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A MATHEMATICAL MODEL OF DETERRENCE

BY

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ABSTRACT

THE PURPOSE OF THIS STUDY IS TO DEVELOP, OPERATIONALIZE, AND APPLY A MATHEMATICAL MODEL WHICH MEASURES THE THEORETICAL CHANGE IN THE DETERRENT EFFECTS OF EACH OF TWO STATES IN A DYAD (AT TIME PERIOD T) PER UNIT CHANGE IN THEIR RESPECTIVE (PREVIOUS PERIOD) WARHEAD DEPLOYMENT LEVELS. THE MODEL IS SPECIFIED SOLELY IN TERMS OF THE VARIABLES AND PARAMETERS OF AN INDEPENDENTLY FORMULATED ACTION-REACTION ARMS ACQUISITION MODEL WHICH DESCRIBES THE WARHEAD DEPLOYMENTS OF THE TWO STATES IN THE DYAD AT TIME PERIOD T. WE THEREFORE SIMPLY ESTIMATE THE PARAMETERS OF THE LATTER MODEL USING ACTUAL WARHEAD DEPLOYMENT DATA AND SUBSTITUTE THE RESULTS INTO THE FORMER THEREBY OPERATIONALIZING IT. WE ESTIMATE THESE PARAMETERS FOR THREE DISTINCT ARMS RACE PERIODS: 1967-1972, 1973-1979 AND 1980-1984.

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NOTWITHSTANDING I ACCEPT SOLE RESPONSIBILITY FOR ANY ERRORS CONTAINED HEREIN.

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Introduction

The development, operationalization and application of consolidated theories is an aim of all practitioners of the mathematical approach to the study of the international relations. The purpose of this study is to develop, operationalize and apply a consolidated Arms-Building-Arms-Using Model (ABAU Model). To begin, an "Arms-Building Model" is one which "directly addresses the arms level question by describing 'how much effort is being spent.' The model itself can vary from a Richardson-type action-reaction process to a pure model of a domestic bureaucratic process; in most cases, it involves ... [simplifying] assumptions about how armament efforts are influenced by various parameters."[1] An "Arms-Using Model," on the other hand, "addresses the arms level question indirectly by attempting to describe 'how much effort is needed' to assure victory, deterrence, second strike capability, national survival, stability, or some other (presumably) rational purpose."[2]

More specifically, we shall develop, operationalize and apply a consolidated mathematical model consisting of two simultaneous differential equations which measure the theoretical changes in the deterrent effects of each of two states in a dyad (at time period t) per unit change in their respective (previous period) nuclear warhead deployment levels, holding constant all other factors herein posited to impinge thereon constant. The value of such an undertaking is clear. Arms Building and Arms Using Models "could be joined together," write Moll and Luebbert, "by a link that defines the national defense criterion. Such a threefold model, never actually implemented in the literature, should provide a new and hitherto unknown analytical power for considering the full range of each nation's security. This model could provide the following unique services: (1) a comprehensive means of considering the interrelationship of goals in the data system (2) a means of evaluating changes in system stability or the security of one's own arms (3) the ability to evaluate armament changes as they occur in 'real time.'"[3] In particular, our model should provide a theoretical and empirical basis from which we may "evaluat[e] changes in system stability or the security of one's own arms."

Normally, at about this point in the introduction of a thesis, one might expect to find a fairly comprehensive literature review and some indication of the place of the thesis within the field. But because a consolidated ABAU Model has never been implemented in the literature we have no basis from which to mount such a review. We could, on the other hand, review the rather substantial literature on Arms-Building Models and the rather substantial literature on Arms-Using Models. Yet this is easier said than done. The reason why no consolidated ABAU Model has to date been implemented in the literature is because developments in the two component fields, arms-building and arms-using modeling, have occurred in isolation of each other. As such no methodological basis, hitherto, existed whereby both models could be consolidated. The essential message here is that we had to rework, from first principles, certain aspects of these component fields in accordance with a consistent cross-cutting methodology. The technical problems involved in such an undertaking were enormous, but, obviously, not insurmountable. For fear that a thorough understanding of these technical problems and the solutions we offer would require a more substantial context than could be provided in this introduction, we have decided, therefore, to merge our literature review with the presentation or rather development of our consolidated ABAU Model. Some preliminary comments can at this point be made, however, regarding the nature of the final result, the consolidated ABAU Model.

The model, to reiterate, consists of two simultaneous differential equations which measure the theoretical changes in the deterrent effects of each of two states in a dyad (at time period t) per unit change in their respective (previous period) nuclear warhead deployment levels, holding constant all other factors herein posited to impinge thereon constant. The critical feature of these equations is that they will be specified entirely in terms of the variables and parameters of a Richardson-type Arms-Building Model. What is critical is this: given time series data on Soviet and American warhead deployments we can estimate the parameters of the Richardson-type model for each state independently of the consolidated ABAU Model. Thereupon we can simply substitute those parameters estimates into the consolidated model thereby operationalizing it. That there is a complete lack of empirical data concerning deterrence levels over time, then, does not hamper our attempt to operationalize the consolidated ABAU Model.

Before this could be attempted, however, it was necessary to develop an Arms-Using Model which functionally related a state's deterrent effect (dependent variable) to its adversary's estimate of the former's deliverable retaliatory capability (independent variable, operationalized as deliverable MTE) and also to its adversary's estimate of the former's intent or willingness to retaliate (independent variable). Developing such a model is the principal concern of the first chapter. It was then necessary to develop an Arms-Building Model showing the adversary's estimate of his potential victim's retaliatory capability (later estimated for warhead deployments). But it was critical that this model be methodologically compatible with the former model such that the two could be readily integrated in the third chapter thus yielding, via partial differential calculus, our consolidated ABAU Model. In the fourth chapter, we estimate the parameters of the model and interpret the results.

The aim of this study is compatible with the efforts of Hollist and Guetzkow (and indeed many others) to in effect begin the process of unifying both overtly compatible and otherwise seemingly disparate pieces of knowledge of international phenomena into a comprehensive whole. "[T]he ultimate aim ... of our research program," writes Hollist, "is to incorporate explanations (models) of arms processes and other empirically researched modules of theory into comprehensive theoretical statements. These models of middle-range theories and the combined, general model of international relations are or will be specified ... as computer simulations. The scheme of the research is to build comprehensive theories (computer simulations) of international affairs by integrating empirically tested, middle-range theories [emphasis included]."[4] No doubt a place in this scheme could be found for a model such as the one proposed here.

Throughout, we will be guided in part by the principles set out in Moll and Luebbert's "Arms Race and Military Expenditure Models: A Review," regarding the development of a consolidated ABAU Model, although to a large degree we will be sailing in uncharted waters, for as they point out, a consolidated Arms-Building-Arms-Using Model has never, yet, been implemented in the literature. In that regard, there are substantial benefits to approaching the task at hand from a mathematical perspective. To begin, Job writes: "[a] model consists of a set of assumptions about behavior from which one may logically derive statements that describe the behavioral properties or outcomes to be expected as results of the process."[5] Statements such as these are what constitute advances in knowledge and we will want to be extremely careful in how we go about deriving them. How can mathematics aid in this effort? A "way in which formal languages [such as mathematics] can aid us in political [or more generally, social scientific] inquiry," writes Gillespie, "is that they help us to specify relationships between ideas [or assumptions]. Often a formal language will give us some precise logical connection between a and b that allows us to see more clearly what we are talking about and how we are thinking."[6] This is no small benefit. As no consolidated ABAU Model has yet been implemented in the literature we lack a standard against which we may measure our progress. Furthermore, writes Gillespie, "the logic of the language [of mathematics] helps us to see that not only can a and b be related, but a and b can be related to c and d through a series of steps. ...[F]ormal languages give us a way of making deductions from our ideas [or assumptions] that we otherwise might not contemplate."[7]

Notes

[1]K.D. Moll and G.M. Luebbert, "Arms Race and Military Expenditure Models: A Review," in *Journal of Conflict Resolution* (Vol. 24, No. 1: March 1980), p. 155.

[2]Moll and Luebbert, p. 155.

[3]Moll and Luebbert, p. 179.

[4]W.L. Hollist, "An Analysis of Arms Processes in the United States and the Soviet Union," in *International Studies Quarterly* (Vol. 21, No. 3: September 1977), pp. 503-504.

[5]B.L. Job, "Two Types of Dynamic Models," in *Missing Elements in Political Inquiry*, eds. J.A. Gillespie and D.A. Zinnes (Beverly Hills: Sage Publications, Inc., 1982), p. 91.

[6]J.A. Gillespie, "Some Basic Puzzles in Political Inquiry," in *Missing Elements in Political Inquiry*, eds. J.A. Gillespie and D.A. Zinnes (Beverly Hills: Sage Publications, Inc., 1982), p. 15.

[7]Gillespie, pp. 15-16.

Chapter 1: Developing an Arms-Using Model

Holsti has posited that at its most basic level a state's deterrent effect is a function of the level of its deliverable destructive capability (we shall be more specific as to the substance of this term presently) and its willingness to retaliate in the event that it becomes the victim of a surprise nuclear strike. But more than that it is dependent not upon the actual levels of those variables, but on an estimate of those level's by an adversary which may or may not equal the actual values. Now "[t]he simplest and most straightforward means of counting [not to be confused with the term estimation (statistical estimator)] capabilities," writes Snow, "is simply to add up the number of strategic delivery vehicles each side has and compare the result."[1] "Although it is the simplest form of comparison, counting launchers can also be the most deceptive. Because of the ability of a single launcher to carry more than one warhead and because the count is not sensitive to the size or accuracy of warheads, this simple count does not yield much precision about the deadlines of an arsenal."[2] Nor do "raw megatonnage figures ... provide an entirely satisfactory measure of destructive capability ... because 'beyond a certain point, larger nuclear bombs do proportionately less damage than smaller weapons,' and 'beyond a certain yield, no more explosive yield is needed to destroy a given target.'"[3] The best measure, which incorporates information regarding the number of launchers, the number warheads, and the megatonnage contained in an arsenal, is megaton equivalent (MTE). "Total MTE [i,e, total destructive capability (as distinct from deliverable destructive capability and as distinct from estimations thereof by an adversary)] for a country is derived by applying the basic formula"

$$MTE = \sum_{i=1}^{n} N_i Y_i^{2/3} \qquad (1)$$

"where N_i is the number of weapons in a class [i] and $Y_i^{2/3}$ is the derivation of yield expressed in megatons (or fractions thereof) and adding the products for each weapons class."[4] Deliverable destructive capability is obtained by multiplying the MTE of each weapons class by a corresponding variable T_i (measured as a probability) which indicates the offensive-defensive technology level of that class and hence the likelihood that the weapons in that class could survive a first strike and subsequently penetrate enemy defenses and then adding the products for each weapons class:

$$(\text{deliverable})\ MTE = \sum_{i=1}^{n} N_i\ Y_i^{2/3} T_i \qquad (2)$$

Furthermore, Holsti posits a state's deterrent effect to be a product of those estimates, not a sum. It is a sensible proposition. The specified relationship is given immediately below.[5]

$$\text{Deterrent Effect} = \text{Estimated Capability} \times \text{Estimated Intent} \quad (3)$$

Immediately after presenting this model Holsti writes: "[a]lthough this formula oversimplifies a complex relationship (a point to which we will return later in considering the deterrent effect of 'overkill' capacity), it does highlight the key point that if either perceived capability or intent is zero, deterrent effect is also zero."[6] In effect Holsti is concerned here that while his specification is structurally accurate (in that the dependent variable depends on a multiple of the independent variables and not, say, the sum thereof, a proposition which we accept throughout this study) it may not have been appropriate to have specified the equation in linear form where each of the independent variables has a directly proportional impact on the dependent variable. This is a critical matter, indeed, the principal matter at issue in this chapter. We must ask: what, in theory, would be the proper functional form of this equation, of the relationship between a state's deterrent effect and an adversary's estimate of the former's deliverable destructive capability and willingness to retaliate? The remainder of this chapter will be spent in addressing this very question.

To begin, Holsti would surely agree that the variable "estimated intent" be measured as a probability.

$$0 \leq \text{estimated intent} \leq 1 \quad (4)$$

Our principal concern in this study, however, is with estimated capability levels and their impact on deterrence. Now given the structure of Equation 3 and the value range just placed on the estimated intent variable, for any values of the estimated intent variable less than one, the "full" impact of estimated capability on deterrent effect will be mitigated and indeed neutralized when estimated intent is equal to zero. Indeed, in the latter circumstance, there would be no basis for the current study (in more ways than one). Accordingly and also because we wish to study the unmitigated effect of estimated capability on deterrent effect we will at this point impose the value of one on the estimated intent variable thereby reducing Equation 3 to the following form.

$$\text{Deterrent Effect} = \text{Estimated Capability} \quad (5)$$

It should be noted that Equation 5 does not constitute a simplified form of Equation 3 as the term is usually applied. Instead Equation 5 continues to respect the importance of estimated intent in regard to deterrent effect. The equation should be read as giving the impact on a state's deterrent effect of the estimate by its adversary of the former's deliverable destructive capability given that the adversary expects (if the need arises) that the former possesses full resolve to respond to any attack (actually to a nuclear first strike). From the perspective of the adversary, this corresponds to the real world practice of planning on the basis of worst case scenarios. We will maintain this interpretation throughout this study.

Now, to continue, we must note that an Arms-Using Model "describe[s] how armaments, military forces, or national resources are consumed in battle."[7] More specifically we are interested in how a potential aggressor might conceive of these matters. How do changes in his estimation of the level of a potential victim's destructive capability affect his decision to launch a first strike? The objective here is to postulate the general functional form of that relationship.

There is an important three-way relationship between the change in a potential aggressor's projection of the level of retaliatory damage that he would have to sustain as a result of launching a first-strike (where DL denotes his damage level projection and dDL the change therein), the change in a potential victim state's deterrent effect (where D denotes its deterrent effect and dD the change therein) and, finally, changes in the potential aggressor's estimation or expectation of the destructive capability of his potential victim (where at estimation is denoted C^e and where dC^e denotes the change therein). How is C^e to be operationalized? Suffice it to say for now that C^e measures an adversary's expectation of his potential victim's deliverable MTE deployment level. (For technical reasons, to be discussed in more detail below, however, C^e uses the average value of offensive-defensive technology, $\bar{T}$. This is necessary in order to remove T_i outside of the summation operator in Equation 2 so that the change in the potential aggressor's estimate of his potential victim's total MTE deployment, dC^e, could be calculated. Accordingly, when, hereafter, we use phrase "holding constant all other factors" we are referring to offensive-defensive technology levels.) To continue, for the sake of simplicity we assume that the change in the potential aggressor's estimate of retaliatory damage levels (dDL) takes into account the effect of only a single retaliatory salvo containing the victim's full retaliatory might and aimed at population centers. (We are in effect assuming a pure MAD framework).

Now each particular retaliatory damage level projection that the potential aggressor makes should correspond to a particular level of deterrence. As projections change (where the change is denoted dDL), so too should the extent to which the potential aggressor is deterred (where this change is denoted dD). The former should cause a corresponding and directly proportional change in the latter. In mathematical terms, this relationship can be expressed as follows.

$$dD = k(dDL), \quad 0 \leq k \leq 1 \tag{6}$$

A proportionality constant (k) has been introduced so that (dDL) and dD) could actually be equated. We will take its value, which ranges between zero and one inclusive, to indicate that there may be a lag in adjustment between changes in damage level projections and changes in the extent of deterrence. This may be due in part to psychological or logistical reasons as discussed in Jervis' "Deterrence and Perception." For example, a decision maker's perceptions may be clouded as a consequence of preconceived and deeply ingrained notions concerning the intentions of his adversary, notions which may distort the objective reality of the situation,[8] or, from a logistical perspective, simply through lack of experience, through lack of precedent upon which to base an understanding of an adversary's behavior.[9] But it will be assumed in this model that adjustment is complete and unimpaired, that (k = 1). Therefore, Equation 2 may be re-written as follows.

$$dD = dDL \tag{7}$$

How do changes in damage level projections come about? Holding all other factors which have been herein posited to impinge on a state's deterrent effect constant, as a potential aggressor perceives that a change has occurred in the amount of <u>total</u> MTE deployment by his prospective victim, (denoted dC^e), he must, accordingly, reformulate his projections of retaliatory damage levels. This is based on the notion that, according to Holsti, "[a]n increase in numbers can, at least temporarily, make it more difficult for the attacker to succeed in a first strike."[10] This can be introduced into Equation 7 by dividing dDL by dC^e and to maintain the equality, by dividing dD by dC^e also.

$$dD/dC^e = dDL/dC^e \tag{8}$$

What does Equation 8 state? It proposes that the change in the level of retaliatory damage (that a potential aggressor may project that he will have to sustain as a consequence of his launching a first strike) as (his estimation of his potential victim's) total MTE deployment level changes (denoted dDL/dC^e) is equivalent to the change in the extent to which the potential aggressor is deterred as his estimation of his potential victim's total MTE deployment level changes (denoted dD/dC^e). (Again, this analysis assumes that all other factors relevant to deterrence are held constant: namely technology levels). To continue, we can, therewith, accommodate the proposition that "[t]he temptation to launch a first strike should diminish as the certainty and probable costs of devastating retaliation increase."[11] And vice versa. What is clear from our argument is that we are treating the potential aggressor as a rational calculator. The degree to which he is prepared to launch a first strike alters <u>objectively</u> with changes in his retaliatory damage level projections (later to be operationalized in terms of percent population destroyed). The matter of what he considers as acceptable or unacceptable (as distinct from devastating) levels of damage poses as interesting dilemma for our study. We shall toward the end of this chapter take up the matter in detail but suffice it to say for now that in what follows there is no notion of levels of acceptable or unacceptable damage. In effect we are, herein, only considering that aspect of deterrent effects which can be <u>objectively</u> assessed. Accordingly, on the basis of the relationship specified in Equation 8 we can determine the functional form of the relationship between a state's deterrent effect and an adversary's estimate of the former's deliverable destructive capability.

Clearly, ($d\mathbf{DL}/d\mathbf{C}^e$) is positive: the level of damage (that a potential aggressor may expect) increases as (his estimation of his potential victim's) total MTE deployments increase. But what of this increase? At the margins, does each additional unit of total MTE, in theory, contribute more (denoted $d^2\mathbf{DL}/d\mathbf{C}^{e2} > 0$), less (denoted $d^2\mathbf{DL}/d\mathbf{C}^{e2} < 0$), or the same (denoted $d^2\mathbf{DL}/d\mathbf{C}^{e2} = 0$) to the level of total damage that the potential victim could inflict on the aggressor than the previous unit of destructive capability? The answer to this question is critical for it will have a bearing on the solution to the problem at hand.

There can be no ex-ante definitive answer to the latter question. In the absence of any empirical data on the destructive effects of multiple blasts, only a tentative answer can be given, an answer based on the speculations of experts in the field. Available studies appear to give support to the contention that each additional unit of destructive capability, in particular total MTE, contributes less ($d^2\mathbf{DL}/d\mathbf{C}^{e2} < 0$) to the level of total damage that the potential victim could inflict on the aggressor than the previous unit of destructive capability. This is made eminently clear in Enthoven and Smith's oft quoted study of defence expenditures which relates various total percentages of Soviet population destroyed with corresponding levels of American MTE.[12] Indeed their data, as shown in Table 1.1, yields an almost perfect logarithmic relationship with total percent of Soviet population destroyed as the dependent variable and total American MTE as the independent variable. Freedman offers a succinct summary of the data: "[a]fter a certain point extra weapons produce few additional effects, because of the concentration of [for example] Soviet society into a finite number of large targets. Doubling the number of 1-megaton warheads from 400 to 800 would increase the destruction of Soviet population and industry by only 9 per cent and 1 per cent respectively. All that was therefore necessary was for US forces sufficient to ride the 'flat of the curve'; anything beyond that would be superfluous."[13] Why diminishing returns to destructive capability? In part, "[o]verlap of the ignition zones from neighbouring explosions would reduce the overall fire area, although this may be counterbalanced by the tendency for nearby conflagrations to generate winds that spread the fires into gaps between them. The dust and smoke caused by one explosion is likely to block some of the heat from a secondary explosion nearby."[14] In any event, one might reasonably assume that a potential aggressor might formulate his damage level projections on the basis of this sort of information. What, then, can this analysis tell us of the functional form of the relationship between a state's deterrent effect and an adversary's estimate of the former's destructive capability?

From Equation 8, $dD/dC^e = dDL/dC^e$, it can be stated that the extent to which a potential aggressor is deterred increases as his estimation of his potential victim's total destructive capability (MTE deployment level) increases (holding all other factors which might impinge on deterrence constant). This is indicated by the fact that since $dDL/dC^e > 0$, it must also be the case that $dD/dC^e > 0$. But at the margins, each incremental increase in a potential aggressor's estimation of the total destructive capability of his potential victim contributes less than each previous unit to the overall extent to which he is deterred. Mathematically, since $dD/dC^e = dDL/dC^e$ and since $d^2DL/dC^{e2} < 0$, it must also be the case that $d^2D/dC^{e2} < 0$. Accordingly, one may employ a reverse in logic to the effect that the functional form of the relationship (primitive) between a state's deterrent effect and its adversary's estimate of the former's deliverable destructive capability is logarithmic. While other functions can have a positive first derivative and a negative second derivative, for example, functions of the form $y = x^{1/a}$ where $a \geq 1$, we will adopt the logarithmic functional form on the basis of the fit of the data in Table 1.1 to the logarithmic model which is almost perfect (see Table 4.1 for the regression analysis results).

Megaton Equivalent (MTE)	Percent Population Destroyed
100	15
200	21
400	30
800	39
1200	44
1600	47

Table 1.1: Soviet Destruction Path Data. Gives percent Soviet population destroyed per given level of American MTE.

Source: Enthoven and Smith

Note we are now back to dealing with (estimated) deliverable destructive capability (MTE) instead of (estimated) total destructive capability (MTE). To reiterate, the difference is that the former measure takes account of offensive-defensive technology levels (measured as a probability) whereas the latter does not. Now when in the foregoing analysis we indicated that "all other factors relevant to deterrence were being held constant" this meant offensive-defensive technology levels thus leaving us with the measure of total destructive capability (MTE). In dealing with a primitive function all variables must be allowed to vary in value. Constancy of technology levels was achieved by means of partial differential calculus, a point which receives substantially more elaboration immediately below.

Before we can present such a set of primitive equations (for a dyad) a few more intermediate steps must be taken. Equation 9 gives the measure of <u>deliverable</u> destructive capability (MTE) for one state, let us call it State A' and Equation 10 gives the measure for another state, let us call it State B, in difference equation form. (We have let **K** serve as the warhead variable for State A and **X** serve as the warhead variable for State B. Distinctions between other variables (and parameters) are indicated with an (') for State B, a practice which will be maintained throughout this study.)

$$CC_t = \sum_{i=1}^{n} K_{i,t} \, Y^{2/3}_{i,t} \, T_{i,t} \tag{9}$$

$$CC'_t = \sum_{i=1}^{n} X_{i,t} \, Y^{2/3'}_{i,t} \, T'_{i,t} \tag{10}$$

Yet Equations 9 and 10 give actual and not expected values and this is a problem because, to reiterate, it is on the expectation of deliverable destructive capability levels that a deterrent effect depends, not on the actual levels. We can, however, find the expected values of these equations thereby allowing us to construct a set of primitive equations describing the deterrent effects of States A and B. There is one technical problem, however. As equations 9 and 10 stand we cannot isolate the individual effects of total MTE (=NY) on deterrent effect from deliverable MTE (=NYT) on deterrent effect. This is because of the summation operator ($\sum_{i=1}^{n}$). If we could, though, for example, remove the offensive-defensive technology variables of States A and B outside of their respective summation operators we could then partial out the individual effect of total MTE. The most straightforward way of doing this is to treat (even proper to differentiation and as distinct therefrom) the offensive-defensive technology levels of States A and B as fixed values (the values would be $\bar{T}$ and $\bar{T}'$ respectively). One way of representing these ideas is by replacing $T_{i,t}$ and $T'_{i,t}$ with their average values $\bar{T}$ and $\bar{T}'$.

Equations 9 and 10 can, then, can be rewritten as follows (with the expectations taken at time t).

$$C^{e\prime}_t = \bar{T}\ E_t\left(\sum_{i=1}^{n} K_{i,t}\ Y^{2/3}_{i,t}\right) \qquad (11)$$

$$C^{e}_t = \bar{T}'\ E_t\left(\sum_{i=1}^{n} X_{i,t}\ Y^{2/3\prime}_{i,t}\right) \qquad (12)$$

The value C^e_t (or alternatively $C^{e\prime}_t$) is the same as that used in Equation 8.

We are, accordingly, now in a position to present a set of equations (primitives) describing the deterrent effects of a pair of rival states. Immediately below, we have set out the relationship for two states, A and B (in difference equation form). Equation 13 gives State A's deterrent effect (D^A_t) as a logarithmic function of State B's estimate of the former's deliverable destructive capability at time t. Similarly , Equation 14 gives State B's deterrent effect (D^B_t) as a logarithmic function of State A's estimate of the former's deliverable destructive capability at time t. Each primitive could, in theory, yield the change in a state's deterrent effect per unit change in its adversary's estimate of the former's <u>total</u> destructive capability (MTE) holding other factors (including offensive-defensive technology levels) constant.

$$D^A_t = \log_b[\bar{T}\ E_t(\sum_{i=1}^{n} K_{i,t}\ Y^{2/3}_{i,t})] \tag{13}$$

$$D^B_t = \log_a[\bar{T}'\ E_t(\sum_{i=1}^{n} X_{i,t}\ Y^{2/3'}_{i,t})] \tag{14}$$

The relationship between a state's deterrent effect and an adversary's estimate of the former's deliverable destructive capability is represented graphically in Figure 1.1. It would be well to take a moment to consider the properties of this relationship, in particular, to consider how they stand up against established theory.

How, for example, is it postulated that changes in State B's expectation of State A's destructive capability impact on State A's deterrent effect (the extent to which State B is deterred)? Figure 1.1 gives a graphical representation of the relationship. Note that the function displays the property of "diminishing returns." Holding all other factors which may impinge upon State A's deterrent effect constant, each additional unit of destructive capability contributes less than the previous unit to the total level of deterrence. This is entirely consistent with the notion of "over-kill." To quote Holsti on this point, "the proposition that the capacity to destroy a potential adversary or coalition of opponents five, ten, or 100 times over increases caution [or deterrence] proportionately seems untenable."[15] On the other hand, as State A's

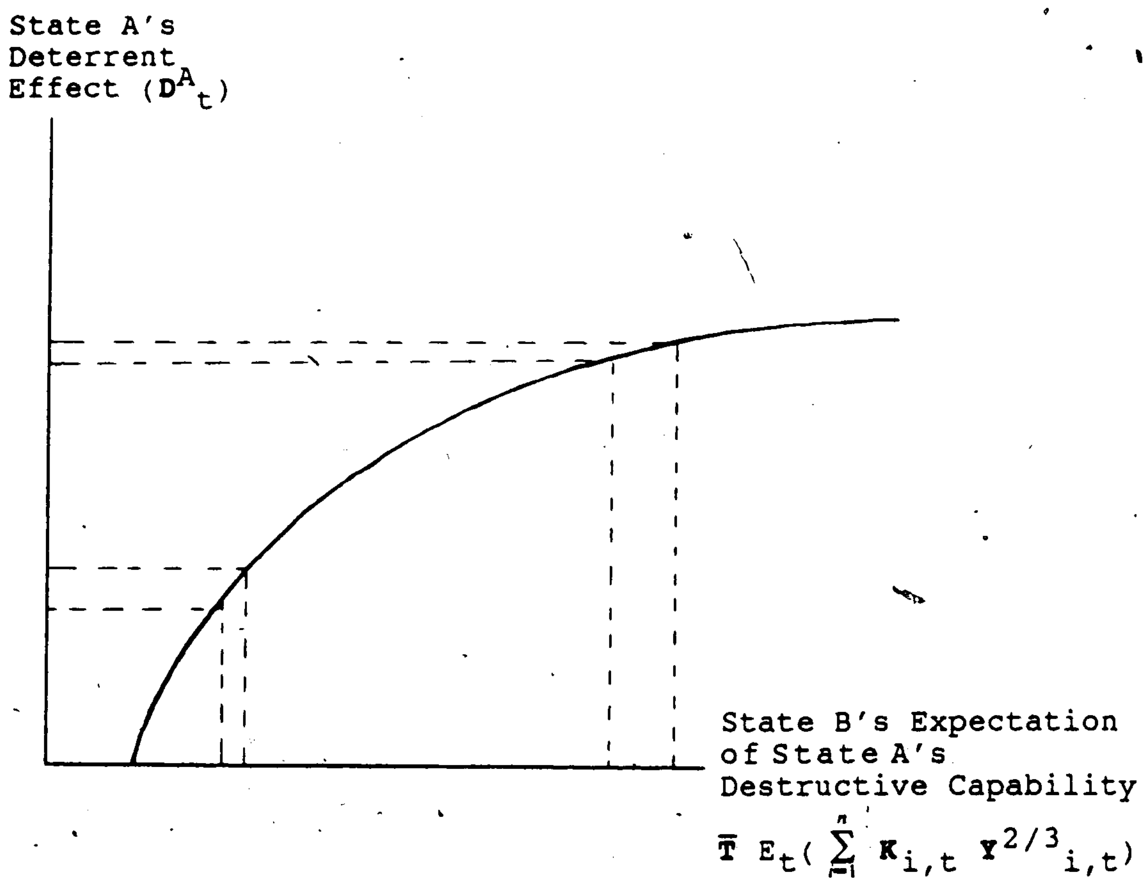

Figure 1.1: Deterrent Effect Curve: The same relationship can be said to hold with respect to State B's deterrent effect and State A's expectation of the former's destructive capability

capability level declines, the extent to which State B is deterred decreases, but at an increasing rate. This tends to agree with, if not confirm, the views of Bull, Kissinger, Schelling, and Snyder to the effect that "changes [decreases] at low levels of forces [capabilities] could be more destabilizing than changes [decreases] at high [initial] levels."[16] The idea here, which is reflected in Figure 1.1, is that changes in initially high capability levels (as estimated by a potential aggressor) cause a smaller than proportionate change in the deterrent effect (of a potential victim) than would be the case if change occurred at initially low capability levels. From this, we can, in mathematical terms, state the following stability conditions.

$$dD^{A}_{t}/dC^{e'}_{t} \longrightarrow 0 \qquad (15)$$

$$dD^{B}_{t}/dC^{e}_{t} \longrightarrow 0 \qquad (16)$$

Holding all else constant, Equation 15 states that stability requires that the change in State A's deterrent effect resulting form a change in State B's estimation of the former's <u>total</u> destructive capability approach zero. Equation 16 advances a similar proposition, but from the perspective of State B. Together, Equations 15 and 16 denote a condition of stable mutual deterrence.

We have, accordingly, reached a critical juncture in the development of our consolidated ABAU Model. To reiterate, our purpose is to develop a consolidated mathematical model which measures the theoretical changes in the deterrent effects of each of two states in a dyad (at time period t) per unit change in their respective (previous period) nuclear warhead deployment leveles, holding constant all other factors herein posited to impinge thereon constant. Yet the primitives, Equations 13 and 14, are not, as they stand, suitable for this task. It was necessary, in order to be able to isolate out the individual effects of changes in expected total MTE levels on deterrent effect, to hold offensive-defensive technology constant (prior to differentiation) by using its average value (refer back to Equations 9 and 10 and subsequently to Equations 11 and 12). Now, in order to be able to isolate out the individual effects of changes in expected warhead deployment levels only we must, in addition to holding technology levels constant, hold yields constant. We can work this into Equation 9 by substituting in for $Y^{2/3}{}_{i,t}$ its average value, $\bar{Y}$, thus treating it as a constant value. This ought not effect the value of deliverable MTE too drastically. In any event, it allows us to remove $Y^{2/3}{}_{i,t}$ outside the summation operator. The same could be done for Equation 10. Accordingly, Equations 9 and 10 could be rewritten as follows. Let lower case c_t denote State A's deployment at time t and c'_t denote State B's deployment at time t.

$$c_t = \bar{Y}\,\bar{T} \sum_{i=1}^{n} K_{i,t} \tag{17}$$

$$c'_t = \bar{Y}'\,\bar{T}' \sum_{i=1}^{n} X_{i,t} \tag{18}$$

Now the sums $\sum_{i=1}^{n} K_i$ and $\sum_{i=1}^{n} X_i$ are simply the total warhead deployments of States A and B respectively at time (t). The notation can therefore be simplified as follows.

$$\sum_{i=1}^{n} K_{i,t} = K_t \tag{19}$$

$$\sum_{i=1}^{n} X_{i,t} = X_t \tag{20}$$

So if we substitute Equations 19 and 20 into Equations 17 and 18 and then take the expected value at time (t) of the result the following will obtain.

$$c^{e\prime}_t = \bar{Y}\,\bar{T}\,K^e_t \tag{21}$$

$$c^e_t = \bar{Y}'\,\bar{T}'\,X^e_t \tag{22}$$

Equations 21 and 22 represent a critical intermediate result in the development of our consolidated ABAU Model. The structure of each is such that upon substitution into a primitive we would have the theoretical basis from which we could calculate the theoretical change in a state's deterrent effect per unit change in its (actual) deployment of warheads. (As we shall see, the consolidated ABAU model will not contain expected values; it will contain only actual (past period) values).

We can thus rewrite Equations 13 and 14 as follows (in difference equation form).

$$D^A_t = \log_b(\bar{Y}\ \bar{T}\ K^e_t) \quad (23)$$

$$D^B_t = \log_a(\bar{Y}'\ \bar{T}'\ X^e_t) \quad (24)$$

Equation 23 gives State A's deterrent effect at time (t), denoted D^A_t, as a function of State B's expectation at time (t) of State A's deliverable destructive capability measured by Equation 21. State B's deterrent effect at time (t), denoted D^B_t, can similarly be postulated to be a logarithmic function of State A's expectation at time (t) of State B's deliverable destructive capability as measured by Equation 22. In each case it is assumed, as per the earliest discussions of this chapter, that the adversary estimates his potential victim's resolve in the face of a first strike to be complete. Accordingly, we can derive the following deterrent stability conditions for changes in expectations of <u>warhead deployment levels</u>.

$$dD^{A}_{t}/dK^{e}_{t} \longrightarrow 0 \qquad (25)$$

$$dD^{B}_{t}/dX^{e}_{t} \longrightarrow 0 \qquad (26)$$

Note, finally, the significance of the logarithmic bases (a) and (b) in Equations 23 and 24 (and indeed Equations 13 and 14). The logarithmic bases (a) and (b) are of key significance. (b), for example, is the logarithmic base of State B's destruction path, that is, the curve relating levels of destruction in State B to levels of State A's destructive capability. As indicated by Equation 13, it is also the logarithmic base of State A's deterrent effect. Thus, State B must deter State A along that path. The base (a) can be similarly interpreted with respect to State A.

In the next chapter we will set about deriving equations describing the expectations of a potential victim's warhead deployments, K^{e}_{t} and X^{e}_{t}, being the expectations of States A and B respectively. This will be the final intermediate step in the derivation of the consolidated ABAU Model.

Before concluding this section, however, it is important that we take a moment to bring to the fore an interesting problem concerning the manner in which we have operationalized the concept of deterrent effect. In attempting to postulate the functional form of the relationship between a State's deterrent effect and its adversary's estimate of the former's destructive capability, we were faced with a dilemma. To begin, one class of Arms-Using Models focusing on deterrence is based upon the notion of levels of unacceptable damage (LUD). Legault and Lindsey take this level to be constant over a nuclear power's stated commitments.[17] They let U_x represent the retaliatory capability level required by State X to deter State Y from a first strike. U_x is based on some notion of what State Y should consider an unacceptable level of damage. The level U_y can be seen in a similar light. The net effect of two opposed arsenals is that there will be either (1) no deterrence if both States X and Y possess capability levels below U_x and U_y, (2) only State X is deterred while State Y is not which occurs when State X's capability level is below U_x and State Y's capability is at or above U_y, (3) the reverse occurs where only State Y is deterred while State X is not which occurs when State Y's capability level is below U_y and State X's capability is at or above U_x, (4) a state of mutual deterrence where State X and State Y both posses second strike capabilities, U_x and U_y, respectively. These four possible outcomes are represented in Figure 1.2.[18] The difficulty with this analysis is that a gives range of capability either deters an adversary or it does not.

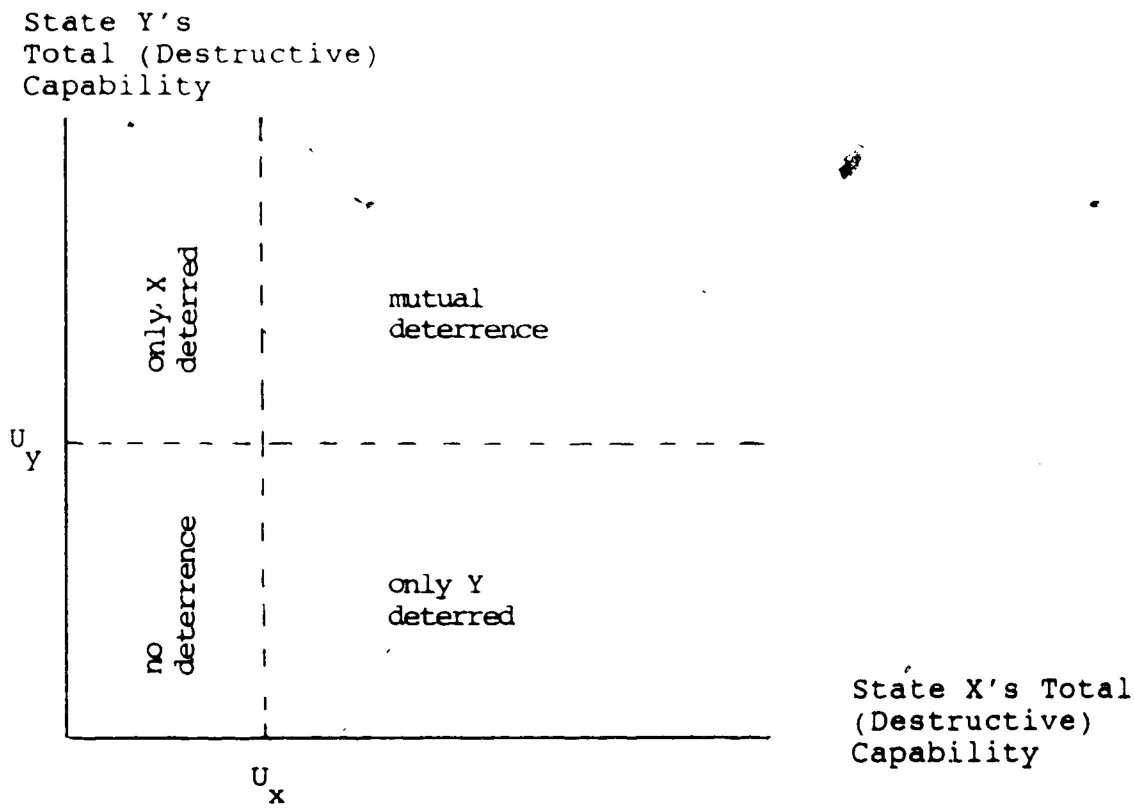

Figure 1.2: Alternative Deterrence Model. Note: Legault and Lindsey measure destructive capability in terms of numbers of missiles

If, for example, U_x was set a 100 megatons, then, in principle, any capability level in the range 1-99 megatons would afford State X no deterrent effect. All other things being equal, for any capability level beyond 100 megatons, State X would effectively be able to deter State Y from a first strike. If one were to plot a curve relating State X's deterrent effect as a function of its destructive capability, the function would be s discontinuous one, a step function, with the discontinuity occurring at the capability level U_x. Either a given capability level deters State Y or it does not. This is shown in Figure 1.3.

Modeling deterrence in this manner makes it impossible to calculate the change in State X's deterrent effect as its destructive capability level changes. Differential calculus cannot be applied to a discontinuous function.

Doran's model of deterrence presents the same problem. He does, however, recognize that in principle estimates of the level of unacceptable damage are not constant "across objectives, as the extension of equivalent commitments to all allies suggests," and that "these levels vary with the value of the objective to the deterrer."[19] Doran does not, however, surmount the discontinuity problem. In recognizing that levels of unacceptable damage (LUD) may vary across objectives, he simply constructs several deterrence models

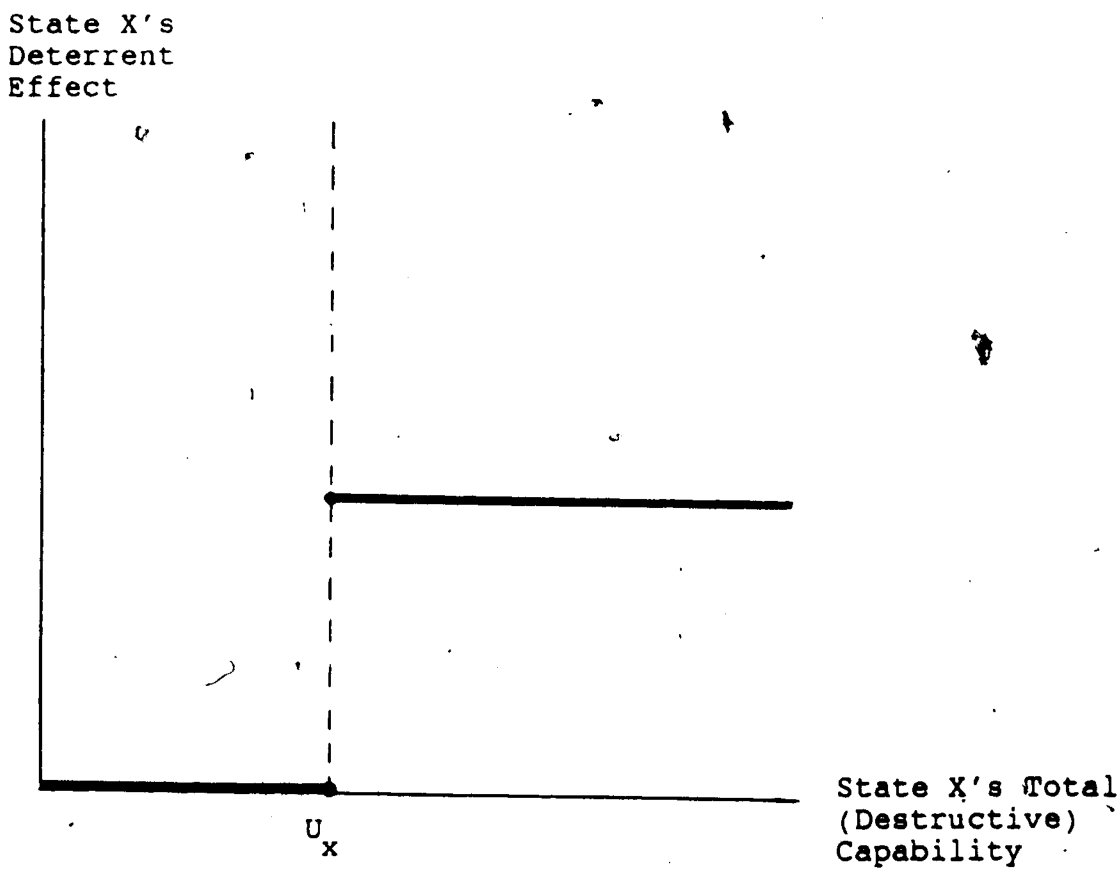

Figure 1.3: The Discontinuity Problem. Note: Legault and Lindsey measure destructive capability in terms of numbers of missiles

each incorporating a different LUD. Indeed the logic of this analysis would seem to preclude the development of a model which would allow both for the notion of LUD's and for the calculation of the change in a state's deterrent effect as its destructive capability level changes. This is the dilemma. In order for this study to progress any further, deterrence cannot be treated as an "either or" concept. At least that aspect of the concept of deterrence must be deemphasized. We must focus exclusively on that aspect of deterrence which provides that different capability levels correspond to different levels of deterrence.

Notes

[1]Snow, Nuclear Strategy in a Dynamic World (Alabama: University of Alabama Press, 1981), p. 108.

[2]Snow, p. 108.

[3]Snow, p. 110.

[4]Snow, p. 100.

[5]K.J. Holsti, International Politics, 4th ed. (Englewood Cliffs: Prentice-Hall Inc., 1983), p. 277.

[6]Holsti, p. 277.

[7]K.D. Moll and G.M. Luebbert, "Arms Race and Military Expenditure Models," in Journal of Conflict Resolution (Vol. 24, No. 1: March 1980), p. 155.

[8]R. Jervis, "Deterrence and Perception," in Strategy and Nuclear Deterrence, ed. S.E. Miller (Princeton: Princeton University Press, 1984), p. 75.

[9]Jervis, p. 64.

[10]Holsti, p. 288.

[11]Holsti, p. 288.

[12]A.C. Enthoven and K.W. Smith, How Much Is Enough? (New York: Harper & Row, 1971), p. 207.

[13]L. Freedman, The Evolution of Nuclear Strategy (New York: St. Martin's Press, 1983), p. 247.

[14]O. Greene, I. Percival, and I. Ridge, Nuclear Winter (Cambridge: Polity Press, 1985), p. 27.

[15]Holsti, p. 290.

[16]J.E. Dougherty and R.L. Pfaltzgraff, Contending Theories of International Relations, 2nd ed. (New York: Harper & Row, 1981), p. 394.

[17]A. Legault and G. Lindsey, The Dynamics of the Nuclear Balance (Ithaca: Cornell University Press, 1974), pp. 167-173.

[18]Taken in modified form from: Legault and Lindsay, p. 173.

[19] C. Doran, "A Theory of Bounded Deterrence," in Journal of Conflict Resolution (Vol. 17, No. 2: June 1973), p. 245.

Chapter 2: Developing an Arms-Building Model

The purpose of this chapter is to derive a set of arms-building equations describing an adversary's expectation of his potential victim's warhead deployment at any time period t. K^e_t and X^e_t will denote these expectations for States B and A respectively. Since we will wish to integrate these equations with those developed in the previous chapter, care must be taken to maintain a logical consistency in formulation across to the present effort. To begin, Moll and Luebbert caution that in attempting to link an Arms-Using Model such as that developed in the previous section and an Arms-Building Model such as that which we are now set upon developing, "a logical criterion for achieving an intermediate objective must link the two types of model. To illustrate, if deterrence is the objective of an armament policy, then some specific criterion for using arms to achieve deterrence must be specified (e.g., second strike counterforce capability or mutually assured destruction). Only through such criterion can the Arms-Using Model be related to requirements of armament policy."[1] Here, the logical criterion linking the two models will be mutually assured destruction. The Arms-Using Model developed in the previous section was indeed based on the premise of mutually assured destruction.

Ultimately we will wish to consolidate the Arms-Building Model developed in this chapter with the Arms-Using Model developed in the first chapter. Recall that for States A and B, respectively, the Arms-Using Model took the form

$$D^{A}_{t} = \log_{b}(\bar{Y}\ \bar{T}\ K^{e}_{t}) \tag{23}$$

$$D^{B}_{t} = \log_{a}(\bar{Y}'\ \bar{T}'\ X^{e}_{t}) \tag{24}$$

where ($\bar{Y}$ and $\bar{T}$) and ($\bar{Y}'$ and $\bar{T}'$) refer to average explosive yield and offensive-defensive technology levels of States A and B respectively. Now, to reiterate, in this chapter, we will take upon ourselves the task of developing a set of equations describing State B's expectation of State A's warhead deployment at time t, denoted K^{e}_{t}, and State A's expectation of State B's warhead deployment at time t, denoted X^{e}_{t}. Thereupon, these equations could simply be substituted into Equations 23 and 24 respectively. We will, therefrom, obtain our consolidated ABAU Model by then taking the partial derivative of D^{A}_{t} with respect to K^{e}_{t} (or more accurately, as will be seen in the next chapter, with respect to K_{t-1}, the actual previous period level of State A's warhead deployment: as we shall soon argue K^{e}_{t} is a function, inter alia, of K_{t-1}). This yields the theoretical change in State A's deterrent effect per unit change in the level of its warhead deployment, holding constant the levels of explosive yield (Y) and offensive-defensive technology (T). Similarly, we will calculate the partial derivative of D^B_t with respect to X^e_t (or more accurately, as will be seen in the next chapter, with respect to X_{t-1}, the actual previous period level of State B's warhead deployment: as we shall soon argue X^e_t is a function, inter alia, of X_{t-1}). This yields the theoretical change in State B's deterrent effect per unit change in the level of its warhead deployment, holding constant the levels of explosive yield (Y') and offensive-defensive technology (T'). But note that partial differentiation can only be properly applied under certain conditions.

"The partial derivative measures the instantaneous rate of change of the dependent variable ... with respect to one of the independent variables [in a multivariable function] ... when the other independent ... variables ... are assumed held constant [emphasis included]."[2] Thus partial differentiation requires that given a multivariable function $y = f(x_1, x_2, x_3, \ldots x_n)$, "the variables x_i ($i = 1,2, \ldots n$) are all independent of one another, so that each can vary by itself without affecting the others [emphasis included]."[3] Otherwise as, say, x_1 varied in dy/dx_1, the assumption that all other x_i are constant (i.e., not changing in value) may not hold. It may be, for example, that x_3 varies as a result of x_1 varying, that variation denoted dx_1. It is, therefore, essential that we be able to formulate $\mathbf{K}^e_t$ and $\mathbf{X}^e_t$ independently of ($\mathbf{Y}$ and $\mathbf{T}$) and ($\mathbf{Y}'$ and $\mathbf{T}'$) respectively such that when we do calculate the partial derivatives $d\mathbf{D}^A_t/d\mathbf{K}_{t-1}$ and $d\mathbf{D}^B_t/d\mathbf{X}_{t-1}$ it can be said that indeed ($\mathbf{Y}$ and $\mathbf{T}$) and ($\mathbf{Y}'$ and $\mathbf{T}'$) are in theory being held constant. The soundest way to proceed on this matter would be to formulate $\mathbf{K}^e_t$ under conditions where $\mathbf{Y}$ and $\mathbf{T}$ are in fact held constant thereby making its absolute value at any time t completely independent of $\mathbf{Y}$ and $\mathbf{T}$ and similarly for $\mathbf{X}^e_t$ with respect to $\mathbf{Y}'$ and $\mathbf{T}'$. Thereupon $\mathbf{K}^e_t$ and $\mathbf{X}^e_t$ can be substituted into Equations 23 and 24, respectively. That this needs to be done is a technical-mathematical requirement of partial differentiation. But is there any real world precedent for holding explosive yield and offensive-defensive technology levels constant over time? Indeed there is.

It has long been known that at one point the composition of American and Soviet MTE had changed over time with the total MTE level remaining fairly constant. Once it was learned that the area of destruction does not increase proportionately with explosive yields, both sides began to deploy more warheads while keeping total yield approximately the same (each warhead was to contain a smaller explosive charge or yield). Under these conditions, any time that one side increased its warhead deployment the other side would in theory had to follow suit in order to maintain a strategic equilibrium. (More will be said on the matter of addressing strategic disequilibriums in conjunction with the following discussion of offensive-defensive technology levels). But note, for now, that we do not wish the particular circumstances of this example to serve as a basis for what follows. Instead, we wish to use it only to show that, in principle, yield levels can in fact, in practice, be held (fairly) constant over time.

The assumption that technology levels are fixed over a period of time is not unreasonable. The R&D process can be a lengthy one, with technological advances having their practical impact only upon deployment. "For a major strategic weapons system," writes Snow, "this process takes eight to ten years from project initiation to weapons delivery (initial operating capability or IOC)."[4] In the mean time, the short-run, then, strategic disequilibriums cannot be redressed by technological developments. In order to maintain a level of destructive capability sufficient in each successive time period to maintain a strategic equilibrium, a potential victim state must link its deployment of destructive capability in time period (t) to that of its adversary in time (t). This formulation is based on the view, supported by Caspary, that "[a]t a given state of military technology there is, in principle, some minimum ratio of forces that is sufficient for defense [or deterrence]."[5] Indeed the ratio should be a positive one such that a state's <u>required</u> level of deployment, denoted K^*_t, and X^*_t, for States A and B respectively, is an increasing linear function of the <u>actual</u> deployment level of that of its adversary, denoted, for States A and B, respectively, by K_t and X_t. This is the simplest possible case. On a related matter, Caspary has noted, a "nation [thus modeled] is represented as not simply seeking arms equality, but striving to maintain at least a constant ratio between its forces and its opponent's. Since a certain ratio of offensive to defensive firepower is generally necessary to win--or defend--an objective, this representation is not unreasonable."[6] Mathematically, the specified relationship is expressed in Equations 27 and 28 representing State A's and State B's required (to maintain equilibrium) deployment levels at time (t).

$$K^*_t = B_0 + B_1 X_t, \quad B_0 \in R, \quad 0 \leq B_1 < 1 \tag{27}$$

$$X^*_t = B'_0 + B'_1 K_t, \quad B'_0 \in R, \quad 0 \leq B'_1 < 1 \tag{28}$$

To continue, we note that in Equation 27, the coefficient B_0 indicates the warhead deployment level required by State A in time period (t) independent of State

B's deployment level. It is equal to some real number, R. The coefficient B_1 is the counter-deployment coefficient. it indicates the degree to which (ratio) State A must redress State B's deployment levels in order to maintain a strategic equilibrium. For example, if $B_1 = 1/3$, this would indicate that for every three units of destructive capability deployed by State B, State A must counter-deploy one unit in order to maintain a strategic equilibrium. The value range imposed on B_1, $0 \leq B_1 < 1$, presumes that exact parity or even superiority is not required to maintain a strategic equilibrium. Schelling has quite ingeniously demonstrated that in order to attain a first strike capability, a state would have to deploy, at any given time period, an amount far in excess ($B_1 > 1$) of that of its adversary.[7] Thus, the value restriction on B_1, namely, $0 \leq B_1 < 1$, is consistent with an armament policy aimed at maintaining a regulatory capability. The coefficients B'_0 and B'_1 can be similarly interpreted with regard to State B.

It is important to emphasize at this point that the variables K^*_t and X^*_t are, respectively, State A and B's required levels of deployment. And accordingly, it must be noted that actual deployment levels, denoted K_t and X_t, may not equal required deployment levels during the time period (t). Account must be taken of this possibility. This can be done by reference to the stock adjustment hypothesis, given immediately below (and adapted for the present analysis with the appropriate notation).

$$\mathbf{K}_t - \mathbf{K}_{t-1} = S[\mathbf{K}^*_t - \mathbf{K}_{t-1}], \quad 0 \leq S \leq 1 \tag{29}$$

$$\mathbf{X}_t - \mathbf{X}_{t-1} = S'[\mathbf{X}^*_t - \mathbf{X}_{t-1}], \quad 0 \leq S' \leq 1 \tag{30}$$

In this model, Nerlove posits that actual changes in stock levels (or for our purposes, changes in deployment levels) over one time period, denoted here for State A by the difference ($\mathbf{K}_t - \mathbf{K}_{t-1}$) may not equal desired changes, given by ($\mathbf{K}^*_t - \mathbf{K}_{t-1}$).[8] The coefficient of adjustment, (S), would indicate the extent to which actual changes in State A's deployment levels mirrored required changes. For example, S = 1, actual changes would in fact be equal to the required change. In fact, adjustment to the required level of deployment, based on a change in an adversary's level of deployment, would occur in the same time period. If S = 0, there would be no change in deployment levels between time (t-1) and (t). For values between zero and one, adjustment, based on changes in an adversary's deployment level, would only be partial thus necessitating a carry over into the next time period. Finally, if the deployment coefficients (S) and (S') took on values between zero and one, this would indicate that there were, perhaps, economic-technological constraints at work preventing the indicated state from meeting its required deployment levels, levels determined on the basis of those of an adversary.

Substituting Equation 27 into Equation 29, a solution can be found for the level $\mathbf{K}_t$, State A's actual deployment level. Given,

$$\mathbf{K}_t - \mathbf{K}_{t-1} = S[B_0 + B_1\mathbf{X}_t - \mathbf{K}_{t-1}] \tag{31}$$

Then, by multiplying out Equation 31 and setting the result in terms of $\mathbf{K}_t$, we obtain,

$$\mathbf{K}_t = SB_0 + SB_1\mathbf{X}_t + (1-S)\mathbf{K}_{t-1} \tag{32}$$

Similarly, a solution can be found for the level $\mathbf{X}_t$, State B's actual deployment level. By substitution,

$$\mathbf{X}_t - \mathbf{X}_{t-1} = S'[B'_0 + B'_1\mathbf{K}_t - \mathbf{X}_{t-1}] \tag{33}$$

Therefore, State B's actual deployment level in time (t) is given by,

$$\mathbf{X}_t = S'B'_0 + S'B'_1\mathbf{K}_t + (1-S')\mathbf{X}_{t-1} \tag{34}$$

From Equations 32 and 34 it becomes evident that if the coefficients of adjustment, respectively, (S) and (S'), were equal to one, then Equation 32 would reduce to Equation 27, the required level of deployment for State A and Equation 34 would reduce to Equation 28, the required level of deployment for State B. For any values of (S) and (S') less than one, the actual level of deployment would be less than the required level of deployment at time (t). The terms $(1-S)\mathbf{K}_{t-1}$ and $(1-S')\mathbf{X}_{t-1}$ are carry-over terms each of which would reduce to zero if the respective state could fully adjust to the changes in its adversary's deployment levels in the same time period. We are now in a position to determine $\mathbf{K}^e_t$ and $\mathbf{X}^e_t$, respectively, State B's and State A's estimation or expectation of the other's deployment level at time (t).

Under the Rational Expectations Hypothesis, the expectations $\mathbf{K}^e_t$ and $\mathbf{X}^e_t$ would be formed at time period (t-1) and would be based on information sets, I_{t-1} and I'_{t-1}, the former being State A's information set, the latter belonging to State B. Mathematically,

$$\mathbf{K}^e_t = E_{t-1}(\mathbf{K}_t \,/\, I'_{t-1}) \tag{35}$$

$$\mathbf{X}^e_t = E_{t-1}(\mathbf{X}_t \,/\, I_{t-1}) \tag{36}$$

"The hypothesis of Rational Expectations asserts that the unobservable subjective expectations of individuals are exactly the true mathematical conditional expectations implied by the model itself. Individuals act as if they know the model [that is Equations 32 and 34] and form expectations accordingly."[9] Indeed Equations 32 and 34, reproduced immediately below, would be contained in the information sets I_{t-1} and I'_{t-1}.

$$K_t = SB_0 + SB_1 X_t + (1-S)K_{t-1} \qquad (32)$$

$$X_t = S'B'_0 + S'B'_1 K_t + (1-S')X_{t-1} \qquad (34)$$

Thus State B's expectation of State A's actual warhead deployment in time period t, given the information contained in set I'_{t-1}, can be determined as follows: it is denoted $K^e_t = E_{t-1}(K_t)$.

$$E_{t-1}(K_t) = SB_0 + SB_1 E_{t-1}(X_t) + (1-S)K_{t-1} \qquad (37)$$

Note at this point that in our formulation of State B's expectations we have avoided committing the "fallacy of the last move." According to Rattinger, one may not validly assume that "[o]nly current and past--but not projected--force levels ... enter into a state's decision"[10] calculus. In Equation 37, State B's expectations at the end of time period (t-1) are comprised of an estimate of its own future armament level $E_{t-1}X_t$ and of an accounting of State A's actual deployment level during time period (t-1), $(1-S)K_{t-1}$, a value which at the end of time period (t-1) would be known to State B. Yet the value $E_{t-1}X_t$ is not unknown to State B. $E_{t-1}X_t$ is simply the expected value or expected level of its own deployment in time (t). This is given by Equation 38.

$$E_{t-1}(X_t) = S'B'_0 + S'B'_1E_{t-1}(K_t) + (1-S')X_{t-1} \tag{38}$$

Accordingly, the value $E_{t-1}(K_t)$ can then be solved for by substituting for $E_{t-1}(X_t)$ in Equation 37. The end result is as follows.

$$E_{t-1}K_t = \frac{(SB_0 + SB_1S'B'_0)}{1-SB_1S'B'_1} + \frac{SB_1(1-S')X_{t-1}}{1-SB_1S'B'_1} + \frac{(1-S)K_{t-1}}{1-SB_1S'B'_1} \tag{39}$$

Of note is the fact that State B's expectation at (the end of) time (t-1) of State A's future warhead deployment level is now a function of values which would at the end of time (t-1) be known to it, namely, its own deployment level during (t-1), X_{t-1}, and State A's deployment level during (t-1), K_{t-1}. The value $E_{t-1}K_t$ therefore has a practical significance. Similarly State A's expectation of State B's actual deployment in time period t, denoted $X^e_t = E_{t-1}(X_t)$, can be so determined, yielding the result,

$$E_{t-1}X_t = \frac{(S'B'_0 + S'B'_1SB_0)}{1-SB_1S'B'_1} + \frac{S'B'_1(1-S)K_{t-1}}{1-SB_1S'B'_1} + \frac{(1-S')X_{t-1}}{1-SB_1S'B'_1} \tag{40}$$

At this point it is worth pausing to note a few additional properties of Equations 39 and 40.

Each equation, as already indicated, is a function of previous period deployment levels. But there is more to it than that. These previous period deployment levels, $\mathbf{K}_{t-1}$ and $\mathbf{X}_{t-1}$, are each multiplied, inter alia, respectively, by the coefficients (1-S) and (1-S') thereby making them, respectively, the carry-over terms of States A and B. What is particularly interesting is that when (S) and (S') each approach one, other things being equal, State B's expectation at time t-1 of State A's current (or rather upcoming) period deployment level, $E_{t-1}\mathbf{K}_t$, and indeed State A's expectation at time t-1 of State B's current period deployment level, $E_{t-1}\mathbf{X}_t$, <u>begin to fall</u> until such time that $\mathbf{K}_{t-1}$ and $\mathbf{X}_{t-1}$ are eliminated from Equations 39 and 40 leaving the constant values,

$$E_{t-1}\mathbf{K}_t = \frac{B_0 + B_1B'_0}{1-B_1B'_1} \tag{41}$$

$$E_{t-1}\mathbf{X}_t = \frac{B'_0 + B'_1B_0}{1-B_1B'_1} \tag{42}$$

These constant values are in fact <u>equilibrium deployment levels</u>. They occur when each side is fully able to adjust, i.e., counter-deploy, to changes in its adversary's deployment in the same time period thereby always maintaining a strategic equilibrium: this would occur when $S = S' = 1$. These properties should be kept in mind for they will be inherited by the consolidated ABAU Model, Equations 39 and 40 being component parts thereof.

There are still other properties of Equations 39 and 40 which must be emphasized. To begin, the notation can be simplified somewhat by letting,

$$P_1 = \frac{(SB_0 + SB_1S'B'_0)}{1-SB_1S'B'_1} \qquad (43)$$

$$P_2 = \frac{SB_1(1-S')}{1-SB_1S'B'_1} \qquad (44)$$

$$P_3 = \frac{(1-S)}{1-SB_1S'B'_1} \qquad (45)$$

Then Equation 39 can be rewritten as

$$E_{t-1}\mathbf{K}_t = f(\mathbf{K}_{t-1}, \mathbf{X}_{t-1}) = P_1 + P_2\mathbf{X}_{t-1} + P_3\mathbf{K}_{t-1} \qquad (46)$$

Similarly, from Equation 40, State A's expectation of State B's actual deployment level at time (t), we can write,

$$P'_1 = \frac{(S'B'_0 + S'B'_1SB_0)}{1-SB_1S'B'_1} \qquad (47)$$

$$P'_2 = \frac{S'B'_1(1-S)}{1-SB_1S'B'_1} \tag{48}$$

$$P'_3 = \frac{(1-S')}{1-SB_1S'B'_1} \tag{49}$$

Thus Equation 40 can be rewritten as

$$E_{t-1}X_t = g(K_{t-1}, X_{t-1}) = P'_1 + P'_2K_{t-1} + P'_3X_{t-1} \tag{50}$$

Equations 46 and 50 reveal quite clearly that the reasoning process followed above has resulted in the derivation of two Richardson-type arms race model in difference equation form (with expectations taken at time (t-1)). That the parameters in Equations 46 and 50 are to be interpreted differently than the parameters in the Richardson model is only a secondary concern, for structurally the equations are the same. Parameters in any event are always open to various interpretations. As such, a number of issues raised in the literature in regard to Richardson's equations must be considered.

Majeski and Jones estimated the parameters of several supposed arms races including the Soviet-American race. The model that they tested was an adaptive expectations model.[11] If one were to truncate this model at (j = 1), to use their notation, it would be identical, structurally, to Equations 46 and 50 in this study and to the Richardson model. More interesting than their parameter estimates was the fact that they discovered the error terms in the Soviet and American equations to be highly cross correlated. This would indicate, among other things, that "[i]f nations react at all to their opponent, they are most likely to respond to the current behavior of their opponent."[12] This would seem to make little intuitive sense, from a practical point of view, as Majeski recognized.[13]

Majeski asks "[i]s this puzzle simply a statistical artifact or ... is there some process at work which can explain ... [it]?"[14] To begin, Equations 27 and 28 both posit instantaneous interaction. Suppose we had encorporated the error terms $\mathbf{U}_t$ and $\mathbf{V}_t$ into Equations 27and 28 respectively. In that case, as a result of the subsequent derivations, Equations 46 and 50 would, respectively, have the following error terms.

$$E_{t-1}\mathbf{Z}_t = m(\mathbf{V}_t, \mathbf{U}_t) = \frac{SS'B_1E_{t-1}\mathbf{V}_t + SE_{t-1}\mathbf{U}_t}{1-SB_1S'B'_1} \qquad (51)$$

$$E_{t-1}\mathbf{Z}'_t = n(\mathbf{V}_t, \mathbf{U}_t) = \frac{SS'B'_1E_{t-1}\mathbf{U}_t + S'E_{t-1}\mathbf{V}_t}{1-SB_1S'B'_1} \qquad (52)$$

Since the error terms $\mathbf{Z}_t$ and $\mathbf{Z}'_t$ are both functions of the same variables, namely, $\mathbf{V}_t$ and $\mathbf{U}_t$, then it is understandable, from a statistical point of view that they should be highly cross correlated. Equations 46 and 50 actually contain the proposition that states are interacting simultaneously as they each contain equations 27 and 28. From an operational perspective it seems that the cross correlation in error terms could be due to the action reaction process itself with the correlation from errors carrying systematically into and between each side's expectations.

Finally, it is worth considering the results of past studies aimed at estimating the parameters of Richardson type models. This would give some indication as to the validity of the reasoning process followed in this study which led to the derivation of Equations 46 and 50. If, for example, parameter estimates were, across studies encompassing many different methodologies, found to be statistically insignificant, some concern should be raised as to the adequacy of our formulation of the arms acquisition process. To begin, there have been a number of problems associated with the estimation of the Richardson model. Hollist's attempt to measure the parameters of the Soviet-American arms race was less than successful. He estimated the parameters of Richardson's reaction, rivalry and submissiveness models.[15] In no case were the parameters significant at the 0.05 level.[16] R^2 values ranged from a low of 0.18 to a high of 0.59.[17] Majeski and Jones, similarly, found, statistically, no interaction in Soviet-American arms acquisitions.[18] In another study, however, under an adaptive expectations hypothesis, Majeski did find that American acquisitions resemble a quasi-Richardson actions-reaction process. But this finding was not paralleled in Soviet acquisitions which in fact appeared to be independent of American acquisitions.[19] Should we be concerned with these results? Should we question the validity of Richardson type arms race formulations?

In each of the cases discussed above, arms expenditures, measured in constant US dollars, served as the operational indicators of arms acquisitions. Moreover, parameters were estimated on the basis of usually very lengthy data series. There are several problems with this methodology. Moll and Luebbert have questioned the efficacy of operationalizing arms expenditures on the basis of aggregate data such as military expenditures data.[20] Aggregate data may lump together opposing acquisition trends for individual weapons systems. "[B]y disaggregating overall military postures into individual services for which multiple indicators are available," writes Rattinger, "it is possible for arms race research to identify reaction processes which not only would have gone unnoticed in aggregate data but also come closer to real-world decision processes."[21] Indeed Rattinger disaggregates arms acquisition data for the Arab-Israeli conflict and thereupon quite successfully estimates the parameters of an action reaction model.[22] Moreover, part of Rattinger's success can be attributed to the recognition that states can engage in temporally distinct arms races. He notes, "[l]umping races together could lead to spurious findings or wash out reaction patterns that are significant but different in both races."[23] Indeed it likely has.

Furthermore, Lucier's ingenious empirical study has revealed that arms race parameters can change over time as a result of changes in armament standard operating procedures (decision rules to be followed by bureaucracies regarding the implementation of armament policy), the replacement of decision makers, or as a result of the occurrence of "a dramatic domestic or international event with manifest implications for armaments."[24] Moll and Luebbert in fact hold that "parameters change fairly frequently."[25] Thus if one were to select a date series spanning, theoretically, the occurrence of a parameter change, it is possible that one would obtain statistically insignificant results. There is no indication in the studies conducted by Hollist and Majeski, nor in the joint study by Majeski and Jones that any regard was paid to the possibility of the selected data series spanning the occurrence of a parameter change.

In summary, when disaggregated data is used and the span of the data series carefully considered so as to contain only data reflecting stable acquisition parameters, the Richardson formulation, as the Rattinger study has revealed, can quite adequately serve as one valid description of arms acquisition processes. We turn now to the consolidation of our Arms-Using and Arms-Building Models.

Notes

[1]K.D. Moll and G.M. Luebbert, "Arms Race and Military Expenditure Models," in Journal of Conflict Resolution (Vol. 24, No. 1: March 1980), p.174.

[2]E.T. Dowling, Mathematics for Economists (New York: McGraw-Hill Book Company, 1980), p. 77.

[3]A.C. Chiang, Fundamental Methods of Mathematical Economics, 3rd ed. (New York: McGraw-Hill Book Company, 1984), p. 174.

[4]D. Snow, Nuclear Strategy in a Dynamic World (Alabama: University of Alabama Press, 1981), p. 17.

[5]W.R. Caspary, "Richardson's Model of Arms Races: Description, Critique, and an Alternative Model," in International Studies Quarterly (Vol. 11, No. 1: March 1967), p. 71.

[6]Caspary, p. 70.

[7]T. Schelling, The Strategy of Conflict (Oxford: Oxford University Press, 1960), p. 236.

[8]D. Gujarati, Basic Econometrics (New York: McGraw-Hill Book Company, 1978), pp. 265-266.

[9]D.K.H. Begg, The Rational Expectations Revolution In Macroeconomics (Baltimore: Johns Hopkins University Press, 1982), p. 30.

[10]H. Rattinger, "From War to War to War," in International Studies Quarterly (Vol. 20, No. 4: December 1976), p. 504.

[11]S.J. Majeski and D.L. Jones, "Arms Race Modeling," in Journal of Conflict Resolution (Vol. 25, No. 2: June 1981), p. 265.

[12]S.J. Majeski, "Expectations and Arms Races," in American Journal of Political Science (Vol. 29, No. 2: May 1985), p. 219.

[13]Majeski, p. 219.

[14]Majeski, p. 219.

[15]W.L. Hollist, "An Analysis of Arms Processes in the United States and the Soviet Union," in

International Studies Quarterly (Vol. 21, No. 3: September 1977), pp. 515-516.

16 Hollist, pp. 514-519.

17 Hollist, pp. 514-519.

18 Majeski and Jones, p. 274.

19 Majeski, p. 232.

20 Moll and Luebbert, pp. 170-171.

21 Rattinger, p. 501.

22 Rattinger, pp. 516-524.

23 Rattinger, p. 513.

24 C.E. Lucier, "Changes in the Values of Arms Race Parameters," in Journal of Conflict Resolution (Vol. 23, No. 1: March 1979), p. 21.

25 Moll and Luebbert, p. 173.

Chapter 3: A Consolidated ABAU Model

The process of consolidating the Arms-Using and the Arms-Building Models developed in the last two chapters is not very complicated. Reproduced immediately below are Equations 23 and 24, Equations 35 and 36, and Equations 46 and 50.

$$\mathbf{D}^A{}_t = \log_b(\bar{\mathbf{Y}}\ \bar{\mathbf{T}}\ \mathbf{K}^e{}_t) \qquad (23)$$

$$\mathbf{D}^B{}_t = \log_a(\bar{\mathbf{Y}}'\bar{\mathbf{T}}'\mathbf{X}^e{}_t) \qquad (24)$$

$$\mathbf{K}^e{}_t = E_{t-1}(\mathbf{K}_t/I'_{t-1}) \qquad (35)$$

$$\mathbf{X}^e{}_t = E_{t-1}(\mathbf{X}_t/I_{t-1}) \qquad (36)$$

$$\mathbf{E}_{t-1}(\mathbf{K}_t) = f(\mathbf{K}_{t-1}, \mathbf{X}_{t-1}) = P_1 + P_2\mathbf{X}_{t-1} + P_3\mathbf{K}_{t-1} \qquad (46)$$

$$\mathbf{E}_{t-1}(\mathbf{X}_t) = g(\mathbf{K}_{t-1}, \mathbf{X}_{t-1}) = P'_1 + P'_2\mathbf{K}_{t-1} + P'_3\mathbf{X}_{t-1} \qquad (50)$$

By Equations 35 and 36, we may substitute Equations 46 and 50 directly into Equations 23 and 24 yielding the following result.

$$\mathbf{D}^A{}_t = \log_b(\bar{\mathbf{Y}}\ \bar{\mathbf{T}}) + \log_b[f(\mathbf{K}_{t-1}, \mathbf{X}_{t-1})] \qquad (53)$$

$$\mathbf{D}^B{}_t = \log_a(\bar{\mathbf{Y}}'\bar{\mathbf{T}}') + \log_a[g(\mathbf{K}_{t-1}, \mathbf{X}_{t-1})] \qquad (54)$$

Taking the partial derivative of $\mathbf{D}^A{}_t$ with respect to $\mathbf{K}_{t-1}$ from Equation 53, denoted, $d\mathbf{D}^A{}_t/d\mathbf{K}_{t-1}$ and the partial derivative of $\mathbf{D}^B{}_t$ with respect to $\mathbf{X}_{t-1}$ from Equation 54, denoted, $d\mathbf{D}^B{}_t/d\mathbf{X}_{t-1}$ we obtain, finally, our consolidated ABAU Model, presented immediately below.

$$\frac{d\mathbf{D}^A{}_t}{d\mathbf{K}_{t-1}} = \frac{(1-S)}{\ln(b)[(SB_0 + SB_1S'B'_0) + SB_1(1-S')\mathbf{X}_{t-1} + (1-S)\mathbf{K}_{t-1}]} \quad (55)$$

$$\frac{d\mathbf{D}^B{}_t}{d\mathbf{X}_{t-1}} = \frac{(1-S')}{\ln(a)[(S'B'_0 + S'B'_1SB_0) + S'B'_1(1-S)\mathbf{K}_{t-1} + (1-S')\mathbf{X}_{t-1}]} \quad (56)$$

Equation 55 gives the theoretical change in State A's deterrent effect, at time period t, denoted $d\mathbf{D}^A{}_t$, per unit change in its (previous period) deployment of destructive capability (warheads), $d\mathbf{K}_{t-1}$. Note that the change in State A's deterrent effect is brought about now only indirectly by a change in expectations (on the part of State B). State A's deterrent effect is directly impacted upon by its own previous period deployment of destructive capability and by its adversary's previous period deployment of destructive capability. State B's expectations remain, however, a function of these values. Similarly, Equation 56 gives the theoretical change in State B's deterrent effect at time period t, denoted $d\mathbf{D}^B{}_t$, per unit change in its (previous period) deployment of destructive capability (warheads),

dX_{t-1}. Note, finally, that Equations 55 and 56 are specified solely in terms of the parameters and variables of Equations 46 and 50 respectively. One may therefore estimate the parameters of Equations 46 and 50 and substitute those values into Equations 55 and 56. That, in fact, is how we intend to operationalize the consolidated ABAU Model. We will in consequence possess a means of assessing change in deterrent effect levels solely on the basis of arms acquisition process parameters. In effect, we can assess changes in deterrent effect levels without ever having any data on actual deterrence levels. But before undertaking this task we ought to consider more thoroughly the properties of Equations 55 and 56 (the consolidated ABAU Model).

Accordingly, one should note the existence of a time lag between the occasion of a change in a potential victim's destructive capability and the subsequent change in the extent to which a potential aggressor is deterred (holding all else constant). This suggests the existence of what might be called a "deterrence-cycle," the period of adjustment to a new level of deterrence once it has been recognized that a change has occurred in a potential victim's level of destructive capability. In this model, a deterrence-cycle lasts for two time periods, since,

$$dD^A{}_t = D^A{}_t - D^A{}_{t-1} \tag{57}$$

$$dK_{t-1} = K_{t-1} - K_{t-2} \tag{58}$$

Similarly,

$$dD^B{}_t = D^B{}_t - D^B{}_{t-1} \tag{59}$$

$$dX_{t-1} = X_{t-1} - X_{t-2} \tag{60}$$

A deterrence-cycle, then, begins at a time (t-2) and ends at a time (t), two periods later.

How are we to interpret the parameters (and variables) of Equations 55 and 56? First of all, we must consider an appropriate context within which to base our interpretation. To begin the stability conditions

$$dD^A{}_t/dK^e{}_t \longrightarrow 0 \tag{25}$$

$$dD^B{}_t/dX^e{}_t \longrightarrow 0 \tag{26}$$

can be refined in light of current developments. In accordance with Equations 55 and 56 the following stability conditions obtain.

$$dD^A{}_t/dK_{t-1} \longrightarrow 0 \qquad (61)$$

$$dD^B{}_t/dX_{t-1} \longrightarrow 0 \qquad (62)$$

Thus stability can be said to exist when the change in State A's deterrent effect (at time period t) per unit change in its (previous period) warhead deployment approaches zero and similarly for State B. We can within this context proceed to interpret the parameters (and variables) of Equations 55 and 56.

Note first of all that each equation contains the previous period warhead deployment levels of States A and B, respectively, K_{t-1} and X_{t-1}. Equations 55 and 56 thus display the property that as the previous period deployment levels of one or both states increase both states' deterrent effects become more stable: both $dD^A{}_t/dK_{t-1}$ and $dD^B{}_t/dX_{t-1}$ approach zero. This is because an adversary's expectation of his potential victim's current period deployment level is a function of both his own and his potential victim's previous period deployment levels. As these levels increase (either one or the other or both) so too does his current period expectation reagrding his potential victim's deployment level and hence the farther up the latter's deterrent effect function he moves with each upward movement tending toward the flat of the curve.

But more than that the variables $\mathbf{K}_{t-1}$ and $\mathbf{X}_{t-1}$ are, inter alia, respectively, multiplied by the coefficients (1-S) and (1-S'), thereby making them, respectively, State A and B's carry-over terms. Thus it becomes apparent that a state's deterrent effect, according to this model, fluctuates not simply on the basis of changes in its own previous period deployment levels or those of its adversary, but on the basis of fluctuations in its carry-over and that of its adversary. It is a reasonable proposition. Recall that carry-over is that portion of an adversary's previous period deployment which the former could not adjust to, i.e., counter-deploy against, so as to maintain a relative second strike capability and it would appear that a state's deterrent effect, to reiterate, fluctuates on the basis of both its own carry-over and that of its adversary.

That the carry over terms $(1-S)\mathbf{K}_{t-1}$ and $(1-S')\mathbf{X}_{t-1}$ occur in the consolidated ABAU Model raises an interesting possibility. Recall that S and S' are, respectively, State A and State B's adjustment coefficients where $0 \leq S, S' \leq 1$. $S = 1$ indicates that State A is able to fully adjust in the same time period to State B's deployments such that State A has no carry-over: $(1-S)\mathbf{K}_{t-1} = 0$. Similarly, $S' = 1$ indicates that State B is able to fully adjust in the same time period to State A's deployments such that State B has no carry-over: $(1-S')\mathbf{X}_{t-1} = 0$. Now focusing on Equation 55 for the moment note that (1-S) occurs in both the numerator and in the dominator (in the latter case multiplied with $\mathbf{K}_{t-1}$). Note also that (1-S') occurs in the denominator (multiplied by $\mathbf{X}_{t-1}$). Accordingly Equation 55 displays the following properties. As S and S' approach one and the carry-over terms in the denominator of Equation 55 approach zero, $d\mathbf{D}^A{}_t/d\mathbf{K}_{t-1}$ becomes less stable: it tends away from zero. Yet at the same time (1-S) in the numerator would approach zero thereby causing $d\mathbf{D}^A{}_t/d\mathbf{K}_{t-1}$ to approach zero. The latter effect would outweigh the former with the net result being that stability, $d\mathbf{D}^A{}_t/d\mathbf{K}_{t-1} = 0$, would obtain when S = 1. But what does all of this mean in substantive terms?

When S and S' approach and ultimately reach one, the carry-over terms $(1-S)\mathbf{K}_{t-1}$ and $(1-S')\mathbf{X}_{t-1}$ disappear from the denominator of Equation 55. The disappearance of these terms from Equation 55 corresponds to their disappearance from Equation 39. We saw in the last chapter that as S and S' approach and ultimately reach one, State B's expectation of State A's current period deployment level, $E_{t-1}\mathbf{K}_t$, begins to <u>fall</u> until such time that it settles at the equilibrium value.

$$E_{t-1}\mathbf{K}_t = \frac{B_0 + B_1 B'_0}{1 - B_1 B'_1} \qquad (41)$$

It settles at a <u>lower value</u> than that which obtained previously (Equation 39 wherein neither S nor S′ equaled one). Now looking to Equation 55, we see that this translates into a destabilization of State A's deterrent effect: dD^A_t/dK_{t-1} tends away from zero: there is a corresponding movement down the logarithmic deterrent effect function away from the flat of the curve as $E_{t-1}K_t$ falls (toward its equilibrium value). In this model, instability prefaces the movement toward equilibrium deployment levels.

To continue, we note an offsetting stabilizing influence, i.e., dD^A_t/dK_{t-1} tends toward zero, as S in the term (1-S) in the numerator approaches one. As S approaches and ultimately comes to equal one, the numerator of Equation 55 would approach and ultimately equal zero thereby causing dD^A_t/dK_{t-1} to approach and ultimately equal zero where stability obtains. Stability would obtain because S = 1 implies that State A can fully adjust to State B's deployment in the same time period thus according it always, for any deployment level State B might select, a relative second strike capability. In this model, this stabilizing effect dominates. But what happens if, say, S = 1 but S′ does not equal one: that is if State A can fully adjust to State B's deployment while State B cannot with respect to State A.

In this case dD^A_t/dK_{t-1} would equal zero but dD^B_t/dX_{t-1} would not. In the former case there would be stability while in the latter there would not. This means that there can be in this model stability without equilibrium. Equilibrium occurs only when both states' deterrent effects are stable: $dD^A_t/dK_{t-1} = dD^B_t/dX_{t-1} = 0$ which occurs only when $S = S' = 1$ (the occurrence of which yields the constant equilibrium deployment values given by Equations 41 and 42).

The coefficients B_1 and B'_1, the counter-deployment coefficients of States A and B, respectively, are contained in the denominators of both Equations 55 and 56. As the values increase, dD^A_t/dK_{t-1} and dD^B_t/dX_{t-1} approach zero. That is deterrence becomes more stable. This is so since the higher the value of these coefficients, the more closely is a state's deployment level tied to that of its adversary, and therefore, the more likely it is that it could retain a second strike capability. Schelling has in fact demonstrated this to be mathematically the case. Arguing under the assumption that technology levels are fixed, he states: "[f]or anything like equal numbers on both sides, the likelihood of successfully wiping out the other side's missiles becomes less and less as the missiles on both sides increase."[1]

Finally we must consider the terms "ln(a)" and "ln(b)."

These terms given an indication of the marginal contribution of a potential victim's destructive capability to the deterrence of the indicated potential aggressor. (b) is the logarithmic base of State B's destruction path, that is, the curve relating levels of destruction (percent population potentially destroyed) in State B for given levels of State A's destructive capability. As indicated by Equation 23 (or alternatively by Equation 53), (b) is also the logarithmic base of State A's deterrent effect function. Thus State A deters State B along that its own destruction path. The proposition is quite reasonable since deterrence rests on degrees of retaliatory damage. In terms of State B's destruction path this means that each successive unit of State A's destructive capability contributes less to the overall level of retaliatory damage that can be inflicted on State B than the previous unit thereof. The greater the value of ln(b) (relative to ln(a)), the less each successive unit of State A's destructive capability contributes to the overall extent to which State B is deterred. And these two effects parallel each other. So it follows that the larger is ln(b) (relative to ln(a)) the more stable State A's deterrent effect should be relative to State B's for any given change in the level of its destructive capability (as fed through State B's expectations into State A's deterrent effect) cause. This is indeed borne-out mathematically.

Notes

[1]T. Schelling, _The Strategy of Conflict_ (Oxford: Oxford University Press, 1960), p. 237

Chapter 4: Operationalizing and Applying the ABAU Model

The consolidated ABAU Model presented in the last chapter measures, for each of two states in a dyad, the theoretical change in their respective deterrent effects at time period t per unit change in the level of their respective previous period warhead deployments, holding constant all other factors which might impinge thereon. It is comprised of two interactive differential equations. What is of extreme value is that each of these equations is specified in terms of the parameters (and variables) of an independently formulated set of (Richardson-type) equations describing the warhead deployments at any time t of the states in the dyad. These parameters can be easily estimated, independently of the consolidated ABAU Model, using actual Soviet and American warhead deployment (over time) data. Thereupon, these parameters can simply be substituted into the consolidated ABAU Model thus operationalizing it. We are not at all constrained by the complete lack of data on deterrent effect levels. Hence the purpose of this chapter.

Using longitudinal data provided by the Stockholm International Peace Research Institute (SIPRI) on Soviet and American warhead deployments from 1967 to 1984 we will estimate the parameters of Equations 39 and 40 (or equivalently Equations 46 and 50). To reiterate these equations constitute a Richardson-type system of action reaction equations: State A's warhead deployment at time t is postulated to be a function of its own warhead deployment level at time (t-1) and that of State B's at time (t-1). (To be completely accurate Equations 46 and 50 can only be so interpreted if we remove the expectations operator E_{t-1}. Indeed we shall do this prior to the parameter estimation). Similarly State B's warhead deployment at time t is postulated to be a function of its own warhead deployment level at time (t-1) and that of State A's at time (t-1). As per the discussion of the third chapter concerning changes in parameter values over time (due to changes in, inter alia, armament policies, the signing of international arms-control agreements) the parameters of Equations 46 and 50 will be estimated for the period 1967-1972, the period 1973-1979 and the period 1980-1984. Our view is, that each of these periods corresponds to a distinct arms race and hence to a distinct set of arms race parameters (as we shall see this has been empirically borne-out as revealed by the statistically significant differences in the parameters estimates across these periods[1]). By implication these three periods also would correspond to distinct deterrent effect periods: since the consolidated ABAU Model is comprised of these parameters (and variables) any time they change there will result a concomitant change in the value of the ABAU Model. We will thus present three temporally successive estimates the parameters of the ABAU Model, for the Soviet-American dyad.

Indeed there is ample justification for believing that the periods 1967-1972, 1973-1979, and 1980-1984 constitute distinct arms race periods. Note first of all that in the period 1967-1972, the year 1967 was imposed upon us by the fact that SIPRI warhead data does not extend back any farther. This does not alter the fact, however, that the period 1967-1972 represents a period of intense Soviet-American arms competition which likely had its origins in the conclusion of the Cuban Missile Crisis. The crisis clearly demonstrated to the Soviets the limited political and military value of a relatively small nuclear arsenal. After the crisis the Soviets embarked on one of the most ambitious strategic deployment programs ever, culminating in the early 1970s with the attainment of rough strategic parity with the United States. Throughout this period the American arsenal also grew substantially. For the Soviet Union warhead deployment increased form 1000 to 2500 in 1972.[2] For the United States warhead deployment increased from 4500 in 1967 to 5700 in 1972.[3] Indeed the Americans were not undisturbed by the Soviet deployments: "[t]he United States, concerned over the rapid rate at which the Soviets were deploying heavy missiles, wondered whether

Moscow could be induced to level off at strategic parity or would try to achieve a strategic superiority which ... might be useful as a political instrument."[4] Hence the SALT I Agreement and the arms-race paramater period 1973-1979.

Note that while the period 1973-1979 covers a period of arms-control (SALT I was signed in 1972) we used, immediately above, the term arms-race in describing it. The term is validly used. A "practice that undermines the objectives of arms control," writes Keating, "is the concerted effort on the part of both the US and the USSR to work to the limits or edges of any agreement reached. In other words, while both parties seem reluctant to violate the privisions of an arms control accord, neither seems reluctant to do absolutely everything that is not explicitly prohibited by the agreement."[5] Thus while SALT I may have placed limits on the number of launchers (delivery vehicles) that each side may possess it did not place limits on the number of warheads that each side could place on those launchers: MIRV technology was at this time just emerging but was being rapidly adopted. Between 1973 and 1979 Soviet warhead deployments rose from 2200 to 5000 and American warhead deployments rose form 6784 to 9200.[6] Moreover, notes Keating, another "feature of the existing arms control process that has encouraged an expansion of weaponry has been the tendency, readily apparent in the United States, to buy political support for arms control agreements by increasing military expenditures in areas not covered by these agreements."[7] While the provisions of SALT I remain in force today (with possibly the exception of the ABM provisions) there occurred in 1979 a fundamental shift in American attitudes toward the Soviet Union which was to manifest itself in stepped-up military deployment programs. The Soviets no doubt were not oblivious to this development.

With an election in the offing the Carter Administration was obliged to demonstrate to an increasingly critical right-wing its commitment to a militarily strong America. Indeed there was strong domestic pressure emanating from the congress, the public at large, and from domestic interest groups to increase military spending. Generally there was a feeling in America, as skillfully articulated by Ronald Reagan, that the country was in decline and than an ambitious military deployment program would be solution to all ills, in particular, to the perceived increasing militancy of the Soviet Union which many believed to be directly attributable to America's military weakness. We put the policy impact of these developments at about 1979 (and we end at 1984 only because SIPRI data beyond that year was unavailable).

In each of these periods we hold that as arms race parameters (relating specifically to warhead deployments) changed so too did the stability of each state's deterrent effect (measured per unit change in respective warhead deployments). We thus now set-out to estimate the arms-race paramaters (warheads) of each of these periods and subsequently the parameters of our consolidated ABAU Model.

But note, first of all, that the consolidated ABAU Model (Equations 55 and 56) is not specified only in terms of Equations 46 and 50. Equation 55, which gives the theoretical change in State A's deterrent effect at time t per unit change in its previous period warhead deployment level, also contains, in the denominator, the variable ln(b), which is the logarithmic base of State B's destruction path, the path relating total percent population destroyed, denoted generally as **DL**, in State B per given level of megaton equivalent, generally, **MTE**, deployed by State A. (Recall that State A must deter State B along this path as per Equation 23). Equation 56 contains the variable ln(a) which has a corresponding interpretation but from the perspective of State B. The logarithmic bases a and b must be calculated if we are to fully operationalize Equations 55 and 56, the consolidated ABAU Model. It is not a very difficult task.

Letting the United States be State A and the Soviet Union State B we can use the data presented in Table 1.1 which related levels of American MTE (independent variable) to total percentages of Soviet population potentially destroyed (dependent variable) to calculate the base b in Equation 55, which now, we will take to measure the theoretical change in the American deterrent effect at time t per unit change in its previous period warhead deployment level. We can do this through the estimation of the following equations.

$$\mathbf{DL}_{(USSR)} = b_1 + b_2 \ln[\mathbf{MTE}_{(USA)}] + \mathbf{E}, \mathbf{E} = \text{error term} \qquad (63)$$

The place of information that we are particularly interested in extracting from Equation 63 is the estimated coefficient b_2. With this piece of information, the logarithmic base b, the logarithmic base of the Soviet Union's destruction path can be found using the simple formula:

$$\text{base } b = e^{1/b_2}, \quad e = 2.71 \qquad (64)$$

Using data contained in Legault and Lindsey's study of deterrence[8] we can likewise calculate the logarithmic base a, the logarithmic base of America's destruction path. The two corresponding equations are given below.

$$DL_{(USA)} = q_1 + q_2 \ln[MTE_{(USSR)}] + F, \quad F = \text{error term} \qquad (65)$$

and

$$\text{base } a = e^{1/q_2}, \quad e = 2.71 \qquad (66)$$

How, exactly, we are to interpret these values, base a and base b, within the context of the consolidated ABAU Model will be addressed presently.

The Tables 4.1 to 4.5 contain the results of the regression analysis performed in this study.

As can be seen from Tables 4.1 and 4.2 the coefficients b_2 and q_2 from Equations 63 and 65 are 11.925 and 28.328 respectively. Applying the formulas given in Equations 64 and 66 allows us to calculate that the logarithmic base of the Soviet Union's destructive path is b = 1.087 and that the logarithmic base of the American destruction path is a = 1.035. It is interesting to note that the larger a logarithmic base is, the less is the marginal contribution of each additional unit of destructive capability (MTE) the overall level of population destroyed (DL). Accordingly, as per our regression results, it would appear that each additional unit of American MTE contributes less to the overall level of Soviet population that can,

Table 4.1: Estimate of parameters of Soviet destruction path given American MTE

$$\mathbf{DL}_{(USSR)} = b_1 + b_2 \ln[\mathbf{MTE}_{(USA)}] + \mathbf{E}$$

$$\mathbf{DL}_{(USSR)} = -40.966 + 11.925 \ln[\mathbf{MTE}_{(USA)}]$$

$$(2.265) \quad (0.3623)$$

$$R^2 = 0.9963, \; n = 6, \; se = 0.87333$$

Table 4.2: Estimate of parameters of American destruction path given Soviet MTE

$$\mathbf{DL}_{(USA)} = q_1 + q_2 \ln[\mathbf{MTE}_{(USSR)}] + \mathbf{F}$$

$$\mathbf{DL}_{(USA)} = -122.08 + 28.328 \ln[\mathbf{MTE}_{(USSR)}]$$

$$(50.653) \quad (8.1492)$$

$$R^2 = 0.9236, \; n = 3, \; se = 16.972$$

potentially, be destroyed by it than does each additional unit of Soviet MTE with respect to the overall level of American population that it could, potentially, destroy. This may be due in part to the relative concentration of American population and the relatively smaller American land mass. Figure 4.1 contains a comparison of the Soviet and American destruction paths. What if we substitute these values into the consolidated ABAU Model?

Letting Equation 55 serve as the measure of the theoretical change in the American deterrent effect at time t per unit change in its previous period warhead deployment level we would, accordingly, be required to substitute into the denominator the value b = 1.087 indicating that the American deterrent effect functions in accordance with the Soviet destruction path, indeed, that it deters the Soviet Union along that path. As Soviet estimates of American warhead deployments change the American deterrent effect changes corresponding to a concomitant change in the level of Soviet population that could, potentially, be destroyed by those warheads. Similarly, the value a = 1.035 would be substituted into the denominator of Equation 56. Its interpretation therein would parallel that of b in Equation 55. What now immediately becomes clear is that, all other things being equal, a change in American warhead deployments (at time t-1) causes (at time t) a less profound change in

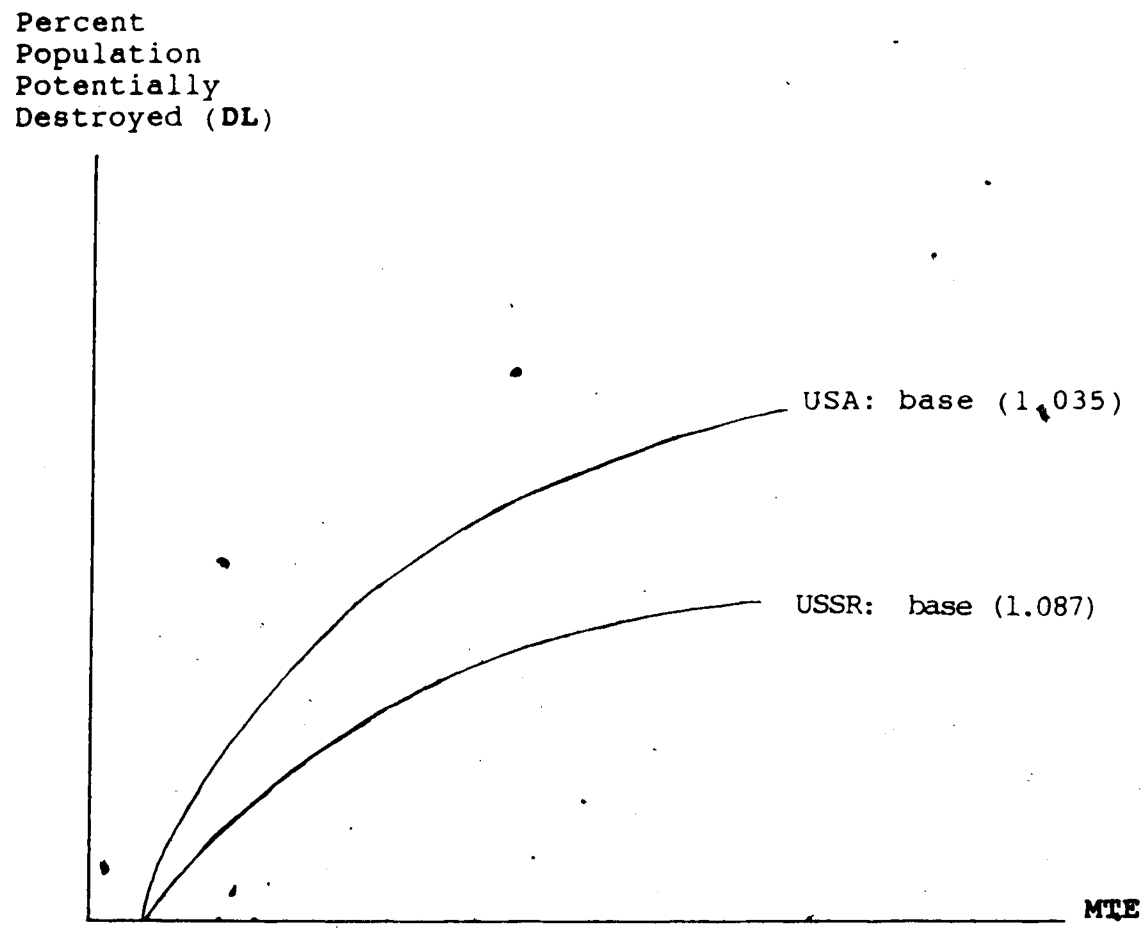

Figure 4.1: US and USSR Destruction Paths. The relative position of the Soviet and American destruction paths depends in large measure on the logarithmic bases of the corresponding equations describing those relationships.

Note: one may wish to compare this plot to that contained in Figure 1.1. The curve relating percent population potentially destroyed in State B for a given level of State A MTE should exactly match (assuming k = 1 from the first chapter) the curve relating State A's deterrent effect to State B's expectation of the former's destructive capability (MTE).

its deterrent effect than does a change in Soviet warhead deployments with respect to its deterrent effect. This is because, to reiterate, the marginal contribution of an additional unit of Soviet destructive capability to overall level of American population that could be destroyed by it is higher than in the reverse case, thus, causing a more profound positive or negative change in the Soviet deterrent (at time t) effect for a given positive or negative change in its previous period warhead deployment level. Finally, it should be noted that there is, in theory, no reason why we should not expect a and b to be constant over the arms race periods being herein examined. But what of the parameter estimates of those arms races?

Tables 4.3, 4.4 and 4.5 reveal an interesting story about Soviet and American warhead deployments over the years 1967 to 1984.[9] To begin, in all three arms race periods 1967-1972, 1973-1979, and 1980-1984 only one variable is significant in the Soviet warhead deployment equations. In each case Soviet warhead deployment at a time period t, X_t, is a function only if its own warhead deployment level in the previous time period t-1, X_{t-1}. This result is not surprising: it indicates that bureaucratic momentum or internal organizational processes account for much of the Soviet Union's military deployment policies, a view which has been amply argued in the literature. Indeed Majeski

Table 4.3: Parameter estimates of Soviet and American warhead deployments at time t for the arms race period 1967-1972.[10]

USA: $\mathbf{K}_t = P_1 + P_2\mathbf{X}_{t-1} + P_3\mathbf{K}_{t-1} + \mathbf{Z}_t$

$$\mathbf{K}_t = -2521 + 1.907\mathbf{X}_{t-1} + 1.2351\mathbf{K}_{t-1}$$

(2237) (0.27212) (0.52047)

$R^2 = 0.9312$, $n = 5$, $se = 235.75$

USSR: $\mathbf{X}_t = P'_1 + P'_2\mathbf{K}_{t-1} + P'_3\mathbf{X}_{t-1} + \mathbf{Z}'_t$

$$\mathbf{X}_t = 535.25 - 0.1181\mathbf{K}_{t-1} + 1.185\mathbf{X}_{t-1}$$

(1299.4) (0.3022) (0.1580)

$R^2 = 0.9657$, $n = 5$, $se = 147.36$

Table 4.4: Parameter estimates of Soviet and American warhead deployments at time t for the arms race period 1973-1979.

USA: $\mathbf{K}_t = P_1 + P_2\mathbf{X}_{t-1} + P_3\mathbf{K}_{t-1} + \mathbf{Z}_t$

$$\mathbf{K}_t = 4904.7 + 0.298\mathbf{X}_{t-1} + 0.33\mathbf{K}_{t-1}$$

(1336) (0.181) (0.212)

$R^2 = 0.8861$, n = 6, se = 235.62

USSR: $\mathbf{X}_t = P'_1 + P'_2\mathbf{K}_{t-1} + P'_3\mathbf{X}_{t-1} + \mathbf{Z}'_t$

$$\mathbf{X}_t = -1959.5 + 0.359\mathbf{K}_{t-1} + 0.841\mathbf{X}_{t-1}$$

(1556) (0.248) (0.211)

$R^2 = 0.9584$, n = 6, se = 274.51.

Table 4.5: Parameter estimates of Soviet and American warhead deployments at time t for the arms race period 1980-1984.

USA: $\mathbf{K}_t = P_1 + P_2\mathbf{X}_{t-1} + P_3\mathbf{K}_{t-1} + \mathbf{Z}_t$

$$\mathbf{K}_t = 14013 + 0.351\mathbf{X}_{t-1} - 0.772\mathbf{K}_{t-1}$$

(1984) (0.055) (0.248)

$R^2 = 0.9816$, n = 4, se = 75.319

USSR: $\mathbf{X}_t = P'_1 + P'_2\mathbf{K}_{t-1} + P'_3\mathbf{X}_{t-1} + \mathbf{Z}'_t$

$$\mathbf{X}_t = 26871 - 2.839\mathbf{K}_{t-1} + 1.054\mathbf{X}_{t-1}$$

(1032) (1.291) (0.290)

$R^2 = 0.9384$, n = 4, se = 391.80

found a similar result using gross military expenditures.[11]

Now the American case is somewhat different. In the first and last arms race periods, 1967-1972 and 1980-1984, American warhead deployment at a time period t, K_t, was/is a function (only) of Soviet warhead deployment in the previous period t-1, X_{t-1}. This indicates a pure arms race on the part of the United States. In the intermediate period, 1973-1979, American warhead deployment at time period t is a constant function, depending neither on its own previous period warhead deployment levels K_{t-1}, nor on that of its adversary, the Soviet Union, X_{t-1}. In the first arms race period America was concerned with the massive Soviet strategic build-up and apparently attempted to counter their warhead deployments directly. In the second period, 1973-1979, which followed on the heels of Vietnam, it seemed that some (basically) constant warhead deployment level was considered sufficient to maintain a strategic equilibrium (perhaps in accordance with SALT I). But, it seems, this view changed in the latest arms race period 1980-1984 where America is again engaged in a pure arms race (in warhead deployments), which may be indicative of the resurgence of the right in American politics since the Vietnam era. Their world view is one of rigid bi-polarity.

Substituting these parameters into the consolidated ABAU Model tells an even more interesting story. Immediately below is the Model fully oprationalized for the

For the period 1967-1972:

USA:

$$\frac{dD_t}{dK_{t-1}} = \frac{0}{\ln(1.087)[0 + 1.1907X_{t-1} + 0]} = 0 \qquad (67)$$

USSR:

$$\frac{dD_t}{dX_{t-1}} = \frac{1.185}{\ln(1.035)[0 + 0 + 1.185X_{t-1}]} = \frac{1}{\ln(1.035)\ X_{t-1}} \qquad (68)$$

For the period 1973-1979:

USA:

$$\frac{dD_t}{dK_{t-1}} = \frac{0}{\ln(1.087)[4904.7 + 0 + 0]} = 0 \qquad (69)$$

USSR:

$$\frac{dD_t}{dX_{t-1}} = \frac{0.841}{\ln(1.035)[0 + 0 + 0.841X_{t-1}]} = \frac{1}{\ln(1.035)X_{t-1}} \qquad (70)$$

For the period 1980-1984:

USA;

$$\frac{dD_t}{dK_{t-1}} = \frac{0}{\ln(1.087)[14013 + 0.351X_{t-1} + 0]} = 0 \qquad (71)$$

USSR:

$$\frac{dD_t}{dX_{t-1}} = \frac{1.054}{\ln(1.035)[0 + 0 + 1.054X_{t-1}]} = \frac{1}{\ln(1.035)X_{t-1}} \quad (72)$$

periods 1967-1972, 1973-1979 and 1980-1984. (Zeros indicate that parameters were not significant at the previously indicated levels. We feel justified in using zeros on the basis of t-statistic values as there was little evidence in the regression results of multicollinearity.)

What is clear from the operationalizaton of Equations 67, 69, and 71, the American ABAU Models, is that there has occurred no theoretical change in the American deterrent effect at time period t, across arms race periods, as its previous period warhead deployment levels changed, holding all else constant. This is indicated by the fact that in each of the USA ABAU equations the numerators equal zero. The corresponding parameter contains the American stock adjustment coefficient S and is structured to equal zero when S = 1. One might recall from the second chapter that when a state's coefficient of adjustment equals one that would indicate that in any time period t the state is capable of fielding sufficient destructive capacility so to maintain in the event that it was to suffer a surprise first strike the ability to inflict correspondingly high levels of damage on the aggressor in retaliation.

On the other hand, Equations 68, 70 and 72 reveal that the Soviet Union was unable to field sufficient capability (warheads) to ensure, in each successive time period, itself a second strike capability. This is indicated by the fact that non-zero values have obtained in the numerators of each of the USSR ABAU equations. In examining the consolidated ABAU Model one notes that, in general, there is a theoretical basis upon which one state may impact the deterrent effect of the other state (i.e., the deterrent effect over its own head, so to speak). For example the equation describing the theoretical change (at time t) in state A's deterrent effect per unit change in its warhead deployment level (at time t-1) is a function not only of its own warhead deployment at time t-1 but also that of its adversary at time t-1, State B, and similarly for State B. Yet, empirically, in the case of the Soviet Union the American warhead deployment coefficient was never significant indicating that America was not and is now still not able to induce changes in the Soviet deterrent effect through its own warhead deployment efforts. Nor can the Soviet Union induce changes in the American deterrent effect through its own warehead deployments, but the reason for this result is not due to the insignificance of the Soviet deployment variable in the American ABAU Models (at least in 1967-1972 and 1980-1984). The problem is that whatever the level of Soviet warehead deployment, America could neutralize its first strike potential through its own counter-deployments (as per the argument of the second chapter). It would appear that one state could only induce a change in the deterrent effect of another through its (warhead) deployments if and only if the other could not directly counter-deploy an amount sufficient to deny the former a first strike potential. The proposition is a reasonable one and the consolidated ABAU Model bears it out. Finally it is clear from Equations 68, 70 and 72 that across arms race periods, the Soviet Union has had the potential to impact its own deterrent effect through increases (or decreases) in its own warhead deployments. Mathematically as the denominator of dD_t/dX_{t-1} increases the ratio tends toward zero or as we have argued it, herein, toward stability. In each arms race period, including the present, an increase in Soviet previous period warhead deployments caused (or causes) the American current period expectation of Soviet deployment levels (the latter being a function of the former) to increase thus moving the USA up along its own destruction path and hence up along the Soviet deterrent effect function toward the flat of the curve and stability.

Notes

[1]The Chow Test was used to test the statistical significance of the parameter estimates across arms race periods. The parameters in the Soviet equations were deemed at alpha = 0.1 to be different across arms race periods. In the American case, the parameter changes across the first two arms race periods was deemed significant at alpha = 0.1 also. But the change across the second and third arms race periods was deemed significant only at alpha = 0.2.

[2]World Armaments and Disarmament 1976 (Stockholm International Peace Research Institute), p.25.

[3]World Armaments and Disarmament 1976. (Stockholm International Peace Research Institute), p.25.

[4]J.E. Dougherty and R.L. Pfaltzgrat, Contending Theories of International Relations, 2nd ed. (New York: Harper & Row, 1981), p.403.

[5]T. Keating, "Abandon arms control talks," in International Perspectives (May/June 1985), p.30.

[6]World Armaments and Disarmament 1981 (Stockholm International Peace Research Institute), p.275.

[7]Keating, p.30.

[8]See A. Legault and G. Lindsey, The Dynamics of the Nuclear Balance (Ithaca: Cornell University Press, 1974), p.170. Note only two data points were given by this source. The third was estimated by the author of this study.

[9]Note: the following discussion will assume a statistical significance level of alpha = 0.10 except in the case of Soviet warhead deployments for the period 1980 to 1984: here we will use a lesser significance level, alpha = 0.20.

[10]Note: The results presented in this table and in the next two are based on the OLS method. In the second chapter we argued that the error terms Z_t and Z'_t should be highly correlated. To be consistent to this chapter we had to consider using the technique of Seemingly Unrelated Regression for the estimations at hand. However because each of the two equations in each of the three arms race periods contains the same variables OLS will suffice.

[11]See S.J. Majeski, "Expectations and Arms Races," in American Journal of Political Science (Vol. 29, No. 2: May 1985).

Chapter 5: Concluding Remarks

We have, accordingly, accomplished what we set out to do: to develop, operationalize, and apply a consolidated mathematical model measuring the theoretical change in a state's deterrent effect (at time period t) per unit change in its (previous period) warhead deployment level, holding constant all other factors which might impinge thereon (for each of two states in a dyad). Insofar as we have been successful we take this accomplishment to constitute a significant advancement in the state of the art of mathematical analyses of interatnional relations. Previous mathematical analyses of arms race stability (or instability) focused on determining the conditions under which the rate of change of armament deployment over time equaled zero, as per the famous (and brilliantly conceived) Richardson model. But these analyses have taken place within a vacuum: no systematic linkage between the conclusions of these studies and the question of the stability (or instability) of the strategic nuclear system could ever made. We may now, however, have at least one window through which to "view the strategic nuclear system ... [from a perspective which allows for the] orderly assessment of changes [in warhead deployments] as they occur and the impact of changes on the overall system."[1] It does, however, remain a weakness of our consolidated ABAU Model that it cannot indicate the approach toward systemic breakdown (or whatever one may wish to call it) because it does not contain, and as we have attempted to argue in the first chapter, no such model could contain, any sense of levels of unacceptable-acceptable damage.

Notwithstanding, much work remains to be done in the area that we have opened up. Indeed the full potential of our work remains yet to be exploited. We must next find the means whereby we can assess the change in a state's deterrent effect allowing all of the variables herein posited to impinge thereon, not simply warhead deployment levels but also destructive yield levels and offensive-defensive technology levels, to vary. Mathematically, this would involve total differentiation rather than partial differentiation. Such a theoretical measure would give a more comprehensive indication of system stability, hence, its value. It will not be an easy task but much of the ground work has already been, herein, laid.

In closing this particular study we leave all of our critics with this one final thought: Richardson once wrote that "[a]ll that can be proved by mathematics is that certain consequences follow from certain abstract hypotheses."[2]

Notes

[1] D. Snow, Nuclear Strategy in a Dynamic World (Alabama: University of Alabama Press, 1981), p. 6.

[2] L.F. Richardson, Arms and Insecurity, eds. N. Rashevsky and E. Trucco (Pittsburgh: The Boxwood Press, 1960), p. 145.

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