University of Alberta

Fundamentals of Segregation

by

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DEDICATION

To
My wife Aparna and my son Aaron

and to
My parents Ayelech and Teklu

ABSTRACT

A common challenge during deposition of slurries is segregation as large particles settle through the matrix of fines and water. Whether segregation occurs or not depends on the grain size distribution of the solids, the void ratio or solids content and the rheological properties of the fines-water matrix.

The rheological characterization of slurry composed of different grain sizes and varying water chemistry was investigated. The vane yield stress was used to characterize different slurries composed of clay, silt and sand materials. Semi-empirical fractal theory showed good agreement with experimental data for fine slurry. Comparison of yield stress at same concentration but different composition showed a decreasing trend as the composition of either silt or sand material increases. The pore-water effect was studied for representative kaolinite slurry. The yield stress was insensitive for pH values in the acidic and neutral range, while in the basic range it showed significant response depending upon the type of the chemical used to achieve the pH: $Ca(OH)_2$ and NaOH.

A modified segmented standpipe was designed and used in a series of experiments to determine concentration profiles during the sedimentation processes. Analyses of the solid content profiles and sand content profiles in the standpipes indicated a capture of sand particles which could be correlated to the yield stress of the fines matrix. Theoretical calculations, however, showed over-prediction of the captured sand size. A correction factor of about 0.2 was applied.

Flume test on a high solid content slurries showed that the dynamic segregation is governed by all the factors governing the static case. Beaching profile shapes were not a necessary indication of segregating and non-segregating type of slurries. Modified version plastic theory for flow slides was used to characterise profile shape.

Computational fluid dynamics approaches based on kinetic theory and bi-viscous model analysis were implemented and showed a reasonable capability in modelling segregation when compared with experimental results. A statistical formulation for segregation index, SI, was proposed. The index accounts for variation in depth of samples. Finally recommendations for future research are proposed based on the observations and findings made from the study.

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CHAPTER 1

1. INTRODUCTION

1.1 General

Segregation is a phenomenon by which certain sizes or components with similar properties tend to preferentially collect in one or another physical zone of the assembly. Segregation commonly occurs in many natural and man-made processes: the sorted layers of magma, rock or ice avalanches, and seasonal stream deposits are a few among many natural events. In industry, where bulk materials are handled and transported, segregation may result in poor product quality as in pharmaceutical, ceramic, cement and food processing. On the other hand ore extraction processes exploit the advantage of differential settling. In oil sands extraction process, the bitumen is separated as a froth which floats to the surface; the coarse sand settles to the bottom and is removed. A portion of this slurry, known as middling, is removed from the central portion of the vessel, and is further processed.

Since segregation is a common problem of profound significance that touches many disciplines, it has been a subject of interest to mathematicians, physicists, engineers and industrial community. The studies are associated with different problems as sedimentation, consolidation, fluidization, erosion and mass transport (Pane and Schiffman, 1997). The study of segregation in civil engineering is relevant in sediment transport (Graf 1973), mud flow (Coussot 1997) and in granular flow problems (Savage 1979). Current development in the subject matter has extended to the application in rock (ice) avalanches, debris flow, sand dunes and oil sands waste management.

While sedimentation and consolidation have received a great deal of treatment in soil engineering, the emphasis has been mainly on fine and uniform soils. The consolidation theory treats the small displacement of soil as a result of water drainage by the excess pore pressure development. The groundwater and seepage studies are concerned with the relative movement of liquids to the soil grains. The flow of the soil grains with the interacting fluid medium as multiphase flow appears to be still on the course of development.

The disposal of high water content soil like materials and the reclamation of disturbed land have been recognized as a major challenge to geotechnical engineers (Krizek, 2000). He further stressed that growing safety issues such as risk-free environment and public safety and the need to integrate reclamation into the planning and design process dictate that much more remains to be done. Two major challenges were identified with mine wastes; the large volume they occupy and the great variability in properties. According to his estimate the annual total worldwide mine waste is in the order of a billion cubic meters or more, and the impoundments for these wastes are among the largest in the world.

With respect to oil sand tailings in Alberta, Morgenstern and Scott (1997) indicated that at the early stage of mine development, the storage volume required by tailings is three times the volume of oil sand mined and processed. These large volumes of tailings and their segregating characteristics were less understood and posed design and planning problems early in the history of oil sands mining. According to their report, the Syncrude tailings pond, which was designed to provide storage for 550×10^6 m^3 of sand, $370 \times 10^6 m^3$ of fine tails and 50×10^6 m^3 of water, required a construction of 18km dyke ranging from 32 m to 90 m in final height occupying a surface area of $22km^2$.

The most convenient and economical method for mining waste disposal is to impound them hydraulically. However such disposal schemes commonly yield settling of coarse and large size particles on the dyke beaches and transport of fine particles farther in the pond forming fine tailings. The major problems associated with a segregating type of tailings are:

- High operating costs
- Large volume to store and high lease costs
- Liquefaction susceptibility of sand beaches and low strength
- Toxicity and high environmental risks

- Slow settling rate and thus less quantity of release water to recycle
- Monitoring cost and difficulty in long term reclamation of the land
- Difficulty of mining the ores underneath the pond.

Cooper (1988) pointed out that the successful management of large dam tailings storage implicitly requires an understanding of the mechanics of slurry flow, the principles of drainage and consolidation and importantly, the development of the strength of the tailings. He also listed the most relevant geotechnical parameters which influence tailings storage construction: bearing capacity, moisture content/compactive effort relationship, permeability, shrinkage, consolidation, rate of loss of moisture (from beach area). It is necessary that the planning, design, operation, and reclamation of impoundment areas containing tailings be dealt not necessarily with traditional geotechnical approaches but rather it should involve expertise from other disciplines.

1.2 Motivation of Study

For the oil sands mining process, approximately 15 to 20 percent of the ore results in bitumen concentrate; the rest of the bulk material ends up as waste material. Such large quantity of waste cannot be disposed to the local environment, due to the presence of some constituents, which are not compatible with the environment. Thus safe disposal is a critical requirement and in most cases, the waste materials or tailings, are discharged into a tailings pond. The major concern upon deposition is that the coarse materials segregate and form a beach close to the dyke while the fine materials are transported further to the middle of the pond where they remain in suspension and undergo extremely slow sedimentation. This disposal scheme is accompanied by some consequences; such as leachate, liquefaction susceptibility and poor reclamation of release water for recycling due to the very slow settling rate in the pond.

Moreover the large volume occupied by the tailing ponds incur significant cost on lease and monitoring of the surroundings for any associated risks such as stability of dykes and seepage to the local groundwater system. Caughill (1992) remarked that the solution to such tailing problems should ideally include, one-step disposal, a reclaimable surface, low cost and high safety, reclaim of water, leachate control and reduction in total storage volume.

Hence, the handling of mine waste seeks an optimal disposal scheme. Production of non segregating tailings is currently viewed as a solution to either the existing tailing management challenges or future planning. Achievement of non-segregating tailings requires a fundamental understanding of the behavior of the constituents and the governing mechanisms involved.

Generally, the mechanisms which govern the segregation process are still not well defined, owing to the complex nature of the process where particle features such as particle size and distribution, shape, density, chemical affinity and many others contribute to the complexity. The subject has been regarded as an engineering frontier by some (Savage 1979) and as the focus of debate among others (Edwards and Grinev 2003). Boogerd et al. (2001) have gone further to question whether something is missed. There exists no unified model which can predict the occurrence of segregation while incorporating all relevant factors over the wide range of flow regimes. Attempts, however, have been made to characterise dominant mechanisms over narrow ranges of conditions.

The segregation process, primarily, occurs under subaqueous and subaerial environment. The sedimentation process refers closely to the former and granular flow to the latter. When solid phases interact with fluid, e.g., water, sedimentation / segregation occur by the action of gravity due to size or density differences among particles. The subject of sedimentation involving suspensions containing mixed particle sizes is not well developed, and reliable relationships have not been available. This is because of the wide range of particles sizes, the variety of particles shapes, and the complex nature of the hydrodynamic and physicochemical phenomena which governs particle-fluid and particle-particle behaviour (Selim et al., 1983). Nevertheless, it is common among engineers to deal with their complex problems in some pragmatic way based on experimental evidence and past experience. For example, Terzaghi et al. (1995) stated that success in geotechnical engineering, more than any other field of civil engineering, depends on practical experience. i.e., experience has tailored the practice.

It appears that the study of sedimentation with application to industrial process has received wide treatment in chemistry or chemical engineering and mining engineering. The works in these areas can be seen mostly as experimental study supplemented by semi-empirical models or simulation (Richardson and Zaki 1954); (Barnea and Mizrahi 1973); (Lockett and Al-Habboby 1973); (Lockett and Al-Habboby 1974); (Garside and Al-Dibouni 1977); (Mirza and Richardson 1979); (Selim et al. 1983); (Zimmels 1983). Some study in the subject matter has also been made in the geotechnical field with emphasis, however, on certain soil types, namely, fine clays (McRoberts and Nixon 1976); (Been and Sills 1981); (Tan et al. 1990); (Toorman 1996); (Pane and Schiffman 1997); (Toorman 1999).

Soil, as encountered in nature, is rarely uniform and dealing with this heterogeneity in mine waste management is an issue where the mine processing industry produces large volumes of waste. The development of effective disposal schemes for this situation has become a major challenge to the geotechnical engineer. For example, Eckert et al. (1996) described the nature of the sedimentation/consolidation process in oil sand tailings as very complex, due to the presence of ultra-fines and some chemicals, and the sedimentation/consolidation process is expected to take very long time (centuries).

While most of the forgoing studies presented are associated with a quasi-static environment, the sedimentation /segregation phenomenon is also common in the process of mixing, transportation and deposition. These processes are collectively referred to as dynamic segregation. Some valuable contributions on dynamic segregation are found in sediment transport studies in hydraulic engineering (Graf,1973), (Vanoni et al. 1975), (Garde and Ranga Raju, 1977), and (Choux and Druitt 2002).

The investigation made by Kuepper (1991) has indicated the existence of segregation (hydraulic sorting) of the hydraulic fill both in laboratory and field experimentation.

While her work typically dealt with the hydraulic deposition of segregating slurries, it was mentioned that non-segregating slurries exhibit non-Newtonian rheology. Morgenstern and Kuepper (1988) stressed that the ability to forecast and control grain size separation is still limited. They further suggest the need for greater understanding of the process. Such insights have notable significance in this research as to identify which rheological characteristics, under segregating and non-segregating regimes, slurries may exhibit, and their significance in the prediction models. Consequently, the stimulus for this study comes from a desire to make available fundamental principles which contribute to the basic understanding of the segregation process.

1.3. Objective of the Research Program

The objective of this research program is to establish the fundamental factors controlling segregation mechanisms in oil sand tailings under static and dynamic (shear) conditions. The principal factors that will be studied include:

- The viscous and chemical nature of the fluid medium (Pore fluid).
- The grain-size distribution.
- Mechanical/shearing action.
- Flow mechanism.

1.4 Statement of the Problem

The gap-graded nature of oil sand tailings stream accounts for the segregation of fines from the sand grains. Accumulation of segregated fine tailings results in a large volume of stable suspension with little release water and insignificant consolidation, incurring increased operating costs and long term reclamation (Fine Tailings Fundamentals Consortium 1995). The very stable nature of fine tailings is attributable to the gelation characteristics of the ultrafines (<3μm) and the chemical composition of the pore water.

The effect of ultra fines on the macroscopic behaviour of fine tailings is manifested by the rheological properties of the slurry. Rheological studies suggest that fine tailings exhibit non-Newtonian characteristics. The significance of non-Newtonian behaviour has also been indicated by Scott et al. (1985), who have explained in their experimental result that the rheological (‘gel’, their term) strength of the slurry contributes to the capture of sand grains resulting in non-segregating mixes.

Research dealing with the application of rheology of fines in segregation studies is not well established and rarely linked to the hydrodynamics of the process. While categorisation of the grain sizes of the tailing materials into two major divisions as fines and sands appears simple and easy from practical point of view, the impact of such bulk division is, however, unexplained. Since segregation by size is the major phenomena, study of wide spectrum of sizes is necessitated. Nevertheless, there exists no adequate research tool to provide data in the process of segregation. The different flow regimes under which segregation occurs need also further investigation.

1.5. Organization of the thesis

A summarized outline is presented briefly as follows. The thesis comprises the following chapters:

Chapter 1 Introduction
Chapter 2 Literature review
Chapter 3 Geotechnical and rheological characterization
Chapter 4 Static sedimentation/segregation experiments
Chapter 5 Flume segregation test
Chapter 6 Numerical modeling studies
Chapter 7 Quantitative analysis
Chapter 8 Conclusion and recommendation

1.6 Scope of the thesis

The material and rheological characterization tests are carried out with the available laboratory equipment. All static standpipe tests are completed with one or two litre standpipes and a custom-designed standpipe developed during this research program. Even though the chemical interactions have influence on the macroscopic behaviour, the microscopic influences due to chemical flocculent or coagulant addition are precluded in this study. Moreover the water to be used in the standpipe test is tap water. Tailing release water is used in all experiments involving tailing materials only. In the case of examining some particular phenomena, the materials to be used may not be exactly similar to the tailing materials, substitute materials are used as an option, keeping the properties as closely similar as possible to that of tailing materials. In the dynamic experimentation, the flume test is used. The study of segregation under pipe flow conditions are not in the scope of this study. The concept of 'similarity of process' as justified by Kuepper (1991) is applied in the experimental program of this research. Time-dependent rheological properties, like creep and thixotropy are known to have some influence on the long-term process of segregation; however, since the focus of the current research is on immediate or short-term segregation mechanism, these rheological characteristics are not investigated in this research study.

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CHAPTER 2

2. LITERATURE REVIEW

2.1 General

The utilization of natural resources has enabled humankind to reach the current level of development. All the inorganic part of the resources are derived from the earth's crust, the thin shell that coats our planet to a depth of 13km (Kelly and Spottiswood 1982). The ores, however, are not ready-to-use in their original form. Their extraction and process involves different level of effort and operation. Only small portion of the ore results in concentrate, the remaining bulk of material is disposed. The mineral processing industry, in general, has put a greater demand on solid-liquid separation equipment in recent years. This trend has been partly due to environmental consideration and partly due to technical issues and cost efficiencies (Pearse et al. 2001).

Most of the attention in geotechnical engineering has been focused on the behaviour of mass of soil grains. Terzaghi's early works primarily deal with the post-sedimentation behaviour of the soil i.e., after the formation of soil, or more specifically after the development of effective stress. The flow of liquid, through porous media has been another part where considerable development has been achieved. When it comes to the movement of the solid grains with the fluid, a multi-phase, multi-component flow, little emphasis is observed as to the involvement of geotechnical engineers. Lately, the area has attracted attention in the resource development and management in highly demanding areas such as handling and depositing mine tailings. Some of the subjects of interest are sedimentation or consolidation of fine slurry, stability of impoundment, ground water contamination, liquefaction susceptibility and slurry handling (segregation), and reclamation.

Numerous disciplines are concerned with the relative motion which can be established between a fluid phase and a suspended solid phase. These disciplines include geotechnical engineering, sedimentology and chemical (mechanical) engineering and environmental engineering, and the associated problems relate to sedimentation, consolidation, fluidization, erosion and mass transport (Pane and Schiffman 1997).

The phenomena of segregation, though unnoticed, may trace back to the era of earth crust formation, where the main geologic event, volcanic eruption, exploded off the magma and then fall-out scoria, the lava flow and the ashes form a segregated deposit (Fisher and Schmincke 1984). The other most common event is sediment transport by streams. The streams carry large load of sediments whereby small sediment loads are transported long distance while the larger ones settle to the bed en-route. As aggradations (rise in bed level) occur usually at downstream sides and such events takes place over seasons form a bed which is sorted of different particle sizes. More common is also the occurrence of segregation as debris flow takes place. The dangers caused by debris flow are attributable to the large boulders which segregate due to their high inertia.

When we consider the segregation phenomena in the industrial sector, the importance of segregation occurrence cannot be understated. Its occurrence is either desired or vice versa. The mineral industry exploits the phenomena of segregation to separate different particle sizes from the ore slurry, whereas, some industries, like the pharmaceutical, ceramic, paints and some others, do not want the mixed slurry to segregate. This becomes evident when one simply recognizes the fact that particulates are universally found as constituents of most commonly used items, which are produced within extensive complex industries, i.e., agriculture, ceramics, chemicals, energy, geological systems, mining, pharmaceuticals, plastics, pollution control systems, and powder metallurgy.

One may define the term segregation as a tendency for certain sizes or components with similar properties to preferentially collect in one or another physical zone of collective (de Silva et al. 1999). The study of segregation may be viewed from two perspectives, namely the sedimentation process and granular flow. In both scenarios, the segregation phenomena occur when there is a size, density or other physical property (say angle of repose) difference among the mixed constituents, (more than or equal to two parts).

The process of sedimentation of particles dispersed in a fluid is one of great practical importance (Kynch 1952). Sedimentation is involved to various degrees of importance. For example, transportation and agitation of slurries depend on the prevention of settling of the suspended solids. Classification, fluidization and elutriation operations are designed to meet the sedimentation characteristics of the particulates. Phase separation in solvent extraction depends on the distribution of dispersed phase in the mixer settler. Thickening and centrifugation are controlled by sedimentation. Density of reactors that utilize counter-current flow of phases involve consideration of sedimentation (Zimmels 1983).

Lockett and Al-Habboby (1973) stated that there seems little prospect at the present time of dealing with the hydrodynamics of binary particles-liquid mixture in a fundamental way. This notion is still shared today as the current developments could not provide us with the thorough theoretical background. Different factors account for the settling characteristics in a suspension, such as hydrodynamic effect of the system, concentration, or geometric packing of the suspended particles and interaction between the liquid and the particles.

Some semi-empirical approaches have been provided to describe the mechanism of segregation. Such models, however, are specific to the test conditions and subjected to different limitation. For example, no sedimentation model is available for predicting the sedimentation characteristics of dense suspension. Selim et al. (1983) stated that sedimentation in a concentrated suspension of particles is a broad subject because of the wide range of particle sizes, the variety of particle shapes, and the complex nature of the hydrodynamic and physicochemical phenomena which govern particle-fluid and particle-particle behaviour. The subject of sedimentation involving suspensions containing mixed particle sizes is not well developed, and reliable relationships are not available.

Natural and industrial processes generally involve many particles with wide distribution of sizes. Sedimentation in such systems results in particle classification by size, and models capable of describing settling in such concentrated mixed particle size system are needed in assessing industrial operations such as separation, and particle fractionation and natural processes involving sedimentation.

Despite an overwhelming appearance of literature in the last five decades, relationships between identified mechanisms are ambiguous, experimental data is scarce and is subject to some limitation, and there is no generally accepted model capable of predicting the occurrence of segregation over the wide spectrum of possible flow regimes. These issues bring to bear scientific as well as technical questions, such as the existence of "universal" mechanism of segregation, their measurement and quantification, the effect of mean flow and particle fluctuation, the evolution of microstructure, and the feasibility of developing a unified model.

While the underlying focus of the research is the study of segregation, with particular attention on oil sands tailings, an effort has also been made to base the study from an integrated theoretical background, experimental studies to date and numerical studies. This literature review attempts to cover some of the developments made so far in an extensive manner in an effort to bridge the whole spectrum of the segregation process.

2.2 Theoretical Background

2.2.1 Suspension properties

Suspensions are a heterogeneous mix of fluid and solid grains exhibiting different interactions like liquid molecule interaction, fine solid particle interaction, friction or collision between grains, particle-water flow, etc. (Coussot 1997).

The interaction within water, commonly known as hydrodynamic interaction, takes place due to momentum transfer as a result of molecular motion. When colloidal particles (1nm to 10μm in size range), collide with the fluid molecules surrounding them, a chaotic motion called Brownian motion results. Van der Waals forces are major forms of interaction between atoms, molecules, or particles. These forces result from dipole or induced-dipole interactions at the atomic level. Colloidal particles have a large number of atoms or molecules and thus exhibit larger van der Waal forces. These forces consist of three major categories known as Keesom interactions (permanent dipole/permanent dipole interactions), Debye interactions (permanent dipole/induced dipole interactions), and London interactions (induced dipole/ induced dipole interactions) (Hiemenz and Rajagopalan 1997).

Electrical interaction among colloidal particles in a suspension influences a particle stability, and interaction with surrounding particle or fluid. Electrical double layer is formed due to non-uniform distribution of ions around a charged particle (Elimelech et al. 1995). At the surface of the mono-layers of clay particles, adsorbed exchangeable cations may slightly diffuse in the liquid (water), while remaining attracted by, and thus close to, the particle surface. In parallel some ions of opposite charges which should be repelled from the surface, will tend to get closer to the surface in order to compensate for the diffusion of the exchangeable cations. This leads to the formation of an electric double-layer made up of the charged surface and a neutralizing excess of counter-ions (exchangeable cations) over the co-ions distributed in a diffuse manner (Coussot 1997).

When particles come into contact, they aggregate and/or deposit and surface contacts between grains will result in deformation at contact locations. The deformation at contact location can be elastic, viscous, plastic or other complex deformation types.

2.2.2 Theory of Sedimentation

The process of sedimentation involves the dispersion of particulate materials in a fluid and the settling process that the particles undergo due to different action such as viscous, traction and particle interaction.

The settling of a single sphere in an unbounded fluid represents the simplest case of solid/liquid sedimentation (Chen 1994). When a single spherical particle is suspended at rest in a liquid, it experiences two opposing forces: buoyant force, B_F, and gravitational force, G_F as shown in Figure 2-1. The unbalanced force, called drag force, F_D, is caused due to difference between the density of solids and the liquids. The drag force increases with an increase in particle velocity.

The drag reduces the acceleration and finally the particle settles at constant terminal velocity. The particle then comes to equilibrium due to driving force ($G_F - B_F$) and the drag force, F_D. The drag force is given by:

$$F_D = V(\rho_s - \rho_l)g \tag{2.1}$$

This relationship holds for a particle in an infinite fluid and depends on the Reynolds number. Reynolds number is given by:

$$Re = \rho_l u_{sr} d_p/\mu \tag{2.2}$$

where

d_p=diameter of particle;

u_{sr}= relative velocity between particle and liquid;

ρ_l = density of liquid; and

μ = viscosity of liquid.

For laminar flow the drag force is given by Stokes as:

$$F_D = 3\pi\mu d_p u_\infty \ . \tag{2.3}$$

Equation (2.3) gives an approximate result. For better accuracy, reference could be made to an equation provided by Proudman and Pearson, provided in literature by (Chen 1994):

$$F_D = 3\pi\mu d_p u_\infty [1 + 3/16 * Re + 9/160 * (Re)^2 \ln\{Re/2\} + \ldots] \ . \tag{2.4}$$

The drag coefficient C_D is obtained by dividing the drag force by $\rho_l u_{sr}^2/2$ and by the area of the body projected onto the plane normal to $u_{sr.}$

$$C_D = \frac{F_D}{\frac{\rho_l u_{sr}^2}{2}\frac{\pi d_p^2}{4}} \tag{2.5 a}$$

Which together with Equations.(2.2) and (2.4) and setting $u_{sr} = u_f$, becomes

$$C_D = \frac{24}{\mathrm{Re}} + \frac{9}{2} + \frac{27}{20}\mathrm{Re}\ln\left\{\frac{\mathrm{Re}}{2}\right\} + \ldots \tag{2.5.b}$$

For normal sedimentation equipment, the second and third terms can be ignored , thus the equation reduces to,

$$C_D = \frac{24}{\mathrm{Re}} \quad . \tag{2.5.c}$$

From Equations (2.4) and (2.5c) one can get the following expression,

$$u_\infty = \frac{d_p^2(\rho_s - \rho_l)g}{18\mu} \tag{2.6}$$

This equation gives the terminal velocity of a single particle in an infinite fluid medium, in laminar flow condition. Equation (2.6) is commonly used to estimate the size of a particle in the sedimentation test.

When other particles are present, the settling of a single particle is affected by the neighbouring particles and the concentration in the liquid. Group effects are presented in the following sections.

2.2.3 Kynch theory of batch sedimentation

Kynch (1952) provided a theory of sedimentation for incompressible materials based entirely on the continuity equation. An analogy of sound propagation has been used to derive the equation. The theory was developed based on the following assumptions:

- The suspension is ideal
- The concentration of particles is constant at any horizontal cross section in the column.
- The suspension has a homogeneous initial concentration ϕ_o.
- At the bottom of the container there is a continuous but extremely rapid increase of concentration from ϕ_o to the final concentration of ϕ_∞.

The main advantage of the Kynch equation is that the results obtained from a single batch test can be used to describe the settling process of the slurry at different initial solids concentration.

The Kynch theory does not account for the effect of the compression zone on the settling behaviour. Tiller (1981) modified Kynch theory to include the effect of the rising sediment layer at the bottom of the cylinder by considering the characteristics, which propagate within the second falling rate region and originate from the sediment-suspension interface. Later works by Tiller (1981) and Fitch (1983) dealt with relations for average solid concentration within the suspension region. The major drawback of the theory is that it requires prior knowledge of the height of variation of the mud line and sediment-suspension interfaces with time. Such information, however, is not known beforehand and is typically part of the solution (Diplas and Papanicolaou 1997). It also deals only with mono-disperse suspensions (Figure 2-2).

2.4. Experimental and model studies

2.4.1 Monodisperse Suspension

In the forgoing discussion the settling velocity of single particle was shown to depend on the diameter, the density of particles and density and viscosity of the fluid. When more particles are present, the settling velocity will also depend on the concentration of the particles, due to the mutual interactions among particles. And earlier experimental work indicated that as the concentration increases, the settling velocity decreases. Mono-disperse suspension refers to the presence of relatively similar particle sizes in the suspension.

When closely sized particles are distributed in a suspension, there is development of a visible interface after the commencement of settling. As the concentration of particles increases, the particles exhibit joint descent. Such phenomenon is known as hindered settling.

The sedimentation rate of particles in a concentrated suspension is always less than the settling rate of a single particle in isolation. This is partly because the downward movement of particles causes an equal volumetric flow rate of displaced fluid relative to which the particles must move. Furthermore, for a given relative velocity, the average velocity gradients, and hence shear stresses, will be greater in a concentrated suspension (Mirza and Richardson 1979). The hindrance effect in a multi-particle system is due to the following major effects (Barnea and Mizrahi 1973):

(i) The pseudo-hydrostatic effect that occurs when the average effective hydrostatic pressure gradient of the suspension is greater than that of the fluid alone, and consequently the effective buoyancy is greater;

(ii) The momentum transfer effect that occurs when the presence of other particles affects the mechanism between each particle and the fluid medium. This effect is related to the increase of the "apparent" bulk viscosity of the suspension; and

(iii) The "wall hindrance" effect that occurs when significant wall effects are detectable even when a single particle is settling in a vessel whose diameter is larger than the particle size by one or two orders of magnitude.

The most familiar expression for settling velocity is the Richardson and Zaki equation (Richardson and Zaki 1954). The equation relates the settling velocity of the interface(u_s) with the terminal (free settling) velocity of a single particle(u_t) given by Equation (2.7):

$$u_s = u_t(1-\phi_s)^n \quad . \tag{2.7}$$

where

u_s= mean settling rate of particles (particle-supernatant interface) in a container in the presence of many others,

u_t= terminal (free settling) velocity of a single representative particle, that is ,u_s, when ϕ_s=0,under otherwise similar conditions. (It is constant for a given solid-liquid system),

ϕ_s= volumetric concentration of particles. It is equal to C/ρ_s,

C= mass concentration of particles,

ρ_s = density of particles

n= a constant.

Richardson and Zaki (Richardson and Zaki 1954) gave the different expression to calculate "n" for different range of Reynolds number.

$n = 4.65 + 19.5\, d_p/D$ for $Re<2.0$

$= (4.35+17.5 d_p/D)Re^{-0.03}$ for $0.2<Re<1.0$

$= (4.45+18 d_p/D)Re^{-0.1}$ for $1<Re<200$

$= 4.45 Re^{-0.1}$ for $200<Re<500$

$= 2.39$ for $Re> 500$

The Richardson and Zaki equation is perhaps the most widely used form of equation, which predicts correct behaviours for most regions. The main drawback is that it over predicts values of u_s/u_t values and discontinuity of the u_s/u_t versus Re at Reynolds numbers of 0.2, 1.0 and 500 with further discontinuity at 200 if the wall effect is taken into account (Garside and Al-Dibouni 1977).

Barnea and Mizrahi (1973) presented a review of different preceding researches and provided the following expression for the creeping flow range, based on their analysis of data from various authors:

$$\frac{u_s}{u_t} = \frac{(1-\phi)^2}{\left(1+\phi^{1/3}\right)\exp\left\{5\phi/3(1-\phi)\right\}} \qquad . \tag{2.8}$$

Further attempts have been made by Garside and Al-Dibouni (1977) to achieve greater accuracy and convenient use. They presented two correlations: correlations based on logistic curve and a correlation of Richardson and Zaki type.

2.4.2 Bi-disperse suspension

In binary suspension, where two distinct particle sizes of equal density are involved, the sedimentation process will results in four zones, from top to bottom, clear liquid, suspension of smaller particle size only, suspension of both particle size (with concentration equal to initial concentration) and at the bottom, the sediment (Mirza and Richardson 1979). A typical sedimentation process for bi-disperse suspension is shown in Figure 2-3.

For such suspensions a number of experimental works and corresponding model works have been completed (Smith 1965; Smith 1966); (Mirza and Richardson 1979); (Selim et al. 1983); (Patwardhan and Tien 1985); (Davis and Gecol 1994). Cheung et al. (1996) provided different models which were used to predict experimental data. The models are limited to dilute concentration of suspension and subject to the experiment condition.

Since the above model developments consider mostly ideal conditions for which surface properties of fine particles are not present, their application need to be considered with care if particles of colloidal nature and large density are considered.

2.4.3 Poly-disperse suspension

Poly-disperse suspensions consist of more than two species of particle sizes. Though some extension of models from bi-disperse suspension were made, no experimental result exists to compare the prediction. Some of the literature providing models can be referred to (Smith 1967);(Masliyah 1979);(Mirza and Richardson 1979);(Selim et al. 1983). Typical schematic for ideal sedimentation of poly-disperse suspension is shown in Figure 2-4.

Other than the above phenomenological kinematic approach, model studies based on particulate multiphase fluid dynamics have been available in recent years. Some developments in this respect could be referenced to the works of Gidaspow (1994), Montante et al.(2001), Sha et al. (2001) and Loth et al.(2006) to mention only the few.

Moreover, discrete element models, another branch of study which deals with discontinuum approaches to modeling, are also available in the study of segregation. The main feature of DEM is that the collision and interaction of particles with their environment is captured by the contact force law of physics. The DEM has great application in study of mixing and segregation of granular particles. Also with the development of commercial software packages there is growing interest in this direction and their application tend to be promising. Recent developments in advanced computation make the study based on numerical modeling more promising. Particularly, the development of discrete element codes as applied to granular flow and segregation have received the latest attention.

Most of the DEM models ignore the hydrodynamic drag effect of the fluid, as they deal on either quasi-static situation or granular flow cases where the air drag effect is neglected. Lately, some models have attempted to account for hydrodynamic effects (Kenji Iimura et al. 1998), (Iimura et al. 1998), (Asmar et al. 2002). The work of Asmar et al. (2002) introduces an index, generalized mean mixing index, GMMI, to quantify the degree of segregation.

2.5 Granular Flow and Segregation

One of the challenges of geotechnical engineers is the problem associated with granular motion state, which are commonly categorized in three regimes: Static, slowly deforming, and rapidly flowing. Although much effort have been put into the research, determination of the regime of motion remains far from perfect mainly due to the complexity of the problem.

Granular materials have been the subject of much research since the time of Coulomb. Some success has been achieved in understanding the mechanisms involved in the physical interaction of the grains as a discrete solids. The theoretical and experimental works so far are subject to some limitation and their application is restrictive. The problem of granular materials when stripped down to fundamental physics, it is quite complex and novel (Edwards and Grinev 2003).

Granular materials behave as a solid and liquid. For example, when placed on inclined plane they resist flow and stay at equilibrium, whereas when the angle is greater than the angle of repose then, the grains start to flow. de Silva et al. (1999) explained the phenomena of segregation in granular flow under three major circumstances: segregation mechanism, a localized event which leads to the separation of components; a segregating process, a situation in which the mechanisms become active; and handling regimes as circumstances which promote or reduce the effect of various segregation mechanisms.

As different mechanisms of segregation, they listed rolling, sieving, push-away, angle of repose, percolation, displacement, trajectory, air current, fluidization, impact, concentration driven displacement, agglomeration and Embedding. Detailed explanation can be found in de Silva et al.(1999).

The mechanism of percolation may be activated by spontaneous conditions, vibrations or local shear (de Silva et al 1999), (Vallance and Savage, 1999). Spontaneous inter-particle percolation is defined as the drainage of small particles through a static arrangement of larger particles. In a gravitational field the fine-grained particles will move downward so that the upper portions of a material will have relatively small proportion of fine grained particles and lower part of the material will have large proportions. The fine-grained particle will be concentrated in the direction of flow, if a fluid is forced through the material. Consideration of triangular arrangement of three large particles having diameter, D, indicates the upper limit of 0.155D as the maximum size of the smaller percolating particles.

Referring to the dimensional analysis of Bridgwater and others, Vallance and Savage (1999) provided the following relationship for percolation velocity was developed:

$$\frac{c}{(2gD)^{\frac{1}{2}}} = fn\left(\frac{d}{D}, \varepsilon, \nu\right) \tag{2.9}$$

where

c is the percolation velocity of small particles;

g is the gravitational constant;

d/D is the diameter ratio of the percolating to static particles;

ε is the coefficient of restitution; and;

ν is the solid fraction.

Generally, c increases as d/D increases; c goes to zero as ε goes to 1 and c increases in the radial direction as ν increases.

They compared their experimental observation, that denser particle of the same size as the bed particles are migrating downward on shaking whereas Williams (1963) observed that large particles migrate to the top of the bed and increasing the density of the particles enhance the upward migration. But the observation by Rippie and others (Rippie et al. 1964) showed that threefold contrast in density resulted in no segregation. The contradictory observation maybe due to the experimental methods adopted.

Vallance and Savage (1999) stated that segregation patterns and velocity profiles observed in steady, uniform, chute-flow experiments, having solid fraction of 0.4 or greater, results from kinetic sieving, attenuated by diffusive mixing in high energy flow and by viscosity or buoyancy in flows with liquid present. Kinetic sieving comprises two processes: 1) flow subjected to shear strain, small and large particles percolate downwards into the void spaces that periodically open beneath them. The small particles percolate downward much more frequently than the large particles; and 2) the expulsion of individual particles out of their layers into adjacent layers owing to contact forces. Expulsion can occur in either direction and does not have to be size preferential.

The well known problem of size segregation is the so called "Brazil Nut effect" where large particles placed at the bottom of the vibrating bed tends to come to the top, while the smaller particles percolate to the bottom. Important discussion about the different hypotheses, for example, regarding the upward migration of larger particle in vibrational segregation have been presented by Vallance and Savage (1999).

The two hypothesis presented are of intuitive and probabilistic explanation. The first one, according to Williams, suggests that as the particles are jostled small voids periodically open beneath the large particle and small particles owing to interlocking and compression of them by the large particle, the small particles are unlikely to move once in place beneath the large particles.

The other hypothesis, purported by Rosato and coworkers Rosato et al. (1986), is that several small particles beneath the large particle would have to be moved simultaneously in order to create a void large enough for it to fit into, but only one particle would have to be moved to create such a void beneath a small particle. Therefore, the probability of opening a void space that is big enough for large particles to fall into is considerably less than that of opening a void big enough a small particle to fall into. Thus, the larger particles will tend to migrate upward because probability favours small particles filling voids beneath them.

2.6. Segregation Study in Tailing Management

Virtually no mineral is extracted as a final product. It needs to pass through many steps generally referred to as mineral processing. The methods of extraction involve mostly chemical and physical processes. The mining industry produces huge volume of mineral wastes. Thus waste disposal has become a major concern in the mineral processing. The cost involved in waste disposal has become a challenge, due to a growing environmental concern marked by stringent regulation.

The waste material from processing comes mostly in the form of slurry. Disposal of these materials requires construction of large impoundments. The disadvantage of such impoundment includes deeper burial of potential ore bodies, high cost of construction and maintenance of the dam, slow rate of consolidation of fines and lack of environmental appeal (Scott and Cymerman 1984).

The tailing stream is transported by pipeline to the tailings pond, where upon disposal the coarse particles settle close to the beach while the fine particles are carried to the centre of the pond. Such deposition results in segregation of the tailings materials. The deposition mechanism is shown in Figure 2-5.

Geotechnical engineers are involved in a variety of processes such as waste handling methods, waste embankment stability, pollution control and reclamation. The common deposition methods of hydraulic fill construction are (1) spigotting and single point discharge, (2) cycloning, and (3) cell construction. Details of the discussion can be found in (Morgenstern and Scott, 1999). Such deposition method mostly results in segregation of mixtures, which is not desirable in many aspects. The innovative method introduced by Robinsky (1999), referred to as thickened tailing disposal, is a major success in mitigating the problem of segregation. The solid content at which thickened tailings are discharged is about 65% solids.

Fine tailings are characterized by high moisture content and low permeability. Such fine tails may be resulting from segregation in a settling pond. And the geo-environmental problem associated with such wastes is of considerable challenge. Large volumes of fine tailings are produced when sedimentary deposits as oil sands, bauxite and phosphate ore are produced (Morgenstern and Scott 1999).

After bitumen extraction, the tailings are transported hydraulically as slurries of process water, bitumen-free solids, and unrecovered hydrocarbons to the tailing retention ponds. The coarser solids settle out relatively rapidly and are used for the construction of containment dykes and beaches for the resulting ponds. Water and fines, not retained with coarse solids, enter the pond where further settling and slow consolidation process continue, with the release water for recycle back to the plant (MacKinnon and Sethi 1999).

2.6.1 Oil Sand Fine Tails

Oil sands are unconsolidated sandstone deposits of a very heavy hydrocarbon: bitumen. Oil sands deposits are present in many locations around the world and appear to be similar in many respects, occurring along the rim of major sedimentary basins, mainly in either fluvial or deltaic environment containing sands of high porosity and permeability (Shaw et al. 1996).

The oil sands contribute about one-third of Canada's supply of oil (Fine Tailings Fundamentals Consortium, 1995). The province of Alberta has the largest reserve of oil sands with the major oil sand deposits in Athabasca, Cold Lake, Wabasca and Peace River areas. The deposits in Fort McMurray area occur at shallow depth which allowed surface mining for recovery. The Suncor mine, Syncrude mine and Albian Sands are the major operators in the area, with other operators to follow.

The oil sands consist of sands with silt, clay, water and bitumen filling the intergranular spaces. The silts and clay particles together with the barren interbeds become dispersed during the Clark Hot Water Extraction (CHWE) extraction process and largely accumulate as fine tails deposits in the tailings settling basins.

The Athabasca Oil Sand deposit is the largest of Alberta’s four oil sand deposits and it contains approximately one trillion barrels of bitumen in place. Based on surface mineable areas only, the recoverable reserves are estimated at 33 billion barrels of bitumen. Commercial surface mine operations in the Athabasca oil sands currently produce over 700 million barrels of Synthetic Crude Oil (SCO) per year. This is undertaken by: 1) Suncor Inc. which began operation in 1967 and currently produces about 5 million cubic meters of SCO per year and (2) Syncrude Canada Ltd. which began operation in 1978 and currently produces approximately 10 million cubic meters of SCO per year. Albian sand has started operation in 2002. Canadian Natural Resources Limited (CNRL) started production in 2009. Shell Jackpine mines and Imperial Oil, Kearl Oil Sands Project will join the operation in the coming years. All companies employ surface mining methods to obtain the oil sand ore and then transport it to a plant for extraction.

For oil sand extraction, about 80% of the ore is waste solid, the largest component of which is sand, with the remaining fines, composed of clays silts and very fine sands. From tailings management perspective, it is the fines fraction and its resulting stable suspension (fine tails) that are posing major concerns regarding oil sands development. The suspended solids fraction of the tailing ponds has been the focus of most of the long term environmental concern regarding tailings management (MacKinnon and Sethi 1999).

The major geo-environmental concern is the presence of complex mixture of carboxylic acids known as naphthenic acid and selenium. If the deposit is segregated and such toxic substances may percolate into the ground water system causing ground water pollution.
Continuing research that has been taking place at the University of Alberta, CANMET and Syncrude Canada Ltd. are the most significant one so far and considerable development has been attained.

Scott and Cymerman (1984) also introduced a schematic diagram called Sand-Fines-Water (SFW) diagram, in order to analyze and explain the different properties and behaviour of slurries composed of coarse and fine mineral matter. The diagram is used for predicting and planning disposal methods for most slurry. In the diagram, a specific amount of water (fluid), fine solids and coarse solid in slurry are plotted as a point.

2.6.2 Soil Structure-Behaviour Diagram

In soil engineering, the USCS classification use of ternary diagram is common. The ternary diagram is subdivided into different categories based on the composition of clay, silt and sand in a given soil sample. The application of ternary diagram for clay-water-sand slurry characterization appeared in the work of Charles and Charles (1971). The three axes plot the three compositions of the ingredients (clay, water, sand) of the slurry. Any point on the diagram sum up to give 100%.

The ternary diagram illustrated in Figure 2-6 appears to be first introduced at the University of Alberta by Scott and Chichak (1983) with a substitute of clay by fines. Scott and Cymerman (1984) initially named this schematic diagram as Sand-Fines-Water (SFW) diagram. Later the ternary diagram was modified to a tailing design diagram (TDD), with a replacement of water content axis by fines-water ratio. It is also named *Slurry Properties Diagram* by Morgenstern and Scott (1999). It is now know generally as the Soil Structure-Behavior Diagram (SSBD).

It has become a valuable tool to analyze and explain the different properties and behavior of slurry composed of coarse and fine mineral matter. The diagram is applied for predicting and planning disposal methods for most slurry. The single ternary diagram eliminates the need for multiple two dimensional plots (Morgenstern and Scott 1999); (Scott 1999). In the diagram, the solid content is plotted on the left side, the fine water ratio on the right side and fine content at the bottom axes, ranging from 0 to 100%.

There is no collective agreement on the dividing line between the fines and coarse particles. Some suggestion to mention are the #200 sieve (74 micron), #325 sieve (44 micron) and 22 micron at Syncrude. (Chalaturnyk and Scott 2001) and (Whipple 1997) put the silt and clay range of the grain size distribution curve as that of fines. It has been observed that the boundary definitions vary depending upon the operational significance.

With the help of the SSBD, it is possible to show the sedimentation/ consolidation boundary, segregating and non-segregating boundary, pumpable and nonpumpable boundaries, liquid-solid boundary, sand matrix-fine matrix boundaries, For example, the segregation limit is taken as the line above which coarse particles settle preferentially with respect to the clay particle.

Other properties which could be explained through the SSBD are sedimentation /consolidation, compressibility, permeability, and undrained shear strength. Its application is reported to have contributed to the understanding of different properties and enhanced some planning activities. It is reported that the use of such diagram by Syncrude has enhanced the research and testing of tailing materials and economic advantage thereof.

The tailing stream composition is falling in the segregating part of the SSBD. To mitigate segregation, methods needs to be adopted say by lowering of the water content of the tailing (thickening) or by increasing the fine contents, or doing both.

The amount of fine particles in tailing has been shown to be of important significance, such as permeability, liquefaction susceptibility and some chemical effect on flocculation, thixotropic and rheological properties of the slurries (Chalaturnyk and Scott 2001). They provided detailed explanation of TDD. They also indicated that determination of segregation boundary is an issue of fine capture.

In defining the segregation boundary, the definition of average solid content and segregation index has been introduced. The average solids contents with time is given by

$$S_{avg} = \frac{1}{\frac{1}{S_o} - \left(1 - \frac{H}{H_o}\right)\left(\frac{G_W}{G_S} + \frac{1}{S_o} - 1\right)} \quad (2.10)$$

where S_{avg} = average solids content at time t

S_o= initial solids content

H = height of slurry at time t

H_o = initial height of slurry

G_s and G_w are specific gravity of solids and water

And a quantitative index, called segregation index showing the distribution of solids within the sedimented material is given as follows,

$$I_S = \sum_{i=1}^{n} \left\{ \frac{1}{2} \left| \left[\frac{S_i}{S_{avg}} - 1 \right] \right| \left[\left(\frac{H}{H_f} \right)_{i+1} - \left(\frac{H}{H_f} \right)_{i-1} \right] \right\} * 100\% \quad (2.11)$$

And Fines Capture, FC, is expressed as 100% - I_S

2.6.3. Use of admixtures

The addition of flocculent (natural or polymer) is observed to shift the segregation boundary. The procedure for adding chemicals to the tailings to make them nonsegregating, deposit and consolidate under their own weight faster has been studied at the University of Alberta. For example, lime treated tailings showed to consolidate at a faster rate (Caughill et al, 1993). Flocculation or aggregation of tailing streams by use of chemical agent has been reported to have produced non-segregating deposit commonly called composite (consolidated) tailings.

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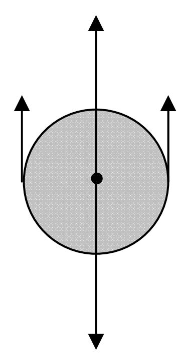

Figure 2-1 Forces acting on spherical particle in a liquid

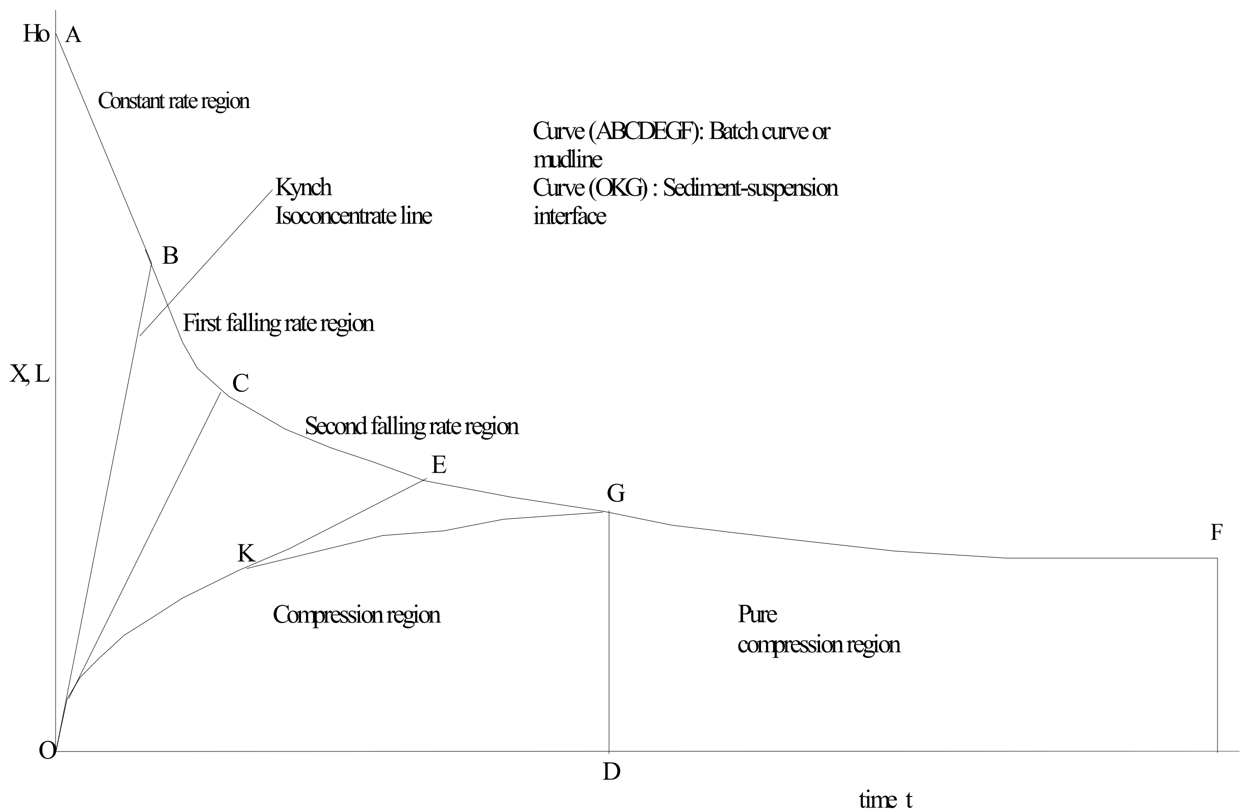

Figure 2-2 Batch settling curve (Diplas and Papanicolaou 1997)

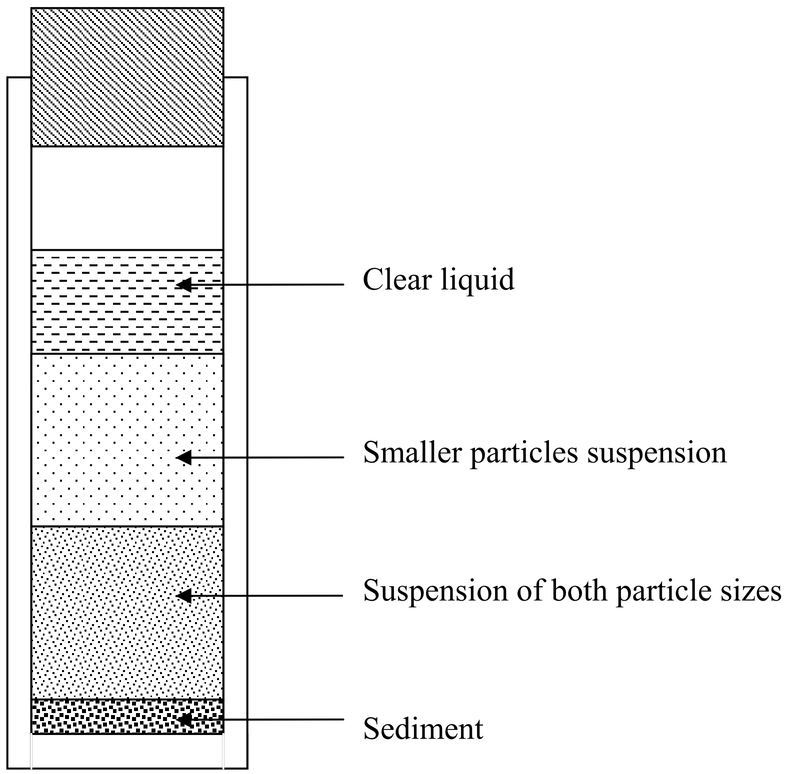

Figure 2-3 Segregation process during sedimentation of binary species suspension

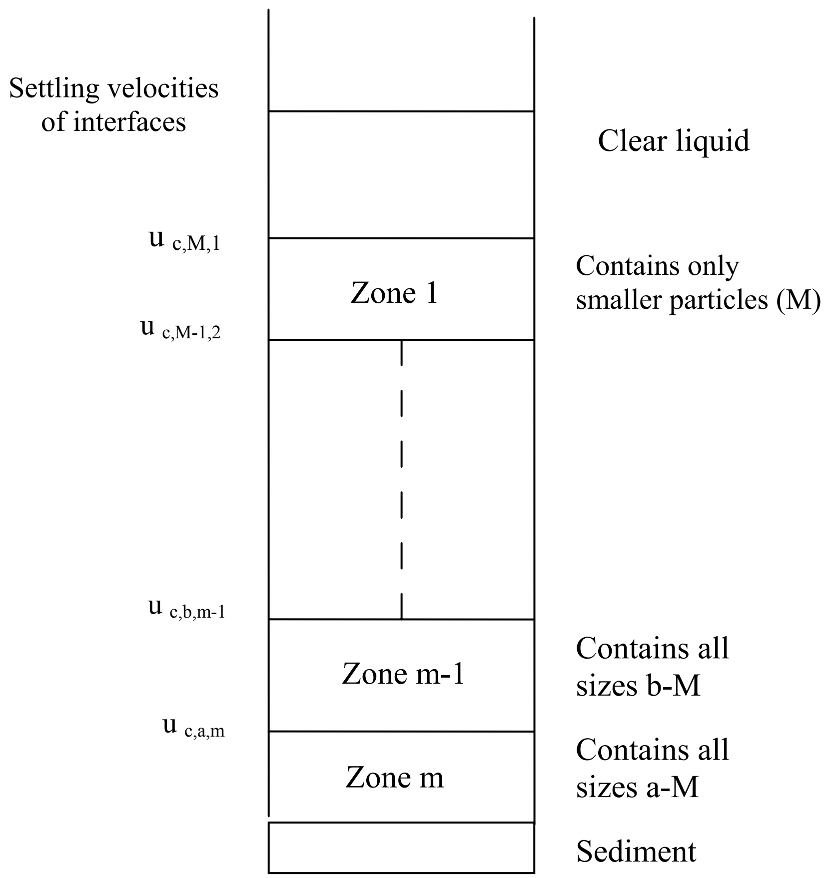

Figure 2-4 Formation of zones in sedimentation of poly-disperse suspension (after Mirza and Richardson 1973)

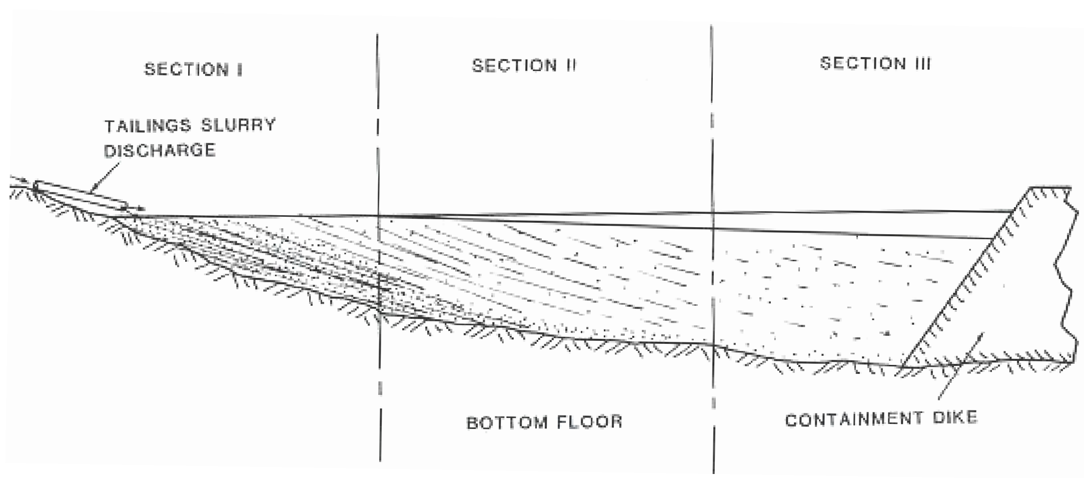

Figure 2-5 Deposition mechanism in tailings impoundment (after Yong 1984)

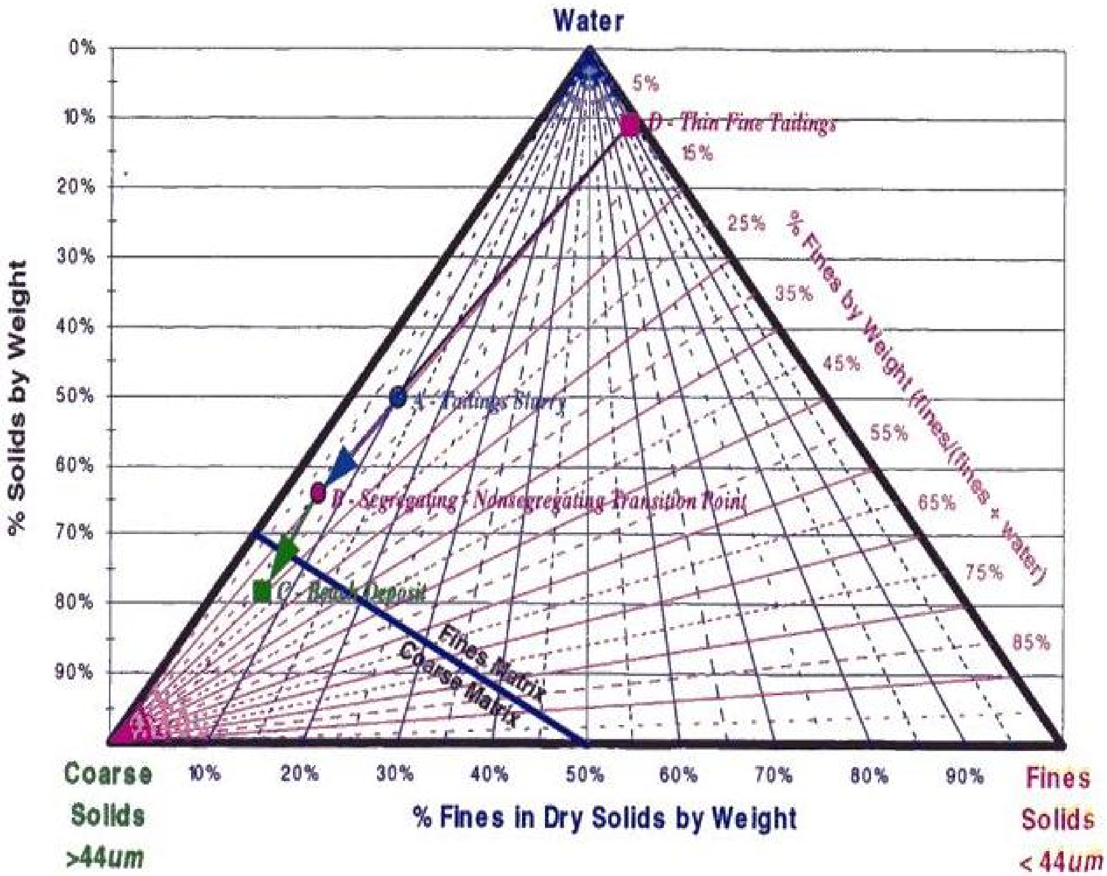

Figure 2-6 Tailing Design Diagram showing different Boundaries

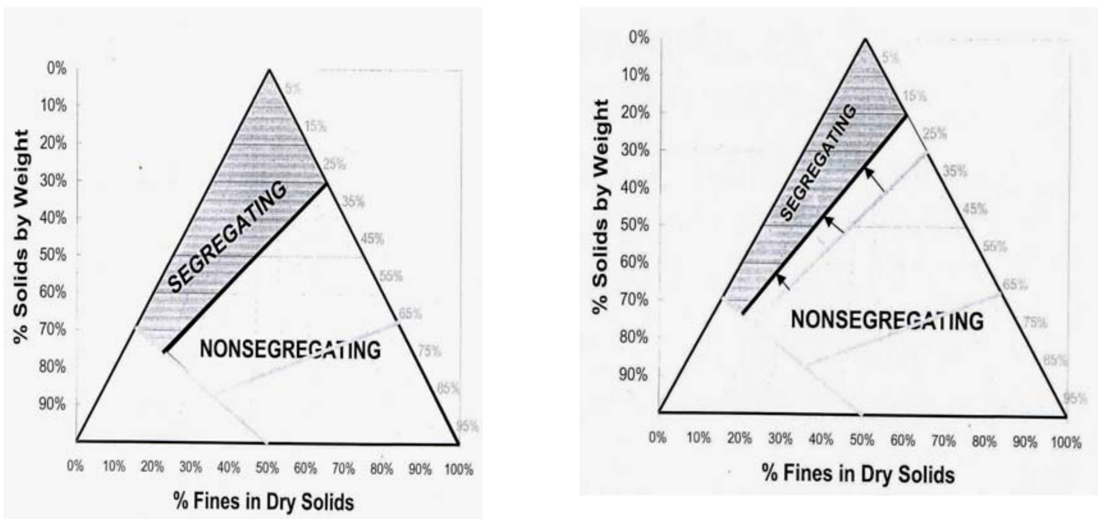

Figure 2-7 Shifting of Segregation Boundary due to addition of flocculent (after (Chalaturnyk and Scott 2001))

CHAPTER 3[1]

3. GEOTECHNICAL AND RHEOLOGICAL CHARACTERIZATION

3.1 Introduction

The constitutive behaviour of mixtures involving different phases is a subject of great complexity. The complex mixtures of solids and fluids exhibit behaviour different from Newtonian fluids. Fluids which deviate from Newtonian behaviour are referred to commonly as non-Newtonian fluids. These materials may involve two or three phases. The study of these mixed phases has shown significance in many fields: the economical and efficient transportation of solids in mining industry, hydraulic dredging, debris and mud flow researches, chemical and processing industries, biomedical and genetic engineering process (Charles and Charles (1971), Govier and Aziz (1972), Coussot (1997), Whipple (1997), Kessel and Fontijn (2000) and Chhabra (1993).

3.2 Previous works (Literature review and Background)

Characterization of the rheological behaviour of non-Newtonian fluids has become very important from an engineering perspective. Rheological characterization provides necessary tools for flow behaviour prediction, quality control and comparison. As Zhou et al. (2001) described, understanding the rheological behaviour of concentrated suspensions is of importance in the analysis and control of the properties of the suspensions so that they display well-defined and desired flow behaviour. One application is to establish criteria for controlling rheological properties and physical stability of a suspension so as to increase the maximum packing volume with desired

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fluidity, while maintaining sufficient stability against sedimentation, aggregation, and agglomeration of the particles. They also provided different aspects of application of rheological studies.

For such purpose, quite a number of models relating shear stress with shearing rate are available. The parameters that are usually required in the models are the apparent yield stress, plastic viscosity and flow indices. The required parameters are determined from laboratory or field experiments and curve fitting techniques. It appears that, compared to other parameters in the model, there is remarkable interest in measuring yield stress.

Houwink (1937) reported that even in crystalline bodies, migration of atoms takes place without application of any tension (stress). Such migration results in plastic deformation. And he concludes that no yield stress exists. Vocadlo and Charles (1971) discussed the difficulties involved in defining yield stress from a "*strength of materials*" perspective and signified the distinction between solids and fluids. Chhabra (1993) concluded that viscoplastic fluids, which are characterized by the existence of yield stress, are not really a fluid according to strict physical definition. Barnes and Walter (1985) argued that the yield stress is a "myth" which has provided a point of controversy among researchers.

Hartnett and Hu (1989) conducted a falling ball experiment in an aqueous 2500wppm Carbopol 960 solution. They tracked the movement of the nylon ball for several months and observed that the ball is essentially at rest from an engineering perspective. Thus they concluded that yield stress is an engineering reality. Their conclusion contradicted the hypothesis of Barnes and Walter (1985) and leads to further debate on the subject in the literature (Astarita (1990), Schurz (1990), Evans (1992), Schurz (1992).

Later, Barnes (1999) provided a comprehensive review of the yield stress where he emphasized that there is no true yield stress and flow exists beyond the yield stress. Reference was even made to Reiner (1969) :" *In reality there is no rigid boundary between solids and liquids. Παντα ρει (Panta rhei), everything flows, as Heraclitus (495B.C.) said*." in reinforcing his argument. According to Barnes (1999), the definition of (apparent) yield stress is a mathematical curve-fitting constant, used along with other parameters to produce an equation to describe the flow curve of a material over a limited range of shear rates. While Vocadlo and Charles (1971) have marked the value of distinction between “fluid” and “solid” “if it enable us to place common materials unambiguously in one category or the other on the basis of experimental tests. However, this is only possible interpretation through personal judgment based on time of observation, time of service, predicted permanent deformation, etc.”

Despite the controversy resident in the subject of yield stress, it can be seen that there is an apparent agreement with regard to the time frame to which test conditions are predominantly considered. For example, Houwink (1937) highlighted that in a practical experiment carried out rapidly, a yield value can be observed. He further stressed the significance of the inclusion of the rate of testing in the reporting of the experimental results. Barnes (1999) reported his consent with the argument of Harnett and Hu (1989) that liquid could be assumed to have yield stress when it comes to a reasonable length and time scale. Astarita (1990) provided a review of the arguments of Barnes and Walter (1985) and Harnett and Hu (1989) and provided a discussion on the relative nature of time and length scales.

As Evans (1992) concluded, the dispute is more of accepting one’s definition or not. Thus in common engineering practice, the yield stress is pragmatically significant and the application is investigated with the intent of characterizing the flow behaviour of suspension in short test duration. The concept of creep appears to be of less significance at least in the context of studies which involve short duration of experimentation. The number and variety of attempts to measure yield stress signifies how the pragmatic approach is prevalent in these types of experiments.

The most common method for computing the yield stress is back extrapolation of the flow curve to zero shear rate from rheometer data. As such, extrapolated results have been used to check the yield stress measurement made by other apparatus. Because this method is a curve fitting procedure, direct experimental measurement of yield stress has received attention amongst researchers.

Charm and Kurland (1967) have devised a glass capillary tube to measure the yield stress of blood. They found good agreement with the back extrapolated yield value of rheometer tests.

Scott et al. (1985) conducted rheological tests on sludge from oil sands. They used a conventional rheometer and back extrapolated the flow curve to zero shear rate to determine yield stress (or gel strength, as they called it). They observed over the range of their study that the addition of sand effects in reducing the apparent viscosity and yield stress. They remarked that the development of gel strength can be beneficial if it is desirable to suspend a granular phase such as sand in the sludge by mixing.

Locat and Demers (1988) studied the rheological properties of sensitive clays. They were able to establish a relationship: plastic viscosity vs. liquidity index and remoulded shear strength (yield stress) vs. liquidity index.

Wuensch (1990) used a falling ball viscometer to find the two parameters of a Bingham fluid: yield stress and plastic viscosity simultaneously. While the results compare well with the working theory, it is unclear why comparison was not made with those of other conventional rheological experiments. Schurz (1990) introduced a rolling ball method of yield stress measurement. He showed that when a ball rolls down a curved groove, the acting shear stress is proportional to the position at which the ball comes to standstill.

Rajani and Morgenstern (1991) conducted slump test to measure the yield stress of mortar and Devon silt. They compared their result with the falling cone tests. They concluded that slump test is a viable method for determining the yield stress of geotechnical material of Bingham fluid type. Zreik et al. (1995) devised a falling cone test to measure undrained shear strength (yield stress) of remolded marine clay. They found good match between the results from their experiment and theoretical prediction.

Coussot and Boyer (1995) conducted inclined plane tests to determine the yield stress of mud mixtures at different solid concentrations. They found good agreement between inclined plane test and rheometrical test data for Herschel-Bulkley model fitting fluids.

Kessel and Fontijn (2000) devised a miniature sounding probe to measure the yield stress of freshly deposited sediments. They measured as small a yield stress value as 1 Pa. Their rationale not to use conventional rheometer techniques were that: 1) the increase in effective stress with depth cannot be accounted for; and 2) the sample is reconstituted while injecting into the rheometer.

The vane shear method, which was initially devised to measure the undrained shear strength of cohesive soils in geotechnical engineering (Osterberg 1956), has been adopted to the yield stress measurement of soft solids and slurry (Keentok, 1982); (Nguyen and Boger, 1985); (Turian et al. 1993). The method has demonstrated itself to be a reliable technique for the measurement of yield stress.

By use of similar device, Turian and coworkers, Turian et al.,(1992, 1993, 2002), were able to characterize and control the properties of coal-water fuel mixtures. They indicated that yield stress is an effective property which indicates the existence of structure within the composite material and, therefore, its value provides a measure of relative strength of the two phase structure. They added further that, in general, effective properties of composite materials depend upon pertinent properties of the continuous and the dispersed phases, their concentrations and the geometrical arrangement of the dispersed phase within the mixture.

Many slurry flows in nature (e.g. mudflow, debris flow, erosion), in industrial processing (mixing handling and processing) and in industrial waste management (tailing transport and disposal) involve different grain size composition and water chemistry. In the literature, however, little emphasis has been given to the study of rheological properties of such material. This may be due to the fact that measurement difficulties are predominantly hindering. Kuepper (1991) has indicated that non-segregating slurries are characterized by an increased viscosity and display a non-Newtonian rheology. Mining wastes (tailings) need to be handled and transported before they are deposited into a pond. It is desirable for the tailings disposal scheme to have rheological properties which are stable, easily transportable and yielding low volume at storage. The measurement of yield stress has become very significant parameter in evaluating the geotechnical suitability of deposits. Thus proper characterization of rheological behaviour is valuable tool in all processes involving their management.

The purpose in this chapter is to investigate the significance of yield stress, as rheological parameter, in characterizing the properties of different mixes of grains sizes (clay, silt and sand) when they form a non-segregating (stable) slurry. Thus the composition, concentration and porewater chemistry effects are examined.

3.3 Theoretical Considerations

3.3.1 Rheological Characteristics

Fluids of any type may exhibit one of the following flow behaviour types: Newtonian, non-Newtonian or Viscoelastic. Detailed discussions about these materials can be found in Wilkinson (1960), Shamlou (1988) or Chhabra (1993). A brief explanation of flow behaviour is presented below.

Newtonian fluids are fluids in which the viscosity is constant and satisfy the complete Navier-Stokes equations (i.e. in simple shear, deviatoric normal stresses are identically zero). Non-Newtonian fluids are those whose flow curve exhibit non-linear relationship. Linear flow curve that does not pass through the origin are also aligned in the non-Newtonian category.

The most common non-Newtonian flow behaviour can be put in three categories

1. Time independent (viscous) fluids:

Fluid of this type can be described by flow properties independent of time. The constitutive rheological equation can be written as

$$\tau = f(\dot{\gamma}) \tag{3.1}$$

These fluids maybe further subdivided into

(a) Shear thinning or pseudoplastic

For this fluids, the ratio of instantaneous shear stress to shear rate, called 'apparent viscosity' drops as the shear rate increases. Most commonly encountered time independent fluids fall in this category. Some examples are inks, paints, clay suspensions, paper pulp, emulsions, oils and greases.

(b) Shear thickening (dilatant)

The 'apparent viscosity' increases as shear rate increases and there is no yield stress. Some paints, concentrated suspension of solids, plastisols and quicksands are of the dilatant type.

(c) Viscoplastic

These fluids are characterized by yield stress which must be exceeded before flow commences. The mathematical models to characterize such flow behaviour are mainly the Bingham, Hershel-Bulkley, or Casson models.

Figure 3-1 shows the rheograms for these different fluid types.

2. Time dependent fluids:

For these types of materials, the shear flow properties depend on both the shear rate and the time of shearing. They are mainly subdivided into:

(a) Thixotropic (breakdown of structures by shear)

If the apparent viscosity decreases with time for a given shear rate, then the fluid is said to exhibit thixotropy.

(b) Rheopectic (antithixotropic) (formation of structure by shear)

If the apparent viscosity increases with time for a given shear rate, then the fluid is said to exhibit (display) rheopexy.

3. Viscoelastic fluids:

These fluids show some properties similar to those of solids. Many such fluids show both elastic and viscous properties.

3.3.2 Empirical models for flow curve

There are many equations to which rheological data can be fitted. Some common equations are provided in Table 3-1, where τ is the shear stress corresponding to a shear rate of $\dot{\gamma}$, τ_y, τ_c are the yield stress corresponding to zero shear rate, and K and n are curve fitting parameters.

Table 3-1 Models of fluid characterization

Model's name	**Constitutive Equation**
Newtonian	$\tau = K\dot{\gamma}$
Bingham Plastic	$\tau = \tau_y + K_b\dot{\gamma}$
Power-law model	$\tau = K\dot{\gamma}^n$
Herschel-Bulkley (HB) model	$\tau = \tau_y + K_{HB}\dot{\gamma}^n$
Casson model	$\tau^{1/2} = \tau_c^{1/2} + (K_c\dot{\gamma})^{1/2}$
Sisko model	$\tau = K_{s1}.\dot{\gamma} + K_{s2}\dot{\gamma}^n$

Hybrid model

Chilton and Stainsby (1999) developed a hybrid model which is partly a HB model and partly a Bingham model:

$$\tau = \tau_y + K_{HB}\dot{\gamma}^n \qquad \text{for } \dot{\gamma} < \dot{\gamma}_T \quad ; \qquad (3.2.a)$$

$$\tau = \tau_y + K_b\dot{\gamma} \qquad \text{for } \dot{\gamma} > \dot{\gamma}_T \quad ; \text{ and} \qquad (3.2.b)$$

$$\tau_T = \tau_y + K_{HB}\dot{\gamma}_T^{\ n} = \tau_y + K_b\dot{\gamma}_T \quad ; \qquad (3.2.c)$$

where the transitional shear strain rate ($\dot{\gamma}_T$) is derived from the rheometer data.

The first three parameters τ_y, K_{HB} and n are obtained from the best fit of HB model to all of the data and using the standard method. The two additional parameters, τ_y and K_b are obtained by fitting a straight line to the high-strain tail of the viscometer data (Chilton and Stainsby 1999).

3.3.3. The principle of yield stress calculation based on vane method

The vane shear apparatus is made of four thin blades welded to a cylindrical shaft at right angle to each other as shown in Figure 3-2. The shaft is coupled to a motor at the top which drives the system. The yield value is the shear stress corresponding to the observed maximum torque. The yield stress is calculated by assuming that the sample yields along a cylindrical surface (and two circular ends) circumscribed by the length and diameter of the vane.

In performing the test, the vane is immersed into the sample slowly and a torque is applied to rotate the vane. Measurement of torque as a function of time is made. If the sample has yield stress, a maximum torque is observed after some time has elapsed. The maximum torque corresponds to the critical force required to overcome the strength of the sample. The maximum developed reaction (force) per unit area along the shear surface is thus the rheological yield stress.

The total torque (T) experienced by the shaft, when the vane is set in motion at a constant speed, can be given by the sum of the reactions at cylindrical surface, top and bottom face of the cylinder as follows:

$$T = \int_0^H dT_s + \int_0^{D/2} dT_b + \int_0^{D/2} dT_t \qquad (3.3.a)$$

$$= \int_0^H \frac{D}{2}(\tau_s \pi D)dH + \int_0^{D/2} r(\tau_b 2\pi r dr) + \int_0^{D/2} r(\tau_t 2\pi r dr) \quad ; \qquad (3.3.b)$$

where T_s is due to shearing on the cylindrical area formed by the shear surface and T_b and T_t are due to shearing on the bottom and top ends of the cylindrical shear surface respectively. And τ_s, τ_b and τ_t are the corresponding shear stress on the respective surfaces.

It is assumed that the shear stress on the cylindrical surface be constant. Also the shear stress at the top and bottom surfaces are taken to be the same, i.e., $\tau_b = \tau_t = \tau_e$ thus Equation (3.3 b) results in:

$$T = \frac{\pi}{2} D^2 H \tau_s + 2 * (2\pi \int_0^{D/2} \tau_e r^2 dr) \qquad (3.4)$$

The exact variation of the shear stress at top and bottom are not exactly known. If an arbitrary power relationship is assumed (Nguyen and Boger 1985), τ_e can be written as

$$\tau_e(r) = \left(\frac{2r}{D}\right)^m \tau_s \quad , \quad \text{for } 0 \le r \le \frac{D}{2} \qquad (3.5)$$

Substituting Equation (3.5) in (3.4) and integrating one gets

$$T = \frac{\pi}{2} D^2 H \tau_s + \frac{\pi D^3}{2(m+3)} \tau_s \qquad (3.6)$$

or

$$T = \frac{\pi}{2} D^3 \left[\frac{H}{D} + \frac{1}{m+3}\right] \tau_s \qquad (3.7)$$

If the shear stress on the ends is assumed to be constant and the same as the cylindrical surface shear, then m=0. This has been proven experimentally by Nguyen and Boger (1985) for flocculated bauxite residue slurries.

Thus, equation (3.7) reduces to

$$T = \frac{\pi}{2} D^3 \left[\frac{H}{D} + \frac{1}{3} \right] \tau_s \qquad (3.8)$$

Equation (3.8) is the widely used equation in the calculation of yield stress, $\tau_y = \tau_s$. If H/D is set to 2:1, the yield stress in terms of maximum torque and D is given as:

$$\tau_y = \frac{6}{7} \frac{T_{max}}{\pi D^3} \qquad (3.9)$$

where τ_y is the yield stress, T_{max} is the maximum torque, and D is the vane diameter.

3.4 Materials and Experimental Methods

The choice of material for this experimentation is based on the following criteria: availability, quality and repeatability of the working sample, and proximity to the real material, in particular tailings from the oil sands industry. Consequently, the materials chosen for geotechnical/rheological experiments are kaolinite, sil-sand rock flour, and sand. The kaolinite material and the sil-sand rock flour are obtained from Sil-Industrial Minerals Inc. The material properties for the studied materials are summarized in Table 3-2 below:

Table 3-2 Material Properties

Material	**Specific Gravity**	**D_{50}(μm)**
Kaolinite	2.70	~<1
Sil flour	2.67	~16
Sand	2.66	~200

Moreover, the liquid limit and the plastic limit of kaolinite are (55.2%) and (29.5%) respectively. The silt size sil-sand rock flour (Sil flour) exhibits no plasticity. The ambient temperature during the test is 20$\pm$1^{o}C. The grain size distributions of the materials used in the experiment are shown in Figure 3-3.

Initially a sample from each material is taken to check for moisture content. The initial moisture content of the kaolinite is about 1% while that of the silt size material and sand is negligible. The material is weighed on a scale (0.01g accuracy) in 5-litre plastic bucket. Water is added to achieve desired solid content. The water used throughout the test is the City of Edmonton tap water, the pH of which is 7.7$\pm$ 0.1. The mixture is then thoroughly mixed manually using a spatula for about 30 minutes. Then the thoroughly mixed slurry is transferred to a 600ml beaker. The beaker with slurry inside is immediately placed on a base of RS rheometer and fixed in a position. The shear apparatus is connected to a computer via R232 ports. All communications are made on computer by use of supplier software-Rheo2000. After adjusting the vane speed by feeding instructions through the incorporated Rheo2000 software, the vane is slowly lowered into the slurry up to the groove mark on the shaft. While the shearing test proceeds, the software allows the data to be retrieved and to simultaneously plot the variation of measured results as a function of time (displacement).

The vane apparatus used in this test is R/S SST2000 Soft Solid Tester manufactured by Brookfield. The different vane geometries used in the test are given in Table 3.3.

Table 3-3 Dimensions of the vanes used in yield stress measurement

Vane Designation	D, Diameter(m)	H, Height(m)
V30-15	0.015	0.030
V40-20	0.020	0.040
V80-40	0.040	0.080

The choice of the vane geometry is conveniently made by using large sizes for low solid content slurry; the medium size for intermediate solid content and smaller size for high solid content. By doing so, the recommended minimum torque (10% of the maximum torque capacity) as suggested by the supplier is maintained. The supplier also suggests that the error may be large for measurement taken below the minimum torque. The dimension of the vane that the height is two times the Diameter of the vane was adopted from the original vanes used by Cadling and Odenstad (1950) referred in Flaate(1966). It is for the purpose of having as small area-ratio as possible. The area ratio is defined as the cross-sectional area of the vane cross-section and stem in percentage of the cross-sectional area of circumscribed cylinder. It is suggested that the area ratio should not be higher than about 15 percent (Flaate 1966).

In some cases also viscosity measurements are taken to plot flow curve from which the yield stress is approximated by extrapolating the flow curve to zero shear rates. The viscometer used for this test is the Brookfield DV-II Viscometer. Unlike the vane shear apparatus, the viscometer is not supplied with software. However, quality data acquisition is achieved through Microsoft windows Hyper Terminal Communication Program. Further discussion about viscosity calibration and measurements is provided in Appendix A.

The slurry composition was prepared in three ingredient categories (i) kaolinite only, (ii) kaolinite and Sil flour mix and (iii) kaolinite and sand mixes. The solid content and proportions are also varied.

The shear rate is set to 6 speeds, i.e., 0.01, 0.02, 0.03, 0.1, 0.2, 0.3 /s. After each test is completed, samples are taken to check the design solid content. The comparison is carried out based on designed solid content and achieved solid content. Figure 3-4 provides a representative plot of these comparisons and illustrates that excellent control on sample solid contents can be achieved during the experimental program.

Table 3-4 Test schedule for vane shear experiment

Materials	Composition	Test solid contents (%)
Kaolinite	100%	30,40,50,55,60,65,70
Kaolinite : Sil flour	80:20	30,40,50,55,60,70
	65:35	25,30,40,50,55,60,65,70,75
	50:50	25,30,40,50,55,60,65,70,75
Kaolinite : Sand	80:20	30,40,50,55,60,70
	65:35	40,50,60,70
	50:50	35,40,50,55,60,65,70

3.5 Results and discussion Section

3.5.1 Test of the working principle

The calculation of yield stress as generally formulated in Equation (3.4) may be analyzed to see the effect of geometry. Equation (3.4) can be written in the following form:

$$T = \left(\frac{\pi}{2} D^2 H\right)\tau_y + 4\pi \int_0^{D/2} \tau_e r^2 dr \quad . \tag{3.10}$$

If it is assumed that the effect of changing the vane geometry does not bring significant change in the end shear stress contribution (the second term on the right side of Equation (3.4)), the slope of the plot of maximum torque versus ($\pi D^2 H/2$) would approximate the yield stress. For the three vane geometries (V30-15, V40-20 and V80-40) vane shear tests were carried out on kaolinite slurry at 50%s(w/w). The tested shear rates are 0.01/s and 0.02/s. The results are depicted in Figures 3-5(a) and (b), respectively.

The measured yield stresses are summarized in Table 3-5. Comparison of the slopes in Figure 3-5 and the average yield stress in Table 3-5 shows that the slope over-predicts the yield stress by about 10%. The linear fit along the three points suggests that the end contribution is constant instead of being a functional relationship with yield value. Alternatively, Bowles (1977) has suggested that the diameter of the shear surface associated with the vane may be larger than the vane diameter by about 5%. If this suggestion is examined with the geometry of the shear surface and the corresponding graph similar to Figure 3-5 is shown in Figure 3-6.

Table 3-5 Summary of yield stress based on measured maximum torque

	Yield Stress (Pa)	
Vane	**Shear Rate at 0.01/s**	**Shear Rate at 0.02/s**
V30-15	81.36	83.07
V40-20	78.27	78.83
V80-40	74.09	74.85
Average	77.91	78.92

Comparison of average measured value (Table 3-5) and the slopes in Figure 3-6 show a very good agreement. While the foregoing analysis indicates the top and bottom ends contribute about 10% of the total resistance; this effect however can be ignored if the diameter of shear surface is at 5% more diameter than the vane diameter as illustrated in Figure 3-6.

(1) Kaolinite Slurry

Since yield-stress is defined as the shear stress that must be exceed prior to flow initiation, attempts to measure such quantity would ideally be made at static condition (zero shear rate). However, it is technically impossible to undertake static test. The convenient and indirect way in any rheometrical experimentation is to back extrapolate the shear stress-shear rate plot to zero shear rate to determine the yield stress. The closer the shear rates are towards zero the better is the yield stress approximation. Similarly, direct yield stress measurements using the vane method need to be conducted at lower speed. Tests at lower speed minimizes the influence of viscous resistance and instrument inertia on the measured maximum torque (Dzuy and Boger 1985). They suggested that the rotational speed be less that 10rpm and showed experimental results at 0.1rpm. Common vane test in geotechnical engineering practice use rotating speed of 0.167 rpm or 1% /minute. Osterberg (1956) and Turian et al. (1993) employed a rotational speed of 0.3rpm in their studies. Coussot (1997) suggested a speed, in terms of shear rate, of 0.01/s. In this research, tests were conducted at six speeds between 0.01/s and 0.3/s.

For kaolinite, Figure 3-7 shows the variation of yield stress with solids content over the six rotational speeds chosen for this research. At 70.4% solid content it was not possible to measure yield stress at shear rate higher than 0.02/s, since the maximum torque capacity of the machine was exceeded. Except the readings at 66.3% solid content, the yield stress shows an increasing trend as the shear rate increases.

Additional insight into the behavior of these slurries can be found by plotting the variation of torque with time for specific solid contents. Figures 3-8 and 3-9 show the variation of the torque and shear stress of 30%s-kaolinite slurry at different shear rates. In Figure 3-8, the torque is expressed as the percentage of the reference torque, which is 5mNm in all cases. The readings are more reliable when the torque is greater than 10% of the reference, i.e., 0.5mNm (according to manufacturer recommendation). These figures show that with minor exceptions, there is an increasing trend in the torque and shear stress with the shearing rate. A similar trend has also been reported by Biscontin and Pestana (2001).

A closer examination of Figures 3-8 and 3-9 shows that the pattern of variation of shear stress is similar to that of the torque. The shear stress is automatically calculated according to the principle discussed in Section 3.3.3 and illustrated in Equation (3.8) or (3.9). The torque is plotted together to check the compliance with minimum torque requirement. Otherwise the shear stress plot is sufficient to determine the yield stress. Thus the yield stress is approximated as the maximum shear stress from the shear stress vs. time plot. A typical plot for 50%s kaolinite slurry is shown in Figure 3-10.

Figures 3-11 to Figures 3-22 show the plots of shear stress and moment at different shear rates and different solid content. What is remarkably observable in these figures is that the time to yield at lower shear rates is longer than those of the faster rates. The longer time is obviously due to slow movement of the vane to bring the material to shear. Moreover, as the shear rate increases and the solid content increases the stress vs. time variation tends to be nearly constant, indicative of stable plastic behaviour. The increase of yield stress with increasing shear rate shows that yield stress is not unique with respect to shear rate. However, the yield stress corresponding to lower shear rates are nearly the same indicating that the measurements should be conducted at lower shear rates as far as possible.

Based on these experimental results, a relationship between the yield stress and solid content can be established. Figure 3-23 shows the measured yield stress for different solid content of kaolinite slurry at a shear rate of 0.01/s. The smallest solid content at which the yield measurement starts is 30%. At this solid content, it is possible to keep the suspension stable with insignificant settlement during the shearing experiment. As the solid content increase, the yield strength shows a faster increasing trend.

There are different ways in which yield stress is correlated to the solid concentration of the slurry. Kao et al. (1975) provided a theoretical equation for Bingham type fluids that dimethyldichlorosilicate glass beads in glycerol show dependence of yield stress on the square of solid concentration. Smith and Bruce (1979) provided an empirical equation which showed the following a cubic relationship:

$$\tau_o = k' (\phi - \phi_o)^3 \quad , \qquad (3.11)$$

where k' and ϕ_o are empirical constants. Other types of empirical equations have been discussed in Turian et al. (1992) and Turian et al (1993).

The influence of fractal structure of suspended flocs on the yield stress has been presented by Kranenburg (1994). He provided the following relationship of yield stress with excess density and fractal dimension D.

$$\tau_y \sim \Delta\rho_a^{\frac{2}{3-D}} \tag{3.12}$$

For this experimental work, in which the yield stress is conveniently plotted as a function of the solid content (s) of the slurry, the above relation can be rewritten in terms of solid content and specific gravity (Gs) as follows:

$$\tau_y = A\left[\frac{S(G_s - 1)}{S + G_s(1-S)}\right]^{\frac{2}{3-D}} \quad , \tag{3.13}$$

where A is fitting parameter.

The above equation is fitted into the yield stress solid content relationship as shown in Figure 3-23 with A= 13620 and a fractal dimension (D=2.67). Excellent agreement is observed between the experimental results and the theoretical fit. Kessel and Fontijn (2000) have estimated the fractal dimension of china clay as D=2.68 which corresponds very closely to the fractal dimension obtained for kaolinite (D=2.67).

(2) Kaolinite and Sil flour

This component of the research was completed to examine the effect of silt size particles on the rheological strength of a kaolinite-sil flour slurry mix. Commonly, clay and silt are categorized as fine particles. Different categories of fine classifications are available in different references. Some of the classifications of fines by grain size are shown in Table 3-6 below. The particles in a suspension may have different grain size composition. There is a common trend to consider sand sizes as coarse and silt and clay size as fines.

Table 3-6 Range of fine sizes according to different references

Size (micron) of fines	Reference
< 22	Syncrude Canada Ltd.
< 30	(Stein 1986) ; (Shamlou 1988)
< 40	(Coussot 1997)
< 44	U of A, Geotechnical Centre
<75	Unified Soil Classification System

Figures 3-24 to 3-36 depict the shear stress and % torque of kaolinite-sil flour (1:1) mix at different solid content. It is observed that at 30% and 40% solid content, the shear stress measurement show very scattered readings (Figures 3-24 and 3-25). The scatter in measurement is attributed to the unstable (segregating) nature of the slurry. Also the measured % torque was less than 10% for solid contents below 50% (inclusive) which contrasts the kaolinite only tests where reliable torque measurements were obtained at the same solid contents. While the yield stress is generally less than that of kaolinite-only slurry, it is possible to achieve, however, workable solid content up to 75% solids. In case of kaolinite only slurry, the 70% solids is very stiff and measurements were not possible.

The dependence of yield stress on the shear rate is also clearly visible as the solid content increases. Here also there is a trend of an increase in yield stress as shear rate increases. Except at 75% solids, the higher shear rates result in the gentle slope, in some of the data nearly horizontal, are visible.

Also the proportion of kaolinite and sil-flour has been varied to a proportion 65:35 and 80:20 (kaolinite: sil-flour). A comparative plot is summarized in Figure 3-37 for tests conducted at a shear rate 0.02/s. In general, similar trends are observed at other shear rates. The addition of more silt size particles show a decreasing effect on the yield stress at the same time the maximum solid content at which yield stress reach 'infinite' value is shifted toward high solid content. This is consistent with the observation made by Kleinecke (1988) who has shown a tendency toward more shear thinning with increased filler content. He has also observed a decrease of elastic properties with increasing filler content.

(3) Kaolinite and Sand

Since the objective of this research is studying the fundamentals of segregation, it is relevant to study the rheological behavior of typical segregating mixtures. Consequently, kaolinite mixed with sand at mixture ratios of 50:50, 65:35 and 80:20 (kaolinite:sand) have been tested. The solid content at which segregation began to cease was observed to be 35%. Figure 3-38 shows the shear stress variation with time for this solid content. While scatter in the data begins to stabilize (due to the minimized segregation), Figure 3-38 clearly shows the very small shear stress magnitudes that were measured and the cyclic behaviour in the shear stress measurements. A modest increase in solids content, from 35% to 40%, produces more stable shear stress readings, as illustrated in Figure 3-39. For both these solids contents, it is important to note that the magnitude of the measured shear stresses are very small and are less than the suggested minimum of 10% of the maximum torque.

Increasingly stable (from a segregation perspective) slurry is attainable at 50% solid content. Figure 3-40 shows the shear stress variation at this solid content. Above 50% s, all the samples tested, i.e., 55% 60%, 65% and 70% show very stable shear stress reading, a representative plot is shown at 60%s in Figure 3-41.

The variation of yield stress as a function of solid content is shown in Figure 3-42. The solid composition is varied at different kaolinite and sand proportion. The results illustrated in Figure 3-42 show that as the proportion of sand in the kaolinite-sand mix increases the yield stress of the mixture decreases.

Comparing Figure 3-29 and Figure 3-42, it is apparent that the yield stress is more dependent on the finest size of the grains present in the slurry indicating that the clay sizes have a predominant effect on the yield stress characteristics of a slurry. Zhou et al.

(2001) have indicated that the yield stress is inversely proportional to the square of particle size. The current research results are in support of their statement. The coarse grains are in essence “inactive” relative to their contribution to the yield stress of the slurry. Suspension yield stress depends on solid concentration, particle size, shape and size distribution and in general, suspension chemistry (Nguyen and Boger, 1985).

Also to observe is the magnitude of variation with time at nearly similar solid concentration, a plot is shown in Figure 3-43. A comparison is made at 60%s (w/w) for which the addition of sil flour and sand substitute 50% of the kaolinite material. The silt size addition reduces the yield stress by nearly half.

Similarly, the addition of the sand reduces the yield stress by nearly a factor of 20. This result is consistent with the work of Scott et al. (1985) who indicated that the yield stress and apparent viscosity reduce with the addition of sand materials.

The significant application is thus in order to get significant yield stress the presence of fines, particularly the clay size is very important. At the same time addition of larger sizes makes the slurry flow easy at high solid content, suggesting that filler addition is more workable than kaolinite slurry alone.

Another significant contribution of yield stress is the minimization of settling of larger grains suspended in the fine matrix. Scott et al. (1985) have discussed such application in their study of oil sand sludge. They indicated that the yield stress development is beneficial to suspend a granular phase such as sand in sludge by mixing. They noted that a more profound understanding of the microscopic behaviour of the sludge system is worthy of pursuit.

3.5.2 Comparison of vane yield stress with rheological model

In the conventional rheological tests, the viscometer data are used to plot a flow curve of shear stress vs. shear rate. The extrapolation of the flow curve to zero shear rate provides an estimate of the yield stress. The yield stress obtained using this technique can also be compared with the vane yield stress.

This comparison was completed for kaolinite only slurry at 40 % s(w/w). The viscometer data are plotted in Figure 3-44 and a Herschel-Bulkley model was fitted to the data. For comparison, another model, Casson's model, has also been fitted to the data and the result is shown in Figure 3-45. Both models fit well with the viscometer data. The yield stress as measured by vane apparatus is about 12 Pa.

Similar attempt of model comparison has been carried out for kaolinite and sil-flour mix (1:1 mix) and 50%s. Figures 3-46 and 3-47 show the rheological data and the HB model and Casson's model fit, respectively. A summary of comparison of yield stress as measured by vane apparatus and model fits is presented in Table 3-7.

Table 3-7 Comparison of yield stress as obtained from different methods

Method	**Slurry yield stress (Pa)**	
	Kaolinite only (40%s)	**Kao-sil flour(1:1) (50%s)**
Vane apparatus	13.28	12.58
Herschel-Bulkley model fit	11.49	11.00
Casson's Model model fit	12.15	10.13

Both models fit to the experimental data well. It is necessary to compare these models to identify which model is the better one. A statistical comparison, called corrected Akaike's Information Criterion (AICc), was used (Motulsky and Christopoulos 2003). The detailed calculation using Excel spreadsheet is shown in Appendix D. The summary of AICc calculation is shown in Table 3-8.

Table 3- 8 Statistical comparison of rheological model fits

Slurry Description	**AICc**		
	H-B Model	**Casson's Model**	**Evidence Ratio**
Kao-Slurry (40%s)	9.022	7.465	0.211
Kao-Sil Slurry(50%s)	2.992	2.409	0.558

Table 3-8 shows that for both slurries, Casson's model has lower AICc that Herschel-Bulkley (H-B) model. Therefore, Casson model fit the data better than H-B model. Furthermore, the evidence ratio shows that the Casson's model is 21% better that H-B model for the Kao-slurry at 40%solids and 56%.better than H-B model, for Kao-sil slurry at 50% solids.

Such an outcome seems to be in support of some studies made by different researchers. For example, Want et al. (1982) reported that excellent agreement has been observed in the yield stress values measured directly by the vane in comparison with other indirect techniques, for concentrated red mud suspensions at solids concentration higher than 62% by weight. Dzuy and Boger (1985) measured the yield stress of flocculated bauxite residue slurries. They found the yield stress measured with vane test to be in good agreement with the results obtained by the more conventional rheological methods. They suggested further that the dubious nature of the true yield stress often associated with the laborious but indirect rheological approach can be avoided.

3.5.3 The effect of pore water chemistry

Different observations are made with regard to the effect of pore water chemistry on the yield stress. Turian et al. (1993) have tested the yield stress of coal water mixture in the acidic range and showed that the yield stress increases with acidity. Bourret et al. (1988) studied the pH effect on a narrow range of 6 to 8. They observed an increase in shear stress for increase in pH.

Whereas Zhou et al. (2001) have shown that the yield stress of a zirconia suspension is maximum as the pH approach to the neutral pH region and decreases as pH moves to lower and higher values.

Tseng and Wu (2003), when comparing the settling rates of Al_2O_3 suspensions at pH 2 and pH 11, found that the faster settling rate at the higher pH is due to prevailing attractive van der Waals force over the interparticle potential. They also reported that the interparticle attraction would compete with the gravitational force during sedimentation.

The kaolinite slurry at different pH level is tested to see the effect pore water chemistry on the slurry yield stress. The pH is varied by adding diluted HCl acid solution to achieve lower pH and $Ca(OH)_2$ /NaOH solution to achieve higher pH.

The shear stress variation at different pH condition is shown in Figure 3-48. Figure 3-49 show yield stress as a function of pH at a solid content of 50% kaolinite slurry. The yield stress seems to show moderate increase by decrease in the pH. This observation is in some way in support of the data presented on Leda Clays by (Torrance and Pirnat, 1984). They stated that the greater yield stress below about pH 7 exhibited by lower salinity material suggest that the hydrogen ions and the polyvalent ions released by acid attack on the mineral had a greater effect when the competition from other ions was weakest. They further discussed that the edges of clay minerals are positively charged below the isolectric point pH. The interaction with negatively charged mineral particles increases below the isolectric point. Such a reaction, along with changing ion saturation, partially explain the increased yield stress as the pH decreased. The edge faces of kaolinite are positively charged in acidic condition (Schofield and Samson, 1953). The high yield value and viscosity of slightly acidic, salt-free suspensions is ascribed to exhibit high yield and viscosity was ascribed to attraction between positive crystal edges and negative crystal faces, possibly giving rise to a cubic card-house structure (Houwink and De Decker 1971).

The increase in yield stress at pH 12 as achieved by the addition of Ca^{2+}, can be explained through the ion exchange properties of the kaolinite slurry. The source of ion exchange can be (i) broken bonds that give rise to unsatisfied negative charges to be balanced by adsorbed cations and (ii) the hydrogen of exposed hydroxyls, which may be replaced by exchangeable cation.

The diffuse double layer at clay surfaces consists of the lattice charge and compensating counterions, which reside in the liquid immediately adjacent to the particles. Counterions are subject to two opposing tendencies (i) electrostatic attraction to negatively charged clay surface and (ii) diffusion from high concentration at the particle surface to low concentration in the bulk solution. Diavalent ions are attracted to the surface with a force twice as great as that of monovalent ions. Thus, in a divalent ion system, the diffuse double layer is more compressed. With an increase in the electrolyte concentration of the solution, the tendency of counterions to diffuse away from the surface is diminished and the diffuse double layer is further compressed. Consequently, the thin diffuse double layer enhance further attraction of clay particles and then flocculation.

The clay slurry prepared at the pH 12 using NaOH shows different shear stress values. The shear stress is almost diminished and shows no distinct yield stress point. Instead the shear stress shows a slight increase in shear stress with time. The addition of sodium hydroxide causes the number of positive charges to be reduced to the point where the edge faces on which they occur are no longer capable of attracting the negatively charged cleavage surface of adjacent crystals. The interparticle repulsive forces, due to the net negative charges, are then adequate to maintain complete dispersion. ((Schofield and Samson 1953). The gradual rise in shear thickening could be due to the mechanical action that realign the particle arrangement and give rise to some attraction between adjacent particles.

Zhou et al. (2001) stated that the nature and magnitude of forces acting in the flow system and the resulting microstructure are responsible for the complicated rheological responses. They explained the existence of three kinds of forces which coexist to various degrees in flowing suspensions: hydrodynamic forces, Brownian forces and colloidal force. It is believed that the complex interaction of these forces account for the remarkably different rheological behaviour at higher pH.

3.5.4 Rheological measurements of oil sands tailings

Oil sands tailings samples were obtained from Suncor Energy Inc (Suncor). and Albian Sands Energy (Albian). The tailing sample obtained from Suncor is a mature fine tailings (MFT) from pond 6 at three depths and the sample obtained from Albian is thickener underflow.

The test data are presented in Appendix A5. The data indicate that rheological properties are dependent on temperature, grain size distribution and porewater chemistry. These observations indicate that the surrogate tailings material used can be further used to understand the fundamental factors and mechanisms in the study.

The Albian thickener underflow at 35% solids content (Figure A-33) exhibited a different rheological response versus the kaolinite slurry (Figure 4-58) when treated with $Ca(OH)_2$ and NaOH at pH 12. This drop in yield stress when for the sample treated with $Ca(OH)_2$ can be ascribed to the process used to produce bitumen. Albian sands use non-caustic sodium citrate to extract the bitumen from the oil sand. The tailing from this non-caustic process thus include sodium citrate. Sodium citrate forms a complex with calcium, apparently so stable that adsorption of this calcium by the clay particles is drastically depressed. The anticoagulant action of sodium citrate was due to the depression of calcium ions (Hussey et al. 1950).

The yield stress measurement shows different pattern depending upon grain composition and concentration. Such properties can be used in the process involving the transport and deposition of slurry, where control of the quality of materials need to be quickly achieved by comparing the yield stress measurement with the set target yield stress. In this respect, the sensitivity of the yield stress to any ingredients change and relatively very short duration of the test suggest that it can be used as a viable and quick controlling means in large scale slurry handling processes. In settling behaviour of slurry, the significance of yield stress accounts for the effect of fine matrix on the rheological models and that need to be accounted for in the flow characteristics. This application is discussed in detail in the ensuing chapters.

3.6 Summary and Conclusion

The yield stress as a parameter of rheological characterization has been investigated using the vane shear apparatus. A test on the working principle of the apparatus has indicated that there exists good repeatability of measurements and less effect of the ends (about 10% of the total yield stress). The yield stress is shown to be predominantly dependent on the surface property of the very fine particles, namely clay size. Changing the composition of slurry by adding silt size or sand size particle accounts for reduced yield stress and consequent decrease in the stiffness of the slurry at very high solid content. By addition of more sand, however, at higher solid contents, results in dilatant behaviour of the slurry. Consideration of fines as silt and clay appears insufficient unless the clay and silt composition are specified in the fines.

As there exists no standard rate of shearing, different rates have been tested. Increasing the shearing rate has shown generally an increase in yield stress. However at low shear rates between 0.01/s and 0.03/s the yield stress shows good repeatability. The experiments suggest that shear rates below 0.03/s can be used to measure the yield stress. Fractal theory fit has been applied to fit to experimental results and very good match is observed.

For the solid content tested (50%s), an important observation is made that kaolinite slurry is affected by the pH of the pore fluid medium. Reference to pH alone is not sufficient without stating the mineralogy of the solution. It is observed that, at similar pH of 12 but prepared from different solution have shown very different shear properties, thus mineralogy need also be specified together with pH.

3.7. References

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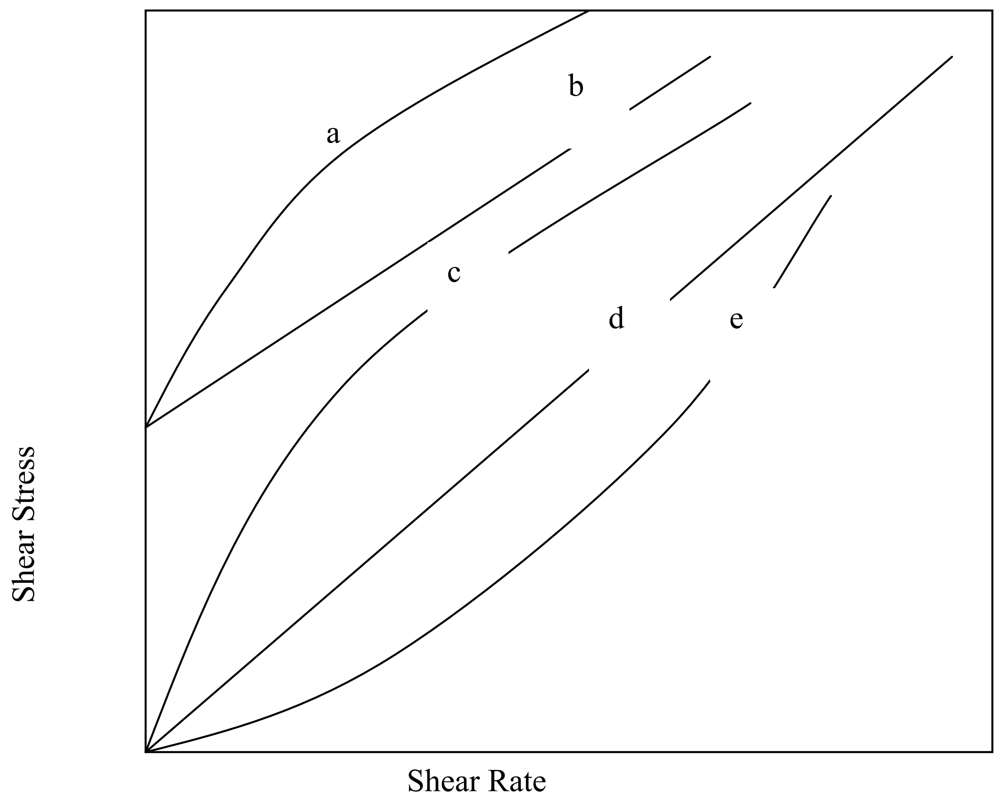


Figure 3-1 Flow Curves for different non-Newtonian materials. (a) Viscoplastic; (b) Bingham plastic(c) Pseudoplastic (d) Newtonian (e) Dilatant

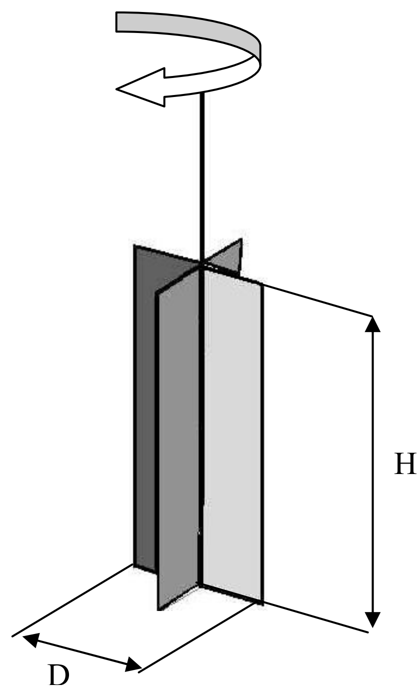


Figure 3-2 Vane Geometry

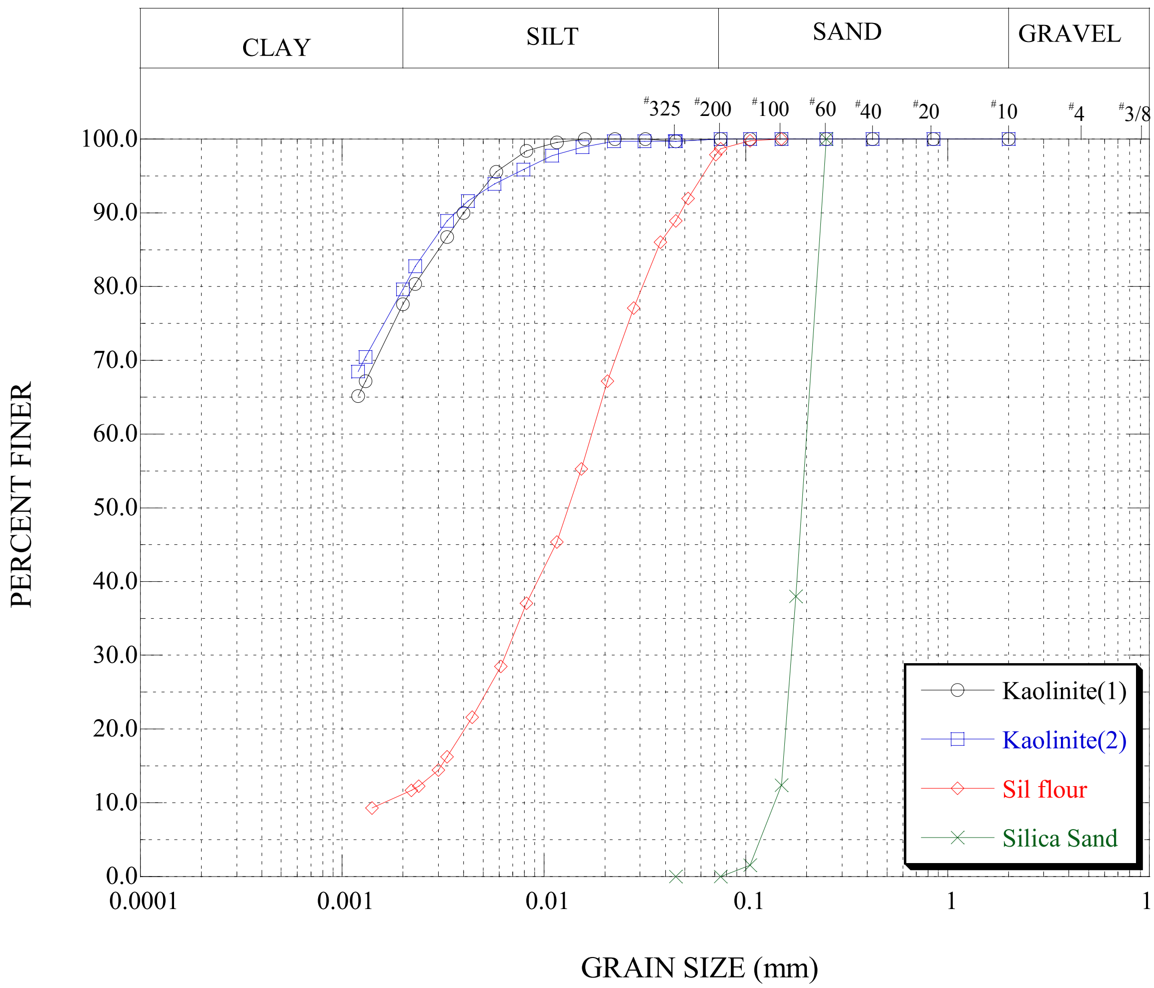


Figure 3-3 Grain size distribution of the material used to prepare the slurry

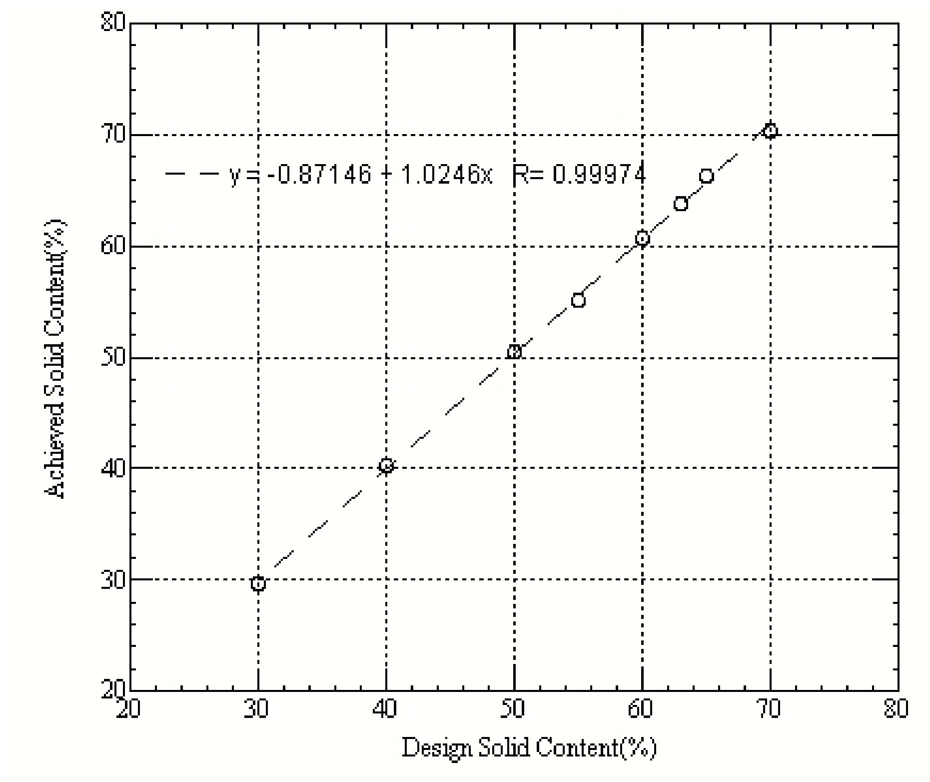
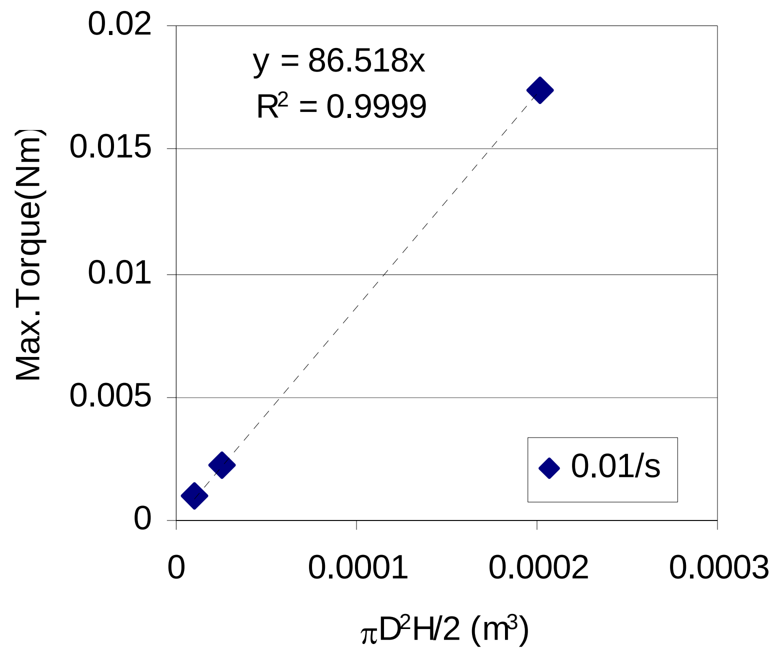
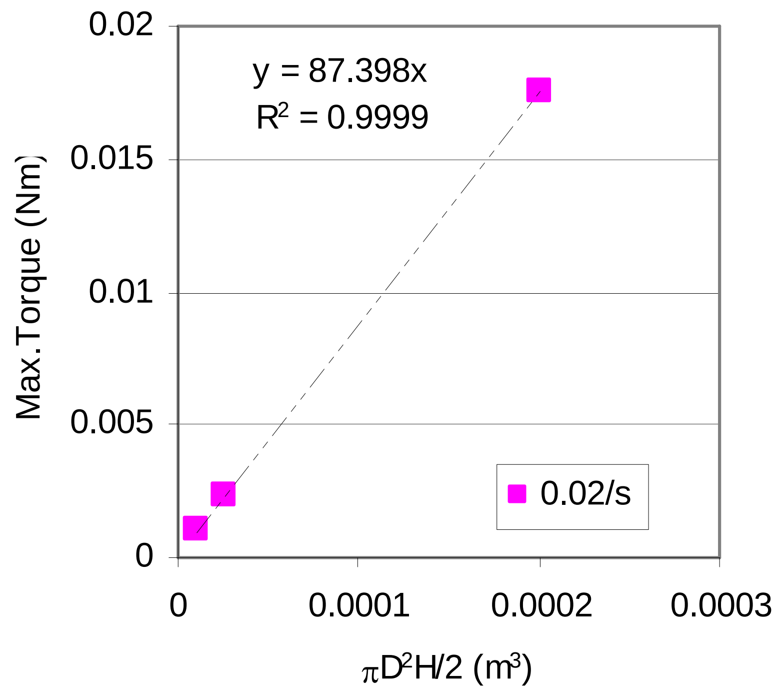


Figure 3-4 Comparison of design and achieved sample solid content for kaolinite slurry

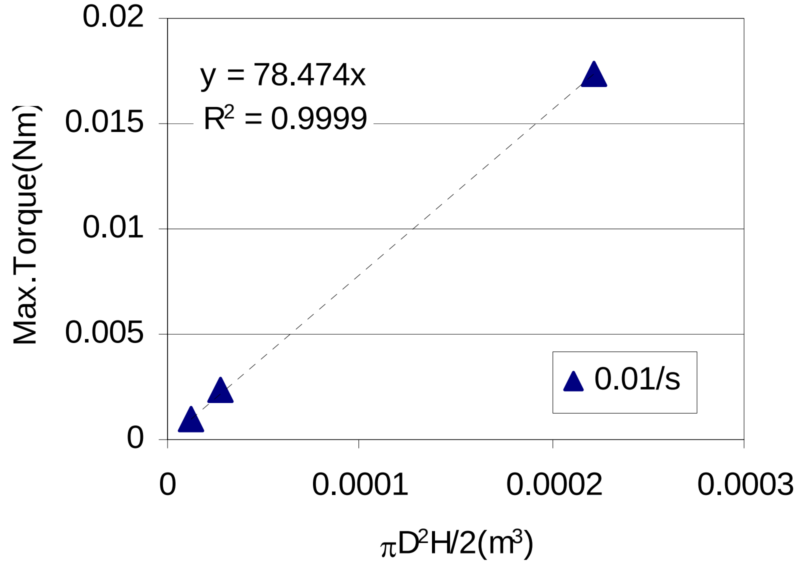


(a)

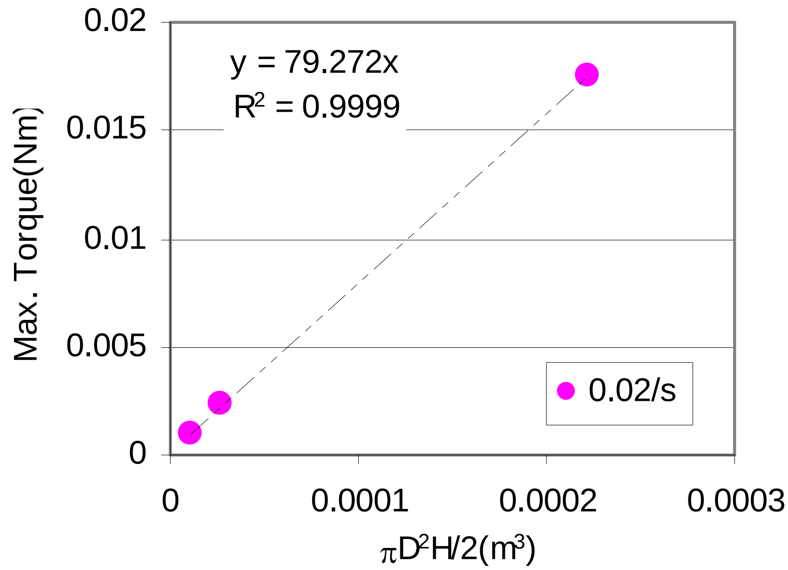


b)

Figure 3-5 Relationship between maximum torque and geometrical parameter for kaolinite slurry(50%w/w) at shear rates (a) 0.01/s and (b) 0.02/s



(a)



(b)

Figure 3-6 Relationship between maximum torque and modified geometrical parameter for kaolinite slurry (50%w/w) at shear rates 0.01/s and 0.02/s, when the shear surface is at diameter 5% larger than the vane diameter.

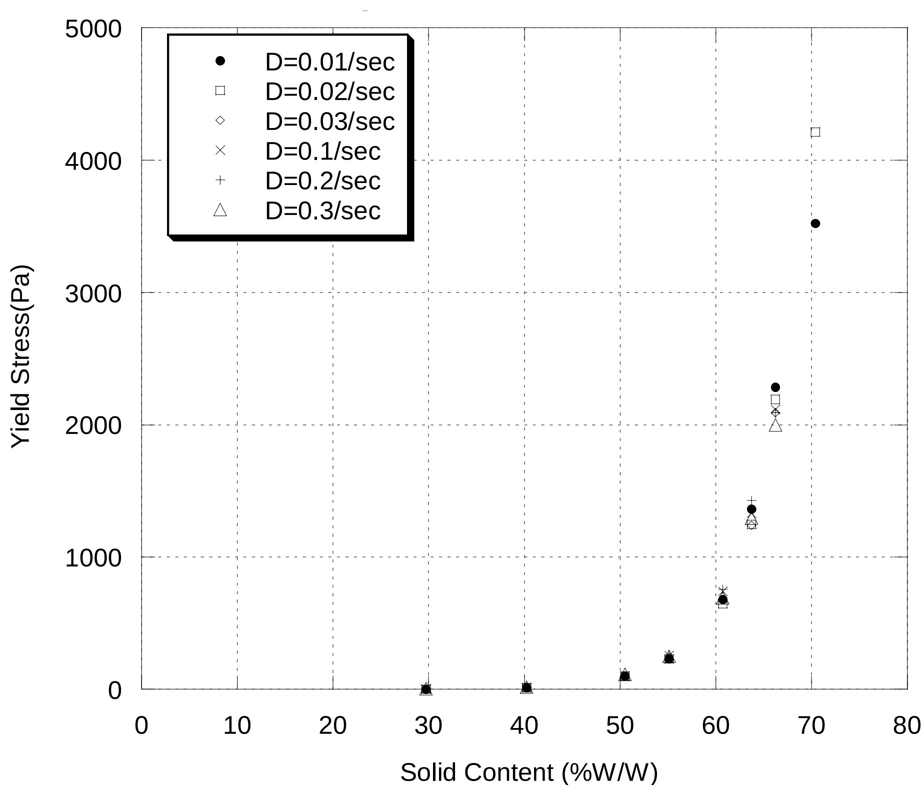


Figure 3-7 Yield stress variation with solid content at different shearing rate

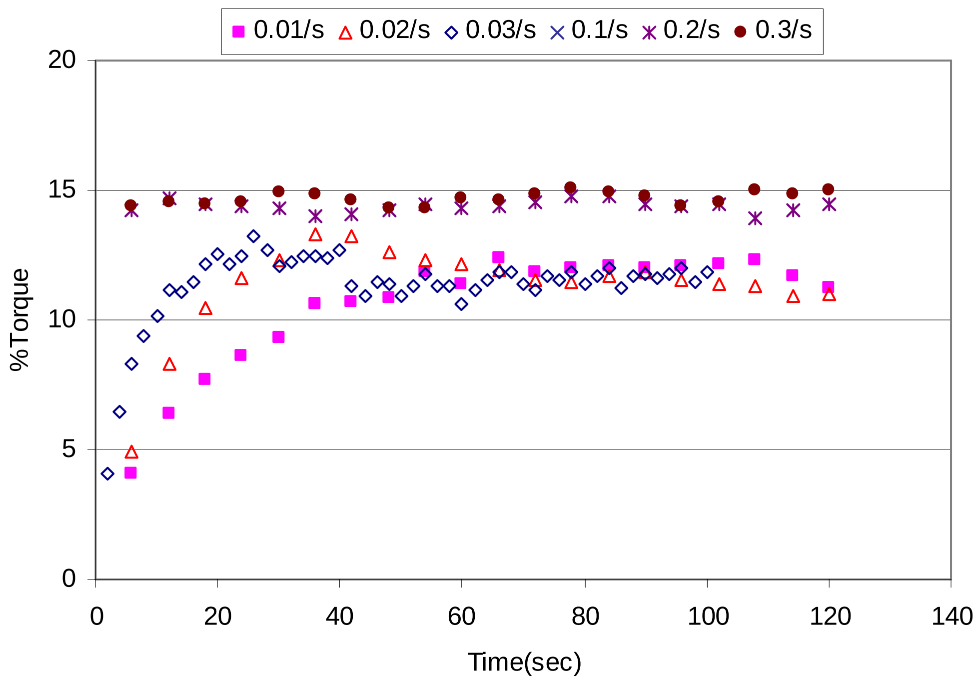


Figure 3-8 Variation of %Torque (reference 5mNm) with time for 30%s, kaolinite slurry at different rates

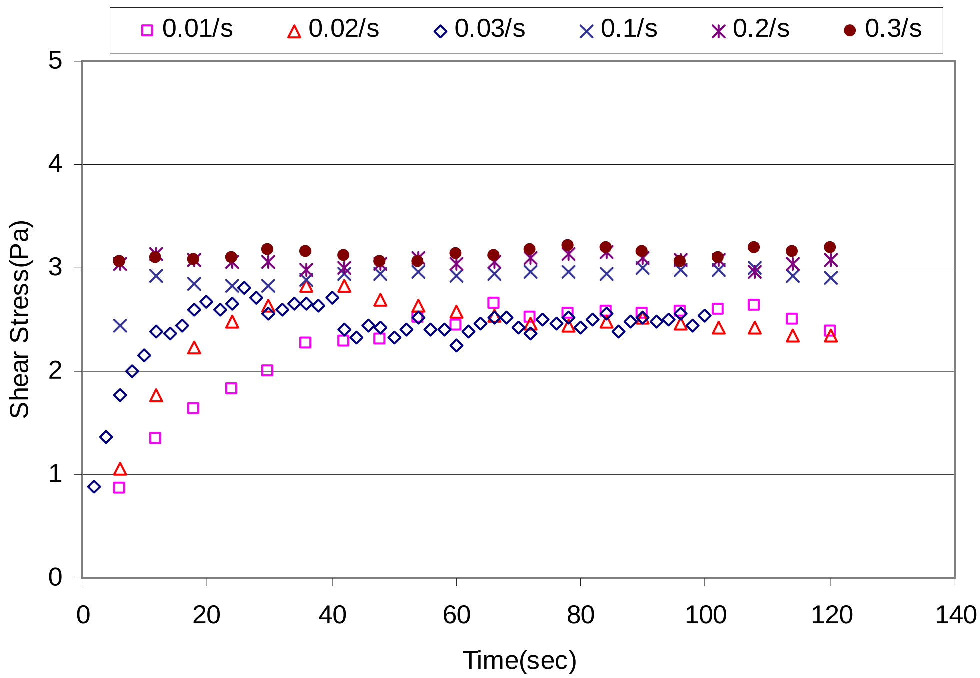


Figure 3-9 Variation of shear stress with time for kaolinite slurry (30%s) at different rates

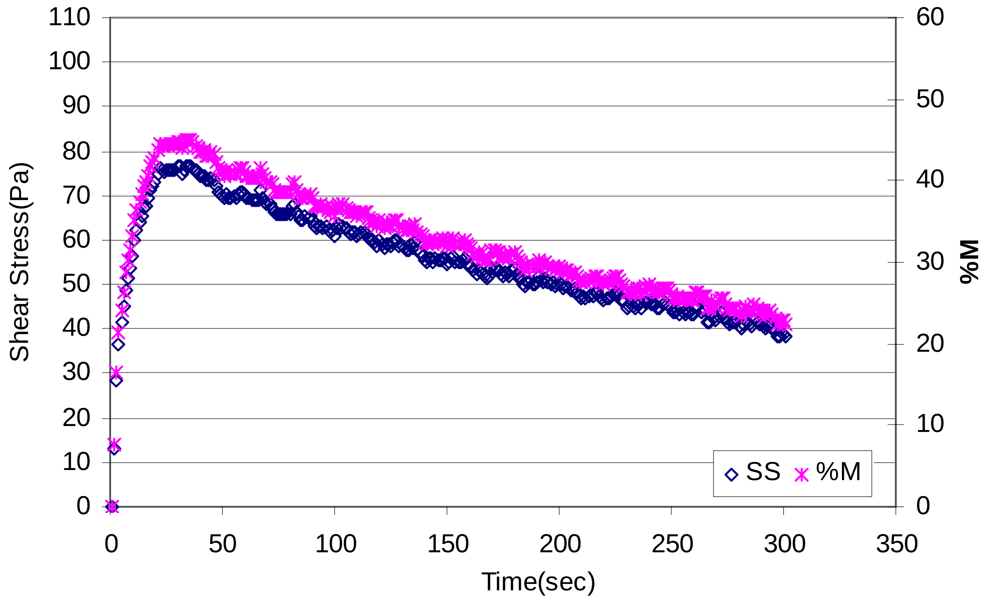


Figure 3-10 Variation of shear stress(SS) and % Torque(%M) with time for 50%-kaolinite slurry @ a rate 0.01/s

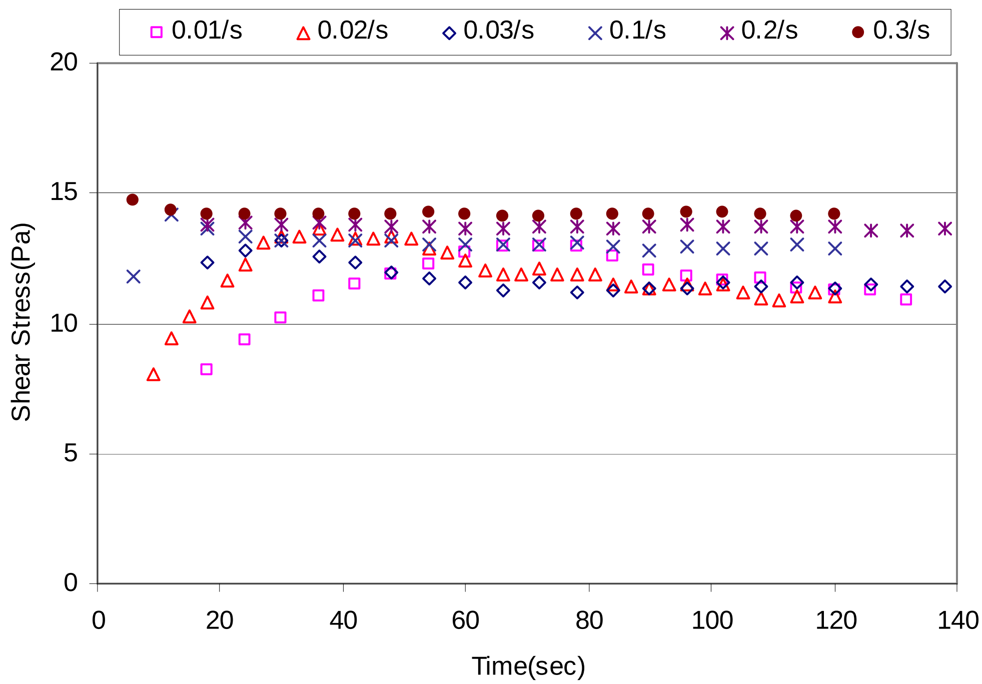


Figure 3-11 Variation of shear stress with time for kaolinite slurry (40%s) at different rates

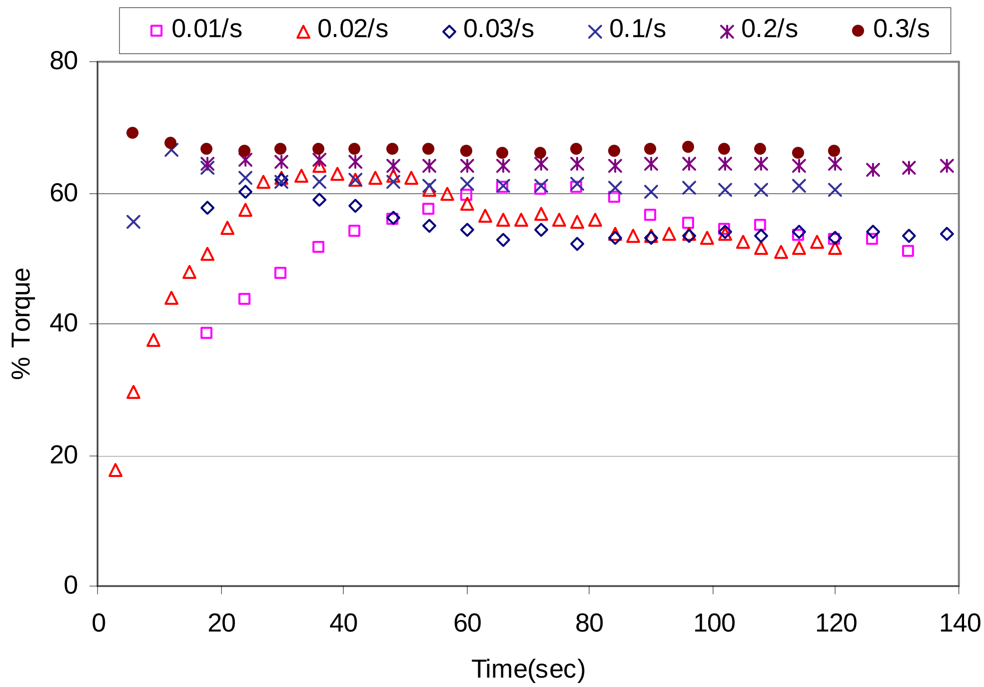


Figure 3-12 Variation of % Torque with time for kaolinite slurry (40%s) at different rates

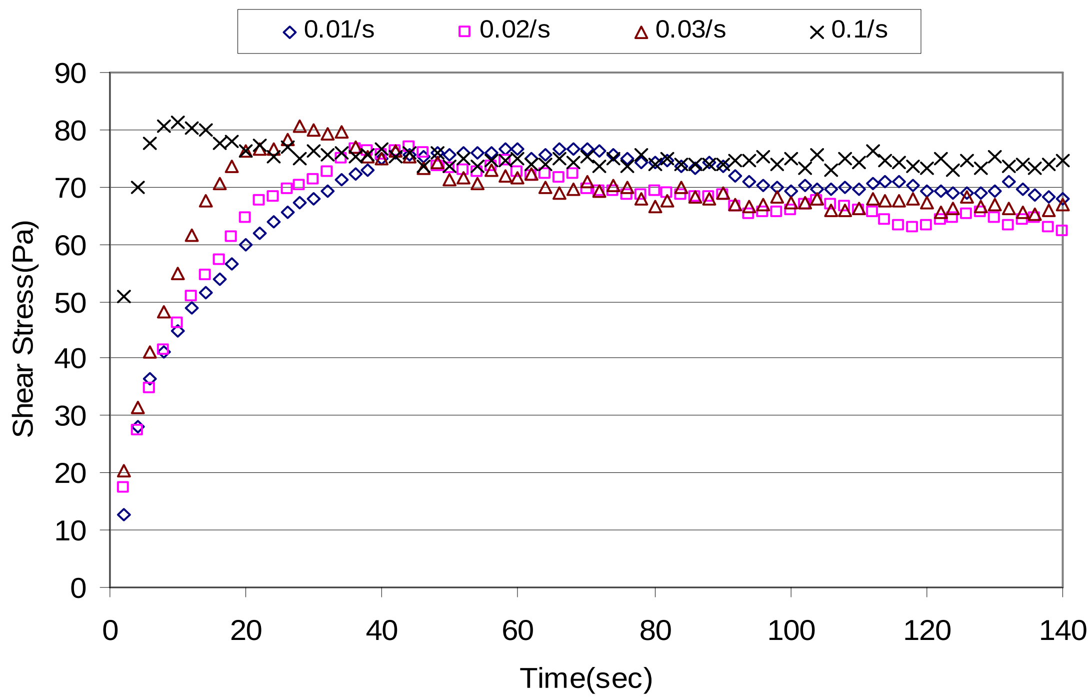


Figure 3-13 Variation of shear stress with time for kaolinite slurry (50%s) at different rates

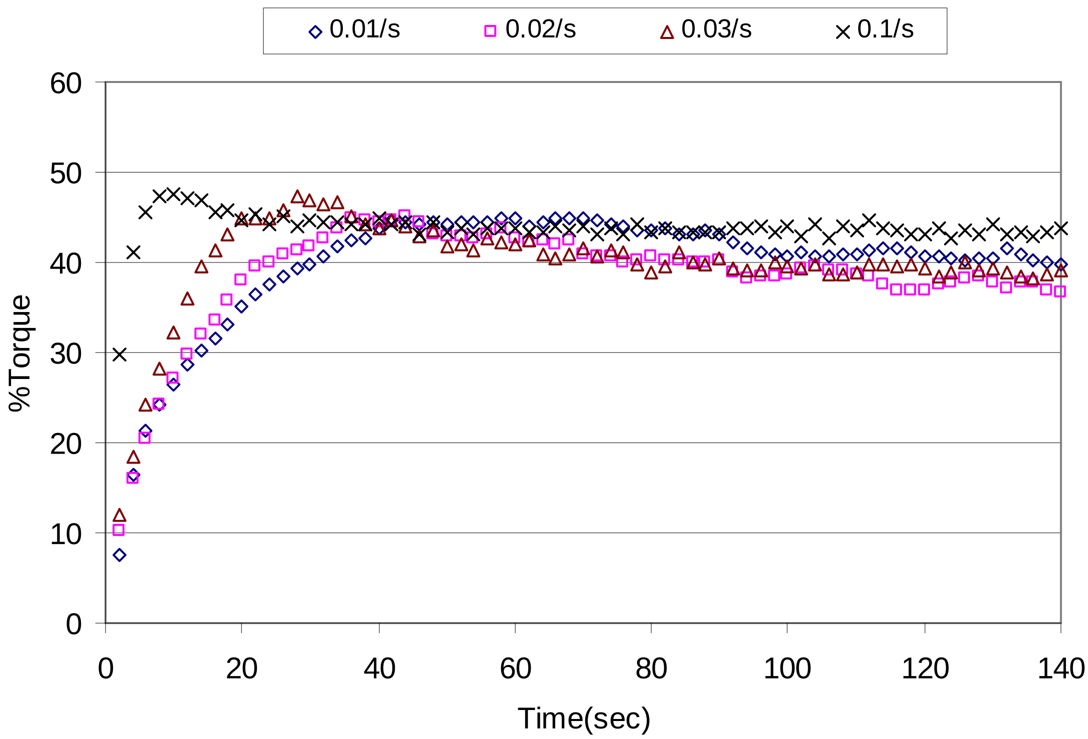


Figure 3-14 Variation of % Torque with time for kaolinite slurry (50%s) at different rates

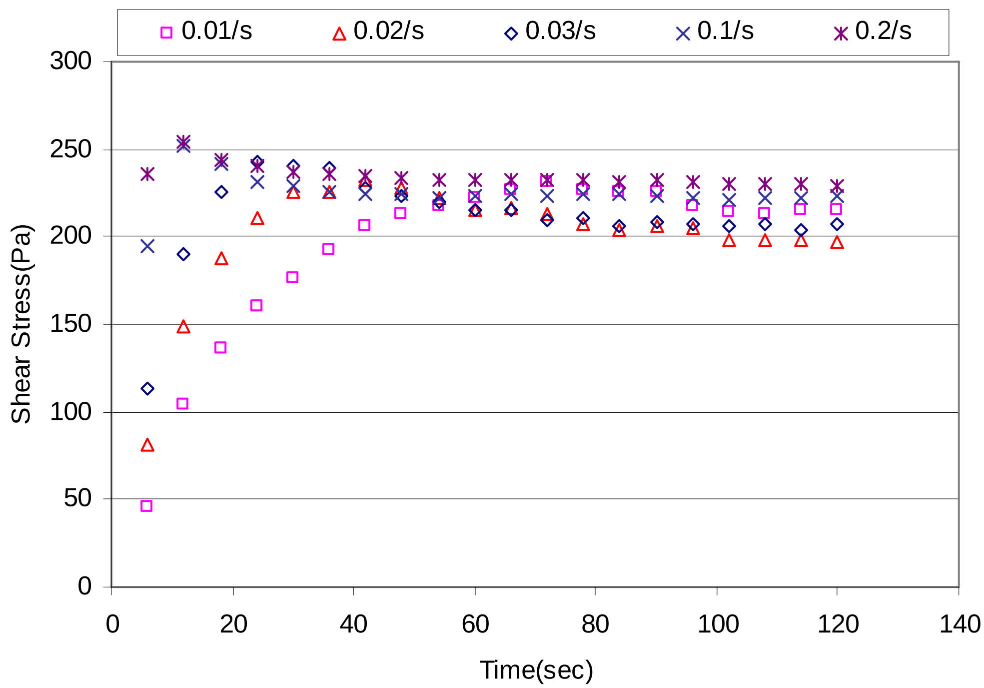


Figure 3-15 Variation of shear stress with time for kaolinite slurry (55%s) at different rates

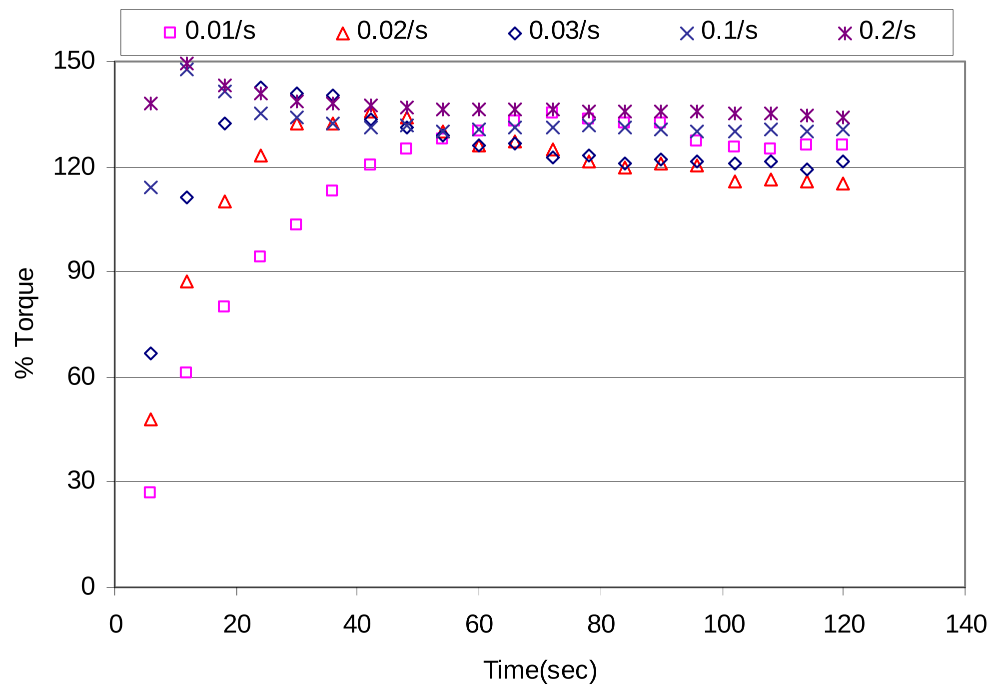


Figure 3-16 Variation of % Torque with time for kaolinite slurry (55%s) at different rates

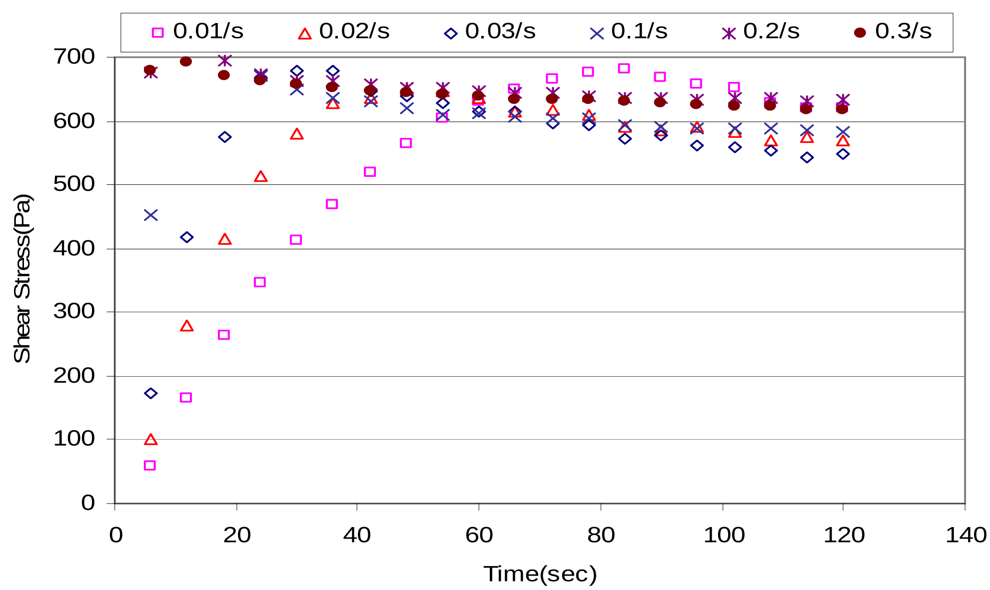


Figure 3-17 Variation of shear stress with time for kaolinite slurry (60%s) at different rates

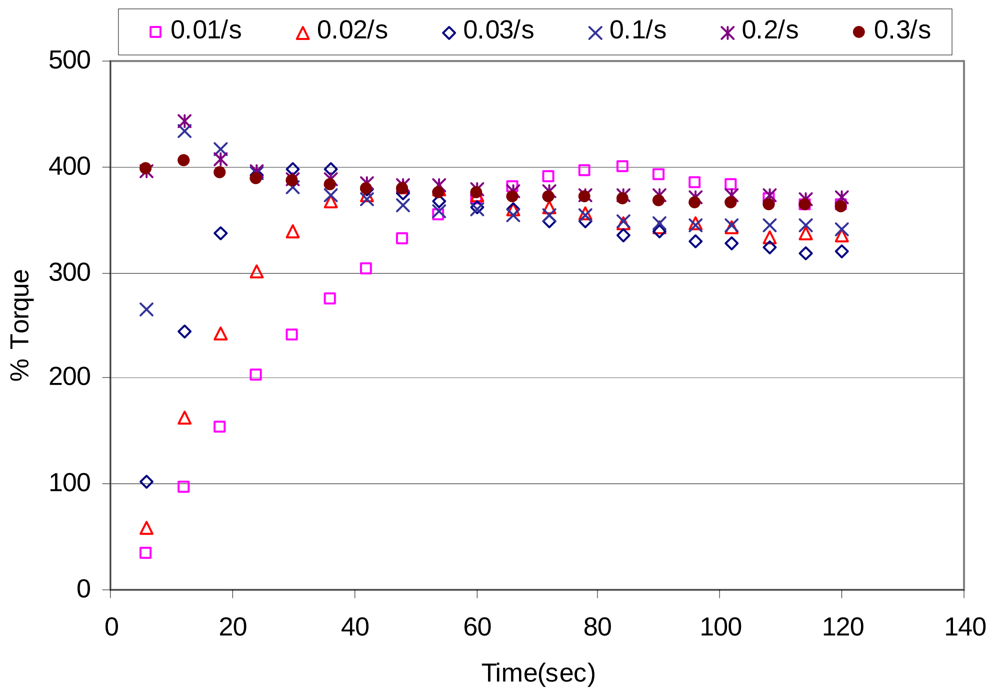


Figure 3-18 Variation of % Torque with time for kaolinite slurry (60%s) at different rates

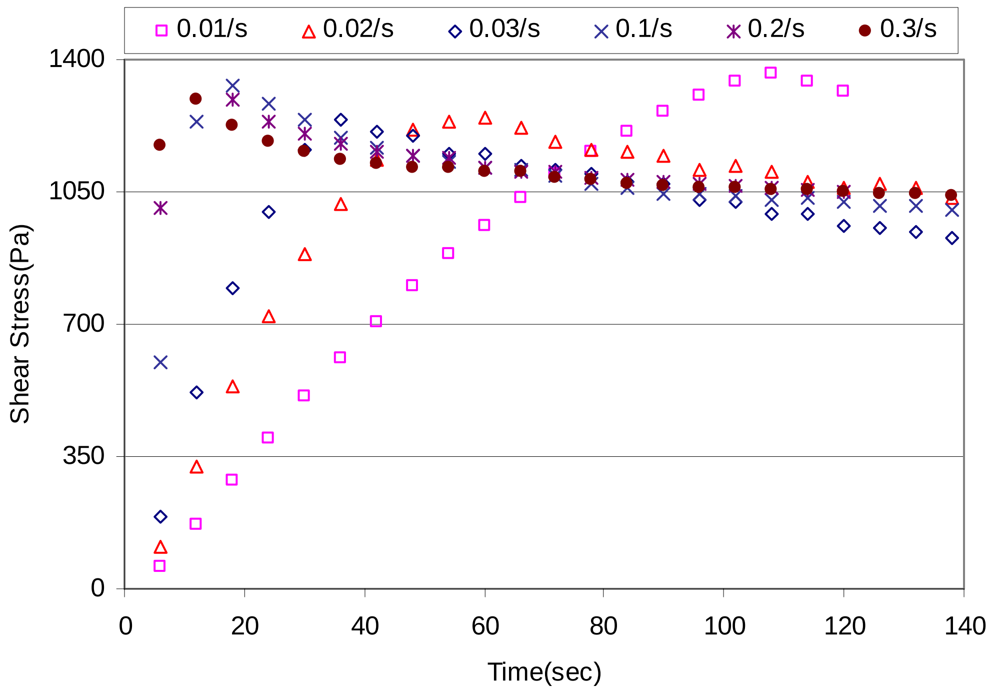


Figure 3-19 Variation of shear stress with time for kaolinite slurry (60%s) at different rates

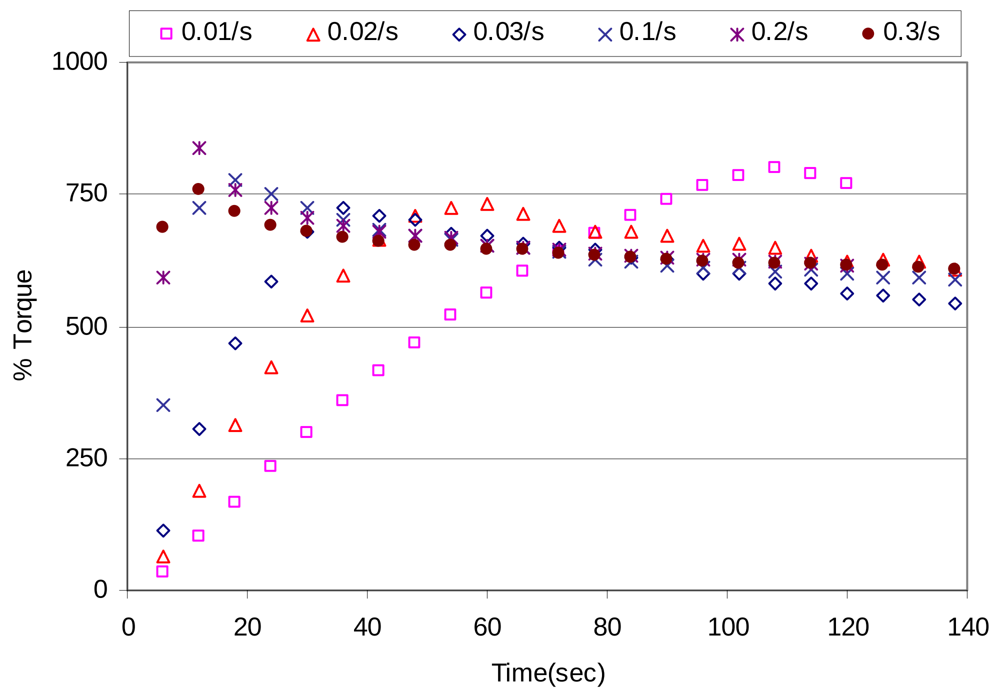


Figure 3-20 Variation of % Torque with time for kaolinite slurry (63%s) at different rates

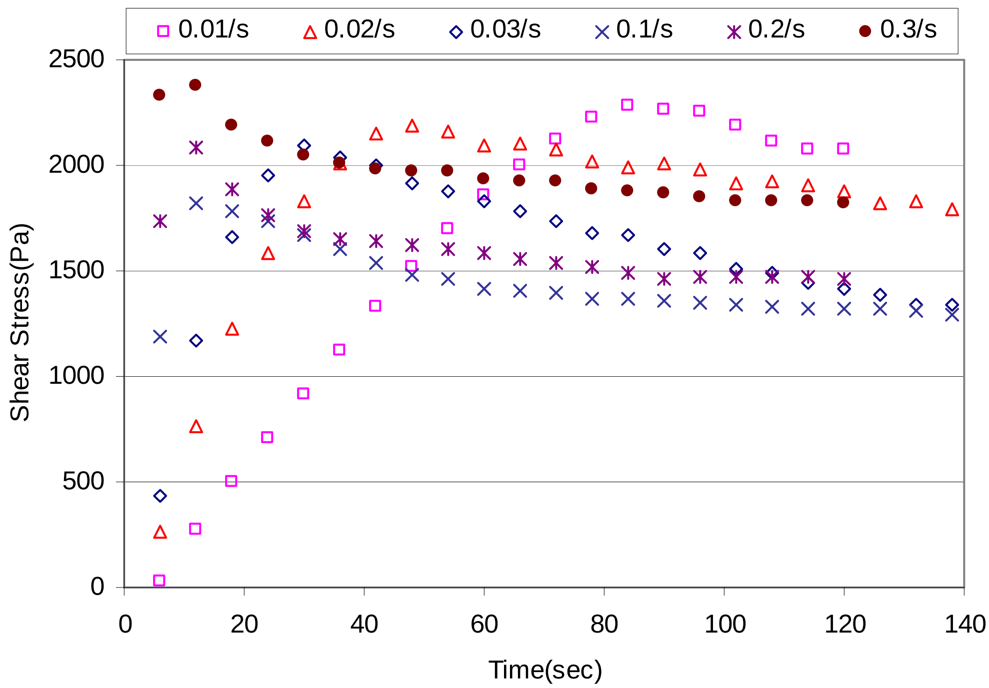


Figure 3-21 Variation of shear stress with time for kaolinite slurry (66%s) at different rates

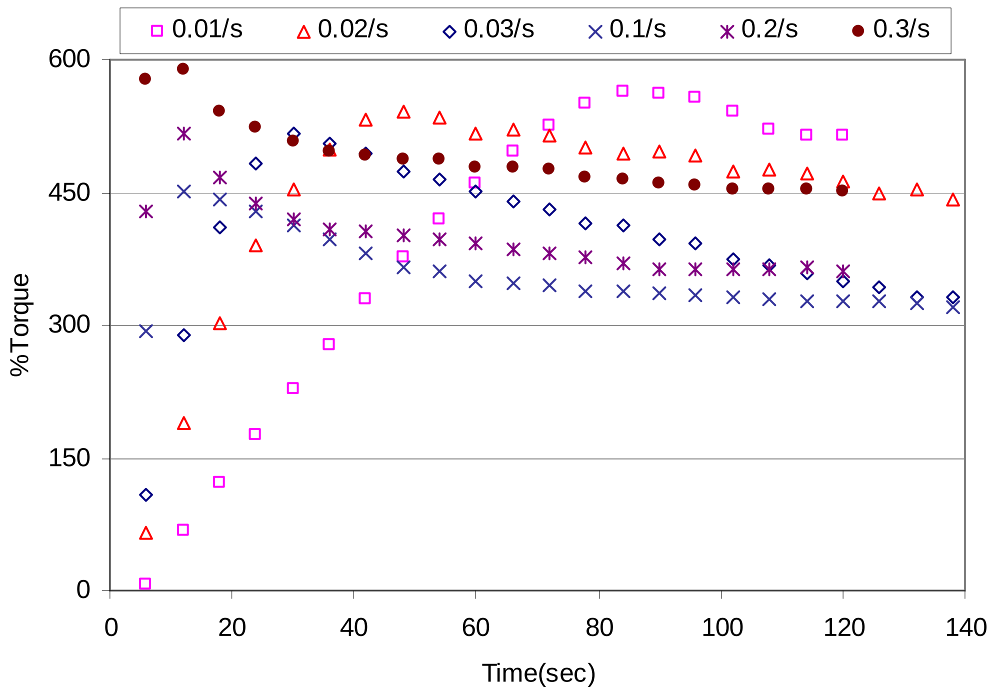


Figure 3-22 Variation of % Torque with time for kaolinite slurry (66%s) at different rates

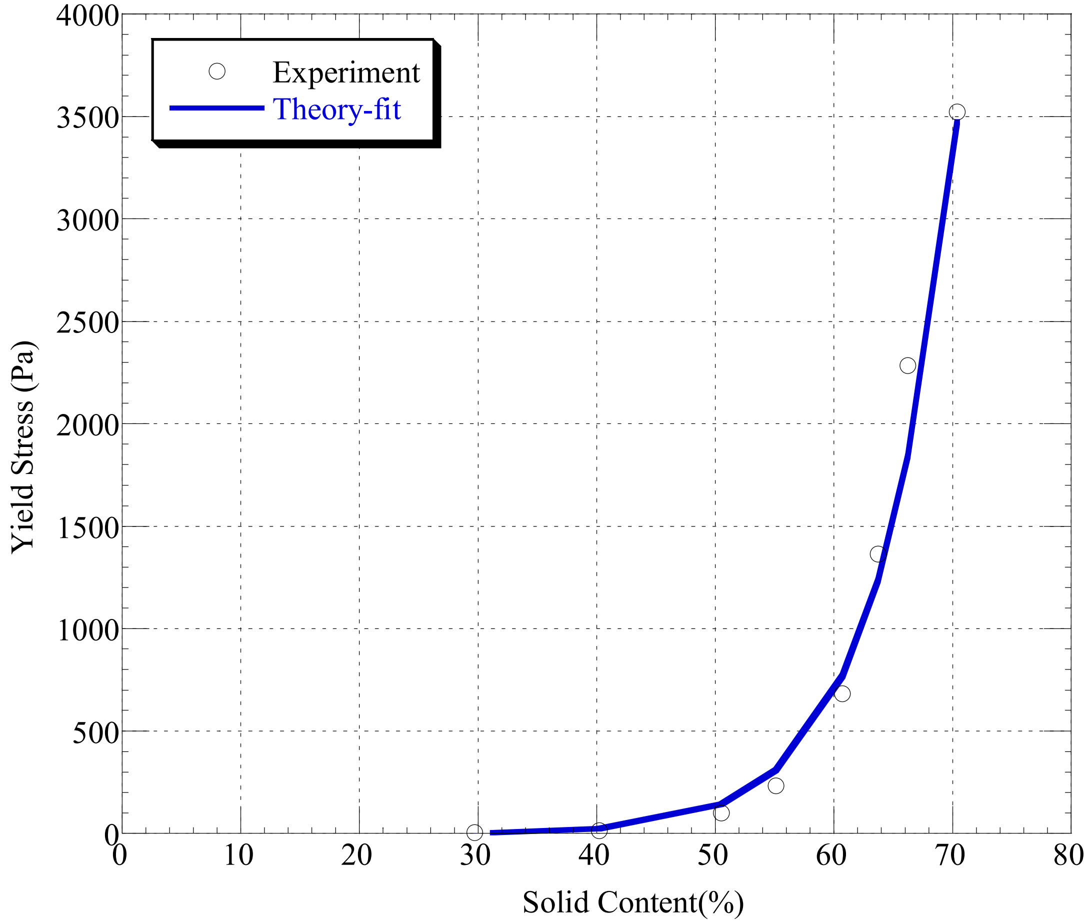


Figure 3-23 Yield stress-solid concentration (%W/W) plot for kaolinite-water mixture at shear rate of 0.01/s

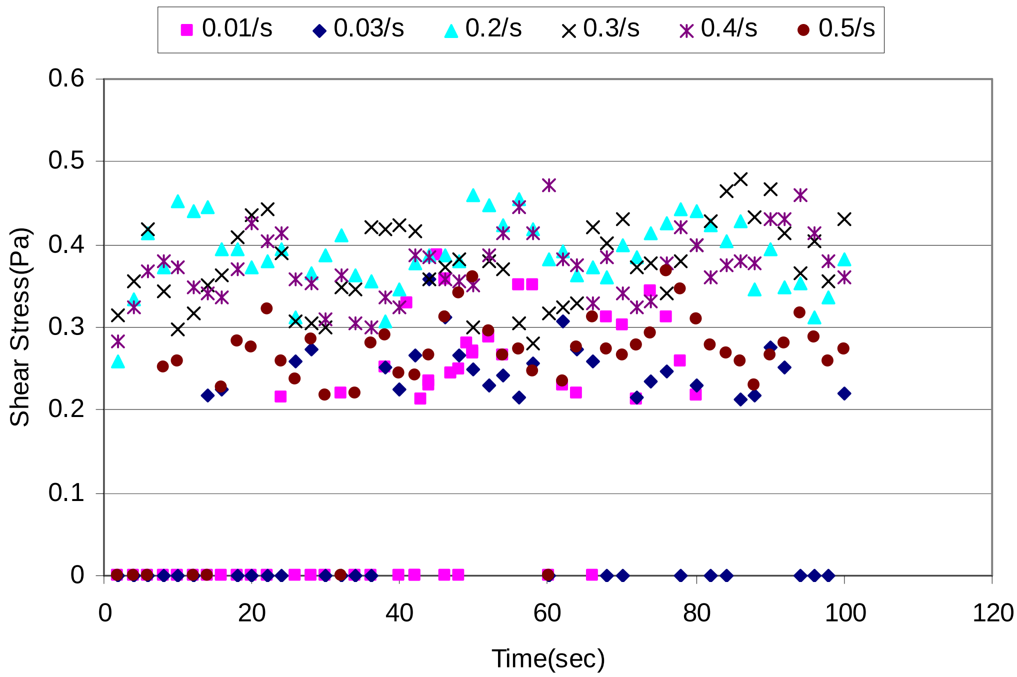


Figure 3-24 Variation of shear stress with time for kao-sil flour slurry (1:1), 30% s, at different rates

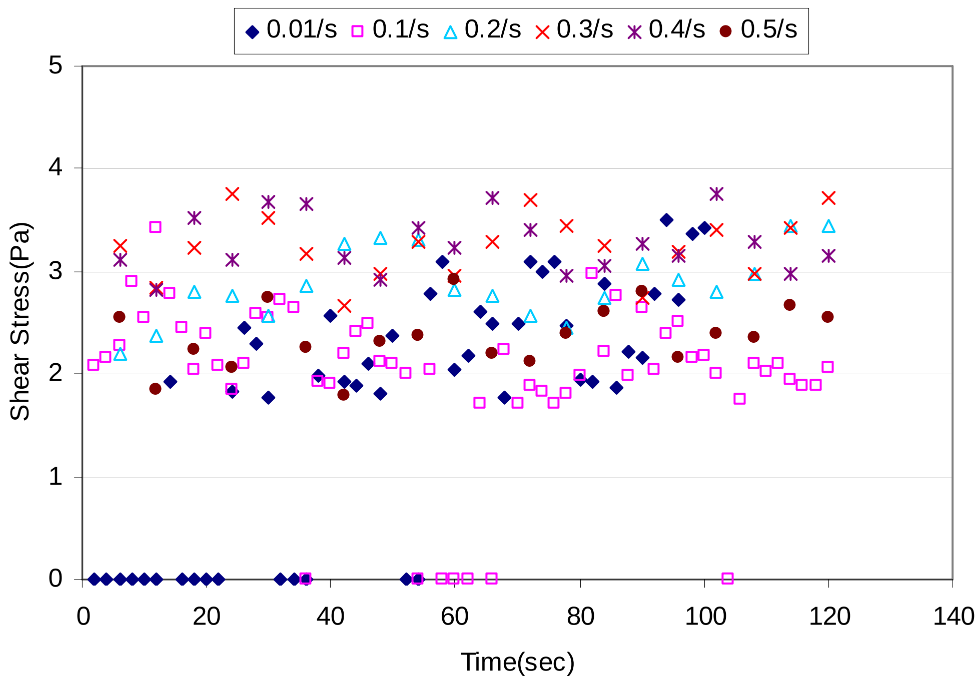


Figure 3-25 Variation of shear stress with time for kao-sil flour slurry (1:1), 40% s, at different rates

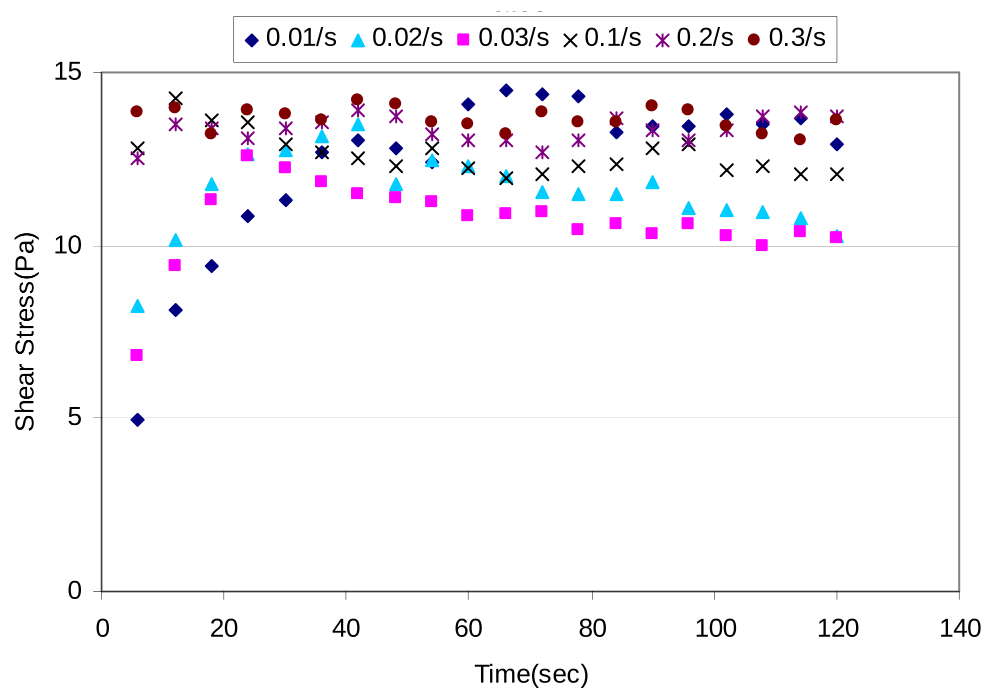


Figure 3-26 Variation of shear stress with time for kao-sil flour slurry(1:1), 50% s, at different rates

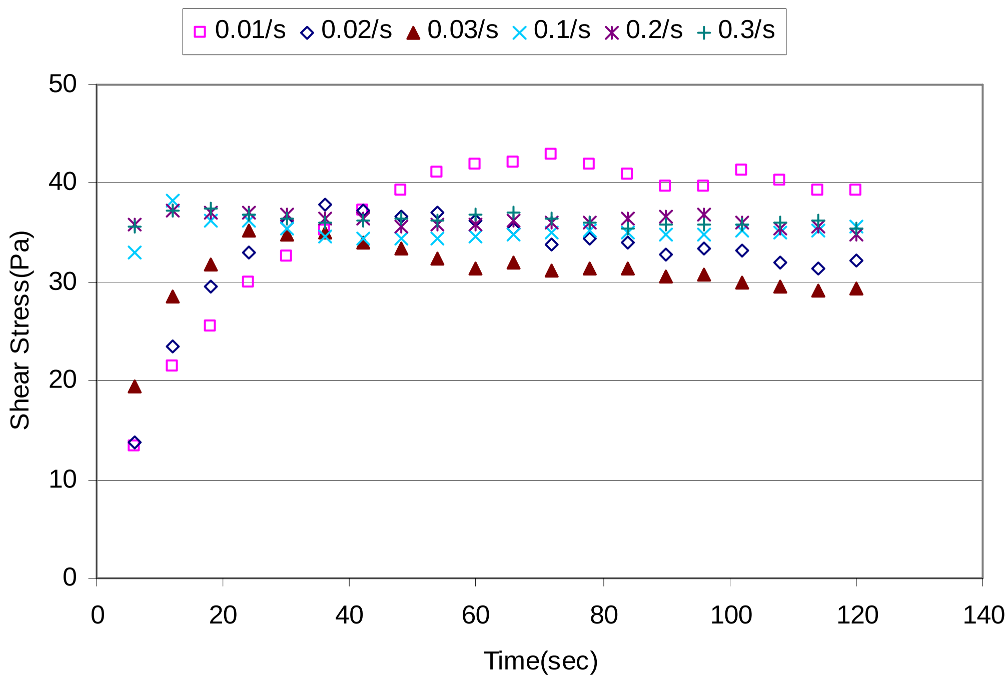


Figure 3-27 Variation of shear stress with time for kao-sil flour slurry(1:1), 55% s, at different rates

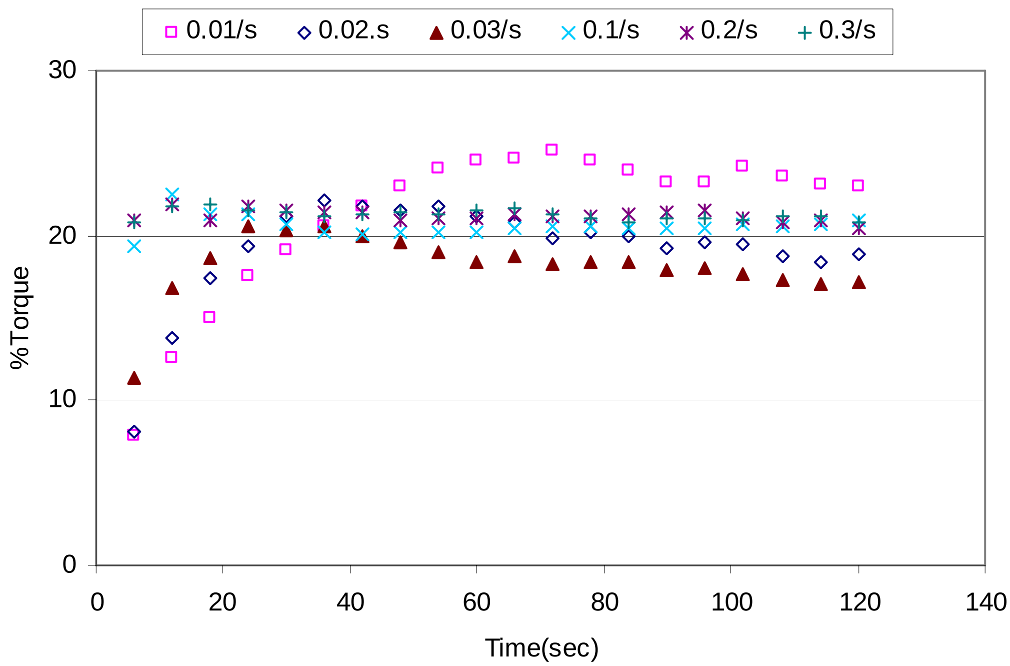


Figure 3-28 Variation of % Torque with time for kaolinite slurry(1:1) (55%s) at different rates

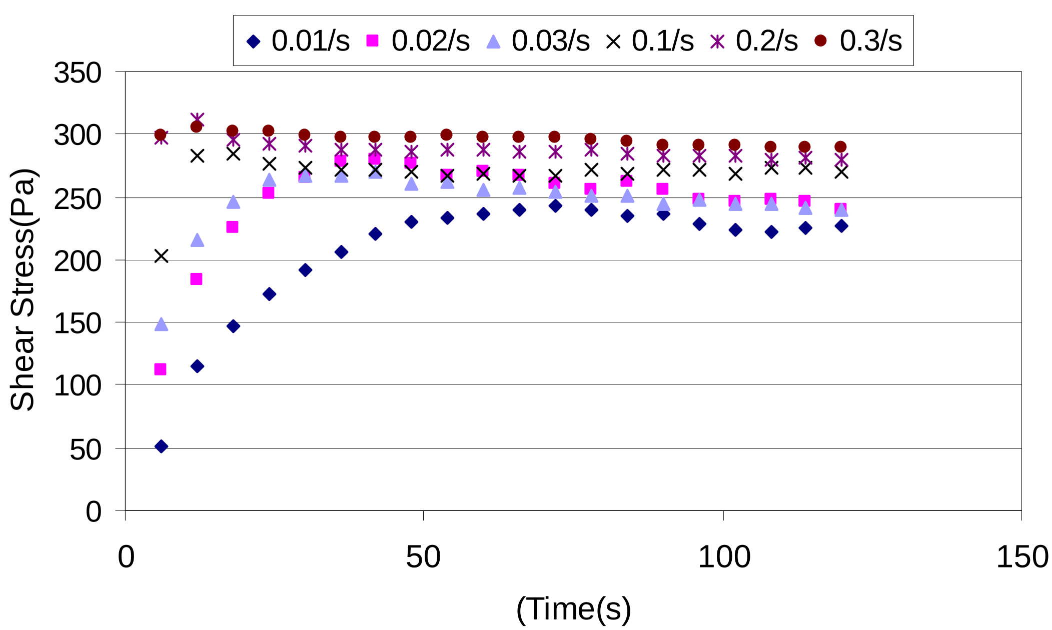


Figure 3-29 Variation of shear stress with time for kao-sil flour slurry(1:1), 62.5% s, at different rates

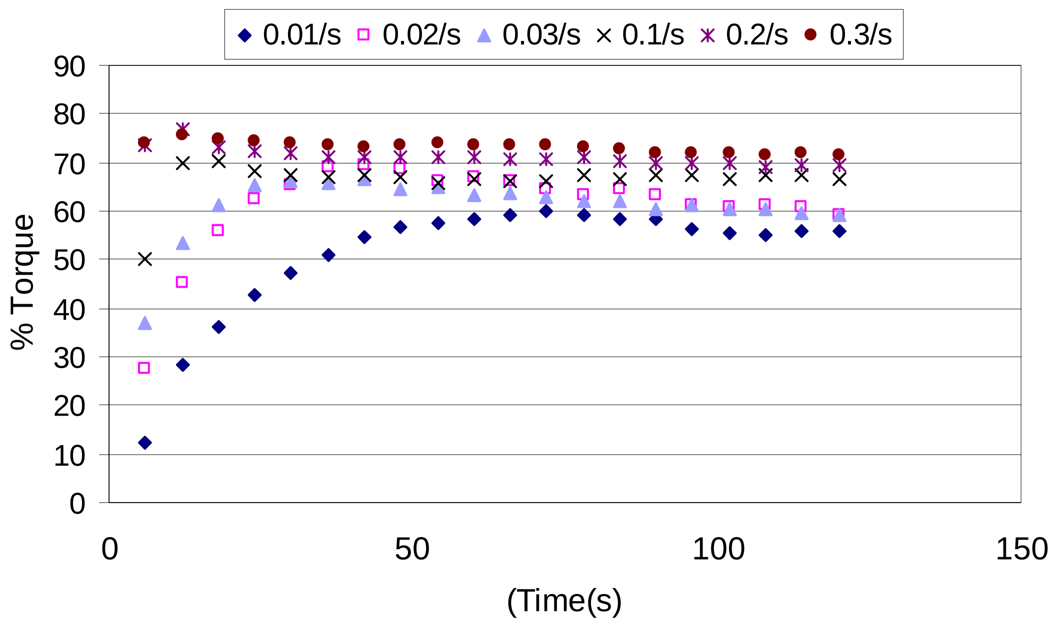


Figure 3-30 Variation of % Torque with time for kao-sil flour slurry (1:1)(62.5%s) at different rates

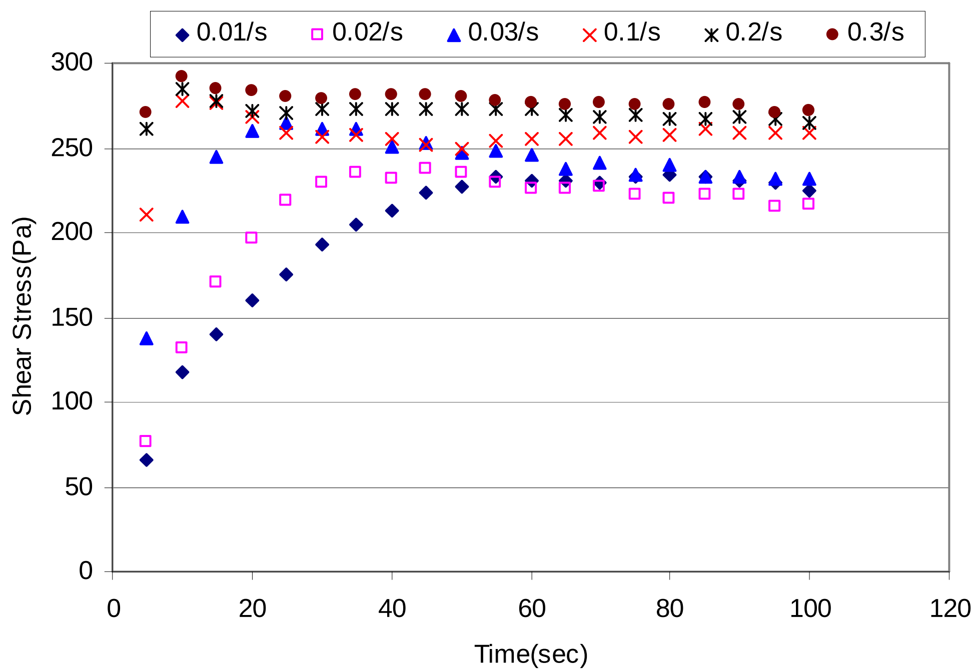


Figure 3-31 Variation of % Shear Stress with time for kao-sil flour slurry(1:1) (65%s) at different rates

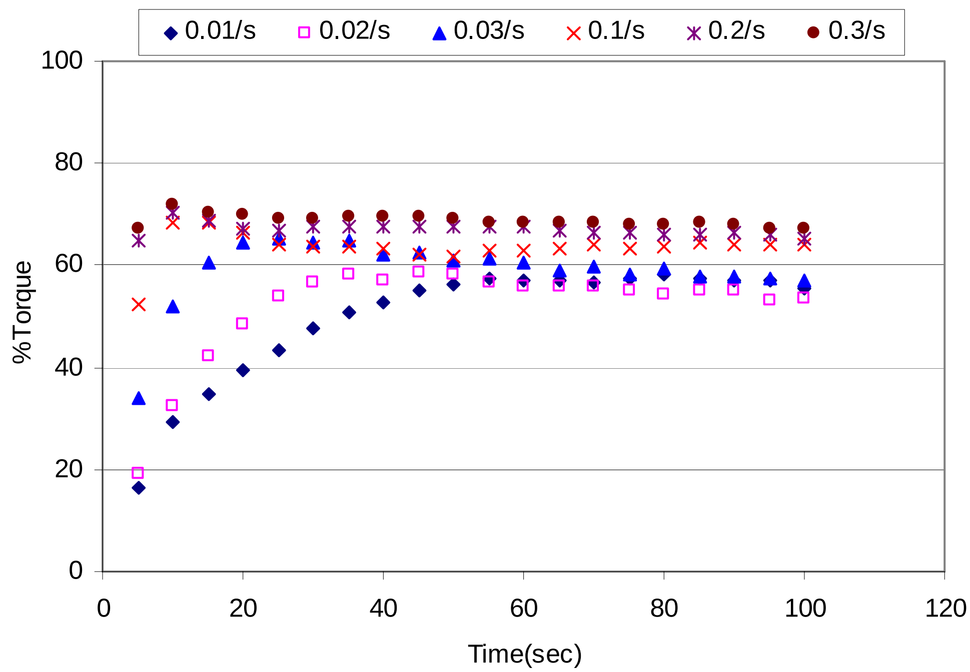


Figure 3 - 32 Variation of % Torque with time for kao-sil flour slurry (1:1) (65%s) at different rates

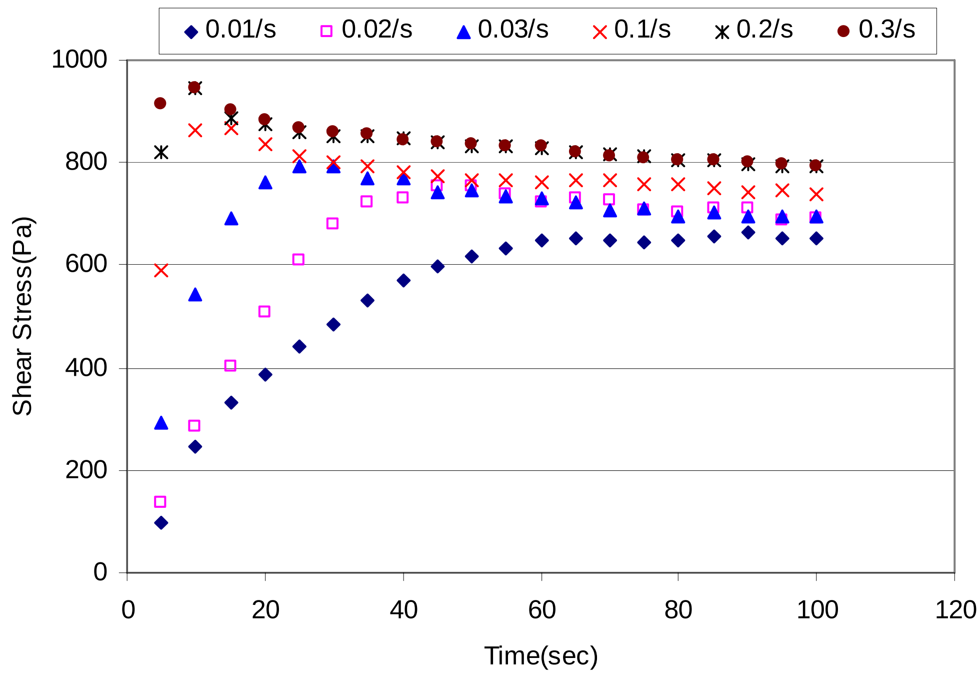


Figure 3-33 Variation of % Shear Stress with time for kao-sil flour slurry(1:1) (70%s) at different rates

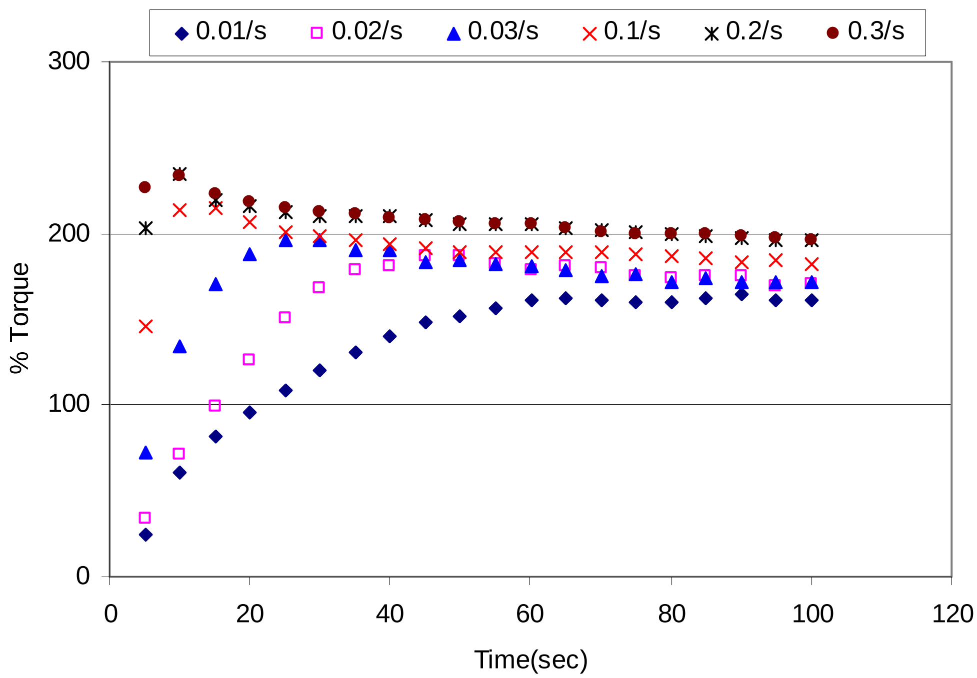


Figure 3-34 Variation of % Torque with time for kao-sil flour slurry (1:1)(70%s) at different rates

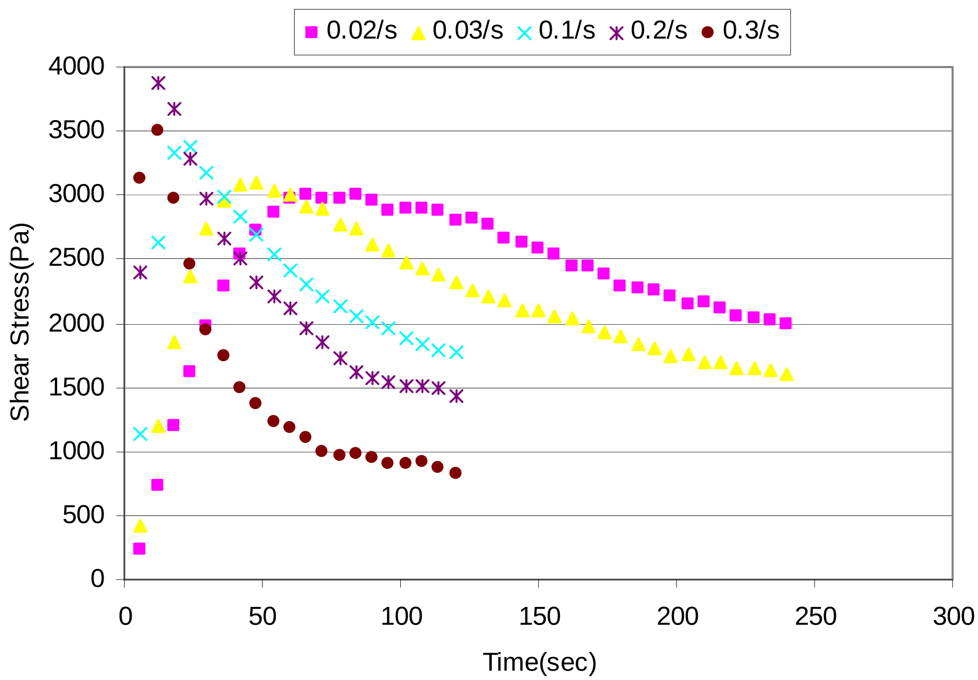


Figure 3-35 Variation of Shear Stress with time for kao-sil flour(1:1) (75%) at different rates

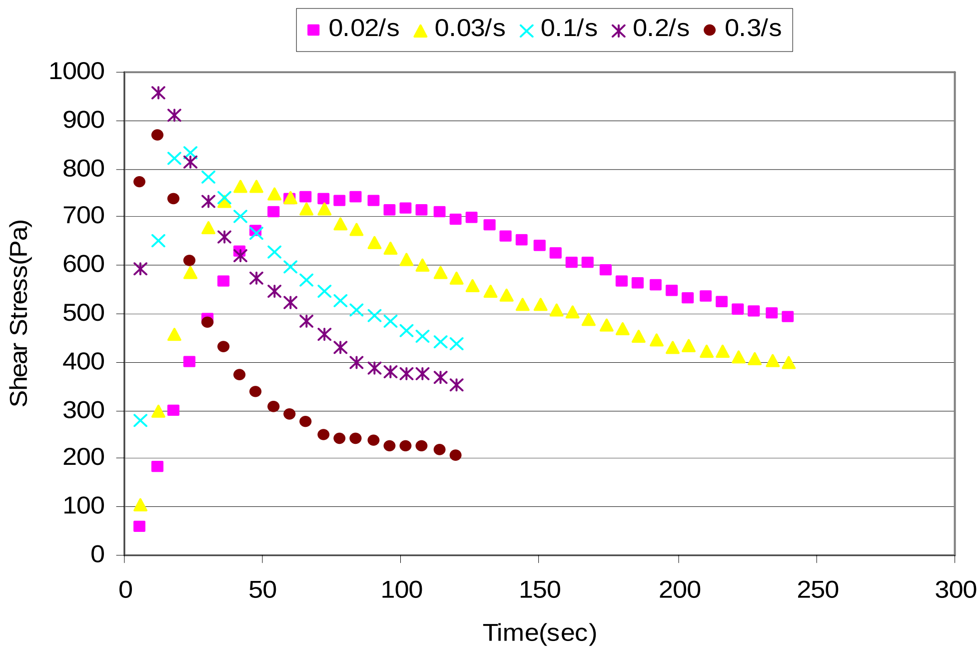


Figure 3-36 Variation of % Torque with time for kao-sil flour slurry (75%s) at different rates

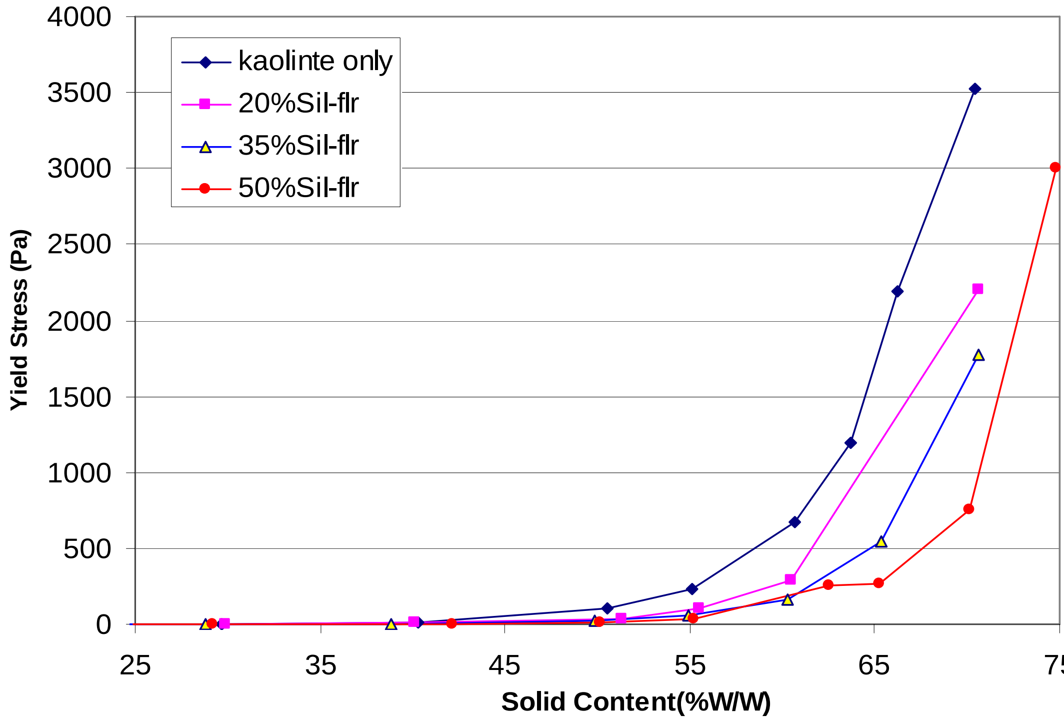


Figure 3-37 Effect of Sil-flour addition on the yield stress of kaolinite slurry, at shear rate of 0.02/s

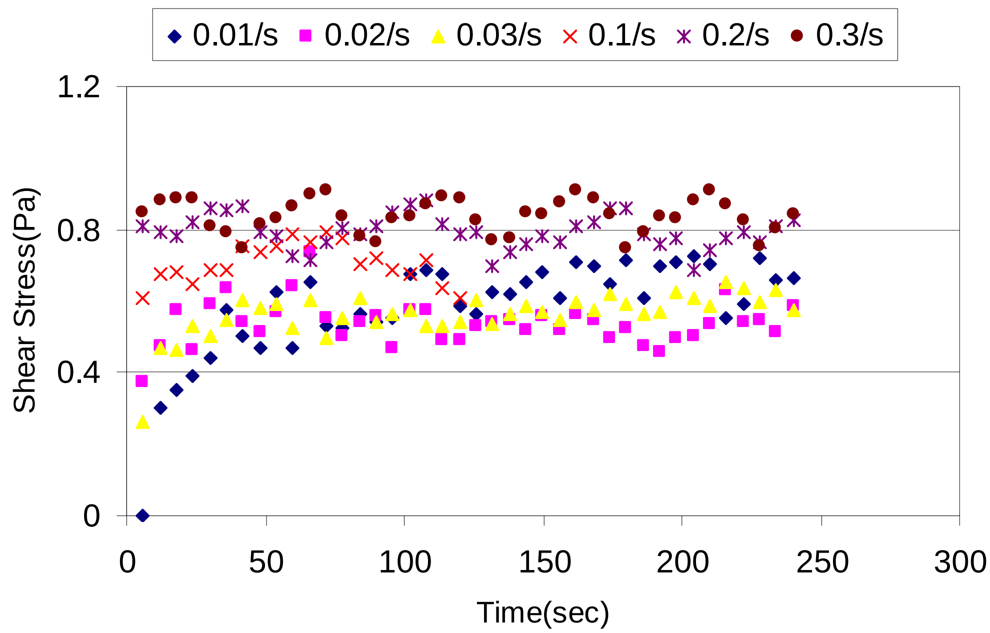


Figure 3-38 Variation of shear stress with time for kao-sand slurry (1:1) (35%s),at different rates

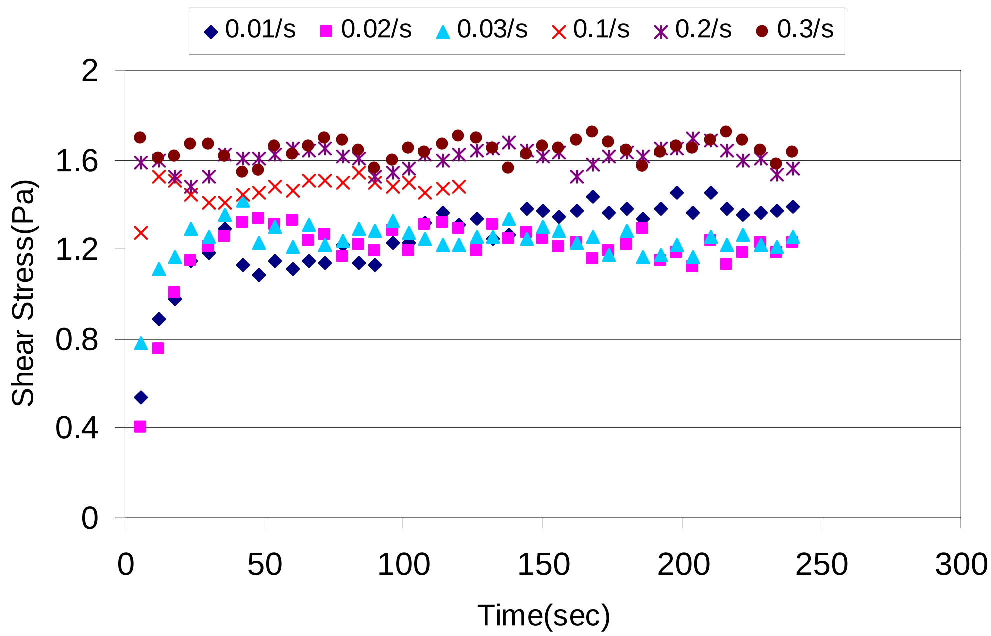


Figure 3-39 Variation of shear stress with time for kao-sand slurry (1:1) (40%s),at different rates

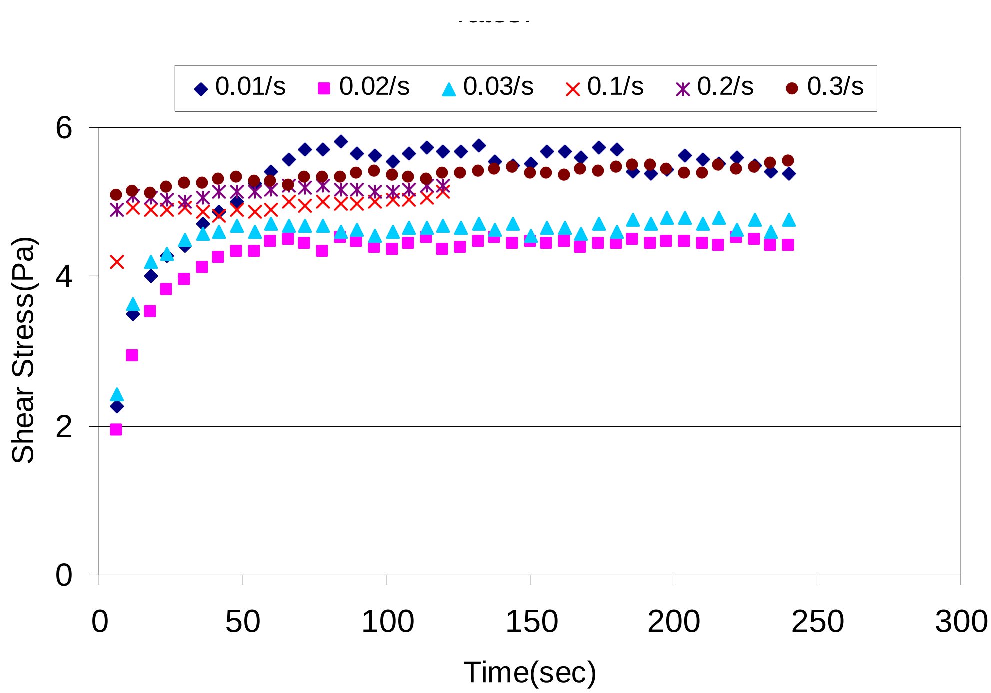


Figure 3-40 Variation of shear stress with time for kao-sand mix (1:1), 50%s, at different shear rates

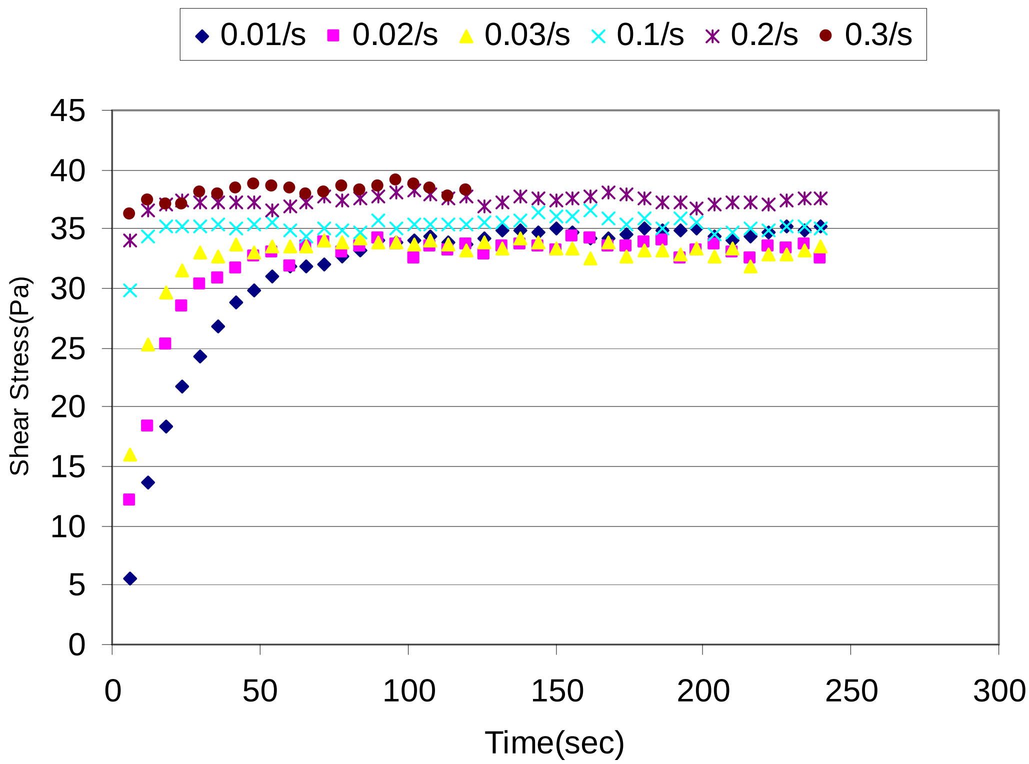


Figure 3-41 Shear stress variation with time for kosand mix(1:1), 60%s, at different rates

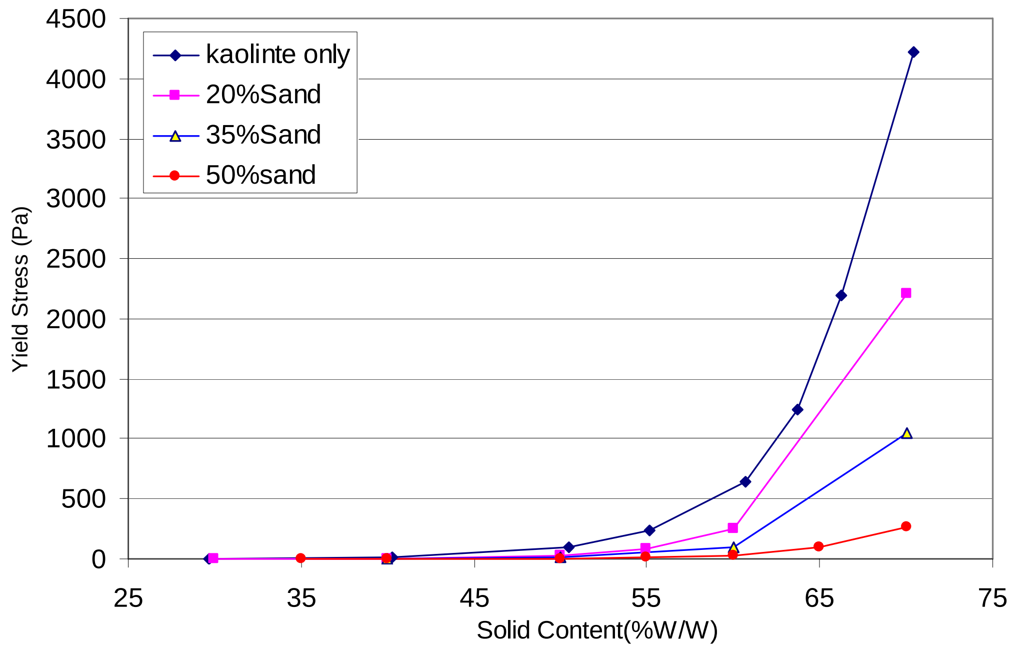


Figure 3-42 Effect of Sand addition on the yield stress of kaolinite slurry, at shear rate of 0.02/s

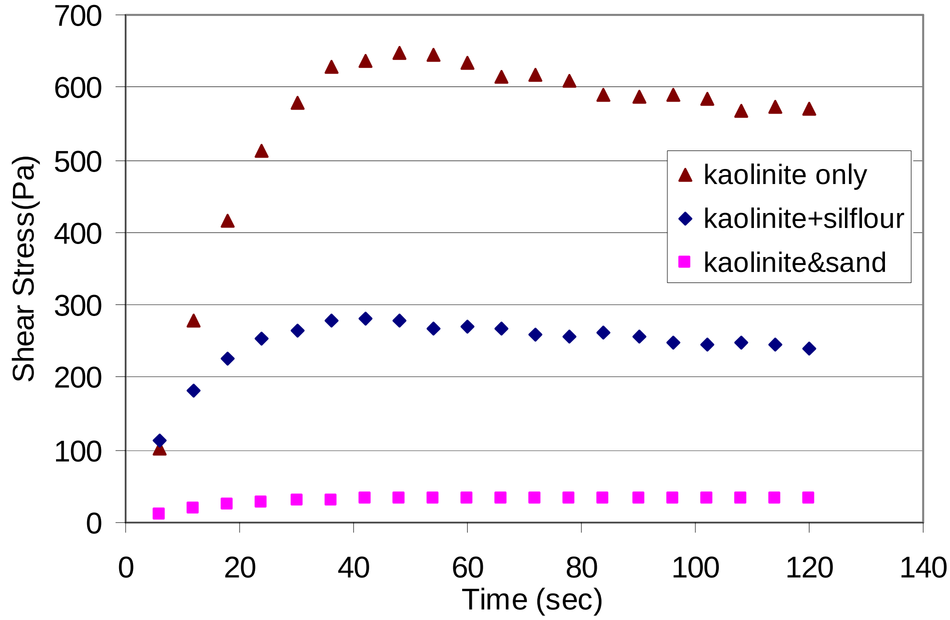


Figure 3-43 Shear Variation with time for different compositions at 60% solids @ 0.02/s

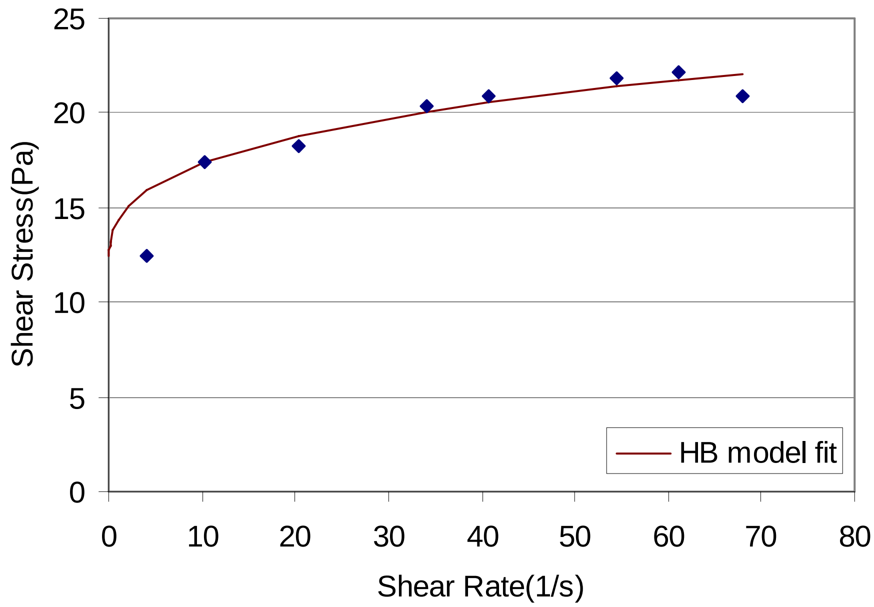


Figure 3-44 Flow curve for kaolinite slurry 40%s(w/w) and Herschel-Bulkley (HB)model fit with K_{HB}=2.855, n=0.3105 and τ_y=11.49 Pa

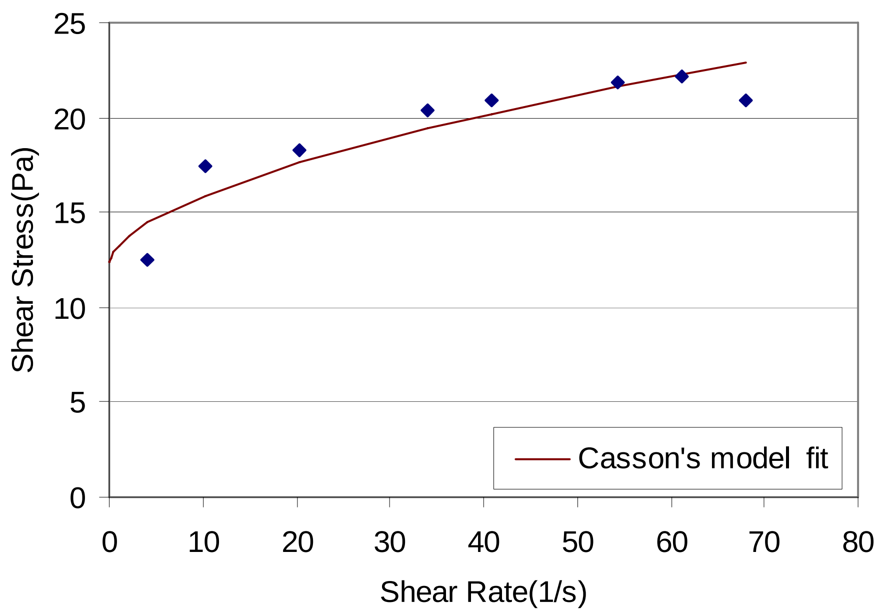


Figure 3-45 Flow curve for kaolinite slurry 40%s(w/w) and Casson's model fit with Kc=0.0248, and τ_c=12.1466 Pa

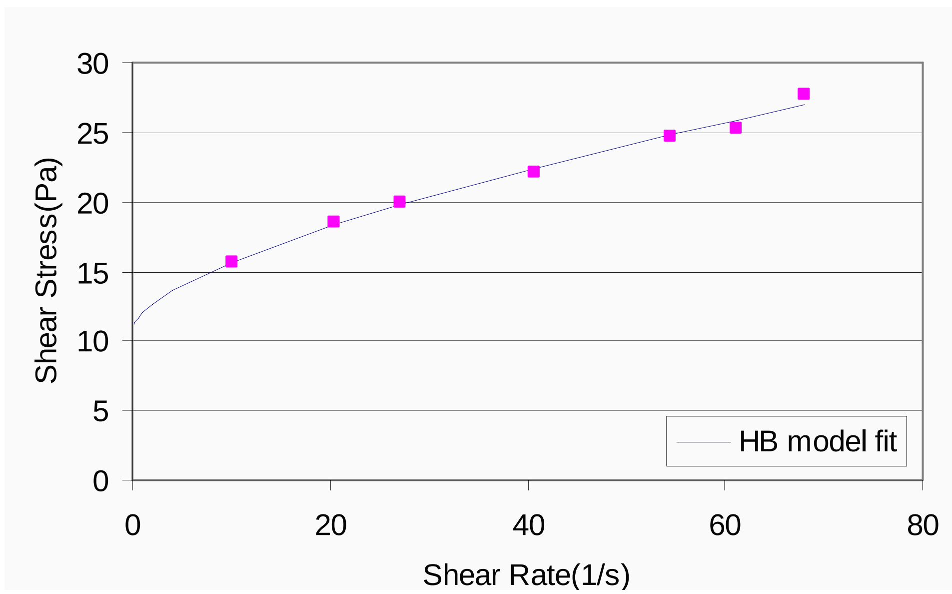


Figure 3-46 Flow curve for kaolinite Sil flour(1:1) slurry 50%s(w/w) and Herschel-Bulkley (HB)model fit with K_{HB}=1.056, n=0.6429 and τ_y=11.0 Pa

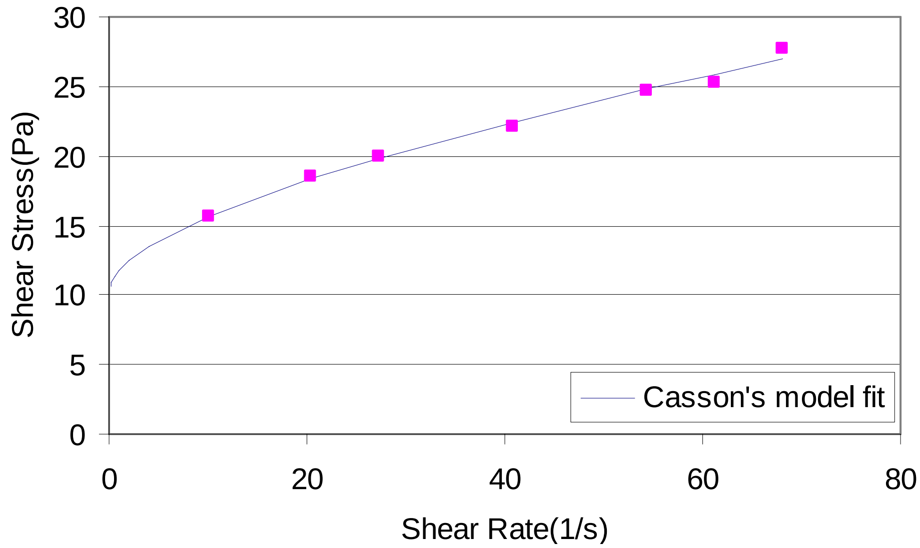


Figure 3-47 Flow curve for kaolinite Sil-flour(1:1) slurry 50%s(w/w) and Casson’s model fit with K_c=0.0593, and τ_c=10.131 Pa

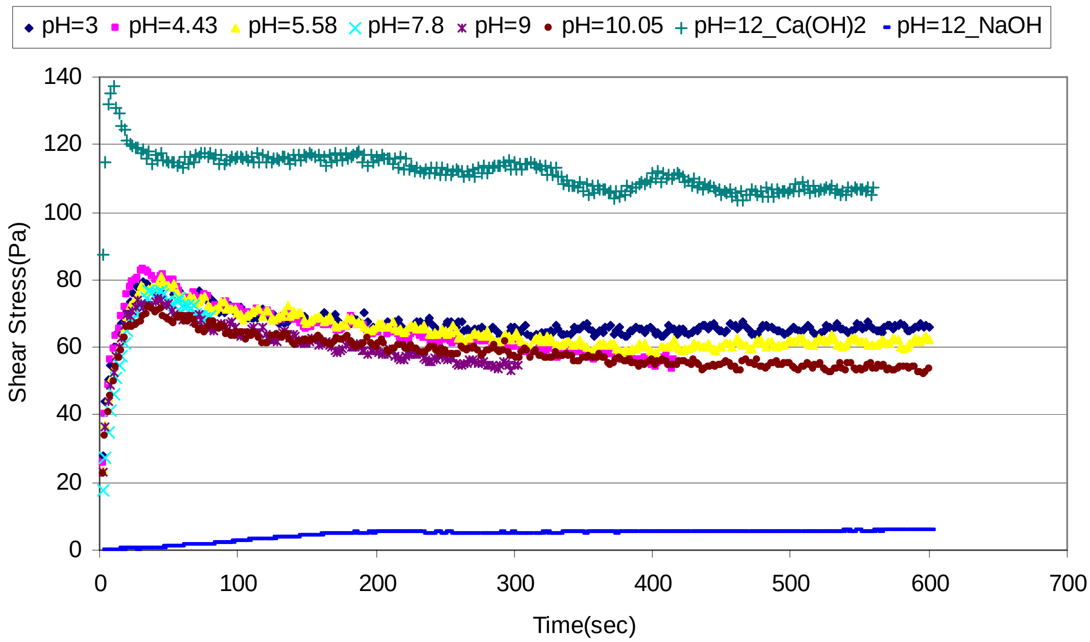


Figure 3-48 Variation shear stress at different pH values for kaolinite slurry @50% s

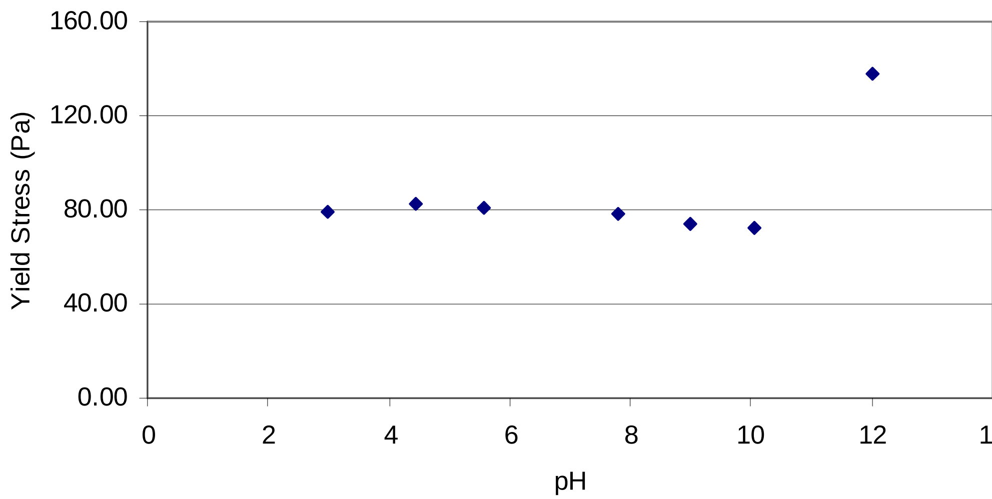


Figure 3-49 The variation of yield stress with pH for kaolinite slurry 50% s

CHAPTER 4[2]

4. EXPERIMENTAL INVESTIGATION OF SEGREGATION IN CLAY-SAND SLURRY

4.1. Introduction

Segregation, as a ubiquitous phenomenon, occurs in many natural and man-made processes: the sorted layers of magma fall out, stone rings in permafrost, rock avalanches, and seasonal stream deposits are few among many natural events. In industrial processes, where bulk materials are handled, transported, and stored segregation usually results in poor quality of product as in pharmaceutical, ceramic, cement and food processing. On the other hand ore extraction process exploits the advantage of differential settling. In oil sands extraction process, for example, the bitumen is separated as a froth which floats to the surface; the coarse sand settles to the bottom and is removed. A portion of this slurry, known as middling, is removed from the central portion of the vessel, and is further processed.

Segregation, in the context of its use here, may be described as a phenomenon by which certain sizes or components with similar properties tend to preferentially collect in one or another physical zone of a collective (de Silva, 1999). It may occur due to differences in size, density ,shape, surface roughness, elastic modulus, voidage and many others, among which size difference is the most dominant Williams (1976), Rhodes (1998), Vallance and Savage (1999).

The segregation process, primarily, occurs under sub-aerial and sub-aqueous environment. The granular flow refers closely to the former and sedimentation process to the latter. The granular flow, where the bulk volume of the particulate materials are involved, is by itself a wide and complex subject, and it is not in the intent of this work to

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[2] A version of this chapter has been published. Mihiretu, Y.M., Chalaturnyk, R.J., Scott, J.D. Proc. of 59th Canadian Geotechnical Conference, Vancouver, British Columbia, 2006.

cover it. However, some theoretical and experimental treatments may be found in the works of Savage and his co-workers; Savage and Lun (1988), Savage (1993), Vallance and Savage (1999), de Silva et al.(1999), Williams (1976) and others.

The study of sedimentation is important, because it is either desired (bitumen extraction, filtration, thickening, consolidation, tailings management) or unwanted (industrial applications: pharmaceutical, ceramic, paints). Sedimentation process takes places when particles settle/segregate through a fluid medium mainly under the action of gravity. This process has been understood since the early times when human beings started to crush and wash mineral ores to extract the concentrate. Later the invention of thickeners and their wide use has in the meantime facilitated the scientific studies in the subject. A comprehensive historical survey of the development has been provided by Buerger and Wendland (2001).

The subject of sedimentation involving suspensions containing mixed particle sizes is not well developed, and reliable relationships have not been available. This is because of the wide range of particle sizes, the variety of particle shapes, and the complex nature of the hydrodynamic and physicochemical phenomena which govern particle-fluid and particle-particle behaviour (Selim et al., 1983). In their critical review on this subject, Boogerd et al. (2001) remarked that although much effort has been applied in past decades, no fundamental breakthrough has been achieved. Nevertheless, it is not uncommon among engineers/ researchers to deal with their complex problems in some pragmatic way based on experimental evidences, past experience and theoretical and empirical models. Mitigation or control of either natural or industrial processes involving segregation thus requires a fundamental understanding.

In a fresh formation of deposits in practice and in most laboratory experimentation, sedimentation and consolidation are concurrent processes which may be difficult to treat independently. However, occasionally the term sedimentation and consolidation are used with ambiguity. In the commonest use of geotechnical engineering, we refer to sedimentation as a tendency of the solid phase to settle to a deposition state in the direction of dominant acting force (mainly gravity, centrifuge) whereas consolidation refers to the settlement of the deposited material as a result of removal of liquid material (water) due to the dissipation of excess pore pressure and development of effective stress.

Study of segregation as it is dealt with in this work focuses on the fundamental aspects of the sedimentation/ consolidation process which involve wide size range. It will be worthwhile to recapitulate the research made in these aspects in different areas and then proceed with the objectives outlined for this work.

4.2. Literature & background

When solid phases interact with fluids, sedimentation/segregation occurs by the action of gravity due to size or density differences among particles. The classic study of sedimentation is attributed to G.G. Stokes in 1851. He derived an equation relating the terminal velocity of a sphere in fluid with the density difference between the solid and liquid, diameter of sphere and viscosity of the fluid. Though his equation was derived for a single sphere, it is applied to very dilute suspension, where the inter-particle interactions are assumed to be negligible and free settling of particles takes place. It is this basic idea that is still being used today as a means of grain size determination in the basic soil mechanics (Bowles 1986).

When the concentration of particles increases, the interaction among the particles also increase and a hindered settling mode is observed. After the invention of thickener in 1905, the application of hindered settling process received wide attention. However the operational systems were more of an art than a practice based on basic scientific understanding (Concha and Buerger 2002). It appears that the study of sedimentation with application to industrial processes has received greater treatment in chemistry or chemical engineering and mining engineering, probably driven by the demand for optimal design and operation in the filtration and separation industry. The works in these areas can be seen mostly as experimental study supplemented by semi-empirical models or simulation.

Coe and Clevenger (1916) identified different settling zones in batch settling of slimes containing wide range of sizes and they used the observed experimental data in the design of thickener. There existed no systematic theory until the time when Kynch (1952) introduced his new theory of sedimentation. The principle of continuity and the assumption that the velocity of propagation of concentration waves depends only on the local concentration constitute the main framework of his theoretical formulation. His theory is later revisited and revised by Tiller (1981) to account for the effect of rising sediment.

Richardson and Zaki (1954) summarized preceding studies which applied modified forms of Stokes' equation in the prediction of the settling rates of suspension of fine and uniform particles. They argued that the previous assumptions that the effective gravitational force acting on a particle in a suspension is determined by the density of the suspension and that the drag on the particle is a function of the apparent viscosity cannot be true for a suspension of uniform particles. They stated that in the suspension of uniform particle, each particle settles at the same rate and displaces its own volume of liquid and they conclude that for a suspension of uniform particles, the use of an effective viscosity of the suspension is not valid. They provide the widely cited equation which relates the settling velocity of uniform suspension to the terminal settling velocity of a single particle ('relative velocity', a term used by Happel (1958)-'velocity ratio' seems more appropriate- and the porosity raised to a constant. The constant is a slope obtained from the log-log plots of settling velocity versus the porosity.

Happel (1958) developed a mathematical free surface model, in which the relative velocity is dependent upon some function of the diameter ratio of the spherical bulb encasing the particle to the diameter of the particle. He showed that the model fitted experimental data in the dilute and concentrated suspension ranges.

Barnea and Mizrahi (1973) produced a comprehensive survey of published works and introduced an empirical equation for the relative velocity also as a function of particle volume fraction. The relative velocity matches a unit value when the concentration reduced to infinite dilution, which is the theoretical value. Most important assumptions in their study were: no flocculation and aggregation were considered, the relative positions of the particles in the clouds are completely random, with segregation and wall effects being ignored.

Garside and Al-Dibouni (1977) also analyzed the experimental results of their predecessors and conducted experiments. They noticed significant curvature on a log-log plot of settling velocity versus porosity at high voidage (volume fraction of non-solids) values and in the turbulent regions. They provide a logistic curve fit for the Richardson and Zaki's constant as a function of the Reynolds number.

Studies have been extended to systems of the bi-disperse and poly-disperse suspension, which are practically the situations encountered in many industrial operations. In these types of suspensions, the significance of segregation becomes apparent. Lockett and Al-Habboby (1973) conducted experiments on bi-disperse system. Their hypothesis was that in the settling of the bi-disperse system, containing particles with the same density but different in size, different zones would appear: from top to bottom, clear liquid zone, zone containing only smaller particles, and a zone containing both large and small particles. They extended the Richardson-Zaki equation to the bi-disperse case, and found a good fit to their experimental data; however, significant over-prediction is visible when they compared the same model with previously published experimental data. Mirza and Richardson (1979) extended this principle to poly-disperse cases.

Masliyah (1979), developed a model by applying steady state momentum balance to poly-disperse suspensions, as it was derived for mono-disperse suspension in Wallis (1969). The model applies for both size and density differences of particles in the suspension. The density difference for a particle size under consideration is its density minus the density of a mixture of the pure fluid and smaller particles present locally. Selim et al. (1983) also developed a model for poly-disperse suspension based on the Garside and Al-Dibouni (1977) approach. They tested their model with bi-disperse suspension and found good agreement. However their model was criticized for some deviations in limiting cases (Doheim et al. 1997) and an error in the formulation (Stamatakis and Tien 1988). Stamatakis also examined some of the cited models in literature. They commented that the models have little difference and it is the ease of computation that mandates the merits of the correlations. Other experimental and model studies in bidisperse and polydisperse suspensions, also available in literature, are due to (Zimmels 1983) (Patwardhan and Tien 1985); (Law, 1987); (Zimmels, 1988); (Stamatakis and Tien 1988); (Smith 1997); (Buerger et al. 2000).

Batchelor (1982), from a colloidal science perspective, developed a theory of sedimentation based on two-sphere hydrodynamics accounting for effects of relative movement of particles due to gravity, the inter-particle force and Brownian diffusion. He investigated the effect of interaction and inter-particle forces between pairs of particle and obtained formula for the average speed of particles (which are correct to the first power of the volume fraction of the particles). His equation introduces dimensionless sedimentation coefficient which is a function of size ratio and reduced density ratio. The theory has been also interpreted/ analyzed numerically (Batchelor and Wen 1982).

A comprehensive review on the development of the non-colloidal sedimentation at low Reynolds number is presented by Davis and Acrivos (1985). They also give attention to a case of enhanced sedimentation in inclined channels which is a rare case appearing in literature. They conclude in their review by recommending further research to reach a relation of the macroscopic properties of a sedimenting polydisperse suspension to the local microphysical interaction between the different particles. Davis and Gecol (1994) proposed a hindered settling function which looks like a hybrid/coupling between Richardson-Zaki model and Batchelor's model. By putting the sedimentation coefficient of Batchelor's theory in to the exponent of Richardson and Zaki's exponential term, they managed to avoid the empirical parameter involved. They compared their model with dilute bi-disperse experimental results and found good agreement. They also suggest that their model be tested for complex cases involving broader range of particle size and densities. The above survey indicates that the emphasis is mainly on the sedimentation of non-colloidal particles in a liquid.

Shen et al. (1994) clearly indicated that the most readily understood situation is of particles without an appreciable affinity for one another. This situation is not always realized, however. The flocculation of particles into aggregates or gel-like networks complicates the processing of a variety of materials including foods, soils, ceramic powders, and chemical/biological waste product.

Coe and Clevenger (1916) noticed that, in metallurgical practice, slime pulp consists of water, finely divided sand or granular particles, and colloidal material. In their different settling zones, they generally took that the granular material falls directly to the bottom immediately, and eliminated from consideration, and implicitly, not contribute to the mechanism. They identified different settling zone in batch settling of slimes containing wide range of sizes and used the observed experimental data in the design of thickener.

For flocculated suspensions, Michaels and Bolger (1962) correlated experimental data by means of a modified form of the Richardson-Zaki equation. They suggested a structural model for the settling of fine kaolin particles in which primary particles are grouped into small clusters (called flocs). They assumed that each floc was spherical in shape, and that the water within it moved with it and there was flow of liquid around the flocs, but no flow through them. While it is clear that these assumptions cannot exactly represent the actual situation, they lead nevertheless to an equation which is useful for data correlation, and which indicates, at least qualitatively, how the flocculation affects the settling process. Michaels and Bolger (1962) analyses also show why critical concentration is much lower for flocculated than for non-flocculated suspensions. If the flocs are truly spherical and closely-sized, the critical concentration would be expected to correspond to a floc volume fraction in the order of 0.64 (random-packed equal spheres).

Landman and White (1992, 1994) discussed the phenomenological approach to the study of flocculated suspensions. They highlighted the importance of rheology and stressed the importance of three rheological parameters: compressive yield stress, the hindered settling factor and the dynamic compressibility. They have shown the application of phenomenological compressional rheology of flocculated suspensions to gravity thickening and pressure filtration. Green et al. (1996) presented an experimental procedure of determining the compressive yield stress.

Phillips (1997) reviewed briefly the sedimentation behaviour of colloidal particles. He based his discussion on Batchelor's sedimentation theory. He discussed that attraction between colloidal particles reduces the concentration dependence of velocity. He noticed also that the concentration dependence of the sedimentation dynamics in the case of non-spherical particles has hardly been investigated, and according to him, is a major challenge in colloidal sedimentation.

Li and Mehta (1998) assessed the hindered settling formulation to fluid mud-like suspension and derived an analytical equation from the continuity equation by applying Darcy's law to the seepage velocity of upward moving displaced water. They estimated the permeability from the Hagen-Poiseulle viscous flow equation.

In the geotechnical field, the study of sedimentation seems to have got relatively little attention. Traditionally soil mechanics is related to the post-sedimentation stage, i.e. after the formation of soils. Primarily, since the early works of Karl Terzaghi, the treatment of soil-water interaction is mainly considered as the water is moving through the pore space of stationary soil grains, namely consolidation or/and seepage. Imai (1980) wrote that our attention has been hitherto concentrated on the soil behaviour after soil formation, and the behaviour before the soil formation has been overlooked in geotechnical engineering, although its knowledge may be invaluable to practical designs of land-filling work, disposal work by sedimentation and so on. The processes which govern the sedimentation process are either implicitly related to the Stokes' law or simply overlooked (McRoberts and Nixon 1976). However, as the geotechnical engineers encounter more complex phenomena, it has necessitated revisiting the basic process to understand the involved mechanisms.

The well-established consolidation theory in geotechnical engineering has been basically developed for solving settlement of foundations on clays (Terzaghi 1943). In essence, the theory is subjected to many simplifying assumptions such as constant permeability, constant compressibility, and validity of Darcy's law. These assumptions are only satisfied approximately in practice (Gibson et al. 1967). In a latter study, Gibson et al. (1967) developed a finite non-linear consolidation equation of saturated clays which accounts for the changes in soil compressibility and permeability.

It is through the early work of McRoberts and Nixon (1976) that the theory of sedimentation was brought to the attention of geotechnical engineers. They introduced, in extended form, the Kynch theory of sedimentation to settling of soil particles. They related the development of effective stress and then the consolidation process with formation of soil and at the same time questioned which concentration level of dispersion is referred to as 'soil' in geotechnical sense. They argue that the sedimentation theory governs the behaviour of soil grains-water mixtures at concentration ranges immediately less than the concentration range appropriate for consolidation. Pane and Schiffman (1997) have estimated a void ratio in the range of 30 to 40 for the formation of soil from their experimentation in Kaolin sedimentation.

Been and Sills (1981) completed laboratory tests on sedimentation of soft soils (kaolin and clayey silt mud). They used a non-destructive method (X-ray) to obtain the density profile of a settling mud. They assessed segregation during settling. They observed that significant proportion of the silt particles settled through the mud to the base. The segregation is very visible when the mud was dispersed with sodium hexametaphosphate, showing that flocculation plays an important role in segregation. In their theoretical explanation to their experimental results, they introduced imaginary overburden to the consolidation equation. However, this approach was challenged by Tan et al (1990), due to the situation that the process is hindered settling rather than a consolidation process. Buerger et al. (2000) were able to capture the shape of the concentration profile by use of the phenomenological sedimentation theory, though involving rough estimates. On the other hand, their measurements show the development of excess pore pressure, which is a sufficient condition to assume the mud as a soil and then consequently apply the consolidation theory.

Imai (1980, 1981) has completed experimental studies on the behaviour of soft soil. He examined the settling characteristics as affected by clay mineralogy, concentration and salt content of water. He identified the major settling stages in three zones: zone settling, flocculated free settling and consolidated settlings. Tan et al. (1990) presented the effect of grain size distribution, pH and flocculent on the settling characteristics of clay slurry. They observed non-Newtonian behaviour from rheological study conducted on the sediment. By spreading sand on the surface of the sediment, they demonstrated that capture of the sand due to gain in strength of sediment. In another study, Tan et al. (1990) modified the Kynch theory to flocculated clay sedimentation by use of momentum equation and empirical formulation for permeability.

The traditional consolidation theory deals with situations where the initial conditions are well known. The question [to geotechnical engineers] would be what if the initial conditions are by themselves undergoing a process, such as is the case in freshly deposited sediments. Then necessity of sedimentation (hindered settling) with the consolidation process becomes evident. Studies which combine sedimentation and consolidation process appeared in the 1980's. Pane and Schiffman (1985) proposed a simplified constitutive formulation for the total stress by introducing interaction coefficient to the effective stress. Incorporating the constitutive equation into Gibson et al.'s (1967) finite strain consolidation theory, they form an equation which can model simultaneous sedimentation and consolidation of saturated soils.

In a separate study in the fluid mechanics area, Auzerais et al. (1988) developed a phenomenological theory of the sedimentation and consolidation theory of concentrated suspensions using the equation of motion for the fluid-particle system, including all the forces active in both the settling and sediment phase. Their analysis predicts concentration profiles of the whole process (sedimentation/consolidation) for both stable and flocculated suspensions. Eckert et al. (1996) have shown that the governing equation for consolidation as developed by Gibson et al. (1967) in soil mechanics and the fluid dynamic equation of Auzerais et al. (1988) are the same equations formulated from a different frame of reference.

Similarily, Toorman (1996,1999), Diplas and Papanicolaou (1997), Karl and Wells (1999) and Buerger (2000) have developed phenomenological models to approach the sedimentation/ consolidation problem. Stamatakis and Tien (1992) also presented an approximate approach of sedimentation/ consolidation dealing with polydispersity. Fitch (1975) offered an opinion that although some success was achieved in the development of sedimentation/consolidation equations, that the literature of this era should be understood as primarily elaboration of Kynch theory, rather than an explanation of reality.

The wide grain size distribution influencing the size segregation is long understood in practice. These effects are counteracted by the increase of particles' concentration (Barnea and Mizrahi 1973). The brief review above show that a great deal of effort has been put into the sedimentation /consolidation study. The effect of grain size was considered somehow by account of poly-dispersity. Falk and Umberto (2002), in their review, noted that previous researchers considered size ratio of about 2 to 3. In their experimental work they considered a wider grain size distribution ranging in four decades. Shih et al. (1987), for example, mentioned a typical size distribution of particles in coal liquids to range from 2 through 20 micron. They formulated a general multiphase hydrodynamic model for multi-sized particles. They attempted to include, fluid-particle interaction, solid compressive stress, particle to particle interaction and other forces. They were somehow able to capture the concentration profiles with some deviations. In related studies, Stamatakis and Tien (1988) and Al-Naafa and Selim (1992) reported that the effect of fine particles presence in the suspension, are accounted for by considering equivalent viscosity of suspension of slow settling particles and fluid medium.

Colloidal systems display complex rheological behaviour related to their thermodynamic non-ideality. These systems depart from Newtonian behaviour, often exhibiting yield stress, before deforming continuously as a liquid. Flocculated suspensions respond elastically to small deformations with moduli that depend strongly on the volume fraction of particles (Russel et al. 1989). It is also apparent that the studies in colloidal sedimentation mainly emphasize on the compressive yield stress in conformance to the hindered settling/ consolidation approach (Bergsroem et al. (1992), Landman et al. (1988), Landman and White (1994), Shen et al. (1994) and Chu et al. (2002).

The important observation from the above review of the literature indicates that the rheological properties of the flocculated slurry are considered in the sediment zone only. A remaining question is how the rheological characteristics of a slurry affect the settling zone. In a different study in non-Newtonian fluid mechanics there is a clear account of different rheological approach in that the significant presence of fine particles contribute to the rheological characteristics of the whole suspension system Valentik and Whitmore (1965), Ansley and Smith (1967), Chhabra et al. (1992), Saha et al. (1992), and Coussot (1997).

The properties of mine tailings, particularly, oil-sands tailings in Northern Alberta, Canada, present complex practical challenges due to the fact that there exists segregating disposal behavior (Kuepper 1991), a large volume of waste materials (Fine Tailings Fundamentals Consortium 1995), and very slow sedimentation /consolidation (Eckert et al. 1996). At the same time safe disposal being a major environmental requirement, handling, storing and monitoring have posed major challenges to the geotechnical engineers. Consequently, it has become necessary to understand the major physical process involved in the tailing disposal schemes.

Despite the fact that important developments have been seen in the study of sedimentation/ segregation in particulate form or in colloidal form separately, there exists very little information in the literature on the results of studies on the constitutive influence of a slurries composition (i.e. fine and coarse particles) on the sedimentation/segregation behaviour. Moreover, information on the interaction or progress with time of the segregation process involving slurries with a wide particle size distribution is hard to find in the literature.

The theme of this work is thus to examine the segregation phenomena experimentally and theoretically with emphasis on identifying the effect of grain size composition commonly classified in geotechnical engineering as clay, silt and sand. It will also be attempted to investigate the concentration level at which the rheological properties of fines play a significant role.

4.3. Materials and Methods

The materials used in the experimental program are mainly kaolinite clay, silica sand (180-250 micron size) and sil-flour which was used as a silt size material. The particle size distribution for each material is shown in Figure 4-1. The liquid and plastic limits of kaolinite are 55.2% and 29.5% respectively. The silt size, sil-flour exhibits no plasticity. The ambient temperature during experimentation is 20 ± 2^{o} C.

The material is weighed on to a scale (0.01g accuracy) in 5-litre plastic pail. The water, which is added to achieve a desired solid content, is Edmonton City tap water, the pH of which is 7.7 ± 0.2. Two types of standpipes are used in this experiment: a modified type 5-litre standpipe or 2-litre standpipes. The choice is made in such a way that slurries that are expected to exercise relatively high settling rates are tested with the modified standpipe, whereas those slurries expected to have a relatively slow settling rate are tested with 2-litre standpipes.

4.3.1 Description of the modified standpipe test

The modified stand pipe test designed for this study is shown schematically and in picture in Figure 4-2. The material used to manufacture the modified standpipe is transparent Plexiglas, with a thickness of 10 mm. The standpipe was designed to enable the development of segregation with time to be studied and to provide sufficient sample volume of each "element" for concentration profiling. Wall effects have been minimized by specifying relatively large dimensions for the width and depth of the cell.

4.3.2 Test procedure

After initial mixing of all the ingredients of the slurry, the sample is thoroughly mixed for about 15 to 20 min in a pail. The mixed slurry is transferred into the modified standpipe and stirred again in the standpipe for about 5 to 10 minutes to insure uniform distribution of the slurry. Then the sliding plates are pushed into the standpipe at a chosen time. The driving of the sliding plates takes about 10 sec to completely form different sections of the height for sampling. Then each section from top down is pumped out of the standpipe and transferred into aluminum tray. Whenever pumping becomes difficult due to high solid content, samples were scooped with large spoons fitted on extension rods. After wet mass measurement, the material is put into the oven overnight. The following day dry mass is measured. After that the material is soaked in water in order that the fines can be easily washed out. The fine washing is carried out through the 44micron sieve. After ensuring that all fines are washed out, by visual observation that clear water come through the bottom of the sieve, the retained sand is transferred into the tray and put in the oven overnight. The next day, dry mass measurement is conducted. Material loss is encountered occasionally due to splashing during stirring or leak while driving the slicing gates. Finally the mass balance before and after the test is checked. If the loss of material is greater than 5 %, the test is repeated.

The test programs as well as interpretation of the results will be described using the ternary diagram (or soil structure behaviour diagram) as shown in Figure 4-3. The definitions for the variables used in the soil structure behavior diagram (SSBD) are the following:

Solid content, s = (weight of dry sample–weight of tray)/(weight of wet sample-weight of tray)*100

Sand content, SC	=	(weight of dry sample after washing- weight of tray)/ (weight of wet sample-weight of tray) *100
Solid content	=	weight of solid in a slurry/ (weight of wet slurry)
Sand content (%)	=	(sand/(sand + fine))*100 (weight basis)
Fine content:	=	fine /(sand +fine) = 100-SC
Sand Fine Ratio (SFR)		= weight of Sand/ weight of Fine (weight basis)
Volumetric solid content		=Volume of Solid/ total volume of slurry.

4.3.3 Results and Discussion

The compositions for all tests are identified in Figure 4-3 as small red points. For modified standpipe experimentation the design of testing is made according to SFR 1, 2, 4 and FWR at 10, 15, 20, 30 and 40%.

(i) Test of Sand Fine Ratio, SFR=1

Figure 4-4 shows the test results for the temporal behavior of slurry with a target initial solid content of 18.8%s. Figure 4-4 illustrates the change in solids content and sand content profiles at 15 second intervals from an elapsed time of 15 seconds to 90 seconds. The solid content profile shows that there exist two distinct regions of concentration, mainly a high concentration region at the bottom and an upper low concentration region. The corresponding sand content profile reveals that the sand has settled almost immediately to the bottom. Though not easily visible in the solid content profile, the sand content shows a decreasing trend in the upper segments. Also visible in the sand profile is that the change in sand content takes place in the intermediate section and the top section remains nearly the same.

With slight increase in the fine content to 15% FWR, Figure 4-5 shows the concentration variation clearly both in terms of solid content and sand content. The time progress of sedimentation shows the increase in concentration at the bottom and drop in concentration at the upper section. Also observed here is that the trend of the time progress of sedimentation showing the concentration profile with respect to the total solid and sand show similar trend.

Figure 4-6 shows the results at FWR of 20 and solid content of 33.3%. The profile at this test condition shows an overlapping tendency for both solid content and sand content profiles for the times of test. Unlike the previous testing, there is no sharp increase at the bottom in this test. Also the gradient of the profiles seem to be smooth throughout the depth. The solid concentration profile for the 30sec test seem to be deviating towards a less concentration level, this could be attributed to experimental errors.

As the initial solid content as well as FWR increases to 46.2% and 30% respectively, the solid content and sand content profiles show nearly vertical profile as shown in Figure 4-7. Despite the extended time of testing to 180sec, the sedimentation process during this time does not produce any visible segregating tendency.

When the FWR is raised to 40% and the solid content to 57.1%, the slurry is very dense and longer testing time up to 24hrs is allowed. As shown in Figure 4-8, the solid content profiles and sand content profiles are nearly vertical in all cases. A small deviation of solid content profile at 18hrs reading has been attributed to experimental errors.

(ii) Test of Sand Fine Ratio, SFR = 2

As the sand proportion is increased (i.e. SFR increased from 1 to 2), evolution of the sedimentation/ segregation process, as illustrated in Figure 4-9, remains visible for the first 30 seconds but thereafter, the solid concentration profile remains essentially unchanged. In contrast, the evolution in the sand content profile continues to show changes well past 60 seconds, indicating that sand segregation is occurring with relative negligible changes in solids content. For the case with an SFR = 1 (Figure 4-4), the evolution of the sand content profile was not as apparent as for SFR = 2. This is likely just due to the higher sand concentration at an SFR = 2 allowing this behavior to be captured in the modified standpipe.

When the FWR is raised to 15% (Figure 4-10), the profiles at different times get closer and become a gentle slope, both with respect to solid content and sand content. A similar trend is also to see for FWR of 20 (Figure 4-11). Nearly vertical profile appears when the FWR is raised to 30% (Figure 4-12). The sand content shows very little deviation vertical-wise for the test duration of 240sec (4min).

(iii) Test of Sand Fine Ratio, SFR=4

For the three tests conducted at SFR=4, the effect of sand content is clearly magnified at the low solid content as shown in Figure 4-13. The increase of FWR by small amount produced gentle slope in both solid and sand content profiles. This result indicate that the effect of granular packing contribution to the interaction and consequent reduction in segregation trend (Figure 4-14).

Due to high solid content, longer testing interval was allowed to the test condition shown in Figure 4-15. There appears no segregation within the first 4hrs of testing time. For longer testing times, 12hrs and 24 hrs, the departure from verticality is noticeable.

However, this is due to settlement of the whole suspension and clear water formation at the top section, and increase in solid content. While the solid concentration profile shows significant increase in solid content of the deposit after 12 hrs, only slight change is visible at the bottom section with respect to the sand content. This maybe an indication that the sand is captured in the fine matrix and the sedimentation takes place en-masse.

4.4 Effect of fine matrix

At low solid concentration, there is immediate settling of the sand and at the same time there are some particles retained in the top sections. The question would be why do not all sand particles do not settle. If we assume the fine particles form part of the fluid medium, the equivalent viscosity of the suspension may be estimated using the Einstein's

(1906, 1911) equation, described as Equation (4.1) or that of Thomas (1965) described as Equation 4.2 which was widely used and tested to fit available data reasonably. For 10%w/w solid content the difference is about 2.0% [1.87 exact]. Consequently, Thomas's equation was used in this research.

$$\mu_{eff} = \mu(1 + 2.5\phi) \tag{4.1}$$

Thomas (1965) provided an extended empirical form of the equation as:

$$\mu_{eff} = \mu(1 + 2.5\phi + 10.05\phi^2 + 0.00273\exp(16.6\phi)) \tag{4.2}$$

where $\mu_{eff.}$ the suspension viscosity , μ is the viscosity of the medium (water in this case) and ϕ is the volume fraction of the solids in the suspension.

For laminar motion (Re<1), the transition region (1 < Re>1000) and the turbulent region (Re>1000) the settling velocity is defined as follows:

Laminar motion [$\mathrm{Re} = \dfrac{\rho d V}{\mu} < 1$],

$$u_o = \frac{g(\rho_s - \rho)d^2}{18\mu} \quad \text{(=>Stokes' law)} \qquad ; \tag{4.3}$$

Transition region [1<Re<1000],

$$u_o = 0.20\left[\frac{g(\rho_s - \rho)}{\rho}\right]^{0.72} \frac{d^{1.18}}{(\mu/\rho)^{0.45}} \qquad ; \tag{4.4}$$

and turbulent motion [Re> 800],

$$u_o = 1.74\left[g\frac{(\rho_s - \rho)}{\rho}\right]^{0.50} d^{0.5} \qquad . \tag{4.5}$$

Since cases of dilute suspensions are being examined, an assumption that sand particles settle individually may be acceptable and the settling velocity calculation for a particle may be used conveniently. The Reynolds number and velocity for sand particle settling through kaolinite+water suspension are calculated to be 6.34 and 3.13cm/s respectively, which show that the settling takes place beyond the creeping flow regime in the transition zone. The velocity is computed according to Allen (1900), as referenced in Govier and Aziz, (1972, pp 6 Equations 4.3-4.5). With a velocity of 3.13cm/s the approximate depth a sand particle could have travelled in 90 sec would theoretically be about 280cm. Thus all the sand particles should have been in the 3rd segment from the top (each segment is 10cm high). However the experimental results show that there are some sand concentrations even in the upper segment after 90sec. The capture of sand particles in the top section may imply that there exists interplay of other interaction mechanisms taking place in the suspension different from well known hydrodynamic interactions.

For FWR of 30 and above, the fines matrix shows a rheological property which is different from Newtonian fluid. A typical flow diagram is shown in Figure 4-16. The rheological data experimental data show that the fine slurry exhibits a yield stress value of about 2.8 Pa. This yield stress must be exceeded before any permanent deformation due to settling takes place. And the equilibrium condition of yield stress results in a threshold particle size which can be captured by yield stress. As shown by Dedegil (1986), the diameter of the particle is given by Equation (4.6). The maximum size used in the experiment is nearly 250micron (0.25mm), and the computed size according to Equation (4.6) is about 1.2mm. The nonsegregating tendency of the slurry above 30% FWR and with maximum sand size of 0.25mm can be attributable to the rheological response of the fine matrix, namely the yield stress, though the calculated size appears larger than experimentally expected size. This condition will be further examined in Section 4.7.

$$d = \frac{3\pi}{2}\frac{\tau_y}{g(\rho_s - \rho)} \qquad (4.6)$$

Where d is the diameter, τ_y is the yield stress, g is acceleration due to gravity, ρ_s is the density of sand particles and ρ is the density of fluid medium. It is to note that the measurement of yield stress is made with two different methods, one with vane shear apparatus and the other with conventional viscosity measurement with a rheogram plot extrapolating to zero shear, gave similar yield stress value.

4.5 Effect of silt size presence

In soil mechanics, clay and silt sizes are generally grouped as fine soils. Such classification was examined in rheological characterization part in the previous chapter. The presence of silt size is observed to decrease the yield strength of the fine matrix. As yield stress is more attributed to the ultra fine clay particles, inclusion of larger sizes tend to minimize the bonding of clay particles and consequently reduce the yield strength. The different composition of clay and silt at initial total solid content of 33.3% is shown in Figure 4-17. The clay and silt size particle composition is varied at 100% clay, 80% clay & 20% silt size, 65% clay & 35% silt size and 50% clay & 50% silt size. The figure clearly shows that despite similar solid content in all tests, the sedimentation results in a segregating concentration profile both in solid content and sand profiles. This kind of phenomena is indirectly observed in the rheological characterization of slurries. Thus it can be ascribed that the sedimentation process to be in direct relation to the rheological property of the suspension (slurry). This is a clear demonstration that the fine clay size presence contribute to the settling properties quite significantly depending upon the concentration level. At dilute concentration ranges (about FWR < 20) the effect is not visible. As the fine concentration gets larger, the dominant effect of clay particle presence comes into play.

The results discussed above clearly indicate the contribution of fines (clay) in the sedimentation process. It is worthwhile then to investigate further the fine matrix effect on the sedimentation process with significant presence of fines and also allowing longer testing time. Thus some more tests were conducted on a 2-litre standpipe. Details of the results will be presented as follows.

4.6. 2-Litre Standpipe Test

To examine further the effect of fine on the sedimentation process, 2 litre standpipe tests were conducted. At low FWR there is clear indication that sand particles easily settle through the suspension leading to two distinct regions: sand with little clay particle presence at the bottom and clay particles at the top. This is clearly visible at FWR less than 20% (Figure 4-18). However, as the FWR is increased the sediment profile seems to be controlled by the fine particle presence. This is seen in Figures 4-19 and 4-20 where the profile at same FWR but different solid content shows similar pattern in profile indicating the FWR after certain limits (i.e. FWR value of 30) controls the sedimentation process. The sand particles are captured throughout the fine matrix.

A remarkable observation for slurries beyond a FWR of 30% is that the solid content profile at different SFR but same FWR exhibit similar shape in profile. It is also observed that the distance between the curves is in the proportion of the initial solid content. If the different curves are normalized to the initial solid content they all fall into a close band at a single location. Figures 4-21 and 4-22 show the normalized plots for FWR of 30 and 45 respectively.

4.7 Fine capture and yield stress

The theoretical size, as obtained by Equation 4.6, seems to give larger sizes than those observed experimentally. Thus detailed experimentation was carried to check the validity of the theoretical formulation in the testing programs. The sand sizes are divided into different size range in order to minimize averaging over a wide size range.

Also in order to analyze the segregating profile of the sedimentation experiment, a statistical approach, i.e., standard deviation which is referred in here as Segregation Index (SI) is used. Details of its formulation are found in Chapter 7.

An SI with respect to solid and SI with respect to sand can be computed. For this test series, it is assumed that an SI of 5 with respect to solids and an SI of 2.5 with respect to sand in solids can provide a reasonable indicator of non-segregation.

Detailed experimental observations on four size ranges of sand indicate that the size captured is actually less than the one calculated from yield stress Equation (4.6). Thus a correction factor is applied as indicated in Equation (4.7). It can be seen from the summary of results provided in Table 4-1; the correction factor is always less than one indicating that the theoretical value over-predicts the size that can be held by yield stress. The difference gets larger as the size increases.

$$d_{measured} = K * d_{theoretical}. \quad (4.7)$$

Table 4-1 Fine capture at different grain size range of sand particles

Size range (μm)	**SI (total solid)**	**SI (sand in solids)**	**FWR**	**K**
150-180	5.0	0.75	30	0.24
250-300	3.7	1.23	32	0.25
600-850	3.4	0.42	38	0.16
1180-2000	1.8	0.44	42	0.15

The correction factor K as in Equation (4.7) is higher at lower FWR and vise versa. It is clear that the capture size for fine slurry increases with increase in fines solid content. However the theoretical prediction is found to over-predict the size of sand that can be captured. This is shown by the value of K found for the kaolinite slurry. On the average the K value is 0.2.

Such experimental results cast doubt on the purely theoretical formulation. In order to help us compare our result with those found in literature a yield parameter defined in Equation (4.8) was chosen to give better comparison.

$$Y = \frac{\tau_y}{gd(\rho_s - \rho)} \quad (4.8)$$

Where d is diameter, τ_y is the yield stress, g is acceleration due to gravity, ρ_s density of sand particles and ρ is the density of fluid medium. So the summary of data collected from different literature together with this work is put in Table 4-2.

Table 4-2 Comparison of Yield Parameter

Reference	**Yield Parameter, Y**
This work	1.06
Cardwell (1941)	0.17
Dedegil (1986) Julien (1995)	0.21
Coussot (1997)	0.05
Raudkivi (1998)	0.17
Chhabra and Richardson (1999)	0.2 (group 1) 0.06+0.02 (group 2)
He et al. (2001)	0.06

It appears that there is large difference among the yield parameters found in the literature. The smaller the yield parameter, the more likely that a particle settles. It is observed that it is only this experimental result that yields conservative results as compared to other references with the theoretical parameter as a reference value. Group1 and group 2 in Table 4-2 refer to different sources cited. This is a clear indication that there exists insufficient theoretical understanding of the mechanism of yield stress capture in non-Newtonian fluid.

According to Chhabra and Richardson (1999) the large discrepancy between the two groups of values they reported could possibly be explained by the fundamental difference in the underlying mechanism inherent in the respective approach, and the different methods used to measure yield stress. It is however more doubtful if the yield parameter value is far less than the theoretical one.

Based on major features of the physical process investigated in this test program, some preliminary classification of a slurry property can be made. Three main regions are shown in Figure 4-3. Region A is a dilute zone where the inertia of sand particles is very large and consequently settles to the bottom of the standpipe immediately. In this region conventional Newtonian particulate fluid mechanics may be applied. This is an area where most sedimentation studies fall. It is in this area that segregation is more prevalent. In region B the surface properties of fine particles, namely clays, come into play. In this region non-Newtonian behaviour is predominantly observed. The microscopic interaction can be explained in terms of a collective macroscopic behaviour in some rheological form. For the fine slurry we tested the rheological model which better suit for the fine slurry is Herschel-Bulkley model. Slurries in this region are apparently less studied.

The bottom region C is a solid and semi-solid region where more soil-like behaviour is exhibited. This region is a high solid content area and consequently has less likelihood of segregation. Long term effects, such as consolidation, creep or stability issues that are significant in this region.

In the experimental observations made in this study, for low solid contents and a wide size distribution, segregation is almost instantaneous occurring in the first minutes following mixing (deposition). As the solids content increases, the effect is counteracted by particle interactions. These phenomena have been also understood in practice and the most readily understood situation is the interaction of particles without an appreciable affinity for one another. This situation is not always realized.

In a study of oil sands tailings, Scott et al. (1985) explained that the sand grains remain suspended due largely to gel strength. They present rheological characteristics of the material they investigated. From their results, the material shows pseudo-plastic behaviour. The significance of understanding the sedimentation and strength characteristics of slurry has also been discussed by Tan et al. (1990). They observed that at the end of settling, the water content of slurry is still much higher that the liquid limit (to be called soil in soil mechanics sense) and the slurry are still fluid-like. In their study about the behaviour of clay, they noticed that fines slurry exhibit non-Newtonian behaviour with some threshold yield strength. They demonstrated the significance of yield stress by pouring sand on to the surface of sediments and observed retained sand particles. They also emphasized that yield strength, though small in conventional soil mechanics sense, is significant in soil treatment. Despite such an understanding, the mechanistic explanation of the phenomena was not available. It was Dedegil (1986) who presented the equation which explain the phenomena theoretically. However, our experimental results indicate that the theoretical formulation generally give larger size. A correction factor in the range of 0.15 to 0.25 needs to be applied for the sand particle size ranges in this experiment.

Russel et al. (1989) have indicated that flocculated suspensions respond elastically to small deformations with moduli that depend strongly on the volume fraction of particles. During sedimentation involving fine colloidal particles sizes, the concentration profile of a sediment changes with depth and time. During such process water is displaced and particles rearrange and settle into a more structured and stable form. At the same time, it is also apparent that the yield stress would change with depth and time. How the development of the network structure affects the yield stress of the fine matrix and also the interaction mechanism with the coarse particle present is far from clear, and merits further research.

An important practical case in the direction of this study will be the oil sands tailings, in northern Alberta, Canada. Since the beginning of commercial extraction of bitumen there exist a large volume of tailings material disposed into ponds that is filling rapidly and dewatering slowly. As conventional disposal schemes produce segregating tailings, the produced fine tailing in the pond are settling or consolidating at a very slow rate and may even take longer time even estimated in centuries (Eckert et al. 1996).

The Fine Tailings Fundamentals Consortium (1995) has reported on the unusual properties of fine tailings and concluded that it is predominantly associated with the gelation characteristics of an ultra-fine, colloidal(< 0.3 micron) solids’ fraction. And the presence of ultra-fines in oil sands fine tailings account for more than 90% of its water holding capacity. In the same compendium it was lucidly put that complete understanding of tailing behaviour requires knowledge of ultra-fines content, the in-situ environment in the parent ore, and process or pond water chemistry. Some efforts to create non-segregating tailings were focused on changing the floc structure of the fine tailings such as addition of divalent ions (flocculants), freeze-thaw and other treatment process.

It is apparent that despite some understanding about some rheological characterization, the application to the fine tailings sedimentation/consolidation behaviour should be a subject of further study. Experimental observations made in this study with some testing on actual tailings materials indicate that such phenomena may be significant. It is believed that proper treatment of rheological properties could give better understanding of the existing challenges in tailings. One of the possible solutions to the existing fine tailings problems is, for example, the in-situ treatment of the fine tailings or mature fine tailings (MFT). In order to improve the consolidation process, internal surcharge by sand grains could be applied if the rheological properties of fine tailings are adequately recognized.

4.7.1 Non Newtonian modelling Issues

Experimental study of sedimentation of a single particle in non-Newtonian fluid was initially made by Valentik and Whitmore (1965) Further developments by Ansley and Smith (1967), Dedegil (1986), and Machac et al. (1995) were significant contributions in the field. An experimental investigation of hindered settling in power-law liquids was conducted by Chhabra et al. (1992) and they extended Newtonian models to interpret their experimental data. Although the subject has been brought to attention fundamentally for almost four decades, very little information is available in the literature. This is likely attributable to insufficient experimental data in the area and/or to the lack of adequate research tools.

In the chemical engineering literature, consolidation of flocculated suspensions has received a great deal of attention. Field studied and model developments have also been shown to match experimental results Auzerais et al. (1988), Buscall and White (1987), Landman and White (1994), Eckert et al. (1996), and Green et al. (1996)). There is a tendency to believe that a linear relationship exists between the compressive yield stress and the rheological yield stress. Floc formation and restructuring mechanism and the process taking place remains not well understood. The presence of a yield stress, a subject of debate though, at the early stage of sedimentation before the onset of consolidation seems to be unaccounted for. Also the presence of coarse, surface free grains adds to the complication the sedimentation mechanism. At current stage, thus, a sedimentation/ consolidation model comprising wide grain size range and incorporating surface properties of fine particles is seemingly unavailable. Also experimental data on the consolidation of fine and coarse mixtures at different composition are not available in the surveyed geotechnical references.

4.8 Summary and Conclusion

In this study of segregation experimental results show the early stages of segregation process. It is observed that at low solid content and low FWR the sand settles to the bed immediately. However, there exist some sand particles within the slurry after some time has elapsed which can not be explained by available hydrodynamic analysis.

The sedimentation tests on different grain-size composition are exhibiting some kind of direct response analogy to the yield stress rheological characterization observation made in accompanying test program.

It is also observed that in all test ranges conducted, there is a tendency that the fine content controls the concentration profile of the sedimentation involving clay and sand particle concentrations.

The impact of constant Fine Water Ratio are significant above the FWR value of 30% where the slurry begins to exhibit yield stress and the effects are remarkably demonstrated through the normalized plot which fairly bring all test point, at different solid content but similar FWR, to the same curve(band).

The theoretical size from yield stress equilibrium, though conservative, is shown to over-estimate the size observed through experiment. Also a quantitative interpretation tool has been applied to analyze the experimental observation.

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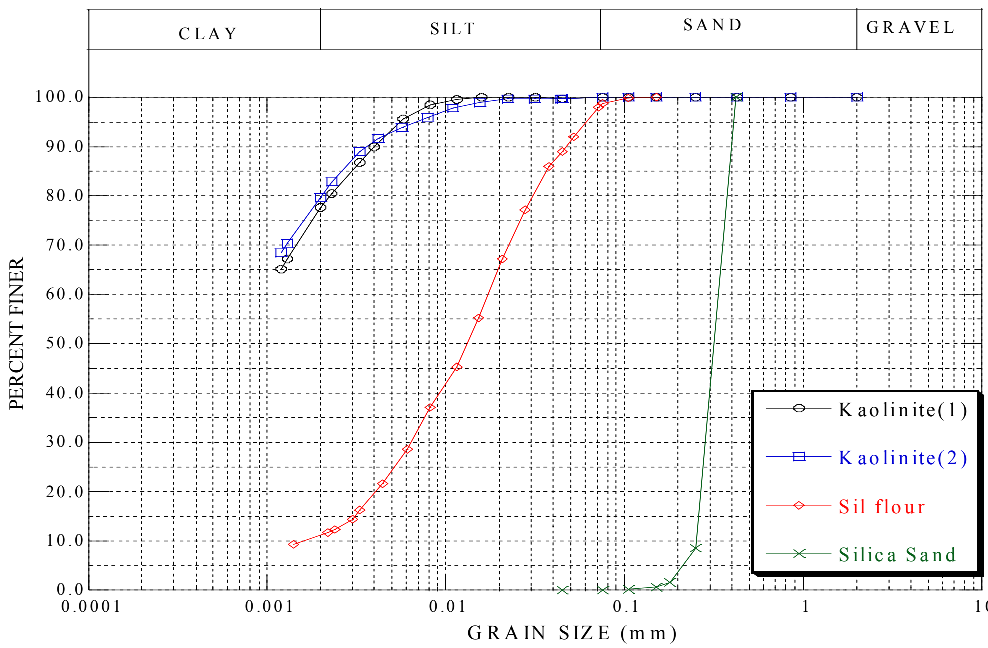


Figure 4-1Grain size distribution of test materials

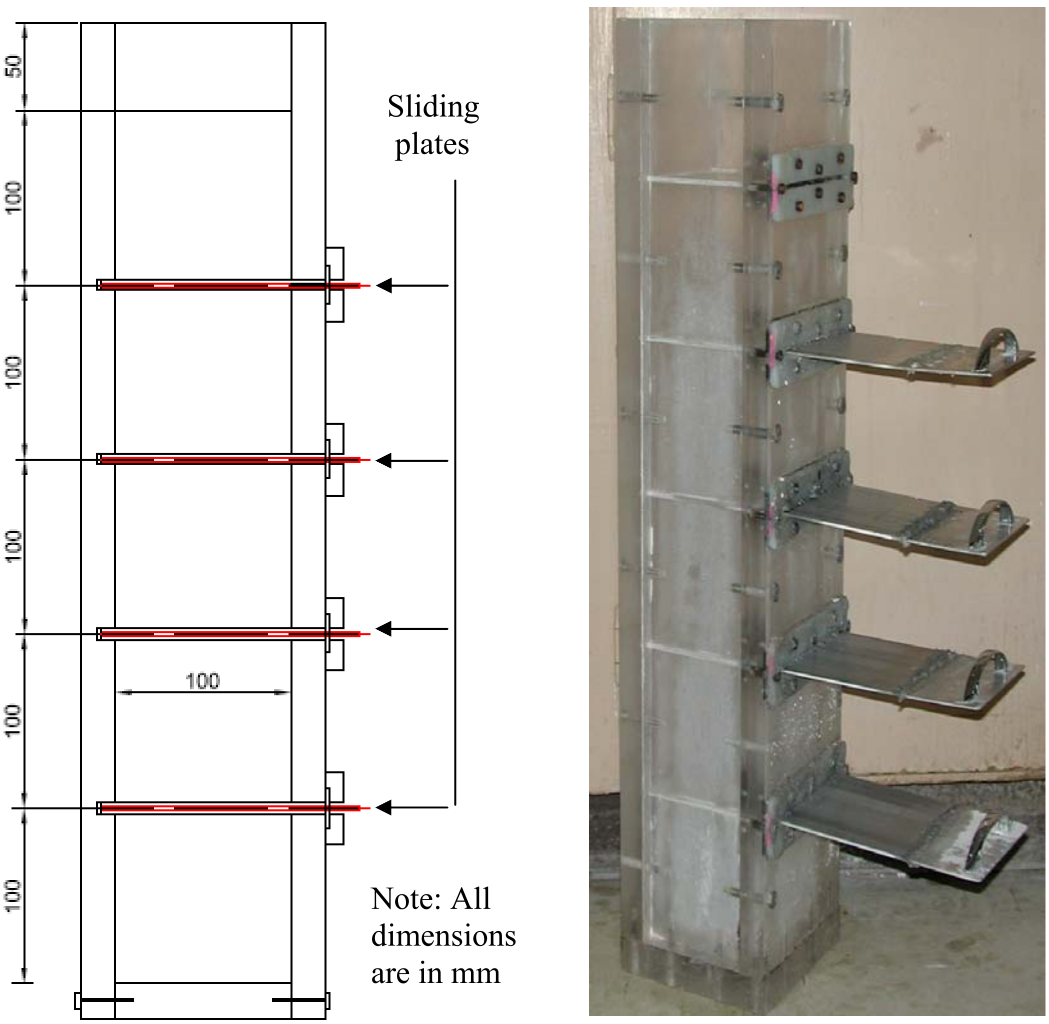


Figure 4-2 Schematic drawing (left) and picture (right) of modified standpipe

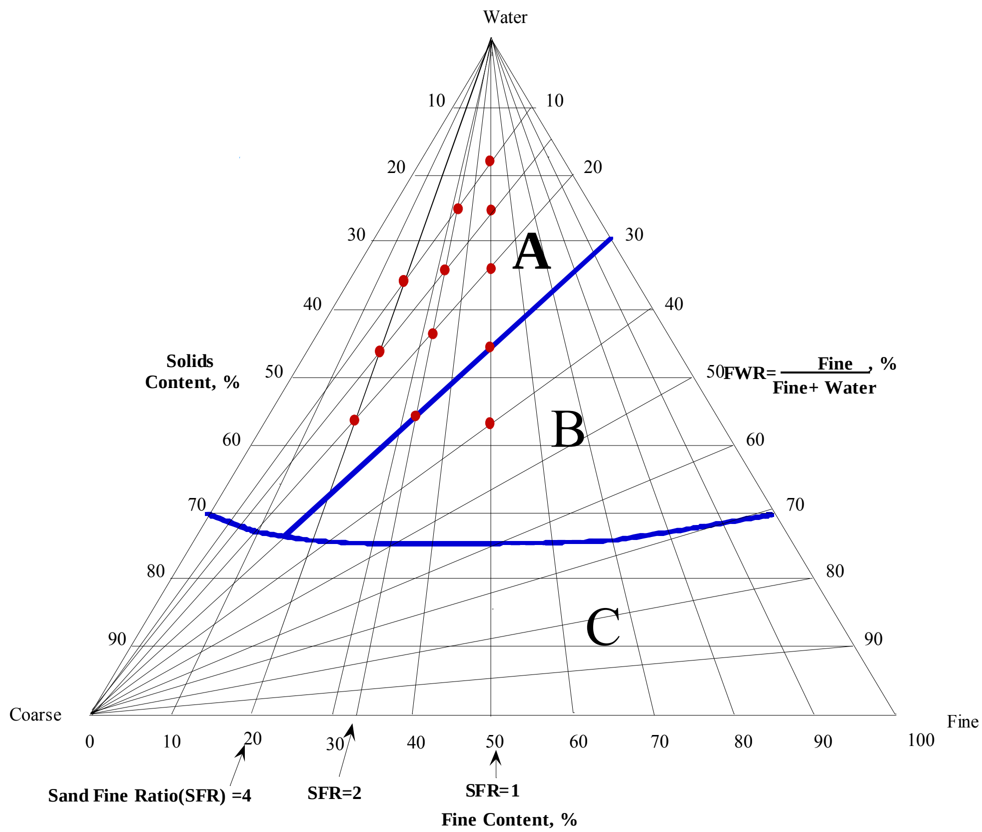


Figure 4-3 Ternary diagram for a slurry

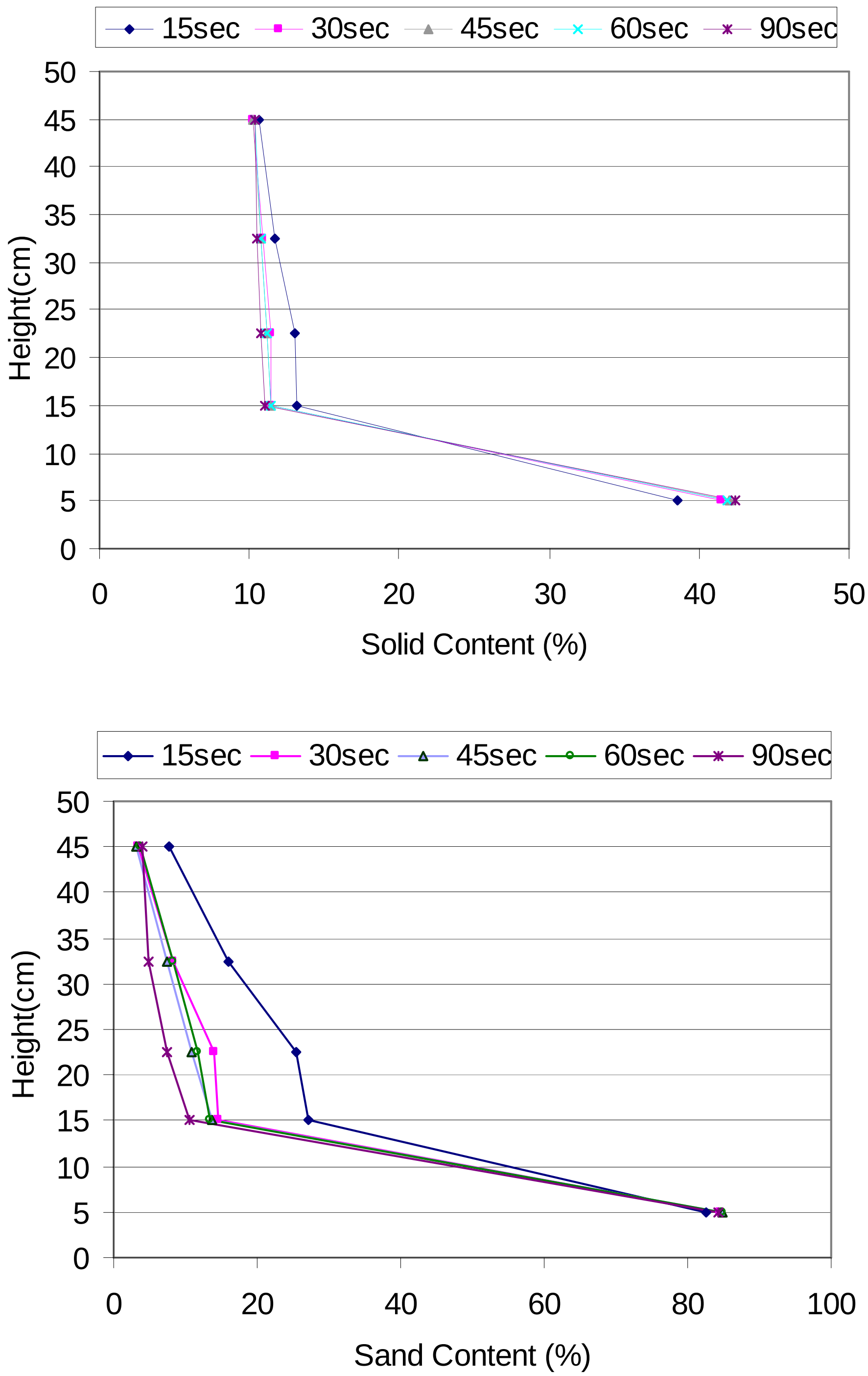


Figure 4-4 Solid and sand content profile of slurry at SFR=1, FWR=10 and an initial solids content of 18.8% at different elapsed times

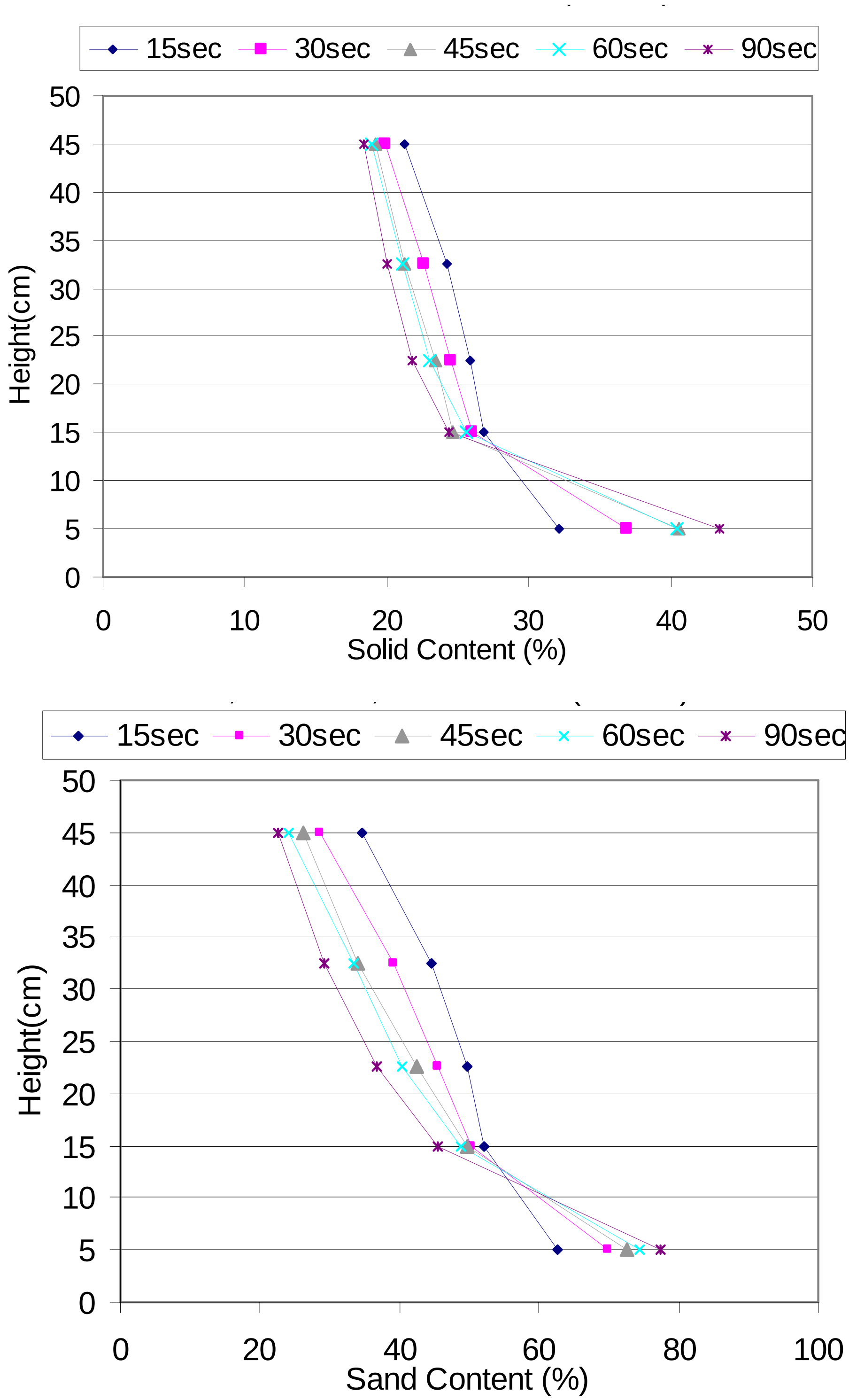


Figure 4-5 Solid and sand content profile of slurry at SFR=1, FWR=15 and an initial solids content of 26.1% at different elapsed times

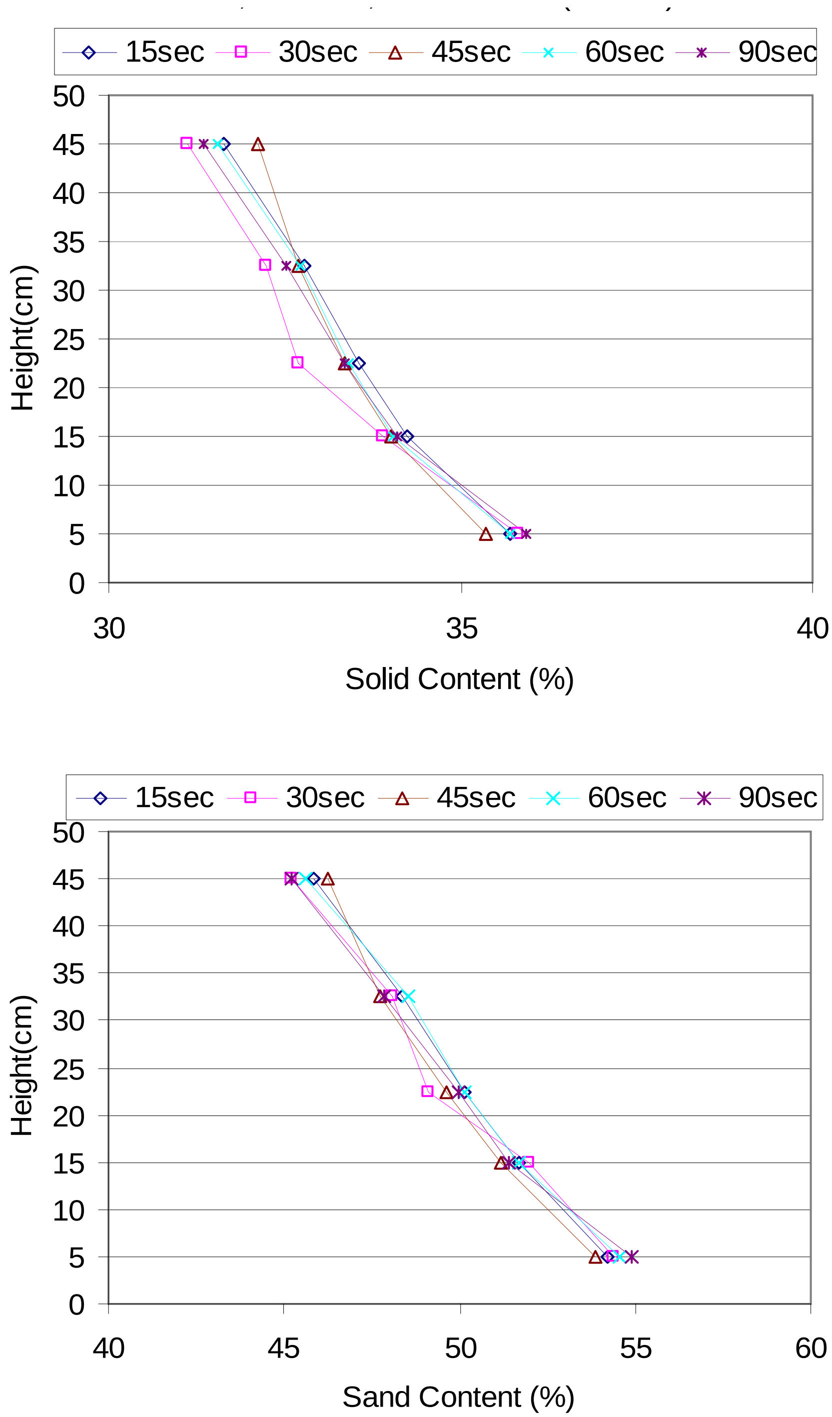
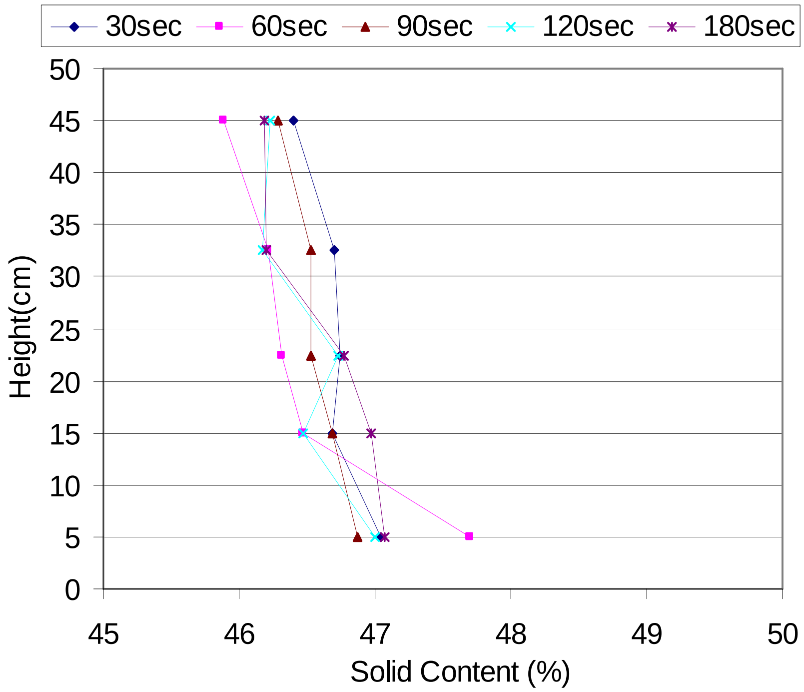


Figure 4-6 Solid and sand content profile of slurry at SFR=1, FWR=20 and an initial solids content of 33.3% at different elapsed times



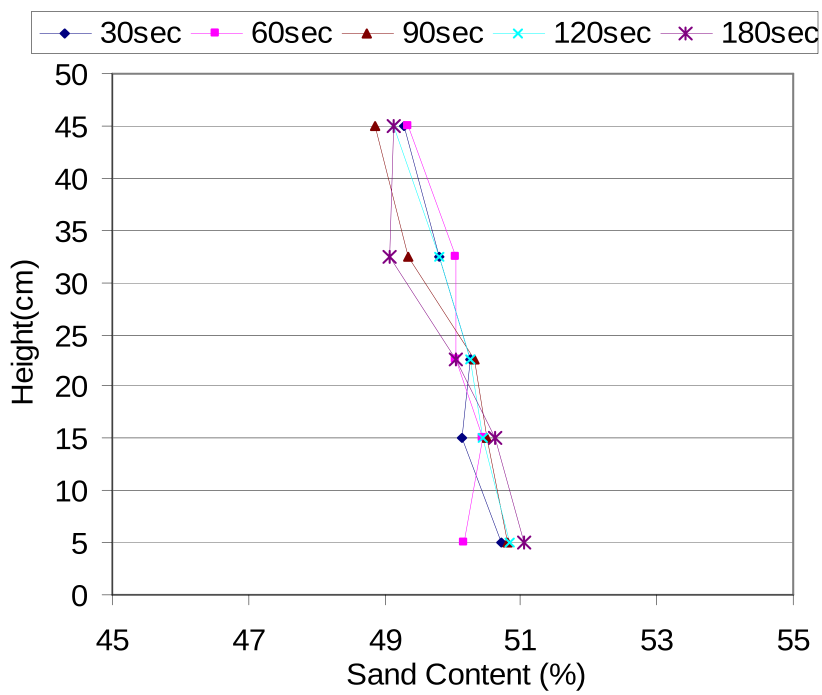
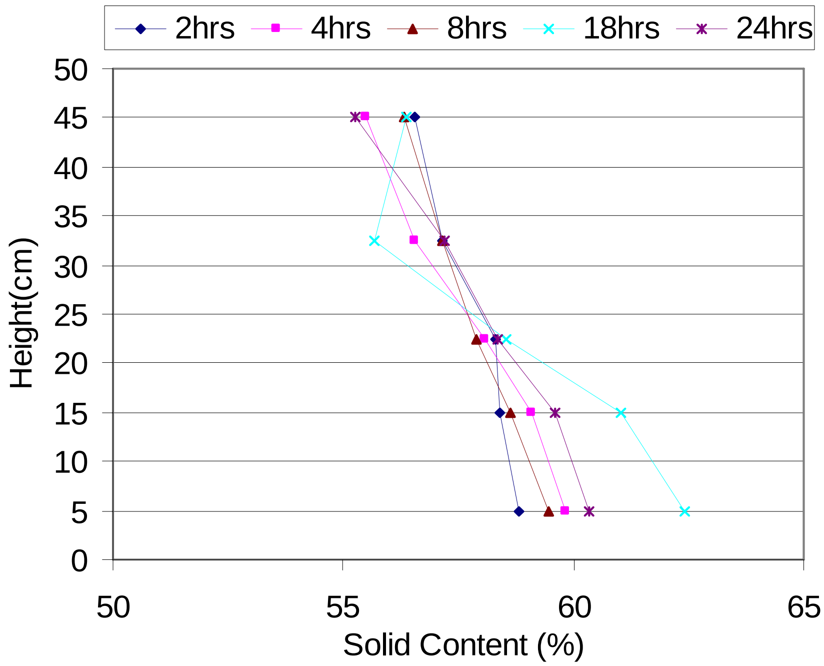


Figure 4-7 Solid and sand content profile of slurry at SFR=1, FWR=30 and an initial solids content of 46.2% at different elapsed times



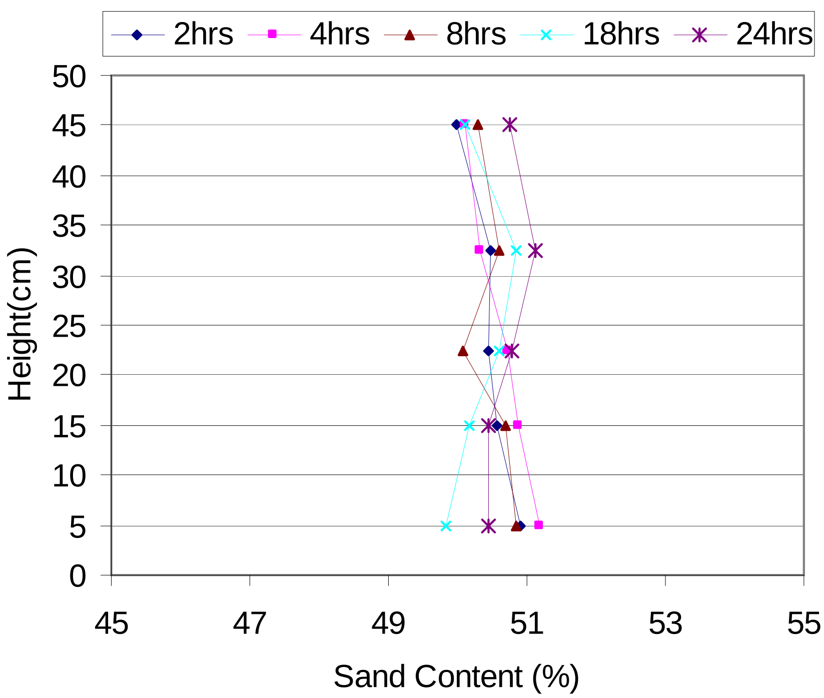
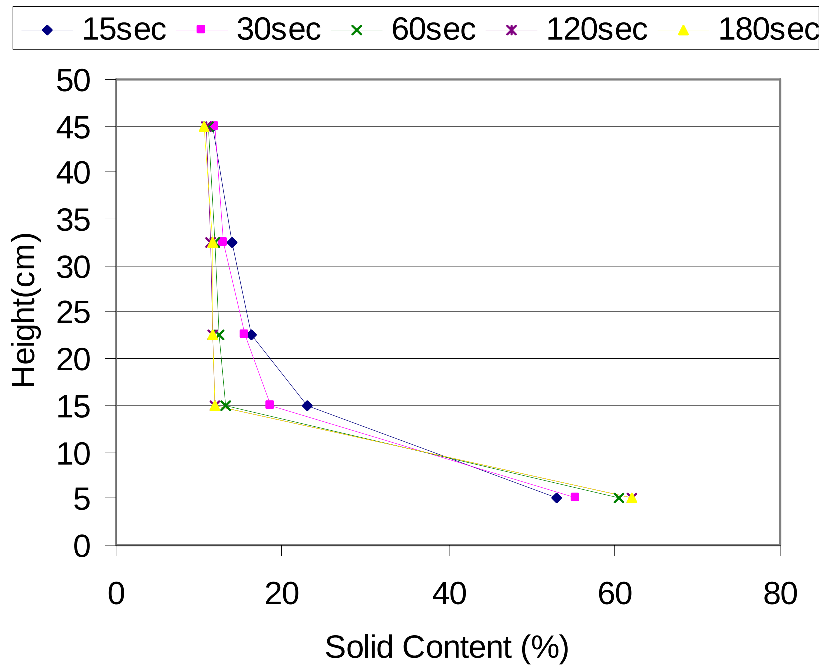


Figure 4-8 Solid and sand content profile of slurry at SFR=1, FWR=40 and an initial solids content of 57.1% at different elapsed times



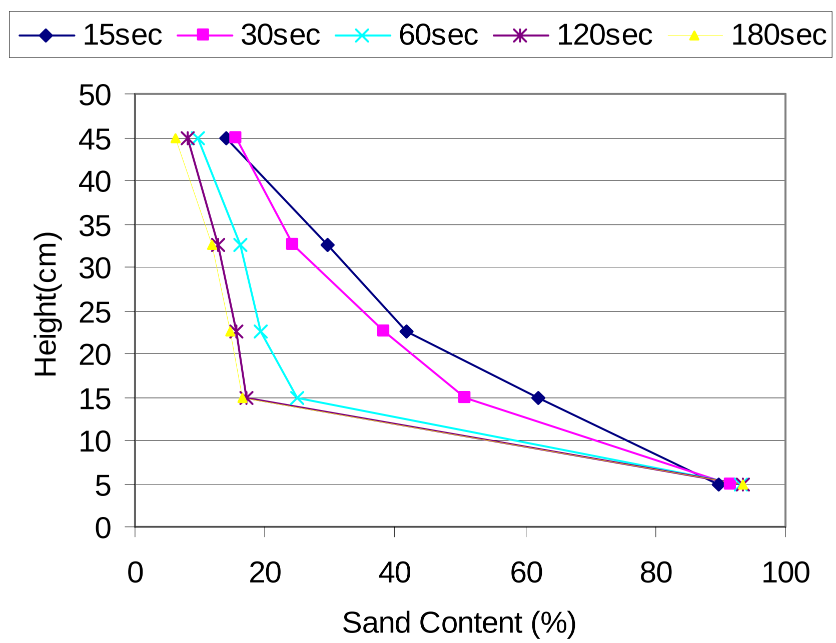
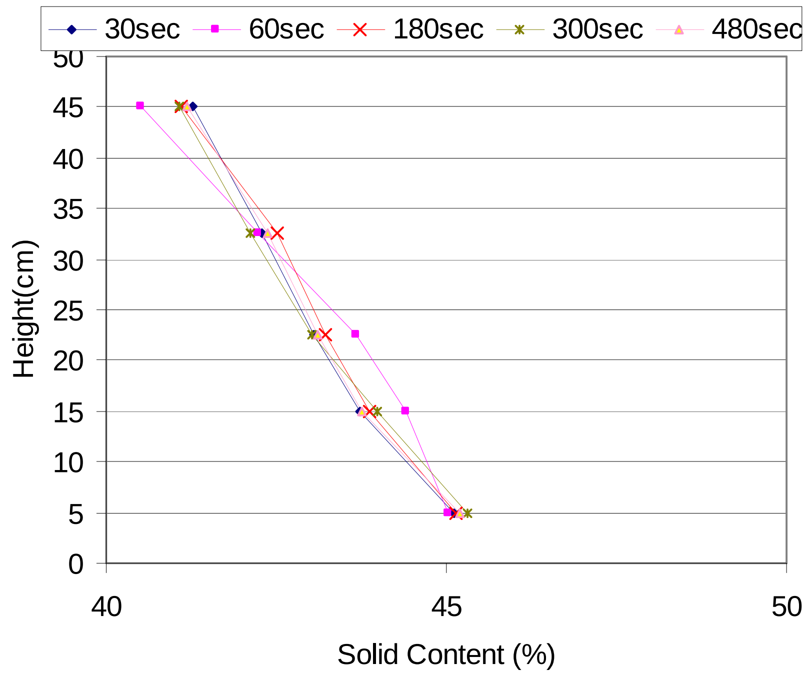


Figure 4-9 Solid and sand content profile of slurry at SFR=1, FWR=10 and an initial solids content of 25.0% at different elapsed times



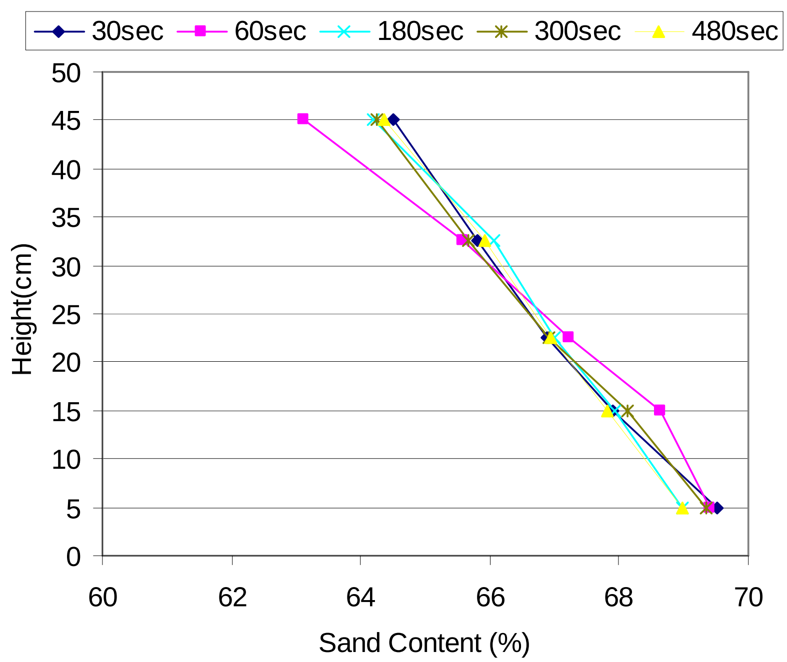


Figure 4-10 Solid and sand content profile of slurry at SFR=2, FWR=15 and an initial solids content of 34.6% at different elapsed times

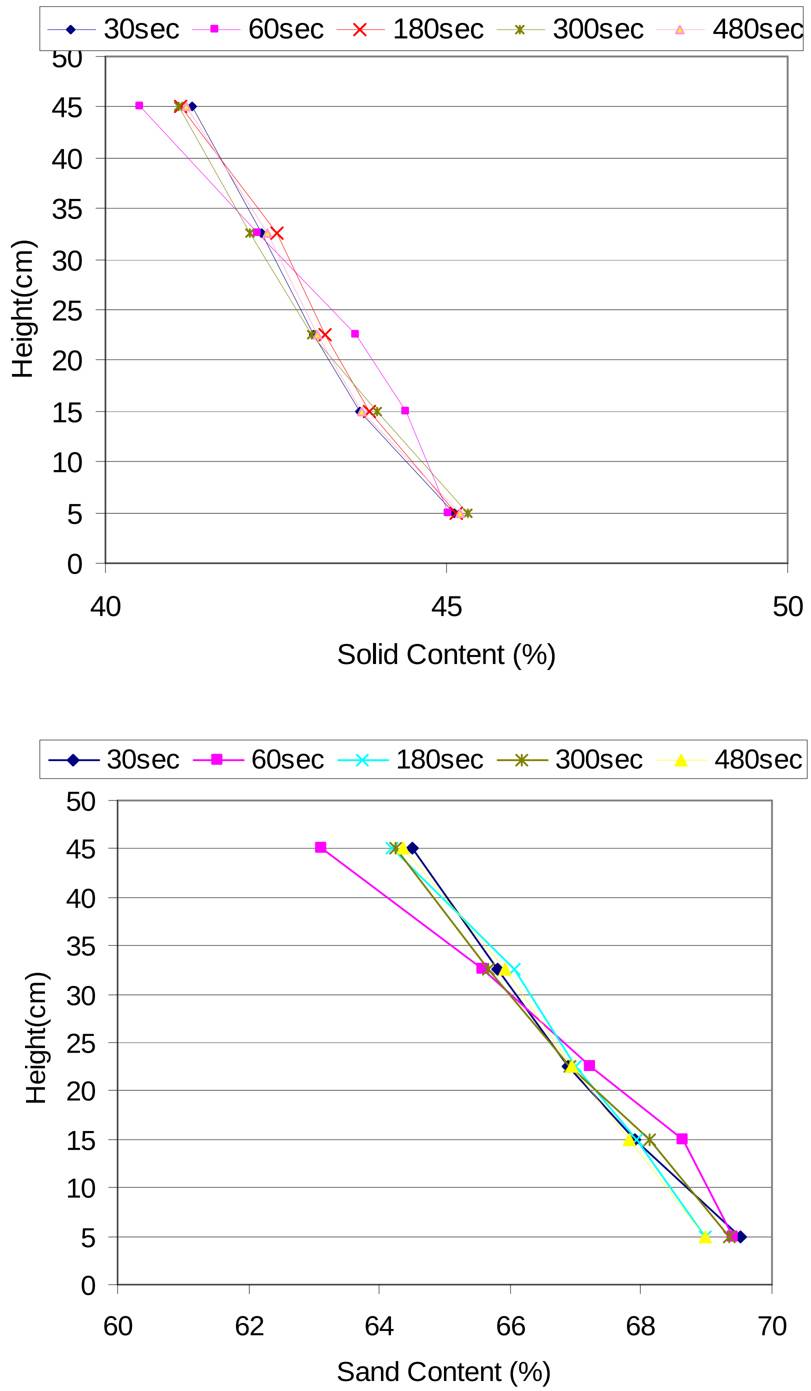


Figure 4-11 Solid and sand content profile of slurry at SFR=2, FWR=20 and an initial solids content of 42.9% at different elapsed times

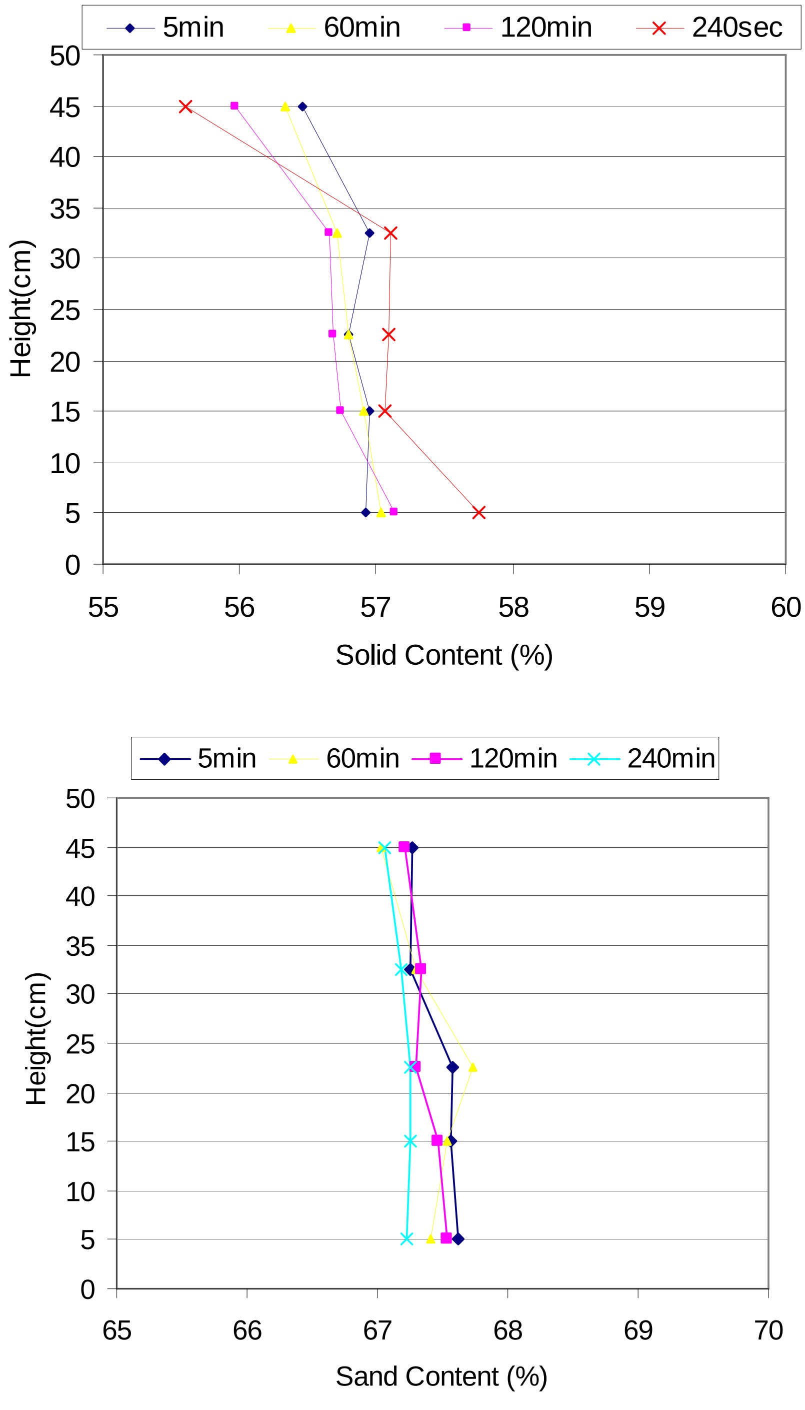
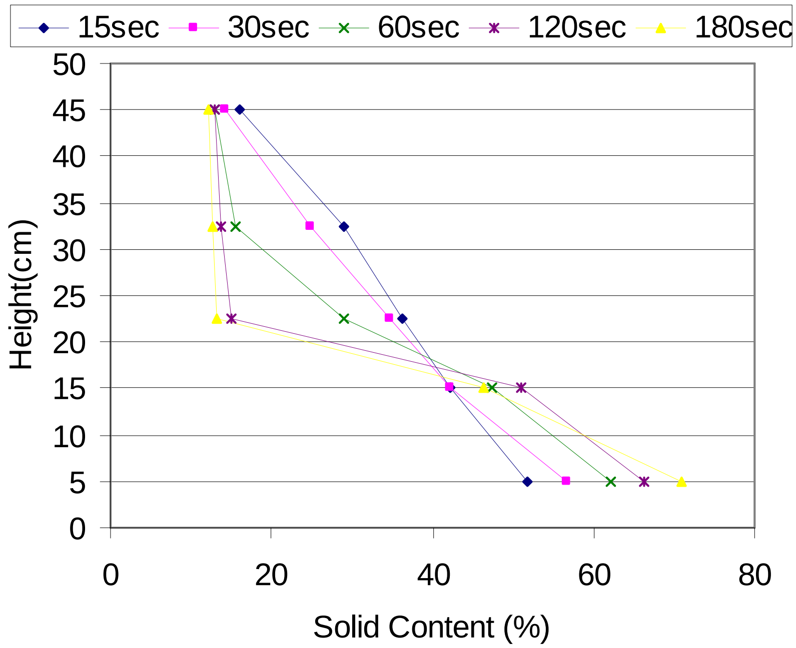


Figure 4-12 Solid and sand content profile of slurry at SFR=2, FWR=30 and an initial solids content of 52.3% at different elapsed times



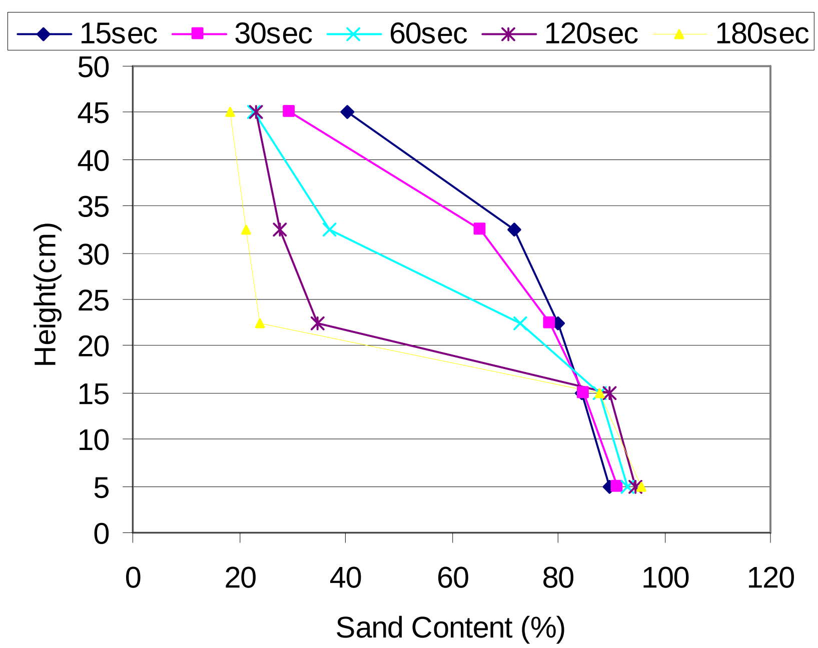
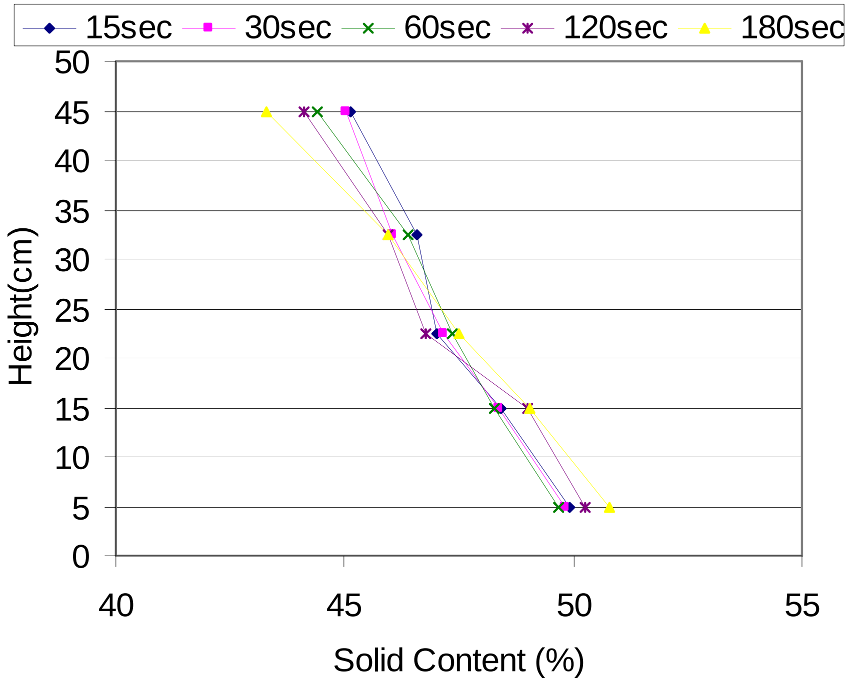


Figure 4-13 Solid and sand content profile of slurry at SFR=4, FWR=10 and an initial solids content of 35.7% at different elapsed times



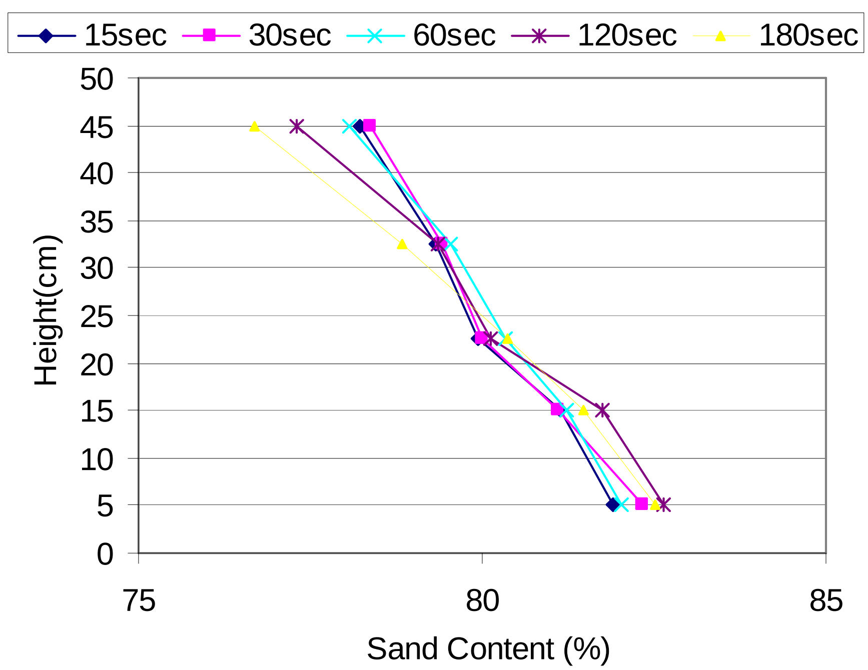
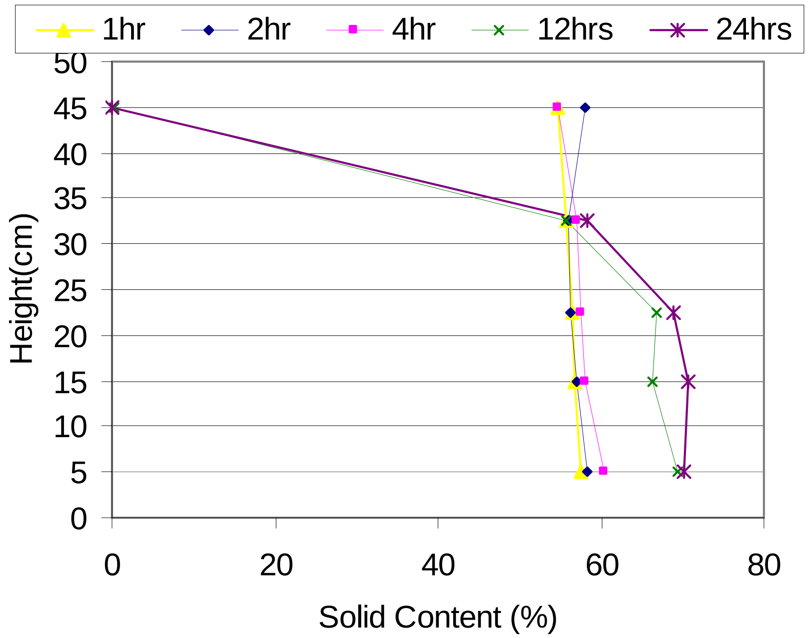


Figure 4-14 Solid and sand content profile of slurry at SFR=4, FWR=15 and an initial solids content of 46.9% at different elapsed times



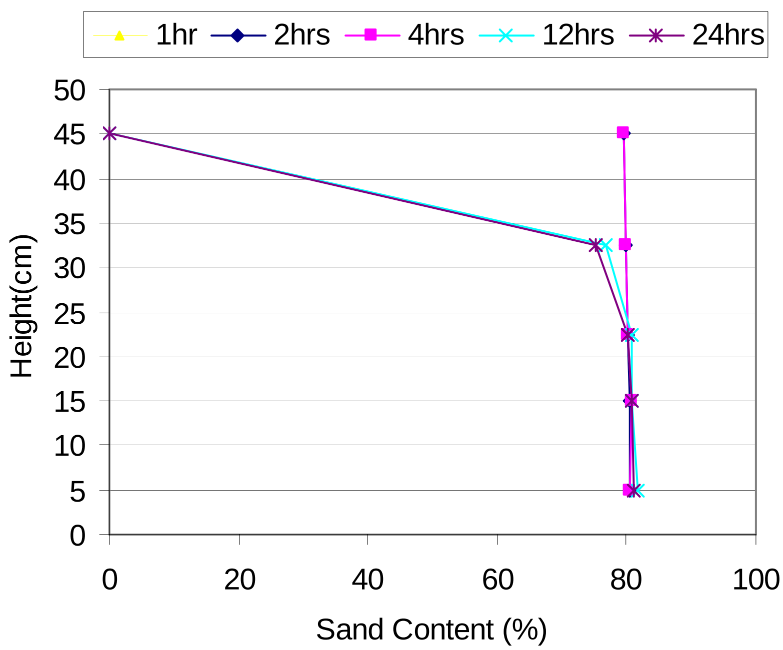


Figure 4-15 Solid and sand content profile of slurry at SFR=4, FWR=20 and an initial solids content of 55.6% at different elapsed times

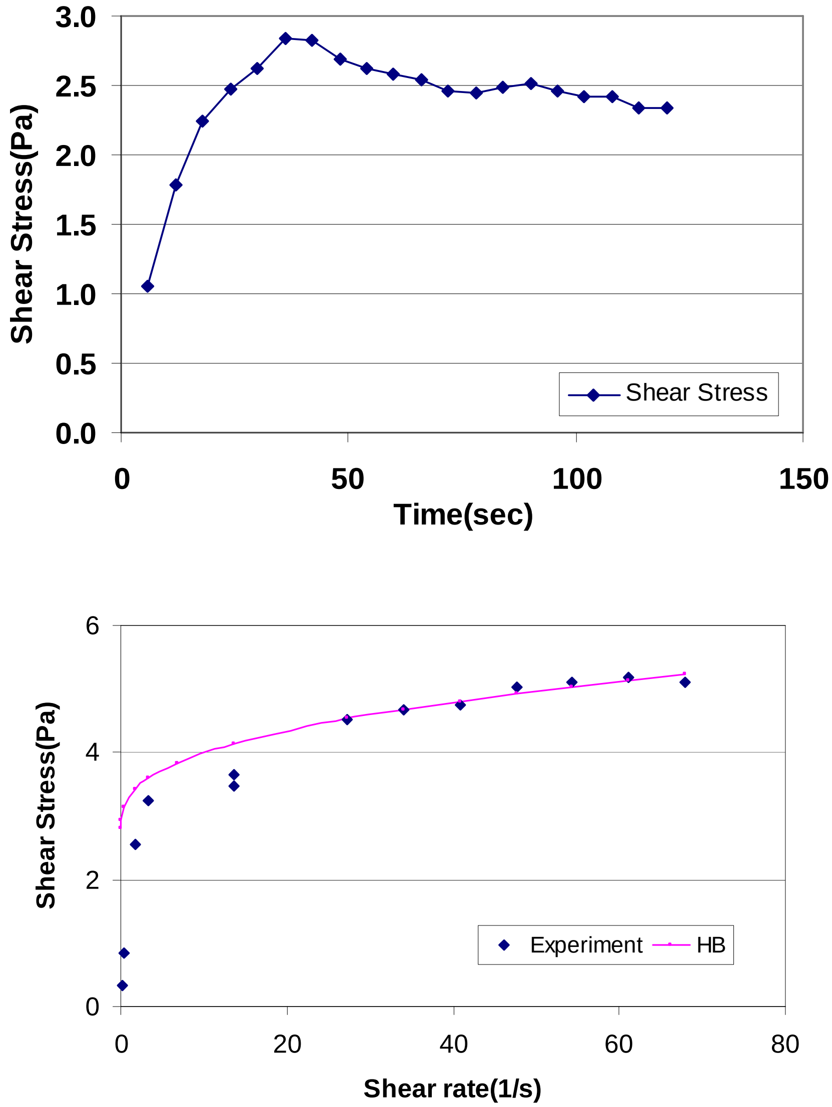
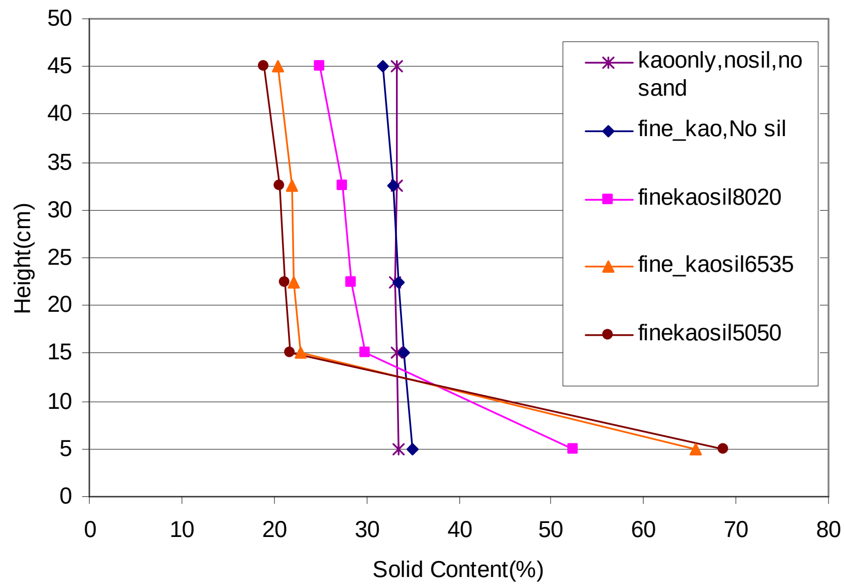


Figure 4-16 Yield stress measurement (left) and rheogram (right) with Herschel-Bulkley model fit



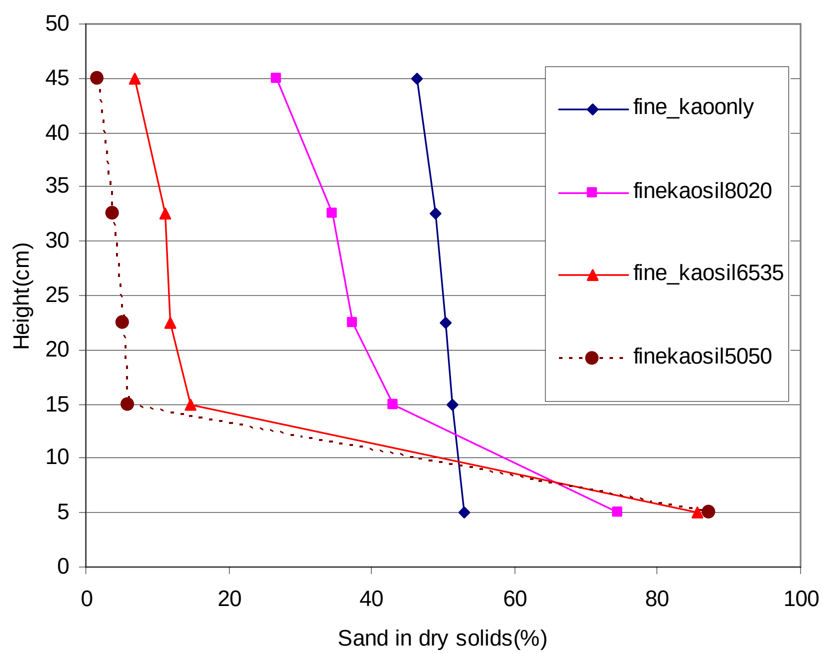
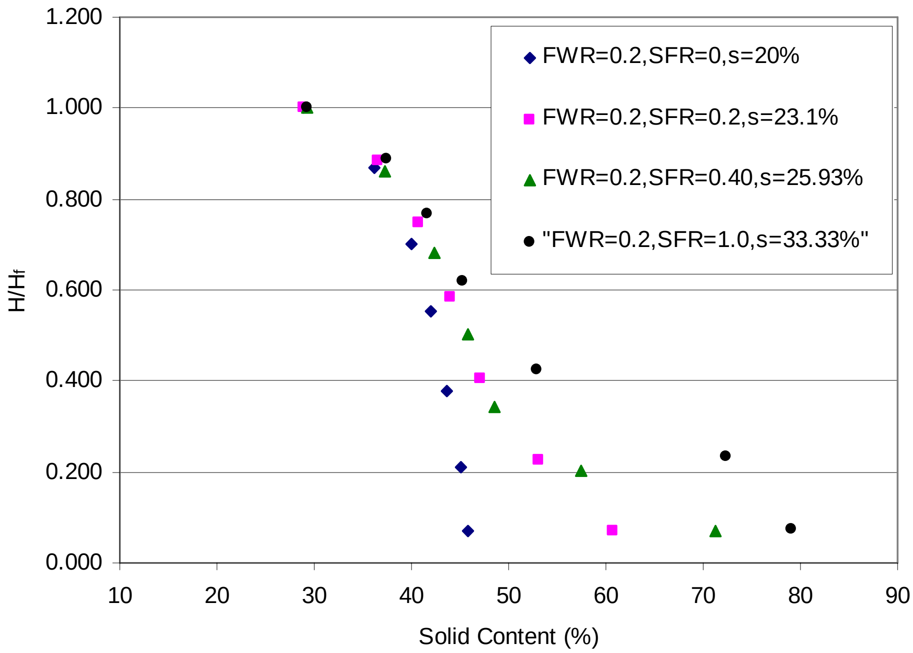


Figure 4-17 Solid and sand content profile in different composition of clay and silt size composition at a time of 15 min



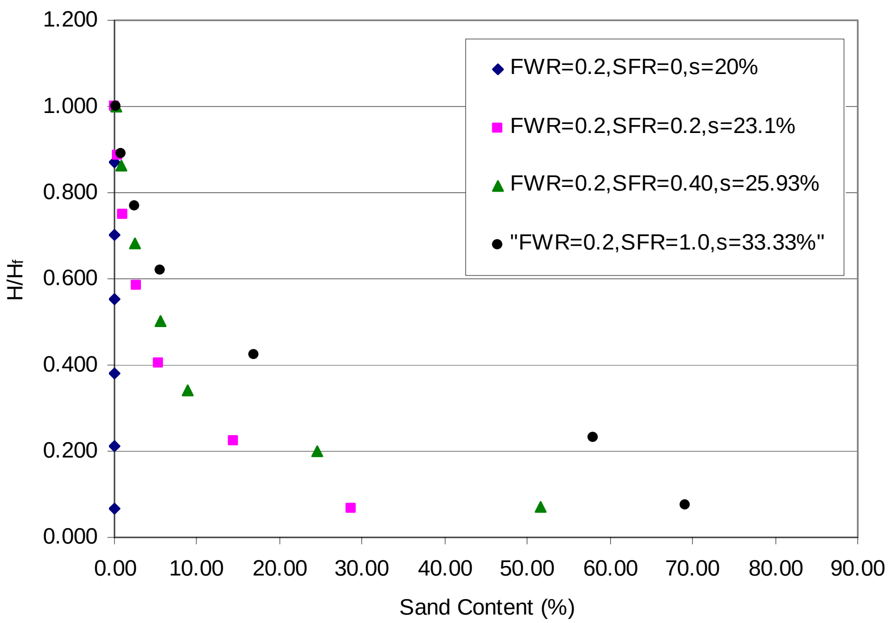


Figure 4- 18 Solid and sand content profile of kaolin sand slurry at constant fine water ratio of 20% and different sand fine ratio

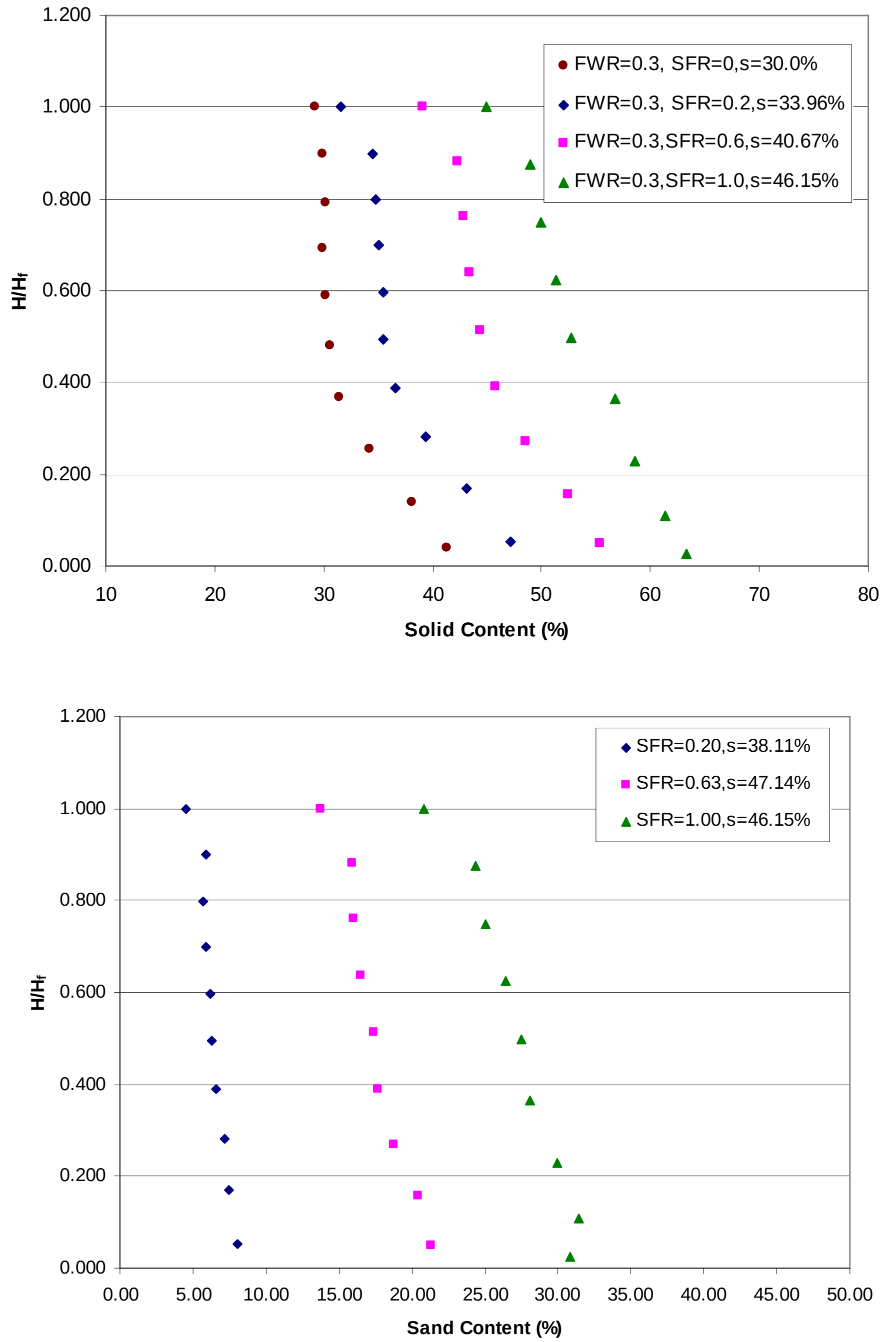


Figure 4- 19 Solid and sand content profile of kaolin sand slurry at constant fine water ratio of 30%and different sand fine ratio

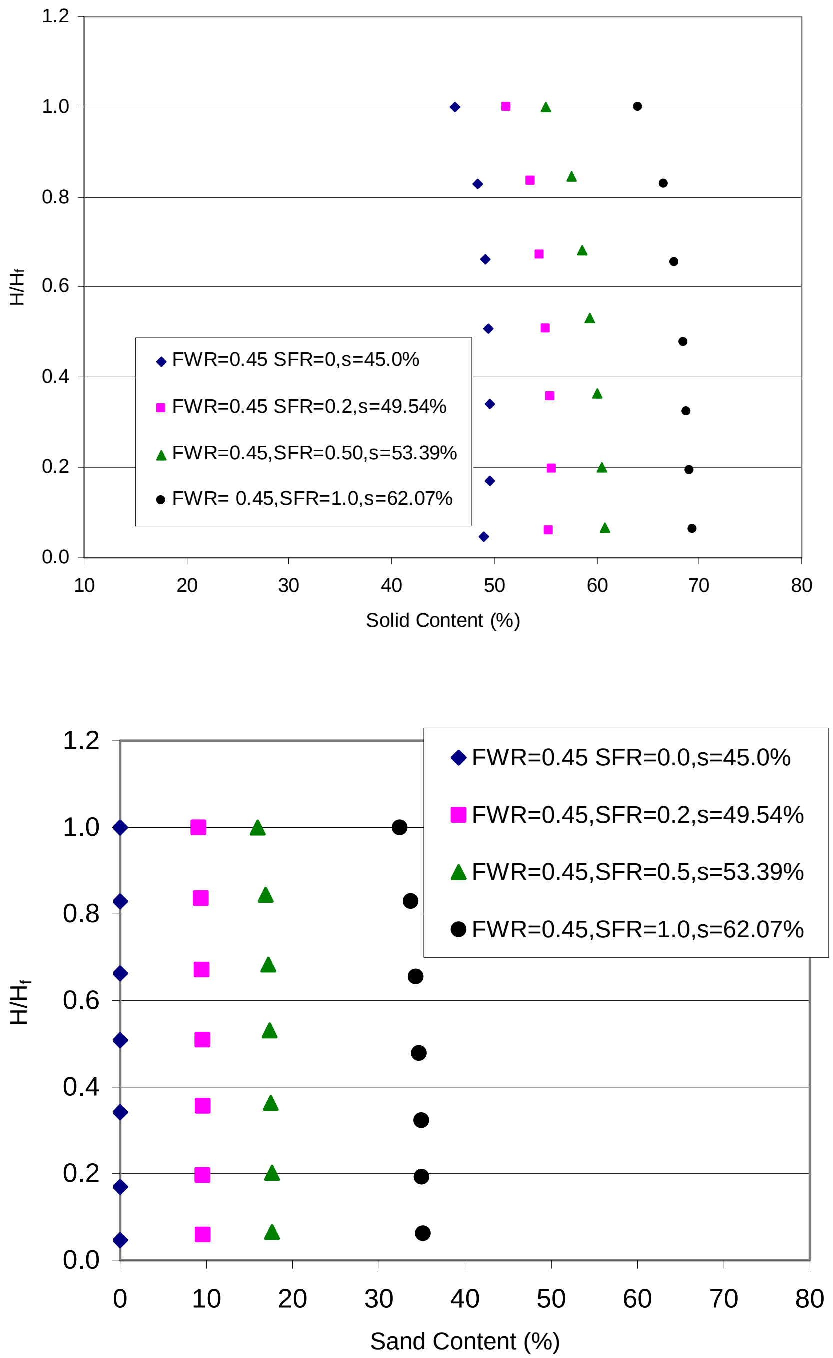


Figure 4- 20 Solid and sand content profile of kaolin sand slurry at constant fine water ratio of 45% and different sand fine ratio

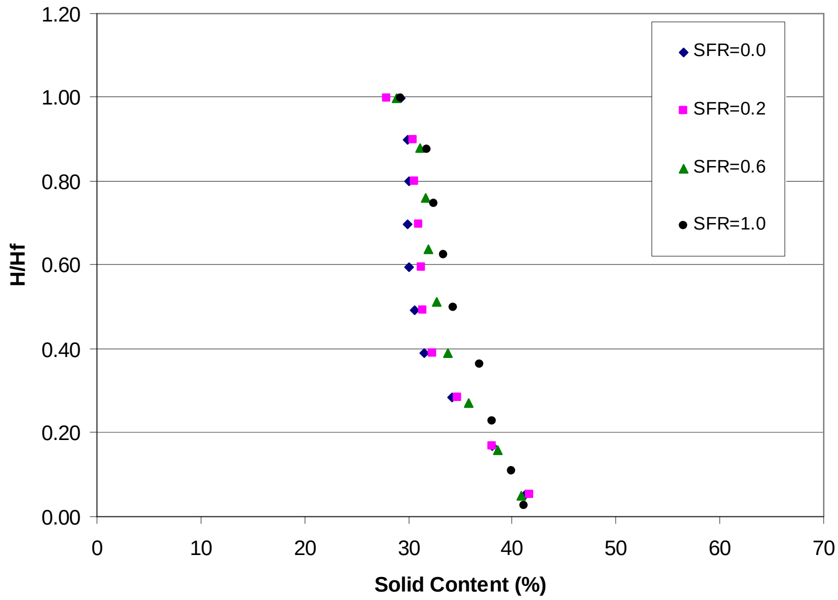


Figure 4-21 Solid content profiles of Figure 4.19 after being renormalized with respect to the initial solid content ratio

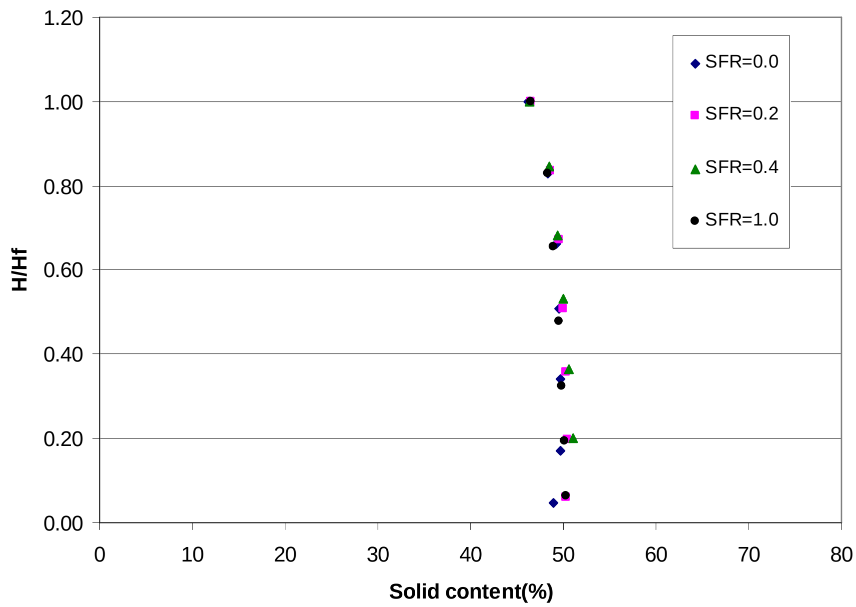


Figure 4-22 Solid content profiles of Figure 4.20 after being renormalized with respect to the initial solid content ratio

CHAPTER 5[3]

5. FLUME SEGREGATION TEST

5.1 General

Segregation through the process of transport or flow commonly occurs in both natural and man-made processes. Natural processes include rock avalanches, debris flows, pyroclastic flows and sediment transport by rivers. These processes are commonly of devastating nature and are the cause of many disasters in mountainous communities. Because of the high and intermittent socio-economic impacts involved, some effort have been made to understand the formation, transport and deposition of solid particle-fluid mixtures (Hungr and Morgenstern 1984), (Iverson 1997), (Iverson and Delinger 2001) (Coussot 1997), (Choux and Druitt 2002), (McLeod et al. 1999) to name only a few.

In man-made processes, the study of complex mixtures is of great significance in many aspects. In mining operation, for example, the disposal scheme involves hydraulic transport of the slurry to a disposal site. The transported material is then deposited into an impoundment where it will eventually settle and consolidate under self weight and surcharge of subsequently placed materials. Hydraulic transport is the most common way of conveying of solid particles. Proper mix proportion, solid concentration, and other geometrical and flow parameters are important component to be studied for efficient transport of bulk material.

Regardless of the environment of occurrence, the mechanism for the segregation process appears to be governed by the same physics of either grain-grain interaction or grain-fluid interaction. The understanding of the mechanics is still in the realm of limited success.

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Morgenstern and Kuepper (1988) defined hydraulic segregation as the process of deposition of particles of different sizes at different distances from the deposition point. Larger particles tend to deposit soon after being discharged, while smaller particles can be carried by the flow and deposited further down-slope. They also stressed that the ability to forecast and control grain size separation is still limited and greater understanding of this process is needed.

In hydraulic engineering solid-liquid mixtures are studied mainly in the area of sediment transport. Sediment transport involves bed load or suspension. Sediments are further studied into two broad classes: coarse or cohesionless and fine or cohesive types. The coarse sediment refers to mainly sand and gravel while the fine (cohesive) sediment is mainly a mixture of silt and clay, for which cohesive bonds between particles are significant and affect the sediment properties. The cohesive forces are due to attractive intermolecular forces and electrochemical forces. Partheniades and Paaswell (1970) explained that cohesive forces in the sediment are dependent on the fluid quality and have time dependent behaviour. Cohesive sediments behaviour in a flow field is a subject of complexity and most sediment transport studies relate almost exclusively to non-cohesive sediments (Raudkivi 1998). From practical point of view, sediment transport rate is of great importance to assess sediment yield, siltation of a reservoir, stability of beds and banks, and dredging in different water bodies. Sediment transport rate is the total weight of grains passing through a section per unit time, and is the product of the weight of moving grains present in the water over a unit area times the velocity at which they move (Dyer 1986).

Quantification of the relationship between water flow and sediment transport rate have been made from studies made on rivers, channels and flumes. According to Dyer (1986) the formulae for sediment transport rate fall into three basic groups:

1. Experimental (e.g. Meyer-Peter and Mueller 1948)–number of flume test and empirical relationship.

2. Theoretical. (e.g. (Einstein 1950) and (Bagnold 1956, 1966)–basic physics of the movement of individual grains. –relations developed.

3. Dimensional analysis (Yalin 1977), (Ackers and White 1973)-sediment flow variables are grouped together in a dimensionless numbers.

More detailed discussion on sediment transport rate is available in some standard text books like (Graf 1971), (Yalin 1977), (Chanson 1999) and (Chang 1988). While the sediment transport problems are conventionally associated with land erosion, rivers and coastlines, the same physical concepts apply to many industrial and chemical processes as well (Raudkivi 1998).

Graf (1973) showed that for purely cohesive material the Shield's parameter can be equated with the coefficient of cohesion of the material (C_0):

$$\frac{\tau_0}{(\gamma_s - \gamma)d} = C_0 \quad , \tag{5.1}$$

where τ_0 is $\gamma * R_h * S$, γ_s is the solids unit weight, γ is the unit weight of the fluid , d is the diameter of particle, R_h is hydraulic radius and S is the slope of the bed.

Graf also discussed that the critical shear stress for cohesionless material is a function of particle size whereas for cohesive soils the critical shear stress is a function of both particle size and coefficient of cohesion. "…*The shear stress of cohesive material depends on the composition of the different fractions (clay, and non-clay minerals and organic matter), the particle-size distribution and particles' shape, the packing, and probably on other items. Also geologic events, such as compression or stratification, may be of importance.*" He also asserts that the critical shear stress depends on the Plasticity Index (PI). In a conclusion, he remarked that erosion of cohesive soils takes place in aggregates rather than particle by particle. It was observed that a well-defined interface exist between the flocculated cohesive material-frequently referred to as fluid mud-and the overlying water. Raudkivi (1998) mentioned that only about 10% of clay will suffice to assume control of the soil properties.

Partheniades and Paaswell (1970) defined critical shear stress as the stress for which the bed material was in general motion and provided a relationship between critical shear stress and plasticity index.

$$\tau_0 = 0.0034(PI)^{0.84} \qquad (5.2)$$

with τ_o in psf.

Flume studies have been major experimental means to study sediment transport behaviour (de Groot et al. 1988), (McLeod et al. 1999), (Parson et al. 2001). Wilson (1991) describes flume as any artificial channel carrying flow with a free surface, which is commonly known by open –channel flow.

de Groot et al. (1988) conducted field and laboratory investigation to study slope and density of dams constructed by hydraulic fill of fine sand. An important result of their study is a formula for the equilibrium slope, defined as the slope at which sedimentation and erosion are in balance. They also reported that the viscosity at concentration of 30%(vol), the apparent viscosity is about 4 times the viscosity of clear water. In a laminar flow the sand grains are supported by mutual collisions and in turbulent regions the grains are kept in suspension by turbulent diffusion. In their study of density current, sedimentation and flow slide processes, using laboratory prototype test, they found out that the underwater slope is mainly a function of grain size and sand transport rate. They stressed that grain size is of primary importance.

Kuepper (1991) studied the deposition mechanism of hydraulic fills both in the laboratory and field experimentation. She adopted ‘similarity of process’ approach for the laboratory system in which the performance of the laboratory study is made similar in terms of process to the general system being studied. This kind of approach mainly serves the purpose of studying the deposition mechanics rather than extrapolation of results from laboratory to field. She also discussed that hydraulic sorting (segregation) is more pronounced for high flow rates, lower slurry concentration and relatively small flow velocity on the beach.

Blight (1987) discussed two important aspect of depositional profile of slurry material. At lower solid concentration, larger particles settle out higher up the beach while finer particles will travel further toward the pool. Such deposition formed can be shown in a master profile (Figure 5-1) which is given by the Equation (5.3)

$$\frac{y}{H} = \left(1 - \frac{x}{L}\right)^n \tag{5.3}$$

He also observed that beyond a certain solids concentration, the flow regime changes and instead of progressive particle sorting occurring, with the average particle size decreasing towards the pool, the slurry starts to flow as a homogeneous material. He found for fly ash slurry that 40%s marks the change from a 'particle settling' to a 'mud flow' condition. Below 40%s is a particle settling regime, under which most conventional tailings dam operate; and above this solid content is the mud-flow regime, under which thickened discharge type dams operate.

Fourie (1988) also conducted flume beaching tests on three tailings, i.e., bauxite tailings, nickel ore slurry and coal tailings. Beach profiles as obtained from his experiment are convex upward unlike those reported by Blight (1987) as concave upward. He discussed that tailings which produced a value for 'n' less than unity (resulting in convex upward profiles), little or no segregation occurs and vise versa.

Kuepper's (1991) flume experimental data also show concave upward beach profile. She also observed that the beach profile is steeper for larger slurry concentrations, smaller flow rates and larger mean grain diameter. Also the field test beach profile show concave upward shape. In a discussion about the occurrence of segregating behavior, she described that the fluid and solid phases interact by retaining separate identities, and the slurry viscosity remains similar to that of the carrier fluid. This latter remark is, however, in contradiction with the experimental observations in this study, as discussed in Chapter 3.

Some similarity of beaching profile can be observed from the flow of ice sheet as presented in (Paterson 1969). He showed a slightly different form of Equation (5.3).

For granular flow at uniform steady condition, Pouliquen (1999) proposed semi-empirical normalized differential equation to predict the shape of granular front on an inclined plane. The basis of the formulation lies in empirical friction factor at the base, and steady equilibrium condition along the flume length.

The study of dynamic processes that involve fluid-particle and particle-particle interaction is greatly relevant. Morgenstern (1985) pointed out that the design of tailings handling systems requires an appreciation of processes at the boundary between fluid mechanics and soil mechanics.

It is of an interest to study whether high concentration slurry would exhibit a segregating behavior when the slurry is subjected to flow condition. The objective of this experimentation is to study the occurrence of segregation in a dynamic environment. Since segregating type experiments both in the laboratory and in the field are done extensively by other researchers e.g. (Kuepper 1991), the subject of this study here emphasizes on the segregation behavior of relatively non-segregating slurry when subjected to a dynamic flow mechanism.

5.2 Equipment and Test Procedure

The flume has an overall length of 244cm and 11.5cm wide. It is wholly made of plexi-glass. At one end of the flume, a compartment was segmented to serve as a reservoir with a sluicing gate at the downstream side. The dimension of the reservoir was 33.4cm length, 21.8cm deep and 11.5cm wide. The schematic showing the flume set up is shown in Figure 5-2. An overflow pipe which limits the depth of the slurry level at 20cm is provided at the middle of the reservoir. Thus a reservoir can handle about 7.7 liter of slurry volume, when the gate is closed. The slope of the flume bed can be adjusted by the jack table that supports one edge of the flume.

The test materials are kaolinite clay material, sand and tap water. Similar procedure as in standpipe test was followed in the preparation of the slurry. A total volume of 15litre slurry was prepared for the initial flume experiment. For the subsequent test the material left over after sampling is collected and remixed after adjusting the proportion to meet the required solid content.

The slurry was poured into the reservoir while the gate was closed. Then the gate was opened to 2.3cm height and the slurry material was discharged. At the same time the reservoir was fed with slurry until the total of 15litre is poured into it. The discharge was so instantaneous that it makes it measurement of the flow rate very difficult.

The depth of profile is measured along the flume length at selected positions. Then after the deposited material is sectioned by cork plates and from each section samples are withdrawn at the top half and bottom half for solid and sand content determination. The design solid contents and SFR are summarized in Table 5-1. The major distinction among the three test set is the SFR as the solid content is slightly varied.

Table 5-1 Design test solid contents and sand fine ratio for flume segregation test

Test Set No.	Design Solid (%)	SFR	Bed slopes (degrees)
1	57.14	1	0, 5, 10
2	56.25	2	0, 5, 10
3	55.56	4	0, 5, 10

5.3 Results and Discussion

The solid and sand content profiles at three bed slopes are shown from Figure 5-3 to Figure 5-5. There is no observable segregation along the flume length both in term of solid content and sand content. Figure 5-6 show the beach profile for test set 1 at three bed slopes. The negative value in distance corresponds to the profile height at the middle of reservoir behind the gate. For 0% slope, the material is fully deposited inside the flume without reaching the flume end.

As the bed slope is increased, the beach height is reduced and the material reaches the flume end. The profiles generally exhibit convex upward shape which is consistent with other observation made on non-segregating type slurries (Fourie 1988). Moreover the slurry material that left the flume were collected into standpipe and allowed to stay overnight and tested for solid content and sand content. Representative standpipe data from slurry material that left the 10%-slope flume is indicated in Figure 5-7.

Similar beach profile comparison for test set 2 (SFR=2) is shown in Figure 5-8. While the effect of the bed slope can be clearly seen by the remarkable difference in the profiles, the effect of SFR is remarkably seen by the less amount of material left in the flume (or simply reduced flume height as compared to SFR=1)

The standpipe test solid content and sand content profile on the slurry material which left the flume for 5% and 10% slopes are indicated in Figure 5-9 and Figure 5-10 respectively. These profiles also show that the slurry material is relatively non-segregating after 24 hrs.

For SFR equals to 4 a comparison of beach profile at different slope is shown in Figure 5-11. Reduced height was observed for zero slope bed set up. Also the difference between zero slope beach profile and 5% profile is only up to about half-way of flume length then overlaps for the rest of flume length up to the exit point. The 10% slope is exhibiting a humpy profile towards the end of the flume. This phenomenon appears to be an indication of ripple formation. If the flume had enough length and large volumes of slurry material were input, the ripple formation might have been possible. A single formation of such ripple may be because of limited amount of slurry. During the experimentation small wavy patterns were visible.

After beaching on the flume, water release at the top surface is visible. The solid and sand content profiles for zero bed-slope along the flume length are shown in Figure 5-12. In this profile there is an indication of segregation at the sampling positions. The increase in solid content is related to the surface water release after deposit formation.

Similar profile trends are observed for 5% and 10% bed slopes which are shown in Figure 5-13 and Figure 5-14 respectively. Furthermore the standpipe solid and sand content profile, after 24hrs on the slurry material that exited at the flume end, are shown in Figure 5-15, for zero-bed slope and in Figure 5-16, for 10% bed slope. The segregating trend was remarkably seen in both cases.

5.4 Comparison of beach profiles at zero bed slope for different SFR's

The design solid contents of the three test sets summarized in Table 5-1 are in close proximity, the maximum difference between solid contents of test set 1 and test set 3 is about 2.8%. This would allow us in an approximate manner to compare the beach profile at similar bed slope but different sand fine ratios. Figure 5-17 shows the beach profile comparison at zero bed slope but different SFR. The information that can be tractable from such comparison is that it is the fine content that controls the shape of the profile. On a normalized scheme the three test plot in a banded location and equation (a) has also been fitted to the normalized plot. The values of the exponent 'n' are 0.4929 for TS1, 0.4706 for TS2 and 0.4762 for TS3. Such banded profile has been explained by Blight

(1987) as a master profile. Such a master profile plot obscure the information about the ingredient composition by normalizing the plot and fitting exponential curve to the plot.

Blight (1987) also discussed two phenomena which are closely related to the master profile concept. (i) The profile appears to be generated by gravitational sorting of particle sizes (ii) the resultant gradient of particle sizes down the beach produces a gradient of permeability.

The depiction of the test results as a master profile gives a vital information about the segregating/ non-segregating condition. When the master profile is convex upward, the slurry is of non-segregating type, whereas, when the beach profile is concaving upward, there is a tendency of segregation. This phenomena has been reported by Fourie (1988). The experimental results in this test program agree with these observations partially. It can be seen that for the sand-fine ratio of one and two, there is almost no segregation. When the sand-fine ratio is four there is segregation both in the beach deposit and standpipe test. The shape of the beach profile looks convex upward and is in close band on master profile though.

Such process can be related to the rheological behavior of the slurry in that the flow and deformation nature of slurry is governed by fluid-particle and particle-particle interactions. At large concentration and significant presence of fine material, mainly clay, the slurry behaves as pseudo-one-phase fluid. The major contributor to the deformation nature in such a case will be the yield stress. Such effects have been reported in some natural debris flow. For example, Parson et al. (2001), reported that the Acuabona debris-flow channel in the Alps exhibit the grain-fluid transition as silty fine material extracted from the eroded marl. They explained that the fine material acts to lubricate for the flow, increasing run out exhibiting non-Newtonian behaviour.

Iverson (1997) showed different depositional characteristics as influenced by grain composition and water saturation. He also asserted that grain size segregation mechanism in debris flows may be complicated and may involve more than one process. The general flow condition in this experiment is unsteady and of short residence time. If the flow is assumed to be a function of instantaneous depth in the reservoir, the discharge velocity is approximately a function of square root of depth in the reservoir. With increase in bed slope, the flow velocity increase. Consequently the shear stress and turbulence intensity induced by the flow reduces the yield stress further, leaving small volume of material left in the flume. The decrease of yield stress due to increase in velocity was also reported by Song and Chiew (1997).

Franzini (2002) wrote that from a mechanical point of view, clays and silts concentration increases the cohesion of the water-sediment mixture, provided by the electrochemical interaction of very fine particles so that coarser particles are held in suspension by matrix strength. He dealt however with the sediments without cohesive particles. The fall velocity of sand particles can also be reduced by the presence of silt or clay particles as explained in de Groot et al. (1988). In case of tailing disposal schemes also, the volume of tailings to be stored is sensitive to the fines content and the densities that these fines will attain (Morgenstern 1985). Hence, the experimental observations clearly demonstrate that presence of fines both in the static and dynamic condition influences the segregation mechanism.

5.5 Applicability of the theory of plasticity

Most of the observed beach profile formed from deposition of non-segregating slurries, have shapes that look similar to mud flow and glacier flow tongues. It is of practical interest to see if the theory developed for the study of flow tongues work for this experimental study.

Brueckl and Scheidegger (1973) applied theory of plasticity to study slow mud flows. The theory they used was a modified version of the theory of (Nye 1951) for glaciers. Based on Mohr-Coulomb constitutive relationship and equilibrium and assumptions they derived equation for the profile of the mud flow. The formulation they arrived is of the form given in Equation (5.4):

$$\frac{dh}{dx}\left(h \pm \frac{rc}{\gamma}\right) - h\varepsilon + \frac{C_s}{\gamma} = 0 \quad , \tag{5.4}$$

where

$$\varepsilon = \beta - \left(1 - \frac{\gamma_w \gamma_h}{\gamma h}\right)\phi_s \qquad \text{, and} \tag{5.5}$$

c is the internal cohesion of the flow material, c_s, cohesion at slip plane β angle of inclination of the slip plane, γ, bulk unit weight and γ_w, weight of pore water, h, height of flowing material measured normal to slip plane and r is a factor which is π /2 when the shear stress at the base is equal to c or 2 when the base shear is much less than c.
Equation (5.4) may be rewritten as:

$$\frac{dh}{dx} = \frac{ah + b}{h + d} \qquad , \tag{5.6}$$

where a is ε, b is c_s/γ and d is $\pm rc/\gamma$.

This is a non-linear ordinary differential equation if the parameters a, b and d are assumed constant. Some kind of such assumption appeared in the works of Brueckl and Scheidegger (1973).

Equation (5.6) was tested for one of the 5% slope experimental result on Figure 5-6., with solid content 57.1% and SFR=1. A software program called FlexPDE 5 was used to solve Equation 5.6, with assumed values for the constants a, b, and d. Figure 5-19 shows a comparison of results from the solution of Equation (5.6) with experimental data.

Equation (5.6) appears to under-predict the beach profile close to the exit point and match fairly well at the other locations. While at this stage the equation was fitted with assumed constant parameter values and it is encouraging to capture the profile in part.

Due to scale effect and short retention time the degree of dynamic shearing /segregation might have been limited and instantaneous. From practical point of view, the master profile could provide information about actual deposition from small scale laboratory tests.

Wilson (1976) describes flume transport with homogeneous slurry flow usually involving longer distances and flatter slopes than the coarse-particle case. These slurries are comprised of partially processed materials or, more typically, tailings being transported to retention areas. Observation in this testing program is not completely in support of his statements. As the sand fine ratio increases the flow exhibit fast flow rate and flatter slope at deposition are also observed.

As Mehta (1984) pointed two decades ago, the linkage between sediment rheology, aggregation dynamics and the settling behavior needs to be explored further at a basic level, in order to reduce the current level of empiricism in evaluating the settling velocity. The test conducted is primarily an initiative to understand the occurrence of segregation in non-Newtonian regime. Due to the complexity of the process and limited resources at the time of testing, flow characteristics and segregation impacts were not examined to full extent. Rather it is recommended that further work be extended with more details.

5.6. Summary and Conclusion

All beach profiles at zero bed-slope testing condition exhibit a convex upward and higher beach profile with non-segregating trend. As the slope is increased the beach profile exhibits nearly parallel to bed profile. The less segregation nature may be attributed not only to the fine-matrix but also to the length of the flume i.e., shorter flume length imply less residence time, less interaction and reduced segregation. The material flows farther as the sand fine ratio is increased indicating that the presence of fine influence highly both the beach profile and flow characteristics of the slurry. That a master beach profile is convex upward is not necessarily an indication on non-segregating slurry mix. As the sand fine ratio increases segregation is more likely.

Non-linear differential equations based on the theory of plasticity show good comparison with experimental data, nonetheless, the parameter need to be studied further to minimize the level of simplifications adopted.

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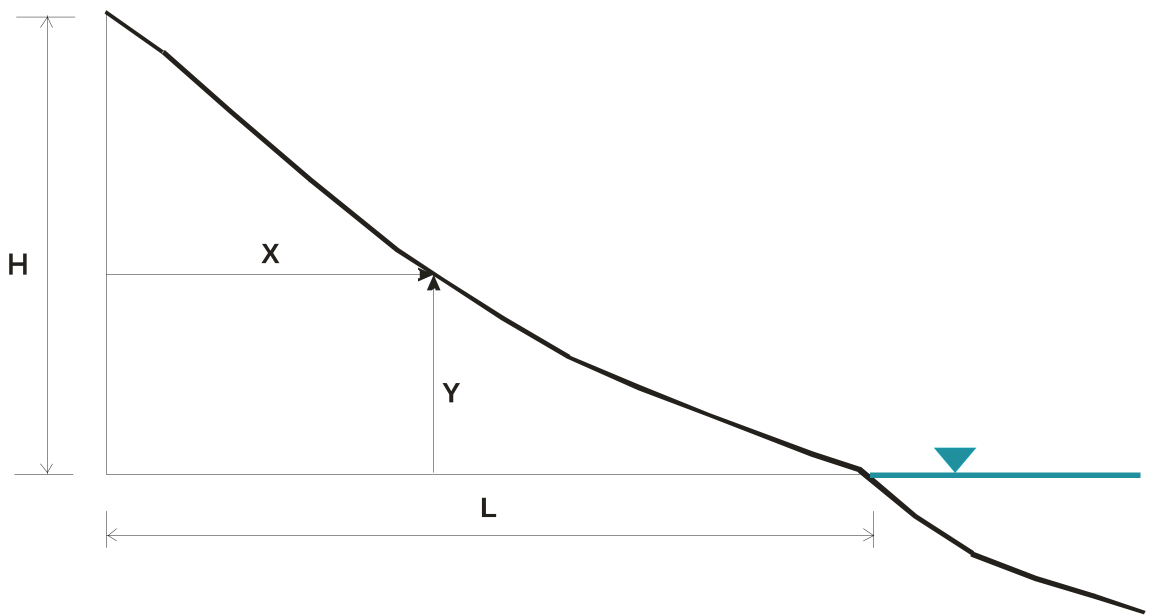

Figure 5-1 Master Beach Profile for Equation 5.3

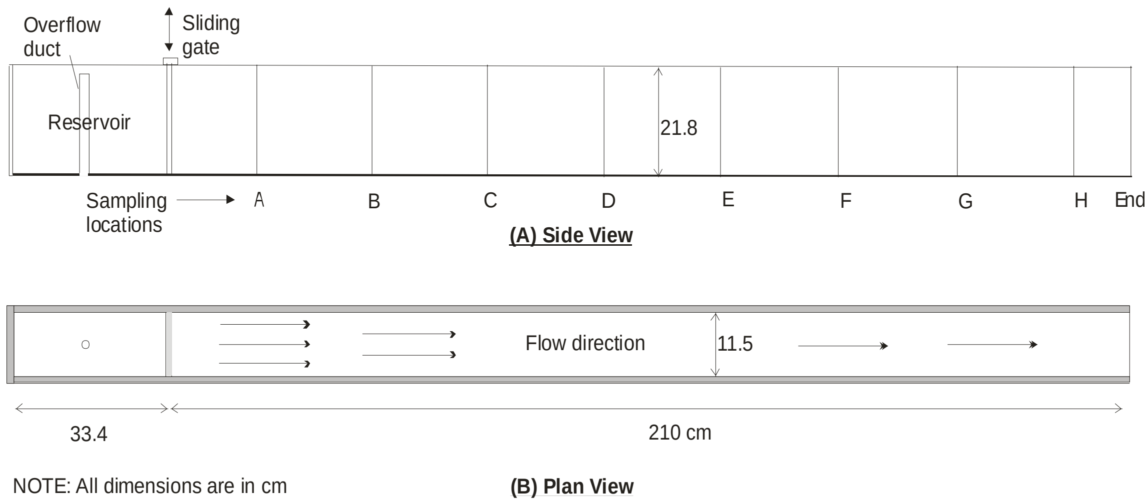

Figure 5-2 Schematic of flume apparatus

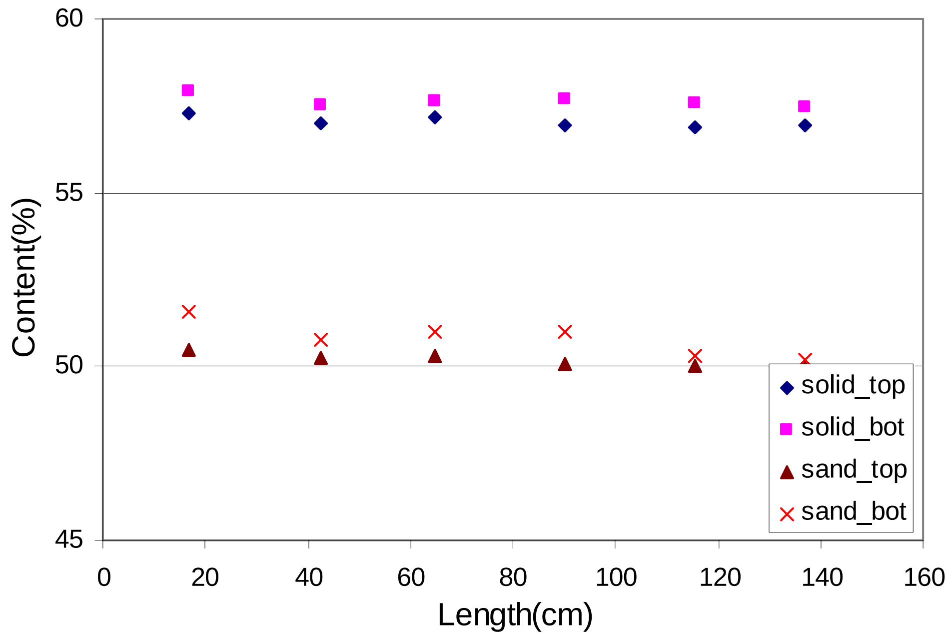

Figure 5-3 Solid and Sand content profile along the flume length test1, 57.17%s, SFR=1, zero-slope

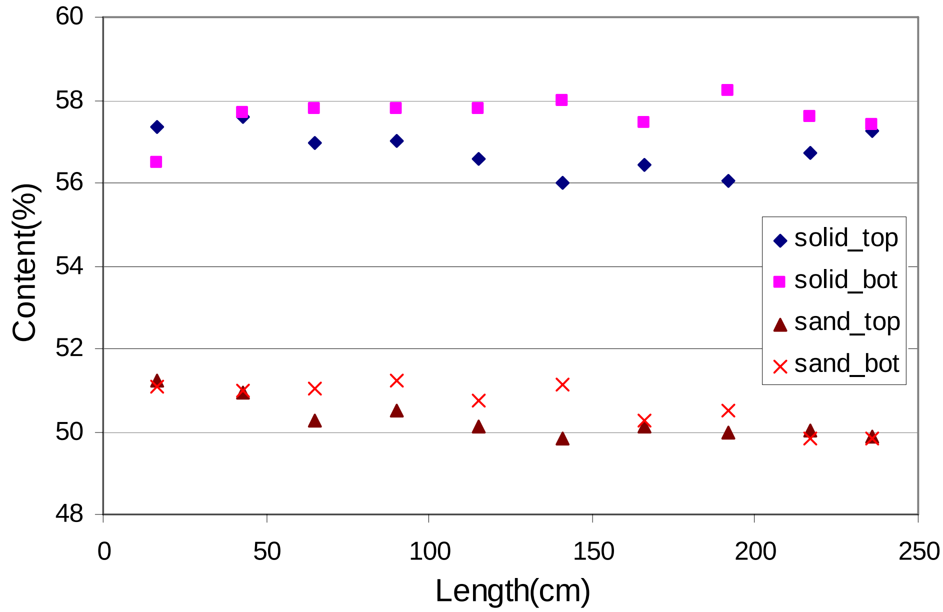

Figure 5-4 Solid and Sand content profile along the flume length test1, 57.2%s, SFR=1, slope 5%

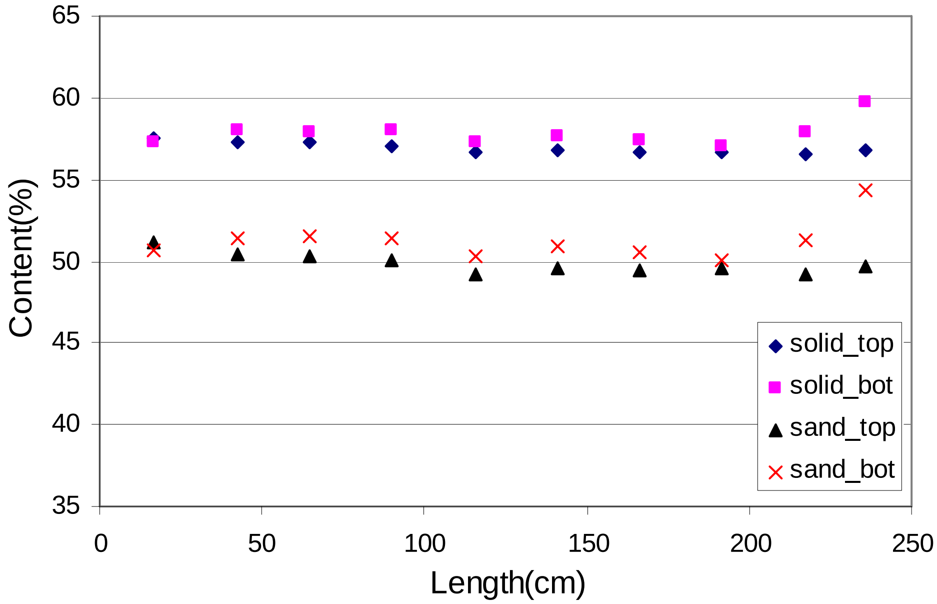

Figure 5-5 Solid and Sand content profile along the flume length test1, 57.14%s, SFR=1, slope =10%

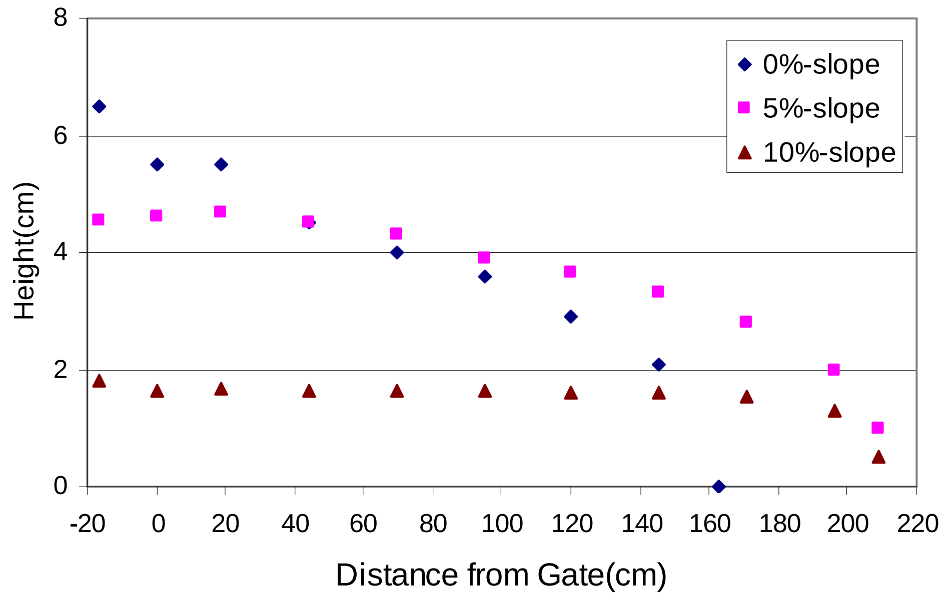

Figure 5- 6 Beach profile along the flume length test set 1 at different slopes

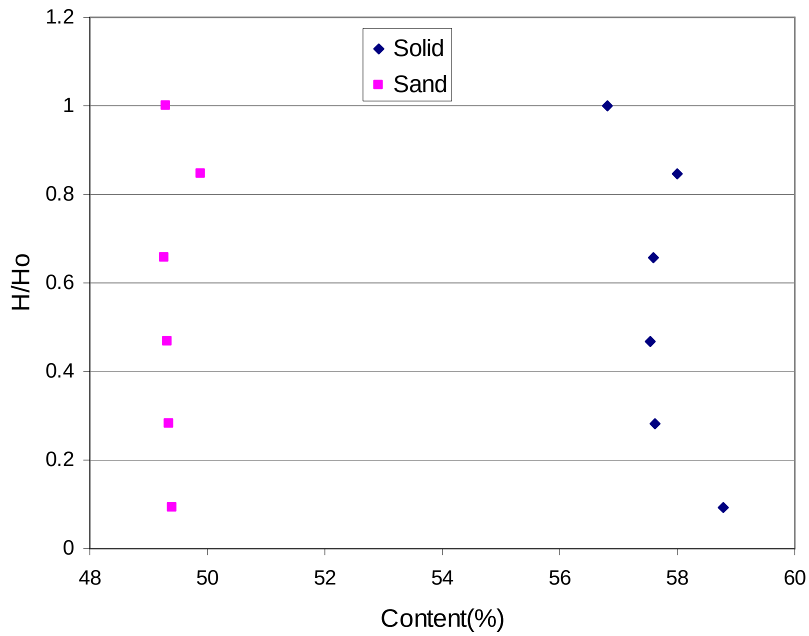

Figure 5-7 Solid and sand content profiles of standpipe test on material collected from flume downstream end

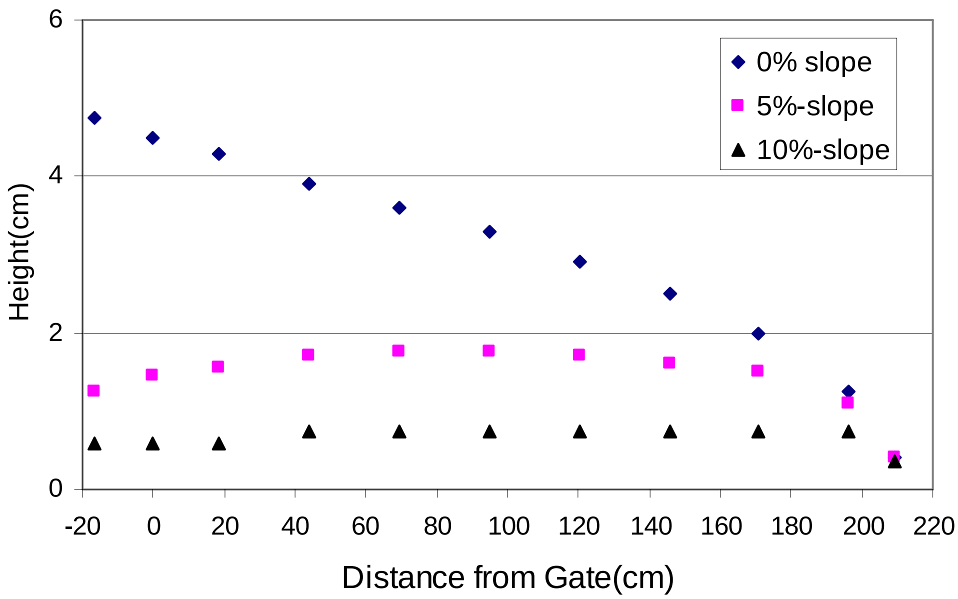

Figure 5-8 Beach profile along the flume length for test set 2 (57.75%s, SFR=2) at different slopes

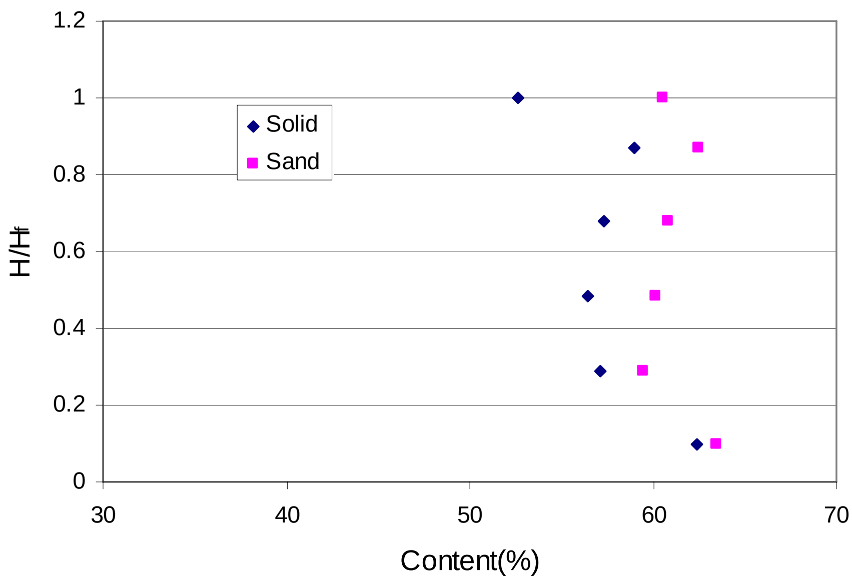

Figure 5-9 Solid and sand content profile for sample deposit in a standpipe at the 5% slope- flume end

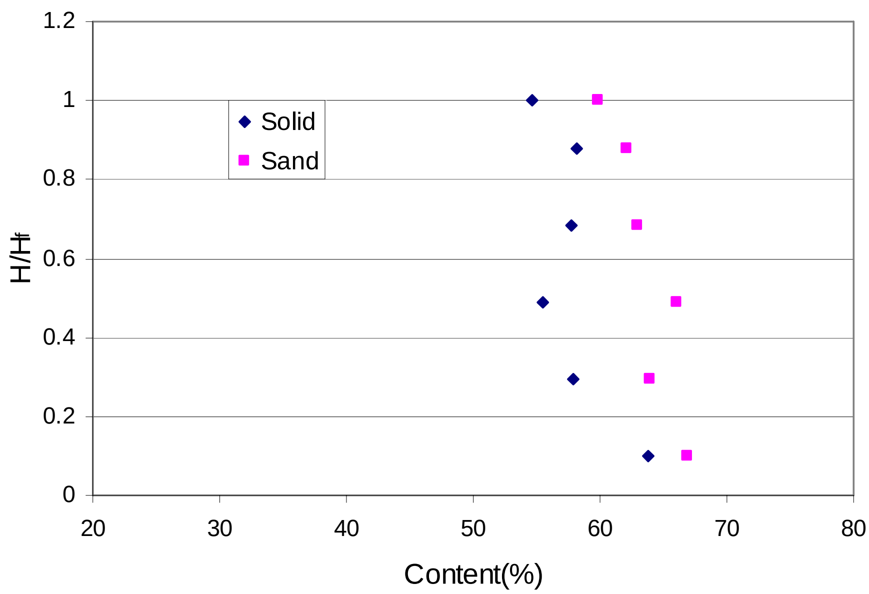

Figure 5-10 Solid and Sand content profile for sample deposited in a standpipe after exited from the 10% slope-flume end

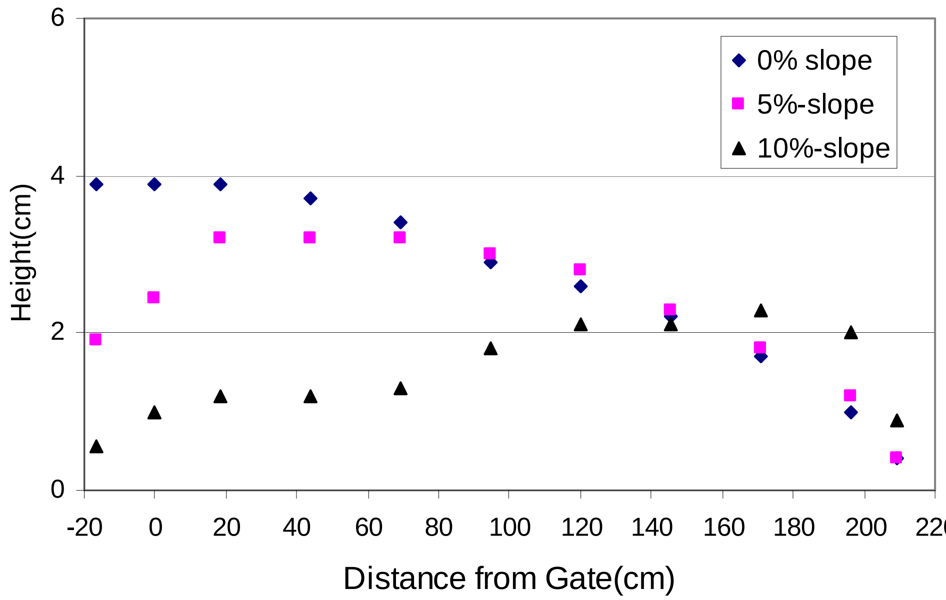

Figure 5-11 Beach profile along the flume length test set 3 (55.6%s, SFR=4) at different slopes

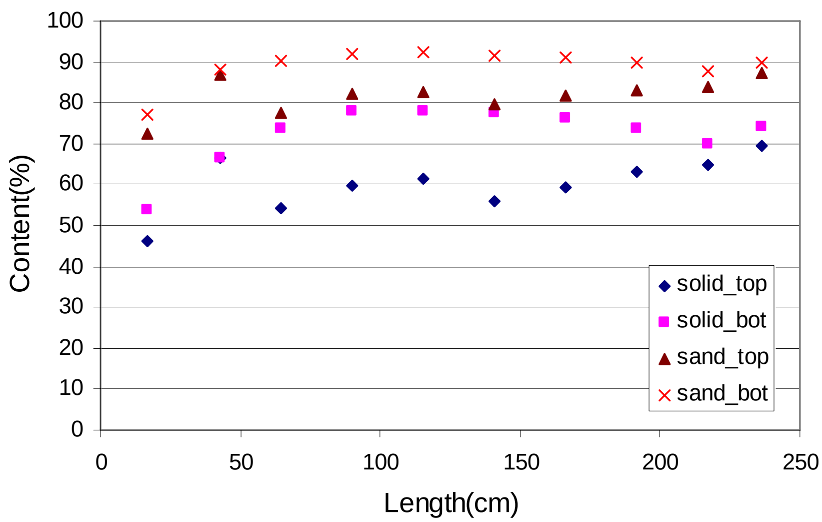

Figure 5-12 Solid and sand content profile along the flume length test set 3 and zero bed slope

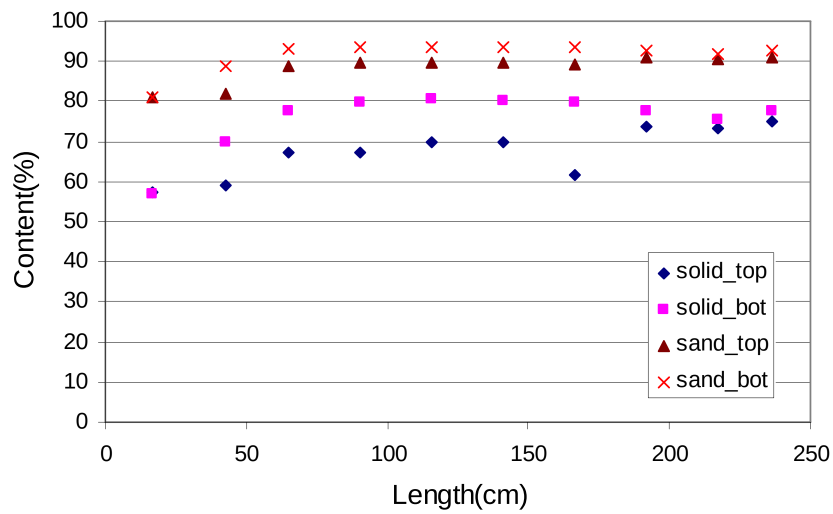

Figure 5-13 Solid and sand content profile along the flume length test set 3 and 5% bed slope (55.6%s, SFR=4)

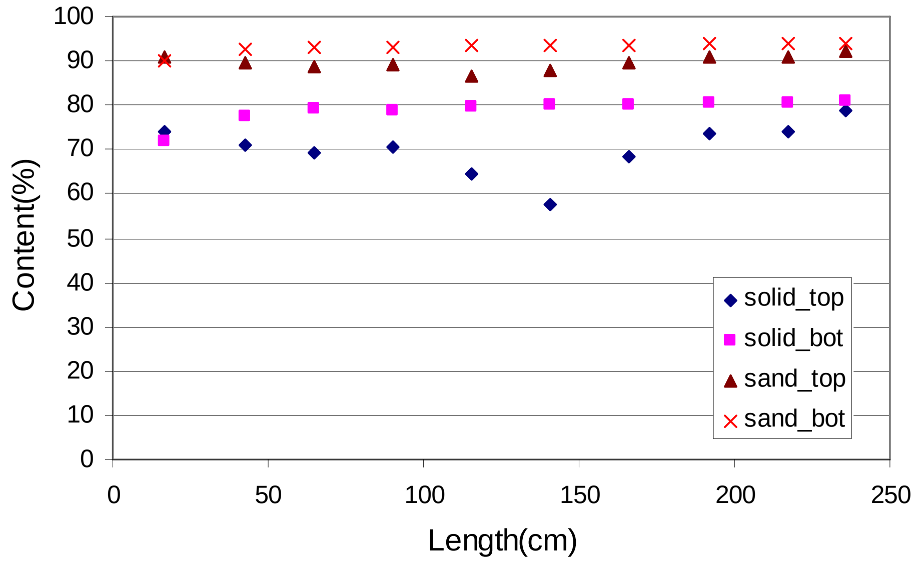

Figure 5-14 Solid and sand content profile along the flume length test set 3 and 10% bed slope (design slurry: 55.6%s, SFR=4)

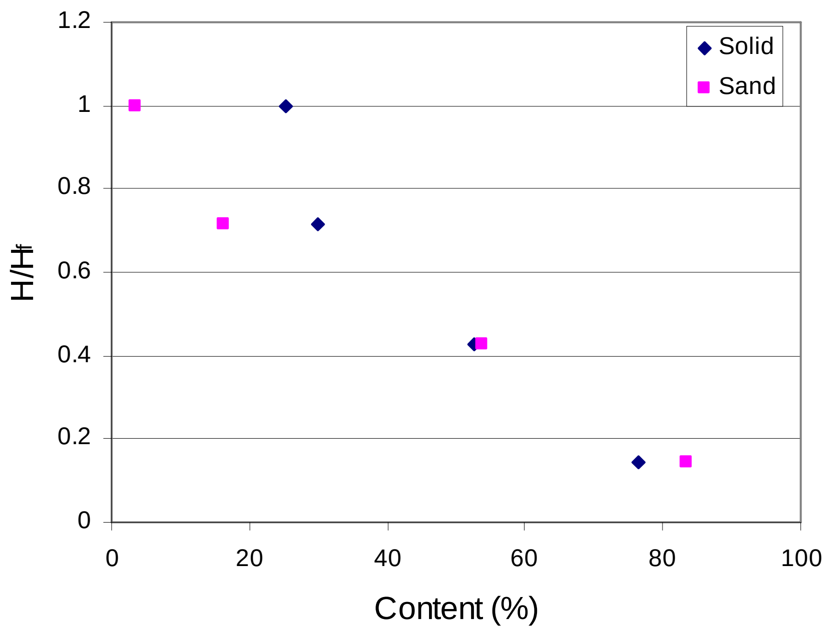

Figure 5-15 Solid and sand content profile for sample deposit in a standpipe at the zero- slope- flume end. (design slurry: 55.6%s, SFR= 4)

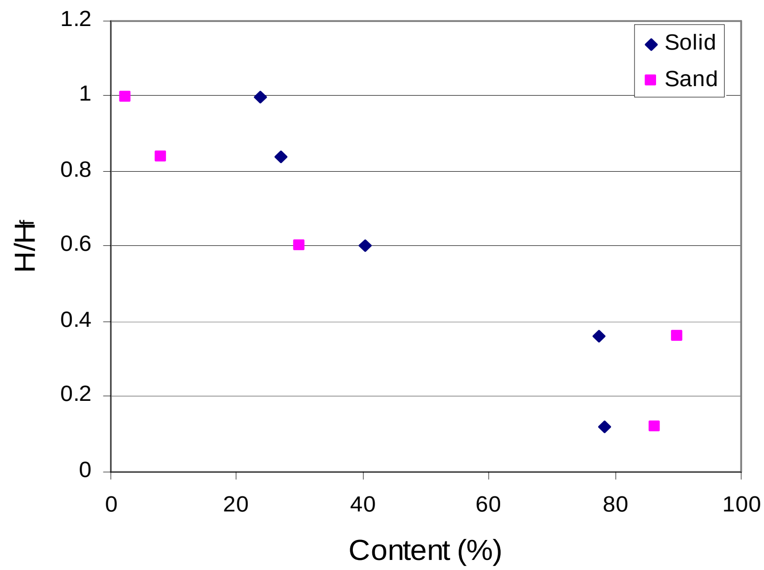

Figure 5-16 Solid and sand content profile for sample deposit in a standpipe at the 10%- slope- flume end. (design slurry: 55.6%s, SFR= 4)

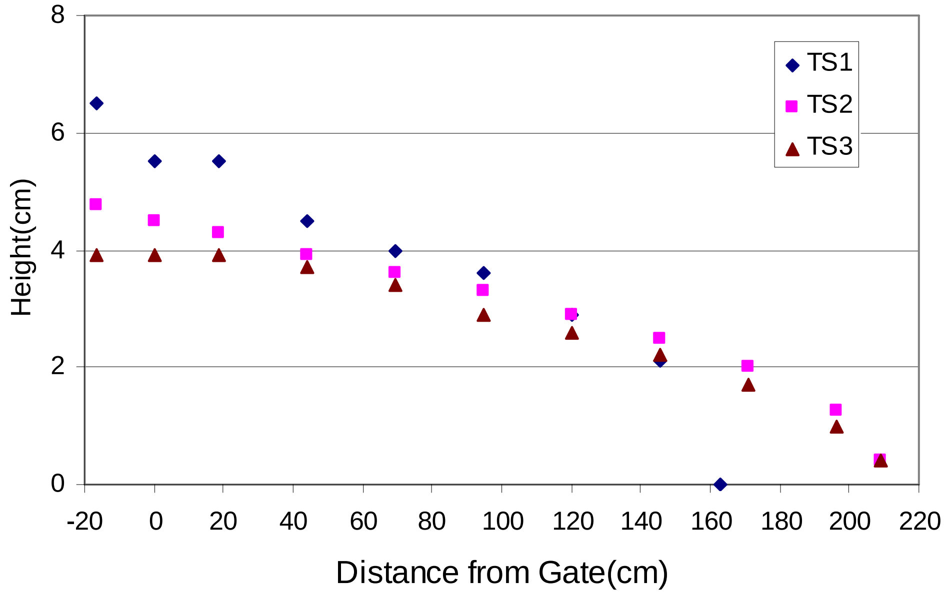

Figure 5-17 Beach profile comparison at zero bed slope, and for three test sets with different sand fine ratios

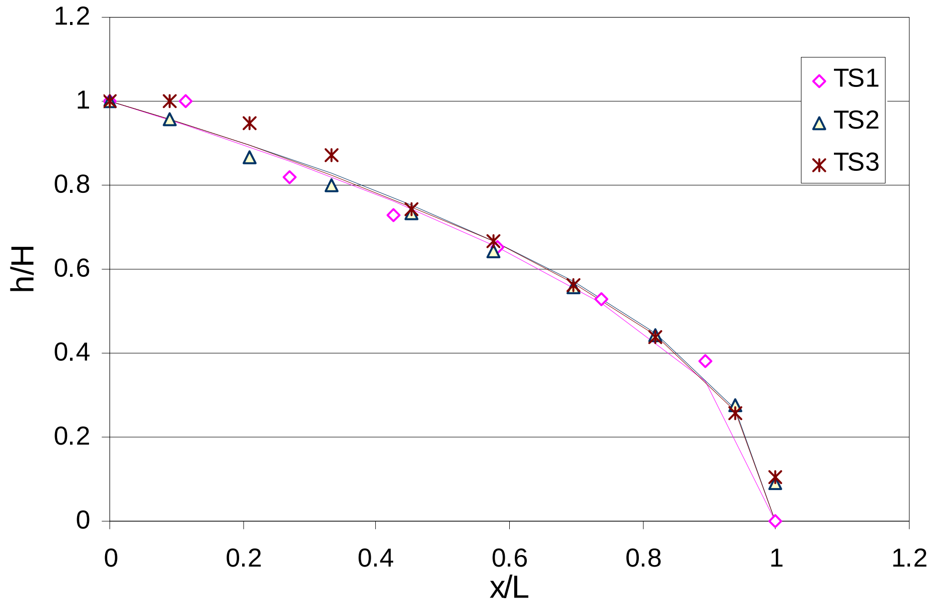

Figure 5-18 Normalized beach profile comparison from Figure 5-17

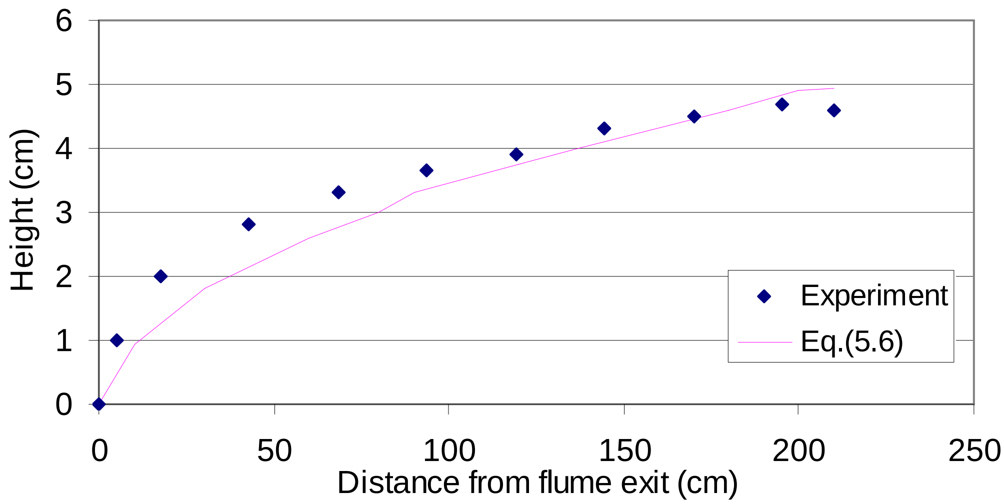

Figure 5-19 Comparison of experimental data with plastic theory equation results with a=0.06, b=1.0e-5 and d= 0.1

CHAPTER 6

6. NUMERICAL MODEL STUDY

6.1 General

Numerical modeling is essentially an exercise by which we seek to provide some form of functional relationship among important governing variables. Numerical modeling has become a great tool in modern science and engineering. When appropriately formulated, modeling has some powerful features which are impossible otherwise to deal with by experimental means, such as: when experimental data give limited results, rather than variable physical process, when sampling methods used in the experiment disturb the physical process, and when future prediction of the physical process is of interest.

In the 42nd Rankine lecture, Potts (2003) addressed the different applications of numerical analysis in geotechnical engineering. He discussed that numerical analysis has led to economical design. He pointed out that the ability for a numerical analysis to reflect field condition depends on the ability of the constitutive models to represent real soil behaviour.

The range of applications in geotechnical engineering is very wide but the emphasis of this study will be focused on the sedimentation/consolidation or more specifically segregation which involves multiphase system. Multiphase flow consists of mixtures of different phases. The different phases refer to solid, liquid and gases. Multiphase flow forms can be gas-liquid, gas-solid, liquid-solid or a mixture of the three phases. They occur almost in any natural and man-made processes.

It is the liquid-solid flow that would be a primary interest here where such flows are regarded as slurry flows. The solid-liquid slurry mix commonly consists of particles of different size and density. The solid particles may segregate due to differences in size or density. Due to the complexities in modeling multiphase fluids, some simplifying assumptions are commonly made to overcome the theoretical and computational difficulties (Elimelech et al. 1995). Model studies, providing systematic information on well-defined systems, can usefully contribute to the greater understanding of real process.

The modeling works carried out to capture or simulate the segregation phenomena are mainly developed based on the continuum or discontinuum approach. When the interstitial fluid presence is dominant, the continuum approach is the common one. Whereas discontinuum methods appears to dominate the cases of granular flows mechanism. In the continuum case, the early models appear with the sedimentation, which are based on the theories presented in Chapter 2. There is a report by Selim et al. (1983) that the sedimentation of poly-disperse system were made by one of the co-worker - Kothari(1981). Zimmels (1988), provided a computer model for non-steady sedimentation of polydisperse mixture. The model uses the principle of tracing positions and concentrations of all particle fractions which constitute a distribution, given the initial conditions and elapsed time.

Concha et al. (1992) presented a model which simulates the settling of mixtures of ideal polydisperse suspension based on Kynch sedimentation process. In their model, the solid concentration and solid-fluid flux relationships are derived from the Kynch sedimentation process and the constitutive equation for the relative solid-fluid velocities from the well known Richardson and Zaki equation as proposed by Masliyah (1979). Most of the existing models for sedimentation deal with the settling velocity. However, it is the solid concentration profile that provides more information for characterizing sedimentation process as a function of time (Quispe et al. 2000).

Stamatakis and Tien (1992) provided an algorithm which predicts the sedimentation of the polydisprsed suspension. Eckert et al. (1996) developed a model predicting sedimentation and consolidation of fine tails using experimental results from centrifuge. They used curve fitting techniques to determine the state functions. They also showed that the governing equation for consolidation used in soil mechanics area and the fluid dynamics equations can be shown to yield the same formulation. The model applies to only fine tailings part where the coarse grains were discarded. Masala (1998) developed the coupled sedimentation consolidation model. The inherent derivation in the approach is based on the assumption that the suspension is ‘slow-settling’ (‘non-segregating’ dispersions, colloidal suspension, etc). It is developed for mono-dispersed type of suspensions only and requires calibration with experimental data.

The case of aerodynamic consideration as fluid was seen in a work by Mudryy et al. (1999). They used finite element code based on distributed Lagarangian multiplier/method (DLM) to study the motion of flowing powders in the regime where the aerodynamic forces are important. They modeled the air-flow using the Navier-Stokes equation. They have demonstrated that aerodynamic forces cause mixing when the particles are identical, and segregation when the particles are different in density. The non-Newtonian flow of open-channel flow modeling appeared in the work of Sadd and Gao (1997) and Whipple (1997).

In the discontinuum approach, the most widely used method is the distinct or discrete element method (DEM). The method has proved its power of application in modeling mixing and segregation. The DEM has been applied for quasi-static cases (Cundall and Strack 1979) and for granular flows by (Yamane 1999); (Ristow 1999);(Kenji Iimura et al. 1998); (Cleary et al. 1998); (Cleary 2001);(Asmar et al. 2002). The works of Ristow, Yamane and Cleary have focused on a rotating cylinder (drum). Cundall (2001) presented a comparison of continuum and discontinuum with emphasis on the geotechnical example of soils and rocks.

The main feature of DEM is that the collision and interaction of particles with their environment is captured by the contact force law of physics. The particles are regarded as soft or hard, depending upon the modelers’ preference. The soft-particle model allows simultaneous and prolonged contact between the particles, which is applicable in quasi-static and rapid flow problems. It was applied first for soil mechanics by Cundall and Strack (1979). Others contributions appear in different literature (Oda 1997), (Sadd and

Gao 1997). The hard-particle model takes the interaction of particles as binary and instantaneous collision, which uses the principle of impact dynamics. The model was first applied for granular flow by Campbell (Shen and Babic 1999). Backgrounds of the important developments in DEM models can be found in (Cundall and Strack 1979); (Kenji Iimura et al. 1998);(Shen and Babic 1999);(Cleary et al. 1998); (Cleary 2001).

Since granular particles which are subjected to segregation are individual grains the consideration of the discrete element method which bases the system of solution on discontinuum appears to be promising. The DEM has made its way from research tool to a vital tool in industrial simulation. The problem associated with DEM is the high computational time and difficulty of properly representing the field condition. The computation time is directly proportional to the displacement modeled (Cundall 2001). For large mobility involved in segregation situations, this may be a bad news. But the development of powerful computers may overcome such difficulties.

Most of the DEM models ignore the fluid drag effect of the fluid, as they deal on either quasi-static situation or granular flow cases where the air drag effect is neglected. Lately some models have attempted to account for hydrodynamic effects (Kenji Iimura et al. 1998), (Asmar et al. 2002), (Limtrakul et al. 2003).

A discrete element method approach to model the behaviour of clay has been presented by Anandarajah (1994). The model considers the inter-particle mechanical and physicochemical repulsive force while ignoring the attractive forces. The model indicates some insight into the application of DEM in simulating the fundamental stress-strain behaviour of cohesive soils.

Even though the phenomenon of segregation in sedimentation process is one of the little understood subjects, there has been considerable effort to explain the process mathematically.

It will be shown here the application of available models to match experimental results. In this respect the approach for the investigation of numerical studies of segregation is the model developed in particulate fluid mechanics. The underlying principle in the theory is the conservation of mass (continuity) and conservation of momentum according to Gidaspow (1994) with supplements from ANSYS CFX manual will be presented hereunder in view of kinetic theory.

6.2 Working Model Description

6.2.1 Continuity

$$\frac{\partial \phi_i \rho_i}{\partial t} + \frac{\partial}{\partial x_i}(\phi_i \rho_i U_i) = \dot{m}_i \qquad (6.1)$$

where ϕ_i and ρ_i are the volume fraction and density of the ith component, U_i is the velocity vector and the term on the right side m_i is user specified mass source.

6.2.2 Momentum

$$\frac{\partial(\phi_i \rho_i U_i)}{\partial t} + \frac{\partial}{\partial x_j}(\phi_i \rho_i U_i U_j) = \phi_i \rho_i F_i + \nabla.\tau_i + \beta_B (U_j - U_i) + \dot{m}_j U_j \qquad (6.2)$$

where β_B is the fluid-particle friction coefficient.

6.2.3 Constitutive equation for continuous phase stress and buoyancy

$$\tau_c = \left[-P_c + \xi_c \nabla.U_c\right] I + 2\mu_c S_c \qquad (6.3)$$

where

$$S_c = \frac{1}{2}\left[\nabla U_c + (\nabla U_c)^T\right] - \frac{1}{3}\nabla.U_c I \qquad (6.3a)$$

$$F_c = g / \phi_c \qquad (6.3b)$$

6.2.4 Constitutive equation for solid phase stress and buoyancy

$$\tau_s = \left[-P_s + \xi_s \nabla . U_s\right] I + 2\mu_s S_s \tag{6.4}$$

$$S_s = \frac{1}{2}\left[\nabla U_s + (\nabla U_s)^T\right] - \frac{1}{3}\nabla . U_s I \tag{6.4a}$$

$$F_c = g(1 - \rho_c / \rho_s) \tag{6.4b}$$

6.2.5 Solids Phase Stress:

A) Solid Phase Pressure –Empirical Constitutive Equation

$$P_s = P_s(\phi_s) \Rightarrow \nabla P_s = G(\phi_s)\nabla \phi_s \tag{6.5}$$

$$G(\phi_s) = G_o e^{C(\phi_s - \phi_{sm})} \tag{6.5a}$$

Where $G(\phi_s)$ is the Elastic Modulus G_o is the reference Elasticity Modulus, C is the compaction modulus, and ϕ_{sm} is the maximum packing parameter.

In ANSYS-CFX the Gidaspow model is implemented with the option of specifying the reference Elasticity Modulus and Compaction Modulus. Direct option for Elasticity modulus is also available.

B) Solid Phase Pressure -Kinetic Theory Models

The kinetic theory model for solid pressure is a modified form of the equation of state for ideal gases by accounting for inelastic collisions, and maximum solid packing.

$$P_s = \rho_s \phi_s \Theta[1 + 2(1+e) g_o \phi_s] \tag{6.6}$$

where Θ is the granular temperature, e is the coefficient of restitution for solid-solid collisions and g_o is the radial distribution function given as:

Gidaspow (1994)

$$g_o = 0.6(1-(\phi_s/\phi_{sm})^{1/3}]^{-1} \tag{6.7}$$

Lun and Savage (1986)

$$g_o = (1-(\phi_s/\phi_{sm})]^{-2.5\phi_{sm}} \tag{6.8}$$

As ϕ_s approaches ϕ_{sm}, g_o tends to infinity, such singularity is being removed in ANSYS CFX by the form of equation as

$$g_o = C_o + C_1(\phi_s-\phi_c) + C_2(\phi_s-\phi_c) + C_3(\phi_s-\phi_c)]^3 \qquad (\phi_s \geq \phi_c) \tag{6.9}$$

Where $\phi_c=\phi_{sm}-0.001$, $C_0=1079$, $C_1= 1.08 \times 10^6$, $C_2=1000C_1$ and $C_3=1000C_2$

C) Solids Phase Bulk Viscosity:

$$\xi_s = \frac{4}{3}\phi_s^2\rho_s d_p g_0(1+e)\sqrt{\frac{\Theta}{\pi}} \tag{6.10}$$

D) Solids Phase Shear Viscosity:

$$\mu_s = \frac{2\mu_{s_{dil}}}{(1+e)g_0}\left[1+\frac{4}{5}(1+e)g_0\phi_s\right]^2 + \frac{4}{5}\phi_s^2\rho_s d_p g_0(1+e)\sqrt{\frac{\Theta}{\pi}} \tag{6.11}$$

where μ_{sdil} is Solid Phase Dilute Viscosity given as:

$$\mu_{s_{dil}} = \frac{5\sqrt{\pi}}{96}\rho_s d_p\sqrt{\Theta} \tag{6.12}$$

6.2.6 Granular Temperature:

(A) Algebraic Equilibrium Model

Granular temperature may be computed from the assumption of local equilibrium in a transport equation model, i.e.

$$production = Dissipation \Rightarrow \tau_{sij} \frac{\partial U_i}{\partial x_j} = \gamma_s \qquad (6.13)$$

Where τ_{sij} denotes the shear stress tensor (Equation 6.4), and

$$\gamma_s = 3(1-e^2)\phi_s^2 \rho_s g_0 \Theta \left(\frac{4}{d_p} \left(\sqrt{\frac{\Theta}{\pi}} - \nabla.U \right) \right) \qquad (6.14)$$

Thus

$$\tau_{sij} = -P_s \nabla.U + \mu_s \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \frac{\partial U_i}{\partial x_j} + \left(\xi_s - \frac{2}{3}\mu_s \right) (\nabla.U)^2 \qquad (6.15)$$

$$\Rightarrow Production = -P_s D + \mu_s S^2 + \lambda_s D^2 \qquad (6.16)$$

Where

$$\lambda_s = \xi_s - \frac{2}{3}\mu_s \text{ , } D = \nabla.U \text{ and } S^2 = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)^2 \qquad (6.16a)$$

In order to determine Θ_s from (Equation 6.13), the dependence of solids pressure and shear bulk viscosities on Θ_s is taken into account, i.e.,

$$P_s \propto \Theta, \mu_s \propto \sqrt{\Theta}, \xi \propto \sqrt{\Theta} \qquad (6.17)$$

So one may write:

$$P_s = P_{s0}\Theta,\ \mu_s = \mu_{s0}\sqrt{\Theta},\ \xi_s = \xi_{so}\sqrt{\Theta} \text{ and } \lambda_s = \lambda_{so}\sqrt{\Theta} \qquad (6.18)$$

Substituting (6.18) into (Equation 6.16), one gets

$$Production = (\lambda_{s0}D^2 + \mu_{s0}S^2)\sqrt{\Theta} - P_{s0}D\Theta$$
$$= A_P\sqrt{\Theta} - B_P\Theta \qquad (6.19)$$

Where:

$$A_P = \lambda_{s0}D^2 + \mu_{s0}S^2 \geq 0 \text{ and } B_P = P_{s0}D \qquad (6.20)$$

Equation (6.14) can also be expressed as

$$Dissipation = A_D\Theta^{3/2} - B_D\Theta \qquad (6.21)$$

Where:

$$A_D = \frac{4}{d_s\sqrt{\pi}}C_D \geq 0,\ B_D = C_D D \text{ and } C_D = 3(1-e^2)\phi_s^2\rho_s g_0 \geq 0 \qquad (6.22)$$

Equating (Equation 6.19) and (Equation 6.21) gives

$$A_P\sqrt{\Theta} - B_P\Theta = A_D\Theta^{3/2} - B_D\Theta \qquad (6.23)$$

Dividing both sides of Equation (6.23) by $\sqrt{\Theta}$ yields the following quadratic equation

$$A_D\Theta + (B_P - B_D)\sqrt{\Theta} - A_P = 0 \qquad (6.24)$$

For the strict case of coefficient of restitution less than unity and when Ap> 0, a positive solution for $\sqrt{\Theta}$ can be found as:

$$\sqrt{\Theta} = \frac{B_D - B_P + \sqrt{(B_D - B_P)^2 + 4A_DA_P}}{2A_D} \qquad (6.25)$$

or

$$\Theta = \left(\frac{B_D - B_P + \sqrt{(B_D - B_P)^2 + 4A_DA_P}}{2A_D}\right)^2 \qquad (6.25a)$$

It has been observed that the algebraic equilibrium can give ‘unphysically’ large granular temperatures in regions of low solid particle volume concentration. ANSYS CFX recommends that user specified upper bound be set.

(B) Zero-Equation Model

An algebraic model for granular temperature as proposed by Ding and Gidaspow is given as:

$$\Theta = \frac{1}{15(1-e)} d_p^2 S^2 \qquad (6.26)$$

In the following section numerical simulation for a sedimentation experiment on 5 litre segmented stand pipe will be presented.

6.3 Results and Discussion

ANSYS-CFX software is applied for this purpose. Such experimentation would enable us to observe the segregation process in a simulated manner. In the case of complex rheological model, for which it is difficult to model using the conventional CFD software method, approximate methods such as bi-viscous methods may be applied. Figure 6-1 shows short duration numerical analysis result of sand clay slurry mixed at a total solid content of about 46% and sand fine ratio of 1. The figure indicates clay content profile at different times. The analysis time was up to about 3min.

It is observed that during the analysis time there is little change in solid content with respect to clay. It shows only slight changes: a decrease at the bottom and an increase at the top sections. In the middle it keeps the initial solid content. Also similar trend but reverse profile is shown in Figure 6-2. The analysis result indicate that there is segregation, i.e. an increase in sand content and decrease in clay content at bottom and vice versa at the top part.

At increased solid content and fine ratio but keeping the analysis time the same 3 min was conducted for a solid content of about 53% with the sand fine ratio increased to 2. Figure 6-3 and Figure 6-4 show clay content and sand content profiles with depth, respectively.

It appears that despite an increase in solid content and sand fine ratio the tendency to segregate is still limited at the bottom and top sections only.

A very high solid content of about 62% and sand fine ratio of 2 is run for 24hrs. The analysis results for clay and sand are indicated in Figure 6-5 and Figure 6-6 respectively. Figure 6-6 indicates that the clay particle size is highly concentrated in the middle section and the sand is nearly deposited at the bottom. Though the solid content is very high and the slurry is of non-segregating nature, the numerical model indicates full segregation behaviour. This tendency shows that the particulate fluid dynamics analysis does not account for the fine matrix rheological influences. Thus the modelling should be made to account this by considering the fine slurry as a pore medium with rheological properties found from experiment and apply a different method of approach. For this reason bi-viscous model approach is adopted and is described in the following section.

6.4 ***Bi-viscous model analysis***

The treatment of visco-plastic fluids having yield stress is numerically difficult due to the condition that yield stress is ideally a stress at zero shear rate at which point there is a singularity condition for viscosity. In order to overcome such a difficulty a bi-viscous model approach is commonly used Lipscomb and Denn (1984); Lipscomb and Denn (1984), Gartling and Phan-Thien (1984), O'Donovan and Tanner (1984), Atapatu et al. (1995), and Balmforth et al.(2000).

The viscosity equation of bi-viscous system for Herschel-Bulkley model is given by Equation (6.27). For a shear rate less than $\dot{\gamma}_o$, a constant viscosity is assumed. For shear rates greater than $\dot{\gamma}_o$ the viscosity is equated from the model expression.

$$\begin{aligned} \eta &= \eta_o \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\dot{\gamma} < \dot{\gamma}_o) \\ \eta &= \tau_y \dot{\gamma}^{-1} + K\dot{\gamma}^{n-1} \ldots\ldots\ldots\ldots(\dot{\gamma} \geq \dot{\gamma}_o) \end{aligned} \tag{6.27}$$

For such kind of analysis, a rheological model from the experimental data of 40%s(w/w) slurry has been used.

The choice of cap-viscosity (η_o), i.e. the viscosity at a very low shear rate, is difficult to make. It ideally marks the position where flow begins. (Cardwell 1941) specified the shear rate of 10^{-3}(1/s) as the point at which the experimental recognition of flow begins. The corresponding viscosity from the rheological model of 40%s kaolinite slurry, is about $1.2*10^4$Pas. For this analysis, a cap-viscosity of $5*10^3$Pas is chosen.

For the analysis, multi-phase fluid dynamics software ANSYS CFX has been implemented. For the viscosity option of bi-viscous situation, Equation (6.27) has been put in the form shown in Equation (6.28), which accounts for the viscosity variation for the whole shear rate range.

$$\eta = \min\left[\eta_0, \tau_y \dot{\gamma}^{-1} + K\dot{\gamma}^{n-1}\right] \tag{6.28}$$

Brown (1991) puts that the shear rate experienced by the fluid at the surface of a sphere of diameter d, settling at a velocity u, is of the order u/d; the maximum occurs at the equator and is 3u/d. And for this work the 1.5u/d is adopted. The kaolinite slurry is taken as a continuous fluid and the sand particles as a suspended solid.

The specific gravity of sand and that of kaolinite are assumed to be nearly equal, i.e, 2.7, then volume fraction of sand is computed as 0.165, and the density of homogeneous kaolinite slurry is about 1337kg/m^3. It is observed that the cap-viscosity predominate the viscosity used in the model, indicating that the shear rate is very small to take up the rheological model. Thus one may increase the cap-viscosity value in case of unsatisfactory results. Such action was not needed in this work.

Figure 6-8 shows the comparison of the solid content profile for 24hrs simulation. The simulation results indicate that it is only at the top and bottom parts that small variation in concentration results. Also comparison with experimental data in terms of sand solid content is shown in Figure 6-9. From this figure it is to observe that both experimental and model result fall in a close band except at the top and bottom. The trend indicated by the bi-viscous model analysis indicates that at the top decrease in the sand content and at the bottom an increase in sand content. This is somehow reasonable.

Even though the choice of viscosity was made about half of the definition value, it still gives satisfactory results. The other limitation to the choice of bi-viscous model is that at a very small shear rate, the viscosity may be very high that the numerical scheme becomes very stiff yielding no result. Thus the choice of cap-viscosity as indicated in Equation (6.28) depends not only on the defined value but also on the computational limits of applied schemes. From this work, it is encouraging to have a comparable bi-viscous model to the experimental result. It is also not fully conclusive at this stage since one needs to deal within the limits of using this approach..

It is apparent from the results that the fine matrix contribute to the capture of larger size particles (sand) with regard to the time span to which the test has been carried to identify the effect of stress degradation at the sedimentation stage and compaction as consolidation takes place require further examination.

The study of sand settling in fine slurry has two major applications such as transport of coarse particles carried by fine medium and treatment of poorly consolidating tailings ponds. The transport of coarse particle with fine medium produces easily transportable, non-segregating and relatively high solid content material, which require reduced impoundment volume. On the other hand tailings ponds are known for their very slow consolidation and very little release water. In such cases understanding the mechanism of inter-particle and fluid-particle interaction helps one to implement an effective way of material handling and deposition.

For example, internal surcharge of fine tailings by coarse sand material assist the consolidation process. At the same time optimum application can be made if the rheology of the mixture is properly understood. The same may apply also in case of capping of soft sediment by sand, or handling of dredged material in coastal areas. Also the presence of coarse particle give improved strength of the deposited material which help the reclamation process and closure plans.

The implementation of the mechanism into a numerical scheme is still at developing stage mainly due to the complexity of the mechanism involved and a limited understanding in that respect. With the available knowledge and some simplifying assumption it is possible to model physical phenomena. The use of bi-viscous model is a typical example of a simplifying assumption. Moreover, no theoretical account has been given for some visible physical phenomena such as stress degradation with time and channelling, indicating that there exist still more research need in the area.

The numerical simulation of the motion of particles in a fluid during sedimentation process is a very difficult problem that until today has not been solved to entire satisfaction (Quispe et al. 2000). This statement appears to be very true and concurring.

6.5 Summary and Conclusion

The principles of particulate fluid mechanics were applied to match the experimental results in this study. The numerical simulations as developed in computational fluid mechanics were the basis of the study. Commercial software package, ANSYS-CFX, was implemented for this study.

The kinetic theory model was implemented in the solid phase pressure in the study. For slurries exhibiting yield stress a bi-viscous approach was applied to account for the effect of yield stress.

Numerical simulations involve some simplifying assumptions either due to lacking physics or computational difficulties. Comparison of numerical results with experimental data showed encouraging outcomes.

It is recommended that additional numerical study be implemented to further explain the mechanics involved such as concurrent development stress degradation and stress build-up during sediment formation.

Shaughnessy et al. (2005) states that although the use of CFD may become widespread it will never totally replace physical experimentation. This notion is applicable and shared in this research work.

6.5 References

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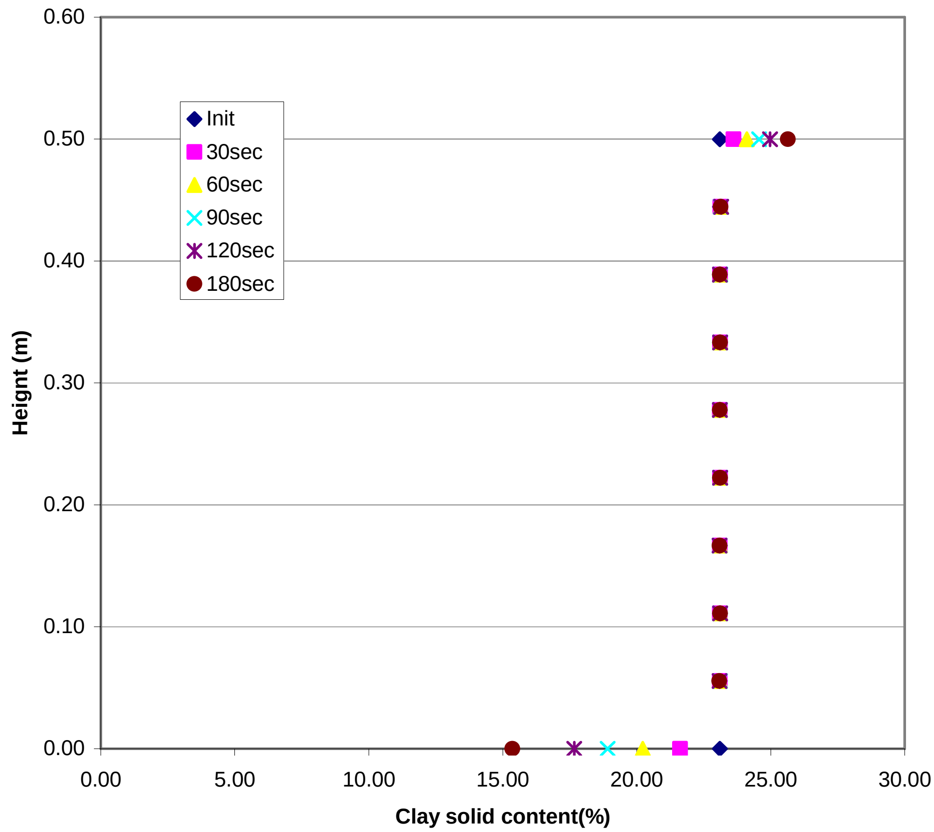

Figure 6-1 Analysis result of clay content for slurry mix at initial solid content of 46.12% and SFR=1

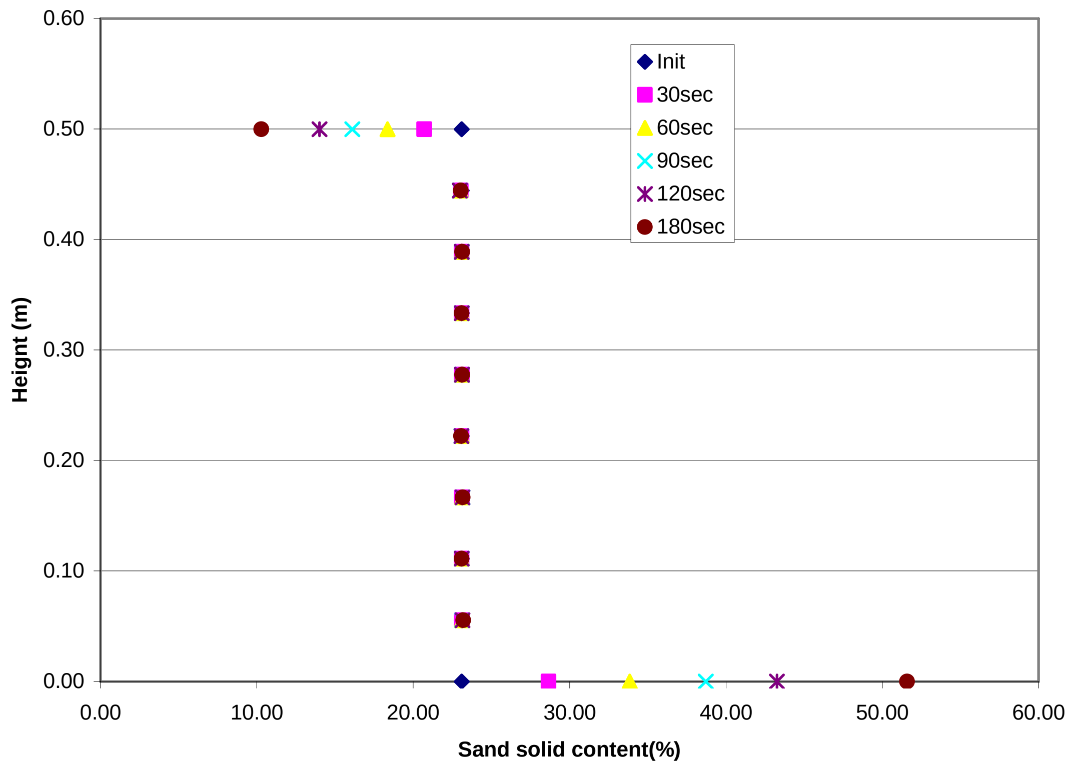

Figure 6-2 Analysis result of sand content for slurry mix at initial solid content of 46.12% and SFR=1

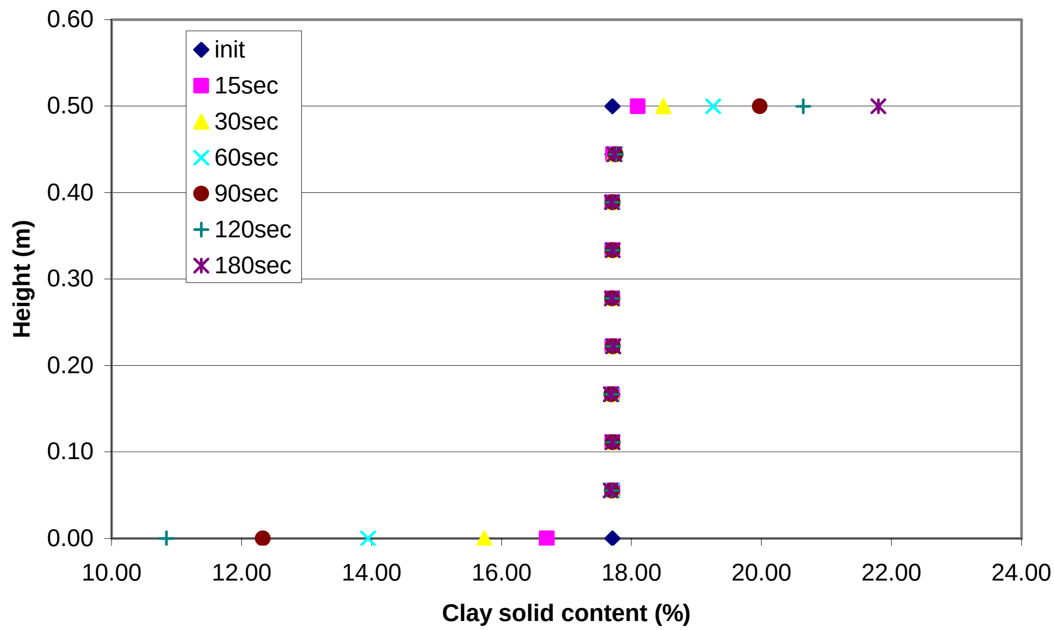

Figure 6-3 Analysis result of clay content for slurry mix at initial solid content of 53.12% and SFR =2

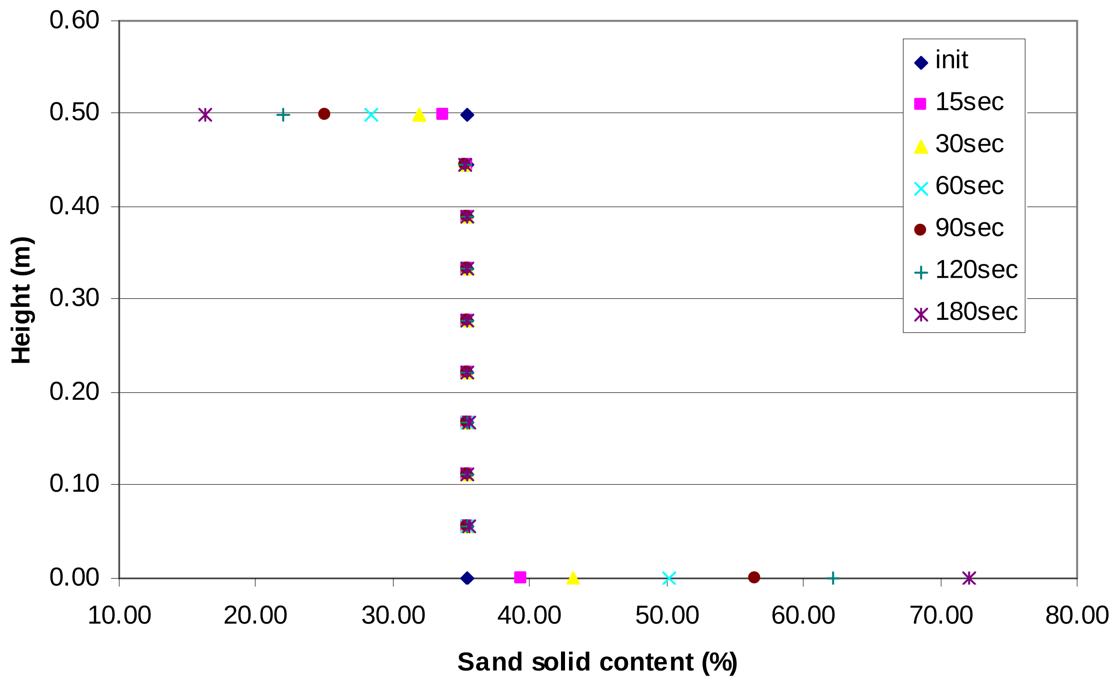

Figure 6-4 Analysis result of sand content for slurry mix at initial solid content of 53.12% and SFR =2

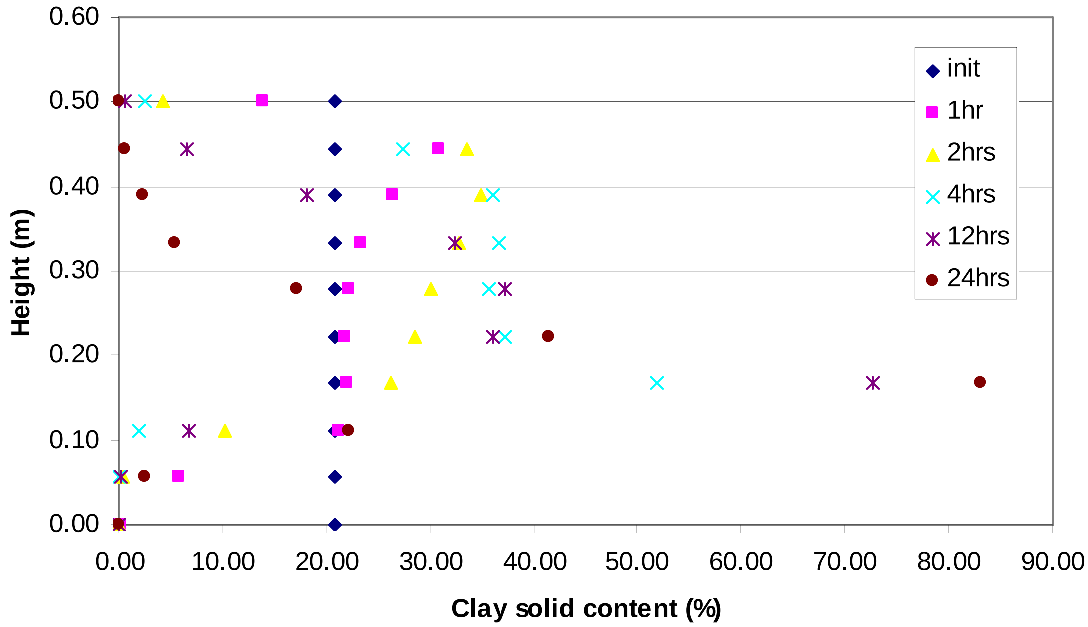

Figure 6-5 Analysis result of clay content for slurry mix at initial solid content of 62.12% and SFR = 2

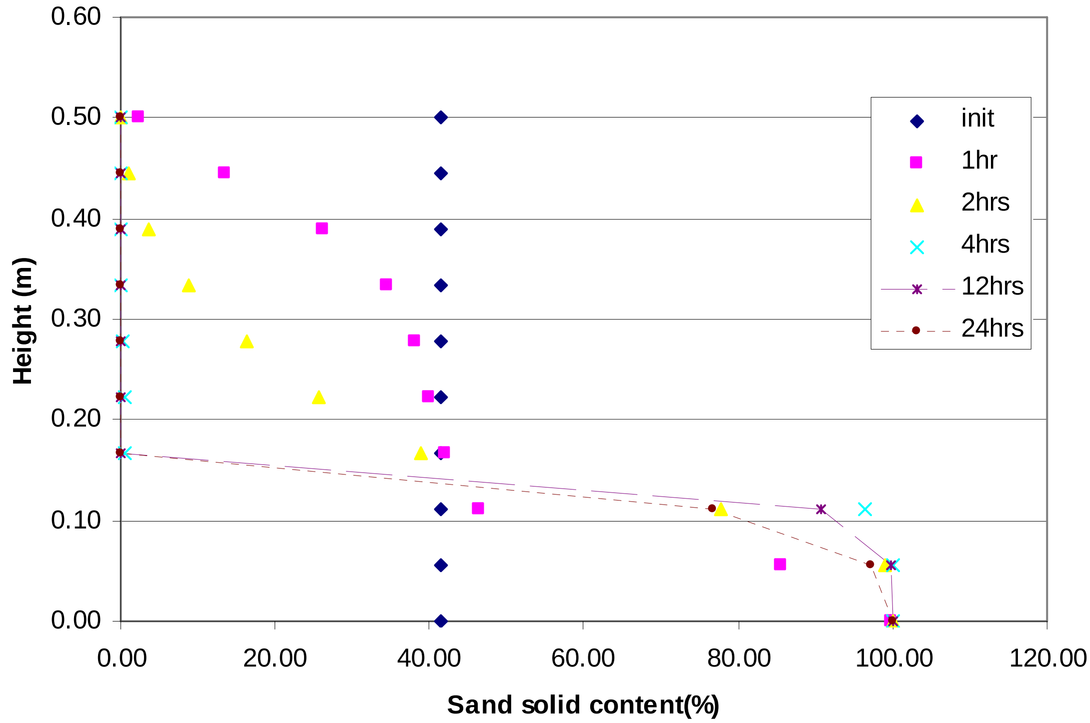

Figure 6-6 Analysis result of sand content for slurry mix at initial solid content of 62.12% and SFR = 2

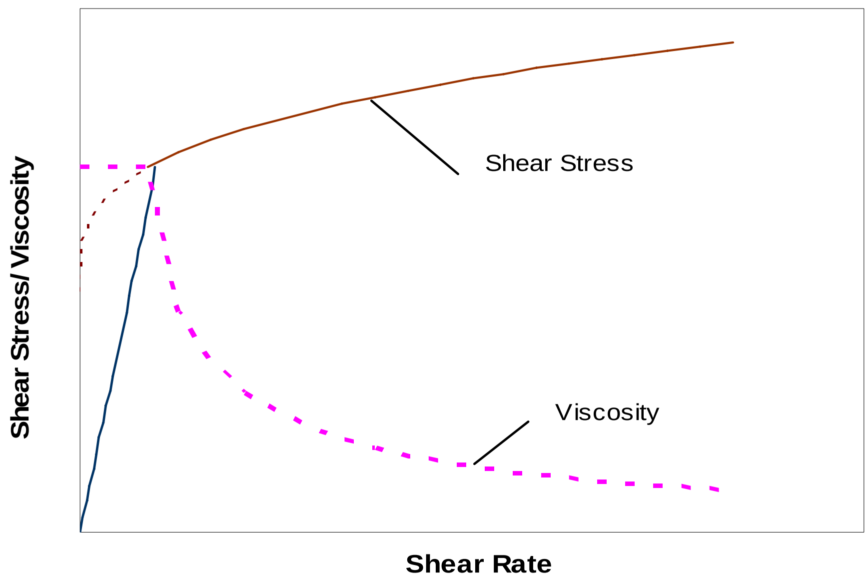

Figure 6-7 Shear stress-Shear rate function for a bi-viscous fluid model to approximate yield-stress fluid

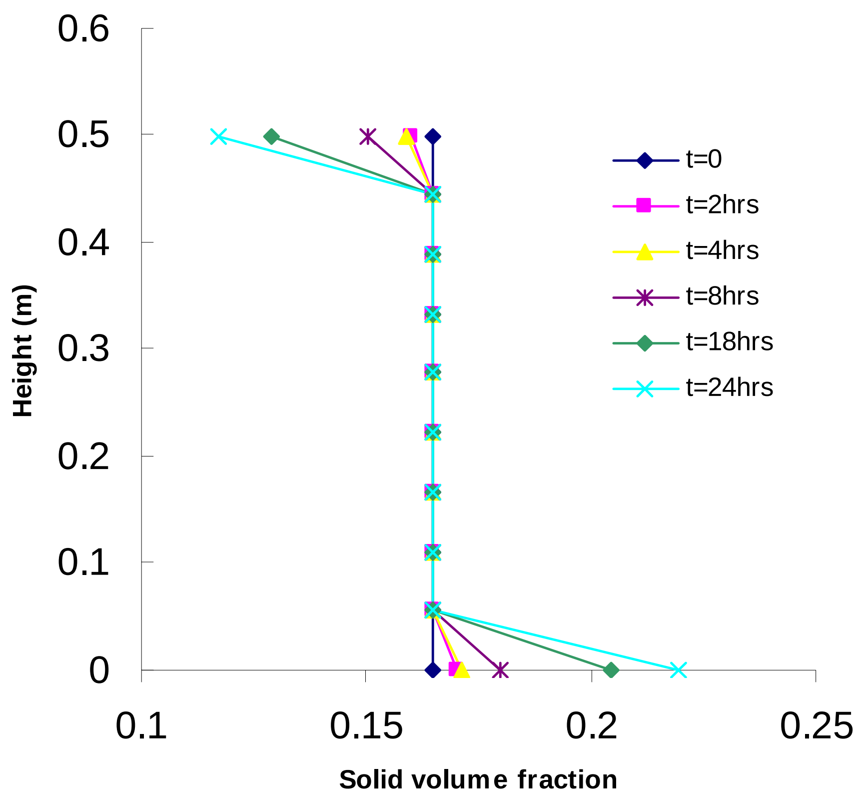

Figure 6-8 Solid (sand) volume fraction profile of sand-kaolinite slurry at 57.1%s and sand fine ratio (SFR) of 1

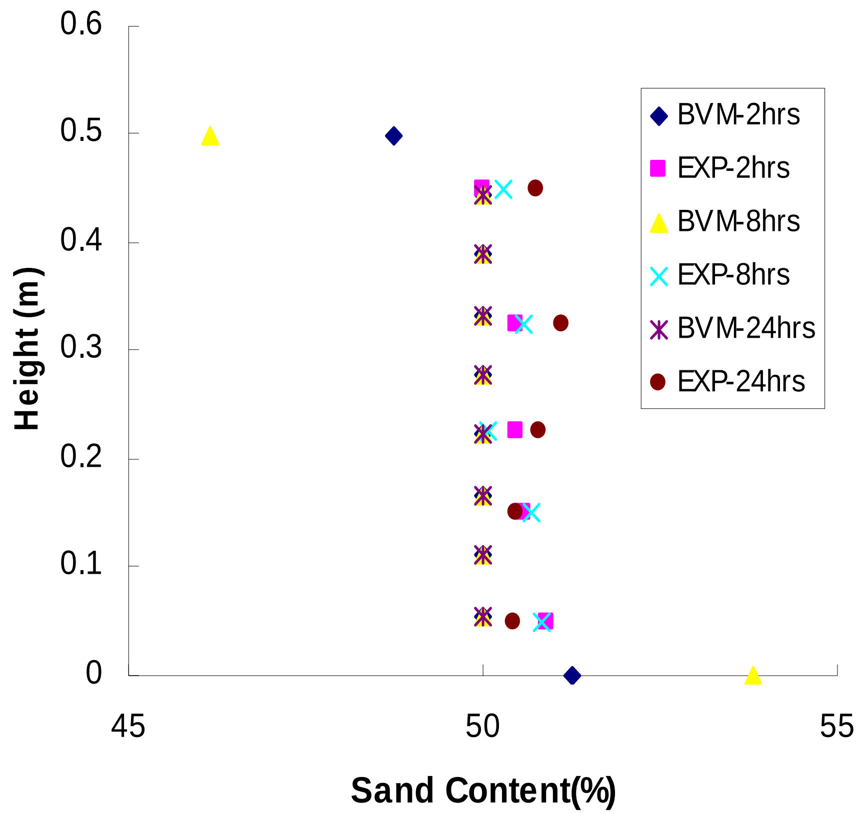

Figure 6-9 Comparison of Bi-Viscous model (BVM) and Experimental (EXP) results at different times

CHAPTER 7[4]

7. QUANTITATIVE STUDY OF SEGREGATION

7.1 General

As there is limited understanding in the subject matter of segregation, there is a need to quantify the degree of segregation or mixing so that some form of quality control or common way of describing the effect of different mechanisms is achieved. The degree of mixing or segregation is expressed commonly by measuring the statistical variation of composition using the standard deviation (Buslik (1950), Lacey (1954), Fuerstenau and Fouladi (1967) and Liss et al. (2004)).

Segregation and mixing, as two opposing phenomena, have prevailed in many industrial processes which are involved in the separation or blending of different ingredients. In the industry it is necessary to have some means of quantifying the quality of mixing so that, for example, the required performance of a mixer can be specified and then measured (Asmar et al. 2002).

According to Davies (1971) mixing of solids is defined as an operation by which two or more solid materials in particulate form are intermingled randomly in the mixer by the random movement of particles. Lacey (1954) assumed that in all mixers, mixing is achieved by one or more of the following:

(a) Convective mixing: Transfer of groups of particles from one location to another
(b) Shear mixing: Setting up of slipping planes within the mass of material
(c) Diffusive mixing: Diffusion of individual particles over newly developed surfaces.

These mechanisms control mixing to a varying degree depending on the modes of operation of the mixer. De-mixing usually accompanies mixing and two mechanisms are

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proposed to describe these effect. The first is the percolation mechanism relating to the slipping of smaller particles through the voids created by the larger particles. The other mechanisms results from differences in trajectories of materials falling at an angle during free flight.

In the case where different phases are involved in the mixtures, differences in mobility of the particles usually yields the sorting of particles. The sedimentation process and multiphase flows in which different composition of properties involved, the occurrence of segregation is more likely depending upon the concentration and flow characteristics.

Differences in size, density or shape are the main factors which contribute to the segregation of the mixture. It is widely accepted that difference in size may be the most dominant factor (Fan et al. 1970); (Rhodes 1998); (Vallance and Savage 1999).

Gravitational settling consisting of particles of different mobility results in segregation of particles. It would be of practical significance to express the degree of segregation in somewhat quantitative manner. An expression as such may provide a common basis for comparison and assessment of the quality of data output.

A quantitative index, called Segregation Index, has been in use by research group at University of Alberta, Suthaker (1995), Tang (1997) and Chalaturnyk and Scott (2001). The index is calculated based on total solid content for the depth profile after sedimentation test is completed. Another term related to Segregation Index (SI) is the fines capture, FC, given as100-SI (refer to Equations (2.10) and (2.11)). The segregation boundary, which divides the segregating type of slurry and the otherwise, is the fine capture value of 95. A value of fine capture less than 95 is categorized as segregating mix. Such an index calculation is in agreement with the definition of segregation as given by Kuepper (1991) where segregation was defined as "*the tendency of solid fraction (or part of it) to settle, creating a concentration gradient within the mass*". The segregation index constructed on the overall solid content may somehow indicate the size segregation indirectly, one finds, however, some difficulty in communicating with other experimental results (Kenney and Westland 1992) (Kaushal and Tomita 2002).

Davies (1971) presented an index for degree of mixing for samples of similar size as:

$$M = S / \sigma_R \quad , \tag{7.1}$$

where S is the estimated standard deviation and σ_R is standard deviation for completely random mixtures. The standard deviation S (for unbiased estimation) given by Equation (7.1) is

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - x)^2}{n-1}} \quad . \tag{7.2}$$

Amongst the most popular indices using standard deviation is Lacey's index (Lacey 1954), which defines a mixing index M, based on the number of particles in a sample, as

$$M = \frac{S_o^2 - S^2}{S_o^2 - S_R^2} \quad , \tag{7.3}$$

where S_o^2 is the variance for completely segregated mixture, S_R^2 is the variance for completely random mixture and S^2 is the variance of the mixture between fully random and completely segregated mixtures. This index suffers from three drawbacks: it is limited to same size particles, very sensitive near the completely segregated state, and relatively insensitive in the final stage of mixing (Fan et al. 1970);(Davies 1971); (Williams 1976) and (Asmar et al. 2002).

Asmar et al. (2002) introduced an index called Genaralised Mean Mixing Index (GMMI) in their Discrete Element Method (DEM) simulations. The GMMI in one of the axes direction, say z, is defined as:

$$GMMIz_i = \left[\frac{\sum_{j=1}^{n} (z_j - z_{ref})}{n} \right] \Big/ \left[\frac{\sum_{k=1}^{N} (z_k - z_{ref})}{N} \right] \tag{7.4}$$

where n is the number of particles of type I, N is the total number of particles, z is the z-coordinate of the position of the particle centre and z_{ref} is the reference z-coordinate.

The GMMI will be the average of the calculations in three directions. The use the index appears simple, however, its application is limited to DEM simulation where the position of individual particles is known.

In the study of segregation of granular filter materials, Kenney and Westland (1992) applied the term *Segregation Index* and *Relative Segregation Index* to determine the extent of segregation. They carried out the rotary-drum test, in which the drum is half filled with soil sample and tumbled. After the test, they divide the drum into three parts, inner 25% volume, the middle 50% and the outer 25%. Grain size analysis is carried out for samples taken from each component and plotted as percent finer vs. grain size. They determined the logarithmic mean particle size (using Popovics' equation), and they defined Segregation Index as follows:

$$SI = \log(d_c/d_f) \tag{7.5}$$

Where d_c and d_f are logarithmic mean particle sizes of the coarse and fine zones, respectively. They defined Relative Segregation Index (RSI), as the ratio of SI (test) to the Segregation Index calculated for the state of perfect segregation, SI_p.

$$RSI = SI(test)/SI_p \tag{7.6}$$

They found that the middle 50% volume is characterised by similar grain size distribution as that of the initial sample. The definition of fines and coarses, according to their work, is relative to test material, (e.g., sand may be fine for gravel and sand mix) as the division contains all particles which lie in the inner 25% and the outer 25% volume respectively. They reported that it is only under perfect segregation that complete sorting according to size may be achieved. The application of their approach to design of a filter in embankment dam is discussed by Milligan (2003). It is also shown that for grain size distribution varying in two decades; segregation is independent of grain size and grain size distribution. The idea of separating in different zones after the test and undertaking the grain size distribution seems attractive to be applied for other similar cases as well.

Other similar studies in bulk material handling (granular mixing) were using index calculation based on statistical means. According to Lacey (1954), it is generally agreed that the most useful way of expressing degree of mixture is by measuring the statistical variation of composition among samples drawn from it. A summary of different mixing indexes as used by different authors are presented by Hastie and Wypych (1999). Fan et al. (1970), referring the works of Lawrence and Beddow (1969), stated that particle density and shape were found to have little effect upon the extent of segregation.

A term degree of mixedness, as it is used by Fuerstenau and Fouladi (1967) for the study of packed particles of two different sizes, is defined as

$$M = 1 - \frac{\sigma}{\sigma_o} \tag{7.7}$$

where M is the degree of mixedness,. σ, is the observed standard deviation of sample taken from the mixture; σ_o is the expected standard deviation of samples taken before any mixing had occurred and is given by

$$\sigma_o = \sqrt{p(1-p)} \tag{7.8}$$

where p is the mass fraction of larger particles in the mixture.

Yamane (1999) defined Segregation Index, which is a different version of Equation (7.7). His index is equivalent to 1-M.

A standard deviation approach was followed by Liss et al. (2004). They introduced Relative Standard Deviation (RSD) to characterize the relative degree of segregation. The form of the expression is shown in Equation (7.9):

$$RSD = \sqrt{\frac{\sum_{i=1}^{n}(\mu - x_i)^2}{n-1}} \quad , \qquad (7.9)$$

where n is the number of samples taken, μ is the mean value of all the samples, and x_i is the value of the i^{th} sample. RSD ranges between the limits 0, perfect mix, and 1, completely segregated.

The above survey of literature shows that there is no universally accepted index. Most of the proposed mixing indices are developed for binary solid-mixtures and are based on statistical analysis, mostly the standard deviation, variance or coefficient of variation.

7.2 Suggested Methods in the Experimentation

The uniformly mixed slurry is poured into standpipe then after the sedimentation process is complete, the release water is decanted and the sediment is divided into different layers and samples for moisture content and grain size analysis. From the grain size analysis, it is possible to estimate the composition of each size in the different layers.

The profile of concentration of each particle size is plotted, which indicate the extent of segregation with respect to size groups. It is believed that a quantitative description of the extent of segregation may give some form of comparison for different test conditions. The following index, Segregation Index (SI), is proposed for the works that follow.

$$SI = \sqrt{\frac{\left(\sum h_i S_i^2 - \frac{\left(\sum h_i S_i\right)^2}{\sum h_i}\right)}{\sum h_i}} \tag{7.10}$$

where S_i is the solid content of sample section with height of h_i. This equation takes the variability of sampling depths into account by weighing them in their separate section (h_i) by assuming that the sample represents a section height from which it is taken

Such an index calculation can also be used for a test which involves different size group. It can also be applied to dynamic segregation cases.

The advantages of the proposed equation are:

- simple and very convenient to calculate
- it can be applied to multi-size particle presence
- it can account the sampling depth variation.

It is to notice that indices which are statistical in nature are basically different forms of mean deviation, standard deviation or variance. Representative spread sheet calculations are presented in Appendix D.

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CHAPTER 8

8. CONCLUSION AND RECOMMENDATION

8.1 Conclusions

A convenient and economical method of mine wastes disposal is to impound them hydraulically. However such disposal methods commonly yield segregation, i.e, settling of coarse and large size particles on the dyke beaches and transport of fine particles farther in the pond forming fine tailings. The research was aimed at establishing the fundamental factors that control the segregation mechanism in the oil sands tailings.

Rheological characterization was carried out on slurry to examine the effect of grain size, void ratio and porewater chemistry. Rheological properties were shown to be highly influenced by grain size composition, solid content (void ratio) and porewater chemistry. Generally, yield stress increases with increase in total solid content. Slurries at the same solid content but different sand-to-fine ratio show difference in yield stress, the higher the sand fine ratio the lesser the yield stress.

Comparisons of yield measurement using vane method and model fit of the conventional viscometer rheogram was showed satisfactory results. Herschel-Bulkley and Casson's model were used to fit the rheogram data. A statistical analysis was carried out to choose the better model. And for the test conditions, the Casson's model appear to fit the data better than Herschel-Bulkley model.

The relationship between yield stress and solid content for fine slurry was presented. with a model fit based on the fractal theory. Porewater chemistry effect was studied by varying the pH of the pore fluid medium. The yield stress was mildly influence in the acidic range by showing a moderate increase with increase in acidity. A significant change in yield stress was observed at pH 12 when a slurry was treated with $Ca(OH)_2$ and NaOH., indicating that higher yield stress in the case of $Ca(OH)_2$ addition versus diminished or no distinct yield stress in the case of NaOH-based pore fluid medium. Thus surface properties of clay particles and the surrounding fluid medium chemistry influence the flow/rheological properties. Reference to pH alone is not sufficient without stating the mineralogy of the solution. The higher yield stress is an indication of the potential of the fine matrix to capture coarse particle (sand) and mitigate segregation.

Rheological studies with oil sands tailings presented similar rheological trends as those of kaolinite slurries. It is therefore suggested that proceeding with the surrogate tailings materials for the rest of the experimental program would be reasonable to understand the segregation mechanism. The potential drawback for the surrogate tailing material is the lack of bitumen traces that is found oil sands tailings.

A custom-designed standpipe was used in a series of experiments to determine concentration profiles during the sedimentation process. Standpipe tests at low solid content and low Fine Water Ratio indicated that the sand particles settle to the bed immediately. However, there existed some sand particles even at the upper section after some time has elapsed. The capture mechanism is correlated to the rheological property of the fine matrix. The fine content was found to control the shape of concentration profile.

It was observed that surface properties of the fine matrix was modified by the addition of intermediate size (silt size) particles in that the settling rate (water release) was improved and the degree of segregation was minimized. The intermediate size serves as surface property modifier and internal surcharge.

The sediment profile of the segregating type of slurries is controlled by the clay water ratio. Segregating slurries at same clay water ratio exhibit similar deposit profile after sedimentation.

Theoretical formulation for coarse size capture by the yield stress of the fine matrix was presented. It was found that the theoretical formulation over-predicted the size of coarse particle that can be captured by the yield stress of the fine matrix. A correction factor was used to the theoretical size to match with experimental observation. The correction factor was compared with other data in the literature and there exist wide difference. Thus the correction factor is rather an indication of a “missing physics” that need yet to be explained well.

Flume test was conducted to study the effect of flow and shearing on relatively non-segregating slurry mixes. The shape at deposition and flow rate were observed to be governed by the solid content, sand fine ratio, bed inclination, and discharge from the reservoir. As the sand fine ratio increases, the beach profile followed nearly a parallel trend to bed slope. The tendency to segregate was also high when the sand fine ratio increases. A simplified form of non-linear first order differential equation derived from the theory of plasticity was applied to study the profile of a beach formed from flume test.

Numerical simulation using Computational Fluid Dynamic (CFD) software was used to supplement the experimental work. The CFD method implemented the kinetic theory approach and resulted in segregating slurry for all range of test. A different approach using a bi-viscous model was adopted for non-segregating type slurries and reasonable match was achieved.

A general purpose quantitative index was introduced to identify whether slurry is segregating or not. The important feature of the index is that it accounts for the thickness the sample is drawn from. Furthermore, the application of the proposed index can be extended to a dynamic segregation conditions.

8.2 Recommendations

Rheological studies indicate the huge impact they have on the segregation properties of tailings. To practical cases, it is suggested that tailings materials are rheologically calibrated comparing with their sand fine ratio, and solid contents. Once this is done, it may provide in-situ quality control method identifying whether the slurry has segregation potential or not.

The basic mechanism of segregation is still not well understood. It appears that the motion of individual particles within a group settling needs to be understood. It is suggested that small beads or balls with embedded signal/frequency emitter, which is varied according to different size groups involved, be allowed to settle in the slurry. The motion of individual particles then can be tracked by their respective signals. This method could be used both in the static and dynamic environment. Such approach could supply the data we need to understand the process of segregation. It may require collaboration with expertise in electrical signal system.

When the solid content of a slurry is above 40% and it is allowed to settle, there are channels along the height of standpipe and lava domes at the interface between release water and suspension. There is no study made on the impact of these micro-channels on the process of sedimentation/ consolidation. Detailed examination of the effects is need.

The fine particles are the major contributors of the rheological properties that govern the segregation process. Application of fractal theory in rheological characterization need further study to minimize the level of empiricism involved. The same recommendation also goes for the application of plastic theory in the study of beach profile.

The effect of pore fluid chemistry on the segregation properties of slurries need to be further investigated in terms of ion exchange capacity in the range of slurry solid contents. Furthermore, the effect of trace bitumen in the segregation mechanism of oil sands tailings needs to be studied.

Also a sedimentation and consolidation model which work for segregating slurries need to be developed. The challenging issue in the modeling would be the moving boundary problem between the sediment and the suspension and the different size distribution involved in the process.

Moreover, in sediment formation the concurrent process of stress degradation and stress build-up are not well understood and further research to understand the physics involved is recommended.

There have been reports that pipeline transport of non-segregating slurry results in segregating slurry at the deposition. More study to better understand the flow mechanism in pipeline transport is required. Moreover, the idea of mobile thickener that avoids transporting high solid content by pipeline should be a subject of future research.

The theory of sedimentation and consolidation which involve segregation and the complexities of fine matrix rheology require more study and further theory improvement.

APPENDIX A

A.1. Viscosity Measurement

ASTM D 4016-02 defined coefficient of viscosity as the ratio between the applied stress and the rate of shear. This coefficient is a measure of the resistance to flow of the liquid. It is commonly called the viscosity of the liquid. The cgs unit of viscosity is 1g/cm.c (1dyne/s cm^2) and is called a poise (P). Viscosities of thin liquids are normally given in hundredth of a poise or centipoises (cP). The SI unit of viscosity is 1Pa.s (1 N.s./m^2) and is equal to 10P, or 1000cP.

A.2. Procedure

Brookfield manual (No. M/97-164-D1000) provides the following general procedure for viscosity measurements. Brookfield recommends the use of a 600 ml Low Form Griffin beaker when using LV/RV/HA/HB spindles.

1. Mount the guardleg on the DV-II+ Viscometer (LV and RV series).

2. Insert and center spindle in the test material until the fluid's level is at the immersion groove on the spindle's shaft. With a disc-type spindle, it is necessary to tilt the spindle slightly while immersing to avoid trapping air bubbles on its surface. Attach the spindle to the lower shaft of the viscometer. Lift the shaft slightly, holding it firmly with one hand while screwing the spindle on with the other (note left-hand thread). Avoid putting side thrust on the shaft. Verify the proper spindle immersion depth and that the viscometer is level.

3. The process of selecting a spindle and speed for an unknown fluid is normally trial and error. An appropriate selection will result in measurements made between 10-100 on the instrument % torque scale. Two general rules will help in the trial and error process.

1) Viscosity range is inversely proportional to the size of the spindle.
2) Viscosity range is inversely proportional to the rotational speed.

To measure high viscosity, choose a small spindle and/or a slow speed. If the chosen spindle/speed results in a reading above 100%, then reduce the speed or choose a smaller spindle.

Experimentation may reveal that several spindle/speed combinations will produce satisfactory results between 10-100%. When this circumstance occurs, any of the spindles may be selected. Non-Newtonian fluid behavior can result in the measured viscosity changing if the spindle and/ or speed is changed.

Allow time for the indicated reading to stabilize. The time required for stabilization will depend on the speed at which the Viscometer is running and the characteristics of the sample fluid. For maximum accuracy, readings below 10% should be avoided.

4. Press the MOTOR ON/OFF/ESCAPE key and turn the motor “OFF” when changing a spindle or changing samples. Remove spindle before cleaning.

5. Interpretation of results and the instrument's use with non-Newtonian and thixotropic materials is discussed in the booklet, "More Solutions to Sticky Problems", and in Appendix C, Variables in Viscosity Measurements.

A.3. Calibration

The Calibration was carried out with the standard fluid provided by the manufacturer, Brookfield Engineering Laboratories. A calibration template obtained from manufacturer website was used in analyzing calibration results. Table A-1 shows the calibration analysis of the viscometer used in the measurement program. Figure A-1 shows a calibration check chart, that shows viscosity measurement at different rate. The data plot falls well within the manufacturer's acceptable ranges, validating the use of the viscometer for test.

A.4. Viscosity measurement

After the calibration check viscosity measurements took place and sample results are presented below. Figure A-2 shows viscosity versus shear rate relationships for a range of solid contents from 25% to 45 % solid contents of kaolinite slurry. And Figure A-3 shows a corresponding shear stress versus shear rate relationship for same range of solid contents of kaolinite slurry used through the experimental program.

TABLE A1:CALIBRATION TEMPLATE FOR ANALYZING CALIBRATION RESULTS

INSTRUCTIONS : *ENTER VALUES IN* **BOLD** *FIELDS ONLY*

Enter value of calibration fluid in column A

Enter instrument model: **RVDV-II+**

serial number: RT70177

Enter spindle code in column D

Enter RPM used In column E

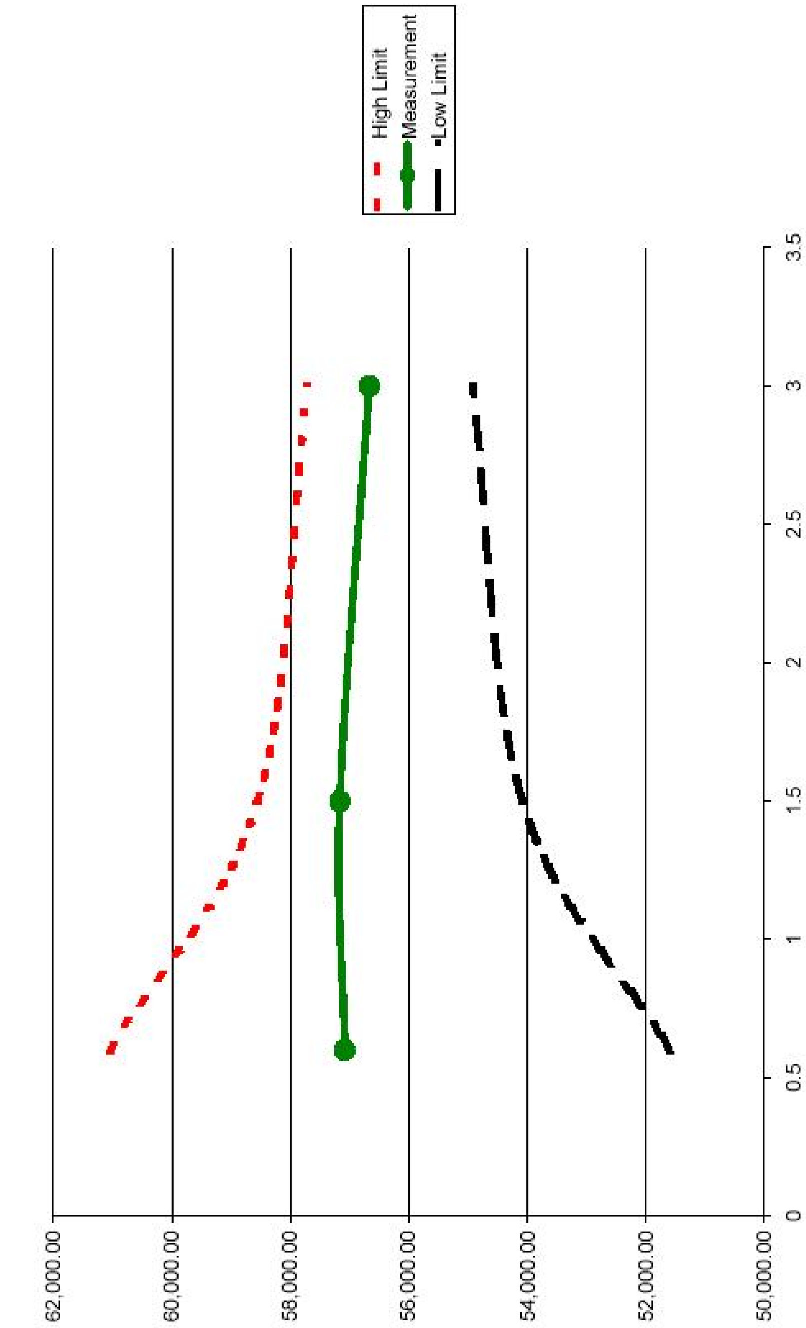
Enter Full Scale Range in column F

Enter viscosity reading in column I

A Actual Fluid Value (cP) from Label or Certificate	B 1% of Fluid Value (cP)	C Instrument Model	D Spindle	E RPM	F FSR cP (depress and hold AUTORANGE key)	G 1% of FSR cP	H % Torque (must be >10%)	I Viscosity Reading in cP
56320	563.2	RVDV-II+	**27**	**0.6**	**4.17E+05**	4170.000	13.7%	**5.71E+04**
56320	563.2	RVDV-II+	27	**1.5**	**1.67E+05**	1670.000	34.2%	**5.72E+04**
56320	563.2	RVDV-II+	27	**3**	**8.33E+04**	833.330	68.0%	**5.67E+04**

Low Limit = A – (B + G)	Reading I	High Limit = A + (B + G)	Calibration Passed?
51,586.80	57083	61,053.20	TRUE
54,086.80	57167	58,553.20	TRUE
54,923.47	56667	57,716.53	TRUE

Figure A1: Calibration Check



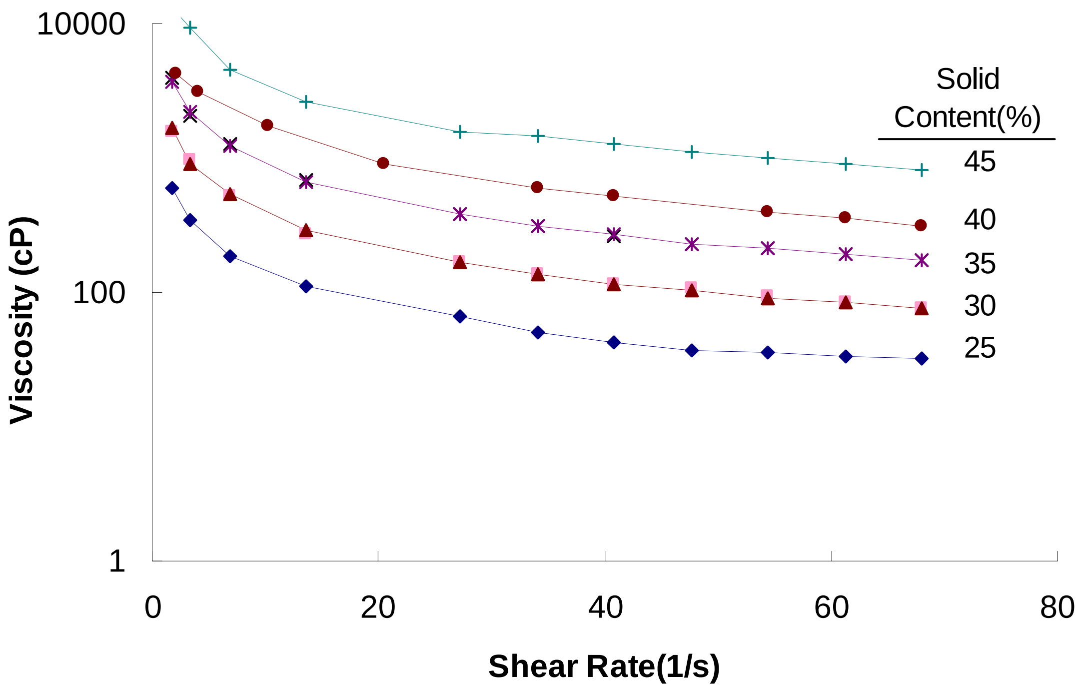


Figure A-2 Viscosity versus shear rate at different solid content of Kaolinite slurry

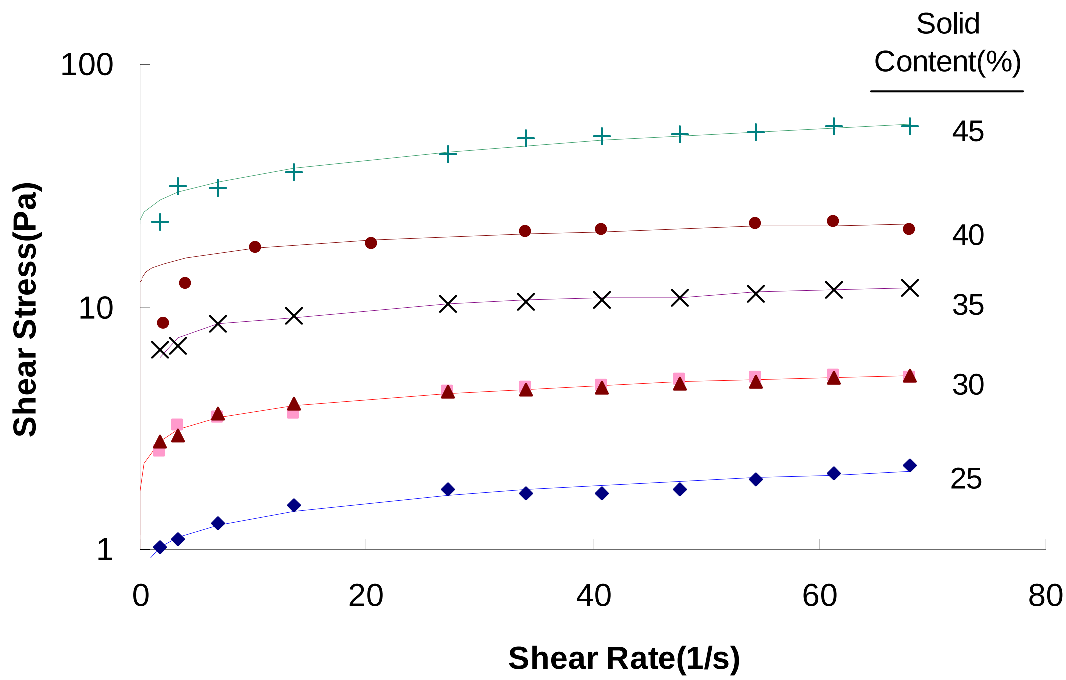


Figure A-3 Shear stress versus shear rate at different solid content of Kaolinite slurry

A5 Rheological Measurements of Tailings Materials

i) Suncor Mature Fine Tailings

Mature fine tailings (MFT) samples were received from Suncor MFT (Pond 6) and Pond 2/3. Three set of samples identified as S20, S30 and S42, the digits after the letter S designate the depth in feet from which the samples were taken. Table A-2 shows sample properties as-received.

Table A-3 summarizes the measured solid content at targeted bulk density of tailings slurry for the three set of samples. Tailings release water sample was received to prepare a slurry mix with a desired slurry mix. The measured pH of the release water was 8.73. Particle size distributions of the tailings materials at different depths were carried out and are shown in Figure A-4.

Table A- 2 Mature Fine Tailings sample properties as-received.

Sample Description	pH	Solid content (by weight)
S20	8.07	23.07
S30	7.71	31.75
S42	7.49	38.15

Table A- 3 Solid content of samples at different bulk densities.

Bulk Density	S20	S30	S42	Average
1.5	57.3	57.8	57.8	57.6
1.4	48.0	51.4	49.2	49.5
1.3	41.5	43.6	41.6	42.2
1.2	29.7	30.0	30.4	30.1
1.1	16.3	15.7	16.8	16.3

Viscosity measurement

Temperature Effect

The effect of temperature was studied on S30 (middle depth, 30ft) sample at three temperatures, 10, 20 and 30 degree Celsius. The samples have a constant bulk density of 1.3g/cc. Figure A-5 shows the data plots.

Figure A-6 shows another rheogram for samples obtained from Suncor pond 2/3 tested at different temperature at bulk density of 1.3 g/cc.

Figures A-7 to A-30 show the rheological data plots at different depths and bulk densities ranging from 1.1 g/cc to 1.4 g/cc tested temperature of 10 degree Celsius.

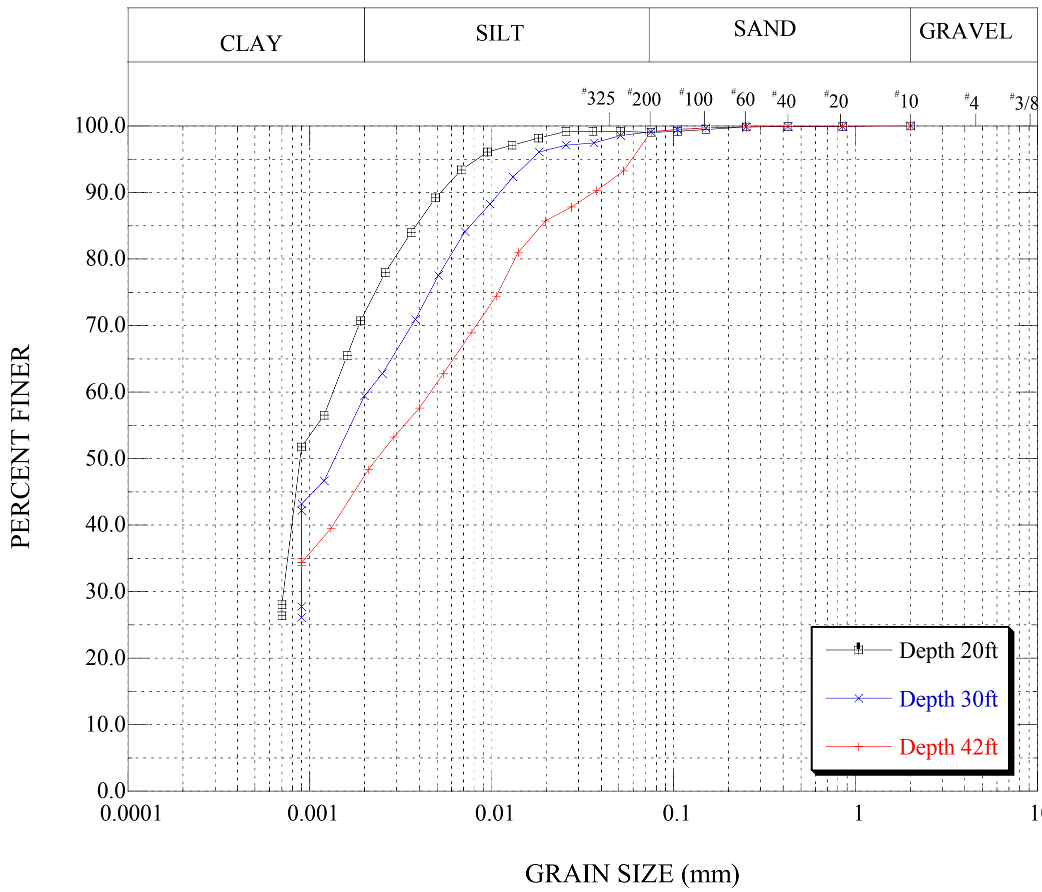


Figure A- 4 Particle size distribution of the MFT samples at different depths.

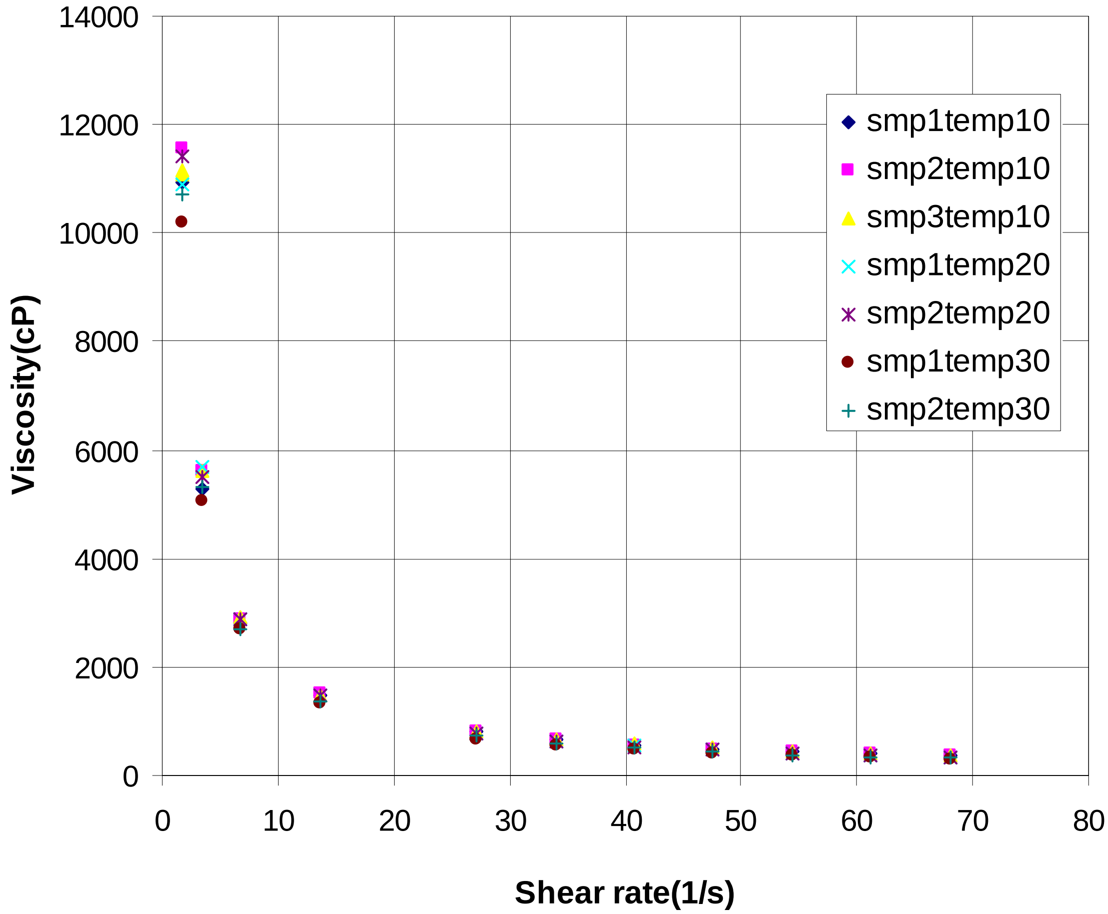


Figure A- 5 Viscosity measurement of S3 MFT (bulk density 1.3 g/cc) at different temperatures.

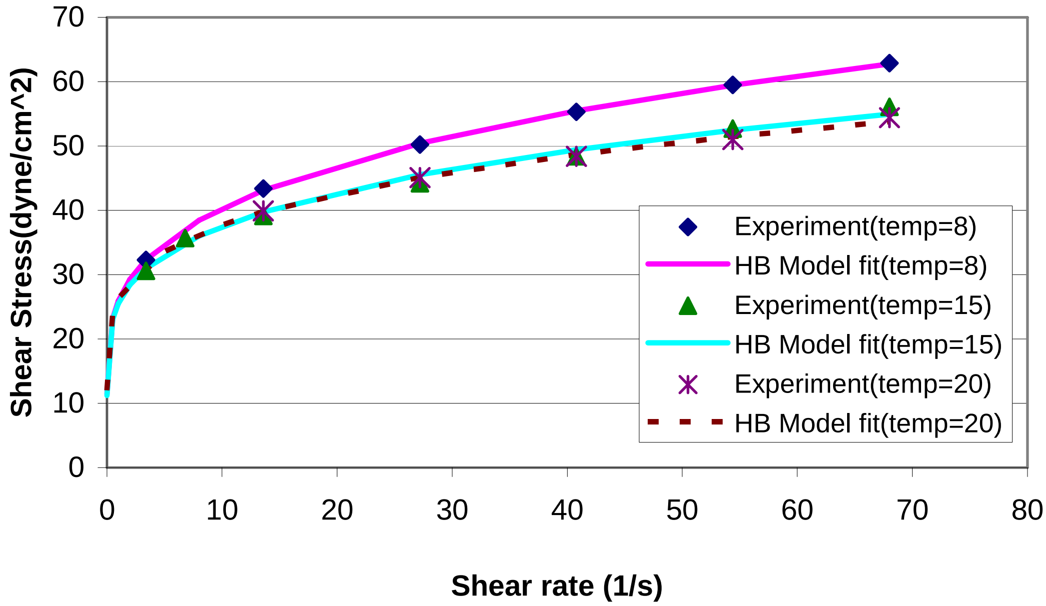


Figure A- 6 Flow Curve of pond 2/3 MFT at different temperatures and Herschel Bulkley model fits

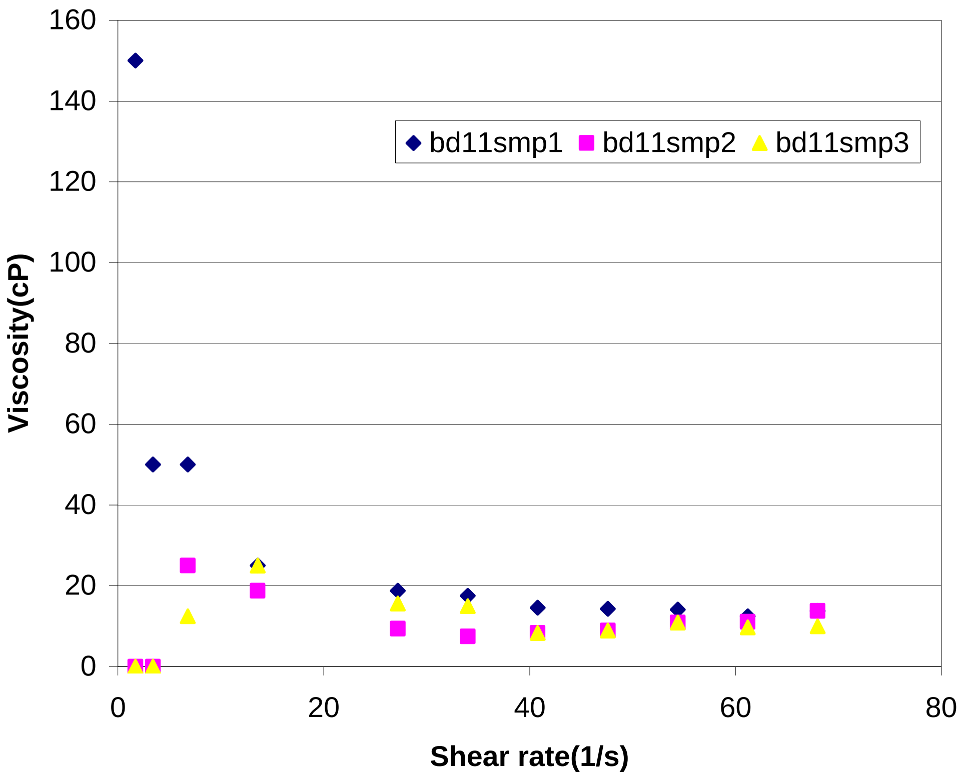


Figure A- 7 Viscosity versus shear rate of Pond 6 MFT (S20) bulk density 1.1 g/cc

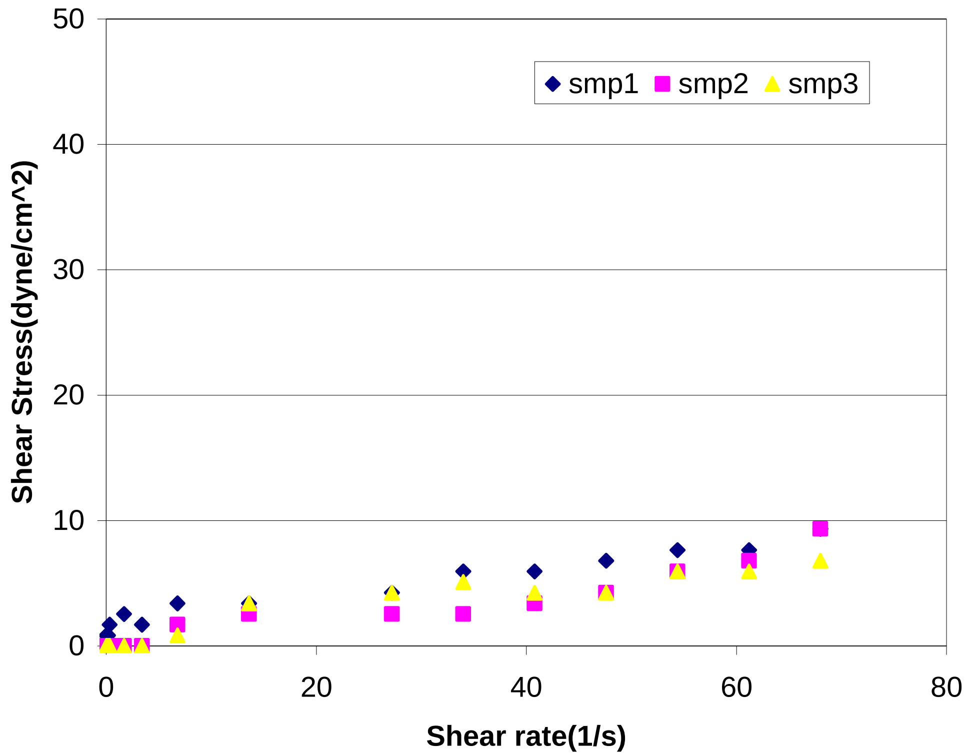


Figure A- 8 Shear stress versus shear rate of Pond 6 MFT (S20) bulk density 1.1 g/cc

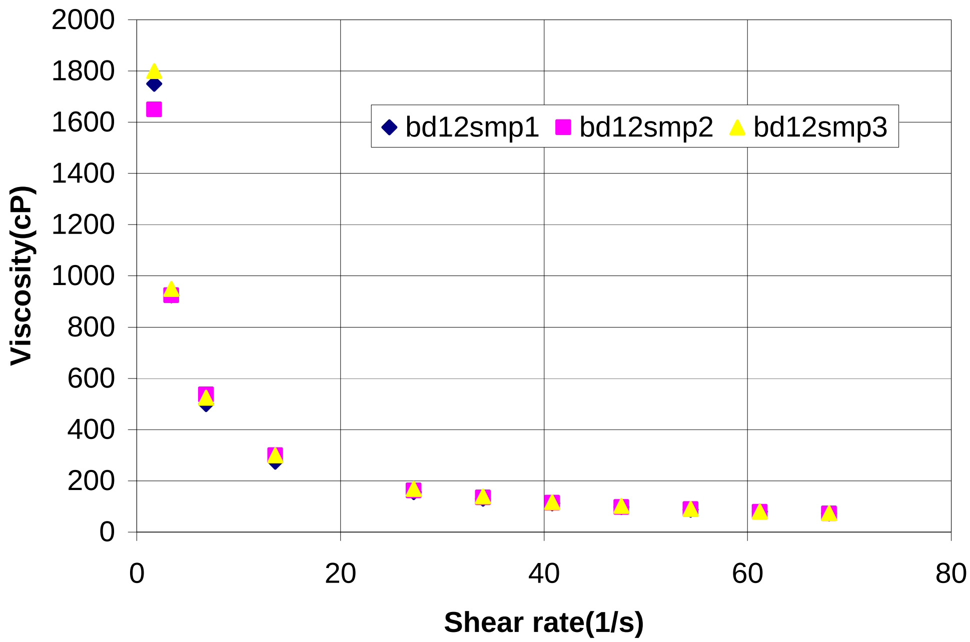


Figure A- 9 Viscosity versus shear rate of Pond 6 MFT (S20) bulk density 1.2 g/cc at temperature of 10 deg. Celsius.

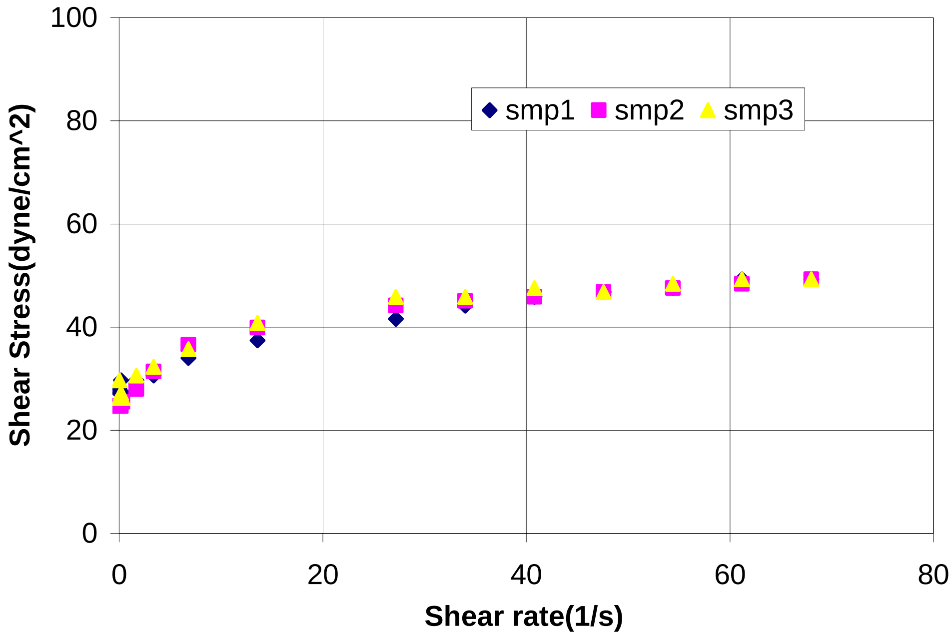


Figure A- 10 Shear stress versus shear rate of Pond 6 MFT (S20) bulk density 1.2 g/cc at temperature of 10 deg. Celsius.

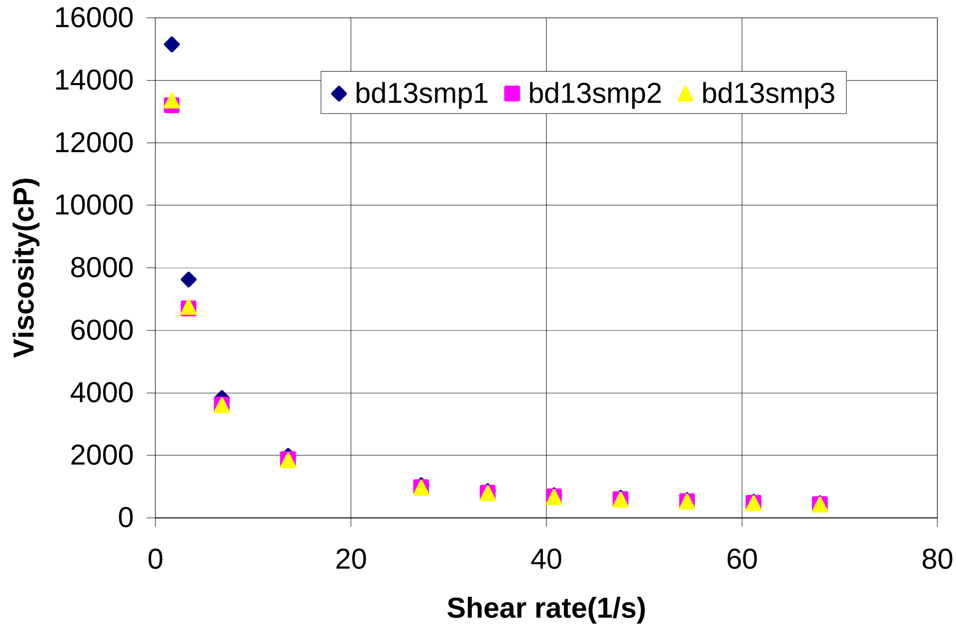


Figure A- 11 Viscosity versus shear rate of Pond 6 MFT (S20) bulk density 1.3 g/cc at temperature of 10 deg. Celsius.

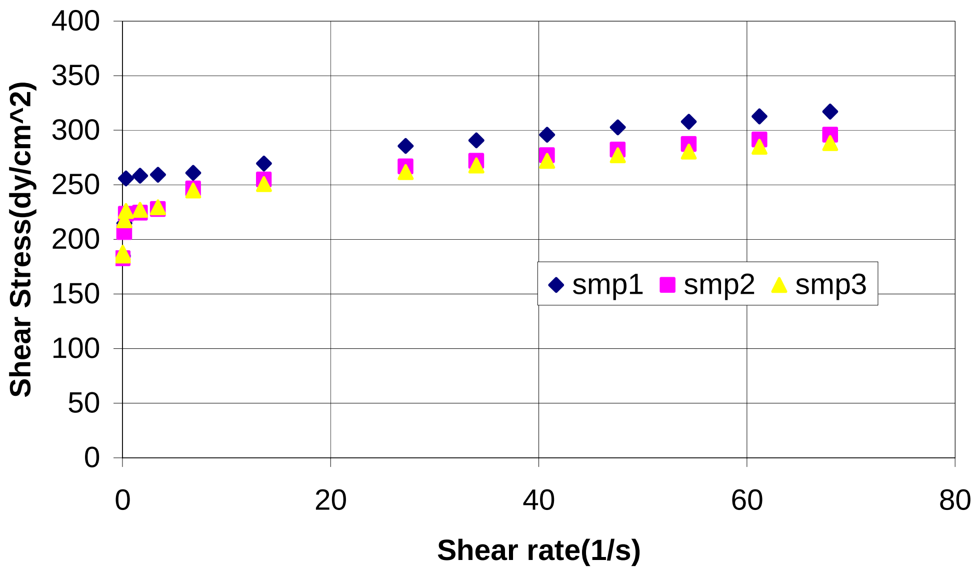


Figure A- 12 Shear stress versus shear rate of Pond 6 MFT (S20) bulk density 1.3 g/cc at temperature of 10 deg. Celsius.

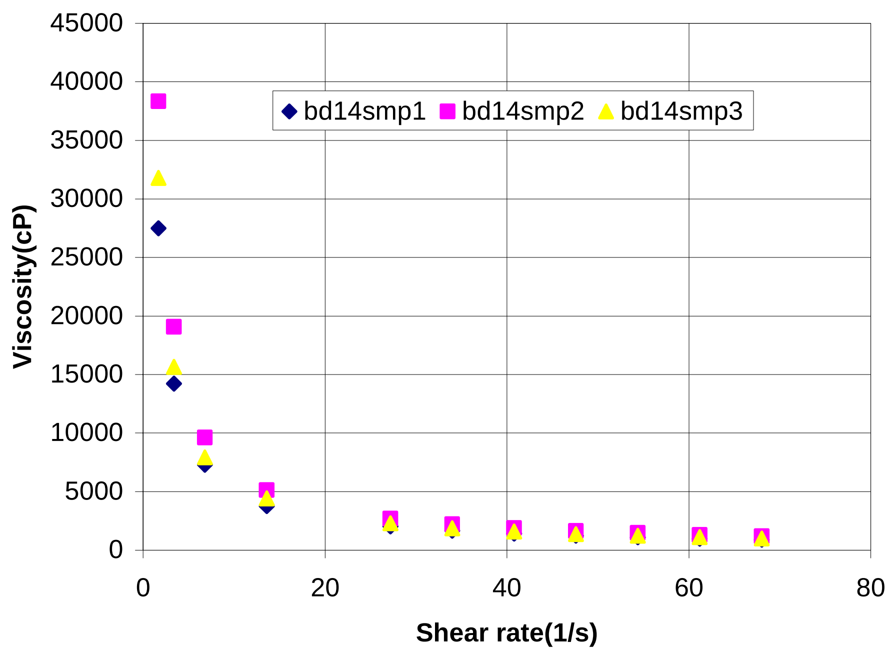


Figure A- 13 Viscosity versus shear rate of Pond 6 MFT (S20) bulk density 1.4 g/cc at temperature of 10 deg. Celsius.

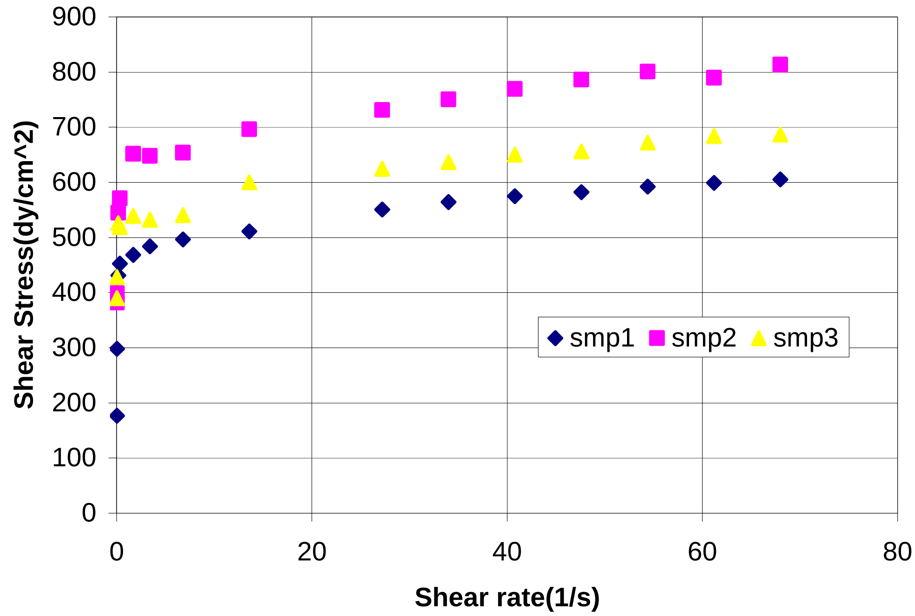


Figure A- 14 Shear stress versus shear rate of Pond 6 MFT (S20) bulk density 1.4 g/cc at temperature of 10 deg. Celsius.

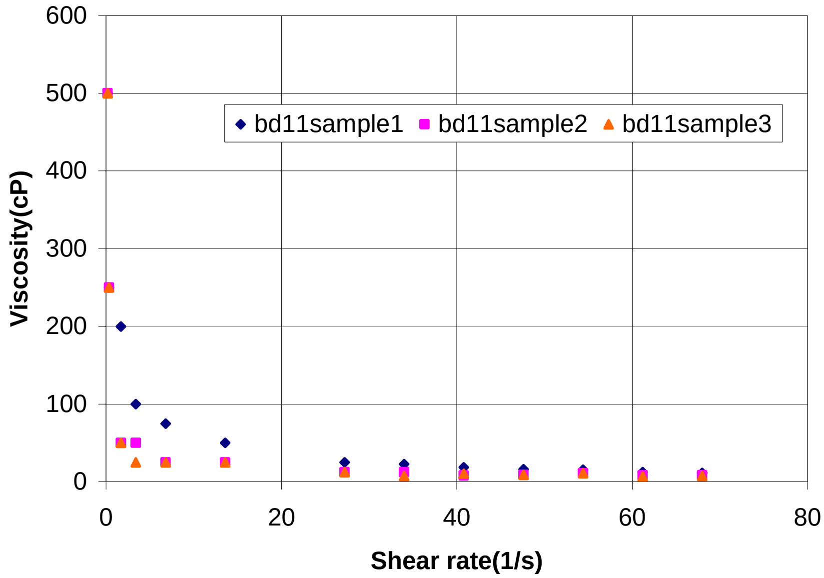


Figure A- 15 Viscosity versus shear rate of Pond 6 MFT (S30) bulk density 1.1 g/cc at temperature of 10 deg. Celsius.

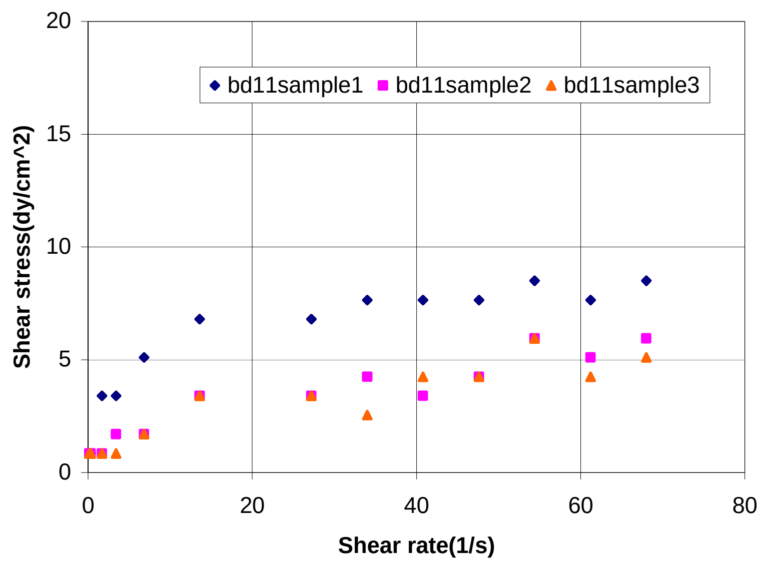


Figure A- 16 Shear stress versus shear rate of Pond 6 MFT (S30) bulk density 1.1 g/cc at temperature of 10 deg. Celsius.

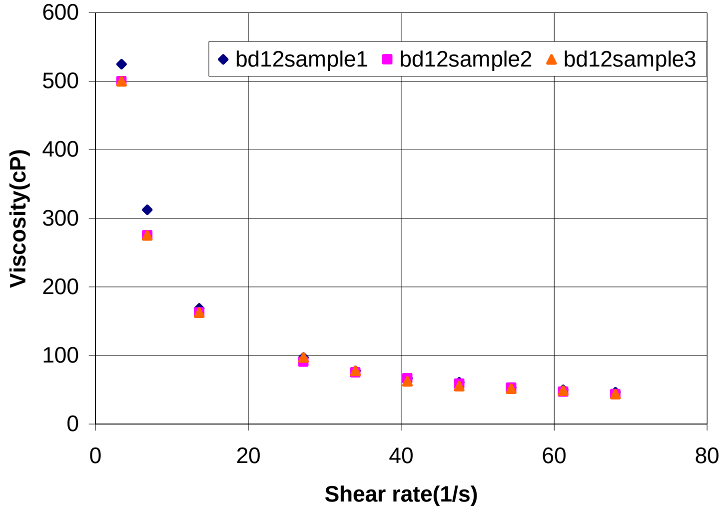


Figure A- 17 Viscosity versus shear rate of Pond 6 MFT (S30) bulk density 1.2 g/cc at temperature of 10 deg. Celsius.

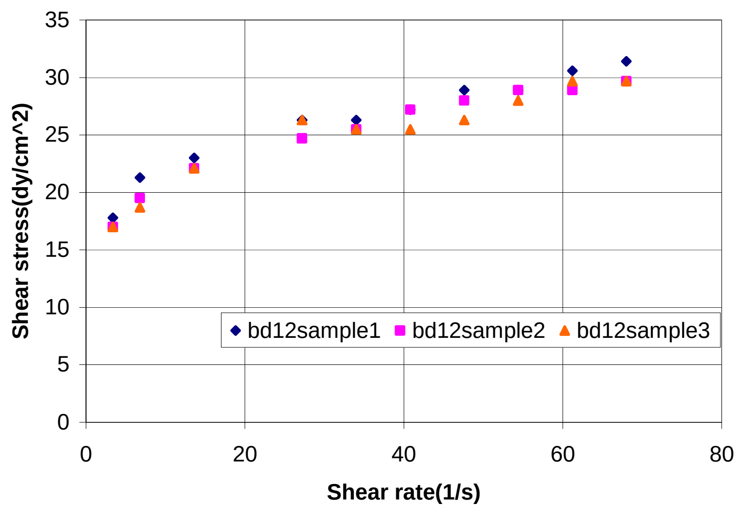


Figure A- 18 Shear stress versus shear rate of Pond 6 MFT (S30) bulk density 1.2 g/cc at temperature of 10 deg. Celsius.

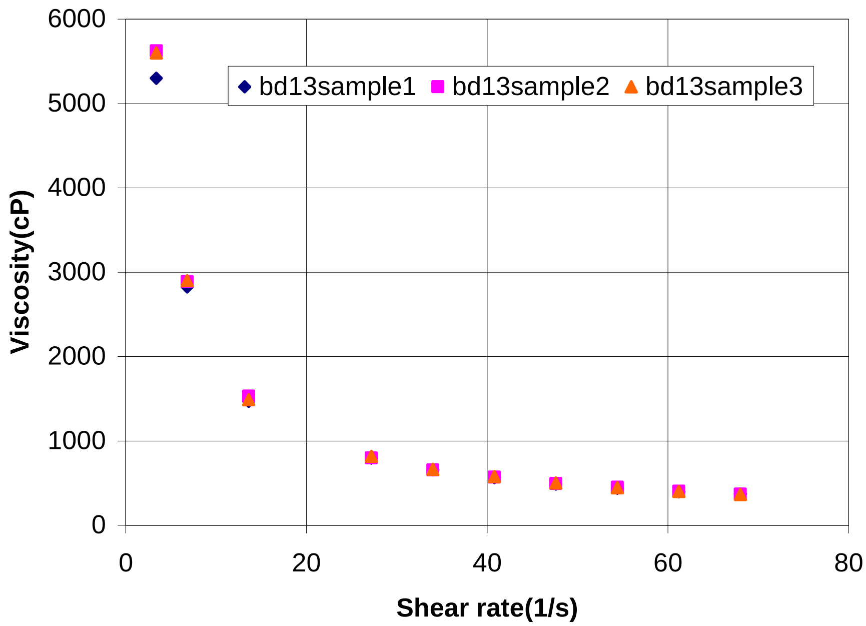


Figure A- 19 Viscosity versus shear rate of Pond 6 MFT (S30) bulk density 1.3 g/cc at temperature of 10 deg. Celsius.

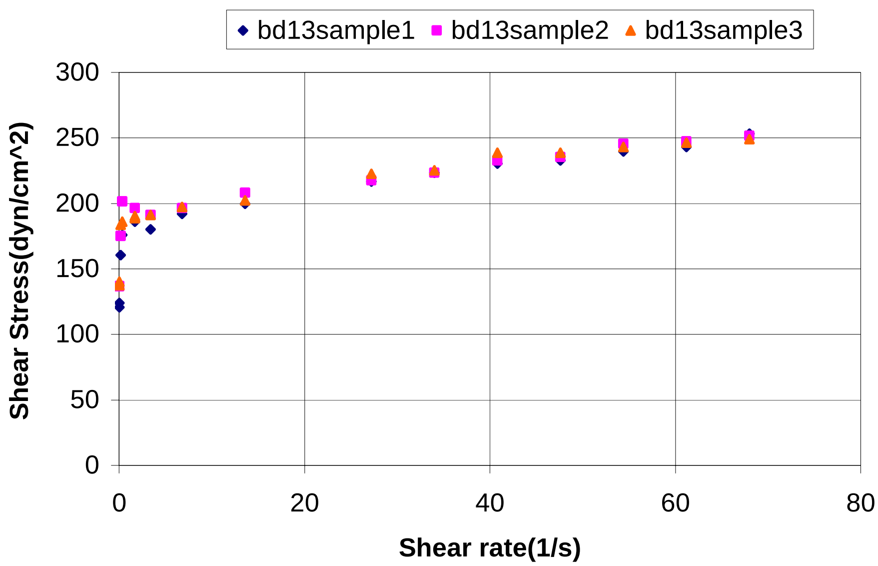


Figure A- 20 Shear stress versus shear rate of Pond 6 MFT (S30) bulk density 1.3 g/cc at temperature of 10 deg. Celsius.

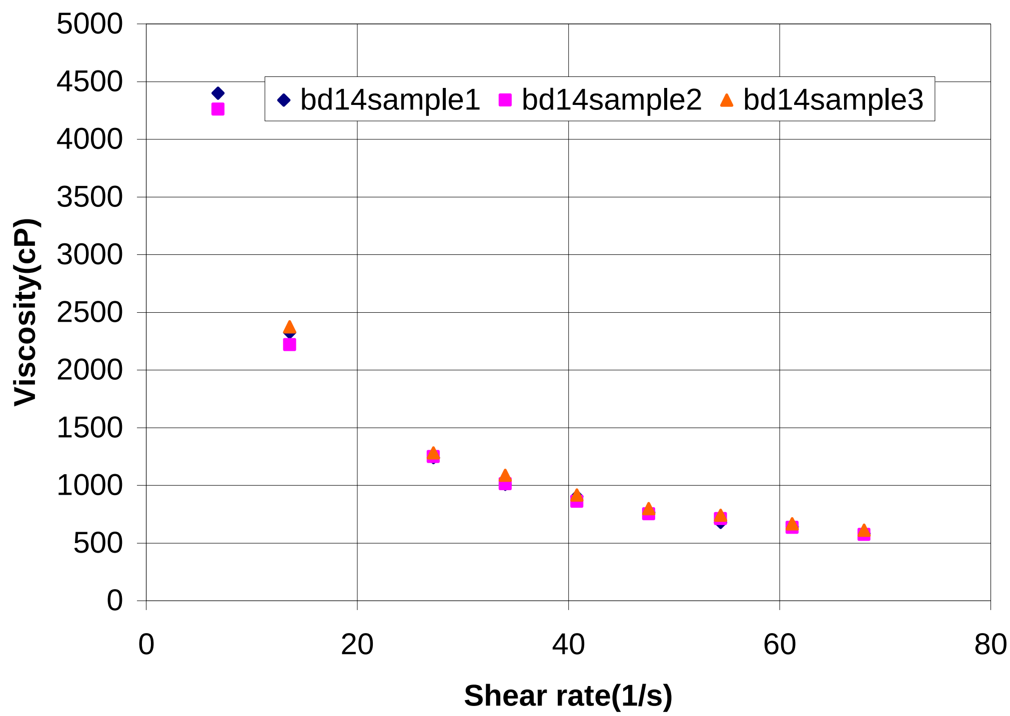


Figure A- 21 Viscosity versus shear rate of Pond 6 MFT (S30) bulk density 1.4 g/cc at temperature of 10 deg. Celsius.

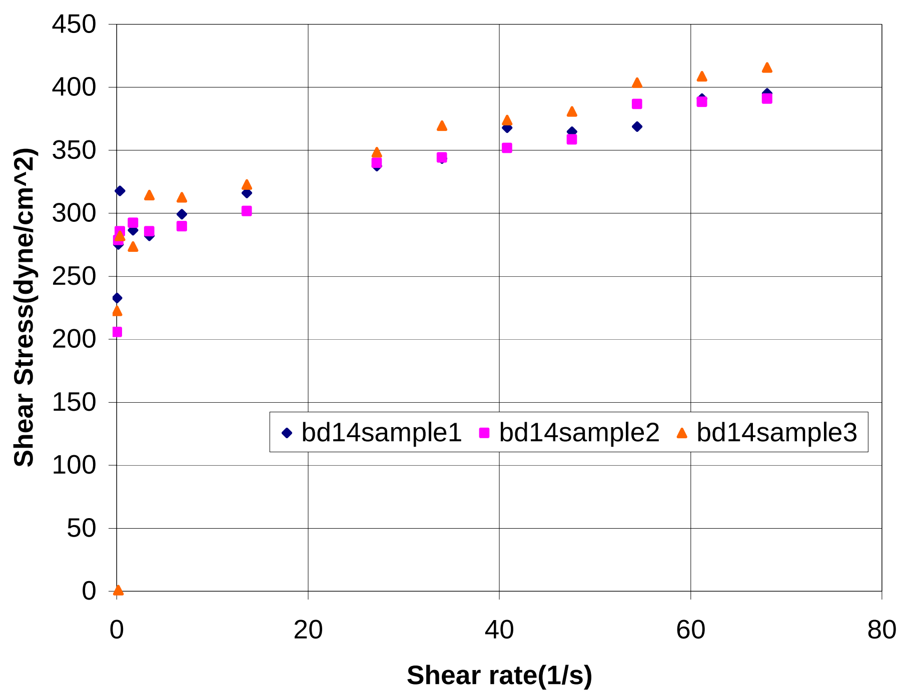


Figure A- 22 Shear stress versus shear rate of Pond 6 MFT (S30) bulk density 1.4 g/cc at temperature of 10 deg. Celsius.

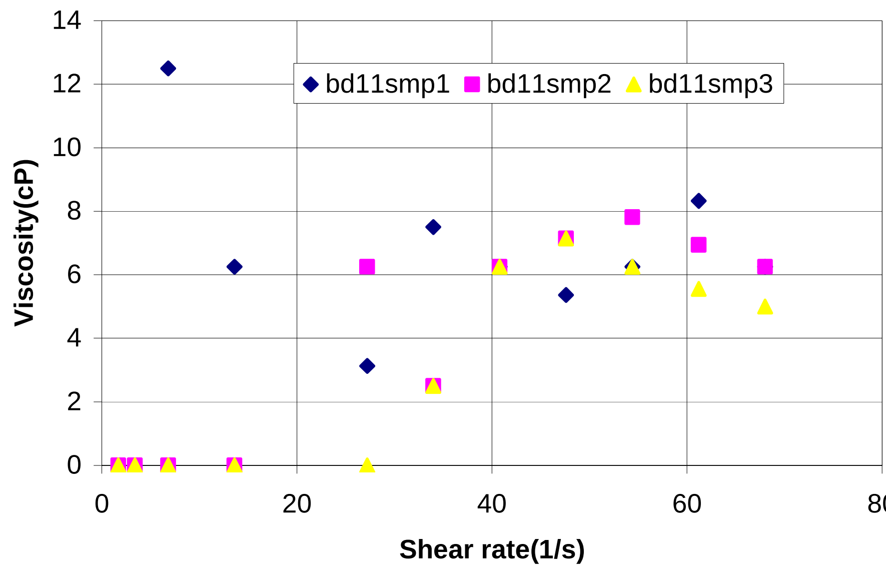


Figure A- 23 Viscosity versus shear rate of Pond 6 MFT (S42) bulk density 1.1 g/cc at temperature of 10 deg. Celsius.

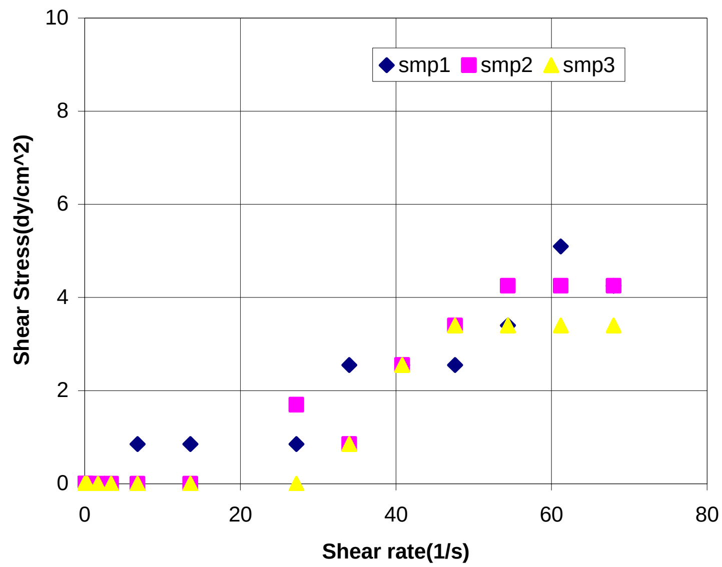


Figure A- 24 Shear stress versus shear rate of Pond 6 MFT (S42) bulk density 1.1 g/cc at temperature of 10 deg. Celsius.

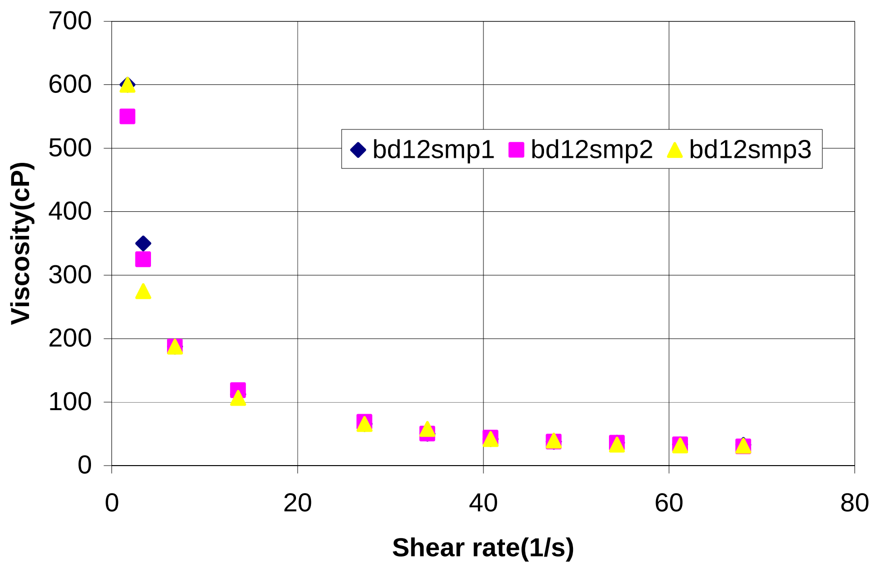


Figure A- 25 Viscosity versus shear rate of Pond 6 MFT (S42) bulk density 1.2 g/cc at temperature of 10 deg. Celsius.

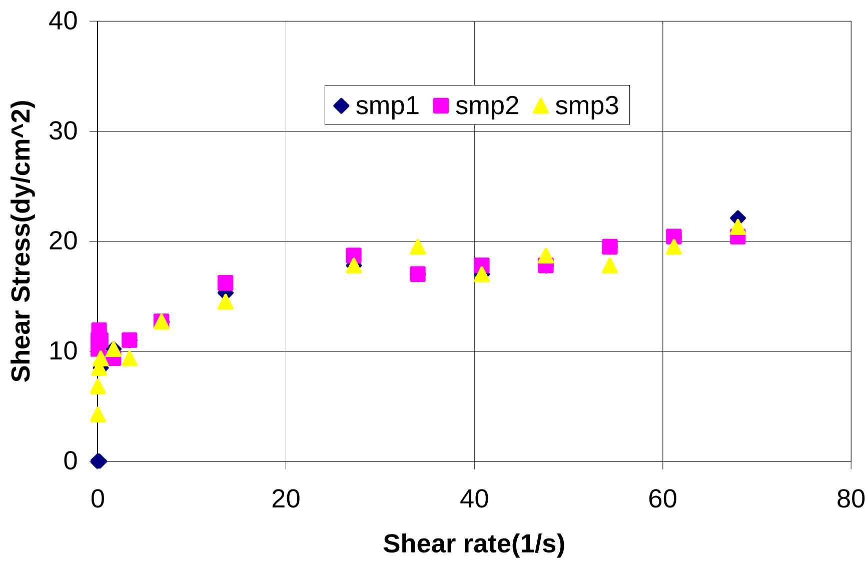


Figure A- 26 Shear stress versus shear rate of Pond 6 MFT (S42) bulk density 1.2 g/cc at temperature of 10 deg. Celsius.

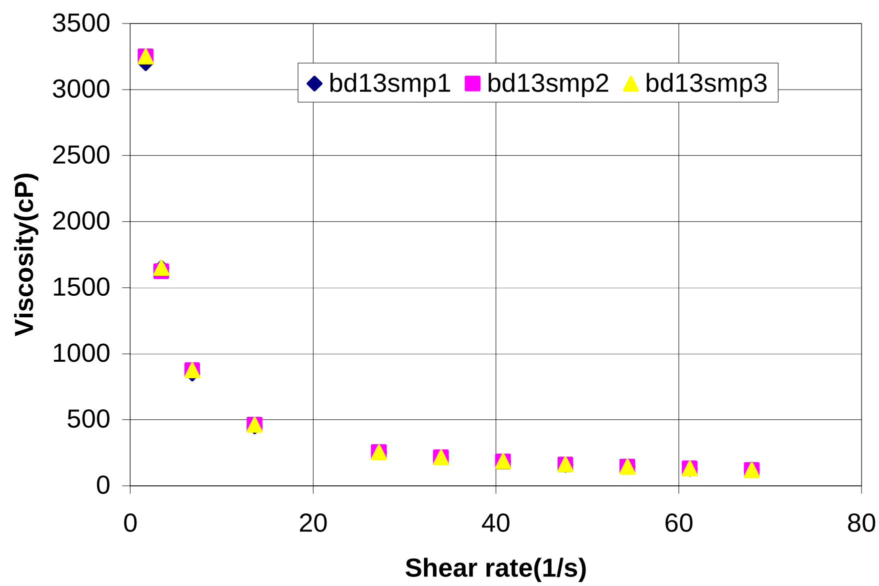


Figure A- 27 Viscosity versus shear rate of Pond 6 MFT (S42) bulk density 1.3g/cc at temperature of 10 deg. Celsius.

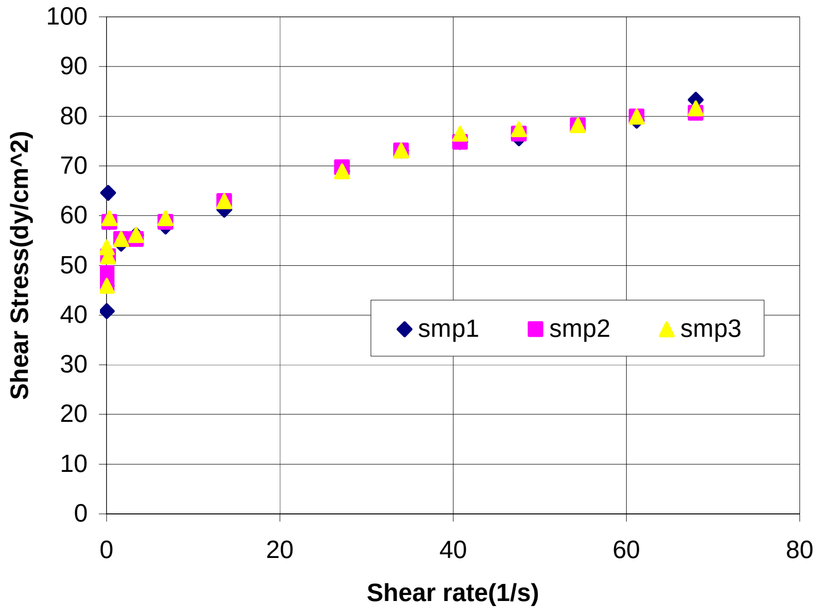


Figure A- 28 Shear stress versus shear rate of Pond 6 MFT (S42) bulk density 1.3 g/cc at temperature of 10 deg. Celsius.

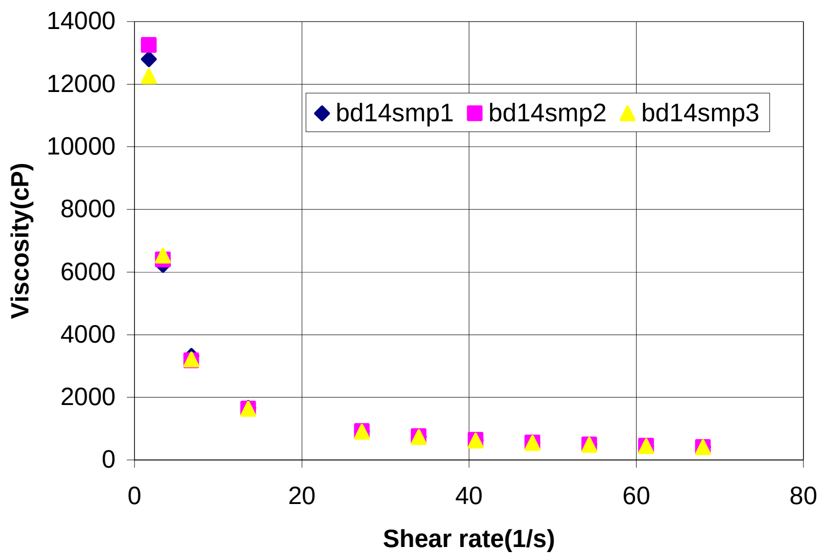


Figure A- 29 Viscosity versus shear rate of Pond 6 MFT (S42) bulk density 1.4g/cc at temperature of 10 deg. Celsius.

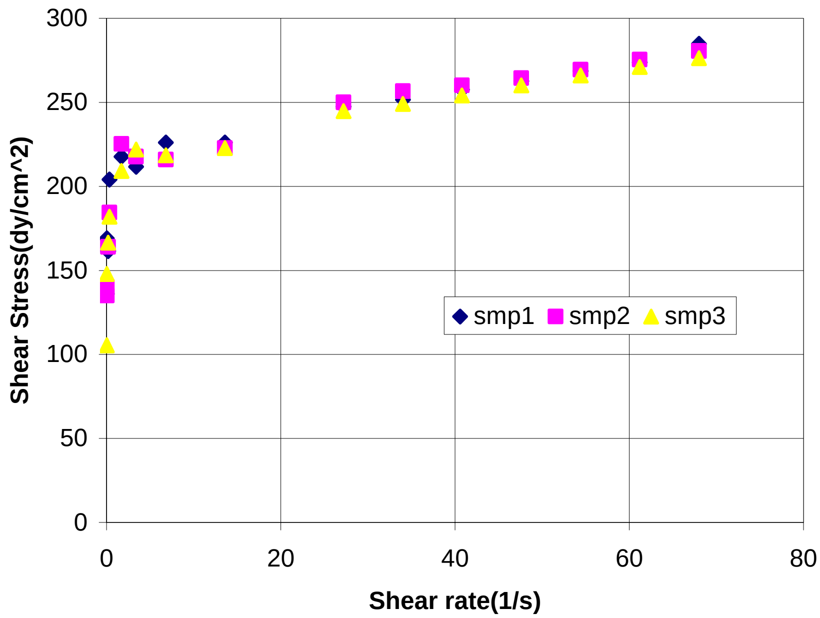


Figure A- 30 Shear stress versus shear rate of Pond 6 MFT (S42) bulk density 1.4 g/cc at temperature of 10 deg. Celsius.

b) Albian Sands, Thickener underflow fines.

A thickener underflow sample was received from Albian Sand and rheological testing was conducted on the fines (less than 44 Micron) at room temperature conditions.
Figure A-31 and Figure A-32 show respectively the viscosity versus shear rate and shear stress versus shear rate data plots at different solid contents.

Yield stress measurement of the thickener underflow fines at a solid content of 35% and treated with NaOH and CaO are shown as comparison in Figure A-33.

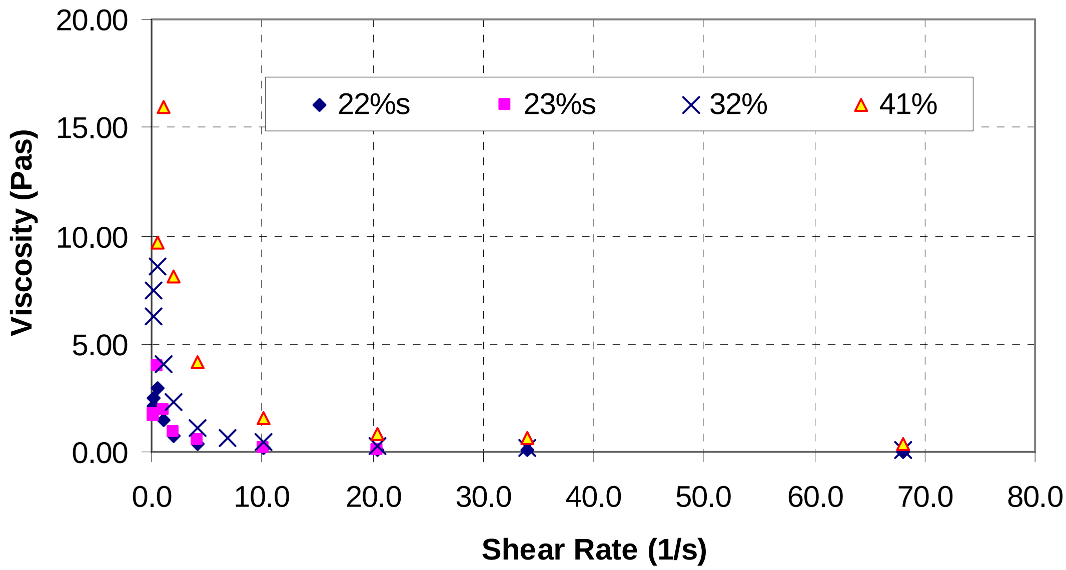


Figure A- 31 Viscosity versus shear rate of thickener underflow fines at different solid content.

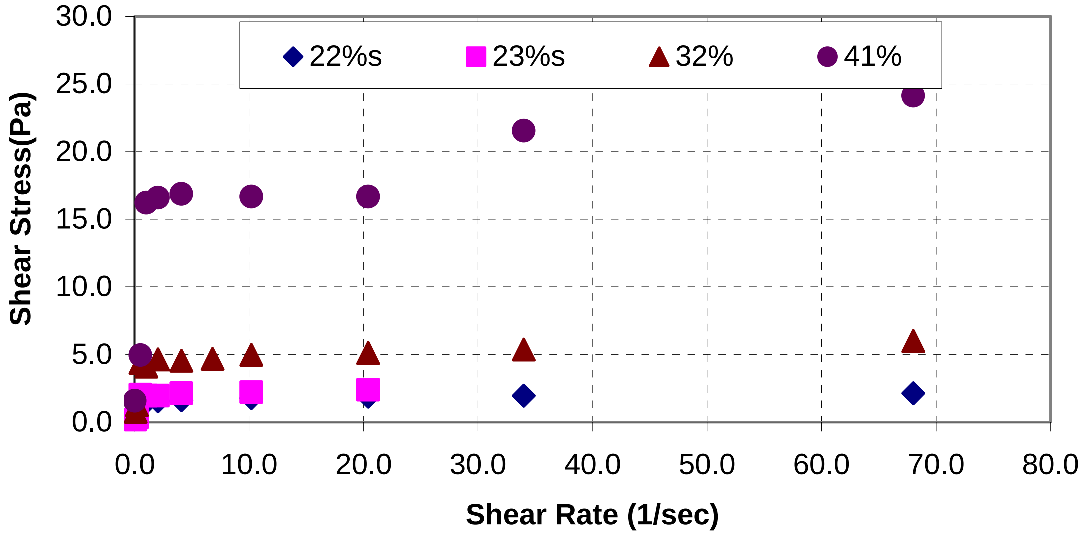


Figure A- 32 Shear stress versus shear rate of thickener underflow fines at different solid contents.

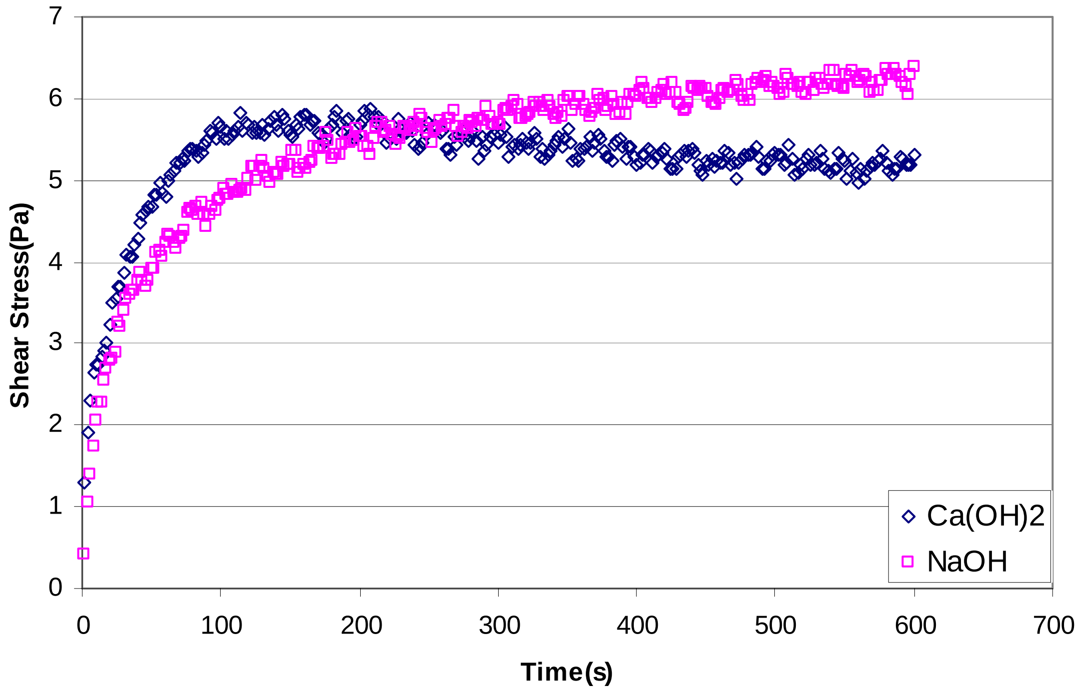


Figure A- 33 Yield stress measurement of thickener underflow fines at solid content of 35% and pH of 12 achieved by $Ca(OH)_2$ and NAOH additions.

APPPENDIX B

Equation derivation for settling of sphere in a fluid with yield stress

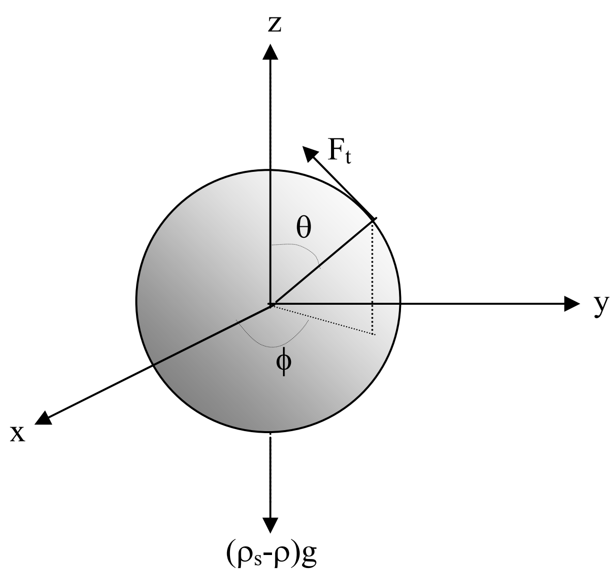


Figure B-1 Free body diagram of a sphere of radius R suspended in a fluid

Integration of the tangential force over the surface gives:

$$F_t = \int_0^{2\pi}\int_0^{\pi}(\tau \sin\theta)R^2 \sin\theta d\theta d\phi$$
$$= \tau R^2 \int_0^{2\pi}\int_0^{\pi} \sin^2 \theta d\theta d\phi$$
$$= 2\pi\tau R^2 \int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$
$$= \pi^2 \tau R^2 \quad (B.1)$$

Or in terms of the yield stress τ_y

$$= \pi^2 \tau_y R^2 \quad (B.2)$$

The buoyant weight of the sphere is given as

$$=\frac{4}{3}\pi R^3(\rho_s-\rho)g \tag{B.3}$$

Where ρ_s is the density of the sphere and ρ is the density of the fluid.

Equating Equation (B.2) to Equation (B.3) results in:

$$R=\frac{3\pi}{4}\frac{\tau_y}{(\rho_s-\rho)g} \tag{B.4}$$

Or in terms of Diameter

$$D=\frac{3\pi}{2}\frac{\tau_y}{(\rho_s-\rho)} \tag{B.5}$$

Equation (B.5) is the same as Equation 4.6 in Chapter 4.

Dedegil (1986) examined Equation (B.5) with data of Valentik and Whitmore (1965) from experiment on settling of spheres in Bingham fluids. The relationship between drag coefficient (C_D) and Reynolds number (Re) from the same data are shown in Figure B-2 without taking yield stress into account and in Figure B-3 by taking the yield stress into account.

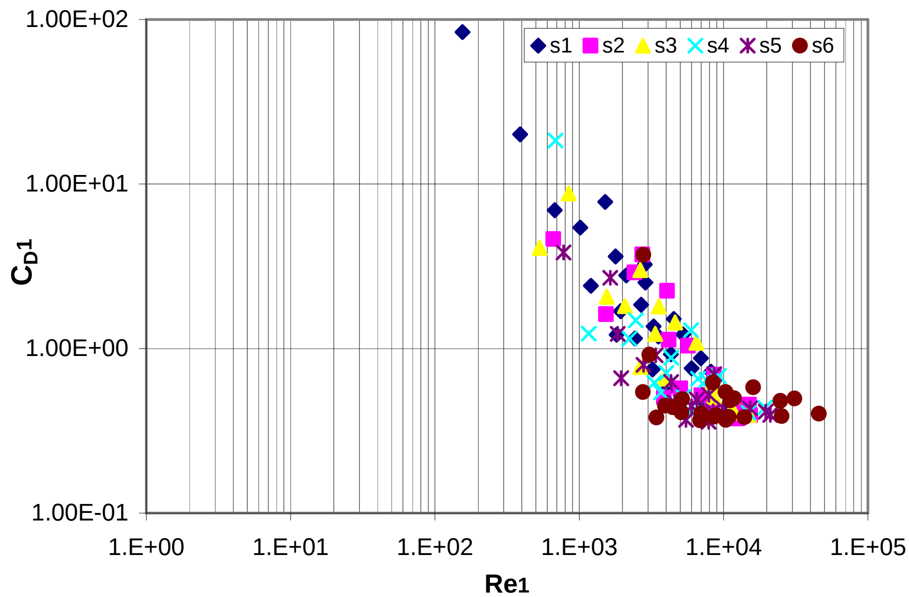


Figure B-2 Drag coefficient versus Reynolds number without taking yield stress into account (Data from Dedegil, 1986, Valentik and Whitmore, 1965)

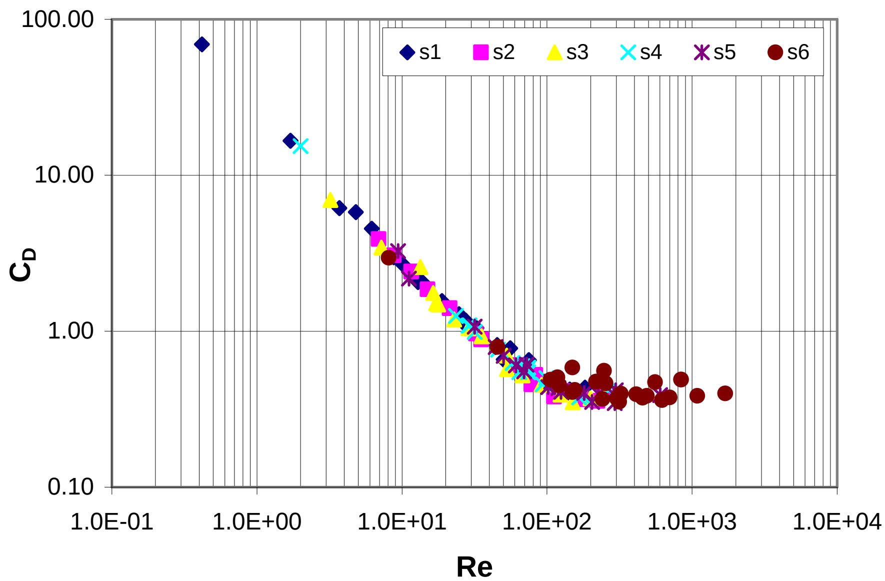


Figure B-3 Drag coefficient versus Reynolds number taking yield stress into account (Data from Dedegil, 1986, Valentik and Whitmore, 1965)

APPENDIX C

C.1. Effect of Mixing on Segregation

The objective of the testing program is to examine the effect of initial turbulent mixing on static (standpipe) segregation on relatively non-segregating slurries. For this testing, three test schedules as shown in Table C-1 were made. The test was to investigate the effect of thorough mixing of slurry at different mixing duration followed by standpipe test.

C.2. Procedure

The slurry is initially mixed to a desired solid content and sand fine ratio in a 5 litre pail manually using spatula until homogeneity was achieved and then the mixed slurry was transferred to a 1 litre beaker, which was part of the Philadelphia Mixer. The mixer speed was controlled by computer. For a given speed a measured torque was recorded by manufacturer-installed software in the controlling computer. The shaft was rotating at constant speed for a specified period of mixing; the slurry is then transferred to a standpipe and allowed to settle until the sedimentation process is near completion. Different mixing times were adopted to see the effect of duration of mixing. On the other hand, a separate standpipe test was conducted on the similar slurry sample with no mixing to serve as a reference.

Figure C-1 to Figure C-3 show the Scan Electron Microscopy (SEM) image of sand particles at different sizes ranges as retained on certain sieves from grain size analysis.

The speed of the impeller was set at 1080 rpm for all tests. And only mixing time was varied on a sample with same solid content and sand fine ratios.

The following table shows the sample used in the testing.

Table C-1 Test schedule for turbulent mixing and standpipe test

Solid Content (%)	**Sand Fine Ratio (SFR)o**	**Mixing time (min.)**
57.14	1	0,10, 20,40
56.25	2	0,10,20,40
55.56	4	0,10,20,40

C.3. Results and discussion

It is apparent that at SFRs of 1 and 2, the turbulent pre-mixing has least impact on the solid and sand content profile after sedimentation. Whereas pre-mixing produced non-segregating profile at SFR of 4 as shown in Figure C-4 (e) and (f). It is to observe also that the mixing time above 10 minutes didn't produce any improvement with respect to both solid content and sand content profiles.

The standpipe sedimentation results for the test samples in Table C-1 after mixing are shown in Figures C-5, to C-7. For SFRs of 1 and 2, samples that are less sheared exhibit relatively faster settling. However, at the SFR of 4 the settling rate for no shear shows faster settling rate at the start and less settling rate after about 24hrs. Those samples which were sheared for less duration are showing faster settling rate as compared to those samples which were sheared for longer time.

It is apparent that the mixing has caused the slurry to settle at settling rate. It is believed that the turbulent mixing generally caused homogenized mixture and as a result some fine particles could have formed a matrix through out the whole slurry that would take longer time for the settling to take place. On the other hand the turbulent mix could also cause the edges of coarse particles to break and reduce the size of the sand particle and then result in lower settling rate. The breakage of sand particle sizes from larger size to smaller size were confirmed by sample sieving before and after the mixing and it was found out that the proportion of larger size sand particles were slightly changed.

C.4. Summary and conclusion

Non-segregating slurry samples at different SFRs show no segregating trend when they are turbulently mixed at different duration. The mixing generally produces improved non-segregating profiles as compared to no mixing cases. It was found out from sedimentation tests that turbulent mixing produce a slow water release rate as compared to static tests. However, as the SFR increase such the above conclusion may not be concurred.

The effect of turbulent mixing is a combination of the following:(i) homogenizing the slurry to a better non-segregating one (ii) breakage of large size sand grains into smaller sizes and further assist in homogenizing.

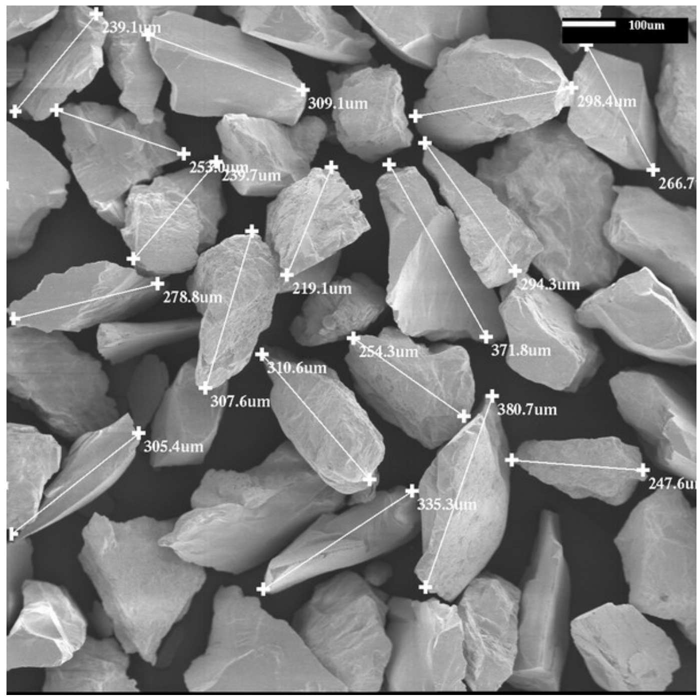


Figure C-1 SEM image of sand particles passing 250micron sieve and retained in 125 mm sieve

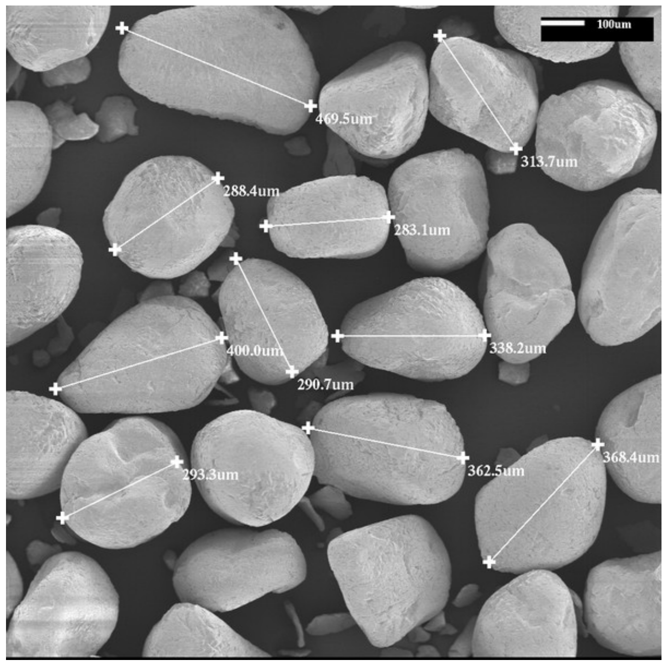


Figure C-2 SEM image of sand particles passing 425 micron sieve and retained in 250micron sieve

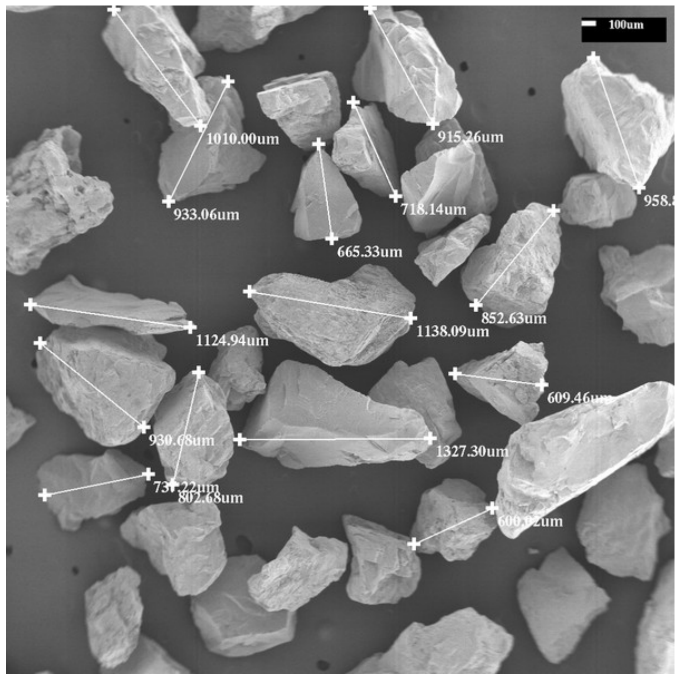


Figure C-3 SEM image of sand particles passing 2 mm sieve and retained in 425 mm sieve

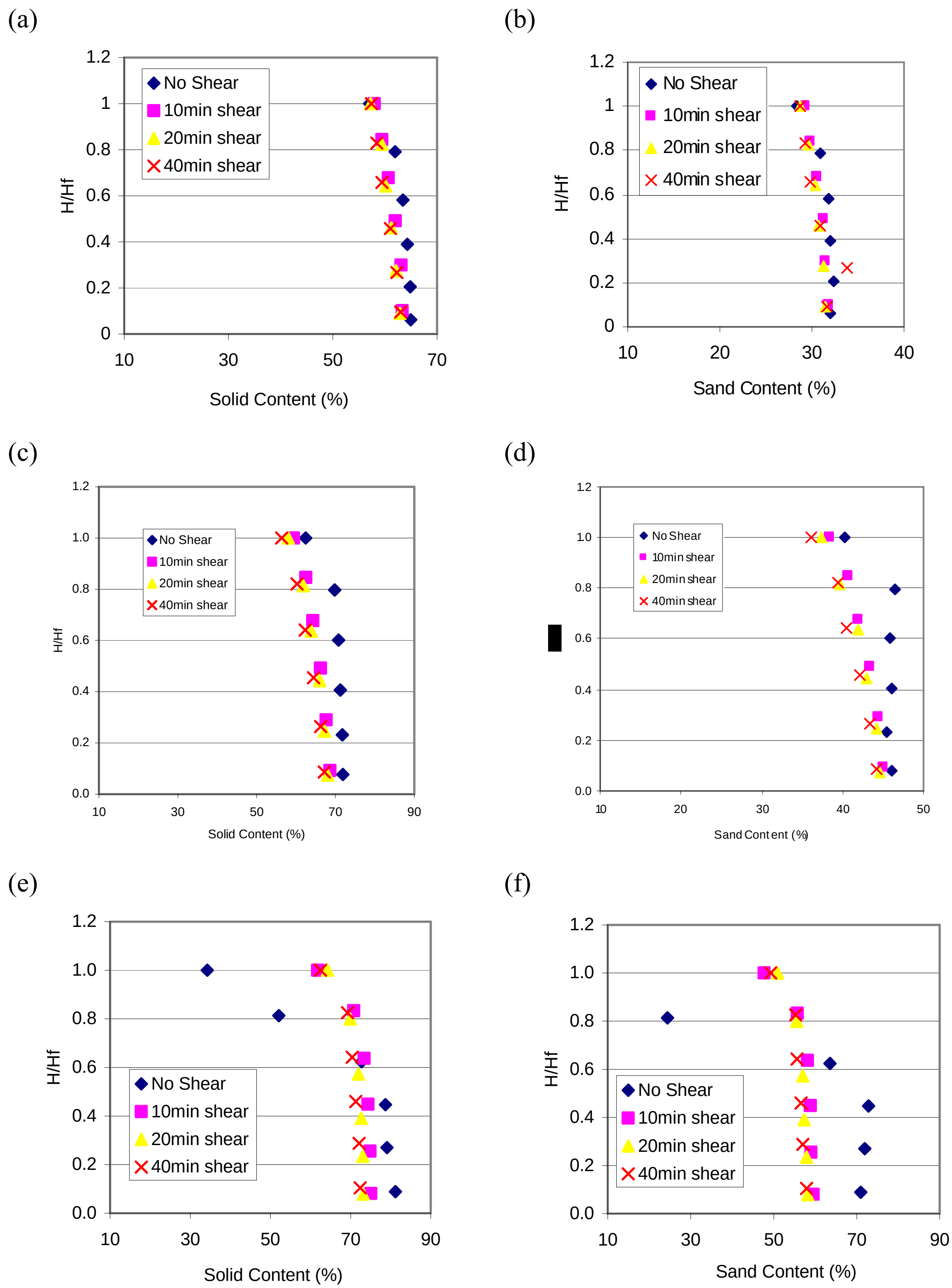


Figure C-4 Solid and Sand Content Profiles; (a) and (b) for DS1, (c) and (d) for DS2, (e) and (f) for DS3

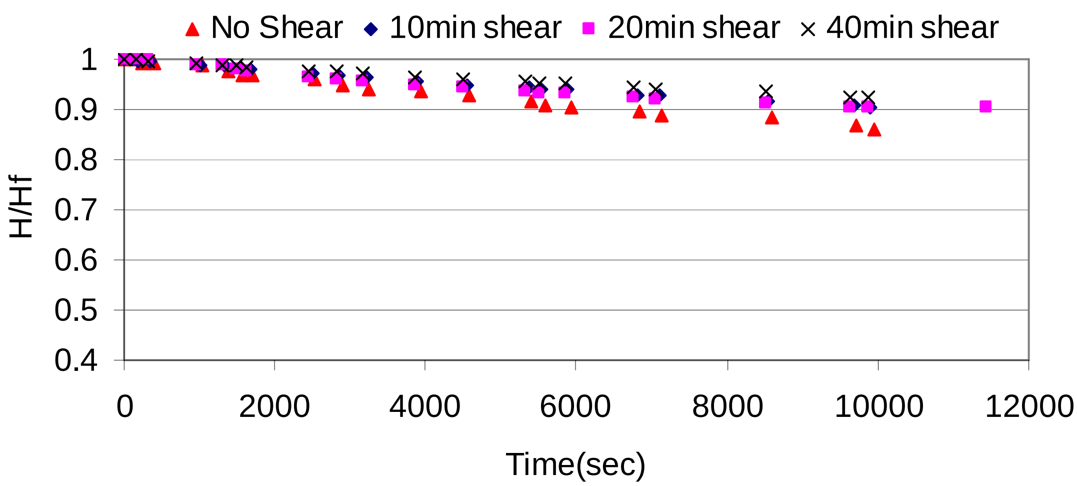


Figure C-5 Sedimentation of sample for test sets DS1 after mixing

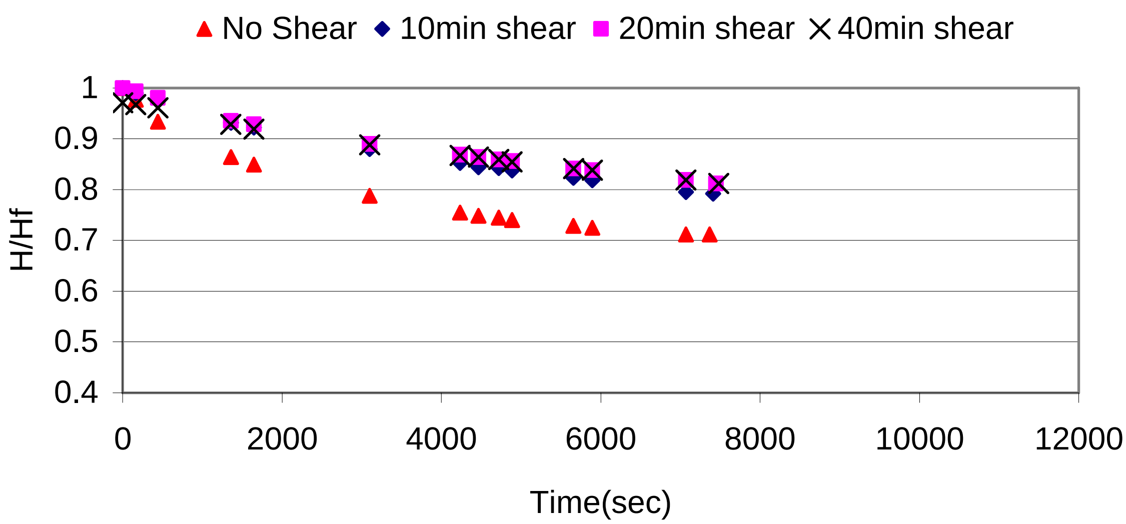


Figure C-6 Sedimentation of sample for test sets DS2 after mixing

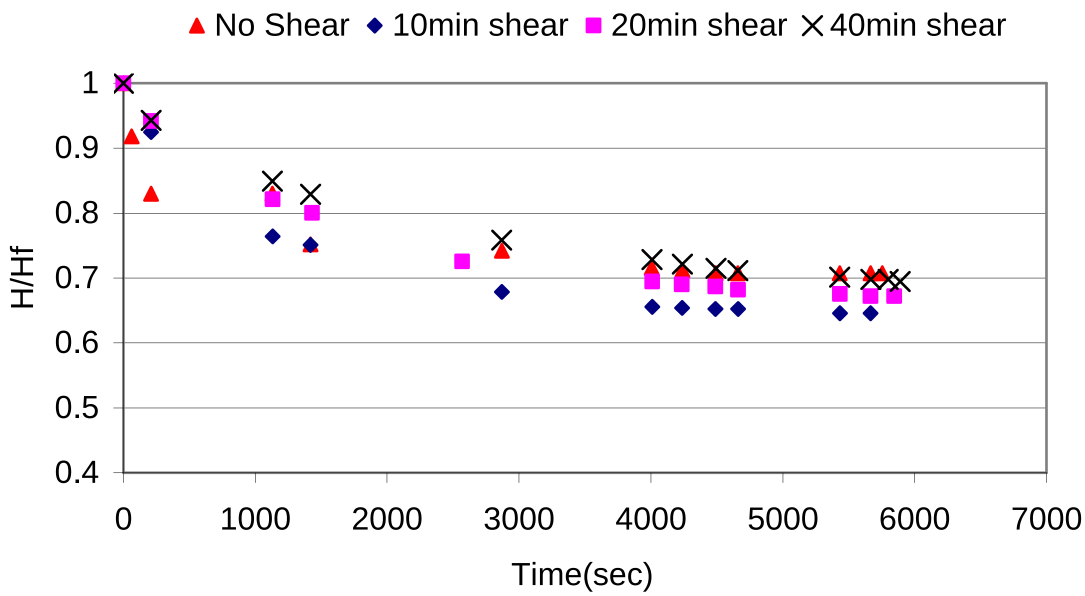


Figure C-7 Sedimentation of samples for test sets DS3 after mixing

Appendix D

Segregation Index Calculation

The proposed Segregation Index calculation in Chapter 7 is:

$$SI = \sqrt{\frac{\left(\sum h_i S_i^2 - \frac{\left(\sum h_i S_i\right)^2}{\sum h_i}\right)}{\sum h_i}} \qquad (7.10)$$

Equation 7.10 may also be expressed as:

$$SI = \sqrt{\frac{\sum H_i \left(S_i - S_{avg}\right)^2}{\sum H_i}} \qquad (7.10\ a)$$

Both Equation 7.10 and Equation 7.10.a can be worked out in a spreadsheet and similar results will serve as a check.

The following spreadsheet example illustrates the calculation of SI for sand content profile shown in Figure 4-4.

Table D.2 provides summary of SI calculation for flume segregation test in Chapter 5. The Hi in this case refers to the sectional length of the flume from which sample was taken.

Table D-1 Segregation index calculation for testing at 15 sec

Si	Hi	SiHi	$SiHi^2$	$Hi(Si-Savg)^2$
7.619912	10	76.19912	580.6307	5802.338
15.9997	10	159.997	2559.904	2467.493
25.37764	10	253.7764	6440.248	400.7284
27.02631	10	270.2631	7304.213	219.1783
82.51621	10	825.1621	68089.25	25814.79
Sum	50	1585.398	84974.25	34704.53

S_{avg}	31.70796			
SI_1 =	**26.3456**	SI_2 =	**26.3456**	Check

Table D-2 Flume test Segregation Index (SI) calculations

Solid Content(%w)	Flume Slope (degree)	SFR	SI
57.6	0	1	0.5
57.6	5	1	0.5
57.6	10	1	1.0
57.9	0	2	1.2
57.9	5	2	1.8
57.9	10	2	3.6
55.5	0	4	3.7
55.5	5	4	3.2
55.5	10	4	5.5

Statistical Comparisons of Model fits

The following procedure was modified from (Motulsky and Christopoulos 2003).

1. Fit the one of the models using non-linear regression.
2. Look at the results non-linear regression and write down the sum-of-squares (SS)
3. Define N to be the number of data points. Be sure to account for replicates properly.
4. Define K to be the number of parameters fit by non-linear regression plus 1. Donot count parameters that are constrained to constant values. If in doubt, count the number of distinct Standard Error (SE) reported by non-linear regression, then add 1 to get the value of K.
5. Compute AICc

$$AIC_c = N \cdot \ln\left(\frac{SS}{N}\right) + 2K + \frac{2K(K+1)}{N-K-1}$$

6. Repeat steps 1-5 with the other model
7. The model with lower AICc is more likely to be correct
8. Calculate the evidence ratio from the difference in AICc scores

$$Evidence.Ratio = \frac{1}{e^{-0.5\Delta AIC_c}}$$

Table D-3 presents the summary of AICc computation for kao-slurry at 40% solids using Excel spread sheet. And Table D-4 presents the summary of AICc computation for kao-sil slurry at 50% solids.

In summary ,the AICc value is lower for Casson model in both tables, indicating that Casson’s model represent the data better that Hershel-Bulkley (H-B) model.

Table D- 3 AICc Calculation for 40% kao slurry

Shear Rate (1/s)	SS (Pa)	HB Model	Casson's Model	SS(H-B)	SS(Casson)
4.08	12.49	15.91	14.47	11.682	3.902
10.2	17.42	17.36	15.91	0.003	2.293
20.4	18.27	18.77	17.61	0.252	0.434
34	20.4	20.02	19.39	0.142	1.017
40.8	20.91	20.52	20.17	0.152	0.546
54.4	21.85	21.36	21.59	0.236	0.066
61.2	22.19	21.73	22.25	0.210	0.004
68	20.91	22.07	22.89	1.352	3.906
SUM				14.028	12.168
N	8				
SE	1.777	0.544	0.431		
K		1.544	1.431		
AIC		7.581	6.216		
AICc		9.022	**7.465**	Lower AICc	
Evidence Ratio			0.211		

Table D- 4 AICc Calculation for 50% kao-sil slurry

Shear Rate (1/s)	SS (Pa)	H-B Model	Casson's Model	SS(H-B)	SS(Casson)
4.08	11.31	13.61	13.50	5.279	4.816
10.2	15.64	15.70	15.69	0.004	0.002
20.4	18.53	18.34	18.34	0.037	0.035
34	19.97	19.83	19.83	0.020	0.020
40.8	22.1	22.46	22.45	0.129	0.124
54.4	24.74	24.79	24.79	0.002	0.003
61.2	25.25	25.87	25.89	0.387	0.406
68	27.71	26.91	26.95	0.633	0.584
SUM				6.491	5.989
N	8				
SE	1.185	0.580	0.596		
K		1.580	1.596		
AIC		1.487	0.876		
AICc		2.992	**2.409**	Lower AICc	
Evidence Ratio			0.558		