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APPLICATION OF STOCHASTIC MODELS IN SCRAP PROCESSING PROBLEM UNDER JIT

by

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ABSTRACT

In this thesis, we study the scrap processing problem under the Just-In-Time setting. We establish a kind of developing model system to handle the scrap rate data under different situations through the analysis about the stochastic properties of the data for the MS/MP/DP case.

For the truncated quality data, we adopt the **Gaussian-Poisson Mixed Model**; for the censored quality data, we adopt the **Proportional Hazard Regression Model**; and for the complete quality data, we adopt the **Log-Linear Regression Model**. Based on these models, we derive a simple Linear Programming algorithm for the optimal order quantity.

Further, we use a case study about a typical manufacturing company to illustrate the convenience and the flexibility of the developing model system from both technical and managerial aspects. Finally, we discuss the implementation of the developing model system through the design of an Automatic Order Generating System under the Enterprise Resources Planning environment.

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To my friends in China and Germany

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List of Abbreviations

ABC:	Activity Based Cost
AOGS:	Automatic Order Generating System
CIM:	Computer Integrated Manufacturing
CRM:	Customer Relationship Management
DP:	Dynamic Programming
DSS:	Decision Support System
EF:	Estimation Function
ERP:	Enterprise Resources Planning
IWLSE:	Iterative Weighted Least Squares Estimation
GLM:	Generalized Linear Model
JIT:	Just In Time
LP:	Linear Programming
LSE:	Least Squares Estimation
MCMC:	Markov Chain Monte Carlo
MLE:	Maximum Likelihood Estimation
MSE:	Mean Squared Error
OR:	Operation Research
PH:	Proportional Hazard
QC:	Quality Control
SCM:	Supply Chain Management
SPC:	Statistical Process Control
TQM:	Total Quality Management
WIP:	Work In Process
WLSE:	Weighted Least Squares Estimation

Chapter 1

Introduction

1.1 Background

In the manufacturing industry, customer's requirements have always been the driver for the competition, and the quality is the only answer for the competition. Under the Quality Control(QC) arena, scrap processing has always been the critical topic. Even under the modern manufacturing environment, with the most advanced technology, reducing scraps is, unfortunately, still a target for the quality control system. Generally speaking, the happening of the scraps will at least affect two aspects of the manufacturing system: the demand from the supplier, and the delivery time to the customer. Under the Just-In-Time(JIT) manufacturing environment, with non-redundant inventory and zero-defect objective, the scrap processing becomes especially important.

The critical point for the scrap processing is, hence, the decision making under uncertainty. And the situation becomes more complex when the happening of the scraps is random during various stages under the manufacturing process and depends on some unpredictable latent factors. The attempt of using statistical models seems to be a kind of reasonable approach to transfer the uncertainty into limited certainty. But, the drawback of this approach is: while most models can provide beautiful theoretical results under related assumptions, only a few of them can be applied to the practice due to the difficulty of finding solutions after the collapse of the assumptions.

1.2 Motivation

Consider a kind of typical mechanical production process under the JIT setting, say, pump manufacturing. To make things a little bit simpler, assume that there is only one customer, with orders for batches of customized pumps from time to time. Also suppose that there are several available suppliers as well as an internal casting plant within the company to provide the pump bodies. The quality and the price of the pump bodies from different suppliers are different. Furthermore, due to the property of the casting material and the limits from the manufacturing technique, the scraps can be found at any stage before delivery and the reworking is impossible. As a manger of the company, of course, you will face the following questions:

1. Vendor selection: What should be the optimal evaluation criteria?
2. Order quantity: How many pump bodies should be ordered to promise the on-time delivery for the customer order with minimal costs?
3. Out-sourcing strategy: Which option will be better concerning the supply of the required pump bodies, out-processing or self-manufacturing?

The answers for those questions may be more difficult at the strategy level than at the tactical level. Yet, quantitative and qualitative analysis based on suitable statistical models will surely help understand the problems.

1.3 Objectives

As directed by the topic and the motivation, the major objective of this thesis is to derive optimal stochastic models for the scrap processing problem under JIT based on detail analysis from both managerial and statistical aspects.

Though the qualitative analysis related to the management issues is important and interesting to the topic, the focus will be on the quantitative analysis of the models under the scope. As we hope, the results of the thesis can provide some useful information for the following management activities:

1. Continuous quality improvement through time series analysis of the scrap within the integrated supply chain.

2. Establishment of a dynamic supplier evaluation system based on the scrap related performance.

3. Optimal order quantity combination that minimizes the related costs.

4. Out-sourcing strategy for the supply of the critical manufacturing components.

1.4 Examples

To illustrate the importance of the decision making originated from the scrap processing, and to facilitate the following discussion, we provide here with two examples related to the topic for the readers as a kind of rehearsal.

Example 1.1. Vendor Selection/Optimal Ordering Quantity

Consider the following scenario: You just received a customer order of 100 pumps. You are required to deliver the products in three months, which is the exact manufacturing lead-time. There are two available suppliers with the same price for pump bodies, say, 50 dollars. Further, you have the historical data about the scrap rates for those suppliers as well as the cost structure for the product series. There are five major stages under the production process, stage four uses a Numerical Control Center operated by a specialist, hence has relatively higher cost allocation. Detailed data are illustrated as the follows: (suppose that both suppliers have the capacity of producing over 100 pump bodies.)

Operation Stage	1	2	3	4	5	Overall
Cost Structure(in Dollars)	10	110	120	230	240	240
Scrap Rate(Vendor #1)	1%	4%	2%	1%	0	8%
Scrap Rate(Vendor #2)	1%	1%	2%	3%	0	7%

Table 1.1: Cost Structure and Scrap Rate

Simple calculation shows:

- If choose vendor #1:
  - Order quantity = 108, Overall manufacturing cost = 30320.
- If choose vendor #2:
  - Order quantity = 107, Overall manufacturing cost = 30400.

Conclusion: the optimal order quantity is not the minimal order quantity. Here, one can see that the optimal solution depends on both the cost structure and the scrap rate.

- Based on order quantity, choose vendor #2.
- Based on overall manufacturing cost, choose vendor #1.

Example 1.2. Out-sourcing Strategy

As the General Manager of this pump company, now suppose that you are facing another paradox. There is an internal casting plant within the company. The function of this plant is to provide high-quality pump bodies for the company. There are no competitive advantages to the external suppliers except the control of the quality. The reason that the company maintains this plant is that sometimes there are big pump project with tight delivery time. Therefore, you can't afford the risk of losing important customer, but ordering pump bodies from the external supplier may not satisfy your requirements for the delivery time or quality. In a special case, for a big pump product, you can use the internal casting plant to produce one pump body for the pump but has to order two pump bodies to promise the quality of the big pump. So, if you have to use the external supplier, you may finish the project but lose money. The optimal solution for this paradox is more complex with qualitative and quantitative analysis. One option is adopting the out-sourcing strategy. The scraps processing model, along with the analysis of the product structure, can help you choose the right partner and find the break-even point for the decision.

1.5 Structure

Although the scrap processing is an important topic in the QC area, most articles discussed the inventory models for the optimal order quantity based on undetermined demand from the market, see Erlebacher(1999), Hadley and Whitin(1963) and Lau and Lau(1996). Further, the articles of Dunsmore and Wright(1985), Wetherill and Chiu(1975) and Guenther(1977) focused on the optimal sampling plan for the scrap. To the best of our knowledge, there is no article discussing the statistical analysis of the optimal scrap processing problem under the JIT setting. This thesis will use the widely used statistical and Operational Research(OR) methods to address the solutions. The structure of the thesis is as follows: In chapter 2, we will review the management issues related to the scrap processing problem and generalize the problem. In chapter 3, we will review the related statistical and OR methods that will be used in the modeling and analysis. In chapter 4, we will use the results from chapter 2 and chapter 3 to derive the optimal model in detail. In chapter 5, we will process the simulation study for the derived models on a case study and discuss some numerical examples. Finally, in chapter 6, we will summarize the analysis, discuss about the design and implementation of the related QC systems and point out the direction for further research on this topic.

Chapter 2

Process Description

2.1 Related Management Concepts

2.1.1 Quality Control Systems

From early inspection, to QC, to Total Quality Management(TQM), quality control systems have become an integrated part of any manufacturing system. The design of the quality control system needs to consider various important issues, including:

1. Criteria to evaluate performance.

2. Procedure to implement the measuring system.

3. Quality objectives.

4. Implementation and controlling.

5. Continuous Improvement.

Obviously, the successful operation of the quality control system depends on sophisticated information system as well as advanced quantitative analysis tools.

Quality objectives represent the philosophy of a company. Those objectives may be different from one company to another in quantities but are always similar in contents. In most cases, quality objectives include:

1. Reduce wastes, like scraps, reworks or returns from the customer.
2. Maintain the desired degree of uniform to product design.
3. Maintain the consistency of quality in product or service output.
4. Improve the quality of the materials from the supplier.
5. Optimize the inspection procedure to minimize the costs related to QC.

Under the JIT production, quality control systems have special meanings to the management. Zero-defect means the priority of QC is to avoid defects rather than compensate for the defects. Thus, accurate forecast for the uncertainty becomes critical. The problem at certain stage in the manufacturing processes needs to be solved in short time to avoid being the bottleneck for the following stages. Optimality can not be reached at one step, so continuous improvement is always needed, and the partnership with the supplier and the customer is strongly recommended.

2.1.2 Available Statistics Tools

There are numerous articles discussing about the relationship between quality and statistics. Actually, many statistical theories and methodologies are the products of research in application problems, as was well established by the work of Sir Ronald A. Fisher, and the introduction of the statistical tools to quality management by Deming led a revolution from Japan in the industry. Harry V. Roberts surveyed textbooks used in business applications and provided a relatively detailed list(Roberts, 1990). His list includes Sampling and Survey Methodology, Experiment Design and Analysis, Time Series Analysis, Bayesian Decision Theory, Simulation Study and Graphical Technique.

Recent important developments in Control Charts, Taguchi Method and Statistical Process Control(SPC) also gain wide acceptance and applications in the industry.

From the statistical aspects of quality control(Chase and Aquilano, 1989), there are two things to consider: the sampling plan and the process control. Optimal sampling plans like double sampling plan and sequential sampling plan can minimize the cost related to the inspection while promising the quality. Control charts, like P-Chart, $\overline{X}$-Chart and R-Chart are useful tools for SPC. Further, the graphical tools can provide convenience to the TQM through cause-and-effect analysis.

2.1.3 Just-In-Time

The JIT production is a brand new manufacturing philosophy developed by the Japanese. From the point of cost accounting, JIT is revolutionary due to its re-definition of the waste: non-value-added items or time. Thus, JIT targets zero-defect, zero-inventory and single lot size. This is just the ideal situation(in practice, it's usually difficult to reach this target), or the JIT philosophy, and the JIT systems are designed based on this philosophy. JIT and its implementation deeply depends on the statistical tools because the pull system can only be triggered without interrupt through 100% accurate information. If the zero-defect is impossible, then the optimal supply should be defined in the Kanban to satisfy the demand(For detail about Kanban system, see p723, Chase and Aquilano, 1989). On the other hand, JIT sets the ultimate target for a company, so only through continuous improvement, within the company and in partnership with the supplier and the customer, can the company near this target. The greatest advantage is the overall effects during this procedure like the reduction of the reworks, scraps and the returns from the customer, the reduction in the manufacturing lead time, and the improvement in the manufacturing flow. As an important step to the Computer Integrated Manufacturing(CIM), JIT will obviously change the operations throughout the whole supply chain.

2.1.4 Cost Structure

Cost structure for the manufacturing system is a big topic. For the convenience of our discussion in the modeling part, we introduce here only some related concepts in this area.

1. Category of the manufacturing cost:

   - Material: Purchasing price plus transportation and insurance.
   - Labour: Wage and benefits of employees direct related to the production.
   - Overhead: Other costs and expenses related to the products like administration, quality control, sales and utilities.

2. Accounting Units:

   - Cost Center: Unit to allocate costs in a company, like a machine or a test center.
   - Cost Item: type of cost to be allocated to the cost center, like utilities.

3. Cost Accounting Methods:

   - Actual Cost Accounting: Allocate cost to products according to the actual costs.
   - Standard Cost Accounting: allocate cost to products according to the standard costs decided beforehand.
   - ABC Accounting: Activity based cost(ABC) allocation, i.e., allocate costs according to the activities produced.

4. Costs related to QC: (Kaplan and Atkinson 1989, p. 380)

- Prevention: The costs of designing, implementing, and maintaining all active quality assurance and control systems, like quality planning, quality controlling, and quality training.

- Appraisal: The costs of ensuring that materials and products meet quality insurance standards, like inspection, lab tests, quality audits, and field tests.

- Internal Failure: The costs of managing losses from materials and products that do not meet quality standards, like scrap, rework, repair, upgrade, downtime, and discount.

- External Failure: The costs of shipping inferior quality products to customers, like warranty, replacement, freight and repairs for returned merchandise.

5. Cost Structure under JIT

Under the new manufacturing environment of JIT production, new concepts in cost accounting are introduced to facilitate the effects to the zero-inventory target. Miltenburg(1990) described time-overhead concept in the new cost structure under JIT to effectively measure the new manufacturing system. Under this structure, the costs related to the lead-time of the raw materials and the Work-In-Process(WIP) are allocated to the total cost during the manufacturing procedure. The advantages are: Clear evaluation of performance in turn-over rate; clear evaluation in lead-time; and clear evaluation in quality control. Further, the management can adopt the target costing approach to optimize the marketing strategy.

2.1.5 Supplier Evaluation

Under most cases, the quality control systems are initialized at the raw materials from the supplier. Good relationship with the supplier or securing the qualified supplier is critical for the success of a typical manufacturing company.

Thus, supplier evaluation is always an important task for the purchasing department. Most companies have used the computer systems to establish a measuring system to track the performance of the suppliers for the materials and services they provided. Generally speaking, the tracking system should include the following criteria: (often being called "Four Elements of Purchasing")

1. Price: The price of the suppliers as compared with the marketing price.
2. Quality: The quality of the suppliers as indicated by the return rate, rework rate or scrap rate.
3. Time: The delivery time of the order as described in the contracts.
4. Service: The after delivery services which include repairs, supplements and warranties provided.

Those criteria are related to one another but the focus can be chosen through suitable assessment in weights when designing an evaluation system. For the discussion in this thesis, we will consider only the first and the second criteria for the vendor evaluation purpose.

2.1.6 Supply Chain Management

As the development in the manufacturing industry is going toward the new environment like JIT and CIM, the optimization of the external processes becomes equally important to that of the internal processes. Recent progress in Supply Chain Management(SCM) and Customer Relationship Management(CRM) represents some efforts in those areas. Concerning the quality control, the life cycle should not only be limited to the manufacturing procedure within a company. It is a company's best interests to establish a win-win partnership relationship with both its supplier and customer, in order that the information provided from both external sides can be shared by others to improve the quality and to upgrade the services. For instance, the quality data collected by the manufacturing company should be not only the evaluation criteria for the supplier evaluation, but also the inputs to the quality system in the supplier's side in order that the supplier can improve the quality based on the analysis of the data from their client, the manufacturing company. The models discussed in the thesis are intended to assist the related quality control procedure across the company boundary for the SCM.

2.2 Process Generalization

The definition of the waste under the JIT setting is the costs related to unnecessary activities, i.e., inventory, quality inspection, reworks, scraps, etc. Thus, the target of the QC is the optimum of the process and the data. The optimum of the process includes three components:

1. Total Quality Control.
2. JIT Production.
3. Employee Involvement.

Of those three components, the key point is the accuracy of the data. An optimal data model can provide important information to the optimal design of the process. Of most are, first, the accurate requirements for the material, labour and facility, and second, the reasons resulting to the happening of the defects.

Generally speaking, there are two kinds of manufacturing processes: the discrete production like those in the machinery industry, and the continuous production like those in the pharmaceutical industry. Further, we can consider the general scrap processing problem under different products with multi-supplier and multi-customer. That is near the practical scenario. But for the convenience of the model building, we will consider only some specific cases under the general setting. As we know from the introduction, optimal order quantity is related to the scrap rate and the cost structure. If we suppose that there are no scrap caused due to the errors in operations during the manufacturing process itself, then we can say that the scrap rate depends on the supplier, while the cost structure depends on the product. Thus, we can choose the following situations as the suitable starting points:

Case I: MS/MP/DP

Multi-supplier, Multi-product, Single customer, Discrete production

Case II: MS/SP/DP

Multi-supplier, Single product, Single customer, Discrete production

Case III: MS/MP/CP

Multi-supplier, Multi-product, Single customer, Continuous production

Case IV: MS/SP/CP

Multi-supplier, Single product, Single customer, Continuous production

In this thesis we will mainly focus on the MS/SP/DP case. The possible approach for other cases will be discussed in the extension part.

Chapter 3

Related Theoretical Results

For the convenience of the following discussion, we review in this chapter the related statistical concepts and theories.

3.1 Stochastic Process(Karlin, 1975)

Definition 3.1. *[Stochastic Process] A stochastic process* $\{X_t; t \in T\}$ *is a family of random variables* $X_t, t \in T$*, where, it depends on: (i) State space* S*, the space in which the possible value of each* X_t *lies. (ii) Index parameter* $T, T = (0, 1, \cdots)$*, i.e., discrete, or* $T = [0, \infty)$*, i.e., continuous. (iii) Family of joint distributions. Under the probability space* $(\Omega, \mathcal{F}, \mathcal{P})$*,* X *can also be considered as a function on* $[0, \infty) \times \Omega$*. The sample paths of* X *are the real-valued function* $X(\cdot, \infty)$ *on* $[0, \infty)$*.*

Category of the stochastic processes:

1. Stationary Process: Examples include physical movement with repeated cycles, like the position of waves depending on the time, and the economic time series, like the unemployment rate in different economic cycles.

2. Martingale: A typical example is the fair game, like the gambling.

3. Markov Process: Concerning the happening of the scraps in different stages under the manufacturing process, it has the property of forgetting the past, hence can be considered as a kind of Markov process.

4. Renewal Process(Counting Process): Examples can be found like the Poisson process for the unit lifetimes.

5. Point Process: In the real world, the point process can be used to describe the distribution of stars in space, the planer distribution of plants and animals, etc.

Here, of most importance to our discussion are Martingale, Counting Process, and Markov Process, which we will explore in further detail later.

3.2 Martingale

Definition 3.2. *[Martingale] A stochastic process $X_n, n = 0, 1, \cdots$ is a martingale, if, for $n = 0, 1, \cdots,$ i) $E[|X_n|] < \infty$; ii) $E[X_{n+1}|X_0, \cdots, X_n] = X_n$.*

Martingale originated from gambling. Bremaud(1981) gave a more general definition that also included submartingale and supermartingale using the concept of information history.

Properties:

1. If X_n is a (super)martingale w.r.t. Y_n, then, $E[X_{n+k}|Y_0, \cdots, Y_n](\leq) = X_n, \forall k \geq 0$.

2. If X_n is a (super)martingale, then for $0 \leq k \leq n$, $E[X_n](\leq) = E[X_k](\leq) = E[X_0]$.

3. Suppose that X_n is a (super)martingale w.r.t. Y_n, and g is a (nonnegative)function of $Y_0, \cdots, Y_n$, for which the expectations that follow exist. Then, $E[g(Y_0, \cdots, Y_n)X_{n+k}|Y_0, \cdots, Y_n](\leq) = g(Y_0, \cdots, Y_n)X_n$.

There are two important theorems related to the properties of martingale. But first, we need to introduce the important concept of stopping time.

Definition 3.3. *[Stopping Time] A random variable T is called a stopping time, or Markov time w.r.t. Y_n if T takes values in $0, 1, \cdots, \infty$ and if, for every $n = 0, 1, \cdots$, the event $\{T = n\}$ is determined by $(Y_0, \cdots, Y_n)$.*

More generally, in Bremaud(1981), T is called an $\mathcal{F}_t$-stopping time if for all $t \geq 0$, $T \leq t \in \mathcal{F}_t$, where $\mathcal{F}_t$ is the information history under the probability space $(\Omega, \mathcal{F}, \mathcal{P})$.

Properties:

1. *If S and T are Markov times, then so is $S + T$.*

2. *The smaller of two Markov times S, T, denoted as $S \wedge T = min\{S, T\}$ is also a Markov time.*

3. *If S and T are Markov times, then so is the larger $S \vee T = max\{S, T\}$.*

Theorem 3.1. *[Optimal Sampling Theorem] Let X_n be a martingale and T a Markov time. If: i) $P_r\{T < \infty\} = 1$; ii) $E[|X_T|] < \infty$; iii) $\lim_{n \to \infty} E[X_n I_{\{T>n\}}] = 0$. Then, $E[X_T] = E[X_0]$.*

The Optimal Sampling Theorem is useful in calculating boundary probabilities related to the stochastic processes.

Theorem 3.2. *[Martingale Convergence Theorem] Let X_n be a martingale w.r.t. Y_n satisfying, for some constant K, $E[X_n^2] \leq K < \infty$, for all n, then X_n convergence as $n \to \infty$ to a limit random variable X_∞ both with probability one and in mean square. That is: $P_r\{\lim_{n \to \infty} X_n = X_\infty\} = 1$, and $\lim_{n \to \infty} E[|X_n - X_\infty|^2] = 0$ prevail. Finally, $E[X_0] = E[X_n] = E[X_\infty]$.*

The Martingale Convergence Theorem also provides powerful tools for the boundary probability calculation in the stochastic process.

3.3 Counting Process(Fleming and Harrrington, 1991)

Definition 3.4. *[Counting Process] A counting process is a stochastic process* $\{N_t : t \geq 0\}$ *adapted to a filtration* $\{\mathcal{F}_t\}$ *with* $N(0) = 0$*, and* $N(t) < \infty$ *a.s., and whose paths are with probability one right continuous, piecewise constant, and have only jump discontinuities, with jump "size + 1".*

Here, depending on t, we have discrete counting processes and continuous counting processes. The Poisson process is the most prevalent example of a counting process.

3.4 Markov Process(Karlin, 1975)

Definition 3.5. *[Markov Process] A Markov process is a stochastic process* $\{X_t, t \in T\}$*that has the property:* $P_r\{a < X_t \leq b | X_{t_1} = X_1, X_{t_2} = X_2, \cdots, X_{t_n} = X_n\} = P_r\{a < X_t \leq b | X_{t_n} = X_n\}, \forall t_1 < t_2 < \cdots < t_n < t.$

An important concept related to the Markov process is the transition probability function. Using this concept, we can write the proceeding equation as: $P_r(X_n, t_n, t, A) = P_r\{a < X_t \leq b | X_{t_1} = X_1, X_{t_2} = X_2, \cdots, X_{t_n} = X_n\}$, where $A = \{\xi | a < \xi \leq b\}$. If the transition probability only relates to the time period, we call the Markov process stationary. As we know, the Poisson process is not a stationary process.

Definition 3.6. *[Markov Chain] A discrete time Markov chain* $\{X_n\}$*is a Markov stochastic process whose state space is a countable or finite set, where* $T = (0, 1, 2, \cdots)$.

Markov chain has many important properties that will be useful in analyzing the related problems.

Properties:

1. *A Markov chain is completely defined by its one-step transition probabil-*

ity matrix and the specification of a probability distribution on the state of the process at initial status. Further, we have: $P_{ij}^n = \sum_{k=0}^{\infty} P_{ik}^r P_{kj}^s$, *for any fixed pair of nonnegative interger* r *and satisfying* $r + s = n$ *and* $P_{ij}^0 = I_{\{i=j\}}$.

2. *State* j *is said to be accessible from state* i *if for some integer* $n \geq 0, P_{ij}^n > 0$. *Two states* i *and* j *are said to communicate if they are accessible to each other, write* $i \leftrightarrow j$. *Communication has the property of reflexivity, symmetry and transitivity.*

3. *A process is irreducible if all states communicate with each other. The period of state* i, *written* $d(i)$, *is the greatest common division(g.c.d.) of all integers* $n \geq 1$ *for which* $P_{ii}^n > 0$.

4. *A state* i *is recurrent if and only if, starting from state* i, *the probability of returning to state* i *after some finite length of time is one. A non-recurrent state is said to be transient.*

3.5 Generalized Linear Model

Historically, there are two approaches to process the count data. The first approach is the adoption of the Generalized Linear Model(GLM) as indicated in McCullagh and Nelder(1989), Cameron and Trivedi(1997) and Agresti(1990). GLM was developed based on the properties of the count data. The power of the linear model is limited due to the discreteness of the count data. As a result, regressions are introduced using a kind of link functions. As summarized in Cameron and Trivedi(1997), in general:

$$f_{GLM}(y|\theta, \phi) = exp\{\frac{\theta y - b(\theta)}{a(\phi)} + c(y, \phi)\} \tag{3.1}$$

is a member of a linear exponential family, with canonical parameter θ and nuisance parameter ϕ. Define the linear predicator $\eta = X'\beta$. Then, the link function $\eta = \eta(\mu)$ relates the linear predictor to the mean μ. Especially, the

Poisson model with mean corresponds to the log link function $\eta = ln\mu$. This is usually called the log linear model. Agresti(1990) introduced relationship between models for the categorical data and GLM, which is a kind of special to general relationship. Further, he summarized the components of a GLM:

1. Random component: Independent observations of $Y = (Y_1, Y_2, \cdots, Y_n)'$ from a distribution on a natural exponential family.

2. Systematic component: A vector $\eta = (\eta_1, \eta_2, \cdots, \eta_n)'$ to a set of explanatory variable through a linear model $\eta = X'\beta$.

3. Link between the random and systematic components: $\eta_i = g(\mu_i)$, where $\mu_i = E(Y_i)$ is any monotonic differential function.

3.6 Proportional Hazard Model

Another approach for the count data analysis is the survival analysis approach. There are numerous articles and textbooks on this topic. Due to its applications in medical trials and quality control areas, survival analysis has been considered as a very useful tool for the analysis of the failure data, especially, when there are censors happening. There are two ways to handle the censored count data. The traditional approach uses the conditional distributions and the hazard functions concepts, see Kalbfleisch & Prentice(1980), Cox & Oakes(1983) and Miller(1981). The martingale approach uses the multiplicative intensity model and thus extends the scope of the survival analysis. Representative publications about this approach include Fleming & Harrington(1991), Bremaud(1981) and Gill(1980).

3.6.1 Traditional Approach

1. Censorship:

Censorship mechanism is an important part of the survival analysis. There are two types of censorship. Type I censorship is the censor with fixed time, while type II censorship is the censor with fixed failed item number. Gill(1980) provided formal definitions for them.

Definition 3.7. *[Type I Censorship] Suppose that* $X_1, X_2, \cdots, X_n$ *are i.i.d. r.v.s with distribution function F.* $\mathcal{F} = \mathcal{F}_\theta$, *where* $\{\mathcal{F}_\theta\}$ *is some parameterized family of distributions.* $X_i, i = 1, 2, \cdots, n$ *are failed times, and* $\mu > 0$ *is a fixed time, at which not all* $X_i < \mu$. *Then,* $(\tilde{X}_i, \delta_i) = (X_i \wedge \mu, I_{\{X_i \leq \mu\}}), i = 1, 2, \cdots, n$ *are the observations.*

Definition 3.8. *[Type II Censorship] Suppose that* $X_1, X_2, \cdots, X_n$ *are i.i.d. r.v.s with distribution function F.* $\mathcal{F} = \mathcal{F}_\theta$, *where* $\{\mathcal{F}_\theta\}$ *is some parameterized family of distributions.* $X_i, i = 1, 2, \cdots, n$ *are failed times, and* $X_{\{i\}}, i = 1, 2, \cdots, n$ *are the order statistics of* $X_i, i = 1, 2, \cdots, n$. *The observations are terminated at some fixed* $r \leq n$, *and the data* $(\hat{X}_i, \delta_i) = (X_i \wedge X_{(r)}, I_{\{X_i \leq X_{(r)}\}}), i = 1, 2, \cdots, n$ *are the observations.*

2. Failure Time Distribution

Assuming that T is an arbitrary continuous non-negative *r.v.* representing the failure time with distribution function $F(t) = P\{T \leq t\}$, and the density function $f(t) = \frac{dF(t)}{dt}$. Then, we also have following useful functions to describe the distribution:

Hazard Function: $\lambda(t) = \lim_{\Delta t \downarrow 0} \frac{1}{\Delta t} P\{t \leq T < t + \Delta t | T \geq t\} = -[\frac{d}{dt}\{S(t)\}]/S(t) = f(t)/S(t).$

Survival Function: $S(t) = P\{T > t\} = 1 - F(t).$

Cumulative Hazard Function: $\Lambda(t) = \int_0^t \lambda(\mu) du.$

3. Estimation of the Survival Function

Kaplan & Meier(1958) derived the Kaplan-Meier estimator for the survival function using the "nonparametric maximum likelihood estimation" approach:

$$\hat{S}(t) = \prod_{j | t_j < t} (\frac{n_j - d_j}{n_j}), \tag{3.2}$$

where $t_1 < t_2 < \cdots < t_k$ represent the observed failure times in a sample of size n from a homogeneous population. With survival function $S(t)$, d_j items fail at $t_j (j = 1, 2, \cdots, k)$, and m_j items are censored in $[t_j, t_{j+1})$. Also, we have: $t_0 = 0, t_{k+1} = \infty$, and $n_j = (m_j + d_j) + \cdots + (m_k + d_k)$ is the number of items at risk at the time just prior to t_j.

4. Proportional Hazard Model

The Proportional Hazard(PH) model(Cox, 1972) has been widely used in the survival analysis area. It specifies that: $\lambda(t; Z) = \lambda_0(t)e^{Z\beta}$, where $\lambda(t; Z)$ represents the hazard function at time t for an individual with covariate vector Z, and $\lambda_0(t)$ is an arbitrary unspecified baseline hazard function for continuous t. In this model, β is a column vector of s regression parameters which addresses the major attention to be estimated.

3.6.2 Martingale Approach

Martingale approach is a recent development in the methods for the survival analysis. Fleming & Harrignton(1991) introduced this approach systematically. The key point of this approach is the use of the Doob-Mayer Decomposition. We indicate here the major results without proof.

1. Process $M = N - A$

Theorem 3.3. *Let $\mathcal{T}$ be an absolutely continuous failure time random variable and $\mathcal{U}$ a censoring time variable with an arbitrary distribution. Set $X = min(\mathcal{T}, \mathcal{U})$, $\delta = I_{\{\mathcal{T} \leq \mathcal{U}\}}$, and let λ denote the hazard function for $\mathcal{T}$. Define now $N_{(t)} = I_{\{x \leq t, \delta=1\}}$, $N^u_{(t)} = I_{\{x \leq t, \delta=1\}}$, $\mathcal{F}_t = \sigma\{N_{(\mu)}, N^u_\mu : 0 \leq \mu \leq t\}$. Then, the process μ given by $\mu(t) = N(t) - \int_0^t I\{x \geq u\}\lambda(u)du$ is an $\mathcal{F}_t$-martingale if and only if:*

$$\lambda(t) = \frac{-\frac{\partial}{\partial u} P\{T \geq u, \mu \geq t\}|_{u=t}}{P\{T \geq t, \mu \geq t\}}, \tag{3.3}$$

whenever $P\{x > t\} > 0$.

Note: Theorem 3.3. is used for known hazard function cases.

Theorem 3.4. *Let $\mathcal{T}$ be an absolutely continuous failure time random variable and $\mathcal{U}$ a censoring time variable with an arbitrary distribution. Let $X = min(\mathcal{T}, \mathcal{U})$, $\delta = I_{\{\mathcal{T} \leq \mathcal{U}\}}$, $N_{(t)} = I_{\{x \leq t, \delta=1\}}$, $N^u_{(t)} = I_{\{x \leq t, \delta=1\}}$, $\mathcal{F}_t = \sigma\{N_{(\mu)}, N^u_\mu :$*

$0 \leq \mu \leq t\}$. *Then the process* $\mathcal{M}$ *given by* $\mathcal{M}(t) = \mathcal{N}(t) - \int_0^t I\{x \geq u\} d\Lambda(u)$ *is a martingale with respect to* $\mathcal{F}_t$ *if and only if:*

$$\frac{dF(z)}{1 - F(z)} = -\frac{dP\{T \geq z; \mu \geq T\}}{P\{T \geq z; \mu \geq z\}}, \tag{3.4}$$

for all z *such that* $P\{T \geq z, \mu \geq z\} > 0$.

Note: Theorem 3.4. is used when the cumulative hazard function is known.

Theorem 3.5. *[Doob-Meyer Decomposition] Let* X *be a right-continuous non-negative sub-martingale with respect to a stochastic basis* $(\Omega, \mathcal{F}, \{\mathcal{F}_t \colon t \geq 0\}, P)$*, then there exists a right-continuous martingale* μ *and an increasing right-continuous predictable process* A *such that* $E[\mathcal{A}(t)] < \infty$ *and* $X(t) = \mu(t) + \mathcal{A}(t) a.s.$ *for any* $t \geq 0$. *If* $\mathcal{A}(0) = 0$ *a.s., and if* $X = M' + \mathcal{A}'$ *is another such decomposition with* $\mathcal{A}'(0) = 0$*, then, for any* $t \geq 0$, $P\{M'(t) \neq M(t)\} = P\{\mathcal{A}'(t) \neq \mathcal{A}(t)\} = 0$. *If, in addition,* X *is bounded, then* M *is uniformly integrable and* $\mathcal{A}$ *is also integrable.*

2. Process $\int H dM$

Another expression for many censored data statistics is of the form $\sum_i \int H_i dM_i$. The following theorem establishes its formal conditions.

Theorem 3.6. *Let* N *be a counting process with* $E[N(t)] < \infty$ *for any* t*, and let* $\{\mathcal{F}_t; t \geq 0\}$ *be a right-continuous filtration such that:*

1. $M = N - A$ *is an* $\mathcal{F}_t$*-martingale, where* $A = \{A(t); t \geq 0\}$ *is an increasing* $\mathcal{F}_t$*-predictable process with* $A(0) = 0$*;*

2. H *is a bounded,* $\mathcal{F}_t$*-predictable process;*

Then, the process $\mathcal{L}$ *given by* $\mathcal{L}(t) = \int_0^t H(u) dM(u)$ *is an* $\mathcal{F}_t$*-martingale.*

3.7 Parameter Estimation

Traditionally, there are two approaches for the statistical inference: the classical statistical estimation and the Beyesian statistical estimation. Also, there are different methods for both parametrical and non-parametrical estimation. The statistical inference which includes point estimation, interval estimation and hypothesis testing can be drawn from both "finite sample" results and "asymptotic" results(Karr, 1986). For the interests of our discussion, we make here a brief review about the major methods used in the parametrical point estimation, including Least Squares Estimation(LSE), Maximum Likelihood Estimation(MLE) and Estimation Function(EF) under the classical arena, as well as the general Bayesian inference approach.

3.7.1 Least Squares Estimation

LSE and MLE are two most commonly used methods for the point estimation. Under most situations, they have the same results, and the same properties like unbiasedness, consistency and sufficiency. Further, new methods are being developed based on the basic idea of LSE to optimize the estimators, such as Weighted Least Squares Estimation(WLSE), Iterative Weighted Least Squares Estimation(IWLSE) and Conditional LSE(Karr, 1986).

Considering a linear regression model $Y = X'\beta + \epsilon$, LSE uses the Mean Squared Error(MSE) value $Q = (Y - X'\beta)^T(Y - X'\beta)$ as the criterion to obtain the estimator for β as $\hat{\beta}_{LSE} = (X^TX)^{-1}(X^TY)$. WLSE can be used for the cases with unequal error variances. With the weights matrix $\mathcal{W}$ estimated from the results of LSE, the adjusted estimator becomes $\hat{\beta}_{WLSE} = (X^TWX)^{-1}X^TWY$. It can be shown that this is a Best Linear Unbiased Estimator(BLUE).

3.7.2 Maximum Likelihood Estimation

MLE is another popular method in finding estimators. Considering one parameter case, suppose that the observations from a population with i.i.d. pdf $f(Y;\theta)$ are: $y_1, \cdots, y_n$, then the joint pdf is: $g(y_1, \cdots, y_n, \theta) = \prod_{i=1}^{n} f(y_i;\theta)$. This function can also be viewed as a function of θ, so we can write it as $L(\theta) = g(y_1, \cdots, y_n, \theta) = \prod_{i=1}^{n} f(y_i;\theta)$. We call this function the likelihood function, and can maximize $L(\theta)$ to yield the maximum likelihood estimator $\hat{\theta}_{MLE}$.

Cox developed the method of the partial likelihood for the proportional hazard model $\lambda(t;z) = \lambda_0(t)exp(Z^T\beta)$, which had actually no direct relationship with MLE. The partial likelihood function is:

$$L(\beta) = \prod_{i=1}^{R} \left(\frac{exp(Z_i^T\beta)}{\sum_{l\in R(t_{(i)})} exp(Z_l^T\beta)}\right), \tag{3.5}$$

which doesn't depend on time related baseline hazard $\lambda_0(t)$.

3.7.3 Estimation Function Approach

Godambe(1960) introduced an innovative approach for the parametric point estimation, the Estimation Function approach. This approach considers a group of functions of both observations and parameters instead of functions only related to the observations. Through studying the optimal criteria directly in EF, which now contains all the information, the parameter estimators can be obtained. This provides convenience for the situations when the functions of observations are difficult to express. The major results are as the follows(Godambe, 1960):

Definition 3.9. *[Estimation Function] An Estimation Function is a function $g(X,\theta)$ of both the observation X and the parameter θ. The Estimation Function is called unbiased if $E[g(X,\theta)] = 0$ for all $F \in \mathcal{F}$ such that $\hat{\theta}(F) = \theta$, where $X = (x_1, x_2, \cdots, x_T)$ is a vector random variable on a probability space. $\theta = (\theta_1, \theta_2, \cdots, \theta_p)$ is the parameter, and the distributional family $\mathcal{F}$ has relations with the parameter.*

Definition 3.10. *[Optimal EF] Within the class ζ of all regular unbiased EFs, a function g belonging to ζ is an optimal EF for θ if, for any $F \in \mathcal{F}$ with $\theta = \theta(F)$, it minimizes the quotient $\frac{E(g^2)}{\{E[\frac{\partial g}{\partial \theta}]\}^2}$.*

As a kind of unified approach, EF approach can be used to derive other estimation methods, like LSE or MLE as special cases. For example, MLE uses the score function as EF under the EF approach.

In summary, the advantages of the EF approach to other point estimation appraches are as the follows:

1. Starting from the estimation functions instead of the functions related only to the observations, hence can provide more information and convenience for the point estimation.

2. Generalizing the common point estimation approaches like LSE and MLE under a unified approach, hence can provide simplicity for the point estimation method.

3.7.4 Bayesian Estimation Approach

Bayesian analysis is important for the decision theory. The Bayesian estimation approach is completely different to the classical estimation approach, and the Bayesian estimators have some specific properties. Box and Tiao(1993) described this approach in detail with applications in Normal theory.

1. Bayesian Approach

The verification of the tentative conjections depends on the designed experiment and the collected data. There are two methods for the statistical analysis purpose: sampling theory and Baye's Theorem. Using the sampling theory, the parameters are considered to be unknown but fixed, and the estimators are functions of both observations in the sample and the parameters. Under the Bayesian approach, the parameters are considered as the random variables that follow the prior distribution. Sometimes the sample from the population that follows the prior distribution is called the reference set or training data, and the Bayesian analysis can be viewed as a kind of learning procedure from the training data.

2. Bayes' Rule

Bayes' Rule plays a critical role in Bayesian analysis. The relationship between the priori and posteriori shown in the rule requires the practitioners to avoid pitfalls in using the Bayesian approach by choosing the proper model structure and the appropriate non-informative prior carefully.

Suppose that $Y' = (y_1, y_2, \cdots, y_n)$ is a vector of n observations with pdf $P(Y|\theta)$, where $\theta' = (\theta_1, \theta_2, \cdots, \theta_n)$ is the parameter. Suppose also that θ has a pdf $P(\theta)$, then we have:

$$P(Y|\theta)P(\theta) = P(Y,\theta) = P(\theta|Y)P(Y). \tag{3.6}$$

Given the observed data Y, we have:

$$P(\theta|Y) = \frac{P(Y|\theta)P(\theta)}{P(Y)}. \tag{3.7}$$

Without loss of generality, suppose θ continuous, as $P(Y)$ is the value related only to the observations, we can denote:

$$P(Y) = \int P(Y,\theta)d\theta = \int P(Y|\theta)P(\theta)d\theta = E[P(Y|\theta)] = C^{-1}.$$

Then, we can write:

$$P(\theta|Y) = CP(Y|\theta)P(\theta). \tag{3.8}$$

This equation is usually referred as the Bayes' Theorem. $P(\theta)$ here is the prior distribution of θ, which provides information about θ without the data. Also, $P(\theta|Y)$ provides us the related information about θ with the data, which is usually called as the posterior distribution of θ given Y. Now if we consider $P(Y|\theta)$ in the expression as a function of θ instead of Y, which is called by Fisher, as the likelihood function of θ for the given Y, then we can rewrite the Bayes' Rule as:

$$P(\theta|Y) = l(\theta|Y)P(Y). \tag{3.9}$$

Here, $l(\theta|Y)$ is defined up to a multiplicative constant. The definition of the likelihood function $l(\theta|Y)$ is important as it represents the information about θ that comes from the data.

3. Non-informative Prior

Now we define the non-informative prior and derive its actual expression, so we can apply the Bayesian approach in practice.

Suppose that $\Phi(\theta)$ is a one-to-one transformation of θ, if the likelihood function can be expressed as $l(\theta|Y) = g[\Phi(\theta) - f(Y)]$, then we can say that the prior distribution of θ which is locally proportional to $|d\Phi/d\theta|$ is non-informative for the parameter θ. For the exponential family, as observations Y are drawn from a distribution $P(Y|\theta)$ of the form $P(Y|\theta) = h(Y)w(\theta)exp[c(\theta)a(Y)]$, it can be shown that the non-informative prior is just the Fisher information. For the other cases, the Jeffrey's Rule indicates that the non-informative prior is proportional to the square root of the Fisher's information measure.

4. Bayesian Estimation

Following the Bayes' Theorem and using the non-informative prior, we can obtain the suitable Bayesian estimators, which contain the information from both the prior distribution and the data. In many cases, we can find conjugate family as a class of the prior distribution to the posterior distribution.

3.8 Optimization

Optimization is a widely used tool for decision making under uncertainty which has been developed with the OR techniques. The major objective of the optimization is to find an optimal solution for a function under a group of constraints. There are many papers in the literature in this area. Among them, of most interests to our discussion are: Drefus and Law(1977), which introduced general method of dynamic programming; Bertsckus(1976), which provided special treatments for the dynamic programming(DP) method under the un-deterministic situations; and Bensausson, Crouhy and Proth(1983), which established a unified framework for the production planning problems with the dynamic programming method.

3.8.1 Decision Theory

A typical decision problem under uncertainty can be abstracted in the following mathematical format:

1. A function $f : \mathcal{D} \times \mathcal{N} \rightarrow \Theta$ for three non-empty sets $\mathcal{D}$, $\mathcal{N}$ and Θ.

2. A complete and transitive relation $\ll$ on Θ.

Where:

$\mathcal{D}$: the set of possible decisions.

$\mathcal{N}$: indexes of the uncertainty in the problem and may be called the set of "states of nature".

Θ: the set of outcomes of the decision problem.

f: the function that determines which outcome will result from a given decision and states of nature.

$\ll$: a relationship determining our preference among the outcomes.

If there exists a real-valued function $G : \Theta \rightarrow R$ with the property $G(\theta_1) \leq G(\theta_2) \rightleftharpoons \theta_1 \leq \theta_2, \forall \theta_1, \theta_2 \in \Theta$, then we can define function $J : D \times N \rightarrow R$ as $J(d, n) = G[f(d, n)]$, which is usually called the *payoff function*. When N contains more than one element, the order on Θ induces only a partial order on D through:

$$d_1 \ll d_2 \rightleftharpoons J(d_1, n) \ll J(d_2, n) \rightleftharpoons f(d_1) \ll f(d_2), \forall n \in N$$

A decision $d^* \in D$ is called a *dominant decision*, if $d \ll d^*, \forall d \in D$.

There are two approaches in general to formulate the decision problems:

1. Min-Max(or Max-Min) Approach:

Min-Max approach takes a pessimistic attitude toward the possible outcomes by maximizing over D the numerical function $F(d) = \inf_{n \in N} J(d, n)$.

2. Utility(or Risk) Function Approach:

Suppose we know that each decision $d \in D$ specifies the probability of each outcome via the function $f(d, \cdot)$ and $P_d(\theta) = P(\{n|f(d,n) = 0\}|d), \forall\theta \in \Theta$. Then, we can define the utility function $u[f(d,n)]$ satisfying:

$$d_1 \ll d_2 \rightleftharpoons P_{d_1} \ll P_{d_2} \rightleftharpoons E\{u[f(d_1,n)]|d_1\} \ll E\{u[f(d_2,n)]|d_2\}, \forall d_1, d_2 \in D.$$

The optimal decision problem is thus transformed to the problem of maximizing over $\mathcal{D}$ the expect value of the numerical function u. This function is also called the risk function or the loss function. A widely used loss function in Statistics is the MSE.

There are two types of decision problems: non-sequential decision problems and sequential decision problems. For the non-sequential decision problems, we usually use the Linear programming(LP) technique, and for the sequential decision problems, DP technique. Under both situations, the Lagarange multipliers can be used to reduce the dimension of the formulation.

3.8.2 Linear Programming

The LP model includes an objective function and a group of resource constraints, both in linear format. The objective function is to be minimized(or maximized) while satisfying all the constraints. Besides the Lagarange multipliers approach, the graphical linear programming approach is often used for simple problems. For those sophisticated problems, *Simplex Method* can be adopted with the aid of computation. When solutions are limited to the integers, the LP problems become more specific as the Integer Programming(IP) problems. The commonly used approaches for the IP problems are the *Transportation Method* and the *Assignment Method*. Related algorithms are also developed, like the forward and backward approach, the search in depth approach and the search in width approach.

3.8.3 Dynamic Programming

The DP approach follows the "Principle of Optimality" as stated. The optimal solution has the property that, whatever the initial value assigned to a step, the remaining steps should hold the optimal value for the solution. In practice, we have both discrete-time sequential decision model and continuous-time sequential model. Under the normal situation, we usually can consider only the discrete case to avoid un-necessary complexity.

Bertsekas(1976) addressed the sequential decision model in this way: Suppose that a system is characterized by three sets: the input set U, the uncertainty set W, and the output set Y through a system function $S : U \times W \rightarrow Y$. Define $\Pi : Y \rightarrow U$ as a feedback controller for the system if for each $w \in W$, the equation $U = \Pi[S(u, w)]$ has a unique solution for u. Now we construct the decision problem as the following: Take Π as the decision set, W as the set of the states of nature, and $Q = (U \times W \times Y)$ as the outcome set, where $U = \Pi[S(u, w)]$ and $Y = S(u, w)$. If G is a numerical function ordering the outcome Q, and J the corresponding payoff function for the decision problem, then the problem can be formulated as:

Finding $\pi \in \Pi$ to minimize $F(\Pi) = \inf_{w \in W} J(\pi, w) = \inf_{w \in W} G(u, w, y)$. Given a known probability measure on Π, we can define $F(\pi) = E\{J(u, w, y)\}$. Thus, the DP decomposes the task of minimizing $F(\pi)$ into a sequence of much simpler optimization problems and then can be solved backwards in time.

For the discrete-time sequential decision model:

$$x_{k+1} = f_k(x_k, u_k, w_k), k = 0, 1, \cdots, N-1, \tag{3.10}$$

the system operates over a finite number of states N(a finite horizon). We look for the feedback controller $\{u_0(x_0), u_1(x_1), \cdots, u_{N-1}(x_{N-1})\}$ as the solution.

Write $u = \{u_0, u_1, \cdots, u_{N-1}\}$ as the system input, $w = \{w_1, w_2, \cdots, w_{N-1}\}$ as the uncertainty quantity, and $y = \{x_0, x_1, \cdots, x_N\}$ as the system output. Further, assume that the utility function has an additive structure of the form $U(u, w, y) = U_N(x_N) + \sum_{k=0}^{N-1} U_k(x_k, u_k, w_k)$. Then, the problem becomes:

Maximize $J_\pi = J(u_0, u_1, \cdots, u_{N-1}) = E\{U_N(x_N) + \sum_{k=0}^{N-1} U_k[x_k, u_k(x_k), w_k]\}$, subject to the system equation constraints:

$$x_{k+1} = f_k[x_k, u_k(x_k), w_k], \quad k = 0, 1, \cdots, N-1. \tag{3.11}$$

Under the specific case of finite state Markov Chain, if the set of the transient probability $P(x_{k+1}|x_k, u_k)$ is given, then by defining $f_k(x_k, u_k, w_k) = w_k$, we can simplify the equation to: $x_{k+1} = w_k$.

For example, if we consider the correlations among different stages in the manufacturing process, we may adopt the model to find the solutions for the feedback controller. This is usually the actual situation under the manufacturing environment. As the components pass from one stage to the next, the operator at the next stage will have some quality information about the components and hence can estimate the quality status of the components under the current stage.

3.9 Simulation

The simulation technique has been developed with the digital computing technology. It has been proved to be a useful tool in analyzing industry systems(Schmidt & Taylar, 1970). The concept of simulation is simple: Instead of operating the real system, a simulator, the model for the real system is operated to obtain the data and analyze the results. As a common sense, every model has error tolerance from the real system. Hence, the simulation technique has disadvantages as well as advantages from the beginning of its application.

Advantages:

1. Simplify the real system with a model, thus provide guidance to the operation.

2. Save money and time for decision making about investment in the real system.

3. Provide more straightforward results than that from the pure mathematical analysis.
4. Provide possibilities for training environment of unrealistic system operations.

Disadvantages:

1. Provide no guarantee for the reality under the real system.
2. Depend heavily on the computing technology.
3. May take more efforts to build a simulation system for the real system.

The simplest simulation is the random number generator. However, more advanced simulation tools are also available for the statistical analysis. There are data simulation, process simulation and Monte Carlo simulation. Most recently, Markov Chain Monte Carlo(MCMC) simulation has been widely used in simulating the sophisticated random processes.

The software used for simulation can be put into two categories: general and specific. The general software can be developed using the general programming language, like Fortran and C. But there are also specific simulation software packages, like GPSS, BUGS and CODA for applications in certain areas.

Chapter 4

Developing Model System

4.1 Basic Notations

Consider the following typical scenario for a manufacturing company:

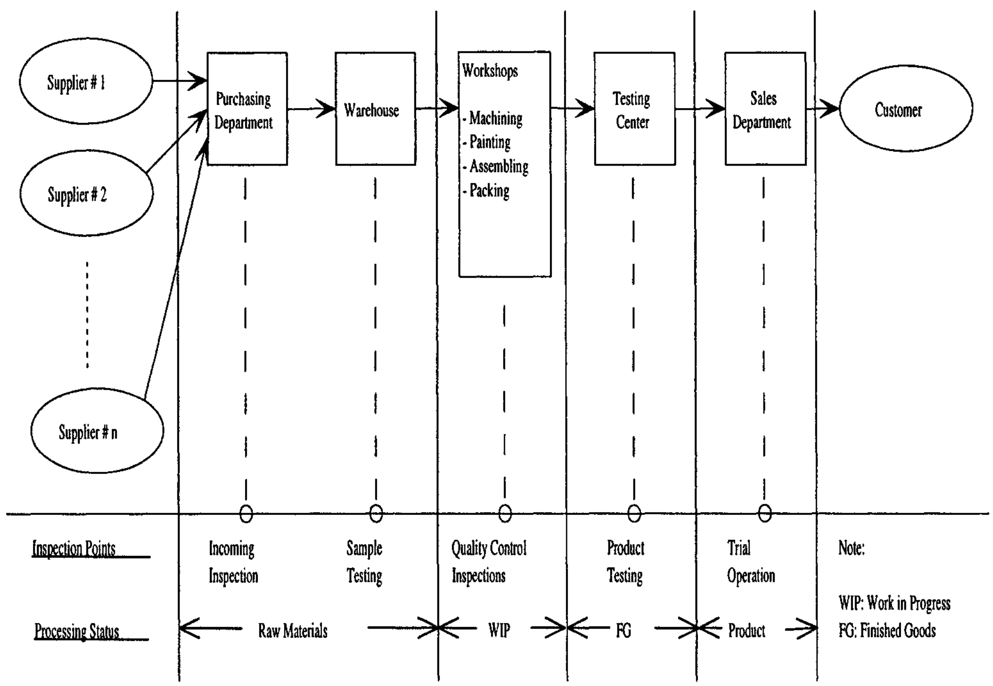

Figure 4.1: Typical Manufacturing Process under MS/SP/DP

Under this scenario(belonging to MS/SP/DP), we assume:

1. There is a sales order existing from a customer for a kind of customized product with demand "D".

2. There are "n" suppliers available for the major component used for the product. The supply capacity of each supplier is: "S_i'', $i = 1, 2, \cdots, n$.

3. There are "m" inspection points during the manufacturing process that may cause the scraps. The cost related to the scraps at each point is: "C_j'', $j = 1, 2, \cdots, m$.

4. No defective component can be used through reworking, hence becomes the scrap.

5. There are ample manufacturing capacity in the manufacturing company for the sales order.

6. The delivery time is fixed in the sales order, delay in delivery will cause the loss of sales.

7. Components from different suppliers have the same manufacturing cost structure.

8. All defects after delivery are considered as the normal tear and wear, and thus belong to the scope of repair/guarantee.

Further, we define:

1. "R_{ij}'', $i = 1, 2, \cdots, n; j = 1, 2, \cdots, m$ as the scrap rate of components from supplier "i" at the inspection point "j". Note here that the scrap rates are small values between '0' and '1', depending on factors from both the supplier and the manufacturing process.

2. "Q_i'', $i = 1, \cdots, n$ as the purchasing quantity from supplier "i".

4.2 Data Structure

The available data for our statistical analysis are the "Historical Quality Data" and the "Empirical Scrap Rate" as the follows:

Supplier\Stage	1	2	$\cdots$	$m-1$	m	Total
1	X_{11}	X_{12}	$\cdots$	$X_{1,m-1}$	$X_{1,m}$	X_1
2	X_{21}	X_{22}	$\cdots$	$X_{2,m-1}$	$X_{2,m}$	X_2
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
n	$X_{n,1}$	$X_{n,2}$	$\cdots$	$Xn, m-1$	$X_{n,m}$	X_n

Table 4.1: Historical Quality Data

Supplier\ Stage	1	2	$\cdots$	$m-1$	m
1	R_{11}	R_{12}	$\cdots$	$R_{1,m-1}$	$R_{1,m}$
2	R_{21}	R_{22}	$\cdots$	$R_{2,m-1}$	$R_{2,m}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
n	$R_{n,1}$	$R_{n,2}$	$\cdots$	$R_{n,m-1}$	$R_{n,m}$

Table 4.2: Empirical Scrap Rate

Note that the relationship between "R" and "X" is: $R_{ij} = X_{ij}/X_i$, where X_{ij} is the actual scraps happened at stage "j" in batch from supplier "i". So, R_{ij} is just the empirical scrap rate.

4.3 Analysis

Based on the objective of our study as well as the data structure, we can draw the following conclusions:

1. From Table 4.1 we learn that we have typical categorical data in the format of '$n \times m$' table. Thus we can use the regular methods for the categorical data

analysis to do our job. But, we may transfer the original data to the format as in Table 4.2. Then, we get the "empirical scrap rate" data. If optimal order quality is the only objective, then, as most industry practitioners did, we may directly use Table 4.2 to make our decision. But, actually the optimal order quantity is only one objective of our study. So, we need to take a deeper look at the data.

2. Considering the factors that may affect the scrap rate, we can construct a non-parametric model under the Bayesian approach as the important first step. There are two major factors: the supplier and the manufacturer. The supplier affects the overall scrap rate, or more accurately, the distribution of the scrap rate during the manufacturing process in average. On the other hand, the manufacturer affects the scrap rate at each manufacturing stage. Further, we consider the properties of the scrap under the JIT setting. We have reason to believe that the happening of the scraps, under most situations, is a kind of rare event. Thus, we can use the Poisson distribution as the prior distribution for the average scrap rate per supplier. Also, we may use the Gaussian distribution as the prior distribution for the scrap rates at each manufacturing stage. We can do this because we can believe that the scrap rates vary randomly at each manufacturing stage due to the environment. Hence, we can adopt the Bayes' Rule to make inference about the scrap rate through the historical quality data as the training data.

Following the notation, suppose that supplier "i" has supplied the component "l" times, with scrap rate "$R''_{ijk}, i = 1, 2, \cdots, n; j = 1, 2, \cdots, m; k = 1, 2, \cdots, l$. Then, $R_{ijk} \sim N(\mu_{ij}, \sigma^2_{ij})$, where $\mu_{ij} \sim Poisson(\lambda_i)$. Further, we can estimate μ_{ij}, σ^2_{ij} and λ_i through LSE or MLE, i.e., $\hat{\lambda}_i = E_j(\overline{R}_{ij})$, where $\overline{R}_{ij} = \frac{1}{l}\sum_{k=1}^{l} R_{ijk}$, and $\hat{\mu}_{ij} = E_k(R_{ijk})$, $\hat{\sigma}^2_{ij} = Var_k(R_{ijk}) + \hat{\lambda}_i$.

Under the Bayesian framework, we consider:

$$``R''_{ijk}, i = 1, 2, \cdots, n; j = 1, 2, \cdots, m; k = 1, 2, \cdots, l$$

as the training data. Then,

$$P(\lambda_i, \mu_{ij}, \sigma_{ij}^2 | R_{ij(l+1)}) \propto P(\lambda_i, \mu_{ij}, \sigma_{ij}^2) \times P(R_{ij(l+1)} | R_{ijk}, k = 1, 2, \cdots, l; \lambda_i, \mu_{ij}, \sigma_{ij}^2).$$

In other words, the posterior distribution of $(\lambda_i, \mu_{ij}, \sigma_{ij}^2)$ given the training data is proportional to the production of its prior distribution and its likelihood. So, the historical quality data is important for the estimation in the model building. For new supplier without historical quality data, we may have to find alternatives for the inference.

Here, we assume that each manufacturing stage is independent. But, under the actual situation, some stages may be correlated. For example, two machines may be operated by the same operator at two different stages. Under this case, we may need to consider the correlation among the multi-Gaussian processes.

3. On the other hand, we can adopt the survival analysis approach to do the analysis. Considering the product life cycle of those components, the scrap rates during the manufacturing process represent a kind of incomplete data with both left and right censoring mechanism. We know that this is a type I censoring, as the scrap rates before the delivery of the components from the supplier are unknown to the manufacturer, and the scrap rates after the delivery of the products to the customer are also out of control from the manufacturer. Of course, the analysis based on the incomplete data may raise concerns about the reliability, so we should either develop specific methods for the incomplete data analysis or avoid incomplete data. The second approach can be achieved through some managerial efforts, especially, through SCM. However, our interests here are mainly focused on the first approach.

Let us consider the states of a component from the supplier "i". We may define $\{y_t^{(i)}\}$ as a stochastic process for the quality states of this component. Here, we have a non-stationery finite-state delivery-time Markov process. The state space $S = \{0, 1\}$, where "0" represents the state of defective item, "1" the non-defective item. The time space $T = \{1, 2, \cdots, m\}$ as the "m" stages. Note that defects prior "1" or after "m" are considered to be censored.

As we have decided that under the JIT setting all scraps can not be reworked, so the state "0" becomes recurrent, while state "1" remains as the transient state. And the transient probability, as defined in the Markov chain, is just the scrap rate during the manufacturing stages.

Now we can define $\tau^{(i)} = min\{t : t = 1, 2, \cdots, m; y_t^{(i)} = 0\}$ as a stopping time for the components. $\{\tau^{(i)}\}$ becomes a random variable representing the survival time for the components. Further, since under general situation, the scrap rates decrease as the components are proceeding, we can use the martingale, or more accurately, the super-martingale definition to describe the variation of the scrap rates. Then, we can adopt the traditional statistical methods for the martingale oriented survival analysis to handle the inference.

4. One important principle in quality control is the continuous improvement. As the feedback information about the quality during the manufacturing process are returned back to the supplier from the manufacturer, the supplier may improve the quality of the components in the next batch to the manufacturer, to be the surviver in the competition. In this case, we can consider a kind of time series model with the auto-correlation between $Y_{t+1}^{(i)}$ and $Y_t^{(i)}$ as the improvement of our developing model system. Then, the quality improvement trend can be expressed through the model. However, this only applies to the situation when the manufacturer adopts the SCM approach and bonds the major suppliers with the long-time relationship.

5. The next step after the modeling is the optimization. As we have shown in the introduction, the optimal order quantity depends on both the cost structure of the product, which is a deterministic factor in our study, and the scrap rate, which is the un-deterministic factor. After the inference about the scrap rate, we can use the LP technique to determine the optimal order quantity for all existing suppliers and evaluate the ranking of the potential suppliers based on the model. In practice, a safe buffer may need to be defined to promise the uninterrupted operation under unexpected emergency situations.

4.4 Stochastic Models

4.4.1 Optimization

Our model for the optimal order quantity includes three related components: the scrap rate estimation, the regression, and the optimization. Let's follow the notation and reverse the consequence to construct the optimal model for the MS/SP/DP case.

Objective: Minimize $\sum_{i=1}^{n}\sum_{j=1}^{m} Q_i R_{ij} C_j$ subject to

constraints: 1. $\sum_{i=1}^{n} Q_i = D$

2. $0 \leq Q_i \leq S_i, i = 1, 2, \cdots, n$

Here, we need to know the following about this model:

1. $\sum_{i=1}^{n} Q_i = D$ is the loss function for the optimal order quantity.

2. R_{ij} is the estimated scrap rates for the components from supplier "i" at the manufacturing stage "j". It can come from the empirical data, the regression results, or the posterior distribution based on the prior distribution and the training data. However, for new suppliers, the only option is the regression results.

3. The optimization process is itself a process of vendor selection. All potential suppliers are considered but those with the result "$Q_i = 0$" are automatically excluded from the actual transaction.

4. The cost structure is decided together by both the product and the manufacturing process. The standard cost accounting method is preferred, but if the actual cost accounting method or the ABC accounting method is applied, then the historical data are required to establish the cost structure for each new sales order.

4.4.2 Regression

For the regression, we need to know more than the scrap rate. As deciding the covariates(or regressors) as the important first step, we will discuss this in detail later in the implementation part. The regression model should be established through considering the data structure and the covariate structure. In practice, the factors that may affect the scrap rate may vary from the numerical values to the nominal values. For example, the year in purchasing the components is a numerical value, but the status of the quality certification is a nominal value(Y/N). Following McCullagh and Nelder(1983), Cameron and Trivelli(1997), and Agresti(1990), we can use the log-linear regression model. On the other hand, from the survival analysis approach, following Kallefleisch and Prentice(1980), Cox and Oakes(1984) and Miller(1981), we can directly apply the PH regression model.

4.4.3 Scrap Rate Estimation

For the scrap rate estimation, besides adopting the methods from the regression, we can also consider the direct distribution model. There are descriptions on various Poisson models and extensions for the count regression in Cameron and Trivelli(1997). In particular, they have discussed many departures from the standard Poisson regression. Related to the scrap processing problem, we may consider the extra-Poisson variation(over dispersion or under dispersion), truncation and censoring, as well as the time-dependent trend.

1. Extra-Poisson Variation:

For both the direct distribution model and the regression model, we can consider the Gaussian-Poisson mixture model. We have discussed the direct distribution model in the analysis part. As we will see here, the idea for the regression case is similar. In the standard Poisson regression model, the conditional distribution of the "i"th observation "$(y_i|x_i)$" has a conditional mean function as a non-stochastic function of x_i. In our mixture model, the conditional distribution of the "i"th observation "$(y_i|x_i, \nu_i)$" has a conditional mean function with respect to ν_i, i.e., $p(\mu)$, where μ is a stochastic function of ν_i. A commonly used functional form is $E[y_i|x_i, \nu_i] = exp(x_i'\beta)\nu_i$, where the stochastic item ν_i is independent of the regressors x_i.

As for the standard Poisson distribution $p(\mu)$, we have: $E(x) = Var(x) = \mu$. But in the mixture model, $\mu_i = exp(x_i'\beta)$, and $\mu*_i = E[y_i|x_i, \nu_i] = \mu_i\nu_i$, where the unobserved heterogeneity term $\nu_i = exp(\epsilon_i)$ may affect the mean and the variation. There are two possibilities. In case $E(x) < Var(x)$, we call it *over-dispersion*; otherwise, *under-dispersion*.

2. Truncation and Censoring:

Due to the property of the scrap processing problem, we only need to consider the type I censoring or the interval truncation. The direct distribution approach uses the pseudo-distribution for the components' whole product life cycle and derives related distribution for the scrap rate during the manufacturing process. However, from the regression approach, the typical method used is the survival model. In some cases, the accelerate survival model can also be considered for the analysis.

3. Time-dependent Trend:

The solution for the time-dependent trend is usually the time series analysis approach. But in our discussion, this is not the key factor that affects the scrap rate, as only under the SCM scenario will this be a factor to be considered. So, the simple solution is to consider a random affect factor related to the time as an independent part in the regression model or the mean value of the scrap rates in the direct distribution model.

4.4.4 Managerial View

In practice, as the actual situation varies from time to time, from place to place, there is not a universal model for all cases in the scrap processing problem. Hence, the best approach is to establish a model database and match the information to provide the suitable solutions case by case. We will discuss this in more detail in the implementation part. For the model building, we consider a developing model system as the integrated solution. This developing model system combines the available models we have discussed early in this section, and can provide managerial solutions for the company for advancing to the ideal status. This includes three important stages:

Step I, Preliminary Stage

In this stage, the information about the vendors are incomplete or unreliable. Also, there are no consistent feedback from both the supplier and the customer. The only reliable information for the scrap rate estimation is the historical quality data. Hence, the only available model at this stage is the direct distribution model, like the Gaussian-Poisson mixture model. For the new supplier case, the quick-and-dirty method that can be used is to adopt the worst possible policy based on the direct distribution model for the existing suppliers to promise the operation under JIT.

Step II, Mature Stage

As the suppliers become the routine suppliers for the components, more reliable information can be obtained from them. Yet, no accurate quality information about the components at the supplier's or the customer's side is available at this stage. This is usually the regular situation in most manufacturing companies. Thus, under this stage, the regression model for the incomplete data can be used to predict the scrap rates for the new suppliers, to provide the analytical results for the vendor evaluation procedure and to give feedback information to the suppliers for further quality improvement. From the management aspect, this is a very important developing stage, as the comparison of the estimation results from the direct distribution model as well as from the regression model can be useful criteria to improve the accuracy of the prediction. New suppliers with related information at this stage can also be evaluated for the scrap rates under the regression model. This can be critical when the manufacturer need to decide the order shift from the an existing supplier to a new supplier.

Step III, Ideal Stage

With the establishment of the SCM system, information about the components among the supplier, the manufacturer and the customers can be shared by each of them. Thus, the incomplete data becomes the complete data, and the regression model for the count data can be used to better represent the accurate prediction. Also, the comparison between the regression models for the incomplete data and those for the complete data will lead the system to a learning model system.

4.4.5 Developing Model System

In summary, the managerial aspect of the developing model system is to use the higher level model to improve the lower level model, as the regression model vs. the direct distribution model, and the regression model for the complete data vs. the regression model for the incomplete data. More importantly, the mixture of those different models and the developing model system can provide the manufacturer more flexibility and convenience for the implementation of the JIT system. Now we summarize the models and inferences under our developing model system as the following:

1. Direct Distribution Model for the Truncated Data

Following the notation for the multivariate random variables, suppose that a long-time supplier has provided the manufacturer several batches of components for the same product. Thus, the historical quality record has the scrap rates for each batch of components at each processing stage. Let R_{ijk} denotes the scrap rate for the components of batch "k" from supplier "i" at stage "j", then, according to our analysis at the beginning of the modeling part, we can establish a Gaussian-Poisson mixed model: $(R_{ijk}) = \mathbf{R_{ij}} \sim N(\mu_{\mathbf{ij}}, \sum_{ij})$, where $\mu_{\mathbf{ij}} = (\mu_{\mathbf{0}} P_k(\mu_{\mathbf{0}})), P_k(\mu_{\mathbf{0}})$ is the probability of $P(x = k|\mu_{\mathbf{0}})$ and $\sum_{ij}$ is the covariance matrix. Thus, this model can satisfy our analysis results for the distribution and the properties of the scrap rate as the following:

(1). The average scrap rate at each process stage depends only on the supplier, with a trend to decrease under the most situations.
(2). The covariance matrix structure depends on the manufacturing process(or the manufacturer), with the variances depending on each processing stage and the covariances depending on related processing stages.
(3). If we don't consider the time related trend for the average scrap rate, then the model can be simplified into the standard multivariate Gaussian process.
(4). On the other hand, we may also directly consider the Poisson process if assuming that the scrap rates are i.i.d.

The next step is the inference. Here we may use the usual estimation approach, like MLE, to obtain the parameter estimates for μ_{ij} and $\sum_{ij}$. But we can also adopt the Bayesian approach to obtain the posterior distribution based on the prior distribution and the likelihood function. For the so called conjugated family, this is usually convenient to do. Thus, we may use the historical quality data as the training data to do the prediction.

The direct distribution model is the starting point for our developing model system. The objective of this step is to provide quick-and-dirty solutions for the prediction under the relatively poor resources. Still, the model can be improved through the historical quality data and the empirical properties of the distributions. Other related distributions, like exponential, Gamma, or more generally, Weibull can be used to better fit the data.

2. Cox PH Regression Model for the Censored Data

Suppose that (Y_i, X_i) are observations for subject "i", where Y_i are the direct observations, and X_i are the regressor vectors. Under the situations that we don't know X_i, and only obtained observations on a certain interval, we call it the *interval truncated* data. Under the situations that we know X_i, but only obtain observations on a certain interval, we call it the *interval censored* data.

Usually, censoring involves loss of information less than that of the truncation. Here, we may consider the truncation for the direct distribution model and the censoring for the PH regression model.

From the proceeding analysis related to the survival analysis, we have already known that the hazard function λ_k under the discrete case is just the scrap rate at processing stage "k''". Thus, we can simply adopt the results for our analysis. If T is a discrete random variable taking values at stage "k" with probability function $f(k) = P(T = k), k = 1, 2, \cdots, m$, then the survival function is:

$$F(k) = \sum_{j \geq k} f(j) = \sum f(j) H(j - k), \tag{4.1}$$

where $H(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$. And the hazard function is:

$$\lambda_k = P(T = k | T \geq k) = \frac{f(k)}{F(k)}, \quad k = 1, 2, \cdots, m. \tag{4.2}$$

Hence, we have:

$$F(t) = \prod_{j|j \geq k} (1 - \lambda_j), \quad f(t) = \lambda_j \prod_{i=1}^{j-1} (1 - \lambda_i). \tag{4.3}$$

And the discrete version of the PH model(Cox,1972) is:

$$F(t; Z) = F_o(t) exp(Z'\beta). \tag{4.4}$$

If $F_o(t) = \prod_{j|j \geq k} (1 - \lambda_j)$, then

$$F(t; Z) = \prod_{j|j \geq k} (1 - \lambda_j) exp(Z'\beta), \tag{4.5}$$

and

$$\lambda_k = 1 - (1 - \lambda_k)^{exp(Z'\beta)}. \tag{4.6}$$

We can also write the hazard function for the covariates Z as:

$$\lambda(t;Z)dt = 1 - [1 - \lambda_d(t)dt]^{exp(Z'\beta)}, \tag{4.7}$$

where the discrete baseline hazard function is $\lambda_d(t)dt = \sum \lambda_k \delta(t-k)dt$.

Cox(1972) also proposed another discrete failure time regression model using the linear log-odds approach. With the same notation, the linear logistics model can be written as:

$$\frac{\lambda(t;Z)dt}{1-\lambda(t;Z)dt} = \frac{\lambda_d(t)dt}{1-\lambda_d(t)dt} exp(Z'\beta). \tag{4.8}$$

The estimation of the parameters for the Cox regression model starts from the concept of partial likelihood proposed by Cox(1975). However, the partial likelihood is actually not a likelihood in the usual sense as proportional to the conditional or marginal probability of the observed events. For example, for the discrete logistics model, the partial likelihood for β is:

$$L(\beta) = \prod_{k=1}^{m} \left(\frac{exp(S_k\beta)}{\sum_{l \in R_{d_k}(k)} exp(S_l\beta)} \right), \tag{4.9}$$

where S_k is the sum of the covariates associated with d_k at stage "k", $S_l = \sum_{j=1}^{d_k} Z_{l_j}, l = (l_1, \cdots, l_{d_k})$. The computation for the partial likelihood in this case is extremely difficult.

For the discrete version of the Cox regression model, related to the hazard function, we can take $\gamma_k = log[-log(1-\lambda_k)]$ to improve the computation of ML. The log likelihood then can be written as

$$LogL(\gamma,\beta) = \sum_{k=1}^{m} \left(\sum_{l \in D_k} log\{1 - exp[-exp(\gamma_k + Z_l\beta)]\} - \sum_{l \in D_k} exp(\gamma_k + Z_l\beta) \right).$$

where $\gamma = (\gamma_1, \cdots, \gamma_m)$, and the score statistics are:

$$\frac{\partial logL}{\partial \gamma_k} = \sum_{l \in D_k} b_{kl} - \sum_{l \in D_k} h_{kl}, \quad \frac{\partial logL}{\partial \beta_k} = \sum_{k=1}^{m} \left(\sum_{l \in D_k} Z_{kl} b_{kl} - \sum_{l \in D_k} Z_{kl} h_{kl} \right).$$

where $\gamma_{kl} = exp(\gamma_k + Z_l\beta), b_{kl} = h_{kl}e^{-h_{kl}(1-e^{-h_{kl}})^{-1}}$.

Under the Newton-Raphson approach, $(\hat{\gamma}, \hat{\beta})$ can be obtained through iteration from the initial value (γ_0, β_0).

Also, the "observed" Fisher information is:

$$\mathcal{H} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \begin{pmatrix} \frac{-\partial^2 logL}{\partial\gamma\partial\gamma} & \frac{-\partial^2 logL}{\partial\gamma\partial\beta} \\ \frac{-\partial^2 logL}{\partial\beta\partial\gamma} & \frac{-\partial^2 logL}{\partial\beta\partial\beta} \end{pmatrix}.$$

where,

$$\frac{-\partial^2 logL}{\partial\gamma_k\partial\gamma_k} = \sum_{l\in D_k} g_{kl} + \sum_{l\in R_k} h_{kl},$$
$$\frac{-\partial^2 logL}{\partial\gamma_k\partial\beta_k} = \sum_{l\in D_k} Z_{kl}g_{kl} + \sum_{l\in R_k} Z_{kl}h_{kl},$$
$$\frac{-\partial^2 logL}{\partial\beta_j\partial\beta_m} = \sum_{k=1}^{m} (\sum_{l\in D_k} Z_{jl}Z_{nl}g_{kl} + \sum_{l\in R_k} Z_{kl}Z_{ml}),$$

and where, $g_{kl} = b_{kl}(e^{-h_{kl}} + h_{kl} - 1)(1 - e^{-h_{kl}})^{-1}$.

If we take initial value$(\gamma_0, \beta_0)' = (0, \hat{\gamma}(0))$, and iterate

$$\begin{pmatrix} \gamma_1 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} \gamma_0 \\ \beta_0 \end{pmatrix} + H_0^{-1}C_0 \tag{4.10}$$

until convergence, where

$$\hat{\gamma}(0) = (\hat{\gamma}_1(0), \hat{\gamma}_2(0), \cdots, \hat{\gamma}_m(0)), and\ \hat{\gamma}_k(0) = log[-log(1 - \frac{d_k}{n_k})],$$

where d_k is the failure number in stage "k". Then, the estimator of the survival function can be obtained by

$$\hat{F}(k; Z) = \prod_{j=1}^{k-1} exp[-exp(\hat{\gamma}_j - Z\hat{\beta})]. \tag{4.11}$$

To simplify the estimator, we can adopt the asymptotic approximation results and apply to $\hat{X} = log[-log\hat{F}(k; Z)]$, with mean $X = log[-logF(k; Z)]$ and variance $\sigma_x^2 = W'H^{-1}W$, where: $W = (\frac{\partial X}{\partial\gamma}, \frac{\partial X}{\partial\beta})'$,

$$\frac{\partial X}{\partial\gamma_j} = \begin{cases} \frac{h_j}{\sum_1^{k-1} h_j} & j < k \\ 0 & j \geq k \end{cases} ; \quad \frac{\partial X}{\partial\beta_k} = \frac{\sum_{j=1}^{k-1} Z_j h_j}{\sum_{j=1}^{k-1} h_j};$$

and where,

$$h_j = exp(\gamma_j + Z\beta).$$

At $\beta = 0$, $\hat{F}(k; Z) = \prod_{j=1}^{k-1}(1 - \frac{d_j}{n_j})$, becomes just the *Kaplan-Meier Estimator.*

3. Log-linear Regression Model for the Complete Data

At this final stage, we have all the quality information about the components through their product life cycle, as well as related information about the suppliers. Thus, we can use the log-linear regression model to fit the data and predict the scrap rate. Especially, we can consider the scrap rate at the supplier's side and the defective rate at the customer's side during a certain period as the counterparts of the additional manufacturing stages. Then, we will have "$m+2$" stages in total with related historical quality data available to establish the model, but may only use the predicted scrap rates during the "real" manufacturing stages for the optimal ordering quantity calculation. The managerial foundation for the model at this stage is the SCM system, which we have introduced in Chapter Two, and will discuss in more detail during the implementation part.

Following the GLM setting, define:

$$\mu_{ij} = E(\overline{R_{ij}}), i = 1, 2, \cdots, n; j = 1, 2, \cdots, m+2;$$

and

$$ln\mu_{ij} = \eta_{ij} = \beta^T X_{ij},$$

where $X_{ij} = (x_{ij1}, x_{ij2}, \cdots, x_{ijp})'$ is the covariate vector for μ_{ij}. This establishes a regression model for the scrap rates related to supplier "i" at stage "j". More generally, we can use the model:

$$ln\mu_{ij} = \phi_j + X_{ij}^T\beta \tag{4.12}$$

to separate the factors related to the manufacturing stages where β of length p is the parameter vector of our interests and ϕ of length "$m+2$" is the stage factor vector. Then, we can obtain the log likelihood function:

$$\begin{aligned} l_R(\phi, \beta) &= \sum [R_{ij}(\phi_i + X_{ij}^T\beta) - exp(\phi_i + X_{ij}^T\beta)] \\ &= \sum_i \phi_i \overline{R_{i\cdot}} + \sum_{ij} R_{ij} X_{ij}^T \beta - \sum_{ij} exp(\phi_i + X_{ij}^T\beta). \end{aligned}$$

Now write $r_i = \overline{R_{i\cdot}}$, i.e., $\tau_i = \sum_j \mu_{ij} = \sum_j exp(\phi_i + X_{ij}^T\beta)$, then,

$$
\begin{aligned}
l_R(\tau,\beta) &= \sum_i (r_i m\tau_i - \tau_i) - \sum_i [\sum_j \overline{R_{ij}} X_{ij}^T\beta - n_i ln(\sum_i exp(X_{ij}^T\beta))] \\
&= l_n(\tau; n) + l_{R|n}[ln(\beta; R)].
\end{aligned}
$$

Note that when 'm' becomes too large, the MLE may not guarantee the asymptotically optimal properties of the estimator $\hat{\beta}$.

4.5 Algorithm

For MS/SP/DP, the algorithm for the optimal ordering quantity is surprisingly simple:

Step I. We use the historical data to estimate the scrap rate distribution for all potential suppliers. For those existing suppliers, we use the direct distribution model or the regression model; while for the new suppliers, we use the regression model only to establish the risk structure.

Step II. Based on the estimated scrap rates and the known cost structure for the product, we can calculate the weighted risk $\sum_{j=1}^{m} R_{ij}C_j$ for each supplier. Then, we can just rank the suppliers with the ranks in this risk.

Step III. We allocate then the sales order to the suppliers according to their supply capacities for the components based on their risk ranks. The allocation procedure will stop when there is $k \leq n$ satisfying:

$$
\sum_{i=1}^{k-1} Q_i < D, \quad \sum_{i=1}^{k} Q_i \geq D.
$$

Step IV. In case there are ties in the ranks for the suppliers, and the ties happen for the last available supplier in the sales order, we need to use other criteria to rank the tied suppliers. For example, we can define the priority level for all suppliers, with the existing suppliers, the suppliers with larger supply capacity having the higher priority, etc.

4.6 Discussion

The developing model system has both managerial and technical advantages to us. In detail, it breaks through the border between the state of art technology and the practice. We discuss here several general important aspects related to the modeling technique under the developing model system.

4.6.1 Modeling and Data

Modeling is an important task for the data analysis. Usually, this seems to be a kind of single-direction approach. We establish the model based on the analysis of the existing data. Without data, or without necessary data, modeling becomes a mission impossible. Yet, the developing model system can provide guidance for the data collection itself. The historical quality data for the scrap rates is the important first step for the modeling, and the system will keep the advantages of those quality data and guide the extension of the data collection to the related covariates. Thus, the more advanced parts of the system can provide value-added solutions for the original objectives. In other words, modeling and the data collection procedure under our developing model system become a kind of interactive process. For example, at the preliminary stage, even though we only need the the historical quality date within the company for the the mixed model to forcast the scrap rate, we know that our target is to include the related information at both customer's and supplier's side. So, the data collection procedure should improve the quality to satisfy the requirments for this target. Consequently, the improvement of the data collection procedure will at the same time help the company to move to the higher stage under the developing model system, hence improve the modeling procedure.

4.6.2 Modeling and Management

The ultimate objective of modeling is the management improvement. Yet, under most situations, there is always a gap between the models and the management targets. In our case, different companies are at different stages for their management and technical preparations to adopt the quantitative analysis methods. The developing model system, on the other hand, can provide flexibility for the implementation of the models under various manufacturing environment for any company. We recognize that the world is changing, so good modeling technique should be able to bring the improvement for the *Status Quo* under the consideration. The developing model system can provide the possibilities for the company to establish the quantitative analysis system even at its initial management stages. Further, the system has the capacity to satisfy the growing requirements from the company at higher management stages. That means, the developing model system can help improve the management with continuous efforts, instead of waiting for the management to prepare every thing needed to adopt the models. At different management stages, we have different models available under the same modeling system to fit the requirements of different types of data, which is now the scenario for most companies in the manufacturing industry.

4.6.3 Modeling and Information Systems

The relationship between modeling and the information system oriented from the relationship between modeling and data. Under the modern manufacturing environment, the information system is the foundation for the implementation of the modeling technique. In our case, the developing model system is just an integrated part of the manufacturing information system. Various models are stored in the system and interact with the data to provide the information needed for the decision making, like the optimal order quantity. On the other hand, the information system can also provide the modeling technique powerful support for the simulation. For example, we may use the computer to simulate the results from different direct distribution models to improve the accuracy of the developing model system. Also, data can be organized in such a way that the historical quality data and the data generated from the models be compared to improve the models. At this stage, we prone to include the developing model system as a kind of DSS plug-in for the existing information systems to promise the value-added properties of our solutions. But in the future, we may see the improvements in the framework of the information systems themselves through the modeling approach at the system design stage.

4.6.4 Modeling and System Integration

The developing model system, as we have discussed, is suitable for the changing world. The successful applications of the model systems depend on many external factors. The manufacturing system includes not only machines, but also processes, human beings and external resources. So, a good model should consider the factors related to the system integration. If there is no strategic advance plan to the SCM system, then the rationales behind the developing model system itself can be of question. As a result, modeling technique becomes a part of the objectives for the systems integration. For example, the adoption of the regression model requires that we collect not only the quality data, but also the data related to the supplier and calculate the cost structure for the product based on the engineering data. If we have already installed the information system, then we can simply retrieve the data, and the accuracy of the prediction from the models depends on model assumptions. This can be accomplished by the operators through separating the fault data. This is just an example how the improvement in the model system and the improvement in the systems integration can be a kind of interactive activity all the time. Actually, the developing model system itself is just a kind of realization of the continuous improvement under the integrated system.

Chapter 5

Numerical Examples and Simulation Study

In this chapter, we will use the developing model system of Chapter 4 to analyze a case study, and construct several numerical examples under the manufacturing company in the case study to find the optimal order quantity through the simulation study.

5.1 Case Study

ABC Company is a large pump manufacturing company, its major products are advanced centrifugal pumps widely used in the industry, agriculture and environment protection area. During the past five years, the company has improved its management through establishing an up-to-date computer system and obtaining the TQM certificate for the manufacturing industry. However, under the growing marketing competition for the standard pump products with smaller pump manufacturers, the company decided to introduce the JIT system for most of its standard pump product series to maintain the competitive advantages in both the price and the quality.

As a key point to successful implementation of the JIT system, the logistic and production control division is reviewing its procedure for handling the purchasing orders. A major issue that comes to their attention is the instability in the quality of the pump bodies which are made out of the casting iron materials during the manufacturing process. Usually, this can be solved through inventory control as there are always enough pump bodies available in the warehouse to replace the scraps happened during the manufacturing process from time to time. However, with the implementation of the new system, this may become a bottleneck for the system as the order quantity will be decided by the sales order(plus a safe buffer) to minimize the stock level for those pump bodies.

The company has three product series, including a new pump series. For all products, there are about five manufacturing stages. And for the pump bodies, scraps can be happened at any stage. The quality problems at the customer's side after the delivery of the products are the responsibility of the repair and maintainence group, a group belonging to the sales division. The replacements for the guarantee are regarded as a kind of internal sales order proceeded under the same scheme as the customer order. The company has about ten major suppliers for the pump bodies, including an internal supplier as the subsidiary casting plant attached to the company. For the external suppliers, two of them are long-time partners with shared quality information through the connected computer system, five of them are regular suppliers with complete supplier's information in the company's computer system, this includes an overseas supplier, and the other two are newly-developed suppliers with only incomplete supplier's information in the company's computer system. The company currently has three major customers for its standard pump products. One of them is a long-time partner with shared information through connected computer system, another is a regular customer with only customer's information in the company's computer system, and the other is a new customer with only incomplete customer's information in the company's information system.

Now the company's chief controller is facing at least the following three questions:

1. How to estimate the scrap rate for the pump bodies during the manufacturing process?

2. How to establish a vendor evaluation system based on the estimated scrap rates?

3. How to rank potential suppliers and allocate the purchasing orders with the optimal order quantity?

5.2 Numerical Examples

Based on the case study, we can construct some numerical examples to illustrate the applications of the developing model system.

Suppose that we can obtain the following related information from various resources:

1. About the customer's order: suppose that we have three sales orders from several customers for product A, product B and product C.

Order Number	Product	Order Quantity
#1	A	220
#2	B	500
#3	C(new product)	200

Table 5.1: Existing Sales Order

2. About the customer:

Customer	Relationship
A	SCM partner
B	regular
C	new

Table 5.2: Customer's Category

3. About the supplier: Supply capacity

Supplier	Relationship	Region	Supply Capacity(for all products)
A	SCM partner	local	200
B	SCM partner	domestic	20
C	regular	local	50
D	regular	domestic	40
E	regular	domestic	50
F	regular	local	350
G	regular	overseas	50
H	special	internal	20
I	new	domestic	100
J	new	overseas	30

Table 5.3: Supplier's Supply Capacity

4. About the supplier: Other related information

We know that the following related information may be good regressors for the regression analysis.

- Category: Special, Partner, Regular Value(0, 1, 2)
- Region: Local, Domestic, Overseas

- Supply history: $1 \sim 5$ year, $5 \sim 10$ year, > 10 year
- Yearly output: < 100, $100 \sim 500$, $500 \sim 1000$, > 1000
- Employee number: < 100, $100 \sim 500$, $50 \sim 1000$, > 1000
- Technical staff #/Employee #: < 15, $15 \sim 25$, $25 \sim 35$, > 35
- R&D budget/Total budget: < 10, $10 \sim 20$, > 20
- Credit rating: AAA, BBB, CCC
- TQM certification: Y, N Value(0, 1)

We can also retrieve those supplier's information from the computer system.

Info.\Supplier	A	B	C	D	E	F	G	H
Category	P	P	P	P	R	P	R	S
Region	L	D	L	D	D	L	O	L
History	15	4	20	10	4	2	20	10
Output	1500	500	300	100	400	50	1000	50
Employee	800	200	80	30	100	60	400	40
Tech %	25	31	15	40	10	12	35	10
R&D %	15	10	18	20	5	12	15	10
Credit	AAA	BBB	CCC	BBB	AAA	AAA	BBB	BBB
TQM Cert.	Y	N	Y	Y	N	Y	N	N

Table 5.4: Related Supplier's Information

For the products, we have the cost structure information for product A and product B, as well as the estimated value for the new product C.

Product	Stage I	Stage II	Stage III	Stage IV	Stage V
A	100	20	10	50	10
B	200	30	20	10	30
C	150	50	50	20	10

Table 5.5: Product's Cost Structure(In US Dollars)

Also, we know the historical quality information about product A and product B. Based on those historical scrap rates and the related supplier's, customer's and product's information retrieved from the computer system, we can construct the following specific numerical examples to show the implementation of the developing model system in the real world.

Example 5.1. Direct Distribution Model for the Truncated Quality Data

When only the empirical scrap rates are available, we may be interested in the distribution of the scrap rates at each stage with different suppliers and products. Then, we can compare the risks from each supplier for interested products and rank the purchasing priority. Yet, under such situation, we can not rank those suppliers without the supply history for the same product.

Now suppose that we have the related information about the supply history for product A and product B(See Table 5.6). To decide the optimal order quantity for sales order in product A, we need to rank the available suppliers.

We can establish the related Gaussian-Poisson Mixed Model for product A from supplier A, B, C, D and F, and hence abtain the 95% upper bound value for the estimated scrap rates(See Table 5.7 and Table 5.8).

Based on this model, we can use the simulation to obtain the 95% upper bound value for the estimated costs related to the scrap risk(See Table 5.9).

Then, we can rank the supliers based on those values. The result is: F-A-C-D-B. Thus, the optimal order quantity is just: 350 to Supplier F, 200 to Supplier A and 43 to Supplier C.

Example 5.2. PH Regression Model for the Censored Quality Data

From Table 5.4 and Table 5.6, we have both the supplier's information and the supply quality history about the product A from supplier A, B, F and H. Now, we can establish a PH regression model based on those censored quality data and use this model to predict the scrap rate for the new supplier group. In this case, we can rank the purchasing priority for supplier A, B, C, D, F and the new supplier group(E,G,H).

First, we use the PH regression model to predict the average risk for the new supplier group. We run the regression on four critical factors: Category(C), History(H), Output(O) and TQM Status(T), and have the following result:

$$h(t)=h_o(t)\exp\{-9.68462(\mathrm{C}) + 0.0717(\mathrm{H}) - 0.00726(\mathrm{O}) - 5.0896(\mathrm{T})\}$$

We know from this regression model that "Category" and "TQM Status" are two most important factors related to the quality of the pump body.

Also, we can obtain the 95% upper bound value of the predicted scrap rates for the new supplier group through simulation(See Table 5.10).

Based on this, we can further get the 95% upper bound value for the costs related to the scrap risk for the new supplier group. The result in this case is: 291.52. Then, we have the new rank: F-A-New Supplier Group(E,G,H)-D-C-B. And the optimal order quantity can be applied through allocating 350 to Supplier F, 200 to Supplier A, and 46 to the new supplier group(E,G or H). Thus, we extend the order scope from the existing supplier group to the new supplier group.

Example 5.3. Log-linear Regression Model for the Complete Quality Data under SCM

As supplier A, B and customer A are the SCM partners of the manufacturer, also suppose that Supplier C, D and F are now SCM partners, then the quality data at both supplier's and customer's side can be shared by the manufacturer through the computer system. Thus, more accurate regression model can be established based on the complete quality data to predict the scrap rates for potential suppliers with related supplier's information. In this case, if we suppose that the sales order # 1 comes from customer A, then we can use the related historical quality data about the product A from supplier A, B, C, D, F and customer A(See Table 5.11).

Here, the scrap rates at the customer's side are just the defect rates within the guarantee period after the installation and on-site testing.

We use the Loglinear Regression Model to predict the scrap rate for all new suppliers. Based on the supply quality data, we first run the regression on three critical factors: History(H), Output(O) and TQM Status(T). The result,

$$m = \exp\{1.97119 - 0.00389(\mathrm{H}) + 0.00004(\mathrm{O}) - 0.36676(\mathrm{T})\},$$

where 'm' is the mean value of the scarp rate for the new supplier group, shows that "TQM Status" is the most important factor related to the quality of the pump bodies. Based on this, we can further predict the mean value of the Poisson Distribution for the scrap rates with Supplier E, G and H. Then, we adopt the simulation to obtain the estimated scrap rates and then get the estimated costs of risk(See Table 5.12).

Combining with the simulation results from the Direct Dirstibution Model under SCM, (See Table 5.12) comparing the 95% upper bound value, we can obtain the new rank for all available suppliers under SCM. The result is: F-G-A-H-E-C-D-B, and hence we can obtain the optimal order quantity by allocating 350 to Supplier F, 50 to Supplier G, and 176 to Supplier A.

Example 5.4. Integrated Model for the Mixed Quality Data

Under most situations, we have mixed quality data. That means, for some suppliers, like supplier A and supplier B for product A in Example 5.3, we have the complete quality data to run the regression; but for others, like supplier F and supplier H for product A, we only have the censored quality data to run the regression; and for those suppliers that don't have supplier's information yet in the computer system but do have some supply quality history for the product A, we can at least start with the direct distribution model. Those models are related and can be transfered into each other according to the changing status of the data. For example, to supplier A for product A, we can use the direct distribution model, the PH regression model and the log-linear regression model to estimate the scrap rates, and we can also compare the results of the estimates from different models to improve the prediction.

From Example 5.1 to Example 5.3, we have shown that the improvment in prediction can be achieved through effective utilization of the available information. In Example 5.1, we can only rank the existing suppliers: Supplier A, B, C, D, and F. In Example 5.2, we can rank not only the existing suppliers, but also the new suppliers as a group. In Example 5.3, we are be able to rank all available suppliers, whether existing or new.

Further, we noticed that the PH Regression Model and the Loglinear Regression Model provided us the same conclusion about the most important factor for the quality of the pump bodies, the TQM Status. Also, in general, the upper bound values of the estimated scrap rates in the Direct Distribution Model decrese from the regular case to the SCM case, as the consideration about the quality status at both the supplier's and the customer's side smoothed the overall scrap rates. This in some aspects gives us more accurate information about the actual scrap rates and can help us avoid unnecessary over-estimation of the scrap rates.

Product	Supplier	Batch	Quantity	I	II	III	IV	V
A	A	1	40	3	1	0	2	1
		2	60	5	2	1	1	2
		3	100	8	3	4	3	1
		4	50	2	0	1	1	0
		5	100	7	2	0	2	0
		6	40	4	2	1	0	0
A	B	1	15	1	0	2	1	0
		2	20	1	1	0	1	1
		3	10	0	1	0	2	0
		4	10	1	2	1	0	2
A	C	1	10	1	0	1	0	0
		2	10	0	1	0	0	1
		3	10	0	0	1	0	0
A	D	1	40	1	4	0	2	1
		2	20	0	1	1	0	2
		3	10	1	2	0	1	0
		4	20	1	0	2	1	0
A	F	1	150	8	3	4	1	0
		2	100	6	2	1	0	2
		3	100	5	4	2	1	1
		4	50	3	2	3	0	0
A	H	1	20	1	0	0	2	0
		2	10	1	1	0	0	0
B	H	1	20	2	1	1	0	0
		2	10	0	1	0	2	0

Table 5.6: Supply History: Product A and Product B

Stage\Supplier	A	B	C	D	F
I	N(0.02953,0.030838)	N(0.06382,0.070069)	N(0.03843,0.043333)	N(0.0453,0.048125)	N(0.0247,0.025358)
II	N(0.00045,0.030716)	N(0.00218,0.075625)	N(0.00077,0.043333)	N(0.00108,0.049323)	N(0.000313,0.025467)
III	N(4.56E-06,0.3068)	N(4.97E-05,0.073056)	N(1.02E-05,0.043333)	N(1.7E-05,0.054792)	N(2.64E-06,0.025358)
IV	N(3.47E-08,0.030717)	N(1.16E-08,0.075625)	N(8.2E-10,0.043333)	N(1.92E-09,0.049792)	N(1.67E-08,0.025425)
V	N(2.11E-10,0.030656)	N(1.16E-08,0.077292)	N(8.20E-10,0.043333)	N(1.92E-09,0.04974)	N(8.48E-11,0.025425)

Table 5.7: Gaussian-Poisson Mixed Model $N(\mu, \sigma^2 + \lambda)$ for the Truncated Quality Data

Stage\Supplier	A	B	C	D	F
I	0.373724	0.582644	0.446438	0.475270	0.336820
II	0.343960	0.541180	0.408775	0.436368	0.313095
III	0.343315	0.529814	0.408017	0.458806	0.314842
IV	0.343517	0.539001	0.408007	0.437127	0.312116
V	0.343174	0.544907	0.408007	0.437127	0.312526

Table 5.8: 95% Upper Bound Value: Estimated Scrap Rate

Supplier	95% Upper Bound Value
A	250.3145
B	392.6344
C	297.7015
D	321.3139
F	227.7434

Table 5.9: 95% Upper Bound Value: Costs of Risk in the Supplier

Stage	95% Upper Bound Value
1	0.6675
2	0.7875
3	0.4575
4	0.1875
5	0.1950

Table 5.10: 95% Upper Bound Value: Scrap Rate of the New Supplier Group

Product	Supplier	Batch	Quantity	S	I	II	III	IV	V	C
A	A	1	40	0	3	1	0	2	1	1
		2	60	1	5	2	1	1	2	0
		3	100	2	8	3	4	3	1	2
		4	50	1	2	0	1	1	0	0
		5	100	1	7	2	0	2	0	1
		6	40	0	4	2	1	0	0	1
A	B	1	15	0	1	0	2	1	0	1
		2	20	1	1	1	0	1	1	0
		3	10	0	0	1	0	2	0	0
		4	10	1	1	2	1	0	2	1
A	C	1	10	0	1	0	1	0	0	1
		2	10	0	0	1	0	0	1	0
		3	10	1	0	0	1	0	0	0
A	D	1	40	2	1	4	0	2	1	1
		2	20	1	0	1	1	0	2	0
		3	10	0	1	2	0	1	0	0
		4	20	1	1	0	2	1	0	1
A	F	1	150	2	8	3	4	1	0	3
		2	100	1	6	2	1	0	2	2
		3	100	1	5	4	2	1	1	1
		4	50	0	3	2	3	0	0	1

Table 5.11: Supply History of Product A from the SCM Partners

Stage\Supplier	A	B	C	D	F
S	0.336523	0.546261	0.43561	0.449766	0.310833
I	0.314106	0.419867	0.399637	0.409586	0.289694
II	0.313049	0.50892	0.398947	0.414311	0.290181
III	0.312828	0.499047	0.398939	0.438921	0.292396
IV	0.313055	0.508886	0.398939	0.416466	0.289462
V	0.312678	0.515138	0.398939	0.416225	0.289904
C	0.312217	0.490466	0.398939	0.409462	0.289462

Table 5.12: 95% Upper Bound Value: Estimated Scrap Rate for the Supplier under SCM

Supplier\Stage	I	II	III	IV	V	Cost of Risk
E	0.250	0.425	0.425	0.500	0.250	268.75
G	0.325	0.375	0.400	0.250	0.375	245.75
H	0.250	0.375	0.575	0.600	0.175	286.00

Table 5.13: Estimated Scrap Rate for the Supplier under SCM

Chapter 6

Conclusion and Extensions

6.1 Conclusion

In this thesis, we discussed the application of stochastic models for the scrap processing problem under the JIT setting. We developed a kind of integrated developing model system to satisfy the requirements from the changing available data status. In detail, we established three types of model for the quality data and combined the cost structure of the product to rank the supplier and hence obtain the optimal order quantity for the coming customer orders.

- Truncated Quality Data: Gaussian-Poisson Direct Distribution Model
- Censored Quality Data: PH Regression Model(Cox)
- Complete Quality Data: Log-linear Regression Model

Further, we analyzed the models from both technical and managerial points of view. We concluded that the relationship between modeling technique and management are really two sides of a knife. On one hand, the management requirements decide the modeling technique. On the other hand, modeling technique can push the management for continuous improvement on decision making procedure. To better adopt the modeling technique, the management need to implement the technical infrastructure for the JIT system.

We focused our discussion on analysis about the scrap rate estimation in this thesis. But, we also considered the optimal solution for the allocation of the customer order to the order quantity of the available suppliers. We addressed this problem through the simple LP technique, assuming that there was no direct relationship between the scrap rate and the cost structure of the product. However, if relax this assumption, the DP technique will be needed to derive the optimal solution.

As we can see, the extension of the scrap processing problem from the traditional manufacturing environment to the JIT/ERP/SCM environment and the integrated modeling system approach provided us some innovative ideas for the research in this area. We consider those as our major contributions in this thesis and are looking forward to developing further solutions in the future.

6.2 Extensions

In this thesis, we considered only a special case in the scrap processing problem under JIT. Still, many interesting further work can be done on this topic. At least, we believe that the following extensions are good examples of such kind of future efforts:

6.2.1 Other Processes

Considering the extension from MS/SP/DP to MS/MP/DP, MS/SP/CP and MS/MP/CP, we can see that the extension from the single-product case to the multi-product case is relatively easier than the extension across the category of the manufacturing processes, ie., from the discrete manufacturing process to the continuous manufacturing process. Under the first situation, we need to consider the weighted value of the costs related to the product for the ranking of the suppliers. However, under the second situation, we need to find the suitable continuous distribution for the random process of the component quality status as the starting point for further statistical analysis.

6.2.2 Serial Correlated Data

There are two typies of correlated data. One type is the serial correlated scrap rate data among different stages within the manufacturing process, another is the scrap rate data with correlation to the cost structure of the product. For both situations, the multivariate distribution for the random process of the component quality status needs to be adapted for further statistical analysis, and the solution will be more complex either for the modeling or for the optimization. Anyway, this is a very interesting topic for future study.

6.2.3 Robustness and Model Validity

When we talk about robustness or model validity, we always try to prove that the removal of certain assumptions under the model will technically not affect the results in the practice. In our case, the developing model system is derived under certain assumptions, like the normal distribution of the scrap rate data in a specific stage under the manufacturing process. This is the ideal situation. Under the actual manufacturing environment, it is possible that there are unexpected disturbances that may contributed by the systematic or random factors. Then, the approach for better modling is to consider other available distributions, like Weibull distribution, or those within the exponential family. The comparison of the simulated results with the practical data will lead to the optimal models for different manufacturing processes.

6.2.4 Time Series Data

The developing model system uses the historical quality data to predict the scrap rate. This is similar to the adaptive-learning mechanism. Yet there may be a forgetting factor for the time-series data. The most recent data may differ a lot from the not-so-recent data and are usually more important. To balance the most recent data and the historical data, the commonly used smoothing technique in the time series analysis can be adapted. Further, the factors that may cause the struture changes of the scrap rate data can be detected to improve the developing model system. More study can be done in this area.

Also, we can relate this topic to SPC. This is very important for the continuous process case, because the scrap rates at the prior stage may affect the scrap rates at the following stages. Under this situation, the management need to decide whether adopt the optimal allocation or the optimal sampling plan.

6.3 Implementation

The next step after the modeling and the optimization is the implementation. We discuss here some important issues in this step, including: System Design, Data Collection, Software and Integration.

6.3.1 System Design

A manufacturing control system using the modeling and optimization techniques is always established on the basis of the information systems. In our case, a stand-alone DSS including the model database and the optimization algorithm can be designed to retrieve and support the information from and to the information systems and the quality control system for the implementation of the optimal scrap processing under JIT. The objective of the control system is to design and develop a learning system, which may also be called the Automatic Order Generating System(AOGS).

from the statistical aspect, we use the historical data and related information to establish the suitable models, and then use the simulation and the optimization techniques to allocate the purchasing order. From the managerial aspect, we improve the prediction through the implementation of the SCM system. The replacement of the complete data to the incomplete data for the regression will nevertheless increase the accuracy of the estimated values and provide better feedback information to the system.

For detail information about the AOGS, see Appendix.

6.3.2 Data Collection

Data collection is an important step in the implementation. In our case, all of the data are proceeded through the information systems. The information about the regressors can be obtained from the "supplier", "customer" database, the cost structure from the "product" database, and the scrap rates from the "quality" database. Further, the information about the sales order and the purchase order can also be retrieved and returned to the related transaction databases during the process.

Under the SCM scenario, the related quality information about the components can be obtained directly through the information systems as the inputs for the DSS. This paperless data collection approach is both flexible and effective. The Data are collected following the principles as defined in the system design. The establishment of the related measuring systems can also be used to improve the modeling process.

6.3.3 Software

Based on the existing ERP system, we can develop some interface software systems to connect the decision-making functions to the major information systems as a kind of DSS plug-in. The interface system can include the following functions:

- Retrieve data from the information systems.
- Return data to the information systems.
- Connect to standard Statistical packages to process the simulations.
- Rank the potential suppliers for their priority in the order allocation for a certain product.

- Generate the optimal purchase orders for the sales orders.

The section of the software packages may depend on the purpose of the designed system. For a system used for academic research and education, we can use ACCESS as the database system, S-Plus as the Statistical package, and Fortran and C as the programming language for the interface systems. For a practical system, we may use the large relational database like ORACLE and Informix, advanced Statistical package like SAS, as well as the object-oriented programming language like C++ and Java to develop the interface systems.

6.3.4 Integration

The successful implementation of the developing model system depends on the integration. We consider the integration at two different levels:

1. Systems Integration

At this technical level, the developing model system can be worked as an integrated part of the manufacturing control system. The entered data coming from the information systems are proceeded through the DSS plug-in and the critical information for the decision-making are collected and returned to the information systems as the next generation training data. Without the information systems and the related JIT environment, without the SCM approach, the implementation of the developing model system can be impossible. More importantly, the decision-making related information flow can be considered as a part of the original information flow to help improve the design of the information systems. This can promise the strategic and tactical flexibility for the future development.

2. Business Integration

Any system contains both hardware and software. While the integration at the hardware or the technical level is important, the integration at the software level is usually more important. Here, the meaning of the "software" is more general than the traditional definition. For a manufacturing company, quantitative analysis tools are only the important first step, further steps after that are not such straightforward but usually more important for the implementation. In our case, the development in the JIT system, the efforts to the SCM and the training of the TQM concept to the employees are all critical factors for the operation of the system when using the developing model system. In a changing world, standardization is the king, while the flexibility is the queen. If we can integrate the technology with the system and the human being, then the modeling of the scrap processing problem under JIT can be the driver for the optimization of the processes and the quality system.

In other words, starting from the data, a developing model system can bring back the data with value-added properties to the system and simplify the manufacturing process toward the JIT under the SCM environment. That's the most important point of the system. As we are looking at the models, we should really take care of something outside the modeling technique to best invest our efforts for the future.

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Appendix A

A DSS Plug-in Program: Overall Framework

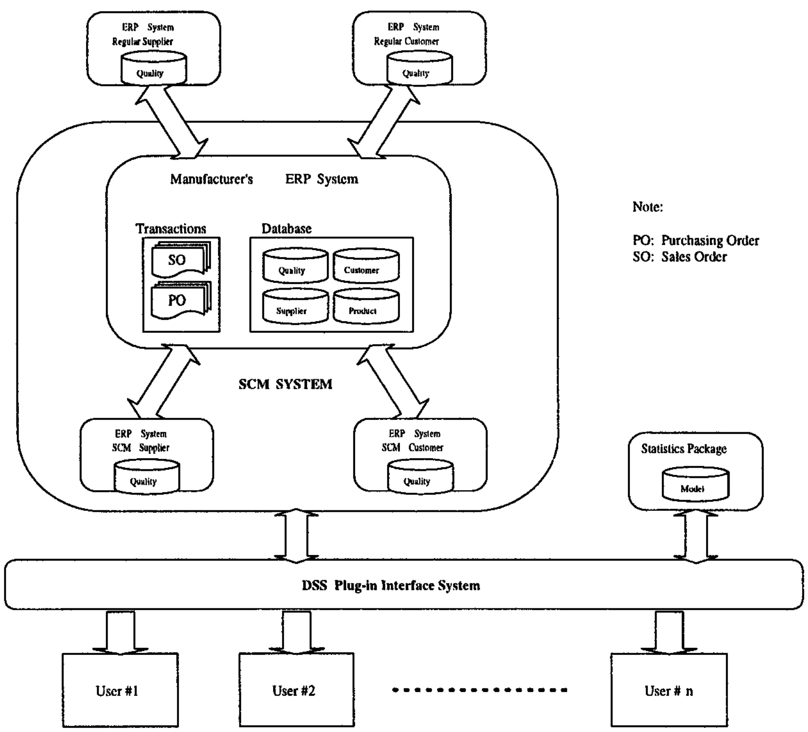

Figure A.1: AOGS: Overall Framework

Appendix B

A DSS Plug-in Program: Generic Flow Chart(Part I)

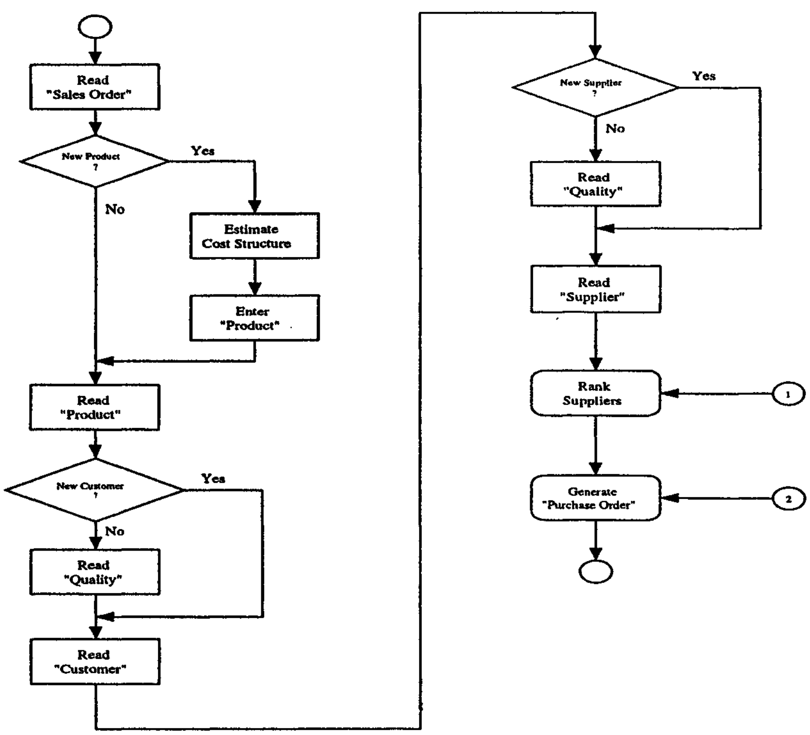

Figure B.1: AOGS: Generic Flow Chart(Part I)

Appendix C

A DSS Plug-in Program: Generic Flow Chart(Part II)

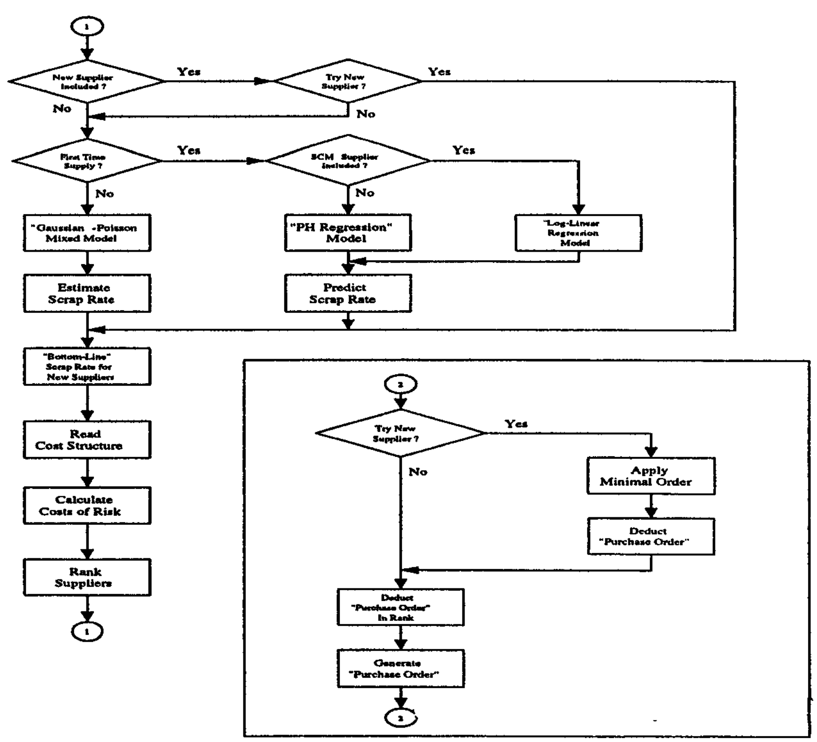

Figure C.1: AOGS: Generic Flow Chart(Part II)