Energy and Reserve Dispatch with Renewable Generation Using Data-Driven Distributionally Robust Optimization

Zhichao Shi, Hao Liang, Venkata Dinavahi
Department of Electrical and Computer Engineering
University of Alberta, Edmonton, AB, Canada
Email: zhichao1@ualberta.ca, hao2@ualberta.ca, dinavahi@ualberta.ca

***Abstract*—With the increasing penetration of renewable generation such as wind power in modern power systems, there are many new challenges arising in power system operation with respect to reliability and economy. In this work, we study a two-stage data-driven distributionally robust (DR) energy and reserve dispatch problem with uncertain wind power. Different from the general moment-based ambiguity set, we design a new distance-based ambiguity set to describe the uncertain probability distribution of wind power, which can be constructed in a data-driven manner from historical data. Base on this new ambiguity set, the second-stage worst-case expectation of the problem is reformulated to a combination of conditional value-at-risk (CVaR) and an expected cost with respect to a reference distribution. Thus, the proposed two-stage DR model becomes a two-stage stochastic optimization problem which can be readily solved. Case studies are carried out to verify the effectiveness of the proposed method based on the IEEE 6-bus test system and modified IEEE 118-bus test system. Simulation results show the value of data in controlling the conservatism of the problem, and the DR problem converges to the stochastic problem with fixed distribution as the data size goes to infinity.**

***Index Terms*—Data-Driven, Distributionally Robust Optimization, Energy and Reserve Dispatch, Renewable Energy**

NOMENCLATURE

The main notations used in this paper are listed below for quick reference.

A. Parameters

a_i^m, b_i^m	Piecewise linear cost coefficients for unit i.
c_i^u, c_i^d	Up and down re-dispatch cost coefficients for unit i.
c_j^w	Wind power curtailment cost coefficient.
c_k^{Lc}	Load shedding cost coefficient.
d_i^u, d_i^d	Up and down reserve cost coefficients.
f_0^s/f^s	Reference/true probability for scenario s.
f_u^s/f_l^s	Upper/lower bound of the unknown probability for scenario s.
$F_i()$	Generation cost function for unit i.
F_l	Power flow limit for line l.
N_g, N_w, N_L	Number of generators, wind farms and loads.
$p_i^{\min}, p_i^{\max}$	Minimum and maximum output of unit i.
p_k^L	Load demand k.
R_i^u, R_i^d	Ramp-up and ramp-down limit for unit i.
w_j^f	Forecasted output for wind generator j.
π_{il}^g	Power transfer distribution factor (PTDF) from unit i to line l.
π_{jl}^w	PTDF from wind generator j to line l.
π_{kl}^L	PTDF from load k to line l.
θ	Tolerance level in the ambiguity set.

B. Variables

p_i^g	Output of unit i in first stage.
p_i^{gd}	Down re-dispatch power of unit i.
p_i^{gu}	Up re-dispatch power of unit i.
p_k^{Lc}	Load shedding in second stage.
r_i^d	Down reserve of unit i in first stage.
r_i^u	Up reserve of unit i in first stage.
w_j^c	Wind power curtailment in second stage.
$\tilde{p}_i^g$	Actual power output of unit i.
$\tilde{w}_j$	Actual power output of wind generator j.
ξ_j	Forecast error of wind generator j.

I. INTRODUCTION

Renewable energies, especially wind power, have been increasingly integrated into existing power systems in recent years due to their clean and environmentally friendly advantages. However, with a close relation to the variable weather or other external factors, they are also inherently uncertain and hard to forecast [1]. Although short-term wind power forecast has been well studied in many works and organizations, the forecast error cannot be totally eliminated. Consequently, the large-scale integration of stochastic renewable generation will pose new challenges to power system operational problems such as the security constrained unit commitment (UC) and economic dispatch (ED) problem [2]. To ensure the reliability of power system operation, new types of reserves are considered to compensate the uncertain renewable generation, which makes it important to co-optimize the energy and reserve dispatch from the economic respective [3].

To cope with the uncertainty involved in power system operation, two major approaches, stochastic optimization and robust optimization, have been widely investigated. In stochastic optimization method, the probability distribution of random variables is usually assumed to be known in advance, and the objective is to minimize the expected cost based on sampling technique [4]. However, this scenario-based stochastic

978-1-7281-0407-2/19/$31.00 © 2019 IEEE

method will lead to a large computational burden, and the true probability distribution is hard to estimate in practice. For robust optimization method, less distribution information is required with the introduction of uncertainty sets [5]. Since it aims to minimize the cost for the worst-case scenario over the uncertainty set which rarely occurs in practice, the solution of this method may be over-conservative.

Considering the deficiencies of stochastic and robust optimization methods, another new technique, distributionally robust optimization (DRO), has been proposed recently [6] to deal with uncertainty according to partial distribution information. In DRO method, the worst-case expected cost is minimized over a set of probability distributions called ambiguity set which is characterized by partial distribution information such as the moments and support of random variables. DRO has been applied to different power system optimization problems recently including unit commitment[7], optimal power flow [8], microgrid energy management [9], and energy and reserve co-dispatch [10]. In [11], a distributionally robust reserve model is proposed to minimize the cost of generation and reserves, and the uncertainty from wind power forecast error is captured by an ambiguity set based on first and second-order moment information. In [12], a two-stage hydro-thermal-wind economic dispatch model considering distributional robustness is studied, and the uncertain probability distribution of wind power is also described by moment-based ambiguity set.

In addition to the moment-based DRO approach mentioned above, statistical-distance-based DRO method has also been reported in the literature. With the application of a certain statistical distance calculated from historical data, this method can use more distributional information than the moment-based method. In [13], a distributionally robust UC model is studied based on Kullback-Leibler (KL) divergence which is used to measure the distance between unknown true distribution and an estimated reference distribution. By constructing confidence sets based on two norms, a data-driven risk-averse stochastic UC model is studied in [14]. Similarly, distance-based DRO methods have also been studied for energy and reserve dispatch problems. In [15], a two-stage data-driven distributionally robust energy and reserve scheduling model with wind power is studied, and the Wasserstein ball based ambiguity set is used to contain all possible probability distributions. Also, a two-stage risk-averse stochastic model is proposed for energy and reserve dispatch problem in [16], and Kernel density estimation is used to estimate the probability distributions of wind power captured by the L_1-norm based ambiguity set.

In this work, we study a data-driven two-stage energy and reserve dispatch problem using DRO method. First, we formulate a two-stage model which minimizes the generation and reserve cost with forecasted wind power in the first stage and minimizes the expected re-dispatch cost considering the worst-case probability distribution in the second stage. Second, an ambiguity set based on L_∞ norm is designed to capture the uncertain probability distribution of wind power which is different from those studied in previous literature. We focus on the distance-based DRO method here since it can use more distributional information from historical data. Third, based on the proposed ambiguity set, the second-stage worst-case expectation is reformulated into a combination of the conditional value-at-risk (CVaR) and expected cost with respect to a reference distribution, thus the original two-stage DRO problem becomes a two-stage stochastic linear program problem which can be readily solved. In summary, the main contributions of this work are as follows: (1) a data-driven two-stage distributionally robust model is proposed for energy and reserve dispatch problem with wind power, and we design a new ambiguity set based on L_∞ norm to describe the uncertainty of wind power probability distribution; (2) we reformulate the second-stage worst-case expectation into a convex combination of CVaR and an expected cost so that the original problem can be solved as a stochastic linear program problem, the effectiveness of the proposed method, especially the value of data, is validated by experiments based on IEEE 6-bus test system and 118-bus test system.

The remainder of this paper is organized as follows. In Section II, we formulate the two-stage DR model for energy and reserve dispatch problem and design the ambiguity set. In Section III, the solution methodology based on reformulation technique is presented. Case studies are carried out to verify the effectiveness of the proposed method in Section IV. Finally, Section V concludes the paper.

II. Problem Formulation

In this section, we first formulate the studied two-stage DR energy and reserve dispatch model with uncertain renewable generation. Without loss of generality, we consider wind power as the renewable generation here. Then, the ambiguity set construction is introduced.

A. Two-stage Dispatch Model

Different from the previous stochastic or robust economic dispatch problem [17], we study a DR dispatch problem in this work. As mentioned above, the proposed two-stage dispatch model determines the generation and reserve with forecasted wind power in the first stage and finds re-dispatch decisions against any uncertainty realization in the second stage. The first-stage decision can be seen as a base-case dispatch plan. As done in the references [15] [16], we also consider an hourly-ahead dispatch problem in this work. Mathematically, the two-stage DR energy and reserve dispatch model can be expressed as follows:

$$\min_{\boldsymbol{p}^g, \boldsymbol{r}^u, \boldsymbol{r}^d} \; [\sum_i F_i(p_i^g) + d_i^u r_i^u + d_i^d r_i^d] + \max_{P \in \mathcal{D}} E_P[Q(\boldsymbol{p}^g, \boldsymbol{r}^u, \boldsymbol{r}^d, \boldsymbol{\xi})] \tag{1}$$

$$\text{s.t.} \; \sum_i^{N_g} p_i^g + \sum_j^{N_w} w_j^f = \sum_k^{N_L} p_k^L \tag{2}$$

$$p_i^{\min} \le p_i^g - r_i^d, \; p_i^g + r_i^u \le p_i^{\max}, \; \forall i \tag{3}$$

$$-F_l \leq \sum_{i}^{N_g} \pi_{il}^g p_i^g + \sum_{j}^{N_w} \pi_{jl}^w w_j^f - \sum_{k}^{N_L} \pi_{kl}^L p_k^L \leq F_l, \ \forall l \quad (4)$$

$$0 \leq r_i^d \leq R_i^d, \ 0 \leq r_i^u \leq R_i^u, \ \forall i \quad (5)$$

$$F_i(p_i^g) \geq a_i^m p_i^g + b_i^m, \ \forall i, \ \forall m \quad (6)$$

where the objective function in (1) includes the fuel cost, spinning reserve cost and second-stage worst-case expected cost. $Q(\cdot)$ represents the second-stage cost and $\mathcal{D}$ is the ambiguity set which will be introduced later. Constraint (2) is the power balance equation. Constraint (3) limits the generation output. Constraint (4) ensures the DC power flow limit. Constraint (5) limits the up and down reserve capacities. Constraint (6) is the piecewise linear approximation with m pieces of the common quadratic fuel cost function.

In the second stage, when the forecast errors $\boldsymbol{\xi}$ of wind generation are revealed, we can adopt corrective actions by re-dispatching. In this case, the real wind power and generation output can be expressed as below:

$$\tilde{w}_j = w_j^f + \xi_j, \ \forall j \quad (7)$$

$$\tilde{p}_i^g = p_i^g + p_i^{gu} - p_i^{gd}, \ \forall i \quad (8)$$

Furthermore, the second-stage problem can be stated as follows:

$$Q(\boldsymbol{p}^g, \boldsymbol{r}^u, \boldsymbol{r}^d, \boldsymbol{\xi}) = \min \ \sum_{i}^{N_g} (c_i^u p_i^{gu} + c_i^d p_i^{gd}) + \sum_{j}^{N_w} c_j^w w_j^c + \sum_{k}^{N_L} c_k^{Lc} p_k^{Lc} \quad (9)$$

$$\sum_{i}^{N_g} \tilde{p}_i^g + \sum_{j}^{N_w} (\tilde{w}_j - w_j^c) = \sum_{k}^{N_k} (p_k^L - p_k^{Lc}) \quad (10)$$

$$-F_l \leq \sum_{i}^{N_g} \pi_{il}^g \tilde{p}_i^g + \sum_{j}^{N_w} \pi_{jl}^w (\tilde{w}_j - w_j^c) - \sum_{k}^{N_L} \pi_{kl}^L (p_k^L - p_k^{Lc}) \leq F_l, \ \forall l \quad (11)$$

$$0 \leq p_i^{gu} \leq r_i^u, \ 0 \leq p_i^{gd} \leq r_i^d, \ \forall i \quad (12)$$

$$0 \leq p_k^{Lc} \leq p_k^L, \ 0 \leq w_j^c \leq w_j^f, \ \forall k, \ \forall j \quad (13)$$

where the second-stage cost in (9) includes the re-dispatch cost of generators, wind power curtailment cost and load shedding cost. Constraint (10) limits the power balance in the second stage problem. Constraint (11) is the transmission line flow limit. Constraint(12) ensures that the generator re-dispatch capability is limited by the first-stage reserve capacity. Constraint (13) restricts the load shedding and wind power curtailment.

In this study, the uncertainty source is the wind power generation, which is demonstrated by $\boldsymbol{\xi}$ in above formulations. In addition, we assume that the uncertain wind power has a finite support [14], i.e., the number of possible realizations ξ^s is finite (e.g., $\xi^1, \xi^2, ..., \xi^S$). However, the true probability distribution is unknown and is restricted by the constraints in the ambiguity set $\mathcal{D}$.

B. Ambiguity Set Construction

As mentioned above, the true probability distribution of wind power is unknown. In this study, we design a distance-based ambiguity set to constrain the true distribution, i.e., we use a distance measure between two distributions to describe the ambiguity set. Specifically, the general ambiguity set can be defined as: $\mathcal{D} = \{P \in \mathcal{P} : dist(P||P_0) \leq \theta\}$, where $\mathcal{P}$ is the set of all distributions, $dist()$ is a distance measure between the true distribution P and a reference distribution P_0, and θ is the tolerance level. In this work, we adopt the L_∞ norm as the distance function which has not been studied for energy and reserve dispatch problem before. Accordingly, the studied ambiguity set is given as follows:

$$\mathcal{D}_\infty = \{P \in \mathcal{P} : ||P - P_0||_\infty \leq \theta\}. \quad (14)$$

The advantage of using the L_∞ norm is that the convergence between the true distribution and reference distribution can be guaranteed as the data size goes to infinity.

In above ambiguity set, the reference distribution can be derived with the histogram method from historical data [14] [18]. For example, we assume that there are N data samples in total, and the sample space can be partitioned into S bins. Then, we can count the frequency of samples in each bin (e.g., N^s) and the reference distribution can be obtained as $P_0 = (f_0^1, f_0^2, ..., f_0^S)$ with the element $f^s = N^s/N$. In addition, the tolerance level θ can be adjusted to control the conservatism. According to the Proposition 9 in [19], θ can be determined as follows:

$$\theta = (z_{\alpha/2}/\sqrt{N}) \ \max_{s=1,...,S} \sqrt{f_0^s(1 - f_0^s)} \quad (15)$$

where $z_{\alpha/2}$ is the $100(1-\alpha/2)$th percentile of standard normal distribution. In this case, the ambiguity set contains the true distribution with a $(1 - \alpha)$ confidence level.

III. Solution Methodology

For better description, we first express the two-stage DR dispatch model in a compact form as follows:

$$\min \ \boldsymbol{c}^\top \boldsymbol{x} + \max_{P \in \mathcal{D}_\infty} \ E_P[Q(\boldsymbol{x}, \boldsymbol{\xi})] \quad (16)$$

$$\text{s.t. } Q(\boldsymbol{x}, \boldsymbol{\xi}) = \min \ \boldsymbol{q}^\top \boldsymbol{y} \quad (17)$$

$$\boldsymbol{Ax} \leq \boldsymbol{h} \quad (18)$$

$$\boldsymbol{Bx} + \boldsymbol{Cy} + \boldsymbol{D\xi} \leq \boldsymbol{d} \quad (19)$$

$$\boldsymbol{Ex} + \boldsymbol{Fy} \leq \boldsymbol{e} \quad (20)$$

$$\boldsymbol{x} \geq 0, \ \boldsymbol{y} \geq 0 \quad (21)$$

where $\boldsymbol{x}$ represents the first-stage decision variables including p_i^g, r_i^u and r_i^d, $\boldsymbol{y}$ is the second-stage variables including $p_i^{gu}, p_i^{gd}, w_j^c$ and p_k^{Lc}. In addition, constraint (18) represents the first-stage constraints, (19) includes (10)-(11), and (20) includes (12)-(13).

To solve the above complex two-stage DR dispatch problem, a general idea is to reformulate the worst-case second-stage expected cost, and this is also the basic idea of our solution method. For the ambiguity set introduced in (14), we can write it equivalently as $f_0^s - \theta \le f^s \le f_0^s + \theta$, for all $s = 1, 2, ..., S$. Furthermore, let $f_l^s = f_0^s - \theta$ and $f_u^s = f_0^s + \theta$ by introducing two auxiliary variables f_l^s and f_u^s, we can get the following equivalent L_∞ norm based ambiguity set:

$$\mathcal{D}_\infty = \{f \ge 0 : f_l^s \le f^s \le f_u^s, \forall s, \sum_{s=1}^{S} f^s = 1\} \tag{22}$$

where f_l and f_u are the lower and upper bound of the unknown probability distribution, respectively. In addition, it is assumed that $P_l := \sum_{s=1}^{S} f_l^s \in (0, 1)$ and $P_u := \sum_{s=1}^{S} f_u^s > 1$ to avert trivial cases.

Based on the above ambiguity set, we can induce two new probability measures from f_l and $f_u - f_l$, respectively, which are given below:

$$\mathbb{P}_l = \sum_{s \in \{1,...,S\}} \frac{f_l^s}{P_l} \tag{23}$$

$$\mathbb{P}_{u-l} = \sum_{s \in \{1,...,S\}} \frac{f_u^s - f_l^s}{P_u - P_l}. \tag{24}$$

With these two probability measures, we can reformulate the worst-case second-stage expectation in (16) to a convex combination of an expectation and the CVaR [19] as follows:

$$\begin{aligned}\max_{P \in \mathcal{D}_\infty} E_P[Q(\boldsymbol{x}, \boldsymbol{\xi})] &= P_l E_{\mathbb{P}_l}[Q(\boldsymbol{x}, \boldsymbol{\xi})] \\ &+ (1 - P_l)\text{CVaR}^{\mathbb{P}_{u-l}}_{(P_u - 1)/(P_u - P_l)}[Q(\boldsymbol{x}, \boldsymbol{\xi})].\end{aligned} \tag{25}$$

Considering the definition of CVaR [20], the right-hand second term of (25) can be further written as below:

$$(1 - P_l)\text{CVaR} = \min_{\phi \in \mathbb{R}} (1 - P_l)\phi + (P_u - P_l) E_{\mathbb{P}_{u-l}}[Q(\boldsymbol{x}, \boldsymbol{\xi}) - \phi]^+ \tag{26}$$

where $[\tau]^+ = \max\{\tau, 0\}$.

Combining (25) and (16), we can find that the original two-stage DR problem is transformed into a two-stage stochastic linear program problem. In addition, since there is only min operator in this two-stage problem, the problem can be readily solved by available solvers without decomposition. More specifically, we need to solve a scenario-based stochastic problem for the second-stage cost and CVaR term in (26) [16], which can be described as follows:

$$\min_{\phi, \beta_s, \boldsymbol{y}_s} \quad (1 - P_l)\phi + (P_u - P_l) E_{\mathbb{P}_{u-l}}(\beta_s) \tag{27}$$

$$\beta_s \ge 0, \beta_s \ge \boldsymbol{q}^\top \boldsymbol{y}_s - \phi, \ \forall s \tag{28}$$

$$\boldsymbol{Bx} + \boldsymbol{Cy}_s + \boldsymbol{D\xi}_s \le \boldsymbol{d}, \ \forall s \tag{29}$$

$$\boldsymbol{Ex} + \boldsymbol{Fy}_s \le \boldsymbol{e}, \boldsymbol{y}_s \ge 0, \ \forall s \tag{30}$$

where β_s is the auxiliary variable for scenario s. Different from the common two-stage DR or robust problem solved by decomposition method such as Benders decomposition and column and constraint generation method, the proposed two-stage model is reformulated to a two-stage stochastic linear program problem which can be solved directly based on scenarios.

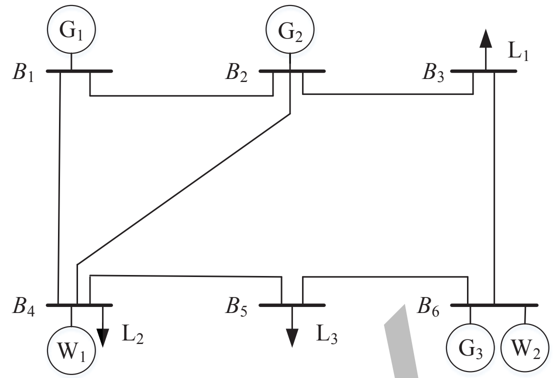

Fig. 1. Structure of 6-bus test system

IV. Case Studies

In this section, we carry out case studies based on IEEE 6-bus test system and 118-bus test system to verify the effectiveness of the proposed method. The problem is programmed in Matlab with YALMIP toolbox and solved by GUROBI solver. All the simulation experiments are implemented on a Windows-based PC with an Intel Core i7-6700 CPU 3.40 GHz and 8 GB of RAM.

A. IEEE 6-bus system

The case study with IEEE 6-bus system is used as an illustrative example and the structure of 6-bus system is shown in Fig. 1. There are 3 generators connected to buses 1, 2 and 6, seven transmission lines and three loads in this test system. Detailed data about this system can be found in [21]. The generation units are assumed to be on for dispatch problem. For the cost coefficients of each unit, d_i^u and d_i^d in the first stage are assumed to be 10% of the first-order coefficients of the quadratic fuel cost function, c_i^u and c_i^d are equal to the per-unit production cost at maximum output [3]. The cost coefficients of wind power curtailment and load shedding are 100 \$/MW and 200 \$/MW, respectively. Three loads at buses 3, 4 and 5 have a demand of 100MW, 100MW and 150MW, respectively.

In addition, we assume that there are two wind farms connected to buses 4 and 6, and each has a forecasted output of 50 MW [15]. To generate historical data of wind power, normal distribution is used where the mean equals to the forecasted value and the variance is 10% of the mean. Note that historical data can be collected directly in practice. In addition, the number of bins is set to be 5 [14]. Then we can estimate the reference distribution with the histogram method. For the piecewise linear function, the number of pieces is 3 in this study.

1) Effects of historical data size: Based on the above parameter settings, we first validate the value of data in influencing the conservatism of the problem. We set α in (15) to be 0.05 and the data size ranges from 100 to 5000. The numerical results of distributionally robust ED (DRED) are summarized in Table I. In addition, we also give the cost result

TABLE I
EFFECTS OF DATA SIZE FOR 6-BUS SYSTEM

Data size	Cost ($)	θ	SED cost ($)
100	7969.51	0.0821	7744.02
500	7837.29	0.0367	7744.02
1000	7807.93	0.0258	7744.02
2000	7777.29	0.0183	7744.02
5000	7768.88	0.0116	7744.02

TABLE II
EFFECTS OF DATA SIZE FOR 118-BUS SYSTEM

Data size	Cost ($)	θ	SED cost ($)
100	105278.57	0.0821	102405.63
500	103803.66	0.0367	102405.63
1000	103397.21	0.0258	102405.63
2000	103003.38	0.0183	102405.63
5000	102771.85	0.0116	102405.63

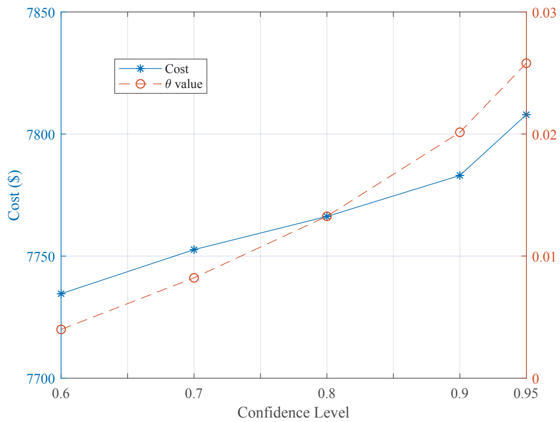

Fig. 2. Influence of confidence level for 6-bus system

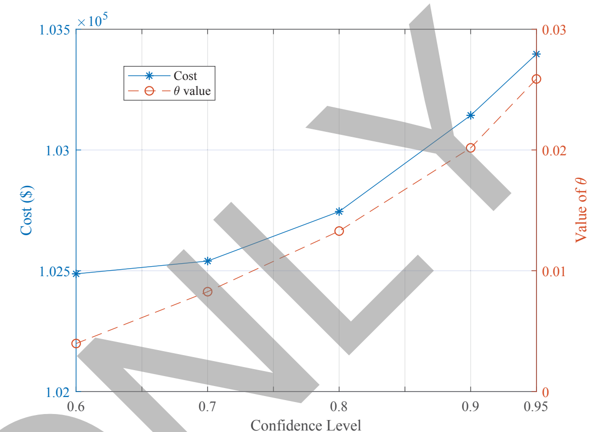

Fig. 3. Influence of confidence level for 118-bus system

with 50000 data size as a benchmark which can be seen as the stochastic economic dispatch (SED) problem with perfect information, i.e., we use this case to estimate the problem with true distribution. From this table, we can see that the total average cost decreases as the data size increases, thus the problem becomes less conservative. Considering the change of θ value, it can also be derived that the ambiguity set shrinks with more data information involved, and the DRED problem is expected to converge to the SED problem with perfect information when the data size goes to infinity. In summary, the results show the value of additional data in controlling the conservatism of the problem.

2) Influence of confidence level: In this section, we test the influence of the confidence level in ambiguity set on the conservatism of the problems. From (15), we can see that the confidence level will influence the θ value which indicates the size of the ambiguity set. In this case study, we fix the number of data as 1000 and adjust the value of confidence level $(1-\alpha)$ from 0.6 to 0.95. The results of corresponding cost and θ value are illustrated in Fig. 2. As can be seen from this figure, both the total cost and the θ value increase with the increment of confidence level. This is reasonable since higher confidence level means a higher chance that the ambiguity set contains the true distribution. Therefore, θ should become larger to enlarge the ambiguity set when confidence level increases, which also results in a more conservative problem.

B. IEEE 118-bus system

In this section, we further test the performance of the proposed method with a larger system, a modified IEEE 118-bus test system, which can also be used to verify the potential application of the approach in practice. The detailed data about this system can be found in [22] and the total load demand is 4720 MW. Related parameters such as the cost coefficients are the same with those in 6-bus system case study. In addition, we assume that six wind farms are connected to the system at buses 12, 17, 49, 59, 80 and 92, and each has a 100 MW forecasted output [15]. The wind farms are assumed to have the same support set for simplicity, and the historical data are also generated from normal distribution.

With a similar simulation experiment, we also validate the effectiveness of the proposed method by checking the influence of historical data size and confidence level. The simulation results are given in Table II and Fig. 3, respectively. From analyzing these results, we can derive the same conclusions as before which are omitted. The computation time for this case study is only several seconds, and this also confirms the potential application of the proposed method on a practical power system.

In addition, we test the proposed method with more data sets, i.e., Weibull distribution and log-normal distribution are also used to generate historical data except for the above normal distribution. For these various distributions, we study the influence of the data size similarly and compare the obtained average total costs with the cost of SED with perfect information. More specifically, the gap between the DRED cost (e.g., z_{dr}) and optimal SED cost (z^*) is calculated which is defined as $(z_{dr} - z^*)/z^*$. The results are illustrated in Fig. 4. As can be seen from this figure, the cost of DRED

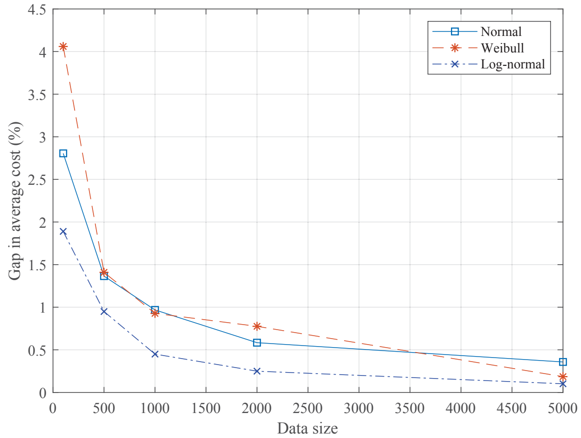

Fig. 4. Influence of data size under different distributions

problem converges to that of perfect SED problem with the increase of historical data size under different distributional settings. Consequently, the conservatism of the DRED problem decreases with more data information.

V. Conclusion

A two-stage data-driven distributionally robust energy and reserve dispatch problem is studied in this work, which considers the base-case dispatch plan and reserve in the first stage and worst-case re-dispatch in the second stage. Unlike the common moment-based ambiguity set, we design a new distance-based ambiguity set, i.e., L_∞ norm based set, which has not been studied for such problem before. Based on this new ambiguity set, the second-stage worst-case expected cost is reformulated into a combination of CVaR and an expectation with respect to a reference distribution, which makes the proposed two-stage distributionally robust model become a two-stage stochastic optimization problem. The results of simulation experiments validate the effectiveness of the proposed approach, especially the value of data in controlling the conservatism of the problem. We also show that the distributionally robust problem converges to the stochastic problem with perfect information as the historical data size goes to infinity.

References

[1] H. Holttinen et al, "Methodologies to determine operating reserves due to increased wind power," *IEEE Trans. Sustain. Energy*, vol. 3, no. 4, pp. 713–723, 2012.
[2] J. Wu, B. Zhang, K. Wang, J. Shao, J. Yao, D. Zeng, and T. Ge, "Optimal economic dispatch model based on risk management for wind-integrated power system," *IET Gener., Transm. Distrib.*, vol. 9, no. 15, pp. 2152–2158, 2015.
[3] W. Wei, F. Liu, and S. Mei, "Distributionally robust co-optimization of energy and reserve dispatch," *IEEE Trans. Sustain. Energy*, vol. 7, no. 1, pp. 289–300, 2016.
[4] L. Wu, M. Shahidehpour, and Z. Li, "Comparison of scenario-based and interval optimization approaches to stochastic scuc," *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp. 913–921, 2012.
[5] Y. Guan and J. Wang, "Uncertainty sets for robust unit commitment," *IEEE Trans. Power Syst.*, vol. 29, no. 3, pp. 1439–1440, 2014.
[6] E. Delage and Y. Ye, "Distributionally robust optimization under moment uncertainty with application to data-driven problems," *Oper. Res.*, vol. 58, no. 3, pp. 595–612, 2010.
[7] X. Peng, P. Jirutitijaroen, and C. Singh, "A distributionally robust optimization model for unit commitment considering uncertain wind power generation," *IEEE Trans. Power Syst.*, vol. 32, no. 1, pp. 39–49, 2017.
[8] Y. Zhang, S. Shen, and J. Mathieu, "Distributionally robust chance-constrained optimal power flow with uncertain renewables and uncertain reserves provided by loads," *IEEE Trans. Power Syst.*, vol. 32, no. 2, pp. 1378–1388, 2017.
[9] Z. Shi, L. Hao, S. Huang, and V. Dinavahi, "Distributionally robust chance-constrained energy management for islanded microgrids," *IEEE Trans. Smart Grid*, vol. 10, no. 2, pp. 2234–2244, 2019.
[10] X. Lu, K. W. Chan, S. Xia, B. Zhou, and X. Luo, "Security-constrained multi-period economic dispatch with renewable energy utilizing distributionally robust optimization," *IEEE Trans. Sustain. Energy*, 2018.
[11] W. Zhen, Q. Bian, H. Xin, and D. Gan, "A distributionally robust co-ordinated reserve scheduling model considering cvar-based wind power reserve requirements," *IEEE Trans. Sustain. Energy*, vol. 7, no. 2, pp. 625–636, 2017.
[12] C. Yue, W. Wei, L. Feng, and S. Mei, "Distributionally robust hydro-thermal-wind economic dispatch," *Appl. Energy*, vol. 173, pp. 511–519, 2016.
[13] Y. Chen, Q. Guo, H. Sun, Z. Li, W. Wu, and Z. Li, "A distributionally robust optimization model for unit commitment based on kullbackleibler divergence," *IEEE Trans. Power Syst.*, vol. 33, no. 5, pp. 5147–5160, Sep. 2018.
[14] C. Zhao and Y. Guan, "Data-driven stochastic unit commitment for integrating wind generation," *IEEE Trans. Power Syst.*, vol. 31, no. 4, pp. 2587–2596, July 2016.
[15] L. Yao, X. Wang, C. Duan, J. Guo, X. Wu, and Y. Zhang, "Data-driven distributionally robust reserve and energy scheduling over wasserstein balls," *IET Gener. Trans. Distrib.*, vol. 12, no. 1, pp. 178–189, 2018.
[16] X. Xu, Z. Yan, M. Shahidehpour, Z. Li, M. Yan, and X. Kong, "Data-driven risk-averse two-stage optimal stochastic scheduling of energy and reserve with correlated wind power," *IEEE Trans. Sustain. Energy*, pp. 1–1, 2019.
[17] A. Lorca and X. A. Sun, "Adaptive robust optimization with dynamic uncertainty sets for multi-period economic dispatch under significant wind," *IEEE Trans. Power Syst.*, vol. 30, no. 4, pp. 1702–1713, July 2015.
[18] A. Bagheri, C. Zhao, and Y. Guo, "Data-driven chance-constrained stochastic unit commitment under wind power uncertainty," in *Proc. IEEE Power Energy Soc. Gen. Meeting*, July 2017, pp. 1–5.
[19] R. Jiang and Y. Guan, "Risk-averse two-stage stochastic program with distributional ambiguity," *Oper. Res.*, vol. 66, no. 5, pp. 1390–1405, 2018.
[20] R. T. Rockafellar and S. Uryasev, "Optimization of conditional value-at-risk," *J. Risk*, vol. 2, pp. 21–42, 2000.
[21] IEEE 6-bus test system data. Available: http://motor.ece.iit.edu/data/JEAS6bus.doc.
[22] IEEE 118-bus system data. Available: http://motor.ece.iit.edu/data/JEAS118bus.doc.