An Aggregation-Based Framework for Construction Risk Assessment with Heterogeneous Groups of Experts

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Abstract

Construction companies continuously seek to improve risk analysis techniques to determine the contingency of projects. Construction risk assessment relies on a group decision-making (GDM) process, whereby a heterogeneous group of experts provides their opinions in order to determine the probabilities and impacts of project risks. In this paper, risk probabilities and impacts are expressed as linguistic terms, which are then represented by fuzzy sets to account for the uncertainty in these assessments. Current GDM processes help experts obtain collective agreement through the use of a consensus-reaching process (CRP), which has several limitations, such as being a time-consuming procedure. The main contributions of this paper are to introduce a list of criteria and a set of metrics to evaluate risk assessment expertise. Additionally, this paper discusses the development of a method for weighting the importance of experts' opinions according to their expertise levels. This research will also serve to improve GDM processes in construction risk assessment by introducing a structured framework that combines assessments from a heterogeneous group of experts through aggregation.

CE Database Subject Headings / Key Words: Construction; Risk management; Fuzzy sets; Decision making; Heterogeneity.

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1. Introduction

Construction projects take place in dynamic environments and involve constantly changing variables, which increases the amount risks to construction stakeholders. To manage risks, construction companies rely on risk analysis techniques and contingency determination procedures. Different techniques have been proposed to analyze risks, such as the probabilistic approach (Ezell et al. 2010) and the traditional deterministic approach (Modarres et al. 2016). The probabilistic approach includes methods such as decision tree analysis (Ahmed et al. 2007), fault tree analysis (Ardeshir et al. 2014), Monte Carlo simulation (MCS) (Salah and Moselhi 2015), failure mode and effect analysis (Mohammadi and Tavakolan 2013), and system dynamics (Nasirzadeh et al. 2008). However, lack of historical data stemming from the uniqueness of each construction project limits the applicability of probabilistic methods, such as the ones utilized in MCS, since it causes difficulties in the estimation of probability distributions for costs (Salah and Moselhi 2015).

On the other hand, the deterministic approach analyzes risk through a single point estimate of potential impacts by assessing the probability and impact of risk and opportunity events (CII 2012). The contingency determination procedure proposed by the Construction Industry Institute (CII) (2012) follows a deterministic (Level 2) approach for calculating risk severity as a product of the probability and impact of risk and opportunity events. However, due to the uncertainty inherent in risk analysis, it is challenging to assess the degree of exposure and the appropriate contingency when using only a single value to determine risk probability and impact in construction projects (Mak and Picken 2000, Elbarkouky et al. 2016). Consequently, input from experts is frequently involved in processes such as risk identification, probability and impact assessment, and contingency determination.

The acquisition and representation of domain knowledge from experts is a critical step in accurately assessing project contingency. Deterministic and probabilistic risk analysis techniques have limited capacity to account for the imprecision and subjectivity present in experts' assessments (Ardeshir et al., 2014); in this context, fuzzy logic (Zadeh 1965) can serve as a valuable tool to handle subjectivity and imprecision inherent in human assessment. In order to account for subjective uncertainties in expert assessments, Elbarkouky et al. (2016) proposed an approach based on research conducted by CII (2012); instead of using single values for risk probabilities and impact, the proposed approach allows experts to provide their assessment using linguistic terms, which are in turn represented by fuzzy numbers.

Involving experts in a group decision making (GDM) process with the purpose of achieving a common solution requires accounting for the heterogeneity with regard to experts' backgrounds, points of view, and levels of expertise (Herrera-Viedma et al. 2014). Due to this heterogeneity, structured GDM processes are very important for achieving collective risk assessment and risk contingency estimation results. There are two approaches commonly used in GDM techniques, the consensus-reaching process (CRP) and the aggregation process. A CRP is a negotiation process conducted iteratively in multistage settings, where the experts discuss and change their opinions or preferences in order to reach a common agreement (Perez et al. 2014). However, CRP is a very time consuming and expensive procedure for construction companies. Also, since the aim of consensus is attaining group consent rather than achieving group agreement, full consent does not necessarily infer that the experts are in full agreement, which can lead to biased CRP results (Butler and Rothstein 2006).

On the other hand, in an aggregation process, a heterogeneous group of experts individually assesses the problem and alternatives, and provides personal opinions as solution inputs (Cabrerizo et al. 2010). In order to determine the influence of each expert's opinion on the final decision, the common approach for addressing the heterogeneity of a group is to assign relative importance weights to each expert (Perez et al., 2014). Then, weighted aggregation operators are applied to combine heterogeneous experts' opinions according to each expert's importance weight. The aggregation process thus serves to facilitate GDM by helping to avoid the biases and discrepancies that are involved in reaching a collective solution during the CRP, which facilitates the group decision making process.

In this research, a multi-step framework was developed to improve the construction risk assessment GDM process by implementing the aggregation process. The proposed framework combines the opinions of a heterogeneous group of experts, based on the experts' importance weights, which are in turn derived from an evaluation of their expertise level in risk assessment contexts. The main contributions of this paper are as follows: to introduce a clear and consistent list of criteria, as well as metrics and scales to evaluate experts' risk assessment expertise; to develop a method for weighting experts' levels of importance in risk assessment; and to improve construction risk assessment GDM by introducing a structured framework that combines expert opinions through aggregation.

This paper is organized according to the following structure. The first section covers results from, a literature review, which highlights gaps in construction risk assessment GDM research. Next, a method for evaluating the expertise levels of experts involved in construction risk assessment is proposed. A new method for assigning importance weights to experts using the fuzzy analytic hierarchy process (FAHP) is then presented. Next, experts' importance weights are used in the aggregation process to determine the influence of each expert's opinion on the final aggregated values for the probability and impact of risks and opportunities. The developed risk assessment framework is then illustrated in a case study, and the most suitable aggregation operator is tested through sensitivity analysis. Finally, in the last section, conclusions and opportunities for future research are discussed.

2. Literature Review

This section outlines the gaps in construction risk assessment that are addressed in this research. First, a review of research on the evaluation of level of expertise in construction risk assessment is presented, followed by a review of previous methods for assigning relative importance weights to experts.

2.1 Assessing experts' levels of expertise in construction risk assessment

Experts possess a large amount of background knowledge and often have cultivated a sensitivity to the relevance of their knowledge in various applications (Cornelissen et al. 2003). Thus, experts are able to provide quick access to information in decision-making contexts. However, there is little consensus in the literature on the definition of an expert. Past research has seen definitions of an expert as an "informed individual", "specialist in field", or "someone who has knowledge about a specific subject" (Baker et al. 2006).

Although there is limited consensus on what an expert is, it should be emphasized that expertise is not related to whom each person is, but rather it concerns the attributes they possess (Sun et al 2008). Key qualification attributes related to the classification and assessment of expertise include knowledge, experience, ability to influence policy, educational background, professional reputation, status among his or her peers, years of professional experience, self-appraisal of relative competence in different areas, and, where appropriate, publication record (Farrignton-Darby and Wilson 2006). All of these qualification attributes form criteria that determine the relevance and credibility of an individual in their field of expertise. However, there is a lack of a clear and consistent list of criteria to evaluate expertise level for the purpose of construction risk assessment, including both quantitative and qualitative attributes. This research addresses the aforementioned gap by proposing a list of criteria, as well as scales of measure for evaluating level of expertise in construction risk assessment.

2.2 Methods for assigning importance weights to experts

There are several methods proposed in the literature for assigning importance weights to experts. For example, a moderator or manager may assign weights directly to the experts (Perez et al. 2011). Although this is a commonly used approach, it is highly biased towards the opinion of the moderator. In addition, consistency methods may be used, whereby weights are determined according to the consistency of the experts' preferences (Perez et al., 2014). However, consistency methods are limited in that experts are evaluated according to their opinions and not in regards to their expertise.

In construction, different methods have been applied in order to assess experts' levels of expertise. For example, Elbarkouky and Fayek (2011a, 2011b) used fuzzy expert systems (FES) to determine experts' importance weights based on their qualification attributes in order to aggregate experts' opinions regarding roles and responsibilities in project delivery systems. In addition, Awad and Fayek (2012a, 2012b) used a multi-attribute utility function (MAUF) to determine the consensus weight factor for each expert, which were based on utility values and relative weight of experience measures; this approach was used in the context of contractor prequalification for surety bonding. However, both these approaches have limitations when dealing with a large number of criteria.

In order to develop a method that assigns weights to experts based on their expertise level and that is also able to handle a large number of criteria, the research discussed in this paper involved a two-step approach. First, a generalization of the analytic hierarchy process (AHP) (Saaty 1987), known as the fuzzy analytic hierarchy process (FAHP), was applied to determine the weight of each qualification criterion used to assess the experts; next, each expert's relative importance weight was derived using the criteria weights provided by the FAHP.

AHP is a logical and clear theory of measurement (Saaty 1987) that has been successfully applied in construction (Askari et al. 2014). Moreover, AHP is able to handle a large number of criteria by hierarchically reducing the number of necessary comparisons. However, standard AHP is unable to handle the uncertainties associated with expert's assessment. To address this limitation, Buckley (1985) proposed FAHP, a generalized version of AHP that allows the experts to provide their assessment using linguistic terms, which are represented by fuzzy numbers.

Aggregation operators are applied in order to use FAHP for the assessment of a group of experts as well as to obtain the experts' relative importance weights. Though there are a wide range of aggregation operators that can be used, since the experts' opinions are represented by fuzzy numbers, only fuzzy aggregation operators were considered for the purpose of this work. Several fuzzy aggregation operators have been proposed in literature, such as fuzzy weighted average (FWA) (Sadiq et al. 2004), fuzzy ordered weighted average (FOWA) (Yager 2004), fuzzy number-induced ordered weighted average (FN-IOWA) (Merigó and Casanovas 2009), fuzzy weighted geometric operator (FWG) (Gohar et al. 2012), and fuzzy similarity aggregation method (FSAM) (Hsu and Chen 1996). However, the choice of aggregation operator is dependent on the application, and there are no clear guidelines on how to choose the most appropriate operator. For the purpose of this research, FWA, FWG, and FOWA operators were considered, since they have been successfully applied in construction risk assessment (Liu et al. 2013).

For FWA, FWG, and FOWA operators, consider n fuzzy numbers, $\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n$. Let $\boldsymbol{w} = (w_1, \dots, w_n)$, such that $w_i \in (0,1)$, and let $\sum_{i=1}^{n} w_i = 1$, be the weighting vector. Equations 1, 2, and 3 illustrate the FWA operator (Dong and Wong 1987; Xu and Da 2003), the FWG operator (Buckley 2001, Ramik and Korviny 2010), and the FOWA operator (Merigó 2011) respectively.

$$\mathrm{FWA}_{\boldsymbol{w}}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_{\mathrm{n}}) = \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{w}_{\mathrm{i}} \tilde{a}_{\mathrm{i}} \tag{1}$$

$$FWG_{\boldsymbol{w}}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_{\mathrm{n}}) = \prod_{i=1}^{n} \tilde{a}_i^{\,w_i} \tag{2}$$

$$FOWA_{\boldsymbol{w}}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \sum_{j=1}^{n} w_j \tilde{b}_j \tag{3}$$

where $\tilde{b}_j$ is the jth largest element of $\{\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n\}$. These operators were also used to aggregate the probabilities and impacts of risks and opportunities in a later step of the proposed framework.

3. Development of a framework for construction risk assessment through aggregation of heterogeneous experts' opinions

In order to develop a framework for construction risk assessment that aggregates experts' opinions based on their expertise level, it is necessary to first determine how to assess expertise level in risk assessment. For this purpose, a list of relevant qualification criteria was developed specifically for construction risk assessment. However, since not all the qualification criteria have the same relevance in assessing expertise level, FAHP was used to determine weights for each criteria. Once the weights of the qualification criteria were determined, the experts involved in the decision-making process were evaluated on the basis of their expertise to determine the weights of their opinions. Next, the experts provided their assessment on the probability and impact of risks and opportunities, which were then aggregated using the weights determined in the previous step. Finally, the aggregated assessment was used to obtain a final contingency value. Figure 1 illustrates the steps of the proposed framework.

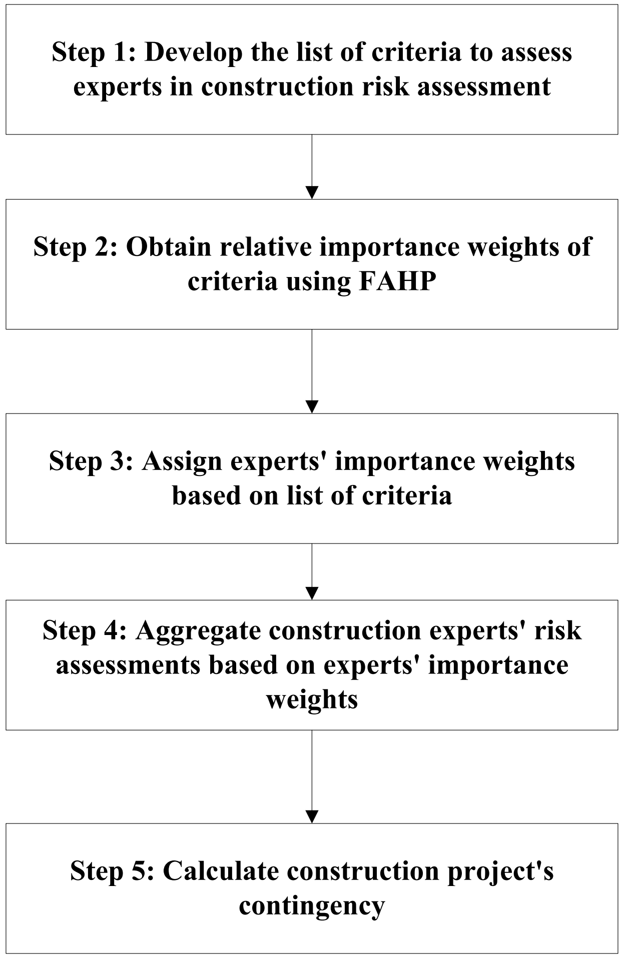

Figure 1. Steps in developing proposed framework for construction risk assessment.

3.1 Step 1: Develop the list of criteria to assess level of expertise in construction risk assessment

In order to develop a list of relevant qualification criteria to evaluate expertise level in construction risk assessment, a comprehensive list of qualification criteria was compiled from the literature (Hoffmann et al. 2007, Wang and Yuan 2011). Next, through a survey, the initial list of criteria was presented to eight experts in the field of construction risk assessment to obtain their level of agreement with each qualification criteria.

The questionnaires asked experts about their level of agreement with each criteria and sub-criteria using a rating scale from 1–5 (Table 1), to assess expertise level in risk assessment. After obtaining input from each of the eight experts, their opinions were aggregated. At this stage, the experts were considered homogenous since they had similar levels of expertise, and the majority prevailed. Those sub-criteria that did not have majority agreement from experts were removed.

The final list of criteria was organized into seven categories, each of which contained between three to seven sub-criteria (i.e., qualification attributes). In total, 32 sub-criteria were selected to assess level of expertise in construction risk assessment (Monzer et al. 2017). The criteria categories and sub-criteria are shown in Figure 2. The questionnaires also asked experts for their level of agreement with the scale of measure for quantitative criteria, and the majority of the experts expressed agreement with its use in this context. However, for the qualitative criteria, the experts provided input for the reference variables. These reference variables (see Table 1 "crisis management" scale) were used to develop a predetermined rating scale from 1–5 to measure qualitative criteria. By utilizing a predetermined rating scale, it is thus possible to better quantify a qualitative sub-criterion and model the decision-making process more accurately (Marsh and Fayek 2010; Awad and Fayek 2012a).

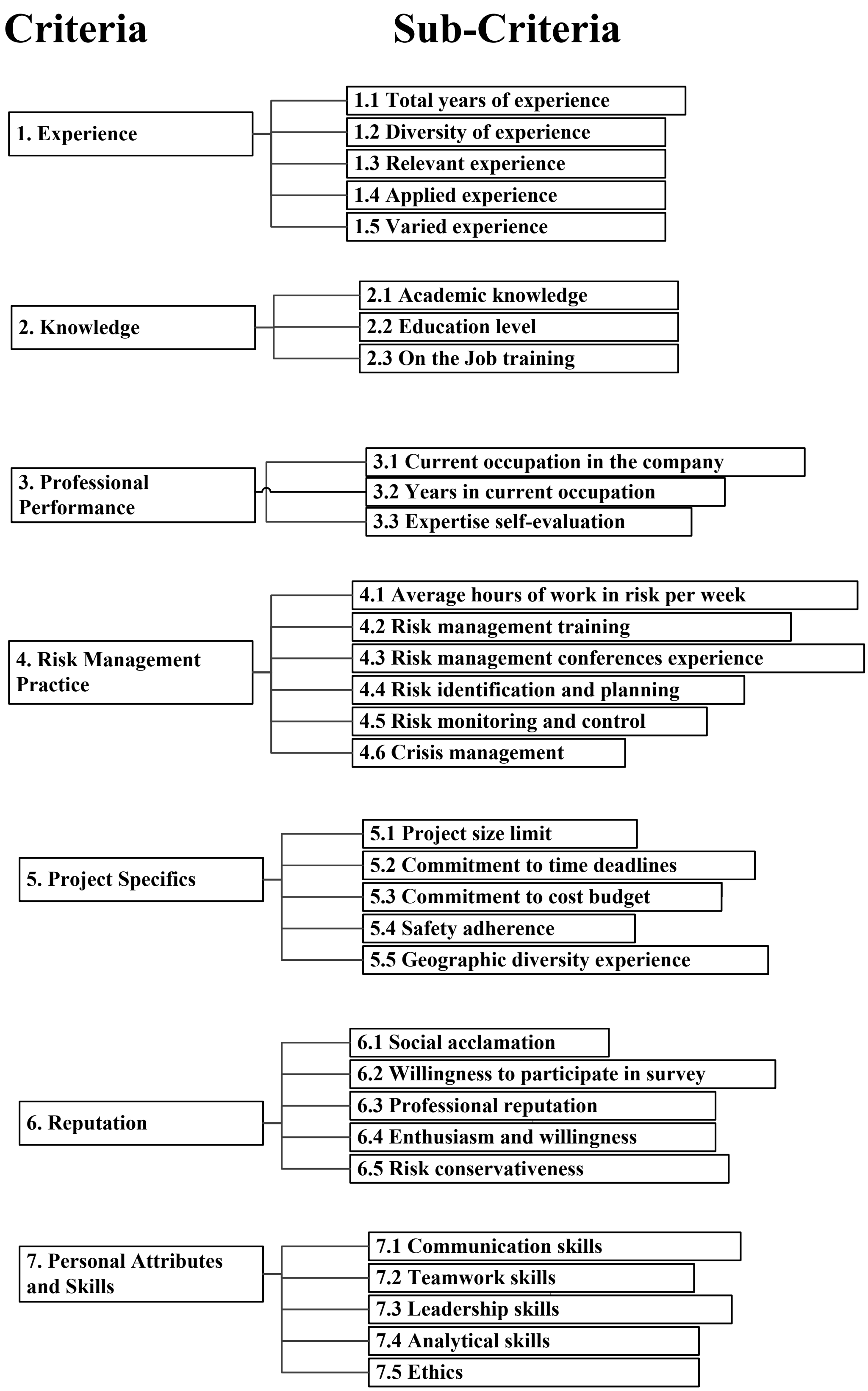

Figure 2. Criteria for expertise in construction risk assessment.

Table 1. Examples of criteria, including their variable types and description, for evaluating level of expertise in construction risk assessment.

Criteria	**Sub-criteria**	**Description**	**Range of values**
1. Experience	1.1 Total years of experience	Number of years expert has been working in his/hers discipline	$\mathbb{R}^+$
2. Knowledge	2.1 Academic knowledge	Number years of study in expert's discipline	$\mathbb{R}^+$
3. Professional performance	3.1 Current occupation in the company	Occupation in company currently working for	Project engineer, Senior engineer, Project manager, Manager, Senior manager
4. Risk management	4.2 Crisis management	Experience handling the time phase of crisis (to be reactive or proactive), and having effective systems to prevent/control/manage crisis	1. Reactive, very poor systems to prevent crisis 2. Reactive, poor systems to prevent crisis 3. Reactive, fair systems to prevent crisis 4. Proactive, good systems to prevent crisis 5. Proactive, very good systems to prevent crisis
5. Project specifics	5.1 Commitment to time deadlines	Percentage of projects finished on time by all projects experts has been involved in	[0, 100]
6. Reputation	6.2 Risk conservativeness	Tendency towards conservative risk assessments	1. Very aggressive risk-taking, 2. Aggressive risk-taking, 3. Moderate, 4. Conservative, 5. Very conservative

3.2 Step 2: Obtain relative importance weights of criteria using FAHP

Once the list of qualification criteria was determined, the relative importance of each criterion for assessing level of expertise was evaluated. In this study, the FAHP was applied to derive the qualification criteria weights.

FAHP presents a clear format for information elicitation in the form of pairwise comparison matrices; each entry a_{ij} of a pairwise comparison matrix represents how much more the element i is preferred over element j with respect to the parent criteria in the level above. In FAHP, the entries of the pairwise comparison matrices are fuzzy numbers; more specifically, they are commonly triangular fuzzy numbers (TFNs) (Van Laarhoven and Predrycz 1983, Chang 1996). TFNs are a special case of trapezoidal fuzzy number. A fuzzy number $\tilde{a}$ is said to be a trapezoidal fuzzy number if its membership function can be represented as shown below in Equation 4.

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{(x-l)}{m_1-l}, \text{when } l \leq x \leq m_1 \\ 1, \text{when } m_1 < x \leq m_2 \\ \frac{(u-x)}{u-m_2}, \text{when } m_1 < x \leq u \\ 0, \text{otherwise} \end{cases} \tag{4}$$

where some $l, m_1, m_2, u \in \mathbb{R}$: $l \leq m_1 \leq m_2 \leq u$. Hereafter, a trapezoidal fuzzy number is represented by the tuple (l, m_1, m_2, u) of its parameters. If $m_1 = m_2 = m$, the fuzzy number is said to be triangular fuzzy number, and it is represented by the tuple (l, m, u) of its parameters.

Consequently, a fuzzy scale based on TFNs is required. Table 2 displays a fuzzy linguistic scale for the pairwise comparisons (Demirel et al. 2008). In addition, for the reciprocity of the pairwise comparison matrices, the fuzzy inverse formula (Equation 5) is applied to represent the reciprocal TFNs.

$$(l, m, u)^{-1} = \left({}^{1}/_{u}, {}^{1}/_{m}, {}^{1}/_{l} \right) \tag{5}$$

Table 2. Linguistic scales for pairwise comparison in the fuzzy analytic hierarchy process (FAHP) model (adapted from Demirel et al. 2008)

Linguistic scale for relative importance	**Triangular fuzzy scale**	**Reciprocal of triangular fuzzy scale**
Exactly the same	(1,1,1)	(1,1,1)
Approximately the same importance	(1/2,1,3/2)	(2/3,1,2)
Weakly more important	(1,3/2,2)	(1/2,2/3,1)
More important	(3/2,2,5/2)	(2/5,1/2,2/3)
Strongly more important	(2,5/2,3)	(1/3,2/5,1/2)
Absolutely more important	(5/2,3,7/2)	(2/7,1/3,2/5)

The fuzzy pairwise comparison matrices were developed based on the expert's input. In cases where more than one expert is involved, it is necessary to aggregate their fuzzy pairwise comparison matrices for each of the hierarchical positions. Let $\tilde{A}_m$ be the pairwise comparison matrix from the mth expert in a specific hierarchical position, as shown in Equation 6.

$$\tilde{A}_m = \left[\tilde{a}_{ij}^{(m)}\right] = \begin{bmatrix} (1,1,1) & \tilde{a}_{12}^{(m)} & \cdots & \tilde{a}_{1n}^{(m)} \\ 1/\tilde{a}_{12}^{(m)} & (1,1,1) & \dots & \tilde{a}_{2n}^{(m)} \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{a}_{1n}^{(m)} & 1/\tilde{a}_{2n}^{(m)} & \cdots & (1,1,1) \end{bmatrix}, m = 1, \dots, d \tag{6}$$

Next, the aggregated fuzzy pairwise comparison matrix $\tilde{A}$ was obtained by aggregating the respective entries of the experts' fuzzy pairwise comparison matrices, as shown in Equation 7.

$$\tilde{A} = \begin{bmatrix} (1,1,1) & f\left(\tilde{a}_{12}^{(1)}, \dots, \tilde{a}_{12}^{(d)}\right) & \cdots & f\left(\tilde{a}_{1n}^{(1)}, \dots, \tilde{a}_{1n}^{(d)}\right) \\ f\left(1/\tilde{a}_{12}^{(1)}, \dots, 1/\tilde{a}_{12}^{(d)}\right) & (1,1,1) & \dots & f\left(\tilde{a}_{2n}^{(1)}, \dots, \tilde{a}_{2n}^{(d)}\right) \\ \vdots & \vdots & \ddots & \vdots \\ f\left(1/\tilde{a}_{1n}^{(1)}, \dots, 1/\tilde{a}_{1n}^{(d)}\right) & f\left(1/\tilde{a}_{2n}^{(1)}, \dots, 1/\tilde{a}_{2n}^{(d)}\right) & \cdots & (1,1,1) \end{bmatrix} \tag{7}$$

where f stands for the aggregation operator. One of the most commonly used aggregation operators for combining fuzzy pairwise comparison matrices is the fuzzy weighted geometric operator (FWG). In this research, the FWG operator (see Equation 2) was applied, since all experts that participated in data collection possessed similar expertise levels (i.e. made up a homogeneous group), and thus were assigned equal weights.

Once the aggregated fuzzy pairwise comparison matrices were obtained for all hierarchical positions, the FAHP was applied to determine the relative importance weights for each criterion and sub-criterion. Several FAHP calculation approaches are discussed in the literature (e.g., Van Laarhoven and Predrycz (1983), Buckley (1985) and Chang (1996)). The approach developed by Chang (1996) is commonly used, since it involves considerably simpler computational efforts than the other methods, and it has been successfully applied in many fields (Ding et al. 2008). Following the approach developed by Chang (1996), there are three main steps for obtaining the relative importance weights of the criteria and sub-criteria in FAHP, which must be performed for each fuzzy pairwise comparison matrix. First, for each element i, $i = 1, \dots, n$, which is represented by the fuzzy pairwise comparison matrix, the value of the fuzzy synthetic extent $\tilde{S_i}$ is computed by applying the algebraic operations of multiplication and summation to the TFNs, as shown below in Equation 8.

$$\tilde{S} = \begin{bmatrix} \tilde{S}_1 \\ \vdots \\ \tilde{S}_n \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{n} \tilde{a}_{1j} \otimes \left(\sum_{k=1}^{n} \sum_{j=1}^{n} \tilde{a}_{kj} \right)^{-1} \\ \vdots \\ \sum_{j=1}^{n} \tilde{a}_{nj} \otimes \left(\sum_{k=1}^{n} \sum_{j=1}^{n} \tilde{a}_{kj} \right)^{-1} \end{bmatrix} =$$

$$\begin{bmatrix} \left(\sum_{j=1}^{n} l_{1j}, \sum_{j=1}^{n} m_{1j}, \sum_{j=1}^{n} u_{1j} \right) \otimes \left(\frac{1}{\sum_{k=1}^{n} \sum_{j=1}^{n} u_{kj}}, \frac{1}{\sum_{k=1}^{n} \sum_{j=1}^{n} m_{kj}}, \frac{1}{\sum_{k=1}^{n} \sum_{j=1}^{n} l_{kj}} \right) \\ \vdots \\ \left(\sum_{j=1}^{n} l_{nj}, \sum_{j=1}^{n} m_{nj}, \sum_{j=1}^{n} u_{nj} \right) \otimes \left(\frac{1}{\sum_{k=1}^{n} \sum_{j=1}^{n} u_{kj}}, \frac{1}{\sum_{k=1}^{n} \sum_{j=1}^{n} m_{kj}}, \frac{1}{\sum_{k=1}^{n} \sum_{j=1}^{n} l_{kj}} \right) \end{bmatrix} \quad (8)$$

where $\otimes$ represents the fuzzy arithmetic multiplication of the TFNs.

Next, in the second step, the non-fuzzy values that represent the relative preference of one element over the others are calculated using the fuzzy synthetic extent values. Therefore, in order to approximate the fuzzy priorities in the pairwise comparison matrices, it is necessary to compute the degree of possibility of $\tilde{S}_{\mathrm{i}} = (l_i, m_{\mathrm{i}}, u_i) \geq \tilde{S}_{\mathrm{j}} = (l_{\mathrm{j}}, m_{\mathrm{j}}, u_{\mathrm{j}})$, as shown in Equation 9.

$$V(\tilde{S}_i \geq \tilde{S}_j) = \begin{cases} 1, & if\ m_j \geq m_i \\ 0, & if\ l_i \geq u_j \\ \frac{l_i - u_j}{(m_j - u_j) - (m_i - l_i)}, & otherwise \end{cases}, \quad i, j = 1, \ldots, n_c \tag{9}$$

In order for the degree of possibility for some TFN $\tilde{S}_i$ to be greater than all n TFNs in $\{\tilde{S}_1, \ldots, \tilde{S}_{n_c}\}$, it must be possible to represent the that TFN using the following equation (Equation 10).

$$V = \begin{bmatrix} v_1 \\ \vdots \\ v_{n_c} \end{bmatrix} = \begin{bmatrix} \min\limits_{k \in \{1,2,\ldots,n_c\}} V(\tilde{S}_1 \geq \tilde{S}_k) \\ \vdots \\ \min\limits_{k \in \{1,2,\ldots,n_c\}} V(\tilde{S}_{n_c} \geq \tilde{S}_k) \end{bmatrix} \tag{10}$$

Each component v_i of V represents the relative non-fuzzy weight of the i^{th} element over the other elements under consideration. However, these weights must be normalized in order to be analogous to the classical AHP criteria weights. Finally, in the third step, the vector V must be normalized using Equation 11 to get the final non-fuzzy normalized weight vector W.

$$W = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} v_1 / \sum_{i=1}^{n} v_i \\ \vdots \\ v_n / \sum_{i=1}^{n} v_i \end{bmatrix} \tag{11}$$

The vector W is the weight vector with respect to the immediate parent element among the elements of the fuzzy pairwise comparison matrix. Let $w_{C_1}, w_{C_2}, \ldots, w_{C_7}$ denote the weights of the seven criteria in Figure 2, and let $w_{s_{ij}}$, $i = 1, \ldots, 7$ and $j = 1, \ldots, n_{C_i}$, be the weight of sub-criterion j with respect to criterion i, where n_{C_i} is the number of sub-criterion under criterion i.

3.3 Step 3: Assign experts' importance weights based on list of criteria

Once the qualification criteria and their relative importance weights are obtained, it is possible to determine importance weights for experts based on their level of expertise. First, each expert involved in the decision-making process is evaluated according to each sub-criterion in the list of criteria (Figure 2). The evaluation data is then normalized to the interval [0,1]. Next, the weights obtained for the criteria and sub-criteria are applied to calculate each expert's score (ES_j), as shown below in Equation 12.

$$ES_j = \sum_{i=1}^{n} \sum_{k=1}^{n_{C_i}} w_{C_i} w_{S_{ik}} I_{S_{ik}}^{(j)}, \ j = 1, \ldots, d \tag{12}$$

where $I_{S_{ik}}^{(j)}$ represents the normalized evaluation of the j^{th} expert according to the k^{th} sub-criterion of criterion C_i, w_{C_i} is the weight of criterion C_i and $w_{S_{ik}}$ is the weight of k^{th} sub-criterion of criterion C_i, as defined above in Section 3.2. In addition, d is the number of experts, n represents the number of criteria, and n_{C_i} is the number of sub-criteria under criterion C_i.

The experts' scores cannot be directly used as weights since they are not normalized. Therefore, after the individual ES_j is calculated for all experts in the group, the importance weight (IW) of each expert is calculated using Equation 13.

$$IW_j = {ES_j}\Big/{\sum_{p=1}^{d} ES_p}, \ j = 1,\ldots,d \tag{13}$$

The importance weight IW of the experts is based on each individual's level of expertise, and is also used weight the experts' risk assessments. The higher an individual's level of expertise is, the higher his/her importance weight will be, and consequently, the greater the impact of his/her assessment on the outcome of the risk analysis process.

3.4 Step 4: Aggregate experts' risk assessments based on their importance weights

In order to calculate the contingency of a construction project, the risk and opportunity events must first be identified. The experts' assessments for both probability and impact are provided by means of linguistic terms, which are represented by trapezoidal fuzzy numbers (TFNs). Once all the experts' assessments of each risk or opportunity event were gathered, they were aggregated into a unique value, reflecting the group's opinion. The experts' importance weights, $\boldsymbol{IW} = (IW_1, \dots, IW_d)$, were used as the weight vector for the experts' assessments to represent level of expertise, and a fuzzy weighted aggregation operator was applied.

Let $E = \{E_1, \dots, E_h\}$ be h risk or opportunity events identified across all work packages of a construction project. For each $E_j, j = 1, \dots, h$, the experts must provide a linguistic assessment of the probability and impact of the event. Let $\tilde{P}_i^{(j)}$ and $\tilde{I}_i^{(j)}$, $i = 1, \dots, d$, be, respectively, the probability and impact assessments of event E_j provided by the i^{th} expert. Next, the aggregated probability value, $\tilde{P}^{(j)}$, and the aggregated impact value, $\tilde{I}^{(j)}$, which represent the group's opinion on the probability and impact of the event E_j are given by $f_{\boldsymbol{IW}}\left(\tilde{P}_1^{(j)}, \dots, \tilde{P}_d^{(j)}\right)$ and $f_{\boldsymbol{IW}}\left(\tilde{I}_1^{(j)}, \dots, \tilde{I}_d^{(j)}\right)$, respectively, where $f_{\boldsymbol{IW}}$ stands for the fuzzy aggregation operator f, using $\boldsymbol{IW}$ as the weighting vector. For example, if the FWA operator, which was presented in Equation 1 is used, then $\tilde{P}^{(j)} = \mathrm{FWA}_{\boldsymbol{IW}}\left(\tilde{P}_1^{(j)}, \tilde{P}_2^{(j)}, \dots, \tilde{P}_\mathrm{d}^{(j)}\right) = \sum_{\mathrm{i}=1}^{\mathrm{d}} \mathrm{IW_i}\tilde{P}_\mathrm{i}^{(j)}$ and $\tilde{I}^{(j)} = \mathrm{FWA}_{\boldsymbol{IW}}\left(\tilde{I}_1^{(j)}, \tilde{I}_2^{(j)}, \dots, \tilde{I}_\mathrm{d}^{(j)}\right) = \sum_{\mathrm{i}=1}^{\mathrm{d}} \mathrm{IW_i}\tilde{I}_\mathrm{i}^{(j)}$. The aggregated probabilities $\{\tilde{P}^{(1)}, \dots, \tilde{P}^{(h)}\}$ and impacts $\{\tilde{I}^{(1)}, \dots, \tilde{I}^{(h)}\}$ of all events are then used to obtain the project's contingency in the next step of the framework.

3.5 Step 5: Calculate the contingency of a construction project

In order to determine the contingency of a construction project, the severity of each event $E_1, \dots, E_h$, must be determined as a percentage value. The severity of a risk or opportunity event is given by Equation 14.

$$\tilde{R}_j = \tilde{P}^{(j)} \times \tilde{I}^{(j)} \ , j = 1, \dots, h \tag{14}$$

where $\tilde{R}_j$ denotes the severity of event E_j and $\tilde{P}^{(j)}$ and $\tilde{I}^{(j)}$ are the aggregated probability and impact of event E_j. Once the severity of each event is obtained, the net severity, $\tilde{O}$, is calculated, as shown in Equation 15.

$$\tilde{O}_j = \tilde{R}_j \times U^{(j)}, \ j = 1, \dots, h \tag{15}$$

where $U^{(j)}$ is the cost of the work package, indicated as dollar value ($) associated with event E_j. Finally, the project's contingency value, $\tilde{V}$, is calculated, as shown in Equation 16.

$$\tilde{V} = \sum_{i \in H_R} \tilde{O}_i - \sum_{i \in H_O} \tilde{O}_i \tag{16}$$

where $H_R = \{i: E_i \text{ is a risk event}\}$ and $H_O = \{i: E_i\}$ is an opportunity event.

Since the aggregated probability and impact, $\tilde{P}^{(j)}$ and $\tilde{I}^{(j)}$, are fuzzy numbers, the operations shown in Equations 14 to 16 involve fuzzy arithmetic. There are two methods available for performing fuzzy arithmetic calculations: the α-cut method and the extension principle. In the α-cut method, interval arithmetic is performed at each α-level cut of the fuzzy numbers to obtain the α-cut of the output. On the other hand, the extension principle generalizes functions from the crisp domain to the fuzzy domain, allowing the generalization of conventional mathematical operators to be applied in the fuzzy domain. A more detailed discussion on fuzzy arithmetic can be found in Hanss (2005).

Considering that the project's contingency, $\tilde{V}$, is a fuzzy number, it is possible to obtain interval ranges for the contingency with different levels of confidence using the α-cut. The α-cut V_α of $\tilde{V}$ represents the confidence interval of the contingency values at a confidence level of $1-\alpha$. If a single crisp value for project contingency is desired, instead of obtaining the project contingency as a fuzzy number, defuzzification operators, such as center of area (COA), smallest of maxima (SOM), middle of maxima (MOM), or largest of maxima (LOM), can be applied. Generally, COA represents the output shape as the "center of gravity". In contrast, SOM and LOM represent the smallest and the largest values of the project contingency when $\alpha = 1$; MOM is the middle value of the range of contingencies when $\alpha = 1$.

In order to illustrate the developed framework, a case study of risk assessment on a real construction project is presented in the next section. The proposed framework was applied to process risk assessments from a heterogeneous group of experts, and the results were compared with a consensus-based approach and the Monte Carlo simulation approach.

4. Testing and validating the construction risk assessment framework: Case study

The proposed framework was applied in a case study to conduct the risk assessment of a wind farm power generation construction project in Kansas, USA. The risk assessment was based on the balance of plant (BOP) construction work packages (CWP), which were valued at approximately \$65 million. The CWP consisted of eight work breakdown structures (WBS), ranging in cost from approximately \$800 thousand to \$16 million. The risk assessment involved a group of four experts who had more than 20 years of experience and held various managerial positions in a Canadian construction company located in Alberta.

In order to apply the proposed framework to this case study, the same eight experts who participated in validating the list of criteria in Step 1(presented in Figure 2) were provided with the refined list of criteria and sub-criteria. Next, for Step 2, each expert provided his/her pairwise comparison of the criteria and sub-criteria, which were collected using questionnaires. The criteria and sub-criteria questionnaires served to gather pairwise comparison data by asking questions such as, “How important is *Knowledge* when compared to *Experience* to evaluate expert’s risk assessment expertise?” The scales used are presented in Table 1. Once all the pairwise comparisons matrices were obtained, the fuzzy pairwise comparison matrices in each hierarchical position were aggregated using Equation 7, along with the FWG aggregation operator (Equation 2). Finally, Equations 8 to 11 were applied to each aggregated pairwise comparison matrix to obtain the relative importance weights of the criteria and sub-criteria. Table 3 shows hypothetical examples of the criteria and sub-criteria weights obtained through this procedure. The actual data for this case study are not presented in order to maintain confidentiality. Note that the weights of the sub-criteria in this example are derived with respect to the parent criterion Experience (shown in Table 3); these weights produce a sum of one when combined together. In addition, the weights of the criteria are derived with respect to the overall parent criterion, which is the *goal* (i.e., to assess level of expertise in risk assessment); these weights also produce a sum of one when combined together.

Table 3. Hypothetical examples of sub-criteria and criteria weights obtained from the fuzzy analytic hierarchy process (FAHP) model.

Criteria	Weights	Subcriteria	Weights
1.Experience	0.11	1.1 Total years of experience	0.34
		1.2 Diversity of experience	0.22
		1.3 Relevant experience	0.28
		1.4 Applied experience	0.05
		1.5 Varied experience	0.11
2.Knowledge	0.17	2.1 Academic knowledge	0.25
		2.2 Education level	0.23
		2.3 On-the-job training	0.52
	0.14	3.1 Current occupation in the company	0.27

Criteria	Weights	Subcriteria	Weights
3.Professional performance		3.2 Years in current occupation	0.32
		3.3 Self-evaluation of expertise	0.41
4.Risk management practices	0.23	4.1 Average hours of work in risk per week	0.11
		4.2 Level of risk management training	0.30
		4.3 Risk management conferences experience	0.13
		4.4 Risk identification and planning	0.07
		4.5 Risk monitoring and control	0.15
		4.6 Crisis management	0.24
5.Project Specifics	0.09	5.1 Project size limit	0.30
		5.2 Commitment to time deadlines	0.27
		5.3 Commitment to cost budget	0.19
		5.4 Safety adherence	0.13
		5.5 Geographic diversity experience	0.11
6.Reputation	0.09	6.1 Social Acclamation	0.31
		6.2 Willingness to participate in survey	0.31
		6.3 Professional reputation	0.17
		6.4 Enthusiasm and willingness	0.12
		6.5 Risk conservativeness	0.09
7.Personal attributes and skills	0.17	7.1 Communication skills	0.09
		7.2 Teamwork skills	0.17
		7.3 Leadership skills	0.40
		7.4 Analytical skills	0.10
		7.5 Ethics	0.24

The criteria and sub-criteria weights were then used to calculate the experts' scores (ES) and importance weights (IW) using Equations 12 and 13, respectively. The results are displayed in Table 4.

Table 4. Case study participants' scores and importance weights obtained from fuzzy analytical hierarchical process (FAHP) model.

Expert	Expert Score (ES)	Importance Weight *(IW)*
1	0.87	0.26
2	1.07	0.32

3	0.79	0.23
4	0.66	0.20

Next, the experts' assessments of probability $\tilde{P}_i^{(j)}$ and impact $\tilde{I}_i^{(j)}$, $i = 1, \ldots, 4$, were aggregated, resulting in aggregated probability $\tilde{P}^{(j)}$ and impact $\tilde{I}^{(j)}$ values for each risk and opportunity event j, $j = 1, \ldots, 17$ in the project. The aggregation operators FWA, FWG, and FOWA, (shown in Equations 1 to 3) were applied, taking into consideration the weighting vector IW for each expert, as shown in Table 4.

Once the aggregated probability $\tilde{P}^{(j)}$ and impact $\tilde{I}^{(j)}$ of all $j = 1, \ldots, 17$ risk or opportunity events were obtained, the project's risk contingency was calculated, . First, Equation 14 was applied to obtain the severity of each risk or opportunity event; next, Equation 15 was used to obtain the net severity of each event. Finally, Equation 16 was used to obtain the project's contingency value. However, Equation 16 provides the project's contingency value as fuzzy number, therefore an additional step was necessary to produce a more interpretable result. As noted in Section 3.5, the α-cuts or the defuzzification formulae can be applied in this context. For the purpose of comparison, the defuzzification strategy was used to obtain the project's contingency value in this case study.

To perform the necessary calculations involved in Step 5 of the framework, the Fuzzy Contingency Determinator© (FCD) software was utilized. FCD automates fuzzy arithmetic procedures to determine the risk contingency of a construction project, based on linguistic assessments of the probability and impact of risk and opportunity events (ElBarkouky et al. 2016).

In order to validate the case study, the project contingency results of the proposed framework were compared with results produced using Monte Carlo simulation (MCS). MCS is used as benchmark, since it is commonly used in the field of construction risk assessment to determine project contingency. The MCS project contingency value in this case study was calculated at P50, representing a confidence level of 0.5 (analogous to the α-cut confidence level discussed in Step 5). In addition, for the purpose of comparison, the experts were also asked to reach a consensus on the probabilities and impacts of the same risk and opportunity events previously assessed through the aggregation process. Therefore, the results of the proposed framework were also compared to the results of the consensus-reaching process.

The error measure applied is the symmetric mean absolute percentage error (SMAPE). SMAPE addresses problems, including asymmetry and the impact of outliers, which are commonly associated with other error measurements, such as mean absolute error and root mean square error (Willmott and Matsuura 2005). The SMAPE ranges from 0% to 200%, and a value of 0% implies perfect agreement between the two approaches being tested (i.e. the proposed risk assessment framework and MCS). The SMAPE measure is expressed in Equation 17.

$$SMAPE = \frac{100}{n} \frac{|P_i - O_i|}{(P_i + O_i)/2} \tag{17}$$

where P_i is the project contingency value predicted by the model under consideration, and O_i is the benchmark value. Again, in this case, the benchmark is the MCS P50 estimate.

Many different combinations of fuzzy aggregation operators, fuzzy arithmetic methods, and defuzzification methods were tested for use in the proposed framework. Table 5 shows the SMAPE for these configurations against the consensus approach.

Table 5. Comparison of case study results using aggregation operators with results from Monte Carlo simulation using SMAPE error calculation.

SMAPE values	Defuzzification method	α-cut	Minimum t-norm	Product t-norm	Drastic t-norm	Bounded t-norm
CONSENSUS	COA	95.78	95.78	86.00	72.78	74.93
	MOM	72.69	72.69	72.69	72.69	72.69
	SOM	43.22	43.22	43.22	43.22	43.22
	LOM	92.83	92.83	92.83	92.83	92.83
FWA	COA	110.53	110.53	107.60	104.20	104.40
	MOM	104.22	104.22	104.22	104.22	104.22
	SOM	84.98	84.98	84.98	84.98	84.98
	LOM	117.95	117.95	117.95	117.95	117.95
FWG	COA	68.46	68.46	46.88	8.00	19.57
	MOM	7.85	7.85	7.85	7.85	7.85
	SOM	45.89	45.89	45.89	45.89	45.89
	LOM	42.32	42.32	42.32	42.32	42.32
FOWA	COA	24.43	24.43	12.81	7.56	1.43
	MOM	**0.08**	**0.08**	**0.08**	**0.08**	**0.08**
	SOM	46.33	46.33	46.33	46.33	46.33
	LOM	0.20	0.20	0.20	0.20	0.20

An analysis of the SMAPE results presented in Table 5 shows that using the FOWA operator with the MOM defuzzification formula in the proposed framework provides the smallest error with respect to the MCS risk contingency results (0.08), independently of the fuzzy arithmetic method used. In addition, it can be seen from Table 5 that both the aggregation operators and the defuzzification methods chosen have a great impact on the resulting SMAPE value. Also, different defuzzification formulae might be more appropriate for different aggregation operators. In general, the FWA aggregation operator results in the highest SMAPE values; during the analysis, all FWA values were higher than 80%. The FWG operator also exhibited poor performance in terms of SMAPE when compared to the FOWA operator: all FWG values were higher than 7%. The FWA and FWG results were thus not in agreement with the MCS results, and were considered unsuitable for use in the case study.

On the other hand, the fuzzy arithmetic methods do not greatly impact SMAPE values in most cases, except when the COA defuzzification formula is used. In the latter case, the impact of the fuzzy arithmetic method is considerable, and the method that provides the smallest error is either the extension principle using the drastic t-norm or the bounded t-norm, depending on the aggregation operator used. It should be noted that with the right choice of parameters, the proposed framework hugely improves the SMAPE in comparison to the best result obtained by the consensus approach: 0.08 as compared to 43.22.

The findings of this case study show that applying the aggregation process to GDM in construction risk assessment provides results that are in higher agreement with the MCS project contingency values than are the results obtained through consensus. Furthermore, among the three aggregation operators tested, the FOWA demonstrated results with the highest MCS agreement for this specific case study, and the fuzzy arithmetic methods used did not affect the results when defuzzification formulae other than COA were used. The proposed risk assessment framework will assist researchers and industry leaders in advancing GDM approaches for construction risk assessment by providing a systematic, transparent, and flexible aggregation-based methodology.

5. Conclusions and Future Research

Assessment of risks and opportunities on construction projects is a very complex topic, and the process frequently involves multiple experts with different levels of expertise. This paper has proposed new risk assessment framework. The proposed framework provides a systematic, multi-step methodology that assesses expertise level in construction risk assessment, and assigns weights to expert's opinions according to their level of expertise. Experts' opinions for both the qualification criteria assessment and the risk assessment are captured by linguistic terms, which are modelled using fuzzy numbers. For this reason, the framework is also able to process the subjectivity and vagueness inherent in human assessments.

The framework was applied in a case study of a real construction projects and compared with the results obtained by the MCS P50. The framework was able to obtain similar results to the MCS approach; however the proposed framework offers a quicker process and does not depend on the availability of historical data for probabilistic distribution estimation. The performance of the framework was also superior to that of the consensus process. Some guidelines for selecting the most appropriate aggregation operator and defuzzification formula were also discussed, which in the context of this case study were the FOWA operator and the MOM formula.

In summary, the main contributions of this paper are as follows: to introduce a clear and consistent list of criteria, metrics, and scales to evaluate risk assessment expertise; to develop a method for weighting level of expertise in risk assessment; and to improve construction risk assessment GDM processes by introducing a structured framework that combines assessments from a heterogeneous group of experts through aggregation.

Future research will explore expansion of the proposed framework to other construction applications that require expert assessments. This goal can be achieved by adjusting the list of criteria to assess expertise level in other fields, and by following the proposed rationale for assigning importance weights during the aggregation process in GDM. Future work will also include a comparison of the risk assessment framework results and the actual project contingency results to better validate the proposed framework. Another topic for future research includes the development of a method to adjust experts' weights according to the work package under evaluation; for example, in the work package "underground collection", experts that have a geotechnical background have higher levels of expertise and thus the weight of their assessments should be adjusted accordingly.

Data Availability Statement

All data generated or analyzed during the study are included in the submitted article or supplemental materials files.

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