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Measurement of Scoliosis Deformity Using Moire Topography

by

Joanne Wegner

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF Master of Science

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Abstract

The accepted method for scoliosis detection involves visual observation. Such observation of the patient over a long term in order to monitor possible progression of scoliosis deformity would be highly subjective and unreliable. Although x-rays are used for long term monitoring of the spinal shape, an alternative method is desirable because of the risk associated with x-rays. This thesis deals with the development of a technique using moire topography to monitor a scoliosis patient over a long term. The technique is both convenient and non-invasive. The thesis describes features of the moire technique in relation to accuracy, reproducibility and reliability. Also, correlations between the internal spinal deformity and the external back shape of scoliotic patients were established.

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1. Introduction

Scoliosis is defined as an abnormal lateral deviation of the spine. Associated with this lateral deviation is vertebral rotation which is the major cause of trunk deformity. Over 80% of the cases of scoliosis found in children today, are of the idiopathic type, meaning the cause of the spinal deformity is not known.

The earlier scoliosis is diagnosed, the more effective is the treatment; and in an effort to achieve this, clinical screening is currently done in schools. This process involves visual observation using the forward bend test where the rib hump is observed as the child bends forward. The rib hump height is used as a measure of deformity, and is defined as the difference in height between the sides of the thoracic cage.

Often a child diagnosed with scoliosis has a mild curve not yet requiring treatment. It is then necessary to monitor the progression of the curve. This is currently done using x-rays, at the rate of two to four roentgenographs per year. From each roentgenograph, the severity of the curve is measured by using the Cobb method which measures the lateral deviation of the spine.

The clinical management of scoliosis depends crucially upon the detection of the progression of the deformity, and if treatment has been started, it is valuable to have a sensitive technique for the detection of change in the deformity. Currently, orthopaedic management tends to focus upon the spinal deformity as measured by the Cobb angle; whereas the surface deformity and its cosmetic implications may be of greater interest to the patient. Moire topographic techniques are receiving considerable attention as a possible method for detecting and monitoring scoliosis (1). The real grating or shadow moire technique, and the grid projection method are both advocated (2,3).

The projection grating techniques for the analysis of the cosmetic deformity is used in this investigation. Algebraic relationships between the moire fringe patterns and surface contours have been well established by others (4,5). Although moire topographic techniques are thoroughly described in literature, the effectiveness of the tool with regard to monitoring scoliosis has not yet been demonstrated.

The purpose of this project was twofold. The first, was to establish the errors and inaccuracies of a particular projection moire system that was already being used clinically. Features of the system and procedure, which may lead to errors in the described topography, were considered. This approach was intended to lead to the identification of critical steps in the process and possibly direct the investigators to procedural or system improvements. Secondly, the results of this investigation were intended to demonstrate how effectively moire topography can be used as a replacement for x-ray monitoring of scoliosis.

In this regard, it is essential to understand the following aspects of the application of moire topography:

1. the inherent accuracy of the moire technique,
2. the reproducibility of patient orientation in the postural frame,
3. the smallest measure of scoliosis deformation that the system is to detect (sensitivity),
4. the ability to predict changes in spinal shape by observing changes in back surface shape.

For the moire topographic technique to be effective, postural variability must be minimized so as to be considerably less than the magnitude of changes in scoliotic shape. The postural frame is required to eliminate postural variability while, at the same time, permitting changes in scoliotic shape to be measured.

2. Moire Methods

2.1 Formation of Fringes

The term moire phenomenon describes the patterns which occur when two or more periodic structures overlap. In experimental mechanics, the technique utilizes moire patterns to measure displacements, or the shape of a surface. This technique is used extensively in research and industrial applications.

The periodic structures most commonly used in the moire technique are line grids consisting of parallel dark and bright lines. The grid pitch s is defined as the distance between the centre of the lines. The ratio of the transparent line width to the pitch can range between 0 and 1. A direction perpendicular to the lines in the plane of the grid is termed the principal direction, while a direction parallel to the lines is termed the secondary direction. If the density of the grid is greater than 40 lines/mm, then light diffraction effects become dominant, and coherent light is required for observation of moire patterns. In this situation, the periodic structure is referred to as a grating. In this thesis, the term grid is used as this particular description deals only with incoherent optics, and a grid spacing considerably greater than 40 lines/mm. The ratio of transparent line width to pitch is equal to unity.

When two grids are superimposed, the interference moire fringes are the result of a difference in pitch, Fig 2.1a, or in orientation, Fig 2.1b. The frequency of the moire pattern is the beat frequency of the two grids and is less than the frequency of the individual grids. Therefore, if the pattern is placed far enough from the observer, the observer is only able to resolve the light and dark fringes and not the individual grid lines. The dark fringes are due to the dark lines of one grating falling into the spaces of the other, whereas the light fringes are the result of the dark lines coinciding with those of the other. The distance between two successive moire fringes formed by the superposition of the two similar grids is called the interfringe spacing. This distance may be measured between either the two darkest or two brightest centre lines of the successive moire fringes.

The moire technique involves the use of a subject grid which characterizes the state of deformation of a surface. A second grid, or reference grid is introduced. The superposition of these two grids produce a moire pattern which is formed on the image plane of the observation system. From the relationship between the subject and reference grids, the observation system, and the surface under study, it is possible to determine the strain, or shape of a surface.

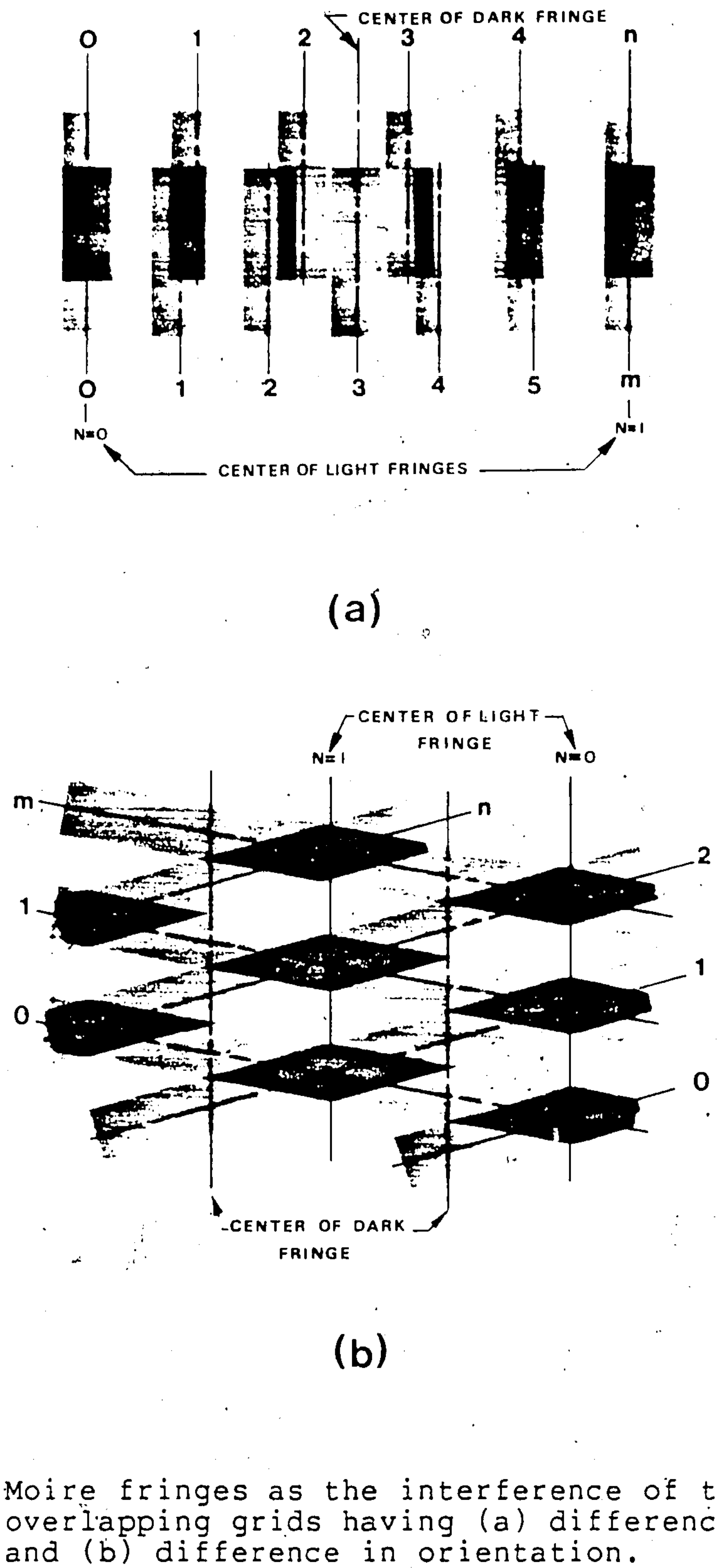

Fig. 2.1. Moire fringes as the interference of two overlapping grids having (a) difference in pitch and (b) difference in orientation.

Moire patterns are produced by several different methods. One obvious method is by directly superposing the grids and observing the resulting fringe pattern. In this case, light which is incident on the first grid is modulated by the structure's transmittance function. The second structure also allows through only certain parts of the modulated light beam. The resultant light intensity is therefore the incident light intensity multiplied by the first structures transmittance function and then by that of the second. The moire pattern as observed is termed a multiplicative moire.

Another common method of producing moire fringes is by double exposure. A photograph is taken of the first grid which may be the subject grid describing the deformed surface. The subject grid is then replaced by a reference grid and the same film is exposed. When the film is developed, the image is a record of the sum of the two grids. The moire pattern produced in this way is termed additive moire.

Subtractive moire occurs when the final image contains the difference of the two grids light intensities and not their sum, and may be obtained by polarization techniques. This moire is difficult to produce and currently is of no practical use to the engineer (6).

Bryngdahl (6) showed mathematically that the multiplicative process has inherently higher contrast than the additive process. However, any difference in contrast may be overcome by using development procedures in the darkroom.

2.2 Indicial Method

The indicial representation of moire patterns is the most widely used of all methods of analysis. From Fig. 2.2, the centre lines of the dark lines of one grid are indexed, relative to the origin, with m; while the other grid lines are indexed with n. The two grids are superimposed with a difference in orientation. The centre lines of the light fringes are denoted by a number 0 to N, where N is referred to as the fringe order. A light fringe occurs when the dark centre lines of the two grids are coincident. The pattern of the two superimposed grids is seen to consist of quadrangles which have their corners lying on a bright moire fringe. The diagonals of each quadrangle correspond to moire fringes satisfying either

$$N = m + n \quad (2.1)$$

$$N = m - n \quad (2.2)$$

Points A,B [illegible] correspond to a moire fringe for which eq. 2.1 is valid, while points E,F,G,H correspond to a moire fringe for which eq. 2.2 is valid.

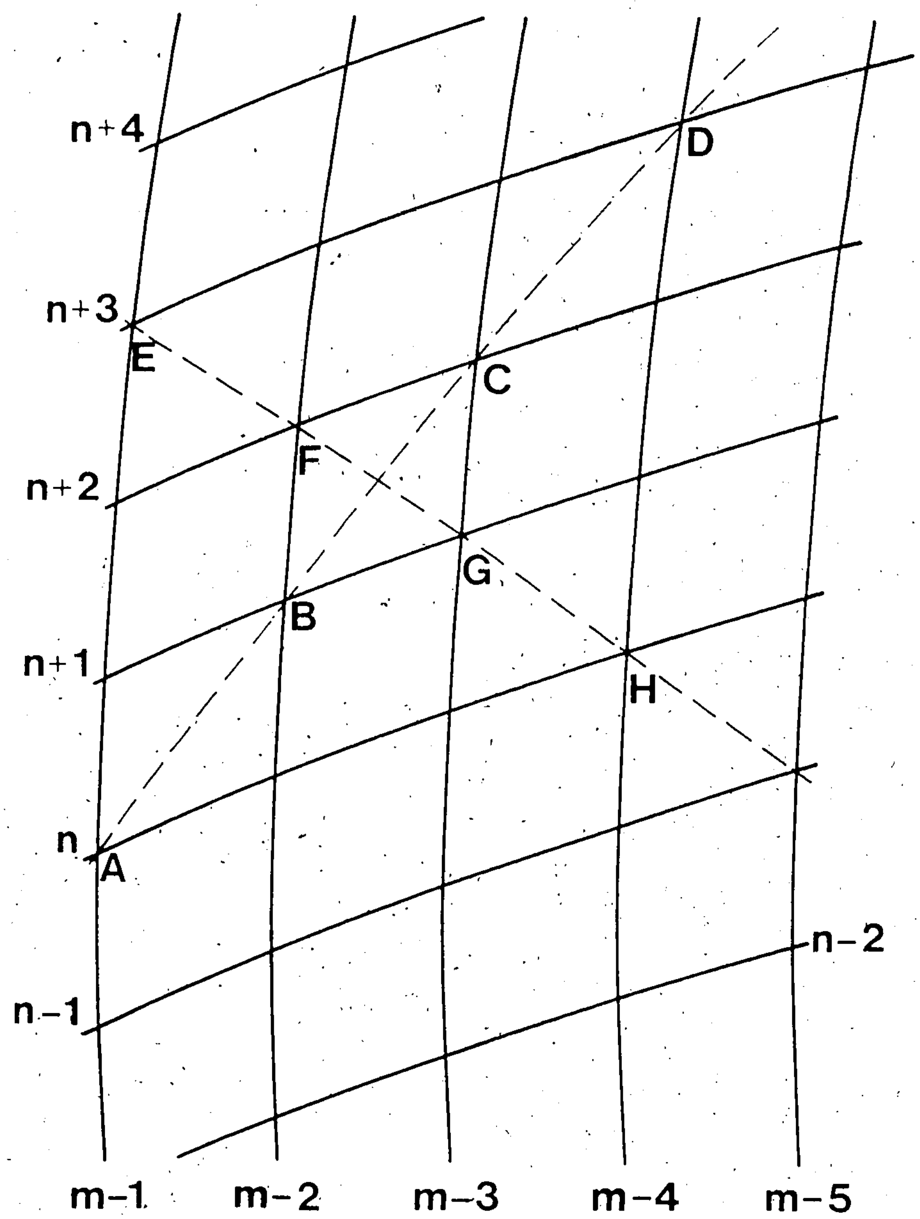

Fig. 2.2. Schematic representation of indicial formation of subtractive and additive moire patterns.

The family of fringes for which eq. 2.1 is valid is called the additive moire pattern, while the family of moire fringes for which eq. 2.2 is valid is called the subtractive moire pattern. The visible moire pattern is the pattern in which the interfringe spacing is the longest; this is the pattern in which the fringes coincide with the shortest disgonals of the individual quadrangles. The visible moire pattern in Fig. 2.2, is the pattern containing the fringe EFGH and belongs to a subtractive moire pattern.

Note that here, the terms additive and subtractive refer to the parametric equations describing the patterns. Previously, the terms additive and subtractive were applied to describe the behaviour of the light intensities in the formation of the patterns.

2.3 Moire Fringe Orientation and Spacing

The pitch of the specimen grating is s', and the pitch of the reference grid is s (Fig. 2.3). The orientation between the two grids is measured by the angle ϕ which is defined as the angle measured from the reference grid line to the subject grid line. ϕ is positive when measured counterclockwise. Before deformation of the structure and subject grid, the pitch of the two grids are assumed to be equal, and the grids are assumed to be perfectly aligned.

A moire fringe pattern results after deformation of the structure and s' no longer equals s, and ϕ is not equal to

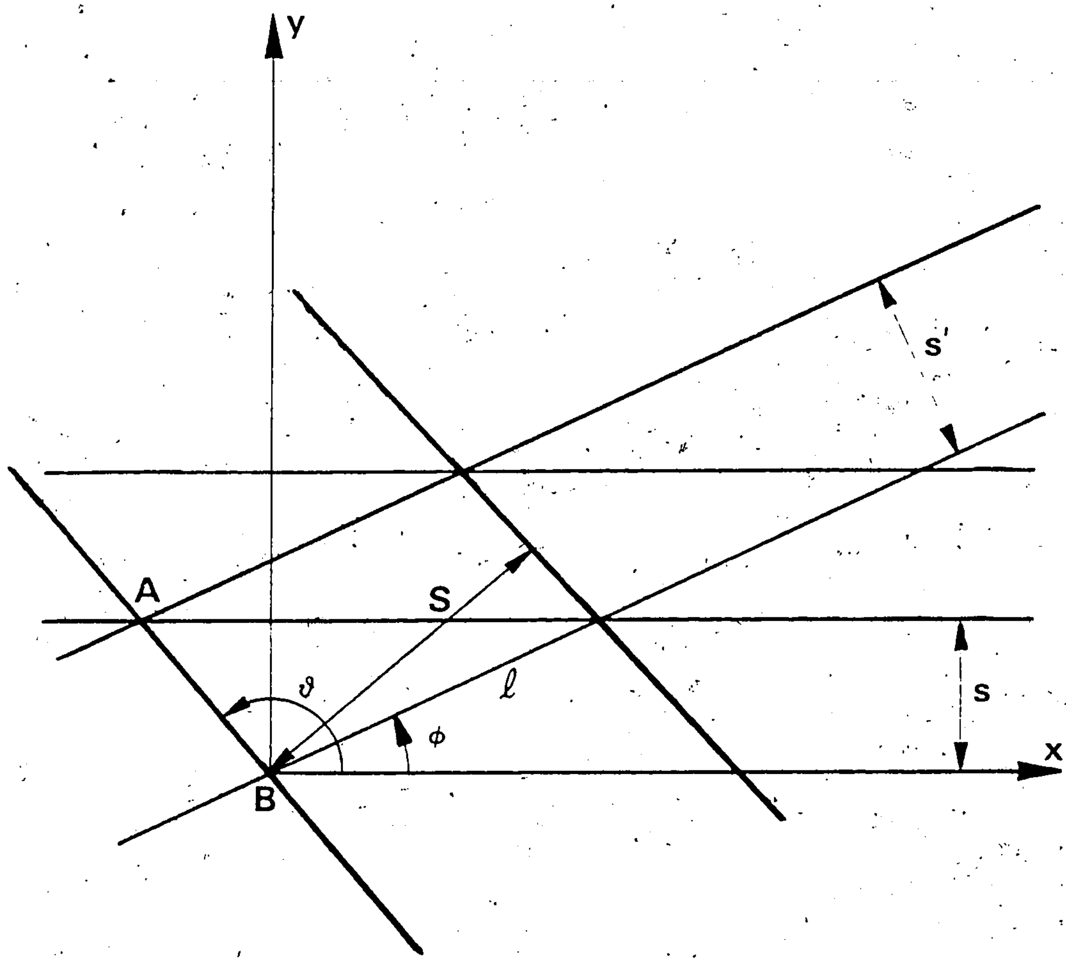

Fig. 2.3. Fringe orientation and spacing as a function of interfering grids.

zero, Fig. 2.3. The interfringe spacing S, is defined to be the perpendicular distance between two neighboring fringes. The angle θ is the orientation of the fringes as measured from the secondary direction of the reference grid which defines the x axis. The principal direction of the reference grid defines the y axis.

Following the analysis of Chiang (7); then from Fig. (2.3)

$$\overline{AB} = s / \sin\theta \qquad (2.3)$$

and

$$\overline{AB} = s' / \sin(\theta-\phi) \qquad (2.4)$$

Equating equations 2.3 and 2.4 and solving for s';

$$s' = s(\sin(\theta-\phi))/\sin\theta = s(\sin\theta\cos\phi-\cos\theta\sin\phi)/\sin\theta \qquad (2.5)$$

Solving for θ, the fringe orientation;

$$\theta = \tan^{-1} (s(\sin\phi)/(s\cos\phi-s')) \qquad (2.6)$$

Also from Fig. 2.3;

$$s = l\sin\phi \qquad (2.7)$$

and

$$S = l\cos(\theta-\pi/2-\phi) = l\sin(\theta-\phi) \qquad (2.8)$$

Substituting equations (2.7,8) into (2.5);

$$s' = l\sin\phi\sin(\theta-\phi)/\sin\theta = S(\sin\phi)/\sin\theta \qquad (2.9)$$

and solving for S, the interfringe spacing;

$$S = s'(\sin\theta)/\sin\phi \qquad (2.10)$$

From equation (2.6),

$$\tan\theta = s(\sin\phi)/(s\cos\phi-s') \qquad (2.11)$$

and therefore,

$$\sin\theta = s(\sin\phi)/\sqrt{(s^2(\sin^2\phi)+(s\cos\phi'-s')^2)} \qquad (2.12)$$

Finally, the interfringe spacing S is given as,

$$S = s'(s)\ /\ \sqrt{(s^2(\sin^2\phi)+(s\cos\phi-s')^2)} \qquad (2.13)$$

Consider the following cases;

1. the reference grid and subject grid are aligned (ϕ=0) and s is not equal to s'. From equation 2.6, θ=0 and the fringes are parallel to the grid lines.
2. when ϕ is small and s equals s'. From equation 2.6, θ approaches 90° and the fringes are approximately perpendicular to the reference grid lines.

2.4 Mismatch Methods

There are various ways of increasing the sensitivity of the moire method. One way is to decrease the pitch of the grid which will increase the number of resulting fringes. Diffraction effects limit this method to 40 lines/mm as described earlier.

Another method which can be used to increase the sensitivity is the mismatch technique. This technique involves the use of either an initial difference in pitch between the two grids which is known as linear mismatch; or an initial difference in orientation which is known as rotational mismatch.

As was shown in section (2.4) by eq. 2.6, fringes which are created from a difference in pitch are parallel to the grid lines. Therefore, the linear mismatch technique can be used to increase the sensitivity in the principal direction. Conversely, fringes created from a small rotational angle φ were shown through the use of eq. 2.6 to be nearly perpendicular to the grid lines. Therefore, rotational mismatch increases the sensitivity in the secondary direction of the reference grid. These two techniques can be combined to increase the sensitivity in both directions.

2.5 Moire Methods

Currently, there are four methods which can be used in the moire technique. These methods are as follows:

1. intrinsic moire for deformation analysis,
2. reflection moire, which is used to obtain the slopes of the surface,
3. shadow moire, which is used to obtain the shape of the surface,

4. projection moire, also used to determine the shape of the surface.

Both the shadow and projection techniques are applicable to the problem of measuring the shape of a person's back. The theory of these two techniques is described in this thesis, and the comparative advantages and disadvantages of each applied to this problem are discussed.

2.5.1 Shadow Moire

This method uses a reference and subject grid. However, the subject grid is not a separate grid, but is the shadow of the reference grid as cast onto the subject's surface. The shadow, or subject grid, is distorted by the out-of-plane depth of the surface, and when it is viewed together with the reference grid, moire fringes are created.

This analysis follows the one developed by Chiang(7), in which the light source and camera are assumed to be a finite distance away from the structure. Both the light source and aperture of the camera are approximated by a point and are assumed to be located at the same distance away from the grid plane, as illustrated in Fig. (2.4). The surface of the subject is assumed to have a point touching the grid surface at a, as illustrated. A fringe order N is observed at point f on the surface through point d of the grid, and is the result of interference between the lines contained in ad of the reference grid, and lines in ab, as

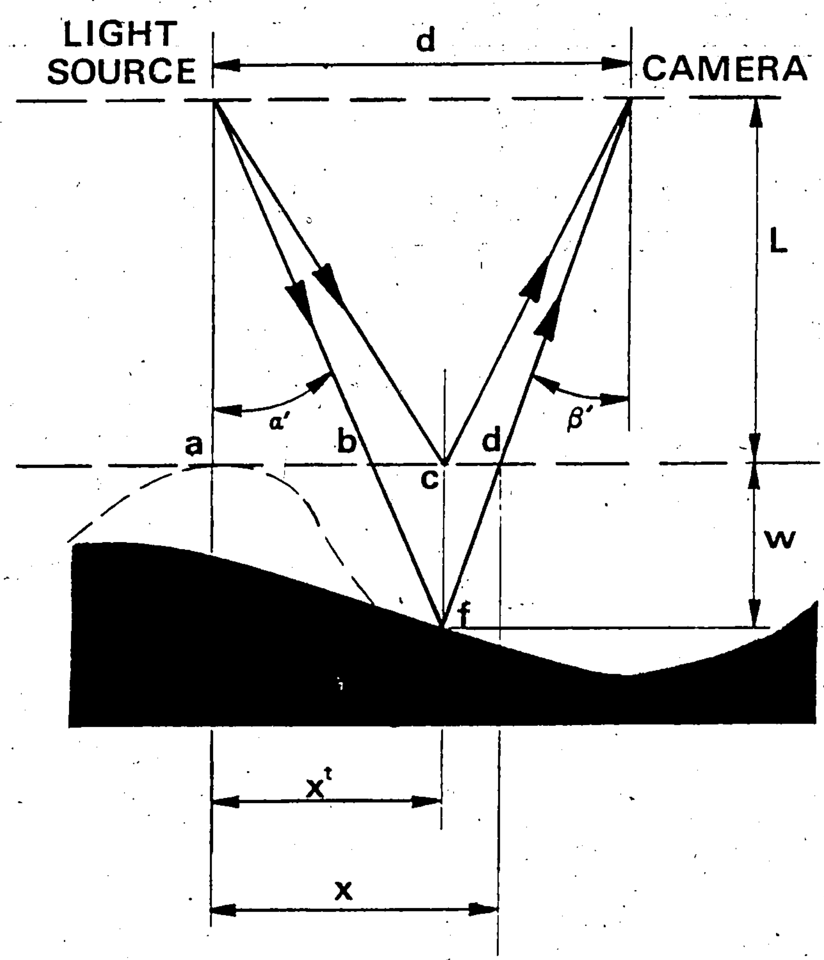

Fig. 2.4. Shadow moire method.

projected onto the surface, of the shadow subject grid. If the lines of the reference and subject grids are indexed by m and n respectively, then ad=ms, and ab=ns, and,

$$bd = ad - ab = (m-n)s = Ns \quad (2.14)$$

where s is the pitch of the grid.
But

$$bd = W\ (\tan\alpha' + \tan\beta') \quad (2.15)$$

then

$$W = Ns\ /\ (\tan\alpha' + \tan\beta') \quad (2.16)$$

or

$$W = Ns/(x'/(L+W)+(d-x')/(L+W)) = Ns(L+W)/d \quad (2.17)$$

where d is the distance separating the light source and the camera and L is their distance from the grid plane.
Equation (2.17) can be simplified to;

$$W = Ns\ /\ (d/L - Ns/L) \quad (2.18)$$

For greater accuracy, it is necessary to correct for the perspective effect of the coordinates. From Fig. (2.4)

$$(x-x')/W = (d-x)/L \quad (2.19)$$

or

$$x' = x - W(d-x)/L \quad (2.20)$$

Also,

$$y' = y - W(d-y)/L \qquad (2.21)$$

where (x,y) are the apparent coordinates, and (x',y') are the actual coordinates.

2.6 A Comparison of the Shadow and Projection Moire Techniques

One advantage of the shadow system is that the sensitivity may easily be changed by altering the position of the light source. The sensitivity increases as the distance between the camera and the light source increases. Increasing the obliquity of the light source may result in shadowing part of the subject. This may be easily resolved by placing light sources symmetrically about the camera.

For the projected grid method, the angle of illumination is fixed for a given camera object distance. Also, for the single beam projection method, the obliquity of the light source, and therefore the sensitivity, is limited. Although a double beam projection method is possible, as proposed by Halisua et al (8), this method is impractical as there must be precise alignment of the individual grids to assure coincidence of the two fringe patterns.

Also, for the shadow techniques, only the moire fringes have to be resolved by the camera. Whereas the projective technique requires the individual grid lines to be resolved which further reduces the attainable sensitivity.

A major aspect of the process is the quantification of data. The shadow technique has several disadvantages in this area. In a flat region of the surface, there are few fringes resulting in poor local sensitivity. Also, it is difficult to determine the sign of fringes and therefore differentiate between concave and convex regions. This would pose a problem when automation of the system is achieved.

The projected grid method, in which both the subject and reference grids are available, can overcome these problems. There are ways of eliminating height ambiguities and interpolating fringes simultaneously.

The first method involves the shift of the reference grid. The fringes shift in particular directions depending on the slope of the surface and the direction of the shift and therefore the direction of the surface's slope can be deduced (7). Interpolation can also be done by shifting the grid by known amounts and any fringe can be brought into coincidence with any point of the surface.

The other two methods involve the use of either linear mismatch which involves changing the pitch of the reference grid; or rotational mismatch which involves changing the orientation of the reference grid.

The projection grid technique is adaptable to automation. With video systems, such as those described by

Hormiere and Mathieu (9) and Idesawa et al (10), the shift, rotation or change in pitch of the reference grid are easily performed electronically, making such systems very versatile.

3. Application of Moire Topography, to Measurement of Scoliotic Deformity

3.1 School Screening

In 1970, Takasaki (11) described the application of the shadow moire technique to contouring a human body. In a subsequent paper (12), he suggested methods of overcoming the problems associated with this application; such as poor contrast of the fringe pattern on a human body. He showed mathematically, that the moire fringe pattern remains stationary against any translational movement of the grid in its own plane. Such a movement smooths the structure of the grid lines, and the moire which forms between higher harmonics of shadow and grid, or higher harmonics of the grid and shadow. This leaves a pattern with improved clarity and ease of interpretation. He also showed that shadow free illumination was possible by placing lights symmetrically about the viewing aperture.

Takasaki's shadow technique has been used extensively in Japan since 1976, when a school screening program was started on a national basis. In Japan there are 15 million primary school and junior high students. To conduct school screenings for scoliosis by visual examination on a national scale requires a large number of skilled examiners and a large number of working hours for each examiner. Use of the moire method for spinal examinations permits objective and uniform criterea for fringes the abnormal, conserves manpower, and promotes efficiency of the initial screening step. The shadow moire method is particulary useful in this regard, as no quantification of data is required; the child needs only to stand square behind the grid and asymmetry of the pattern is noted for abnormal spinal development.

Since 1976, the shadow technique has been applied to the school screening of scoliosis in Japan. For example, the total number of children screened at one centre (Chiba University) had reached 30,000 by 1980. Good results have been obtained in accuracy and efficiency as reported by Ohtsuka (13).

The shadow technique has also been applied to school screening in North America. Adair (14) screened a population of school children. The results from the examination showed it to be more effective for detecting scoliosis than the clinical examination of the child during the forward bend test. The apparatus, especially developed for this purpose is now available commercially [Atomic Energy of Canada Ltd., Commercial Products, Ottawa.] The apparatus consists of a falling grid screen connected to a framework to mount the camera and light source. Haack (15) of the University of Nebraska, adapted this basic design, and Huurman (16) successfully used this device in a school screening program in Nebraska.

Bio-Tek Instruments Inc. (17) has also designed a portable moire contourgraph for the purpose of school screening, and is currently manufacturing the Bio-Tek M-500 Scoliosis Screening System consisting of a positioning device, a stationary screen, and a framework to mount the camera and light source.

The projection technique has also been used for school screening. Suzuki et al (18) first announced the use of the projection moire technique in 1971. Since then, Suzuki et al (19) have commercialized a moire topography camera (FM40) for the purpose of detecting scoliosis, and is produced by the Fuji Photo Optical Co., Ltd. The basic unit of this system is a contour moire fringe generating camera. This is provided with projection type optical system and a moire fringe generating optical system for photographing transformed grids. It also incorporates a relay optical system for the option of transmitting the generated moire pattern to a recording camera or to a TV monitor. The system is contained on one of two stands; one is light weight and pc table and is used for school screening, while the other is more durable and less portable. The interval of the contour moire fringes is designed to be every 5mm of depth when the camera is placed 180mm away from the subject. This projection system does not allow the use of mismatch techniques or interpolation by movement of the reference grid, and, similiar to the shadow technique, the local sensitivity is poor. This system was designed to accompany the positioning device designed by Inoue et al (20). This positioning device consists of a flat plate hinged at its bottom end to a horizontal base board so that the plate may be inclined at any desired angle. The examinee stands on the base board close to the plate with the full length of the body leaning face down on the plate. When the plate is inclined at an angle of 5 to 10° to the vertical, and the examinee is positioned onto the holder, the body can be held motionless. The FM40 projection system was designed to be capable of adjusting the plane of the camera's grid to be parallel to the plane of the holder plate.

3.2 Monitoring the Progression of the Scoliotic Curve

The application of moire topography to monitoring the progression of the scoliotic curve utilizes the fact that in most structural curves of the spine, there is an accompanying asymmetry of the back. To measure the deformity of the patient, it is necessary to extract from the moire topogram, parameters describing the deformity which are independent of the coordinate system and the patient's position.

Willner (21), used the shadow method to study asymmetry in moire f patterns, and used no positioning device. He use the data qualitatively, and did no quantification of data. For each pattern, he judged the level of maximum asymmetry. He then chose two points on the same horizontal line, at the same distance to the vertical mean line. These two points should, in a symmetric spine, be found on the same contour line. By recording the number of fringes which differentiate these points, the difference in distance to the grid between these points can then be calculated. He compared this to the Cobb angle with no significant results. Emans et al (22) used the above technique at Children's Hospital Medical Center, Boston. They studied 72 patients and found that no patients showed a progression of 5 or more degrees by x-ray, without also showing greater than one fringe interval of moire. The authors concluded that moire topography may be sufficiently accurate to partially supplant x-rays in the follow up of scoliosis.

Some researchers, such as Roger (5) and Stokes (23) use a paremeter known as the hump angle which measures the trunk rotation. This is obtained by measuring the slope of the line tangent to the two peaks of the rib humps. Stokes used the FM40 projection moire camera to study the relationship between the major Cobb angle and the maximum angle of trunk rotation. Although there was a wide scatter of results, and a low correlation coefficient between the two parameters, he made a significant discovery. The maximum value of hump angle occurred on the average at a level of 3 vertebrae below the apex for curves in the thoracic region. This is due to the downward sloping of the ribs away from the vertebral column.

Shinoto et al (24), also studied the relation between the maximum value of hump angle and the major Cobb angle. Using the FM40 moire system in conjunction with the positioning device designed by Inoue as described earlier, they studied the effectiveness of treatment by noting the change in value of the maximum hump angle and the corresponding change in Cobb angle. They observed that in some cases the value of the hump angle was reduced remarkably in comparison with the correction of the Cobb angle. Conversely, there were some cases whose hump angle increased in spite of an improvement of Cobb angle. However, when the moire technique is being used to compare to results obtained from x-rays, it is important that the patient be placed in the same position for each process. The positioning device used in this study requires the patient to lie forward which could decrease the lateral curvature of the spine as seen when the patient is x-rayed in the upright position. It would need to be shown that no changes occur by taking x-rays when the patient is in the forward position.

Armstrong et al (25) have proposed an index to measure the asymmetry of the surface of the back, which is based on a large number of points taken from the complete surface of the back and uses a single longitudinal axis about which the asymmetry is assessed. Thirteen patients were assessed with this index, and the results were compared to the major Cobb angle. The results were inconclusive due to the small sample size.

3.3 Automation of the Moire Projection Method

There have been several attempts to automate this process. Yatagai Fuji et al (11) describe a semi-automatic fringe analysis system called Rifran (Riken Interactive FRinge ANalyzer). This system provides for automatic peak detection and allows for interactive correction of fault data points. Most of the operations are interactively performed by the use of a light pen connected to a TV monitor as an input device. A TV camera makes image data aquisition. A TV monitor displays not only the image to be analyzed, but also displays resultant data and the information required for interactive operations. A TV I/O controller transfers video signals to a computer, and results of the analysis to the TV monitor, and also makes interface with the light pen. Software has been developed to construct the 3-D shape of the surface being measured. Shinoto et al (24) have used this system in conjunction with the FM 40 moire system.

Idesawa et al (26) have proposed a system which provides for automatic generation and analysis of a moire fringe pattern. A grid is cast on the subject, and is observed with an electronic scanning device, such as a TV camera and photodiode array. A moire fringe pattern is observed on a monitor TV. The procedure of scanning and-sampling in the image input device corresponds to superposition of a reference grid in the conventional projection type moire topography. In the scanning method it is possible to change the phase, the pitch, and the direction of the scanning lines, and therefore, various contours can be immediately generated. This allows the sign determination and interpolation of moire contours to be made. The authors have modified the commercially available moire camera (FM40, Fuji Photo Optical Co.) for the scanning moire method. The developed software system provides for two modes, the automatic mode, and the man-machine interactive mode (Rifran, as described earlier).

However, the inherent accuracy, and therefore the effectiveness of these systems with regard to monitoring scoliosis, has not yet been demonstrated.

4. Theory

This analysis follows the one developed by Yoshino et al (27) and further developed by Roger (5), where the subject and reference grids are located on a plane, and two coordinates axes (x,y) are defined on this plane.

4.1 Analysis of Moire Fringe Formation

In this analysis, the camera nodal point (C), and the projector nodal point (P), are placed at distances L_C and L_P respectively from the reference plane (S-S) (the plane on which the reference grid is projected to an image of parallel vertical grid lines, Fig (4.1)). The optical axis of the camera intersects the reference plane at the origin (0) of the coordinate system, and defines the z axis. The x and y axes are in the reference plane with

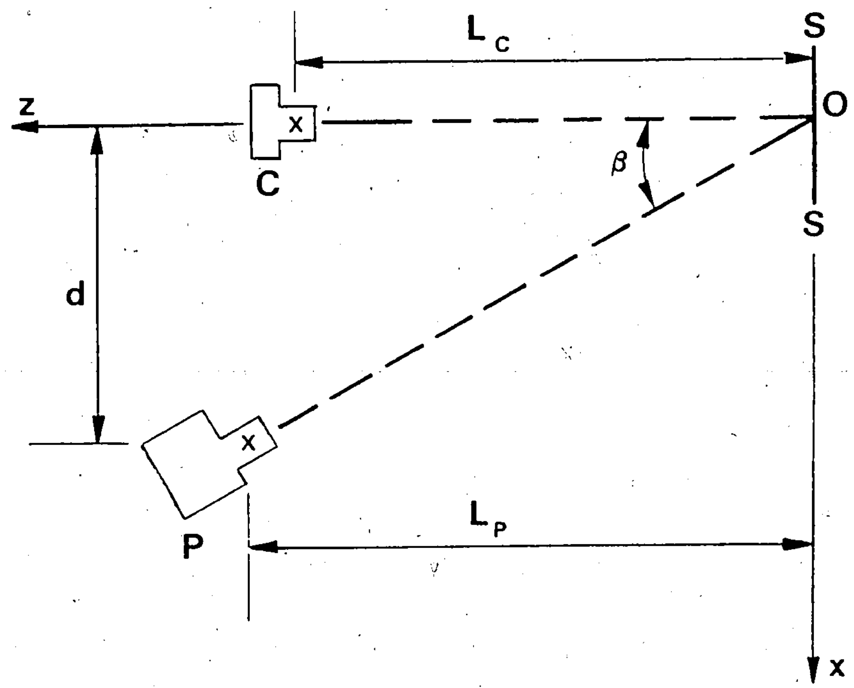

Fig. 4.1. Physical layout of projector (P), camera (C) and reference plane (S-S). The z axis is coincident with the optical axis of camera and the x axis is perpendicular to the optical axis of the camera. The y axis parallel to the projected grid lines.

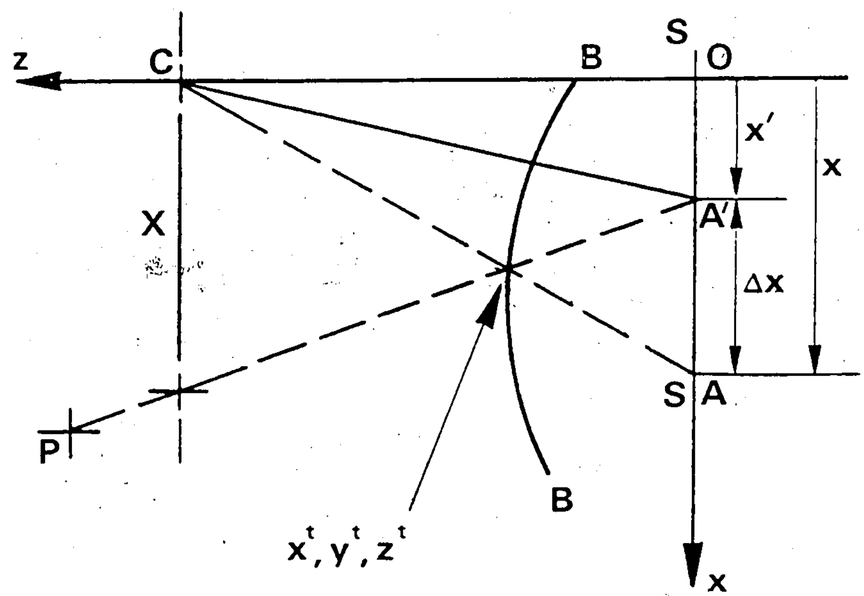

Fig. 4.2. Configuration with subject profile B-B.

indexed by n relative to the origin. The grid line appears to the camera to be projected at A on the reference plane, a distance x from the origin (Fig. 4.2).

A moire fringe pattern is then produced by superposition of the two transparencies. To increase the local sensitivity of this technique, rotational mismatch is used (Chapter 2), and the reference grid is rotated an angle ϕ to the subject grid.

The moire fringe pattern produced can be described by the indicial equation (Chapter 2) as;

$$N = m - n \tag{4.1}$$

where $N = \pm 0, 1, 2, \ldots\ldots$

If

$$x - x' = \Delta x \tag{4.2}$$

then the x coordinate in the subject grid is represented by

$$x = ms + \Delta x \tag{4.3}$$

The x' coordinate of the rotated reference grid can be expressed in terms of the (x,y) coordinates of the subject grid;

$$x' = x\cos\phi + y\sin\phi = ns' + \lambda \tag{4.4}$$

where λ is the linear mismatch of the grids at the origin for the two photographs. Solving equations 4.4, 4.3 for m and n, and substitution into the indicial equation yields;

$$N = (x\cos\phi + y\sin\phi - \lambda)/s' - (x - \Delta x)/s \qquad (4.5)$$

Solving for Δx:

$$\Delta x = s(N + \lambda/s') + x(1 - s\cos\phi/s') - ys(\sin\phi)/s' \qquad (4.6)$$

However, the fringe indexed by N in the indicial equation derived in Chapter 2, is a bright fringe as it is the curve of intersection where the lines of the two grids coincide. To create a dark fringe along this curve, the reference grid would move one-half of its pitch with respect to the subject grid. As it is usually easier to locate a dark rather than bright fringe, equation 4.6 can be corrected to allow for the measurement of dark fringes as follows;

$$\Delta x = s(N + \lambda/s' + 1/2) + x(1 - s\cos\phi/s') - ys(\sin\phi)/s' \qquad (4.7)$$

In practice, the same grid is projected onto the reference plane as the subject, and therefore s=s' and

$$\Delta x = s(N + \lambda/s + 1/2) + x(1 - \cos\phi) - y\sin\phi \qquad (4.8)$$

It is shown later that the correction term $(\lambda/s' + 1/2)$ is accounted for directly with a single measurement.

4.2 Determination of the Topographic Height

The z coordinate, or height, of a point on the back of the patient can be derived in terms of Δx. From the geometry of Fig (4.2);

$$\Delta x/z' = X/(L_C - z')$$

or

$$z'(X+\Delta x/X\Delta x) = L_C/X$$

Solving for z';

$$z' = L_C\Delta x/(X+\Delta x) = L_C/(1+X/\Delta x) \qquad (4.9)$$

Now it is possible to derive a relationship for X;

$$X = (d-x')L_C/L_P + x' \qquad (4.10)$$

Substituting

$$\Delta L = L_P - L_C \text{ and } x' = x+\Delta x$$

into equation 4.10 and rearranging yields;

$$1+X/\Delta x = (dL_C + x\Delta L + \Delta x L_C)/(xL_P) \qquad (4.11)$$

Substituting equation 4.11 into 4.9;

$$z' = L_C/(dL_C + x\Delta L + \Delta x L_C)/\Delta x L_P$$

or

$$z' = L_P/(1+d/\Delta x(1+x\Delta L/dL_C))$$

(4.12)

In this experiment, $L_P=L_C$ and,

$$z' = L/(1+d/\Delta x) \quad (4.13)$$

The x and y coordinates, corrected for perspective are:

$$x' = x(1-z'/L) \quad (4.14)$$

$$y' = y(1-z'/L) \quad (4.15)$$

4.3 Determination of Absolute Fringe Orders

One of the first steps after the photographs have been taken involves the identification of specific moire fringe order values. To achieve this, reference lines are marked on the projection grid (Fig. 5.1). The identification of a reference line on the reference grid and object grid allows Δx to be measured directly. One such reference line passes through the origin of the local coordinate system (0), and is approximately in the center of the subject's back. At this location both x' and y equal zero. From equation 4.8;

$$sN' = x_0 \cos\phi \quad (4.16)$$

or

$$N' = x_0 \cos\phi / s \quad (4.17)$$

where N' is the absolute fringe order at the origin of the subject grid and x_0 is the apparent displacement of the origin of the subject grid.

Because the location of the reference point can be expected to fall between specific fringe orders, interpolation along a line perpendicular to the fringe contours is used to establish numerical values for each fringe in the whole pattern. These fringe order values occur in increments of one and include $(\lambda/s+1/2)$ as a fractional value.

4.4 Physical Interpretation of Contour Surfaces

(i) For no rotational mismatch $(\phi=0)$ and $d/\Delta x>>1$, eqn. (4.13) reduces to;

$$z' = z = (sN+\lambda)L/d \qquad (4.18)$$

The contour planes are parallel to the reference plane, and are equally spaced. The fringes as observed, are lines of intersection of the subject with the contour planes.

ii) With rotational mismatch, $(\phi\neq0)$ and $d/\Delta x>>1$,

$$z' = z = s(N+\lambda/s+1/2)^{T}/d - ax - by \qquad (4.19)$$

where:

$$a = -(1-\cos\phi)L/d$$

and

$$b = \sin\phi L/d$$

As the height of a point (z') is now a function of ($N,x,\hat{y}$), the fringe pattern is no longer a representation of contours; however along any fringe, the height is well defined.

Equation 4.19 can be seen to consist of a reference plane.

$$ax + by + z = 0 \qquad (4.20)$$

from which the measurements are made. The orientation β of the reference plane in the (x,y) plane is:

$$\tan\beta = -a/b$$

The inclination, the angle between the reference plane and the (x,y) plane, α of the reference plane is:

$$\tan\alpha = \sqrt{(a^2+b^2)} \qquad (4.21)$$

The spacing, V, between the contour planes is;

$$V = sL/d\sqrt{(a^2+b^2+1)} \qquad (4.22)$$

For a given value of ϕ, the inclination and orientation of the reference plane and contour planes is constant, and therefore the contour planes are parallel. As the rotation angle is altered, the planes are inclined with respect to the x and y axes resulting in a different slicing of the subject and the creation of a new fringe pattern. For a given value of ϕ, the spacing between the contour planes is constant, and the planes are equally spaced.

iii) The last case to be considered is when $\phi \neq 0$, and the (x,y,z) coordinates are corrected for perspective. From equation 4.13,

$$z' = L/(1+d/\Delta x) = 1/(1/L+d/\Delta xL) = 1/(1/L+1/z)$$

or

$$z' = zL/(z+L)$$

and

$$z' = z(1-z'/L) \qquad (4.23)$$

Multiplying equation (4.18) by (4.23) and substituting for (x,y) using equations (4.14), (4.15);

$$z' = C_n(1-z'/L) - ax - by$$

where

$$C_n = s(N+\lambda/s+1/2)$$

Solving for z', the following expression is obtained;

$$z' = C_n/(1+C_n/L) - a/(1+C_n/L)x - b/(1+C_n/L)y$$

or

$$z' = C_n' - a'x - b'y \qquad (4.24)$$

The surfaces contours are not quite parallel or equally spaced as the constants a' and b' depend on the fringe number N.

5. Apparatus and Procedure

5.1 Perspective Grid Slide

The grid slide used in this investigation was a perspective grid slide. A perspective grid slide is not a simple equispaced grid, but is prepared so that when projected onto a reference plane at a particular angle of incidence, it displays an equispaced line grid of pitch s'. The angle of incidence, β, is the acute angle between the optical axis of the projector and the reference plane (Fig. 4.1). The preparation of this slide for the purpose of projection moire topography is described by Miles and Speight(4). The particular grid slide used in this investigation was developed by Roger(5) and produced a uniform grid spacing when projected onto the reference plane at an angle of incidence β, such that;

$$\tan\beta = 0.25$$

In practice the grid was projected separately on to the reference plane and onto the subject resulting in two grid photographs. To permit precise alignment of the two grids, a part of the reference screen (datum screen) appeared in both photographs. Superimposed on the grid lines, were parallel dark reference lines (I,II) as illustrated in Fig. 5.1. The left hand line I appeared on the datum screen in both photographs, and was used for determining the scale factor

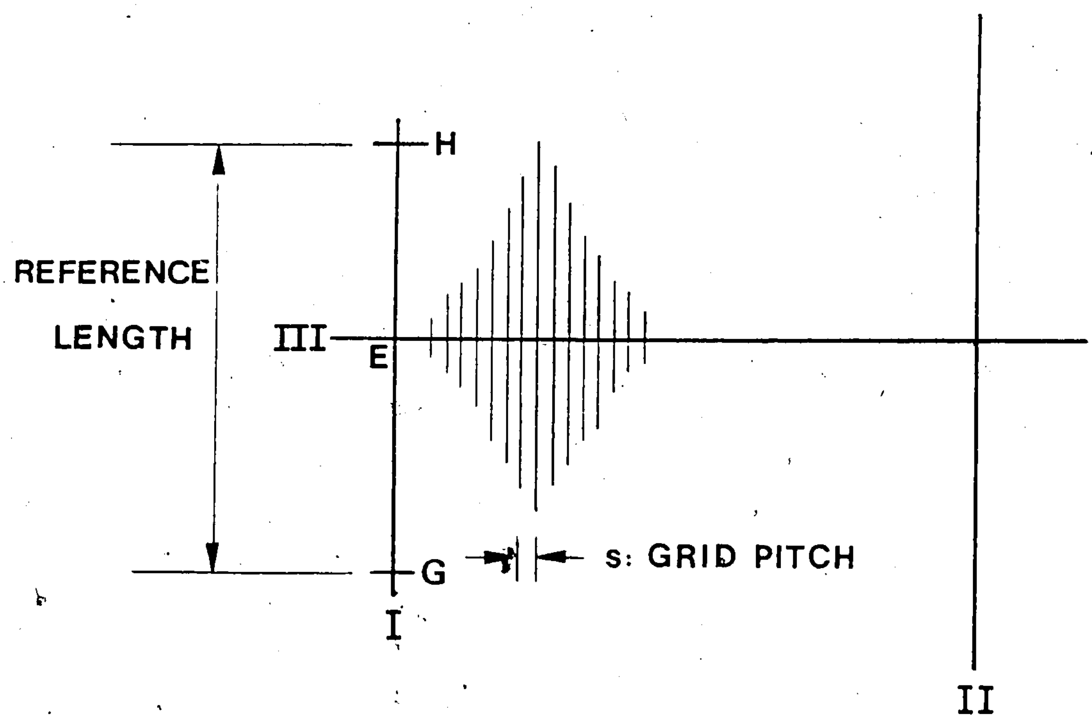

Fig. 5.1. The grid description illustrating the reference line used for determination of scale, I; and the reference line II used for determining the fringe number at the subject origin.

and the rotation of the reference grid with respect to the subject grid. Point O, located at the intersection of line II and the horizontal line III, defines the origin of the coordinate system (x,y,z). The apparent displacement of line II of the subject grid with respect to the reference grid was used to determine absolute fringe orders.

5.2 Arrangement of Apparatus

The arrangement of the camera and projector were as shown in Fig. (4.1). The camera (a Minolta X700) included a 100mm focal length lens; while the projector had a lens with a focal length of 90mm. The camera and projector were a common distance L away from the reference screen and were separated by a distance d. For a constant grid pitch on the reference plane,

$$\tan\beta = 0.25 = d/L$$

The distances d and L were 700 and 2800 mm respectively, producing a grid pitch of 2.34 mm.

5.3 Positioning the Patient

The shape of any surface is independent of the coordinate system or frame of reference in which it is measured. However, each moire contouring system defines a unique frame of reference in which the contours are measured. Each time a patient's back is contoured, a unique set of coordinates discribing that surface is obtained. To measure the progression of the deformity of a patient, it is necessary to determine the relationship between each coordinate set.

This problem is approached in two ways. First, some descriptors of the surface which are independent of the frame of reference are used as reference points. The spinous process of the seventh cervical vertebra(C7) is a conspicuous landmark and is used for a reference point. Another reference point used is the fifth lumbar vertebra(L5) (Plate. 5.1).

For each measurement, it would be ideal if the surface could be placed with the same position and orientation. However, the surface of the back may change due to random voluntary postural changes or due to involuntary structural changes. Therefore, for repeated mesurements it is important to control postural changes and to place the back surface in a fixed position and orientation. A postural frame is used in an effort to achieve standardization of position without altering structural changes.

The postural frame used in this investigation was designed for alignment of the lower limbs (Plate. 5.1) and therefore only provides a minimum amount of postural restraint. Further postural restraint may include hip alignment, sternum location and shoulder alignment. With regard to these considerations, it is not certain whether

Plate. 5.1. A patient standing in the positioning device. Markers are used to locate the L5 and C7 vertebrae.

restrictive postural control serves to mask the structural changes that the moire technique is attempting to detect.

Elsewhere, various degrees of restraint have been applied. Willner(21), uses no positioning device, but only requires the patients to stand in a relaxed, square position. Conversly, Bio-Tek Instruments Incorporated(17), is currently manufacturing a portable moire contourgraph for the purpose of school screening which includes a positioning device that fixes four anatomical landmarks; the midclavicle bony landmarks, and the anterior superior iliac spines over the pelvis. However, some members of the medical community(1), feel that in severe scoliosis the orientation of the thorax is deviant from the orientation of the pelvis. Therefore, by positioning both the pelvis and also the clavicles, a twist may be introduced. Alternatively a three point fixation device, where the hips and sternum are standardized, have been used but it was found that the patient could rotate about the sternal pad(1).

From the engineering standpoint, the goal is to provide a reference surface from which measurements are made. The purpose of the positioning device then, is to permit reproducibility of posture and therefore allow only shape changes due to structural changes of the patient. It needs to be shown that alignment of the shoulders and hips will provide the necessary standardization of position.

Alternatively, a posture other than upright may provide more control over a person's shape. For example, Harada et al(28), required the patient to be in the forward bending position, and Shinoto et al(24) used a positioning device where the patient lies against a surface which is inclined at a small angle to the vertical. However, it is the cosmetic appearance while in the standing position, and not the measured shape which is important to the patient.

5.4 Photographic Procedure

For the first few tests, using Kodak technical Pan Film 2415 with an ISO rating of 125, a set of photos with various exposures (Table 5.1) was taken of the grid projected onto the reference screen to record the reference grid.

Exposure Time	f / number			
1/30(secs)	2.5	4	4.8	5.6
1/15		4	4.8	5.6

Table 5.1 : Exposures used in the photographic procedure

Paper markers were used to locate the reference points C7 and L5 on the patient. The reference screen is removed and the patient positioned in the standing frame with the feet aligned against the lower bar. The patient was asked to grasp the support uprights at shoulder height, and at the same time to stand tall and to look straight ahead (Fig 5.2). The centre of the camera's view finder was aligned to correspond to point O, Fig (5.1), of the grid, and a set of exposures (Table 5.1) was taken.

To develop the film, a high contrast developer (Kodak D-19) was used. For increased contrast, a developing time of two minutes was added to the time specified by the manufacturer. Standard small tank processing procedures, as specified by the manufacturer, followed.

More recently, a reduced range of exposures has been used as better control if the darkroom process has evolved.

5.5 Formation of the Fringe Pattern

Two negatives from the set of exposures, one each of the reference and subject grids, were chosen so as to produce clear fringes when overlayed. These two negatives were then superimposed between two glass plates so that points E, Fig. 5.1, of the two images coincide. The superpositioning of the two images results in the formation of a multiplicative fringe pattern(2.1). As mentioned in (2.1), the multiplicative fringe pattern is preferable over the additive fringe pattern created by using double exposure, as the former has inherently higher contrast.

The fringe pattern produced in this way has poor local sensitivity as the fringes are widely spaced (Plate. 5.2). The sensitivity could be increased by a decrease in grid pitch. However, this is undesirable as any reduction would limit the depth of focus. Therefore, to increase the local sensitivity, rotational mismatch was used (2.4). The reference grid was rotated with respect to the subject grid based on a compromise between two extremes. For a small grid rotation, the fringes are widely spaced, while for extreme rotational mismatch, the fringes are merely intersections of nearly horizontal planes with the back of the patient. For a camera with a horizontal view of the subject, this system is insensitive to topographic detail. For a relative grid rotation of 10 to 15°, approximately 20 to 30 fringes appear on the patient between L5 and C7. This provides acceptable spacing of fringes as well as monotonic change of fringe order with position (Plate 5.3).

5.6 Quantification of the Analysis

The moire fringe slide was projected onto a digitizing slaten and the following points were digitized:

1. a local origin (E) and another point on the positive axis (Fig. 5.1),
2. two points (G and H) on the subject grid for determining the scale factor,
3. two points on the "vertical" axis of the rotated reference grid for the purpose of determining the rotation angle,
4. the projection of a known reference (O) point onto the subject,
5. the anatomical landmarks denoting L5 and C7 vertebrae.

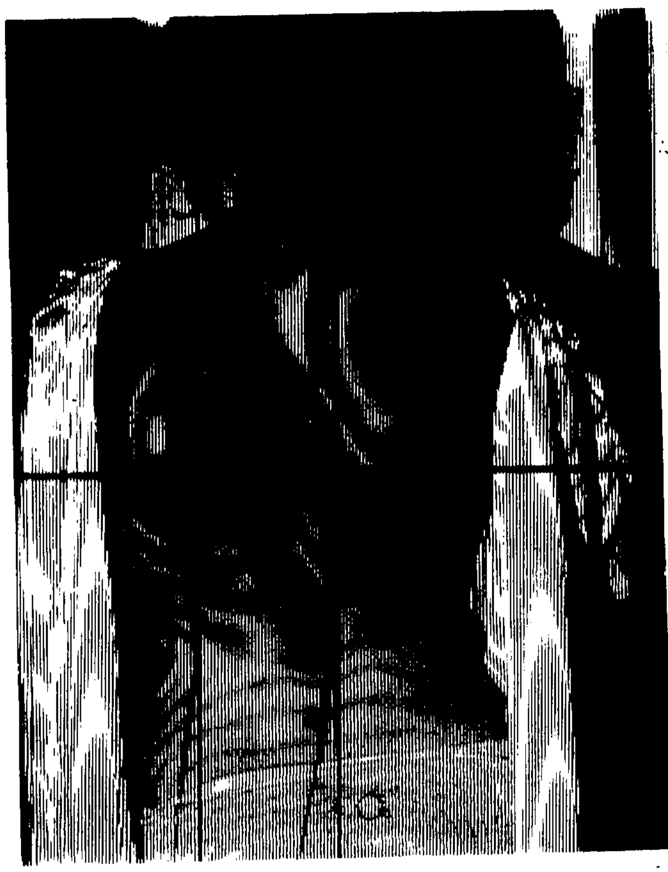

Plate. 5.2. A typical fringe pattern using no rotational mismatch.

Plate. 5.3. A typical fringe pattern when rotational mismatch is applied.

The absolute fringe order at the origin (O) was determined using equation 4.17. The fringes adjacent to the reference point were then determined using interpolation. The remainder of the fringes were numbered incrementally proceeding from the L5 fringe. Each fringe was located by digitizing points along the centre line, and coordinates (x',y',z') of the points were determined by applying equations (4.8, 13, 14, 15). These coordinates were taken relative to the L5 vertebra. A flow chart of the program is included in Appendix (B).

The digitized coordinates include random errors. To minimize this error, a smoothing process was introduced where the algorithm of Dierkx (29) was applied several times to determine the optimum smoothing function. Details of this process is given in Chapter 7.

The coordinates of the contours were then completely defined by applying Dierckx's algorithm. The algorithm determines a smoothing spline function and determines automatically the number of nodes and their position. The number of nodes is less than the number of data points. Compared to a cubic spline algorithm, this algorithm requires less storage space as only the position of nodes and coefficients of the normalized B splines, and not all of the raw data points, need to be stored.

Of particular clinical interest is a transverse profile of the subject which demonstrates asymmetries present in the trunk. To obtain a cross-section, intersection points of a given elevation plane with the digitized points were determined. Fringe values at other points along this line are obtained by interpolating between fringes on either side.

Fig. 5.2 illustrates a typical profile obtained from an analysis of the topography. Two features of this profile are clinically significant, and have been used clinically in monitoring the progression of the deformity by several researchers including Roger(5) and Stokes et al(23). The inclination or slope across the rib humps is a measure of trunk rotation and is obtained by measuring the slope of a line tangent to the two peaks of rib humps. The depth of the valley segment with respect to the two peaks is the parameter of clinical significance. The depth is made non-dimensional through division by the distance between tangent points. In the example of Fig. 5.2, the slope profile is 7° and the depth width ratio is 0.07.

5.7 Establishment of a correlation between internal and external deformities

For the moire technique to be effective in monitoring the progression of the deformity in a scoliotic patient, a relationship between the internal spinal deformity, and the external cosmetic deformity must be established. For this reason, a longitudinal study of patients with idiopathic

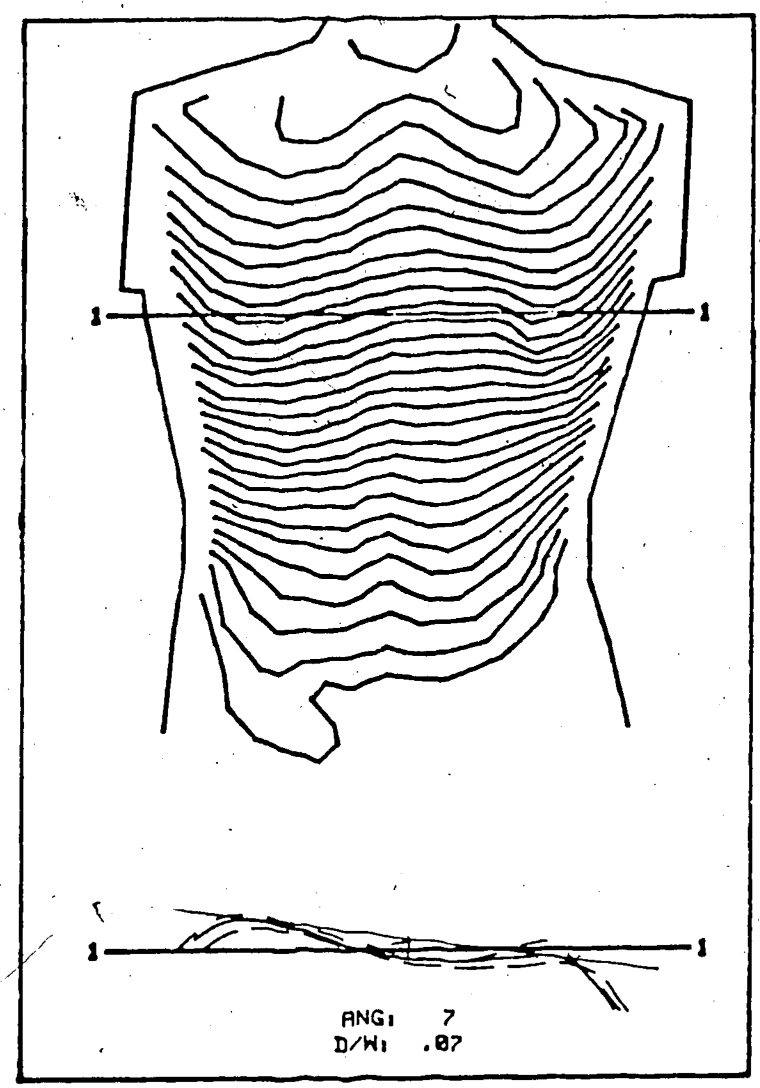

Fig. 5.2. Typical Profile at elevation 1-1. The dashed lines represent range of profile from repeated analysis.

scoliosis was conducted. The patients had both anterior-posterior x-rays and moire photographs taken in the positioning device, described in (5.4), to standardize position. The Cobb angle and the vertebral rotation at the apex from the roentgenographs were measured, and their relative level noted. From the moire analysis, the trunk rotation and a measure of the valley depth to trunk width were determined. These were also measured at a reference profile at approximately the L4 level. Because of the sloping ribs in the thoracic spine, the trunk rotation was measured at two levels below the apex for thoracic curves (23). The relative trunk rotation was determined by noting the change from the reference level.

6. Error Analysis

Unfortunately, experimental error is an inherent feature of any measurement system. These errors arise in the following basic steps which were described in detail earlier in this report:

1. determination of the absolute fringe number, N', using equation 4.17,
2. evaluation of Δx, using equation 4.8,
3. evaluation of the height z' using equation 4.13.

In this investigation the standard deviation (σ), determined through experimentation, is used as a measure of the error in a particular parameter. The error in z' is determined as a function of the error in each physical measurement. From equations (4.8, 13, 17) the following is apparent:

$$z' = f(L, d, \Delta x)$$
$$\Delta x = g(N', s, \phi, x, y)$$
$$N' = h(s, \phi, x_0)$$

therefore,

$$z' = f(L, d, g(h(s,\phi,x_0)\ s,\phi,x,y)) = F(L,d,s,\phi,x_0,x,y)$$

The error in z' is a function of the errors in L, d, s, φ, x_0, x and y. To determine the relationship between the error in z' and its causes, the chain rule of differential calculus is applied (30):

$$(\delta z^t)^2 = \left(\frac{\partial z^t}{\partial L}\,\delta L\right)^2 + \left(\frac{\partial z^t}{\partial d}\,\delta d\right)^2 + \left[\left(\frac{\partial z^t}{\partial \Delta x}\frac{\partial \Delta x}{\partial s} + \frac{\partial z^t}{\partial \Delta x}\frac{\partial \Delta x}{\partial N'}\frac{\partial N'}{\partial s}\right)\delta_s\right]^2$$

$$+ \left[\left(\frac{\partial z^t}{\partial \Delta x}\frac{\partial \Delta x}{\partial \phi} + \frac{\partial z^t}{\partial \Delta x}\frac{\partial \Delta x}{\partial N'}\frac{\partial N'}{\partial \phi}\right)\delta\phi\right]^2 + \left(\frac{\partial z^t}{\partial \Delta x}\frac{\partial \Delta x}{\partial N'}\frac{\partial N'}{\partial x_0}\,\delta x_o\right)^2$$

$$+\left(\frac{\partial z^t}{\partial \Delta x}\frac{\partial \Delta x}{\partial x}\,\delta x\right)^2 + \left(\frac{\partial z^t}{\partial \Delta x}\frac{\partial \Delta x}{\partial y}\,\delta y\right)^2 \qquad (6.1)$$

where σ_L, σ_d, σ_s, σ_ϕ, σ_{xo}, σ_x, σ_y are the respective errors for ∂L, ∂d, ∂s, $\partial\phi$, ∂x_o, ∂x, ∂y. Differentiation and substitution yields:

$$(\sigma_z t)^2 = \left[\frac{z^t}{L}\,\sigma_L\right]^2 + \left[\frac{z^t}{\Delta x + d}\,\sigma_d\right]^2 + \left[\frac{z^t d}{\Delta x(\Delta x + d)}\,\bar{N}\,\sigma_s\right]^2$$

$$+ \left[\frac{z^t d}{\Delta x(\Delta x + d)}\,((x - x_0)\sin\phi - y\cos\phi)\,\sigma_\phi\right]^2$$

$$+ \left[\frac{z^t d}{\Delta x(\Delta x + d)}\,(1 - \cos\phi)\,\sigma_x\right]^2 + \left[\frac{z^t d}{\Delta x(\Delta x + d)}\,\sin\phi\,\sigma_y\right]^2$$

$$+ \left[\frac{z^t d}{\Delta d(\Delta x + d)}\,\cos\phi\,\sigma_{x_0}\right]^2 \qquad (6.2)$$

where:

$$N = \bar{N} - N' = \text{relative fringe number}$$

In addition to errors in physical measurements, other sources of error must be considered. The reproducibility of patient positioning must also be evaluated.

7. Results

Uncertainty in z' depends on systematic errors as well as random errors in physical measurement. Some parameters such as the digital coordinates Xd and Yd, have inherent random errors for each reading taken. Others such as ϕ and the scale factor, are random because a different error exists for each analysis. However, within each analysis these errors are constant and produce a systematic error in z'. Some parameters, such as measurements d and L are systematic and do not change from analysis to analysis. For a particular parameter α, the standard deviation is σ_α, and the contribution of this single parameter to an error in z' is:

$$\sigma_{z'} = (\partial z'/\partial \alpha)\sigma_\alpha \qquad (7.1)$$

7.1 Systematic Errors

Consider the terms which include systematic errors only. Uncertainty in L includes two factors. First, the exact nodal locations of the camera and projector are difficult to determine. These points lie close to the focal distance in front of the film plane. Secondly, any unevenness of the reference plane will also produce a systematic variation in L. Measurements carried out as a part of this investigations have shown that variation in screen flatness is minimal. Uncertainty in L as well as d is primarily dependent on the uncertainty in the location of the nodal points. The estimated error in L and d could not be determined precisely; however, their effect on the result ($\sigma_z{}'$) is insignificant, (Table 7.1).

Variation in grid spacing (s) was estimated by overlapping two reference grids. σ_s was estimated by shifting the reference grids with respect to each other in the (x) direction only. This resulted in the appearance of one fringe over the entire reference area containing 320 grid lines for an estimated error 0.007mm.

Coordinates were measured on the projected image, which involved scaling of the digitized coordinates Xd and Yd. It follows then that the appropriate version of equation (4.8) is:

$$\Delta x = Ns + F(Xd(1-\cos\phi)-Yd\sin\phi) \qquad (7.2)$$

where F is the scale factor.
The error in z' induced by an error in scale factor is:

$$\sigma_z{}' = (\partial Z/\partial \Delta X)\ (\partial \Delta X/\partial F)\ (\sigma_F)$$

$$= (Z'd/\Delta x(\Delta x+d))\ (Xd(1-\cos\phi)-Yd\sin\phi)\sigma_F \qquad (7.3)$$

By repeating digitizing, σ_F was determined to be 0.0004. Similarly, the standard deviation (σ_ϕ) of rotation angle ϕ was determined to be 0.06°. In this experiment, L_P was set equal to L_C. If ΔL is not equal to zero, equation (4.12) results in:

$$\sigma_z{}' = (\partial Z'/\partial \Delta L)\sigma_{\Delta L} = (x\ /\ \Delta x(1+d/\Delta x(1+x\Delta L/dL))^2)\ \sigma_{\Delta L}$$

(7.4)

where $\sigma_{\Delta L}$ is approximately equal to σ_L.

7.2 Random Errors

Conversion of visual data into numerical data introduces random errors in the digitized coordinates of points on each fringe since the fringe centre line location is based on observed light intensity. To determine the standard deviation of this digitized error, a straight line horizontal fringe was projected onto the digitizing platen. Through repeated digitizing, σ_Y was determined to be 0.50mm. This defines the error, measured in a direction normal to the fringe centre line, resulting when fringe coordinates are digitized.

Based on this known value of digitizing error, optimized smooth polynomial splines were used to define each fringe contour by the following method.

1. a polynomial was used to represent a typical fringe in terms of its centreline location;
2. simulated data points were generated and scattered about the defined curve using a standard deviation of 0.50 mm (perpendicular distance from curve centreline);
3. the algorithm of Dierkx was applied several times to determine the optimum smoothing function.

In applying the above technique, the most effective smoothing was achieved when the standard deviation of perpendicular distance between data points on the Dierckx polynomial and the corresponding points on the initial seventh order polynomial was minimized. The optimum number of data points was determined to be 100 for which σ_x was determined to be 0.014mm and σ_y was determined to 0.13mm. The error in x was small since the fringe was predominantly parallel to the x axis. Errors in z' for points between fringes can be expected to be slightly higher due to interpolation required.

Ten percent of the data points at each end of the fringe were not included in the above calculation of standard deviations because this portion of each fringe is not of clinical significance and the ends of polynomial splines tend to have the poorest fit.

Based on error information before and after the above smoothing technique is applied, it is apparent that the procedure effectively reduces the random error introduced by digitizing.

7.3 Reproducibility of Patient Positioning

One feature of clinical significance is the difference in height of the rib humps ($\bar{z}'$). To evaluate the reproducibility of patient positioning, a series of photographs was taken where the patient was placed in the standing frame, photographed, and removed. Figure (5.2) illustrates the resulting range of a typical profile. An exact anlaysis of the difference in z' values between rib humps showed the repeatability to be ±4.4mm ($\sigma_{\bar{z}}'$), which includes the error due to posture changes and errors in the physical measurements. The resulting error in $\bar{z}'$ caused by particular parameters are tabulated in Table 7.1. The total uncertainty in $\bar{z}'$ from equation (10) is 0.81 mm. The largest errors in $\bar{z}'$ are caused by errors in s and ϕ. The other parameters contribute negligibly to the error in $\bar{z}'$. (Table 7.1)

7.4 Correlations Between Internal and External Deformities

A total of 63 curves from 48 patients were analyzed. The curves consisted of 47 thoracic curves and sixteen lumbar curves. For the purpose of this study, the curves were divided into two groups consisting of curves with the Cobb angle less than 30 degrees, and of curves with Cobb angle greater than 30 degrees. Each of these groups was further subdivided into thoracic curves and lumbar curves.

Overall, there was a wide range of Cobb angles with a mean of 28°. Details of each group are given in Table 2. For each group, the vertebra rotation at the apex of the curve was plotted against the trunk rotation at the same level; and for thoracic curves, at a level of 2 vertebrae below the apex (Figs. 7.1-7.4). There was generally a wide scatter of results, with generally poor correlation coefficients (ranging from 0.6 to 0.7). Stokes (23), who did a similar

Parameter (α)	Error (σ_α)	Error in [illegible] due to α ($\sigma_z'(\sigma_\alpha)$), mm
L	4 mm	0.026
d	4 mm	0.10
s	0.007 mm	0.64
ΔL	5.7 mm	0.004
F	0.00041	0.07
ϕ	0.001 rad	0.42
x_0	0.04 mm	0.21
x	0.014 mm	0.0014
y	0.13 mm	0.14

Table 7.1. Summary of Errors Contributing to Errors in Topographic Height.

Comments: Parameters L, d, s and ΔL include systematic errors only. F, ϕ, and x_0 include random errors but are systematic for a given trial. Coordinates x and y include fully random errors.

study, noted a correlation coefficient of 0.7 and concluded that there was no relationship between the apical vertebra rotation and the external rib hump.

However, when the trunk rotation was taken relative to a trunk rotation at the L5 level, the scatter was significantly reduced (Figs. 7.4-7.8) showing significant relationships (P=0.005) between these parameters. Although there was also a high correlation coefficient for these variables in the lumbar spine, there was insufficient data to properly test for a significant relationship.

Several researchers, such as Stokes et al (23), Shinoto et al (24), Armstrong et al (25), have attempted to observe a relationship between the topographic measurements and the Cobb angle. However, in this study no significant relationships between the Cobb angle and the trunk deformity were found. It was not expected that a relationship would exist, as the topographic measurements measure the rib hump, or the axial rotation deformity of the back surface, whereas at the skeletal level, the Cobb angle is primarily a measure of the lateral deviation of the spine.

		Cobb Angle, degrees	
	No. of Curves	Range	Mean
Overall	63	0 - 53	26.5
Cobb angle < 30°	52	0 - 29	14.5
Cobb angle > 30°	11	32 - 53	42.5
Thoracic	47	0 - 50	25.0
Lumbar	16	8 - 53	30.5

Table 7.2 : Details of the 63 structural curves in the 48 patients studied by the means of moire topography and standing AP xrays. For the purposes of this study, curves were divided into four groups.

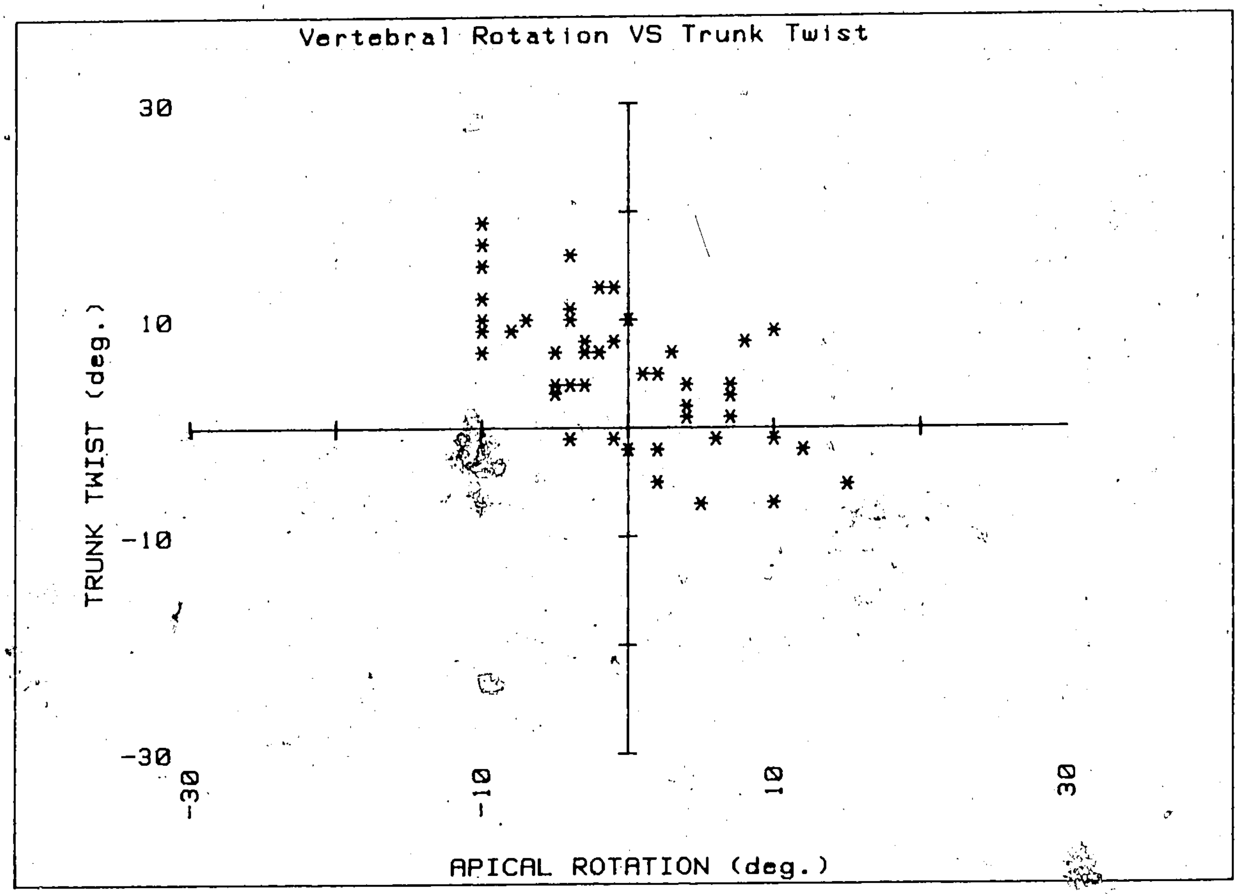

Fig. 7.1. Apical vertebral rotation versus trunk rotation at the curve apex for curves with a Cobb angle less than 30 deg.. Thoracic and Lumbar data are both included.

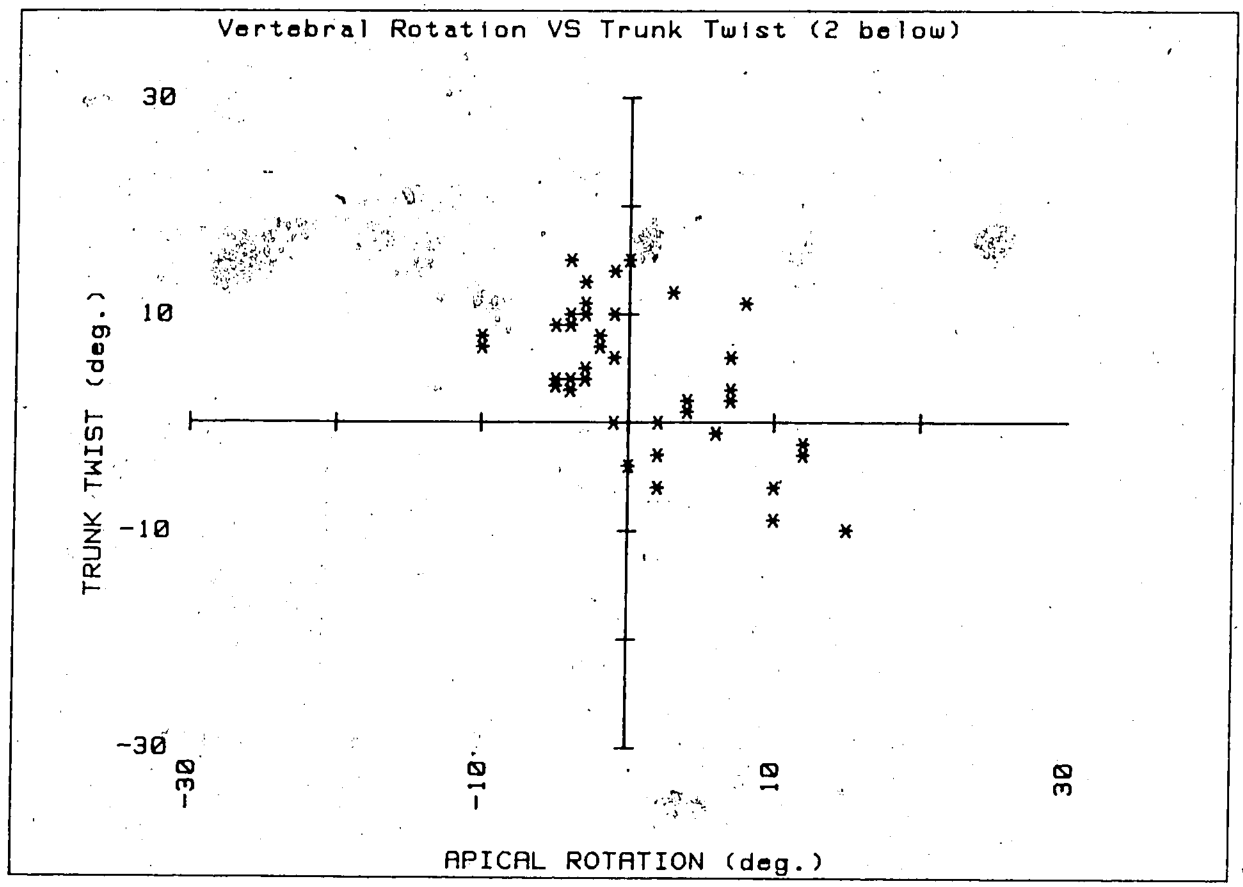

Fig. 7.2. Apical vertebral rotation versus trunk rotation at 2 vertebrae below the curve apex for thoracic curves with a Cobb angle less than 30 deg..

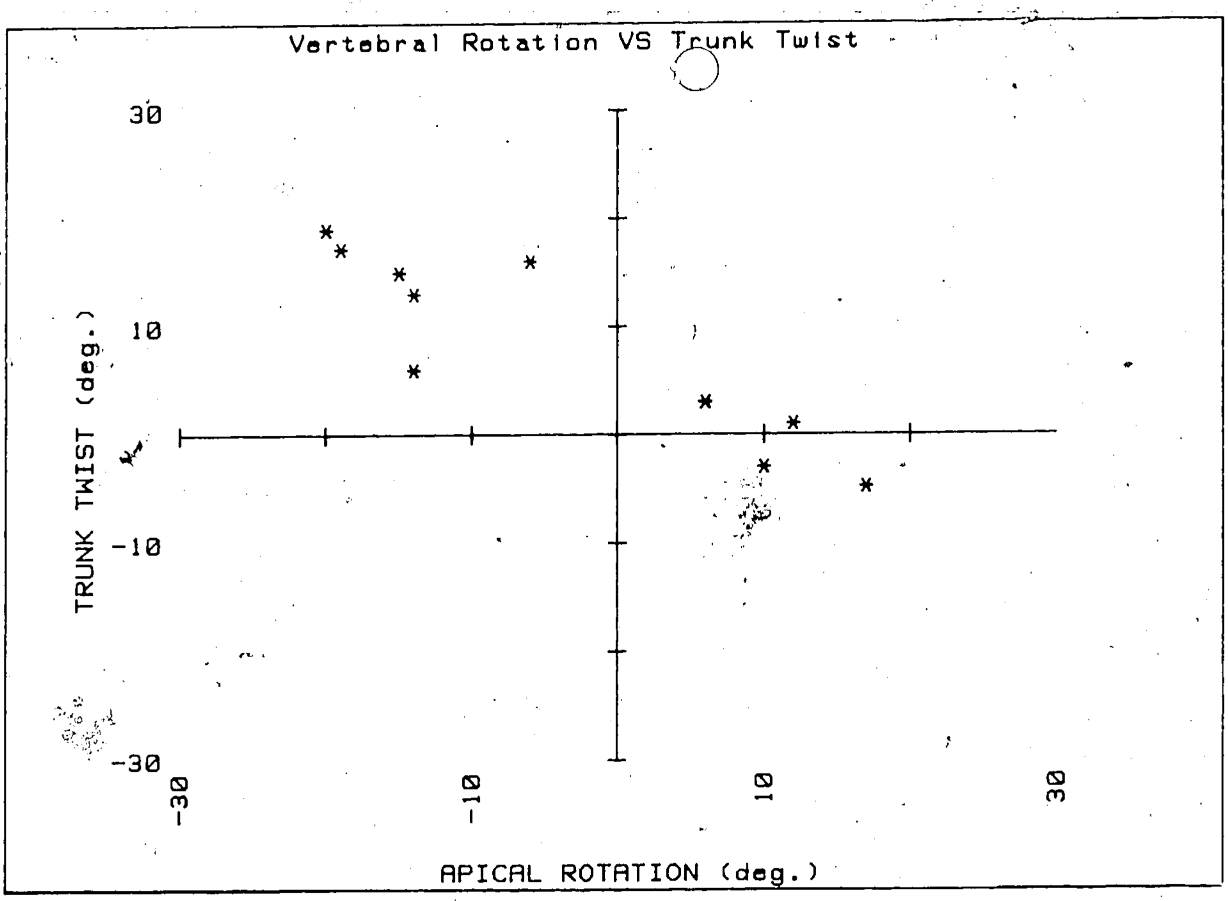

Fig. 7.3. Apical vertebral rotation versus trunk rotation at curve apex for curves with a Cobb angle greater than 30 deg.. Thoracic and lumbar data are both included.

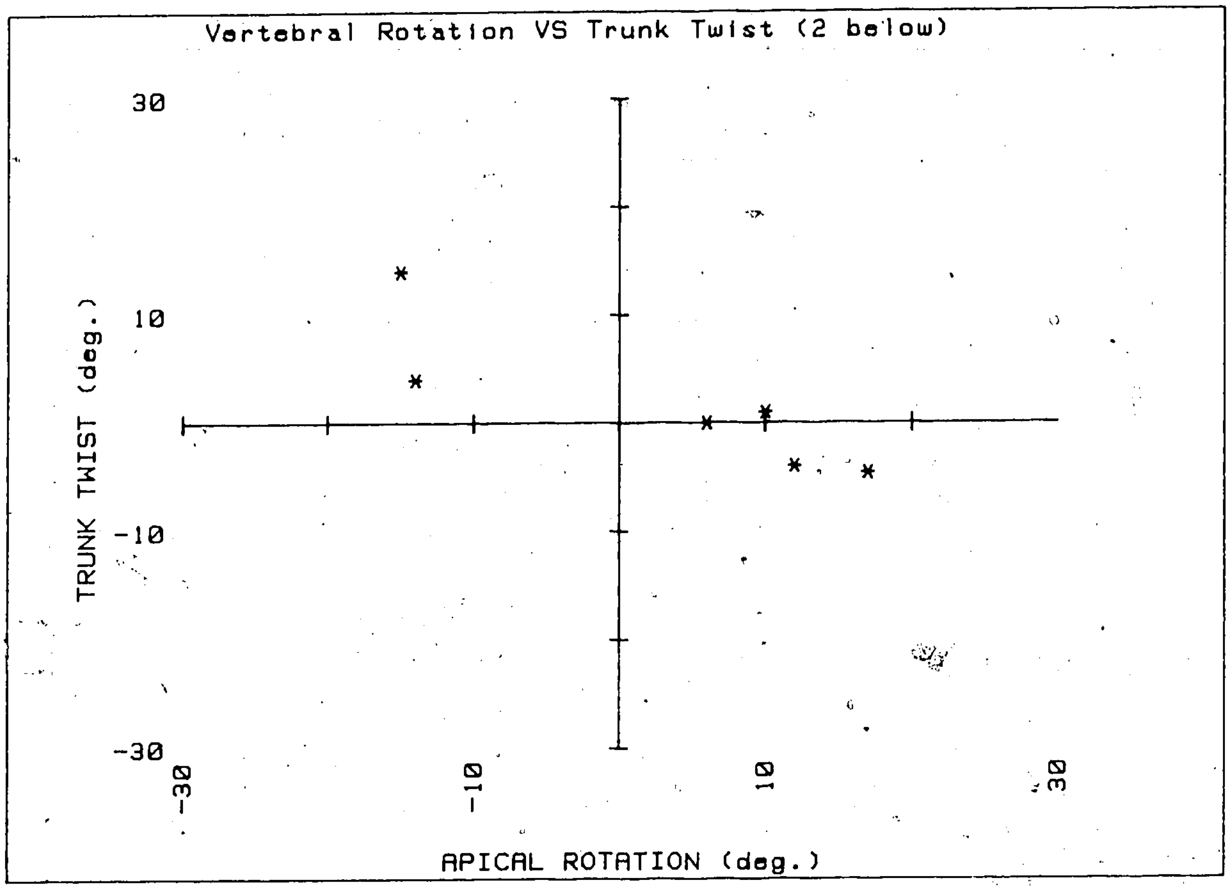

Fig. 7.4. Apical vertebral rotation versus trunk rotation at 2 vertebrae below the curve apex for thoracic curves with a Cobb angle greater than 30 deg..

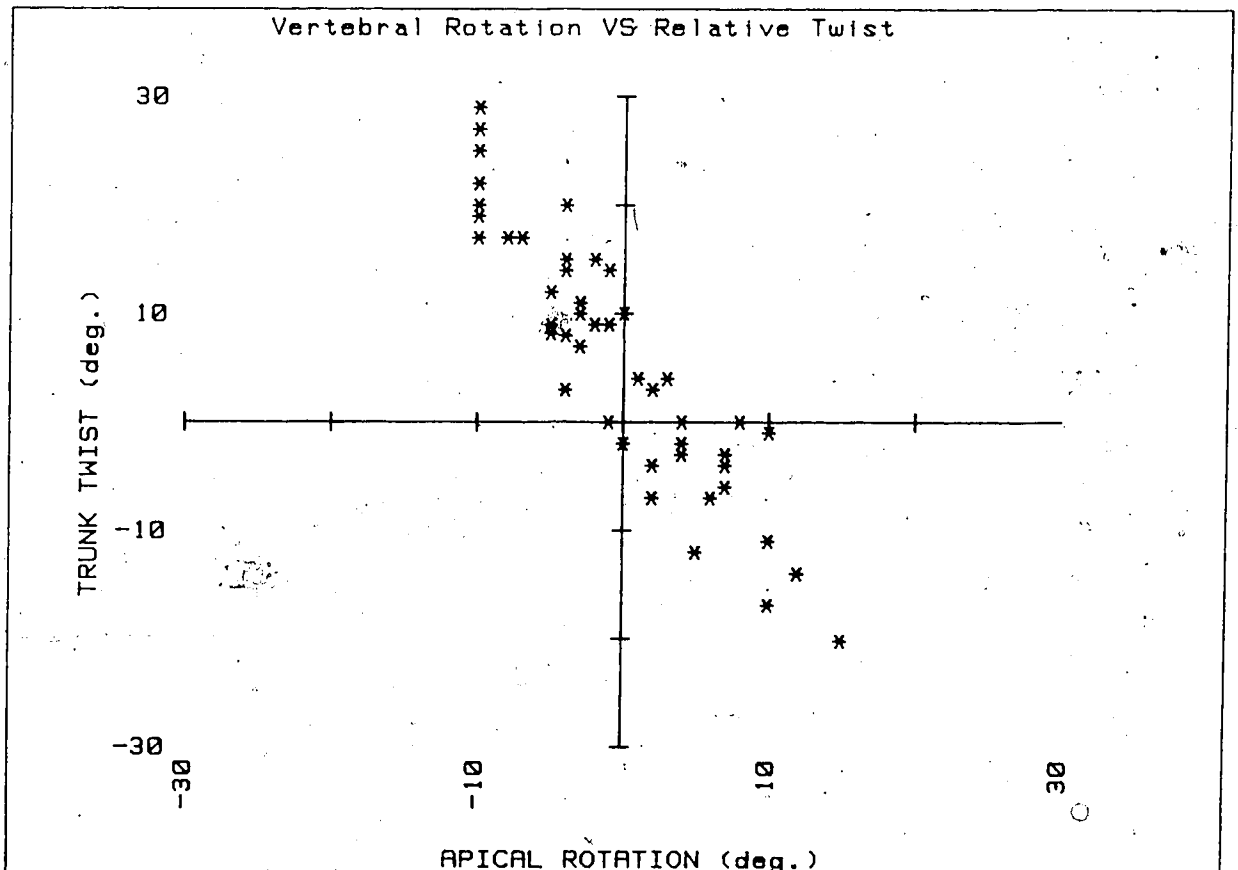

Fig. 7.5. Apical vertebral rotation versus relative trunk rotation at curve apex for curves with a Cobb angle less than 30 deg. Thoracic and lumbar data are both included.

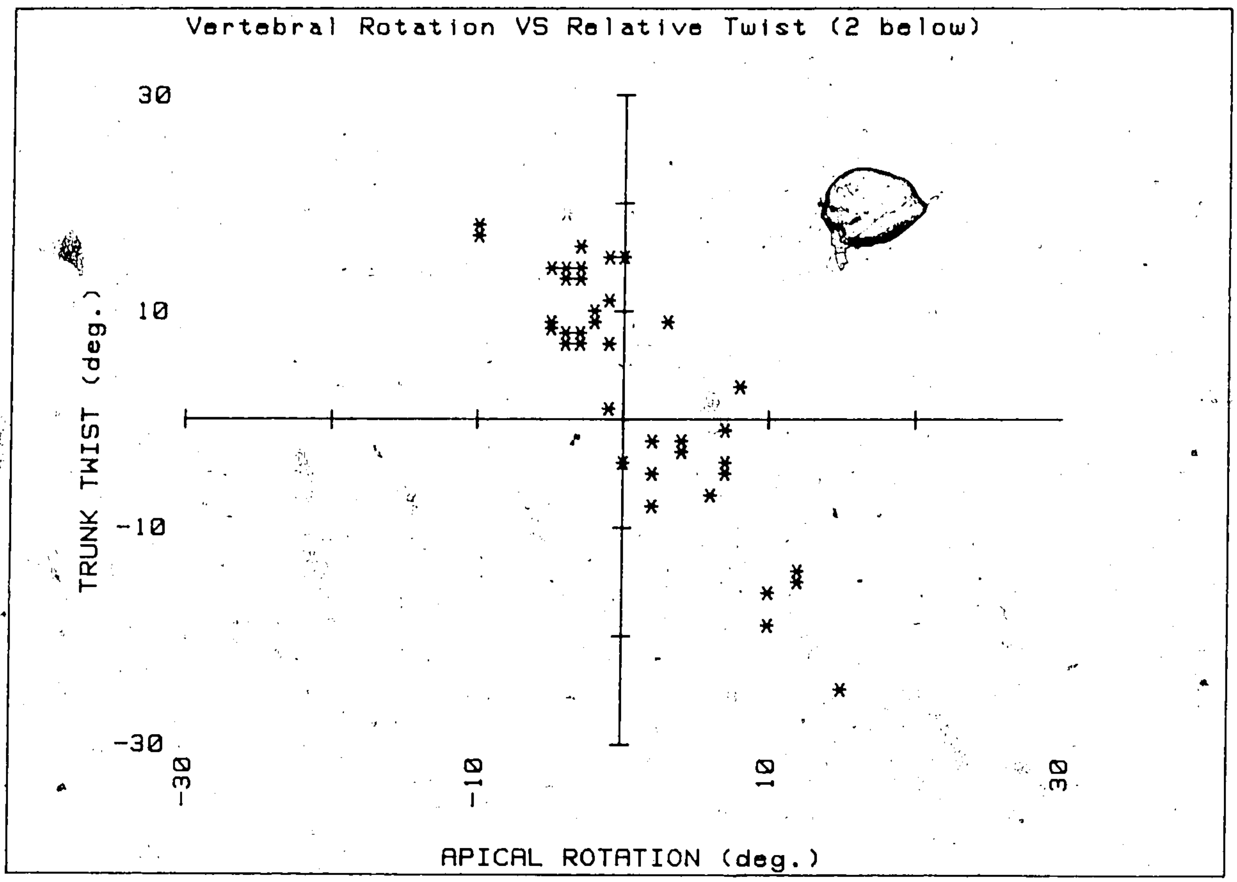

Fig. 7.6. Apical vertebral rotation versus relative trunk rotation at 2 vertebrae below the curve apex for thoracic curves with a Cobb angle less than 30 deg..

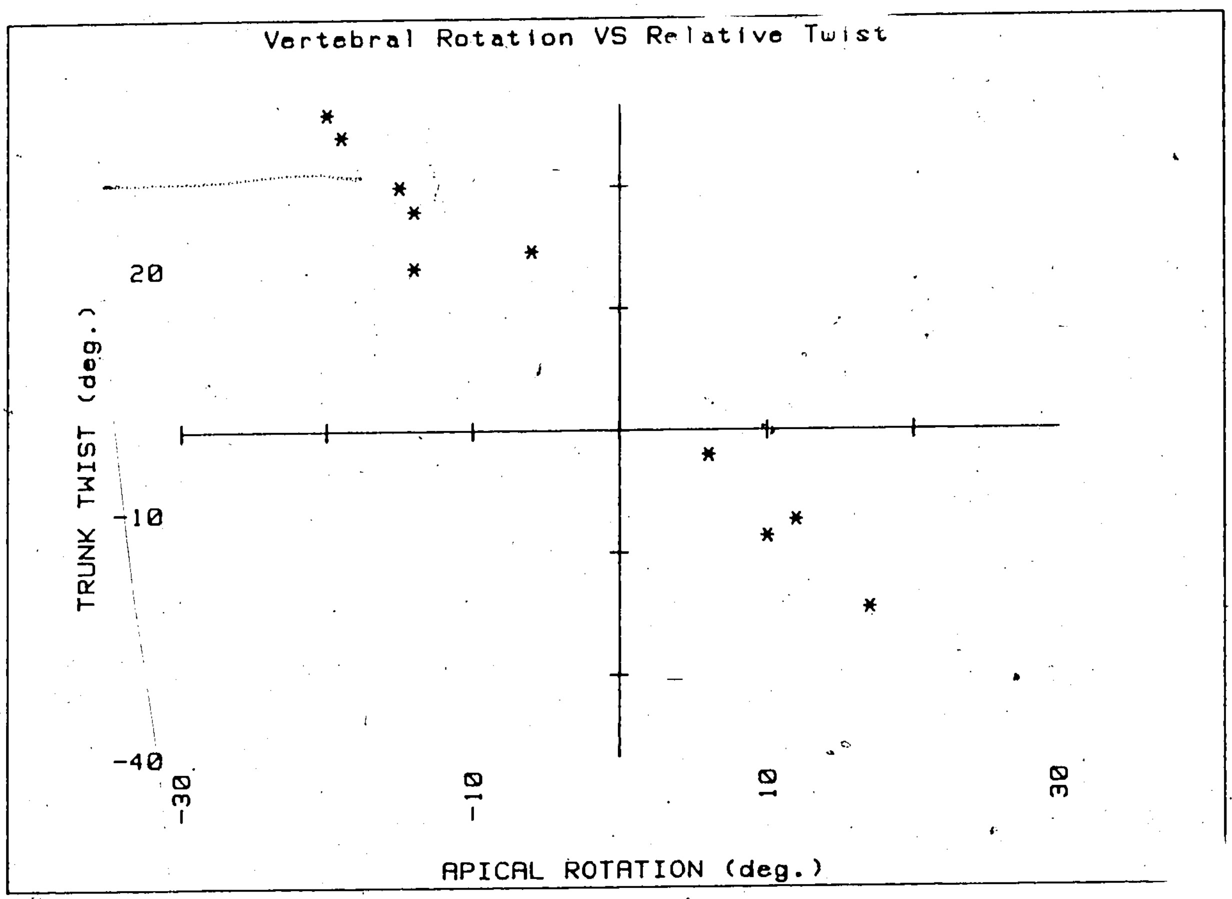

Fig. 7.7. Apical vertebral rotation versus relative trunk rotation at curve apex for curves with a Cobb angle greater than 30 deg. Thoracic and lumbar data are both included.

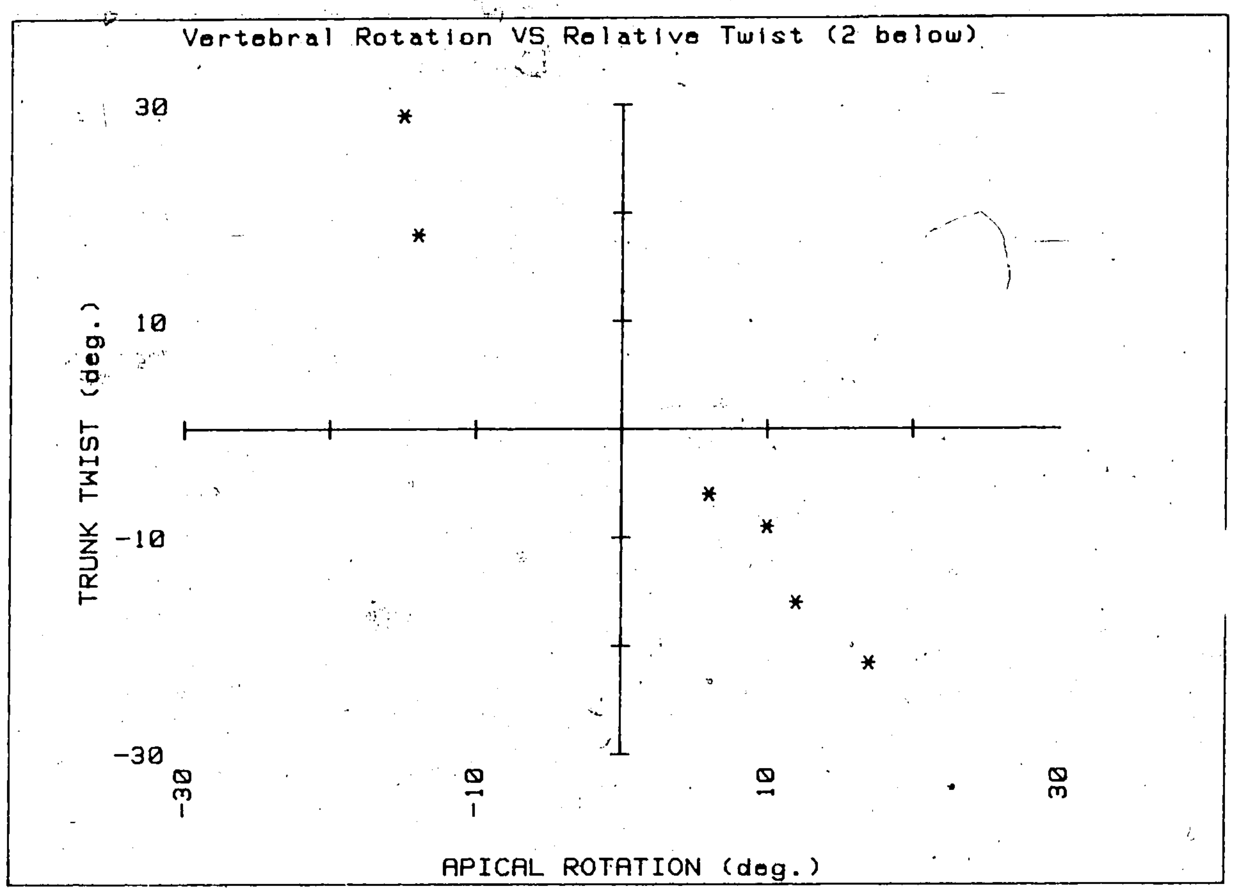

Fig. 7.8. Apical vertebral rotation versus relative trunk rotation at 2 vertebrae below the curve apex for thoracic curves with a Cobb angle greater than 30 deg..

Conclusions

The projection moire method used in this investigation is capable of determining the topographical contour of a patient to within a standard deviation of less than one millimetre. Further improvement can be achieved by an improved grid slide and by improving the method of determining the rotation of the reference grid with respect to the subject grid. All other parameters that are used to calculate the contour information contribute insignificant errors.

It was found that the minimum restriction postural frame used in this investigation permitted postural variation greatly in excess of the sensitivity of the moire measurement system. The next step in the investigation requires the development of a postural frame that further reduces the freedom of the subject to take an arbitrary stance. It is suggested that both hips and both shoulders should make contact with the postural frame. It would then need to be shown that such a postural frame does not alter the appearance of scoliosis and that changes in the trunk deformity due to progression of the scoliosis can still be detected. In work published by Turbourne et al.(31), the changes in the trunk deformity over time can be expected to be in the range of 10 to 20 mm; based on measurements taken in a forward bend position. In this regard, the moire topographical procedure is a promising method of monitoring scoliosis. However, automation of the process is a requirement for fast, accurate processing of the patients. Automation would require sophisticated equipment and would be justified on an economic basis only if a large number of patients were processed daily. The cost of automation then, may be prohibitively high for smaller centers.

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Appendix A : Nonmenclature

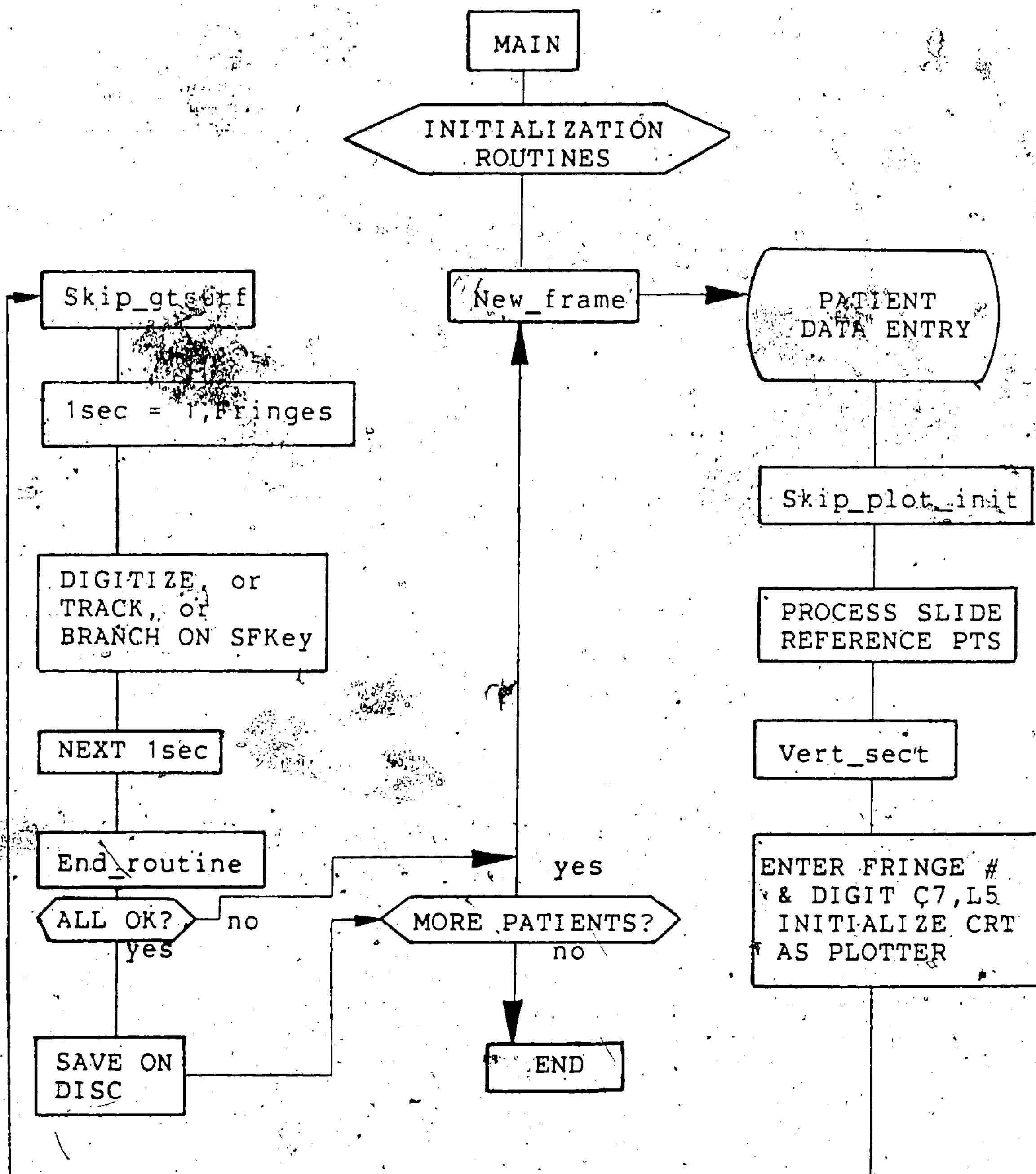
LIST OF SYMBOLS

C	Camera nodal point
d	Horizontal distance which separates light source and camera
L_P	Distance from reference plane to projector nodal point
L_c	Distance from reference plane to camera nodal point
L	Common value for L and L in this experiment
$\bar{N}$	Relative fringe order
N'	Absolute fringe order of reference point
N	$\bar{N}$ + N', absolute fringe order
O	Local origin
P	Projector nodal point
S_o	Interfringe spacing which is the perpendicular distance between adjacent fringes
s	Pitch of the subject grid
s'	Pitch of the reference grid
x'	Horizontal distance from the origin to position of a grid line onto the reference plane
x_o	The horizontal distance between the origin of the reference grid with respect to the origin of the subject grid
x,y	The horizontal and vertical distances from the origin to the apparent position of a subject grid line in the reference plane
x',y',z'	Surface coordinates
$\bar{z}'$	The difference in height between the sides of the thoracic cage
ΔL	Estimated difference between L and L
Δx	x - x'
β	The angle of incidence of the projector defined as the acute angle between the optical axis of the projector and the reference plane
θ	The orientation of the fringes as measured counterclockwise from the secondary direction of the reference grid
λ	Linear mismatch of reference grid with respect to subject grid
φ	Rotation of reference grid with respect to subject grid
σ_d	Estimated error in the determination of d
σ_F	Estimated error in the determination of the scale factor
σ_L	Estimated error in the determination of L
σ_s	Estimated error in the grid pitch s
σ_{xo}	Estimated error in the determination of x_o
σ_x', σ_y', σ_z'	The estimated errors in the coordinates

	x', y', z'
$\sigma_{\bar{z}'}$	Estimated error in the determination of $\bar{z}'$
σ_{ϕ}	Estimated error in the determination of the rotation angle

Appendix B : Flowchart

FLOWCHART - PROGRAM DIGIT



ANALYSIS OF MOIRE FRINGES

PROGRAM DIGIT : USERS MANUAL

PROGRAM SEGMENT : MAIN

GOSUBS

1. DEBUG: Routine which set up to eight variables: Debug 1 to Debug 8. Setting any of these flags to 1 cause particular values to be dumped to the thermal printer.

2. DATA_INIT: Initialization of variables used in the analysis of digitized data. This routine also sets up the rotation matrix to produce the oblique view.

3. TITLE: This subprogram produces a title block which outlines the limits of the program for the user.

PROGRAM DIGIT

PROGRAM SEGMENT: NEW_FRAME

GOSUBS

1. START: This routine guides the user through the beginning of the program. It prompts the user to power on the disc drive and to insert the appropriate discs into the drive.

   After this is done a list of files is displayed and the user is prompted for the patient name and history number and whether this is a clinical patient or a test set model. Any previous information on this patient is displayed along with a prompt to enter the appropriate examination number If this is a new patient then a file based on his hospital number is created and the user is asked for the following information:

   a) Birthdate (yy/mm/dd)
   b) Sex (m/f)
   c) Date of Photo (yy/mm/dd)
   d) Comments (up to 18 characters).

For previous patients (c) and (d) are repeated.

2. CHECK_DATE: Checks for too many days or months that may give in response to a date prompt.

3. DIGIT: Prompts the user to power on the digitizer and align the platten axis with the positive X axis of the slide.

PROGRAM DIGIT

PROGRAM SEGMENT: SKIP_PLOT_INIT

GOSUBS

1. SETUP: Guides the user in digitizing the correct reference points from the Moire slide. These reference points are needed to establish frame work within which the subsequent analysis is done.

2. DISPLAY: Sends a message to the LED on the digitizer. NOTE: This is the only actual subroutine in the program. As a subroutine it cannot be easily eliminated.

3. DIGIT: Sends the appropriate signals to the digitizer telling it is ready to receive data. One of three responses can occur:

   a) The digitizer bar is pressed on the cursor (this set BIT (STATUS, 2)) in which case an x and y coordinate are received in digitizer units. These coordinates are then converted to millimeters and used with fringe data to calculate spatial coordinates of the point.

   b) One of the keys of the digitizer keypad is pressed (BIT (STATUS,[illegible]) #0). If this occurs a special condition exists, eg. end of fringe, and causes the program to branch off accordingly.

   c) If neither (a) nor (b) exist then the x and y coordinates are received and converted to millimeters and used to position the cross on the CRT. This acts to track the digitizer cursor.

4. GTSURF: Converts the corrected x and y coordinates into x, y, z coordinates using the fringe number and the data determined in S/R SETUP.

PROGRAM DIGIT

PROGRAM SEGMENT: VERT_SECT

GOSUBS

1. DISPLAY - see program segment SKIP_PLOT_INIT.

2. DIGIT - see program segment SKIP_PLOT_INIT.

3. GTSURF - see program segment SKIP_PLOT_INIT.

4. PRINT_KEY_VALUES - routine causes the function of Fa to Fe on the digitizer keypad to be printed.

5. PLOT_INIT - sets the CRT to be the graphics output device. It goes on to set the limits and scale of the display.

PROGRAM DIGIT

PROGRAM SEGMENT: SKIP_GTSURF

GOSUBS

1. DIGIT - see program segment SKIP_PLOT_INIT.

2. GTSURF - see program segment SKIP_PLOT_INIT.

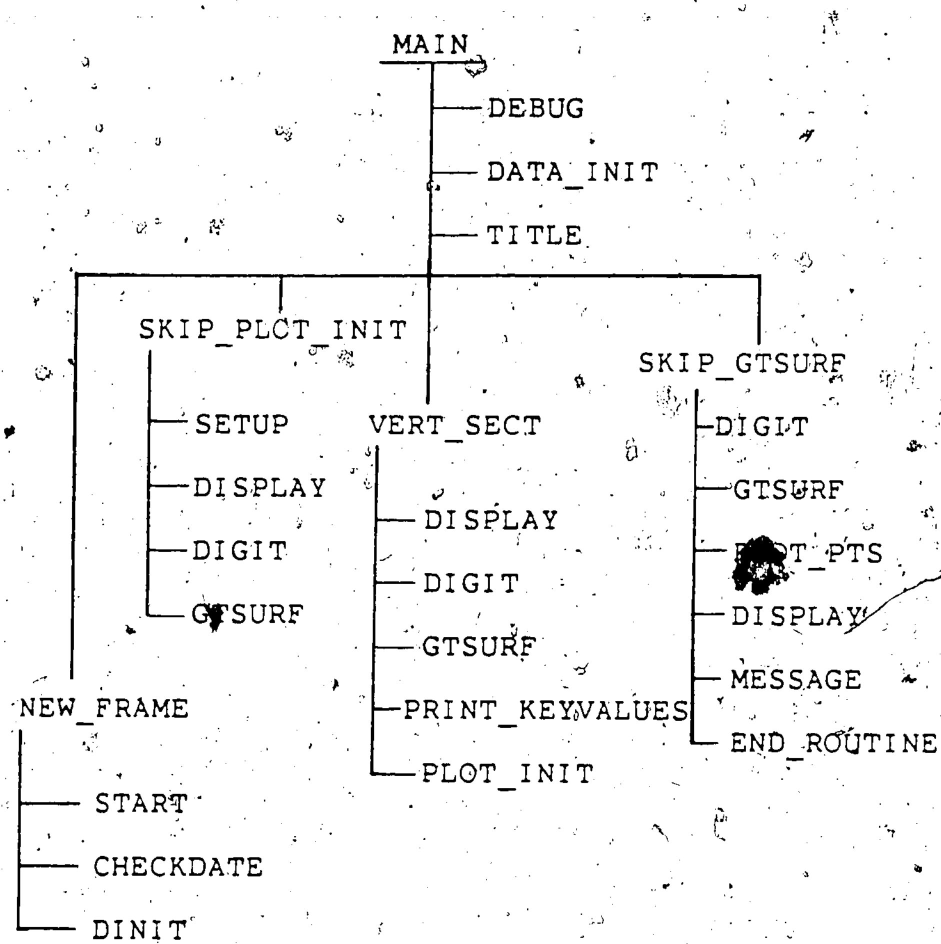
3. PLOT_PTS - plots on the CRT the point just digitized.

4. DISPLAY - see program segment SKIP_PLOT_INIT.

5. MESSAGE - displays a message indicating that the fringes are complete and to start digitizing the body outline.

6. END_ROUTINE - converts Xm(*), Ym(*) and Zm(*) to digitizer units and stores them in the integer array Surf(*). This matrix, along with an index of fringe points is stored on disc for future reference.

PROGRAM Digit
DIAGRAM OF ROOT SEGMENTS



Appendix C

```
10    REM       "DGT.V1:T15"
20    REM       MOIRE PHOTOGRAPH DIGITIZING ROUTINE UNDERGOING
30    REM       RENOVATIONS TO DIGITIZE IN CONTINUOUS MODE.
40    REM       In this version, the No. of points digitized (per fringe)
50    REM       may be in the range of    [ 0 to 500 ]
60    REM       Of these points, a VARIABLE No. of points will be stored.
70    REM       The final (say 25) points will be determined by an algorithm in
80    REM       the section called  'Point_calc'
90    REM       SEPT. 13, 1984
100   OPTION BASE 1
110   PRINTER IS 16
120   NORMAL
130   OVERLAP
140   EXIT GRAPHICS
150   ! ==================================================================
160   COM Fil$[6],INTEGER Rec_no
170   INTEGER Nf,Nc,Nt
180   DIM Xm(850),Ym(850),Zm(850),C(35,6),T(35,10),Xa(200),Ya(200)
190   DIM Rot(3,3),A(3),B(3),Xvictor(800),Yvictor(800),Xadd(3),Yadd(3),Npoints(1
00)
200       INTEGER Surf(800,3),Index(40),Nhighs,Linde ,Ntotal,Lnk
210       DIM Name$(15),Date$(13)[6]
220   DIM Pd(2),Xref(2),Yref(2),Orig(2),Landmark$(4)[2]
230   DIM Messl$[60],Title$(2)[121],Instring$[80],Histor$(36)[6],Cat$(25)[42]
240   ! ==================================================================
250             GOSUB Debug              ! SET FLAGS FOR TRACKING VARIABLES
260             GOSUB Data_init          ! INITIALIZE DATA
270             GOSUB Title
280 New_frame:  GCLEAR                   ! ENTER NEW DATA
290             EXIT GRAPHICS
300             GOSUB Start              ! QUERY USER FOR NAME AND NO. OF PATIENT
310 Dig_init:   GOSUB Dinit              ! INTIALIZES DIGITIZER & ALIGNE +X AXIS O
F GRATING
320 Skip_plot_init:   GOSUB Setup
330                   Factor=Tr/Dist
340                   A$="ROTATED GRATING"
350                   CALL Display(Sc,A$)
360                   A$="LOWER TICK"
370                   CALL Display(Sc,A$)
380                   GOSUB Digit
390                         Xref(1)=Xd-Tx1
400                         Yref(1)=Yd-Ty1
410                         EXIT GRAPHICS
420                   A$="UPPER TICK."
430                   CALL Display(Sc,A$)
440                   GOSUB Digit
450                         Xref(2)=Xd-Tx1
460                         Yref(2)=Yd-Ty1
470                         Dxref=Xref(2)-Xref(1)
480                         Dyref=Yref(2)-Yref(1)
490                         R_ref=SQR(Dxref^2+Dyref^2)
500                         Sinn=Dxref/R_ref
510                         Coss=Sinn*Dyref/Dxref
520                         A$="CENTER ON BACK"
530                         CALL Display(Sc,A$)
540                   GOSUB Digit
550                         Pd(1)=Xd-Tx1
560                         Pd(2)=Yd-Ty1
570                         Xdo=Pd(1)/Factor
580                         Dx1=Xdo-Orig(1)
590                         Nlines=Dx1/S
600              Delx=Nlines*Coss          ! -Dorig/S   CALCULATE DELX
610              IF Debug4=0 THEN 700
620              PRINTER IS 0
630              PRINT USING "4(K,MDDD.DD,2X)";"TX1= ";Tx1,"TY1= ";Ty1,"TX2= ";T
x2,"TY2= ";Ty2
```

```
640              PRINT USING "3(K,MDDD.DD,2X)";"TR= ";Tr,"COSN= ";Cosn,"FACTOR=
";Factor
650              PRINT USING "4(K,MDDD.DD,2X)";"XREF(1)= ";Xref(1),"YREF(1)= ";Y
ref(1),"XREF(2)= ";Xref(2),"YREF(2)= ";Yref(2)
660              PRINT USING "4(K,MDDD.DD,2X)";"XD= ";Xd,"YD= ";Yd,"PD(1)= ";Pd(
1),"PD(2)= ";Pd(2)
670              PRINT USING "3(K,MDDD.DD,2X)";"XDO= ";Xdo,"DX1= ";Dx1,"NLINES=
";Nlines
680              PRINT USING "3(K,MDDD.DD,2X)";"COSS= ";Coss,"DORIG= ";Dorig,"DE
LX= ";Delx
690              PRINTER IS 16
700              PRINT " C7/T1/MM LANDMARK "
710 Vert_sect:    !
720 Vs:           N=1
730               OUTPUT Sc;"DF"          ! CLEAR ANNOTATION BIT
740               Lsect=0
750               Iout=0
760               Fringes=2               ! ZERO ORDER FRINGE + BODY OUTLINE FRING
E
770               FOR N=1 TO 3
780                 A$="FRINGE AT "&Landmark$(N)
790                 CALL Display(Sc,A$)
800 Get_status:         OUTPUT Sc;"OS"
810                     ENTER Sc;Status
820                     IF BIT(Status,0)<>0 THEN Entry
830                     GOTO Get_status
840 Entry:          OUTPUT Sc;"ON"
850                 ENTER Sc;Ainc
860                 BEEP
870                 IF N=3 THEN Cont_15
880                 Fringes=Fringes+Ainc                 ! COUNT TOTAL FRINGES
890                 IF Fringes<=Max_fringes THEN Cont_15
900                 ! **
910                 EXIT GRAPHICS
920                 BEEP
930                 PRINT LIN(5)
940                 PRINT "TOTAL NO. OF FRINGES = ";Fringes
950                 PRINT "MAXIMUM ALLOWABLE = ";Max_fringes-2
960                 PRINT ""
970                 PRINT "ADJUST SO THAT THERE ARE LESS THEN ";Max_fringes-1;"
 FRINGES"
980                 PRINT "RUN ME AGAIN WHEN YOU HAVE DONE SO!"
990                 STOP
1000 Cont_15:       !
1010                A$="DIGITIZE "&Landmark$(N)
1020                CALL Display(Sc,A$)
1030                GOSUB Digit
1040                PRINTER IS 0
1050                IF Debug4=1 THEN PRINT "Xd=  ";Xd;"    Yd=  ";Yd
1060                PRINTER IS 16
1070                IF Yd>=0 THEN Ainc=-Ainc
1080                Delfr=Delx+Ainc
1090                Bel=Delfr
1100                IF N=2 THEN Temp_delfr=Delfr    ! SAVE THIS FRINGE NUMBER
1110                EXIT GRAPHICS
1120          !     PRINT LIN(2);"AINC=  ";Ainc;"   DELX=  ";Delx;"   BEL=  ";B
el
1130                GOSUB Gtsurf
1140                IF Debug4=0 THEN 1210
1150                PRINTER IS 0
1160                PRINT "AINC= ";Ainc;"   DELX=  ";Delx;"  DELFR=  ";Delfr
1170                PRINT "X0= (Pd(1)) ";Pd(1);"   XM= (Xm(N)) ";Xm(N);"   N=
";N
1180                PRINT "Y0= (Pd(2)) ";Pd(2);"   YM= (Ym(N)) ";Ym(N);"   N=
";N
1190                PRINT "ZM= (Zm(N)) ";Zm(N);"   N=  ";N
```

```
1200                   PRINTER IS 16
1210                 NEXT N
1220                 Delfr=Temp_delfr
1230                 Bel=Delx+Delfr
1240               GOSUB Beep
1250               GOSUB Print_keyvalues
1260               X15=Xm(2)
1270               Z15=Zm(2)
1280               GOSUB Plot_init
1290               Ierase=0
1300               FOR I=1 TO 3
1310                   X1=Xm(I)-X15
1320                   Y1=Ym(I)
1330                   Z1=Zm(I)-Z15
1340                   PLOT X1,Y1,-2
1350                   PRINT USING 1360;Landmark$(I),X1,Y1
1360                   IMAGE 5X,18A,2X,"X1=   ",10D.DD,2X,"Y1=   ",10D.DD
1370                   LABEL USING "K";Landmark$(I)
1380                NEXT I
1390                GOSUB Beep
1400                Ierase=1
1410 Skip_gtsurf:       !
1420                    Frm1=Fringes-1
1430                    Maxpts=Max_pts-Bodypts-3   ! MAXPTS - WILL BE UPDATED AFTER
                                                          EACH FRINGE
1440                    P_fringe=1
1450                    Isec1=0                    ! FLAG USED IN RESETTING [Maxpts]
1460                    !
1470                    FOR Isec=1 TO Fringes
1480 Select:                !
1490 S:                     IF Isec>1 THEN Isec1=0
1500                        IF Isec-1=Frm1 THEN Z              ! BODY OUTLINE CONDITION
1510                        Frleft=Frm1-(Isec-1)               ! FRINGES LEFT
1520                        Ppfring=Maxpts/Frleft              ! POINTS-PER-FRINGE
1530                        Ppout=Bodypts+(Maxpts-INT(Ppfring)*Frleft)  ! POINTS-PE
R-OUTLINE
1540                        EXIT GRAPHICS
1550                        IF Isec<=Frm1 THEN PRINT LIN(5);" NOTE:   No. of POI
NTS per FRINGE should not exceed: ";INT(Ppfring);"."
1560                        PRINT LIN(1);" NOTE:   There is room for: ";INT(Ppout
)-1;"  POINTS  in the BODY CONTOUR."
1570 Z:                     !
1580                        Index(Isec)=Iout=Nt=El_flag=0   !
1590                        MAT Xvictor=ZER                 ! ZERO THESE VARIABLES
1600                        MAT Yvictor=ZER                 !
1610                        OUTPUT Sc;"SK0"
1620                        A$="FRNG="&VAL$(Isec)&" N="&VAL$(N)&"."
1630                        CALL Display(Sc,A$)
1640                        IF Isec<=Frm1 THEN Digit_line
1650 Body_out:                !
1660                          WAIT Time
1670                          Mess1$="DIGITIZE BODY OUTLINE..."
1680                          A$="BODY OUTLINE..."
1690                          CALL Display(Sc,A$)
1700                          GOSUB Message
1710                          Iout=1
1720                          GOSUB Beep
1730 Digit_line:            !
1740                        Index(Isec)=N
1750                        Lsect=Isec
1760                        IF Iout<>1 THEN Call_gtsurf        ! DOING FRINGES
1770                        Lsect=Lsect-1
1780 Dig_bod:                 ! DIGITIZING ROUTINE FOR BODY OUTLINE
1790                          GOSUB Digit
1800                          DISP "POINTS REMAINING:";Maxpts+Bodypts-Nt;" ..."
1810                          IF El_flag=1 THEN Check_bod
```

```
1820                         IF Nt>=Maxpts+Bodypts THEN GOSUB Err2
1830                         GOTO Dig_bod
1840 Check_bod:              !
1850                         WAIT Time*5
1860                         EXIT GRAPHICS
1870                         Y$="Y"
1880                         GOSUB Beep
1890                         INPUT "Are you SATISFIED with the BODY OUTLINE ? (Y
,N) ",Y$
1900                         IF UPC$(Y$[1])="Y" THEN Last_section
1910                         Maxpts=Maxpts-Npoints(Isec)
1920                         Isec=Isec+1
1930                         Bel=Bel-1
1940                         GOTO Redo_line
1950 Call_gtsurf:           !
1960                        GRAPHICS
1970                        GOSUB Digit_fr
1980                        Maxpts=Maxpts-Npoints(Isec)
1990                        IF (Isec=Frm1) AND (Max_pts-N Bodypts-10) THEN GOSUB Er
r1
2000 Next_section:           ! NOT ENOUGH BODY POINTS HAVE BEEN LEFT...(NOT FATAL
2020 Last_section:      NEXT Isec
2030 Check_end:         !
2040                    Y$[1;1]="Y"
2050                    OUTPUT Sc;"SK0"
2060                    OUTPUT Sc;"DC"
2070                    Y$="Y"
2080                    GOSUB Beep
2090                    INPUT "IS ALL OK ? (YES/NO)",Y$[1,1]
2100                    Y$=UPC$(Y$)
2110                    IF Y$[1;1]="Y" THEN GOTO End_routine
2120                    PRINT " REDO ENTIRE FRAME"
2130                    Y$="Y"
2140                    P_fringe=0
2150                    GOSUB Beep
2160                    INPUT ". IS PATIENT PERSONNAL DATA OK ? (YES/NO)",Y$
2170                    Y$=UPC$(Y$)
2180                    IF Y$[1;1]="Y" THEN GOTO Dig_init
2190                    GOTO New_frame
2200                    ! ===========
2210 End_routine:       ! EXIT ROUTINE
2220 Endr:            EXIT GRAPHICS
2230                  Lindex=Lsect+Iout+1
2240                  Index(Lindex)=N
2250                  Ntotal=N-1                    ! TOTAL NUMBER OF POINTS DIGITIZED
2260                  Nhighs=Index(Lsect)-1         ! NUMBER OF HEIGHT POINTS DIGITIZED
2270                  DISP " WORKING  ..."
2280                  FOR I=1 TO Ntotal
2290                      Surf(I,1)=Xm(I)*40
2300                      Surf(I,2)=Ym(I)*40
2310                      Surf(I,3)=Zm(I)*40
2320                  NEXT I
2330                  Y$="Y"
2340                  READ #1,Exam+1
2350                  PRINT #1,Exam+1;Index(*),Ntotal,Nhighs,Lindex,Surf(*)
2360                  ASSIGN #1 TO *
2370                  Y$="N"
2380                  Fil$=Hist$                      ! THESE VARIABLES ARE
2390                  Rec_no=Exam+1                   ! COMMON TO  "NDF_MT"
2400                  INPUT "Put this data into  NEW DATA FILE ? (Y,N)",Y$
2410                  IF UPC$(Y$[1;1])<>"Y" THEN 2440
2420                  DISP "  LOADing     DATA  TRANSFER  PROGRAM "NDF_MT]..."
2430                  LOAD Trans_prog$&Tape$,1
2440                  Y$="Y"
2450                  INPUT "MORE FRAMES ? (Y/N)",Y$
2460                  IF UPC$(Y$[1;1])="Y" THEN New_frame
```

```
2470                REWIND ":T15"
2480 End1:     !
2490           STOP
2500           END
2510 ! ===============     E N D   O F   M A I N    ===========================
2520  REM         SECTION "SUB3:A7,2"
2530 Dinit:         !
2540                PRINT LIN(10)
2550                PRINT " POWER ON DIGITIZER,PRESS  CONT "
2560                PAUSE
2570                PRINT LIN(10)
2580                PRINT ""
2590                PRINT "ALIGNE AXIS BY :"
2600                PRINT "            A) DIGITIZING POINT AT LEFT END OF HORIZONTAL AXI
S"
2610                PRINT " "
2620                PRINT "            B) DIGITIZING POINT TO RIGHT  ON SAME AXIS",LIN(3
)
2630                PRINT "            PRESS  CONT  TO CONTINUE"
2640                OUTPUT Sc;"IN"
2650                OUTPUT Sc;"AA"
2660                PAUSE
2670                PRINT LIN(5)
2680                RETURN
2690 Setup:         ! SET UP THE DIGITIZER
2700               A$="SUBJECT GRATING"
2710               CALL Display(Sc,A$)
2720               A$="LOWER TICK MARK"
2730               CALL Display(Sc,A$)
2740                          GOSUB Digit
2750                          Tx1=Xd
2760                          Ty1=Yd
2770               A$="UPPER TICK MARK"
2780               CALL Display(Sc,A$)
2790                          GOSUB Digit
2800                          Tx2=Xd
2810                          Ty2=Yd
2820               Deltx=Tx2-Tx1
2830               Delty=Ty2-Ty1
2840               Tr=SQR(Deltx^2+Delty^2)
2850               Sine=Deltx/Tr
2860               Cosn=1
2870               IF ABS(Deltx)<=10^(-8) THEN RETURN        ! CHECK FOR SMALL ANGLE
2880               Cosn=Sine*Delty/Deltx
2890               RETURN
2900               ! ====
2910 Digit:        ! DIGITIZING IN SINGLE SAMPLING MODE
2920 Ds:           OUTPUT Sc;"SK0"
2930               OUTPUT Sc;"SG"
2940 Begin:        !
2950               OUTPUT Sc;"OS"
2960               ENTER Sc;Status
2970               IF BIT(Status,7)<>0 THEN Check_pad
2980               IF BIT(Status,2) THEN Tk_pt
2990               IF P_fringe=0 THEN Begin
3000               OUTPUT Sc;"OC"
3010               ENTER Sc;Xd,Yd
3020               Xd=Xd/40
3030               Yd=Yd/40
3040               GOSUB Gtsurf
3050               Xc=Xmn-X15+Xadd(1)
3060               Yc=Ymn-Y15+Yadd(1)
3070               POINTER Xc,Yc,2
3080               GOTO Begin
3090 Tk_pt:          ! TAKE A POINT
3100                 OUTPUT Sc;"OD"
```

```
3110              ENTER Sc;Xd1,Yd1,Penn,Ann
3120              Xd=Xd1/40
3130              Yd=Yd1/40
3140              IF Iout<>1 THEN Bep
3150              Nt=Nt+1
3160              Xvictor(Nt)=Xd1
3170              Yvictor(Nt)=Yd1
3180 Bep:         BEEP
3190              RETURN
3200 Digit_fr:     ! DIGITIZING A FRINGE IN CONTINUOUS MODE
3210 Df:           ! CONSTRAINTS:
3220               !      a. Move the platen slowly to pick up more points
3230               !      b. The objective is to pick up around 120+ points
3240 Dc:           OUTPUT Sc;"CN32767,5"     ! delta T=32.767sec, delta D=0.5mm
3250 Digit_cont:  !
3260               OUTPUT Sc;"OS"
3270               ENTER Sc;Status
3280               IF BIT(Status,7)<>0 THEN Check_pad
3290               IF BIT(Status,2)<>0 THEN Tk_ptc
3300               GOTO Digit_cont
3310    !             OUTPUT Sc;"OC"
3320    !             ENTER Sc;Xd,Yd
3330    !             Xd=Xd/40
3340    !             Yd=Yd/40
3350    !             GOSUB Gtsurf
3360    !             Xc=Xmn-X15+Xadd
3370    !             Yc=Ymn-Y15+Yadd
3380    !             POINTER Xc,Yc,2
3390    !             GOTO Digit_cont
3400 Tk_ptc:       OUTPUT Sc;"OD"
3410               ENTER Sc;Xvictor(Nt+1),Yvictor(Nt+1),Penn,Ann
3420               Nt=Nt+1
3430               PRINT Xvictor(Nt),Yvictor(Nt)
3440               GOTO Digit_cont
3450                 ! THESE (2) TESTS ARE USED TO  DIGITIZE  IN ONE DIRECTION
3460    !            IF (FRACT(Isec/2)<>0) AND (Xvictor(Nt+1)<Xvictor(Nt)) THEN Di
git_cont
3470    !            IF (FRACT(Isec/2)=0) AND (Xvictor(Nt+1)>Xvictor(Nt)) THEN Dig
it_cont
3480    ! =============
3490 Point_calc:    ! POINT CALCULATIONS ALGORITHM
3500 Pc:            ! This is where the points are determined for storage
3510                !       For a look at this algorithm, RUN the program
3520                !       "CALC_N" on micro-floppy - varying Nreq
3530                EXIT GRAPHICS
3540                Nreq=Ppfring                         ! Variable No. required
3550                Npoints=Nt                           ! No. pts. taken 0 - 400
3560                Nn1=1
3570                Nn2=Nt
3580                Nstep=1
3590                IF Iout=1 THEN Skip_calc
3600                IF Nt<=Ppfring+1 THEN Skip_calc
3610                Nn1=MAX(1,INT(.05*Nt))
3620                Nn2=MAX(1,INT(.95*Nt))
3630                Nstep=MAX(1,INT(Nt/Nreq))
3640                Npoints=MAX(0,INT(.9*Nt/Nstep))+1 ! (PLUS  1  END POINT)
3650 Skip_calc:     !
3660   !            PRINT LIN(2);"    No.         Xd           Yd"
3670                Ii=1
3680                FOR I=Nn1 TO Nn2 STEP Nstep
3690                  Xd=Xvictor(I)/40
3700                  Yd=Yvictor(I)/40
3710   !              PRINT USING 3620;Ii,Xd,Yd
3720                  IMAGE 5D,2(3X,10D.DD)
3730                  GOSUB Gtsurf
3740                  N=N+1
```

```
3750                 Ii=Ii+1
3760               NEXT I
3770               Npoints(Isec)=Ii-1
3780            PRINTER IS 16
3790               PRINT "FRINGE No.:   ";Isec;"   N=   ";N;"   NPOINTS=   ";Npoints(
Isec);"   NT=   ";Nt;"   MAXPTS=   ";Maxpts
3800            PRINTER IS 16
3810 Pc_ret:       RETURN
3820              ! ======
3830 Grsurf:      Pd(1)=Xd-Tx1                              ! PERSPECTIVE
3840              Pd(2)=Yd-Ty1
3850              Pd(1)=Pd(1)/Factor-Orig(1)
3860              Pd(2)=Pd(2)/Factor-Orig(2)
3870              Delta=S*Bel+Pd(1)*(1-Coss)-Pd(2)*Sinn
3880              IF Iout=1 THEN Delta=0
3890              Sss=Delta/(Projec+Delta)
3900              Zmn=Sss*Camera
3910              Zm(N)=Zmn
3920              Xmn=Pd(1)*(1-Sss)
3930              Xm(N)=Xmn
3940              Ymn=Pd(2)*(1-Sss)
3950              Ym(N)=Ymn
3960              RETURN
3970              ! ====
3980 Plot_init:   ! INTIALIZE PLOT MODE
3990              PLOTTER IS 13,"GRAPHICS"
4000 Pi:          LIMIT 0,184,0,149
4010              SHOW -175,575,-450,350
4020              AXES 100,100,0,0
4030              FRAME
4040              Xlbl=-250
4050              Ylbl=345
4060              Dl=-25
4070              MOVE 200,300
4080              LABEL USING "#,K";"PATIENT: "&Hist$[1;6]
4090              MOVE 200,275
4100              LABEL USING "K";"DATE: "&D$[1;8]
4110              RETURN
4120              ! ====
4130 Err1:      ! ERROR: TOO MANY POINTS TAKEN IN THE LAST FRINGE
4140            BEEP
4150            EXIT GRAPHICS
4160 E1:        PRINT LIN(5);" ERROR #1 :    Too many points have been taken in
this, the last fringe."
4170            PRINT LIN(0);"               You can CONTINUE on - keeping in mind
 the No. of points"
4180            PRINT LIN(0);"               remaining [";Max_pts-N;"] . OR..."
4190            PRINT LIN(0);"               You MAY CHOOSE to RE-DIGITIZE this
fringe; taking fewer points."
4200            PRINT LIN(1);"               This can be done in two ways:"
4210            PRINT LIN(0);"                    a. DIGITIZING   V E R Y   S L
O W L Y  "
4220            PRINT LIN(0);"                    b. DIGITIZING, quickly  - BUT
 CAREFULLY"
4230            PRINT LIN(0);"               Both of these techniques should produ
ce fewer points digitized."
4240            PRINT LIN(1);"               Don't be concerned if you are require
d to do this more than once."
4250            PRINT LIN(2);" REMINDER:     There are ONLY: ";Max_pts-N;"  poin
ts remaining for the BODY OUTLINE."
4260            PRINT LIN(0);"                              ****"
4270            PRINT LIN(2);"               PRESS  CONT  to go on."
4280            PAUSE
4290            RETURN
4300 Err2:      ! ERROR: TOO MANY BODY POINTS HAVE BEEN DIGITIZED
4310            A$="ERROR 2 CONT.."
```

```
4320            CALL Display(Sc,A$)
4330            BEEP
4340            EXIT GRAPHICS
4350 E2:        PRINT LIN(2);" ERROR #2 :   Too many points have been taken in
the body outline."
4360            PRINT LIN(1);"              All points digitized after this will
not be stored in the"
4370            PRINT LIN(0);"              permanent file."
4380            PRINT LIN(0);"              You may continue digitizing though, i
f nearly finished."
4390            PRINT LIN(1);"              If more points are needed, you should
 go back one fringe"
4400            PRINT LIN(0);"              and digitize fewer points. This would
 leave more for the"
4410            PRINT LIN(0);"              body outline."
4420            PRINT LIN(2);" PRESS  CONT  to go on."
4430            PAUSE
4440            RETURN
4450 Warn1:   PRINT LIN(5)
4460          EXIT GRAPHICS
4470          BEEP
4480          PRINT LIN(5)," WARNING:    We are using: ";Npoints(Isec);" POINT
S per FRINGE."
4490          RETURN
4500          ! ====
4510 Message:   PRINT LIN(20)
4520            EXIT GRAPHICS
4530            PRINT Mess1$
4540            WAIT 1000
4550            GRAPHICS
4560            RETURN
4570            ! ====
4580 Print_keyvalues: PRINT LIN(5)
4590                ! PRINT " PRESS Fa TO REDIGITIZE A POINT (NOT a point on a
 fringe!!)"
4600                  PRINT "   ''  Fb to REDIGITIZE A LINE - FRINGE OF BODY OUT
LINE"
4610 Pk:              PRINT "   ''  Fc  '' INDICATE END OF LINE"
4620                ! PRINT "   ''  Fd  ''      ''      ''  ''  FRAME"
4630                ! PRINT "   ''  Fe IF ALL IS OK (SO FAR)";LIN(5)
4640                  PRINTER IS 16
4650                  Ipkeys=1
4660                  RETURN
4670                  ! ====
4680 Check_pad:       !
4690                  OUTPUT Sc;"OK"
4700 Cp:              ENTER Sc;Pad_status
4710 ! IF Pad_status=1 THEN GOTO Error_pt   ! INDICATES LAST POINT INCORRECT - Fa
4720  IF Pad_status=2 THEN GOTO Redo_line   !    ''       ENTIRE FRINGE        - Fb
4730  IF Pad_status=4 THEN GOTO End_line    !    ''       END OF               - Fc
4740 ! IF Pad_status=8 THEN GOTO End_frame  !    ''       END OF FRAME         - Fd
4750  IF Pad_status<>16 THEN GOTO Trouble   !             OKAY SIGN            - Fe
4760               ! =
4770 Error_pt:     PEN Erase                !<--- SET PEN TO ERASE LINE
4780 Ep:           Nisec=Index(Isec)        !  THIS IS 1st POINT OF SECTION
4790               Nm2=N-2                  !   ''  '' LAST GOOD POINT; IF IT IS
IN THE PREVIOUS
4800               IF Nm2<Nisec THEN Nm2=Nisec ! SECTION THEN START WITH 1ST PT O
F SECT.
4810               Nm1=N-1                  ! PT. TO BE REMOVED
4820               IF Nm1<Nisec THEN Nm1=Nisec ! CANNOT GO BACK TO PREVIOUS SECT.
4830               PLOT Xm(Nm2)-X15,Ym(Nm2),-2 ! MOVE TO THE LAST GOOD PT
4840               PLOT Xm(Nm1)-X15,Ym(Nm1),-1 ! ERASE LINE
4850               PEN Pdraw
4860               PLOT Xm(Nm2)-X15,Ym(Nm2),-2 ! MOVE BACK TO GOOD PT.
4870               X0=Xm(Nm2)-X15
```

```
4880             Y0=Ym(Nm2)
4890             Z0=Zm(Nm2)-Z15
4900             N=Nm1
4910             OUTPUT Sc;"SK0"
4920             OUTPUT Sc;"SG"
4930             GOTO Call_gtsurf
4940             ! ==============
4950 Redo_line:   ! RE-DO A LINE (IN CONTINUOUS MODE) OF BODY OUTLINE
4960 R1:          EXIT GRAPHICS
4970              DISP
4980              GOSUB Beep
4990              IF (Isec=1) AND (Isec1=1) THEN Dontredo_line
5000              Maxpts=Maxpts+Npoints(Isec-1)
5010              Isec=Isec-1                          ! RESET FRINGE No.
5020              Bel=Bel+1
5030              PRINT LIN(5);" NOTE:   You are now going back   ONE FRINGE.
"
5040              PRINT LIN(1);"          You will now be on  FRINGE No.:  ";Ise
c;" ."
5050              GOSUB Beep
5060              WAIT Time*1
5070  !           TRACE ALL
5080  !           TRACE WAIT 150
5090                  GRAPHICS
5100                  Nel=N-1
5110                  N=Index(Isec)                    ! RESET N
5120                  FOR Ivu=1 TO 3
5130  !                 IF (Ivu=2) AND (Iout=1) THEN Dontredo_line
5140                    X0=Xm(N)-X15+Xadd(Ivu)
5150                    Y0=Ym(N)+Yadd(Ivu)
5160                    IF Ivu=2 THEN Y0=Zm(N)-Z15+Yadd(Ivu)
5170                    Dv=N
5180                    IF Ivu=3 THEN GOSUB Rot_vu
5190                    PEN Erase
5200                    PLOT X0,Y0,-2
5210                    POINTER X0,Y0,2
5220                    FOR Iclear=N+1 TO Nel
5230                      X0=Xm(Iclear)-X15+Xadd(Ivu)
5240                      Y0=Ym(Iclear)+Yadd(Ivu)
5250                      IF Ivu=2 THEN Y0=Zm(Iclear)-Z15+Yadd(Ivu)
5260                      Dv=Iclear
5270                      IF Ivu=3 THEN GOSUB Rot_vu
5280                      PLOT X0,Y0,-1
5290                    NEXT Iclear
5300                  NEXT Ivu
5310              Ierase=1
5320              OUTPUT Sc;"SK0"
5330              POINTER 0,0,2
5340 Dontredo_line:   !
5350                  IF Isec=1 THEN Isec1=1
5360                  GOTO Select
5370                  ! =========
5380 End_line:    !
5390 E1:          A$="FRINGE="&VAL$(Isec)&" "&VAL$(Nt)&".."
5400              CALL Display(Sc,A$)
5410              DISP
5420              E1_flag=1
5430              GOSUB Point_calc
5440  !           TRACE ALL
5450  !           TRACE WAIT 150
5460                          ! DRAW THE 1st VIEW
5470              N1=Index(Isec)
5480              IF Iout=1 THEN N1=N1-1
5490              N2=N-1
5500              POINTER Xadd(1),Yadd(1),2
5510              FOR I=N1+1 TO N2
```

```
5520              PEN Pdraw
5530              IF Ierase=1 THEN PEN Erase
5540              IF Eline=1 THEN PEN 0
5550              X1=Xm(I)-X15
5560              Y1=Ym(I)
5570              Z1=Zm(I)-Z15
5580              IF N>Index(Isec) THEN Skip_nots
5590              X0=X1
5600              Y0=Y1
5610              Z0=Z1
5620 Skip_nots:   PLOT X1,Y1,-1
5630              X0=X1
5640              Y0=Y1
5650              Z0=Z1
5660              Ierase=0
5670              PEN Pdraw
5680              IF Eline=1 THEN Eline=0
5690          NEXT I
5700          IF Iout=1 THEN El_ret   ! DRAW BODY OUTLINE ONLY
5710                                  ! DRAW THE 2nd VIEW
5720         POINTER Xadd(2),Yadd(2),2
5730         Pts_used=N2-N1
5740         X0=Xm(N1)-X15+Xadd(2)
5750         Z0=Zm(N1)-Z15+Yadd(2)
5760         PLOT X0,Z0,-2
5770         FOR I=N1+1 TO N2
5780              X1=Xm(I)-X15+Xadd(2)
5790              Z1=Zm(I)-Z15+Yadd(2)
5800              PLOT X1,Z1,-1
5810         NEXT I
5820                                   ! DRAW THE 3rd VIEW
5830         POINTER Xadd(3),Yadd(3),2
5840         Dv=N1                     ! DUMMY VARIABLE IS SET
5850         GOSUB Rot_vu              ! FOR THIS SUB (Rot_vu)
5860         PLOT X0,Y0,-2
5870         FOR I=N1+1 TO N2
5880              Dv=I                 ! DUMMY VARIABLE IS SET
5890              GOSUB Rot_vu         ! FOR THIS SUB
5900              PLOT X0,Y0,-1
5910         NEXT I
5920         POINTER 0,0,2
5930         IF Debug4=0 THEN Skip_dbg4
5940         PRINTER IS 0
5950         PRINT USING "2(K,MDDD.DD,2X)";"BEL= ";Bel,"ISEC= ";Isec
5960         PRINT LIN(1)
5970         FOR I=N1 TO N2 STEP 2
5980              PRINT USING "2(DDD,4X,3(MDDD.D,2X))";I;Xm(I);Ym(I);Zm(I);I+1
;Xm(I+1);Ym(I+1);Zm(I+1)
5990         NEXT I
6000         PRINTER IS 16
6010 Skip_dbg4: IF Npoints(Isec)>Ppfring+5 THEN GOSUB Warn1
6020         Eline=1
6030         Ierase=1
6040         Bel=Bel-1                            ! FOR DIGITIZING NEXT FRINGE
6050         PLOT 0,1000,-2
6060         IF Isec>Frm1 THEN End_frame
6070 El_ret:  !
6080         RETURN
6090        ! ==============
6100 End_frame:  EXIT GRAPHICS                    ! END OF DATA FOR THIS PATIENT
6110             PRINT LIN(10)
6120 Ef:         PRINT "  END OF FRAME  "
6130             GOTO Check_end
6140             ! ============
6150 Trouble:  EXIT GRAPHICS
6160           PRINT LIN(10)
```

```
6170 Mnt:        PRINT "  NOT A VALID KEYBOARD ENTRY ! "
6180             PRINT ""
6190             PRINT " PRESS AN APPROPRIATE KEY "
6200             WAIT 1000
6210             PRINT " "
6220             GOSUB Print_keyvalues
6230             OUTPUT Sc;"SK0"
6240             WAIT Time
6250             GRAPHICS
6260             RETURN
6270 Rot_vu:      !
6280              A(1)=Xm(Dv)-X15
6290              A(2)=Ym(Dv)
6300              A(3)=Zm(Dv)-Z15
6310              MAT B=Rot*A
6320              X0=B(1)+Xadd(3)
6330              Y0=B(2)+Yadd(3)
6340              RETURN
6350 ! ====================    PATIENT HISTORY SECTION     ====================
6360 Start:   PRINT LIN(10)
6370 Ph:      PRINT "POWER ON DISC DRIVE"
6380 P:       PRINT "ENSURE THAT DATA DISC #1 IS IN H6,0,0"
6390          PRINT "AND DISC #2 IS IN H6,0,1 IF NECESSARY",LIN(1)
6400          PRINT "USE APPROPRIATE PATIENT DATA  OR  TEST DATA DISCS",LIN(1)
6410          PRINT "TO CONTINUE PRESS 'CONT'"
6420          PAUSE
6430 Inpat:    !
6440           INPUT " PLEASE ENTER PATIENT'S LAST NAME,FIRST",Na$[1;12],Na$[13;6
]
6450           Hist$[1;6]=""
6460           INPUT "ENTER PATIENT HOSPITAL #",Hist$[1;6]
6470           Hist$=UPC$(TRIM$(Hist$))
6480           IF Hist$[1;1]="0" THEN Hist$=Hist$[2]
6490           IF Debug2=1 THEN PRINTER IS 0
6500 Cont_2:   Blank=0
6510           New=0
6520           GOSUB Check_disc
6530           IF New=1 THEN GOTO New_pat
6540 Old_pat:   ! HAVE FOUND PATIENT FILE
6550            Ipat=I
6560            File$=Master$
6570            IF Ds=1 THEN File$=Slave$
6580            ASSIGN #1 TO Hist$&File$
6590            BUFFER #1
6600            READ #1,1;Name$(*),Date$(*)
6610                    Exam=VAL(Name$(2)[8;2])+1
6620                    Exam$=VAL$(Exam)
6630                    PRINT Name$(1),Name$(2),LIN(2),SPA(2);"EXAM#";SPA(2);"DATE
"
6640                    FOR I=3 TO Exam+1
6650                    PRINT SPA(3),I-2;"- ";Date$(I-2)[1;6],Name$(I)
6660                    NEXT I
6670                    PRINT LIN(3)
6680                    EDIT "THIS IS THE ASSUMED EXAMINATION NUMBER, EDIT IF REQU
IRED",Exam$
6690                    Exam=VAL(Exam$)
6700                    Icoment=Exam+2
6710                    GOSUB Rotation
6720 Xxdate:   INPUT "ENTER DATE OF PHOTO (YY/MM/DD)",D$[1;8]
6730           GOSUB Check_date
6740           IF D_ok=1 THEN Xxdate_ok
6750           GOTO Xxdate
6760           ! =========
6770 Xxdate_ok: Date$(Exam)[1;2]=D$[1;2]
6780            Date$(Exam)[3;2]=D$[4;2]
6790            Date$(Exam)[5;2]=D$[7;2]
```

```
6800              Name$(Icoment)=RPT$(".",18)
6810              INPUT "ENTER ANY COMMENTS TO THIS EXAM",Name$(Icoment)
6820              Name$(2)[8;2]=VAL$(Exam)
6830              READ #1,1
6840              PRINT #1,1;Name$(*),Date$(*)
6850              RETURN
6860              ! ====
6870 New_pat:   ! NEW PATIENT FOUND
6880              IF Blank>0 THEN I=Blank
6890              Exam=1
6900              Pat_num=I
6910              File$=Master$
6920              IF Ds=1 THEN File$=Slave$
6930              CREATE Hist$&File$,Max_exams+1,256+39
6940              ASSIGN #1 TO Hist$&File$
6950              BUFFER #1
6960 In_birth:  INPUT "ENTER BIRTHDATE (YY/MM/DD)",D$[1;8]
6970            GOSUB Check_date
6980            IF D_ok=1 THEN Bdate_ok
6990            BEEP
7000            GOTO In_birth
7010            ! ===========
7020 Bdate_ok:  Birth$[1;2]=D$[1;2]
7030            Birth$[3;2]=D$[4;2]
7040            Birth$[5;2]=D$[7;2]
7050            Sx$[1;1]="F"
7060            INPUT "ENTER SEX OF PATIENT (F M)",Sx$[1;1]
7070            Sx$=UPC$(Sx$)
7080            Name$(1)=Na$
7090            Name$(2)[1;6]=Birth$[1;6]
7100            Name$(2)[7;1]=Sx$[1;1]
7110            Name$(2)[8;2]=VAL$(Exam)
7120 Xdate:   INPUT "ENTER DATE OF PHOTO (YY/MM/DD)",D$[1;8]
7130          GOSUB Check_date
7140          IF D_ok=1 THEN Xdate_ok
7150          GOTO Xdate
7160          ! ========
7170 Xdate_ok:  Date$(1)[1;2]=D$[1;2]
7180            Date$(1)[3;2]=D$[4;2]
7190            Date$(1)[5;2]=D$[7;2]
7200            Icoment=3
7210            Name$(Icoment)=RPT$(".",18)
7220            PRINT USING "K";"ENTER COMMENTS (MAX. 18 CHARACTERS)"
7230            INPUT "",Name$(Icoment)
7240            READ #1,1
7250            PRINT #1,1;Name$(*),Date$(*)!NOTE DATA FOR EXAM #1 IS STORED IN 
FILE 2
7260            ! DATA FOR EXAM i IS STORED IN FILE i+1
7270            PRINTER IS 16
7280            RETURN
7290            ! ====
7300 End_history:     Y$="Y"
7310                  PRINT " END OF FILE #'s"
7320                  INPUT "WOULD YOU LIKE TO INITIALIZE HISTORY # S (Y N)",Y$
[1;1]
7330                  IF UPC$(Y$[1;1])="N" THEN GOTO End_2
7340                  FOR I=1 TO Numpat
7350                    History$(I)=RPT$(".",6)
7360                  NEXT I
7370                  READ #2,1
7380                  PRINT #2;History$(*)
7390                  GOTO Cont_2
7400 End_2:         !
7410                PRINT LIN(2);"PROGRAM ENDING "
7420                ASSIGN #2 TO *
7430                STOP
```

```
7440              END
7450 File_full:     !
7460 Ff:            PRINT LIN(10);" FILE FULL"
7470                PRINT LIN(2),"STOP "
7480                STOP
7490                END
7500 End_pat:     Y$[1;1]="Y"
7510              PRINT "PATIENT NOT ON FILE : IS THIS A NEW PATIENT ?"
7520              INPUT "YES OR NO",Y$[1;1]
7530              Y$=UPC$(Y$)
7540              IF Y$[1;1]="Y" THEN GOTO New_pat
7550              PRINT "PLEASE CHECK DATA AND RE ENTER "
7560              PRINT "PATIENT NO. GIVEN: ";Hist$[1;6]
7570              WAIT Time
7580              GOTO Start
7590 ! ====================    OTHER  SUBROUTINES ...    ====================
7600 Check_date: IF D$[2;1]="/" THEN D$="0"&D$[1]
7610 Os:         IF D$[5;1]="/" THEN D$=D$[1;3]&"0"&D$[4]
7620 Ss:         IF VAL(D$[4;2])<=12 THEN Check_day
7630             PRINT "TOO MANY MONTHS !"
7640             D_ok=0
7650             RETURN
7660             ! ====
7670 Check_day:   IF VAL(D$[7;2])<=31 THEN Date_ok
7680              PRINT "TOO MANY DAYS !"
7690              D_ok=0
7700 Date_ok:     D_ok=1
7710              RETURN
7720              ! ====
7730 Check_disc:    Ds=0
7740                Mess1$="LEFT HAND DRIVE-:H6,0,0"
7750                D1$=Master$[1;7]
7760                !   ON ERROR GOTO Bomb1
7770 Mas_st:        MASS STORAGE IS D1$[1;7]
7780                CAT TO Cat$(*)
7790                PRINT LIN(1),Mess1$,LIN(1)
7800                FOR I=1 TO 15 STEP 5
7810                    PRINT USING "5(6A,4X)";Cat$(I),Cat$(I+1),Cat$(I+2),Cat$(I
+3),Cat$(I+4)
7820                NEXT I
7830                FOR I=1 TO Files_per_disc
7840                    IF Cat$(I)="" THEN New=1
7850                    IF New=1 THEN Xit
7860                    IF Cat$(I)[1;6]<>Hist$[1;6] THEN X_file
7870                    Ipat=I
7880                    GOTO Xit
7890 X_file:        NEXT I
7900                IF Ds=1 THEN Xit1
7910                D1$=Slave$[1;7]
7920                Mess1$="RIGHT HAND DRIVE-:H6,0,1"
7930                Ds=1
7940                GOTO Mas_st
7950 Bomb1:       BEEP
7960              IF ERRN=80 THEN PRINT "NO DISC IN DRIVE "&Mess1$
7970              IF ERRN<>80 THEN PRINT "ERROR NUMBER #";ERRN
7980              IF ERRN<>80 THEN STOP
7990              WAIT 1000
8000              OFF ERROR
8010              GOTO Start
8020              ! ========
8030 Xit1:       PRINT LIN(5),"HISTORY NUMBER NOT FOUND ON THIS SET OF DISCS. INS
ERT NEXT SET OF DISCS."
8040             PRINT "THEN PRESS 'CONT'"
8050             PAUSE
8060             GOTO Check_disc
8070 Xit:        RETURN
```

```
8080            ! ====
8090 Debug:   Debug1=0    !  CHECK INITIALIZATION OF DATA
8100          Debug2=0    !  CHECK PATIENT FILES
8110 De:      Debug3=0    !  CHECK ORIENTATION OF SLIDES
8120          Debug4=0    !  CHECK INITIAL FRINGE DATA
8130          Debug5=0    !  CHECK DIGITIZING OF FRINGE DATA
8140          Debug6=0    !  CHECK SFKeys
8150          Debug7=0    !  CHECK BODY OUTLINE
8160          Debug8=0    !  CHECK ENDING ROUTINE
8170          RETURN
8180          ! ====
8190 Data_init: !
8200             DISP "WORKING..."
8210             Max_pts=800          ! MAXIMUM NO. OF POINTS
8220 D_init:     Bodypts=25           ! MIN. # OF POINTS BODY OUTLINE
8230 Data:       Max_fringes=40       ! MAXIMUM NO. OF FRINGES INCLUDES BODY OUTLINE
8240 Id:         Numpat=50            ! MAXIMUM NO. OF PATIENTS
8250 Di:         Max_exams=10         ! MAXIMUM NO. OF EXAMINATIONS
8260             Sc=706               !   SELECT CODE FOR DIGITIZER
8270             Lnk=0                ! DO NOT LOAD PLOTTING PROGRAM
8280             Pdraw=1              ! DRAW LINE                    | USE WITH
8290             Erase=-1             ! ERASE LINE                   | PEN STATEMENT
8300             Dist=452.0           !  480.5  DISTANCE BETWEEN SCALE MARKERS
8310             S=2.34               !  2.47   GRATING SPACING AT THE SCREEN
8320             Orig(1)=300          !  321    X COORD. OF GRATING CENTER
8330             Orig(2)=226          !  242    Y COORD. OF GRATING CENTER
8340             Dorig=105      !  X DISTANCE OF DIGITIZED CENTER FROM TRUE CENTER
8350             Camera=2800          !  DISTANCE FROM CAMERA TO SCREEN
8360             Projec=700           !  DISTANCE FROM CAMERA TO PROJECTOR
8370             Master$=":H6,0,0"    !  PRIMARY DISC STORAGE FOR DATA
8380             Slave$=":H6,0,0"     !  SECONDARY DISC     ''      ''
8390             Tape$=":T15"            !  TAPE T15        ''      ''  PROGRAMS
8400             Trans_prog$="NDF_MT"    !  MOIRE DATA TRANSFER PROGRAM
8410             Load_disc$=":A7"
8420             Maxfiles=30             !  MAXIMUM NUMBER OF FILES ON TWO DISCS
8430   !         Files_per_disc=85       !  MAXIMUM FILES PER DISC
8440             Memory=1.135E6          !
8450             File_size=256*41*11     !
8460             Files_per_disc=INT(Memory/File_size)    !  MAXIMUM FILES PER DISC
8470   !         PRINT LIN(5);"FILES=  ";Files_per_disc
8480   !         Files_per_disc=15       !  MAXIMUM FILES PER DISC
8490             Time=1000
8500             DATA 0,0,0,-300,420,0
8510             FOR Id=1 TO 3
8520                 READ Xadd(Id),Yadd(Id)
8530             NEXT Id
8540             RESTORE
8550             Landmark$(1)="C7"
8560             Landmark$(2)="L5"
8570             Landmark$(3)="MM"
8580             FOR Id=1 TO Max_exams
8590                 Date$(Id)=RPT$(".",6)
8600             NEXT Id
8610 Rotation:   DEG
8620             Theta=30              ! ANGLES FOR OBLIQUE VIEW
8630             Beta=75
8640             C1=COS(Theta)
8650             S1=SIN(Theta)
8660             C2=COS(Beta)
8670             S2=SIN(Beta)
8680             Rot(1,1)=C2
8690             Rot(1,2)=-S2*C1
8700             Rot(1,3)=S1*S2
8710             Rot(2,1)=S2
8720             Rot(2,2)=C1*C2
8730             Rot(2,3)=-C2*S1
```

```
8740              Rot(3,1)=0
8750              Rot(3,2)=S1
8760              Rot(3,3)=C1
8770              WAIT Time
8780              DISP
8790              IF Debug1=0 THEN RETURN
8800              ! ====================
8810              PRINTER IS 0
8820              PRINT "INTIAL DATA ",LIN(1)
8830              PRINT "MAX_PTS= ";Max_pts,"BODYPTS= ";Bodypts,"MAX_FRINGES= ";Ma
x_fringes
8840              PRINT "MAX. PATIENTS";Numpat,"MAX. EXAMS= ";Max_exams,"MAX. FILE
S= ";Maxfiles,"FILES PER DISC= ";Files_per_disc,LIN(1)
8850              PRINT "DORIG= ";Dorig,"CAMERA= ";Camera,"PROJEC= ";Projec
8860              PRINT "ORIG(1)= ";Orig(1),"ORIG(2)= ";Orig(2),"S= ";S,"DISTANCE=
 ";Dist
8870              PRINT "DRAW= ";Pdraw,"ERASE= ";Erase,"LINK= ";Lnk,LIN(1)
8880              PRINT "MASTER= ";Master$,"SLAVE= ";Slaue$,"TAPE= ";Tape$,"SELECT
 CODE= ";Sc
8890              PRINTER IS 16
8900              RETURN
8910 Beep:        !
8920            FOR Bp=60 TO 120 STEP 30
8930              BEEP
8940              WAIT Bp
8950            NEXT Bp
8960            RETURN
8970 Title:  PRINT LIN(20)
8980          Title$(1)=RPT$(" ",40)&CHR$(128)
8990          Title$(2)=RPT$("[illegible]",40)
9000          PRINT USING "K";[illegible]le$(1)&Title$(1)&CHR$(128)
9010          PRINT USING "K,60X,K";Title$(1)[1;30]&CHR$(128);Title$(1)[1;30]&CHR$
(128)
9020          PRINT USING "K,60X,K";Title$(1)[1;30]&CHR$(128);Title$(1)[1;30]&CHR$
(128)
9030          PRINT USING "K,18X,23A,19X,K";Title$(1)[1;30]&CHR$(128);"MOIRE SHADO
W TOPOGRAPHY";Title$(1)[1;30]&CHR$(128)
9040          PRINT USING "K,21X,17A,22X,K";Title$(1)[1;30]&CHR$(128);"GLENROSE HO
SPITAL";Title$(1)[1;30]&CHR$(128)
9050          PRINT USING "K,60X,K";Title$(1)[1;30]&CHR$(128);Title$(1)[1;30]&CHR$
(128)
9060          PRINT USING "K,16X,27A,17X,K";Title$(1)[1;30]&CHR$(128);"PROGRAM TO
DIGITIZE FRINGES";Title$(1)[1;30]&CHR$(128)
9070          PRINT USING "K,19X,22A,19X,K";Title$(1)[1;30]&CHR$(128);"VERSION 4.1
 - AUG.1984";Title$(1)[1;30]&CHR$(128)
9080          PRINT USING "K,180A,K";Title$(1)[1;30]&CHR$(128);RPT$("_",180);Tit
le$(1)[1;30]&CHR$(128)
9090          PRINT USING "K,60X,K";Title$(1)[1;30]&CHR$(128);Title$(1)[1;30]&CHR$
(128)
9100          PRINT USING "K,4X,46A,DD,8X,K";Title$(1)[1;30]&CHR$(128);"LIMITATION
S: a) MAXIMUM NO. OF EXAMINATIONS...";Max_exams;Title$(1)[1;30]&CHR$(128)
9110          PRINT USING "K,17X,33A,DD,8X,K";Title$(1)[1;30]&CHR$(128);"b  MAXIMU
M NO. OF FRINGES........";Max_fringes-1;Title$(1)[1;30]&CHR$(128)
9120          PRINT USING "K,17X,32A,DDD,8X,K";Title$(1)[1;30]&CHR$(128);"c) MAXIM
UM NO. OF POINTS........";Max_pts;Title$(1)[1;30]&CHR$(128)
9130          PRINT USING "K,60X,K";Title$(1)[1;30]&CHR$(128);Title$(1)[1;30]&CHR$
(128)
9140          PRINT USING "K,1X,58A,1X,K";Title$(1)[1;30]&CHR$(128);"NOTE: DO NOT
DIGITIZE MORE POINTS ON FRINGE THAN INDICATED";Title$(1)[1;30]&CHR$(128)
9150          PRINT USING "K,180A,K";Title$(1)[1;30]&CHR$(128);RPT$("_",180);Tit
le$(1)[1;30]&CHR$(128)
9160          PRINT USING "K,60X,K";Title$(1)[1;30]&CHR$(128);Title$(1)[1;30]&CHR$
(128)
9170          PRINT USING "K,19X,24A,19X,K";Title$(1)[1;30]&CHR$(128);"TO CONTINUE
 PRESS CONT ";Title$(1)[1;30]&CHR$(128)
9180          PRINT USING "K,60X,K";Title$(1)[1;30]&CHR$(128);Title$(1)[1;30]&CHR$
```

```
(128)
9190          PRINT USING "K";Title$(1)&Title$(1)&CHR$(128)
9200          PAUSE
9210          RETURN
9220          ! ====
9230   SUB Display(Select;A$)
9240   OPTION BASE 1
9250   DATA 238,254,156,252,158,142,190,110,96,112,0,28,0,42,252,206,230,10,182,3
0,124,0,0,0,118,0
9260   DATA 59,62,26,122,222,142,246,46,32,112,0,96,0,42,58,206,230,10,182,30,56,
0,0,0,118,0
9270   DATA 0,96,218,242,102,182,62,224,254,230,252,1,156,240,190,2,32,68,64,202
9280                 INTEGER A(72)
9290                 DIM D$[240],Alpha$[72]
9300                 OUTPUT Select;"BP100,100"
9310                 READ A(*)
9320                 D$=""
9330 Display:  IF LEN(A$)>15 THEN A$="LINE TOO LONG"
9340              Alpha$="ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz 1234
567890.[]=-, '?"
9350              Alpha$[70,70]=CHR$(34)
9360              FOR I=1 TO LEN(A$)
9370                 P=POS(Alpha$,A$[I,I])
9380                 IF P=0 THEN P=53
9390                 D$=D$&";DD"&VAL$(I)&","&VAL$(A(P))
9400              NEXT I
9410              OUTPUT Select USING "K";D$
9420   SUBEND
```

Appendix D

```
C THIS PROGRAM GENERATES A FRINGE USING SEVENTH ORDER
C POLYNOMIAL
      LOGICAL*1 FREEIO(1)/'*'/
      DIMENSION XIN(800),YIN(800),X(800)
      DIMENSION R(800),XW(8000),Y(800)
      DIMENSION IR(800),YW(800),A(800)
      DIMENSION SL(200),XS(200),YS(200)
      DIMENSION N1(800),SY1(200)
      DIMENSION XS1(200,200),YS1(200,200)
      DIMENSION SX1(200),SR1(200)
      INTEGER N,LA,IR
      REAL INCR,R,A
      DOUBLE PRECISION DSEED,XO,YO
      READ(5,FREEIO)M,K,D
      A0=-417.6
      A1=36.931
      A2=-.7459
      A3=.005337
      A4=2.59E-6
      A5=-2.1807E-7
      A6=1.012E-9
      A7=-1.4721E-12
      XI=81.075
      XE=209.05
      N=M
      INCR=(XE-XI)/M
      DO 10 I=1,M
   10 XIN(I)=0.5*INCR+(I-1)*INCR+XI
      DSEED=325017.D0
      CALL GGNML(DSEED,N,R)
      DO 20 I=1,M
      Y1=A0+A1*XIN(I)+A2*(XIN(I)**2.)+A3*
     &(XIN(I)**3.)+A4*(XIN(I)**4.)
      Y2=A5*(XIN(I)**5.)+A6*(XIN(I)**6.)+A7*(XIN(I)**7.)
      YIN(I)=Y1+Y2
      SM1=A1+2*A2*XIN(I)+3*A3*(XIN(I)**2.)
     &+4*A4*(XIN(I)**3.)
      SM2=5*A5*(XIN(I)**4.)+6*A6*(XIN(I)**5.)
     &+7*A7*(XIN(I)**6.)
      SM=SM1+SM2
      R(I)=R(I)*D
      THETA=ATAN(-1./SM)
      X(I)=COS(THETA)*R(I)+XIN(I)
      Y(I)=SIN(THETA)*R(I)+YIN(I)
   20 CONTINUE
C NOW THE X VALUES ARE SORTED INTO INCREASING ORDER
      LA=N
      DO 50 J=1,M
      A(J)=X(J)
   50 IR(J)=J
      CALL VSRTR(A,LA,IR)
       DO 40 KK=1,M
```

```
        J=IR(KK)
      YW(KK)=Y(J)
   40 XW(KK)=A(KK)
      II=1
      I1=M*.1
      I2=M*.9
      M=I2-I1+1
      DO 45 I=I1,I2
      Y(II)=YW(I)
      X(II)=XW(I)
   45 II=II+1
      WRITE(4,1000)M
      DO 51 I=1,M
   51 WRITE(4,100)X(I),Y(I)
C NOW THE SMOOTHING FUNCTION IS CALLED.THE OPTIMUM
C VALUE OF THE SMOOTHING FACTOR S IS
C FOUND IN THE RANGE OF M+.75M,M-.75M
      XM=M
      S1=M-.75*M
      S2=M+.75*M
   55 XMULT=(S2-S1)/5.
      L=6
      DO 70 KK=1,L
      SL(KK)=S1+(KK-1)*XMULT
      S=SL(KK)
      CALL SMOOTH(X,Y,M,K,D,S,XS,YS,SX,SY,SR,N)
      WRITE(4,1000)M
      DO 90 K=1,200
   90 WRITE(4,100)XS(K),YS(K)
      SR1(KK)=SR
      SX1(KK)=SX
      SY1(KK)=SY
      N1(KK)=N
      DO 60 J=1,M
      XS1(J,KK)=XS(J)
      YS1(J,KK)=YS(J)
   60 CONTINUE
   70 CONTINUE
C DETERMINE THE OPTIMUM VALUE OF THE SMOOTHING FACTOR
      DIFF=SR1(1)
      DIFF2=SR1(2)
      LJ=L+1
```

```
      DO 80 J=3,LJ
      IF(DIFF2.GT.DIFF)GO TO 79
      DIFF=DIFF2
   79 IF(J.EQ.LJ)GO TO 81
   80 DIFF2=SR1(J)
   81 DO 91 J=1,L
      IF(DIFF.NE.SR1(J))GO TO 91
      WRITE(6,600)M,N1(J)
      WRITE(6,500)S1,S2,SL(J)
      WRITE(6,300)SX1(J),SY1(J),SR1(J)
      IF(J.GT.1)S1=SL(J-1)
      IF(J.EQ.1)S1=SL(J)
      IF(J.EQ.6)S2=SL(J)
       IF(J.LT.6)S2=SL(J+1)
       GO TO 92
   91 CONTINUE
   92 CONTINUE
      IF(XMULT.GE.0.25)GO TO 55
  100 FORMAT(2F11.3)
  200 FORMAT(3F11.3)
  300 FORMAT(3F11.4)
  400 FORMAT(F7.4)
  500 FORMAT(3F14.7)
  600 FORMAT(2I6)
 1000 FORMAT(I4)
      STOP
      SUBROUTINE SMOOTH(X,Y,M,K,D,S,XS,YS,SX,SY,SR,N)
      DIMENSION X(800),Y(800),W(800),T(800)
      DIMENSION SC(800),XS(200),YS(200)
      DIMENSION DELX(200),DELR(200),DELY(200)
      DIMENSION A(8),R(7),Z(7)
      INTEGER B,NDEG,IER
      REAL DERIV,YVAL,INCR,SM,A,R,MEANX,MEANY,MEANR
      COMPLEX Z
      NU=1
      A0=-417.6
      A1=36.931
      A2=-.7459
      A3=.005337
      A4=2.59E-6
      A5=-2.1807E-7
      A6=1.012E-9
```

```
      A7=-1.4721E-12
C CALCULATE N,W
      N=800
      DO 10 I=1,M
      W(I)=1./(D**2.)
      IF(W(I).LE.0.)RETURN
   10 CONTINUE
      XI=X(1)
      XE=X(M)
      CALL SMOOT(X,Y,W,M,XI,XE,K,S,N,T,NK1,C,IER)
      IF(IER.GT.0)GO TO 80
C DIVIDE SMOOTH INTO 200 INTERVALS
      INCR=(XE-XI)/200.
      DO 40 I=1,200
   40 XS(I)=.5*INCR+(I-1)*INCR+XI
      DO 79 I=1,200
       ARG=XS(I)
       DO 76 J=K,NK1
        IF(ARG .GE. T(J+1)) GOTO 76
        L=J
        GOTO 77
   76 CONTINUE
   77 CONTINUE
      SM=DERIV(T,N,C,NK1,NU,ARG,L)
      SM=-1./SM
      NDEG=7
      YS(I)=YVAL(T,N,C,NK1,ARG,L)
      A(8)=A0-YS(I)+SM*ARG
      A(7)=A1-SM
      A(6)=A2
      A(5)=A3
      A(4)=A4
      A(3)=A5
      A(2)=A6
      A(1)=A7
      CALL ZPOLR(A[illegible]EG,Z,IER)
      DO 82 JJ=1,7
   82 R(JJ)=REAL(Z([illegible]))
C DETERMINE WHICH PO[illegible] IS CLOSEST TO ARG, =XP
      DIFF=(R(1)-ARG
      DIFF2=(R(2)[illegible]ARG
      DO 78 II=3,8
```

```
      IF(ABS(DIFF2).GT.ABS(DIFF)) GO TO 75
      DIFF=DIFF2
   75 IF(II.EQ.8)GO TO 83
   78 DIFF2=(R(II)-ARG)
   83 XP=DIFF+ARG
      YP1=A0+A1*XP+A2*(XP**2.)+A3*(XP**3.)+A4*(XP**4.)
      YP2=A5*(XP**5.)+A6*(XP**6.)+A7*(XP**7.)
      YP=YP1+YP2
      DELX(I)=ABS(XP-ARG)
      DELY(I)=ABS(YP-YS(I))
      DELR(I)=SQRT((DELX(I)*DELX(I))+(DELY(I)*DELY(I)))
   79 CONTINUE
      SUMX=0.
      SUMY=0.
      SUMR=0.
      DO 89 I=1,200
      SUMX=SUMX+DELX(I)
      SUMY=SUMY+DELY(I)
   89 SUMR=SUMR+DELR(I)
      MEANX=SUMX/200
      MEANY=SUMY/200
      MEANR=SUMR/200
C CALCULATE THE STANDARD DEVIATION
      SX=0.
      SY=0.
      SR=0.
      DO 81 I=1,200
      SX=(DELX(I)-MEANX)**2+SX
      SY=(DELY(I)-MEANY)**2+SY
      SR=(DELR(I)-MEANR)**2+SR
   81 CONTINUE
      SX=SQRT(SX/199.)
      SY=SQRT(SY/199.)
      SR=SQRT(SR/199.)
   80 CONTINUE
  100 FORMAT(2F11.3)
  200 FORMAT(3F11.4)
  300 FORMAT(3F12.5)
  400 FORMAT(3F13.6)
  500 FORMAT(3F14.7)
  600 FORMAT(F11.3)
      RETURN
```

```
      END
      SUBROUTINE SMOOT(X,Y,W,M,XI,XE,K,S,N,T,NK1,C,IER)
C SMOOTH DETERMINES A SMOOTHING SPLINE APPROXIMATION
C (NORMALISED B-SPLINE REPRESENTATION)
C OF A GIVEN DISCRETE FUNCTION.
      DIMENSION X(1000),Y(1000),W(1000),T(1000)
      DIMENSION C(800),A(800,20), H1(10),H2(10)
      DIMENSION B(800,10),G(800,10),Z(800),V(800),H(12)
C DATA INITIALIZATION STATEMENT TO SPECIFY
C TOL: THE REQUESTED RELATIVE ACCURACY FOR
C THE ROOT OF F(P)=S
C MAX: THE MAXIMAL NUMBER OF ITERATIONS ALLOWED IN THE
C NEWTON PROCESS
      DATA TOL/0.01/,MAX/20/
C BEFORE STARTING COMPUTATIONS A DATACHECK IS MADE
C IF THE INPUTDATA ARE INVALID CONTROLE IS REPASSED TO
C THE DRIVER PROGRAM (IER=10)
      IER=10
      IF(K.LT.2.OR.M.LT.2*K) RETURN
      IF(XI.GT.X(1).OR.X(M).GT.XE) RETURN
      WMAX=W(1)
      IF(WMAX.LE.0.) RETURN
      DO 20 I=2,M
      IF(X(I-1).GE.X(I).OR.W(I).LE.0.) RETURN
      IF(W(I).GT.WMAX) WMAX=W(I)
   20 CONTINUE
C COMPUTATIONS ARE STARTED
      IF(S.LT.0.) S=0
      S2=SQRT(S)
      K1=K+1
      M1=M-1
C WE CHOOSE THE INITIAL VALUE FOR THE NUMBER OF KNOTS, I.E.
C S.NE.0 : N=3K+1
C S.EQ.0: N=M+K+1 (INTERPOLATION)
      IFIB1=1
      IFIB2=2
      NMIN=3*K+1
      NMAX=M+K1
      NN=N
      N=NMIN
      IF(S.EQ.0.) N=NMAX
C WE CHECK WHETHER THE REQUIRED STORAGE SPACE EXCEEDS THE
```

```
C AVAILABLE STORAGE SPACE
  100 IF(N.GT.NN) GO TO 930
C WE CHOOSE THE KNOTS T(K),......T(NK1+1) IN
C THE RANGE (X1,XE)
      NK1=N-K1
      M2=N-2*K1+1
      L=M1/M2+1
      IR=M1-M2*(L-1)
      MI=L+1
      ME=M-L
      NI=K1+1
      NE=NK1
      DO 160 JJ=1,2
      IR1=IR/2
      IF(IR1.EQ.0) GO TO 140
      DO 120 J=1,IR1
      T(NI)=X(MI)
      T(NE)=X(ME)
      NI=NI+1
      MI=MI+L
      NE=NE-1
      ME=ME-L
  120 CONTINUE
  140 IF(JJ.EQ.2) GO TO 180
      IR2=2*IR1-IR+1
      L=L-1
      IR=M2-IR-IR2
      MI=MI-1
      ME=ME+1
  160 CONTINUE
  180 IF(IR1*2.EQ.IR) GO TO 200
      T(NI)=X(MI)
      IF(IR2.EQ.1) GO TO 200
      T(NI)=(X(MI)+X(MI+1))*0.5
  200 T(K1)=XI
      T(NK1+1)=XE
C WE CHOOSE 2K ADDITIONAL KNOTS FOR OUR
C B-SPLINE REPRESENTATION
      F1=T(K1+1)-XI
      F2=XE-T(NK1)
      DO 220 J=1,K
      IZ=K1-J
```

```
      IN=N-IZ
      T(IZ)=T(IZ+1)-F1
      T(IN+1)=T(IN)+F2
  220 CONTINUE
C THE ELEMENTS OF A AND Z ARE COMPUTED
      DO 240 IR=1,NK1
      Z(IR)=0.
      DO 240 IK=1,K1
      A(IR,IK)=0.
  240 CONTINUE
      L=K1
      DO 360 IT=1,M
      ARG =X(IT)
      IF(ARG.GE.T(L+1).AND.L.NE.NK1) L=L+1
C IF T(L) <=X<T(L+1) ONLY THE NORMALISED
C B-SPLINES NL-K,K(X),....NL,K(X) HAVE
C A VALUE DIFFERENT FROM ZERO. WE COMPUTE
C H1(K1-I)=NL-I,K(X),I=0,1,....K
      H1(1)=1.
      DO 300 J=1,K
      H2(1)=0.
      J1=J+1
      DO 260 I=1,J
      LI=L+I
      LJ=LI-J
      F1=H1(I)/(T(LI)-T(LJ))
      H2(I)=H2(I)+F1*(T(LI)-ARG)
      H2(I+1)=F1*(ARG-T(LJ))
  260 CONTINUE
      DO 280 I=1,J1
      H1(I)=H2(I)
  280 CONTINUE
  300 CONTINUE
C A IS A (2K+1) BANDED POSITIVE DEFIINITE MATRIX.
C THE ELEMENTS ARE STORED IN COMPACT FORM.
      LK=L-K
      DO 360 L1=LK,L
      K5=L1-L+K1
      Z(L1)=Z(L1)+H1(K5)*Y(IT)*W(IT)
      DO 360 L2=L1,L
      K6=L2-L+K1
      IR=L2
```

```
      IK=K1-L2+L1
      A(IR,IK)=A(IR,IK)+H1(K5)*H1(K6)*W(IT)
  360 CONTINUE
C WE FIRST DETERMINE THE LEAST SQUARES
C SPLINE FUNCTION (P=INFIN.)
      ITER=0
      DO 380 IR=1,NK1
      DO 380 IK=1,K1
  380 G(IR,IK)=A(IR,IK)
      LS=K1
C DECOMPOSITION OF THE POSIITIVE DEFINITE BANDMATRIX G=A+B/P
  400 CALL BANDET(G,LS,NK1,IER)
      IF(IER.EQ.0) GO TO 420
C G WAS FOUND TO BE NOT POSITIVE DEFINITE
C ITER=0 (G=A) THIS RESULT IS THEORETICALLY IMPOSSIBLE
C ITER=1 OUR INITIAL CHOISE OF P WAS TOO SMALL (B SINGULAR)
      IF(ITER.EQ.0) GO TO 960
      P=P*10.
      GO TO 700
C WE SOLVE THE SYSTEM OF EQUATIONS G*C=Z AND FIND THE
C COEFFICIENTS OF THE B-SPLINE REPRESENTATION
  420 CALL BANSOL(G,LS,NK1,Z,C)
      IF(S.EQ.0.) GO TO 910
C WE COMPUTE FP=F(P)
      FP=0.
      L=K1
      DO 440 IT=1,M
      ARG=X(IT)
      IF(ARG.GE.T(L+1).AND.L.NE.NK1)L=L+1
      DO 425 I=1,K1
      IK=L+I-K1
  425 H(I)=C(IK)
      DO 435 J=2,K1
      DO 435 JJ=J,K1
      I=J+K1-JJ
      LI=L+I-K1
      LJ=L+I-J+1
  435 H(I)=((ARG-T(LI))*H(I)+(T(LJ)-ARG)*H(I-1))/
     &(T(LJ)-T(LI))
  440 FP=FP+W(IT)*(Y(IT)-H(K1))**2
C TEST ON CONVERGENCE
      IF(ABS((FP-S)/S).LT.TOL) RETURN
```

```
      IF(ITER.NE.0) GO TO 600
C TEST WHETHER THE NUMBER OF KNOTS MUST BE INCREASED
      IF(FP.GT.S)GO TO 800
C THE ELEMENTS OF B ARE COMPUTED. B IS A (2K+3)
C BANDED POSITIVE SEMIDEFINITE MATRIX.
C THE ELEMENTS ARE STORED IN COMACT FORM
      LS=LS+1
      DO 460 IR=1, NK1
      DO 460 IK=1,LS
  460 B(IR,IK)=0.
      DO 560 L=LS,NK1
      DO 480 J=1,K1
      L1=L+J
      L2=L1-LS
      K5=K1+J
      H(J)=T(L)-T(L2)
  480 H(K5)=T(L)-T(L1)
      F1=-H(LS)*H(K1)
      DO 560 J=1,LS
      DO 560 I=J,LS
      F=1.
      DO 500 LL=1,K1
      L1=J+LL-1
      L2=I+LL-1
      F2=H(L1)*H(L2)/F1
  500 F=F*F2
      IR=I-1+L-K1
      IK=LS-I+J
      L1=L+J-1
      L2=L+I-1
      L3=L1-K1
      L4=L2-K1
  560 B(IR,IK)=B(IR,IK)+(T(L1)-T(L3))*(T(L2)-T(L4))/(F*F1)
C WE CHOOSE THE INITIAL VALUE OF P
      ITER=1
      P=0.0001/WMAX
      GO TO 700
C TEST WHETHER S>F(P)
C ITER=1: THE LEAST SQUARES POLYNOMIAL OF DEGREE K IS THE
C TRIVIAL SOLUTION OF OUR SMOOTHIING PROBLEM
C ITER>1: F(P)**-0.5 IS NOT CONCAME. (THEORETICALLY
C IMPOSSIBLE)
```

```
  600 IF ((S.GT.FP) .AND. (ITER.EQ.1)) GOTO 920
      IF ((S.GT.FP) .AND. (ITER.GT.1)) GOTO 950
C TEST ON THE NUMBER OF ITERATIONS
      IF(ITER.EQ.MAX) GO TO 940
C COMPUTATION OF DFP=FIRST DERIVATIVE OF F(P)
      DO 680 I=1,NK1
      L1=I-K1
      IF(L1.LT.1) L1=1
      F=0.
      DO 620 I2=L1,I
      I3=I2-I+LS
  620 F=F+C(I2)*B(I,I3)
      IF(I.EQ.NK1) GO TO 680
      L1=I+K1
      IF(L1.GT.NK1) L1=NK1
      I1=I+1
      DO 640 I2=I1,L1
      I3=I+LS-I2
      F=F+C(I2)*B(I2,I3)
  640 CONTINUE
      V(I)=F
  680 CONTINUE
      CALL BANSOL(G,LS,NK1,V,C)
      DFP=0.
      DO 690 I=1,NK1
      DFP=DFP+C(I)*V(I)
  690 CONTINUE
      DFP=-2.*DFP*PINV**3
C WE CARRY OUT ONE MORE STEP OF THE NEWTON PROCESS
      FP2=SQRT(FP)
      P=P+2.*FP2/S2*(S2*FP2-FP)/DFP
      ITER=ITER+1
C THE ELEMENTS OF G=A+B/P ARE COMPUTED.
C G IS A (2K+3) BANDED POSITIVE DEFINITE MATRIX.
C THE ELEMENTS ARE STORED IN COMPACT FORM
  700 PINV=1./P
      DO 720 IR=1,NK1
      G(IR,1)=PINV*B(IR,1)
      DO 720 IK=2,LS
  720 G(IR,IK)=A(IR,IK-1)+PINV*B(IR,IK)
      GO TO 600
C WE INCREASE THE NUMBER OF KNOTS AND
```

```
C RESTART THE COMPUTATIONS
  800 IF(N.EQ.NMAX) GO TO 910
      IFIB3=IFIB1+IFIB2
      IFIB1=IFIB2
      IFIB2=IFIB3
      N=NMIN+IFIB3
      IF(N.GT.NMAX) N=NMAX
      GO TO 100
C BORDERLINE CASES
  910 IER=-1
      RETURN
  920 IER=-2
      RETURN
C ERROR CODES
  930 IER=1
      RETURN
  940 IER=2
      RETURN
  950 IER=3
      RETURN
  960 IER=4
      RETURN
      END
C
C
C
C
C
C
      SUBROUTINE BANDET(G,LS,NK1,IER)
C BANDET DECOMPOSES THE 2*LS-1 BANDET NK1*NK1
C POSITIVE DEFINITE MATRIX G IN AN UPPER TRIANGULAR
C MATRIX AND ITS TRANSPOSE USING THE METHODS OF
C CHOLESKY. THE TRIANGULAR MATRIX IS RETURNED IN G.
      INTEGER P,Q,R,S
      DIMENSION G(800,10)
C ATTENTION: MATRIX G MUST HAVE THE SAME DIMENSIONS
C AS SPECIFIED IN THE DRIVER PROGRAM
      DO 500 I=1,NK1
      P=1
      IF(I.LE.LS) P=LS-I+1
      R=I-LS+P
```

```
      DO 300 J=P,LS
      S=J-1
      Q=LS-J+P
      Y=G(I,J)
      IF(P.GT.S) GO TO 200
      DO 100 K=P,S
      Y=Y-G(I,K)*G(R,Q)
      Q=Q+1
      100 CONTINUE
  200 IF (J.EQ.LS) GO TO 400
      G(I,J)=Y*G(R,LS)
      R=R+1
  300 CONTINUE
  400 IF(Y.LE.0.) GO TO 600
      G(I,J)=1./SQRT(Y)
  500 CONTINUE
C NORMAL RETURN TO DRIVER PROGRAM
      IER=0
      RETURN
C MATRIX G WAS FOUND TO BE NOT POSITIVE DEFINITE
  600 IER=-1
      RETURN
      END
      SUBROUTINE BANSOL(G,LS,NK1,Z,C)
C BANSOL SOLVES THE SYSTEM DECOMPOSED BY BANDET WITH RIGHT
C HAND SIDE Z. THE SOLUTION IS RETURNED IN C
      INTEGER P,Q,R
      DIMENSION G(800,10),Z(NK1),C(NK1)
C ATTENTION: MATRIX G MUST HAVE THE SAME DIMENSIONS AS
C SPECIFIED IN THE DRIVER PROGRAM
      L=LS-1
      DO 300 I=1,NK1
      Y=Z(I)
      IF(I.EQ.1) GO TO 200
      P=1
      IF(I.LE.LS) P=LS-I+1
      Q=I
      DO 100 J=P,L
      K=P+L-J
      Q=Q-1
      Y=Y-G(I,K)*C(Q)
  100 CONTINUE
```

```
  200 C(I)=Y*G(I,LS)
  300 CONTINUE
      DO 600 I=1,NK1
      R=NK1+1-I
      Y=C(R)
      IF(R .EQ.NK1) GOTO500
      P=1
      IF(NK1-R.LT.LS) P=LS-NK1+R
      Q=R
      DO 400 J=P,L
      K=P+L-J
      Q=Q+1
      Y=Y-G(Q,K)*C(Q)
  400 CONTINUE
  500 C(R)=Y*G(R,LS)
  600 CONTINUE
      RETURN
      END
C
C
C
      REAL FUNCTION YVAL(T,N,C,NK1,ARG,L)
C S(X) =ARG.
      DIMENSION T(N),C(NK1),H(6)
      YVAL = 0.
      K1=N-NK1
      DO 100 I=1,K1
       IK=L+I-K1
  100 H(I)=C(IK)
      DO 400 J=2,K1
       DO 400 JJ=J,K1
        I=J+K1-JJ
        LI=L+I-K1
        LJ=L+I-J+1
        H(I)=((ARG-T(LI))*H(I)+(T(LJ)-ARG)*H(I-1))/
     & (T(LJ)-T(LI))
  400 CONTINUE
      YVAL=H(K1)
      RETURN
      END
      FUNCTION DERIV(T,N,C,NK1,NU,ARG,L)
C GIVEN THE NORMALIZED B-SPLINE REPRESENTATION OF A
```