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INFLUENCE OF SPECIMEN SIZE AND SHAPE ON THE STRENGTH OF COAL

by

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ABSTRACT

Multiple failure state triaxial compression tests on both mudstone and coal were carried out, to investigate a proposed failure criterion for weak sedimentary rocks. On the basis of proving the multiple failure state technique is valid and reliable for the tests, experiments on mudstone showed that the "Hoek-Brown" parameters m, s and σ_c were not significantly affected by the varying sizes, shapes and testing times of the mudstone specimens. The Hoek-Brown failure criterion represents the peak stress failure data well and the assumption that the mudstone is intact was justified. While showing the Hoek-Brown failure criterion is not appropriate for the residual strength test data, the linear regression model was proved to agree well with the test results. Furthermore, a brittle-ductile transition point for the mudstone was found and the failure criterion was verified.

A theoretical/empirical relationship between coal specimen size, shape and its uniaxial compressive strength was studied in detail, which enabled the establishment of a new approach to determine the parameters needed in the failure criterion for coal. By this approach the parameters of m_i and σ_{pi} for intact coals and, the m and s for *non-intact coals* (including the coal mass) can be calculated.

Multiple state failure triaxial compressive strength test on different sizes and shapes of a sub-bituminous coal from the Whitewood Mine was carried out. Statistical analyses on the test data proved that this approach is correct and can be used to find the intact coal parameters from tests on *non-intact* specimens.

Effort was made to collect as much of the published and non-published triaxial test data on coal specimens of different sizes, shapes, ranks as was possible. Utilizing the approach developed, the parameters for these coals were found out. A correlation between the intact coal parameter m_i and the coal rank was found.

This investigation on a failure criterion for coal will be helpful in the coal mining industry, especially saving the time-consuming and extremely expensive experiments on testing large size *in-situ* specimens in order to determine a coal mass strength. After having the appropriate coal parameters available, the coal strength of any size, any shape or any rank under any confining pressure can easily be found.

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LIST OF SYMBOLS

Symbol	Description	Units
σ_1	Major Principle Stress at Failure	*MPa*
σ_3	Minor Principle Stress at Failure	*MPa*
σ_c	Intact Uniaxial Compressive Strength of the Material	*MPa*
r^2	Coefficient of Determination	
r	Correlation Coefficient	
m	Material Parameter in the Hoek-Brown Failure Criterion	
s	Material Parameter in the Hoek-Brown Failure Criterion	
RMR	Rock Mass Rating	
m_i	m Value for Intact Material	
Q	Tunnelling Quality Index	
C	Compressive Strength at the Confining Pressure P_H	*kb*
P_H	Confining Pressure	*kb*
β	Tangent of the Slope of the Brittle and Ductile Region Division Line = $(1+\sin\phi)/(1-\sin\phi)$	
ϕ	Angle of Sliding Friction of the Failed Material	*Degree*
ϕ_B	Angle of Sliding Friction of Broken Material	*Degree*
γ	Residual Uniaxial Compressive Strength	*MPa*
σ_{1T}	Major Principle Stress at Brittle-Ductile Transition Point	*MPa*

Symbol	Description	Units
σ_{3T}	Minor Principle Stress at Brittle-Ductile Transition Point	MPa
σ_{pi}	Uniaxial Compressive Strength of Intact Coal	MPa
m_{mass}	m Parameter for the Coal Mass	
s_{mass}	s Parameter for the Coal Mass	
P_1	First Peak Failure Stress	MPa
P_2	Second Peak Failure Stress	MPa
P_3	Third Peak Failure Stress	MPa
P_4	Forth Peak Failure Stress	MPa
R_1	First Residual Strength	MPa
R_2	Second Residual Strength	MPa
R_3	Third Residual Strength	MPa
D	Diameter or the Smaller Lateral Dimension of Rectangular Cross Section of Specimens	$meter$
ε_1	Strain at Peak Failure Stress	
E	Modulus at Peak Strength Testing Stage	MPa
H	Height of Specimen	$meter$
σ_r	Residual Strength	MPa
σ_{3pr}	Previous Confining Pressure	MPa
σ_{1A}	Peak Failure Strength of Group A Specimens	MPa
σ_{1B}	Peak Failure Strength of Group B Specimens	MPa
$\bar{\sigma}_{1A}$	Sample Mean of σ_{1A} for Group A Specimens (etc.)	MPa

Symbol	Description	Units
$\bar{\sigma}_{1B}$	Sample Mean of σ_{1B} for Group B Specimens (etc.)	*MPa*
$V[\sigma_{1A}]$	Variance of σ_{1A} of Group A Specimens	$(MPa)^2$
$V[\sigma_{1B}]$	Variance of σ_{1B} of Group B Specimens	$(MPa)^2$
s^2_A	Sample Variance of Group A Specimens	
s^2_B	Sample Variance of Group B Specimens	
$V[E_A]$	Variance of *E* for Group A Specimens	$(MPa)^2$
$V[E_B]$	Variance of *E* for Group B Specimens	$(MPa)^2$
$V[\varepsilon_{1A}]$	Variance of ε_{1A} of Group A Specimens	
$V[\varepsilon_{1B}]$	Variance of ε_{1B} of Group B Specimens	
s_i	The *s* Parameter of the Intact Material	
m_B	*m* Parameter for Broken Material	
s_B	*s* Parameter for Broken Material	
σ_p	Uniaxial Prism Strength of Non-intact Coal	*MPa*
σ_{cube}	Uniaxial Compressive Strength of Coal Cube	*MPa*
σ_{rc}	Residual Strength for Coal	*MPa*
$V[m_1]$	Variance of *m* for Group 1 Specimens	
$V[m_2]$	Variance of *m* for Group 2 Specimens	
s^2_1	Sample Variance of Group 1 Specimens	
s^2_2	Sample Variance of Group 2 Specimens	
S_T	Total Variability of the Data	
S_H	Differences among the Specimen Heights	

Symbol	Description	Units
S_D	Differences among the Specimen Diameters	
S_I	Joint Effects (Interactions)	
S_E	Experimental Error	
R	E.C.E International Class Number for Coal	

1. INTRODUCTION

1.1 Introduction

In an underground excavation (in either mining or civil engineering), the designer must ensure the stability of the working place while employing the most cost effective methods. This is not an easy task, since the stability of an excavation depends on so many factors, such as the structural conditions in the rock mass, the stress field around the excavation and the response of the rock mass to it. One approach is to measure and calculate the stress field around the proposed excavation, then to predict the response of the rock mass to these stresses, i.e. is the excavation going to fail or is it safe? To do this it is necessary to establish a failure criterion which enables the designer to predict how the rock mass is going to behave under various stress conditions and the influence of discontinuities in the rock. Many people have worked on the search for a suitable failure criterion and a large part of rock mechanics research has concentrated on this goal; yet the work is not nearly finished. New developments on it are reported continually.

The problem with defining a failure criterion is that the experimental difficulties increase significantly as the rock specimen size increases. The strength varies greatly as the *intact* rock (small size specim[illegible]ts to the *rock mass* (large size specimen), while the stability of an underground excavation may involve all the stages of this transition: the strength of the intact rock decides which kind of excavation method (drilling and blasting or tunnel boring machine) should be used; the strength of the rock in the immediate vicinity of the excavation, which is related to the existing discontinuities and the fractures induced by excavation, decides whether or what kind of support is needed; the strength of the rock mass influences the whole structure.

While testing the strength of intact rock with small size specimens may be relatively easy, testing the strength of non-intact rock with large specimens is very difficult. The quality and quantity of experimental data decreases rapidly once testing moves from the intact rock specimen to the rock mass. Small specimens of intact rock are easy to prepare and test under a whole range of stress conditions. Large size specimens of non-intact rock may contain one or more sets of discontinuities, which influence the strength and make the testing more difficult. Moreover, full scale tests on a heavily jointed rock mass are not only extremely difficult to prepare and load, but also are very costly. On the other hand, testing small size specimens can be a difficult problem with some low strength sedimentary rocks, e.g. coal, since the specimen size has to be very small to be *intact*; often times too small to test.

Despite all the difficulties in finding a failure criterion that will meet all the requirements mentioned above, some excellent research has been carried out, even though many of the failure theories provide a good criterion in some stress ranges but fail to work in other stress ranges. Faced with the task of providing a more realistic failure criterion upon which the underground excavation designer can rely, Hoek and Brown [1] drew on their experience and developed, by a process of trial and error, an empirical strength criterion for both intact and heavily jointed or weathered rock masses given by:

$$\sigma_1=\sigma_3+\sqrt{m\sigma_3\sigma_c+s\sigma_c^2} \qquad (1.1)$$

where σ_1 and σ_3 are the major and minor principle stresses at failure, σ_c is the *intact* uniaxial compressive strength of the rock and *m* and *s* are two dimensionless parameters that depend on the properties of the rock and the degree to which the rock had been broken prior to the application of the failure stresses σ_1 and σ_3 (s = 1 for intact rock).

These workers showed that for tests on intact rocks, the parameter *m* can be determined from laboratory triaxial strength tests on intact rock specimens and, further, that for any particular *rock type* the value of *m* is approximately constant (see Table 1.1).

Table 1.1 Values of Strength Parameter m for Intact Rock Materials (s = 1.0) (after [2]).

Rock type	Number of Data points	Range of σ_c in pounds per square inch	m	Coefficient of determination, r^2
Limestone	84	6,380-29,200	5.4	0.68
Dolomite	25	21,500-73,400	6.8	0.90
Mudstone	34	-*	7.3	0.82
Marble	105	7,210-19,300	10.6	0.90
Sandstone	375	5,790-57,800	14.3	0.87
Dolerite	51	42,600-83,000	15.2	0.97
Quartzite	59	32,900-47,500	16.8	0.84
Chert	24	84,000	20.3	0.93
Norite	17	-*	23.2	0.97
Quartz-diorite	10	27,300 (saturated)-50,900 (dry)	23.4	0.98
Gabbro	10	29,700 (saturated)-50,900 (dry)	23.9	0.97
Gneiss	10	26,700 (parallel foliation)-25,900 (perpendicular foliation)	24.5	0.91
Amphibolite	10	21,300 (parallel foliation)-29,200 (perpendicular foliation)	25.1	0.98
Granite	109	16,900-50,000	27.9	0.99

*Source data in Reference given in normalized form. Note: 1 *psi* = 6.9 *KPa*.

Hence, if the rock type is known and a value of the uniaxial compressive strength can be estimated or determined, then equation (1.1) gives an approximate empirical strength criterion for the rock. This has been proved to be exceedingly useful in *preliminary* design considerations since it requires no testing other than the determination of the uniaxial compressive strength.

For heavily jointed rock masses (four or more discontinuity sets), Hoek and Brown showed that the rock mass is approximately isotropic; they then examined empirically the manner in which the values of *m* and *s* varied with the rock mass characteristics. The presence of discontinuities in the rock mass results in a decrease in both *m* and *s* below the intact values. Recognizing that the characteristics of the rock mass controlling its strength and deformation behaviour are similar to those adopted by the C.S.I.R. (Council for Scientific and Industrial Research) [3] and the N.G.I. (Norwegian Geotechnical Institute) [4] rock mass classifications, Hoek and Brown proposed a table [1] relating the variation in the *m* and *s* parameters to both of the classification ratings. This table was well accepted by the geotechnical community and has been used on a large number of projects. Experience gained from these applications showed that, while giving reasonable results for slope stability studies, it tends to be conservative in applications involving underground excavations where the confining pressures do not permit as much loosening of the rock mass as in a slope.

Incorporating the lessons learned from practical applications, Hoek and Brown [5] revised a set of relationships between the *RMR* (Rock Mass Rating) from the C.S.I.R. classification and the parameters *m* and *s*. They presented the relationships as:

Disturbed rock masses:

$$\frac{m}{m_i} = e^{\frac{RMR-100}{14}}, \qquad s = e^{\frac{RMR-100}{6}}.$$

Undisturbed or interlocking rock masses:

$$\frac{m}{m_i} = e^{\frac{RMR-100}{28}}, \qquad s = e^{\frac{RMR-100}{9}}.$$

where m and s are rock mass parameters and m_i is the value of m for the *intact* rock. The value of the Tunnelling Quality Index Q from the N.G.I. classification was calculated according to the formula proposed by Bieniawski [6]:

$$RMR = 9\,Log_e Q + 44 .$$

From these equations they were able to prepare their well known Table 1.2 below describing the approximate relationship between rock mass quality and the Hoek-Brown material parameters.

Table 1.2 Approximate Relationship between Rock Mass Quality and Material Parameters (after [5])

Disturbed rock mass m and s values						*undisturbed rock mass m and s values*
EMPIRICAL FAILURE CRITERION $\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2}$ σ_1 = major principal effective stress σ_3 = minor principal effective stress σ_c = uniaxial compressive strength of intact rock, and m and s are empirical constants.		CARBONATE ROCKS WITH WELL DEVELOPED CRYSTAL CLEAVAGE *dolomite, limestone and marble*	LITHIFIED ARGILLACEOUS ROCKS *mudstone, siltstone, shale and slate (normal to cleavage)*	ARENACEOUS ROCKS WITH STRONG CRYSTALS AND POORLY DEVELOPED CRYSTAL CLEAVAGE *sandstone and quartzite*	FINE GRAINED POLYMINERALLIC IGNEOUS CRYSTALLINE ROCKS *andesite, dolerite, diabase and rhyolite*	COARSE GRAINED POLYMINERALLIC IGNEOUS & METAMORPHIC CRYSTALLINE ROCKS – *amphibolite, gabbro gneiss, granite, norite, quartz-diorite*
INTACT ROCK SAMPLES						
Laboratory size specimens free	m	7.00	10.00	15.00	17.00	25.00
from discontinuities	s	1.00	1.00	1.00	1.00	1.00
CSIR rating: RMR = 100	*m*	*7.00*	*10.00*	*15.00*	*17.00*	*25.00*
NGI rating: Q = 500	*s*	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*
VERY GOOD QUALITY ROCK MASS						
Tightly interlocking undisturbed rock	m	2.40	3.43	5.14	5.82	8.56
with unweathered joints at 1 to 3m	s	0.082	0.082	0.082	0.082	0.082
CSIR rating: RMR = 85	*m*	*4.10*	*5.85*	*8.78*	*9.95*	*14.63*
NGI rating: Q = 100	*s*	*0.189*	*0.189*	*0.189*	*0.189*	*0.189*
GOOD QUALITY ROCK MASS						
Fresh to slightly weathered rock, slightly	m	0.575	0.821	1.231	1.395	2.052
disturbed with joints at 1 to 3m.	s	0.00293	0.00293	0.00293	0.00293	0.00293
CSIR rating: RMR = 65	*m*	*2.006*	*2.865*	*4.298*	*4.871*	*7.163*
NGI rating: Q = 10	*s*	*0.0205*	*0.0205*	*0.0205*	*0.0205*	*0.0205*
FAIR QUALITY ROCK MASS						
Several sets of moderately weathered	m	0.128	0.183	0.275	0.311	0.458
joints spaced at 0.3 to 1m.	s	0.00009	0.00009	0.00009	0.00009	0.00009
CSIR rating: RMR = 44	*m*	*0.947*	*1.353*	*2.030*	*2.301*	*3.383*
NGI rating: Q = 1	*s*	*0.00198*	*0.00198*	*0.00198*	*0.00198*	*0.00198*
POOR QUALITY ROCK MASS						
Numerous weathered joints at 30-500mm,	m	0.029	0.041	0.061	0.069	0.102
some gouge. Clean compacted waste rock	s	0.000003	0.000003	0.000003	0.000003	0.000003
CSIR rating: RMR = 23	*m*	*0.447*	*0.639*	*0.959*	*1.087*	*1.598*
NGI rating: Q = 0.1	*s*	*0.00019*	*0.00019*	*0.00019*	*0.00019*	*0.00019*
VERY POOR QUALITY ROCK MASS						
Numerous heavily weathered joints spaced	m	0.007	0.010	0.015	0.017	0.025
<50mm with gouge. Waste rock with fines.	s	0.0000001	0.0000001	0.0000001	0.0000001	0.0000001
CSIR rating: RMR = 3	*m*	*0.219*	*0.313*	*0.469*	*0.532*	*0.782*
NGI rating: Q = 0.01	*s*	*0.00002*	*0.00002*	*0.00002*	*0.00002*	*0.00002*

This table probably represents the best approach to date that allows the estimation of rock mass properties for preliminary design purposes.

Bieniawski and Bauer [7] believed that such an empirical strength criterion would also be of major practical importance in coal mining for estimating the *in-situ* strength of coal pillars. Since Hoek and Brown had not studied coal as one of their rock types, Bieniawski and Bauer attempted to extend this work to cover coal by considering a range of triaxial strength data obtained for eight different U.S. coals. Their calculations for the parameter m were performed by assuming the coal was *intact*, thus putting $s = 1$. The mean values of m, the corresponding values of the uniaxial compressive strength σ_c and the range of the confining pressures σ_3 are shown in Table 1.3.

Table 1.3 Mean Values of Parameters m and σ_c for Eight Coal Regions in the United States (after [7])

Coal region	Number of tests	σ_C (*MPa*)	Range of σ_3 (*MPa*)	m
A	14	18.27	0-20.7	20.058
B	15	22.52	0-20.7	12.128
C	49	19.03	0-6.9	27.413
D	106	13.39	0-6.9	21.052
E	155	15.70	0-6.9	21.531
F	17	16.42	0-1.4	4.402
G	25	26.41	0-1.4	9.786
H	8	33.51	0-2.8	3.835

From this table, they concluded that, contrary to Hoek and Brown's experience, the *m* value for coal could not be regarded as constant but covered a very wide range (from about 3.8 to 27.4) for these "intact" coal specimens. Bieniawski and Bauer then tried to explain this phenomenon by proposing that the parameter *m* should be related to coal strata quality utilizing the C.S.I.R. rock mass classification. This proposal was listed in Table 1.4, in which the nomenclature is: *I* = intact rock; *VG* = very good rock mass; *G* = good rock mass; *F* = fair rock mass; *P* = poor rock mass; and *VP* = very poor rock mass.

Table 1.4 Parameters *m* and *s* for Coal for Various Rock Mass Classes (after [7])

Rock class	m	s	m	s	m	s	m	s	m	s
I	5.00	1.00	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00
VG	1.67	0.20	3.33	0.20	5.00	0.20	6.67	0.20	8.33	0.20
G	0.67	0.02	1.33	0.02	2.00	0.02	2.67	0.02	3.33	0.02
F	0.33	0.002	0.67	0.002	1.00	0.002	1.33	0.002	1.67	0.002
P	0.17	0.0002	0.33	0.0002	0.50	0.0002	0.67	0.0002	0.83	0.0002
VP	0.03	0.00	0.17	0.0	0.25	0.0	0.33	0.0	0.42	0.0

To calculate the strength of full size coal pillars, they proposed to interpolate the coal mass parameters *m* and *s* according to Table 1.4, using the *m* and *s* for intact coal and incorporating the shape effect.

Commenting on the large variation of *m* values, Hoek and Brown [8] noted that:

(a) the coals were of various ranks, and

(b) the most likely explanation for this unexpectedly wide range in *m* values was that the specimens were not truly intact and that the assumption $s = 1$ was not justified.

Bieniawski, Bauer, Hoek and Brown all believe that the empirical rock mass strength criterion could be used with effect in studies of pillar strength and the formation of yield zones around tunnels or roadways and, the research on this topic will assist in applying this failure criterion to coal mining problems.

Amplifying comment (a), Hoek and Brown agreed that both the parameters m and s should be related to coal strata quality, or rock mass quality, as they did in their original paper. The author also notes that, differences in coal rank and moisture content will likely affect these values.

Expanding on comment (b), it is well established that the strength of laboratory size coal specimens is very sensitive to the size and shape of the specimen tested [9]. This sensitivity to specimen size and shape is a clear, but indirect, indication that the specimens contain discontinuities causing the strength variations. Consequently, in the size range represented by the usual laboratory size test specimens, the coal cannot be regarded as "intact" and thus s cannot be unity. If the above proposed failure criterion for coal is to be useful for predicting the strength behaviour of coal in the field, some evaluation of how this failure criterion is affected by specimen size must be made in order to determine the coal mass properties (as opposed to the specimen properties). It is the coal mass properties that must be used to evaluate the stability of excavations in coal.

The problem therefore remains, **how can the intact m values for coal be determined from triaxial strength tests on *non-intact* specimens?** If this can be done then it should be possible to follow a procedure similar to that of Hoek and Brown to examine the variation of *m* and *s* with degree of fracturing (or specimen size) to define coal mass failure criteria. Further the intact *m* values could be examined to investigate any correlation with coal rank and/or moisture content.

To determine the intact *m* values for coals from tests on laboratory size specimens requires some additional information that is currently lacking. It is believed that this additional information can be obtained by making use of the independently established empirical relationships for the size/shape dependency of uniaxially loaded coal prisms [9]. **It is therefore proposed to use this information to develop the necessary theoretical/empirical relationships and then to carry out a triaxial testing program on specimens of one coal over as wide a range of specimen sizes as is practical, in order to prove or disprove these concepts.**

If these concepts are proven to be valid, then a logical next step would be to carry out the tests over a range of coals of different ranks and moisture content. However, it is not proposed to carry out such studies until the basic concepts have been validated from the tests on one individual coal. Nevertheless a preliminary evaluation of the effects of rank and moisture content may be able to be obtained from the evaluation of data from triaxial tests on coals that have been reported in the literature; **it is proposed to carry out this evaluation of this data reported in the literature.**

1.2 Proposed Failure Criterion for Sedimentary Rocks

1.2.1 The failure mechanism

Figure 1.1 illustrates typical stress-strain curves for triaxial tests on rock specimens.

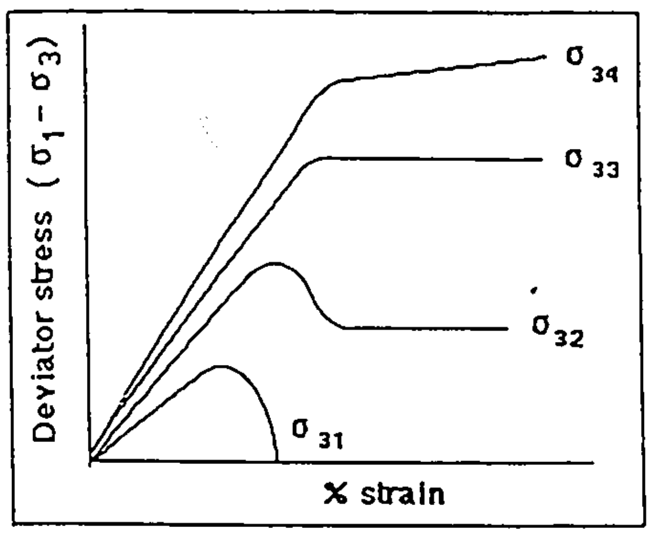


Figure 1.1 Typical Stress-Strain Curves (after [10])

At low confining pressures fracture occurs at peak differential stress, when the shear resistance to failure is exceeded. After fracture, a normal stress continues to act across the failure plane and the peak stress drops to the residual value where deformation continues by sliding on the fracture surface.

At high confining pressures the frictional resistance to sliding exceeds the shear resistance to fracture, thus it is easier to form a new fracture surface than to produce displacement on existing fractures.

At an intermediate confining pressure, the shear resistance to fracture just equals the frictional resistance to sliding on the fracture surface; there is no drop from the peak to the residual stress level and large deformations can take place with little or no increase in axial stress. Such deformation is called ductile when associated with metals in which there is no loss of cohesion. However, as explained above, similar deformation behaviour, with a loss in cohesion, can be caused by brittle fracture. This has been called "pseudo-ductile" [10] failure to distinguish it from pure plastic behaviour, which has no loss in cohesion during the deformation.

Mogi [11] studied the relationships between compressive strength and confining pressure for different rock types and established a formula for transition from brittle fracture to ductile flow. He noted that pressure dependence of strength is very different for different types of fractures and continuously changes from a brittle stage to a ductile stage as confining pressure is increased. The transition pressure is higher in stronger rocks. In silicate rocks, the brittle failure region and the ductile failure region are clearly divided (Figure 1.2), by a straight boundary line defined by

$$C=3.4P_n,$$

or

$$\sigma_1=3.4\sigma_3 \qquad (1.2)$$

where C was defined by Mogi as the compressive strength at the confining pressure

P_H. The dotted line in Figure 1.2 is the boundary between brittle region and ductile region; the closed circles, semi-closed circles and open circles represent brittle, transitional and ductile points respectively. That is, the failure is brittle when $\sigma_1 > 3.4\sigma_3$, otherwise ductile. Moreover, Mogi found that the slope of the boundary line is steeper for carbonate rocks; especially, the transition pressure of weaker marbles and

Figure 1.2 has been removed due to the unavailability of copyright permission.

Figure 1.2 Fracture Behaviour of Silicate Rocks at Various Strengths and Confining Pressures (after [11]).

limestones is appreciably lower than that of silicate rocks and, the boundary line is not linear, but slightly concave downward.

Hoek and Brown placed practical limitations on the use of their equation (1.1), due to the change from brittle to ductile behaviour for most rocks. They used Mogi's definition of the transition point equation (1.2) and stated that equation (1.1) is only valid for the brittle failure of rocks at stresses below the brittle-ductile transition point, i.e. the equation is valid provided that:

$$\sigma_1 > 3.4\sigma_3.$$

At high confining pressures, when σ_1 is less than the value given by equation (1.2), the curve bends over more severely due to ductile behaviour and departs from the relationship given by equation (1.1) (Figure 1.3).

Other work (Orawin [12], Maurer [13], Byerlee [14]) suggests that this brittle-ductile transition point occurs due to pseudo-ductile behaviour for hard rocks at great depth and may be expressed in terms of the sliding coefficient of friction of the failed rock, as:

$$\sigma_1=\beta\sigma_3,$$

where

$$\beta=\frac{(1+\sin\phi)}{(1-\sin\phi)}$$

and ϕ is the angle of sliding friction of the failed rock. (Note: if $\beta = 3.4$, as suggested by Mogi, then this corresponds to a ϕ value of 33°).

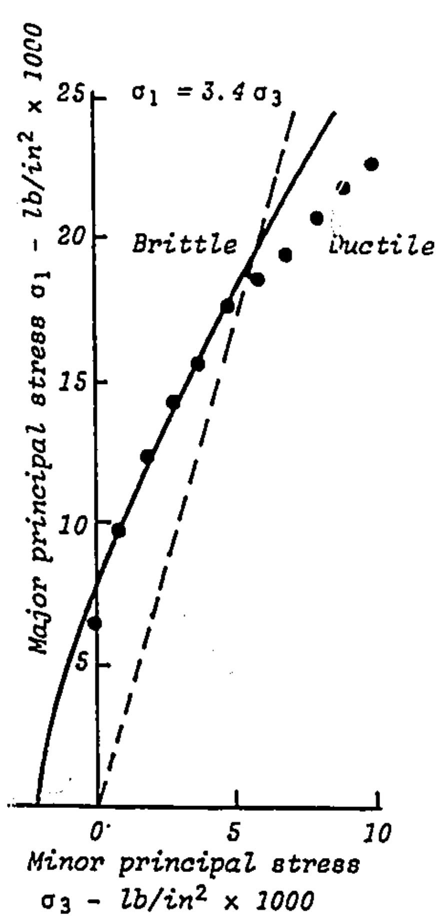


Figure 1.3 Transition from Brittle to Ductile Failure Illustrated by Test Data Obtained by Schwartz for Indiana Limestone (after [1]).

It was suggested by Barron [15, 16] that such failure can occur in coal, and in other weak sedimentary rocks (such as mudstone), at confining pressures normally encountered at coal mining depths.

In this thesis, the brittle-ductile transition line is going to be defined according to the test pressure data of the broken rock sliding on the failed plane. Since both mudstone and coal, which were tested in the experiments, are weak materials, it is anticipated that the slope of the transition line β will be greater than 3.4 in equation

(1.2), as stated by Mogi.

The point of transition between brittle fracture and pseudo-ductile yield is given by the point of intersection of the curves for the peak stress and the residual stress plotted against confining pressure. Such an intersection can only occur if the peak stress curve is non-linear. Further, it is assumed that the residual stress curve varies linearly with the confining pressure (i.e. a constant "coefficient of sliding friction"). Thus if the peak stress-confining pressure curve is given by:

$$\sigma_1 = f(\sigma_3)$$

and the residual stresses are assumed to obey Coulomb-Navier theory and are related by:

$$\sigma_r = \gamma + \beta \sigma_3, \qquad (1.3)$$

where β can be shown to be equal to $(1 + \sin\phi_B)/(1 - \sin\phi_B)$, which is Wilson's "triaxial stress factor" [17] and ϕ_B is the angle of sliding friction of the broken rock and γ is the residual uniaxial compressive strength of the broken rock.

Experience has shown that a good fit to rock strength triaxial test data from laboratory size specimens can be obtained from the Hoek and Brown expression, equation (1.1). The transition point, σ_{1T}, σ_{3T}, from brittle to pseudo-ductile failure occurs at the intersection of these two curves as illustrated in Figure 1.3, is given by putting $\sigma_1 = \sigma_r = \sigma_{1T}$ and $\sigma_3 = \sigma_{3T}$ in equation (1.1) and equation (1.3):

$$\sigma_{1T}=\sigma_{3T}+\sqrt{m\sigma_c\sigma_{3T}+s\sigma_c^2}=\gamma+\beta\sigma_{3T}$$

i.e.

$$m\sigma_c\sigma_{3T}+s\sigma_c^2=[\gamma+(\beta-1)\sigma_{3T}]^2$$

or

$$(\beta-1)^2\sigma_{3T}^2+[2(\beta-1)\gamma-m\sigma_c]\sigma_{3T}+[\gamma^2-s\sigma_c^2]=0$$

and putting

$$\alpha=[2(\beta-1)\gamma-m\sigma_c]$$

then this equation becomes:

$$(\beta-1)^2\sigma_{3T}^2+\alpha\sigma_{3T}+[\gamma^2-s\sigma_c^2]=0$$

Hence

$$\sigma_{3T}=\frac{-\alpha+\sqrt{\alpha^2-4(\beta-1)^2(\gamma^2-s\sigma_c^2)}}{2(\beta-1)^2}$$

and from equation (1.3):

$$\sigma_{1T}=\gamma+\beta\sigma_{3T}$$

This failure criterion has important implications because it allows for brittle failure at low confining pressures and for pseudo-ductile yielding of the rock at high confining pressures. It has been applied with some success to determine the strength of coal pillars [15, 16]. However, **direct proof of the validity of this proposed failure criterion, based on direct specimen testing, has yet to be obtained.**

Because of the difficulties of sampling and preparing coal specimens for laboratory testing, and because coal strengths are expected to be affected by specimen size, it was decided to attempt to verify the failure criterion by tests on mudstone. It was anticipated that laboratory size mudstone samples could be regarded as being intact; thus no size effects would be expected. Hence the proposed failure criterion could be tested without the additional complication of size effects. However, this is an assumption and thus it will be necessary to verify this assumption by assessing whether or not the mudstone test results are significantly influenced by specimen size and shape.

If this failure criterion can be verified for a typical weak sedimentary rock such as mudstone, then it can be applied to coal. With coal it would be anticipated that laboratory size specimens cannot be regarded as being intact, and hence the influence of size and shape on these results must be evaluated.

1.2.2 Objectives

The objectives of the research are:

1. To verify the validity of the proposed failure criterion for weak sedimentary rocks by performing triaxial tests (to determine the peak and residual stress

relationships at failure) on mudstone, in which is assumed there will be no size/shape effects on laboratory size specimens. It is therefore essential to test whether this assumption is true.

2. To develop the appropriate theoretical/empirical relationships whereby triaxial tests on non-intact coal specimens can be interpreted to allow the determination of intact *m* values for coal and the m_{mass} and s_{mass} values for the coal mass.

3. To carry out triaxial tests on specimens of one coal (determination of peak and residual strengths) over a range of specimen sizes, in order to assess the validity of the theoretical/empirical relationships developed in objective 2 above, and to examine the influence of size/shape effects, on the failure criterion.

4. To gather from the literature as many suites of triaxial test data on various coals as is possible and to evaluate the intact *m* values for these coals, using the concepts developed in objective 2 above.

5. If possible, to investigate whether or not there is a correlation between the intact *m* values so determined and the coal rank and moisture content.

6. If possible, to derive approximate coal mass strength criteria for coals of different ranks.

1.3 Significance of the Research

The failure criterion presented by Hoek and Brown is an attractive and useful way to predict the *in situ* strength of rock masses. This is very important because the strength criteria developed in rock mechanics, so far, emphasize the strength of rock materials instead of rock masses and therefore, have limited practical use.

Since coal is playing an increasing important part on the current world's energy stage, extending the failure criterion for practical use in coal strata would bring considerable potential benefit to coal mining.

At present it is not possible to use laboratory strength test data for coal to evaluate or estimate the coal mass strength. However, if analytical models are to be successfully used to predict the stability of excavations in coal, it is essential to use a valid failure criterion for the coal mass as input to the model.

If successful, the above program would validate the proposed failure criterion for sedimentary rocks (including coal) and would also allow the coal mass properties to be estimated from tests on laboratory size specimens. This would represent a major breakthrough in the ability to use analytical models to design safe excavations in coal. This, in turn, would result in improved mine safety and, in specific cases, may result in increased resource recovery and reduced mining costs.

1.4 Organization of the Thesis

The organization of the thesis is as follows:

Chapter 1 introduces the concepts and backgrounds of the proposed Hoek-Brown failure criterion and, its potential benefits if it can be adopted in the coal mining industry.

Chapter 2 describes the experimental technique used and gives example data from the tests on mudstone.

Chapter 3 examines in detail, whether and to what extent the factors such as the multiple failure state triaxial test technique, different specimen size and shape (including their combinations) and time of testing influence the mudstone test results. The results are compared to the proposed failure criterion.

Chapter 4 studies the previous research work on specimen size and shape effects on the uniaxial compressive strength of coals and derives a general relationship between the specimen size and shape and the uniaxial strength. An approach is then developed whereby triaxial tests on non-intact coal specimens can be interpreted to determine the intact m_i value and uniaxial compressive strength σ_{pi} of the intact coal, the m and s values of non-intact coal, and m_{mass} and s_{mass} values for the coal mass.

Chapter 5 describes triaxial tests with coal specimens and gives a statistical examination of the effects of pertinent factors on the parameters for coal. Using the test results with coal, the concepts developed in Chapter 4 were verified and the coal parameters were determined. As many suites as possible of triaxial test data on various ranks of coal were then gathered from literature. Using the approach developed in Chapter 4, the parameters of these different ranks of coals were evaluated. The correlation between the parameters so determined and the coal rank was investigated.

Chapter 6 summarizes the research work carried out in this project, the main conclusions, and recommendations for further research to improve this failure criterion established for the coal mining industry.

2. EXPERIMENTS ON MUDSTONE

2.1 Multiple Failure State Triaxial Test Introduction

Triaxial compression tests have long been used to determine the compressive strength of the rock under confining pressure. Conventionally, each test specimen yields only one point on the failure envelope. The test begins with the axial pressure σ_1 and confining pressure σ_3 both being increased at the same rate until a pre-determined value of σ_3 is reached. Then σ_3 is kept constant while the axial pressure

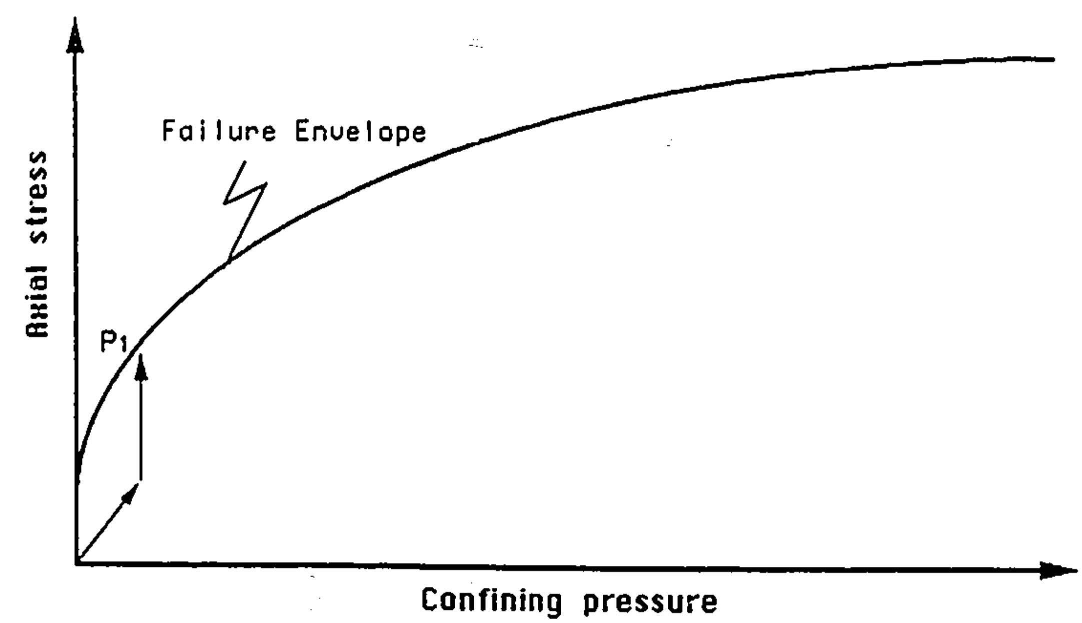


Figure 2.1 Conventional Triaxial Test.

σ_1 is increased until the specimen fails. The axial pressure σ_1 at the failure point is called the compressive strength of the specimen under that specific confining pressure σ_3. Therefore these two stress values, σ_1 and σ_3 at failure, define a point P_1 on the "σ_1/σ_3" failure envelope. (Fig. 2.1)

In the experiments with mudstone described in Section 2.2, the so called "Multiple Failure State Triaxial Test" [18] was used. In this test four stress points (P_1, P_2, P_3, P_4) of the peak strength failure envelope and three points (R_1, R_2, R_3) of the residual strength envelope were obtained with the test on a *single* specimen. In this test, the confining pressure was increased in four increments and the corresponding peak strength at each stage was recorded on the Load-Deformation graph. After the fourth peak strength point was reached, the confining pressure was decreased in three

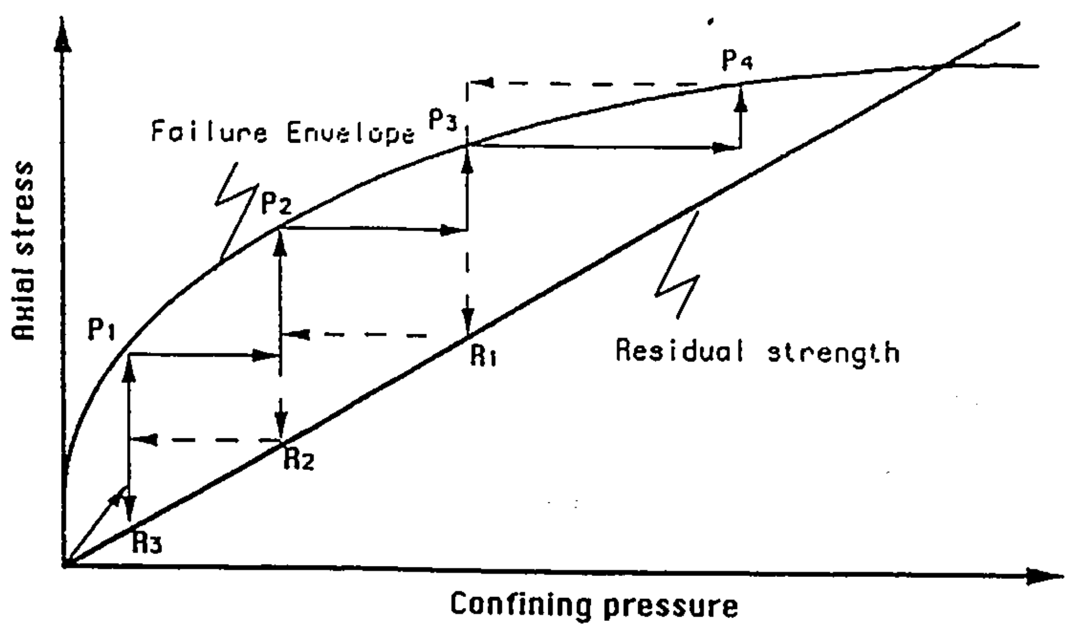


Figure 2.2 Multiple Failure State Triaxial Test.

stages and therefore three residual strengths corresponding to three confining pressures were determined (Fig. 2.2). The solid arrows indicate the increasing path of the confining pressure. The dotted arrows indicate the decreasing path of the confining pressure.

The question then arises as to whether or to what an extent the test results are influenced by the previous *failures* created in the Multiple Stage Failure test, compared with the Single Stage Failure test results in which no previous failure occurred. The test results must therefore be examined to see if there are significant differences resulting from the two different test techniques. Kovári and Tisa [18] showed that in their tests the differences were insignificant. However, this fact will also be checked using the results reported in this thesis. This was done in Section 3.1. The conclusion was that there was no significant difference between the results obtained from "single stage" and "multiple stage" tests.

2.2 Experiment Description

2.2.1 Scope

In situ rock is confined both laterally and vertically, except on the surface of an excavation. The strength of a cylindrical specimen is substantially increased when a confining pressure is applied around its cylindrical surface, compared with the unconfined strength. This increase in strength due to confinement is of great practical significance in mining engineering. The multiple failure state triaxial test with mudstone is intended to measure the strength of cylindrical specimens under confinement, i.e. to measure the *triaxial strength* as it is commonly called. This provides the values necessary to determine the peak strength envelope, the residual strength envelope and the transition point from brittle to ductile failure.

2.2.2 Apparatus

The apparatus consists essentially of three parts (see Fig. 2.3): a triaxial cell, a loading device and a device for generating confining pressure.

(1) A triaxial cell. This comprises:

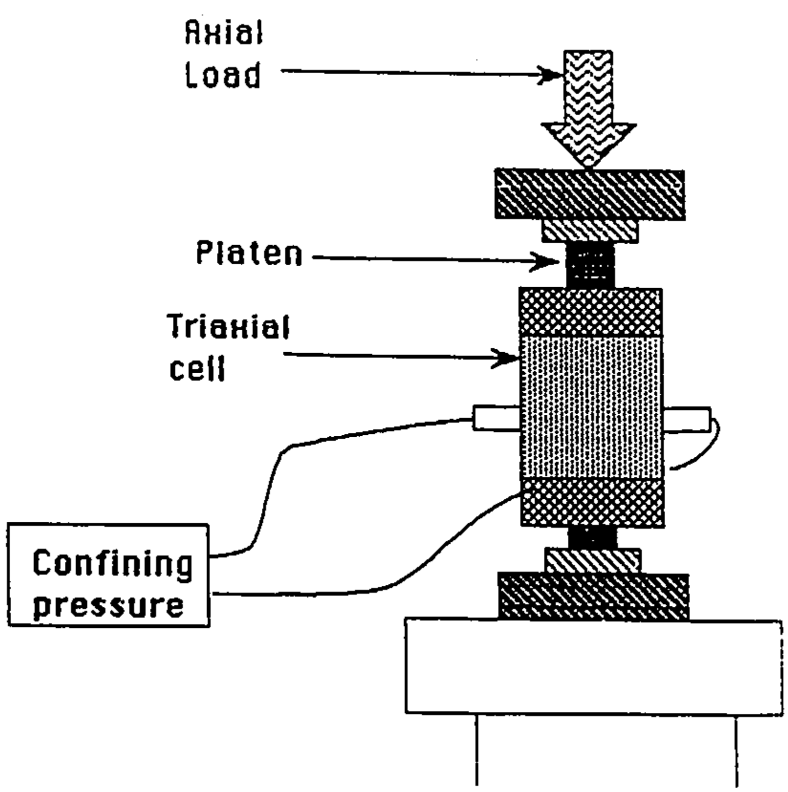


Figure 2.3 Block Diagram of the Triaxial Test Arrangement.

a) A flexible plastic sleeve to prevent the hydraulic fluid from entering the specimen.

b) A steel cell into which the specimen contained in the sleeve was placed. The body of the cell has an air bleeder hole and a connection for a hydraulic line applying the confining pressure.

c) Two platens were placed at the specimen ends. The diameter of the platens are between *D* and *D*+2*mm*, where *D* is the diameter of the specimen. The space between the cell and the sleeve was filled with hydraulic oil.

d) Spherical seats which were incorporated in each of the platens. The curvature centre of the seat surfaces coincided with the centre of the specimen ends.

(2) An X-Y Recorder

The recorder is a precision instrument designed to plot cartesian coordinate graphs of specimen axial force versus deformation in the tests.

(3) A 4137.8 *MPa* (600,000 *pounds*) capacity in compression MTS servo-controlled testing machine for applying and controlling axial load.

Four load capacity ranges of 413.8 *MPa*, 827.6 *MPa*, 1655.1 *MPa*, and 4137.8 *MPa* can be pre-selected, as appropriate, according to the specimen condition and test objective. The load was plotted directly in the Y axis of the X-Y recorder. The specimen deformation was plotted on the X axis of the X-Y recorder.

The peak load, and residual load where appropriate, could also be read directly from the peak voltage output from the control system memory. This can be recorded on the corresponding output graph in the X-Y recorder by the experimenter.

A good cross check is therefore possible between the loads as recorded by the calibration of the X-Y recorder and the direct output of the cell. The load can be applied at a rate conforming to the requirements of each test.

(4) Equipment for generating and measuring the confining pressure. This includes:

a) A hydraulic pump and a pressure intensifier of sufficient capacity and capable of maintaining constant confining pressure within 2% of the desired value.

b) A pressure gauge which is accurate enough to allow the above to be observed.

2.2.3 Machine settings

(1) The Load Range for the small diameter (D = 0.029 *meter*) specimens was 413.8 *MPa* (60,000 *lb/sq in*); the load range for the large diameter (D = 0.053 *meter*) specimens was 827.6 *MPa* (120,000 *lb/sq in*).

(2) The Stroke Range for both diameters were 0.0127 *meter* (0.5 *in*).

(3) The stroke rate was set to be 0.1524 *mm/min* throughout the mudstone tests.

(4) The X-Y Recorder:

a) Horizontal Graph scale for both diameters: 2% *of range*/2.54 *cm* (2% *of range/in*), where 1% = 0.1 *volt*. So, 2.54 *cm* (1 *in*) *of graph* = 2% *of range* = 0.2 *volts*. With 10-*volt* full scale, 1 *volt* = 1.27 *cm*/10 (0.5 *in*/10) = 0.127 *cm* (0.05 *in*). Therefore, 2.54 *cm* (1 *in*) *of graph* = 0.2 × 0.127 = 0.0254 *cm* (0.01 *in*) *of deformation*. 1 *cm of graph* = 0.01 *cm of deformation*.

b) Vertical Graph scale for the small diameters: 10% *of range*/2.54 *cm* (10% *of range/in*), where 1% = 0.1 *volt*. So, 2.54 *cm* (1 *in*) *of graph* = 10% *of range* = 1 *volt*. With 10 *volt* full scale, 1 *volt* = 413.778 *MPa*/10 = 41.4 *MPa* (6,000 *lb/sq in*). Therefore, 2.54 *cm* (1 *in*) *of graph* = 1 × 41.4 = 41.4 *MPa* (6,000 *lb/sq in*). 1 *cm of graph* = 16.3 *MPa* (2362 *lb/sq in*).

c) Vertical Graph scale for the large diameters: 10% *of range*/2.54 *cm* (10% *of range/in*), where 1% = 0.1 *volt*. So, 2.54 *cm* (1 *in*) *of graph* = 10% *of range* = 1 *volt*. With 10 *volt* full scale, 1 *volt* = 827.6 *MPa*/10 = 82.8 *MPa* (12,000 *lb/sq in*). Therefore, 2.54 *cm* (1 *in*) *of graph* = 1 × 82.8 = 82.8 *MPa* (12,000 *lb/sq in*). 1 *cm of graph* = 32.6 *MPa* (4724 *lb/sq in*).

2.2.4 Preparation of the test specimens

The specimens, drilled with water-flushed hollow diamond coring bits, were of right circular cylinder shape having two diameters: one was about 0.029 *meter* (referred to as the small diameter) and one was about 0.053 *meter* (referred to as the large diameter). After coring the specimens were wrapped and numbered according to the blocks from which they were drilled and the two diameter categories (whenever a letter was used it refers to a small diameter specimen, otherwise a large diameter one). The first number indicates the number of the rock block to which it belonged. The letter or number after the dash indicates the order in which the specimen was cored. The specimens were numbered alphabetically or in ascending numerical order. For instance, specimen # 1-B means it came from block number 1 and it is the second core of this block; also it is a small diameter specimen since B is a letter. Specimen # 3-8 was from block number 3 and it is the eighth specimen from the block; it is a large diameter core since there are no letters in the numbering. Sometimes a specimen was too long for the testing, it was broken in two. Then either a letter or a number would be added to the end of the specimen number to distinguish them, for instance, "1-E1" and "1-E2" or "1-LA" and "1-LB".

Before testing the specimen was cut to certain height using a thin high-speed carborundum splitting wheel. The ends of each specimen were ground three times to the required height (with coarse, medium and fine carborundum belt respectively). The sides of the specimen were smooth and free of abrupt irregularities.

The diameter of each specimen was measured five times to the nearest 0.1 *mm* at about 72° to each other: two at the top part, one at the mid-height and two at the bottom part of the specimen. The minimum diameter was used for calculating the cross sectional area. The height of the specimen was also measured five times to the nearest 0.1 *mm* at about 72° to each other. The smallest height was used in the calculations.

In order to minimize the test biased error, the test dates for all the specimens and the confining pressures for each specimen were randomized. Consequently specimens from the same block having different sizes and different shapes were tested on randomized dates at randomized confining pressures. A table of the randomized test arrangement was listed in Appendix 1. Each line, from left to right, lists the specimen number, the four confining pressures used to test the four corresponding peak strengths. The three confining pressures used to determine the residual strengths were the same as the first three confining pressure values used in the peak strength testing (refer to Fig 2.2). From top to bottom of the list, three specimens were scheduled to be tested every day. But because of some testing problems the actual testing did not follow exactly the arranged schedule. The original testing records for each specimen are available to reference, but they are not compiled in the thesis because of the restriction of volume.

2.2.5 Test Procedure

The cell was assembled with the specimen aligned between steel platens and surrounded by the sleeve. The specimen, the platens and the spherical seats were aligned accurately so that each was coaxial with the other.

The triaxial cell was filled with hydraulic oil, allowing the air to escape through an air bleeder hole. When there were no air bubbles, the air bleeder hole was closed. The cell was then placed onto the axial loading device.

The axial load and the confining pressure were increased simultaneously and in such a way that axial stress and confining pressure were approximately equal, until the predetermined test level for the confining pressure was reached. Subsequently, the confining pressure was maintained to within 2% of the prescribed value.

The axial load on the specimen was then increased continuously at a constant stroke rate. The onset of fracture was normally marked by the load remaining constant while the deformation increases, although a small drop in load sometimes occurred (refer Fig. 2.4). As soon as a failure was observed from the Load-Deformation curve, the axial load was quickly put on hold because that means a failure plane had been created. The maximum axial load at that particular confining pressure on the specimen was read in terms of voltage and recorded on the graph. In these experiments special attention was paid when the confining pressure was zero or a very small value (e.g. 3.45 *MPa*), because the axial load should be put on hold immediately with no delay when there was a small drop in the curve; otherwise the specimen might undergo a large slip on the failure plane and, there would be a large, sharp drop in the curve because of the small confining pressure. If this occurred the experiment was aborted since no large sliding on the failure plane was supposed to happen in the multiple failure test in the peak stress testing range. After all the peak loads were obtained at the different confining pressures, the confining pressure was reduced to a certain level while the axial load was continuously applied. The curve became flat after an initial sharp drop, then the residual load which the specimen can sustain at that particular confining pressure was recorded. Afterwards the confining pressure was reduced to another level and the corresponding residual strength was similarly obtained, until all the residual strengths of the specimen at each prescribed confining pressure were tested. Then the broken specimen was taken out of the cell and put into a cup or a plastic bag with its number on it. The test for that specimen was finished and the next one begun. This procedure continued until all the specimens scheduled for test on that day were finished.

2.2.6 Calculations and plotting

For each specimen, the compressive strength at every confining pressure was calculated by dividing the maximum axial load calculated from the corresponding voltage recorded on the graph, with the original cross-sectional area calculated from the minimum diameter measured before the testing. The residual strength was calculated by dividing the residual load with the same original cross-sectional area used in the peak stress calculation. The deformation and strain at every peak strength of the specimen were also calculated. The original minimum height of the specimen was used in the calculation of strain ε_1 at peak failure stress, assuming that all deformation occurred within this specimen height. The Young's modulus, *E*, at each failure stage during the peak strength testing, was calculated from the tangent of the linear portion of the load-deformation recording curve prior to failure at the corresponding stage. Furthermore, the peak strength and the residual strength versus the confining pressure were plotted on one graph for each specimen.

2.3 Data from the Experiments

The test date, test time, original measurements, machine settings, recording graph, calculations for all the parameters and the plotting mentioned above were recorded in detail for each specimen. These materials are not included in the thesis, but are available upon request. The data were recorded in imperial units since originally all the testing equipment was calibrated in these units; but in the thesis all the units have been converted into the SI units. A table has been prepared for each specimen to list its number, test start time, test end time, test date, minimum values of diameter D and height H measurements, confining pressure σ_3, corresponding peak stress σ_1 and residual stress σ_r, strain ε_1 at peak failure stress and estimated modulus E at different loading stages, and the material parameters of m and uniaxial compressive strength σ_c calculated individually for each specimen using linear regression analysis of $(\sigma_1 - \sigma_3)^2$ against $m\sigma_c$ (Note: intact rock is assumed, so $s = 1$ for every specimen; the method determining the m and σ_c is the same as that used in Section 3.4, except this was done separately for each specimen using its own test data). The following Table 2.1 is the one for specimen # 1-A. Such tables for all the specimens are given in Appendix 2. If there is a question mark in a cell of a table, it means the experiment failed at that point because of an oil leak or some other problem and therefore, the correct value was in doubt.

Note that not all the specimens were listed in Appendix 2, only the ones with relatively complete test results were included. The ones in which a large part of tests were aborted or failed because of technical problems were excluded. However, a few test data of the specimens which were not included in Appendix 2 were made use of in the following analyses of the test. Those data can be referenced if necessary.

Table 2.1 Example Test Results - Specimen # 1-A

1-A Time: Start: 9:44 End: 9:59 Date: July 31, 1989

D (m)	0.030		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.071		6.9	121.4	49.4	0.01376	12413.3
m	13.27		27.6	205.9	148.2	0.01792	22337.1
σ_c (MPa)	73.7		48.3	274.5	234.5	0.02263	25778.4
r^2	1.00		69.0	341.3		0.02715	28612.7

Figure 2.4 is a copy of the experiment recording graph with specimen # 1-A. The figures on the graph are the voltages read from the machine for the calculation of axial loads at those points. From the curve it can be seen that the load dropped slightly during loading stage when a peak strength was reached. When the confining pressure was increased the curve began to rise again until reached the next peak strength at that specific confining pressure. When reducing confining pressure to determine the residual strength the curve dropped sharply, then flattened when the residual strength at that point was reached; i.e. although the strain continued to increase, no further loss of strength occurred.

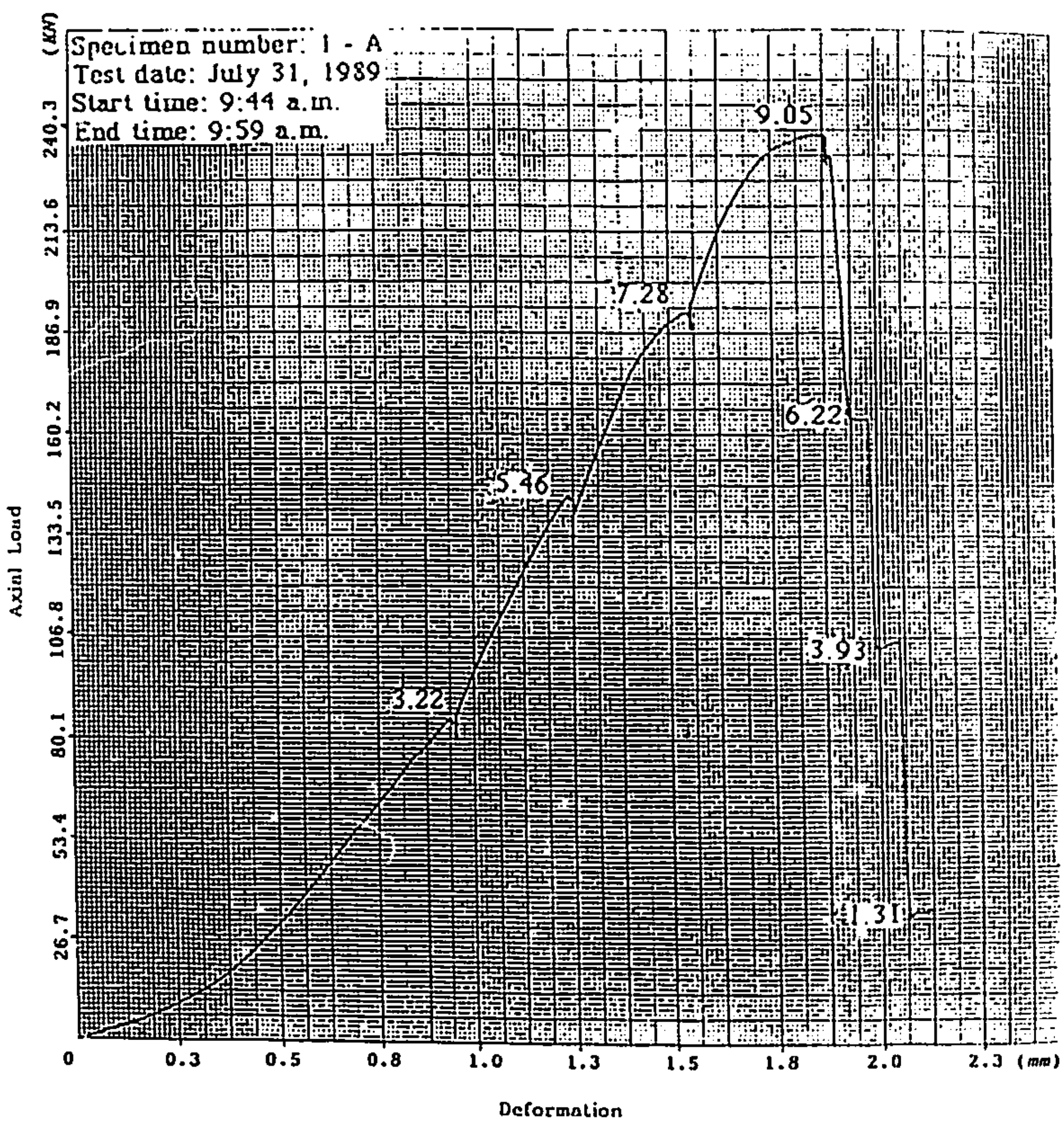


Figure 2.4 Recording Graph of Axial Load versus Deformation.

3. ANALYSIS OF MUDSTONE TEST RESULTS

3.1 The Effect of Previous Failure on σ_1, ε_1 and E Values

3.1.1 The effect of previous failure on σ_1 values

Since the Multiple Failure State Triaxial Test technique was used in the experiment, it is important to know whether, or to what an extent, the strength test results of σ_1 are influenced by the previous failures created in the multiple stage failure test, compared to the single stage test in which no previous failure occurred. The answer was given by performing the following two-sample t test of the multiple failure triaxial test data with mudstone. Referring to Appendix 2, choose two groups of data, Group A and Group B. In Group A the specimens failed the first time at a confining pressure $\sigma_3 = 13.8$ MPa, while in Group B the specimens first failed at a confining pressure $\sigma_{3_{pr}} = 3.4$ MPa, and failed a second time at $\sigma_3 = 13.8$ MPa, as is shown in Table 3.1. This test will examine whether there is a significant difference between Group A strength σ_{1A} and Group B strength σ_{1B}. It will be proven in later sections that different specimen sizes or different specimen shapes, different test times and, different combinations among them have no significant effect on the test results. So if there is a noticeable difference between Group A strength and Group B strength, it must be the previous failure which resulted in the difference.

Table 3.1 σ_1 Values of the Two Groups of Test Specimens: with and without Previous Failure.

Group A			Group B		
σ_3 (*MPa*)	σ_1 (*MPa*)		σ_{3pr} (*MPa*)	σ_3 (*MPa*)	σ_1 (*MPa*)
13.8	111.1 (2-E)		3.4	13.8	137.8 (5-H)
13.8	151.6 (5-I)		3.4	13.8	126.9 (1-6)
13.8	107.4 (2-3)		3.4	13.8	118.8 (2-B)
13.8	117.0 (1-E1)		3.4	13.8	111.5 (5-M)
13.8	137.6 (1-2)		3.4	13.8	101.8 (1-J)
13.8	179.0 (2-A)		3.4	13.8	138.6 (4-G)
13.8	140.3 (1-H)				
13.8	123.9 (3-3)				

Regard the σ_1 data from Group A and Group B as two independent random samples from two normal populations with means of μ_A and μ_B and variances of $V[\sigma_{1A}]$ and $V[\sigma_{1B}]$ respectively. The sample sizes of Group A and B are $n_A = 8$, $n_B = 6$. The sample means are:

$$\overline{\sigma}_{1A}=\frac{1}{n_A}\sum_{i=1}^{n_A}\sigma_{1A_i}=\frac{1067.8}{8}=133.5\ (MPa),$$

$$\overline{\sigma}_{1B}=\frac{1}{n_B}\sum_{i=1}^{n_B}\sigma_{1B_i}=\frac{735.3}{6}=122.5\ (MPa)\ .$$

The sample variances are:

$$s_A^2=\frac{\sum_{i=1}^{n_A}(\sigma_{1A_i}-\overline{\sigma}_{1A})^2}{n_A-1}=\frac{4010.1}{8-1}=572.9\ (MPa)^2,$$

$$s_B^2=\frac{\sum_{i=1}^{n_B}(\sigma_{1B_i}-\overline{\sigma}_{1B})^2}{n_B-1}=\frac{1078.0}{6-1}=215.6\ (MPa)^2.$$

First, test the null hypothesis that variances of the two populations are equal, i.e. $V[\sigma_{1A}] = V[\sigma_{1B}]$, against the two-sided alternative $V[\sigma_{1A}] \neq V[\sigma_{1B}]$ with an error probability of

$$\alpha=0.05.$$

where the common choice of the numerical value of 0.05 is also known as the *level of significance* of the test. Throughout this thesis, unless otherwise stated, all the tests of statistical hypotheses were constructed to control a 0.05 probability of committing a Type I error. Since

$$\frac{s_A^2}{s_B^2}=\frac{572.9}{215.6}=2.7,$$

which is less than

$$F_{\alpha/2}(n_A-1,\ n_B-1)=F_{0.025}(7,\ 5)=6.9$$

the null hypothesis has to be accepted, i.e. the two populations have the same variance and the use of the following two-sample t test is justified.

Now test the null hypothesis $\mu_A - \mu_B = \delta = 0$ against the two-sided alternative $\mu_A - \mu_B \neq 0$. Since

$$t=\frac{|\bar{\sigma}_{1A}-\bar{\sigma}_{1B}-\delta|}{\sqrt{\frac{(n_A-1)s_A^2+(n_B-1)s_B^2}{n_A+n_B-2}}\sqrt{\frac{1}{n_A}+\frac{1}{n_B}}}$$

$$=\frac{|133.5-122.5-0|}{\sqrt{\frac{7\times572.9+5\times215.6}{8+6-2}}\sqrt{\frac{1}{8}+\frac{1}{6}}}$$

$=1.0$, which is less than $t_{\alpha/2}(n_A+n_B-2)=t_{0.025}(8+6-2)=2.2$,

hence the null hypothesis must be accepted; or, looking upon the test as a *test of significance*, it can be concluded that the observed differences between the sample means may reasonably be attributed to chance. Hence the strength values obtained in multiple stage failure test with previously existing failure are not significantly different from the strength results where no previous failure exists.

3.1.2 The effect of previous failure on ε_1 and E values

The examination of previous failure effects on peak failure strain ε_1 and modulus E can be done in the same way as with σ_1 above. Referring to Appendix 2, choose the same two groups of specimens: Group A and Group B. The calculated strain and modulus values are listed in Table 3.2

Table 3.2 ε_l and E Values of Two Groups of Test Specimens: with and without Previous Failure.

Group A			Group B			
σ_l (MPa)	ε_l	E (MPa)	σ_{lpr} (MPa)	σ_l (MPa)	ε_l	E (MPa)
13.8	0.01084 (2-E)	12040.9 (2-E)	3.4	13.8	0.02540 (5-H)	11682.3 (5-H)
13.8	0.01849 (5-I)	11523.7 (5-I)	3.4	13.8	0.02112 (1-6)	15806.3 (1-6)
13.8	0.01053 (2-3)	11316.8 (2-3)	3.4	13.8	0.01717 (2-B)	10827.2 (2-B)
13.8	0.03312 (1-E1)	4199.8 (1-E1)	3.4	13.8	0.04057 (5-M)	6496.3 (5-M)
13.8	0.02052 (1-2)	9130.7 (1-2)	3.4	13.8	0.01989 (1-J)	12751.3 (1-J)
13.8	0.02618 (2-A)	10178.9 (2-A)	3.4	13.8	0.02977 (4-G)	8537.6 (4-G)
13.8	0.01720 (1-H)	13509.9 (1-H)				
13.8	0.01398 (3-3)	13806.3 (3-3)				

The calculations of the two sample t tests are given in Appendix 3. According to the t test results, the previous failures in the multiple stage tests do not affect the ε_l and E values significantly as compared to the results without previous failures. The Multiple Failure State Test is therefore a reliable technique.

3.2 Size and Shape Effects on the m and σ_c Values

3.2.1 Height variation effect on the m values

Does the change of the specimen height H have a significant influence on the m value? The following examines the small diameter test data since there are more small diameter test data than the large diameter. The combined influence of different diameters together with the variance of the heights will be analyzed later using two-way analysis of variance. Firstly, examine the height variation effects on the m value using one-way analysis of variance.

Assume there are k groups of independent observations of m values

$$m_{11},\ m_{12},\ \ldots,\ m_{1n_1};\ m_{21},\ \ldots,\ m_{2n_2};\ m_{k_1},\ \ldots,\ m_{kn_k},$$

from normally distributed populations with means μ_1, ..., μ_k, all with the same variance $V[m]$. Thus the model is

$$m_{ij}=\mu_i+\epsilon_{ij};\qquad i=1,\cdots,\ k,\quad j=1,\cdots,\ n_i, \qquad (3.1)$$

where the μ_i are fixed constants and ϵ_{ij} are independent normal deviates with zero mean and variance $V[m]$.

Reparameterize the model (3.1) for convenience. Define μ as the weighted average of the μ_i:

$$\mu=\left(\sum_{i=1}^{k} n_i\right)^{-1}\left(\sum_{i=1}^{k} n_i\mu_i\right)=\frac{\sum_{i=1}^{k} n_i\mu_i}{n},$$

where

$$n=\sum_{i=1}^{k} n_i,$$

and introduce the new parameters δ_i defined as

$$\delta_i=\mu_i-\mu.$$

Then model (3.1) can now be written as:

$$m_{ij}=\mu+\delta_i+e_{ij}.$$

The weighted sum of δ_i is zero:

$$\sum_{i=1}^{k} n_i\delta_i=\sum_{i=1}^{k} n_i(\mu_i-\mu)=\sum_{i=1}^{k} n_i\mu_i-\mu\sum_{i=1}^{k} n_i=0.$$

The null hypothesis which needs to be tested is that the means of the k populations are all equal or, in other words,

$$H_0:\ \delta_i=0, \qquad i=1,\ 2,\ \cdots,\ k.$$

The alternative hypothesis is that the means are not all equal or, in other words,

$$H_A:\ \delta_i\neq 0$$

for at least one value of i. Define the following identities:

$$\overline{m}=\frac{\sum_{i=1}^{k}\sum_{j=1}^{n_i} m_{ij}}{n},$$

$$\overline{m}_{i}.=\frac{1}{n_i}\sum_{j=1}^{n_i} m_{ij},$$

$$S_i=\sum_{j=1}^{n_i} m_{ij}, \qquad SS_i=\sum_{j=1}^{n_i} m_{ij}^2,$$

$$S_T=\sum_{i=1}^{k}\sum_{j=1}^{n_i}(m_{ij}-\overline{m})^2=\sum_{i=1}^{k} SS_i-\frac{\left(\sum_{i=1}^{k} S_i\right)^2}{n},$$

$$S_E=\sum_{i=1}^{k}\sum_{j=1}^{n_i}(m_{ij}-\overline{m}_{i}.)^2=\sum_{i=1}^{k} SS_i-\sum_{i=1}^{k}\frac{S_i^2}{n_i},$$

$$S_B=\sum_{i=1}^{k} n_i(\overline{m}_{i}.-\overline{m})^2=\sum_{i=1}^{k}\frac{S_i^2}{n_i}-\frac{\left(\sum_{i=1}^{k} S_i\right)^2}{n}.$$

where S_T is referred to as the total sum of squares, S_E as the error sum of squares and, the S_B as the between sample sum of squares. It can be easily proven [19] that

$$S_T=S_E+S_B.$$

Therefore S_T, a measure of the total variability of the m, has been decomposed into the sum of two components: the first component, S_E, measures the chance variation (the variability within the samples) regardless of whether the null hypothesis is true; the second component, S_B, measures the chance variation if the null hypothesis is true, but it can be attributed in part to differences among the population means if the null hypothesis is false.

From the experiments, Table 3.3 can be obtained (referring to Appendix 2):

Table 3.3 m Values of Different Specimen Height Groups.

H	0.044	0.048	0.051	0.054	0.056	0.060	0.067	0.068	0.071	Total
m	21.8 (4-E)	19.6 (1-B)	12.7 (5-I)	3.9 (1-G2)	10.2 (1-D)	5.9 (1-E2)	8.4 (1-J)	6.3 (1-H)	13.3 (1-A)	
	10.1 (5-B)	7.5 (1-LB)	16.6 (6-A)	7.0 (4-H)	8.8 (1-E1)	14.0 (2-D)	7.0 (2-E)	14.3 (4-A)	6.5 (1-K)	
	11.2 (5-H)	15.5 (4-F)		19.3 (5-M)		12.0 (4-B)	12.0 (3-A1)			
		13.7 (5-F)								
Σ	43.0	56.3	29.3	30.2	19.0	31.9	27.4	20.6	19.8	277.4

Here, the number of different heights $k = 9$. Under each height the number of tests are $n_1 = 3$, $n_2 = 4$, $n_3 = 2$, $n_4 = 3$, $n_5 = 2$, $n_6 = 3$, $n_7 = 3$, $n_8 = 2$, $n_9 = 2$ respectively. Total $n = 3 + 4 + 2 + 3 + 2 + 3 + 3 + 2 + 2 = 24$. Denote μ_1, μ_2, μ_3, μ_4, μ_5, μ_6, μ_6, μ_7, μ_8, μ_9 as the mean values of m of the k populations respectively corresponding to the specimen heights of H = 0.044, 0.048, 0.051, 0.054, 0.056, 0.060, 0.067, 0.068, 0.071

meter, then examine:

$$H_0: \mu_1=\mu_2=\mu_3=\mu_4=\mu_5=\mu_6=\mu_7=\mu_8=\mu_9$$

The squares of the *m* values in Table 3.3 are shown in Table 3.4 below:

Table 3.4 Squares of the *m* Values of Different Specimen Height Groups.

H	0.044	0.048	0.051	0.054	0.056	0.060	0.067	0.068	0.071	Total
m^2	473.9 (4-E)	383.8 (1-B)	160.3 (5-I)	15.0 (1-G2)	104.2 (1-D)	34.9 (1-E2)	70.6 (1-J)	40.1 (1-H)	176.1 (1-A)	
	101.4 (5-B)	56.3 (1-LB)	275.9 (6-A)	49.4 (4-H)	77.8 (1-E1)	195.4 (2-D)	48.4 (2-E)	203.6 (4-A)	42.6 (1-k)	
	124.8 (5-H)	240.9 (4-F)		371.7 (5-M)		144.0 (4-B)	143.8 (3-A1)			
		187.4 (5-F)								
Σ	700.1	868.3	436.2	436.1	182.0	374.4	262.8	243.7	218.7	3722.3

Hence

$$\sum_{i=1}^{k}\frac{S_i^2}{n_i}=\frac{43.0^2}{3}+\frac{56.3^2}{4}+\frac{29.3^2}{2}+\frac{30.2^2}{3}+\frac{19.0^2}{2}+\frac{31.9^2}{3}$$
$$+\frac{27.4^2}{3}+\frac{20.6^2}{2}+\frac{19.8^2}{2}=3318.6,$$

$$\frac{\left(\sum_{i=1}^{k}S_i\right)^2}{n}=\frac{277.4^2}{24}=3207.0,$$

$$\sum_{i=1}^{k} SS_i = 3722.3$$

$$S_B = \sum_{i=1}^{k} \frac{S_i^2}{n_i} - \frac{\left(\sum_{i=1}^{k} S_i\right)^2}{n} = 3318.6 - 3207.0 = 111.6,$$

$$S_E = \sum_{i=1}^{k} SS_i - \sum_{i=1}^{k} \frac{S_i^2}{n_i} = 403.7,$$

$$S_T = \sum_{i=1}^{k} SS_i - \frac{(\sum_{i=1}^{k} S_i)^2}{n} = S_B + S_E = 515.3.$$

The degrees of freedom are:

$$f_B = k-1 = 9-1 = 8, \quad f_E = n-k = 24-9 = 15, \quad f_T = n-1 = 24-1 = 23.$$

The analysis of variance table is shown as Table 3.5:

Table 3.5 Analysis of Variance for the Specimen Height Effect on the *m* Values.

Sources of variation	Sum of squares	Degrees of freedom	Mean square	F
Between heights	111.6	8	14.0	
Error	403.7	15	26.9	0.5
Total	515.3	23		

Because $F_{0.05}(8, 15) = 2.6$ is greater than 0.5, under the error probability of 0.05, the null hypothesis H_0 is accepted. It may therefore be concluded that different heights of the samples have no significant effect on the m value.

3.2.2 Height variation effect on the σ_c values

A similar statistical analysis of the result has been carried out to examine the influence of height on the σ_c values. The calculations are given in Appendix 4, Section A4.1. It is seen from these calculations that it can be concluded that the σ_c values are not significantly affected by specimen height.

3.2.3 The combined effect of both height and diameter variation on the *m* values

To investigate the effect of k different heights and s different diameters of the specimens on the m value of the mudstone, one specimen was tested for each possible combination of heights and diameters. The testing of ks specimens was randomized, and m_{ij} is obtained with the ith height and jth diameter. A model was set up looking upon m_{ij} as values assumed to be independent random variables having normal distributions with the means μ_{ij} and the variance *V[m]*, where

$$\mu_{ij}=\mu+\beta_i+\gamma_j$$

and

$$\sum_{i=1}^{k}\beta_i=0, \quad \sum_{j=1}^{s}\gamma_j=0. \qquad (3.2)$$

These assumptions can also be specified by writing

$$m_{ij}=\mu+\beta_i+\gamma_j+\epsilon_{ij}, \qquad i=1,\ 2,\ \cdots,\ k; \quad j=1,\ 2,\ \cdots,\ s.$$

where the ε_{ij} are values assumed by independent random variables having normal distributions with zero means and the common variance *V[m]*, while μ, β_i, γ_j, and *V[m]* are unknowns. Note in this model the ***effects*** of the two variables, β_i and γ_j are added to μ; the restrictions in equation (3.2) arise from the fact that it is required for the *mean* of the μ_{ij} for a fixed value of j to equal $\mu + \gamma_j$ and for a fixed value of i to equal $\mu + \beta_i$. The null hypotheses to be tested are:

$$H_{0_1}:\ \beta_1=\beta_2=\cdots=\beta_k=0,$$
$$H_{0_2}:\ \gamma_1=\gamma_2=\cdots=\gamma_s=0.$$

The corresponding alternative hypotheses are that the respective parameters β_i and γ_j are not all equal to zero. The test of these hypotheses are based on the analysis of the total variability of the m data:

$$S_T=\sum_{i=1}^{k}\sum_{j=1}^{s}(m_{ij}-\overline{m})^2.$$

where

$$\overline{m}=\frac{1}{ks}\sum_{i=1}^{k}\sum_{j=1}^{s}m_{ij}.$$

The total variability of m_{ij} can be decomposed into differences among the heights S_H, differences among the diameters S_D, and chance (experimental error) S_E, i.e.

$$S_T=S_H+S_D+S_E.$$

Let

$$S_i.=\sum_{j=1}^{s}m_{ij},\qquad S._j=\sum_{i=1}^{k}m_{ij},$$

$$SS_i.=\sum_{j=1}^{s}m_{ij}^2,\qquad SS._j=\sum_{i=1}^{k}m_{ij}^2,$$

$$S=\sum_{i=1}^{k}S_i.=\sum_{j=1}^{s}S._j=\sum_{j=1}^{s}\sum_{i=1}^{k}m_{ij},$$

$$SS=\sum_{i=1}^{k} SS_{i\cdot}=\sum_{j=1}^{s} SS_{\cdot j}=\sum_{i=1}^{k}\sum_{j=1}^{s} m_{ij}^2.$$

With k and s chosen to be 2 respectively in the experiment, corresponding m values were obtained as shown in the following table:

Table 3.6 m Values of Different Specimen Height and Diameter Combinations.

Specimen diameter	(*meter*)	0.029	0.053	$S_{i\cdot}$	S
Height (*meter*)	0.060	14.0 (2-D)	14.4 (1-6)	28.4	
	0.075	6.1 (3-C)	9.2 (2-7)	15.3	
	$S_{\cdot j}$	20.1	23.6		43.7

The squares of the m values in Table 3.6 are shown in Table 3.7

Table 3.7 Squares of the m Values of Different Specimen Height and Diameter Combinations.

Specimen diameter	(*meter*)	0.029	0.053	$SS_{i\cdot}$	SS
Height (*meter*)	0.060	195.4 (2-D)	207.9 (1-6)	403.4	
	0.075	37.1 (3-C)	84.3 (2-7)	121.4	
	$SS_{\cdot j}$	232.5	292.2		524.7

With the following calculation:

$$S_H=\frac{1}{s}\sum_{i=1}^{k} S_{i.}^{2}-\frac{1}{ks}S^2=\frac{1}{2}\times(28.4^2+15.3^2)-\frac{1}{2\times2}\times43.7^2=43.1,$$

$$S_D=\frac{1}{k}\sum_{j=1}^{s} S_{.j}^{2}-\frac{1}{ks}S^2=\frac{1}{2}(20.1^2+23.6^2)-\frac{1}{2\times2}\times43.7^2=3.1,$$

$$S_T=SS-\frac{1}{ks}S^2=524.7-\frac{1}{2\times2}\times43.7^2=48.0,$$

$$S_E=SS-\frac{1}{s}\sum_{i=1}^{k} S_{i.}^{2}-\frac{1}{k}\sum_{j=1}^{s} S_{.j}^{2}+\frac{1}{ks}S^2$$
$$=S_T-(S_A+S_B)=48.0-(43.1+3.1)=1.8,$$

The analysis of variance table for this two-way analysis is presented as Table 3.8:

Table 3.8 Analysis of Variance for the Effect of Different Height and Diameter Combinations on the *m* Values.

Sources of variation	Sum of squares	Degrees of freedom	Mean square	*F*
Between heights	43.1	1	43.1	13.8
Between diameters	3.1	1	3.1	1.8
Error	1.8	1	1.8	
Total	48.0	3		

Since $F_{0.05}(1, 1) = 161.4$ is greater than $F_H = 13.81$ and $F_{0.05}(1, 1) = 161.4$ is greater than $F_D = 1.8$, neither of the null hypotheses can be rejected. It can be concluded that the *m* value of the mudstone is *not* affected by the combinations of different heights and different diameters.

3.2.4 The combined effect of both height and diameter variation on the σ_c values

A similar analysis was carried out for the σ_c data and is reported in Appendix 4, Section A4.2. It is shown in this appendix that the σ_c values are not significantly affected by the combined effect of height and diameter variances.

The overall conclusion can therefore be drawn that neither size nor shape, nor their different combinations, of the specimens have a significant effect on the *m* and σ_c values (to a 95% confidence level).

3.3 Time Effect on the m and σ_c Values

All the test specimens were drilled within a few days of each other; however, the tests were performed over a period from July to October 1989. Although the specimens were carefully wrapped and stored, nevertheless it is conceivable that the properties (m and σ_c) might have changed with time due, for example, to change in moisture content over this period.

Now, because the specimen testing was fully randomized, it is possible to examine whether time of testing had any significant influence on the test results.

These analyses were detailed in Appendix 4, Section A4.3 and A4.4. The conclusion was that the time of testing had no significant influence on the m values (at 95% confidence level) and the σ_c values appear to have been influenced by time at a 95% confidence level. However, at a 97.5% confidence level of not committing the *Type I* error, the time influence on the σ_c values can be neglected according to the statistical test.

3.4 Regression Analysis on All the σ_1 and σ_3 Data

Since it has been proven above that there is no size or shape effect on the experimental data, all the σ_1 and σ_3 data can now be lumped together and used to calculate σ_c and m. The peak failure stresses at various confining pressures were plotted in Figure 3.1.

According to Hoek and Brown [1] the empirical criterion is given by equation

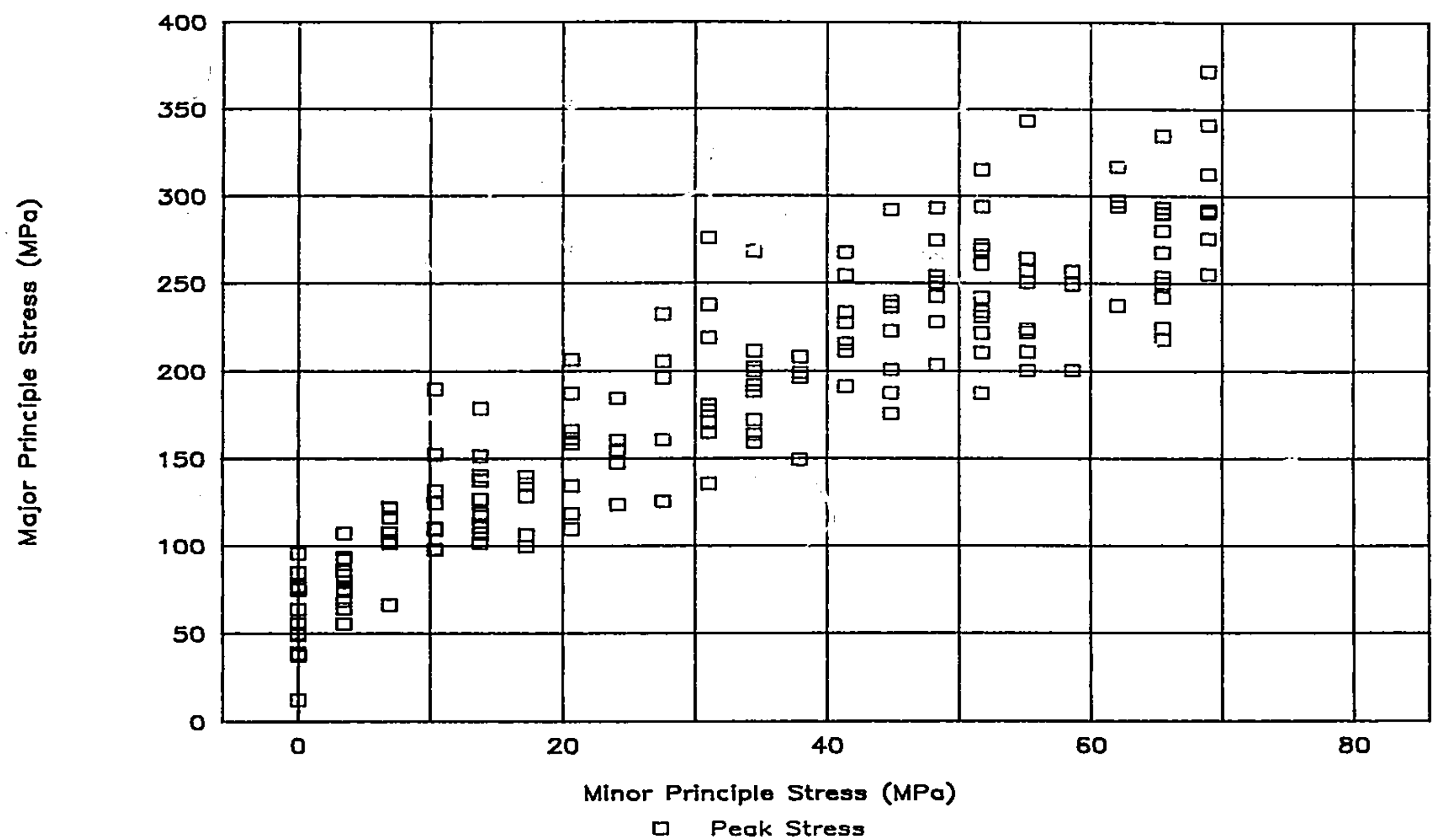


Figure 3.1 Peak Failure Stresses at Different Confining Pressures for Mudstone.

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_c . \sigma_3 + s\sigma_c^2}$$

which may be rewritten as:

$$y = m\sigma_c . x + s\sigma_c^2 \qquad (3.3)$$

where

$$y = (\sigma_1 - \sigma_3)^2, \qquad x = \sigma_3 .$$

From the experiment, there are now available paired data

$$(x_1,\ y_1),\ (x_2,\ y_2),\ \ldots,\ (x_i,\ y_i),\ \ldots,\ (x_n,\ y_n),$$

where $n = 161$ denotes the total number of such data pairs. Because values of x_i were selected by the experimenter, they are considered constants. On the other hand, values of y_i were observed at those values of x_i, they are assumed by independent random variables having the conditional probability densities

$$f(y_i|x_i) = \frac{1}{\sqrt{2\pi V[y_i]}} e^{-\frac{1}{2}\left(\frac{y_i-(m\sigma_c . x+s\sigma_c^2)}{V[y_i]}\right)^2}, \qquad i=1,2,\ldots,n.$$

for $-\infty < y_i < \infty$; i.e. y_i is distributed normally about an expected value y with variance $V[y_i]$, and that all observations are independent. The mudstone can be regarded as intact rock, so $s = 1$. Therefore linear regression analysis can be used to obtain, from the data, sample estimates $\hat{\sigma}_c$, $\hat{m}$, and $\hat{V}[y_i]$ of σ_c, m and $V[y_i]$ and to determine the distribution of these estimates. The estimated regression equation is

$$\hat{y}=\hat{m}\hat{\sigma}_c\cdot x+s\hat{\sigma}_c^2.$$

The maximum likelihood estimates for the uniaxial compressive strength σ_c and the material constant m are given by:

$$\hat{\sigma}_c^2=\frac{\sum_{i=1}^{n} y_i}{n}-\left[\frac{\sum_{i=1}^{n} x_i y_i-\frac{\sum_{i=1}^{n} x_i\sum_{i=1}^{n} y_i}{n}}{\sum_{i=1}^{n} x_i^2-\frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}\right]\frac{\sum_{i=1}^{n} x_i}{n}=4469.6\ (MPa)^2,$$

$$\hat{\sigma}_c=66.9\ (MPa),$$

$$\hat{m}=\frac{1}{\hat{\sigma}_c}\left[\frac{\sum_{i=1}^{n} x_i y_i-\frac{\sum_{i=1}^{n} x_i\sum_{i=1}^{n} y_i}{n}}{\sum_{i=1}^{n} x_i^2-\frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}\right]=10.0.$$

The sample correlation coefficient r is given by:

$$r=\frac{n\cdot\sum_{i=1}^{n}x_iy_i-\left(\sum_{i=1}^{n}x_i\right)\left(\sum_{i=1}^{n}y_i\right)}{\sqrt{n\cdot\sum_{i=1}^{n}x_i^2-\left(\sum_{i=1}^{n}x_i\right)^2}\sqrt{n\cdot\sum_{i=1}^{n}y_i^2-\left(\sum_{i=1}^{n}y_i\right)^2}}=0.79$$

where r is a measure of the strength of the linear relationship between x and y. The closer the value of r is to 1.00, the better the fit of the empirical equation to the triaxial test data. The so called coefficient of determination r^2 is therefore 0.62.

The unbiased estimate $\hat{V}[y_i]$ of $V[y_i]$, which is the variance of the variable y_i, is:

$$\hat{V}[y_i]=\frac{1}{n-2}\sum_{i=1}^{n}(y_i-\hat{y}_i)^2=1.3\times10^8\ (MPa)^4.$$

The maximum likelihood estimator $\widehat{m\sigma}_c$ has a normal distribution with an expectation

$$E(\widehat{m\sigma}_c)=m\sigma_c,$$

and a variance

$$V[\widehat{m\sigma}_c]=\frac{\hat{V}[y_i]}{\sum_{i=1}^{n}(x_i-\bar{x})^2}.$$

The estimate of which can be obtained from

$$\hat{V}[\hat{m}\hat{\sigma}_c] = \frac{\hat{V}[y_i]}{\sum_{i=1}^{n} (x_i-\bar{x})^2} = \frac{1.3\times10^8}{75947.4} = 1711.7 \ (MPa)^2$$

The estimate $\hat{\sigma}_c^2$ has a normal distribution with an expectation

$$E(\hat{\sigma}_c^2) = \sigma_c^2,$$

and a variance

$$V[\hat{\sigma}_c^2] = \frac{V[y_i]\left(\frac{\sum_{i=1}^{n} (x_i-\bar{x})^2}{n} + \bar{x}^2\right)}{\sum_{i=1}^{n} (x_i-\bar{x})^2} = \frac{1.3\times10^8\times\left(\frac{75947.4}{161} + 32.3^2\right)}{75947.4}$$

$$= 2.6\times10^6 \ (MPa)^4.$$

where

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i.$$

To test whether the assumed model (3.3) is appropriate or not, carry out a test of the null hypothesis

$$H_0: \ m\sigma_c = 0.$$

This null hypothesis is equivalent to the null hypothesis that the correlation coefficient is zero. Since $\hat{m}\hat{\sigma}_c$ and $\hat{V}[y_i]$ are independent of each other, therefore:

$$\frac{(\hat{m}\hat{\sigma}_c - m\sigma_c)}{\sqrt{\frac{\hat{V}[y_i]}{V[y_i]}}\sqrt{\frac{V[y_i]}{\sum_{i=1}^{n}(x_i-\bar{x})^2}}}$$

has the t distribution with n - 2 degrees of freedom. That is,

$$\frac{\hat{m}\hat{\sigma}_c - m\sigma_c}{\sqrt{\hat{V}[y_i]}}\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2} \sim t(n-2),$$

Considering the null hypothesis, since

$$\frac{|\hat{m}\hat{\sigma}_c|}{\sqrt{\hat{V}[y_i]}}\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2} = \frac{10.0\times 66.6}{11412.1}\sqrt{75947.4} = 16.1,$$

and

$$t_{\alpha/2}(n-2) = t_{0.025}(161-2) \approx z_{0.025} = 2.0,$$

which is less than 16.1, the null hypothesis must be rejected. In other words, there is a significant correlation between the y and x from which the sample was obtained.

The Hoek-Brown failure criterion for the mudstone can therefore be written as

$$\sigma_1=\sigma_3+\sqrt{10.0\times66.9\sigma_3+4469.6s}$$

Considering that $s = 1$ in this experiment, this Hoek-Brown failure criterion curve was plotted in Figure 3.2, together with the test data.

3.5 Linear Regression Analysis on All the σ_r and σ_3 Data

3.5.1 The estimates of parameters and significance of regression

The residual strength values σ_{ri} corresponding to various confining pressures from the experiment were plotted in Figure 3.3. Assume σ_{ri} is distributed normally about an expected value σ_r with variance $V[\sigma_{ri}]$, and that all observations are independent. Moreover, σ_r is assumed to be a linear function of σ_3:

$$\sigma_r=\alpha+\beta(\sigma_3-\overline{\sigma}_3) . \qquad (3.4)$$

The estimated regression equation is

$$\hat{\sigma}_r=a+b(\sigma_3-\overline{\sigma}_3) . \qquad (3.5)$$

where

$$\overline{\sigma}_3=\sum_{i=1}^{k}\frac{\sigma_{3_i}}{k} .$$

Equations (3.4) and (3.5) were written in the forms given instead of as

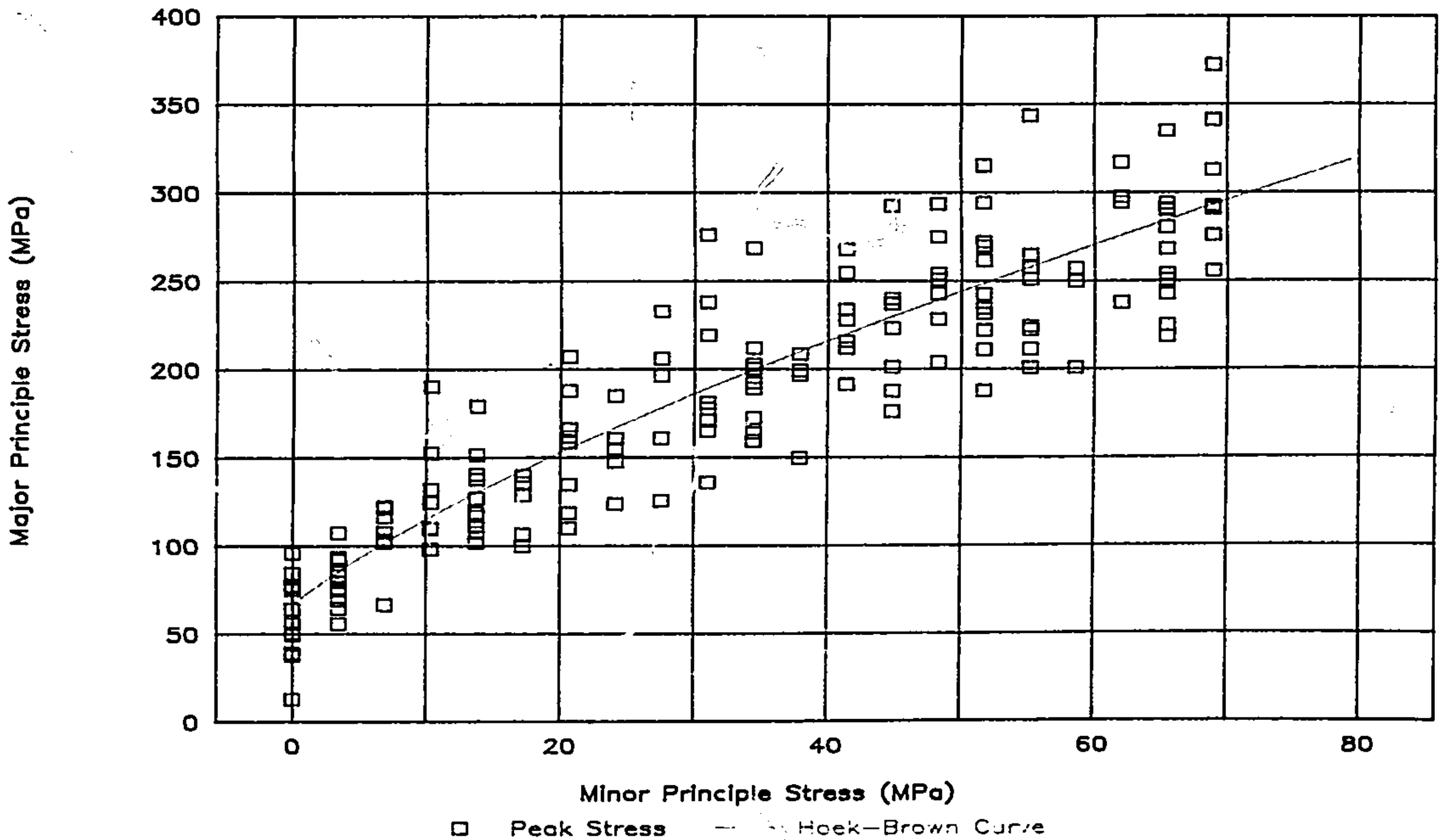


Figure 3.2 Peak Stress Experiment Data and Hoek-Brown Failure Criterion for Mudstone.

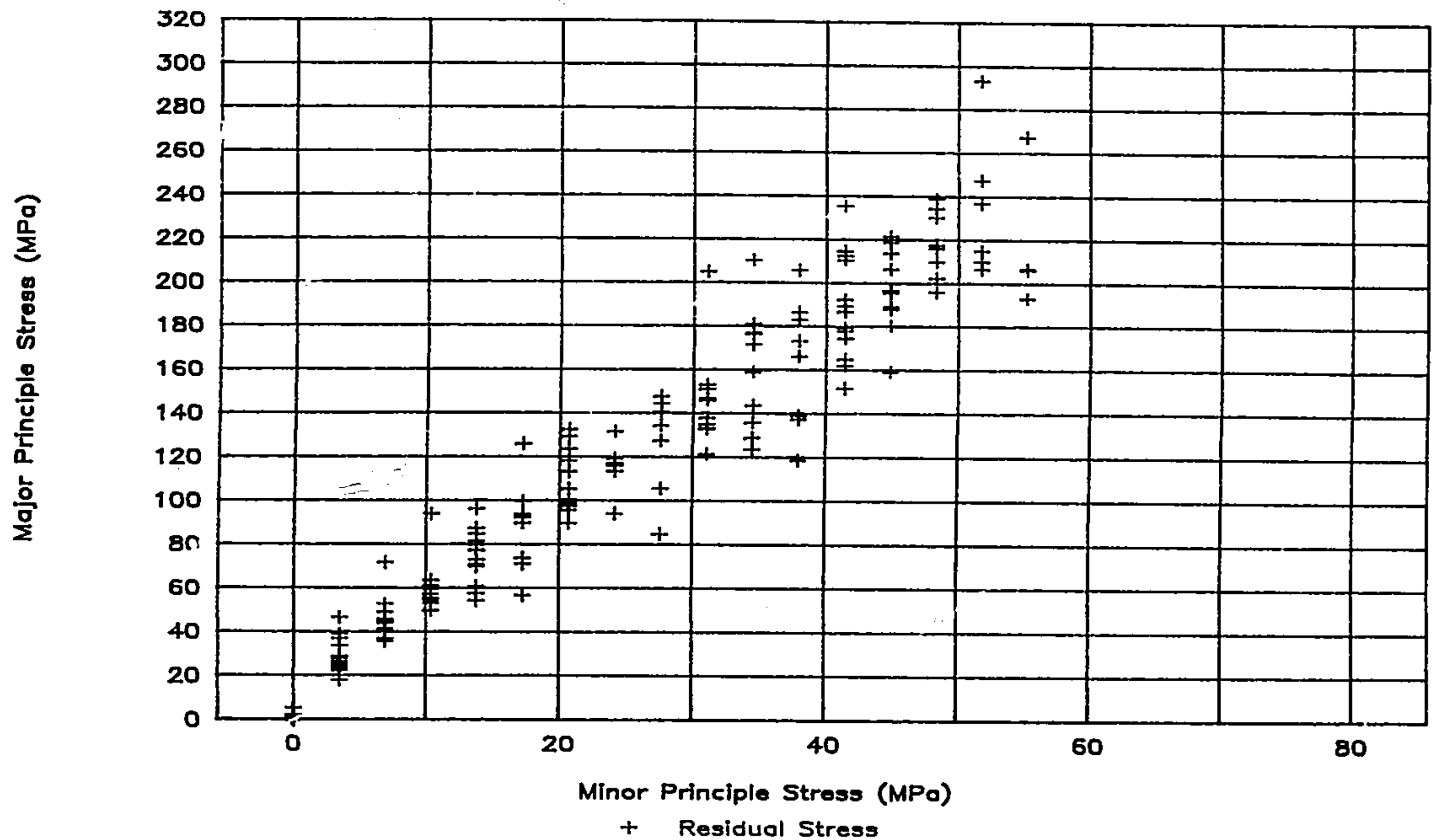


Figure 3.3 Residual Stresses at Different Confining Pressures for Mudstone.

$$\sigma_r=\beta\sigma_3+\alpha', \qquad \hat{\sigma}_r=b\sigma_3+a'$$

because in this latter form a′, b are dependent whereas in the form (3.5) a and b are independent. This property is convenient when considering $V[\hat{\sigma}_r]$, the variance of $\hat{\sigma}_r$.

From the experiments there are k = 165 pairs of σ_{ri}, σ_{3i}, therefore:

$$\sum_{i=1}^{k}\sigma_{3_i}=3989.5\ (MPa), \qquad \sum_{i=1}^{k}\sigma_{r_i}=18929.5\ (MPa),$$

$$\sum_{i=1}^{k}(\sigma_{3_i})^2=142902.8\ (MPa)^2, \qquad \sum_{i=1}^{k}(\sigma_{r_i})^2=3043214.1\ (MPa)^2,$$

$$\sum_{i=1}^{k}\sigma_{3_i}\sigma_{r_i}=651739.1\ (MPa)^2.$$

from which it can be derived

$$\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)^2=\sum_{i=1}^{k}(\sigma_{3_i})^2-\frac{\left(\sum_{i=1}^{k}\sigma_{3_i}\right)^2}{k}=142902.8-\frac{3989.5^2}{165}=46441.0,$$

$$\sum_{i=1}^{k}(\sigma_{r_i}-\overline{\sigma}_r)^2=\sum_{i=1}^{k}(\sigma_{r_i})^2-\frac{\left(\sum_{i=1}^{k}\sigma_{r_i}\right)^2}{k}$$

$$=3043214.1-\frac{18929.5^2}{165}=871550.7,$$

$$\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)(\sigma_{r_i}-\overline{\sigma}_r)=\sum_{i=1}^{k}\sigma_{3_i}\sigma_{r_i}-\frac{\sum_{i=1}^{k}\sigma_{3_i}\sum_{i=1}^{k}\sigma_{r_i}}{k}$$

$$=651739.1-\frac{3989.5\times18929.5}{165}=194046.6$$

The sum of squares due to regression is:

$$\sum_{i=1}^{k}(\hat{\sigma}_{r_i}-\overline{\sigma}_r)^2=\frac{\left[\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)(\sigma_{r_i}-\overline{\sigma}_r)\right]^2}{\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)^2}=\frac{194046.6^2}{46441.0}=810793.0,$$

$$\sum_{i=1}^{k}(\sigma_{r_i}-\hat{\sigma}_{r_i})^2=\sum_{i=1}^{k}(\sigma_{r_i}-\overline{\sigma}_r)^2-\frac{\left[\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)(\sigma_{r_i}-\overline{\sigma}_r)\right]^2}{\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)^2}$$

$$=871550.7-810793.0=60757.7$$

The sample correlation coefficient

$$r=\frac{k\cdot\sum_{i=1}^{k}\sigma_{3_i}\sigma_{r_i}-\left(\sum_{i=1}^{k}\sigma_{3_i}\right)\left(\sum_{i=1}^{k}\sigma_{r_i}\right)}{\sqrt{k\cdot\sum_{i=1}^{k}(\sigma_{3_i})^2-\left(\sum_{i=1}^{k}\sigma_{3_i}\right)^2}\sqrt{k\cdot\sum_{i=1}^{k}(\sigma_{r_i})^2-\left(\sum_{i=1}^{k}\sigma_{r_i}\right)^2}}$$

$$=\frac{165\times651739.1-3989.5\times18929.5}{\sqrt{165\times142902.8-3989.5^2}\sqrt{165\times3043214.1-18929.5^2}}=0.96$$

Now assemble Table 3.9, finding the residual sum of squares by difference.

Table 3.9 Analysis of Variance for the Significance of Regression of σ_r against σ_3.

Source of variance	Sums of squares	Degrees of freedom	Mean squares	F
Due to regression	810793.0	1	810793.0	2174.9
Residual	60757.7	163	372.8	
Total	871550.7	164		

Under the null hypothesis $\beta = 0$ the variance ratio 2174.9 is distributed as $F(1, 163)$. Hence the result is obviously significant because 2174.9 is greater than $F_{0.05}(1, 163) = 3.8$ and the null hypothesis rejected.

An alternative test would be to use the fact that

$$\frac{b-\beta}{\sqrt{\hat{V}[b]}}\sim t(k-2),$$

for which

$$b=\frac{\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)(\sigma_{r_i}-\overline{\sigma}_r)}{\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)}=\frac{194046.6}{46441.0}=4.18,$$

Since the variance of b

$$V[b]=V\left[\frac{\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)\sigma_{r_i}}{\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)^2}\right]=\frac{\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)^2V[\sigma_{r_i}]}{\left[\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)^2\right]^2}=\frac{V[\sigma_{r_i}]}{\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)^2},$$

To calculate *V[b]*, substitute

$$V[\sigma_{r_i}]=\hat{V}[\sigma_{r_i}]=\frac{1}{k-2}\sum_{i=1}^{k}(\sigma_{r_i}-\hat{\sigma}_{r_i})^2. \qquad (3.6)$$

therefore

$$\hat{V}[b]=\frac{\hat{V}[\sigma_{r_i}]}{\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)^2}=\frac{372.8}{46441.0}=0.008.$$

A test of the null hypothesis that $\beta = 0$ is given by

$$\frac{b-0}{\sqrt{V[b]}}=\frac{4.18-0}{\sqrt{0.008}}=46.65,$$

which will be distributed as $t(163)$. Obviously the null hypothesis has to be rejected. Of course, $t^2 = 46.65^2 = 2174.9 = F$ of the previous test. In this experiment there is no reason to doubt the significance of the regression. It is possible to construct 95 per cent confidence limits for β; for these,

$$t_{0.025}(163)=2.0, \quad \sqrt{V[b]}=0.1, \quad t\sqrt{V[b]}=0.2,$$

and the 95 per cent confidence limits are 4.2 ± 0.2 or (4.0, 4.4).

Since

$$\bar{\sigma}_r=\frac{18929.5}{165}=114.7 \ (MPa), \qquad \bar{\sigma}_3=\frac{3989.5}{165}=24.2 \ (MPa), \qquad b=4.$$

the estimated regression line can constructed as

$$\hat{\sigma}_r=114.7+4.2(\sigma_3-24.2)$$

or

$$\hat{\sigma}_r=4.2\sigma_3+13.7 \qquad (3.7)$$

At $\sigma_3 = 0$, $\hat{\sigma}_r = 13.7$ (MPa). Because

$$V[a]=V\left[\frac{\sum_{i=1}^{k}\sigma_{r_i}}{k}\right]=\frac{1}{k^2}\sum_{i=1}^{k}V[\sigma_{r_i}]=\frac{V[\sigma_{r_i}]}{k},$$

$$V[\hat{\sigma}_r]=V[a+b(\sigma_3-\overline{\sigma}_3)]=V[a]+(\sigma_3-\overline{\sigma}_3)^2V[b]$$

$$=V[\sigma_{r_i}]\left[\frac{1}{k}+\frac{(\sigma_3-\overline{\sigma}_3)^2}{\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)^2}\right].$$

Therefore when $\sigma_3 = 0$

$$\hat{V}[\hat{\sigma}_r]=372.8\left[\frac{1}{165}+\frac{(0-24.2)^2}{46441.0}\right]=7.0\ (MPa)^2.$$

Under the null hypothesis that $\sigma_r = 0$ when $\sigma_3 = 0$, the ratio

$$\frac{(\hat{\sigma}_r)_{\sigma_3=0}-0}{\sqrt{\hat{V}[\hat{\sigma}_r]_{\sigma_3=0}}}=\frac{13.7-0}{\sqrt{7.0}}=5.2$$

is distributed as $t(163)$: clearly the null hypothesis of a zero intercept has to be rejected. The reason for non-zero intercept may be attributed to the fact that the specimen could still support some axial load at zero confining pressure even after it has virtually failed because the plastic sleeve was holding the parts of specimen together. The 95% confidence limits for the intercept on the σ_r axis are

$$13.7 \pm 2.0\sqrt{7.0} = (8.5,\ 18.8).$$

The estimated variance of an observation around its true value is

$$\hat{V}[\sigma_{r_i}] = 372.8\ (MPa)^2,$$

corresponding to a standard deviation of 19.3 *MPa*. The residual strength equation (3.7) and the test points were plotted in Figure 3.4.

3.5.2 Prediction for the residual strength

For a specified σ_3, the corresponding observation of σ_r is independent of $\hat{\sigma}_r$. Therefore, given

$$\sigma_3 = \sigma_{3_0},$$

then

$$\sigma_{r_0} = \alpha + \beta(\sigma_{3_0} - \overline{\sigma}_3),$$

$$\hat{\sigma}_{r_0} = a + b(\sigma_{3_0} - \overline{\sigma}_3).$$

Because

$$E[\sigma_{r_0} - \hat{\sigma}_{r_0}] = E[\sigma_{r_0}] - E[\hat{\sigma}_{r_0}] = 0,$$

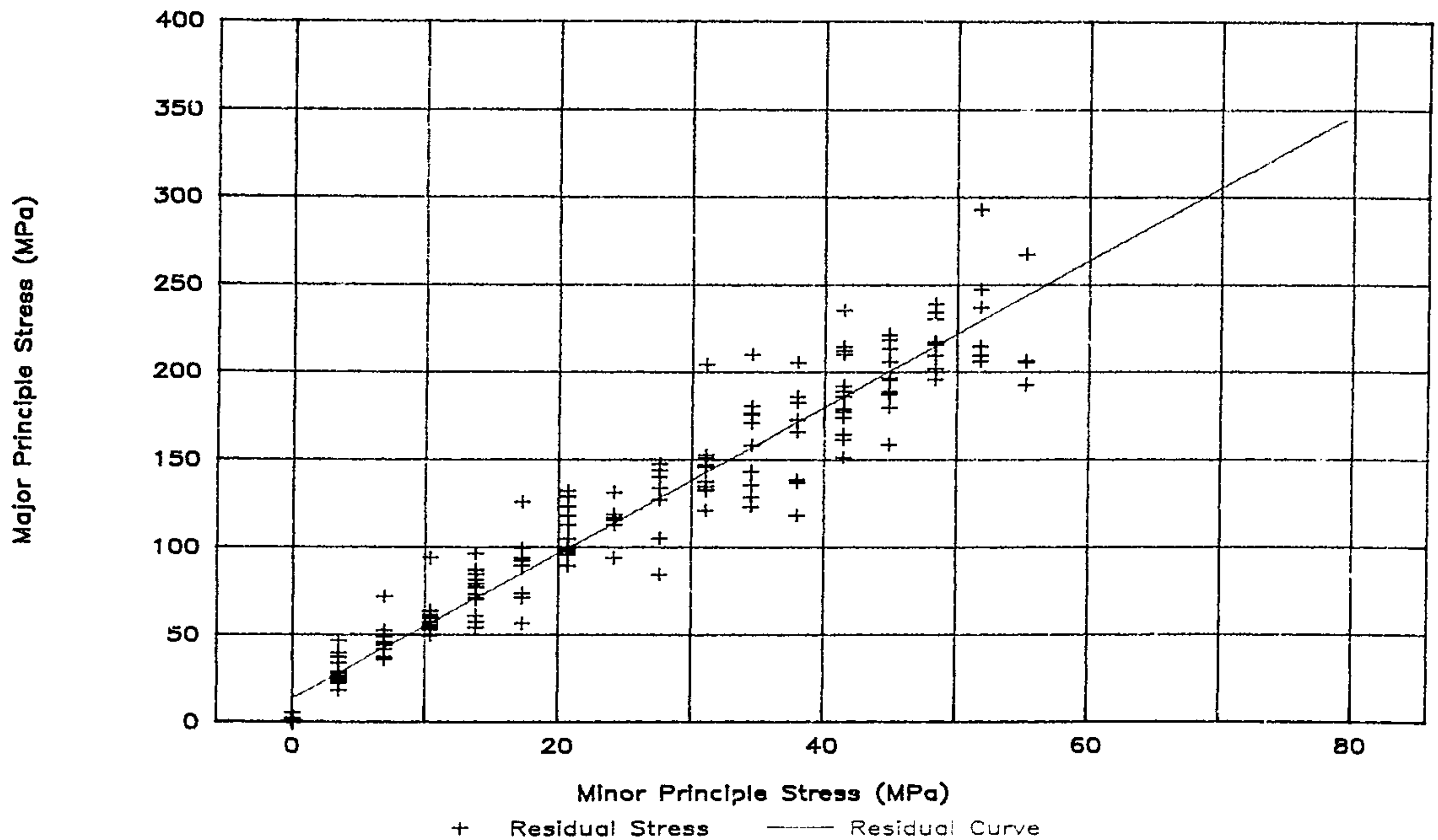


Figure 3.4 Residual Strength Curve and Experiment Data for Mudstone.

$$V[\sigma_{r_0}-\hat{\sigma}_{r_0}]=V[\sigma_{r_0}]+V[\hat{\sigma}_{r_0}]=V[\sigma_{r_i}]\left[1+\frac{1}{k}+\frac{(\sigma_3-\overline{\sigma}_3)^2}{\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)^2}\right].$$

and making the substitution of equation (3.6), then

$$\frac{\sigma_{r_0}-\hat{\sigma}_{r_0}}{\sqrt{\hat{V}[\sigma_{r_0}-\hat{\sigma}_{r_0}]}}=\frac{\sigma_{r_0}-\hat{\sigma}_{r_0}}{\hat{V}[\sigma_{r_i}]\sqrt{1+\frac{1}{k}+\frac{(\sigma_{3_0}-\overline{\sigma}_3)^2}{\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)^2}}}$$

has t distribution with (k-2) degrees of freedom. Hence the 95% confidence prediction interval for the observation at this given point is

$$\left(\hat{\sigma}_{r_0}\pm t_{0.025}(k-2)\,\hat{V}[\sigma_{r_i}]\sqrt{1+\frac{1}{k}+\frac{(\sigma_{3_0}-\overline{\sigma}_3)^2}{\sum_{i=1}^{k}(\sigma_{3_i}-\overline{\sigma}_3)^2}}\right). \qquad (3.8)$$

This interval differs from the confidence interval in that the latter is an interval for a parameter while the former represents the limits between which it is 95% confident that the new single observation at the given point will lie. Denote (3.8) as

$$(\hat{\sigma}_{r_0}\pm\delta(\sigma_{3_0})).$$

According to the experiment data, plot the two curves

$$\sigma_{r_1}=\hat{\sigma}_r-\delta\,(\sigma_3)\ ,$$

and

$$\sigma_{r_2}=\hat{\sigma}_r+\delta\,(\sigma_3)\ ,$$

together with data points and the regression line in one figure (Fig. 3.5). The upper and lower limit curves formed a band containing the regression line of equation (3.7). At the place where σ_3 takes its mean value, the band is the narrowest.

3.6 Transition Point Between Brittle and Ductile Failure

The point of transition between brittle fracture and pseudo-ductile yield is given by the point of intersection of the curves for the peak stress and residual stress plotted against confining pressure. Figure 3.6 shows the test data, the Hoek-Brown curve and the residual stress curve obtained above from the experiment data using regression analysis. As illustrated in the figure, the transition point, σ_{3T} = 64 MPa, σ_{1T} = 270 MPa, occurs at the intersection of these two curves.

Therefore, when evaluating m and s from triaxial test data, care should be taken to try to distinguish at what stress level brittle-ductile transition occurs and the data above this transition point should not be used in the evaluation of the parameters.

Based on this, discard those data above the transition point in Figure 3.6 and re-

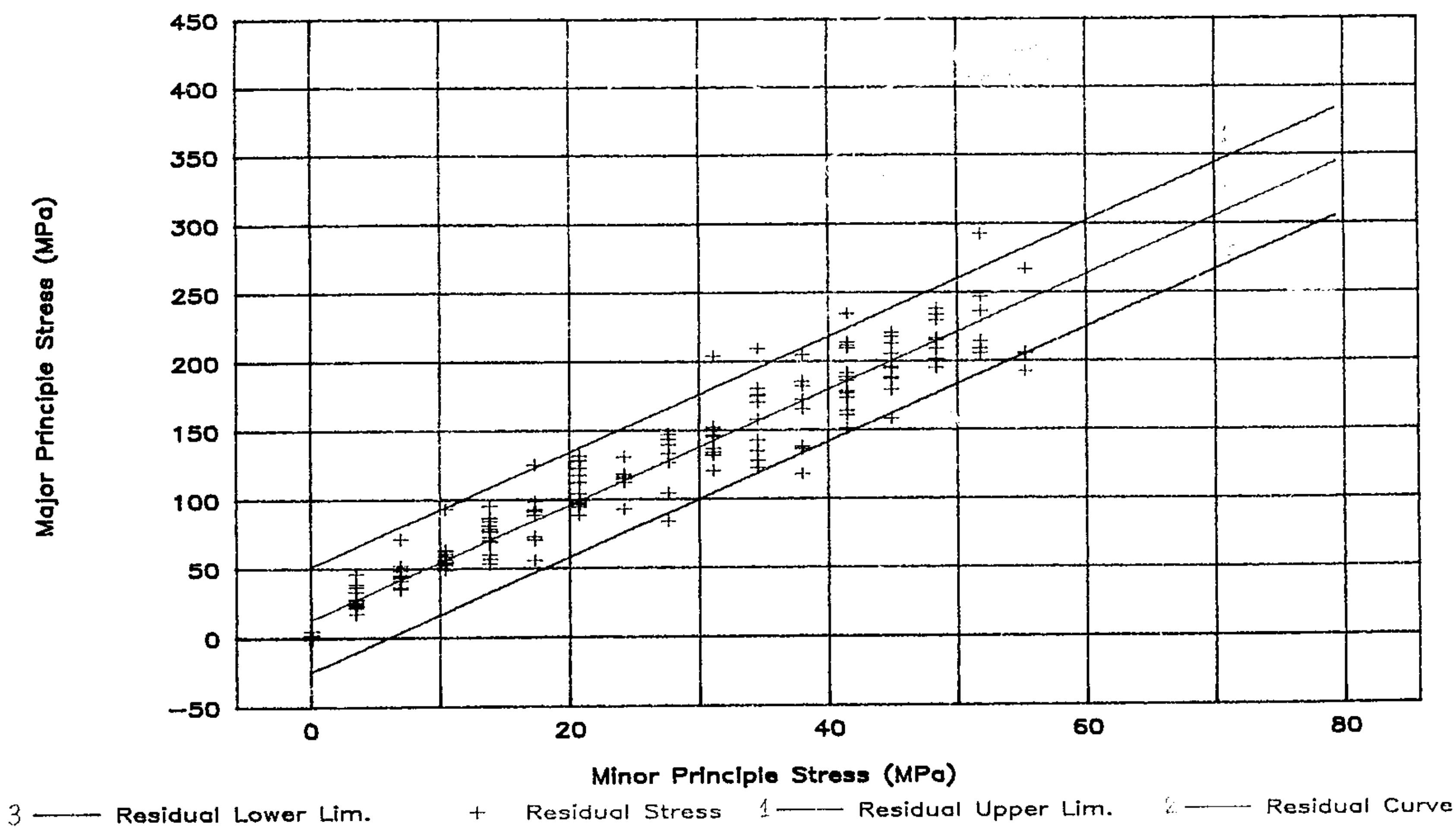


Figure 3.5 The 95% Confident Prediction Interval for Residual Strength Variable of Mudstone.

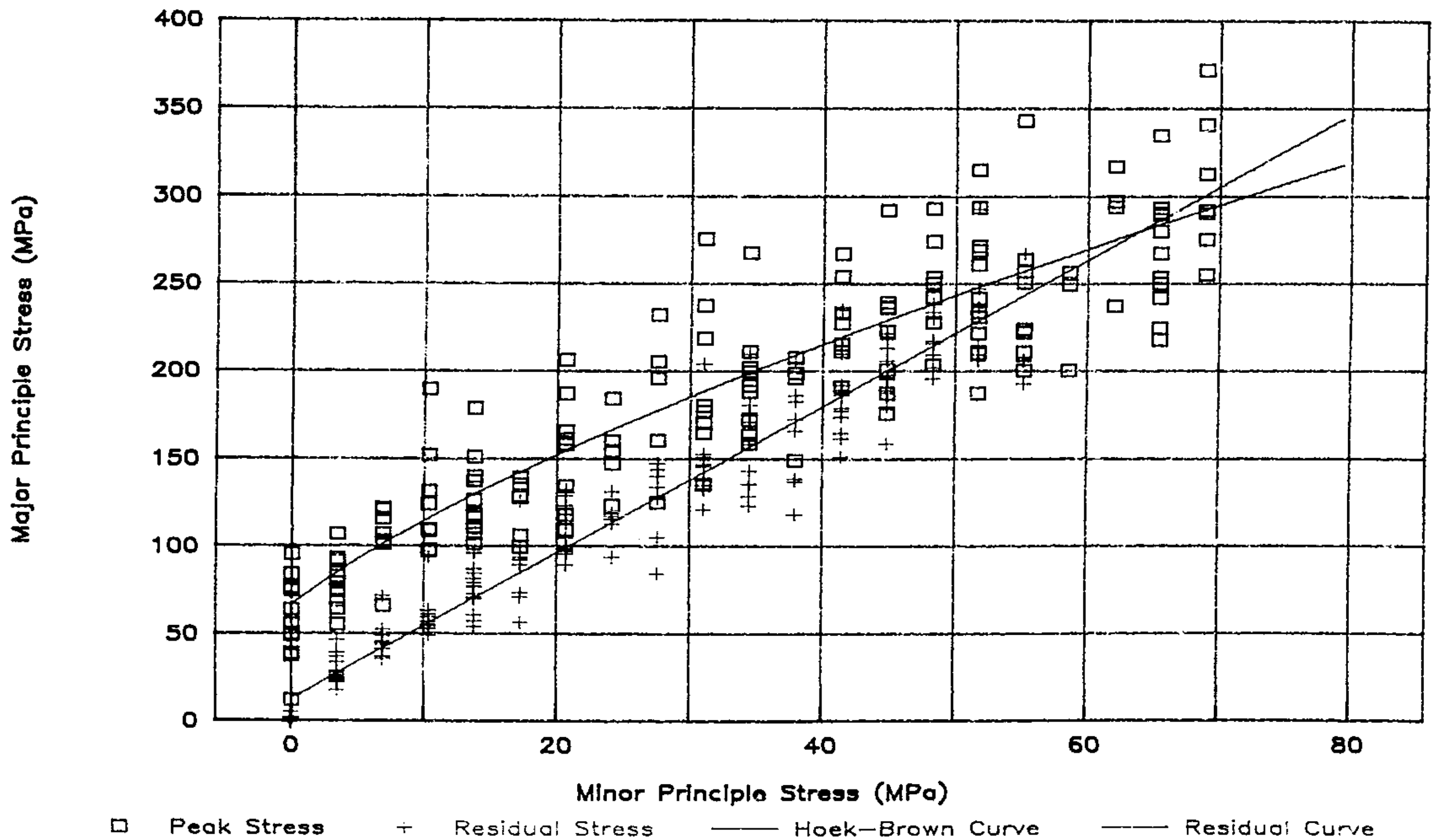


Figure 3.6 Transition from Brittle to Ductile Failure for Mudstone.

do all the analyses.

There are now n = 143 data pairs of (σ_1, σ_3) left. Similar to the steps in Section 3.4, obtain

$$\hat{\sigma}_c=65.9\ (MPa),\quad \hat{m}=10.3,\quad r^2=0.60,\quad r=0.78$$

$$\hat{V}[y_i]=\frac{1}{n-2}\sum_{i=1}^{n}(y_i-\hat{y}_i)^2=\frac{1}{141}\sum_{i=1}^{143}(y_i-\hat{y}_i)=13,637,237.8\ (MPa)^4.$$

$$\frac{|\hat{m}\hat{\sigma}_c|}{\hat{V}[y_i]}\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2}=\frac{10.3\times 65.9}{3692.9}\sqrt{5.2\times 10^4}=41.6.$$

Since $t_{0.025}(143\text{-}2) \approx z_{0.025} = 2.0$ is less than 41.6, the null hypothesis H_0: $m\sigma_c = 0$ must be rejected. That is, there is a significant correlation between the y and x.

Now use the new Hoek-Brown equation

$$\sigma_1=\sigma_3+\sqrt{10.3\times 65.9\sigma_3+65.9^2 s},$$

noticing s = 1, i.e.

$$\sigma_1=\sigma_3+\sqrt{10.3\times 65.9\sigma_3+65.9^2}, \qquad (3.9)$$

and plot the transition from brittle to ductile failure as Figure 3.7, as compared with Figure 3.6. From the figure, it can be seen that the new estimate of the transition point is σ_{3T} = 64 MPa, σ_{1T} = 280 MPa.

Similar to equation (3.8) in Section 3.5, given any fixed σ_3, the 95% confident

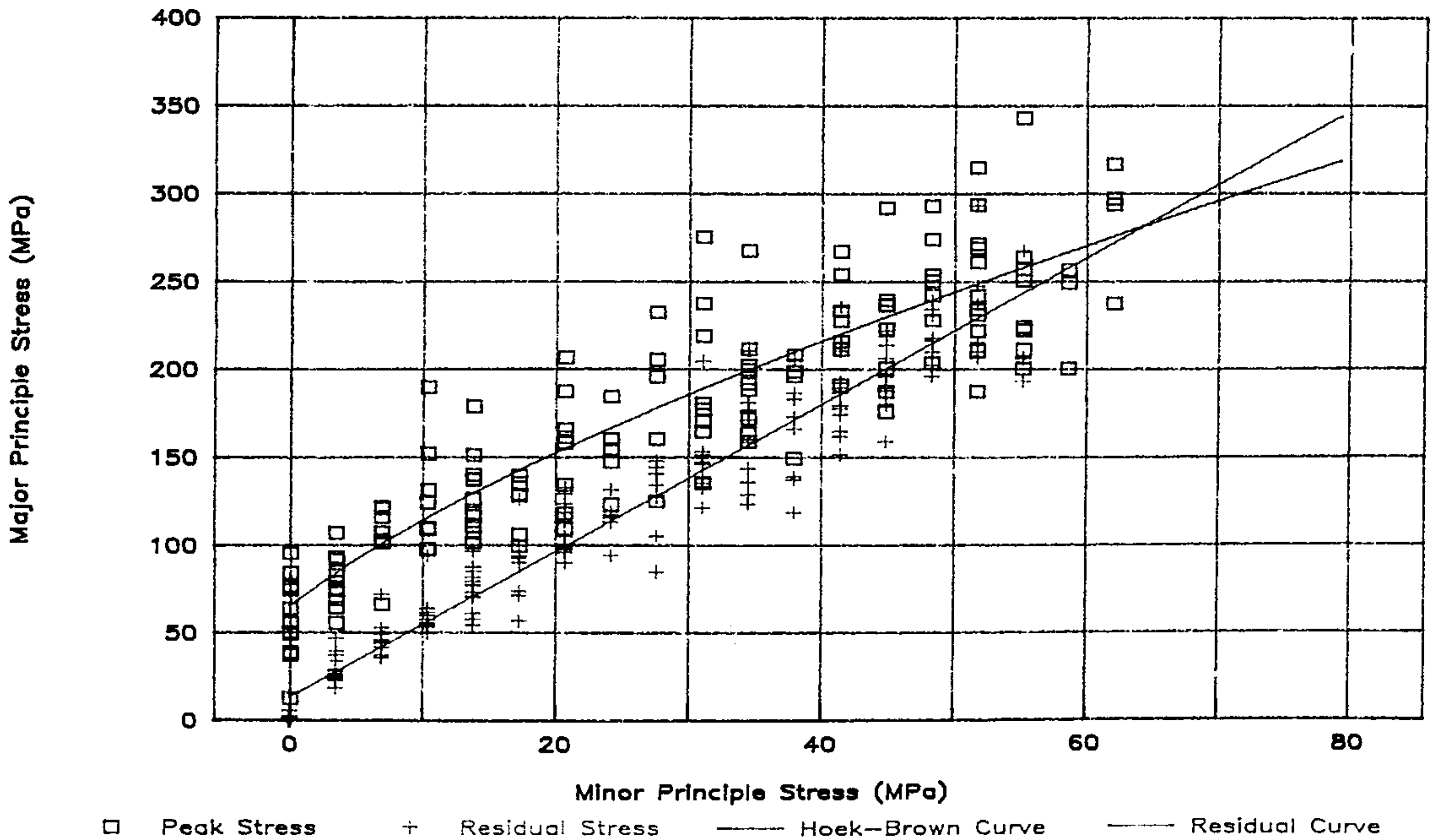


Figure 3.7 New Plot of Transition from Brittle to Ductile Failure for Mudstone.

prediction interval for σ_1 is

$$\left(\sigma_3+\sqrt{m\hat{\sigma}_c\sigma_3+s\hat{\sigma}_c^2\pm t_{0.025/2}(n-2)\sqrt{\hat{V}[y_i]}\sqrt{1+\frac{1}{n}+\frac{(\sigma_3-\overline{\sigma}_3)^2}{\sum_{i=1}^{n}(\sigma_{3_i}-\overline{\sigma}_3)^2}}}\right)$$

i.e., the lower and upper limit curves of the prediction interval are

$$\sigma_{1_{LOWER}}=\left(\sigma_3+\sqrt{10.3\times65.9\sigma_3+65.9^2-1.96\times\sqrt{13637237.8}\sqrt{1+\frac{1}{143}+\frac{(\sigma_3-28.0)^2}{51722.9}}}\right),$$

$$\sigma_{1_{UPPER}}=\left(\sigma_3+\sqrt{10.3\times65.9\sigma_3+65.9^2+1.96\times\sqrt{13637237.8}\sqrt{1+\frac{1}{143}+\frac{(\sigma_3-28.0)^2}{51722.9}}}\right).$$

The graph showing the peak strength test data, the new Hoek-Brown curve, together with the 95% confident lower and upper prediction limits for the peak strength at various confining pressures, is shown as Figure 3.8.

The graph with both the peak strength and residual strength test data, Hoek-Brown curve of equation (3.9) with its 95% confident prediction limits, and the residual strength regression line is shown as Figure 3.9.

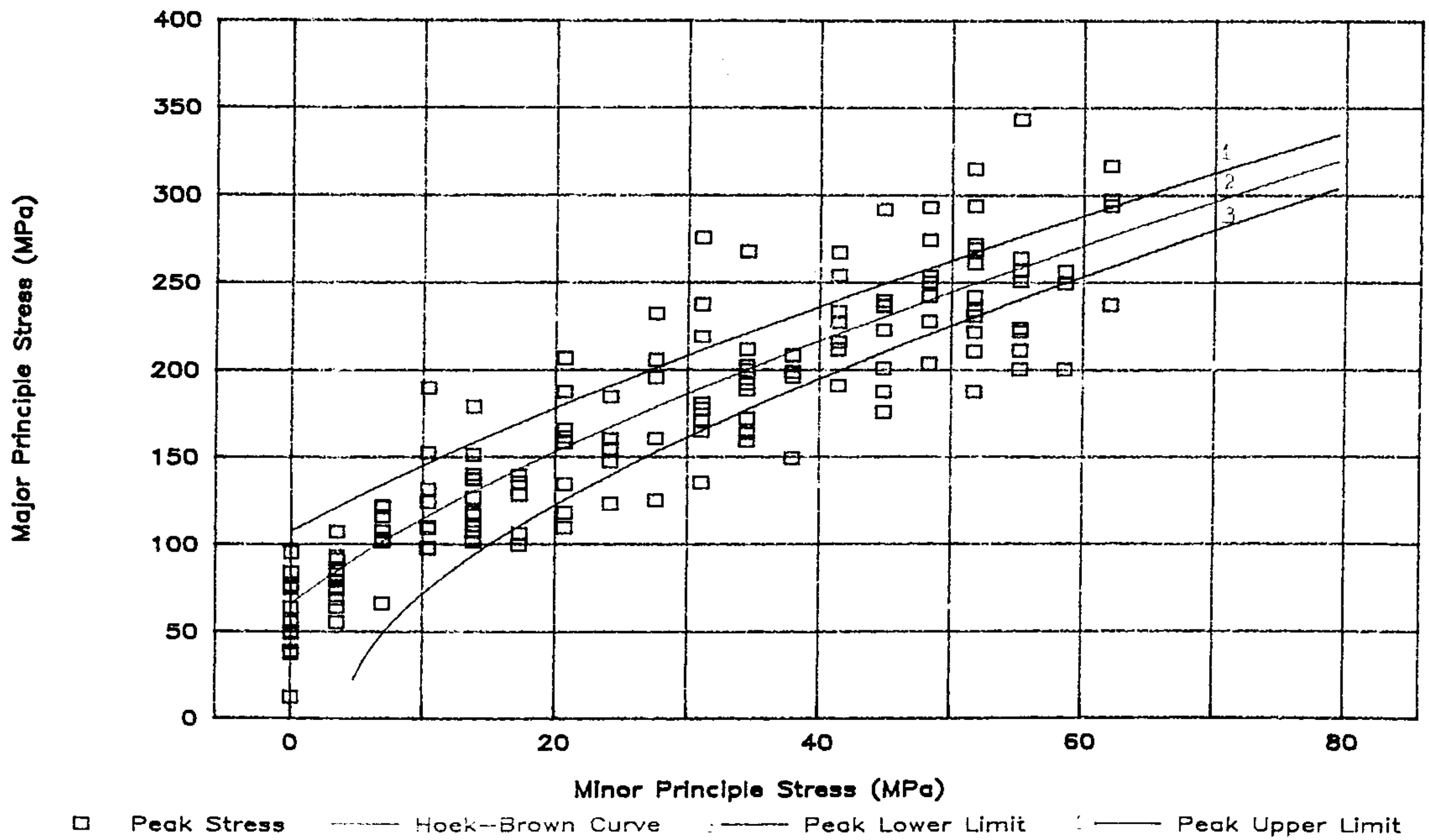


Figure 3.8 Test Data, Hoek-Brown Curve and the Lower and Upper 95% Confident Prediction Limits for Peak Failure Stresses.

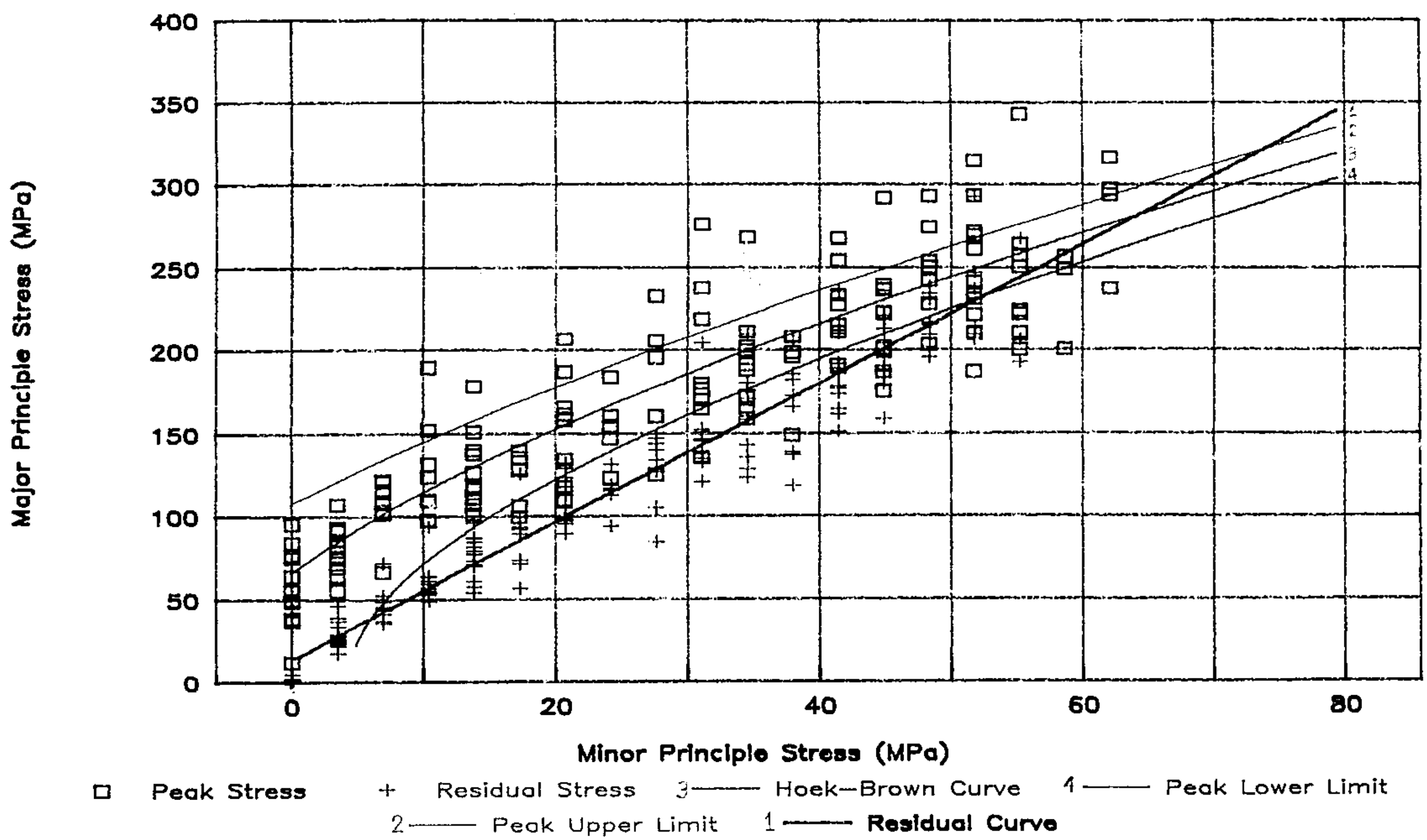


Figure 3.9 Transition from Brittle to Ductile, Together with the Prediction Limits for the Peak Strength.

3.7 Examination of σ_r and σ_3 Relationship in Hoek-Brown Equation Form

After the peak strength tests are finished, each specimen can be regarded as a piece of broken or heavily jointed rock, so the residual strength test data with mudstone could be treated as the strength of broken or jointed mudstone. Since Hoek and Brown have studied failure criterion in cases of broken and heavily jointed rocks, it would be of interest to examine the σ_r and σ_3 relationship in Hoek-Brown equation form, to see whether it differs from the results obtained above and, what influence it will have upon the brittle-ductile transition, or if the correlation between the proposed empirical equation and the test data gets better.

In their study of the failure criterion for heavily jointed rock masses, Hoek and Brown noted that the presence of one or more discontinuities in a rock specimen causes a reduction in the values of both *m* and *s*; since the rock mass is composed of a number of interlocking pieces of intact rock, it is considered to be logical to use the uniaxial compressive strength of this intact material as the value of σ_c for the rock mass. This approach gives the advantage that the strength of the heavily jointed rock mass is related back to the strength of intact rock specimens tested in the laboratory. The strength of the intact mudstone pieces, σ_c, has been determined from the analysis in Section 3.6, which gives $\hat{\sigma}_c$ = 65.9 *MPa*. The linear regression calculating the estimated values of *m* and *s*, denoted as $\hat{m}_B$ and $\hat{s}_B$ for broken or heavily jointed rock, is similar to that used above for intact rock. Therefore

$$\hat{m}_B=\frac{1}{\hat{\sigma}_c}\left[\frac{\sum_{i=1}^{k}x_iy_i-\frac{\sum_{i=1}^{k}x_i\sum_{i=1}^{k}y_i}{k}}{\sum_{i=1}^{k}x_i^2-\frac{\left(\sum_{i=1}^{k}x_i^2\right)^2}{k}}\right]=8.9$$

$$\hat{s}_B=\frac{\sum_{i=1}^{k}y_i-\hat{m}_B\hat{\sigma}_c\sum_{i=1}^{k}x_i}{k\hat{\sigma}_c}=-0.6$$

where

$$\hat{\sigma}_c=65.9\ (MPa),\quad k=165,\quad y_i=(\sigma_{r_i}-\sigma_{3_i})^2,\quad x_i=\sigma_{3_i}.$$

When the value of s is very close to zero, the calculation of $\hat{s}_B$ will sometimes give a negative value, as it did above. In this case, according to Hoek and Brown [1], put $\hat{s}_B = 0$ and calculate $\hat{m}_B$ as follows

$$\hat{m}_B=\frac{\sum_{i=1}^{k}y_i}{\hat{\sigma}_c\sum_{i=1}^{k}x_i}=7.2 \qquad (3.10)$$

But if performing the linear regression through the origin after putting $\hat{s}_B = 0$, i.e. for the true regression line to be

$$y = m_B \sigma_c . x$$

then the estimated regression equation is:

$$\hat{y} = \hat{m}_B \hat{\sigma}_c . x .$$

The sum of squares of deviations between the observed values y_i and the predicted values of $\hat{y}_i$ is

$$R = \sum_{i=1}^{k} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{k} (y_i - \hat{m}_B \hat{\sigma}_c x_i)^2 .$$

Differentiating with respect to $\hat{m}_B$ ($\hat{\sigma}_c = 65.9\ MPa$ has been determined) and equating to zero to make R a minimum gives

$$\frac{dR}{d\hat{m}_B} = -2 \sum_{i=1}^{k} (y_i - \hat{m}_B \hat{\sigma}_c x_i) \hat{\sigma}_c x_i = 0 ,$$

whence

$$\hat{m}_B = \frac{\sum_{i=1}^{k} x_i y_i}{\hat{\sigma}_c \sum_{i=1}^{k} x_i^2} = 7.7 \qquad (3.11)$$

and the correlation coefficient notated as r_B is

$$r_B = \frac{k\cdot\sum_{i=1}^{k} x_i y_i - \left(\sum_{i=1}^{k} x_i\right)\left(\sum_{i=1}^{k} y_i\right)}{\sqrt{k\cdot\sum_{i=1}^{k} x_i^2 - \left(\sum_{i=1}^{k} x_i\right)^2}\sqrt{k\cdot\sum_{i=1}^{k} y_i^2 - \left(\sum_{i=1}^{k} y_i\right)^2}} = 0.88$$

The correlation coefficient of the Hoek-Brown model for the residual strength is a little lower compared with the straight line model (3.4) in Section 3.5, which gives a correlation coefficient of 0.96. The estimated regression equation, using the $\hat{m}$ in equation (3.10), is

$$\hat{\sigma}_1 = \sigma_3 + \sqrt{7.2 \times 65.9\sigma_3}$$

or, using the $\hat{m}$ in equation (3.11), it becomes

$$\hat{\sigma}_1 = \sigma_3 + \sqrt{7.7 \times 65.9\sigma_3}.$$

Add the two Hoek-Brown curves for the residual strength above (one with $m = 7.2$ and one with $m = 7.7$) to Figure 3.7, which is shown as Figure 3.10. As can be seen, neither of the two curves can intersect with the peak strength curve, therefore no brittle-ductile transitional point exists. This is obviously unreasonable, i.e. using the Hoek-Brown equation to fit the residual strength data gives non-logical results. The straight line model assigned previously for the residual strength and confining pressure relationship is much more appropriate.

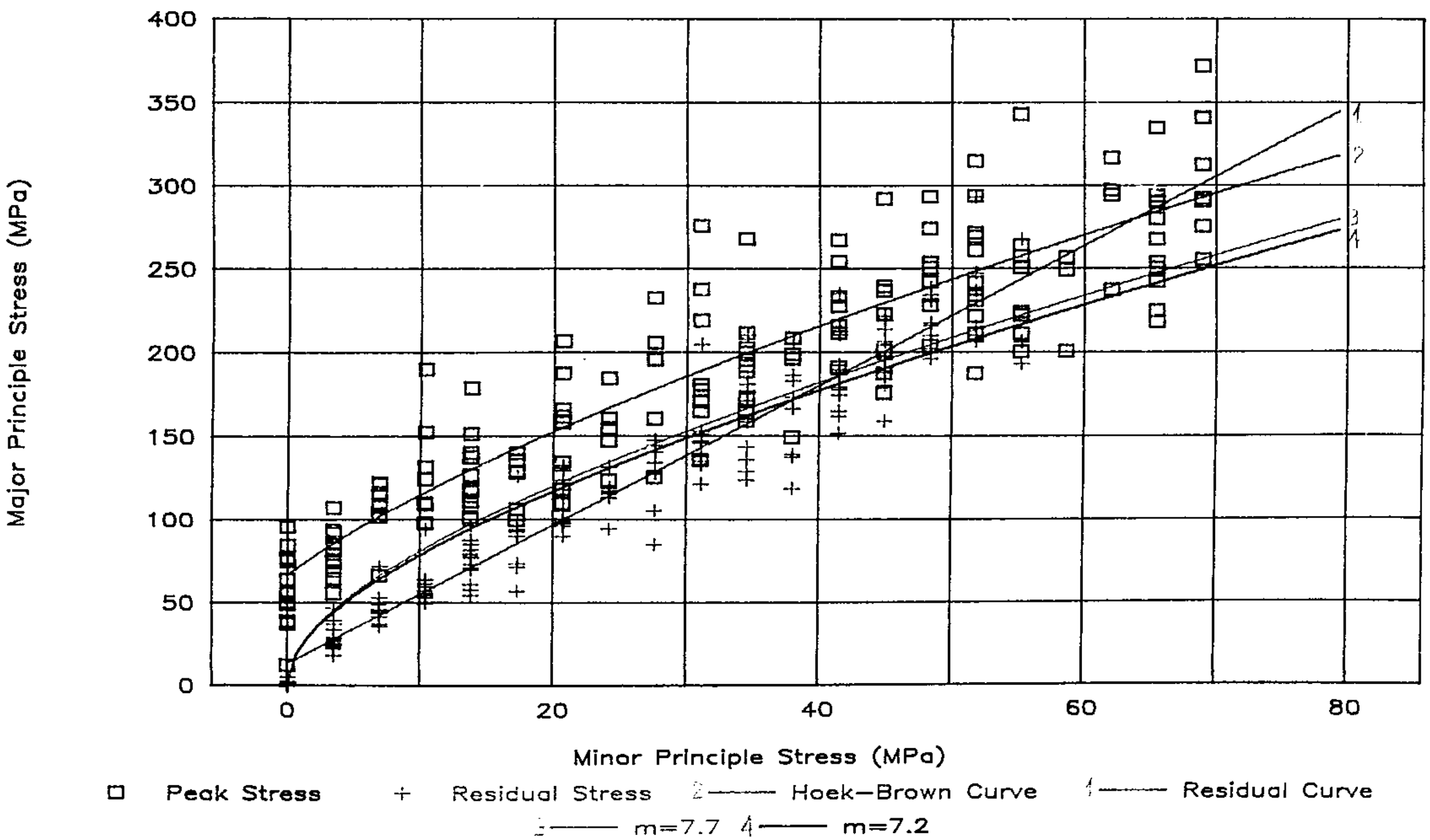


Figure 3.10 Adding Hoek-Brown Failure Curves for the Residual Strength; One with m=7.7 and One with m=7.2.

The graph with test data, 95% confident prediction limits for both peak and residual strength is shown as Figure 3.11.

The graph with the regression models curves, their 95% confident prediction limit curves for both peak and residual strength is shown as Figure 3.12. From the figure, the regression models for both peak and residual stresses, the brittle-ductile transition point and their 95% confidence prediction limits can easily be identified.

3.8 Summary of Chapter 3

This chapter is of unusual length and has lots of calculations, therefore the following summary was compiled:

a) There is no significant difference between the test results of multiple stage and single stage triaxial tests, such as the σ_1, strain ε_1, and modulus E at failure.

b) Specimen height variation does not affect m or σ_c values significantly.

c) The combined effect of both height and diameter variation, including the variation of specimen shape, has no significant influence on either m or σ_c values.

d) Test time has no significant effect on either m or σ_c values.

e) The conclusions stated in (a), (b), (c) and (d) all have a 95% degree of confidence except for the time effect on σ_c values which used a 97.5% degree of confidence level.

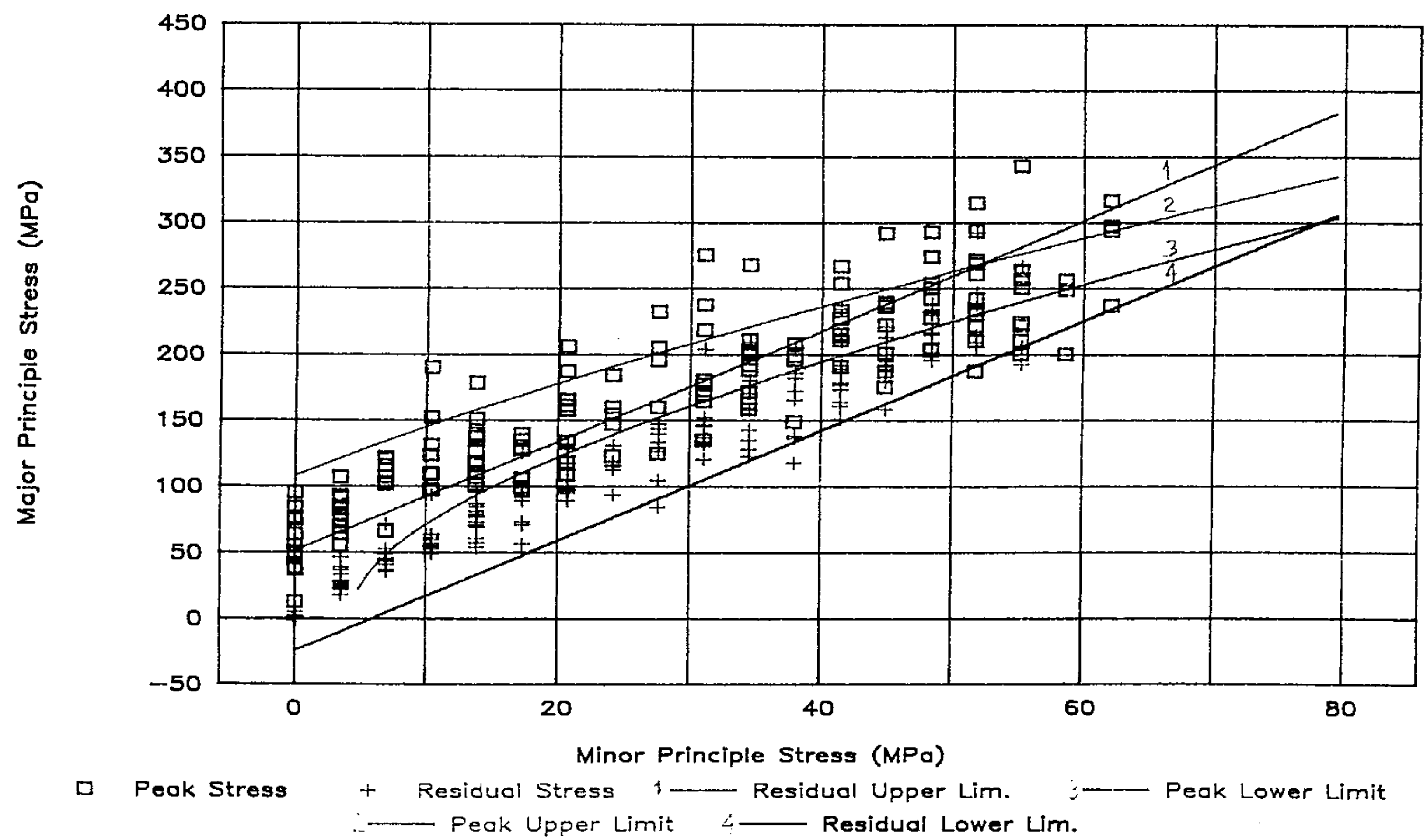


Figure 3.11 Test Data, 95% Confident Prediction Limits for Both Peak and Residual Strengths.

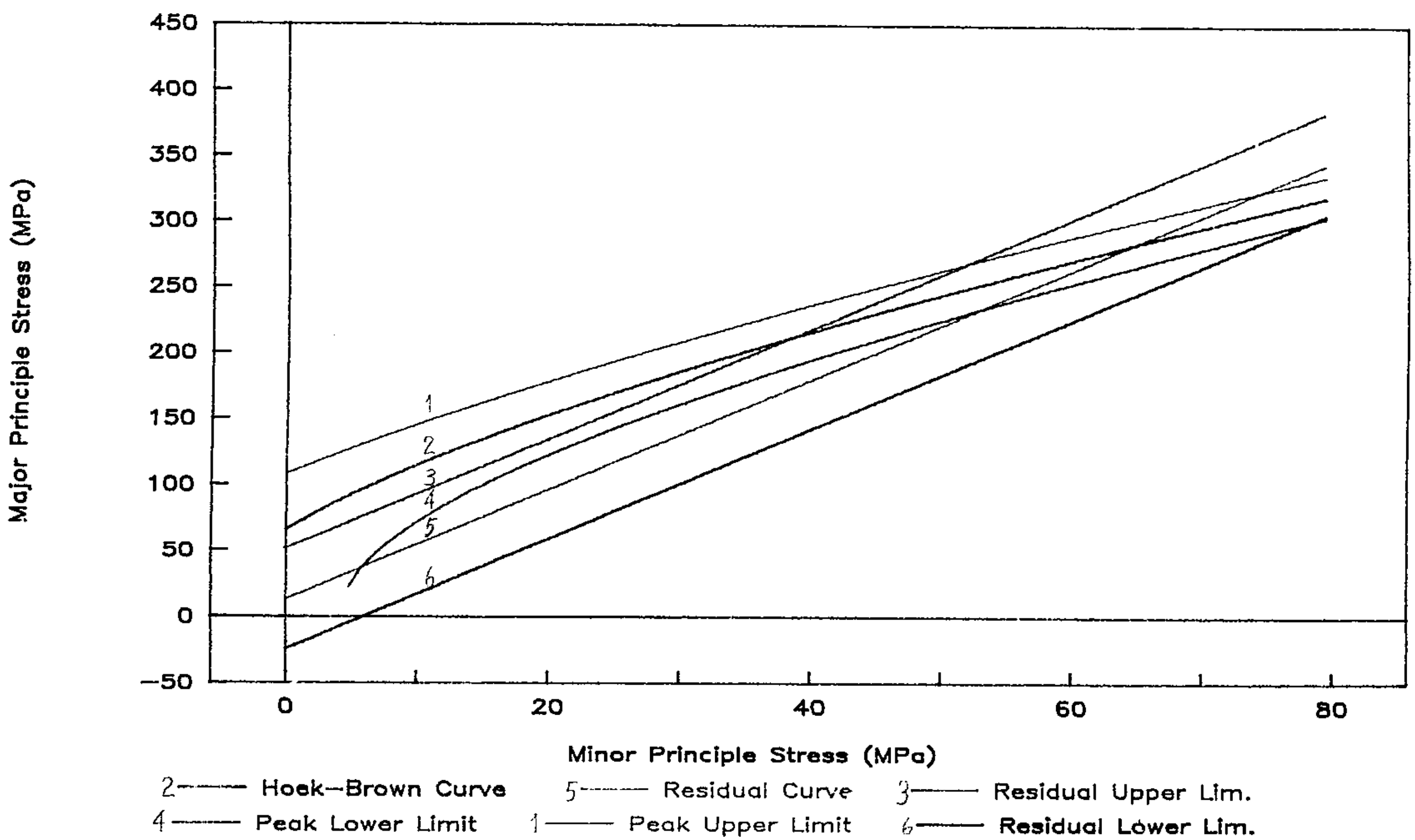


Figure 3.12 The Regression Model Curves and Their 95% Confident Prediction Limit Curves for Both Peak and Residual Strengths.

f) Linear regression analysis on all the σ_1 and σ_3 data yielded that for the mudstone tested: σ_c = 66.9 *MPa*, m = 10.0, the coefficient of determination r = 0.79. There is a 95% degree of confidence that the assumed regression model is appropriate, i.e. there is a significant correlation between the two regression variables x and y. The σ_1 and σ_3 relationship is described as:

$$\sigma_1=\sigma_3+\sqrt{10.0\times66.9\sigma_3+4469.6s}$$

g) Linear regression analysis on all the σ_r and σ_3 yielded the following relationship:

$$\hat{\sigma}_r=4.2\sigma_3+13.7$$

The correlation is obviously significant and a null hypothesis of zero intercept has to be rejected. The estimated confidence interval for the intercept is (8.5, 18.9) with an error probability of 5%. The estimated variance of an observation σ_r around its true value is 372.8 $(MPa)^2$, equivalent to a standard deviation of 19.3 *MPa*. The 95% confident prediction limits for the residual strength were obtained.

h) The brittle-ductile transition point was found, i.e. the failure of mudstone changes from brittle to ductile at the point of σ_{3T} = 64 *MPa* and σ_{1T} = 270 *MPa*.

i) After having discarded the stress points which are above the brittle-ductile transition point ($\sigma_1 < 4.2\sigma_3 + 13.7$), the modified calculation of regression analysis yields: $\sigma_c = 65.9\ MPa$, $m = 10.3$, $r = 0.78$. The new Hoek-Brown failure criterion becomes:

$$\sigma_1=\sigma_3+\sqrt{10.3\times65.9\sigma_3+65.9^2 s},$$

It can again be proved that the correlation is significant. The 95% confident prediction limits for the peak strength at various confining pressures were also obtained.

j) The examination of the σ_r and σ_3 relationship in Hoek-Brown equation form gives unreasonable results, since no brittle-ductile transition point can be found. The linear relationship proposed earlier for the σ_r and σ_3 is much more realistic.

The major conclusions are therefore:

1) The initial assumption that mudstone can be regarded as intact rock when laboratory size specimen are tested has been proven. There are no significant size or shape effects on the mudstone properties.
2) The Hoek-Brown failure criterion can be used to define the peak strength failure of mudstone at stress levels below the brittle-ductile transition point.
3) The residual strength of mudstone is best represented by a linear relationship between the residual strength and the confining pressure. Treating the data in the form of a Hoek-Brown relationship is unsatisfactory.
4) The brittle-ductile transition point is defined by the intersection of the peak stress versus the confining pressure and the residual strength versus

confining pressure curves. The best estimate for brittle-ductile transition point for mudstone is $\sigma_{3T} = 64$ *MPa* and $\sigma_{1T} = 280$ *MPa*.

5) The proposed failure criterion for sedimentary rocks has therefore been verified for one such rock. It is therefore not unreasonable to apply it to other sedimentary rocks.

4. APPROACH TO THE EVALUATION OF INTACT *m* VALUES FROM TRIAXIAL STRENGTH TESTS ON NON-INTACT COAL SPECIMENS

4.1 Introduction

Research on a failure criterion for coal is essential for the design of colliery lay outs and the economical extraction of coal. However, the various discontinuities such as bedding planes, cleats and cracks contained in the coal make the strength of coal greatly affected by the specimen size and shape, because the strength depends on how many and what type of discontinuities are present in the specimen. Ideally, to determine the strength of coal mass, large size specimens should be tested. Even if it is possible to perform few *uniaxial* compression tests on specimens as large as 60 inch square in cross section [30] (although extremely costly and time-consuming), such large size *triaxial* compression tests are virtually impossible. So far no such triaxial tests on coal have been reported. Moreover, strength measurements on coal specimens subjected to a uniaxial stress have a limited application to the understanding of the behaviour of coal in the seam, for the state of stress in the seam changes from biaxial at the coal face to triaxial in the solid coal. As a basis of estimating the failure strength of a coal mass from small size laboratory specimen tests, the Hoek-Brown failure criterion is appealing because not only might it describe the strength of coal under uniaxial stress but also the strength of coal of any size under any confining pressure. To do this the coal parameters of *m*, *s* and σ_c are needed. But the equations used to calculate *m* and σ_c for mudstone in Chapter 3 cannot now be used since coal specimens are believed to be *non-intact*, and therefore *s* no longer equals 1 but will vary with the specimen size and shape. The question is, therefore, how can intact *m*, σ_c values for coal be determined from triaxial tests on non-intact specimens?

4.2 The Approach to the Evaluation of Intact *m* Values

4.2.1 The conditions

Let σ_{pi} be the *intact* uniaxial prism strength of coal, let m_i be the *intact* "*m*" value and $s_i = 1$ be the *intact* "*s*" value for a particular coal. Assume that the failure criterion for intact coal is given in the form derived by Hoek and Brown as:

$$\sigma_{1i}=\sigma_{3i}+\sqrt{m_i\sigma_{3i}\sigma_{pi}+s_i\sigma_{pi}^2} \qquad (4.1)$$

where $s_i = 1$, σ_{1i} and σ_{3i} are the principle stresses at failure for the intact coal. Assume that triaxial tests are carried out on a suite of laboratory size specimens of height *H*, and diameter (width), *D*. Further, **assume that these specimens cannot be regarded as being *intact*.** Assume also that the results of these strength tests can be described by the following relationship of stresses at failure:

$$\sigma_1=\sigma_3+\sqrt{m\sigma_3\sigma_{pi}+s\sigma_{pi}^2} \qquad (4.2)$$

hence, the uniaxial prism strength, σ_p, of a non-intact coal specimen of this size is given by:

$$\sigma_p=\sqrt{s\sigma_{pi}^2} \qquad (4.3)$$

where s is not equal to unity.

σ_p can be measured, but s and σ_{pi} are unknown. To determine s, some additional information is required, i.e. the relationship between the compressive strength of coal σ_p and its size and shape.

4.2.2 Size effect on the uniaxial compressive strength of coal cubes

The specimen size effect on the uniaxial compressive strength of coal has long been recognized. Size effects have been investigated by considering the strength of ***cubical*** specimens of different dimensions. Daniels and Moore [20] tested bituminous and anthracite specimens of cubical blocks and concluded that the uniaxial compressive strength σ_p increases with increasing edge dimension D (Note: In this thesis D is used to denote the least lateral dimension of the specimen cross section. It represents therefore the *width* for a rectangular cross section; while the longer lateral dimension is called the *length*. The vertical dimension is denoted by the *height* H. For a cylindrical specimen, D become the diameter).

Lowall and Holland [21] studied the compression characteristics of West Virginia coal cut into cubes approximately 0.076 *meter* on a side and also into pieces varying in height from 0.006 meter up to about 0.064 meter. They found that the size of the cubical test piece has a very marked influence upon the ultimate compressive strength and that the larger the cube, the smaller the value of σ_p. However, no specific formula for such a relationship was developed.

Steart [22] reported, after performing small specimen tests and from mine observation, that for cubical specimens with $D = H$, the uniaxial compressive strength σ_p and the specimen dimension have the following relationship:

$$\sigma_p \propto \frac{1}{\sqrt{D}}.$$

Millard et al [23] studied the variation of crushing strength with size by testing cubical specimens of Llandebie anthracite and irregular lump specimens from Bettesanger and Langwith. They found the relationship between σ_p, the specimen weight W, and cube edge D could be expressed respectively as:

$$\sigma_p \propto W^a.$$

and

$$\sigma_p \propto D^{-d}.$$

where a and d are constants. a = 0.52, 0.51, 0.49 and d = 0.44, 0.47, 0.53 respectively for Liandebie, Betteshanger, and Langwith coals.

Gaddy [24] concluded, from experience of testing a wide range of different coal specimens of different sizes from 0.051 *meter* to 1.626 *meters*, that the ultimate unit strength of a coal cube in compression varies inversely as the square root of the edge dimension of the cube, and is given by:

$$\sigma_p = k\frac{1}{\sqrt{D}} \qquad (4.4)$$

where k is a coefficient depending upon the chemical and physical properties of the coal, and is numerically (but not dimensionally) equal to the unit cube strength.

Evans et al [25] confirmed that the uniaxial cube strength was dependent on a power function of D, but they obtained the exponent of -0.32 from tests on Deep Duffryn and -0.17 from tests on Barnsley Hards coal.

Bieniawski [26, 27, 28, 29, 30] described both laboratory and *in-situ* test results from cubical coal specimens from Witbank Colliery, with sizes from 0.019 *meter* to 2.012 *meter* in side dimension. He then stated that three different equations were needed for three different cases of specimen size:

Case (a): an initial constant strength relationship, when D is less than a certain value. He did not give this specific value (probably 0.076 meter according to his test results), neither did he establish this constant strength relationship since he believed this case had little practical importance.

Case (b): a subsequent strength reduction relationship when D is less than 1.524 *meters* and greater than that specific value mentioned in Case (a)

$$\sigma_p = 4.8\frac{D^{0.16}}{H^{0.55}} \quad (MPa)$$

which, for cubical specimens, reduces to

$$\sigma_p = 4.8D^{-0.39} \quad (MPa)$$

Case (c): a final constant strength relationship when D is greater than 1.524 *meters*

$$\sigma_p = 2.8 + 1.5\frac{D}{H} \quad (MPa)$$

where D and H are all in *meters*.

Hustrulid [31] questioned the meaning of the results from small cubes less than 0.025 *meter* which Evans *et al* [25] tested, because technically it is extremely difficult to prepare and test such small specimens. He then showed that a better fit could be obtained for Evans's test results in the following form if the data of cubes less than 0.025 *meter* were excluded:

$$\sigma_p \propto \frac{1}{\sqrt{D}}.$$

By comparing the strength data of Pittsburgh coal obtained by a number of investigators, with the curves of the form of equation (4.4) and based on 0.051 and 0.102 *meter* cube strengths obtained by Gaddy [24], he found the quite good agreement between the predicted strength values from equation (4.4) and the actual values. This means that the cube strengths for large specimens can be predicted with reasonable accuracy using equation (4.4) from small specimen tests. He also noted that there was only a small difference for cubes larger than 0.610 *meter* in Bieniawski's tests. The strength decreases only 23% as D increases from 0.610 to 2.007 *meters*, and the strength decreases only 9% as D varies from 0.914 to 2.007 *meters*. He then suggested that, for practical purposes, strengths for cubes larger than 0.914 *meters* can be regarded as constant and equal to that of a 0.914 *meter* cube.

Evans [32], commenting on Bieniawski's conclusions, noticed that three cases correspond to three specimen size groups tested by Bieniawski: 0.019 to 0.076 *meter*; 0.152 to 0.457 *meter*; 0.610 to 1.524 *meters*, which, in turn, correspond to three different test methods he employed. Since significant variations of strength with size in Case (a) had been observed by other investigators, Evans questioned the conclusion in the Case (a) size range.

As shown from the research review above, the most generally accepted strength and size relationship can be expressed as

$$\sigma_p = \frac{k}{\sqrt{D}},$$

where k is a constant.

There is some concern as to the upper size limit beyond which this equation does not apply, since logically, σ_p will not decrease *ad infinitum* as size continuously increases. Several workers have proposed somewhat different upper size limits. Hustrulid suggests 0.914 *meter*, Bieniawski suggests 1.524 *meters* while Gaddy indicates the strength continues decreasing even after 1.626 *meters*.

There may also exists a lower size limit beyond which the formula no longer applies, for instance, for specimens much less than 0.025 *meter*. However, this was not considered to be an important practical concern since usually k is determined from testing laboratory size specimens which are usually between 0.025 to 0.102 *meters*.

4.2.3 Shape effect on the compressive strength of coal

The shape effect on the uniaxial compressive strength is characterized by the varying ratio of D/H. The discussion in this section will consider mostly prismatic specimens having square cross sections, however, some results from core specimens and rectangular section specimens will also be included.

The shape effect may be reflected by either holding the width constant and varying the height, or fixing the height while varying the width. Obviously changing width while fixing height is more pertinent to this study, since in a coal seam the height of the coal pillar (usually the seam thickness) is generally consistent and it is the width of the pillar that changes. In some cases the formulae from previous research were expressed in normalised form to assist in comparisons, i.e. the two sides of the strength formula are divided by the cube strength σ_{cube}. Previous researchers have not been consistent in their choices of the specimen dimension in determining the cube strength of coal, in some cases they normalised their equations by dividing by the strength of a cube equal to the specimen height, whereas in others they divided by the strength of a cube equal to the width. This has been largely responsible for the variation of different formulae.

According to Hustrulid [31], Baushinger performed a series of compression tests on Swiss sandstone prisms and the following normalised equation was derived:

$$\frac{\sigma_p}{\sigma_{cube}}=0.778+0.222\frac{D}{H}. \qquad (4.5)$$

The deviation of the actual test data from the predicted values calculated by this formula becomes large as *D/H* is greater than 1. This, however, is the range of greatest interest in coal pillar strength prediction.

Bunting [33], based on tests on anthracite coal specimens with various dimensions from several mines, characterised the following equation:

$$\sigma_p=12.1+5.2\frac{D}{H}\ (MPa) \qquad (4.6)$$

and, using an average compressive strength 17.3 *MPa* for coal cubes of edge size between 0.051 to 0.152 *meter*, the equation was normalised as

$$\frac{\sigma_p}{\sigma_{cube}}=0.7+0.3\frac{D}{H}. \qquad (4.7)$$

By plotting the "squeezing" loads calculated from the overburden weight and the extraction ratio for six mine pillars of varying *D/H* ratios and applying a safety factor of 2.5 to equation (4.6), it can be shown that the following expression:

$$\sigma_p=4.8+2.1\frac{D}{H}$$

describes the field data very well. Further, assuming the strength of a cubical pillar with edge dimension equal to the seam height is 6.9 *MPa*, the relationship between prism and cube strengths would be expressed the same as equation (4.7).

Holland [34] pointed out that if cubical specimens where $D/H = 1$ for Bunting's results are considered, they also fit closely the following form:

$$\sigma_p \propto \sqrt{\frac{D}{H}}.$$

Griffith and Conner [35] decided that the following relationship exists for anthracite specimens:

$$\sigma_p \propto \frac{1}{\sqrt{H}}$$

all other things being equal (i.e. constant D).

Greenwald *et al* [36] tested seven pillars from about 0.813 to 1.626 *meters* in horizontal dimension and about 0.787 to 1.626 *meters* high. The pillars were loaded vertically to their failure strength (about 3.4 to 6.3 *MPa*). The following relationship was proposed:

$$\sigma_p = k\sqrt{\frac{D}{H}}$$

where k is a constant.

Rice [37] reported that, in 1900, the Scranton Engineer's Club appointed a committee to determine the strength of anthracite. For this study, 423 small specimens were collected from several localities and different Pennsylvania anthracite beds. Each specimen had a 0.051 by 0.051 *meter* base, but there were three heights: 0.025, 0.051, and 0.102 *meter*. From the tests the committee concluded that in general, other things being equal, σ_p would vary inversely as the square root of the height H:

$$\sigma_p \propto \frac{1}{\sqrt{H}}.$$

Rice pointed out that, this conclusion seemed to apply only to mine pillars of lateral dimensions that are small with respect to the height. If the ` teral dimensions are large in proportion to the height, differences in height would make a significant difference to σ_p; but he did not give specific limits.

Greenwald *et al* [38] reported, in 1941, the testing results at an underground coal mine of 10 small square pillars in the Pittsburgh bed. The following relationship was found:

$$\sigma_p = 5.7\frac{D^{0.5}}{H^{0.833}} \quad (MPa)$$

where D and H are in *meters*. However, as Hustrulid [31] stated, equation (4.5) also describes the relationship between σ_p/σ_c and D/H very well.

Holland [39] stated that pillar dimensions were one of the factors affecting the strength of coal pillars in addition to the physical properties, joint formations and other factors. He showed [34], with tests on coal pillars of 4 different coals with width to height ratios of 4.9 to 12.0, that the following relation exists:

$$\sigma_p \propto \sqrt{\frac{D}{H}}$$

for which *D/H* should be between about 3 and 7, and that it only applied to the specimens which failed abruptly. Applying the method of averages to Holland's data, Hustrulid [31] obtained the relationship

$$\frac{\sigma_p}{\sigma_{cube}}=0.775+0.225\frac{D}{H},$$

which is essentially identical to the original Bauschinger formula. It is noted also that in these experiments the *H* was kept constant and *D* was varied.

Steart [22], from compression tests on prisms of coal in which the width *D* was held constant and the height *H* changed, stated three "rules", which can be summarized as:

Rule 1: The compressive strengths of coal pillars of constant width vary in inverse ratio to height.

$$\sigma_p \propto \frac{1}{H}. \qquad (D\ constant)$$

Rule 2: The strengths of pillars of constant height vary as the square root of their widths, i.e.:

$$\sigma_p \propto \sqrt{D}. \qquad (H\ constant)$$

Rule 3: In those cases in which height and width are equal (pillars of cube form), the strength is given by:

$$\sigma_p \propto \frac{1}{\sqrt{D}}. \qquad (cubes)$$

The rules seem to fit the data of other tests.

Holland and Gaddy [41] established the general pillar strength formula as:

$$\sigma_p = k\frac{\sqrt{D}}{H}$$

which can be regarded as an application of Steart's rules.

As for the effect on σ_p of the length L of the rectangular cross section of a prism, Evans et al [25] conducted experiments on prisms for which D and H were kept constant while L was varied. They found that, when H remains constant, the prism strength was controlled only by the least lateral dimension D.

Salamon and Munroe [42, 43], based on data obtained from a survey of actual mining dimensions, derived the following formula which defines approximately the strength of coal pillars in South African collieries:

$$\sigma_p = k\frac{D^{0.46}}{H^{0.66}} = 7.2\frac{D^{0.46}}{H^{0.66}} \quad (MPa),$$

whereas D and H are in *meters*. The constant multiplier k, the powers of D and H were estimated by the method of maximum likelihood. This equation is very similar in form to that of Holland [40].

Hustrulid [31] conducted a comparison between above the equation and the equation below:

$$\sigma_p = 7.1 + 2.0\frac{D}{H},$$

or using the normalised form:

$$\sigma_p = 0.778 + 0.222\frac{D}{H}.$$

He found that they were not very much different. The maximum difference between the two curves over the D/H range of 1 to 9 is only about 15%.

Bieniawski [26, 27, 28, 29, 30] stated that according to his studies, the strengths of coal cubes with edge dimensions 1.524 *meters* or larger were constant. From his data for pillars with a square base of 1.524 *meters* and various heights, he derived:

$$\sigma_p = 2.8 + 1.5\frac{D}{H},$$

or in the normalised form:

$$\frac{\sigma_p}{\sigma_{cube}}=0.645+0.355\frac{D}{H},$$

which is normalised by a cube of strength with edge dimension equal to the constant width. This normalised form has been constructed from experiments in which D was held constant and H varied from 0.610 to 1.524 *meters*. Because the strength difference between 0.610 and 1.534 meter cubes are relatively small, the normalised form of Bieniawski is similar to equation (4.5). Hustrulid then plotted Bieniawski's data for specimens with constant H but varying D. Using the cube strength of that equal in size to the of the pillar height, he showed that this data were well described by the normalised equation:

$$\frac{\sigma_p}{\sigma_{cube}}=0.778+0.222\frac{D}{H}.$$

Hustrulid [31] noted that this equation fits most of the data well. He recommended that for specimen sizes having size effects, σ_{cube} should be determined from cubes with edge dimension H instead of D.

4.2.4 Discussion on σ_p and size/shape relationship

Summarizing the past research work reviewed above, the various coal strength formulae can be classified into three basic forms:

(a) The *general* form:

$$\sigma_p = k\frac{\sqrt{D}}{H},$$

which for cubes, reduces to:

$$\sigma_p = \frac{k}{\sqrt{D}}, \quad or: \quad \sigma_p \propto \frac{1}{\sqrt{D}},$$

and is a special case of the general form.

(b) The ***linear*** form:

$$\sigma_p = a + b\frac{D}{H},$$

where a and b are constants.

(c) The ***power*** form:

$$\sigma_p = k\sqrt{\frac{D}{H}},$$

The following form, derived by Salamon and Munroe [42, 43]:

$$\sigma_p = k\frac{D^{\alpha}}{H^{\beta}}, \qquad \beta \neq 1.$$

can be regarded as a variation of the power form.

Bieniawski [27] argued that the strength of pillars cannot be expressed as a power form as there exists an upper size limit above which the strength no longer changes with an increase in pillar dimensions. This is conceivably correct since coal is an extremely weak material containing many discontinuities such as bedding planes, cleats and joints. The chances of containing discontinuities in small size specimens are less than those in larger size specimens. Therefore small size specimens show higher strength than large ones. This explains the large scatter of test data with small specimens. However, when the specimen size increases above some upper limit, the probability of containing discontinuities becomes stable and the strength no longer changes significantly.

Bieniawski's argument also challenged the other formulae given above except for those in which some size limits were specified. His upper size limit for cubic coal strength was 1.524 *meters*, while Hustrulid [31] used 0.914 *meter*. It is logical that an upper size limits exists, but the exact value of it is still arguable since not enough tests on large size specimens have been carried out.

Hustrulid [31] emphasized the advantage that the normalized linear form:

$$\frac{\sigma_p}{\sigma_{cube}} = a' + b' \frac{D}{H}$$

was dimensionally correct. According to Hustrulid, the *compressive strength-shape* relationship can be expressed by equation (4.5); and this equation applies only to cases where H is kept constant and D varied. If the specimen size is large enough to show a size effect, the value of σ_{cube} has to be determined from cubes having an edge dimension equal to the value of H instead of D. As for cubes, for which shape effect does not exist, Hustrulid suggested the following equations:

$$\sigma_p=\frac{k}{\sqrt{H}}, \qquad H < 0.914\ (meter);$$

and

$$\sigma_p=\frac{k}{\sqrt{0.914}}, \qquad H > 0.914\ (meter).$$

These two sets of equations compiled by Hustrulid fit most of the test data reasonably well and more over, an upper size limit was specified, except Bieniawski puts 1.524 *meters* as this limit while other workers stated no limits due to the lack of upper size limit specimen tests.

However, one concern about the conclusion made by Hustrulid above is that it requires two separate equations to describe the combined size and shape effects and consequently, it is different in shape effects when changing D while H is fixed or vice versa. Also the so called advantage of dimensional correctness of this formula is lost when it comes to the cube strength case in which $D = H$, i.e.:

$$\sigma_p=\frac{k}{\sqrt{D}}.$$

Obviously the constant k does not have the dimension of strength. Moreover, a cube is in fact a special case of a prism and for which the strength formula should also be a special case of the prism strength formula in which $D = H$. This challenges the statement made by Hustrulid [31] about the attractiveness of this linear form formula.

On the other hand, the general form:

$$\sigma_p = k\frac{\sqrt{D}}{H} \qquad (4.8)$$

agrees with several important conclusions. For example, when D is concerned it states:

$$\sigma_p \propto \sqrt{D}.$$

This complies with conclusions made by Greenwald [36] and [38], Holland [39, 40, 44], Hustrulid [31], Gaddy [24], Holland and Gaddy [41], Steart [22], etc.. Some other workers also concluded that σ_p is proportional to a power of D, but they obtained different powers of D, some of them are quite close to 0.5. Note some workers obtained

$$\sigma_p \propto \frac{1}{\sqrt{D}}, \qquad or: \quad \sigma_p \propto \frac{k}{\sqrt{D}}$$

for cubes. Obviously, this does not contradict equation (4.8), as a matter of fact, this agrees fully with equation (4.8) when applying equation (4.8) to cubes.

According to equation (4.8)

$$\sigma_p \propto \frac{1}{H}, \qquad (4.9)$$

when H is concerned. This agrees with the conclusions made by Bunting [33], Hustrulid [31], Gaddy [24], Holland and Gaddy [41], Steart [22], Holland [40, 44], Bieniawski [26, 27, 28, 29, 30], etc.. Similarly, some people concluded σ_p varies inversely as a power of H close to 1.

Consequently, equation (4.8) is believed to be the best representation of the research results of the strength and size/shape relationship and furthermore, more reasonable when it comes to combining various sizes and shapes of specimen under one strength formula.

4.2.5 Evaluation of the general form formula using all available test data

In order to determine whether or not this single equation in the so called ***general*** form can truly represent the strength and size/shape relationship, a detailed statistical analysis of all available data was carried out by Barron and Das [9]. The following paragraphs (a) to (c) reference some of their results.

(a) A linear correlation between σ_p and $D^{1/2}/H$ was performed for each set of test data of every coal and the following form was obtained:

$$\sigma_p = a_0 + b_0 \frac{\sqrt{D}}{H},$$

where a_0 and b_0 are constants. From this formula the uniaxial compressive strength of a 1 *meter* cube coal, σ_{cube}, was determined and the data was then normalised by dividing each σ_p by the corresponding σ_{cube} of that coal, i.e.:

$$\frac{\sigma_p}{\sigma_{cube}} = a_0' + b_0' \frac{\sqrt{D}}{H} .$$

A linear correlation on this normalised data yielded the following formula:

$$\frac{\sigma_p}{\sigma_{cube}} = 0.689 + 0.311 \frac{\sqrt{D}}{H} ,$$

where D and H are in *meters*. The correlation coefficient $r = 0.82$ (refer to Fig. 4.1).

The correlation is considered good if the variations of coal ranks and the different testing methods were taken into account.

Noticing that the test data consisted mostly of cube coal tests and the $1/D^{1/2}$ form has been well established previously, Barron and Das examined the adequacy of this formula for the non-cubi, or prism specimens by excluding the cube data, which resulted in the following formula:

$$\frac{\sigma_p}{\sigma_{cube}} = 0.685 + 0.315 \frac{\sqrt{D}}{H} ,$$

where D and H are in *meters*, with a correlation coefficient $r = 0.74$ (refer to Fig. 4.2).

It can been seen, comparing the correlations for the total data and the prism data, that there is no significant difference between the two. It can then be concluded that the general form formula is proper for both cubes and prisms and using two separate equations for the size/shape effect, as Hustrulid did, is not necessary.

(b) From Figure 4.1, Barron and Das noted that a disproportionate number of

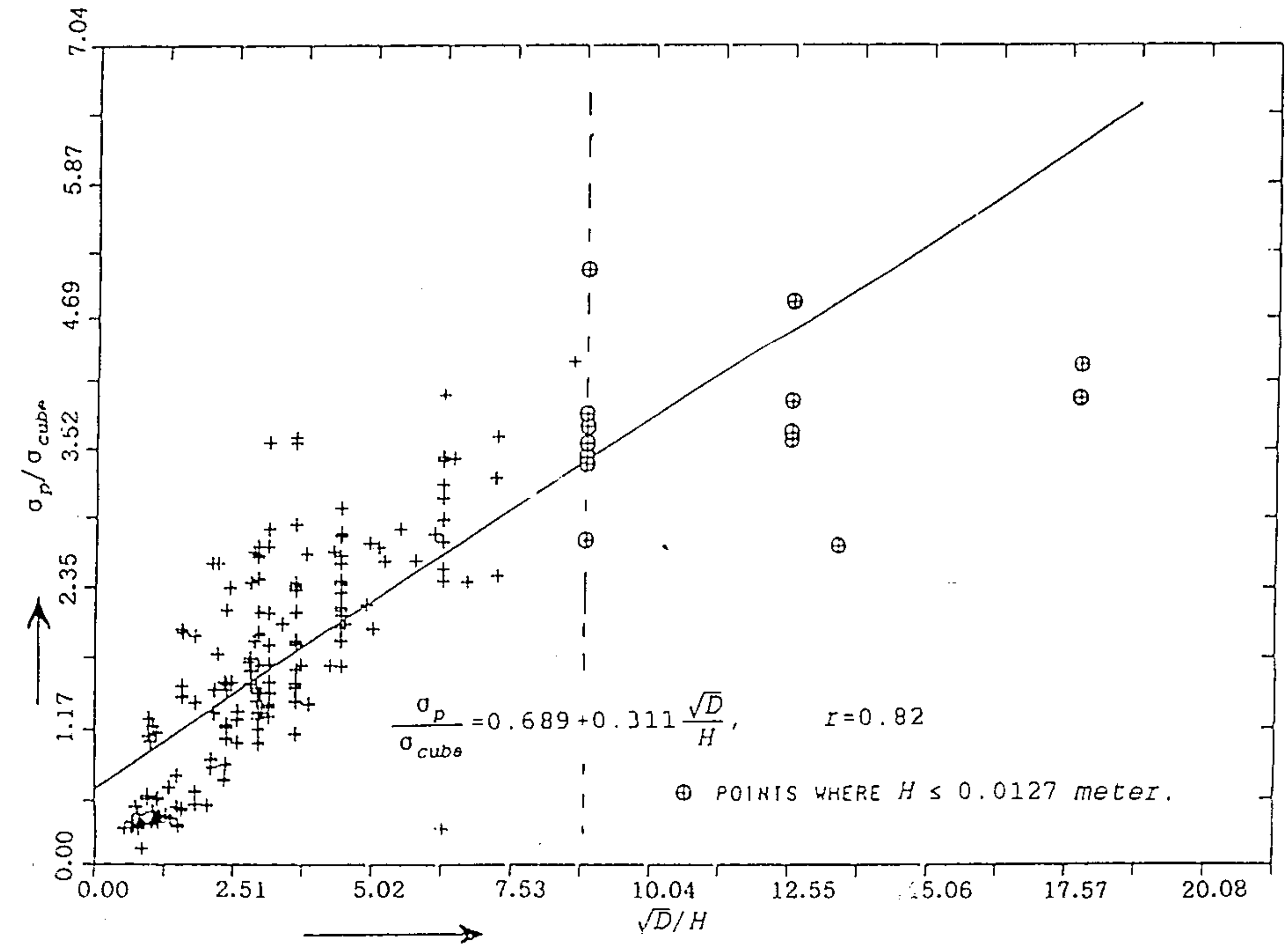


Figure 4.1 Uniaxial Compressive Strength Correlations - All Coals - All Data (after [9])

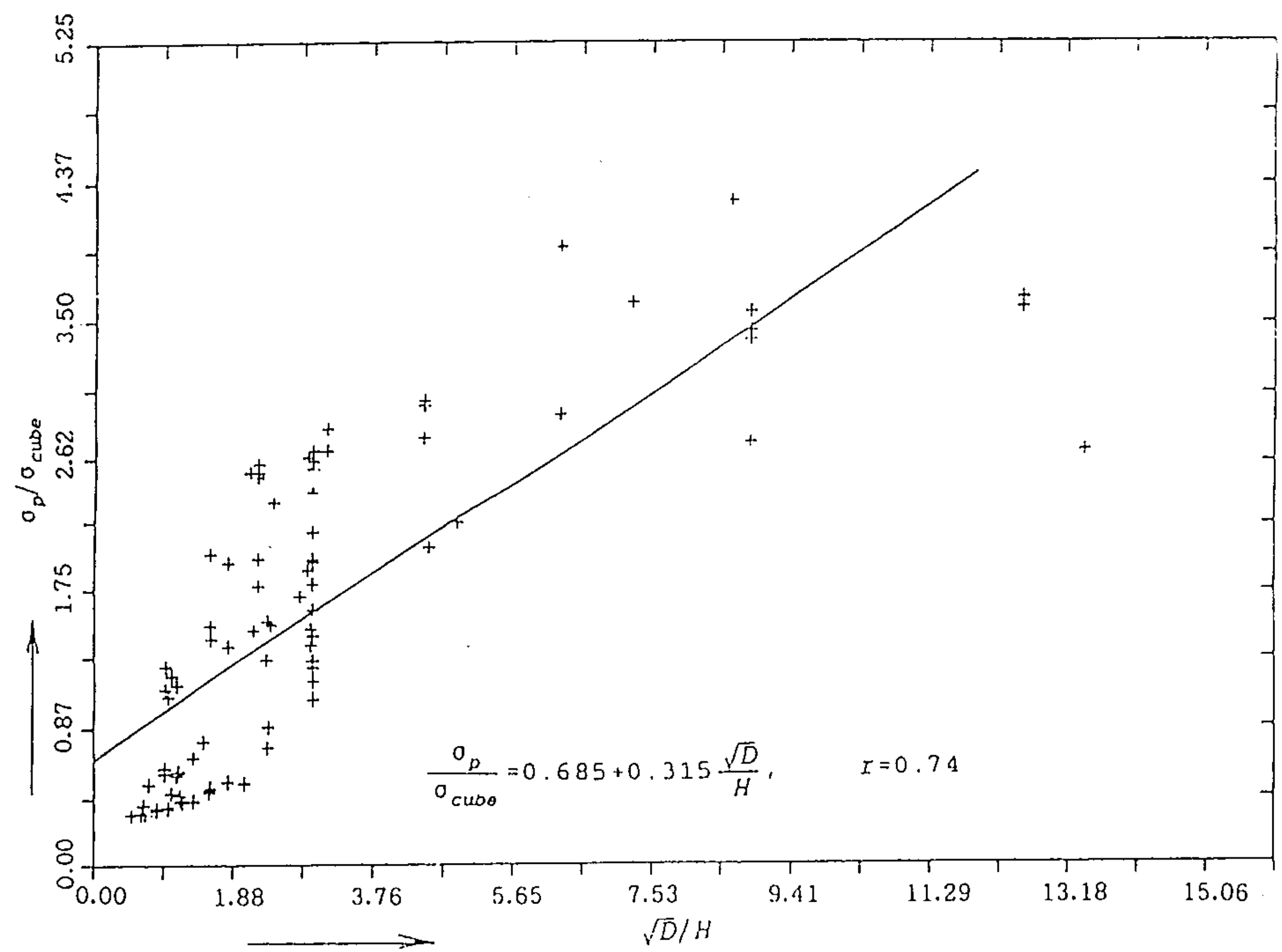


Figure 4.2 Uniaxial Compressive Strength Correlations - All Coals - Prisms Only (after [9])

points deviate from the correlation line when the $D^{1/2}/H > 8.784$ $(meters)^{-1/2}$. Further, it was noted that these points were all associated with test specimens where the height was less than or equal to 0.013 *meter*, i.e. the "general form" of equation describing the size effects breaks down when $D^{1/2}/H > 8.784$ $(meter)^{-1/2}$ and the specimens are very small. This implies that beyond this limit the size effects disappear for very small specimens and that, below some critical dimension, the strength becomes constant (and could be regarded as the "intact" uniaxial coal strength). This is in accordance with Hustrulid's comments on the doubtfulness of the application of his equation to Evan's results the cube specimens below 0.013 *meter* edge dimension.

Consequently, Barron and Das eliminated all data where $D^{1/2}/H > 8.784$ and re-analyzed the remaining data. Their new normalised equation was given by:

$$\frac{\sigma_p}{\sigma_{cube}} = 0.414 + 0.586\frac{\sqrt{D}}{H},$$

with an improved correlation coefficient of $r = 0.87$, as shown in Figure 4.3.

(c) Barron and Das then investigated the possibility that there might also exist a lower limit of the $D^{1/2}/H$ ratio, below which the "general form" of the equation describing size effects also breaks down. Since on Figure 4.1 and 4.3 all the points for small values of $D^{1/2}/H$ are concentrated near the region, it is hard to distinguish deviations from the correlation line. They therefore plotted the inverse correlation σ_{cube}/σ_p versus $H/D^{1/2}$, as shown in Figure 4.4. It is seen from the figure that all the data points fell on one side of the correlation line when $H/D^{1/2} > 1.275$ $(meter)^{1/2}$. In

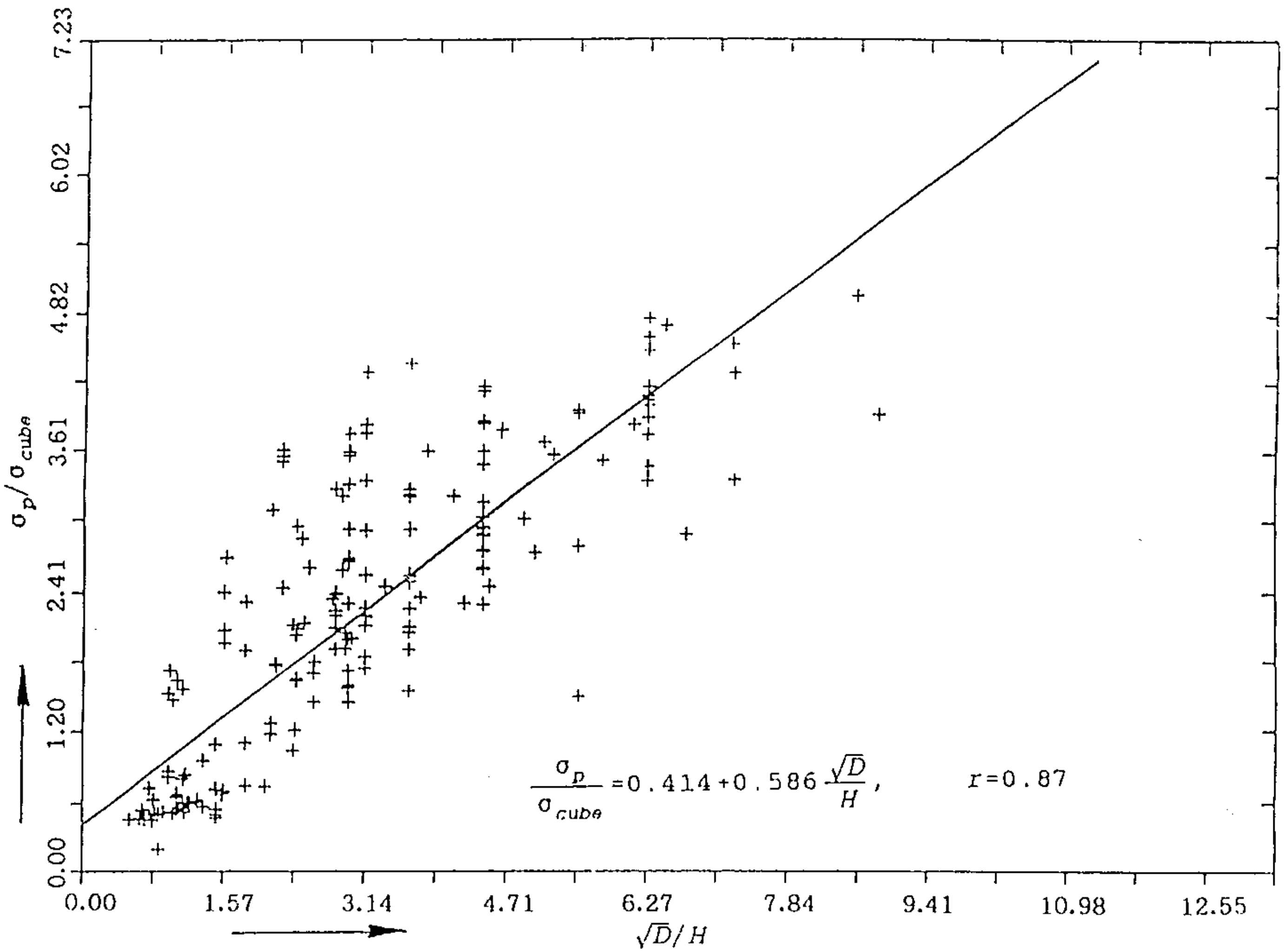


Figure 4.3 Correlations - All Coals When $D^{1/2}/H < 8.784$ (after [9])

other words, there exists a lower limit $D^{1/2}/H < 0.784$ (*meter*)$^{-1/2}$, below which the general formula again breaks down. For a cube, this occurs when $D = H = 1.626$ *meters*, i.e. the general form of equation describing the size effects again breaks down when $D^{1/2}/H < 0.784$ (*meter*)$^{-1/2}$. This implies that, beyond this limit, the size effects disappear for very large specimens and that, above some critical dimension, the strength again becomes a constant (and could be regarded as the uniaxial coal mass strength). This is in close agreement with Bieniawski's size limit which, for cubes, was 1.524 *meters* (60 *inches*).

Thus they eliminated all data where $D^{1/2}/H < 0.784$ (*meter*)$^{-1/2}$ and again re-analyzed the data, obtaining the final "normalised equation":

$$\frac{\sigma_p}{\sigma_{cube}} = 0.452 + 0.548\frac{\sqrt{D}}{H},$$

with a correlation coefficient $r = 0.85$, as shown in Figure 4.5.

Barron and Das therefore concluded that:

(i) If $D^{1/2}/H > 8.784$ (*meter*)$^{-1/2}$, the uniaxial coal strength is constant and can be regarded as that for intact coal.

(ii) If $8.784 > D^{1/2}/H > 0.784$, the uniaxial compressive strength is size and shape dependant and is a function of $D^{1/2}/H$ given by:

$$\frac{\sigma_p}{\sigma_{cube}} = 0.452 + 0.548\frac{\sqrt{D}}{H} \quad (MPa)$$

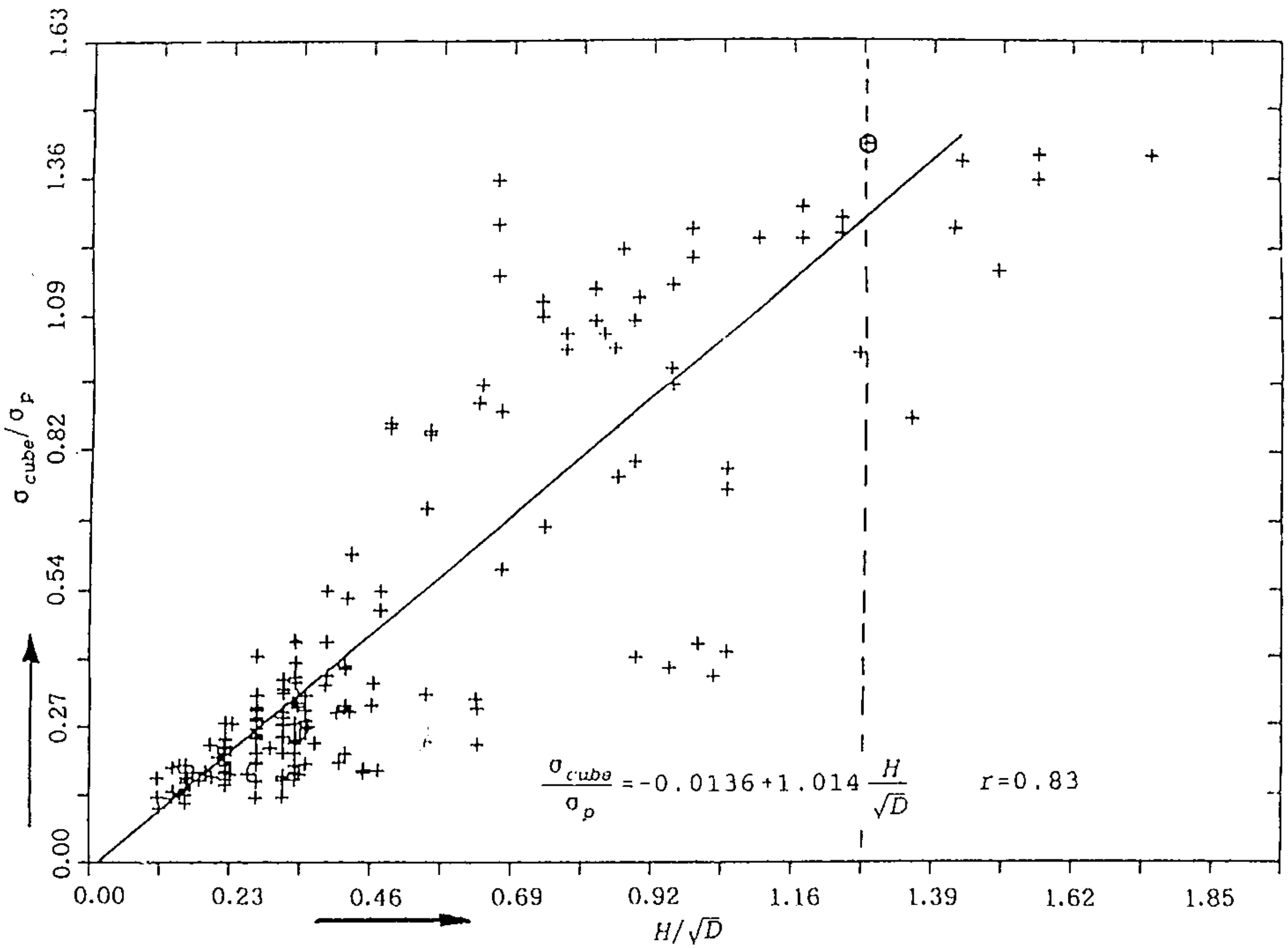


Figure 4.4 Correlations - All Coals in Inverse ($D^{1/2}/H$) Form (after [9])

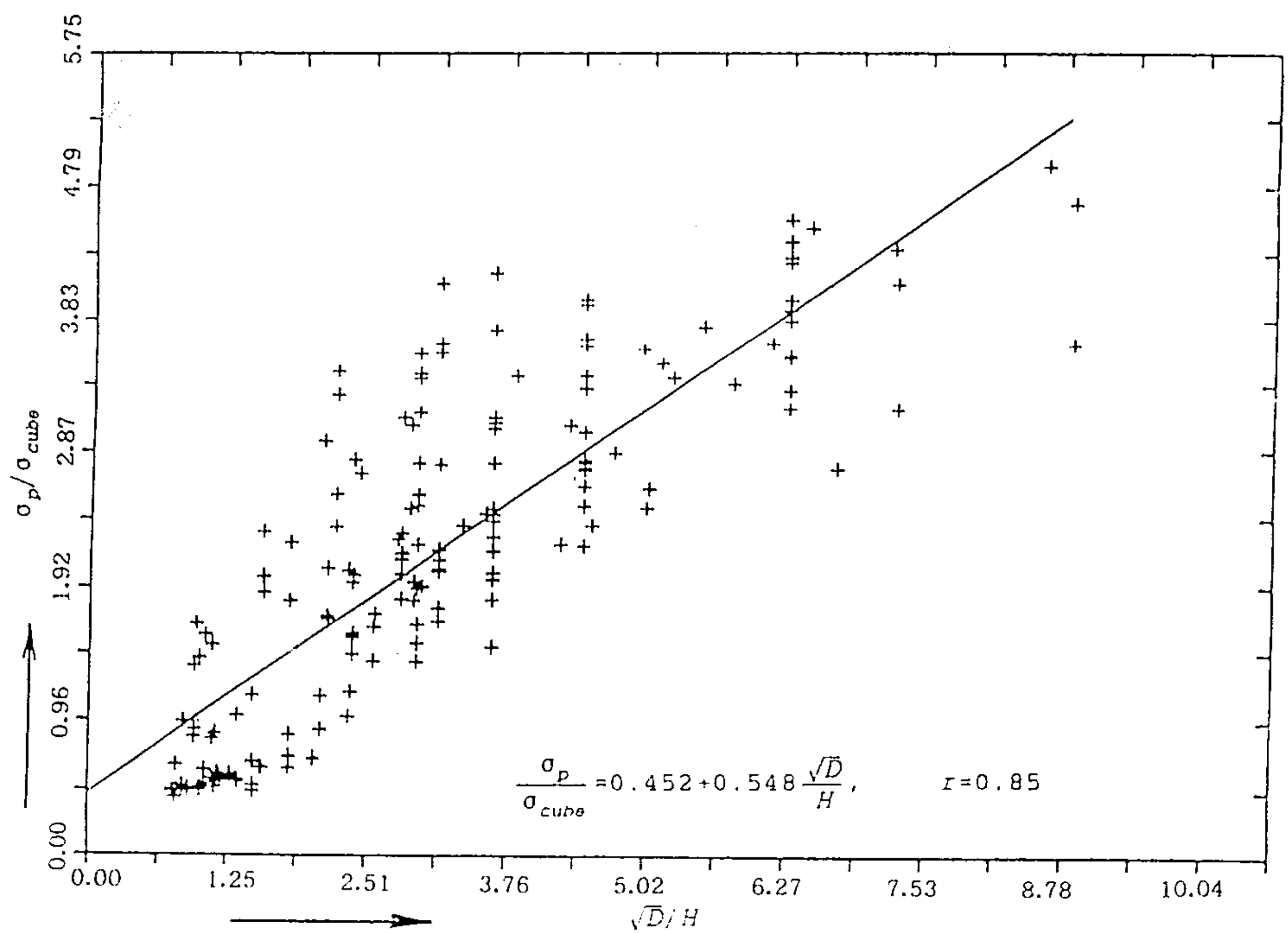


Figure 4.5 Correlations σ_p/σ_{cube} versus $D^{1/2}/H$, Where $0.784 \le D^{1/2}/H \le 8.784$ (after [9])

where D and H are in *meters* and σ_{cube} is numerically (but not dimensionally) equal to the strength of a unit meter cube.

(iii) If $D^{1/2}/H < 0.784$, the uniaxial strength of coal is again a constant and can be regarded as the coal mass uniaxial compressive strength.

4.2.6 Further limitations to the Barron/Das relationship

The author has carried out a more detailed examination of the upper and lower size limits determined by Barron and Das with the objective of examining more closely the critical specimen dimensions at these limits. This has enabled additional limits to be established as follows:

(a) $D^{1/2}/H \geq 8.784$ and $D \leq 0.013$ *meter*.

The limit $D^{1/2}/H \geq 8.784$ can be expressed as $D^{1/2} \geq 8.784H$ or $H^2 \leq D/(77.16) = 0.013\ D$. Now if $D \leq 0.013$ *meter*, then $H^2 \leq (0.013)^2$, then H must also be ≤ 0.013 *meter* for all D/H ratios, i.e. there is a critical width $D \leq 0.013$ *meter* beyond which the strength must be constant. Thus it can be stated that the prism strength becomes constant for *small* specimens (generally below the size that would normally be tested in the laboratory). This therefore could be regarded as the size below which the likelihood that the specimen contains discontinuities is remote and consequently this could be regarded as the intact uniaxial compressive strength.

(b) $D^{1/2}/H \leq 0.784$ or $D \geq 1.626$ *meters*

Firstly, examine the cube special case of $D = H$, $D^{1/2}/H = 0.784$ means $D = 1.626$ *meters*, corresponding to an H of 1.626 *meters*. Restricted by the condition of

$D^{1/2}/H \leq 0.784$, $D/H \geq 1$ when $D \geq 1.626$ *meters* and $D/H \leq 1$ when $D \leq 1.626$ *meters*. If $D \geq 1.626$ *meters* then $H^2 \geq (1.626)^2$ and hence H must also always be ≥ 1.626 *meters* for all D/H ratios. Clearly, these are quite large size specimens. Therefore the observed break down of the formula in the general form can easily be attributed to the fact that when the specimen sizes grow beyond an upper size limit the probability of containing discontinuities in them become stable and hence the strength becomes fixed. Bearing the common pillar or specimen shape (i.e. the D/H ratio) in mind, it is safe to make a conclusion that when the least lateral dimension $D \geq 1.626$ *meters*, both size and shape effect disappears and the coal strength reduces to a constant (the strength of the coal mass). Secondly, examine the practical meaning of $D^{1/2}/H < 0.784$ when D is less than 1.626 *meters*. For this a table of some D values and the corresponding H value limits beyond which the formula breaks down was prepared as shown below:

Table 4.1 D and H Value Limits of Breaking Down ($D < 1.626$ *meters* and $D^{1/2}/H = 0.784$).

D (*meter*)	H (*meter*)	D/H
0.013	0.144	0.090
0.025	0.203	0.125
0.127	0.455	0.279
0.254	0.643	0.395
0.508	0.909	0.559
0.762	1.113	0.684

From the table it is conceivable that the shape effect is very strong when $D^{1/2}/H <$ 0.784. This is especially true when D is small. Actually, such specimens with very small D values hardly exist and they must be weak strength specimens if there are any. Therefore it is proper to assume that the strength of such specimens would be a fixed value because of the as equally strong shape effects in this size range as that when D > 1.626 *meters*. It then can be assumed that this fixed strength value is the same as the constant strength of specimens for which $D \geq 1.626$ *meters*. In summary, it can be said that when specimen dimension satisfies either $D \geq 1.626$ *meters* or $D^{1/2}/H \leq 0.784$, the specimen strength becomes a constant. In practice, this lower limit $D^{1/2}/H \leq 0.784$ will only be experienced for large specimens (pillars). For pillars with fixed H while D is greater than 1.626 *meters*, it implies that pillars above some critical width will have a constant strength, for instance, cases like thin flat coal seam pillars.

(c) $D^{1/2}/H \geq 8.784$ and 1.626 *meters* > D > 0.013 *meter*

For this a table has been prepared for some values of D (1.626 *meters* > D > 0.013 *meters*) and the corresponding H limit values below which the formula breaks down, as is shown in Table 4.2.

Table 4.2 D and H Value Limits of Breaking Down (1.626 *meters* > D > 0.013 *meters* and $D^{1/2}/H = 8.784$).

D (*meter*)	H (*meter*)	D/H
0.025	0.018	1.400
0.127	0.041	3.130
0.254	0.057	4.427
0.508	0.081	6.261
0.762	0.099	7.668
1.016	0.115	8.584
1.270	0.128	9.900

It can be seen from this table that, specimens with H less than the limit values for breaking down could only possibly be met when D is very small, as soon as D grows larger the D/H ratio becomes unrealistic under the breaking down condition. As a matter of fact, specimens of such dimensions as

$$0.013\ (meter) < D < 1.626\ (meters)$$

and

$$\frac{\sqrt{D}}{H} \geq 8.784$$

represent a very small and extreme size/shape variation and are unlikely to occur in practice, therefore are of little study value. So the strength of coal specimen in this range will not be considered.

(d) The single relationship described by the formula in the general form is intrinsically more satisfying, e.g. it allows the cubes be reasonably treated as a special case of a prism. Furthermore, with this general form one does not have to differentiate between the D variation for a fixed H or vice versa. This formula simultaneously reflects both effects.

Hence, summarizing the above conclusions and adding them to the limits established by Barron and Das, the following criteria have been established:

In Category 1, i.e.:

$$0.784 < \frac{\sqrt{D}}{H} < 8.784$$

and

$$0.013\ (meter) < D < 1.626\ (meter).$$

The uniaxial compressive strength of a coal prism, σ_p, is given by:

$$\frac{\sigma_p}{\sigma_{cube}} = 0.452 + 0.548\frac{\sqrt{D}}{H}, \qquad (4.10)$$

where D and H are in *meters* and σ_{cube} is numerically (but not dimensionally) equal to the uniaxial compressive strength of a 1 *meter* cube in *MPa*.

In Category 2, i.e.:

$$\frac{\sqrt{D}}{H} \geq 8.784$$

and

$$D \leq 0.013 \ (meter),$$

the uniaxial compressive strength of a coal becomes constant and can considered to be the ***intact*** uniaxial compressive strength, σ_{pi} and is equal to the value when $D^{1/2}/H$ = 8.784, i.e.:

$$\begin{aligned} \sigma_{pi} &= constant \\ &= \sigma_{cube}[0.452+0.548(8.784)] = 5.266\sigma_{cube} \qquad (4.11) \end{aligned}$$

where σ_{cube} is defined as above.

In **Category 3**, i.e. when

$$\frac{\sqrt{D}}{H} \leq 0.784$$

or

$$D \geq 1.626 \ (meters),$$

the uniaxial compressive strength of a coal prism becomes constant and can be considered to be the uniaxial compressive strength of the coal mass, σ_{pm} and is equal to the value when $D^{1/2}/H$ = 0.784, i.e.:

$$\sigma_{pm} = constant$$
$$= \sigma_{cube}[0.452+0.548(0.784)] = 0.88\sigma_{cube} \qquad (4.12)$$

where σ_{cube} is defined as for **Category 1** above.

4.2.7 Evaluation of the σ_{pi}, m_i, m and s values.

With the conclusion of coal strength formulae made above, it is now possible to evaluate the intact m values of coal m_i, the various s values and the uniaxial compressive strength σ_{pi} from triaxial tests on non-intact coal specimens.

According to equation (4.11)

$$\sigma_{cube}=\frac{\sigma_{pi}}{5.266} \qquad (4.13)$$

Hence, substituting from equation (4.13) into equation (4.10), the following expression is obtained:

$$\sigma_p=\left(\frac{\sigma_{pi}}{5.266}\right)\times\left(0.452+0.548\frac{\sqrt{D}}{H}\right) \qquad (4.14)$$

but now equations (4.3) and (4.14) can be equated. hence:

$$\sigma_p=\left(\frac{\sigma_{pi}}{5.266}\right)\times\left(0.452+0.548\frac{\sqrt{D}}{H}\right)=\sqrt{s\sigma_{pi}^2}$$

which gives:

$$s=\left(0.086+0.104\frac{\sqrt{D}}{H}\right)^2 \qquad (4.15)$$

Thus substituting from equation (4.15) for s into equation (4.2) gives:

$$\sigma_1=\sigma_3+\sqrt{m\sigma_3\sigma_{pi}+\left(0.086+0.104\frac{\sqrt{D}}{H}\right)^2\sigma_{pi}^2}$$

Dividing this equation by s and re-arranging, the following expression is obtained:

$$\frac{(\sigma_1-\sigma_3)^2}{s}=\left(\frac{m}{s}\right)\sigma_3\sigma_{pi}+1.\sigma_{pi}^2 \qquad (4.16)$$

and comparing equation (4.11) and equation (4.1) it is seen that:

$$(\sigma_{1i}-\sigma_{3i})^2=\frac{(\sigma_1-\sigma_3)^2}{s}$$

and

$$m_i=\frac{m}{s}, \qquad s=\left(0.086+0.104\frac{\sqrt{D}}{H}\right)^2,$$

i.e.

$$\frac{(\sigma_1-\sigma_3)^2}{s}=m_i\sigma_3\sigma_{pi}+\sigma_{pi}^2 \qquad (4.17)$$

Hence, following the procedure of Hoek and Brown, as was done earlier, put:

$$\frac{(\sigma_1-\sigma_3)^2}{s}=y, \qquad \sigma_3=x,$$

then equation (4.17) becomes:

$$y=m_i\sigma_{pi}x+\sigma_{pi}^2,$$

and m_i and σ_{pi} can be evaluated from:

$$\hat{\sigma}_{pi}^2=\frac{\sum_{i=1}^{n}y_i}{n}-\left[\frac{\sum_{i=1}^{n}x_iy_i-\frac{\sum_{i=1}^{n}x_i\sum_{i=1}^{n}y_i}{n}}{\sum_{i=1}^{n}x_i^2-\frac{\left(\sum_{i=1}^{n}x_i\right)^2}{n}}\right]\frac{\sum_{i=1}^{n}x_i}{n} \quad (MPa)^2$$

$$\hat{m}_i=\frac{1}{\hat{\sigma}_{pi}}\left[\frac{\sum_{i=1}^{n}x_iy_i-\frac{\sum_{i=1}^{n}x_i\sum_{i=1}^{n}y_i}{n}}{\sum_{i=1}^{n}x_i^2-\frac{\left(\sum_{i=1}^{n}x_i\right)^2}{n}}\right],$$

and the correlation coefficient is given by:

$$r=\frac{n\cdot\sum_{i=1}^{n}x_iy_i-\left(\sum_{i=1}^{n}x_i\right)\left(\sum_{i=1}^{n}y_i\right)}{\sqrt{n\cdot\sum_{i=1}^{n}x_i^2-\left(\sum_{i=1}^{n}x_i\right)^2}\sqrt{n\cdot\sum_{i=1}^{n}y_i^2-\left(\sum_{i=1}^{n}y_i\right)^2}}$$

where $\hat{\sigma}_{pi}$ and $\hat{m}_i$ are the maximum likelihood estimates for the σ_{pi} and m_i respectively; x_i and y_i are the data pairs; n is the total number of the data pairs. Thus m_i and σ_{pi} for the intact coal can be evaluated.

If specimen sizes and shapes fall into Category 2, the way to determine the coal parameters is obviously no different from the way treating the mudstone in Chapter 3.

In Category 1, given n sets of triaxial test data (σ_1, σ_3) at failure of coal specimens with a common $D^{1/2}/H$ ratio:

$$s=\left(0.086+0.104\frac{\sqrt{D}}{H}\right)^2, \qquad m=\frac{m_i}{s},$$

thus the intact uniaxial prism strength, σ_{pi}, the intact m_i value, the intact s_i value (= 1, by definition) for this kind of coal can be determined; whereby the m and s values of specimens of this particular $D^{1/2}/H$ ratio can then be determined.

In Category 3, making use of equation (4.13) the uniaxial compressive strength of the coal mass can be expressed as

$$\sigma_{pm}=0.88\sigma_{cube}=\left(\frac{0.88}{5.266}\right)\sigma_{pi}=0.167\sigma_{pi} \qquad (4.18)$$

and equating equation (4.18) and (4.3) gives:

$$\sigma_{pm}=0.167\sigma_{pi}=\sqrt{s\sigma_{pi}^2},$$

i.e.

$$s=0.167^2=0.0279 \qquad (4.19)$$

i.e. for large specimens, there is a minimum value of s_{mass} = 0.0279, beyond which s remains constant and the strength also remains constant.

This also implies that the strength of a large coal mass (large enough that the dimensions fall into Category 3) is given by putting s = 0.0279 in equation (4.2), i.e. for coal mass:

$$\sigma_1=\sigma_3+\sqrt{m_{mass}\sigma_3\sigma_{pi}+s_{mass}\sigma_{pi}^2} \qquad (4.20)$$

where

$$m_{mass}=0.0279m_i \qquad (4.21)$$

5. RESEARCH ON THE FAILURE CRITERION FOR COAL

5.1 Triaxial Test with Coal Specimens

5.1.1 Introduction

In order to assess the validity of the theoretical/empirical relationship developed in Chapter 4, to examine whether or to what an extent the size and shape affect the failure criterion, and to determine the intact m_i, m and s values for the coal, multiple failure state triaxial compression tests were carried out on specimens of different sizes and shapes of coal obtained from the Whitewood Mine of the Fording Coal Ltd., which is located to the north of Lake Wabamun and seventy Kilometres west of Edmonton, Alberta. The Wabamun Lake district is of low relief at elevations between 700 and 800 *meters* within the plains area of central and eastern Alberta.

5.1.2 Geology and structure of the coal

The geology and structure of the coal tested is briefly summarized as below with reference to Keele [56]:

The coal bearing region in Western Alberta extends eastward from the lower slopes of the Rockies. The coal becomes younger in age and lower in rank in an easterly direction. The Wabamun Lake area is characterized by a high water table, rolling coal seams and an overburden that consists of muskeg, clay and various sedimentary layers of sand, gravel and fine-grained sandstone. A large extent of coal has outcropped along a northwest-southeast running line. The seams occur within the Scolland Member of the Paskapoo Formation of Upper Cretaceous age. Although the average dip of the seams is about 1% to the west, the beds have been deformed by glaciation into severely rolling configurations. As a result, changes in overburden depth can occur suddenly and be quite significant. At the Whitewood Mine five seams of coal having a total thickness of 6.5 *meters* are economically mineable. According to the A.S.T.M. (American Society for Testing Materials) Classification, the coal belongs to sub-bituminous C, a satisfactory grade for use as a fuel source for thermal power generation. The formations covering the seams consist of sedimentary deposits of sandstone, siltstone, bentonite shales and clays. The typical as-received properties of the coal are: moisture 22%; ash 16%; volatile matter 27%; fixed carbon 35%; sulphur 0.2% and thermal value 17.8 *MJ/kg*.

5.1.3 Test procedure

The apparatus and technique used in the coal strength tests are the same as that with mudstone, except for the triaxial cells. The coal blocks were collected from run-of-mine supplies and were sealed with plastic bags to prevent weathering and cracking of the coal after removal from the mine atmosphere, due to changes in humidity and temperature. Specimen drilling was done in the laboratory with core barrels of 0.056 and 0.068 *meter* diameter respectively. After drilling, each specimen was trimmed and ground to its required height. Like the mudstone tests, each specimen diameter and height was measured five times respectively and the minimum values were used in the following analyses. Before testing, the specimens were sealed in plastic bags to keep the humidity and moisture from changing.

Also, the testing was protected by randomization. The specimens of different sizes drilled from different blocks were applied randomized confining pressures, similar to the mudstone tests in Chapter 3. Even though the order of test runs of the two different diameters of specimens was not be able to randomized due to equipment availability restriction, the order of test runs for each diameter specimen was randomized. Having randomized the experiment in this fashion, it is believed that extraneous factors such as the equipment deterioration, experimenter fatigue will not upset the results. Since all the testing was performed in a relatively short span of time, the effect of testing time on the result is neglected and will not be analyzed.

Again the multiple failure state test technique was used. For each specimen, four confining pressure values ranging from 0 to 70.0 *MPa* were applied during the peak stress testing stage and three confining pressure values equal to the first three peak stress confining pressure values were used to determine the residual stresses. Load-deformation data were both plotted directly on the X-Y recorder and recorded in digital form at every failure point. Figure 5.1 is a copy of the test recording graph for specimen # Coal1-C. For each specimen, the test date, test start time, test end time, specimen measurements before testing, machine settings, recording graph, calculations of parameters such as peak stress σ_1, residual stress σ_r and estimated tangential modulus E at every failure stage were recorded, calculated and plotted in detail. These

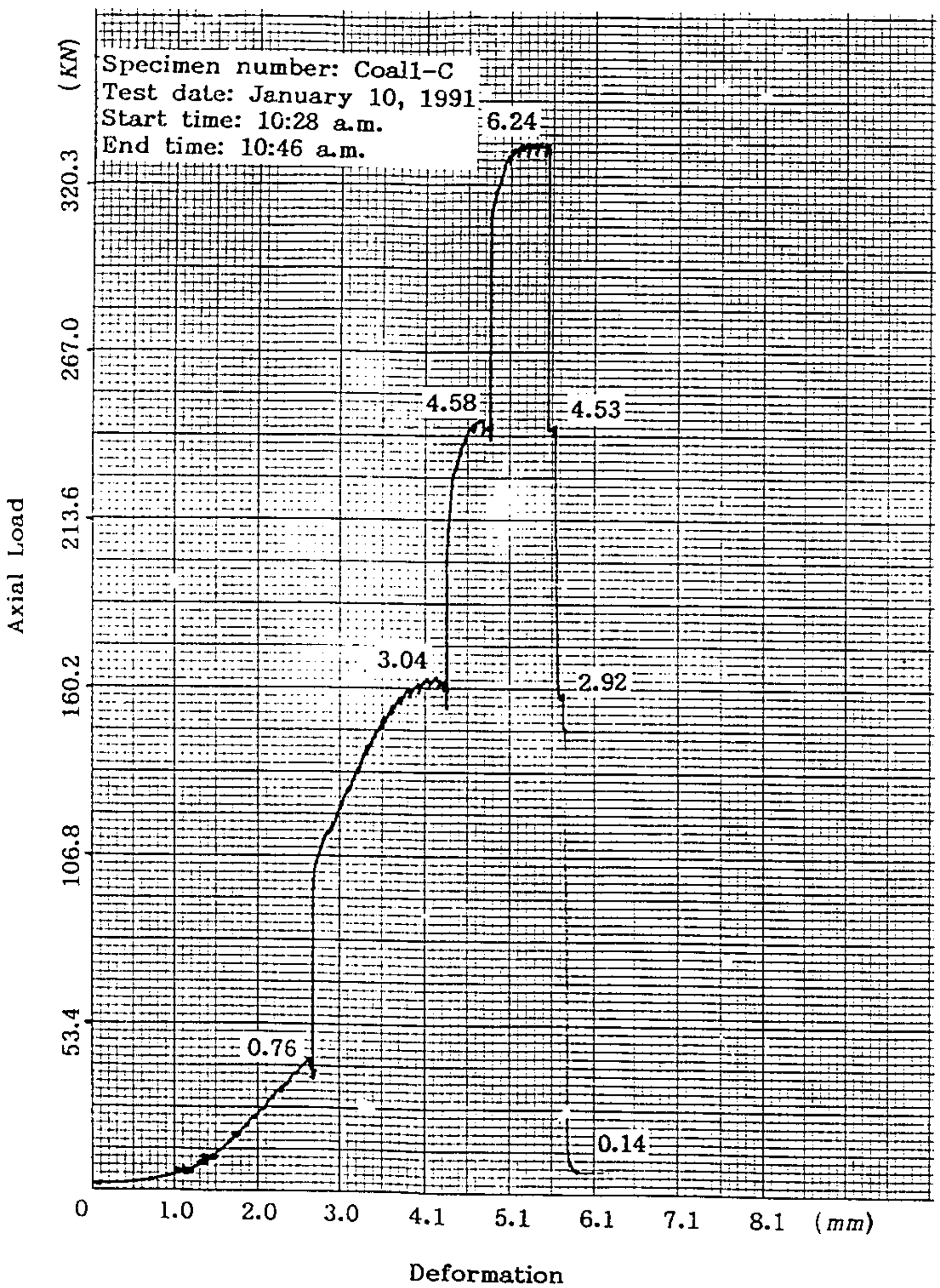


Figure 5.1 Recording Graph of Axial Load versus Deformation for Coal.

are not included in the body of the thesis due to its volume restriction, but are available for reference. Nevertheless, most of the information above can be found from Appendix 5 or Appendix 6, which will be mentioned in the following sections.

5.2 The Transition Point Assuming Non-Intact Coal Specimens

5.2.1 Determination of the intact m_i and σ_{pi} values

Assuming the approach developed in Chapter 4 is correct (as will be proved later in this chapter), the parameters such as σ_{pi}, m_i, and m were calculated (assuming non-intact coal specimens), and listed in Appendix 5.

5.2.2 Determination of the transition point for the non-intact specimens

(1) Referring to Appendix 5, the coal specimens with a diameter of 0.056 *meter* and a height of 0.080 *meter* have a common s value of 0.155, a mean m value of 0.3 and a mean σ_{pi} of 58.2 *MPa*. The following Figure 5.2 is prepared for these specimens with their peak and residual stress points, the Hoek-Brown curve for the peak stress and the linear regression line for the residual stress on it.

The linear regression line for the residual stresses in Figure 5.2 has the following form:

$$\sigma_{rc}=1.6\sigma_3+11.0 \ (MPa),$$

with a correlation coefficient of $r = 0.94$, where σ_{rc} is the residual stress for the coal.

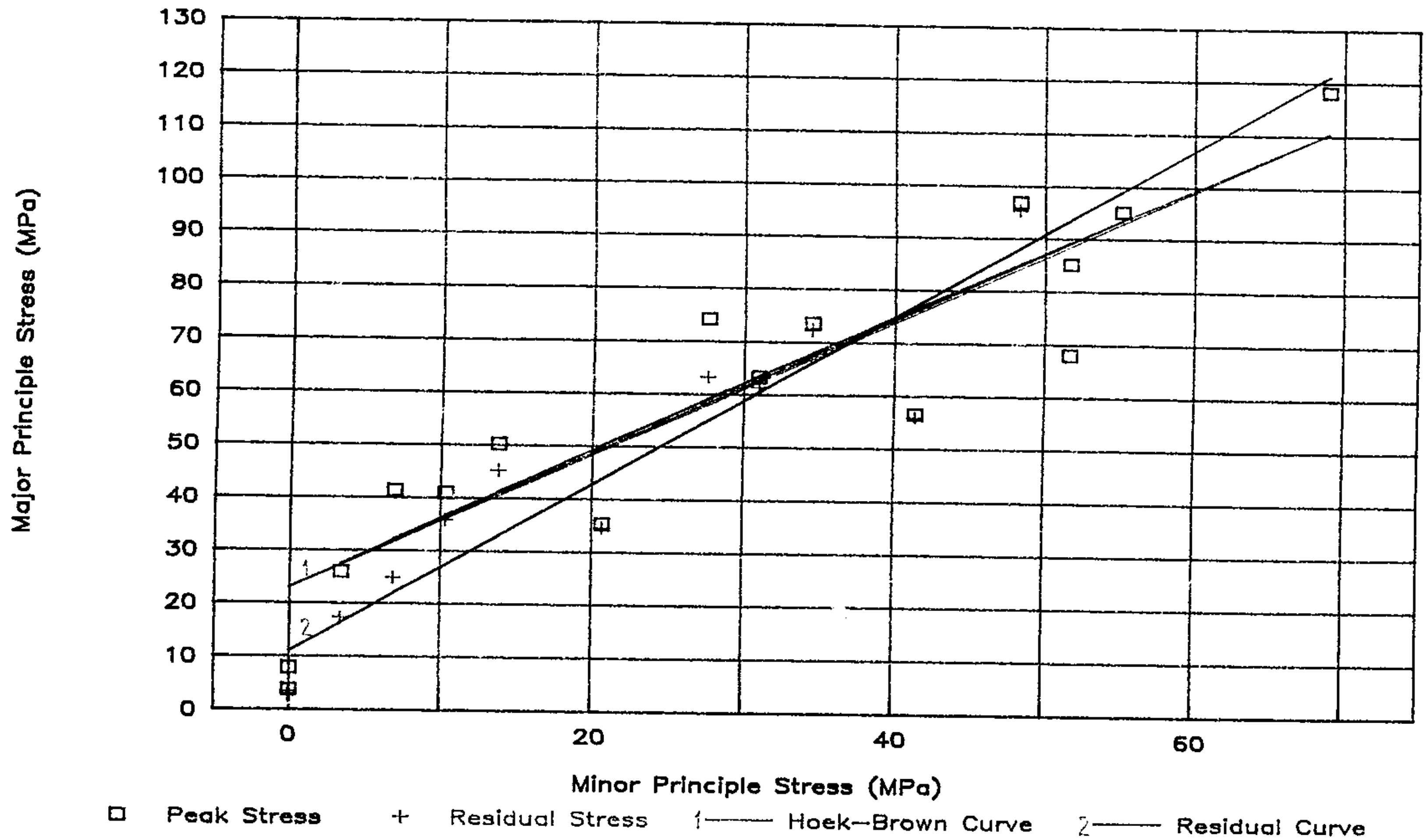


Figure 5.2 Brittle-Ductile Transition for Coal Specimens (D = 0.056 meter, H = 0.080 meter)

From this figure it can be seen that when the confining pressure exceeds about 37 *MPa* (about 5400 *psi*), the transition occurs.

(2) Similarly, for specimens with a diameter of 0.056 *meter*, a height of 0.090 *meter* and a common s of 0.129, their mean m value is calculated to be 0.09 and a mean σ_{pi} is 57.7 *MPa*. Figure 5.3 shows the test points and the transition point where the curves intersect.

The linear regression line for the residual stresses in Figure 5.3 has the following form:

$$\sigma_{rc}=1.2\sigma_3+16.7 \quad (MPa)$$

with a correlation coefficient of r = 0.95. This figure shows that the transition point occurs at a confining pressure about 45 *MPa* (about 6500 *psi*).

(3) For specimens with a diameter of 0.056 *meter*, a height of 0.120 *meter* and a common s of 0.085, their mean m value is 0.04 and the mean σ_{pi} is 66.5 *MPa*. Figure 5.4 shows the test points and the corresponding curves.

The linear regression line for the residual stresses in Figure 5.4 has the following form:

$$\sigma_{rc}=1.2\sigma_3+13.3 \quad (MPa)$$

with a correlation coefficient of r = 0.93. This figure shows that the transition point occurs at a confining pressure about 47 *MPa* (about 6800 *psi*).

(4) For specimens with a diameter of 0.068 *meter*, a height of 0.080 *meter* and

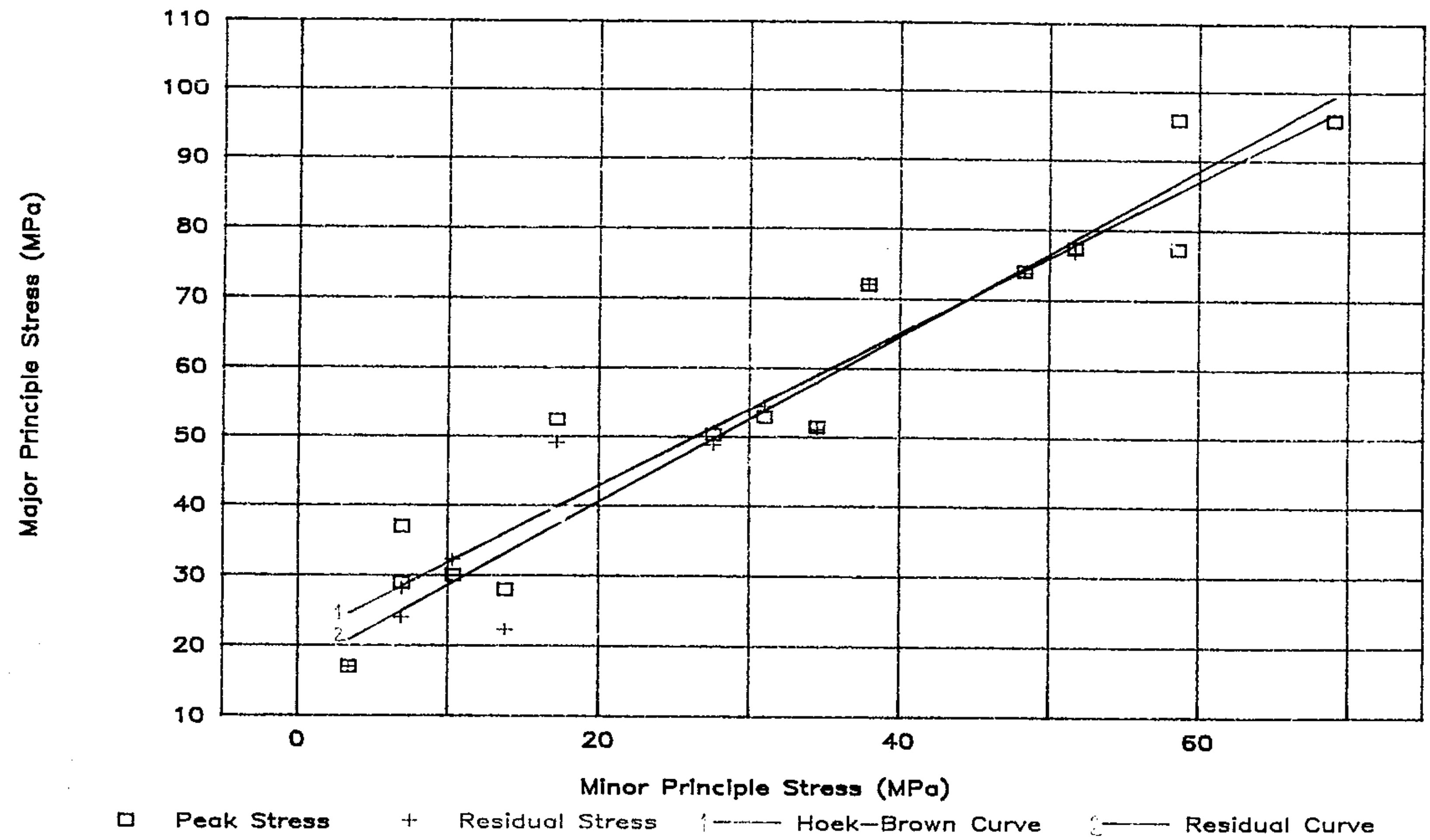


Figure 5.3 Brittle-Ductile Transition for Coal Specimens (D = 0.056 meter, H = 0.090 meter)

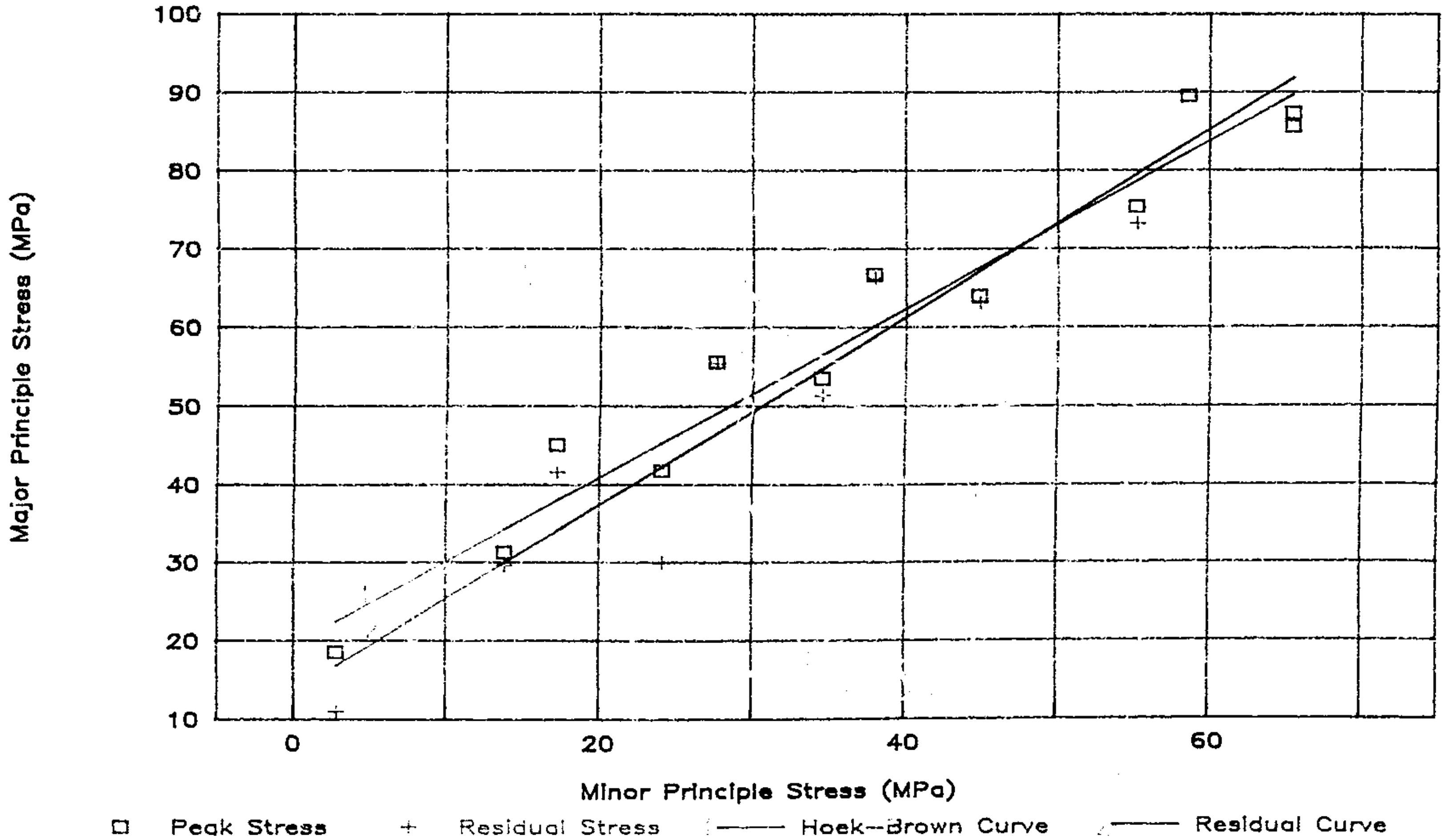


Figure 5.4 Brittle-Ductile Transition for Coal Specimens (D = 0.056 meter, H = 0.120 meter)

a common s of 0.181, their mean m value is 0.2 and the mean σ_{pi} is 58.0 *MPa*. Figure 5.5 shows the test points and the transition point.

The linear regression line for the residual stresses has the following form:

$$\sigma_{rc}=1.4\sigma_3+14.4 \quad (MPa)$$

with a correlation coefficient of r = 0.94. This figure shows that the transition point occurs at a confining pressure about 48.5 *MPa* (about 7000 *psi*).

(5) For specimens with a diameter of 0.068 *meter*, a height of 0.120 *meter* and a common s of 0.097, their mean m value is 0.06 and the mean σ_{pi} is 95.6 *MPa*. Figure 5.6 shows the test points and the corresponding curves.

The linear regression line for the residual stresses has the following form:

$$\sigma_{rc}=1.3\sigma_3+19.5 \quad (MPa)$$

with a correlation coefficient of r = 0.95. This figure shows that the transition point occurs at a confining pressure about 48.4 *MPa* (about 7000 *psi*).

5.3 Size/Shape Effect Examination Assuming *Intact* Coal Specimens

5.3.1 Determination of the parameters assuming intact coal specimens

As stated before, it is assumed that the coal can not be regarded as intact, therefore s is not equal to one and the way to determine the coal parameters such as

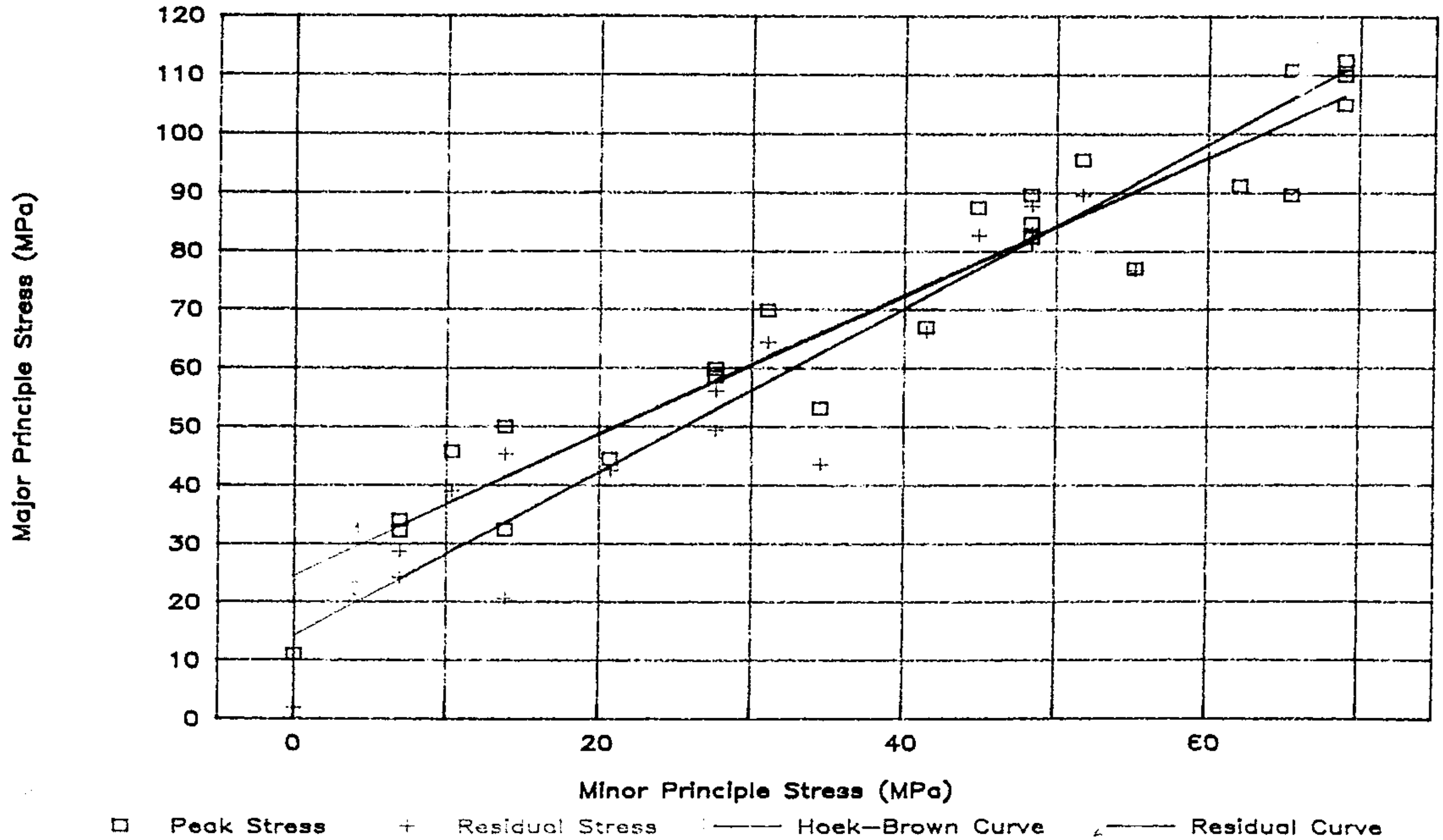


Figure 5.5 Brittle-Ductile Transition for Coal Specimens (D = 0.068 meter, H = 0.080 meter)

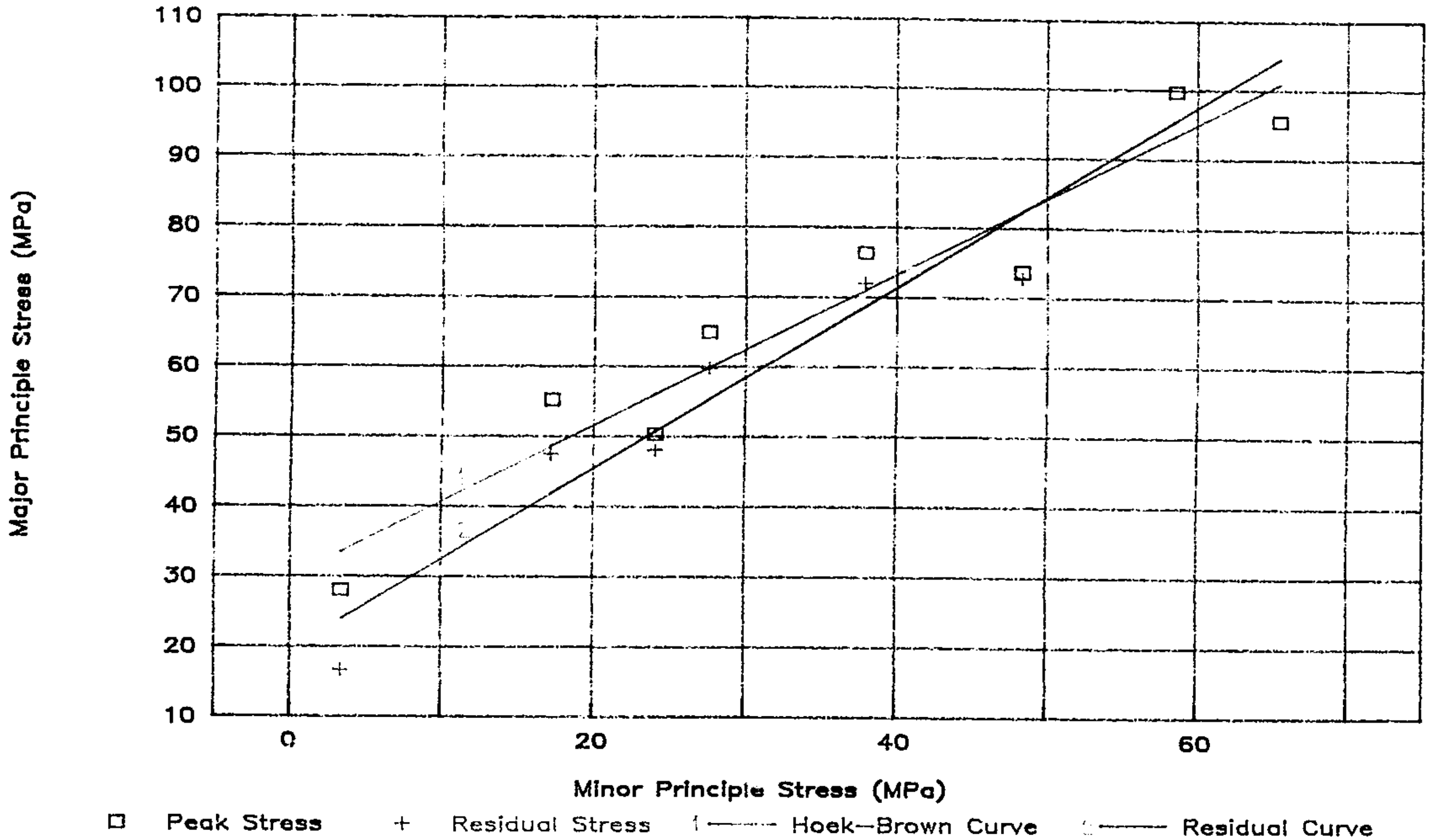


Figure 5.6 Brittle-Ductile Transition for Coal Specimens (D = 0.068 meter, H = 0.120 meter)

m has to resort to the approach developed in chapter 4. However, it is very important to verify first the assumption that the coal is *non-intact* and $s = 1$ is not justified. For this purpose a table has been prepared for each specimen to list its experimental conditions and the parameters determined assuming it is ***intact***. Appendix 6 contains such tables for all the specimens, with D and H being the minimum of the five measured diameters and heights, m and σ_{pi} being the coal parameters so determined (with s assumed to be one), ε_1 being the peak strain at the peak stress testing stage and r the correlation coefficient. Note if a stress point exceeds the transition limit determined earlier, this point is not used in the determination of the m and σ_{pi}. If more than two stress points are discarded for a specimen, the test points from that specimen are not used for the calculation and hence, a question mark is placed at the corresponding parameter position.

5.3.2 Height variation effect on the *m* values assuming intact coal

By assuming the coal specimens are *intact* ($s = 1$), the m and σ_{pi} can be found from Appendix 6 for each specimen. The coal specimen height variation effect on the m values is evaluated below.

Table 5.1 *m* Values of Different Height Groups for Intact Specimens.

H (*meter*)	0.080	0.090	0.120	Total
m	1.6 (Coal3-1)	0.3 (Coal7-4)	0.2 (Coal6-3)	
	1.2 (Coal5-2)	0.2 (Coal6-4)	0.09 (Coal7-2)	
Σ	2.8	0.5	0.29	3.6

As can be seen, the number of different heights $k = 3$. Under each height the number of tests are $n_1 = 2$, $n_2 = 2$, $n_3 = 2$ respectively. Total $n = 2 + 2 + 2 = 6$. Denote μ_1, μ_2, μ_3 as the mean values of m of the k populations corresponding to the specimen heights of H = 0.080, 0.090, 0.120 *meter*, then examine:

$$H_0:\ \mu_1=\mu_2=\mu_3$$

The squares of the m values in Table 5.1 are shown in Table 5.2:

Table 5.2 Squares of the m Values of Different Height Groups for Intact Specimens.

H (*meter*)	0.080	0.090	0.120	Total
m^2	2.3 (Coal3-1)	0.09 (Coal7-4)	0.04 (Coal6-3)	
	1.4 (Coal5-2)	0.04 (Coal6-4)	0.008 (Coal7-2)	
Σ	4.0	0.1	0.05	4.2

Hence

$$\sum_{i=1}^{k} \frac{S_i^2}{n_i} = \frac{2.8^2}{2} + \frac{0.5^2}{2} + \frac{0.3^2}{2} = 4.1,$$

$$\frac{\left(\sum_{i=1}^{k} S_i\right)^2}{n} = \frac{3.6^2}{6} = 2.2,$$

$$\sum_{i=1}^{k} SS_i = 4.2,$$

$$S_B = \sum_{i=1}^{k} \frac{S_i^2}{n_i} - \frac{\left(\sum_{i=1}^{k} S_i\right)^2}{n} = 4.1 - 2.2 = 1.9,$$

$$S_E=\sum_{i=1}^{k} SS_i-\sum_{i=1}^{k} \frac{S_i^2}{n_i}=4.2-4.1=0.1,$$

$$S_T=\sum_{i=1}^{k} SS_i-\frac{(\sum_{i=1}^{k} S_i)^2}{n}=S_B+S_E=2.0.$$

The degrees of freedom are:

$$f_B=k-1=3-1=2,\quad f_E=n-k=6-3=3,\quad f_T=n-1=6-1=5.$$

The analysis of variance table is shown as Table 5.3:

Table 5.3 Analysis of Variance of Height Effect on the *m* Values for Intact Specimens.

Sources of variation	Sum of squares	Degrees of freedom	Mean square	F
Between heights	1.9	2	1.0	
Error	0.1	3	0.03	33.3
Total	2.0	5		

Because $F_{0.05}(2, 3) = 9.6$ is less than 33.3, under the error probability of 0.05, the null hypothesis H_0 is rejected. That is, if $s = 1$ is assumed, different heights of specimens with the same diameter will give significantly different *m* values due to the height variation effect.

5.3.3 Height variation effect on the σ_{pi} values assuming intact coal

A similar analysis is performed as is put in Section A7.1 of Appendix 7. The result shows that the σ_{pi} values so determined is not significantly affected by the specimen height variation, unlike the *m* values. This may mean that the σ_{pi} values is less sensitive to the two approaches of calculation as described earlier.

5.3.4 Diameter variation effect on the *m* values assuming intact specimens

Referring to Appendix 6, choose specimens with a height of 0.080 *meter* and divide them into two groups, Group 1 and Group 2. In Group 1 all the specimens have a diameter of 0.056 *meter* while in group 2, 0.068 *meter*. Their corresponding *m* values (denoted as m_1 and m_2 respectively) from Appendix 6 are listed in Table 5.4.

Table 5.4 The *m* Values of Two Diameter Groups for Intact Specimens

Group 1				Group 2		
D (meter)	H (meter)	m_1		D (meter)	H (meter)	m_2
0.056	0.080	1.6 (Coal3-1)		0.068	0.080	1.0 (Coal1-C)
0.056	0.080	1.2 (Coal5-2)		0.068	0.080	0.6 (Coal2-A)
				0.068	0.080	0.4 (Coal4-B)
				0.068	0.080	0.5 (Coal4-E)

Regard the *m* data from Group 1 and Group 2 as two independent random samples of size $n_1 = 2$ and $n_2 = 4$ from two normal populations with means of μ_1 and μ_2 and the known variances of $V[m_1]$ and $V[m_2]$ respectively. The sample means are:

$$\bar{m}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} m_{1_i} = \frac{2.8}{2} = 1.4,$$

$$\bar{m}_2 = \frac{1}{n_2}\sum_{i=1}^{n_2} m_{2_i} = \frac{2.5}{4} = 0.6.$$

The sample variances are:

$$s_1^2 = \frac{\sum_{i=1}^{n_1} (m_{1_i} - \overline{m}_1)^2}{n_1 - 1} = \frac{0.08}{2-1} = 0.08,$$

$$s_2^2 = \frac{\sum_{i=1}^{n_2} (m_{2_i} - \overline{m}_2)^2}{n_2 - 1} = \frac{0.2}{4-1} = 0.07.$$

Before actually performing the t test concerning the means, the equality of the two population variances, i.e. the null hypothesis $V[m_1] = V[m_2]$, has to be tested. The two-sided alternative is $V[m_1] \neq V[m_1]$. The error probability chosen is

$$\alpha = 0.05.$$

Since

$$\frac{s_1^2}{s_2^2} = \frac{0.08}{0.07} = 1.1,$$

which is less than

$$F_{\alpha/2}(n_1 - 1,\ n_2 - 1) = F_{0.025}(1,\ 3) = 17.4,$$

the null hypothesis can be accepted, i.e. the two populations have the same variance and the use of the following two-sample t test is justified.

Now test the null hypothesis $\mu_1 - \mu_2 = \delta = 0$ against the two-sided alternative $\mu_1 - \mu_2 \neq 0$. Since

$$t=\frac{|\bar{m}_1-\bar{m}_2-\delta|}{\sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$=\frac{|1.4-0.6-0|}{\sqrt{\frac{1\times0.08+3\times0.07}{2+4-2}}\sqrt{\frac{1}{2}+\frac{1}{4}}}$$

$=3.4$, *which is greater than* $t_{\alpha/2}(n_1+n_2-2)=t_{0.025}(2+4-2)=2.8$,

hence the null hypothesis must be rejected, i.e. the two different groups of specimens give significantly different *m* values because of the diameter difference. In other words, specimen diameter, just like the height, has a significant effect on the *m* values so determined.

5.3.5 Diameter variation effect on σ_{pi} values assuming intact specimens

The examination of the diameter variation effect on the σ_{pi} values for assumed intact specimens, as listed in Appendix 6, can be performed in the same way as with the *m* above. The statistical analysis of the two sample *t* test is given in Section A7.2 of Appendix 7. According to the *t* test results, the differences between the σ_{pi} values of the two different diameter coal specimen groups are not significant.

5.3.6 The combined effect of height and diameter variation on *m* values assuming intact specimens

As is shown in Appendix 6, coal specimens having the same heights (0.080 or 0.120 *meter*) with different diameters (0.056 or 0.068 *meter*) and each possible height and diameter combination was tested and the m values were calculated, assuming intact specimen. Note that there is a possibility that a particular height might yield a very high or very low m value if and only if it has a particular diameter, that is, there might be joint effects , or interactions, of the two variables: height H_i and diameter D_j, where $i = 1, 2, ..., r$ and $j = 1, 2, ... s$. To account for the joint effects, look upon the m value obtained with height H_i and diameter D_j as a value assumed by a random variable having a normal distribution with a variance $V[m]$ and the mean

$$\mu_{ij}=\mu+\beta_i+\gamma_j+\epsilon_{ij}$$

where

$$\sum_{i=1}^{r}\beta_i=0, \quad \sum_{j=1}^{s}\gamma_j=0,$$

while

$$\sum_{i=1}^{r}\epsilon_{ij}=0, \quad j=1,\ 2,\ \ldots,\ s,$$

$$\sum_{j=1}^{s}\epsilon_{ij}=0, \quad i=1,\ 2,\ \ldots,\ r.$$

In order to test the hypotheses concerning the β_i, the γ_j, as well as the ε_{ij}, it is necessary to replicate, i.e. to take more than one observation of each combination of heights and diameters; otherwise there would not be an estimate of the experimental error. The test results are shown in Table 5.5

Table 5.5 *m* Values of Different Height and Diameter Combinations for Intact Specimens

		Diameter (*meter*)	
		0.056	0.068
Height (*meter*)	0.080	1.6 (Coal3-1)	0.6 (Coal2-A)
		1.2 (Coal5-2)	0.5 (Coal4-E)
	0.120	0.2 (Coal6-3)	0.04 (Coal1-D)
		0.09 (Coal7-2)	0.05 (Coal5-B)

Denote m_{ijk} as the kth value obtained with height H_i and diameter D_j, and taking t observations of each kind, the model considered becomes

$$m_{ij}=\mu+\beta_i+\gamma_j+\epsilon_{ij}+\eta_{ijk},$$

$$i=1,\ 2,\ \ldots,\ r;\quad j=1,\ 2,\ \ldots,\ s,\quad k=1,\ 2,\ \ldots,\ t.$$

where the η_{ijk} are values assumed by independent random variables having normal distributions with zero means and the common variance $V[m]$, while μ, β_i, γ_j, ε_{ij} and $V[m]$ are unknowns. The null hypotheses to be tested are:

$$H_{0_1}:\ \beta_1=\beta_2=\cdots=\beta_r=0,$$

$$H_{0_2}:\ \gamma_1=\gamma_2=\cdots=\gamma_s=0,$$

$$H_{0_3}:\ \epsilon_{11}=\cdots=\epsilon_{1s}=\cdots=\epsilon_{rs}=0.$$

The corresponding alternative hypotheses are that the respective parameters are not all equal to zero. The tests of these hypotheses are based on the analysis of the total variability of the m data, decomposing it into terms which can be attributed to differences among the heights S_H, differences among the diameters S_D, interactions (joint effects) S_I, and chance (experimental error) S_E. First denote

$$\bar{m}=\frac{1}{rst}\sum_{k=1}^{t}\sum_{j=1}^{s}\sum_{i=1}^{r}m_{ijk},$$

$$\bar{m}_{ij.}=\frac{1}{t}\sum_{k=1}^{t}m_{ijk},$$

$$\bar{m}_{i..}=\frac{1}{st}\sum_{k=1}^{t}\sum_{j=1}^{s}m_{ijk},$$

$$\bar{m}_{.j.}=\frac{1}{rt}\sum_{k=1}^{t}\sum_{i=1}^{r}m_{ijk}.$$

therefore

$$\begin{aligned}S_T&=\sum_{k=1}^{t}\sum_{j=1}^{s}\sum_{i=1}^{r}(m_{ijk}-\bar{m}_{ij.})^2\\&+st\sum_{i=1}^{r}(\bar{m}_{i..}-\bar{m})^2+rt\sum_{j=1}^{s}(\bar{m}_{.j.}-\bar{m})^2\\&+t\sum_{j=1}^{s}\sum_{i=1}^{r}(\bar{m}_{ij.}-\bar{m}_{i..}-\bar{m}_{.j.}+\bar{m})^2\\&=S_E+S_H+S_D+S_I.\end{aligned}$$

Let

$$S_{ij\cdot}=\sum_{k=1}^{t} m_{ijk},$$

$$S_{i\cdot\cdot}=\sum_{k=1}^{t}\sum_{j=1}^{s} m_{ijk}, \qquad S_{\cdot j\cdot}=\sum_{k=1}^{t}\sum_{i=1}^{r} m_{ijk},$$

$$S=\sum_{k=1}^{t}\sum_{j=1}^{s}\sum_{i=1}^{r} m_{ijk}, \qquad SS=\sum_{k=1}^{t}\sum_{j=1}^{s}\sum_{i=1}^{r} m_{ijk}^{2}.$$

Hence

$$S_{11\cdot}=2.8, \qquad S_{12\cdot}=1.1,$$

$$S_{21\cdot}=0.3, \qquad S_{22\cdot}=0.09,$$

$$S_{1\cdot\cdot}=3.9, \qquad S_{2\cdot\cdot}=0.4,$$

$$S_{\cdot 1\cdot}=3.1, \qquad S_{\cdot 2\cdot}=1.2,$$

$$S=4.3, \qquad SS=4.7.$$

and

$$S_{H}=\frac{1}{st}\sum_{i=1}^{r} S_{i\cdot\cdot}^{2}-\frac{1}{rst}S^{2}=3.8-2.3=1.5,$$

$$S_{D}=\frac{1}{rt}\sum_{j=1}^{s} S_{\cdot j\cdot}^{2}-\frac{1}{rst}S^{2}=2.8-2.3=0.5,$$

$$S_I=\frac{1}{t}\sum_{j=1}^{s}\sum_{i=1}^{r}S_{ij\cdot}^2-\frac{1}{st}\sum_{i=1}^{r}S_{i\cdot\cdot}^2-\frac{1}{rt}\sum_{j=1}^{s}S_{\cdot j\cdot}^2+\frac{1}{rst}S^2$$
$$=4.6-3.8-2.8+2.3=0.3,$$

$$S_E=SS-\frac{1}{t}\sum_{j=1}^{s}\sum_{i=1}^{r}S_{ij\cdot}^2=4.7-4.6=0.1,$$

$$S_T=SS-\frac{1}{rst}S^2=4.7-2.3=2.4.$$

The analysis of variance table for this kind of two way analysis is presented as Table 5.6

Table 5.6 The Analysis of Variance Table for the m Values Assuming Intact Specimens

Source of variation	Sum of squares	Degrees of freedom	Mean square	F
Between H's	1.5	1	1.5	$F_H = 50.0$
Between D's	0.5	1	0.5	$F_D = 16.7$
Interaction	0.3	1	0.3	$F_I = 10.0$
Error	0.1	4	0.03	
Total	2.4	7		

Because

$$F_\alpha((r-1), rs(t-1)) = F_{0.05}(1, 4) = 7.7 < F_H = 50.0,$$

the null hypothesis H_{0_1} must be rejected.

$$F_\alpha((s-1), rs(t-1)) = F_{0.05}(1, 4) = 7.7 < F_D = 16.7,$$

the null hypothesis H_{0_2} must be rejected.

$$F_\alpha((r-1)(s-1), rs(t-1)) = F_{0.05}(1, 4) = 7.7 < F_I = 10.0,$$

the null hypothesis H_{0_3} must be rejected.

Therefore, after having considered the different height and diameter combinations and their joint effects on the *m* values, it can now be concluded that both specimen height and diameter have a significant effect on the *m* values. The fact that there exists significant interactions means that there are differences in the magnitude of *m* values due to particular combinations of heights and diameters. Hence one has to be careful not to say that one diameter gives larger *m* values that the other, since this may be true only if this diameter specimen happens to have a particular height. Another important feature of the conclusion is that most of the variability of the *m* values can be interpreted as being attributed to interactions, not just to chance. It is very likely, in tne author's opinion, that if more test data were available, this joint effect may continue to prove one idea to be true: the larger the diameter and the height, the smaller the *m* values, or in other words, the specimen with the combination of the largest diameter and the largest height gives the smallest *m* value, as is exactly the case in Table 5.5.

5.3.7 The combined effect of height and diameter variation on σ_{pi} values assuming intact specimens

A similar analysis can be carried out for the σ_{pi} data. The statistical test is put Section A7.3, Appendix 7. None of the height, diameter or joint effects were found. This is by no means surprising since it has been known that the σ_{pi} values usually vary over a wide range as compared to the *m* values and specimen size and shape differences over a small range may not show the influence on the magnitude of the σ_{pi} values.

5.4 Size/Shape Effect Examination Assuming *Non-Intact* Coal Specimens

5.4.1 Re-determination of the parameters for non-intact specimens

Discarding the stress points exceeding the transition point for each size of specimen, the m_i and σ_{pi} values for each specimen were re-calculated. If more than one peak stress points exceed the transition limit because of the selected high confining pressures, the data from this specimen will not be used. The new results are listed in Appendix 8, under the section of E.C.E. International Class Number 12.

5.4.2 Height variation effect on the m_i values

Referring to the section of E.C.E. International Class Number 12 in Appendix 8, the m_i values are used for the non-intact coal specimens to do the following examination. The tables and calculations are listed in A7.4 of Appendix 7. Statistical examination showed that m_i values are still significantly influenced by the specimen heights. However, the re-determined m_i values yielded an F ratio of 29.5, which is less than the F ratio 33.3 of m values calculated in Section 5.3.2. This means that by assuming the non-intact coal the variation of the m parameter has been reduced. If the approach of calculating the m_i values is further refined or improved, it is possible that statistical analysis may prove that different specimen heights do not influence the m_i values significantly.

5.4.3 Height variation effect on the σ_{pi} values for non-intact specimens

The calculations are listed in A7.5 of Appendix 7. The analyses proved that the coal specimen height variations do not have an significant effect on the σ_{pi} values so determined with this approach.

5.4.4 Diameter variation effect on the m_i values

The statistical test is put in A7.6 of Appendix 7. The result showed that the null hypothesis that the mean m_i values of the specimens from two diameter groups are equal has been rejected. Because of the size and shape effect (reflected by the diameter variation) on the m and m_i parameters, it is again considered important to investigate further the cause of this effect and the possible treatment toward it, e.g. the approach to determine these parameters, or the effort to obtain more test data in order to study it in more detail.

5.4.5 Diameter variation effect on the σ_{pi} values for non-intact specimens

The analysis is put in A7.7 of Appendix 7. No Significant differences was found between the mean values of σ_{pi} for two different diameters.

5.4.6 The combined effect of height and diameter variation on the m_i values

The analysis is in A7.8 of Appendix 7. According to the test, heights and diameters still showed some combined effect on the m_i values and there are some joint effects between the different combinations of heights and diameters. However, by comparing the F_H and F_D ratios with those in Section 5.3.6, it is clear that both F_H and F_D have decreased. This means that the influences of height and diameter on the m_i values are not as strong as they are on the m values determined assuming intact specimens. Ideally, the m_i values of specimens with the same rank and other test conditions should not be affected by the specimen sizes and shapes, since the approach to calculated m_i has accounted for the size and shape effects already. If more test specimen data are available, it is quite likely that the statistical analyses will prove that the size and shape effects really affect the m values, not the m_i values. The number of coal tests in this thesis is limited by the difficult coring, testing condition as well as the low success rate of the testing.

5.4.7 The combined effect of height and diameter variation on the σ_{pi} values for non-intact specimens

The statistical test is put Section A7.9, Appendix 7. All the null hypotheses are accepted according to the test, i.e. no height, diameter or joint effects were found. According to the idea proposed in Chapter 4, there should be no height, diameter or the joint effects of their different combinations on the σ_{pi} values so determined as non-intact specimens having the same rank and other testing conditions. Although this result seems to agree with the idea, further proof with more test data is recommended.

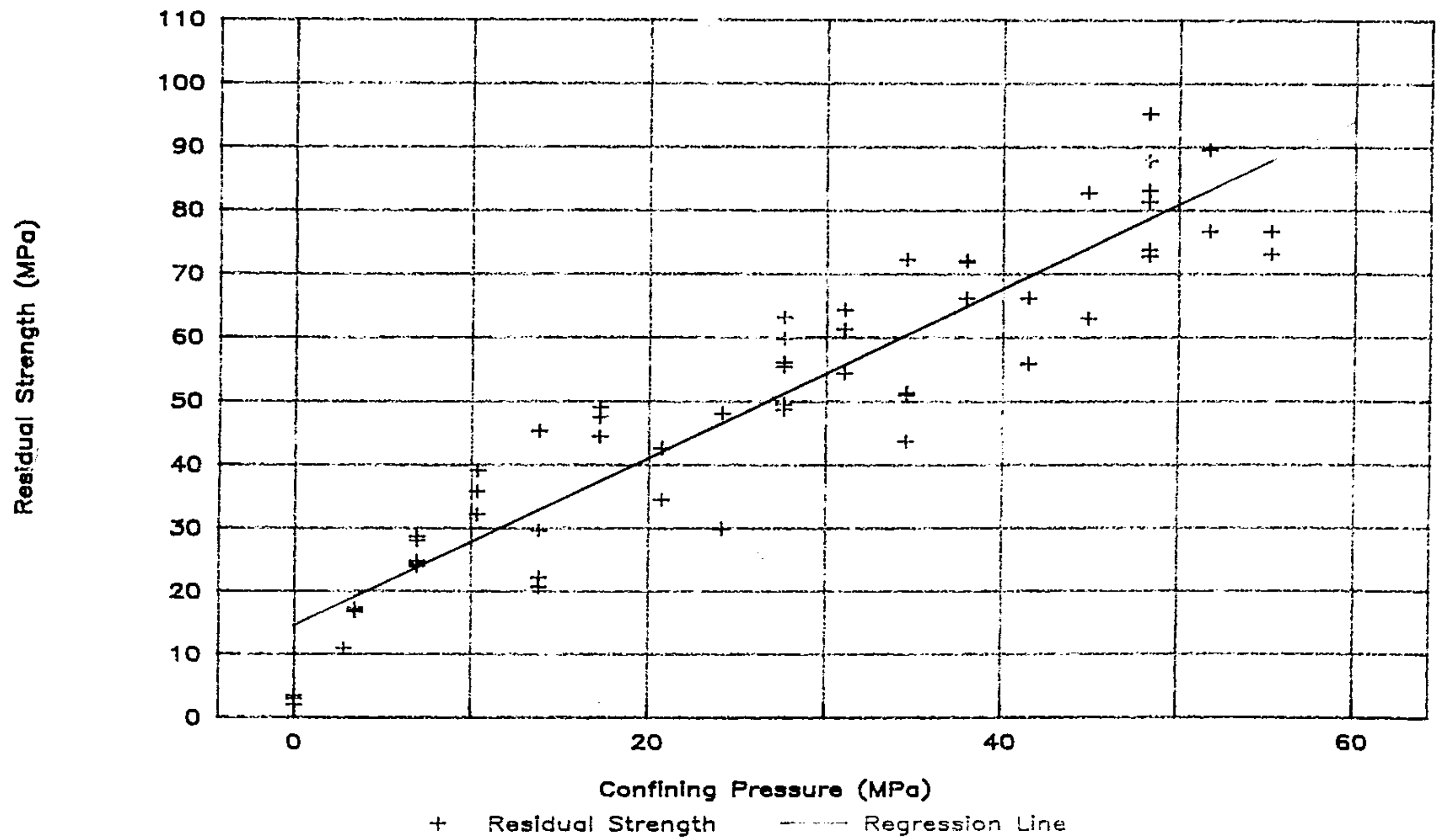
5.5 Analysis of Test Data for All the Size Groups

5.5.1 Analysis of all residual strength data

Since the previous test data examination based on the parameters calculated from five size groups defined by s = 0.085, 0.097, 0.129, 0.155 and 0.181 respectively, it is important to investigate the outcome of the testing when considering the whole data from all size groups together. For the residual strength test, a linear regression was carried out for all the data points from all the test specimens (referring to Appendix 6). Figure 5.7 shows the test data points for all the specimens and the regression line for the residual strength. The equation for the regression line is

$$\sigma_{rc}=1.3\sigma_3+14.5 \ (MPa),$$

with a correlation coefficient r = 0.93.


Residual Strength (MPa)
110
100
90
80
70
60
50
40
30
20
10
0
0
20
40
60
Confining Pressure (MPa)
+ Residual Strength
Regression Line

5.5.2 Analysis of all peak strength data ($s \neq 1$)

According to the relationship established in Chapter 4, use all the data to regress $(\sigma_1 - \sigma_3)^2/s$ versus σ_3 for the strength values and their corresponding s values. From this the following result was obtained

$$m_i=1.0, \qquad \sigma_{pi}=66.1 \ (MPa),$$

with a correlation coefficient of $r = 0.34$. Notice that the mean value of m_i for all the coal specimens from Appendix 5 is 1.0, which agrees well with the m_i value just determined above. The mean value of σ_{pi} for all specimens from Appendix 5 is 63.3 (MPa), which is also very close to the σ_{pi} calculated by lumping all the specimens. Figure 5.8 shows the transition point for the ***intact*** coal, i.e. the intersection of the Hoek-Brown curve with parameters of m_i and σ_{pi} calculated above and the linear regression line defined in Section 5.5.1. From the figure it is seen that the transition point for intact coal is approximately

$$\sigma_{3T_i}=670 \ (MPa), \qquad \sigma_{1T_i}=900 \ (MPa),$$

which are much higher strength values than that of the non-intact coal.

5.5.3 Transition point for each size group

Since $m = m_i/s$, the m values corresponding s = 0.085, 0.097, 0.129, 0.155 and 0.181 are 0.085, 0.097, 0.129, 0.155 and 0.181 respectively. According to these m and s values, the transition point for each size group can be found from the intersections

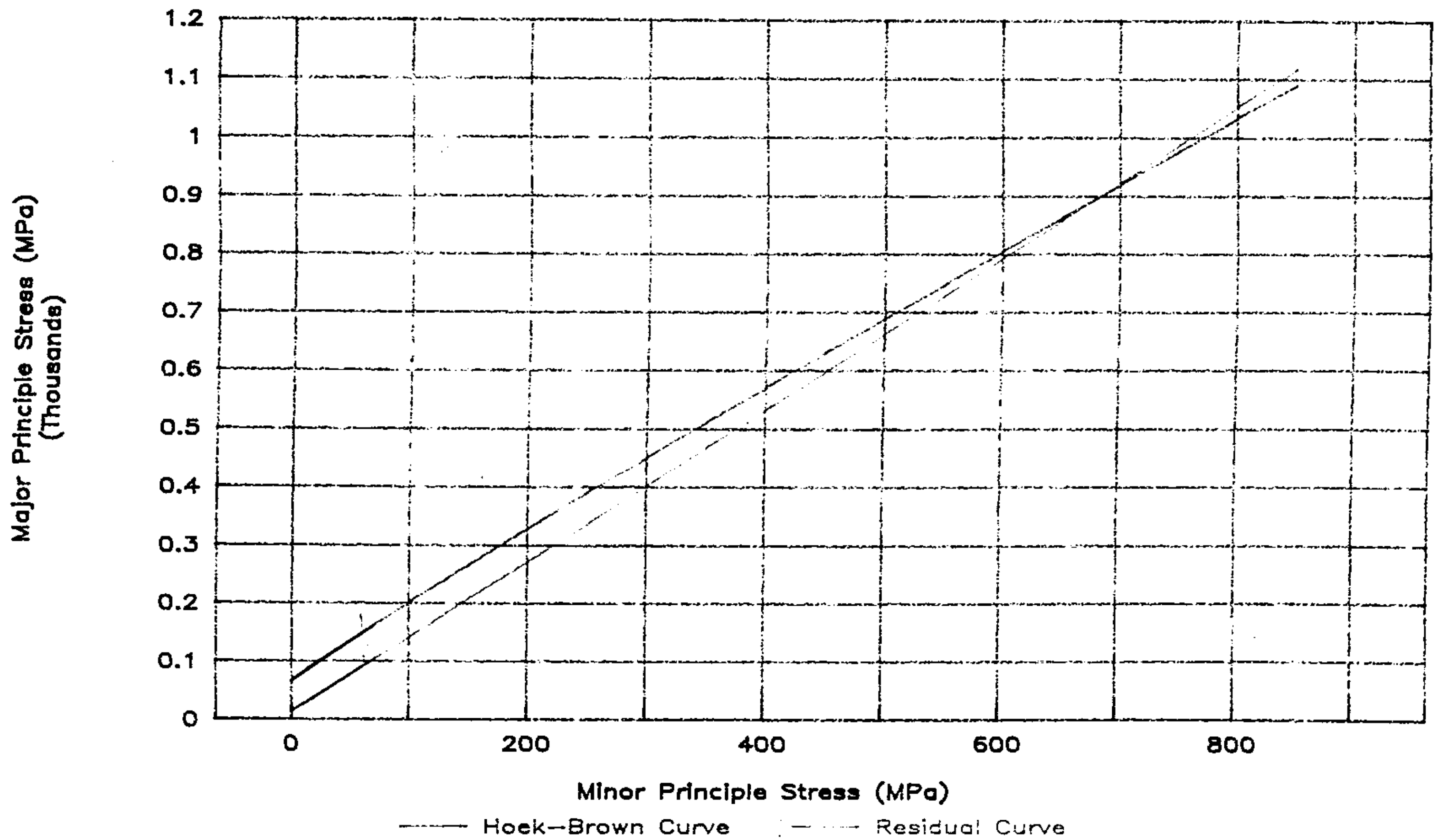


Figure 5.8 The Transition Point of the Intact Coal for All the Specimens.

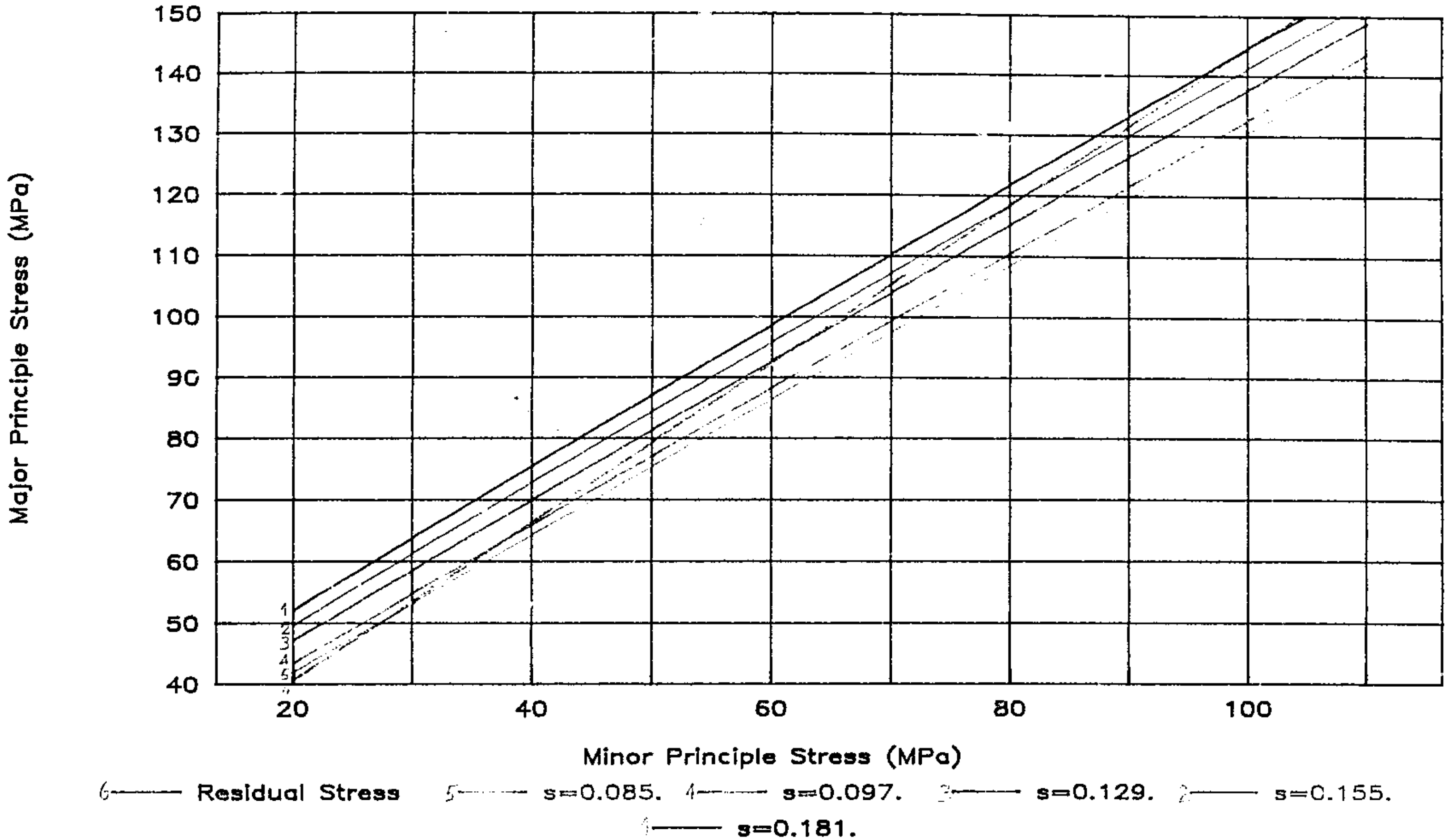


Figure 5.9 The Transition Points for Each Size Group of the Coal Specimens.

between the Hoek-Brown curves and the regression line for residual stresses, as is shown in Figure 5.9.

From the Figure 5.9, it is determined that the transition points are approximately:

$s = 0.085, \quad \sigma_3 > 28\ (MPa).$

$s = 0.097, \quad \sigma_3 > 37\ (MPa).$

$s = 0.129, \quad \sigma_3 > 62\ (MPa).$

$s = 0.155, \quad \sigma_3 > 81\ (MPa).$

$s = 0.181, \quad \sigma_3 > 100\ (MPa).$

5.5.4 The revised parameters

Rejecting those peak stress points above the transition limits, the parameters of m_i and σ_{pi} are re-calculated as m_i = 1.1 and σ_{pi} = 64.5 MPa, with a correlation coefficient r = 0.37. The revised m values corresponding to s = 0.085, 0.097, 0.129, 0.155 and 0.181 are 0.094, 0.107, 0.142, 0.171 and 0.199 respectively.

5.6 The Justification of Assuming Non-Intact Specimens

From the statistical test performed above, it can be seen that the specimen size and shape showed a strong effect on the m values calculated as intact specimens (s = 1). While by employing the approach in Chapter 4 and treating the specimens as non-intact, the specimen size and shape effects on the intact m_i values are greatly reduced.

This agrees with the theory that, after have accounted for the size and shape effect in the calculation, the m_i for all the coal specimens with the same rank and other conditions should virtually be the same. Of course, the m values determined on the basis of m_i and s vary with the specimen size and shape. This also agrees with the phenomenon observed in practice, because different size and shapes of coal specimens do have different strengths or, in other words, the strength of coal is size and shape dependant because of the size and shape dependant m and s values. So far it has been proved that the approach calculating the coal parameters as non-intact specimens is much more reasonable than the one as "intact" specimens. The author feels that it is correct to say that the coal is non-intact and the coal strength calculated by using the approach developed in Chapter 4 is more reasonable and more accurate. However, some refinement of the equations developed in Chapter 4 may still be required in order to totally eliminate size effects.

5.7 Evaluation of the Parameters for Different Coals and the Correlation with Their Ranks

5.7.1 Classification of the rank of coals

The rank of coal can be defined as its position in the coalification or carbonification series, or the degree of metamorphism to which it has been subjected. This ranges from peat (the lowest rank), through lignite and bituminous coal, to anthracite (the highest rank). The physical and chemical characteristics most used for determining coal rank are the carbon content, volatile content, moisture content, and hydrogen content, together with the calorific values and reflectance. Numerous coal rank classification systems are in current use, since many national classifications have been proposed for commercial purposes and are accordingly designed to represent the particular range of coals existing in the various countries. Consequently, different classification systems were used in the reports on coal strength tests. In order to investigate the correlation between coal parameters and its rank while making a good use of the coal strength research work from literature, it is necessary to convert various classification systems into one system. Such a conversion table has been reported by Williamson [45] and the table was reproduced as Table 5.7.

Table 5.7 Approximate Correlations of Some Coal Classification Systems (after [45])

The E.C.E (Economic Commission for Europe) International Classification system was chosen to be the one used in this thesis and all the ranks in other systems were converted into this system. This E.C.E. International Classification system was established after the Second World War by a committee appointed by the United Nations Economic Commission for Europe, as an international coal classification to enable the accurate evaluation of coals produced in different countries for purposes of international trade. Two classification systems have been proposed for hard and soft coals with calorific values respectively above and below 10260 Btu calculated on a moist, ash-free basis. Both systems employ numerical codes of three digits. The detailed classification scheme can be referenced from Williamson [45]. Because this classification system has both international characteristics and numerical class numbers, which is easy to use to correlate with coal parameters, it was therefore decided to employ this system to do the following analyses.

5.7.2 Triaxial coal strength test data collection and determination of the parameters

In order to determine the s, m_i and m values for various coals, as much coal strength triaxial test data as is possible has been gathered from literature. The collection of the data was put into Appendix 8 and was classified according to the coal rank and their size and shape categories. Note D is the diameter or the width of the specimen, L is the bigger lateral dimension of the specimen cross section (applicable to non-cylindrical specimens) and H is the height of the specimen. The tables in

Appendix 8 list the strength test data of coal gathered from the literature and the parameters calculated according to the approach developed in Chapter 4. During the recording of the data, effort was made to collect and describe as much as is practical the information about those test conditions in which the data were generated. Even though some of the information was not utilized in the following analyses, it will be a very useful collection of reference material for further studies for any purpose. Note also that although no triaxial compression test data were obtained for specimens falling into Category 2 and Category 3 (so no m_i, σ_{Pi}, or m values were so determined), still some uniaxial test data in these two categories were collected to make the collection more complete and easier to compare.

5.7.3 Correlation between rank and the intact m_i values

A linear correlation between the intact m_i values and the coal rank was done, which yielded the following equation:

$$m_i=-4.3R+55.0,$$

where m_i is the intact m value and R is the rank number of the coal. The correlation coefficient is 0.68. The test data points of m_i against their ranks and the correlation line is plotted in Figure 5.10.

It can be seen from the figure that the m_i value decreases as the coal rank

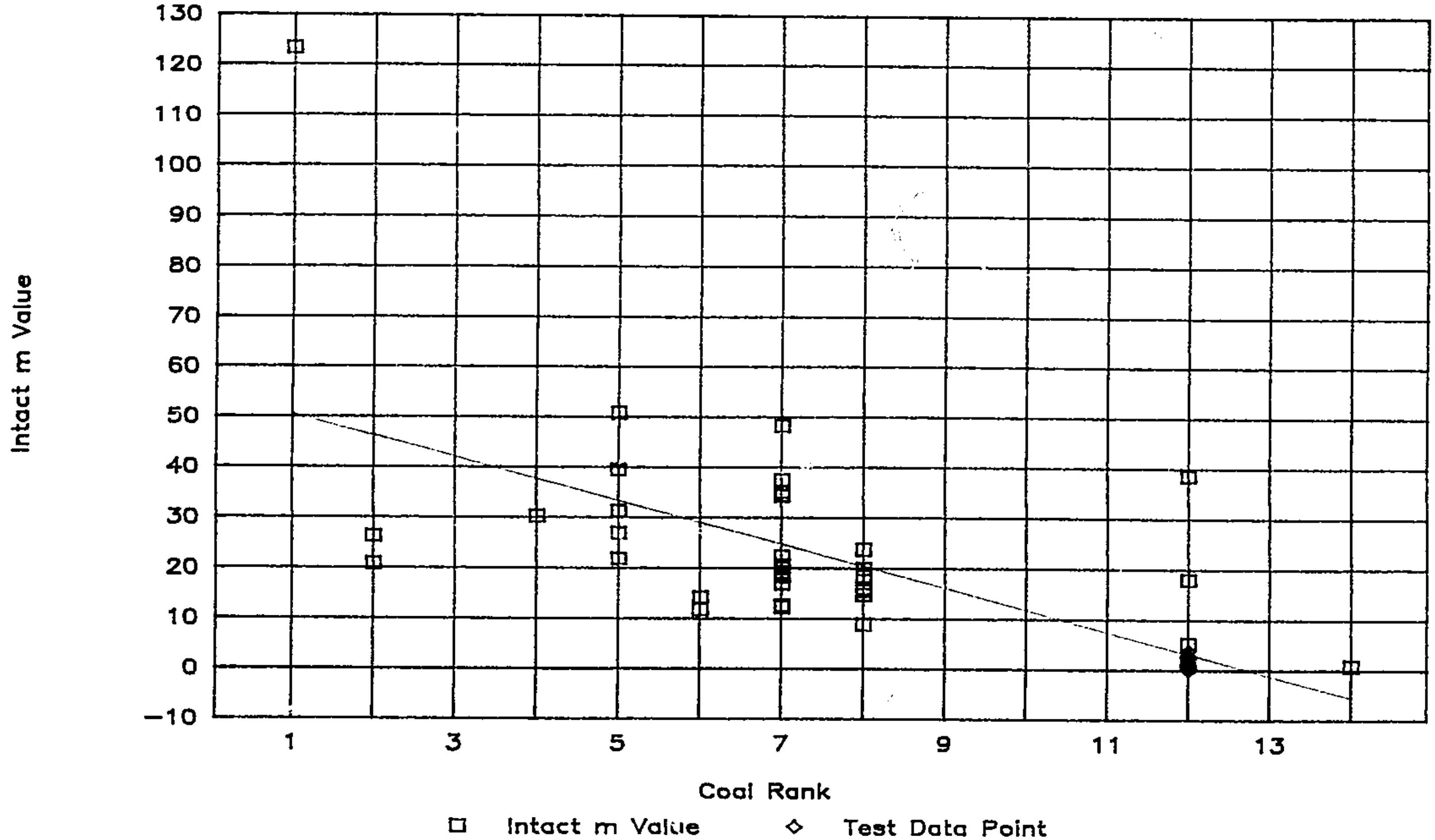


Figure 5.10 Relationship between Coal Rank and the m_i Values for Intact Coals.

number increases.

5.7.4 Correlation between rank and σ_{pi} values

The values of uniaxial compressive strength σ_{pi} of intact coals of different ranks from Appendix 8 are plotted against their ranks in Figure 5.11.

From the figure it is hard to distinguish any relation between the coal rank and the σ_{pi} values, therefore, no attempt was made to find the correlation between the σ_{pi} and the rank.

5.7.5 Influence of moisture content

It would seem likely that the strength of various coals would also be influenced by the moisture content. However, test results reported in the literature seldom, if ever, report the moisture content. Hence it was not possible to examine the influence of this parameter. This could be a useful area of future study.

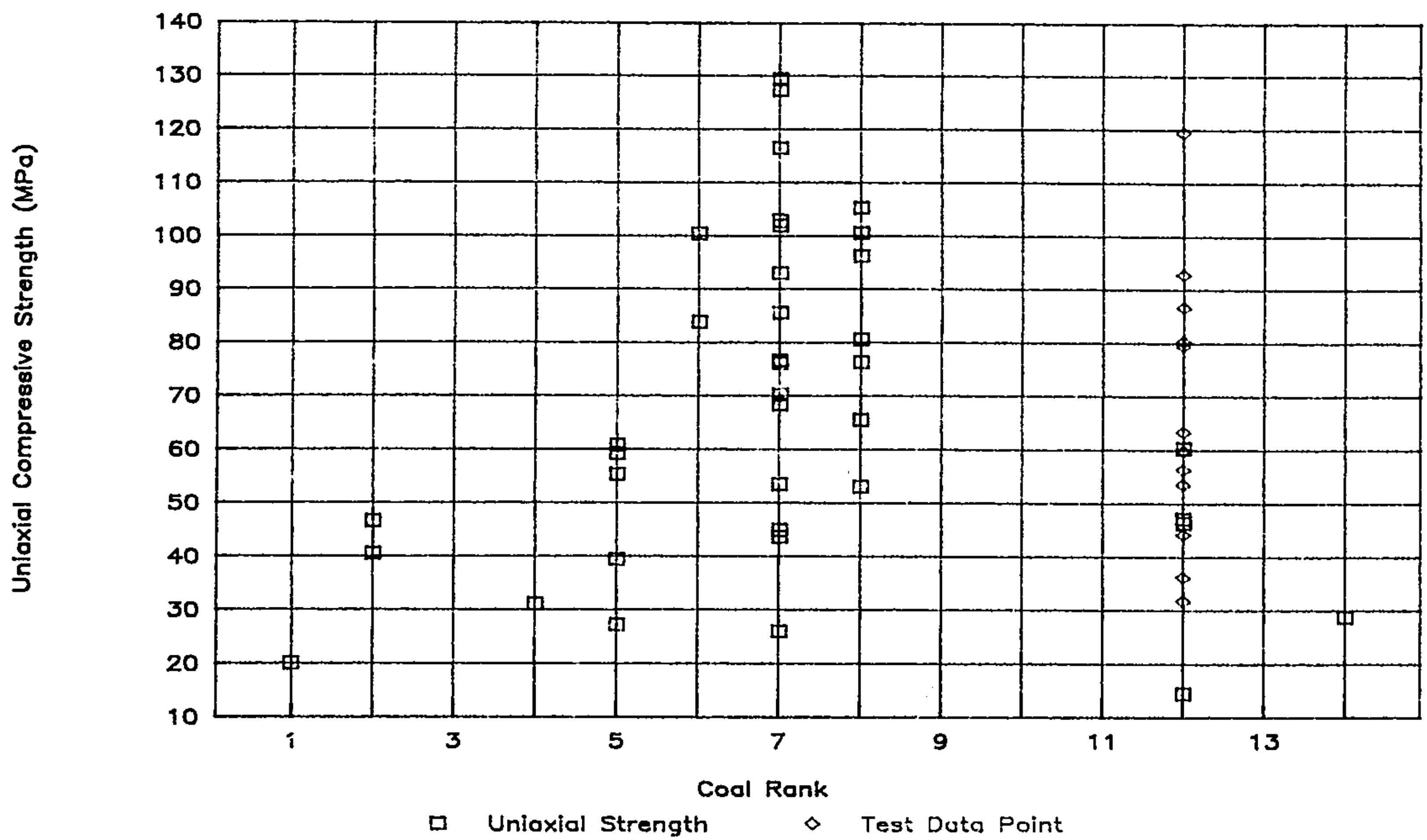


Figure 5.11 Coal Rank and the Uniaxial Compressive Strength for Intact Coals.

6. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER INVESTIGATION

6.1 Summary

The main research work carried out through this project is summarized as follows:

A comprehensive laboratory study, using the multiple failure state triaxial compressive test, of mudstone and coal specimens was carried out. Both kinds of specimens (mudstone and coal) were carefully prepared to reflect the specimen size effect, shape effect and testing time effect on the failure characteristics or the material parameters in the failure criterion. The test sequence and test conditions were randomized. For every specimen, the peak failure stress and strain, the residual stress and modulus at each failure stage were obtained; the failure stresses (σ_t and σ_r) versus the confining pressures were plotted and the material parameters were calculated. Complete statistical analyses were performed for specimens having the same diameters while the height varied, to evaluate the significance of height difference on the mechanical characteristics of the materials. Similarly, the diameter variation effects were tested statistically for specimens having the same height. Finally, specimens having different combinations of the various diameters and heights were examined simultaneously to test the significance of the effects of different combinations and joint effects on the establishment of the failure criterion. For mudstone, the testing time effect was also examined. The transition points from brittle to ductile failure were found for each kind of specimens.

A theoretical/empirical relationship between the coal specimen shape, size and the uniaxial compressive strength was established, based on which a new approach to the evaluation of coal parameters from tests on non-intact coal specimens was proposed. Subsequent coal test data and the statistical analyses supported this theory. A thorough literature review of the triaxial compressive strength test data for all rank ranges of coals were performed. A classification of the coal ranks were conducted using one ranking system by converting the ranks in other systems into this system. Utilizing the approach developed and incorporating the size and shape effects, the parameters for all the coals collected were able to be determined. A linear correlation between the coal ranks and the parameter m_i so determined was established.

6.2 Conclusions

From the investigation concerning the establishment of a failure criterion for weak sedimentary rocks, the following conclusions can be drawn:

The multiple state failure triaxial test is a reliable new technique. It can let the experimenter get several strength values from a ***single*** specimen, therefore is much more economical and time-saving than the conventional "single stage" tests. Statistical analyses on mudstone have proved that the existence of the previous failure in the multiple state failure test exerts no significant effect on the test results compared with the conventional single stage tests.

Experiments on a typical weak sedimentary rock - mudstone - have shown that rocks, as compared with coal, can be regarded as intact. The specimen size and shape, including their various combinations and testing times do not affect their mechanical properties significantly. Therefore in the determination of the material parameters in the Hoek-Brown failure criterion, s can be put into one. There exists a strong correlation between the Hoek-Brown failure model and the peak failure stresses for mudstone. Also, while proving the same Hoek-Brown failure model does not fit the residual strength data, a linear regression model

$$\hat{\sigma}_r = 4.2\sigma_3 + 13.7$$

showed significant correlation between the model and the residual strength test data. This result agrees with Mogi's research results and further, the transition pressure for the brittle-ductile failure transition point was identified to be $\sigma_{3T} = 64\ MPa$ and $\sigma_{1T} = 280\ MPa$. After discarding the stress points beyond the transition point, the Hoek-Brown failure criterion for mudstone was found to be:

$$\sigma_1 = \sigma_3 + \sqrt{10.3 \times 65.9\sigma_3 + 65.9^2 s}.$$

The Hoek-Brown failure criterion was verified to describe the mudstone test data very well.

Through a literature review and statistical analysis, the uniaxial compressive strength of coal and the specimen size and shape relationship was established as following:

In Category 1, i.e.:

$$0.784 < \frac{\sqrt{D}}{H} < 8.784$$

and

$$0.013\ (meter) < D < 1.626\ (meter),$$

The uniaxial compressive strength σ_p is defined as:

$$\frac{\sigma_p}{\sigma_{cube}} = 0.452 + 0.548\frac{\sqrt{D}}{H}\ (MPa)$$

In Category 2, i.e.:

$$\frac{\sqrt{D}}{H} \geq 8.784$$

and

$$D \leq 0.013\ (meter),$$

The strength formula for σ_p becomes:

$$\sigma_p = \sigma_{pi} = constant$$
$$= \sigma_{cube}[0.452 + 0.548(8.784)] = 5.266\sigma_{cube}.$$

In **Category 3**, i.e. when

$$\frac{\sqrt{D}}{H} \leq 0.784$$

or

$$D \geq 1.62\text{[illegible]}\ (meters),$$

the strength formula becomes

$$\sigma_p = \sigma_{Pm} = constant$$
$$= \sigma_{cube}[0.452 + 0.548(0.784)] = 0.88\sigma_{cube}.$$

With the establishment of these empirical relationships between the uniaxial compressive strength of coal and the specimen size and shape, it was then possible to evaluate s for ***non-intact*** specimens:

In **Category 1**,

$$s = (0.086 + 0.104\frac{\sqrt{D}}{H})^2.$$

In **Category 2** (intact),

$$s = 1.$$

In **Category 3** (mass),

$$s = 0.0279.$$

where D and H are in *meters*. Given s, it was shown that it is then possible to calculate the intact, m_i, and intact σ_{pi} values for the coal from the triaxial test results on non-intact specimens of convenient laboratory size and shape.

Through triaxial compressive strength tests with high confining pressures on sub-bituminous coal specimens of different sizes and shapes, a brittle-ductile transition point for this kind of coal was found to be at a confining pressure around $\sigma_{3T} = 44.8$ *MPa* (6500 *psi*) and $\sigma_{1T} = 74$ *MPa* (10700 *psi*). Statistical analyses showed significant specimen size and shape effects (and their joint effects) on the coal parameter m determined by assuming $s = 1$. Utilizing the approach developed in Chapter 4, the coal parameters were re-calculated. Statistical analyses proved that specimen size and shape effects on the parameter m_i of intact coal have been greatly reduced. This proved to some extent that the assumption - coal is non-intact and $s \neq 1$ due to size effects and shape effects - is correct and the approach calculating m_i, s, m and σ_{pi} is reasonable, if not perfect.

By employing this approach, the parameters for various coals collected from literature available were determined. A correlation between the coal ranks and the parameter m_i so determined for the intact coals were investigated and the following relationship was established:

$$m_i = -4.3R + 55.0.$$

After having incorporated the specimen size and shape effects, the failure criterion developed in this research project has been proved to describe the failure stress state of coals reasonably well. This failure criterion offers many potential advantages for use by coal mining industry to determine coal strengths under various confining pressures and of different ranks. It is easy to use and it can represent not only uniaxial but also triaxial stress states, which are more commonly encountered in practical mining.

It is believed that this investigation has contributed to the research and establishment of a failure criterion for coals.

6.3 Recommendations for Further Investigation

It is believed that the Hoek-Brown failure criterion can be adopted by the coal mining industry to calculate the coal strength as long as the coal specimens size and shape effects are taken into account, i.e. the approach developed in this thesis should be used to modify the determination of the coal parameters. Even though the experiments carried out in this project for mudstone and coal both supported the assumptions and theories proposed, direct field application data have yet to be obtained to verify or improve further this theory. The investigation of the correlation between the coal rank and its parameters is a new attempt which has not been practised before and, there is a scarcity of triaxial test data to work with because of the difficulties of performing these tests, the conclusion reached therefore is a preliminary one. Further investigation regarding this correlation relationship is recommended to modify and perfect this conclusion. For this, more triaxial compression tests on much more varied ranks of coals are needed, including the first-hand data collection from mine site observations. The more mature this theory becomes the more convenient it will be to use it and more economical benefit can it bring to the mining industry.

In addition, it has not been possible to examine the influence of moisture content on the strength of coal because this information has not been reported in the literature. Since it would seem likely that moisture content would also be a parameter affecting coal strength, it is recommended that this be studied in the future.

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APPENDIX 1. RANDOMIZED ARRANGEMENT FOR THE MULTIPLE STAGE TRIAXIAL TESTS

Specimen #	Confining pressures (*MPa*)	Day
1-L	0.0; 20.7; 41.4; 62.1	1
5-E	3.4; 24.1; 44.8; 65.5	
2-1	0.0; 20.7; 41.4; 62.1	
5-G	6.9; 27.6; 48.3; 69.0	2
1-E2	0.0; 10.3; 31.0; 51.7	
5-H	3.4; 13.8; 34.5; 55.2	
2-4	3.4; 24.1; 44.8; 65.5	3
3-E	6.9; 17.2; 37.9; 58.6	
4-I	10.3; 31.0; 51.7; 69.0	
2-E	13.8 ; 34.5; 55.2; 65.5	4
5-L	17.2; 27.6; 37.9; 58.6	
5-B	0.0; 20.7; 41.4; 51.7	
2-6	6.9; 27.6; 48.3; 69.0	5
5-I	13.8; 44.8; 48.3; 65.5	
1-4	0.0; 10.3; 31.0; 51.7	
3-D	0.0; 20.7; 41.4; 62.1	6
4-F	3.4; 24.1; 44.8; 65.5	
5-F	6.9; 27.6; 48.3; 69.0	
1-G1	0.0; 10.3; 31.0; 51.7	7
1-6	3.4; 13.8; 34.5; 51.7	
2-B	3.4; 13.8; 34.5; 55.2	

Specimen #	Confining pressures (MPa)	Day
3-8	6.9; 17.2; 37.9; 48.3	8
3-2	10.3; 38.5; 51.7; 69.0	
2-3	13.8; 34.5; 55.2; 65.5	
2-5	17.2; 27.6; 37.9; 58.6	9
1-D	6.9; 17.2; 37.9; 58.6	
1-3	0.0; 20.7; 41.4; 51.7	
3-C	10.3; 31.0; 51.7; 69.0	10
1-F	13.8; 34.5; 55.2; 65.5	
5-C	17.2 ; 27.6; 37.9; 58.6	
4-C	0.0; 20.7; 41.4; 51.7	11
1-E1	13.8; 44.8; 48.3; 65.5	
1-B	0.0; 20.7; 41.4; 62.1	
3-7	13.8; 44.8; 48.3; 65.5	12
6-B	0.0; 20.7; 41.4; 62.1	
4-E	3.4; 24.1; 44.8; 65.5;	
1-A	6.9; 27.6; 48.3; 69.0	13
1-C	0.0; 10.3; 31.0; 51.7	
3-5	0.0; 20.7; 41.4; 62.1	
5-M	3.4; 13.8; 34.5; 55.2	14
3-A2	6.9; 17.2; 37.9; 58.6	
2-D	10.3; 31.0; 51.7; 69.0	
1-I1	13.8; 34.5; 55.2; 65.5	15
4-D	17.2; 27.6; 37.9; 58.6	
1-5	3.4; 24.1; 44.8; 65.5	

Specimen #	Confining pressures (*MPa*)	Day
1-I	6.9; 27.6; 48.3; 69.0	16
3-1	0.0; 10.3; 31.0; 51.7	
5-N	0.0; 20.7; 41.4; 51.7	
3-3	13.8; 44.8; 48.3; 65.5	17
2-7	3.4; 13.8; 34.5; 55.2	
3-A1	6.9; 17.2; 37.9; 62.1	
3-6	10.3 ; 31.0; 51.7; 69.0	18
1-K	3.4; 24.1; 44.8; 65.5	
5-J	6.9; 27.6; 48.3; 69.0	
4-A	0.0; 10.3; 31.0; 51.7	19
1-2	13.8; 34.5; 55.2; 65.5	
1-J	3.4; 13.8; 34.5; 55.2	
4-H	6.9; 17.2; 37.9; 58.6	20
1-G2	10.3; 31.0; 51.7; 69.0	
2-A	13.8; 34.5; 55.2; 65.5	
3-B	17.2; 27.6; 37.9; 58.6	21
4-B	0.0; 20.7; 41.4; 51.7	
1-H	13.8; 44.8; 48.3; 65.5	
2-2	17.2; 27.6; 37.9; 58.6	22
5-D	0.0; 20.7; 41.4; 62.1	
3-4	0.0; 20.7; 41.4; 51.7	
5-A	3.4; 24.1; 44.8; 65.5	23
2-C	6.9; 27.6; 48.3; 69.0	
1-I2	0.0; 10.3; 31.0; 51.7	

Specimen #	Confining pressures (*MPa*)	Day
4-G	3.4; 13.8; 34.5; 55.2	24
1-7	13.8; 44.8; 48.3; 65.5	
2-8	0.0; 20.7; 41.4; 62.1	

APPENDIX 2. RESULTS OF EXPERIMENTS ON MUDSTONE

1-A Time: Start: 9:44 End: 9:59 Date: July 31, 1989

D (m)	0.030		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.071		6.9	121.4	49.4	0.01376	12413.3
m	13.27		27.6	205.9	148.2	0.01792	22337.1
σ_c (MPa)	73.7		48.3	274.5	234.5	0.02263	25778.4
r^2	1.00		69.0	341.3		0.02715	28612.7

1-B Time: Start: 16:47 End: 17:25 Date: July 21, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.048		0.0	38.4	0.0	0.01005	4510.2
m	19.59		20.7	166.2	105.5	0.02460	9310.0
σ_c (MPa)	43.3		41.4	227.7	187.0	0.03069	19337.2
r^2	0.99		62.1	297.6		0.03704	20351.0

1-C Time: Start: 10:08 End: 10:23 Date: July 31, 1989

D (m)	0.030		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.050		0.0	49.3	5.3	0.01633	3461.9
m	9.90		10.3	110.3	64.1	0.02297	10606.5
σ_c (MPa)	57.5		31.0	177.8	147.4	0.02782	16675.3
r^2	1.00		51.7	231.2		0.03292	18378.6

1-D Time: Start: 10:25 End: 10:50 Date: July 19, 1989

D (m)	0.030		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.056		6.9	101.8	43.9	0.01508	10227.2
m	10.21		17.2	128.6	90.1	0.01804	17764.9
σ_c (MPa)	58.6		37.9	196.4	166.5	0.02330	21578.5
r^2	0.99		58.6	256.9		0.02832	22757.8

1-E1 Time: Start: 16:50 End: 16:31 Date: July 21, 1989

			σ_3 (*MPa*)	σ_1 (*MPa*)	σ_r (*MPa*)	ε_1	*E* (*MPa*)
D (*m*)	0.029						
H (*m*)	0.056		13.8	117.0	77.6	0.03312	4199.8
m	8.82		44.8	187.6	180.3	0.04461	11130.6
σ_c (*MPa*)	52.4		48.3	203.9	196.1	0.04687	16951.1
r^2	0.94		65.5	253.8		0.05408	17689.0

1-E2 Time: Start: 10:00 End: 10:40 Date: July 13, 1989

			σ_3 (*MPa*)	σ_1 (*MPa*)	σ_r (*MPa*)	ε_1	*E* (*MPa*)
D (*m*)	0.029						
H (*m*)	0.060		0.0	38.9	1.6	0.01022	4648.1
m	5.91		10.3	97.7	49.9	0.01711	8806.6
σ_c (*MPa*)	51.2		31.0	135.8	121.6	0.02222	13027.1
r^2	0.96		51.7	187.7		0.02667	15799.4

1-G2 Time: Start: 10:53 End: 11:11 Date: July 25, 1989

			σ_3 (*MPa*)	σ_1 (*MPa*)	σ_r (*MPa*)	ε_1	*E* (*MPa*)
D (*m*)	0.029						
H (*m*)	0.054		10.3	124.5	59.6	0.01901	10289.3
m	3.87		31.0	170.7	138.2	0.02441	17702.8
σ_c (*MPa*)	92.8		51.7	210.8	206.7	0.03028	18516.6
r^2	0.98		69.0	255.4		0.03427	19392.4

1-H Time: Start: 19:00 End: 19:18 Date: Oct. 7, 1989

			σ_3 (*MPa*)	σ_1 (*MPa*)	σ_r (*MPa*)	ε_1	*E* (*MPa*)
D (*m*)	0.029						
H (*m*)	0.068		13.8	140.3	69.9	0.01720	13509.9
m	6.33		44.8	237.2	188.1	0.02355	19737.2
σ_c (*MPa*)	93.4		48.3	242.5	202.4	0.02505	23461.2
r^2	0.99		65.5	280.1		0.02804	25771.5

1-J Time: Start: 10:50 End: 11:05 Date: July 24, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.067		3.4	64.5	22.3	0.01458	3896.4
m	8.40		13.8	101.8	61.2	0.01989	12751.3
σ_c (MPa)	47.3		34.5	159.3	143.9	0.02652	16482.2
r^2	1.00		55.2	211.2		0.03390	17813.1

1-K Time: Start: 17:04 End: 17:19 Date: Oct. 7, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.071		3.4	107.4	25.2	0.01126	12840.9
m	6.53		24.1	184.7	117.6	0.01654	19240.7
σ_c (MPa)	97.0		44.8	239.6	196.9	0.02023	24792.2
r^2	1.00		65.5	290.1		0.02287	27240.4

1-LA Time: Start: 10:00 End: 11:00 Date: July 12, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.051		0.0	12.6	2.0	0.01115	2524.1
m	29.85		20.7	109.9	98.9	0.02985	5992.9
σ_c (MPa)	12.6		41.4	?	179.4	?	?
r^2	1.00		62.1	?		?	?

1-LB Time: Start: 11:15 End: 11:43 Date: July 31, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.048		0.0	55.5	1.2	0.01143	7537.7
m	7.50		20.7	134.6	90.0	0.01792	12489.2
σ_c (MPa)	63.9		41.4	211.6	168.2	0.02494	14675.3
r^2	0.93		62.1	237.6		0.02857	18385.5

2-A Time: Start: 11:20 End: 11:38 Date: July 25, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.043		13.8	179.0	84.8	0.02618	10178.9
m	14.61		34.5	268.2	176.9	0.03441	13475.4
σ_c (MPa)	92.3		55.2	343.4	267.8	0.04059	15427.0
r^2	1.00		65.5	?		?	?

2-B Time: Start: 10:41 End: 10:58 Date: July 27, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.047		3.4	55.7	24.409	0.01019	6592.9
m	36.89		13.8	118.8	71.192	0.01717	10827.2
σ_c (MPa)	19.8		34.5	192.0	159.063	0.02361	14261.5
r^2	1.00		55.2	257.5		0.02977	17033.9

2-D Time: Start: 10:40 End: 11:00 Date: July 22, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.060		10.4	109.5	61.2	0.01520	9606.5
m	13.98		31.0	180.8	146.3	0.02111	16633.9
σ_c (MPa)	47.9		51.7	242.0	237.2	0.02638	22626.8
r^2	1.00		69.0	290.7		0.03166	26385.2

2-E Time: Start: 10:10 End: 10:45 Date: July 17, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.067		6.9 13.8	111.1	36.9	0.01084	12040.9
m	6.96		34.5	172.3	136.2	0.01510	19688.9
σ_c (MPa)	62.0		55.2	224.2	206.3	0.01936	22447.5
r^2	0.99		65.5	242.4		0.02130	25909.4

3-A1 Time: Start: 15:40 End: 16:00 Date: Oct. 7, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.067		0.0	76.6	0.0	0.02048	3848.1
m	11.99		20.7	158.5	132.8	0.02882	11751.3
σ_c (MPa)	62.8		41.4	215.8	215.0	0.03375	17468.3
r^2	0.97		62.1	294.4		0.04115	19895.8

3-A2 Time: Start: 10:04 End: 10:30 Date: July 22, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.062		6.9	102.2	45.0	0.01253	11765.1
m	3.86		17.2	128.5	92.8	0.01602	17095.9
σ_c (MPa)	82.9		37.9	?	183.2	?	?
r^2	1.00		58.6	?		?	?

3-C Time: Start: 10:25 End: 10:47 Date: July 20, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.075		10.3	190.2	63.5	0.01327	21461.2
m	6.09		31.0	276.0	153.4	0.01684	30991.9
σ_c (MPa)	157.2		51.7	315.2	247.7	0.01956	32688.3
r^2	0.97		69.0	372.3		0.02296	32688.3

4-A Time: Start: 10:14 End: 10:30 Date: July 24, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.068		0.0	50.5	2.0	0.01217	3958.5
m	14.27		10.3	98.2	57.5	0.01816	9799.6
σ_c (MPa)	41.2		31.0	165.0	151.6	0.02509	16758.0
r^2	0.99		51.7	234.3		0.03146	18847.6

4-B Time: Start: 18:38 End: 18:56 Date: Oct. 7, 1989

			σ_3 *(MPa)*	σ_1 *(MPa)*	σ_r *(MPa)*	ε_1	*E (MPa)*
D (m)	0.029						
H (m)	0.060		0.0	63.9	0.0	0.01945	3110.2
m	12.00		20.7	206.9	113.5	0.03206	12716.8
σ_c *(MPa)*	86.8		41.4	267.5	210.6	0.03741	19647.6
r^2	0.97		51.7	294.2		0.03955	23716.4

4-E Time: Start: 11:25 End: 11:40 Date: July 30, 1989

			σ_3 *(MPa)*	σ_1 *(MPa)*	σ_r *(MPa)*	ε_1	*E (MPa)*
D (m)	0.029						
H (m)	0.044		3.4	74.6	28.7	0.01311	6330.8
m	21.77		24.1	160.6	131.9	0.02185	14413.3
σ_c *(MPa)*	34.2		44.8	222.9	219.2	0.02768	16564.9
r^2	0.99		65.5	293.7		0.03497	19316.5

4-F Time: Start: 10:30 End: 10:50 Date: July 18, 1989

			σ_3 *(MPa)*	σ_1 *(MPa)*	σ_r *(MPa)*	ε_1	*E (MPa)*
D (m)	0.029						
H (m)	0.048		3.4	83.0	28.2	0.02708	8420.4
m	15.52		24.1	155.0	116.1	0.03385	16840.8
σ_c *(MPa)*	47.6		44.8	236.8	206.5	0.04115	21571.6
r^2	0.99		65.5	290.8		0.04583	22806.1

4-H Time: Start: 10:30 End: 10:46 Date: July 25, 1989

			σ_3 *(MPa)*	σ_1 *(MPa)*	σ_r *(MPa)*	ε_1	*E (MPa)*
D (m)	0.029						
H (m)	0.054		3.4	91.6	46.6	0.01638	8737.6
m	7.03		17.2	135.8	94.1	0.02012	13985.7
σ_c *(MPa)*	74.9		37.9	199.2	173.4	0.02761	17151.1
r^2	1.00		58.6	249.5		0.03159	24012.9

4-I Time: Start: 10:50 End: 11:14 Date: July 14, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.030		10.3	131.9	53.1	0.02252	8144.5
m	4.13		31.0	219.2	133.5	0.03128	12116.8
σ_c (MPa)	122.1		51.7	271.9	210.3	0.03753	13447.8
r^2	0.76		69.0	275.6		0.04337	13447.8

5-B Time: Start: 12:00 End: 12:21 Date: July 17, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.044		0.0	77.0	1.2	0.01994	7875.6
m	10.07		20.7	161.8	123.9	0.03333	11861.6
σ_c (MPa)	74.0		41.4	233.5	212.7	0.03869	18813.1
r^2	1.00		51.7	261.2		0.04226	21068.2

5-F Time: Start: 10:59 End: 11:13 Date: July 18, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.048		6.9	103.5	45.8	0.01105	9261.7
m	13.69		27.6	160.7	134.6	0.01684	19233.8
σ_c (MPa)	48.2		48.3	228.2	216.3	0.02237	21350.9
r^2	0.98		69.0	292.4		0.03184	21350.9

5-H Time: Start: 10:50 End: 11:20 Date: July 13, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.044		3.4	79.5	33.6	0.01790	5103.3
m	11.17		13.8	137.8	81.9	0.02540	11682.3
σ_c (MPa)	63.8		34.5	200.3	176.3	0.03176	15054.6
r^2	0.99		55.2	264.3		0.03897	17378.7

5-I Time: Start: 9:18 End: 9:36 Date: July 26, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.051		13.8	151.6	73.0	0.01849	11523.7
m	12.66		44.8	292.4	221.8	0.02774	17302.8
σ_c (MPa)	84.5		48.3	293.6	239.2	0.02874	21019.9
r^2	0.96		65.5	335.1		0.03198	22171.6

5-J Time: Start: 16:37 End: 16:56 Date: Oct. 7, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.062		6.9	122.1	52.8	0.02454	10089.3
m	22.94		27.6	232.7	140.7	0.03333	13882.3
σ_c (MPa)	60.7		48.3	?	217.8	?	?
r^2	1.00		69.0	?		?	?

5-[illegible] Time: Start: 9:15 End: 9:40 Date: July 22, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.054		3.4	69.2	26.4	0.02547	2972.3
m	19.28		13.8	111.5	79.7	0.04057	6496.3
σ_c (MPa)	34.6		34.5	188.8	171.7	0.05094	12785.7
r^2	1.00		55.2	251.0		0.05849	15558.1

6-A Time: Start: 14:04 End: 14:22 Date: Oct. 7, 1989

D (m)	0.029		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.051		0.0	37.3	0.0	0.00662	6930.8
m	16.61		20.7	187.7	100.5	0.01791	13751.2
σ_c (MPa)	60.6		41.4	254.6	189.7	0.02257	20447.5
r^2	0.99		62.1	317.0		0.02699	21523.4

1-1 Time: Start: 10:40 End: 11:00 Date: July 23, 1989

D (m)	0.053		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.106		6.9	116.3	41.4	0.01139	14434.0
m	9.12		27.6	196.1	127.7	0.01631	21675.1
σ_c (MPa)	82.2		48.3	250.6	210.1	0.01978	22702.6
r^2	1.00		69.0	312.9		0.02350	27771.4

1-2 Time: Start: 9:20 End: 9:50 Date: July 24, 1989

D (m)	0.053		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.115		13.8	137.6	57.5	0.02052	9130.7
m	2.79		34.5	202.6	129.2	0.02839	15544.3
σ_c (MPa)	114.1		55.2	222.4	206.9	0.03106	20468.2
r^2	0.84		65.5	250.4		0.03239	25640.4

1-3 Time: Start: 11:58 End: 12:29 Date: July 19, 1989

D (m)	0.053		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.109		0.0	75.1	1.5	0.01211	10110.0
m	7.80		20.7	118.5	97.8	0.02189	10841.0
σ_c (MPa)	59.2		41.4	191.1	178.0	0.03329	21468.2
r^2	0.95		51.7	221.8		0.03632	24668.1

1-5 Time: Start: 19:58 End: 20:14 Date: Sept. 17, 1989

D (m)	0.053		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.061		3.4	85.9	37.0	0.01323	9296.2
m	9.46		24.1	147.7	119.5	0.01926	13689.2
σ_c (MPa)	57.1		44.8	201.2	195.9	0.02303	15964.9
r^2	0.97		65.5	267.9		0.02889	16875.2

1-6 Time: Start: 11:10 End: 11:40 Date: July 27, 1989

			σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
D (m)	0.053						
H (m)	0.060		3.4	93.3	39.1	0.01648	9965.2
m	14.42		13.8	126.9	87.5	0.02112	15806.3
σ_c (MPa)	57.8		34.5	211.7	181.3	0.03063	15806.3
r^2	0.99		51.7	269.3		0.03676	20068.2

2-3 Time: Start: 10:06 End: 10:27 Date: July 29, 1989

			σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
D (m)	0.053						
H (m)	0.099		13.8	107.4	54.2	0.01053	11316.8
m	3.63		34.5	164.0	124.0	0.01619	16447.7
σ_c (MPa)	76.2		55.2	200.6	193.3	0.02004	20364.8
r^2	0.97		65.5	218.2		0.02569	20364.8

2-4 Time: Start: 12:00 End: 12:40 Date: July 14, 1989

			σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
D (m)	0.053						
H (m)	0.094		3.4	75.2	18.057	0.01120	6813.5
m	5.91		24.1	123.7	94.434	0.01834	10778.9
σ_c (MPa)	55.5		44.8	175.9	159.342	0.02293	15668.4
r^2	0.99		65.5	224.7		0.02832	18544.2

2-5 Time: Start: 11:00 End: 11:45 Date: July 19, 1989

			σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
D (m)	0.053						
H (m)	0.109		17.2	106.5	71.2	0.01163	10896.2
m	2.19		27.6	125.5	105.7	0.01430	18226.9
σ_c (MPa)	72.3		37.9	?	139.2	?	?
r^2	1.00		58.6	?		?	?

2-7 Time: Start: 0:55 End: 1:23 Date: Sept. 17, 1989

D (m)	0.053		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.075		6.9	66.4	35.7	0.01744	4406.7
m	9.18		17.2	99.8	74.0	0.02381	5937.7
σ_c (MPa)	34.5		37.9	149.5	137.7	0.03169	7082.5
r^2	0.99		58.6	200.7		0.03890	8144.5

3-2 Time: Start: 9:14 End: 9:36 Date: July 29, 1989

D (m)	0.053		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.102		10.3	152.5	54.4	0.01184	16668.4
m	11.55		31.0	237.9	135.3	0.01658	19730.3
σ_c (MPa)	94.5		51.7	?	215.4	?	?
r^2	1.00		69.0	?		?	?

3-8 Time: Start: 8:56 End: 9:17 Date: July 28, 1989

D (m)	0.053		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.062		6.9	107.8	71.9	0.01684	8475.6
m	13.25		17.2	139.9	126.3	0.02095	16302.9
σ_c (MPa)	57.2		37.9	208.4	206.0	0.02773	17233.9
r^2	0.97		48.3	254.0		0.03657	16302.9

APPENDIX 3. DIFFERENCES BETWEEN MEANS OF STRAIN AND MODULAE FOR MUDSTONE TESTS

A3.1 Two sample t test of differences between means of ε_1 values

Suppose that the strain values in Group A and Group B listed in Table 3.2 are two independent random samples drawn from two normal distributions, with means of μ_A and μ_B, variances of $V[\varepsilon_{1A}]$ and $V[\varepsilon_{1B}]$ respectively. The sample sizes of Group A and B are $n_A = 8$, $n_B = 6$. The sample means are:

$$\overline{\epsilon}_{1A}=\frac{1}{n_A}\sum_{i=1}^{n_A}\epsilon_{1A_i}=\frac{0.15086}{8}=0.01886,$$

$$\overline{\epsilon}_{1B}=\frac{1}{n_B}\sum_{i=1}^{n_B}\epsilon_{1B_i}=\frac{0.12852}{6}=0.02565.$$

The sample variances are:

$$s_A^2=\frac{\sum_{i=1}^{n_A-1}(\epsilon_{1A_i}-\overline{\epsilon}_{1A})^2}{n_A-1}=\frac{0.00042}{8-1}=0.00006,$$

$$s_B^2=\frac{\sum_{i=1}^{n_B}(\epsilon_{1B_i}-\overline{\epsilon}_{1B})^2}{n_B-1}=\frac{0.00393}{6-1}=0.00079.$$

Before actually performing the two sample t test concerning the means, the null hypothesis that variances of the two populations are equal, i.e. $V[\varepsilon_{1A}] = V[\varepsilon_{1B}]$ has to be tested. Since

$$\frac{s_A^2}{s_B^2}=\frac{0.00006}{0.00079}=0.08,$$

which is less than

$$F_{\alpha/2}(n_A-1,\ n_B-1)=F_{0.025}(7,\ 5)=6.9$$

the null hypothesis can be accepted with an error probability of 0.05. The variances of the two populations are equal and it is reasonable to use the following two-sample t test.

With the t test, test the null hypothesis $\mu_A - \mu_B = \delta = 0$. The two-sided alternative is $\mu_A - \mu_B \neq 0$. Since

$$t=\frac{|\bar{\varepsilon}_{1A}-\bar{\varepsilon}_{1B}-\delta|}{\sqrt{\frac{(n_A-1)s_A^2+(n_B-1)s_B^2}{n_A+n_B-2}}\sqrt{\frac{1}{n_A}+\frac{1}{n_B}}}$$
$$=\frac{|0.01886-0.02565-0|}{\sqrt{\frac{7\times0.00006+5\times0.00079}{8+6-2}}\sqrt{\frac{1}{8}+\frac{1}{6}}}$$

$=0.7$, *which is less than* $t_{\alpha/2}(n_A+n_B-2)=t_{0.025}(8+6-2)=2.2,$

hence the null hypothesis must be accepted. This means that the observed differences between the sample means may reasonably be attributed to chance and the strain values obtained in multiple stage test with previously existing failure are not significantly different from the results where no previously failure exists.

A3.2 Two sample *t* test of differences between means of *E* values

Suppose the modulus values in Group A and Group B listed in Table 3.2 are two independent random samples obtained from two normal distributions, with means of μ_A and μ_B, variances of $V[E_A]$ and $V[E_B]$ respectively. The sample sizes of Group A and B are $n_A = 8$, $n_B = 6$. The sample means are:

$$\bar{E}_A = \frac{1}{n_A}\sum_{i=1}^{n_A} E_{A_i} = \frac{85707.0}{8} = 10713.4 \quad (MPa)\ .$$

$$\bar{E}_B = \frac{1}{n_B}\sum_{i=1}^{n_B} E_{B_i} = \frac{66101.0}{6} = 11016.8 \quad (MPa)\ .$$

The sample variances are:

$$s_A^2 = \frac{\sum_{i=1}^{n_A} (E_{A_i} - \bar{E}_A)^2}{n_A - 1} = \frac{65386991.1}{8-1} = 9340998.7 \quad (MPa)^2,$$

$$s_B^2 = \frac{\sum_{i=1}^{n_B} (E_{B_i} - \bar{E}_B)^2}{n_B - 1} = \frac{53007991.8}{6-1} = 10601598.4 \quad (MPa)^2.$$

First, test the null hypothesis that variances of the two populations are equal: $V[E_A]$

$= V[E_B]$, against the two-sided alternative $V[E_A] \neq V[E_B]$. Since

$$\frac{s_A^2}{s_B^2} = \frac{9340998.7}{10601598.4} = 0.9,$$

which is less than

$$F_{\alpha/2}(n_A-1,\ n_B-1) = F_{0.025}(7,\ 5) = 6.9$$

the null hypothesis has to be accepted. The two populations have the same variance. The prerequisite of the use of the following t test is satisfied.

Now test the null hypothesis $\mu_A - \mu_B = \delta = 0$ against the two-sided alternative $\mu_A - \mu_B \neq 0$. Since

$$t = \frac{|\bar{E}_A - \bar{E}_B - \delta|}{\sqrt{\frac{(n_A-1)s_A^2 + (n_B-1)s_B^2}{n_A+n_B-2}}\sqrt{\frac{1}{n_A}+\frac{1}{n_B}}}$$
$$= \frac{|10713.4-11016.8-0|}{\sqrt{\frac{7\times9340998.7+5\times10601598.4}{8+6-2}}\sqrt{\frac{1}{8}+\frac{1}{6}}}$$
$$=0.2,\ \textit{which is less than}\ t_{\alpha/2}(n_A+n_B-2) = t_{0.025}(8+6-2) = 2.2,$$

obviously the null hypothesis must be accepted. It can then be concluded that the observed differences between the two sample means may be attributed to chance; the previously existing failures in multiple stage tests have no significant effect on the moduli, as compared with the moduli obtained in single stage tests.

APPENDIX 4. STATISTICAL ANALYSIS OF THE EFFECTS OF SPECIMEN SIZE, SHAPE AND TEST TIME FOR MUDSTONE

A4.1 Statistical analysis of the height variation effect on the σ_c values

Denote μ_1, μ_2, μ_3, μ_4, μ_5, μ_6, μ_6, μ_7, μ_8, μ_9 as the mean values of σ_c from $k = 9$ populations corresponding to sample heights of $H = 0.044$, 0.048, 0.051, 0.054, 0.056, 0.060, 0.067, 0.068, 0.071 *meter* respectively, then what needs to be examined is

$$H_0:\ \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = \mu_7 = \mu_8 = \mu_9 .$$

With the following table:

Table A4.1 σ_c Values of Different Specimen Height Groups.

H (meter)	0.044	0.048	0.051	0.054	0.056	0.060	0.067	0.068	0.071	Total
σ_c *(MPa)*	34.2 (4-E)	43.3 (1-B)	84.5 (5-I)	92.8 (1-G2)	58.6 (1-D)	51.2 (1-E2)	47.3 (1-J)	93.4 (1-H)	73.7 (1-A)	
	74.0 (5-B)	63.9 (1-LB)	60.6 (6-A)	74.9 (4-H)	52.4 (1-E1)	47.9 (2-D)	62.0 (2-E)	41.2 (4-A)	97.0 (1-K)	
	63.8 (5-H)	47.6 (4-F)		34.6 (5-M)		86.8 (4-B)	62.8 (3-A1)			
		48.2 (5-F)								
Σ	172.0	203.1	145.2	202.3	111.0	185.8	172.1	134.6	170.7	1496.6

Under each height the number of tests are $n_1 = 3$, $n_2 = 4$, $n_3 = 2$, $n_4 = 3$, $n_5 = 2$, $n_6 =$

3, n_7 = 3, n_8 = 2, n_9 = 2. Total n = 3 + 4 + 2 + 3 + 2 + 3 + 3 + 2 + 2 = 24. The squares of the σ_c values in Table A4.1 above are shown in Table A4.2 below:

Table A4.2 Squares of the σ_c Values of Different Specimen Height Groups.

H (meter)	0.044	0.048	0.051	0.054	0.056	0.060	0.067	0.068	0.071	Total
σ_c^2 $(MPa)^2$	1171.0 (4-E)	1877.5 (1-B)	7145.3 (5-I)	8602.6 (1-G2)	3428.1 (1-D)	2616.3 (1-E2)	2234.5 (1-J)	8716.1 (1-H)	5437.6 (1-A)	
	5473.0 (5-B)	4087.0 (1-LB)	3676.0 (6-A)	5614.5 (4-H)	2747.9 (1-E1)	2290.6 (2-D)	3849.0 (2-E)	1700.7 (4-A)	9401.2 (1-k)	
	4064.1 (5-H)	2269.6 (4-F)		1195.1 (5-M)		7536.0 (4-B)	3943.8 (3-A1)			
		2321.3 (5-F)								
Σ	10708.1	10555.4	10821.3	15412.2	6176.0	12442.9	10027.3	10416.8	14838.8	101398.8

$$\sum_{i=1}^{k} \frac{S_i^2}{n_i} = \frac{172.0^2}{3} + \frac{203.1^2}{4} + \frac{145.2^2}{2} + \frac{202.3^2}{3} + \frac{111.0^2}{2}$$

$$+ \frac{185.8^2}{3} + \frac{172.1^2}{3} + \frac{134.6^2}{2} + \frac{170.7^2}{2} = 95505.3,$$

$$\frac{\left(\sum_{i=1}^{k} S_i\right)^2}{n} = \frac{1496.6^2}{24} = 93330.5,$$

$$\sum_{i=1}^{k} SS_i = 101398.8,$$

$$S_B=\sum_{i=1}^{k}\frac{S_i^2}{n_i}-\frac{\left(\sum_{i=1}^{k}S_i\right)^2}{n}=95505.3-93330.5=2174.9,$$

$$S_E=\sum_{i=1}^{k}SS_i-\sum_{i=1}^{k}\frac{S_i^2}{n_i}=101398.8-95505.3=5893.4,$$

$$S_T=\sum_{i=1}^{k}SS_i-\frac{\left(\sum_{i=1}^{k}S_i\right)^2}{n}=S_B+S_E=2174.9+5893.4=8068.3.$$

Therefore the degrees of freedom are:

$$f_B=k-1=9-1=8,\quad f_E=n-k=24-9=15,\quad f_T=n-1=24-1=23.$$

The analysis of variance table is shown as Table A4.3:

Table A4.3 Analysis of Variance for Specimen Height Effect on the σ_c Values.

Sources of variation	Sum of squares	Degrees of freedom	Mean square	F
Between heights	2174.9	8	271.9	0.7
Error	5893.4	15	392.9	
Total	8068.3	23		

Because $F_{0.05}(8, 15) = 2.6$ is greater than 0.7, the null hypothesis H_0 is accepted. That is, different heights of the specimens have no significant effect on the σ_c value.

A4.2 Statistical analysis of the combined effect of both height and diameter variation on the σ_c values

To investigate the combined effect of k different heights and s different diameters on the σ_c value of the mudstone, make the null hypotheses and alternative hypotheses similar to that with the m data. According to the experiment it can be shown as the following table:

Table A4.4 σ_c Values of Different Specimen Height and Diameter Combinations.

Specimen diameter	(*meter*)	0.029	0.053	$S_{i\cdot}$	S
Height (*meter*)	0.060	47.9 (2-D)	57.8 (1-6)	105.7	
	0.075	157.2 (3-C)	34.5 (2-7)	191.7	
	$S_{\cdot j}$	205.0	92.4		297.4

The squares of the σ_c values in Table A4.4 are shown in Table A4.5

Table A4.5 Squares of the σ_c Values of Different Specimen Height and Diameter Combinations.

Specimen diameter	(*meter*)	0.029	0.053	$SS_{i\cdot}$	SS
Height (*meter*)	0.060	2290.6 (2-D)	3345.5 (1-6)	5636.1	
	0.075	24699.3 (3-C)	1190.9 (2-7)	25890.2	
	$SS_{\cdot j}$	26989.9	4536.4		31526.3

With the following calculation:

$$S_H=\frac{1}{s}\sum_{i=1}^{k} S_{i\cdot}^{2}-\frac{1}{ks}S^2$$

$$=\frac{1}{2}\times(105.7^2+191.7^2)-\frac{1}{2\times2}\times297.4^2=1847.7,$$

$$S_D=\frac{1}{k}\sum_{j=1}^{s} S_{\cdot j}^{2}-\frac{1}{ks}S^2$$

$$=\frac{1}{2}(205.0^2+92.4^2)-\frac{1}{2\times2}\times297.4^2=3173.6,$$

$$S_T=SS-\frac{1}{ks}S^2$$

$$=31526.3-\frac{1}{2\times2}\times297.4^2=9419.0,$$

$$S_E=SS-\frac{1}{s}\sum_{i=1}^{k} S_{i\cdot}^{2}-\frac{1}{k}\sum_{j=1}^{s} S_{\cdot j}^{2}+\frac{1}{ks}S^2=S_T-(S_A+S_B)$$

$$=9419.0-(1847.7+3173.6)=4397.7,$$

The analysis of variance table for this two-way analysis is presented as Table A4.6

Table A4.6 Analysis of Variance for the Effect of Different Height and Diameter Combinations on σ_c Values.

Sources of variation	Sum of squares	Degrees of freedom	Mean square	F
Between heights	1847.7	1	1847.7	0.4
Between diameters	3173.6	1	3173.6	0.7
Error	4397.7	1	4397.7	
Total	9419.0	3		

Since $F_{0.05}(1, 1) = 161.4$ is greater than $F_H = 0.4$ and $F_{0.05}(1, 1) = 161.4$ is greater than $F_D = 0.72$, neither null hypotheses can be rejected. The conclusion is that the σ_c value of the mudstone is *not* affected by the combination effects of different heights and different diameters. Therefore the differences between m and σ_c values among different specimens were resulted from random error only. Note that when either the height or the diameter of the specimen varies both the size and shape of the specimen vary. In other words, it has been proven that neither the size nor the shape, or their different combinations, of the specimen have a significant effect on the m or σ_c values.

A4.3 Time effect on the *m* Values

The randomization of the specimen numbers, confining pressures and test dates made it possible to examine whether the time differences of the tests have an significant effect on both m and σ_c data. As can be seen from Appendix 2, the first test began on July 12, 1989 and the last test was finished on October 7, 1989. Since it has been shown that size and shape have no noticeable effect on m and σ_c, the time effect can therefore easily be evaluated.

Choose eight groups of specimens tested on eight different dates as shown on Table A4.7. The following null hypothesis needs to be tested:

$$H_0: \ \mu_1=\mu_2=\mu_3=\mu_4=\mu_5=\mu_6=\mu_7=\mu_8$$

where μ_1, μ_2, ..., μ_8 are the mean values of m of the k = 8 populations respectively, corresponding to test dates of July 13, July 17, ..., October 7.

Table A4.7 m Values of Different Test Date Groups.

Test date	July 13	July 17	July 19	July 21	July 25	July 31	Sep. 17	Oct. 7	Total
m	5.9 (1-E2)	7.0 (2-E)	10.2 (1-D)	8.8 (1-E1)	7.0 (4-H)	13.3 (1-A)	9.5 (1-5)	16.6 (6-A)	
	11.2 (5-H)	10.1 (5-B)	7.8 (1-3)	19.6 (1-B)	3.9 (1-G2)	9.9 (1-C)	9.2 (2-7)	12.0 (3-A1)	
					14.6 (2-A)	7.5 (1-LB)		6.5 (1-K)	
								12.0 (4-B)	
Σ	17.1	17.1	18.0	28.4	25.5	30.7	18.6	47.1	202.5

On each test date the number of tests are n_1 = 2, n_2 = 2, n_3 = 2, n_4 = 2, n_5 = 3, n_6 = 3, n_7 = 2, n_8 = 4. Total n = 20. The squares of the m values in Table A4.7 are listed in Table A4.8.

Table A4.8 Squares of the *m* Values of Different Test Date Groups.

Test date	July 13	July 17	July 19	July 21	July 25	July 31	Sep. 17	Oct. 7	Total
m^2	34.9 (1-E2)	48.4 (2-E)	104.2 (1-D)	77.8 (1-E1)	49.4 (4-H)	176.1 (1-A)	89.5 (1-5)	275.9 (6-A)	
	124.8 (5-H)	101.4 (5-B)	60.8 (1-3)	383.8 (1-B)	15.0 (1-G2)	98.0 (1-C)	84.3 (2-7)	143.8 (3-A1)	
					213.5 (2-A)	56.3 (1-LB)		42.6 (1-K)	
								144.0 (4-B)	
Σ	159.7	149.9	165.1	461.6	277.9	330.4	173.8	606.3	2324.5

Similar to the one way analysis of height variation effects, it follows that:

$$\sum_{i=1}^{k}\frac{S_i^2}{n_i}=\frac{17.1^2}{2}+\frac{17.0^2}{2}+\frac{18.0^2}{2}+\frac{28.4^2}{2}+\frac{25.5^2}{3}+\frac{30.7^2}{3}+\frac{18.6^2}{2}+\frac{47.1^2}{4}=2116.1,$$

$$\frac{\left(\sum_{i=1}^{k}S_i\right)^2}{n}=\frac{202.5^2}{20}=2049.9,$$

$$\sum_{i=1}^{k}SS_i=2324.5,$$

$$S_B=\sum_{i=1}^{k}\frac{S_i^2}{n_i}-\frac{\left(\sum_{i=1}^{k}S_i\right)^2}{n}=2116.1-2049.9=66.2,$$

$$S_E=\sum_{i=1}^{k} SS_i-\sum_{i=1}^{k} \frac{S_i^2}{n_i}=2324.5-2116.1=208.3,$$

$$S_T=\sum_{i=1}^{k} SS_i-\frac{\left(\sum_{i=1}^{k} S_i\right)^2}{n}=S_B+S_E=66.2+208.3=274.5.$$

The degrees of freedom are:

$$f_B=k-1=8-1=7,\quad f_E=n-k=20-8=12,\quad f_T=n-1=20-1=19.$$

Table A4.9 is the analysis of variance table.

Table A4.9 Analysis of Variance for the Effect of Time on the m Values.

Sources of variation	Sum of squares	Degrees of freedom	Mean square	F
Between dates	66.2	7	9.5	0.4
Error	208.3	12	24.0	
Total	274.5	19		

$F_{0.05}(7, 12) = 2.9$ which is greater than 0.4, therefore under the error probability of 0.05, the null hypothesis H_0 is accepted. Note that even if taking the probability of making *Type I error* as 0.10, $F_{0.10}(7, 12) = 2.28$ is still greater than 0.4 and the null hypothesis still can to be accepted. This means different test times have no significant effect on the m values observed.

A4.4 Time effect on the σ_c values

Again, choose the same eight groups of specimens tested on eight different dates as shown on Table A4.10 and test the null hypothesis

$$H_0: \ \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = \mu_7 = \mu_8$$

where μ_1, μ_2, ..., μ_8 are the mean values of σ_c of the $k = 8$ populations corresponding to test dates of July 13, July 17, ..., October 7.

Table A4.10 σ_c Values of Different Test Date Groups.

Test date	July 13	July 17	July 19	July 21	July 25	July 31	Sep. 17	Oct. 7	Total
σ_c (MPa)	51.2 (1-E2)	62.0 (2-E)	58.6 (1-D)	52.4 (1-E1)	74.9 (4-H)	73.7 (1-A)	57.1 (1-5)	60.6 (6-A)	
	63.8 (5-H)	74.0 (5-B)	59.2 (1-3)	43.3 (1-B)	92.8 (1-G2)	57.5 (1-C)	34.5 (2-7)	62.8 (3-A1)	
					92.3 (2-A)	63.9 (1-LB)		97.0 (1-K)	
								86.8 (4-B)	
Σ	114.9	136.0	117.7	95.8	259.9	195.1	91.6	307.2	1318.3

On each test date the number of tests are $n_1 = 2$, $n_2 = 2$, $n_3 = 2$, $n_4 = 2$, $n_5 = 3$, $n_6 = 3$, $n_7 = 2$, $n_8 = 4$. Total $n = 20$. The squares of the σ_c values in Table A4.10 are listed in Table A4.11.

Table A4.11 Squares of σ_c Values of Different Test Date Groups.

Test date	July 13	July 17	July 19	July 21	July 25	July 31	Sep. 17	Oct. 7	Total
σ_c^2 $(MPa)^2$	2616.3 (1-E2)	3849.0 (2-E)	3428.1 (1-D)	2747.9 (1-E1)	5614.5 (4-H)	5437.6 (1-A)	3258.1 (1-5)	3675.1 (6-A)	
	4064.1 (5-H)	5473.0 (5-B)	3502.3 (1-3)	1877.5 (1-B)	8602.6 (1-G2)	3301.7 (1-C)	1190.9 (2-7)	3943.8 (3-A1)	
					8510.1 (2-A)	4087.0 (1-LB)		9401.2 (1-K)	
								7536.0 (4-B)	
Σ	6680.4	9322.0	6930.4	4625.4	22727.1	12826.3	4449.1	24557.1	92117.6

$$\sum_{i=1}^{k}\frac{S_i^2}{n_i}=\frac{114.9^2}{2}+\frac{136.0^2}{2}+\frac{117.7^2}{2}+\frac{95.8^2}{2}+\frac{259.9^2}{3}$$

$$+\frac{195.1^2}{3}+\frac{91.6^2}{2}+\frac{307.2^2}{4}=90366.4,$$

$$\frac{\left(\sum_{i=1}^{k}S_i\right)^2}{n}=\frac{1318.3^2}{20}=86889.2,$$

$$\sum_{i=1}^{k}SS_i=92117.6,$$

$$S_B=\sum_{i=1}^{k}\frac{S_i^2}{n_i}-\frac{\left(\sum_{i=1}^{k}S_i\right)^2}{n}=90366.4-86889.2=3477.2,$$

$$S_E=\sum_{i=1}^{k}SS_i-\sum_{i=1}^{k}\frac{S_i^2}{n_i}=92117.6-90366.4=1751.3,$$

$$S_T=\sum_{i=1}^{k}SS_i-\frac{\left(\sum_{i=1}^{k}S_i\right)^2}{n}=S_B+S_E=3477.2+1751.3=5228.5.$$

Table A4.12 is the analysis of variance table.

Table A4.12 Analysis of Variance of Time Effect on the σ_c Values.

Sources of variation	Sum of squares	Degrees of freedom	Mean square	F
Between dates	3477.2	7	496.7	3.4
Error	1751.3	12	145.9	
Total	5228.5	19		

$F_{0.05}(7, 12) = 2.9$ which is less than 3.4, therefore the null hypothesis H_0 should be rejected. However, if the probability of *Type I error* is lowered to 0.025, then $F_{0.025}(7, 12) = 3.6$. which is greater than 3.4 and the null hypothesis should be accepted. It may therefore be concluded that there is a slight influence of the time of testing on the σ_c value, however, this influence is relatively small and will be assumed negligible.

APPENDIX 5. COAL PARAMETERS ASSUMING NON-INTACT SPECIMENS

Coal1-4 Time: Start: 11:36 End: 12:08 Date: December 18, 1990

D *(meter)*	H *(meter)*	σ_3 *(MPa)*	σ_1 *(MPa)*	# of tests	Ref.	s	σ_{ci} *(MPa)*	m_i	m	r
0.056	0.080	6.9	41.5	1	[Test]	0.155	92.5	1.3	0.2	0.88
0.056	0.080	27.6	74.2	1						
0.056	0.080	48.3	96.9	1						
0.056	0.080	69.0	118.3	1						

Coal3-1 Time: Start: 15:33 End: 16:00 Date: December 18, 1990

D *(meter)*	H *(meter)*	σ_3 *(MPa)*	σ_1 *(MPa)*	# of tests	Ref.	s	σ_{ci} *(MPa)*	m_i	m	r
0.056	0.080	0.0	8.0	1	[Test]	0.155	51.3	2.1	0.3	0.77
0.056	0.080	10.3	41.1	1						
0.056	0.080	31.0	63.3	1						
0.056	0.080	51.7	85.2	1						

Coal5-2 Time: Start: 9:25 End: 10:02 Date: December 19, 1990

D *(meter)*	H *(meter)*	σ_3 *(MPa)*	σ_1 *(MPa)*	# of tests	Ref.	s	σ_{ci} *(MPa)*	m_i	m	r
0.056	0.080	3.4	26.0	1	[Test]	0.155	70.4	1.6	0.3	0.82
0.056	0.080	13.8	50.4	1						
0.056	0.080	34.5	73.5	1						
0.056	0.080	55.2	95.2	1						

Coal5-3 Time: Start: 14:50 End: 15:19 Date: December 18, 1990

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{ci} (MPa)	m_i	m	r
0.056	0.090	6.9	29.0	1	[Test]	0.129	58.5	0.6	0.07	0.97
0.056	0.090	27.6	50.4	1						
0.056	0.090	48.3	74.1	1						
0.056	0.090	69.0	96.0	1						

Coal6-2 Time: Start: 10:56 End: 11:21 Date: December 19, 1990

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{ci} (MPa)	m_i	m	r
0.056	0.090	10.3	30.1	1	[Test]	0.129	49.9	0.9	0.1	0.99
0.056	0.090	31.0	53.0	1						
0.056	0.090	51.7	77.4	1						
0.056	0.090	69.0	96.0	1						

Coal6-3 Time: Start: 14:08 End: 14:32 Date: December 20, 1990

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{ci} (MPa)	m_i	m	r
0.056	0.120	2.8	18.6	1	[Test]	0.085	52.2	0.8	0.07	0.99
0.056	0.120	24.1	41.8	1						
0.056	0.120	44.8	64.0	1						
0.056	0.120	65.5	87.4	1						

Coal6-4 Time: Start: 11:15 End: 11:55 Date: December 20, 1990

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{ci} (MPa)	m_i	m	r
0.056	0.090	6.9	37.0	1	[Test]	0.129	86.0	0.7	0.09	0.82
0.056	0.090	17.2	52.5	1						
0.056	0.090	37.9	72.0	1						
0.056	0.090	58.6	96.0	1						

Coal7-2 Time: Start: 10:30 End: 11:00 Date: December 20, 1990

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{ci} (MPa)	m_i	m	r
0.056	0.120	17.2	45.1	1	[Test]	0.085	89.4	0.6	0.05	0.97
0.056	0.120	27.6	55.7	1						
0.056	0.120	37.9	66.7	1						
0.056	0.120	58.6	89.6	1						

Coal7-4 Time: Start: 15:20 End: 15:45 Date: December 20, 1990

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{ci} (MPa)	m_i	m	r
0.056	0.090	3.4	17.1	1	[Test]	0.129	36.5	0.7	0.09	0.99
0.056	0.090	13.8	28.1	1						
0.056	0.090	34.5	51.5	1						
0.056	0.090	58.6	77.4	1						

Coal9-1 Time: Start: 14:24 End: 14:47 Date: December 20, 1990

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{ci} (MPa)	m_i	m	r
0.056	0.120	13.8	31.3	1	[Test]	0.085	58.0	0.4	0.04	0.97
0.056	0.120	34.5	53.6	1						
0.056	0.120	55.2	75.4	1						
0.056	0.120	65.5	85.7	1						

Coal9-2 Time: Start: 14:53 End: 15:14 Date: December 20, 1990

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{ci} (MPa)	m_i	m	r
0.056	0.080	0.0	3.9	1	[Test]	0.155	18.7	1.6	0.2	0.91
0.056	0.080	20.7	35.4	1						
0.056	0.080	41.4	56.6	1						
0.056	0.080	51.7	68.1	1						

Coal1-C Time: Start: 10:28 End: 10:46 Date: January 10, 1991

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{ci} (MPa)	m_i	m	r
0.068	0.080	0.0	11.1	1	[Test]	0.181	33.9	1.8	0.3	0.95
0.068	0.080	20.7	44.5	1						
0.068	0.080	41.4	67.0	1						
0.068	0.080	62.1	91.3	1						

Coal1-D Time: Start: 11:23 End: 11:45 Date: January 10, 1991

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{ci} (MPa)	m_i	m	r
0.068	0.120	3.4	28.1	1	[Test]	0.097	77.1	0.5	0.05	0.79
0.068	0.120	24.1	50.4	1						
0.068	0.120	48.3	73.8	1						
0.068	0.120	65.5	95.6	1						

Coal2-A Time: Start: 11:33 End: 11:50 Date: January 24, 1991

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{ci} (MPa)	m_i	m	r
0.068	0.080	6.9	34.2	1	[Test]	0.181	59.1	1.4	0.3	1.00
0.068	0.080	27.6	59.9	1						
0.068	0.080	48.3	84.8	1						
0.068	0.080	69.0	110.1	1						

Coal2-B Time: Start: 10:01 End: 11:17 Date: January 10, 1991

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{ci} (MPa)	m_i	m	r
0.068	0.080	10.3	45.8	1	[Test]	0.181	80.3	0.8	0.2	0.95
0.068	0.080	31.0	69.9	1						
0.068	0.080	51.7	95.6	1						
0.068	0.080	69.0	112.6	1						

Coal4-B Time: Start: 12:21 End: 12:36 Date: January 24, 1991

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.068	0.080	13.8	50.1	1	[Test]	0.181	78.8	1.0	0.2	0.98
0.068	0.080	44.8	87.5	1						
0.068	0.080	48.3	89.7	1						
0.068	0.080	65.5	110.9	1						

Coal4-E Time: Start: 9:28 End: 9:49 Date: January 24, 1991

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.068	0.080	6.9	32.3	1	[Test]	0.181	58.7	1.0	0.2	0.98
0.068	0.080	27.6	58.6	1						
0.068	0.080	48.3	82.6	1						
0.068	0.080	69.0	105.0	1						

Coal5-A Time: Start: 9:53 End: 10:07 Date: January 24, 1991

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.068	0.080	13.8	32.5	1	[Test]	0.181	37.1	0.7	0.1	0.91
0.068	0.080	34.5	53.2	1						
0.068	0.080	55.2	77.1	1						
0.068	0.080	65.5	89.7	1						

Coal5-B Time: Start: 10:16 End: 10:40 Date: January 24, 1991

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.068	0.120	17.2	55.2	1	[Test]	0.097	114.1	0.6	0.06	0.90
0.068	0.120	27.6	65.0	1						
0.068	0.120	37.9	76.4	1						
0.068	0.120	58.6	99.8	1						

APPENDIX 6. COAL PARAMETERS ASSUMING INTACT SPECIMENS

Note: The stress values in brackets are not used in the calculation of the parameters because they exceeded the brittle-ductile transition limit. If more than one stress points of a specimen are beyond the transition limit because of the high confining pressure, a question mark is placed at the corresponding parameter position.

Coal1-4 Time: Start: 11:36 End: 12:08 Date: December 18, 1990

D (m)	0.056		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.080		6.9	41.5	24.9	0.04128	1324.1
m	?		27.6	74.2	63.2	0.06034	1572.4
σ_{pi} (MPa)	?		(48.3)	(96.9)	95.4	0.06875	1827.5
r	?		(69.0)	(118.3)		0.07383	2082.7

Coal3-1 Time: Start: 15:33 End: 16:00 Date: December 18, 1990

D (m)	0.056		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.080		0.0	8.0	3.5	0.02779	593.1
m	1.6		10.3	41.1	35.9	0.05399	1200.0
σ_{pi} (MPa)	17.4		31.0	63.3	61.4	0.06034	1572.4
r	0.81		(51.7)	(85.2)		0.06319	2082.7

Coal5-2 Time: Start: 9:25 End: 10:02 Date: December 19, 1990

			σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
D (m)	0.056						
H (m)	0.080		3.4	26.0	17.3	0.04368	717.2
m	1.2		13.8	50.4	45.4	0.06591	1200.0
σ_{pi} (MPa)	24.8		34.5	73.5	72.3	0.07465	1448.2
r	0.86		(55.2)	(95.2)		0.08180	1572.4

Coal5-3 Time: Start: 14:50 End: 15:19 Date: December 18, 1990

			σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
D (m)	0.056						
H (m)	0.090		6.9	29.0	24.0	0.03898	1075.8
m	?		27.6	50.4	48.8	0.04237	1489.6
σ_{pi} (MPa)	?		(48.3)	(74.1)	73.9	0.04944	1765.5
r	?		(69.0)	(96.0)		0.05508	2048.2

Coal6-2 Time: Start: 10:56 End: 11:21 Date: December 19, 1990

			σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
D (m)	0.056						
H (m)	0.090		10.3	30.1	32.2	0.02753	1627.5
m	?		31.0	53.0	54.5	0.03741	2199.9
σ_{pi} (MPa)	?		(51.7)	(77.4)	76.8	0.04715	2344.7
r	?		(69.0)	(96.0)		0.05364	2489.6

Coal6-3 Time: Start: 14:08 End: 14:32 Date: December 18, 1990

			σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
D (m)	0.056						
H (m)	0.120		2.8	18.6	11.0	0.02030	1075.8
m	0.2		24.1	41.8	30.0	0.02434	1086.8
σ_{pi} (MPa)	15.6		44.8	64.0	63.1	0.02781	3324.0
r	1.00		(65.5)	(87.4)		0.03099	3124.0

Coal6-4 Time: Start: 11:15 End: 11:55 Date: December 20, 1990

			σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
D (m)	0.056						
H (m)	0.090		6.9	37.0	28.1	0.06011	800.0
m	0.2		17.2	52.5	49.1	0.07037	1765.5
σ_{pi} (MPa)	31.3		37.9	72.0	72.0	0.07567	1910.3
r	0.58		(58.6)	(96.0)		0.09689	2565.4

Coal7-2 Time: Start: 10:30 End: 11:00 Date: December 20, 1990

			σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
D (m)	0.056						
H (m)	0.120		17.2	45.1	44.5	0.03028	2351.6
m	0.09		27.6	55.7	55.5	0.03865	1799.9
σ_{pi} (MPa)	27.0		37.9	66.7	66.3	0.05083	3117.1
r	0.96		(58.6)	(89.6)		0.05559	3310.2

Coal7-4 Time: Start: 15:20 End: 15:45 Date: December 20, 1990

			σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
D (m)	0.056						
H (m)	0.090		3.4	17.1	16.9	0.02515	937.9
m	0.3		13.8	28.1	22.3	0.02854	1489.6
σ_{pi} (MPa)	13.0		34.5	51.5	51.1	0.03659	1627.5
r	0.99		(58.6)	(77.4)		0.05199	2048.2

Coal9-1 Time: Start: 14:24 End: 14:47 Date: December 20, 1990

D (m)	0.056		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.120		13.8	31.3	29.6	0.02434	1613.7
m	?		34.5	53.6	51.4	0.02858	2351.6
σ_{pi} (MPa)	?		(55.2)	(75.4)	73.2	0.03281	2730.9
r	?		(65.5)	(85.7)		0.03895	3317.1

Coal9-2 Time: Start: 14:53 End: 15:14 Date: December 20, 1990

D (m)	0.056		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.080		0.0	3.9	3.0	0.01682	475.8
m	?		20.7	35.4	34.6	0.03269	1448.2
σ_{pi} (MPa)	?		(41.4)	(56.6)	56.0	0.03681	1827.5
r	?		(51.7)	(68.1)		0.04126	1951.7

Coal1-C Time: Start: 10:28 End: 10:46 Date: January 10, 1991

D (m)	0.068		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.080		0.0	11.1	2.0	0.03427	641.4
m	1.0		20.7	44.5	42.7	0.05332	1082.7
σ_{pi} (MPa)	13.5		41.4	67.0	66.3	0.05966	1586.1
r	0.93		(62.1)	(91.3)		0.06855	1772.3

Coal1-D Time: Start: 11:23 End: 11:45 Date: January 10, 1991

D (m)	0.068		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.120		3.4	28.1	16.8	0.03133	1434.4
m	0.04		24.1	50.4	48.2	0.04149	1937.9
σ_{pi} (MPa)	25.0		48.3	73.8	72.9	0.04742	2372.3
r	0.50		(65.5)	(95.6)		0.05165	2503.4

Coal2-A Time: Start: 11:33 End: 11:50 Date: January 24, 1991

D (m)	0.068		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.080		6.9	34.2	24.4	0.05843	993.1
m	0.6		27.6	59.9	49.6	0.07621	1579.3
σ_{pi} (MPa)	25.5		48.3	84.8	83.3	0.08257	2006.8
r	1.00		(69.0)	(110.1)		0.08892	2310.3

Coal2-B Time: Start: 10:01 End: 11:17 Date: January 10, 1991

D (m)	0.068		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.080		10.3	45.8	39.1	0.05712	1296.5
m	?		31.0	69.9	64.5	0.06728	1586.1
σ_{pi} (MPa)	?		(51.7)	(95.6)	89.7	0.08378	1586.1
r	?		(69.0)	(112.6)		0.08886	1682.7

Coal4-B Time: Start: 12:21 End: 12:36 Date: January 24, 1991

D (m)	0.068		σ_3 (MPa)	σ_1 (MPa)	σ_r (MPa)	ε_1	E (MPa)
H (m)	0.080		13.8	50.1	45.4	0.04188	1599.9
m	0.4		44.8	87.5	82.9	0.04949	2524.0
σ_{pi} (MPa)	33.8		48.3	89.7	87.9	0.05584	2737.8
r	0.96		(65.5)	(110.9)		0.05964	2993.0

Coal4-E Time: Start: 9:28 End: 9:49 Date: January 24, 1991

			σ_3 (*MPa*)	σ_1 (*MPa*)	σ_r (*MPa*)	ε_1	*E* (*MPa*)
D (*m*)	0.068						
H (*m*)	0.080		6.9	32.3	28.7	0.04827	1372.4
m	0.5		27.6	58.6	56.2	0.06732	1599.9
σ_{pi} (*MPa*)	24.0		48.3	82.6	81.5	0.07621	1689.6
r	0.99		(69.0)	(105.0)		0.08384	1786.1

Coal5-A Time: Start: 9:53 End: 10:07 Date: January 24, 1991

			σ_3 (*MPa*)	σ_1 (*MPa*)	σ_r (*MPa*)	ε_1	*E* (*MPa*)
D (*m*)	0.068						
H (*m*)	0.080		13.8	32.5	20.7	0.03935	1041.3
m	?		34.5	53.2	43.8	0.05839	1503.4
σ_{pi} (*MPa*)	?		(55.2)	(77.1)	76.8	0.06982	1586.1
r	?		(65.5)	(89.7)		0.07642	2013.7

Coal5-B Time: Start: 10:16 End: 10:40 Date: January 24, 1991

			σ_3 (*MPa*)	σ_1 (*MPa*)	σ_r (*MPa*)	ε_1	*E* (*MPa*)
D (*m*)	0.068						
H (*m*)	0.120		17.2	55.2	47.5	0.07788	1027.5
m	0.05		27.6	65.0	59.9	0.08466	1496.5
σ_{pi} (*MPa*)	37.3		37.9	76.4	72.2	0.08889	2034.4
r	0.48		(58.6)	(99.8)		0.09312	2248.2

APPENDIX 7. SIZE EFFECT ANALYSES ON m_i AND σ_{pi} PARAMETERS

A7.1 Height variation effect on the σ_{pi} values assuming intact specimens

Assuming $s = 1$, the σ_{pi} values for different specimen height groups are listed in Table A7.1 (refer to Appendix 6).

Table A7.1 σ_{pi} Values of Different Height Groups Assuming Intact Specimens.

H (*meter*)	0.080	0.090	0.120	Total
σ_{pi} (*MPa*)	17.4 (Coal3-1)	13.0 (Coal7-4)	15.6 (Coal6-3)	
	24.8 (Coal5-2)	31.3 (Coal6-4)	27.0 (Coal7-2)	
Σ	42.2	44.3	42.6	129.1

The number of different heights $k = 3$. Under each height the number of tests are $n_1 = 2$, $n_2 = 2$, $n_3 = 2$ respectively. Total $n = 2 + 2 + 2 = 6$. Denote μ_1, μ_2, μ_3 as the mean values of σ_{pi} of the k populations corresponding to the specimen heights of $H = 0.080$, 0.090, 0.120 *meter*, then examine:

$$H_0: \mu_1=\mu_2=\mu_3$$

The squares of the σ_{pi} values in Table A7.1 are shown in Table A7.2:

Table A7.2 Squares of the σ_{pi} Values of Different Height Groups Assuming Intact Specimens.

H (*meter*)	0.080	0.090	0.120	Total
σ_{pi}^2 $(MPa)^2$	302.8 (Coal3-1)	169.0 (Coal7-4)	243.4 (Coal6-3)	
	615.0 (Coal5-2)	979.7 (Coal6-4)	729.0 (Coal7-2)	
Σ	917.8	1148.7	972.4	3038.9

Hence

$$\sum_{i=1}^{k}\frac{S_i^2}{n_i}=\frac{42.2^2}{2}+\frac{44.3^2}{2}+\frac{42.6^2}{2}=2779.0,$$

$$\frac{\left(\sum_{i=1}^{k}S_i\right)^2}{n}=\frac{129.1^2}{6}=2777.8,$$

$$\sum_{i=1}^{k}SS_i=3038.9,$$

$$S_B=\sum_{i=1}^{k}\frac{S_i^2}{n_i}-\frac{\left(\sum_{i=1}^{k}S_i\right)^2}{n}=2779.0-2777.8=1.2,$$

$$S_E=\sum_{i=1}^{k}SS_i-\sum_{i=1}^{k}\frac{S_i^2}{n_i}=3038.9-2779.0=259.9,$$

$$S_T=\sum_{i=1}^{k}SS_i-\frac{(\sum_{i=1}^{k}S_i)^2}{n}=S_B+S_E=261.1.$$

The degrees of freedom are:

$$f_B=k-1=3-1=2,\quad f_E=n-k=6-3=3,\quad f_T=n-1=6-1=5.$$

The analysis of variance table is shown as Table A7.3:

Table A7.3 Analysis of Variance of Height Effect on the σ_{pi} Values for Intact Specimens.

Sources of variation	Sum of squares	Degrees of freedom	Mean square	F
Between heights	1.2	2	0.6	0.007
Error	259.9	3	86.6	
Total	261.1	5		

$F_{0.05}(2, 3) = 9.6$, which is greater than 0.007, therefore the null hypothesis H_0 has to be accepted.

A7.2 Diameter variation effect on σ_{pi} values assuming intact specimens

Referring to Appendix 6, choose the same two groups of specimens: Group 1 and Group 2. The σ_{pi} values are listed in Table A7.4

Table A7.4 σ_{pi} Values of Two Diameter Groups for Intact Specimens

Group 1				Group 2		
D (meter)	*H (meter)*	σ_{pi1} *(MPa)*		*D (meter)*	*H (meter)*	σ_{pi2} *(MPa)*
0.056	0.080	17.4 (Coal3-1)		0.068	0.080	13.5 (Coal1-C)
0.056	0.080	24.8 (Coal5-2)		0.068	0.080	25.5 (Coal2-A)
0.056	0.080			0.068	0.080	33.8 (Coal4-B)
0.056	0.080			0.068	0.080	24.0 (Coal4-E)

Regard the σ_{pi} data from Group 1 and Group 2 as two independent random samples of size $n_1 = 2$ and $n_2 = 4$ from two normal populations with means of μ_1 and μ_2 and the known variances of $V[\sigma_{pi1}]$ and $V[\sigma_{pi2}]$ respectively. The sample means are:

$$\overline{\sigma}_{pi1}=\frac{1}{n_1}\sum_{i=1}^{n_1}\sigma_{pi1_i}=\frac{42.2}{2}=21.1,$$

$$\overline{\sigma}_{pi2}=\frac{1}{n_2}\sum_{i=1}^{n_2}\sigma_{pi2_i}=\frac{96.8}{4}=24.2.$$

The sample variances are:

$$s_1^2=\frac{\sum_{i=1}^{n_1}(\sigma_{pi1_i}-\overline{\sigma}_{pi1})^2}{n_1-1}=\frac{27.4}{2-1}=27.4\ (MPa)^2,$$

$$s_2^2=\frac{\sum_{i=1}^{n_2}(\sigma_{pi2_i}-\overline{\sigma}_{pi2})^2}{n_2-1}=\frac{208.4}{4-1}=69.5\ (MPa)^2.$$

Before performing the t test concerning the means, the equality of the two population variances, i.e. the null hypothesis $V[\sigma_{pi1}] = V[\sigma_{pi2}]$, has to be tested. The two-sided alternative is $V[\sigma_{pi1}] \neq V[\sigma_{pi2}]$. The error probability chosen is

$$\alpha=0.05.$$

Since

$$\frac{s_1^2}{s_2^2}=\frac{27.4}{69.5}=0.4,$$

which is less than

$$F_{\alpha/2}(n_1-1,\ n_2-1)=F_{0.025}(1,\ 3)=17.4$$

the null hypothesis must be accepted, i.e. the two populations have the same variance and the use of the following two-sample t test is justified.

Now test the null hypothesis $\mu_1 - \mu_2 = \delta = 0$ against the two-sided alternative $\mu_1 - \mu_2 \neq 0$. Since

$$t=\frac{|\overline{\sigma}_{pi1}-\overline{\sigma}_{pi2}-\delta|}{\sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$=\frac{|21.1-24.2-0|}{\sqrt{\frac{1\times27.4+3\times69.5}{2+4-2}}\sqrt{\frac{1}{2}+\frac{1}{4}}}$$

$=0.5$, *which is less than* $t_{\alpha/2}(n_1+n_2-2)=t_{0.025}(2+4-2)=2.8$,

therefore the null hypothesis must be accepted, i.e. the differences of the σ_{pi} values between the two different diameter groups of coal specimens are not significant.

A7.3 The combined effect of height and diameter variation on σ_{pi} values assuming intact specimens

Referring to Appendix 6, the following table is obtained.

Table A7.5 The σ_{pi} Values of Different Height and Diameter Combinations for Intact Specimens

		Diameter (*meter*)	
		0.056	0.068
Height (*meter*)	0.080	17.4 (Coal3-1)	25.5 (Coal2-A)
		24.8 (Coal5-2)	24.0 (Coal4-E)
	0.120	15.6 (Coal6-3)	25.0 (Coal1-D)
		27.0 (Coal7-2)	37.3 (Coal5-B)

Denote x_{ijk} as the kth value of σ_{pi} obtained with height H_i and diameter D_j, and taking t observations of each kind, the model considered becomes

$$x_{ij}=\mu+\beta_i+\gamma_j+\epsilon_{ij}+\eta_{ijk},$$
$$i=1,\ 2,\ \ldots,\ r;\quad j=1,\ 2,\ \ldots,\ s,\quad k=1,\ 2,\ \ldots,\ t.$$

where the η_{ijk} are values assumed by independent random variables having normal distributions with zero means and the common variance $V[\sigma_{pi}]$, while μ, β_i, γ_j, ϵ_{ij} and $V[\sigma_{pi}]$ are unknowns. The null hypotheses to be tested are:

$$H_{0_1}:\ \beta_1=\beta_2=\cdots=\beta_r=0,$$
$$H_{0_2}:\ \gamma_1=\gamma_2=\cdots=\gamma_s=0,$$
$$H_{0_3}:\ \epsilon_{11}=\cdots=\epsilon_{1s}=\cdots=\epsilon_{rs}=0.$$

The corresponding alternative hypotheses are that the respective parameters are not all equal to zero. The tests of these hypotheses are based on the analysis of the total variability of the x data, decomposing it into terms which can be attributed to differences among the heights S_H, differences among the diameters S_D, interactions (joint effects) S_I, and chance (experimental error) S_E. First denote

$$\bar{x}=\frac{1}{rst}\sum_{k=1}^{t}\sum_{j=1}^{s}\sum_{i=1}^{r}x_{ijk},$$

$$\bar{x}_{ij.}=\frac{1}{t}\sum_{k=1}^{t}x_{ijk},$$

$$\bar{x}_{i..}=\frac{1}{st}\sum_{k=1}^{t}\sum_{j=1}^{s}x_{ijk},$$

$$\bar{x}_{.j.}=\frac{1}{rt}\sum_{k=1}^{t}\sum_{i=1}^{r}x_{ijk}.$$

therefore

$$\begin{aligned}S_T=&\sum_{k=1}^{t}\sum_{j=1}^{s}\sum_{i=1}^{r}(x_{ijk}-\bar{x}_{ij.})^2\\&+st\sum_{i=1}^{r}(\bar{x}_{i..}-\bar{x})^2+rt\sum_{j=1}^{s}(\bar{x}_{.j.}-\bar{x})^2\\&+t\sum_{j=1}^{s}\sum_{i=1}^{r}(\bar{x}_{ij.}-\bar{x}_{i..}-\bar{x}_{.j.}+\bar{x})^2\\=&S_E+S_H+S_D+S_I.\end{aligned}$$

Let

$$S_{ij\cdot}=\sum_{k=1}^{t} x_{ijk},$$

$$S_{i\cdot\cdot}=\sum_{k=1}^{t}\sum_{j=1}^{s} x_{ijk}, \qquad S_{\cdot j\cdot}=\sum_{k=1}^{t}\sum_{i=1}^{r} x_{ijk},$$

$$S=\sum_{k=1}^{t}\sum_{j=1}^{s}\sum_{i=1}^{r} x_{ijk}, \qquad SS=\sum_{k=1}^{t}\sum_{j=1}^{s}\sum_{i=1}^{r} x_{ijk}^2.$$

Hence

$$S_{11\cdot}=42.2, \qquad S_{12\cdot}=49.5,$$

$$S_{21\cdot}=42.6, \qquad S_{22\cdot}=62.3,$$

$$S_{1\cdot\cdot}=91.7, \qquad S_{2\cdot\cdot}=104.9,$$

$$S_{\cdot 1\cdot}=84.8, \qquad S_{\cdot 2\cdot}=111.8,$$

$$S=196.6, \qquad SS=5132.7.$$

and

$$S_H=\frac{1}{st}\sum_{i=1}^{r} S_{i\cdot\cdot}^2-\frac{1}{rst}S^2=4853.2-4831.4=21.8,$$

$$S_D=\frac{1}{rt}\sum_{j=1}^{s} S_{\cdot j\cdot}^2-\frac{1}{rst}S^2=4922.6-4831.4=91.2,$$

$$S_I=\frac{1}{t}\sum_{j=1}^{s}\sum_{i=1}^{r}S_{ij\cdot}^2-\frac{1}{st}\sum_{i=1}^{r}S_{i\cdot\cdot}^2-\frac{1}{rt}\sum_{j=1}^{s}S_{\cdot j\cdot}^2+\frac{1}{rst}S^2$$

$$=4963.6-4853.2-4922.6+4831.4=19.2,$$

$$S_E=SS-\frac{1}{t}\sum_{j=1}^{s}\sum_{i=1}^{r}S_{ij\cdot}^2=5132.7-4963.6=169.1,$$

$$S_T=SS-\frac{1}{rst}S^2=5132.7-4831.4=301.3.$$

The analysis of variance table for this two way analysis is presented as Table A7.6.

Table A7.6 The Analysis of Variance Table for the σ_{pi} Values Assuming Intact Specimens

Source of variation	Sum of squares	Degrees of freedom	Mean square	F
Between H's	21.8	1	21.8	$F_H = 0.5$
Between D's	91.2	1	91.2	$F_D = 2.2$
Interaction	19.2	1	19.2	$F_I = 0.5$
Error	169.1	4	42.3	
Total	301.3	7		

Because

$$F_\alpha((r-1),\ rs(t-1))=F_{0.05}(1,\ 4)=7.7 > F_H=0.5,$$

the null hypothesis H_{0_1} must be accepted.

$F_{\alpha}((s-1),\ rs(t-1))=F_{0.05}(1,\ 4)=7.7 > F_D=2.2,$

the null hypothesis H_{0_2} *must be accepted.*

$F_{\alpha}((r-1)(s-1),\ rs(t-1))=F_{0.05}(1,\ 4)=7.7 > F_I=0.5,$

the null hypothesis H_{0_3} *must be accepted.*

A7.4 Height variation effect on the m_i values

Referring to Appendix 8, the following table is obtained.

Table A7.7 m_i Values of Different Non-Intact Specimen Height Groups.

H (*meter*)	0.080	0.090	0.120	Total
m_i	4.0 (Coal3-1)	0.7 (Coal7-4)	0.6 (Coal6-3)	
	3.0 (Coal5-2)	0.6 (Coal6-4)	0.3 (Coal7-2)	
Σ	7.0	1.3	0.9	9.2

The number of different heights $k = 3$. Under each height the number of tests are $n_1 = 2$, $n_2 = 2$, $n_3 = 2$ respectively. Total $n = 2 + 2 + 2 = 6$. Denote μ_1, μ_2, μ_3 as the mean values of m_i of the k populations corresponding to the specimen heights of $H = 0.080$, 0.090, 0.120 *meter*, then examine:

$$H_0:\ \mu_1=\mu_2=\mu_3$$

The squares of the m_i values in Table A7.7 are shown in Table A7.8:

Table A7.8 Squares of the m_i Values of Different Height Groups for Non-Intact Specimens.

H (*meter*)	0.080	0.090	0.120	Total
m_i^2	16.0 (Coal3-1)	0.5 (Coal7-4)	0.4 (Coal6-3)	
	9.0 (Coal5-2)	0.4 (Coal6-4)	0.09 (Coal7-2)	
Σ	25.0	0.9	0.5	26.3

Hence

$$\sum_{i=1}^{k}\frac{S_i^2}{n_i}=\frac{7.0^2}{2}+\frac{1.3^2}{2}+\frac{0.9^2}{2}=25.8,$$

$$\frac{\left(\sum_{i=1}^{k}S_i\right)^2}{n}=\frac{9.2^2}{6}=14.1,$$

$$\sum_{i=1}^{k}SS_i=26.3,$$

$$S_B=\sum_{i=1}^{k}\frac{S_i^2}{n_i}-\frac{\left(\sum_{i=1}^{k}S_i\right)^2}{n}=25.8-14.1=11.7,$$

$$S_T=\sum_{i=1}^{k}SS_i-\frac{(\sum_{i=1}^{k}S_i)^2}{n}=S_B+S_E=12.2.$$

The degrees of freedom are:

$$f_B=k-1=3-1=2,\quad f_E=n-k=6-3=3,\quad f_T=n-1=6-1=5.$$

The analysis of variance table is shown as Table A7.9:

Table A7.9 Analysis of Variance of Height Effect on the m_i Values for Non-Intact Specimens.

Sources of variation	Sum of squares	Degrees of freedom	Mean square	F
Between heights	11.7	2	5.9	29.5
Error	0.5	3	0.2	
Total	12.2	5		

Because $F_{0.05}(2, 3) = 9.6$ is less than 29.5, under the error probability of 0.05, the null hypothesis H_0 is rejected.

A7.5 Height variation effect on the σ_{pi} values for non-intact specimens

According to Appendix 8, the σ_{pi} values determined for non-intact specimens are listed in the following table.

Table A7.10 σ_{pi} Values of Different Height Groups for Non-Intact Specimens.

H (*meter*)	0.080	0.090	0.120	Total
σ_{pi} (*MPa*)	44.3 (Coal3-1)	36.2 (Coal7-4)	53.5 (Coal6-3)	
	63.4 (Coal5-2)	86.7 (Coal6-4)	92.9 (Coal7-2)	
Σ	107.7	122.9	146.4	377.0

The number of different heights $k = 3$. Under each height the number of tests are $n_1 = 2$, $n_2 = 2$, $n_3 = 2$ respectively. Total $n = 2 + 2 + 2 = 6$. Denote μ_1, μ_2, μ_3 as the mean values of σ_{pi} of the k populations corresponding to the specimen heights of $H = 0.080$, 0.090, 0.120 *meter*, then examine:

$$H_0:\ \mu_1=\mu_2=\mu_3$$

The squares of the σ_{pi} values in Table A7.10 are shown in Table A7.11:

Table A7.11 Squares of the σ_{pi} Values of Different Height Groups for Non-Intact Specimens.

H (*meter*)	0.080	0.090	0.120	Total
σ_{pi}^2 $(MPa)^2$	1962.5 (Coal3-1)	1310.4 (Coal7-4)	2862.3 (Coal6-3)	
	4019.6 (Coal5-2)	7516.9 (Coal6-4)	8630.4 (Coal7-2)	
Σ	5982.1	8827.3	11492.7	26302.0

Hence

$$\sum_{i=1}^{k}\frac{S_i^2}{n_i}=\frac{107.7^2}{2}+\frac{122.9^2}{2}+\frac{146.4^2}{2}=24068.3,$$

$$\frac{\left(\sum_{i=1}^{k}S_i\right)^2}{n}=\frac{377.0^2}{6}=23688.2,$$

$$\sum_{i=1}^{k}SS_i=26302.0,$$

$$S_B=\sum_{i=1}^{k}\frac{S_i^2}{n_i}-\frac{\left(\sum_{i=1}^{k}S_i\right)^2}{n}=24068.3-23688.2=380.1,$$

$$S_E=\sum_{i=1}^{k} SS_i-\sum_{i=1}^{k} \frac{S_i^2}{n_i}=26302.0-24068.3=2233.7,$$

$$S_T=\sum_{i=1}^{k} SS_i-\frac{(\sum_{i=1}^{k} S_i)^2}{n}=S_B+S_E=2613.8.$$

The degrees of freedom are:

$$f_B=k-1=3-1=2,\quad f_E=n-k=6-3=3,\quad f_T=n-1=6-1=5.$$

The analysis of variance table is shown as Table A7.12:

Table A7.12 Analysis of Variance of Height Effect on the σ_{pi} Values for Non-Intact Specimens.

Sources of variation	Sum of squares	Degrees of freedom	Mean square	F
Between heights	380.1	2	190.1	0.3
Error	2233.7	3	744.6	
Total	2613.8	5		

$F_{0.05}(2, 3) = 9.6$, which is greater than 0.03, therefore the null hypothesis H_0 has to be accepted.

A7.6 Diameter variation effect on the m_i values

Referring to Appendix 8, choose specimens with a height of 0.080 *meter* and divide them into two groups, Group 1 and Group 2. In Group 1 all the specimens has a diameter of 0.056 *meter* while in group 2, 0.068 *meter*. Their corresponding m_i values (denoted as m_{i1} and m_{i2} respectively) from Appendix 8 are listed in Table A7.13.

Table A7.13 The m_i Values of Two Diameter Groups for Non-Intact Specimens

Group 1				Group 2		
D (*meter*)	*H* (*meter*)	m_{i1}		*D* (*meter*)	*H* (*meter*)	m_{i2}
0.056	0.080	4.0 (Coal3-1)		0.068	0.080	2.2 (Coal1-C)
0.056	0.080	3.0 (Coal5-2)		0.068	0.080	1.3 (Coal2-A)
				0.068	0.080	0.9 (Coal4-B)
				0.068	0.080	1.3 (Coal4-E)

Regard the m_i data from Group 1 and Group 2 as two independent random samples of size $n_1 = 2$ and $n_2 = 4$ from two normal populations with means of μ_1 and μ_2 and the known variances of $V[m_{i1}]$ and $V[m_{i2}]$ respectively. The sample means are:

$$\overline{m}_{i1}=\frac{1}{n_1}\sum_{i=1}^{n_1} m_{i1_i}=\frac{7.0}{2}=3.5,$$

$$\overline{m}_{i2}=\frac{1}{n_2}\sum_{i=1}^{n_2} m_{i2_i}=\frac{5.7}{4}=1.4.$$

The sample variances are:

$$s_1^2=\frac{\sum_{i=1}^{n_1}(m_{i1_i}-\overline{m}_{i1})^2}{n_1-1}=\frac{0.5}{2-1}=0.5,$$

$$s_2^2=\frac{\sum_{i=1}^{n_2}(m_{i2_i}-\overline{m}_{i2})^2}{n_2-1}=\frac{0.9}{4-1}=0.3.$$

First, examine the equality of the two population variances, i.e. the null hypothesis $V[m_{i1}] = V[m_{i2}]$. The two-sided alternative is $V[m_{i1}] \neq V[m_{i1}]$. Since

$$\frac{s_1^2}{s_2^2}=\frac{0.5}{0.3}=1.7,$$

which is less than

$$F_{\alpha/2}(n_1-1,\ n_2-1)=F_{0.025}(1,\ 3)=17.4,$$

the null hypothesis can be accepted, i.e. the two populations have the same variance and the use of the following two-sample t test is justified.

Now test the null hypothesis $\mu_1 - \mu_2 = \delta = 0$ against the two-sided alternative $\mu_1 - \mu_2 \neq 0$. Since

$$t=\frac{|\bar{m}_{i1}-\bar{m}_{i2}-\delta|}{\sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$=\frac{|3.5-1.4-0|}{\sqrt{\frac{1\times0.5+3\times0.3}{2+4-2}}\sqrt{\frac{1}{2}+\frac{1}{4}}}$$

$=4.1$, *which is greater than* $t_{\alpha/2}(n_1+n_2-2)=t_{0.025}(2+4-2)=2.8$

hence the null hypothesis must be rejected.

A7.7 Diameter variation effect on σ_{pi} values for non-intact specimens

Referring to Appendix 8, choose the two groups of specimens: Group 1 and Group 2. The σ_{pi} values are listed in Table A7.14

Table A7.14 σ_{pi} Values of Two Diameter Groups for Non-Intact Specimens

Group 1				Group 2		
D (*meter*)	H (*meter*)	σ_{pi1} (*MPa*)		D (*meter*)	H (*meter*)	σ_{pi2} (*MPa*)
0.056	0.080	44.3 (Coal3-1)		0.068	0.080	31.8 (Coal1-C)
0.056	0.080	63.4 (Coal5-2)		0.068	0.080	59.9 (Coal2-A)
0.056	0.080			0.068	0.080	79.5 (Coal4-B)
0.056	0.080			0.068	0.080	56.3 (Coal4-E)

Regard the σ_{pi} data from Group 1 and Group 2 as two independent random samples of size $n_1 = 2$ and $n_2 = 4$ from two normal populations with means of μ_1 and μ_2 and the known variances of $V[\sigma_{pi1}]$ and $V[\sigma_{pi2}]$ respectively. The sample means are:

$$\overline{\sigma}_{pi1}=\frac{1}{n_1}\sum_{i=1}^{n_1}\sigma_{pi1_i}=\frac{107.7}{2}=53.9,$$

$$\overline{\sigma}_{pi2}=\frac{1}{n_2}\sum_{i=1}^{n_2}\sigma_{pi2_i}=\frac{227.5}{4}=56.9.$$

The sample variances are:

$$s_1^2=\frac{\sum_{i=1}^{n_1}(\sigma_{pi1_i}-\overline{\sigma}_{pi1})^2}{n_1-1}=\frac{182.4}{2-1}=182.4,$$

$$s_2^2=\frac{\sum_{i=1}^{n_2}(\sigma_{pi2_i}-\overline{\sigma}_{pi2})^2}{n_2-1}=\frac{1150.1}{4-1}=383.4.$$

Before performing the *t* test concerning the means, the equality of the two population variances, i.e. the null hypothesis $V[\sigma_{pi1}] = V[\sigma_{pi2}]$, has to be tested. The two-sided alternative is $V[\sigma_{pi1}] \neq V[\sigma_{pi2}]$. Since

$$\frac{s_1^2}{s_2^2}=\frac{182.4}{383.4}=0.5,$$

which is less than

$$F_{\alpha/2}(n_1-1,\ n_2-1)=F_{0.025}(1,\ 3)=17.4$$

the null hypothesis must be accepted, i.e. the two populations have the same variance and the use of the following two-sample *t* test is justified.

Now test the null hypothesis $\mu_1 - \mu_2 = \delta = 0$ against the two-sided alternative $\mu_1 - \mu_2 \neq 0$. Since

$$t=\frac{|\overline{\sigma}_{pi1}-\overline{\sigma}_{pi2}-\delta|}{\sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$=\frac{|53.9-56.9-0|}{\sqrt{\frac{1\times182.4+3\times383.4}{2+4-2}}\sqrt{\frac{1}{2}+\frac{1}{4}}}$$

$=0.2$, *which is less than* $t_{\alpha/2}(n_1+n_2-2)=t_{0.025}(2+4-2)=2.8$, therefore the null hypothesis must be accepted, i.e. the mean values of σ_{pi} between the two different diameter groups of non-intact specimens are not significantly different.

A7.8 The combined effect of height and diameter variation on m_i values for non-intact specimens

Referring to Appendix 8, the following two-way analysis of variance is carried out to account for the possible joint effect that a particular height might yield a very high or very low m_i value if and only if it has a particular diameter. There are two variables: height H_i and diameter D_j, where $i = 1, 2, ..., r$ and $j = 1, 2, ... s$. Look upon the m_i value obtained with height H_i and diameter D_j as a value assumed by a random variable having a normal distribution with a variance $V[m_i]$ and the mean

$$\mu_{ij}=\mu+\beta_i+\gamma_j+\epsilon_{ij}$$

where

$$\sum_{i=1}^{r}\beta_i=0, \quad \sum_{j=1}^{s}\gamma_j=0,$$

while

$$\sum_{i=1}^{r}\epsilon_{ij}=0, \quad j=1,\ 2,\ \ldots,\ s,$$

$$\sum_{j=1}^{s}\epsilon_{ij}=0, \quad i=1,\ 2,\ \ldots,\ r.$$

In order to test the hypotheses concerning the β_i, the γ_j, as well as the ε_{ij}, each possible combination of the heights and diameters were tested more than once. The test results are shown in Table A7.15

Table A7.15 m_i Values of Different Height and Diameter Combinations for Non-Intact Specimens

		Diameter (*meter*)	
		0.056	0.068
Height (*meter*)	0.080	4.0 (Coal3-1)	1.3 (Coal2-A)
		3.0 (Coal5-2)	1.3 (Coal4-E)
	0.120	0.6 (Coal6-3)	0.1 (Coal1-D)
		0.3 (Coal7-2)	0.2 (Coal5-B)

Denote x_{ijk} as the kth value obtained with height H_i and diameter D_j, and taking t observations of each kind, the model considered becomes

$$x_{ij}=\mu+\beta_i+\gamma_j+\varepsilon_{ij}+\eta_{ijk},$$
$$i=1,\ 2,\ \ldots,\ r;\quad j=1,\ 2,\ \ldots,\ s,\quad k=1,\ 2,\ \ldots,\ t.$$

where the η_{ijk} are values assumed by independent random variables having normal distributions with zero means and the common variance $V[m]$, while μ, β_i, γ_j, ε_{ij} and

$V[m]$ are unknowns. The null hypotheses to be tested are:

$$H_{0_1}: \ \beta_1=\beta_2= \cdots =\beta_r=0,$$

$$H_{0_2}: \ \gamma_1=\gamma_2= \cdots =\gamma_s=0,$$

$$H_{0_3}: \ \epsilon_{11}= \cdots =\epsilon_{1s}= \cdots =\epsilon_{rs}=0 .$$

The corresponding alternative hypotheses are that the respective parameters are not all equal to zero. The tests of these hypotheses are based on the analysis of the total variability of the m_i data, decomposing it into terms which can be attributed to differences among the heights S_H, differences among the diameters S_D, interactions (joint effects) S_I, and chance (experimental error) S_E. First denote

$$\bar{x}=\frac{1}{rst}\sum_{k=1}^{t}\sum_{j=1}^{s}\sum_{i=1}^{r}x_{ijk},$$

$$\bar{x}_{ij\cdot}=\frac{1}{t}\sum_{k=1}^{t}x_{ijk},$$

$$\bar{x}_{i\cdot\cdot}=\frac{1}{st}\sum_{k=1}^{t}\sum_{j=1}^{s}x_{ijk},$$

$$\bar{x}_{\cdot j\cdot}=\frac{1}{rt}\sum_{k=1}^{t}\sum_{i=1}^{r}x_{ijk}.$$

therefore

$$S_T=\sum_{k=1}^{t}\sum_{j=1}^{s}\sum_{i=1}^{r}(x_{ijk}-\overline{x}_{ij.})^2$$
$$+st\sum_{i=1}^{r}(\overline{x}_{i..}-\overline{x})^2+rt\sum_{j=1}^{s}(\overline{x}_{.j.}-\overline{x})^2$$
$$+t\sum_{j=1}^{s}\sum_{i=1}^{r}(\overline{x}_{ij.}-\overline{x}_{i..}-\overline{x}_{.j.}+\overline{x})^2$$
$$=S_E+S_H+S_D+S_I.$$

Let

$$S_{ij.}=\sum_{k=1}^{t}x_{ijk},$$

$$S_{i..}=\sum_{k=1}^{t}\sum_{j=1}^{s}x_{ijk},\qquad S_{.j.}=\sum_{k=1}^{t}\sum_{i=1}^{r}x_{ijk},$$

$$S=\sum_{k=1}^{t}\sum_{j=1}^{s}\sum_{i=1}^{r}x_{ijk},\qquad SS=\sum_{k=1}^{t}\sum_{j=1}^{s}\sum_{i=1}^{r}x_{ijk}^2.$$

Hence

$S_{11.}=7.0,\qquad S_{12.}=2.6,$

$S_{21.}=0.9,\qquad S_{22.}=0.3,$

$S_{1..}=8.6,\qquad S_{2..}=1.2,$

$S_{.1.}=7.9,\qquad S_{.2.}=2.9,$

$S=10.8,\qquad SS=28.9.$

and

$$S_H=\frac{1}{st}\sum_{i=1}^{r}S_{i..}^2-\frac{1}{rst}S^2=18.9-14.6=4.3,$$

$$S_D=\frac{1}{rt}\sum_{j=1}^{s}S_{.j.}^2-\frac{1}{rst}S^2=17.7-14.6=3.1,$$

$$S_I=\frac{1}{t}\sum_{j=1}^{s}\sum_{i=1}^{r}S_{ij.}^2-\frac{1}{st}\sum_{i=1}^{r}S_{i..}^2-\frac{1}{rt}\sum_{j=1}^{s}S_{.j.}^2+\frac{1}{rst}S^2$$
$$=28.3-18.9-17.7+14.6=6.3,$$

$$S_E=SS-\frac{1}{t}\sum_{j=1}^{s}\sum_{i=1}^{r}S_{ij.}^2=28.9-28.3=0.6,$$

$$S_T=SS-\frac{1}{rst}S^2=28.9-14.6=14.3.$$

The analysis of variance table is presented as Table A7.16

Table A7.16 The Analysis of Variance Table for the m_i Values for Non-Intact Specimens

Source of variation	Sum of squares	Degrees of freedom	Mean square	F
Between H's	4.3	1	4.3	$F_H = 21.5$
Between D's	3.1	1	3.1	$F_D = 15.5$
Interaction	6.3	1	6.3	$F_I = 31.5$
Error	0.6	4	0.2	
Total	14.3	7		

Because

$$F_\alpha((r-1),\ rs(t-1))=F_{0.05}(1,\ 4)=7.7 < F_H=21.5,$$

the null hypothesis H_{0_1} must be rejected.

$$F_\alpha((s-1),\ rs(t-1))=F_{0.05}(1,\ 4)=7.7 < F_D=15.5,$$

the null hypothesis H_{0_2} must be rejected.

$$F_\alpha((r-1)(s-1),\ rs(t-1))=F_{0.05}(1,\ 4)=7.7 < F_I=31.5,$$

the null hypothesis H_{0_3} must be rejected.

Therefore, after having considered the different height and diameter combinations and their joint effects on the m_i values, it can now be concluded that both specimen height and diameter have a significant effect on the m_i values and there exist joint effects between the different combinations of heights and diameters.

A7.9 The combined effect of height and diameter variation on σ_{pi} values for non-intact specimens

Obtain Table A7.17 form Appendix 8 for the σ_{pi} values calculated as non-intact specimens.

Table A7.17 The σ_{pi} Values of Different Height and Diameter Combinations for Non-Intact Specimens

		Diameter (*meter*)	
		0.056	0.068
Height (*meter*)	0.080	44.3 (Coal3-1)	59.9 (Coal2-A)
		63.4 (Coal5-2)	56.3 (Coal4-E)
	0.120	53.5 (Coal6-3)	80.4 (Coal1-D)
		92.9 (Coal7-2)	119.5 (Coal5-B)

Denote [illegible] as the kth value of σ_{pi} obtained with height H_i and diameter D_j, and taking t observations of each kind, the model considered becomes

$$x_{ij}=\mu+\beta_i+\gamma_j+\epsilon_{ij}+\eta_{ijk},$$
$$i=1,\ 2,\ \ldots,\ r;\quad j=1,\ 2,\ \ldots,\ s,\quad k=1,\ 2,\ \ldots,\ t.$$

where the η_{ijk} are values assumed by independent random variables having normal distributions with zero means and the common variance $V[\sigma_{pi}]$, while μ, β_i, γ_j, ϵ_{ij} and $V[\sigma_{pi}]$ are unknowns. The null hypotheses to be tested are:

$$H_{0_1}:\ \beta_1=\beta_2=\cdots=\beta_r=0,$$
$$H_{0_2}:\ \gamma_1=\gamma_2=\cdots=\gamma_s=0,$$
$$H_{0_3}:\ \epsilon_{11}=\cdots=\epsilon_{1s}=\cdots=\epsilon_{rs}=0.$$

The corresponding alternative hypotheses are that the respective parameters are not all equal to zero. Omit the mathematical derivation, the following is obtained:

$$S_{11\cdot}=107.7,\qquad S_{12\cdot}=116.2,$$
$$S_{21\cdot}=146.4,\qquad S_{22\cdot}=199.9,$$
$$S_{1\cdot\cdot}=223.9,\qquad S_{2\cdot\cdot}=346.3,$$
$$S_{\cdot1\cdot}=254.1,\qquad S_{\cdot2\cdot}=316.1,$$
$$S=570.2,\qquad SS=44976.8.$$

and

$$S_H=\frac{1}{st}\sum_{i=1}^{r}S_{i\cdot\cdot}^2-\frac{1}{rst}S^2=42513.7-40641.0=1872.7,$$

$$S_D=\frac{1}{rt}\sum_{j=1}^{s}S_{\cdot j\cdot}^2-\frac{1}{rst}S^2=41121.5-40641.0=480.5,$$

$$S_I=\frac{1}{t}\sum_{j=1}^{s}\sum_{i=1}^{r}S_{ij\cdot}^2-\frac{1}{st}\sum_{i=1}^{r}S_{i\cdot\cdot}^2-\frac{1}{rt}\sum_{j=1}^{s}S_{\cdot j\cdot}^2+\frac{1}{rst}S^2$$
$$=43247.4-42513.7-41121.5+40641.0=253.2,$$

$$S_E=SS-\frac{1}{t}\sum_{j=1}^{s}\sum_{i=1}^{r}S_{ij\cdot}^2=44976.8-43247.4=1729.4,$$

$$S_T=SS-\frac{1}{rst}S^2=44976.8-40641.0=4335.8.$$

The analysis of variance table for this two way analysis is presented as Table A7.18

Table A7.18 The Analysis of Variance Table for the σ_{pi} Values for Non-Intact Specimens

Source of variation	Sum of squares	Degrees of freedom	Mean square	F
Between H's	1872.7	1	1872.7	$F_H = 4.3$
Between D's	480.5	1	480.5	$F_D = 1.1$
Interaction	253 2	1	253.2	$F_I = 0.6$
Error	1729.4	4	432.4	
Total	4335.8	7		

Because

$$F_\alpha((r-1),\ rs(t-1))=F_{0.05}(1,\ 4)=7.7 > F_H=4.3,$$

the null hypothesis H_{0_1} *must be accepted.*

$F_{\alpha}((s-1),\ rs(t-1))=F_{0.05}(1,\ 4)=7.7 > F_{D}=1.1$,
the null hypothesis H_{0_2} *must be accepted.*

$F_{\alpha}((r-1)(s-1),\ rs(t-1))=F_{0.05}(1,\ 4)=7.7 > F_{I}=0.6$,
the null hypothesis H_{0_3} *must be accepted.*

APPENDIX 8. THE PARAMETERS FOR COAL OF VARIOUS RANKS, SIZES AND SHAPES

Category 1

In Category 1, the dimensions of the specimens satisfy both of the following two conditions (a) and (b):

$$0.784 < \frac{\sqrt{D}}{H} < 8.784 \qquad (a)$$

$$0.013\ (meter) < D < 1.626\ (meter) \qquad (b)$$

The test data were further classified according to the E.C.E. International Class Number and the ratio of $D^{1/2}/H$.

Note: Sometimes the test data give a negative value for the square of σ_{pi} during the regression analysis, the calculation therefore cannot proceed any more to give a reasonable result. In such cases the test data was still kept in this appendix since these test data may be a valuable reference material for some other studies in the future, while the positions of the corresponding parameters were left blank

E.C.E. International Class Number 1A

$D^{1/2}/H$=3.137 ($D^{1/2}/H$=0.5 in *inches*) and *meters* 0.013<D<1.626 *meters*.

[Anthracite, lightly cleated. Volatile (dry, ash-free) 5%. Pumpquart seam, Pentremawr Colliery. Orientation of weakness planes to specimen axis: Bedding planes parallel, main cleat planes parallel.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.025	0.051	0	18.1	8	[46]	0.170	20.1	123.4	21.6	0.97
0.025	0.051	6.9	66.5	8						
0.025	0.051	13.8	93.2	8						
0.025	0.051	20.7	98.5	8						
0.025	0.051	27.6	137.4	8						
0.025	0.051	32.8	155.4	8						

[Anthracite, lightly cleated. Volatile (dry, ash-free) 5%. Pumpquart seam, Pentremawr Colliery. Orientation of weakness planes to specimen axis: Bedding planes perpendicular, main cleat planes parallel.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.025	0.051	0	31.6	7	[46]	0.128				
0.025	0.051	6.9	76.5	7						
0.025	0.051	13.8	97.9	7						
0.025	0.051	20.7	128.5	7						
0.025	0.051	27.6	158.1	7						
0.025	0.051	32.8	204.9	7						

E.C.E. International Class Number 2

$D^{1/2}/H$=3.137 ($D^{1/2}/H$=0.5 in *inches*) and *meters* 0.013<D<1.626 *meters*.

[Friable, two well-defined systems of cleat planes. Volatile (dry, ash-free) 12%. Five feet (Gellideg) seam, Deep Duffryn Colliery. Orientation of weakness planes to specimen axis: Bedding planes parallel, main cleat planes parallel.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.025	0.051	0	7.0	7						
0.025	0.051	3.4	35.0	7						
0.025	0.051	6.9	45.4	7						
0.025	0.051	13.8	65.2	7	[46]	0.170	40.6	26.3	4.5	0.99
0.025	0.051	20.7	86.8	7						
0.025	0.051	27.6	104.2	7						
0.025	0.051	34.5	111.3	7						

[Friable, two well-defined systems of cleat planes. Volatile (dry, ash-free) 12%. Five feet (Gellideg) seam, Deep Duffryn Colliery. Orientation of weakness planes to specimen axis: Bedding planes perpendicular, main cleat planes parallel.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.025	0.051	0	6.9	7						
0.025	0.051	0.7	19.0	7						
0.025	0.051	1.7	26.1	7						
0.025	0.051	3.4	34.6	7						
0.025	0.051	6.9	50.7	7	[46]	0.170	46.6	20.9	3.6	0.99
0.025	0.051	13.8	68.8	7						
0.025	0.051	20.7	80.1	7						
0.025	0.051	27.6	99.7	7						
0.025	0.051	34.5	110.8	7						

E.C.E. International Class Number 4

$D^{1/2}/H$=3.137 ($D^{1/2}/H$=0.5 in *inches*) and *meters* 0.013<*D*<1.626 *meters*.

[Extremely friable, two well-defined systems of cleat planes. Volatile (dry, ash-free) 22%. Meadow seam, Oakdale Colliery. Orientation of weakness planes to specimen axis: Bedding planes parallel, main cleat planes parallel.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pi} (*MPa*)	m_i	*m*	*r*
0.025	0.051	0	2.2	8	[46]	0.170				
0.025	0.051	3.4	25.4	8						
0.025	0.051	6.9	30.9	8						
0.025	0.051	13.8	47.9	8						
0.025	0.051	20.7	68.8	8						
0.025	0.051	27.6	95.0	8						
0.025	0.051	34.5	105.7	8						

[Extremely friable, two well-defined systems of cleat planes. Volatile (dry, ash-free) 22%. Meadow seam, Oakdale Colliery. Orientation of weakness planes to specimen axis: Bedding planes perpendicular, main cleat planes parallel.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pi} (*MPa*)	m_i	*m*	*r*
0.025	0.051	0	5.3	8	[46]	0.170	31.1	30.3	5.1	0.95
0.025	0.051	0.7	19.9	8						
0.025	0.051	1.7	24.2	8						
0.025	0.051	3.4	41.3	8						
0.025	0.051	6.9	42.5	8						
0.025	0.051	13.8	56.3	8						
0.025	0.051	20.7	76.6	8						
0.025	0.051	27.6	87.4	8						
0.025	0.051	34.5	117.2	8						

[Extremely friable, two well-defined systems of cleat planes. Volatile (dry, ash-free) 22%. Meadow seam, Oakdale Colliery. Orientation of weakness planes to specimen axis: Bedding planes parallel, main cleat planes perpendicular.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{ri} (*MPa*)	m_i	m	r
0.025	0.051	0	5.7	8	[48]	0.170				
0.025	0.051	3.4	28.1	8						
0.025	0.051	6.9	44.7	8						
0.025	0.051	13.8	46.5	8						
0.025	0.051	20.7	71.9	8						
0.025	0.051	27.6	85.0	8						
0.025	0.051	34.5	115.9	8						

E.C.E. International Class Number 5

$D^{1/2}/H$=3.137 ($D^{1/2}/H$=0.5 in *inches*) and *meters* 0.013<D<1.626 *meters*.

[Friable, two well-defined systems of cleat planes. Volatile (dry, ash-free) 30%. Garw seam, Cwmtillery Colliery. Orientation of weakness planes to specimen axis: Bedding planes parallel, main cleat planes parallel.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{ri} (*MPa*)	m_i	m	r
0.025	0.051	0	7.0	5	[46]	0.170	39.5	39.5	6.7	1.00
0.025	0.051	3.4	40.1	5						
0.025	0.051	6.9	50.5	5						
0.025	0.051	20.7	99.9	5						
0.025	0.051	34.5	130.0	5						

[Friable, two well-defined systems of cleat planes. Volatile (dry, ash-free) 30%. Garw seam, Cwmtillery Colliery. Orientation of weakness planes to specimen axis: Bedding planes perpendicular, main cleat planes parallel.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pi} (MPa)	m_i	m	r
0.025	0.051	0	13.2	5	[46]	0.170	60.8	21.8	3.7	0.99
0.025	0.051	0.7	29.7	5						
0.025	0.051	1.7	30.0	5						
0.025	0.051	3.4	42.8	5						
0.025	0.051	6.9	58.8	5						
0.025	0.051	20.7	94.4	5						
0.025	0.051	34.5	125.2	5						

[Friable, two well-defined systems of cleat planes. Volatile (dry, ash-free) 30%. Garw seam, Cwmtillery Colliery. Orientation of weakness planes to specimen axis: Bedding planes parallel, main cleat planes parallel.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pi} (MPa)	m_i	m	r
0.025	0.051	0	5.8	8	[47]	0.170	27.3	50.8	8.6	0.99
0.025	0.051	6.9	50.3	8						
0.025	0.051	13.7	69.8	8						
0.025	0.051	20.6	94.1	8						
0.025	0.051	27.4	107.1	8						

[Friable, two well-defined systems of cleat planes. Volatile (dry, ash-free) 30%. Garw seam, Cwmtillery Colliery. Orientation of weakness planes to specimen axis: Bedding planes perpendicular, main cleat planes parallel.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pi} (MPa)	m_i	m	r
0.025	0.051	0	10.7	8	[47]	0.170	59.3	26.9	4.6	0.98
0.025	0.051	6.9	57.0	8						
0.025	0.051	13.7	84.8	8						
0.025	0.051	20.6	101.6	8						
0.025	0.051	27.4	113.4	8						

[Friable, two well-defined systems of cleat planes. Volatile (dry, ash-free) 30%. Garw seam, Cwmtillery Colliery. Orientation of weakness planes to specimen axis: Bedding planes parallel, main cleat planes perpendicular.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{ci} (*MPa*)	m_i	m	r
0.025	0.051	0	7.9	8	[47]	0.170	55.4	31.3	5.3	0.98
0.025	0.051	6.9	50.3	8						
0.025	0.051	13.7	75.5	8						
0.025	0.051	20.6	95.3	8						
0.025	0.051	27.4	103.5	8						

E.C.E. International Class Number 6

$D^{1/2}/H$=3.137 ($D^{1/2}/H$=0.5 in *inches*) and *meters* 0.013<D<1.626 *meters*.

[Fragile bright, heavily cleated. Volatile (dry, ash-free) 39%. Black Shale seam, Markham Colliery. Orientation of weakness planes to specimen axis: Bedding planes parallel, main cleat planes parallel.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{ci} (*MPa*)	m_i	m	r
0.025	0.051	0	15.9	8	[46]	0.170	100.5	11.8	2.0	0.96
0.025	0.051	3.4	58.5	8						
0.025	0.051	6.9	69.6	8						
0.025	0.051	13.8	82.5	8						
0.025	0.051	20.7	99.9	8						
0.025	0.051	27.6	111.7	8						
0.025	0.051	34.5	125.7	8						

[Fragile bright, heavily cleated. Volatile (dry, ash-free) 39%. Black Shale seam, Markham Colliery. Orientation of weakness planes to specimen axis: Bedding planes perpendicular, main cleat planes parallel.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{p} (*MPa*)	m_i	m	r
0.025	0.051	0	21.8	5	[46]	0.170	83.9	14.2	2.4	0.97
0.025	0.051	0.7	24.5	5						
0.025	0.051	1.7	46.0	6						
0.025	0.051	3.4	52.1	6						
0.025	0.051	6.9	57.6	6						
0.025	0.051	13.8	83.2	6						
0.025	0.051	20.7	99.0	6						
0.025	0.051	27.6	105.5	6						
0.025	0.051	34.5	123.4	6						

E.C.E. International Class Number 7

$D^{1/2}/H$=2.136 ($D^{1/2}/H$=0.340 in *inches*) and *meters* 0.013<*D*<1.626 *meters*.

[Blind Canyon seam, Beehive Mine, American Coal Co. Emery Co., Utah. Moisture 4.8%, BTU Value (moist) 13590, Volatile 44.6%, Fixed Carbon 48.0%, Ash 7.2%, Sulphur 0.4%. Specimens were cored both perpendicular and parallel to the bedding planes.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_p (*MPa*)	m_i	*m*	*r*
0.055	0.110	1.4	12.4	1	[55]	0.095	70.2	34.4	3.3	0.66
0.055	0.110	3.1	18.3	1						
0.055	0.110	0.6	20.6	1						
0.055	0.110	0.3	22.4	1						
0.055	0.110	2.1	21.4	1						
0.055	0.110	0.7	23.4	1						
0.055	0.110	0.7	23.9	1						
0.055	0.110	3.6	24.7	1						
0.055	0.110	1.7	28.6	1						
0.055	0.110	1.4	29.0	1						
0.055	0.110	1.7	34.8	1						
0.055	0.110	0.7	36.6	1						
0.055	0.110	3.4	34.5	1						
0.055	0.110	2.2	36.4	1						
0.055	0.110	1.4	38.6	1						
0.055	0.110	3.4	37.2	1						
0.055	0.110	3.8	38.3	1						
0.055	0.110	3.4	46.2	1						
0.055	0.110	6.6	45.2	1						
0.055	0.110	6.9	48.3	1						
0.055	0.110	7.2	52.8	1						
0.055	0.110	3.1	58.3	1						
0.055	0.110	6.9	56.5	1						
0.055	0.110	7.2	58.3	1						

[Pittsburgh seam, Federal No. 2 Mine, Eastern Assoc. Coal Corp., Marion Co., West Va.. Moisture 2.3%, BTU Value (moist) 13930, Volatile 37.5%, Fixed Carbon 54.4%, Ash 8.0%, Sulphur 1.7%. Specimens were cored perpendicular to the bedding planes.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{ft} (MPa)	m_i	m	r
0.055	0.110	2.1	29.7	1	[55]	0.095	76.7	37.5	3.5	0.60
0.055	0.101	2.1	33.8	1						
0.055	0.101	3.8	34.8	1						
0.055	0.110	4.1	37.2	1						
0.055	0.110	3.8	38.3	1						
0.055	0.110	4.1	45.5	1						
0.055	0.110	7.6	46.2	1						
0.055	0.110	3.4	51.0	1						
0.055	0.110	3.5	51.7	1						
0.055	0.110	6.9	53.8	1						
0.055	0.110	6.9	57.9	1						
0.055	0.110	7.6	60.0	1						
0.055	0.110	6.2	68.3	1						

[Pittsburgh seam, Bruceton Exp. Mine, U.S. Bureau of Mines, Bruceton, Pennsylvania. Moisture 3.2%, BTU Value (moist) 13640, Volatile 36.7%, Fixed Carbon 54.5%, Ash 8.7%, Sulphur 1.6%. Specimens were cored perpendicular to the bedding planes.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_m (*MPa*)	m_i	m	r
0.055	0.110	0.7	26.9	1						
0.055	0.101	3.5	28.3	1						
0.055	0.101	0.7	33.8	1						
0.055	0.110	2.1	35.2	1						
0.055	0.110	1.0	36.9	1						
0.055	0.110	4.8	40.7	1						
0.055	0.110	1.4	45.5	1						
0.055	0.110	3.5	46.2	1	[55]	0.095	102.9	34.4	3.3	0.65
0.055	0.110	1.4	49.7	1						
0.055	0.110	1.4	53.8	1						
0.055	0.110	6.9	52.4	1						
0.055	0.110	4.1	62.1	1						
0.055	0.110	6.9	63.5	1						
0.055	0.110	6.2	69.7	1						
0.055	0.110	7.6	77.9	1						

[York Canyon Bed, York Canyon No. 1 Kaiser Steel Corp., Raton, New Mexico. Moisture 1.7%, BTU Value (moist) 14340, Volatile 35.7%, Fixed Carbon 54.9%, Ash 8.8%, Sulphur 0.6%. Specimens were cored both perpendicular and parallel to the bedding planes.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.055	0.110	0	5.0	1						
0.055	0.101	0.3	6.6	1						
0.055	0.101	0.3	7.9	1						
0.055	0.110	0.7	9.0	1						
0.055	0.110	1.7	10.0	1						
0.055	0.110	0.3	12.1	1						
0.055	0.110	1.0	12.8	1						
0.055	0.110	1.4	13.8	1						
0.055	0.110	1.4	15.2	1						
0.055	0.110	0	16.6	1						
0.055	0.110	1.0	19.0	1						
0.055	0.110	3.1	19.7	1						
0.055	0.110	0.7	22.8	1	[55]	0.095	44.9	48.3	4.6	0.53
0.055	0.110	1.4	27.6	1						
0.055	0.110	3.8	25.9	1						
0.055	0.110	1.7	29.3	1						
0.055	0.110	3.8	27.9	1						
0.055	0.110	0.7	31.0	1						
0.055	0.110	3.1	29.3	1						
0.055	0.110	3.8	31.4	1						
0.055	0.110	3.1	32.8	1						
0.055	0.110	2.8	35.9	1						
0.055	0.110	3.5	37.9	1						
0.055	0.110	1.7	46.6	1						
0.055	0.110	3.8	45.2	1						

$D^{1/2}/H$=3.137 ($D^{1/2}/H$=0.5 in *inches*) and *meters* 0.013<*D*<1.626 *meters*.

[Dull and hard, two systems of cleat planes often widely spaced. Volatile (dry, ash-free) 37%. Barnsley (Top Hards seam), Rossington Colliery. Orientation of weakness planes to specimen axis: Bedding planes parallel, main cleat planes parallel.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pi} (*MPa*)	m_i	*m*	*r*
0.025	0.051	0	30.3	6	[46]	0.170				
0.025	0.051	3.4	52.3	6						
0.025	0.051	6.9	67.4	6						
0.025	0.051	13.8	96.1	6						
0.025	0.051	20.7	99.2	6						
0.025	0.051	27.6	130.5	6						
0.025	0.051	34.5	138.8	6						

[Dull and hard, two systems of cleat planes often widely spaced. Volatile (dry, ash-free) 37%. Barnsley (Top Hards seam), Rossington Colliery. Orientation of weakness planes to specimen axis: Bedding planes perpendicular, main cleat planes parallel.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pi} (*MPa*)	m_i	*m*	*r*
0.025	0.051	0	51.4	6	[46]	0.170	93.0	18.4	3.1	0.91
0.025	0.051	0.7	55.5	6						
0.025	0.051	1.7	58.0	6						
0.025	0.051	3.4	64.3	6						
0.025	0.051	6.9	96.7	6						
0.025	0.051	13.8	103.0	6						
0.025	0.051	20.7	113.3	6						
0.025	0.051	27.6	123.7	6						
0.025	0.051	34.5	138.1	6						

[Dull and hard, two systems of cleat planes often widely spaced. Volatile (dry, ash-free) 37%. Barnsley (Top Hards seam), Rossington Colliery. Orientation of weakness planes to specimen axis: Bedding planes parallel, main cleat planes parallel.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.025	0.051	0	24.4	8	[47]	0.170	85.6	20.1	3.4	0.92
0.025	0.051	6.9	67.5	8						
0.025	0.051	13.7	96.5	8						
0.025	0.051	20.6	93.9	8						
0.025	0.051	27.4	126.1	8						

[Dull and hard, two systems of cleat planes often widely spaced. Volatile (dry, ash-free) 37%. Barnsley (Top Hards seam), Rossington Colliery. Orientation of weakness planes to specimen axis: Bedding planes perpendicular, main cleat planes parallel.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.025	0.051	0	42.8	5	[47]	0.170	129.4	12.3	2.1	0.93
0.025	0.051	6.9	76.7	5						
0.025	0.051	13.7	105.0	5						
0.025	0.051	20.6	112.8	5						
0.025	0.051	27.4	123.9	5						

[Dull and hard, two systems of cleat planes often widely spaced. Volatile (dry, ash-free) 37%. Barnsley (Top Hards seam), Rossington Colliery. Orientation of weakness planes to specimen axis: Bedding planes parallel, main cleat planes perpendicular.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.025	0.051	0	32.1	5	[47]	0.170	101.9	20.6	3.5	0.99
0.025	0.051	6.9	78.8	5						
0.025	0.051	13.7	95.0	5						
0.025	0.051	20.6	116.9	5						
0.025	0.051	27.4	133.1	5						

[Dull and hard, two systems of cleat planes often widely spaced. Volatile (dry, ash-free) 37%. Barnsley (Top Hards seam), Rossington Colliery. Orientation of weakness planes to specimen axis: Bedding: 0°; Main cleat: 90°; Cross cleat: 0°.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pl} (MPa)	m_i	m	r
0.025	0.051	0.1	17.9	24	[49]	0.170	68.4	20.3	3.5	0.98
0.025	0.051	3.5	50.4	24						
0.025	0.051	13.8	78.0	24						
0.025	0.051	20.7	95.2	24						

[Dull and hard, two systems of cleat planes often widely spaced. Volatile (dry, ash-free) 37%. Barnsley (Top Hards seam), Rossington Colliery. Orientation of weakness planes to specimen axis: Bedding: 15°; Main cleat: 75°; Cross cleat: 0°.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pl} (MPa)	m_i	m	r
0.025	0.051	0.1	16.5	24	[49]	0.170	43.7	35.1	6.0	0.99
0.025	0.051	3.5	37.6	24						
0.025	0.051	13.8	79.8	24						
0.025	0.051	20.7	94.5	24						

[Dull and hard, two systems of cleat planes often widely spaced. Volatile (dry, ash-free) 37%. Barnsley (Top Hards seam), Rossington Colliery. Orientation of weakness planes to specimen axis: Bedding: 30°; Main cleat: 60°; Cross cleat: 0°.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pl} (MPa)	m_i	m	r
0.025	0.051	0.1	8.5	24	[49]	0.170	26.0	16.9	2.9	0.98
0.025	0.051	3.5	22.3	24						
0.025	0.051	13.8	50.6	24						
0.025	0.051	20.7	59.7	24						

[Dull and hard, two systems of cleat planes often widely spaced. Volatile (dry, ash-free) 37%. Barnsley (Top Hards seam), Rossington Colliery. Orientation of weakness planes to specimen axis: Bedding: 45°; Main cleat: 45°; Cross cleat: 0°.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pl} (MPa)	m_i	m	r
0.025	0.051	0.1	19.2	24	[49]	0.170	53.5	19.3	3.3	0.98
0.025	0.051	3.5	36.7	24						
0.025	0.051	13.8	71.4	24						
0.025	0.051	20.7	82.7	24						

[Dull and hard, two systems of cleat planes often widely spaced. Volatile (dry, ash-free) 37%. Barnsley (Top Hards seam), Rossington Colliery. Orientation of weakness planes to specimen axis: Bedding: 60°; Main cleat: 30°; Cross cleat: 0°.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pi} (*MPa*)	m_i	*m*	*r*
0.025	0.051	0.1	25.0	24	[49]	0.170	76.2	18.6	3.2	0.99
0.025	0.051	3.5	51.3	24						
0.025	0.051	13.8	79.8	24						
0.025	0.051	20.7	97.4	24						

[Dull and hard, two systems of cleat planes often widely spaced. Volatile (dry, ash-free) 37%. Barnsley (Top Hards seam), Rossington Colliery. Orientation of weakness planes to specimen axis: Bedding: 75°; Main cleat: 15°; Cross cleat: 0°.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pi} (*MPa*)	m_i	*m*	*r*
0.025	0.051	0.1	41.9	24	[49]	0.170	127.4	12.6	2.1	0.93
0.025	0.051	3.5	71.5	24						
0.025	0.051	13.8	99.4	24						
0.025	0.051	20.7	108.4	24						

[Dull and hard, two systems of cleat planes often widely spaced. Volatile (dry, ash-free) 37%. Barnsley (Top Hards seam), Rossington Colliery. Orientation of weakness planes to specimen axis: Bedding: 90°; Main cleat: 0°; Cross cleat: 0°.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pi} (*MPa*)	m_i	*m*	*r*
0.025	0.051	0.1	44.0	24	[49]	0.170	116.5	22.4	3.8	0.99
0.025	0.051	3.5	68.0	24						
0.025	0.051	13.8	108.5	24						
0.025	0.051	20.7	126.1	24						

E.C.E. International Class Number 8

$D^{1/2}/H$=3.137 ($D^{1/2}/H$=0.5 in *inches*) and *meters* 0.013<D<1.626 *meters*.

[Fragile bright, heavily cleated. Volatile (dry, ash-free) 37%. Barnsley (Brights) seam, Rossington Colliery. Orientation of weakness planes to specimen axis: Bedding planes parallel, main cleat planes parallel.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.025	0.051	0	23.6	7	[46]	0.170	100.6	14.7	2.5	0.98
0.025	0.051	3.4	60.5	7						
0.025	0.051	6.9	69.4	7						
0.025	0.051	20.7	106.6	7						
0.025	0.051	34.5	134.3	7						

[Fragile bright, heavily cleated. Volatile (dry, ash-free) 37%. Barnsley (Brights) seam, Rossington Colliery. Orientation of weakness planes to specimen axis: Bedding planes perpendicular, main cleat planes parallel.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.025	0.051	0	23.9	6	[46]	0.170	96.3	15.6	2.6	0.98
0.025	0.051	0.7	43.1	6						
0.025	0.051	1.7	47.6	6						
0.025	0.051	3.4	50.6	6						
0.025	0.051	6.9	72.1	6						
0.025	0.051	20.7	108.3	6						
0.025	0.051	34.5	133.0	6						

[Fragile, heavily cleated. Volatile (dry, ash-free) 37%. Dunsil seam, Teversal Colliery. Orientation of weakness planes to specimen axis: Bedding planes parallel, main cleat planes parallel.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pi} (MPa)	m_i	m	r
0.025	0.051	0	13.5	7	[46]	0.170	80.6	23.8	4.1	0.99
0.025	0.051	3.4	54.8	7						
0.025	0.051	6.9	70.3	7						
0.025	0.051	20.7	109.6	7						
0.025	0.051	34.5	144.9	7						

[Fragile, heavily cleated. Volatile (dry, ash-free) 37%. Dunsil seam, Teversal Colliery. Orientation of weakness planes to specimen axis: Bedding planes perpendicular, main cleat planes parallel.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pi} (MPa)	m_i	m	r
0.025	0.051	0	15.7	5	[46]	0.170	76.3	19.9	3.4	0.98
0.025	0.051	3.4	46.4	5						
0.025	0.051	6.9	63.6	5						
0.025	0.051	20.7	106.3	5						
0.025	0.051	34.5	130.4	5						

[Fragile, heavily cleated. Volatile (dry, ash-free) 39%. High Main seam, Linby Colliery. Orientation of weakness planes to specimen axis: Bedding planes parallel, main cleat planes parallel.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pi} (MPa)	m_i	m	r
0.025	0.051	0	10.4	8	[46]	0.170	65.5	16.0	2.7	0.97
0.025	0.051	3.4	38.6	8						
0.025	0.051	6.9	56.9	8						
0.025	0.051	20.7	92.1	8						
0.025	0.051	34.5	114.3	8						

[Fragile, heavily cleated. Volatile (dry, ash-free) 39%. High Main seam, Linby Colliery. Orientation of weakness planes to specimen axis: Bedding planes perpendicular, main cleat planes parallel.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{ci} (*MPa*)	m_i	m	r
0.025	0.051	0	21.7	7						
0.025	0.051	0.7	37.7	8						
0.025	0.051	1.7	57.7	8						
0.025	0.051	3.4	60.5	8	[46]	0.170	105.3	18.4	3.1	0.98
0.025	0.051	6.9	80.0	8						
0.025	0.051	20.7	118.2	8						
0.025	0.051	34.5	146.4	8						

$D^{1/2}/H$=3.354 ($D^{1/2}/H$=0.535 in *inches*) and *meters* 0.013<D<1.626 *meters*.

[Dietz No. 1, Decker Coal Co., Deker, Montana. Moisture 20.6%, BTU Value (moist) 12400, Volatile 39.7%, Fixed Carbon 54.8%, Ash 5.5%, Sulphur 0.6%. Specimens were cored perpendicular to the bedding planes.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{ci} (*MPa*)	m_i	m	r
0.022	0.044	0	19.3	1						
0.022	0.044	0	20.0	1						
0.022	0.044	1.0	23.1	1						
0.022	0.	0	25.5	1						
0.022	0.044	1.4	24.8	1						
0.022	0.044	0.3	28.6	1	[55]	0.191	53.1	8.9	1.7	0.72
0.022	0.044	2.4	28.6	1						
0.022	0.044	2.8	31.7	1						
0.022	0.044	3.5	37.2	1						
0.022	0.044	6.6	35.5	1						
0.022	0.044	5.2	40.3	1						

E.C.E. International Class Number 10

$D^{1/2}/H$=2.136 ($D^{1/2}/H$=0.340 in *inches*) and *meters* 0.013<D<1.626 *meters.*

[Geotechnical drilling, RME/Dravo, Hanna, Wyoming. Moisture 10.7%, BTU Value (moist) 11966, Volatile 43.6%, Fixed Carbon 45.5%, Ash 6.7%, Sulphur 0.7%. Specimens were cored perpendicular to the bedding planes.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pi} (*MPa*)	m_i	*m*	*r*
0.055	0.110	0	4.1	1						
0.055	0.101	0	6.9	1						
0.055	0.101	0.3	7.9	1						
0.055	0.110	0	12.4	1						
0.055	0.110	3.5	11.7	1						
0.055	0.110	0	19.3	1						
0.055	0.110	2.4	17.6	1						
0.055	0.110	3.5	17.2	1						
0.055	0.110	1.9	19.2	1						
0.055	0.110	0.3	24.5	1						
0.055	0.110	2.8	29.0	1						
0.055	0.110	1.7	30.7	1	[55]	0.095				
0.055	0.110	4.1	40.0	1						
0.055	0.110	3.5	46.2	1						
0.055	0.110	3.5	47.6	1						
0.055	0.110	6.2	47.6	1						
0.055	0.110	3.5	53.1	1						
0.055	0.110	7.2	57.6	1						
0.055	0.110	5.5	60.7	1						
0.055	0.110	5.5	71.7	1						
0.055	0.110	9.0	71.0	1						
0.055	0.110	9.7	73.1	1						
0.055	0.110	9.7	78.6	1						

E.C.E. International Class Number 12

$D^{1/2}/H$=1.318 ($D^{1/2}/H$=0.210 in *inches*) and *meters* 0.013<*D*<1.626 *meters*.

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Highvale Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular, main cleat planes 30°.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pi} (*MPa*)	m_i	*m*	*r*
0.070	0.200	0.8	8.4	1	[50]	0.050				
0.070	0.200	0.4	3.6	1						
0.070	0.200	1.0	6.3	1						
0.070	0.200	0.4	4.9	1						
0.070	0.200	0.6	5.5	1						
0.070	0.200	0.6	6.8	1						
0.070	0.200	0.9	9.6	1						
0.070	0.200	0.8	9.1	1						

$D^{1/2}/H$=1.972 ($D^{1/2}/H$=0.315 in *inches*) and *meters* 0.013<*D*<1.626 *meters*.

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pi} (*MPa*)	m_i	*m*	*r*
0.056	0.120	13.8	31.3	1	[Test] Coal (9-1)	0.085	?	?		
0.056	0.120	34.5	53.6	1						
0.056	0.120	(55.2)	(75.4)	1						
0.056	0.120	(65.5)	(85.7)	1						

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pi} (MPa)	m_i	m	r
0.056	0.120	17.2	45.1	1	[Test] Coal (7-2)	0.085	92.9	0.3	0.03	0.95
0.056	0.120	27.6	55.7	1						
0.056	0.120	37.9	66.7	1						
0.056	0.120	(58.6)	(89.6)	1						

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pi} (MPa)	m_i	m	r
0.056	0.120	2.8	18.6	1	[Test] Coal (6-3)	0.085	53.5	0.6	0.05	1.00
0.056	0.120	24.1	41.8	1						
0.056	0.120	44.8	64.0	1						
0.056	0.120	(65.5)	(87.4)	1						

$D^{1/2}/H$=2.169 ($D^{1/2}/H$=0.346 in *inches*) and *meters* 0.013<*D*<1.626 *meters*.

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Highvale Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pi} (MPa)	m_i	m	r
0.076	0.127	0	13.8	1	[52]	0.097	47.1	18.0	1.7	0.79
0.076	0.127	0	11.8	1						
0.076	0.127	0	10.7	1						
0.076	0.127	6.9	15.7	1						
0.076	0.127	13.8	60.7	1						
0.076	0.127	13.8	39.6	1						
0.076	0.127	13.8	65.3	1						
0.076	0.127	27.6	78.3	1						
0.076	0.127	27.6	70.6	1						

$D^{1/2}/H$=2.173 ($D^{1/2}/H$=0.347 in *inches*) and *meters* 0.013<*D*<1.626 *meters*.

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pt} (*MPa*)	m_i	*m*	*r*
0.068	0.120	3.4	28.1	1	[Test] Coal (1-D)	0.097	80.4	0.1	0.01	0.45
0.068	0.120	24.1	50.4	1						
0.068	0.120	48.3	73.8	1						
0.068	0.120	(65.5)	(95.6)	1						

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pt} (*MPa*)	m_i	*m*	*r*
0.068	0.120	17.2	55.2	1	[Test] Coal (5-B)	0.097	119.5	0.2	0.02	0.46
0.068	0.120	27.6	65.0	1						
0.068	0.120	37.9	76.4	1						
0.068	0.120	(58.6)	(99.8)	1						

$D^{1/2}/H$=2.231 ($D^{1/2}/H$=0.356 in *inches*) and *meters* 0.013<*D*<1.626 *meters*.

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Highvale Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{ci} (*MPa*)	m_i	*m*	*r*
0.054	0.104	0	13.3	1						
0.054	0.104	0	14.7	1						
0.054	0.104	0	14.8	1						
0.054	0.104	0	16.5	1						
0.054	0.104	0	12.1	1						
0.054	0.104	6.9	35.6	1						
0.054	0.104	6.9	35.8	1						
0.054	0.104	13.8	56.4	1						
0.054	0.104	13.8	37.4	1						
0.054	0.104	13.8	44.8	1						
0.054	0.104	20.7	53.9	1	[52]	0.101	60.4	5.3	0.5	0.82
0.054	0.104	20.7	58.3	1						
0.054	0.104	27.6	56.4	1						
0.054	0.104	27.6	61.2	1						
0.054	0.104	27.6	53.4	1						
0.054	0.104	41.4	89.5	1						
0.054	0.104	41.4	80.1	1						
0.054	0.104	51.7	97.1	1						
0.054	0.104	51.7	93.5	1						
0.054	0.104	69.0	130.7	1						
0.054	0.104	69.0	107.7	1						

$D^{1/2}/H$=2.629 ($D^{1/2}/H$=0.420 in *inches*) and *meters* 0.013<D<1.626 *meters*.

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{m} (*MPa*)	m_i	m	r
0.056	0.090	6.9	29.0	1	[Test] Coal (5-3)	0.129	?	?	?	?
0.056	0.090	27.6	50.4	1						
0.056	0.090	(48.3)	(74.1)	1						
0.056	0.090	(69.0)	(96.0)	1						

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{m} (*MPa*)	m_i	m	r
0.056	0.090	10.3	30.1	1	[Test] Coal (6-2)	0.129	?	?	?	?
0.056	0.090	31.0	53.0	1						
0.056	0.090	(51.7)	(77.4)	1						
0.056	0.090	(69.0)	(96.0)	1						

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{m} (*MPa*)	m_i	m	r
0.056	0.090	6.9	37.0	1	[Test] Coal (6-4)	0.129	86.7	0.6	0.08	0.58
0.056	0.090	17.2	52.5	1						
0.056	0.090	37.9	72.0	1						
0.056	0.090	(58.6)	(96.0)	1						

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{m} (*MPa*)	m_i	m	r
0.056	0.090	3.4	17.1	1	[Test] Coal (7-4)	0.129	36.2	0.7	0.09	0.98
0.056	0.090	13.8	28.1	1						
0.056	0.090	34.5	51.5	1						
0.056	0.090	(58.6)	(77.4)	1						

$D^{1/2}/H$=2.654 ($D^{1/2}/H$=0.423 in *inches*) and *meters* 0.013<D<1.626 *meters*.

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular, main cleat planes 20°- 60°.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.036	0.071	9.3	37.4	1	[50]	0.132	14.4	38.6	5.1	0.96
0.036	0.071	9.3	34.9	1						
0.036	0.071	3.8	20.4	1						
0.036	0.071	7.2	31.6	1						
0.036	0.071	10.7	39.2	1						
0.036	0.071	7.2	29.6	1						
0.036	0.071	3.8	20.0	1						
0.036	0.071	3.8	23.6	1						

$D^{1/2}/H$=2.958 ($D^{1/2}/H$=0.420 in *inches*) and *meters* 0.013<D<1.626 *meters*.

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.056	0.080	6.9	41.5	1	[Test] Coal (1-4)	0.155	?	?	?	?
0.056	0.080	27.6	74.2	1						
0.056	0.080	(48.3)	(96.9)	1						
0.056	0.080	(69.0)	(118.3)	1						

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.056	0.080	0.0	8.0	1	[Test] Coal (3-1)	0.155	44.3	4.0	0.6	0.81
0.056	0.080	10.3	41.1	1						
0.056	0.080	31.0	63.3	1						
0.056	0.080	(51.7)	(85.2)	1						

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pi} (MPa)	m_i	m	r
0.056	0.080	3.4	26.0	1	[Test] Coal (5-2)	0.155	63.4	3.0	0.5	0.86
0.056	0.080	13.8	50.4	1						
0.056	0.080	34.5	73.5	1						
0.056	0.080	(55.2)	(95.2)	1						

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pi} (MPa)	m_i	m	r
0.056	0.080	0.0	3.9	1	[Test] Coal (9-2)	0.155	?	?	?	?
0.056	0.080	20.7	35.4	1						
0.056	0.080	(41.4)	(56.6)	1						
0.056	0.080	(51.7)	(68.1)	1						

$D^{1/2}/H$=3.260 ($D^{1/2}/H$=0.520 in *inches*) and *meters* 0.013<D<1.626 *meters*.

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pi} (MPa)	m_i	m	r
0.068	0.080	0.0	11.1	1	[Test] Coal (1-C)	0.181	31.8	2.2	0.4	0.93
0.068	0.080	20.7	44.5	1						
0.068	0.080	41.4	67.0	1						
0.068	0.080	(62.1)	(91.3)	1						

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pi} (*MPa*)	m_i	*m*	*r*
0.068	0.080	6.9	34.2	1	[Test] Coal (2-A)	0.181	59.9	1.3	0.2	1.00
0.068	0.080	27.6	59.9	1						
0.068	0.080	48.3	84.8	1						
0.068	0.080	(69.0)	(110.1)	1						

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pi} (*MPa*)	m_i	*m*	*r*
0.068	0.080	10.3	45.8	1	[Test] Coal (2-B)	0.181	?	?	?	?
0.068	0.080	31.0	69.9	1						
0.068	0.080	(51.7)	(95.6)	1						
0.068	0.080	(69.0)	(112.6)	1						

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pi} (*MPa*)	m_i	*m*	*r*
0.068	0.080	13.8	50.1	1	[Test] Coal (4-B)	0.181	79.5	0.9	0.2	0.96
0.068	0.080	44.8	87.5	1						
0.068	0.080	48.3	89.7	1						
0.068	0.080	(65.5)	(110.9)	1						

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pi} (*MPa*)	m_i	*m*	*r*
0.068	0.080	6.9	32.3	1	[Test] Coal (4-E)	0.181	56.3	1.3	0.2	0.99
0.068	0.080	27.6	58.6	1						
0.068	0.080	48.3	82.6	1						
0.068	0.080	(69.0)	(105.0)	1						

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Whitewood Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pt} (MPa)	m_i	m	r
0.068	0.080	13.8	32.5	1	[Test] Coal (5-A)	0.181	?	?	?	?
0.068	0.080	34.5	53.2	1						
0.068	0.080	(55.2)	(77.1)	1						
0.068	0.080	(65.5)	(89.7)	1						

$D^{1/2}/H$=4.200 ($D^{1/2}/H$=0.669 in *inches*) and *meters* 0.013<D<1.626 *meters*.

[Sub-bituminous C coal. Volatile Content 27%. Late Cretaceous and early Tertiary ages seam. Highvale Mine. Orientation of weakness planes to specimen axis: Bedding planes perpendicular.]

D (meter)	H (meter)	σ_3 (MPa)	σ_1 (MPa)	# of tests	Ref.	s	σ_{pt} (MPa)	m_i	m	r
0.054	0.055	0	16.1	1	[52]	0.276	46.4	1.6	0.4	0.71
0.054	0.055	0	9.8	1						
0.054	0.055	0	17.5	1						
0.054	0.055	6.9	38.2	1						
0.054	0.055	6.9	35.9	1						
0.054	0.055	6.9	35.5	1						
0.054	0.055	13.8	45.3	1						
0.054	0.055	13.8	48.1	1						
0.054	0.055	13.8	53.0	1						
0.054	0.055	27.6	45.9	1						
0.054	0.055	27.6	64.8	1						
0.054	0.055	27.5	68.6	1						
0.054	0.055	41.4	76.9	1						
0.054	0.055	41.4	75.6	1						
0.054	0.055	41.4	80.6	1						
0.054	0.055	55.2	94.8	1						
0.054	0.055	55.2	96.9	1						

E.C.E. International Class Number 14

$D^{1/2}/H$=2.352 ($D^{1/2}/H$=0.377 in *inches*) and *meters* 0.013<D<1.626 *meters*.

[Texas Lignite. Orientation of weakness planes to specimen axis: Bedding planes both perpendicular and parallel. At all of the confining pressures used there is no significant difference in the properties with orientation.]

D (*meter*)	H (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	s	σ_{pi} (*MPa*)	m_i	m	r
0.051	0.096	0	8.3	20	[51]	0.109	28.8	0.8	0.09	0.61
0.051	0.096	3.4	14.5	23						
0.051	0.096	6.9	17.9	19						
0.051	0.096	10.3	20.7	14						
0.051	0.096	13.8	24.8	14						

E.C.E. International Class Number: Lacking

$D^{1/2}/H$=3.390 ($D^{1/2}/H$=0.541 in *inches*) and *meters* 0.013<*D*<1.626 *meters*.

D (*meter*)	*H* (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pl} (*MPa*)	m_i	*m*	*r*
0.040	0.059	0	11.8	1						
0.040	0.059	0	13.2	1						
0.040	0.059	0	14.7	1						
0.040	0.059	0	15.7	1						
0.040	0.059	0	16.7	1						
0.040	0.059	0	17.6	1						
0.040	0.059	0	19.6	1						
0.040	0.059	0	21.3	1						
0.040	0.059	1.8	27.4	1						
0.040	0.059	2.0	23.0	1						
0.040	0.059	2.0	27.9	1						
0.040	0.059	2.0	31.9	1						
0.040	0.059	2.0	33.3	1						
0.040	0.059	3.9	28.4	1						
0.040	0.059	3.9	30.9	1	[53]	0.192	39.5	25.5	4.9	0.83
0.040	0.059	4.0	39.7	1						
0.040	0.059	4.3	31.4	1						
0.040	0.059	4.7	44.1	1						
0.040	0.059	4.7	48.0	1						
0.040	0.059	4.9	30.4	1						
0.040	0.059	5.4	33.3	1						
0.040	0.059	5.4	52.9	1						
0.040	0.059	6.2	47.0	1						
0.040	0.059	6.4	49.5	1						
0.040	0.059	6.6	47.0	1						
0.040	0.059	6.9	37.2	1						
0.040	0.059	6.9	42.1	1						
0.040	0.059	7.4	41.2	1						
0.040	0.059	7.4	58.8	1						

0.040	0.059	7.4	56.8	1						
0.040	0.059	8.2	49.5	1						
0.040	0.059	8.3	51.5	1						
0.040	0.059	8.4	49.5	1						
0.040	0.059	8.8	60.8	1						
0.040	0.059	9.8	50.0	1						
0.040	0.059	9.8	62.7	1						
0.040	0.059	11.8	52.9	1						
0.040	0.059	11.8	57.8	1						
0.040	0.059	11.8	70.6	1						

Category 2

In Category 2, the specimen dimensions satisfy both of the two conditions (c) and (d):

$$\frac{\sqrt{D}}{H} \geq 8.784 \qquad (c)$$

$$D \leq 0.013 \ (meter) \qquad (d)$$

E.C.E. International Class Number 1A

$D^{1/2}/H$=12.5 ($D^{1/2}/H$=1.992 in *inches*) and $D \leq 0.013$ *meters*.

[Anthracite. The direction of crushing is perpendicular to the bedding planes.]

D (*meter*)	*H* (*meter*)	L (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pl} (*MPa*)	*M*	*m*	*r*
0.0064	0.0064	0.0064	0	42.3	10	[54]	1.0				

E.C.E. International Class Number 2

$D^{1/2}/H$=13.1 ($D^{1/2}/H$=2.1 in *inches*) and $D \leq 0.013$ *meters*.

[Deep Duffryn. A friable steam coal permeated with cracks and weaknesses that are plainly visible on the surface. Volatile content 12%.]

D (*meter*)	*H* (*meter*)	L (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pl} (*MPa*)	*M*	*m*	*r*
0.0058	0.0058	0.0058	0	25.5	262	[25]	1.0				

E.C.E. International Class Number 6

$D^{1/2}/H$=8.874 ($D^{1/2}/H$=1.414 in *inches*) and $D \leq 0.013$ *meters*.

[Barnsley Hards. A hard, smooth-looking and apparently "uniform" bituminous coal. Volatile content 36%.]

D (*meter*)	*H* (*meter*)	L (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pl} (*MPa*)	*M*	*m*	*r*
0.013	0.013	0.013	0	66.1	29	[25]	1.0				

$D^{1/2}/H$=12.5 ($D^{1/2}/H$=2.0 in *inches*) and $D \leq 0.013$ *meters*.

[Barnsley Hards. A hard, smooth-looking and apparently "uniform" bituminous coal. Volatile content 36%.]

D (*meter*)	*H* (*meter*)	L (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pl} (*MPa*)	*M*	*m*	*r*
0.0064	0.0064	0.0064	0	74.6	445	[25]	1.0				

$D^{1/2}/H$=17.7 ($D^{1/2}/H$=2.8 in *inches*) and $D \leq 0.013$ *meters*.

[Barnsley Hards. A hard, smooth-looking and apparently "uniform" bituminous coal. Volatile content 36%.]

D (*meter*)	*H* (*meter*)	L (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pl} (*MPa*)	*M*	*m*	*r*
0.0032	0.0032	0.0032	0	80.9	159	[25]	1.0				

Category 3

In this Category, the specimen dimensions satisfy either of the following two conditions (e) and (f):

$$\frac{\sqrt{D}}{H} \leq 0.784 \qquad (e)$$

$D \geq 1.626$ (*meter*) (*f*)

E.C.E. International Class Number ?

$D^{1/2}/H$=0.697 ($D^{1/2}/H$=0.111 in *inches*) and $D^{1/2}/H \leq 0.784$.

[Durban Navigation Colliery, Natal, South Africa]

D (*meter*)	*H* (*meter*)	L (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pl} (*MPa*)	*M*	*m*	*r*
0.229	0.686	0.229	0	4.8	1	[22]	1.0				

$D \geq 1.626$ *meters*.

[Pittsburgh coal bed, U.S.A.]*

D (*meter*)	*H* (*meter*)	L (*meter*)	σ_3 (*MPa*)	σ_1 (*MPa*)	# of tests	Ref.	*s*	σ_{pl} (*MPa*)	*M*	*m*	*r*
1.626	1.626	1.626	0	4.8	7	[24]	1.0				

*Note: as can be seen, this specimen size above does have very close mechanical properties as the other specimen in Category 3. As a matter of fact, they have the same strength values. This can be regarded as a support to the theory developed in Section 4.2.6 of Chapter 4 in this thesis.