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Multistage robust energy management for microgrids considering uncertainty

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Abstract: Microgrids play an important role in modern power systems which can integrate different kinds of distributed energy resources (DERs). To deal with the uncertainty from various factors such as renewable generation, robust energy management for microgrids has become a significant problem. In this work, a novel multistage robust energy management model for grid-connected microgrids is developed which considers the uncertainty of renewable generation and load demand. The multistage energy management problem is complex and computationally difficult. To solve this problem, a robust version of dual dynamic programming method is proposed which includes a forward pass and a backward pass procedure and has a similar framework with the common stochastic dual dynamic programming (SDDP) method. Based on real datasets, a case study is carried out to validate the effectiveness of the proposed model and solution methodology. Numerical results show that the proposed approach can effectively achieve the robust optimal solution, and the comparison with other methods also testifies the advantage of the proposed multistage robust model.

1 Introduction

With the progress of advanced communication and information technology, the development and utilisation of renewable energy have attracted wide attention around the world. As important ways of integrating renewable energy generation, microgrids play a significant role in future smart grid and have been studied for a long time. Generally, a microgrid is a small-scale low-voltage power distribution system which is composed of different loads and distributed energy resources (DERs) such as diesel generators (DGs), renewable generation units, and energy storage units [1]. It can operate either in grid-connected mode or islanded mode and address the reliability, safety, and environmental issues by integrating DERs [2]. Due to the high penetration of intermittent and uncertain renewable generation like wind and solar energy, microgrid energy management has become a complex and challenging task. In addition, the uncertain factors from load demand and electricity market also intensify the difficulty of microgrid energy management problems.

In order to deal with the uncertainties in microgrid energy management, different methods have been proposed in existing literature which mainly focus on stochastic programming and robust optimisation. For example, in [3], a two-stage stochastic energy management model is proposed for grid-connected microgrid. The variability of renewable generation is represented by known probability distribution in this work, and the objective is to minimise the expected operational cost. Similarly, a two-stage stochastic model is studied for microgrid short-term scheduling in [4] which considers a wide variety of uncertainties such as the solar irradiance, wind speed, load demand, and outage of components. Also, a risk-constrained two-stage stochastic scheduling model for an islanded microgrid is developed in [5] where the conditional value at risk method is used to model the risk aversion for various uncertainties.

In addition to stochastic programming model, robust optimisation methods are also applied to solve microgrid energy management problems. In [6], a scenario-based robust energy management method is developed for grid-connected microgrids with the uncertainty of renewable generation and load. This is actually a single stage model as the uncertainties are all revealed by generating scenarios. Considering the day-ahead unit commitment schedule of co-generation units, a two-stage adaptive robust energy management model is also studied in [7], where the grid-connection state is also considered uncertain in addition to the uncertain renewable generation. Similarly, the two-stage robust optimisation method is applied to the energy schedule of microgrid system in [8]. The uncertainty budget is used to control the conservativeness of uncertain photovoltaic power. With the development of distributionally robust optimisation method, microgrid energy management considering the distributional robustness of random variables has also been studied recently [9]. However, the relevant research works also focus on single-stage or two-stage models which neglect the non-anticipativity of uncertainty.

As discussed above, single-stage or two-stage energy management models have been widely studied for microgrids. In such models, the first stage is to determine the day-ahead schedule under uncertainty, and the second stage aims to re-adjust the optimal schedule with fixed uncertainty realisation. Despite the prevalence of two-stage models in the literature, we should note that these models have limitations in capturing the uncertainty. With a simple uncertainty description method, it is typically assumed that the information is perfect and the uncertainty is anticipative in the second stage. More specifically, after the determination of day-ahead schedule, the uncertainties are all revealed at the same time for the scheduling horizon (e.g. 24 h) with perfect information assumption in the second stage [10]. However, in practice, the uncertainties can only be revealed gradually and the future information is unknown at current time. Thus, a multistage model would be more practical and proper to capture the inter-temporal uncertainties than a two-stage model.

Compared with two-stage models, multistage models with uncertainty (e.g. multistage stochastic programming models) are more complicated and computationally difficult. To overcome the computational tractability problems of multistage models, various methods have been proposed, including the sample average approximation (SAA) and the popular decomposition method, stochastic dual dynamic programming (SDDP). SDDP method was first proposed in [11] for the hydrothermal generation scheduling problem. Recently, this method has been applied to deal with power system multistage optimisation problems, such as the real-time economic dispatch [12], energy storage management [13], and

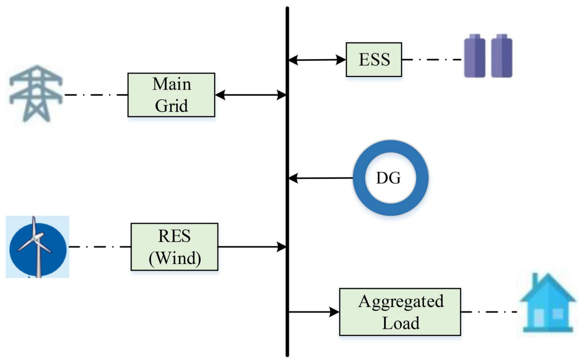

Fig. 1 *Microgrid system model*

DER aggregators operation under multiple sources of uncertainty [10]. Although SDDP method has been demonstrated to solve the computational challenge of multistage stochastic optimisation problems, a limitation is that the assumed distribution of random variables is hard to be known in practice, and it is usually approximated by fitting the historical data. On the other hand, it is well known that the true distribution is not required in robust optimisation model which aims to find the optimal solution under the worst-case scenario. In this case, the so-called uncertainty set is used to capture the uncertainty of random variables. In addition, although different uncertainty modelling methods have been studied in the literature such as probabilistic approach, robust optimisation and information gap decision theory (IGDT) [14], we should note that each method has its own advantages and disadvantages. For the robust optimisation method, it needs less distribution information compared with probabilistic approach and does not need forecasted value as in IGDT method. Moreover, it can guarantee the reliable solution by considering the worst-case scenario uncertainty. In comparison with the study of multistage stochastic programming models, multistage robust models have been seldom studied for microgrid energy management problems. Although multistage robust decision-making has been discussed in other areas such as inventory management or scheduling [15], how to develop microgrid energy management schemes based on this technique still require extensive research.

Based on the above analysis, in this work, we study the energy management problem for grid-connected microgrids from a multistage perspective. The microgrid is usually composed of renewable energy sources (RESs) such as wind power, DGs, energy storage systems (ESSs), and different loads. Compared with previous literature about microgrid energy management, there are several contributions in the present work as given below.

- A new multistage robust energy management model for grid-connected microgrid is developed, which considers the non-anticipative uncertainty from a practical perspective compared with previous two-stage models, and the uncertainty considered comes from renewable generation and load demand. Although robust energy management has been studied in many references, to the best of our knowledge, almost all of them focus on two-stage or single-stage models.
- To deal with the computational difficulty of multistage robust model, a novel decomposition method similar to SDDP method, i.e. the robust dual dynamic programming method (RDDP), is proposed to solve the problem. This robust version of dual dynamic programming makes the complex multistage problem computationally tractable to find the worst-case optimal solutions. It decomposes the multistage problem into small-stage problems and tries to approximate the unknown cost-to-go function with a lower and upper bound.
- A case study with real datasets is carried out to verify the effectiveness of the proposed model and method. In particular, the simulation results are analysed and the comparison with other methods including the common SDDP is discussed.

The remainder of this paper is organised as follows. The multistage robust energy management problem is formulated in Section 2, which consists of microgrid system modelling and a description of multistage robust optimisation. Section 3 demonstrates the solution methodology based on RDDP method for the proposed multistage problem. The case study and corresponding simulation results are provided in Section 4. Finally, the conclusion is given in Section 5.

2 Problem formulation

Generally, a microgrid has two operation modes, i.e. grid-connected mode and islanded mode. In grid-connected mode, the microgrid can exchange power with the main grid to maintain the power balance. In case of main grid fault, it can switch to the islanded mode. We focus on multistage energy management of a grid-connected microgrid with uncertainty in this work, which can also be extended to an islanded microgrid by setting zero power exchange. To formulate the problem, the microgrid system model and uncertainty set considered are first introduced in this section, then the multistage robust optimisation problem is presented.

2.1 System model

Generally, a microgrid is composed of a number of conventional generators, renewable generators (or RESs), ESSs, and a group of loads. The schematic diagram of a typical grid-connected microgrid is shown in Fig. 1 [16]. In this model, all units are connected to one bus, only the nodal power balance is considered and there are no transmission line flows. In addition, it is assumed that the considered distribution lines have sufficiently large capacity and line loss is neglected [6, 7, 16]. The objective of microgrid energy management is to minimise the total system cost by determining the output of different DERs, the charging or discharging schedule of ESSs and the amount of electricity purchased from or sold to the main grid.

There are different kinds of conventional generators in a microgrid such as micro-turbines, DGs, and fuel cells. In this work, we consider DGs as the conventional generators, and the fuel cost is formulated as a linear function at each time period t, which can also be approximated by a piece-wise linear function [6]:

$$C_{i,t}^{dg} = (a_i^{dg} P_{i,t}^{dg} + b_i^{dg})\Delta t, \quad \forall i, t \tag{1}$$

where $P_{i,t}^{dg}$ is the output of ith DG at time t and Δt is the duration of a time slot (e.g. 1 h). At each time period, the output of DGs should satisfy the lower and upper limits and corresponding ramp-up/ramp-down constraints as follows:

$$\underline{P}_i^{dg} \le P_{i,t}^{dg} \le \overline{P}_i^{dg}, \quad \forall i, t \tag{2}$$

$$P_{i,t}^{dg} - P_{i,t-1}^{dg} \le R_i^{up}, \quad \forall i, t \tag{3}$$

$$P_{i,t-1}^{dg} - P_{i,t}^{dg} \le R_i^{dn}, \quad \forall i, t\,. \tag{4}$$

The day-ahead schedule of DGs is assumed to be finished and the DGs have on status in this work. Therefore, the start-up and shutdown costs are also neglected here.

To alleviate the negative impact of renewable generation uncertainty, distributed ESSs such as batteries and flywheels can be utilised in microgrid. We select the battery storage system in this work. Let $E_{j,t}$ denote the stored energy of ESS unit j at the end of time t, then the energy balance of ESS in Δt time slot can be expressed as follows [9]:

$$E_{j,t} = E_{j,t-1} + \eta_j^{+} P_{j,t}^{ch} \Delta t - P_{j,t}^{dch} \Delta t / \eta_j^{-}, \quad \forall j, t \tag{5}$$

where η_j^{+} and η_j^{-} represent the charging and discharging rates, respectively, $P_{j,t}^{ch}$ and $P_{j,t}^{dch}$ are charging and discharging power at time period t for unit j, respectively. In addition, the charging and discharging power should satisfy the following constraints:

$$0 \le P_{j,t}^{ch} \le \overline{P}_j^{ch}, \quad 0 \le P_{j,t}^{dch} \le \overline{P}_j^{dch}, \quad \forall j, t \tag{6}$$

where $\overline{P}_j^{ch}$ and $\overline{P}_j^{dch}$ are maximum charging and discharging limits, respectively. Note that the complementary constraint to avoid simultaneous charging and discharging is not required here as the charging/discharging efficiency is considered [17]. Since each ESS unit has a capacity limit, the following constraint is used to restrict its stored energy:

$$\underline{E}_j \le E_{j,t} \le \overline{E}_j, \quad \forall j, t \tag{7}$$

where $\underline{E}_j$ and $\overline{E}_j$ are minimum and maximum stored energy for unit j. In addition, for periodic use of ESS, the final stored energy level at time T should be equal to the known initial capacity value as follows:

$$E_{j,T} = E_{j,0}. \tag{8}$$

Considering the influence of frequent charging and discharging on storage lifetime, a linear function model is used to describe the ESS degradation or maintenance cost [7]:

$$C_{j,t}^{ess} = c_j^{ess}(\eta_j^+ P_{j,t}^{ch} + P_{j,t}^{dch}/\eta_j^-)\Delta t, \quad \forall j, t \tag{9}$$

where c_j^{ess} denotes the degradation cost coefficient of unit j.

For a grid-connected microgrid, it can trade energy with the external market, i.e. purchase electricity from the main grid or sell the excess power to the main grid. Let c_t^p and c_t^s denote the purchase price and sale price at time period t in electricity market, respectively, then the power exchange cost with the main grid can be expressed as follows:

$$C_t^{grid} = (c_t^p P_t^p - c_t^s P_t^s)\Delta t \tag{10}$$

where P_t^p and P_t^s represent the amount of power purchased from or sold to the external market, respectively. It is usually considered that $c_t^p > c_t^s$ which is reasonable to avert trading arbitrage. Otherwise, we can gain profits from buying and selling power in the market [18]. In addition, the trading power should satisfy the following constraints:

$$0 \le P_t^p \le \overline{P}_t^p, \quad 0 \le P_t^s \le \overline{P}_t^s \tag{11}$$

where $\overline{P}_t^p$ and $\overline{P}_t^s$ are the maximum limits of power purchase and sale, respectively.

Renewable energy generation is an important part of microgrids. Without loss of generality, wind power is considered as the renewable source in this work. Although there are different kinds of demands in a microgrid, aggregated load is considered for simplicity. Taking all the power supply and demand into account, we have the following power balance constraint at each time period:

$$\sum_i P_{i,t}^{dg} + \sum_j (P_{j,t}^{dch} - P_{j,t}^{ch}) + P_t^p - P_t^s + w_t + P_t^{loss} = L_t \tag{12}$$

where w_t and L_t represent the aggregated wind power output and load demand, respectively, P_t^{loss} is an auxiliary variable to balance the load shedding. Wind power and load demand are sources of uncertainty in this work, which can be expressed by a new random vector $\xi_t = (w_t, L_t)$ for convenience.

In robust optimisation problems, the true distribution of random variable is usually unknown and uncertainty set is used to describe the uncertainty. In this work, we use the polyhedral interval uncertainty set for uncertain wind power and load demand as given below:

$$U_\xi = [\xi_t : \underline{\xi}_t \le \xi_t \le \overline{\xi}_t] \tag{13}$$

where $\underline{\xi}_t$ and $\overline{\xi}_t$ are the lower and upper bounds of the variable. The uncertainty set introduced above is bounded and stage-wise rectangularity is assumed. To obtain such uncertainty sets, several methods can be used as shown in the literature. For example, we can use the popular interval prediction method to estimate the uncertainty set [19]. In addition, the uncertainty set can also be attained by point forecast methods which construct the interval by a point forecast value and corresponding confidence level [6]. Note that the random variable is assumed to be deterministic in the first stage of a multistage problem.

2.2 Multistage robust optimisation problem

Based on the microgrid equipment models introduced above, we can define the decision variable x_t and the objective function of a single-stage problem as follows:

$$c_t^\top x_t = \sum_i C_{i,t}^{dg}\Delta t + \sum_i C_{j,t}^{ess} + C_t^{grid} + c^{loss}P_t^{loss} \tag{14}$$

$$x_t = [P_{i,t}^{dg}, P_{j,t}^{ch}, P_{j,t}^{dch}, E_{j,t}, P_t^p, P_t^s, P_t^{loss}] \tag{15}$$

where c^{loss} is the corresponding penalty cost coefficient. Note that $x_t = x_t(\xi_t)$ where ξ_t is omitted for notational simplicity. For a multistage model, the decisions are made sequentially with the gradual disclosure of uncertainty.

Combining the previous constraints and objective functions, we can write the multistage robust problem as below:

$$\begin{array}{ll} \min\limits_{x_1} & c_1^\top x_1 + [\max\limits_{\xi_2} \min\limits_{x_2} c_2^\top x_2 + [\cdots + \max\limits_{\xi_T} \min\limits_{x_T} c_T^\top x_T]\cdots] \\ \text{s.t.} & (2)-(8), (11)-(12). \end{array} \tag{16}$$

By transforming the constraints into a matrix form at each stage, we can also write the multistage problem in a compact form:

$$\begin{array}{l} \min\limits_{\substack{A_1x_1 \ge b_1 \\ x_1 \ge 0}} c_1^\top x_1 + [\max\limits_{\xi_2} \min\limits_{\substack{B_2x_1 + A_2x_2 \ge b_2 \\ x_2 \ge 0}} c_2^\top x_2 + [\cdots + \\ \quad \max\limits_{\xi_T} \min\limits_{\substack{B_Tx_{T-1} + A_Tx_T \ge b_T \\ x_T \ge 0}} c_T^\top x_T]\cdots] \end{array} \tag{17}$$

where A_t and B_t are corresponding coefficient matrices derived from the constraints, and ξ_t is omitted in $b_t(\xi_t)$ which means that b_t is influenced by the random parameter ξ_t. ξ_1 is deterministic in the first stage. The problem (17) is also called the nested problem formulation, which is very difficult or intractable to solve for even a small number of stages.

To solve this problem, we can decompose it into stage problems and write it in a dynamic programming form as follows:

$$\begin{array}{ll} \min & c_1^\top x_1 + \mathcal{Q}_2(x_1) \\ \text{s.t.} & A_1x_1 \ge b_1 \\ & x_1 \ge 0 \end{array} \tag{18}$$

where $\mathcal{Q}_2$ represents the worst-case future cost-to-go function in the first-stage problem and the stage- t worst-case cost-to-go function $\mathcal{Q}_t$ is defined as $\mathcal{Q}_t(x_{t-1}) = \max_{\xi_t}\{\mathcal{S}_t(x_{t-1};\xi_t)\}$ with the inner problem $\mathcal{S}_t(x_{t-1};\xi_t)$ defined as follows:

$$\begin{array}{lll} \mathcal{S}_t(x_{t-1};\xi_t) = \min & c_t^\top x_t + \mathcal{Q}_{t+1}(x_t) \\ \text{s.t.} & B_tx_{t-1} + A_tx_t \ge b_t \\ & x_t \ge 0, \; t = 2, \ldots, T \end{array} \tag{19}$$

and at the last stage, we have $\mathcal{Q}_{T+1}(x_T) = 0$ or it can be some known convex polyhedral function [20]. Thus, the nested multistage robust problem can be replaced by the dynamic programming equations, i.e. the stage-1 problem and stage- t

problem as shown in (18) and (19). The solution methodology is also developed based on the dynamic programming form.

3 Solution methodology

In multistage stochastic programming problem, we need to identify the expected future cost function to solve the problem and the SDDP method is thus developed with an outer approximation method for the expected future cost. By contrast, it is the worst-case future cost that needs to be considered in multistage robust problem, and this difference hinders the direct application of SDDP method. To solve the multistage robust energy management problem introduced above in this work, a robust version of dual dynamic programming method is proposed which is also called robust dual dynamic programming (RDDP) method [21]. This method has a similar framework with the popular SDDP method which also consists of a forward pass and a backward pass procedure. Generally, the forward pass is used to search for the worst-case realisations of random variables and corresponding optimal recourse decisions which will generate upper and lower bounds of the worst-case cost-to-go function at each stage, while the backward pass aims to refine the obtained upper and lower bounds.

Compared with the SDDP method, the RDDP method introduces an upper bound to approximate the worst-case cost-to-go function in addition to the common lower bound. Based on the assumption of a convex affine worst-case cost-to-go function, the upper bound is obtained by inner approximation method [22], and the lower bound is attained by outer approximation which will be discussed in detail below.

3.1 Upper bound problem

Recall that the multistage robust problem in (19), the worst-case cost-to-go function $\mathcal{Q}_{t+1}$ in this problem is unknown at current stage which makes the evaluation of $\mathcal{Q}_t$ difficult. A good idea is to find the approximation of this function so that the robust optimal solution can be obtained. Therefore, in this work, we approximate each worst-case cost-to-go function $\mathcal{Q}_t, t = 2, \dots, T$ by iteratively constructing its lower and upper bounds which satisfy $\underline{\mathcal{Q}}_t(x_{t-1}) \le \mathcal{Q}_t(x_{t-1}) \le \overline{\mathcal{Q}}_t(x_{t-1})$. To get the upper bound $\overline{\mathcal{Q}}_t(x_{t-1})$, we can define and solve the following upper bound problem:

$$\begin{aligned} \overline{\mathcal{S}}_t(x_{t-1};\xi_t) = \max_{\xi_t}\ & \min_{x_t} c_t^{\mathrm{T}} x_t + \overline{\mathcal{Q}}_{t+1}(x_t) \\ \text{s.t.}\ \ & B_t x_{t-1} + A_t x_t \ge b_t(\xi_t) \\ & x_t \ge 0,\ t = 2, \dots, T \end{aligned} \tag{20}$$

where $\overline{\mathcal{Q}}_{t+1}(x_t)$ is the upper bound of $\mathcal{Q}_{t+1}(x_t)$. Generally, we have $\overline{\mathcal{S}}_t(x_{t-1};\xi_t) \ge \mathcal{Q}_t(x_{t-1})$ since the upper bound $\overline{\mathcal{Q}}_{t+1}(x_t)$ is considered in $\overline{\mathcal{S}}_t(x_{t-1};\xi_t)$. Likewise, if $\overline{\mathcal{Q}}_{t+1}(x_t) = \mathcal{Q}_{t+1}(x_t)$, then the optimal value of $\overline{\mathcal{S}}_t(x_{t-1};\xi_t)$ would be the same with that of $\mathcal{Q}_t(x_{t-1})$. Hence, the optimal solution of problem $\overline{\mathcal{S}}_t(x_{t-1};\xi_t)$ can be used to approximate $\mathcal{Q}_t(x_{t-1})$ from above.

It is obvious that the problem $\overline{\mathcal{S}}_t$ is a common max-min problem which can be solved by dual method or vertex enumeration method in condition that $\overline{\mathcal{Q}}_{t+1}(x_t)$ is a convex piecewise affine function. By dualising the inner minimisation problem, the problem can be transformed into a single-level problem and solved by mixed-integer linear programming method. Another method is the vertex numeration method [21]. Since it has been demonstrated that the inner minimisation problem in (20) is convex in the parameter ξ_t, the optimal solution is obtained at a certain extreme point of the uncertainty set U_ξ. Thus, we can try to identify the extreme points of the set U_ξ and solve a finite number of linear programming problems. The vertex enumeration method can be much easier when the number of extreme points is small in the problem or the parallel computation technique can be used. Considering the problem scale and the interval uncertainty set, we will use the vertex enumeration method to solve the upper bound problem in this work.

As the explicit expression of $\overline{\mathcal{Q}}_{t+1}(x_t)$ is unknown, we use the inner approximation method [22] for it in this work so that we can solve the upper bound problem approximately. Assuming that J_t points $(\underline{x}_t^j, \overline{\mathcal{S}}_{t+1}^j(\underline{x}_t)), j = 1, \dots, J_t$ have been collected for the upper bound $\overline{\mathcal{Q}}_{t+1}$ of stage $t+1$ worst-case cost-to-go function, then we can approximate the upper bound function with the lower convex envelop or convex hull of these points, and the upper bound problem in (20) can be expressed approximately as follows:

$$\begin{aligned} \overline{\mathcal{S}}_t(x_{t-1};\xi_t) = \max_{\xi_t}\ & \min_{x_t} c_t^{\mathrm{T}} x_t + \sum_{j=1}^{J_t} \lambda^j \overline{\mathcal{S}}_{t+1}^j \\ \text{s.t.}\ \ & B_t x_{t-1} + A_t x_t \ge b_t(\xi_t) \\ & x_t = \sum_{j=1}^{J_t} \lambda^j x_t^j,\ \sum_{j=1}^{J_t} \lambda^j = 1 \\ & \lambda^j \ge 0,\ x_t \ge 0,\ t = 2, \dots, T. \end{aligned} \tag{21}$$

where λ^j is the auxiliary variable. Note that the critical point of using inner approximation is the convex assumption of the worst-case cost-to-go function and the inner approximation property can be inherited from stage to stage.

3.2 Lower bound problem

To obtain the lower bound $\underline{\mathcal{Q}}_t(x_t)$, we can solve the following lower bound problem with any one fixed uncertainty realisation $\tilde{\xi}_t$:

$$\begin{aligned} \underline{\mathcal{S}}_t(x_{t-1};\tilde{\xi}_t) = \min\ & c_t^{\mathrm{T}} x_t + \underline{\mathcal{Q}}_{t+1}(x_t) \\ \text{s.t.}\ \ & B_t x_{t-1} + A_t x_t \ge b_t(\tilde{\xi}_t) \\ & x_t \ge 0,\ t = 2, \dots, T \end{aligned} \tag{22}$$

where $\underline{\mathcal{Q}}_{t+1}(x_t)$ is the lower bound of $\mathcal{Q}_{t+1}(x_t)$. Compared with $\mathcal{Q}_t(x_{t-1})$, only one realisation of the random variable and the lower bound $\underline{\mathcal{Q}}_{t+1}$ of stage $(t+1)$ worst-case cost-to-go function are considered here, thus the optimal value of this problem can bound $\mathcal{Q}_t(x_{t-1})$ from below. With a convex and piecewise affine function $\underline{\mathcal{Q}}_{t+1}(x_t)$, the lower bound problem can be solved as a linear programming problem, and we can find the lower bound of $\mathcal{Q}_t(x_{t-1})$ by the supporting hyperplanes or outer approximation of $\underline{\mathcal{S}}_t$ during the iteration process. More specifically, we can define the approximate problem by replacing $\underline{\mathcal{Q}}_{t+1}(x_t)$ with a variable θ_{t+1} constrained by a set of cutting planes [20] as follows:

$$\begin{aligned} \underline{\mathcal{S}}_t(x_{t-1};\tilde{\xi}_t) = \min\ & c_t^{\mathrm{T}} x_t + \theta_{t+1} \\ \text{s.t.}\ \ & B_t x_{t-1} + A_t x_t \ge b_t(\tilde{\xi}_t),\ [\pi(\tilde{\xi}_t)] \\ & \theta_{t+1} + \pi_{t+1,k}^{\mathrm{T}} B_{t+1} x_t \ge \overline{g}_{t+1,k},\ k = 1, \dots, K \\ & x_t \ge 0,\ t = 2, \dots, T \end{aligned} \tag{23}$$

where the variable $\pi(\tilde{\xi}_t)$ is the Lagrange multiplier vector corresponding to the constraint and $\overline{g}_{t+1,k}$ is the intercept of k th cut defined as follows:

$$\overline{g}_{t+1} = \underline{\mathcal{S}}_{t+1}(\underline{x}_t;\tilde{\xi}_{t+1}) + \pi_{t+1}^{\mathrm{T}} B_{t+1} \underline{x}_t \tag{24}$$

where $\underline{x}_t$ is the optimal solution of problem $\underline{\mathcal{S}}_{t+1}(x_t;\tilde{\xi}_{t+1})$. The cuts can also be generated and expressed by the cut calculation algorithm in [23] which has the same principle of outer approximation method.

3.3 RDDP method

The upper bound and lower bound problems introduced above are the main components of the proposed RDDP method. They are frequently constructed and refined in the forward pass and backward pass during the iteration process until we find the optimal worst-case cost-to-go function. Based on this idea, the procedure of RDDP method can be summarised as follows:

(1) *Initialisation*: Set the lower bound and $\underline{Q}_t(x_{t-1}) = -M$ and the upper bound $\overline{Q}_t(x_{t-1}) = +M$, for $t = 2, \dots, T$, where M is a large enough number and it can also be set to ∞. Note that this is for the first iteration before the update of lower and upper bounds. In addition, set $\underline{Q}_{T+1}(x_T) = \overline{Q}_{T+1}(x_T) = 0$ for the last stage.
(2) *First-stage problem*: Solve the lower bound problem for the first stage as follows with a deterministic realisation of the random variable and store the optimal solution $\underline{x}_1$. If the difference between the upper bound $\overline{Q}_2$ and the lower bound $\underline{Q}_2$ is less than a small predefined parameter or the maximum iteration is reached, then terminate the algorithm, else go to step 3).

$$\begin{aligned} \min \quad & c_1^{\mathrm{T}} x_1 + \underline{Q}_2(x_1) \\ \text{s.t.} \quad & A_1 x_1 \ge b_1 \\ & x_1 \ge 0. \end{aligned} \tag{25}$$

(3) *Forward pass*: For $t = 2, \dots, T$, solve the upper bound problem $\overline{\mathcal{S}}_t(\underline{x}_{t-1};\xi_t)$ in (20) and get the optimal solution $\overline{\xi}_t^{fw}$. Then based on the worst-case uncertainty realisation $\overline{\xi}_t^{fw}$, solve the lower bound problem $\underline{\mathcal{S}}_t(\underline{x}_{t-1};\overline{\xi}_t^{fw})$ in (22) and store the optimal solution $\underline{x}_t$.
(4) *Backward pass*: For $t = T, \dots, 2$, solve the upper bound problem $\overline{\mathcal{S}}_t(\underline{x}_{t-1};\xi_t)$ based on $\underline{x}_{t-1}$ and obtain the new optimal solution $\overline{\xi}_t^{bw}$. If $\overline{\mathcal{S}}_t(\underline{x}_{t-1}) < \overline{Q}_t(\underline{x}_{t-1})$, then collect the point $(\underline{x}_{t-1}, \overline{\mathcal{S}}_t(\underline{x}_{t-1}))$ to update the upper bound approximation in (21). Based on $\overline{\xi}_t^{bw}$ and $\underline{x}_{t-1}$, we can solve the lower bound problem $\underline{\mathcal{S}}_t(\underline{x}_{t-1};\overline{\xi}_t^{bw})$, and let π_t be the dual optimal solution. If $\underline{\mathcal{S}}_t(\underline{x}_{t-1};\overline{\xi}_t^{bw}) > \underline{Q}_t(x_{t-1})$, then update the lower bound problem in (23) by adding the following cut into the previous stage problem:

$$\theta_t + \pi_t^{\mathrm{T}} B_t x_{t-1} \ge \underline{g}_t \tag{26}$$

$$\underline{g}_t = \underline{\mathcal{S}}_t(\underline{x}_{t-1};\overline{\xi}_t^{bw}) + \pi_t^{\mathrm{T}} B_t \underline{x}_{t-1}. \tag{27}$$

This lower bound update method has the same idea with that in the common SDDP method, which iteratively adds Benders' cuts to approximate a convex and piecewise affine function. After completing step 4), go back to step 2).

To better illustrate the RDDP solution process for multistage robust energy management, a schematic diagram based on the above procedure is given in Fig. 2. The idea is to express the multistage problem in a nested formulation. Take the first-stage problem as an example, the uncertainty is revealed in the first stage, and different cost-to-go functions are used to capture the uncertain parameters in the following stages. As shown in this figure, we need to obtain the estimated $Q_2(x_1)$ function for the first-stage problem. Similarly, the corresponding cost-to-go function $Q_{t+1}(x_t)$ should also be estimated at stage t. These cost-to-go functions can be approximately attained by solving the upper bound problem and lower bound problem iteratively. With the above RDDP method, we can solve the proposed multistage robust energy management problem, more specifically, we can find the worst-case scenario and get the robust optimal solution. The case study and numerical results will be presented in next section.

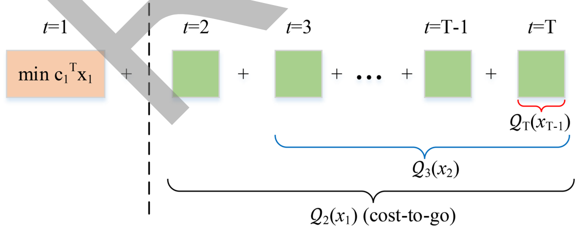

Fig. 2 *Schematic diagram of solution process*

4 Case study

In order to validate the effectiveness of the proposed multistage robust model and solution methodology, a case study for a grid-connected microgrid with real-world data is conducted in this section. First, we give a description of the studied microgrid and related parameters and data. Then, the simulation results of the robust optimal solution from RDDP method are presented. The comparison between the proposed method and other methods is also analysed.

4.1 Microgrid description and data

In this work, we study a grid-connected microgrid composed of three diesel generators, an ESS, a wind turbine, and uncertain loads. Without loss of generality, wind power is considered as the renewable generation. The time horizon for the energy management problem is set to 24 h, i.e. the number of stages T is 24, and the time step Δt is 1 h. For three diesel generators, the involved parameters are collected from related references [6, 24] and summarised in Table 1. The considered storage system has a maximum capacity of 90 kWh, and the minimum storage level is set to be 20 kWh to avoid overdischarging. The limits for charging and discharging rate are both set to 50 kWh, and we assume the same charging and discharging efficiency which is 0.95. In addition, the initial energy storage level is half of the maximum capacity, and the degradation cost coefficient of the storage system is set to 0.0035 $/kWh [7].

Renewable generation and load demand are considered to be uncertain factors. As mentioned before, the uncertainty set for these random variables can be obtained by different methods. Here, we use the point forecast values to generate the interval uncertainty set. The point forecast data for wind power and load are collected from the IESO website [25], which are properly scaled. Based on the point forecast values, we create the interval sets by setting up a certain deviance (e.g. 10%) from the nominal values. The generated intervals and the point forecast values are given in Fig. 3. Note that the interval bounds are assumed to be known in this work. If the bounds are also uncertain, we can try to estimate them using some popular interval prediction methods [19]. In addition, it can be a potential future research topic to consider the bounds as decision variables.

As the microgrid operates in grid-connected mode, we also need to consider the exchange power with the main grid and the market electricity price. The maximum power exchange including the purchase and sale is set to 100 kWh in this work, and the electricity prices are acquired from the NYISO website [26]. The appropriately scaled day-ahead electricity price is used as the purchase price from the market, and the electricity sale price is set to 80% of the purchase price for simplicity [6], which are both presented in Fig. 4. In electricity market, the electricity price can

Table 1 Parameters for the generators

Unit	$\underline{P}_i^{dg}$, kW	$\overline{P}_i^{dg}$, kW	R_i^{up}/R_i^{dn}, kW	a_i^{dg}, $/kWh	b_i^{dg}, $/h
G1	10	50	30	0.13	30
G2	8	45	25	0.2	50
G3	15	70	40	0.25	80

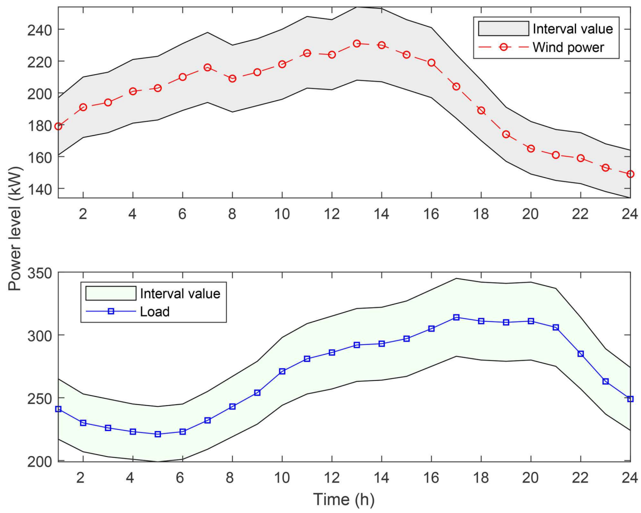

Fig. 3 *Wind power and load data*

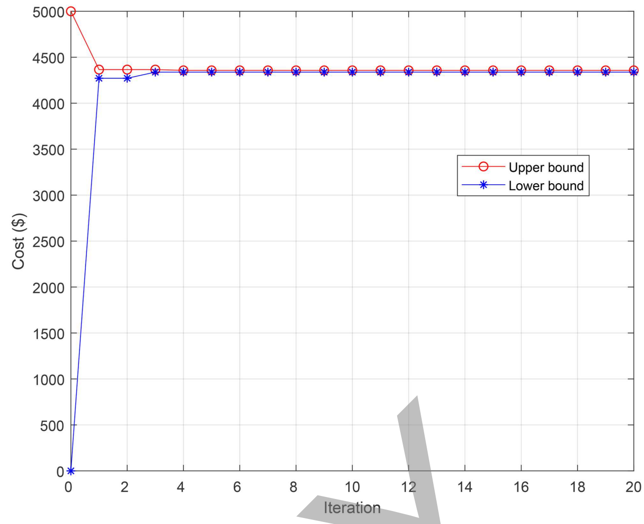

Fig. 5 *Evolution of lower and upper bounds*

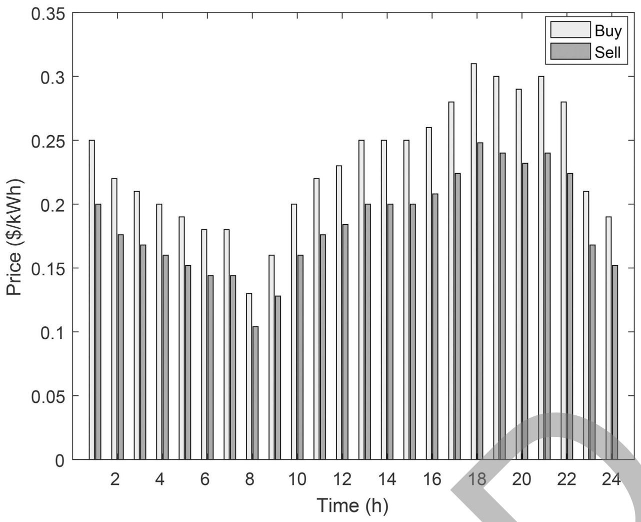

Fig. 4 *Electricity exchange price*

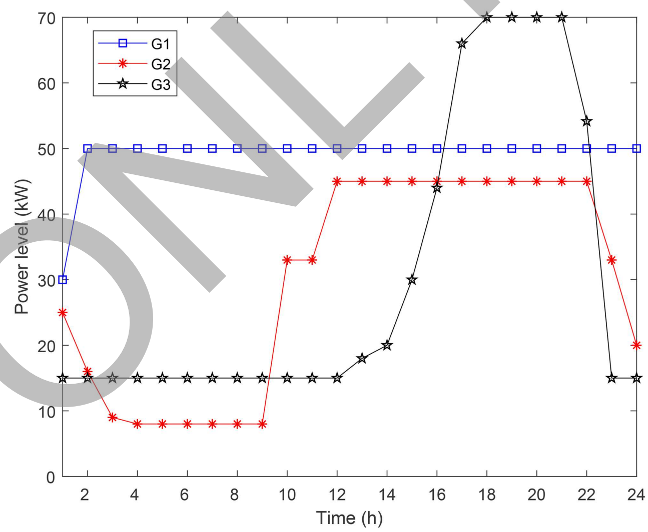

Fig. 6 *Power generation level of DGs*

be determined by static or dynamic pricing scheme. Since the renewable generation and load are usually main factors of uncertainty in a microgrid system, we adopt the static prices in this work which includes fixed prices and time-of-use prices [16]. For the effect of uncertain prices, it will lead to uncertain coefficients in the objective function, and this may be studied in future work. In addition, a linear penalty cost function is used for possible load shedding and the unit penalty cost is equal to 10 \$/kWh in this study [7].

4.2 Simulation results

Based on the above relevant parameters and data, the multistage robust energy management problem can be solved with the RDDP method. All the experiments are implemented in MATLAB environment with Gurobi solver on a desktop with an Intel Core i7-6700 CPU 3.40 GHz and 8 GB of RAM.

We first analyse the convergence of the algorithm. In this experiment, the maximum iteration is used as the termination criterion. The evolution of the lower and upper bounds for the worst-case cost-to-go function value in the first stage problem is shown in Fig. 5. As can be seen from this figure, the lower and upper bounds almost converge to the same value and their difference is very small (0.44%). Thus, the obtained solution can be considered as the optimal robust solution when the algorithm terminates. Note that we can also set a small positive parameter ϵ (e.g. 10^{-2}) to terminate the method, which may need less iterations. The execution time for solving this problem with this method is about 255 s which is acceptable in practice. For the proposed multistage robust problem, the critical point is that we need to determine the unknown cost-to-go functions in this method. It is supposed that the approximate optimal cost-to-go functions are obtained when the algorithm terminates. Accordingly, with the estimated cost-to-go functions, optimal decisions or implementable policies can be attained which can guard against any uncertainty realisation in the uncertainty set.

After the termination of the method, the optimal robust solution including the schedule of DGs, ESS, and the electricity exchange with the market can be obtained corresponding to the worst-case realisations of uncertainty. The optimal schedule of three DGs is given in Fig. 6. From this figure, we can find out that G1 always has the maximum output since the second time period. For G2 and G3, they both have the minimum output at the first few time slots, then the output increases gradually until the maximum output is reached which corresponds to the change of load demand. Taking the generation cost function into account, we can see that G1 always maintains the maximum output due to its lowest cost, while the other two units have larger generation cost and only increase their output when the load becomes larger.

Similarly, we can also attain the optimal schedule of the ESS and the power exchange with the main grid as shown in Fig. 7. Note that the blue line in this figure represents the storage level of ESS at each time slot. As shown in this figure, the ESS first discharges to the minimum value considering the initial storage level (half of the maximum capacity), then it charges almost to the maximum capacity and stay unchanged for some time periods. When the load demand increases at later time stages, the ESS starts to discharge. For the electricity exchange between the microgrid and main grid, it is influenced by the electricity prices. For

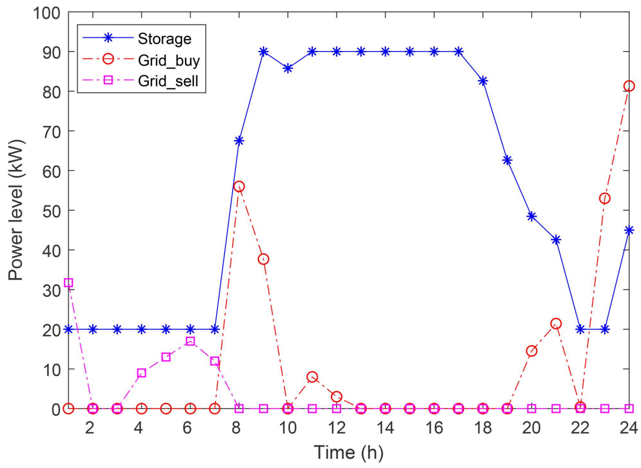

Fig. 7 *Schedule of storage and power exchange*

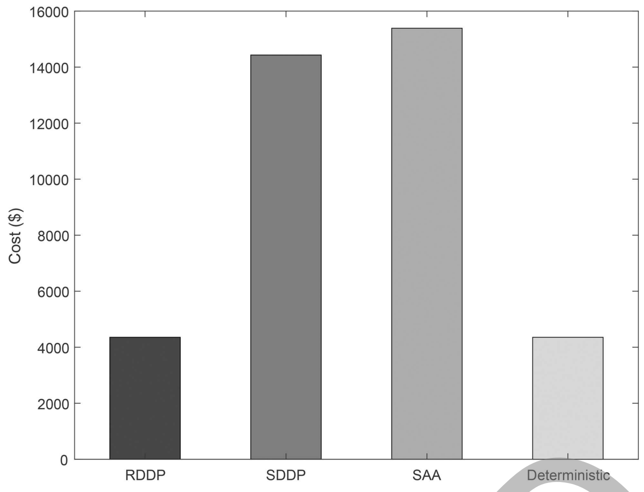

Fig. 8 *Total cost of different methods*

example, from time period 13 to 22 when the electricity purchase price is >0.25, there is no electricity purchase unless the load demand is large enough and cannot be satisfied by the cheaper DGs and ESS. When the electricity purchase price decreases, more power will be purchased from the main grid to substitute the generator output, i.e. the power supply from the main grid would be cheaper than the generation of unit G3. In this case, the total cost for the robust optimal solution is $4356.17. Note that the above optimal robust solution is obtained with the given uncertainty set. If the uncertainty set is adjusted, the corresponding optimal solution will also change.

4.3 Comparison with other methods

In this subsection, we compare the proposed method with some other methods to verify its effectiveness and advantage. Since the objective of multistage robust energy management problem is to find the robust optimal solution, we can focus on the comparison of solutions from different methods regarding the system reliability. The comparison methods include SDDP method, SAA method and the deterministic method with perfect information.

SDDP method is usually used to solve the multistage stochastic programming problems as shown in (28) and only the outer approximation for the expected future cost-to-go function is considered. It has a similar framework with the proposed method which mainly consists of a forward pass to generate a statistical upper bound and a backward pass to refine the lower bound. There are different convergence criteria for this method, for instance, the method terminates when the difference between the lower bound and the upper bound is very small, and here we use the improved convergence criterion reported in [27]. The uncertain wind power and load are assumed to follow uniform distribution in the interval set. Note that the uniform distribution here is used to generate

Table 2 Detailed cost comparison for different methods

Method	RDDP	SDDP	SAA	Deterministic
generation cost	4316.34	4220.14	4209.79	4313.80
storage cost	0.68	0.65	0.67	0.67
exchange cost	39.15	−68.74	−82.92	39.97
penalty cost	0	10,279.70	11,260.35	0
total cost	4356.17	14,431.75	15,387.90	4354.44

some scenarios from the interval set for SDDP method. Some other distributions such as Weibull distribution or normal distribution can also be utilised. In addition, IGDT method [28] may also be investigated to deal with the uncertainty in multistage microgrid energy management problem. SAA is a common method to solve stochastic programming problems. In this work, SAA is actually used to solve a two-stage robust problem as we generate scenarios from the interval uncertainty set and the uncertainties are all known with the scenario. Both SDDP and SAA method belong to stochastic approach, and we will check them with the worst-case scenario for system reliability concern. In addition, we also study a deterministic two-stage problem with perfect information, i.e. the worst-case scenario and the cost function at each stage are all known, and the problem can be solved without the approximation of future cost-to-go function.

$$\begin{aligned} &\min_{\substack{A_1x_1 \ge b_1 \\ x_1 \ge 0}} c_1^{\top}x_1 + \mathbb{E}\Big[\min_{\substack{B_2x_1 + A_2x_2 \ge b_2 \\ x_2 \ge 0}} c_2^{\top}x_2 + [\cdots + \\ &\mathbb{E}\Big[\min_{\substack{B_Tx_{T-1} + A_Tx_T \ge b_T \\ x_T \ge 0}} c_T^{\top}x_T]\cdots]\Big] \end{aligned} \tag{28}$$

The total cost comparison of the solutions from different methods is given in Fig. 8. From this figure, we can see that the total cost of RDDP method for the worst-case scenario is much lower than that of SDDP method and SAA method which validates the advantage of the robust method. Moreover, it is very close to the cost of the deterministic problem with perfect information. Note that the cost of SDDP method is the average value of five simulation experiments and the number of scenarios is 100 for SAA method. For better comparison, we also list the corresponding detailed cost of these methods in Table 2. As can be seen from this table, with larger generation cost and exchange cost, both the RDDP and deterministic method have no penalty cost, while SDDP and SAA method generate very large penalty cost for the unexpected load shedding in the worst-case scenario. Note that the negative exchange cost represents the profit from selling excess power. Therefore, the RDDP method can achieve robust optimal energy management by introducing more generator output and electricity purchase from the main grid. Also, the comparison results show the advantage of the proposed multistage model, i.e. the system is more reliable in the worst-case realisations of uncertainty.

Although the above analysis focuses on the comparison of solution methods, the comparison of system models may also be investigated. As mentioned before, microgrid system has been widely studied in the literature, and most of the system models have a similar structure. In this work, the microgrid system model is developed based on previous literature. However, we study the energy management problem in a multistage perspective which is a significant difference compared with the previous single-stage or two-stage problems. In addition, we mainly focus on the distributed microgrid system here, it is also worth studying the proposed approach with a large realistic system in the future, e.g. the economic dispatch problem in transmission system.

5 Conclusion

A novel multistage robust energy management model for grid-connected microgrids is developed in this work, which considers the uncertainty of renewable generation and load demand. Compared with traditional two-stage models, the multistage model can deal with the non-anticipativity of uncertainty. To solve the multistage problem which is computationally difficult, a robust version of dual dynamic programming named the RDDP method is proposed which combines the outer approximation and inner approximation in the forward and backward pass. A case study with real datasets is conducted and the experimental results verify the effectiveness of the method in achieving robust energy management. In addition, the comparison with other methods including SDDP, SAA, and deterministic method also demonstrates the advantage of the proposed multistage model and solution method with respect to the robust optimal solution.

For future research, the proposed multistage energy management model can be further improved. For example, the incorporation of time correlation of the random variable and some other convex uncertainty sets are worth studying. In addition, new solution method may also be investigated for non-convex optimisation problems.

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