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THE UNIVERSITY OF ALBERTA
IDENTIFICATION OF UTILITY FUNCTIONS THROUGH AN INVESTMENT GAME

by

 RALPH B. YOUNG

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
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UNIVERSITY OF ALBERTA
FACULTY OF GRADUATE STUDIES
AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Identification of Utility Functions Through an Investment Game" submitted by Ralph B. Young in partial fulfilment of the requirements for the degree of Master of Business Administration.


.....
Supervisor


.....
Stephen D. Lewis

Date April 27, 1973

ABSTRACT

This thesis utilizes an investment game to observe the behavior of several distinct participant groups and to test the predictive ability of alternate utility functions. The study analyses the responses of the groups to determine whether significant differences in behavior are evident. The study utilizes the responses to determine the degree to which alternate utility functions are descriptions of actual behavior. Although participant groups exhibited varied behavior to the investment game, all groups supported the predictive ability of the Logarithmic function of wealth plus a constant.

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I would also add thanks to Mrs. Teresa Lindstrom who provided excellent service in performing the typing duties associated with the thesis.

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CHAPTER I

INTRODUCTION

The March 1972 edition of The American Economic Review carried an article entitled "Experimental Evidence of Alternative Decision Rules" which was written by M.J. Gordon, G.E. Paradis and C.H. Rorke. The authors were on the faculties of the University of Rochester, Laval University and Queen's University, respectively. The article reported a research study conducted as a test of alternative utility functions in a situation involving the reactions of a group of business administration students to a game requiring portfolio decisions. Gordon, Paradis and Rorke point out in their study that although there has been considerable study of how the portfolio decisions of an investor should be effected under alternative utility functions "there has been no empirical research on optimal portfolio policy".¹ The Gordon, Paradis and Rorke study has overcome that deficiency and in so doing has opened the door to further research and analysis.

This thesis is an extension of the work of Gordon, Paradis and Rorke and stems directly from a draft of their article, cited above, which was available to the author prior to publication. The thesis evolved from the author's concern regarding the implications

¹M.J. Gordon, G.E. Paradis and C.H. Rorke, "Experimental Evidence of Alternative Decision Rules" The American Economic Review, (Vol. 62, No. 1, 1972) p. 107.

of the Gordon, Paradis and Rorke research. Although empirical support for an optimal portfolio policy was found, the results are limited to responses of a test group, presumably with many similar psychological and sociological characteristics and presumably far removed from the real world decision making process. It was the author's belief that a logical extension of the Gordon, Paradis and Rorke study would be to involve a group of professional investment managers to determine whether their responses would differ significantly from MBA students and whether both would achieve a common optimal portfolio policy. K.H. Borch in his book The Economics of Uncertainty paraphrased the problem very clearly.

"We can also construct a utility function based on a student's statements as to how he would decide in situations which could lead to losses of thousands of dollars, which he does not have. It is however, unlikely that this utility function will give us any useful information about the economic behavior of people with money or about the behavior of the student in later years when he has some money to lose."²

This position has previous support. As early as 1951, Mosteller and Nogee³ in a series of controlled experiments found

²K.H. Borch, The Economics of Uncertainty, (Princeton, Princeton University Press, 1968) p. 72.

³Frederick Mosteller and Philip Nogee, "An Experimental Measure of Utility" The Journal of Political Economy, (Vol. 59, No. 5, 1951) p. 371-404.

significant differences between the utility functions of a group of students and a group of military personnel.

The purpose of the current study was to achieve two things:

1. To replicate the Gordon, Paradis and Rorke study using a group of MBA students from the University of Alberta and to establish these results as control for further research.

2. To add an additional dimension to the study as a pilot for further research, that being the addition of a test group of professional investment managers to test the following research hypotheses.

- (i) The behavior of the test group and control group would be significantly different in the relationships of their responses to the Gordon, Paradis and Rorke investment game.

- (ii) The utility functions of the participant groups would be significantly different.

The limited population from which to draw data in the immediate Edmonton area presented a problem to the author in assessing the statistical significance of the potential results. In light of this, two additional participant groups were selected which related to the test and control groups by educational background and relative station in life. A group of second year undergraduate students in the Department of Business Administration was selected as one participant group. A group of Professors of Finance/Economics in the Department of Business Administration was selected as the second group.

The conclusions of the thesis verified the need for further research into the behavioral implications of the Gordon, Paradis and Rorke study. In replicating the Gordon, Paradis and Rorke study, significant differences in responses were observed between their test group and the control group of this study. The control group did however support the general analysis of alternative utility functions of the previous test group.

The inclusion of several participant groups provided strong evidence for acceptance of both hypotheses stated earlier for some groups, while results led to the rejection of the hypotheses in others. The predictive power of the logarithmic utility function plus a constant was strongly supported in all cases.

Paul Slovic a noted researcher in the field of psychology paraphrases the essence of what the conclusions indicate; "risk taking behavior appears to be multidimensional in nature. It has substantial subjective components and is susceptible to a variety of motivational and other influences. Research attempting to establish the validity and consistency of risk taking measures has generally neglected these factors."⁴

⁴P. Slovic "Assessment of Risk Taking Behavior,"
Psychological Bulletin, (Vol. 61, No. 3, 1964) p. 230.

CHAPTER II

THEORETICAL FRAMEWORK OF STUDY

The thesis and the research of Gordon, Paradis and Rorke center on an attempt to rationalize actual behavior, within a portfolio decision making framework, with alternative utility functions. Portfolio theory provides a normative approach to the selection of optimal portfolio given a series of plausible assumptions. The alternate utility functions provide a quantitative means of describing actual behavior.

Portfolio theory although only recently accepted has taken many years to formulate. As early as Adam Smith, economists had accepted the notion of expected value as a criterion for decision making under risky conditions. Irving Fisher¹ later suggested that the risk of an option was in some way related to the dispersion or variance of its outcomes.

Harry Markowitz² formulated a theory of portfolio selection concerning investment decisions under conditions of risk. The Markowitz model hypothesized that optimal portfolios of securities could be identified by analyzing the securities on the basis of their

¹Irving Fisher, The Nature of Capital and Income, (New York: The Macmillan Company, 1923)

²H.M. Markowitz, Portfolio Selection: Efficient Diversification of Investments (New York: John Wiley and Sons, 1959)

expected value, standard deviations and correlation coefficients. The Markowitz model implied that the optimal portfolio could be determined by finding a point on a line joining all optimal portfolios which maximized the investor's utility.

William Sharpe³ and others expanded the work of Markowitz to show that given the rate of return on a risk free security, an optimal combination of risky securities could be determined regardless of utility considerations. The selection of an optimal portfolio for a particular individual was reduced to the allocation of wealth between the risky portfolio and a risk free security.

Research into the analysis of alternative utility functions has an equally long history. Daniel Bernoulli⁴ was the first to formulate a theory of utility. Bernoulli observed two critical deviations from classical expected value determination of risky decision making. He noted that an individual's wealth affected his desire to accept risks. Bernoulli attributed this to a misconception of the meaning of wealth, stating that value must be related to utility not price. Secondly, Bernoulli observed deviations in decisions from those anticipated based on expected value. To rationalize these observations, Bernoulli hypothesised a utility function; the relationship which he suggested was a logarithmic function.

³William F. Sharpe, Portfolio Theory and Capital Markets, (New York: McGraw Hill Book Co. 1970)

⁴Daniel Bernoulli, Exposition of a New Theory on the Measurement of Risk, (Gregg Press, 1967)

John von Neumann and Oskar Morgenstern⁵ proved mathematically the existence of utility measurable on an interval scale for an individual satisfying certain axioms. Friedman and Savage⁶ attempted to provide a crude empirical test of the von Neumann and Morgenstern findings, by observing behavior of individuals choosing among risky alternatives. The authors' observations were entirely consistent with the earlier hypothesis, if a rather special shape was given to the total utility curve of money.

Mosteller and Nogee⁷ expanded on the Friedman and Savage work and made the first attempt to measure utility with controlled experimentation. Their research was conducted utilizing college undergraduates and members of the United States National Guard as subjects. Although the subjects lacked consistency in their choices, the theory based on the original Bernoulli function had considerable predictive power.

Numerous authors have explored the implications of alternative utility functions and their implications for an investor's portfolio decisions; the work of Hakansson, Latane and Tuttle, Freimer and Gordon, Neave, and Yaari has been cited in the Gordon, Paradis and

⁵J. von Neumann and O. Morgenstern, Theory of Games and Economic Behavior, (Princeton: Princeton University Press, 1944)

⁶M. Friedman and L.J. Savage, "The Utility Analysis of Choices Involving Risk", Journal of Political Economy, (Vol. 56, No. 4, 1948) p. 279-304.

⁷F. Mosteller and P. Nogee, "An Experimental Measurement of Utility", The Journal of Political Economy, (Vol. 59, No. 5, 1951) p. 371-404.

Rorke paper.

There has been considerable literature in the area of social and mathematical psychology which relates directly and indirectly to the Gordon, Paradis and Rorke study.

Nathan Kogan and M.A. Wallach⁸ in a major research study determined that the various cognitive judgemental behaviors (i.e. general confidence of judgement, breadth of categorizing, extremity concerning judgements about external events and extremity concerning selfreferent judgements) do possess risk-conservatism implications for particular subgroups of individuals.

Scodel, Ratoosh and Minas,⁹ studying risk taking in a gambling situation determined that intelligence was related to the variability of risk taking. The authors also found need achievement and fear of failure to influence significantly the type of gamble selected.

Y. Rim¹⁰ in a series of studies found that radicalism and extroversion were significant factors influencing risky decisions particularly in group situations.

⁸Nathan Kogan and M.A. Wallach, Risk Taking: A Study in Cognition and Personality, (New York: Holt, Rinehart and Winston, 1967) p. 199

⁹A. Scodel, P. Ratoosh and J.S. Minas, "Some Personality Correlates of Decision Making under Conditions of Risk", Behavioral Science, (Vol. 3-4, 1958-59), p. 28

¹⁰Y. Rim, "Social Attitudes and Risk Taking," Human Relations, (Vol. 17, No. 3, 1964) p. 259-265.

Y. Rim, "Personality and Group Decisions Involving Risk", Psychological Record, (Vol. 14, No. 1, 1964) p. 37-45.

Paul Slovic¹¹ in a study of gambling situations found that decision strategies differed depending on whether gains and losses were real or hypothetical. When subjects were not committed to the consequences of their actions, they tended to maximize gain and discount potential hypothetical losses. When subjects knew the gambles were real, they were more cautious, preferring relatively higher probability of winning, lower probability of losing and lower potential losses.

Pollatsek and Tversky¹² provide a quantitative explication of the concept of risk in the form of a psychological theory and conclude that under certain assumptions, the risk of an option is a linear combination of its mean and variance. Coombs and Pruitt¹³ stated that a gamble can be characterized by the expectation, variance and skewness of its probability density distribution over money. While subjects always prefer more expectation to less, he may have a utility for risk which will exhibit itself as a preference for certain amounts of variance and skewness.

¹¹P. Slovic, "Real vs. Hypothetical Payoffs in Choices of Gambles", Journal of Experimental Psychology, (Vol. 80, 1969)

¹²Pollatsek and Tversky, "A Thoery of Risk", Journal of Mathematical Psychology, (Vol. 7, 1970) p. 540-553.

¹³Coombs and Pruitts, "Components of Risk in Decision Making", Journal of Experimental Psychology, (Vol. 40, No. 2, 1967)

The vast majority of the research in the area on psychology supports the complexity of risk taking behavior and underlies the potential wide variations in responses to risk taking situations depending on a host of psychological variables. In light of these research studies, it was felt there could exist amongst participant groups wide variations in their responses to similar situations involving risky decisions and in fact could result in significantly different utility functions.

CHAPTER III

METHODOLOGY

Research Strategy

The Research strategy comprised defining three broad stages of the thesis:

1. Conceptual Stage.
2. Implementation Stage.
3. Analysis and Conclusion Stage.

1. Conceptual Stage

This initial stage consisted of the following components.

- (i) Conceptual statement of Problem.
- (ii) Detailed review of the Gordon, Paradis and Rorke study which was available to the author in draft form.
- (iii) Review of literature relating to the problem statement.
- (iv) Detailed formulation of purpose and hypotheses to be tested.

2. Implementation Stage

This secondary stage consisted of the following components.

- (i) Construction of an investment game based upon information from the Gordon paper.

The game was designed from descriptions of the game in the Gordon, Paradis and Rorke paper. An aspect which was not carried through in this thesis was the attempt by the previous authors to

provide an imaginary setting in which the participants were to imagine themselves while participating in the game. This aspect is further expanded upon under the heading "Research Techniques". It was felt by the author that this approach might lead to adverse reaction from some participants, notably those professional investment managers. The game was distributed by hand or by mail with a covering letter describing the purpose of the game and the procedures to be followed in completing it. It was the author's belief that distribution in this manner would allow participants maximum freedom to complete the game and anonymity in returning the completed forms.

(ii) Selection of Participants

The control group was selected as the class of first year MBA students at the University of Alberta. The tests were conducted after a half year of classes where the students had been through courses in basic statistics and were beginning an introductory course in Finance and had been exposed to Cost of Capital determination and had an introduction to portfolio theory. As the studies were conducted independently, no attempt to pair participants was made. The control group was necessitated due to possible deviations between the two research studies.

The main test group was selected from professional investment managers within the City of Edmonton. The author contacted managers of local mutual funds, collective mutual funds, speculative mutual funds, local institutional and government investment trust and pension funds, local trust company security portfolios, local mortgage companies and local banks. The author had previously interviewed a

number of the participants in an earlier class project. The educational background of the participants varied considerably although this information was not specifically requested. The type of portfolio managed varied from predominantly bonds or mortgages to predominantly more speculative securities. The common thread amongst the participants was that their primary function involved the investing of large amounts of other people's money in alternate risky assets.

The two test groups which were added to the study consisted of an undergraduate class of second year students in Business Administration and a group of professors of Finance/Business Economics within the Faculty of Business Administration, all from the University of Alberta at Edmonton.

The undergraduate students were selected because they had been exposed to introductory courses in economics and statistics sufficiently to appreciate some of the basic concepts underlying the investment game. It was the author's belief that due to their limited educational background and limited permanent work experience in relation to the MBA students, the undergraduates would represent a distinctive group in their responses to the investment game.

The group of professors were all Ph. D's in Finance or Economics. It was the author's belief that their more advanced educational background and their working knowledge of the concepts underlying the investment game would result in distinctive responses to the game.

(iii) Implementation of Game

The preliminary contact with the majority of investment managers was by way of interview as described earlier. The investment game was distributed to the investment managers by mail with a covering letter outlining the basic purpose of the study and instructions on how to complete and return the game. A self addressed and stamped, return envelope was provided. The participants were requested to complete the exercise at their convenience; however, a two week limit was suggested.

The investment game was distributed by hand to the other groups and all were given identical instructions to those the managers had received.

The implementation of the game provided basic differences from the Gordon, Paradis and Rorke study. Firstly, it was not believed practical to replicate the method utilized by Gordon, Paradis and Rorke. Secondly, by providing participants with the game individually, the author believed would ensure a more real life type of response and help minimize the amount of collusion amongst responses. It must also be recognized that there may be some bias between studies, as the author did not have the same control over ensuring a total response. It was the author's belief that these alterations related exclusively to implementation and would not significantly effect the desired type of response from the participants.

(iv) Collection and Transformation of Data

The responses were collected on a totally voluntary basis

by way of self addressed envelopes; and deposit at a central location by individuals at the University. The raw data to be utilized in the analysis was segregated from the body of the investment game and collated into groupings applicable to various stages of analysis.

In reviewing the raw data, it was decided to completely disregard the results of the eleventh period for all participant groups for two reasons. Firstly, several participants indicated that their responses to the last period were not realistic because they chose to gamble "the works". Secondly, the Gordon, Paradis and Rorke group were not told when their participation in the game would cease; by dropping the final period it was believed would result in a more comparable relationship between the studies.

For analysis and comparison of the behavior of the participants, the data was transformed into groupings which corresponded to those of the Gordon, Paradis and Rorke study. For analysis of alternative utility functions, the data was transferred to key punched cards for a computer regression analysis. The analysis is further detailed in Chapter IV.

3. Analysis and Conclusion Stage

This tertiary stage consisted of the following:

- (i) Analysis of data.
- (ii) Conclusions.

This stage is described and expanded upon in Chapters IV and V.

Research Techniques

1. Investment Game

The basic research technique employed was an investment game for the collection of data. The game was patterned after one used by Gordon, Paradis and Rorke in their research.

The Gordon, Paradis and Rorke game Aipotu (Utopia spelled backwards) centers around a fictitious nation Aipotu where an individuals' only source of income is his wealth. Periodically Aipotuans make decisions regarding the level of consumption desired during the period as well as the size and type of investment to be made during the period. Investment decisions involve the selection of one of five possible investments each yielding different possible returns and with varying degrees of dispersion.

Investment Alternatives in Aipotu				
Gamble Number	Payoffs		Probability	
	Red	Black	Red	Black
1	\$ 1.30	\$.80	.5	.5
2	1.50	.70	.5	.5
3	1.90	.40	.5	.5
4	2.50	.00	.5	.5
5	100.00	.00	.005	.995

Borrowing is allowed in the game without charge; however, it can only be used for investment and must be repaid immediately upon the outcome of the gamble. Aipotuans are not

allowed to have negative levels of wealth at any time and hence borrowing is limited to $W \times B \div (1-B)$ (where W represents wealth and B represents the lower possible outcome of the investment).

Consumption involves a choice between expenditures of \$5,000 (which provides bare necessities) \$10,000 (which provides a comfortable standard of living) and \$20,000 (which provides for luxury). Aipotuans must always consume at one of these levels each period unless their wealth falls to zero at which time they receive welfare payments of \$3300. \$3000 of this payment must go for consumption and \$300 can be used for investment.

Each participant in the Gordon, Paradis and Rorke study was given an initial wealth between \$110,000 and \$190,000 and attempted to accumulate enough wealth to provide his family with an adequate endowment for living in Aipotu.

Of the five alternative gambles available the fifth was simply a lottery to detect the presence of risk lovers. The other four were all profitable although gamble two provided a lower degree of risk per dollar invested than the other three which were constant. For any conceivable outcome there was an investment involving gamble two and some degree of borrowing which would yield the same average return while assuming a lower level of risk.

The author adopted several modifications to the Gordon, Paradis and Rorke game which apply more to the mechanics than to the underlying concepts of the game.

(i) As described earlier the method of implementing the game

was altered for the reasons previously outlined.

(ii) There was no reference to an imaginary setting as it was believed that this might jeopardize the credibility participants placed in the game, particularly the managers. Participants were requested to react as though the game represented a real life situation.

(iii) Each participant was provided with an initial wealth of \$150,000 for simplicity. The only plausible reason for varying the initial level of wealth would be to ensure a greater number of responses within each wealth class.

(iv) As described earlier, due to the nature of implementation the results of the last period were disregarded.

(v) Due to the nature of implementation, an unbiased means was provided for determining the outcome of gambles. Gordon, Paradis and Rorke handled this aspect by the flip of a coin under supervised conditions. The author handled the situation in the following manner:

Two outcomes "R" (favorable) and "B" (unfavorable) were placed on the games sheets. A self adhesive tab was placed over each outcome prior to the game being distributed. The participant lifted one tab only, to determine his success. The letters "B" and "R" were placed on successive sheets based upon a random distribution and the distribution was unique for each game. The lottery was constructed by placing a random number between 0 to 200 beneath a tab and requesting the participant to write down a number between 0 to 200 prior to lifting the tab. Although this method left some room for cheating, none of the participants won on

the lottery, confirming the fairness of their participation.

Statistical Analysis Employed

1. Behavior of the Participants

The groups were observed to study the relationship between wealth levels and risk taking. More specifically the relationship between consumption/investment decision and wealth level was explored. Within the investment decision, gamble preferences were studied. No attempt was made in the design of the study, to attempt to rationalize various strategies. The relative small sizes of the participant groups and the nature of the data resulted in employing two basic means of analysis neither of which lead to strong statistical conclusions.

(i) Gamble preferences

The χ^2 test of two independent samples¹ was utilized to determine whether significant differences existed in the participants' choices of gamble. The χ^2 test requires no specific assumptions about the groups and thus the statistical inference relates to whether groups differ between themselves in any manner.

Null Hypothesis $H_0: p_1 = p_2$

i.e. There is no difference between the probability that the control group will select gamble two and that the test group will select

¹Sidney Siegel, Nonparametric Statistics for the Behavioral Sciences, (New York: McGraw-Hill Book Company, 1956) p. 104-111.

gamble two.

$$\chi^2 = \frac{N(AD-BC)^2}{(A+B)(C+D)(A+C)(B+D)} \quad df = 1$$

Where	Gamble Two	Others	Total
Control Group	A	B	A + B
Test Group	C	D	C + D
Total	A + C	B + D	N

A, B, C and D are observed frequencies relating to the group and the gamble selected. The test relates to whether the observed frequencies in the 2x2 contingency table could have occurred under H_0 .

(ii) Relationship of Consumption Decision and Wealth Level

The function mean G/W versus wealth level was plotted based upon the aggregated results of all participant groups. For comparison purposes, the results of the Gordon, Paradis and Rorke were plotted on the same graph.

(iii) Relationship of Investment Decision and Wealth Level

The function mean G/W-C versus W-C was plotted for each gamble to observe the relative behaviors of each participant group. The results for each gamble were plotted to increase the credibility of the relative configurations of the functions for each group.

The function mean G/W-C versus W-C was plotted for each participant group to observe how the particular gamble selected affected the responses.

NOTE - Where mean G/W-C equals the value of investment during the period divided by net wealth after consumption for the specific Wealth Class.

2. Analysis of Alternative Utility Functions

(i) Predictive Ability of Alternate Functions

Five basic utility functions were analyzed using data generated from the investment game. The functions analyzed were those detailed in the unpublished draft of Gordon, Paradis and Rorke.

(a) The first function was one in which utility is a quadratic function of wealth. The expected value of utility is:

$$E(U) = \sum P_j W_j + \frac{\alpha}{2} \sum P_j W_j^2$$

Where P_j = probability that wealth is W_j

$\alpha < 0$ implying risk aversion.

(b) The second function, the Bernoulli Logarithmic function referred to earlier, differs from the quadratic function in the level of investment is independent of the participants personality. The expected value of utility is a Logarithmic function of wealth.

$$E(U) = \sum p_j \ln W_j$$

(c) The third function referred to is the power function is of the form

$$E(U) = \sum p_j W_j^\delta$$

Where δ implying risk aversion is constrained by

$$0 < \delta < 1.$$

It should be noted that the logarithmic function is a special case of the power function where δ equals zero.

(d) The fourth function common in decision theory is the exponential function of the form.

$$E(U) = \sum p_j (1 - e^{-\frac{W}{C}})$$

Where C = Positive constant

e = Exponential factor

In their final draft published following the completion of the analysis of this thesis, Gordon, Paradis and Rorke substituted a different function in place of the exponential function. Under the Pratt implications an adjusted power function had more plausible appeal than the exponential function.

$$\text{i.e. } E(U) = \sum p_j (W_j + k)^\delta$$

(e) The fifth function analyzed has been designated as the logarithmic plus a constant function of the form.

$$E(U) = \sum \ln(W + k)$$

Where k = some arbitrary constant

In investigating the Pratt implications of the various utility functions, Gordon, Paradis and Rorke found intuitive support for this function.

The analysis of the functions was conducted utilizing a

statistical program of least squares regression. The regression analysis was performed using the "ECON" program developed by M.R. Norman at the Wharton School of Finance and Commerce, the University of Pennsylvania and revised by Elizabeth B. Iwan from Cornell University. The "ECON" program provides estimates of the parameters of single-equation models by the method of least squares, and includes estimates of various related statistics. The basic output utilized in this study was:

- a. Mean and standard deviation of variables.
- b. Estimated regression coefficients.
- c. Standard error of coefficient.
- d. Estimate of intercept and its standard error.
- e. R-squared coefficient.

The mean and standard deviation of variables are utilized in the general overview of data following in Chapter IV. The regression coefficients and standard errors are utilized in the comparison of the characteristics of the participant groups for specified utility functions.

(ii) Pratt Implications

Pratt proposed a system for classifying utility functions according to their degree of risk aversion.

"If the amount invested in the risky asset increases (decreases) with wealth, the investor has decreasing (increasing) absolute risk aversion. If the fraction of wealth invested in the risky asset increases (decreases) with wealth, the investor has decreasing (increasing)

relative risk aversion."²

Pratt's Classification of Utility of Wealth Functions.³

	<u>Absolute Risk Aversion</u>	<u>Relative Risk Aversion</u>
Quadratic	increasing	increasing
Logarithmic	decreasing	increasing
Power	decreasing	constant
Exponential	constant	decreasing
Logarithmic (w+k)	decreasing	increasing

(iii) Comparison of Participant Group Responses to Individual Functions.

The participant groups were compared by their regression coefficients generated from the analysis of each utility function. The analysis of the groups was achieved by applying the χ^2 approximation of the Fisher Test to the regression coefficients of the control and test group. The basis of the test is as follows:

Null Hypothesis

Ho: There is no difference between the regression coefficients of utility functions representing the responses of the control group and the regression coefficients representing the responses of the test group for alternate utility functions (and thus no difference in the utility preferences of the groups).

²M.J. Gordon, G.E. Paradis and C.H. Rorke, Experimental Evidence on Alternative Portfolio Decisions Rules, unpublished, p. 25

³Ibid., p. 25

Statistical Test⁴

$$b_i^* = \frac{\sum_{t=1}^T b_{it} / S_{it}^2}{\sum_{t=1}^T 1 / S_{it}^2} \quad Y = \sum_{t=1}^T \frac{(b_{it} - b_i^*)^2}{S_{it}^2}$$

$$Y \approx \chi^2 \quad \text{d.f.} = T-1$$

Where b_{it} = regression coefficients

S_{it} = standard error of coefficients

The R-squared coefficient was utilized to determine whether any of the alternate utility functions exhibited a high degree of predictive power for investor behavior for the groups tested. The R-squared coefficient is an estimate of the percentage of the test data which is explainable by the function being analyzed.

⁴Lawrence Fisher "Determinants of Risk Premiums" Journal of Political Economy, (Vol. 67, No. 3, 1959) p. 230.

CHAPTER IV

SUMMARY AND ANALYSIS OF DATA

The data has been segmented into two areas of analysis: the behavior of the participants and the analysis of alternative utility functions.

Behavior of the Participants

The behavior of participants was analyzed within four categories.

1. Overview of data and general relationships of data within participant groups.
2. Analysis of gamble preferences.
3. Analysis of relationship of consumption decision to wealth level.
4. Analysis of relationship of investment decision to disposable wealth level.

1. Overview and General Relationships of Data.

The data summarized in Table 4-1 has been presented as a general overview of some of the basic results of the responses. In reviewing these results, considerable care must be taken not to draw conclusions without consideration of various related factors. For example, the relationship of Mean G/W might lead to the conclusion that undergraduates invest a lower percentage of their wealth because they are more risk averse than other groups. Although the observation is

correct the cause is certainly not obvious. The mean level of wealth at the beginning of each period was generally higher for undergraduates which would yield generally smaller values of G/W if one believes that relative level of investment declines with increasing wealth. Undergraduates also tended to choose gambles with higher variances and thus higher potential rates of return which intuitively might explain lower values of G/W if one believes that the level of investment decreases with increasing variance of the gamble.

It is of interest to note that those participants who utilized borrowing did so predominantly when selected gamble two. This observation lends some support to the belief that those participants selecting gamble two did so because of their understanding of its relative superiority. As outlined earlier for any outcome resulting from an investment there is an investment in gamble two combined with borrowing/lending which yields an identical return with a smaller variance.

e.g. (i) Invest \$100,000 in gamble four

$$\text{Expected return} = 2.50(100,000) \div 2 = \$125,000$$

$$\text{Variance} = 1.0$$

(ii) By investing \$100,000 in gamble two and borrowing

\$150,000 the same expected return can be achieved with a smaller variance.

$$\begin{aligned} \text{Expected return} = & [1.50(250,000) - 150,000 + \\ & .70(250,000) - 150,000] \div 2 = \end{aligned}$$

$$[\$225,000 + \$25,000] \div 2 = \$125,000$$

$$\text{Variance} = 0.8$$

OVERVIEW OF DATA

Table 4-1

	CONTROL GROUP	TEST GROUPS		
	MBA	INVESTMENT MANAGERS	UNDER- GRADUATES	PROFESSORS
Number of Participants Contacted	17	21	30	17
Number of Participants Responding	13	9	21	9
Mean Level of Wealth At End of Ten Periods	284,000	370,000	258,000	102,000
Mean Level of Wealth At Start of Each Period (W)	174,000	174,000	198,000	131,000
Mean Level of Consumption (C)	10,600	8,500	12,100	11,600
Mean C/Mean W	.061	.049	.061	.089
Mean (W-C)	162,700	163,200	171,800	119,100
St. Deviation (W-C)	124,500	162,600	154,500	67,300
Mean Level of Investment (G)	137,000	90,000	62,000	71,000
Mean (G/W)	.764	.589	.325	.590
Mean Expected Rate of Return Based on Gambles Selected	.112	.104	.153	.145
Mean Level of Borrowing	94,500	54,500	111,500	47,500
Percent of Periods Borrowing Was Utilized	.31	.21	.08	.16
Number of Periods Borrowing Was Utilized - Gamble 2	17	15	1	14
Number of Periods Borrowing Was Utilized - Other	23	4	15	--

2. Analysis of Gamble Preferences

The responses summarized in Table 4-2 make one aspect immediately clear. In relation to their recognition of gamble 2 as a superior alternative, very significant differences amongst the groups and the Gordon study group were exhibited. Although most participants recognized and reacted to dispersion and expected value considerations, very few attempted any explicit determination of the relative desirability of the alternate gambles. The only conclusion which can be drawn from the deviation in responses between the two study groups is that some external stimulus led the Gordon, Paradis and Rorke study group to carry out calculations of the relationship of rate of return and standard deviation.

A somewhat surprising result was the high frequency with which investment managers chose gamble two, relative to other participant groups. The frequency with which investment managers selected gamble two was sufficiently high to yield statistical differences amongst the participant groups of the thesis. The frequency was not however high enough to relate statistically with the results of the Gordon, Paradis and Rorke study. One might suppose that business managers have some intuitive ability to follow wealth maximizing precepts based upon experience or resulting from survival in the market place.

To determine whether significant differences existed amongst the groups in their gamble preferences, the groups were analyzed utilizing the χ^2 test based upon the groups choice or non-choice of gamble two. A stronger test would have analyzed their

Table 4-2

GAMBLE PREFERENCES
PERCENT OF TIME CHOOSING ALTERNATE GAMBLES

GAMBLE PARTICIPANT GROUP		1	2	3	4	5
MBA's	(130)	.278 (36)	.238 (31)	.254 (33)	.215 (28)	.015 (2)
INVESTMENT MANAGERS	(90)	.067 (6)	.500 (45)	.145 (13)	.255 (23)	.033 (3)
UNDERGRADUATES	(210)	.072 (15)	.224 (47)	.357 (75)	.338 (71)	.009 (2)
PROFESSORS	(90)	.200 (18)	.312 (28)	.106 (15)	.322 (29)	0 (0)
GORDON, PARADIS AND RORKE	(372)	.027 (10)	.677 (253)	.054 (20)	.208 (78)	.029 (11)

NUMBER OF PERIODS CHOOSING ALTERNATE 2
VERSUS OTHER ALTERNATES

GAMBLE PARTICIPANT GROUP		GAMBLE 2	OTHER
MBA's		31	99
INVESTMENT MANAGERS		45	45
UNDERGRADUATES		47	103
PROFESSORS		28	62
GORDON, PARADIS AND RORKE		253	119

reactions to all gambles; however, it was the author's belief that this might have yielded biased results. Firstly, a participant's choice of gambles would be profoundly effected by his ability to discover the relative superiority of gamble two. Secondly, it appeared intuitively plausible that a participant's selection of gamble might be influenced by his level of wealth, and it has been pointed out that the mean level of wealth for the various groups differed. The details of analysis are as follows:

(a) Null Hypothesis

H₀: There is no difference in the behavior of the control group and the test group as reflected in their selection of type of gamble.

H₁: The behaviors are different.

(b) Statistical Test¹

The χ^2 test for two independent samples was chosen because the two groups are independent and the data under study are frequencies in discrete categories.

$$\chi^2 = \frac{N(AD-BC)^2}{(A+B)(C+D)(A+C)(B+D)}$$

This is a test of whether an observed breakdown of frequencies in a 2x2 contingency table could have occurred under H₀.

	Gamble No. 2	Others	Total
Control Group	A	B	A + B
Test Group	C	D	C + D
Total	A + C	B + D	N

¹Siegel, Op. Cit. p. 107

(c) Significance Level

Due to the small sample sizes involved in the study a significance level of twenty percent was chosen.

(d) Region of Rejection

The region of rejection consists of all values of which are so large that the probability associated with their occurrence is equal to or less than $\alpha = .20$. Table C in Siegel indicates a region of rejection of all $\chi > 1.64$

(e) Decision

- (i) χ^2 Investment Managers versus MBA's.
 $\chi^2_{1-2} = 14.95$
- (ii) χ^2 Undergraduates versus MBA's.
 $\chi^2_{1-3} = .032$
- (iii) χ^2 Professors versus MBA's.
 $\chi^2_{1-4} = 1.05$
- (iv) χ^2 Gordon, Paradis and Rorke versus MBA's.
 $\chi^2_{1-5} = 74.67$
- (v) χ^2 Gordon, Paradis and Rorke versus Managers.
 $\chi^2_{2-5} = 9.49$

The behavior of investment managers and MBA's in their selection of gambles was significantly different. The behavior of all participant groups was significantly different from the Gordon, Paradis and Rorke test group.

3. Analysis of Relationships of Consumption Decision and Wealth Level.

The wealth position at the start of each period and the consumption for the period were segregated into intervals of wealth

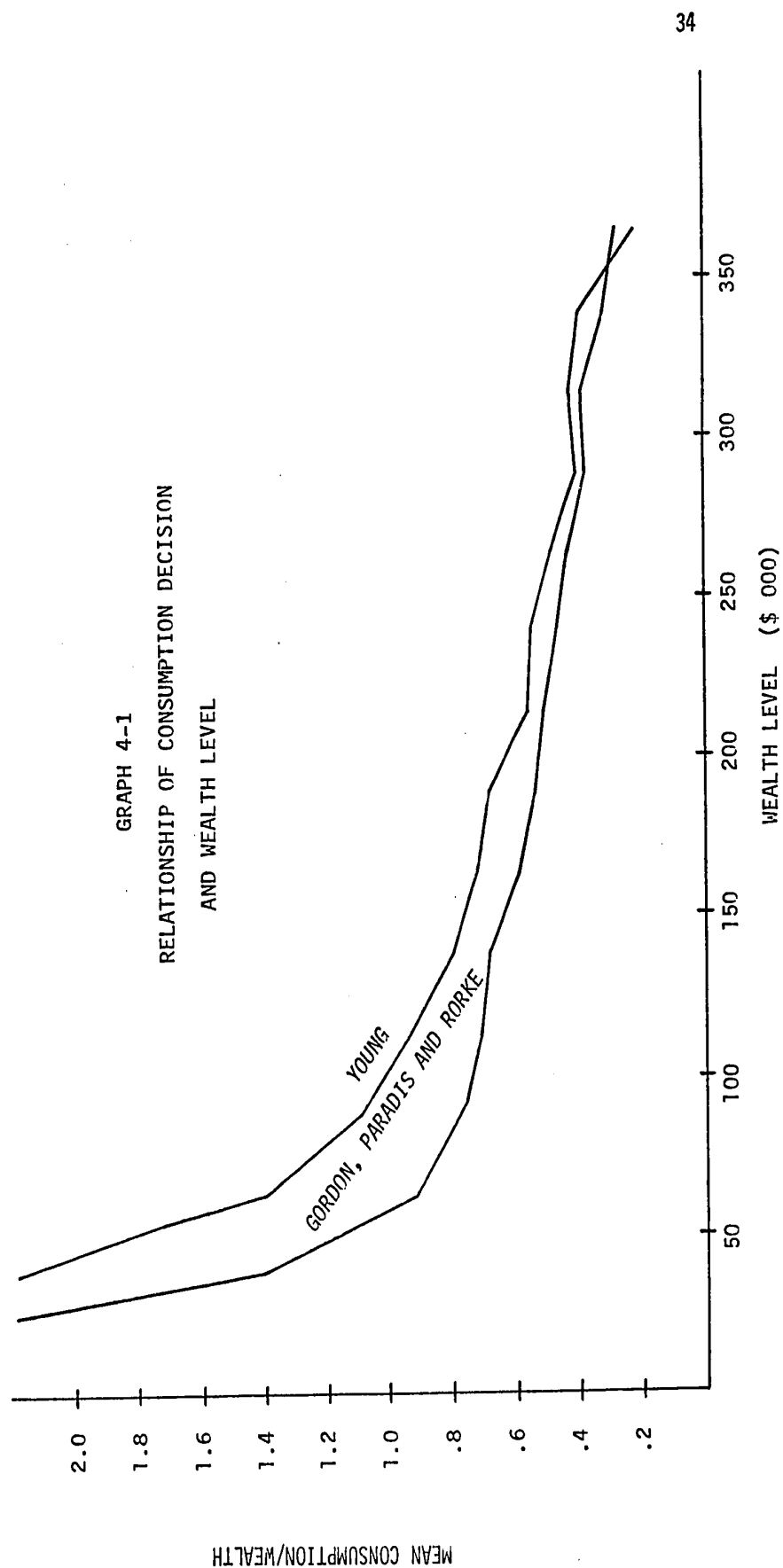
in a like manner to those outlined in the Gordon, Paradis and Rorke study. The value of consumption divided by wealth level was determined for each period and the mean of all values was determined for each interval. The results have been summarized in Graph 4-1 along with the Gordon, Paradis and Rorke study results for comparison purposes.

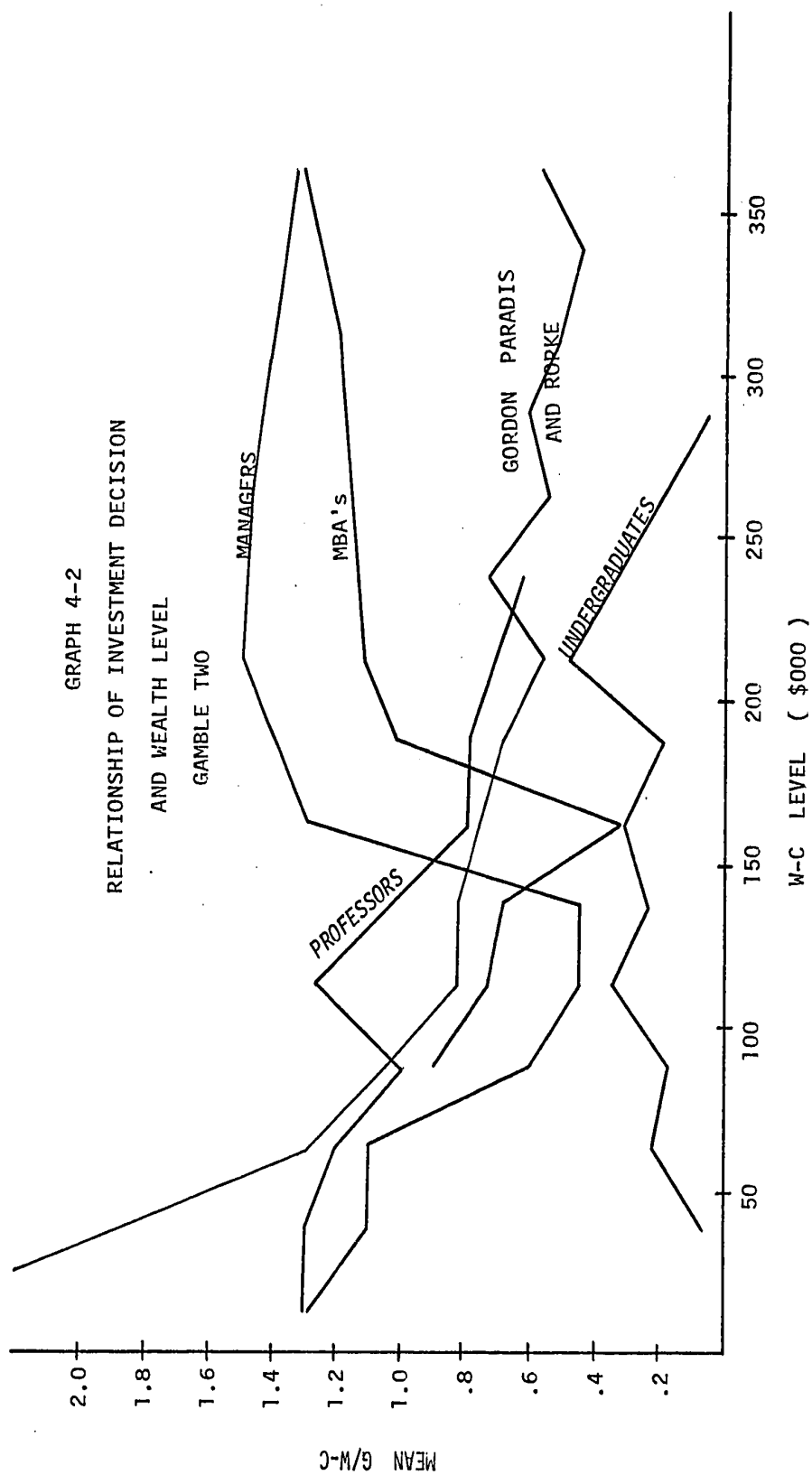
Although the two groups have a close correlation, the results are significantly different in that the Gordon, Paradis and Rorke results show an almost constantly smaller relationship. The results could reflect a difference in the test itself or in fact that the groups were drawn from different populations. It is the author's belief that both factors influenced the outcomes significantly. It is becoming increasingly evident that the participant groups do reflect different populations. The wide variations in responses between the Control group and the Gordon, Paradis and Rorke group provide strong support for the belief that the tests themselves were significantly different.

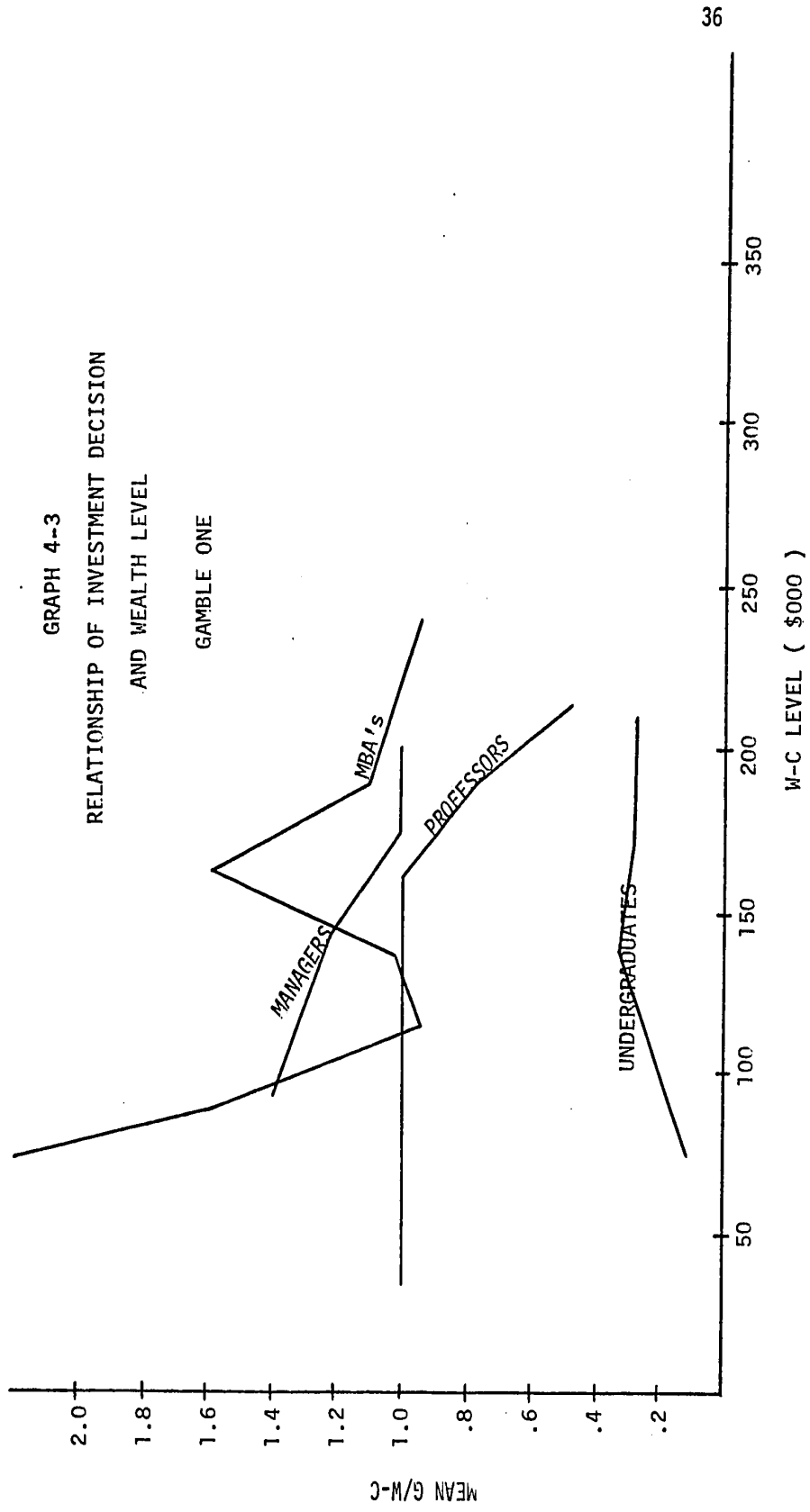
4. Analysis of Relationship of Investment Decision and Disposable Wealth Level.

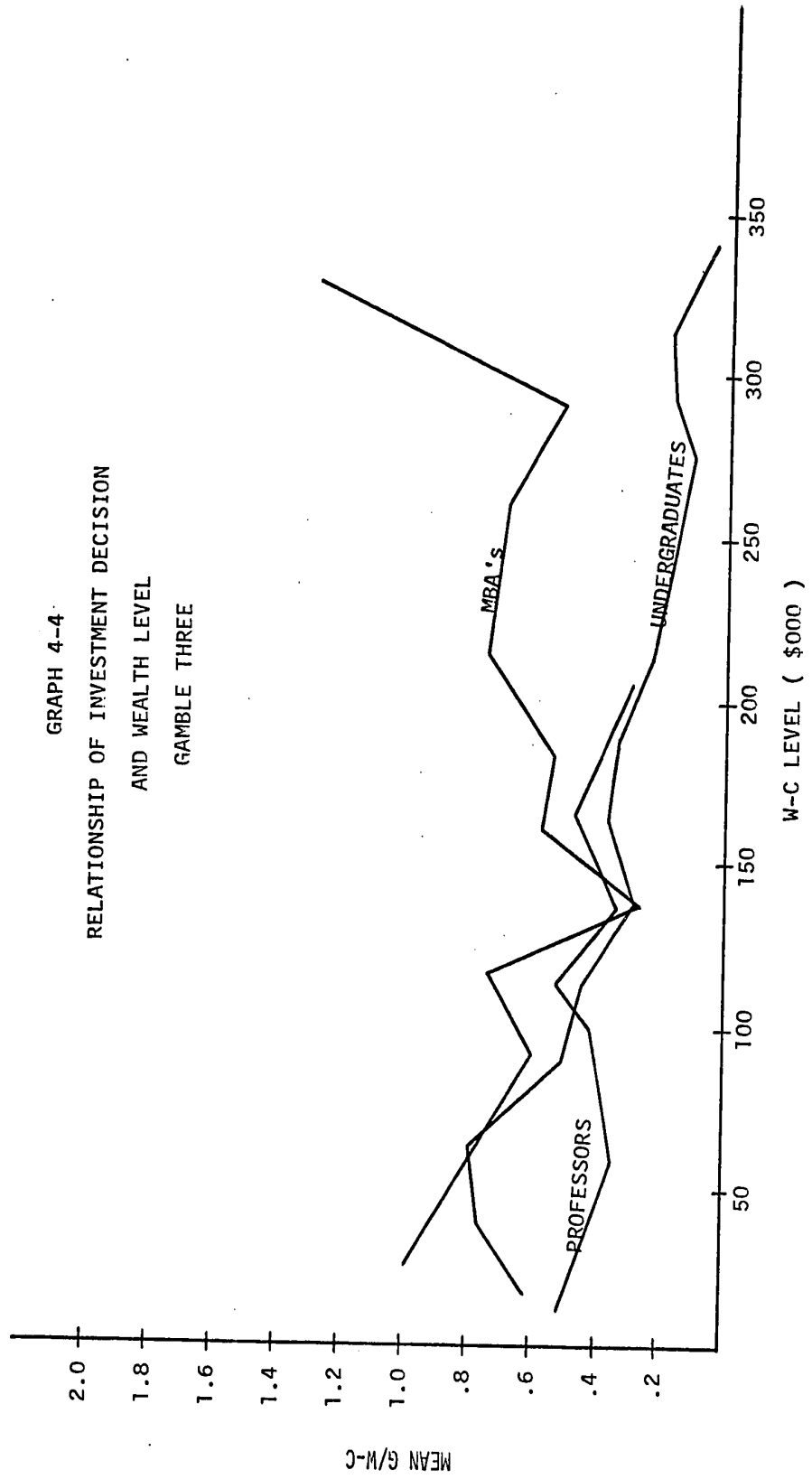
The Wealth position after consumption and the level of investment for each period were put through the same transformation as described immediately above. The results were analyzed in the series of graphs which follows. (Graph 4-2 to Graph 4-9)

A graph representing all four participant group was drawn for each gamble to remove possible effects of the selection of alternate gambles. i.e. A participant might react entirely differently in his investment decision if he selected gamble one as opposed to gamble

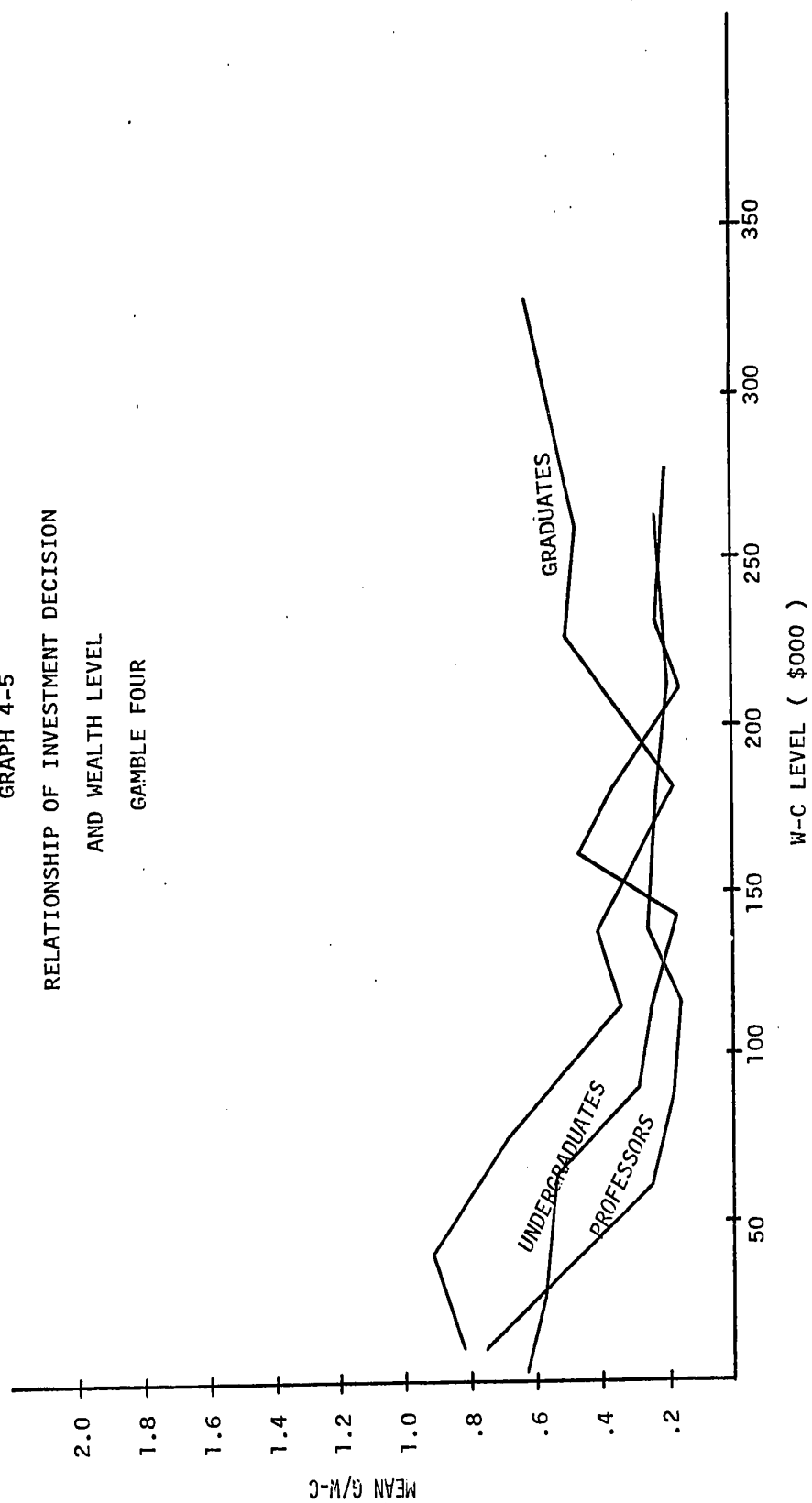


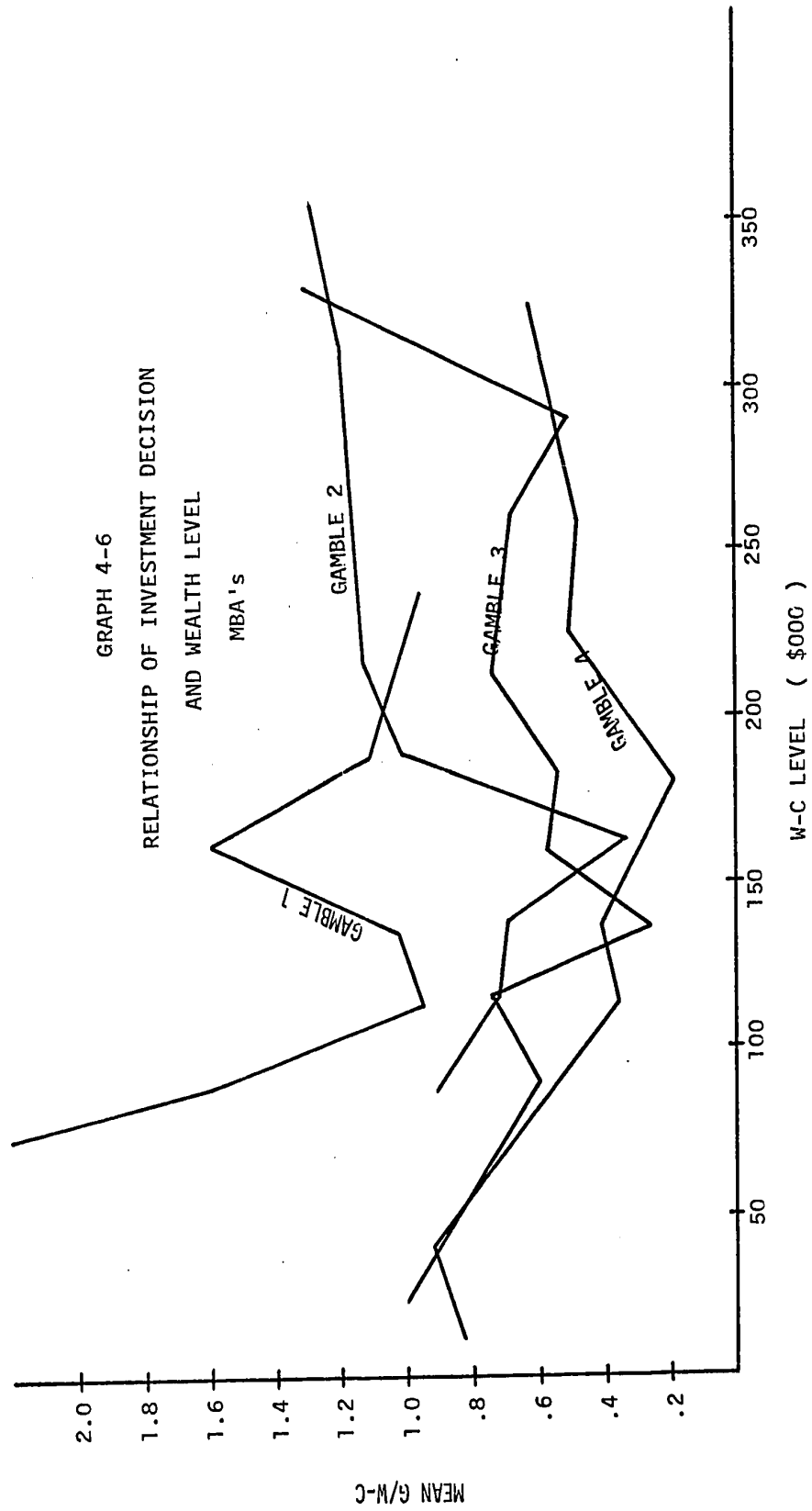


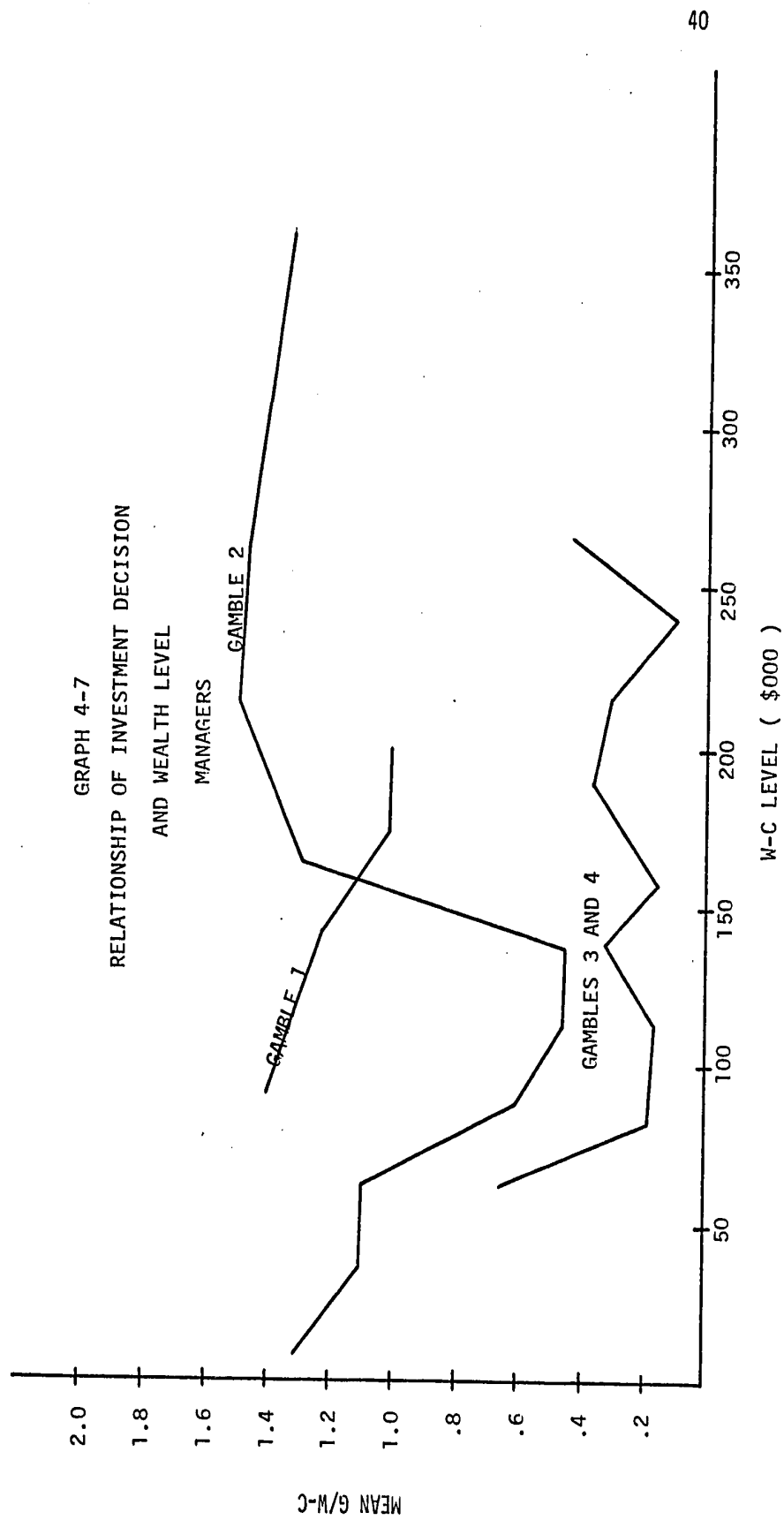




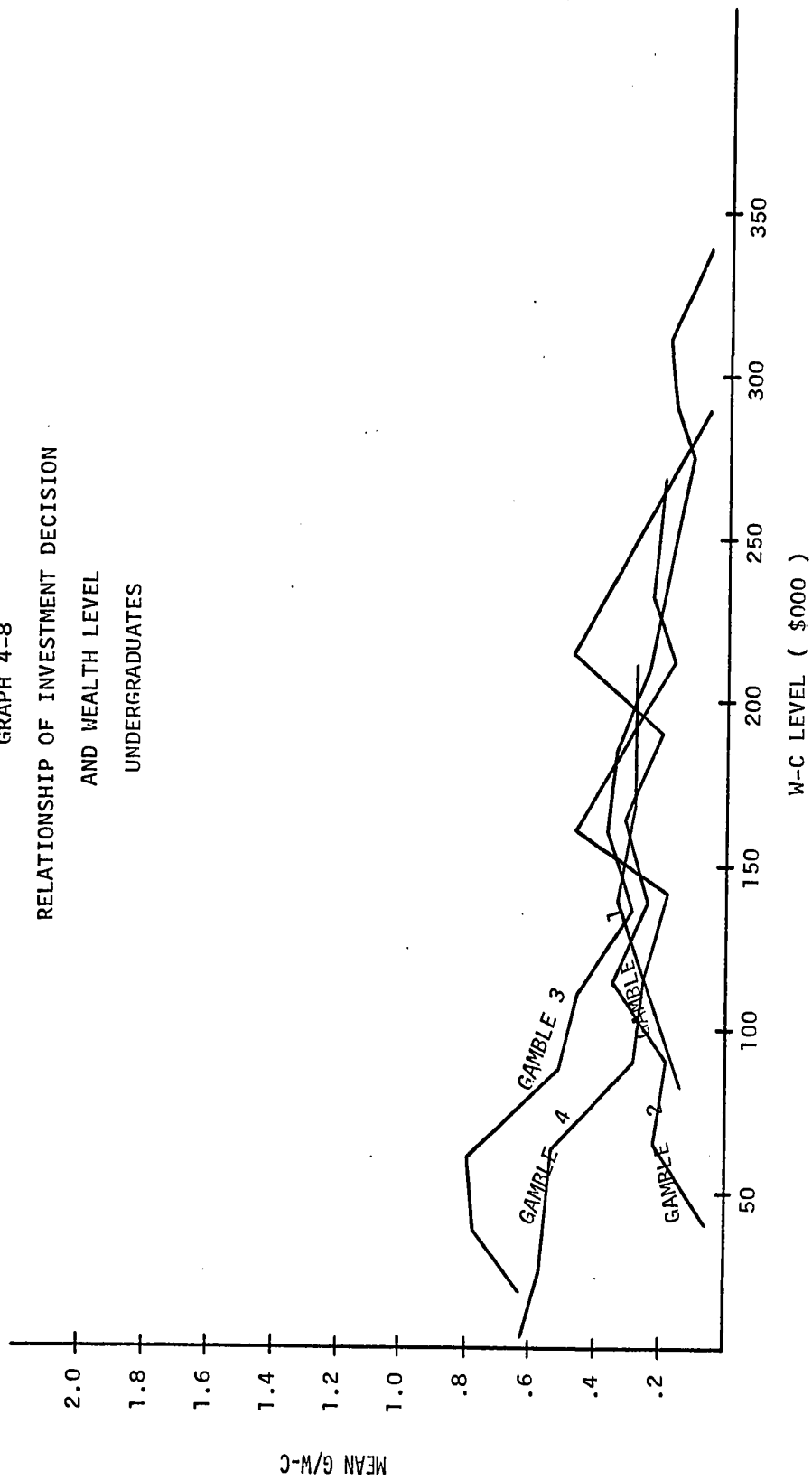
GRAPH 4-5
RELATIONSHIP OF INVESTMENT DECISION
AND WEALTH LEVEL
GAMBLE FOUR

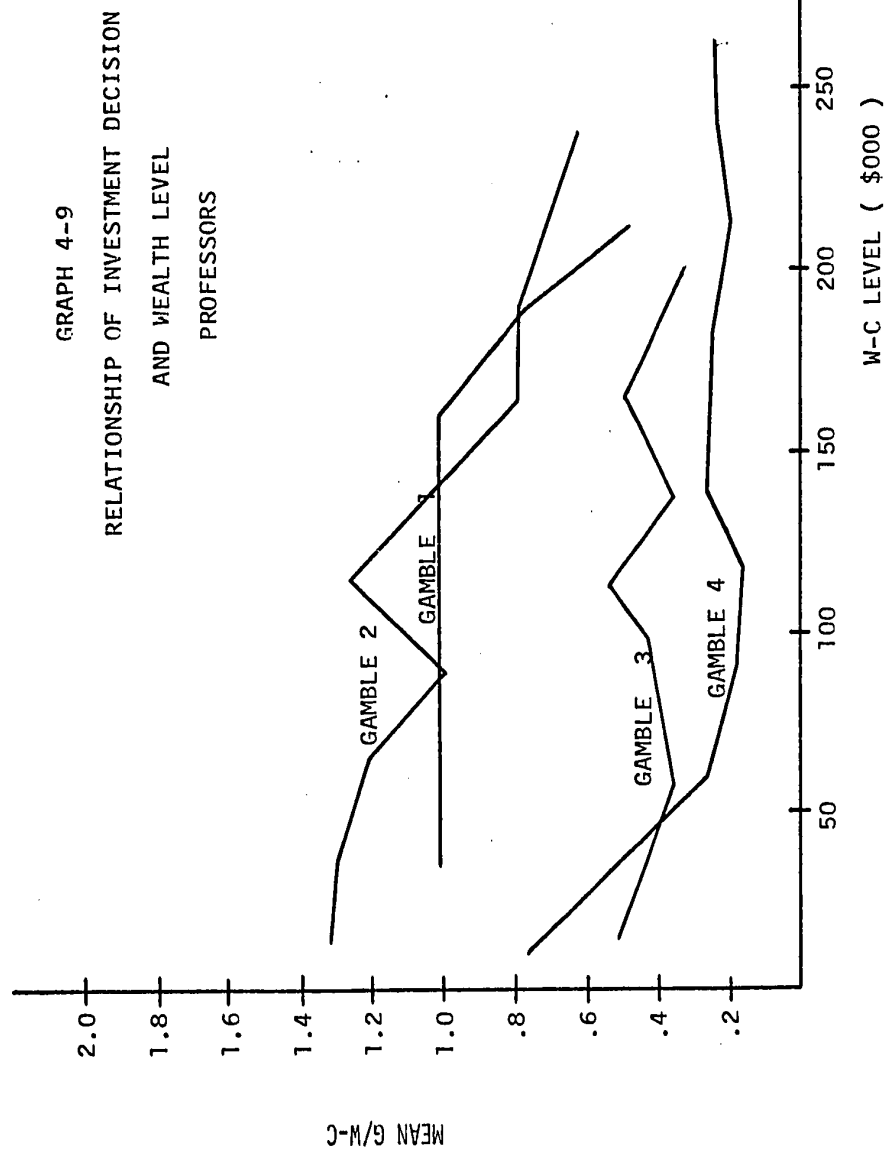






GRAPH 4-8
RELATIONSHIP OF INVESTMENT DECISION
AND WEALTH LEVEL
UNDERGRADUATES





four.

A graph representing all four gambles was drawn for each participant group to reflect these differing responses as a result of the gamble selected. The data for the groups based on responses to gamble number two, was plotted against the Gordon, Paradis and Rorke results as the only data available for comparison.

The relationships summarized in Graph 4-2 were quite dramatic in relation to those anticipated based upon the Gordon, Paradis and Rorke study group. The Gordon, Paradis and Rorke results indicate a function which is downward sloping to the right but at a decreasing rate.

The responses of Group 1 (Control group / MBA students) indicated a function which decreased to a minimum at wealth class \$150,000 to \$175,000 at which point it increased sharply to a plateau.

The responses of Group 2 (Investment Managers) had a similarly shaped function; however, the minimum point fell within the wealth classes \$100,000 to \$150,000.

The responses of Group 3 (Undergraduate students) indicated a function which was relatively constant between wealth classes \$50,000 to \$250,000.

The responses of Group 4 (Professors) indicated a function similar to the function of the Gordon, Paradis and Rorke study group.

To test the strength of the general shape of the functions representing each participant group, graphs were drawn for each

gamble and for each participant group. The results tended to support the general shapes of the functions for all groups except the investment managers where the shape of the function varied widely depending upon the gamble selected.

Based upon observation of the graphs, the following conclusions were drawn:

- (i) Undergraduates (Group 3) exhibited significantly different behavior to the other participant groups in their responses to the investment game based on the following criterion.
 - (a) slope of the function.
 - (b) relative magnitude of G/W-C versus other groups.
 - (c) lack of sensitivity of G/W-C relationship to alternate gambles.
- (ii) MBA students (Group 1) exhibited significantly different behavior to Groups 3 and 4 in their responses to the investment game based on the following criterion.
 - (a) slope and shape of the function.
 - (b) relative magnitude of G/W-C versus other groups.
- (iii) Professors (Group 4) exhibited significantly different behavior to Groups 3 and 1 as outlined above.
- (iv) Investment Managers (Group 2) exhibited significantly different behavior to Group 3 as outlined above.

Alternate Utility Function Analysis

The analysis of alternative utility functions involved attempting to fit the raw data generated from the game into the

alternate functions utilizing a process of least squares regression analysis. The framework established within the Gordon, Paradis and Rorke study was closely adhered to. The five functions (Quadratic, Bernoulli, Logarithmic, Power, Exponential and Logarithmic plus a constant) were analyzed to determine their predictive power and to determine whether significant differences in the functions were evident for the various participant groups.

1. The Quadratic Function

There are two hypotheses relating to quadratic functions. The most common of the two hypotheses is that utility is a quadratic function of an investor's wealth. With future wealth uncertain the expected value of utility is:

$$E(U) = W + \alpha/2 W^2$$

$$E(U) = \sum p_j W_j + \alpha/2 \sum p_j W_j^2$$

Where p_j = the probability that future wealth will be W_j

$$W_j = \sum G(R_j - 1) \text{ for any profitable gamble.}$$

α = risk preference parameter and constrained by $0 > \alpha > (-1/W)$

Taking the first derivative of the resulting expression with respect to G and setting it equal to zero, $E(U)$ is maximized by the investment.²

$$G^* = \frac{\bar{R} - 1}{-\alpha V} - \frac{(\bar{R} - 1)}{V} W$$

Where \bar{R} = expected value of gamble's payoff.

V = variance of the gamble

²Gordon, M.J, Paradis, and Rorke Op. Cit. p. 113

The second hypothesis states that investor behavior is described by a utility function which is quadratic in the rate of return on the investor's wealth.

$$E(U) = \sum p_j r_j + \alpha/2 \sum p_j r_j^2$$

Where p_j = the probability that the rate of return on the investment is r_j

$$r_j = \sum G(R_j - 1)/W \quad \text{for any profitable gamble.}$$

Proceeding as previously, $E(U)$ is maximized by the investment.³

$$G = -\alpha \frac{(\bar{R}-1)}{V} W$$

Assuming that all participants within the individual groups had the same value for α , the hypotheses were both tested by the regression.

$$G/Q = \beta_0 + \beta_1 (W-C)$$

Where $Q = \frac{\bar{R}-1}{V}$ for the gamble selected.

C = Consumption prior to the investment decision.

Q_1 = .210 for gamble one

Q_2 = .275 for gamble two

Q_3 = .230 for gamble three

Q_4 = .250 for gamble four

The results of the regression analysis are summarized in Table 4-3.

The R-squared coefficients for the regression analysis of the Quadratic function gave modest support for its predictive power

³Ibid., p. 113

for all the participant groups excepting that group comprised of professors. These results are of interest in that no support was found for the Quadratic function in the Gordon, Paradis and Rorke study.

2. The Bernoulli Logarithmic Function And Power Function

The Bernoulli logarithmic function states that the expected value of utility is a logarithmic function of an investor's wealth.

$$E(U) = \ln(W)$$

$$E(U) = \sum p_j \ln W_j$$

Where p_j = the probability the future wealth is W_j .

$$W_j = \sum G(R_j - 1) \quad \text{for any profitable gamble.}$$

Proceeding as before, $E(U)$ is maximized by the investment.⁴

$$G^* = \frac{(\bar{R} - 1)}{-(R_r - 1)(R_b - 1)} W$$

Where R_r and R_b are the two possible payoffs.

The Bernoulli logarithmic function is a special case of the Power function of the form.

$$E(U) = W^\delta$$

$$E(U) = \sum p_j W_j^\delta$$

Where p_j = the probability that future wealth is W_j

$$W_j = \sum G(R_j - 1) \quad \text{for any profitable gamble.}$$

δ = a risk preference parameter constrained by

$$0 < \delta < 1.0 \quad \text{for risk aversion.}$$

⁴Ibid., p. 114

Proceeding as before, $E(U)$ is maximized by the investment.

$$G^* = \frac{1 - \frac{Pb(1-Rb)}{Pr(Rr-1)}^{\frac{1}{1-\delta}}}{(1-Rb) + (Rr-1) \frac{Pb(1-Rb)}{Pr(Rr-1)}^{\frac{1}{1-\delta}}} \frac{W}{Pr(Rr-1)}^{\frac{1}{1-\delta}}$$

With $\delta = 0$, the expression is similar to that for the Logarithmic function. The Power function was tested for values of δ of 0.2, 0.4, 0.6 and 0.8 by the regression.

$$G/W = \beta_1 + \beta_2 (X)$$

$$\text{Where } X = \frac{1 - \left[\frac{Pb(1-Rb)}{Pr(Rr-1)} \right]^{\frac{1}{1-\delta}}}{(1-Rb) + (Rr-1) \left[\frac{Pb(1-Rb)}{Pr(Rr-1)} \right]^{\frac{1}{1-\delta}}}$$

	$\delta = 0.0$	$\delta = 0.2$	$\delta = 0.4$	$\delta = 0.6$	$\delta = 0.8$
X_1	.825	1.042	1.398	2.061	3.617
X_2	.667	.835	1.119	1.638	2.719
X_3	.227	.351	.465	.687	1.207
X_4	.375	.462	.593	.781	.976

The Logarithmic function was tested by setting $\delta = 0.0$.

The results are summarized in Table 4-4.

The R-squared coefficients for both the Logarithmic and Power functions showed no strength for the predictive ability of the functions for any of the groups tested. Modest support was found for the predictive power of the power function at $\delta = 0.6$ for the test group comprised of professors.

3. The Exponential Function

The Exponential function, which was not written up in the Gordon, Paradis and Rorke published paper, implies that the expected value of utility is exponentially related to an investor's wealth.

$$E(U) = 1 - e^{-\frac{W}{C}}$$

$$E(U) = \sum p_j 1 - e^{-\frac{W_j}{C}}$$

Where p_j = the probability that future wealth will be

$W_j = \sum G(R_j - 1)$ for any profitable gamble.

C = positive constant.

Proceeding as described previously, $E(U)$ is maximized by the investment.⁵

$$G^* = \frac{C}{R_b - R_r} \ln \left[\frac{1 - R_b}{R_r - 1} \right]$$

Where R_b and R_r are the two possible payoffs.

The hypothesis that the Exponential function would have a high degree of predictive power for the participant groups was tested by the regression.

$$G = \beta_0 + \beta_1 (V)$$

$$\text{Where } V = \frac{\ln \left[\frac{1 - R_b}{R_r - 1} \right]}{R_b - R_r}$$

⁵Ibid., p. 115

$$\begin{aligned}
 V_1 &= .815 \text{ for gamble one.} \\
 V_2 &= .638 \text{ for gamble two.} \\
 V_3 &= .272 \text{ for gamble three.} \\
 V_4 &= .163 \text{ for gamble four.}
 \end{aligned}$$

The results of the regression analysis are summarized in Table 4-3.

The R-squared coefficients indicate no support for the Exponential function as predictive of investor behavior for any of the participant groups. This finding is of interest in that Gordon, Paradis and Rorke found weak support for the function.

4. The Logarithmic Function of Wealth Plus a Constant

The Logarithmic function of wealth plus a constant is related to the Bernoulli function and has some intuitive appeal in view of the Pratt implications of the functions.

$$E(U) = \ln(W + k)$$

$$E(U) = p \ln(W_j + k)$$

Where k is some arbitrary constant.

Proceeding as with the Bernoulli function $E(U)$ is maximized by the investment.⁶

$$G^* = \frac{(k + W)(2 - R_b - R_r)}{2(R_r - 1)(R_b - 1)}$$

$$\frac{G}{Z} = k + W$$

$$\text{Where } Z = \frac{(2 - R_b - R_r)}{2(R_r - 1)(R_b - 1)}$$

⁶Ibid. p. 115

The hypothesis that the Logarithmic function plus a constant would have a high degree of predictive power for the participant groups was tested by the regression.

$$G/Z = \beta_0 + \beta_1 (W-C)$$

Where $Z_1 = .833$ for gamble one.

$Z_2 = .667$ for gamble two.

$Z_3 = .278$ for gamble three

$Z_4 = .167$ for gamble four.

The results of the regression analysis are summarized in Table 4-3.

The R-squared coefficients indicate strong support for the Logarithmic function of wealth plus a constant as predictive of investor behavior for all participant groups. The findings lend strength to the support found for the function by Gordon, Paradis and Rorke.

5. Comparison of Participant Group Responses to Individual Functions.

The regression coefficients of the Quadratic and Logarithmic function plus a constant were analyzed to determine whether there were significant differences between the control group and the test group for each function. Differences in the coefficients reflect differences in the characteristics of the functions and thus differences in behavior. The details of analysis are as follows:

(a) Null Hypothesis

Ho: There is no difference between the characteristics of the utility functions for the control group and the test group for each alternate function tested.

Table 4-3

ALTERNATE UTILITY FUNCTION ANALYSIS

PARTICIPANT GROUP \ FUNCTION	QUADRATIC		EXPONENTIAL		LOGARITHMIC PLUS CONSTANT	
	β_0	β_1	β_0	β_1	β_0	β_1
Gordon, Paradis and Rorke						
(i) Mean	83,210	.692	-42.54	1880.6	82126	.606
(ii) Standard error	8,365	.049				
MBA's						
(i) Mean	49,205	3.058	76.86	108.35	-184,857	3.118
(ii) Standard error	58,648	.268	24.15	43.51	47,332	.231
(iii) R ²	.4685		.0391		.8337	
Investment Managers						
(i) Mean	41,507	1.981	76.13	34.13	-175,396	2.728
(ii) Standard error	48,428	.210	28.31	54.04	30,150	.131
(iii) R ²	.5039		.0070		.8337	
Undergraduates						
(i) Mean	-172,880	2.466	78.14	-50.64	-139,412	2.142
(ii) Standard error	29,002	.126	14.34	34.08	25,061	.109
(iii) R ²	.6532		.0059		.6555	
Professors						
(i) Mean	117,339	1.493	9.57	136.90	28,368	1.117
(ii) Standard error	49,292	.361	10.57	20.30	14,380	.105
(iii) R ²	.1536		.3332		.5566	

Table 4-4

ALTERNATE UTILITY FUNCTION ANALYSIS

PARTICIPANT GROUP	FUNCTION	BERNOULLI LOGARITHMIC		POWER					
				$\alpha = .2$		$\alpha = .4$		$\alpha = .6$	
		β_0	β_1	β_0	β_1	β_0	β_1	β_0	β_1
Gordon, Paradis and Rorke									
(i)	Mean	-.106	1.815	-1.04	1.445	-1.04	1.081	-.107	.738
(ii)	Standard error								
MBA's									
(i)	Mean	.167	1.093	.168	.870	.180	.641	.196	.429
(ii)	Standard error	.125	.211	.123	.166	.121	.122	.115	.080
(iii)	R ²	.1680		.1705		.1726		.1784	
Investment Managers									
(i)	Mean	.037	1.031	-.021	.899	-.003	.656	.029	.430
(ii)	Standard error	.136	.240	.144	.201	.139	.146	.130	.094
(iii)	R ²	.1686		.1807		.1826		.1875	
Undergraduates									
(i)	Mean	.434	-.259	.435	-.199	.428	-.141	.412	-.084
(ii)	Standard error	.395	.085	.044	.074	.043	.054	.041	.036
(iii)	R ²	.0394		.0300		.0277		.0212	
Professors									
(i)	Mean	-.090	1.281	-.144	1.086	-.125	.798	.030	.345
(ii)	Standard error	.084	.146	.086	.119	.083	.086	.059	.032
(iii)	R ²	.4590		.4785		.4871		.5677	

H_1 : The characteristics of each alternate function are different for the two groups.

(b) Statistical Test⁷

The χ^2 approximation of the Fisher test was utilized as the regression coefficients as well as their standard errors were available from the data generated from the "ECON" program.

$$b_i^* = \frac{\sum_{t=1}^T b_{it} / S_{it}^2}{\sum_{t=1}^T 1 / S_{it}^2}$$

$$Y = \sum_{t=1}^T \frac{(b_{it} - b_i^*)^2}{S_{it}^2} \quad Y \approx \chi^2 \quad \text{d.f.} = T-1$$

Where b_{it} = regression coefficients

S_{it} = standard error of coefficients

The regression coefficients and standard errors are summarized in Table 4-3.

(c) Significance Level

Due to the small sample sizes involved in the study a significance level of twenty percent was chosen.

(d) Region of Rejection

The region of rejection consists of all values of χ^2 which are so large that the probability associated with their occurrence is equal to or less than $\alpha = .20$. Table C in Siegel indicates

⁷Fisher, Op. Cit., p. 230

a region of rejection of all $\chi^2 > 1.64$

(e) Decision

(i) Quadratic Function

The Gordon, Paradis and Rorke group was utilized as the control group in this case.

1. MBA's versus Gordon, Paradis and Rorke

$$\chi^2_{\beta_0} = .37 \quad \chi^2_{\beta_1} = 66.51$$

2. Investment Managers versus Gordon, Paradis and Rorke

$$\chi^2_{\beta_0} = .72 \quad \chi^2_{\beta_1} = 31.90$$

3. Undergraduates versus Gordon, Paradis and Rorke

$$\chi^2_{\beta_0} = 71.98 \quad \chi^2_{\beta_1} = 173.13$$

4. Professors versus Gordon, Paradis and Rorke

$$\chi^2_{\beta_0} = .55 \quad \chi^2_{\beta_1} = 5.84$$

The characteristics of the utility functions of the participant groups were difference from the control group of Gordon, Paradis and Rorke.

(ii) Logarithmic plus a Constant Function

The MBA group was utilized as the control group.

1. Investment Managers versus MBA's

$$\chi^2_{\beta_0} = .03 \quad \chi^2_{\beta_1} = 2.15$$

2. Undergraduates versus MBA's

$$\chi^2_{\beta_0} = .720 \quad \chi^2_{\beta_1} = 14.58$$

3. Professors versus MBA's

$$\chi^2_{\beta_0} = 18.58 \quad \chi^2_{\beta_1} = 62.17$$

The characteristics of the utility functions of the test groups were different from the control group; however, investment managers exhibited regression coefficients similar to the MBA's.

CHAPTER V

CONCLUSIONS

The primary purpose of the thesis was to provide a pilot study for further research into the implications of experimental research in the formulation and testing of alternate portfolio decisions rules. The thesis was a direct but independent extension of the Gordon, Paradis and Rorke paper cited earlier. The function of the study was to provide information on the effects of the addition of distinct participant groups into the framework of the Gordon study.

The following represents a summation and conclusion of the results as they relate to the purpose of the thesis outlined in Chapter I.

1. In replicating the Gordon, Paradis and Rorke study the basic findings relating to alternate utility function analysis was supported. The strength of the predictive power of the Logarithmic of Wealth plus a constant function was supported lending credence to the universality of its use in predicting investor behavior.

Several deviations were observed in the behavior of the participant groups as opposed to the Gordon, Paradis and Rorke group. These differences were reflected in the choices of gamble selected and the general character of the relationship $G/W-C$ versus $W-C$. The participants of the Gordon, Paradis and Rorke group selected gamble two a significantly higher percentage of the time than the

groups in this thesis. The general character of the relationship of investment decision to wealth level was not consistent with the findings of Gordon, Paradis and Rorke.

2. (i) The hypothesis that the behavior of the test groups and the control group would be significantly different in their responses to the investment game was supported. It has been shown that the responses of the investment managers in their choices of gamble differed significantly from the MBA students.

It has also been shown that the general nature of the relationship of investment decision to wealth level varied significantly amongst the participant groups. Undergraduates exhibited unique relationship in comparison to all other groups. MBA students exhibited a unique relationship in comparison to undergraduates and professors. Conversely professors exhibited a unique relationship in comparison to undergraduates and MBA's. Investment managers exhibited a unique relationship in comparison to undergraduates.

2. (ii) The hypothesis that the utility functions of the participant groups would be significantly different was supported. In the analysis of regression coefficients of the Logarithmic plus a constant utility function undergraduates and professors were found to exhibit significantly different behavior in comparison to MBA's. Investment managers did not exhibit significantly different behavior. These conclusions lend support to the graphical presentations which indicated similar behavior in investment decisions between the investment managers and MBA's.

The results of the thesis, recognizing the limitations on statistical significance, might be generalized as follows. It is clear that significant differences in behavior exist between unique participant groups in relation to their investment decisions at distinct levels of wealth. Regardless of these differences, the utility function based upon Logarithmic of wealth plus a constant precepts provided the best description of actual behavior for all participant groups of the functions tested.

BIBLIOGRAPHY

- ARROW, K.J. Aspects of the Theory of Risk Taking. Helsinki: Yrjö Jahnssonin Saatio, 1965.
- BERNOULLI, Daniel. Exposition of a New Theory on the Measurement of Risk. Translated by Louise Sommer. Gregg Press, 1967.
- BORSCH, K.H. The Economics of Uncertainty. Princeton: Princeton University Press, 1968.
- COOMBS, C.H. and Pruitt, D.G. "Components of Risk in Decision Making," Journal of Experimental Psychology, 1960, 60, p. 265-277.
- EDWARDS, Ward. "Behavioral Decision Theory," Annual Review of Psychology, 1961, p. 473-498.
- , "Theory of Decision Making." Psychological Bulletin, 1964, Vol. 61, No. 3, p. 22-233.
- FISHER, Irving. The Nature of Capital and Income. New York: The Macmillan Company, 1923.
- FRIEDMAN, M. and Savage, L.J. "The Utility Analysis of Choices Involving Risk," Journal of Political Economy, Vol. 56, No. 4, August 1948, p. 279-304.
- GORDON, M.J., Paradis, G.E., and Rorke C.H. "Experimental Evidence on Alternative Portfolio Decision Rules," The American Economic Review, Spring 1972, p. 107-118.
- HESLER, D.D. and Tobin, J. eds. Risk Aversion and Portfolio Choice, ("Cowles Foundation Monograph No. 19") New York: John Wiley and Sons Inc., 1967.
- KOGAN, Nathan and Wallch, Michael A. Risk Taking: A Study in Cognition and Personality. New York: Holt, Rinehart and Winston, 1964.
- MARKOWITZ, H.M. Portfolio Selection: Efficient Diversification of Investments. ("Cowles Foundation Monograph No. 16") New York: John Wiley and Sons, 1959.
- MOSTELLER, Frederick and Nogee, Philip. "An Experimental Measurement of Utility." The Journal of Political Economy, 1951, Vol. 59, No. 5, p. 371-404.

- MYERS, J. L., Reilly, R.E. and Taub, H.A. "Differential Cost, Gain, and Relative Frequency of Reward in a Sequential Choice Situation," Journal of Experimental Psychology, 1961, Vol. 62, No. 4, p. 351-366.
- MYERS, J.L., Suydam, M.M., and Gambino, B. "Contingent Gains and Losses in a Risk Taking Situation," Journal of Mathematical Psychology, 1965, 2, p. 363-370.
- POLLATSEK, A. and Tversky, A. "A Theory of Risk," Journal of Mathematical Psychology, 1970. Vol. 7, p. 540-553.
- PRATT, John W. "Risk Aversion in the Small and in the Large," Econometrica, 1964, Vol. 32, No. 1-2, p. 122-136.
- RAIFFA, Howard. Decision Analysis: Introductory Lectures on Choices Under Uncertainty. Reading, Mass.: Addison, Wesley Publishing Co., 1968.
- RAPOPORT, A. and Jones, L.V. "Gambling Behavior in Two Outcome Multi-stage Betting Games," Journal of Mathematical Psychology, 1970, Vol. 7, p. 163-187.
- RIM, Y. "Personality and Group Decisions Involving Risk," Psychological Record, Vol. 14, No. 1, January 1964, p. 37-45.
- _____, "Social Attitudes and Risk Taking," Human Relations, Vol. 17, No. 3, 1964, p. 259-265.
- SCODEL, A., Ratoosh, P. and Minas, J.S. "Some Personality Correlates of Decision Making under Conditions of Risk," Behavioral Science, Vol. 3-4, 1958-59, p. 19-28.
- SHARPE, William F. Risk Aversion in Stock Market: Some Empirical Evidence. : The Rand Corporation, March 1965.
- _____, Portfolio Theory and Capital Markets. New York: McGraw Hill Book Co., 1970.
- SLOVIC, Paul. "Assessment of Risk Taking Behavior," Psychological Bulletin, 1964, Vol. 61, No. 3, p. 220-233.
- _____, "Real vs. Hypothetical Payoffs in Choices of Gambles." Journal of Experimental Psychology, 1969, Vol. 80, p. 434-437.
- STARBUCK, W.H. and Bass, F.M. "An Experimental Study of Risk Taking and the Value of Information in a New Product Context," The Journal of Business of the University of Chicago, Vol. 40, No. 2, April 1967.

TAUB, H.A. and Myers, J.L. "Differential Monetary Gains in a Two-Choice Situation," Journal of Experimental Psychology, Vol. 61, No. 2, p. 157-162.

VAN NEUMANN, J. and Morgenstern, O. Theory of Games and Economic Behavior, Princeton, N.J.: Princeton University Press, 1944.

APPENDIX A - INVESTMENT GAME

Faculty of Business Administration
And Commerce
The University of Alberta
Edmonton 7, Canada

March, 1971

Dear Sir:

I would like to request your assistance for a study which I am undertaking about the characteristics of professional investment decisions. The study is a portion of a thesis for a Master of Business Administration degree at the University of Alberta.

In an attempt to obtain information on the nature of investment decisions, I have devised an investment game which I hope you will participate in. In order to draw meaningful conclusions, I have also given the game to a group of students to determine how their decisions differ from those of professional investors.

The object of the study is not to evaluate the results on the basis of winning or losing, but rather to evaluate the results in terms of how the types of decisions of the two groups differ. Obviously, the game is too naive to measure any type of performance.

When the results have been obtained and tabulated, I will forward a copy to all the participants. I hope you will find the game and the results interesting and possibly useful. One reason for making the game simple was to minimize the amount of your time which is required. Should you find it too time consuming, please return the portion completed as it will still be of use to the study. I would only ask that you play the game honestly and attempt to measure your decisions as if real money was involved.

Space has been provided at the end of the game for any comments which you might have on your strategy in playing the game.

As I have a limited number of participants, your co-operation in this study will be greatly appreciated. Please complete the game and return it at your convenience, although I would like to have the results no later than March 30, 1971. I have enclosed an anonymous self-addressed envelope for your use.

Thank you in advance for your participation.

Yours sincerely,

Ralph B. Young

Encl:

P.S. Should you have any problems, please feel free to contact me at 432-5778 during the day.

The Game - Instructions and Information

The game involves making decisions over eleven imaginary years. During this period you are expected to make a series of decisions about spending and investing. The object is to try to achieve a desirable balance between your aspired living style and a financial position through the accumulation of wealth. The following points should outline how to participate in the game:

1. You start the game with an imaginary \$150,000. This is the only income you will receive other than possible earnings from investments or possible welfare payments.

2. Decisions are made for one period at a time. At the beginning of the first year, you must first decide how much of your \$150,000 to consume. You have three choices:

- (i) \$5,000 consumption - a modest living
- (ii) \$10,000 consumption - a comfortable living
- (iii) \$20,000 consumption - an affluent living

3. The remainder of your initial \$150,000 can be either held in cash, or committed to any one of the following investment alternatives.

Investment No.	Outcomes
1.	even chance of winning either \$.80 or \$1.30 on each dollar invested.
2.	even chance of winning either \$.70 or \$1.50 on each dollar invested.
3.	even chance of winning either \$.40 or \$1.90 on each dollar invested.
4.	even chance of winning either 0 or \$2.50 on each dollar invested.
5.	one chance in 200 of receiving \$100.00 or 199 chances out of 200 of receiving nothing on each dollar invested.

The alternatives are listed in terms of annual return per dollar played. For example, if you Invest \$50,000 in investment no. 1, your investment will either grow to $1.30 \times 50,000 = 65,000$ or shrink to $.80 \times 50,000 = 40,000$ by the end of the year. Following the first year the types of decisions remain the same, but your wealth becomes the wealth of the preceding year.

4. Once you have decided how much to invest and in which alternative you would like to invest, the outcome can be determined in the following manner:

(i) for any investment in nos. 1, 2, 3 or 4 lift off one of the tabs under the title "outcomes". If the letter "R" appears, your return is the higher of the two possibilities. If "B" appears your investment has been relatively unsuccessful and your return is the lower of the two possibilities.

(ii) for any investment in no. 5, pick a number between 0 and 200, and write it beneath the tab entitled "Lottery". Then lift the tab and if the numbers correspond your return is \$100 per dollar invested. If the numbers don't correspond you lose the whole investment.

5. You can invest more money than you presently have, by borrowing at an interest free rate. This is accomplished by simply writing the amount you would like to borrow in the appropriate blank and repaying it as soon as your investment outcome has been determined.

There is one limit to borrowing: you must be sure that you do not go into debt by not being able to repay the loan. We determine the maximum amount that can be borrowed for any particular

investment, you can calculate the maximum loan

$$\text{Maximum loan} = \frac{\text{Maximum wealth available for investment} \times \text{the lower value of the two possible outcomes}}{1 - \text{the lower value of the two possible outcomes}}$$

For example, suppose that in year 1, you consumed \$10,000; you would then have a maximum of \$140,000 left for investment. Suppose you wanted to invest in investment 3, but felt that you might wish to invest more than \$140,000. The maximum loan which you could obtain would be:

$$\begin{aligned} \text{Max. loan} &= \$140,000 \times .40 \div (1-.40) \\ &= \$ 56,000 \div .60 \\ &= \$ 93,333. \end{aligned}$$

6. When you have completed the play for one year, you can identify your wealth at the beginning of the next year by summing the values on the calculation sheet using the signs which are indicated. See the example calculation sheet provided.

7. It is apparent that depending on your decisions and your luck, you could become very wealthy or possibly very poor. As in the real world this game contains some special provisions for the poor.

- (i) if your wealth falls below \$5,000 but above \$3,300; you may get along by consuming only \$3,000 in that period.
- (ii) if your wealth falls below \$3,300; you are given a welfare payment to bring you up to \$3,300. Of this you must consume \$3,000 and have \$300 for possible investment.

8. The number of periods involved in the game is arbitrary; thus you should play the game as though it might continue beyond the eleventh period.

The Game - Sample Calculation

Period I

1.	Initial Wealth	(N)		150,000
2.	Welfare Payment			
	(0 to 3300)		+	<u>0</u>
3.	Consumption			
	(3,000, 5,000, 10,000, 20,000)		-	<u>10,000</u>
4.	Investment Number			
	(1, 2, 3, 4, 5, Nil)	<u>3</u>		
5.	Borrow			
	Max. Loan = $\frac{N \times B}{T - B}$		+	<u>NIL</u>
6.	Size of Investment		-	<u>50,000</u>
7.	Investment Outcome			
	Outcome	Lottery		
	<u>B</u> R	194	+	<u>20,000</u>
	.4 x 50,000 = 20,000			
8.	Return Amount Borrowed		-	<u>NIL</u>
9.	New Wealth	(N)	=	<u>110,000</u>
	(Carry Fwd. to Current Wealth Next Period)			

The Game - Calculation Sheet

Period I

1.	Initial Wealth	(N)		150,000
2.	Welfare Payment	(0 to 3300)	+	_____
3.	Consumption	(3,000 5,000 10,000 20,000)	-	_____
4.	Investment Number	(1, 2, 3, 4, 5, Nil)		_____
5.	Borrow			
	Max. Loan =	$\frac{N \times B}{1 - B}$	+	_____
6.	Size of Investment		-	_____
7.	Investment Outcome			
	(note: remove only one tab per investment)			
	Outcome	Lottery		
			+	_____
8.	Return Amount Borrowed		-	_____

9.	New Wealth	(N)	=	_____
	(Carry Fwd. to Current Wealth			_____
	Next Period)			_____

The Game - Calculation Sheet

Period 2

1.	Current Wealth	(N)	+	_____
2.	Welfare Payment			
	(0 to 3300)		+	_____
3.	Consumption			
	(3,000 5,000 10,000 20,000)		-	_____
4.	Investment Number			
	(1, 2, 3, 4, 5, Nil)	_____		
5.	Borrow			
	Max. loan = $\frac{N \times B}{1 - B}$		+	_____
6.	Size of Investment		-	_____
7.	Investment Outcome			
	(note: remove only one tab per investment)			
	Outcome	Lottery		
			+	_____
8.	Return Amount Borrowed		-	_____
9.	New Wealth	(N)		_____
	(Carry Fwd. to Current Wealth		=	_____
	Next Period)			=====

The Game - Calculation Sheet

Period 3

1.	Current Wealth	(N)	+	_____
2.	Welfare Payment			
	(0 to 3300)		+	_____
3.	Consumption			
	(3,000 5,000 10,000 20,000)		-	_____
4.	Investment Number			
	(1, 2, 3, 4, 5, Nil)	_____		
5.	Borrow			
	Max. Loan = $\frac{N \times B}{1-B}$		+	_____
6.	Size of Invesmtnet		-	_____
7.	Investment Outcome			
	(note: remove only one tab per investment)			
	Outcome	Lottery		
			+	_____
8.	Return Amount Borrowed		-	_____
9.	New Wealth	(N)		
	(Carry Fwd. to Current Wealth		=	_____
	Next Period)			_____

The Game - Calculation Sheet

Period 4

1.	Current Wealth	(N)	+	_____
2.	Welfare Payment (0 to 3300)		+	_____
3.	Consumption (3,000 5,000 10,000 20,000)		-	_____
4.	Investment Number (1, 2, 3, 4, 5, Nil)	_____		
5.	Borrow Max. Loan = $\frac{N \times B}{1 - B}$		+	_____
6.	Size of Investment		-	_____
7.	Investment Outcome (note: remove only one tab per investment)			
	Outcome	Lottery	+	_____
8.	Return Amount Borrowed		-	_____
9.	New Wealth	(N)	=	_____
	(Carry Fwd. to Current Wealth Next Period)			_____

The Game - Calculation Sheet

Period 5

1.	Current Wealth	(N)	+	_____
2.	Welfare Payment		+	_____
	(0 to 3300)			
3.	Consumption		-	_____
	(3,000 5,000 10,000 20,000)			
4.	Investment Number			
	(1, 2, 3, 4, 5, Nil)	_____		
5.	Borrow		+	_____
	Max. Loan = $\frac{N \times B}{1-B}$			
6.	Size of Investment		-	_____
7.	Investment Outcome			
	(note: remove only one tab per investment)			
	Outcome	Lottery	+	_____
8.	Return Amount Borrowed		-	_____
9.	New Wealth	(N)	=	_____
	(Carry Fwd. to Current Wealth			_____
	Next Period)			_____

The Game - Calculation Sheet

Period 6

1.	Current Wealth	(N)	+	_____
2.	Welfare Payment		+	_____
	(0 to 3300)			
3.	Consumption		-	_____
	(3,000 5,000 10,000 20,000)			
4.	Investment Number			_____
	(1, 2, 3, 4, 5, Nil)			
5.	Borrow		+	_____
	Max. Loan = $\frac{N \times B}{1-B}$			
6.	Size of Investment		-	_____
7.	Investment Outcome			
	(note: remove only one tab per investment)			
	Outcome	Lottery	+	_____
8.	Return Amount Borrowed		-	_____

9.	New Wealth	(N)	=	_____
	(Carry Fwd. to Current Wealth			_____
	Next Period)			_____

The Game - Calculation Sheet

Period 7

1.	Current Wealth	(N)	+	_____
2.	Welfare Payment			
	(0 to 3300)		+	_____
3.	Consumption			
	(3,000 5,000 10,000 20,000)		-	_____
4.	Investment Number			
	(1, 2, 3, 4, 5, Nil)	_____		
5.	Borrow			
	Max. Loan = $\frac{N \times B}{T - B}$		+	_____
6.	Size of Investment		-	_____
7.	Investment Outcome			
	(note: remove only one tab per investment)			
	Outcome	Lottery		
			+	_____
8.	Return Amount Borrowed		-	_____
9.	New Wealth	(N)		
	(Carry Fwd. to Current Wealth		=	_____
	Next Period)			_____

The Game - Calculation Sheet

Period 8

1.	Current Wealth	(N)	+	_____
2.	Welfare Payment			
	(0 to 3300)		+	_____
3.	Consumption			
	(3,000 5,000 10,000 20,000)		-	_____
4.	Investment Number			
	(1, 2, 3, 4, 5, Nil)	_____		
5.	Borrow			
	Max. Loan = $\frac{Nx B}{1-B}$		+	_____
6.	Size of Investment		-	_____
7.	Investment Outcome			
	(note: remove only one tab per investment)			
	Outcome	Lottery		
			+	_____
8.	Return Amount Borrowed		-	_____
9.	New Wealth	(N)		
	(Carry Fwd. to Current Wealth		=	_____
	Next Period)			_____

The Game - Calculation Sheet

Period 9

- | | | | | |
|----|--|---------|---|-------|
| 1. | Current Wealth | (N) | + | _____ |
| 2. | Welfare Payment | | | |
| | (0 to 3300) | | + | _____ |
| 3. | Consumption | | | |
| | (3,000 5,000 10,000 20,000) | | - | _____ |
| 4. | Investment Number | | | |
| | (1, 2, 3, 4, 5, Nil) | | | |
| 5. | Borrow | | | |
| | Max. Loan = $\frac{Nx B}{1-B}$ | | + | _____ |
| 6. | Size of Investment | | - | _____ |
| 7. | Investment Outcome | | | |
| | (note: remove only one tab per investment) | | | |
| | Outcome | Lottery | | |
| | | | + | _____ |
| 8. | Return Amount Borrowed | | - | _____ |
| 9. | New Wealth | (N) | | |
| | (Carry Fwd. to Current Wealth | | = | _____ |
| | Next Period) | | | _____ |

The Game - Calculation Sheet

Period 10

1.	Current Wealth	(N)	+	_____
2.	Welfare Payment			
	(0 to 3300)		+	_____
3.	Consumption			
	(3,000 5,000 10,000 20,000)		-	_____
4.	Investment Number			
	(1, 2, 3, 4, 5, Nil)	_____		
5.	Borrow			
	Max. Loan = $\frac{N \times B}{1 - B}$		+	_____
6.	Size of Investment		-	_____
7.	Investment Outcome			
	(note: remove only one tab per investment)			
	Outcome	Lottery		
			+	_____
8.	Return Amount Borrowed		-	_____
9.	New Wealth	(N)		
	(Carry Fwd. to Current Wealth		=	_____
	Next Period)			_____

The Game - Calculation Sheet

Period 11

1.	Current Wealth	(N)	+	_____
2.	Welfare Payment			
	(0 to 3300)		+	_____
3.	Consumption			
	(3,000 5,000 10,000 20,000)		-	_____
4.	Investment Number			
	(1, 2, 3, 4, 5, Nil)	_____		
5.	Borrow			
	Max. Loan = $\frac{N \times B}{1-B}$		+	_____
6.	Size of Investment		-	_____
7.	Investment Outcome			
	(note: remove only one tab per investment)			
	Outcome	Lottery		
			+	_____
8.	Return Amount Borrowed		-	_____
9.	New Wealth	(N)		
	(Carry Fwd. to Current Wealth		=	_____
	Next Period)			_____

COMMENTS

(I would appreciate any comments which you might have concerning your strategy during the game.)