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UNIVERSITY OF ALBERTA

Mathematical Comprehension of Adults with Learning Difficulties

by

Lesley J. LeMaître

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfilment of the requirements for the degree of Master of Education.

IN

DEPARTMENT OF EDUCATIONAL PSYCHOLOGY

Edmonton, Alberta

FALL, 1993

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Date: September 22, 1993

I would like to dedicate this thesis to all of the hard-working students who, in one way or another, helped to make it possible.

Abstract

In order to better understand the mathematical comprehension of adults having difficulties in mathematics, five students were asked to "think aloud" as they worked through fifty-two fraction questions during four or five "math sessions" over several weeks. For the purposes of this study one of the student's results was examined in greater detail. Emphasis was given to the first sixteen questions which consisted of equal combinations of addition, subtraction, proper, and improper fractions, each with like and unlike denominators. The remaining questions and other students' results were used to supplement this material. Data were also gathered from i) an initial interview, including a short questionnaire; ii) the Woodcock-Johnson Psycho-Educational Battery (WJPEB) (Woodcock & Johnson, 1978); iii) the Raven's Standard Progressive Matrices (Raven, 1960); iv) documented material; and v) classroom observations. Results from a pilot study were also reported and discussed.

Each session resembled the dynamic assessment model that Lidz (1991) described because it centred "on learning processes, in contrast to the traditional assessment focus on already learned products" (p. 3). Once the questions had been completed and the tapes transcribed, a process corresponding in many ways to Giorgi's (1975) and Colaizzi's (1978) was employed to review the transcripts, extract, and then to cluster pertinent information into units of solution procedures, tactics, and errors. Other behaviours noted during the session were also incorporated. These results were later re-matched with the transcripts that were themselves re-checked against the tapes.

The outcome indicated a number of similarities in the difficulties demonstrated by these students and those manifested by children. Many related to aspects of mathematics such as the students' lack of factual knowledge, attention, and

organizational ability. However, unlike their younger counterparts, the errors made by these adults could not be attributed to a lack of time for memory or cognitive development. As well as resulting from a low level of understanding and poor metacognitive ability, they appeared to be a reflection of what and how these adults were taught.

ACKNOWLEDGEMENTS

I wish to convey my sincere appreciation to Dr. Michael Rodda, my supervisor. Thank you for your guidance and encouragement but most of all for your obvious confidence in my ability.

My gratitude is also extended to Dr. Mulcahy for his helpful comments and Dr. Kieren for being there. Thank you.

I also want to express my indebtedness to Dr. Bisanz who was an inspiration to me. Thank you for taking the time.

Many thanks as well to the students and staff at both the Continuing Education Annex Building and the Alberta Vocational Centre (AVC), both in Edmonton. Thank you for letting me into your world and helping to make my goal a reality. Thanks especially to Arnold, Barb, and Harpal.

I particularly want to thank the students who participated in my studies. You taught me a lot. Thank you for your time and effort.

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Chapter I

Introduction

Estimates on the percentage of adults in Canada with some type of learning disability vary from lows of around 1% to highs such as the British Columbia government's figure of 15% (B.C. Ministry of Education, 1984). The lack of a common definition and problems with identification procedures are among the reasons for this variability (Woods, Sedlacek, & Boyer, 1990; Wood, 1991; Morrison, McMillan, & Kvale, 1985; Morris 1988; & Hamill, 1990). Often it is suggested that those with learning disabilities have minimal neurological dysfunctions. The Learning Disabilities Association of Canada (LDAC), for instance, defines a learning disability as "a specific neurological inefficiency that affects a particular area of intellectual functions...or that impedes acquisition or learning of specific information at an average speed" (1991, p. 3). Gerald Coles (1987) and others, however, found little evidence to support this contention and believe that learning disabilities result because these individuals try to process information in ways that are different. DeBettencourt (1987) thinks that many people with learning disabilities are inactive learners who do not possess strategies that could help them.

The decision by leaders in the field to use the term "learning disabilities" was important because it shifted the emphasis away from medical causation towards educational relevance (Smith, 1983) and, although the cause may be unknown, the effect is that these individuals are seriously impaired and function academically at a level significantly below their expected potential in one or possibly more of the basic academic areas (Palincsar & Brown, 1987; Whyte, Kovach, & Vosahlo, 1992). They may lack metacognitive abilities, be unable to receive and process information, or have difficulty using strategies and symbols (Goldstein, 1989; B. C. Ministry of Education, 1987; Hoy & Noel, 1984).

Most people associate learning disabilities with problems in reading but a notable proportion of this population has difficulties in mathematics and many of these are severe and persistent.[1] Attention, however, has only recently been turned to adults with LD and research is particulary lacking in the area of mathematics (Montague, 1988; Sinicrope and Mick, 1983). The few educational diagnostic tools used have often been adapted from those developed for children (Goldstein, 1989).

Children who have problems learning mathematics are far from a homogeneous population (Giordano, 1987). Each has a unique set of impairments. These are not systematic or predictable (Johnston, 1987). Results of this study indicate a similar finding with adults, who showed greater variability of problems within fewer, broader categories. It is probably because the resulting problems are so diverse that a standard classification system has not yet been developed (Garnett, 1987).

In 1986 Snowman expressed concern that learning strategy research was lacking and that more was needed. Still today relatively little has been done in this important area. Since cognitive psychologists believe that instruction should focus on process and because evidence indicates that students may use entirely different knowledge structures to generate a correct response (Mayer, 1982), research must also consider this aspect of problem solving. Qualitative methods are particularly suited to answering questions about procedures used in mathematics and the dynamic interview process employed in this study is especially useful. Its primary purpose was to examine the procedures used, and errors made, by adults having difficulties in mathematics, including learning disabilities, in order to better understand these phenomena and the related comprehension of individuals who manifest them. The accomplishment of this goal provides the reader with information that was not likely to have otherwise been revealed.

Chapter II

Literature Review

The recognition and acceptance of individuals with learning disabilities (LD), or at least learning difficulties, that were seen as other than those resulting from school phobia, mental retardation, or a specific disabilitating condition, is a relatively recent occurrence. Schools in Canada began in the late 1960's and early 1970's to add special programs to their curriculum to help children with LD (Hoy & Noel, 1984; B. C. Ministry of Advanced Education & Job Training, 1987). In 1975 the United States mandated special services for learning disabled students to be funded by federal law (Smith, 1983) and seven years later the Canadian government formally recognized learning disabilities by including it in section 15 of the Charter of Rights and Freedoms. Unfortunately, in both Canada and the United States aid did not reach those already out of the school system and for many others it came too late.

Although learning disabilities are much more understood today than twenty years ago, there is still a great deal of discrimination against people with LD. Miller (1987) thinks that poor public attitudes is the greatest barrier to the success of handicapped students. Adults with learning difficulties are particularly vulnerable and may lack awareness of resources and assistance available to them or the means to attain these. Even with advanced programs developed to assist these individuals, many are still having problems in learning. In some instances these programs are not being utilized because of a lack of funding or trained personnel. Weiss and Weiss (undated) made an observation several years ago that still far too often holds true today. They noted "that our schools as a whole are not meeting the survival needs of learning disabled adolescents...by the time the learning-different student has reached secondary school he has often turned off to the school experience" (p. 1). Society is missing the benefit of the resources of these individuals, as few of them ever realize their full potential and many have trouble obtaining and maintaining employment.

Those with learning disabilities have difficulty acquiring, storing, and retrieving information. As well, they have trouble relating it to previous knowledge and using it repeatedly in new and different ways (Hoy & Noel, 1984; B.C. Ministry of Advanced Education and Job Training, 1987). Some of these adults are not able to generalize, or even generate, learning strategies or may be unable to make good use of those they do possess, so even if they have the information they may still not be able to perform a required task. Goldman (1989) notes that good learners must not only possess the strategies and knowledge but they must also recognize when to access and use these as efficiently as possible. Unfortunately, many students with learning difficulties are "passive" learners who have trouble selecting and applying appropriate strategies (Montague, 1988).

The old, and now obsolete, Alberta Curriculum Guide for senior high school mathematics emphasized the importance of being able to quantify information and perform mathematical operations (Alberta Education, 1983). Although true, it is not as revealing as the very much updated observation made in the most recent guide (Alberta Education, 1990) which noted that "emphasis has shifted from the memorization of mathematical formulae and algorithms toward a more dynamic view of mathematics as a precise language, used to reason, interpret and explore" (p. 3). Knowledge and understanding are the keys to success in this increasingly technological and information-oriented era. Regrettably, many adults are floundering because they lack these essentials in mathematics. Garnett (1987) cited a study by Johnson and Blalock that found students to be "significantly more handicapped in their daily routine by their lack in mathematics than by their poor reading" (p. 4) and England's Advisory Council for Adult and Continuing Education noted that even simple mathematics problems could induce feelings of anxiety, helplessness, guilt, and inadequacy (Hughes, 1986). In order to cope, some of the individuals discussed by this council would pay by cheque while others would buy predetermined quantities of an item (e.g., £10.00 worth of gasoline). Participants of this thesis research as well as those in the pilot study also expressed or demonstrated anxiety. In the report "Mathematics Counts", committee members discovered that many pupils experienced extreme difficulty translating problems into appropriate mathematical terms and the writers believed that more discussion should take place between the students themselves as well as with their teachers (Cockcroft, 1982).

Algozzine, O'Shea, Crews, and Stoddard's (1987) assessment of employers' ratings of the relative importance of various mathematics skills indicated that addition, subtraction, multiplication, and division were highly rated and they went on to comment that students with learning disabilities demonstrated mastery on very few of these. Hughes (1986) also noted that while "children's performance of basic calculations was superficially adequate, their understanding of these calculations, their ability to apply their skills to new problems, and their ability to interpret their findings were frequently inadequate" (p. 10). Student Evaluation Director, Frank Horvath, expressed his concern over recent findings that math 30 students appeared "to lack the necessary depth of understanding" (Alberta Education, 1993, press release, front page) and had difficulty generalizing what they knew. Almost 33% actually failed the June examination.[2] Birenbaum and Shaw (1985) uncovered evidence that showed many junior high school age students probably lacked conceptual understanding of, and had little computational skill with, fractions. It is also one of the most difficult areas to teach (Kelly, Gersten, & Carnine, 1990). Cawley (1985a), citing a survey by McLeod and Armstrong involving 104 middle and high school teachers of the learning disabled, noted that "the six most commonly reported deficit concerns were (1) division, (2) operations with fractions, (3) decimals, (4) percent, (5) fraction terminology, and (6) multiplication of whole numbers" (p. 3). Like Hughes (1986) and Cockcroft (1982), they also found that the simplest manipulations could create overwhelming stress. My own recent work (LeMaitre, 1991, 1992) seems to support this contention.

Much of the research that has been done, almost always with children, is based on data utilizing scores obtained from achievement tests rather than on patterns of responses or procedures used (Ashlock, 1976; Harnisch, 1983; Tatsuoka & Tatsuoka, 1983). As noted by Pellegrino and Goldman (1987), "relatively little consideration has been devoted to characterizing the performance of learning disabled students on tasks involving quantitative thinking or computational skills" (p. 23) and little emphasis has been given to the diagnosis of learning disabilities in mathematics (Hollander, 1988), even though it is of great concern to educators (Montague, 1988). Cawley (1987) also suggested less reliance on paper and pencil tests which cannot inform educators of the seriousness of the errors made. As well "intelligence tests are static measures of ability...[that] do not predict the ability to learn" (Lidz, 1987, p. vii). "When product is the outcome there is no information regarding the reason for failure or the learner's ability to achieve. There are also no guidelines or implication for intervention to connect the assessment with intervention and to making assessment relevant for an educational setting" (Lidz, 1991, p. 4). Consequently, a substantial amount of helpful information is often never utilized. Nickerson, Smith, & Perkins (1985) began their chapter on problem solving by emphasizing the importance of the task and stating that without its careful analysis, problem solving could not be fully understood. A case study in Daniels (1988) may help to illustrate the value not only of analyzing the task but also of examining the procedures used to bring it to resolution. David could correctly answer subtraction problems involving any two numbers less than one hundred. However, when the numbers exceeded this figure, he was unable to do the calculation. Because he had never been asked to demonstrate his procedure for the easier problem, it had been incorrectly assumed that he was using the same method as the other students. Not until evidence showed otherwise, could his difficulties be dealt with. He had developed a unique method for solving subtraction problems that, as with many students, grew out of his experiences. It was only through the actual analysis of the task and David's procedures that special educators were able to understand the reasons for his difficulty and thus begin to plan a means of helping him through appropriate and effective teaching techniques. Baroody and Hume (1991) and Mack (1990) emphasized that teaching techniques should make better use of children's "informal strengths" rather than direct instruction and paper and pencil tests which rely almost entirely on test scores. Information in this study reveals some methods used by the adults that do intermittently produce correct results. This illustrates that unless an analysis of the procedures is performed, these kinds of misunderstandings could easily go unnoticed.

Seamon (1982) suggested concrete techniques that could help the researcher achieve "genuine contact" with the phenomenon. These include reflection, in-depth descriptions, and careful observations. He saw the "idiosyncracies" of researchers as beneficial since they could lead to a variety of perspectives. Valle and King (1978) would seem to agree when they state that "only after seeing these different reflections and varied appearances on repeated occasions does the constant, unchanging crystalline structure [the perceived phenomenon] become known to us" (p. 15). Informal assessments through interviews, observations, and analysis of a student's daily work can do much to pinpoint the learning needs of the individual (Van Devender & Harris, 1987).

Summary

Individuals with learning disabilities are generally seen as those who have severe and permanent difficulties in one or more of the basic academic or social areas. None of the common causes normally associated with these deficits are identified (e.g., physical, psychological, or emotional problems) although it is often linked to a central nervous disorder. In any case these people do have serious problems and they need to be helped.

Learning disabilities are usually associated with reading but many disabled students suffer acalculia and related difficulties, especially when abstract reasoning is involved. There has been little research in diagnosing mathematical disabilities, particularly with adults. Most rely on pencil and paper tests which consider the scores only; they cannot distinguish between strategies used, errors made, or the combinations of right and wrong answers that produce the scores (Harnisch, 1983). Whether the student possessed the strategies but did not comprehend the problem, or understood the problem but either did not have the skills or was unable to select and use those that were appropriate, cannot be determined.

It is becoming increasingly clear that further research into the problems of adults with learning difficulties in mathematics is needed. If done with due care, this study could prove beneficial to all concerned with the welfare of others and those who want to see the "freeing-up" of this great natural resource. With the greater likelihood of increased mainstreaming and the extra burden of more responsibilities falling on teachers and other academic staff, the results of this effort should be of particular interest to those directly involved.

By using a dynamic process in order to encourage and observe the methods and thinking procedures followed by the adult students, strategies such as summarizing, reviewing, and looking for main ideas which Mulcahy, Short, and Andrews (1991) noted distinguish between good and poor learners, were considered and specific methods examined. As well, it helped to identify where processing problems existed. No biological cause was assumed. Although the major requirement was that the individual had difficulties solving fraction problems, an attempt was made to enlist students for whom no other of the more commonly accepted explanation could be found.

The design of the set of questions was a compromise between covering all contingencies and consequently having a prohibitive number of questions--some of which would yield little of relevance--or being unable to adequately cover a worthwhile variety of situations. It was apparent that when dealing with proper and improper fractions, for instance, the possibility of creating corresponding categories of questions (e.g., those with answers less than one) would be impossible. The addition of two improper fractions would never yield an answer of less than two while the subtraction of one proper fraction from another would always give an answer of less than one. Those selected from a number of devised questions were thought to yield the most interesting and meaningful results.

By determining the procedures used and the comprehension demonstrated by adults with learning difficulties, not only can improvements be made in providing information to assist teachers adjust their instructional styles and select tasks that more closely match these students' skills but, by knowing where and perhaps more importantly why they are having problems, other benefits may also result. Aid can be directed specifically at the areas of most need, more effective programs designed, alternate forms of support developed, and additional helpful teaching tools and resources made available.

Definition of Terms

Acalculia

An inability to perform simple arithmetic.

Addend

A number that is being added to another.

Associative Property of Multiplication

$a(b \times c) = (a \times b)c$

Augend

A number to which another number is being added.

Common Denominator

A whole number that is a multiple common to both denominators.

Common Factor (CF)

A number k is a *common factor* of two other numbers m and n if it is a factor of both numbers (i.e., it divides evenly into both m and n (Hunter, Shmyr, 1990).

Commutative Property of Multiplication

$a \times b = b \times a$

Conceptualization

The integration of new information with previous information to form new relationships and knowledge structures (Hoy & Noel, 1984, P. 12).

Discalculia

A flawed ability to do calculations.

Distributive Property of Multiplication over Addition

$a(b + c) = ab + ac$

Dividend

The number being divided in a division.

Divisor

In a division the number that is divided into another.

Error Analysis

Each question is reviewed and incorrect responses made by students are identified and categorized.

Fraction

A numerical representation of two numbers in the form $^a/_b$ where *b* is not equal to zero. The top number (i.e., *a*), is called the numerator while the bottom number (i.e., *b*), is called the denominator.

Improper fraction

A fraction where *a*, the numerator, is greater than, or equal to *b*, the denominator.

Learning Tactic

A specific technique used in the service of a solution procedure while confronted with a task.

Least Common Denominator

The smallest whole number that is a multiple common to both denominators.

Meaningful Learning

The procedure of solving problems is related to other knowledge, and understood (Mayer, 1982, p.4).

Metacognition

Knowing about knowing. Being aware of one's own cognitive abilities. The act of pondering or deliberation and the setting aside of time to ponder or deliberate (Cawley, 1985b). Knowledge about *all* cognitive processes, their products, and anything related to them (Zechmeister, Nyberg, 1982). It includes "the active monitoring and consequent regulation and orchestration of these [cognitive] processes (Flavell, 1977, p.232).

Minuend

The number from which another is subtracted during a subtraction.

Mixed Number

An improper fraction expressed as a whole number and fraction (Silbert, Carnine, & Stein, 1990). It is of the form $C^a/_b$ where C represents the whole number.

Multiplicand

A number to be multiplied.

Proper Fraction

A fraction where a, the numerator, is less than b, the denominator.

Quotient

The result of a division.

Reversibility Property of Multiplication

If a ÷ b = c, then b x c = a.

Rote Learning

The correct response or procedure is memorized without learner understanding (Mayer, 1982, p.4).

Routine

A mechanical performance of an established procedure (Webster's New Collegiate Dictionary, 1981, p. 1001).

Strategy

A procedure that is invoked in a flexible, goal oriented manner and that influences the selection and implementation of subsequent procedures (Bisanz & Lefevre's, 1990, p. 236).

Subitizing

The automatic recognition of small quantities of an object through visualization.

Subtrahend

The number to be subtracted to another.

Task Analysis

The task (i.e., solving a fraction question), is broken down into its basic steps. In this instance a flowchart was created, based on the format presented, to show choices available and their resulting outcomes.

Think-aloud

The process of individual's verbalizing their thoughts as they come to them while working through, in this case, fraction questions.

Chapter III

Method

In order to determine the feasibility of the present study and to "iron out" any problems that could occur, a pilot study was first conducted during the months of April through June, 1992 at the Continuing Education Annex Building (CEAB), in Edmonton, Alberta. It began with an introduction to staff and students alike, continued as time was spent with students both while attending the library where they worked and by conducting the sessions some of them attended, and ended when classes were dismissed for the summer. A brief summary of this research will be presented first. This will be followed by a synopsis of the current study.

Summary of Pilot Study

Students were asked to think aloud as they worked through several two-fraction addition, subtraction, multiplication, and division questions. The first seven were the same for every participant but those remaining were not necessarily presented in the same order. There were also a number of oral questions asking for estimations to the answers or requiring the student to choose the bigger of the two fractions shown. The actual number completed by each adult depended on how many he or she was able to solve in the time available. Some did not progress to the multiplication and division questions.

Seventeen students were involved. Six completed one session and nine completed a second. Two students participated in a portion of the first session, one for fifty minutes and the other just long enough to attempt the seventh question. Full sessions were ninety minutes in duration.

The results suggested that there were many common areas of difficulty but that the problems within these categories were varied. There were some areas with which most of the students had trouble. None of the students, for instance, were able to estimate answers to the fraction questions or use estimation to simplify their calculations and most of them had difficulty reducing fractions, especially proper fractions--they seemed to know what was required when simplifying improper fractions. Many tended to think "light" of operation and equal signs and would often leave these out as well as the whole numbers in mixed number answers. These difficulties could be divided into about twelve different problem types.

Many had trouble attending to more than one "piece" of information and would lose track of their place in the solution process. Part of the difficulty seemed to be related to the less efficient use of their short-term memory (STM) store, probably because their calculations tended to be more cumbersome and time consuming. Very few, if any seemed to understand part/whole relationships. They tended not to monitor or review their progress and more than a few errors in computation resulted because of poor figures and signs, and cramped work. It was not the purpose of this study to determine whether the adults possessed the information but were unable to retrieve it or simply lacked the factual knowledge. Some may have lacked confidence due to inexperience utilizing this knowledge or had trouble expressing themselves. Most of those who did know how to solve a problem or perform an operation were unable to explain why it was done that way. For example, they memorized the rule to invert and multiply when dividing fractions but did not understand the reason for this, a finding similar to Baroody and Hume's (1991) for children. Many adults, even those without learning difficulties, would likely have the same problem since explanations are often lacking within the educational system.

For a more detailed analysis of these results as well as summaries of two of the students' work during the first session, see Appendix E. A transcript of the first seven questions completed by Danny can be found in Appendix F.

Current Study

Four teachers of the Fraction/Decimal classes at the Alberta Vocational College (AVC) in Edmonton presented their students with details about the study based on an information sheet (see Appendix A), copies of which had been given to them earlier. In it participants' rights, benefits, and inconveniences were set out as stipulated by the Ethics Review Committee. Each instructor read this sheet to his or her students and copies were made available to those interested. A seminar was organized to provide these adults with an opportunity to ask more questions and interviews were arranged during this event for those who wanted to proceed. AVC was chosen over some other institutions because of its Strategies for Effective Learning (SEL) facility and the presence of a group which came to be known as the Karen Plourde Society for Learning Disabled Adults.

Although time and financial constraints did not permit any extensive formal assessment, intake interviews were conducted. During these, each adult was given a copy of the information sheet to read, and sign once it was evident that they understood it and were still interested. These individuals were also asked to sign a consent form for participation and fill out a short questionnaire in order to get some student background and to help continue the process of establishing rapport begun during the group seminar (see Appendix A). This was also a way of allowing the students a chance to talk about the obstacles they had encountered in their quest to improve their mathematical skills. The Woodcock-Johnson Psycho-Educational Battery (WJEPB)--Part Two: Tests of Achievement (Woodcock & Johnson, 1977) was also administered at this time and permission was obtained to examine and discuss the students' current records (see Appendix A). The four relevant mathematics classes were observed and later those participating in this research were asked to write the Raven's Standard Progressive Matrices Test (Raven, 1960).

Participants

The most important considerations when soliciting participants were that they had experienced difficulties in learning, particularly mathematics, and were able and willing to share these experiences as fully as possible as they worked through the fraction questions. These interviews, tests, and reviews were carried out with the individuals' permission in order to identify those students with an average or potentially average intellectual ability who best demonstrated this phenomenon but for whom no reason for their difficulty was clearly apparent. This allowed the students each an opportunity to change their minds about participating, particularly with the commitment necessary. Those for whom problems were identified (e.g., physical handicaps, language problems) were offered help but were not included in the study.

Five students from AVC completed all of the sessions. Two were in their late twenties, two in their mid thirties, and one was fifty years old. An in depth analysis was conducted using one of the student's results while summaries were made of the others' work and corresponding tapes. A brief synopsis has been given for each of the participants and their work reviewed. Names and other identifying information have been changed in order to ensure the anonymity of the participants.

Material

The focus of this study was on two-fraction addition and subtraction questions, selecting from the initial research those which yielded the most interesting and relevant results while eliminating others which seemed too simplistic (i.e., most of the students solved these without difficulty). Some questions were added in order to have at least two examples of the same type of item. The original set was developed using an elaborated version of Birenbaum and Shaw's *Task Analysis Chart* (TAC) as well as other research and documented material. Suggestions from *Direct Instruction Mathematics* (Silbert, Carnine, & Stein, 1990), for instance, were taken into consideration.

The final set of questions consisted of twenty-two that were of an oral format and fifty-two that were written. Forty-eight of those written had equal numbers of proper and improper fractions as well as mixed number items (i.e., sixteen of each), each with the same number of like and unlike denominators. In the remaining four written questions, one of the two fractions being operated on was proper while the other was improper. Three of these had unequal denominators and one had denominators which were equal. Overall there were twenty-seven items with unlike denominators and twenty-five with like denominators. Two examples of oral questions were when Donald was asked to 1) estimate the answer to ${}^{20}/_{4}$ - ${}^{20}/_{19}$ = ? and 2) identify which was the bigger fraction in question #42 (${}^{3}/_{4}$ + ${}^{19}/_{20}$) before beginning to work out the answer.

Each of the fifty-two questions was written horizontally with a green felt marker in the upper left corner of a 3" X 5" (i.e., 76mm X 127mm) blank card. Consideration was given to the size and quality of print to ensure that each question would look about the same. The cards were presented one at a time beginning with those questions thought to be the easiest. This ranking was based on earlier research (LeMaitre, 1991, 1992), mathematics texts, curriculum guides, and the *Direct Instruction Mathematic's* chapter on fractions (Silbert, Carnine & Stein, 1990). Oral questions were asked as they were encountered (see MathQuestion questions, Appendix B). Care was taken to assure a fairly equal distribution of even and uneven numbers and to use some that were larger, as well as to have questions that could be more easily estimated than worked out. Some of the types of considerations can be seen in the examples of mixed number items. The fraction parts (which were mostly proper), for example, consisted of equal numbers of those with and without the same denominators where borrowing was required for five of the eight cases involving subtraction.

The answers fell into three major categories, 1) those where the numerator was less than the denominator 2) those where the numerator and denominator were equal, and 3) those where the numerator was larger than the denominator. Within each of these categories were subcategories. Some of the resulting answers had common factors while others contained denominators, or numerators, which were multiples of one another. There was a fairly even distribution of fractions that could or could not be reduced throughout the groups of like and unlike denominators because each had an equal number of proper and improper fractions. There were also two negative answers and two that were equal to zero. As much as possible the last half of the questions tended to mirror the first half.

Although the students completed all of the items, a detailed analysis has been presented for only one of the students while summaries of the other students' work, with emphasis on the first sixteen questions, was used to supplement these results. The first eight of these sixteen involved fractions with like denominators, beginning with four addition and, next, four subtraction questions. The last eight questions were a mixture of these previous types. There were equal numbers of addition and subtraction questions as well as proper and improper fractions. All of these questions had the same number of like denominators as unlike denominators (see Table 1) and none of these results were negative or equal to zero or one. Seven related oral questions were also asked.

Table 1.

<u>Combinations of Fraction-Types</u>

	Addition Fractions		Subtraction Fractions	
Proper Fractions	Unlike Denominators	Like Denominators	Unlike Denominators	Like Denominators
Improper Fractions	Like Denominators	Unlike Denominators	Like Denominators	Unlike Denominators

Interviewing

I believed that one of "the most fruitful way[s] of gaining [a] rich portrayal of the phenomenon" (Becker, 1986, p. 102) being investigated in this study was through interviewing and, like Kvale (1983), accepted that the participant and researcher would influence one another, but agreed that this information in itself could be useful because it could enable one to obtain a more comprehensive understanding than would otherwise be possible.

The purpose of the study was explained and students were told how they could be of most help by thinking aloud as they worked. It was suggested that if they had trouble with this, they try to think of me as a student whom they were assisting and to explain things to me as they would to this student. These exchanges were audio-taped.

There were similarities between the dynamic exchange that occurred during the math sessions and the dynamic assessment process that Lidz (1991) discussed in that 1) the focus was on the process rather than the product, 2) it produced information about the modifiability of the student, and 3) the interviewer acted, where necessary, as a facilitator, encouraging active participation. As little direct influence as possible, however, was used in order to allow each participant's comprehension to reveal itself. The focus of the mathematics sessions was on the students' use of processes as they actually took place in order to understand them in this context. The interviews took place in AVC's Learning Resources Centre's quiet study room. It was hoped that the atmosphere created would encourage these individuals to freely express what they were thinking as it occurred to them.

Thinking Aloud

Aanstoos (1983) used the "think-aloud" process quite successfully to analyze the thinking processes of expert chess players. For the purposes of this study, Donald and the other participants were asked to think-aloud as they worked and were encouraged to attempt all of the questions.

In their well researched article about verbal reporting, Ericsson and Simon (1980) noted that verbalization could be the participant's primary task or simply an incidental part of what he or she was doing. It could be either concurrent or retrospective. If the conditions were favourable, then "the additional cognitive load imposed by the instruction to verbalize would be negligible" (p. 218). They referred to a human information processing model when they distinguished between three levels of coding. If the coding was direct, or level one, then the information which was to be reproduced was of the form acquired from the central processing unit. Levels two and three occurred "when one or more mediating processes occurred between attention to the information and its delivery" (p. 219).

Ericsson and Simon's paper also referred to three different classes of probing. The first was simply to ask the participants to think-aloud, the second involved asking the subjects for specific information as they worked on a task, and the third probed the individual for information about a task they had already completed. They made clear their belief that retrospective probing was unreliable as a source of data obtained directly from the individual's actual thought processes and noted the importance of using undirected probes when appropriate because participants could interpret a request for particular information as an indication of the importance of that part of the task.

Although many believe that the act of verbalizing itself will change the course and structure of the cognitive processes being analyzed, evidence indicates that the additional processing required in level two verbalization may slow down these processes but that "thinking aloud will not change the course and structure of the cognitive processes" (Ericsson & Simon, 1980, p. 227) if the information being reported is already part of the person's knowledge base. If the participant is asked to verbalize information that would not normally be attended to, say by asking individuals to explain their reasons for their actions, then the course of processing would probably be altered.

In this study help was given to any student who seemed unable to proceed. This was usually done in such a way as to minimally affect his or her own processes while hoping to discover some of the reasons for the difficulties. To begin, an attempt was made to use very general, undirected probes. On some occasions, however, it was necessary to take deliberate steps to influence each adult's thinking and there is no doubt that in these instances, and those where specific information was requested, some of their cognitive processes would have been altered. These intentional actions were only taken when the student did not seem to have the strategies with which to even begin and it was believed little would have been gained by accepting exclamations such as Donald's exclamations of "I can't, I can't". In some instances probing may have drawn attention to part of a question but in most cases a difficulty would already have caused this. Where the probe was more direct or a demonstration given, the effects were judged by analyzing the taped exchange. As suggested by Kvale (1983), these exchanges could also provide additional information about what may or may not have benefitted the student. To be able to examine this dynamic process in action is in itself a necessary step, since it is under these types of conditions that most students would be working once they seek help to move forward.

The term solution procedures was chosen over that of learning strategy in an attempt to reflect the complex interactions involved when individuals try to find solutions to mathematical problems. As defined by Snowman (1986) and Bisanz and Lefevre (1990) a learning strategy implies that the decision was made before any action was taken and was selected from many options. Clearly neither Donald nor any of the other participants demonstrated that their procedures involved this kind of selection since they appeared to have few, if any, options and their procedures seemed quite inflexible. They were not decided upon after careful examination of the task but were, rather, simply a reaction to it, relying on memorized rules and procedures--knowing the "how" but not the "why" of it.

Despite the fact that the term solution procedure was used rather than that of strategy, I still chose to borrow the term learning tactic from Snowman (1986) because I saw some of the actions as tactics as he defined them except that they were used in aid of a procedure instead of a strategy. The use of long division was seen both as a procedure and a tactic since on the one hand it was employed when the division was more complex while on the other used automatically to reduce all improper fractions.

Limitations of Study

The purpose of this study was not to compare groups of adult students or to look for causes of their mathematical difficulties. The purpose was to simply analyze the participants' work in order to determine and better understand their comprehension of this subject. The number of individuals participating in this research was small, so it was not possible to generalize to any great extent the results that were found. However, the fact that these results were consistent and repeated throughout both this and the pilot study gives credence to the conclusions drawn. In order to examine what might be the causes of these adults difficulties, more studies would have to be done.

It would be interesting to have a control group in order to determine the likelihood that the mathematical sessions had an effect on the students' abilities in the area of mathematics. The nature of the questions and their presentation would naturally have had an affect on the approaches that the students took to answer the questions and their responses would be somewhat more limited than, say, if material that could have been manipulated were also used. However, that was not the concern with this research and other studies using methods which employ concrete items can be, and have been used to examine this aspect of problem solving.

Chapter IV

Results

Data Analysis

An attempt was made in this study to "allow [the] general patterns to appear in their own time and fashion" (Seamon, 1986, p. 122). In Aanstoos's (1986) words, "This reduction results...in the suspension of all narrowly confining interests...in order to become fully interested in the phenomenon itself" (p. 185).

The process used to analyze the material collected was similar in some respects to those used by Colaizzi (1978) and Giorgi (1975). It began with a transcription of the tapes (see Appendix D). This in itself was the start of a review as I began to very quickly re-familiarize myself with, and form an impression of, what had taken place earlier. Working with one question at a time, the transcripts were checked against the tapes, notes, and the student's original work. Any information that was thought to be relevant was noted so that the process of extracting and then grouping like material could begin. The extracted data fell into categories of strategies, learning tactics, errors, and other pertinent information (e.g. organizational skills, neatness).[3] Having developed the Task Analysis Chart used in preparing this study helped me to become aware of the different alternatives and allowed, in Goldin's (1982) words, "the meaning of particular observations to be understood in relation to other outcomes which might have occurred" (p. 89). Because the emphasis was on examining the procedures used to solve problems, Tables 2 and 3 do not include errors or information not directly employed in answering the questions. These will, however, be discussed while reviewing the procedures and processes utilized.

The analysis began with an in-depth examination of Donald's work, including a general overview. A summary of Helen's and the other students' work follows. The general discussion will include observations made of all of the students' results, including some interesting features that stood out during the pilot study conducted earlier. Table 2 shows the summary of the solution procedures used by Donald for the first sixteen questions.

Donald

Donald completed Grade 8 in British Columbia almost seventeen years ago at the age of eighteen. He eventually left school because he "got to [sic] old". He wrote a placement test in 1991 and began his academic upgrading the following year. Donald said that he was weak in mathematics and that he always had problems in this area but never received any extra help. His mathematics teachers noted that, although a hard worker, he was easily distracted. Donald stated no physical impairments or psychological problems, but did comment that he had experienced some emotional turmoil in his home life. He did not however, consider this to be the cause for his difficulties and was accepted by SEL at AVC for extra assistance. A staff member confirmed that she considered him to be learning disabled. His goal was to be a computer programmer.

Donald said that he did not know if he enjoyed solving fraction problems but considered himself to be "okay" at doing them. The Woodcock Johnson Psycho-

Educational Battery (WJPEB) (Woodcock & Johnson, 1978) results suggested that this student was moderately deficient at reading and written language and severely deficient in mathematics for those in his age range. This was based on Donald's mathematics grade score of 7.4 and corresponding percentile in the range of 7-15. There is a considerable difference in the mathematics results and his reading and written language results. Records at AVC indicated that this student's grade equivalent for reading was 12.9. They also suggested that Donald did better if he talked aloud and I noticed that he did this during the comprehension segment of the WJPEB. The overall reading results of this test were seriously reduced because of his inability to pronounce nonexistent words as required in the *Word Attack* portion of it. Although the results for the Raven's Standard Progressive Matrices may have been slightly elevated due to following a "think-aloud" process, they signified that Donald was well within the range of average intellectual ability for those of his age since, despite having the opportunity to reflect on his choices, he did not seem to give much thought as to why he made them. He had very definite ideas as to what the answers were.

As can be seen from Tables 2 and 3, Donald used some distinct solution procedures and tactics to solve the first sixteen fraction questions presented. Those directly employed to perform the operations of addition, subtraction, multiplication, and division will be discussed first. It should be noted that he almost always used his fingers and tapped as he counted although this was not always evident on the tape. He also talked to himself a great deal. The figures presented of Donald's work were reduced to 64% of their actual size.

Table 2

Summary of Donald's Solution Procedures

Solution Procedures	Examples
1. Counting On to Add Student would add on using his fingers from the larger, and sometimes smaller number. For two digit numbers, he used this same technique but would write the numerals down and begin by adding to the units place first.	Q.1, Q. 2, Q.10, Q.16
2. Counting Up to Subtract Student would count from the subtrahend to the minuend using his fingers and monitoring how many were involved. He would write two digit numbers as he did with the addition.	Q.1, Q.5, Q.12, Q.16
3. Fingers as Objects to Subtract Student held up the number of fingers representing the minuend and bent the number representing the subtrahend.	Q.16
4. Adding Multiplicand to Multiply For simple, single digit numbers student would add repetitions of the multiplicand while keeping track on his fingers.	Q.1, Q.9, Q.10, Q.11, Q.12, Q.13, Q.15
5. Multiplying Longhand Student would multiply two digit numbers by writing them out if they had to be multiplied two or more times and his multiplication tables were unavailable.	Q.16

6. Adding Divisor to Divide Student would add a series single digit divisors, keeping track on his fingers until they equalled or surpassed the dividend. Two digit divisors would be written.	Q.2, Q.8, Q.9, Q.10, Q.12, Q.13, Q.15, Q.16
7. Multiplying Denominators to find CD With few exceptions, student would automatically multiply denominators that differed in order to find the common denominator.	Q.12, Q.15
8. Converting Up Even when a fraction had a denominator that was a multiple of the other, the student always "rounded" the "factor" fraction "up" rather than reduce "multiple" fraction.	Q.10, Q.13, Q.14

Table 3

<u>Summary of Tactics used by Donald</u>

Tactics	Examples
1. Vertical Presentation	Q.2, Q.5, Q.6, Q.7, Q.8, Q.10, Q.11, Q.12, Q.13, Q.15, Q.16
2. Little Multipliers	Q.11, Q.12, Q.13, Q.15, Q.16
3. Little Divider	Q.1, Q.3, Q.4, Q.6, Q.8, Q.10, Q.12, Q.13, Q.15, Q.16
4. Self-talk	Q.1, Q.2, Q.4, Q.9, Q.11, Q.12, Q.15
5. Longhand Division	Q.1, Q.2, Q.4, Q.9, Q.10, Q.12, Q.15, Q.16
6. Counting Around Numerals	Q.16
7. Little Arrows	Q.15, Q.16
8. Zeros as PLace Holders in a Division	Q.5, Q.12, Q.15

Counting-On to Add

Donald added most of his numbers by "counting on", usually with the fingers of one hand (with or without tapping) from the larger, and sometimes the smaller, number. He would often say the first number but not tap or not tap as loudly (i.e., he would not count this on his finger). He would start with his little finger as number one and count toward his thumb. If more than five fingers were needed, Donald would simply return to this starting position and continue by counting previously "used" fingers. On a couple of occasions Donald kept his hands from view, either by bending over them or by keeping them under the table, making it difficult to determine the exact method he used to count.

When at least one of the numbers being added was larger than one digit, he would write both of them down and incorporate this same finger technique by beginning in the units place and carrying to the next place if necessary. For example, in question #1, he added fourteen and seven by first counting on and tapping from four to eleven. The "one" was then carried and added to the other in the tens position, giving the total of twenty-one. Donald could have added seven directly to fourteen which would have been more efficient but he always resorted to writing these calculations regardless. A possible explanation may be that this student too often found instances where he encountered difficulties when trying to add two digit numbers and simply resorted to always writing them down so that he could deal with one digit at a time. It would be interesting to see how he would add a series of numbers containing two or more digits. If he had to carry, this number was always added after the addition of the addend and augend.[4] This was demonstrated in the transcript for questions #2 and #11 (see

Appendix D). This increased the chance of error because of forgetting to add this "carry-over".

While counting, Donald would have to monitor his thinking processes in order to keep track of how many "fingers" had been added. Clearly this method of addition was less efficient because of the greater number of steps and, thereby, more to remember, more confusion, and more chances for error. This is quite typical of children (see Fuson, 1988). By starting in most cases with the larger number, though, Donald required the use of fewer fingers and decreased slightly these risks.

He sometimes employed the tactic of counting around the physical features of the numeral (see Figure 1). He did this around the numerals two and five in question #16 (session #1), around the numeral four in questions #17 and #28, and in question #21 around the numeral three. He usually marked the numeral one and in question #45 he counted around a six in the same way as he did with the five by simply adding an extra point in the centre.[5]

Figure 1. Donald counted "around" some of the physical features of the numerals as indicated.

There were a couple of exceptions to this student's use of this finger method for addition. In question #2, for instance, he added six and six automatically (in twenty-six plus twenty-six). The cases in which he used this method involved the addition of two of a number less than ten. Donald seemed to realize that ten plus ten was twenty as shown when he added the whole numbers eleven and nine in question #20. He added the one to the nine--this is much easier for children than adding the other way around and something that Donald always did--and then added the resulting ten to the other ten rather then carry to the tens place as he did with other two digit sums. In multiplication Donald also used a series of additions that were automatic to a point but, of course, he was always adding the same number to itself.

By counting on, Donald had to represent the addend with his fingers. That is, he had to know how many fingers to count on and where to stop this process. He seemed to do this by knowing the correct finger position of this representation and counting on to this point. His kinaesthetic sense may also have been involved. When children first learn to count, they "count all". That is, they begin with zero and add each addition from this point. Donald's method is a step up from this process.

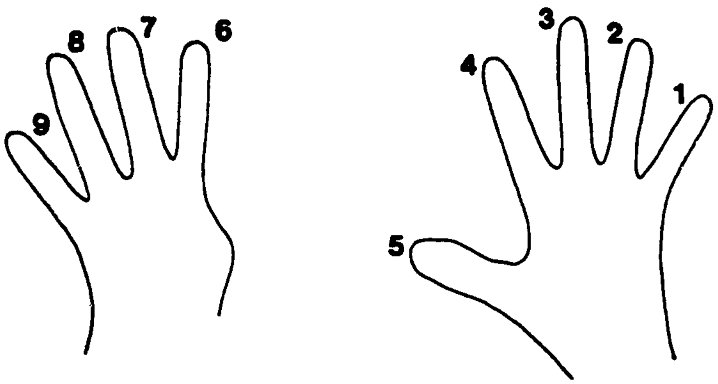
Counting-Up to Subtract

Donald subtracted single digit numbers by "counting up" using his fingers, usually with tapping, from the subtrahend to the minuend. For numbers of two or more digits, he would still count up but would write the numerals out and begin with the units place even if the two numbers were close. For example, in question #1 he subtracted eighteen from twenty-one by borrowing from the tens place and then counting up from eight to eleven in the units column. Only then would he go on to the tens place. As with addition, he wrote these numbers down. Sometimes he went back to count the number of fingers represented between the subtrahend and the minuend but at others this information seemed almost automatic as he subitized by looking at the fingers counted. His kinaesthetic sense perhaps also guided him at times as occasionally he did not appear to even look at his fingers. Donald could still have counted on his fingers but increased his ease of doing so by counting up from eighteen to twenty-one. However, by beginning with the unit place, he would never have a difference greater than could be represented with the fingers of his two hands. This is an indication of some thought either at a conscious or unconscious level. As with children, it was probably easier for him to count up rather than count backwards which would have been the case had he started from the minuend and counted down to the subtrahend. Counting aloud in this procedure allowed him to simply listen for the minuend, in this case eleven, and then observe or count the finger pattern, each finger representing a difference of one. Naturally this student would have had to know what numbers to start and end with. For example, if in question #5 (i.e., $^{22}/_{29}$ - $^{17}/_{29}$), he had started by counting seven in the units column, Donald would have had to end with eleven. However, he began at eight (i.e., he said but did not include seven) and counted to twelve, inclusively. Of course it would have been easier to start with the whole two digit number rather than in the units place.

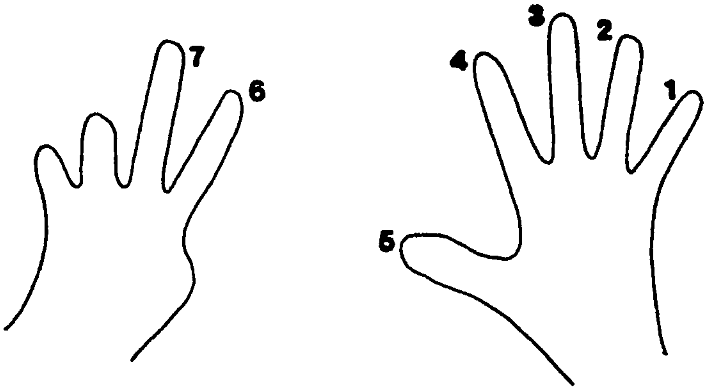
As with addition, there were a couple of exceptions to this method of subtraction. In questions #16 and #22 Donald held up the number of fingers representing the minuend and then "took-away" by bending the number of fingers representing the subtrahend to reveal the pattern left. Figure 2 illustrates how he subtracted two from nine in the first question. The second question is a good example of how Donald used his fingers as objects to subtract four from seven when trying to determine the total difference between twenty-seven and four. This method is more primitive as children first learn to subtract by manipulating objects.

Donald used an interesting strategy in question #23 to subtract twelve from twenty-three in his estimate by utilizing the fact that twelve and twelve were twenty-four, so that twenty-three minus twelve had to be equal to eleven. This required some thought and level of abstract thinking since he would have had to recognize and assimilate the difference in operation and be able to reason that if twenty-three was one less than twenty-four, the answer had to be one less than twelve. When he worked this question through for the second time (the first being the estimate), he reverted back to writing the numbers down and simply subtracting as usual. Clearly he could have utilized the information he had just obtained but either he chose not to or he did not think to do so. He seemed to completely separate the act of estimating from that of calculating.

Figure 2. Subtracting two from nine.



a) Donald began by holding up nine fingers using both hands.



b) Donald "takes away" two fingers to leave a "seven" pattern.

Adding Multiplicand to Multiply

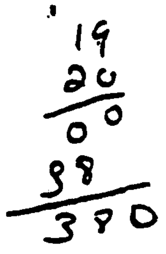
Like most children with learning disabilities (Cardoni, 1987), Donald could not remember his multiplication tables and had to resort to other methods to perform this task. He used different procedures depending upon the number of the digits involved.

If the multiplicand was a single digit of five or less, Donald would add it as necessary by usually tapping and keeping track of the number of times on his fingers. Sometimes he would add the larger number fewer times but on other occasions he would add the smaller number more often. An example of the first instance would be where he multiplied six times three in question #1 by adding six three times. Adding five seven times in question #11 is an example of the latter (see Appendix D). It seemed unusual for him to choose to add the six rather than the three and in one instance he even chose to add three nine times rather than the other way around, probably because this "larger" number was too difficult for him to handle as it would have very quickly involved two digits. It seems unlikely that he would have been able to make use of the fact that nine was one less than ten. Still he had trouble adding threes once he reached the sum of eighteen. Part of the explanation may relate to the fact that twos and fives are amongst the first numbers elementary students learn to add[6] (Alberta Education, 1990) and they are numbers Donald seemed to manipulate most easily. Rather than making a conscious choice of which number to add, however, he probably chose simply to add what he considered to be the multiplicand in the question as the commutative property of multiplication was one with which he still appeared to be struggling. This was suggested by his surprise that nineteen times four in question #16 was the same as four times nineteen and his lack of utilization of this property and knowledge of reversibility.

Donald also multiplied nineteen by twenty by multiplying first by the zero and then by the two and adding these totals in the usual longhand method (see Figure 3). He could have added nineteen twice which would have required fewer steps but again, was not aware that he could do this and then "add on" a zero to multiply by ten. This shows a lack of knowledge of the associative property of multiplication.

Multiplying single digits by adding these and keeping track on his fingers meant that this student not only had to know how many he had added, but he also had to be able to sum this number easily so that he could keep track and determine the new totals as he proceeded. Donald seemed to be unable to think of new approaches to solve a multiplication. In question #17, for example, he thought that five times nine was fifty. When asked what five times ten was, he realized his mistake but rather than simply subtract five from fifty to get the answer, Donald resorted to adding a series of nine fives.

Figure 3. Donald used longhand multiplication to multiply multiples of ten.



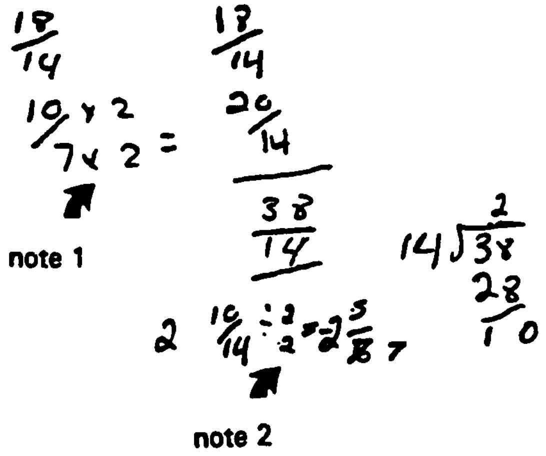
When this student had trouble multiplying six and eight each by three in question #20, he resorted to another method to accomplish the task. He began with twelve and sixteen respectively and counted on six and eight for each, one at a time, using his fingers. It is interesting that when he multiplied eight times three this way and arrived at the answer of twenty-four, he immediately noted that "six times four is twenty-four". He seemed to momentarily forget this because a short time later he said that six times two was twenty-four (he did correct himself). It appeared that the former calculation definitely had a positive effect but shows his lack of aptitude with multiplication.

Donald often resorted to his tables--one of which contained many two digit numbers--or multiplied numbers out longhand if they were otherwise unmanageable. For example in question #12 he multiplied eighty times two this way and used the result to check his earlier written addition of eighty to itself. He seemed to want to make sure that eight times two was really sixteen but was unable to make use of the fact that eighty was simply eight times ten.

Donald would verbally state the multiplicand and the number by which it was to be multiplied, even for very simple calculations such as in question #2 where he multiplied three by one, and he always used the tactic of writing out little multiplication signs beside the numerators and denominators when multiplying fractions. This would have helped keep him on track and prevent confusion. It was interesting how he used two of these signs when multiplying and only one division sign for both the numerators and denominators when reducing fractions (see Figure 4). This figure also illustrates Donald's somewhat haphazard use of operation signs. It may have been a factor leading to his incorrect use of a negative sign in front of his final answer since the question was one of addition.

Donald utilized a form of multiplication when dividing but these will be discussed in the section to follow. The difference between the two was that with multiplication he knew how many times the multiplicand had to be added, whereas in division he had to determine how many divisors were required.

Figure 4. Donald's use of little multipliers and a little divider.



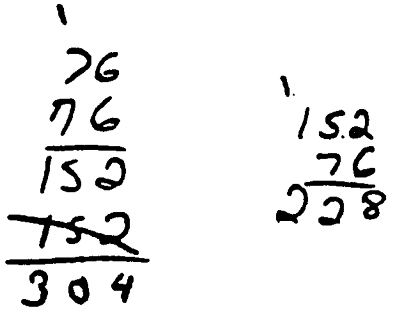
Adding Divisor to Divide

As mentioned earlier, Donald employed various multiplication procedures in his overall division method depending on the size of the divisor. If the divisor was small he would use his fingers to add it to itself until its total equalled, or was greater than the dividend, but divisors of two or more digits were written down.

Donald used longhand division for every instance where an answer resulted in an improper fraction, even in cases where it was obviously unnecessary such as in question #4 where he divided four into six, in question #9 where fifteen was divided into nineteen, and in question #25 where it was used to divide twenty-six into twenty-seven. Perhaps simply subtracting the divisor from the dividend to obtain the remainder (and thus the numerator) without first dividing was too big a step in abstract reasoning for this student. The use of long division was not automatic for improper fractions imbedded within the questions, even in some instances where it could have been advantageous to do so as in question #37 $({}^{1}/_{5} + {}^{4}/_{2})$ or question #33 $({}^{43}/_{4} - {}^{12}/_{2})$. Admittedly the latter requires more reasoning ability. This neglect is probably typical of many adults since all individuals experience "mind sets" to some extent and because searching for details within the questions themselves appears to be something not taught in the schools. The majority of people examining these questions would probably not even notice, for example, that the second fraction of each question could be reduced.

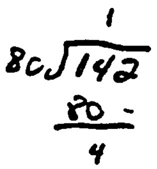
Larger divisors were usually added one at a time in order to determine how many would go into the dividend. In question #2 he used this procedure to divided seventy-eight by twenty-six by adding twenty-six twice, obtaining a subtotal, and then adding twenty-six to this total when he found it less than the dividend. He then went back and counted how many times he had added twenty-six to itself. Donald varied this method slightly in question #16 by adding the divisor to itself and then adding this result to the subtotal. When he found this too large, he went back and added seventy-six to the first total (see Figure 5).

Figure 5. Determining how many times 76 went into 300.



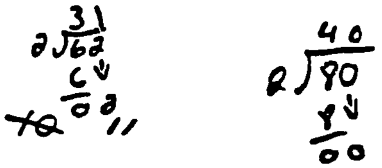
Donald seemed unable to manipulate much information without writing it down or using his fingers. He also sometimes had difficulty using the longhand method. For example, in question #12 he misaligned the vertical columns of numbers when he put the number eighty under the hundreds and tens digit places instead of the tens and units places (see Figure 6).

Figure 6. Misaligning the digits while dividing eighty into one hundred forty-two using the longhand method.



Using little arrows to indicate numbers that had been "brought down" and zeros as place holders in front of remainders as he usually did, helped to overcome some of the chances for error by keeping Donald in control (see Figure 7 and Figure 11, note 2).

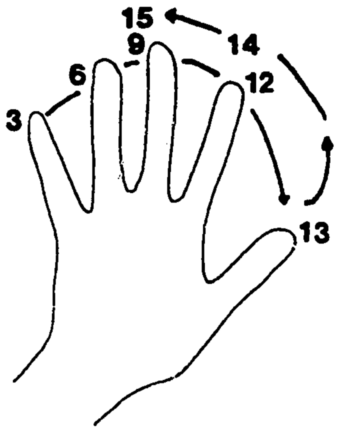
Figure 7. Donald's use of arrows and zeros in longhand division helped keep him in control.



Donald sometimes became confused because he did not know where to begin to divide when the number of digits in the divisor did not match that of the dividend, he and was unable to break away from some routines to think for himself. In question #2, for instance, he proceeded with the division although he had already determined the answer of three after adding twenty-sixes three times as previously discussed (see work next to question #4 in Appendix D, Figure 1).

Sometimes, if Donald had trouble adding the series of dividends, he would vary his method by adding ones to the total he had already counted until he reached or surpassed the dividend. In an attempt to see if three would divide into fifteen, for example, he began to add a series of threes. When he had added four, reaching a total of twelve, he resorted to adding three ones to this, each represented by another finger, giving him the correct sum of fifteen but the wrong number of fingers. Figure 8 shows how this was done and the two errors that resulted. One mistake was in forgetting that three ones was equal to only one three. This mistake was compounded when he failed to count his thumb twice as would be necessary to get an accurate total when counting the fingers of one hand only. If Donald had started with his thumb, then he would have had to count his little finger twice to help ensure a correct figure.[7] Donald concluded from this method that three divided into fifteen eight times. His reasoning can be deduced from the finger pattern indicated.

Figure 8. The method Donald used to divide fifteen by three. He concluded that the answer was eight.



In question #12 Donald attempted to divide two into sixty-two by counting by twos. It appeared that he was wanting to track how many groups of ten he would need. He counted up to twenty, stated "that's ten" but gave up at this point because he simply could not "do it". Undoubtedly this was far too much for him to manage. Possibly writing something down may have helped, but he apparently did not think of this, or decided against it, and finally resorted to using longhand. In question #10 (see Figure 4), Donald managed to multiply seven by two without any trouble (see note 1) but only a short time later was unable to count accurately using his fingers (it sounded as though he missed a count) and ended up with an answer of six when dividing fourteen by two (see note 2) Obviously he also lacked knowledge of the property of reversibility for multiplication.

Self-talk

Donald almost always resorted to talking to himself while attempting to answer the questions. This was quite distinct from his verbalization of thoughts in which he stated what he was doing. He seemed almost to get lost in these other deliberations as he whispered very quietly. Most of the time he seemed to be verbalizing in order to "think more clearly" but in question #1 he used it to correct a mistake. A study by Meichenbaum and Goodman (1971) indicated impulsive children's performance improved as they were taught to talk to themselves. He suggested that this was because it increased each child's self control. It was believed that Donald performed better when allowed to verbalize and this was the reason he was given permission to write exams secluded from others. I noted that he also talked to himself during various segments of the WJPEB.

Searching for a Common Denominator

Whenever the denominators differed and when one was not a multiple of the other, Donald always obtained the common denominator (CD) by multiplying the two together. He never made any attempt to find the lowest common denominator (LCD) although this would not have been very difficult in many instances. For example, the LCD in question #12 ($^7/_8$ + $^9/_{10}$) would have been forty but Donald simply multiplied eight times ten to get a CD of eighty. He did not appear to have any notion of the concept of lowest common denominator which meant that he was sometimes working with numbers that were very large and difficult for him to manipulate. It may have also been an indication of greater reliance on memorized steps requiring less thought and comprehension. Even when one denominator was a factor of the other and the other fraction could be reduced so that both denominators were the same, Donald always chose to "round" up the fraction with the smaller denominator. As mentioned earlier, this seems typical of many people, learning disabled or not, and may be more a reflection of the way students are taught to handle these problems rather than the students' abilities. We are all rule bound to some extent; it is just a matter of degree that each of us is affected.

In question #11 ($^5/_9$ + $^3/_7$) Donald insisted that there was no CD since neither two nor three would divide *into* both of the denominators. This was probably the most difficult question to that point since the denominators were different, odd numbers that were relatively large and harder for Donald to manipulate and, unlike question #10, neither was a multiple of the other. On three occasions, when asked what he was looking for, Donald stated that he was looking for a number that would divide into both denominators. When asked if he knew what a common denominator was, Donald said, "a prime number" even after a brief explanation telling him that he needed a number that both denominators would divide into and not a factor of both of them. It is possible that students were shown a procedure for finding the LCD's which involved the identification of prime numbers, but there was no substantive evidence to show this to be the case here.

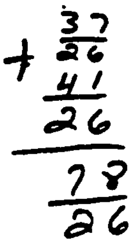
Vertical Presentation

When asked if he found it easier to set up the question vertically, Donald responded with a simple, "Sometimes". It was noted, however, that he re-wrote most of the questions this way. Among the questions he did not re-write vertically were those for which he attempted to give an estimate (i.e., #1, #9, and #14) and some of the most simple questions with the same denominators (i.e., #3, #4, #25, #30, #35, #41, and #46). Still, he re-wrote even a few of these latter type vertically. A good example of this is question #52 (i.e., $^4/_5 - {}^1/_5$).

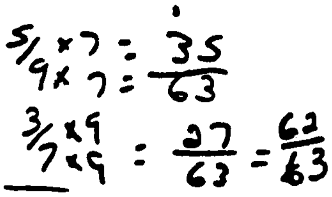
Re-stating the questions vertically may have been a good tactic for Donald as he and some of the other students suggested. Perhaps the proximity and alignment of the numerals facilitated the performance of the operation by allowing both numbers to be seen at one time and by lessening any possible confusion about which place values were being added or subtracted. However, it probably would have been even more helpful if he had been consistent in the way he did it. Sometimes Donald wrote the correct answer below the question while at other times he would write it next to the lower fraction with an equal sign in between this fraction and this answer. This was somewhat risky since he could have mistaken it as an equivalent fraction rather than the answer. Occasionally he simply wrote his answer next to his last calculation, sometimes in the middle of the page, beside other work which made it often difficult to see. In two of the questions, he overcame this problem by identifying the answer by putting a circle around one and a square around the other. I thought the most reasonable format was one where Donald aligned the second fraction under the first with the answer, or intermediate answer, written just below this (see Figure 9a).

However, any format chosen by him would have been acceptable if it had been done in a consistent and systematic fashion (i.e., neat, organized, well spaced, well written numerals) that was helpful to him (See Figures 9b, 9c for other examples of formats used by this student). Working in this somewhat less systematic and consistent way may not have been a concern for students who had little difficulty, but Donald became confused when juggling material and needed to be especially careful. I believe that greater congruence in this student's work would have helped him to increase his understanding by allowing him to see more clearly how things "fit" and to think about what he was doing.

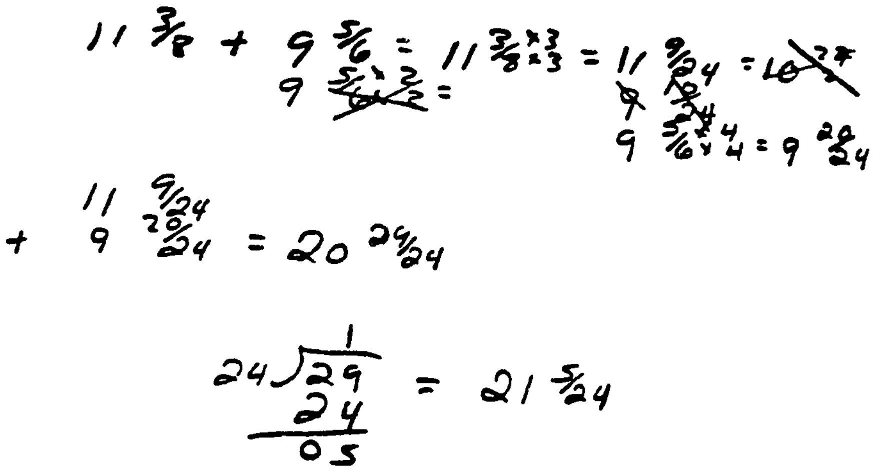
Figure 9 Examples of Donald's vertical formats.



a) A reasonable vertical presentation.



b) The way Donald typically presented his work.



$$11\tfrac{3}{8} + 9\tfrac{5}{6} = 11\tfrac{3\times3}{8\times3} = 11\tfrac{9}{24}$$

$$9\,\cancel{\tfrac{5\times2}{6\times2}} =$$

$$9\tfrac{5\times4}{6\times4} = 9\tfrac{20}{24}$$

$$\begin{array}{rr} & 11\tfrac{9}{24} \\ + & 9\tfrac{20}{24} \end{array} = 20\tfrac{29}{24}$$

$$\begin{array}{r} 1 \\ 24\overline{)29} \\ \underline{24} \\ 05 \end{array} = 21\tfrac{5}{24}$$

c) A confusing format for anyone retracing Donald's steps. Again the final answer was written next to the last division.

General Overview of Donald's Work

Some of Donald's difficulties stemmed from what many would probably have labelled a result of carelessness. Others were related to operation, multiplication facts, conceptualization, procedural knowledge, and organization.

One of Donald's biggest problems was due to his lack of mathematical factual knowledge and understanding. For example, he could recall very few multiplication facts and seemed at times unsure or unable to make use of the commutative and associative properties of multiplication as well as the reversibility principle. These, along with the ability to multiply first by tens and hundreds, are usually mastered by the end of grade four (Alberta Education, 1990). When questioned about his answer of thirty-four when seven and five were multiplied in question #11 (i.e., $^5/_9 + ^3/_7$), for instance, he said that he was not aware that numbers multiplied by five had to end in either zero or five. This need affected his ability to solve the questions efficiently because he could not use the knowledge to estimate answers and had to resort to more cumbersome and less efficient methods with which to do the work. He identified multiples by counting as much as possible by small numbers but tended to use his tables when these were available. Because the capacity of the short term memory (STM) store is limited, much of it was used manipulating material required in the performance of Donald's inefficient procedures. This meant that he was often unable to accurately manipulate and keep track of other data involved in the overall solution process. A disorganized format and dependence on rote learning only exasperated this situation. He spent a much longer time than would have otherwise been needed to find quotients and solutions in general. In question #29, for example, he had a hard time trying to decide how many times thirty went into two hundred twelve because his tables did not go as high as thirty. He began by multiplying it by four before proceeding with five through seven using the longhand method. It did not seem to occur to him to use three instead of thirty as the number being multiplied and then multiply this result by ten, probably because he did not understand the associative property. Of course he had to also have the understanding to "put it all together" and be able to remember and organize what he had done, particularly if he was doing the actual calculation rather than just making a guess. There were many instances where the problem could have been made simpler--question #2 (i.e., $^{37}/_{26}$ + $^{41}/_{26}$)--if Donald had been able to do the multiplication. His lack of multiplication facts also made reducing difficult. He did not realize that $^{4}/_{15}$ nor $^{4}/_{19}$

could not be reduced until he added five fours to establish that this numerator would go into neither fifteen nor nineteen.

In the second session, where question #16 was again presented, Donald used longhand division to determine how many times nineteen went into twenty. He did this when prompted to find an easier method of answering the question. Unfortunately, he seemed to remember little of the question itself and nothing of the review from the previous week.

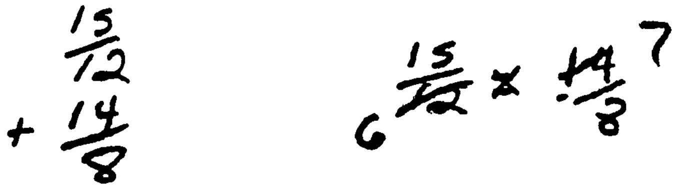
The whole concept of estimation seemed foreign to this student. Whenever asked to estimate, he would always begin by looking for a common denominator even though a more direct, and to most probably a simpler, method was demonstrated. In some cases he made no attempt at estimating because he claimed that there was no CD. In question #16 (i.e., $^{20}/_{4}$ - $^{20}/_{19}$) he eventually concluded that the answer would be a whole number since twenty minus twenty was zero. Donald was prone to making more mistakes this way as illustrated in question #9 ($^{11}/_{5}$ + $^{8}/_{3}$) where he simply added the numerators without regard to changes in the denominators. He was confident enough with this estimate that he proceeded with it when doing the calculation.

Although it was true that the nature of the questions may have fostered a somewhat limited approach to answering each question, there was much more "room" for thought and understanding than was demonstrated. For instance, Donald could have chosen to reduce some fractions before adding or subtracting. Rather than having the ability to plan a strategy to a particular question, he relied more on routines, processes that he performed habitually but did not fully comprehend and to which he gave little thought. Some steps or items were recognized by him which suggested that he understood these parts more readily than others in the process. This was especially evident on a few occasions when he stressed that his result was an improper fraction. It seemed to be a very important point, "triggering" the next steps he was to take. In these cases he always knew that he had to divide the denominator into the numerator. It was one thing of which he was certain. He attempted to monitor his work but was often overcome by the burden of being forced to juggle and retain a great deal of information.

Donald's work was somewhat disorganized and his terminology was not always correct. For instance, in question #6 he said, "thirty *from* fifteen minus twenty *from* fifteen" when he meant thirty and twenty each *over* fifteen. Sometimes he would say things like, "bring it to the whole" when he meant "reduce" the fraction. Occasionally he would say the wrong thing but write it correctly and vice versa. Two examples are where he states, "multiply six hundred and eight" instead of "subtract six hundred and eight "and when he said, "ten and twenty-four" instead of ten twenty-fourths. I believe many students are "slack" in the terminology they use, at least on some occasions. Donald may simply have been demonstrating this phenomenon. However, it could also indicate an incomplete understanding of part/whole relationships or reduction of fractions altogether. His "confusion" when writing or speaking could suggest something more "going on", but the fact that he did this only seldom probably means that it was nothing more than a simple form of error. However, further investigation would be required to confirm this. Another interesting observation was his general use of the terminology "divided into" rather than "divided by". Many other students, especially in the pilot study, also seemed to find these terms easier to understand.

In question #48 $({}^{15}/_{12} - {}^{14}/_{8})$ Donald cancelled between the twelve and fourteen because he misread a "cross" he had used to "delete" an equal sign which he had initially written by mistake (see Figure 10). Sometimes he would write over numbers rather than erase them if they were wrong, especially when he was anxious; at least on one occasion his operation sign was covered up with other work. He accidentally circled part of the mixed number rather than the question number for question #28. Generally, however, his work was quite neat and well spaced.

Figure 10. Donald tried to cross cancel in an addition question. Here is an instance where use of an eraser would have been advantageous.

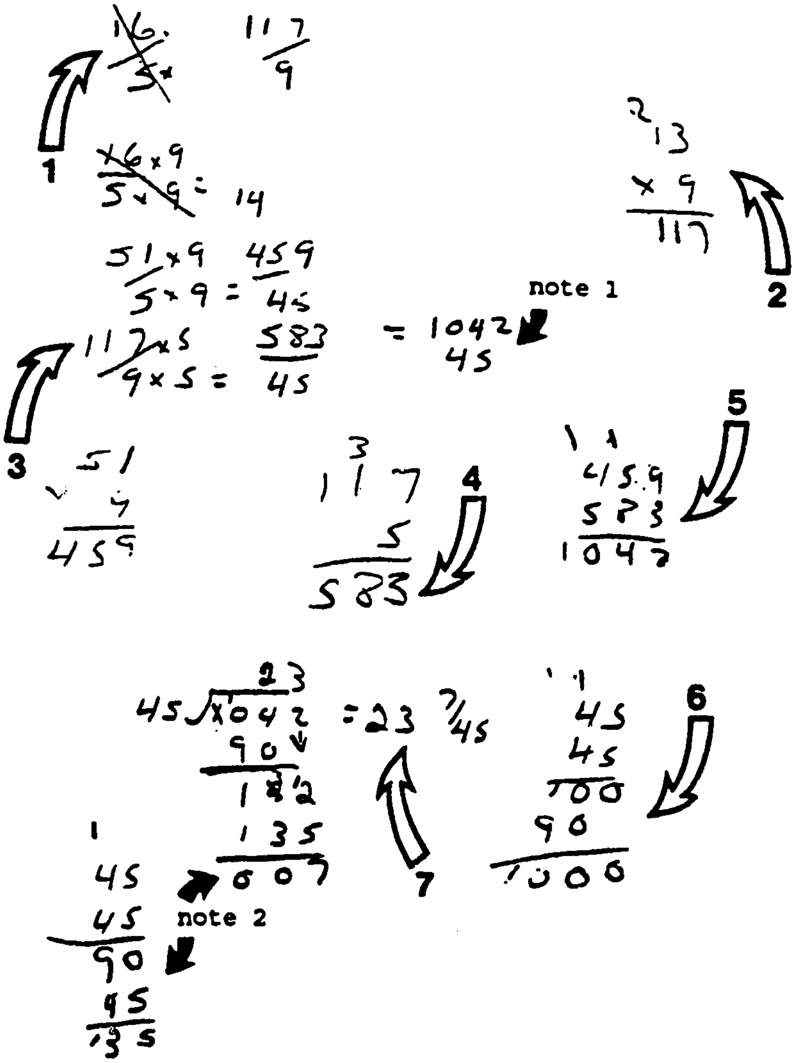


Many of Donald's errors could have been avoided if he had been able to retain the "whole picture" instead of just some aspects of it. Sometimes he seemed to be distracted by the other information impinging on his memory capabilities. Some wrong answers resulted because he copied incorrectly, multiplied only part of the number (e.g., 15 X 4 = 20), or performed the wrong operation as was the case in questions #7, #10, and #12. Occasionally he thought he knew the multiplication without working it out (e.g., 4 X 3 = 16).

Figure 11, which shows this student's work on question #17, illustrates many of the problems he had along with the types of errors that resulted. To begin, it probably would have benefitted Donald to have written the question out. This may have helped him to avoid at least some of the errors. His first statement when presented with this question was, "Ooooooooo ten times five is fifteen and one is sixteen over five"--also his first mistake (first arrow). His next was multiplying thirteen by nine without adding the numerator of seven (see second arrow). He was at this point asked to check his work and multiplied five times ten by adding five ten times. He corrected his mistake but neglected to check his other figure (see third arrow), and did not include an operation sign. Donald proceeded to add these fractions by multiplying each by the denominator of the other. His next error resulted because of a poorly written numeral. This occurred as he multiplied one hundred seventeen times five (see fourth arrow). He obtained the correct answer for this multiplication but his five looked so much like a three that he added five hundred eighty-three, instead of five hundred eighty-five, to four hundred fifty-nine (see fifth arrow). He may not have made this error if he had an easier time with the multiplication. After struggling with each part of the calculation of one hundred seventeen times five, Donald hesitated in order, it seemed, to get his thoughts together in an attempt to recall what he was supposed to do next. He used this result (i.e., one thousand forty-two) over forty-five (note without the dividing line) (see note 1), to represent his intermediate figure.

At this point Donald needed to determine "1042 ÷ 45". He tried his usual strategy of writing and then adding the dividend to itself as necessary. It is not totally clear what he did, but it seems as though he was using a multiplication format to do the addition. His end result was "1,000" (see sixth arrow). He realized this was wrong but used the figure anyway. It was suggested that he do the calculation again after he stated that forty-five times two was one hundred but hesitated when was asked the result of fifty times two because he recognized this as one hundred. He managed to complete the question but left the answer beside his last calculation (see seventh arrow).

Figure 11. Many mistakes were made while Donald worked out the question $10^1/_5 + 13^7/_9$.

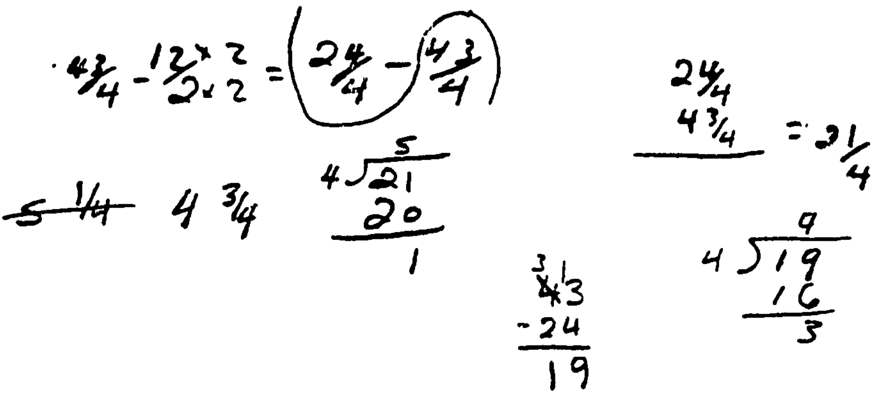


Donald had a very difficult time and became quite frustrated while working on questions #11, #12, #15, and #16. He became even disoriented at times. During the course of working out question #12, he lost track of what he was doing a few times, as he became engrossed in the more tedious work. He was confused when reducing $^{62}/_{80}$ and noted that "eighty could not go into sixty-two" but thought that was what he had to do since eighty was the denominator. This relates back to his recognition of improper fractions and "reflexively" dividing by the bottom number. At one point he began to multiply eight times two instead of dividing eighty by two in order to reduce. As mentioned earlier, he asked if sixty-two was divisible by two and then wanted to know by how many times.

Donald did not automatically reduce fractions and had to be reminded in questions #1, #2, #3, #10, and #13 to do so. He did not know that twenty-eight from thirty-eight was ten until he actually carried out the subtraction starting with the units. Perhaps this relates to a lack of confidence. He could not identify which fraction was bigger in each of these questions and it is questionable as to whether he understood part/whole relationships.

Figure 12 illustrates the trouble this student had subtracting fractions. He began by first writing the fractions in reverse and then trying to subtract by "taking" the smaller digits away from the larger regardless of whether they were part of the subtrahend or minuend (i.e., first three from four and then two from four). It is difficult to find his final answer in this figure.

Figure 12. Donald made a couple of errors while attempting to answer this subtraction question.



Although Donald made several errors, these tended not to be systematic. Even so, most were related to his use of less efficient methods and lack of, or incomplete knowledge, especially of multiplication facts. Except in the odd case, they were not a result of wrong procedures but were, rather, probably because these procedures required more memory space, manipulation, and time. They presented more opportunity for error to occur and allowed less opportunity for understanding.

There were other examples of Donald's difficulties but the emphasis was on his methodology since this was what became most apparent in the data analysis. Obviously there could be other interpretations of his work and a copy of it for the first sixteen questions (see Appendix D, Figure 1), along with the corresponding transcripts (see Appendix D) will allow the reader to draw his or her own conclusions.

Donald was a hard worker, persevering despite the obstacles that limited his understanding. I believe he could make tremendous improvements with the aid of a calculator and some short-term assistance.

Summaries of the other participants' work follows.

Helen

Although failing Grade 7 three times, Helen went on to complete Grade 11 and part of Grade 12 in 1981, reaching the math 15 level. She even attended the Northern Alberta Institute of Technology (NAIT) in 1989 but an accident led her to take a placement test at AVC in 1992. That year she began her academic upgrading.

Helen had just turned twenty-eight when she wrote the WJPEB in September, 1992. The results indicated that she was moderately deficient in reading and mathematics and severely deficient in written language with percentile ranges of 16-22, 12-23, and 4-6 respectively and corresponding grades of 9.8, 8.4, and 4.7. Her mark on the written test was considerably reduced because of a low score on the *Usage* portion of the *Dictation* subtest.

Results of tests done by an outside agency showed Helen's FSIQ on the WAIS-R to be in the low average range. Numeracy on the GATB (form B) was her lowest score. The WRAT-R showed spelling to be Helen's worst area. Raven's Standard Progressive Matrice's results indicated that this student had a low average to below average intellectual capacity with a score around the 25th percentile. Helen, however, seemed a much more conversant and capable person than these results suggested.

Helen stated that she enjoyed solving fraction problems and considered herself to be fair at doing so. Although she suffered no physical or psychological problems, she did note an inhibiting test anxiety. This student had received some assistance from SEL.

Her goal, like many attending AVC, was to become a social worker.

Helen's Work Summary

All of the questions were written by Helen horizontally and, except for question #16 and #28, the actual figuring was done without writing anything down; she usually retrieved the answers directly from memory.

Helen did not always know what to do when reducing proper fractions. She said that she knew she had to reduce for sure if the numerator was bigger but was not sure if it was smaller. This was confusing for Helen and sometimes she would divide the numerator into the denominator and arrive at a mixed or whole number rather than an equivalent fraction. The first signs of this were in question #3 when she initially "reduced" $^{5}/_{15}$ to three and did not know, in question #6, if she should reduce $^{10}/_{15}$ because the numerator was smaller than the denominator. In question #11 Helen reduced $^{62}/_{63}$ to $1^{1}/_{63}$. She divided sixty-two into sixty-three but it seems that she knew the denominator should have been sixty-three so she left this in place. In question #12 she commented on "$^{72}/_{80}$" saying that "it could be reduced but I'm not sure whether I should do it or not because I always think that if the number isn't bigger than the denominator it doesn't have to be reduced". Helen did not reduce $^{6}/_{30}$, possibly thinking that thirty was an odd number. She sometimes spent quite awhile trying to find a factor greater than two for both the numerator and denominator even though she knew it would work and, in question #17 ($10^{1}/_{5}$ + $13^{7}/_{9}$), tried two, three, and four as divisors before declaring that $^{44}/_{45}$ could not be reduced. She also had problems deciding if $^{19}/_{60}$ could be simplified. Lack of factual knowledge was perhaps part of the problem but so was a poor reasoning ability. At first Helen thought that $^{21}/_{24}$ in question #20 would not reduce since one was odd and the other even but she checked her work and realized that it could be simplified. She had no problem reducing some simple fractions such as $^{4}/_{6}$ and did so without hesitation. The students were asked at the start of the sessions to reduce all fractions and the reasons for doing so were discussed at some point with those who demonstrated difficulty for doing so.

It is interesting to note, and perhaps there is a connection, that Helen often would verbally reverse the numerator and denominator. Rather than saying eighteen fourteenths and twenty fourteenths in question #10, for example, she said fourteen eighteenths and fourteen twentieths. She reversed $^{35}/_{63}$ in question #11 and $^{19}/_{20}$ in question #16 (in both session #1 and #2). In question #20, she said eleven twenty-fourths over nine when she meant eleven and nine twenty-fourths, she changed $^{27}/_{26}$ to $1^{1}/_{27}$, and she commented that she was thinking of $^{2}/_{4}$ when she gave an estimate of $^{1}/_{2}$. Once in awhile she also appeared to get other numbers confused. In question #20, for instance, she stated that $9^{5}/_{6}$ was equal to $9^{36}/_{24}$. It may be that she was thinking of nine times four instead of five times four as the numerator. For the same question she stated that she had put "eleven twenty-four over nine" when she was referring to $11^{9}/_{24}$.

Helen's addition methods were much more efficient than those used by Donald and most of the other students. Usually by truncating one or more of the numerals (and occasionally rounding up) to the nearest tens place, before adding these results plus the numbers she had previously "cut off". This seems to be very similar to what Bisanz and Lefevre (1990) refer to as decomposition. In question #2 $^{37}/_{26}$ + $^{41}/_{26}$), for instance, she added thirty-seven and forty-one by first adding forty to thirty-seven and then adding the one. Often she would total single digit numbers before adding these to the larger result. An example of this was when Helen added the numerators of thirty-five and twenty-seven by first grouping their unit totals as follows. She started with thirty, then added this to ten (five from the thirty-five, plus five from the seven in twenty-seven) to get forty, added another ten (from twenty-seven) to get fifty, and finally added twelve (the last ten and two from the twenty-seven), which resulted in the correct answer of sixty-two. Although the addition was done in error, she also added twenty-two and seventeen by first adding ten to twenty-two and then adding the seven.

When dividing, Helen would alter the figures to make them easier for her to work with. An illustration of this was when, in question #2 she viewed the denominator as twenty-five, added it to itself first to get first fifty, then seventy-five before adding the three which was a result of adding the one (1) she had taken off, three times. This involved keeping track of considerable information but was much easier to work out than by actually trying to divide by twenty-six. In question #16, Helen divided seventy-six into three hundred by knowing that seventy-five twice was one hundred fifty and that one hundred fifty times two was three hundred. This meant that the answer was less than, but close to, four. She must have kept tally ot how many times she added the number and whether she needed to add or subtract from the total to obtain the final answer. In this case, Helen knew that she was three short of seventy-five, so her numerator would be seventy-two. To reduce $^{72}/_{78}$, she saw that nine times eight was seventy-two, realized it would not divide into seventy-six but seemed to have the idea that maybe four would work as a divisor and was able to simplify the fraction by reasoning that if four times twenty was eighty, then four less was seventy-six and another four less was seventy-two so that the denominator and numerator would be nineteen and eighteen respectively. It is intriguing that she originally multiplied nineteen, and then its result, by two to get seventy-six but wondered if seventy-six was divisible by four. When finally reducing ${}^{72}/_{76}$ she did not seem to remember this fact or realize that it could be useful, perhaps another possible indication of a poor reasoning ability.

When this student could not remember the result of a multiplication, she would employ one that she did know and would work from there by adding the multiplicand as many times as necessary. In question #12 (${}^{7}/_{9}$ + ${}^{9}/_{10}$), for instance, she needed to multiply eight times eight but resorted to starting with eight times seven and adding eight onto fifty-six. She continued to add eight to first make seventy-two and then eighty in order to determine the common denominator for her estimate. When trying to figure nine times five in question #17 ($10^{1}/_{5}$ + $13^{7}/_{9}$), she started with nine plus nine as eighteen and then added successive nines to this total before stating that nine times five was forty-five. In another question, she wanted to know nine times six and simply added six to forty-eight because she remembered that six times eight was forty-eight (at first she stated that six times seven was forty-eight).

When adding ${}^{3}/_{4}$ + ${}^{3}/_{4}$ in question #3, Helen helped herself by relating these to real life experiences. She retrieved the answer immediately from memory because she recognized it as familiar and knew that three quarters coinage was $1.50 or, in fraction terms, $1^{1}/_{2}$. She was apparently used to manipulating this "${}^{3}/_{4}$" figure, even in different forms as suggested by her use of the divisor seventy-five rather than seventy-six as discussed earlier. This demonstrated at least some ability for abstract reasoning.

Helen often checked her calculations at the end of the question but tended to neglect to examine other areas, often resulting in mistakes. She seldom reviewed, for example, work in progress or paid much attention to what she was supposed to be doing such as illustrated in question #5 ($^{22}/_{29}$ - $^{17}/_{29}$) when she performed the wrong operation. It was the first subtraction question and she continued to add. Helen almost forgot the whole number in question #9 ($^{11}/_{5}$ + $^{8}/_{3}$) and would occasionally leave out equal signs, explaining in question #10 ($^{18}/_{14}$ + $^{10}/_{7}$), for instance, that she was "too busy figuring it out". She also had difficulties with questions that had "odd" denominators like $^{1}/_{3}$ and $^{1}/_{9}$ and explained in question #19 that they were odd "because it's not $^{1}/_{4}$ or $^{1}/_{2}$". Some of this student's mistakes suggest that she was following rote procedures, paying little attention to what she was doing since some answers were very clearly incorrect. Question #12 ($^{7}/_{8}$ + $^{9}/_{10}$) contains several repeated mistakes which resulted as she attempted to add these fractions. It took her four attempts to get the correct numerators for the first fraction and this was after suggestions that she check her work. Some of these errors probably resulted from her "twisting" numerals around. Since Helen chose a CD of eighty, her numerator for the first fraction should have been seventy. She used, in order of occurrence, fifty-six (8 X 7), eighty (8 X 10), and sixty-three (9 X 7). She noted the correct answer after being told that sixty-three was still not right. Reasoning should have allowed her to see that eighty was incorrect since $^{80}/_{80}$ was obviously not equivalent to the first fraction and the answer of $1^{9}/_{10}$ was also clearly wrong, but she was moving along very rapidly and thinking little about what she was doing. This may not be a reflection of her reasoning ability at all. It may just be that she did not allow herself the privilege to stop and consider what she had done. Everyone makes these kinds of errors and sometimes needs to return to a question that has caused difficulties in order to scrutinize it more closely. This is where Helen and the other students were lacking.

Estimation was also a problem for Helen. She usually found a common denominator and simply added or subtracted numerators while ignoring changes that would result from the use of a common denominator. Questions #9 and #12 are examples of this. She hesitated when asked to give an estimate for question #14 ($^{22}/_{4}$ - $^{10}/_{2}$) since the numerators were larger than the denominators. Her words help to illustrate her thinking, "Ten over two. Two is the whole and ten is the part, so it's harder to figure out". This occurred after she had been shown a way of estimating and demonstrated her inability to generalize to a slightly varied situation. She seemed to be confused by the fact that the fraction was greater than one. Even questions that should have been easy to estimate, such question #37 ($^{1}/_{5}$ + $^{4}/_{2}$), caused problems (she guessed $^{1}/_{2}$). Helen could probably have worked the answer out almost as easily, but seemed to be stuck on attempting this previously demonstrated method. Upon later reflection, however, she said that she was inverting the second fraction, which would have made her estimate reasonable. Her guess for $^{5}/_{9}$ + $^{3}/_{7}$ was $^{63}/_{10}$. Apparently she used the product of the denominators as the numerator but how she obtained ten as the denominator, (perhaps she added the seven and the three), was not clear. Her estimate for question #33 ($^{43}/_{4}$ - $^{12}/_{2}$) was $^{1}/_{8}$. A contributing factor may also be that these students were not taught to search for details or make estimates for fraction questions.

Helen was given a demonstration of how to estimate question #16 ($^{20}/_{4}$ - $^{20}/_{19}$) but was unable to recall any of this information one week later. Her first estimate for this question was $1^{3}/_{7}$ but she changed this to $1^{1}/_{4}$. Like many others, she insisted on putting whole numbers which resulted within the fraction over "something". In this instance Helen wanted to put the five (i.e., $^{20}/_{4}$) over four. It was interesting that she continued to have problems estimating this answer despite recognizing that these fractions resulted in numbers that were close to five and one respectively. Of course memory was also implicated. This was again indicated by the fact that she could not remember using the numbers eight and six as denominators when only one question separated her present work (in which she was trying to determine a CD for these numbers) and a question that she had just solved which also had these denominators.

Generally Helen was not able, when asked, to determine which fraction was bigger than another. Sometimes she would say they were close as she did for question #11 ($^{5}/_{9}$ + $^{3}/_{7}$). Part of the problem was that they were, in her words, odd. Viewing fractions in this way was another consideration with which she seemed unfamiliar.

Helen had trouble knowing how to handle a fraction with zero as the numerator as was the case for the answer in question #26 ($^{152}/_{8}$ - $^{608}/_{32}$). Eventually she put $^{0}/_{0}$ and concluded, hesitantly, that the answer was probably zero. She had the same difficulty in question #38 ($^{3}/_{15}$ - $^{4}/_{20}$) but in this case said that the answer to $^{0}/_{60}$ was equal to sixty.

Some errors could not be accounted for. When Helen subtracted $^{4}/_{24}$ from $^{9}/_{24}$ to get $^{12}/_{24}$, it was not clear even to her why she made this mistake. Others were more obvious such as when she subtracted $3^{4}/_{24}$ from $4^{3}/_{24}$ to get $1^{1}/_{24}$. She simply subtracted the smaller numbers from the larger ones. She also did this with question #49 ($3^{1}/_{10}$ -$1^{2}/_{5}$). Some of her reducing errors were probably due to poor attention or an inability to focus as when, in question #32 ($^{7}/_{4}$ - $^{5}/_{4}$) she reduced $^{2}/_{4}$ to $^{1}/_{4}$, in question

#25 (${}^{13}/_{26}$ + ${}^{14}/_{26}$) where she simplified ${}^{27}/_{26}$ to $1{}^{1}/_{27}$, and in question #28 ($4{}^{11}/_{12}$ + $6{}^{5}/_{9}$) when she wrote the equivalent fractions as the same for the two different fractions. In question #12 (${}^{7}/_{8}$ + ${}^{9}/_{10}$) Helen multiplied the numerator, seven, by its denominator, eight, instead of by the second denominator, ten. Lack of understanding of the associative principle for multiplication inhibited her in some of her computations. This was the case in question #16 (${}^{20}/_{4}$ - ${}^{20}/_{19}$). Although she knew the answer to nineteen times two, Helen resorted to multiplying out nineteen times twenty using the long method rather than employing this principle and simply "adding" zero to "38".

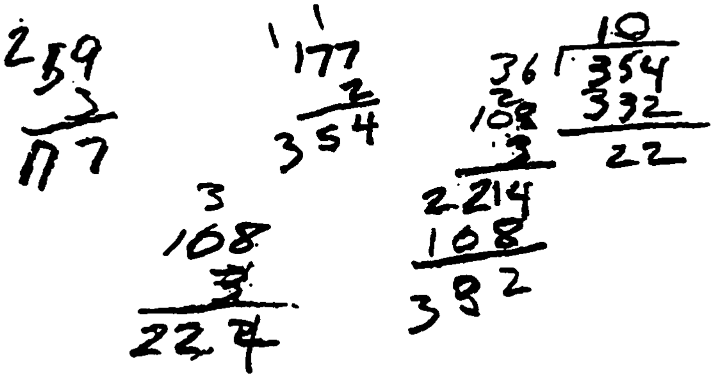
Even though Helen could add or subtract mixed numbers without converting to improper fractions, she continued to do just that--in spite of working with numbers that became almost unmanageable, as was the case with question #44 ($10{}^{1}/_{5}$ + $13{}^{7}/_{9}$)--because she reasoned that it would somehow be cheating not to do so.

This student did not utilize information within the questions that could have been helpful. Information, such as the fact that if two denominators were even then their product must also be divisible by two. This could have been very useful when determining the lowest common denominator. In some instances, searching the question for details would have allowed her, and the other students, to make very fine estimates (e.g., the result of $11{}^{3}/_{8}$ + $9{}^{5}/_{6}$ would have to have been between 21 than $21{}^{1}/_{2}$). These oversights resulted because students were not searching for details. Most people would probably not have noticed these kinds of details unless first made aware of them. So, although it may suggest a lack of reasoning ability, it could also be a reflection of the insignificance placed on this type of knowledge by our schools in general, particularly when these students would have been attending junior high school.

Usually her work was fairly neat but besides leaving out operation signs she sometimes left out whole numbers. Occasionally she left answers as improper fractions even though instructed to reduce all answers to their simplest form. She was confused by zeros as numerators; subtraction, if borrowing was required; and negative answers--often leaving out negative signs. This may also have been an indication of a poor understanding. She also tended to use the wrong terminology for certain things such as when she kept referring to improper fractions as mixed numbers.

Sometimes Helen talked to herself and counted on her fingers, probably because she also had problems with multiplication facts, either accessing, retrieving, or remembering these. For instance, she had difficulty dividing three hundred fifty-four by thirty-six in question #28 (see Figure 13). She used her fingers to determine that thirty-six times three equalled one hundred eight (counting thirty, sixty, ninety, plus eighteen) but then multiplied one hundred eight by three incorrectly to get two hundred twenty-four. To this figure she added one hundred eight--at this point Helen figured the number of thirty-sixes to be ten by counting incorrectly three for each "one hundred eight", multiplying this three by another three (because there were three one hundred eights), but then added only one (instead of three) for the last one hundred eight. She concluded that thirty-six went into three hundred fifty-four ten times or, according to her calculations, thirty-six times ten equalled three hundred thirty-two. Helen made several mistakes on this question. It was obviously more difficult for her and was one of two for which she wrote out the calculations. She did not check her answer to see that it made sense. This maybe due to a lack of ability to focus or of being easily distracted.

Figure 13. Helen had difficulty dividing thirty-six into three hundred fifty-four.



This student demonstrated many abilities but sometimes had problems with concepts. Many of her problems may have been a result of number reversal or confusion. others perhaps were due to a lack of experience. She tried hard, but did not seem to be able to think clearly about what she was doing.

Roger

Roger completed his grade 9 ten years ago in rural Alberta. He began his academic upgrading in 1992 but had been attending AVC since 1987 and was a part-time student at the time of the sessions. He was not taking any mathematics classes. He said that he sometimes found the classes too fast and became depressed when he did not understand the lesson. He stated that he did not enjoy working on fraction problems and accurately considered himself to be poor at it.

Previous reports suggested that his weaknesses were in written expression and attention to detail. He apparently reversed letters and numbers. His reading comprehension was stated as very weak. The reports also suggested that he had good visual and auditory memory for patterns.

The results of the WJPEB written at the beginning of October, 1992, indicated that Roger was severely deficient in reading, mathematics, and written language with percentile rank ranges based on age of 9-13, 7-14, and 8-13 respectively. His grade range for mathematics was between six and nine. Roger said that he had reading problems but did not think that he had any physical or other problems which could affect his performance in mathematics although he did mention that he had operations on his ears. He was twenty-seven at the time or writing.

Although this student said that he had been assessed as learning disabled several years ago, no records could be found to confirm these results. An assessment done by an outside agency concluded that Roger did not appear to demonstrate a learning disability based on a discrepancy between I.Q. and achievement which were both in the low average range. These results would seem to coincide with those of the WJPEB and Raven's Standard Progressive Matrices. His intake test showed that he appeared to have a good grasp of whole numbers but was poor with fractions and decimals. He was referred to SEL.

Roger's goal was to become a professional musician.

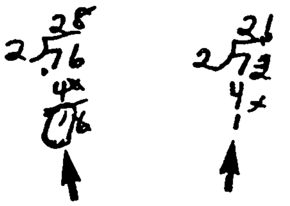
Roger's Work Summary

Because Roger said that he did not think he would be able to verbalize his thoughts as he worked, he was asked to do the calculations and then explain what he had done right after.

He made numerous mistakes, many of which involved copying errors or writing the wrong figures as well as performing the incorrect operation. He also made a number of multiplication mistakes because he lacked this type of factual knowledge. Roger spotted many of his errors as he was explaining what he had done and corrected these. He seldom seemed to take the initiative to check his work otherwise.

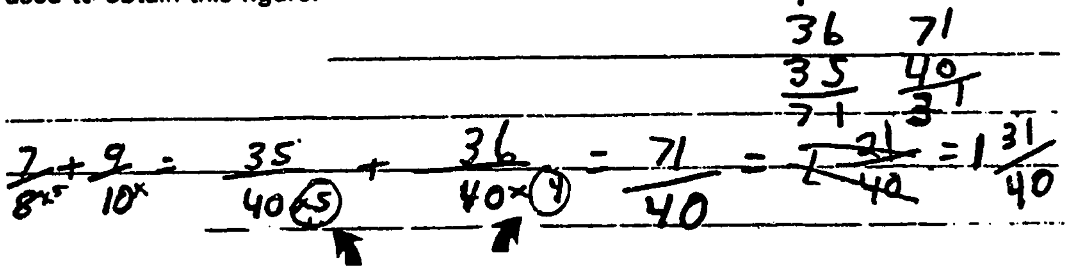
Like Donald, Roger also counted on his fingers. However, in some respects he seemed more advanced in this regard. For instance, in determining the difference between a minuend and subtrahend he did not have to begin with the units place. When dividing both seventy-six and seventy-two by two he made errors in subtracting four from seven by putting one instead of three for each, which is why he reduced $3^{72}/_{76}$ to $3^{26}/_{28}$ (see Figure 14).

Figure 14. Roger subtracts four from seven incorrectly.



In a couple of instances Roger began to multiply his common denominator by the number he had used to obtain the CD(see Figure 15).

Figure 15. Roger became confused and began to multiply his CD by the number he had used to obtain this figure.

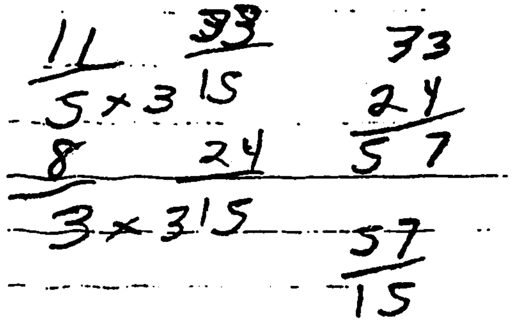


In the question #13 ($^2/_3$ - $^3/_9$), he wrote $^2/_9$ - $^{18}/_9$ as equivalent fractions for $^2/_3$ and $^3/_9$ respectively. At this point he lost track of what he was doing "because everything went backwards". Roger was given assistance to solve the question. Perhaps, like Donald, he was also having trouble keeping track of data and "mixed up" some of the numbers he was manipulating.

Reducing fractions caused many problems for this student. Having difficulties with multiplication was a contributing factor in some cases such as when in question #11 he reduced $^8/_{54}$ to $^1/_{17}$. He was trying to divide by four. After being questioned, he corrected this. Roger reduced $3^3/_6$ in question #1 to $3^1/_6$ and said that $^5/_{15}$ in question #3 was equal to three. At times he did not bother to reduce items at all, even though instructed to reduce all fractions and specifically reminded of this on some occasions where he had not done so. Some reductions that seemed relatively straight forward (i.e., $^{10}/_{15}$), took him a very long time to do.

Roger did not always use the correct terminology. It was difficult at these times to know what he was talking about. A good example was when he referred to fractions with different denominators as improper fractions or at least not proper fractions. In question #9 ($^{11}/_5$ + $^8/_3$) he began with, "It's not a proper fraction". When asked to explain, he went on to say that both "bottom numbers had to be even" and that he "would put a bottom number fifteen". He then said, "somethin's missin' there. I don't know how to do that." At this point Roger incorrectly multiplied both fractions by three but still got a CD of fifteen (see Figure 16). The meanings of proper and improper fractions had been explained but he continued to refer to questions that involved differing denominators using this terminology. When he did encounter improper fractions, Roger always divided using longhand. Although he had difficulties with this question, probably because it was the first one with different denominators, it was really relatively easy since one of the denominators was a multiple of the other. Perhaps his problem was that he relied on rote learning but did not quite have the procedure memorized completely or accurately.

Figure 16. Roger used an incorrect procedure to obtain a CD of fifteen.



When Roger wanted to determine the result nineteen times four, he stated that he had to figure how many times four went into nineteen. It became apparent that this was not what he meant when he proceeded to do the multiplication longhand. He also said that he preferred the horizontal format but wrote all except the first nineteen questions out vertically. This lead one to question his metacognitive ability since he seemed to understand the terms.

Roger made several different kinds of mistakes. An interesting illustration of an error he made when writing down a wrong numeral occurred after he had worked the answer out to question #2 to be three, the correct answer, but wrote it as the numeral "2". The question number was written as three instead of two. Another error was when, in question #10, he added ${}^{18}/_{14}$ + ${}^{10}/_{7}$ to get ${}^{28}/_{14}$. He also subtracted forty from seventy-one and got twenty-one. Sometimes he left out operation or equal signs. Although Roger noticed and corrected many of these mistakes, he continued to make them. In question #11 (${}^{5}/_{9} + {}^{3}/_{7}$) he again multiplied *both* fractions by ${}^{3}/_{3}$ in order to determine the common denominator. However, he realized his mistake and tried to correct it by multiplying the denominators together to get a CD. Unfortunately, he incorrectly thought the product of nine times seven was fifty-four. When it was pointed out to him that this was not correct, Roger immediately gave the right answer but then said he was thinking of eight times nine which suggested faulty reasoning or perhaps anxiety. When asked which was the bigger fraction in this question, his response was based on the fact that one denominator was larger than the other. It was difficult at this point to determine whether or not his reasoning was valid, but later questions revealed that he had a poor understanding of the part/whole relationships. His estimate of the answer was ${}^{8}/_{27}$.

Roger had difficulties estimating. Usually he would look for a common denominator and attempt to "work out" a guess without writing anything down--the other students did this as well--but sometimes he would make an unrealistic guess after saying he had no idea. In question #12 his estimate for ${}^{7}/_{8} + {}^{9}/_{10}$ was ${}^{1}/_{8}$. When questioned about what each fraction was equal to, he said one. He then gave an estimate of two but laughed nervously and commented that "these are so backwards, it's unbelievable". When asked what he meant, Roger said, "People try to make ya guess at what the answer is and then ya gotta [pause] and there ain't no way you can do that". After pointing out that he had made a estimate of two, he commented, "yea, I know, but there's no way it's going to be two". Roger's immediate response to the request for an estimate of $^{20}/_{4}$ - $^{20}/_{19}$ was $1^{1}/_{2}$. He did not know but took it for granted and said that because "it was an improper fraction, it had to go into each other at least once or better". Like many of the other students, he wanted to put his estimate "over something".

On a couple of occasions, his guesses were good but his reasoning did not seem to make much sense. Roger's estimated $^{22}/_{4}$ - $^{10}/_{2}$ to be $^{1}/_{2}$. However, his explanation did not justify this answer. It may have been something he made up. When first questioned about how he arrived at his estimate, Roger responded that he made it "because everything was just about half of what it was out of". When questioned what he meant by this, Roger answered that "two is one half of ten" and "I can put four into twenty-two and both have two". When he was asked to clarify he said he "put twenty-two and four down [pause]. I can do that in my head but I can't write it down. Four times six is twenty-four, so I figured six. So I figured half way through if you know what I mean." Some of what he said was logical but he seemed to have a difficult time "putting" his thoughts together and making himself understood. He may have had an idea of what he was doing but one would have to read quite extensively into what he said to determine the extent of this. The statement that he could do the work in his head but could not write it down suggested some other factors could have been interfering.

Roger laughed nervously whenever he was having a difficult time. He laughed often. At one point, when asked if it was really that funny, he said that "it was totally ridiculous". It seemed that he was very self conscious and did not like to be seen making any mistakes. He lost his place a few times and explained that there was too much figuring. When an error was pointed out, he almost always responded with "yea, I know". An attempt was made not to indicate these directly in order to minimize interference as well as student embarrassment.

His work was untidy because he tended to scribble over mistakes and do all of his calculation at the top of the page on which he was working. One must consider, however, the possibility that Roger had some insights and knowledge but was either simply unable to explain or write the answer. He may also have had problems retrieving the information from his long-term memory store. He said that he used to work things out by talking to himself but, unfortunately, was unable to do this during a class, undoubtedly because it would have been distracting to other students. Roger also lacked confidence.

Garcia

Garcia completed grade 8 in South America seventeen years ago. Several years later he continued his education in Canada by taking business mathematics through Adult Education in 1984. Although english was not his first language, he was quite proficient and felt confident speaking it after six years of study. He stated no physical or other problems that could affect his performance in mathematics and appeared to be a well adjusted, happy person. He was not receiving any assistance from SEL.

Garcia said that he sometimes enjoyed solving fraction questions and that he was sometimes good at it. The results of the WJPEB which he wrote last September 25, 1992 when he was thirty-five, suggested that he was severely deficient in reading and especially mathematics. He was only slightly below the average range in written language for those within his age group. On a percentile basis, he fell within the 11-15, 4-8, and 16-26 range for reading, mathematics, and written language respectively. His average was reduced in the reading area because of a low *Comprehension* score. Garcia did poorly in both the *Applied Problem Solving* and *Calculation* subtests.

The results of the Raven's Standard Progressive Matrices Test indicated that this student possessed an average intellectual ability with a score that placed him around the 40th percentile.

Garcia's Work Summary

Like the other students, Garcia's estimates were usually the result of attempts to work out the answer to questions covertly by finding a common denominator for the fraction involved. In many questions, such as #9 ($^{11}/_5 + ^{8}/_3$), he took for granted that the addition or subtraction of the original numerators over a CD would result in a guess that would be close to the correct answer. This was despite the fact that he was aware of changes that would have to be made to these numbers as indicated when he worked on the question. His first estimate for question #11 ($^5/_9 + ^3/_7$) was $^{15}/_{27}$ but he changed this to $^8/_{27}$, the same answer given by Roger. Both seemed to think that twenty-seven was a common multiple of these denominators. Garcia thought these mistakes were due to his fast pace and commented, "I'm not thinking. I'm just going too fast". Nevertheless, for most of the questions Garcia continued to estimate by obtaining a common denominator and adding or subtracting numerators without regard to these changes. Question #14 ($^{22}/_4 - ^{10}/_2$) was an exception. It took him a long time to estimate the answer but finally he guessed $2^4/_5$. When asked how he obtained this, Garcia said that he "just made it up". He went on to work it out correctly using a CD of four. Part of the difficulty may have been that Garcia was not familiar with looking at fractions this way and even commented to this effect. He seemed to have trouble deciding that an answer of $^{62}/_{63}$ was almost equal to a given estimate of one. Even when he knew the approximate value for each fraction, as was the case in question #16 ($^{20}/_{4}$ - $^{20}/_{19}$), he was unable to combine these to give an estimate to the answer. At one point Garcia asked if estimating was like finding a common denominator. Perhaps this is why he never utilized his estimate to check his work or to aid him in calculations such as finding quotients during a division.

Although Garcia automatically used long division to solve all improper fractions, including very simple one such as $^{27}/_{26}$ or $^{21}/_{6}$, he really had few problems until he came to question #9 ($^{11}/_{5}$ + $^{8}/_{3}$) which was the first one involving different denominators. He obtained an answer quite different from his estimate. This was due in part to his procedure of determining the estimate as already mentioned but was also because he reversed one of the fractions when writing it out. He realized that the CD was not the same as that used in his guess but ignored this and continued without checking. When a discrepancy between his guess and answer was indicated, he decided to redo the question. It is possible that Garcia made this copy error because his attention was already focused on the unlike denominators. In another question he hesitated when dividing forty into seventy-one. He was not sure if the whole number in the answer should have been one or ten but eventually made the correct choice for quotient.

Usually this student thought that fractions composed of bigger numbers were larger than those with smaller ones. When asked, however, which was the bigger of $^{20}/_{30}$ or

$^{4}/_{5}$, he said that they were both the same since each was "different" by one. He may have said this because he suspected that his previous assumption that $^{5}/_{9}$ was larger than $^{3}/_{7}$ was wrong when confronted with this impromptu second comparison. Still, it was clear that he did not comprehend part/whole relationships. Nevertheless, he was concerned enough to ask how to determine which was the larger fraction in a comparison.

Garcia generally did not have much difficulty with multiplication or division but when reducing $^{72}/_{76}$, he added thirty-five and thirty-five to see what seventy-six divided by two would be. He had already determined that seventy-two divided by two would be thirty-six. Nevertheless, he then said that since thirty-five and thirty-five equalled seventy, the answer would have to be $32^{1}/_{2}$. He changed this to thirty-three but realized that this was also incorrect and continued, eventually working out the answer. Perhaps Garcia was "not thinking" as he had mentioned earlier because he clearly had not been able to use the fact that the answer had to be greater than thirty-six, possibly because he was not efficient at manipulating large amounts of information. It also could have been a case of subtracting one half of the difference between seventy-two and seventy-six instead of adding this figure, but this was not apparent from his verbalizations.

There were a number of "little" things that Garcia did which may have hindered him somewhat or at least made his efforts less efficient. For instance, some properties of mathematics seemed to be foreign to him. He did not understand the associative principle of multiplication and consequently could not take advantage of simply adding zero to figures multiplied by ten or a multiple thereof. This is why he multiplied nineteen times twenty using the long method. He also did not comprehend the reversibility property and wondered after calculating that nineteen times four was seventy-six, how many times four went into seventy-six. Garcia also had trouble reducing fractions and sometimes finding a common denominator. In some cases it could have been advantageous to reduce a fraction before performing the operation, but like all of the others, he either chose not to do this or did not recognize the possibility. He thought, for instance that a fraction composed of odd and even numbers could not be reduced. Occasionally he used CD's that were bigger than needed by simply multiplying the denominators. He did not always say things correctly either. He would abbreviate a fraction by leaving out a word such as "over", like twenty-one six or seventy-eight twenty-six instead of twenty-one *over* six and seventy-eight *over* twenty-six. He might also have said something like twenty-two nines rather than twenty-two twenty-ninths. as he did in question #5 ($^{22}/_{29}$ - $^{17}/_{29}$).

Although there were one or two things that Garcia did inconsistently, usually he was very systematic and neat, with well spaced work. This helped to keep him organized and focused (i.e., in control). He noticed the first change in operation sign from addition to subtraction that others missed. Garcia used two "little dividers" when reducing fractions, even for very simple fractions such as $^{2}/_{4}$, explaining that he needed to see everything in black and white. He only used one "little multiplier" when finding equivalents. He also wrote all of the questions in vertical fashion and circled his answers. Sometimes he used two equal signs when showing equivalents next to the original fractions. Some monitoring and planning were suggested by the work done.

It was enjoyable working with this student who, although he continued having difficulties in some areas, did show some improvement (e.g., in estimating) by the end of the sessions.

Paula

Paula's academic upgrading began in late 1992 with a grade 11 education that she had completed thirty years before while attending a rural school. The highest level of mathematics attained at this time was math 15. She said that she was not good at solving fraction problems but was beginning to enjoy doing them since her multiplication knowledge had improved.

No physical impairments or other problems which could have affected Paula's mathematical performance were stated, although the length of time away from school no doubt contributed to her difficulties and at one point she mentioned having a thyroid problem. This may have indirectly affected her work but Paula did not think so. She noted that she always had difficulties in this area and it was clear that the grade 9 level standing obtained in this subject was below that of her other subjects. She specifically requested help in mathematics.

The results of the WJPEB indicated that Paula was below average in reading, moderately deficient in mathematics and severely deficient in written language. She especially had trouble on the *Word Attack* subtest in the reading area and difficulty in the *punctuation*, *capitalization*, and *usage* portions of the subtests involving dictation and proofing. Her lowest score was on the *Calculation* subtest. Her percentile ratings fell within the ranges of 18-25, 15-24, and 13-19 for reading, mathematics, and written language respectively. Not surprisingly, Paula commented that she had difficulty with word problems.

This student did not write the Raven's Standard Progressive Matrices test but it is likely that her intellectual capacity would have fallen well within the average range for her age group. She was fifty years old.

Her goal was to become a practical nurse.

Paula's Work Summary

There may be a connection between Paula's written language deficiency and the severe problems she had trying to express herself, sometimes to the point of being very difficult to understand. It was initially unclear what she was attempting to convey, for instance when, in question #11 ($^5/_8$ + $^3/_7$), she said, "eight goes into ten twice, five, seven". After much consideration, it became obvious that she was trying to subtract seven from fifteen (i.e., looking at the units of the equivalent fractions), by first counting up from what she thought was the subtrahend to ten--she began with eight instead of seven--and then adding this result to the remaining five, which came from the minuend, to obtain a total of seven. Question #5 ($^{22}/_{29}$ - $^{17}/_{29}$) was confusing because she counted "three, four, five" and put the answer over twenty-nine. When asked to explain, Paula responded with, "seven into twenty is three and two is five". Upon further questioning, she explained that seven (she really meant seventeen) up to twenty was three and "then two". In other words, she counted up to twenty and then added the remaining two from the minuend.

She would quite often say things that she did not mean or use "abbreviations", as some of the other students did, by leaving out words as was the case in the very first question ($^{7}/_{6}$ + $^{14}/_{6}$) which she stated as "seven six, fourteen six". In one instance she said the opposite of what she meant when she commented that "somethin's not wrong". She meant that something did not look right. It was somewhat difficult to sort out what Paula meant by, "Like two times nine is eighteen, and I'll write down, um, like um, add an eighteen, and then I'll, um, that before, and then the six, then the six, and then I'll just add one nine...", as she tried to put into words how she would sometimes add in order to multiply. In this case she was trying to explain that when attempting to multiply a multiplicand of nine, she would begin with eighteen and add to this until it equalled or exceeded a dividend. If appropriate she would resort to simply adding nine by itself. Paula also explained that what she had in mind to write down was not necessarily what went on the paper. She had to watch her copying. On the *Letter-Word Identification* and *Word-Attack* subtests of the WJPEB, she tended to add letters or pronounce different letters than were given. In question #16 ($^{20}/_{4}$ - $^{20}/_{19}$) she said "sixteen plus sixteen" when she meant "eighteen plus eighteen". Of course these difficulties would negatively affect her ability to comprehend and solve problems.

This student began the questions by doing all of her calculations covertly and writing down just the answer which, if necessary, would be reduced or simplified. At times this required her to write down part of the calculation. In question #2 ($^{37}/_{26}$ + $^{41}/_{26}$), for example, she wrote in very small numerals adjacent to the denominator the result of this figure (i.e., twenty-six) first multiplied by two and then by three. Most of her calculations were done at the side or on top of the page rather than near the problem itself. She was more advanced than some of the students in that it was not mandatory for her to use longhand division when converting improper fractions to mixed numbers. A simple example is in question #4 ($^3/_4$ + $^3/_4$), where she easily simplified $^6/_4$ first to $1^2/_4$ and then to $1^1/_2$. Some of the calculations required more effort but many of the reductions seemed to involve only retrieval from memory. She talked to herself when working out some of the more complex answers.

This student's work tended to look sloppy because she had a habit of writing over her mistakes and sometimes cramming it together. Otherwise the size and formation of her numerals was good. She usually had a separation between each question. Her work appearance improved after it was suggested that she use an eraser and try to space it out more.

Paula did, on occasion, leave out the whole numbers as well as operation and equal signs. In question #30 ($^2/_{20}$ + $^4/_{20}$), she reduced by dividing the numerator into the denominator and writing the result as a mixed number. In question #4 (as above) Paula automatically multiplied the numerator to get the improper fraction $^9/_4$ which she simplified to $2^1/_4$. Reflection on this question allows one to see that the answer must be less than two since each fraction is less than one. This may have been an inadvertent error since she changed the answer immediately when asked for an explanation. Another example which suggests a lack of reasoning ability was when she was required to divide fourteen into thirty-eight and began by multiplying the divisor by five to see if that would work as a quotient. It should have been clear that this result would have been too great even if truncating the divisor and using a figure of ten. It would have probably worked best to use a figure of fifteen instead of fourteen since it was a number with which this student seemed to be familiar.

When subtracting one two-digit number from another, Paula usually began with the subtrahend and counted up to the nearest ten of the minuend. She would then add any difference between the ten and the minuend onto this result. Question #5 ($^{22}/_{29}$ - $^{17}/_{29}$), as discussed earlier is a good example. There were three instances which suggested that this student had learned that five and three equalled seven and that five from eight was two. First, in question #7 ($^{7}/_{3}$ - $^{5}/_{3}$) she stated that the answer was equal to three over three or one; second, for question #9 ($^{11}/_{5}$ - $^{8}/_{3}$), she subtracted seventy-five from seventy-eight and said this answer was two; third, when comparing fractions in question #11 ($^{5}/_{9}$ + $^{3}/_{7}$), Paula said that there were five parts "left" out of the whole for the second fraction. In the first instance she corrected this immediately after writing it down. She did have trouble at times with subtraction and even had to subtract twenty-eight from thirty-eight by writing it out. To add she simply counted on from one augend to another, beginning with the units if this involved two or more digits.

When asked which fraction was bigger in question #11, Paula initially chose the second, commenting that it had four parts left out of nine while the second had five. Her words trailed off as she apparently realized that both had four parts left and decided that they must be the same size. Although her reasoning was inaccurate, she was correct in her observation that the same number of parts were left; it was this fact that made the comparison, and most of the others, so much easier to do in the first place. The fact that she recognized and focused on the number of parts perhaps is an indication that she could benefit from help in this area.

Paula did not always use the lowest common denominator possible which meant that she was sometimes working with very large numbers. In some instances she could have reduced one of the fractions to get the LCD but chose not to or, as is more likely the case, was not aware of this choice. In other cases she simply multiplied the denominators together. Two examples of the first scenario are in questions #10 ($^{18}/_{14}$ + $^{10}/_{7}$) where the first fraction could have been reduced, and #13 ($^{2}/_{3}$ - $^{3}/_{9}$) where the second fraction could have been reduced. Paula did comment at one point that she thought all improper fractions should be simplified. She apparently was watching for these but not for proper fractions which could also have been reduced. Two examples of the second instance are where she used a CD of eighty in question #12 ($^{7}/_{8}$ + $^{9}/_{10}$). She wondered if a smaller CD could be used but could not "see" one. In question #15 ($^{3}/_{8}$ - $^{1}/_{6}$) Paula used an incorrect common denominator of sixteen. It should be noted, however, that she went on to use the proper LCD in other cases where these denominators were used. In fact, it seemed to be one that most students were familiar with and used with little hesitation.

Occasionally Paula had trouble reducing as when she reduced $^{3}/_{16}$ in question #15 to $^{1}/_{3}$--even after stating that three times six equalled sixteen--but usually she did not have much trouble. Question #16 ($^{20}/_{4}$ - $^{20}/_{19}$) caused her difficulty though, particularly because the numerators were identical. When asked to give an estimate, she stated that the common denominator "would be four times nineteen since these [the numerators] are the same. It would have to be four times this, twenty". Although she later explained that the fact that the numerators were the same did not matter, she nevertheless had difficulty and concluded that the answer of $^{0}/_{76}$ was equal to seventy-six. She had given an estimate of $^{100}/_{99}$ since, according to her, twenty times four was one hundred and nineteen was almost the same as twenty (i.e., it would also be close to one hundred).

Although some improvement was seen by the end of the sessions, this student almost always performed the required operation on the numerators without taking into account changes due to the use of a common denominator. Her estimate for question #9 ($^{11}/_{5}$ + $^{8}/_{3}$), was $^{19}/_{15}$. Paula knew that the numerators would be altered but, like the others, did not utilize this information. She also commented that she had a difficult time estimating for some questions--she seemed to be referring to questions that had different denominators--and that she did not like estimating. She always estimated by first finding a common denominator and, for question #11 ($^{5}/_{9}$ + $^{3}/_{7}$), said that she could not give an estimate since it was difficult for her to find a CD for the denominators.

Paula stated that her multiplication "past the fives" was poor. This did appear to be confirmed. Her facts on the fives times table were also incomplete as illustrated in question #9 when she directly retrieved five times eight and said that this was forty-five and in question #16 ($^{20}/_{4}$ - $^{20}/_{19}$), when she stated that nine times four was twenty-seven. She usually seemed to be aware of these multiplication facts but the connection was obviously shaky. Paula also repeated that four times twenty was one hundred but eventually worked it out using longhand. Still with the same question, she added a series of three fifteens in order to multiply this number. She then resorted to multiplying fifteen, first by four and finally by five using longhand because it was becoming too difficult to add fifteens covertly. Two of the following examples show how she employed multiplication facts that she had previously retrieved to help herself. In question #12 ($^7/_8$ + $^9/_{10}$), she added forty-five and thirty-six to get the total of nine times eight because she knew that nine times five was forty-five and that nine times four was thirty-six. In the first question she determined that six times three was eighteen by first multiplying this multiplicand by two and then adding another six onto the intermediate result. When adding two-digit numbers, carry-overs were added after the addition of the other digits.

It was clear that Paula was monitoring at least some of her work and that she realized that she had some weak areas that needed work. She stated, for instance, that she thought she should slow down and watch more carefully what she was copying. She also recognized how much easier it would have been for her had she a better grasp of multiplication facts. Paula even commented that having to spend as much time as she did was a waste.

The difficulties experienced by this student for the first sixteen questions were consistent with those she had with the rest of the questions. She did seem more advanced than some of the students, but had many problems which may have been related to an attention deficit as well as the language problem. Poor multiplication skills and even some trouble adding and subtracting also caused problems. Paula tended to become "flustered" if she made a mistake which compounded her difficulties. However, she never allowed herself to become angry or lose control.

Paula always worked hard and wanted to improve. Unfortunately she did not follow her own advice to slow down and allow herself more time to think about what she was doing. Her difficulties with written language and in expressing her thoughts were definite barriers.

Chapter V

Discussion

Difficulty with mathematics is not a new phenomenon. People have probably struggled with figures since they first began to tally their possessions. No doubt it was easier when each item could be represented by a mark on a tree or by one's finger. However, man has developed much more efficient ways to count. Still many have trouble comprehending this sometimes complex language.

Apparently educators once believed that supplying students with rules and teaching them to apply these in an algorithmic-like fashion was sufficient for the learner to progress and generalize what he or she had learned. Teachers, and those in related professions are now, however, becoming increasingly concerned with the lack of mathematical understanding shown by students and are beginning to emphasize teaching methods that consider learner needs and readiness and that will instill what Baroody and Hume (1991) referred to as "meaningful learning". At least ten years ago educators began to realize that students taught rules without meaning often did not understand the approach taken and were unable to relate it to other similar problems (Cherkes-Julkowski, 1985a; Lester, 1982). Even math 30 students who knew how to solve particular problems on the June, 1993 provincial examinations could not explain why the processes they used worked (F. Glanfield, personal communication, August 11, 1993). Unfortunately, little progress seems to have been made in this regard as the tendency is still for educators to focus on narrowly defined concepts and tasks as noted by Schoenfeld (1982) and more recently by Frank, (1992). In fact Baroody and Hume (1991) used David Elkind's term "curriculum disabled" to refer to students whom they said "were victims of instruction that was not suited to how children think and learn" (p. 55). Of course the concern should not just be related to how pupils think and learn; it should also be reflected in what they are taught. I believe this applies to anyone attempting to learn within our educational systems. Naturally there are several factors, including the individual's background, intellectual capacity, and motivation, that affect his or her readiness and ability to learn with understanding.

Although the overall comprehension of the adults in this study was generally poor, they may have been capable of much more understanding than they exhibited. Donald, for instance, managed to manipulate and keep track of a great deal of information even though he had to resort to writing much of it down. This limited the resources he had available with which to experience learning that would perhaps have been more meaningful to him. His use of inefficient methods was primarily a consequence of his inability to learn the multiplication tables (with a few exceptions). Helen showed signs of number reversals. This may have caused some of her difficulties. She also lacked confidence as demonstrated by her decision to continue converting mixed numbers to improper fractions although she found it easier to work with mixed numbers. Despite her difficulties, however, she displayed proficiency in some areas. Paula had a great deal of trouble expressing herself and in many ways seemed to know much more than she was able to explain. Increased verbalization would probably have helped her and Roger, who tended to "twist" numbers around but who found and corrected many mistakes while verbalizing. He was very self-conscious and did not want to appear ignorant, perhaps a result of past failures. Garcia was the most systematic and neat but like the others, he relied quite extensively on rote learning. He did, though, make a greater effort to examine the questions and evaluate his work, unfortunately not to any extent that proved very beneficial.

The reasons for these adults difficulties were almost as varied as the number and types of impediments exhibited, not only amongst themselves but also for each individual. Nevertheless, as with those in the pilot study (LeMaitre, 1992) (see Appendix E), there were commonalities and many of the errors made were the result of a lack of, or incomplete knowledge in a particular area. Often their difficulties appeared to be attentional or related to focusing problems, many may also have been a reflection of how and what they were taught. The following highlights the most recurring problems:

1. None of the students in either the pilot study or present research were able to estimate. They did not seem to even understand its purpose and did not use it as a means of verifying their answers or to aid them in their calculations.
2. Most did not monitor or check their work, or did so only on a limited scale, perhaps checking their last calculation. They did not search for details which may have helped them and sometimes they even ignored evidence that there was a problem.
3. The students in this study and most of those in the pilot study seemed to lack understanding in many areas, depending very much on rote learning. Some were unable to generalize even to very similar questions.
4. Everyone in the present study, and many participants in the research just prior to it, had difficulty recalling multiplication facts. Some members of both groups were unfamiliar with the associative law of multiplication as well as the

reversibility principle. For instance, many resorted to multiplying by ten, or a multiple of it, using the longhand method.

5. Almost all of the students believed that fractions with larger numbers were bigger than those with smaller numbers. Only a couple in the pilot study judged the relative size of fractions by finding a common denominator. None of the students spontaneously used diagrams to help themselves and some had difficulty with those drawings they were encouraged to make. They all chose to use circles and did not seek alternatives even when expressing difficulty dividing this shape into the appropriate number of segments.
6. The participants had trouble with fractions with different denominators; negative numbers; and subtraction, particularly with mixed numbers, when borrowing was required.
7. The five students in this study, and nearly all those in the pilot study, had difficulty simplifying proper fractions. Most resorted to using longhand when reducing improper fractions.
8. Students in both studies routinely left out operation and equal signs as well as whole numbers. A few even asked if this was important. Their work was often crowded and numbers were sometimes written one over another. Several errors made by these individuals resulted from their "sloppiness". It should be noted, however, that the neat workers did not necessarily have any greater understanding than those who were untidy.
9. When the denominator of one fraction was a multiple of the denominator of the other fraction, every participant chose to use the larger number as the LCD even

though it could have been reduced to match the other denominator. This relates to searching for details and suggests that students develop mind sets, probably because they are not typically shown alternatives.

I would agree with Cawley (1985a) who believed that many of these kinds of problems arise because there is no time for thinking in the class and instruction materials are drill and practice using a paper and pencil format almost exclusively. He also noted that "most regular class teachers proceeded at the rate embedded in the text" (p. 4). Lester (1982) found that some learners used less efficient or inappropriate solutions because they became "locked into" a rigid approach that resulted from pre-training. A slight variation of this perspective is to view the dependence on rote learning as a result of a lack of exposure to other possibilities. I believe that most individuals attempting the questions in this study would have demonstrated many of the same "choices" as the participants and think that they would also have had trouble explaining why their methods worked. These kinds of outcomes result because students learn that the methods they are taught work consistently. Pupils are often not told why the procedures work and they are seldom ever shown other choices. Consequently, these individuals have no reason to question these methods. Although this may seem unimportant to some, explanations help to demonstrate the connections between concepts and processes and are more likely foster greater understanding, retention, and application (Baroody and Hume, 1991).

Estimation is an important aspect in business as well as in everyday life. How else does one judge the reasonableness of one's answers or decide if they have enough money for three chocolate bars or four? The participants of this present research were unable to estimate reasonable answers to the questions and did not employ those "guesses" which were made, in any other of their computations. They actually attempted to work the answer out covertly by finding a common denominator. Some began adding or subtracting numbers without concern for changes caused by using this CD because they thought that the result would be a close approximation to the answer. Unfortunately, the inability to estimate does not seem out of the ordinary. Cherkes-Julkowski (1985a) found this generally to be true of children with learning disabilities and the second National Assessment of Educational Progress (NAEP) showed that 55% of thirteen year olds surveyed thought the answer to $^{12}/_{13} + {}^{7}/_{8}$ would be close to either nineteen or twenty-one (Hughes, 1986). Not everyone agrees with this assessment, however. Kieren (personal communication, September 22, 1993) reported that 85% of LD students with whom he had worked could do the estimation. This may be a reflection of the methodology he employed or perhaps an indication that improvements are being made within the education system.

The lack of factual information on mathematics, or at least problems retrieving it, especially multiplication facts, was a major source of difficulty for the participants. It limited them tremendously in every facet of this type of work. How could they become cognizant of the concepts so important to understanding all areas of this subject, while still grappling with mundane tasks like multiplying seven times nine? Because these adults lacked understanding and spent so much time and effort using more tedious solution procedures to do the work that others would simply retrieve directly from memory, they often became "lost" in their endeavours.

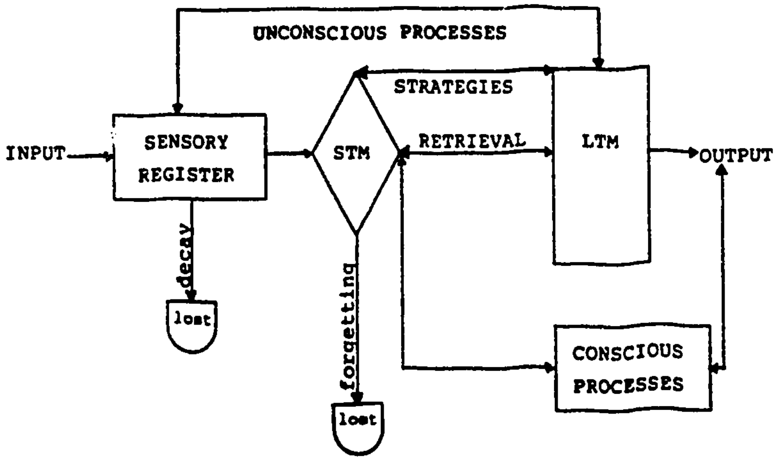
The counting procedures used by Donald and some of the other students are typical of those in early primary grades who first learn to add by manipulating objects but usually progress to counting all, then counting on, and finally to retrieving the information directly from memory. This transition normally takes place by the fourth grade, improving as more facts are added to their knowledge base and others reinforced (Pellegrino & Goldman, 1987). None of the students appeared to use the counting all procedure but in two cases Donald did appear to manipulate objects when, for example, he subtracted two from nine. Although no reason for this was obvious, it was probably because he found the usually more efficient counting down procedure more difficult and the counting up procedure less efficient. Perhaps it was better for him since he probably found, as young children do, that counting down was more difficult because it involved counting backwards from the minuend an amount equal to the subtrahend. The incrementing procedure required only that he count up from the subtrahend to the minuend. There is some evidence that suggests that as children become older they are generally able to choose the more efficient of both of these procedures (Pelligrino & Goldman, 1987). The method chosen would depend on the problem itself. For example, it would be much easier to use the decrementing procedure if a large difference existed between the minuend and subtrahend since there would be less to count down. On the other hand, if this difference was small, then it would make more sense to use the incrementing procedure. In Donald's case, it would have been more efficient to count down from nine to take away two. He chose neither of these procedures since he held up his fingers and took away directly by bending two of them and using recognition to establish that there were seven left.

Using the finger method to add or subtract involved the ability to monitor one's activity but otherwise few of the students in this or the pilot study (LeMaitre, 1992) monitored their work or seemed to spend much time evaluating either what was presented or their conclusions based on it. Those who did usually only did so on a superficial level, seldom ever noticing or even considering whether or not their answers made sense. For many this was probably a consequence of not being able to recognize what was acceptable. If a student has difficulty multiplying forty times three, how is she or he to judge that a quotient of six is unrealistic when dividing one hundred nine by thirty-eight? Clearly this poor skills base for multiplication also negatively affected their ability to make estimates. None of the adults searched for detail so, consequently, never utilized helpful information within the problem itself. This, however, may be more the norm than the exception. Probably few individuals working on the questions would have been cognizant of this information even though many of them would have the thinking capacity to allow for greater flexibility and ability to make use of these facts.[8] Apparently searching for details is not something commonly taught in the classroom.

The reliance on memorization of the steps was another reason that many of the students became confused and had problems establishing where they had "left off" before beginning a calculation especially as they became "bogged down" in their computations, often because these were less efficient, requiring more time and energy. Bidwell (1983) also found that students performed better on rote calculations than on problems requiring even simple thought. This was supported by Denckla & Heilman's (1979) study which showed that students who functioned at a low conceptual level were more dependent on rules and tended not to generate their own ideas or consider other possibilities. They cited a study by McLachlan & Hunt which indicated that these individual's performance improved when rules were presented before an example. Teaching the students to go slower only helped reduce errors if they were shown how to search for details. Some of the students in this thesis research and the prior pilot study worked quite slowly. This did not ensure improvement in their work, however, because they did not look for the things that could have helped them and most tended not to notice errors they had previously made. They knew how to follow rules but a greater understanding would probably have helped improve their performance. Van Erp and Heshusius (1986) suggested that the development of the "internalization" processes of perception, visualization and thinking depended on the guided actions of those who had already mastered these. Perhaps students having difficulties have not had the help they require to realize these internalization processes. Even with memorized formulae, participants of this present study did not have a systematic approach. Some of them could remember the steps, or at least aspects of them, but they seemed unsure about the order or had difficulty coordinating these. Cherkes-Julkowski (1985a) found this to be an area in which children were also weak.

Mental computations required memory processes and keeping track of the steps involved necessitated use of both their short-term memory (STM) store while retrieval of procedural, declarative, and conceptual knowledge involved their long-term memory (LTM) store. A brief overview of the memory processes should help to illuminate some of the areas where, and reasons why, difficulties were encountered. Structural models of memory like Atkinson & Shriffrin's (1971) multistore model perhaps best illustrate some of the concepts of memory that are important to the learning processes. They distinguish between structural components and processes that may later alter the way memory functions. The prototype used in this paper should also help to illustrate the three stages of memory (i.e., registering, retaining and recalling) and provide a clearer picture when explaining the pitfalls encountered by those with learning difficulties (see Figure 17).

Figure 17. A diagram of memory functions derived from Atkinson and Shriffrin's (1971) and other's structural models.



Most environmental information enters the sensory register through visual and auditory stimuli (Hergehahn, 1988). Although it holds vast amounts of data, usually each "item" only lasts for milliseconds and, unless attended to, it is lost (Connell, 1987). The capacity of this registry does not seem to change, but the processes involved in getting the information into the STM do. Registration is affected by attitude, interest, attention, developing senses, and organization.

The STM has a limited capacity, holding 7 (+ or - 2) units of material for adults (Miller, 1956). Besides holding information from the sensory register, it may also contain items from the LTM that it may be processing. Anything in consciousness is in the STM. "Traces" (a term coined by the Greeks) last about 15 seconds in this store but through various methods, such as elaboration, rehearsal, chunking, or item sequencing, may be maintained long enough to be encoded (i.e., transformed into a representational form that can be stored in the LTM. This process, called consolidation, is thought to take about $1^1/_2$ hours (Hergenhahn, 1988). These aids help to make better use of the memory capacity.

Studies have shown that adults usually make better use of the space that is available than children, which allows them to take in and manipulate more information. They are normally more systematic about what is taken in, have more efficient procedures to apply to a problem, have increased knowledge about the world with which to understand problems, and are more able to monitor the workings of their cognitive processes. The adults in this and the pilot study seemed to lack at least some of these advantages. Donald, for instance, failed to transform existing strategies as Swanson and Rhine's (1985) study indicated was also the case with learning disabled children. What they referred to as "method replacement" is an example. Donald would use sequential addition to perform a multiplication rather than "replace this procedure with, for example, the direct retrieval of the product.

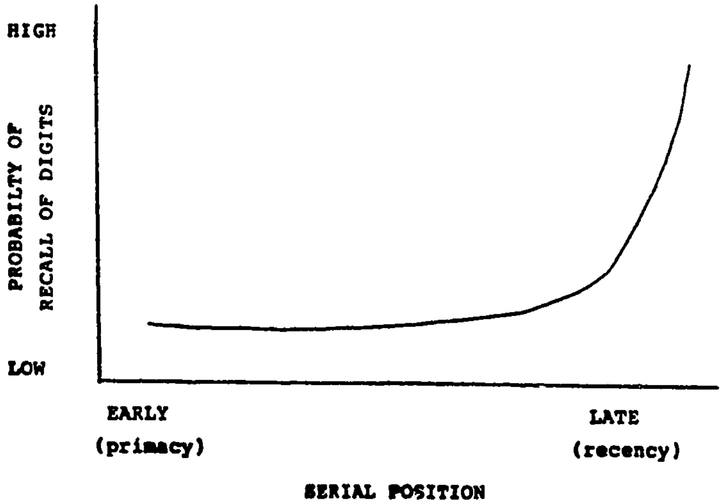
The LTM contains procedural ("know how") and declarative ("know what") knowledge. Included in this storehouse of knowledge are the methods with which material is manipulated in order to aid memory. The extent of the knowledge already in the LTM influences the ease with which new information is stored and later retrieved (Briars, 1982; Bjorklund, 1989).

Tasks which at first require a lot of attention become more automatic with experience. This creates less demand on working memory which frees it for other things. Adults with learning difficulties are less adept at processing information, grouping and chunking items. Things that are meaningful, pleasurable, interesting, and obviously useful will be most easily registered, retained, and recalled.

"Thought...entails the use of memory and the ability to organize and categorise [sic] information" (Connell, 1987). Memory is continually being reorganized as new understandings and information are experienced and become accessible for recall. The adult organizes the world on the basis of consistencies and links bits of information about objects. Because they have more experiences, adults can see commonalities and slowly develop goal-directed procedures to aid them in their memory performance. The sophistication and effectiveness of these cognitive operations normally increases with age and they are used spontaneously when appropriate. These individually unique procedures condense, summarize, and associate old and new information. Most adults have built a solid foundation on which to base new learning. This did not seem to be the case with the participants of this and the pilot study (LeMaitre, 1992), most of whom, for example, had trouble remembering the multiplication tables.

The adults who were given repeats of previous questions did not seem to recall accurately any of the related information. For instance, Helen could remember that part of the answer for question #16 $({}^{20}/_{4} - {}^{20}/_{18})$ was a whole, but she could not recall where it "fit in" and could not make an estimate. She was referring to "the bottom" as the whole when she explained, "so I'm trying to think of a whole for that and that would be thirty-eight and subtract twenty from thirty-eight [pause] I can't remember". Recall, which is more difficult than recognition, involves retrieving items that may or may not be related. When asked to recall items on a list, adults show distinct serial effects^9 (Ornstein, Nause, & Liberty, 1975; Huttenlocher & Burke, 1976). The primacy effect results because items that are presented first are thought to be rehearsed more and put into LTM. The recency effect is present because the last three or four items on the list are still in STM and are, therefore, easier to retrieve. Although individuals with learning disabilities show recency effects, their lack of differentiation or recall across early and middle serial position (ie., no primacy effect) shows the inability to use rehearsal systematically and reliably to get material into LTM (Swanson, 1987) (see Figure 18). This appears to be the case with some of the participants of this study who testified that they simply could not learn their multiplication tables. This should not be confused with those other students who did have the knowledge but seemed unable to retrieve it, perhaps because of the method by which it was stored or decoded.

Figure 18. Idealized diagram of serialized position curves for adults with learning disabilities.

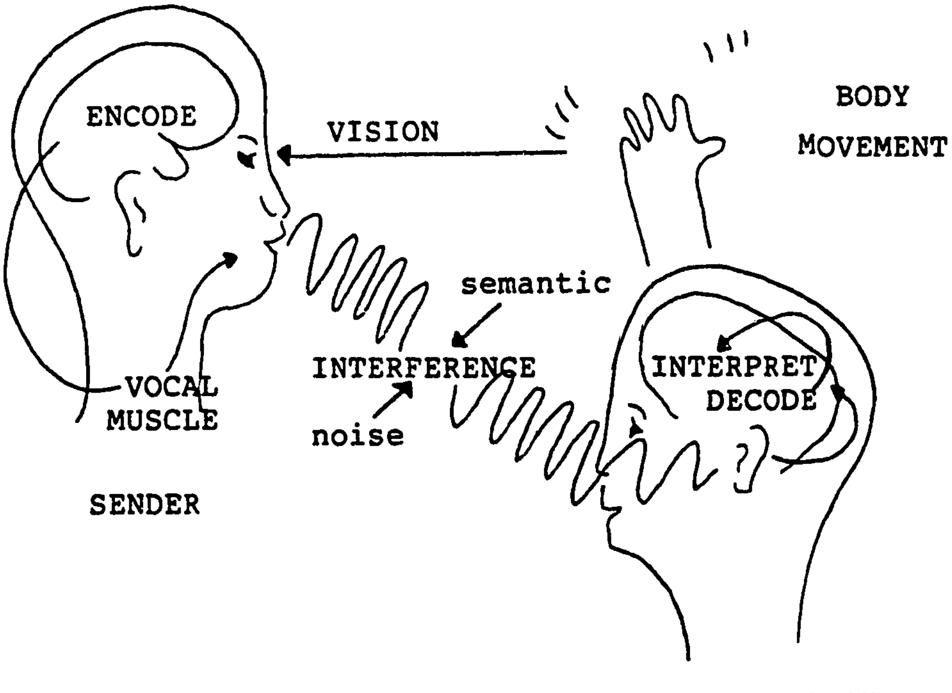


Reasons for Helen's inability to recall information about estimating and solving question #16 could include the fact that she did not fully understand what she was shown and was probably inhibited by old information infringing on the acquisition of this new knowledge.

Since adults with learning disabilities are easily distracted, and their attention span limited, they often have trouble executing more than one demanding task at a time and even acquiring the skills necessary to perform other tasks. Anything not attended to or maintained through memory aids either does not enter the system or is very quickly forgotten. Material that is not rehearsed, or in some way maintained, will begin to decay in about three seconds and will be almost entirely lost within fifteen seconds (Hergenhahn, 1988). Information is coming at them from all directions and in many modalities (e.g., through sight, sound, and smell). For some this can be especially confusing (see Figure 19).

The participants of this and the pilot study did not have a complete grasp of part/whole relationships and usually found the manipulation of fractions with different denominators more difficult than with those that were the same. The addition and subtraction of fractions with differing denominators is more complex since it involves finding a common denominator and converting fractions to their equivalents. Some of the participants in the pilot study were not even sure where to begin with these questions. Adults in the present study found these types of fractions more difficult because they had to "juggle" much more information. None of the students in either study used diagrams to help them determine which fraction was bigger unless this was suggested. They all chose to use circles when diagrams were incorporated although some found it difficult to divide these up into sections other than fourths, thirds, or some multiple thereof. It did not occur to them to use some other shape and when rectangles were drawn for them, many expressed surprise. Despite being given no restrictions on the means participants could use to determine which fraction was bigger, very few spontaneously incorporated any method other than looking at the size of the numbers. Using a common denominator to do the comparison was not something most considered, even after solving the problem.

Figure 19. A diagram adapted from various models (Ross's, 1965; Bryars', 1971; Darnell's, 1971) in an attempt to reflect a two-way exchange and various elements which affect it.



The majority of adults had at least some problems reducing proper fractions. These students had less trouble simplifying improper fractions. They also had difficulties with negative numbers and subtraction, particularly of mixed numbers when borrowing was necessary, but even when the numerators were identical since a zero "on top" resulted. Age could have been a factor, at least for a few of the students, who may have started to experience some memory related errors due to the aging process. Thinking slows because searching for and recalling data take longer; the best memory aids, such as organization, association and imagery are used less; the individual becomes more susceptible to distractions; learning something new takes longer, especially since old learning can interfere; and more cues like music, pictures, words or smells are needed to recall previously learned material (West, 1985). The rate and time of these various changes vary within individuals and from one person to the next, but they can begin in individuals as young as thirty-five years of age.

Some of the students experienced difficulty and anxiety when trying to solve even the easiest questions while others demonstrated trouble verbalizing knowledge that they seemed to be able to apply. Many had trouble assimilating and making accommodations for new material. They seemed to have poor metacognitive abilities and some definitely used inappropriate or inefficient strategies. As was the case in Bachor's (1985) study, many also tended to react negatively to their difficulties by becoming frustrated rather than by looking for their mistakes. He recommended that these individuals should be praised more often as long as it was contingent, specific and credible. Unfortunately, many individuals experiencing difficulties in learning become trapped in a cycle of learned helplessness with repeated failures leading to anxiety, low self-esteem and self-blame. It was clear that some students involved in this and the pilot study were experiencing anxiety and probably attributed much of their difficulty to something wrong with themselves. One of the reasons these poor achievers blame themselves is because they do tend to be blamed. Educators must also begin to look beyond the student for answers.

Lidz (1987) identified good learners as those with an adequate knowledge and skills base who, among other characteristics, were able to spontaneously select and apply relevant strategies and processes in a flexible manner while monitoring and evaluating the results of their endeavours. One of the major stumbling blocks for the participants of this thesis research was their lack of a solid knowledge and skills foundation. It was clear, for instance, that the students understood the process of multiplication but that they simply were unable to recall or remember these facts. I believe that use of a calculator may have freed them to focus on the more abstract concepts and given them greater opportunity for deeper understanding. These helpful tools are used extensively in the work force and I think calculators should be accessible to pupils within the classroom.

Borkowski, Estrada, Milstead, and Hale (1989) pointed out that children with learning disabilities have problems acquiring higher order metacognitive skills such as those described by Alley (1977) and Snowman (1986) (e.g., monitoring, evaluating, analyzing), but said that the training of strategy use alone was insufficient for strategy generalization. They believed that direct instruction related to what they referred to as metacognitive acquisition procedures and attributions would be necessary. Students need to see and accept the benefits of strategy use. Teaching these acquisition procedures and following Baroody and Hume's (1991) and van Erp and Heshusius's (1986) respective recommendations that students be shown the connections between concepts and guided in their use of visual, perceptual, and thinking processes could help adults with learning difficulties. Lessons that are more interesting and meaningful to the students will probably be better understood and, thus, remembered and recalled by them more easily. Consideration should also be given to the task, language usage, the environment, and learning and teaching styles.

The importance of being systematic and neat should be emphasized. This does not mean following an inflexible routine. It means being organized and consistent, always making sure to include equal and operation signs as well as the whole number parts of mixed numbers, so that the equations accurately reflect the statements one is trying to make through mathematics. Many of the errors that these adults made could probably have been eliminated had they been neater. Being neat helps to "free" the mind to focus on greater understanding.

The skill and benefits of being able to estimate cannot be overemphasized. It is a necessary part of everyday life. The ability to multiply is a crucial aspect of estimating. Without it one would be forced in a division, for example, to go through a series of "multiplications" in order to determine the starting divisor. Calculators would also help to alleviate this difficulty and should be allowed as long as students understand the principles for which they are being used.

The approach to solving fraction and other types of mathematical problems should be expanded. Students had problems reducing proper fractions because there was not a quick "solution" as they found with improper fractions (i.e., they must divide the numerator into the denominator). The participants in the study lacked understanding about prime numbers, what to do when one number in the fraction was odd and the other even. Perhaps more of these types of items need to be addressed. Searching for details would have helped most the participants when answering many of the questions. Instructors must distance themselves from the rigid approaches that influence all learners. There are choices one can make when solving fraction problems.

The adults in both the pilot study and this present research stated that they were amenable to any method that would help them to grasp whatever it was that they had to learn. During the first group session, copies of Bennett's (1989) "*Tower of Bars*" were distributed. This helped the students to relate the relative sizes of two or more fractions (i.e., whatever combinations they could devise) because it gave them something concrete to see and touch. They also enjoyed the physical exercises we did before beginning work on the fractions. It seemed to help reduce some of their anxiety and is a technique that has been used in Japan to alleviate stress on the job.

It became clear during the interviews with students partaking in the pilot study (LeMaitre, 1992) that they required an area in which to work "together". This was necessary because the only place they had to work also functioned as the library and they were unable to communicate effectively since it was the policy that students not "talk" while in this facility. This rule was established because instructors found that this communication could become loud and disturbing. These students had many and varied talents. Working together in small groups may help to utilize these abilities and bring students together in a cooperative and beneficial fashion. Pupils, even adults, are reluctant and sometimes frightened to be coerced into "performing" in front of the class and often there seems to be little benefit when individuals are singled out. Working as a team member could help to encourage the students to participate.

Concluding Remarks

The purpose of this study was to better understand the mathematical comprehension of adults having difficulties in learning by examining the procedures used and errors made. Other data was also considered. The results indicated a number of similarities between the adults in this and the pilot study, and those manifested by children as discussed. Some of the difficulties seemed to reflect on the curriculum content and approach.

The reader has been provided with a great deal of information, much of which would not have otherwise been disclosed. It is hoped that this may eventually help instructors to adjust their teaching techniques and select tasks that more closely match the learning styles and abilities of their students as well as the environment in which they all operate. Perhaps by knowing some of the areas and reasons for these adults difficulties aid can be directed to meet these needs, more effective programs designed, alternate forms of support developed and additional teaching tools and resources be made available. I believe it will be beneficial to everyone concerned with the welfare of those within the educational system and will lead to further studies of this important, but somewhat neglected, area.

Footnotes

1. The Learning Clinic Inc. in Edmonton, which performs academic testing, relayed this information to me during a telephone interview on April 9, 1992. The individual I spoke to noted that 30% of their clients had difficulties in mathematics and that 20% of these were severe. This was later confirmed by Dr. Meek, the clinic's director.

   Bachor (1985) also noted that about 50% of the clients seen at the Learning Assistance Center at the University of Victoria were referred for mathematics.

2. Mr. Horvath's comments were aired on CBC's Edmonton A. M. radio news broadcast on August 6, 1993. These findings were confirmed on August 11, 1993 in a telephone conversation with Ms. Glanfield, the Examinee Manager, and later with documented material she sent me, including the news release from which the quote was taken.

3. I did not know at the time that these categories would result.

4. Obviously this could not be proven for cases where the carry-over was one and the place numeral was also one, but since all other instances demonstrated this technique, it was assumed in these instances as well.

5. A discussion with another AVC student revealed that he used this counting around procedure also. In fact he used if for all numbers less than ten.

6. The Curriculum Department of Alberta Education confirmed that this applied to all four western provinces.

7. By trying to add five and seven using my fingers, I found that I would normally use both hands. I would say five to "fix" my position and then begin with my thumb counting it as six and moving towards by little finger before going on to the other hand. I immediately recognized when I had added seven, more because of the pattern than anything else. However, when counting on the fingers of one hand, I double counted the little finger and recognized when to stop because I knew I had counted on another two after using all of the fingers of my right hand and had started with my thumb as "six".

8. I did ask individuals who were not learning disabled and they demonstrated the same neglect as those with learning difficulties.

9. This has not been shown to be the case with all material but serial effects have been demonstrated with digits

10. I also found that the two participants in my independent study had great difficulty understanding what was requested of them depending on the words chosen to make the request (e.g., of or times, divide by or divide into).

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Appendix A

INFORMATION SHEET FOR *MATHQUEST*

The main purpose of this study is to identify and categorize strategies used by adults with learning difficulties when solving fraction-type mathematical problems. It is hoped that this knowledge will assist educators and other professionals to better understand and help these and other individuals with difficulties in learning.

The study will involve the researcher observing students (and helping if necessary) as they work through a number of mathematical questions. Before the students actually begin these mathematics sessions, however, they will be asked to sign consent forms and complete the WJPEB as well as a short questionnaire. This could take up to $1^1/_2$ hours.

Each participating student will work on solving written subtraction and addition fraction problems in addition to answering a few related oral questions during a session. The number of sessions will be dependant upon the length of time each individual needs to answer the questions. It is anticipated that this could involve up to a maximum of four sessions. With the student's permission, these will be tape-recorded. They should last just over one hour in duration so as to fit into the individual's break period. Once all of the sessions have been completed and upon approval of the chairman and teachers involved, the researcher will visit and observe some of the mathematical classes while in session.

Although it would be appreciated if adults taking part in the study would attempt as many questions as possible, they are under no obligation and may withdraw at any time without penalty. Personal information will be kept confidential and the researcher will obtain consent before eliciting any material which may be of a confidential nature. The study's results will be used for research purposes only under the supervision of an academic staff member at the University of Alberta.

If you think that you may want to participate in this study and would like to meet with the researcher, plan on attending the September 9th, 1992 meeting at 1:30 p.m. in the Learning Resources Centre's Orientation Room on the 5th Floor. Everyone, regardless of whether they believe they have a difficulty or not, may attend this information session. See your math teacher if you want to put your name on a list of those interested.

CONSENT TO PARTICIPATE

I, ___________________________________ have read the attached *Information Sheet* and understand the costs as well as benefits expected. I realize that I may withdraw at any time without penalty and that personal information will be kept confidential. The researcher will obtain my consent before any information which may confidential is elicited by her. I also understand that the results of the study may be used in ongoing research. Having read and understood this, I would like to participate in Lesley LeMaitre's study as my signature below indicates.

Date: _________________________________

Signature _________________________________

Witness _________________________________

CONSENT FOR OBTAINING CONFIDENTIAL INFORMATION

I, ______________________________ hereby authorize Lesley LeMaitre, Masters Degree student at the University of Alberta, to request and receive information concerning myself from the Alberta Vocational College as it directly pertains to her study and which by law or otherwise might be considered to be confidential or privileged. I authorize this information to be used, where appropriate, for research purposes under the direction of a University academic staff member.

SIGNED: ______________________________

WITNESS: ______________________________

DATE: ______________________________

CONSENT TO TAPE-RECORD

I, ______________________________________, a participant in Lesley LeMaitre's study involving adults with difficulties in mathematics, give my permission to participate in sessions which are tape-recorded. I understand that these recordings will be used for research purposes only.

SIGNED: ______________________________________

WITNESS: ______________________________________

DATE: ______________________________________

INTERVIEW SUMMARY

for *MATHQUEST*

Please answer the following questions to the best of your ability and as completely as possible.

Student identification:

Name: Male ☐ Female ☐

Date of birth:

What was the level of education you last completed? When was it completed?

What was the last grade in mathematics you completed? When?

Do you enjoy solving fraction questions?

Do you consider yourself good at solving fraction questions?

Do you have any physical impairments which might adversely affect your performance in solving mathematical questions?

Are there any other problems which might affect your performance (e.g., a history of mental illness) in solving the mathematical questions?

Have you ever been assessed as having a learning disability? When and where was the assessment done?

If applicable who is your mathematics instructor?

Do you have any objections to having the mathematical sessions in which you will participate tape recorded?

Do you have any objections to the researcher (Lesley LeMaitre) accessing or discussing your school files or other pertinent records? Please feel free to state any objections or queries. Information will remain confidential.

Comments:

Date:

Signature ______________________________

Witness ______________________________

Appendix B

MathQuest Questions

These questions are printed on 3" X 5" blank cards in horizontal fashion using a green felt marker.

Two examples of oral questions are to estimate the answer to $^{20}/_{5} - {}^{20}/_{19}$ and to state which is the bigger fraction in the equation $^{3}/_{4} + {}^{19}/_{20}$.

1. $^{7}/_{6} + {}^{14}/_{6}{}^{*}$
2. $^{37}/_{26} + {}^{41}/_{26}$
3. $^{4}/_{15} + {}^{1}/_{15}$
4. $^{3}/_{4} + {}^{3}/_{4}$
5. $^{22}/_{29} - {}^{17}/_{29}$
6. $^{30}/_{15} - {}^{20}/_{15}$
7. $^{7}/_{3} - {}^{5}/_{3}$
8. $^{5}/_{6} - {}^{1}/_{6}$
9. $^{11}/_{5} + {}^{8}/_{3}{}^{*}$
10. $^{18}/_{14} + {}^{10}/_{7}$
11. $^{5}/_{9} + {}^{3}/_{7}{}^{**}$
12. $^{7}/_{8} + {}^{9}/_{10}{}^{*}$
13. $^{2}/_{3} - {}^{3}/_{9}$
14. $^{22}/_{4} - {}^{10}/_{2}{}^{*}$
15. $^{3}/_{8} - {}^{1}/_{6}$
16. $^{20}/_{4} - {}^{20}/_{19}{}^{*}$
17. $10^{1}/_{5} + 13^{7}/_{9}$
18. $16^{9}/_{10} + 10^{5}/_{12}{}^{*}$
19. $26^{1}/_{3} - 8^{1}/_{9}$
20. $11^{3}/_{8} + 9^{5}/_{6}$
21. $3^{1}/_{4} + 7^{1}/_{12}$
22. $4^{1}/_{8} - 3^{1}/_{6}$
23. $23^{3}/_{4} - 12^{1}/_{4}{}^{*}$
24. $15^{1}/_{6} - 5^{1}/_{2}$

25. ${}^{13}/_{26} + {}^{14}/_{26}$
26. ${}^{152}/_{8} - {}^{608}/_{32}{}^{*}$
27. $7{}^{1}/_{8} - 6{}^{7}/_{8}$
28. $4{}^{11}/_{12} + 6{}^{5}/_{9}{}^{*}$
29. $2{}^{2}/_{3} + 4{}^{4}/_{10}$
30. ${}^{2}/_{20} + {}^{4}/_{20}$
31. $5{}^{2}/_{9} - 4{}^{2}/_{8}$
32. ${}^{7}/_{4} - {}^{5}/_{4}$
33. ${}^{43}/_{4} - {}^{12}/_{2}{}^{*}$
34. ${}^{20}/_{19} + {}^{3}/_{2}{}^{*}$
35. ${}^{18}/_{12} + {}^{6}/_{12}$
36. $11{}^{1}/_{2} - 7{}^{3}/_{5}$
37. ${}^{1}/_{5} + {}^{4}/_{2}{}^{*}$
38. ${}^{3}/_{15} - {}^{4}/_{20}{}^{*}$
39. $9{}^{4}/_{5} + 5{}^{7}/_{6}{}^{*}$
40. ${}^{2}/_{9} - {}^{5}/_{6}$
41. ${}^{6}/_{3} + {}^{5}/_{3}{}^{*}$
42. ${}^{3}/_{4} + {}^{19}/_{20}{}^{*}$
43. ${}^{16}/_{15} - {}^{11}/_{12}{}^{*}$
44. $10{}^{1}/_{5} + 13{}^{7}/_{9}$
45. ${}^{9}/_{10} + {}^{8}/_{7}{}^{*}$
46. ${}^{2}/_{4} - {}^{3}/_{4}$
47. ${}^{40}/_{14} - {}^{18}/_{14}$
48. ${}^{15}/_{12} + {}^{14}/_{8}{}^{*}$
49. $3{}^{1}/_{10} - 1{}^{2}/_{5}$
50. ${}^{29}/_{7} + {}^{18}/_{7}$
51. ${}^{2}/_{3} + {}^{3}/_{9}{}^{*}$
52. ${}^{4}/_{15} - {}^{1}/_{15}$

Note: * and ** designate additional questions of an oral nature.

Appendix C

Guidelines for Interviewer

In this study the interviewer will meet with potential participants in an informal setting before the actual interview or mathematics sessions begin. This will help both student and researcher feel more comfortable and will be the start to establishing good rapport.

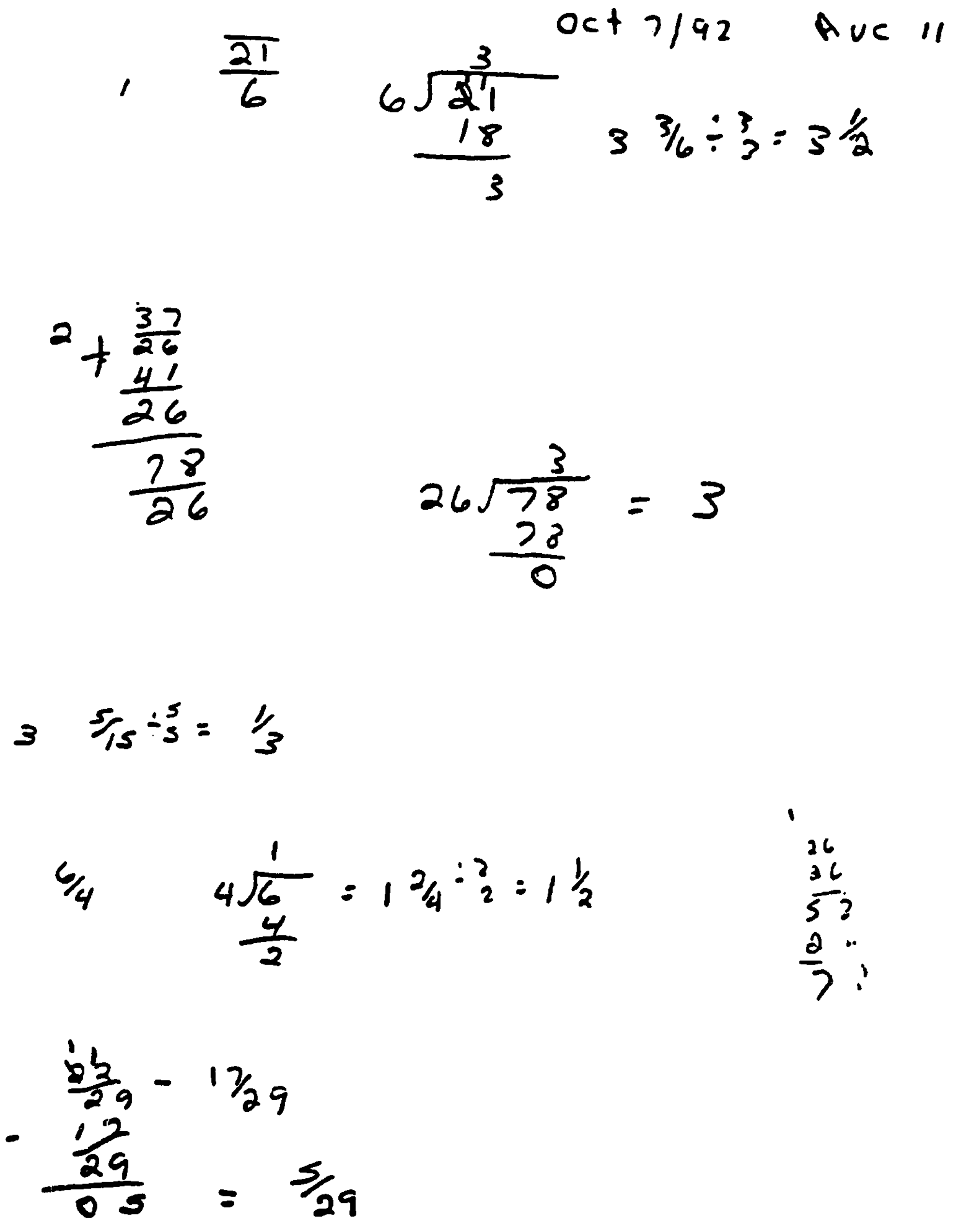
The first individual interview will continue this process even though the Woodcock-Johnson Psycho-Educational Battery--Part Two: Tests of Achievement will be conducted at this time since the researcher will use this opportunity to get to know and understand the student better and try to make him or her feel more relaxed while still maintaining some formality.

During the mathematics sessions, the interviewer will be seated to the individual's left if she or he is right-handed or to his or her right if left-handed. She may encourage the student to verbalize but must be careful not to direct this person towards any particular strategy use except in special circumstances where the student is not progressing at all. Careful notes will be made indicating when this has transpired. Before the interviewer does this, she will attempt to spur the student by asking questions such as, "What do you think you should do?" or "What do you think is the first thing you should do?" In the case where the participant hesitates at what to do with an improper fraction the question might be, "Is the top number bigger than the bottom?" If the student asks if his or her work is okay, the interviewer will first respond with "What do you think?" or some neutral answer. When it is clear to the interviewer that the student is consistently making the same mistake or the student asks for help and does not respond to this type of questioning, the interviewer will point out the students error or demonstrate how the problem may be tackled. Depending how difficult the participant is finding the process and how much aid will be given must be judged by the interviewer, but she will try to limit her assistance to the point where the individual can take over. If the student is becoming frustrated, she will step in.

Encouragement will be given to have the participant do as much work on his or her own and anything else will be noted and utilized in the study. Praise will be given freely but not unduly. Students who seem to find it difficult to verbalize at the same time as performing the operations on the fractions, will be instructed to do the work first and then attempt to explain what they have done and what they were thinking.

Appendix D

Figure 1. Donald's Work for the First Sixteen Questions



Oct 7/92 Avc 11

1 $\frac{21}{6}$ $\begin{array}{r} 3 \\ 6\overline{)21} \\ \underline{18} \\ 3 \end{array}$ $3\ \frac{3}{6} \div \frac{3}{3} = 3\frac{1}{2}$

2 $\begin{array}{r} \frac{37}{26} \\ + \frac{41}{26} \\ \hline \frac{78}{26} \end{array}$ $\begin{array}{r} 3 \\ 26\overline{)78} \\ \underline{78} \\ 0 \end{array} = 3$

3 $\frac{5}{15} \div \frac{5}{5} = \frac{1}{3}$

$\frac{6}{4}$ $\begin{array}{r} 1 \\ 4\overline{)6} \\ \underline{4} \\ 2 \end{array} = 1\frac{2}{4} \div \frac{2}{2} = 1\frac{1}{2}$

$\begin{array}{r} 26 \\ \underline{36} \\ 52 \\ \underline{2} \\ 7 \end{array}$

$\frac{22}{29} - \frac{17}{29}$

$\begin{array}{r} \frac{22}{29} \\ - \frac{17}{29} \\ \hline 05 \end{array} = \frac{5}{29}$

a) Page 1, questions #1 - #5

2

6) $\frac{30}{15}$

$- \frac{20}{15} = \frac{10}{15} \div \frac{5}{5} = \frac{2}{3}$

7) $\frac{7}{3}$

$\frac{5}{3} = \frac{2}{3}$

8 $\frac{5}{6}$

$- \frac{1}{6} = \frac{4}{6} \div \frac{2}{2} = \frac{2}{3}$

9) $\frac{19}{15}$ $\quad \begin{array}{r} 1 \\ 15\overline{)19} \\ \underline{15} \\ 4 \end{array} = 1\ \frac{4}{15}$

10 $\frac{18}{14}$ $\quad \frac{18}{14}$

$\frac{10 \times 2}{7 \times 2} = \frac{20}{14}$

$\frac{38}{14}$ $\quad \begin{array}{r} 2 \\ 14\overline{)38} \\ \underline{28} \\ 10 \end{array}$

$2\ \frac{10}{14} \div \frac{2}{2} = 2\frac{5}{8}\ 7$

41
111
28
~~41~~
2

b) Page 2, questions #6 - #10

③

Oct 7, 1992 AVC 11

1●) Not sure

D

$\frac{5}{9}\times\frac{7}{7} = \frac{35}{63}$

$\frac{3}{7}\times\frac{9}{9} = \frac{27}{63} = \frac{62}{63}$

12 $\frac{7}{8}\times\frac{10}{10} = \frac{70}{80}$

$\frac{9}{10}\times\frac{8}{8} = \frac{72}{80} = \frac{142}{80}$

$80\overline{)142}^{\,1}$

80 -

4

$$\begin{array}{r} 80 \\ 80 \\ \hline 160 \end{array}$$

$$\begin{array}{r} 80 \\ 2 \\ \hline 160 \end{array}$$

$80\overline{)142}^{\,1}$

$$\begin{array}{r} 80 \\ \hline 62 \end{array} = 1\ \frac{62}{80} \div \frac{2}{2} = 1\frac{3}{4}$$

$2\overline{)62}^{\,31}$

6

~~+2~~ 02 11

$2\overline{)80}^{\,40}$

8

00

$80\overline{)}$

c) Page 3, questions #11 - #12

④ AVC 11

13 $\frac{2 \times 3}{3 \times 3} = \frac{6}{9}$

$- \quad \frac{3}{9} = \frac{3}{9} \div \frac{3}{3} = \frac{1}{3}$

14) $\frac{1}{2}$

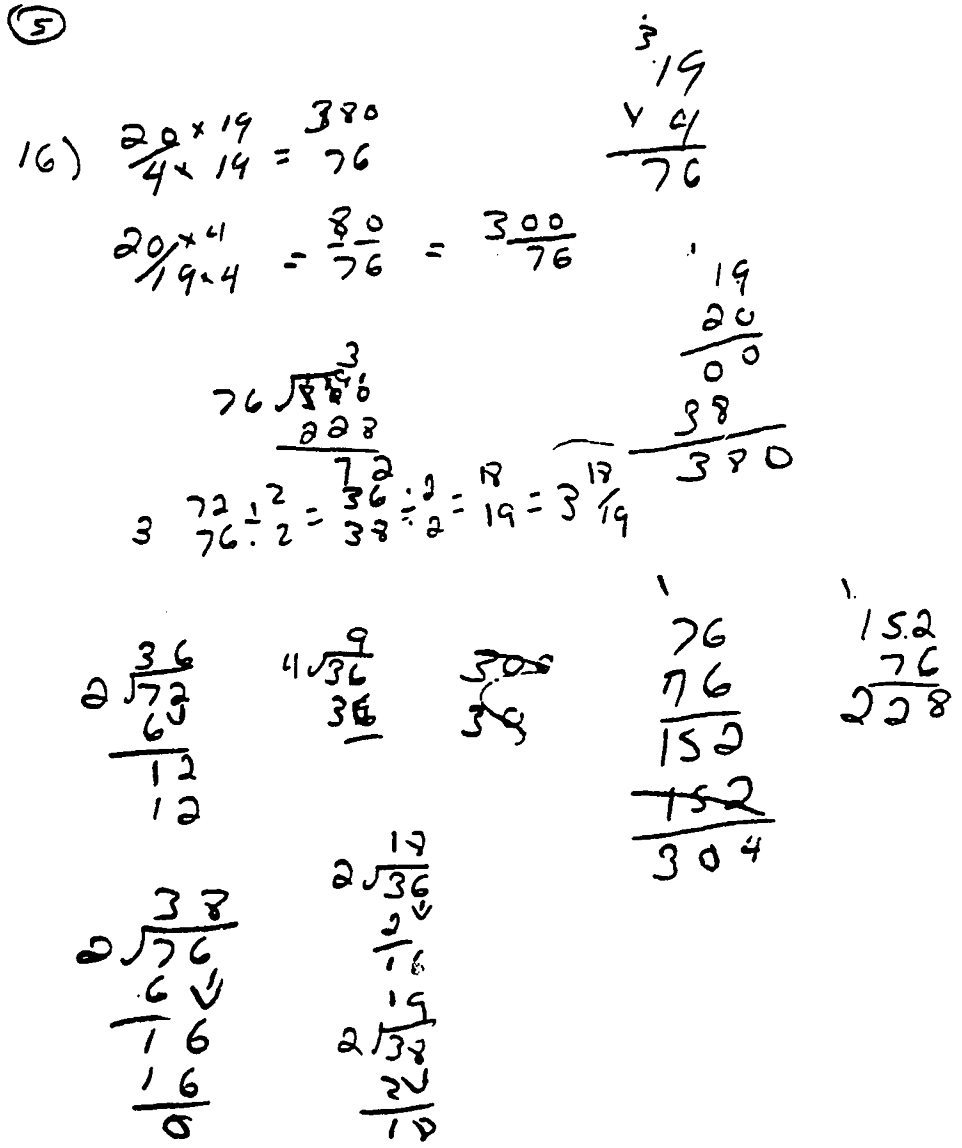
15 $\frac{3 \times 6}{8 \times 6} = \frac{18}{48}$

$- \quad \frac{1 \times 8}{6 \times 8} = \frac{8}{48}$

$$\begin{array}{r} 1\frac{5}{48} \\ \underline{\frac{8}{48}} \end{array} = \frac{10}{48} \div \frac{2}{2} = \frac{5}{24}$$

$$\begin{array}{r} 24 \\ 2\overline{)48} \\ \underline{4}\downarrow \\ 08 \\ \underline{-} \\ 0 \end{array}$$

d) Page 4, questions #13 - #15



⑤

16) $\frac{20 \times 19}{4 \times 14} = \frac{380}{76}$

$\frac{20 \times 4}{19 \times 4} = \frac{80}{76} = \frac{300}{76}$

$76\overline{)380}$ 3

228

72

$3 \quad \frac{72 \div 2}{76 \div 2} = \frac{36 \div 2}{38 \div 2} = \frac{18}{19} = 3\,^{17}/_{19}$

$19 \times 4 = 76$

19
20
00
38
380

$2\overline{)72}$ 36
6
12
12

$4\overline{)36}$ 9
36

~~308~~
38

76
76
152
~~152~~
304

15.2
76
228

$2\overline{)76}$ 38
6
16
16
0

$2\overline{)36}$ 17
2
16

$2\overline{)38}$ 19
2
18

e) Page 5, question #16

Transcript of Session #1 for Donald

SESSION #1, October 7, 1992

Question #1

$^{7}/_{6} + ^{14}/_{6}$

Lesley: Here's the first question. Can you give me an estimate of what the answer will be before you actually work it out?

Donald: [Pause then taps on table seven times with fingers.] It's twenty-one over six. [D. writes twenty-one with a line over it and below it (for the division) over the six.] That's an improper fraction.

Lesley: Okay, so go ahead and work it out then.

Donald: [Student begins to talk to himself quietly as he works it out.] Six goes into twenty-one [D. begins dividing longhand] six, twelve. It's eighteen. eight, nine, ten, eleven. [D. taps for each number as he counts up.] Three and three sixths. Divide that by three [each part of the fraction] equals three and one half. Oops. one half Donald, one half. [D. had written $^{1}/3$ but erased the three and put two.] There.

Lesley: Good. There's the next one [as he is handed question #2]. Oh. And can you put number one here [beside question #1] next to the question so that I know which one you are working on?

Question #2

$^{37}/_{26} + ^{41}/_{26}$

Donald: Thirty-seven over twenty-six plus forty-one over twenty-six is eight [seven and one]. Four, five, six, seven, [pause] twenty-six. [D. is adding thirty-seven and forty-one to get seventy-eight which he puts over twenty-six.] Twenty-six goes into seventy-eight. [D. begins to talk very quietly as he writes out the longhand division and then, in a different area on the page, begins to add twenty-six and twenty-six.] Twelve, five. [D. now adds another twenty-six to this subtotal.] One, two three times [D. is now counting the number of twenty-sixes that he added together and then goes back to complete the division.] Zero. It goes three.

Lesley: Okay. No don't. [D. is about to erase his computation.] It's okay because I like to see what you are doing and if you erase it, then I don't see what you've done.

Question #3

$^{4}/_{15} + ^{1}/_{15}$

Donald: Five over fifteen.

Lesley: Okay. I want you to reduce all fractions to lowest terms.

Donald: [D. works for a few seconds.] One third.

Lesley: Okay.

Question #4

$^{3}/_{4} + ^{3}/_{4}$

Donald: [D. begins to divide six by four longhand and talks aloud as he works this out. He is talking very softly to himself as he subtracts four from six and then divides $^{2}/_{4}$ by "^{2} $_{2}$"]. Two, four, five, six [pause] two. Goes one and one half.

Lesley: Can you speak up just a little bit because I am not sure that your voice is being picked up and I can't quite hear what you're saying. [pause] Does it bother you to do that or does it interfere?

Donald: [pause] These weren't bad.

Lesley: Okay.

Question #5

$^{22}/_{29} - ^{17}/_{29}$

Lesley: Now you're talking quieter [said with a touch of humour].

Donald: I'm sorry. I'm sorry [researcher laughs and student smiles as he begins to talk louder]. Twenty-two minus seventeen over twenty-nine. Okay [sighs], borrow this one [the two in the tens place] makes twelve, seven. Eight, nine, ten, eleven, twelve...five [D. taps his fingers very loudly as he counts to subtract by 'counting up.] Zero. Equals five over twenty-nine. Can't reduce it. [D. wrote question out horizontally but re-wrote $^{17}/_{29}$ vertically when subtracting. Put zero five below subtraction and answer next to this using an equals sign.]

Lesley: Right. That's good.

Question #6

$^{30}/_{15} - ^{20}/_{15}$

Donald: Five [Incorrect question number].

Lesley: Put the page number here so I know it's on the reverse side.

Donald: Thirty from fifteen minus twenty from fifteen [D. meant over fifteen]. Equalsssss [D. makes a whistle-like sound as he continues to sound the "s"] ten, fifteen divided by five [writes little fives beside the ten and the fifteen with a division sign in the middle]. That goes into ten twice and that goes into fifteen three times. [He wrote the answer beside the subtracted fraction.] Number six. [D. is correcting the question number by writing over it.]

Lesley: Do you find it easier to write the question vertically as opposed to horizontally?

Donald: Sometimes.

Lesley: Okay. It depends? [Non verbal response.]

Question #7

$^{7}/_{3} - ^{5}/_{3}$

Donald: Seven over three minus five, three equals two thirds. [D. did not put in operation sign.]

Lesley: Yea. It wasn't very tough was it.

Question #8

$^{5}/_{6} - ^{1}/_{6}$

Donald: Five sixths minus one sixth equals four sixths divide by two equals...Two goes into four twice. Two, four, six [taps finger]. Three times. Two thirds. [Wrote $^{2}/_{2}$ beside the fraction with a division sign in between.]

Question #9

$^{11}/_{5} + ^{8}/_{3}$

Lesley: Can you give me an estimate of this answer before you work it out?

Donald: [pause] Three, six, nine, twelve, fifteen. Times this by five [the three] makes it fifteen. [D. was multiplying three times five by adding three five times.] Times this one [the denominator of five]. I think it's fifteen. Eight. Minus nineteen over fifteen. Divide that. I can't. I've got to do it [meaning that he could not give me an estimate without writing it out].

Lesley: Okay, so what did you figure out there? You just gave me an answer of something.

Donald: Nineteen over fifteen.

Lesley: Okay. But you think that that's not quite right?

Donald: It's an improper fraction. Because the denominator's lower than the nunominator [his word].

Lesley: Well just put that down as your guess and then go ahead and work it out.

Donald: Okay. Nineteen over fifteen. Fifteen goes into nineteen once. fifteen swwwww [makes this sound as he seems to be searching for the answer in his memory for the subtraction of fifteen from nineteen]. What denominator? [This is what it sounds like but D. is talking more to himself.] Four. Equals one and four fifteenths. Four, eight, twelve [as he taps fingers] Sixteen. twenty. I can't.

Lesley: Okay, so what were you just doing there? Oh you're just trying to see if it will go into the fifteen. Right. Okay. [pause] So that's one and four fifteenths?

Donald: Uh hmm. Sorry. Do you want me to change it? [D. incorrectly wrote $^1/_{15}$.]

Lesley: Good. But I didn't see you really do anything here [D. simply worked from his estimate of nineteen over fifteen].

Donald: Right here.

Lesley: Oh. I see.

Donald: Fifteen goes into nineteen once. One times fifteen is fifteen. Minus fifteen from nineteen is one [a mistake already corrected above].

Lesley: Okay.

Donald: Why, is it wrong?

Lesley: You'll see. Just do this other one here first.

Question #10

${}^{18}/_{14} + {}^{10}/_{7}$

Donald: [D. makes some noises to himself.] Eighteen over fourteen. Ten over seven times two [D. writes out "X 2" beside the ten and "X 2" beside the seven. He does not put in subtraction sign] equals twenty over fourteen. Eighteen over fourteen. Thirty-eight over fourteen [erases because he had written eleven down instead of fourteen]. Something's wrong here. [D. talks to himself as he re-adds eighteen and twenty.] Fourteen into thirty-eight [D. wants to divide thirty-eight by fourteen longhand but writes a different number than thirty-eight. He corrects this. He then adds fourteen and fourteen. To this answer he begins to add another fourteen but stops at the addition of the ones place values when he sees that the answer will be too large (i.e., forty-two). He adds the eight and four using his fingers to count up from eight]. No it only goes twice. That's twenty-eight [he writes this under the thirty-eight in the long division]. Eight from eight is zero. Two from three is one. That's two and ten fourteenths. [D. is now talking very quietly].

Lesley: I lost track of what you were doing here. So you've got...Oh okay, Right. Can you reduce it any further?

Donald: Two, four, six, eight [pause] Two, four, six, eight, ten [as he taps out his fingers.] That's five. Two, four, six, eight, ten, twelve, fourteen [taps loudly but doesn't tap on the ten and twelve counts]. That's two and five sixths. [D. writes little two's beside the ten and the fourteen respectively with one division sign as with all the fractions that were reduced before. He used six instead of seven as the denominator and put a negative sign in front of the answer.]

Lesley: Just show me how you did that again.

Donald: I divided by two.

Lesley: Yea but just do it for me again. The process.

Donald: Two, four, six, eight, ten. That's five. Two goes into ten five times.

Lesley: Okay.

Donald: Two, four, six, eight, ten, twelve, fourteen [an extra loud tap on fourteen]. Two goes into fourteen seven times.

Lesley: Okay. Just put a cross through it [the six that he had written before].

Donald: Okay. Seven.

Lesley: Okay.

Donald: Two, four, six, eight, twelve, fourteen. [D. taps loudly on last two.]

Lesley: Okay. I'm just wondering how you got six the first time.

Donald: I just wrote down the wrong number.

Lesley: Okay. You just stopped at the wrong number? Because I heard you say fourteen but you...[I was hoping he would fill in if necessary]. Do you find that easier to do it that way than, say, divide ten directly by two and just...You know taking it into half and saying half and half?

Donald: Yuh.

Lesley: Because you have trouble with that multiplication?

Donald: Hm [nodding head].

Lesley: Okay. Try this one here.

Question #11

$^5/_9 + {}^3/_7$

D. put #10 instead of #11 but filled in the zero when he realized his mistake.

This is probably the most complex question for Donald. The denominators are odd and different, one is not a multiple of the other and neither is easy to count.

Lesley: I have two question for you on this one here. First of all, do you know which is the bigger fraction, this one or this one? Five ninths or three sevenths?

Donald: No. Uh uh.

Lesley: Okay, so can you just put down "not sure which is bigger". Now can you give me an estimate of what the answer will be?

Donald: No.

Lesley: Okay, so go ahead and work it out then.

Donald: [pause] I can't.

Lesley: Sure you can.

Donald: I can't find a CD.

Lesley: Okay. So you know that's the first thing you're supposed to do, right?

Donald: Yea. There is no common denominator. [pause]

Lesley: What makes you think there isn't?

Donald: 'Cause two can't go into it.

Lesley: Uh hu.

Donald: Two, four, six, eight.

Lesley: Right.

Donald: Three, six, nine. No, because three won't go into seven. [D. is looking for number to divide into the denominators rather than a number that both denominators will divide into.]

Lesley: Okay. But do you know what you are looking for when you are finding a common denominator?

Donald: Prime number.

Lesley: Is that what you've been taught? [No verbal response.] Okay. Let me show you what you have to do here. You don't need a number that is going to divide into both of these. What you need is to find a number that both of these will divide into. Do you see?

Donald: There is nothing [sounding very puzzled].

Lesley: Sure there is. There has to be. Even if you multiply the both of them together. Then they both go into that number, right? Because you've multiplied both of them...[I was hoping student would fill in rest of sentence]. Do you get that? [pause] Here, let's look on your multiplication table here. Okay, we've got nine, right? Times seven [finding number and then following column nine down and row seven across until they intersect].

Donald: Sixty-three.

Lesley: So nine goes into sixty-three seven times, right. And seven goes into sixty-three nine times. So that's your common denominator. The number that both of these...this will go into and this will go into [pointing to each in turn]. It is not a number that will divide into those. [pause] Okay? [Big sigh response.] [pause] Is that a confusing point for you?

Donald: [D. doesn't answer question but instead begins to count by fives] Five, ten, fifteen, twenty. Five, ten, fifteen, twenty [taps both times but counts faster second time. Short pause.] Twenty-five, thirty, thirty-five. [D. doesn't tap on twenty-five. He is multiplying out the first numerator of five by seven.] Yea. [In answer to my question.] That's [illegible] sixty-three [the new denominator from seven times nine which he writes down]. Three, six, nine, twelve, fifteen, eighteen, twenty [pause then says very quietly what sounds like twenty-one] Twenty-one. [speaks louder] Twenty-one, twenty-four [pause] twenty-six, [talks quietly to self saying what sounds like eight times nine, then pauses] that's twenty- [pause] Twenty-seven?

Lesley: Um hm. You can double check it using your multiplication table here.

Donald: I don't think so. [D. says this very quietly.]

Lesley: Are you sure?

Donald: [Sighs] Seven, twelve [talking quietly to himself] twelve [pause] five, six. Sixty-two over sixty-three? [D. sounds very hesitant.]

Lesley: [pause] Um hm. In fact that's right answer. That's the right answer. Now I could tell you were getting real frustrated there. Do you find it upsetting?

Donald: Um hm.

Lesley: Okay, so is it just because it's so confusing or? [Mumbles, sounds like, "I don't know."]

Lesley: I mean, you know, this is not something that you can't improve, right?

Donald: I know.

Lesley: So, rather than get frustrated, you should look at it as a turning point. Try and utilize your table if you can. Okay, now that you've done that I'm going to go back to that one answer.

Question #9 (reviewing question)

$^{11}/_{5} + {}^{8}/_{3}$

Lesley: This one here. I want to show you something. Okay, this was #9. Now, #10 you did correctly, okay. You see how you got a number they would both go into [the denominators]? Fourteen goes into fourteen once. Seven goes into fourteen twice. So do you see that process that you used?

Donald: Um hm.

Lesley: Okay. Now what you did here, [question #9] is that you found the right common denominator. But you have to keep in mind, like you did here [question #10], you had to multiply this [the #10 numerator] because you had to multiply the bottom [the denominator of seven] by two, right, to get fourteen?.

Donald: Uh hu.

Lesley: Well, when you've got fifteen here...

Donald: Forgot to do the top.

Lesley: Right. Okay, so that was your mistake. I could see that you could do it but I needed to see whether, what kind of difficulties you have when the denominators aren't the same. So that means that's something we can work on, right?

Donald: Mm hm.

Lesley: Okay. Try and use you past examples. If you see something and you're stopping and you're having confusion about it, look on an answer that you know you got right.

Donald: Uh hu.

Question #12

$^7/_8 + {}^9/_{10}$

Lesley: Can you give me an estimate for the answer for this one? Got any ideas? [No response.] Okay, I'll tell you how you can make an estimate. $^7/_8$, that's almost a whole, right?

Donald: Mm hm.

Lesley: It would be one. And $^9/_{10}$ is also almost a whole. So that's like saying one plus one. So the answer should--it's not going to be exact because it's just a guess--

Donald: Mm hm.

Lesley: The answer's going to be close to two. Okay? So go ahead and work it out. [D. begins to work on paper but says very little. He has no difficulty finding the common denominator and obtaining the improper fraction although it takes awhile.] Your addition seems to be pretty good.

Donald: Mm hm. [D. continues to work out by dividing one hundred forty-two by eighty using the longhand method. D. Talks quietly to himself as he does this. When dividing, he determines that the eighty goes into one hundred forty-two once but puts the eighty under the hundreds and the tens position instead of the tens and the ones position (i.e., the one and the four instead of the four and the two). He realizes that something is wrong as he begins to subtract the eighty from the fourteen.] Zero from four's four. Somethin's wrong here.

Lesley: Okay. Just go ahead and do it over again here [a different spot.]

Donald: Eighty goes into it once because it's one forty-two. It can only go into it once because if it goes twice, it's a hundred and sixty. [D. knew this because he added eighty twice to get that answer. He also then multiplied it by two to double check].

Lesley: Right, that's good thinking. Okay.

Donald: So once!

Lesley: There you go. Now you've got it.

Donald: Mm hmmm. [D. sounds happy with himself and begins his longhand division but now subtracts eighty correctly from one hundred forty-two to get sixty-two as a remainder]. Zero from two is two. Eight from fourteen. eight, nine, ten, eleven, twelve, thirteen, fourteen [counting on fingers but not tapping] Six times. Equals one and sixty-two over eighty. Okay, now wait a minute.

Lesley: Mm hm.

Donald: I can mult...lower that. [Pause]

Lesley: Okay, and how do...

Donald: Does two go into sixty-two?

Lesley: Right. Yea.

Donald: How many times does two go into sixty-two?

Lesley: Um. Okay. Do it longhand. Just do the division like you did here [the previous division].

Donald: Okay. [short pause] Eighty can't go into sixty-two.

Lesley: Is that what you need to do?

Donald: Yea, because eighty is my denominator.

Lesley: Mm hm. But is it a proper or improper fraction [the $^{62}/_{80}$]?

Donald: It's a proper fraction [D. is beginning to sound frustrated].

Lesley: Right. So do you need to divide eighty into sixty-two?

Donald: Well I've got to find out how many... [D. stops and begins to count to himself by two's.] Two, four, six, eight, ten, twelve, fourteen, sixteen, eighteen, twenty. [D. stops here but talks very quietly to himself.] That's ten. [D. then continues talking to himself very quietly and then pauses.] Oh, I can't do it.

Lesley: Here, yes you can. Okay. You're telling me that you want to divide by two, right?

Donald: Well, I've got to find the [pause] 'cause it can go lower.

Lesley: Right.

Donald: So its got to go two because it's two there.

Lesley: Right. So tell me what you want to do here to make it lower.

Donald: I've got to divide.

Lesley: Are you going to divide both of them by two? [D. writes one divide sign followed by "$^{2}{}_{2}$" without a dividing line.]

Donald: Okay, Just a minute. [D. begins to divide two into sixty-two using longhand.] Two goes into three [dividing two into six but sees answer almost immediately so states answer rather than bother saying six].

Lesley: Okay, there. That's what I meant when I said divide by longhand.

Donald: Two [D. puts a little arrow under the two from sixty-two and brings it down. In front of this he puts zero to indicate that six from six is zero.] Two times zero is zero. Two times eight is sixteen. [pause] What am I doing now? [This is almost inaudible. D. is trying to divide eighty by two but begins to multiply instead.]

Lesley: Okay, I think that you're getting yourself confused. Let's stop and take a look at this. Okay. Look at what you did when you divided eighty into one hundred forty-two, right.

Donald: I know. I should have moved it over [under the eighty].

Lesley: Yea, and you did. You corrected that mistake. Okay, now you want to divide two...

Donald: Right. Right.

Lesley: There you go.

Donald: Two, four, six, eight [pause]

Lesley: Right.

Donald: Four times is eight. [Puts an arrow to bring down the zero in the one's place, puts a zero on top and writes two zeroes under the subtraction as he talks quietly to himself. This takes a few seconds.] Zero.

Lesley: Okay.

Donald: Now what was I doing?

Lesley: Okay.

Donald: Two goes into eighty forty times?

Lesley: Right. Right. Okay.

Donald: Divide that by two equals forty.

Lesley: If you get confused, just stop and take it one step at a time.

Donald: Okay, two goes into sixty-two. Two goes into six, two, four, six, three times. Two times three is six, that's zero. Bring down the two. Uhhh [big sigh]. Two goes into two [pause] once. [D. is going over his previous work. He raises the pitch of his voice as he says "once".]

Lesley: Umm hm.

Donald: That's thirty-one times [again raising the pitch of his voice seeming to indicate that it is coming together and he sees where the answer came from and where it belongs. He is now back on track].

Lesley: Right. Okay.

Donald: So that's one and thirty-one sixtieths [but wrote the correct fraction $^{31}/_{40}$]. Can that go lower? I don't think so.

Lesley: So you got it. So that wasn't so tough after all, was it?

Donald: I know. I just get lost. [D. says with an acknowledging chuckle.]

Lesley: Well you see, you're letting yourself get really frustrated too. When that starts to happen, just stop and take a deep breath and take a look--go back to the original question. Because once you divided this into here, then this part [looking at the result of dividing eighty into one hundred forty-two] becomes a proper fraction and that means that this [the denominator] doesn't go into that [the numerator] does it. So that's what you have to look at.

Donald: Okay.

Lesley: Okay, so you now stopped and you took your time--you were getting yourself all worked up when in fact you knew how to do it--but you had to stop and take a look at it. Go back again and look at some examples you had previously.

Donald: Mm hm.

Lesley: Okay, so now you've got the answer.

Donald: Right.

Question #13

$^{2}/_{3} - ^{3}/_{9}$

Lesley: Okay, try that one.

Donald: Two thirds, three, six, nine. Equals six ninths minus three ninths. Equals three sixths three ninths. Woh, tricky question.

Lesley: Okay. [Said with a chuckle.] So can you reduce that any further?

Donald: Three, six, nine [This is followed by tapping]. Three goes into three. Three goes into nine [put one division sign followed by three above three next to fraction. Wrote $^{1}/_{3}$.]

Question #14

$^{22}/_{4} - ^{10}/_{2}$

Lesley: Can you give me an estimate of this answer?

Donald: Times that by four. Times that by two. That'd be twenty over four minus [figures quietly what this figure should be]. Twenty-two from twenty [should be the other way around] equals two over four.

Lesley: Okay.

Donald: I'll right down one half [to this point he had not written anything down].

Lesley: Right. Yup. Okay. So you don't even need to work out the answer because that's right.

Donald: Mm hm.

Lesley: So you can just write down that that's the answer then. One half. Okay. So let's go on to the next one.

Question #15

$^3/_8 - ^1/_6$

Donald: [pause] Sigh. Three eighths minus one sixth [pause]

Lesley: So what are you looking for now?

Donald: Something that will go into them both.

Lesley: Okay, not something that will go into them. Something that both of will go into. Okay?

Donald: That's what I meant.

Lesley: Yea. But that's important. That's crucial. You've got to think about that the right way because that's why you got frustrated on that last question [question #11.]

Donald: Okay.

Lesley: Alright.

Donald: Eight times six, right?

Lesley: You know that that's got to work.

Donald: Forty-eight.

Lesley: Okay.

Donald: [D. writes "X 6" and "X 8" beside each numerator and denominator respectively] three, three, six, nine, twelve, fifteen, eighteen. [D. taps for each count after first number three. D. rewrites converted fractions off to one side.]

Okay. Okay Eight and eighteen [Repeats figures to himself as he writes them down.] Eight oh. Two, four, six, eight [taps as he goes] One, two, three, four. [D. is dividing two into forty-eight using longhand.] Two, four. Two, four. Tap, tap [long pause] Tap, tap, tap, tap [D. talks to himself. Followed by a long pause.] What am I...Oh.

Lesley: Okay? [D. has flipped page to begin next question.]

Question #16

$^{20}/_{4} - ^{20}/_{19}$

Lesley: Here's a good one to give me an estimate on. [D. begins to look back on other questions.] Are you comparing it?

Donald: Uh uh. Number sixteen.

Lesley: Okay, number sixteen. What page is this? That's four, so this is going to be number five.

Donald: Can't give an estimate.

Lesley: No.

Donald: It will be a whole number anyway.

Lesley: Okay, how do you know that?

Donald: Because both tops are that.

Lesley: Because they're both twenty?

Donald: Twenty from twenty is nothing.

Lesley: Okay, so how would that make it a whole number then?

Donald: Just a second.

Lesley: Okay.

Donald: I don't know. Sigh.

Lesley: Okay. Now you're doing a lot of thinking but you're not telling me what you are thinking.

Donald: I don't know yet.

Lesley: Okay.

Donald: Sigh. Mm hm. [long pause] Thirty-six, four, five, six, seven [D. is multiplying nineteen times four.] Seventy-six.

Lesley: Mm Hm.

Donald: I don't know [pause] know what to do. [pause] Four times two is eight.

Lesley: Right.

Donald: [D. takes a deep breadth.] Four times nineteen issss seventy-six. Ha! [This wasn't automatic.] Nineteen times [the 20 fades out] is [pause] Zero times nine is zero. Zero times one is zero. Two times nine is eighteen. Nine and nine is eighteen. Two times one is two and one is three. Zero, eight, three [D. is adding the row of zero's to the row containing thirty-eight.] Whooooh. Three hundred eighty. Now what am I supposed to do? Minus eighty. Equals three hundred. Improper fraction.

Lesley: Mm hm.

Donald: And seventy-six goes into three hundred. [pause] I don't know.

Lesley: Okay. Well, um. Does it go into thirty?

Donald: No.

Lesley: Okay. So we have to look at three hundred.

Donald: Mm. Three hundred...

Lesley: Okay.

Donald: Seventy-six and seventy-six is twelve. Seven and seven. That's fourteen. That's fifteen.

Lesley: Mm hm.

Donald: Two, two, four [D. is now adding one hundred fifty-two to itself.] That's four. That's ten. Mmmmm. Slpp. One fifty-two. Seventy-six. [One hundred fifty-two plus seventy-six.] Seven, eight [six plus two]. Seven, eight, nine, ten, eleven, twelve [seven plus five]. That's two hundred and twenty-eight.

Lesley: Okay. Well that was good what you did here because in essence you multiplied it by two. You added them twice and then you said, well you're going to try and add this twice and you knew then it was seventy-six four times, right?

Donald: Mm hm. But it's only three times. It goes in there two hundred and twenty-eight times.

Lesley: [pause] Is that right?

Donald: Sigh. Six, seven, eight.

Lesley: Mm hm.

Donald: Twelve [short pause] two. [D. is checking the addition of one hundred fifty-two and seventy-six.]

Lesley: Okay. Show me what...

Donald: So it's seventy-six goes into three hundred [pause] three times.

Lesley: Okay.

Donald: One, two, three times.

Lesley: Right. Okay.

Donald: Whooo!

Lesley: You kind of scared me there for a second. [D. gives a relieved laugh.]

Donald: Okay. That's one, ten. That's minus ten. Eight, nine, two. Eight, nine, ten's two. [D. is subtracting two hundred twenty-eight from three hundred.] Minus two. [D. holds up nine fingers and "takes away" two when doing the following subtraction.] Seven. Three, one. [D. crosses out the three in three hundred and puts a little one above it when subtracting the two hundred twenty-eight.] Seventy-two over seventy-six. [D. writes this and the resulting reduced fractions without a dividing line between them.] Just a minute.

Lesley: Um hm.

Donald: Two goes into seventy-two. Two, four, six, eight [tapping loudly for each number.] I wonder what else goes better. Two doesn't go into seven.

Lesley: Doesn't go into what?

Donald: Two, four, six, [tapping loudly] three. Six.

Lesley: Mm hm.

Donald: Minus one. Two, four, six, eight, ten, twelve [tapping] six. [D. is dividing two into seventy-two.] That's twelve. Soowheeee. Thirty-six. [D. taps loudly three times as he begins to divide two into seventy-six using the longhand method as before.] Three. Six. One. Two, four, six, eight, ten, twelve, fourteen, sixteen. Goes eight times. Sixteen. Zero. [pause] Thirty-eight. Oh come on. This is getting silly. [D. recognizes that he can divide both results again by two and makes some hissing sounds through his nose followed by a long pause. Yawns as he starts to divide thirty-six by four.] F... you. Two, four [Tap, tap]. Four doesn't go into three, so it's got to go into thirty-six. Nine. Nine times. [D. says this in a very animated way which he follows by a pause.] Oh Christ. Nine. Four won't go into thirty-eight. [D. makes one loud tap followed by a series of taps on the table not associated with counting.] It doesn't go into thirty-eight.

Lesley: No.

Donald: F....

Lesley: Well, you know how you know it doesn't go anyway?

Donald: Uhhhhh.

Lesley: See because if it goes in there [the thirty-six], there's only two left over.

Donald: Um hm.

Lesley: Four can't go into two. You know, a whole number. It would have to be a fraction, right?

Donald: I know. I've got to get this low one yet.

Lesley: Okay, so...[pause] your best bet is to...

Donald: Stick with two's.

Lesley: Right.

Donald: Yup. Okay. Four go into thirty-six [slight pause] nine times. Haaa. Two no go no. [erases the "4's" he had put beside each part of the fraction with one dividing sign in between.]

Lesley: Because you are dividing by two, not four.

Donald: Yea. I did it here somewhere. [He remembers doing a division and is looking for the work but can't find it because he hadn't divided thirty-six by two before. He doesn't seem to remember that he had divided it by four and begins tapping table.] I'll try it again. Two goes into thirty-six. Two, four [tap, tap, then pause]

Lesley: Use your...

Donald: Nine times.

Lesley: Maybe your table will help you a bit.

Donald: No. [pause] Two, four, six [His voice is fading].

Lesley: Okay...

Donald: Five [Almost inaudible.]

Lesley: ...does two go into three?

Donald: No. Once.

Lesley: Okay, there you go. [He now has started the long division of thirty-six by two.]

Donald: One, two, once. Two go into sixteen [pause] eight times. One, eight, eighteen. Two go into thirtyyyyy [short pause] once. Two's from [pause] eighteen. [As with the other longhand division, D. uses and arrow below the ones place to show that it should be brought down.] Nine times. And two in...equals...ohhh...forget it. [D. realizes that he has already divided the two into the eighteen which he created when he brought down the eight from the ones place.] Eighteen over nineteen.

Lesley: Um hm. Okay. And that's right. Now I'll show you how you could have made a guess at this, okay.

Donald: I could have done it by four.

Lesley: Yea, you could have. But, you know... The thing is if you know it goes twice, and you do because it's an even number, right?...

Donald: Um hm.

Lesley: ...it doesn't hurt to stop and divide it by two 'cause...you know that if you're going to spend a lot of time to try and figure out what to divide it by, you're faster just to go ahead and divide it by two. Okay. See, like, the way I would have estimated here is I would have said [looking at the original question]...

Donald: Haaaaa [big sigh].

Lesley: ...four goes into twenty five times.

Donald: Um hm. [D. taps table a couple of times.]

Lesley: But you see, this is a problem if you have difficulty with your multiplication [D. sighs again.] table. But you know you can always utilize this thing, right [the actual table]. Okay, there's twenty.

Donald: [tap] Um hm.

Lesley: So you can find the number and then look to see what...So five. So it's divided by four.

Donald: Haaaaa. [big sigh]

Lesley: So that goes in there five times.

Donald: Um hm.

Lesley: And nineteen [pause] twenty nineteenths. That's just over one, right? It's almost one.

Donald: Yea.

Lesley: So my estimate would have been about four and this is...You can see your really close because three and eighteen nineteenths. That's almost four, isn't it?

Donald: Um hm. Whoooah! [D. begins making a lot of noise with the pages.]

Lesley: Has it been a long day today?

Donald: Uh hu.

Lesley: Lots of concentrating?

Donald: Yea. [Response is almost inaudible and is followed by a pause.]

Lesley: Do you find it's really tiring after awhile?

Donald: Oh yea [almost inaudible].

Summary of General Information

Question #1

1. Work is somewhat crowded and a little sloppy (e.g., some sevens look like the number two).
2. Forgot to put question number and when he did put it in when reminded, it looked like a simple mark on the page--not the number one.
3. Unable to estimate. Seems to think estimate means doing it in your head without resorting to writing anything out. D. makes an estimate by working out the problem (e.g., $^{21}/_{6}$). Although in this case it may have been almost as easy to do so since the numbers were so easily added. (Unfortunately Donald sometimes had difficulty with this and division, so both actions would have been difficult for him.)
4. Student recognized the subtotal as an improper fraction.
5. Erased mistakes.
6. Did not write question, just did work (i.e., it seems he found this question easier).
7. Did not automatically reduce intermediate fraction.
8. Could not multiply or divide easily.

Question #2

1. Did not automatically reduce fraction.
2. No equal sign.
3. Did calculation on a different part of the page, not near the problem.
4. Donald was going to erase the calculation he used to determine how many times twenty-six went into seventy-eight. This meant that he would not be able to utilize this information again and that it would take time out of his allotment.
5. He carried on with the longhand division even though he knew the answer already. He didn't seem to accept this answer until he subtracted in the longhand division.

Question #3

1. Did not write out question, just work.

Question #4

1. Did not write out question, just work.

Question #5

1. Utilized first fraction in horizontal and vertical equation. However, this could be detrimental to this student.
2. Wrote the numerator part of the answer with a zero in front, below the subtraction and then said this was equal to this difference (without the zero), over the denominator.

Question #6

1. Said thirty from fifteen and twenty from fifteen instead of thirty and twenty over fifteen.
2. Wrote the numeral six over five when correcting question number from five to six.
3. Wrote equal sign beside second fraction and put answer to question here. Since he also wrote equivalent fractions next to each fraction, he could encounter problems, especially since he was really confused.

Question #7

1. Left out the operation sign.
2. Bracketed the question number.
3. Wrote answer beside second fraction so that it looked as if the fraction was equal to the answer instead of continuing the "vertical" process.

Question #8

1. Wrote answer beside second fraction instead of continuing the "vertical" process.
2. Used a single division sign and two little two when reducing four over six but this time included a "dividing" line between the twos.

Question #9

Note: This was the first question with different denominators and student was unable to answer it correctly.

1. Did not seem to understand what an estimate is. When asked for an estimate, he immediately began to try and find a common denominator. But he probably could not estimate very well anyway since he could not divide. It did not occur to him to look at the difference between the two numbers in order to make a guess.
2. Gave an intermediate answer of minus nineteen over fifteen (changed to $^{19}/_{5}$).
3. Bracketed question number.
4. Incorrectly called numerator nunominator.
5. Did not recognize that four fifteenths could not be reduced without "multiplying" (by performing a series of additions of fours up to a total of twenty) until it surpassed 15 without ever equalling it.
6. Seemed unsure whether common denominator was the addition of the two denominators or their multiplication.
7. In doing the division said, "Minus fifteen from nineteen is one [instead of four]." He probably meant "goes in" once.
8. Seemed a little hesitant with the commutative law of multiplication.

Question #10

1. Student did not automatically realize that thirty-eight less twenty-eight was ten and that fourteen could not possibly go into thirty-eight more than twice.
2. Student made no attempt to reduce fraction.
3. He seemed to miss the ten and twelve counts and came up with the answer that two went into fourteen six times.
4. Did not seem to recognize how he made mistake and said that he just wrote the answer down wrong but he definitely seemed to have kept track of the number of twos incorrectly when dividing fourteen by two. It may be related to the fact that he did not seem to tap on the ten and twelve counts--perhaps he was thrown off.
5. Student clearly has difficulties with multiplication and other number manipulations.
6. Used eraser to correct one mistake but not another (i.e., wrote a seven over a six).
7. Wrote a negative sign in front of the answer.
8. Did not check work until asked by researcher what he had done.
9. Left out operation sign.
10. Put an equal sign between the lower fraction and its equivalent fraction but did not do this for the upper fraction.
11. No equal sign was used in front of the intermediate answer.
12. Could not divide even simple numbers by two without using fingers and counting out (e.g., ten and fourteen).
13. Made strange noises to himself.

Question #11

Note: This was perhaps the most complex question to date. The denominators are relatively large, odd numbers. They differ from one another and it was probably more difficult to find a common denominator because of this and the fact that they are not multiples of one another.

1. Wrote the wrong question number down. Instead of erasing and writing the correct question number, student simply filled in the zero and wrote numbers over others.
2. Student bracketed question number.
3. Student did not know which fraction was bigger and could not estimate.
4. Student made no attempt on his own to work out problem because he said that there was no common denominator. When asked why he thought that there was no common denominator student's response was, "'Cause two can't go into it."
5. Student seemed to lack a strategy to find a common denominator and became frustrated. He said one looked for a prime number when finding a common denominator.
6. Left out operation sign.
7. Could not use past knowledge.
8. Used eraser to erase one mistake but not others.
9. Seemed unable or unwilling to check work using multiplication table.
10. Had trouble adding three's once eighteen was reached.

11. Did not use past examples or information that was handy (e.g., the multiplication table).
12. Questionable whether he knew what a prime number was.
13. Used little multiplication signs.
14. Seemed to have a mind set. When told that he needed a number that both the denominators went into he said that there was nothing and he used the example of finding a common denominator for almost every other question.

Question #12

1. Began to get frustrated when he could not estimate.
2. Wasn't sure if two went into sixty-two.
3. Lost track of what he was doing and became confused. "Eighty can't go into sixty-two’ when he was trying to reduce sixty-two over eighty".
4. Brought down zeros in division.
5. Unable to utilize the fact that eighty was ten times eight when multiplying eighty by two.
6. Utilized information that eighty times two was greater than one hundred twenty to determine how many times it went into one hundred twenty.
7. Did not know how to estimate.
8. Put eighty under fourteen in the hundreds and tens place in the division of one hundred forty-two by eighty.
9. Recognized that it reduced.
10. Ready to give up easily.
11. Became confused and started to multiply eighty by two instead of dividing it by two.
12. In the confusion wanted to divide sixty-two by eighty.
13. Really struggled with this question.
14. Writing answers beside the equivalent fractions could be especially confusing since he also wrote these next to the originals.

Question #13

1. Could have reduced three ninths but chose to multiply two thirds by three over three.
2. D. thought that this was a tricky question.
3. Had to be asked to reduce fraction.
4. Did not notice that three ninths could be reduced to two thirds (i.e., he was unable to utilize previous information).
5. Had to add three, six, nine to see if three would go into nine.
6. Multiplied by adding three's three times even though he had performed this operation before.

Question #14

1. Estimated by trying to add in head.

2. Was going to cross multiply but didn't seem to do this.
3. Reduced right away.
4. Got numbers mixed up but came up with correct answer.
5. Was going to multiply denominators together but it looks as if he recognized that two was a multiple of four and multiplied this fraction to get equivalent denominators.

Question #15

1. Struggled with work.
2. Said that he was looking for something that would go into both denominators when looking for a common denominator when he should have been looking for a common multiple. Said that this was what he meant.

Question #16 (From sessions #1 and #2.)

1. Could not estimate but said it would be a whole number because twenty minus twenty (the numerators) was zero.
2. Tried to compare with a question before but unsuccessful.
3. Writing a little sloppy and careless (e.g., forgot division line in fraction and left out whole number until end but still said "equal to"14. Wrote over other numbers).
4. Double checked that nine times two was eighteen by adding nine and nine (emphasis on checking work).
5. Got lost in what he was doing.
6. Recognized improper fraction.
7. Did not know how to approach dividing seventy-six into three hundred.
8. Got mixed up and said seventy-six goes into three hundred two hundred and twenty-eight times but got back on track when questioned.
9. Very inefficient when dividing seventy-six into three hundred. Many steps to keep track of.
10. Circled answer.
11. Became frustrated.
12. Wondered what to do after multiplying nineteen times four. Then, "eureka", realized that four times nineteen was the same as nineteen times four.
13. Divided four into thirty-six but upset when four did not divide into thirty-eight.
14. Unable to make use of the fact that if four went into thirty-six nine times, then two would go into it eighteen times. Abstract thinking was difficult.
15. Always divided "into" not "by".
16. Sometimes said the wrong thing but did the right one (e.g., said seventy-six went into three hundred, two hundred twenty-eight times).
17. Knew addition of some numbers (i.e., six and six; seven and seven).
18. Questioned what would go better than two into seventy-two because two "didn't go" into seven. After dividing two into seventy-two, divided it into seventy-six, beginning the process all over.
19. Added fours to see if it would divide into twenty. Made a line for every four he counted and then counted the lines.

20. Said that $4^{19}/_{19} - {}^{20}/_{19} = {}^{(20 - 19)}/_{(19 - 19)} = {}^{1}/_{0}$.
21. Divided twenty by nineteen longhand.
22. Converted to vertical format.
23. When subtracting two twenty-eight from three hundred, crossed off the three and put one instead of two and simply ignored fact that this did not make sense. He probably did not realize what he had done but this could have posed problems later.
24. Said that ${}^{20}/_{19}$ was about twenty.
25. Great difficulty with abstract ideas.
26. Could not reduce ${}^{20}/_{4}$ (during one of the sessions).
27. Confused when asked for an exact answer.
28. Even after working through question and obtaining answer in session #2 (i.e., that is for the second time), still could not understand how it was done.

Summary of Solution Procedures and Tactics

Question #1

$^{7}/_{6} + ^{14}/_{6}$

1. Student "counted on" from the larger number while tapping with his fingers to do addition of seven and fourteen.
2. Donald talked quietly to himself as he tried to work through problem. For example he told himself off when he made a mistake (i.e., "One half, Donald, one half").
3. Used a series of additions to multiply (e.g., 6 X 3 = 6 + 6 + 6).
4. Used longhand even for simple division. (e.g., 21 ÷ 6).
5. Counted and tapped using fingers when subtracting by counting up from the number being subtracted beginning with the "ones" column (as opposed to counting up from eighteen to twenty-one). For example when subtracting eighteen from twenty-one, he counted up from eight to eleven.
6. Tapped on fingers and kept [illegible] of "how many" this way.
7. Wrote with one division sign [illegible] numbers for which he was dividing numerator and denominator.

Question #2

$^{37}/_{26} + ^{41}/_{26}$

1. Added three and four by counting up from three using his fingers.
2. Talked to himself.
3. Re-wrote question vertically.
4. Performed the division of seventy-eight by twenty-six by adding twenty-six twice and then adding another on to this subtotal.
5. After adding 26 three times, he went back and counted how many times he had added these.
6. Used longhand division.

Question #3

$^{4}/_{15} + ^{1}/_{15}$

1. Used one division sign and two little fives beside the numerator and denominator respectively when reducing fiver over fifteen.

Question #4

$^{3}/_{4} + ^{3}/_{4}$

1. Divided six by four using longhand.
2. Talked to himself quietly.

4. He added two to four and then double checked that the answer was six by counting up from four and seeing that this was two.
5. Wrote one division sign and two twos for the numerator and denominator respectively when reducing two over four (i.e., he did not automatically recognize this as one half).

Question #5

$^{22}/_{29} - {}^{17}/_{29}$

1. Initially wrote this question out horizontally but re-wrote the second fraction and subtraction sign below the first to convert it to a vertical format.
2. When subtracting, counted up from seven to twelve using his fingers and tapping for each count. He keeps track on his fingers the difference between the two numbers.

Question #6

$^{30}/_{15} - {}^{20}/_{15}$

1. Re-wrote question vertically.
2. Used a single division sign and two little fives to reduce ten fifteenths.

Question #7

$^{7}/_{3} - {}^{5}/_{3}$

1. Re-wrote question vertically.

Question #8

$^{5}/_{6} - {}^{1}/_{6}$

1. Re-wrote question vertically.
2. Divided six by two by tapping out three twos on his fingers as he added these. This was even though he had just figured that two went four twice.
3. Unable to utilize prior knowledge.
6. Single division sign.

Question #9

$^{11}/_{5} + {}^{8}/_{3}$

1. Obtained the common denominator by "multiplying" the two denominators together. This was appropriate in this case.

2. Added three five times in order to find the common denominator. He could have added the five three times which would have been easier and faster.
3. He added the two numerators (to get nineteen), without regard to the changes made to the denominators.
6. Gave an intermediate answer as minus nineteen over fifteen but said he could not give an estimate without working it out.
7. Divided nineteen by fifteen using longhand.
8. Attempted to see if four fifteenths could be reduced by adding fours until he reached twenty. None of the numbers equalled fifteen, so he knew that it could not be reduced.

Question #10

$^{18}/_{14} + {}^{10}/_{7}$

1. Re-wrote question out vertically.
2. Made noises to himself (e.g., humming).
3. Wrote equivalent fractions to the right of original fractions.
4. Divided fourteen into thirty-eight by using the longhand method.
5. He added fourteen and fourteen and then added another fourteen to the resulting answer of twenty-eight to see if fourteen will go into thirty-eight a third time.
6. Student used his fingers to add eight and four beginning with eight and adding four to this.
7. He subtracted twenty-eight from thirty-eight beginning with the ones place. He did not recognize the answer automatically as ten.
8. Counted as he tapped on fingers to see how many times two would divide into ten.
9. Counted as he tapped on fingers to divide two into fourteen. He seemed to miss the ten and twelve counts and came up with the answer that two went into fourteen six times.
10. D. did not realize that thirty-eight minus twenty-eight was ten and subtracted by writing it out.
11. Rounded $^{10}/_{7}$ up rather than reduce $^{18}/_{14}$.

Question #11

$^{5}/_{9} + {}^{3}/_{7}$

1. Talked to himself.
3. He tried to see if two would go into either denominator by adding a series of twos until he reached ten. He then tried adding three's but found it did not go into the seven.
4. Found common denominator by automatically multiplying both denominators which, in this case, would have given the least common denominator.
5. Added smaller number nine times rather than larger number fewer times when multiplying nine times three. Probably could not add large number as well.
6. Added carry-over after adding other numbers.

Question #12

$^{7}/_{8} + {}^{9}/_{10}$

1. Found common denominator by automatically multiplying both denominators.
2. Added eighty twice to get one hundred sixty and then multiplied out eighty times two to double check.
3. Talked to himself.
4. Counted up from eight to fourteen to subtract.
5. Divided two into sixty-two longhand.
6. During his first attempt at dividing sixty-two by two began to add two's. He went up to twenty. It seemed that he wanted to keep track of how many twenty's (i.e., ten twos) but that this was too complex.
7. Used little arrows to "bring down" the numbers in the long division.
8. Divided by a series of additions even when dividing six by two.

Question #13

$^{2}/_{3} - {}^{3}/_{9}$

1. Added three three times to see if it would go into nine and to see how many times.
2. Repeated the identical action when reducing the first answer of three ninths.

Question #14

$^{22}/_{4} - {}^{10}/_{2}$

1. Found a CD and subtracted as usual.

Question #15

$^{3}/_{8} - {}^{1}/_{6}$

1. Multiplied three by six by adding three six times.
2. Divided eight by two by adding four two's and then counting how many fingers.
3. Automatically multiplied denominators together to find the common denominator.

Question #16

$^{20}/_{4} - {}^{20}/_{19}$

1. Added three and four by counting up from four. He appeared to count the "knobs" on the figure 3.
2. Multiplied nineteen times twenty longhand (i.e., was not able to make use of the fact that twenty was ten times two).

3. Donald added seven and five by counting up from seven. He created points around the five and in the middle of it when counting it on.
4. Counted up from eight to subtract eight from ten. Usually said this first number but did not count it on his fingers.
5. To subtract two from nine, Donald held up nine fingers and then bent two down. He noted that there are seven fingers left (i.e., five plus two).
6. To divide two into sixteen, Donald counted eight two's on his fingers.
7. Double checked that two times nine was eighteen by adding nine and nine.
8. Began to divide two into thirty-six but said that it did not go into three (counted two, four) and, therefore, it had to go into thirty-six as a whole. Earlier I had said that seventy-six did not go into thirty so that it must then go into three hundred.
9. Divided three hundred by seventy-six by adding seventy-six twice and then adding this subtotal to itself. He saw that it was too large so subtracted seventy-six from this result. He then counted the number of seventy-sixes that he had to add to get two hundred and twenty-eight.
10. Divided two into seventy-six longhand by first adding two's four times and then three times. He saw that four was too much and three times was one less than seven.
11. When dividing longhand, Donald used little arrows to bring down the next appropriate number.

Appendix E

Methodology of Pilot Study

After explaining generally what the study was about, arrangements were made to meet with those interested in more specific information. One of the classrooms was made available for this purpose. Almost thirty students signed up as potential participants but only seventeen actually booked for a mathematical session. About one hour was spent with each participant the week prior to the start of the sessions. The first ninety minute session included time to allow the student to become as comfortable as possible and to fill out a short questionnaire.

Material

The eighty-six written questions and numerous oral questions were developed using an elaborated version of Birenbaum and Shaw's (1985) *Task Analysis Chart* (TAC) as well as other research and documented material such as the *Direct Instruction Mathematics* manual. Most of those completed by the majority of the students were of subtraction and addition but individuals participating in two sessions also completed several multiplication and division types. The numbers of oral questions (i.e., those pertaining to the size of fractions or involving estimation of the answer) as well as those written, varied because each person did as many questions as possible during the time given.

Tests were not conducted as these adults had written the Canadian Achievement Test (CAT) the previous year and most were due to write a second test shortly after the sessions were to end. With the participants' permission, the CAT results were relayed to me the individual in charge of testing.

Participants

The students ranged in age from twenty to fifty years with an average age of thirty-two years for both groups. Two thirds of the students were in their twenty's and thirty's but one third were in their forty's (one was fifty years). Although most had been attending school for the last year at least, on average they had been out of the system for fourteen years. However, these fell into distinct groups as follows. Six had been away for two to three years, one for seven years, three for twelve to sixteen years and seven between thirty to thirty-two years. One student did not provide any of this information and his age was estimated at close to fifty years. It was assumed he would have been out of school for at least twenty years. No student was eliminated due to a physical handicap because these were not deemed to have caused, or greatly impacted on the types of difficulties in mathematics manifested by them.

Results

Data Analysis

Summaries of participants' results were made using available tapes, notes, and each student's actual work. From these information was extracted for each person. Most of this fell into categories related to difficulties encountered by these individuals. These were then amalgamated for all of the participants and Table 1 was created highlighting those difficulties experienced by the greatest numbers of students.

An average of forty-three questions were completed by students attending two sessions while this figure was twenty for those participating in one full session. One student completed only six questions before becoming frustrated during the seventh and leaving. Up to that point she had answered all of the questions correctly despite struggling with whole number division and an inability to multiply or reduce. Another left after 50 minutes due to family concerns. Six completed one session and nine went on to complete a second three weeks later. The student who attended for fifty minutes completed fourteen question. Most of the participants completing the same number of questions also completed the same questions but not necessarily in the same order, except for the first seven. The first six involved the addition and subtraction of fractions with the same denominator. The seventh fraction involved a subtraction of improper fractions having different denominators and an oral question asking for an estimate. A five minute break half way through the session was given to any student desiring one. Most chose to continue without stopping. Those completing two sessions did not appear to be any more capable that those completing only one and visa versa. However most of them did seem to be more determined to improve their understanding.

Nineteen of the twenty-six sessions were taped and an attempt to provide elaborate notes, particularly if taping was not possible, was made. Summaries were made based on these tapes, notes and the student's own work. From these the problems which became apparent were noted along with any unique methods used. Table 1 highlights the more obvious and consistent difficulties experienced by more of those taking part.

Three group sessions were also held for any student wishing to take part. These were more for the benefit of the students but they also provided some interesting insights. Brief summaries of two of the students' work for the first session follow and a transcript for one of the participants (i.e., Danny) is included in Appendix F. For obvious reasons, fictitious names have been used.

Fred

The CAT for mathematics, level 15, was administered to Fred in May, 1991. He achieved an inflated score of 7.5 and went on to a power of 8.7. Fred's grade level for this subject was estimated at between five and seven. He was forty-six years old and had been out of school twenty-seven years prior to attending Continuing Education.

Initially Fred sounded hesitant when answering questions, raising his voice so it sounded as if he was always looking for confirmation of his work. As the session proceeded, however, this changed to become more positive.

Fred had difficulty estimating fractions with differing denominators. His estimate for $^{20}/_{4}$ - $^{20}/_{19}$ was zero. He arrived at this answer by subtracting twenty from twenty and using this as the numerator over nineteen minus four. This created a fraction of $^{0}/_{15}$ or zero. At this point a demonstration of how he could solve this problem by converting the improper fractions to mixed numbers was given. He commented that he had not seen fractions like this and that it helped to know that whole numbers could be operated on separately. His answer to $^4/_3$ + $^2/_4$ was $^6/_7$. Asking Fred if he remembered how to find common denominators (CD) seemed to be all that he needed to proceed properly. He was not told how to find the CD. Perhaps this was a case of not knowing when to apply a certain procedure or maybe he just needed a little reminder. Sometimes his CD was not the lowest he could have used. In adding $^{18}/_{14}$ + $^{10}/_7$, for instance, he used a CD of twenty-eight. He made several small mistakes in reducing the fraction. He always rounded fractions with smaller denominators up even if the other fraction could have been reduced to make the question easier to solve. For example the question $^7/_3$ + $^3/_9$ became $^{21}/_9$ + $^3/_9$. Although he determined the lowest common denominator (LCD) for $^3/_8$ - $^1/_6$ to be twenty-four Fred, unfortunately, added the fractions instead of subtracting them. Initially he said that $^3/_4$ was bigger than $^{19}/_{20}$ but went on to say that $^{19}/_{20}$ was bigger once he found a CD for both fractions. Only a couple of the students thought to use a common denominator in order to determine which fraction was larger than the other.

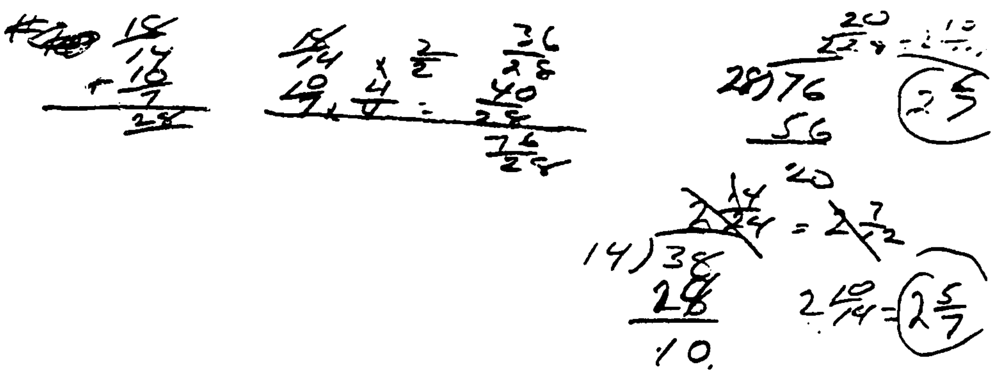
Some of the simpler questions that Fred seemed to have little difficulty with were skipped in order to move on to some mixed numbers. He also had few problems with these although he was not completely confident. He said that the demonstration of how to solve $^{20}/_4$ - $^{20}/_{19}$ had helped as he previously could not remember how to do fractions with different denominators. He then remembered doing them before although, as noted earlier, he still initially performed the wrong procedure.

Subtracting a larger number from a smaller one was difficult for this student. For instance, his answer to $4^{11}/_{12} - 6^{5}/_{9}$ was $-2^{13}/_{36}$. He had simply subtracted the smaller number from the larger (i.e., "6 - 4" and "$^{33-20}/_{36}$"). After showing him how to work this problem out, he was given some division questions.

This student could not solve division problems. To divide $^{3}/_{4}$ by $^{1}/_{2}$ he put both fractions over a CD of eight and proceeded to subtract one from the other to get an answer of $^{1}/_{4}$. He did the next division question in the same manner which tended to confirm that this was not a simple mistake of performing the wrong operation because of inattention. His answer to $^{5}/_{3} \div {}^{2}/_{3}$ was one. He said he had trouble remembering. He was given the rule about inverting and multiplying and a diagram to demonstrate that $^{1}/_{2}$ went into $^{3}/_{4}$ at least once. When asked how many more times it would go, he correctly responded "$^{1}/_{2}$" more times. One of his problems was in knowing when to cancel. He was then instructed to cancel during multiplication only. Fred managed to work out the next division question properly although he was hesitant about "turning the fraction around". In solving $^{7}/_{3} \div {}^{9}/_{21}$ however, he simply divided twenty-one by seven and nine by three to get an answer of $^{3}/_{3}$ or one and the answer to $^{3}/_{4} \div {}^{1}/_{4}$ was six. Even though he was able to correct this mistake after being asked if it was what he had done previously, a lack of abstract reasoning was apparent. Like many other students, he would say, for questions like "8 ÷ 2", "divided by eight" instead of eight divided by two.

Fred's work tended to be small and cramped with poorly formed numbers. He wrote the equivalent fractions next to the originals in a vertical fashion. His answers were usually left where he last did the calculation (see Figure 2). Because he wrote his operation sign next to the lower fraction but did not use equal signs between equivalencies, it was somewhat difficult to see what had been done, especially at first glance. These factors together would have made it even harder for him to check his work. He did, however, make an attempt to be neat and allow more room for questions. He completed twenty questions during the first session but left Continuing Education before the second, apparently because he had found employment.

Figure 2. It was sometimes difficult to find Fred's answer.



Yolanda

Yolanda wrote the CAT, level 16 for mathematics, in March, 1991. It was too difficult and she got a somewhat deflated score of 5.6 with a power of 6.4. She was 40 years old when she participated in the mathematics sessions. Prior to attending CEAB she had been out of school for about 25 years.

This student's work was neat, well spaced and easy to follow. She performed systematically and tended to check what she had done. For example, when writing out a mixed number, Yolanda always wrote the whole number first followed by the denominator and then the numerator. This, no doubt, helped her to keep things clear and allowed her to spot and correct errors without panicking.

The first question confused this student. She actually knew what to do but seemed to be somewhat unsure and lacked the confidence to begin. With just a few very general questions to assist her (e.g., Do you have any idea where to start?) she was able to answer the question and go on to solve many more. This could suggest a difficulty choosing from various procedures possible. Another problem was in estimating.

Yolanda's estimate of $^{20}/_{4}$ - $^{20}/_{19}$ was one. She had tried to cross cancel the denominator of the first fraction with the numerator of the second. It was explained to her that this could not be done for subtraction and it was suggested that she look at each fraction separately to solve this particular problem. She did not know what $^{20}/_{4}$ meant and asked if it meant to divide. Her first estimate for this fraction was $^{5}/_{4}$. A demonstration was given on how she could get an estimate and, although Yolanda stated that the question represented five minus one, her second estimate was $^{4}/_{4}$. In her, and apparently others' minds, all approximations were seen necessarily as fractions which, consequently, had to be over denominators. When she was asked to give an estimate for the addition of $^{9}/_{10}$ + $^{8}/_{7}$ she made a guess of $1^{1}/_{10}$ in spite of the fact that she already had noted that each fraction was about equal to one. Eventually she gave an estimate of two.

The greatest difficulty for Yolanda was in reducing fractions. It took this student a very long time to work out $^{4}/_{3}$ + $^{2}/_{4}$. Her answer of $^{22}/_{12}$ was correct as was the mixed number $1^{10}/_{12}$. While dividing the denominator correctly by two, however, Yolanda divided the numerator by what would have been the answer to ten divided by two (i.e., five) to get an intermediate answer of $1^2/_6$. She did not notice her error. Others also made this type of error. It seemed as though she had determined the answer but wrote the divisor down as the numerator by mistake. The end result was $1^1/_3$. In the question $^{18}/_{14}$ + $^{10}/_7$ she worked the answer out to $2^{10}/_{14}$ but began to multiply fourteen out first by one, then two, and finally three. When asked what she had to do to reduce the fraction, she explained that she needed a number that would divide into both the numerator and denominator. This, obviously, was not what Yolanda was doing. She may have become caught up in the routine required to find a CD but she was not alone in this misconception, if that was what it was. It should be noted that she had also done this in the previous question before reducing (i.e., she multiplied twelve first by one and then two before realizing this was incorrect). Once she stated what she needed to do, Yolanda was able to proceed. When working out the final sum of $^7/_8$ + $^9/_{10}$ (i.e., $^{71}/_{40}$), however, she began to multiply forty first by two and then by three. She eventually realized that she had become sidetracked and could see that forty went into seventy-one once. Interestingly, this student did not subtract as one might have expected. Instead she kept adding numbers to forty in order to find the one that would give her a sum of seventy-one. This, she knew, would be the difference. Some of Yolanda's attempts were far "off base" and showed a real lack of reasoning in this area. She began by adding twelve to forty. Despite that this was not even close to what she wanted, she next added thirteen followed by nineteen, twenty-one and thirty. Even though thirty and forty did not total seventy-one, this was the figure used by Yolanda as the numerator in her final answer of $1^{30}/_{40}$. No attempt was made to reduce this further.

She also used this technique to obtain the differences between one hundred thirteen and seventy, and twenty-one and three. Because she knew that seventeen plus three equalled twenty, this participant began by adding three to eighteen. She wanted to make sure that the total was indeed twenty-one.

Yolanda did not say too much as she worked through the twenty-nine questions she completed but there was a consistency in what she did. For instance, in order to find an LCD she would multiply the bigger denominator first by one, then two, three, etcetera until she came to an answer that was a multiple of the other denominator. When asked how she found the CD, though, she said that she looked "for numbers that would go evenly with the bottoms." She was not alone in this explanation. Yolanda followed this routine even in some cases where it hardly seemed necessary. For example, when determining the LCD for $^7/_8 + ^9/_{10}$, she multiplied ten out by two, three, and four consecutively to arrive at an LCD of forty. She had some trouble determining the LCD for $^9/_{10} + ^8/_7$ and had to multiply seven by ten using the longhand method in order to work it out as seventy. It was then necessary for her to multiply the first numerator by the second denominator. Since she could not remember this product, she began with one that she could remember (i.e., nine times five) and added consecutive nines to each of these subtotals until she reached the desired result. It either did not occur to her that she could simply subtract seven from seventy or Yolanda chose the other method because of her difficulties with subtraction. Eventually she worked the subtotal out correctly to $^{143}/_{70}$ but incorrectly wrote the answer down as $^{17}/_{70}$ even though she knew the denominator went into the numerator twice with a remainder of three. She gradually resolved this problem. Yolanda also found which fractions were bigger by putting them over a common denominator. There was no clear evidence to demonstrate that she really had a good understanding of part/whole relationships and, in fact, other data seemed to suggest otherwise.

This student usually wrote any multiplicands faintly next to each denominator. Still in the question $^3/_4$ + $^3/_4$, it appeared that she multiplied the numerators in order to get a final answer of $^9/_4$ which she reduced to $2^1/_4$. Other students also had difficulty with this question which will be discussed in more detail later.

A second session was completed by Yolanda three weeks after the first.

Table 1

Summary of CEAB Student Difficulties

Difficulty or Inefficiency	Examples
1. Trouble Attending, Easily Distracted, became Disoriented, Did Not Monitor or Check work	• Left out equal or operation signs (or used wrong one). • Left out whole numbers. • Performed wrong operation. • Lost track. • Made copying errors. • Erased part of the answer by mistake. • Performed the operation using the wrong numerals. • Misalignment of numerals.
2. Inability to Estimate	• Students were unable to estimate. answers to any of the questions. • None of the students used estimation to simplify any of the operations required.
3. Problems Reducing Fractions	• Instead of using the quotient as the numerator (usually), the student used the divisor, dividend or a related number when reducing. • Students divided the numerator into the denominator and used this result as the answer. • Unless they could find a "large enough" divisor, some students would not reduce the fraction. • Many students believed that only fractions with numerators larger than the denominators could be reduced. • Students were unsure which parts of the division went where when the result was a mixed number.
4. Lack of Confidence	• Many students hesitated and had trouble getting started/finished, although many seemed to know what to do.

5. Not Understanding the Parts/Whole Relationship	• Students thought that fractions with larger numbers were always bigger. • Some students meticulously drew and shaded diagrams but did not worry about sizes or shapes when comparing portions. • Students were confused by the significance of the difference between the piece that was left and the "piece" that made up the fraction.
6. Unable to Choose/Retrieve or did not Understand what Procedures to use	• Students did not know when they could "cross cancel". • Students did not know which procedures to use for a particular operation. • Students did not know how to divide fractions and would: •inverted the wrong fraction •divided numerators and denominators separately •thought that division and multiplication were the same. • Students added/subtracted numerators and denominators without a CD. • Students found a CD but ignored it and added/subtracted numerators.
7. Procedures were Inefficient	• Using fingers (even joints) to count. • Used longhand to multiply by one, ten, or a multiple of ten. • Students' always rounded one fraction instead of lowering the other. • Worked in head when better to write things out. • Tried to be faster than was beneficial. • Work was disorderly. • Found the CD by multiplying denominators or using another when there was one given. • Many mistakes resulted directly from use of less efficient methods.

8. Lack of Metacognition and Information Utilization	• Students did not utilize information available within the question. • Students did not access information they had with them. • Students did not recognize results as unrealistic. • Students did not check their work even though some spotted mistakes when asked related questions. • Students did not recognize own abilities or lack thereof. • Mixed procedures and language when trying to explain.
9. Lack of Factual Knowledge and Understanding	• Some did not know what improper/proper fractions were • Students lacked basic facts on multiplication and division. • Students did not understand commutative or associative property. • Students depended on memorized formulas. • Students were confused by subtraction procedures, negative results and different denominators. •Sometimes they twisted numbers around to make them fit. • Could not multiply or divide easily.
10. Errors in Computation	• Most errors due to inefficient methods or lack of understanding. • Poor figures, signs, and cramped work. • Misalignment of numerals.
11. Memory Deficits	• Some students could not remember material presented from one session to the next or even within the same session although they seemed to grasp it initially.
12. Difficulty Conveying Thoughts	• Did not say what they meant, or had trouble expressing thoughts.

Discussion

The first thing to note about these categories is that they are not mutually exclusive and some examples may not be a perfect match with the ones they are listed under. However, it is hoped that a better understanding will result as each is explained more fully.

Lack of Attention

In a few of the instances stated, the example may not have occurred because of the participant's lack of attention or because of his or her distractibility but in most cases it seemed to be that this was at least related. For example, many students neglected to put in equal signs or signs indicating the operation to be performed. They sometimes forgot whole numbers. In one case the participant actually asked if it was necessary to put in the whole numbers. A lot of the problems stemmed from the lack of importance some students placed on these "minor" things.

Participants sometimes performed the wrong operation. This usually appeared to be because they were not attending as was the situation where students continued to add even though the operation sign had changed to one of subtraction, but occasionally it resulted because of a lack of understanding or for some other reason. It was not always clear why a student had performed another operation. One student appeared to have multiplied numerators in the question $^3/_4 + {}^3/_4$. This was a particularly puzzling question for a few of the adults and will be discussed in more detail later. At other times students performed the correct operation but used the wrong numbers. These instances were not necessarily due to copy errors which they made occasionally.

Many students became "caught-up" in what they were doing and lost sight of the "bigger" picture. They seemed to become overloaded with information, often because they had to resort to less efficient methods to do the work and could not remember why they were doing what they were doing. Three participants began to incorporate decimals into their format while dividing fractions as if they had become disoriented to the fact that they were working with fractions and somehow wanted to make their new "representation" fit the mold for which they seemed to be more accustomed. All were working on the same question. Two examples will help to illustrate this last point. In the first, one student began to divide one hundred eight into one thousand thirty-nine. She did not stop with the quotient of eleven with its remainder but continued on using a decimal, obtaining an end result of 11.146 with still a remainder of thirty-two. While she was not totally confident about it, her final answer was $11^{146}/_{108}$. Another student was dividing four hundred thirteen by thirty-six. She had originally obtained an answer of one thousand forty-five because she kept dividing. She realized this mistake, saying that she had forgotten the decimal. Her final answer became $11^{45}/_{36}$ with a remainder of seventeen. This illustrates how confusing it can be for students to change mode. Their work also showed how they sometimes had to struggle with division of larger numbers.

Some students worked very quickly and made mistakes in their rushed and often cramped work. A couple of students accidentally erased part of their answers by mistake and did not notice.

Inability to Estimate

Estimating was an aid that none of the students were able to utilize. Only one of these adults had any success at estimating and this was very limited. Only a very few were partially successful when assistance was provided. None of the students, for instance was able to make a guess at the answer to question #7 ($^{20}/_{4}$ - $^{20}/_{4}$). There were various reasons for this kind of difficulty. One student said that $^{9}/_{10}$ + $^{8}/_{7}$ was close to

$1^1/_{10}$ in spite of already stating that each was about equal to one. Another had written her whole numbers so small that when she returned to it after being temporarily distracted (by a copy error), she re-wrote the answer incorporating the whole number into the numerator (i.e., $1^{10}/_{12}$ became $^{110}/_{12}$). Many of these adults could not accept that a fraction could be represented by a whole number within the question and, unless it was a final answer, insisted on putting it over a "denominator". A few stated that they did not know that "you could do that". Even students who knew that four went into twenty, five times, would keep putting this result over four because that was the denominator (see question #7 of the transcript provided for Danny in Appendix F).

The estimates for question #7 ($^{20}/_4$ - $^{20}/_4$) varied. One of zero was quite typical and was justified by one of the more capable adults when he explained that twenty minus twenty was zero and that it did not matter that a CD was not considered since zero over anything would still be zero. He realized that his thinking was in error once it was pointed out that numerators could change with a change in denominator, other did not. One student first guessed the answer at nineteen since the numerator would be zero and nineteen was the largest denominator. She then amended this to an answer of thirty-six. Another student was able to get the figure of four but put this over four to get a final answer of one. Most students tried to get an estimate by finding a CD and working the problem out in their heads. Their answers could be very complicated fractions or quite simple as when $^1/_5$ + $^4/_2$ was estimated at $^{22}/_{10}$ (the actual answer). It was quite amazing some of the complicated calculations they could do without writing anything out. A few students could sometimes work out the answers (or results that were close) faster in their heads than they could writing out the calculations. Some students simply added or subtracted the numerators without regard to the denominator such as when $^3/_8$ was subtracted from $^7/_3$ to get $^4/_8$. They seemed to think that this method would give them a close enough estimate of the answer. Most went on to do the calculation correctly. These students did not comprehend the concept of estimation as demonstrated by their attempts at very accurate "guesses" such as $^{143}/_{70}$ which they usually would not have reduced.

These participants were not only unable to estimate answers but they either chose not to, were incapable of, or simply did not think to use these to aid them in carrying out intermediate procedures such as division which could have simplified the work of most of the students. Estimation appears to be something that was not taught within our school systems until a later period. These students stated that it was a new and surprising experience.

Reducing Problems

One of the biggest problems for these students was in reducing fractions once they had obtained an intermediate answer. A few of them had the belief that only fractions with numerators that were larger than the denominators (i.e., improper fractions) needed to be reduced. Fractions like $3^3/_6$ or $^{10}/_{14}$, and $^3/_9$, would, therefore be left. Two participants divided the numerators into the denominators and put those quotients as the answers (e.g., $^{10}/_{48}$ became $4^8/_{10}$. Two students did not reduce the fraction because they could not find a large enough factor to use even when both the numerator and denominator were even numbers which they knew were divisible by two. Many of the students had trouble because they put down a number related to the answer but not the answer itself. Some examples will make this more clear. In one case the fraction being reduced was $^6/_8$. The student reduced this to $^2/_4$. The divisor was often used in place of the correct figure. This was also the case when $1^{10}/_{12}$ was reduced to $1^2/_6$. It was not always clear whether the student simply used this figure without actually doing any calculation with it or had the answer worked out and divided this into the dividend. One student reduced three different fractions (i.e., $^{3}/_{15}$, $^{5}/_{15}$, and $^{10}/_{15}$), presented consecutively, to the same result of $^{1}/_{5}$. She was only partly successful at altering this after being asked to look again at the questions to see if she noticed anything unusual. She was unable to make use of the fact that since they all had the same denominator but different numerators, the results had to be different.

Some students had the mistaken belief that odd and even number combinations, one each for the numerator and denominator, could not be reduced. One commented, for example, that neither three nor two would go into a fraction of this type. Fractions like $^{10}/_{15}$ would not be reduced. One student seemed to thing that 30 was an odd number. A few students had problems reducing even simple fractions like $^{7}/_{3}$ and one asked if the divisor had to fit into both the "top" and the bottom. Some were just not sure if a fraction could be reduced (e.g., $^{13}/_{210}$). They did not seem to have much knowledge about prime numbers and were poor "reasoners" who seldom appeared able to think of alternate means to find out if a fraction could be reduced. One student stated that she did not know what to do with two after trying to reduce $^{2}/_{30}$ and determining that the denominator was fifteen by dividing two into it. Another participant made an incomplete reduction when she worked $^{35}/_{3}$ out to be $10^{5}/_{3}$. She even checked her work when asked and still got the same answer and insisted that it added to $^{35}/_{3}$. Perhaps she failed to see because her mind was already "set". Three other students were not sure what part of the division (usually the divisor or remainder) to use as what part of the fraction. For instance $^{47}/_{7}$ was written as $6^{7}/_{5}$. Only one of the participants reduced any fractions--one--embedded within the questions.

Lack of Confidence

Many students seemed to have an idea of what they were supposed to do but were a' to act. They had to confirm many of the steps. Two students had great difficulty even getting started but progressed well once they verbalized the procedures they thought were needed and were told to put their words into action.

Part/whole Relationships

More than half of the students were unable to state which fraction was the bigger of two given. Some believed that any fraction with larger numbers would be bigger. Because this is true in some instances, students with incorrect beliefs might not ever be challenged and their thinking altered. It is a fact, for instance that $^9/_{10}$ is larger than $^7/_8$ but it is definitely not true that $^{20}/_{30}$ is larger. This also tended to reinforce their belief that they had the right method but were simply making "careless" errors "somewhere".

Two of the students asked to draw pictures were very meticulous about making sure they had the correct number of pieces and that each was painstakingly shaded but they gave no concern about the size or shapes of the obstacles they were comparing

These adults were confused by whether to consider the piece(s) left behind or those "taken" and more than one student believed that $^7/_8$ and $^9/_{10}$ were the same size since each had one piece missing.

A few students figured which fraction was larger by putting them both over the same denominator but it still appeared that they lacked an understanding of part/whole relationships.

Inability to Choose or Use a Correct Procedure

When one of the students stated that, "sometimes you're supposed to leave it and sometimes you're supposed to change it", she was demonstrating a confusion many students had when faced with choosing what procedures to use to solve the addition or subtraction of fractions with different denominators, but it was not only with differing denominators that these participants had difficulties. Many wondered whether or not they could cross cancel and some tried to cancel one denominator with the numerator of another fraction when these fractions were being added or subtracted). About half of the students added numerators or denominators or both without regard to changes which resulted by using a CD. For instance $^2/_3 + {}^5/_3$ became $^7/_6$ and $^4/_3 + {}^2/_4$ became $^6/_7$. One student said that she thought she had to divide 3 into 11 in the question $^{11}/_5 - {}^3/_5$ because the operation changed to one of subtraction and this meant that something different had to be done.

Some students had difficulty understanding the division of fractions and a very few insisted that it and the multiplication of fractions were the same, requiring identical procedures. Others simply did not know how to do the division of fractions. One inverted the wrong fraction before multiplying. Baroody and Hume (1991) discussed identical types of problems experienced by children which they attributed to a lack of meaningful understanding. They believed that teaching should become less dependant upon pencil and paper formats and begin to focus on encouraging greater comprehension.

Inefficient, Error Prone Methods

Many of the errors made resulted because of the less efficient and more memory utilizing methods incorporated by these students while attempting to answer the questions. Some used their fingers. One from Trinidad even used her finger joints. Probably the greatest barrier for these adults was the fact that they did not know their multiplication tables and had to do most of the calculations by writing them down or looking the answers up in their tables (okay if they had them). One third of the students multiplied by one, ten or a multiple of ten by writing them out longhand. Although all of the students had been given tables, some never used this resource.

Inefficient methods to find CD's were often used. One third of the students multiplied denominators automatically or used one that was larger than necessary.

A small number of students needed to slow down. They often performed better when trying to estimate than when actually doing the calculation. They attempted to work more quickly than was desirable or required and errors and confusion frequently resulted. A few tried to do a lot of the calculations without the benefit of writing anything down.

More than half of the participants who dealt with mixed numbers converted these to improper fractions. The result was that they were sometimes dealing with very large, cumbersome numbers.

Only one student reduced one of the fraction in a question in order to add it more easily. She did this with only one of the questions before resorting to raising the "lower" fraction as was done by all the others. In ${}^{7}/_{3}$ - ${}^{3}/_{9}$ for instance, the questions was always written as ${}^{21}/_{9}$ - ${}^{3}/_{9}$ rather than ${}^{7}/_{3}$ - ${}^{1}/_{3}$ which would have probably been easier for these adults.

Only a couple of the students had systematic approaches to finding solutions. One student in particular stands out as being very neat and organized. She also checked her work. This helped a great deal in spite of the difficulties encountered because of her failings in other related areas. One third of the students' calculations either were not written near the question they were working on or they were so cramped and untidy that they sometimes led to errors. They could not be used to locate mistakes.

Some of these pupils were quite resourceful and had developed unique and interesting strategies to circumvent their inabilities in all facets of this work. In order to divide by two at least a couple of the students would pick a "round" number that they thought would "fit" and added this to itself. For example one student picked thirty-five as the number to be added in order to divide seventy-six by two. This number would be adjusted if necessary once the "multiplication" had been performed. Two students used the same technique to subtract three from twenty-one. These students began by adding seventeen to three since they both knew this to be equal to twenty. One of the students very quickly determined the answer to be eighteen but the other tried several numbers, unfortunately, not the correct one. The successful student in this case was not always so lucky as can be seen from her attempts at finding the difference between seventy and forty. She began by adding first twelve, then thirteen, next she tried nineteen and then twenty-one. At this point she knew a larger number was required and concluded her calculation using thirty as the result even though she could see that forty plus thirty was still only equal to seventy. This student appeared to be lacking in understanding of the concept of sizes of numbers.

Another student would simply add and subtract as follows. If both numbers were larger than ten she would subtract by first taking away ten, or a multiple thereof, before subtracting what was then left over. When adding numbers, she would first round the augend to the nearest tens place if possible and then add what was left. Sometimes this became a little complicated. Some examples will help to illustrate these two related procedures. To subtract eighteen from twenty-nine she first took away ten from twenty-nine, leaving a remainder of nineteen. From this she took away the eight, giving a final answer of eleven. To add eighteen and four, she first added two to the eighteen and then added the last two to twenty to get twenty-two. Adding fifteen and nineteen was a little more involved. She did not add one to nineteen as one might have thought. Rather, she reduced both numbers to ten which gave her a total of twenty. To this and the consequent intermediate sums she then added five, four, and five consecutively.

This student also had various methods which she used to divide. To begin with, she divided "backwards" by seeing first how many times the divisor went into the units of the dividend. As mentioned earlier, to divide by two, she would add two numbers that she thought would give a result that would be close to the dividend. Another method she used to divide by two was to count by twos to the nearest ten and divide what was left over by two. Her answer became then the total of five times the number of tens, plus the result of the division of what was left over. For instance, "34 ÷ 2" would have been three tens, and therefore three times five) plus two (i.e., "4 ÷ 2") which would give a result of fifteen plus two or seventeen. Another student also used this technique. As a last resort to dividing by two this student would divide directly in half. Once she got tired of trying to do the calculation and simply stated what the answer would be. It was not clear why the student would use other, less efficient methods when she appeared able to retrieve the answer directly from memory. It can only be assumed that this was something that she did not think of or it was not easier for this student. She also added sixes to divide this number into twenty-four.

In order to try and find a lower CD one student attempted to break down both denominators into their prime numbers. He was the only student to used this unique technique (after being asked to find a lower CD). He usually just multiplied the two denominators together. Unfortunately, he was unsuccessful because he really did not understand the purpose of what he was doing and did not cancel the primes both denominators had in common.

One student had an interesting method for division of fractions. She would always divide the smaller numerator and denominator of one fraction into the numerator and denominator of the other respectively. If this resulted in a quotient with a remainder, the remainder was simply ignored. This kind of error would be difficult to detect since it would be successful some of the time. To divide $^3/_4$ by $^1/_4$, for instance, the three was divided by the one and this result became the numerator over the denominator of "4 ÷ 4". The answer: three. This was obviously correct. The next example, however, illustrates both misunderstandings mentioned. Her answer to $^7/_3 \div {}^9/_{21}$ was $^1/_7$, clearly incorrect. This was obtained by dividing seven into nine and ignoring the remainder so putting one over the result of "21 ÷ 7" or three.

Several students would utilized information they knew in order to complete a calculation. One student could not remember what nine times seven was equal to so used nine times five and added another two nines consecutively to each of these result to obtain the final answer. Another student added nineteen to itself and intermediate totals to get nineteen times four. One multiplied nineteen times three by first adding ten and nine to thirty-eight which she knew was nineteen times two. In order to add thirty-five and thirty-six, one student broke this down into two thirties, one ten, and one one. The total was then "30 + 30 + 10 + 1".

Sometimes these methods led directly to errors because the student would count one too many fingers or add numbers which they should have been subtracting. An example of the last case was when one student added instead of subtracting two from twenty-nine after she attempted to solve the question $^{18}/_7 + {}^{29}/_7$ by first changing the eighteen to twenty. She also made a mistake by adding the denominators so that her final answer became ${}^{51}/_{14}$ rather than ${}^{47}/_{7}$.

Lack of Metacognition and Information Utilization

Some students did not access information they had available with them, such as multiplication tables or they could only use these in a limited way. For example, even if they could figure out the result of a multiplication they may not have been able to reverse the process. Most did not recognize an answer as unrealistic partly because they seldom checked their work. For instance it should have been clear that "613 ÷ 36" could not be one hundred seven or that $10{}^{5}/_{12} + 16\,{}^{9}/_{10}$ could not be $32{}^{5}/_{12}$. One adult said "$16{}^{9}/_{10} = 220$" and another "${}^{280}/_{76} = 38{}^{12}/_{12}$". Students also seemed to be "blind" to information within the question itself that could have been helpful. One student said that ${}^{1}/_{5} + {}^{4}/_{2} = {}^{1}/_{2}$. Another added ${}^{4}/_{3}$ and ${}^{2}/_{4}$ to get ${}^{6}/_{12}$ or ${}^{1}/_{2}$. She was asked what ${}^{2}/_{4}$ was equal to and eventually accepted that the answer had to be greater than ${}^{1}/_{2}$. He next answer of ${}^{5}/_{6}$, which resulted after she had accidentally erased the whole number, was also unrealistic.

These kinds of problems may also be a consequent of the fact that students are taught very limited methods for solving these kinds of problems. They do not appear to be diligent in their work as also indicated by the fact that only a few students bothered to systematically check their work. Even these students only usually went back one or two steps.

It was also interesting how students, in an attempt to explain what they were doing, would twist their words around or explain a procedure other then the one they were doing or needed to do. The one example illustrated when discussing Fred's work was when he said, "divided by eight" instead of eight divided by two. Another occurred often when students would explain that to find a common denominator one needed to find a number that would go into both denominators.

Lack of Factual Knowledge and Understanding

Students lacked declarative knowledge and understanding in many areas. They seemed to be more dependent upon memorized formulas and did not have a grasp the commutative (i.e., $a \times b = b \times a$), or associative (i.e., $a(b \times c) = (a \times b)c$ properties of multiplication, or the reversibility (i.e., If $a \div b = c$, then $b \times c = a$) principle. Some of them looked for a common denominator even when the question involved multiplying or dividing the fractions. Two students figured that nineteen times four equalled seventy-six but wondered how many times four went into seventy-six. They also seemed to be confused by the language used to request multiplication or division of one fraction by another. Mick & Sinicrope's (1989) article reviews some aspects of this.[10]

Basic multiplication facts illuded these students which is why they usually had to resort to other means to do the calculation. Some students did not understand division of fractions. Two of them thought division and multiplication were the same. Most of the students who completed only one session did not get as far as the multiplication and division questions. Some of the adults had difficulty with the terminology and one asked if an improper fraction was greater than or less than one. Abstract reasoning was difficult as illustrated by the case of one student who could not comprehend that $^{20}/_{19}$ was close to one. Adding $^{3}/_{4}$ and $^{3}/_{4}$ was difficult for at least three of the students. They thought that it had to be dealt with in a different way than all of the addition problems with the same denominator even though they had already answered many of these types of questions correctly. One student could not figure the right answer by calculating it but told me that it was $1^{1}/_{2}$. She knew this because it was a measurement she often used in baking. Many of these errors would not be easily detected by the students because they lacked the factual knowledge to recognize their answers as incorrect.

Errors in Computation

Although most of the errors did not appear to be systematic, they were often related to the same fundamental problem within specific areas. Some of the students solutions procedures may have worked some of the time, giving the overall pattern the appearance of randomness. These students were simply confused by what they were supposed to do. As discussed earlier, many of the errors were due to less efficient methods where more steps meant more chances for making errors.

The benefits of well organized, spaced-out work were not clear to all of the students. A few errors resulted because of cramped work, poorly formed figures which were haphazardly written down.

Memory Deficits

I believe that many of these students' difficulties resulted because they had trouble manipulating and storing so much material due to their less efficient procedures. Some of them were unable to remember information presented from one session to the next or even within the same session. Many researchers suggest this to be a consequence of the lack of memory aids such as rehearsal (these are often referred to as strategies). Memory deficits have been discussed in more detail in the text of this paper.

Appendix F

Transcript of First Seven Questions for Danny

Question #1, May 4, 1992

Lesley: This is the microphone so it should be able to pick both of us up fairly well.

Danny: Oh, okay.

Lesley: Now here's an eraser in case you want a bigger one. I am going to present you with a series of cards. There is only one question on each card. I'd like you to do your best to see how you can work through the problem. What I want you to do is to let me know what you are thinking, what you are doing so I can follow the process. I might make a few notes while I am sitting here.

Danny: Okay.

Lesley: Okay. I'll just give you one card at a time and then you can do what you want. I've got a sample which I'll give to you. But you can try it first. Here's the first one so see if you can just work out what the answer might be.

Question #1

$^{2}/_{3} + ^{5}/_{3}$

Danny: That'd be just seven over three, wouldn't it?

Lesley: Okay.

Danny: Because these are even. [He seems to be indicating that they were the same.]

Lesley: Uh huh.

Danny: Or you could reduce. Okay?

Lesley: Your seven over three sounds good to me. [Student hadn't written anything yet]. If you could just put this number down. Perfect. Okay and ...

Danny: Can't reduce it. [Pause while looking over work] No. That's about all I can do with it.

Lesley: Is seven bigger than three?

Danny: Yuh.

Lesley: Could you divide it?

Danny: Into that [pointing to seven]. That'd be. [Repeats] That'd be one and one half wouldn't it? I think.

Lesley: Maybe if you try making your letters, your numbers a bit bigger that might help.

Danny: Yea.

Lesley: That's Okay. [After student begins to erase previous work]. Mm hm. You're right so far.

Danny: Okay, so that's it? [Has as seven over three]

Lesley: If you want you can leave it in that form. If you want, try and reduce it.

Danny: Okay, so that's just divide three into seven.

Lesley: Right. Good, now you're getting the hang of it.

Danny: [Student divides correctly but is not sure what to put down.] So that would be three and one half wouldn't it? No three, no two over one. I don't know the order that goes.

Lesley: Yes, Okay. The three went into seven twice, right?

Danny: Uh huh.

Lesley: So that's what you are looking for. Remember you are dividing this [pointing to the three] into that [pointing to the seven] so you have the answer right and you have one left over, right? Well three would be a decimal [meaning three X three into one with one brought down], but we're going to leave it in fraction form, so that's one third. Okay?

Danny: Oh, Okay. That's the where it goes. Okay.

Lesley: Mm Hmm. Now you probably will remember that for the next time. Let's see how it goes. Whatever you want to do with these. It's up to you. Okay? Here's the next one.

Question #2

$^{18}/_{7} + {}^{29}/_{7}$

Danny: Okay. So you would have to. [Pause] Hm.

Lesley: What do you think would be the easiest thing to do?

Danny: Probably uh try and reduce these. No. You can't. Three won't go into that or this.

Lesley: That's right.

Danny: Nor seven.

Lesley: You're right.

Danny: Six. [pause]

Lesley: Why don't you. Um. They're both the same, right?

Danny: Yea.

Lesley: So maybe you should just start off by adding the top ones and then see what happens from there.

Danny: Okay, so that would be forty-seven over seven. [Did some calculations and then hesitates when division worked out.]

Lesley: Okay. Do you remember what we did with this one here [pointing to first problem]?

Danny: Yuh. That would be six and five sevenths. Woops. I put that backwards [wrote five backwards]

Lesley: Have you ever been assessed as having a learning disability?

Danny: Hm. Assessed as having a learning disability? Well, yea. I went to a special ed school for a few years.

Lesley: Mm Hm. Okay. Because that makes it a little tougher, but you are doing pretty good so far.

Lesley: We'll call this one [pause] [I hadn't numbered this question]

Danny: Number three.

Question #3

$^{11}/_5 - ^{3}/_5$

Lesley: Right.

Lesley: Maybe you should allow yourself a little bit more room because you're going to get yourself a little bit cramped and it gets confusing. There's lots of paper, don't worry about that. If we run out of paper, I'll steal some. [Both laugh].

Lesley: [Student works quietly.] Is that right?

Danny: So far, yea. To me anyway.

Lesley: Okay. Is there anything different about that sign?

Danny: Oh [laughs]. I overlooked it. [Student had been adding instead of subtracting]. That would be eight.

Lesley: Tell me what you are thinking because you are doing a lot of it in your head. If it is easier for you, tell me after you've done it.

Danny: I'm not sure exactly what to do actually. Well these you don't minus anything. [Pointing to denominator]

Lesley: Mm hm.

Danny: They're the numerator?

Lesley: Those are called denominators.

Danny: Okay.

Lesley: Okay, so.

Danny: Okay, so this would be the numerator. You never do anything with the numerators unless you are dividing or something. You probably could reduce them.

Lesley: You mean denominators.

Danny: Denominators, yea.

Lesley: Well okay, look. You are on the right track here. It's just that you performed the wrong operation, Okay. So [pause]

Danny: So that would be eight over five.

Lesley: Right.

Danny: Okay.

Lesley: Okay, [divides] Right.

Question #4

$^{11}/_{5} + ^{3}/_{5}$

Lesley: Now what does this look like?

Danny: Well its eleven fifths plus three fifths That'd be fourteen over five

Lesley: Hm.

Danny: And that can be reduced?

Lesley: Yes, just what you started to do before.

Danny: It's pretty simple actually.

Lesley: Mm hm. Yea its not too bad so far. [student works] Okay.

Danny: Is that right?

Lesley: Mm hm. Maybe you should give yourself a little more room because you are going to get yourself confused with these numbers I think [referring to above figures in previous question]. Student starts to give himself one extra space]. Maybe you should give yourself three spaces instead of just one.

Danny: Yea.

Lesley: Or however many you want. Just so long as you have lots of space to do your work. Okay.

Question #5

$^{29}/_{7} - ^{18}/_{7}$

Danny: Okay, so that's, let's see.

Lesley: What does that say?

Danny: It's eleven. Right here [twenty-nine minus eighteen].

Lesley: Okay. [Pause] It sounds okay to me.

Danny: Yea. So that would be...

Lesley: There you have a lot more space now.

Danny: Yea. Wait a minute. Is that the way it is supposed to be? Let's see. No. [Drew five backwards]. Right [corrected].

Lesley: Maybe you could just cross it over instead.

Question #6

$^{7}/_{6} + ^{14}/_{6}$

Lesley: [D. works without saying anything.] Okay what are you doing there. You're so fast. Okay. Well that looks Okay. You just added them up and you divided?

Danny: Yuh.

Lesley: Does that look wrong? Of funny?

Danny: Well let's see. It goes in this order, doesn't it? Like this. So that would go on top. [Had forgotten which part of division answer to put down for answer to question]

Lesley: Well your first one is right [the whole number part]. I'll tell you that.

Danny: Okay.

Lesley: Somehow that three got erased. I don't know how. I didn't notice that. Okay.

Danny: Oh. Okay. So [pause]

Lesley: So does that other one look right?

Danny: No.

Lesley: I think you're on the right track here, you know. You realize there is something wrong. Just cross it off because then, when we talk about it later, it will give me an opportunity to show you where you might be able to improve your math.

Danny: Oh, Okay. [D. finishes]

Question #7

$^{20}/_{4} - ^{20}/_{19}$

Lesley: Okay. This is a tough one [giving next question which has a mixed denominator].

Danny: I don't really know this procedure. I haven't really learnt these.

Lesley: You haven't? Maybe I can find one to give you an example.

Danny: Do you have to [pause] let's see.

Lesley: You could probably give it a try.

Danny: You'd probably have to reduce this, wouldn't you?

Lesley: Well that's an idea. Okay, I want to ask you a question about this. Could you give me an estimate of what that answer would be [20/4 - 20/19]?

Danny: An estimate?

Lesley: Mm hm. Just a guess?

Danny: A guess?

Lesley: Yea, an educated guess.

Danny: Mm.

Lesley: You've already said something to me which indicates that you would have a pretty good idea of how to do that.

Danny: Probably about [pause] five. No five and one third or something. [laughs] Its just a guess. No.

Lesley: Well I think you are on the right track. What does this say [pointing to the twenty over four].

Danny: Well that's twenty over four.

Lesley: Twenty over four. Okay. What does that say [pointing to twenty over nineteen].

Danny: Twenty over nineteen.

Lesley: Yea that's right. Okay, so just think about that, those two numbers. You already mentioned something that was good.

Danny: Okay, so. [pause] The way of doing it would be five and then you could reduce this one. That goes into four and that goes into one.

Lesley: You're on the right track because you said that this could be reduced to 5, right?

Danny: Yea.

Lesley: Okay. Now this nineteen doesn't go evenly into twenty does it?

Danny: No.

Lesley: Then how many times does it go into twenty.

Danny: It goes into there once.

Lesley: Now its pretty close to one isn't it? So could you, if I'm just saying make a guess, then what would you guess. Because you've told me approximately what this is equal to [the twenty over nineteen] and you know exactly what that's equal to [the twenty over four] so what's it going to be pretty close to?

Danny: Hm.

Lesley: What is this [20/4] minus what, approximately?

Danny: Oh. Four minus nineteen?

Lesley: No, no. I mean this fraction minus this one [20/4 - 20/19].

Danny: That fraction minus this one?

Lesley: Mm hm.

Danny: Ahhhhh.

Lesley: What did you say this was equal to? [the 20/4]

Danny: Five fourths, I think.

Lesley: No. How many times does it go into there?

Danny: Well it goes into there five times.

Lesley: Okay. So stop while you're ahead. Okay, because you've got it. You're just confusing yourself a little bit. Okay, and so this goes into there how many times, about?

Danny: Ah, can't discern.

Lesley: No, I mean what is that fraction equal to? This one.

Danny: I don't know.

Lesley: Well, okay. We said that nineteen goes into twenty once with a very small remainder right?

Danny: Yea.

Lesley: Okay, so isn't that pretty close to one?

Danny: So that would be...

Lesley: So this was what? What was that answer? What did you say? This one here. This fraction. What was that equal to?

Danny: So that was equal to five fourths, I think.

Lesley: No.

Danny: Or...

Lesley: It's a whole number.

Danny: It's equal to five.

Lesley: Right. Okay. Stop there. Okay, now what was that equal to [the other fraction], about?

Danny: Uh one and...

Lesley: One's a pretty good guess, because there is such a small remainder that it really doesn't matter too much. One nineteenth. We won't worry too much about that because I'm only asking you to make a guess, okay? So what is that [the whole thing]? What minus what?

Danny: Uh twenty. Well five minus twenty. No five, mmm.

Lesley: Minus. What did you just tell me that was approximately equal to?

Danny: Five minus, uh...

Lesley: Okay. How many times does nineteen go into twenty

Danny: Uh, once.

Lesley: Okay.

Danny: It's five, one.

Lesley: Five minus...?

Danny: One.

Lesley: Right. Okay, so now you know what five minus one is, right?

Danny: Mm.

Lesley: Okay, so give me a guess as to what the answer is now.

Danny: Okay, so that's [pause] uh...

Lesley: Okay, think of [didn't complete] Put this in terms of Write it out if you have to okay? What is that about? What is that approximately equal to?

Danny: Uhhh.

Lesley: Just this fraction.

Danny: Oh one, one fifth. One fifth because don't you have to reduce this one as well as the top. Don't you have to do to the top what you do to the bottom?

Lesley: Okay, let me show you how this works. Now remember, what did you do when you figured out this answer here [pointing to a previous question]?

Danny: Uh, I divided.

Lesley: Right. You divided [pause]

Danny: Once. Divided the top, the bottom, the denominator into the numerator.

Lesley: Right. Okay, so now you told me that was approximately equal to...What's that approximately equal to?

Danny: Uh one, one fifth.

Lesley: How many times does four go into twenty?

Danny: Oh five times, five fourths.

Lesley: No. Okay, write down twenty over four. See if you can work out this problem, okay? First you give me an estimate, okay? Now what you can do is figure this one out [one of the fractions] right. And then you can figure the other one out, approximately. Okay, so what does that equal?

Danny: Five fourths or...

Lesley: Okay, well do it like you did the other one.

Danny: Oh, okay. So that's five and don't you have to reduce this by one [he meant reduce to one by dividing the four by four] since you did that to the other?

Lesley: Right, you do.

Danny: That's one.

Lesley: Right. Okay.

Danny: That would go one.

Lesley: What is five divided by one?

Danny: Five divided by one?

Lesley: Mm hm.

Danny: Five.

Lesley: Okay, so then you've got your answer. Okay, so maybe put equals five. Okay, so then what's this one here [the 20/19] approximately?

Danny: Oh. That one would be one. That'd be ah, uh one [student sounds hesitant]

Lesley: One sounds pretty close to me, because nineteen into twenty, you've only got one left over, right?

Danny: Yea.

Lesley: Okay. So maybe write this fraction out.

Danny: Okay, so...

Lesley: Okay [as student writes it out]. Right.

Danny: I don't know the procedure now.

Lesley: Sure you do. You've done it over here, Okay?

Danny: Okay, I was just [continues working]

Lesley: Right. That's good [as student divides twenty by nineteen]

Danny: So that would by one and one nineteenth.

Lesley: Right, good. Okay, so maybe write that down so that you don't forget. Okay, so now I'm going to say one nineteenth is quite a small number, so just for now let's just forget about that, okay? Because I just want you to give me an approximate answer to this question. So what would that be approximately [the 20/19]? This is what number? That equals five, right?

Danny: Right.

Lesley: Okay.

Danny: I kind of see what it is now.

Lesley: Yea, I think you're going to start to get the hang of it here pretty soon.

Danny: Now I have to uh, I have to uh [pause] Okay, lets see 1 goes into...

Lesley: Okay, that's approximately one, okay? Because twenty follows just after nineteen and they're fairly large numbers so that means that if there's a remainder it's going to be small. Okay so, that's the same as...maybe you should try writing things out because you'll see it a little bit better.

Danny: So just [pauses as he proceeds to write figures down but still somewhat hesitant]

Lesley: What did that equal?

Danny: That's equal to five.

Lesley: Okay. Maybe write the five down. Okay, now put minus. Okay, now what did this equal to? Approximately. Just a guess.

Danny: One.

Lesley: Yea, okay. Okay, so what's the answer then?

Danny: Four [still sounds hesitant but doesn't continue even though I give him the opportunity]

Lesley: Okay so that's it. Okay so I'll give you the next one then.

Danny: So that's the answer, just four?

Lesley: Sure, that's a guess.

Danny: Oh.

Lesley: That's about five and that's about one, right? five minus one. It's this one, this whole fraction, which actually is a whole number. Right? Because four goes exactly into twenty five, times, right? and this is so close to one that will say that. If I want you to do it exactly then it's a little more complicated, okay. And you have to start trying to figure that part out [the part with a remainder]. Do you want to see if you can figure that part out?

Danny: Ah. Let's see, I have to.

Lesley: Okay. Try and give me the exact answer then. May I should try and give you and easier question first?

Danny: Yea, sure.

Lesley: Well, it's not too bad, but it does take a lot of thinking here.

Danny: Okay, uh [and proceeds but gets confused] The numbers one times nineteen, no is it one times one is one. Let's see [pause]

Lesley: Okay, I think I might have a sample here. This might help you.

Danny: Uh so that's the beginning of it.

Lesley: Mm hm. Actually, you know if you write this out again, if you write this out as the exact answer. You know it's one and one nineteenth, right. So instead of one, put down the exact answer because now I'm going to ask you for the exact answer. So you know what this is and you know what that is, so maybe try writing that out and take it from there, okay. You really don't need this [the sample]. You'll start to see.

Danny: Okay, so, ah...

Lesley: So this was the approximate answer. So now see if you can put down the exact answer.

Danny: The exact answer, oh.

Lesley: This is a guess [the four]. Okay, this is exact [wrote word exact on page]. Okay what is the first fraction equal to?

Danny: Five.

Lesley: Okay. You could start off by writing that down. You know, you can always break it down if you are having difficulty.

Danny: Okay, so probably five and one nineteenth.

Lesley: But this was a minus.

Danny: Oh boy.

Lesley: So maybe we should put a minus in front of here. Okay. [as student puts in sign]

Danny: [Student yawns] Okay, uh [pause] five. I'm not really sure how to minus though.

Lesley: Okay. Well just put down what you are minussing for now, okay.

Danny: Minussing.

Lesley: You worked it out.

Danny: This [pointing].

Lesley: Okay, and what does that say?

Danny: Uh one and one nineteenth.

Lesley: That's right and it's a subtract so maybe put that down. Okay, but you're subtracting this okay. So I want to see if you can now figure that out.

Danny: Okay, you have to go five and put a one over it, like that?

Lesley: No.

Danny: No, I almost made a mistake. pause Okay, uh...

Lesley: You have to change this five, okay. In order to subtract one nineteenth. So if you make this four right and five is equal to four plus one right? Are you tired? [as student yawns again]

Danny: Yea. I feel light-headed today.

Lesley: Okay, so four and if I say nineteen over nineteen that's the same as one, right?

Danny: Yea.

Lesley: So if I put four and nineteen nineteenths, that's equivalent to five.

Danny: Four and nineteen nineteenths?

Lesley: Yea. Because that is four plus nineteen nineteenths and nineteen nineteenths is one Okay? And now you can subtract your one and one nineteenth, okay. Does that look a little bit easier.

Danny: Uh, no.

Lesley: No. Okay. Well, what you can do now is just deal with this part and this part [the whole numbers and fractions separately] okay? So you have nineteen nineteenths minus one nineteenth and what does that equal to?

Danny: Well that's just eighteen nineteenths?

Lesley: Right, Okay. You've got the answer there. Okay, now you still have the 4 here and the one and subtract those.

Danny: Uh...

Lesley: What's four minus one?

Danny: Three.

Lesley: Okay, so now you take three plus whatever the answer is here, okay?. I'll just put that in there. Now you can use this if you want to.

Danny: Uh...

Lesley: What's your answer?

Danny: That's the correct answer.

Lesley: Mm hm. We said the answer was approximately four right? Well three and eighteen nineteenths is pretty close to four isn't it?

Danny: Uh...

Lesley: Because you've got eighteen out of nineteen. If I said to you that you've got eighteen out of nineteen on a test you'd think you did really well right?

Danny: Yea.

Lesley: So that means that you got almost one hundred percent which means that this is almost one. One out of one. So this means three plus whatever this is, so that's almost four which is what your guess was. So that was a good guess. Okay, so here's the next question.