Cut-off Grade Optimization in Open-pit Mines Considering Two Processing Streams and Rehabilitation Cost

by

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ABSTRACT

Cut-off grade is one of the important aspects of mining engineering. It distinguishes which material is worth mining and processing or stockpiling and which one should be left or dumped as waste. The success of many mining operations now days is measured by the net present value generated. Maximization of NPV, therefore, becomes an important process. This goal is inherently achieved by a cut-off grade policy which involves establishing a planned sequence of cut-offs and the associated amounts of material that will flow through different stages of the operation over time. To maximize NPV, the cut-off grade policy ensures that high grade materials are mined earlier to generate high cash flows earlier.

Environmental protection is one of the critical elements required to achieve sustainable development. This aspect has therefore gained high priority in modern mining. Mining industry produces solid, liquid, gaseous wastes. Such by-products are of uneconomic value and they pose a threat to the environment. Mine planning should integrate the strategy and cost related to rehabilitation of the mine wastes.

A need to incorporate rehabilitation cost of the waste rock in cut-off grade optimization process has been addressed in some past studies. However, the proposed models are limited to a single source of material, single processing stream, and single refinery. Furthermore, Lane's method has been extended to form algorithms that aid determination of cut-off grades in operations that utilize multiple processing streams. Uninterestingly, such studies share a common drawback; ignoring rehabilitation cost of the mine wastes and more specifically, waste rock

This study aims to develop and implement an integrated cut-off grade optimization framework that takes into account the rehabilitation cost of the waste rock and geological uncertainty of the orebody for an operation employing multiple processing streams in

exploiting the ore resource to maximize the NPV. By using fifteen (15) simulated grade tonnage curves, three scenarios, 1, 2, and 3 related to different consideration of the rehabilitation cost of the waste rock in optimizing the cut-off grades were analyzed. By fully incorporating the rehabilitation cost in the whole process of deriving the optimum cut-off grade strategy, two benefits are realized over the method of deducting the cost from the annual profits. First, more material is classified as ore resulting in the decrease of the amount of waste that is rehabilitated by 4.33%. Second, the average NPV of the operation is improved by 1.41% compared to the counterpart approach.

Incorporating geological uncertainty finds more application in the prefeasibility stage, where the project's risk-profile can be studied by applying the model across several equally probable grade-tonnage curves, aiding decision makers to make well-informed decisions at the pre-feasibility stage when little information about the orebody is available.

This Thesis is Proudly Dedicated To:

My father, Sylvester Kalitenge, who molded me into the way I am today. I will always miss his care, love, and support in my life. May his soul rest in eternal peace.

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LIST OF ABBREVIATIONS

BCOG	Break-even cut-off grade
CIC	Carbon-In-Column
CIL	Carbon-In-Leach
HL	Heap Leach Facility
LOM	Life of Mine
MCOGO	Modified Cut-Off Grade Optimization
NPV	Net Present Value
NSR	Net Smelter Return
POX	Pressure Oxidation
SR	Stripping Ratio

LIST OF NOMENCLATURE

Indices and Sets

$k \in \{1,\ldots,K\}$	Index and set for all grade categories.
$p \in \{1,2\}$	Index and set for all processing streams.
$T \in \{1,\ldots,T_N\}$	Index and set for all production periods.

Parameters

c_p	Processing cost per tonne of ore by stream p (C \$/t).
f	Annual fixed cost (C \$/yr).
$\bar{g}_p$	Average grade of ore processed by stream p (g/t).
d	Discount rate (%).
h	Rehabilitation cost per tonne of waste rock (C \$/t).
m	Mining cost per tonne of material (C \$/t).
S	Commodity selling price (C \$/oz).
r	Refining unit cost (C \$/oz).
y	Metallurgical recovery (%).
y_p	Metallurgical recovery associated with processing stream p (%).
g	Grade of material (g/t).
g_m	Mining limiting cut-off grade (g/t).

g_c	Process limiting cut-off grade (g/t).
g_r	Refining limiting cut-off grade (g/t).
g_{mc}	Mine-mill balancing cut-off grade (g/t).
g_{cr}	Mill-refinery balancing cut-off grade (g/t).
g_{mr}	Mill-refinery balancing cut-off grade (g/t).
$\overline{g}_c$	Average grade of ore above cut-off grade g_c (g/t).
$\overline{g}_{cp}$	Average ore grade above cut-off grade g_c for processing stream p (g/t).
g_k	Lower grade boundary in the grade category k (g/t).
g_{k+1}	Upper grade boundary in the grade category k (g/t).
$TO(g_c)$	Total amount of material above cut-off grade g_c (t).
$TO(g_{cp})$	Amount of ore corresponding to processing stream p (t).
$TO(g_{cpm})$	Total amount of ore with reference to the minimum cut-off grade (t).
$TW(g_{cpm})$	Total amount of waste below minimum cut-off grade (t).
T_k	Tonnage of material in grade category k (t).
G_{mc}	Effective optimum cut-off grade satisfying simultaneously mining and concentrator capacity constraints (g/t)
G_{mr}	Effective optimum cut-off grade satisfying simultaneously mining and refining capacity constraints (g/t).
G_{rc}	Effective optimum cut-off grade satisfying simultaneously refining and concentrator capacity constraints (g/t).
G	Optimum cut-off grade (g/t).
X	Ongoing capital expenditure (C$).

B	Minimum profit or net cashflow (C$).
P	Annual profit (C$/yr).
P_T	Cash flow generated during period T (C$).
v	Increment in present value per unit of resource used.
V	Present value (C$).
C	Maximum annual processing capacity (t/y).
C_p	Maximum annual processing capacity for stream p (t/yr).
M	Maximum annual mining capacity (t/yr).
R	Maximum annual refining capacity (t/yr).
Q	Total quantity of material remaining in the resource (t).
Q_c	Quantity of ore processed (t).
Q_{cp}	Quantity of ore processed by stream p (t).
Q_m	Quantity of material mined in period (t).
Q_{mT}	Quantity of material mined in period T (t).
Q_r	Quantity of refined product (oz).
Q_{rT}	Quantity of refined product in period T (oz).

CHAPTER 1

INTRODUCTION

This chapter presents the background and the general structure of this study. The definition of the problem pertaining to this research is stated. Furthermore, a summary of the literature review, objectives of this study, limitations and scope of the study, and research methodology are addressed.

1.1 Background

Mining is the first operation in the exploitation of mineral or energy resources. It aims to extract material from the ground that bears one or more components of interest from the mined material upon processing. Mining operations produce metallic or industrial minerals from the mineralized bodies called deposits. Natural processes and minerals within them form the deposits are unevenly distributed. The concentration of the mineral in a volume of material is simply referred to as the grade. Some material available in the deposit does not have enough concentration of mineral to be economically processed. The decision as to whether the material should be left in place or mined and get processed, stockpiled, or dumped is made by the so-called cut-off grade (Hall, 2014; Lottermoser, 2010; Rendu, 2014; Thompson and Barr, 2014).

Different mining operations release different types of wastes. The three core activities of the mining industry are mining, processing, and metallurgical extraction. They all produce solid, liquid, and gaseous wastes. Such by-products are of uneconomic value. Open-pit and underground mining generate waste rocks, overburden, spoils, mining water, and atmospheric emissions in metal mines. Mineral processing produces tailings and mill water. Pyrometallurgy, hydrometallurgy, and electrometallurgy generate slags, leached ores, and process water. All activities commonly cause atmospheric emissions, mainly in the form of greenhouse gases (Lottermoser, 2010)

While exploration costs have been increasing sharply over the past six decades, the gold mining industry has been facing volatile prices and a significant reduction in funds available for exploration. Because of these reasons, the industry is experiencing a rapid decline in the average head grade of ore mined and consequently the quantity of gold being mined, suggesting that the inventory of high-grade, high-quality deposits is being depleted. Furthermore, poor investments in recent years have narrowed the ability of the industry to carry out effective explorations resulting in few new deposits being reported (McKeith et al., 2010).

In pursuit of solving this alarming challenge, mine operations should focus on the technology that reduces capital and operating expenditures while ensuring that metal recoveries and the values of the operations are simultaneously maximized. The application of multiple processing streams has become an important strategy in recovering gold from various deposits as the mineralized body can be reclassified into zones depending on the ore type or mineral content, and each zone can then be routed to a more suitable recovery method (Asad and Dimitrakopoulos, 2013; Kappes, 2002; Pettingell, 2017).

The environmental aspect is one of the elements encompassed by sustainable development and due to this, environmental protection has gained high priority in modern mining. Waste disposal on the surface in the form of tailings and waste rock, waste material containments such as tailings dams, and waste dumps are of much concern in mine waste management. Due to mining activities, tailings may release toxic chemicals to water bodies and pose a survival threat to aquatic organisms. In addition, oxidation of sulfidic waste rock at elevated temperatures in the presence of meteoric water releases low pH water to the environment, leading to acidic drainages accompanied by mobilized toxic, corrosive, or radioactive metals (Craw, 2001; Liu et al., 2017).

The net present value (NPV) which is obtained by summing all future profits discounted by using an appropriate interest rate, is the standard and common criterion in dealing with unsteady economic conditions (Minnitt, 2004). Subject to mine, mill, and refinery capacity constraints, the maximization of this parameter requires using the optimum cut-off grades determined by the most popular algorithm proposed by Lane (Lane, 1988). While mines spend quite a lot of dollars rehabilitating the wastes generated during commodity production, Lane's method ignores these costs during mine design and planning. That being the case, the established cut-off grades are unrealistic, so is the NPV (Ramirez-Rodriguez and Rozgonyi, 2004). Efforts to incorporate the cost in Lane's model have been made in the past two decades. However, all modifications done to the original Lane's algorithm are constrained to a single processing stream.

As stated earlier, one solution of exploiting the finite resource to maximize commodity recovery and the value of the operation, whenever economically and technically practical, is through using multiple processing streams. Multiple processing streams reduce blending requirements, mitigate geologic uncertainty, allow the mine to processing more material at a given time, and push forward the revenue due to processing each ore type using an appropriate method (Pettingell, 2017). Asad and Dimitrakopoulos (2013) and Pettingell (2017) provide the models based on Lane's theory that establish the optimum cut-off grade policies for mine operations that employ multiple processing streams. Uninterestingly, the extended models do not consider rehabilitation costs of the mine wastes and hence they suffer the problem of giving sub-optimal cut-off grades and the NPV.

This research proposes a methodology that considers the rehabilitation cost of waste rock in optimizing cut-off grades for an open-pit mine operation with multiple processing streams.

1.2 Statement of the Problem

Traditional mine plans rely on break-even cut-off grades to decide the destination of material from the open-pit. Such cut-off grades are solely derived by considering price and costs to define the discrimination point between ore and waste. They are clearly constant and fail to consider the geology of the deposit and the processing capacities of various components of the mining system. Lane (1964, 1988) provides an algorithm that considers the financial parameters, geology of the deposit, and the capacities of the components of the mining system. The primary goal of many mining operations is to maximize the value. Lane (1964) points out three possible definitions that are compatible with this goal:

a) Maximize total profits. Optimization to fulfill this goal is done by using a zero discount rate.

b) Maximize the present value of all future profits. The optimization process that aims to maximize the NPV applies the appropriate discount rate and a series of profits of the given operation.

c) Maximize short-term profits. This case uses an arbitrarily high discount rate.

By observing objectives (a) through to (c), one can notice that objectives (a) and (c) are special cases of definition (b). Hence, all three objectives are fulfilled by optimizing objective (b).

Mining operations focus on extracting and processing material to get a product of interest that is finally sold to generate revenue. Therefore, operations may use one or multiple processing streams to recover interest's metal(s). The decision to use multiple processing streams is mainly influenced by the existence of:

a) Different ore types with single or multiple metals of interest. In this case, different processing methods are applied to separate ore types to achieve optimum metal recovery.

b) Existence of a deposit with homogeneous ore type that warrants amenability of the available ore to more than one processing stream in line with maximization of metal recovery.

The existence of multi-metal deposits may necessitate an operation to use multiple refinery streams to get the desired marketable products. In addition to the situations above, processing the ore by employing multiple methods should maximize the NPV of the operations. Further description of what multiple processing streams entails and specific examples of mines that use several recovery streams to recover the available metals of interest is presented in section 3.2 of this thesis.

Many stakeholders benefit from the mine operations in varied ways but most commonly in revenue, including shareholders, the mineral owner, employees, customers, regional authority, local community, and lenders (Sinding and Larsen, 1995).

From the perspectives of individual stakeholders, Sinding and Larsen (1995) assert that:

a) The parent company or major shareholders are more concerned with the overall financial performance of the company.

b) Lenders are interested in the debt service rather than the optimal cut-off grades, and hence they are attracted by stable cash flows over the repayment period.

c) The mineral owner, most commonly the state in the form of a regional authority, collects tax revenues from the operations using different tax instruments based on either profit, calculated net rent, or revenue. Rent is defined by Ricardo (1821) as "that portion of the produce of the earth, which is paid to the landlord for the use of the original and indestructible powers of the soil." Other tax instruments may base on the physical mineral output or fixed annual payments.

d) The operation creates environmental externality effects on the surrounding community simply termed as neighbors. The level of benefits moderates these effects in the form of environmental problems both direct and indirect the neighbors derive from the mining activity. The direct benefit is the employment of the residents in the production, while the indirect benefit is through transfers from the regional authority.

e) The customers of a mine are smelters and refiners, and they carry out the downstream processing of their output. Long-term contracts govern the relationship between the mine and its customers.

While the company may be required to serve the interests of different stakeholders, more often from the generated revenue, the model proposed by Lane (1988) ignores some of the stakeholders' needs, and hence relying on it may substantially give sub-optimal results. In other words, the mine operation is said to be optimal if the benefit derived by the stakeholder is figured in the cut-off grade optimization process. Sinding and Larsen (1995) emphasize that each stakeholder's need uniquely affects the cut-off grade policy. For instance, debt service, the company's share performance on the stock market, or internal finance needs may lead to a different cut-off grade policy, typically higher than the one determined by normal

Lane's algorithm. The mineral owner and employees/local residents/neighbors prefer a cut-off grade that maximizes the mine life. From the environmental point of view, the level of environmental disturbance is a function of the duration of the mining operations and the extent of the operations. The cut-off grade policy determined by the initial Lane's method results in a shorter and more concentrated operation, followed by closure. Under this practice, a lower quantity of high-grade ore is expected by a smaller amount of waste.

The effect of different stakeholders' needs on Lane's cut-off grade policy has been identified. Sinding and Larsen (1995) have collectively named these needs as factors influencing cut-off grade policy. Figure 1-1 is a complex loop showing the interaction between such factors.

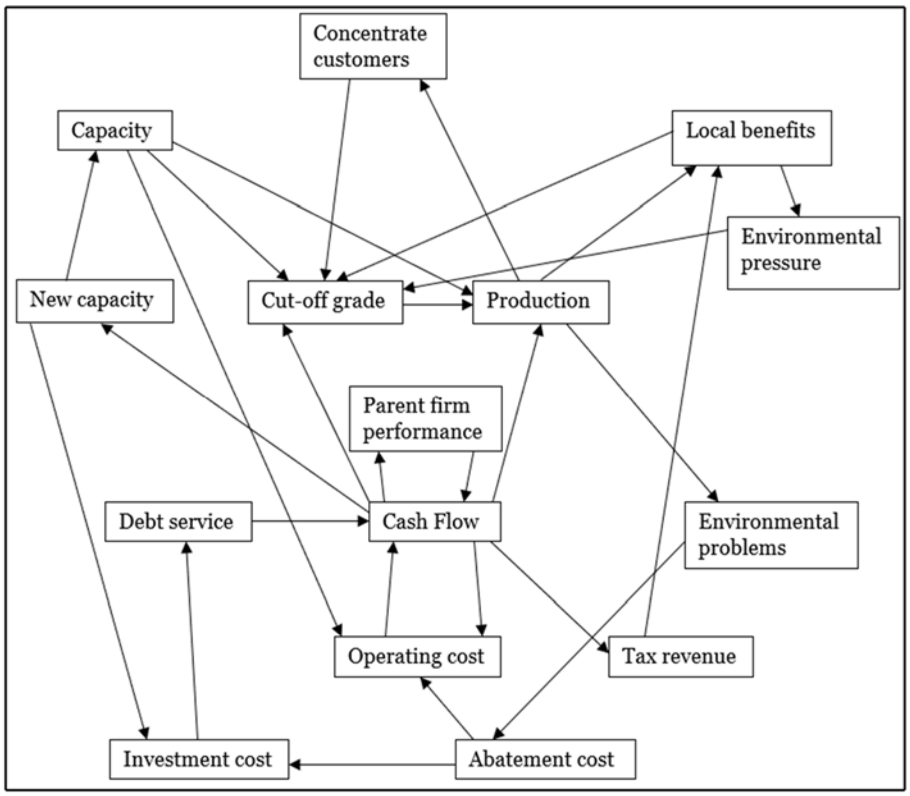

Figure 1-1: Complex loop diagram of factors influencing cut-off grade policy (after Sinding and Larsen,1995).

Despite its ability to facilitate achieving results that align with the goals of many mining operations, the application of the original Lane's method is limited to a single source of material and a single processing stream. Also, the method does not consider the rehabilitation cost of various mine wastes. The overall NPV of the operation reported from the application of this method is unrealistic and sub-optimal since it is subject to deduction of a one-time cost required to rehabilitate the waste. The one-time cost deducted is not figured in the cut-off grade policy used to generate the overall NPV. Owing to these failures, Osanloo et al. (2008) and Gholamnejad (2009) extended Lane's method by incorporating environmental costs. By applying the modified models, lower cut-off grades relative to those determined using the original Lane's algorithm are realized. With lower cut-off grades, the amount of material mined as ore increases, and as a result, the quantity of waste generated decreases.

Furthermore, the overall NPV of the operations is maximized and realistic. In the context of multiple processing streams, the original Lane's model can not directly be used to optimize their cut-off grades. Asad and Dimitrakopoulos (2013), and Pettingell (2017) present modified Lane's algorithms capable of establishing cut-off grade policies for operations employing multiple processing streams. The study done by Pettingell (2017) shows a significant increase in recovered metal and NPV when multiple processing streams are used relative to exploiting the same finite resource by using a single processing stream.

Asad and Dimitrakopoulos (2013), Pettingell (2017), and Githiria and Musingwini (2019) emphasize the importance of optimizing the cut-off grades and consequently the NPV by using a set of simulated grade-tonnage distributions of the orebody. The simulated grade-tonnage distributions account for the geological uncertainty of the orebody due to few samples and variability of the grade and tonnage parameters across the deposit, creating the uncertainty of the grade and tonnage estimates of the resource.

The aforementioned modified models that incorporate environmental costs are limited to a single processing stream. Also, the models used to optimize cut-off grades in the presence of multiple processing streams ignore the environmental cost of the wastes generated in mining operations.

An integrated cut-off grade optimization model that considers the rehabilitation cost of the waste rock and valid to multiple processing streams is developed in this research aiming primarily to maximize the NPV of the mine operation.

This research seeks to answer the following question:

Can an integrated model for an open-pit mining operation consisting of a single source of material, two processing streams, and one refinery be developed to optimize the cut-off grade policy under consideration of the rehabilitation cost of the waste rock and geological uncertainty of the deposit?

1.3 Summary of Literature Review

Cut-off grade is a decision criterion that is used to discriminate between ore and waste within a mineralized material. Material with a grade above the cut-off grade has sufficient economic value to cover the associated production costs and is classified as ore. Once mined, ore is sent to the processing facility to get a salable product or is stockpiled to cater to future needs. Contrary, the material below the cut-off grade, termed as waste, has a mineral content that can not cover the costs when mined and processed and is left in place or mined to expose the ore. Excavated waste is dumped in the waste dump. Besides distinguishing which material should be mined and processed/stockpiled or which material should be wasted, a cut-off grade is used to estimate the size of the reserve as well as for deciding the most appropriate processing method (Dagdelen, 1992; Hall, 2014; Lane, 1988; Pettingell, 2017; Rendu, 2014).

Three different theories propose the way of determining this criterion for an open-pit mining operation. The break-even analysis used by many companies to determine cut-off grades considers the commodity price, costs, and metallurgical recoveries to establish a point where the costs of producing that commodity equal the revenue generated from the commodity (Hall, 2014). This model ignores the grade-tonnage distribution of the mineralization, the time value of money, and the capacities of the components of the mining system. Furthermore, this method is suitable when the objective of an operation is to maximize undiscounted profits. However, it does not guarantee if the desired profit will be achieved.

Mortimer (1950) addresses two important conditions that material should meet to be classified as ore. First, the average grade of rock must provide a certain minimum profit per tonne treated. The average grade is obtained from the grade-tonnage distribution of a given orebody and hence contrary to break-even, this method honors the grade distribution of the orebody. Second, the lowest grade must pay for itself. Under this case, it is the break-even cut-off grade (BCOG) that is determined. Once the two cut-off grades required to evaluate each condition are known, the larger of the two is chosen as the final cut-off grade, and it will be used to determine the average grade of the material that should be sent to subsequent stages of the mining operation (Hall, 2014).

The NPV is a standard and widely accepted criterion in dealing with unsteady economic conditions. It is calculated as the sum of all future cash flows discounted by an appropriate rate of interest, which should at least equal the cost of capital. The process of developing an optimum cut-off grade strategy is complex because it can not be easily determined nor measured with precision by using a single parameter (Minnitt, 2004; Osanloo et al., 2008). Lane (1964, 1988) addressed the flaws associated with the BCOG model and proposed a pioneering cut-off grade optimization technique that maximizes the NPV of the operation subject to the capacities of different stages of the mining system. The inputs in the model proposed by Lane include the financial parameters, grade-tonnage distribution of the deposit, and the capacities of the components of the mining system.

A mine may adapt different cut-off grades to fulfill specific purposes. Pasieka and Sotirow (1985) point out different cut-off grades used for operational and strategic planning. Different cut-off grades are used to delineate ore and waste in operational planning to achieve different short-term objectives. On the contrary, strategic planning helps to establish the project's overall life in conjunction with long-term goals. Four cut-off grades for operational planning are identified, and they include break-even, minimum(marginal), budget, and accounting cut-off grades. The cut-off grades used for strategic planning include geological and planning cut-off grades (Pasieka and Sotirow, 1985).

Since the seminal work presented by Lane (1964) and his subsequent book in 1988, numerous studies have been undertaken in cut-off grade optimization, and most of them have focused on modifying this formulation.

Osanloo and Ataei (2003) challenge the methods used to determine cut-off grades in multi-metal deposits, such as using value per ton of ore calculated from the net smelter return (NSR), critical level method, single cut-off grade approach, and dollar value cut-off grade approach. These methods ignore the grade-tonnage distribution of the deposits, capacities of the mining system, and the time value of money. Lane’s methodology is restricted to a single metal deposit, and only six cut-off grades are possible candidates of the optimum cut-off grade. In their study, Lane's model is extended to incorporate the effect of the presence of multiple metals in deposits so that real optimal cut-off grades are obtained while grade distribution, capacities, and the time value of money are honored.

Barr (2012) and Thompson and Barr (2014) propose a cut-off grade model that considers the price dynamics by using a full stochastic future curve to predict future prices rather than a stochastic spot price model. Their studies lie on the fact that many mines operate for several years such that the commodity price and cash costs associated with the production change with time. Moreover, Asad (2005a) and Barr (2012) assert that operators are unable to decide on the commodity price. The only option they have is to respond to what is determined by international markets. Barr (2012) and Thompson and Barr (2014) find that not integrating price uncertainty in cut-off grade optimization leads to higher cut-off grades, resulting in wasting valuable resources. These studies were preceded by a study done by Asad (2007) where

Lane's based model was developed to optimize the cut-off grades under stochastic price and costs.

Githiria and Musingwini (2019) extended deterministic Lane's algorithm by developing a stochastic cut-off grade model that simultaneously considered variability in both commodity price and grade-tonnage distribution to establish dynamic cut-off grades over the life of the mine.

Asad and Dimitrakopoulos (2013) extended Lane's algorithm to form a model that can optimize cut-off grades at an open-pit mining project with multiple processing streams.

The studies done by Githiria and Musingwini (2019) and Asad and Dimitrakopoulos (2013) uses several equally but probable simulated grade-tonnage curves of the deposit to account for the effect of geological uncertainty on maximum NPV. For instance, by using 15 simulated grade-tonnage curves, Asad and Dimitrakopoulos (2013) observe a difference of 13.8% between the maximum and minimum NPV across the simulated grade-tonnage curves. The two extensions can be cited as risk-based models as they can quantify the risk associated with relying on a single grade-tonnage distribution of a deposit by providing alternative policies as part of the feasibility studies.

The mine wastes generated in the mining industry include but are not limited to waste rocks and tailings. Waste rocks, especially those rich in sulfide but poor in carbonate content, tend to release low pH water to the environment, leading to acidic drainages once oxidized at elevated temperatures and in the presence of meteoric water. In addition to AMD, the process may release to the environment metals of various concentrations (Craw, 2001; Vriens et al., 2019). In 1987, the World Commission on Environment Development WCED (1987) defined sustainable development in the so-called Burntland report as a development that "meets the needs of the present without compromising the ability of the future generation to meet their own needs". The concept requires mining systems to be re-engineered by incorporating economic, environmental, and social issues.

Some of the studies have put efforts in developing models that maximize the NPV of the operations and honor the policy of modern mining; mining for sustainable development. Osanloo et al. (2008) incorporated environmental costs related to the extraction and processing of porphyry copper deposits into cut-off grades modeling for a single source of material, single processing stream operation. The costs taken into account are those pertaining to waste rocks and tailings. Similarly, Gholamnejad (2009) extended Lane's model by integrating the rehabilitation cost of the waste rock. The two models show an increase in NPV relative to that obtained by using cut-off grade policies established by the original Lane's method. Furthermore, results from both studies reveal that incorporating rehabilitation costs related to mine wastes leads to lower cut-off grades compared to the cut-off grades established by the traditional Lane's technique. Therefore, the amount of material mined and treated as ore increases while the amount of waste rock that will be taken to the waste dumps decreases.

While a lot of researches have been conducted on cut-off grade optimization in open-pit mines, the following drawbacks can be evidenced:

1) Most of the proposed cut-off grade models are modifications to Lane's theory, and they are limited to an operation that makes use of a single processing stream.
2) Many mining operations use a fixed BCOG model to decide the destination of the material. This model fails to consider the production capacity constraints and the grade-tonnage distribution and may substantially fail to schedule material in a way that maximizes the value of the operation.
3) Few studies have been conducted on multiple processing streams. Still, none of them considers the environmental cost related to mine wastes in the process of determining the optimum cut-off grade strategies.
4) Most of the studies assume fixed prices and costs over the entire life of the mine. The cut-off grade policy and NPV generated by the models are sub-optimal as suggested by Barr (2012) and Thompson and Barr (2014) since prices and cost are subject to change with time (Asad, 2005a, 2007).
5) Almost all models assume a single source of material and a single refinery system. Consideration should be to develop and implement a mathematical formulation that accounts for multiple mines and refineries.

1.4 Objectives of the Study

This research aims to develop and implement a modified cut-off grade optimization method that considers the rehabilitation cost of the waste rock to maximize the NPV of the open-pit mining operation that uses two processing streams to exploit a low-grade homogeneous gold deposit. The proposed model incorporates concurrently two processing streams; a heap leach facility (HL) and carbon in leach plant (CIL) to exploit a set of equally probable simulated grade-tonnage curves to establish the cut-off grade policy and decide a suitable destination of the material subject to capacity constraint of the processing facilities.

The following are the specific objectives of this research:

1) Apply extended Lane's method that ignores rehabilitation cost to develop an optimum cut-off grade strategy that maximizes the NPV of the operation that employs two processing streams to exploit the orebody.

2) Identify the optimum cut-off grade policy for an operation using two processing streams to maximize the NPV by applying the proposed Modified Cut-Off Grade Optimization model.

3) Assess the impact of incorporating rehabilitation cost on the NPV and production scheduling for an operation employing two processing streams.

4) Assess the impact of considering the stochastic grade tonnage data of the orebody in determining the value of the operation.

1.5 Scope and Limitations of the Study

This research focuses on developing a Modified Cut-Off Grade Optimization (MCOGO) model that considers rehabilitation cost of the material below the cut-off grade for an operation with a single source of material, two processing streams, and single refinery. The final pit limit is presumably known. The model gives a production schedule in terms of optimum cut-off grade strategy, quantities of waste-rock to be dumped and rehabilitated, and the amount of ore to be delivered to each processing facility considering the best combination that maximizes the NPV of the operation. The model proposed in this research is valid under the following conditions:

1) There is a single source of material for the operation.

2) The deposit consists of a homogeneous ore type with a single metal of economic interest.

3) Economic and operational parameters are deterministic.

4) The cost to rehabilitate mine wastes other than the material below COG will not be considered in the COG determination process.

5) Option to stockpile material is not part of this study. Hence, it is assumed that mining will take place in a manner that allows every ton of ore mined to be delivered directly to the designated processing facility for further beneficiation/treatment. However, it should be noted that determining cut-off grades by using Lane's based models leads to higher grading of material. Therefore, the material between break-even and optimum cut-of grades is likely to be ignored if no additional strategy is incorporated in the mining operation. Interestingly, such material can be worth processing when there is a spare capacity at the processing plant or latter stages when the resource is depleted.

This suggests that material between the break-even and optimum cut-off grades should be stockpiled. The cashflows generated by processing material from the stockpile push forward the NPV of the mine operation.

6) Processing capacity is the only factor limiting the operation.
7) The model assumes that a resource and an ultimate pit limit with no multiple pushbacks had been defined along with the stated economic, operational, and simulated grade-tonnage data.

1.6 Research Methodology

Rehabilitation of mine wastes is an important and integral part of modern mining. A traditional Lane formulation is improved in this research to form a so-called MCOGO model. The proposed model aims to maximize the NPV of an operation that utilizes multiple processing streams and considers the relevant cost to rehabilitation waste rock subject to capacity constraints.

The following tasks were undertaken to achieve the objectives of this study:

1) Develop a heuristic cut-off grade optimization model for two processing streams that do not incorporate a rehabilitation cost of the waste rock based on Lane's theory.
2) Test and verify the developed cut-off grade optimization models stated in task 1 of section 1.6 on a hypothetical gold deposit presented in (Pettingell 2017).
3) Extending the heuristic cut-off grades optimization models developed in task 1 of section 1.6 by incorporating the rehabilitation cost.
4) Test and verify a modified cut-off grade optimization model of an operation with a single processing stream on the hypothetical deposit presented in (Gholamnejad, 2009).
5) Creating a total of 15 equally probable grade-tonnage realizations of the hypothetical gold deposit.
6) Apply the modified cut-off grade optimization model on the simulated grade-tonnage data representing the hypothetical gold deposit for the sake of data analysis in this research.

1.7 Scientific Contributions and Industrial Significance of the Research

The main contribution of this research lies in the development of a modified Lane's model that incorporates the rehabilitation cost of the waste rock and geological uncertainty of the deposit to establish the cut-off grade policy in operations utilizing two processing streams. In summary, the main scientific contributions and motives behind this research are as follows:

- By incorporating the relevant cost related to the rehabilitation of waste rock in the proposed model, the industry will have an opportunity to establish the cut-off grade strategy of open-pit mining operations utilizing two processing streams along with the resulting profits and NPV that align with the goal of operating a mining business that honors the concept of sustainable development as proposed by (Gholamnejad, 2009; Osanloo et al., 2008; WCED, 1987).
- The developed model in this thesis provides an initial production schedule for the mine's defined life, which is further used to set short-term and medium-term production planning goals.

1.8 Organization of Thesis

Chapter 1 of this thesis describes the background and the general structure of this study. The definition of the problem related to this research is stated. Furthermore, a summary of the literature review, objectives of this study, limitations and scope of the study, and research methodology are addressed.

Chapter 2 contains the review of different theories on cut-off grades. Different types of cut-off grades and their respective application in different phases of the mining project are described. A brief description of wastes generated in the process of operating a mining business is highlighted. A review of the previous cut-off optimization models and their application in mine design and planning is given. In this chapter, limitations of some past developed models are highlighted and it is from them a basis of this research is built.

Chapter 3 discusses the theoretical framework for the proposed optimization model. Lane's theory presents a description of the mathematical problem of cut-off grade optimization and a review of general procedures used to determine the optimum cut-off grade policy. Furthermore, a review of the past cut-off grade optimization frameworks on environmental aspects and multiple processing streams that form the proposed model's basis is presented. A description of the steps and their corresponding formulations of the algorithm used to optimize dynamic cut-off grades of an operation that simultaneously considers environmental aspects related to waste rock and uses using multiple processing streams is presented.

Chapter 4 discusses the application of the integrated model to a hypothetical deposit of an open-pit gold mine. A detailed description of the simulation of the grade-tonnage distributions of the particular hypothetical deposit is given. A sensitivity analysis of different economic and operational parameters is conducted to optimize the cut-off grades and the operation's NPV. Finally, the strengths and limitations of the proposed model are stated in this chapter.

Chapter 5 contains the summary and conclusions of this thesis. Contributions of this research and suggestions for future work are also discussed.

CHAPTER 2

LITERATURE REVIEW

This chapter reviews different theories on cut-off grade. Applying an appropriate cut-off grade is one of the key factors driving the success of any mining operation. To achieve this, it is paramount to understand the nature of costs associated with mining operations. A brief description of different costs related to the operations and their applicability in cut-off grade derivation is given. A mine may be required to operate with different cut-off grades at different operation phases to serve different needs. Different cut-off grades and their applications during the mine life are demonstrated. Owing to the environmental impacts caused by mining activities, an overview of mine wastes in conjunction with the need to integrate costs related to mine waste rehabilitation in cut-off determination is highlighted. Furthermore, a lot of researches have been done since the publication of work on cut-off grade theory introduced by K. Lane in 1988. A general review of some prominent research is presented, and some form the basis of this research.

2.1 Cut-Off Grade Theories Transitions

Cut-off grade is a decision criterion that is used to discriminate between ore and waste within a mineralized material. Material with a grade above the cut-off grade has sufficient economic value to cover the associated production costs and is classified as ore, while material with a grade below the cut-off grade has a mineral content that can not cover the costs when mined and processed and is termed as waste Ore is mined and processed to get a saleable product or stockpiled to cater for future processing needs. Conversely, waste is left in place or mined to expose the ore. Mined waste is dumped in the waste dump. Besides distinguishing which material should be mined and processed/stockpiled or which material should be wasted, a cut-off grade is used to estimate the size of the reserve as well as for deciding the most appropriate processing method (Dagdelen, 1992; Hall, 2014; Lane, 1988; Pettingell, 2017; Rendu, 2014).

Cut-off grade is a major economic driver of any mining operation, and owing to this importance, there have been a number of efforts to develop models that can sufficiently be used for planning purposes. Many of the developed models are modifications of the following discussed fundamental cut-off grade theories:

2.1.1 Break-Even Theory

Break-even analysis is a one-dimensional process that is essentially based on financial parameters only. A combination of prices and metallurgical recovery is compared with the costs incurred and if the grade of the mineralized material can generate the revenue that pays for the costs associated with the chain of production from extraction to refining, that material is classified as ore. The break-even grade can therefore be defined as the grade at which the costs of producing a product equal the revenue obtained from the product (Hall, 2014). The financial parameters taken as inputs in the calculation of break-even cut-off grades are selling price (s), refining, market and/or sales cost (r), mining cost (m), processing cost (c), metal content or grade (g), and metallurgical recovery (y). The profit (P) per tonne of material can be derived as follows (Asad et al., 2016).

$$Profit(P) = (S - r) * g * y - m - p \tag{2.1}$$

At break-even, the profit in equation (2.1) equals zero and the break-even cut-off grade (BCOG) is therefore expressed as follows (Asad et al., 2016):

$$BCOG = \frac{m + c}{(S - r)y} \tag{2.2}$$

Since break-even cut-off grade appears in many variations, the general equation is stated as (Hall, 2014):

$$BCOG = \frac{Costs}{Product\,Unit\;Price \times Recovery} \tag{2.3}$$

Hall (2014) describes a number of variations of the break-even cut-off grades and the costs that should be included in their determinations. However, different areas of the project may apply different break-even cut-off grades. Based on the costs considered in the calculations, the following break-even cut-off grades and their related costs are identified:

- Marginal break-even which includes mining and milling variable costs.
- Mine operating break-even which includes total mining costs and total milling costs.
- Site operating break-even which includes total mining costs, total milling costs, total site administration, and services costs.
- Full cost break-even which includes total mining costs, total milling costs, total site administration and services costs, head office charges, and capital allowance.

Many operations use a combination of two or more of these cut-off grades with the first cut-off grade used as an operational cut-off grade for specifying the marginal ore that can be treated when spare capacity in the treatment facility is available. In this case, the marginal ore should not displace the ore originally planned to be processed to avoid incurring an opportunity cost (Hall, 2014; Rendu, 2014). Cut-off grades calculated similarly to third and fourth fashions are used for identifying ore and waste in the long-term mine plan.

The break-even model is widely used in the industry despite its limitations. Some of the very noticeable features of the model include ignoring the grade distribution of the mineralization and the capacities of the components of the mining production system. For that reason, some costs which are incurred during production are not included in the cut-off grade determination. One of the serious consequences that mines can face by relying on the method that does not consider the geology of the orebody and capacities of different components of the mining system is the failure to establish the optimum cut-off grades that can maximize the net present value of the operations. Furthermore, while the method can be used to ensure the profitability of the operations, there is no guarantee if the desired profit will be achieved (Hall, 2014).

2.1.2 Mortimer's Definition

Mortimer (1950) developed an important theory about cut-off grade determination known as Mortimer's Definition. The theory can be cited as a pillar towards other successful methodologies. It established a fundamental principle that material mined at the lowest grade must pay for its extraction cost, but the average grade of the material should provide a certain profit per ton processed.

According to this cut-off grade methodology, the material is classified as ore if it meets the following criteria:

- The average grade of rock must provide a certain minimum profit per tonne treated.
- The lowest grade of rock must pay for itself.

The first part of Mortimer's Definition focuses on identifying the minimum head grade required to deliver the specified minimum profit. However, this grade does not assure that the targeted level of profitability will be achieved. It just closely aligns with the goal of many companies where operations must ensure that a certain level of profit is realized. The grade required to achieve this goal is solely the minimum average grade of all material classified as an ore for a given planning time frame, not just a single block. For that reason, the computed grade is not used directly as a cut-off grade. The grade-tonnage curve of the given orebody during a specified time is used to determine the cut-off grade needed to give the required head grade (Hall, 2014). Figure 2-1 is used to explain the aforementioned concept with the grades expressed in g/t.

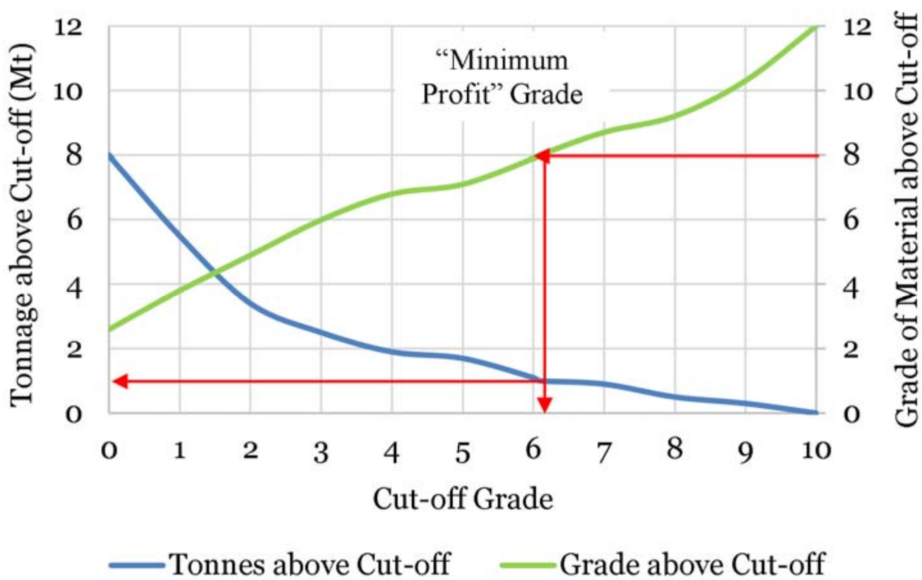

Figure 2-1. Grade-tonnage curve for determining the break-even cut-off grade from an average grade that provides minimum profit (after Hall, 2014).

Consider the red lines in Figure 2-1 , if the average grade required to deliver the minimum profit is 8 g/t, based on the grade-tonnage curve of the deposit, this average grade is achieved by mining material with a minimum grade of 6.1 g/t. Therefore, at this cut-off grade, 1.0 Mt of material is classified as ore.

The second part of the definition concentrates on identifying the break-even cut-off that will distinguish ore and waste. This cut-off ensures that each tonne mined pays for itself, and once determined, the average grade that will balance the costs and revenues can be determined from the grade-tonnage curve. In Figure 2-2, the required break-even cut-off grade is 3 g/t. This cut-off grade delivers 2.5 Mt of the material at an average grade of 6 g/t.

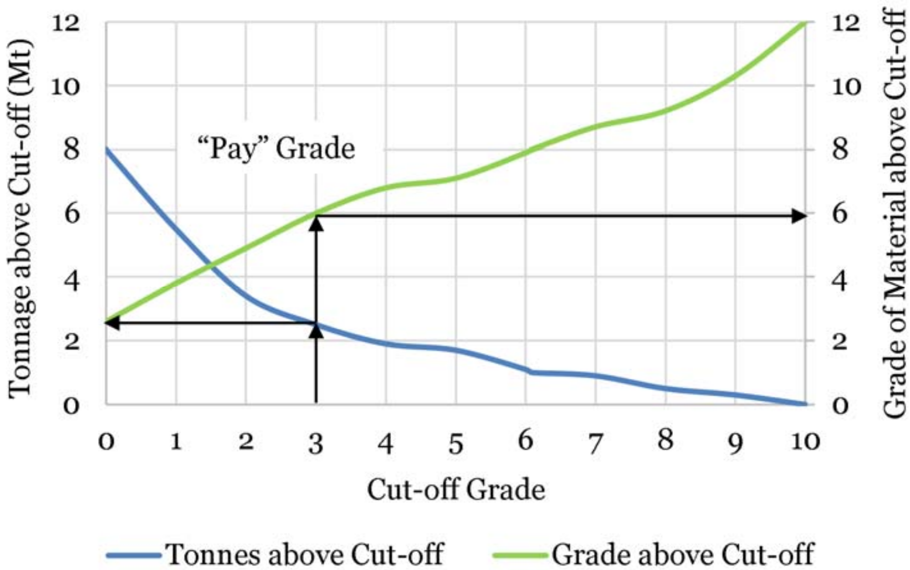

Figure 2-2: Grade-tonnage curve to identify the cut-off grade at which every tonne pays for itself (after Hall, 2014).

Now that two cut-off grades have been determined. A single cut-off grade that meets all requirements is chosen, and it is the larger of the two. For that case, 6.1 g/t will ensure that every tonne mined pays for itself while the required profit margin is also delivered.

Each criterion is associated with its own break-even cut-off grade because of different costs included in the determination of each. To achieve the minimum profit in the first part of Mortimer's Definition, the costs to be covered in the minimum profit break-even cut-off grade includes total mining costs, total milling costs, total site administration and services costs, depreciation and amortization, head office charges, interest on the debt, and the required profit margin. In contrast, the costs required for every ton mined to pay for itself comprises the differential costs of mining ore relative to waste, variable costs associated with ore and product, including sustaining capital, and fixed costs (Hall, 2014).

As stated above, both parts of Mortimer's Definition lead to break-even calculations, with the first part having more cost components. It will finally have higher costs and a larger break-even cut-off grade than the second part. The profit margin in the first part is treated as a cost in determining the minimum profit break-even cut-off grade.

The introduction of the concept of targeting some profits in Mortimer's Definition leads to an extension of the simple break-even model to consider grade distribution of the orebody in addition to costs and prices. Therefore, the theory can be regarded as a two-dimensional model of cut-off grades since it considers both financial parameters and mineralization's nature, especially the orebody's grade distribution.

It is very common for many mines to experience high charges on the capital invested in their early operations. At this point, the cut-off grade required to bring minimum profit is expected to be larger than the cut-off grade at which every tonne mined pays for itself. Some years later when the capital has been fully depreciated, some costs used initially to calculate the break-even cut-offs may be excluded, and any of the two cut-off grades may be larger than the other. Regardless of the operation phase, if Mortimer's Definition was used to determine the cut-off grade, the larger of the two cut-offs would be chosen (Hall, 2014).

2.1.3 Lane's Theory

The net present value is a standard and the most widely accepted criterion in dealing with the unsteady economic conditions. For that reason, it has become the most useful criterion in determining the optimum cut-off grades. It is calculated as the sum of all future cash flows discounted by an appropriate rate of interest, which should at least equal the cost of capital The process of getting optimum cut-off grade is complex because it can not be easily determined nor measured with precision by using a single parameter (Minnitt, 2004; Osanloo et al., 2008). In 1964, Lane published a technical paper on cut-off grade optimization that seeks to maximize the value of the operation. It was followed by the publication of a textbook in 1988. In his remarkable works, Lane honors the two basic concepts introduced in Mortimer's Definition. To maximize the value of the operation, Lane's model extensively includes capacities of various production stages and their associated costs in the determination of the optimum cut-off grade.

Additionally, the opportunity cost is another important parameter in cut-off grade optimization in this theory. Lane's methodology has been state-of-the-art in cut-off optimization for several decades. However, its use has been limited and criticized because of causing high grading of the deposit. Its explicit goal is compatible with the goal of many mining operations where the main target has been to maximize the net present value (Hall, 2014; Lane, 1964, 1988).

A proposed cut-off grade optimization model in this thesis is based on Lane's theory. Readers are advised to go through section 3.1 for more formulations and explanations on this theory.

2.2 Strategic and Operational Planning Cut-Off Grades

Most mines operate with different cut-off grades each intended to fulfill a specific purpose for a given circumstance. It is imperative to define appropriately the type of cut-off grade to be applied for a given circumstance (Hall, 2014). While the mine project starts from exploration to closure and reclamation stage, each phase of the project may require a unique type of cut-off grade. Pasieka and Sotirow (1985) point out different cut-off grades that are used for operational and strategic planning. Different cut-off grades are used in operational planning to delineate ore and waste aiming to achieve different short-term objectives. On the contrary, strategic planning helps establish the project's overall life in conjunction with long-term goals. Since the success of each planning activity is attributed to a specific cut-off grade, it is important to understand the part of the planning cycle which is being carried out in a given time to make use of the appropriate cut-off grade. Figure 2-3 summarizes different types of cut-off grades and the areas they fit in the planning cycle.

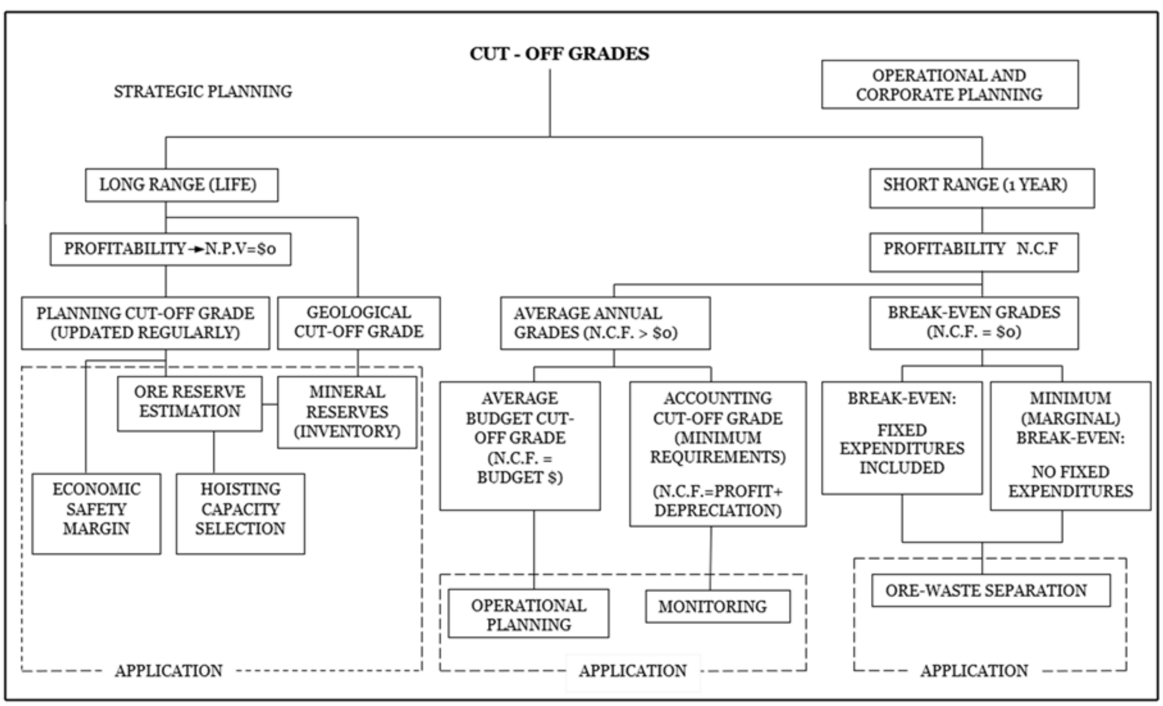

Figure 2-3: Cut-off grades and their respective purposes (after Pasieka and Sotirow, 1985).

2.2.1 Break-Even Cut-Off Grade

This cut-off grade generates an annual revenue equal to all costs incurred to generate that revenue, including fixed and variable operating costs, corporate and mining taxes, and all allocated capital expenditures in the whole operating year. Operating a mine at this cut-off grade results in zero net cash flow. It further finds application in defining the optimum pit outline and is referred to as mining cut-off grade or external cut-off grade if it is used for this purpose (Asad et al., 2016; Baird and Satchwell, 2001; Dagdelen, 1992; Pasieka and Sotirow, 1985). A typical break-even cut-off grade is defined as (Mugwagwa, 2017):

$$Break-even\ cut\text{-}off\ grade = \frac{m + c + \left(\dfrac{f + X}{C}\right)}{\left(S - r\right)y} \tag{2.4}$$

2.2.2 Minimum (Marginal) Cut-Off Grade

The marginal cut-off grade determines the lowest grade at which material can be mined and processed without making losses if no other material is available for the specified mining or processing capacity to generate positive net cash flow. It intends to meet the variable operating costs only and differs from the normal break-even cut-off grade in the sense that its calculation and consequently the net cash flow excludes administrative and other fixed and operating. Furthermore, all mining costs related to drilling, blasting, loading, and hauling are not included where these costs were spent to mine more profitable material. This cut-off grade is also referred to as milling cut-off grade in a situation where there is an excess milling capacity in open-pit operations. Once the ultimate pit limit is defined, the marginal cut-off grade can be used to decide whether a block should be mined and sent to the processing facility or mined and sent to the waste dump. The cut-off grade applied to distinguish blocks as ore or waste within the optimum pit shell has been named an internal cut-off grade (Baird and Satchwell, 2001). While Lane (1988) proposes that processing material at this cut-off grade helps to maximize the operation's undiscounted profits, the processing of more lucrative material is deferred and the operation incurs an opportunity cost. Because of this, applying this cut-off grade will not result in the maximum net present value (Asad et al., 2016; Baird and Satchwell, 2001; Hall, 2014; Lane, 1964; Pasieka and Sotirow, 1985).

Marginal cut-off grade is expressed as follows (Mugwagwa, 2017):

$$Marginal\ cut\text{-}off\ grade = \frac{c}{\left(S - r\right)y} \tag{2.5}$$

2.2.3 Budget Cut-Off Grade

This is the required yearly head grade for a specified capacity at which the mine should operate to generate a budgeted net cash flow that meets a certain corporate objective for the budgeted year (Pasieka and Sotirow, 1985). It is mathematically expressed as follows (Mugwagwa, 2017):

$$Budget\ cut\text{-}off\ grade = \frac{m + c + \left(\frac{f + Vd + B}{C}\right)}{(S - r)y} \tag{2.6}$$

2.2.4 Accounting Cut-Off Grade

This is the annual cut-off grade needed to generate sufficient revenue to pay for all operating costs, depreciation of fixed assets, and the minimum required profit for a specified capacity. Its use is very limited in scope as it finds application mainly in monitoring the economic utilization of a company's assets from an accounting point of view. Depreciation is a non-cash cost and its use may negatively affect the operational cut-off grade strategy, ore reserves assessment, and mine life (Pasieka and Sotirow, 1985). Accounting cut-off grade is stated as follows (Mugwagwa, 2017):

$$Accounting\ cut\text{-}off\ grade = \frac{m + c + \left(\frac{f + Vd + X + B}{C}\right)}{(S - r)y} \tag{2.7}$$

2.2.5 Geological Cut-Off Grade

This type of cut-off grade is used to generate grade frequency distributions from which grade-tonnage curves of the mineralized material are plotted. Geological cut-off grade is exclusively used to separate mineralized material from waste and to assess mineral reserves that form a basis for determining ore reserves (Pasieka and Sotirow, 1985). Hustrulid and Kuchta (2006) define ore reserve as a portion of the measured and/or indicated mineral resource that can be mined economically.

2.2.6 Planning cut-off grade

Besides developing mine plans and designs, this cut-off grade is used in long-term studies like feasibility and strategic studies. The NPV of the mining operations applying this cut-off grade is zero. Its calculation considers that each ton with this grade must generate revenue equal to all capital, operating, and financial costs, plus all taxes and royalties that will be paid from the operation during the project's entire life. Extensively, this type of cut-off grade can quantify the ore grade safety margin of the operation and define the minimum minable grade used for ore reserves estimation from which the mine or processing capacity can be determined (Hall, 2014; Pasieka and Sotirow, 1985). It is defined by the following formula (Mugwagwa, 2017):

$$Planning\ cut\text{-}off\ grade = \frac{m + c + \left(\dfrac{f + Vd + X}{C}\right)}{(S - r)y} \tag{2.8}$$

2.3 Classification and Behavior of Costs Used in Cut-off Grade Calculations.

As described in section 2.2, numerous types of cut-off grades are dedicated to achieving a specific purpose. What differentiates these cut-off grades are the cost components that each dwell on during its determination. Therefore, it is important to develop a clear understanding of the nature and types of costs that should go into different cut-off grades calculations.

Since cut-off grade simply distinguishes material in a mineralized body, Hall (2014) identifies rock, ore, and product as three types of material we usually deal with in the mining industry. In the context of classifying material, rock refers to all the material in place that is intended to be mined. The application of cut-off grade on the rock gives the distinction of the ore from the waste within the rock mass. The relationship between rock, ore, and waste is described using the following equation (Hall, 2014):

$$\text{Rock} = \text{Ore (treated or stocpiled)} + \text{Waste} = \text{Total material moved}$$

Hall (2014) defines ore as the mineralized material which is treated at the time of mining or stockpiled to serve for later needs. Processing an ore result in the metal concentrate or saleable product once the concentrate is taken through the refinery.

An overall value of the operation is largely influenced by the production level and the production costs associated with each type of material. Hence, rock, ore, and product are regarded as cost drivers. The behavior of costs varies between rock, ore, and product. Therefore, to determine the cut-off grade appropriately, costs must be separated and allocated based on the physical parameters that drive them (Hall, 2014).

Operating costs form a group of costs that are incurred during the operation of the mine when dealing with rock, ore, and product. Operating costs are classified into two groups; fixed and variable costs. Fixed costs are expenses that remain unchangeable within a given time regardless of the level of activity. They are stated in form of dollars per unit time; normally a year or month. Each time elapsed requires spending a certain amount of dollars irrespective of the production capacity or level of activity that has taken place. Equipment depreciation, administration, wages and salaries, marketing, pumping, interest expenses, business licenses, insurance, and permit fees are some common examples of fixed costs. In contrast, variable costs change in direct proportion to the level of activity. Variable costs are normally expressed in dollars per unit of production. For instance, the unit of production can be tonne mined, tonne processed, ounce produced, metres drilled, and kilograms of explosive consumed. Each unit dealt with represents spending of the respective unit cost and hence the higher the production capacity the higher the variable costs expended. Some examples of variable costs include drilling, blasting, loading, crushing and grinding, flotation, filtering and shipping, and smelting and refining (Hall, 2014; Mugwagwa, 2017; Rendu, 2014). Hall (2014) presents that fixed costs should be included in break-even cut-off grades since their use in classifying a piece of rock as ore results in extending the life of the operation. This extension happens when the processing facility or refinery is operating at capacity. Any extra tons of ore added in the stream will require extra time to be processed and refined and the extra time demands additional fixed cost. This fixed cost increment needs to be covered by the revenue generated from the additional ore, and hence this justifies a reason why fixed costs should be included in the break-even calculation. Rendu (2014) points out that fixed costs are always valid within a certain level of activity or the prescribed time. Suppose changes are made to the cut-off grade in such a way that the life of the mine is changed beyond the initially expected life or expansion of one or more of the mine facilities is required. In that case, the new fixed cost should be determined and taken into account in cut-off grade determination by considering which fixed costs have been affected. Pasieka and Sotirow (1985) highlight that fixed operating costs should not be included in the minimum (marginal) break-even cut-off grade calculations regardless of the fact that they are included in many cut-off grade calculations.

Costs incurred during pre-stripping, plant construction, and construction of the permanent infrastructure are not used to establish the cut-off determination. They are collectively termed as sunk costs. Sunk costs are expenses that were incurred in the past and are independent of the level of activity. Including them may result in higher cut-off grade values which in turn leads to lower reserves. The possibility of recovering the capital costs incurred is narrowed when there are lower reserves, making an operation uneconomical (Mugwagwa, 2017; Pasieka and Sotirow, 1985; Rendu, 2014). In conjunction with the initial capital cost, such costs only influence cut-off grade during the feasibility study (Rendu, 2014).

Costs incurred to maintain the current production level must be included in cut-off grade determination by adding them to an appropriate cost category. For instance, if the capital is used to replace a plant or to purchase a new truck that provides support or replace a portion of the existing fleet, the expected life span of the equipment and discount rate should be used to determine the annual cost that must be taken into account to recover the capital used to purchase that equipment. A combination of this annual cost and the production capacity that is expected to be delivered by the equipment is used to determine the cost per tonne associated with such equipment. This unit cost must be added to mining costs. The same holds where costs associated with a leach pad or tailings dam expansions must be added to leaching costs and milling costs respectively. Regular, ongoing costs incurred to maintain the current level of production are collectively termed as sustaining capital (Hall, 2014; Rendu, 2014).

Non-operating costs such as capital costs and working capital are deterministic by nature and can therefore be discounted and subtracted from the discounted cash flows to determine the final NPV(Barr, 2012).

Mining operations are always subjected to capacity constraints. The constraints happen in the mine, processing facility, or refinery mainly because of financial limitations or wrong designs. In the course of operating a mining business, unscheduled ore may be processed in preference to the scheduled ore. Because of the capacity constraint of the processing facility, processing new material may displace and hence postponing the originally scheduled ore from being processed. The consequence of this displacement is the delay of realization of the income from the initially scheduled ore. The decrease in net present value as a result of this delay is termed as the opportunity cost. It is important to understand that the same phenomenon and consequence may happen when rescheduling is done to extract material in the mine or refine the concentrate in the refinery (Rendu, 2014). Opportunity cost is a loss experienced by an investor for typing up capital in the present mining operation rather than investing in a more profitable option available at a given time. The process foregoes the profits that would have been received by investing in the best available option and hence incur a cost (Minnitt, 2004). High opportunity costs may make the operation uneconomical since it leads to a high cut-off grade which may significantly lower the amount of ore reserve (Rendu, 2014).

Lane (1988) indicates that there is no opportunity cost incurred if adding material to the initial mining, processing, or refining schedule will in no way cause any displacement. This is because the expected cash flow from the previous schedule is not affected by incorporating a tonne or more of unscheduled material.

2.4 The Concept of Grade-tonnage Distribution

In section 2.1, the required economic and operational inputs for cut-off grade determination were identified. In addition to the two aforementioned input categories, calculations of cut-off grades are possible in the presence of geological data. A block model is a primary geological input and is defined as a three-dimensional array of several minable blocks, each defined by spatial coordinates, grade, and the quantity of material. Data about grade and tons of the mineral inventory are used to construct a histogram where grade categories and their corresponding tons are presented (Asad et al., 2016; Pettingell, 2017).

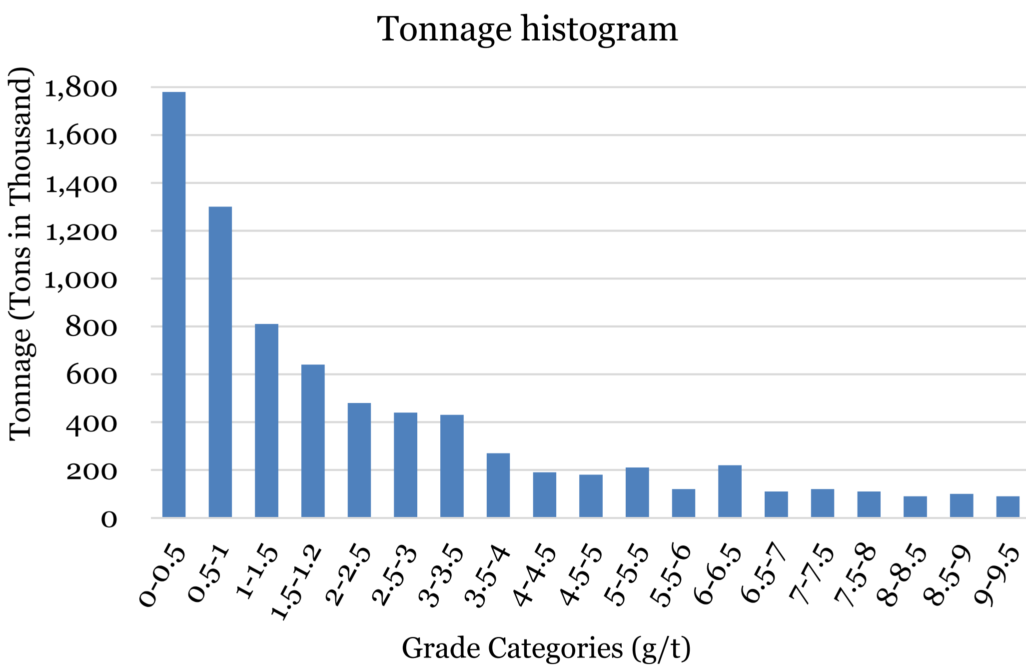

Figure 2-4: Tonnage histogram of a hypothetical deposit.

From Figure 2-4, the lower boundaries of the grade categories are taken as the cut-off grades. By knowing the cut-off grade g_c and the amount of material T_k within each grade category k, the cumulative tonnage $TO(g_c)$ is calculated by considering tons in grade categories k^* above the cut-off grade as per equation (2.9) (Asad, 2007):

$$TO(g_c) = \sum_{k \geq k^*}^{K} T_k \tag{2.9}$$

Where; k^* in equation (2.9) is the grade bin from which g_c is found.

Similarly, the weighted average grade $\overline{g}_c$ of the material above the cut-off grade g_c is computed as follows (Asad, 2007):

$$\overline{g}_c = \frac{\sum_{k \geq k^*}^{K} T_k \left(\frac{g_k + g_{k+1}}{2} \right)}{TO(g_{cp})} \tag{2.10}$$

The term in the brackets in equation (2.10) signifies the average grade corresponding to the material in a specific grade category.

Having determined different values of cut-off grades, tons above such cut-off grades, and the corresponding average grades, two graphs, one showing cumulative tonnages above cut-off grades and another showing the average grades of such tonnages above cut -off grades are presented in one curve which is simply referred to as grade-tonnage curve. Figure 2-5 is a plot reflecting how the cut-off grade influences cumulative tonnage and the average grade of a given deposit.

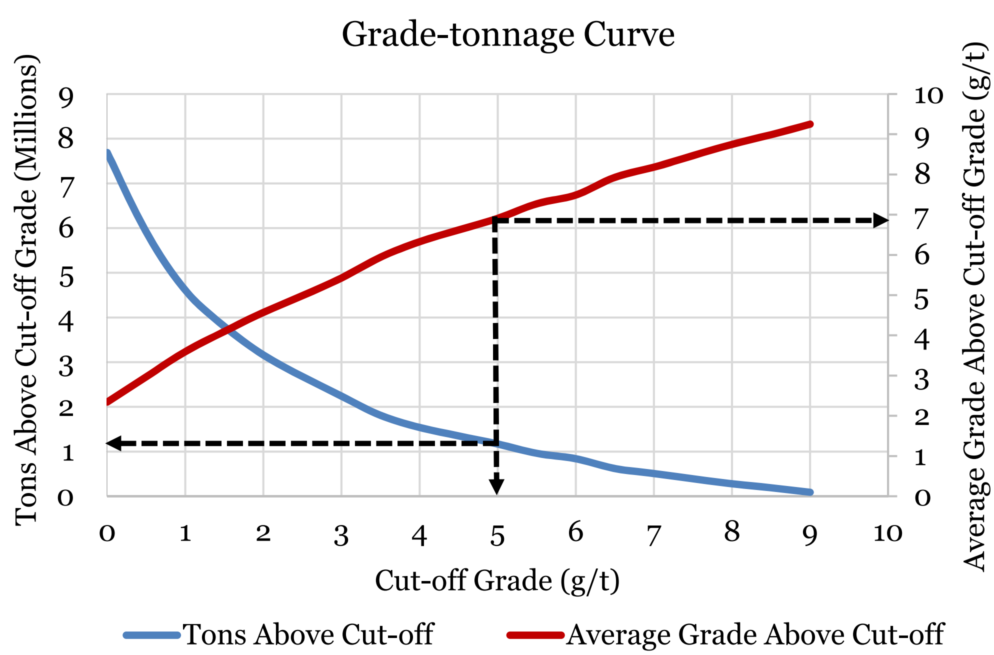

Figure 2-5: Grade-tonnage curve of a hypothetical deposit from Figure 2-4.

In Figure 2-5, if 5 g/t is the calculated cut-off grade, the cumulative tonnage and average grade of the material above this cut-off grade are roughly estimated as 1.17 Mt and 6.9 g/t respectively.

2.5 Mine Wastes

Mining is the first operation in the exploitation of mineral or energy resources. It aims to extract material from the ground which bears one or more components of interest from the mined material upon processing. Mining operations produce metallic or industrial minerals. Metallic minerals are found in metal-bearing ore while industrial minerals are found in rock of economic value. Most metallic ores do not contain the element of interest only. Metals tend to appear combined chemically with other elements in their ores (Lottermoser, 2010).

Different mining operations release different types of wastes. The three core activities of the mining industry are mining, processing, and metallurgical extraction. They all produce solid, liquid, and gaseous wastes. Such by-products are of uneconomic value. Remarkably, the mining industry is cited as a major producer of solid waste among many industries with approximately 20,000–25,000 Mt being produced annually (Lottermoser, 2010).

Open-pit and underground mining generate waste rocks, overburden, spoils, mining water, and atmospheric emissions in metal mines. Mineral processing produces tailings and mill water. Pyrometallurgy, hydrometallurgy, and electrometallurgy generate slags, leached ores, and process water. All activities commonly cause atmospheric emissions (Lottermoser, 2010).

Mining wastes are heterogeneous geological material and they include rocks (sedimentary, metamorphic, or igneous), soils, and lose sediments. The increase of demand for mineral products for different uses and the exploitation of lower-grade deposits necessitates processing large volumes of material. To uncover such ore material, a large volume of waste rocks needs to be displaced. Both open-pit and underground mining operations contribute to the generation of waste rocks but disproportionately with the open-pit mining standing as the major source. Processing of material to recover the valuable component may or may not result in the generation of wastes. Extraction and production of sand and gravel generally do not produce waste. Conversely, only a small portion constitutes the metal of interest in metallic ores, leaving a large volume of wastes after processing (Hassinger, 1997; Lottermoser, 2010).

Geological processes involved in the formation of different deposits give each of them a set of unique characteristics in terms of the amount and type of metals formed, the kind and grain size of minerals formed, and the type of rocks associated with the deposit. Different ore deposits and rock types are therefore associated with different occurrences of elements. While the proportion of sulfide minerals in the earth crust is relatively small compared to other minerals, in some geological settings, phosphate ores, coal seams, oil shales, and metallic ore deposits especially of copper, lead, zinc, gold, nickel, uranium, and iron may contain significant amounts of sulfides. Geological aspects in line with weathering rates determine the impact that the deposits will pose on the environment (Hammarstrom et al., 2003; Lottermoser, 2010; Seal et al., 2008). For instance, mesothermal gold deposits that are formed at intermediate temperature (200 to 300 Celcius) hydrothermal solutions are rich in acid buffering calcite. Calcite buffers any acid generated from the oxidation of sulfides and hence oxidation of mesothermal deposits will not result in (Craw and Pacheco, 2002). On the contrary, epithermal gold deposits formed at low temperatures (50 to 200 Celcius) hydrothermal solution are rich in pyrite and poor in carbonate. Oxidation of these sulfidic wastes at elevated temperatures in presence of meteoric water releases to the environment low pH water leading to acidic drainages accompanied by mobilized toxic, corrosive, or radioactive metals (Craw, 2001; Liu et al., 2017). Lottermoser (2010) highlights that exposing sulfidic wastes to an oxidizing environment makes the material chemically unstable. Instability destroys the equilibrium between mineral assemblages and the environment and hence a series of chemical weathering reactions begin. The presence of meteoric water, atmospheric gases, and microorganisms speed up the rate of chemical weathering. Vriens et al. (2019) point out that fine-grained, waste rock rich in sulfide with low carbonate content exhibits high sulfidic oxidation rates leading to acidic drainage specifically known as Acid Mine Drainage (AMD) associated with metals of various concentrations. They further point out that coarse-grained, waste rock rich in carbonates does not contribute to acidic and metal-laden drainage. The release of metals is inhibited by sorption and the type of secondary minerals formed from these wastes.

In 1987, the World Commission on Environment Development WCED (1987) defined sustainable development in the so-called Burntland report as a development that “meets the needs of the present without compromising the ability of the future generation to meet their own needs”. The concept requires mining systems to be re-engineered by incorporating economic, environmental, and social issues. Environmental protection has become an integral and important part of modern mining due to mine wastes' impacts posed to the environment. This aspect becomes more important because of the volume and diverse compositions of the wastes generated by mining activities compared to other human activities. While monitoring and rehabilitation should be an ongoing task, operational constraints do limit sufficient rehabilitation to take place parallel to other operations. Monitoring should focus on understanding the composition, characteristics, and impacts of wastes, reducing or recycling waste, and ensuring minimal or no impacts on the environment because of the wastes generated. To prevent AMD, sulfide oxidation should be controlled by eliminating at least one reactant that interacts with the sulfidic waste to initiate and perpetuate the reaction. Exclusion of water, exclusion of oxygen, pH control, control of bacteria activities, removal or isolation of sulfides are important goals that oxidation control strategies seek to achieve. Some of the techniques used to control the formation of AMD include the use of wet covers, dry covers (unsaturated, saturated, and store-and-release covers), encapsulation, in-pit disposal, and mixing, co-disposal and blending, the addition of organic wastes, and the use of bactericides. The techniques help to control water and oxygen movement, pH, and microbial activity. Generally, upon completion of the project, the disturbed land due to mining and the wastes generated should be rehabilitated to restore vegetation, topography, hydrology, and stability of the landscape destroyed by the operations (Amos et al., 2015; Lottermoser, 2010).

Rehabilitation of the mine site is a costly process demanding long-term commitment (Vriens et al., 2019). In pursuit of minimizing environmental impacts caused by mine waste, Gholamnejad (2009) extends Lane's model to account for the rehabilitation need whereby a new cost parameter known as rehabilitation cost is included in cut-off derivation. Including rehabilitation cost in cut-off grade determination has proved a decrease in the cut-off grades, resulting in more material being classified as ore. Classification of more material as ore maximizes utilization of the available resource thereby lessening the amount of waste to be disposed of in the waste dump. Rehabilitation cost is the unit cost of rehabilitating material of a particular type of waste rock after it has been dumped.

2.6 Extensions to Lane's Method

Lane's model is the most practical and pioneer work that has gained acceptance over the break-even model in mine planning. While the break-even cut-off grade model considers financial parameters only, Lane's model additionally incorporates realistic cut-off grade determination parameters some of which are ignored by the break-even model like grade-tonnage distribution, and mining system capacities. However, the model cannot consider some important aspects of real mining operations such as multi-processing streams, environmental concerns, the uncertainty of some inputs, multi-metal deposits, and stockpiling policy. Depending on the goal that a particular cut-off grade model should deliver, different modifications have been done to Lane's theory by incorporating different parameters initially not considered (Lane, 1988). Some of the remarkable studies include:

2.6.1 Multi-Metal Deposits

Osanloo and Ataei (2003) reviewed different methods that are used to determine the cut-off grade in multi-metal deposits like the use of value per ton of ore calculated from the net smelter return (NSR), critical level method, single cut-off grade approach, and dollar value cut-off grade approach. These methods ignore grade distribution of the deposits, capacities of the mining system, and the time value of money. It should be noted that the original Lane's theory is restricted to cut-off grade determination in single metal and can't handle more than six cut-off grades at a time. Contrary, in multiple metal deposits, an infinite number of cut-off grades are possible members for the optimum cut-off grade at any time, and the objective function evaluation of these infinite cut-off grades cannot be determined by using normal Lane's approach. They instead extended Lane's theory to incorporate the effect of the presence of multiple metals in deposits in such a way that real optimal cut-off grades are obtained while grade distribution, capacities, and the time value of money are honored. The profit equation was modified and parameters like price, selling cost, average grade, and recovery of additional metal were included. The equivalent factor was derived by considering recoveries, selling prices, and refining costs of each metal. Individual grades and the equivalent factor were combined to form a single equivalent grade which was then factored in the cut-off grade calculation formula. The simple Lane's algorithm was then used to determine the cut-off grade policy.

Cetin and Dowd (2016) established a cut-off grade optimization model for a multi-mineral deposit. The deposit used in their case study consisted of gold, lead, and zinc metals. A genetic algorithm was used, and its results were compared with those obtained by using grid search and dynamic programming methods. Each metal was associated with its value of refining capacity, refining cost, recovery, and lower and upper limit of cut-off grades. Common parameters to all minerals include mining capacity, mineral processing capacity, the variable cost of the material mined, variable cost of the processed material, fixed cost and discount rate.

2.6.2 Dynamic Price and Costs

The original Lane's model assumes that economic parameters are deterministic and fixed over the project's life. Hence, relying on this assumption implies that cut-off grades should be determined based on the fixed metal price and operating costs. Owing to the longer life of many operations, the price and cash cost associated with the components of the mining system tend to change with time. Also, mine operators are always unable to decide the price at which they should sell their mineral commodities and instead, they have to respond to what is determined by international markets (Asad, 2005a; Barr, 2012). Barr (2012) and Thompson and Barr (2014) propose a cut-off grade model that takes into account the price dynamics by using a full stochastic future curve to predict future prices rather than a stochastic spot price model. Both Asad (2005a) and Barr (2012) agree that operational strategies developed by not taking into account the stochasticity behavior of the economic parameters are unrealistic. For instance, Barr (2012) and Thompson and Barr (2014) find that not incorporating price uncertainty in cut-off grade optimization leads to higher cut-off grades, resulting in wasting valuable resources.

Asad (2007) developed a cut-off grade model based on Lane's theory to maximize NPV under dynamic price and costs escalations. Two scenarios were observed. In the first case, the price was escalated at a single fixed rate. The fixed mining, milling, and refining operations costs escalated at another fixed rate per annum. Thus, both price and costs rates were dynamically changing per annum. It was found that the maximum NPV dropped instantly in one year compared to a scenario when no escalation is considered. In the second scenario, constant price escalation per annum was used while the costs were dynamically changing. It was found that NPV is most and least sensitive to an increase in escalation rates of milling and fixed costs respectively. It was further deduced that a 5.0% escalation rate per annum of operating and fixed costs would make the operation uneconomic.

Johnson et al. (2011) used a partial differential equation model to determine and operate an optimal dynamic cut-off grade of a short-time scale mining problem that would give the optimal processing strategy subject to stochastic price and deterministic geological model. The mine was divided into 60,000 blocks and they were assumed to be extracted in sequential order as long as the mineral content of each was accurately known. The option to process or not to process under price fluctuations was valued, helping the mining company to react to future market conditions. Considering the reality, Barr (2012) asserts two shortcomings in the problem definition by Johnson et al. (2011). First, several blocks can be exposed at any given time and they can be extracted in any order by using multiple pieces of machinery and for that reason, there can be an infinite number of extraction orders. Second, it is not easy to tell that sampling of all blocks has been done sufficiently in such a way to declare the mineral content of each block with certainty. The mineral content of each block can be stated with certainty once it is exposed

2.6.3 Stochastic Price and Grade-tonnage Distribution

Githiria and Musingwini (2019) extended deterministic Lane's algorithm by developing a stochastic cut-off grade model that simultaneously considered variability in both commodity price and grade-tonnage distribution to establish dynamic cut-off grades over the life of the mine. Six possible grade-tonnage curves for the deposit were used to account for the effect of grade-tonnage uncertainty on maximum NPV from a deposit. To avoid overestimation or underestimation of grades, volumes or tonnages, and other parameters exhibited in deterministic methods, Monte Carlo simulation was used to simulate the grade-tonnage distribution of the orebody. Gold price uncertainty applied was assumed to increase by 2% yearly. Results obtained by the stochastic method were compared against those obtained by the break-even cut-off grade model, Cut-off Grade Optimiser, OptiPit®, and Maptek Evolution® approaches. The stochastic optimization approach produced an NPV ranging between 7% and 186% higher than that obtained from deterministic approaches. The study done by Asad and Dimitrakopoulos (2013) shows the difference of 13.83% between the minimum and maximum

NPV generated across 15 simulated grade-tonnage curves. Findings from the two studies demonstrate the risk of ignoring geological uncertainty in open-pit mining operations.

2.6.4 Multiple Processing Options

Asad and Dimitrakopoulos (2013) used a heuristic approach to extend Lane's algorithm for optimizing cut-off grade at an open-pit mining project with multiple processing streams. They also considered geological uncertainty by simulating several different, but equally probable grade-tonnage curves where the optimal cut-off grade policy and corresponding production rates, cash flows, and NPV were determined by using the new algorithm they developed. The difference between the minimum and maximum NPV generated from different grade-tonnage curves was 13.8%. They concluded that it is vital to consider geological uncertainty during planning to avoid severe economic impacts on a project with this significant difference. However, their study suffered a problem of underutilization of the processing streams where none of the four streams operated at even half of the designed capacity.

Rahimi et al. (2015) developed a Lagrange multiplier-based cut-off grade optimization model that can be used to determine cut-off grades between concentrating ad heap leaching processes under different operating conditions. Heap leaching has gained special attention particularly in extracting copper from low grade ores and a variety of oxides attributed by low operating costs, low investment cost, and its ability to handle ambiguities arising from waste management and anticipated impacts on the environment. Comparably, the concentration method has proved little success in copper oxides as high recovery can not be achieved. Heap leaching becomes a reliable hydrometallurgical method in this circumstance. The effect of price change was observed on cut-off grades of both processes and the significant impact was noted in the concentration process.

2.6.5 Environmental Considerations

Osanloo et al. (2008) incorporated environmental costs related to extracting and processing porphyry copper deposits into cut-off grades modeling. Considering the adverse impacts of copper's production on the environment, the costs relevant to the environmental consideration were included in the optimization process. A mine with two waste dumps (WD1 & WD2) and one processing plant with two tailing disposal facilities (TD1 & TD2) were considered. The tailings facilities were designed to handle non-acid (NA) and acid-generating material (AG). Costs and proportions parameters related to each type of waste rock and tailings were introduced in Lane's model. The proposed approach by Osanloo et al. (2008) is similar to what Gholamnejad (2009) did. The only difference between them is that Gholamnejad (2009) didn’t consider the cost of rehabilitating tailings, and instead, the only rehabilitating cost he considered was that of rehabilitating waste rock. It should be noted that using the traditional methods (break-even and Lane's models) in determining the cut-off grades forces operations to incur a one-time rehabilitation cost and this practice tends to decrease the NPV (Osanloo et al., 2008; Rendu, 2014). Mugwagwa (2017) in his study illustrates a practice at one of the active mines in the Democratic Republic of Congo (DRC) whereby rehabilitation is done along with mining. For that reason, rehabilitation commences whenever the dumping area is proved full. Budgeting the costs is done annually by deducting it from the profits generated each year. Therefore, the proposed models developed by Osanloo et al. (2008) and Gholamnejad (2009) comply with the industrial practice. Results from both models reveal that including rehabilitation costs in cut-off grade optimization leads to a decrease in cut-off grades and hence the amount of material mined and treated as ore increases. The study done by Gholamnejad (2009) shows a significant decrease in the amount of waste when a new model is employed. The application of a modified cut-off grade optimization model in the study done by Osanloo et al. (2008) shows an increase in the net present value by 3.6% relative to that obtained by using the traditional Lane's model. Comparably, the net present values exhibit a similar trend when a modified model in the study conducted by Gholamnejad (2009) is implemented in optimizing the cut-off grades. Generally, models that take into account the rehabilitation cost in optimizing cut-off grades have proved better and realistic net present values and cut-off grades in comparison to the traditional Lane's model. Both studies focused on a single source of material and a single processing stream.

Rahimi et al. (2014) developed a cut-off determination Lagrange model by considering the environmental costs of copper production. The deposit has two distinct ore types; one rich in oxide and another rich in sulfide. The two types of ores make the mine to opt for two processing methods. Environmental costs included in the optimization model include mining waste disposal costs, leached waste disposal costs, tailings from heap leaching, concentration and refining tailings, environmental protection costs of hydrometallurgical, and pyrometallurgical processes. An iterative procedure based on NPV maximization was used to determine the optimum concentration and leaching cut-off grades separately in the same way the cut-off grades for a single process are determined.

2.7 Summary and Remarks

The break-even cut-off grade method has been widely used in many operations to discriminate between ore and waste. The cut-off grade established by this method ensures every ton of ore mined pays for itself. This method ignores the capacities of the components of the mining system and the geology of the orebody. In addition to every ton pays for itself, Mortimer’s Definition brings in a new concept in cut-off grade derivation by considering the geology of the deposit. Neither the break-even nor Mortimer's Definition cut-off grade models honor the capacities of different stages of the mining system. Limitations addressed in the break-even and Mortimer’s Definition are solved by Lane’s method. The nature of mineralization explained in terms of grade distributions helps to quantify a profit level that can be achieved under given economic and operational conditions.

Misallocation of costs in cut-off grade calculations leads to sub-optimal cut-off grades and, once applied may result in devastating economic losses to the operations. Some of the immediate consequences of this phenomenon include underestimating or overestimating the reserves whereby worth material may be excluded, or worthless material may be counted respectively in the reserve size estimation. Likewise, mining and processing some material below the cut-off grade affects the profits and consequently the overall value of the operation.

Many studies on cut-off grade determination have been conducted in the past decades by improving the method proposed (Lane, 1988). Researchers have incorporated more parameters in the initial Lane’s method to get new models that aim to achieve some specific goals in efforts of maximizing the net present value of the operations.

While environmental cost has been considered in some studies, none of them addresses the integration of the cost in multiple processing streams. This research, therefore, establishes a cut-off grade optimization framework that incorporates a rehabilitation cost in multiple processing streams, the primary purpose of which is to determine the cut-off grade policy and the NPV of the operation that takes the cost into account.

CHAPTER 3

THEORETICAL FRAMEWORK

This chapter presents an overview of the cut-off grade optimization model proposed by Lane (1988), whereby steps to determine the limiting, balancing, and optimum cut-off grades are elaborated. A general review of an environmental-based cut-off grade optimization model and multiple processing streams model as an extension to Lane's method is given. Some classic examples of mine operations utilizing multiple streams, rationale, and key considerations are presented. Finally, the proposed model that considers a rehabilitation cost of the waste rock in establishing the optimum cut-off grade strategy in operations with multiple processing streams is developed.

3.1 Lane's Method

According to Lane (1964, 1988) theory of cut-off grades, a mining operation is considered to have three components: mining, milling, and refining components. In NPV maximization, optimum cut-off grades tend to rely on product price and cash costs associated with the three stages of the mining operation, the capacity of one or more stages of the operation, and the grade-tonnage distribution of the mineral deposit. In the mining stage, materials with various grades are mined up within permitted capacity. Material below cut-off grade is left unmined or dumped as waste. The one above cut-off grade is sent to the processing plant for milling and concentrating within the designed capacity or stockpiled for future use. Lastly, the concentrate is smelted and refined to get the saleable product. Each of the three stages has its own capacity and associated costs Gholamnejad (2009). Lane (1988) and Dagdelen (1993) highlight that a cut-off grade establishes the means of deciding the quantity of material that should be mined, processed, and refined at a given period. For that reason, it directly affects the cash flows of a mining operation since the higher the cut-off grade the higher grades per tonne of ore, hence, depending on the grade distribution of the mineral deposit, the higher net present value is realized.

Limiting and balancing cut-off grades are two main groups of cut-off grades that a mining operation can have. At a given period, an individual stage's capacity or the capacities of the stages existing in pairs may limit the operation. Three cut-off grades, commonly known as limiting cut-off grades, are formed when the capacity of each stage is a bottleneck to the throughput of the mining operation. Similarly, when the capacities of two components concurrently limit the throughput, the second group of cut-off grades referred to as balancing cut-off grades is formed. Each group consists of three cut-off grades, and hence, six cut-off grades are formed. One of the six cut-off grades is the best cut-off grade that will maximize NPV while ensuring each stage's capacity constraint is honored. Therefore, given the economic, operational, and grade-tonnage distribution of the mineral deposit, defining the optimum cut-off grade strategy that will maximize the NPV for a given set of constraints becomes an inherent solution to the problem (Asad, 2002; Hustrulid and Kuchta, 2006; Lane, 1964, 1988).

Assuming that a deposit's grade-tonnage distribution is known, the economic and operation parameters are defined. Determination of the optimum cut-off grade g from period T to period T_N will be the ones that will maximize the NPV of the mining project subject to mining, processing, and refining capacity constraints. According to Lane (1988), the general NPV maximization mathematical formulation model is presented as:

$$\text{Max } NPV = \sum_{T=1}^{T_N} \frac{P_T}{(1+d)^T} \tag{3.1}$$

Subject to:

$$Q_{cT} \le C \text{ for } T = 1,\ldots,T_N \tag{3.2}$$

$$Q_{mT} \le M \text{ for } T = 1,\ldots,T_N \tag{3.3}$$

$$Q_{rT} \le R \text{ for } T = 1,\ldots,T_N \tag{3.4}$$

Where; P_T is the cash flow or profit generated, Q_{mT} is the quantity of material mined, Q_{cT} is the amount of ore processed, and Q_{rT} is the amount of metal refined at time T.

The constraints defined by equations (3.2) to (3.4) guide the amount of material that should be processed, mined, and refined at time T, respectively.

For the purpose of this thesis, maximization of the NPV assumes that processing capacity is the only factor limiting the operation.

Assume V in Figure 3-1 is the possible maximum present value of the future cash flows at time zero. Assume also that the amount of material in a reserve before mining commences is Q.

The future cash flows are divided into two parts. The first part P_T is the profit earned at the end of period T by mining the quantity of material Q_m, processing the quantity of ore Q_c, and refining the quantity of metal Q_r. After mining the quantity Q_m, the quantity of reserve $Q-Q_m$ is left, and if it is scheduled to be mined during time periods $T+1$ to T_N, W becomes the maximum possible value of the future profits P_{T+1} to P_{T_N} realized by mining the stated remaining quantity of reserve.

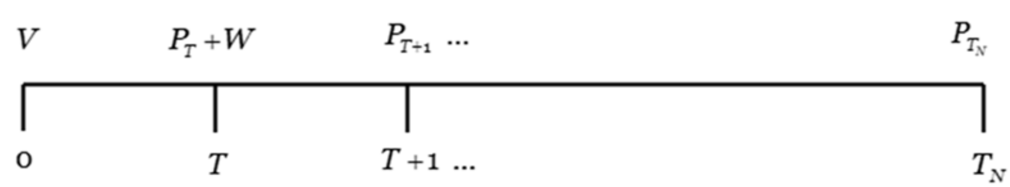

Figure 3-1. Diagrammatic presentation of the cash flows and net present value in Lane's model (after Asad 2005b).

The overall possible maximum NPV of all cash flows generated from time T to T_N is stated as follows:

$$V = \frac{W}{(1+d)^T} + \frac{P}{(1+d)^T} = \frac{W+P}{(1+d)^T} \tag{3.5}$$

Equation (3.5) can further be written as:

$$W + P = V(1+d)^T \tag{3.6}$$

Binomial expansion to the term $(1+d)^T$ gives:

$$(1+d)^T = 1 + Td + \frac{T(T-1)d^2}{2!} + \frac{T(T-1)(T-2)d^3}{3!} + \ldots \tag{3.7}$$

For a small value of d, equation (3.7) becomes:

$$(1+d)^T \approx 1 + Td \tag{3.8}$$

Combining equations (3.6) and (3.8) results in the following equation:

$$W + P = V(1+Td) = V + VTd \tag{3.9}$$

The difference between the present value of the remaining reserves at time $t = T$ and $t = 0$ is equal to:

$$v = V - W = P - VTd \tag{3.10}$$

In equation (3.1), $P = P_T$ is the profit obtained by mining the quantity of material Q_m in time T for a single processing stream and is stated as:

$$P = (S-r)Q_r - cQ_c - mQ_m - fT \tag{3.11}$$

3.1.1 Limiting Cut-Off Grades

It was explained in the previous sections that mining, processing facility, and refinery/market are the three stages of the mining operation. The capacity of each stage limits the throughput of the mining system. The mining rate (for open-pit mining) or the development rate (for underground mine) decides the amount of material flow from the mine. It is merely affected by the size of the truck fleet, the number of shovels, and drilling equipment. In the processing stage, the throughput is limited by either the ore handling facilities or the concentrating plant. The market restriction is due to the sales contract or the limiting capacity of the refinery or smelter (Lane, 1988).

Three cut-off grade formulas arise when the capacity of each stage is individually limiting the operation and the following procedures derive them:

Applying equation (3.11) in equation (3.10) gives the following expression:

$$v = (S-r)Q_r - cQ_c - mQ_m - (f + Vd)T \tag{3.12}$$

The quantity Q_r is related to the average grade $\bar{g}$, recovery y, and Q_c as $Q_r = Q_c \bar{g} y$, hence, equation (3.12) becomes:

$$v = \left[(S-r)\bar{g}y - c \right] Q_c - mQ_m - (f + Vd)T \tag{3.13}$$

In equation (3.13), v is the increment in net present value per unit of resource utilized. Since later benefits get discounted more than those earned earlier, mining should be scheduled in such a way that a decline in present value occurs rapidly. Maximizing NPV requires maximizing the value of v (Hustrulid and Kuchta, 2006; Lane, 1988).

Considering the three stages of a mining system, the capacity of each can be a limiting factor in maximizing the value of v and consequently the NPV.

If each stage is considered independently, three equations for v can be developed from which the respective limiting cut-off grades can be determined.

Depending on the limiting stage, the duration T corresponding to mining, processing, and refining can be defined as $T_m = \frac{Q_m}{M}$, $T_c = \frac{Q_c}{C}$, and $T_r = \frac{Q_r}{R}$ respectively.

By applying the durations above in equation (3.13), the following functions related to mining, processing, and refining are derived, respectively:

$$v_m = \left[(S-r)\bar{g}y - c \right] Q_c - \left[m + \frac{f + Vd}{M} \right] Q_m \tag{3.14}$$

$$v_c = \left[(S-r)\bar{g}y - \left(c + \frac{f + Vd}{C} \right) \right] Q_c - mQ_m \tag{3.15}$$

$$v_r = \left[\left((S-r) - \frac{f + Vd}{R} \right) \bar{g}y - c \right] Q_c - mQ_m \tag{3.16}$$

The three limiting cut-off grades corresponding to mining, processing, and refining stages are determined by first maximizing the three preceding functions where the derivative of v with respect to grade is found and set equal to zero. It should also be noted that the quantity mined Q_m is independent of the grade and hence its derivative with respect to grade is zero. The limiting cut-off grades are therefore stated as follows (Lane, 1988):

$$g_m = \frac{c}{(S-r)y} \tag{3.17}$$

$$g_c = \frac{\left(c + \dfrac{f + Vd}{C}\right)}{\left(S - r\right)y} \tag{3.18}$$

$$g_r = \frac{c}{\left(\left(S - r\right) - \dfrac{f + Vd}{R}\right)y} \tag{3.19}$$

The value of V in equations (3.18) and (3.19) is initially unknown for new operations as it depends on the cut-off grades. An iterative process is required to get the optimum cut-off grades where the parameter V is first assumed to be equal to zero. The optimum cut-off grade is obtained by repeating this process. The value V obtained in the preceding iteration becomes an initial V value in the next iteration, and the process stops when the difference between two successive V values becomes zero or lies within a predetermined tolerance.

From the three preceding equations, one can learn the following salient features:

- The formula for calculating the cut-off grade as presented by equation (3.17) is certainly the same as the break-even cut-off grade formula. They both propose that mineralized material should be classified as ore once its implicit value, $(S - r)yg$, exceeds the cost of further processing, c. The formula for g_m has two significant features. First, the implicit value of the mineralized material needs only to cover the variable cost of treatment. Time costs and mining costs are not relevant. This is because the derivation of the formula assumes that the decision to operate beyond the existing time horizon has already been taken. Second, it does not make any reference to present values due to the reason that there is no trade-off of future losses versus present gains to modify the current policy (Lane, 1988).

- The present value term Vd in equations (3.18) and (3.19) is the opportunity cost. It is an additional time cost of classifying the material as ore. Since the NPV of the operation keeps on declining with resource depletion, the cut-off grades will also decline with time. This is contrary to the break-even method which gives constant cut-off grades. Methods that establish the optimum cut-off grades consider the opportunity cost of not receiving future cashflows earlier due to the capacity constraints of the stages of the mining system. The two formulas have a common behavior that the fixed costs are distributed according to the limiting capacity and added to the corresponding variable cost and for that reason, the treatment and refining costs become $c+\frac{f+Vd}{C}$ and $r+\frac{f+Vd}{R}$ respectively (Asad, 1997; Asad, 2005b; Lane, 1964, 1988; Pettingell, 2017).

3.1.2 Balancing Cut-Off Grades

Three cut-off grades considering that each component limits the throughput of the mining system individually, have been established in section 3.1.1. The capacity of a single stage will seldomly limit the throughput of any mining system like other operations having many stages. Instead, at least two stages of the mining operation limit the throughput in a given period. It was described earlier that the time when mining, processing, and refinery limit the operation independently are Q_m / M, Q_c / C, and Q_r / R respectively. If two stages are concurrently limiting the throughput of the mining operation, a single balancing cut-off grade that will keep each of the stages operating at its full capacity should be determined. It is the one that balances the aforementioned ratios. For three components, three possible pairs of components and consequently three balancing cut-off grades should be derived.

Consider a hypothetical deposit with a certain grade-tonnage distribution. The cumulative tonnage above that grade is simply referred to as ore or mineralized material at every cut-off grade. The total quantity of material that should be mined can be calculated once the cut-off grade is known. The ratios expressed as the quantity of ore per unit of material mined can be plotted versus their corresponding cut-off grades as shown in Figure 3-2. There is a point along the declining line presented in this figure at which the quantity of ore per unit of material mined/ the proportion of mineralized material equals the ratio C / M.

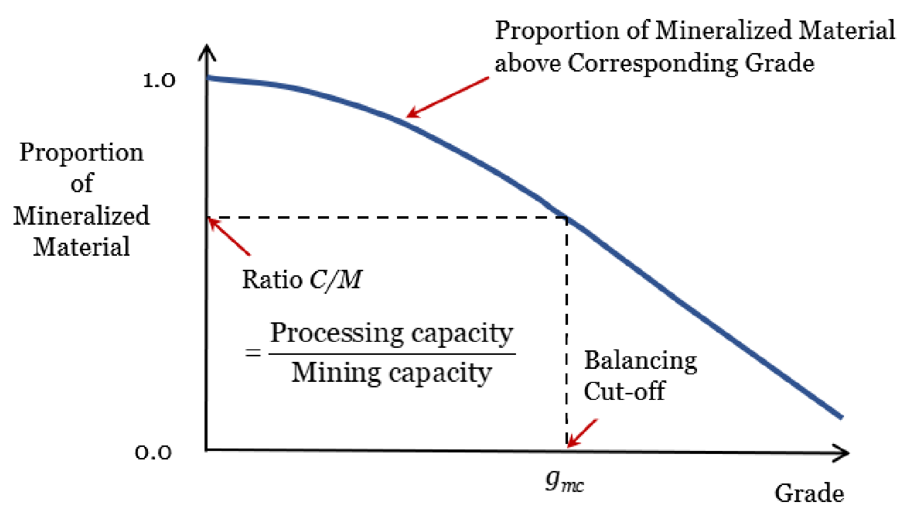

Figure 3-2: Cumulative Grade Distribution for a Mine Planning Increment (after Lane, 1988).

The grade required to bring the balance between the two ratios is called mine-mill balancing cut-off grade and is simply denoted by g_{mc}. At this cut-off grade, both the mining and processing stages simultaneously operate at their maximum capacities. For that case, the following mathematical formula is satisfied:

$$\frac{Q_c}{Q_m} = \frac{C}{M} \text{ or } \frac{Q_m}{M} = \frac{Q_c}{C} \tag{3.20}$$

The amount of recoverable mineral content for each cumulative tonnage above the cut-off grade is determined for the hypothetical deposit. Then, the ratio between it and the total amount of material mined is calculated. Figure 3-3 is a plot of the ratios versus cut-off grades. There is a point along the line of this graph where the ratio of recoverable minerals to the total tonnes of mined material equals the ratio R / M. The grade at which these two ratios are equal is called mine-refinery balancing cut-off grade and is denoted as g_{mr}. Both the mine and the refinery operate at their full capacities at this cut-off grade. The following expression hold at mine-refinery balancing cut-off grade:

$$\frac{Q_r}{Q_m} = \frac{R}{M} \text{ or } \frac{Q_m}{M} = \frac{Q_r}{R} \tag{3.21}$$

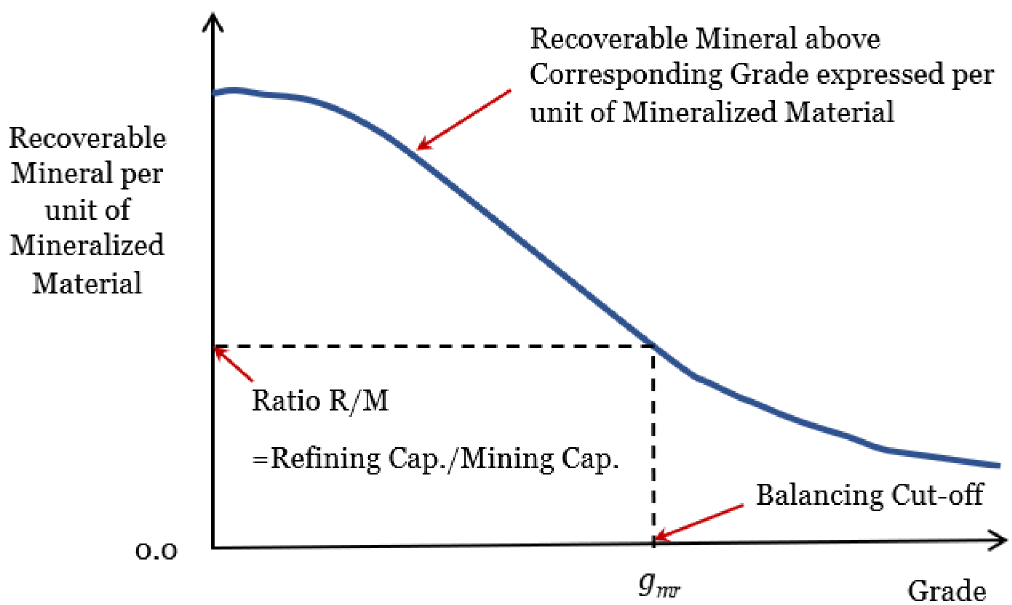

Figure 3-3: Recoverable Mineral per unit of Mineralized Material as a function of Grade (after Lane, 1988).

At different cut-off grades, the cumulative tonnage above them can be deduced. The recoverable minerals for each cumulative tonnage can also be computed. The ratios of the recoverable mineral content per unit of ore can be plotted versus their corresponding cut-off grades to form Figure 3-4. There is a point along the resulting line at which the ratio of recoverable mineral content to cumulative tonnage equals the ratio R/C. At this point, the two stages are said to be simultaneously operating at their maximum capacities, and the grade required to achieve this is called mill-refinery balancing cut-off grade and is denoted as g_{rc}. The following equation is satisfied if both refinery and concentrating plant limit the throughput of the mining system:

$$\frac{Q_r}{Q_c} = \frac{R}{C} \text{ or } \frac{Q_r}{R} = \frac{Q_c}{C} \tag{3.22}$$

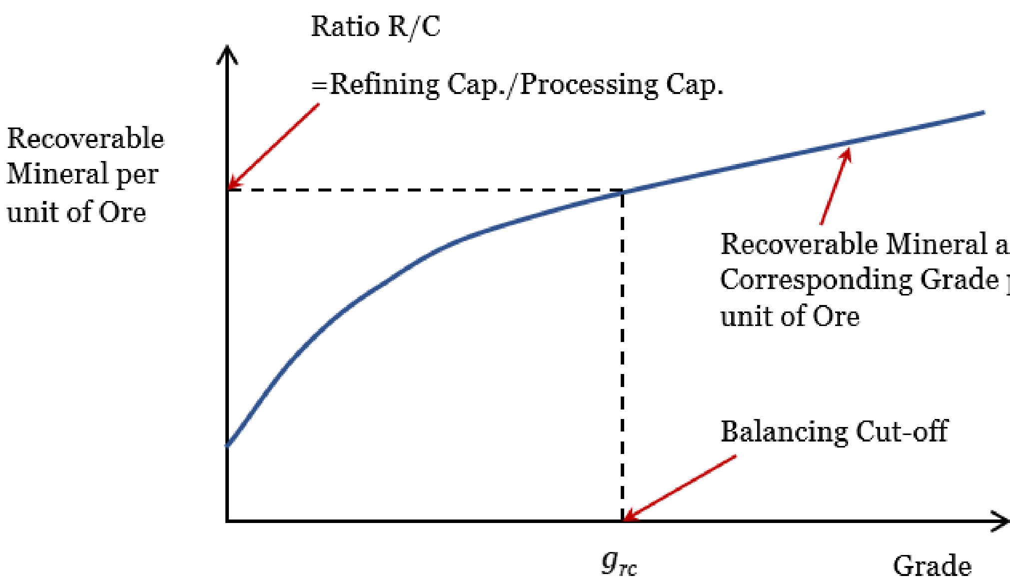

Figure 3-4: Recoverable Mineral per unit of Ore as a function of Grade (after Lane, 1988).

The grade distribution data, columns of mineral content, cumulative tonnage, cumulative mineral content, and the ratios discussed in this section are tabulated to obtain balanced cut-off grades. A balancing point is then determined by inspection (Lane, 1988).

3.1.3 Effective Optimum Cut-Off Grade

Six possible cut-off grades have been established, three limiting cut-off grades and the other three balancing cut-off grades. Limiting cut-off grades are purely based on the capacities of the components, costs associated with each component, and the product price. Conversely, the balancing cut-off grades depend on the grade-tonnage distribution of the mineral deposit (Hustrulid and Kuchta, 2006; Pettingell, 2017).

The optimum cut-off grade is selected among the six cut-off grades. Since the intention of optimizing the cut-off grades is to maximize the NPV of the operation, the determination of optimum cut-off grades is achieved by maximizing the rate of change of the net present value with respect to resource usage. This was discussed in section 3.1.1 where v was observed to take three forms. Such forms were corresponding to the mining, concentrating, and refining stages. Since all six cut-off grades are the possible candidates in the process of deriving the optimum cut-off grade, it is important to understand the nature of the three forms of function v. To get through this purpose, a graphical method is used. Figure 3-5 is plotted to show how v varies with the cut-off grades when mining is the only limiting factor. The values corresponding to the function v appear to take three segments with increasing cut-off grades by examining this graph. In the first segment, the value of the function increases as the cut-off grade increases. In the second segment, the maximum value is reached. In the last segment, the value of the function starts to decline with increasing the cut-off grades. The cut-off grade corresponding to the maximum value becomes the limiting cut-off grade of the particular stage. Generally, individual graphs of the functions presented by equations (3.14) through to (3.16) versus cut-off grade resemble one another, and they are convex upwards with a single maximum (Lane, 1988).

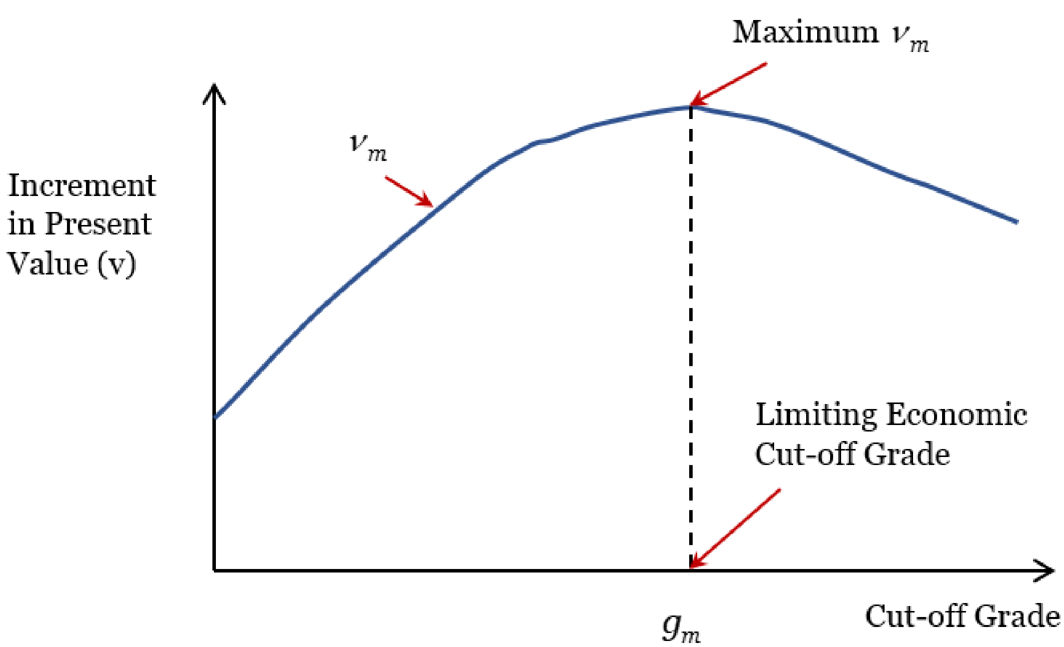

Figure 3-5: Increment in Present Value versus Cut-off Grade for Single Component (M); Limiting Economic Optimum (after Lane, 1988).

Consider the case when mining and milling stages are simultaneously limiting the throughput of the operation. Two graphs similar to Figure 3-5 for v_m and v_c are plotted together. Figure

3-6 is an example of the graph that results when two graphs are superimposed. The shaded part is the feasible region of the two curves from which infinite feasible values can be deduced. The maximum feasible point appears at the intersection of the two curves and it corresponds to the mine-mill balancing cut-off grade g_{mc}. By observing Figure 3-6, it is revealed that when a cut-off grade less than g_{mc} is chosen, the feasible value is taken along the line corresponding to v_c. This implies that the milling plant is the limiting factor as long as the cut-off grades are less than g_{mc}. In contrast, mining is the limiting factor when cut-off grades greater than g_{mc} are chosen since the feasible values lie on the curve corresponding to v_m. From the choice of the feasible values along the two curves, it can be concluded that the feasible form of v, at any cut-off grade, is always the lower of the two curves (Lane, 1988).

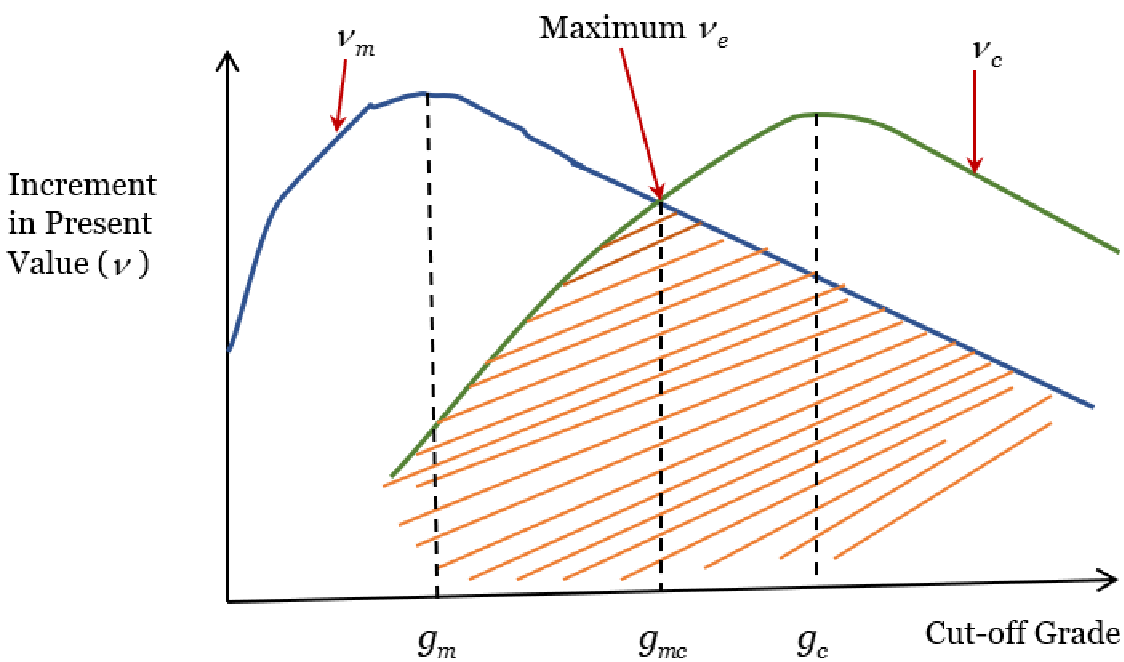

Figure 3-6: Increment in Present Value versus Cut-off Grade for Two Components (M & C); Balancing Optimum (after Lane, 1988).

Asad (2007) and Asad and Topal (2011) present an approximate method of finding the balancing cut-off grades. Consider a grade-tonnage curve with K categories defined as $\left[[g(1),g(2)],[g(2),g(3)],\ldots,[g(K-1),g(K)]\right]$ with each category having Q tonnes. Let the lower grade $g_k=\lambda$ in the grade category $[g_k,g_{k+1}]$ be chosen as the initial estimate of the cut-off grade.

If mining and processing stages are limiting the operation, then the quantity of ore per unit of material mined at the cut-off grade $g_k = \lambda$ can be defined as follows:

$$T_{mc}(\lambda) = \frac{Q_c(\lambda)}{Q_m(\lambda)} = \frac{Q_c(\lambda)}{Q_c(\lambda) + Q_w(\lambda)} \tag{3.23}$$

If it is further assumed that the balancing cut-off grade will be somewhere between $g_k = \lambda$ and $g_{k+1} = \lambda + 1$. At the later cut-off grade, the amount of ore per unit of material mined is defined as follows:

$$T_{mc}(\lambda+1) = \frac{Q_c(\lambda+1)}{Q_m(\lambda+1)} = \frac{Q_c(\lambda+1)}{Q_c(\lambda+1) + Q_w(\lambda+1)} \tag{3.24}$$

The balancing cut-off grade for mining-milling stages constraining the throughput has been defined as g_{mc} in the previous section. Let λ^* represent this cut-off grade. The amount of ore per unit of material mined can be stated as follows:

$$T_{mc}(\lambda^*) = \frac{Q_c(\lambda^*)}{Q_m(\lambda^*)} = \frac{Q_c(\lambda^*)}{Q_c(\lambda^*) + Q_w(\lambda^*)} \tag{3.25}$$

The three cut-off grades and the three ratios of ore per unit of material mined that are presented in equations (3.23), (3.24), and (3.25) can be used to prepare a simple sketch as shown in Figure 3-7.

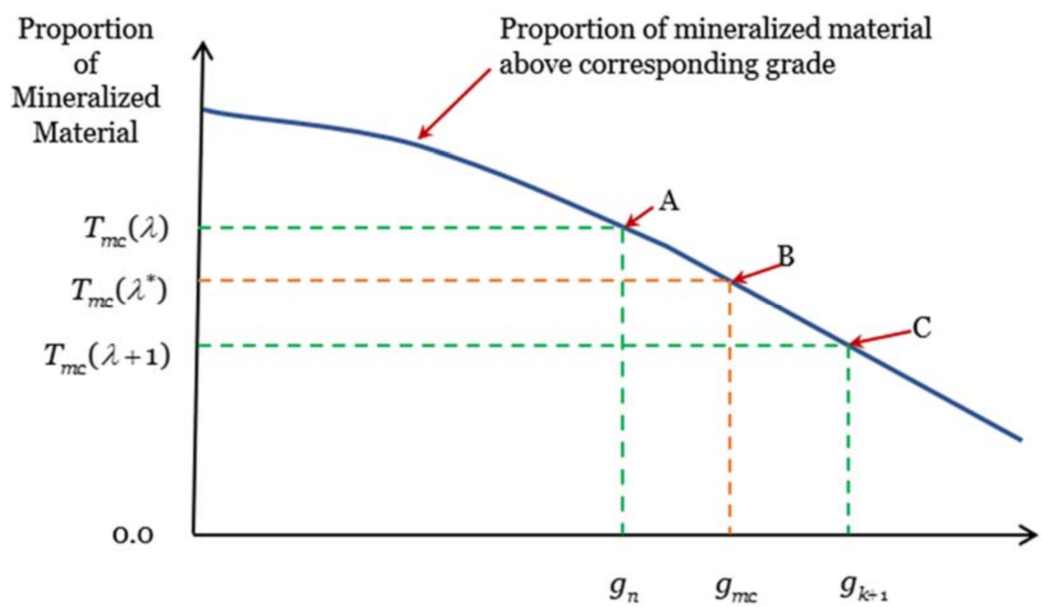

Figure 3-7: Approximation of a mine-mill balancing cut-off grade.

The coordinates of points A, B, and C can be defined as A(g_k , $T_{mc}(\lambda)$), B(g_{mc} , $T_{mc}(\lambda^*)$), and C(g_{k+1} , $T_{mc}(\lambda+1)$), respectively. The only unknown parameter among the six parameters is the balancing cut-off grade g_{mc} . Assuming the three points are collinear, the linear interpolation technique can be used to approximate the value of g_{mc} as follows:

$$\frac{T_{mc}(\lambda^*)-T_{mc}(\lambda)}{g_{mc}-g_k}=\frac{T_{mc}(\lambda+1)-T_{mc}(\lambda)}{g_{k+1}-g_k} \tag{3.26}$$

At balancing cut-off grade, the ratio $T_{mc}(\lambda^*)$ equals $C\,/\,M$. Equation (3.26) can be modified to:

$$g_{mc}=\frac{\dfrac{C}{M}-T_{mc}(\lambda)}{\left[\dfrac{T_{mc}(\lambda+1)-T_{mc}(\lambda)}{g_{k+1}-g_k}\right]}+g_k \tag{3.27}$$

If mining and marketing/refining are limiting the operation, the recoverable metal per unit of material mined at the cut-off grade $g_k=\lambda$ is defined as:

$$T_{mr}(\lambda)=\frac{Q_r(\lambda)}{Q_m(\lambda)}=\frac{Q_c(\lambda)*\bar{g}*y}{Q_c(\lambda)+Q_w(\lambda)} \tag{3.28}$$

The mine-refinery balancing cut-off grade is obtained by following the same procedures used to derive equation (3.28) and it is expressed as follows:

$$g_{mr}=\frac{\dfrac{R}{M}-T_{mr}(\lambda)}{\left[\dfrac{T_{mr}(\lambda+1)-T_{mr}(\lambda)}{g_{k+1}-g_k}\right]}+g_k \tag{3.29}$$

If refining and processing stages are limiting the operation, the recoverable metal per unit of ore processed at lower grade is defined as:

$$T_{rc}(\lambda)=\frac{Q_r(\lambda)}{Q_c(\lambda)}=\frac{Q_c(\lambda)*\bar{g}*y}{Q_c(\lambda)}=\bar{g}*y \tag{3.30}$$

The mill-refinery balancing cut-off grade is obtained by following the same procedures used to derive equation (3.28) , and it is stated as follows:

$$g_{rc} = \frac{\frac{R}{C} - T_{rc}(\lambda)}{\left[\frac{T_{rc}(\lambda+1) - T_{rc}(\lambda)}{g_{k+1} - g_k}\right]} + g_k \qquad (3.31)$$

In Figure 3-8, all three forms of v are superimposed, and the six cut-off grades explained previously are presented. The maximum value is found at the intersection of v_m and v_r. For this case, g_{mr} is the optimum cut-off grade of the operation as far as six cut-off grades are concerned. This figure has been used only to describe how the graphical method aids the selection of the optimum cut-off grade. The maximum value can appear anywhere in the feasible region and not necessarily at the vertices as shown in this figure. In other words, individual components or pairs of the components may limit the throughput of the operation. This makes the optimum cut-off grade to take any value of the six cut-off grades discussed so far.

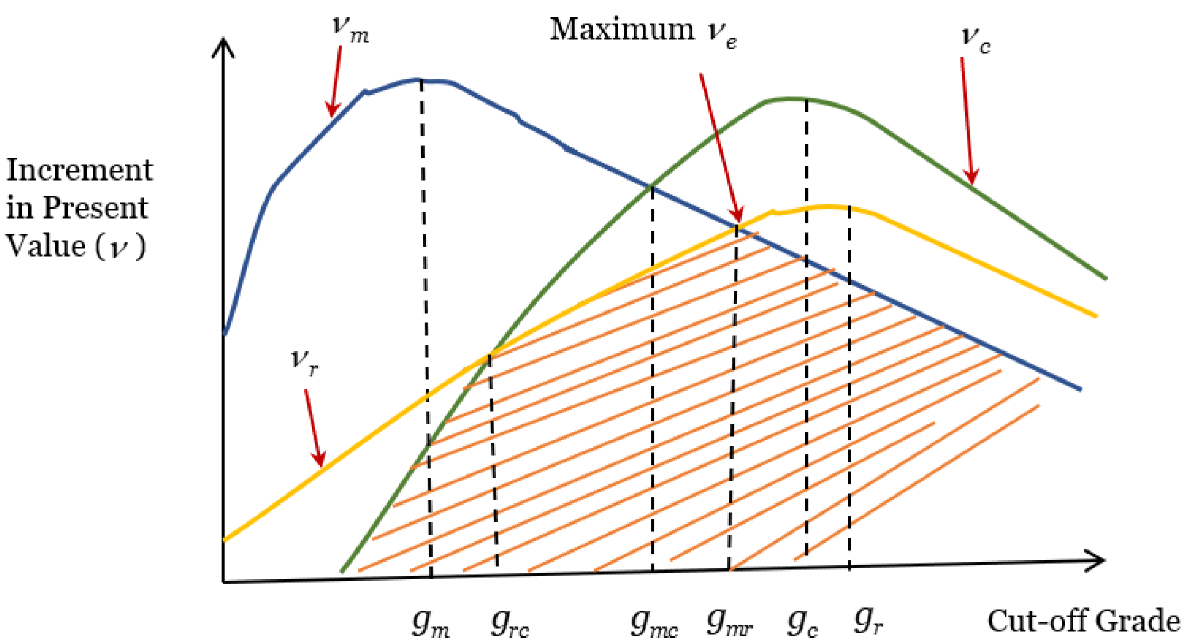

Figure 3-8: Increment in Present Value versus Cut-off Grade for Three Stages (after Lane, 1988).

A careful examination of the limiting cut-off grade equations reveals that the optimum cut-off grade will never be less than g_m since it is the break-even cut-off grade. Also, the optimum cut-off grade will never be higher than g_c, since this will lead to throwing some valuable ore in waste dumps. Hence, the following relationship holds (Asad and Dimitrakopoulos, 2013; Dadgelen, 1993; Osanloo et al., 2008):

$$g_m \le g_r \le g_c \tag{3.32}$$

Therefore, the optimum cut-off grade (G) required to maximize the objective function is the value between g_m and g_c. This is expressed as (Asad and Dimitrakopoulos, 2013; Dadgelen, 1993; Osanloo et al., 2008):

$$g_m \le G \le g_c \tag{3.33}$$

The optimum cut-off grade of the six cut-offs is found by first finding the effective cut-off grade of each pair of the three mining components in balance, which satisfies the corresponding pair. Then, referring to the arguments summarized in equations (3.32) and (3.33), three effective optimums cut-off grades for components limiting the operation in pair and the optimum cut-off grade (G) are determined as follows (Lane, 1988):

- If G_{mc} is the effective optimum cut-off grade satisfying simultaneously mining and concentrator capacity constraints, then:

$$G_{mc} = \begin{cases} g_m & \text{if } g_{mc} \le g_m \\ g_c & \text{if } g_{mc} \ge g_c \\ g_{mc} & \text{otherwise} \end{cases} \tag{3.34}$$

- If G_{mr} is the effective optimum cut-off grade satisfying simultaneously mining and refining capacity constraints, then:

$$G_{mr} = \begin{cases} g_m & \text{if } g_{mr} \le g_m \\ g_r & \text{if } g_{mr} \ge g_r \\ g_{mr} & \text{otherwise} \end{cases} \tag{3.35}$$

- If G_{rc} is the effective optimum cut-off grade satisfying simultaneously refining and concentrator capacity constraints, then:

$$G_{rc} = \begin{cases} g_r & \text{if } g_{rc} \le g_r \\ g_c & \text{if } g_{rc} \ge g_c \\ g_{rc} & \text{otherwise} \end{cases} \tag{3.36}$$

The optimum cut-off grade (G) is the middle value of G_{mc}, G_{mr}, and G_{rc}.

$$\text{i.e. Optimum cut-off grade } (G) = \text{Middle value}(G_{mc}, G_{mr}, G_{rc}) \tag{3.37}$$

Circumstances may arise where locating the maxima of the economic model becomes impossible. Under that situation, the technique of identifying the effective optimum cut-off grade from the limiting and balancing cut-off grades is no longer enforceable. A search technique turns to be an alternative method (Ataei and Osanloo, 2003; Lane, 1988).

3.2 Multiple Processing Streams

To decide the processing method, ore material is classified based on the geology or ore type of the mineral deposit (Pettingell, 2017). Mines may have single or varying ore types with single or multiple minerals of economic interest. Since different ore types may require different processing techniques, applying multiple processing streams is the ultimate solution to achieving a specific degree of beneficiation. Likewise, mines with a homogeneous ore type with single or multiple minerals of interest may use multiple processing streams each dedicated to a specific grade, to achieve a specific degree of beneficiation that maximizes the operation's value (Pettingell, 2017). In the latter case, the low-cost, low-recovery method is used to high tonnages of low-grade ore. Conversely, the high cost, high recovery processing technique is applied to low tonnages of high-grade material. To have multiple processing streams for a homogeneous ore type, considerations must be given to the ore's amenability to the selected processing techniques, the climate, and the project's location, especially when heap leaching is one of the possible techniques (Kappes, 2002). Consider the following projects:

Mulatos Project

The Mulatos project is a combination of the Mulatos Mine Area and the San Carlos Area. The Mulatos mine, located in a rural area of the State of Sonora, Mexico, is one of the mines in the world that benefits from multiple processing streams. As of December 31, 2011, the measured and indicated mineral resources for the Mulatos project totaled 84.99 Mt with 2.77 Moz of gold contained. Proven and probable mineral reserves contain approximately 1.5 Moz with 150,000 oz as annual production. The mineral reserve for the Mulatos project mounts to 63.45 Mt with 2.34 Moz of contained gold. The project initially started processing its open-pit ore by heap leaching in 2005. Currently, the heap leach facility processes the lower grade ore at a maximum rate of 17,500 t/d. In early 2012, the project constructed a high-grade milling plant that was intended to process high-grade ore from the open-pit mining by using a gravity concentrator at 500 t/d. The San Carlos and Escondida areas started supplying high-grade ore for the milling operation in 2013. The gravity concentrator can process a maximum of 1000 t/d (MulatosProject, 2012).

Aurizona Project

The Aurizona Project is located in the state of Maranhão in northeastern Brazil. It operates three open-pits and one underground mine. Measured and Indicated mineral resources for this project are around 8,649,000 tonnes and 712,000 tonnes as an inferred mineral resource. The mine schedule is based on 2019 reserves. A total of 19.8 million tonnes of proven and probable ore grading 1.51 g/t is delivered to the process plant over a current mine life of 6.5 years. The mineral reserves declared on December 31, 2019 include 12.4 million tonnes of proven mineral reserves grading 1.51 g/t gold and 7.4 million tonnes of probable mineral reserves grading 1.51 g/t gold. This is a total of 19.8 million tonnes of mineral reserves grading 1.51 g/t gold for a contained 958,000 ounces of gold. In addition, the ore tonnage includes 0.7 Mt of ore at 1.07g/t gold currently in the stockpile from 2019 mining activity. The process plant consists of a crushing, grinding, gravity, cyanide leach, and cyanide destruction facility. The crushing and grinding circuits deliver ore to the gravity and cyanide leach/CIP circuits. A gravity circuit is integrated with an intensive leach reactor, an electrowinning cell, and associated equipment. A cyanide leach/CIP circuit consists of gold recovery and carbon handling circuits, including pre-leach thickening, leach, and CIP tanks, acid wash and elution, carbon reactivation, gold electrowinning, and melting. The plant was originally designed to treat soft saprolitic ores at a rate of 5,500 t/d through gravity and cyanide leach /CIP circuits. It was subsequently upgraded in 2018-2019 and the upgraded plant started commercial production in 2019 with a nominal processing rate of 8000t/d (AurizonaProject, 2020).

Cortez Mine

The mine is located 62 miles southwest of Elko, Nevada, USA, in Eureka and Lander Counties. It consists of three open-pits and one underground mine The mine is a joint venture between two wholly-owned subsidiaries of Barrick, Barrick Cortez Inc. (60%), and Barrick Gold Finance Inc. (40%). As of December 31, 2018, the deposit for the open-pit had 126 Mt equivalent to 4.04 Moz of gold in a reserve. The total reserve of the Cortez operations is 145.05 Mt.

Ore from Cortez occurs in oxide or refractory form. Low-grade oxide material is leached as ROM ore on three prepared double-lined leach pads. Pregnant solution from the leach pads is fed to CIC columns for gold recovery. The loaded carbon from the heap leach operation is transported to the mill for gold recovery. High-grade material is processed by mill/CIL method whose average throughput is 15,000 st/d. This high-grade oxide processing plant is made up of crushing, a semi-autogenous grinding (SAG) mill, a ball mill, grind thickener, a carbon-in-column (CIC) circuit for the grind thickener overflow solution, a CIL circuit, tailings counter-current-decantation (CCD) wash thickener circuit, carbon stripping and reactivation circuits, and a refinery to produce gold doré. There are no appropriate processes for single and double refractory ore at Cortez. Therefore, these ore types are shipped to a nearby operation; Goldstrike for processing. Current limits to the transportation rates imposed by environmental permits restrict the amount of ore that can be shipped to 1.8 million tons per year. If additional refractory ore is mined, it must be stockpiled. At Goldstrike, the ore will be processed in the roaster followed by a CIL circuit or in the total carbonaceous material (TCM) process, which includes pressure oxidation (POX) followed by resin-in-leach with Calcium thio-sulphate (CaTS). In 2018, the heap leaching facility processed 66,181 st/d at 0.012 oz/st with 58.1% recovery. The quantity, cut-off grade, and recovery for the mill/CIL circuit were 13,005, 0.131 oz/st, and 82.40%, respectively. Furthermore, the roaster circuit's corresponding quantity, cut-off grade, and recovery were 1,865 st/d, 0.198 oz/st, and 88.60%, respectively. The processing costs for low-grade oxide, high-grade oxide, and refractory ore types in $ /st are 1.57, 9.87, and 22.12, respectively (CortezMine, 2019).

Castle Mountain Project

The Castle Mountain Project is located in the historic Hart Mining District, at the southern end of the Castle Mountains, San Bernardino County, California, approximately 70 miles south of Las Vegas, Nevada. It started production in 1991 and ceased in 2001 due to local wall instability. Throughout this period, heap leaching was the only existing processing technique. Efforts to resume production have been undertaken between 2014 and 2018 whereby exploration, drilling activities, and recovery tests have been conducted. As of June 29, 2018, a deposit with 197.6 Mt in reserves equivalent to 3.56 Moz of gold has been declared. Since production is expected to start soon, the processing plan has been divided into two stages. At stage 1 (Years 1-3), the mine is expected to process 12,700 t/d of lower-grade ore at a cut-off grade of 0.343 g/t by the heap leaching method. At stage 2 (Years 4+), the project will employ two processing streams and its preparation will start during year 3 and involves expanding the stage 1 leach pad, adsorption, and desorption circuits to process 38,600 t/d of lower-grade ore at 0.435 g/t cut-off grade, and adding a 2,360 tonnes crushing system and mill for high-grade ore (at 3.17 g/t cut-off grade) with a Carbon-in-Leach (CIL) circuit for recovery of gold and silver. Recoveries are 72.40% for heap leaching at both stages and 94% for mill/CIL facility (CastleMountainProject, 2018).

Other operations applying multiple processing streams include but are not limited to:

- Carlin complex-Nevada Gold Mines located near the towns of Carlin and Elko Nevada, USA.

- Los Filos Mine Complex located in the Municipality of Eduardo Neri, Guerrero State, Mexico approximately 180 km southwest of Mexico City.
- Mesquite Gold Mine located in California, U.S.A.
- Santa Luz Project located within the Maria Preta mining district, Bahia state, Brazil.
- Fazenda Brasileiro Mine located in Bahia State, Brazil.
- Pueblo Viejo located in the central part of the Dominican Republic on the Caribbean island of Hispaniola in the province of Sanchez Ramirez.
- Loulo-Gounkoto Gold Mine Complex located west of the capital city of Bamako, Mali

Most of the projects described above rely heavily on open-pits to obtain ore. Because of variation in ore types and the abundance of low-grade material in several deposits, the use of multiple processing streams and the appropriate cut-off grade policy helps to maximize the overall value of a deposit. The Cortez mine is an example of operations with different ore types. The selection of an appropriate processing method is of paramount importance to maximize the gold recovery and consequently the value of this operation.

Low-grade deposits generally call for applying low-cost, low recovery processing methods such as heap leaching, which has proved effective production of gold under this circumstance. The addition of higher cost higher recovery methods like mill/CIL circuits to process the little available high-grade material in such deposits helps maximize the value of operations by maximizing gold production.

Many companies realize a cost advantage as they treat a large volume of material by heap leaching. Kappes (2002) describes that a basic heap leach (3,000 t/d) is $3,500 to $5,000 per daily ton of ore treated. Large operations (15,000 – 30,000 t/d) cost $2,000 to $4,000 per daily tonnes. This justifies that small operations with low-grade material will likely not benefit from using the heap leaching method.

Asad and Dimitrakopoulos (2013) developed a risk-based cut-off grade optimization model of multiple processing streams by extending Lane's algorithm. By applying the model, the material is classified and sent to two distinct processing plants. The material should be assigned and treated by a more lucrative option since different processing methods offers different contributions to the net values when they process a tonne of material with the same average grade. Implementation of the model on several but equally probable grade-tonnage curves shows the significant difference between the minimum and maximum NPV generated from different curves. Not taking into account the geological uncertainty of the deposit may pose severe economic implications to the operation.

This section aims to review the procedures used to determine the limiting, balancing, and effective optimum cut-off grade for operations using multiple processing options as proposed by (Asad and Dimitrakopoulos, 2013).

The profit $P = P_T$ obtained through mining the quantity of material Q_m in time T for multiple processing streams is stated as (Asad and Dimitrakopoulos, 2013):

$$P = \left(S - r\right)Q_r - \sum_p c_p Q_{cp} - mQ_m - fT \tag{3.38}$$

Inserting equation (3.38) in equation (3.10) results in the following expression

$$v = (S - r)Q_r - \sum_p c_p Q_{cp} - mQ_m - (f + Vd)T \tag{3.39}$$

The quantity refined Q_r is related to the average grade $\overline{g}$, recovery y, and Q_c as, $Q_r = \sum_p Q_{cp} \overline{g_p} y_p$ hence, equation (3.39) becomes:

$$v = (S - r)\sum_p Q_{cp} \overline{g_p} y_p - \sum_p c_p Q_{cp} - mQ_m - (f + Vd)T \tag{3.40}$$

It was discussed in the previous sections that maximizing NPV requires maximizing the value of v.

Considering the three stages of a mining system mentioned earlier, the capacity of each can be a limiting factor in maximizing the value of v and consequently the NPV.

If each stage is considered independently, the three equations for v can be developed from which the respective limiting cut-off grades can be determined.

Depending on the governing constraint, the duration T corresponding to mining, concentrator, and refining can be defined as $T_m = \frac{Q_m}{M}$, $T_c = \frac{Q_{cp}}{C_p}$, and $T_r = \frac{Q_r}{R}$ respectively.

Applying the durations above in equation (3.60), the following functions related to mining, processing, and refining are derived respectively by considering process p:

$$v_{mp} = \left(S - r\right)\sum_p Q_{cp} \overline{g_p} y_p - \sum_p c_p Q_{cp} - \left[m + \frac{f + Vd}{M}\right]Q_m \tag{3.41}$$

$$v_{cp} = \left(S - r\right)\sum_p Q_{cp} \overline{g_p} y_p - \sum_p c_p Q_{cp} - mQ_m - \left[\frac{f + Vd}{C_p}\right]Q_{cp} \tag{3.42}$$

$$v_{rp} = \left[\left(\left(S - r\right) - \frac{f + Vd}{R}\right)\right]\sum_p Q_{cp} \overline{g_p} y_p - \sum_p c_p Q_{cp} - mQ_m \tag{3.43}$$

The three limiting cut-off grades corresponding to each processing streams are defined as follows (Asad and Dimitrakopoulos, 2013; Pettingell, 2017):

$$g_{mp} = \frac{c_p}{(S-r)y_p} \tag{3.44}$$

$$g_{cp} = \frac{\left(c_p + \dfrac{f+Vd}{C_p}\right)}{(S-r)y_p} \tag{3.45}$$

$$g_{rc} = \frac{c_p}{\left((S-r) - \dfrac{f+Vd}{R}\right)y_p} \tag{3.46}$$

The mine-mill, mine-refinery, and refinery-mill balancing cut-off grades are respectively denoted as g_{mcp}, g_{mrp}, and g_{rcp}. The balancing cut-off grades and consequently the optimum cut-off grade for a particular processing stream are calculated by following the same procedure used in subsection 3.1.3.

3.3 Modified Lane's Method

Some of the prominent studies and rationale behind considering the environmental aspect of mining operations in cut-off grade optimization were thoroughly discussed in sections 1.1, 1.2, 1.3, 2.5, and subsection 2.6.5. In the following sections, some important equations used to establish the optimum cut-off grade model for a single processing stream considering the rehabilitation cost by Gholamnejad (2009) are reviewed. Subsection 3.4.2 is a proof of some important formulas that are used in the model proposed in this research to determine the optimum cut-off grade strategy for an operation applying multiple processing streams with consideration of a rehabilitation cost of the waste rock generated in the course of mining.

3.3.1 Single Processing Stream with Rehabilitation Cost

Gholamnejad (2009) considered the quantity $Q_m - Q_c$ as the amount of material that is sent to the waste dump. He applied cost $'h'$ and finally came out with the following profit equation:

$$P = SQ_r - \left[mQ_m - cQ_c - rQ_r - h(Q_m - Q_c) - fT\right] \tag{3.47}$$

Collecting like terms in equation (3.47) gives:

$$P = (S-r)Q_r - (m+h)Q_m - (c-h)Q_c - fT \tag{3.48}$$

Substituting equation (3.48) in equation (3.10) gives the following expression:

$$v = (S-r)Q_r - (c-h)Q_c - (m+h)Q_m - (f+Vd)T \tag{3.49}$$

The quantity refined Q_r is related to the average grade $\bar{g}$, recovery y, and Q_c as $Q_r = Q_c\bar{g}y$, hence, equation (3.49) becomes:

$$v = \left[\left(S-r\right)\bar{g}y - \left(c-h\right)\right]Q_c - \left(m+h\right)Q_m - \left(f+Vd\right)T \tag{3.50}$$

The capacity of each component can be a limiting factor in maximizing the value of v and consequently the NPV. If each component is considered independently, three equations for v can be established from which the respective limiting cut-off grades can be determined.

The durations corresponding to mining, concentrator, and refining when each is independently limiting the throughput of the operation were defined in subsection 3.1.1 as $T_m = \frac{Q_m}{M}$, $T_c = \frac{Q_c}{C}$, and $T_r = \frac{Q_r}{R}$ respectively. By inserting these durations in equation (3.50), the following functions related to mining, processing, and refining are derived respectively :

$$v_m = \left[(S-r)\bar{g}y - (c-h)\right]Q_c - \left[(m+h) + \frac{f+Vd}{M}\right]Q_m \tag{3.51}$$

$$v_c = \left[\left(S-r\right)\bar{g}y - \left((c-h) + \frac{f+Vd}{C}\right)\right]Q_c - \left(m+h\right)Q_m \tag{3.52}$$

$$v_r = \left[\left(\left(S-r\right) - \frac{f+Vd}{R}\right)\bar{g}y - (c-h)\right]Q_c - \left(m+h\right)Q_m \tag{3.53}$$

The three limiting cut-off grades corresponding to mining, processing, and refining are determined by first maximizing the three preceding functions where the derivative of v with respect to grade is found and set equal to zero. The quantity mined Q_m is independent of the grade and hence its derivative with respect to grade is also zero. The respective limiting cut-off grades are therefore stated as follows (Gholamnejad, 2009):

$$g_m = \frac{c-h}{\left(S-r\right)y} \tag{3.54}$$

$$g_c = \frac{\left(\left(c-h\right) + \frac{f+Vd}{C}\right)}{\left(S-r\right)y} \tag{3.55}$$

$$g_r = \frac{c-h}{\left(\left(S-r\right)-\dfrac{f+Vd}{R}\right)y} \tag{3.56}$$

Determination of the balancing cut-off grades does not rely on economic parameters. Rather, they are determined by using the grade-tonnage distribution of the mineral deposit (Asad and Topal, 2011; Lane, 1964; Pettingell, 2017). The balancing cut-off grades and the effective optimum cut-off grade are derived using the same procedures discussed in sections 3.1.2 and 3.1.3, respectively.

3.3.2 Multiple Processing Streams with Rehabilitation Cost

As an extension to the studies done by Pettingell (2017) and Gholamnejad (2009), this section focuses on establishing a modified cut-off grade model for multiple processing streams in which the rehabilitation cost is incorporated. The model developed is based on Lane's model since previous studies from which this model is deduced underlies on Lane's method. The development of this model and its application to determine the optimum production schedule is the key objective which this research seeks to achieve.

The profit $P = P_T$ obtained through mining the quantity of material Q_m in time T for multiple processing streams is stated as (Asad and Dimitrakopoulos, 2013):

$$P = (S-r)Q_r - \sum_p c_p Q_{cp} - mQ_m - fT \tag{3.57}$$

If multiple processing streams and rehabilitation cost are concurrently considered, the following overall profit equation can be deduced from equations (3.48) and (3.57):

$$P = (S-r)Q_r - \sum_p Q_{cp}(c_p - h) - (m+h)Q_m - fT \tag{3.58}$$

Inserting equation (3.58) in equation (3.10) results in the following expression:

$$v = \left(S-r\right)Q_r - \sum_p Q_{cp}(c_p - h) - (m+h)Q_m - (f+Vd)T \tag{3.59}$$

The quantity refined Q_r is related to the average grade $\overline{g}$, recovery y, and Q_c as $Q_r = \sum_p Q_{cp}\overline{g_p}y_p$, hence, equation (3.59) becomes:

$$v = \left(S-r\right)\sum_p Q_{cp}\overline{g_p}y_p - \sum_p (c_p - h)Q_{cp} - (m+h)Q_m - (f+Vd)T \tag{3.60}$$

It was discussed in the previous sections that maximizing NPV requires maximizing the value of v.

Considering the three components of a mining system mentioned earlier, the capacity of each can be a limiting factor in maximizing the value of v and consequently the NPV.

If each component is considered independently, the three equations for v can be developed from which the respective limiting cut-off grades can be determined.

Depending on the governing constraint, the duration T corresponding to mining, concentrator, and refining can be defined as $T_m = \frac{Q_m}{M}$, $T_c = \frac{Q_{cp}}{C_p}$, and $T_r = \frac{Q_r}{R}$ respectively.

Applying the durations above in equation (3.60), the following functions related to mining, processing, and refining when a process p is considered are respectively derived :

$$v_{mp} = (S-r)\sum_p Q_{cp}\overline{g_p}y_p - \sum_p (c_p - h)Q_{cp} - \left[(m+h) + \frac{f+Vd}{M}\right]Q_m \quad (3.61)$$

$$v_{cp} = \left(S-r\right)\Sigma_p Q_{cp}\overline{g_p}y_p - \Sigma_p (c_p - h)Q_{cp} - (m+h)Q_m - \left[\frac{f+Vd}{C_p}\right]Q_{cp} \quad (3.62)$$

$$v_{rp} = \left[\left(\left(S-r\right) - \frac{f+Vd}{R}\right)\right]\Sigma_p Q_{cp}\overline{g_p}y_p - \Sigma_p (c_p - h)Q_{cp} - (m+h)Q_m \quad (3.63)$$

For each processing stream in consideration, the limiting cut-off grades related to mine, mill, and refinery are defined as follows:

$$g_{mp} = \frac{c_p - h}{(S-r)y_p} \quad (3.64)$$

$$g_{cp} = \frac{\left(\left(c_p - h\right) + \frac{f+Vd}{C_p}\right)}{\left(S-r\right)y_p} \quad (3.65)$$

$$g_{rp} = \frac{c_p - h}{\left(\left(S-r\right) - \frac{f+Vd}{R}\right)y_p} \quad (3.66)$$

The mine-processing, mining-refining, and refining-processing balancing cut-off grades are respectively denoted as g_{mcp}, g_{mrp}, and g_{rcp}. The balancing cut-off grades and consequently the optimum cut-off grade for a particular processing stream are calculated by following the same procedure used in section 3.1.3.

While a general overview about cut-off grade optimization considering multiple processing streams and rehabilitation cost has included analysis of different grades from limiting to balancing and finally to the selection of an overall optimum cut-off grade, computation of cut-off grades in this thesis assumes that processing capacity is the only bottleneck in the mine operation.

3.4 Steps of the Algorithm

The model uses a set of simulated grade-tonnage distributions to establish an optimum cut-off grade policy. As described in subsection 3.3.2, computation of cut-off grades by using the proposed model assumes that mining and refining capacities will at any point not be the bottlenecks. For this case, balancing cut-off grades are not computed, and the dynamic limiting cut-off grades for each process are the only grades calculated to maximize NPV. Each grade-tonnage curve gives an NPV from which the average of the NPVs of all grade-tonnage curves gives the expected NPV of the operation for a given mineral deposit. The procedure is computation-intensive; hence the iterative steps of the algorithm are established in a MATLAB environment, and it ceases when the resource has completely depleted. Figure 3-9 illustrates the steps of the algorithm.

The steps in the flow chart can be described as follows:

- **Economic, Operational, and Geological Data**

This is the first step of the algorithm, and it involves entering economic, operational, and geological data. The economic data required in determining the cut-off grade policy include metal price, unit processing costs, unit rehabilitation cost, fixed cost, and the discount rate. Geological data is simply constituted of grade-tonnage distribution. Operational data include processing capacities and metallurgical recoveries.

- **Total Reserve**

The total amount of material available in the deposit before mining and as mining progresses is computed at this step. No distinction of material as ore or waste at this stage until optimal cut-off grades are determined.

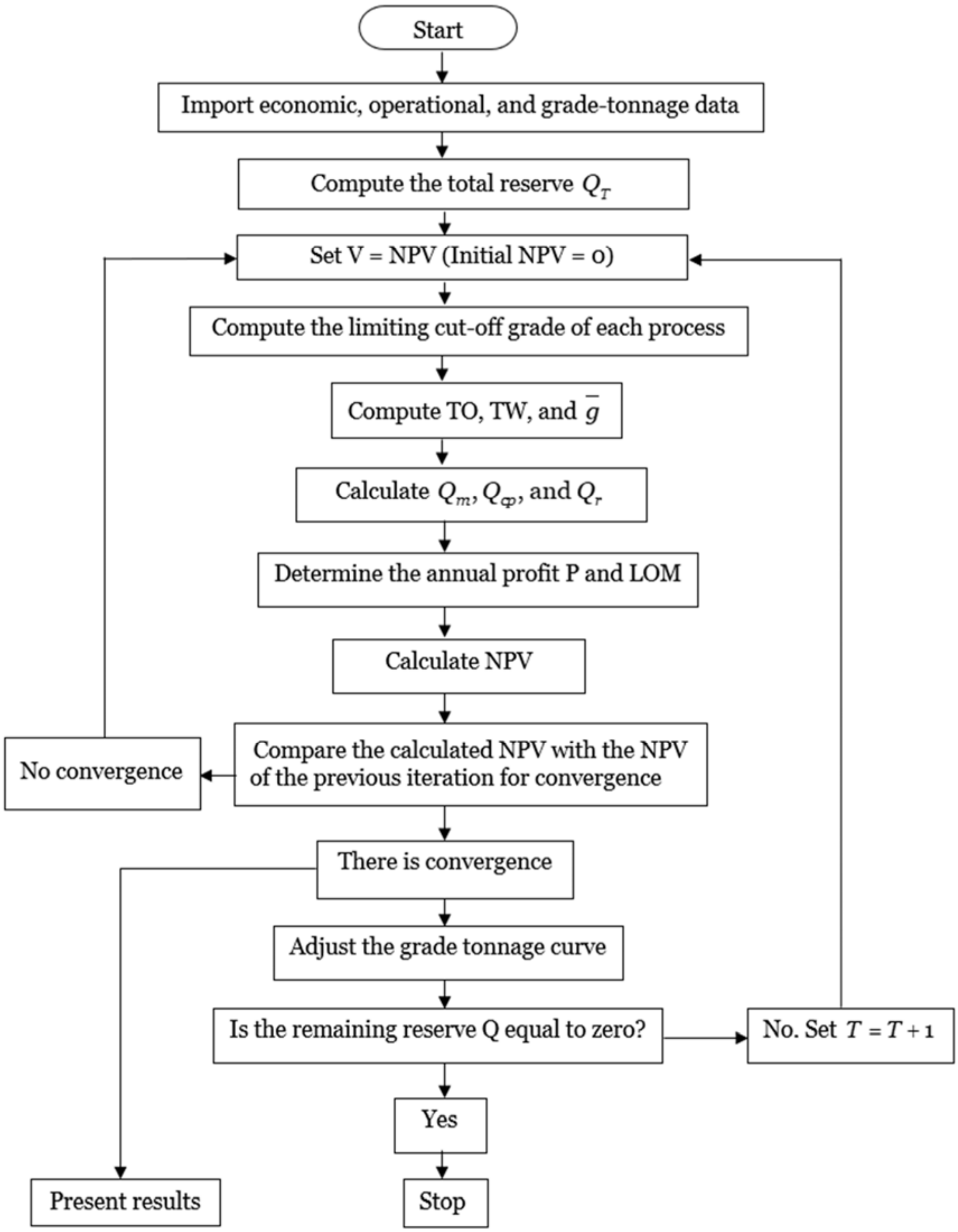

Figure 3-9: A flowchart of the algorithm used for cut-off grade optimization.

- **Initial NPV Estimates**

Determination of dynamic cut-off grades requires a predetermined NPV. The NPV of new operations is initially unknown since it depends on the cut-off grade. For this reason, an initial NPV estimate equivalent to zero is assumed.

- **Limiting Cut-off Grades Determination**

The cut-off grade related to each process when rehabilitation cost is taken into account is computed as follows:

$$g_{cp} = \frac{\left(\left(c_p - h \right) + \dfrac{f + Vd}{C_p} \right)}{\left(S - r \right) y_p} \tag{3.67}$$

If parameter $'h'$ is set to zero, equation (3.67) reduces to an expression that can be used to calculate the cut-off grade under no rehabilitation cost consideration.

- **Amount of Ore, Waste, and Average Grades**

When the cut-off grade of each process is known, the amount of ore, waste, and average grades corresponding to each process can be computed by using equations (3.68)-(3.70)respectively.

$$TO(g_{cp}) = \sum_{k \geq k^*}^{K} T_k \tag{3.68}$$

$$TW(g_{cpm}) = \sum_{k < k^*}^{K} T_k \tag{3.69}$$

$$\overline{g_{cp}} = \frac{\sum_{k \geq k^*}^{K} T_k \left(\dfrac{g_k + g_{k+1}}{2} \right)}{TO(g_{cp})} \tag{3.70}$$

A deposit consists of minerals that are not distributed evenly, and as a result, the concentration of the mineral in a given volume of the material varies within the deposit. At any given time, the classification of the material as ore or waste is determined by the cut-off. To understand how much material will be treated as ore or waste, the material within a deposit is divided into several groups, and a range of grades defines each group. The group will therefore have lower and upper-grade boundaries. A range of grades defining a group will be in the form $\left[g_k, g_{k+1} \right]$ simply read as a grade g_k to grade g_{k+1}. The frequency related to each grade category accounts for the tonnage of material in that group.

In equations (3.68)-(3.70), k represents the individual grade categories and T_k is the tonnage of material corresponding to each group. Furthermore, k^* is the grade category from which the lower grade boundary is an optimum cut-off grade. That being the case, if $[g_k, g_{k+1}]$ is the grade category from which the optimum cut-off grade is deduced, the value g_k is the ultimate optimum cut-off grade.

For two processing streams, two optimum cut-off grades are often determined. Each processing stream will have a specific amount of material that can be treated economically as defined in equation (3.68). The total amount of ore in the deposit is the algebraic sum of all material above the minimum of the two optimum cut-off grades. The material below the minimum cut-off grade is simply termed as waste and the quantity $TW(g_{cpm})$ is referred to as waste tonnage. The combination of all grade categories with their respective tonnages and the average grades of material for a given deposit defines the grade-tonnage curve.

The average grade of material in each grade category is a term in brackets in equation (3.70), and it defines the average concentration of metal in every tonne of material present in that particular category. The term $\overline{g}_{cp}$ is the weighted average grade of material classified as an ore for the process p. It signifies the concentration of metal present in every tonne of material sent to the processing unit.

- **Quantity Mined, Processed, and Refined**

Once the cut-off grades of all processes are determined, the quantity of material mined Q_m, the quantity of material sent to the processing facilities Q_c, and the quantity of refined product Q_r can successfully be determined.

The quantity of material mined Q_m consists of both ore and waste. In this thesis, it is assumed that no stockpiling of material takes place. The material above cut-off grades is sent directly to their respective processing facilities while material below the minimum cut-off grade will be routed to the waste dump. The quantity Q_m is related to the amounts of ore and waste mined as follows:

$$Q_m = Q_c + TW(g_{cpm}) \tag{3.71}$$

Processing facilities always have limited capacities they can handle annually. Based on the determined optimum cut-off grades, the total amount of material declared as an ore for each processing facility may be larger than the maximum processing capacity of that facility.

Conversely, the amount of material classified as ore may drop below the processing capacity near the end of the mining project. Due to this variation of the amount of material above the cut-off grades during the life of the mine, the amount of ore sent to the processing stream p can be defined as follows:

$$Q_{cp} = \begin{cases} TO(g_{cp}) \text{ if } < C_p \\ C_p \text{ otherwise} \end{cases} \quad (3.72)$$

In equation (3.72), Q_{cp} is the amount of ore processed by stream p. Q_c in equation (3.71) is the total amount of material processed by all facilities and is related to the quantities processed by individual facilities as:

$$Q_c = \sum_p Q_{cp} \quad (3.73)$$

The quantity of material mined Q_m described in equation (3.71) can be rewritten as:

$$Q_m = \left[\sum_p Q_{cp}\right] * \left[1 + SR\right] \quad (3.74)$$

As far as this research is concerned, SR in equation (3.74) is the ratio between the total amount of waste mined to the total amount of ore processed and is simply termed as stripping ratio. The amount of product refined is defined as:

$$Q_r = \sum_p Q_{cp} \overline{g_p} y_p \quad (3.75)$$

Here, $\overline{g_p}$ and y_p are the average grade and average metal recovery respectively for the process p. The quantity Q_r is the quantity of saleable metal obtained by refining products from all processing streams for the same production period.

- **Annual Profit and Life of Mine (LOM)**

Determination of the limiting cut-off grades aids computation of the quantities mined, processed, and refined. Once these quantities are known, the profit associated with a given operation can simply be calculated using equation (3.76) below:

$$P = (S - r)Q_r - \sum_p Q_{cp}(c_p - h) - (m + h)Q_m - fT \quad (3.76)$$

The proposed model assumes one year period in profit calculations. This duration may change near the end of the mining project where the available material above cut-off grades can no longer satisfy the required capacities of the processing facilities for the entire operational year.

The value of T to be used in equation (3.76) can mathematically be expressed as $0 < T \leq 1$. The choice of the value of T depends on the remaining life of the mine (LOM) which is basically computed using the quantity of ore present in the deposit and the quantity of ore processed by all facilities.

The life of the mine is calculated as shown in equation (3.77).

$$LOM = \frac{TO(g_{cpm})}{\sum_p Q_{cp}} \tag{3.77}$$

Now that LOM is determined, a specific condition can be set to decide which value of T should be used in equation (3.76). The following expression defines different situations and their corresponding values of T.

$$T = \begin{cases} 1 \text{ if } LOM > 1 \\ LOM \text{ otherwise} \end{cases} \tag{3.78}$$

- **Net Present Value (NPV)**

The NPV is calculated by discounting the annual cashflows P over the life of the mine LOM using the discount rate d as shown in equation (3.79).

$$NPV = P * \left[\frac{1-(1+d)^{-LOM}}{d} \right] \tag{3.79}$$

- **Comparison of NPVs for Convergence**

As stated previously, an initial estimate of NPV is taken as zero, and the whole process of determining the optimum cut-off grade is iterative. A specific value is set and used to check convergence between net present values obtained in two consecutive iterations. Having V_0 as an initial estimate of NPV in equation (3.67), the cut-off grade, and NPV for $T = 1$ denoted as V_1 is obtained after several intermediate steps within the first iteration. In the next iteration, the previous NPV (V_1) is set as an initial estimate of V in calculating the cut-off grade in equation (3.67) and the associated NPV (V_2) for $T = 2$. V_1 and V_2 are said to converge if their difference is within the predefined range. If V_1 and the next value of V in the intermediate steps of the second iteration do not converge, V replaces V_1 in equation (3.67), and the process repeats until convergence occurs.

The limiting cut-off grade determined by completely setting NPV equal to zero eliminates the opportunity cost in equation (3.67) , and the value of the cut-off grade obtained in this case is sub-optimal and will consequently not maximize the NPV of the project.

- **Grade-tonnage Curve Adjustment and the Remaining Reserve**

Once convergence between two successive NPVs has occurred and results of the current operational year $'T'$ are available, the year is then set to $T = T + 1$, and the new grade-tonnage distribution corresponding to this new operational year is obtained by subtracting the mined material from the preceding grade-tonnage curve in proportionate amounts in such a way that the overall grade distribution is maintained, otherwise it will turn to a different grade-tonnage distribution leading to wrong outputs.

Adjustment is done by deducting $TW(g_{cpm})$ tonnes from the intervals below the minimum optimum cut-off grade and individual Q_{cp} tonnes above each optimum cut-off grade in the intervals corresponding to each processing stream (Asad, 2005b; Asad and Dimitrakopoulos, 2013; Bascetin and Nieto, 2007; Githiria and Musingwini, 2019; Pettingell, 2017). Figure 3-10 shows an example of adjusted curves at every specific time. The amount of reserve available at any time and the average grade of the material in reserve can be computed once the cut-off grade is determined and the adjusted grade-tonnage is available.

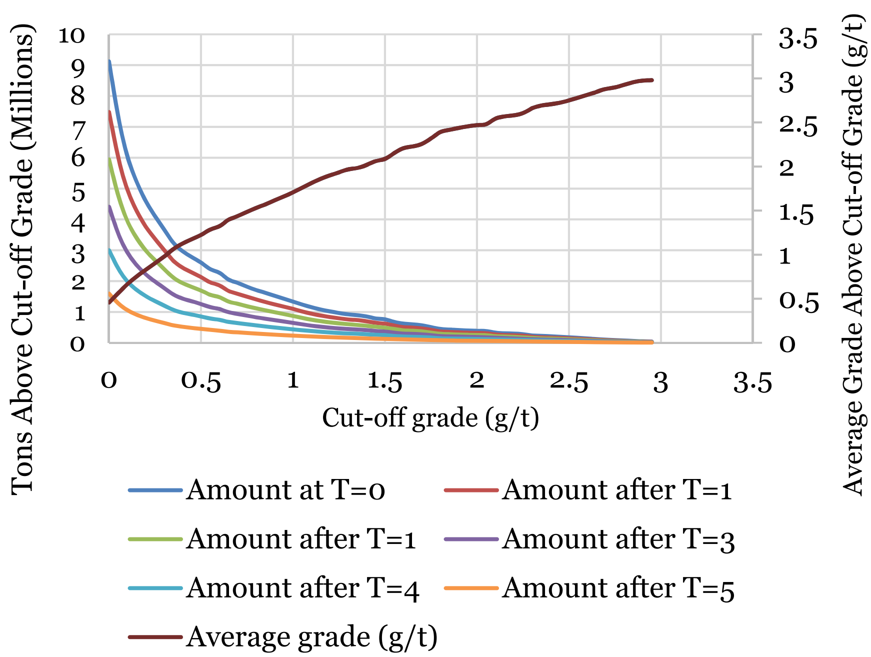

Figure 3-10. Grade-tonnage adjustment to show resource depletion with time.

After adjusting the grade-tonnage curve, the algorithm checks the remaining reserve. The iterative process continues until the resource is exhausted, there is no more profitable material available in the deposit, or a predetermined amount of material remains (Pettingell, 2017).

- **Presentation of Results**

After the resource has been exhausted, results such as optimum cut-off grades and related quantities mined, processed, refined, stripping ratios, average grades, profits, and net present values are presented.

3.5 Summary and Conclusion

In this chapter, a review of the previous studies that form the basis of this research was presented. An integrated model that additionally considers a rehabilitation cost to optimize the cut-off grades for operations using multiple metal recovery methods was developed.

A detailed explanation of the steps of the algorithm that serve as an optimization procedure is given. The steps of the proposed model are implemented in the MATLAB environment to define the cut-off grade policy and the production schedule based on the given economic, operational, and grade-tonnage data of a hypothetical low-grade gold deposit for an operation that uses multiple processing methods.

CHAPTER 4

VERIFICATION, EXPERIMENTS AND DISCUSSION OF RESULTS

This chapter presents the application of the developed model on a hypothetical gold deposit. Owing to the limited availability of information pertaining to the orebody, a means of generating several equally probable grade-tonnage curves is highlighted. In addition, details about the economic and technical parameters used to implement the model are presented. In establishing the schedule of cut-off grades and the flow of material for an operation employing multiple processing streams, three different cases based on consideration of the rehabilitation cost of the waste rock are analyzed, and their results are compared. Furthermore, the response of the NPV on some parameters is analyzed across all simulated grade-tonnage curves.

4.1 Introduction

This chapter focuses mainly on applying the developed model on the hypothetical gold deposit to establish the cut-off grade policy of an operation utilizing multiple processing streams. A set of simulated grade-tonnage curves are developed to establish the LOM production schedule, evaluate the value of the operation, and quantify the risk associated with relying on a constant grade-tonnage curve. Most of the economic and operational parameters used in this thesis are obtained from different literatures, and the rest are estimated with reference to the past studies.

While the determination of the optimum cut-off grades by incorporating rehabilitation cost considering an operation with multiple processing streams is a key purpose of this study, three different scenarios are considered to assess the impact of rehabilitation cost on the value of the project.

- **Scenario (1):** this scenario ignores the rehabilitation cost of the waste rock when defining the optimum cut-off grades. However, doing this necessitates operations to set a one-time rehabilitation cost. To avoid misreporting when this approach is practiced, Osanloo et al. (2008) propose that the NPV obtained from the application of the model across the simulated grade-tonnage curves is subject to deduction of this cost. This thesis does not determine the one-time cost that should be deducted from the NPV obtained by this approach. The NPV obtained is simply used to describe the impact of not incorporating the rehabilitation cost of the waste rock on the operation's value.
- **Scenario (2):** in this scenario, the cost of rehabilitating the waste rock is deducted from the annual profits but not included in the whole process of optimizing the cut-off grades.
- **Scenario (3):** this scenario takes into account the full rehabilitation cost when establishing the cut-off grade strategy. This scenario is taken as the base scenario to compare different outputs.

4.2 Modeling the Grade-tonnage Data

One of the primary sources of risks affecting mining projects is the uncertainty in the grade-tonnage distributions of the orebody. The uncertainty is due to limited information on grade and other parameters pertaining to the orebody (Botin et al., 2015). The grade-tonnage data and the economic and technical parameters are used to evaluate the economic potential of the mining project (Pettingell, 2017). To assess the impact of geological uncertainty on the value of the operation, in this thesis, a set of several equally probable grade-tonnage curves are developed by using Monte Carlo simulation.

Monte Carlo simulation is a mathematical technique used to model and simulate a probabilistic system where several random scenarios are generated for the sake of evaluating the performance of a decision policy or value of an asset (Brandimarte, 2014). The use of Monte Carlo simulation requires a defined probability function which in such is substituted into the random uncertain variable and the probability of an outcome is defined (Pettingell, 2017). To establish the grade-tonnage data, grades are assumed to follow a lognormal distribution with the mean and variance of 1.3 and 5.5, respectively. A lognormal distribution is defined by the function defined by equation (4.1) below:

$$f(x) = \begin{cases} \dfrac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{\ln(x)-\mu}{\sigma}\right)^2} & \text{if } x > 0 \\ 0 \text{ if } x \le 0 \end{cases} \tag{4.1}$$

If Y is a normal random variable with mean μ and standard deviation σ, the variable $X = e^Y$ will follow a lognormal distribution. If X is a lognormal distribution, then:

$$E(X) = e^{\left(\mu + \frac{1}{2}\sigma^2\right)} \tag{4.2}$$

$$Var(X) = e^{(2\mu+2\sigma^2)} - e^{(2\mu+\sigma^2)} \tag{4.3}$$

Table A1 and Table (APPENDIX A) show a set of 15 simulated grade-tonnage curves.

The cut-off grade policy of a hypothetical gold mine was established by applying the model described in CHAPTER 4. It has been assumed in this research that the hypothetical gold deposit will be exploited concurrently by the heap leach (HL) and carbon-in-leach (CIL) facilities. As stated earlier, the optimum cut-off grades were determined for three different scenarios of the rehabilitation cost by considering that the operation uses multiple processing streams. If a single processing stream was to be employed to exploit the ore resource, the HL facility would be the one.

4.3 Economic and Operational Parameters

While the importance of incorporating dynamic economic parameters has been described by Barr (2012) and Thompson and Barr (2014), in this study, all economic and operational parameters were assumed constant and highlighted in Table 4-1. As stated earlier, the economic and technical parameters used to establish the cut-off grade policy in this study were obtained from different literatures and estimates from the past studies.

Table 4-2 shows the literatures from which some parameters used in this study originate.

Table 4-1. Economic and technical parameters of a hypothetical gold mine.

Parameter	Value	Unit	Notation
Gold price	1,500.00	C$/oz	S
Processing cost HL	5.00	C$/t	c1
Processing cost CIL	16.00	C$/t	c2
Mining cost (both ore and waste)	2.70	C$/t	m
Refining cost	5.50	C$/oz	r
Recovery HL	70.00	%	y1
Recovery CIL	90.00	%	y2
Processing capacity HL	640,000.00	t/yr	C1
Processing capacity CIL	350,000.00	t/yr	C2
Mining capacity	5,000,000.00	t/yr	M
Refining capacity	100,000.00	oz/yr	R
Annual fixed cost	1,300,000.00	C$/yr	f
Discount rate	10.00	%	i
Rehabilitation cost	0.95	C$/t	h

Table 4-2. Source of some economic and technical parameters.

Parameter	**Source**
Gold price	Pettingell (2017) and Levinson and Dimitrakopoulos (2020)
Processing cost HL	Senécal and Dimitrakopoulos (2020)
Refining cost	Senécal and Dimitrakopoulos (2020)
Recovery HL	Pettingell (2017) and Moosavi and Gholamnejad (2016)
Discount rate	Levinson and Dimitrakopoulos (2020)
Processing cost CIL	Pettingell (2017)
Recovery CIL	Pettingell (2017), Senécal and Dimitrakopoulos (2020), Khan and Asad (2020), and Githiria and Musingwini (2019)

4.4 Results of the Application of the Model

The model assumes that a resource and an ultimate pit limit with no multiple pushbacks had been defined along with the stated economic, operational, and simulated grade-tonnage data. Furthermore, the model used in this research holds true for an operation whose throughput is only limited by the capacities of the processing facilities. Mining and refining capacities are set too large so that they may not constrain the operation's throughput at any time, as shown in Table 4-1.

The steps of the model presented in this thesis were coded in the MATLAB environment. They were tested on the data presented by Pettingell (2017), where rehabilitation cost had to be set equal to zero. This is the same approach of defining cut-off grades as scenario 1 in this thesis. The average NPV obtained upon application of the steps of the algorithm was found to be less than the average NPV presented in such a study by 1.23%. Furthermore, the model was applied on the data presented in the study done by Gholamnejad (2009). In this case, a model was modified to suit a single processing stream that incorporates rehabilitation costs when defining the cut-off grade policy. The NPV obtained when the model was applied perfectly matches with the NPV presented in such a study.

It has been described in section 4.2 that a total of 15 equally probable simulated grade-tonnage curves were established with the intention of capturing the geologic uncertainties associated with the deposits. The simulated grade-tonnage data presented in APPENDIX A (Table A1 and Table) is used as the grade-tonnage curve throughout the life of the mine. However, in the real world situation, for the entire life of the mine, an operation may have multiple mining phases, each having a unique grade-tonnage curve (Barr, 2012; Pettingell, 2017).

Subject to the capacity constraints, the model is applied to each of the 15 simulated grade-tonnage curves to establish the cut-off grade policy for a multiple processing streams operation.

Scenario (1):

In this scenario, rehabilitation cost was not considered in determining the cut-off grade strategy. Referring to the steps of the proposed algorithm shown in section 3.4 of this thesis, this approach excludes the cost in both the cut-off grade formula and the profit function represented by equations (3.67) and (3.76) respectively. This represents one of the shortcomings manifested in the traditional Lane's method, where this cost is not part of the parameters required to establish the optimum cut-off grade policy. It was previously explained that one of the key elements of a successful mining business is environmental protection. Not considering rehabilitation cost results in either overreporting the value of the operation as the cost to rehabilitate the waste rock must be deducted from the cashflows; otherwise, the wastes should be left unrehabilitated, which is contrary to the environmental needs as proposed by Gholamnejad (2009), Osanloo et al. (2008), and WCED (1987).

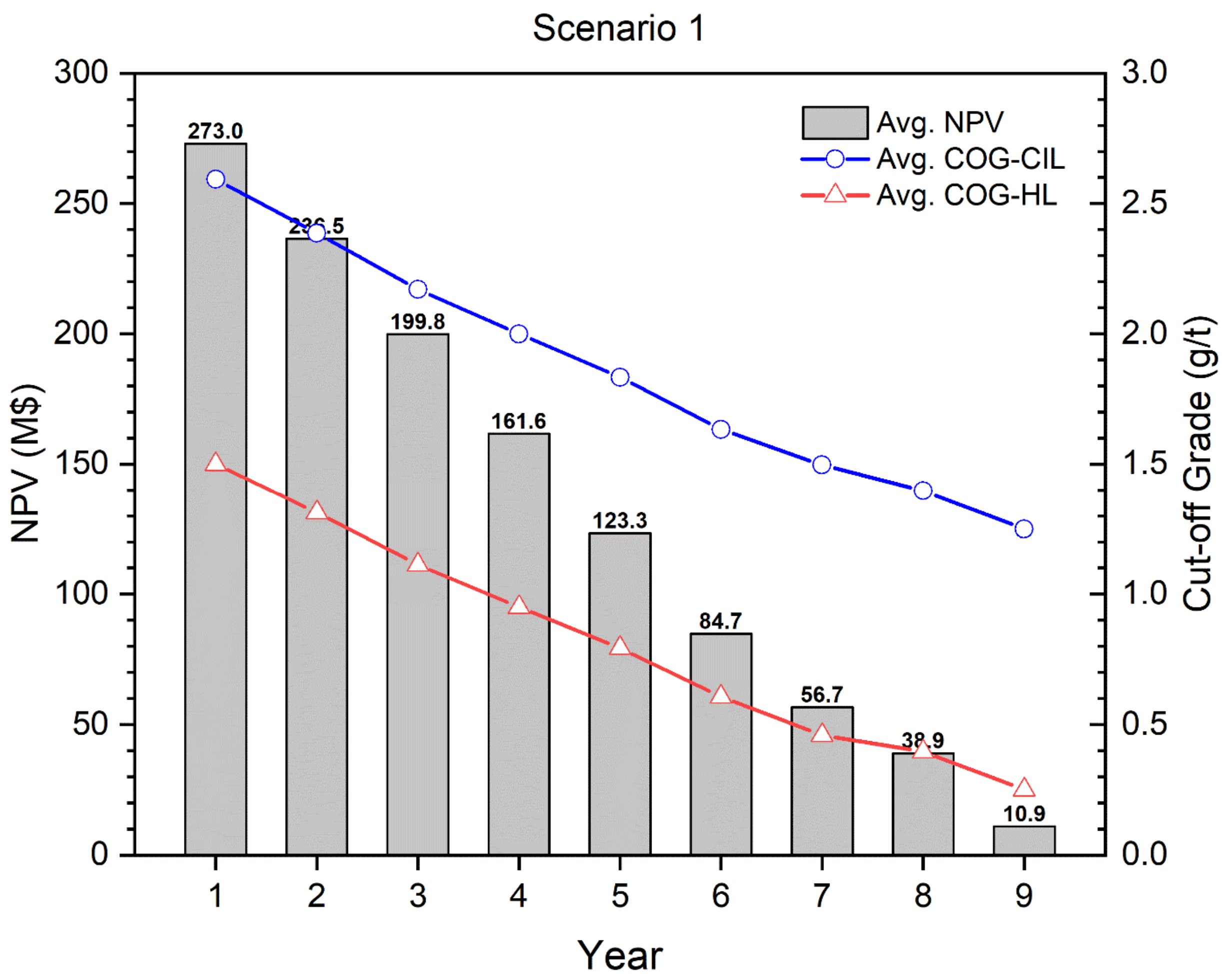

Figure 4-1 summarizes the average NPV and COGs expected from the operation that applies multiple processing streams to exploit the hypothetical gold deposit without considering the waste rock's rehabilitation cost.

Table 4-3 is a cut-off grade strategy for grade-tonnage realization 1 (GTR 1) when rehabilitation cost is not included in the cut-off grade optimization process. A complete summary of the cut-off grade policies determined across individual 15 simulated grade tonnage curves by using this approach is found in Table B1 (APPENDIX B).

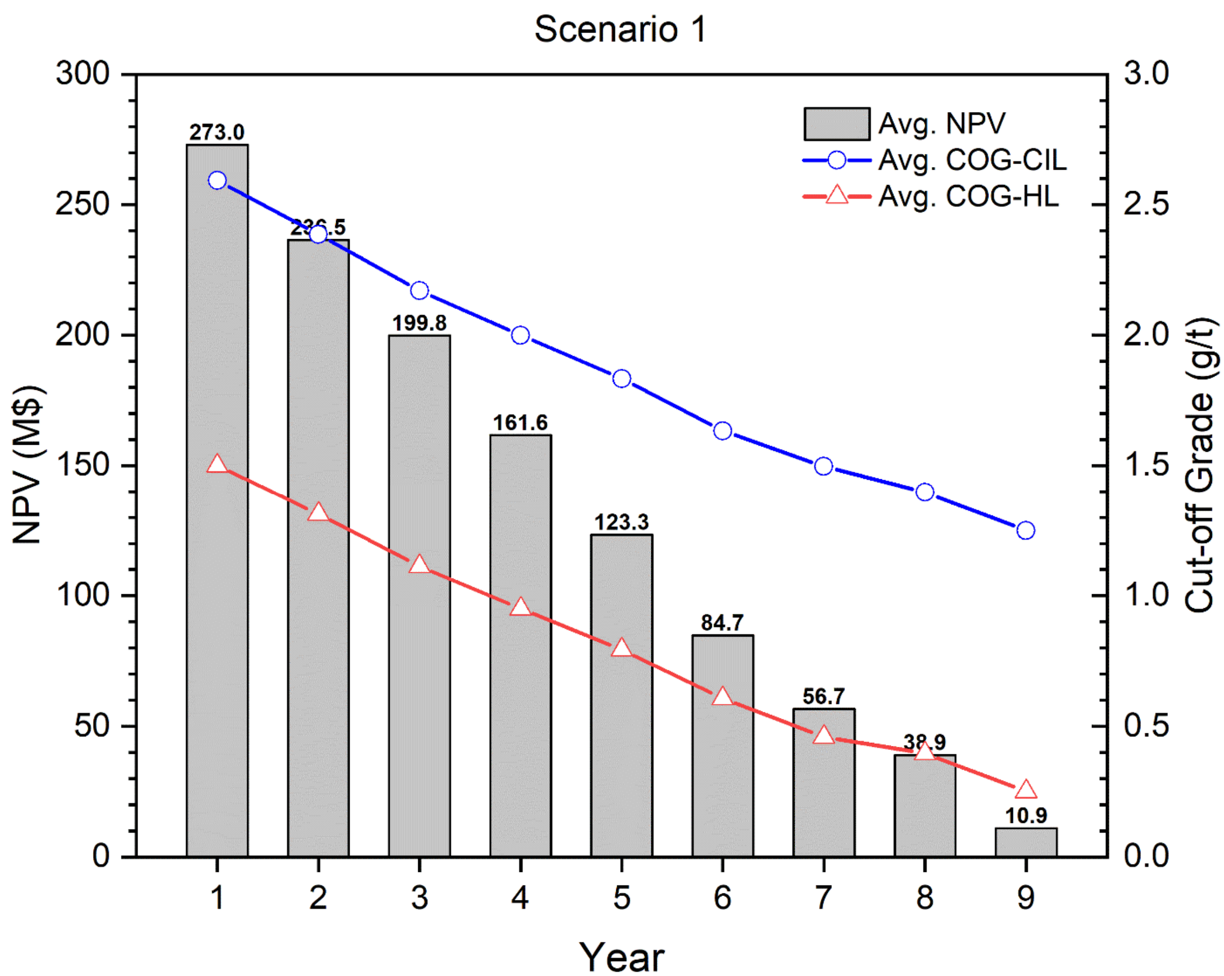

Figure 4-1. Average NPV and COGs during the mine life for scenario 1.

Table 4-3. A complete cut-off grade policy for GTR 1, rehabilitation cost not included in cut-off grade optimization process.

GTR 1	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.60	640,000	350,000	3,582,371	59,536	69,203,541	271,087,998	2,592,371
	2	1.30	2.40	640,000	350,000	3,086,057	55,240	64,124,216	235,916,881	2,096,057
	3	1.10	2.15	640,000	350,000	2,628,517	50,772	58,682,393	200,886,945	1,638,517
	4	0.95	2.00	640,000	350,000	2,357,463	47,587	54,653,127	162,806,617	1,367,463
	5	0.80	1.85	640,000	350,000	2,093,313	44,441	50,664,413	125,238,792	1,103,313
	6	0.60	1.65	640,000	350,000	1,813,622	40,656	45,763,889	87,032,371	823,622
	7	0.45	1.50	398,265	350,000	1,199,275	32,844	36,956,597	56,727,629	451,009
	8	0.40	1.40	22,490	350,000	570,934	25,934	30,203,951	39,155,521	198,444
	9	0.25	1.25	21,658	142,520	218,448	10,629	11,606,965	11,079,376	54,270

Scenario (2):

This scenario does not ignore environmental protection. It budgets the cost related to rehabilitation of the waste rock from the profits generated on an annual basis. However, this cost is not full-figured in the cut-off grade optimization process in a sense that in the proposed model, the cost is not incorporated in the cut-off grade function defined by equation (3.67) but mining operations honoring environmental protection budgets the cost related to waste rock rehabilitation by deducting it from equation (3.76). The average NPV and COGs expected from the operation that applies multiple processing streams to exploit the hypothetical gold deposit while budgeting rehabilitation cost of the waste rock from the annual profits without figuring the cost in the cut-off grade function are presented in Figure 4-2. Table 4-4 shows a cut-off grade strategy for GTR 1 under this scenario. A complete summary of the cut-off grade policies determined across 15 simulated grade tonnage curves using this approach is found in Table B2 (APPENDIX B).

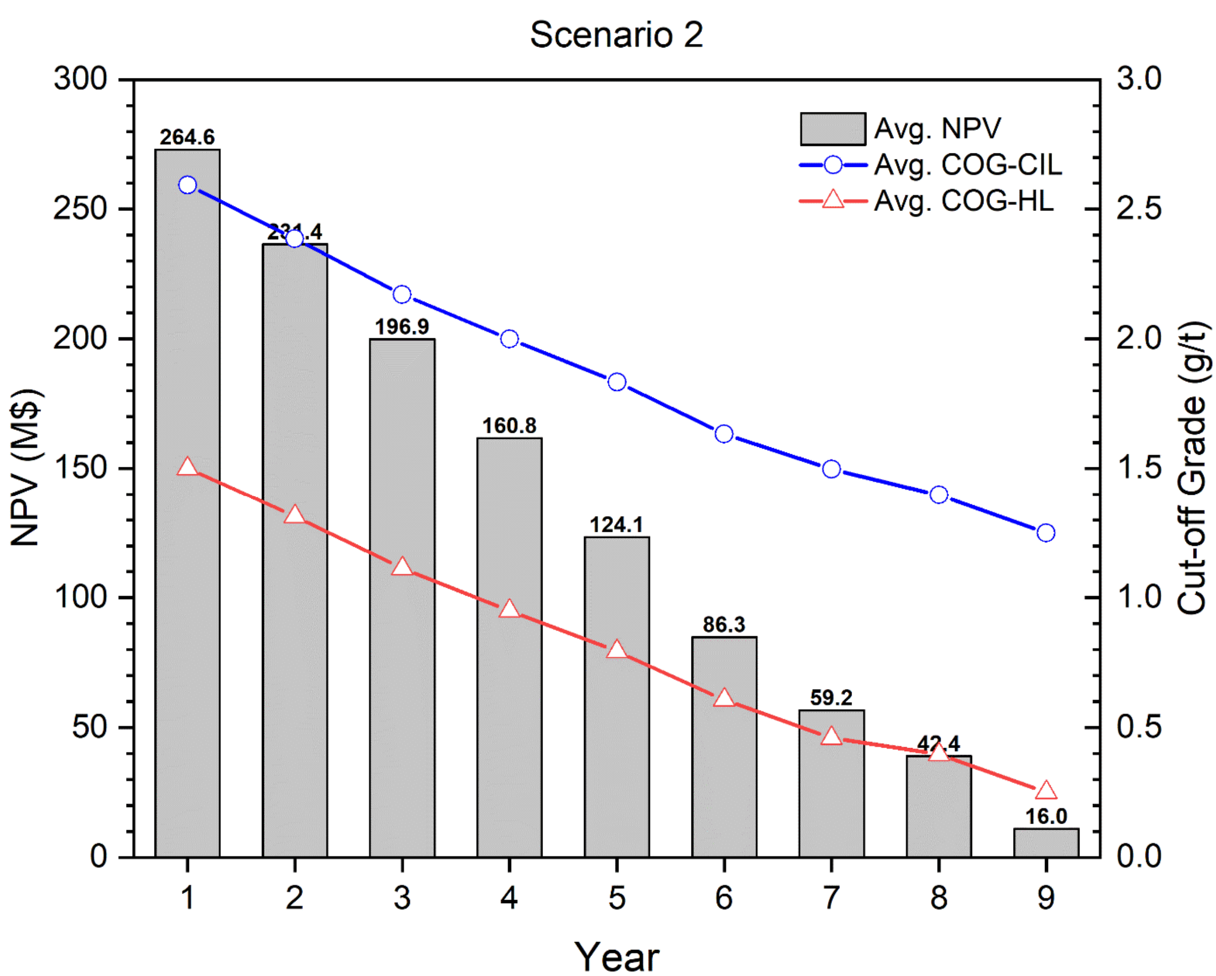

Figure 4-2. Average NPV and COGs during the mine life for scenario 2.

Table 4-4. A complete cut-off grade policy for GTR 1, rehabilitation cost partially figured in cut-off grade optimization process.

GTR 1	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.45	2.55	640,000	350,000	3,467,964	58,593	65,750,045	264,175,633	2,477,964
	2	1.25	2.30	640,000	350,000	2,949,830	53,663	60,273,041	231,344,245	1,959,830
	3	1.10	2.15	640,000	350,000	2,628,517	50,767	57,116,993	199,184,853	1,638,517
	4	0.95	2.00	640,000	350,000	2,357,463	47,591	53,360,034	163,000,675	1,367,463
	5	0.80	1.85	640,000	350,000	2,093,313	44,452	49,633,722	127,212,231	1,103,313
	6	0.65	1.65	640,000	350,000	1,882,394	41,201	45,544,197	88,719,579	892,394
	7	0.50	1.50	370,911	350,000	1,210,716	32,698	36,377,815	59,994,427	489,805
	8	0.40	1.40	54,690	350,000	620,288	26,347	30,322,360	43,656,462	215,599
	9	0.30	1.30	21,280	221,506	339,513	16,490	18,685,939	17,836,578	96,727

Scenario (3):

In this scenario, rehabilitation cost is figured in the whole cut-off grade optimization process by incorporating it into the cut-off grade and profit functions defined by equations (3.67) and (3.76), respectively. Table 4-5 shows a cut-off grade policy for GTR 1 when a multiple processing streams operation exploits the deposit under this scenario. A complete summary of the cut-off grade policies determined across all 15 simulated grade tonnage curves by using this approach is found in Table B3 (APPENDIX B). In addition, Figure 4-3 below shows the average values of the COGs and the associated NPVs during the mine life for an operation that fully incorporates rehabilitation cost of the waste rock during cut-off grade optimization. It can be depicted that cut-off grade declines with time. In this case, high-grade material is mined in the early years to generate large cash flows that contributes much to the NPV as they are discounted less. The declining trend of the cut-off grades and quantities of material mined, processed, and refined is in accordance with Lane's theory of dynamic cut-off grades. It has further been found that it takes 9 years to exploit each of the 15 grade-tonnage curves (see APPENDIX B).

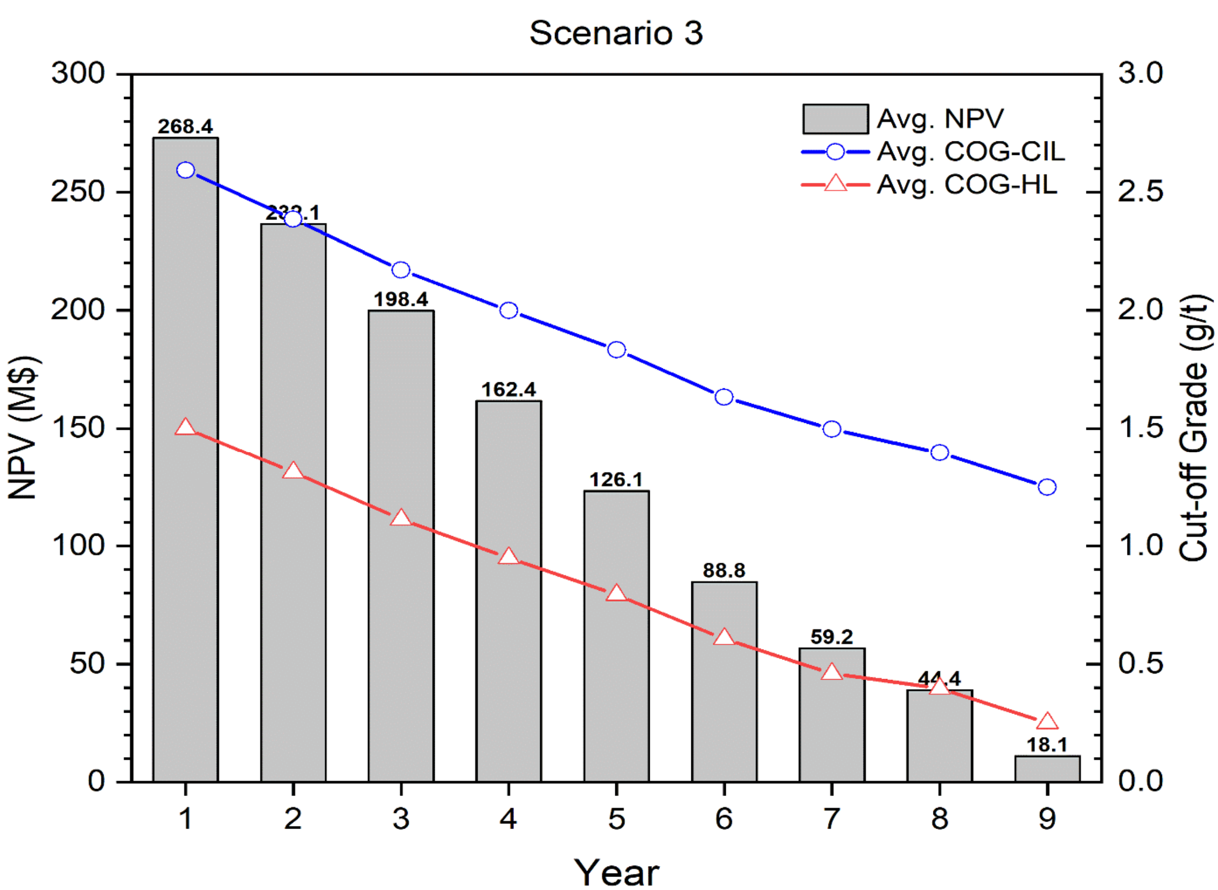

Figure 4-3. Average NPV and COGs during the mine life for scenario 3.

Table 4-5. A Complete cut-off grade policy for GTR 1, rehabilitation cost figured in the whole cut-off grade optimization process.

GTR 1	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.40	2.50	640,000	350,000	3,315,744	57,409	64,536,513	268,461,074	2,325,744
	2	1.30	2.35	640,000	350,000	3,086,057	54,719	61,354,275	229,158,053	2,096,057
	3	1.10	2.15	640,000	350,000	2,628,517	50,773	57,126,346	199,449,721	1,638,517
	4	0.95	2.00	640,000	350,000	2,357,463	47,604	53,379,244	163,316,241	1,367,463
	5	0.80	1.85	640,000	350,000	2,093,313	44,472	49,663,101	127,574,457	1,103,313
	6	0.60	1.65	640,000	350,000	1,813,622	40,711	45,063,673	91,092,015	823,622
	7	0.45	1.45	466,918	350,000	1,309,306	33,988	37,558,111	59,707,690	492,389
	8	0.40	1.40	26,951	350,000	577,772	25,936	29,976,481	45,476,616	200,821
	9	0.25	1.30	36,506	240,225	368,205	17,889	20,327,432	19,403,458	91,474

From Table 4-6, ignoring rehabilitation cost in cut-off grade optimization results in an average NPV of $273.00M. This exceeds the NPV of the base case (scenario 3) by $4.57M, equivalent to 1.70%. On the other hand, if rehabilitation cost is budgeted from the annual profits (scenario 2) without figuring it appropriately in the cut-off grade optimization, the average NPV of the operation is $264.64M. This represents a loss of $3.79M, which is equivalent to 1.41%.

Table 4-6. Comparison of NPVs across 15 simulated grade-tonnage curves for multiple processing streams under different considerations of the rehabilitation cost.

	NPV (M$)		
Realization	Rehabilitation cost not considered	Rehabilitation cost partially considered	Rehabilitation cost fully considered
GTR 1	271.09	264.18	268.46
GTR 2	271.59	265.23	268.33
GTR 3	274.16	261.54	264.74
GTR 4	272.48	267.73	269.77
GTR 5	270.58	264.13	267.80
GTR 6	281.64	267.04	272.40
GTR 7	268.93	263.46	266.55
GTR 8	272.87	259.91	266.36
GTR 9	271.06	266.20	268.40
GTR 10	270.19	262.88	269.60
GTR 11	278.81	263.66	265.62
GTR 12	279.06	263.28	266.37
GTR 13	274.01	265.35	268.44
GTR 14	263.43	266.10	269.78
GTR 15	275.18	268.95	273.82
Average	273.00	264.64	268.43

The work presented by Osanloo et al. (2008) suggests that if rehabilitation cost is not included in establishing the cut-off grade strategy, it will be deducted from the calculated NPV, and if this is not done, there is an overreporting of the NPV of the operation. While rehabilitation cost is budgeted from the annual profits in the second scenario without including it in the whole process of establishing the optimum cut-off grade policy, the consequence of this practice is the misclassification of resources. For example, consider the cut-off grade policies for GTR 1 presented in Table 4-4 and Table 4-5. One may find that incorporating rehabilitation cost in the whole cut-off grade optimization process results in lower cut-off grades than when the cost is not fully figured in the process of defining the cut-off grade strategy. This makes more material to be classified as ore. For the same grade-tonnage curve (GTR 1), the amounts of material regarded as ore by completely and partially including the rehabilitation cost are 7.41M and 7.31M tonnes, respectively. Likewise, the amounts of refined metal are 373,502 and 371,801 ounces, respectively. Not considering the rehabilitation cost in the whole cut-off grade optimization process results in classifying 102,213 tonnes of ore as waste leading to a drop of the quantity of metal refined by 1,701 ounces over the life of the mine. While this piece of analysis is only for GTR 1, the same trend is observed throughout 15 realizations and the effect of these variations on the NPVs is reflected in Table 4-6 and Figure 4-4. A full trend of the cut-off grades, flow of material, and the NPVs across 15 grade-tonnage curves for three scenarios is presented in APPENDIX B (Table B1-B3).

The amount of waste rock generated in the course of mining is another important element of this study. The analysis was done on GTR 1 by completely and by partially including the rehabilitation cost results in a release of 2.33M and 2.48M tonnes of waste rock, respectively. From Table 4-7, the complete analysis done on 15 simulated grade-tonnage curves shows an average of 2.40M and 2.50M tonnes of waste rock, respectively. The variation of the amounts of waste generated by considering individual grade-tonnage curves under different scenarios is clearly presented in Table 4-7 and Figure 4-5.

Table 4-7. Comparison of waste tonnage across 15 simulated grade-tonnage curves for multiple processing streams under different considerations of the rehabilitation cost.

	WASTE TONNAGE (Mt)		
Realization	Rehabilitation cost not considered	Rehab cost partially considered	Rehab cost fully considered
GTR 1	2.59	2.48	2.33
GTR 2	2.58	2.45	2.33
GTR 3	2.55	2.55	2.42
GTR 4	2.57	2.43	2.32
GTR 5	2.61	2.48	2.34
GTR 6	2.36	2.52	2.36
GTR 7	2.62	2.48	2.36
GTR 8	2.56	2.56	2.44
GTR 9	2.59	2.46	2.33
GTR 10	2.66	2.54	2.41
GTR 11	2.45	2.56	2.45
GTR 12	2.41	2.53	2.53
GTR 13	2.63	2.49	2.49
GTR 14	2.86	2.49	2.49
GTR 15	2.62	2.49	2.35
Average	2.58	2.50	2.40

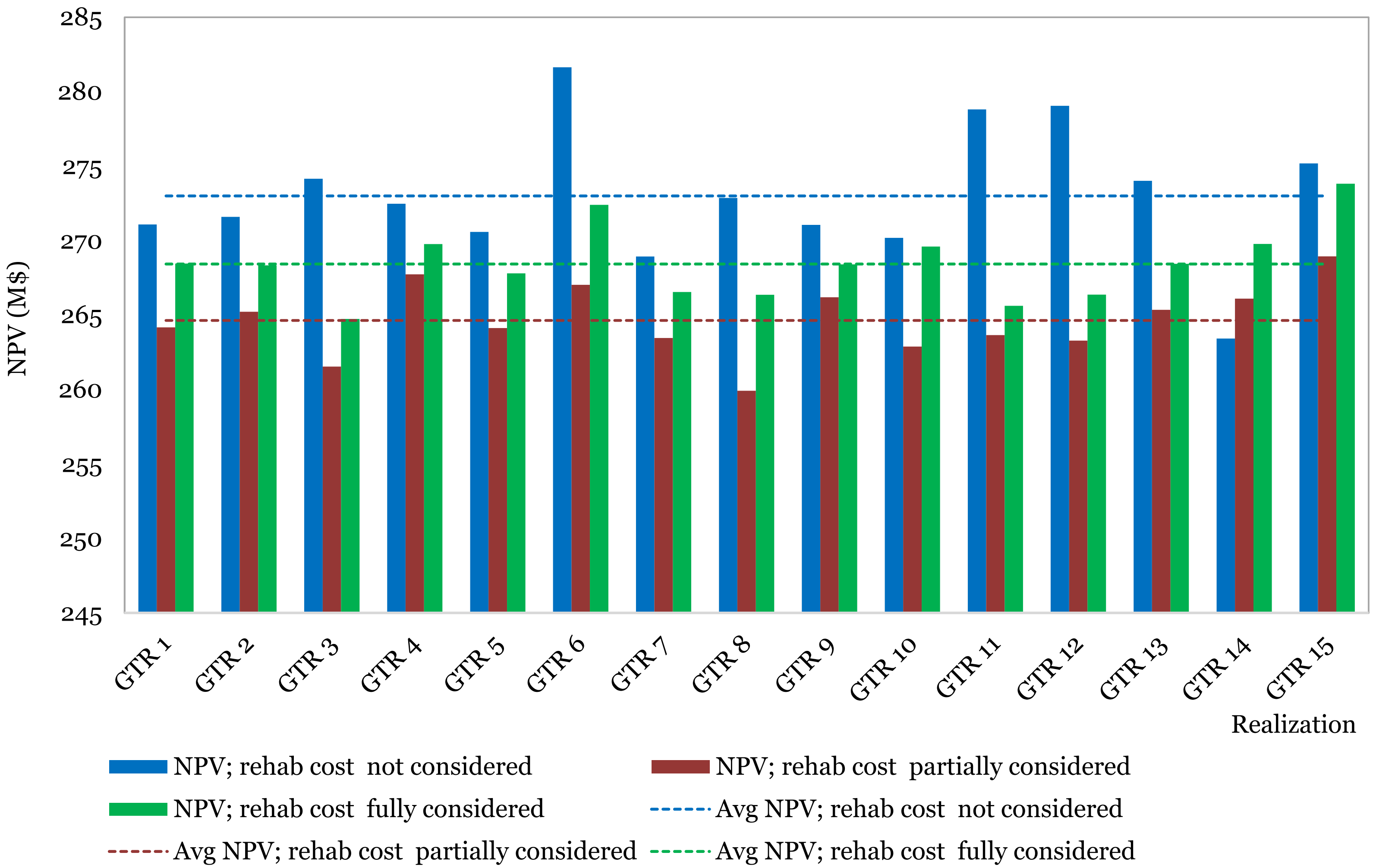

Figure 4-4. Comparison of NPV of different grade-tonnage curves under different considerations of the rehabilitation cost

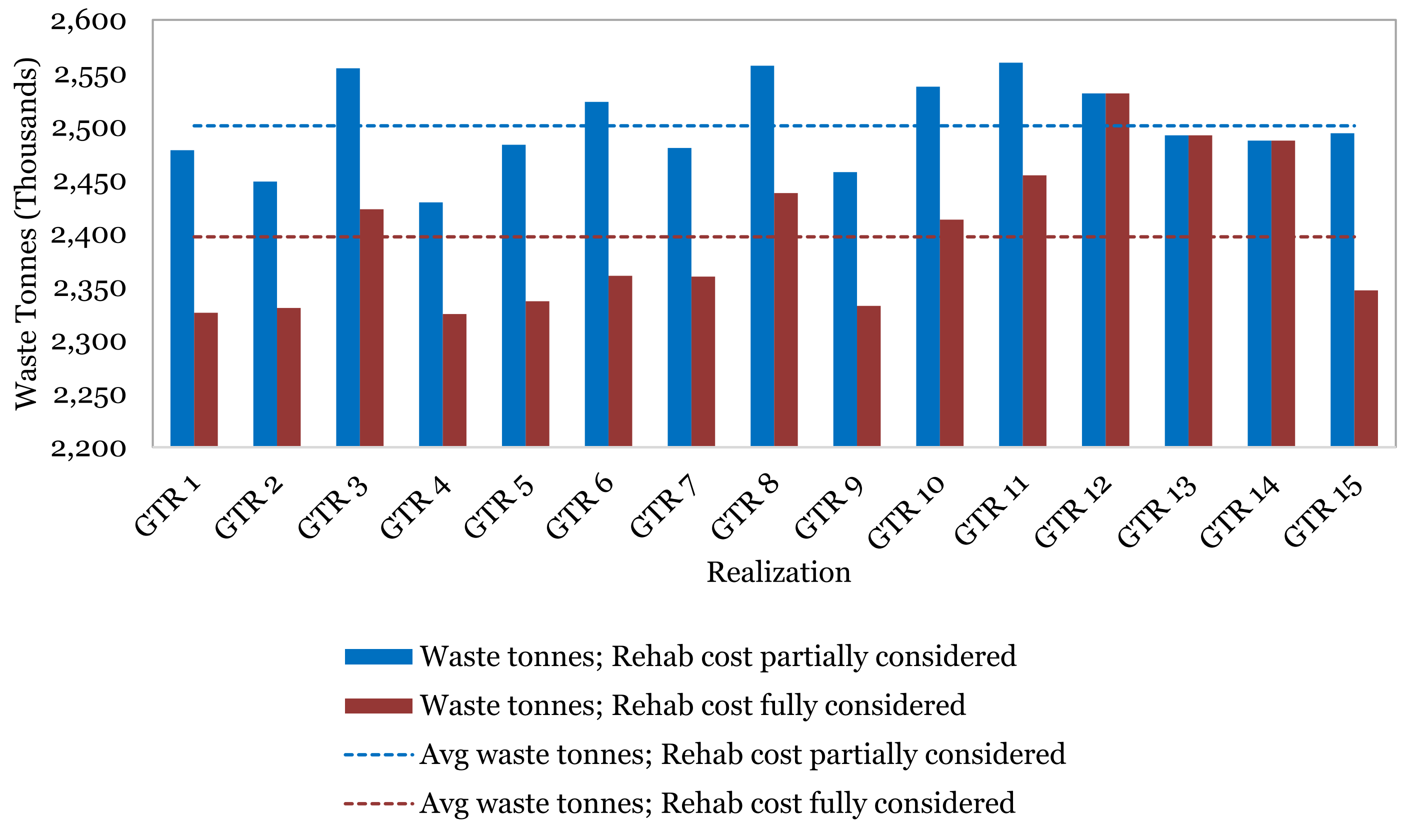

Figure 4-5. Waste tonnes generated under different considerations of the rehabilitation cost.

4.5 Grade-tonnage Curves and Risk Quantification

The application of the model has revealed variations in trends of the cut-off grades, supply of ore to the processing facilities, the overall amount of material mined, quantities of metal refined, amount of waste rock generated, profits, and NPV obtained between 15 simulated grade-tonnage curves. Regarding the NPVs between different scenarios, not including rehabilitation cost gives an average of $273.00M. The minimum NPV expected from the 15 grade-tonnage curves is $263.43M, representing a difference of $9.58M between it and the average value. This deviation is equivalent to 1.75%. Likewise, if rehabilitation cost is partially included in the process of establishing the cut-off grade policy, the average NPV is $264.64M. The minimum and maximum NPVs from 15 simulated curves represent the decrease and increase of these NPVs by 1.79% and 1.63%, respectively, from the mean NPV. Furthermore, while the average NPV considering all curves is $268.43M as rehabilitation cost is completely taken into account when deriving the optimum cut-off grades, the minimum and maximum NPV differ from this average value by 1.37% and 2.01%, respectively. Readers are advised to refer to Table 4-8 and Figure 4-6 to have a look at a complete summary of NPVs described in this part.

By considering the cut-off grade policies derived under the previously discussed scenarios, the NPV that will be generated in each production year can be computed as the average of all NPVs from 15 simulated grade-tonnage curves for a particular year.

Table 4-8. Average, minimum, and maximum NPVs of different scenarios.

	NPV (M$); rehab cost not considered	NPV (M$); rehab cost partially considered	NPV (M$); rehab cost fully considered
Minimum	263.43	259.91	264.74
Average	273.00	264.64	268.43
Maximum	281.64	268.95	273.82

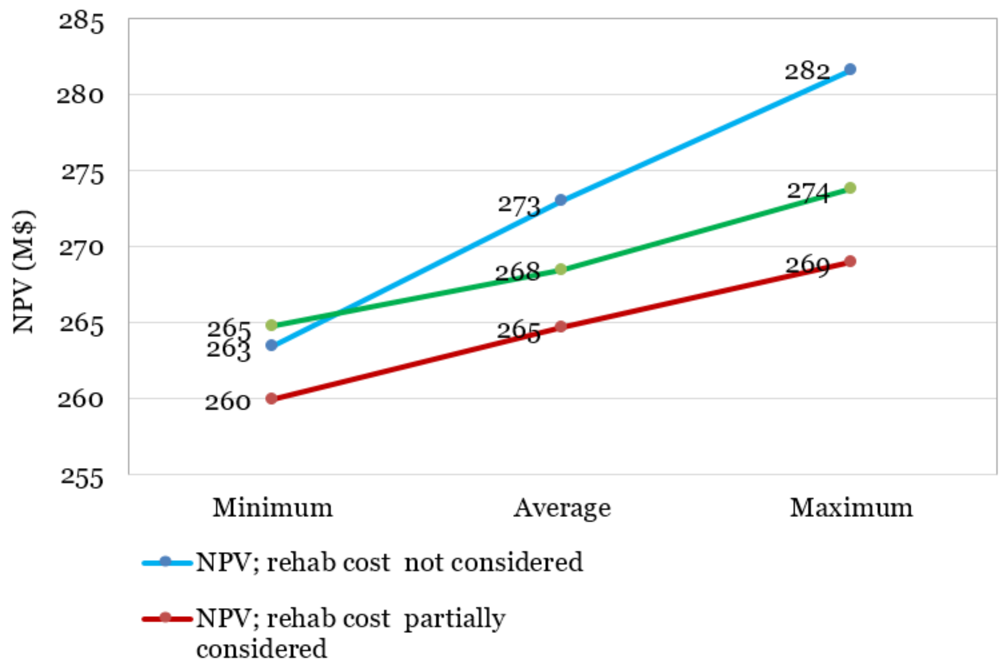

Figure 4-6. Graphical presentation of the average, minimum, and maximum NPVs across 15 curves.

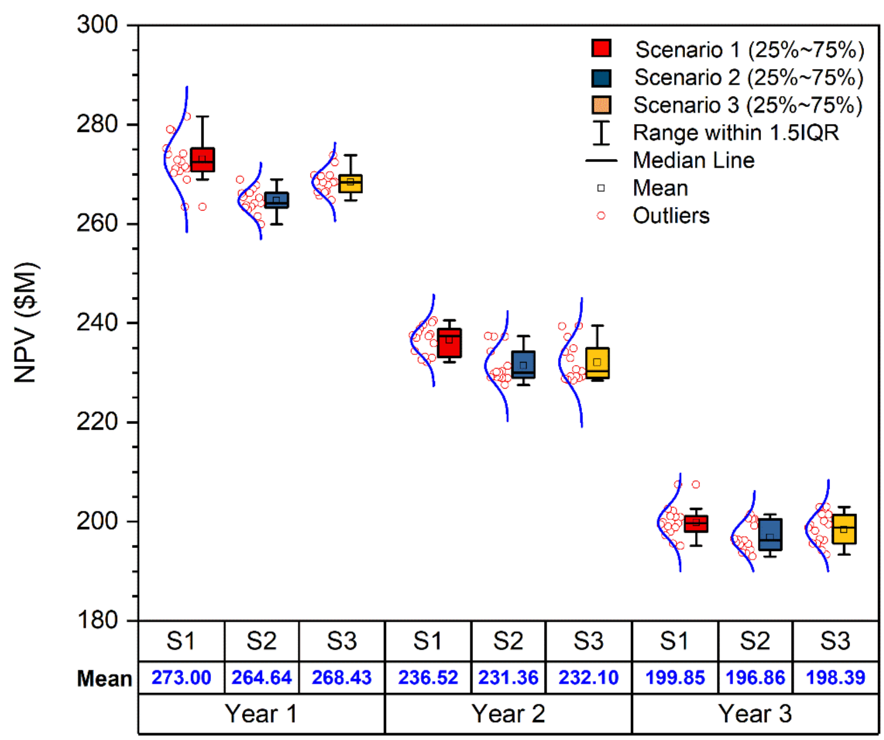

Figure 4-7. Trend of the NPVs across different grade-tonnage curves for all considerations of the rehabilitation cost from year 1 through to 3.

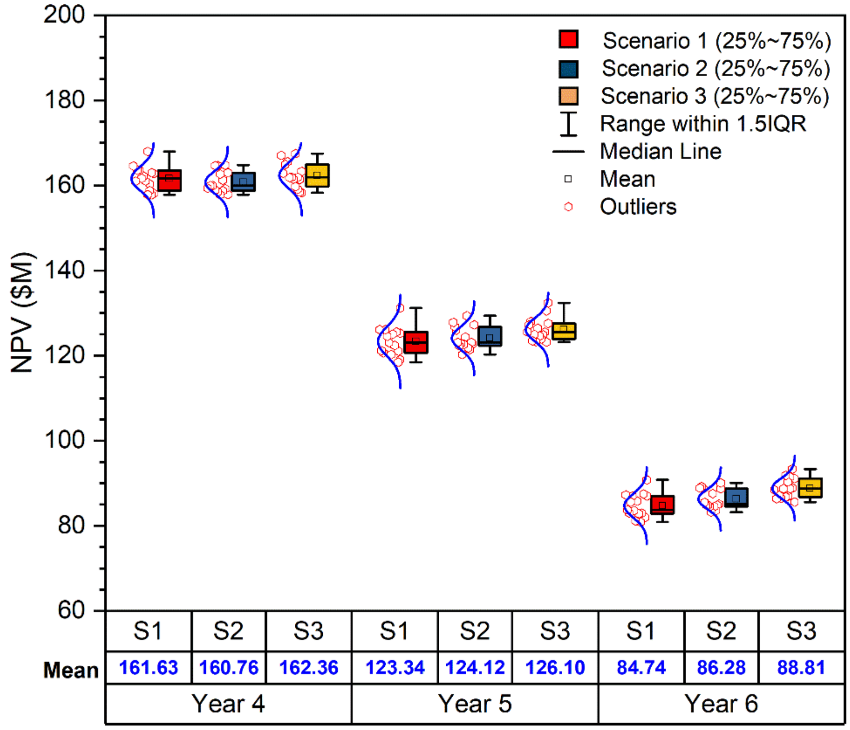

Figure 4-8. Trend of the NPVs across different grade-tonnage curves for all considerations of the rehabilitation cost from year 4 through to 6.

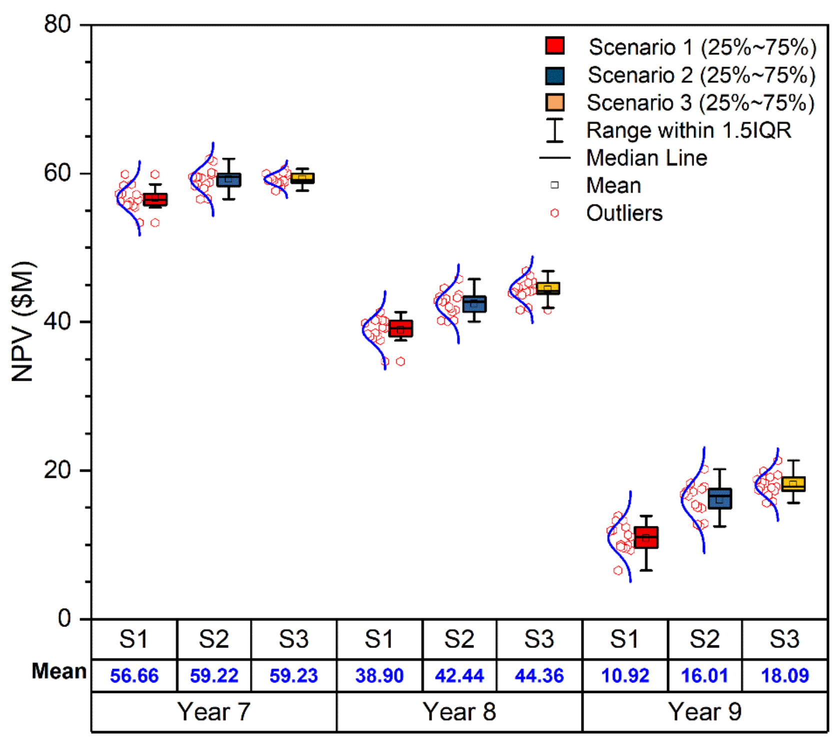

Figure 4-9. Trend of the NPVs across different grade-tonnage curves for all considerations of the rehabilitation cost from year 7 through to 9.

The variation in the quantities of material mined, processed, and refined across 15 simulated curves along with their corresponding NPVs emphasizes the inherent risk of relying on a constant and known grade-tonnage curve in traditional approaches as highlighted by Asad and Dimitrakopoulos (2013). Due to the geological uncertainty associated with deposits, Godoy and Dimitrakopoulos (2011) highlight that given a cut-of grade, material classified as ore in one grade-tonnage curve can be classified as waste in another grade-tonnage curve. Using traditional methods that ignore uncertainty in the orebody's geological behavior may lead to undervaluing or overvaluing the operation. The trend presented in Table 4-6, Table 4-8, Figure 4-4, Figure 4-6, Figure 4-7, Figure 4-8, and Figure 4-9 in line with the results presented in APPENDIX B (Table shows a risk profile that would not be observed if a stochastic behavior associated with grade-tonnage curves was not incorporated in the analysis.

4.6 Sensitivity Analysis

Different parameters considered during the optimization of cut-off grades are expected to have varying impacts on the value of the operation.

In this section, the effect of changing some economic and operational parameters on the NPV is examined. The following are parameters analyzed:

- Metal price
- Recovery of the heap leach facility
- Recovery of the CIL facility
- Discount rate
- Refining cost

Each of the parameters mentioned above was varied by $\pm$ 10% with reference to the base case value, and the proposed model was applied across all 15 simulated grade-tonnage curves by using the new values of the selected parameters. The NPVs expected from each grade-tonnage curve at different values of the chosen parameters are presented in APPENDIX C (Table C1-C5). As far as sensitivity analysis is concerned, the mean NPVs calculated from 15 realizations are summarized in Table 4-9 and Figure 4-10.

Table 4-9. Average NPVs (M$) across 15 simulated grade-tonnage curves as a result of sensitivity analysis of different economic and operational parameters.

	Change of the parameter from its base value (0%)		
Parameter analysed	-10%	0%	10%
Metal price	234.42	268.43	299.92
HL recovery	239.80	268.43	296.26
CIL recovery	263.00	268.43	275.50
Discount rate	280.15	268.43	256.52
Refining cost	268.56	268.43	268.09

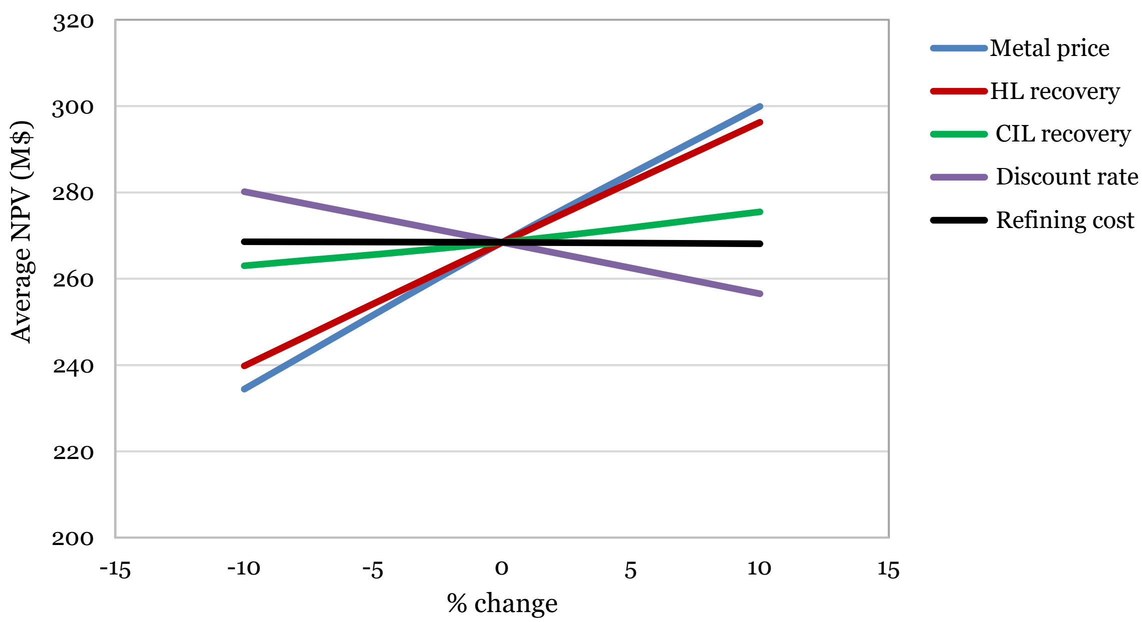

Figure 4-10. Sensitivity of the NPV to some economic and operational parameters.

The analysis done on the metal price indicates that if it is decreased by 10%, the average NPV drops by 12.67%. Similarly, increasing the metal price by the same rate increases the average NPV by 11.73%.

Regarding the recoveries of the processing facilities, decreasing and increasing this parameter for HL by 10% leads to the decrease and increase of the average NPV by 10.66% and 10.37%, respectively. On the other hand, if the recovery of the CIL facility is reduced and increased by 10%, the mean NPV decreases and increases by 2.02% and 2.63% correspondingly. The NPV is, therefore, less sensitive to the capacity of the CIL facility.

It can also be examined that the average NPV of the operation increases by 4.37% if the discount rate is lowered by 10%. On the contrary, increasing the discount rate by 10% leads to 4.44% decrease in the average NPV.

Investigation of the impact of changing refining cost on the NPV shows an increase of the NPV by 0.05% as the cost is reduced by 10%. Likewise, a 0.13% drop of the NPV is experienced when the cost is raised by 10%.

Generally, one can observe the steepest slope in Figure 4-10 once the metal price is varied at different rates. Augmented by the percentage changes in the NPV described in this section, the steepest slope suggests that the metal price has a big influence on the NPV of the operation and a small change on this parameter may have a major impact on the overall economy of the mine. On the other hand, a very gently slope is observed when the refining cost is changed at the previously mentioned rate. This implies that among the chosen parameters for sensitivity analysis, the refining cost poses no significant impact on the NPV compared to the rest.

In order to decrease the sensitivity of the parameter on the NPV, the sequence becomes metal price, HL recovery, discount rate, CIL recovery, and finally, the refining cost.

4.7 Summary and Conclusion

In this chapter, 15 equally probable grade-tonnage curves were developed by using Monte Carlo Simulation. The model developed in CHAPTER 4 was applied to the simulated grade-tonnage curves to develop the cut-off grade policy of the hypothetical gold deposit that utilizes multiple processing streams to exploit the deposit. Three different considerations of the rehabilitation cost were analyzed. In the first scenario, the cost related to rehabilitating the waste rock was not included in developing the cut-off grade strategy. In the second case, rehabilitation cost was budgeted from the annual profits, but it was not fully considered in establishing the optimum cut-off grade policy. In the third case, rehabilitation cost was fully incorporated in the optimization of cut-off grades. Taking the latter case as the base case scenario, the average NPV corresponding to the first scenario was $4.57M, larger than the base case average NPV. This is equivalent to 1.70%. Likewise, budgeting the rehabilitation cost without including the cost appropriately in establishing the cut-off grade policy has promised $3.79M equivalent to 1.41% less than the base case average NPV. Therefore, not including rehabilitation cost leads to an overestimation of the value of the operation.

On the other hand, deducting the rehabilitation cost from the annual profits without incorporating it in the whole process of optimizing the cut-off grades has proved misclassification of the resource where some material that would be treated as ore if the cost was fully considered is ignored. This is due to higher cut-off grades associated with this practice, as observed in Table 4-4 and Table 4-5. The operations must consider the rehabilitation cost appropriately when developing their cut-off grade policies to avoid the problems of overvaluing the operation and wastage of some valuable material due to mistreatment of the rehabilitation cost observed in the second scenario.

Each of the 15 grade-tonnage curves analyzed has shown unique values of the quantities of material mined and processed or wasted, amount of metal refined, series of profits, and the associated NPVs throughout the mine life. Table 4-6, Table 4-7, Figure 4-6Figure 4-6., and APPENDIX B show the summary and variation of the stated outputs across the 15 curves. Due to the deposits' geological uncertainty, relying on a single grade-tonnage curve may also pose a risk while valuing the project.

The sensitivity analysis done on five (5) different parameters has exhibited different impacts on the NPV. It should be noted that changing any economic or technical parameters will change the cut-off grade policy. For this reason, the NPV will consequently also be affected. Of the assessed parameters, the NPV is very sensitive to the metal price. On the other hand, refining cost has been found to have minimum impact on the NPV.

CHAPTER 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Chapter 5 contains the summary and conclusion of this thesis. The contributions of this research are emphasized, as well as recommendations for future studies.

5.1 Summary of Research

The importance of applying multiple processing streams was well described in the study done by Pettingell (2017). This research mainly focused on developing a cut-off grade optimization framework that establishes the cut-off grade strategy of an operation that employs multiple streams to exploit the mineral deposit and considers the rehabilitation cost of the waste rock and the geological uncertainty of the orebody.

The economic and technical inputs used in this study are fixed and adapted from past studies and personal experience from existing operations. Analysis done in this study if for a hypothetical gold mine. Its grade-tonnage data were generated by using Monte Carlo simulation. Grades in the hypothetical deposit were assumed to follow a lognormal distribution with the mean and variance of 1.3 and 5.5 respectively. A total of 15 grade-tonnage realizations were formed and they aid establishing LOM production schedule, determining the value of an operation, and quantifying the risk that mine operations can face by relying on single and fixed grade-tonnage curve especially when little information pertaining to the ore body is known.

Three different scenarios based on consideration of the rehabilitation cost were analyzed by using the model proposed in this research to determine the implication of each option on the flow of material and, consequently the NPV of the operation. The first scenario involved a complete exclusion of the rehabilitation cost in deriving the optimum cut-off grades. The second scenario involved assessing the consequence of the practice of budgeting the rehabilitation cost from the profits generated annually without including it in some steps involved in the process of establishing the optimum cut-off grade policy on the NPV of the operation. In the last scenario, rehabilitation cost was included in the whole cut-off grade optimization process. The latter scenario was taken as the base case for the purpose of comparing different findings. The model developed in CHAPTER 4 and the input information were modified in different fashions to analyze the three scenarios accordingly. In line with 15 simulated grade-tonnage curves, the same model was used to study the response of the NPV of the operation on some economic and operational parameters.

Incorporating geological uncertainty in optimizing the cut-off grades offers an opportunity of studying different variables in greater detail by examining the effect of each on the value of the operation.

5.2 Conclusions

As far as optimization of the cut-off grades is concerned, several modifications to traditional Lane's theory have been done by many researchers. Unfortunately, many of the available literatures consider a single source of material and a single processing stream. Few pieces of literature on optimization frameworks that consider rehabilitation cost is available. The most prominent one is the work done by Gholamnejad (2009). Regarding multiple processing streams, very few studies have been made on this area. The known works are those done by Pettingell (2017) and Asad and Dimitrakopoulos (2013). It should be noted that none of the work done on multiple processing streams has incorporated rehabilitation cost in establishing the cut-off grade strategy. The model presented in this research has been used to assess the NPV of the operation under geological uncertainty by considering three different scenarios, each linked to a unique consideration of the rehabilitation cost, with one scenario representing the practice proposed by Pettingell (2017) and Asad and Dimitrakopoulos (2013), and the other two representing an extension to these studies.

- Higher cut-off grades were observed when rehabilitation cost was ignored in defining the cut-off grade strategy. While this leads to classifying more material as waste, it has shown higher NPVs across all 15 grade-tonnage curves compared to other assessed scenarios. Thus, the major impact of relying on the cut-off grades that have been derived without considering rehabilitation cost is over-estimation of the value of an operation.

- Referring to the two scenarios that include rehabilitation cost, higher cut-off grades were observed when the cost was simply deducted from the annual profits without including it in the whole cut-off grade optimization. Likewise, less amount of material processed was observed compared to the scenario when rehabilitation cost was fully incorporated. This is because high cut-off grades result in classifying more material as waste. Taking the scenario that incorporated the cost in the whole optimization process as the base case, budgeting the rehabilitation cost from the annual profits without figuring it in the whole process of defining the cut-off grades resulted in diminishing the NPV by 1.41%. Due to high cut-off grades experienced when the cost is partially taken into account, the amount of waste rock generated is 4.33% greater than the amount generated in the base case.

- Results have shown a variation of the quantities of material mined, processed, wasted, refined along with profits and NPVs across 15 different grade-tonnage curves. Due to the geological uncertainty of the deposits, relying on a fixed grade-tonnage distribution may lead to overvaluation or undervaluation of the mineral deposit. Therefore, it is important to create several grade-tonnage curves once the statistical behaviors of the orebody are available to develop a range of alternative cut-off grade policies.

- Through sensitivity analysis done on five different parameters, the metal price has shown a far significant impact on the value of the operation compared to other parameters assessed. One of the direct consequences of changing the metal price is changing the trend of cut-off grade for a given grade-tonnage curve; for instance, lowering the metal price results in high optimum cut-off grades. This leads to classifying more material as waste. In this study, lowering the price by 10% has shown a decrease of the average NPV by 12.67%. Likewise, increasing the metal price by 10% has promised an increase of the average NPV by 11.73%. In contrast, refining cost has shown the least influence on the average NPV among the parameters examined.

5.3 Contributions of the Research

The main contribution of this research is the development of an integrated cut-off grade optimization framework that incorporates geological uncertainty and rehabilitation cost of waste rock to define the cut-off grade policy of multiple processing operations.

Not incorporating the rehabilitation cost in deriving the optimum cut-off grades has been pointed to cause a challenge of over-estimating the value of the operation. The model established in this study can be modified in different ways to estimate the average NPV of the mining project under different considerations of the rehabilitation cost. As such, the model can be used to establish the cut-off grade of a mining project with a single source of material, single or multiple processing streams, and a single refinery.

The presented method can be used to study the risk profile of the project once applied on several equally probable grade-tonnage curves, hence, aiding decision-makers to make well-informed decisions, especially at the pre-feasibility stage when little information about the orebody is known.

5.4 Recommendations for Future Research

The cut-off grade optimization framework developed in this thesis provides a new method of deriving the cut-off grade strategies closer to the real-world situation. Despite this benefit, numerous assumptions were used in this model. Owing to these assumptions, the author of this thesis believes that the model is still limited, and more improvements can be made. The following areas address the limitations of the model used and hence require future research:

1. Economic, technical, and grade-tonnage data are the three basic inputs in evaluating the mine operation's potential. In this study, economic and technical inputs were assumed deterministic. However, the real situation is that there are uncertainties

related to these parameters because many mines have longer life spans. In particular, there is the high volatility of the metal price and costs with time. Therefore, while the geological uncertainty was considered in this study, it is crucial to develop a new risk-based framework that will capture the anticipated changes of the input data to develop a cut-off grade strategy of a mine that uses multiple processing streams while considering rehabilitation cost.

2. The model used in this thesis assumes a single source of the material. For the same source of material, the model assumes a single mining phase to define the cut-off grade strategy over the life of the mine. Therefore, all 15 simulated grade-tonnage curves represent an orebody of a mine with a single mining phase. The proposed model can be extended to form a new framework that optimizes the cut-off grades of multiple processing operations having several mining phases or relying on multiple sources of materials.
3. The cost to rehabilitate the tailings was not included in this study. The proposed model can be modified to take into account this cost in order to have a more realistic cut-off grade strategy.
4. Considering an option to stockpile material since in practice, it is not easy to carry out mining activities in such a way that every tonne of ore mined is delivered directly to the designated processing facility for further beneficiation/treatment. Studies done by Asad (1997) and Asad (2005b) are limited to a single source of material and single processing stream but with two metals of interest, and therefore two refineries are used. This brings a need to integrate the model proposed in this research with the one presented by Asad (1997) and Asad (2005b) in order to develop a new approach of deriving the optimum cut-off grades that reflect the real-world situation where stockpiling is in most cases inevitable. Besides the proposed combination of studies, an alternative would be to develop a new optimization framework that exploits the resource by multiple processing streams while exercising the stockpiling strategy. It should be borne in mind that the determination of cut-off grades by Lane's based models leads to higher material grading. For that reason, mineralized material with the grade between the break even and the optimum cut-off grades should be stockpiled during the mine life. Stockpiled material can be utilized parallel to mining if they warrant increase in NPV as they will displace some tonnes of ore determined by the established optimum cut-off grade. Alternatively, the stockpile can be utilized after mining ceases. Under this practice, the stockpile behaves as a pushback or simply a

source of ore. The cashflows generated by processing material from the stockpile pushes forward the NPV. This justifies why this area is worth future research.

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APPENDIX A

SIMULATED GRADE-TONNAGE DATA

Table A1. Simulated grade-tonnage data for GTR 1 to GTR 9.

	Realization and tonnage of material in individual grade categories								
Grade Category (g/t)	GTR 1	GTR 2	GTR 3	GTR 4	GTR 5	GTR 6	GTR 7	GTR 8	GTR 9
0.00 - 0.05	1160000	1180000	1230000	1200000	1190000	1130000	1180000	1200000	1190000
0.05 - 0.10	1000000	1040000	1040000	1090000	1030000	980000	1010000	1010000	1040000
0.10 - 0.15	810000	880000	840000	880000	850000	840000	920000	830000	840000
0.15 - 0.20	760000	780000	820000	760000	830000	800000	720000	790000	740000
0.20 - 0.25	630000	610000	680000	600000	650000	680000	710000	700000	700000
0.25 - 0.30	640000	700000	580000	600000	640000	590000	600000	610000	640000
0.30 - 0.35	590000	590000	530000	580000	550000	550000	580000	560000	620000
0.35 - 0.40	510000	540000	520000	530000	500000	540000	500000	560000	490000
0.40 - 0.45	500000	490000	420000	470000	440000	430000	460000	440000	450000
0.45 - 0.50	500000	460000	480000	440000	500000	400000	520000	510000	430000
0.50 - 0.55	400000	410000	440000	370000	420000	430000	400000	380000	460000
0.55 - 0.60	470000	390000	370000	410000	370000	400000	410000	380000	420000
0.60 - 0.65	350000	380000	420000	370000	370000	360000	370000	320000	380000
0.65 - 0.70	330000	360000	300000	370000	350000	310000	310000	360000	400000
0.70 - 0.75	330000	290000	340000	350000	370000	360000	340000	330000	330000
0.75 - 0.80	270000	320000	290000	370000	330000	300000	350000	310000	300000
0.80 - 0.85	320000	360000	320000	290000	320000	310000	310000	320000	320000
0.85 - 0.90	300000	300000	320000	310000	290000	350000	290000	310000	300000
0.90 - 0.95	310000	270000	290000	270000	300000	290000	290000	280000	270000
0.95 - 1.00	280000	280000	310000	280000	260000	260000	260000	290000	280000

Table A1. Cont.

Grade Category (g/t)	GTR 1	GTR 2	GTR 3	GTR 4	GTR 5	GTR 6	GTR 7	GTR 8	GTR 9
1.00 - 1.05	260000	280000	230000	250000	260000	280000	280000	300000	250000
1.05 - 1.10	220000	220000	240000	220000	270000	240000	230000	240000	240000
1.10 - 1.15	230000	200000	230000	210000	220000	240000	240000	200000	220000
1.15 - 1.20	260000	210000	240000	260000	220000	240000	210000	210000	220000
1.20 - 1.25	230000	210000	210000	200000	200000	220000	240000	200000	210000
1.25 - 1.30	260000	180000	240000	230000	210000	220000	210000	200000	220000
1.30 - 1.35	190000	200000	190000	220000	200000	220000	210000	210000	200000
1.35 - 1.40	200000	180000	180000	150000	180000	220000	220000	220000	160000
1.40 - 1.45	230000	180000	180000	160000	220000	160000	180000	190000	190000
1.45 - 1.50	160000	190000	190000	200000	180000	240000	200000	170000	190000
1.50 - 1.55	180000	190000	190000	150000	160000	180000	150000	180000	180000
1.55 - 1.60	140000	180000	170000	200000	180000	160000	170000	160000	180000
1.60 - 1.65	190000	140000	180000	160000	160000	200000	140000	180000	190000
1.65 - 1.70	150000	190000	180000	150000	150000	180000	170000	190000	130000
1.70 - 1.75	160000	160000	170000	160000	150000	170000	140000	180000	160000
1.75 - 1.80	140000	160000	140000	150000	180000	170000	190000	170000	130000
1.75 - 1.80	150000	130000	150000	190000	130000	170000	150000	180000	150000
1.80 - 1.85	150000	170000	150000	160000	160000	140000	150000	130000	170000
1.85 - 1.90	160000	160000	130000	160000	150000	150000	150000	130000	160000
1.90 - 1.95	140000	130000	160000	150000	120000	130000	170000	130000	130000
1.95 - 2.00	160000	130000	120000	140000	150000	110000	130000	140000	170000
2.00 - 2.05	150000	110000	140000	140000	130000	130000	140000	140000	140000
2.05 - 2.10	140000	140000	130000	160000	160000	190000	110000	140000	140000
2.10 - 2.15	130000	140000	140000	130000	120000	130000	150000	150000	130000
2.15 - 2.20	100000	120000	140000	130000	150000	120000	130000	130000	130000
2.20 - 2.25	110000	140000	130000	140000	130000	150000	100000	130000	110000
2.25 - 2.30	110000	130000	120000	110000	120000	120000	120000	110000	110000
2.30 - 2.35	110000	130000	110000	110000	140000	140000	130000	120000	120000

Table A1. Cont.

Grade Category (g/t)	GTR 1	GTR 2	GTR 3	GTR 4	GTR 5	GTR 6	GTR 7	GTR 8	GTR 9
2.35 - 2.40	140000	130000	150000	120000	100000	120000	60000	100000	120000
2.40 - 2.45	130000	110000	140000	120000	130000	110000	150000	110000	120000
2.45 - 2.50	130000	110000	130000	120000	140000	110000	110000	130000	150000
2.50 - 2.55	110000	130000	100000	90000	120000	120000	110000	110000	80000
2.55 - 2.60	110000	90000	130000	100000	100000	90000	120000	100000	140000
2.60 - 2.65	120000	130000	100000	120000	140000	100000	110000	110000	90000
2.65 - 2.70	120000	100000	90000	100000	90000	100000	120000	100000	120000
2.70 - 2.75	100000	120000	90000	100000	100000	100000	100000	150000	90000
2.75 - 2.80	120000	100000	120000	100000	80000	110000	100000	100000	90000
2.80 - 2.85	100000	120000	120000	100000	90000	110000	130000	100000	110000
2.85 - 2.90	110000	90000	90000	120000	120000	100000	90000	90000	90000
2.90 - 2.95	120000	120000	110000	90000	90000	70000	110000	90000	110000
2.95 - 3.00	90000	90000	110000	80000	120000	110000	100000	110000	100000
3.00 - 3.05	100000	100000	110000	100000	100000	130000	110000	80000	100000
3.05 - 3.10	90000	80000	110000	90000	90000	100000	90000	90000	120000
3.10 - 3.15	80000	90000	70000	80000	80000	90000	90000	100000	60000
3.15 - 3.20	80000	100000	80000	100000	90000	70000	80000	90000	90000
3.20 - 3.25	100000	70000	80000	100000	80000	100000	90000	100000	100000
3.25 - 3.30	100000	90000	90000	80000	80000	80000	90000	70000	80000
3.30 - 3.35	80000	90000	90000	100000	90000	80000	90000	80000	90000
3.35 - 3.40	70000	80000	80000	80000	60000	110000	100000	80000	80000
3.45 - 3.50	80000	70000	70000	90000	90000	90000	70000	110000	90000

Table A2. Simulated grade-tonnage data for GTR 10 to GTR 15.

	Realization and tonnage of material in individual grade categories					
Grade Category (g/t)	GTR 10	GTR 11	GTR 12	GTR 13	GTR 14	GTR 15
0.00 - 0.05	1280000	1200000	1330000	1170000	1230000	1230000
0.05 - 0.10	1040000	1090000	1120000	1000000	1040000	1010000
0.10 - 0.15	900000	880000	870000	940000	890000	830000
0.15 - 0.20	750000	760000	710000	760000	820000	780000
0.20 - 0.25	650000	600000	630000	710000	670000	720000
0.25 - 0.30	580000	650000	620000	610000	620000	620000
0.30 - 0.35	550000	520000	590000	520000	590000	510000
0.35 - 0.40	530000	480000	500000	460000	530000	500000
0.40 - 0.45	500000	520000	450000	480000	450000	450000
0.45 - 0.50	420000	440000	420000	460000	420000	450000
0.50 - 0.55	410000	440000	390000	440000	390000	420000
0.55 - 0.60	360000	410000	350000	380000	390000	410000
0.60 - 0.65	330000	370000	360000	400000	390000	300000
0.65 - 0.70	390000	370000	320000	350000	320000	380000
0.70 - 0.75	320000	340000	370000	330000	330000	280000
0.75 - 0.80	300000	350000	310000	340000	290000	300000
0.80 - 0.85	320000	320000	270000	320000	290000	280000
0.85 - 0.90	290000	280000	340000	280000	270000	280000
0.90 - 0.95	300000	260000	300000	270000	240000	290000
0.95 - 1.00	240000	250000	240000	270000	240000	250000
1.00 - 1.05	240000	230000	220000	230000	230000	290000
1.05 - 1.10	270000	280000	230000	220000	250000	240000
1.10 - 1.15	240000	270000	230000	210000	260000	250000
1.15 - 1.20	220000	210000	230000	170000	220000	220000
1.20 - 1.25	200000	200000	220000	220000	220000	260000
1.25 - 1.30	220000	230000	190000	240000	190000	220000

Table A2. Cont.

Grade Category (g/t)	GTR 10	GTR 11	GTR 12	GTR 13	GTR 14	GTR 15
1.30 - 1.35	220000	190000	240000	200000	180000	190000
1.35 - 1.40	180000	190000	200000	190000	210000	190000
1.40 - 1.45	180000	190000	210000	210000	200000	200000
1.45 - 1.50	180000	150000	170000	180000	190000	220000
1.50 - 1.55	170000	180000	180000	190000	180000	180000
1.55 - 1.60	180000	180000	190000	170000	190000	190000
1.60 - 1.65	140000	150000	190000	200000	180000	180000
1.65 - 1.70	150000	170000	160000	160000	160000	170000
1.70 - 1.75	140000	160000	170000	180000	190000	160000
1.75 - 1.80	170000	140000	170000	190000	150000	160000
1.75 - 1.80	150000	150000	160000	150000	150000	130000
1.80 - 1.85	180000	160000	160000	130000	160000	170000
1.85 - 1.90	140000	180000	120000	140000	160000	120000
1.90 - 1.95	150000	150000	120000	140000	140000	160000
1.95 - 2.00	140000	150000	150000	150000	120000	150000
2.00 - 2.05	150000	140000	150000	140000	160000	180000
2.05 - 2.10	130000	120000	120000	180000	150000	140000
2.10 - 2.15	130000	120000	140000	140000	120000	140000
2.15 - 2.20	140000	140000	130000	140000	100000	130000
2.20 - 2.25	120000	120000	150000	90000	140000	120000
2.25 - 2.30	100000	130000	120000	110000	120000	120000
2.30 - 2.35	130000	150000	110000	110000	120000	100000
2.35 - 2.40	130000	170000	120000	130000	120000	120000
2.40 - 2.45	120000	130000	120000	110000	130000	120000
2.45 - 2.50	120000	120000	120000	110000	120000	140000
2.50 - 2.55	140000	90000	100000	100000	120000	110000

Table A2. Cont.

Grade Category (g/t)	GTR 10	GTR 11	GTR 12	GTR 13	GTR 14	GTR 15
2.55 - 2.60	120000	100000	100000	110000	100000	120000
2.60 - 2.65	100000	90000	110000	110000	120000	130000
2.65 - 2.70	90000	120000	100000	130000	120000	90000
2.70 - 2.75	120000	80000	110000	90000	100000	100000
2.75 - 2.80	110000	80000	80000	120000	120000	130000
2.80 - 2.85	120000	100000	100000	90000	100000	110000
2.85 - 2.90	80000	90000	100000	110000	100000	100000
2.90 - 2.95	100000	90000	100000	100000	90000	80000
2.95 - 3.00	100000	90000	120000	80000	90000	90000
3.00 - 3.05	100000	90000	130000	100000	110000	80000
3.05 - 3.10	80000	100000	100000	120000	110000	90000
3.10 - 3.15	90000	90000	100000	80000	100000	120000
3.15 - 3.20	80000	90000	80000	80000	90000	90000
3.20 - 3.25	100000	90000	100000	110000	70000	110000
3.25 - 3.30	110000	90000	100000	100000	90000	100000
3.30 - 3.35	90000	100000	90000	120000	100000	80000
3.35 - 3.40	110000	100000	80000	90000	90000	100000
3.45 - 3.50	100000	110000	90000	90000	120000	80000

APPENDIX B

CUT-OFF GRADE POLICIES

Table B1. Cut-off grade policy for GTR 1 - GTR 15 when rehabilitation cost is not considered.

GTR 1	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.60	640,000	350,000	3,582,371	59,536	69,203,541	271,087,998	2,592,371
	2	1.30	2.40	640,000	350,000	3,086,057	55,240	64,124,216	235,916,881	2,096,057
	3	1.10	2.15	640,000	350,000	2,628,517	50,772	58,682,393	200,886,945	1,638,517
	4	0.95	2.00	640,000	350,000	2,357,463	47,587	54,653,127	162,806,617	1,367,463
	5	0.80	1.85	640,000	350,000	2,093,313	44,441	50,664,413	125,238,792	1,103,313
	6	0.60	1.65	640,000	350,000	1,813,622	40,656	45,763,889	87,032,371	823,622
	7	0.45	1.50	398,265	350,000	1,199,275	32,844	36,956,597	56,727,629	451,009
	8	0.40	1.40	22,490	350,000	570,934	25,934	30,203,951	39,155,521	198,444
	9	0.25	1.25	21,658	142,520	218,448	10,629	11,606,965	11,079,376	54,270

GTR 2	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.60	640,000	350,000	3,572,963	59,530	69,220,302	271,592,122	2,582,963
	2	1.30	2.35	640,000	350,000	3,095,294	54,825	63,478,535	232,975,715	2,105,294
	3	1.10	2.15	640,000	350,000	2,708,986	50,771	58,463,133	195,114,032	1,718,986
	4	0.95	2.00	640,000	350,000	2,415,104	47,706	54,676,072	158,107,313	1,425,104
	5	0.75	1.80	640,000	350,000	2,057,417	43,476	49,319,772	121,054,830	1,067,417
	6	0.60	1.60	640,000	350,000	1,833,643	40,005	44,737,069	81,987,806	843,643
	7	0.45	1.50	289,875	350,000	1,045,983	30,591	34,544,590	56,452,450	406,109
	8	0.40	1.40	22,645	350,000	582,549	25,899	30,119,364	39,280,147	209,904
	9	0.25	1.25	23,794	145,886	228,060	10,876	11,885,851	11,345,585	58,380

Table B1. Cont.

GTR 3	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.60	640,000	350,000	3,544,644	59,543	69,315,935	274,156,376	2,554,644
	2	1.30	2.40	640,000	350,000	3,080,389	55,329	64,271,898	237,725,006	2,090,389
	3	1.10	2.15	640,000	350,000	2,649,041	50,603	58,373,311	199,656,880	1,659,041
	4	0.95	2.00	640,000	350,000	2,367,918	47,593	54,633,676	163,017,645	1,377,918
	5	0.80	1.85	640,000	350,000	2,101,957	44,457	50,666,021	125,575,956	1,111,957
	6	0.60	1.65	640,000	350,000	1,807,290	40,459	45,486,625	87,474,270	817,290
	7	0.45	1.50	401,715	350,000	1,210,179	32,810	36,858,905	57,150,311	458,464
	8	0.40	1.40	19,557	350,000	572,910	25,811	30,030,462	40,144,915	203,354
	9	0.25	1.25	22,778	158,471	245,672	11,768	12,974,077	12,384,346	64,422

GTR 4	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.60	640,000	350,000	3,559,528	59,582	69,334,518	272,479,027	2,569,528
	2	1.30	2.40	640,000	350,000	3,095,518	55,665	64,733,675	237,342,966	2,105,518
	3	1.10	2.15	640,000	350,000	2,666,908	50,982	58,891,602	198,830,391	1,676,908
	4	0.95	2.00	640,000	350,000	2,391,021	47,767	54,831,950	160,328,483	1,401,021
	5	0.80	1.80	640,000	350,000	2,134,840	43,891	49,731,348	119,072,239	1,144,840
	6	0.60	1.65	640,000	350,000	1,809,489	40,169	45,046,986	82,978,413	819,489
	7	0.45	1.50	320,177	350,000	1,086,555	31,192	35,182,545	55,411,100	416,378
	8	0.40	1.40	20,565	350,000	575,740	25,802	30,003,629	37,526,305	205,175
	9	0.25	1.25	18,594	122,549	190,402	9,101	9,732,955	9,290,548	49,259

Table B1. Cont.

GTR 5	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.60	640,000	350,000	3,602,614	59,711	69,411,255	270,580,099	2,612,614
	2	1.30	2.35	640,000	350,000	3,100,821	54,817	63,451,514	232,148,745	2,110,821
	3	1.10	2.15	640,000	350,000	2,692,186	50,731	58,448,984	195,544,422	1,702,186
	4	0.95	2.00	640,000	350,000	2,398,425	47,464	54,359,557	157,816,017	1,408,425
	5	0.80	1.80	640,000	350,000	2,130,626	43,919	49,784,683	118,483,921	1,140,626
	6	0.60	1.60	640,000	350,000	1,814,483	39,836	44,535,858	80,881,360	824,483
	7	0.45	1.50	285,093	350,000	1,025,740	30,439	34,395,786	55,647,268	390,647
	8	0.40	1.40	19,444	350,000	573,455	25,828	30,053,850	38,143,085	204,012
	9	0.25	1.25	19,429	129,911	201,650	9,658	10,413,443	9,940,105	52,309

GTR 6	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.45	2.55	640,000	350,000	3,350,328	58,517	68,308,464	281,642,843	2,360,328
	2	1.35	2.40	640,000	350,000	3,121,349	56,015	65,186,067	240,533,119	2,131,349
	3	1.15	2.25	640,000	350,000	2,686,486	52,352	60,886,153	207,463,435	1,696,486
	4	0.95	2.00	640,000	350,000	2,320,147	47,736	54,976,973	167,986,246	1,330,147
	5	0.80	1.85	640,000	350,000	2,058,683	44,696	51,140,264	131,207,717	1,068,683
	6	0.65	1.65	640,000	350,000	1,846,245	41,147	46,408,745	90,770,836	856,245
	7	0.50	1.50	426,787	350,000	1,285,842	33,719	37,886,847	58,521,841	509,056
	8	0.40	1.40	40,762	350,000	599,830	25,861	29,926,132	40,174,173	209,068
	9	0.25	1.25	25,022	170,088	261,089	12,550	13,903,887	13,271,892	65,980

Table B1. Cont.

GTR 7	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.60	640,000	350,000	3,614,220	59,524	69,100,333	268,928,667	2,624,220
	2	1.30	2.35	640,000	350,000	3,093,310	54,885	63,573,822	233,160,814	2,103,310
	3	1.10	2.15	640,000	350,000	2,666,319	50,809	58,634,875	197,962,306	1,676,319
	4	0.95	2.00	640,000	350,000	2,384,691	47,844	54,964,314	161,044,564	1,394,691
	5	0.80	1.85	640,000	350,000	2,125,232	44,642	50,880,000	122,386,615	1,135,232
	6	0.60	1.60	640,000	350,000	1,820,356	40,209	45,077,219	82,871,551	830,356
	7	0.45	1.50	320,292	350,000	1,081,831	31,371	35,461,249	56,162,994	411,539
	8	0.40	1.40	20,277	350,000	573,374	26,044	30,372,913	38,503,542	203,097
	9	0.25	1.25	19,198	129,588	200,666	9,710	10,500,453	10,023,159	51,881

GTR 8	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.60	640,000	350,000	3,546,994	59,476	69,210,442	272,866,115	2,556,994
	2	1.30	2.40	640,000	350,000	3,053,662	55,475	64,562,435	239,613,368	2,063,662
	3	1.10	2.15	640,000	350,000	2,672,542	50,914	58,775,026	199,055,359	1,682,542
	4	0.95	2.00	640,000	350,000	2,369,508	47,653	54,720,287	162,204,307	1,379,508
	5	0.80	1.85	640,000	350,000	2,107,509	44,704	51,019,955	125,036,707	1,117,509
	6	0.60	1.65	640,000	350,000	1,816,209	40,499	45,522,664	85,793,978	826,209
	7	0.45	1.50	380,946	350,000	1,183,565	32,443	36,485,668	55,766,772	452,619
	8	0.40	1.40	19,338	350,000	574,672	25,790	29,994,560	37,758,203	205,334
	9	0.25	1.25	19,289	125,543	195,338	9,325	10,003,284	9,548,590	50,507

Table B1. Cont.

utoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
1.50	2.60	640,000	350,000	3,582,371	59,531	69,196,536	271,060,560	2,592,371
1.30	2.35	640,000	350,000	3,108,140	54,915	63,579,084	232,580,135	2,118,140
1.10	2.15	640,000	350,000	2,689,551	50,952	58,785,426	197,169,462	1,699,551
0.95	2.00	640,000	350,000	2,403,112	47,662	54,642,617	158,796,560	1,413,112
0.80	1.85	640,000	350,000	2,139,717	44,454	50,559,265	120,260,732	1,149,717
0.60	1.60	640,000	350,000	1,823,137	39,999	44,755,758	81,169,724	833,137
0.45	1.50	289,284	350,000	1,035,003	30,660	34,679,770	55,733,884	395,719
0.40	1.40	19,717	350,000	574,715	25,976	30,270,355	38,054,387	204,998
0.25	1.25	19,370	124,964	194,254	9,347	10,048,123	9,591,390	49,919

utoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
1.55	2.60	640,000	350,000	3,653,621	60,273	70,112,889	270,185,356	2,663,621
1.30	2.40	640,000	350,000	3,055,405	55,593	64,734,701	238,816,410	2,065,405
1.10	2.15	640,000	350,000	2,645,534	50,981	58,947,823	199,821,531	1,655,534
0.95	2.00	640,000	350,000	2,374,104	47,875	55,038,484	161,690,421	1,384,104
0.80	1.85	640,000	350,000	2,111,277	44,608	50,866,042	123,058,216	1,121,277
0.60	1.60	640,000	350,000	1,815,345	40,280	45,196,477	83,582,765	825,345
0.50	1.50	231,921	350,000	987,519	30,227	34,448,123	59,866,767	405,598
0.40	1.40	46,561	350,000	617,931	26,487	30,782,965	41,337,320	221,369

Table B1. Cont.

GTR 11	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.45	2.55	640,000	350,000	3,444,416	59,127	68,965,710	278,814,003	2,454,416
	2	1.30	2.40	640,000	350,000	3,095,071	55,728	64,829,524	240,104,206	2,105,071
	3	1.10	2.15	640,000	350,000	2,663,752	50,838	58,685,337	200,942,736	1,673,752
	4	0.95	2.00	640,000	350,000	2,386,049	47,758	54,832,479	163,565,353	1,396,049
	5	0.80	1.85	640,000	350,000	2,134,270	44,485	50,620,896	124,582,226	1,144,270
	6	0.60	1.65	640,000	350,000	1,815,689	40,355	45,308,786	86,980,534	825,689
	7	0.50	1.50	337,563	350,000	1,158,244	32,013	36,128,167	58,378,061	470,681
	8	0.40	1.40	47,673	350,000	613,442	26,166	30,311,042	40,347,921	215,770
	9	0.25	1.25	24,329	167,943	259,066	12,500	13,873,291	13,242,687	66,794

GTR 12	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.45	2.55	640,000	350,000	3,403,973	58,673	68,396,118	279,055,029	2,413,973
	2	1.35	2.40	640,000	350,000	3,151,141	55,981	65,055,865	238,051,758	2,161,141
	3	1.15	2.20	640,000	350,000	2,717,859	51,874	60,088,156	202,157,279	1,727,859
	4	0.95	2.00	640,000	350,000	2,376,270	47,881	55,041,772	163,601,051	1,386,270
	5	0.80	1.85	640,000	350,000	2,113,524	44,878	51,263,674	126,130,337	1,123,524
	6	0.65	1.65	640,000	350,000	1,884,540	41,299	46,532,933	85,576,035	894,540
	7	0.45	1.50	324,826	350,000	1,102,949	31,536	35,627,973	57,265,794	428,122
	8	0.40	1.40	20,995	350,000	581,999	26,144	30,496,480	40,226,295	211,003
	9	0.25	1.25	23,139	151,551	237,746	11,401	12,556,488	11,985,738	63,056

Table B1. Cont.

GTR 13	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.55	2.65	640,000	350,000	3,619,688	60,485	70,522,370	274,014,164	2,629,688
	2	1.35	2.40	640,000	350,000	3,119,300	55,961	65,112,127	236,995,977	2,129,300
	3	1.15	2.20	640,000	350,000	2,714,766	52,233	60,632,040	201,071,026	1,724,766
	4	0.95	2.00	640,000	350,000	2,370,327	47,969	55,189,999	161,023,290	1,380,327
	5	0.80	1.80	640,000	350,000	2,118,841	44,448	50,607,318	120,658,814	1,128,841
	6	0.60	1.65	640,000	350,000	1,806,081	40,540	45,610,910	83,011,198	816,081
	7	0.45	1.50	292,466	350,000	1,034,428	30,893	35,014,653	56,215,446	391,963
	8	0.40	1.40	20,966	350,000	572,096	26,122	30,489,517	38,381,742	201,130
	9	0.25	1.25	17,619	126,102	194,473	9,466	10,216,481	9,752,095	50,751

GTR 14	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.55	640,000	350,000	3,851,141	58,932	67,576,296	263,427,033	2,861,141
	2	1.30	2.40	640,000	350,000	3,302,936	55,464	63,873,167	234,359,493	2,312,936
	3	1.10	2.20	640,000	350,000	2,847,009	51,405	59,037,728	199,435,128	1,857,009
	4	0.95	2.00	640,000	350,000	2,585,306	47,920	54,535,829	157,868,388	1,595,306
	5	0.75	1.80	640,000	350,000	2,201,957	43,712	49,282,810	121,172,808	1,211,957
	6	0.60	1.65	640,000	350,000	1,945,755	40,523	45,207,768	83,728,277	955,755
	7	0.45	1.45	367,970	350,000	1,247,631	31,920	35,596,384	53,346,353	529,661
	8	0.35	1.35	39,335	350,000	621,279	25,751	29,710,195	34,682,982	231,944
	9	0.25	1.25	9,015	92,111	146,985	6,745	6,864,478	6,552,456	45,859

Table B1. Cont.

GTR 15	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.55	2.65	640,000	350,000	3,614,220	60,599	70,706,701	275,180,421	2,624,220
	2	1.35	2.40	640,000	350,000	3,104,357	55,827	64,951,084	237,532,883	2,114,357
	3	1.15	2.20	640,000	350,000	2,678,644	51,932	60,279,791	202,579,528	1,688,644
	4	0.95	2.00	640,000	350,000	2,311,755	47,636	54,849,712	164,576,608	1,321,755
	5	0.80	1.85	640,000	350,000	2,076,989	44,641	51,007,778	126,113,985	1,086,989
	6	0.60	1.65	640,000	350,000	1,805,234	40,795	45,993,945	87,247,285	815,234
	7	0.45	1.50	370,825	350,000	1,160,191	32,376	36,499,589	57,232,299	439,366
	8	0.40	1.40	20,722	350,000	573,052	25,898	30,154,073	39,842,143	202,331
	9	0.25	1.25	21,866	152,388	235,557	11,348	12,475,687	11,908,611	61,304

Table B2. Cut-off grade policy for GTR 1 - GTR 15 when rehabilitation cost is partially considered.

GTR 1	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.45	2.55	640,000	350,000	3,467,964	58,593	65,750,045	264,175,633	2,477,964
	2	1.25	2.30	640,000	350,000	2,949,830	53,663	60,273,041	231,344,245	1,959,830
	3	1.10	2.15	640,000	350,000	2,628,517	50,767	57,116,993	199,184,853	1,638,517
	4	0.95	2.00	640,000	350,000	2,357,463	47,591	53,360,034	163,000,675	1,367,463
	5	0.80	1.85	640,000	350,000	2,093,313	44,452	49,633,722	127,212,231	1,103,313
	6	0.65	1.65	640,000	350,000	1,882,394	41,201	45,544,197	88,719,579	892,394
	7	0.50	1.50	370,911	350,000	1,210,716	32,698	36,377,815	59,994,427	489,805
	8	0.40	1.40	54,690	350,000	620,288	26,347	30,322,360	43,656,462	215,599
	9	0.30	1.30	21,280	221,506	339,513	16,490	18,685,939	17,836,578	96,727

GTR 2	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.45	2.55	640,000	350,000	3,438,535	58,424	65,604,940	265,227,110	2,448,535
	2	1.25	2.30	640,000	350,000	2,999,067	53,836	60,351,440	228,880,716	2,009,067
	3	1.10	2.15	640,000	350,000	2,708,986	50,775	56,835,805	192,973,254	1,718,986
	4	0.95	2.00	640,000	350,000	2,415,104	47,711	53,330,119	157,891,848	1,425,104
	5	0.75	1.80	640,000	350,000	2,057,417	43,488	48,323,517	122,741,664	1,067,417
	6	0.60	1.60	640,000	350,000	1,833,643	40,029	43,971,606	85,126,950	843,643
	7	0.50	1.50	303,019	350,000	1,115,283	31,296	34,906,541	59,877,948	462,264
	8	0.40	1.40	52,643	350,000	629,444	26,214	30,099,359	43,252,619	226,801
	9	0.30	1.30	22,067	219,104	342,521	16,251	18,349,508	17,515,440	101,350

Table B2. Cont.

utoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
1.50	2.55	640,000	350,000	3,544,644	59,032	66,126,602	261,542,017	2,554,644
1.30	2.35	640,000	350,000	3,080,389	54,817	61,520,685	227,548,986	2,090,389
1.10	2.15	640,000	350,000	2,649,041	50,618	56,820,358	194,345,247	1,659,041
0.95	2.00	640,000	350,000	2,367,918	47,620	53,366,433	159,236,404	1,377,918
0.80	1.85	640,000	350,000	2,101,957	44,501	49,674,750	123,119,086	1,111,957
0.60	1.60	640,000	350,000	1,807,290	40,066	44,122,947	84,851,813	817,290
0.50	1.50	313,846	350,000	1,117,856	31,565	35,255,254	58,800,067	454,010
0.40	1.40	46,633	350,000	614,887	26,162	30,098,064	41,614,644	218,253
0.30	1.30	17,680	190,947	296,019	14,153	15,826,543	15,107,155	87,391

utoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
1.45	2.55	640,000	350,000	3,419,112	58,656	66,022,000	267,733,263	2,429,112
1.30	2.35	640,000	350,000	3,095,518	55,131	61,935,254	228,908,543	2,105,518
1.10	2.15	640,000	350,000	2,666,908	50,985	57,303,456	195,511,854	1,676,908
0.95	2.00	640,000	350,000	2,391,021	47,780	53,520,313	158,756,414	1,401,021
0.80	1.85	640,000	350,000	2,134,840	44,568	49,655,242	121,405,278	1,144,840
0.60	1.60	640,000	350,000	1,809,489	39,908	43,878,472	83,625,328	819,489
0.45	1.50	364,041	350,000	1,157,672	32,026	35,595,724	56,562,392	443,631
0.40	1.40	22,425	350,000	578,630	25,797	29,783,689	40,205,228	206,205
0.25	1.25	25,080	165,292	256,811	12,264	13,502,055	12,888,325	66,439

Table B2. Cont.

GTR 5	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.45	2.55	640,000	350,000	3,472,920	58,667	65,841,532	264,132,133	2,482,920
	2	1.25	2.30	640,000	350,000	2,988,744	53,816	60,359,857	229,095,376	1,998,744
	3	1.10	2.15	640,000	350,000	2,692,186	50,737	56,840,382	193,628,648	1,702,186
	4	0.95	2.00	640,000	350,000	2,398,425	47,472	53,033,211	157,817,627	1,408,425
	5	0.80	1.85	640,000	350,000	2,130,626	44,364	49,366,197	121,799,979	1,140,626
	6	0.60	1.60	640,000	350,000	1,814,483	39,933	43,897,346	84,532,271	824,483
	7	0.50	1.50	317,203	350,000	1,129,609	31,585	35,228,689	58,559,463	462,405
	8	0.40	1.40	48,930	350,000	619,224	26,137	30,036,339	41,347,035	220,294
	9	0.30	1.30	17,587	189,268	293,784	14,003	15,635,897	14,925,174	86,929

GTR 6	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.60	640,000	350,000	3,513,097	59,670	67,194,876	267,035,554	2,523,097
	2	1.30	2.40	640,000	350,000	3,002,543	55,564	62,921,832	237,274,812	2,012,543
	3	1.10	2.15	640,000	350,000	2,590,254	50,805	57,314,233	200,525,939	1,600,254
	4	0.95	2.00	640,000	350,000	2,320,147	47,695	53,651,947	164,794,212	1,330,147
	5	0.80	1.85	640,000	350,000	2,058,683	44,649	50,054,927	129,383,032	1,068,683
	6	0.65	1.65	640,000	350,000	1,846,245	41,096	45,519,205	90,079,006	856,245
	7	0.45	1.50	486,115	350,000	1,333,675	34,383	37,981,456	57,996,667	497,561
	8	0.40	1.40	21,227	350,000	569,843	25,615	29,547,648	41,806,225	198,616
	9	0.30	1.30	18,375	192,513	295,513	14,122	15,754,927	15,038,794	84,625

Table B2. Cont.

GTR 7	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.45	2.55	640,000	350,000	3,469,940	58,479	65,572,777	263,463,394	2,479,940
	2	1.25	2.30	640,000	350,000	2,981,887	53,876	60,474,419	230,245,605	1,991,887
	3	1.10	2.15	640,000	350,000	2,666,319	50,805	57,037,056	196,264,771	1,676,319
	4	0.95	2.00	640,000	350,000	2,384,691	47,840	53,633,495	161,273,465	1,394,691
	5	0.80	1.85	640,000	350,000	2,125,232	44,639	49,796,416	124,376,110	1,135,232
	6	0.60	1.65	640,000	350,000	1,820,356	40,573	44,832,857	87,586,563	830,356
	7	0.50	1.50	353,084	350,000	1,191,713	32,607	36,384,399	59,363,978	488,629
	8	0.40	1.40	51,336	350,000	621,470	26,407	30,421,233	42,027,599	220,133
	9	0.30	1.30	18,352	192,697	298,392	14,395	16,150,104	15,416,009	87,343

GTR 8	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.55	640,000	350,000	3,546,994	58,903	65,925,196	259,913,846	2,556,994
	2	1.30	2.35	640,000	350,000	3,053,662	54,901	61,743,632	229,151,820	2,063,662
	3	1.10	2.15	640,000	350,000	2,672,542	50,936	57,209,141	193,752,124	1,682,542
	4	0.95	2.00	640,000	350,000	2,369,508	47,690	53,464,748	158,482,583	1,379,508
	5	0.80	1.85	640,000	350,000	2,107,509	44,754	50,033,428	122,618,984	1,117,509
	6	0.60	1.60	640,000	350,000	1,816,209	40,113	44,160,309	83,226,425	826,209
	7	0.45	1.50	346,592	350,000	1,127,938	31,764	35,383,309	56,535,219	431,346
	8	0.40	1.40	20,735	350,000	576,845	25,886	29,929,465	40,074,793	206,110
	9	0.25	1.25	24,567	159,897	248,792	11,914	13,091,729	12,496,650	64,328

Table B2. Cont.

GTR 9	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.45	2.55	640,000	350,000	3,447,321	58,675	65,946,894	266,199,399	2,457,321
	2	1.25	2.30	640,000	350,000	2,990,448	53,927	60,519,407	230,057,433	2,000,448
	3	1.10	2.15	640,000	350,000	2,689,551	50,948	57,165,090	195,377,133	1,699,551
	4	0.95	2.00	640,000	350,000	2,403,112	47,656	53,290,978	158,904,896	1,413,112
	5	0.80	1.80	640,000	350,000	2,139,717	43,940	48,698,159	120,261,408	1,149,717
	6	0.60	1.60	640,000	350,000	1,823,137	39,906	43,826,272	84,480,937	833,137
	7	0.50	1.50	305,381	350,000	1,104,893	31,252	34,868,442	59,519,451	449,512
	8	0.40	1.40	47,727	350,000	618,255	26,172	30,095,960	43,127,194	220,529
	9	0.30	1.30	21,097	214,584	333,566	15,915	17,951,861	17,135,868	97,884

GTR 10	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.55	640,000	350,000	3,527,378	59,146	66,359,820	262,882,361	2,537,378
	2	1.30	2.35	640,000	350,000	3,055,405	54,976	61,849,562	229,828,302	2,065,405
	3	1.10	2.15	640,000	350,000	2,645,534	50,979	57,372,543	196,332,749	1,655,534
	4	0.95	2.00	640,000	350,000	2,374,104	47,887	53,742,001	159,936,175	1,384,104
	5	0.80	1.85	640,000	350,000	2,111,277	44,636	49,842,281	122,870,300	1,121,277
	6	0.60	1.60	640,000	350,000	1,815,345	40,338	44,499,450	84,836,782	825,345
	7	0.50	1.50	255,241	350,000	1,027,093	30,768	34,632,686	61,623,473	421,852
	8	0.40	1.45	51,110	350,000	625,018	26,635	30,749,842	44,558,258	223,908
	9	0.30	1.30	21,492	223,874	348,845	16,876	19,191,900	18,319,541	103,479

Table B2. Cont.

GTR 11	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.55	640,000	350,000	3,549,857	59,475	66,768,431	263,659,658	2,559,857
	2	1.25	2.30	640,000	350,000	2,973,385	53,910	60,555,731	230,168,618	1,983,385
	3	1.10	2.15	640,000	350,000	2,663,752	50,840	57,097,878	195,741,941	1,673,752
	4	0.95	2.00	640,000	350,000	2,386,049	47,773	53,528,351	159,936,157	1,396,049
	5	0.80	1.85	640,000	350,000	2,134,270	44,518	49,581,825	122,314,042	1,144,270
	6	0.60	1.60	640,000	350,000	1,815,689	40,021	44,023,898	84,835,879	825,689
	7	0.50	1.50	307,956	350,000	1,108,368	31,430	35,112,371	59,505,826	450,413
	8	0.40	1.40	51,285	350,000	619,015	26,283	30,246,073	42,735,228	217,730
	9	0.30	1.30	18,191	207,195	319,615	15,396	17,351,164	16,562,475	94,229

GTR 12	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.55	640,000	350,000	3,521,113	59,058	66,250,222	263,281,455	2,531,113
	2	1.35	2.40	640,000	350,000	3,151,141	55,984	63,006,598	229,023,272	2,161,141
	3	1.10	2.15	640,000	350,000	2,623,575	50,844	57,250,855	196,525,224	1,633,575
	4	0.95	2.00	640,000	350,000	2,376,270	47,887	53,734,318	159,343,910	1,386,270
	5	0.80	1.85	640,000	350,000	2,113,524	44,884	50,204,700	123,108,583	1,123,524
	6	0.60	1.65	640,000	350,000	1,813,796	40,770	45,151,043	85,494,077	823,796
	7	0.50	1.50	297,770	350,000	1,101,773	31,600	35,431,164	58,307,611	454,003
	8	0.40	1.40	43,020	350,000	616,550	26,387	30,442,798	40,172,658	223,530
	9	0.25	1.25	24,551	160,802	252,257	12,101	13,344,372	12,737,810	66,905

Table B2. Cont.

GTR 13	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.55	640,000	350,000	3,481,864	58,960	66,247,935	265,348,453	2,491,864
	2	1.30	2.40	640,000	350,000	3,011,179	55,553	62,873,800	237,239,911	2,021,179
	3	1.10	2.15	640,000	350,000	2,628,517	51,236	57,819,098	200,525,486	1,638,517
	4	0.95	2.00	640,000	350,000	2,370,327	47,980	53,894,702	162,666,318	1,380,327
	5	0.80	1.85	640,000	350,000	2,118,841	44,961	50,301,521	125,984,567	1,128,841
	6	0.60	1.65	640,000	350,000	1,806,081	40,658	45,011,392	88,709,535	816,081
	7	0.50	1.50	355,473	350,000	1,185,925	32,576	36,348,734	60,130,436	480,452
	8	0.40	1.40	50,737	350,000	618,008	26,387	30,406,416	43,394,518	217,271
	9	0.30	1.30	18,386	213,502	329,258	15,927	18,013,758	17,194,951	97,370

GTR 14	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.55	640,000	350,000	3,476,880	59,005	66,333,187	266,104,473	2,486,880
	2	1.30	2.35	640,000	350,000	3,007,682	54,944	61,976,311	234,267,017	2,017,682
	3	1.10	2.15	640,000	350,000	2,606,357	51,042	57,609,251	201,449,030	1,616,357
	4	0.95	2.00	640,000	350,000	2,352,422	48,005	53,997,512	164,680,374	1,362,422
	5	0.80	1.85	640,000	350,000	2,122,637	44,958	50,282,027	126,795,764	1,132,637
	6	0.60	1.65	640,000	350,000	1,826,092	40,798	45,148,196	89,202,250	836,092
	7	0.50	1.50	324,827	350,000	1,150,481	32,101	35,892,505	61,955,791	475,655
	8	0.40	1.45	50,409	350,000	629,470	26,503	30,538,973	45,782,229	229,061
	9	0.30	1.30	24,746	246,795	387,978	18,523	21,151,472	20,190,042	116,438

Table B2. Cont.

GTR 15	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.60	640,000	350,000	3,483,848	59,566	67,146,024	268,945,647	2,493,848
	2	1.30	2.40	640,000	350,000	3,002,487	55,441	62,738,663	237,380,875	2,012,487
	3	1.15	2.20	640,000	350,000	2,678,644	51,943	58,691,596	200,690,476	1,688,644
	4	0.95	2.00	640,000	350,000	2,311,755	47,640	53,600,426	164,658,690	1,321,755
	5	0.80	1.85	640,000	350,000	2,076,989	44,641	49,974,830	127,812,825	1,086,989
	6	0.65	1.65	640,000	350,000	1,863,280	41,245	45,680,167	88,874,909	873,280
	7	0.50	1.50	365,077	350,000	1,200,454	32,567	36,244,380	59,541,934	485,377
	8	0.40	1.40	48,308	350,000	615,694	26,140	30,055,221	42,843,652	217,386
	9	0.30	1.30	18,799	211,447	326,849	15,644	17,628,878	16,827,566	96,603

Table B3. Cut-off grade policy for GTR 1 - GTR 15 when rehabilitation cost is fully considered.

GTR 1	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.40	2.50	640,000	350,000	3,315,744	57,409	64,536,513	268,461,074	2,325,744
	2	1.30	2.35	640,000	350,000	3,086,057	54,719	61,354,275	229,158,053	2,096,057
	3	1.10	2.15	640,000	350,000	2,628,517	50,773	57,126,346	199,449,721	1,638,517
	4	0.95	2.00	640,000	350,000	2,357,463	47,604	53,379,244	163,316,241	1,367,463
	5	0.80	1.85	640,000	350,000	2,093,313	44,472	49,663,101	127,574,457	1,103,313
	6	0.60	1.65	640,000	350,000	1,813,622	40,711	45,063,673	91,092,015	823,622
	7	0.45	1.45	466,918	350,000	1,309,306	33,988	37,558,111	59,707,690	492,389
	8	0.40	1.40	26,951	350,000	577,772	25,936	29,976,481	45,476,616	200,821
	9	0.25	1.30	36,506	240,225	368,205	17,889	20,327,432	19,403,458	91,474

GTR 2	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.40	2.50	640,000	350,000	3,320,191	57,463	64,600,997	268,334,811	2,330,191
	2	1.30	2.35	640,000	350,000	3,095,294	54,833	61,491,332	228,938,514	2,105,294
	3	1.10	2.15	640,000	350,000	2,708,986	50,786	56,852,908	193,345,509	1,718,986
	4	0.95	2.00	640,000	350,000	2,415,104	47,725	53,350,073	158,301,841	1,425,104
	5	0.75	1.80	640,000	350,000	2,057,417	43,503	48,346,365	123,193,784	1,067,417
	6	0.60	1.60	640,000	350,000	1,833,643	40,044	43,994,063	85,603,305	843,643
	7	0.45	1.50	368,387	350,000	1,174,326	32,092	35,615,201	58,841,239	455,938
	8	0.40	1.40	26,121	350,000	587,983	25,926	29,926,556	44,192,999	211,862
	9	0.25	1.30	36,209	222,005	347,056	16,547	18,675,632	17,826,739	88,842

Table B3. Cont.

GTR 3	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.45	2.50	640,000	350,000	3,412,588	57,947	64,985,979	264,739,867	2,422,588
	2	1.30	2.35	640,000	350,000	3,080,389	54,818	61,522,511	229,261,923	2,090,389
	3	1.10	2.15	640,000	350,000	2,649,041	50,623	56,827,302	196,275,969	1,659,041
	4	0.95	2.00	640,000	350,000	2,367,918	47,628	53,377,106	161,395,051	1,377,918
	5	0.80	1.85	640,000	350,000	2,101,957	44,512	49,690,800	125,538,462	1,111,957
	6	0.60	1.65	640,000	350,000	1,807,290	40,545	44,838,575	88,896,966	817,290
	7	0.45	1.50	427,596	350,000	1,251,845	33,411	37,064,567	59,036,498	474,248
	8	0.40	1.40	21,716	350,000	576,258	25,990	30,083,428	44,091,254	204,542
	9	0.25	1.25	30,849	214,617	332,714	16,034	18,093,387	17,270,961	87,247

GTR 4	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.40	2.50	640,000	350,000	3,314,512	57,681	64,946,783	269,771,111	2,324,512
	2	1.30	2.35	640,000	350,000	3,095,518	55,136	61,942,137	230,289,149	2,105,518
	3	1.05	2.15	640,000	350,000	2,579,598	50,575	57,008,639	201,382,777	1,589,598
	4	0.95	2.00	640,000	350,000	2,391,021	47,801	53,551,714	161,924,061	1,401,021
	5	0.80	1.85	640,000	350,000	2,134,840	44,583	49,677,385	124,873,211	1,144,840
	6	0.60	1.65	640,000	350,000	1,809,489	40,342	44,527,646	88,711,301	819,489
	7	0.45	1.50	447,941	350,000	1,293,698	33,721	37,291,657	58,220,574	495,757
	8	0.35	1.40	50,904	350,000	594,900	26,051	29,988,554	41,902,100	193,997
	9	0.25	1.25	19,976	199,760	296,424	14,743	16,563,740	15,810,843	76,688

Table B3. Cont.

GTR 5	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.40	2.50	640,000	350,000	3,326,552	57,456	64,566,806	267,798,115	2,336,552
	2	1.30	2.35	640,000	350,000	3,100,821	54,828	61,462,777	228,425,476	2,110,821
	3	1.10	2.15	640,000	350,000	2,692,186	50,750	56,859,192	194,182,057	1,702,186
	4	0.95	2.00	640,000	350,000	2,398,425	47,488	53,057,792	158,434,397	1,408,425
	5	0.75	1.80	640,000	350,000	2,047,712	43,439	48,285,256	123,928,716	1,057,712
	6	0.60	1.60	640,000	350,000	1,814,483	39,968	43,949,270	86,945,089	824,483
	7	0.45	1.50	415,288	350,000	1,236,018	32,935	36,460,501	58,732,841	470,730
	8	0.40	1.40	23,174	350,000	579,245	25,786	29,761,900	44,067,296	206,071
	9	0.25	1.30	33,198	221,978	344,557	16,453	18,556,680	17,713,195	89,380

GTR 6	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.45	2.55	640,000	350,000	3,350,328	58,517	66,066,152	272,397,561	2,360,328
	2	1.30	2.40	640,000	350,000	3,002,543	55,567	62,925,616	239,471,218	2,012,543
	3	1.10	2.15	640,000	350,000	2,590,254	50,812	57,324,721	202,959,197	1,600,254
	4	0.95	2.00	640,000	350,000	2,320,147	47,704	53,665,202	167,491,583	1,330,147
	5	0.80	1.85	640,000	350,000	2,058,683	44,660	50,070,803	132,399,373	1,068,683
	6	0.65	1.65	640,000	350,000	1,846,245	41,107	45,535,537	93,358,032	856,245
	7	0.45	1.50	540,603	350,000	1,420,589	35,448	39,034,683	60,061,954	529,986
	8	0.40	1.40	23,087	350,000	572,699	25,608	29,520,100	44,940,428	199,612
	9	0.25	1.30	35,317	240,069	368,512	17,671	20,007,593	19,098,157	93,127

Table B3. Cont.

GTR 7	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.40	2.50	640,000	350,000	3,349,595	57,503	64,552,236	266,551,637	2,359,595
	2	1.30	2.35	640,000	350,000	3,093,310	54,887	61,578,258	229,258,455	2,103,310
	3	1.10	2.15	640,000	350,000	2,666,319	50,814	57,050,409	196,439,135	1,676,319
	4	0.95	2.00	640,000	350,000	2,384,691	47,852	53,651,284	161,470,282	1,394,691
	5	0.80	1.85	640,000	350,000	2,125,232	44,654	49,819,377	124,592,945	1,135,232
	6	0.60	1.60	640,000	350,000	1,820,356	40,230	44,320,126	86,761,777	830,356
	7	0.45	1.50	406,397	350,000	1,220,802	33,007	36,659,949	58,729,936	464,405
	8	0.40	1.40	23,568	350,000	578,471	26,045	30,150,133	43,615,389	204,903
	9	0.25	1.25	30,732	207,441	321,223	15,526	17,484,411	16,689,665	83,050

GTR 8	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.45	2.55	640,000	350,000	3,427,826	58,518	65,784,221	266,360,657	2,437,826
	2	1.30	2.35	640,000	350,000	3,053,662	54,897	61,737,887	230,686,485	2,063,662
	3	1.05	2.15	640,000	350,000	2,577,236	50,466	56,855,089	200,088,200	1,587,236
	4	0.95	2.00	640,000	350,000	2,369,508	47,685	53,457,559	161,921,620	1,379,508
	5	0.80	1.85	640,000	350,000	2,107,509	44,736	50,006,198	126,437,505	1,117,509
	6	0.60	1.65	640,000	350,000	1,816,209	40,535	44,790,365	88,732,664	826,209
	7	0.45	1.45	440,061	350,000	1,279,286	33,347	36,818,100	57,661,859	489,225
	8	0.35	1.40	50,728	350,000	593,973	25,935	29,818,665	41,613,659	193,245
	9	0.25	1.25	19,686	198,882	294,790	14,612	16,388,700	15,643,759	76,221

Table B3. Cont.

GTR 9	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.40	2.50	640,000	350,000	3,322,084	57,479	64,617,466	268,403,218	2,332,084
	2	1.30	2.35	640,000	350,000	3,108,140	54,922	61,576,070	228,602,107	2,118,140
	3	1.10	2.15	640,000	350,000	2,689,551	50,962	57,186,365	195,558,124	1,699,551
	4	0.95	2.00	640,000	350,000	2,403,112	47,677	53,322,656	159,119,305	1,413,112
	5	0.75	1.80	640,000	350,000	2,063,480	43,496	48,313,602	123,270,092	1,073,480
	6	0.60	1.60	640,000	350,000	1,823,137	40,028	44,007,442	86,478,890	833,137
	7	0.45	1.50	394,477	350,000	1,205,311	32,575	36,118,346	59,052,143	460,834
	8	0.40	1.40	23,979	350,000	581,340	25,901	29,922,283	44,660,339	207,361
	9	0.25	1.30	35,284	227,630	353,845	16,951	19,172,496	18,301,019	90,931

GTR 10	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.45	2.55	640,000	350,000	3,402,882	58,722	66,180,161	269,604,722	2,412,882
	2	1.30	2.40	640,000	350,000	3,055,405	55,592	62,770,480	234,900,913	2,065,405
	3	1.10	2.15	640,000	350,000	2,645,534	50,978	57,370,225	198,142,006	1,655,534
	4	0.95	2.00	640,000	350,000	2,374,104	47,869	53,715,626	161,872,606	1,384,104
	5	0.80	1.85	640,000	350,000	2,111,277	44,601	49,790,418	124,984,525	1,121,277
	6	0.60	1.60	640,000	350,000	1,815,345	40,275	44,405,855	87,144,499	825,345
	7	0.45	1.50	359,765	350,000	1,157,412	32,244	35,939,151	60,590,939	447,647
	8	0.40	1.40	27,611	350,000	588,402	26,197	30,324,174	46,209,619	210,791
	9	0.25	1.30	35,950	243,636	379,639	18,314	20,871,646	19,922,935	100,053

Table B3. Cont.

GTR 11	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.45	2.50	640,000	350,000	3,444,416	58,504	65,702,237	265,620,461	2,454,416
	2	1.30	2.35	640,000	350,000	3,095,071	55,016	61,765,211	228,755,142	2,105,071
	3	1.10	2.15	640,000	350,000	2,663,752	50,855	57,120,794	195,585,632	1,673,752
	4	0.95	2.00	640,000	350,000	2,386,049	47,794	53,559,182	159,767,104	1,396,049
	5	0.75	1.80	640,000	350,000	2,046,388	43,491	48,368,559	123,473,344	1,056,388
	6	0.60	1.60	640,000	350,000	1,815,689	40,064	44,088,195	86,378,411	825,689
	7	0.45	1.50	381,785	350,000	1,182,839	32,376	35,955,157	59,248,487	451,054
	8	0.40	1.40	27,696	350,000	582,626	25,968	30,002,436	44,715,504	204,931
	9	0.25	1.30	33,166	228,947	353,169	17,052	19,314,865	18,436,917	91,056

GTR 12	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.60	640,000	350,000	3,521,113	59,578	67,027,539	266,370,551	2,531,113
	2	1.30	2.35	640,000	350,000	3,019,844	54,952	61,944,144	232,945,770	2,029,844
	3	1.10	2.20	640,000	350,000	2,623,575	51,432	58,129,085	201,494,780	1,633,575
	4	0.90	2.00	640,000	350,000	2,282,717	47,351	53,275,348	165,570,442	1,292,717
	5	0.75	1.80	640,000	350,000	2,036,803	43,848	48,937,485	128,008,190	1,046,803
	6	0.60	1.65	640,000	350,000	1,813,796	40,718	45,072,726	91,444,593	823,796
	7	0.45	1.50	467,702	350,000	1,336,468	34,249	37,844,871	59,443,382	518,765
	8	0.40	1.40	23,965	350,000	586,657	25,782	29,725,081	44,015,508	212,692
	9	0.25	1.30	33,969	222,488	349,028	16,489	18,582,936	17,738,257	92,571

Table B3. Cont.

GTR 13	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.60	640,000	350,000	3,481,864	59,477	67,020,074	268,441,168	2,491,864
	2	1.30	2.40	640,000	350,000	3,011,179	55,552	62,871,624	237,231,700	2,021,179
	3	1.15	2.20	640,000	350,000	2,714,766	52,239	59,003,395	199,314,212	1,724,766
	4	0.90	2.00	640,000	350,000	2,286,118	47,491	53,471,444	164,928,737	1,296,118
	5	0.75	1.80	640,000	350,000	2,034,485	43,916	49,046,602	127,210,440	1,044,485
	6	0.60	1.65	640,000	350,000	1,806,081	40,640	44,984,057	90,318,758	816,081
	7	0.45	1.50	458,549	350,000	1,301,839	33,934	37,538,168	59,016,313	493,289
	8	0.40	1.40	24,989	350,000	578,301	25,850	29,853,227	43,819,437	203,311
	9	0.25	1.25	30,384	217,464	335,368	16,132	18,188,890	17,362,122	87,521

GTR 14	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.50	2.60	640,000	350,000	3,476,880	59,618	67,248,530	269,776,497	2,486,880
	2	1.30	2.35	640,000	350,000	3,007,682	54,936	61,964,893	234,223,858	2,017,682
	3	1.15	2.20	640,000	350,000	2,712,075	52,041	58,717,077	198,779,450	1,722,075
	4	0.95	2.00	640,000	350,000	2,352,422	47,986	53,968,594	162,866,285	1,362,422
	5	0.75	1.80	640,000	350,000	2,050,047	43,954	49,047,229	125,559,701	1,060,047
	6	0.60	1.65	640,000	350,000	1,826,092	40,770	45,106,321	88,454,121	836,092
	7	0.45	1.50	371,904	350,000	1,182,521	32,468	36,132,995	59,972,713	460,618
	8	0.40	1.40	24,404	350,000	588,588	26,089	30,176,022	45,278,245	214,184
	9	0.25	1.30	36,038	231,347	363,693	17,352	19,677,499	18,783,067	96,308

Table B3. Cont.

GTR 15	Year	CutoffHL	CutoffCIL	QcHL	QcCIL	Qm	Qr	P	NPV	TW
	1	1.45	2.55	640,000	350,000	3,336,737	58,518	66,116,843	273,822,528	2,346,737
	2	1.30	2.40	640,000	350,000	3,002,487	55,443	62,740,839	239,352,345	2,012,487
	3	1.15	2.20	640,000	350,000	2,678,644	51,946	58,696,992	202,878,715	1,688,644
	4	0.95	2.00	640,000	350,000	2,311,755	47,645	53,608,732	167,092,084	1,321,755
	5	0.80	1.85	640,000	350,000	2,076,989	44,647	49,985,019	130,509,912	1,086,989
	6	0.65	1.65	640,000	350,000	1,863,280	41,254	45,693,496	91,831,174	873,280
	7	0.45	1.45	456,436	350,000	1,297,985	33,568	37,013,029	60,154,128	491,549
	8	0.40	1.40	25,425	350,000	580,322	25,737	29,675,336	46,850,969	204,897
	9	0.30	1.30	23,686	266,404	411,801	19,591	22,370,684	21,353,835	121,711

APPENDIX C

SENSITIVITY ANALYSIS

Table C1. Effect of price change on NPV across 15 simulated grade-tonnage curves.

Realization	Percentage change and corresponding NPV ($)		
	-10%	0%	10%
GTR 1	236,283,146	268,461,074	295,160,556
GTR 2	235,621,862	268,334,811	296,961,626
GTR 3	232,901,023	264,739,867	304,516,796
GTR 4	232,053,932	269,771,111	299,287,806
GTR 5	235,986,921	267,798,115	294,758,812
GTR 6	236,206,529	272,397,561	298,524,776
GTR 7	228,218,895	266,551,637	294,986,898
GTR 8	230,819,649	266,360,657	301,901,666
GTR 9	236,862,631	268,403,218	296,624,794
GTR 10	233,721,410	269,604,722	305,488,035
GTR 11	233,531,589	265,620,461	305,243,302
GTR 12	233,790,760	266,370,551	301,885,325
GTR 13	235,939,440	268,441,168	300,772,282
GTR 14	236,911,308	269,776,497	301,610,575
GTR 15	237,469,616	273,822,528	301,055,950
Average	234,421,247	268,430,265	299,918,613

Table C2. Effect of HL recovery change on NPV across 15 simulated grade-tonnage curves.

Realization	Percentage change and corresponding NPV ($)		
	-10%	0%	10%
GTR 1	232,801,647	268,461,074	292,234,274
GTR 2	240,597,858	268,334,811	293,744,488
GTR 3	242,461,596	264,739,867	296,795,025
GTR 4	242,422,057	269,771,111	295,237,720
GTR 5	240,108,026	267,798,115	292,995,323
GTR 6	241,146,071	272,397,561	302,874,086
GTR 7	239,703,368	266,551,637	287,911,423
GTR 8	242,462,780	266,360,657	294,042,681
GTR 9	240,883,740	268,403,218	293,573,871
GTR 10	238,130,961	269,604,722	298,739,479
GTR 11	235,768,485	265,620,461	296,082,860
GTR 12	245,915,991	266,370,551	297,261,693
GTR 13	238,295,382	268,441,168	297,262,353
GTR 14	238,525,771	269,776,497	301,873,875
GTR 15	237,818,059	273,822,528	303,214,355
Average	239,802,786	268,430,265	296,256,234

Table C3. Effect of CIL recovery change on NPV for GTR 1 - GTR 15.

Realization	Percentage change and corresponding NPV ($)		
	-10%	0%	10%
GTR 1	266,889,792	268,461,074	274,976,029
GTR 2	257,351,777	268,334,811	275,141,276
GTR 3	260,083,527	264,739,867	277,230,874
GTR 4	258,697,128	269,771,111	274,593,150
GTR 5	256,693,563	267,798,115	274,598,462
GTR 6	266,226,899	272,397,561	274,456,682
GTR 7	265,542,152	266,551,637	274,140,558
GTR 8	260,813,044	266,360,657	277,197,737
GTR 9	257,193,954	268,403,218	275,456,140
GTR 10	264,615,160	269,604,722	278,981,120
GTR 11	263,247,966	265,620,461	276,806,659
GTR 12	263,590,868	266,370,551	270,047,062
GTR 13	267,098,428	268,441,168	276,602,277
GTR 14	268,353,796	269,776,497	276,507,135
GTR 15	268,640,092	273,822,528	275,694,223
Average	263,002,543	268,430,265	275,495,292

Table C4. Effect of changing discount rate on NPV for GTR 1 - GTR 15.

Realization	Percentage change and corresponding NPV ($)		
	-10%	0%	10%
GTR 1	274,345,011	268,461,074	254,993,953
GTR 2	278,640,357	268,334,811	250,906,554
GTR 3	274,184,120	264,739,867	257,974,617
GTR 4	273,240,232	269,771,111	256,383,806
GTR 5	277,457,727	267,798,115	254,500,462
GTR 6	286,650,241	272,397,561	256,234,646
GTR 7	276,885,948	266,551,637	252,872,806
GTR 8	284,887,232	266,360,657	252,889,079
GTR 9	277,643,659	268,403,218	254,967,191
GTR 10	282,363,021	269,604,722	260,470,359
GTR 11	282,610,091	265,620,461	259,852,558
GTR 12	281,016,805	266,370,551	255,904,467
GTR 13	283,824,908	268,441,168	261,681,716
GTR 14	284,262,280	269,776,497	259,207,312
GTR 15	284,304,498	273,822,528	258,971,334
Average	280,154,409	268,430,265	256,520,724

Table C5. Effect of changing rehabilitation cost on NPV for GTR 1 - GTR 15.

Realization	Percentage change and corresponding NPV ($)		
	-10%	0%	10%
GTR 1	268,592,422	268,461,074	268,329,727
GTR 2	268,466,089	268,334,811	265,097,201
GTR 3	264,869,702	264,739,867	264,610,032
GTR 4	269,902,886	269,771,111	269,639,336
GTR 5	267,929,183	267,798,115	267,667,047
GTR 6	272,530,261	272,397,561	272,264,860
GTR 7	266,682,230	266,551,637	266,421,043
GTR 8	266,490,974	266,360,657	266,230,340
GTR 9	268,534,532	268,403,218	268,271,904
GTR 10	269,736,295	269,604,722	269,473,150
GTR 11	265,750,546	265,620,461	265,490,376
GTR 12	266,500,772	266,370,551	266,240,331
GTR 13	268,572,193	268,441,168	268,310,142
GTR 14	269,908,037	269,776,497	269,644,956
GTR 15	273,955,822	273,822,528	273,689,234
Average	268,561,463	268,430,265	268,091,979

APPENDIX D

COMPUTER CODES AND INSTRUCTION

This appendix presents the MATLAB codes used to implement the proposed cut-off grade optimization model in this study. Two Microsoft EXCEL files containing separately the economic, technical and grade-tonnage data are prepared to run the codes successfully. The data are imported in MATLAB environment by using the functions shown in this section.

Five functions have been prepared to establish the cut-off grade policies under the proposed model. Before running the codes, the following should be done:

1) Create the Microsoft EXCEL file that defines economic and technical parameters of the mine project. Parameters should be arranged sequentially as read in function **f1_readPar.**
2) Formulate the grade tonnage data by using Monte Carlo Simulation and define:
    a) The lower and upper grade boundaries of each grade category.
    b) The average grade of each grade category.
    c) The quantity of material initially available in each grade category.

Having created the two files described above, the following functions are run sequentially to determine the cut-off grade strategies.

1) Function 1 defined as **f1_readPar** reads the Microsoft EXCEL file containing economic and technical parameters of the hypothetical open-pit gold mine.
2) Function 2 named as **f2_ReadGRTON** prompts the user to select the file containing the grade-tonnage data. Once the file is selected, the function reads the lower, upper, and average grades along with the tonnes of material present in each grade category of the hypothetical deposit.
3) Function 3 defined as **f3_CutoffOptimizer** is the main function containing the steps of the algorithm from which the optimum cut-off grades in line with the quantities of material flow and the NPV of the mining operation are determined.

In addition, there are two other functions built in the preceding function to fulfill some specific purposes.

The function defined as **f_GrTonAdjuster** is used to adjust the tonnage of material in the deposit by subtracting the quantity of mined material in proportionate amount. The adjusted grade tonnage curve becomes the grade-tonnage input data of the new operational year.

The function **f_Plot_GradeTonnage(data)**is used to plot the grade tonnage data.

In using the codes, the function **f3_CutoffOptimizer** should be modified to fit the three scenarios described previously as follows:

1) For scenario (1), rehabilitation cost "h" is ignored in both the cut-off grade and profit functions.
2) For scenario (2), rehabilitation cost "h" is ignored in the cut-off grade equation but included in the profit function.
3) For scenario (1), rehabilitation cost "h" is included in both the cut-off grade and profit functions.

f1_readPar

```
function f1_readPar


    [fileName,path,indx] = uigetfile({'*.xlsx'},'Data Selector');

    InputDataRead = xlsread(fileName);
    numData = length(InputDataRead);

    Data = xlsread(fileName);
    Params = struct('S',Data(1),'c1',Data(2),'m',...
        Data(3),'r',Data(4),'y1',Data(5),...
        'C1',Data(6),'M',Data(7),'R',Data(8),...
        'f',Data(9),'i',Data(10),'e',Data(11),'c2',Data(12),...
        'y2',Data(13),'C2',Data(14),'h',Data(15),'q',Data(16),...
        'k',Data(17));

   save('Params','Params')
```

f2_ReadGRTON

```
function f2_ReadGRTON

[fileName,path,indx] = uigetfile({'*.xlsx'},'Data Selector');


CurveInputData = xlsread(fileName);
numData = length(CurveInputData);

%Data is saved in cell arrays. {1,1} first 1 indicates it is a
horizontal
% cellarray and second 1 indicates period 1.
GradeTonData.From{1,1} = CurveInputData(:,1);
GradeTonData.Middle{1,1} = CurveInputData(:,2);
GradeTonData.To{1,1} = CurveInputData(:,3);
GradeTonData.Ton{1,1} = CurveInputData(:,4);
GradeTonData.Grade{1,1} = CurveInputData(:,5);

% Cutoff grades
Cutoff = CurveInputData(:,1);

% Initial Cumulative Ton
IniCumulativeTon = cumsum(CurveInputData(:,4),'reverse');

% Tons * grade
tonbygrade = CurveInputData(:,4).* CurveInputData(:,5);

% Average grade
    AverageGrade = zeros(numData,1);
    for iLoop = 1:numData
        AverageGrade(iLoop,1) =
sum(tonbygrade(iLoop:end))/IniCumulativeTon(iLoop,1);
    end

    GradeTonData.CumTon{1,1} = IniCumulativeTon;
    GradeTonData.AvgGrade{1,1} = AverageGrade;

    helpVector = zeros(length(Cutoff),1);
    helpVector(1,1) = 1;
    PeriodInfo = helpVector;

    GradeTonData.PLOT{1,1} =
[Cutoff,IniCumulativeTon,AverageGrade,PeriodInfo];

   save('GradeTonData','GradeTonData');

  Plot_GradeTonnage(GradeTonData.PLOT{1,1})


  CurrentPeriod = 1;
  save('CurrentPeriod','CurrentPeriod');
```

f3_CutoffOptimizer

```
function f4_CutoffOptimizer

%% Load data
load('GradeTonData.mat');
load('Params.mat');
load('CurrentPeriod.mat');

CurrentPeriod = CurrentPeriod;

       % Total available material
       Q = sum(GradeTonData.Ton{1,CurrentPeriod});

      % Measure of convergence
        e = Params.e/100;
        Results =[];

 while Q > 10


    GradeData = GradeTonData.Grade{1,CurrentPeriod};
    CutoffGrades = GradeTonData.From{1,CurrentPeriod};
    TonsData = GradeTonData.Ton{1,CurrentPeriod};
    V = 0;
    PV = V;

    CheckCriteria = 1;
    newPV = 0;


     while CheckCriteria > e

        V = newPV;


             %Cut-off grade calculation for HL.

             CutoffCal1 =((Params.c1 - Params.h + (Params.f + V *
Params.i/100)/Params.C1)*31.1035)/((Params.S -
Params.r)*(Params.y1/100));


            %CutoffCal2 =((Params.c2 + (Params.f + V *
Params.i/100)/Params.C2)*31.1035)/((Params.S -
Params.r)*(Params.y2/100));
            CutoffCal2 =((Params.c2 - Params.h + (Params.f + V *
Params.i/100)/Params.C2)*31.1035)/((Params.S -
Params.r)*(Params.y2/100));
```

```
            CutoffCal2 = CutoffCal2 + (Params.q/100)*(Params.k -
CutoffCal2);



            %Check for HL.
            ID1 = find(CutoffGrades == CutoffCal1);

                    if isempty(ID1) == 1

                         % find the first cut-off grade which is
greather than the calculated one.
                         Cutoff_UP_ID1 = find (CutoffGrades >
CutoffCal1,1);
                         % find the first cut-off grade which is less
than the calculated one.
                         Cutoff_LW_ID1 = Cutoff_UP_ID1 - 1;


                         %Find the average grade of the lower and
upper
                         %grades bounding the calculated grade.
                         AvgBin1 = (CutoffGrades(Cutoff_UP_ID1) +
CutoffGrades(Cutoff_LW_ID1))/2;

                         if CutoffCal1 < AvgBin1
                             Cutoff1 = CutoffGrades (Cutoff_LW_ID1);
                             ID1 = find(CutoffGrades == Cutoff1);
                         else
                             Cutoff1 = CutoffGrades (Cutoff_UP_ID1);
                             ID1 = find(CutoffGrades == Cutoff1);
                         end
                    else
                             Cutoff1 = CutoffCal1;
                    end

                     %%Check for CIL.
                    ID2 = find(CutoffGrades == CutoffCal2);

                    if isempty(ID2) == 1

                         Cutoff_UP_ID2 = find (CutoffGrades >
CutoffCal2,1);
                         Cutoff_LW_ID2 = Cutoff_UP_ID2 - 1;

                         AvgBin2 = (CutoffGrades(Cutoff_UP_ID2) +
CutoffGrades(Cutoff_LW_ID2))/2;

                         if CutoffCal2 < AvgBin2
                             Cutoff2 = CutoffGrades (Cutoff_LW_ID2);
                             ID2 = Cutoff_LW_ID2;
                         else
```

```
                  Cutoff2 = CutoffGrades (Cutoff_UP_ID2);
                  ID2 = Cutoff_UP_ID2;
              end
          else
                  Cutoff2 = CutoffCal2;
          end


      %Tonnage for HL.
      HLoreTons = TonsData(ID1:ID2-1);
      HLGrade=GradeData(ID1:ID2-1);
      TO1 = sum(HLoreTons);


      %Tonnage for CIL.
      CILoreTons = TonsData(ID2:end);
      CILGrade = GradeData(ID2:end);
      TO2 = sum(CILoreTons);



       if TO1 > 1
          TO1 = sum(HLoreTons);
      else
          TO1 = 0;
      end

       if TO2 > 1
          TO2 = sum(CILoreTons);
      else
          TO2 = 0;
       end


      % Calculation of Qc, Qm, and Qr

      %Quantity and average grade of material processed by Heap
Leaching, Qc1.
      if TO1 > Params.C1
              Qc1 = Params.C1;
              g1=(HLoreTons'*HLGrade)/sum(HLoreTons);
      elseif TO1 < 1
              TO1 = 0;
              Qc1 = 0;
              g1 = 0;
      else
          Qc1 = TO1;
          g1=(HLoreTons'*HLGrade)/sum(HLoreTons);
      end
```

```
            %Quantity and average grade of material processed by CIL.
             if TO2 > Params.C2
                 Qc2 = Params.C2;
                 g2 = (CILoreTons'*CILGrade)/sum(CILoreTons);
             elseif TO2 < 1
                  TO2 = 0;
                  Qc2 = 0;
                  g2 = 0;
             else
                 Qc2 = TO2;
                 g2 = (CILoreTons'*CILGrade)/sum(CILoreTons);
             end


           %Total ore tonnage
            TO = TO1 + TO2;

            % Total waste calculation.
            TW = Q - TO;

            % Stripping Ratio.
            SR = TW/TO;


            % Computing the total quantity of material  processed.
              Qc = Qc1+Qc2;

            % Computing the quantity of material mined.
               Qm = Qc*(1+(TW/TO));


            %Quantity of product produced from HL.
               Qr1 = (Qc1*g1*(Params.y1/100))/31.1035;

             %Quantity of product produced from CIL.
               Qr2 = (Qc2*g2*(Params.y2/100))/31.1035;

              % Computing the quantity of product produced from both
streams.
               Qr = Qr1+Qr2;

            % Profit calculation.

            if Qc1 < 1
               Qc1 = 0;
               P =((Params.S-Params.r)*Qr) - ((Params.c1 - Params.h)
* Qc1+(Params.c2 - Params.h) * Qc2) - ((Params.m + Params.h) * Qm) -
Params.f;
            elseif Qc2 < 1
               Qc2 = 0;
```

```
                P =((Params.S-Params.r)*Qr) - ((Params.c1 - Params.h)
* Qc1+(Params.c2 - Params.h) * Qc2) - ((Params.m + Params.h) * Qm) -
Params.f;
            else
                P =((Params.S-Params.r)*Qr) - ((Params.c1 - Params.h)
* Qc1+(Params.c2 - Params.h) * Qc2) - ((Params.m + Params.h) * Qm) -
Params.f;
            end

            % Life of the mine, processing capacity limiting.

            LOM = TO/Qc;

            % waste Tons
            TW = Qm - Qc;

            % Present value of the profits generated over this life
of mine.
            PV = V;
            newPV = P*1.05*((1-(1+(Params.i/100))^-
LOM)/(Params.i/100));
            CheckCriteria = abs(newPV-PV)/PV;

     end


            %if Results existed load that.

            Results(CurrentPeriod,1) = CurrentPeriod;
            Results(CurrentPeriod,2) = Cutoff1;
            Results(CurrentPeriod,3) = Cutoff2;
            Results(CurrentPeriod,4) = Qc1;
            Results(CurrentPeriod,5) = Qc2;
            Results(CurrentPeriod,6) = Qm;
            Results(CurrentPeriod,7) = Qr;
            Results(CurrentPeriod,8) = P;
            Results(CurrentPeriod,9) = newPV;
            Results(CurrentPeriod,10) = TW;

            CurrentPeriod = CurrentPeriod +1;

            save(('Results'),'Results');

            clear GradeTonData.mat


            % Adjust Grade-Tonnage Curve
            f5_GrTonAdjuster(CurrentPeriod,Cutoff1,Cutoff2, Qm,
Qc1,Qc2);

            % Update Q
            load('GradeTonData.mat');
```

```
            Q = sum(GradeTonData.Ton{1,CurrentPeriod});

end
```

f_GrTonAdjuster

```
Function f5_GrTonAdjuster(period,Cutoff1,Cutoff2, Qm, Qc1,Qc2)

load('GradeTonData.mat');

CutoffGrades = GradeTonData.From{1,period-1};
TonsData = GradeTonData.Ton{1,period-1};
AverageGrade = GradeTonData.AvgGrade{1,period-1};
GradeData = GradeTonData.Grade{1,period-1};

ID1 = find(CutoffGrades == Cutoff1);
ID2 = find(CutoffGrades == Cutoff2);

          %Waste tons data adjustment
            Qc = Qc1 + Qc2;
            Qw = Qm - Qc;
            WasteTonData = TonsData(1:ID1-1);
            WasteCoeff = Qw /sum(WasteTonData);

            Waste_Vvector = zeros(length(WasteTonData),1);

            for iLoop = 1:length(WasteTonData)
                Waste_Vvector(iLoop,1) = WasteTonData(iLoop,1)*(1 -
WasteCoeff);
            end

          %Ore tons data adjustment for HL
            OreTonData1 = TonsData(ID1:ID2-1);
            OreCoeff1 = Qc1 / sum(OreTonData1);

            Ore_Vvector1 = zeros(length(OreTonData1),1);

            for jLoop = 1:length(OreTonData1)
               Ore_Vvector1(jLoop,1) = OreTonData1(jLoop,1)*(1 -
OreCoeff1);
            end

           %Ore tons data adjustment for CIL
            OreTonData2= TonsData(ID2:end);
            OreCoeff2 = Qc2 / sum(OreTonData2);

            Ore_Vvector2 = zeros(length(OreTonData2),1);

            for mLoop = 1:length(OreTonData2)
               Ore_Vvector2(mLoop,1) = OreTonData2(mLoop,1)*(1 -
OreCoeff2);
            end


            GradeTonData.From{1,period} = CutoffGrades;
            Tonnage = [Waste_Vvector;Ore_Vvector1;Ore_Vvector2];
```

```
            GradeTonData.Ton{1,period} = Tonnage ;


            %Cumulative Ton
            CumulativeTon = cumsum(Tonnage,'reverse');


            GradeTonData.CumTon{1,period} = CumulativeTon;
            GradeTonData.AvgGrade{1,period} = AverageGrade;
            GradeTonData.Grade{1,period} = GradeData;

            helpVector = zeros(length(GradeData),1);
            helpVector(1,1) = period;
            PeriodInfo = helpVector;

            GradeTonData.PLOT{1,period} =
[CutoffGrades,CumulativeTon,AverageGrade,PeriodInfo];

            Plot_GradeTonnage(GradeTonData.PLOT{1,period})

            save('GradeTonData','GradeTonData');
```

f_Plot_GradeTonnage(data)

```
function f3_Plot_GradeTonnage(data)

% load total material type here

load('GradeTonData.mat');
iniData = GradeTonData.CumTon{1,1};
iniTon = iniData(1,1)/1000000;

x = data(:,1);
y1 = data(:,2)/1000000;
y2 = data(:,3);
period = data(1,4);

yyaxis left
plot(x,y1);
axis([0 max(x) 0 iniTon*1.2]);
title(['Grade-Tonnage Curve for period  ',num2str(period)])
xlabel('Cut-off Grade (g/t)')
ylabel('Ore Tonnage (MTons)')

yyaxis right
plot(x,y2);
ylabel('Average Grade (g/t)')


filename = ['GrTonCurve_t ' num2str(period) '.tif'];



saveas(1,filename)
```