

Supplementary Material for

An early warning indicator trained on stochastic disease-spreading models with different noises

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Data Characteristics

White noise data, represented by equation (2.5), simulates random fluctuations that are uniform across time. Environmental noise, represented by equation (2.8), reflects external factors such as environmental disruptions. Demographic noise, represented by equation (2.9), captures temporal variations within population processes, including births, deaths, immigration, emigration, and state transitions. While synthetic data allows for controlled experiments and extensive training sets, real-world data is more complex due to its inherent variability and numerous unpredictable factors. For instance, real COVID-19 data exhibits noise and fluctuations and to mimic these fluctuations, we added random noise intensity to the synthetic data, incorporating different levels of fluctuations. Transition times were randomly chosen between 0 to 1500 in these synthetic time series. Since the exact time of transition in real-world diseases like COVID-19 is unknown, these random transition points enable the deep learning model to anticipate critical transitions despite the uncertainty in transition times.

Supplementary figures

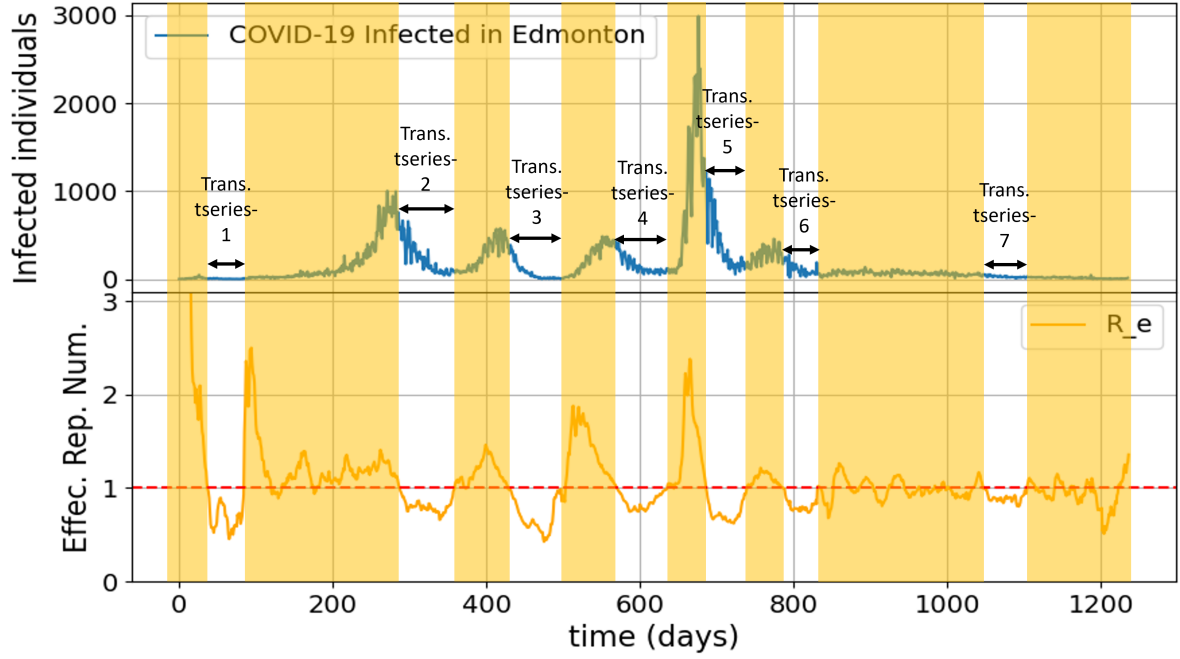


Figure S1: Empirical data of COVID-19 in Edmonton and the calculated effective reproduction number using EpiEstim package. Based on $R_e < 1$, we extracted daily cases until $R_e = 1$. Each sub-series is labeled as transcritical, representing the cases just before the bifurcation point, i.e., $R_e = 1$.

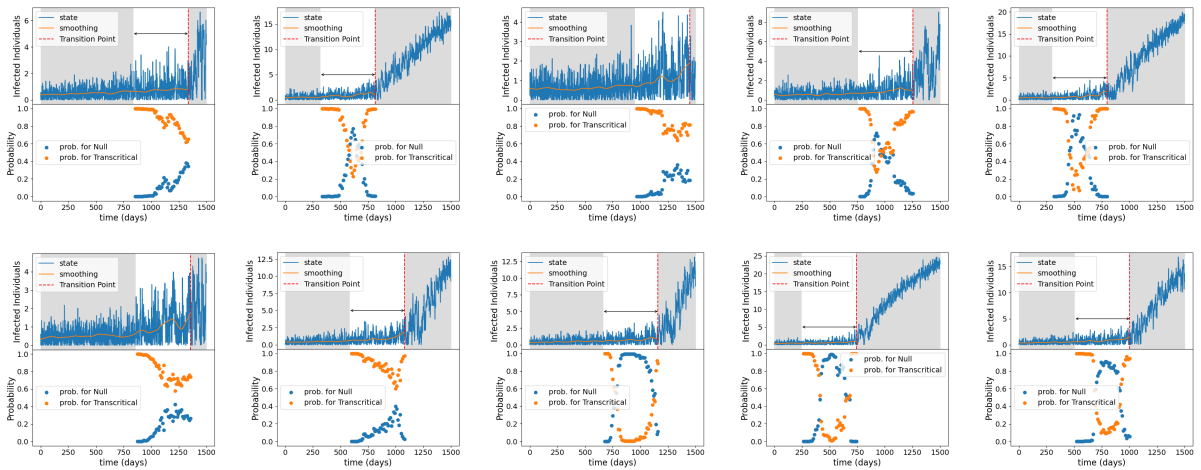


Figure S2: Probabilities for a transition assigned by the SIDATR-500 DL model based on the subset of observations of transcritical simulations of the SIR model with additive white noise.

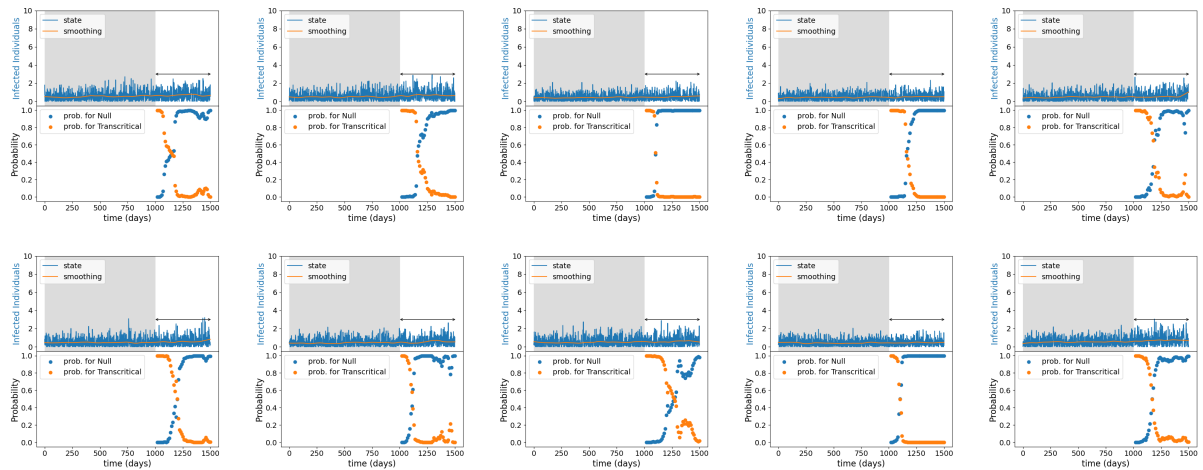


Figure S3: Probabilities for a transition assigned by the SIDATR-500 DL model based on the subset of observations of null simulations of the SIR model with additive white noise.

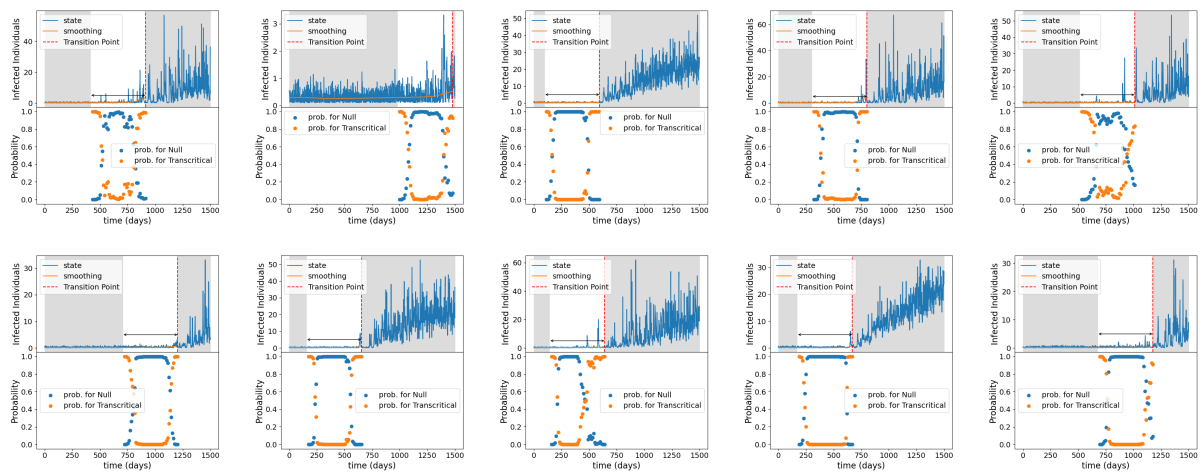


Figure S4: Probabilities for a transition assigned by the SIDATR-500 DL model based on the subset of observations of transcritical simulations of the SIR model with multiplicative environmental noise.

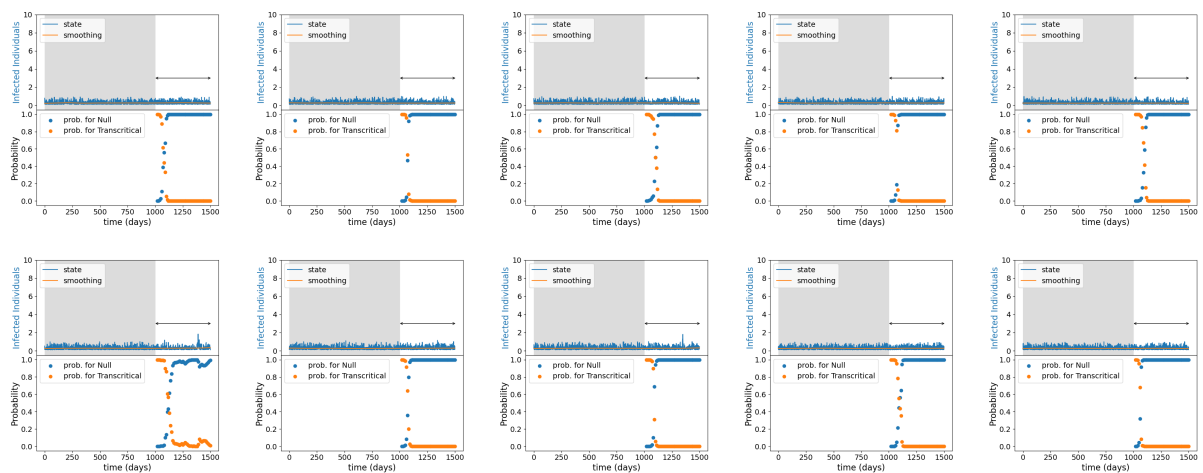


Figure S5: Probabilities for a transition assigned by the SIDATR-500 DL model based on the subset of observations of null simulations of the SIR model with multiplicative environmental noise.

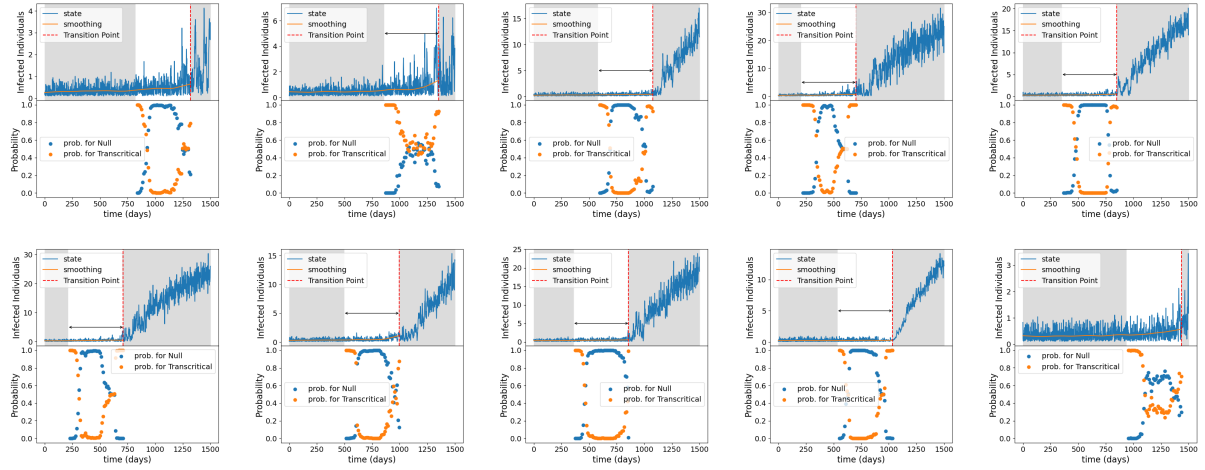


Figure S6: Probabilities for a transition assigned by the SIDATR-500 DL model based on the subset of observations of transcritical simulations of the SIR model with demographic noise.

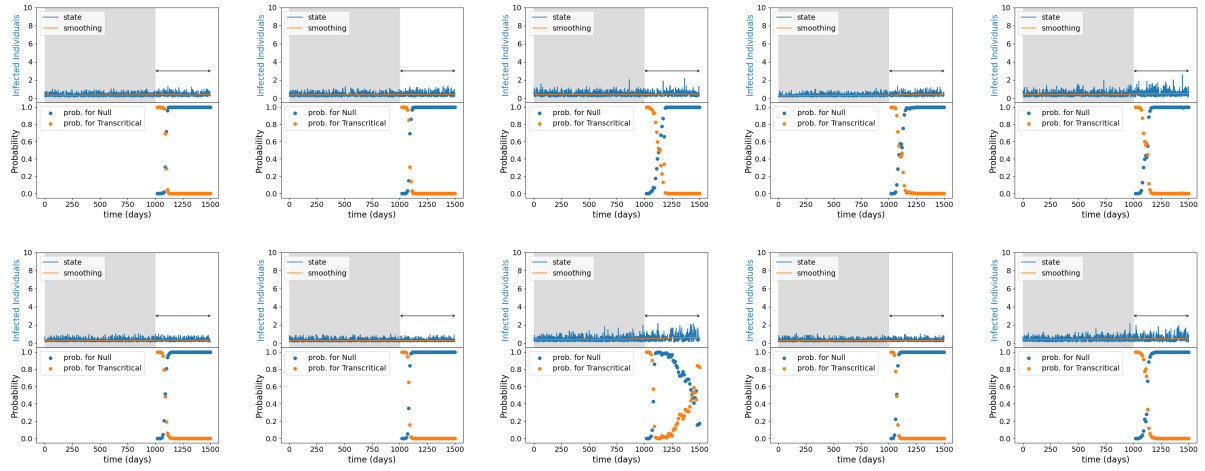


Figure S7: Probabilities for a transition assigned by the SIDATR-500 DL model based on the subset of observations of null simulations of the SIR model with demographic noise.

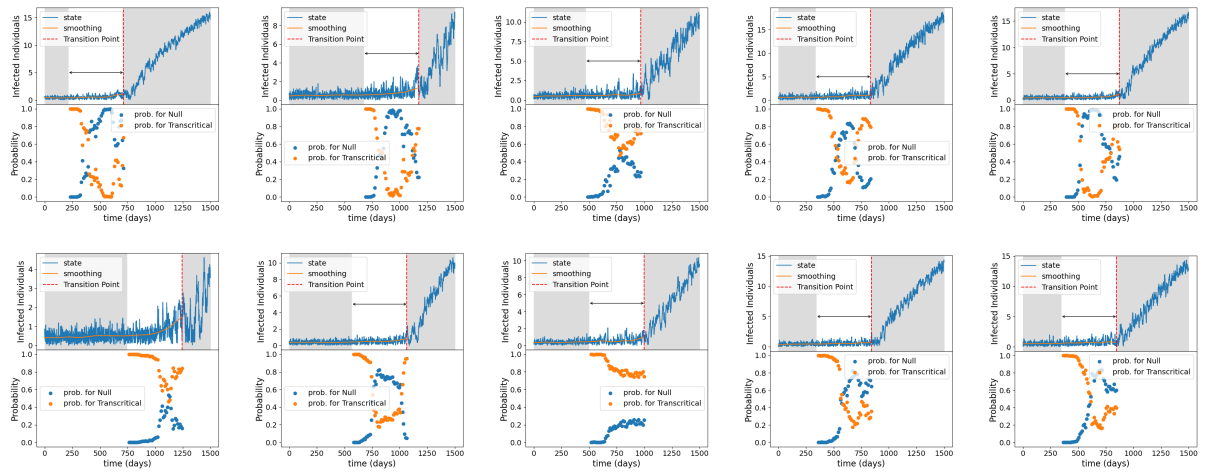


Figure S8: Probabilities for a transition assigned by the SIDATR-500 DL model based on the subset of observations of transcritical simulations of the SEIR model with additive white noise.

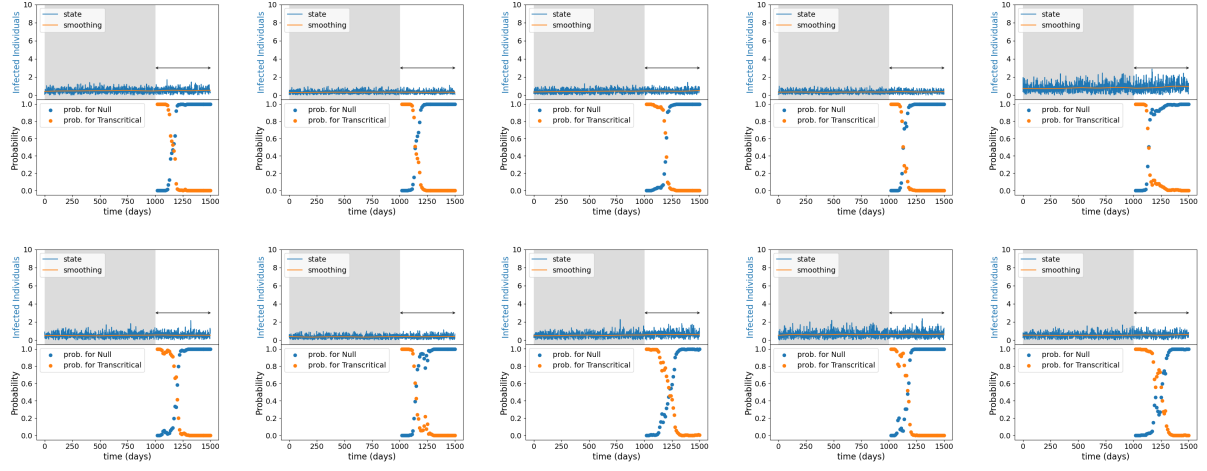


Figure S9: Probabilities for a transition assigned by the SIDATR-500 DL model based on the subset of observations of null simulations of the SEIR model with additive white noise.

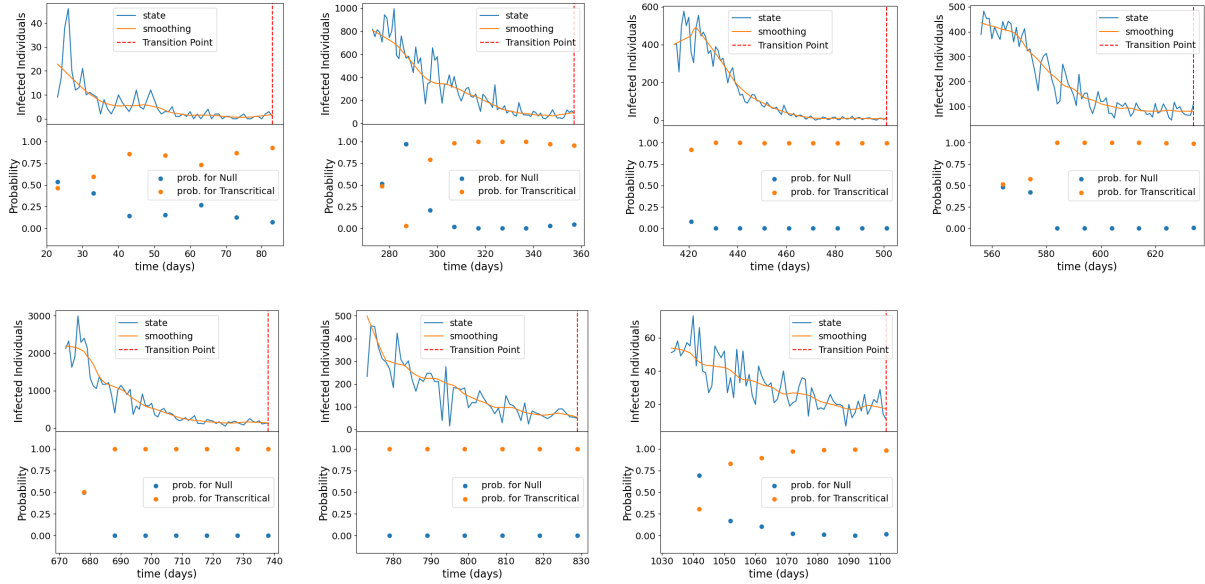


Figure S10: Probabilities for a transition assigned by the SIDATR-100 DL model based on the transcritical time series of the COVID-19 dataset of Edmonton.

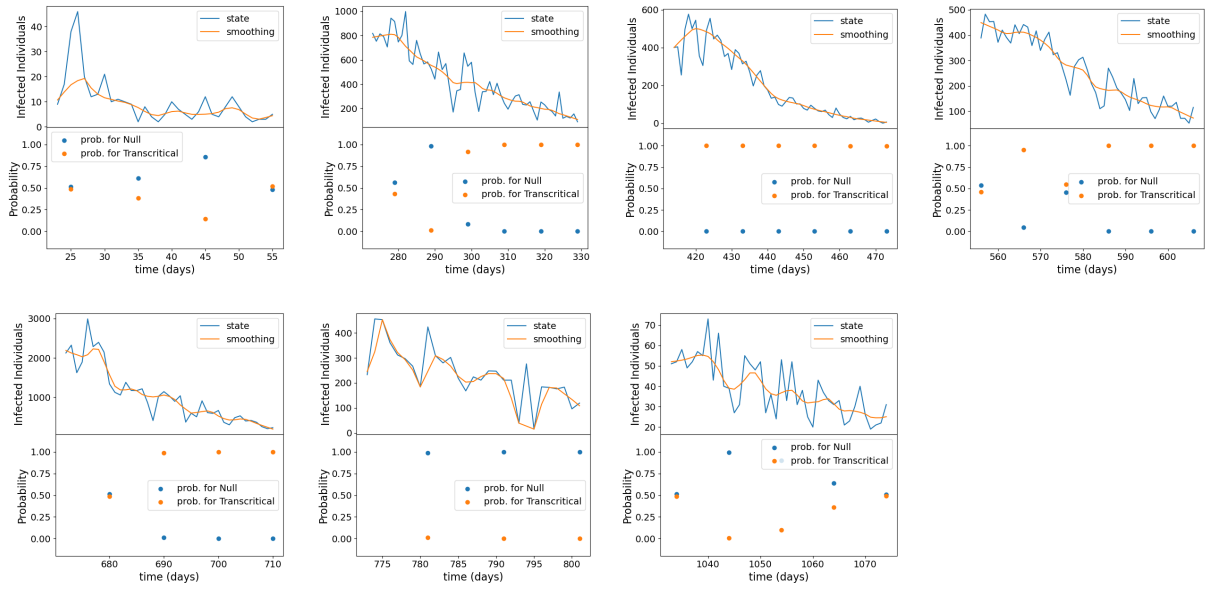


Figure S11: Probabilities for a transition assigned by the SIDATR-100 DL model based on the null time series of the COVID-19 dataset of Edmonton.