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MODELING OF SOME COMPONENTS OF THE HUMAN BALLISTOCARDIOGRAM

by

© IVAN TOMEK

A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled MODELING OF SOME COMPONENTS OF THE HUMAN BALLISTOCARDIOGRAM submitted by Ivan Tomek in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

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Abstract

This thesis is concerned with the development of analytical methods for the simulation of the complete ballistocardiogram (BCG).

The first chapter of this thesis contains a derivation of the basic equation of BCG, definition of components of BCG which can be treated individually, and a critical note on BCG simulation as used by other authors.

The second chapter presents a new method of simulation of the contribution of arterial circulation to the BCG. This method based on the assumption of linearity and using the frequency domain, is formulated as a digital computer program. Results obtained from this program can easily and economically be used for BCG simulation. The method is accurate and can be used for the investigation of various linear models of arterial circulation or as a diagnostic tool for the design of analogs. Two of the results obtained in this chapter are : Modeling of the system as having distributed parameters. A new representation of small vessels for the purpose of BCG simulation, more realistic than models used by analogs and easy to handle by digital computers.

The third chapter contains an analysis of factors causing the motion of the heart. It is more complete and realistic than known methods and it is completely analytical (it requires knowledge of mechanical properties of the tissue surrounding the heart and some hemodynamic variables). Some of the results obtained in this chapter agree well with reality, one factor,

however, does not seem to be represented adequately. It is suggested that better knowledge of some hemodynamic variables and a more accurate geometrical model account for discrepancies between these results and reality.

The conclusion of the research presented here is that the description of components of the BCG is now complete and their simulation possible.

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Introduction

Ballistocardiography is a recording technique based on the fact that during the heart cycle, the distribution of mass in the human body is changing. At present it is generally accepted that the best way in which to make use of this phenomenon is to measure the displacement velocity or acceleration of a bed supporting the subject in such a way that the subject is practically freely floating with the bed without much restraining mechanical coupling to the support of the bed. This technique is called ultralow frequency ballistocardiography.

Ballistocardiography (BCG) is an old noninvasive method. It gives a quantitative picture of the pumping function of the heart and the resulting circulation - as opposed to electrocardiography, (ECG), which measures the electrical phenomena associated with the heart cycle. This is a good reason why BCG should be a widely accepted clinical method - to the same extent as ECG. In reality, however, relatively few hospitals use BCG, while ECG is generally accepted. One of the main reasons why this is so is that the BCG record (BCG) is not so well understood as the ECG. This is partly because the shape of BCG is more complex than ECG and its variability is larger, and partly because BCG represents the integrated effect of heart function on the whole circulatory system and individual peaks of the record are, therefore, not immediately associable with definable single causes. This is the reason why a relatively large number of researchers have in the last ten years

been attempting BCG simulation by various methods. Their work has already given many useful results but is far from being complete. The best known results have been obtained by Dr. Noordergraaf and his students from the Department of Biomedical Engineering at the University of Pennsylvania. Their approach is simulation of the human circulation by an electrical analog adapted to BCG simulation. It is described in (1, 2, 3). (In the rest of this chapter analog A always means this particular analog). Analog A, however, cannot simulate all components of the BCG and gives only what its designers agree is the essential part of BCG. This fact has lead the author of this thesis to an investigation of an important component of the BCG not simulated by A - the contribution of the heart itself. This investigation is described in the third chapter of this thesis. Before the study of this component was undertaken, the basic equations of BCG simulation were reformulated and various components of BCG defined. This forms the first chapter of this thesis. The second chapter examines some of these components - those simulated by A. A new method of simulation is suggested - using a digital rather than analog computer. Since this approach is more accurate, it can also be used as a tool for design and check of accuracy of analogs. It is used here to examine the accuracy of A. It is also shown, as a by-product of this analysis that

1) One of the assumptions upon which A is based is false.
2) The BCG contribution of one group of the signals available from A has been ignored.

This chapter also includes a new, simple, and more realistic model of groups of branching small vessels. The third part is, as mentioned above, a study of the contribution of the heart to the BCG. This problem is

equivalent to an investigation of the motion of the heart during the heart cycle. Two such attempts have already been made: one by Dr. Noordergraaf et al (4), which is basically synthetic (and cannot, therefore, be easily and reliably used for routine simulation) - it is based on experimental data not available from A and on several simplifying assumptions. The other, proposed by Hooks (5), is analytical but has two serious drawbacks: it is oversimplified and completely omits two out of four essential factors which cause the motion of the heart. In the approach suggested here, physically more accurate assumptions are made and all four factors are considered.

It is difficult to evaluate the accuracy of the approach proposed in the third chapter. There are three possible ways of doing it. One is comparison with results obtained in previous studies (mainly (4) since (5) does not seem to be realistic). The correspondence is not complete; reasons for discrepancies are discussed in the appropriate place. Another is a comparison of a real and simulated BCG - this is of limited value only, mainly because of the large variability of BCG's as mentioned earlier. It would be more valuable in some physiologically abnormal cases - but in these cases data necessary for calculations are not available. The third way is an analysis of the physical factors behind the simulated phenomenon. This too will be done later.

There are two encouraging results: the contribution of the heart can be simulated under even more simplifying assumptions without affecting results noticably and some parameters, which are not known to an accuracy normally considered as sufficient, do not affect results substantially.

The CGS system of units will be used throughout this thesis

because it is the standard system used in this field.

To conclude this introduction let it be stressed here that this thesis presents methods rather than results.

This work, although somewhat critical of previous research in this field, would not have been possible without data and results gathered and formulated by other people - mainly Dr. Noordergraaf and his students.

It is the hope of this author that methods presented here will eventually help in advancing the understanding of BCG and so lead to a broader application of this potentially very valuable method.

CHAPTER 1

Basic equations and definitions

In this chapter basic equations of BCG simulation are derived and used to divide the total BCG into several components which can be calculated separately. Although these equations are very simple, it seems that they have not been analyzed carefully enough by previous authors. It will be shown that this has lead to a conclusion which could possibly result in serious errors in simulation (see end of this chapter).

To simplify notation only the head-foot (y) axis will be considered. Similar equations hold in other directions. The reason why this axis is discussed is that it is generally believed to be the most important one, in that y BCG's seem to contain more information than records taken in other directions. Also, it is easier, for technical reasons, to obtain reproducible records in this direction.

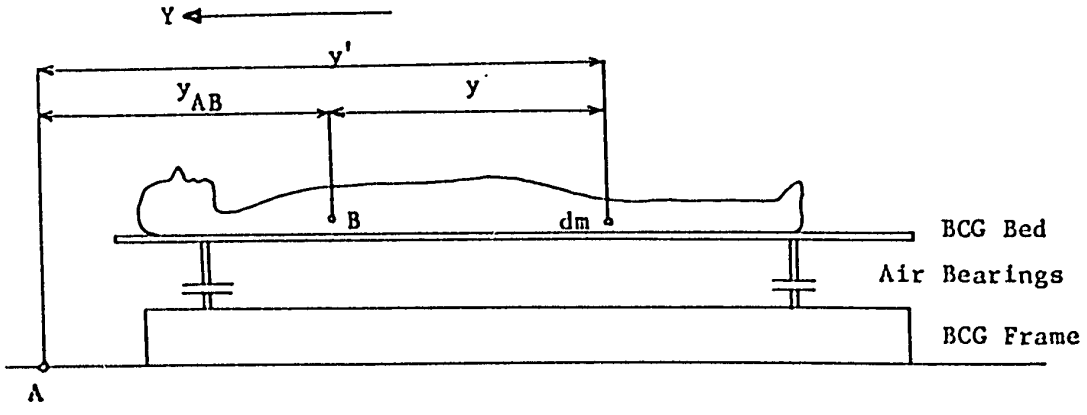


Fig. 1 BCG Bed and Notation

Notation (see also Fig. 1):

m_s total mass of the subject.

m_f total mass of the frame of subject's body.

m_b mass of subject's blood.

A a fixed point in the external frame.

B a fixed point on the steady part of subject's frame.

y' position of an infinitesimal volume of subject's body at 'rest' (corresponds to a fictitious state when the subject is not breathing and his heart is not moving but gives an output corresponding to the DC component of blood pressure) with respect to A.

$\Delta y'$ position of an infinitesimal volume with respect to the resting state at time t. (In general symbols without a prime have the same meaning as symbols with a prime but the reference point is B).

y_f' position of the center of gravity (CG) of the frame at rest with respect to A.

y_b' position of the CG of the total volume of blood at rest with respect to A.

V_b part of the body volume corresponding to m_b.

V_f part of the body volume corresponding to m_f.

Basic equations

Essentially, BCG records reflect changes of the position of the CG of the subject's body due to breathing and circulatory events. In this thesis, only changes due to circulatory events will be analyzed, although BCG records are also influenced by mechanical coupling between the subject's body and the BCG bed, properties of measuring devices etc. These effects

can be represented with sufficient accuracy by a linear block in series with the source of BCG as described here.

Let us examine the equation for the position of the CG of the subject lying on a freely floating BCG bed, assuming that at rest (as defined above) the CG is not moving. Since there are no external forces acting upon the body, the position of the CG w.r.t. A must be constant at all times:

$$\int_{V_b} (y' + \Delta y')(dm + d\Delta m) + \int_{V_f} (y' + \Delta y')(dm + d\Delta m) = \text{Const} \qquad (1\text{-}1)$$

here dm is an infinitesimal mass in the infinitesimal volume dV at rest, dm + dΔm is the infinitesimal mass in this volume at time t.*

Introducing y_{AB} - the position of A w.r.t. B gives ($y' = y + y_{AB}$ and $\Delta y' = \Delta y$)

$$y_{AB}\left[\int_{V_b} (dm + d\Delta m) + \int_{V_f} (dm + d\Delta m)\right] + \int_{V_b+V_f} \Delta y.dm + \int_{V_b+V_f} \Delta y.d\Delta m + \int_{V_b+V_f} y.d\Delta m + \int_{V_b+V_f} y.dm = \text{Const} \qquad (1\text{-}2)$$

---

* Since the method of calculation of the center of gravity represented by equation (1-1) is not quite common a more detailed explanation is given in an appendix to this chapter.

or

$$m_s y_{AB} + \int_{V_b} \Delta y.(dm + d\Delta m) + \int_{V_f} \Delta y.(dm + d\Delta m) + \int_{V_b} y.d\Delta m + \int_{V_f} y.d\Delta m$$

$$+ m_b y_b + m_f y_f = \text{Const} \qquad (1\text{-}3)$$

y_{AB} is the variable that we measure. By a proper choice of points A and B all constant terms can be cancelled out (they are of no importance anyway since it is the derivatives we usually measure). Let us assume that A and B have been chosen in this way. Then

$$y_{AB} = -\frac{1}{m_s}\left[\int_{V_b} y.d\Delta m + \int_{V_b} \Delta y.(d\Delta m + dm) + \int_{V_f} y.d\Delta m + \int_{V_f} \Delta y.(dm + d\Delta m)\right] \qquad (1\text{-}4)$$

Let us define

$$y_{blood} = -\frac{1}{m_s}\int_{V_b} y.d\Delta m \qquad (1\text{-}5)$$

$$y_{mf} = -\frac{1}{m_s}\left[\int_{V_f} \Delta y.(d\Delta m + dm) + \int_{V_f} y.d\Delta m\right] \qquad (1\text{-}6)$$

$$y_{mb} = -\frac{1}{m_s}\int_{V_b} \Delta y.(dm + d\Delta m) \qquad (1\text{-}7)$$

then

$$y_{AB} = y_{blood} + y_{mf} + y_{mb} \qquad (1\text{-}8)$$

y_{blood} is due to changing volumes of blood in vessels and the heart - assumed not to be moving.

y_{mb} is the contribution due to the motion of some vessels (e.g. the aorta) and their variable volume.

y_{mf} is the contribution of the moving parts of the frame (e.g. the moving heart filled with variable volume of blood etc).

It is not claimed here that these contributions are equally important - this remains to be evaluated.

Let us now examine these rather obvious results from the point of view of simulation. It is generally believed that y_{blood} constitutes by far the most important component of the total BCG. This is the main reason why attempts to simulate BCG by electrical analogs have been limited to simulation of this term (and in fact only a part of it: that caused by the arterial parts of systemic and pulmonary circulation). Another reason is that this component can be simulated by linear passive elements without difficulties. One of the assumptions designers of this analog have made is that the choice of the reference point B is irrelevant. This is not correct when the quantity to be simulated is defined as it has been defined here (which seems to be the only reasonable way of doing it): assuming that y_{blood} represents the total BCG accurately enough means that all displacements are neglected. In particular the motion of the heart is neglected too. The heart then is assumed to be a pump which is not moving. But its volume is still changing. If this factor is not included in the model this leaves us only one possible choice of point B, namely B must be placed in the center of gravity of the pump. In the next chapter a

quantity representing the effect of a different choice of B is defined and evaluated.

The y_{blood} and y_{mf} components defined above will be analyzed in detail in the following chapters of this thesis. Investigation of y_{mb} would require a more complicated model. It can, in fact, be analyzed once an acceptable way of analyzing y_{mf} has been found.

APPENDIX TO CH. 1

Calculation of the center of gravity

The center of gravity of a body is usually calculated from one of the following equations:

$$y_{CG}(t) = \frac{1}{m} \int_M y(t)\rho dV = \frac{1}{m} \int_M y(t)dm \qquad (1\text{-}9)$$

or

$$y_{CG}(t) = \frac{1}{m} \int_V y\rho(v,t)dv, \qquad (1\text{-}10)$$

where

$$m = \int_M dm = \int \rho(v)dv,$$

y is distance to a fixed point on the external frame and v denotes volume.

In (1-9) y is a function of time and ρ constant, referring to a fixed element of mass $dm = \rho dv$.

In (1-10) y is a constant (it is the position of the fixed infinitessimal valume dv) and ρ is a function of time.

A third approach, which is a combination of the two described above, may be advantageous. It will be explained by an example, the case of elastic tubing in motion.

Let us consider elastic tubing and a pulsating flow. Each infinitessimal segment of the tubing contains in general a time dependent volume of liquid and its distance to the reference point is a function of time. We can take these segments as the fixed elements of integration and write

$$y_{CG}(t) = \frac{1}{m} \int_V y(t)dm(t) \qquad (1\text{-}11)$$

In equation (1-1) this approach is combined with the division of the considered system into two complimentary sub-systems denoted V_b and V_f. In this case (1-11) becomes

$$y_{CG} = \frac{1}{m} \left[\int_{V_b} y(t)dm(t) + \int_{V_f} y(t)dm(t) \right] \qquad (1\text{-}12)$$

This is the meaning of equation (1-1).

CHAPTER 2

Contribution of blood moving in the arterial part of the circulation.

As stated in the preceding chapter, the component y_{blood} is believed to be the most important one. The venous part of the circulation is believed to have small effect because the pressure changes there are relatively small - and these changes are the decisive factor for BCG, as will be shown later. This chapter contains an outline of a new method of simulation of y_{blood} with two possible applications:

1) it can replace the analog A - i.e. it fulfills the same functions but more accurately and more economically.
2) it can be used as a tool for the design of new analogs or a check of accuracy of existing ones. This last application is shown in this chapter and the accuracy of A is critically examined.

This chapter also contains a simple and relatively realistic model of groups of small vessels, to be used in the method of BCG simulation described here.

Only the y axis will be investigated for two reasons - no information necessary for the evaluation of contributions in other directions is available and the contribution of this part of the circulation (almost completely symmetrical in the head-foot axis) is believed to be negligible as far as the lateral (x) BCG (the only other BCG practically measureable) is concerned.

In the rest of this chapter the symbol BCG will denote only the contribution to be studied here.

General considerations

Several models, suitable for computer realization, will be described later in this chapter. All these programs are general in that the user can prescribe his own description of the system to be analyzed. All the models are based upon the assumption that the circulatory system (or, rather, those parts of it which are to be analyzed) is linear and that its distributed parameters are described by the following equations

$$\frac{dP(s,z)}{dz} = - Z'_{\ell}(s,z)F(s,z) \tag{2-1}$$

$$\frac{dF(s,z)}{dz} = - \frac{P(s,z)}{Z'_{t}(s,z)} \tag{2-2}$$

in the s-domain (Laplace transform w.r.t. time, zero initial conditions). Here $P(s,z)$ is the Laplace transform of the pressure at distance z from the source.

$$P(s,z) = \int_0^{\infty} e^{-st} p(t,z)dt \tag{2-3}$$

$F(s,z)$ is the Laplace transform of the flow at z.

Z'_{ℓ}, Z'_{t} are the distributed longitudinal and transversal impedances per unit length.

An infinitesimal segment of vessel (parallel to axis y) of length dy gives the following contribution to the total BCG.

$$d(Bcg(t)) = -\frac{\rho}{m_s}\int\left(y.dy.\ \frac{\partial f}{\partial y}\right).dt \qquad (2-4)$$

(Integration with respect to t)

This means that the BCG is a linear function of f.

If we assume that the pressure (and, therefore, the flow) is harmonic

$$p(t,z) = P(\omega,z).e^{j\omega t} \qquad (2-5)$$

$$f(t,z) = F(\omega,z).e^{j\omega t} \qquad (2-6)$$

this means that the distributed BCG (BCG') will have a similar form

$$d(Bcg(t)) = BCG'(\omega,z).e^{j\omega t}.dy \qquad (2-7)$$

After substitution the following expression for the amplitude of the distributed BCG is obtained.

$$BCG'(\omega,z) = \frac{\rho}{m_s}.y.\frac{P(\omega,z)}{j\omega Z'_t} \qquad (2-8)$$

Whatever the representation of this branching system the model has to be segmented and therefore the following equation has to be used to obtain the total BCG.

$$BCGT(\omega) = \sum_{\text{all segments}} \Delta BCG \qquad (2-9)$$

where ΔBCG is a BCG contribution of one segment at frequency ω.

This variable which is a function of frequency will be called BCG impedance, since it will be calculated from the assumption that the flow from the source is

$$F(\omega) = 1 \tag{2-10}$$

If the actual flow is

$$f(t) = \sum_{-N}^{N} F(n\omega).e^{j\omega t} = 2.\text{Real}\left(\sum_{n=1}^{N} F(n\omega).e^{jn\omega t}\right) + F(0) \tag{2-11}$$

(The Fourier expansion into a sufficient number of terms)

the total BCG is

$$Bcgt(t) = 2.\text{Real}\left(\sum_{n=1}^{N} BCGT(n\omega).F(n\omega).e^{jn\omega t}\right) \tag{2-12}$$

(The DC component is not considered since this constant is of no practical interest).

The variable BCGT(ω) thus gives all the information necessary to obtain Bcgt for periodic input flows. This is quite sufficient since transients are of no practical interest either. Also any complex of input flows can be approximated by Fourier coefficients as closely as desired.

If p(t) is given instead of f(t) then

$$F(\omega) = \frac{P(\omega)}{Z(\omega)} \tag{2-13}$$

and the impedance Z(ω) is calculated by the program along with BCGT(ω).

To evaluate the effect of the choice of position of the reference point B upon the variable BCG'(ω), let us shift B by a unit of length. This results in a change of BCG' by

$$Q'(\omega) = \frac{\rho.P(\omega)}{m_s.j.\omega.Z'_t} \tag{2-14}$$

and the effect on the total BCG is described by

$$QT(\omega) = \int_{V_b} Q'(\omega)\,dz \qquad (2\text{-}15)$$

If B is shifted by $L_{(cm)}$ the new value of BCGT ($\overline{BCGT}$) will be

$$BCGT(\omega) = \overline{BCGT}(\omega) + L.QT(\omega) \qquad (2\text{-}16)$$

If the correct value $BCG(\omega)$ is to be obtained $L.QT(\omega)$ must be subtracted from $\overline{BCGT}$ according to (2-16).

The variable Q is evaluated by the program along with $Z(\omega)$ and $BCGT(\omega)$.

The physical meaning of this variable can be understood from equations (2-14),(2-15). It corresponds to the surplus of blood accumulated in the part of the system being considered at the given frequency. When this variable is left out a change of position of the reference point B causes a change in the shape of the BCG frequency characteristic. This, in turn, leads to a different shape of the BCG calculated from the frequency characteristic. But the apparatus is always measuring the same BCG. The variable $QT(\omega)$is, therefore, very important.

Development of the approach

The chronological development of various models will be described here - it shows which modifications (there are 8 different versions altogether) have been progressively introduced and why.

The first decision that had to be made was whether the time domain or the frequency domain should be used. It is evident from the previous section that it was decided to use the frequency domain. The reasons for

this choice are the following:

1) the frequency characteristic of the system can be calculated once and for all and the desired studies can then be made via Fourier coefficients. This saves a lot of computer time and even makes it possible to study BCG without a computer. On the other hand transients caused by changes of parameters cannot be studied since the source - the heart - is connected to the system by nonlinear elements. This disadvantage is not important even when the method is compared with the analog A - the analog works 1000 times faster than reality and it is, therefore, hardly possible to study transient phenomena by its use (the system being a distributed one). The analog, however, has the advantage that, to some extent at least, it generates its own input (flow).

2) the frequency response is a good and accepted characteristic of the circulatory system. Several studies published so far make it possible to compare models and real systems via frequency characteristics (as far as the circulatory system itself is concerned).

3) since the circulatory system is represented by a branching system of uniform tubes, each of which is an element with distributed parameters, the frequency domain is the only approach which allows a fast and general exact solution. It will be seen that this advantage over analogs is an important one. This feature of the proposed method also makes possible its use as a diagnostic tool.

4) as a consequence of 1 and 2 a computer program using the frequency domain offers an efficient method for comparison of various physical models and their simplifications (provided they are linear) - also

possibly in the first phase of design of analogs.

A. Original Model.

This model is called C when complex value of E (modulus of elasticity of vessel wall) is used, R when real E is used.

This first version is a complete analog of A in digital form and frequency domain - i.e. the topology and parameters are identical. Also, the system is divided into segments which are assumed to have lumped parameters.

The system to be simulated is assumed to be described by distributed impedances

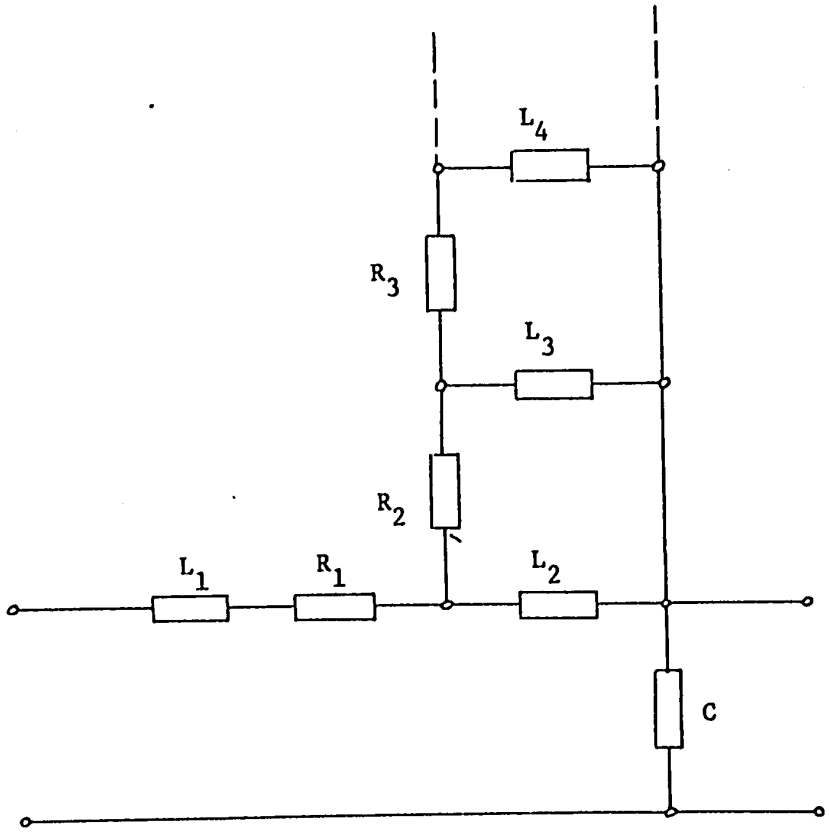
$$Z'_{\ell}(\omega) = - \frac{1}{F(\omega)} \frac{\partial P}{\partial z} \qquad (2\text{-}17)$$

$$Z'_{t}(\omega) = - P(\omega) / \frac{\partial F}{\partial z} \qquad (2\text{-}18)$$

z is the distance along the vessel measured in the direction from the source as mentioned above. In versions C and R, these impedances are lumped and a segment of length L is presented by overall longitudinal and transversal impedances (see Figs. 2 and 15).

$$Z_{\ell} = Z'_{\ell} . L \qquad (2\text{-}19)$$

$$Z'_{t} = Z'_{t} / L \qquad (2\text{-}20)$$



L = Inductance
R = Resistance
C = Capacitance

Fig. 2 One segment of vessel as represented in (1).

Expressions for Z'_ℓ and Z'_t are taken from (1) and are as follows

$$Z'_\ell = \frac{4\mu}{\pi . r^4} \quad \frac{\sum_0^\infty \lambda^m/(m!)^2}{\sum_0^\infty \lambda^m/[m! . (m+2)!]} \qquad (2\text{-}21)$$

$$Z'_t = \frac{1}{j\omega} \frac{E.h}{3\pi r^2} \frac{2r + h}{(r + h)^2} \tag{2-22}$$

where

$$\lambda = j \frac{r^2 \omega}{4\nu} \tag{2-23}$$

here

r is the internal radius of the vessel when the internal pressure is equal to the DC component of the actual pressure.

ρ is the density of blood.

ω is the frequency $\omega = 2\pi.f$

h is the thickness of the wall of the vessel.

E is its modulus of elasticity.

Equation (22) contains a ratio of two infinite series (representing Bessel functions). A fast converging continued fraction expansion is given in (1). A few terms are then used to obtain a desired accuracy. The whole segment is then represented (in the electrical analog of which this version is a computer model) by the network in Fig. 2 of total impedance Z_ℓ and Z_t.

Although the accuracy of individual blocks is good, the large number of segments (over 100) and tolerances of the values of individual passive elements, as well as the effect of connections etc., limit the overall accuracy of the analog. The number of passive elements in individual segments was chosen such as to give 2% accuracy. In the computer program this was replaced by 1%.

Physical limitations of analog A are to some extent replaced by roundoff errors. These were minimized by the use of double precision

arithmetic for important variables. The accuracy of the computer program has been checked by a simple branching system resembling the circulatory system but having uniform parts. Exact solutions were then compared with solutions obtained by subdividing this system into more and more lumped segments. The convergence was excellent and thus it was concluded that the program is accurate. Results obtained from this version of computer program were compared with published frequency characteristics of the analog A (see (1)) and the agreement of hemodynamic variables has been found good. This is shown in Fig. 6 and confirms the relative accuracy of the analog, at least as far as hemodynamic variables are concerned. No comparable characteristic of its BCG performance is available. The original version of the analog A was assuming real values of E. This assumption has then been replaced by complex F (physically elastic modulus of viscoelastic material of vessel walls). Both these alternatives were tested on the computer program and it was found that the second version of the analog gives results closer to those obtained from the digital computer.

B. Distributed version of Original Model.

This version is denoted by CD or RD for complex and real value of E respectively.

Unlike the analog model or the time domain approach on digital computers, the use of frequency domain makes it possible to obtain accurate results even for quite general linear systems with distributed parameters (by 'general' is meant generality of expressions for distributed impedances). It must be remembered, however, that even this solution will only be an

approximation because the continuous and general way in which physical parameters are changing must be approximated.

When the basic equations

$$\frac{dP}{dz} = - Z'_{\ell} . F \tag{2-24}$$

$$\frac{dF}{dz} = - \frac{P}{Z'_t} \tag{2-25}$$

are solved (e.g. (7)) (Z'_{ℓ},Z'_t are assumed to be constant throughout the individual segments), we obtain

$$Z_{in} = \frac{\gamma . Z_{out} . Z'_{\ell} + Z'^{2}_{\ell} . tgh(\gamma . L)}{\gamma^2 . Z_{out} . tgh(\gamma . L) + \gamma . Z'_{\ell}} \tag{2-26}$$

for $s \neq 0$ and

$$Z_{in} = Z_{out} + L . Z'_{\ell} \tag{2-27}$$

for $s = 0$

Here

$$\gamma = \sqrt{\frac{Z'_{\ell}}{Z'_t}} \tag{2-28}$$

$$P(\ell,s) = P_{out} . \left(\cosh(\gamma . \ell) + \frac{Z'_{\ell}}{\gamma . Z_{out}} . \sinh(\gamma . \ell)\right) \tag{2-29}$$

Here

Z_{out} is the loading impedance at the end of the segment.

Z_{in} is the loading impedance presented by the segment and its load.

L is the length of the segment

$P(\ell,s)$ is the Laplace transform of the pressure at distance ℓ from the end of the segment.

For the calculation of BCG the artery is assumed to have parameters indicated in Fig. 3.

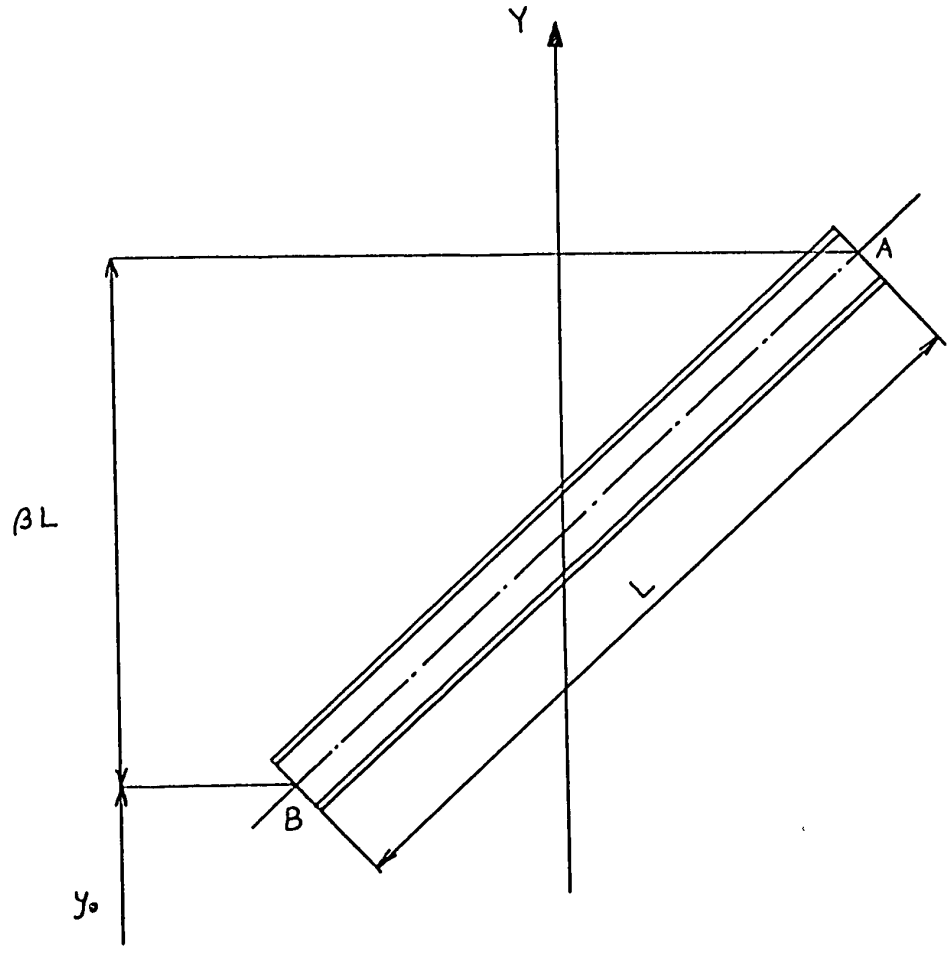


Fig. 3 Meaning of symbols used in the text.

In this figure $\beta > 0$ when $Z_A < Z_B$ (measured from the source).

For $s = j\omega$

$$\Delta BCG(\omega) = \int_0^L BCG'(\omega)\, d\ell = \frac{\rho}{m_s . j\omega . Z'_t} \cdot \int_0^L P(\ell,\omega) y . d\ell \qquad (2\text{-}30)$$

but $y = y_o + \beta . \ell$ so that

$$\Delta BCG(\omega) = \frac{\rho . P_{out}}{m_s . j\omega . Z'_t} \int_0^L (\cosh\ (\gamma . \ell) + \frac{Z'_\ell}{\gamma . Z_{out}} \cdot \sinh\ (\gamma . \ell))(y_o + \beta . \ell) . d\ell \qquad (2\text{-}31)$$

Finally

$$\Delta BCG(\omega) = \frac{\rho . P_{out}}{m_s . j\omega . \gamma^3 . Z'_t . Z_{out}} \cdot \Big[\{Z_{out} . (y_o + \beta L)\gamma^2 - \beta . Z'_\ell\} . \sinh\ (\gamma L) + \{(y_o + \beta L) Z'_\ell - \beta Z_{out}\} . \cosh\ (\gamma L) + \gamma . (\beta Z_{out} - y_o Z'_\ell) \Big] \qquad (2\text{-}32)$$

and ΔQ (substituting $\beta = 0$ and $y_o = 1$).

$$\Delta Q = \frac{\rho . P_{out}}{m_s . j\omega . \gamma^2 . Z'_t . Z_{out}} \cdot \Big[\gamma Z_{out} . \sinh\ (\gamma L) + Z'_\ell . \{\cosh\ (\gamma L) - 1\} \Big] \qquad (2\text{-}33)$$

At this point only was it realized that the choice of the reference point is important and the variable Q was introduced. The analog was then examined more closely and it was concluded that there are two other questionable points. Both concern the part of analog A representing the small vessels:

1) In the analog small vessels are represented by resistors. This is adequate for hemodynamic variables (which was confirmed later when a

new model was tried) but perhaps not for BCG simulation. The reason is that resistors replace a system of elastic branching vessels by one element with zero time delay and no elasticity. The contribution of small vessels is (in the analog A) calculated from

$$B_{cg}(t) = \frac{3V.\{r.(r + h)\}^2}{E.(2r + h)h^2}\, y.\Delta p(t) \qquad \Delta p(t) = p(t) - p_{DC} \tag{2-34}$$

with $\frac{r}{h} = 3$

and $E = 16 \times 10^6$ (see (4)).

V is the volume of the considered system of small vessels.

The result of this approach is that the contribution of small vessels far from the reference point is suspiciously large.

2) the analog does not take into account one important factor at all, perhaps intentionally, but then the model lacks physical meaning. The model is at all terminations grounded by resistors. Some current (the electrical analog of flow), however, is flowing out or into the system through these resistors. Since the amount of blood in the system cannot change - the physical system being closed - the model represents a system terminated by 'reservoirs'. This fact is not considered at all. It was decided here to view the reservoirs as localized at ends of resistors representing small vessels (see Fig. 4). This is rather arbitrary, but it is simple and it at least considers the neglected effect. If f(t) denotes the bloodflow from the reservoir then its contribution to the BCG is

$$\Delta Bcg(t) = \Delta BCG(\omega).e^{j\omega t} = \frac{\rho}{m_s}\int f(t)y.dt = \frac{\rho y}{m_s}\int F(\omega).e^{j\omega t}.dt = \frac{\rho y F(\omega)}{j\omega}\, e^{j\omega t}$$

(2-35)

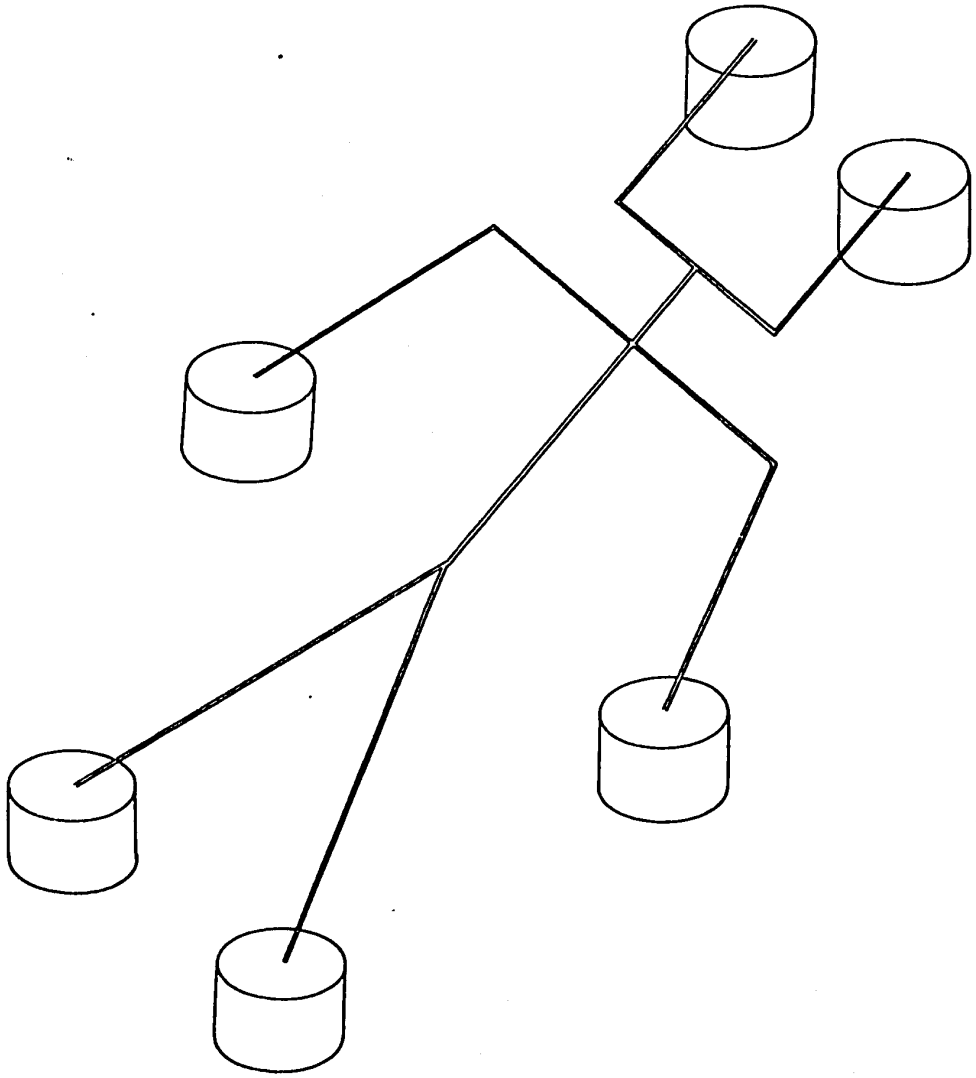


Fig. 4 Completion of the system by 'reservoirs'.

$$\Delta BCG(\omega) = \frac{\rho y F(\omega)}{j\omega} \qquad (2\text{-}36)$$

If the total effect of this factor were negligible, its omission would be justified, but this is not the case.

For these two reasons it was decided that the model had to be modified. This is described in the next section.

C. Model with modified representation of small vessels.

This version is denoted by C1 and R1 for lumped version and CD1 and RD1 for distributed version.

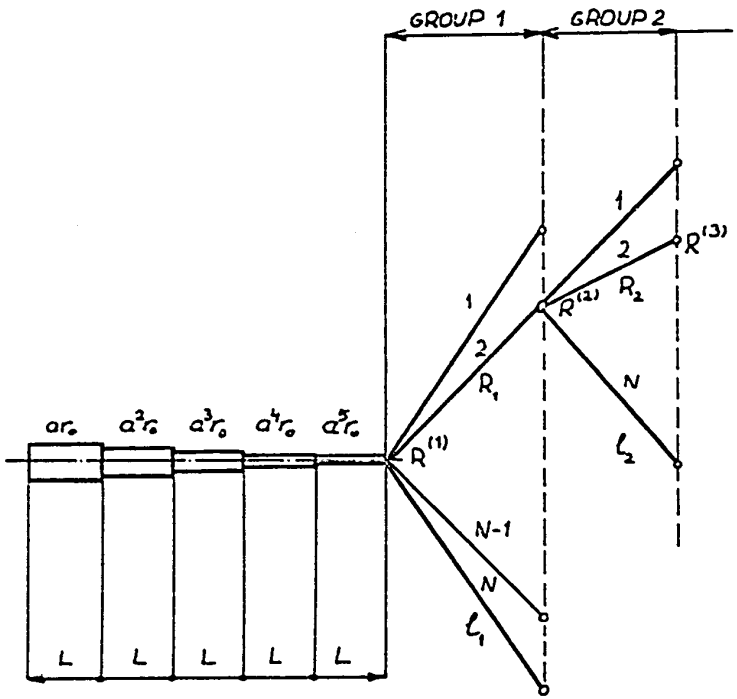
This new description is partly based on greatly simplified physiological data, partly on data used in the original model, and partly on the formula for distributed longitudinal impedance given earlier. Fig. 5 shows the geometric configuration assumed for branching small vessels. It is evident that, compared to the nonsymmetrical vessel anatomy, the model is very simplified. On the other hand the number of branches and groups as well as the ratio of volumes of successive groups were kept as close to reality as possible (for comparison see (6)).

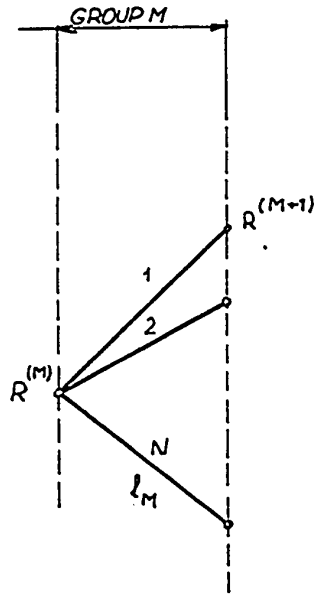
The symmetry of the system and some further assumptions make it easy to find parameters describing these systems of vessels.

Let us assume that

$$R_n = \frac{R_{n+1}}{N} \qquad (2\text{-}37)$$

Here R_n is the resistance of one branch of the n-th group, N is the number of branches. Since the number of branches of the n-th group





$L = .15\ell$

$\ell_1 = c_\ell . \ell$

$\ell_{n+1} = c_\ell . \ell_n$

$M_{n+1} = c_r . r$

$M = 6$

$N = 11$

Fig. 5 Branching small vessels

is 1/N times the number of branches of the (n + 1)-st group, this means that the overall resistance of groups is constant from one group to another.

Since

$$R = \frac{8\mu}{\pi r^4} \ell \tag{2-38}$$

(This can be obtained by substituting $\lambda = 0$ into (2-21) section A). For one vessel, (2-37) gives

$$\frac{\ell_n}{r_n^4} = \frac{1}{N} \frac{\ell_{n+1}}{r_{n+1}^4} \tag{2-39}$$

When we define

$$c_\ell = \frac{\ell_{n+1}}{\ell_n} \qquad c_r = \frac{r_{n+1}}{r_n} \tag{2-40}$$

this means that

$$c_\ell = N c_r^4 \tag{2-41}$$

If $R^{(n)}$ is the loading resistance of a branch of the (n-1)-st group (see Fig. 5)

$$R^{(n)} = \frac{1}{N}(R_n + R^{(n+1)}) \tag{2-42}$$

we have

$$R^{(1)} = \frac{1}{N}(R_1 + \frac{1}{N}(R_2 + \ldots\ldots)) = \frac{1}{N} R_1 + \frac{1}{N^2} R_2 + \ldots + \frac{1}{N^M} R_M \tag{2-43}$$

since at the end $R^{(M+1)} = 0$.

But according to (1)

$$\frac{1}{N} R_1 = \frac{1}{N^2} R_2 = \ldots = \frac{1}{N^M} R_M \tag{2-44}$$

so that

$$R^{(1)} = \frac{M}{N} R_1 = \frac{8\mu\ell}{\pi r^4} \frac{M}{N} \tag{2-45}$$

For the first 5 sections (included to obtain a smoother transition from large to small diameters) let

$$\frac{r_{n+1}}{r_n} = a \tag{2-46}$$

with

$$r_1 = a\, r_o \tag{2-47}$$

here r_o is ideally equal to the radius of the vessel immediately preceding the considered system of small vessels. Let

$$r_6 = c_r \cdot r_5 \tag{2-48}$$

then

$$R_{total} = \frac{8\mu\ell}{\pi r_o^4} \cdot \left[\left(\frac{1}{a^4} + \frac{1}{a^8} + \ldots + \frac{1}{a^{20}}\right) \times .15 + \frac{M}{Na^{20} c_r^4} \right] \tag{2-49}$$

By a similar reasoning we obtain the following expression for the total volume of the sytem:

$$V_{total} = \pi r_o^2 \ell . \left[(a^2 + a^4 + .. + a^{10}) x.15 + N a^{10} c_r^2 \frac{1 - (N^2 c_r^6)^M}{1 - N^2 c_r^6} \right] \qquad (2\text{-}50)$$

The ratio of total areas of cross sections of consecutive groups is

$$c_s = \frac{s_{n+1}}{s_n} = \frac{N r_{n+1}^2}{r_n^2} = N c_r^2 \qquad (2\text{-}51)$$

Expressions (2-41)-(2-50) and data about P_{total} and R_{total} used in previous models are then used to obtain values for unknowns. There are 3 equations and 7 unknowns ($r_o, \ell, a, c_\ell, c_r, M, N$). It is therefore necessary to choose some of them arbitrarily - but as close to physical reality as possible (see (6)). It was decided to set

$c_r = .5 \rightarrow c_\ell = .6875 \quad c_s = 2.75$

$M = 6$

$N = 11$

A digital computer program was then used to find acceptable values for r_o by varying the value of a.

The result of this approach is that the total volume and resistance remain almost identical (the difference is due only to the roundoff error introduced into the program) with those used in the analog and the above mentioned versions.

The 'reservoir effect' is calculated from outflows from the last

group according to (2-36) in the previous section.

The model presented here is believed to be more realistic than those suggested by other authors. It satisfies the requirements for which it was designed as will be seen later in this chapter.

It should be mentioned that a physical analog with these characteristics is practically unrealizable because of the enormous number of elements involved. The computer model, however, requires little computer time because of its symmetry.

D. A simple model based on Poisseuille's Flow.

This version is denoted LL for lumped and LD for distributed parameters. Only complex E was used for calculations.

Welkovitz and Fich in (7) derived and used the following expressions for distributed impedances:

$$Z'_{\ell} = \frac{8\nu}{\pi r^4} + j\omega \cdot \frac{\rho}{\pi r^2} = R + j\omega L \qquad (2\text{-}52)$$

$$Z'_{t} = \frac{hE}{j\omega 2\pi r^2} = \frac{1}{j\omega C} \qquad (2\text{-}53)$$

ν in the first equation is apparently a printing error and should be replaced by μ. These equations were, with some modifications, used by the authors to describe a part of the aorta. If these equations could be used instead of those used in previous sections, the result would mean a substantial saving of computer time or a significant reduction of the number of elements in the analog. For this reason, it was decided to compare results from other programs with two versions using these equations - one using lumped parameters, the other using distributed parameters.

In addition to the change made in the first equation mentioned above it was decided to replace C by

$$C = \frac{3\pi r^2 (r + h)^2}{E\, h(2\, r + h)} \qquad (2\text{-}54)$$

This is the formula used in previous sections taken from (1). Its use makes it easier to compare results from various versions of the original program. The last mentioned formula seems to be generally accepted as being better justified than the one used in (7).

Only the modified model of small vessels described in the previous section was used.

Results and Conclusions

Frequency characteristics obtained from different versions are in Figs. 7, 8. It should be mentioned that 'force BCG' and its integrals rather than the more common acceleration BCG (acceleration of the BCG bed) and its integrals are used. This is because the author believes that dividing the force BCG by the mass of the subject (which is the transformation through which the acceleration BCG is obtained from the force BCG) introduces a somewhat modifiable scaling factor and thus obscures results of simulation. Only complex value of the modulus of elasticity E is considered. Real value of E was originally investigated by the author mainly for the purpose of checking the accuracy of the analog.

Examination of results leads to the following immediate conclusions:

1) the overall hemodynamic impedance calculated by the program is quite close to that measured on the analog. This is true both for lumped (C) and distributed (CD) parameters versions. It means that from the

point of view of circulation itself the number of segments into which the simulated system is divided is sufficient.

2) BCG frequency characteristics C and CD differ significantly in amplitude in the lower frequency range. This means that for BCG simulation the number of segments should be increased.

3) BCG frequency characteristics of the modified lumped (C1) and distributed (CD1) model differ from C and D mostly at high frequencies. The modified version gives generally smoother results in agreement with reality. This difference is probably smaller when C1 and CD1 are compared with the analog A because the analog works 1000 times faster than reality and various unwanted effects can affect its frequency characteristic at higher frequencies.

4) The relative contribution of small vessels is much smaller in C1 and CD1 than in C and CD. This can be seen in Fig. 9. This confirms the original impression that the contribution of small vessels is exaggerated in A. Evidently the way in which these vessels are simulated plays an important role.

5) Comparison of normal BCG records obtained in the way described above (see Figs. 10, 11) and clinical records (e.g. (8)) shows that amplitudes obtained from versions with distributed parameters are closer to real values than those obtained for lumped parameters. The same observation is valid for results obtained from A (see (3)). This supports the earlier conclusion that for accurate BCG simulation the number of segments should be increased if the system is to be simulated by an analog.

6) the simplified versions LL and LD give essentially the same results as

C1 and CD1 which means that a simpler representation of the hemodynamic phenomena is acceptable when details of local physical phenomena are not essential.

It should be noted that the results presented do not cover the contribution of the pulmonary circulation which can be obtained by the same program with different data (available from (2)).

Summary of Chapter 2

A new approach to BCG simulation is presented. The author feels that it is more realistic, economical and easier to use than the existing analog model simulating the same phenomenon (i.e. a part of the human BCG). Results, once obtained, make it possible to simulate various BCG's even without the use of computers. This approach makes easy comparison between various linear physical models of the same physiological system and can be used as a tool when an analog of the circulation is designed or critically analyzed.

The approach described in this chapter is used to investigate the accuracy of an analog built at the University of Pennsylvania and described in (1,2,3). It is concluded that although this analog simulates the human BCG quite well, it is not physically complete, simulation of small vessels is not quite satisfactory and the number of segments into which the circulatory system is divided is not large enough. A conclusion that this kind of simulation is not ideal is put forward. A new model of representation of small vessels both physically more realistic than the one used in the analog and easy to handle on a digital computer is proposed. Introduction

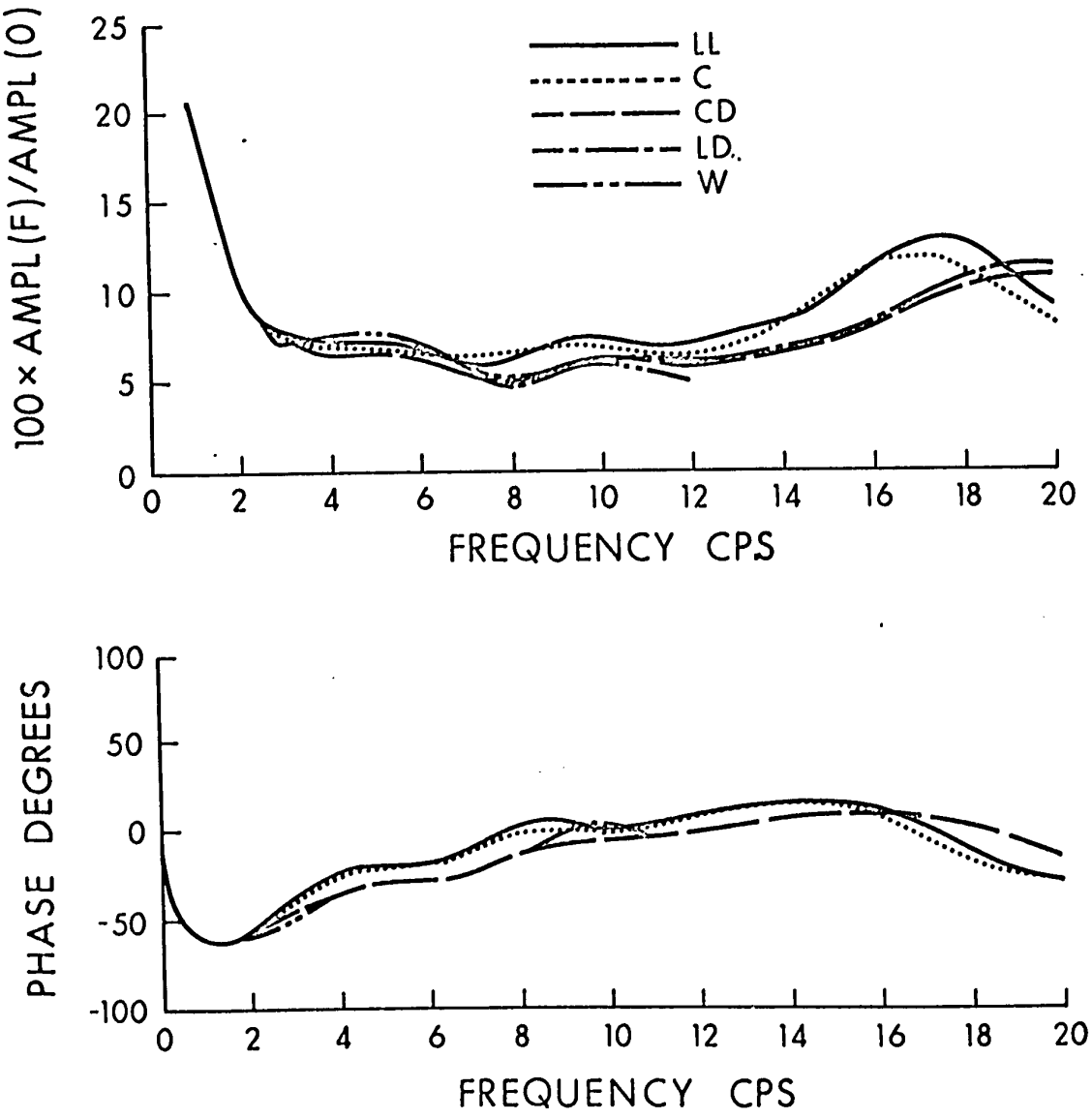


Fig. 6 Hemodynamic Impedance. Notation as in text. W - Westerhof.

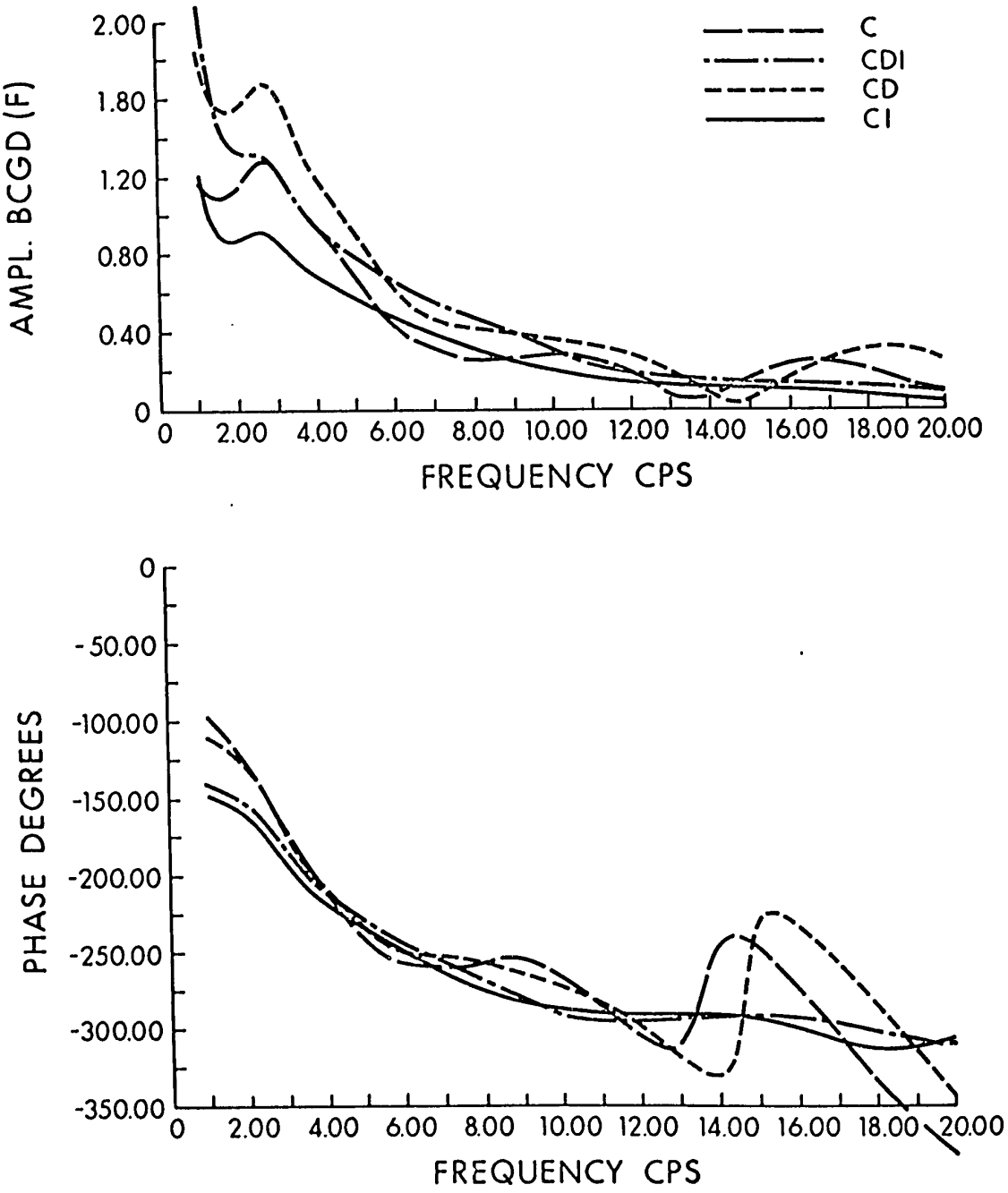


Fig. 7 BCG Displacement Impedance.

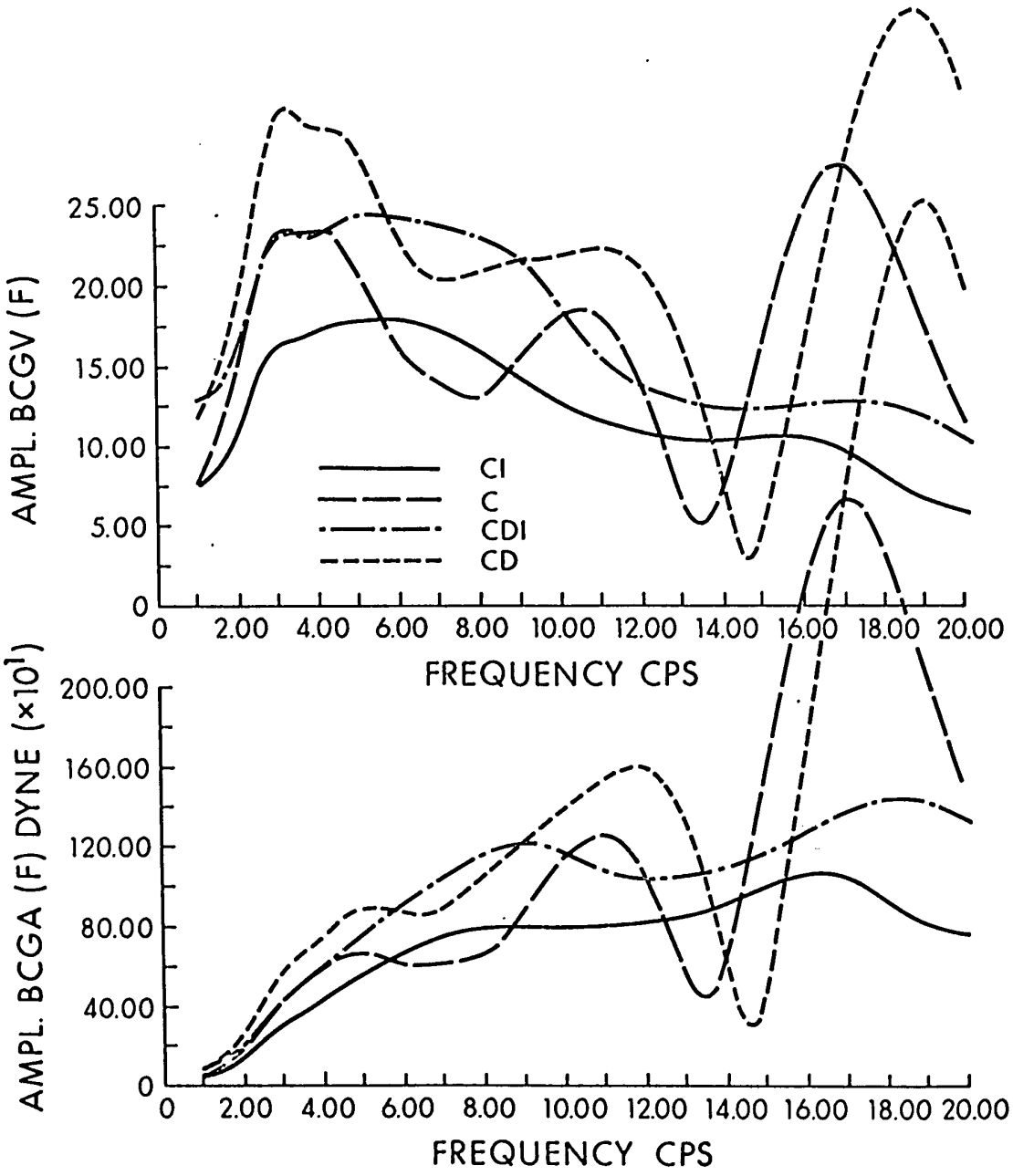


Fig. 8 BCG Velocity and Acceleration Impedance.

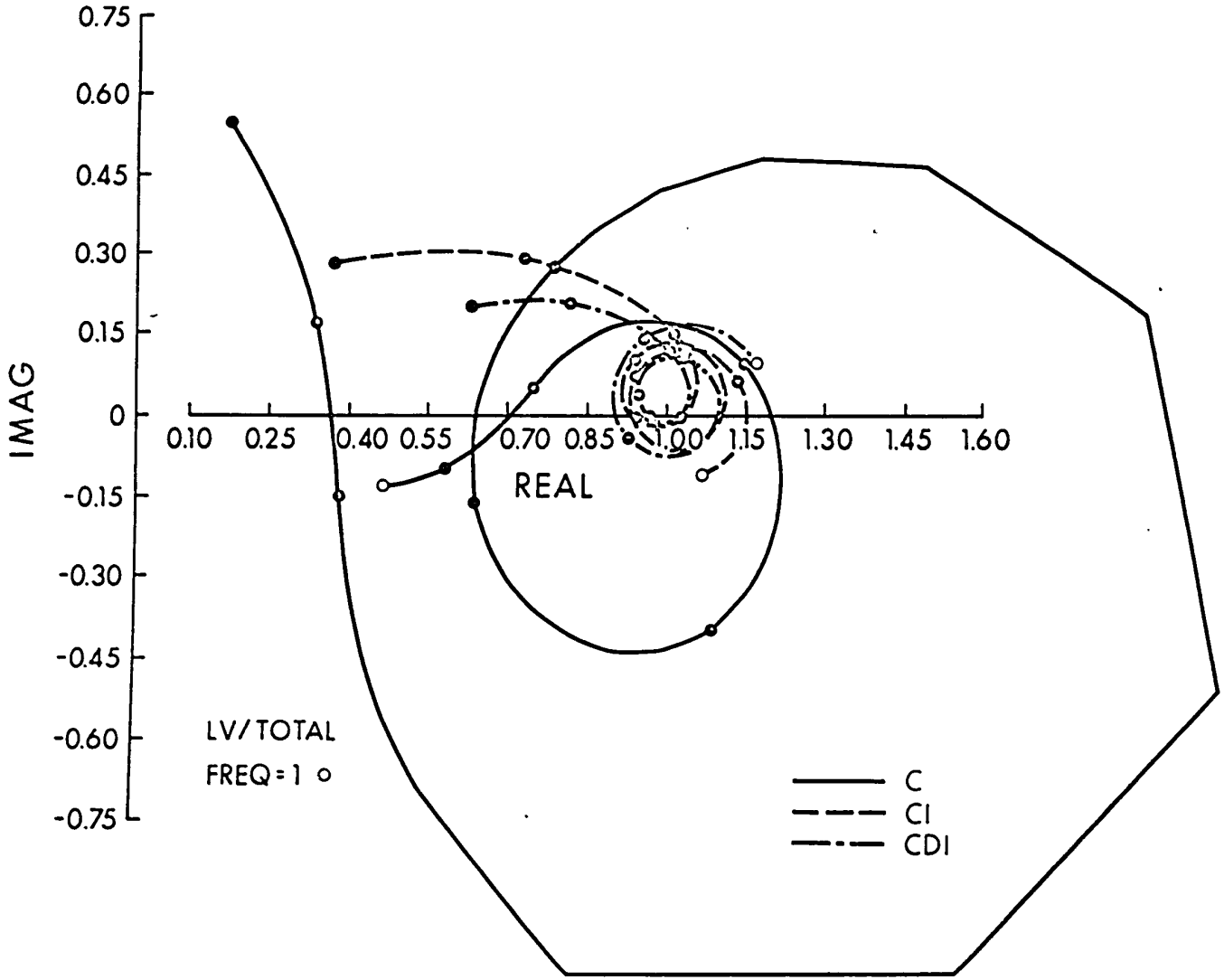


Fig. 9 LV - Contribution of Large Vessels.
Total - Total BCG Impedance. Full circles denote 2,4 ... cps

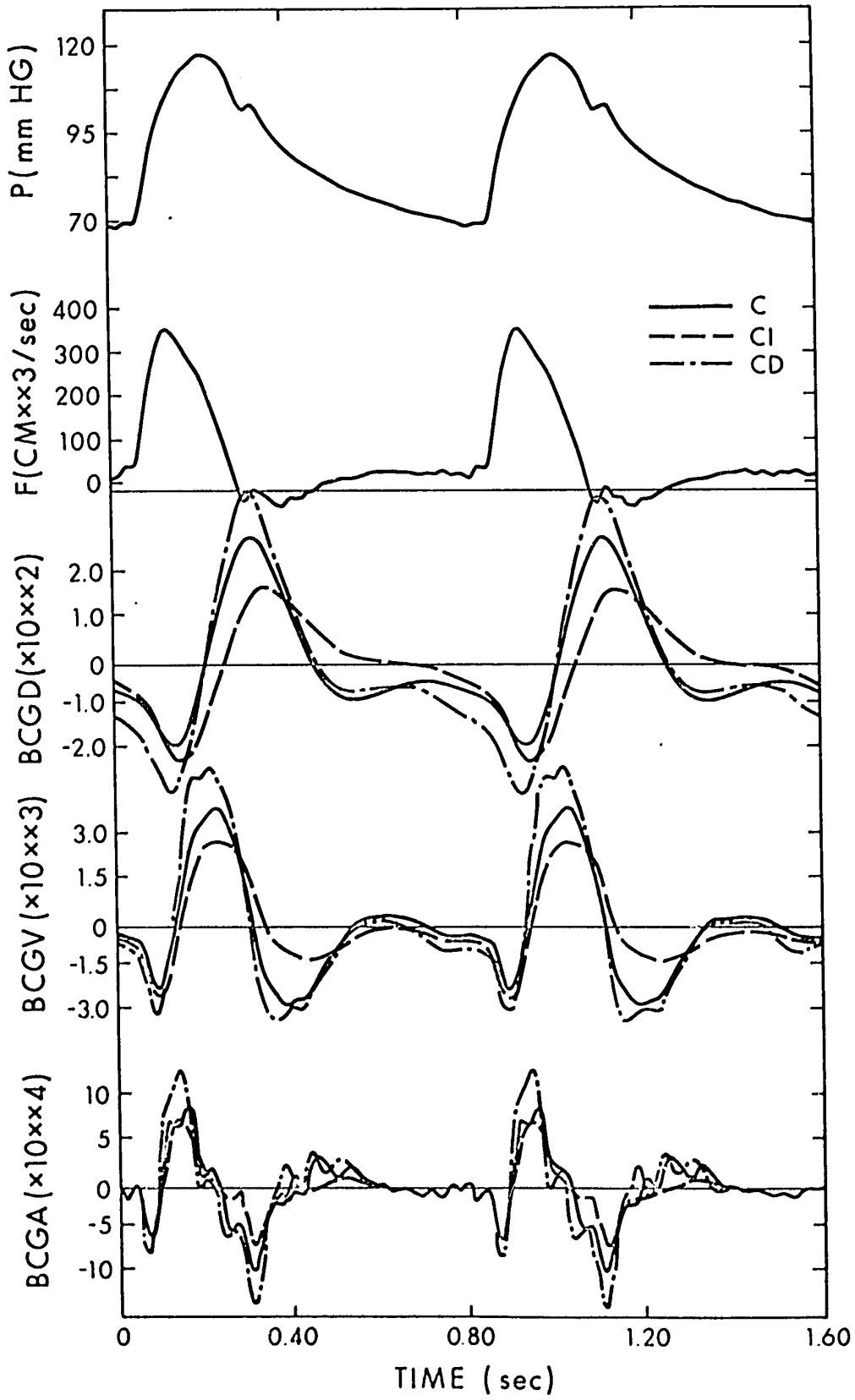


Fig. 10 Simulated BCG (20 terms of Fourier Series).

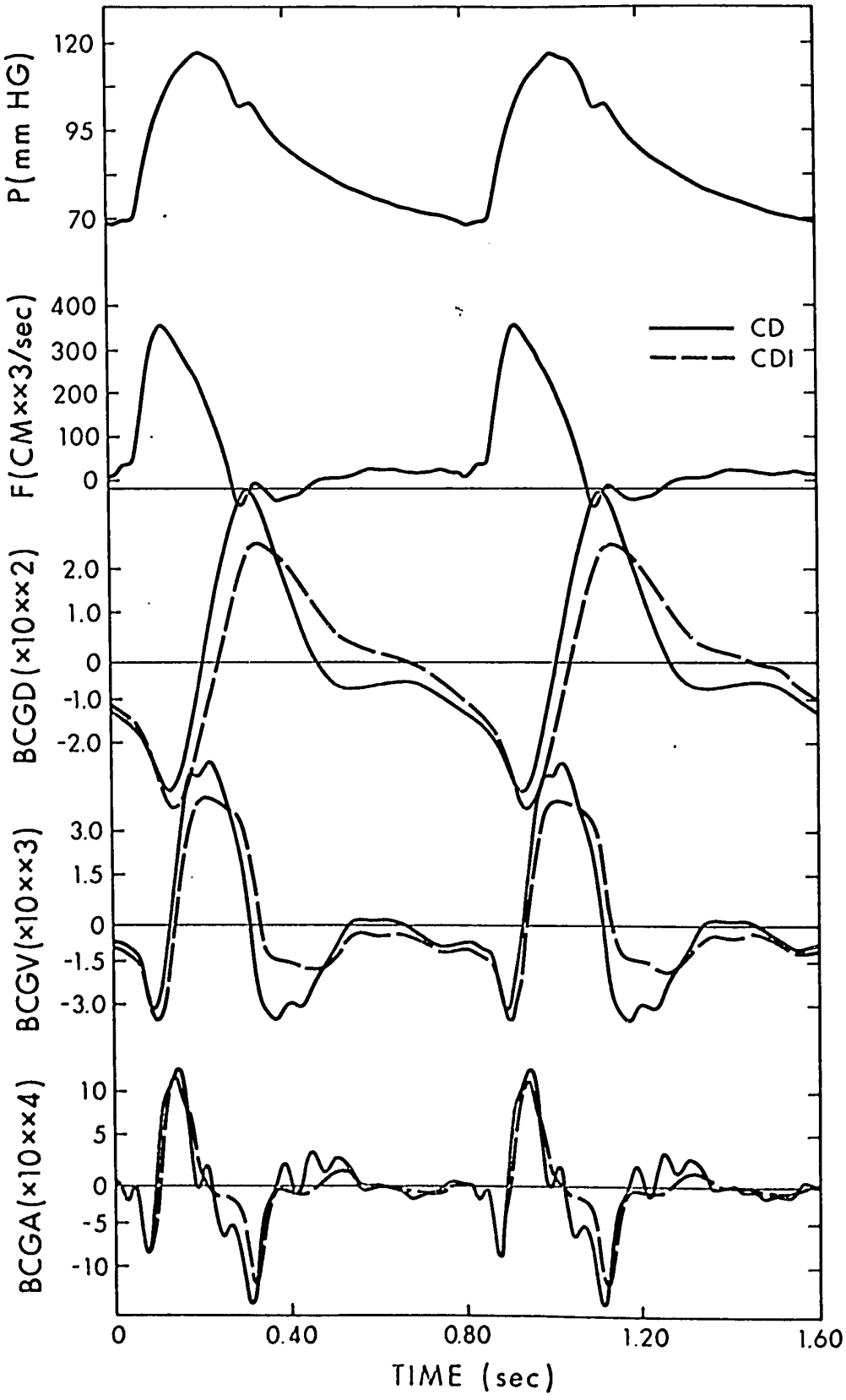


Fig. 11 Simulated BCG.

of this modification leads to a smaller contribution of small vessels and a smoother BCG in agreement with reality.

Appendix to Chapter 2

In the first place the organization of data giving information about the system to be analyzed will be described. Then the block diagram of the whole program will be presented and some blocks described in more detail. The program was written to be used via terminal under the MTS system implemented at the Computing Center at the University of Alberta.

A. Data Description

The system is assumed to be branching in the direction away from the source only. The following rules and definitions are accepted for the description of the system:

1) 'Distal' nodes are nodes farther from the source. 'Proximal' nodes are closer to the source.
2) All branches have both distal and proximal nodes (this is important for the terminal nodes).
3) Every node and every branch has its number.
4) Every branch has the same number as its distal node.
5) Numbering starts from an arbitrary terminal node and proceeds in the way as one would proceed when calculating impedances of the system.

These rules are illustrated by an example presented in Fig. 12.

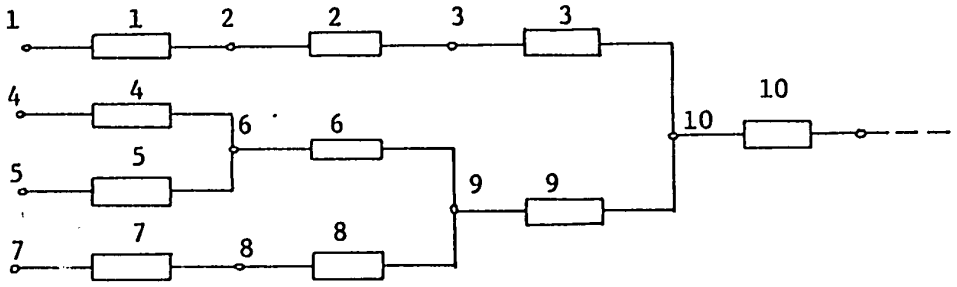


Fig. 12 An Example Illustrating the Use of Some Rules from Section A.

Block of segments (group of segments) number I is described by an integer vector B(I.J).

B(I,1) = 0 for terminal nodes, 1 otherwise.

B(I,2) = number of distal node of the I-th segment.

B(I,3) = number of subdivisions into which the segment is divided.

B(I,4) = 1 for the aorta, zero otherwise. (The reason for this distinction is that complex E is described by a different expression for the aorta than for other vessels).

Parameters of the J-th subsegment of the I-th block are stored in the vector S(I,J,K). Components of this vector are the inner radius, wall thickness, modulus of elasticity, length and distance from the reference point of the segment etc.

B. The Block Diagram of the Program and a Description of some of Its Blocks.

The block diagram of the program is given in Figs. 13, 14.

PART 1

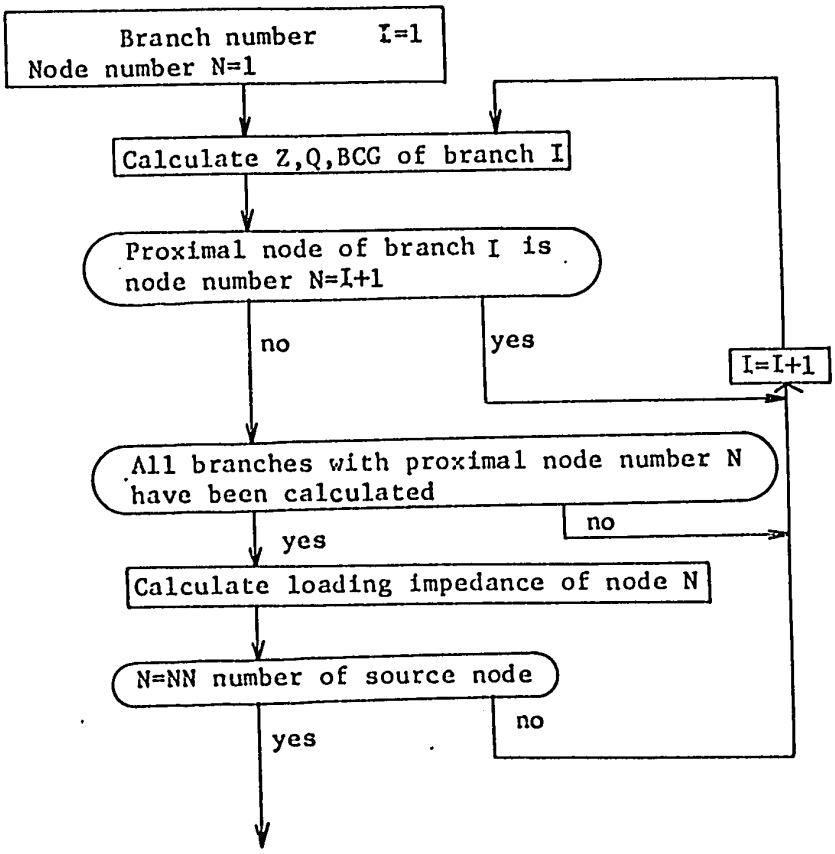


Fig. 13 Part 1 of Block Diagram

PART 2

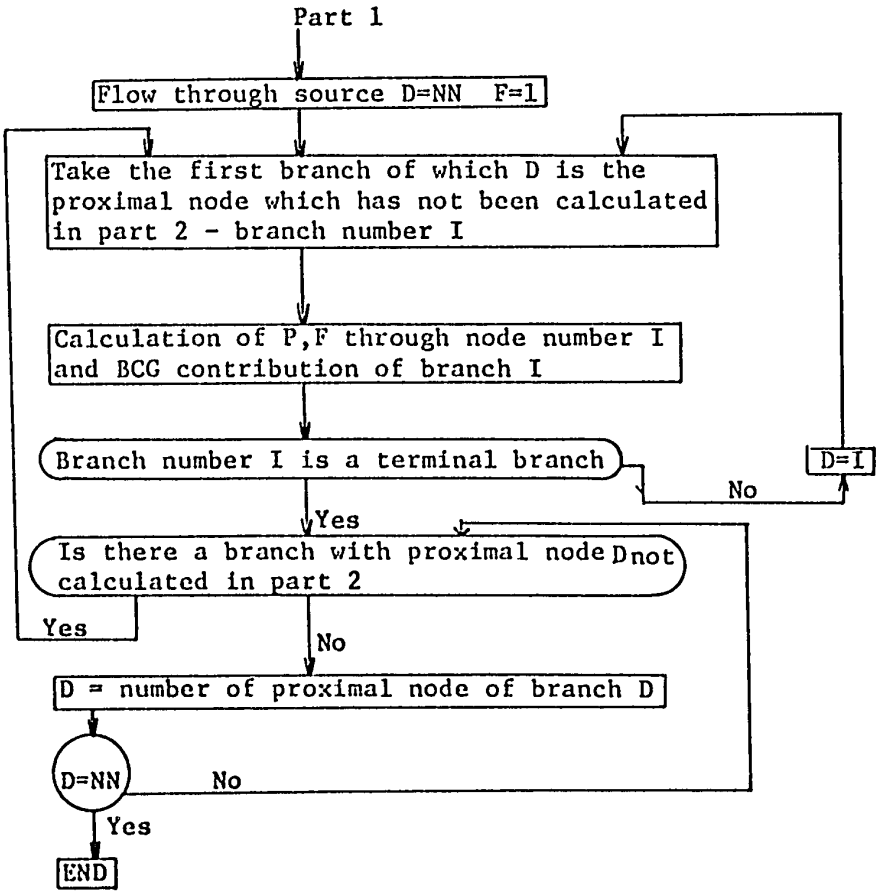


Fig. 14 Part 2 of Block Diagram

1) Impedances of the lumped model are calculated from the configuration illustrated in Fig. 15. This leads to the following expressions:

$$Z_{in} = \frac{Z_\ell + Z_\ell Z_{ti} Z_{out} + Z_{out}}{1 + Z_{ti} Z_{out}} \qquad (2\text{-}55)$$

with

$$Z_{ti} = \frac{1}{Z_t}$$

$$U_{in} = U_{out} \frac{Z_\ell + Z_{out} + Z_{ti} Z_{out} Z_\ell}{Z_{out}} \qquad (2\text{-}56)$$

U is the electrical analog of P (voltage - pressure).

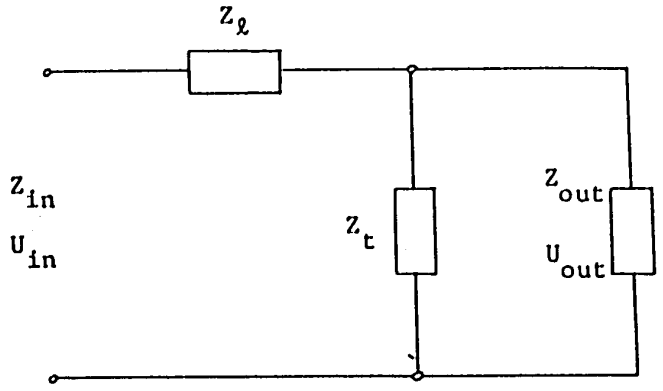


Fig. 15 One segment of a vessel

2) A note on the order of calculations performed by the program.

In each segment calculation proceeds in the following order:

At the beginning of calculations it is assumed that the outflow from the segment considered is F = 1. This means that in the case of lumped parameters Q and BCG are calculated from P and F at the distal end of the segment. For this reason calculations of the contribution of the terminal group in the modified model of small vessels (which is very important because of the number of branches involved) have to be performed using the assumption that parameters in this group are not lumped but distributed (unlike the rest of the system), since P = $\emptyset$ at the distal end of this group.

3) In part 3 of the program actual values of variables are found from assumed values (see previous section) from the condition of linearity:

$$F_{real} = F \frac{P_{real}}{P} = \frac{P_{real}}{P} \qquad (2\text{-}57)$$

Here F_{real} is the flow through the considered node when the flow from the source is

$$F_0 = 1 \qquad (2\text{-}58)$$

It is calculated in part 2 of the program. F is the flow through this node assumed in part 1. P is the pressure calculated in part 1.

A similar notation will be used for other variables throughout this section.

$$P_{real} = F_{real} \cdot Z_{out} \qquad (2\text{-}59)$$

(pressure in the considered node) and

$$BCG_{real} = F_{real} \cdot BCG \qquad (2\text{-}60)$$

$$Q_{real} = F_{real} \cdot Q \qquad (2\text{-}61)$$

4) Modified model of small vessels. Using the same expressions as in section 3 and the fact that there are 11 branches and 6 groups

$$F_{real} = 11^6 \cdot F_{real_1} = 11^6 \frac{P_{real}}{P} \qquad (2\text{-}62)$$

where F_{real_1} is the real outflow through one terminal node and F_{real} the total real outflow from terminal nodes of the last group).

In the calculation of BCG and Q the number of branches in each group is considered in part 2 of the program. For this reason

$$BCG_{real} = F_{real_1} \cdot BCG \qquad (2\text{-}63)$$

$$Q_{real} = F_{real_1} \cdot Q \qquad (2\text{-}64)$$

CHAPTER 3

Contribution of the Heart

This chapter examines the most significant contribution of the component y_{mf} - the contribution of the moving heart.

It is generally accepted that the contribution of the heart is the second most important single component of the human BCG. Two attempts to estimate the motion of the heart have been made so far. One of them (4) has been made specifically for the purpose of BCG analysis, the other one for other purposes, but its possible application for BCG analysis has been expressed in (5). The first of these two studies is of a more synthetic nature - it is based on experimental data which are used to obtain the motion of the center of gravity of the heart via simplifying assumptions. The other is analytical in that it starts from a physical description of the system, makes some simplifying assumptions and arrives at the motion of the CG of the heart. Results of these two studies are quite different. Both these studies consider only the head-foot component of the BCG. The character of the approach presented in this chapter is analytical. It uses some simplifications suggested in (5). It is, however, physically more complete.

It is assumed in (5) that the heart is a sphere with a single outlet, the cross-section of which is equal to the total cross-section of vessels leaving or entering the heart ('heart vessels' from now on). It is oriented in the head-foot (y) direction as is the direction of the blood-

flow. The bloodflow through this outlet is equal to the sum of bloodflows leaving or entering the heart. This means that the directions in which individual heart vessels are leaving or entering the heart are not considered, which is a very drastic simplification. As for other simplifications (the shape of the heart and its mechanical coupling to the chest cavity), these are made necessary by the lack of data and by physical complications which can hardly be overcome at this moment. The heart is assumed to be surrounded by a viscoelastic tissue which is, however, represented by lumped mechanical coupling (see Fig. 16). The tissue itself is enclosed in a cylindrical cavity (see Fig. 17). The only force (except for the force due to coupling and acceleration of the heart) considered is the force due to the flow through the only outlet. The dynamic equation governing the motion of the center of gravity of the heart is then

$$m_H \ddot{y} + d\dot{y} + cy = F_f \tag{3-1}$$

with

$$F_f = - v_{rel} \frac{dm_H}{dt} \tag{3-2}$$

here

m_H is the mass of the heart.

c,d are viscoelastic constants (see Fig. 16). Their derivation in (5) is described later.

F_f is the force due to outflows.

v_{rel} is the relative velocity with which the blood leaves the heart.

Comparison of results from (4) and (5) (Figs. 28,29) shows that they are completely different.

The first approach used by this author considered all six degrees of freedom (i.e. not only the displacement but also the rotation of the heart were simulated). It will be described here although it has been abandoned later for reasons that will be explained at an appropriate place in the text.

Results obtained from this first model were used as a justification for simplifications introduced into models investigated later.

The following description of the first model could, perhaps, be used as a starting point of another study.

a) the heart is a spherical cavity the volume of which is given by the instantaneous volume of blood it contains and the volume of the heart muscle. It is realized that this assumption is rather arbitrary since the heart will be considered as a solid body coupled to the external frame by mechanical springs and dashpots which neglects the motion of the vibrating tissue surrounding the heart.

b) for simplicity it is assumed that the density of the heart muscle and of the blood is 1.

c) the heart vessels considered are the same as the real ones. Their positioning on the surface of the sphere and directions in which blood flowing through them is leaving or entering the heart are chosen in such a way as to approximate the reality in an acceptable way.

d) flows through various heart vessels are taken from (8) - presumably typical normal flows.

e) the coupling of the heart to the walls of the thoracic cavity (which is assumed to be at rest - since the frequency of breathing is normally much lower than that of the heart, this seems to be an assumption which

should not affect results significantly) is viscoelastic. (In (5) only forces are considered).

(f) coupling parameters are calculated from assumed viscoelastic parameters of the tissue in the same way as in (5) for forces and from simple assumptions for moments.

It is obvious that these assumptions simplify the reality significantly. The main reasons why they were accepted are, again, the complexity and poor documentation of the real system. Without drastic simplifications simulation of this problem would not be possible at all.

a) Parameters

The mechanical coupling of the heart to the chest wall is assumed to be as in Fig. 16.

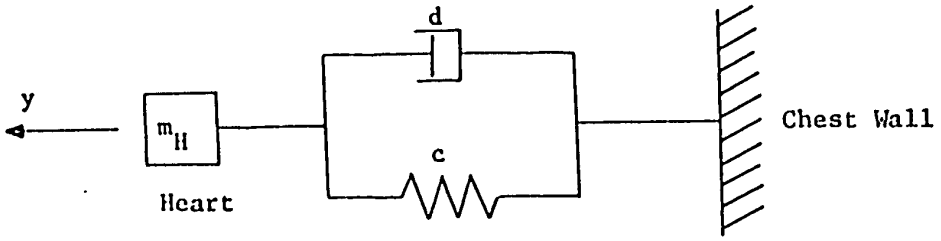


Fig. 16 Mechanical coupling Heart - Chest Wall

Index I = F is used for force coupling, M is used for rotational coupling.

The geometry assumed in (5) is accepted as the basis for the calculation of constants c and d (Fig. 17). Constants for force coupling were derived from measurements published in (9)and the assumption that the

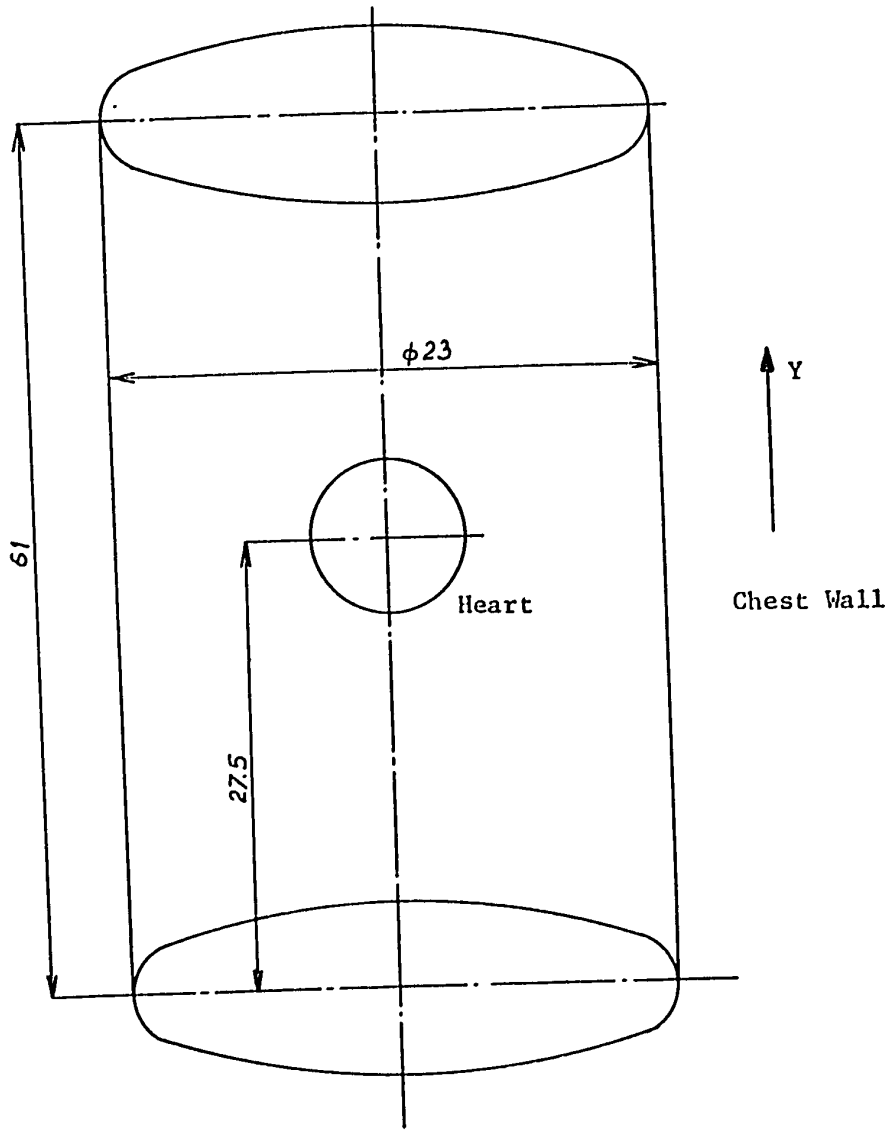


Fig. 17 Geometry assumed in (5)

heart is supported by a cylinder of a homogeneous tissue, the cross-section of which is equal to the cross-section of the heart. On the basis of these assumptions two sets of constants are calculated in (5) - one for head-foot and one for lateral coupling. These assumptions and constants were accepted here with the only change that constants were assumed to be the same in all directions. The reason for this simplification is that the geometry is so simplified that making the distinction between the two directions is hardly justified and also because the way in which coupling constants are calculated for momentum makes a similar assumption. For these reasons instead of

$$c_{FH} = 6.346 \times 10^5 \qquad d_{FH} = 1.929 \times 10^4$$

$$c_{FL} = 17.6278 \times 10^5 \qquad d_{FL} = 1.929 \times 10^4$$

(head-foot and lateral)

$$c_{FH} = c_{FL} = c_F = 6.346 \times 10^5$$

$$d_{FH} = d_{FL} = d_F = 1.93 \times 10^4$$

was assumed.

To derive c_M, d_M the geometry illustrated in Fig. 18 was accepted.

It is assumed that the heart is a cylinder coupled to a fixed outer ring by a viscoelastic material. (In the dynamic equation of the heart the mass of the surrounding tissue will again be neglected). Let us assume that under small deformation planes perpendicular to the axis of rotation remain planar and perpendicular to the axis of rotation and that

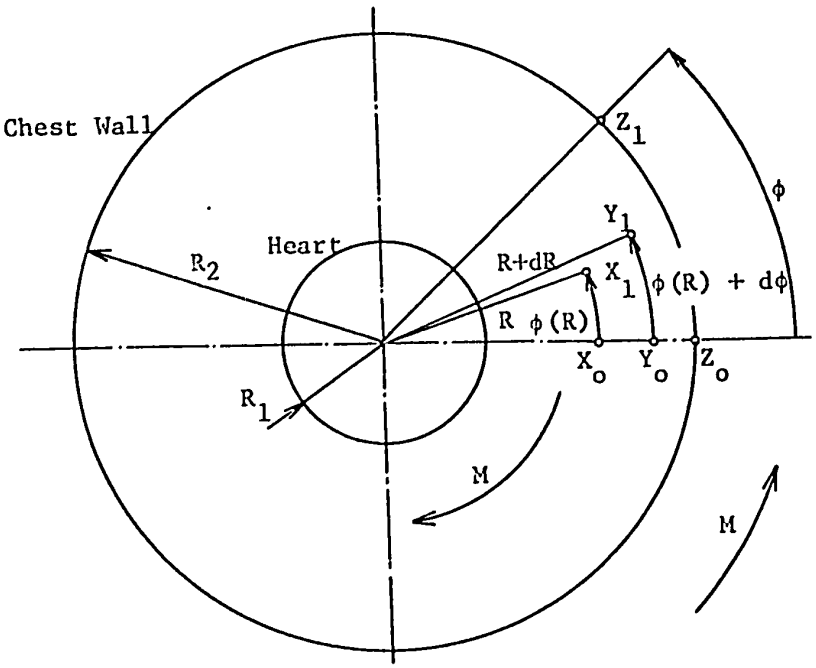


Fig. 18 Derivation of constants c_M, d_M.

radial lines in these planes remain radial under the considered deformation. Then (see Fig. 18)

$$\tau . 2\pi R\ell = M(R) = M \rightarrow \tau = \frac{M}{2\pi R\ell} \tag{3-3}$$

$$d\phi = \frac{\tau}{G} \quad \text{with} \quad G = \frac{E}{2(1 + \frac{1}{m})} \tag{3-4}$$

$$\phi = \int_{R_1}^{R_2} d\phi = \int_{R_1}^{R_2} \frac{M}{2\pi R\ell G}\, dR = \frac{M}{2\pi \ell G} \ln \frac{R_2}{R_1} \tag{3-5}$$

here

G is the modulus of rigidity of the tissue.

$\frac{1}{m}$ is Poisson's ratio which for incompressible material is .5 (this is a rather arbitrary assumption but the effect of change of parameters proved to be small)

τ is shearing stress.

At the same time we have (see Fig. 16 for coupling)

$$M = c_M \cdot \phi \tag{3-6}$$

or

$$c_M = \frac{M}{\phi} \tag{3-7}$$

so that

$$c_M = \frac{M}{\frac{M}{2\pi \ell G} \ell n \frac{R_2}{R_1}} = \frac{2}{3} \frac{\pi \ell E}{\ell n \frac{R_2}{R_1}} \tag{3-8}$$

here R_1 is the radius of the heart, R_2 the radius of the chest cavity.

For $E = 10^5$, $\ell = 10$, $R_1 = 5$, $R_2 = 12$ we obtain $c_M = 1.17 \times 10^6$.

From

$$d_F = \frac{2\pi R_1^2 \nu}{R_2 - \frac{1}{2} R_1} \tag{3-9}$$

(here R_2 is the radius of the chest cavity).

(This equation follows from the assumption that the heart with a total cross-section πR_1^2 is supported by a cylinder of tissue (viscosity ν) of effective length $R_2 - \frac{1}{2} R_1$).

and

$$d_F = 1.26 \times 10^4$$

(calculated in (5) from data published in (9)).

we get

$$\nu = 800$$

and

$$d_M = c_M \frac{\nu}{E} = 9.35 \times 10^3 \tag{3-10}$$

As stated before the same constants are assumed in all directions. Summary of parameters:

$$c_F = 6.346 \times 10^5$$
$$c_M = 1.17 \times 10^6$$
$$d_F = 1.93 \times 10^4$$
$$d_M = 9.35 \times 10^3$$

b) Dynamic Equations

The heart is assumed to have 6 degrees of freedom. The motion of the heart will be fully described by three equations for the displacement of the center of gravity of the heart, and three equations for rotation. These have to contain or be accompanied by equations for forces and moments. Let us start with equations governing the motion of the center of gravity of a body with changing mass. It is well known that the following equation

$$m_H \ddot{\bar{r}} + \sum_{\text{outlets}} \bar{v}_{rel_i} \frac{dm_i}{dt} = \bar{F}_{ext} \qquad (3\text{-}11)$$

describes the displacement of the CG of such a body.

$\bar{r}$ is the displacement of the center of gravity of the body.

m_H is the time dependent mass of the body.

v_{rel_i} is the relative velocity with which the mass is leaving the body through the I-th outlet with the rate of mass flow.

$\bar{F}_{ext}$ is the vector sum of external forces acting upon the body.

A similar equation can be derived for rotation:

Let $\bar{L}$ denote the angular momentum of the body:

$$\bar{L} = \int_{\text{volume}} \bar{r} \times \bar{v}\, dm = \int_{\text{volume}} \bar{r} \times \bar{\omega} \times \bar{r}\, dm \qquad (3\text{-}12)$$

In our case of spherical body the situation is simple since we have

$$\bar{L} = J\bar{\omega}. \quad \text{with} \quad J = \frac{8}{15}\pi r^5 \qquad (3\text{-}13)$$

$\bar{\omega}$ is the angular velocity of the body.

The equation for the change of angular momentum during an infinitesimal time interval dt under the action of external moment $\bar{M}_{ext}$ is

$$\bar{L}_H(t + dt) + \bar{L}_B(t + dt) - \bar{L}_H(t) = \bar{M}_{ext}.dt \qquad (3\text{-}14)$$

(subscript H for the heart, B for the blood that left the heart in the interval dt) or in our case

$$(J + dJ)(\bar{\omega} + d\bar{\omega}) + \bar{r}_{out} \times \bar{u}\frac{dm}{dt} - J\bar{\omega} = \bar{M}_{ext} \qquad (3\text{-}15)$$

here $\bar{r}_{out}$ is the position vector of an outlet with respect to the center of the sphere, $\bar{u}$ the velocity of blood leaving the heart. We will assume only one outlet for simplicity. This leads to

$$\frac{dJ}{dt}\bar{\omega} + J\frac{d\bar{\omega}}{dt} + \bar{r}_{out} \times \bar{u}\frac{dm}{dt} = M_{ext} \qquad (3\text{-}16)$$

or

$$J\dot{\bar{\omega}} + \bar{\omega}\frac{dJ}{dt} + \bar{r}_{out} \times \bar{u}\frac{dm}{dt} = M_{ext} \qquad (3\text{-}17)$$

when we replace $\bar{u}$ by

$$\bar{u} = \bar{u} - \bar{v}_{rel} + \bar{v}_{rel} \qquad (3\text{-}18)$$

this becomes

$$J\dot{\bar{\omega}} + \bar{r}_{out} \times \bar{v}_{rel} \frac{dm}{dt} = \bar{M}_{ext} \qquad (3\text{-}19)$$

External Forces

We have two kinds of external forces - one is due to the mechanical coupling, the other is due to the fact that the heart is connected to heart vessels which pull at the heart because of the forces generated in them by the changing direction of blood flowing in them. This concerns mainly the aorta and the pulmonary artery.

Forces due to coupling are described by

$$\bar{F}_c = c\,\bar{r} + d\,\dot{\bar{r}} \qquad (3\text{-}20)$$

A similar equation is used for moments.

The last component of the external force to be considered here is the force developed by the changing direction of flow in the heart vessels which is transmitted to the heart through the walls of these vessels. Only the force developed in the aorta and in the pulmonary artery will be used, the others being considered negligible.

A. The Aorta

It will be assumed that the shape of the aorta is circular right from the aortic root. Since a part of the blood flowing through the aorta is leaving it approximately at the highest point of the aortic arch (see Fig. 19) an outlet will be assumed at this point and the amount leaving here will be taken as

$$f_{out}(t) = \varepsilon\, f(t) \qquad (3\text{-}21)$$

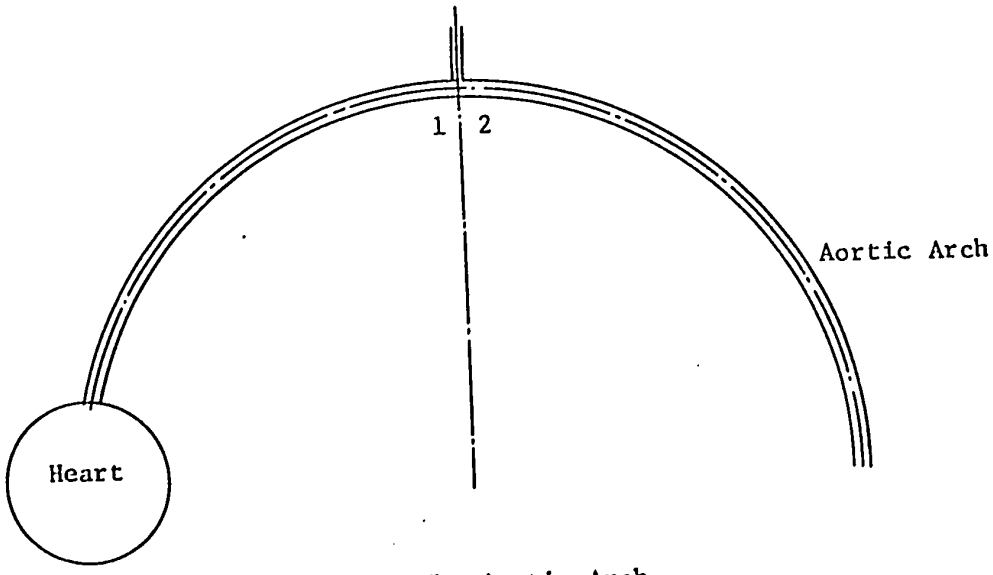


Fig. 19 The Aortic Arch

here f_{out} is the bloodflow at point 2.

f is the bloodflow at point 1 immediately before the outlet.

ε is taken to be equal to .4.

It is realized that this is a simplification.

An infinitessimal mass dm moving along an arch of radius R with velocity v causes a force

$$dF = \frac{v^2 . dm}{R} \tag{3-22}$$

which is pulling radially from the center of the arch.

Let us assume that the velocity of flow at time t and point (R,ϕ) (polar coordinates) (see Fig. 20) is (assuming an aorta with uniform parameters)

$$v(t,\phi) = v(t - \frac{R(\phi-\alpha)}{c_p}, 0) = v(t - \frac{R(\phi-\alpha)}{c_p}) \text{ for } \phi < \pi \tag{3-23}$$

and

$$v(t,\phi) = v(t - \frac{R(\phi-\alpha)}{c}).(1 - \varepsilon) \quad \text{for } \phi > \pi \tag{3-24}$$

where c_p is the velocity of transmission of a pressure pulse. Let us take $c_p = 500$ cm/sec.

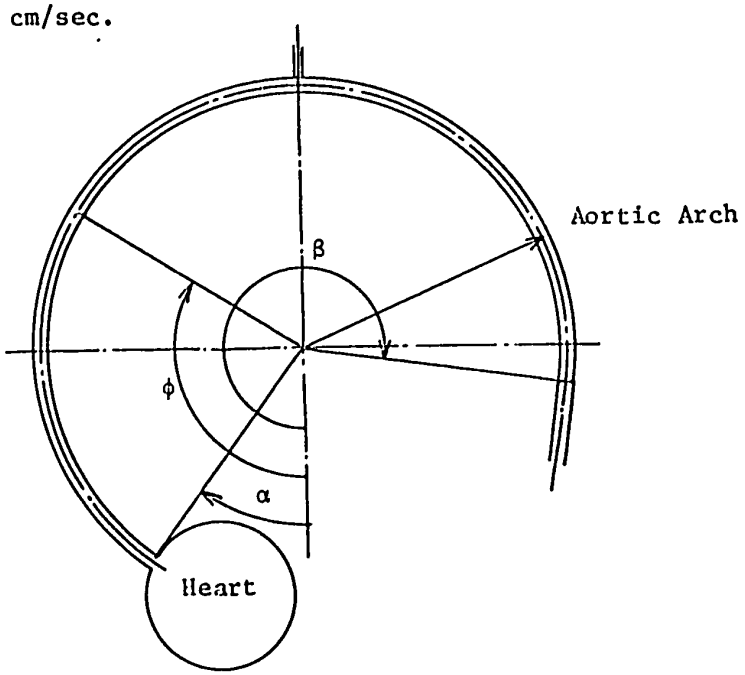


Fig. 20 The Aortic Arch

We have

$$dF = \frac{v^2.dm}{R} = \frac{v^2 \rho A R d\phi}{R} = v^2 \rho A.d\phi \tag{3-25}$$

where A is the area of the cross-section of the vessel, assumed to be constant.

For the two components of the total force in the plane of the aortic arch we have (assuming $\rho = 1$)

$$F_H = \int_\alpha^\beta dF_H = -\int_\alpha^\beta v^2(t,\phi) A \sin\phi \, d\phi \tag{3-26}$$

$$F_y = \int_\alpha^\beta dF_y = -\int_\alpha^\beta v^2(t,\phi)A \cos\phi \, d\phi \qquad (3\text{-}27)$$

Expressions (23,24) are substituted into (26) and (27) and the resulting integrals evaluated numerically. The force F_H is then decomposed into its components F_x, F_z according to Fig. 21. (Z is the vertical axis, X horizontal oriented to the right hand side of the subject).

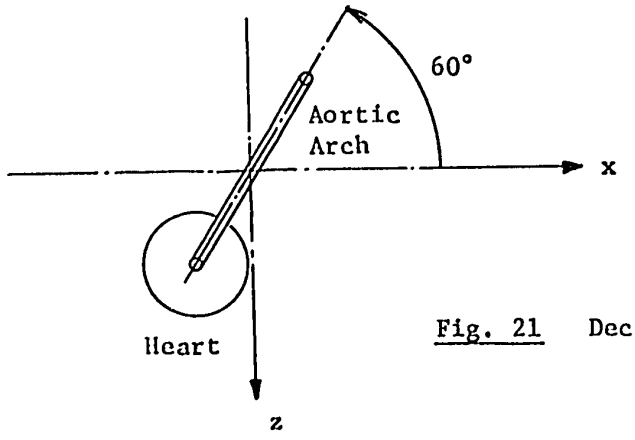


Fig. 21 Decomposition of F_H.

The next step is to obtain the force acting upon the heart. To do this it is assumed that the physical situation can be simplified to that illustrated in Fig. 22 (zero moment)

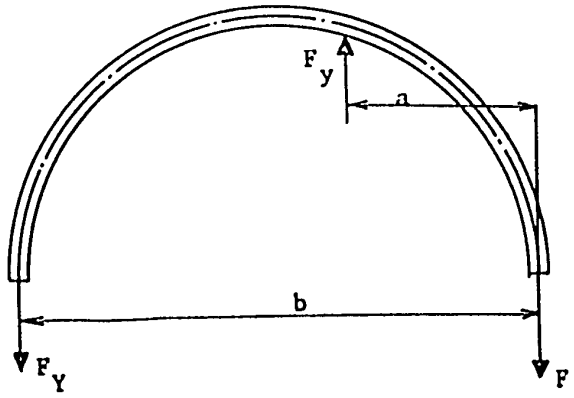


Fig. 22 Decomposition of F_y.

The component F_Y is then obtained from

$$F_Y = F_y \frac{a}{b} \tag{3-28}$$

F_X, F_Z are assumed to act directly upon the heart.

B. The Pulmonary Artery

With the notation of Fig. 23 we can derive the force due to the change of direction of flow in the following way:

$$dm.v_y + F_{yp}.dt = dm.v_y(t + dt) = 0 \tag{3-29}$$

dt is the time in which the velocity of dm changes from v_y to zero, i.e. changes direction by 90 degrees. This gives

$$F_{yp} = - v_y.\frac{dm}{dt} \tag{3-30}$$

This force acts upon the heart directly and is parallel to the y axis.

We assume again

$$v(t,\ell) = v(t - \frac{\ell}{c_p},0) \tag{3-31}$$

and similarly

$$\frac{dm}{dt}_{t,\ell} = \frac{dm}{dt}_{t - \frac{\ell}{c_p},0} \tag{3-32}$$

ℓ is the length of the main pulmonary artery.

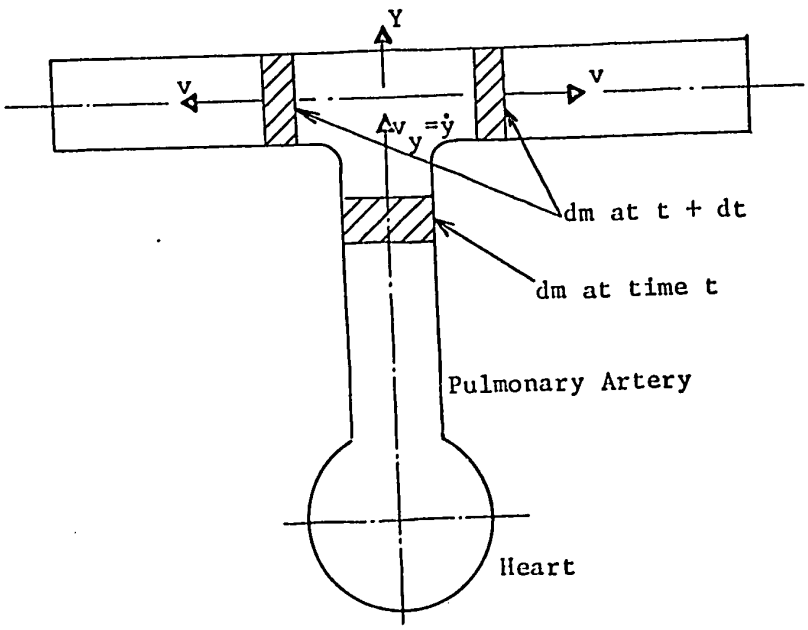


Fig. 23 Flow in the Pulmonary Artery.

This concludes the description of forces acting upon the heart. Let us note that because of the effect of smallness of distances with respect to c the effect of forces due to outflow and forces due to the change of direction of the flowing blood in the y direction practically cancel one another except for the first part of ejection - see Fig. 24.

C. Physiological and Anatomical Data

It was not easy to gather the necessary data and some new assumptions had to be made. Flows in various heart vessels (see Fig. 25) are from (8,13) and represent a normal heart.* Data about the mean volume of atria and ventricles as well as the mass of the heart muscle were taken from (15). The following table (Table 1) summarizes cross-sections and positions of outlets and directions of flows, which are deduced from

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* Flows measured in dogs are extrapolated to man

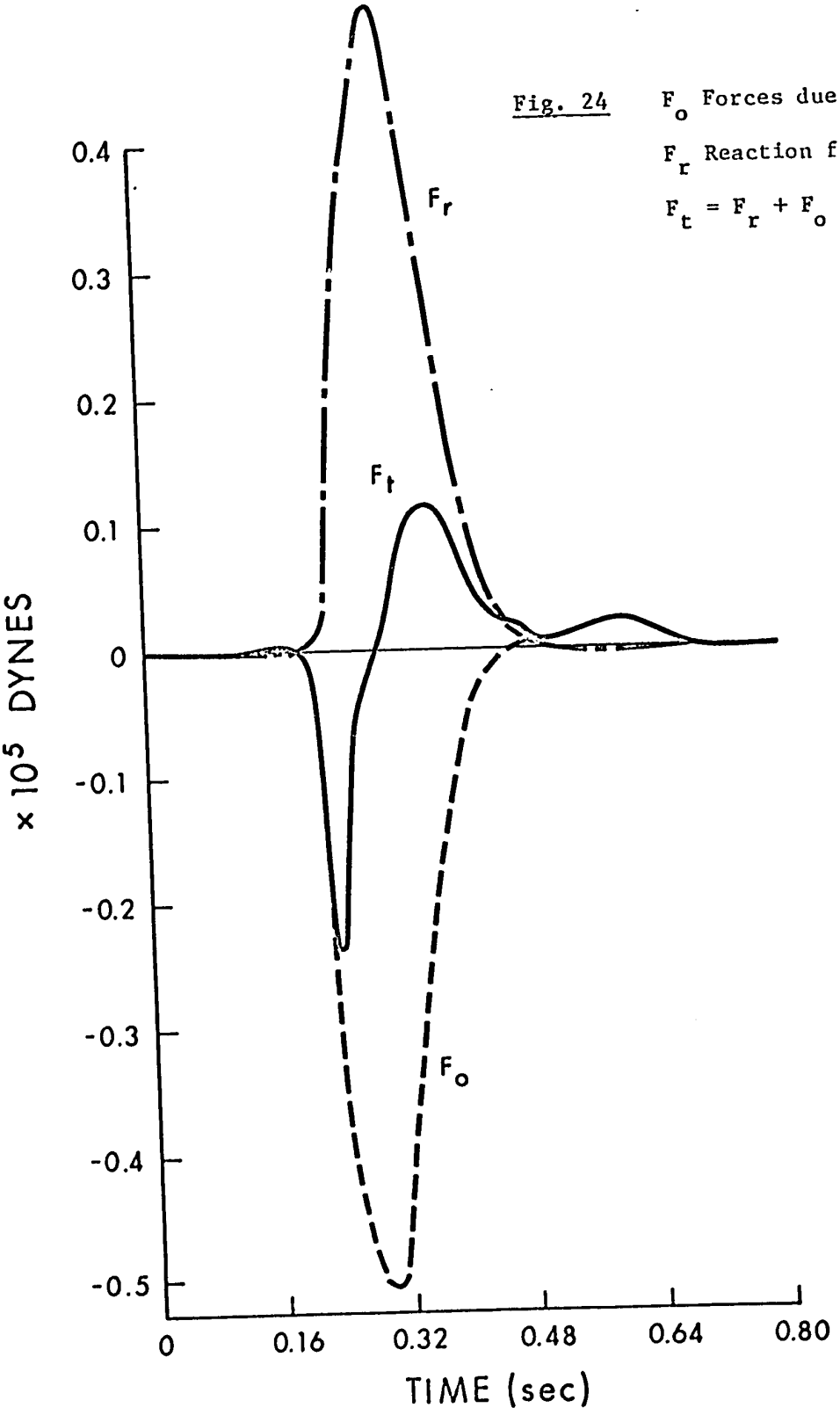


Fig. 24

F_o Forces due to outflow

F_r Reaction from vessels

$F_t = F_r + F_o$

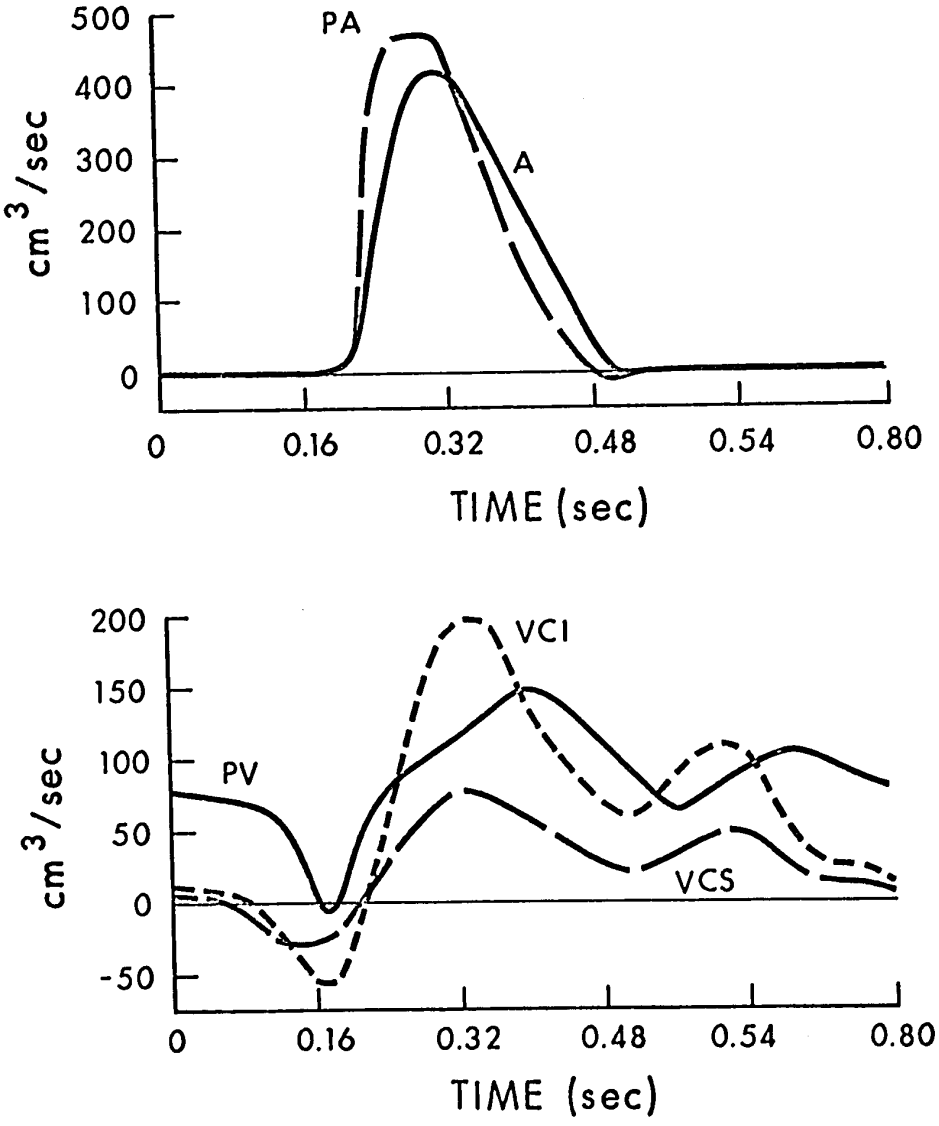


Fig. 25 Flows in: PA Pulmonary artery
A Aorta
VC_I Vena Cava Inferior
VC_S Vena Cava Superior
PV Pulmonary Veins

illustrations in various anatomy textbooks. Positions of outlets are given as positions of points on a sphere, the diameter of which is the mean diameter of the heart corresponding to its mean volume. Directions of flows are given by end points of vectors of unit length (this is convenient for calculations).

(axis z is perpendicular to the sagital plane, y is head-food).

Table 1. Some anatomical data.

		vena cava		aorta	pulmonary artery	pulmonary veins	
		superior	inferior				
Position of outlets	x	-3.9	-3.9	0	-1	1	-1
	y	3.9	-3.9	5.4	5.5	4.5	4.5
	z	-1.6	-1.6	-1.5	-1	-3.3	-3.3
Direction of flow	x	0	0	0	.22	.67	-.63
	y	-1	.99	.93	.72	.17	0
	z	0	-.17	-.37	-.65	-.72	-.77
Cross-section (cm^2)		3.02	3.83	6.8	7.1	4.64	

To solve the described system of equations a computer program using the HPCG subroutine from the scientific subroutine package of IBM (14) was written. In order to include the effect of rotation of the heart, the motion of three points (at time t = 0 unit vectors in the x, y and z directions) connected to the moving heart is calculated. We thus obtain a set of 21 simultaneous differential equations: 6 for the position and velocity of the CG of the heart, 6 for angular position and velocity

describing the rotation of the heart and 9 for the position of three reference points. The last three vectors were continuously normalized in order to prevent the effect of numerical error from distorting the mutual positions of these orthonormal vectors. The details of this program will not be described.

When this set of equations was solved it was clear that the resulting rotation (as described by the three reference vectors) is relatively so small (the largest component - originally of length 1, has never been changed by more than .08) that in view of other simplifications it seems admissible to consider only the motion of the CG of the heart instead of the whole rotating sphere. This is very important since the number of equations to be solved simultaneously is reduced from 21 to 6. Since these equations are decoupled (the effect of variables from one component of the three vector components does not influence equations for the two remaining components) we can solve just 2 or 4 of these equations if we are not interested in all components (which is usually the case).

Only displacement (in the y axis) was investigated from this point on. As it was decided to simulate the effect of several factors (to be discussed later) separately it would have been expensive to do it on the complete model and the simplified model neglecting rotation was simulated. For this reason only results obtained from this simplified model will be shown and discussed here.

Another change introduced at this point concerns the value of mechanical constants. The way in which constants, c,d, were obtained in

(5) is not very reliable because of the uncertainty of the value of the complex modulus of elasticity. (10) contains values of the natural frequency and damping of the heart inside the thoracic cavity (f_r = 8 cps, δ = .25). If it is accepted that the system can be described by

$$m_H\ddot{y} + d\dot{y} + cy = F_{ext} \tag{3-33}$$

then free oscillations are given by

$$y(t) = a\, e^{\alpha t} \cos(\omega t + \phi) \tag{3-34}$$

with

$$\alpha = -\frac{d}{2m_H} \tag{3-35}$$

and

$$\omega = \frac{1}{2m_H}\sqrt{4m_H c - d^2} = 2\pi f_r \tag{3-36}$$

Since

$$\delta = \left|\frac{y_2}{y_1}\right| \tag{3-37}$$

(y_1 and y_2 are amplitudes of two waves separated by time $\frac{\pi}{\omega}$)

m_h, f_r, δ can be used to calculate c and d. The values obtained and used from here on are

$$c = 6 \times 10^5 \qquad d = 2.12 \times 10^4$$

When assumptions described above (except for rotation) were used to simulate the y BCG, results in Fig. 27 were obtained. It was then decided to try to incorporate the effect of the changing distribution of

blood in the heart and the shape of the heart itself during the heart cycle. It seemed possible to do this by changing equations for the coupling forces since the tissue is touching the surface of the heart, assumed to be spherical, the force generated by the mechanical coupling is related to the position and velocity of the center of the sphere. This is a good opportunity to incorporate the changing distribution of mass inside the heart.

Let us accept the following model of the heart:

The spherical heart is hollow and divided into compartments - let us consider only two compartments for simplicity (one has a volume equal to the combined volume of both atria, the other has volume equal to the combined volume of the two ventricles). Let us assume that these volumes are constant (equal to the mean volumes) but that the mass contained in them is changing according to real volumes of the respective chambers. From this model the instantaneous position of the heart's CG with respect to the center of the sphere can be easily calculated. If $\bar{\ell}$ denotes the position of the CG of the heart along its longitudinal axis w.r.t. the center of the heart, $\bar{\ell}_1$, $\bar{\ell}_2$ the fixed distances of CG's of the two compartments to the center and m_1, m_2 masses of blood in these compartments; m_{HB} the total mass of blood inside the heart, we have

$$m_{HB}\bar{\ell} = m_1\bar{\ell}_1 + m_2\bar{\ell}_2 \tag{3-38}$$

Since the two compartments have nearly equal mean volumes (their centers of gravity are assumed to lie on the axis of the heart) we have

$$\bar{\ell}_1 \doteq -\bar{\ell}_2 = \bar{\ell}_0 \tag{3-39}$$

so that

$$m\bar{\ell} \doteq (m_1 - m_2)\bar{\ell}_0 \tag{3-40}$$

The force due to mechanical coupling is then calculated from the position of the center of the sphere as

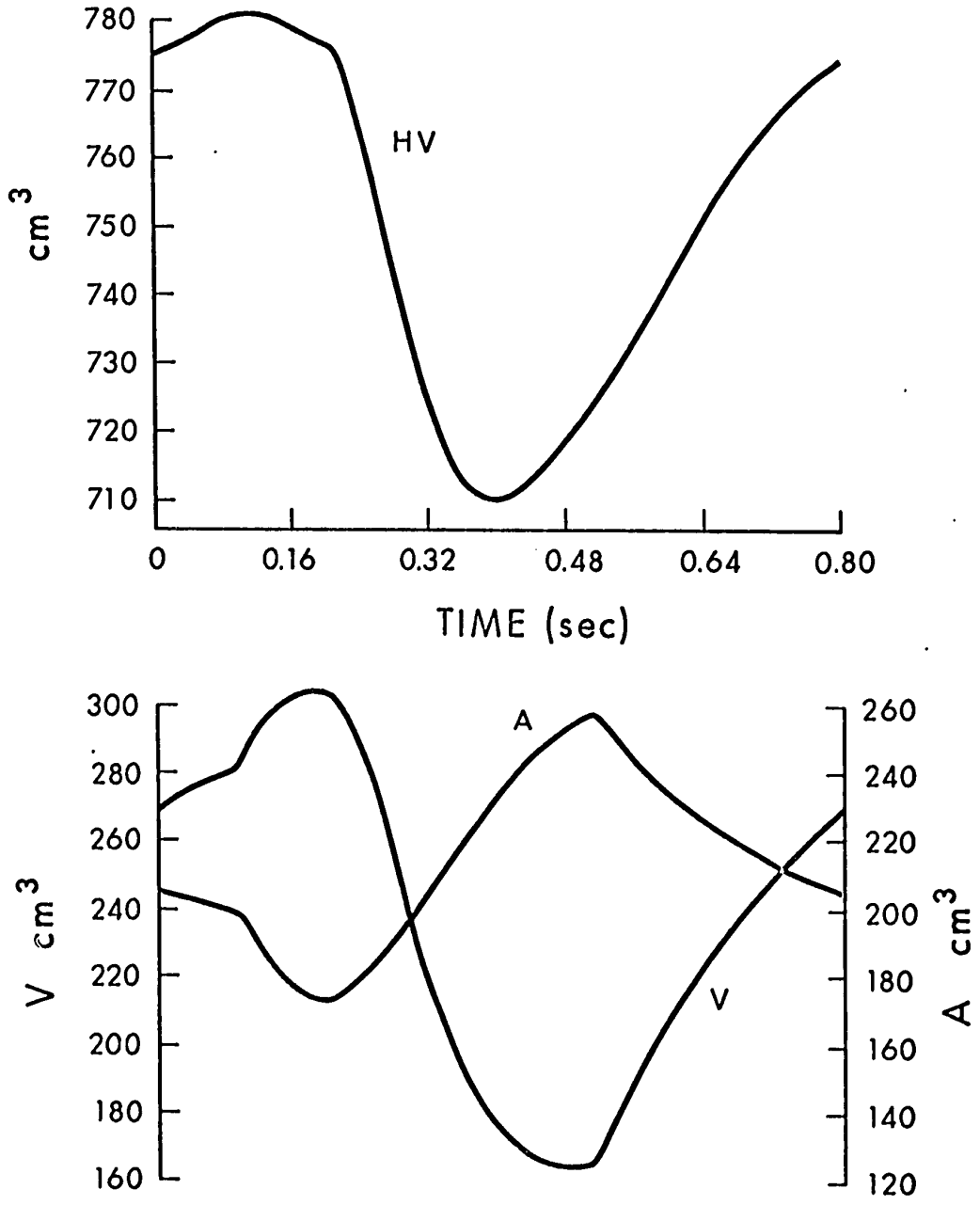
$$F_c = c(\bar{r} - \bar{\ell}) + d(\dot{\bar{r}} - \dot{\bar{\ell}}) \tag{3-41}$$

To recapitulate: the equation of motion contains coordinates of the center of gravity of the heart, which is not identical with the geometric center of the spherical heart. The force due to coupling is calculated from the position of the center of the sphere, which is obtained from the position of the CG calculated from the equation of motion and from the calculated position of the CG w.r.t. the center of the heart.

No experimental values have been found for the time-dependence of the flow between the atria and the ventricles. For this reason a calculated shape of the time dependence of the volume of the ventricles has been taken from (11) and the volume of the atria calculated from this and flows through the heart vessels (Fig. 26).

Results obtained from this modified model are shown in Fig. 27, which also includes the acceleration due to flows and the combination of these two factors.

The following modification was then considered: as stated above, the heart does not really behave as an isolated single mass but rather as a mass surrounded by a continuum moving with it. It would not be



HV = Total heart volume

A = Atrial volume

V = Ventricular volume

Fig. 26

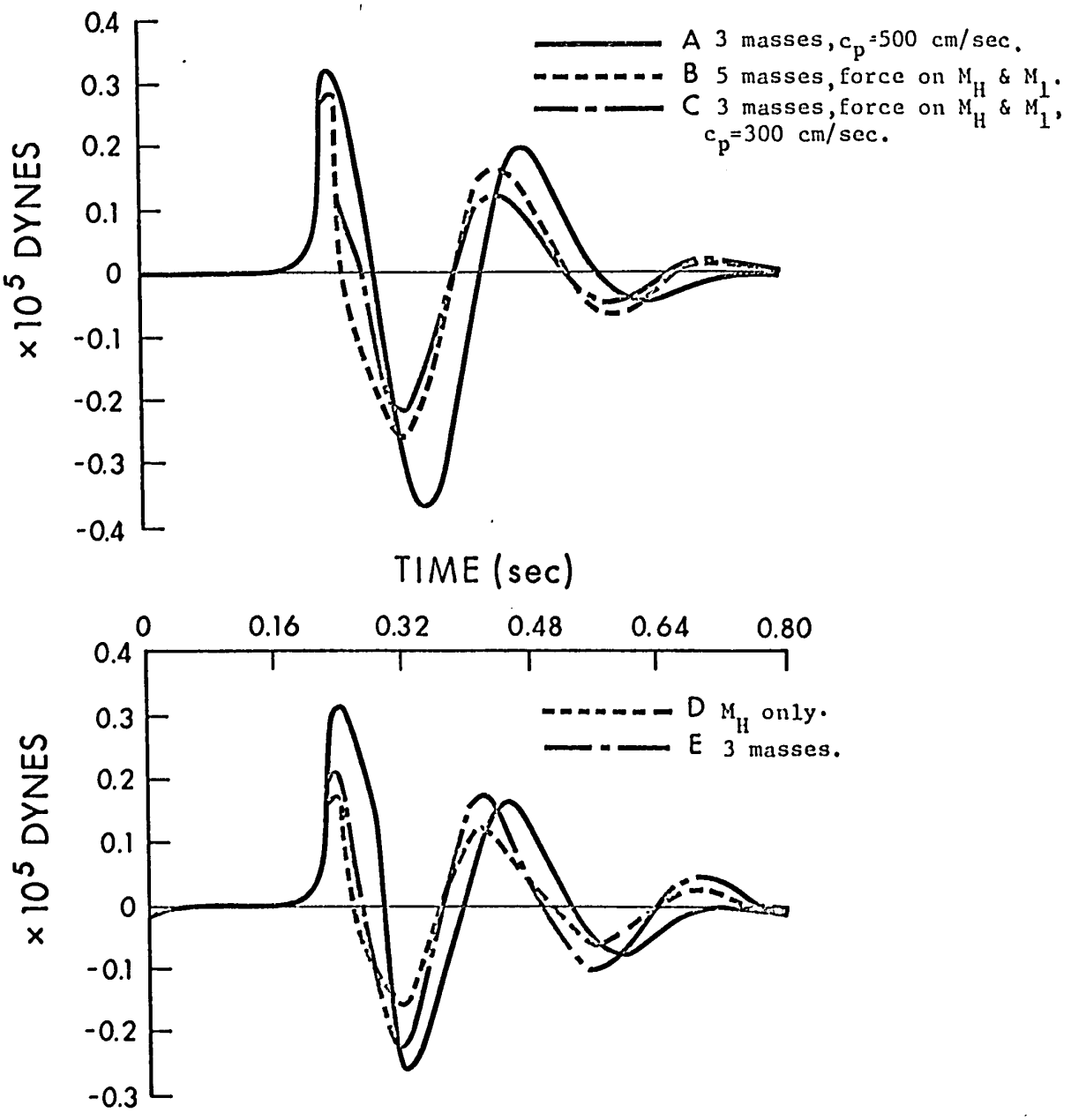


Fig. 27 BCG Contribution due to outflow only; c_p = 500 cm/sec (except for C).
A - Force due to outflow only.

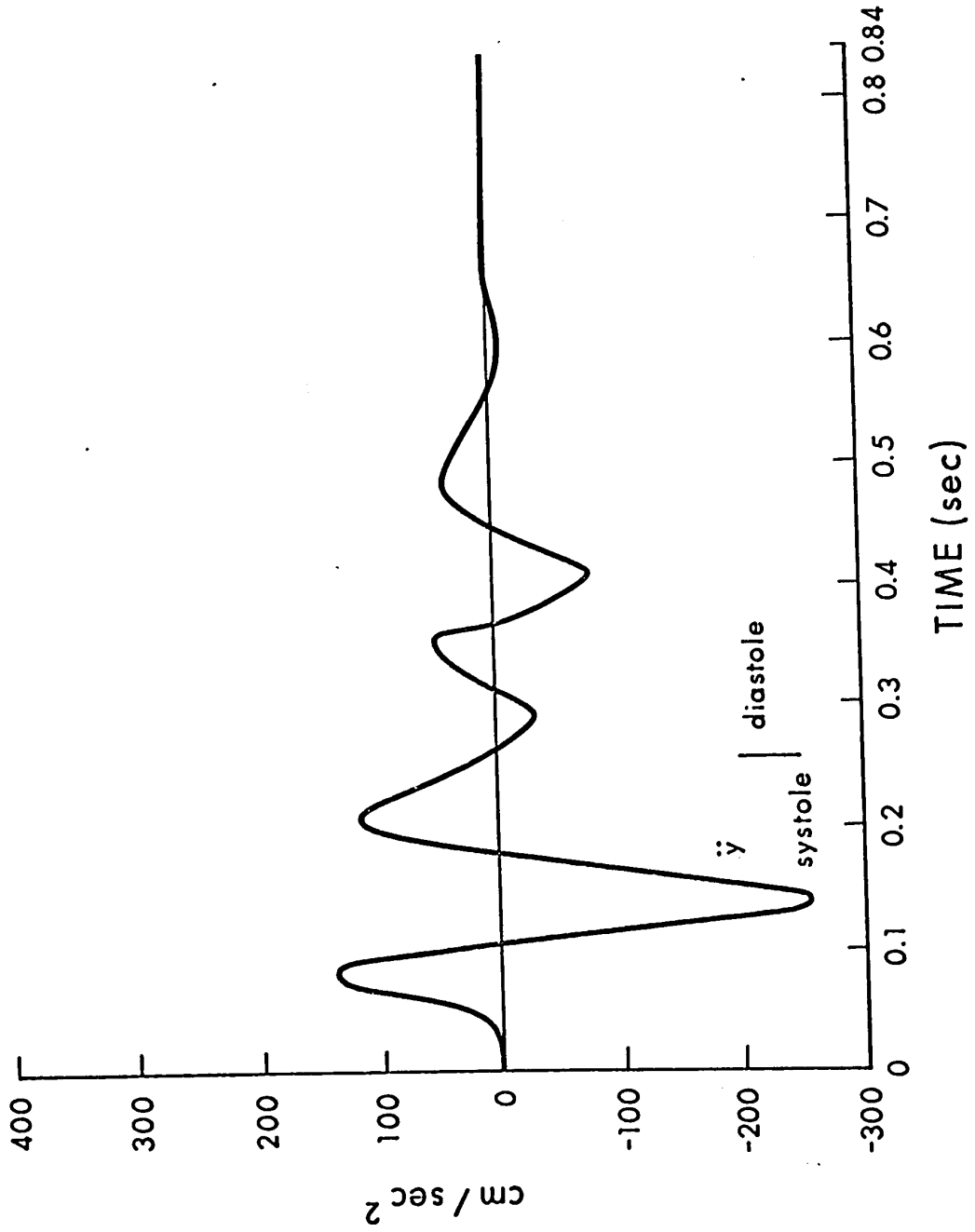


Fig. 28 Acceleration of the Heart (from (5)).

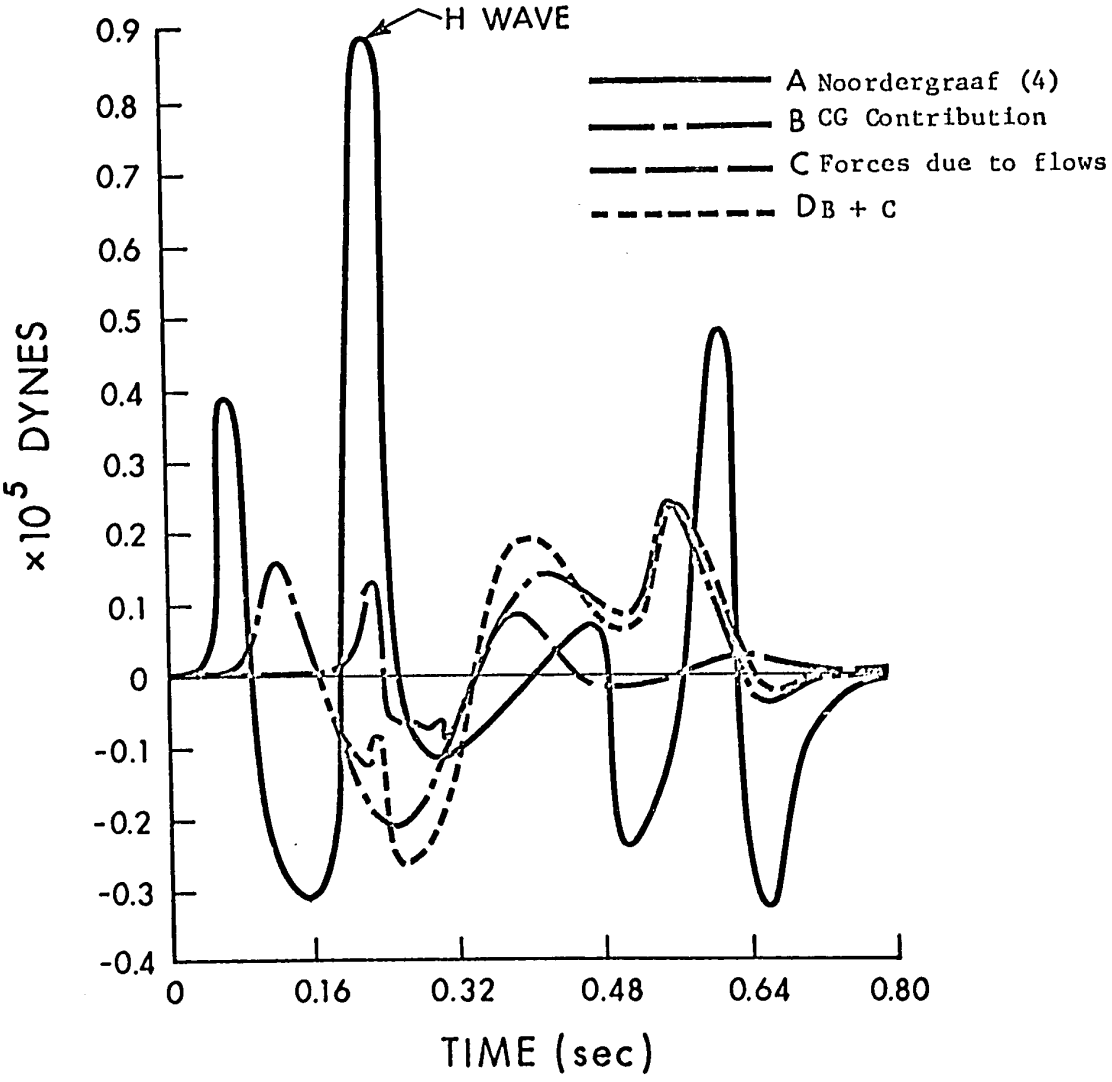


Fig. 29 BCG Contribution without continuum.

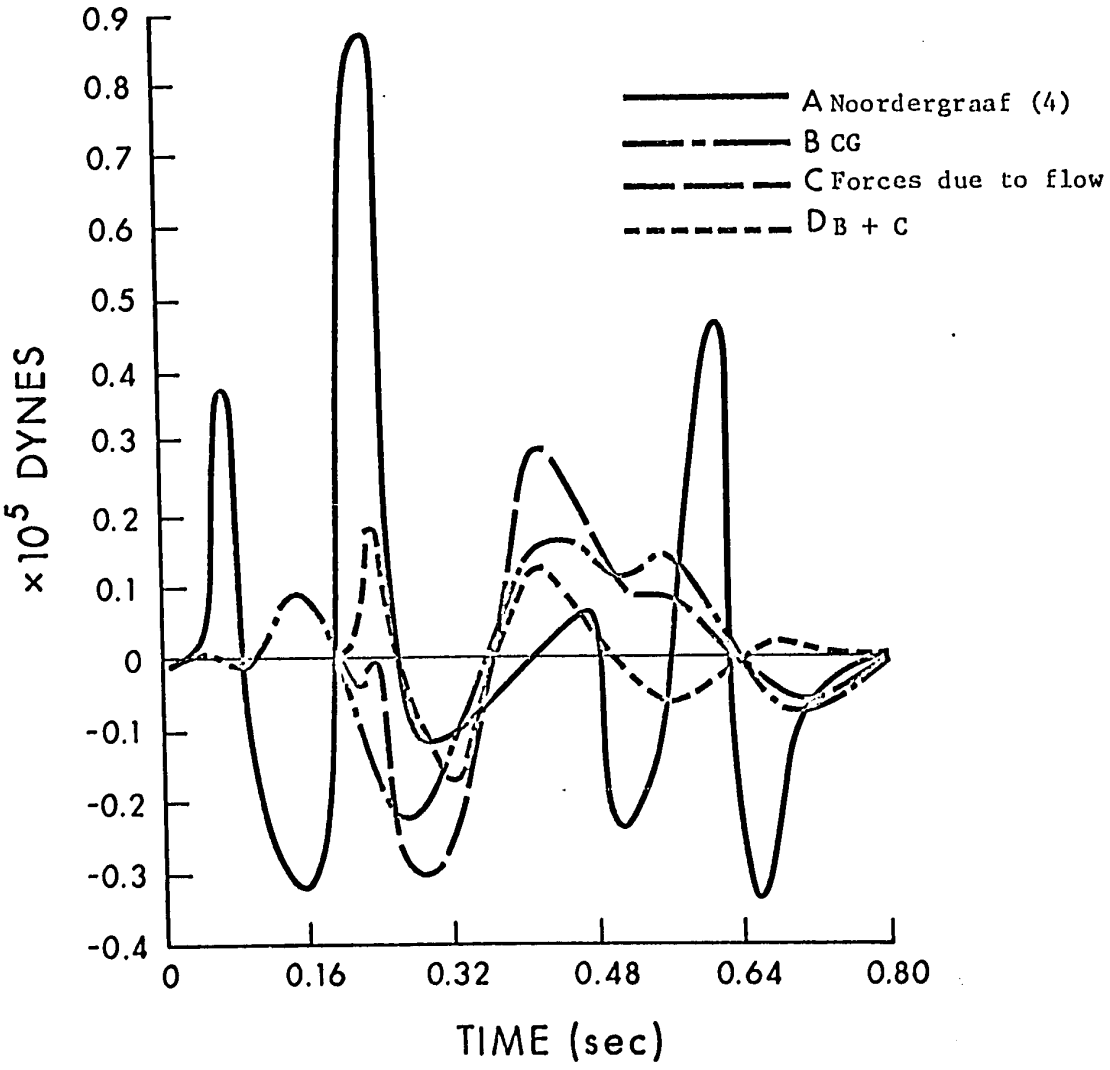


Fig. 30 BCG Contribution with Continuum.

practically possible to investigate this situation in this form. It seems, however, that the model could be made more realistic by lumping the continuum in a way illustrated in Fig. 31.

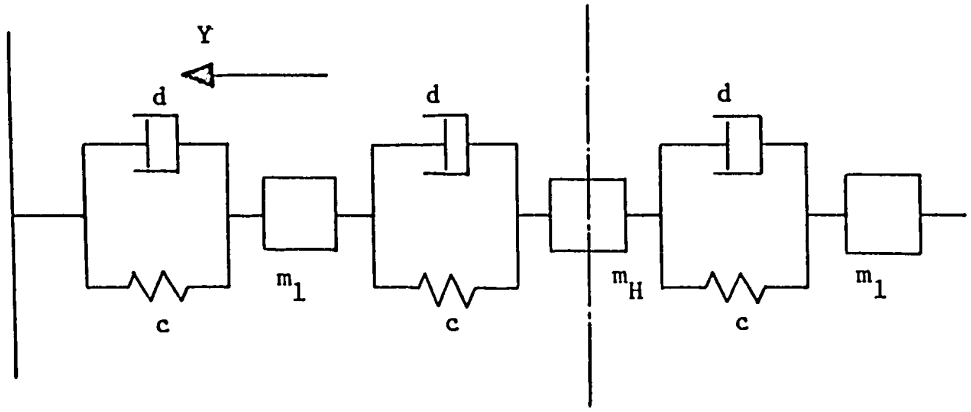


Fig. 31 A Symmetrical Model of the Surrounding Tissue.

Physically this means that the thoracic cavity (or that part of it which is assumed to be affected by the motion of the heart) is divided into sections, which are coupled to one another by springs and dashpots. The coupling was chosen as symmetrical and constants such that the overall spring and dashpot constants remained unchanged (Fig. 32).

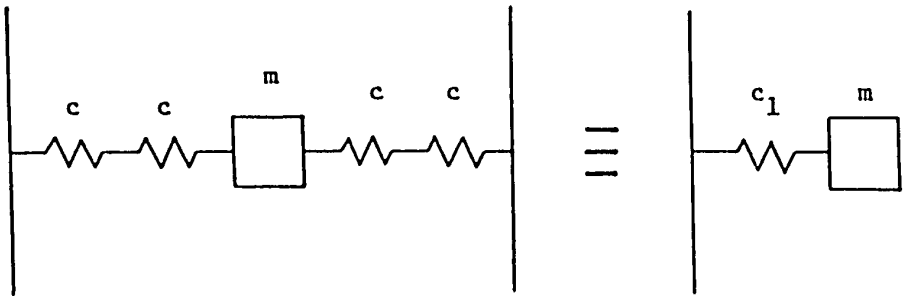


Fig. 32 Equivalent Springs.

$$c_1 = c \tag{3-42}$$

$$d_1 = d \tag{3-43}$$

Results obtained from this model are in Figs. 27,30.

It was decided to try several sections to see the effect of lumping. For this purpose the situation illustrated in Fig. 33 was simulated.

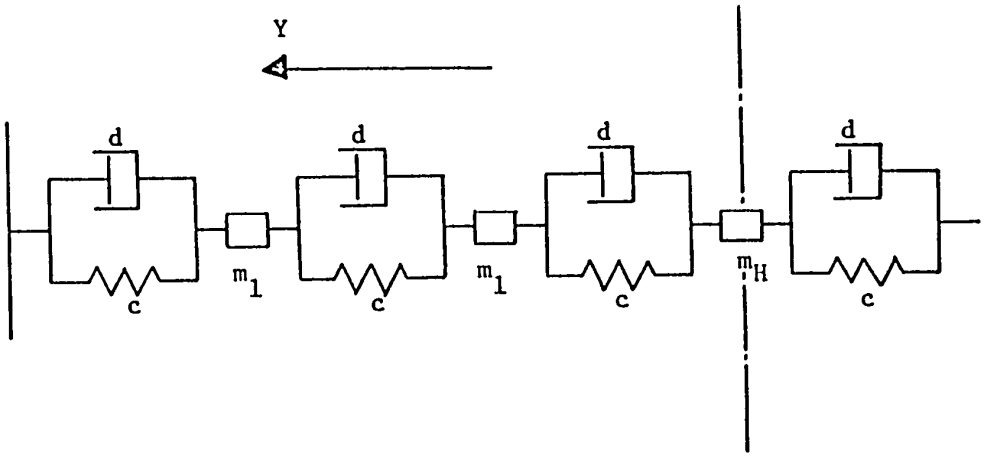


Fig. 33 Lumped Continuum.

In this case

$$c_1 = \frac{2}{3} c \tag{3-44}$$

$$d_1 = \frac{2}{3} d \tag{3-45}$$

The last modification was that the force due to the changing direction of bloodflow was taken as acting upon the section immediately above the heart (Fig. 34) and not the heart itself. Results are in Fig. 27 which also includes the contribution of the heart due to outflow

only.

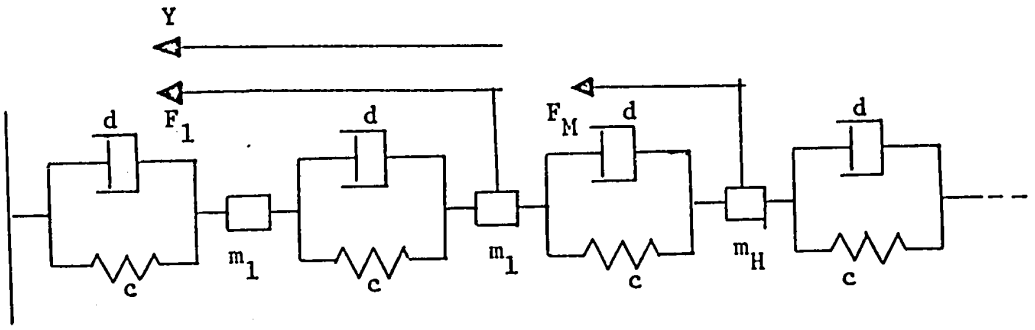


Fig. 34 Lumped Continuum.

Discussion of Results

Before results obtained from the described models are compared and discussed, some general notes must be made:

Probably the only part of the BCG contribution of the heart that can be compared with experimental results is the H wave. This is because the H wave occurs around the beginning of the ventricular systole (12) before any significant effect of the remaining part of the circulation can be observed and also because the contribution of the normal heart in other parts of the BCG is relatively small. When the result published in (4) (Fig. 29) is examined from this point of view and compared with normal BCG's (11), it must be concluded that the amplitude of the H wave obtained in (4) is about twice the normal one (taking into account the variability of the H wave in normal BCG's).

Let us now compare results obtained from the model, considering forces due to outflow and reactions but not the effect of the CG of the heart, with results from (4). It can be seen from Fig. 28 that the timing of the H wave and its orientation agree. The rest of the two results do not agree quite so closely but, as already mentioned, it is difficult to decide about the contribution of the heart except for the H wave. The amplitude of the H wave as calculated here is smaller than that obtained in (4) and also smaller than amplitudes normally observed (between 30,000 and 50,000 dynes). Reasons for the last discrepancy could be in the shape of flows used for calculations (the rate of change of flow plays an important role as can be judged from Fig. 24), in the accepted geometry of the aortic arch (larger distances lead to longer delays between the two forces in Fig. 24) (this seems to be in agreement with the fact that people with otherwise similar BCG records may have quite different amplitudes of the H wave), in the assumed dependence of flow velocity upon the distance from the heart, the way in which the reaction from the aorta and the pulmonary aorta are transmitted to the heart and the way in which the continuum was represented by segments in our model. Also the effect of the changing shape has not been included yet. It should be noted here that the contribution of the heart calculated in (4) has been redrawn for this thesis from a small picture with a different time scale so that some differences can have origin in inaccuracies resulting from this approximation.

Results obtained from assumptions made about the effect of the changing blood distribution inside the heart cavity are not acceptable when the criterion of comparison with the H wave is used.

This may have two reasons:

1) assumptions used for the calculation of the CG of the heart are not realistic (for example the real distribution of the heart muscle is not considered at all).
2) knowledge of time courses of ventricular and atrial volumes used for the calculation of the CG of the heart is not good (this is an important factor because we are differentiating these variables twice and thus local details are significant).

The assumption of 'lumped continuum' not only seems to be realistic but also improves results of calculations.

Results from (5) are summarized in Fig. 28. They will not be discussed here since the author thinks that they were obtained from an overly simplified description of the system. (see page 50).

Summary of Chapter 3

This chapter contains the first complete enumeration and discussion of factors associated with the motion of the heart - forces causing it and reactions. It is suggested that a model considering only displacements and neglecting the rotation of the heart is sufficient for the simulation of the BCG contribution of the heart since the effect of rotation is small.

Factors affecting the motion of the heart are:

1) forces due to the blood leaving the heart and entering it.
2) forces due to mechanical coupling: heart - surrounding tissue - chest wall.

3) forces due to the changing directions of flow through the aortic arch and the pulmonary artery.
4) forces due to the changing shape (and position of the CG) of the heart.
5) forces due to the mechanical coupling with the tissue surrounding the heart and moving with it as a continuum.

Equations describing factors 1 to 5 are derived and used for the simulation of the motion of the heart. For the simulation of factor 4, a simple model making use of the coupling of the surface of the heart with the surrounding tissue is suggested. Factor 5 is simulated by a lumped representation of the motion of the continuum.

Individual factors and their effect on the motion of the heart are examined separately (factors 1 and 3 are considered together) because the differential equations representing the motion of the heart are linear. This makes it possible to estimate the validity of description of individual factors.

It is concluded from results obtained from this simplified approach that all factors, except for factor 4, are described adequately and that a reformulation of their representation will not be necessary in future research. Factor 4, on the other hand, is not described adequately and more research will be needed to simulate its effect. This author believes, however, that the method of representation of this factor as proposed here is valid and that only a more sophisticated geometrical description and a better knowledge of the hemodynamic variables affecting its influence on the motion of the heart will be needed to obtain realistic results. This will require a complete model of the circulation. For this purpose

analogs seem to be better suited than digital computers. The amount of experimentation involved in simulation could be too expensive if a digital computer were used. In fact, once a linearized description of the problem at hand is accepted it is no longer justifiable to use a digital computer for its simulation.

Summary

In the first part of this thesis a new method for the calculation of the largest part of the human BCG (due to the arterial part of the circulation) is proposed. It is easy and economical to use. It can also be used as a diagnostic tool for the design of analogs or for checking the accuracy of existing analogs. It is used for this purpose here and an analog of the human circulation is evaluated from the point of view of BCG simulation. A new way of representation of small vessels for BCG simulation is suggested.

In the second part (chapter 3) the contribution of the heart to the total BCG is simulated. Two methods for the solution of this problem have been published - (4,5), one of them (4) cannot be used for direct simulation (by an analog or digital computer). The other is over-simplified and incomplete. The approach suggested here is complete in that it considers all physical factors causing the motion of the heart. More research will be needed to describe more accurately those of the considered factors which are not described adequately. It has been concluded that a relatively simple approach can be used to simulate the contribution of the heart motion by neglecting the rotation of the heart. This is very important since nonlinear simulation is difficult by analogs and a detailed study of various factors (e.g. the calculation and effect of changing ventricular and atrial volumes) impossible unless a model of the complete circulation is used - and this would be very expensive on digital computers.

It is the impression of the author that all components of the

human BCG have now been adequately described and that the simulation of the complete BCG is now possible. This is very important since detailed analysis of BCG records is the first condition for its complete understanding.

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