M^{\circ}-WL_2)K_2\} + (1-L_2)[X_2-\dot{R}-(1-u)BX_1](E_2B/i^2p-E_2) - [(1-u)B(E_1+X_1)+WE_2-X_2]K_2]$$

The price level clearly goes up, but the sign of the other three multipliers is uncertain.

Regarding the balance of payments multiplier in IV-9(c), the terms of capital flows K_2 in the first and second terms are in conflict with each other. If the interest rate goes up (i.e. the predominant fiscal effect), the capital inflow will result; otherwise the capital outflow will occur. The unambiguous rise of the price level deteriorates the balance of payments. Whether the balance of payments improves or deteriorates depends on the elasticity of the balance of payments with respect to a change in the price level and the interest rate.

The sign of the endogenous policy variable multiplier is determined by a change in the value of interest payments and the non-sterilized value of the balance of payments. Since the price level goes up, the real value of interest payments goes down so that the government gains a budget surplus. If the balance of payment surplus results by the effect of money financed government expenditures, the marginal tax rate will be raised to adjust the surplus. Otherwise it will be lowered.

2. Bond Financed Government Expenditures

The system of the matrix form is expressed as:

$$\text{IV-9(ii)} \quad \begin{bmatrix} b_{11} & b_{12} & 0 & b_{14} \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & 0 & -s & b_{44} \end{bmatrix} \begin{bmatrix} dp/p^2 \\ di \\ dR \\ dU \end{bmatrix} = \begin{bmatrix} [(1-u)(E_1+X_1)i + E_2+1]dB/ip \\ -L_2 dB/ip \\ -(1-u)X_1 idB/ip \\ (1-u)idB/ip \end{bmatrix}.$$

where b_{11}.........b_{44} are the same as IV-9. Solving for endogenous variables we obtain the following multipliers:

$$\text{IV-9(e)} \quad dp/(dB/i) = p/\Delta_4 \{(1+sX_1)[(L_2B/i^2p-L_2)-E_2L_2+L_2E_2] - sL_2K_2(E_1+X_1)\}$$

$$\text{IV-9(f)} \quad di/(dB/i) = 1/p\Delta_4 \{L_2[(1+sX_1)WE_2-(1-sE_1)X_2] + (1+E_2)(1+sX_1)(M^o-WL_2)\}$$

$$\text{IV-9(g)} \quad dR/(dB/i)=1/p\Delta\{(1+E_2)[(M^{\circ}-WL_2)K_2+X_2(L_2B/i^2p-L_2)]$$
$$+L_2[(WE_2-X_2)K_2-X_2(E_2B/i^2p-\dot{E}_2)]\}$$

Iv-9(h)

$$dU/(dB/i)=1/p\Delta[-(1-u)B\{[(1-u)(E_1+X_1)i+E_2+1](L_2B/i^2p-L_2)$$
$$-L_2(E_2B/i^2p-E_2)\}$$
$$+s[(1+E_2)\{[X_2-\dot{R}-(1-u)BX_1](L_2B/i^2p-L_2)+(M^{\circ}-WL_2)K_2\}$$
$$-L_2[X_2-\dot{R}-(1-u)BX_1](E_2B/i^2p-E_2)$$
$$-[(1-u)B(E_1+X_1)+WE_2-X_2]K_2]$$
$$+(1-u)i\{WE_2(L_2B/i^2p-L_2)+(M^{\circ}-WL_2)[K_2+(E_2B/i^2p-E_2)]\}]$$

In IV-9(e) there exist two negative terms, both of which crowd out an increase in the price level. If these crowding-out effects are assumed not to exceed the positive effect of the increased government expenditues, the price level will go up unambiguously.

The unambiguous increase in the interest rate causes the inflow of capital, but the rise of the price level lowers net exports. Hence, whether the balance of payments results in surplus or deficit is uncertain.

The endogenous policy variable multiplier is determined by three elements. The first term is the effect of the price level change, the second term the effect of the balance of payments and the third term the effect of interest payments. As under the floating exchange rate system, an increase in interest payments causes the marginal tax rate to go up, but since the rise of the price level affects the real value of interest payments, there is some trade off between these two elements. If the balance of payment surplus results, the marginal tax rate will go up; otherwise it will go down.

3. Open Market Purchases

The system of the matrix form is expressed as:

$$\text{IV-9(iii)}\quad \begin{bmatrix} b_{11} & b_{12} & 0 & b_{14} \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & 0 & -s & b_{44} \end{bmatrix} \begin{bmatrix} dp/p \\ di \\ dR \\ dU \end{bmatrix} = \begin{bmatrix} -(1-u)(E_1+X_1)idM^{\bullet}/p \\ dM^{\bullet}/p \\ (1-u)X_1 idM^{\bullet}/p \\ -(1-u)idM^{\bullet}/p \end{bmatrix}$$

where b_{11}.........b_{44} are the same as IV-9.

Solving for endogenous variables we obtain the following multipliers:

$$\text{IV-9(j)}\quad dp/dM^{\bullet}=p/\Delta_4\,[(1+sX_1)(E_1B/i^2p-E_2)+sK_2(E_1+X_1)] \quad >0$$

$$\text{IV-9(k)}\quad di/dM^{\bullet}=-1/p\Delta_4\,[(1+sX_1)WE_2-X_2(1-sE_1)] \quad <0$$

IV-9(l) $dR/dM^{*}=1/p\Delta \; [(X_2-\dot{R})(E_2B/i^2p-E_2)-(WE_2-X_2)K_2] \quad <0$

IV-9(m)

$$
\begin{aligned}
dU/dM^{*}=1/p\Delta \; [& -(1-u)B(1+sX_1)(E_2B/i^2p-E_2) \\
& -s\{[(1-u)B(E_1+X_1)+WE_2-X_2]K_2-(X_2-\dot{R})(E_2B/i^2p-E_2)\} \\
& -(1-u)i\{[(1-u)B(E_1+X_1)+(1+sX_1)WE_2 \\
& \quad -(1-sE_1)(X_2-R)](L_2B/i^2p-L_2) \\
& \quad +[(1+sX_1)(E_2B/i^2p-E_2)+s(E_1+X_1)K_2](M^{*}-WL_2)\}]
\end{aligned}
$$

The analysis of each multiplier for open market purchases under the pegged exchange rate system is analogous to that under the floating exchange rate system. The only significant difference lies in the effect of the balance of payments on the endogenous policy variable multiplier under the pegged exchange rate system expressed in the second term in IV-9(m). In addition to the effects of the price level rise and lower interest payments, the unambiguous balance of payment deficit reinforces the drop in the marginal tax rate.

4. Exchange Rate Change Policy

The system of the matrix form is expressed as:

$$\text{IV-9(iv)} \quad \begin{bmatrix} b_{11} & b_{12} & 0 & b_{14} \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & 0 & -s & b_{44} \end{bmatrix} \begin{bmatrix} dp/p \\ di \\ dR \\ dU \end{bmatrix} = \begin{bmatrix} X_2 dr \\ 0 \\ -X_2 dr \\ 0 \end{bmatrix}$$

where b_{11}.........b_{44} are the same as IV-9.

Solving for endogenous variables we obtain the following multipliers:

IV-9(n) $dp/dr = 1/\Delta_4 \cdot X_2(1-sE_1)(L_3B/i^2p-L_2) \quad >0$

IV-9(o) $di/dr = 1/\Delta_4 \cdot X_2(1-sE_1)(M^*-WL_1) \quad >0$

IV-9(p) $dR/dr = 1/\Delta_4 \cdot X_2\{WE_3(L_3B/i^2p-L_2)$

$+(M^*-WL_1)[K_2+(E_3B/i^2p-E_2)]\} \quad >0$

IV-9(q) $dU/dr = 1/\Delta_4 \cdot X_2[-(1-u)B(1-sE_1)(L_3B/i^2p-L_2)$

$+s\{WE_3(L_3B/i^2p-L_2)+(M^*-WL_1)[K_2+(E_3B/i^2p-E_2)]\}]$

The price level and the interest rate clearly go up due to the effect of the exchange rate devaluation. The balance of payments results in surplus. The balance of payment surplus and the price level rise are the forces in conflict on the endogenous policy variable multiplier.

E. Summary

With regards to the flexible price level, Blinder-Solow(1973) state:

> The result would be that expansionary fiscal policy causes some inflation of the price level which reduces the value of the multiplier for (at least) three reasons:.... While each of these serves to reduce the absolute value of the fiscal multiplier, none of them has any bearing on its sign, ...[43]

Our results prove the statement above to be true. The sign of the price multipliers for every policy is consistent with that of the income multipliers in chapter III. This implies that an increase in the income level for each government policy is not as high as the magnitude derived under the assumption of the rigid price level in chapter III.

The sign of the interest rate and that of the external balance multipliers are also not different from that obtained in chapter III.

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[43]A.S. Blinder and R.M. Solow, "Does Fiscal Policy Matter?" <u>Journal of Public Economics</u> (November 1973): p.324.

The determination of the endogenous policy variable is also similar to that in chapter III. However, now that the income level is assumed to be rigid, there is no value of the induced tax revenue. In lieu of the induced tax revenue, a change in the real value of interest payments is one of the factors that determines the endogenous policy variable multiplier under the assumption of the flexible price level. Other factors are the same as those observed in chapter III; that is, the interest payments of the increased government bonds and the balance of payments under the pegged exchange rate system.

V. CONCLUSION

The objective of this thesis was to build a simple macro model with the government budget restraint in an open economy in the context of a static analysis. This attempt was made to eradicate the deficiencies of recent developments on the government budget restraint in an macro open economy.

There were three main concerns in building the model. The first was to include the variable of interest payments of government bonds paid out to the private sector in the model. The second was to include a sterilization coefficient of the balance of payments. Finally, the endogenous variables which are determined in the private sector were explicitly recognized in the government budget restraint. This meant that two endogenous policy variables must be recognized in order to satisfy two government budget restraints. The first endogenous policy variable was determined by a policy specification; the second was chosen from the marginal tax rate. The second policy variable was determined to equilibrate changes in variables on the government budget restraint.

Inclusion of interest payments is not common as far as the analysis of government policies in the context of comparative statics is concerned. However, interesting results were observed by including the variable of interest payments explicitly in the model. Particularly in bond financed government expenditures where an increase in government expenditures is financed by an increase in government bonds, the effect of interest payments on the income level could be observed. When only the first endogenous policy variable was recognized in chapter II, the effect of interest payments on the income level was similar to the conclusion made by Blinder-Solow(1973). That is, an increase in interest payments to the private sector has a multiplier effect on private expenditures so that the income level can increase higher than in the case of money financed government expenditures. When the two endogenous policy varialbes were recognized in the government budget restraint in chapter III; however, the effect of interest payments on the income level was observed different from the conclusion made by Blinder-Solow(1973). That is, by recognizying the second endogenous policy variable, the multiplier effect of interest payments disappears from the income multiplier. This is because the endogenous policy variable must be raised to finance the deficit caused by increased interest payments. Thus an increase in private expenditures due to the increased interest payments is totally offset by an increase in the marginal tax rate. Only if the government issues new government bonds which can finance not only an increase in government expenditures, but also an increase in net interest payments, the multiplier effect of increased interest payments can be seen on the income level.

Recognition of the second endogenous policy variable did not show any significant change in sign of endogenous multipliers in contrast to the multipliers obtained in chapter II. But we learned that it is important to recognize changes in variables in the government budget restraint other than a specified policy variable in order to satisfy the government budget restraint. But we learned that it is important to recognize changes in variables in the government budget restraint other than a specified policy variable in order to satisfy the government budget restraint. Under the floating exchange rate system, a change in the income level and interest payments are the factors to determine the second endogenous policy variable. Under the pegged exchange rate system, the multipliers became complex since the balance of payments was involved in the government budget restraint. The balance of payment surplus (deficit) by the effect of a domestic policy raised (lowered) the second endogenous policy variable. Therefore, the balance of payment surplus was in fact observed as a government deficit similar to an increase in interest payments.

The effects of the balance of payments in the system could be observed only if non-complete sterilization policy was employed. If a complete sterilization policy was adopted, the fluctuation of the balance of payments would be completely insulated from a domestic economy.

As an extension of study, the assumption of the flexible price level was employed and the multipliers of each endogenous variable were observed. As a result, a singificant change in multipliers was not found in contrast to the case of the variable income level. But since all domestic policies increase the price level, we may conclude that an increase in the real income level would not be as high as those obtained under the assumption of the rigid price level. We should also note that an increase in the price level causes a government budget surplus due to a fall in the real value of interest payments.

All the domestic policies raised the income level under the assumptions of rigid price level and non-perfect capital mobility. However, the discovery of the domestic policy that most efficiently increases the income level must be left to empirical investigations.

An external balance tends to deteriorate by the effect of any domestic policy if a current account has a dominant force (i.e. a steep BP curve). On the other hand, an elasticity of capital flow is high (i.e. a flat BP curve), there is a chance for the external balance to improve. It is bond financed government expenditures which raises the interest rate. Therefore, bond financed government expenditures can raise the income level without deteriorating the external balance. All the other policies which increase money supply tend to lead the external balance deficit.

Although this thesis attempted to eradicate the deficiencies of recent developments on the government budget restraint literature, many more improvements must be accomplished. A stability condition of the system was not examined. Inflation was not taken into account in the system. Although the system was expanded to an open economy, the specification of the external balance employed was the simplest case, by following the traditional theory in an open economy. The analysis was limited to the short run, but the long run or the steady state (full adjustment of capital stock) were not analyzed. Recent developments on rational expectation in the macro theory were ignored. These aspects within the context of this thesis remain as my future research topics.

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