MINT 709 Capstone Project

Spectrum Sensing With Energy Detector In Fading Channels With Impulsive Noise

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# ABSTRACT

The noncoherent energy detector (ED), an algorithm that detects a primary signal based on the sensed energy, is one of the simplest devices used in cognitive radio (CR) spectrum sensing. Although the sensing performance of the ED has been extensively analysed in the literature, almost all of the research studies model the noise at the receiver as Gaussian which might not be a valid assumption when the noise exhibits impulsive behavior. Additionally, multipath fading, an inherent phenomenon in wireless propagation, makes the spectrum sensing task more difficult. Motivated by these scenarios, spectrum sensing performance with a single CR deploying the ED is considered in this study under multipath fading and impulsive noise environments. The primary user (PU) to CR channel is modeled as Rayleigh faded, the additive noise at the receiver is modeled as Laplacian, and the sensing performance of the ED is characterized. The ED performance is found to deteriorate with the increase in severity of the Laplacian noise. To mitigate the problem, use of multiple antennas is considered and found to yield a significant performance boost. Further, to obtain possible performance gains and to address the hidden terminal problem in spectrum sensing, a number of cooperative CRs is considered for joint detection of the PU signal. Interestingly, cooperation yields remarkable performance gains even under the aforementioned scenarios. Numerical (simulation) results are presented and discussed to yield valuable insights.

List of Abbreviations

AWGN	Additive white Gaussian noise
BS	Base station
CR	Cognitive radio
CDF	Cumulative distribution function
CSI	Channel state information
dB	Decibel
DTV	Digital television
ED	Energy detector
FCC	Federal communications commission
HF	High frequency
LOS	Line of sight
MRC	Maximal ratio combiner
MS	Mobile station
NLOS	Non-line-of-sight
PU	Primary user
PDF	Probability density function
RF	Radio frequency
ROC	Receiver operating characteristics
SNR	Signal to noise ratio
SH	Spectrum holes
SC	Selection combiner
SLC	Square law combiner
SLS	Square law selection

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List of Symbols

μ	Average value of Laplacian noise
γ	SNR
T	Integration time
n	Noise
K	Number of cognitive radios
P_d	Probability of detection in spectrum sensing
P_f	Probability of false alarm in spectrum sensing
Pe	Probability of error in spectrum sensing
Q_d	Probability of detection in cooperative spectrum Sensing
Q_f	Probability of false alarm in cooperative spectrum Sensing
Q_e	Probability of error in cooperative spectrum Sensing
h	Rayleigh fading channel gain
Pr	Received SNR
Y	Decision variable
H_1	Signal Present
H_0	Signal Absent
b	Scaling parameter
σ	Noise Variance
u	Time bandwidth product
i.i.d.	Independent and identically distributed

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Chapter 1

Introduction

Wireless communication technology has undergone enormous growth recently. The demand for data rate is exponentially growing, while the demand for data fidelity is unprecedented. As forecast by Cisco, the global mobile data traffic demand will exceed 11 exabytes per month by 2017 [1] (Fig 1). To be more specific, the data rate demand will increase at a rate of about 10 exabytes per month in the years 2013 to 2017 [1], thus, an equivalent bandwidth will be needed. To meet this enormous demand, the available radio frequency spectrum needs to be effectively utilized. According to the Federal Communications Commission (FCC), the current licensed radio spectrum is largely underutilized across time and in space [2]. For instance, in New York and Washington D.C., the maximum total spectrum occupancy was found to be as low as 13% and 35%, respectively, of the spectrum below 3 GHz [3].

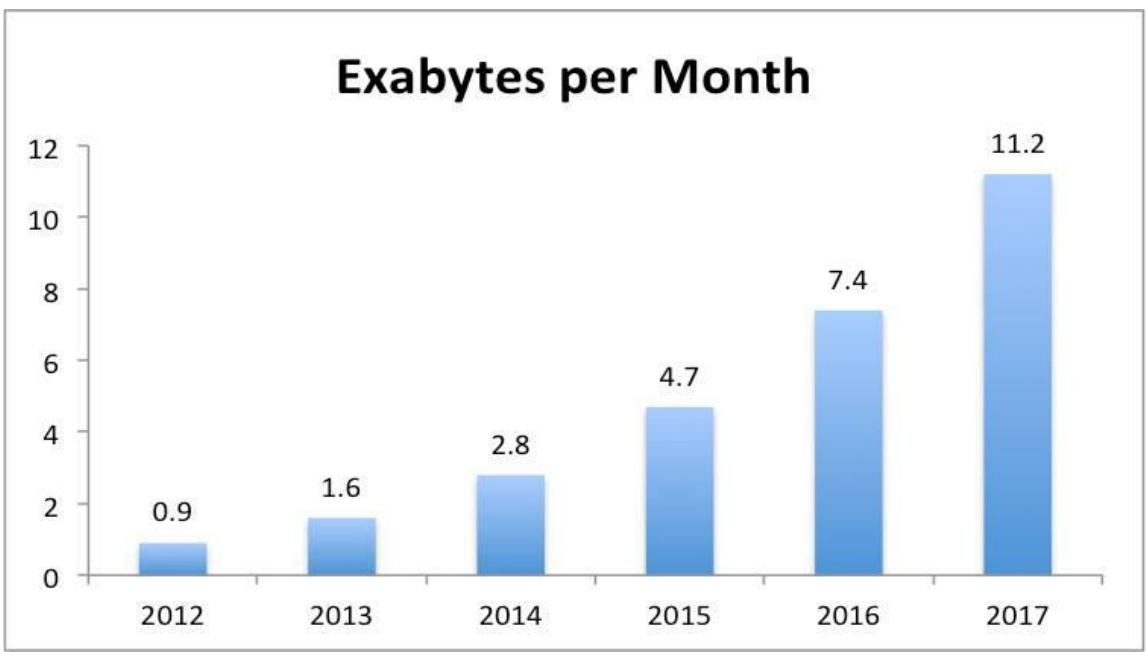

Fig. 1. Global mobile data traffic forecast [1].

Radio spectrum underutilization is commonly spread over a large geographical area. Licensed users use it for a very short period of time which makes the utilization inefficient. If unused spectrum could be detected and utilized opportunistically, bandwidth would not be wasted.

Intelligent devices called cognitive radios (CRs) have emerged [2] with a goal of effectively utilizing licensed spectrum. In the CR communication paradigm, unlicensed users, i.e., secondary users (SUs), can utilize the spectrum allocated to licensed, primary users (PUs) whenever the PUs are inactive. PUs' inactivity produces "spectrum holes" [18], or unused spectrum that can be put to use by secondary users. The detection of spectral holes is termed "spectrum sensing," and this is the work of CR based communications [5]. By sensing and adapting to the environment, a CR is able to fill in spectrum holes and serve additional secondary users without causing harmful interference to the licensed user. This necessitates the detection of PUs that are operating within the communicable range of the CR. Once the PU is detected, the CR should withdraw from the spectrum to minimize the interference it may cause. To avoid interference, the CR must continuously sense the spectrum it is using in order to detect reappearance of the PU. Since the fundamental task of a CR is to detect the presence or absence of spectrum holes, several spectrum sensing algorithms have been developed. The energy detector (ED) algorithm is one of the most popular techniques. It is extensively used in practice due to its simple structure and ease of implementation [4], [5] and will be considered in this research study.

The ED has several challenges to detect the presence of PUs because radio frequency signals propagating through a wireless channel are subject to impairments such as path-loss, shadowing, and multipath fading [6]. For the ED, multipath fading is an inherent challenge. Multipath fading is the random fluctuation of the envelope of the wireless signal due to constructive and destructive additions of signals received via multiple paths. Multipath fading detrimentally affects the performance of wireless systems [6] making signal detection more difficult because of degradation in the transmitted signal strength. Several studies have extensively analyzed the spectrum sensing performance of the ED in fading conditions [4], [32], [33]. However, these analyses considered additive white Gaussian noise (AWGN) at the receiver end. Although AWGN is the most popular noise model according to the literature, there are some situations where noise at the receiver may be better modeled by non-Gaussian noise. The detection performance of an ED is unknown under multipath fading in a non-Gaussian environment and is the interest of this research study. In section 1.1, the main objectives of this study are described.

1.1 Objectives

The main objectives of the proposed project are to:

1. Investigate the quality of spectrum sensing in wireless multipath fading channels and impulsive noise.

2. Extend the scenario in objective 1 to investigate multiple antennas and cooperative spectrum sensing techniques to attain further performance improvement in fading channels with impulsive noise.

1.2 Problems

Objectives 1 and 2 are now briefly discussed as problems 1 and problem 2, respectively.

Problem 1

A received radio signal is affected by noise corruption at the CR receiver. Multipath fading and noise can cause the sensing performance of an ED to deteriorate [4], [5]. The noise at the receiver is popularly modeled as Gaussian due to the simplicity of the model and the ease in its analysis. However, non-Gaussian noise may arise in some power delivery networks and some mobile networks that experience significant interference, and the Gaussian noise model is impractical in these scenarios [7]. This impulsive noise is likely to worsen the sensing performance of the ED, which performs optimally only for Gaussian noise [8]. The problem of spectrum sensing in multipath fading with non-Gaussian noise is a focus of this research study.

Problem 2

The “hidden terminal problem,” which occurs when a CR is shadowed or in severe multipath fading [5], is a challenge in spectrum sensing. For example, in a television broadcast system, if a CR is shadowed from the DTV transmitter, it cannot detect its presence and starts to transmit, harmfully interfering with the PU’s transmission. To address this issue, multiple CRs can be designed to collaborate in spectrum sensing. Recent work has shown that multiple antennas and cooperative spectrum sensing can greatly increase the probability of detection in fading channels. However, the role of cooperative spectrum sensing in multipath fading and non-Gaussian noise is unexplored. Improvement of the ED performance in the problem 1 scenario with the help of cooperative spectrum sensing is the second problem considered in this study.

Chapter 2

Literature Review

2.1 RF spectrum

The term radio frequency (RF) refers to alternating current having characteristics such that, if the current is input to an antenna, an electromagnetic field is generated suitable for wireless broadcasting and or communications. These frequencies cover a significant portion of the electromagnetic radiation spectrum, extending from 9 kHz, the lowest allocated wireless communications frequency, to thousands of gigahertz [16].

Many types of wireless devices operate in the radio frequency (RF) spectrum, such as cordless, cellular telephones, radio and broadcast stations, and satellite communications systems. For example, the Global System for Mobile Communications (GSM) operates at 700 MHz, The Code Division Multiple Access (CDMA) uses 450 MHz to provide a mobile coverage area, and Long Term Evolution (LTE) has a range from 700–800 MHz but can go up to 3600 MHz. Therefore, a specific portion of the RF spectrum is allocated to specific services. However, the RF spectrum is a scarce resource, which is getting scarcer due to ever increasing demand for ubiquitous wireless services.

2.2 Spectrum underutilization

Traditional RF spectrum allocation is based on services and regulations that forbid a device to use an empty portion of the spectrum unless the particular service has been allocated to that spectrum. For example, the FCC will allocate more than 11 exabytes per month by the end of 2017. However, studies have shown that the spectrum being allocated is not being effectively used and the total bandwidth is underutilized due to few licensed users. Thus the regulatory regime results in large portions of unused spectrum. Spectrum underutilization occurs both spatially and temporally. That is, there are a number of instances in which spectrum is used only in certain geographical areas, and a number of instances of spectrum being used only for short periods of time. So, spectrum scarcity is largely due to the inefficiency of available spectrum utilization [17].

2.3 Cognitive Radio

In traditional RF spectrum policy, specific spectrum is allocated to licensed users, or primary users (PU). If the PU does not use the complete spectrum allocated to him/her, the rest is wasted. Thus the spectrum will be underutilized. To alleviate spectrum underutilization, an emerging technology called cognitive radios (CRs) helps secondary users (SUs) to access spectra when it is idle. A cognitive radio is an intelligent device that can detect the available channels in a wireless spectrum and change transmission parameters to enable SUs to operate concurrently in the same RF band. A cognitive radio uses a number of technologies including adaptive radio and software defined radio in which traditional hardware components, including mixers, modulators, and amplifiers, have been replaced with intelligent software. Its transceiver is designed to use the best wireless channels in its vicinity. Such a radio automatically detects available channels in a wireless spectrum, then changes its transmission or reception parameters accordingly to allow more concurrent wireless communications in a given spectrum band at one location. Two fundamental characteristics that a CR must possess are cognitive capability and re-configurability [13], [14].

Cognitive capability

The cognitive capability of a radio technology enables it to sense information from its radio environment. This capability cannot simply be realized by monitoring the power in some frequency band of interest, more sophisticated techniques are required to sense the temporal and spatial variations in radio environments and to avoid interference to other users. Through this capability, the unused spectrum at a specific time or location can be identified. The best spectrum and appropriate operating parameters can then be selected.

Reconfigurability

The architecture of a CR is shown in (Fig. 2), where the main components consist of the front-end and the baseband processing unit. Each component can be reconfigured with the help of a control bus to adapt to a time varying RF environment. The received signal is amplified and converted by the front-end. The baseband processing unit has to operate in different bands under various data rates and must combat adverse channel conditions. Reconfigurability enables a radio to be programmed according to the radio environment. The CR can be programmed to transmit and receive on a variety of frequencies and to use different transmission access technologies supported by its hardware design [15]. The transmission parameters of a CR can be reconfigured at the beginning of and during transmission. According to the spectrum characteristics, these parameters can be reconfigured such that the cognitive radio is switched to a different spectrum band, the transmitter and receiver parameters are reconfigured, and the appropriate communication protocol parameters and modulation schemes are used [34].

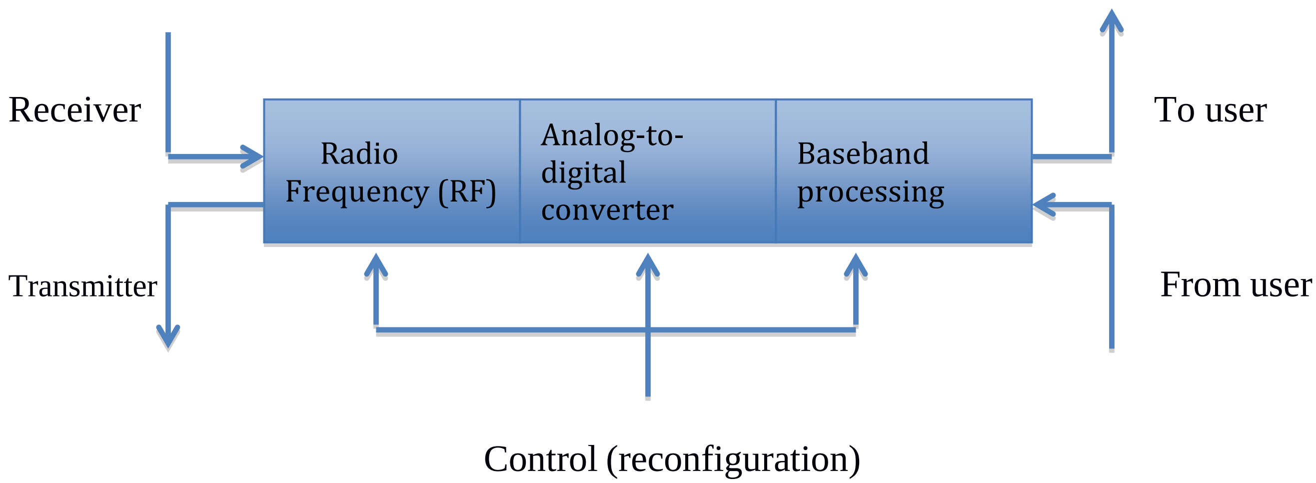

Fig. 2. Cognitive radio architecture [34].

2.4 Dynamic spectrum utilization

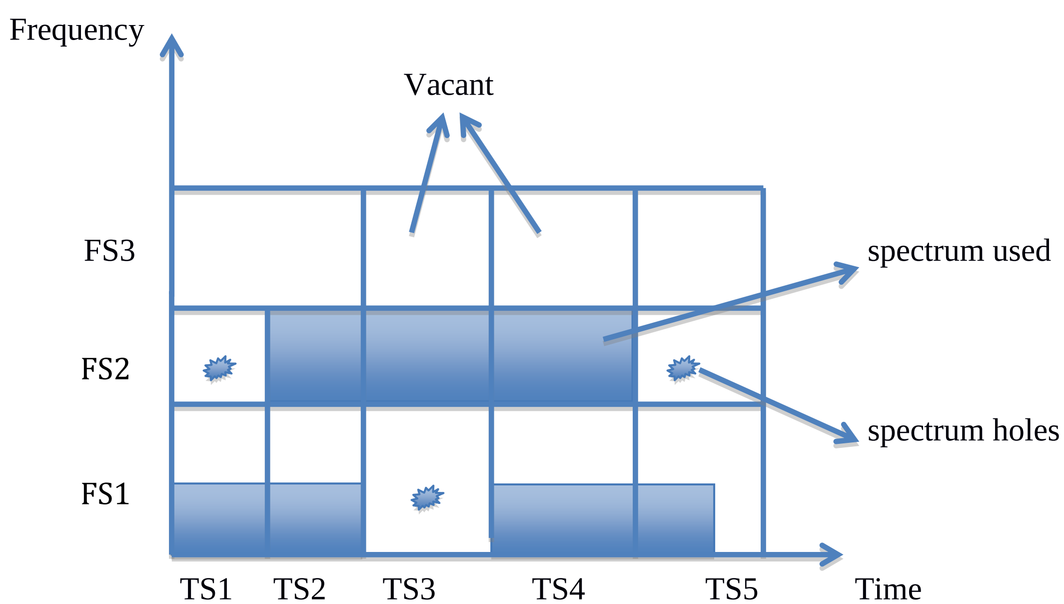

Fig. 3. Spectrum holes concept.

A spectrum hole (SH) is defined as a band of frequencies assigned to a primary user that is not being utilized by the user at a particular time and specific geographic location [18]. If an SU can access the spectrum hole, the utilization of the spectrum is improved significantly. As shown in (Fig. 3), a promising mechanism to improve spectrum utilization by exploiting spectrum holes is based on the CR concept. The figure depicts the frequency-time slot, which has different time slots TS1, TS2, TS3, TS4 and TS5 and the frequency slots FS1, FS2 and FS3. For example, an SU at TS1 and FS2 may be looking for access to an available spectrum. When a PU is idle, the SU takes the opportunity to obtain access to the spectrum hole. When the PU tries to access the slot FS2 at TS1, the SU leaves the channel and skips to another slot where a spectrum hole exists. Most of the existing CRs detect SHs by sensing whether the primary signal is present or absent and then try to access the SHs so that SUs and PUs use the spectrum band either at different time slots or in different geographic regions [18].

2.3 Spectrum sensing

To promote the concept of dynamic spectrum utilization, the foremost task of the CR is to detect the presence of spectrum holes. The CR must have the capability to continuously sense the spectrum to identify any possible return of the PU, which would require the SU to vacate the channel immediately. Spectrum sensing is a detection process that enables the CR to adapt to its environment by detecting spectrum holes. Many spectrum sensing techniques have been reported in the literature; four are discussed in the following sections.

(i) Matched filter detection A matched filter is obtained by correlating a known signal with an unknown signal to detect the presence of the known signal in the unknown signal [19]. The main advantage of matched filter detection is that it needs only a short time to achieve high processing gain due to coherent detection. However, a major drawback of coherent detection is that it requires a dedicated sensing receiver for all PU signal types. In the CR scenario, the use of a matched filter may be impractical since the information about the PU signal is hardly available at the CR. This detection technique can be used only if partial information of the PU signal, such as pilot symbols or preambles, are known, which may not be available at the SUs.

(ii) Energy detection The ED is a noncoherent device, that is, the ED detects the primary signal based on the sensed energy. The ED is therefore easy to implement. The major drawback is that it can detect the signal of the primary user only if the energy is above a certain threshold. This detection approach is chosen over the other detection techniques in this project as it is easy to implement and this detection does not require prior knowledge of a PU signal.

(iii) Cyclostationary detection In cases where the PU signal exhibits strong cyclostationary properties, it can be detected by exploiting the cyclic information embedded in the received signal [5]. This detection approach is robust to random noise and interference from other modulated signals having different cyclic frequencies. However, cyclostationary detection requires prior knowledge of PU signal such as modulation format, which may not always be available, particularly if a wide range of spectrum is being sensed in which PUs may be exploiting various modulation formats in transmission [19].

(iv) Wavelet detection Input signals are decomposed into different frequency components, then each component is studied with resolutions matched to its scale. To identify vacant frequency bands, the entire wide-band is modeled as a train of consecutive frequency sub-bands where the power spectral characteristic is smooth within each sub-band but changes abruptly on the border of two neighbouring bands [5]. Wavelet detection is based on such abrupt changes and it is preferred because of it low cost and easy implementation [20]. However, in CR systems wavelet transform based detection is employed as a coarse sensing stage and temporal signature detection is used as a fine sensing stage. The processing time is large. Wavelet detection also requires prior knowledge of a PU signal.

2.6 Spectrum sensing as a binary hypothesis test

The essence of spectrum sensing is a binary hypothesis-testing problem of the form,

$$H_0 : : Primary\ user\ is\ absent,$$

$$H_1 : Primary\ user\ is\ in\ operation.$$

Thus, the problem of identifying the presence or the absence of a PU is translated into a problem of deciding on one of the hypotheses, H_0 or H_1. The signed model for spectrum sensing is popularly modeled as [4]:

$$y(t) = \begin{cases} w(t) & : H_0 \\ hs(t) + w(t) & : H_1 \end{cases}, \tag{2.1}$$

where $y(t)$ is the observed signal at the CR, $s(t)$ is the signal from the primary transmitter, $w(t)$ is the additive noise, and h is the channel gain of the sensing channel between the PU and the CR. We assume that the sensing channel is time-invariant during the sensing process.

2.7 Energy detector

The structure of the spectrum sensing algorithm ED, which is the detector to be used in this research project as shown in (Fig. 4) [32]. The observed signal y(t) is fed to the detector and the decision is to be made by comparing the signal against the threshold for a given time period. The time bandwidth product can be defined as

$$u = TW, \tag{2.2}$$

where u is the product of integration time T and noise bandwidth W. Measuring the energy of the received signal performs the energy detection. By comparing the energy of the received signal with the threshold, the PU is detected.

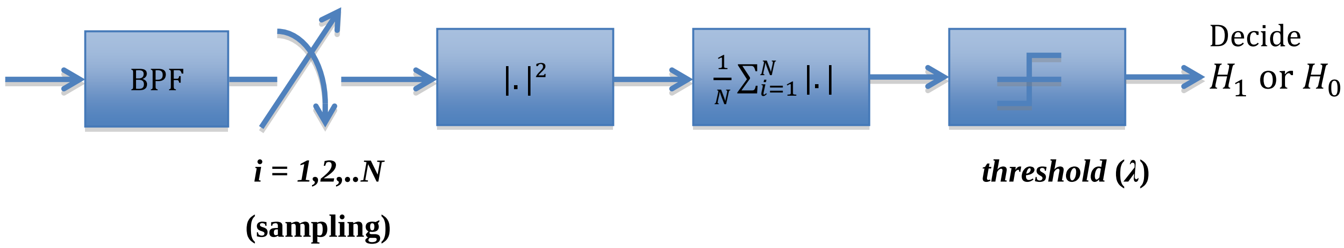

Fig. 4. Digital energy detector.

The ED decision variable equation is:

$$T = \frac{1}{N}\sum_{i=1}^{N} |Y_i|^{\,2} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda. \tag{2.3}$$

2.8 Performance metrics of the energy detector

The detection performance of the ED is characterized mainly by two metrics, the probability of correctly detecting the PU, termed as the "probability of detection (P_{d})" and the probability of incorrectly detecting the PU, called the "probability of false alarm (P_{f})," which are mathematically defined as [5]

$$P_d = Pr\{\widehat{H} = H_1 | \, H_1\} = Pr\{T \geq \lambda \mid H_1\}, \text{ and} \tag{2.4}$$

$$P_{\mathrm{f}} = Pr\left\{\widehat{H} = H_1 \middle| H_0\right\} = Pr\{T \geq \lambda \mid H_1\}, \tag{2.5}$$

where $P_{\mathrm{r}}\{x|y\}$, is the conditional probability of an event x given an event y, $\widehat{H}$ is the hypothesis decision made by the detector, and $H_1|H_0$ are the hypotheses denoting the presence/absence of a PU, respectively. Thus, a higher detection probability and a lower false alarm probability are desirable to ensure reliable ED performance. Another performance metric that jointly characterizes the effect on P_d and P_f is the probability of error:

$$P_e = Pr\{H_o\}P_f + Pr\{H_1\}P_{md}, \tag{2.6}$$

where $P_{md} = 1 - P_d$ is the probability of misdetection. The lower the probability of error, the better is the detection probability.

2.9 Multipath fading

Multipath fading is an inherent phenomenon in wireless propagation; it occurs due to a constructive and destructive combination of randomly delayed, reflected, scattered, and diffracted signal components. (Fig. 5) depicts a multipath fading effect, which occurs due to interference in the atmosphere. Typically, this type of interference occurs due to a non-line-of-sight radio (NLOS) propagation path between the base station (BS) and the mobile station (MS), because of natural and man-made objects that are situated between the BS and MS. At the MS, the waves arrive from many different directions with different delays. If the MS is moving in the scattering environment, then the spatial variations in the amplitude and phase of the composite received signal will manifest themselves as time variations, a phenomenon called envelope fading. Models that describe the behaviour of the multipath fading envelope depend on the nature of the radio propagation environment. Next, we

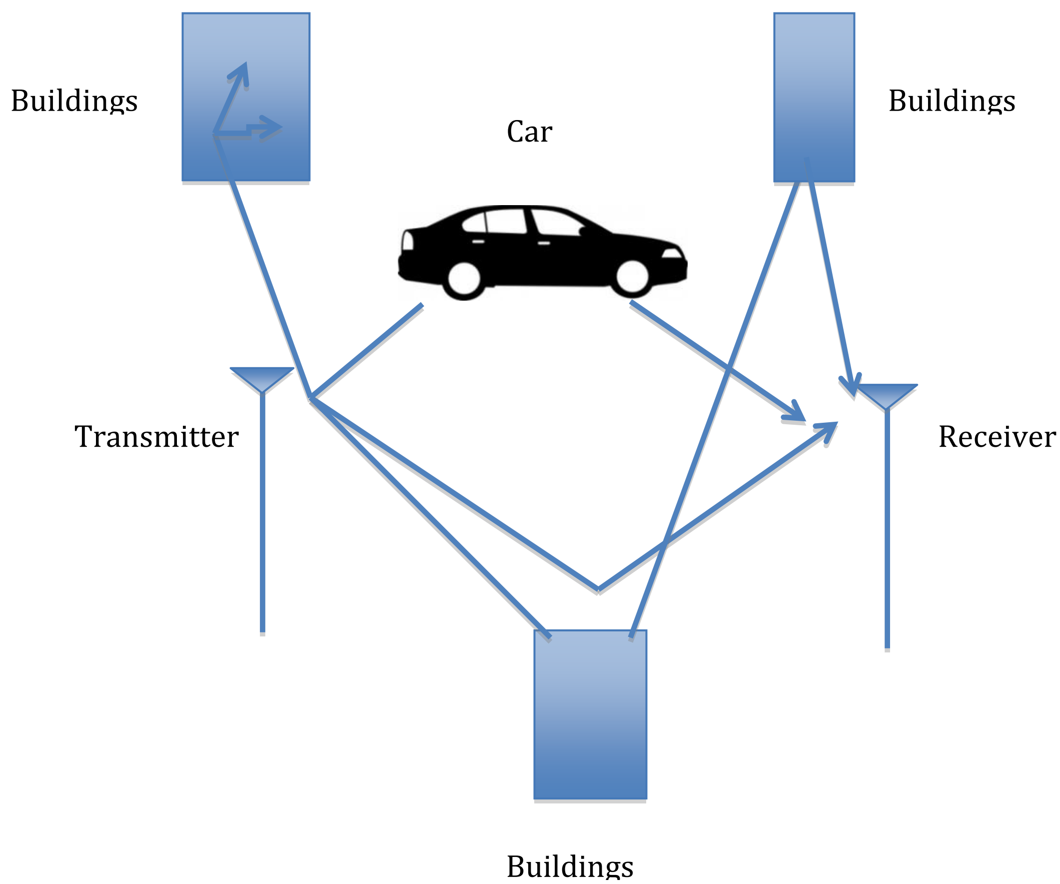

Fig. 5. Reflected, diffracted, and scattered wave components.

discuss one of the most popularly used, multipath fading model in wireless communication system, the Rayleigh fading.

2.9.1 Rayleigh fading

The Rayleigh fading model is the most widely used fading model due to its ability to incorporate the scatter multipath effect and its relatively less complicated mathematical form; it is used in this study. In this fading model, the magnitude of a signal that has passed through such a transmission medium varies randomly, or fades, according to the Rayleigh distribution. The Rayleigh distribution is frequently used to model multipath fading with no direct line-of-sight (LOS) path. In this case, the probability density function (PDF) of the channel fading amplitude (|h|) is given by

$$p(\alpha) = \frac{2\alpha}{\Omega} e^{\frac{-\alpha^2}{\Omega}}, \qquad \alpha \geq 0, \tag{2.7}$$

where α is the channel fading amplitude and Ω is the average fading power.

The Rayleigh fading channel typically agrees well with the experimental data for mobile systems where no LOS path exists between the transmitter and receiver antennas [21]. It also applies to the propagation of reflected and refracted paths through the troposphere [22] and ionosphere [23], [24] and to ship-to-ship radio links [25].

2.10 Antenna diversity

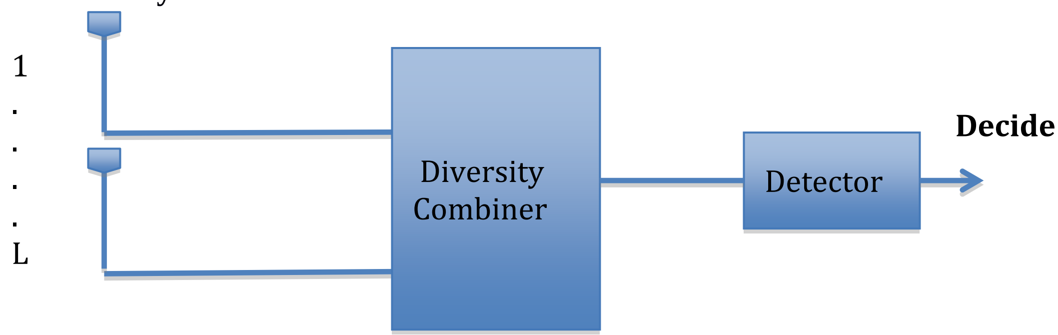

Fig. 6. Detector with multiple antennas.

Antenna diversity (Fig. 6) is popular in wireless systems because spectrum sensing performance is improved when the receiver has multiple antennas. Multiple antennas can cover a large space over a geographical area, enhancing the detection performance of the ED. Antenna diversity requires multiple antennas at the receiver, making the receiver bulkier than in the other diversity systems. Using multiple antennas in CRs is one of the possible approaches for the spectrum sensing. The PU signal is treated as an unknown deterministic noise and based on this model the performance of the ED has been evaluated in fading channels. Antenna diversity can be achieved through spatial separation, pattern configurations, or polarization. Spatial separation is the most common of the three and requires two or more antennas to be separated in space at the terminal. Two antennas that are physically separated in space experience different propagation environments and multipath components sum differently at each antenna. Ideally the antennas are spaced far enough in distance such that the brand signals have high probability of fading independently. This separation could be in order of tens of RF carrier wavelengths [28].

Complexity and system requirements are traded off in terms of different antenna diversity techniques. The most popular ones reported in the literature are discussed in the following sections.

2.10.1 Combining techniques

(i) Maximal ratio combining Maximal ratio combining (MRC) is an optimal coherent signal combining technique that maximizes the signal-to-noise ratio and requires complete channel state information (CSI) of all branches [29]. The branches are co-phased prior to summing to ensure that all branches are added in phase for maximum diversity gain. The summed signals are then used as the received signal and connected to the demodulator. The information on all channels is used with this technique to obtain a more reliable received signal. However, the coherent requirement makes the technique more complicated and it may not be useful for a non-coherent method such as the ED.

(ii) Selection combining In selection combining (SC), the branch with the highest signal-to-noise ratio (SNR) is chosen among all of the collected diversity branches and fed to the detector. Thus, the receiver has to monitor the SNR of all branches all the time which conflicts with the ED requirements for low complexity in software and hardware.

(iii) Square law selection Unlike MRC and SC, square law selection (SLS) (Fig. 7) is a non-coherent technique in which the branch with the maximum decision statistics is selected and the decision is then made to determine the presence or the absence of a PU. The decision variable in this case is given by

$$T_{sls} = \max\{T_1, T_2, \dots T_L\}. \tag{2.8}$$

The resulting probability of detection and probability of false alarm are, respectively, given by,

$$P_{d_{SLS}} = 1 - (1 - P_d)^L, \quad \text{and} \tag{2.9}$$

$$P_{f_{SLS}} = 1 - (1 - P_f)^L. \tag{2.10}$$

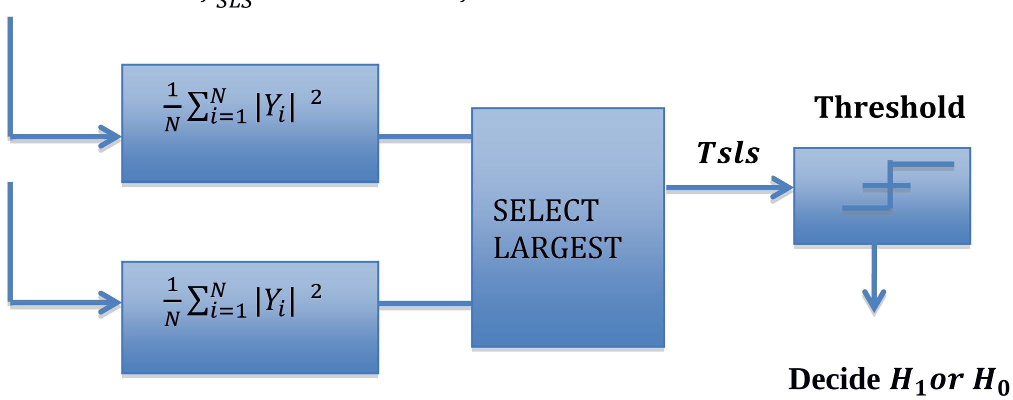

Fig. 7. Square law selection combiner

2.11 Cooperative spectrum sensing

A number of CRs collaborate together by sharing their information in order to improve the performance of spectrum sensing. This technique is known as cooperative spectrum sensing. Cooperative spectrum sensing is shown in (Fig. 8). Spectrum sensing is carried out with the help of the ED and cooperative spectrum sensing is implemented to improve the performance of the ED. At the receiver, two or more cognitive radios are placed. Due to interference caused by buildings, trees, etc., shadowing occurs. This might mislead an SU to use the channel when a PU is still active. To avoid such scenarios, a cooperative spectrum sensing technique is implemented with the help of many CRs. Individual CRs (1, 2, 3) make decisions based on the signals they sense and send their individual decisions to the fusion center. The strongest signal is considered to be the probability of detection or false alarm. Decision making is the critical task in deciding the presence/absence of a PU.

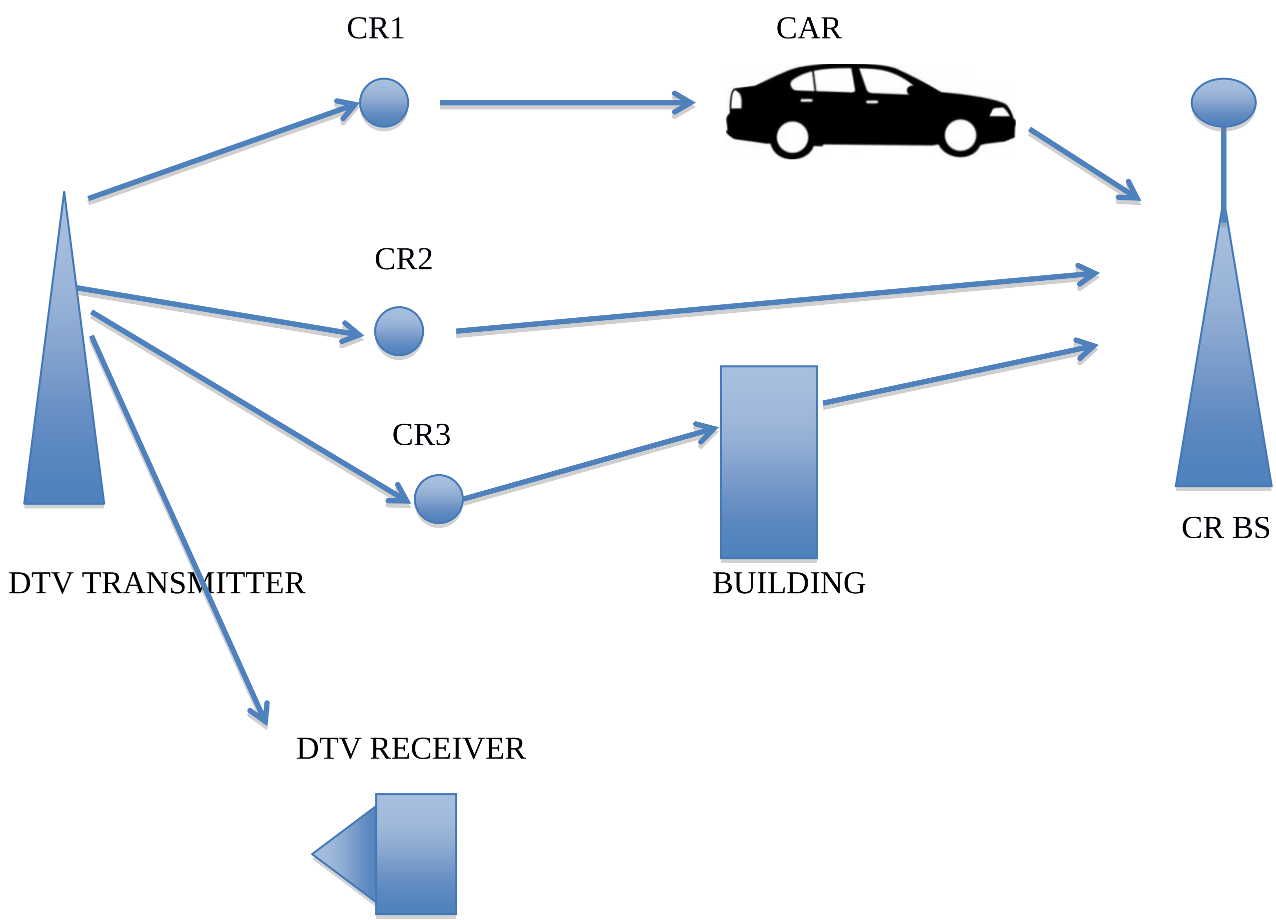

Fig. 8. Cooperative spectrum sensing.

The cooperative spectrum sensing approach can be seen as data fusion and decision fusion protocols as elaborated in the following sections.

Decision fusion

In decision fusion, each cooperating CR partner makes a binary decision based on its local observation and then forwards one bit of the decision to the common receiver. At the common receiver, all 1-bit decisions are fused together [5]. The main advantage of this is that it consumes limited bandwidth. When binary decisions are reported to the common node, two suboptimal rules of decision can be used, which are:

- OR rule – The OR rule decides that a signal is present if any of the users detect a signal.
- AND rule – The AND rule decides that a signal is present if all users have detected a signal.

The false alarm probability of cooperative spectrum sensing based on the OR rule is given by [5]

$$Q_f = 1 - \prod_{i=1}^{K}(1 - P_f^{(i)}), \tag{2.11}$$

where $P_f^{(i)}$ is the probability of a false alarm for each SU i in the coalition set K. The detection probability of cooperative spectrum sensing based on the OR rule is given by

$$Q_d = 1 - \prod_{i=1}^{K}(1 - P_d^{(i)}), \tag{2.12}$$

where $P_d^{(i)}$ is the probability of detection for each SU i in the coalition set K.

Data fusion

Alternatively, instead of transmitting the 1-bit decision to the common receiver, each CR can forward the entire sensing result to the fusion center without performing any local decision. A data fusion scheme is described by taking a linear combination of measurements of various cognitive users to decide between two hypotheses [5]. It is seen that the data fusion combining for spectrum sensing based on the ED achieves more precise detection than the decision fusion combining. However, data fusion consumes a large amount of bandwidth. The OR rule is considered in this study in view of the requirement to consume the least bandwidth. Hence, the OR rule, the decision fusion based approach, is used in this study.

2.12 Noise models

The performance of wireless communication systems is highly affected by noise at the receiver which is modeled as additive. The popular norm is to use with additive white noise (meaning the power spectral density is same over the whole range of frequencies in consideration). In general, the noise models used in wireless communication performance analyses are classified as Gaussian or non-Gaussian.

2.12.1 Gaussian noise models

In wireless communications, Gaussian noise represents statistical noise having probability density function equal to that of the normal distribution, which is also known as Gaussian distribution. Communication channels can be affected by wideband Gaussian noise coming from many natural sources, such as the thermal vibrations of atoms in conductors [35]. Gaussian noise model is widely used and its main advantage is that it helps in characterizing the thermal noise. In the noise channel model, the only impairment to communication is a linear addition of white noise with a constant spectral density and a Gaussian distribution of amplitude. Additive white Gaussian noise (AWGN) noise is additive, that is, the received signal equals the transmitted signal plus some noise, where the noise is statistically independent of the signal. The noise is white, that is, the power spectral density is flat so the auto correlation of the noise in the time domain is zero for any nonzero time offset. The noise has a Gaussian distribution [31]. The probability density function (PDF) of a Gaussian random variable z is given by

$$p_G(z) = \frac{1}{\sigma\sqrt{2\pi}} \, e^{-\frac{(z-\mu)^2}{2\sigma^2}}, \tag{2.13}$$

where μ represents the mean value and σ is the standard deviation.

2.12.2 Non-Gaussian noise models

Due to infrequent but high level noise spikes in radio frequency (RF) noise and low frequency atmospheric noise, the Gaussian noise model is not appropriate for impulsive noise characterization. Commonly used non-Gaussian noise models include the Laplacian noise, the Cauchy noise, and the Gaussian mixture noise [31]. Relative to Gaussian noise, the tail of a non-Gaussian noise PDF typically decays at a slower rate. In certain situations, for example, in power line communications, the signal at the receiver fades due to thermal effect. This can be easily depicted with Laplacian noise model which is considered in this study because of its impulsive nature (it includes impulsive sharp sounds like clicks and pops).

The Laplacian noise PDF is given by

$$p(n) = \frac{1}{\sqrt{2\sigma^2}} \exp\left(-\sqrt{\left(\frac{2}{\sigma^2}\right)}\, |n|\right) \quad , \qquad -\infty < x < \infty \tag{2.14}$$

where σ^2 is the noise variance.

Chapter 3

Performance of the energy detector in fading channels with impulsive noise

The ED's performance is studied in this project with Rayleigh fading as the fading channel model and Laplacian noise as the non-Gaussian noise model to characterize an impulsive noise environment. The quality of spectrum sensing is studied and the simulated results are based on the probability of detecting the presence of a PU under the fading and impulsive noise environment. To improve detection probability, multiple antennas and cooperative spectrum sensing techniques are implemented and their detection performance results are determined. Multiple antennas and cooperation spectrum sensing are used with the ED algorithm to improve the detection performance by taking advantage of spatial diversity so that the primary user can be protected and to reduce false alarms so that the spectrum is utilized more efficiently. Extension to cooperating CRs with multiple antennas and cooperative spectrum sensing techniques, SLS, and OR rule based suboptimal fusion is considered.

The rest of this chapter is organized as follows. The system model is described in section 3.2. The simulation model is explained in section 3.3. Performance of the energy detector in multipath fading and Laplacian noise is characterized in section 3.4. Extension of the scenario to cooperative spectrum sensing is presented in section 3.5.

3.1 System model

Consider a cognitive network, with K cognitive users to sense the spectrum in order to detect existence of a PU [4]. Each CR performs local spectrum sensing independently by using N samples of received signal. As mentioned in section 2.3, the spectrum sensing problem can be formulated as a problem with two possible hypotheses H_0 and H_1.

$$y(t) = \begin{cases} w(t) & : H_0 \\ hs(t) + w(t) & : H_1 \end{cases},$$

where $s(t)$ is the transmitted signal, $w(t)$ is the receiver noise, which is are assumed to be statistically independent of each other and h is the channel gain of the channel between the PU and the CR user which is modeled as Rayleigh faded. The H_0 and H_1 hypotheses stand for signal presence (PU absent) and absence (PU present), respectively. The signal $y(t)$ is fed to the input of the ED such that the detector decision variable is rewritten based on section 2.5 as

$$T = \frac{1}{N}\sum_{i=1}^{N} |Y_i|^{2} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda.$$

The decision variable for SLS, in the case of multiple antennas is given by

$$T_{sls} = \max\{T_1, T_2, \dots T_L\},$$

and the probability of detection and probability of false alarm are given by

$$Pd_{SLS} = 1 - (1 - P_d)^L, \text{ and}$$

$$Pf_{SLS} = 1 - (1 - P_f)^L.$$

When K cognitive radios are installed at the receiver, the false alarm probability based on the suboptimal OR rule is given by

$$Q_f = 1 - \prod_{i=1}^{K}(1 - P_f^{(i)}).$$

where $P_f^{(i)}$ is the probability of a false alarm for each SU *i* in the coalition set K. The detection probability of cooperative spectrum sensing based on the suboptimal OR rule is given by

$$Q_d = 1 - \prod_{i=1}^{K}(1 - P_d^{(i)}).$$

where $P_d^{(i)}$ is the probability of detection for each SU *i* in the coalition set K.

The probability of error is

$$Q_e = (1 - Q_d + Q_f)/2.$$

3.2 Simulation model

The numerical results in the project are based on semi-analytical Monte Carlo simulations. The Monte Carlo simulation is an iterative method, which is a class of computational algorithms that rely on repeated random sampling to obtain numerical results. In this study, Monte Carlo simulation is performed in MATLAB over 10^6 iterations.

Rayleigh fading is a reasonable model where there are many objects in the environment that scatter the radio signal before it arrives at the receiver. The received complex envelope of the Rayleigh fading channel can be treated as Gaussian random process which is given as

$$h(t) = h_i(t) + jh_q(t), \tag{3.1}$$

where $h_i(t)$ and $h_q(t)$ are independent and identically distributed (*i.i.d.*) Gaussian random variables with a mean of 0 and a variance of 1 and $j=\sqrt{-1}$ is the imaginary unit. The amplitude $|h(t)|$ is Rayleigh distributed as equation (2.7) in Chapter 2.

For generating Laplacian noise, its cumulative distribution function (CDF) is utilized, which is given by

$$F(x) = \frac{1}{2} + \frac{1}{2} sgn\,(x - \mu)\,1 - \exp\left(-\frac{|x-\mu|}{b}\right), \tag{3.2}$$

where *sgn* is the sign function, μ is the location parameter, and b is a scale parameter. To obtain Laplacian noise samples, a uniform random variable which lies between 0 and 1 is generated. This generated value is mapped to the inverse CDF of equation 3.2 to the corresponding abscissa which represents the sample which has Laplacian distribution.

3.3 Numerical Results and Discussion: Single CR case

In this section, the effects of various parameters on the spectrum sensing outcome are illustrated in Fig. 9-17. Numerical plots are illustrated to gain understanding on the critical parameters of interest and the performance quantification of the ED is carried out by plotting receiver operating characteristic (ROC) curves such as P_e vs. λ and P_d vs. SNR. The ROC curves are plotted with respect to how the detection probability changes with the false alarm probability for a varying threshold. The probability of detection for different values of SNR has been simulated with constant parameters. One of the most important detector parameters of interest, the probability of error P_e is considered in numerical plots. Similarly, we consider cooperative probability of error for collaborative spectrum sensing.

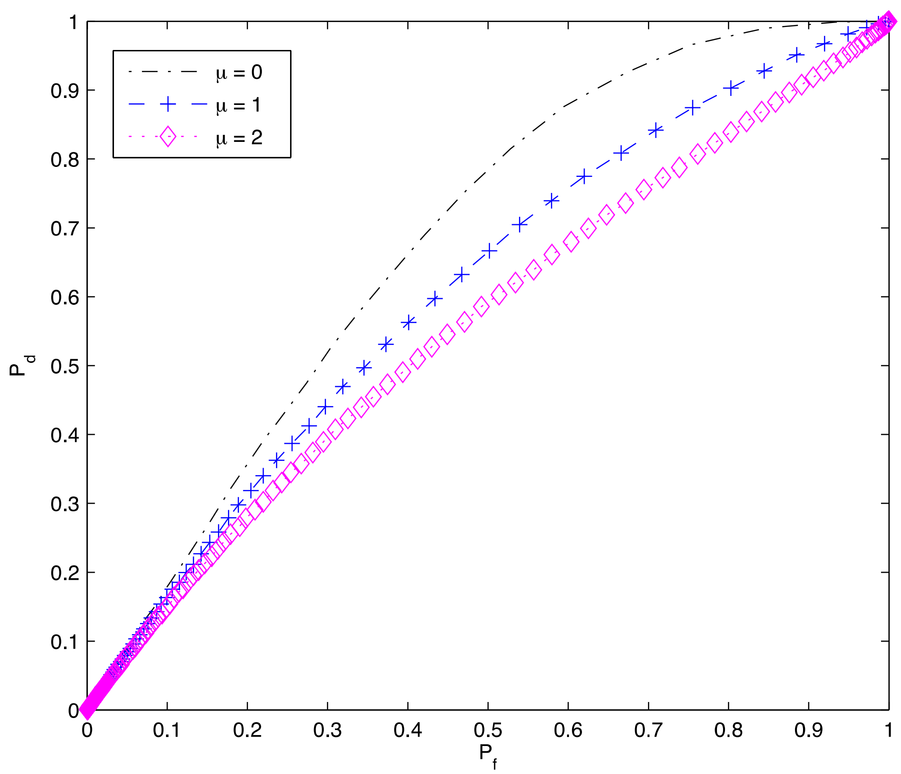

Fig. 9. ROC plot of P_d vs. P_f for different mean values of μ; SNR = 0 dB and σ = 1.

Effect of noise level μ on ED performance

The ROC curves shown in (Fig. 9) characterize the ED performance with variation in the average noise level μ. As depicted in the figure, the increase in the average noise level μ clearly degrades the performance of an ED, thus lower levels of noise is desirable. For instance, when μ varies from 2 to 1, about 14.2% gain in P_d is observed at P_f = 0.4. When μ varies from 1 to 0, about 16.07% gain in P_d is observed at P_f = 0.4. This means 16.07% improvement in detection performance can be attained by keeping μ as 0 and not higher values. We conclude that the lower the value of μ, the better is the detection probability.

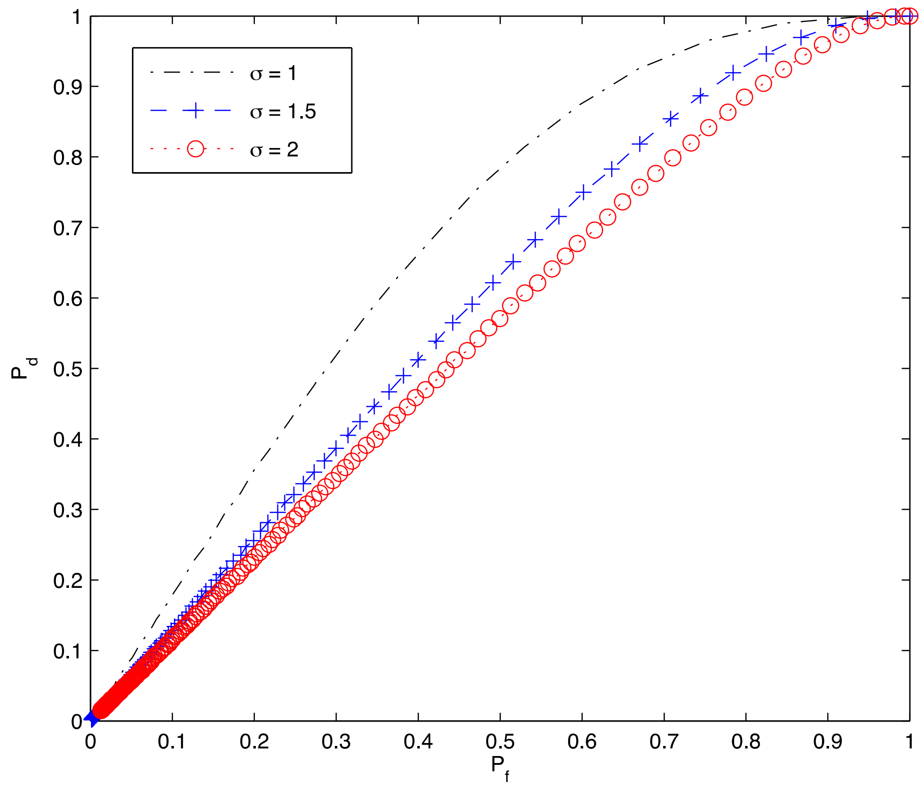

Fig. 10. ROC plot of P_d vs. P_f for different sigma values; μ = 1.

Effect of noise variance σ on detection probability

A ROC curve is plotted in (Fig. 10) for different values of noise variance σ,. The figure depicts the impact of σ; for example, an increase in σ decreases the detection performance. The figure shows that P_d dramatically decreases at higher σ values. Larger σ, results in higher error probability. From the plot we can see that, when σ varies from 2 to 1, about 13.3% gain in P_d is observed for P_f = 0.4.

Again, when σ varies from 1 to 0, about 35.2% gain in P_d is observed for P_f = 0.4 which means 35.2% improvement in the probability of detection performance can be attained by setting the value of σ as 1. We conclude that the lower the noise variance, the higher is the detection probability.

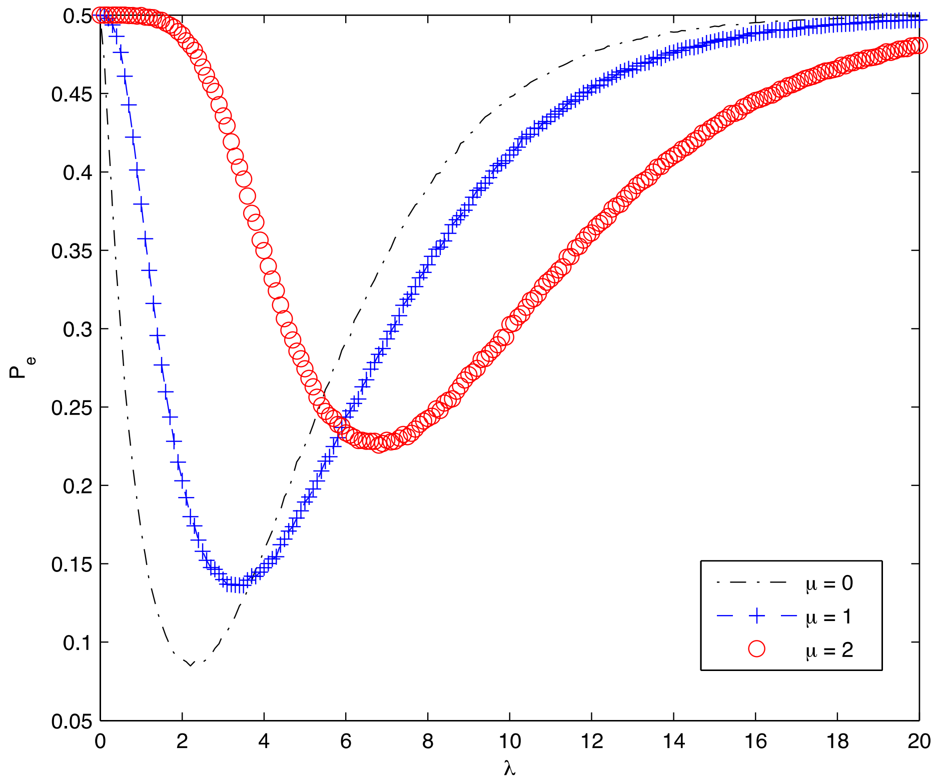

Fig. 11. P_e vs. λ for different mean values of μ; SNR = 0 dB and $\sigma = 1$.

Effect of μ on P_e for varying λ

In (Fig. 11), the impact of μ on P_e for a varying threshold λ at a fixed SNR is evaluated. The probability of error degrades when the value of μ decreases for λ between 0 and 6. For example, when the value of μ varies from 0 to 1, about 51.3% increase in P_e is seen when the threshold λ is fixed at 2. For the same threshold, when μ varies from 1 to 2, about 72.48% increase in P_e is noted. The performance of the detector degrades when μ increases. The optimal threshold λ keeps shifting for each value of μ. The minimal probability of error changes for different values of μ for a shift in threshold λ. When μ is 0, the minimum probability of error is attained when the threshold λ is set to 2. Likewise, when μ is 1 and 2, the shift in threshold is 3.2 and 6.7, respectively. We conclude that, the lower the value of μ, the lesser is the probability of error.

3.4 Numerical Results and Discussion: Antenna diversity and cooperative diversity

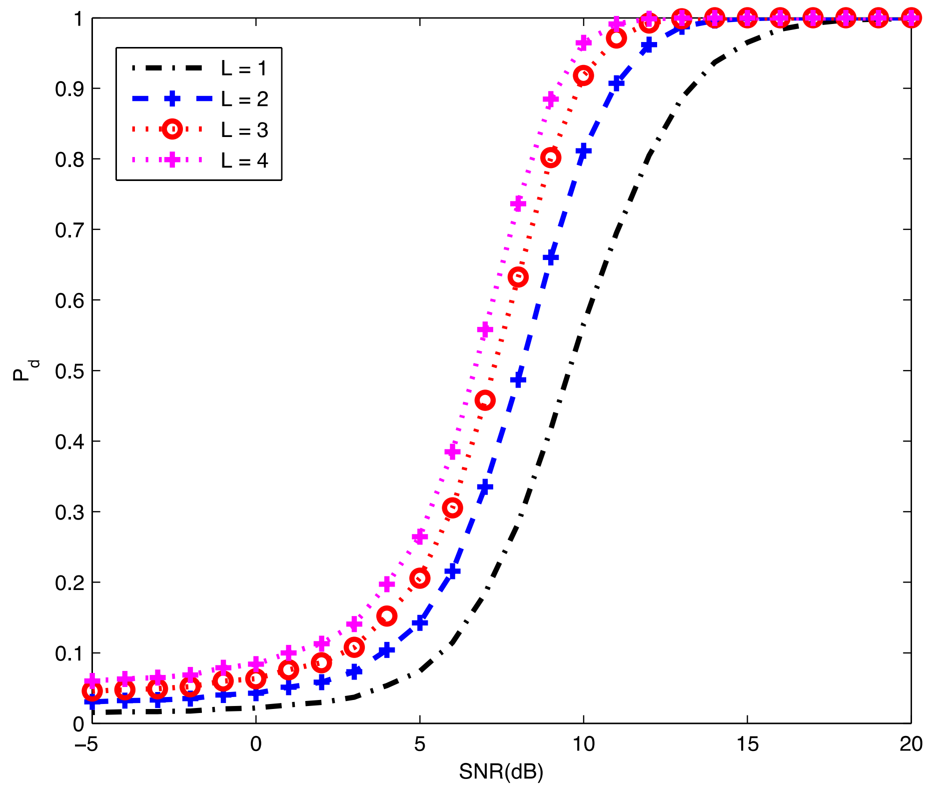

Fig. 12. P_d vs. SNR for multiple antennas; λ = 5 and P_f = 0.01.

Effect of *L*

As shown in (Fig. 12), as the number of antennas L increases, the chance of detecting the PU signal increases. The value of P_f is fixed by varying the threshold λ to attain a fixed threshold at which the ED operates. The number of samples, noise variance, and mean are all set to constant values. At SNR = 10 dB, we can see that effect of increasing the number of antennas approaches saturation. When L varies from 1 to 4, about 80% gain in P_d is observed at P_f = 0.1. This means that the detection performance of an ED using multiple antennas improves by 80%. With 2 or more antennas, detection of the PU can be achieved faster than by using a single antenna. The performance of the detector improves when a multiple antenna technique is incorporated under a fading channel.

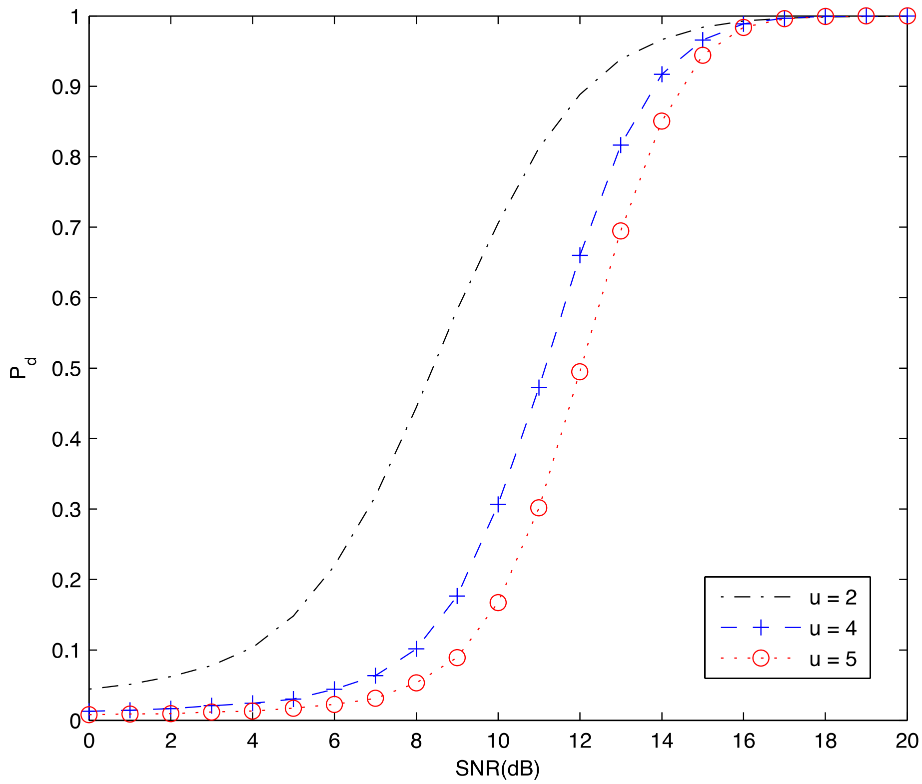

Fig. 13. P_d vs. SNR for multiple antennas with different time bandwidths; λ = 5.25 and P_f = 0.01.

Effect of time bandwidth product

In an energy detector, the time bandwidth product u is an important factor in detecting the presence of a PU. As shown in (Fig. 13), increased incoherence of noise u leads to reduced degradation in detection P_d. From the simulation results we can see that for a constant threshold, when the time bandwidth of u varies from 2 to 5, about 13.6% decrease in the detection probability is estimated. For instance, when u is set to 2, for SNR = 10 dB, the detection probability observed is 0.76. For the same SNR value, when u is set to 4, the detection probability observed is 0.32 and it further decreases to 0.16 when u is 5. Noise incoherence plays an important role is the performance of an ED, thus noise incoherence has a drastic impact on the performance of the detector when they have higher values.

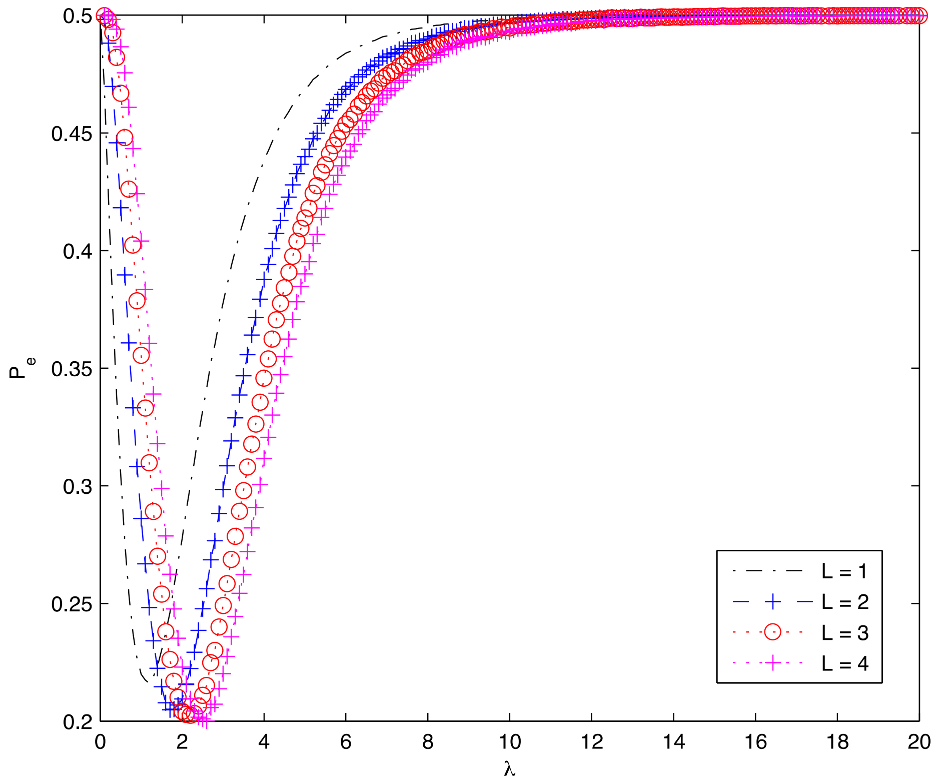

Fig. 14. P_e vs. λ for multiple antennas; SNR = 10 dB.

Effect of L on P_e for varying λ

In (Fig. 14), the effect of multiple antennas on the probability of error P_e in spectrum sensing performance is evaluated. Sensing channels and reporting channels both experience Rayleigh fading with a constant value of SNR. We infer from our analysis that the usage of multiple antennas improves the efficiency of the detector compared to the use of a single antenna. The probability of error can be minimized with multiple antennas. For example, 34.8% gain in P_d is achieved for fixed value of P_f to 0.01. Antennas cannot only increase the channel capacity of the systems but also provide a high spatial diversity gain to combat channel fading. Implementing multiple antennas at each CR might be costly but it is very helpful in sensing the spectrum accurately for better detection probability and a lower error rate.

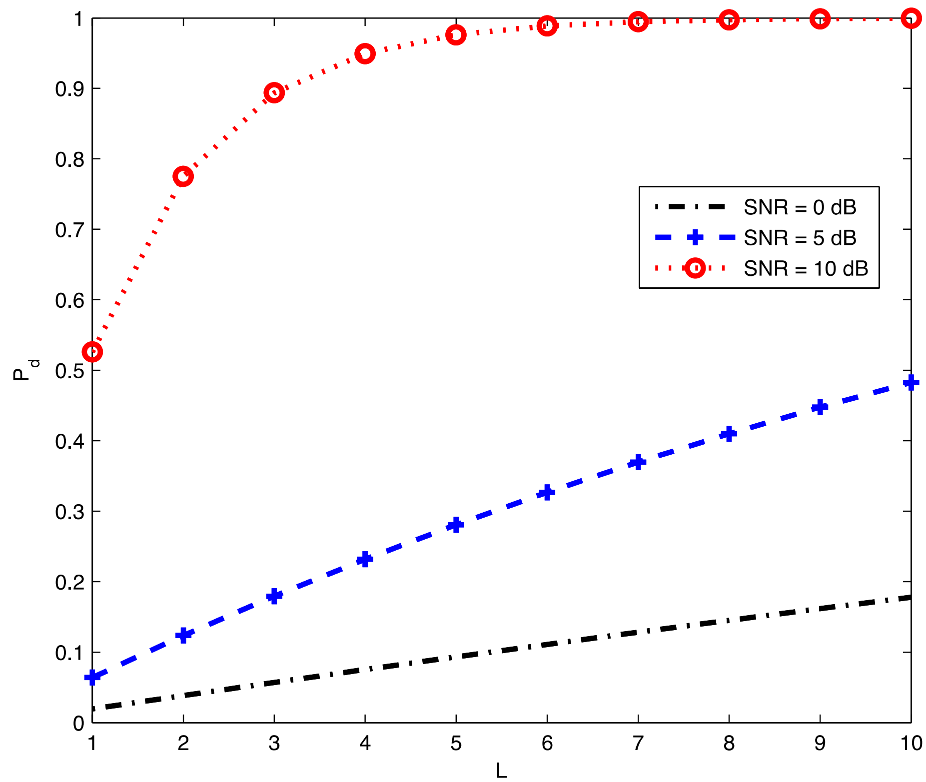

Fig. 15. P_d vs. L for different values of SNR; λ = 6 and P_f = 0.01.

Effect of SNR

As seen before, identifying the presence of a PU's signal under low SNR is difficult. The higher the SNR, the better is the detection probability. As shown in (Fig. 15), when the SNR = 10 dB, the probability of detection approaches saturation, which is the best case for any detector. For lower values of SNR, 0 dB and 5 dB, the probability of detection reaches up to 0.15 and 0.45, respectively. With multiple antennas, low SNR values can be detected, but to achieve a better antenna gain, the SNR should be high and multiple antennas should be used. Using a single antenna at SNR = 10dB, we attain a probability of detection close to 0.52 which is not the best case scenario for an ideal detector. For lower values of SNR, the detection probability is very minimal. From the numerical results we infer that the presence of a PU's signal can be detected with multiple antennas and a higher SNR.

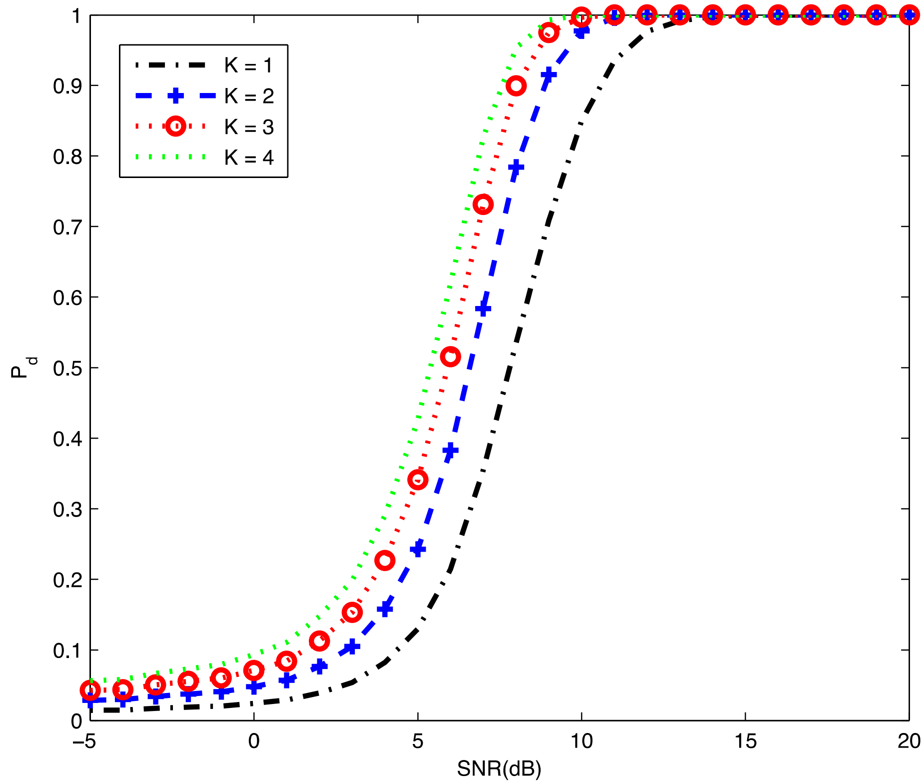

Fig. 16. P_d vs. SNR for cooperative spectrum sensors; λ = 6.03 and P_f = 0.01.

Effect of cooperative spectrum sensors *K*

The spectrum sensing under a Rayleigh fading channel using a cooperative spectrum sensing technique is shown in (Fig. 16) for a constant threshold λ, and a probability of false alarm equal to 0.01. We know that the higher the value of the threshold, the lower is the detection probability, and the higher the value of SNR, the easier it is to detect the presence of a PU's signal. For example, when K varies from 1 to 4, about 35.7% gain in P_d is obtained for P_f = 0.01. This means that 35.7% improvement in the detection performance of an ED when multiple cooperative sensors are used compared to a single cognitive radio. Thus, we determine from the numerical results that the presence of two or more cooperative spectrum sensors characterizes spatial diversity which achieves a higher probability of detection for cooperative spectrum sensing.

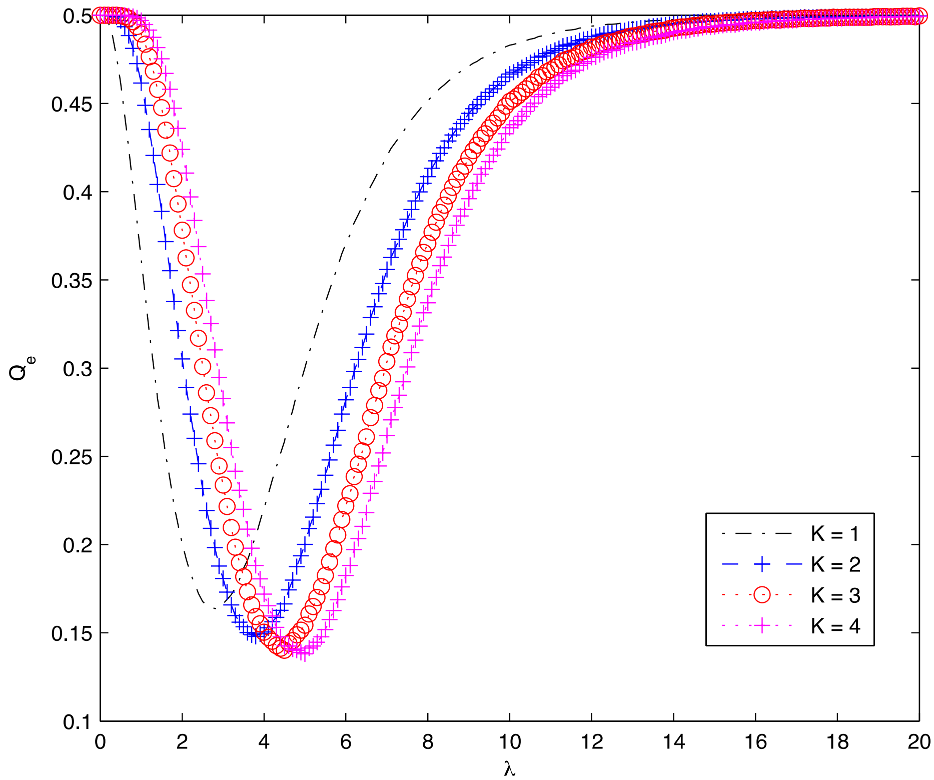

Fig. 17. Q_e vs. λ for cooperative spectrum sensors; SNR = 10 dB.

Effect of K on Q_e for varying λ

In (Fig. 17), cooperative spectrum sensors K are used to improve the detection probability. The probability of error is higher for a single sensor than for multiple sensors. About 28.5% gain in P_d is achieved for P_f = 0.01 when cooperative spectrum sensors are incorporated. The optimal threshold value (λ), shifts as we increase the number of sensors. As discussed previously, the threshold value must be low to attain a better detection probability and low probability of error. For $K = 4$, and the threshold set to 5.03, 28.5% improvement in the detection performance is observed compared to a single spectrum sensor. A recently proposed solution for achieving spatial diversity at any terminal or node is cooperative diversity. It is cost effective and is based on grouping several nodes together into a cluster to form a virtual antenna array. Multiple sensors reduce the error rate and provide better performance of the ED, so the probability of detecting the presence of a PU's signal is higher.

3.5 Conclusion

In this chapter, the spectrum sensing performance of an ED in Rayleigh fading and Laplacian noise is characterized. The effect of Laplacian noise on the detector performance metrics is studied through numerical simulation examples. Our results show the degradation in detection performance with increase in average mean value and variance of noise. To mitigate the impact of severe noise levels in a single CR, specific parameters such as λ, u, SNR were taken into consideration which increases the detection performance by minimizing the probability of error in decision making. Next, the scenario is extended to multiple antenna with L number of antennas each deployed with an ED. Square law selection method is used for multiple antenna in which the branch with the maximum decision statistics is selected and the decision is then made to determine the presence or the absence of a PU. Other than multiple antennas, cooperative spectrum sensing with K collaborating CRs are used and each cooperative spectrum sensors were deployed with an ED. The fusion centre (FC) combines the individual decisions made by each CR according to the OR rule and comes up with a final decision on the presence or the absence of the PU signal. Interestingly, multiple antenna and cooperative spectrum sensing techniques are proven to be beneficial to improve the reliability of spectrum sensing provided the above said parameters are chosen appropriately.

Chapter 4

Conclusion and Future Work

In this study, the spectrum sensing performance of an ED-based CR network is considered. Sensing performance of the ED in multipath fading channel modeled by Rayleigh fading and impulsive noise environment modeled by Laplacian noise is investigated. As expected, the ED performance is found to degrade more as the Laplacian noise mean and or variance larger. To mitigate the degradation in performance, the use of multiple antenna is considered by deploying the SLS combining scheme at the ED. Interestingly, multiple antennas are found to yield huge performance boost compared to single antenna-based CRs in the aforementioned context of Rayleigh fading and Laplacian noise. Motivated by the popular hidden terminal problem in spectrum sensing, which renders the spectrum sensing by a single CR very unreliable, the use of a number of collaborating CRs for detecting the presence or absence of the PU signal is considered next with the help of the suboptimal OR rule for fusing the d. As expected, the cooperation among different CRs improves the overall quality of spectrum sensing by providing remarkable gains compared to a single CR-based spectrum sensing.

Further extension of the work may involve investigation of the quality of spectrum sensing in other types of non-Gaussian noise models such as Gaussian mixture, generalized Gaussian, Cauchy noise etc. Another interesting extension would be to investigate the problem of spectrum sensing in multipath fading and non-Gaussian noise under very low SNR regimes, which may arise when the received signal is very weak and comparable in magnitude to the noise at the receiver.

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