Task-Space Synchronization of Nonlinear Teleoperation with Time-Varying Delays and Actuator Saturation

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ARTICLE HISTORY
Compiled July 28, 2018

ABSTRACT
In a nonlinear teleoperation system controlled for task-space position tracking, while the time-varying delay in the communication channel has been addressed, the actuator saturation has not been taken into account yet. Considering that in practice, the actuator saturation is a serious constraint, disregarding it in the controller design stage can cause problems. In this paper, we have proposed a control framework to ensure end-effectors position tracking while satisfying sub-task control in the presence of the nonlinear dynamics for the telemanipulators, bounded time-varying delays in the communication channels and saturation in the actuators. We have shown that in free motion and when the operator applies a bounded force to the local robot, the proposed controller not only guarantees the position convergence of the end-effectors but also guarantees the accomplishment of the sub-task control. The asymptotic stability of the closed-loop dynamics is studied using a Lyapunov-Krasovskii functional under conditions on the controller parameters and the maximum values of time-varying delays. The efficiency of the proposed teleoperation system and the control algorithm is validated using numerical simulations with a 2-DOF planar local robot and a 3-DOF planar redundant remote robot.

KEYWORDS
Actuator saturation; nonlinear teleoperation; redundant manipulator; semi-autonomous system; time-varying delay

1. Introduction

A bilateral teleoperation system is composed of interconnected local and remote robots, where various signals are exchaned between the two robots via a communication channel. A human operator manipulates the local robot and the controlled coupling between the local and remote robots enables carrying out tasks on a remote environment in which the remote robot operates. The main advantage of a teleoperation system is its capability to provide a stable interaction between the operator and the environment to remotely accomplish tasks in unsafe or hazardous conditions (Arcara & Melchiorri, 2002; Hokayem & Spong, 2006). The aforementioned merits warrant the application of teleoperation systems in areas like outer space manipulation, undersea exploration, and remote medical operation (Hokayem & Spong, 2006; Li, Cao,

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Tang, Li, & Ye, 2013; Rodriguez-Seda et al., 2010; Slawiñski, Mut, Salinas, & García, 2012). The distance between the local and remote robots imposes inevitable communication delays, which can destabilize and degrade the performance of the telerobotic system (Liu & Chopra, 2012b; Richard, 2003).

The stability of teleoperation systems subject to constant time delays has been extensively studied using scattering or wave-variable formulation (Anderson & Spong, 1989; Niemeyer & Slotine, 1991). Control techniques using passivity-based synchronization (Chopra, Spong, & Lozano, 2008), PD-like control (D. Lee & Spong, 2006) and neural networks (Forouzantabar, Talebi, & Sedigh, 2012) have been employed recently to enhance tracking performance. In practice, the communication delay can be time-varying and asymmetric in the forward and backward paths between the operator and the remote environment (Gao & Chen, 2007; Gao, Chen, & Lam, 2008). Also, communication network problems like data congestion and limitations of transmission bandwidth may lead to time-varying delays that substantially compromise the system's performance and even result in instability (Kang, Li, Shang, & Xi, 2010; Nuño, Basañez, Ortega, & Spong, 2009; Ryu, Artigas, & Preusche, 2010).

There are a number of control schemes for time-varying delay compensation in the literature. An LMI-based method in which a proportional plus damping injection controller has been used to address the stability in a teleoperation system with time-varying and asymmetric delays (Hua & Liu, 2010). An algorithm has been presented to improve the stability/performance characteristics of a force-reflecting teleoperation system in the presence of time-varying communication delays (Polushin, Liu, & Lung, 2006). The approach of scattering transformation has been modified to deal with teleoperation systems subject to time-varying delays (Lozano, Chopra, & Spong, 2002; Yokokohji, Imaida, & Yoshikawa, 1999). Passivity-based control algorithms have been proposed for teleoperators facing time-varying communication delays and passive external forces (Fujita & Namerikawa, 2009; Ryu et al., 2010). Delay-dependent (Fujita & Namerikawa, 2009; Islam, Liu, & El Saddik, 2014) and mode-dependent (Kang et al., 2010) control schemes have been proposed to cope with the stability of teleoperation systems confronting time-varying delays. Moreover, PD-like control (D. Lee & Spong, 2006) has been extended to teleoperators with variable time delays (Nuño et al., 2009).

In almost all applications of control systems including teleoperation systems, the actuator output (i.e., the control signal) has a limited amplitude, i.e., is subject to saturation. Controllers that ignore actuator saturation may cause undesirable responses and even closed-loop instability (Kothare, Campo, Morari, & Nett, 1994). In order to address the stability of the position control loop for a single robotic manipulator subjected to bounded actuator output, several approaches have been proposed in the literature (Loria, Kelly, Ortega, & Santibanez, 1997; Morabito, Teel, & Zaccarian, 2004; Zergeroglu, Dixon, Behal, & Dawson, 2000). In the context of teleoperation systems, addressing saturation in the control process has recently received some attention (Hashemzadeh, Hassanzadeh, & Tavakoli, 2013; S.-J. Lee & Ahn, 2010, 2011; Zhai & Xia, 2016). In S.-J. Lee and Ahn (2010), a nonlinear proportional control scheme incorporated with wave variable is proposed to handle actuator saturation for the case where the delay in the communication channel is constant. In S.-J. Lee and Ahn (2011), an anti-windup approach combined with wave variables is used for constant-delay teleoperation subjected to bounded control signals. In Hashemzadeh et al. (2013), nonlinear proportional plus damping (nP+D) control has been proposed to deal with joint-space synchronization problem of nonlinear teleoperation subjected to both time-varying transmission delays and actuator saturation. The nP+D controller is similar to the proportional plus damping (P + D) controller (Hua & Liu, 2010), with the difference that it takes into account the actuator saturation at the outset of control design and alters the proportional term by passing it through a nonlinear function. In Zakerimanesh, Hashemzadeh, and Ghiasi (2017), the nP+D controller was developed for joint-space synchronization problem of cooperative thrilateral nonlinear teleoperation subjected to time-varying delays and bounded inputs. In Zhai and Xia (2016), an adaptive switching control framework is developed for joint-space synchronization problem of nonlinear teleoperation system where actuator saturation and asymmetric time-varying delays have been taken into accout in the proposed scheme.

Given that teleoperation systems mostly involve remote interaction between a human operator and an environment in the end-effector space, the study of task-space teleoperation has become an emerging research topic (Kawada, Yoshida, & Namerikawa, 2007; Liu, 2015; Liu & Chopra, 2011, 2012a, 2013; Malysz & Sirouspour, 2011; Nath, Tatlicioglu, & Dawson, 2009; Wang & Xie, 2012). In Kawada et al. (2007), scaled synchronization has been proposed for bilateral teleoperators with different configurations while the local and remote robots were assumed to be kinematically identical and non-redundant manipulators. Teleoperation of redundant manipulators was studied in Nath, Tatlicioglu, and Dawson (2009), where the robots are assumed to track a desired trajectory in task-space. However, the teleoperation system was developed without considering communication delays and the local and remote robots were required having the same degrees of freedom. Synchronization of heterogeneous robotic manipulators following a desired trajectory in the task-space was presented in Liu and Chopra (2012a). Even though heterogeneity of the robotic manipulators and communication delays were considered, the controller required all agents to have knowledge of a common trajectory, which may not be practical for teleoperation systems. A control algorithm for task-space teleoperation with guaranteed position and orientation tracking has been proposed in Wang and Xie (2012). A control theoretic framework was proposed to ensure the task-space position and velocity tracking between the non-redundant local and redundant remote robots in the presence of constant delays (Liu & Chopra, 2011). However, external forces were not considered and the performance of the force reflection was not studied. A teleoperation framework has been studied in Malysz and Sirouspour (2011), where two local robots are utilized to control different coordinates assigned to the remote robot. Moreover, the issue of communication delays was not considered in the proposed controller. In Liu and Chopra (2013), a semi-autonomous control framework is proposed to deal with a bilateral teleoperation that robots have different configurations. Dynamic uncertainties and asymmetric constant communication delays are considered in this reaserch and the redundancy of the remote robot is utilized to enhance the efficiency of complex teleoperation. In Liu (2015), control algorithms for heterogeneous teleoperation systems has been proposed to guarantee stability and tracking performance in the presence of time-varying communication delays. In Zhai and Xia (2017), an adaptive control of semi-autonomous teleoperation is developed to address the task-space bilateral teleoperation system with asymmetric time-varying delays for heterogeneous local and remote robots to guarantee stability and tracking performance. However, in the proposed task-space teleoperation systems (Liu, 2015; Zhai & Xia, 2017), the actuator saturation have not been taken into account.

Considering that in practice, the actuator saturation is a serious constraint, disregarding it in controllers is problematic. In this paper, a novel nP+D like controller that incorporates gravity compensation and a sub-task-oriented term is proposed for the control problem of task-space nonlinear teleoperation involving heterogeneous revolute-joint robots subjected to the actuator saturation and bounded time-varying delays. A redundant telemanipulator is considered as the remote robot so that the null-space can be exploited to achieve additional missions autonomously or conventionly speaking, besides the main task, to accomplish a sub-task control. The proposed control framework is able to ensure end-effectors convergence while satisfying a sub-task control such as singularity avoidance. It has been substantiated that in free motion or even when the operator applies a bounded force on the local robot, the proposed controller not only guarantees the position convergance of the end-effectors but also guarantees the accomplishment of the sub-task. By employing a Lyapunov-Krasovskii functional, the relationships among the controller parameters and the upper bounds of the time-varying delays are established.

The contributions of this paper can be summarized as follows. In contrast to Liu (2015); Zhai and Xia (2017) in which the actuators saturation are not taken into account, the proposed controller is capable of concurrently addressing the time-varying delay and actuators saturation which both are of paramount importance in the stability study of teleoperation systems. In contrast to Liu (2015); Zhai and Xia (2017) which both are contingent on the assumption that the Jacobian matrix of the local manipulator is full rank, the proposed controller is valid even if the local robot faces singularity. The proposed controller is able to guarantee the position convergence of the end-effectors and the accomplishment of the sub-task control even when the human operator exerts a bounded and uninterrupted force. Moreover, in contact motion, the signals of the system are shown to be ultimately bounded. The obtained conditions of the controller's parameters present the operator with an overall view about the system and its performance, and enables him to tune the system's settling time and its robustness to further upper bound of time-varying delay. In Liu (2015), the real-time rates of change of the delays are necessary for the controller design and are measured by transmitting a known auxiliary function between the local and remote sides. This necessity stems from the time derivative of error signal in the proposed controller. In contrast to Liu (2015), the proposed controller only needs the upper bound of time-varying delays.

This paper is organized in sections as follows. Section 2 gives problem formulation while the proposed controller and its stability analysis are studied in Sections 3 and 4, respectively. In Section 5, sub-task control and in Section 6, simulation results are discussed. In Sections 7 and 8, conclusion and appendix are presented, respectively.

Throughout this paper, we denote the set of real numbers by $\mathbb{R}=(-\infty,\infty)$, the set of positive real numbers by $\mathbb{R}_{>0}=(0,\infty)$, and the set of nonnegative real numbers by $\mathbb{R}_{\geq 0}=[0,\infty)$. Also, $\|X\|_\infty$ and $\|X\|_2$ stand for the ∞-norm and Euclidean 2-norm of a vector $X\in\mathbb{R}^{n\times 1}$, and $|X|$ denotes element-wise absolute value of the vector X. The L_∞ and L_2 norms of a time function $f:\mathbb{R}_{\geq 0}\to\mathbb{R}^{n\times 1}$ are shown as $\|f\|_{L_\infty}=\sup_{t\in[0,\infty)}\|f\|_\infty$ and $\|f\|_{L_2}=\sqrt{\int_0^\infty \|f\|_2^2 dt}$, respectively. The L_∞ and L_2 spaces are defined as the sets of $\{f:\mathbb{R}_{\geq 0}\to\mathbb{R}^{n\times 1}, \|f\|_{L_\infty}<+\infty\}$ and $\{f:\mathbb{R}_{\geq 0}\to\mathbb{R}^{n\times 1}, \|f\|_{L_2}<+\infty\}$, respectively. For simplicity, we refer to $\|f\|_{L_\infty}$ as $\|f\|_\infty$ and to $\|f\|_{L_2}$ as $\|f\|_2$. We also simplify the notation $\lim_{t\to\infty} f(t)=0$ to $f\to 0$.

2. Problem Formulation

With the assumption that the manipulators in the teleoperation system are modeled by Lagrangian systems, driven by actuated revolute-joints and their control signals are subjected to actuator saturation, the dynamics of the local (l) and remote (r) robots are given as (Hashemzadeh et al., 2013)

$$M_k(q_k)\ddot{q}_k+C_k(q_k,\dot{q}_k)\dot{q}_k+G_k(q_k)=\tau_{e_k}+S_k(\tau_k) \tag{1}$$

where for $k\in\{l,r\}$, $q_k,\dot{q}_k,\ddot{q}_k\in\mathbb{R}^{\beta_k\times 1}$ are the vectors of the joint positions, velocities and accelerations of the robots knowing that $\beta_l=n$, $\beta_r=m$. Note that n and m are the number of joints in the local and remote robots, respectively. $M_k(q_k)\in\mathbb{R}^{\beta_k\times\beta_k}$, $C_k(q_k,\dot{q}_k)\in\mathbb{R}^{\beta_k\times\beta_k}$ and $G_k(q_k)\in\mathbb{R}^{\beta_k\times 1}$ are the inertia matrix, the Coriolis/centrifugal matrix and the gravitational vector, respectively. Moreover, $\tau_{e_k}\in\mathbb{R}^{\beta_k\times 1}$ are the exerted torques on the local and remote robots, and $\tau_k\in\mathbb{R}^{\beta_k\times 1}$ are their control signals. Also, saturaion of the control signals are modeled by the vector function $S_k(\tau_k):\mathbb{R}^{\beta_k\times 1}\to\mathbb{R}^{\beta_k\times 1}$ whose elements $s_{k_i}(\tau_{k_i}):\mathbb{R}\to\mathbb{R}$; $i=1,\ldots,\beta_k$, are defined as (S.-J. Lee & Ahn, 2011; Morabito et al., 2004)

$$s_{k_i}(\tau_{k_i})=\begin{cases} B_{k_i} & \text{if} \quad \tau_{k_i}>B_{k_i} \\ \tau_{k_i} & \text{if} \quad |\tau_{k_i}|\leq B_{k_i} \\ -B_{k_i} & \text{if} \quad \tau_{k_i}<-B_{k_i} \end{cases} \tag{2}$$

where $B_{k_i}\in\mathbb{R}_{>0}$ is the saturation level of the corresponding actuator and τ_{k_i} denotes the control signal applied on the i^{th} joint of the corresponding robot. It is imperative to have $0<\Omega_{k_i}<B_{k_i}$, where $|g_{k_i}(q_k)|\leq\Omega_{k_i}$, and $g_{k_i}(q_k)$ is the i^{th} element of the gravity vector $G_k(q_k)$. This condition implies that the actuators of the manipulators are capable of overcoming the gravity of corresponding robots within their workspaces.

Let $X_k\in\mathbb{R}^{z\times 1}$ represents the positions of the robots' end-effectors in the task-space and z represents the dimension of the task-space. The relation between the task-space and the joint-space of the robots are as

$$X_k=h_k(q_k), \qquad \dot{X}_k=J_k(q_k)\dot{q}_k \tag{3}$$

where $h_k(q_k):\mathbb{R}^{\beta_k\times 1}\to\mathbb{R}^{z\times 1}$ denotes the mapping between the joint-space and the task-space and $J_k(q_k)\in\mathbb{R}^{z\times\beta_k}$ is the Jacobian matrix defined as $J_k(q_k)=\frac{\partial h_k(q_k)}{\partial q_k}$. The task-space position errors are defined as

$$\begin{aligned} e_l&\triangleq X_l(t)-X_r(t-d_r(t)), \qquad e_l^0\triangleq X_l(t)-X_r(t) \\ e_r&\triangleq X_r(t)-X_l(t-d_l(t)), \qquad e_r^0\triangleq X_r(t)-X_l(t) \end{aligned} \tag{4}$$

where $d_r(t)$ and $d_l(t)$ are backward (from the remote robot to the local robot) and forward (from the local robot to the remote robot) time-varying delays between the robots. For simplicity, in the rest of the paper, notations M_k, M_k^{-1}, C_k, C_k^T, G_k, J_k, J_k^T, J_r^+, X_k and $\dot{X}_k$ are used instead of $M_k(q_k)$, $M_k^{-1}(q_k)$, $C_k(q_k,\dot{q}_k)$, $C_k^T(q_k,\dot{q}_k)$, $G_k(q_k)$, $J_k(q_k)$, $J_k^T(q_k)$, $J_r^+(q_r)$ ($\mathbb{R}^{m\times 1}\to\mathbb{R}^{m\times z}$, being the pseudo-inverse of J_r defined later), $X_k(t)$ and $\dot{X}_k(t)$, respectively. Also q_{k_i}, $\dot{q}_{k_i}$ and g_{k_i} are the i^{th} element (coressponding to the i^{th} joint) of the vectors q_k, $\dot{q}_k$ and G_k, respectively. Inspired by the works in Liu and Chopra (2013); Zergeroglu, Dawson, Walker, and Setlur (2004), the modified form of the local and remote dynamics are obtained to incorporate the sub-task control in the controller development. To achieve this goal, let define the signals $\zeta_k\in\mathbb{R}^{\beta_k\times 1}$ and

$\varphi_k \in \mathbb{R}^{\beta_k \times 1}$ as

$$\varphi_k \triangleq \dot{q}_k - \zeta_k; \qquad \zeta_k \triangleq \begin{cases} 0 & \text{if} \quad k=l \\ [\mathbb{I}_m - J_r^+ J_r]\Psi_r & \text{if} \quad k=r \end{cases} \tag{5}$$

where $\mathbb{I}_m$ is the identity matrix of size m, $\Psi_r \in \mathbb{R}^{m\times 1}$ ($\Psi_r = -\frac{\partial}{\partial q_r} f(q_r)$, $f(q_r):\mathbb{R}^{m\times 1} \to \mathbb{R}$), is the negative gradiant of an appropriately defined function (for sub-task control), $J_r^+ \in \mathbb{R}^{m\times z}$ is the pseudo-inverse of J_r, which is defined by $J_r^+ \triangleq J_r^T \left(J_r J_r^T\right)^{-1}$ (when J_r has full row rank, i.e., the manipulator is not in a singular configuration) and satisfies $J_r J_r^+ = \mathbb{I}_m$ and $J_r[\mathbb{I}_m - J_r^+ J_r] = 0$. Taking time derivative from the both sides of the equation $\varphi_k = \dot{q}_k - \zeta_k$, premultiplying by the inertia matrix M_k and substituting $M_k \ddot{q}_k$ from (1), the following modified form of the local and the remote dynamics can be found:

$$M_k \dot{\varphi}_k + C_k \varphi_k = \Theta_k + \tau_{e_k} + S_k(\tau_k) \tag{6}$$

such that $\Theta_k \in \mathbb{R}^{\beta_k \times 1} \triangleq -M_k \dot{\zeta}_k - C_k \zeta_k - G_k$ with elements $\theta_{k_i} \in \mathbb{R}$; $i=1,\ldots,\beta_k$. Note that $\Theta_l = -G_l$ and $\tau_{e_k} = J_k^T F_{e_k}$ where $F_{e_k} \in \mathbb{R}^{z\times 1}$ is the applied force vector on the end-effector of the local or the remote manipulator.

3. Proposed Controller

Consider the dynamical system (1) and let the control signals be given by

$$\tau_k = -\Theta_k - J_k^T \Phi P(e_k) - \Sigma_k \varphi_k \tag{7}$$

where $\Sigma_k \in \mathbb{R}^{\beta_k \times \beta_k}$ is a positive-definite diagonal matrix with elements $\sigma_{k_{i=1,\ldots,\beta_k}} \in \mathbb{R}_{>0}$ such that $\sigma_{k_{min}} \triangleq \min_i \{\sigma_{k_i}\}$ and $\sigma_{k_{max}} \triangleq \max_i \{\sigma_{k_i}\}$. Also, $\Phi \in \mathbb{R}^{z\times z}$ is a positive-definite diagonal matrix with elements $\phi_{j=1,\ldots,z} \in \mathbb{R}_{>0}$ such that $\phi_{max} \triangleq \max_j \{\phi_j\}$, and $P(e_k)$: $\mathbb{R}^{z\times 1} \to \mathbb{R}^{z\times 1}$ is a nonlinear vector function with elements $p_j\left(e_{k_j}\right):\mathbb{R}\to\mathbb{R}$; $j=1,\ldots,z$. The nonlinear scalar function $p_j\left(e_{k_j}\right)$ is required to be strictly increasing, bounded, continuous, passing through the origin, concave for positive e_{k_j} and convex for negative e_{k_j} with continuous first derivative around the origin such that $\left|p_j\left(e_{k_j}\right)\right| \leq \left|e_{k_j}\right|$ and $p_j\left(-e_{k_j}\right) = -p_j\left(e_{k_j}\right)$ (Hashemzadeh et al., 2013). For instance, by choosing $p_j\left(e_{k_j}\right) = b_j \tan^{-1}\left(e_{k_j}\right)$; $0 < b_j \leq 1$, all the mentioned properties are satisfied. Note that $\frac{\partial p_j\left(e_{k_j}\right)}{\partial e_{k_j}}$ is bounded and $N_j \triangleq \sup p_j\left(e_{k_j}\right) = b_j \pi/2$. Recalling that the manipulators in the teleoperation system (1) are revolute-joint robots, important properties of the nonlinear dynamic models are revisited here (Kelly, Davila, & Perez, 2006; Spong, Hutchinson, & Vidyasagar, 2006):

Property 1. *The inertia matrix $M_k \in \mathbb{R}^{\beta_k \times \beta_k}$ is symmetric positive-definite and has the following upper and lower bounds:*

$$0 < \lambda_{min}(M_k)\mathbb{I}_{\beta_k} \leq M_k \leq \lambda_{max}(M_k)\mathbb{I}_{\beta_k} < \infty$$

where $\mathbb{I}_{\beta_k}$ is the identity matrix of size β_k.

Property 2. *$\dot{M}_k - 2C_k$ is a skew symmetric matrix.*

Property 3. *The time derivative of C_k is bounded if $\ddot{q}_k$ and $\dot{q}_k$ are bounded.*

Property 4. *The gravity vector G_k is bounded (there exist positive constants Ω_{k_i} such that every element of the gravity vector $g_{k_{i=1,\ldots,\beta_k}}$ satisfies $|g_{k_i}| \leq \Omega_{k_i}$).*

Property 5. *For a revolute-joint manipulator, there exists a positive σ bounding the Coriolis/centrifugal term as follows:*

$$\|C_k(q_k, x)y\|_2 \leq \sigma \|x\|_2 \|y\|_2$$

Also, some assumptions are made as follows

Assumption 1. *The time derivative of the forward and backward time delays in the communication channels are bounded.*

Assumption 2. *The operator and the environment are assumed to be passive, i.e., there exist positive constants $\vartheta_k < \infty$ such that*

$$\vartheta_k + \int_0^t -\dot{q}_k^T(\mu)\tau_{e_k}(\mu)d\mu > 0 \tag{8}$$

Let's introduce a few preliminary lemmas that will be used in the rest of the paper:

Lemma 1. *The following inequality holds (see Appendix for proof):*

$$\left\|\dot{X}_k\right\|_2^2 \leq \beta_k J_{k_{max}}^{(2)} \|\varphi_k\|_2^2 \tag{9}$$

where $J_{k_{max}}^{(2)} \triangleq \max\limits_{i\alpha}\left(\sup\left|J_{k_{i\alpha}}^{(2)}\right|\right)_{i,\alpha=1,\ldots,\beta_k}$ such that $J_k^{(2)} \triangleq J_k^T J_k$. Note that $J_{k_{i\alpha}}^{(2)}$ are the elements of the $J_k^{(2)}$ matrix.

Lemma 2. *The following inequalities hold (see Appendix for proof):*

$$\dot{X}_l^T \Phi\left(P(e_l^0) - P(e_l)\right) \leq 2\left|\dot{X}_l\right|^T \Phi \int_{t-d_r(t)}^t P\left(\left|\dot{X}_r(\tau)\right|\right) d\tau \tag{10}$$

$$\dot{X}_r^T \Phi\left(P(e_r^0) - P(e_r)\right) \leq 2\left|\dot{X}_r\right|^T \Phi \int_{t-d_l(t)}^t P\left(\left|\dot{X}_l(\tau)\right|\right) d\tau \tag{11}$$

Lemma 3. *The following inequalities hold (see Appendix for proof):*

$$\left|\dot{X}_r\right|^T \Phi \int\limits_{t-d_l(t)}^{t} P\left(\left|\dot{X}_l(\tau)\right|\right) d\tau - \int\limits_{t-d_l(t)}^{t} \dot{X}_l^T(\tau)\Phi P\left(\dot{X}_l(\tau)\right) d\tau \leq d_{l_{max}} \dot{X}_r^T \Phi P\left(\dot{X}_r\right) \tag{12}$$

$$\left|\dot{X}_l\right|^T \Phi \int\limits_{t-d_r(t)}^{t} P\left(\left|\dot{X}_r(\tau)\right|\right) d\tau - \int\limits_{t-d_r(t)}^{t} \dot{X}_r^T(\tau) \Phi P\left(\dot{X}_r(\tau)\right) d\tau \leq d_{r_{max}} \dot{X}_l^T \Phi P\left(\dot{X}_l\right) \tag{13}$$

4. Stability Analysis

In this section, the stability and asymptotic performance of the system (1) with the proposed controllers (7) is analyzed. Applying the controllers (7) to the modified dynamics (6), the following closed-loop dynamics can be found:

$$M_k \dot{\varphi}_k + C_k \varphi_k = \Theta_k + \tau_{e_k} + S_k\left(-\Theta_k - J_k^T \Phi P(e_k) - \Sigma_k \varphi_k\right) \tag{14}$$

Theorem 1. *Assume that the operator and the environment are passive. In the bilateral teleoperation system (1) with the controllers (7), the end-effector velocities $\dot{X}_k$ and task-space position errors e_k and e_k^0 (4) are bounded for any bounded forward and backward time delays provided that*

(1) $\sigma_{k_{min}} \geq 2T \beta_k \phi_{max} J_{k_{max}}^{(2)}$

(2) $\phi_{max} \leq \min\limits_k \left\{ \dfrac{\omega_k \left(B_{k_{min}} - \Omega_{k_{max}} - \max\limits_i \left\{ \left| \sum\limits_{\alpha=1}^{\beta_k} \left(M_{k_{i\alpha}} \dot{\zeta}_{k_\alpha} + C_{k_{i\alpha}} \zeta_{k_\alpha} \right) \right| \right\} \right)}{z J_{k_{max}} N_{max} (\omega_k + 2T)} \right\}$

where $J_{k_{max}} \triangleq \max\limits_{ji} \left(\sup \left| J_{k_{ji}} \right| \right)_{j=1,\ldots,z \text{ and } i=1,\ldots,\beta_k}$ *(note that $J_{k_{ji}}$ are the elements of the matrix J_k). $N_{max} \triangleq \max\limits_j \{N_j\}$, $B_{k_{min}} \triangleq \min\limits_i \{B_{k_i}\}$ and $\Omega_{k_{max}} \triangleq \max\limits_i \{\Omega_{k_i}\}$. Also, $M_{k_{i\alpha}}, C_{k_{i\alpha}} \in \mathbb{R}$ are the elements of M_k and C_k matrices, respectively, in which $i,\alpha = 1,\ldots,\beta_k$. Moreover, $\dot{\zeta}_{k_\alpha}, \zeta_{k_\alpha} \in \mathbb{R}$ are the elements of the vectors $\dot{\zeta}_k$ and ζ_k, respectively. Furthermore, $\omega_k \triangleq \frac{\sigma_{k_{min}}}{\sigma_{k_{max}}}$ and T is the maximum round-trip delay defined as $T \triangleq (d_{l_{max}} + d_{r_{max}})$ such that $d_{l_{max}} \triangleq \max\{d_l(t)\}$ and $d_{r_{max}} \triangleq \max\{d_r(t)\}$.*

Proof. To show the stability of the closed-loop dynamics (14), define $x_t = x(t+\varrho)$ as the state of the system (Chopra et al., 2008; Hale & Lunel, 2013), where $x(t) \triangleq \left[\varphi_l \; \varphi_r \; X_l \; \dot{X}_l \; X_r \; \dot{X}_r\right]$, $-d_{max} \leq \varrho \leq 0$ and $d_{max} \triangleq \max\{d_{l_{max}}, d_{r_{max}}\}$ is the maximum delay in the communication channel. Consider the Lyapunov-Krasovskii functional $V(x_t) = \sum\limits_{u=1}^{4} V_u(x_t)$ where

$$V_1(x_t) = \sum_{k=\{l,r\}} \frac{1}{2} \varphi_k^T M_k \varphi_k \tag{15}$$

$$V_2(x_t) = \sum_{k=\{l,r\}} \int_0^t -\dot{q}_k^T(\mu) \tau_{e_k}(\mu) d\mu + \vartheta_k \tag{16}$$

$$V_3(x_t)=\sum_{j=1}^{z}\int_0^{X_{l_j}-X_{r_j}}\phi_j p_j(\gamma_j)d\gamma_j \tag{17}$$

$$V_4(x_t)=2\sum_{k=\{l,r\}}\int_{-d_{k_{max}}}^{0}\int_{t+\gamma}^{t}\dot{X}_k^T(\eta)\Phi P\left(\dot{X}_k(\eta)\right)d\eta d\gamma \tag{18}$$

Considering Property 2, the closed-loop dynamics (14), and the facts that $\tau_{e_k}=J_k^T F_{e_k}$, $\zeta_k^T J_k^T=0$, $\varphi_k^T\tau_{e_k}=\dot{q}_k^T\tau_{e_k}$ and $\varphi_k^T J_k^T=\dot{X}_k^T$, and defining $D_k\triangleq\dot{X}_k^T\Phi P(e_k)$ and $D_k^{'}\triangleq-\dot{X}_k^T\Phi P(e_k)$ such that $D_k+D_k^{'}=0$, we have

$$\sum_{u=1}^{2}\dot{V}_u(x_t)+\sum_{k=\{l,r\}}D_k=\sum_{k=\{l,r\}}\varphi_k^T\left[S_k\left(-\Theta_k-J_k^T\Phi P(e_k)-\Sigma_k\varphi_k\right)-\left(-\Theta_k-J_k^T\Phi P(e_k)\right)\right] \tag{19}$$

Defining

$$\delta_k\triangleq\frac{-1}{\varphi_k^T\Sigma_k\varphi_k}\varphi_k^T\left[S_k\left(-\Theta_k-J_k^T\Phi P(e_k)-\Sigma_k\varphi_k\right)-\left(-\Theta_k-J_k^T\Phi P(e_k)\right)\right] \tag{20}$$

and substituting (20) into (19), we get

$$\dot{V}_1(x_t)+\dot{V}_2(x_t)+\sum_{k=\{l,r\}}D_k=-\sum_{k=\{l,r\}}\delta_k\varphi_k^T\Sigma_k\varphi_k \tag{21}$$

Also, the time derivatives of $V_3(x_t)$ and $V_4(x_t)$ are

$$\dot{V}_3(x_t)=\sum_{k=\{l,r\}}\dot{X}_k^T\Phi P\left(e_k^0\right) \tag{22}$$

$$\dot{V}_4(x_t)\leq 2\sum_{k=\{l,r\}}\left(d_{k_{max}}\dot{X}_k^T\Phi P\left(\dot{X}_k\right)-\int_{t-d_k(t)}^{t}\dot{X}_k^T(\mu)\Phi P\left(\dot{X}_k(\mu)\right)d\mu\right) \tag{23}$$

Now using Lemma 2, we have

$$\dot{V}_3(x_t)+\sum_{k=\{l,r\}}D_k^{'}\leq 2\left|\dot{X}_l\right|^T\Phi\int_{t-d_r(t)}^{t}P\left(\left|\dot{X}_r(\tau)\right|\right)d\tau+2\left|\dot{X}_r\right|^T\Phi\int_{t-d_l(t)}^{t}P\left(\left|\dot{X}_l(\tau)\right|\right)d\tau \tag{24}$$

By adding $\dot{V}_4(x_t)$ to (24) and using Lemma 3, it results in

$$(24)+\dot{V}_4(x_t)\leq\sum_{k=\{l,r\}}2(d_{l_{max}}+d_{r_{max}})\dot{X}_k^T\Phi P\left(\dot{X}_k\right) \tag{25}$$

Finally, with (21) and (25), $\dot{V}(x_t)$ satisfies

$$\dot{V}(x_t) \leq \sum_{k=\{l,r\}} \left(2T\dot{X}_k^T \Phi P\left(\dot{X}_k\right) - \delta_k \varphi_k^T \Sigma_k \varphi_k \right) \leq \sum_{k=\{l,r\}} \left(2T\phi_{max} \dot{X}_k^T P\left(\dot{X}_k\right) - \delta_k \sigma_{k_{min}} \varphi_k^T \varphi_k \right) \tag{26}$$

Defining $\psi_k \triangleq \frac{\dot{X}_k^T P(\dot{X}_k)}{\|\varphi_k\|_2^2}$ and substituting it into (26) leads to

$$\sum_{u=1}^{4} \dot{V}_u(x_t) \leq -\sum_{k=\{l,r\}} (\delta_k \sigma_{k_{min}} - 2T\phi_{max}\psi_k) \varphi_k^T \varphi_k \tag{27}$$

It is obvious from (27) that a sufficient condition for $\dot{V}(x_t) \leq 0$ is

$$\delta_k \sigma_{k_{min}} \geq 2T\phi_{max}\psi_k \tag{28}$$

Now, considering (20) as

$$\delta_k = \frac{-1}{\varphi_k^T \Sigma_k \varphi_k} \sum_{i=1}^{\beta_k} \varphi_{k_i} \left[s_{k_i} \left(-\theta_{k_i} - \sum_{j=1}^{z} J_{k_{ji}} \phi_j p_j\left(e_{k_j}\right) - \sigma_{k_i} \varphi_{k_i} \right) - \left(-\theta_{k_i} - \sum_{j=1}^{z} J_{k_{ji}} \phi_j p_j\left(e_{k_j}\right) \right) \right] \tag{29}$$

where $-\theta_{k_i} = g_{k_i} + \sum_{\alpha=1}^{\beta_k} \left(M_{k_{i\alpha}} \dot{\zeta}_{k_\alpha} + C_{k_{i\alpha}} \zeta_{k_\alpha} \right)$, and using the property (see Appendix)

$$\psi_k \leq \min \left\{ \beta_k J_{k_{max}}^{(2)}, \frac{\sum_{i=1}^{\beta_k} z J_{k_{max}} N_{max} |\varphi_{k_i}|}{\|\varphi_k\|_2^2} \right\} \tag{30}$$

let's find conditions under which the inequality (28) is satisfied.

Case 1. $\left| -\theta_{k_i} - \sum_{j=1}^{z} J_{k_{ji}} \phi_j p_j\left(e_{k_j}\right) - \sigma_{k_i} \varphi_{k_i} \right| \leq B_{k_i}$

Considering (2) and (29), we have $\delta_k = 1$ and given $\psi_k \leq \beta_k J_{k_{max}}^{(2)}$ from (30), the following inequality is found as a sufficient condition for $\dot{V}(x_t) \leq 0$:

$$\sigma_{k_{min}} \geq 2T\phi_{max}\beta_k J_{k_{max}}^{(2)} \tag{31}$$

Case 2. $\left| -\theta_{k_i} - \sum_{j=1}^{z} J_{k_{ji}} \phi_j p_j\left(e_{k_j}\right) - \sigma_{k_i} \varphi_{k_i} \right| > B_{k_i}$

Because (see Appendix)

$$\varphi_{k_i}\left[s_{k_i}\left(-\theta_{k_i}-\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)-\sigma_{k_i}\varphi_{k_i}\right)-\left(-\theta_{k_i}-\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)\right)\right]\leq 0 \tag{32}$$

(29) *can be written as*

$$\delta_k=\frac{1}{\varphi_k^T\Sigma_k\varphi_k}\sum_{i=1}^{\beta_k}|\varphi_{k_i}|\left|s_{k_i}\left(-\theta_{k_i}-\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)-\sigma_{k_i}\varphi_{k_i}\right)-\left(-\theta_{k_i}-\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)\right)\right| \tag{33}$$

Considering (see Appendix)

$$\left|-\theta_{k_i}-\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)\right|<\Omega_{k_{max}}+z\phi_{max}J_{k_{max}}N_{max}+\max_i\left\{\left|\sum_{\alpha=1}^{\beta_k}\left(M_{k_{i\alpha}}\dot{\zeta}_{k_\alpha}+C_{k_{i\alpha}}\zeta_{k_\alpha}\right)\right|\right\} \tag{34}$$

and using the reverse triangle inequality, (2) *and* (34) *we get*

$$\begin{aligned}&\left|s_{k_i}\left(-\theta_{k_i}-\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)-\sigma_{k_i}\varphi_{k_i}\right)-\left(-\theta_{k_i}-\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)\right)\right|\\&\quad\geq\left|s_{k_i}\left(-\theta_{k_i}-\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)-\sigma_{k_i}\varphi_{k_i}\right)\right|-\left|-\theta_{k_i}-\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)\right|\\&\quad>B_{k_{min}}-\Omega_{k_{max}}-z\phi_{max}J_{k_{max}}N_{max}-\max_i\left\{\left|\sum_{\alpha=1}^{\beta_k}\left(M_{k_{i\alpha}}\dot{\zeta}_{k_\alpha}+C_{k_{i\alpha}}\zeta_{k_\alpha}\right)\right|\right\}\end{aligned} \tag{35}$$

Considering (33) *and using* (35)*:*

$$\delta_k>\sum_{i=1}^{\beta_k}\frac{|\varphi_{k_i}|\left(B_{k_{min}}-\Omega_{k_{max}}-z\phi_{max}J_{k_{max}}N_{max}-\max_i\left\{\left|\sum_{\alpha=1}^{\beta_k}\left(M_{k_{i\alpha}}\dot{\zeta}_{k_\alpha}+C_{k_{i\alpha}}\zeta_{k_\alpha}\right)\right|\right\}\right)}{\sigma_{k_{max}}\|\varphi_k\|_2^2} \tag{36}$$

Using (36) *and given* $\psi_k\leq\sum_{i=1}^{\beta_k}\frac{zJ_{k_{max}}N_{max}|\varphi_{k_i}|}{\|\varphi_k\|_2^2}$ *from* (30)*, it is possible to find that*

$$\delta_k\sigma_{k_{min}}\geq\frac{\sigma_{k_{min}}\psi_k\left(B_{k_{min}}-\Omega_{k_{max}}-z\phi_{max}J_{k_{max}}N_{max}-\max_i\left\{\left|\sum_{\alpha=1}^{\beta_k}\left(M_{k_{i\alpha}}\dot{\zeta}_{k_\alpha}+C_{k_{i\alpha}}\zeta_{k_\alpha}\right)\right|\right\}\right)}{z\sigma_{k_{max}}J_{k_{max}}N_{max}} \tag{37}$$

Therefore, the following inequality is found as a sufficient condition for (28) and $\dot{V}(x_t) \leq 0$:

$$\phi_{max} \leq \min_k \left\{ \frac{\omega_k \left(B_{k_{min}} - \Omega_{k_{max}} - \max_i \left\{ \left| \sum_{\alpha=1}^{\beta_k} \left(M_{k_{i\alpha}} \dot{\zeta}_{k_\alpha} + C_{k_{i\alpha}} \zeta_{k_\alpha} \right) \right| \right\} \right)}{z J_{k_{max}} N_{max} (\omega_k + 2T)} \right\} \tag{38}$$

In conclusion, if (31) and (38) are satisfied, we would have $\dot{V}(x_t) \leq 0$, which means all terms in $V(x_t)$ are bounded. Therefore, $\dot{X}_k, \varphi_k, e_k^0 \in L_\infty$. Noting that $e_l = e_l^0 + \int_{t-d_r(t)}^{t} \dot{X}_r(\tau) d\tau$ and $\int_{t-d_r(t)}^{t} \dot{X}_r(\tau) d\tau \in L_\infty$, we can conclude that $e_l \in L_\infty$. Similarly $e_r \in L_\infty$ and the proof of Theorem 1 has been completed. □

Remark 1. *Given controller (7) and the conditions of its parameters (31) and (38), if we lower ϕ_{max}, then it will leave less room for the task-space position difference between the robots to contribute to the control signal, resulting in an increase in the settling time for the end-effectors position tracking response which is not favorable for our control purposes. Therefore, it is better to assign ϕ_{max} near to its permissible upper bound (38). Furthermore, it is beneficial to choose $b_j = 1$; $j = \{1, \ldots, z\}$, which will give the maximum upper and lower bounds for the nonlinear function used in the proposed controller.*

Remark 2. *Considering the proposed controller (7) with Φ equal to the identity matrix of size z, then the conditions of Theorem 1 will change to*

(1) $\sigma_{k_{min}} \geq 2T \beta_k J_{k_{max}}^{(2)}$

(2) $b_{max} \leq \min \left\{ \min_k \left\{ \frac{2\omega_k \left(B_{k_{min}} - \Omega_{k_{max}} - \max_i \left\{ \left| \sum_{\alpha=1}^{\beta_k} \left(M_{k_{i\alpha}} \dot{\zeta}_{k_\alpha} + C_{k_{i\alpha}} \zeta_{k_\alpha} \right) \right| \right\} \right)}{z \pi J_{k_{max}} (\omega_k + 2T)} \right\}, 1 \right\}$

where $b_{max} \triangleq \max_j \{b_j\}$. It is possible to see a trade-off between the robustness to the maximum values of time delays and the task-space synchonization performance. Suppose that we lower the amplitude of the nonlinear function $P(e_k)$ within its permissible bound and therefore the position difference between the local and the remote robots' end-effectors will contribute less to the control signal, resulting in an increase in the settling time for the position tracking response. On the other hand, when b_{max} is lowered, the nonlinear proportional term in the control signal (7) is suppressed, leaving more room for the signals φ_k to contribute to the control signal, i.e., the gains Σ_k are allowed to be larger. The first stability condition above clearly indicates that increasing the Σ_k parameter will improve the robustness of the system stability to larger time delays. Therefore, for a fixed $B_{k_{min}}$, there is a trade-off between stability and performance of the system and this trade-off can be tuned by changing the parameter b_{max}.

Theorem 2. *For the closed-loop teleoperation system described by (14), in free motion ($\tau_{e_k} = 0$) the task-space position errors (4) asymptotically converge to the origin if all conditions in Theorem 1 are satisfied and the redundant remote manipulator is able to avoid the singularities.*

Proof. Integrating both sides of (27), it is possible to see that $\varphi_k \in L_2$. Based on the results of Theorem 1, $\varphi_k, \dot{X}_k, e_k \in L_\infty$. Also, considering Assumption 1 it can be concluded that $\dot{e}_k \in L_\infty$. Considering (5), $\varphi_k \in L_\infty$ and assuming that the redundant remote manipulator is able to avoid the singularities results in $\zeta_r \in L_\infty$ and so $\dot{q}_k \in L_\infty$ which in turn leads to $\dot{M}_k \in L_\infty$. Considering (1), since the term G_k is bounded, using Properties 1 and 5 of the system dynamics and given the boundedness of $S_k(\tau_k)$, it is possibe to see that $\ddot{q}_k \in L_\infty$. Also, $\dot{q}_k, \ddot{q}_k \in L_\infty$ results in $\dot{\Psi}_r, \ddot{\Psi}_r \in L_\infty$. Furthermore, $\dot{q}_k, \ddot{q}_k, \dot{\Psi}_r, \ddot{\Psi}_r \in L_\infty$ and given the assumption that the redundant remote manipulator is able to avoid the singularities, $\dot{\zeta}_k, \ddot{\zeta}_k \in L_\infty$. Considering $\ddot{q}_k, \dot{\zeta}_k \in L_\infty$, it can be concluded that $\dot{\varphi}_k \in L_\infty$. Because $\varphi_k \in L_2$ and $\dot{\varphi}_k \in L_\infty$, using the Barbalat's lemma results in $\varphi_k \to 0$. Considering (5), $\dot{q}_l \to 0$, and using the fact that $\dot{X}_k = J_k \varphi_k$, we get $\dot{X}_k \to 0$. If the redundant remote manipulator is able to avoid the singularities, then it can be obtained that $\dot{q}_r \to 0$. Applying $\tau_{e_k}=0$ into (1) yields $\ddot{q}_k = M_k^{-1}\big(-C_k\dot{q}_k - G_k + S_k\big(-\Theta_k - J_k^T \Phi P(e_k) - \Sigma_k \varphi_k\big)\big)$. Differentiating both sides with respect to time and given $\frac{d}{dt}\big(M_k^{-1}\big) = -M_k^{-1}\big(C_k + C_k^T\big)M_k$, we get

$$
\begin{aligned}
\dddot{q}_k =& \frac{d}{dt}\big(M_k^{-1}\big)\big(-C_k\dot{q}_k - G_k + S_k(-\Theta_k - J_k^T\Phi P(e_k) - \Sigma_k\varphi_k)\big) \\
&+ M_k^{-1}\frac{d}{dt}\big(-C_k\dot{q}_k - G_k + S_k\big(-\Theta_k - J_k^T\Phi P(e_k) - \Sigma_k\varphi_k\big)\big) \\
=& \frac{d}{dt}\big(M_k^{-1}\big)\big(-C_k\dot{q}_k - G_k + S_k\big(-\Theta_k - J_k^T\Phi P(e_k) - \Sigma_k\varphi_k\big)\big) \\
&+ M_k^{-1}\frac{d}{dt}(-C_k\dot{q}_k - G_k) - M_k^{-1}\dot{\Theta}_k\dot{S}_k\big(-\Theta_k - J_k^T\Phi P(e_k) - \Sigma_k\varphi_k\big) \\
&- M_k^{-1}\frac{d}{dt}\big(J_k^T\Phi P(e_k)\big)\dot{S}_k\big(-\Theta_k - J_k^T\Phi P(e_k) - \Sigma_k\varphi_k\big) \\
&- M_k^{-1}\Sigma_k\dot{\varphi}_k\dot{S}_k\big(-\Theta_k - J_k^T\Phi P(e_k) - \Sigma_k\varphi_k\big)
\end{aligned}
\tag{39}
$$

Based on Properties 1 and 5, and given $\dot{q}_k \in L_\infty$, $\frac{d}{dt}\big(M_k^{-1}\big)$ is bounded. Also, given

$$
\frac{d}{dt}\big(J_k^T\Phi P(e_k)\big) = \left(\frac{\partial}{\partial q_k}J_k^T\right)\dot{q}_k\Phi P(e_k) + J_k^T\Phi\left(\frac{\partial}{\partial e_k}P(e_k)\right)\dot{e}_k \tag{40}
$$

it can be concluded that $\frac{d}{dt}\big(J_k^T\Phi P(e_k)\big) \in L_\infty$. Now let's study (39) in two cases as follows:

Case 1. $\left|-\theta_{k_i} - \sum_{j=1}^{z} J_{k_{ji}}\phi_j p_j\big(e_{k_j}\big) - \sigma_{k_i}\varphi_{k_i}\right| \le B_{k_i}$

Considering (2), it is possible to see that $\dot{s}_{k_i}\left(-\theta_{k_i} - \sum_{j=1}^{z} J_{k_{ji}}\phi_j p_j\big(e_{k_j}\big) - \sigma_{k_i}\varphi_{k_i}\right) = 1$ *and given the facts that* $M_k^{-1}\dot{\Theta}_k = -\ddot{\zeta}_k - M_k^{-1}\dot{M}_k\dot{\zeta}_k - M_k^{-1}\frac{d}{dt}(C_k\zeta_k + G_k)$, $\ddot{\zeta}_k \in L_\infty$ *and* $\ddot{\varphi}_k = \dddot{q}_k - \ddot{\zeta}_k$, *and considering Properties 1, 3, 4 and 5, it is straightforward to get* $\ddot{\varphi}_k \in L_\infty$.

Case 2. $\left|-\theta_{k_i} - \sum_{j=1}^{z} J_{k_{ji}}\phi_j p_j\big(e_{k_j}\big) - \sigma_{k_i}\varphi_{k_i}\right| > B_{k_i}$

Considering (2), it is easy to see that $\dot{s}_{k_i}\left(-\theta_{k_i} - \sum_{j=1}^{z} J_{k_{ji}}\phi_j p_j\big(e_{k_j}\big) - \sigma_{k_i}\varphi_{k_i}\right) = 0$ *and from (39) it can be readily concluded that* $\dddot{q}_k \in L_\infty$. *Given* $\dddot{q}_k, \ddot{\zeta}_k \in L_\infty$, *it results in*

$\ddot{\varphi}_k \in L_\infty$.

Therefore, having $\ddot{\varphi}_k \in L_\infty$ and given that $\varphi_k \to 0$, using the Barbalat's lemma we get $\dot{\varphi}_k \to 0$. Considering the closed-loop dynamics (14), noting (A.14) (see Appendix) and having shown that $\varphi_k, \dot{\varphi}_k \to 0$, we get

$$\begin{aligned} J_l^T \Phi P(e_l) &\to 0 \\ J_r^T \Phi P(e_r) &\to 0 \end{aligned} \tag{41}$$

Given $\dot{q}_k \to 0$ and $\dddot{q}_k \in L_\infty$, using the Barbalat's lemma $\ddot{q}_k \to 0$. Considering (41) and noting that in steady state ($\dot{q}_k = \ddot{q}_k = 0$), $P(e_l) = P\left(e_l^0\right) = -P\left(e_r^0\right)$, reforming (41) in matrix form yields

$$\begin{bmatrix} J_l^T \\ J_r^T \end{bmatrix} \Phi P(e_r^0) \to 0 \tag{42}$$

Therefore, if the redundant remote manipulator is able to avoid the singularities, then J_l^T and J_r^T are entirely independent of each other and it can be concluded that $P(e_r^0) \to 0$. Noting that $P(e_r^0)$ passes through the origin, we get $e_r^0 \to 0$. Similarly $e_l^0 \to 0$ and the proof has been completed. □

Remark 3. *Suppose that a human exerts a bounded, continuous and constant force on the end-effector of the local robot and there is no contact force between the remote robot's end-effector and the environment. It can still be concluded that $\varphi_k, \dot{\varphi}_k \to 0$. Therefore,*

$$\begin{bmatrix} J_l^T \\ J_r^T \end{bmatrix} \Phi P(e_r^0) \to \begin{bmatrix} J_l^T F_{e_l} \\ 0 \end{bmatrix} \tag{43}$$

Now if the redundant remote manipulator is able to avoid the singularities, then the task-space position errors (4) asymptotically converge to the origin and $P(e_r^0) \to 0$ will come at the cost of F_{e_l} being in the null space of J_l^T matrix. In other words, local robot will approach singularity.

Remark 4. *If there is a passive contact force (F_{e_r}) between the remote robot's end-effector and the environment, the conclusions $\varphi_k, \dot{\varphi}_k \to 0$ are still valid and we will have bounded task-space position errors such that*

$$\begin{bmatrix} J_l^T \\ J_r^T \end{bmatrix} \Phi P(e_r^0) \to \begin{bmatrix} J_l^T F_{e_l} \\ J_r^T F_{e_r} \end{bmatrix} \tag{44}$$

By the assumption that the remote robot is able to avoid the singularities, the reflected force from the environment on the end-effector of the remote robot would converge to $\Phi P(e_r^0)$ (i.e., $F_{e_r} \to \Phi P(e_r^0)$). To study the tracking between the operator and environment applied forces, two possibilities are considered:

possibility 1. *Local robot is not in its singularity.*

Assume that $F_{e_{l_x}} \le \phi_{max} N_{max}$ and $F_{e_{l_y}} \le \phi_{max} N_{max}$, where $F_{e_{l_x}}$ and $F_{e_{l_y}}$ denote the human's applied forces on the local robot's end-effector in X and Y directions, respectively. Considering (44), $J_l^T (F_{e_l} - F_{e_r}) = J_l^T \begin{bmatrix} \Delta F_{e_x} \\ \Delta F_{e_y} \end{bmatrix} \to 0$, in which $\Delta F_{e_x} \triangleq F_{e_{l_x}} - F_{e_{r_x}}$

and $\Delta F_{e_y} \triangleq F_{e_{l_y}} - F_{e_{r_y}}$. *Therefore, there will be force tracking betweeen applied forces. In other words,* $F_{e_{r_x}} \to F_{e_{l_x}}$ *and* $F_{e_{r_y}} \to F_{e_{l_y}}$. *Note that, for instance,* $F_{e_{r_x}}$ *denotes the exerted force on the remote robot's end-effector in X-direction.*

possibility 2. *Local robot is in its singularity.*

Considering (44), $J_l^T(F_{e_l} - F_{e_r}) = J_l^T \begin{bmatrix} \Delta F_{e_x} \\ \Delta F_{e_y} \end{bmatrix} \to 0$ *and vector* $\begin{bmatrix} \Delta F_{e_x} & \Delta F_{e_y} \end{bmatrix}^T$ *will be in the null sapce of matrix* J_l^T. *For simplicity, consider a 2-DOF local revolute-joint planar robot. It is possible to obtain the null space of the matrix* J_l^T *as* $\cos(q_{l_1})\Delta F_{e_y} = \sin(q_{l_1})\Delta F_{e_x}$. *If* $F_{e_{l_y}} = F_{e_{r_y}} = 0$, *it is straightforward to have* $\sin(q_{l_1})\Delta F_{e_x} = 0$ *where for* $q_{l_1} \neq 0, \pi, \ldots$, $\Delta F_{e_x} \to 0$ *(if* $F_{e_{l_x}} \leq \phi_{max} N_{max}$*) or for* $\Delta F_{e_x} \neq 0$, $q_{l_1} \to 0, \pi, \ldots$. *Also, in case* $F_{e_{l_x}} = F_{e_{r_x}} = 0$, *it is easy to see that* $\cos(q_{l_1})\Delta F_{e_y} = 0$ *where for* $q_{l_1} \neq \pi/2, 3\pi/2, \ldots$, $\Delta F_{e_y} \to 0$ *(if* $F_{e_{l_y}} \leq \phi_{max} N_{max}$*) or for* $\Delta F_{e_y} \neq 0$, $q_{l_1} \to \pi/2, 3\pi/2, \ldots$.

As the human stops applying force on the end-effector of the local robot, the remote robot moves away to vansih the reflected force from the environment and the task-space position errors asymptotically converge to the origin.

5. Sub-Task Control

Redundant manipulators pave the way for achieving sub-task control and provide flexibility and increased manipulability (Nakamura, 1990; Nath, Tatlicioglu, et al., 2009; Yoshikawa, 1984) to execute complex and awkward tasks. Most methods for resolving redundancy in manipulation involve defining a cost function such as manipulability (Yoshikawa, 1984) that has a minimum or maximum value at a desireable configuration. Using the gradient (or its negative) of this function to control the joint velocity in the redundant directions, the manipulator will tend to seek the optimal configuration. The gradiant projection method (Siciliano, 1990) is utilized in this paper with the proposed teleoperation framework in order to achieve the self-motion (semi-autonomous behavior (Liu & Chopra, 2013)) of the remote robot.

The remote robot is assumed to be redundant and the null space of J_r is assumed to have at least $m - z$ dimension. Therefore, if the joint velocity of the remote robot $\dot{q}_r$ is in the null sapce of J_r, then it does not contribute to the task-space velocity $\dot{X}_r$. The manipulator is thus free to move in this configuration-dependent subspace. This type of motion is called self-motion since it is not observed at the end-effector (Hsu, Mauser, & Sastry, 1989). Hence, we can exploit this property to achieve a desired sub-task control by assigning an appropriate Ψ_r vector for the remote robot. Premultiplying φ_r by $[\mathbb{I}_m - J_r^+ J_r]$ and using the property that $[\mathbb{I}_m - J_r^+ J_r][I_m - J_r^+ J_r] = \mathbb{I}_m - J_r^+ J_r$, the relation between the sub-task tracking error (Hsu et al., 1989) and φ_r is obtained as (Liu & Chopra, 2013)

$$\text{sub-task tracking error} \triangleq \left[\mathbb{I}_m - J_r^+ J_r\right] \varphi_r = \left[\mathbb{I}_m - J_r^+ J_r\right] (\dot{q}_r - \Psi_r) \tag{45}$$

Therefore, if $\varphi_r \to 0$ (Theorem 2), the sub-task tracking error approaches the origin and the function $[\mathbb{I}_m - J_r^+ J_r]\Psi_r$ can be considered as the desired velocity in the null space of J_r. The sub-task of the remote robot can be controlled by any differentiable auxiliary function Ψ_r which is expressed in terms of the joint angles or the end-effector position. Hence, we can define a differentiable function $f(q_r): \mathbb{R}^{m \times 1} \to \mathbb{R}$ for which a lower value corresponds to more desirable configurations. Then, the auxiliary function

$\Psi_r = -\frac{\partial}{\partial q_r} f(q_r)$ is utilized for achieving the sub-task control of the remote robot.

Remark 5. *Considering (38), we can draw the conclusion that the term* $\max_i \left\{ \left| \sum_{\alpha=1}^{\beta_k} \left(M_{k_{i\alpha}} \dot{\zeta}_{k_\alpha} + C_{k_{i\alpha}} \zeta_{k_\alpha} \right) \right| \right\}$ *serves as a dissipation parameter, which means when* $\varphi_k, \dot{\varphi}_k \to 0$ *the term will converge to zero. Therefore, let's define* $\phi_{k_{diss}} \triangleq \frac{\omega_k \max_i \left\{ \left| \sum_{\alpha=1}^{\beta_k} \left(M_{k_{i\alpha}} \dot{\zeta}_{k_\alpha} + C_{k_{i\alpha}} \zeta_{k_\alpha} \right) \right| \right\}}{z J_{k_{max}} N_{max} (\omega_k + 2T)}$*, in which the subscript diss denotes dissipation. Also, let's define* $\phi_{k_{fixed}} \triangleq \frac{\omega_k \left(B_{k_{min}} - \Omega_{k_{max}} \right)}{z J_{k_{max}} N_{max} (\omega_k + 2T)}$*, in which the subscript fixed means the parameter* $\phi_{k_{fixed}}$ *is fixed throughout the control process. In other words, we have* $\phi_{max} < \min_k \{ \phi_{k_{fixed}} - \phi_{k_{diss}} \}$.

6. Simulation Results

In this section, the simulation results are presented to verify the theoretical findings. The local and remote manipulators are considered to be 2-DOF and 3-DOF revolute-joint planar robots, respectively. We choose the parameters $g = 9.81 m/s^2$, $m_{1_l} = m_{2_l} = m_{1_r} = m_{2_r} = m_{3_r} = 0.3kg$, $L_{1_l} = L_{2_l} = L_{1_r} = L_{2_r} = L_{3_r} = 0.38m$, $\Omega_{l_{max}} = 3.35$ and $\Omega_{r_{max}} = 6.71$. It is assumed that the control signals are subjected to actuator saturation at levels $+20N.m$ and $-20N.m$ (i.e., $B_{k_{min}} = 20N.m$). The forward and backward time delays are considered to be identical and equal to $d_r(t) = d_l(t) = 0.1 + 0.1\sin(t)$ (i.e., $T = 0.4s$). The nonlinear function $p_j(e_{k_j}) = \tan^{-1}(e_{k_j})$ has been chosen (i.e., $N_j = N_{max} = \frac{\pi}{2}$) to be used in the controllers. Furthermore, $\min_k \{ \phi_{k_{fixed}} \} = 2.06$ and the gains $\phi_1 = \phi_2 = \phi_{max}$ are set. Also, $\sigma_{l_1} \geq 0.92\phi_{max}$, $\sigma_{l_2} \geq 0.92\phi_{max}$, $\sigma_{r_1} \geq 3.11\phi_{max}$, $\sigma_{r_2} \geq 3.11\phi_{max}$ and $\sigma_{r_3} \geq 3.11\phi_{max}$ are considered. So, the controllers become as listed below:

$$\tau_l = G_l - J_l^T \begin{bmatrix} \phi_{max} & 0 \\ 0 & \phi_{max} \end{bmatrix} \tan^{-1}(e_l) - \begin{bmatrix} 0.93\phi_{max} & 0 \\ 0 & 0.93\phi_{max} \end{bmatrix} \dot{q}_l \tag{46}$$

$$\tau_r = -\Theta_r - J_r^T \begin{bmatrix} \phi_{max} & 0 \\ 0 & \phi_{max} \end{bmatrix} \tan^{-1}(e_r) - \begin{bmatrix} 3.12\phi_{max} & 0 & 0 \\ 0 & 3.12\phi_{max} & 0 \\ 0 & 0 & 3.12\phi_{max} \end{bmatrix} \varphi_r \tag{47}$$

Initial conditions $q_l(0) = \left[\frac{2\pi}{3}, \frac{\pi}{2}\right]^T$ and $q_r(0) = \left[\frac{\pi}{6}, \frac{\pi}{6}, \frac{-\pi}{6}\right]^T$ are chosen for the local and remote robots, respectively. Also, it is assumed that $\dot{q}_k(0) = \ddot{q}_k(0) = 0$.

Singularity avoidance is an important objective in choosing the function Ψ_r for the control law. As mentioned earlier, a common method is to define a function, $f(q_r)$, for which a lower value is associated with a more desirable configuration. Motivated by Hsu et al. (1989); Zergeroglu et al. (2004), in simulations $\Psi_r \triangleq -0.01(q_{1_r} - 2q_{2_r} + q_{3_r})[1, -2, 1]^T$ is chosen for singularity avoidance. Note that the selected Ψ_r is the negative of the gradiant of the cost function $f(q_r) = 0.005(q_{3_r} - 2q_{2_r} + q_{1_r})^2$ and the control law will try to minimize the cost function. To show the effectiveness of the proposed controller, simulations are conducted in three sections: **Scenario 1**, **Scenario 2** and **Scenario 3**.

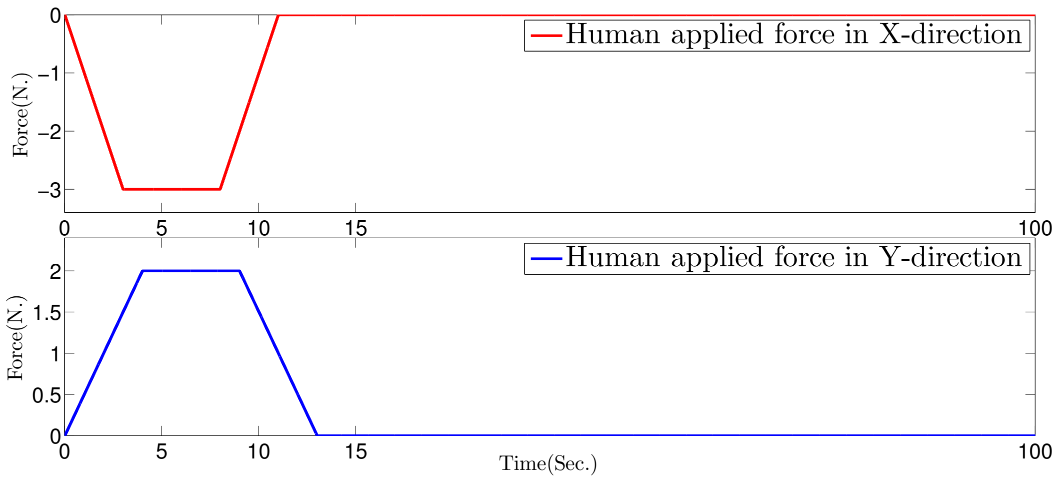

Figure 1. The operator's applied force in Scenario 1.

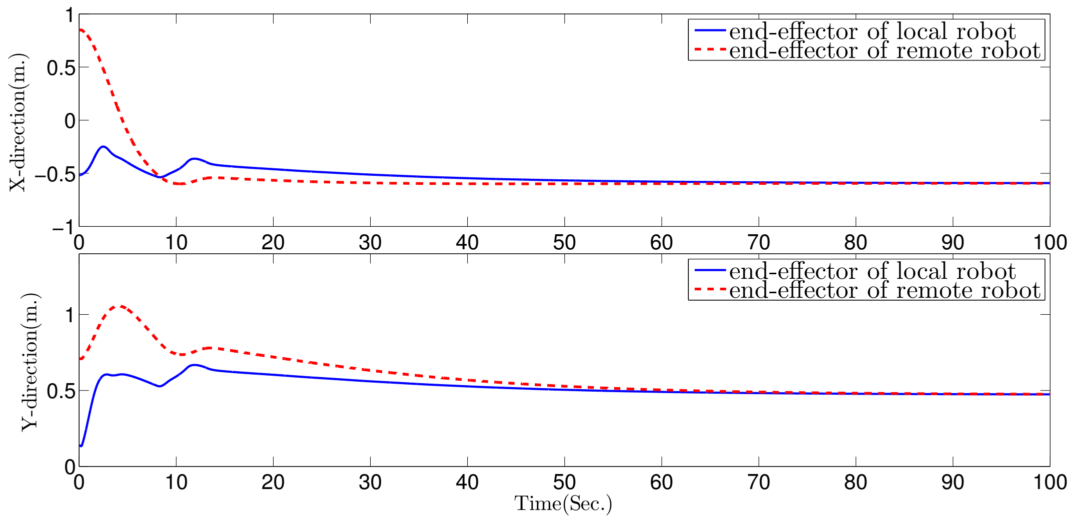

Figure 2. The positions of end-effectors for Scenario 1.

6.1. Scenario 1

Suppose that the operator applies force on the end-effector of the local robot in X and Y-directions as it is shown in Figure 1. Also, suppose that in the remote site, we do not have any obstacle. X and Y-direction positions of the two robots' end-effectors are shown in Figure 2. Also, traversed trajectories of the end-effectors are shown in Figure 3. It is clear from Figures 2 and 3 that with the convergence of the applied force to zero, the end-effectors of the two robots converge to the same position. Noting (1), the input torques are comprised of the terms τ_{e_k} and $S_k(\tau_k)$. Each of which is outweighed, determines the direction of the corresponding robot's movement. For instance, the local robot inclines initially toward the positive X-direction (see Figure 3), even though the operator's applied force in the X-direction is negative. This means that in the mentioned course of time, the term $S_k(\tau_k)$ has outweighed the term τ_{e_k}, making the local robot to move in positive X-direction. In Figure 4, the sub-task tracking error (45) is shown for the three joints of the redundant remote manipulator.

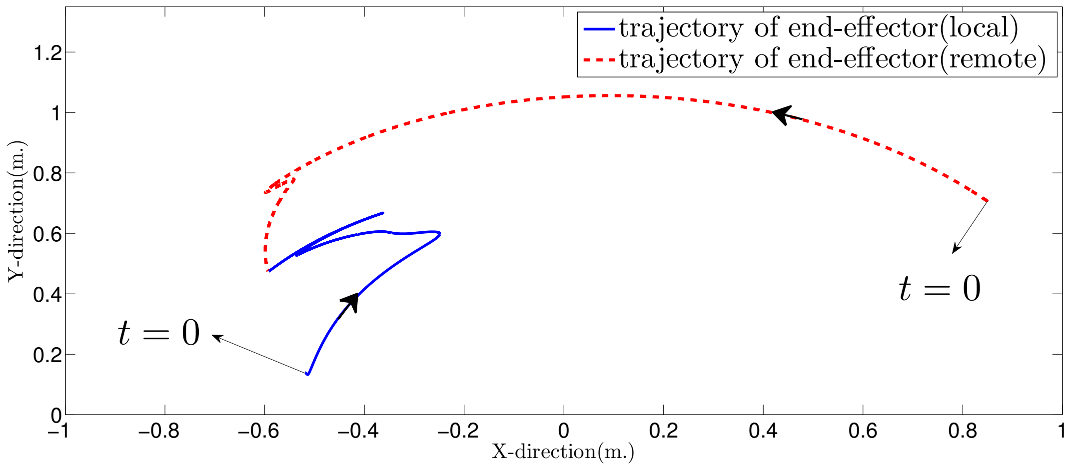

Figure 3. The traversed trajectories of the end-effectors during the control process of Scenario 1.

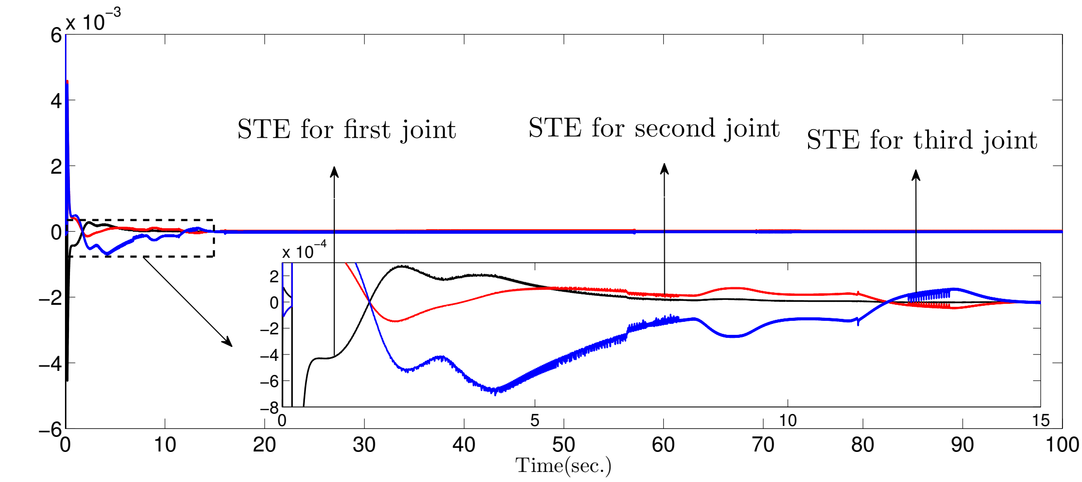

Figure 4. Sub-task Tracking Error (STE) of redundant manipulator for Scenario 1.

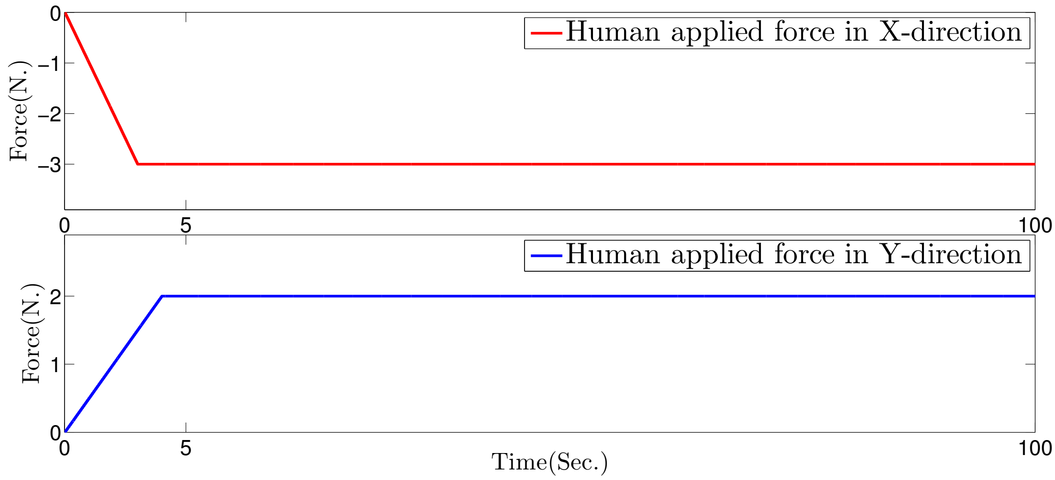

Figure 5. The operator's applied force in Scenario 2.

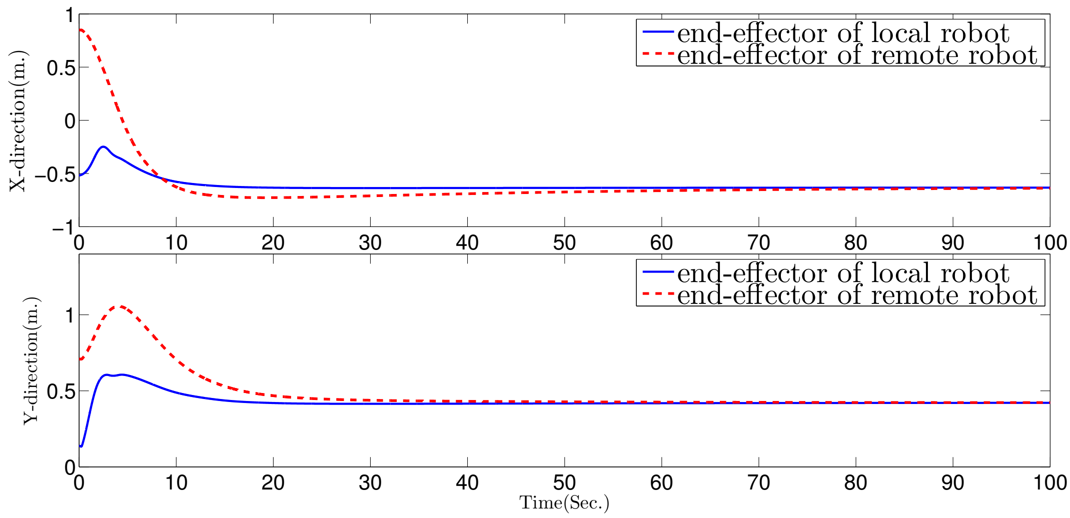

Figure 6. The positions of end-effectors for Scenario 2.

Note that STE is the acronym for Sub-task Tracking Error. In the following, **Scenario 2** and **Scenario 3** are intended to provide insights in what was explained in Remarks 3 and 4.

6.2. Scenario 2

Suppose that the operator applies force on the end-effector of the local robot in X and Y-directions as shown in Figure 5. Also, suppose that in the remote site we do not have any obstacle. X and Y-direction positions of the two robots' end-effectors are shown in Figure 6. Also, traversed trajectories of the end-effectors are shown in Figure 7. It is clear from Figures 6 and 7 that for constant and bounded applied forces, the end-effectors of the two robots converge to the same position. As mentioned in Remark 3, converging the end-effectors' position errors to zero, despite exerting constant force on the local robot's end-effector, come at the cost of approaching the configuration of

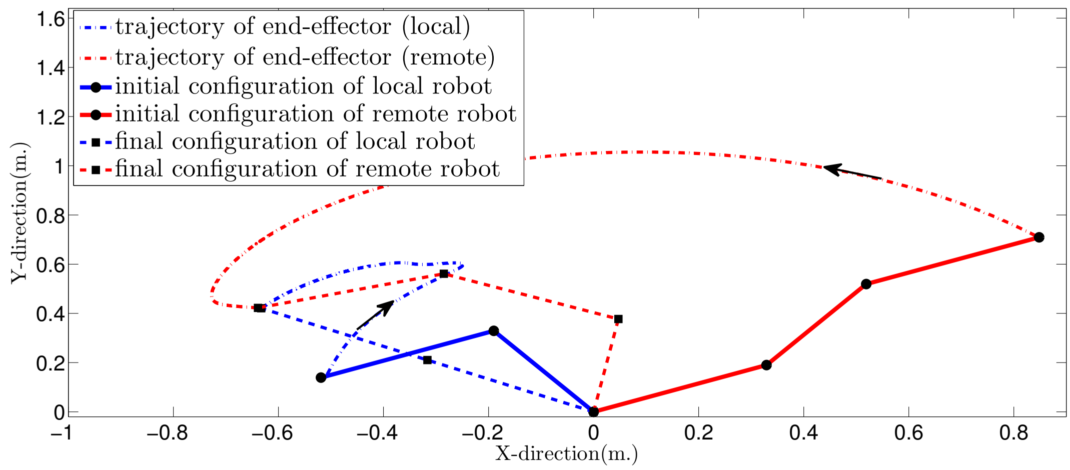

Figure 7. The traversed trajectories of the end-effectors during the control process of Scenario 2.

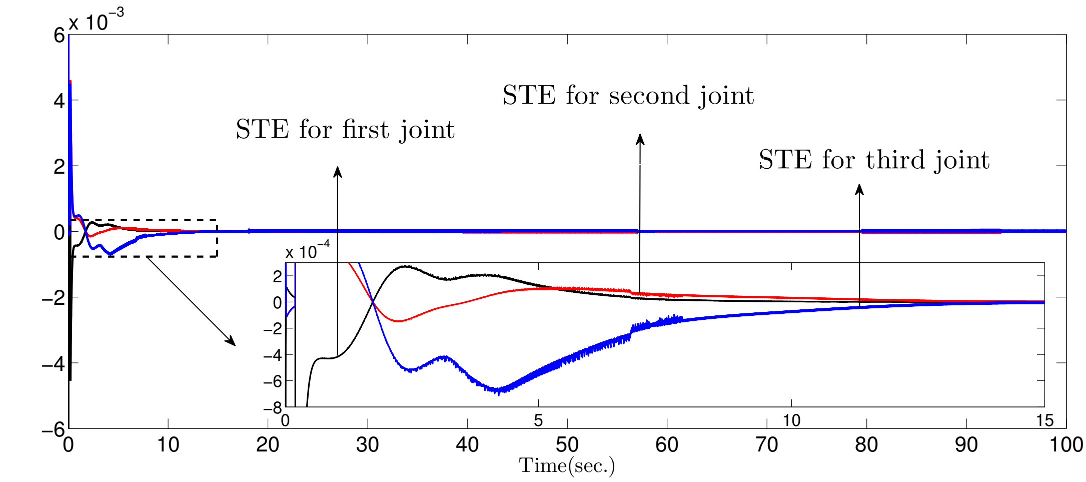

Figure 8. Sub-task Tracking Error (STE) of redundant manipulator for Scenario 2.

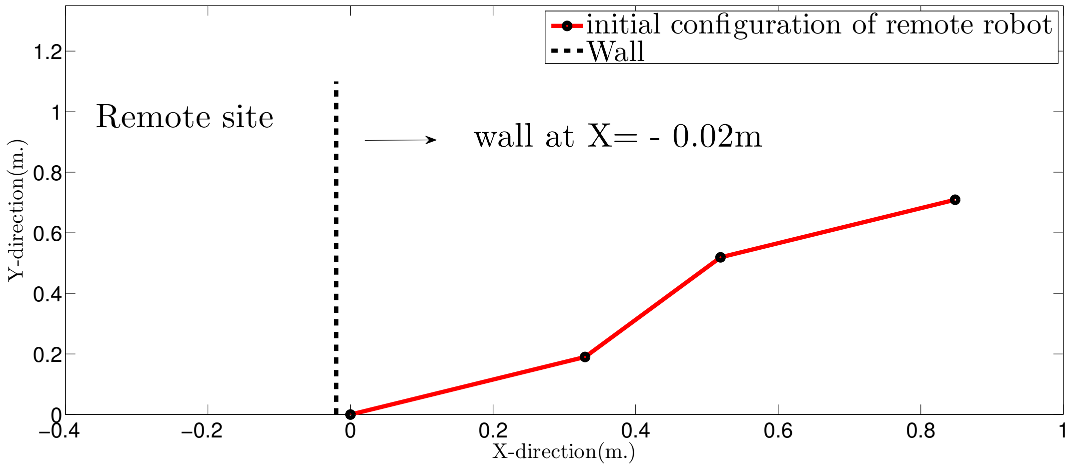

Figure 9. The position of wall in remote site for Scenario 3.

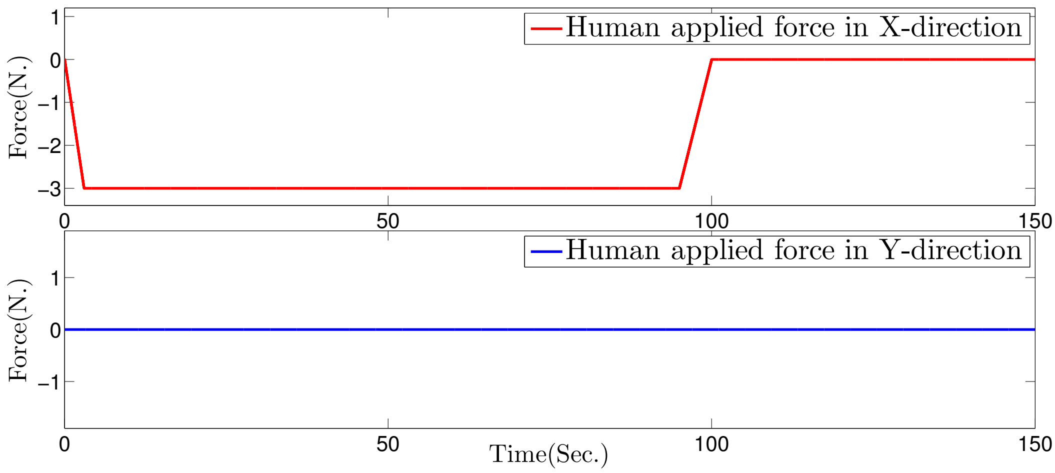

Figure 10. The operator's applied force in Scenario 3.

the local robot to a singular position. Figure 8 indicates that sub-task tracking errors of three joints of the redundant remote manipulator converge to zero.

6.3. Scenario 3

In this section, we study the contact motion of the robot in remote site where there is a stiff wall at $X{=}-0.02m$ (see Figure 9). Suppose that the operator applied force on the end-effector of the local robot is as shown in Figure 10. For the sake of simplicity, it is assumed that the operator applies its force only in the X-direction. It is also assumed that the wall behaves like a stiff spring with a stiffness of 10,000 N/m. Therefore, when the end-effector of the remote robot reaches the wall and tries to move further, the feedback force in the X-direction will be $F_{e_x}{=}10000(X{+}0.02)N$ and in the Y-direction will be $F_{e_y}{=}0$. Consequently, this reflected force inhibits the advance of the remote robot's end-effector through an equivalent torque of $J_r^T[F_{e_x}0]^T$ on the joints of the remote robot. As explained in Remark 4, we see in Figure 11 that after collision, the

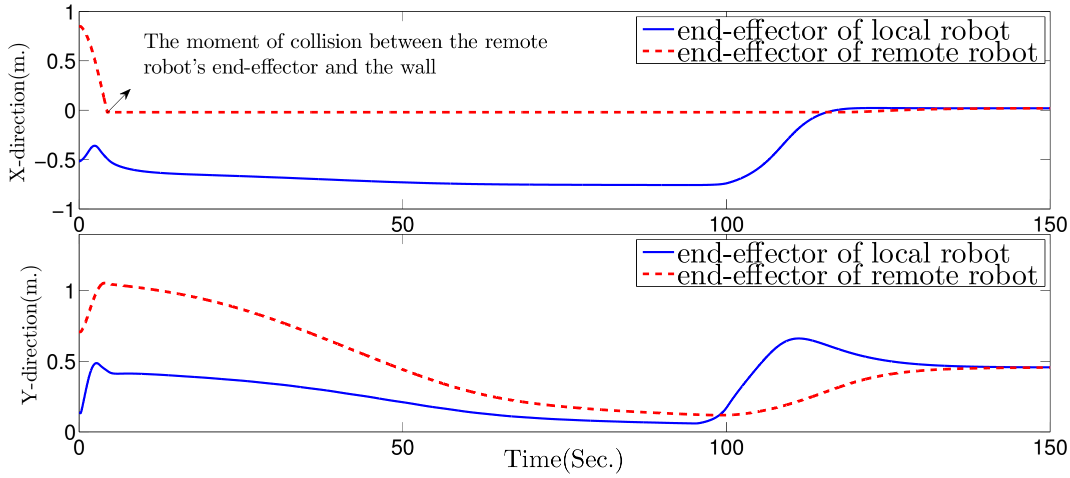

Figure 11. The positions of end-effectors for Scenario 3.

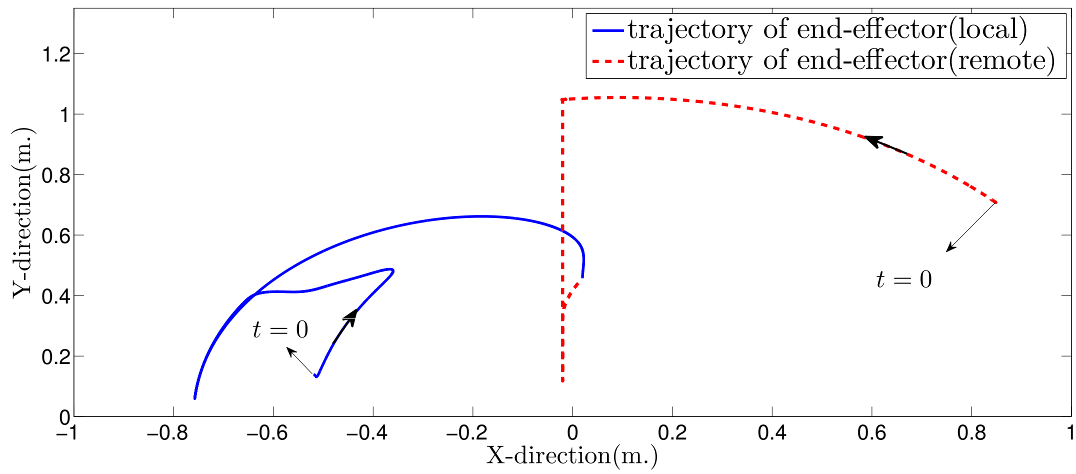

Figure 12. The traversed trajectories of the end-effectors during the control process of Scenario 3.

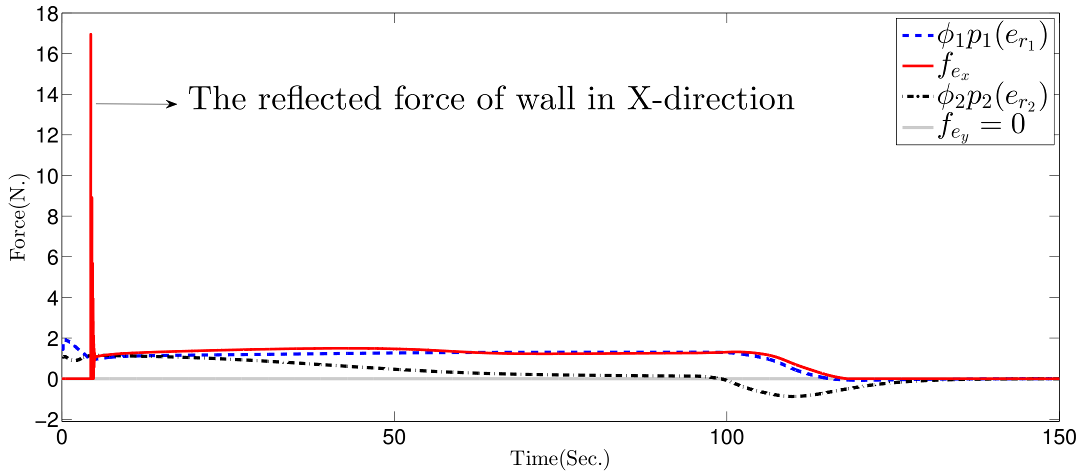

Figure 13. The reflection force of wall in contact motion.

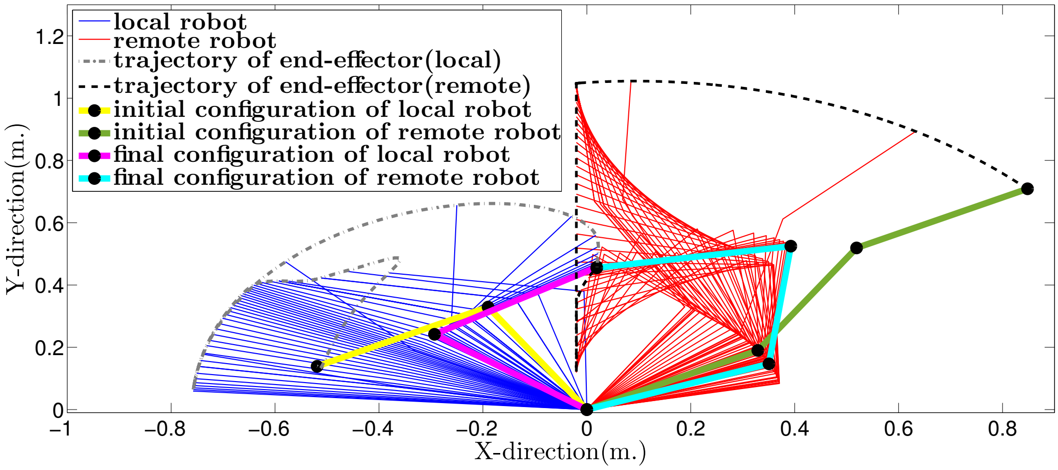

Figure 14. Evolution of links during control process of contact motion Scenario.

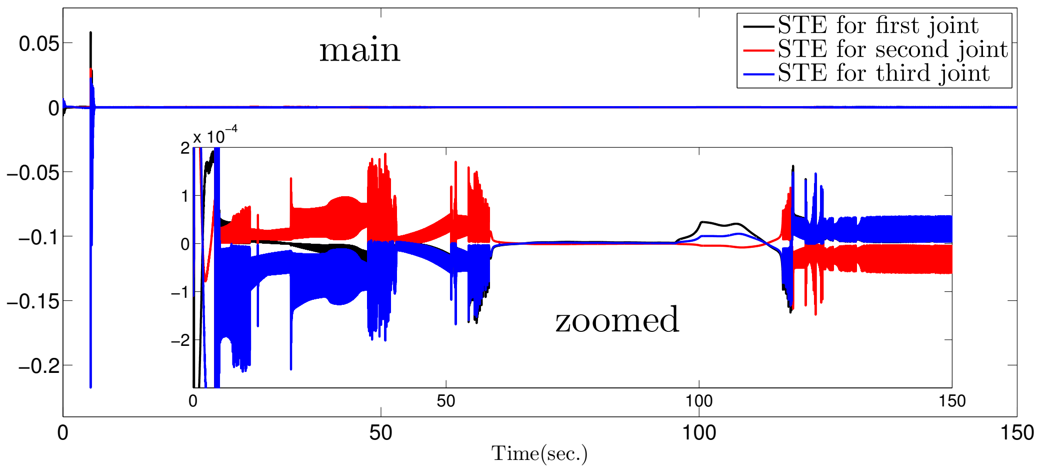

Figure 15. Sub-task Tracking Error (STE) of redundant manipulator for Scenario 3.

period of time during which the force is applied and before its reduction into zero, we have a bounded error between the X-direction positions of two robots' end-effectors and a convergence between the wall reflected force (f_{e_x}) and $\phi_1 p_1(e_{r_1})$ (see Figure 13). Because of no applied force in the Y-direction, the error between the Y-direction positions of the end-effectors tend to converge to zero. As the applied force starts to diminish to zero ($t{\simeq}95$), the reflected force of the wall also starts to dwindle into zero. This, in turn, makes the end-effectors adjust themselves to be coordinated in X and Y-directions (see Figure 11). It is worth noting that, since the wall prevents the advance of the remote robot's end-effector, the local robot tries to coordinate itslef with the remote robot's end-effector (see Figure 11 after reduction of applied force). To shed more light on the issue, the traversed trajectories of the end-effectors and the evolution of the local and remote robots' links are shown in Figures 12 and 14, respectively. Finally, Figure 15 demonstrates how sub-task tracking errors of three joints of the redundant remote manipulator converge to zero. Note that **Scenario 3** is a synthesis of **Scenario 1** and **Scenario 2**, and final oscillations in the zoomed part of Figure 15 ($t{\simeq}150$) will converge asymptotically to zero for $t{>}150$ *sec*. For the sake of clarity in figures, final simulation time for **Scenario 3** has been set to 150 *sec*.

The contributions of the proposed scheme was discussed in the Introduction section. Candidly speaking, the proposed controller has its own downsides. In contact motion, taken the nonsingularity of the local robot as the preliminary given, the tracking of the applied force by the reflected force from environment is contingent on the assumptions $F_{e_{l_x}}{\leq}\phi_{max}N_{max}$ and $F_{e_{l_y}}{\leq}\phi_{max}N_{max}$. Otherwise, it tracks a proportion of error signal which passes through the nonlinear function used in the controller. Also, the proposed controller is developed only for $\Psi_r{\neq}0$. However, covering the case $\Psi_r{=}0$ is not of crucial concern in stability analysis and task-space synchronization of teleoperation systems. It is worth noting that if we assume that the local robot avoids singularity, then the proposed controller can be developed for both $\Psi_r{\neq}0$ and $\Psi_r{=}0$. Please note that, in addition to the simulation results, downloadable **simulation videos** illustrating the aforementioned scenarios accompany this paper[1].

7. Conclusion

In this paper, a novel (nP+D)-like controller that incorporates gravity compensation and sub-task-oriented term was proposed to ensure the end-effectors position convergence while satisfying a subtask control such as singularity avoidance in the presence of the nonlinear dynamics for telemanipulators with bounded time-varying delays in the communication channels and saturation in the actuators. It was shown that in free motion and when the operator applies a bounded force, the proposed controller not only guarantees the position convergance of the end-effectors but also guarantees the accomplishment of the subtask. The asymptotic stability of closed-loop dynamics is studied using a Lyapunov-Krasovskii functional under conditions on the controller parameters and the maximum values of time-varying delays. The efficiency of the proposed teleoperation system and the control algorithm is shown using numerical simulations with a 2-DOF planar local robot and a 3-DOF planar redundant remote robot.

[1] https://www.dropbox.com/sh/j9br3yt81a0s1dm/AAAqbnqO08wBhsdAVlBtpNP-a?dl=0

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8. Appendix

Proof of lemma 1. *Considering the property that* $\dot{X}_k{=}J_k\varphi_k$*, we have*

$$\begin{aligned}\|\dot{X}_k\|_2^2&=\dot{X}_k^T\dot{X}_k=\varphi_k^TJ_k^TJ_k\varphi_k=\varphi_k^TJ_k^{(2)}\varphi_k=\sum_{\alpha=1}^{\beta_k}\sum_{i=1}^{\beta_k}J_{k_{i\alpha}}^{(2)}\varphi_{k_\alpha}\varphi_{k_i}\\&=\sum_{\alpha=1}^{\beta_k}J_{k_{\alpha\alpha}}^{(2)}\varphi_{k_\alpha}^2+\sum_{\alpha=1}^{\beta_k-1}\sum_{i=\alpha+1}^{\beta_k}\left(J_{k_{\alpha i}}^{(2)}+J_{k_{i\alpha}}^{(2)}\right)\varphi_{k_\alpha}\varphi_{k_i}\end{aligned}\tag{A.1}$$

Because $J_{k_{\alpha\alpha}}^{(2)}\le J_{k_{max}}^{(2)}$*,* $J_{k_{\alpha i}}^{(2)}+J_{k_{i\alpha}}^{(2)}\le 2J_{k_{max}}^{(2)}$ *and* $\varphi_{k_\alpha}\varphi_{k_i}\le\frac{1}{2}\left(\varphi_{k_\alpha}^2+\varphi_{k_i}^2\right)$*, (A.1) leads to*

$$\begin{aligned}&\le\sum_{\alpha=1}^{\beta_k}J_{k_{max}}^{(2)}\varphi_{k_\alpha}^2+\sum_{\alpha=1}^{\beta_k-1}(\beta_k-\alpha)J_{k_{max}}^{(2)}\varphi_{k_\alpha}^2+\sum_{\alpha=1}^{\beta_k-1}\sum_{i=\alpha+1}^{\beta_k}J_{k_{max}}^{(2)}\varphi_{k_i}^2\\&=\sum_{\alpha=1}^{\beta_k}J_{k_{max}}^{(2)}\varphi_{k_\alpha}^2+\sum_{\alpha=1}^{\beta_k}(\beta_k-\alpha)J_{k_{max}}^{(2)}\varphi_{k_\alpha}^2+\sum_{\alpha=1}^{\beta_k}(\alpha-1)J_{k_{max}}^{(2)}\varphi_{k_\alpha}^2\\&=\sum_{\alpha=1}^{\beta_k}\beta_kJ_{k_{max}}^{(2)}\varphi_{k_\alpha}^2=\beta_kJ_{k_{max}}^{(2)}\|\varphi_k\|_2^2\end{aligned}\tag{A.2}$$

Therefore, the proof of Lemma 1 has been completed.

Proof of lemma 2. *Equation (10) can be written as follows:*

$$\begin{aligned}\dot{X}_l^T\Phi\left(P(e_l^0)-P(e_l)\right)&=\dot{X}_l^T\Phi[P(X_l-X_r)-P(X_l-X_r(t-d_r(t)))]\\&=\sum_{j=1}^{z}\dot{X}_{l_j}\phi_j\left[p_j(X_{l_j}-X_{r_j})-p_j(X_{l_j}-X_{r_j}(t-d_r(t)))\right]\\&\le\sum_{j=1}^{z}\left|\dot{X}_{l_j}\right|\phi_j\left|p_j(X_{l_j}-X_{r_j}(t-d_r(t)))-p_j(X_{l_j}-X_{r_j})\right|\end{aligned}\tag{A.3}$$

Note that $X_{k_j},\dot{X}_{k_j}\in\mathbb{R}$ *are the elements of* $X_k,\dot{X}_k\in\mathbb{R}^{z\times1}$ *vectors. Using the property that for any* $x_j,y_j\in\mathbb{R}$*,* $|p_j(x_j)-p_j(y_j)|\le 2p_j(|x_j-y_j|)$ *(Hashemzadeh et al., 2013), the* *inequality* (A.3) *can be written as:*

$$\leq 2\sum_{j=1}^{z}\left|\dot{X}_{l_j}\right|\phi_j p_j\left(\left|X_{r_j}-X_{r_j}(t-d_r(t))\right|\right) \tag{A.4}$$

and using the facts that $X_{r_j}-X_{r_j}(t-d_r(t))=\int_{t-d_r(t)}^{t}\dot{X}_{r_j}(\tau)d\tau$ *and* $\left|\int_{t-d_r(t)}^{t}\dot{X}_{r_j}(\tau)d\tau\right|\leq \int_{t-d_r(t)}^{t}\left|\dot{X}_{r_j}(\tau)\right|d\tau$*, the inequality* (A.4) *can be written as*

$$=2\sum_{j=1}^{z}\left|\dot{X}_{l_j}\right|\phi_j p_j\left(\left|\int_{t-d_r(t)}^{t}\dot{X}_{r_j}(\tau)d\tau\right|\right)\leq 2\sum_{j=1}^{z}\left|\dot{X}_{l_j}\right|\phi_j p_j\left(\int_{t-d_r(t)}^{t}\left|\dot{X}_{r_j}(\tau)\right|d\tau\right) \tag{A.5}$$

Using the inequality that (Hashemzadeh et al., 2013),

$$p_j\left(\int_{t-T(t)}^{t}|x_j(\tau)|d\tau\right)\leq\int_{t-T(t)}^{t}p_j(|x_j(\tau)|)d\tau$$

it can be concluded from (A.5) *that:*

$$\leq 2\sum_{j=1}^{z}\left|\dot{X}_{l_j}\right|\phi_j\int_{t-d_r(t)}^{t}p_j\left(\left|\dot{X}_{r_j}(\tau)\right|\right)d\tau=2|\dot{X}_l|^T\Phi\int_{t-d_r(t)}^{t}P\left(\left|\dot{X}_r(\tau)\right|\right)d\tau \tag{A.6}$$

Therefore, the proof of the inequality (10) *has been completed. The proof of the inequality* (11) *can also be concluded in a similiar way.*

Proof of lemma 3. *Given the properties that* $x_jp_j(y_j)\leq x_jp_j(x_j)+y_jp_j(y_j)$, $x_jp_j(x_j)\geq 0$ *and* $y_jp_j(y_j)\geq 0$ *(Hashemzadeh et al., 2013), it can be concluded that* $|x_j|\phi_jp_j(|y_j|)\leq x_j\phi_jp_j(x_j)+y_j\phi_jp_j(y_j)$*. Therefore, it is possible to see that:*

$$\sum_{j=1}^{z}\left[\left|\dot{X}_{r_j}(t)\right|\phi_jp_j\left(\left|\dot{X}_{l_j}(\tau)\right|\right)-\dot{X}_{l_j}(\tau)\phi_jp_j\left(\dot{X}_l(\tau)\right)\right]\leq\sum_{j=1}^{z}\dot{X}_{r_j}(t)\phi_jp_j\left(\dot{X}_{r_j}(t)\right) \tag{A.7}$$

and (A.7) *can be written as:*

$$\left|\dot{X}_r^T(t)\right|\Phi P\left(\left|\dot{X}_l(\tau)\right|\right)-\dot{X}_l^T(\tau)\Phi P\left(\dot{X}_l(\tau)\right)\leq\dot{X}_r^T(t)\Phi P\left(\dot{X}_r(t)\right) \tag{A.8}$$

Integrating the both sides of (A.8) *from* $t-d_l(t)$ *to* t *leads to:*

$$\int\limits_{t-d_l(t)}^{t}\left|\dot{X}_r^T(t)\right|\Phi P\left(\left|\dot{X}_l(\tau)\right|\right)d\tau-\int\limits_{t-d_l(t)}^{t}\dot{X}_l^T(\tau)\Phi P\left(\dot{X}_l(\tau)\right)d\tau\leq\int\limits_{t-d_l(t)}^{t}\dot{X}_r^T(t)\Phi P\left(\dot{X}_r(t)\right)d\tau \tag{A.9}$$

that can be simplified to

$$\left|\dot{X}_r\right|^T \Phi \int_{t-d_l(t)}^{t} P\left(\left|\dot{X}_l(\tau)\right|\right) d\tau - \int_{t-d_l(t)}^{t} \dot{X}_l^T(\tau) \Phi P\left(\dot{X}_l(\tau)\right) d\tau \leq d_{l_{max}} \dot{X}_r^T \Phi P\left(\dot{X}_r\right) \tag{A.10}$$

Therefore, the proof of the inequality (12) *has been completed. The proof of the inequality* (13) *can also be proved in a similar way.*

Proof of inequality (30)**.** *Considering* $P\left(\dot{X}_k\right):\mathbb{R}^{z\times 1}\to\mathbb{R}^{z\times 1}$ *with elements* $p_j\left(\dot{X}_{k_j}\right)$: $\mathbb{R}\to\mathbb{R}$; $j=1,\ldots,z$, *and* $p_j\left(\dot{X}_{k_j}\right)<N_j$, *and given the definition of* ψ_k *we get*

$$\begin{aligned}\psi_k &= \frac{1}{\|\varphi_k\|_2^2}\dot{X}_k^T P\left(\dot{X}_k\right) = \frac{1}{\|\varphi_k\|_2^2}\varphi_k^T J_k^T P\left(\dot{X}_k\right) = \frac{1}{\|\varphi_k\|_2^2}\sum_{j=1}^{z}\sum_{i=1}^{\beta_k}\varphi_{k_i} J_{k_{ji}} p_j\left(\dot{X}_{k_j}\right) \\ &\leq \frac{1}{\|\varphi_k\|_2^2}\sum_{j=1}^{z}\sum_{i=1}^{\beta_k}|\varphi_{k_i}|\left|J_{k_{ji}}\right| N_j \leq \frac{1}{\|\varphi_k\|_2^2}\sum_{j=1}^{z}\sum_{i=1}^{\beta_k}|\varphi_{k_i}| J_{k_{max}} N_{max} = \frac{1}{\|\varphi_k\|_2^2}\sum_{i=1}^{\beta_k} z J_{k_{max}} N_{max}|\varphi_{k_i}|\end{aligned} \tag{A.11}$$

Using the properties that $x_j p_j(x_j)\geq 0$ *and* $|p_j(x_j)|\leq|x_j|$, *it can be concluded that* $0\leq\dot{X}_k^T P\left(\dot{X}_k\right)\leq\dot{X}_k^T\dot{X}_k$. *Using the result of Lemma* 1; $\psi_k\leq\beta_k J_{k_{max}}^{(2)}$, *we get*

$$\psi_k \leq \min\left\{\beta_k J_{k_{max}}^{(2)}, \frac{\sum_{i=1}^{\beta_k} z J_{k_{max}} N_{max}|\varphi_{k_i}|}{\|\varphi_k\|_2^2}\right\} \tag{A.12}$$

and the proof of the inequality (30) *has been completed.*

Proof of inequality (32)**.** *Following* (2), *the second condition of Theorem* 1 *and the fact that* $\frac{\omega_k}{\omega_k+2T}<1$, *it can be concluded that*

$$\phi_{max} \leq \frac{B_{k_{min}} - \Omega_{k_{max}} - \max_i\left\{\left|\sum_{\alpha=1}^{\beta_k}\left(M_{k_{i\alpha}}\dot{\zeta}_{k_\alpha} + C_{k_{i\alpha}}\zeta_{k_\alpha}\right)\right|\right\}}{z J_{k_{max}} N_{max}} \tag{A.13}$$

which results in (discernible from (A.18)*)*

$$\begin{aligned}&\left|-\theta_{k_i} - \sum_{j=1}^{z} J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)\right| \leq B_{k_i} \\ &-\theta_{k_i} - \sum_{j=1}^{z} J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right) = s_{k_i}\left(-\theta_{k_i} - \sum_{j=1}^{z} J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)\right)\end{aligned} \tag{A.14}$$

Due to the strictly increasing property of the saturation function (2), *in the linear region*

$$s_{k_i}\left(-\theta_{k_i}-\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)-\sigma_{k_i}\varphi_{k_i}\right)\leq s_{k_i}\left(-\theta_{k_i}-\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)\right)\quad \text{if}\quad \varphi_{k_i}\geq 0, \tag{A.15}$$

and

$$s_{k_i}\left(-\theta_{k_i}-\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)-\sigma_{k_i}\varphi_{k_i}\right)> s_{k_i}\left(-\theta_{k_i}-\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)\right)\quad \text{if}\quad \varphi_{k_i}< 0 \tag{A.16}$$

Therefore, it can be concluded that

$$\varphi_{k_i}\left[s_{k_i}\left(-\theta_{k_i}-\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)-\sigma_{k_i}\varphi_{k_i}\right)-\left(-\theta_{k_i}-\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)\right)\right]\leq 0 \tag{A.17}$$

and the proof of the inequality (32) *has been completed.*

Proof of inequality (34).

$$\begin{aligned}\left|-\theta_{k_i}-\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)\right| &< \left|-\theta_{k_i}\right|+\left|\sum_{j=1}^{z}J_{k_{ji}}\phi_j p_j\left(e_{k_j}\right)\right| \\ &< \Omega_{k_i}+\left|\sum_{\alpha=1}^{\beta_k}\left(M_{k_{i\alpha}}\dot{\zeta}_{k_\alpha}+C_{k_{i\alpha}}\zeta_{k_\alpha}\right)\right|+\phi_{max}J_{k_{max}}\sum_{j=1}^{z}N_j \\ &< \Omega_{k_{max}}+\max_i\left\{\left|\sum_{\alpha=1}^{\beta_k}\left(M_{k_{i\alpha}}\dot{\zeta}_{k_\alpha}+C_{k_{i\alpha}}\zeta_{k_\alpha}\right)\right|\right\}+z\phi_{max}J_{k_{max}}N_{max}\end{aligned} \tag{A.18}$$

Dynamics informations.

$$J_l=\begin{bmatrix}-L_{1_l}s_{l1}-L_{2_l}s_{l12} & -L_{2_l}s_{l12}\\ L_{1_l}c_{l1}+L_{2_l}c_{l12} & L_{2_l}c_{l12}\end{bmatrix} \tag{A.19}$$

$$J_l^{(2)}\triangleq J_l^T J_l=\begin{bmatrix}L_{l11}+L_{l22}+2L_{l12}c_{l2} & L_{l22}+L_{l12}c_{l2}\\ L_{l22}+L_{l12}c_{l2} & L_{l22}\end{bmatrix} \tag{A.20}$$

$$J_{l_{max}}^{(2)}\triangleq (L_{1_l}+L_{2_l})^2,\qquad J_{l_{max}}\triangleq L_{1_l}+L_{2_l} \tag{A.21}$$

$$J_r = \begin{bmatrix} J_{r_{11}} & J_{r_{12}} & J_{r_{13}} \\ J_{r_{21}} & J_{r_{22}} & J_{r_{23}} \end{bmatrix} \tag{A.22}$$

$$\begin{aligned}
J_{r_{11}} &= -L_{1_r} s_{r1} - L_{2_r} s_{r12} - L_{3_r} s_{r123} \\
J_{r_{12}} &= -L_{2_r} s_{r12} - L_{3_r} s_{r123} \\
J_{r_{13}} &= -L_{3_r} s_{r123} \\
J_{r_{21}} &= L_{1_r} c_{r1} + L_{2_r} c_{r12} + L_{3_r} c_{r123} \\
J_{r_{22}} &= L_{2_r} c_{r12} + L_{3_r} c_{r123} \\
J_{r_{23}} &= L_{3_r} c_{r123}
\end{aligned} \tag{A.23}$$

$$J_r^{(2)} = J_r^T J_r = \begin{bmatrix} J_{r_{11}}^{(2)} & J_{r_{12}}^{(2)} & J_{r_{13}}^{(2)} \\ J_{r_{21}}^{(2)} & J_{r_{22}}^{(2)} & J_{r_{23}}^{(2)} \\ J_{r_{31}}^{(2)} & J_{r_{32}}^{(2)} & J_{r_{33}}^{(2)} \end{bmatrix} \tag{A.24}$$

$$\begin{aligned}
J_{r_{11}}^{(2)} &= L_{r11} + L_{r22} + L_{r33} + 2L_{r12} c_{r2} + 2L_{r13} c_{r23} + 2L_{r23} c_{r3} \\
J_{r_{12}}^{(2)} &= J_{r_{21}}^{(2)} = L_{r22} + L_{r33} + L_{r12} c_{r2} + L_{r13} c_{r23} + 2L_{r23} c_{r3} \\
J_{r_{13}}^{(2)} &= J_{r_{31}}^{(2)} = L_{r33} + L_{r13} c_{r23} + L_{r23} c_{r3} \\
J_{r_{22}}^{(2)} &= L_{r22} + L_{r33} + 2L_{r23} c_{r3} \\
J_{r_{23}}^{(2)} &= J_{r_{32}}^{(2)} = L_{r33} + L_{r23} c_{r3} \\
J_{r_{33}}^{(2)} &= L_{r33}
\end{aligned} \tag{A.25}$$

$$J_{r_{max}}^{(2)} \triangleq (L_{1_r} + L_{2_r} + L_{3_r})^2, \qquad J_{r_{max}} \triangleq L_{1_r} + L_{2_r} + L_{3_r} \tag{A.26}$$

Note that

$$\begin{aligned}
L_{kij} &\triangleq L_{i_k} L_{j_k} \\
c_{kij\epsilon} &= cos\left(q_{k_i} + q_{k_j} + q_{k_\epsilon}\right) \\
c_{kij} &= cos\left(q_{k_i} + q_{k_j}\right) \\
c_{ki} &= cos(q_{k_i}) \\
s_{kij\epsilon} &= sin\left(q_{k_i} + q_{k_j} + q_{k_\epsilon}\right) \\
s_{kij} &= sin\left(q_{k_i} + q_{k_j}\right) \\
s_{ki} &= sin(q_{k_i})
\end{aligned} \tag{A.27}$$