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THE UNIVERSITY OF ALBERTA

ARCHING THEORY
APPLIED TO THE
CORES OF EMBANKMENT DAMS

by

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A REPORT

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a report entitled "Arching Theory Applied to the Cores of Embankment Dams" submitted by G.W. Stevenson in partial fulfillment of the requirements for the degree of Master of Engineering.

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Professor Z. Eisenstein
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Dear Professor Eisenstein:

I enclose the report Arching Theory Applied to the Cores of Embankment Dams, for the completion of my Nonthesis Project for Master of Engineering.

I have revised the report in line with the comments by Dr. Thomson and you. I have also added an additional reference since you reviewed the draft, to the LG4 main dam (Paré et al, Canadian Geotechnical Journal, vol 21, no. 2, May 1984).

Thank you for the opportunity to pursue a topic of my own choice. The research and report preparation were interesting and rewarding. I trust that this report meets the requirements for Nonthesis Project.

Yours very truly,

GW Stevenson

G. W. Stevenson

ABSTRACT

A simple arching theory is used to calculate vertical stresses within the cores of embankment dams. The calculated stresses are compared with published finite element analyses (FEA) and earth pressure measurements.

The comparison with nonlinear FEA shows good agreement for a wide range of soil strengths and hyperbolic parameters in the core and shells. The apparent accuracy of the arching theory, when compared the linear FEA, is poor. Computational details of the FEA do not, in general, influence the results.

Vertical core stresses from the arching theory, compared to earth pressure measurements, exhibit a wide range in the apparent accuracy. However, the agreement is good where design or construction details do not unduly influence the dam behaviour.

The agreement between the arching theory and FEA, particularly nonlinear FEA, or earth pressure measurements, is good for a wide range of core inclinations, thicknesses and material properties.

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1. INTRODUCTION

The phenomenon of arching within earth and rockfill dams, whereby stresses are transferred from a relatively compressible core to stiffer shells, has long been recognized. Several theories have been advanced to permit the calculation of core stresses (Lofquist, 1951; Reinius, 1954; Samsioe, 1955; Trollope, 1957); all involve a number of simplifying assumptions which may limit the application of the theories. The development of the finite element method has permitted a more rigorous analysis of embankment stresses than is possible with simple theories which yield formulas intended for manual calculation.

Several dams have been successfully instrumented with earth pressure cells which provide a direct measurement of total stresses (Schober, 1967; Penman and Mitchell, 1970; Adikari and Parkin, 1982). The measurements provide a basis for assessing the accuracy of various arching theories.

This study will compare the theory of arching as presented by Terzaghi (1943) with the results of several finite element analyses (FEA) of and earth pressure measurements within embankment dams. The FEA include both parametric studies, where ranges of parameters are input to determine their influence on embankment behaviour, and studies of existing dams utilizing parameters determined from laboratory testing. It will be shown that in many cases the arching theory, assuming the active limit state has been fully mobilized at the core boundaries, provides a reasonable estimate of the vertical total stresses within the core. The limitations of the theory will also be demonstrated.

The stresses within the core at the end of construction will be assessed in this study, that is, before nonsymetrical loading due to reservoir filling is imposed on the embankment.

2. BEHAVIOUR OF EMBANKMENT DAMS

The data from several well-instrumented dams demonstrate that movements within their central portions typically result in a lateral transverse spreading of the cores. FEA also indicate similar tensile straining of the core in the horizontal direction.

Figs. 1 and 2 present movement data from two embankments. Fig. 1 shows measured deformations within Llyn Brianne dam at the end of construction (Carlyle,1973). Fig. 2 presents calculated deformations of Talbingo dam (Adikari and Parkin,1982). For both the central core of Llyn Brianne and the inclined core of Talbingo, the horizontal strains within the core are tensile.

Several FEA have demonstrated that a high proportion of the strength of the core material is mobilized at the core boundaries. As an example, Fig. 3 illustrates the strength mobilization calculated for Oroville dam (Kulhawy and Duncan, 1972).

The above observations and predictions suggest that an arching theory which assumes the active strength of the core to be mobilized at its boundaries might provide a reasonable estimate of stresses within the core. It is recognized that the movements within a dam are more complex than the case of rigid shells moving away from the core, that is, the familiar active case for a retaining structure. Despite this simplification, it will be shown that such an approach provides reasonable estimates of vertical stresses in many cases.

3. THEORY OF ARCHING

Terzaghi (1943) presented the theory of arching developed by Janssen (1895) for a column of soil sliding between two vertical surfaces. The theory is summarized herein for the case where the active limit state has been mobilized on the vertical surfaces.

Fig. 4 illustrates the geometry and stresses. Assuming that σ_v is constant across the width 2B, the summation of vertical forces gives

$$[1] \qquad 2B\gamma dz = 2B(\sigma_v + d\sigma_v) - 2B\sigma_v + 2c\,dz + 2\sigma_h \tan\phi\, dz$$

or, simplifying

$$[2] \qquad \frac{d\sigma_v}{dz} = \gamma - \frac{c}{B} - \frac{\sigma_h \tan\phi}{B}$$

For a soil possessing friction and cohesion and where σ_v and σ_h are major and minor principal stresses at the active limit state, respectively

$$[3] \qquad \sigma_h = K_A\sigma_v - \frac{2c\cos\phi}{1+\sin\phi}$$

where

$$[4] \qquad K_A = \tan^2\left(45 - \frac{\phi}{2}\right)$$

Substituting Equation 3 into Equation 1 and solving for the case where σ_v =q when z=0 gives

$$[5] \qquad \sigma_v = \frac{B}{K_A\tan\phi}\left[\gamma - \frac{c}{B}\left(1 - \frac{2\sin\phi}{1+\sin\phi}\right)\right]\left(1 - e^{\frac{-K_A z\tan\phi}{B}}\right) + q\,e^{\frac{-K_A z\tan\phi}{B}}$$

Note that in this development there are no pore pressures, that is, the stresses are treated as total stresses in Equation 1 and as effective stresses in Equation 3.

The arching theory as presented herein assumes full mobilization of soil friction on the vertical surfaces of sliding, as shown on Fig. 4. For dams with relatively compressible cores this would be a reasonable assumption regardless of the general stress state within the cores; the relative core-shell movements would mobilize friction which is to some extent independent of the principal stress ratio within the core.

Fig. 5 shows the vertical core stresses calculated from the arching theory for at-rest and active conditions. Friction at the core boundaries is assumed fully mobilized for both cases. Two soil strength parameters are shown which cover the range commonly encountered in practice. For cores of low strength there is little difference between the active and at-rest cases. The difference is significant for high strength cores, especially when they are narrow.

Fig. 5 presents a simplified case where the core width at the crest is zero. In this case the core stress ratio is dependent only on the core slope (when cohesion = zero). In actual dams the stress ratio would vary with depth; the variation is typically significant only near the crest.

4. ARCHING THEORY APPLIED TO CORES OF DAMS

The Janssen and similar arching theories have been applied to the problem of predicting stresses in the cores of dams by several authors. Their work is reviewed below in the light of results from numerous instrumentation observations and FEA results.

Several methods in addition to those based on the Janssen theory have been developed to estimate core stresses, for example Reinius (1954), Samsioe (1955) and Trollope (1957). However, the arching theory presented by Terzaghi (1943) is the most well known, is relatively simple in development and, as will be shown below, it provides a reasonable result in many cases.

Lofquist (1951) used the arching theory to analyze two dams, their impervious elements comprising thin clay cores behind concrete walls. Lofquist assumed that the active pressure of the core was applicable down to some depth, below which the active pressure of the shells was greater than that of the core. He recognized that the core strength would not, in fact, be fully or uniformly mobilized over the entire height of the dam. He also noted that for the cases studied, the active pressure from the shells assuming full overburden pressure was similar to that of the core assuming stress transfer.

The results from instrumented dams and FEA suggest that, in general, it is unlikely that the shells would reach an active state at depth. This would require that the shells undergo tensile strain, whereas they are typically compressed in the central part of an embankment.

Nonveiller and Anagnosti (1961) developed the arching theory for the case of a symetrical core with sloping boundaries. They assumed that the limit state was fully mobilized over the entire width of the core and they further assumed a parabolic distribution of stress across the core. Their method would be difficult to extend to the case of nonsymetrical cores. The full mobilization of strength across the core need not be assumed; FEA typically demonstrate a variation in strength mobilization across the core as shown, for example, in Fig. 3.

Blight (1973) introduced the concept of effective stress to the arching theory; that is, he introduced the pore pressure parameter $\overline{B}$ to account for the pore pressures which do not contribute to arching. Blight also incorporated the mobilized strengths c/F and $\tan\phi/F$. The result was that some trial and error was required to determine $\overline{B}$, F and the lateral stress parameter K. The factor of safety was assumed to be constant over the full height of the dam.

Unfortunately, Blight ignored the fact that at core boundaries, drainage typically occurs rapidly into the adjacent much more permeable shell or filter zones. Thus, $\overline{B}$ is approximately zero at the core boundaries and the effective stress equals the total stress. Blight's method would be applicable only where drainage from the core is impeded; in such a case the appropriate value of $\overline{B}$ would be that measured at the core boundaries.

5. COMPARISON OF ARCHING THEORY WITH FINITE ELEMENT ANALYSES

5.1 General

In the following, the arching theory will be compared with several published case histories of FEA. The theory will be applied assuming an active stress state at the core boundaries. It will be assumed that the principal stresses are vertical and horizontal and the inclinations of the core boundaries will be ignored.

The geometric terms are defined on Fig. 6. The subscripts c and s will identify properties of the core and shell, respectively.

The arching theory will be compared with the results of FEA using both linear and nonlinear soil moduli.

5.2 Linear Elastic FEA

Although the assumption of linear elastic soil behaviour in the analysis of embankments can lead to errors in the magnitude of stresses (Eisenstein, 1974), a comparison with the arching theory is presented in order to assess the influence of relative core-shell stiffness. Studies such as those by Maksimovic (1973) and Eisenstein and Law (1975) demonstrate that the relative stiffness has an important influence on the degree of stress transfer.

Data from the case histories are summarized in Table 1. Figs. 7 to 14 present the information for each case. Elastic theory and the Jaky equation were used to calculate ϕ from Poisson's ratio ν , for use in the arching equation.

Figs. 7 and 8 summarize the data from Maksimovic (1973) and Eisenstein and Law (1975), respectively. The influence of relative core-shell stiffness is apparent. Fig. 7b illustrates the influence of Poisson's ratio on the degree of stress transfer.

Fig 14 presents data from a parametric study by Cavounidis and Vaziri (1982) on the influence of post-yield behaviour on stress transfer. It is obvious that a simple arching theory can not model the complex behaviour displayed on Fig. 14. It is worth noting, however, that at depth the behaviour of the two dams with wider cores is similar, as is the case of the narrowest core when yield is not modelled.

The information from the linear elastic analyses is summarized on Fig. 15. The accuracy of the arching theory at a depth z=0.8H is plotted against the ratio of the soil moduli E_C/E_S. The depth is arbitrary; except for the data from Cavounidis and Vaziri (1982) the accuracy of the arching theory changes little over the lower portions of the embankments.

Beside each data point or curve on Fig. 15 are shown the core strength and ratio of core base width to height. The curves based on data from Maksimovic (1973) and Eisenstein and Law (1975) suggest a relation between relative core width and accuracy of the arching theory; this may or may not be coincidence.

In order to establish a lower limit for the accuracy of the arching theory, the results from studies of homogeneous embankments can be utilized. Figs. 16 to 18 present analyses of stresses within homogeneous dams by Clough and woodward (1967), Poulos et al (1972) and Lee and Idriss (1975). The dams represent three values of ν . By assuming a 'core' of various widths the stresses within the core can be calculated by the arching theory and compared with the results of the FEA. These values are plotted on Fig. 15 for E_C/E_S=1. Similarly, inclined 'cores' can be assumed to extend the data from Maksimovic (1973) to the case in which E_C/E_S=1.

Fig. 15 indicates the arching theory to be a poor method for estimating core stresses as determined from linear elastic FEA. However, it is considered significant that where linear elastic soil moduli were selected to model the behaviour of real dams, the calculated core stresses agree closely with those calculated from the arching theory (data points 6 and 7 in Fig. 15).

The ratio σ_v from arching theory/ σ_v from FEA is highest for the case of Madin dam (Flores and Auvinet, 1977). A very coarse mesh was used for the analysis of this dam, the core width being represented by a single element. This is considered to be an inappropriate model to study stress transfer, as the entire core 'hangs up' on the stiffer shells. Relative core-shell slip was not permitted by the model.

The greatest range of results on Fig. 15 is for the case in which E_C=E_S, which is typically not the condition in actual dams. For this case the friction mobilized at the core boundaries would be low, because of the compatibility of movements between core and shells. From Equation 5, lower mobilized friction would result in higher vertical core stresses calculated from the arching theory.

Fig. 15 indicates that as ϕ_C decreases. the ratio σ_v from arching/ σ_v from FEA becomes more nearly constant. This behaviour is reasonable; in the limit a fluid core (ϕ_C=0) would transfer the same stress to the shells regardless of the shell stiffness.

5.3 Nonlinear FEA

Core stresses determined from several FEA modelling nonlinear stress-strain behaviour, together with stresses calculated from the arching theory, are shown on Figs. 9, 10 and 19 to 28. Table 2 summarizes the details of the case histories.

A parametric study by Kulhawy and Gurtowski (1976) is particularly useful in assessing the influence of core width and inclination on degree of stress transfer. Data from the study are presented on Fig. 23. Note that for inclined cores the degree of stress transfer is practically the same as for central cores with the same width to height ratio (compare Cases 11 and 12 with Case 5; Case 13 with Case 7). Thus, the arching theory would provide the same accuracy regardless of the core inclination.

As shown on Figs. 9, 10 and 19 to 28, the arching theory provides a reasonable estimate of the vertical stresses in the core for many of the embankments. There is poor agreement near the crests of many of the dams. This is to be expected since the arching theory assumes full mobilization of the active state, which is less likely to occur near the crest where strains are smaller than at depth.

Fig. 29 summarizes the accuracy of the arching theory as a function of the ratio of hyperbolic parameters K (Duncan and Chang, 1970) for core and shell. The majority of the data on Fig. 29 indicate that the arching theory provides estimates of vertical stresses in the core which are within 15 per cent of those determined by nonlinear FEA. This is a significant result, considering the range of soil strengths, core geometries and hyperbolic parameters analyzed in the case histories. Note also that the computational details of the various FEA have been neglected.

No clear relationship can be distinguished on Fig. 29 between accuracy of the arching theory and relative core-shell stiffness as measured by the ratio of hyperbolic parameters K_c/K_s. Other hyperbolic parameters were examined but showed a similar lack of relationship. This is not surprising, considering the range of parameters, which are summarized in Table 2.

It appears from Fig. 29 that as the ratio σ_v from arching theory/ σ_v from FEA increases, ϕ_c decreases and the ratio core base width/ height increases. However, no definite relationships can be established.

The cases for which the arching theory is least accurate are worth examining in more detail. The lowest value of σ_v from arching theory/ σ_v from FEA is for Case 10 of Kulhawy and Gurtowski (1976), shown in Table 2. This case has a high ratio K_c/K_s and the initial Poisson parameter G is greater in the shells than in the core. These conditions would tend to inhibit the movements necessary for full mobilization of friction on the core boundaries and for full mobilization of the active strength.

The maximum ratio of σ_v from arching theory/ σ_v from FEA is for a study of Mica dam by Skermer (1975) shown as data point 6 on Fig. 29. The moduli in this study were found on a trial and error basis, adjusting values until good agreement between calculated and measured settlements was achieved. The elastic moduli used for shell zones were assumed to be constant; this is unlikely to be the case for the actual behaviour. Note that the arching theory gave good agreement with the results of Eisenstein and Law (1975), who used hyperbolic parameters based on laboratory testing of the soils used in Mica dam.

The core stresses determined in El Infiernillo dam by Skermer (1973), data point 3 on Fig. 29, are significantly higher than those calculated from the arching theory. The dam has a relatively thin core, the width of which Skermer modelled with two elements. In addition, very high moduli were used for the transition zones adjacent to the core and no slip was permitted at the core boundaries. This combination of coarse mesh and extreme modulus contrast appears to have influenced the results of the FEA.

A high ratio of σ_v from arching theory/ σ_v from FEA is shown at data point 7 on Fig. 29, from Lee and Idriss (1976). This case has one of the largest contrasts in the Poisson parameter D of data in Table 2. Thus, under increasing stress Poisson's ratio for the shell would increase while that for the core would remain nearly constant. This would confine the core at high stress levels and permit more stress transfer to the shells than the arching theory would indicate.

Vertical stresses in the core of Teton dam (U.S. Department of the Interior, 1976) are shown on Fig. 24. Teton dam featured a narrow core trench beneath a wide core. It is obvious that a simple arching theory can not account for the influence of the trench on the stresses in the overlying core. An empirical method has therefore been used to modify the width over which arching is assumed to occur. As shown on Fig. 24, the width is taken between lines rising a 1 horizontal: 2 vertical from the top of the core. This results in good agreement between the arching theory and the FEA.

The stresses in the core trench of Teton dam were determined by applying the load calculated at the top of the trench as a surcharge q, as in Equation 5.

The summary of nonlinear stress-strain parameters in Table 2 shows that the Poisson parameters can have as important an influence on the degree of stress transfer as the modulus parameters. For example Cole and Cummins (1981) utilize similar modulus parameters for the core and shells, but the Poisson parameters for the core are so much greater than those for the the shells that a significant stress transfer occurs.

6. COMPARISON OF ARCHING THEORY THEORY WITH EARTH PRESSURE MEASUREMENTS

Several dams have been instrumented with earth pressure cells in order to provide a direct measurement of total stresses. Although difficulties can arise in the installation of the cells (Skermer, 1975; DiBiagio et al, 1982), in many cases they provide reliable data.

Earth pressure cell measurements are shown on Figs. 11 to 13, 27 and 30 to 36, together with the vertical stresses calculated from the arching theory. For several cases there is reasonable agreement between theory and measurements. Comments are given below for the cases where significant differences occur.

In Balderhead dam, Fig. 30, the lower part of the shells was compacted to a lower density than the upper part (Kennard et al, 1967; Vaughan et al, 1971). This might be the reason why the arching theory underestimates the measured vertical stress; the lower shell modulus in the lower part of the dam may have resulted in small relative movements between the core and shells and, consequently, little stress transfer from the core.

The earth pressure measurement near the base of Scammonden dam, Fig. 33, is approximately 20 per cent higher than that calculated from the arching theory. The inclined core of the dam is zoned, with the upstream third placed at a higher water content (Penman and Mitchell, 1970). It is possible that the wetter zone is arching to some degree within the core, resulting in a higher stress in the center where the cell is located than would otherwise occur.

Earth pressure measurements in High Aswan dam, Fig. 34, are much higher than those calculated from the arching theory. This dam was constructed in a unique manner (Kinawy and Shenouda, 1973; Kinawy et al, 1973). The lower part of the dam comprises a central zone of sand bounded upstream and downstream by rockfill which was sluiced with sand. The clay core was constructed on top of the sand which was grouted during core placement; grouting caused uplift of the core. Thus, the core was loaded by the relative settlement of the shells, a movement which is opposite to that assumed in the arching theory.

At Takase dam, Fig. 36, the core is approximately the same strength as the shells (Takai et al, 1977). In addition, there was little relative settlement between the core and shells during construction. The net result appears to be that a high degree of shear strength has not been mobilized at the core boundaries.

The comparisons between arching theory and earth pressure measurements display a large range in the apparent accuracy of the theory. However, in the majority of cases where a significant difference between theory and measurement exists, the difference can be explained by reference to the design details and construction methods. Neglecting these cases, the theory gives calculated stresses within about 20 per cent of the measurements.

It should be noted that earth pressure measurements are influenced by three-dimensional effects, design and installation details, local variation of material properties and other conditions which the idealized behaviour modelled by FEA does not take into account. In view of this the general agreement of arching theory and measurements, neglecting the cases described above, is encouraging.

7. SUMMARY

An analysis of several case histories demonstrates that in many instances the arching theory presented by Terzaghi (1943) provides a reasonable estimate of vertical stresses in the cores of embankment dams. The theory has been applied using the following assumptions:

a. the core boundaries are vertical;

b. the vertical stress is uniform across the full width of the core;

c. the principal stress directions are vertical and horizontal;

d. sufficient lateral strain occurs within the core to fully mobilize the active limit state;

e. sufficient shear occurs at the core boundaries to fully mobilize soil friction; and

f. no excess pore pressures exist at the core boundaries.

Core stresses calculated from the arching theory are typically within 15 per cent of those determined from nonlinear FEA, provided that the relative stress-strain behaviour of core and shells are such that settlement of the core relative to the shells and tensile straining of the core can occur. The arching theory can not account for the presence of extremely stiff zones. Abrupt changes in geometry, such as core trenches, can not be accounted for by the theory, but an empirical adjustment of the geometry parameter has resulted in good agreement with FEA in one case.

Comparison of the arching theory with results of linear elastic FEA is much poorer than with nonlinear FEA. However, the poorer accuracy of linear FEA in calculating stress magnitudes is well known.

For both linear and nonlinear FEA the case histories in which actual dams are analyzed generally give core stresses reasonably close to those calculated from the arching theory. Parametric studies, where material properties may have been chosen for computational convenience or to study a wide range of soil properties, show less agreement with the arching theory.

The data summarized in Tables 1 and 2 and on Figs. 15 and 29 illustrate that the Poisson ratio (linear FEA) and Poisson parameters (nonlinear FEA) can have as great an influence on the degree of stress transfer as the modulus parameters. The data in the tables, especially Table 2 for nonlinear FEA where a large number of studies are summarized, can be used as a guide to determine whether the arching theory may be applied to a particular embankment.

The computational details of the various FEA have been ignored in the study and appear, in general, to have little influence on the resulting core stresses. However, very coarse meshes can significantly affect the stress magnitudes, particularly where relative core-shell slip is prohibitted.

A review of published earth pressure measurements reveals a large range in vertical core stresses compared to those calculated by the arching theory. However, the large differences can typically be explained in terms of the material properties or method of construction. Stresses calculated by the arching theory are in reasonable agreement with the measurements where design and construction details have not unduly influenced the embankment behaviour.

Comparisons of the arching theory with both the FEA and the earth pressure measurements suggest that the assumptions regarding strength mobilization have the greatest influence on the accuracy of the theory. The accuracy is generally poorer near the crests of embankments where strains are relatively small and the active state is not fully mobilized.

Despite the simplifying assumptions made in the development of the arching theory, it provides reasonable estimates of vertical stress in many cases for a wide range of core inclinations, thicknesses and material properties.

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No.	Reference	Dam	Height, m	Slopes, 1:- u/s	Slopes, 1:- d/s	Core slopes, 1:-[(a)] u/s	Core slopes, 1:-[(a)] d/s	Core base width / Height	Total density, kN/m^3 Core	Total density, kN/m^3 Shells	E, MPa Core	E, MPa Shells	ν Core	ν Shells	ϕ[(b)] Core	ϕ[(b)] Shells
1	Clough and Woodward(1967)	-	30	2.5	2.5	Homogeneous		-	21.2	21.2	10		0.40		19	
2	Poulos et al (1972)	-	-	1.7	1.7	Homogeneous		-	-	-	-		0.30		35	
3	Maksimovic (1974)	1	100	2.0	1.5	0.75	-0.35	0.52	19.6	19.6	varies	20	varies	0.35	varies	27
		2	100	2.0	1.5	0.50	-0.10	0.52	19.6	19.6	varies	20	varies	0.35	varies	27
		3	100	2.0	1.5	0.25	0.15	0.52	19.6	19.6	varies	20	varies	0.35	varies	27
4	Eisenstein and Law (1975)	-	91	2.5	2.5	0.1	0.1	0.37	22.0	22.0	varies	varies	0.35	0.35	27	27
5	Lee and Idriss (1975)	1	122	1.8	1.8	0.075	0.075	0.15	20.4	20.4	29	239	0.45	0.25	10	42
		2	122	1.8	1.8	0.15	0.15	0.30	20.4	20.4	38	239	0.45	0.25	10	42
		3	32	3.5	2.5	Homogeneous		-	20.1	20.1	12		0.25		42	
6	Cavounidis and Hoeg (1977)	-	61	1.5	1.5	0.5	0.5	1.2	20.6	20.6	96	81	0.40	0.33	19	30
7	Flores and Auvinet (1977)	Madin	77	2.0	2.0	0.25	0.25	0.57	18.1	16.7	9	275	0.29	0.30	36	35
8	Cathie and Dungar (1978)	Llyn Brianne	90	2.0	1.75	0.25	0.25	0.56	23.2	23.6	1.3	27.6	0.495	0.30	1	35
9	Alberro and Moreno (1982)(c)	Chicoasen	257	2.1	2.1	0.125	0.125	0.35	19.6	18.6	59	59	0.45	0.30	11	35
10	Cavounidis and Vaziri (1982)	-	60	1.5	1.5	0.15	0.15	0.45	20.0	20.0	10	20	0.49	0.25	2	42
			60	1.5	1.5	0.275	0.275	0.70	20.0	20.0	10	20	0.49	0.25	2	42
			60	1.5	1.5	0.425	0.425	1.00	20.0	20.0	10	20	0.49	0.25	2	42

(a) Negative sign indicates d/s core boundary slopes toward u/s, ie, core is inclined.

(b) ϕ calculated from ν using elastic theory and Jaky equation.

(c) Shell properties are for wide transition zone.

Table 1. Summary of data from linear elastic finite element analyses.

No.	Reference	Dam	Height,m	Slopes, 1:- u/s	Slopes, 1:- d/s	Core slopes, 1:-[a] u/s	Core slopes, 1:-[a] d/s	Core base width / Height	Total density, kN/m^3 Core/Shell	Hyperbolic parameters K	n	R_f Core/Shell	G	F	D	ϕ Core/Shell	Co[illegible] c,[illegible]
1	Kulhawy and Duncan (1972)	Oroville	235	2.6, 2.75	2.0	0.9	-0.5	0.40	23.6/23.6	345/3780	0.76/0.19	0.88/0.76	0.30/0.43	-0.05/0.19	3.83/14.8	25/44	[illegible]
2	Alberro (1972)	El Infiernillo	148	1.75	1.75	0.09	0.09	0.20	18.9/-			See (b)				10/[c]	[illegible]
3	Skermer (1973)	El Infiernillo	148	1.75	1.75	0.09	0.09	0.20	18.9/17.6			See (d)				19/[c]	[illegible]
4	Valera and Chen (1974)	Lower Van Norman	47	3.5	3.0	0.5	0.0	0.55	19.7/20.9	157/545	0.11/0.55	0.865/0.780		See (g)		26/37	[illegible]
5	Eisenstein and Law (1975)	Mica	240	2.25	2.0	0.4	-0.1	0.36	23.6/23.9	260/893	0.54/0.46	0.72/0.81	0.44/0.46	0.17/0.23	4.2/5.7	31/38	[illegible]
6	Skermer (1975)	Mica	240	2.25	2.0	0.4	-0.1	0.36	23.9/23.9			See (e)				30/45[f]	[illegible]
7	Lee and Idriss (1975)	1	122	1.8	1.8	0.075	0.075	0.15	20.4/20.4	150/1000	0.35/0.50	0.95/0.75	0.44/0.32	-0.10/0.40	0.40/6.0	12[h]/42	[illegible]
		2	122	1.8	1.8	0.15	0.15	0.30	20.4/20.4	250/1000	0.35/0.50	0.95/0.75	0.44/0.32	-0.05/0.50	0.50/6.0	12[h]/42	[illegible]
8	Kulhawy and Gurtowski (1976)	1	122	2.75	2.75	0.10	0.10	0.28	19.1/17.3	100/400	0.55/0.33	0.80/0.70	0.42/0.30	0.10/0.15	3.0/5.0	27/38	[illegible]
		2	122	2.25	2.25	0.10	0.10	0.28				Same as Dam 1					
		3	122	1.75	1.75	0.10	0.10	0.28				Same as Dam 1					
		4	122	1.75	1.75	0.25	0.25	0.58				Same as Dam 1					
		5	122	2.75	2.75	0.10	0.10	0.28	19.1/23.6	100/1200	0.55/0.33	0.80/0.70	0.42/0.40	0.10/0.15	3.0/5.0	27/42	[illegible]
		6	122	1.75	1.75	0.10	0.10	0.28				Same as Dam 5					
		7	122	1.75	1.75	0.25	0.25	0.58				Same as Dam 5					
		8	61	1.75	1.75	0.10	0.10	0.28				Same as Dam 5					
		9	122	1.75	1.75	0.10	0.10	0.28	19.8/23.6	250/1200	0.45/0.33	0.60/0.70	0.35/0.40	0.00/0.15	5.0/5.0	34/42	[illegible]
		10	122	1.75	1.75	0.10	0.10	0.28	19.2/23.6	600.1200	0.35/0.33	0.50/0.70	0.30/0.40	-0.13/0.15	9.5/5.0	36/42	[illegible]
		11	122	1.75	1.75	0.40	-0.20	0.28	19.1/23.6	100/1200	0.55/0.33	0.80/0.70	0.42/0.40	0.10/0.15	3.0/5.0	27/42	[illegible]
		12	122	1.75	1.75	0.60	-0.40	0.28				Same as Dam 11					
		13	122	1.75	1.75	0.60	-0.10	0.58				Same as Dam 11					
		14	122	1.75	1.75	0.10	0.10	0.28				Same as Dam 11					
9	U.S. Dept of Interior (1976)	Teton[i]	123	2.5, 3.5	2.0, 3.0	1.5	1.0	0.88, 0.16[j]	18.4/-	470/-	0.12/-	0.79/-	0.35/-	0.10/-	4.0/-	31/-	[illegible]
10	Cavounidis and Hoeg (1977)	-	61	1.5	1.5	0.50	0.50	1.20	20.6/20.6	200/2000	0.25/0.25	0.60/0.60		See (k)		40/42	[illegible]
11	Sharma et al (1979)	Tehri	260	2.5	2.0	0.25	0.10	0.40	19.2/17.7	500/2500	0.60/0.25	0.90/0.76	0.48/0.43	0.00/0.19	0.0/14.8	27/38	[illegible]
			260	2.5	2.0	0.50	-0.25	0.33	19.2/17.7	500/2500	0.60/0.25	0.90/0.76	0.48/0.43	0.00/0.19	0.0/14.8	27/38	[illegible]

Table 2. Summary of data from nonlinear finite element analyses.

No.	Reference	Dam	Height, m	Slopes, 1:- u/s	Slopes, 1:- d/s	Core slopes, 1:-[a] u/s	Core slopes, 1:-[a] d/s	Core base width / Height	Total density, kN/m^3 Core/Shell	K	Hyperbolic parameters n	R_f Core/Shell	G	F	D	ϕ Core/Shell	Core c, kPa
12	Cole and Cummins (1981)	Dartmouth	180	1.7, 1.8	1.7	0.4, 0.5	0.2	0.67	22.0/19.6	560/450	0.32/0.42	0.88/0.73	0.43/0.21	0.38/0.13	14.4/6.9	20-34/33/45	60
13	Adikari et al (1982)	Dartmouth	180	1.7, 1.8	1.7	0.4, 0.5	0.2	0.67	21.0/22.0	350/600	0.41/0.31	0.84/0.71	0.40/0.22	0.12/0.12	4.2/7.0	29/44	60
14	Adikari and Parkin (1982)	Talbingo	162	2.0	2.0	0.9	-0.2	0.66	18.1/20.4	250/680	0.45/0.65	0.86/0.71	0.315/0.25	0.045/0.06	4.8/7.7	23/45	85
15	Pare et al (1984)	LG4	125	1.7	1.7	0.25	0.25	0.57	23.0/22.8	1000/1200	0.39/0.73	0.55/0.60	0.33/0.45	0.04/0.30	13.5/8.0	37/42	-

(a) Negative sign indicates d/s core boundary slopes toward u/s, ie, core is inclined.
(b) E input in tabular form, ν_s in stress-dependent equation.
(c) ν_c assumed constant. ϕ_c calculated from elastic theory and Jaky equation.
(d) E, ν input in tabular form as function of stress level.
(e) E_c, ν_c and ν_s input in tabular form. E_s assumed constant.
(f) Calculated from initial ν using elastic theory and Jaky equation.
(g) ν_c=0.45; ν_s=0.35 constant.
(h) Calculated from elastic theory and Jaky equation assuming G=initial ν. Reference gave only total stress parameters.
(i) Parameters shown for core before wetting. In core trench wetted parameters used.
(j) To top and bottom, respectively, of core trench at section analyzed.
(k) ν_c=0.40; ν_s=0.33 constant.

Table 2. Continued.

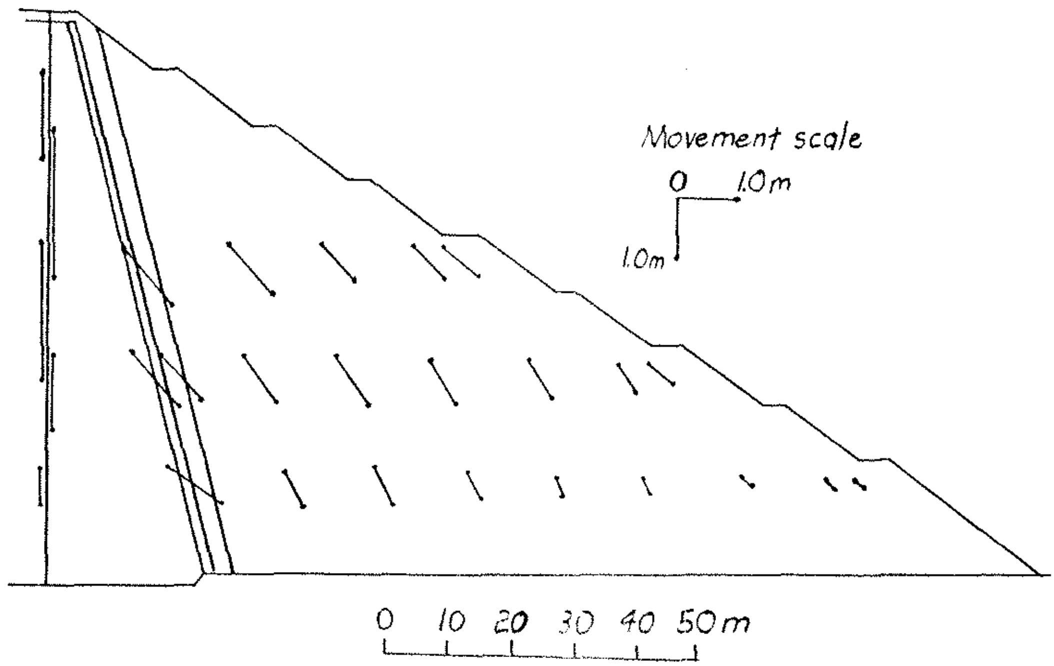


Fig. 1. Deformation of Llyn Brianne dam during construction (after Carlyle, 1973).

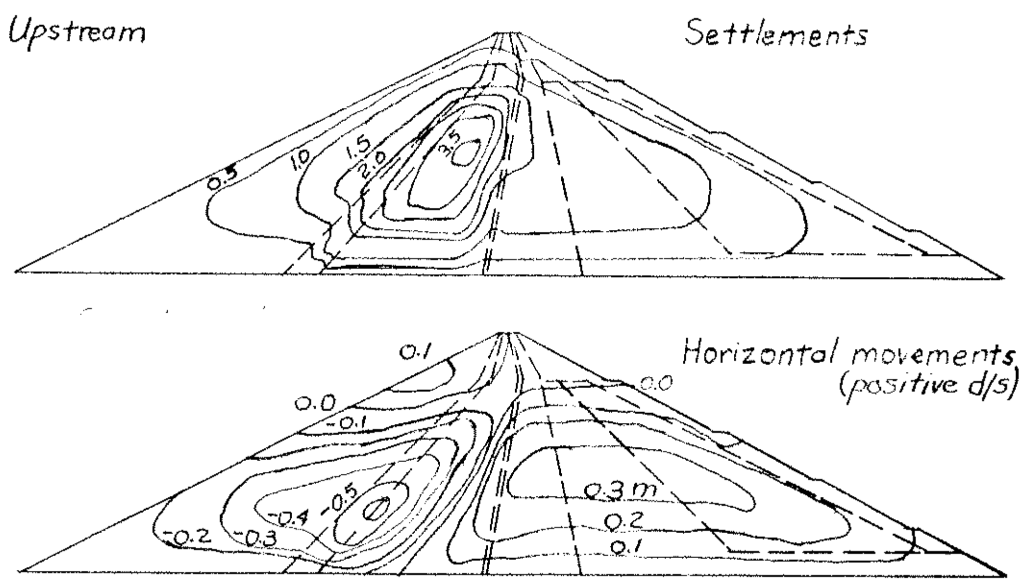


Fig. 2. Predicted deformations of Talbingo dam at end of construction (after Adikari and Parkin, 1982).

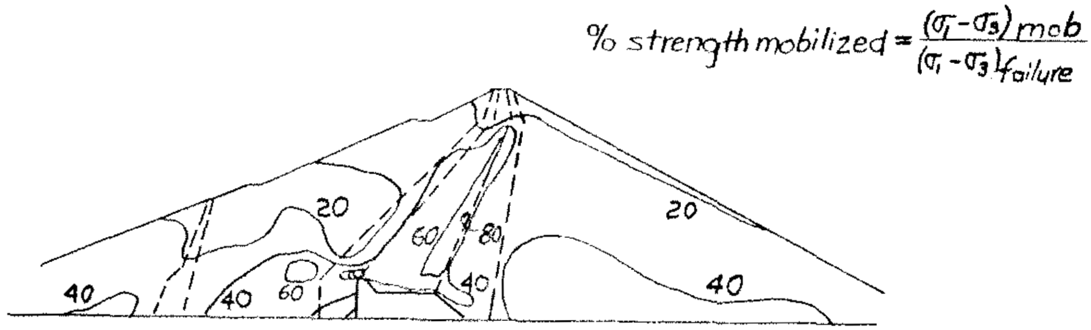


Fig. 3. Strength mobilized in Oroville dam (after Kulhawy and Duncan, 1972). Calculated by nonlinear FEA.

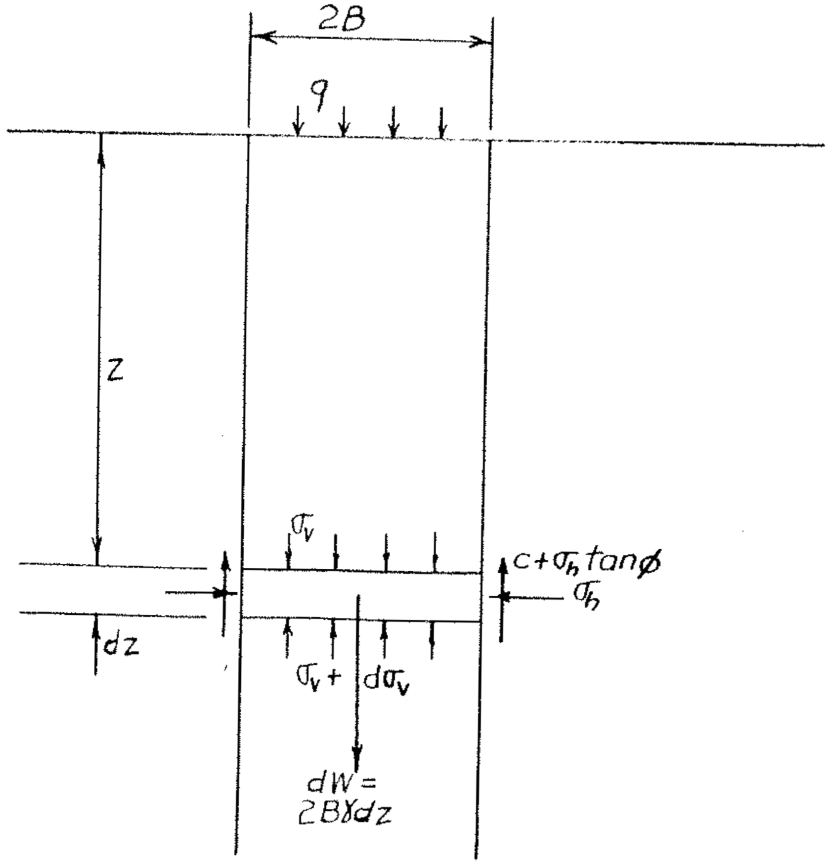


Fig. 4. Geometry and stresses for soil column sliding between two vertical surfaces (after Terzaghi, 1943).

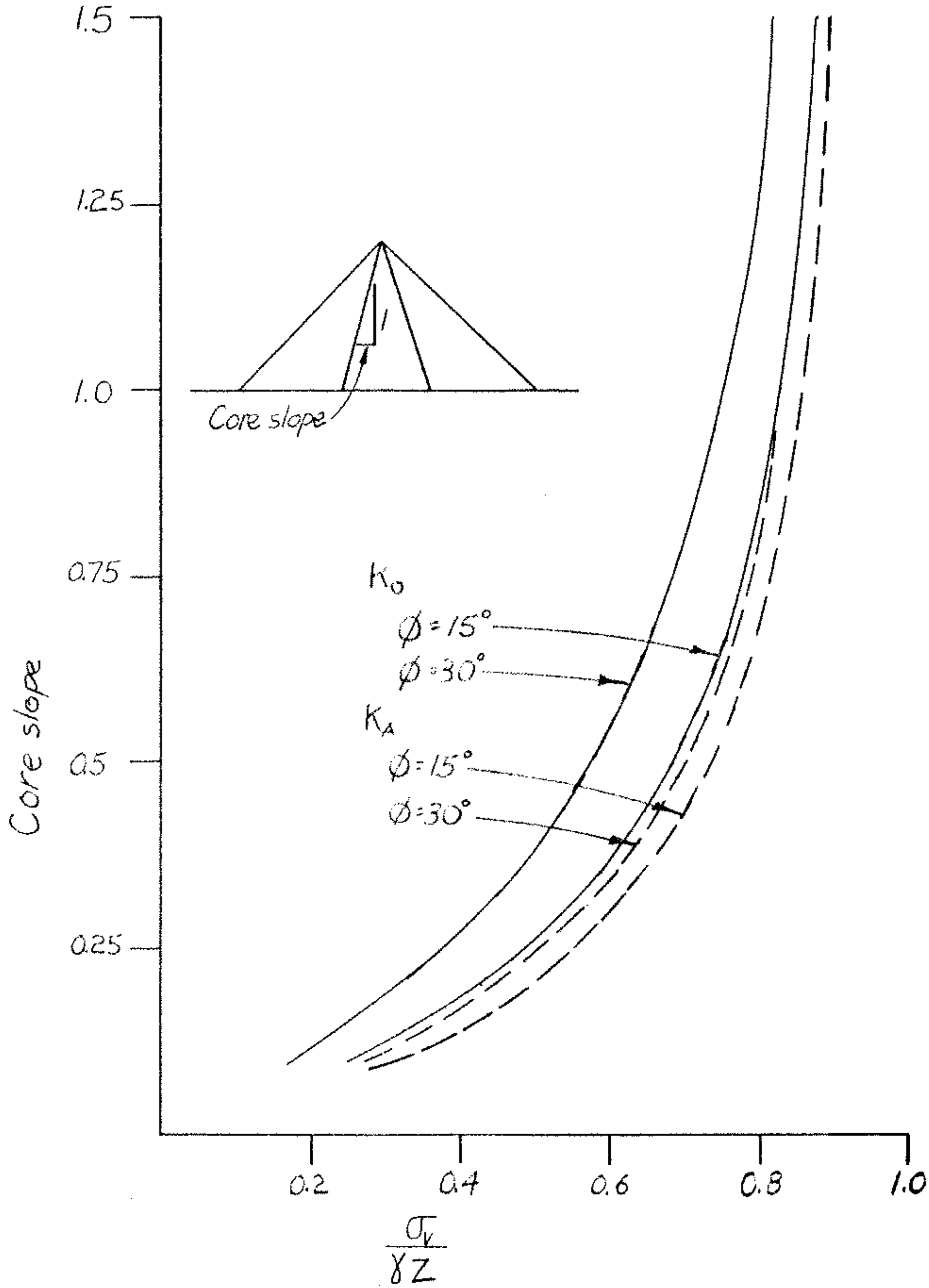


Fig. 5. Vertical stresses in the core calculated from arching theory for at-rest and active stresses on core boundaries.

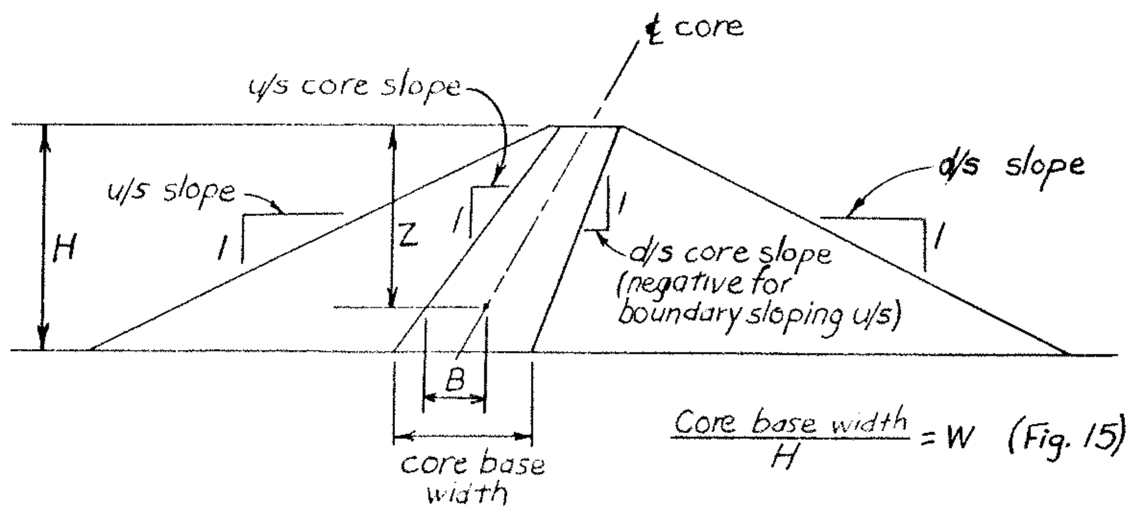


Fig. 6. Geometric terms used in study.

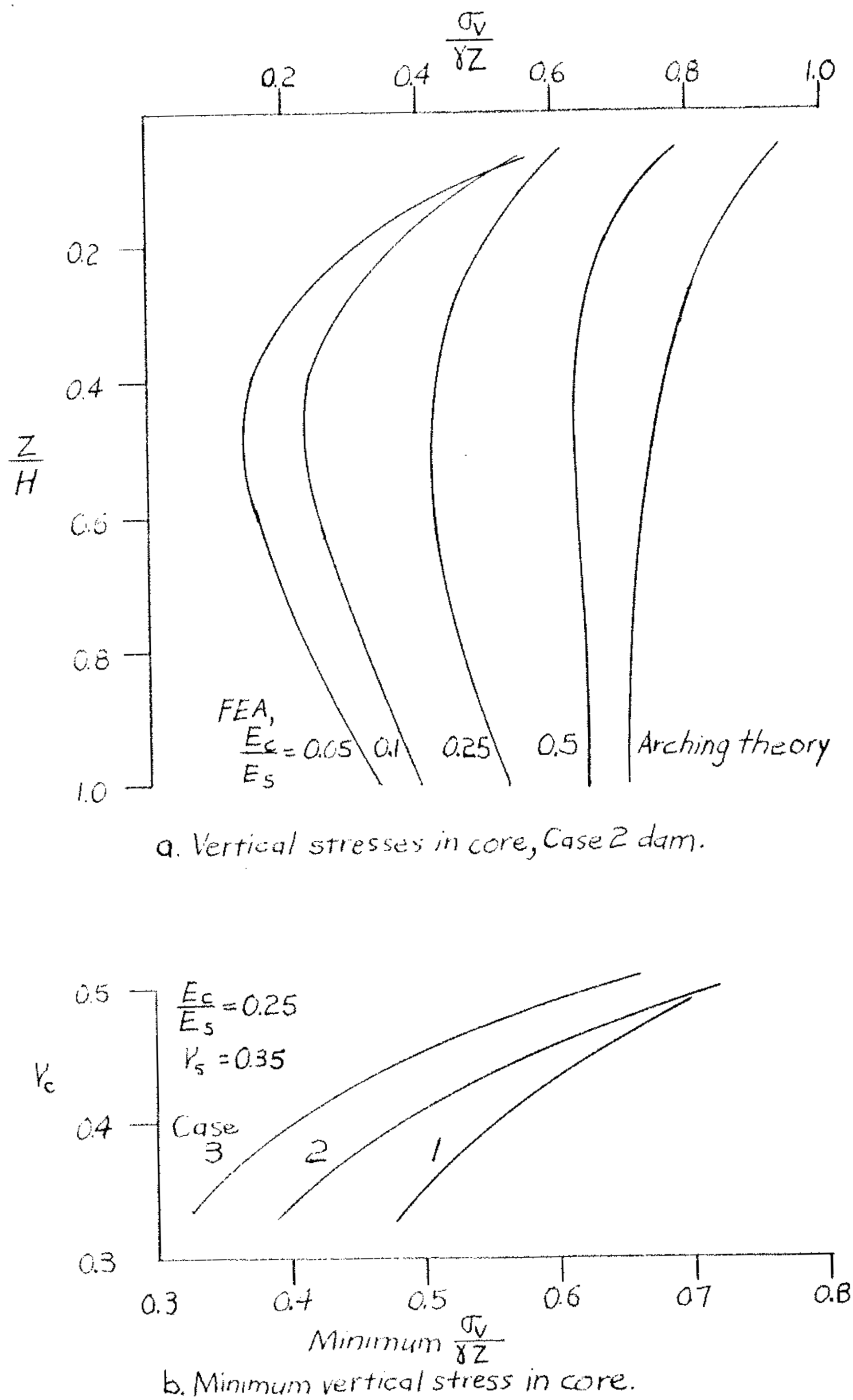


Fig. 7. Core stresses (after Maksimovic, 1973). Linear elastic FEA.

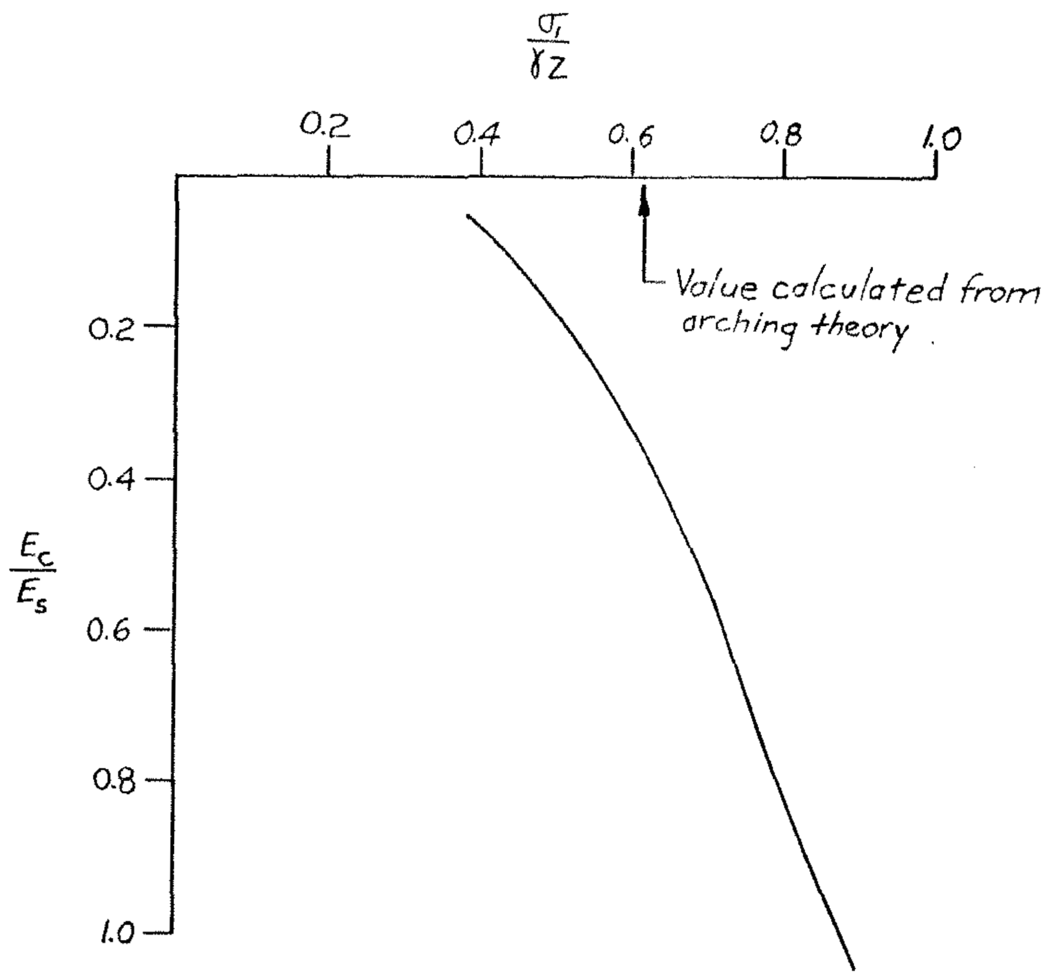


Fig. 8. Major principal stress near the base of a dam (after Eisenstein and Law, 1975). Linear elastic FEA.

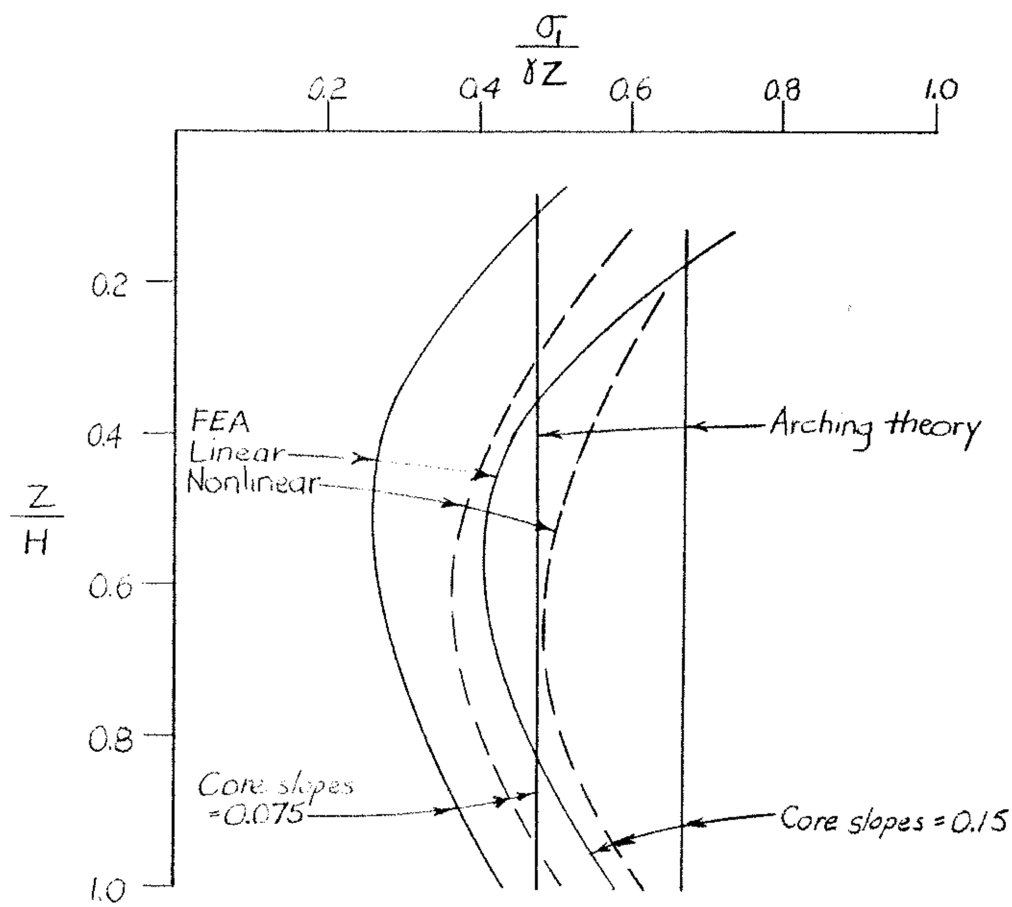


Fig. 9. Core stresses in zoned dams (after Lee and Idriss, 1975).

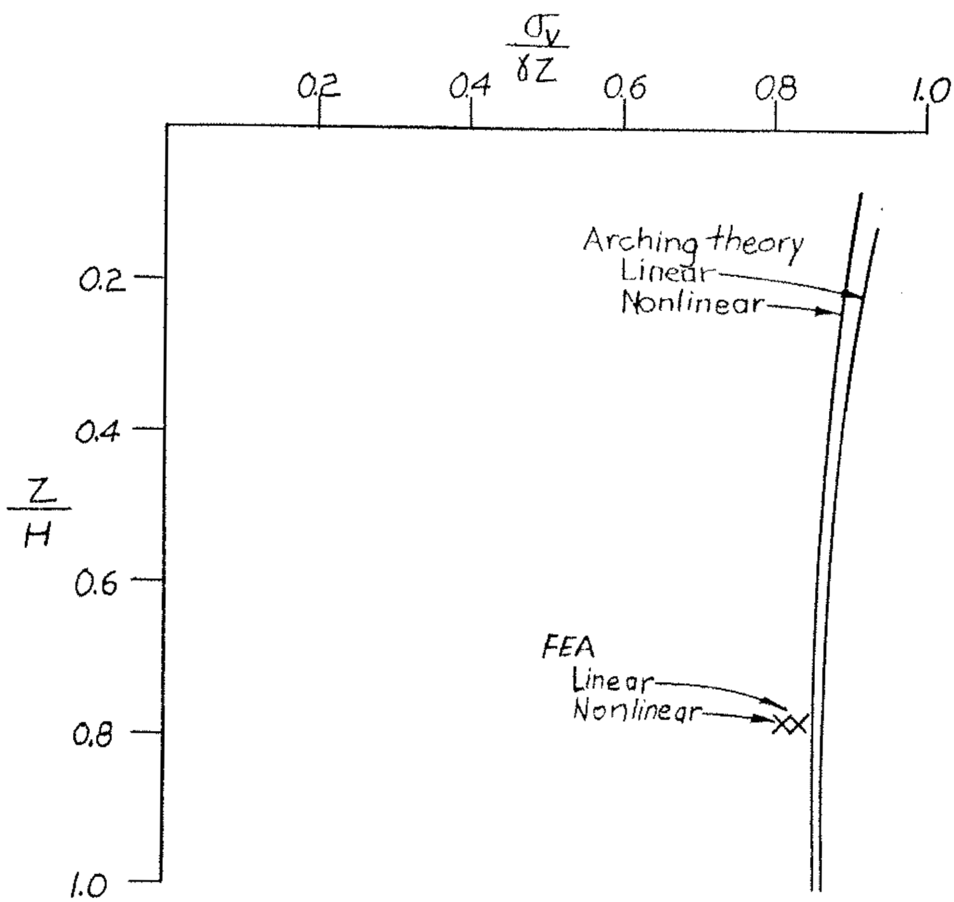


Fig. 10. Core stresses at end of construction (after Cavounidis and Hoeg, 1977).

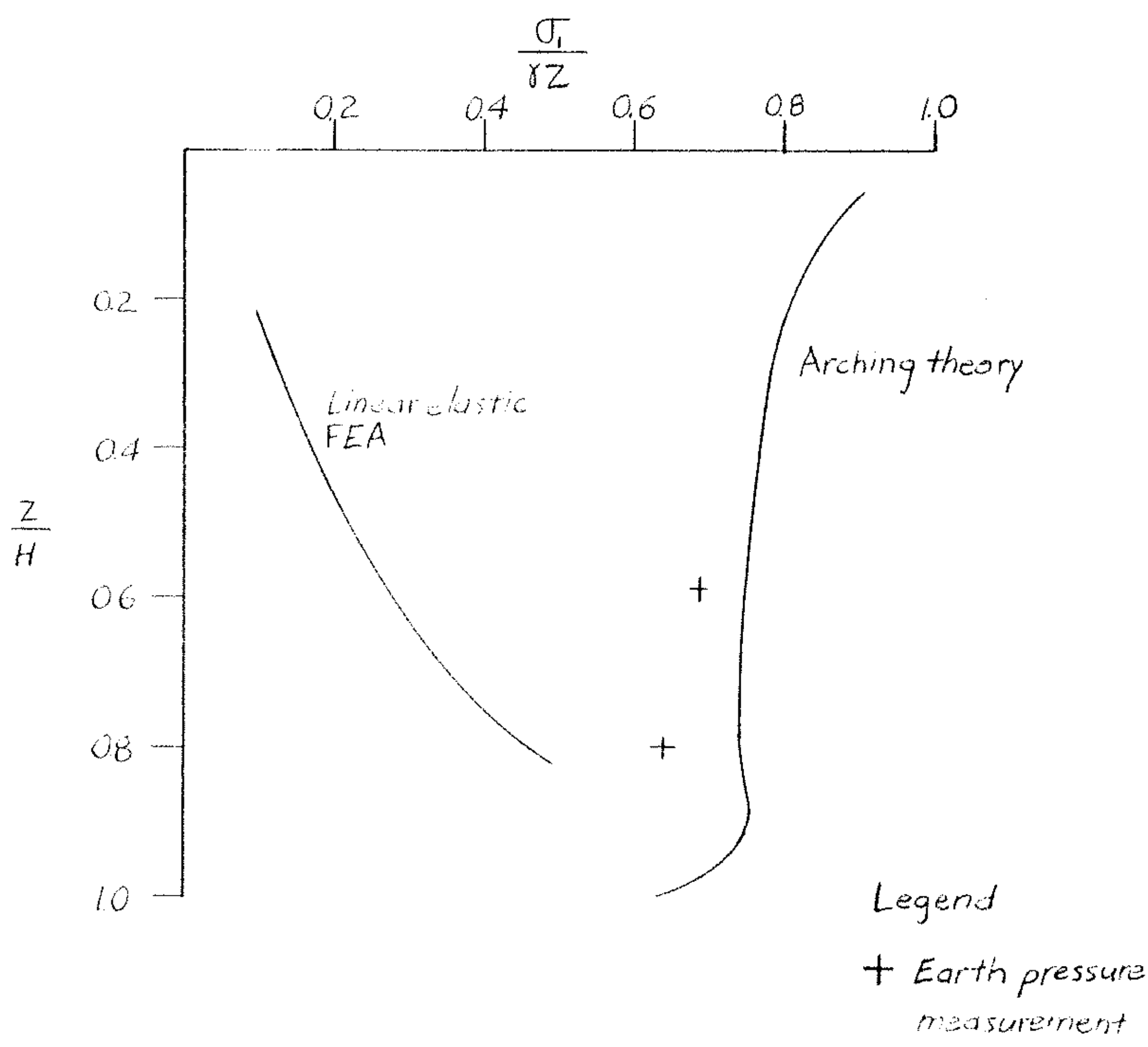


Fig. 11. Core stresses in Madin dam (after Flores and Auvinet, 1977).

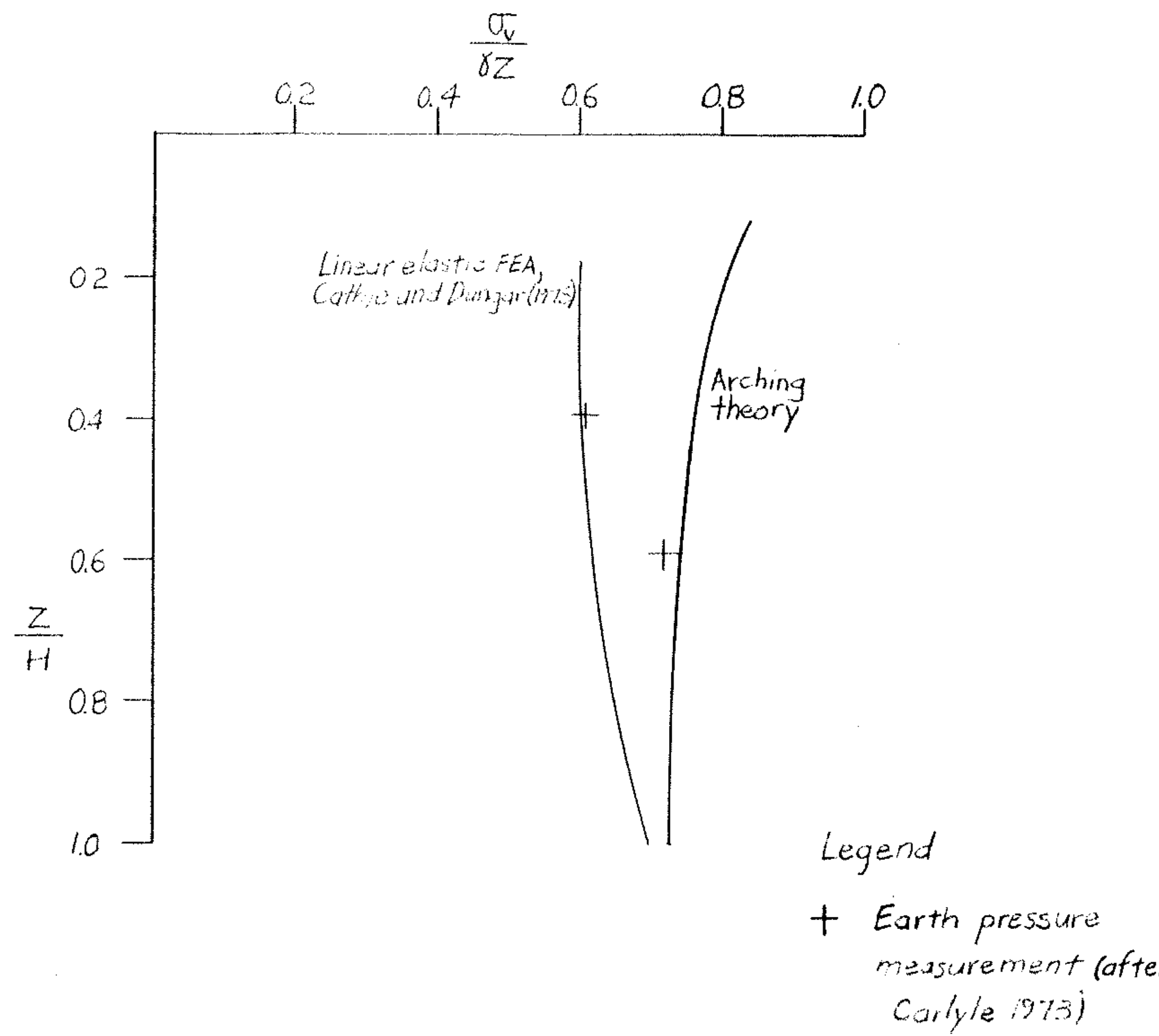


Fig. 12. Core stresses in Llyn Brianne dam.

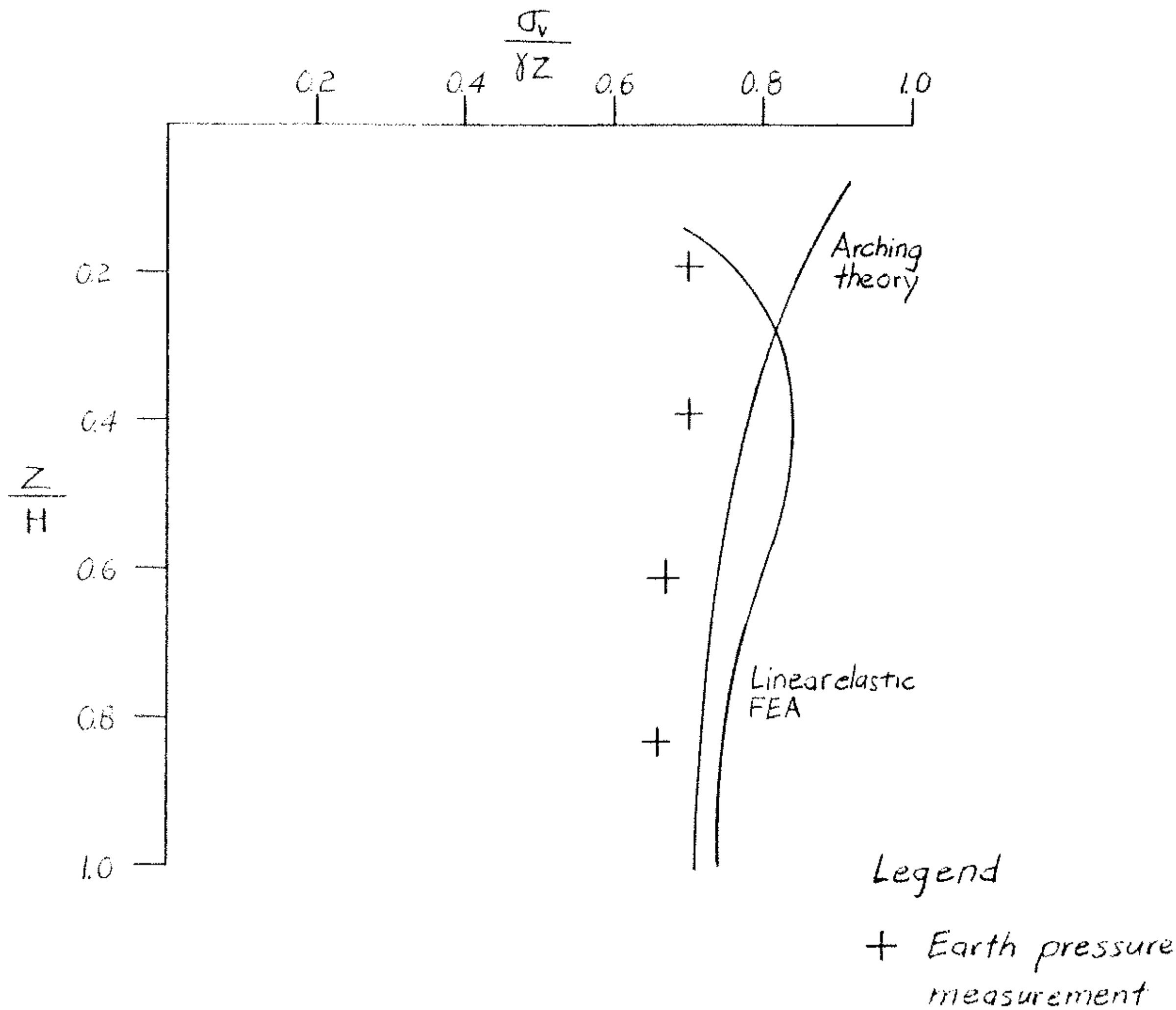


Fig. 13. Core stresses in Chicoasen dam (after Alberro and Moreno, 1982).

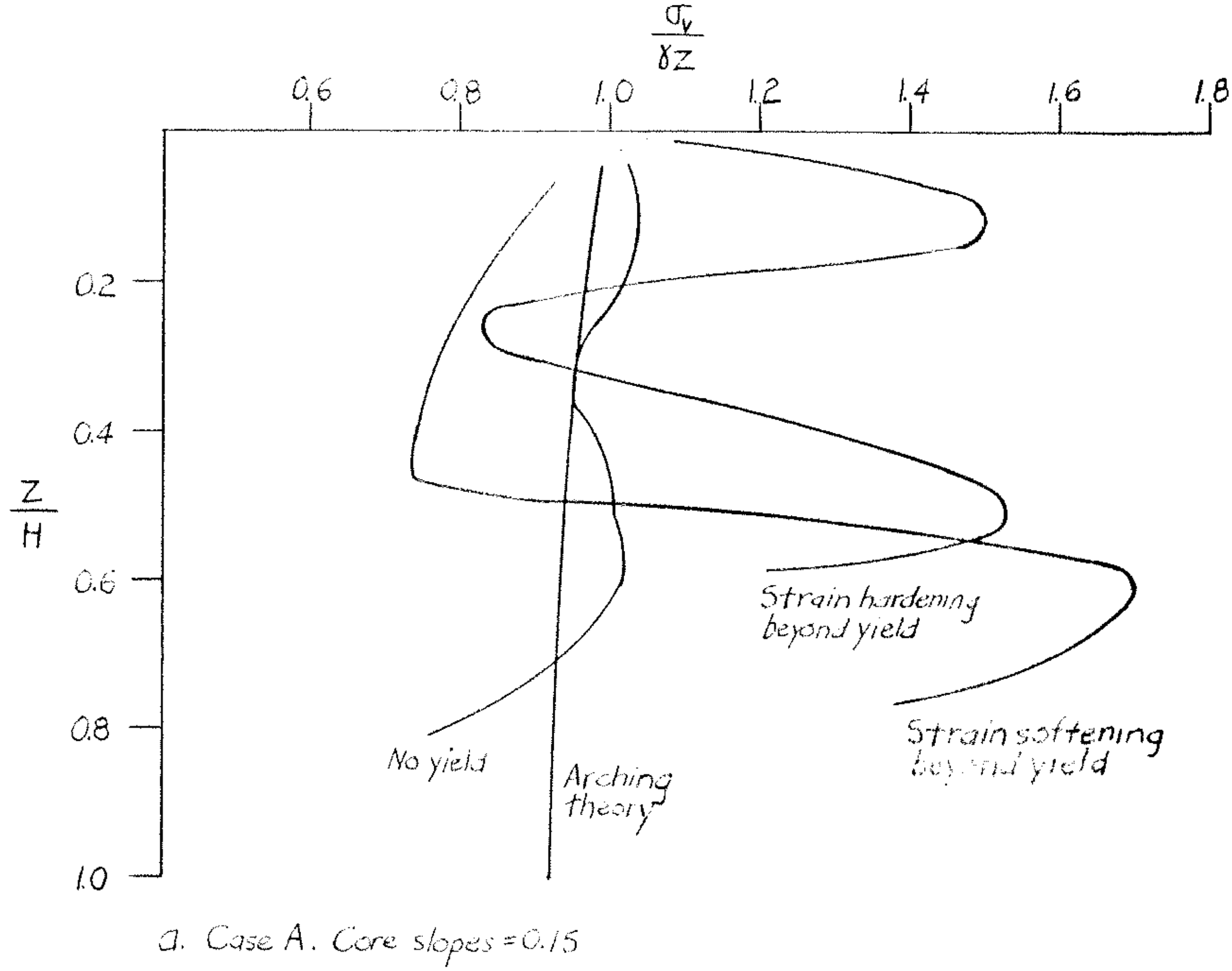
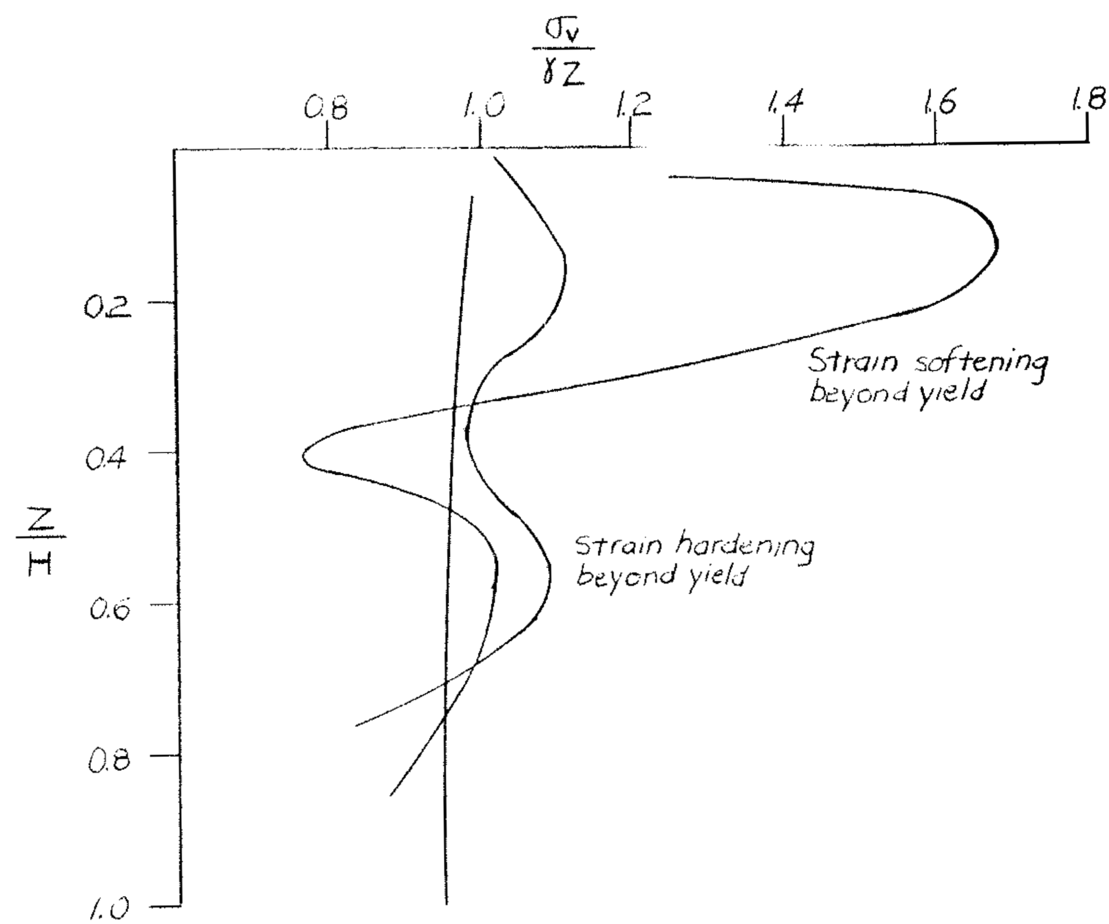


Fig. 14. Core stresses as influenced by post-yield behaviour (after Cavounidis and Vaziri, 1982). Linear elastic FEA. Yield defined by von Mises criterion, Y=0.7γH.



b. Case B. Core slopes = 0.275.

Fig. 14. Continued.

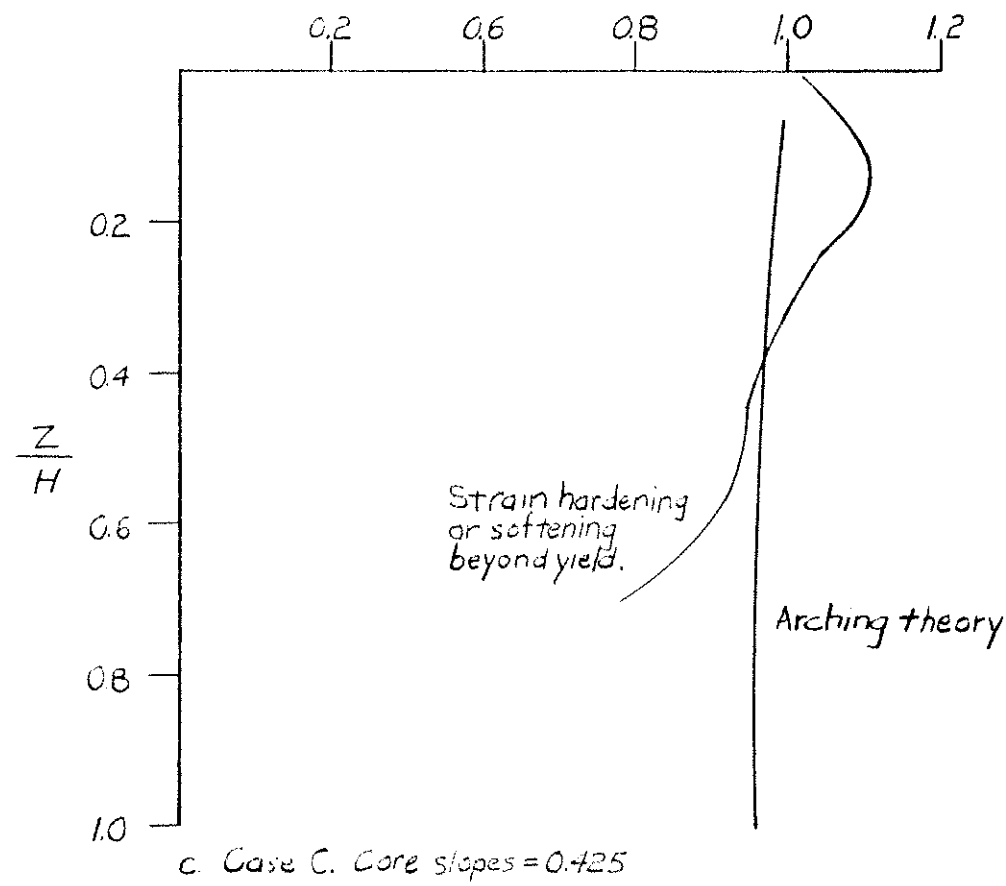


Fig. 14. Continued.

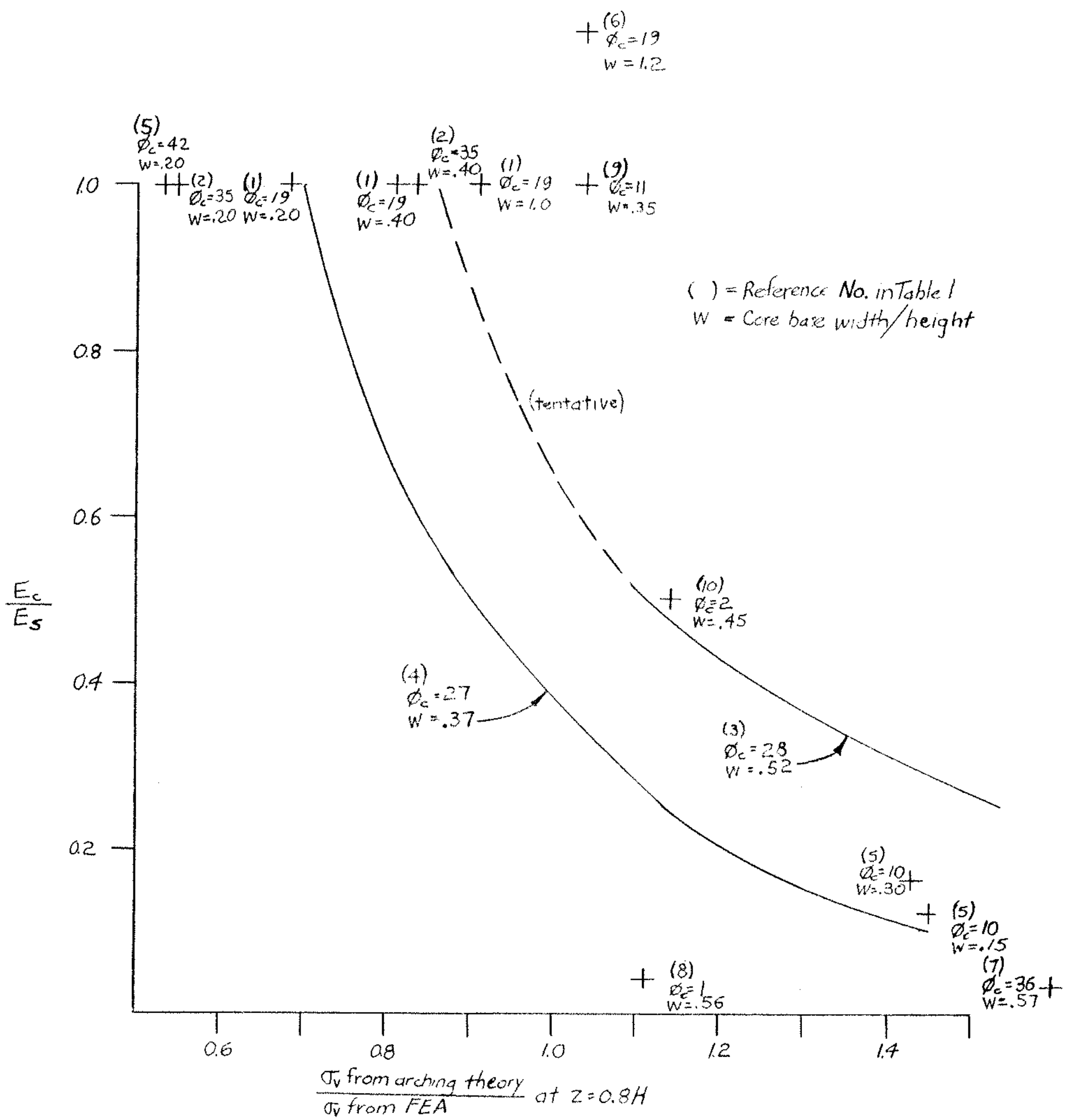
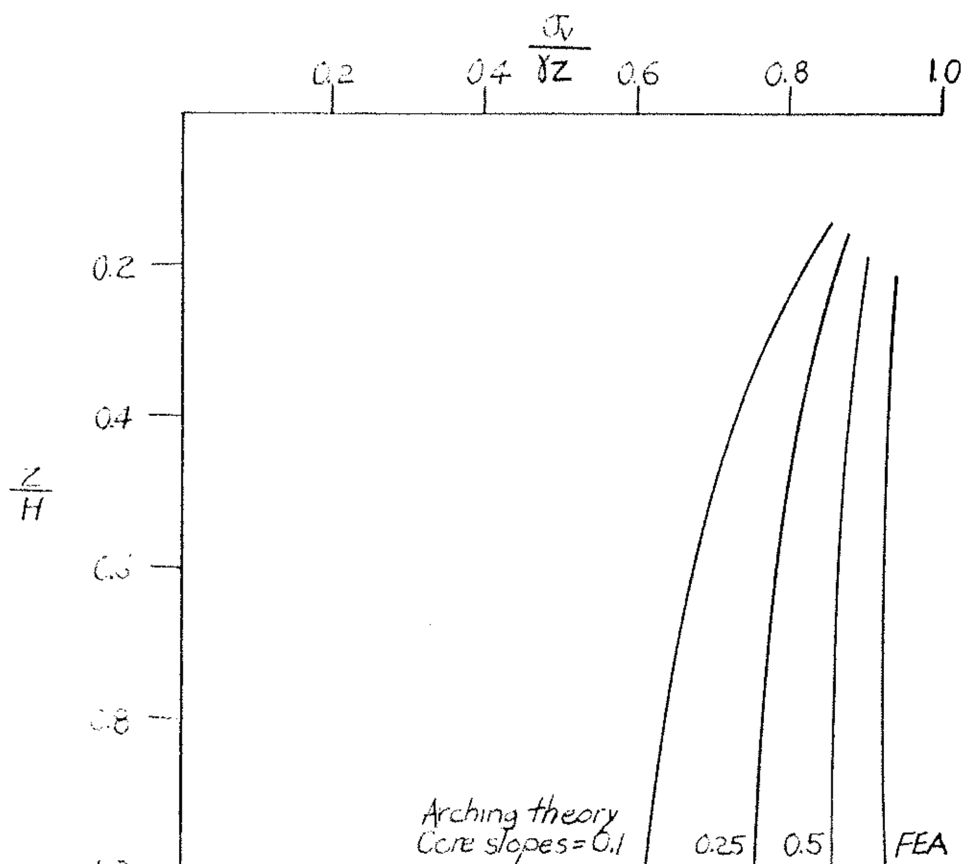
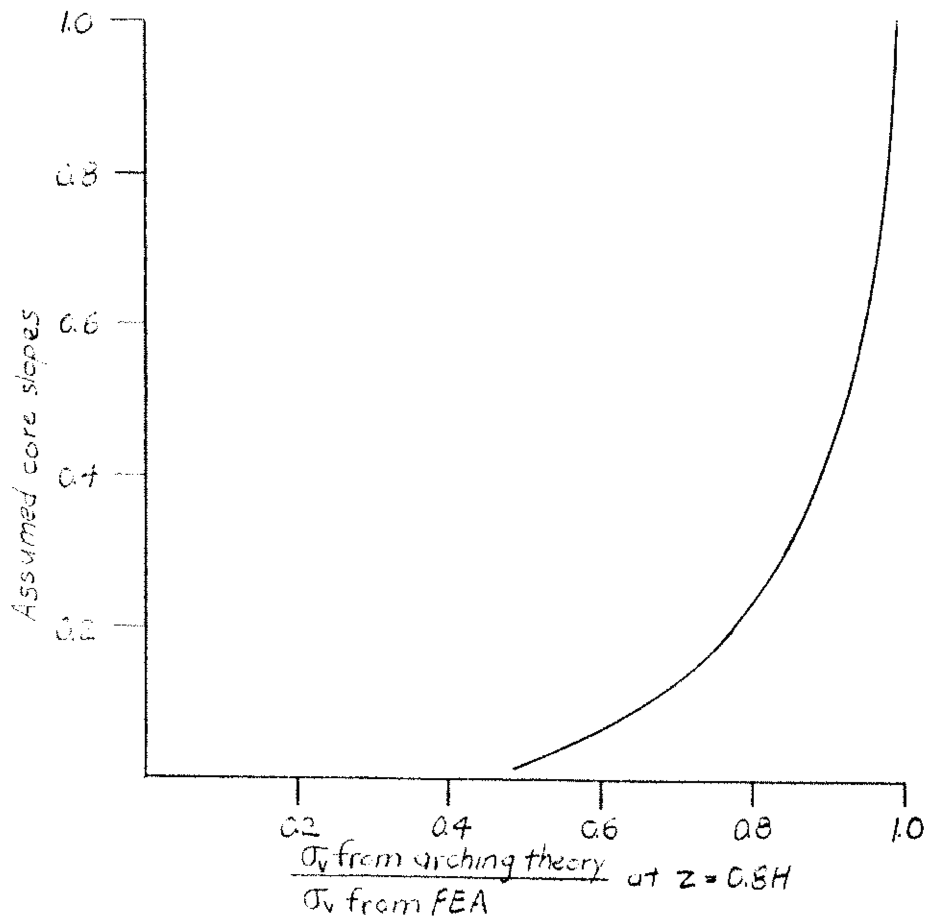


Fig. 15. Comparison of arching theory with linear elastic FEA.



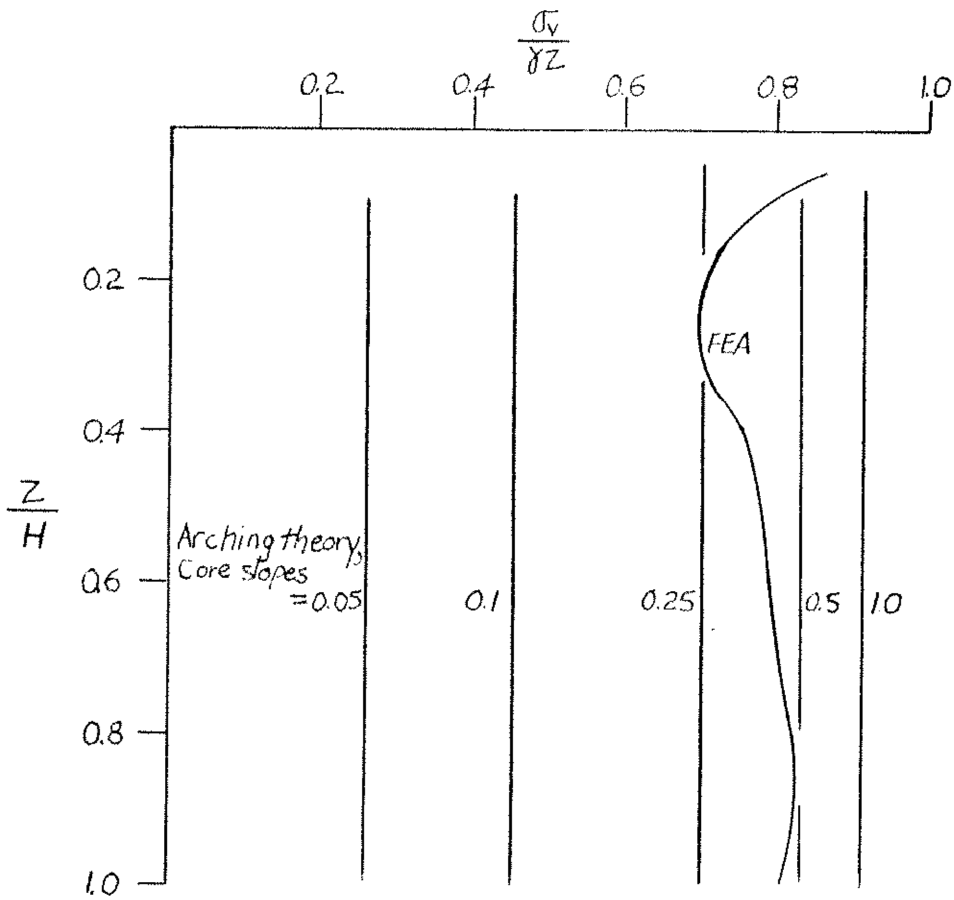
a Core stresses for various assumed core widths.

Fig. 16. Analysis of homogeneous embankment (after Clough and Woodward, 1967). Linear elastic FEA.



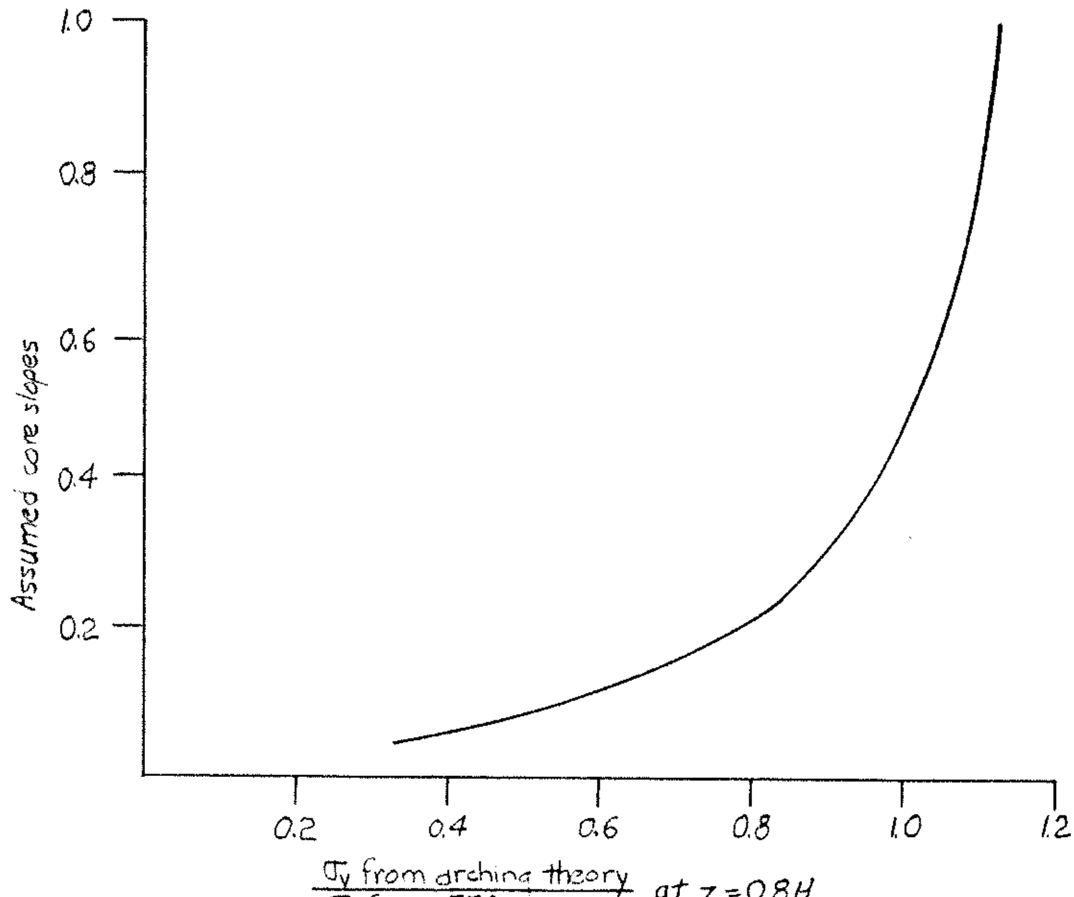
b. Comparison of theory and FEA.

Fig. 16. Continued.



a. Core stresses for various assumed core widths.

Fig. 17. Analysis of homogeneous embankment (after Poulos et al, 1972). Linear elastic FEA.



b. Comparison of theory and FEA.

Fig. 17. Continued.

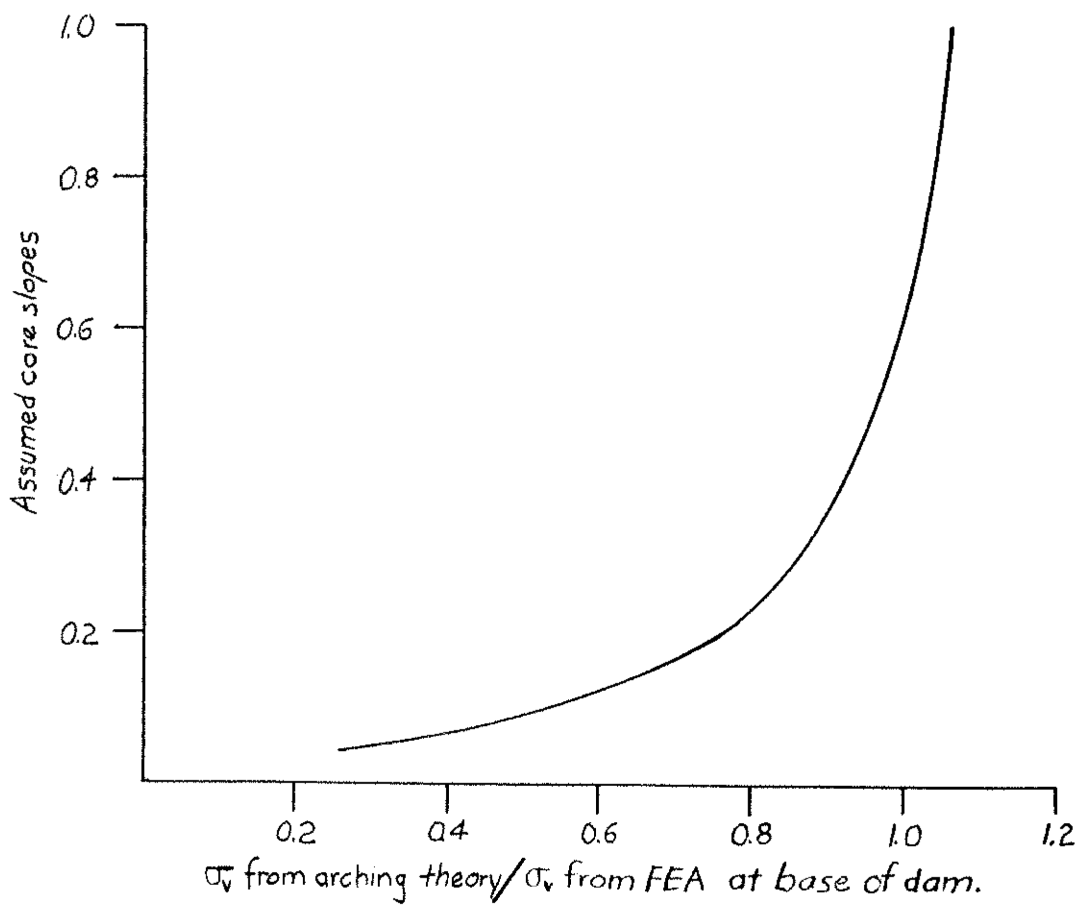


Fig. 18. Analysis of homogeneous embankment (after Lee and Idriss, 1975). Linear elastic FEA.

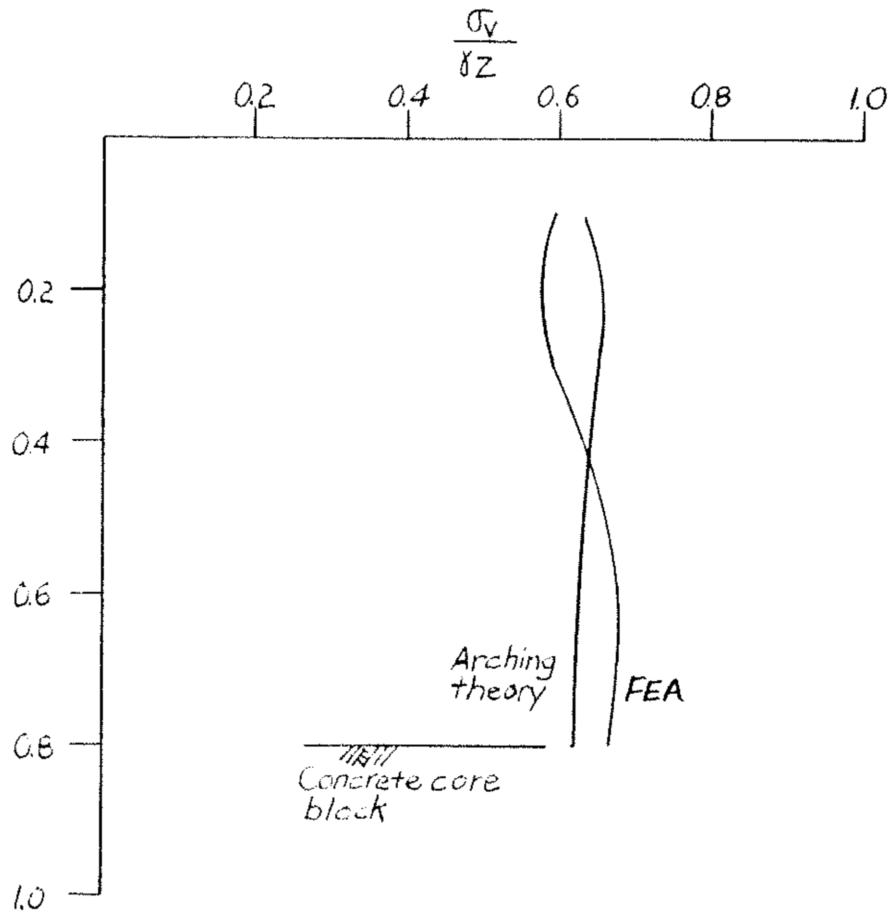


Fig. 19. Core stresses in Oroville dam (after Kulhawy and Duncan, 1972). Nonlinear FEA.

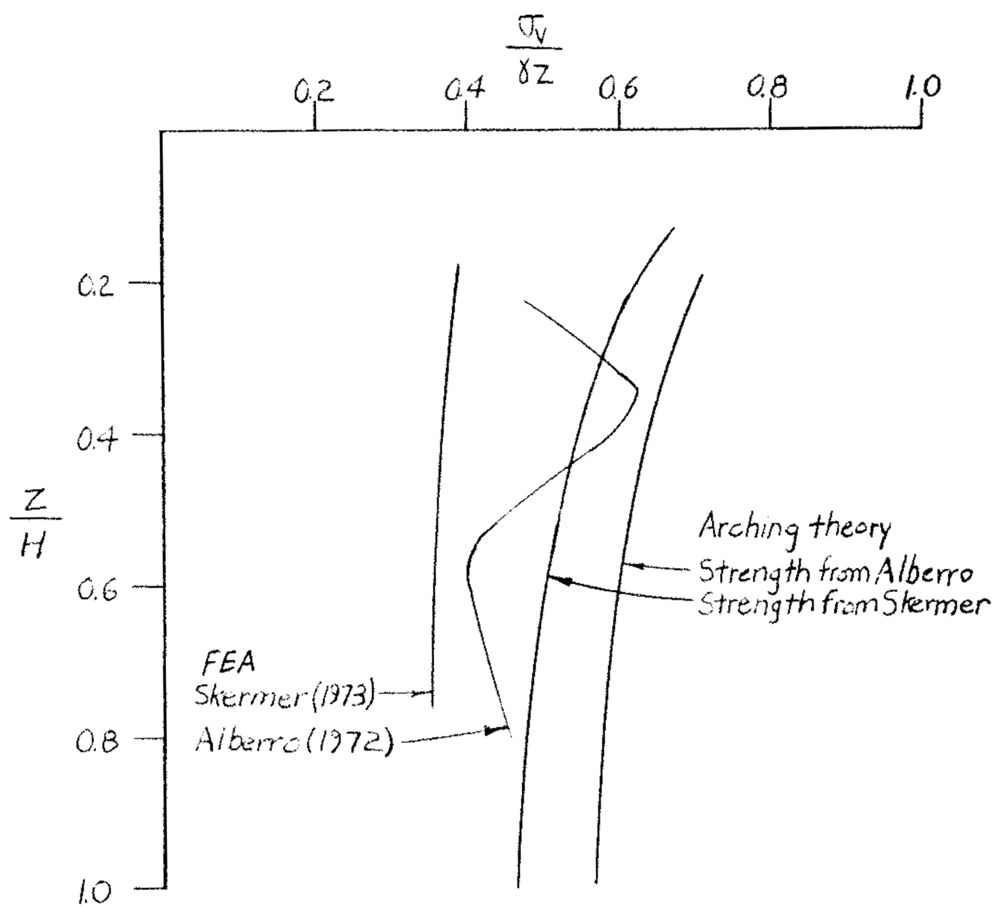


Fig. 20. Core stresses in El Infiernillo dam. Nonlinear FEA.

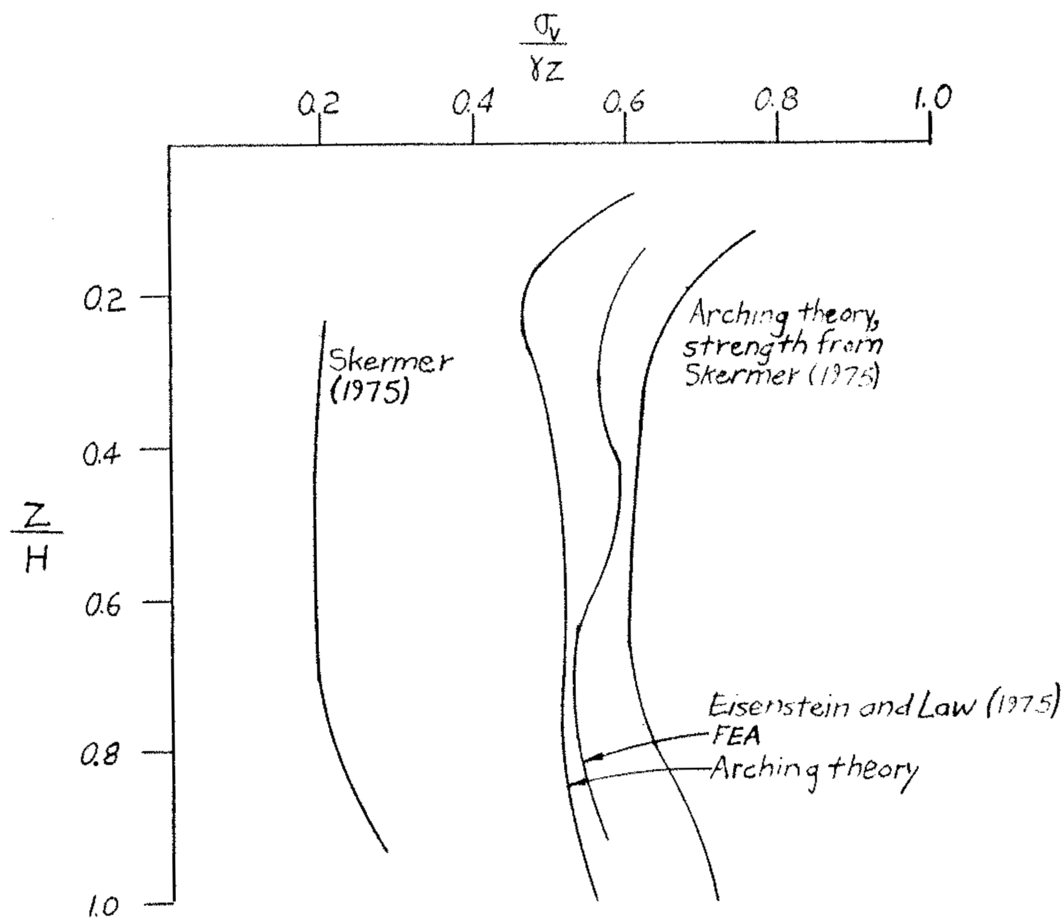


Fig. 21. Core stresses in Mica dam. Nonlinear FEA.

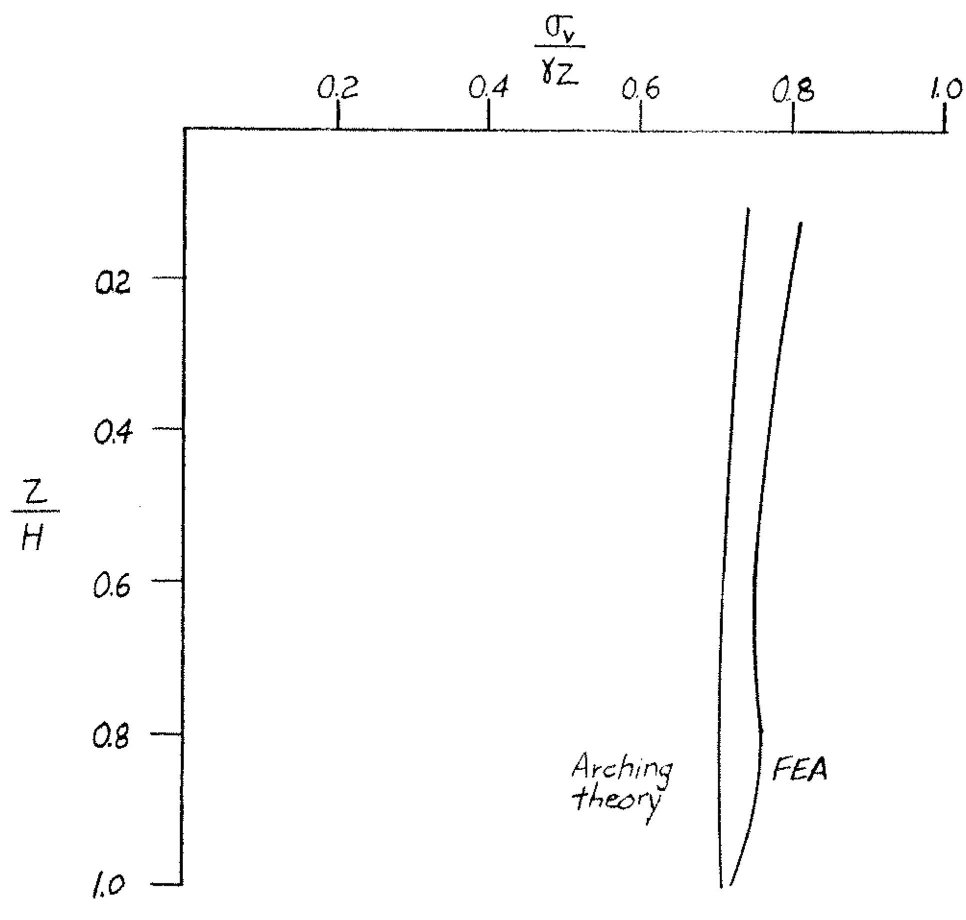


Fig. 22. Core stresses in Lower Van Norman dam (after Valera and Chen, 1974). Nonlinear FEA.

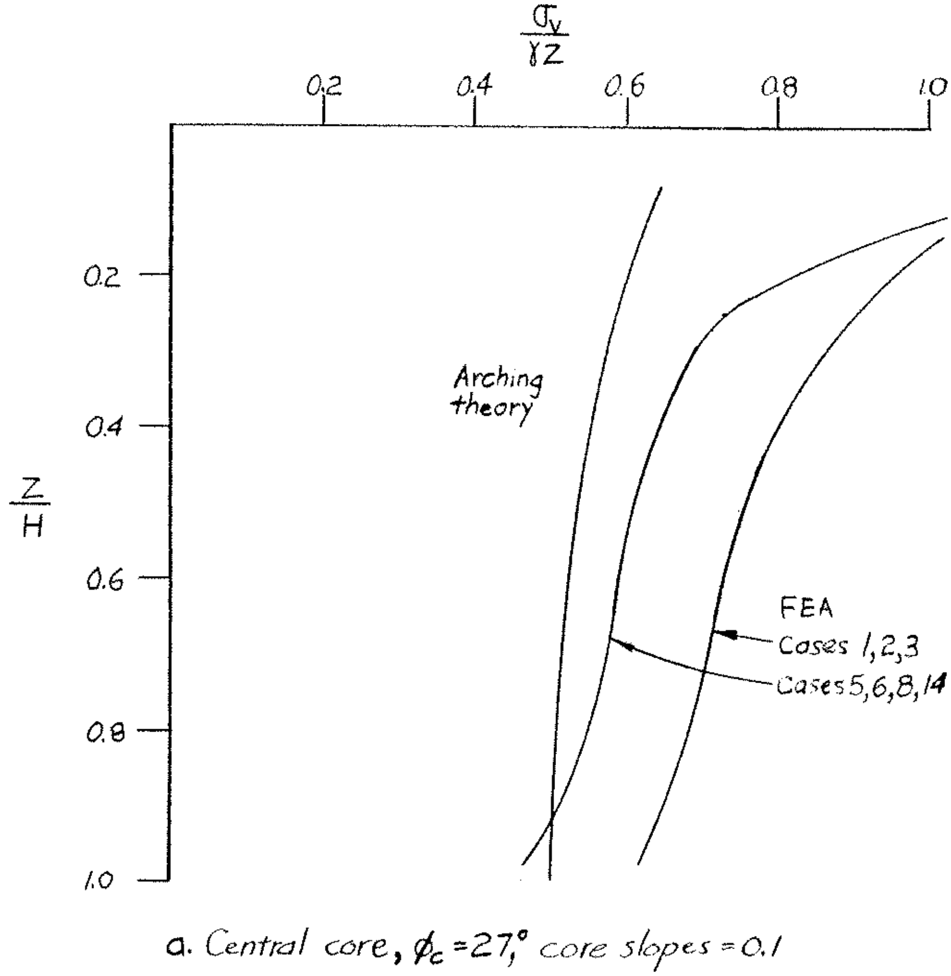
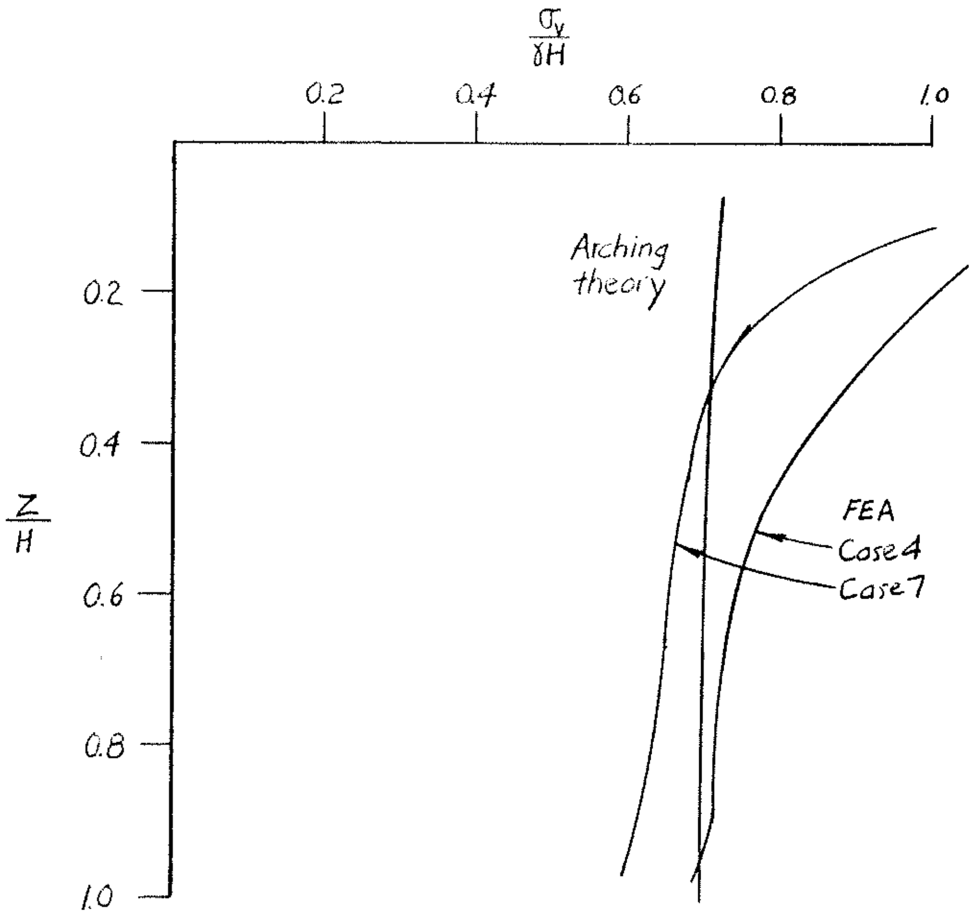
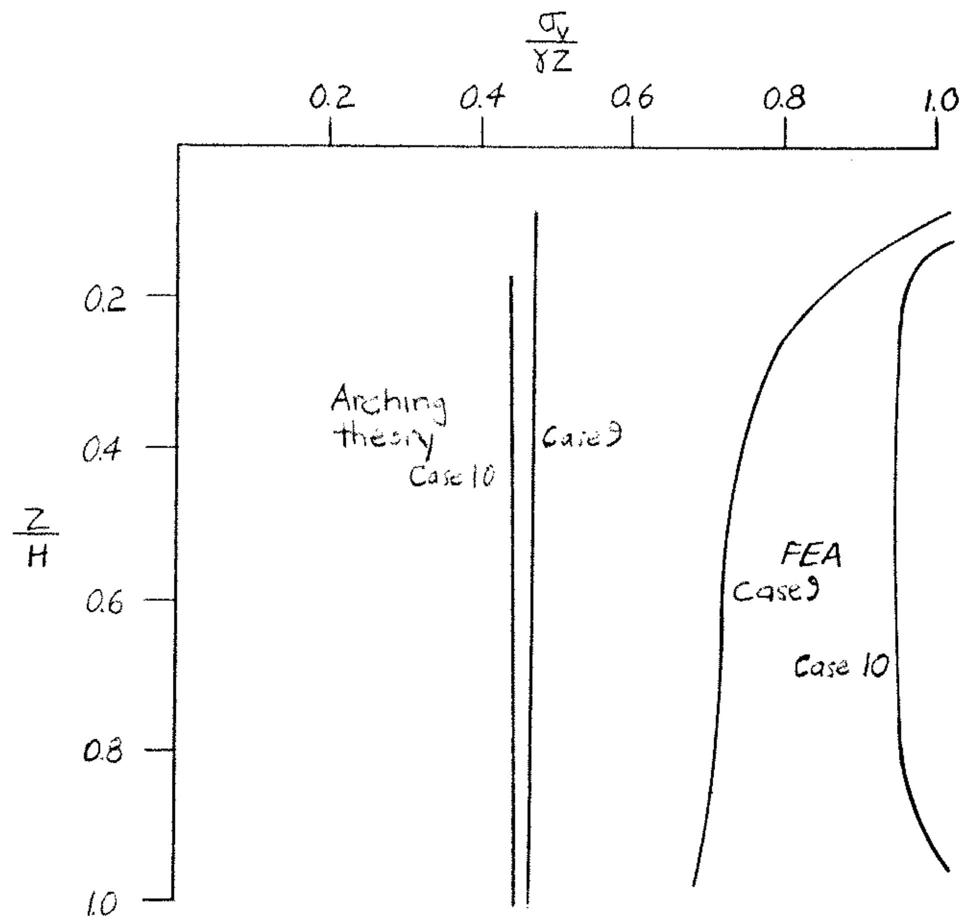


Fig. 23. Core stresses from parametric study (after Kulhawy and Gurtowski, 1976). Nonlinear FEA.



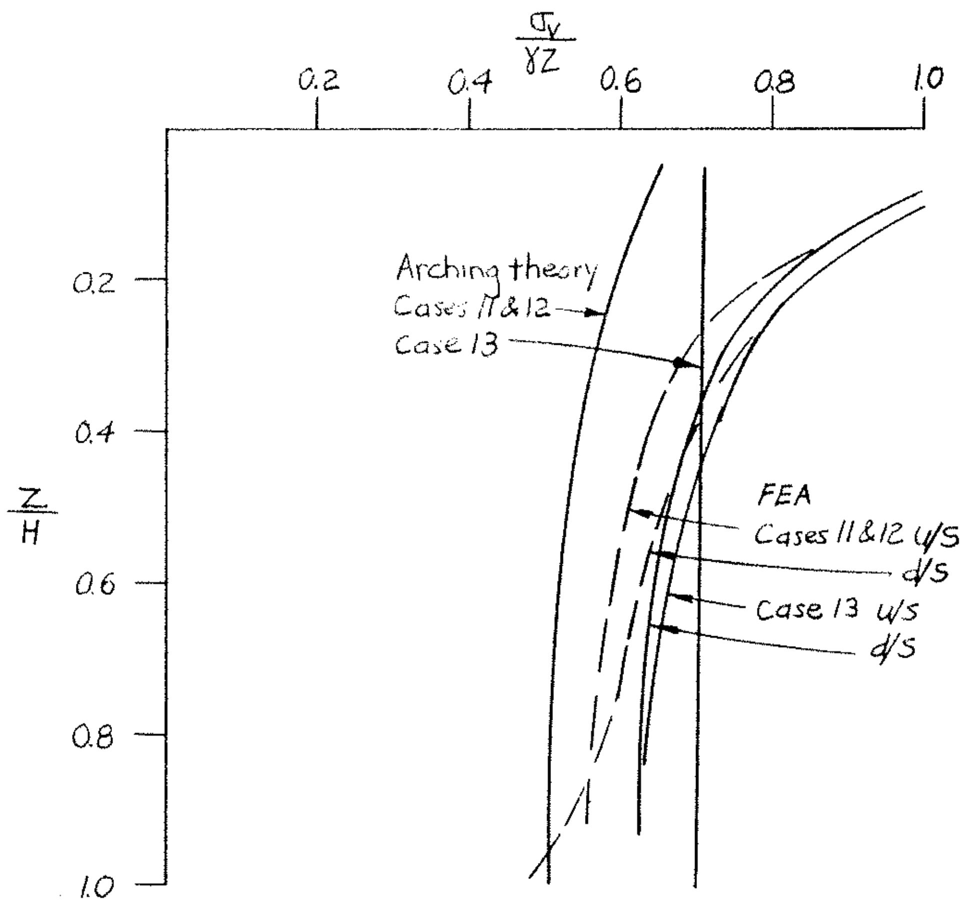
b. Central core, $\phi_c = 27°$, core slopes = 0.25

Fig. 23. Continued.



c. Central core, $\phi = 34° \& 36°$, core slopes = 0.1

Fig. 23. Continued.



d. Inclined cores. Stresses calculated at core boundaries.

Fig. 23. Continued.

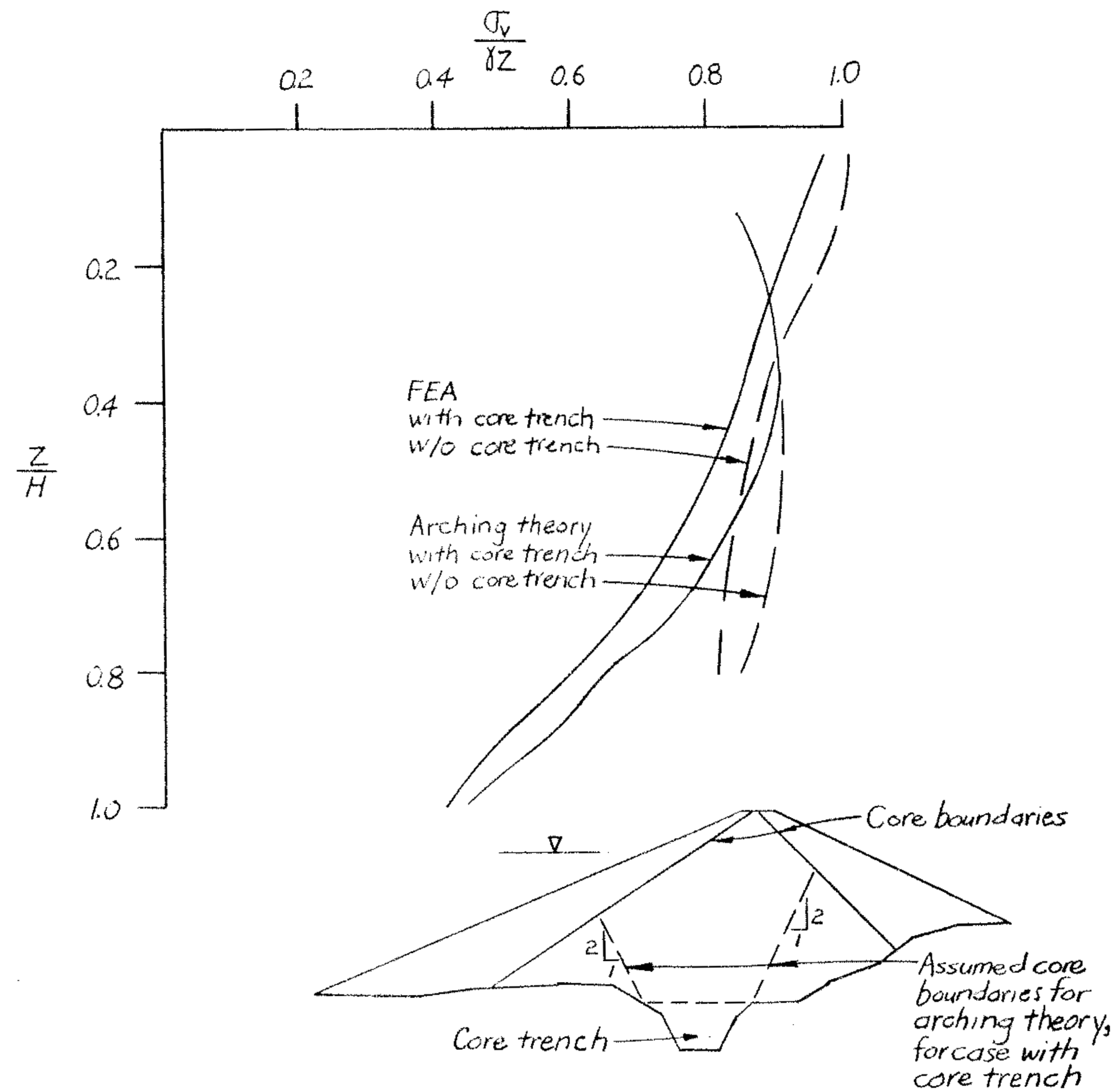


Fig. 24. Core stresses in Teton dam (after U.S. Department of the Interior, 1976). Nonlinear FEA.

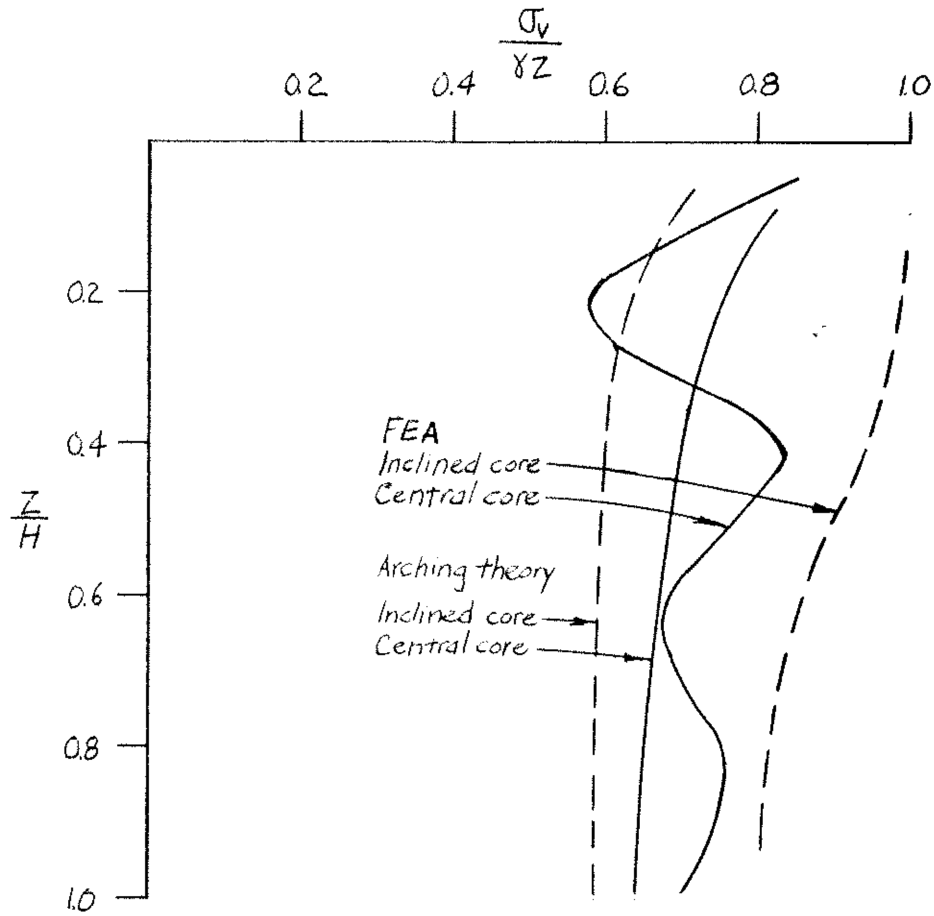


Fig. 25. Core stresses in Tehri dam (after Sharma et al, 1979). Nonlinear FEA.

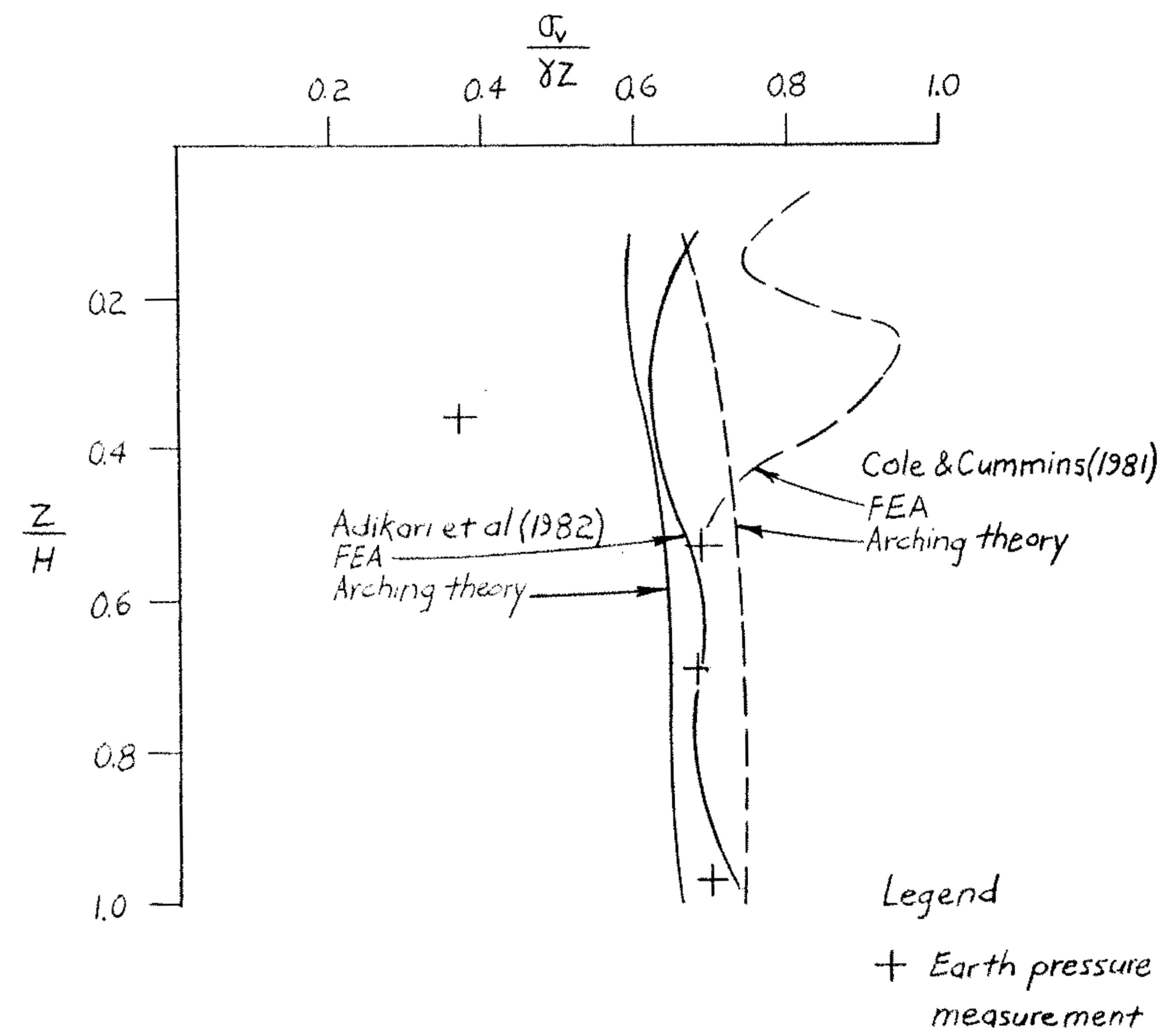


Fig. 26. Core stresses in Dartmouth dam. Nonlinear FEA.

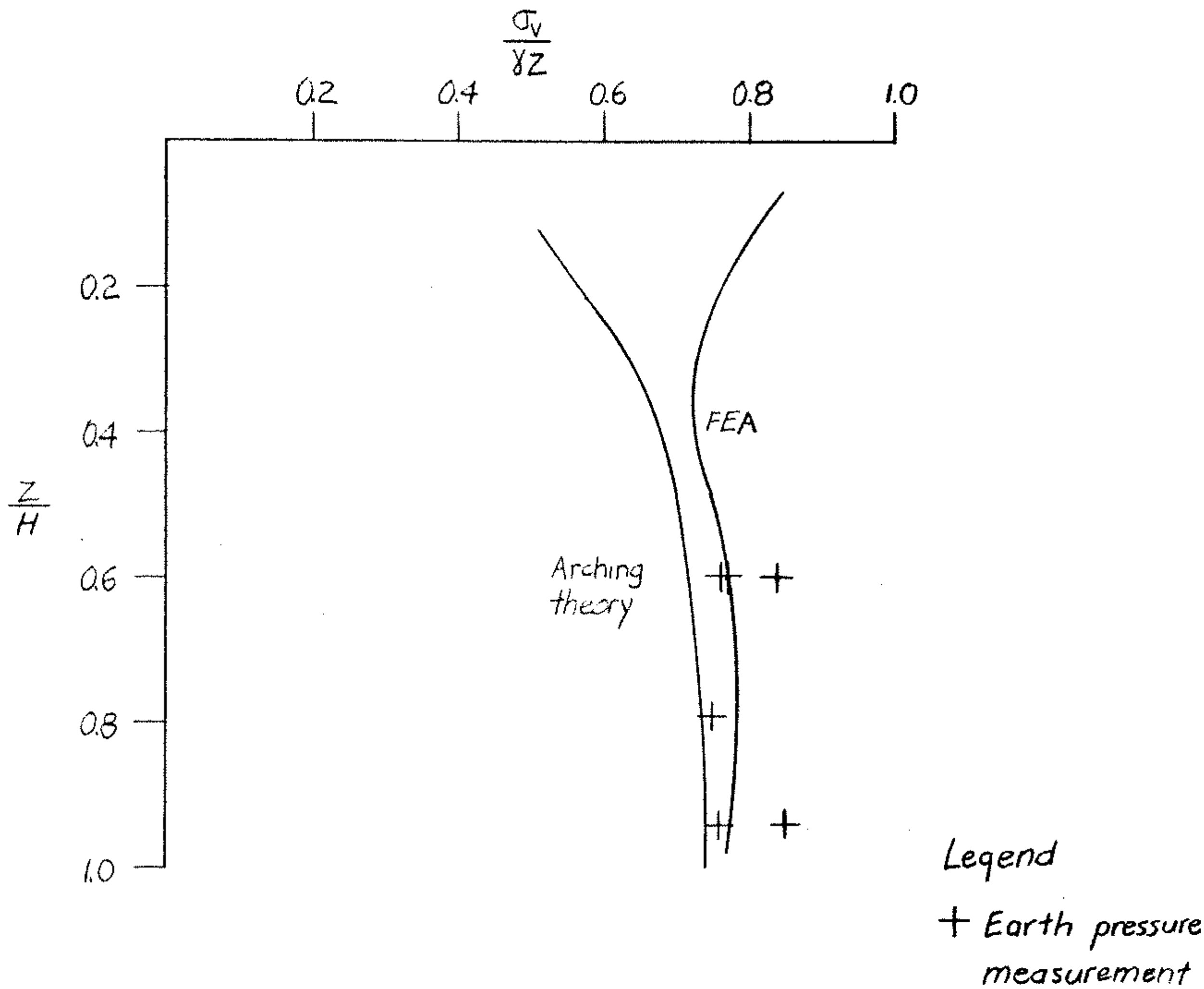


Fig. 27. Core stresses in Talbingo dam (after Adikari and Parkin, 1982). Nonlinear FEA.

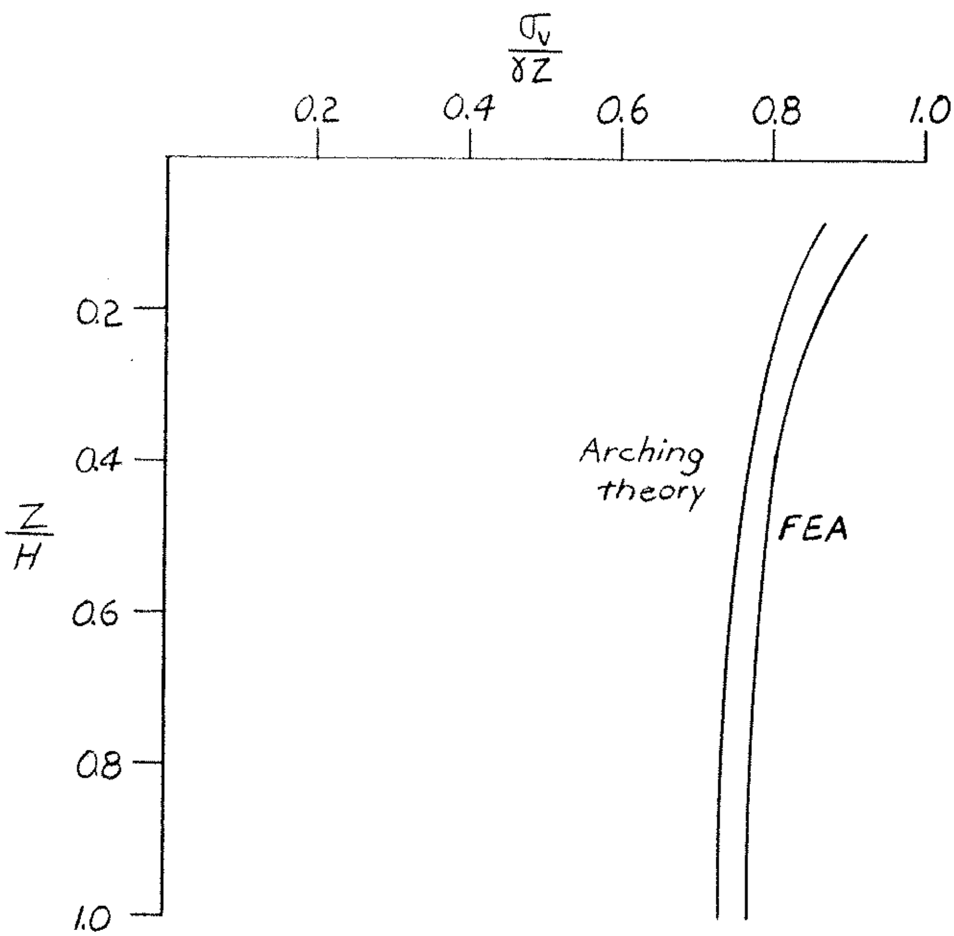


Fig. 28. Core stresses in LG4 main dam (after Paré et al, 1984). Nonlinear FEA.

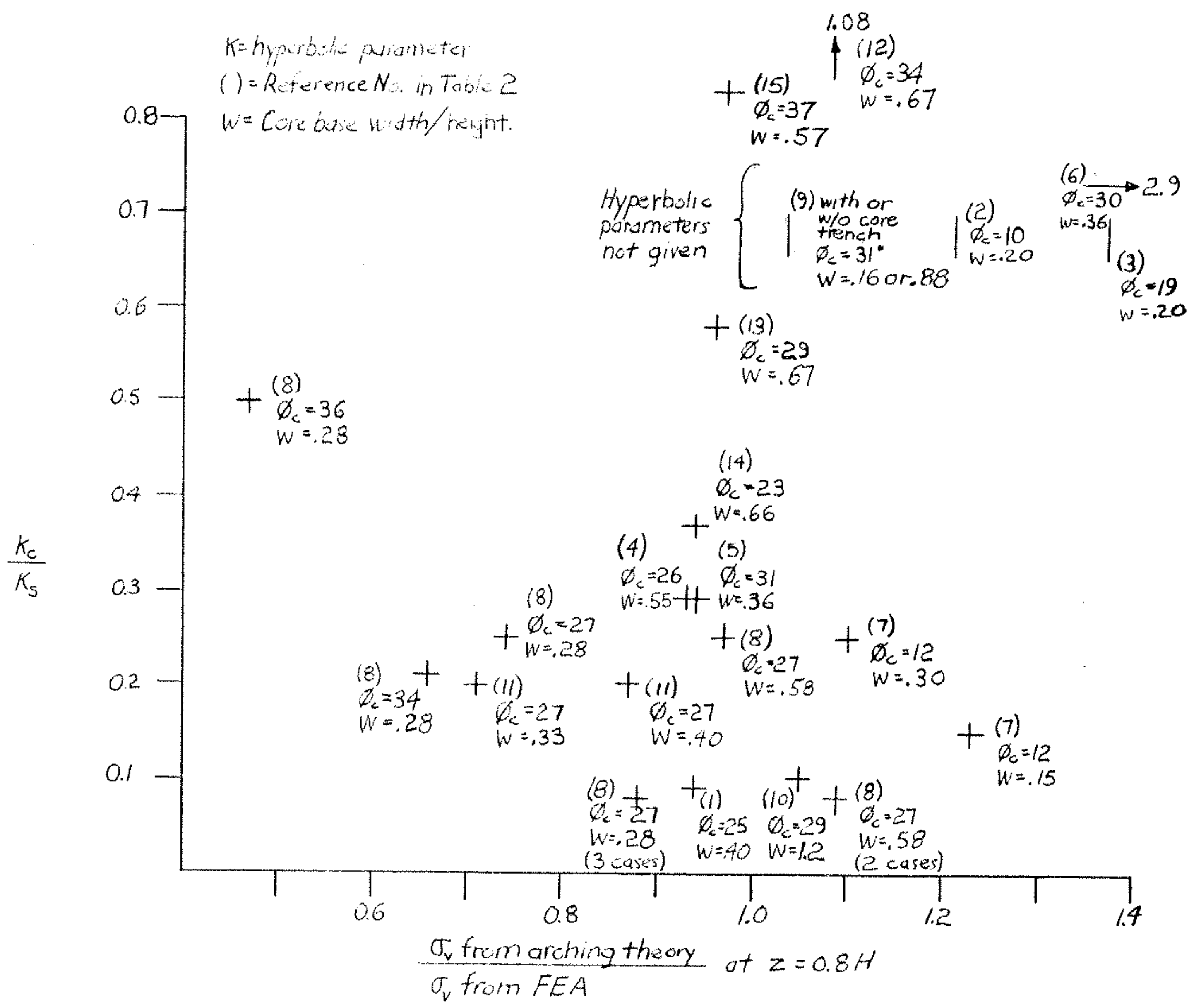


Fig. 29. Comparison of arching theory with nonlinear FEA.

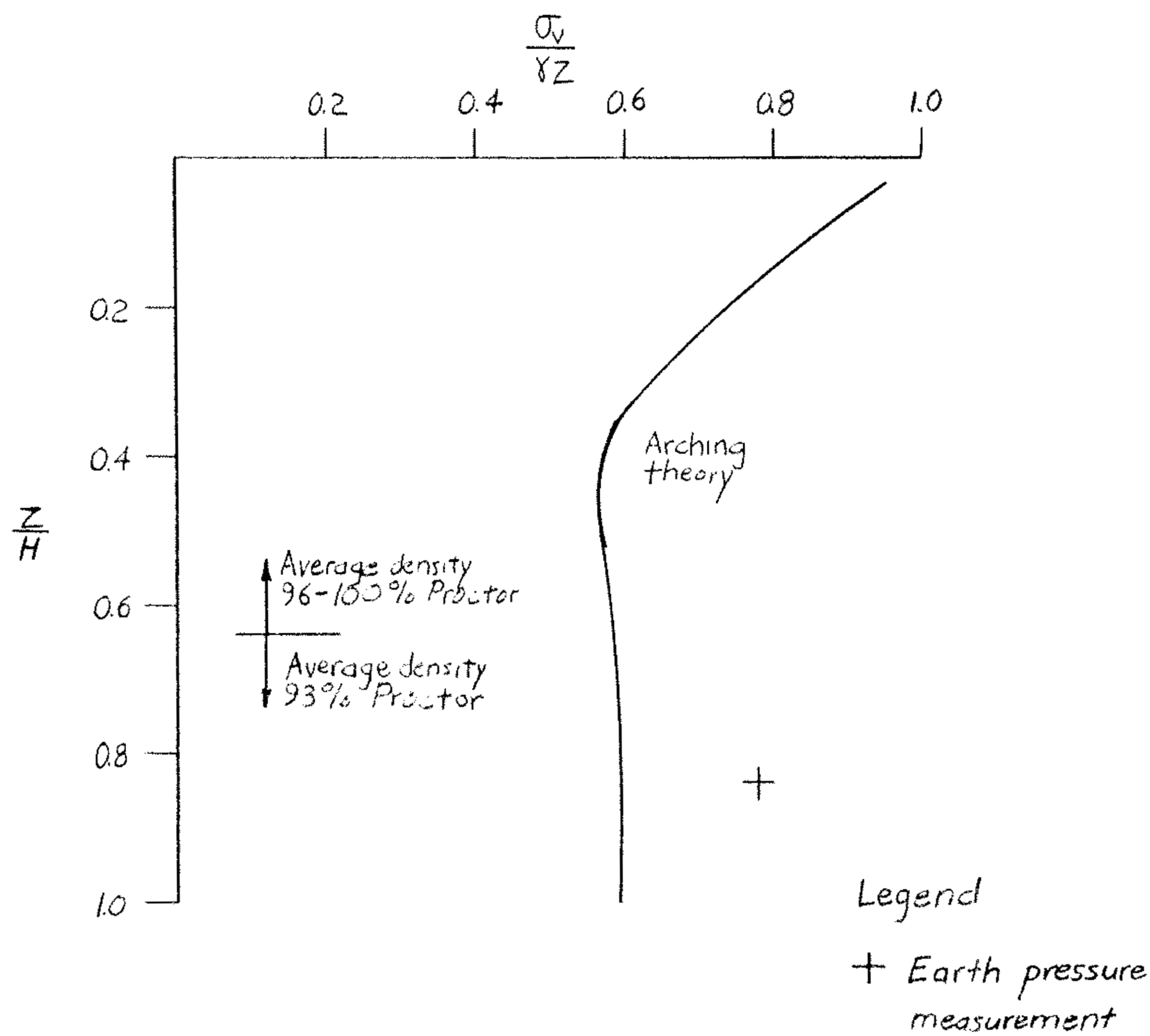


Fig. 30. Core stresses in Balderhead dam (after Kennard et al, 1967).

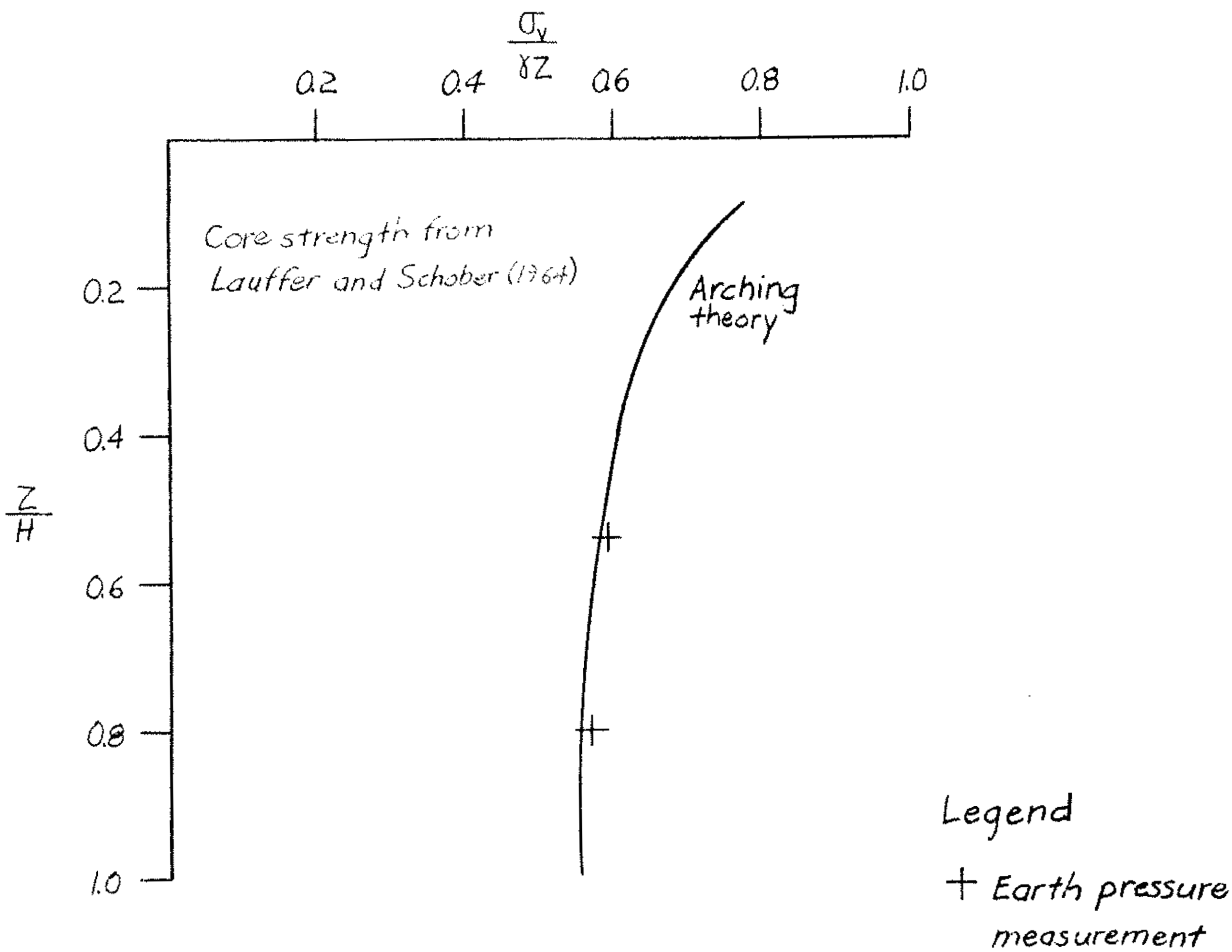


Fig. 31. Core stresses in Gepatsch dam (after Schober, 1967).

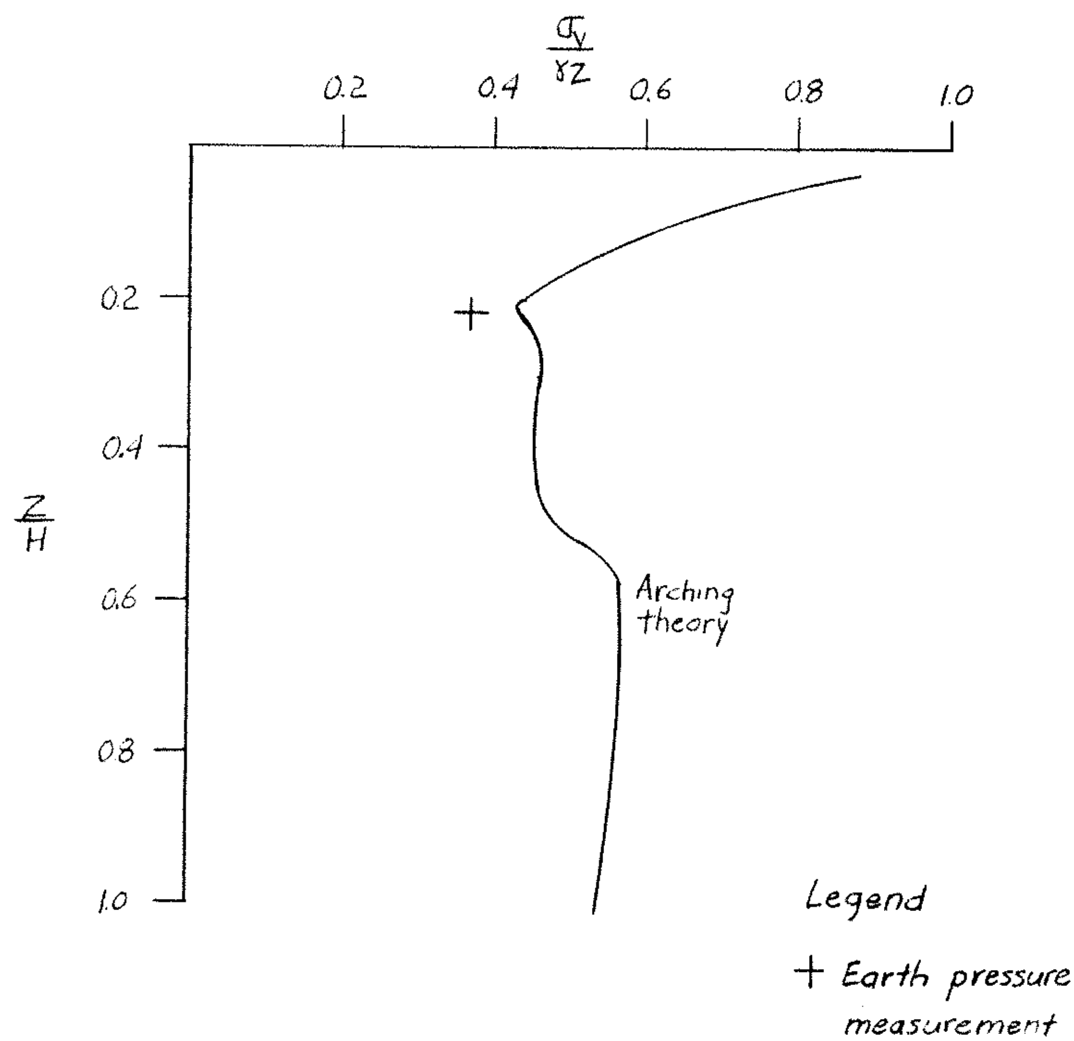


Fig. 32. Core stresses in Hyttejuvet dam (after Kjaernsli and Torblaa, 1968).

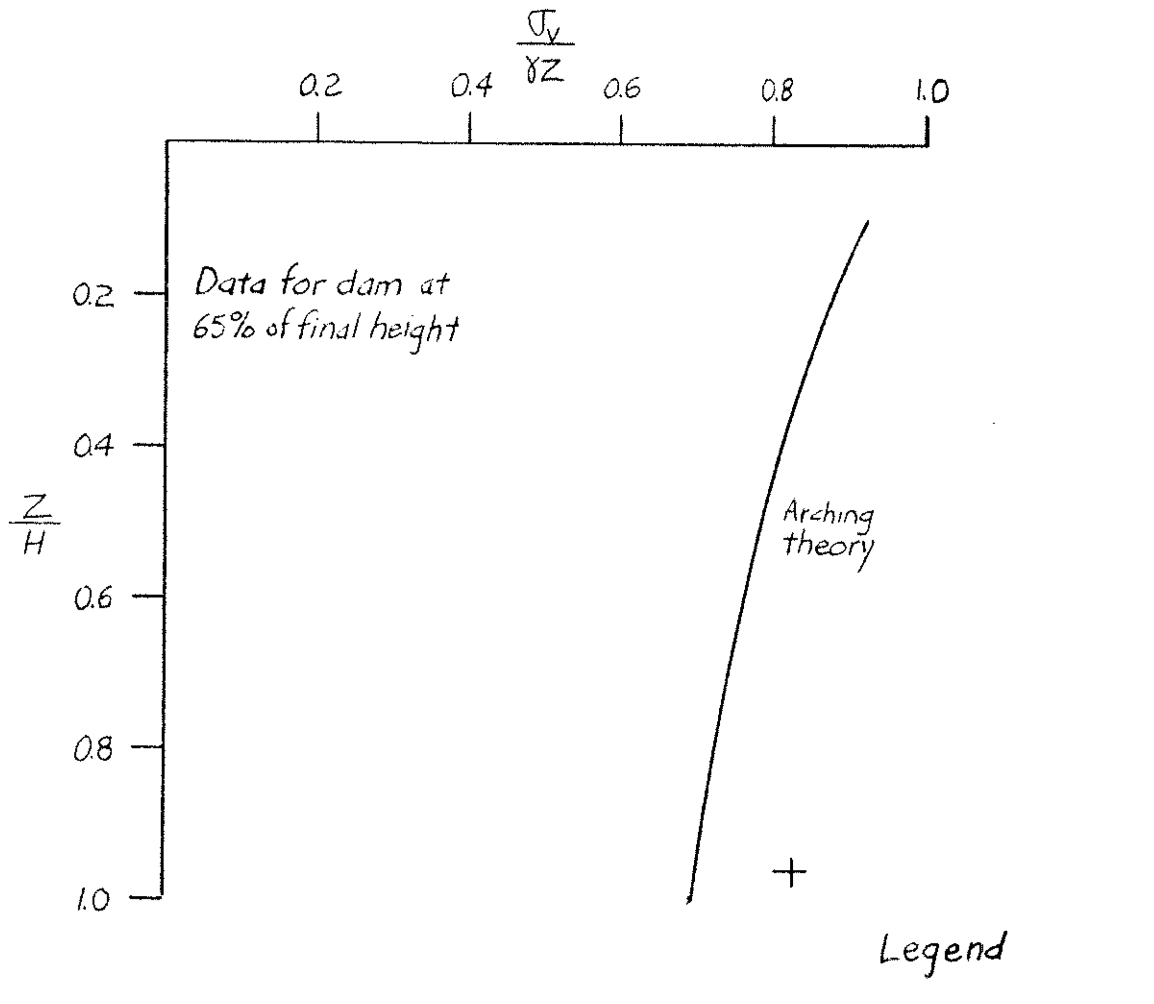


Fig. 33. Core stresses in Scammonden dam (after Penman and Mitchell, 1970).

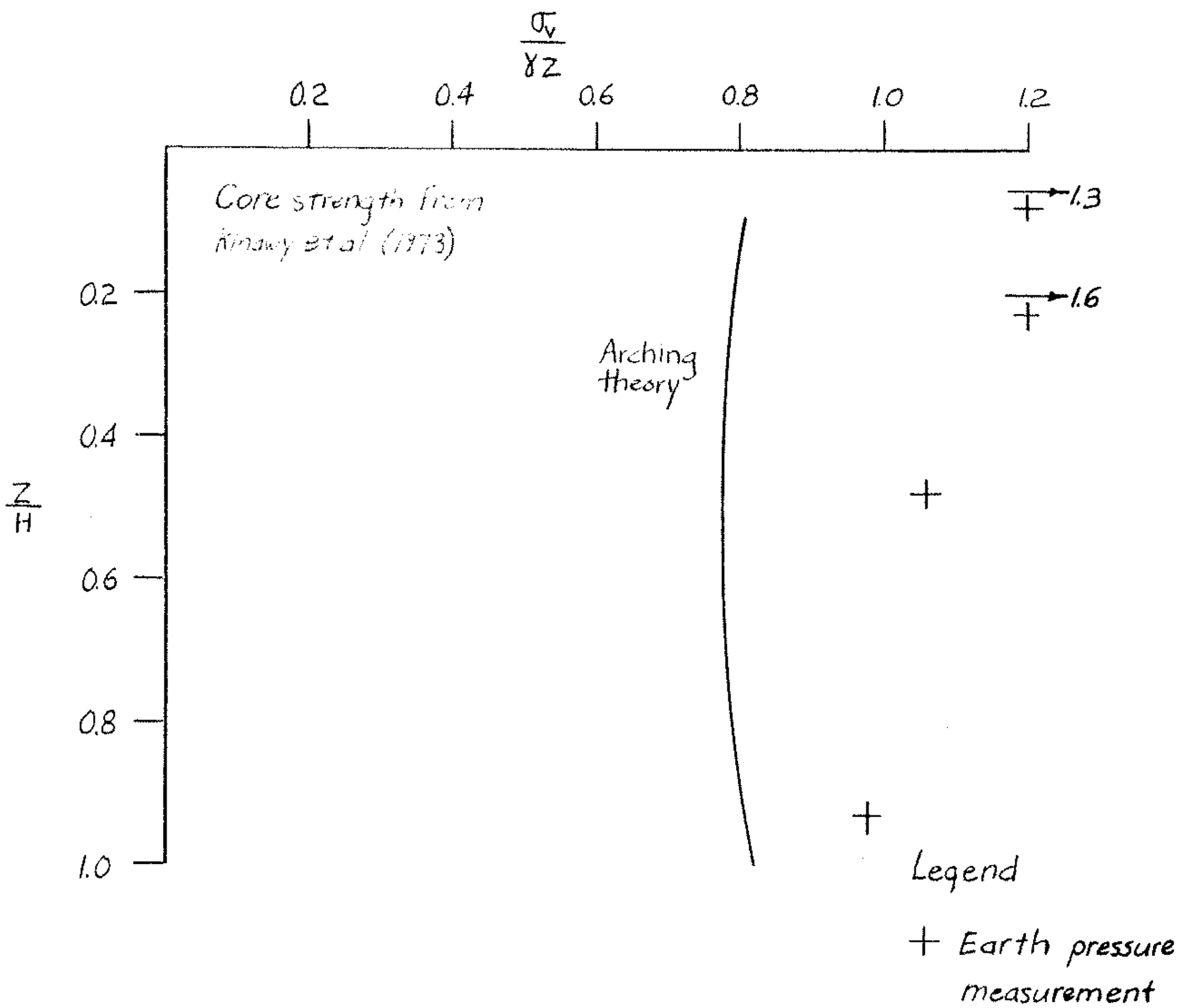


Fig. 34. Core stresses in High Aswan dam (after Kinawy and Shenouda, 1973).

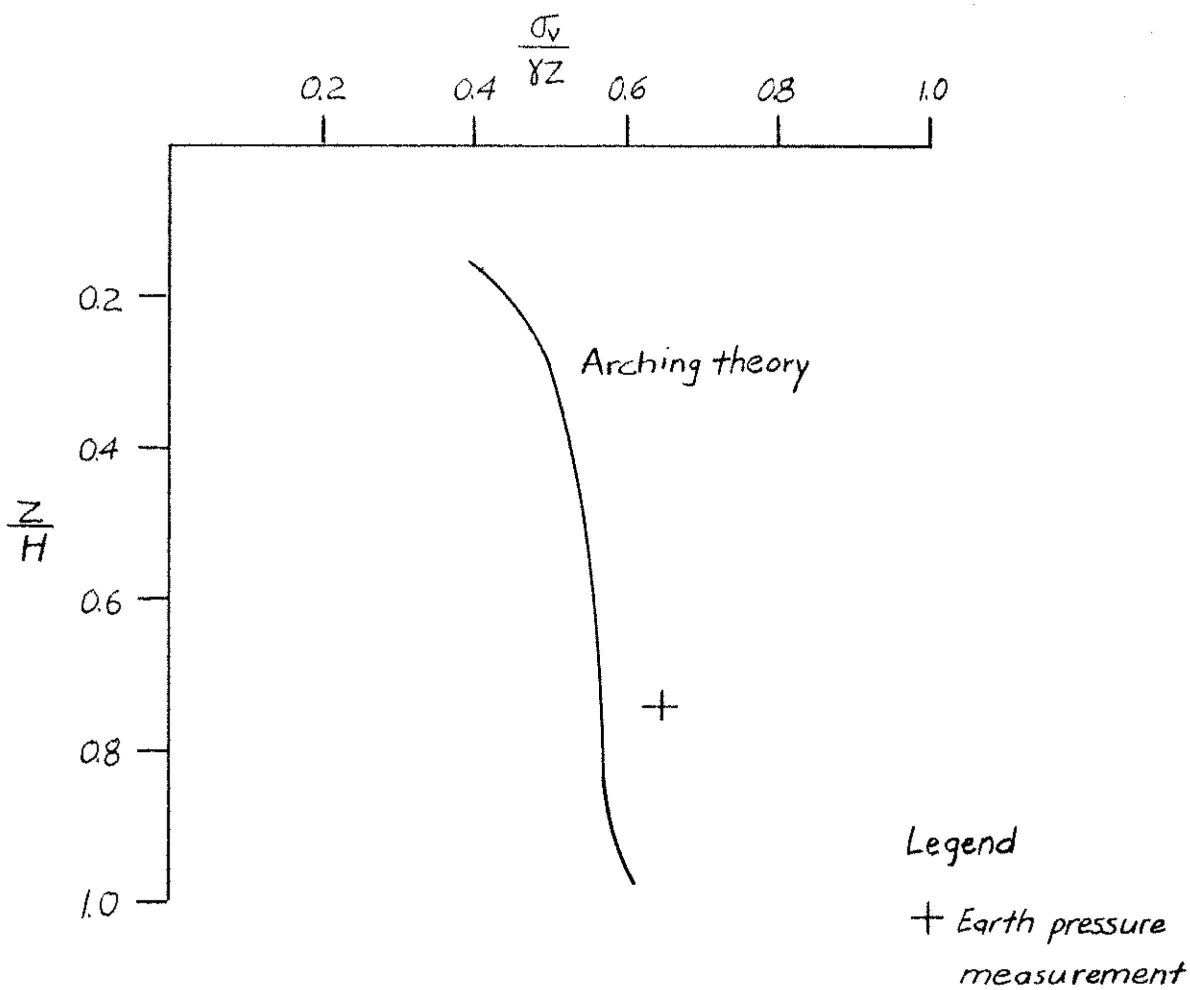


Fig. 35. Core stresses in Mautthaus dam (after Lorenz, 1973).

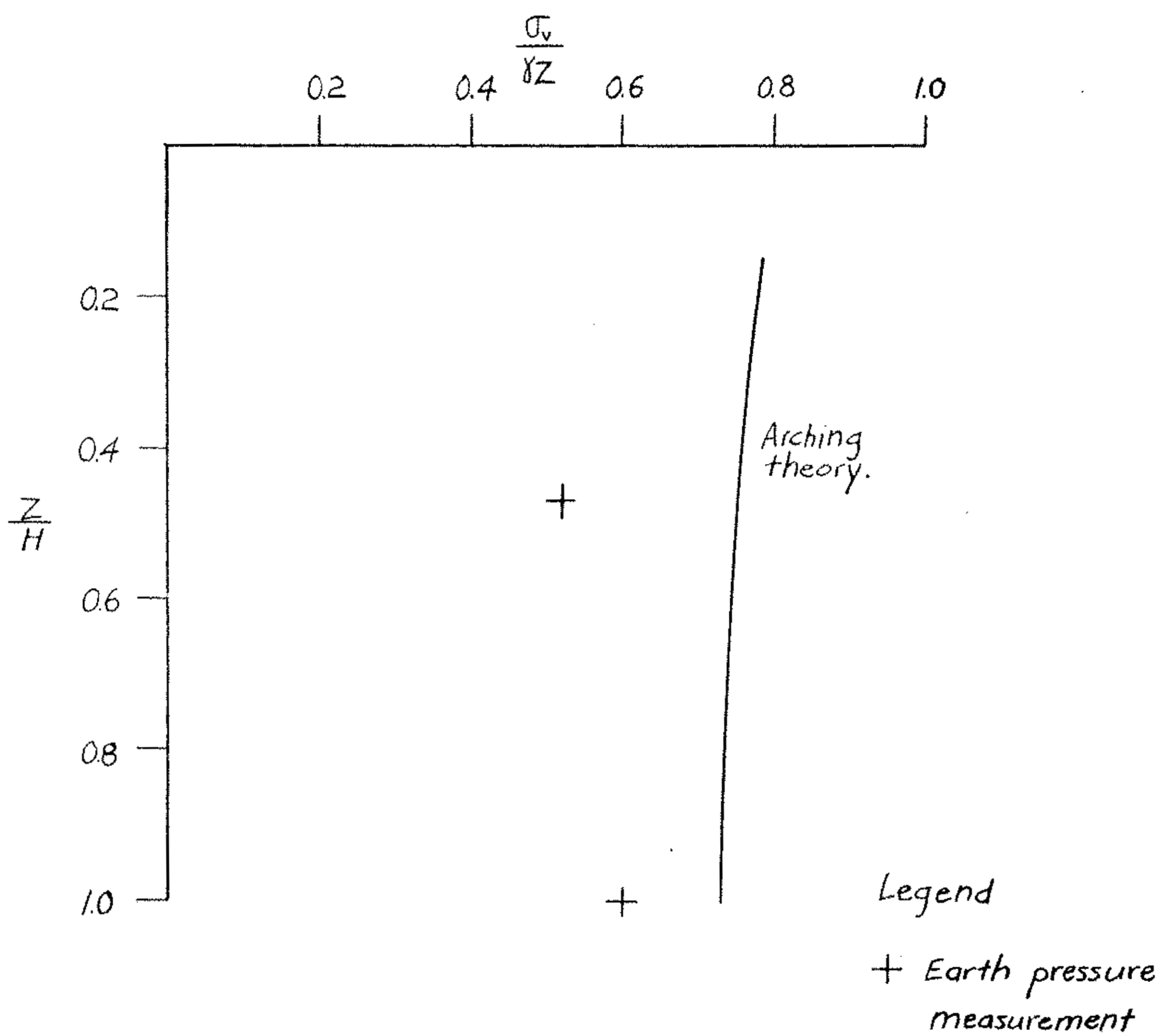


Fig. 36. Core stresses in Takase dam (after Takai et al, 1977).