Selective Maintenance for Multi-state Systems Considering Dependence

by

Cuong Duc Dao

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Abstract

Industrial organizations rely on the safe and efficient operation of their assets to manufacture products and provide services with high quality. As a result of widespread mechanization, maintenance costs have increased significantly over the years. Maintenance is essential to keeping engineering systems (mechanical equipment, vehicles, electrical devices, etc.) reliable at minimum costs. Due to limited resources, e.g. budget, time, it may not be possible to do all desirable maintenance actions. In this context, the maintenance manager has to decide which subsystems or components should be maintained in order to meet the requirements on system's performance. This problem is called selective maintenance.

In traditional reliability theory, the system is considered to be in two possible states of perfect functioning or failed. However, in practice, many systems can degrade and operate in an intermediate working state. As the system deteriorates, its performance may be in several states varying from perfect functioning to complete failure. Such a system is called multi-state system. When the system and its components are multi-state,

the decision making becomes more complicated since imperfect maintenance actions to its intermediate levels are also possible. The maintenance manager has to decide which components to be maintained and the state levels that they should be maintained to.

Existing selective maintenance models usually assume the components to work in a stable condition without considering the inter-relationships between them when maintenance activities are performed as well as when the system is running. However, components interact with each other due to several reasons. For example, the failure of a component may create fire and cause immediate failures of other components due to an induced failure mode, or the performance of a component may dictate the degradation of other components in a specific system design.

This PhD research aims to study selective maintenance modeling for complex systems, specifically selective maintenance for systems with multiple working levels, while considering dependent relationships in the system. This research tackles the maintenance problem when resources are limited by providing maintenance strategies using both systems performance requirements and maintenance resources as input information. Several types of relationships of components in the systems are modelled as follows.

i. When maintaining several components simultaneously, there are savings due to the sharing of resources, e.g. materials, tool, manpower, etc.
ii. In a complex system structure, repairing a component requires actions on other components.

iii. The current health state of a component may affect the performance and degradation of other components.

iv. The degradation rate of a component may change depending on the operating conditions and its current health state.

This research proposes maintenance models for multi-state systems with dependence, which is one of the most challenging topics in the field of reliability and maintainability. It has significant contributions in terms of dependence modelling, reliability analysis, and selective maintenance optimization of multi-state systems. The results can be applied in a wide range of industries where physical mechanical systems exist and maintenance for them is compulsory. The maintenance models not only provide decision makers with a set of maintenance actions to ensure reliable and cost-effective operation of the systems, but also help them wisely utilize their available resources.

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List of Abbreviations

C-K: Chapman-Kolmogorov

DN: Do-nothing

DP: Disassembly Path

GA: Genetic Algorithm

GDD: Gradual Degradation Dependence

IF: Immediate Failure

IFD: Immediate Failure Dependence

IM: Imperfect Maintenance

MCS: Monte-Carlo Simulation

MSS: Multi-state System

PG: Production Gain

PHM: Proportional Hazard Model

PL: Production Loss

SM: Selective Maintenance

Chapter 1

Introduction

Today, engineering systems are sophisticated. The development of science and technology has allowed us to manufacture complex systems with thousands of sub-units and components. In addition, the systems usually involve complicated interactions of elements, hardware, software, humans, and operating conditions. These interactions play critical roles when the systems are analyzed. These complex systems bring challenges to engineers and managers since efficient tools need to be developed for modelling, operating, and maintaining them.

Failures of such systems have substantial impact on the society. The following example demonstrates a complex engineering system's failure with severe consequences, where the relationship between components plays a critical role. The incident happened at midnight in the town of Lac-mégantic, Quebec, Canada on July 06, 2013 [1]. The Montreal, Maine and Atlantic Railway (MMA) train with 74 cars carrying crude oil was parked in Nantes, Quebec with a lead locomotive engine running. The lead locomotive engine drove the air compressor which supplied air to a system of air brakes. A number of hand brakes were also applied to the train. Then, the fuel supply and the electricity breakers were shut down since the locomotive engine was producing excessive smoke and fire was reported due to a failure of "a non-standard repair" [2]. In this case, an automatic penalty brake would have been applied to the entire train, but the electricity breakers were in the off position, and the brake had no power to operate. With the shutdown of the locomotive engine, the air compressor was also off. The air in the brake system started leaking and the air brakes were gradually released. In a steep track from Nantes (515 metres above the sea level) to Lac-mégantic (407 metres above the sea level), the number of hand brakes applied were not sufficient to hold the train. The train started moving downhill, reached 101 km/h, derailed, and exploded in the town of Lac-mégantic in hot weather. The train derailment and explosion in Lac-mégantic is one of the worst accidents in Canada's transportation history. Sixty-three tank cars were damaged, and about six million litres of crude oil was released. The fire and several explosions began immediately, causing 47 deaths, leveling buildings; about 2000 people were forced from their homes, and much of the downtown core was destroyed [1].

The Lac-mégantic disaster was due to several contributing factors. The failure of the locomotive engine, insufficient awareness of the relationships between different operating units, e.g. engine, air compressor, air brakes, and hand brakes, and the changes in operating conditions, e.g. steep track, hot weather, and the special characteristics of the carried good - crude oil, all contributed to the accident.

Figure 1.1 The Lac-mégantic Train derailment and explosion [3]

One of the best ways to keep systems in service and to prevent failures is to perform proper maintenance. Undoubtedly, the relationships within complex engineering systems need to be considered when maintaining the systems. On the other hand, industrial organizations often have limited resources for performing maintenance, which include budget, time, and maintenance personnel, etc. Maintenance optimization plays a critical role in the business objective of assuring safe operations and contributing to the total profit. This thesis presents a study of maintenance optimization for complex systems while considering the reliability, dependency, and economical factors in maintenance of the systems.

1.1 Systems and components

In this dissertation, the system is considered to be a collection of smaller and less complex sub-units (e.g. sub-systems, components) to perform a specific function. Component is defined as a self-contained unit that is not further sub-divided. This does not mean that a component cannot be made of smaller parts; instead, it is the smallest unit considered in maintenance analysis of the system.

Since the system can be decomposed into a set of sub-systems and components, the system configuration describes how the components are connected. In a complex system, components can be connected in different types of configuration or arrangement such as series, parallel, series-parallel, k-out-of-n, bridge, etc. [4]. When the system is in operation, the system state can be determined by the states of its components. When the system is to be maintained, all the components are considered simultaneously.

In traditional reliability theory [4], [5], the system and its components are considered to be in two possible states of *functioning* – state 1 or *failed* – state 0. However, in practice, many systems and components can degrade and operate in an intermediate working state. The system may be in several states varying from perfect functioning to complete failure (Figure 1.2). Such a system is called multi-state system (MSS) [6], [7]. Several examples of multi-state systems in practice such as power generation systems [6], [7], pipe-line systems [7], and coal transportation systems [8], [9] have been introduced.

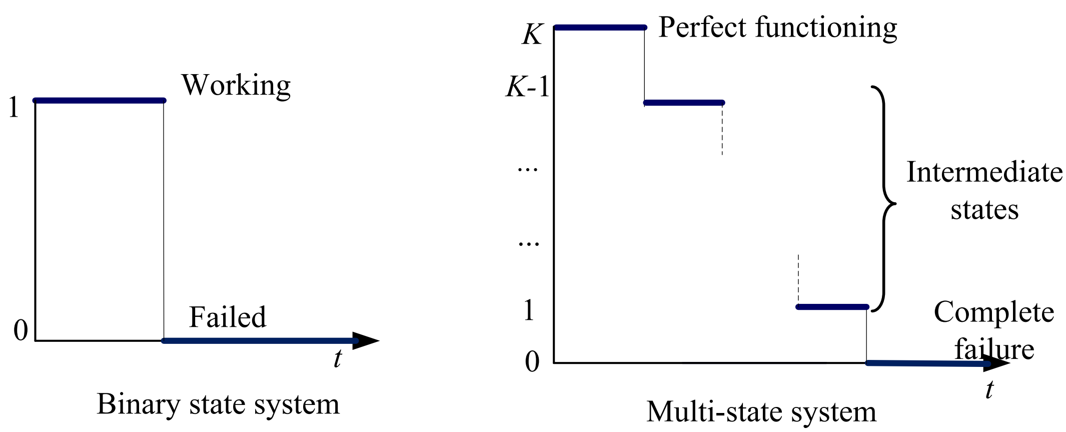


Figure 1.2 Binary and multi-state systems

When the system and its component are multi-state, the decision making process becomes more complicated since imperfect maintenance for a component to its intermediate level is also possible. The maintenance manager has to decide which components to maintain and the state level that it should be maintained to.

1.2 Reliability and maintenance

1.2.1 Reliability

Reliability is an important measure in engineering system design, analysis, and maintenance. Reliability of an item is defined as "the probability that it will perform its intended functions for a given period of time when used under specified operating conditions" [4]. The *item* in this definition can be an individual component or a collection of components, i.e. the system. When considering a system of n components, a structural function, which defines the system state from its component states, can be used to describe the relationship of components and determine the system reliability.

In the binary state context, the reliability of an item can be defined as the probability that it is in the functioning state - state 1 for a specified time duration and under specified operating conditions, i.e. $r(t) = \Pr(x(t) = 1)$, where $x(t)$ is the state of the item at time t. For a multistate item with $K+1$ levels of performance in an increasing order of $0,1,\ldots,K$, the reliability of the item is the probability that the item is in a state k, $k \in \{0,1,\ldots,K\}$ or above, $r(t) = \Pr(x(t) \geq k)$. This definition implies that all the states which are greater than or equal to k are satisfactory states.

1.2.2 Maintenance

In the classical view, maintenance is simply reactive tasks to repair or replace broken items. This old approach is known as corrective maintenance or breakdown maintenance. With the introduction of new maintenance technologies, maintenance is now defined as "all actions necessary for retaining an item in or restoring it to a physical state considered satisfactory for its fulfillment of its production functions" [10]. This new concept widens the range of maintenance tasks including, but not limited to, cleaning, lubrication, adjustment, inspections, performance tracking, minor repair, major repair, overhauls, and replacements.

According to Kececioglu [10], four to forty times of the purchase cost is needed for maintenance to keep a system operating satisfactorily through its lifetime. For industrial organizations, maintenance is not only to prevent failure but also to extend the life of their assets. As stated in Jardine and Tsang [11], “maintenance excellence is concerned with balancing performance, risks, and the input resources to achieve optimal solutions”. Therefore, maintenance optimization deals with crucial decisions to be made on the system such as what maintenance actions to perform, when to perform them, how much resources to spent, and what the restored system performance level to be. Ultimately, optimizing maintenance can contribute to profits by keeping an efficient and safe operation and utilizing their resources in terms of manpower and materials effectively.

1.3 Dependence and maintenance

A definition of multi-component system maintenance models was given by Cho and Parlar [12] as “multi-component maintenance models are concerned with optimal maintenance policies for a system consisting of several units of machines or many pieces of equipment, which may or may not depend on each other”. Since 1980s, the topic of maintenance for multi-component systems has been attracting the attention of researchers and it has been recognized that the interaction between components should be taken into consideration in maintenance decision making [13]. In this dissertation, dependence in maintenance of multi-component systems is classified into four types, i.e. economic, structural, stochastic, and operational dependence, as illustrated in Figure 1.3. This classification is based on the relationships between components’ maintenance actions, between components’ lifetimes, and between components’ lifetimes and operating conditions.

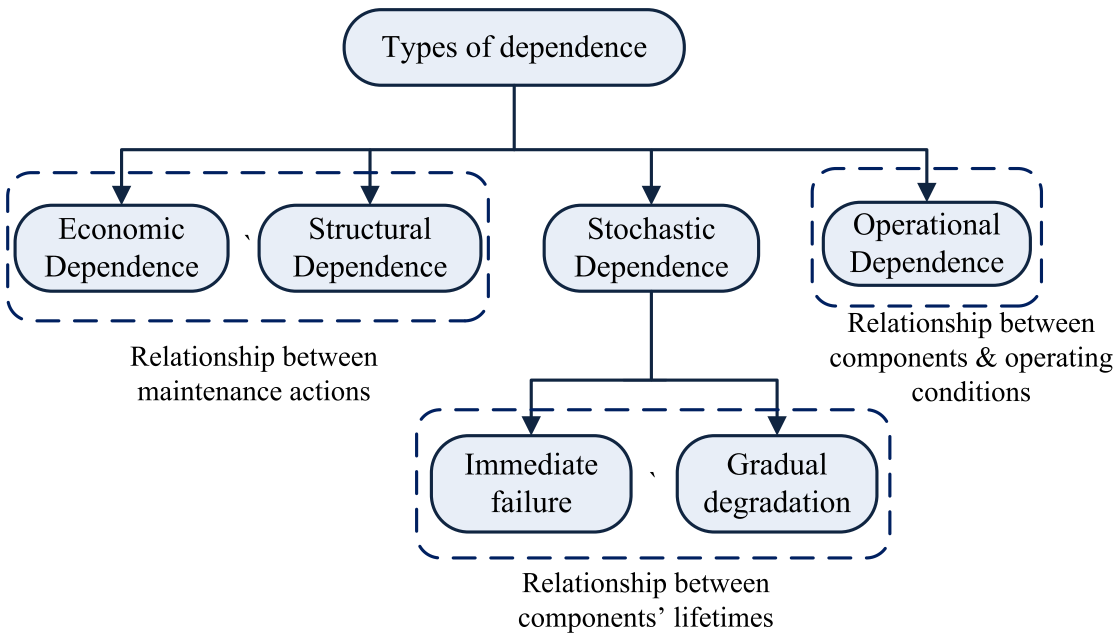


Figure 1.3 Types of dependence in maintenance of multi-component systems

Economic dependence implies that the cost of maintaining multiple components can be lower in comparison with individually maintaining each component. Structural dependence applies when the components form a group and the maintenance of a component in the group requires an extra intervention of other components or at least disassembling them. Stochastic dependence means that the state of a component may affect the state of other components in the multi-component system. Operational dependence depicts the effect of changes in operating conditions on the degradation of components.

Economic dependence and structural dependence are related to implementation of maintenance activities, and the component's deterioration is not affected by these types of dependence. The main difference between economic and structural dependence is that economic dependence is related to the economies of scale where multiple maintenance actions on multiple components together are cheaper than doing maintenances on these components individually, while structural dependence is related to the inter-relationship of connected components that requires extra actions to other components in order to maintain a specific component.

Stochastic and operational dependence, on the other hand, relate to components' deterioration. Stochastic dependence defines the relationship between components when a component fails or changes its current state, e.g. a component's failure induces failures or causes shocks to other components. The state and the degradation of the affected component can both be influenced by an influencing component and therefore the system reliability is subjected to change when stochastic dependence exists. Operational dependence reflects the effect of variable operating conditions on the degradation of the components. Generally, a component degrades faster under more severe conditions such as heavier workload or higher temperature. Since the system reliability depends on the degradation of its components, the maintenance policy for the system needs to be investigated depending on the component-component or component-operating condition relationships.

1.4 Selective maintenance

In the maintenance literature, various models have been proposed [11]–[17]. In these reported studies, preventive replacement, imperfect maintenance, and inspection policies for repairable systems were investigated. However, the majority of these maintenance models ignore the limitation of resources to perform maintenance actions. Many systems have to perform several missions with limited resources, such as time and budget, for maintenance between successive missions. It is often impossible to do all desirable maintenance actions within the available resources. Thus, the maintenance manager has to decide which components to maintain and how to perform maintenance in order to meet the requirements on system performance. This problem is called selective maintenance [18].

Selective maintenance has been studied for several industrial applications [19]–[21]. A production system may be in operation 24 hours a day during the weekdays and be maintained at weekend; a computer system is utilized heavily during daytime and maintained at night; airplanes and ships are maintained within a few hours to a few days before the next journey. In another example, a power generation plant often has a scheduled outage, i.e. maintenance break, every year for maintenance. During the scheduled outage, ranging from one to three weeks, there are several subsystems and components that need to be maintained. The plant manager wants to ensure reliable operation of the plant and satisfy its power generating demand while time, budget, and personnel for maintenance are limited. Selective maintenance can help determine which components and what maintenance actions to perform to accommodate this need. The decision is made based on the available resources for maintenance and the requirement of the system performance in the next mission.

Industrial organizations today are required to manage their assets more effectively with limited expenses to accommodate the advancements of technologies and global market competitiveness. Selective maintenance is a powerful solution, which can help system managers by providing optimal maintenance strategies within limited resources.

Selective maintenance was first introduced by Rice et al. [18] in 1998. Since then, it has been studied many researchers. Reported studies on selective maintenance mainly focuses on binary systems [18]–[36]. Some investigations of selective maintenance of multi-state systems are also reported in [9], [37], [38]. A detailed literature review of selective maintenance is to be presented in Chapter 2. The dependence relationships are mostly ignored in these reported studies. This dissertation will focus on modeling dependency in complex multi-state systems while looking for an optimal selective maintenance strategy of the systems. The next section will provide the scope and objectives of this research.

1.5 Thesis scope and objectives

This PhD research aims to study selective maintenance modeling for complex systems with multiple performance levels, while considering the dependence relationships in the system. This research tackles the maintenance problem when resources are limited by providing maintenance strategies using both system performance requirements and maintenance resources as input information. The two main goals of this research are as follows.

1. To model the dependence within multi-state systems, including the interactions between components' maintenance actions, component-component lifetimes, and components' lifetimes and the operating conditions.
2. To construct and solve the selective maintenance optimization models for multi-state systems with dependence. Benefits of the model, consequences of ignoring dependence, and the optimal selective maintenance strategy are discussed and compared in several illustrative examples to emphasize the importance of dependence.

The proposed research considering dependence is one of the most challenging topics in the field of reliability and maintainability. The results can be used in a wide range of applications where maintenance of physical systems is compulsory. The maintenance models can provide the system managers with a set of maintenance actions to ensure reliable and cost-effective operation of the systems, and also help them wisely utilize their available resources.

Within this dissertation, the relationships within the systems to be addressed can be described as follows.

1. When maintaining several components simultaneously, there are savings due to the share of resources, e.g. materials, tool, manpower, etc. The differences in savings between maintaining identical and non-identical components are also considered.
2. In a complex system structure, repairing a component requires intervention actions to other components, thus extra cost may be needed for maintaining a component. This is described by precedence relations in assembly and disassembly of components in the systems.
3. When a component fails or changes its current state, it may affect the performance and degradation of other components. This often occurs when the components are inter-connected or functionally dependent.
4. The components operate in dynamic loading conditions, and when the load condition changes, the degradations of components are affected.

When the dependence is related to the degradation of components, methods for the system reliability evaluation of the system are to be developed. The selective maintenance models will employ results of reliability evaluation methods to find the satisfactory system reliability for the next operating mission.

1.6 Organization of the thesis

With the proposed scope and objectives in Section 1.5, this thesis contains 7 chapters which are organized as follows. Chapter 1 presents the challenges in selective maintenance modelling of multi-state systems considering dependence, followed by the scope, objectives, and structure of this thesis. Relevant concepts of system, component, reliability, maintenance, and dependence are also included in Chapter 1. Chapter 2 introduces the fundamentals of selective maintenance including some general notations and basic concepts, a detailed literature review, and a general selective maintenance optimization model for multi-state systems with independent components.

Chapter 3 discusses the economic dependence between maintenance actions of dependent components. An extended selective maintenance model based on Chen et al. [37] with two mechanisms of time and cost savings when repairing non-identical and identical components in multi-state series-parallel systems will be developed.

Chapter 4 examines the structural dependence of components in a multi-state series system. We formulate the structural dependence relationships between components when disassembling the system. The directed graph [39] is used to represent the precedence relation in the disassembly of components. A back-ward search procedure for determining the system disassembly path is developed. Both economic dependence and structural dependence are considered in the selective maintenance model.

In Chapter 5, we will model two types of stochastic dependence for multi-state components. The first type relates to the failure of a component induced by another component. The second type reflects the degradation rate change of a component to the degradation of another component. A reliability evaluation method based on stochastic process is proposed for analysis of the system. The output performance of the system is considered and integrated in a cost-based selective maintenance model to maximize the total system profit in the next mission.

Chapter 6 is devoted to modeling the dependence of components' degradations on the operating conditions. We propose a load-dependent degradation model for multi-state components operating in dynamic loading conditions. The development of the model is based on the component's degradation in the k-out-of-n load-sharing systems [40]. The system reliability is estimated by Monte-Carlo simulation method. Analysis of optimal selective maintenance scenarios for different levels of budget limitation is provided.

The summary of contributions of this research is presented in Chapter 7. Possible future research directions are also discussed in this chapter.

Chapter 2

Fundamentals of Selective Maintenance

In this chapter, we introduce the fundamentals of the selective maintenance problem including general notation, basic concepts, and a selective maintenance problem statement. A detailed literature review of reported work on this topic in the last two decades and a general selective maintenance model will also be presented.

General notation:

n:	Number of components in the system
i, j:	Component indices
K:	Maximum state
a, b, k, l:	State indices
τ:	Operating mission duration
Y_i:	Given state of component i at the end of the previous mission

Y: State vector of all components in the system at the end of the previous mission

X_i: State of component i at the end of the maintenance period

X: State vector of all components in the system at the end of the maintenance period

ϕ_s: The system state function in the next operating mission, $\phi_s \in \{0, 1, ..., K\}$

$t_i(a,b)$: Required time of a single repair of component i from state a to state b, $b>a$

T_i: A $(K+1)\times(K+1)$ repair time matrix of component i with an element in row a, column b being $t_i(a,b)$ if $b>a$ and being 0 if $b \leq a$

T: Total time needed of all selected maintenance activities

T_0: Duration of the maintenance break

$c_i(a,b)$: Cost of a single repair of component i from state a to state b, $b>a$

C_i: A $(K+1)\times(K+1)$ repair cost matrix of component i with an element in row a, column b being $c_i(a,b)$ if $b>a$ and being 0 if $b \leq a$

C: Total cost of all selective maintenance activities

C_0: Available budget for all selective maintenance activities

$p_i(a,b)$: The probability of component i being in state b at the end of an operating mission given that it is in state a at the beginning of the mission.

P_i: A $(K+1)\times(K+1)$ probability transition matrix of component i with an element in row a, column b being $p_i(a,b)$

D: Demand of the system in the next mission

$R_s(t,D)$: The reliability of the system at time t for demand D

R_0: Minimum reliability requirement of the system

2.1 Basic concepts

2.1.1 Mission and maintenance break

In selective maintenance, a *mission* is a task that the system has to perform. A mission is associated with a time duration τ and a specified operating condition. For example: an aircraft has to perform a mission which can be a flight between two airports under certain weather conditions; a truck needs to carry a certain load between two depots. Between two missions, there is a time break for doing the maintenance called the *maintenance break*, e.g. sojourn time at an airport or depot. The maintenance break is also associated with a specified duration of T_0. The job of the maintenance decision maker is to find the optimal maintenance plan to ensure the success of the next mission.

2.1.2 Mission reliability

Mission reliability is the probability that the system will successfully accomplish the next mission. It implies system reliability in the time duration of the next mission, thus is taken into consideration prior to the start of the mission. Mission reliability is an important part of selective maintenance decision making. It represents the system requirement for the next mission. For instance, the maintenance manager has to optimize the selective maintenance decisions so that the system reliability is greater than or equal to a required threshold.

2.1.3 Maintenance resources

Maintenance resources refer to available supplies for system maintenance, e.g. time, materials, staff, energy, etc. Since maintenance activities must be performed prior to the next mission, *maintenance time* is a special limited resource in selective maintenance problem. Other types of resources will be represented in a single type, namely, the *maintenance cost*. Thus, in many cases, only two types of resources, i.e. time and cost, are considered when modelling the selective maintenance problem.

In selective maintenance, the resources are limited. All the maintenance activities must be performed within the maintenance break duration of T_0. Similarly, total system maintenance costs must be less than or equal to a *maintenance budget*, C_0, scheduled for the break.

2.1.4 States of components at the maintenance depot

Maintenance activities are performed at the maintenance depot. When the system arrives at the maintenance depot, the health state of each component can be detected. Selective maintenance often assumes that the state of each component is known right after the system enters the maintenance depot. In an n component system, the set of states of all the components at the time of entering the maintenance depot can be represented by a vector $\mathbf{Y} = [Y_1\ Y_2\ \ldots\ Y_n]$, where Y_i, $i = 1,2,\ldots,n$ is the state of component i.

When the system exits the maintenance depot, the vector of components' states is $\mathbf{X} = [X_1\ X_2\ \ldots\ X_n]$, where X_i, $i = 1,2,\ldots,n$, is the state of component i at the time of exiting the maintenance depot, and X_i, $i = 1,2,\ldots,n$, is also the state of component i at the beginning of the next mission. In selective maintenance, X_i is a variable that needs to be determined in order to obtain the best maintenance strategy.

2.1.5 Selective maintenance strategy

Selective maintenance optimization seeks an optimal maintenance strategy utilizing available resources. Selective maintenance strategy includes a set of components in the system and the corresponding maintenance actions to be performed on these components. Since the health states of components are known at the time of entering the maintenance depot, selective maintenance strategy can also be represented by a set of all component states at the time of exiting the maintenance depot.

2.2 Selective maintenance problem statement

In this research, the selective maintenance problem can be defined as follows: given the states of components at the time of entering the maintenance depot, the resources needed for different levels of maintaining each component, and the total available resources, the maintenance decision maker has to find a maintenance strategy including a set of components to be maintained and the corresponding level of maintenance for each selected component to ensure the system's reliable operation in the next mission.

2.3 Literature review on selective maintenance

2.3.1 Selective maintenance for binary systems

The first selective maintenance model was introduced in 1998 by Rice et al. [18]. A binary integer optimization model for optimizing the reliability of series-parallel systems was proposed. At the maintenance depot, there are only two available maintenance options on a failed component: replace or do nothing. Afterwards, different selective maintenance models based on Rice's model were developed for binary systems in [19]. Cassady et al. [20] further extended the work of Rice et al. by considering components whose lifetimes follow the Weibull distribution and different maintenance actions such as replacement and minimal repair of failed components, and preventive replacement of surviving components. Schneider and Cassady [24], [33] extended the model by Rice et al. by considering selective maintenance for a fleet comprised of multiple binary series-parallel systems. Pandey et al. [21], [22] studied selective maintenance with an imperfect repair model, in which the health of a component may be not "as good as new" and depends on the cost spending in the maintenance break. Pandey et al. [23] later extended the model in [21] by considering components with two types of failure modes, namely maintainable and non-maintainable failure modes. Ali et al. [27] assumed that the cost of repairing or replacing a component are random variables following the Normal distributions. Khatab and Aghezzaf [28] considered a selective maintenance model with random quality of imperfect maintenance. The age-reduction factor in the imperfect repair model was a random variable with a known probability density function. Zhu et al. [29] studied the selective maintenance model considering different cost factors in a manufacturing line, including maintenance cost and possible production loss due to system unavailability.

Maaroufi et al. [31] first investigated the inter-relationship between components in selective maintenance of binary systems. They considered a special case of stochastic dependence where the failure of a component can cause other components to fail immediately. A fixed "set-up" cost for dismantling and reassembling the system was incurred only once when more than one component was replaced. A set of rules to reduce the solution space was later proposed by the same authors in [32]. The selective maintenance study in [31], [32] has not fully addressed the inter-relationships between components when maintaining a complex system. First, other types of stochastic dependence may be presented, e.g. when the failure of a component does not cause other component to fail immediately but affects the degradation rates of other components. Second, the impacts of economic dependence on the optimal maintenance strategy have not been investigated. Investigation of the effects of economic dependence on selective maintenance decision making would help the maintenance department in utilizing their resource effectively. In addition, modeling other types of dependence such as structural and operational dependence and their effects on maintenance decision also needs further examination.

Some researchers extended the traditional single-mission selective maintenance into the multi-mission model, where the system has to perform multiple missions consecutively in a finite or infinite planning horizon. Maillart et al. [35] studied selective maintenance for a system with multiple identical missions in both finite and infinite planning horizons. They claimed that the differences between the policies of a single mission and multiple identical missions were minimal. Pandey et al. [36] extended the study of Maillart by considering non-identical missions in a finite planning horizon. The model in [36] jointly optimized the number of maintenance intervals and the selective maintenance schedule in each interval for series-parallel systems.

2.3.2 Selective maintenance for multi-state systems

Not many researchers have studied selective maintenance when the system is multi-state. Chen et al. [37] reported a preliminary work on selective maintenance for multi-state systems. A selective maintenance model for a series-parallel system with the objective of minimizing the total maintenance cost subject to reliability constraints was proposed. Liu and Huang [38] presented a selective maintenance model for systems with multi-state corresponding to cumulative performance of N binary components and considering the imperfect maintenance that may restore the condition of the system to an intermediate state. Pandey et al. [9] investigated selective maintenance for a system with multi-state independent components, where multiple intermediate repair actions are available on a component between do-nothing and replacement.

Existing selective maintenance models for multi-state systems with multi-state components rely on the independence assumption between components in the system. This motivates the author to investigate and model different types of dependence and explore their effects on the selective maintenance problem for multi-state systems.

2.3.3 Solution methodologies for selective maintenance optimization

In terms of solution methodology, Rice et al. [18] solved the selective maintenance problem based on the enumeration method. Rajagopalan and Cassady [26] improved the selective maintenance solution approach for series-parallel systems with identical components in the same sub-system by assigning bounds to the possible number of components to be repaired on the basis of available resources. Lust et al. [25] proposed a heuristic based on the Tabu search approach and an exact method based on the branch and bound procedure. They found that the time taken by the heuristic was less, but the result was inferior compared to the exact method. In recent selective maintenance studies [9], [34], [36], [38], [41], evolutionary algorithms such as genetic algorithm and differential evolution were used for solving the selective maintenance problem and they have proved to be more efficient methods.

2.4 A general selective maintenance optimization model

In this section, a baseline model for selective maintenance optimization problems for general multi-state systems of independent components is presented. Unless otherwise stated, the materials in this section are based on Rice et al. [18] and Dao et al. [42].

Selective maintenance looks for the best maintenance strategy for multi-component systems with given input data such as the states of components at the time of entering the maintenance depot and resources needed for various maintenance activities. It is generally a non-linear integer optimization problem. Assume that the state vector of all components at the time entering the maintenance depot is known as $\mathbf{Y}= [Y_1 \ Y_2 \ \dots \ Y_n]$. We need to determine the state vector of components at the time exiting the maintenance depot $\mathbf{X}= [X_1 \ X_2 \ \dots \ X_n]$. In the following selective maintenance model, we need to find the maintenance activity associated with each component to be performed to achieve the maintenance objective of increasing the system reliability under limitation of time and cost.

P: $$\textit{Maximize } f = R_s(\mathbf{X}) \tag{2.1}$$

Subject to: $$T(\mathbf{X}) \leq T_0 \tag{2.2}$$

$$C(\mathbf{X}) \leq C_0 \tag{2.3}$$

$$Y_i \leq X_i \leq K, \ i=1,2,...,n \tag{2.4}$$

$$X_i \text{ is integer}, \ i=1,2,...,n \tag{2.5}$$

In this model **P**, the objective function (2.1) is to maximize the system reliability in the next mission, i.e. the probability that the system will successfully complete the mission of duration τ and the demand level D. There are two constraints in (2.2) and (2.3) which restrict the total time and the total cost for all maintenance activities within available time, T_0, and budget, C_0. If the repairs of components in the system are independent, the total time and cost for maintaining the system can be calculated as in (2.6) and (2.7). The decision variables, X_i, are integer variables between Y_i and the maximum state K for all i = 1,2, ..., n.

$$T(X) = \sum_{i=1}^{n} t_i(Y_i, X_i) \tag{2.6}$$

$$C(X) = \sum_{i=1}^{n} c_i(Y_i, X_i) \tag{2.7}$$

In Equations (2.6) and (2.7), $t_i(Y_i, X_i)$ and $c_i(Y_i, X_i)$ are the required time and cost of repairing of component i from state Y_i to state X_i. Equations (2.6) and (2.7) indicate that the total system maintenance time (cost) can be obtained by taking the sum of all components' maintenance times (costs).

2.5 Concluding remarks

The topic of selective maintenance has been active for the last two decades, ever since it was first introduced in 1998. The majority of reported studies on selective maintenance is for binary systems. In addition, most of them rely on an assumption of independent components in the system, though engineering systems are complicated and the components within a system are inter-related and their dependence relationships cannot be ignored. The advancement in computing technology enables us to find the solution for complicated selective maintenance optimization problems. Thus, the focus of this PhD research is on modeling dependence in complex multi-state systems aiming at the optimal selective maintenance strategy of the systems. Four types of dependence presented in Section 1.3 and their effects on the selective maintenance problem will be analyzed in the remaining chapters of this thesis.

Chapter 3

Selective Maintenance for Multi-state Systems under Economic Dependence

This chapter studies economic dependence in selective maintenance for multi-state series-parallel systems. We propose two mechanisms of savings due to simultaneous repair of identical and non-identical components in the series-parallel system. Both time and cost savings can be achieved when several components are simultaneously repaired in a selective maintenance strategy. Several optimization models are derived and genetic algorithm is used to solve the models. Versions of this chapter have been published as a book chapter [43] and a journal paper in Reliability Engineering and System Safety [42].

3.1 Introduction

A preliminary work on selective maintenance for multistate systems with multi-state components was performed by Chen et al. [37]. A selective maintenance model for series-parallel system with the objective of minimizing the total maintenance cost subject to reliability constraints was proposed. However, in maintenance management and particularly in selective maintenance, the time allocated for maintenance activities is one of the most critical constraints. This chapter extends the model by Chen et al. [37] to a situation in which the maintenance crew has to consider the available time for total maintenance activities, T_0, which is limited. The total maintenance time, cost or the system reliability in the next mission can be either treated as objectives or constraints in selective maintenance modelling.

Multiple maintenance actions are usually selected in a selective maintenance scenario. In many industrial systems such as aircrafts, medical equipment, automotive machines, and power plants, etc., repairing multiple components, especially identical components, is always more economical due to the share of resources for maintenance. In this case, the components in such systems are considered to be economically dependent. For example, if a power plant has two turbine units, the joint maintenance of both turbines would be more economical since their maintenance is very similar in terms of inspection, repairing tool, labor and materials and thus, they tend to be scheduled for maintenance at the same time considering this economic dependence.

The idea that performing a group of maintenance activities may require only one set-up was employed in a series of papers on corrective and preventive maintenance [44]–[48] and reviewed in [13] and [49]. More recently, Laggoune et al. [50] investigated an

N-component series system, where the failure of a component prevents the system from operation and the system downtime can bring opportunistic replacement for non-failed components. Besnard et al. [51] and Ding and Tian [52] studied opportunistic maintenance for wind power systems, where the transportation cost to bring a maintenance crew to a wind farm can be saved if preventive maintenance is performed once failure occurs to a wind turbine. Nourelfath and Chatelet [53] investigated economic dependence between components of a parallel system in the production and preventive maintenance planning problem with the objective of minimizing the total production and maintenance cost. In selective maintenance, Maaroufi et al. [32] considered maintenance for binary systems with propagated failures and economic dependence where a fixed "set-up cost" for dismantling and reassembling the system is incurred only once when more than one component is replaced. In these papers, authors considered that the same set-up cost is incurred each time a corrective, preventive or opportunistic replacement is needed. The components are assumed to be "as good as new" after replacement, rather than the possibility of repairing components to different intermediate states as in multi-state systems. Moreover, these papers did not consider time savings when performing maintenance on multiple components in the systems. In this study, the system and its components may be in any state from a set of all possible states, $S =$ K where state K is perfect functioning, state 0 is complete failure, others are intermediate states.

In this chapter, we focus on modeling two types of economic dependence between multi-state components based on the share of setting up and the advantage of repairing multiple identical components in each subsystem of the multi-state series-parallel system. Both time and cost savings can be realized when several components are selected to be repaired in a selective maintenance strategy. The multi-state series-parallel systems are assumed to have the following characteristics.

- All the components are s-independent.
- The maintenance activities do not make the condition of components or the system worse, i.e. the states of the system and its components are not lower after going out of the maintenance depot.
- The resource requirement for a single maintenance activity is deterministic and known.
- The probability for a component at any pre-specified state b degrading to all possible states $a<b$ is known.

3.2 Problem formulation

3.2.1 The system descriptions

The series-parallel system consists of M s-independent subsystems connected in series and in subsystem i, $i = 1, 2, ..., M$, there are N_i identical components connected in parallel as shown in Figure 3.1. Each component in the system may be in any state from perfect functioning, i.e. state K, to complete failure, i.e. state 0. The state of a subsystem is the maximum state among its components and the system state is determined by the minimum state among M subsystems. In the selective maintenance problem, the system has to work consecutively identical missions with break interval of T_0 - time from the end of previous mission to the beginning of the next mission. The maintenance crews have to decide what and how to maintain each component in the system within available budget C_0 and available time T_0.

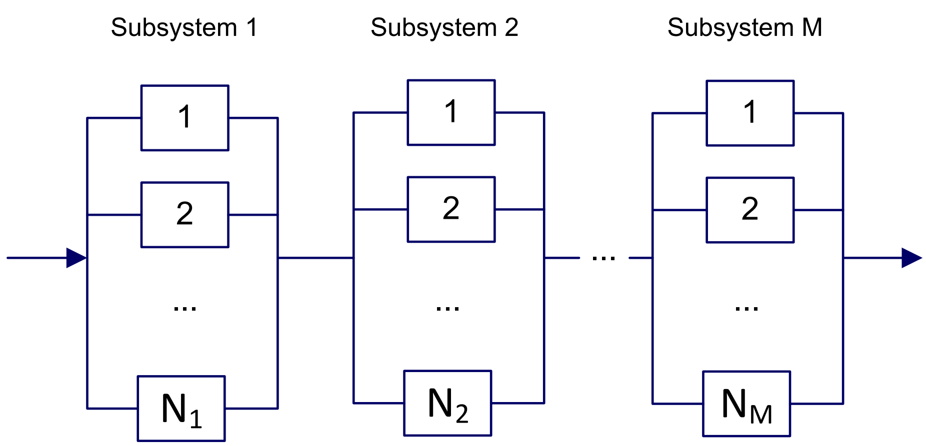


Figure 3.1 A series-parallel aystem

For each j^{th} component in subsystem i, Y_{ij} $\leq Y_{ij} \leq K$, represents its state at the time of entering the maintenance depot and X_{ij} is its state after the selective maintenance break. Because the maintenance activities do not make the condition of components and the system worse, it is clear that Y_{ij} X_{ij} K. In the selective maintenance problem, the state vector of all components in the system at the time of entering the maintenance depot, $Y = Y_{ij}$ $i = 1, 2, ..., M, j = 1, 2, ..., N_i$, is known; we have to find its state vector at the time of exiting the maintenance depot, $X = X_{ij}$, $i = 1, 2, ..., M, j = 1, 2, ..., N_i$.

3.2.2 The system reliability

A component degrades with use, and its state at the end of an operating mission is a random variable. Xue and Yang [54] and Sheu and Zhang [55] investigated the measure of evaluating components' performance degradation in reliability analysis and optimal replacement of multi-state systems. These may serve as a good reference for the state transition analysis of multi-state components and systems. Here, we will not focus on the component's degradation process in the operating mission time τ. In this chapter, we assume that the probabilities for a component in subsystem i at any pre-specified state a degrading to all possible state b $(0 \le b \le a)$ after the operating mission, $p_i(a,b)$, are already known. When $a = 0, 1, ..., K$, these probabilities form a $(K+1) \times (K+1)$ transition probability matrix of each component in subsystem i for completing a mission as given in Equation (3.1).

$$P_i = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ p_i(1,0) & p_i(1,1) & & 0 \\ \cdots & \cdots & \cdots & \cdots \\ p_i(K,0) & p_i(K,1) & & p_i(K,K) \end{bmatrix}, \quad i = 1, \ldots, M \tag{3.1}$$

In Equation (3.1), the state of components cannot rise after an operating mission, i.e. $p \;\; a \; b \; = \quad if \; a < b$ If a component begins in state 0, it will remain in that state after the next mission since it cannot degrade further, hence $p_i(0,0) = 1$. If a component is in state a, $0 < a \leq K$ its state after the next mission can be any value from the set of $\{a \; a - \qquad 0$. Therefore,

$$\sum_{b=}^{K} p \;\; a \; b \; = \sum_{b=}^{a} p \;\; a \; b \; = \quad for \; a = \qquad K \tag{3.2}$$

In order to deal with the selective maintenance problem, we need to find the system reliability at the end of the next mission at a specified level d, i.e. the required reliability level, $R \;\; d \qquad\qquad d$. In series-parallel structures, a subsystem is in a state less than d, $\phi_{subi} < d$, when all of its components are in states less than d. The event that the subsystem i is in state d or above at the end of the next mission, $\phi_{subi} > d$, is the complement event of $\phi_{subi} < d$. Thus, the reliability of the system at level $d, R \;\; d$, can be computed by using Equation (3.3) [37].

$$\begin{aligned} R \;\; d \; &= \quad \phi \geq d \; = \prod_{i=}^{M} \quad \phi \quad \geq d \; = \\ &= \prod_{i=}^{M} (1 - \Pr(\phi_{subi} < d \quad = \prod_{i=}^{M} \Big(\quad - \prod_{j=} \sum_{=}^{d-} p \;\; X \;\; b \; \Big) \end{aligned} \tag{3.3}$$

3.2.3 The system repair time and cost

When entering the maintenance depot, components may be in any state from the set of state space, i.e. each elements of Y_{ij} can be any number from $\{0,1,...,K$. Within the maintenance break duration, T_0, and the available budget, C_0, the system and its components are subjected to be maintained to "properly working" states so that the system will meet the reliability requirement in the next operating mission. Here, the required time and cost for single repair (individual repair) of an independent and identically distributed (*i.i.d.*) component in subsystem i from state a to any state b which is greater than a are known. The single repair time and cost of each component in subsystem i from an arbitrary state a to state b are arranged in matrix form as in (3.4) and (3.5) respectively.

$$T \quad \begin{bmatrix} 0 & t & & & t & K \\ 0 & 0 & \cdots & & t & K \\ & & & & & \\ 0 & 0 & \cdots & t & K & K \\ 0 & 0 & \cdots & & 0 & \end{bmatrix} \quad i \qquad M \tag{3.4}$$

$$C \quad \begin{bmatrix} 0 & c & & & c & K \\ 0 & 0 & \cdots & & c & K \\ & & & & & \\ 0 & 0 & \cdots & c & K & K \\ 0 & 0 & \cdots & & 0 & \end{bmatrix} \quad i \qquad M \tag{3.5}$$

In (3.4) and (3.5), the time and cost of repairing a single component from state a to state

b are non-zero values when a is less than b. In general, $t\ a\ b \quad t\ a\ b$ and $c\ a\ b \quad c\ a\ b \quad if\ b \quad b$; $t\ a\ b$ and $c\ a\ b \quad if\ a \quad b$. Without doubt to the calculation of maintenance time and cost, an element in these matrices is set equal to zero if a is greater than b, i.e. $t\ a\ b \quad c\ a\ b \quad if\ a \quad b$

In order to calculate the total repair time and cost of the entire system, it is necessary to analyze the relationship of the repair time and cost between components. The time and cost for improving the state of a component may not affect that of other components - independent repair time for each component. However, the same setting up, maintenance process, equipment, etc. may be utilized when multiple components are simultaneously selected to be repaired in a maintenance strategy. In this case, time and cost savings will be achieved by repairing multiple components in the selective maintenance modeling.

3.2.2.1 Independent repair time and cost

When the repair time for each component is independent, the total time required to do the selective maintenance for the system is simply a summation of all the individual repair time of its components in (3.4). If we know the components' state vector at the time of entering the maintenance depot $Y \quad Y \quad i \quad M\ j \quad N$, the total system maintenance time corresponding to a vector of component state at the time of exiting the maintenance depot, $X \quad X$, can be computed and represented as in Equation (3.6).

$$T(X) = \sum_{i}^{M} \sum_{j} T(Y_{ij}, X_{ij}) \tag{3.6}$$

Similarly, the total system maintenance cost can be obtained by taking the sum of all single components' repair costs.

$$C(X) = \sum_{i}^{M} \sum_{j} C(Y_{ij}, X_{ij}) \tag{3.7}$$

3.2.2.2 Dependent repair time and cost

In most realistic systems, time and cost savings are achieved when multiple components are selected in a selective maintenance strategy, especially for identical components in each subsystem of series-parallel systems. Maintaining multiple components requires similar initial setting up, labor and equipment. Here, the concept of "set-up cost" [49] is employed to both cases of time and cost. In addition, it is even more economical when repairing multiple identical components in the same current state (condition) a to the same properly working state b. This is because there are not only the share of setting up but also the advantages of ordering materials (batch order) and using the same process of performing maintenance on those identical components. Additional cost and time savings for this type of repair should be addressed. Therefore, we consider two types of time and cost savings of repairing multiple components in series-parallel systems as follows:

1. Time and cost savings due to the share of setting up.

2. Additional reduction time and cost of repairing multiple identical components in a subsystem from the same current state a to the same properly working state b.

In the section below we will focus on formulating the total actual repair cost for a selective maintenance strategy based on the single repair cost matrix in (3.5) with the consideration of two types of cost saving above. Once the total repair cost is determined, the total maintenance time of the system can be calculated accordingly.

The savings due to the share of setting up are assumed to be fixed per component since it is associated with the process of preparation for maintenance such as erecting, dismantling and reassembling the system, etc. Thus, in the first type of dependency, a fixed amount of "set-up cost", Δc , is saved whenever an additional component is selected in a selective maintenance strategy (Figure 3.2). The more components to be maintained, the more money is saved due to the share of setting up. If N_r components in the system are maintained in a selective maintenance strategy, the total amount of money saved due to the share of setting up will be $(N$ c .

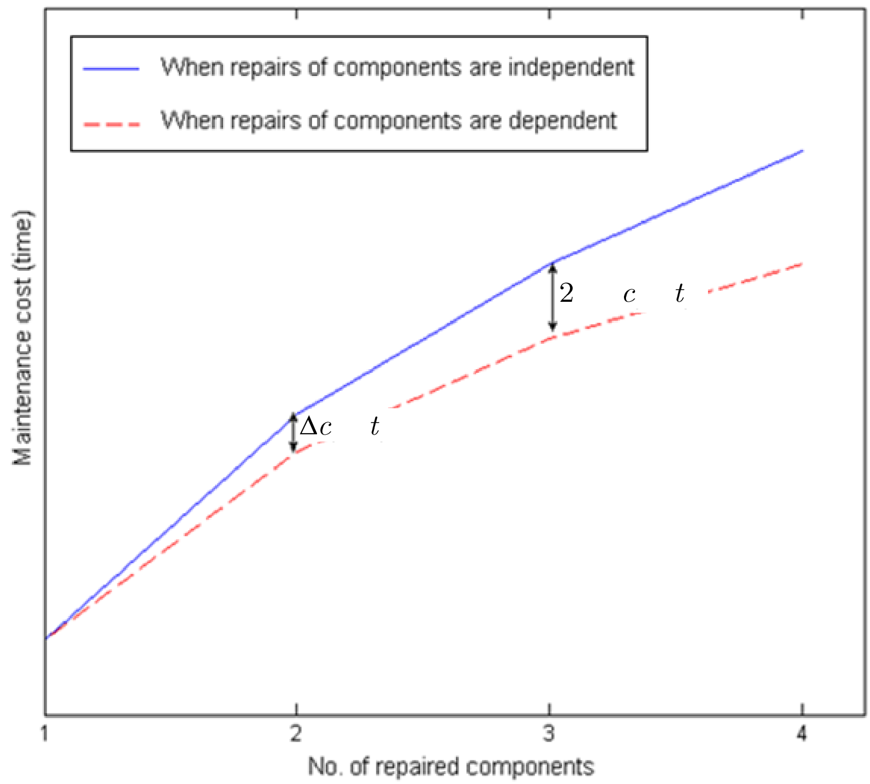


Figure 3.2 The maintenance cost (time) vs. number of repaired components when there is the share of setting up

Now, let's consider repairing m identical multi-state components in a subsystem i with $c \;\; a \; b$ being the cost for individually repairing each component from state a to state b. In multiple repairs of identical components, the same technology and equipment can be utilized, and the maintenance is more efficient in repetitive tasks. Thus, in addition to the cost saved by setting up sharing, we introduce a cost saving coefficient, $f \;\; a \; b$, to represent the repairing cost dependency of multiple identical components in subsystem i. Denote $c \;\; a \; b$ to be the adjusted repair cost for an additional component in subsystem i from state a to state b. The calculations of total maintenance cost and the saved amount due to each type of dependencies are illustrated in Figure 3.3. The adjusted repair cost and the saved amount in addition to a fixed set-up cost for repairing a component from state a to state b are calculated by (3.8) and (3.9) respectively.

$$c \;\; a \; b \qquad f \;\; a \; b \qquad c \;\; a \; b \qquad c \tag{3.8}$$

$$c \;\; a \; b \qquad c \;\; a \; b \qquad c \qquad f \;\; a \; b \qquad c \;\; a \; b \tag{3.9}$$

From (3.8), we can form an adjusted repair cost matrix for a component in each subsystem i, C ; each element in this matrix is a function of f and Δc . The total adjusted cost for repairing m identical components from state a to state b can be computed as in (3.10).

$$\begin{aligned} C \qquad & c \;\; a \; b \qquad m \qquad c \;\; a \; b \\ & = c \;\; a \; b \qquad m \qquad f \;\; a \; b \qquad c \;\; a \; b \qquad m \qquad c \end{aligned} \tag{3.10}$$

When maintaining identical components simultaneously, we need a constraint that the adjusted repair time, c' , is greater than or equal to 0. This is equivalent to $f \;\; a \; b \geq \dfrac{\Delta c}{c \;\; a \; b}$. Thus, $f \;\; a \; b$ can take any value between $\dfrac{\Delta c}{c \;\; a \; b}$ and 1.

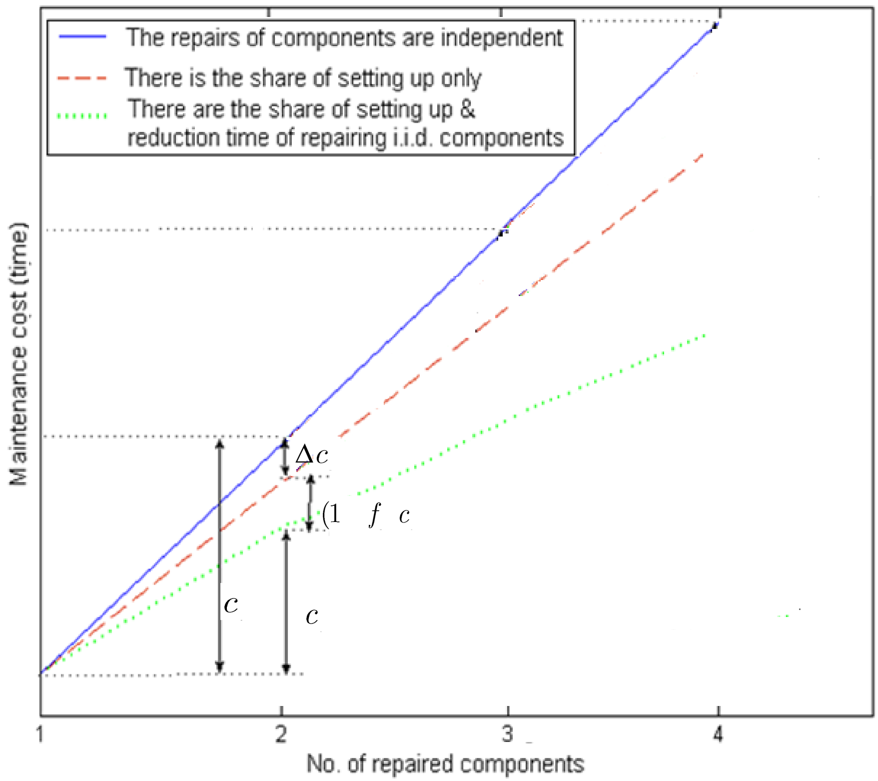


Figure 3.3 The maintenance cost when repairing identical components

The total system maintenance cost is, then, the summation of total adjusted repair cost of each component. It is a function of the decision variables X_{ij}, the amount of saved money due to the share of setting up - Δc_i, the cost saving coefficients - f_i and the components' state at the time of entering and exiting the maintenance depot.

$$C(X) = \sum_{i}^{M} \sum_{j} C_{ij}(X_{ij}, Y_{ij}, \Delta c_i, f_i) \tag{3.11}$$

In (3.11), $C_{ij}(X_{ij}, Y_{ij}, \Delta c_i, f_i)$ is the adjusted repair cost of component (i,j) which is equivalent to $c\ a\ b$ in the explanation above. In the selective maintenance problem for the series-parallel system considering economic dependence, we need an assumption that the components in the same subsystem have to be maintained in a group to gain the benefits of time and cost savings. However, in the process of calculation, it is no doubt to assume a subsystem is repaired before another sub-system since all the results on the system reliabilities, cost and time are not affected. With this assumption, if there are multiple components in different sub-systems subjected to be maintained in a selective maintenance strategy, the value of the adjusted repairing for the first component is always equal to the cost of the single repair for that component. For the next component, $c\ a\ b$ is calculated as in (3.12).

$$c\ a\ b \quad \begin{cases} f \quad c\ a\ b \quad c & \text{...e is identical repair} \\ \quad c\ a\ b \quad c & \text{Otherwise} \end{cases} \tag{3.12}$$

Similarly, the total system maintenance time for a selective maintenance strategy can be obtained if the time saved due to the share of setting up - Δt , the time dependent coefficients - f , and the state of component at the time of entering the maintenance depot are known.

$$T\ X \quad \sum_{i}^{M} \sum_{j} T\ X\ Y \quad t\ f \tag{3.13}$$

where T is the adjusted repair time for component j in subsystem i when the repairs of components are dependent.

In order to understand more about the proposed saving model, we also investigate other saving mechanisms and their effects on the selective maintenance for the multi-state series-parallel system. The saving patterns of performing multiple activities when task repetitions take place will be investigated and compared to our saving model. The basic saving model is known as "power model" or "Log-linear model" [56]. It has the following mathematical representation:

$$C\ m\ = C\ \times m^{-\beta} \text{ or } \log C\ m\ = \quad C - \beta \quad m, \tag{3.14}$$

where C is the cost (time) of the first unit, $C(m)$ is the average cost (time) of m units, and β is an exponent indicating the rate of productivity improvement, 0 1. $\beta = 0$ means that there is no advantage of performing multiple tasks and when β is close to 1, the benefit of multiple tasks execution is larger.

From the basic model in Equation (3.14), several models have been developed to measure the average unit cost/time as a function of the number of units produced. These models are summarized in [57]. Here, we will consider three typical saving patterns: the Log-linear, Plateau and Exponential models.

$$C\ m\ = C\ + C\, m^{-\beta} \tag{3.15}$$

$$C\ m\ = C\, m^{-\beta} e \tag{3.16}$$

The Plateau model in (3.15) is often applied when there is a limit of the unit cost as x

increases to a very big value. C_p is a parameter representing the plateau limit of the average unit cost. In (3.16), c is another parameter, usually a constant that reflects the upturn effect on the cost when the number of units increases to a considerably large value.

From the average unit cost, we can find the total cost of repairing m identical components, C , in a sub-system of the series-parallel system. Table 3.1 summarizes C calculated from the proposed model and three saving models in the literature.

Table 3.1 Summary of different cost saving models
of repairing m identical components

Model	Average cost of m identical components	Total cost of repairing m identical components
Log-linear	$C\,m^{-\beta}$	$C \quad C\,m$
Plateau	$C + C\,m^{-\beta}$	$C \quad Cm \quad C\,m$
Exponential	$C\,m^{-\beta}e$	$C \quad C\,m \quad e$
Proposed model	$\left[C + m - C\right] m$	$C \quad C \quad m \quad C$

In the calculation of average cost of x units for the proposed model in Table 3.1, we use C as the adjusted repair cost, which can be computed in the same way as in (3.12). We use three saving patterns in Table 3.1 for describing the dependent relationship between the cost/time of multiple repairs and the number of components involved in each sub-system. The results of selective maintenance for multi-state series-parallel systems and discussions will be provided in Section 3.4.

3.2.4 Selective maintenance optimization models

Cassady et al. [19] proposed mathematical programming models for optimizing selective maintenance of binary systems. In this section, we extend those models to the multi-state case. The selective maintenance problem for multi-state series parallel system is a non-linear integer programming problem with known components' characteristics such as the state of components at the time of entering the maintenance depot, the time and cost of single repair, time and cost savings of multiple repair, state probability distribution of each component in the next mission.

In a very popular selective maintenance problem, the maintenance crews have to find what maintenance activities associated with each component to be performed to achieve the maintenance objective of increasing the system reliability under limitation of resources such as time and cost. This type of problem can be formulated as follows:

P3.1: $$\textit{Maximize } f = R\ \ d\ = \prod_{i=}^{M}\left(\ -\prod_{j=}^{N}\sum_{=}^{d-} p\ \ X_{\ \ } \ b\ \right) \tag{3.17}$$

Subject to: $$T(X) \leq T_0 \tag{3.18}$$

$$C(X) \leq C_0 \tag{3.19}$$

$$Y_{ij} \leq X_{ij} \leq K \tag{3.20}$$

$$X_{ij} \text{ is integer}, \; i=1,2,\ldots,M, \; j = 1,2,\ldots,N_i \tag{3.21}$$

In the model, the objective function (3.17) is to maximize the reliability of the system at a specified working level *d,* which has been formulated in Section 3.2.2. There are two types of constraints in (3.18) and (3.19) which restrict the total time and cost for all maintenance activities within available time, T_0, and budget, C_0. The component repair time and cost may be or may be not independent. We can find the total time and cost for maintaining the system to desired working level as explained in Section 3.2.3. The decision variables, $X_{ij}, i \quad M \; j \quad N$ are the states of components at the time of exiting the maintenance depot. Since the maintenance activities do not worsen the state of the components, X_{ij} must be integer value between Y_{ij} and the maximum state K for all $i \quad M \; j \quad N$.

The decision makers can use the problem **P3.1** when they have information about their available time and budget and the system reliability is very critical. The solution of **P3.1** gives us the most reliable system with on-hand resources. In practice, we may face many other situations of the principal objective and resource availability. When the total completion time is the most important issue it is treated as the objective of the selective maintenance problem. Then, the system reliability and cost are two constraints, and we have an alternative **P3.2**. Similarly, another derivation, **P3.3**, can be obtained as follows:

P3.2: $\quad \mathit{Minimize} \quad f \quad T \; X \quad (3.22)$

Subject to: $R_s(X) \geq R_0$ (3.23)

$C(X) \leq C_0$ (3.19)

$Y_{ij} \leq X_{ij} \leq K$ (3.20)

X_{ij} is integer, $i=1,2,...,M,\ j = 1,2,...,N_i$ (3.21)

P3.3: *Minimize* $f \quad C\ X$ (3.24)

Subject to: $R_s(X) \geq R_0$ (3.23)

$T(X) \leq T_0$ (3.18)

$Y_{ij} \leq X_{ij} \leq K$ (3.20)

X_{ij} is integer, $i=1,2,...,M,\ j = 1,2,...,N_i$ (3.21)

In the problems **P3.2** and **P3.3**, the constraint (3.23) requires the reliability of the system to be greater than or equal to a specified level R_0. Both $R_s(X)$ and R_0 are vectors with K-dimension; each dimension is the probability that the multi-state system is at the corresponding state or above.

For an extremely urgent situation, we have to maintain the system to attain a certain reliability level as soon as possible regardless of how costly it is. In this case, the constraints on cost will be released and a special case of problem **P3.2** can be derived:

P3.4: *Minimize* $f \quad T\ X$ (3.22)

Subject to: $R_s(X) \geq R_0$ (3.25)

$Y_{ij} \leq X_{ij} \leq K$ (3.20)

X_{ij} is integer, i=1,2,...,M, j = 1,2,...,N_i (3.21)

It is also noted that many other derivations of these models can be obtained. Depending on the principal purpose of the maintenance and available information, the maintenance decision makers can select the most appropriate models and thereby understand what actions should be done based on his/her on-hand conditions.

The selective maintenance optimization problem is to find the optimal state combination of all components in the MSS. Its complexity lies in the size of the solution space, i.e. the number of possible state combinations. For each component i, we need to search for all possible states between Y_i and K inclusive. Thus, the total number of possible solutions is $\prod_{i=}^{n}(K-Y+ \quad = O\ K$. For small size problems, e.g. $n \leq \quad K <$, it is possible to enumerate all possible solutions and find the optimal one. However, for larger size problems, it may take a long time to enumerate all possible solution. Heuristic or evolutionary computational methods are recommended to find a relatively good solution with less computational effort.

3.3 Solution approach

In view of the selective maintenance problem complexity, it is apparent that the enumeration approach can only be used for a small-sized selective maintenance problem for MSS. However, when the size of the problem is large, it is not practical to enumerate all the possible solutions and compare them. Heuristic or meta-heuristic methods should be used in this case to find optimal or near optimal solutions.

In this thesis, Genetic Algorithm (GA) is used to solve the proposed selective maintenance models. GA was also used in many previous maintenance optimization and selective maintenance studies and it has proved to be efficient [6] [34] [38]. The reason for using GA as the solution approach is that GA is relatively simple and easy to use for many practitioners. In addition, the proposed selective maintenance model is a combinatorial optimization problem and GA is very well suited for combinatorial optimization with a high dimensional searching space [59]. Lastly, GA is also highly adapted to problems with integer variables while the selective maintenance problem belongs to a class of non-linear integer optimization model.

In this Section, we will provide a detailed solution representation for the selective maintenance problem, a general procedure of GA, and parameters setting in GA program. Further details on GA searching evolution and genetic operators' mechanisms can be found in [58] and [59].

3.3.1 Solution representation

Solution representation is one of the most important parts in the implementation of genetic algorithm. Let n be the total number of components in the system, n N N N The detail of solution representation is illustrated in Figure 3.4. Each chromosome consists of n genes; each gene is an integer number between 0 and K which represents the state of the corresponding component at the end of the maintenance break.

All elements in the vector of components' state are ordered from subsystem *1* to subsystem M and the decision variable, X $i =$ N $j =$ M , can be transformed to the state vector X $i =$ n. Each state vector is considered as a solution of the proposed selective maintenance problem. If the states of components at the time of entering the maintenance depot are known and can be rewritten in vector form, Y i n, we can use GA to find the best combination of the components' outcome states after the maintenance break for the proposed optimal selective maintenance problem.

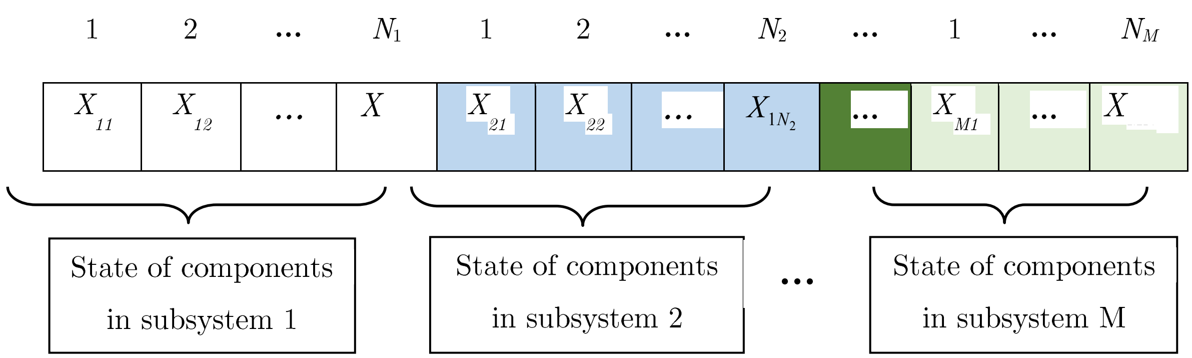


Figure 3.4 Solution representation

3.3.2 General procedure and implementation of GA

GA starts by randomly initializing a population of N chromosomes in the first generation. Each chromosome represents a possible solution to the selective maintenance problem. Since each gene of a chromosome represents the state of the corresponding component, we use a random generator to create each integer number in each gene within the range from Y_i to M. The fitness function, i.e. the objective function, is used to evaluate the chromosomes, which allows a particular chromosome to be ranked against all the others. After computing the fitness values of the individuals, existing solutions are recombined using crossover, mutation, and reproduction procedures, to obtain new ones. The genetic algorithm terminates when a pre-specified number of generations is attained or a given time limit is over.

In the GA program, four important parameters need to be tuned, namely population size, number of GA iterations, crossover probability, and mutation probability. The only crossover type considered is one-point crossover. Uniform mutation and tournament selection are used in the GA program. While testing a parameter, the other parameters are fixed using the best known value. The crossover and mutation probabilities were tuned with increment of 0.05 between 0.7 to 0.95 and 0.05 to 0.15 respectively. The best probability with best performance was selected. The population size and the number of GA iterations are selected in order to balance the efficiency and the accuracy of the program rather than trying to find the optimal solution. After many trials, the reasonably good parameters setting is shown in Table 3.2.

Table 3.2 GA parameters setting

GA Parameters	**Value/ Probability**	**Type**
Population size	50	-
Crossover	0.8	Single point
Mutation	0.1	Uniform
Selection	-	Tournament
Number of GA iterations (Stopping criteria)	100	-

3.4 Illustrative examples, results and discussion

Example 3.1: Considering a multi-state series-parallel system (Figure 3.5) with $K = 3$, $M = 3$, $N_1 = 3$, $N_2 = 2$, $N_3 = 4$, the given states of components when entering the maintenance depot are:

$$Y = \begin{bmatrix} 2 & 0 & 1 & \\ & & & \\ 0 & 2 & 2 & 1 \end{bmatrix} \quad or \; Y = \begin{bmatrix} 1 & 2 & 2 & 0 & 2 & 2 & 1 \end{bmatrix}$$

For each *i.i.d.* component in subsystem i, i=1, 2, 3, the transition probability matrices and corresponding cost and time matrices for each individual maintenance activity are:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0 & 0 \\ 0.1 & 0.3 & 0.6 & 0 \\ 0.05 & 0.1 & 0.1 & 0.75 \end{bmatrix} \quad P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.1 & 0.9 & 0 & 0 \\ 0.1 & 0.3 & 0.6 & 0 \\ 0.05 & 0.05 & 0.1 & 0.8 \end{bmatrix} \quad P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 \\ 0.05 & 0.15 & 0.8 & 0 \\ 0.05 & 0.1 & 0.15 & 0.7 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 7 & 8 & 10 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 3 & 9 & 10 \\ 0 & 0 & 6 & 8 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 5 & 6 & 10 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & 2 & 5 & 7 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad T = \begin{bmatrix} 0 & 3 & 4 & 6 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad T = \begin{bmatrix} 0 & 1 & 3 & 6 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The repairing time and cost for each component are independent. The required level of the system at the end of the next operating mission is $d = 3$. We want to find the maximum reliability of the system in the next mission with available time $T_0 = 25$ time units, and total allowed budget $C_0 = 45$ cost units.

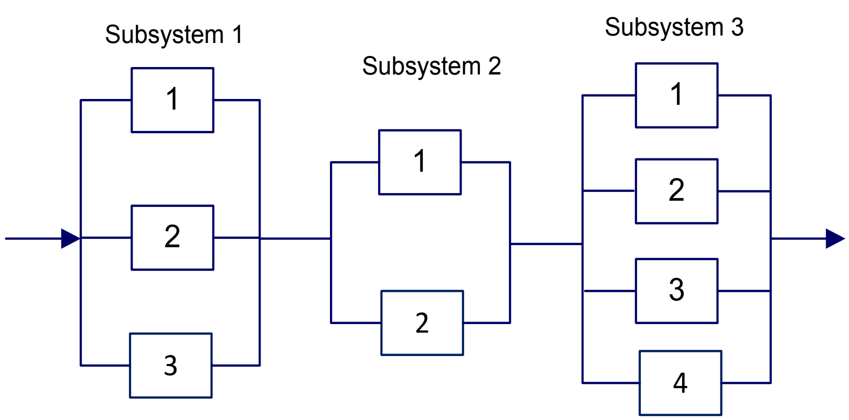


Figure 3.5 Series-parallel system in Example 3.1

By running the GA program to solve problem **P1** to find the optimal reliability of the system at state $d = 3$, we obtain $X = (3 \quad 3 \quad 3 \quad 3 \quad 0 \quad 2 \quad 3 \quad 3)$, that is, we select to repair all components in subsystem 1 and 2 to state 3, do nothing to the first two components in subsystem 3 and repair the last two components in subsystem 3 to state 3.

Table 3.3 The optimal maintenance actions for all components when repairs of components are independent (Problem P3.1)

Subsystem i	**Component j**			
	1	**2**	**3**	**4**
1	2→3	0→3	1→3	-
2	2→3	2→3	-	-
3	0 - 0	2 - 2	2→3	1→3

The total cost for all maintenance activities is 44 cost units (≤ 45) and the total maintenance time is 22 (≤ 25) time units. Recall that $R_s(d)$ is the probability of the system state being in state d or above in the next operating mission, we obtained the system optimal reliability at level 3, $R_s(3) = 0.85995$. The probabilities that the system will be at least in state 1 and 2 in the next mission are 0.99725 and 0.98222 respectively.

Table 3.4 The probability of the system being in state k or above in the next operating mission (Problem P3.1)

d	1	2	3
$R_s(d)=\Pr(\phi_s \geq d)$	0.99725	0.98222	0.85995

To analyze the quality of the GA results, we have run the GA 10 times with the setting in Table 3.2 for Example 3.1. In addition, the enumeration approach of all possible solutions is also used to find the optimal solution in this example since the size of the problem in this example is not too big. A summary of results is shown in the following Table 3.5.

Table 3.5 Summary on the quality of GA results

Best objective function	**Worst objective function**	**Optimality**	**Ave. difference in objective function**	**Ave. CPU time (seconds)**
0.85995	0.8190	8/10	0.00818	2.67283

It is seen that GA can produce very good results in a reasonable short amount of time.

In Example 3.1, solving problem **P3.1** helps us find the most reliable system within available resources. Now, we want to investigate what maintenance actions should be implemented when the total maintenance time is the most critical issue. This situation is usually encountered in practice when a tight deadline is required in production or the next mission needs to be started as soon as possible. Example 3.2 is devoted to the illustration of this situation. With this example, the decision makers can find the best maintenance actions when time is the most critical while the required system reliability level is still close to the achieved results in Table 3.4.

Example 3.2: Consider the system in Example 3.1. Find the selective maintenance strategy that we can finish as soon as possible without any requirement on budget, but the system reliability at each level must be greater than $R_0(1) = 0.99$, $R_0(2) = 0.96$, $R_0(3) = 0.85$.

Here, the cost constraint is released and we deal with problem **P3.4** to minimize the total completion time of all selective maintenance activities with the constraint of achieving the required reliability. The solution of GA program results the optimal completion time of **18 time units**. The vector state of all components at the end of the maintenance break is: X 1 3 3 3 0 3 3 3.

Table 3.6 The optimal maintenance actions for all components when repairs of components are independent (Problem P3.4)

Subsystem *i*	**Component *j***			
	1	**2**	**3**	**4**
1	2→3	0→1	1→3	-
2	2→3	2→3	-	-
3	0 - 0	2→3	2→3	1→3

In comparison with Problem **P3.1** in Example 3.1, there are two different maintenance activities on component (1,2) and (3,2). Component (1,2) is repaired to state 1 and component (2,3) is repaired to the perfect working state 3; the other components in the system receive similar maintenance actions as in the previous example. The total cost of this strategy is 48 cost units, which indicates that we need more budget for all maintenance actions in comparison with the solution in Problem **P3.1**. The reliability of the system at level $d = 3$ is $R_s(3) = 0.8757$ which is, interestingly, greater than the optimal reliability in Example 3.1. The achieved reliability at each level satisfying the reliability constraints is provided in Table 3.7.

Table 3.7 The probability of the system being in state k or above when repairs of components are independent (Problem P3.4)

d	1	2	3
$R_s(d) = \Pr(\phi_s \geq d)$	0.99688	0.96446	0.8757

Example 3.3: Consider the problem in Example 3.1 again, but there are advantages of repairing multiple components, i.e. the components are economically dependent. The time and cost savings for each component due to the share of setting up of 0.4 time units and 0.8 cost units; the cost saving coefficients of components in subsystem 1, 2, 3 are 0.7, 0.6, 0.45; and the time saving coefficients of components in subsystem 1, 2, 3 are 0.5, 0.4, 0.3 respectively.

Now, we have to deal with Problem **P3.1** to find the most reliable system with the available time and cost of 25 and 45 respectively. The GA program gives the state vector of components at the time of exiting the maintenance depot as follows:

$X \quad 3\ \ 3\ \ 3\ \ 3\ \ 1\ \ 3\ \ 3\ \ 3$

Table 3.8 The optimal maintenance actions for all components when repairs of components are dependent (Problem P3.1)

Subsystem *i*	**Component** *j*			
	1	**2**	**3**	**4**
1	2→3	0→3	1→3	-
2	2→3	2→3	-	-
3	0 →1	2→3	2→3	1→3

In this strategy, we repair the first component in subsystem 3 to state 1 and all other components to the best condition. In comparison with problem **P3.1** in Example 3.1, there are two more maintenance actions which can be taken on components (3,1) and (3,2) within available resources.

Due to the time and cost savings of repairing multiple components, the achieved reliability of the system at each level (see Table 3.9) is much higher than the results in Example 3.1.

Table 3.9 The probability of the system being in state k or above when repairs of components are dependent

d	1	2	3
$R_s(d) = \Pr(\phi_s \geq d)$	0.99734	0.98333	0.91949

Example 3.4: In this example, we use three different saving models in section 3.2.3.2 for

modeling the economic dependence when repairing multiple identical components in a sub-system. The input data from Example 3.3 is used.

We use the Matlab program to solve the selective maintenance problem to get the most reliable system in the next mission within available time of $T_0=25$ units and budget of $C_0=45$ units. In this example, the exponents for the cost and time savings, $\beta_C^i \quad {}_T^i$, are set based on $f \quad f \quad c \quad t$ so that the adjusted repair cost/time of a component from the proposed model is equivalent to the amount of cost/time for repairing an additional identical component in a sub-system from Equation (3.14). The obtained results of vector **X** from GA, the total time and cost used for maintenance and the system reliabilities with corresponding parameters used in each model are shown in Table 3.10.

The first five elements in state vectors, $X(1)$ to $X(5)$, at the time of exiting the maintenance depot are similar in all four models, i.e. the maintenance actions for these components are exactly the same. In the maintenance strategy from the proposed model, one additional maintenance action (corresponding to $X(6)$) can be performed in comparison with Log-linear and Plateau models and two more maintenance actions (corresponding to $X(6)$ and $X(9)$) can be performed in comparison with the Exponential model. The total maintenance cost and time spending in the proposed model is higher than in the other models (43.75 cost unit and 20.7 time units), and this brings the system to a state with higher system reliabilities of 0.99734, 0.98333, 0.91949 at levels 1, 2, and 3 respectively.

Table 3.10 Results of different saving models

Model	**Vector** X	$C(X)$	$T(X)$	$R_s(1)$	$R_s(2)$	$R_s(3)$
Log-linear	[3 3 3 3 3 0 3 3 3]	41.15	19.9	0.9973	0.98333	0.91949
Plateau (C_p=0.1C_1)	[3 3 3 3 3 0 3 3 3]	43.55	20.2	0.9973	0.98333	0.91949
Exponential (c=0.1)	[3 3 3 3 3 0 3 3 2]	41.21	18.69	0.99725	0.98222	0.85995
Proposed model	[3 3 3 3 3 1 3 3 3]	43.75	20.7	0.99734	0.98333	0.91949

To explain the differences in the results in Table 3.10, it is observed that the cost saving of a component in sub-system i is characterized by two parameters, f and Δc, in the proposed model. Meanwhile, the other saving models describe the economic dependence relationship using a power function and parameter β. These models can only describe the saving mechanism within a sub-system (when the components are identical), while the proposed model also addresses the economic dependence when repairing components in different sub-systems. This is close to the practical situation since the time and cost of doing common preparation and maintenance activities on the serial-parallel system such as erecting, cleaning, doing inspections, lubricating, etc. can be saved even with different types of components.

To investigate the time and cost savings versus the number of components involved in a maintenance strategy, we study the selective maintenance strategy resulting from Example 3.3 for independent repair and dependent repair cases. Figures 3.6 and 3.7 show how the maintenance time and cost for the series-parallel system are saved.

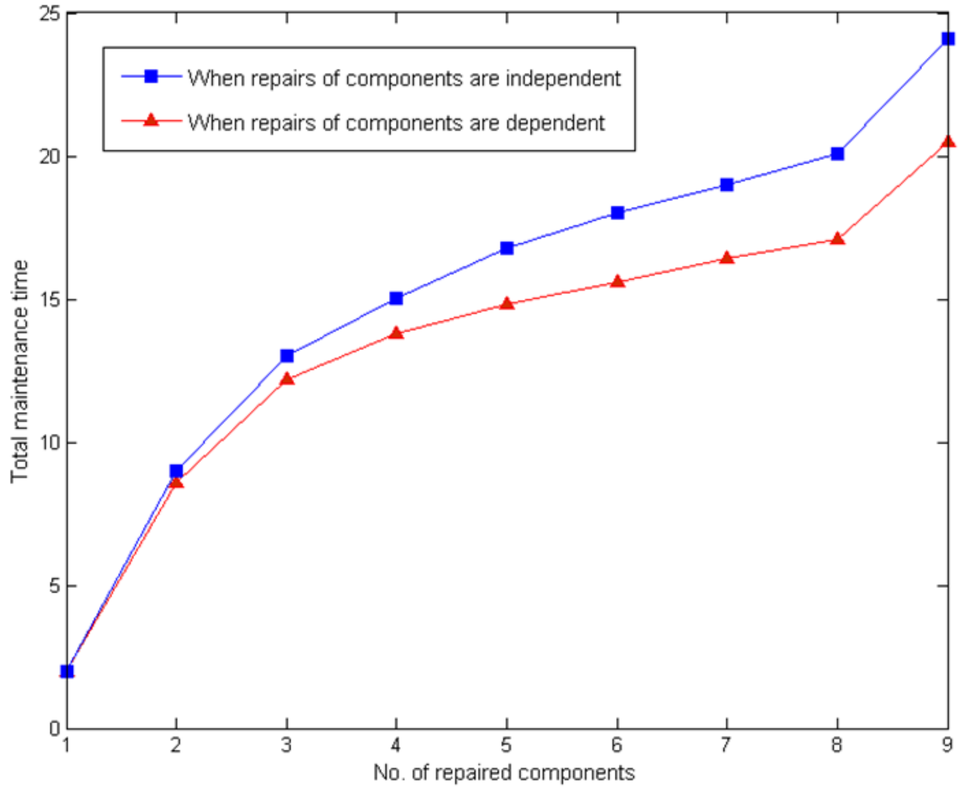


Figure 3.6 Total maintenance time of repairing dependent components vs. independent components

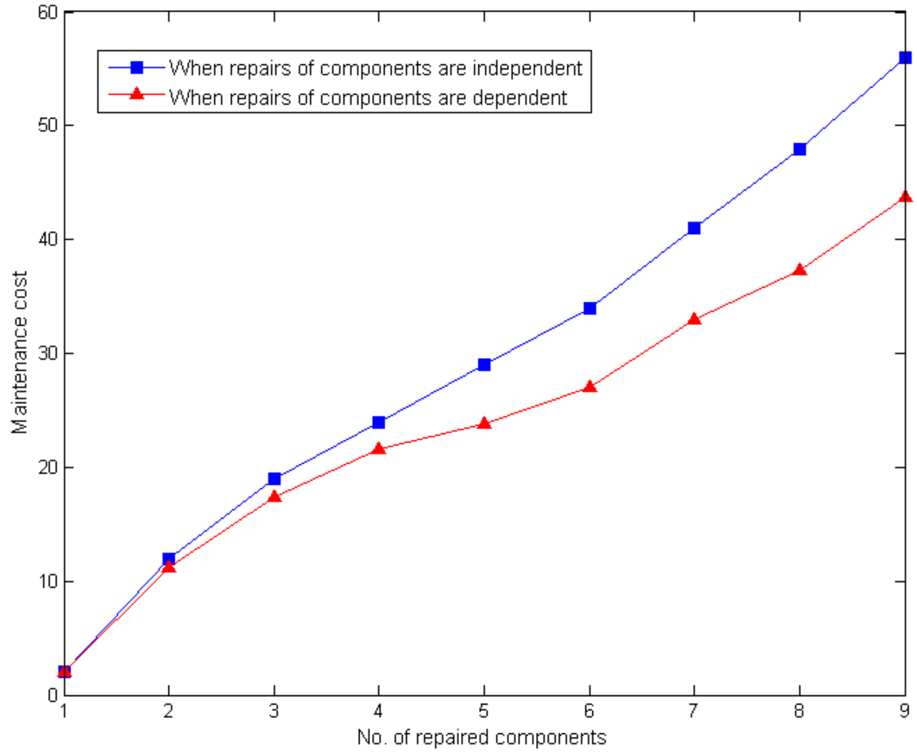


Figure 3.7 Total maintenance cost of repairing dependent components vs. independent components

Generally, we only need an assumption that the components in the same sub-system have to be maintained in a group to gain the benefits of time and cost savings. In Figures 3.6 and 3.7, we also assume that the components are maintained in an order from the first sub-system to the last sub-system. In this example, there are 9 components which are subjected to be maintained in the maintenance scenario. When the repairs of components are dependent, the maintenance time and cost are smaller than those in the case of independent repairs. The different amounts of required resources between the two cases are more considerable as the number of repaired components increases. To repair all components in this maintenance strategy, the total required time and cost are just

20.7 time units and 43.75 cost units when repairs of components are dependent in comparison with 24 time units and 56 cost units in the case of independent repairs between components.

3.5 Concluding remarks

This chapter documents the selective maintenance model for multi-state series-parallel system with economic dependence. In the optimization models, total system maintenance cost, completion time or system reliability can be treated either as objective or as constraint; several alternative optimization models are derived. Both time and cost savings can be realized when several components are maintained together in a selective maintenance strategy. Economic dependence is analyzed in multi-state contexts based on two types of time and cost savings: *(i.)* the share of setting up and *(ii.)* the advantage of repairing multiple identical components in series-parallel systems simultaneously. The illustrative examples show that the maintenance schedulers may perform different maintenance actions on the system depending on the main objective and the availability of resources.

In selective maintenance of multi-component systems, the amount of resource savings is system and component specific, i.e. it depends on the nature of the system, number of components and type of components involved. By dividing the type of time and cost savings into fixed (the share of the setting up) and variable (multiple identical repairs), the proposed model can capture both the system features and the component specific features. Our main objective is to investigate the selective maintenance problem for multi-state systems under the effect of this saving mechanism. In general, the selective maintenance model in this chapter can help the maintenance manager determine the best maintenance strategy to achieve a reliable system and allocate the resources effectively.

Chapter 4

Selective Maintenance for Multi-state Systems with Structural Dependence

This chapter focuses on the selective maintenance problem for multi-state systems with structural and economic dependence. In a multi-component system, components can structurally form multiple hierarchical levels and dependence groups. We formulate the structural dependence between components considering assembly precedence relationships using a directed graph. A backward search algorithm is proposed to determine an assembly sequence for a selective maintenance scenario. This chapter is based on a submitted paper to Reliability Engineering and System Safety [60].

4.1 Introduction

In maintenance of multi-component systems, components may interact with each other in different ways. Based on the effect on the lifetime distribution of components, the interaction or dependence can be classified into two main categories. In the first category, dependence does not affect the degradation of components. This type of dependence represents the relationships between components during the implementation of maintenance activities and often occurs in maintenance models with economic or structural dependence. As investigated in Chapter 3, economic dependence in maintenance has been addressed in several maintenance models [13], [31], [42]–[48], [50]–[52], [61]. *Structural dependence* exists when the components form a structural group and the maintenance of a component implies the maintenance of other components or at least dismantling them [15], [39], [62]–[66]. In the second category, dependence relates to the degradation of components when they are in operation, and thus, the lifetime distributions of components will be affected. The reason for this degradation dependence may come from another component in the system or from variable operating conditions.

In this chapter, we will focus on the first case of dependence as described above. The selective maintenance problem for multi-state systems with both structural and economic dependence will be investigated. The degradations of components in an operating mission are assumed to be independent.

The remainder of this chapter is organized as follows. Existing studies on maintenance of systems with structural dependence are reviewed in Section 4.2. The system model and a selective maintenance model are presented in Section 4.3. Illustrative examples, numerical results, and analysis are provided in Section 4.4 and a concluding remark of this chapter is presented in the Section 4.5.

4.2 Literature review

Structural dependence means that components form a structural group, and repairing a component requires intervention actions to other components. Early studies on structural dependence focus on replacement policy [15], [62]. In these studies, it is suggested that the system is built in a vertical structure of modules, and replacing a component at a higher level requires replacing all the components at its lower levels. The problem is whether to replace the whole system or replace a sub-assembly or just a single component when that single component fails. Zhou et al [39] considered a more realistic system model given the relationship of components in a hierarchical structure, where the components can have relationships with others at both higher or lower levels and at the same level. A preventive maintenance time for the system is optimized using the Monte Carlo simulation method. Jia [63] divided a multi-component system into m modules and assumed that replacing a component requires dismantling all components in the same module, so structural dependence is the shared cost of dismantling and re-assembling components. The shared cost of dismantling and re-assembling in [63] can also be considered as the "set-up" cost in other economic maintenance models. Horenbeek and Pintelon [64] added another cost element to the set-up cost to represent the structural dependence. The limitation of [63] and [64] is the simplification of the concept of structural dependence so that it can be treated using a single parameter. However, the system becomes more and more complicated and it may be decomposed into sub-systems, several sub-assemblies, parts and components. Thus, the idea of using a tree-like structure to represent multiple hierarchy levels of set-up was presented in [67]. There also exists a disassembly sequence between components in the system, so disassembling all preceding components in a disassembly sequence is required before maintaining a component. The disassembly sequence planning was also investigated and summarized in [68]–[70]. However, these studies [67]–[70] focus on the grouping or the disassembly sequencing problems given a pre-scheduled maintenance action rather than an optimal maintenance model with difference maintenance activities on the system.

There have also been specific maintenance models with structural dependence. Yan et al. [65] considered structural dependence in maintenance scheduling problems. When a component fails, a number of related components need to be shut down for replacing the failed component. This model is specifically applied to a system with the assumption that the number of shutdown components proportionally increases the downtime cost of the system. Dekker et al. [66] studied the road maintenance problems, where the road is divided into carriage ways, lanes and small segments. Different failure mechanisms and maintenance actions can be applied to a small segment, but some maintenance packages are restricted to "large-scale" or "junction-to-junction" maintenance. Thus, once the maintenance package is needed for a segment, the whole set of segments have to be considered.

In the literature, structural dependence is tied with specific applications or simplified such that another form of set-up cost can be used. For those considering the replacement policies, components are assumed to be “as good as new” after replacement and a single preventive replacement time is optimized. None of the papers considered different maintenance actions or different working levels a component may be at. In addition, when the resources for maintenance are limited and there are multiple hierarchical levels with a disassembly sequence in the system, the questions raised are which components or group of components should be selected, and what maintenance actions should be performed to optimize the system performance in the next mission. Thus, in this chapter, we will explore the selective maintenance of multi-state series systems with structural dependence. Selective maintenance policy applies when the whole system is maintained at the maintenance depot and the joint maintenance of components naturally exists. Thus, economic dependence is also considered. In summary, the main focus of this Chapter is as follows.

- Both economic dependence and structural dependence are considered in the system. Economic dependence simply means the share of the set-up, while structural dependence relates to the sequence of disassembling components when implementing maintenance activities on the system.
- The multi-level hierarchical relationship of disassembling components is to be recognized in the maintenance model.

- The optimal selective maintenance policy for multi-state components and structural dependence is investigated.

4.3 Problem formulation

4.3.1 The system model

The multi-state system in this study has n multi-state components connected in series (Figure 4.1). The system state is determined as the minimum state of its components.

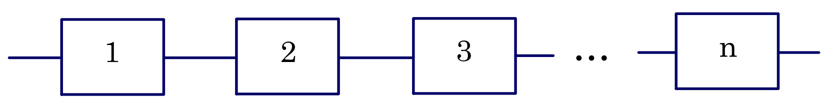


Figure 4.1 A series system

Some components may be structural dependent so that they form multiple hierarchy levels. Each level may contain several groups of components and there is disassembly sequence of components/groups of components. The precedence relations govern the order of disassembling components, which is represented by a directed graph [39] as in Figure 4.2.

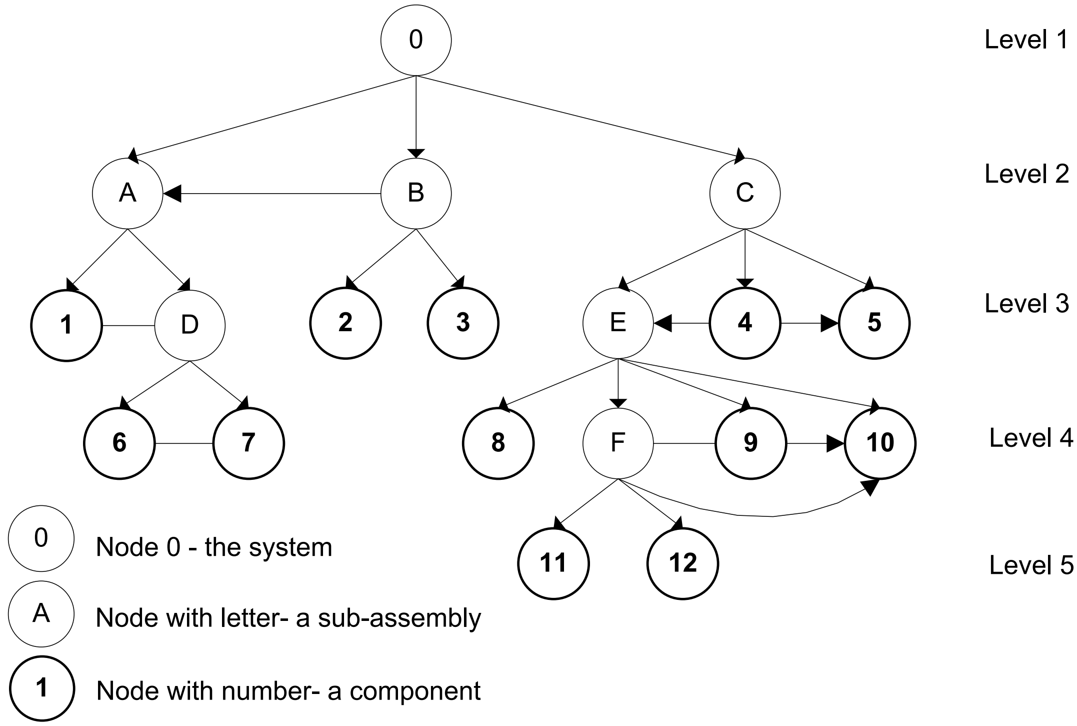


Figure 4.2 A directed graph representing the precedence relations

The directed graph is a representation graph with nodes and edges modeling the system and precedence relations between its components. There are three different types of nodes, including root node or node "0" – which represents the system, intermediate nodes or nodes with letters – which represent sub-assemblies, and leaf nodes or nodes with numbers – which represent the components. The root and intermediate nodes can be further broken down into their children nodes, but there is no further breakdown for leaf nodes. The edges may be directed or undirected to represent the relationship between nodes. A directed edge is used when there is a sequence of assembly relationship. It is a "father-son" relationship between two nodes in different levels or a sequence order from one node to another in the same level following the direction of the arrow. The undirected edge between two nodes at the same level means the two nodes are mutually restricted and disassembling one means disassembling the other.

In the selective maintenance problem, the maintenance manager has to determine which components to maintain and how to maintain each component in the system to ensure the completion of the next mission within available budget C_0 and available time T_0. In order to formulate the selective maintenance problem, we need to determine the system reliability as well as the maintenance resources, i.e. cost and time, associated with a selective maintenance scenario. These are to be presented in Sections 4.3.2. and 4.3.3 respectively.

4.3.2 Component state distribution and the system reliability

The multi-state component degrades along with the time of use. Similar to Chapter 3, we assume that the probability for component i at a pre-specified state a at the beginning of the next mission degrading to a state b at the end of the next mission, $p_i(a,b)$, is given. the transition probability matrix of component i in the operating mission is as in Equation (4.1).

$$P \quad \begin{bmatrix} 1 & 0 & \dots & 0 \\ p & p & & 0 \\ \dots & \dots & \dots & \dots \\ p \;\; K & p \;\; K & & p \;\; K \;\; K \end{bmatrix} \quad i \qquad n \tag{4.1}$$

We also have:

$$\sum_{b=}^{K} p \quad a \; b \; = \sum_{b=}^{a} p \quad a \; b \; = \quad for \; a = \qquad K \tag{4.2}$$

The system reliability at the end of the next mission meeting a specified demand level d, $R_s(d)$, is the probability that the system will complete the next mission at the required working level or state level d. In other words, the system reliability is the probability that the state of the system at the end of the next mission, ϕ_s, is greater or equal to d. With an assumption that the degradations of components are independent, it can be computed using the following equation.

$$R \;\; d \; = \qquad \phi \; \ge d \; = \prod_{i=}^{n} \quad s \; \ge d \; = \prod_{i=}^{n} \quad - \qquad s \; < d \quad = \prod_{i=}^{n} \left(\; - \sum_{=}^{d-} p \;\; X \;\; b \; \right) \tag{4.3}$$

where s_i is the state of component i at the end of the next mission.

4.3.3 Maintenance cost and time

3.3.2.1 Individual component maintenance time and cost

In the selective maintenance problem, the system and its components are maintained at the maintenance depot within the available resources. The maintenance decision maker has to select components and maintain them to states so that the system will meet the reliability requirement in the next operating mission. If components are already disassembled, the required time and cost for individually maintaining a component i from state a to state b, $b > a$, are known. Let $t_i(a,b)$ and $c_i(a,b)$ be the individual maintenance time and cost of component i from state a to state b. Similar to Chapter 3, they are arranged in matrix form as in (4.4) and (4.5) respectively.

$$T \begin{bmatrix} 0 & t & & & t & K \\ 0 & 0 & \cdots & & t & K \\ & & & & & \\ 0 & 0 & \cdots & t & K & K \\ 0 & 0 & \cdots & & 0 & \end{bmatrix} \quad i \qquad n \tag{4.4}$$

$$C \begin{bmatrix} 0 & c & & & c & K \\ 0 & 0 & \cdots & & c & K \\ & & & & & \\ 0 & 0 & \cdots & c & K & K \\ 0 & 0 & \cdots & & 0 & \end{bmatrix} \quad i \qquad n \tag{4.5}$$

The maintenance time and cost can be treated similarly. In the following sections, we will formulate the total system maintenance time related to the selective maintenance problem; the maintenance cost can be formulated in the same way if there is no further explanation.

3.3.2.2 Disassembly time and cost

Different from Chapter 3, here we consider the disassembly and assembly time for each component subjected to maintenance in a selective maintenance strategy. The maintenance of a component includes a process of disassembling the component, repairing/replacing the component, and putting it back to the system. Here, we use the term "disassembly time" to represent the time related to both processes of disassembling and re-assembling the component. Considering two nodes i and j, which can be either letters or numbers, they are said to have a precedence relationship if there exists a directed or undirected edge between them. In the directed graph, we define the precedence relation between two nodes i and j, $r(i,j)$, as follows.

$$r\ i\ j = \begin{cases} 0, & \text{if there is no edge between } i \qquad j \\ -1 & \text{if there is a directed edge from } i \quad j \\ 1, & \text{if there is a directed edge from } j \quad i \\ 2, & \text{if there is an undirected edge between } i \qquad j \end{cases} \tag{4.6}$$

For each component i, the t is the disassembly time for component i itself. A set of nodes that needs to be disassembled from the root node to node i inclusive forms a disassembly path for node i. it is noted that a leaf node, i.e. a component, always appears in its own disassembly path. Two leaf nodes are structurally dependent if they **both** appear in at least one assembly path among the two assembly paths.

To maintain a component i, we need to consider the cumulative disassembly time of all nodes in its disassembly path. The method to find the disassembly path of a component can be found in [39]. However, in selective maintenance, we need to calculate the total disassembly time for the system with a given maintenance scenario. Thus, one needs to find the disassembly path of all components in each possible selected maintenance strategy, DP_s. We propose a backward search algorithm to find the disassembly path for the system to perform a maintenance strategy, DP_s, as follows.

1. Starting from a leaf node i representing a component in the selective

maintenance scenario, search for nodes j in the same and upper levels.

2. If i is not in DP_s, add i to DP_s
3. Check if $r(i,j) > 0$, i.e. node j is in the assembly path of node i. If $r(i,j) > 0$ and j is not in DP_s, then add j to a position before i in the system disassembly path DP_s.
4. If j is in the disassembly path of node i, the same search is performed from node j for all its precedence nodes in the same and upper levels.
5. When the root node is reached, we move to the next leaf node in the selective maintenance strategy in the next level and set the searching set of all nodes in the directed graph excluding DP_s and return to step 2.
6. The procedure stops when all nodes in the SM scenario are considered.

The key part of the algorithm is to use the precedence relation, $r(i,j)$, defined in Equation (4.6), to search for nodes to be added in the system disassembly path, DP_s. If $r(i,j) > 0$, i.e. $r(i,j) \in \{1, 2\}$, node i must either be a child of node j or be mutually restricted to j. Thus, j must be disassembled before i, i.e. j is in the disassembly path if i is maintained. On the other hand, if $r(i,j) < 0$, i.e. $r(i,j) \in \{-1, 0\}$, there is no requirement of disassembling j prior to disassembling i. For each component i in a selective maintenance scenario, the closed loop from step 2 to step 5 is performed with the stopping criteria that the root node is reached, i.e. a correct disassembly sequence of component i is found. All the components in the selective maintenance scenario are searched to complete the whole system disassembly path. The procedure is illustrated in

Figure 4.3.

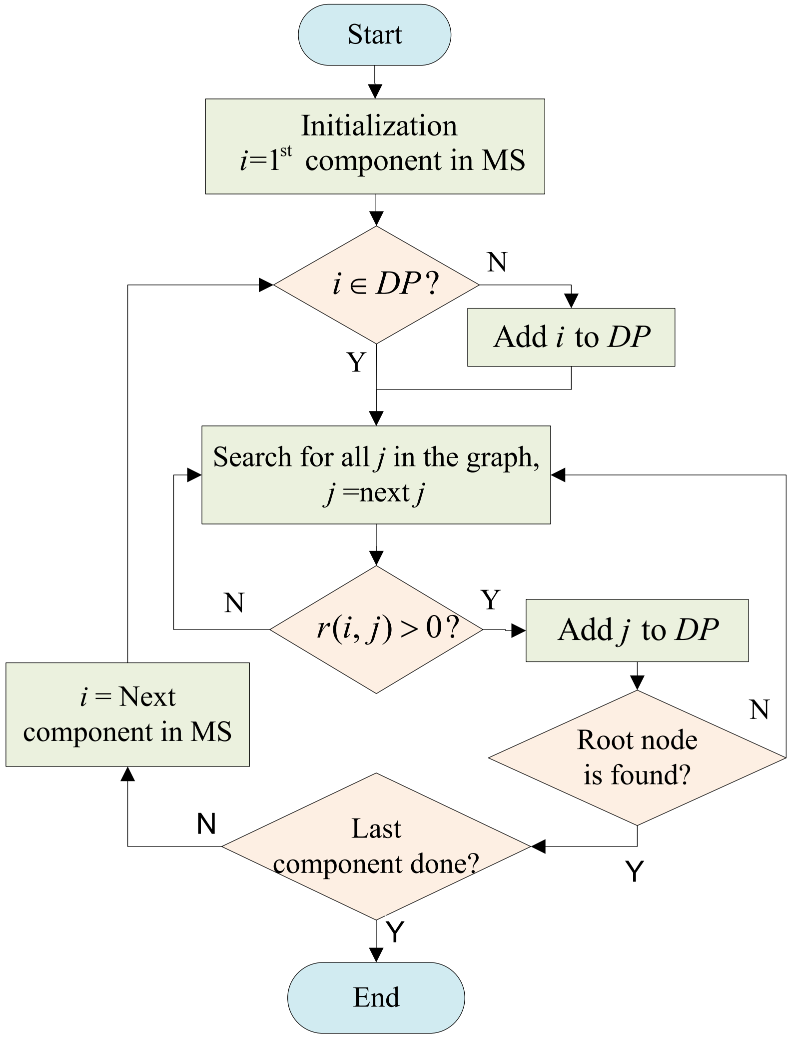


Figure 4.3 The Back-ward search algorithm

Once the disassembly path for a selective maintenance strategy is determined, the total disassembly time of the system, then, can be calculated using Equation (4.7).

$$T = \sum_{j \in DP} t_j \qquad (4.7)$$

where t is the disassembly time of node j itself. Once the disassembly time is determined, the disassembly cost can be calculated accordingly based on the disassembly cost per unit time c_d.

$$C \qquad c\,T \tag{4.8}$$

3.3.2.3 The total system maintenance time and cost

In selective maintenance, there are often multiple components to be maintained at the same time. Since the precedence relations exist in the system, the maintenance of component i requires disassembling all components in its precedence relations. This directly affects the maintenance resources and the selective maintenance decision-making. On the other hand, it may be a good idea for joint-maintenance of the components which can save resources related to the maintenance process such as labor, materials, etc. Thus, one needs to determine the total resources for maintaining the whole system including the maintenance, disassembly as well as the savings related to maintaining multiple components in the system. Specifically, the total system maintenance time, T, is the total maintenance time of each individual component, T_M, plus the system disassembly time, T_d, and minus the time savings as an advantage of maintaining multiple components in the system, ΔT , as in Equation (4.9).

$$T \quad T \quad T \qquad T \tag{4.9}$$

In Equation (4.9), T_M is the summation of all individual maintenance times of all components in the system, which can be calculated using Equation (4.10). The disassembly time is explained in Equation (4.7) and the total time savings, ΔT represents the advantage of the shared utilization of the maintenance process, personnel, and materials when repairing multiple components together. For simplicity, we assume that the amount of time savings depends on the number of selected components in a selective maintenance scenario and a fixed time savings per component Δt . We can calculate ΔT using Equation (4.11).

$$T \qquad {}_{i}^{n}\; t \;\; Y \;\; X \tag{4.10}$$

$$\Delta T \; = \left(\sum_{i=}^{n} \delta \;\; Y \;\; X \;\; - \;\; \right) \Delta t \tag{4.11}$$

where $\delta_i{}' Y \;\; X \;\; = \begin{cases} 1 \text{ if } t \;\; Y \;\; X \;\; > \\ 0 \text{ if } t \;\; Y \;\; X \;\; = \end{cases}$

Delta $\delta_i{}' Y \;\; X$ is defined as being equal to 1 if $t \;\; Y \;\; X \;\; >$, i.e. component i is selected for maintenance, and being equal to 0 if $t \;\; Y \;\; X \;\; =$, i.e. component i is not selected for maintenance. With this definition, the summation $\sum_i \delta_i{}' Y \;\; X$ is the total number of selected components for maintenance. If there are more than one selected component, and a fixed setup time, Δt , is saved per additional component, the total amount of saving will be evaluated exactly using Equation (4.11).

Equation (4.9) can be rewritten as:

$$T\ X\ = \sum_{i=}^{n} t\ Y\ X\ + T\ \ Y\ X\ - \left(\sum_{i=}^{n} \delta\ Y\ X\ - \ \right) \Delta t\ , \tag{4.12}$$

where the $T\ \ Y\ X\ = \sum_{j \in DP} t$

Similarly, the total system maintenance cost includes the total maintenance cost of individual components, C_M, plus the system disassembly cost, C_d, and minus the cost savings due to maintaining multiple components together.

$$C\ X\ = C\ \ + C\ \ - \Delta C\ = \sum_{i=}^{n} c\ Y\ X\ + c\ T\ \ Y\ X\ - \left(\sum_{i=}^{n} \delta\ Y\ X\ - \ \right) \Delta c \tag{4.13}$$

4.3.4 Selective maintenance optimization model

Assume that the state vector of all components at the time entering the maintenance depot is known as **Y**= $[Y_1\ \ Y_2\ \ldots\ \ Y_n]$. We need to determine the state vector of components at the time exiting the maintenance depot **X**= $[X_1\ X_2\ \ldots\ X_n]$. The selective maintenance model in this chapter is for finding the best set of components and maintenance actions to maximize the system reliability in the next mission under limitation of resources including time and cost. The selective maintenance model is formulated as follows:

$$\textbf{P:}\ \ \textit{Maximize} \qquad R\ \ d\ = \prod_{i=}^{n} \left(\ - \sum_{b=}^{d-} p\ \ X\ \ b \right) \tag{4.14}$$

$$\textit{Subject to:} \quad \sum_{i=}^{n} t \; Y \; X \; + T \; Y \; X \; - \left(\sum_{i=}^{n} \delta \; Y \; X \; - \; \right) \Delta t \; \leq T \tag{4.15}$$

$$\sum_{i=}^{n} c \; Y \; X \; + c \, T \; Y \; X \; - \left(\sum_{i=}^{n} \delta \; Y \; X \; - \; \right) \Delta c \; \leq C \tag{4.16}$$

$$Y_i \leq X_i \leq K \tag{4.17}$$

$$X_i \text{ is integer, } i{=}1{,}2{,}\ldots{,}n \tag{4.18}$$

In the model, the objective function (4.14) is to maximize the system reliability at a specified working level *d,* which has been formulated in Section 4.3.2. The two constraints of (4.15) and (4.16) restrict the total maintenance time and cost for the system within available time, T_0, and budget, C_0, respectively. The total maintenance time and cost of the system are explained in Section 4.3.3. The states of components at the time of exiting the maintenance depot, X_i, are decision variables. X_i must be an integer value and must be between Y_i and the maximum state K for all $i =$ n as stated in (4.17).

We use genetic algorithm (GA) [58], [71], a widely used meta-heuristic, to solve the selective maintenance optimization model. In GA, we use a chromosome consisting of n genes to represent a solution of the selective maintenance problem. Each element of the state vector corresponds to each gene in GA solution representation. X is the targeted state of the corresponding component i in the maintenance period. If the states of components at the time of entering the maintenance depot, Y, are known, the multistate system reliability, maintenance cost and time can be calculated for each vector **X**= [X_1 X_2 … X_n]. Thus, GA can be used to find the best combination of the components' outcome states after the maintenance break for the proposed optimal selective maintenance problem.

4.4 Examples, results and analysis

4.4.1 Example 4.1

This example is for illustration of the backward search algorithm to find the disassembly path of the system for a maintenance scenario. Considering the multi-state system in Figure 4.2 with 12 multi-state components connected in series, each component has 5 possible states, i.e. K=4; the state of each component at the time of entering the maintenance depot and the maintenance scenario of interest are given in Table 4.1; we have to find the disassembly path for the given maintenance scenario.

Table 4.1 Initial and end states of components at the maintenance depot

Compnent # i	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
State Entering the depot Y	1	2	0	3	2	1	2	3	2	2	1	3
State exiting the depot X	**4**	2	**3**	3	2	**4**	**4**	3	**3**	**3**	**4**	3

From Table 4.1, the set of selected components in the given selective maintenance scenario is $\{1,3,6,7,9,10,11\}$. The following results are obtained using the backward search algorithm for each component in this set.

- For i=1, DP $B\ A\ D$
- For i=3, DP $B\ A\ D$
- For i=6, DP $B\ A\ D$ 6
- For i=7, DP $B\ A\ D$ 6
- For i=9, DP $B\ A\ D$ C $E\ F$
- For i=10, DP $B\ A\ D$ C $E\ F$
- For i=11, DP $B\ A\ D$ C $E\ F$

The final disassembly path is DP $B\ A\ D$ C $E\ F$, which means that we disassemble the system following the order of the disassembly path from 0→B→A→D and 1→3→6 and 7→C→4 and E→F and 9→10→11 to perform all maintenance activities in this maintenance scenario. If the disassembly time for each individual node is known, the total disassembly time for the system can be calculated using Equation (4.7).

4.4.2 Example 4.2

This example is to analyze the selective maintenance problem for the same system in Example 1 considering economic and structural dependence. The states of components at the time of entering the maintenance depot are the same as in Table 4.1. Individual components' maintenance time, cost and state probability distribution are given in Table 4.2. The information on disassembly time, time and cost savings per component and available resources for maintenance are presented in Tables 4.3 and 4.4. We need to find the maintenance strategy, i.e. the states of all component at the time exiting the maintenance depot X_1, X_2,..., X_n, to maximize the system reliability in the next mission within the given available resources.

The problem is coded in Matlab R2014a. The analysis is performed for two cases of the system with assembly precedence relations and without assembly precedence relations. In the first case, the disassembly times in Table 4.3 and backward search algorithm are used to find the disassembly time for the system. For the second case, the summation of disassembly times of all selected individual components in a maintenance strategy is used as the disassembly time of the system. The results on maintenance strategies, resource allocation, and the system reliability at different resource availability are to be discussed in this section.

Table 4.5 presents the main results on the optimal maintenance strategies and the corresponding system reliability at a required state level $d = 2$. X is the optimal exiting state vector when the disassembly of components is restricted by the precedence graph in Figure 1, i.e. the actual assembly dependence is considered, and X is the optimal exiting state vector for the system when the maintenance of a component requires disassembling itself only.

Table 4.2 Component characteristics

Component #	state	Time (time units) 0	1	2	3	4	Cost (cost units) 0	1	2	3	4	Probability 0	1	2	3	4
i=1	**0**	0	1	3	5	8	0	3	4	7	10	1	0	0	0	0
	1	0	0	2	4	7	0	0	1	4	7	0.2	0.8	0	0	0
	2	0	0	0	2	5	0	0	0	3	6	0.05	0.1	0.85	0	0
	3	0	0	0	0	3	0	0	0	0	3	0.01	0.02	0.05	0.92	0
	4	0	0	0	0	0	0	0	0	0	0	0.002	0.01	0.038	0.12	0.83
i=2	**0**	0	1	2.5	4.5	6	0	2	4	6	8	1	0	0	0	0
	1	0	0	1.5	3.5	5	0	0	2	4	6	0.1	0.9	0	0	0
	2	0	0	0	2	3.5	0	0	0	2	4	0.01	0.09	0.9	0	0
	3	0	0	0	0	1.5	0	0	0	0	2	0.005	0.015	0.03	0.95	0
	4	0	0	0	0	0	0	0	0	0	0	0.001	0.005	0.024	0.11	0.86
i=3	**0**	0	2	3	6	8	0	3	4	7	9	1	0	0	0	0
	1	0	0	1	4	6	0	0	1	4	6	0.15	0.85	0	0	0
	2	0	0	0	3	5	0	0	0	3	5	0.03	0.08	0.89	0	0
	3	0	0	0	0	2	0	0	0	0	2	0.01	0.01	0.02	0.96	0
	4	0	0	0	0	0	0	0	0	0	0	0.005	0.005	0.04	0.15	0.8
i=4	**0**	0	1.5	3	4	5	0	3.5	6	9	11	1	0	0	0	0
	1	0	0	1.5	2.5	3.5	0	0	2.5	5.5	7.5	0.3	0.7	0	0	0
	2	0	0	0	1	2	0	0	0	3	5	0.02	0.08	0.9	0	0
	3	0	0	0	0	1	0	0	0	0	2	0.005	0.012	0.1	0.883	0
	4	0	0	0	0	0	0	0	0	0	0	0.003	0.005	0.012	0.06	0.92
i=5	**0**	0	1.5	4	6	8	0	4	7	10	14	1	0	0	0	0
	1	0	0	2.5	4.5	6.5	0	0	3	6	10	0.15	0.85	0	0	0
	2	0	0	0	2	4	0	0	0	3	7	0.02	0.03	0.95	0	0
	3	0	0	0	0	2	0	0	0	0	4	0.01	0.015	0.025	0.95	0
	4	0	0	0	0	0	0	0	0	0	0	0.002	0.008	0.04	0.1	0.85
i=6,7	**0**	0	1.5	2	3.5	5	0	2.5	4	6	8	1	0	0	0	0
	1	0	0	0.5	2	3.5	0	0	1.5	3.5	5.5	0.25	0.75	0	0	0
	2	0	0	0	1.5	3	0	0	0	2	4	0.04	0.06	0.9	0	0
	3	0	0	0	0	1.5	0	0	0	0	2	0.01	0.02	0.09	0.88	0
	4	0	0	0	0	0	0	0	0	0	0	0.005	0.005	0.03	0.06	0.9
i=8	**0**	0	1	2.5	3	4.5	0	2.5	4	6	7	1	0	0	0	0
	1	0	0	1.5	2	3.5	0	0	1.5	3.5	4.5	0.1	0.9	0	0	0
	2	0	0	0	0.5	2	0	0	0	2	3	0.06	0.08	0.86	0	0
	3	0	0	0	0	1.5	0	0	0	0	1	0.005	0.015	0.05	0.93	0
	4	0	0	0	0	0	0	0	0	0	0	0.003	0.006	0.01	0.05	0.931

		Time (time units)					Cost (cost units)					Probability				
	state	**0**	**1**	**2**	**3**	**4**	**0**	**1**	**2**	**3**	**4**	**0**	**1**	**2**	**3**	**4**
i=9	**0**	0	2	4	5	6.5	0	3	5	7	11	1	0	0	0	0
	1	0	0	2	3	4.5	0	0	2	4	8	0.1	0.9	0	0	0
	2	0	0	0	1	2.5	0	0	0	2	6	0.03	0.06	0.91	0	0
	3	0	0	0	0	1.5	0	0	0	0	4	0.005	0.02	0.1	0.875	0
	4	0	0	0	0	0	0	0	0	0	0	0.001	0.002	0.05	0.12	0.827
i=10	**0**	0	1.5	4	5	6	0	2	5	7	10	1	0	0	0	0
	1	0	0	2.5	3.5	4.5	0	0	3	5	8	0.2	0.8	0	0	0
	2	0	0	0	1	2	0	0	0	2	5	0.02	0.05	0.93	0	0
	3	0	0	0	0	1	0	0	0	0	3	0.002	0.015	0.12	0.863	0
	4	0	0	0	0	0	0	0	0	0	0	0.001	0.013	0.056	0.08	0.85
i=11	**0**	0	1.5	3	4	5	0	2	5	7	9	1	0	0	0	0
	1	0	0	1.5	2.5	3.5	0	0	3	5	7	0.15	0.85	0	0	0
	2	0	0	0	1	2	0	0	0	2	4	0.03	0.05	0.92	0	0
	3	0	0	0	0	1	0	0	0	0	2	0.008	0.015	0.02	0.957	0
	4	0	0	0	0	0	0	0	0	0	0	0.001	0.006	0.023	0.1	0.87
i=12	**0**	0	1	3	5	6.5	0	2.5	6	9	12	1	0	0	0	0
	1	0	0	2	4	5.5	0	0	3.5	6.5	9.5	0.25	0.75	0	0	0
	2	0	0	0	2	3.5	0	0	0	3	6	0.02	0.12	0.86	0	0
	3	0	0	0	0	1.5	0	0	0	0	3	0.01	0.08	0.1	0.81	0
	4	0	0	0	0	0	0	0	0	0	0	0.005	0.01	0.065	0.11	0.81

Table 4.3 Disassembly time (time units)

Node	***A***	***B***	***C***	***D***	***E***	***F***	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***
t_i^d	2	1	2.5	1.2	1.6	1.5	1.2	2.4	2	1.5	2	1	1	2.5	1.5	1	1.8	2.5

Table 4.4 Input data on cost, time, and required working level of the system

C *(Budgeted cost)*	T *(Break duration)*	Δt *(Time saving per component)*	Δc *(Cost saving per component)*	c *(Assembly cost per unit time)*	d *(Required working level)*
75	50	0.5	1	1.7	2

Table 4.5 Selective maintenance results in Example 4.2

Componnet $\#i$	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	$\boldsymbol{R_s(d)}$
Entering state Y	1	2	0	3	2	1	2	3	2	2	1	3	
Exiting state X (Strategy 1)	**3**	4	4	**4**	**2**	4	4	3	**3**	3	4	4	0.80997
Exiting state X (Strategy 2)	**4**	4	4	**3**	**4**	4	4	3	**4**	3	4	4	0.87116

From Table 4.5, it is seen that there are 10 components selected in both strategies. That is, whether structural dependence is considered (X) or not (X), 10 out of 12 components of the system are selected for maintenance. However, the set of selected 10 components in X is a bit different from the set in X. The resulting component states in X and X are not the same either.

In strategy 1, components 2, 3, 4, 6, 7, 11, 12 should be maintained to their best state (state 4); components 1, 9, 10 should be maintained to state 3 and nothing is suggested for components 5 or 8. The projected system reliability at d =2 is 0.80997 for this maintenance strategy.

Comparing X and X, there are 4 out of 10 maintenance actions, which are different in the two strategies. Component 4 is selected in strategy 1 but not in strategy 2, and component 5 is selected in strategy 2 but not strategy 1. Components 1 and 9 are both maintained to state 3 in strategy 1, but they are suggested to be maintained to their maximum state 4 in strategy 2.

In general, maintenance strategy 2 recommends repairing components in the system to higher states than strategy 1 does. This result is because of ignoring the precedence relations between components. For example, strategy 2 recommends repairing component 5 but not component 4 while there is a directed edge from 4 to 5 in the precedence graph. In strategy 2, the resources for disassembly and maintaining this component are $t \quad t$ 6 time units and $c\,t \quad c$ 10.4 cost units. In fact, there is a sequence of assembly and to maintain component 5, we need to disassemble following a disassembly path {0, C, 4, 5}, i.e. sub-assembly C and component 4 need to be disassembled sequentially before component 5. It turns out to be $t \quad t \quad t \quad t$ 10 time units and $c\ t \quad t \quad t \quad c$ 17.2 cost units to maintain component 5 in actual condition. This omission results in more maintenance activities allocated to repair components to higher states in strategy 2 within the same available resources. Consequently, the estimated system reliability at the end of the next mission in strategy 2 is 0.87116, which is 7.55% overestimated compared to strategy 1. These results demonstrate that the maintenance planner may make an incorrect maintenance selection and over-estimate the system reliability when not taking the assembly sequence into consideration.

In terms of maintenance resources allocation, Table 4.6 and Figure 4.4 and Figure 4.5 show the time and cost elements for each strategy. The time and cost elements are broken down using Equations (4.12) and (4.13).

Table 4.6 Resource allocation results in Example 4.2

	Maintenance		Assembly		Saving		Total	
Strategy	T_M	C_M	T_d	C_d	ΔT_s	ΔC_s	T	C
Strategy 1	30	42.5	24.2	41.14	4.5	9	49.7	74.64
Strategy 2	37.5	54.5	16.4	27.88	4.5	9	49.4	73.38

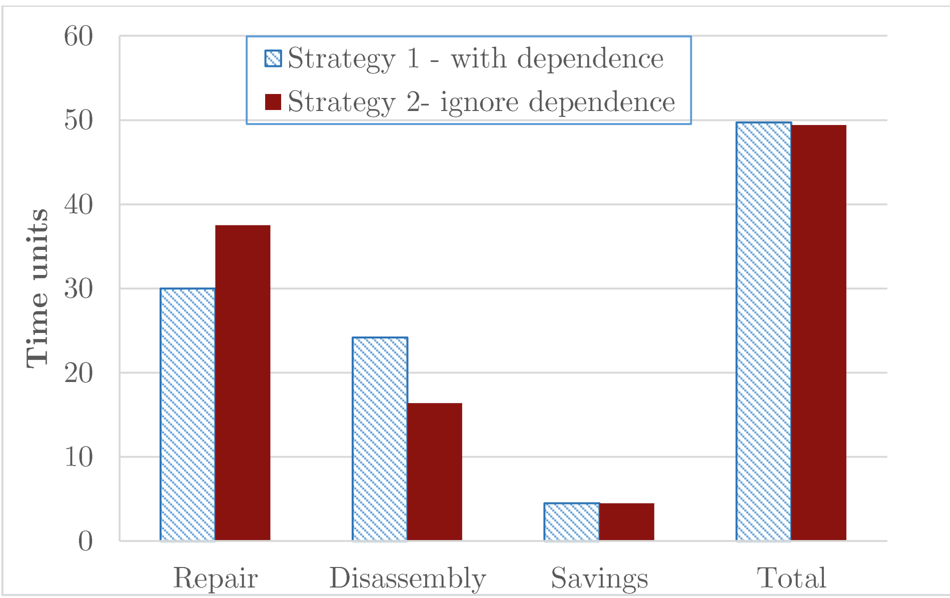


Figure 4.4 Maintenance time in Example 4.2

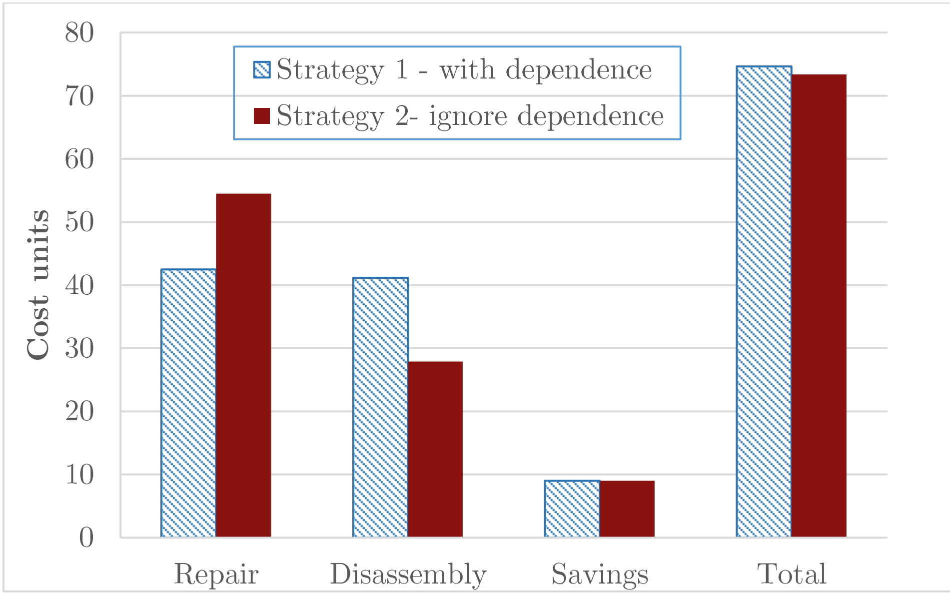


Figure 4.5 Maintenance cost in Example 4.2

From these figures, the total maintenance time and cost and the time and cost savings are not significantly different for the two maintenance strategies. However, the repair time and cost spending for strategy 1 is significantly less than that for strategy 2, which implies that more improvement maintenance actions are suggested in strategy 2. This also explains the reliability gap between the two strategies. Another result that can be drawn from the figures is that the resources for disassembly contribute a considerable amount when disassembly precedence relation is considered. This result implies that ignoring structural dependence can lead to a mistake in resource allocation in the process of maintenance implementation.

The deviation of the system reliability between considering and ignoring structural dependence is investigated when the maintenance budget varies. To do this, constraint (4.15) is deleted and the optimization model is run repeatedly for different total maintenance budget, C_0, varying from 30 to 100 cost units. The results on the system reliability are shown in Figure 4.6.

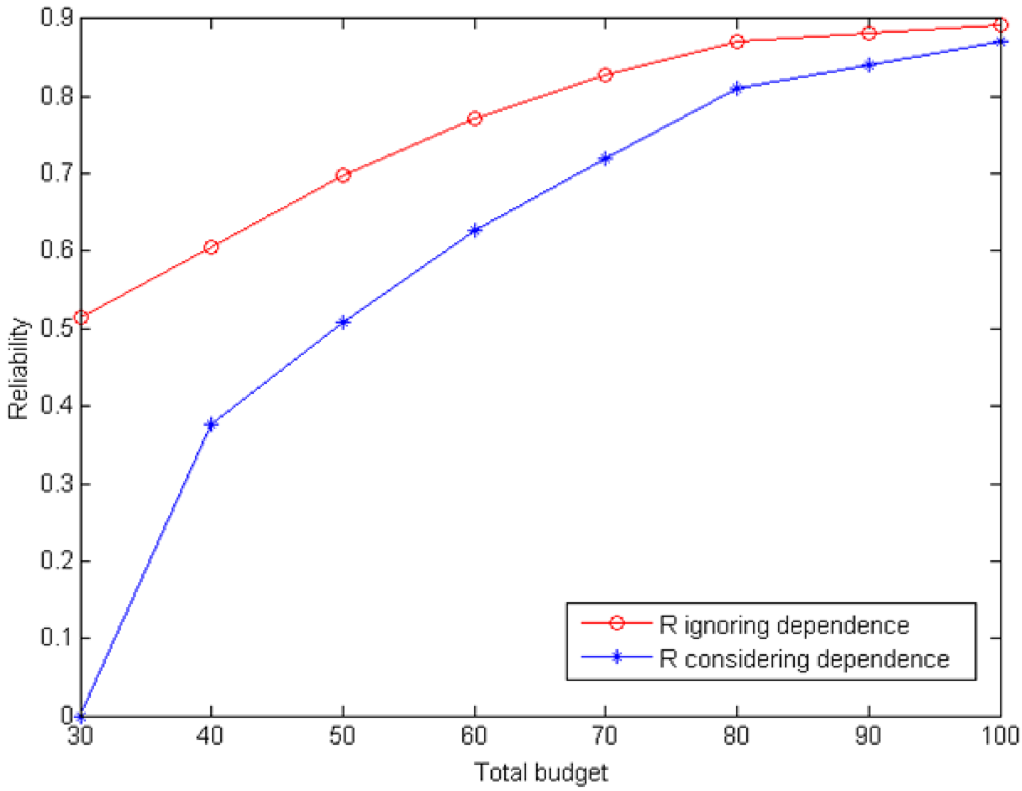


Figure 4.6 System reliability vs. total maintenance budget

An obvious result of Figure 4.6 is that the system reliability at the end of the next mission is overestimated when ignoring structural dependence. This matches the above explanations when examining maintenance strategies and resource allocation results. In addition, the gap of the system reliability estimation between the two cases of considering structural dependence and not considering structural dependence is bigger when there are tighter budgets. For instance, when $C_0 = 100$, the estimated system reliability for case 2 is 0.892, while the actual system reliability (case 1) is 0.871, down by 2.4%; and when $C_0 = 50$, the estimated system reliability for case 2 is 0.698, while the actual system reliability (case 1) is 0.508, down by 27.2%. This experiment indicates that the tighter the resource limitation is, the more severe the mistake caused by ignoring structural dependence.

4.5 Concluding remarks

A selective maintenance problem for multi-state systems with multi-state components and precedence relations of component assembly sequence is investigated in this chapter. We have formulated the relationships between components using a directed graph, and a backward search algorithm is proposed to calculate the total disassembly time for a maintenance scenario. The developed selective maintenance optimization model is to maximize the system reliability in the next mission under both time and cost constraints. The illustrative examples show that not considering structural dependence between components results in an incorrect maintenance scenario and an over-estimated reliability of the system in the next mission. This error is magnified when the resource limitation is more strict.

Chapter 5

Selective Maintenance for Multi-state Systems with Stochastic Dependent Components

In this chapter, we investigate the stochastic dependence (s-dependence) between multi-state components and its effects on the selective maintenance problem. S-dependence implies that the current health state of a component affects the state and degradation of other components. Since s-dependence relates to components' degradations, the system reliability needs to be evaluated for selective maintenance modelling. An approach based on stochastic process is proposed to evaluate the system reliability. We develop a cost-based selective maintenance model to maximize the total multi-state system profit, which includes the production gain and loss in the next mission as well as possible maintenance costs for the system. Versions of this chapter have been published as a conference paper [72] and a journal paper in IEEE Transactions on Reliability [73].

5.1 Introduction

In the literature of selective maintenance, the assumption of stochastic independence between components is frequently made. However, in a specific system design, the current health condition of a component may affect the performance of other components, and stochastic dependence exists in most real and complex systems. As stated in [74], the system reliability may be underestimated or overestimated when there is existence of s-dependence, and thus maintenance optimization for those systems needs to be thoroughly investigated. In the literature on selective maintenance, the selective maintenance model for systems with s-dependent components was studied in [31], [32], in which the components are binary and the failure of a component can cause other selected components to fail immediately. In these papers, the discussion on the effects of s-dependence on selective maintenance decision making was not investigated. In addition, the dependence relationship discussed in this study is a special type of s-dependence (type 1), which will be reviewed in the remaining part of this section.

In binary systems, s-dependence is often referred to as "failure dependence" or "failure interaction". In [75], failure interaction in multi-component systems is classified into two types, i.e. when a component fails: (i). it may cause immediate failure to the other component (type 1), or (*ii*). it may cause damage by increasing the deterioration rate of the affected component (type 2).

In type 1 s-dependence, a component may fail and cause the other component to fail with probability *p,* or leave no effect on the other component with probability $1-p$ Many papers have provided maintenance analysis of systems with type 1 s-dependence, including component replacement policy [76]–[78] and block replacement policy [79]. Regarding type 2 s-dependence, Barros et al. [80] studied a two-unit parallel system where the failure of one component modifies the deterioration rate of the other component. A preventive replacement model was given to find the time of maintenance, T^*, that minimizes the failure and maintenance costs of the system. Lai and Chen [81] studied a two-unit series system where the failure of unit 1 causes damage to unit 2 by increasing the failure rate of unit 2 by a certain amount, while the failure of unit 2 will cause immediate failure of the system. Sung et al. [82] extended the two-unit system in [81] by introducing an external shock affecting both components. Albin and Chao [83] studied an n-component system in series where the deterioration of the first component changes the failure rate of all *n-1* subsequent components. However, they did not discuss in details how the failure rate is affected. Rasmekonen and Parlikad [84] considered a system consisting of M parallel non-critical components feeding a critical component. Parallel elements are independent but the performance of the critical components decreases as a parallel component fails. Optimal preventive maintenance is determined to maximize the performance of the critical component.

Few efforts have been made on s-dependence existing in multi-state systems (MSS). Researchers so far have only focused on the reliability analysis of MSS with s-dependence. In [80] and [81], type 1 s-dependence is explored in MSS where the complete failure originating from a multi-state component immediately causes complete failures of other dependent components. In Levitin [87], the universal generating function approach is shown to be efficient to evaluate the reliability of MSS when the conditional probability distributions of the dependent components are explicitly known.

The discussion on s-dependence between multi-state components is still limited. Only type 1 s-dependence has been discussed for multi-state systems. In addition, existing maintenance models for binary systems with type 2 s-dependent components are restricted to 2-component systems or systems with two stages of dependent components. Therefore, the first objective of this chapter is to consider both types of s-dependence in an n-component multi-state series system. An approach based on the stochastic process will be used to evaluate the system reliability. Secondly, existing literature on selective maintenance has not taken both the costs and the profit associated with MSS performance rates into consideration. In this Chapter, both the production loss and gain corresponding to multiple working levels of the system and the maintenance costs are integrated into the selective maintenance cost model. To sum up, a cost-based selective maintenance model will be developed for multi-state systems with two types of s-dependence. The objective of the maintenance model is to maximize the total system profit considering resources and reliability constraints. The proposed selective maintenance model is expected to help the maintenance decision maker find the most economical maintenance strategy under reliability and resource constraints.

The remaining part of this chapter is organized as follows. Section 5.2 describes the MSS with two types of dependence. A method of reliability analysis for the system with s-dependent components is presented in Section 5.3. Selective maintenance policies and a cost-based maintenance optimization model are developed in Section 5.4. Illustrative examples, results and discussion will be presented in Section 5.5 and conclusion of this chapter is provided in Section 5.6.

5.2 Multi-state systems with s-dependent components

The multi-state system in this chapter consists of n multi-state components connected in series (Figure 5.1). Each multi-state component i can be in a state in a set of K+1 possible discrete states, $S =$ K The state k is associated with a corresponding working performance rate g G g g g . The sets of state S and performance rates G are assumed in ascending order, i.e. g g k l.

At time t, the state of component i, denoted by s t is a random variable, s t S K . The performance rate of component i is denoted by g t and the probability that component i is in state k at time t is defined as in Equation (5.1). The system performance rate, $G(t)$, is defined as the minimum performance rate of its components as in Equation (5.2). The system's state associated with performance rate $G(t)$ at time t is $\phi(t)$.

$$p \quad t \qquad g \; t \quad g \tag{5.1}$$

$$G \, t \quad Min \; g \;\; t \;\; g \;\; t \qquad g \;\; t \tag{5.2}$$

$$\phi(t \quad Min \; s \;\; t \;\; s \;\; t \qquad s \;\; t \tag{5.3}$$

The state of the system and its performance rate can be determined by the combination of its components' states from Equations (5.2) and (5.3). The state probability of the system is:

$$P \;\; t \qquad t \quad k \qquad G \, t \quad g \tag{5.4}$$

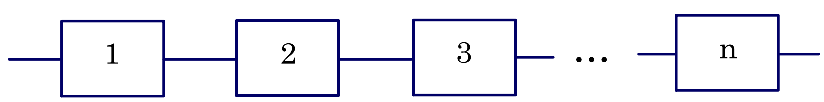


Figure 5.1 A series system with n components

In the presented MSS, there might be two types of dependence when a multi-state component changes its state as follows.

- Type 1 - Immediate failure dependence (IFD): when a component (influencing component) fails, it may cause immediate failures of some other affected components. For example, a component fails and creates fire that causes some nearby components to fail immediately; or an electrical component fails and causes a voltage spike that triggers the failure of some other components in the system that do not have protective device.

- Type 2 - Gradual degradation dependence (GDD): as the multistate component deteriorates with time and once it degrades to a lower working performance rate, it does not cause affected components to fail immediately but increases or decreases the degradation of the affected components. In our model, when a component degrades to a lower state, i.e., its output performance rate decreases, both the state and the degradation rate of the next component in the series system are affected. This dependence mechanism occurs when the components have direct connection to each other and the output of a component controls the load on the affected component. In a steam turbine generation unit, a system consisting of a water feed pump station and a steam turbine can serve as an illustrative example. This is a two-component series system by considering the water feed pump station as component 1 and the turbine as component 2. When the pump station works at its maximum performance rate of x (ton/h), the healthy turbine can generate its maximum power corresponding to x. When the pump's health condition deteriorates, the amount of water per unit time delivered to the next component (the turbine) decreases. This will result in a decrease in the degradation rate of the turbine due to the reduced load that the turbine needs to handle, and for the same reason, the output performance of the turbine decreases too. Thus, we assume that both the performance state and the degradation rate of a component may be affected by the degradation of the influencing component.

The two types of dependence in multi-state series systems are represented in Figure 5.2.

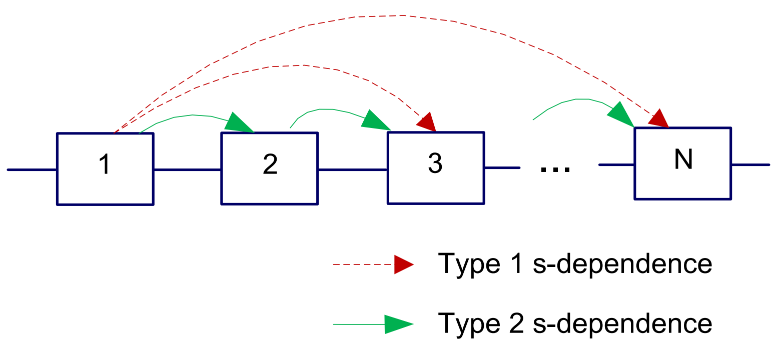


Figure 5.2 Two types of s-dependence in a multi-state series system

In MSS, it is seen that type 1 s-dependence can be defined in the same way as in binary systems, i.e. the failure originating from a multi-state component can immediately cause complete failures of the affected components. In Figure 5.2, the red-dash arrows represent type 1 s-dependence relationship, e.g. the failure of component 1 can cause immediate failures (IFs) to component 3 and component N only. As stated in [76], a component may cause type 1 s-dependence failure with probability p, or leave no effect on the other components with probability $1-p$ It is assumed that the influencing component can fail in two failure processes: one is its local degradation process that has no effect on other components, and the other failure process, called induced failure process, that causes immediate failures of affected components.

On the other hand, type 2 s-dependence in multi-state context describes the "degradation dependence" of a multi-state component on the degradation of other components in the system. The transition rate between states of an affected component varies depending on the instantaneous performance rate of the influencing component. In MSS, the performance rate is often defined as productivity or capacity. In series systems, the input of a component is the output performance rate of its predecessor and the output performance rate of a component is the input of its subsequent component. Thus, type 2 s-dependence has a chain effect (green-solid arrows in Figure 5.2). Thus, a component i, i=2, 3,…,n-1, can be both an influencing component to its subsequent components and an affected component from its predecessors.

In summary, the MSS with s-dependent components has the following characteristics.

- The system consists of n components connected in series.
- Some components in the system can fail in a failure mode that causes immediate failures of affected components (type 1 s-dependence). Once an affected component fails, it cannot cause IF to any other component in the system.
- Components may degrade to a lower working performance rate and it affects the degradation process of subsequent components by increasing or decreasing the degradation rate of the subsequent components (type 2 s-dependence).

- A component can cause both types of s-dependence and a component can be subjected to both types of s-dependence.

5.3 Multi-state system reliability analysis

5.3.1 Multi-state systems with independent components

As explained in Section 5.2, all components in the system degrade due to a local degradation process from the perfect functioning state to complete failure state. Without considering s-dependence, the probabilities associated with different states of a component i at time t due to its local degradation process can be represented by a set:

$$\mathbf{p} \quad t \;\; p \;\; t \;\; p \;\; t \quad p \;\; t \tag{5.5}$$

Figure 5.3 shows the transition diagram that represents the degradation of a multi-state component without considering s-dependence.

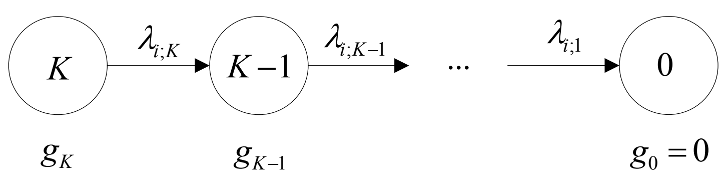


Figure 5.3 Transition diagram of a multi-state component without s-dependence

In Figure 5.3, component i has $K +$ possible states from perfect functioning (state K) to complete failure (state 0) with nominal transition rate from state k to state $k -$ of $\lambda_{i\,k}$. For simplicity, we assume that the sojourn time at each state of the component follows the Exponential distribution, and the component degrades gradually, i.e. if the component is currently in state k, it will degrade to state $k -$ before reaching a worse state $k -$. With these assumptions, the next transition of a component only depends on its current state, and the stochastic process governing the local degradation process is a Markov chain. The state probabilities can be determined by solving the Chapman-Kolmogorov (C-K) system of differential equations as follows.

$$\begin{cases} \frac{dp \quad t}{dt} = -\lambda_{i\,K} p \quad t \\ \dots \\ \frac{dp \quad t}{dt} = \lambda_{k} \; p \quad t \qquad p \quad t \\ \dots \\ \frac{dp \quad t}{dt} = \lambda \; p \quad t \end{cases} \tag{5.6}$$

The last equation based on transition diagram is $\frac{dp \quad t}{dt} = \lambda \; p \quad t$. However, it can be replaced by $\sum_{k}^{K} p \quad t$ since the sum of all state probabilities of a component is always equal to 1. Further discussions on modeling the degradation of multi-state independent components can be found in [55]. Regarding the method to solve the system of equations, there are several methods including numerical and analytical methods. Here, we solve the system of equations using a method described in [88] analytically.

5.3.2 Modeling two types of s-dependence of components

- *Type 1 s-dependence*: In the system, we denote I as the set of components that can fail and cause IFs of other components and A as the set of components affected by the induced failure of component i $i \in I$. When component i fails, it will either cause all the affected components in A to fail with probability p, or leave no effect on these components with probability 1-p. As a result, an influencing component i may fail following two failure processes: local failure and induced failure. The first failure process is the local degradation process that does not affect any other components as described in Section 5.3.1. The induced failure process causes the affected components to fail immediately. In previous studies, the probability p is assumed to be a predetermined value, but in this chapter, the probability of induced failure from component i, $p \quad t$, is defined as a function of time:

$$p \quad p \quad t \qquad \left\{ \int_0^t \qquad s\, ds \right\} \tag{5.7}$$

Where λ_i^{IF} is the induced failure rate of component i. In this case, a state $0'$ is added into the set of states of the influencing component to represent the induced failure state related to type 1 s-dependence. The performance rates of influencing and affected components at state $0'$ are g . The state probability vector of the influencing component i takes the following form:

$$\mathbf{p} \qquad t \;\; p \;\; t \;\; p \;\; t \qquad p \;\; t \tag{5.8}$$

In Equation (5.8), $p \quad t$ is the probability that the influencing component i fails and causes IF. Thus, $p \quad t \qquad p \quad t$ and $p \quad t \quad k \qquad\qquad K$ is the state probability of component i in the case that it does not fail due to the induced failure process. We assume that the local and induced failure processes are stochastically independent, then:

$$p \quad t \; = p \quad t \qquad \left\{ -\int_0^t \lambda \qquad s \, ds \right\} \tag{5.9}$$

where $p \quad t$ is the state probability obtained by its local degradation process which can be obtained from solving the system of equations (5.6).

The state probability distribution of an affected component $j \in A$ depends on the state of component i. We assume that an affected component j cannot cause immediate failure to any other component in the system. This also means that a component in the set of influencing components, I, cannot be affected by another component in the same set I.

If the induced failure does not occur, the state probability vector of component j is $\mathbf{p} \qquad t \;\; p \;\; t \qquad p \;\; t$, where $p \quad t \quad k = \qquad\qquad K$ is the state probability of component j obtained by solving (5.6). If the induced failure of component i occurs, the state of component j is 0, i.e. $\mathbf{p} \qquad\qquad ..,0$. Thus, we have the conditional probability distribution of component j as follows.

$$\mathbf{p} = \begin{cases} \{p(t), p(t), \ldots, p(t)\} & \text{if no IF occur} \\ \{1,0,\ldots,0\}, & \text{if IF from at least 1 influencing component occurs} \end{cases} \qquad (5.10)$$

There is another way to explain Equations (5.9) and (5.10) in conditional probabilities by defining a stochastic process $\{s(t), t\}$ with a state space of $\{0,1, \ldots, K\}$ as the stochastic process governing the local degradation process. Another stochastic process $\{s(t), t\}$ with a state space of $\{0',0,1,\ldots,K\}$ is defined to represent type 1 s-dependence of influencing component i as follows.

$$s(t) = \begin{cases} s(t) & t < T \\ 0' & \text{if } T \le t \end{cases}, \qquad (5.11)$$

where T_i is the time to IF of component i.

For $0 \le k \le K$,

$$\begin{cases} \Pr(s(t)=k \mid T>t) = \Pr(s(t)=k \mid T>t) = \Pr(s(t)=k) = p(t) \\ \Pr(s(t)=k \mid T \le t) = 0 \end{cases} \qquad (5.12)$$

Hence,

$$\Pr(s(t)=k) = \Pr(s(t)=k \mid T>t)\Pr(T>t) = p(t)\exp\left\{-\int_0^t \lambda(s)\,ds\right\} \qquad (5.13)$$

The above equation (5.13) is the same as equation (5.9).

When $k = 0'$,

$$
\begin{aligned}
&\Pr\ s\ \ t \qquad T\ \ t \qquad T\ \ t\ T\ \ t \\
&\Pr\ s\ \ t \qquad T\ \ t
\end{aligned}
\tag{5.14}
$$

Similarly, we can define a stochastic process $\{s\ \ t\ \ t$ with a state space $\{0,1,...,K\}$ to represent the type 1 s-dependence for an affected component j as follows.

$$
s\ \ t\ = \begin{cases} \ \ t \quad s\ \ t \neq \qquad i \in I \qquad j \in A \\ 0, \quad \text{if } s\ \ t\ = \qquad i \in I \qquad j \in A \end{cases}
\tag{5.15}
$$

The above Equation (5.15) is equivalent to Equation (5.10).

- *Type 2 s-dependence:* Type 2 s-dependence exists when components have direct connection to each other and the output of the influencing components controls the performance of the affected components. In series systems, a component i, $i = \quad n-$, can be both an influencing component to its subsequent component and an affected component from its predecessor. We assume that the degradation of component i, $i > 1$ depends on the output performance rate of its preceding component $i-$. When its predecessor $i-$ is at the perfect functioning state, i.e. $g\ \ t\ \ \ g$, component i degrades at its nominal transition rate, i.e. the transition rate of the local degradation process $\lambda_{i\ k}$. When component $i-$ degrades and $g_{-}\ \ t < g_{\ }$, type 2 s-dependence between component i and its immediate predecessor is described as follows.

- The performance rate dependence: The performance rate of component i is automatically adjusted to the output of performance rate supply to it, i.e. $g \; t \quad g \quad t \quad g$

- The degradation rate dependence: The transition rate between states of the affected component i, $\lambda_{i\,k}^{(d}$ also depends on $g \quad t$. We use $\varphi_i($ $)$ as a dependence function to describe type 2 s-dependence of component i on its immediate predecessor.

$$\lambda_{i\,k}^{(d} \; g \quad t \qquad g \quad t \qquad \begin{cases} g \quad t \\ g \end{cases} \qquad k \qquad K \tag{5.16}$$

In (5.16), $\varphi_i($ $)$ is a function of the instantaneous performance rate of the component $i \quad g \quad t$, and a dependent exponent, m_i, characterizing the degree of dependence between component i and its immediate predecessor. When $m_i > 0$, the reduced performance of the influencing component decreases the degradation process of the affected component, i.e. $\lambda_{i\,k}^{(d} \qquad {}_{i\,k}$; when $m_i < 0$, the reduced performance of the influencing component increases the degradation process of the affected component, i.e. $\lambda_{i\,k}^{(d} \qquad {}_{i\,k}$. When $m_i = 0$, we have $\lambda_{i\,k}^{(d} \qquad {}_{i\,k}$, i.e. the transition rate doesn't change but the performance of the affected component still depends on the performance of its preceding component.

A power law with m_i being an exponent is used to represent the degree of dependence of component i on its preceding component. In general, the transition rate of an affected component may also depend on state k. If this is the case, the component type and state effect can be characterized by an exponent $m_{i,k}$, where $m_{i,k}$ represents the dependence of component i at state k on its preceding component. We assume that the performance rate associated with each state of a component is uniformly distributed and the dependence effects are equal for each state, i.e. $m_{i,k}$ can be considered as m_i for all k. In addition, the same methods of reliability analysis and maintenance optimization in the next sections can be applied for either case where m_i or $m_{i,k}$ is used.

5.3.3 The system reliability

In this section, we analyze the system reliability when both types of dependence exist. To calculate the system reliability, the system state probabilities is first obtained using a Markov analysis for the system with type 2 *s*-dependence. Then, type 1 *s*-dependence is incorporated using the law of total probability based on the event that an immediate failure occurs.

5.3.3.1 Markov analysis for MSS with type 2 s-dependence

In Type 2 s-dependence, we see that the degradation of a component i, $i > 1$, depends not only on its current state but also on the state of its immediate predecessor. If we consider simultaneously the state of component i and the state of its immediate predecessor at the same time, the degradation of the subsystem of components i - 1 and i can be modeled as a Markov chain.

To illustrate the Type 2 s-dependence as a Markov chain, we take an example of 2 components connected in series, wherein each component's state space is $\{0,1,2,...,K$. Although the series system state can be represented as min s s K , we denote the system state as a pair of $(s$ s . Since the state of component 2 is always less than or equal to the state of component 1, the system state space is $\{(K\ K$ $K\ K$.,(1,0),(0,0) . With this notation, the system states denoted by (k, k-1) and (k-1, k-1) are considered to be different although the output performance of the system is identical. The first element in system state $(s$ s determines the state and the associated output performance rate of component 1. The second element in the pair $(s$ s depends on the value of the first element. However, the stochastic process utilizing the pair notation is a Markov chain since the next state of the system in pair notation $(s$ s only depends on its current state. The stochastic process with the local degradation process is $\{(s\ t\ s\ t$ with transition rates $(\lambda_{1,k}$ l . When type-2 dependence is introduced, the stochastic process is modified to $\{(s\ t\ s\ t$ with additional requirements of $s\ t$ $s\ t$ and the transition rates of $(\lambda_{1,k}$ $^{d}_{l}$ $k\ l$ K l k to represent type 2 s-dependence between the two components as described in Section 5.3.2. The state transition diagram for the group of two components is shown in Figure 5.4.

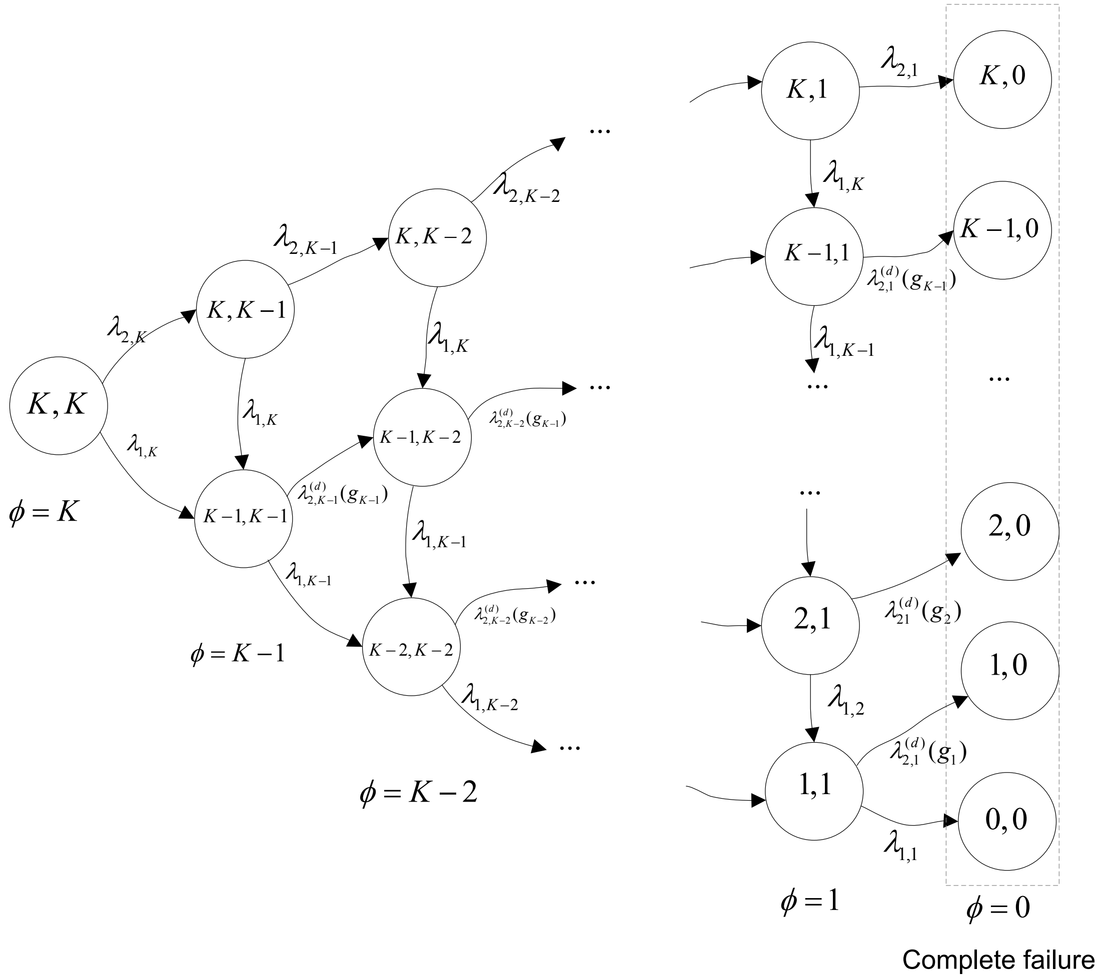


Figure 5.4 Transition diagram of two components with type 2 s-dependence

In Figure 5.4, type 2 s-dependence happens whenever the influencing component changes its state, which may result in state dependence and transition rate dependence. Transition rate dependence always happens when the influencing component changes its state, but state dependence occurs only if the transition of the influencing component causes its state to be less than the state of the affected component. For example, there are two transition paths from state $(K\ K$ to state $(K-\ \ K-$, which are $(K\ K$ $\rightarrow$

$(K\ K- \quad \to (K- \quad K- \quad$, and $(K\ K \to (K- \quad K- \quad$. In the first transition path, we can understand that there is no dependence in the first transition $(K\ K \to (K\ K- \quad$. In the system state $(K\ K- \quad$, the second component has its nominal transition rate $\lambda_{2,K}$. From this system state, when component 1 degrades from state K to $K-1$, i.e. the system state becomes $(K- \quad K- \quad$, the transition rate of component 2 is modified to $\lambda_{2,K}^{d} \quad g$ since the performance rate of component 1 has changed. Although state dependence does not take place, type 2 s-dependence still occurs in this transition in terms of transition rate dependence. In the second path, there is no transition from the state (K,K) to state $(K-1, K)$, because when component 1 enters state $K-1$ the state of component 2 is instantly adjusted to $K-1$ and its transition rate is also adjusted to $\lambda_{2,K}^{d} \quad g$ due to type 2 *s*-dependence. It is noted that state dependence only happens in the second path. This is because state dependence happens if the transition causes the state of the affected component to be greater than the state of the influencing component.

When any component degrades to state “0”, the system is in complete failure state. In Figure 5.4, we can combine all such states with at least one component in state “0” as the “*F*” state or complete failure state. “*F*” is an absorbing state during the mission, i.e. no transition can happen further when the system is failed. The C-K system of equations for type 2 s-dependence is written as in (5.17).

$$
\begin{cases}
\frac{dP \quad t}{dt} = -(\lambda_{1,K} \quad \quad _{K})P \quad t \\
\frac{dP \quad \quad t}{dt} = \lambda_{2,K} P \quad t \quad \quad \quad \quad P \quad t \\
\frac{dP \quad \quad \quad t}{dt} = \lambda_{1,K} P \quad t \quad \quad P \quad t \quad \quad \quad \quad g \quad P \quad t \\
\cdots \\
\frac{dP \quad t}{dt} = \quad _{,1} P \quad t \quad \quad g \quad P \quad t \quad . \quad (g \quad P \quad t \quad \quad P \quad t
\end{cases}
$$

(5.17)

In (5.17), the last equation can be replaced by $\sum_{k}^{K}\sum_{l}^{k} P \quad t \quad P \quad t$ since the sum of all system state probabilities is 1. Since the system will degrade gradually from a higher state to a lower state in the next mission, the state (s_1, s_2) is lexicographically ordered in decreasing numerical order. With this order of the system state, the system of C-K equations can be rewritten in matrix form as [88]:

$$\frac{d\mathbf{P}}{dt} - \mathbf{P} \tag{5.18}$$

In (5.17), there are a total of f equations; f is the number of state combinations of all components in the system. Then, $\mathbf{P}$ is a row vector, $1 \times f$, and Λ is the transition rate matrix, $f \times f$. For Equation (5.18), we can write matrix Λ as:

$$\Lambda = \begin{bmatrix} -(\lambda_{1,K} \quad K) & 0 & 0 & \cdots & 0 & 0 \\ \lambda_{2,K} & (\quad K \quad K) & 0 & \cdots & 0 & 0 \\ \lambda_{1,K} & K & (\quad K \quad {}^{d}_{K} \quad g & & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & {}^{d} \quad g & \end{bmatrix} \tag{5.19}$$

It is noted that the sum of all elements in a column of matrix Λ is always equal to zero. Given $\mathbf{P}$ as a row vector of the initial condition, we can solve the system of C-K equations and determine the system state probabilities. The solutions to the system of equations (5.18) take the following form [88].

$$\mathbf{P} \quad \mathbf{P} \quad V \tag{5.20}$$

where V is a matrix of eigenvectors of matrix Λ and D is a diagonal matrix with diagonal elements being eigenvalues of Λ.

When the system has more than 2 components, the components' states are grouped and denoted in a similar way. Each system state has to be denoted by N-tuple for an N-component series system. Markov properties and analysis for the system can be carried out similarly. The degradation of component i can be explicitly determined when the current states of the group of components $\{1,2,...,i$ are included in the system state. The next state of the system denoted by $\{s \;\; s \;\; s$ only depends on its current state. The state transition diagram and the system of C-K equations can be obtained similarly.

The state space of the Markov model depends on the number of components and the number of health states of each component. We will discuss the state space of the system model and illustrate the complexity of the system model in this Section

Consider a system of n multi-state components with type 2 s-dependence. Each component may be in K+1 possible states. We use $f \quad n \; K \geq$ to represent the number of states of such a system with n components and K is the common maximum state of each component.

- When $n =$, i.e. the system has only one component, and we have: $f \qquad K \qquad .$
- When $n =$, the state dependence requirement dictates that $s \geq s$ where s_1 is the state of component 1 and s_2 is the state of component 2. Considering s_1 may take integer values from 0 to K, inclusive and s_2 may take integer values from 0 to s_1, inclusive, the number of states of the system in $(s \quad s$ is then equal to

  $$f \quad f \quad f \qquad f \qquad f \qquad (K \quad \frac{K}{2} .$$

- When there are n components in the system, there is a cascading dependence requirement such that $K \geq s \geq s \geq \quad \geq s \geq$. Using similar reasoning, we can find the number of states of the system in $(s \quad s \qquad s$ as

  $$f \quad = \sum_{k=}^{K} f_{-} \quad = \prod_{i=}^{n} \left(\frac{K + i}{i} \right).$$

We can see that $f \quad = \prod_{i=}^{n} \left(\frac{K}{i} + \right) \gg \left(\quad \right)$ and $\frac{K}{n} + \; > \;$. Thus, the number of system states increases with the order of at least $\mathrm{O}\left(\left(\frac{K}{n} + \right) \right)$ when $n \to \infty$ and $n \to \infty$. This exponential increase in the number of system states limits the model for systems with large n and K values. In the future, more efficient methods for solving the system model need to be developed to handle the relationships between components in the proposed model.

5.3.3.2 Combining two types of s-dependence in system reliability analysis

When both types of dependence are considered, the stochastic process governing the degradation of each component cannot be defined individually since there is a chain effect in type 2 s-dependence from the first component to the last component in a series system. Thus, all components in the system are considered at the same time. The system state probabilities can be first obtained using the Markov analysis for the system with type 2 *s*-dependence as outlined in Section 5.3.3.1. Then, type 1 *s*-dependence is incorporated in the event that the system is in a failure state, which is caused when at least one immediate failure occurs.

A stochastic process for the system with n components considering both types of *s*-dependence is defined as follows. For a system of n components, we use $S \quad t = (s \quad t \; s \quad t \quad\quad s \quad t)$, which is an *n-tuple,* to represent the system state considering type 2 s-dependence. The superscript refers to type 2 s-dependence and the subscript is the component in the system. Each *n-tuple,* $S \quad t = s \quad t \; s \quad t \quad\quad s \quad t$, exactly defines the state of the system and $\{S \quad t \quad t$ is the stochastic process representing the system degradation due to type 2 s-dependence. Then, the stochastic process governing both types of s-dependence of the system is $\{S \quad t \quad t$, where:

$$S \quad t = \begin{cases} S \quad t \qquad s \quad t \neq \qquad \forall\ i \in I \\ 0, \quad \text{if } s \quad t = \qquad\qquad\qquad i \in I \end{cases} \tag{5.21}$$

In the system of equations (5.17), the state probabilities of the system are in the form of $P \quad t$. Given the initial state of each component in the system, the system of C-K equations can be solved using the method presented in [88] to obtain the state probabilities $P \quad t$. Denote $Q \quad t$ as the probability that the system is in state k at time t without considering IF. We can easily transfer the system state probabilities $P \quad t$ into $Q \quad t \quad k \qquad K$ by applying Equation (5.22).

$$Q \quad t = \sum_{\min s \ = k} P \qquad t \tag{5.22}$$

When IF from component i happens, both influencing and affected components are in failed state, and the system is also in failed state. In addition, if a component is in state 0, IF failure cannot happen to the system. Thus, the state probabilities of the system considering type 1 s-dependence can be represented as follows.

For k= 1,2,...,K,

$$P \ t \ = Q \ t \ \times \qquad\qquad = Q \ t \ \times \prod_{i \in I} p \quad t \ , \tag{5.23}$$

and

$$\begin{aligned} P \ t \ &= Q \ t \ \times \quad \text{here is no IF} \ + \Pr \ \text{at least one IF occurs} \\ &= Q \ t \ \times \prod_{i \in I} p \quad t \ + \left(\ - \prod_{i \in I} p \quad t \ \right) \end{aligned} \tag{5.24}$$

From (5.23) and (5.24), we can calculate the system state probabilities $P \ t$, for all k=0,1,2,...,K. The system reliability is the probability that the system can operate and satisfy a required demand D. Once the state probabilities $P \ t$ are determined, the system reliability can be obtained using a zero-one function, $\delta(G \ \ D$.

$$R \ t \ D \qquad G \ t \qquad D \qquad {}^{K}_{k} \ P \ t \quad g \ \ D \ , \tag{5.25}$$

where $\delta(g \ \ D \quad \begin{cases} 0 \ \ if \ g \quad\ D \\ 1 \ \ otherwise \end{cases}$.

5.4 Selective maintenance model

In our selective maintenance model, no repair action is allowed within a mission. During the next mission, if an IF happens or when a component degrades to a state that cannot satisfy the demand, the system is considered failed and the mission is failed. Thus, the maintenance decision maker must take the probability of successfully completing the next mission, i.e. the system reliability, into consideration prior to the start of the mission. The proposed maintenance model will optimize the maintenance decisions before a mission starts so that the system reliability is greater than or equal to a required threshold.

5.4.1 Maintenance actions on a multi-state component

In the selective maintenance problem, one needs to find the optimal combination of maintenance actions on the components when they arrive at the maintenance depot under the limited resources. Similar to Chapter 3, we denote Y and X as the states of component i at the time of entering and exiting the maintenance depot, respectively. It is assumed that the system is inspected at the time of entering the maintenance depot and the components' states are detected instantaneously, i.e. the state vector at the time of entering the maintenance depot is explicitly known. The maintenance actions do not worsen the condition of the component, i.e. $Y \leq X$. In general, multiple maintenance options are available on a multi-state component when it is in the maintenance depot as follows.

- Do-nothing (DN): the current health state of the component does not change at the maintenance depot, i.e. $X \quad Y$.
- Replacement: The component is renewed to the state "as good as new", i.e. state K.

- Imperfect maintenance (IM): $Y \quad X \quad K$.

In a selective maintenance modeling for the system consisting of multiple s-dependent components, it is necessary to consider the effect of s-dependence on the selection of maintenance activities. We take s-dependence between components into account by considering the following aspects when modeling the maintenance activities of components.

- At the maintenance depot, if the influencing component is in induced failure state, the maintenance action of its affected components is valid only when the influencing component is selected, i.e. when the affected component is selected in a maintenance strategy, the influencing component in type 1 s-dependence is also selected.
- Among maintenance actions for type 2 s-dependence components, the repair of the affected components cannot bring it to a state greater than the influencing component's state. Because of the state dependence between components, the repair action to bring the affected component to a state greater than the state of the influencing component requires more maintenance resources but does not improve the system reliability and stochastic degradation in the next mission in comparison with repairing the affected component to the state equal to the state of the influencing component.

5.4.2 MSS profit and cost evaluation

In this chapter, our aim is to build a cost-based selective maintenance model to identify the most economical maintenance strategy when s-dependence exists. To do so, we analyze the profit and costs for the system during the maintenance period and in the next operating mission, including:

- The system production gain and loss associated with its performance rate in the next mission.
- The maintenance cost in the maintenance depot.

5.4.2.1 The system production gain and loss

In MSS, the performance rate often represents productivity or capacity. Thus, the MSS production gain and loss are evaluated using the multiple output performance rates it can produce in the next mission. Let c_g and c_l be the average production gain and loss per unit of performance. It is assumed that c_g and c_l are given. This is a reasonable assumption since the values of c_g and c_l can be determined based on the sale price of a unit, labor costs, material costs and penalty per shortage unit in customer contracts in real operation context. The total expected system profit in the next mission is defined as the production gain, C_{PG}, minus the production loss, C_{PL}.

$$\text{Production Profit} = \text{Production gain} - \text{Production loss} = C_{PG} - C_{PL} \tag{5.26}$$

$$C \quad c_{g \; D} \quad g \; \frac{1}{\_} \int_{0} P \;\; t \, dt \qquad (5.27)$$

$$C \quad c_{g \; D} \quad D \quad g \; \frac{1}{\tau} \int_{0} P \;\; t \, dt \qquad (5.28)$$

where $P \;\; t$ is the state probability of the system at time t in the next mission, τ is the next mission duration, and D is the demand level of the system in the next mission.

In multi-state systems, each state k associates with a performance rate g_k. In many practical applications, g_k is related to the output capacity of the system and we can say that at state k, the system produces g_k units of performance rate. For example, a multi-state power generating unit can produce several performance rates of 100 MW, 50 MW, and 0 MW corresponding to state 2, 1, and 0 respectively. Then, one unit of performance rate is 1 MW.

In Equation (5.27), the production gain, C_{PG}, is calculated for each unit of performance rate that satisfies demand ($g \;\; g \quad D$). It is a product of the average production gain per unit of performance rate, c_g, the output performance rate of the system at state k, $g \;\; g \quad D$, and the expected probability that the system is in state k in the next mission, $\frac{1}{\tau} \int_{0} P \;\; t \, dt$. The production loss, C_{PL}, is calculated in a similar way but it is only counted per unit of unsupplied demand ($D \quad g \;\; g \quad D$.

5.4.2.2 Maintenance cost and time in the maintenance break

For each component i, an amount of $c\ Y\ X$ is required to repair it from state Y to state X in the maintenance break. If no maintenance action is performed on component i, i.e. $X\quad Y$, $c\ Y\ Y$ When $Y\quad X\quad K$, the maintenance cost of maintenance actions on component i: $c\ Y\ X\quad c\ Y\ K$. The maintenance cost of a component i can be arranged in matrix form as in Chapter 3.

$$C = \begin{bmatrix} 0 & c & & c\ K \\ 0 & 0 & \dots & c\ K \\ & & & \\ 0 & 0 & \dots & c\ K - \ K \\ 0 & 0 & \dots & 0 \end{bmatrix} \quad i = \qquad n \tag{5.29}$$

The total system maintenance cost is the summation of maintenance costs of all components.

$$C\ X = \sum_{i=}^{n} c\ Y\ X \tag{5.30}$$

Similarly, we define $t\ Y\ X$ as the time of repairing component i from state Y to state X.

$$T = \begin{bmatrix} 0 & t & & t\ K \\ 0 & 0 & \dots & t\ K \\ & & & \\ 0 & 0 & \dots & t\ K - \ K \\ 0 & 0 & \dots & 0 \end{bmatrix} \quad i = \qquad n \tag{5.31}$$

When the components in the system are repaired one by one, the total system maintenance time can be obtained by taking the sum of all components' repair times.

$$T\ X\ = \sum_{i=}^{n} t\ \ Y\ \ X \tag{5.32}$$

In summary, the total system profit (cost) function is calculated from the start of the maintenance break to the end of the next mission as presented in (5.33).

$$C\quad C\quad\ C\quad C \tag{5.33}$$

5.4.3 Selective maintenance modeling and solution methodology

The objective of the selective maintenance problem is to find which components to be maintained and how to maintain them during the maintenance break given their conditions at the time of entering the maintenance depot. Thus, a solution of the selective maintenance is a combination of the states of all components at the time of exiting the maintenance depot. It is in vector form $X = \ X\ \ X\qquad X$.

The selective maintenance problem can be formulated as a non-linear integer programming problem as follows.

$$\textit{Maximize}\qquad C\quad C\quad\ C\quad C \tag{5.34}$$

$$\textit{Subject to}\qquad R\ \ X\quad D\quad R \tag{5.35}$$

$$C \quad X \qquad C \tag{5.36}$$

$$T\,X \qquad T \tag{5.37}$$

$$if\, X \; > \quad then\; X \; > \quad \forall j \in A \tag{5.38}$$

$$X \qquad X \qquad\qquad i \qquad\qquad N \tag{5.39}$$

$$Y \quad X \quad K\;X\; is\; integer\;\; \forall i \qquad\qquad N \tag{5.40}$$

The maintenance manager has to determine maintenance activities associated with each component to achieve the maintenance objective of maximizing the total profit of the system during the whole next mission under limitation of time and budget for maintenance as well as maintaining the requirement of the system reliability in the next operating mission. The objective function is to maximize the total multi-state system profit, which is equal to the production gain in the next mission minus the production loss and minus the maintenance cost. Each element in the cost function, C, is calculated as in Section 5.4.2.

Constraint (5.35) indicates that the required system reliability in the next operating mission at demand D must be greater than or equal to a specified value R_0. $R \;\; X \quad D$ is calculated using the approach discussed in Section 5.3.3. Constraints (5.36) and (5.37) are two resource constraints, which indicate that the resources for maintenance must be within the available budget and time allotted within the break between the last mission and the upcoming mission. Constraint (5.38) describes type 1 s-dependence in the system:

an affected component j belongs to the set of components affected by component i, A then component i cannot be in failure state if the state of component $j > 0$. Constraint (5.39) describes type 2 s-dependence in the system: the state of a component is less than or equal to the states of its predecessors. The decision variables, X_i, are components' states at the beginning of the next mission. Since the maintenance activities do not worsen the state of the components, X_i must be an integer value between Y_i and the maximum state K for all $i = 1,2,...,n$.

Similar to Chapters 3 and 4, we use genetic algorithm (GA) to deal with the non-linear selective maintenance optimization model.

5.5 Examples, results, and discussions

5.5.1 Example 5.1

This example is for illustrating the reliability analysis of MSS with type 2 s-dependent components. Consider a MSS consisting of 3 multi-state components connected in series with type 2 s-dependence as presented in Figure 5.5. Each component can be in one of four possible states, i.e. S={0,1,2,3} with corresponding performance rates in a set G={0,1,2,3}. The nominal state transition rates are provided in Table 5.1 and dependent exponents m m .

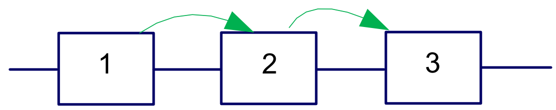


Figure 5.5 Multi-state system in Example 5.1

All components are as good as new at time $t = 0$, find the probability that the system can deliver at least at demand $D = 2$ in a mission time $t = 1$ year.

Table 5.1 Components' transition rates in Example 5.1

Component (i)	*Transition rate (year^{-1})*		
	k=3	*k=2*	*k=1*
1	0.12	0.18	0.15
2	0.09	0.15	0.05
3	0.17	0.12	0.2

Denote the system's state by a set of $(s \;\; s \;\; s$ with s_i being the state of component i, $i =$ 1, 2, 3. The state diagram is constructed as in Figure 5.6.

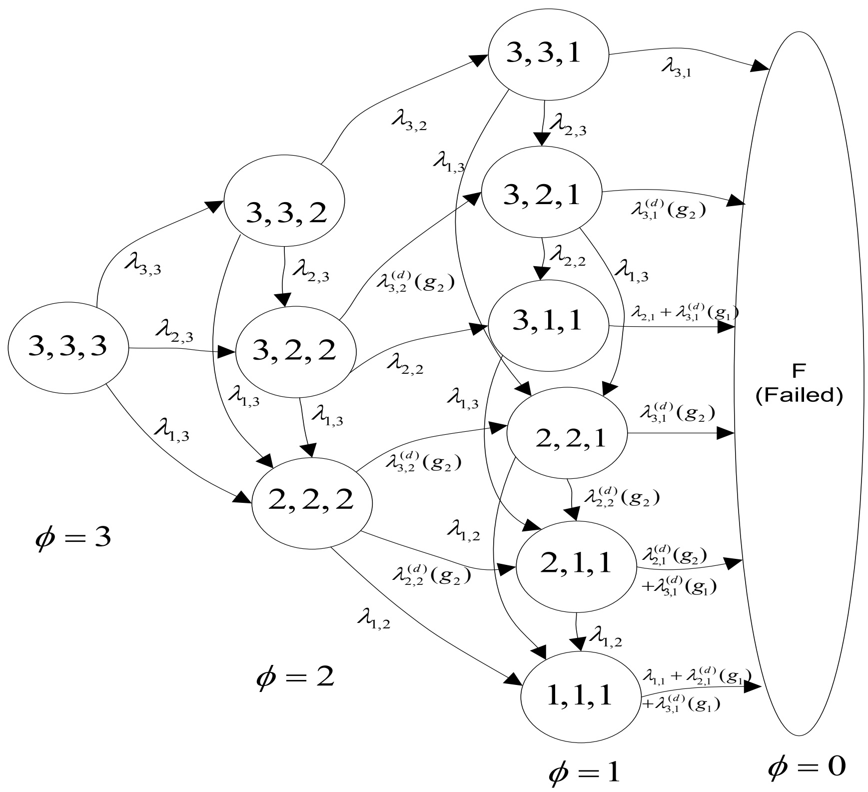


Figure 5.6 Transition diagram of a 3-component system with type 2 s-dependence

In Figure 5.6, state "F" or "Failed" indicates that at least one component in the system is in state 0, i.e. complete failure state. Once a component is in state 0, no other components can degrade further in the mission. The C-K system of equations can be written as follows.

$$\begin{cases} \frac{dP \quad t}{dt} = -(\lambda_{1,3} \quad \lambda_{2,3} \quad \lambda_{3,3}) P \quad t \\ \frac{dP \quad t}{dt} = \lambda_{3,3} P \quad t \qquad P \quad t \\ \frac{dP \quad t}{dt} = \lambda_{2,3} P \quad t \qquad P \quad t \qquad g \quad P \quad t \\ \cdots \\ \frac{dP \quad t}{dt} = \lambda_{,1} P \quad t \qquad g \quad P_{1}(t \qquad g \qquad g \quad P \quad t \end{cases} \tag{5.41}$$

In Example 5.1, the size of matrix Λ is 11 11. The C-K system of equations can be solved with initial condition vector $\mathbf{P}$ $0,0,0,0,0,0,0,0,0,0$ using the method presented in Section 5.3.3. With t=1, we get:

$\mathbf{P}$ 39 0.1192 0.0678 0.0940 0.0070 0.0029 0.0053 0.0039 0.0050 0.0090 0.0020

$\Rightarrow P \quad P$ 5839

$P \quad P \quad P \quad P$ 192 0.0678 0.0940 0.2810

P 70 0.0029 0.0053 0.0039 0.0050 0.0090 0.0331

P 20

Hence, the system reliability at D= 2 is:

$$\Rightarrow R \quad D \quad \sum_{k}^{K} P \quad g \quad D \qquad 839 \quad 0.2810 \quad 0.9649$$

5.5.2 Example 5.2

Consider a MSS consisting of 5 multi-state components in series with their dependence as presented in Figure 5.7. Each component can be in one of 4 possible states S={0,1,2,3} with corresponding performance rates in a set G={0,1,2,3}. The nominal state transition rates and dependence exponents are provided in Table 5.2.

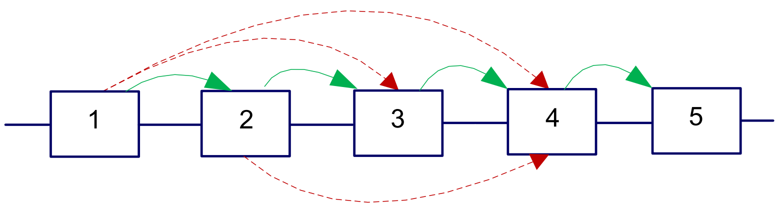


Figure 5.7 MSS in Example 5.2

Table 5.2 Components' characteristics in Example 5.2

Component	*Transition rate (year^{-1})*			m_i
(i)	k=3	k=2	k=1	
1	0.08	0.15	0.1	-
2	0.06	0.11	0.05	1
3	0.14	0.09	0.2	0.5
4	0.18	0.1	0.15	0.5
5	0.11	0.08	0.16	1

Components 1 and 2 may fail due to induced failure processes with $\lambda_1^{IF}(t$ and $\lambda_2^{IF}(t$ t. In the set $I =$, component 1 can cause components 3 and 4 to fail immediately, and component 2 can cause component 4 to fail immediately, i.e. A = A = .

Upon arriving at the maintenance depot, the vector of components' states is $Y \ \left[\ \ ,1,1,0\right]$. The repair time (in days) and cost (in $1,000) matrices of each component are given as follows.

$$C \begin{bmatrix} 0 & 4.5 & 9 & 14.5 \\ 0 & 0 & 4 & 12 \\ 0 & 0 & 0 & 9.5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C \begin{bmatrix} 0 & 3 & 8.5 & 16 \\ 0 & 0 & 6 & 12 \\ 0 & 0 & 0 & 4.5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C \begin{bmatrix} 0 & 2.5 & 6 & 10 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & 3.5 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$C \begin{bmatrix} 0 & 2.5 & 4.5 & 9 \\ 0 & 0 & 3 & 6.8 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C \begin{bmatrix} 0 & 3.5 & 6 & 10 \\ 0 & 0 & 3 & 5.5 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T \begin{bmatrix} 0 & 1 & 2.5 & 3.5 \\ 0 & 0 & 1.5 & 2 \\ 0 & 0 & 0 & 1.5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad T \begin{bmatrix} 0 & 1.5 & 2 & 5 \\ 0 & 0 & 1.5 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad T \begin{bmatrix} 0 & 1 & 2.5 & 5 \\ 0 & 0 & 2 & 3.5 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$T \begin{bmatrix} 0 & 1 & 3 & 4.5 \\ 0 & 0 & 1.5 & 3 \\ 0 & 0 & 0 & 1.5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad T \begin{bmatrix} 0 & 1.5 & 2 & 3 \\ 0 & 0 & 1 & 2.5 \\ 0 & 0 & 0 & 1.5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The maintenance manager has to find selective maintenance actions to be performed on the system under the requirement and resource availabilities as given in Table 5.3.

The Markov analysis for the system is performed by denoting the state of the system by

$(s \;\; s \;\; s \;\; s \;\; s$. We enumerate all states of the system including (3,3,3,3,3), (3,3,3,3,2), (3,3,3,2,2), (3,3,2,2,2), (3,2,2,2,2), (2,2,2,2,2), (3,3,3,3,1), (3,3,3,2,1), (3,3,2,2,1), (3,2,2,2,1), (2,2,2,2,1), (3,3,3,1,1), (3,3,2,1,1), (3,3,1,1,1), (3,2,2,1,1), (3,2,1,1,1), (3,1,1,1,1), (2,2,2,1,1), (2,2,1,1,1), (2,1,1,1,1), (1,1,1,1,1), and state F denoting the complete failure state of the system. The reliability analysis is done in the same way as in Example 5.1.

Table 5.3 The system information and resource availabilities

c_g *($10,000)*	c_l *($10,000)*	R_0	D	τ *(years)*	C_0 *($1,000)*	T_0 *(days)*
20	50	0.85	2	0.5	40	12

Using the given input data, we code and solve the selective maintenance problem for the MSS in MatlabR2012a. Two selective maintenance scenarios for the system with s-dependence components are compared to the scenario where s-dependence is ignored, as presented in Table 5.4.

Table 5.4 Selective maintenance results in Example 5.2

Maintenance scenario for the system with	X_1	X_2	X_3	X_4	X_5	$C \quad X$ *($1,000)*	$T \; X$ *(days)*	$R \; D$	$C \; X$ *($1000)*
S-dependent components	3	3	2	2	2	38.5	12	0.85845	289.1
Independent components	3	2	3	3	2	39	12	0.90474	317.8

In the first maintenance scenario, the system is subjected to both types of s-dependence.

The best maintenance strategy is to repair component 1 and 2 to their perfect functioning state and three other components to state 2. The expected profit for this maintenance strategy in the next mission considering the average production gain, loss and the cost of maintenance is $289.1K.

We also investigate the effect of dependence in the system by solving the selective maintenance problem for the same system and assuming that the components degrade independently. The system reliability and cost elements can be evaluated using the same method but the induced failure rate and dependence exponents are all set to zero; constraints (5.38) and (5.39) are removed from the optimization model. The obtained result indicates that the best maintenance strategy is to repair components 1, 3, and 4 to their perfect condition and components 2 and 5 to state 2. It can be seen that the resources used in the maintenance break for this strategy are not much different from the strategies for dependent systems. However, the system reliability when components are independent is much higher, at 0.90474, i.e. 5% larger; and the total profit is also approximately 10% larger, at $317.8K, in comparison with the independent case. Thus, we can conclude that ignoring s-dependence leads to another selective maintenance decision, and this may overestimate the system reliability and profit.

In order to investigate the differences between two maintenance scenarios in Table 5.4 in terms of cost and profit results, we examine the elements in the cost function as in Equation (5.30) for each optimal strategy. Figure 5.8 shows the profit and cost elements of interest.

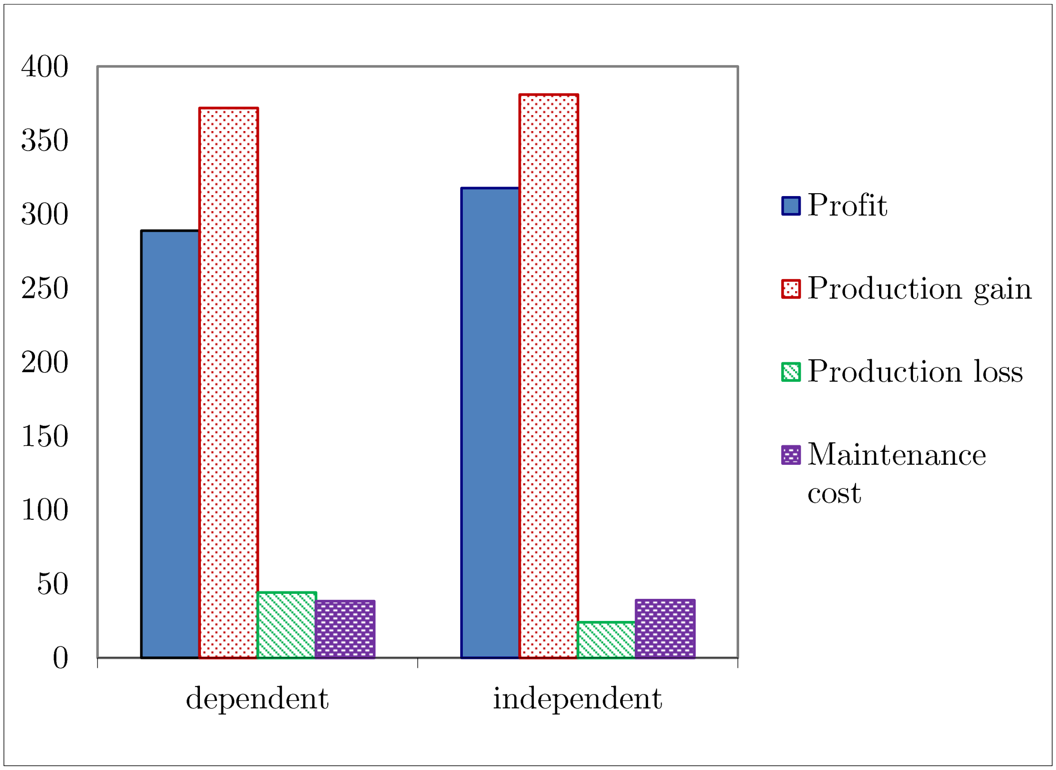


Figure 5.8 MSS profit and cost analysis

In the two selective maintenance strategies, the costs for maintenance in the break when components are dependent and when they are independent are not much different. However, when components are dependent, the production loss is much greater (~$20K) and the production gain is smaller. Thus, the total system profit is significantly different for the two cases. We can conclude that the two types of s-dependence have negative effects on the reliability and performance of the system. It is reasonable since the system reliability when components are independent is greater than that when components are s-dependent. When the probability of the system being in high states is bigger, it can result in a larger total profit. This result implies that ignoring s-dependence may lead to an optimistic estimation of the system profit.

5.6 Concluding remarks

This chapter studies the selective maintenance problem for multi-state systems with multi-state s-dependent components. The two types of dependence relationships between components are discussed in multi-state context. Firstly, the induced failure of a multistate component can immediately cause complete failures of some other components. Secondly, as components in the system deteriorate, the degradation of a component reduces the capacity flowing to the subsequent components, thus affecting the degradation of these components. Both the costs and the profit associated with the MSS performance rates and maintenance actions of s-dependent components are taken into consideration. A selective maintenance model is proposed to maximize the total profit of the multistate system subjected to reliability and resources constraints.

The examples and results imply that s-dependence has a significant effect on the MSS reliability. Also, ignoring s-dependence may lead to a wrong determination of selective maintenance plans for the system and overestimation of the system performance. The selective maintenance model can help maintenance managers make the right decisions to maintain the system with available resources and under a system reliability requirement.

Chapter 6

Optimal Selective Maintenance for Multi-state Systems in Dynamic Loading Conditions

This chapter studies the selective maintenance problem for multi-state systems working in dynamic loading conditions. In the next mission, the load can vary with time and the exact load on a component is not known with certainty. The degradation of a component is affected by the instant load applied on it and similar to Chapter 5, the system reliability evaluation is needed for selective maintenance modeling. We propose a load-dependent degradation model for multi-state components operating in dynamic loading conditions. This model is inspired by the load-sharing model where many components share a common workload and the failure rate of a component depends on the state of other components. A Monte-Carlo simulation method is used to model the multi-state component's degradation and to evaluate the system reliability. A selective maintenance model is provided to maximize the expected system reliability in the next mission within available resources. Materials in this chapter have been published in a conference paper [89] and submitted to Reliability Engineering and System Safety [90] for possible publication.

6.1 Introduction

In traditional reliability and maintenance models, the assumption that a system and its components operate under a normal and stable condition is usually made. In practice, the system and its components operate in variable operating conditions, where the load and environment conditions vary with time, i.e. time-varying load. For example, the load on a power generation unit often varies depending on the time of the year and the time in a day. It can be higher in a hot day compared to a cool day, and it can be very high in the middle of the day and very low at midnight. The health of industrial devices degrades faster under a heavier loading condition and in a more severe environment.

The condition-dependent failure rate was first presented by Cox [91] in analysis of survival data in biostatistics. It was later brought to reliability and maintainability of a component in [53], [92]–[96]. When considering the time-varying load and its effect on component's degradation, two classes of variable loading conditions can be encountered. First, the load varies with time and the loading profile is explicitly known, i.e. deterministic load. Second, the load is totally dynamic, i.e. it varies with time and the loading profile is not explicitly known. The Weibull Cumulative Damage Model [92], the

Proportional Hazard Model (PHM) [93], and the Accelerated Failure Time Model (AFTM) [53] has been used to model the load-dependent degradation of a binary-state component. While deterministic load was implied in [92], [93], dynamic loading was discussed in [94]–[96] to model the dynamic of operating conditions. In dynamic loading, stochastic processes were used to model the change between different load levels. Preventive maintenance was investigated in [94], [96], where two levels of operating environment, i.e. normal and severe conditions, were considered. One state represents the normal condition, and the other represents the severe condition. The hazard rate function jumps when the environment switches from one state to the other.

Selective maintenance has been a significant subject of interest among many researchers recently. However, the majority of the papers on selective maintenance ignore the effect of loading conditions on the degradation of components. Chen et al [97] performed a preliminary work on the deterministic load distribution and selective maintenance of multi-component systems. They jointly optimized the load to distribute and selective maintenance action to perform on each component in the system. The load on each component can be controlled and the component is assigned a fixed load during the next operating mission. In practice, load on a component may change during the mission due to the change of operating demand instead of being controlled.

One of the weaknesses of the existing models on maintenance of systems in dynamic operating conditions is that the condition is often divided into several discrete levels for easy modeling of the condition-dependent failure rate. Generally, the loading condition changes several times in an operating mission. In addition, the deterministic load does not accommodate the uncertainty of the loading conditions. For example, in many cases, we know the general load trend but not the exact load in the next operating mission. Another aspect is that the current modeling of load-dependent failure rate is limited to a binary-state component. A load-dependent degradation model for multi-state components in variable operating conditions with uncertainty is needed for MSS reliability analysis and maintenance decision making.

In this chapter, we will study the selective maintenance problem for a multi-state series system, which is expected to complete a mission of duration τ in dynamic loading conditions. The future loading conditions in the next mission are modelled by a known baseload function and a random variation following the Normal distribution. In the mission, the component's degradation depends on its current state and the load applied on it. A load-dependent degradation model is proposed for multi-state components working in such conditions. The analysis of components' degradations and the system reliability estimation are discussed. The final objective is to determine the best selective maintenance strategy to maximize the expected system reliability in the next mission within available maintenance resources.

The remaining part of this chapter is organized as follows. Section 6.2 describes the system model and the dynamic loading conditions. Section 6.3 proposes a load-dependent degradation model for multi-state components in such loading conditions. This model is inspired by the degradation of components in k-out-of-n load-sharing systems. A Monte-Carlo simulation-based method is presented in this section to model the component degradation and to evaluate the system reliability. A selective maintenance model to maximize the expected system reliability in the next mission is presented in Section 6.4. Section 6.5 consists of an illustrative example and discussions. Concluding remarks for this chapter are given in Section 6.6.

6.2 The system model and the dynamic loading conditions

6.2.1 Multi-state system model

The multi-state system in this chapter consists of n multi-state components connected in series as in Figure 6.1. Each multi-state component i, i=1,2,...,n can work in $K +$ possible states of $K \ K - \quad ,1,0$, where $K \ K - \quad ,1$ are operating states and 0 is a failure state.

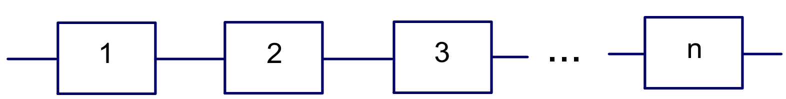


Figure 6.1 A series system

In this chapter, the multi-state components are assumed to degrade gradually, i.e. a component, currently at state k, $k \geq 2$, will degrade to state $k -$ before degrading further to state k . In the next mission, the reliability of component i is defined as the probability that the component successfully completes the next mission, i.e. its state at the end of the mission is greater than or equal to a predefine minimum state level d_i, $1 \le d \le K$ The multi-state series system reliability is the probability that all of its components successfully complete the next mission. Detailed explanations on the multi-state component's degradation and system reliability analysis will be presented in Section 6.3.

6.2.2 Dynamic loading conditions

The multi-state series system is required to work in a mission of duration τ in dynamic loading conditions. In this chapter, we only know the general load trend but not the exact loading profile. The load on the system, $L(t)$, comprises of a base-load profile $L_0(t)$ and a random variation ε_L. The base-load profile is a continuous function of time, which represents the expected load trend in the next mission. The random variation follows the Normal distribution with mean of 0 and standard deviation of σ_L, i.e. $\varepsilon_L \sim$, to reflect the uncertainty associated with the future loading conditions.

$$L\ t\ = L\ \ t\ + \varepsilon \tag{6.1}$$

An example of the variable loading conditions is given in Figure 6.2.

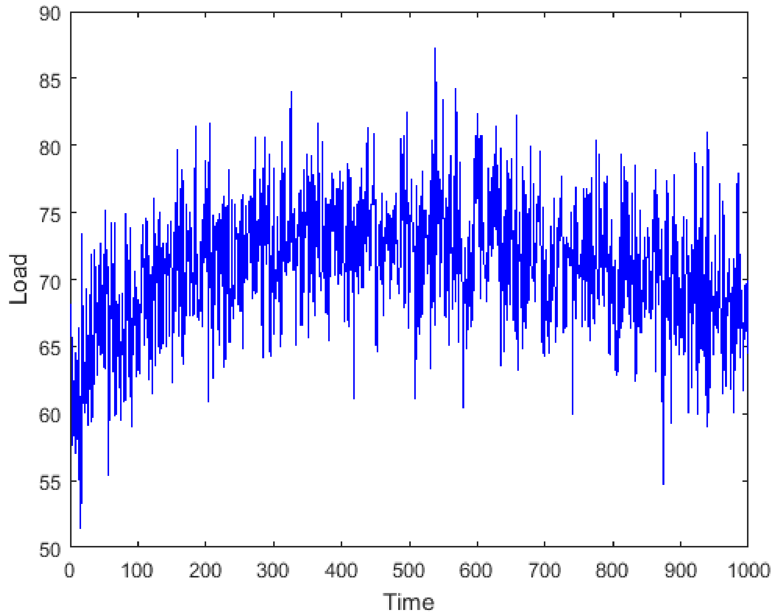
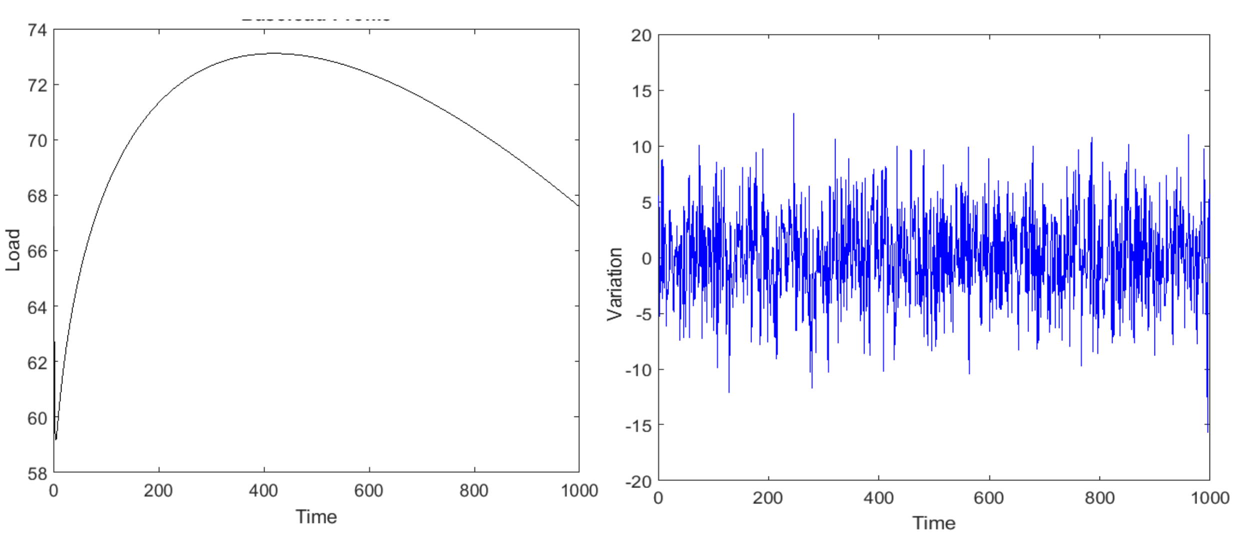


Figure 6.2 A loading condition example

Figure 6.3 shows an example of $L_0(t)$ and ε_L, which generate the variable loading conditions in Figure 6.2.



(a) *(b)*

Figure 6.3 Base-load profile (a) and random variation (b)

In such load conditions, a component's degradation rate at time t depends on the load applied on it. In addition, the multi-state component degrades in the next operating mission and its degradation rate depends on the component's current operating state. The dynamic behavior of the varying loads on the component's degradation and the state degradation of the component itself bring great challenges in reliability and maintenance modeling of such systems. We propose a load-dependent degradation rate model for a multi-state element in the system. Details on the load-dependent degradation rate of a multi-state component are presented in the next section.

6.3 System reliability analysis

6.3.1 Multi-state component degradation

Considering component i with $K +$ possible states. The multi-state degradation model for this component is shown in Figure 6.4.

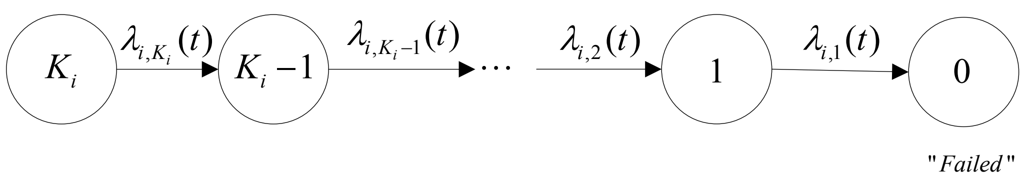


Figure 6.4 Multi-state component degradation

In the component degradation model in Figure 6.4, the component degrades gradually from state K_i to state 0. As mentioned in Section 6.2.1, K K_{-} are all operating states and 0 is the failure state. We denote λ_i $'t$, $s =$ K as the state transition rate or the degradation rate of component i at time t from state s_i to $s_i - 1$. The load and state dependent degradation rate of component i, $\lambda_{i,s_i}(t)$, is proposed as in Equation (6.2).

$$\lambda_{i,s_i}(t) = s_i h_i(t) \exp(\theta_i L(t)/s_i) = s_i h_i(t) \exp\left(\theta_i \frac{L(t)}{s_i}\right) \tag{6.2}$$

In Equation (6.2), $L(t)$ is the load applied on the system at time t, θ_i is a positive exponent, and $h_i(t)$ is the baseline hazard function of component i. The proposed load-dependent degradation model implies that the degradation rate of a multi-state component at time t from its current operating state s_i to the next degraded state $s_i - 1$, $s_i = 1, 2, \ldots, K_i$, depends on both its current state and the load applied on it. The rationale of this model, inspired by a k-out-of-n load-sharing system, is to be presented in Section 6.3.2.

This model is an extension of the proportional hazard model (PHM) [98], which was used to incorporate the effects of environmental covariates on the failure rate of a binary component. The PHM represents the effects of the environmental covariates by an exponential term as in Equation (6.3).

$$h(t; Z(t)) = h_0(t) \exp(\theta Z(t)), \tag{6.3}$$

where $Z(t)$ is a vector of time dependent covariates, and θ is an exponent vector representing the effect of the covariates. In our load-dependent degradation model, we have considered the load as a single covariate. We also assume that the degradation of a component in variable loads follows a model with the exponential based-line hazard function. Equation (6.2) can be rewritten as:

$$\lambda_{i\,s}\,'t \;= s\,\lambda \qquad \left(\theta_i \frac{L\;t}{s}\right), \tag{6.4}$$

where λ_i is a constant baseline hazard rate.

In order to develop a selective maintenance model for the system, we need to estimate the system reliability in the next mission with time duration τ and dynamic loading conditions. The reliability of component i is defined as the probability that the component successfully complete the next mission at a predefined minimum state level d_i, $1 \le d \;\le K$ or above. For series system, the system reliability is the probability that all the components in the system successfully complete the next mission at level d_i or above. The reliability of component i and the reliability of the system relates to the component's state probabilities as shown in Equations (6.5) and (6.6) respectively.

$$R\;\;t \;= \qquad s\;\;t\;\;\ge d \;\; = \sum_{k=d}^{K} \qquad s\;\;t\;\;= k \tag{6.5}$$

$$R\;\;t\;\;= \prod_{i=}^{n} R\;\;t\;\;= \prod_{i=}^{n} \qquad s\;\;t\;\;\ge d \;\;= \prod_{i=}^{n} \sum_{=} \qquad s\;\;t\;\;= k \tag{6.6}$$

Here, we assume that the hazard rate of a component varies stochastically with the load and its current health state. During the operation of the system, the state of component *i*, $s_i(t)$, varies stochastically with time. In a stable and certain loading condition, one can develop a system of differential equations as in [6] to find the state probabilities for component *i*, $\Pr(s_i(t) = k)$. However, the future loading condition varies stochastically with time and the exact loading condition on the system is not known with certainty. The load-dependent degradation rate $\lambda_i'(t)$ is a function of two random variables $L(t)$ and $s_i(t)$, and the method in [6] is not applicable to determine the state probabilities. Thus, we use the Monte-Carlo simulation method to model the degradation of multi-state components and evaluate the system reliability in variable loading conditions with uncertainty. The description of the simulation method will be presented in Section 6.3.3.

6.3.2 Component degradation and failure of a *k*-out-of-*n* load-sharing system

This section explains the rationale of the proposed load-dependent degradation model for a single multi-state component in the targeted series system. The load-dependent degradation model is inspired by a case where many binary components share a common workload. The failure of a component results in an increased workload on the remaining working components [40], [99]–[101]. This section provides the similarities of a multi-state component degradation model in Equation (6.2) and the degradation of a

k-out-of-n load-sharing system.

Consider a system with K identical components sharing a common workload. The system is considered as functioning if there are at least d functioning components, (d-out-of-K system). If each component in the system can be either in working or failure states, we define the system's state $s(t)$ as an indicator of the number of surviving components. In load-sharing systems, if a component fails, the other working components will carry the system's task with an increase in the shared load until all components have failed. There will be $K+$ possible states of $K\ K-\quad ,1,0$, and all states $K\ K-\quad d+\quad d$ are functioning states. The d-out-of-K load-sharing system reliability can be written as in Equation (6.7), which is very similar to the reliability of a multi-state component in Equation (6.5)

$$R\ t \qquad s\ t \quad d \quad \sum_{k\ d}^{K} \quad s\ t \quad k \tag{6.7}$$

A variable load $L(t)$ is applied on the system. When the load is equally distributed among surviving components, the load applied on a surviving component $j\ j=\qquad K$ is a function of the system load, $L(t)$, divided by the total number of surviving components, $s(t)$.

$$L_j\ t = \frac{T\ t}{s\ t} \quad s\ t \in \qquad K \tag{6.8}$$

If the hazard function of a binary component j follows a PHM model with the component based-line hazard rate h_0, and exponent parameter *6*, it can be written as in Equation (6.9).

$$\lambda_j(t = h\ t \qquad \theta L\ t = h\ t \left(\theta \frac{L\ t}{s\ t}\right) \tag{6.9}$$

Since there are $s(t)$ surviving components, the d-out-of-K system is a multi-state element with the degradation diagram similar to Figure 6.4. The degradation rate of the load-sharing system from state s to state s is $s \times \lambda\ t$, which is equivalent to the load-dependent degradation model for a multi-state component proposed in Equations (6.2) and (6.4).

6.3.3 Monte-Carlo simulation method for system reliability estimation

Monte-Carlo Simulation (MCS) method is a powerful tool for analyzing many systems in different areas. The MCS method is based on the generation of random numbers that is repeated many times in a computer model. Afterwards, the occurrence number of a specific condition of interest is counted [102], [103].

In this chapter, MCS is used to generate variable loading conditions which were discussed in Section 6.2. We generate N_L loading profiles. With each generated loading profile, the system reliability is estimated by conducting a series of experiments for its subsystems. We first define $p\ \Delta t\ s$ as the probability that the transition of a subsystem i from the current state s_i to state $s -$ does not occur in the next small operating time interval from t to $t + \Delta t$.

$$p \quad \Delta t \; s \quad = \quad x \;\; t + \Delta t \; = s \quad x \;\; t \; = s \quad = \quad \left(- \int_{t}^{t + \Delta t} \lambda \quad t \right) \tag{6.10}$$

where $x \;\; t$ is the state of component i at time t. In a very small interval $(t \;\; t + \Delta t$, the load during the interval can be assumed to be $L \;\; t$. Since the transition rates of component i in that small interval Δt is determined by Equation (6.10), $p \;\; \Delta t \; s$ can be estimated as follows.

$$p \quad \Delta t \;\; s \quad = \quad \left(-s \, \lambda \quad \left(\theta \;\; {}^{T \; +}_{\;\; s} \right) \Delta t \right) \tag{6.11}$$

In Monte-Carlo simulation, the transition probability is modeled by a random number, r_i, between 0 and 1. If r_i is greater than $p \;\; \Delta t \; s$, the transition occurs in the small interval Δt and vice versa. Whenever the transition is observed, the state of component i, s_i, changes and the transition probability needs to be updated according to Equation (6.11). The simulation runs for all components in the system consecutively. Starting from the initial states of all components at time 0, a simulation run stops if the state of any component reaches a state that is less that the minimum required state $d \;\; i = \qquad n$, i.e. a failure occurs.

The simulation is a continuous procedure for each interval until the time index t reaches the mission duration time τ. We repeat the simulation N times and record the state of the system at time $t = \imath$. A variable, *count*, is set to count the number of failures, and the reliability of the system can be estimated as follows.

$$R = -\frac{count}{N} \qquad (6.12)$$

The Monte-Carlo simulation procedure is presented in Figure 6.5. The key of the simulation program is to generate random numbers following the Uniform distribution, r_i, and compare this number to $p\ \Delta t\ s$ to model the state transition of a multi-state component in the system. Three loops are nested in the procedure to simulate the dynamic loading conditions and estimate the system reliability in the next mission. The most inner loop is created by dividing the mission into several small time intervals Δt for the state transition simulation of each component. The middle loop is to simulate the system degradation N times for the system reliability evaluation. The outer loop is to accommodate the dynamic loading conditions by generating N loading profile as described in Section 6.2.2. The output of the simulation program is a vector, sized $1 \times N$, that stores all the expected system reliability values for N_L generated loading profiles.

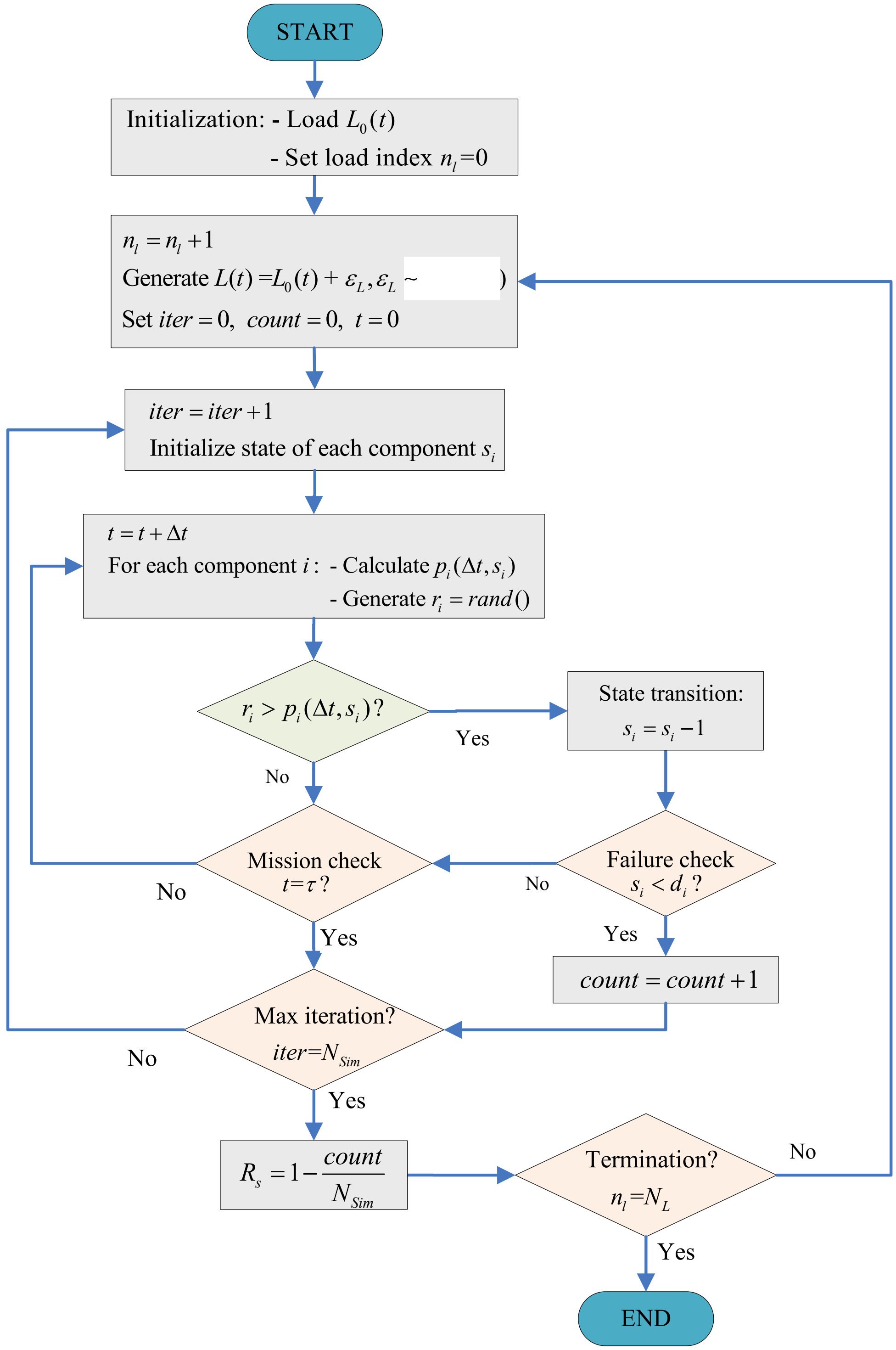


Figure 6.5 System reliability estimation procedure using Monte-Carlo simulation

6.4 Selective maintenance model

The objective of the selective maintenance model in this chapter is to determine the best maintenance scenario for the multi-state series systems working in a mission under the dynamic loading conditions as described in Section 6.2.2. Let $Y \in$ K be the state of component i, and $\mathbf{Y}$ Y Y be the state vector of all components in the system at the time entering the maintenance depot. It is assumed that the states of components at the end of the previous mission are detected and the vector of components states at the time entering the maintenance depot, $\mathbf{Y}$, is known. There is limited time and budget in the scheduled maintenance break, and all components in the system may not be maintained to their maximum state. We will present a selective maintenance model to determine a set of components and the associated maintenance actions to be performed in order to maximize the expected system reliability in the next mission.

The system reliability in the next mission and the maintenance resource utilization in the maintenance break are important elements of the selective maintenance model. Let $\mathbf{X}$ X X be the vector of components' states at the end of the maintenance break. Since we know the state vector of components at the time entering the maintenance depot, the selective maintenance scenario is determined for each vector $\mathbf{X}$. With each vector $\mathbf{X}$, the system reliability in the next mission can be estimated using the Monte-Carlo simulation in Section 6.3.3. The maintenance resources are calculated for each vector **X** and this will be presented in Section 6.4.1. The selective maintenance optimization model will be presented in Section 6.4.2.

6.4.1 Maintenance resources

In the maintenance break, there are two types of resources to maintain the system, which are maintenance time and maintenance cost. Let $t \;\; a \; b$ and $c \;\; a \; b$ be the time and cost for repairing component i from state a to state $b>a$. Similar to Chapter 3, the repair time and cost of component i in the system from any state a to state b are arranged in matrix form as in (6.13) and (6.14) respectively.

$$T = \begin{bmatrix} 0 & t & & t \;\; K \\ 0 & 0 & \ldots & t \;\; K \\ & & & \\ 0 & 0 & \ldots & t \;\; K - \;\; K \\ 0 & 0 & \ldots & 0 \end{bmatrix} \quad i = \qquad n \qquad (6.13)$$

$$C = \begin{bmatrix} 0 & c & & c \;\; K \\ 0 & 0 & \ldots & c \;\; K \\ & & & \\ 0 & 0 & \ldots & c \;\; K - \;\; K \\ 0 & 0 & \ldots & 0 \end{bmatrix} \quad i = \qquad n \qquad (6.14)$$

We assume that the repairs of components are independent in this chapter. The total maintenance time and cost of the system in the maintenance break can be calculated using Equations (6.15) and (6.16) respectively.

$$T\ X\ = \sum_{i=}^{n} t\ \ Y\ \ X \tag{6.15}$$

$$C\ X\ = \sum_{i=}^{n} c\ \ Y\ \ X \tag{6.16}$$

6.4.2 Selective maintenance optimization model

The objective of solving the selective maintenance problem is to find an optimal set of maintenance actions to maximize the expected system reliability in the next mission. This problem can be formulated as follows.

P: $$\mathit{Maximize}\ f = R\ \ X \tag{6.17}$$

$$\mathit{Subject\ to}\quad T\ X\ \leq T \tag{6.18}$$

$$C\ X\ \leq C \tag{6.19}$$

$$Y\ \leq X\ \leq K \tag{6.20}$$

$$X\ \ \mathit{is\ integer}\ \forall i = \qquad\quad n \tag{6.21}$$

In the model, the objective function (6.17) is to maximize the expected system reliability in the next mission, i.e. the probability that the system successfully completes the mission. Since the system operates in variable loading conditions in the next mission, the output of the Monte-Carlo simulation is a vector sized $1 \times N$ for N simulated loading profiles. The expected system reliability is calculated as the average of N values in the output vector of the Monte-Carlo simulation as in Equation (6.22).

$$\bar{R}\ X = \frac{\sum_{=}^{N} R \quad X}{N} \tag{6.22}$$

where $R \quad X$ is the obtained system reliability corresponding to the loading profile L .

In addition to the mean system reliability, we can calculate the standard deviation of the estimated reliability, σ_R, as in Equation (6.23). In the next mission, the system reliability is represented by a pair (R , where σ_R measures the variation of the system reliability in the dynamic loading conditions.

$$\sigma_{R\ X} = \sqrt{\frac{1}{N}\sum_{=}^{N} R \quad X - R\ X} \tag{6.23}$$

In the optimization model, there are two constraints in (6.18) and (6.19) which restrict the total time and cost for all maintenance activities within the break duration T_0, and total maintenance budget C_0. The total maintenance time and cost of the system are calculated as in Section 6.4.1. The decision variables, $X \quad i = \quad n$, are the states of components at the time of exiting the maintenance depot. X is an integer between Y and the maximum state K_i for all $i = \quad N$

6.5 Numerical example and results

In this Section, we consider a multi-state series system with $n = 5$ components as shown in Figure 6.6. The number of states of each component is given in Table 6.1.

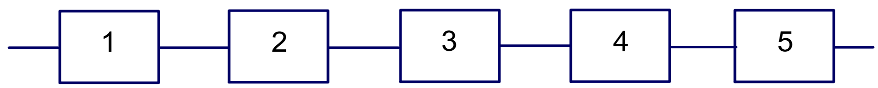


Figure 6.6 The multi-state series system with 5 components

In the next mission, the system operates in variable loading conditions $L\,t = L\;\;t + \varepsilon$ in a mission with $\tau = 1000$ hours. The baseload profile, shown in Figure 3a, is given in Equation (6.24). The random load ε_L follows the Normal distribution with $\mu = 0$ and standard deviation $\sigma = 4.2063$.

$$L\;\;t \;=\; \qquad + \qquad t^{-} \;+ \qquad \bar{t} \;- \qquad t \;+ \qquad\qquad - t \tag{6.24}$$

The vector of components' states at the time entering the maintenance depot is $\mathbf{Y}$ 1 1 2 . Parameters of the load dependent degradation rates, minimum state requirement, and maintenance resources for maintaining each component are given in Tables 6.1 and 6.2 respectively.

Table 6.1 Components' states and parameters in load-dependent degradation model

Component	i =1	i =2	i =3	i =4	i =5
Number of states	4	3	4	3	5
Minimum required state, d_i	1	1	2	1	1
λ_i (/hours)	0.0001	0.00012	0.0001	0.00001	0.0004
θ_i	0.005	0.004	0.008	0.006	0.005

Table 6.2 Time and cost of repairing each component

		Maintenance Cost (in $1000)					**Maintenance Time** (in hours)				
Component	**State**	**0**	**1**	**2**	**3**	**4**	**0**	**1**	**2**	**3**	**4**
i=1	**0**	0	5.5	11	16.5	-	0	12	24	36	-
	1	0	0	5.5	11	-	0	0	12	24	-
	2	0	0	0	5.5	-	0	0	0	12	-
	3	0	0	0	0	-	0	0	0	0	-
i=2	**0**	0	5	10	-	-	0	8	16	-	-
	1	0	0	5	-	-	0	0	8	-	-
	2	0	0	0	-	-	0	0	0	-	-
i=3	**0**	0	6	12	18	-	0	9	18	27	-
	1	0	0	6	12	-	0	0	9	18	-
	2	0	0	0	6	-	0	0	0	9	-
	3	0	0	0	0	-	0	0	0	0	-
i=4	**0**	0	6.5	13	-	-	0	9.5	19	-	-
	1	0	0	6.5	-	-	0	0	9.5	-	-
	2	0	0	0	-	-	0	0	0	-	-
i=5	**0**	0	6.2	12.4	18.6	24.8	0	9	18	27	36
	1	0	0	6.2	12.4	18.6	0	0	9	18	27
	2	0	0	0	6.2	12.4	0	0	0	9	18
	3	0	0	0	0	6.2	0	0	0	0	9
	4	0	0	0	0	0	0	0	0	0	0

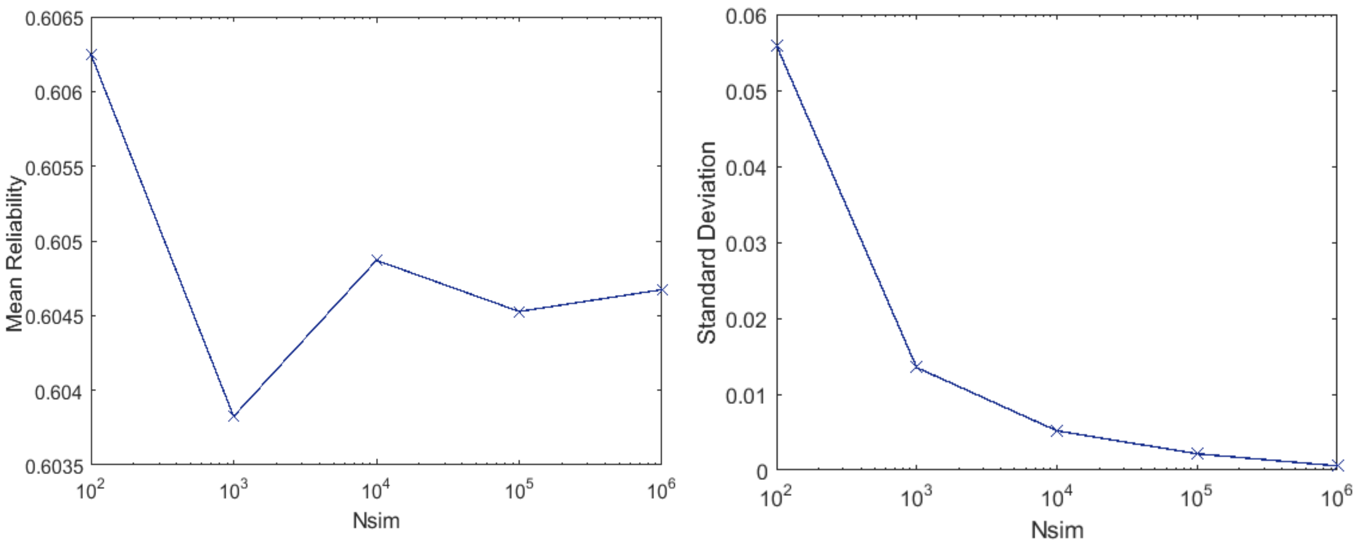
6.5.1 Results on the system reliability estimation

In this section, the states of components at the beginning of the next mission is given as $\mathbf{X}$ 2 2 3 . We built a Monte-Carlo simulation program in Matlab 2015b to investigate the results on the system reliability estimation method presented in Section 6.3.3 and test for N = variable loading profiles. The simulation was run on a computer with CPU Intel Core i7 processor, 3.60GHz, 16GB of RAM, and Windows 7 operating system. The results on the system reliability estimation for different number of simulation runs, N , from 100 to 1,000,000 are presented in Table 6.3.

Table 6.3 Results on the system reliability evaluation

Number of runs, N	$\bar{R}$ $\mathbf{X}$	**Standard deviation**	**Simulation time (seconds)**
100	0.60625	0.055916	2.496
1,000	0.60383	0.01351	25.695
10,000	0.60487	5.178e-3	253.2052
100,000	0.60453	2.158e-3	2.55e+03
1,000,000	0.60467	5.610e-4	2.50e+04

In Table 6.3, besides the mean value of $\bar{R}$ $\mathbf{X}$, the standard deviation and the CPU time are also reported. It is observed that the standard deviation decreases when the number of simulation runs increases, i.e. the mean of the system reliability converges. Figures 6.7a and 6.7b show variations in the system reliability and standard deviation against the number of simulation runs N_{sim} in log scale. The simulation time increases proportionally with the number of runs N_{sim}. The average system reliability with $N_{sim} = 1{,}000{,}000$ is 0.60467, which is considered to be accurate with a standard deviation of 5.61e-4. The simulation with $N_{sim} = 10{,}000$ can give the average system reliability accurately with 3 decimal places.



(a) (b)

Figure 6.7 The mean system reliability (a) and standard deviation (b) versus number of simulation runs

To examine the effect of random variation of the loading conditions, we define the percentage of load variation as $p = \frac{\sigma_L}{L} \times 100\%$, where σ_L is the standard deviation of the random variation and $\overline{L}$ is the mean load calculated from the baseload profile. Three levels of p , 2%, 6%, and 10%, are tested for this example with the number of runs $N = {,}000$. The results are shown in Figure 6.8.

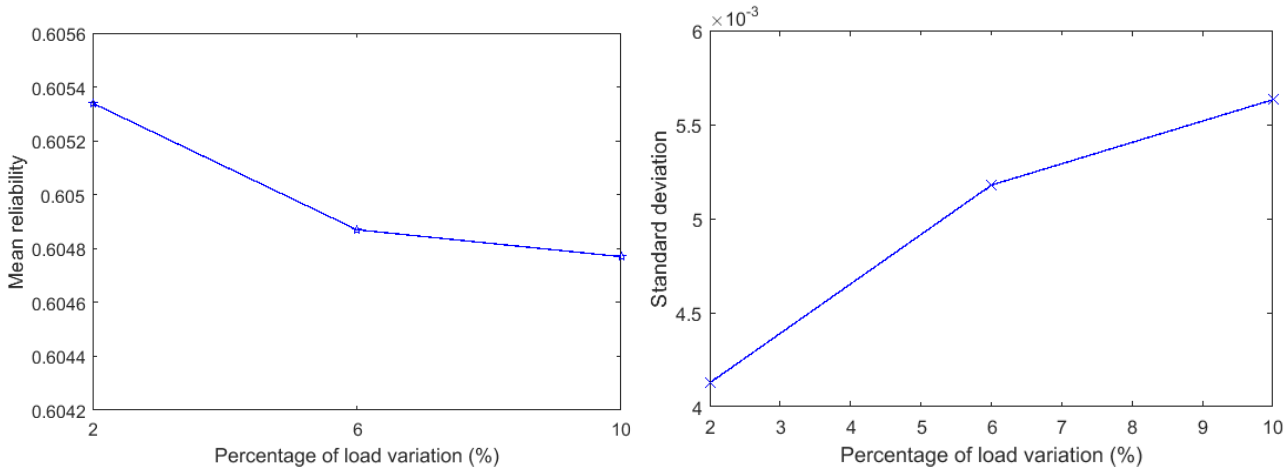


Figure 6.8 The mean system reliability and standard deviation at different levels of load variation

It is seen that the mean system reliability does not change much, roughly 0.0005, when the percentage of load variation varies. The system reliability variation agrees with the analysis on Figure 6.7 that the mean system reliability remains accurate up to 3 decimal places. On the other hand, the standard deviation of the system reliability increases significantly, approximately 0.0015, when the load variation changes from 2% to 10%. This implies that a large variation of the system reliability is predicted when the loading conditions are more uncertain.

6.5.2 Results on the selective maintenance optimization

In this section, the maintenance budget and time limitations are given as $C =$ 000 and $T =$ hours. We need to find the optimal selective maintenance scenario for the system within the available resources. From constraint (6.19) in the optimization model, $Y_i \le X_i \le K_i$, $i = \ldots n$, and $\mathbf{Y}$ $\ldots 1 \;\; 1 \;\; 2$ and $K = \;\; 2 \;\; 3 \;\; 2 \;\; 4$, there are a total of $3 \times 2 \times 3 \times 2 \times 3 = 108$ possible solutions for the selective maintenance optimization problem.

The system reliability is evaluated for each maintenance scenario. We built another program in Matlab 2015b to find the maximum average system reliability $R(\mathbf{X})$. We also ran experiments for different numbers of runs N and observed that the optimal solution of $\mathbf{X}$ converges for this specific example with $N = \,000$. The optimal maintenance scenario is presented in Table 6.4.

Table 6.4 The optimal selective maintenance scenario with $C =$ 000, $T =$ hours

Component i	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
Entering state Y_i	1	1	1	1	2
Exiting state X_i	2	2	3	1	3
Maintenance actions	1→2	1→2	1→3	DN*	2→3

*DN = Do nothing

The maintenance scenario indicates that imperfect repairs are suggested for components 1 and 5 to their intermediate states 2 and 3 respectively; no maintenance action is recommended on component 4; and both of the two components 2 and 3 should be renewed to their maximum states $K =$ and $K =$ respectively. The vector of components' states at the time exiting the maintenance depot is $\mathbf{X}$ 2 3 1 3. For this maintenance strategy, the average system reliability is 0.858 and the total maintenance time and cost are 47 hours ($< T_0 =$ 48 hours) and \$28,700 ($< C_0 =$ \$30,000). The system reliability and maintenance resources consumption are shown in Table 6.5.

Table 6.5 The system reliability and maintenance resources consumption

Ave. Reliability $\bar{R}$ $\mathbf{X}$	Total Cost C $\mathbf{X}$	Total Time T $\mathbf{X}$
0.858	\$28,700	47hrs

We also investigate the sensitivity of the maintenance budget to the system reliability estimations. The time constraint in the optimization model is released, and the budget limitation C_0 is set to vary from \$10,000 to \$50,000 with an increment of \$5,000. The system reliability in the optimal selective maintenance strategy for each case is plotted versus the maintenance budget as in Figure 6.9.

It is noted that the system state corresponding vector $\mathbf{Y}$ 1 1 2 is a failure state since $Y = \; < d =$. At least one maintenance action on component 3 needs to be performed to guarantee a functioning system at the beginning of the next mission.

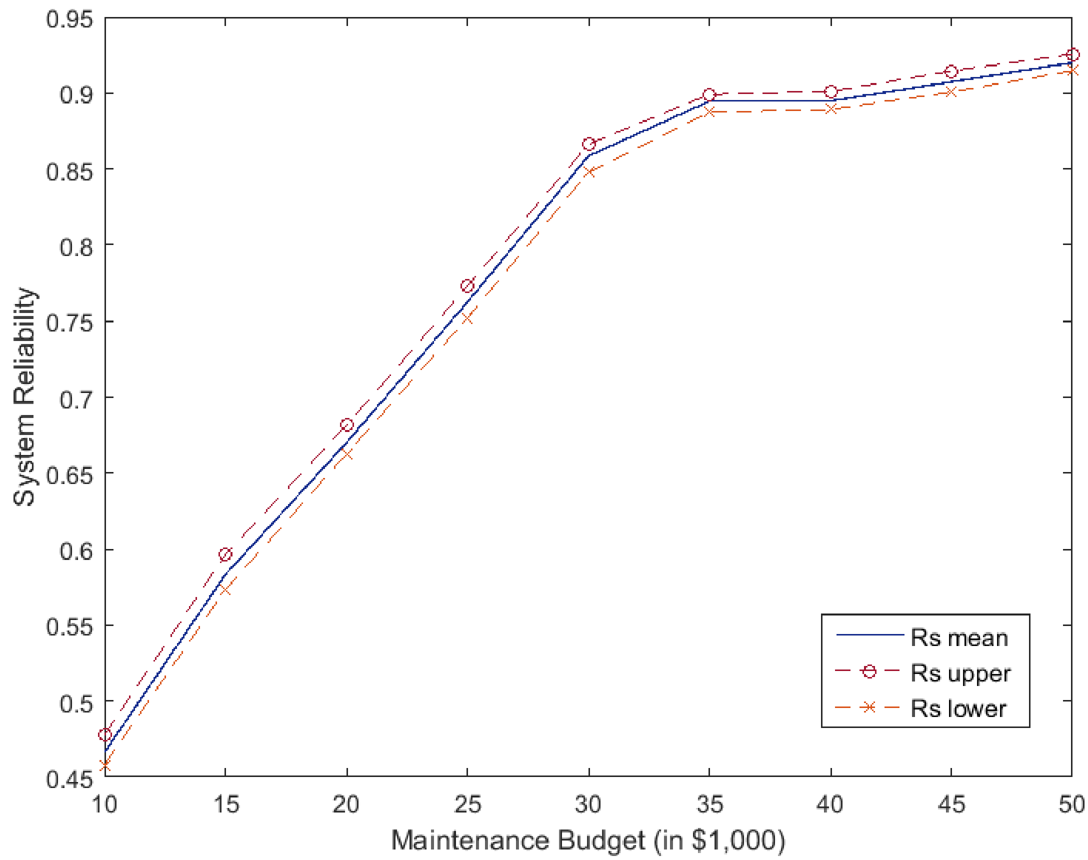


Figure 6.9 The system reliability versus maintenance budget

For each level of maintenance budget, three values of the system reliability are presented, including R $\mathbf{X}$, $R\,upper$ $\mathbf{X}$ $\mathbf{X}$, and $R\,lower$ $\mathbf{X}$ min R $\mathbf{X}$. The range of the mean system reliability is from 0.4661 when $C_0 = 10{,}000$, i.e. a single maintenance action to repair component 3 to state 2 is performed, to 0.9199 when C = 000, i.e. all components are repaired to their best state. The mean system reliability increases as the budget increases, but the improvement on the mean system reliability does not vary at the same rate as the budget increments. The mean system reliability is significantly improved when the budget increases from \$10,000 (R = 61) to \$15,000 (R = 32), \$20,000 (R = 99), \$25,000 (R = 24), and \$30,000 (R = 84). When the budget is greater than \$40,000, the system reliability improves at a much more slowly rate. Interestingly, when the budget increases from \$35,000 to \$40,000, there is no improvement on the system reliability, i.e. the same optimal maintenance strategy is suggested. The range between R_{upper} and R_{lower} for different levels of budget is reasonably stable and it tends to be a bit smaller when more budget is allowed. The sensitivity analysis can help maintenance managers consider the trade-off between the system reliability improvement and maintenance budget spending. This chart is also useful in a situation where a plan is needed for maintenance resource allocation while an expected system reliability in the next mission is required.

6.6 Concluding remarks

In summary, this chapter investigates the selective maintenance problem for multi-state series systems working in dynamic loading conditions. The future loading profile is not known with certainty. We present a load-dependent degradation model for multi-state components operating in variable loading conditions. During the next mission, the load-dependent degradation rate of a multi-state component varies depending on its current state and the instant load applied on it. The model can capture the dynamic behavior of components in a k-out-of-n load-sharing system, where many components share a common work-load and the failure rate of a component depends on the state of other components.

The system reliability is estimated using the Monte-Carlo simulation method. A selective maintenance optimization model to maximize the expected system reliability in the next mission is presented. The sensitivity analysis of the optimized system reliability against various levels of budget limitation indicates that the mean system reliability varies at a different rate of the budget increments while the standard deviation of the system reliability is quite stable at various budget levels.

CHAPTER 7

SUMMARY AND FUTURE WORK

7.1 Summary

Engineering systems today are more and more complex and their failures are critical since they often involve several complicated interactions between elements, hardware, software, and operating conditions. Industrial organizations require their systems to be managed effectively with limited expenses to accommodate the global market competitiveness. Selective maintenance provides an optimal set of maintenance actions to perform on selected components to fulfill the system requirements within available resources. Selective maintenance is a powerful tool that can contribute to industrial organizations by keeping their operation safely and utilizing their resources effectively.

This PhD dissertation investigates the selective maintenance problem for multi-state systems considering dependence relationships when maintaining the systems. This research has significant contributions in terms of dependence modelling, reliability analysis, and selective maintenance optimization of multi-state systems.

7.1.1 Dependence modelling

In this thesis, four types of dependence in the maintenance of multi-state systems are modelled including economic, structural, stochastic, and operational dependence. Economic and structural dependence represent the relationships between maintenance actions; stochastic dependence demonstrates the relationships between components' lifetimes; while operational dependence refers to the relationship between components' degradation and operating environment.

Firstly, we modelled the dependence relationships between maintenance actions by economic and structural dependence. Economic dependence reflects the savings due to the share of resources when maintaining several components simultaneously, e.g. materials, tool, manpower, etc. Two mechanisms of savings in maintaining several identical and non-identical components were modelled and they applied to both types of resources of maintenance cost and time. Structural dependence implies that extra cost is required when maintaining the system since there is a sequence of assembling and disassembling components. We used the directed graph to model the precedence relations in the disassembly sequence of components and presented a backward search algorithm to find the disassembly path for the system.

Secondly, the relationship between components' lifetimes was modelled. Two types of stochastic dependence between components were discussed in multi-state context. First, the failure of a component can immediately cause failures of some other components. Second, as components in the system degrade, the degradation of a multi-state component reduces the flow to the following components and thus affecting the degradation of these components. We used the power law to represent the relationship between the output performance rate of an influencing component and the degradation rate of an affected component.

Thirdly, the relationship between multi-state components' degradations and operating conditions was modelled. The dynamic future load on the system was represented as a known baseload function and a random variation following the Normal distribution. We considered a case where the degradations of components varied stochastically depending on its current state and the dynamic loading conditions. A load-dependent degradation model was proposed to model the transition between states of multi-state components. The model was based on the proportional hazard model and degradation of k-out-of-n load-sharing systems where the failure of a component increases the degradation of other surviving components in the system.

7.1.2 Reliability analysis of MSS considering dependence

When the dependence is related to the degradation of components, it is necessary to evaluate the system reliability prior to selective maintenance modeling. The methods for system reliability analysis were elaborated in the cases that s-dependence and operational dependence exist. In doing analysis of multi-state systems with s-dependence, we considered the states of all n components in the system together and presented the state of the system as an n-tuple including the states of n components. This practice allowed us to use the stochastic process to model the system degradation and evaluate the system reliability. In doing analysis of multi-state systems with load dependence, because the load is uncertain, a Monte-Carlo simulation method was presented to model the multi-state component's degradation and to evaluate the system reliability. In the Monte-Carlo simulation, we divided the operating mission into several small intervals and the dynamic behavior of a multi-state component, including changes of component's state and uncertain loading conditions, was captured in the Monte-Carlo simulation method.

7.1.3 Selective maintenance modeling

In Chapter 3, several selective maintenance optimization models for multi-state systems were generated. In these models, the total system maintenance cost, maintenance time, and the system reliability can be treated either as objective or as constraints. Depending on the principal purpose of the maintenance and available information, the maintenance decision maker can select the most appropriate models and thereby understand what actions should be done based on his/her on-hand conditions.

When dependence relates to maintenance implementation, i.e. economic and structural dependence – Chapters 3 and 4, the relationships between components are reflected in the resource calculations during selective maintenance modeling. Either the savings due to joint maintenance of multi-components due to economic dependence or the precedence relations due to structural dependence were modeled in the total system maintenance time and cost calculations in the maintenance break. When dependence is related to components' degradations, i.e. stochastic and operational dependence – Chapters 5 and 6, the relationships between components were integrated in the reliability evaluation, which could be reflected in a constraint (Chapter 5) and in the objective function (Chapter 6).

In Chapter 5, we developed a cost-based selective maintenance model. This model provided an alternative approach in accordance with the multi-state system performance analysis. In the next mission, the system was assumed to produce a certain benefit or loss proportionally with the state it may be in. The objective function in this model was to maximize the total system profit, which considered both the maintenance cost in the maintenance break and the possible profit or loss related to the expected performance of the multi-state systems in the next mission.

7.2 Future work

From the work in this thesis, several research directions related to the complexity of the systems, methods for reliability analysis, and selective maintenance modeling are proposed for future studies. Details are as follows.

7.2.1 Study more complex systems

In this thesis, we have focused on the multi-state series and series-parallel systems. It is suggested that more complex structures such as bridge, k-out-of-n, network, etc. are investigated in the future. The dependence modeling from this thesis can be used to model the relationships between components in other system structures. However, it is noted that the inter-connection between components in a specific structure affects the system reliability, and the system structure function must be considered in evaluating the system reliability.

In a real and complex system, several types of dependence may co-exist. Except in Chapter 4, where economic and structural dependence were considered simultaneously, different types of dependence in this dissertation are treated separately. In our classification, the dependence can be related to maintenance implementation (economic and structural dependence) or component's degradations (stochastic and operational dependence). The models in this thesis can be extended to the case where a type of dependence related to maintenance implementation and a type of dependence related to components' degradations exist. However, other cases, such as where two types of dependence related to components' degradations or more than two types of dependence exist, still require further elaboration.

7.2.2 Study methods for reliability evaluation of multi-state systems with s-dependence

In Chapter 5, we proposed a reliability evaluation method for multi-state systems with s-dependence components. This method was based on the Markov process that had a state space increasing with an order of $\mathbf{O}\left(\left(\frac{K}{n}+\,'\right)\quad\right)$. This method has limitations since the state space explosion occurs when the number of components n and number of states K increase. Alternative methods for evaluating the system reliability with stochastic dependence are recommended for future investigations.

One of the methods that can be used for evaluating the MSS performance in the next mission is the Universal Generating Function (UGF) technique. UGF technique evaluates the multi-state system reliability by representing the state probability as a u-function and calculating the system's u-function from its components' u-functions [104].

The use of UGF technique can potentially reduce the computational effort to evaluate system reliability since it does not require solving a large system of differential equations as needed by the stochastic process approach. However, it is noted that the traditional UGF technique based on the traditional composition operator is only suitable for systems with independent components. To use this method for analysis of MSS with s-dependent components, an extended version of UGF needs to be developed. The following suggestions are provided when adapting the UGF technique for the MSS considering two types of s-dependence.

i. A new composition operator should be developed to represent the degradation dependence relationship between components.

ii. An alternative representation of component's u-function and/or a two-stage UGF may be needed to accommodate both types of s-dependence in multi-state systems.

7.2.3 Study selective maintenance in a finite planning horizon

This thesis focuses on the single-mission selective maintenance model. An underlying assumption was that the maintenance decisions were for the next mission. Thus, the current available resources for a single maintenance break and the system requirements in a single subsequent mission are input information. In a finite planning horizon, multiple missions are required to be scheduled and several breaks for maintenance can be possible; therefore, the selective maintenance problem in a finite planning horizon should be considered.

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