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MULTIVARIABLE SELF-TUNING CONTROL OF SYNCHRONOUS GENERATOR SYSTEMS

BY

WENYAN GU ©

A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled MULTIVARIABLE SELF-TUNING CONTROL OF SYNCHRONOUS GENERATOR SYSTEMS submitted by Wenyan Gu in partial fulfillment of the requirements for the degree of Doctor of Philosophy

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ABSTRACT

This thesis pursues an extensive study on the application of a multivariable generalized minimum variance (GMV) self-tuning controller to a single-machine-infinite-bus (SMIB) power system. The nonlinear characteristics of a synchronous generator system are discussed from the viewpoint of multi-input/multi-output system, and the multivariable self-tuning controller is designed for such a system. Digital simulations show that the self-tuning algorithm used in this thesis can cope well with wide operating conditions of a synchronous generator system. A theoretical analysis of the convergence property of the multivariable GMV self-tuning algorithm in a deterministic environment is pursued. It is further proved that such a multivariable GMV self-tuning power system is always stable even in a stochastic environment.

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NOMENCLATURE

Principal Symbols

$\mathbf{A}$ = matrix

$\|\mathbf{A}\|$ = induced norm of matrix $\mathbf{A}$

D = moment of inertia, lb-ft^2

det = determinant

E = bus voltage

E(·) = expectation value

F = turbine power fraction

$\mathbf{I}$ = unit matrix

i = instantaneous current

M = machine-inertia constant, kws/kva

p = operator d/dt

P = real power

Q = Var power

q^{-1} = backward shift operator

R^n = the set of all ordered n-tuples of real numbers

$R^{n \times n}$= the set of all n×n matrices with real elements

r = resistance

s = instantaneous machine slip

T = torque

v = instantaneous voltage

$\mathbf{x}$ = vector

$\|\mathbf{x}\|$ = Euclidean norm of vector $\mathbf{x}$

$\mathbf{y}^*(t+k|t)$ = optimal estimation of $\mathbf{y}(t+k)$ at time t

δ = load angle of synchronous machine with respect to bus

θ = instantaneous angular displacement

λ_{max} = maximum eigenvalue

τ = time constant

ω = angular velocity

ψ = flux linkage

Subscripts

c = steam chest

d = direct axis

e = external

fd = field winding

hp = high pressure

m = mechanical

o = open-curcuit

q = quadrature axis

s = synchronous

t = terminal

0 = initial

Superscripts

' = transient quantity

T = transposition operator

^ = estimated value

CHAPTER 1

GENERAL INTRODUCTION

1.1 Introduction

As indicated by the title of this thesis, the aim of this project is to develop a multivariable self-tuning, or adaptive, controller for power systems and prove its stability. The importance of this topic is brought into focus by reviewing control strategies which are in use presently in power systems.

It has long been recognized that a synchronous generator can be modeled by a set of nonlinear equations. However, it is difficult to analyze and synthesize controllers for such a system by using the nonlinear model directly. Therefore, a linearized model is needed if the principle of linear feedback control is used[28,35]. Such a linearized model of a power system is a coupled two-loop system. To further simplify the design procedure of controllers it is common to treat the coupled loops as two individual ones i.e. speed loop (or frequency loop) and voltage loop (or excitation loop). Then the controller in the speed loop, which is usually called Governor (GOV), and the controller in the voltage loop, which is usually called the Automatic Voltage Regulator (AVR), are tuned based on classical feedback control theory such as Bode plots, root locus technique, or Nyquist criterion, etc.[29,31]. The GOV functions as either a shaft speed controller or a power controller depending on the mode of generator operation. The AVR is used to regulate terminal voltage through controlling the excitation of the generator. In many situations a Power System Stabilizer (PSS) is needed to reduce the oscillations caused by load disturbances or short-circuit faults, etc. The basic function of the PSS is to offer damping torque to cancel or reduce the effect of negative damping torque caused by the excitation loop and other factors such as power angle, weak transmission and load characteristics. These controllers are widely used in power systems at present and have simple structures due to relaxed design constraints based on small-signal analysis and intuitive concepts. These PSS configurations cannot achieve satisfactory control results for a wide range of operating conditions.

To improve the control quality much work has been done since 1970's with many interesting results by applying linear optimal feedback control to power systems, especially to the excitation control[46-51]. It seems, however, that it is not very practical when it is applied to a real power system. The reasons perhaps are, in part, the complexity of the theory and the difficulty in accessing state variables. But the most important reason for lack of acceptance in power engineering is that the linear quadratic optimal controller does not offer good control for wide operating ranges of generators.

It is known that both the classical feedback controller and the linear quadratic optimal controller are established on the same assumption that an explicit deterministic mathematical model of the system to be controlled is available. The nonlinear features of a power system cause the linearized system parameters to change in response to different operating points. Some of the parameters may even have large deviations from the assumed operating point. These characteristics cannot be modeled in the design stage of the fixed parameter controllers because the nature of the variations is random and the system is nonlinear. This indicates that on-line parameter identification will be helpful. It is also important for a control policy to involve the random nature of the power system environment. Adaptive control (or self-tuning) theory has the required characteristics needed for good performance in the power system environment. This technique identifies the system parameters and calculates an optimal control action subject to certain design criteria based on the identified parameters at each sampling steps. Therefore, it can "chase" the changes of the system parameters.

Research on adaptive control began in the early 1950's. Unfortunately, there was not much progress until 1970's due to the lack of appropriate hardware as well as adequate theory for analysis. The interest in adaptive control was renewed in 1970's. "The progress in control theory during the previous decade contributed to an improved understanding of adaptive control. The rapid and revolutionary progress in microelectronics has made it possible to implement adaptive regulators simply and cheaply"[52]. Since 1970 several excellent surveys on adaptive control theory and applications have been published, such as Landau[71] (1974) and Astrom[52] (1983).

Research on applications of adaptive control to power systems began in the early 70's. Among the studies on the application of adaptive control in power systems, references [1~26] deal mainly with the synchronous generator controls. Most of them were concentrated on adaptive excitation control such as self-tuning AVR or self-tuning PSS[1,11,14,20,25]. These adaptive AVR's and adaptive PSS's are designed on the basis of single-input/single-output (SISO) system models.

Bonanomi *et al.*[1] (1980) use the technique called "gain scheduling adaptive control" to design an AVR. There are five pairs of off-line calculated gains stored in a look-up table, and the controller will pick-up one of them from the table based on the identified levels of real power, var power and terminal voltage at a fixed period; Outhred *et al.*[18] (1972), Malik *et al.*[16] (1978), Irving *et al.*[9] (1977) use model reference adaptive control to design AVRs; Ghosh *et al.*[4,5] (1984, 1985), Chen *et al.*[2,3] (1986) employ the pole-placement and pole-shifting self-tuner for the excitation control; Sheirah *et al.*[22] (1978), Ledwich[12] (1978), Kanniah *et al.*[11,12] (1984) design their exciters by using adaptive minimum variance controller; And Xia *et al.*[25] (1983), Romero *et al.*[21] (1986) applied self-tuning generalized minimum variance controller to their exciters. All of the research was supported by corresponding simulation results. As a power system is inherently a coupled multi-input/multi-output (MIMO) system, Hanus *et al.*[8] (1983) and Yokokawa *et al.*[26] (1987) developed their combined GOV and AVR self-tuning controllers. Hanus *et al.* developed a digital multivariable self-tuner by using a modified pole-assignment technique. However, divergence or instability of the controlled system was observed in his simulation studies. Therefore, the problem with his algorithm is that it cannot ensure the convergence of the controlled system. Yokokawa *et al.* developed a multivariable self-tuning controller based on a state-space model. The system reactances are estimated through a Kalman filter, then a state feedback gain matrix subject to a linear quadratic performance index is solved on-line by solving the discrete matrix Riccati equation. The optimal controller works effectively in reducing long-period oscillations of the generator. It is a relatively slow algorithm, resulting in updates of the optimal feedback matrix every 3 or 4 seconds. Also the stochastic characteristics are not modelled in their work. Therefore, further probing into the many phases of multivariable adaptive control and the deriviation of a more practical, simpler adaptive controller for a power system presents an interesting development challenge and would also be of practical benefit to improving power system controller performance.

1.2 Scope of the thesis

The main objective of this thesis is concerned with the multivariable self-tuning control of synchronous generators to minimize the variance of specified "quality" variables of the plant.

To meet this objective, an extensive study of a power generation system based on the MIMO system representation was pursued. This is discussed in Chapter 2. The nonlinear model of a turbine-synchronous generator system used in simulation studies of the multivariable self-tuning controller is discussed. The coupling effect between the excitation and the speed loops, and a transfer function matrix for the linearized model is discussed in Section 2.3.

Chapter 3 deals with the multivariable self-tuning controller to minimize the variance of the outputs of the synchronous generator systems. Section 3.2 is used to introduce the basic theory of a multivariable self-tuning controller, called Generalized Minimum Variance (GMV) self-tuning controller. Section 3.2.1 presents the GMV controller for a MIMO system with known parameters, and Section 3.2.2 then develops it into the self-tuning algorithm. In Section 3.3, the multivariable GMV self-tuning controller for the generator systems is developed for the stochastic case.

Convergence analysis of the power system with multivariable GMV self-tuning controller is described in Chapter 4. It first proves the convergence property of the multivariable GMV self-tuning controller for deterministic MIMO systems, then shows a system with unit time delay subject to a stochastic environment is equivalent to the deterministic case, and finally concludes that a power system with the suggested self-tuner is stable under certain conditions.

Chapter 5 offers comprehensive simulation results. The nonlinear model discussed in Chapter 2 is used to simulate the single-machine-infinite-bus (SMIB) system. A variety of operation conditions is simulated to test the multivariable GMV self-tuning controller, and comparision with a SMIB system with properly tuned conventional controllers is made.

Chapter 6 provides a summary of the thesis, conclusions, and suggestions for further research.

CHAPTER 2

MATHEMATICAL MODELS OF SYNCHRONOUS GENERATOR

2.1 Introduction

It is important to investigate the mathematical model of a plant for the purpose of control. There are two kinds of synchronous machine models, i.e., non-linear and linear. For power system control analysis the linear model is very often used. However, the linear model used in power system studies is treated as two independent loops when GOV and AVR are tuned. Therefore, a study of the synchronous machine on both non-linear and linear models from the multivariable system viewpoint will be pursued in this chapter. All the symbols used in this chapter are included in the definitions listed in the nomenclature.

2.2 Non-linear model

Under the assumptions that there are no amortisseur windings and that the resistances and saturation are negligible, a synchronous machine connected to an infinite (or very large) bus can be described by the following set of non-linear equations[34] in p.u.,

$$v_d = p\Psi_d - \Psi_q p\theta \tag{2.2.1}$$

$$v_q = p\Psi_q + \Psi_d p\theta \tag{2.2.2}$$

$$v_d = px_e i_d - x_e i_q p\theta + E\sin\delta + r_e i_d \tag{2.2.3}$$

$$v_q = px_e i_q + x_e i_d p\theta + E\cos\delta + r_e i_q \tag{2.2.4}$$

$$\Psi_d = I_{fd} - x_d i_d \tag{2.2.5}$$

$$\Psi_q = -x_q i_q \tag{2.2.6}$$

$$Mp^2\theta = T_m - \Psi_d i_q + \Psi_q i_d - D(p\theta - 1) \tag{2.2.7}$$

$$\Psi_{fd} = I_{fd} - (x_d - x_d') i_d \tag{2.2.8}$$

$$E_{fd} = I_{fd} + T_{d0}' p\Psi_{fd} \tag{2.2.9}$$

$$v_t^2 = v_d^2 + v_q^2 \tag{2.2.10}$$

If it is further assumed that voltages $p\Psi_d$, $p\Psi_q$, $px_e i_d$, $px_e i_q$ are negligible compared with other terms and $p\theta = \omega_s = 1$, with these assumptions, Eqns.(2.2.1) through (2.2.4) and Eqn.(2.2.7) become,

$$v_d = -\Psi_q \tag{2.2.11}$$

$$v_q = \Psi_d \tag{2.2.12}$$

$$v_d = E\sin\delta - x_e i_q + r_e i_d \tag{2.2.13}$$

$$v_q = E\cos\delta + x_e i_d + r_e i_q \tag{2.2.14}$$

$$Mp^2\theta = T_m - \Psi_d i_q + \Psi_q i_d \tag{2.2.15}$$

This is a set of formulae frequently used in modelling synchronous generators[45]. By defining,

$$E' = \Psi_{fd} \tag{2.2.16}$$

$$E_q = \Psi_d + x_q i_d \tag{2.2.17}$$

$$s = \omega - 1 \tag{2.2.18}$$

a set of nonlinear equations describing a synchronous generator supplying power to an infinite bus (SMIB) system is derived,

$$\dot{\delta} = \omega_0 s \tag{2.2.19}$$

$$\dot{s} = \frac{1}{M}(T_m - T_e - Ds) \tag{2.2.20}$$

$$\dot{E}'_q = \frac{1}{T'_{d0}}\left[E_{fd} - (x_d - x'_d)i_d - E'_q\right] \tag{2.2.21}$$

$$v_d = x_q i_q \tag{2.2.22}$$

$$v_q = E'_q - x'_d i_d \tag{2.2.23}$$

$$T_e = E_q i_q \tag{2.2.24}$$

$$i_d = \frac{(E_q - E\cos\delta)(x_e + x_q)}{r_e^2 + (x_e + x_q)^2} - \frac{r_e E\sin\delta}{r_e^2 + (x_e + x_q)^2} \tag{2.2.25}$$

$$i_q = \frac{(E_q - E\cos\delta) r_e}{r_e^2 + (x_e + x_q)^2} + \frac{(x_e + x_q) E\sin\delta}{r_e^2 + (x_e + x_q)^2} \tag{2.2.26}$$

$$v_t = \sqrt{v_d^2 + v_q^2} \tag{2.2.27}$$

$$E_q = E'_q + (x_q - x'_d) i_d = v_q + x_q i_d \tag{2.2.28}$$

The prime-mover of a steam turbine can be described by,

$$\dot{T}_m = \frac{1}{\tau_c} (F_{hp} u_g - T_m) \tag{2.2.29}$$

where u_g is the control signal of mechanical power, i.e., the turbine valve opening.

Expressions (2.2.19) through (2.2.29) will be used to simulate the turbine-synchronous-generator system in this thesis. The parameters used in simulation studies are given in Appendix A. If practical limitations of the steam valve and exciter are taken into account a block diagram for the nonlinear model of SMIB system can be drawn as in Figure 2.2.1, where u_g and u_f are taken as input signals, ω and v_t output signals of the system, i_d and i_q are determined by Eqns. (2.2.25) and (2.2.26). It is obvious that such a system is a two-input/two-output system.

2.3 Linearized Model

As the linear SMIB model is frequently used in control analysis, it will be briefly discussed in this section.

A linear model of a turbine-SMIB system can be derived from Eqns.(2.2.19) through (2.2.29). Choosing a fixed operating point, and using Taylor expansion around the chosen operation point, one can easily linearize the nonlinear model as shown in Figure 2.3.1.

In Figure 2.3.1, shaft speed ω and terminal voltage v_t are arranged to be the two outputs. The constants K_1~K_6 are defined as follows,

$$k_1 = \frac{E_{q0}E_0}{A}\left[r_e\sin\delta_0+(x_e+x_d')\cos\delta_0\right]+$$

$$+\frac{i_{q0}E_0}{A}\left[(x_q-x_d')\sin\delta_0-r_e(x_q-x_d')\cos\delta_0\right]$$

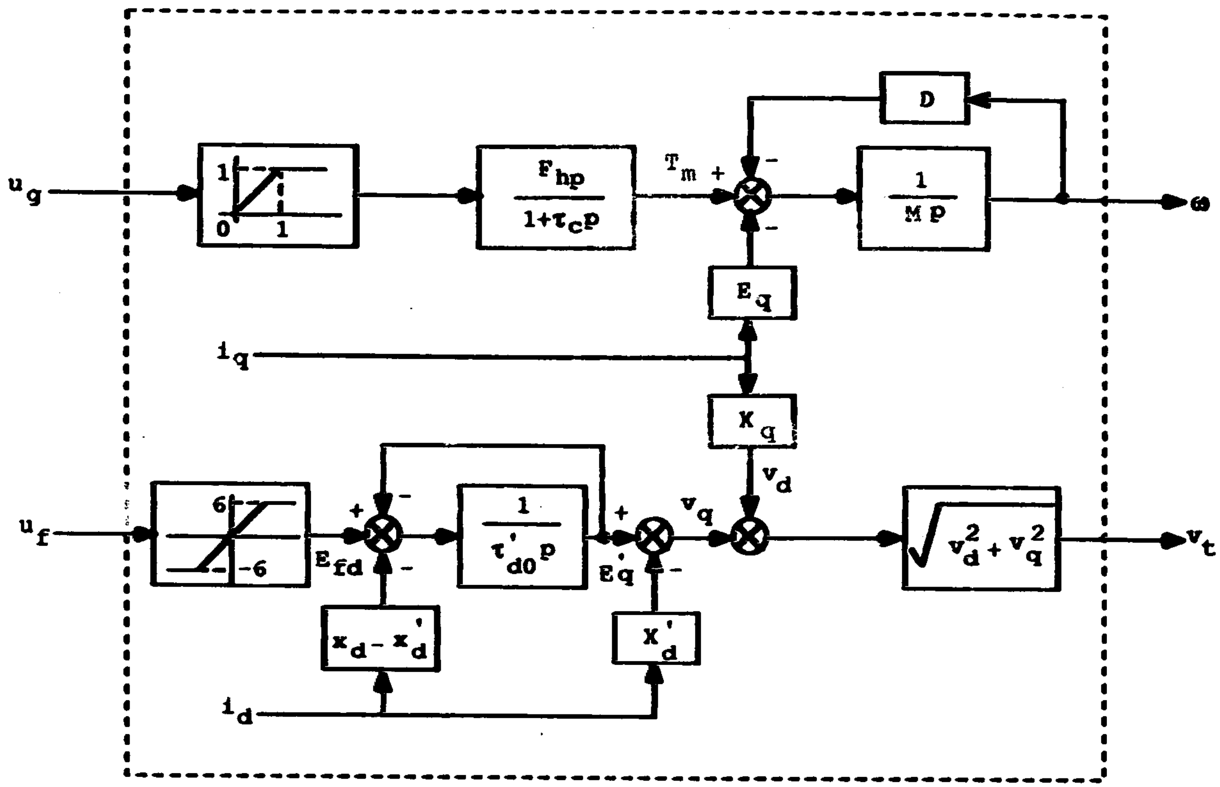


Fig. 2.2.1 Block diagram of nonlinear model of a turbine-SMIB system viewed as a multi-input/multi-output system.

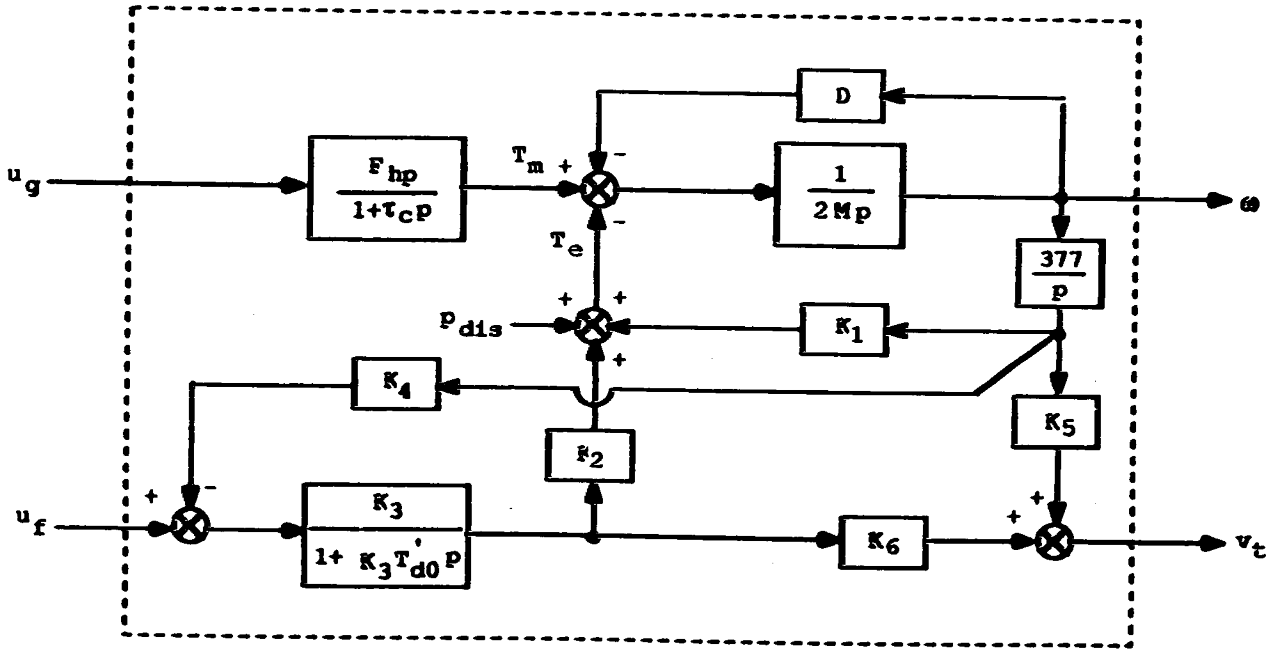


Fig. 2.3.1 Block diagram of linearized model of a turbine-SMIB system viewed as a multi-input/multi-output system.

$$K_2 = \frac{r_e E_{q0}}{A} + i_{q0}\left[1 + \frac{(x_e+x_q)(x_q-x'_d)}{A}\right]$$

$$K_3 = \left[1 + \frac{(x_e+x_q)(x_q-x'_d)}{A}\right]^{-1}$$

$$k_4 = \frac{E_0(x_d-x'_d)}{A}\left[(x_e+x_q)\sin\delta_0 - r_e\cos\delta_0\right]$$

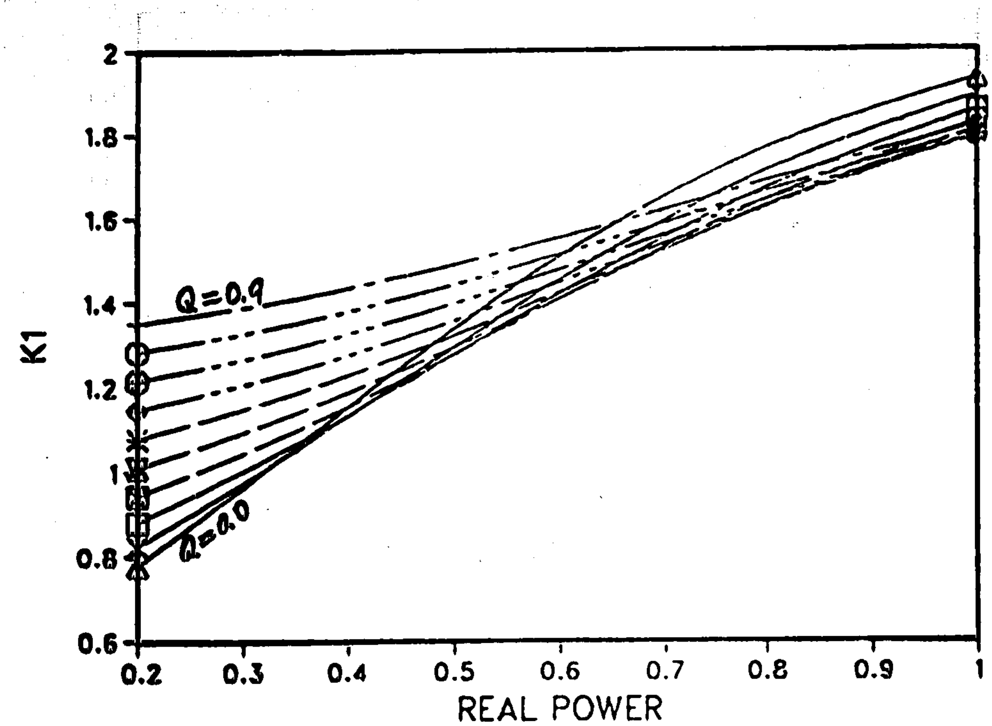
$$k_5 = \frac{v_{d0}}{v_{t0}} x_q \left[\frac{r_e E_0 \sin\delta_0 + (x_e+x'_d) E_0 \cos\delta_0}{A}\right] +$$

$$+ \frac{v_{q0}}{v_{t0}} x'_d \left[\frac{r_e E_0 \cos\delta_0 - (x_e+x_q) E_0 \sin\delta_0}{A}\right]$$

where

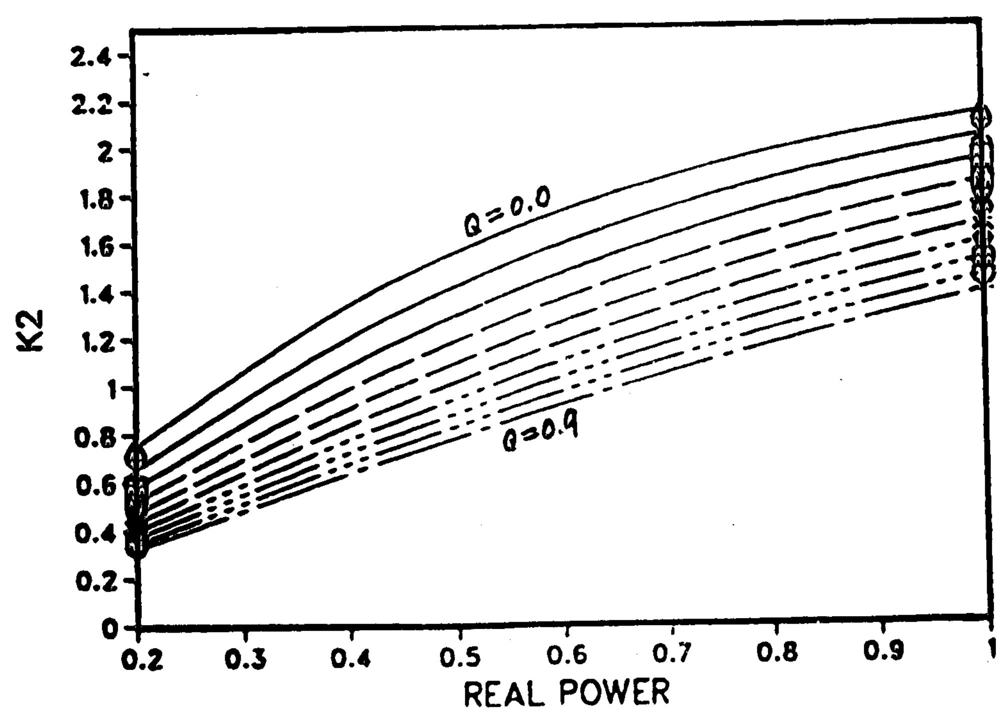
$$A = r_e^2 + (x_e+x'_d)(x_q+x_e)$$

It is helpful to mention the effects of changing operating point on the constants $K_1 \sim K_6$. Figure 2.3.2 shows the locus of $K_1 \sim K_6$ (except K_3) for the simulated system of the thesis when real power P and reactive power Q change. Obviously they change along with the loading conditions, and K_5 even changes its sign under heavy loading conditions.



(a)

Fig.2.3.2 (a~e) The effects of changes of loading conditions on k_1, k_2, k_4, k_5 and k_6 for a synchronous generator.



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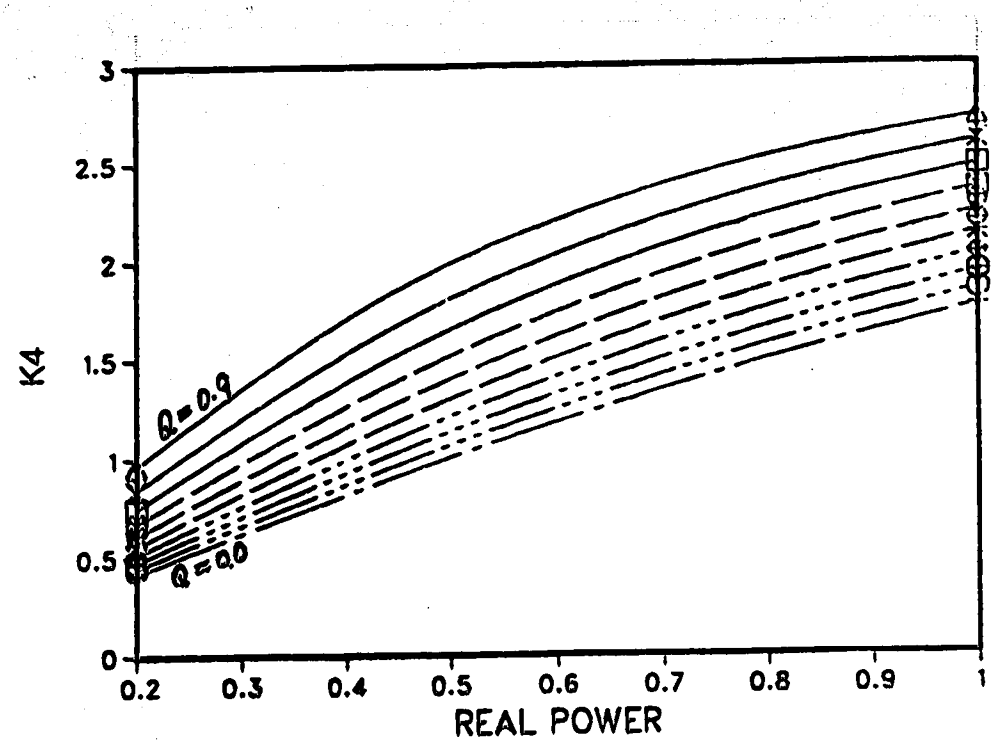

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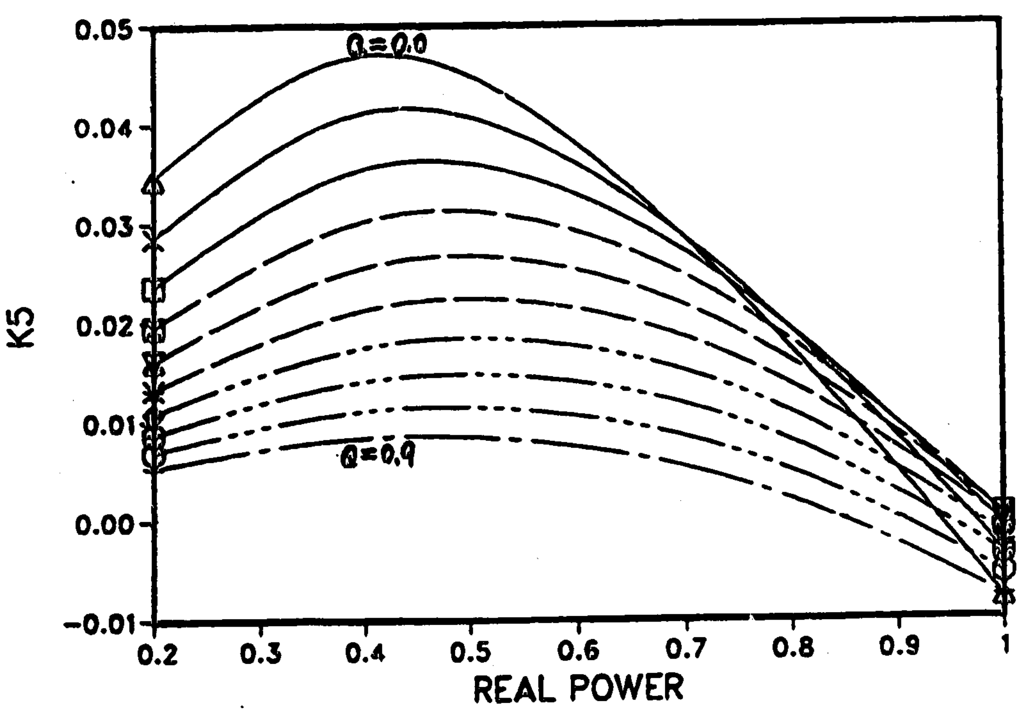

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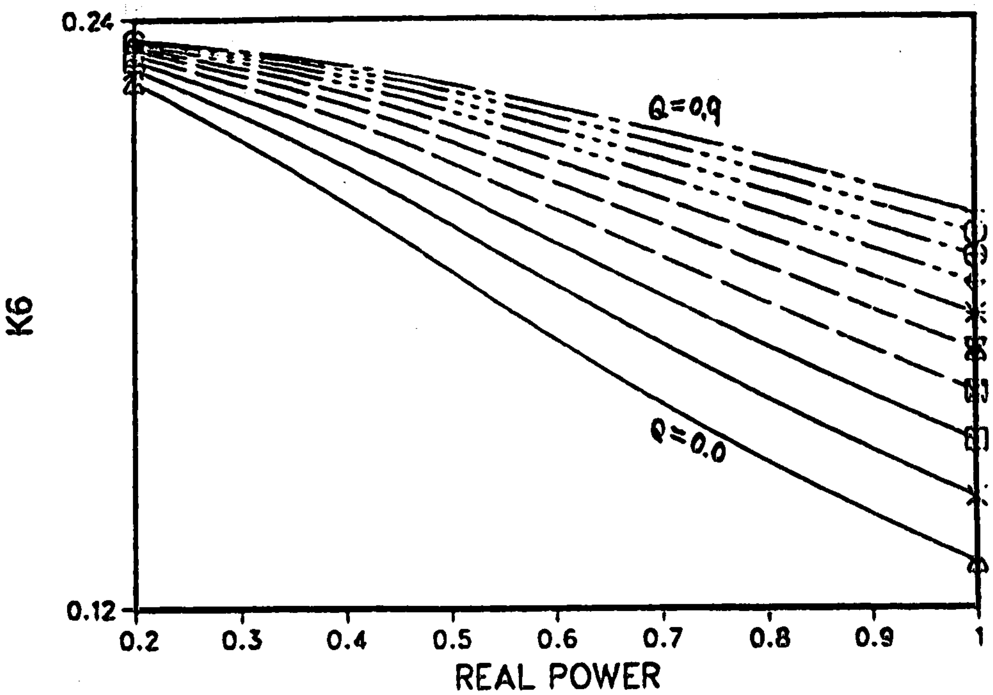

(e)

Though Figure 2.3.1 offers a good view of the SMIB system, it is still difficult to find the general coupling expression between the two loops. Reference [34] analyzes the coupling phenomenon and the transfer function matrix is derived there. It is a 4^{th}-order system with no pure time delay. This indicates that a 4^{th}-order multivariable self-tuning controller with one-step time delay for adaptive control will be adequate for a SMIB system.

CHAPTER 3

MULTIVARIABLE SELF-TUNING CONTROL OF SYNCHRONOUS GENERATOR SYSTEMS

3.1 Introduction

If an explicit system model with invariant properties is available, classical feedback control theory or modern control theory can then be used to design controllers for this system. For example, if the parameters K_1~K_6 were not changing with the load condition, the transfer function matrix in reference [34] could be used for design purposes. However, if the parameters of a system model are unknown, or changing slowly over a long period, the design techniques for the fixed model parameters are then no longer suitable. The self-tuning (or adaptive) control uses on-line identification techniques, and therefore, allows the optimal controller to be updated when the system is subjected to random disturbances over a wide operating range.

There are a variety of self-tuning control strategies and on-line identification techniques documented in the literature. This thesis does not intend to discuss all of the self-tuning control strategies systematically. Instead, it will concentrate on the generalized minimum variance self-tuning strategy and will apply it to the power system considered in this thesis. A general discussion on multivariable Generalized Minimum Variance (GMV) control for known parameter systems will be undertaken in Section 3.2.1, and an extension of this strategy to include self-tuning is presented in Section 3.2.2.

The principle of GMV control strategy was first introduced by Clarke and Gawthrop[56] in 1975 for SISO systems. Koivo[70] extended this technique to MIMO systems in 1980. A self-tuner of power system based on Clarke-Gawthrop-Koivo's multivariable GMV controller with variable forgetting factor is developed in Section 3.3. The convergence property of Clarke-Gawthrop self-tuning controller in SISO case was discused by Tsiligiannis *et al.* in 1986[80]. But the convergence analysis for MIMO case is not available in literature. A proof for the convergence and stability of the multivariable GMV STC is presented in Chapter 4.

3.2 Multivariable Self-Tuning Controller

3.2.1 Multivariable Generalized Minimum Variance Controller

Consider a MIMO system, with m inputs and m outputs, described by an AutoRegressive Moving-Average model with external inputs (ARMAX),

$$\mathbf{A}(q^{-1})\mathbf{y}(t) = \mathbf{B}(q^{-1})\mathbf{u}(t-k)+\mathbf{C}(q^{-1})\xi(t) \qquad (3.2.1)$$

where,

output vector $\mathbf{y}(\cdot)\in R^m$

input vector $\mathbf{u}(\cdot)\in R^m$

noise vector $\xi(\cdot)\in R^m$

and $\{\xi(\cdot)\}$ is a sequence of independent equally distributed random signals with zero mean value and covariance,

$$E\{\xi(\cdot)\xi(\cdot)^T\} = \mathrm{diag}\left(\sigma_1^2\cdots\sigma_m^2\right)$$

q^{-1} is the backward shift operator , i.e.,

$$q^{-1}\mathbf{y}(t) \triangleq \mathbf{y}(t-1)$$

and k is the time delay. Matrix polynomials $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are defined as,

$$\mathbf{A}(q^{-1}) = \mathbf{I}+\mathbf{A}_1q^{-1}+\cdots+\mathbf{A}_nq^{-n}, \quad \mathbf{A}_i \in R^{mxm}$$

$$\mathbf{B}(q^{-1}) = \mathbf{B}_0+\mathbf{B}_1q^{-1}+\cdots+\mathbf{B}_nq^{-n}, \quad \mathbf{B}_i \in R^{mxm}$$

$$\mathbf{C}(q^{-1}) = \mathbf{I}+\mathbf{C}_1q^{-1}+\cdots+\mathbf{C}_nq^{-n}, \quad \mathbf{C}_i \in R^{mxm}$$

where $\mathbf{B}_0$ is non-singular and $\det[\mathbf{C}(q^{-1})]$ has its zeros strictly outside of the unit circle on the q^{-1}-plane. It is assumed that coefficient matrices $\mathbf{A}_i$, $\mathbf{B}_i$ and $\mathbf{C}_i$ are known a priori.

The cost function to be considered is of the form,

$$J(t)=E\left\{ \| \mathbf{S}(q^{-1})\mathbf{y}(t+k)-\mathbf{R}(q^{-1})\mathbf{y}_d(t) \|^2+\| \mathbf{Q}'(q^{-1})\mathbf{u}(t) \|^2 \right\} \tag{3.2.2}$$

where $\mathbf{S}(q^{-1}) \in R^{mxm}$, $\mathbf{R}(q^{-1}) \in R^{mxm}$, and $\mathbf{Q}'(q^{-1}) \in R^{mxm}$ are polynomial matrices, and $\mathbf{S}(q^{-1})$ satisfies,

$$\mathbf{S}(q^{-1}) = s(q^{-1})\ \mathbf{I}$$

where,

$$s(q^{-1}) = 1+s_1q^{-1}+s_2q^{-2}+\cdots$$

$\mathbf{y}_d(t) \in R^m$ is the reference vector, and notation $\|\cdot\|$ means Euclidean norm as defined in the Nomenclature, and $\|\mathbf{x}\|^2 = \mathbf{x}^T\mathbf{x}$.

The k-step-ahead predictor of Eqn.(3.2.1) is derived in Appendix B and is given by Eqn.(B.8),

$$\mathbf{y}^*(t+k|t) = \tilde{\mathbf{C}}(q^{-1})^{-1}\left[\tilde{\mathbf{F}}(q^{-1})\mathbf{y}(t) + \tilde{\mathbf{E}}(q^{-1})\mathbf{B}(q^{-1})\mathbf{u}(t)\right] \qquad (3.2.3)$$

The polynomial matrices $\tilde{\mathbf{C}}(q^{-1})^{-1}$, $\tilde{\mathbf{F}}(q^{-1})$ and $\tilde{\mathbf{E}}(q^{-1})$ can be calculated by using the relationships of (B.1) through (B.4) in Appendix B. Thus $\mathbf{y}^*(t+k|t)$ can be calculated based on the data up to time t,

$$\{\mathbf{y}(t) \ \ \mathbf{y}(t-1) \ \cdots \ \mathbf{u}(t) \ \ \mathbf{u}(t-1) \ \cdots\}$$

There is a prediction error existing between the true value of $\mathbf{y}(t+k)$ and the predicted value of $\mathbf{y}^*(t+k|t)$. This error can be calculated by using Eqn.(B.7) in Appendix B,

$$\mathbf{e}(t+k) = \mathbf{y}(t+k) - \mathbf{y}^*(t+k|t)$$

$$= (\mathbf{I} + \mathbf{E}_1 q^{-1} + \ldots + \mathbf{E}_{k-1} q^{-k+1})\xi(t+k) \qquad (3.2.4)$$

or,

$$\mathbf{e}(t+k) = \xi(t+k) + \mathbf{E}_1\xi(t+k-1) + \ldots + \mathbf{E}_{k-1}\xi(t+1) \qquad (3.2.5)$$

It is obvious that the prediction error $\mathbf{e}(t+k)$ is uncorrelated with $\mathbf{y}^*(t+k|t)$.

To find the optimal control which minimizes the cost function of Eqn.(3.2.2), Eqn.(3.2.4) is substituted into Eqn.(3.2.2) yielding,

$$J(t)=E\left\{ \| \mathbf{S}(q^{-1}) \, [\mathbf{y}^*(t+k|t)+\mathbf{e}(t+k)]-\mathbf{R}(q^{-1})\mathbf{y}_d(t) \|^2 + \right.$$

$$\left. + \| \mathbf{Q}'(q^{-1})\mathbf{u}(t) \|^2 \right\} \qquad (3.2.6)$$

Since $\{\mathbf{S}(q^{-1})\mathbf{e}(t+k)\}$ is uncorrelated with $\{\mathbf{S}(q^{-1})\mathbf{y}^*(t+i|t)\}$ and $\{\mathbf{R}(q^{-1})\mathbf{y}_d(t)\}$ $(i=1,\cdots,k)$, $J(t)$ can be expressed as,

$$J(t) = \| \mathbf{S}(q^{-1})\mathbf{y}^*(t+k|t)-\mathbf{R}(q^{-1})\mathbf{y}_d(t) \|^2 +$$

$$+ \| \mathbf{Q}'(q^{-1})\mathbf{u}(t) \|^2 + E\left\{ \| \mathbf{S}(q^{-1})\mathbf{e}(t+k) \|^2 \right\}$$

Taking partial derivative of $J(t)$ with respect to $\mathbf{u}(t)$ and equating it to zero leads to the optimal control strategy,

$$\frac{\partial J(t)}{\partial \mathbf{u}(t)} = 2\left[\frac{\partial \mathbf{S}(q^{-1})\mathbf{y}^*(t+k|t)}{\partial \mathbf{u}(t)}\right]^T [\mathbf{S}(q^{-1})\mathbf{y}^*(t+k|t)-\mathbf{R}(q^{-1})\mathbf{y}_d(t)] +$$

$$+2(\mathbf{Q}_0')^T\mathbf{Q}'(q^{-1})\mathbf{u}(t)$$

$$= 0$$

By observing that $\mathbf{S}_0=\mathbf{I}$,

$$\mathbf{B}_0^T[\mathbf{S}(q^{-1})\mathbf{y}^*(t+k|t)-\mathbf{R}(q^{-1})\mathbf{y}_d(t)]+(\mathbf{Q}_0')^T\mathbf{Q}'(q^{-1})\mathbf{u}(t)=0 \quad (3.2.7)$$

The control $\mathbf{u}(t)$ can be calculated by using Eqn.(3.2.7). The control vector forms an admissible control law, i.e., the control at time t is a function of the observed outputs up to time t and the previous controls up to time t-1.

Clarke and Gawthrop introduced an "auxiliary" output into the controlled system[56], and Koivo extended this concept to MIMO case. This concept is helpful in developing the self-tuning algorithm and in showing that the optimal control in Eqn.(3.2.7) is equivalent to minimum variance control.

Define a matrix polynomial

$$\mathbf{Q}(q^{-1}) \triangleq (\mathbf{B}_0^T)^{-1}(\mathbf{Q}_0')^T\mathbf{Q}'(q^{-1}) \quad (3.2.8)$$

and an auxiliary output vector,

$$\mathbf{\Phi}(t+k) \triangleq \mathbf{S}(q^{-1})\mathbf{y}(t+k)-\mathbf{R}(q^{-1})\mathbf{y}_d(t)+\mathbf{Q}(q^{-1})\mathbf{u}(t) \quad (3.2.9)$$

Then the k-step-ahead predictor of the auxiliary output is,

$$\mathbf{\Phi}^*(t+k|t)=\mathbf{S}(q^{-1})\mathbf{y}^*(t+k|t)-\mathbf{R}(q^{-1})\mathbf{y}_d(t)+\mathbf{Q}(q^{-1})\mathbf{u}(t) \quad (3.2.10)$$

and the prediction error of $\varepsilon(t+k)$ is

$$\varepsilon(t+k) = \Phi(t+k) - \Phi^*(t+k|t)$$

$$= S(q^{-1})[y(t+k) - y^*(t+k|t)]$$

$$= S(q^{-1})e(t+k) \qquad (3.2.11)$$

Now define a cost function of the form,

$$J(t) = E\left\{ \|\Phi(t+k)\|^2 \right\} \qquad (3.2.12)$$

then it is obvious that this is a minimum variance control[53] problem. Substituting the relationship (3.2.11) into J(t) leads to,

$$J(t) = E\left\{ \|\Phi^*(t+k|t) + \varepsilon(t+k)\|^2 \right\}$$

Notice that $\Phi^*(t+k|t)$ is uncorrelated with the prediction error $\varepsilon(t+k)$. The above expression can be written as ,

$$J(t) = E\left\{ \|\Phi^*(t+k|t)\|^2 \right\} + E\left\{ \|\varepsilon(t+k)\|^2 \right\} \qquad (3.2.13)$$

Since $\varepsilon(t+k)$ is unaffected by $u(t)$, the minimum J(t) happens only when the following fact holds,

$$\mathbf{\Phi}^*(t+k|t)=0 \qquad (3.2.14)$$

Eqn.(3.2.14) is equivalent to Eqn.(3.2.7). Therefore it can be used to calculate the optimal control $\mathbf{u}(t)$.

3.2.2 Self-Tuning Multivariable Generalized Minimum Variance Controller

In Section 3.2.1 the multivariable GMV controller is derived for the systems with known parameters. If the polynomial matrices $\mathbf{A}(q^{-1})$, $\mathbf{B}(q^{-1})$ and $\mathbf{C}(q^{-1})$ are unknown, a self-tuning algorithm has to be developed. This subsection will be used to describe this identification algorithm.

3.2.2.1 Self-Tuning Algorithm

Substituting the predictor given by Eqn.(B.8) in Appendix B into Eqn.(3.2.10) implies that,

$$\mathbf{\Phi}^*(t+k|t)=\mathbf{S}(q^{-1})\tilde{\mathbf{C}}(q^{-1})^{-1}\left[\tilde{\mathbf{F}}(q^{-1})\mathbf{y}(t)+\tilde{\mathbf{E}}(q^{-1})\mathbf{B}(q^{-1})\mathbf{u}(t)\right]-$$

$$-\mathbf{R}(q^{-1})\mathbf{y}_d(t)+\mathbf{Q}(q^{-1})\mathbf{u}(t) \qquad (3.2.15)$$

Since the matrix polynomial $\mathbf{S}(q^{-1})$ is defined as (see Eqn.(3.2.2)),

$$\mathbf{S}(q^{-1}) \triangleq s(q^{-1})\mathbf{I}$$

$\mathbf{S}(q^{-1})$ and $\tilde{\mathbf{C}}(q^{-1})$ are commutative, therefore, Eqn.(3.2.15) can be written as,

$$\tilde{\mathbf{C}}(q^{-1})\boldsymbol{\Phi}^*(t+k|t)=\boldsymbol{\alpha}(q^{-1})\mathbf{y}(t)+\boldsymbol{\beta}(q^{-1})\mathbf{u}(t)+\mathbf{H}(q^{-1})\mathbf{y}_d(t) \quad (3.2.16)$$

where,

$$\boldsymbol{\alpha}(q^{-1}) \triangleq s(q^{-1})\tilde{\mathbf{F}}(q^{-1})$$

$$\boldsymbol{\beta}(q^{-1}) \triangleq s(q^{-1})\tilde{\mathbf{E}}(q^{-1})\mathbf{B}(q^{-1})+\tilde{\mathbf{C}}(q^{-1})\mathbf{Q}(q^{-1})$$

$$\mathbf{H}(q^{-1}) \triangleq -\tilde{\mathbf{C}}(q^{-1})\mathbf{R}(q^{-1})$$

These polynomial matrices will be estimated on-line, so the notation $\hat{}$ will be used to express the estimated parameters. The data vector, or regressor, $\mathbf{x}(t)$ is defined as,

$$\mathbf{x}(t)^T \triangleq [\mathbf{y}(t)^T\ \mathbf{y}(t-1)^T\ \cdots\ \mathbf{u}(t)^T\ \mathbf{u}(t-1)^T\ \cdots$$

$$\mathbf{y}_d(t)^T\ \mathbf{y}_d(t-1)^T\ \cdots] \quad (3.2.17a)$$

and parameter matrix $\hat{\Theta}(t)$ as,

$$\hat{\Theta}(t) = \left[\hat{\theta}_1(t)\ \cdots\ \hat{\theta}_m(t)\right] \quad (3.2.17b)$$

or,

$$\hat{\theta}(t) = \left[\hat{\alpha}_0(t)\ \hat{\alpha}_1(t)\ \cdots\ \hat{\beta}_0(t)\ \hat{\beta}_1(t)\ \cdots\ \hat{h}_0(t)\ \hat{h}_1(t)\ \cdots\right]^T$$

where the column vectors $\hat{\theta}_i(t)$, $i=1,2,\cdots,m$, are of the form,

$$\begin{aligned}\hat{\theta}_i(t) = \Big[\hat{\alpha}^0_{i1}(t)\ \cdots\ \hat{\alpha}^0_{im}(t) \,\vdots\, \hat{\alpha}^1_{i1}(t)\ \cdots\ \hat{\alpha}^1_{im}(t) \,\vdots\, \cdots \\ \vdots\, \hat{\beta}^0_{i1}(t)\ \cdots\ \hat{\beta}^0_{im}(t) \,\vdots\, \hat{\beta}^1_{i1}(t)\ \cdots\ \hat{\beta}^1_{im}(t) \,\vdots\, \cdots \\ \vdots\, \hat{h}^0_{i1}(t)\ \cdots\ \hat{h}^0_{im}(t) \,\vdots\, \hat{h}^1_{i1}(t)\ \cdots\ \hat{h}^1_{im}(t) \,\vdots\, \cdots\Big]^T\end{aligned}$$

The relationship derived from Eqn.(3.2.11) can then be expressed component-wise as,

$$\Phi_i(t+k) = \Phi^*_i(t+k|t) + \varepsilon_i(t+k) \qquad (3.2.18)$$

or,

$$\Phi_i(t+k) = x(t)^T\hat{\theta}_i(t) + \varepsilon_i(t+k) + \left[1-\tilde{C}_{ii}(q^{-1})\right]\Phi^*_i(t+k|t) -$$

$$-\sum_{i\neq j}\tilde{C}_{ij}(q^{-1})\Phi^*_i(t+k|t) \qquad (3.2.19)$$

and the control law is determined by setting

$$\mathbf{x}(t)^T\hat{\theta}_i(t)=0 \qquad (3.2.20)$$

However, it should be noticed that for the case of $\mathbf{C}(q^{-1})\neq\mathbf{I}$, Eqn.(3.2.19) indicates that $\Phi^*(t+k|t)$ is correlated with $\mathbf{x}(t)$. Therefore, if the least-squares estimate is used, the estimation will be biased. But since the control law sets $\Phi^*(t+k|t)$ equal to zero, the last two terms in Eqn.(3.2.19) will vanish. If $\mathbf{C}(q^{-1})=\mathbf{I}$, then $\tilde{\mathbf{C}}(q^{-1})=\mathbf{I}$, Eqn.(3.2.19) becomes

$$\Phi_i(t+k)=\mathbf{x}(t)^T\hat{\theta}_i(t)+\varepsilon_i(t+k) \qquad (3.2.21)$$

It is obvious that the components of $\mathbf{x}(t)$ are uncorrelated with $\varepsilon_i(t+k)$. Therefore, the least-squares estimation is unbiased.

The control law calculated from Eqn.(3.2.20) is,

$$\mathbf{u}(t)=-\hat{\beta}_0^{-1}\left[\sum_{i\geq 0}\hat{\alpha}_i\mathbf{y}(t-i)+\sum_{i\geq 1}\hat{\beta}_i\mathbf{u}(t-i)+\sum_{i\geq 0}\hat{\mathbf{h}}_i\mathbf{y}_d(t-i)\right] \qquad (3.2.22)$$

3.2.2.2 Parameter Estimation

To identify the parameter matrix $\hat{\Theta}(t)$ in Eqn.(3.2.17b), an estimator has to be used. It is well known that the recursive parameter identification (or estimator) plays a crucial role in adaptive control. In fact, the convergence property of an on-line estimator is the key in the proof of stability of a self-tuning algorithm. Therefore, there are a variety of identification schemes for adaptive control developed in the literature. Of the many identification techniques, the recursive least-squares (RLS) is perhaps the best known and most widely used in self-tuning control.

The basic least-squares estimation is derived by minimizing the following cost function with respect to θ_i,

$$J(\theta_i)=\sum_{t=1}^{N}\left[\phi_i(t)-\mathbf{x}(t-k)^T\theta_i\right]^2 \quad i=1,2,\cdots,m \qquad (3.2.23)$$

and the resultant estimation law for a MIMO system in the recursive form is,

$$\hat{\theta}_i(t+1)=\hat{\theta}_i(t)+\mathbf{K}(t)\left[\phi_i(t)-\mathbf{x}(t-k)^T\hat{\theta}_i(t)\right], i=1,2,\cdots,m \qquad (3.2.24)$$

This iteration can be explained by the following expression,

$$\begin{bmatrix}\text{new parameter}\\ \text{estimates}\end{bmatrix}=\begin{bmatrix}\text{previous parameter}\\ \text{estimates}\end{bmatrix}+\begin{bmatrix}\text{algorithm}\\ \text{gain}\end{bmatrix}\begin{bmatrix}\text{prediction}\\ \text{error}\end{bmatrix}$$

It is clear that larger "algorithm gain" will give larger modification on the new estimated parameters. The standard RLS algorithm gain $\mathbf{K}(t)$ is given by,

$$\mathbf{K}(t)=\mathbf{P}(t)\mathbf{x}(t-k)\left[1+\mathbf{x}(t-k)^T\mathbf{P}(t)\mathbf{x}(t-k)\right]^{-1} \qquad (3.2.25)$$

where the covariance matrix $\mathbf{P}(t)$ satisfies,

$$\mathbf{P}(t+1)=\left[\mathbf{P}(t)-\frac{\mathbf{P}(t)\mathbf{x}(t-k)\mathbf{x}(t-k)^T\mathbf{P}(t)}{1+\mathbf{x}(t-k)^T\mathbf{P}(t)\mathbf{x}(t-k)}\right]/\lambda \qquad (3.2.26)$$

with $\lambda=1$.

It can be observed that if the initial value of $\mathbf{P}(0)$ is considered to be the same for all parameter vectors, the corresponding algorithm gain $\mathbf{K}(t)$ will also be the same for all estimators. This gives a significant saving in computations[54].

This standard RLS algorithm has good convergence properties. However, the basic difficulty with this algorithm is that the covariance matrix $\mathbf{P}(t+1)$ in Eqn.(3.2.26) will gradually decay to a small value and therefore the algorithm does not retain its alertness or adaptivity[77]. This is easily seen from the second term on the right of Eqn.(3.2.26). This term is always positive or zero, so $\mathbf{P}(t)$

gets smaller and smaller as time progresses. Smaller $\mathbf{P}(t)$ means smaller algorithm gain $\mathbf{K}(t)$ in Eqn.(3.2.25).

It is logical to consider modifying the covariance matrix $\mathbf{P}(t)$ by setting $\lambda<1$. This algorithm is usually called RLS with Exponential Data Weighting in the literature because it is the resultant recursive form of the optimal estimation which minimized the cost function below,

$$J(\theta_i)=\sum_{t=0}^{N}\lambda^{N-t}\left[\phi_i(t)-\mathbf{x}(t-k)^T\theta_i(t)\right]^2, \quad i=1,2,\cdots,m \qquad (3.2.27)$$

This algorithm works well when the controlled system is excited properly. Otherwise it will lead to covariance matrix $\mathbf{P}(t)$ wind-up.

Fortescue *et al.*[64] (1981) modified the fixed forgetting factor RLS algorithm, changing constant λ to a time varying quantity $\lambda(t)$. This algorithm is called RLS with variable forgetting factor, and it has the following property: $\lambda(t)$ will be set to a small value when the prediction error $\phi_i(t)-\mathbf{x}(t-k)^T\hat{\theta}_i(t)$ is large, and go to 1 when the prediction error is zero. The introduction of $\lambda(t)$ will be able to maintain the alertness of the algorithm, and avoid the wind-up phenomenon for small data vector. Yet an upper bound for $\mathbf{P}(t)$ has to be imposed in case a small $\lambda(t)$ causes exponential increment. For RLS with a variable forgetting factor

algorithm Eqn. (3.2.26) is replaced by the following relationships,

$$\lambda(t) = 1 - \frac{\sum_{i=1}^{m}\left[\phi_i(t) - \mathbf{x}(t-k)^T\hat{\theta}_i(t)\right]^2}{\left[1+\mathbf{x}(t-k)^T\mathbf{P}(t-1)\mathbf{x}(t-k)\right]\sigma} \quad (3.2.28)$$

$$\mathbf{W}(t) = \left[\mathbf{I}-\mathbf{K}(t)\mathbf{x}(t-k)^T\right]\mathbf{P}(t-1) \quad (3.2.29)$$

$$\mathbf{P}(t) = \begin{cases} \frac{\mathbf{W}(t)}{\lambda(t)} & \text{if } \frac{\text{trace}[\mathbf{W}(t)]}{\lambda(t)} \le C \\ \mathbf{W}(t) & \text{otherwise} \end{cases} \quad (3.2.30)$$

where σ is a constant to be chosen to ensure $\lambda(t)>0$ for all t, and C is a constant of the upper bound of $\mathbf{P}(t)$.

Eqn.(3.2.28) has been modified to the MIMO system case. $\lambda(t)$ will decrease when any one of the prediction errors $\varepsilon_i(t)=\phi_i(t)-\mathbf{x}(t-k)^T\hat{\theta}_i(t)$, $i=1,2,\cdots,m$, increases.

3.3 Multivariable Self-Tuning Controller of Synchronous Generator Systems

The basic theory of multivariable GMV self-tuning controller is discussed in the previous section. This section will be used to discuss how to form a self-tuner for synchronous generator systems.

3.3.1 System Model

Since the ARMAX model is based on the input-output characteristics of physical plants, it is widely used in engineering. It is a kind of stochastic linear vector difference equation. The multivariable self-tuning controller discussed in Section 3.2 is based on such a model.

It has been mentioned in Section 2.3.2 that a thermal turbine synchronous generator connected to an infinite bus can be described by a 4^{th}-order transfer function matrix with no pure time delay. The fact suggests that a 4^{th}-order ARMAX model with two-input/two-output will be adequate for such a SMIB system, i.e.,

$$\mathbf{A}(q^{-1})\mathbf{y}(t)=\mathbf{B}(q^{-1})\mathbf{u}(t-1)+\xi(t) \qquad (3.3.1)$$

where,

$$\mathbf{A}(q^{-1})=\mathbf{I}+\mathbf{A}_1q^{-1}+\mathbf{A}_2q^{-2}+\mathbf{A}_3q^{-3}+\mathbf{A}_4q^{-4}, \quad \mathbf{A}_i\in R^{2\times 2}$$

$$\mathbf{B}(q^{-1})=\mathbf{B}_0+\mathbf{B}_1q^{-1}+\mathbf{B}_2q^{-2}+\mathbf{B}_3q^{-3}, \quad \mathbf{B}_i\in R^{2\times 2}$$

$$\mathbf{B}_0\neq 0$$

$$\mathbf{y}(t)=\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix},\quad \mathbf{u}(t)=\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix},\quad \boldsymbol{\xi}(t)=\begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix}$$

$\mathbf{y}(t)$, $\mathbf{u}(t)$ and $\boldsymbol{\xi}(t)$ are the samples of corresponding variable at instant t, and

$y_1(t) = \omega(t)$, the generator shaft speed
$y_2(t) = v_t(t)$, the generator terminal voltage
$u_1(t) = u_g(t)$, the turbine valve opening
$u_2(t) = u_f(t)$, the field exciter voltage
$\{\xi(t)\}$ is a white noise sequence

It should be pointed out that an under-parameterized model could be used in practice, because it saves significant computing time for MIMO systems.

3.3.2 Cost Functions and Choice of $\mathbf{Q}(q^{-1})$

The GMV self-tuning algorithm is designed to minimize the cost function given in Eqn.(3.2.2). To apply it to a power system, one first considers the choice of matrices $\mathbf{R}(q^{-1})$, $\mathbf{S}(q^{-1})$ and $\mathbf{Q}'(q^{-1})$. For simplicity and fast response of the system, $\mathbf{R}(q^{-1})$ and $\mathbf{S}(q^{-1})$ are usually chosen as unit matrices,

$$\mathbf{R}(q^{-1})=\mathbf{S}(q^{-1})=\mathbf{I}$$

A different $\mathbf{Q}'(q^{-1})$ will put a different penalty on control actions. This is important for real industrial plants because nearly all real systems have limitations on control signals. For example, the valve opening and field voltage of a generator are limited to certain ranges. Excessive control actions should be avoided for such systems. To achieve this, a careful choice of $\mathbf{Q}'(q^{-1})$ is helpful. However, it is more convenient to choose $\mathbf{Q}(q^{-1})$ given in Eqn.(3.2.9) directly in practice, instead of $\mathbf{Q}'(q^{-1})$. The relationship between the two matrices is given by Eqn.(3.2.8). If considering the fact that the coefficient matrix $\mathbf{B}_0$ is usually dominated by its diagonal elements under a careful ordering, i.e.,

$$\left| \mathbf{B}^0_{ii} \right| > \sum_{\substack{i=1 \\ i\neq j}}^{m} \left| \mathbf{B}^0_{ij} \right|$$

one can derive the following relationship from Eqn(3.2.8),

$$\mathbf{Q}'(q^{-1}) = \mathrm{diag}(d_1 \ \cdots \ d_m)\mathbf{Q}(q^{-1}) \qquad (3.3.3)$$

where d_i, $i=1,\cdots,m$ are constants. Thus, if,

$$\mathbf{Q}(q^{-1})=\begin{bmatrix} V_1 & 0 \\ 0 & V_2 \end{bmatrix} \qquad (3.3.4)$$

or

$$\mathbf{Q}(q^{-1})=(1-q^{-1})\begin{bmatrix} v_1 & 0 \\ 0 & v_2 \end{bmatrix} \qquad (3.3.5)$$

then the corresponding cost functions for a power system will be,

$$J(t)=E\left\{ \| \mathbf{y}(t+k)-\mathbf{y}_d(t) \|^2 + \left\| \begin{bmatrix} v_1' & 0 \\ 0 & v_2' \end{bmatrix} \mathbf{u}(t) \right\|^2 \right\} \qquad (3.3.6)$$

or,

$$J(t)=E\left\{ \| \mathbf{y}(t+k)-\mathbf{y}_d(t) \|^2 + \left\| \begin{bmatrix} v_1' & 0 \\ 0 & v_2' \end{bmatrix} \Delta\mathbf{u}(t) \right\|^2 \right\} \qquad (3.3.7)$$

where,

$y_{d_1}(t)=\omega_r(t)$ the shaft speed reference signal

$y_{d_2}(t)=v_r(t)$ the terminal voltage reference signal

The difference between these two choices of $\mathbf{Q}(q^{-1})$ is that the one in Eqn.(3.3.5) puts integration action into the system, so that the control action will eliminate the steady state error in output vector $\mathbf{y}(t)$ subject to step changes in $\mathbf{y}_d(t)$. To see this, one can employ the closed-loop characteristic equation. It has been shown in Section 3.2

that the control law sets $\Phi^*(t+k|t)$ to zero in Eqn.(3.2.14). For a power system it means that,

$$\mathbf{y}^*(t+1|t)-\mathbf{y}_d(t)+\mathbf{Q}(q^{-1})\mathbf{u}(t)=0$$

or,

$$\mathbf{u}(t)=\mathbf{Q}(q^{-1})^{-1}[\mathbf{y}_d(t)-\mathbf{y}^*(t+1|t)] \qquad (3.3.8)$$

For the system in Eqn.(3.3.1), the optimal prediction of $\mathbf{y}^*(t+1|t)$ is,

$$\mathbf{y}^*(t+1|t)=\mathbf{y}(t+1)-\xi(t+1) \qquad (3.3.9)$$

Thus Eqn.(3.3.8) can be written as,

$$\mathbf{u}(t)=\mathbf{Q}(q^{-1})^{-1}[\mathbf{y}_d(t)-\mathbf{y}(t+1)+\xi(t+1)] \qquad (3.3.10)$$

Substituting the above expression into Eqn.(3.3.1) leads to the closed-loop behavior,

$$\mathbf{y}(t+1)=[\mathbf{A}(q^{-1})+\mathbf{B}(q^{-1})\mathbf{Q}(q^{-1})^{-1}]^{-1}\mathbf{B}(q^{-1})\mathbf{Q}(q^{-1})^{-1}\mathbf{y}_d(t)+$$

$$+[\mathbf{A}(q^{-1})+\mathbf{B}(q^{-1})\mathbf{Q}(q^{-1})^{-1}]^{-1}[\mathbf{I}+\mathbf{B}(q^{-1})\mathbf{Q}(q^{-1})^{-1}]\xi(t+1)$$

$$(3.3.11)$$

The first term on the right-hand side describes the response behavior to input signals. If $\mathbf{Q}(q^{-1})$ takes the form of Eqn. (3.3.5), there will be an integrator $1/(1-q^{-1})$ between $\mathbf{y}(t+1)$ and $\mathbf{y}_d(t)$. However, no such a segment exists between $\mathbf{y}(t+1)$ and $\xi(t+1)$. It is also interesting to note that the closed-loop characteristic equation is described by,

$$\det\left[\mathbf{A}(q^{-1})+\mathbf{B}(q^{-1})\mathbf{Q}(q^{-1})^{-1}\right]=0$$

or,

$$\det\left[\mathbf{A}(q^{-1})\mathbf{Q}(q^{-1})+\mathbf{B}(q^{-1})\right]=0 \qquad (3.3.12)$$

It is clear that the choice of $\mathbf{Q}(q^{-1})$ will influence the system stability. Assuming that $\mathbf{Q}(q^{-1})$ takes the form of Eqn. (3.3.4), and $\mathbf{B}(q^{-1})$ is minimum phase, then small or zero v_1 and v_2 will ensure the stability of the system. However, if $\mathbf{B}(q^{-1})$ is non-minimum phase, i.e., $\det[\mathbf{B}(q^{-1})]$ has some zeros inside the unit disk on q^{-1}-plane, v_1 and v_2 should take values large enough to ensure that there are no eigenvalues within the unit circle. Although it is assumed that $\mathbf{A}(q^{-1})$ and $\mathbf{B}(q^{-1})$ are unknown for a self-tuning controlled system, the above analysis gives the users directions on how to choose $\mathbf{Q}(q^{-1})$ in practice.

For the specific application to power systems both the forms of $Q(q^{-1})$ in Eqns.(3.3.4) and (3.3.5) are not adequate. Instead, a combined form should be taken, i.e.,

$$Q(q^{-1}) = \begin{bmatrix} v_1 & 0 \\ 0 & v_2(1-q^{-1}) \end{bmatrix} \tag{3.3.13}$$

By using this form of $Q(q^{-1})$, an integrator is installed into the excitation loop. But there is no integrator installed into the shaft speed control loop through the self-tuner in order to avoid saturated $u_g(t)$, the opening of the turbine valve, when subjected to a step change in reference signal $\omega_r(t)$. This characteristic is caused by the fact that the system frequency is fixed in a SMIB system, and it cannot be changed by regulating the single generator. Therefore, the function of the regulator in the frequency loop is to control electric power, as mentioned in Section 1.1.

3.3.3 Self-Tuning Algorithm

Based on the multivariable GMV self-tuning theory discussed in Section 3.2, the self-tuning algorithm for a power system will be derived in this subsection.

Using the notations in Appendix B, one can derive the following relationship,

$$\mathbf{E}(q^{-1}) = \tilde{\mathbf{E}}(q^{-1}) = \tilde{\mathbf{C}}(q^{-1}) = \mathbf{I}$$

$$\mathbf{F}(q^{-1}) = \tilde{\mathbf{F}}(q^{-1}) = -q\left[\mathbf{A}(q^{-1}) - \mathbf{I}\right]$$

Thus the degree of the polynomial matrices $\alpha(q^{-1})$ and $\beta(q^{-1})$ in Eqn.(3.2.16) satisfy the following,

$$\text{degree}\{\alpha(q^{-1})\} = \text{degree}\{\mathbf{A}(q^{-1})\} - 1 \qquad (3.3.14)$$

$$\text{degree}\{\beta(q^{-1})\} = \text{degree}\{\mathbf{B}(q^{-1})\} \qquad (3.3.15)$$

$$\text{degree}\{\mathbf{H}(q^{-1})\} = 0 \qquad (3.3.16)$$

i.e., the parameter matrix $\hat{\Theta}(t)$ can be expressed as,

$$\hat{\Theta}(t) = \left[\hat{\theta}_1(t) \quad \hat{\theta}_2(t)\right]$$

$$= \left[\hat{\alpha}_0 \;\; \hat{\alpha}_1 \;\; \hat{\alpha}_2 \;\; \hat{\alpha}_3 \;\; \hat{\beta}_0 \;\; \hat{\beta}_1 \;\; \hat{\beta}_2 \;\; \hat{\beta}_3 \;\; \hat{h}_0\right]^T \qquad (3.3.17)$$

where the column vectors $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$ can be further written in detail,

$$\hat{\theta}_1(t) = \Big[\hat{\alpha}^0_{11} \; \hat{\alpha}^0_{12} \;\vdots\; \hat{\alpha}^1_{11} \; \hat{\alpha}^1_{12} \;\vdots\; \cdots \;\vdots\; \hat{\alpha}^3_{11} \; \hat{\alpha}^3_{12} \;\vdots\; \hat{\beta}^0_{11} \; \hat{\beta}^0_{12} \;\vdots$$

$$\hat{\beta}^1_{11} \; \hat{\beta}^1_{12} \;\vdots\; \cdots \;\vdots\; \hat{\beta}^3_{11} \; \hat{\beta}^3_{12} \;\vdots\; \hat{h}^0_{11} \; \hat{h}^0_{12}\Big]^T \qquad (3.3.18)$$

$$\hat{\theta}_2(t)=\Big[\hat{\alpha}^0_{21}\ \hat{\alpha}^0_{22}\ \vdots\ \hat{\alpha}^1_{21}\ \hat{\alpha}^1_{22}\ \vdots\ \cdots\ \vdots\ \hat{\alpha}^3_{21}\ \hat{\alpha}^3_{22}\ \vdots\ \hat{\beta}^0_{21}\ \hat{\beta}^0_{22}\ \vdots$$

$$\hat{\beta}^1_{21}\ \hat{\beta}^1_{22}\ \vdots\ \cdots\ \vdots\ \hat{\beta}^3_{21}\ \hat{\beta}^3_{22}\ \vdots\ \hat{h}^0_{21}\ \hat{h}^0_{22}\Big]^T \qquad (3.3.19)$$

Consequently, the data vector should be defined as,

$$x(t)^T=\Big[y_1(t)\ \ y_2(t)\ \vdots\ y_1(t-1)\ \ y_2(t-1)\ \vdots\ \cdots\ \vdots$$

$$y_1(t-3)\ \ y_2(t-3)\ \vdots\ u_1(t)\ \ u_2(t)\ \vdots\ u_1(t-1)\ \ u_2(t-1)\ \vdots\ \cdots\ \vdots$$

$$u_1(t-3)\ \ u_2(t-3)\ \vdots\ y_{d_1}(t)\ \ y_{d_2}(t)\Big] \qquad (3.3.20)$$

where $y_1(t)$, $y_2(t)$, $u_1(t)$ and $u_2(t)$ are defined in Section 3.3.1; $y_{d_1}(t)$ and $y_{d_2}(t)$ in Section 3.3.2. The self-tuning algorithm using the choice of initial conditions $P(0)$, $\hat{\theta}_1(0)$ and $\hat{\theta}_2(0)$ can then be summarized as follows at sampling instant t:

(1) Read new outputs $y_1(t)=\omega(t)$ and $y_2(t)=v_t(t)$, setpoints $y_{d_1}(t)$ and $y_{d_2}(t)$;

(2) Compute $\Phi(t)$,

$$\mathbf{\Phi}(t)=\mathbf{y}(t)-\mathbf{y}_d(t)+\mathbf{Q}(q^{-1})\mathbf{u}(t-1) \qquad (3.3.21)$$

(3) Shift the data to right in vector $\mathbf{x}(t)$ of Eqn.(3.3.20) and thus put $\mathbf{y}(t)$ and $\mathbf{y}_d(t)$ into the appropriate locations;

(4) Update parameter vectors $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$ by using RLS with variable forgetting factor given in Eqns.(3.2.24), (3.2.25), (3.2.28). (3.2.29) and (3.2.30) with k=1;

(5) Calculate new control vector $\mathbf{u}(t)$ by,

$$\mathbf{u}(t)=-\hat{\beta}_0^{-1}\left[\sum_{i=0}^{3}\hat{\alpha}_i\mathbf{y}(t-i)+\sum_{i=1}^{3}\hat{\beta}_i\mathbf{u}(t-i)+\hat{\mathbf{h}}_0\mathbf{y}_d(t)\right] \qquad (3.3.22)$$

(6) Renew data vector $\mathbf{x}(t)$ by putting $\mathbf{u}(t)$ into Eqn.(3.3.20);

(7) Set t=t+1 and go back to step (1).

If $\mathbf{Q}(q^{-1})$ takes the form described in Eqn.(3.3.4) in step (2), the auxiliary output $\Phi(t)$ will be,

$$\Phi_1(t)=y_1(t)-y_{d_1}(t)+v_1u_1(t-1) \qquad (3.3.23a)$$

and,

$$\Phi_2(t)=y_2(t)-y_{d_2}(t)+v_2u_2(t-1) \qquad (3.3.23b)$$

If $Q(q^{-1})$ uses the form described in Eqn.(3.3.13), $\Phi(t)$ will be,

$$\Phi_1(t)=y_1(t)-y_{d_1}(t)+v_1u_1(t-1) \qquad (3.3.24a)$$

$$\Phi_2(t)=y_2(t)-y_{d_2}(t)+v_2\left[u_2(t-1)-u_2(t-2)\right] \qquad (3.3.24b)$$

3.3.4 Constants in the Self-Tuning Algorithm

There are four constants appearing in the self-tuning algorithm discussed in the previous subsection. It is helpful to have a brief discussion of them.

The constant σ appearing in Eqn.(3.2.28) is used to ensure $\lambda(t)>0$ for all t. If $\lambda(t)<0$ for some t, the covariance matrix $P(t)$ would not be positive definite anymore, and consequently, the algorithm will become divergent. Thus, a larger σ will ensure the convergence of the algorithm. However, a large σ will also result in a large $\lambda(t)$ which means less alertness in the identification phase. Therefore, the choice of σ is a compromise between the two constraints.

The constant C in Eqn.(3.2.30) is a switch for controlling the upperbound of the trace of $P(t)$ matrix. To keep the alertness of the algorithm, C can be chosen very large (say 10^5).

The constants V_1 and V_2 in Eqn.(3.3.13) will affect the penalty on control actions and the closed-loop poles of the controlled system. Therefore, much attention should be paid to the choice of V_1 and V_2 for a specific power system. If the controlled system is non-minimum phase, V_1 and V_2 should take large values; otherwise, small V_1 and V_2 will keep the system stable.

3.3.5 Initial Conditions

To start the algorithm, initial values of $\mathbf{P}(0)$ and $\hat{\Theta}(0)$ are needed. Because of the good convergence property of the RLS algorithm, the initial choice of $\hat{\Theta}(0)$ is not crucial. Therefore, there is much freedom in choosing $\mathbf{P}(0)$ and $\hat{\Theta}(0)$ as long as one makes sure that $\hat{\beta}_0(0)$ is non-singular. As a common selection, the following ranges are proposed,

$$\mathbf{P}(0)=(10\sim100)\mathbf{I}$$

where $\mathbf{I}$ is a unit matrix of dimension 18×18, and

$$\hat{\beta}_0(0)=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\hat{h}_0(0) = \begin{bmatrix} -1.5 & 0 \\ 0 & -1.5 \end{bmatrix}$$

$$\hat{\alpha}_i(0) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad i=0,1,2,3$$

$$\hat{\beta}_i(0) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad i=1,2,3$$

CHAPTER 4

CONVERGENCE ANALYSIS OF THE MULTIVARIABLE SELF-TUNING CONTROLLER FOR POWER SYSTEMS

As mentioned previously the proof for convergence of the stochastic multivariable self-tuning algorithm discussed in the previous chapter is still an open problem. Instead of trying to give a proof on the convergence of the GMV self-tuning controller for stochastic MIMO system with arbitrary time delay, this thesis will address the convergence analysis of a deterministic MIMO system. It will also be proven that a stochastic MIMO system with unit time delay is equivalent to a deterministic system, and lastly, the stability proof of the self-tuning controller for power systems will be derived.

4.1 Convergence Analysis of Deterministic Multivariable Generalized Minimum Variance Self-Tuning Controller with Variable Forgetting Factor

It was mentioned in the previous chapter that the proof of the generalized minimum variance self-tuning controller for deterministic SISO system was not given until 1986 by Tsiligiannis and Svoronos[80]. In this section the author will prove the convergence property of the deterministic multivariable GMV self-tuning controller with variable forgetting factor for the MIMO systems. This algorithm is the deterministic version of the one discussed in Section 3.3.2.

The system to be controlled is described by the linear vector difference equation,

$$\mathbf{A}(q^{-1})\mathbf{y}(t)=\mathbf{B}(q^{-1})\mathbf{u}(t-k)+\mathbf{d} \qquad (4.1.1)$$

where,

$\mathbf{y}(t)\in R^m$

$\mathbf{u}(t)\in R^m$

$\mathbf{d}\in R^m$, a constant vector

$\mathbf{A}(q^{-1})=\mathbf{I}+\mathbf{A}_1 q^{-1}+\cdots+\mathbf{A}_n q^{-n}$

$\mathbf{B}(q^{-1})=\mathbf{B}_0+\mathbf{B}_1 q^{-1}+\cdots+\mathbf{B}_{n-1} q^{-n+1}, \qquad |\mathbf{B}_0|\neq 0$

k --- time delay, $k\geq 1$

n --- order of the system

The coefficients matrices $\mathbf{A}_i$ and $\mathbf{B}_i$ are unknown and k and n are known. A quadratic cost function takes the form of,

$$J(t)=\frac{1}{2}\|\mathbf{S}(q^{-1})[\mathbf{y}(t+k)-\mathbf{y}_d]\|^2+\frac{1}{2}\|\mathbf{Q}'(q^{-1})[\mathbf{u}(t)-\mathbf{u}_d]\|^2 \qquad (4.1.2)$$

where $\mathbf{S}(q^{-1})$ is a diagonal polynomial matrix with,

$$\mathbf{S}(0)=\mathbf{S}_0=\mathbf{I}$$

and $\mathbf{Q}'(q^{-1})$ is a polynomial matrix, $\mathbf{y}_d \in R^m$ is a set point vector, and $\mathbf{u}_d \in R^m$ is a control vector corresponding to $\mathbf{y}_d$. If $\mathbf{u}_d$ is unknown in some applications , one can choose $\mathbf{Q}'(1)=\mathbf{0}$ to delete it. Comparing Eqn.(4.1.2) with Eqn.(3.2.2) one can also notice that it takes $\mathbf{R}(q^{-1})=\mathbf{S}(q^{-1})$. This choice is realistic in many applications.

The optimal control which minimizes Eqn.(4.1.2) sets,

$$\mathbf{\Phi}'(t+k)=\mathbf{S}(1)\mathbf{y}_d \tag{4.1.3}$$

where,

$$\mathbf{\Phi}'(t+k)=\mathbf{S}(q^{-1})\mathbf{y}(t+k)+\mathbf{Q}(q^{-1})\left[\mathbf{u}(t)-\mathbf{u}_d\right] \tag{4.1.4}$$

$$\mathbf{Q}(q^{-1})\triangleq(\mathbf{B}_0^T)^{-1}(\mathbf{Q}_0')^T\mathbf{Q}'(q^{-1}) \tag{4.1.5}$$

$$\mathbf{y}(t+k)=\alpha(q^{-1})\mathbf{y}(t)+\beta(q^{-1})\mathbf{u}(t)+\delta \tag{4.1.6}$$

$$\alpha(q^{-1})=\mathbf{F}(q^{-1})=\alpha_0+\alpha_1 q^{-1}+\cdots+\alpha_{n-1}q^{-n+1} \tag{4.1.7}$$

$$\beta(q^{-1})=\mathbf{E}(q^{-1})\mathbf{B}(q^{-1})=\beta_0+\beta_1 q^{-1}+\cdots+\beta_{n+k-2}q^{-n-k+2} \tag{4.1.8}$$

$$\delta=\mathbf{E}(1)\mathbf{d}$$

$\mathbf{E}(q^{-1})$ and $\mathbf{F}(q^{-1})$ are the unique polynomial matrices satisfying,

$$\mathbf{I}=\mathbf{A}(q^{-1})\mathbf{E}(q^{-1})+q^{-k}\mathbf{F}(q^{-1}) \tag{4.1.9}$$

$$\mathbf{E}(q^{-1})=\mathbf{I}+\mathbf{E}_1q^{-1}+\cdots+\mathbf{E}_{k-1}q^{-k+1} \tag{4.1.10}$$

$$\mathbf{F}(q^{-1})=\mathbf{F}_0+\mathbf{F}_1q^{-1}+\cdots+\mathbf{F}_{n-1}q^{-n+1} \tag{4.1.11}$$

Eqn.(4.1.6) is called k-step ahead predictor, and $\alpha(q^{-1})$ and $\beta(q^{-1})$ are polynomial matrices. In the self-tuning case, $\mathbf{A}(q^{-1})$, $\mathbf{B}(q^{-1})$ and $\mathbf{d}$ are unknown, so $\alpha(q^{-1})$, $\beta(q^{-1})$ and δ have to be estimated on-line. The ST controller will set,

$$\Phi'(t+k|t)=\mathbf{S}(1)\mathbf{y}_d \tag{4.1.12}$$

where,

$$\Phi'(t+k|t)=\mathbf{S}(q^{-1})\left[\hat{\alpha}(q^{-1})\mathbf{y}(t)+\hat{\beta}(q^{-1})\mathbf{u}(t)+\hat{\delta}\right]+\mathbf{Q}(q^{-1})\left[\mathbf{u}(t)-\mathbf{u}_d\right] \tag{4.1.13}$$

which is the optimal prediction of $\Phi'(t+k)$ based on the information collected up to and including sampling instant t. Sometimes this is referred to as suboptimal control in the literature because the estimated parameters are used instead of the "true" parameters.

The estimation of coefficient matrices $\hat{\alpha}(q^{-1})$ and $\hat{\beta}(q^{-1})$, as well as $\hat{\delta}$, uses recursive least-squares with variable forgetting factor (Fortescue *et al.* 1981) in this thesis.

Defining data vector $\mathbf{x}(t)$ as,

$$\mathbf{x}(t)=\left[\mathbf{y}(t)^T\ \mathbf{y}(t-1)^T \cdots \mathbf{y}(t-n+1)^T\ \mathbf{u}(t)^T \cdots \mathbf{u}(t-n-k+2)^T\ 1\right]^T$$

$$=\Big[y_1(t)\ \cdots\ y_m(t) \,\vdots\, y_1(t-1)\ \cdots\ y_m(t-1) \,\vdots\, \cdots \,\vdots$$

$$y_1(t-n+1)\ \cdots\ y_m(t-n+1) \,\vdots\, u_1(t)\ \cdots\ u_m(t) \,\vdots\, \cdots$$

$$\vdots\, u_1(t-n-k+2)\cdots u_m(t-n-k+2) \,\vdots\, 1\Big]^T \tag{4.1.14}$$

and parameter matrix,

$$\hat{\mathbf{\Theta}}(t)=\left[\hat{\theta}_1(t)\ \cdots\ \hat{\theta}_m(t)\right]$$

or,

$$\hat{\Theta}(t)=\left[\hat{\alpha}_0(t)\ \cdots\hat{\alpha}_{n-1}(t)\ \hat{\beta}_0(t)\ \cdots\ \hat{\beta}_{n+k-2}(t)\ \hat{\delta}\right]^T \tag{4.1.15}$$

where,

$$\hat{\alpha}_i = \begin{bmatrix} \hat{\alpha}^i_{11} & \cdots & \hat{\alpha}^i_{1m} \\ \vdots & & \vdots \\ \hat{\alpha}^i_{m1} & \cdots & \hat{\alpha}^i_{mm} \end{bmatrix}, \qquad \hat{\beta}_i = \begin{bmatrix} \hat{\beta}^i_{11} & \cdots & \hat{\beta}^i_{1m} \\ \vdots & & \vdots \\ \hat{\beta}^i_{m1} & \cdots & \hat{\beta}^i_{mm} \end{bmatrix}$$

The column vectors $\hat{\theta}_i(t)$, $i=1,2,\cdots,m$, are of the form,

$$\hat{\theta}_i(t) = \Big[\hat{\alpha}^0_{i1}(t) \cdots \hat{\alpha}^0_{im}(t) \ \vdots \ \cdots \ \vdots \ \hat{\alpha}^{n-1}_{i1}(t) \cdots \hat{\alpha}^{n-1}_{im}(t) \ \vdots$$

$$\hat{\beta}^0_{i1}(t) \cdots \hat{\beta}^0_{im}(t) \ \vdots \ \cdots \ \vdots \ \hat{\beta}^{n+k-2}_{i1}(t) \cdots \hat{\beta}^{n+k-2}_{im}(t) \ \vdots \ \hat{\delta}_i \Big]^T$$

Eqn.(4.1.6) can then be expressed as,

$$\mathbf{y}(t) = \hat{\Theta}(t)^T \mathbf{x}(t-k) \tag{4.1.16}$$

or in component form,

$$y_i(t) = \hat{\theta}_i(t)\mathbf{x}(t-k), \qquad i=1,2,\ldots,m \tag{4.1.17}$$

The self-tuning algorithm is,

(1) $\varepsilon_i(t) = y_i(t) - \mathbf{x}(t-k)^T \hat{\theta}_i(t-1), \qquad i=1,2,\ldots,m$

(2) $w(t-1) = \mathbf{x}(t-k)^T \mathbf{P}(t-1)\mathbf{x}(t-k)$

(3) $\mathbf{K}(t) = \dfrac{\mathbf{P}(t-1)\mathbf{x}(t-k)}{1+w(t-1)}$

(4) $\hat{\theta}_i(t)=\hat{\theta}_i(t-1)+\mathbf{K}(t)\varepsilon_i(t)$, $\quad i=1,2,\ldots,m$

(5) $N(t)=\dfrac{\sigma(1+w(t-1))}{\sum_{i=1}^{m}\varepsilon_i(t)^2}$

σ — a constant to ensure $N(t)>1$

(6) $\lambda(t)=1-\dfrac{1}{N(t)}$

(7) $\mathbf{W}(t)=\left[\mathbf{I}-\mathbf{K}(t)\mathbf{x}(t-k)^T\right]\mathbf{P}(t-1)$

$$\mathbf{P}(t)=\begin{cases}\dfrac{\mathbf{W}(t)}{\lambda(t)} & \text{if } \dfrac{\text{trace}[\mathbf{W}(t)]}{\lambda(t)}\le C\\ \mathbf{W}(t) & \text{otherwise}\end{cases}$$

C --- a constant

Before proceeding with the convergence analysis of the algorithm it is convenient to give the following definitions,

$$\tilde{\theta}_i(t)=\theta_i-\hat{\theta}_i(t) \qquad i=1,2,\ldots,m \qquad (4.1.18)$$

$$\bar{\lambda}(t)=\begin{cases}\lambda(t) & \text{if } \dfrac{\text{trace}[\mathbf{W}(t)]}{\lambda(t)}\le C\\ 1 & \text{otherwise}\end{cases} \qquad (4.1.19)$$

$$e_i(t)=y_i(t)-y_{d_i} \qquad i=1,2,\ldots,m \qquad (4.1.20)$$

The analysis of convergence is based on the two assumptions below.

Assumption 4.1.1 The value of k in the diagonal interactor matrix $q^{-k}\mathbf{I}$ and upper bound of the system order n are known. ΔΔΔ

Assumption 4.1.2 Choose diagonal polynomial matrices $\mathbf{S}(q^{-1})$ and $\mathbf{Q}(q^{-1})$ off-line such that the polynomial matrix

$$\mathbf{f}(q^{-1})=\mathbf{A}(q^{-1})\mathbf{Q}(q^{-1})+\mathbf{B}(q^{-1})\mathbf{S}(q^{-1}) \qquad (4.1.21)$$

satisfies

$$\det\left[\mathbf{f}(q^{-1})\right]\neq 0 \qquad \forall\ |q|\leq 1$$ ΔΔΔ

Also, the following lemmas are necessary in proving the convergence property.

Lemma 4.1.1 If

$$\lim_{t\to\infty}\frac{\|\gamma(t)\|^2}{b_1(t)+b_2(t)\sigma(t)^T\sigma(t)}=0 \qquad (4.1.22)$$

where $\{b_1(t)\}$, $\{b_2(t)\}$ are real scalar sequences, $\{\gamma(t)\}$ and $\{\sigma(t)\}$ are real vector sequences. Then subject to

(i) $0<b_1(t)<K<\infty$ and $0<b_2(t)<K<\infty$, $\forall\ t\geq 0$, and

(ii) $\|\sigma(t)\| \leq C_1 + C_2 \max_{0<\tau<t} \|\gamma(t)\|$ where $0 \leq C_1 < \infty$, $0 < C_2 < \infty$

it follows that

$$\lim_{t \to \infty} \|\gamma(t)\| = 0$$

and $\{ \|\sigma(t)\| \}$ is bounded. ΔΔΔ

[proof]

This lemma is an extension from lemma 3.1 in reference [67]. If $\{ \|\gamma(t)\| \}$ is a bounded sequence, then, by condition (ii), $\{ \|\sigma(t)\| \}$ is a bounded sequence too. Thus, by Eqn.(4.1.22) and condition (i), it follows that

$$\lim_{t \to \infty} \|\gamma(t)\| = 0$$

Now assume that $\{ \|\gamma(t)\| \}$ is unbounded. It follows that there exists a sub-sequence $\{t_n\}$ such that

$$\lim_{t_n \to \infty} \|\gamma(t_n)\| = \infty$$

and,

$$\|\gamma(t)\| \leq \|\gamma(t_n)\| \qquad \forall\, t \leq t_n$$

Along the sub-sequence $\{t_n\}$, it has

$$\frac{\|\gamma(t_n)\|}{[b_1(t_n)+b_2(t_n)\sigma(t_n)^T\sigma(t_n)]^{1/2}}$$

$$\geq \frac{\|\gamma(t_n)\|}{[K+K\|\sigma(t_n)\|^2]^{1/2}}$$

$$\geq \frac{\|\gamma(t_n)\|}{K^{1/2}+K^{1/2}\|\sigma(t_n)\|}$$

$$\geq \frac{\|\gamma(t_n)\|}{K^{1/2}+K^{1/2}[C_1+C_2\|\gamma(t_n)\|]}$$

Therefore,

$$\lim_{t_n\to\infty} \frac{\|\gamma(t_n)\|}{[b_1(t_n)+b_2(t_n)\sigma(t_n)^T\sigma(t_n)]^{1/2}}$$

$$\geq \lim_{t_n\to\infty} \frac{1}{\frac{K^{1/2}}{\|\gamma(t_n)\|}+\frac{C_1K^{1/2}}{\|\gamma(t_n)\|}+C_2K^{1/2}}$$

$$=\frac{1}{C_2K^{1/2}}$$

$$>0$$

This is a contradiction to Eqn.(4.1.22), hence the assumption that $\{\|\gamma(t)\|\}$ is unbounded is false. Therefore the conclusion in Lemma 4.1.1 is true.

Lemma 4.1.2 Symmetric matrix $\mathbf{P}(t)$ satisfies,

(i) Positive definite for all t;

(ii) $\|\mathbf{P}(t)\| \leq C, \qquad 0<C<\infty$ ΔΔΔ

[proof]

(i) The definition of the covariance matrix $\mathbf{P}(t)$ gives the conclusion if $\mathbf{P}(0)$ is chosen to be positive definite.

(ii) Based on the fact that $\mathbf{P}(t)$ is symmetric, the induced norm $\|\mathbf{P}(t)\|$ will satisfy the following equality from Lemma C3 in Appendix C,

$$\|\mathbf{P}(t)\| = \lambda_{max}[\mathbf{P}(t)] \tag{4.1.23}$$

Choose $\lambda_{max}[\mathbf{P}(0)] \leq C$, and suppose

$$\|\mathbf{P}(\tau)\| \leq C, \qquad \tau=0,1,2,\ldots,t-1 \tag{4.1.24}$$

then if

$$\frac{\text{trace}[\mathbf{W}(t)]}{\lambda(t)} \leq C \tag{4.1.25}$$

is satisfied in step (7) of the self-tuning algorithm, the norm of $\mathbf{P}(t)$ will satisfy

$$\|\mathbf{P}(t)\| = \frac{1}{\lambda(t)}\|\mathbf{W}(t)\|$$

or,

$$\|\mathbf{P}(t)\| \leq \frac{1}{\lambda(t)}\mathrm{trace}[\mathbf{W}(t)]$$

or,

$$\|\mathbf{P}(t)\| \leq C \qquad (4.1.26)$$

If, on the other hand,

$$\frac{\mathrm{trace}[\mathbf{W}(t)]}{\lambda(t)} > C \qquad (4.1.27)$$

then,

$$\mathbf{P}(t)=\mathbf{P}(t-1)-\frac{\mathbf{P}(t-1)\mathbf{x}(t-k)\mathbf{x}(t-k)^T\mathbf{P}(t-1)}{1+\mathbf{x}(t-k)^T\mathbf{P}(t-1)(t-k)}$$

or, by matrix inversion,

$$\mathbf{P}(t)^{-1}=\mathbf{P}(t-1)^{-1}+\mathbf{x}(t-k)\mathbf{x}(t-k)^T \qquad (4.1.28)$$

Hence, the following inequality is true for any vector $\mathbf{z}\in R^p$, $p=m(2n+k+1)$,

$$\mathbf{z}^T\mathbf{P}(t)^{-1}\mathbf{z} \geq \mathbf{z}^T\mathbf{P}(t-1)^{-1}\mathbf{z}$$

Now choose $\mathbf{z}$ as the eigenvector corresponding to the minimum eigenvalue of $\mathbf{P}(t)^{-1}$, then from the above inequality,

$$\lambda_{min}[\mathbf{P}(t)^{-1}]\,\|\mathbf{z}\|^2 \geq \lambda_{min}[\mathbf{P}(t-1)^{-1}]\,\|\mathbf{z}\|^2$$

or

$$\lambda_{min}[\mathbf{P}(t)^{-1}] \geq \lambda_{min}[\mathbf{P}(t-1)^{-1}] \qquad (4.1.29)$$

Noting the fact that $\mathbf{P}(\cdot)$ is a real symmetric matrix, it leads to,

$$\lambda_{max}[\mathbf{P}(t)] \leq \lambda_{max}[\mathbf{P}(t-1)] \qquad (4.1.30)$$

or, by noting Eqn.(4.1.24),

$$\|\mathbf{P}(t)\| \leq C \qquad (4.1.31)$$

Thus the conclusion in (ii) is true.

Lemma 4.1.3 In recursive least-squares estimation the function,

$$v_i(t) = \tilde{\theta}_i(t)^T\mathbf{P}(t)^{-1}\tilde{\theta}_i(t)$$

is a bounded, nonnegative, nonincreasing function for any $\lambda(t)>0$[67]. ΔΔΔ

[Proof]

By the definition of $\bar{\lambda}(t)$ in Eqn.(4.1.19), $\mathbf{P}(t)$ at step (7) in the self-tuning algorithm can be written as,

$$\mathbf{P}(t) = \frac{1}{\bar{\lambda}(t)}\left[\mathbf{I}-\frac{\mathbf{P}(t-1)\mathbf{x}(t-k)\mathbf{x}(t-k)^T}{1+w(t-1)}\right]\mathbf{P}(t-1) \qquad (4.1.32)$$

By the definition of Eqn.(4.1.18), and subtracting $\tilde{\theta}_i(t-1)$ from $\tilde{\theta}_i(t)$ gives,

$$\tilde{\theta}_i(t)=\tilde{\theta}_i(t-1)-\left[\hat{\theta}_i(t)-\hat{\theta}_i(t-1)\right]$$

$$=\tilde{\theta}_i(t-1)-\mathbf{K}(t)\varepsilon_i(t)$$

$$=\tilde{\theta}_i(t-1)-\frac{\mathbf{P}(t-1)\mathbf{x}(t-k)}{1+w(t-1)}\mathbf{x}(t-k)^T\tilde{\theta}_i(t-1)$$

$$=\left[\mathbf{I}-\frac{\mathbf{P}(t-1)\mathbf{x}(t-k)\mathbf{x}(t-k)^T}{1+w(t-1)}\right]\tilde{\theta}_i(t-1),\quad i=1,\ldots,m \qquad (4.1.33)$$

Noting Eqn.(4.1.32) one can arrive at,

$$\tilde{\theta}_i(t)=\bar{\lambda}(t)\mathbf{P}(t)\mathbf{P}(t-1)^{-1}\tilde{\theta}_i(t-1) \qquad (4.1.34)$$

or expressed as,

$$\mathbf{P}(t)^{-1}\tilde{\theta}_i(t)=\bar{\lambda}(t)\mathbf{P}(t-1)^{-1}\tilde{\theta}_i(t-1) \qquad (4.1.35)$$

Noting the definition of $v_i(t)$ in Lemma 4.1.3 which gives the following fact,

$$v_i(t-1) = \tilde{\theta}_i(t-1)^T P(t-1)^{-1}\tilde{\theta}_i(t-1)$$

and also the facts in Eqns.(4.1.34) and (4.1.35), one can have,

$$v_i(t)-\bar{\lambda}(t)v_i(t-1)=\bar{\lambda}(t)\left[\tilde{\theta}_i(t)-\tilde{\theta}_i(t-1)\right]^T P(t-1)^{-1}\tilde{\theta}_i(t-1)$$

$$=-\bar{\lambda}(t)\frac{\tilde{\theta}_i(t-1)^T x(t-k)x(t-k)^T\tilde{\theta}_i(t-1)}{1+w(t-1)}$$

Recognizing $\varepsilon_i(t)=x(t-k)^T\tilde{\theta}_i(t-1)$, one can express the above relationship as,

$$v_i(t)-\bar{\lambda}(t)v_i(t-1)=-\bar{\lambda}(t)\frac{\varepsilon_i(t)^2}{1+w(t-1)} \qquad (4.1.36)$$

Thus the following relationship holds,

$$v_i(t)-v_i(t-1)=-\bar{\lambda}(t)\frac{\varepsilon_i(t)^2}{1+w(t-1)}+\bar{\lambda}(t)v_i(t-1)-v_i(t-1)$$

$$=-\bar{\lambda}(t)\frac{\varepsilon_i(t)^2}{1+w(t-1)}-\left[1-\bar{\lambda}(t)\right]v_i(t-1) \qquad (4.1.37)$$

Noting the facts below,

(i) $\mathbf{P}(t)$ is positive definite;

(ii) $v_i(t) \geq 0$ and

(iii) $0 < \bar{\lambda}(t) \leq 1$;

one can draw the following conclusion from Eqn.(4.1.37),

$$v_i(t) - v_i(t-1) \leq 0 \qquad (4.1.38)$$

This gives the conclusions in Lemma 4.1.3.

Lemma 4.1.4 Consider the system,

$$\mathbf{f}(q^{-1})v(t) = \mathbf{g}(q^{-1})\rho(t)$$

where $\mathbf{f}(q^{-1})$ and $\mathbf{g}(q^{-1})$ are polynomial matrices in the backward shift operation. $\mathbf{f}(q^{-1})$ is stable, i.e., all its roots are outside the closed unit disc on the q^{-1}-plane. $v(t)$ and $\rho(t)$ are real vector sequences. Then,

(i) There exist positive constants k_1 and k_2 such that,

$$\| v(t) \| \leq k_1 + k_2 \max_{0 \leq \tau \leq t} \| \rho(\tau) \|$$

(ii) If $\lim_{t \to \infty} \| \rho(t) \| = 0$, then,

$$\lim_{t \to \infty} \| v(t) \| = 0, \qquad \Delta\Delta\Delta$$

[Proof]

The results are obvious if it is viewed as a forced multivariable linear system with input vector $\rho(t)$ and output vector $V(t)$.

Lemma 4.1.5 The following is true,

(i) $\varepsilon_i(t)=\mathbf{x}(t-k)^T\tilde{\theta}_i(t-1) \qquad i=1,2,\ldots,m$

(ii) $S_i(q^{-1})\left[e_i(t)-\tilde{\theta}_i(t-k)^T\mathbf{x}(t-k)\right]+Q_i(q^{-1})\tilde{u}_i(t-k)=0$

(iii) $S_i(q^{-1})\left[e_i(t)-\varepsilon_i(t)-\mathbf{x}(t-k)^T\mathbf{a}_i(t)\right]+Q_i(q^{-1})\tilde{u}_i(t-k)=0$

where,

$$\mathbf{a}_i(t) = \sum_{r=2}^{k}\frac{P(t-r)\mathbf{x}(t-k-r+1)}{1+w(t-r)}\varepsilon_i(t-r+1)$$

$$\tilde{u}_i(t) = u_i(t)-u_{di}$$

ΔΔΔ

[Proof]

(i) $\varepsilon_i(t) = y_i(t)-\mathbf{x}(t-k)^T\hat{\theta}_i(t-1)$

$$= \mathbf{x}(t-k)^T\left[\theta_i-\hat{\theta}_i(t-1)\right]$$

$$= \mathbf{x}(t-k)^T\tilde{\theta}_i(t-1)$$

(ii) $$e_i(t) = y_i(t)-y_{d_i}$$

$$= \theta_i^T\mathbf{x}(t-k)-y_{d_i}$$

i.e.,

$$\theta_i^T\mathbf{x}(t-k)-e_i(t) = y_{d_i}$$

Multiplying the above equation by $S_i(q^{-1})$ and noting y_{d_i} is a constant, one can get,

$$S_i(q^{-1})\left[\theta_i^T\mathbf{x}(t-k)-e_i(t)\right] = S_i(1)y_{d_i} \qquad (4.1.39)$$

From Eqns.(4.1.12) and (4.1.13) it can be deduced that,

$$S_i(1)y_{d_i}=\Phi_i'(t+k|t)$$

$$=S_i(q^{-1})\hat{\theta}_i(t-k)^T\mathbf{x}(t-k)+Q_i(q^{-1})\tilde{u}_i(t-k) \qquad (4.1.40)$$

Combining Eqn.(4.1.39) with Eqn.(4.1.40) and rearranging it one has the result of (ii).

(iii) Noting that,

$$\hat{\boldsymbol{\theta}}_i(t)=\hat{\boldsymbol{\theta}}_i(t-1)+\mathbf{K}(t)\varepsilon_i(t) \qquad i=1,2,\ldots,m$$

one can end up with,

$$\boldsymbol{\theta}_i-\hat{\boldsymbol{\theta}}_i(t)=\boldsymbol{\theta}_i-\hat{\boldsymbol{\theta}}_i(t-1)-\mathbf{K}(t)\varepsilon_i(t)$$

i.e.,

$$\tilde{\boldsymbol{\theta}}_i(t)=\tilde{\boldsymbol{\theta}}_i(t-1)-\mathbf{K}(t)\varepsilon_i(t)$$

$$=\tilde{\boldsymbol{\theta}}_i(t-1)-\frac{\mathbf{P}(t-1)\mathbf{x}(t-k)}{1+w(t-1)}\varepsilon_i(t)$$

Therefore one can use direct substitution to get,

$$\tilde{\boldsymbol{\theta}}_i(t-1)=\tilde{\boldsymbol{\theta}}_i(t-2)-\frac{\mathbf{P}(t-2)\mathbf{x}(t-k-1)}{1+w(t-2)}\varepsilon_i(t-1) \qquad (4.1.41)$$

$$\vdots$$

$$\tilde{\boldsymbol{\theta}}_i(t-k+1)=\tilde{\boldsymbol{\theta}}_i(t-k)-\frac{\mathbf{P}(t-k)\mathbf{x}(t-2k+1)}{1+w(t-k)}\varepsilon_i(t-k+1)$$

Thus it follows that,

$$\tilde{\boldsymbol{\theta}}_i(t-1)=\tilde{\boldsymbol{\theta}}_i(t-k)-\sum_{r=2}^{k}\frac{\mathbf{P}(t-r)\mathbf{x}(t-k-r+1)}{1+w(t-r)}\varepsilon_i(t-r+1)$$

$$=\tilde{\boldsymbol{\theta}}_i(t-k)-\mathbf{a}_i(t), \qquad i=1,2,\ldots,m \qquad (4.1.42)$$

Pre-multiplying the above equation by $\mathbf{x}(t-k)^T$, and noting the relation (i) in Lemma 4.1.5, one gets,

$$\varepsilon_i(t)=\mathbf{x}(t-k)^T\tilde{\theta}_i(t-k)-\mathbf{x}(t-k)^T\mathbf{a}_i(t)$$

Adding $e_i(t)$ on both sides of the above equation and rearranging it, one can have,

$$e_i(t)-\mathbf{x}(t-k)^T\tilde{\theta}_i(t-k)=e_i(t)-\varepsilon_i(t)-\mathbf{x}(t-k)^T\mathbf{a}_i(t)$$

Multiplying the last equation by $S_i(t)$ and adding $Q_i(q^{-1})\tilde{u}_i(t-k)$ on both sides, one ends up with,

$$S_i(t)\left[e_i(t)-\mathbf{x}(t-k)^T\tilde{\theta}_i(t-k)\right]+Q_i(q^{-1})\tilde{u}_i(t-k)$$

$$=S_i(t)\left[e_i(t)-\varepsilon_i(t)-\mathbf{x}(t-k)^T\mathbf{a}_i(t)\right]+Q_i(q^{-1})\tilde{u}_i(t-k)$$

The left side of the last equation equals to zero by the result in (ii), so

$$S_i(t)\left[e_i(t)-\varepsilon_i(t)-\mathbf{x}(t-k)^T\mathbf{a}_i(t)\right]+Q_i(q^{-1})\tilde{u}_i(t-k)=0$$

This is the result in (iii).

Lemma 4.1.6 The following is true for the self-tuning algorithm,

$$\lim_{t\to\infty} \frac{\|\varepsilon(t)\|^2}{1+w(t-1)} = 0$$

ΔΔΔ

[Proof]

Noting $0<\bar{\lambda}(t)\leq 1$, it is true from Eqn.(4.3.36) that

$$v_i(t)\leq\bar{\lambda}(t)v_i(t-1)\leq v_i(t-1) \qquad \forall\, t>1$$

Assuming that $v_i(t)$ decreases to a small value $v_i^*\geq 0$ as $t\to\infty$, i.e.,

$$\lim_{t\to\infty} v_i(t) = v_i^*$$

then,

$$\lim_{t\to\infty} \left[v_i(t)-\bar{\lambda}(t)v_i(t-1)\right] = \left[1-\bar{\lambda}(t)\right]v_i^*$$

Now noting that,

$$\left[1-\bar{\lambda}(t)\right]v_i^* \geq 0 \tag{4.1.43}$$

and,

$$v_i(t)-\bar{\lambda}(t)v_i(t-1) \leq 0 \tag{4.1.44}$$

one can get the following conclusion from the above two equations,

$$\lim_{t\to\infty} \left[v_i(t) - \bar{\lambda}(t) v_i(t-1) \right] = 0 \tag{4.1.45}$$

Noticing the relationship in Eqn.(4.1.36), Eqn.(4.1.45) can be expressed as,

$$\lim_{t\to\infty} \left[-\bar{\lambda}(t) \frac{\varepsilon_i(t)^2}{1+w(t-1)} \right] = 0 \tag{4.1.46}$$

or

$$\lim_{t\to\infty} \frac{\varepsilon_i(t)^2}{1+w(t-1)} = 0 \qquad i=1,2,\ldots,m$$

This leads to the result in this lemma.

Lemma 4.1.7 It is true that

$$\lim_{t\to\infty} \|\mathbf{a}(t)\| = 0$$

where $\mathbf{a}(t) = [\mathbf{a}_1(t) \cdots \mathbf{a}_m(t)] \in R^{m\times m}$ and $\mathbf{a}_i(t)$ is the same as defined in Lemma 4.1.5. ΔΔΔ

[Proof]

By the definition of $\mathbf{a}_i(t)$, its norm has the property below,

$$\|\mathbf{a}_i(t)\| = \left\| \sum_{r=2}^{k} \frac{\mathbf{P}(t-r)\mathbf{x}(t-k-r+1)}{[1+w(t-r)]^{1/2}} \frac{\varepsilon_i(t-r+1)}{[1+w(t-r)]^{1/2}} \right\|$$

$$\leq \sum_{r=2}^{k} \frac{\|\mathbf{P}(t-r)\mathbf{x}(t-k-r+1)\|}{[1+w(t-r)]^{1/2}} \frac{\|\varepsilon_i(t-r+1)\|}{[1+w(t-r)]^{1/2}}$$

Note that the first term on the right-hand side satisfies,

$$\frac{\|\mathbf{P}(t-r)\mathbf{x}(t-k-r+1)\|^2}{1+w(t-r)} \leq \frac{\|\mathbf{P}(t-r)\|^2 \ \|\mathbf{x}(t-k-r+1)\|^2}{w(t-r)}$$

$$\leq \frac{\|\mathbf{P}(t-r)\|^2 \ \|\mathbf{x}(t-k-r+1)\|^2}{\lambda_{min}[\mathbf{P}(t-r)] \ \|\mathbf{x}(t-k-r+1)\|^2}$$

$$\leq \frac{C^2}{\lambda_{min}[\mathbf{P}(t-r)]} < \infty$$

Express $\frac{C^2}{\lambda_{min}[\mathbf{P}(t-r)]}$ as C'^2, it thus has,

$$\|\mathbf{a}_i(t)\| \leq \sum_{r=2}^{k} C' \frac{\|\varepsilon_i(t-r+1)\|}{[1+w(t-r)]^{1/2}}$$

By using Lemma 4.1.6 one has

$$\lim_{t\to\infty} \|\mathbf{a}_i(t)\| = 0$$

Hence $\mathbf{a}(t) \to \mathbf{0}$ as $t\to\infty$. This leads to the conclusion of Lemma 4.1.7.

Lemma 4.1.8 The tracking error and input dynamics are described by,

$$[\mathbf{A}(q^{-1})+\mathbf{B}(q^{-1})\mathbf{S}(q^{-1})\mathbf{Q}(q^{-1})^{-1}]\,\mathbf{e}(t)$$

$$=\mathbf{B}(q^{-1})\mathbf{S}(q^{-1})\mathbf{Q}(q^{-1})^{-1}[\varepsilon(t)+\mathbf{a}(t)^{T}\mathbf{x}(t-k)] \qquad (4.1.47)$$

$$[\mathbf{B}(q^{-1})+\mathbf{A}(q^{-1})\mathbf{S}(q^{-1})^{-1}\mathbf{Q}(q^{-1})]\,\tilde{\mathbf{u}}(t-k)$$

$$=\mathbf{A}(q^{-1})[\varepsilon(t)+\mathbf{a}(t)^{T}\mathbf{x}(t-k)] \qquad (4.1.48)$$

where $\mathbf{a}(t)$ is defined in Lemma 4.1.7. ΔΔΔ

[Proof]

The steady state version of the system equation around a specific operating point can be written as follows,

$$\mathbf{A}(q^{-1})\mathbf{y}_d = \mathbf{B}(q^{-1})\mathbf{u}_d+\mathbf{d}$$

and subtracting it from Eqn.(4.1.1) yields,

$$\mathbf{A}(q^{-1})[\mathbf{y}(t)-\mathbf{y}_d] = \mathbf{B}(q^{-1})[\mathbf{u}(t-k)-\mathbf{u}_d]$$

i.e.,

$$\mathbf{A}(q^{-1})\mathbf{e}(t)-\mathbf{B}(q^{-1})\tilde{\mathbf{u}}(t-k)=0 \qquad (4.1.49)$$

Note that item (iii) in Lemma 4.1.5 can be written in vector form,

$$\mathbf{S}(q^{-1})\left[\mathbf{e}(t)-\varepsilon(t)-\mathbf{a}(t)^T\mathbf{x}(t-k)\right]+\mathbf{Q}(q^{-1})\tilde{\mathbf{u}}(t-k)=\mathbf{0} \qquad (4.1.50)$$

where $\mathbf{S}(q^{-1})$ and $\mathbf{Q}(q^{-1})$ are diagonal polynomial matrices. Pre-multiplying Eqn.(4.1.49) by $\mathbf{B}(q^{-1})^{-1}$ and Eqn.(4.1.50) by $\mathbf{Q}(q^{-1})^{-1}$, then adding them together gives,

$$\mathbf{Q}(q^{-1})^{-1}\mathbf{S}(q^{-1})\left[\mathbf{e}(t)-\varepsilon(t)-\mathbf{a}(t)^T\mathbf{x}(t-k)\right]+\mathbf{B}(q^{-1})^{-1}\mathbf{A}(q^{-1})\mathbf{e}(t)=\mathbf{0} \qquad (4.1.51)$$

Rearranging the above expression and noting the fact that,

$$\mathbf{Q}(q^{-1})^{-1}\mathbf{S}(q^{-1})=\mathbf{S}(q^{-1})\mathbf{Q}(q^{-1})^{-1} \qquad (4.1.52)$$

one can derive the expression in Eqn.(4.1.47).

Pre-multiplying Eqn.(4.1.49) by $\mathbf{S}(q^{-1})\mathbf{A}(q^{-1})^{-1}$ and adding $\mathbf{Q}(q^{-1})\tilde{\mathbf{u}}(t-k)$ on both sides, one gets,

$$\left[\mathbf{S}(q^{-1})\mathbf{A}(q^{-1})^{-1}\mathbf{B}(q^{-1})+\mathbf{Q}(q^{-1})\right]\tilde{\mathbf{u}}(t-k)=\mathbf{S}(q^{-1})\mathbf{e}(t)+\mathbf{Q}(q^{-1})\tilde{\mathbf{u}}(t-k)$$

(4.1.53)

From the fact of (iii) in Lemma 4.1.5, it follows that,

$$\mathbf{S}(q^{-1})\mathbf{e}(t)+\mathbf{Q}(q^{-1})\tilde{\mathbf{u}}(t-k)=\mathbf{S}(q^{-1})\left[\varepsilon(t)+\mathbf{a}(t)^T\mathbf{x}(t-k)\right] \quad (4.1.54)$$

Substituting Eqn.(4.1.54) into Eqn.(4.1.53) gives,

$$\left[\mathbf{S}(q^{-1})\mathbf{A}(q^{-1})^{-1}\mathbf{B}(q^{-1})+\mathbf{Q}(q^{-1})\right]\tilde{\mathbf{u}}(t-k)=\mathbf{S}(q^{-1})\left[\varepsilon(t)+\mathbf{a}(t)^T\mathbf{x}(t-k)\right]$$

(4.1.55)

This leads to the result in Eqn.(4.1.48).

Lemma 4.1.9 Let $\{\mathbf{z}(t)\}$, $\{\mathbf{w}(t)\}$ be real vector sequences, $\{\mathbf{a}(t)\}$ real matrix sequence and

$$\lim_{t\to\infty} \|\mathbf{a}(t)\|=0 \quad (4.1.56)$$

then,

$$\|\mathbf{z}(t)\| \le k_1+k_2 \max_{0\le\tau\le t} \left\|\mathbf{w}(\tau)+\mathbf{a}(\tau)^T\mathbf{z}(\tau)\right\| \quad (4.1.57)$$

for fixed k_1, $k_2 \ge 0$ implies that there exist constants k_3, $k_4 \ge 0$ such that,

$$\|\mathbf{z}(t)\| \le k_3+k_4 \max_{0\le\tau\le t} \left\|\mathbf{w}(\tau)\right\| \quad (4.1.58)$$

ΔΔΔ

[Proof]

From condition Eqn.(4.1.57) it follows that,

$$\max_{0\le\tau\le t} \| \mathbf{z}(\tau) \| \le k_1 + k_2 \max_{0\le\tau\le t} \| \mathbf{w}(\tau) + \mathbf{a}(\tau)^T \mathbf{z}(\tau) \|$$

$$\le k_1 + k_2 \max_{0\le\tau\le t} \| \mathbf{w}(\tau) \| + k_2 \max_{0\le\tau\le t} \| \mathbf{a}(\tau)^T \mathbf{z}(\tau) \|$$

Since $\|\mathbf{a}(t)\| \rightarrow 0$ as $t \rightarrow \infty$ there exists a t_n such that,

$$k_2 \|\mathbf{a}(t)\| \le \frac{1}{2} \qquad \forall \ t \ge t_n$$

and let,

$$M = \max_{0\le\tau\le t_n} \| k_2 \mathbf{a}(\tau)^T \mathbf{z}(\tau) \|$$

be a constant, the last inequality can be expressed as,

$$\max_{0\le\tau\le t} \| \mathbf{z}(\tau) \| \le k_1 + k_2 \max_{0\le\tau\le t} \| \mathbf{w}(\tau) \| + \max_{0\le\tau\le t_n} \| k_2 \mathbf{a}(\tau)^T \mathbf{z}(\tau) \| +$$

$$+ \max_{t_n\le\tau\le t} \| k_2 \mathbf{a}(\tau)^T \mathbf{z}(\tau) \|$$

or,

$$\max_{0\le\tau\le t} \| \mathbf{z}(\tau) \| \le (k_1 + M) + k_2 \max_{0\le\tau\le t} \| \mathbf{w}(\tau) \| + \frac{1}{2} \max_{t_n\le\tau\le t} \| \mathbf{z}(\tau) \|$$

or,

$$\max_{t_n \le \tau \le t} \| \mathbf{z}(\tau) \| \le 2(k_1+M)+2k_2 \max_{0 \le \tau \le t} \| \mathbf{w}(\tau) \| \tag{4.1.59}$$

Now let,

$$\max_{0 \le \tau \le t_n} \| \mathbf{z}(\tau) \| = N \qquad \text{(a fixed constant)}$$

then,

$$\max_{0 \le \tau \le t} \| \mathbf{z}(\tau) \| \le 2(k_1+M)+N+2k_2 \max_{0 \le \tau \le t} \| \mathbf{w}(\tau) \| \tag{4.1.60}$$

Defining $k_3=2(k_1+M)+N$, and $k_4=2k_2$, it follows from the inequality of Eqn.(4.1.60) that Eqn.(4.1.58) holds.

The convergence property of the algorithm can now be pursued based on the previous lemmas.

Theorem 4.1 Subject to the assumptions (4.1.1) and (4.1.2) the self-tuning controller applied to system of Eqn.(4.1.1) leads to,

$$\lim_{t \to \infty} \mathbf{y}(t) = \mathbf{y}_d \tag{4.1.61}$$

ΔΔΔ

[Proof]

Noticing the facts that,

$$\det\{\mathbf{A}(q^{-1})+\mathbf{B}(q^{-1})\mathbf{S}(q^{-1})\mathbf{Q}(q^{-1})^{-1}\}$$

$$=\det\{\mathbf{A}(q^{-1})\mathbf{Q}(q^{-1})+\mathbf{B}(q^{-1})\mathbf{S}(q^{-1})\}\det\{\mathbf{Q}(q^{-1})^{-1}\}$$

and,

$$\det\{\mathbf{B}(q^{-1})+\mathbf{A}(q^{-1})\mathbf{S}(q^{-1})^{-1}\mathbf{Q}(q^{-1})\}$$
$$=\det\{\mathbf{A}(q^{-1})\mathbf{Q}(q^{-1})+\mathbf{B}(q^{-1})\mathbf{S}(q^{-1})\}\det\{\mathbf{S}(q^{-1})^{-1}\}$$

it follows from Lemmas 4.1.4, 4.1.8 and assumption 4.1.2 that,

$$\|\mathbf{e}(t-k)\| \le k_0' + k_2' \max_{0\le\tau\le t-k} \|\varepsilon(\tau)+\mathbf{a}(\tau)^T\mathbf{x}(\tau-k)\| \quad (4.1.62)$$

and,

$$\|\tilde{\mathbf{u}}(t-k)\| \le k_0'' + k_2'' \max_{0\le\tau\le t-k} \|\varepsilon(\tau)+\mathbf{a}(\tau)^T\mathbf{x}(\tau-k)\| \quad (4.1.63)$$

Since,

$$\|\mathbf{e}(t-k)\| = \|\mathbf{y}(t-k)-\mathbf{y}_d\|$$

or,

$$\|\mathbf{y}(t-k)\| \le \|\mathbf{e}(t-k)\| + \|\mathbf{y}_d\|$$

it follows from inequality Eqn.(4.1.62) that,

$$\|\mathbf{y}(t-k)\| \le k_0' + k_2' \max_{0\le\tau\le t} \|\varepsilon(\tau)+\mathbf{a}(\tau)^T\mathbf{x}(\tau-k)\| + \|\mathbf{y}_d\|$$

$$\le k_1' + k_2' \max_{0\le\tau\le t} \|\varepsilon(\tau)+\mathbf{a}(\tau)^T\mathbf{x}(\tau-k)\| \quad (4.1.64)$$

where,

$$k_1' = k_0' + \|\mathbf{y}_d\|$$

Also,

$$\tilde{\mathbf{u}}(t-k)=\mathbf{u}(t-k)-\mathbf{u}_d$$

i.e.,

$$\|\mathbf{u}(t-k)\| \leq \|\tilde{\mathbf{u}}(t-k)\| + \|\mathbf{u}_d\|$$

it follows from inequality Eqn.(4.1.63) that,

$$\|\mathbf{u}(t-k)\| \leq k_1'' + k_2'' \max_{0 \leq \tau \leq t} \|\varepsilon(\tau) + \mathbf{a}(\tau)^T \mathbf{x}(\tau-k)\| \tag{4.1.65}$$

where,

$$k_1'' = k_0'' + \|\mathbf{u}_d\|$$

Noticing that data vector $\mathbf{x}(t-k)$ is composed of $\mathbf{y}(t-i)$ and $\mathbf{u}(t-j)$ it follows that,

$$\|\mathbf{x}(t-k)\| \leq \|\mathbf{y}(t-k)\| + \|\mathbf{y}(t-k-1)\| + \cdots + \|\mathbf{y}(t-k-n+1)\|$$

$$+ \|\mathbf{u}(t-k)\| + \cdots + \|\mathbf{u}(t-2k-n+2)\| + 1$$

from inequalities of Eqns.(4.1.64) and (4.1.65), and using the sequential substitution, it leads to,

$$\|\mathbf{x}(t-k)\| \leq n \left[k_1' + k_2' \max_{0 \leq \tau \leq t} \|\varepsilon(\tau) + \mathbf{a}(\tau)^T \mathbf{x}(\tau-k)\|\right] +$$

$$+ (n+k-1)\left[k_1'' + k_2'' \max_{0 \leq \tau \leq t} \|\varepsilon(\tau) + \mathbf{a}(\tau)^T \mathbf{x}(\tau-k)\|\right] + 1$$

or,

$$\|\mathbf{x}(t-k)\| \leq k_1 + k_2 \max_{0 \leq \tau \leq t} \|\varepsilon(\tau) + \mathbf{a}(\tau)^T \mathbf{x}(\tau-k)\| \tag{4.1.66}$$

with,

$$k_1 = nk_1' + (n+k-1)k_1'' + 1$$

$$k_2 = nk_2' + (n+k-1)k_2''$$

From Lemma 4.1.7,

$$\lim_{t \to \infty} \|\mathbf{a}(t)\| = 0 \qquad (4.1.67)$$

the following relationship is true based on the fact of Eqn.(4.1.67) and Lemma 4.1.9,

$$\|\mathbf{x}(t-k)\| \le k_3 + k_4 \max_{0 \le \tau \le t} \|\varepsilon(\tau)\| \qquad (4.1.68)$$

It follows from Lemma 4.1.2 that,

$$w(t-1) = \mathbf{x}(t-k)^T \mathbf{P}(t-1)\mathbf{x}(t-k)$$

$$\le C\mathbf{x}(t-k)^T \mathbf{x}(t-k) \qquad (4.1.69)$$

Using the facts of Eqns.(4.1.60), (4.1.69) and Lemma 4.1.6 and comparing the notations used in Lemma 4.1.1 one can find that,

$$b_1(1) = 1$$

$$b_2(t) = C$$

so the conditions in Lemma 4.1.1 are satisfied. This leads to,

$$\lim_{t \to \infty} \|\varepsilon(t)\| = 0 \qquad (4.1.70)$$

and,

$$\|\mathbf{x}(t-k)\| \text{ is bounded.} \tag{4.1.71}$$

Since,

$$\|\varepsilon(t)+\mathbf{a}(t)^T\mathbf{x}(t-k)\| \le \|\varepsilon(t)\| + \|\mathbf{a}(t)\|\,\|\mathbf{x}(t-k)\|$$

and,

$$\lim_{t\to\infty}\{\|\varepsilon(t)\| + \|\mathbf{a}(t)\|\,\|\mathbf{x}(t-k)\|\} = 0$$

the following holds,

$$\lim_{t\to\infty}\{\|\varepsilon(t)+\mathbf{a}(t)^T\mathbf{x}(t-k)\|\} = 0$$

By Lemma 4.1.8 the above expression means that the following is true in Eqn.(4.1.47),

$$\lim_{t\to\infty}\|\mathbf{e}(t)\| = 0 \tag{4.1.72}$$

This leads to the conclusion in this lemma by noting $\mathbf{e}(t)=\mathbf{y}(t)-\mathbf{y}_d$.

4.2 Convergence Property of the Multivariable Generalized Minimum Variance Self-Tuning Controller with Variable Forgetting Factor for Power System in Stochastic Environment

In the last section the convergence property of the multivariable GMV ST controller with variable forgetting factor in a deterministic environment was proved. However,

the proof of the convergence property of the discussed algorithm in a stochastic environment has not been solved at this stage. This problem will not be solved for the general case, but in this section the author will prove the convergence property of the algorithm for the regulator problem (i.e, $\mathbf{y}_d$ is a constant vector) in a stochastic environment for systems with a unit time-delay diagonal interactor matrix $q^{-1}\mathbf{I}$.

Lemma 4.2.1 For the regulator problem, a stochastic system with diagonal interactor matrix $q^{-1}\mathbf{I}$ is equivalent to the deterministic case except a dc offset vector exists in the estimated parameter matrix if the multivariable GMV STR is used.

ΔΔΔ

[Proof]

A sytem with unit time delay can be expressed by ARMAX model,

$$\mathbf{A}(q^{-1})\mathbf{y}(t)=q^{-1}\mathbf{B}(q^{-1})\mathbf{u}(t)+\mathbf{C}(q^{-1})\boldsymbol{\xi}(t) \qquad (4.2.1)$$

where,

$\boldsymbol{\xi}(t)\in R^m$, white noise vector

$\mathbf{C}(q^{-1})=\mathbf{I}+\mathbf{C}_1q^{-1}+\cdots+\mathbf{C}_nq^{-n}$

Defining an innovation sequence $\{\tilde{\mathbf{y}}(t)\}$,

$$\tilde{\mathbf{y}}(t+1) \triangleq \mathbf{y}(t+1) - \hat{\mathbf{y}}(t+1) = \boldsymbol{\xi}(t+1) \qquad (4.2.2)$$

where $\hat{\mathbf{y}}(t+1)$ is the optimal prediction of $\mathbf{y}(t+1)$ at time t, yields the following relationship,

$$\mathbf{C}(q^{-1})\left[\mathbf{y}(t+1) - \hat{\mathbf{y}}(t+1)\right] = \mathbf{C}(q^{-1})\boldsymbol{\xi}(t+1) \qquad (4.2.3)$$

or, noting the relationship in Eqn.(4.2.1),

$$\mathbf{C}(q^{-1})\hat{\mathbf{y}}(t+1) = q\left[\mathbf{C}(q^{-1}) - \mathbf{A}(q^{-1})\right]\mathbf{y}(t) + \mathbf{B}(q^{-1})\mathbf{u}(t)$$

or,

$$\hat{\mathbf{y}}(t+1) = q\left[\mathbf{C}(q^{-1}) - \mathbf{A}(q^{-1})\right]\mathbf{y}(t) + \mathbf{B}(q^{-1})\mathbf{u}(t) + q\left[\mathbf{I} - \mathbf{C}(q^{-1})\right]\hat{\mathbf{y}}(t) \qquad (4.2.4)$$

The auxiliary output in GMV ST is defined as,

$$\hat{\boldsymbol{\Phi}}(t+1) = \mathbf{S}(q^{-1})\hat{\mathbf{y}}(t+1) + \mathbf{Q}(q^{-1})\mathbf{u}(t) \qquad (4.2.5)$$

or

$$\hat{\boldsymbol{\Phi}}(t) = \mathbf{S}(q^{-1})\hat{\mathbf{y}}(t) + \mathbf{Q}(q^{-1})\mathbf{u}(t-1) \qquad (4.2.6)$$

Substituting Eqn.(4.2.4) into Eqn.(4.2.5), adding and subtracting $q[\mathbf{I} - \mathbf{C}(q^{-1})]\mathbf{Q}(q^{-1})\mathbf{u}(t-1)$, and using Eqn.(4.2.6), give,

$$\hat{\boldsymbol{\Phi}}(t+1) = \boldsymbol{\alpha}(q^{-1})\mathbf{y}(t) + \boldsymbol{\beta}(q^{-1})\mathbf{u}(t) + \boldsymbol{\eta}(q^{-1})\hat{\boldsymbol{\Phi}}(t) \qquad (4.2.7)$$

where,

$$\boldsymbol{\alpha}(q^{-1})=q\mathbf{S}(q^{-1})\left[\mathbf{C}(q^{-1})-\mathbf{A}(q^{-1})\right]$$

$$=\alpha_0+\alpha_1 q^{-1}+\cdots+\alpha_{n-1}q^{-n+1}+\cdots$$

$$\beta(q^{-1})=\mathbf{S}(q^{-1})\mathbf{B}(q^{-1})+\mathbf{C}(q^{-1})\mathbf{Q}(q^{-1})$$
$$=\beta_0+\beta_1 q^{-1}+\cdots+\beta_{n-1}q^{-n+1}+\cdots$$

$$\eta(q^{-1})=q[\mathbf{I}-\mathbf{C}(q^{-1})]=\eta_1+\cdots+\eta_n q^{-n+1}$$

Defining the following data vector and parameter matrix and using extended RLS[66], polynomial matrices $\alpha(q^{-1})$, $\beta(q^{-1})$ and $\eta(q^{-1})$ can be estimated on line,

$$\mathbf{x}(t)=\left[\mathbf{y}(t)^T\ \mathbf{y}(t-1)^T\cdots\ \mathbf{u}(t)^T\ \mathbf{u}(t-1)^T\cdots\ \hat{\boldsymbol{\Phi}}(t)^T\ \cdots\right]^T \tag{4.2.8}$$

$$\hat{\Theta}(t)=\left[\hat{\alpha}_0\ \hat{\alpha}_1\ \cdots\ \hat{\beta}_0\ \hat{\beta}_1\cdots\ \hat{\eta}_1\ \hat{\eta}_2\cdots\right]^T \tag{4.2.9}$$

For the regulator problem the GMV control sets,

$$\hat{\boldsymbol{\Phi}}(t+1)=\mathbf{S}(1)\mathbf{y}_d \tag{4.2.10}$$

thus if substituting $\hat{\Phi}(t)$, $\hat{\Phi}(t-1)$, $\cdots$ by 1 in Eqn.(4.2.8), and consequently substituting $\hat{\eta}_1$, $\hat{\eta}_2$, $\cdots$ by a column vector $\hat{\delta}$ in Eqn.(4.2.9), then $\hat{\delta}$ satisfies,

$$\hat{\boldsymbol{\delta}}=[\mathbf{I}-\mathbf{C}(1)]\mathbf{S}(1)\mathbf{y}_d \qquad (4.2.11)$$

and it is a constant vector. This means that for the regulator problem the adaptive GMV algorithm developed in a stochastic framework coincides with the deterministic algorithm with a "one" in the data vector to account for dc offsets. This ends the proof of Lemma 4.2.1.

Proposition 4.2 The synchronous generator system using the multivariable GMV self-tuning controller with a variable forgetting factor as designed by Eqns.(3.28) through (3.30) is stable provided Assumption 4.1.2 is satisfied. ΔΔΔ

[Proof]

Noting that the synchronous generator system has a unit time-delay diagonal interactor matrix $q^{-1}\mathbf{I}$ (or in other words "with no pure time delay") in discrete time domain, and combining Theorem 4.1 and Lemma 4.2.1, one comes to the result given in this proposition.

CHAPTER 5

SIMULATION STUDIES OF MULTIVARIABLE SELF-TUNING CONTROL TO A THERMAL SYNCHRONOUS GENERATOR SYSTEM

5.1 Introduction

The algorithm of the multivariable self-tuning controller and its convergent property have been discussed in Section 3.3 and Chapter 4. It is proved that a power system with multivariable GMV self-tuning controller is stable in a stochastic environment provided polynomial matrices $\mathbf{S}(q^{-1})$ and $\mathbf{Q}(q^{-1})$ are carefully chosen to meet Assumption 4.1.2. In this chapter an extensive simulation study of this self-tuning algorithm will be undertaken. The computer simulation of a SMIB system is based on the non-linear model discussed in Chapter 2. A single-line diagram of the simulated SMIB system is shown in Figure 5.1.1, where the local bus B_l of a thermal-turbine-synchronous-generator is connected to an infinite bus B_i through two parallel short transmission lines. In Figure 5.1.1, b_1~b_4 are breakers. The parameters of the simulated SMIB system are given in Appendix A.

The installation of the multivariable GMV self-tuning controller to this SMIB system is shown in Figure 5.1.2. The

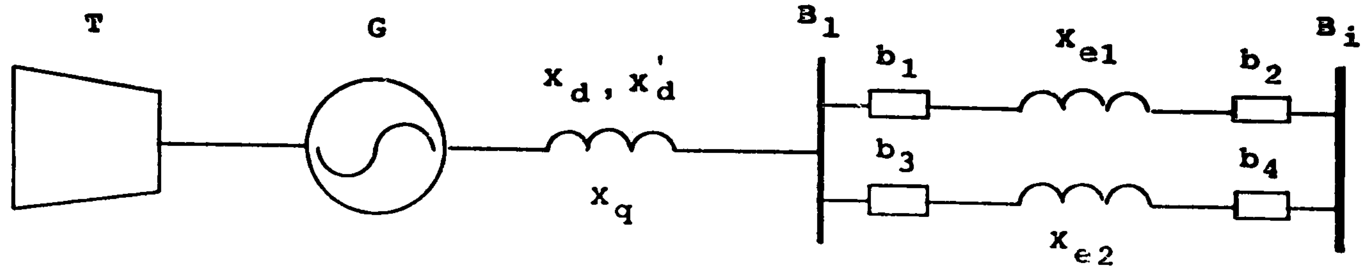

Fig. 5.1.1 Diagram of simulated SMIB system

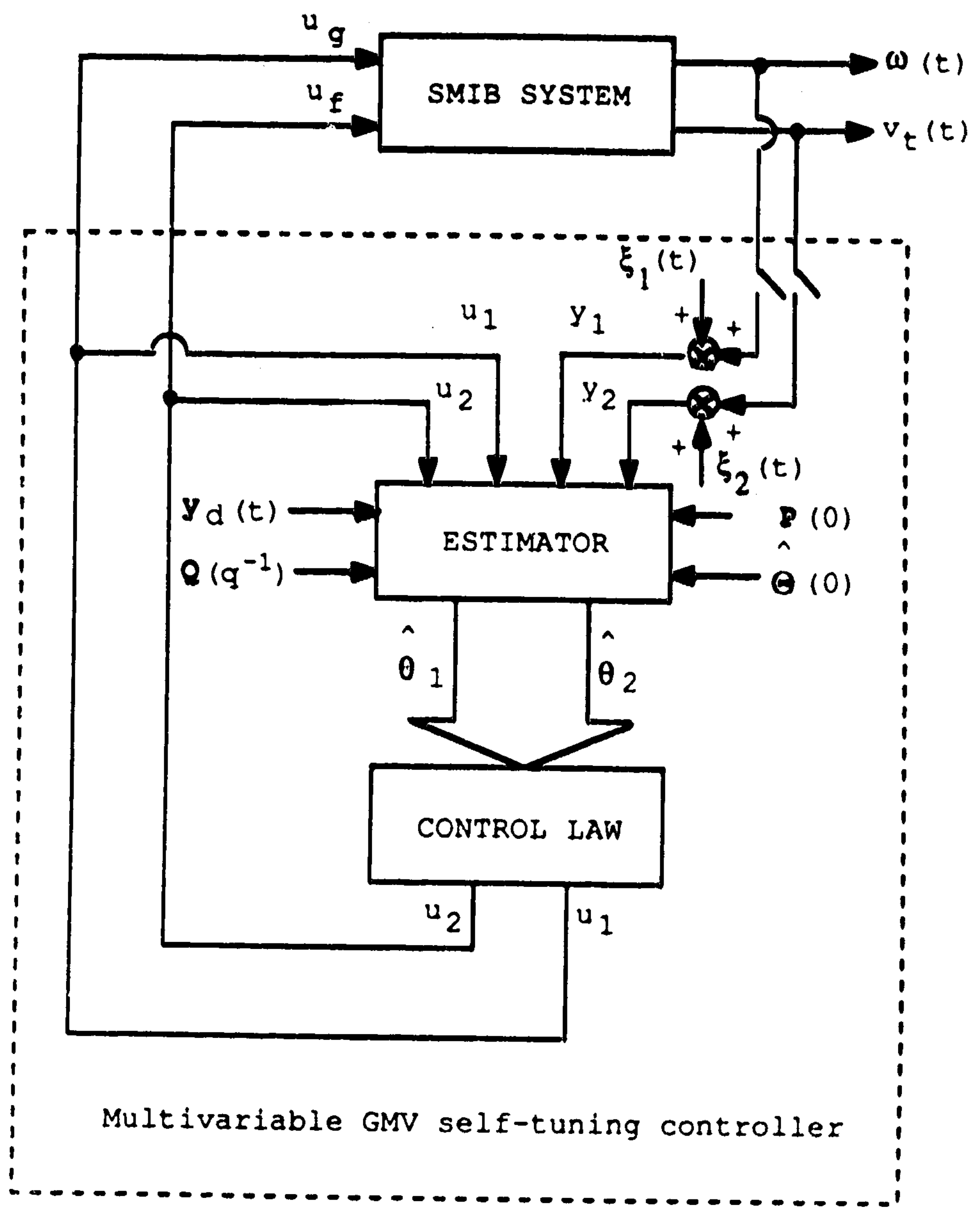

Fig. 5.1.2 Block diagram of simulated multivariable STR of SMIB system.

outputs $\omega(t)$ and $v_t(t)$ are sampled and two uncorrelated Gaussian noises $\xi_1(t)$ and $\xi_2(t)$ are added to them, and fed into the estimator at every sampling instant. The estimated parameter vectors $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$ are used to calculate the control actions $u_g(t)$ and $u_f(t)$. The signals $u_g(t)$ and $u_f(t)$ are used to control the turbine valve and the field voltage respectively. At the same time, they are fed into the estimator to renew the data vector. The initial conditions $\mathbf{P}(0)$ and $\hat{\Theta}(0)$ are used to start the algorithm. The command signal $\mathbf{y}_d(t)$, e.g., $\omega_r(t)$ and $v_r(t)$, is read by the estimator at every sampling step.

Simulation studies will include the effects of different weighting functions $\mathbf{Q}(q^{-1})$, and the behaviors of the adaptive controlled system to various load disturbances, e.g., three phase fault, transmission line switching and electrical power disturbance.

A comparison with conventional controllers will also be made. To make the comparison between adaptive and conventional systems, the same SMIB system is used for both cases. Figure 5.1.3 depicts the diagram of the conventional power system with properly tuned GOV and AVR. The parameters of GOV and AVR are given in Appendix A.

During the simulations, a sampling time T=0.05 sec. is used. Gaussian random noise vector $\{\xi(t)\}$ with zero mean value and $Cov\{\xi(t)\}$=diag(0.0001,0.0001) is generated by using the Fortran subroutine GGNSM of the International Mathematical and Statistical Library (IMSL).

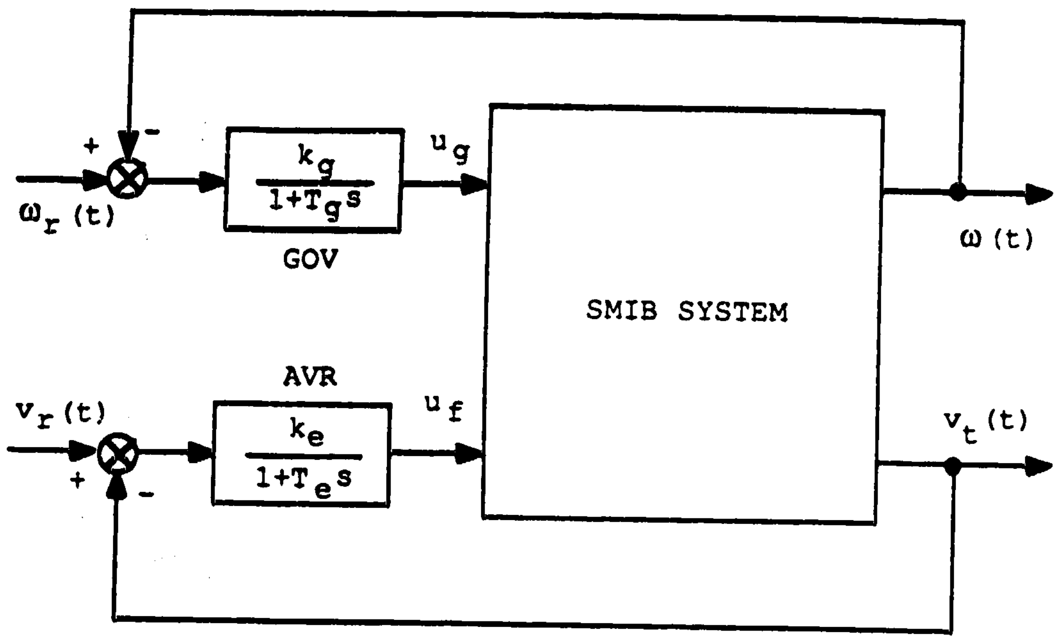

Fig. 5.1.3 Block diagram of simulated conventional SMIB system .

5.2 The Effect of Different Weighting Matrix $Q(q^{-1})$ on Steady State Error

5.2.1 Introduction

It has been mentioned in Section 3.3.2 that the choice of weighting matrix $\mathbf{Q}(q^{-1})$ will affect the steady-state error of the outputs. In this section the simulation results will be used to show the previous analysis.

5.2.2 Weighting Matrix $Q(q^{-1})$ for Equivalent Integral Action

The simulation results shown in Figure 5.2.1 are obtained by using a ±0.1 p.u. step change in $v_r(t)$ at operating point P=0.8 p.u., Q=0.3 p.u.. Initial values of the covariance matrix and parameter vectors are taken to be the same as those introduced in Section 3.3.5. The polynomial matrix $\mathbf{Q}(q^{-1})=\text{diag}[0.1,\ 0.01(1-q^{-1})]$.

As mentioned previously, if integration action is to be installed in the excitation loop, $\mathbf{Q}(q^{-1})$ should take the form of Eqn.(3.3.13). Thus there will be no steady-state error existing in $v_t(t)$ for step change in reference signal $v_r(t)$. This is shown by the simulation result in Figure 5.2.1b.

The estimated parameters of $\hat{\alpha}_i(t)$, $\hat{\beta}_i(t)$, $\hat{h}_i(t)$, the variable forgetting factor $\lambda(t)$ and the trace of $P(t)$ matrix are also given in Figure 5.2.1. It can be observed that the algorithm has good convergence properties for parameter estimation, and $\lambda(t)$ keeps varying along with the prediction error.

It should be pointed out here that though the identification shown in Figure 5.2.1 was done with noises $\xi_1(t)$ and $\xi_2(t)$, it does not mean that the identification could be only performed with the applied signals $\xi_1(t)$ and $\xi_2(t)$. It can be done in a deterministic environment, i.e., with $\xi_1(t)=\xi_2(t)=0$, as long as there exists system disturbances, such as changes in reference signals, or load disturbances. The identification with system disturbances in a deterministic case will be shown in Section 5.4.

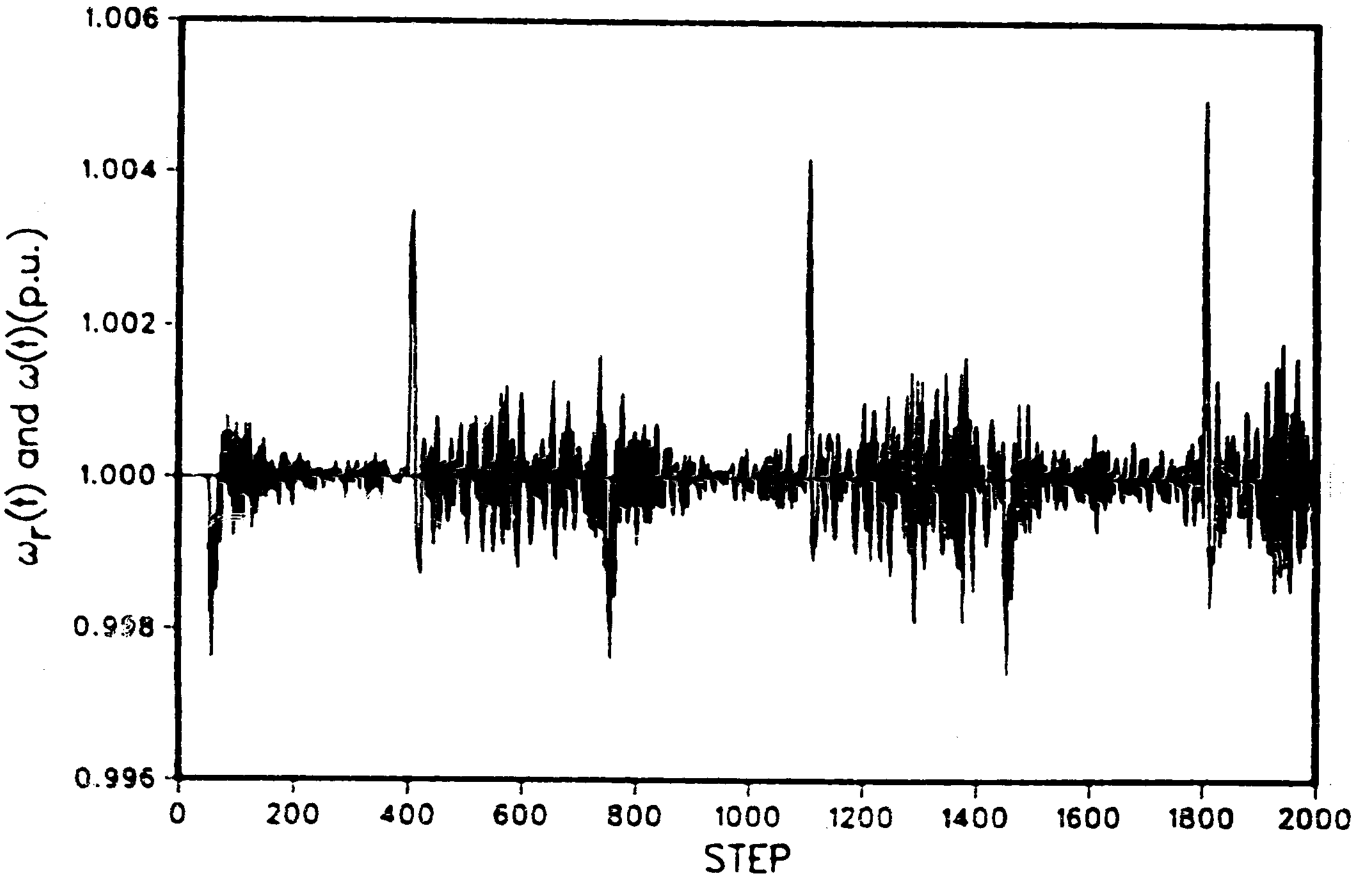

Fig. 5.2.1a Shaft speed reference signal and shaft speed.

Fig. 5.2.1 (a-o) Time responses of the SMIB system using GMV STC, subject to ±0.1 p.u. step change in $v_r(t)$. Operating point: P=0.8 p.u., Q=0.3 p.u..

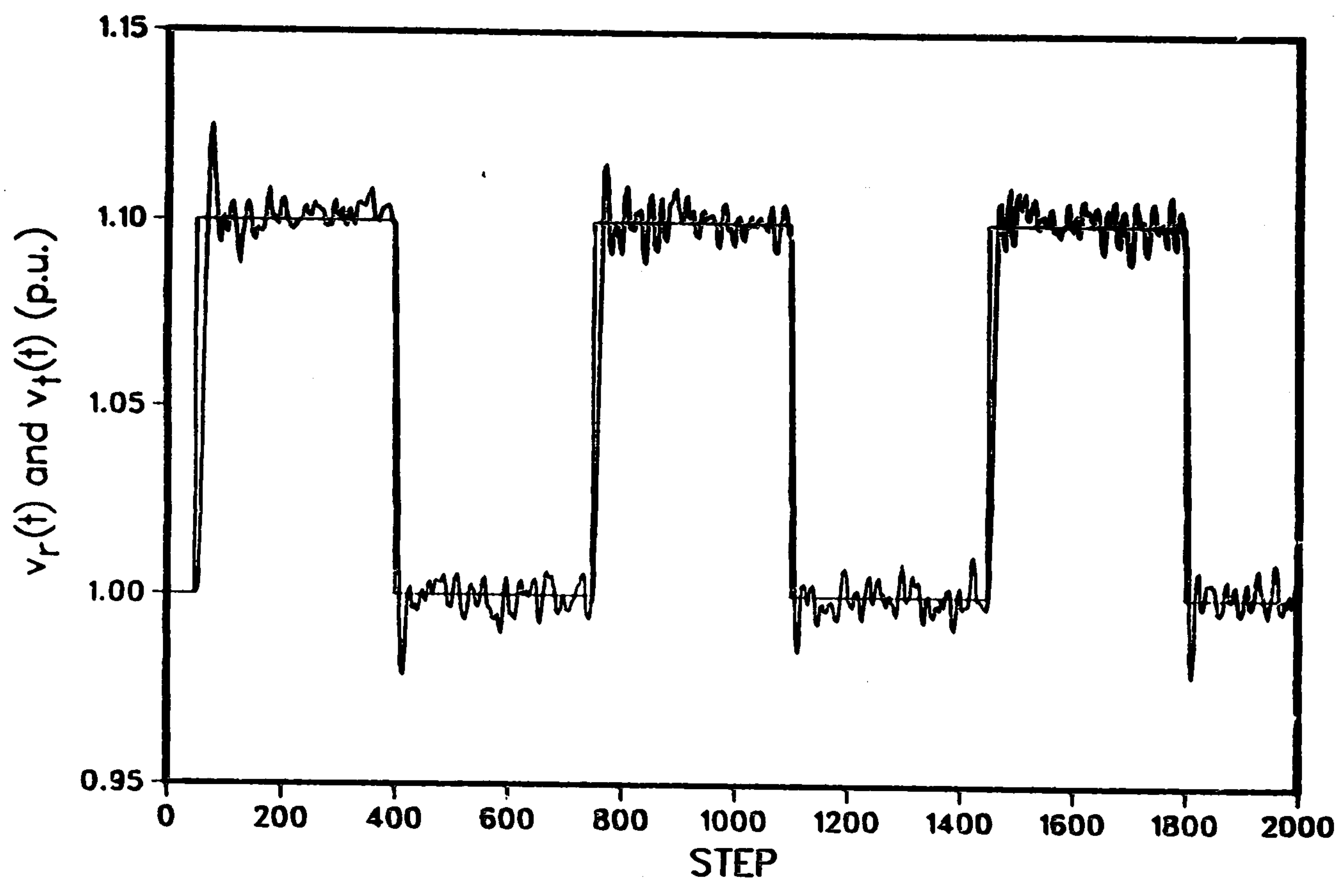

Fig. 5.2.1b Terminal voltage reference signal and terminal voltage.

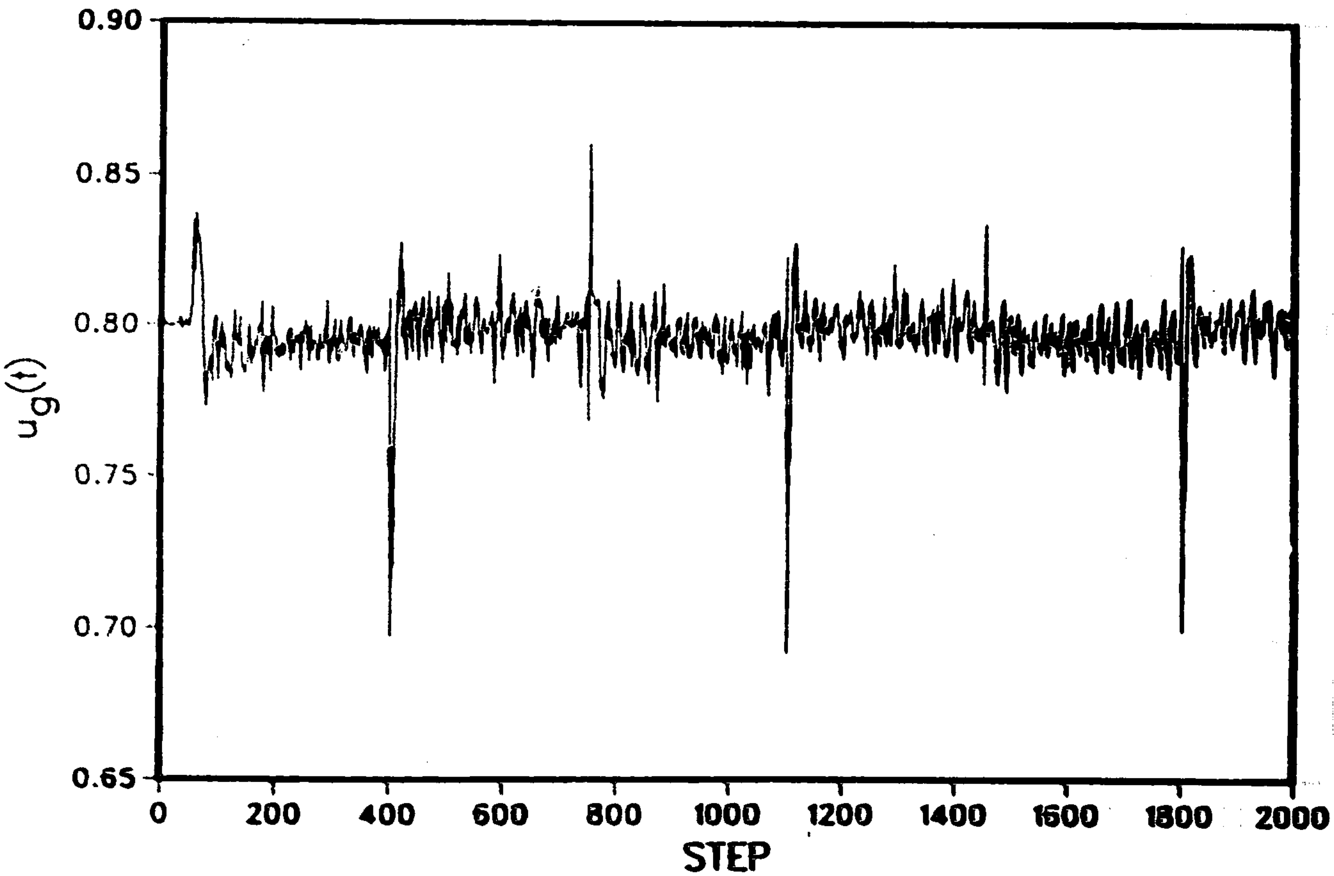

Fig. 5.2.1c Control $u_g(t)$.

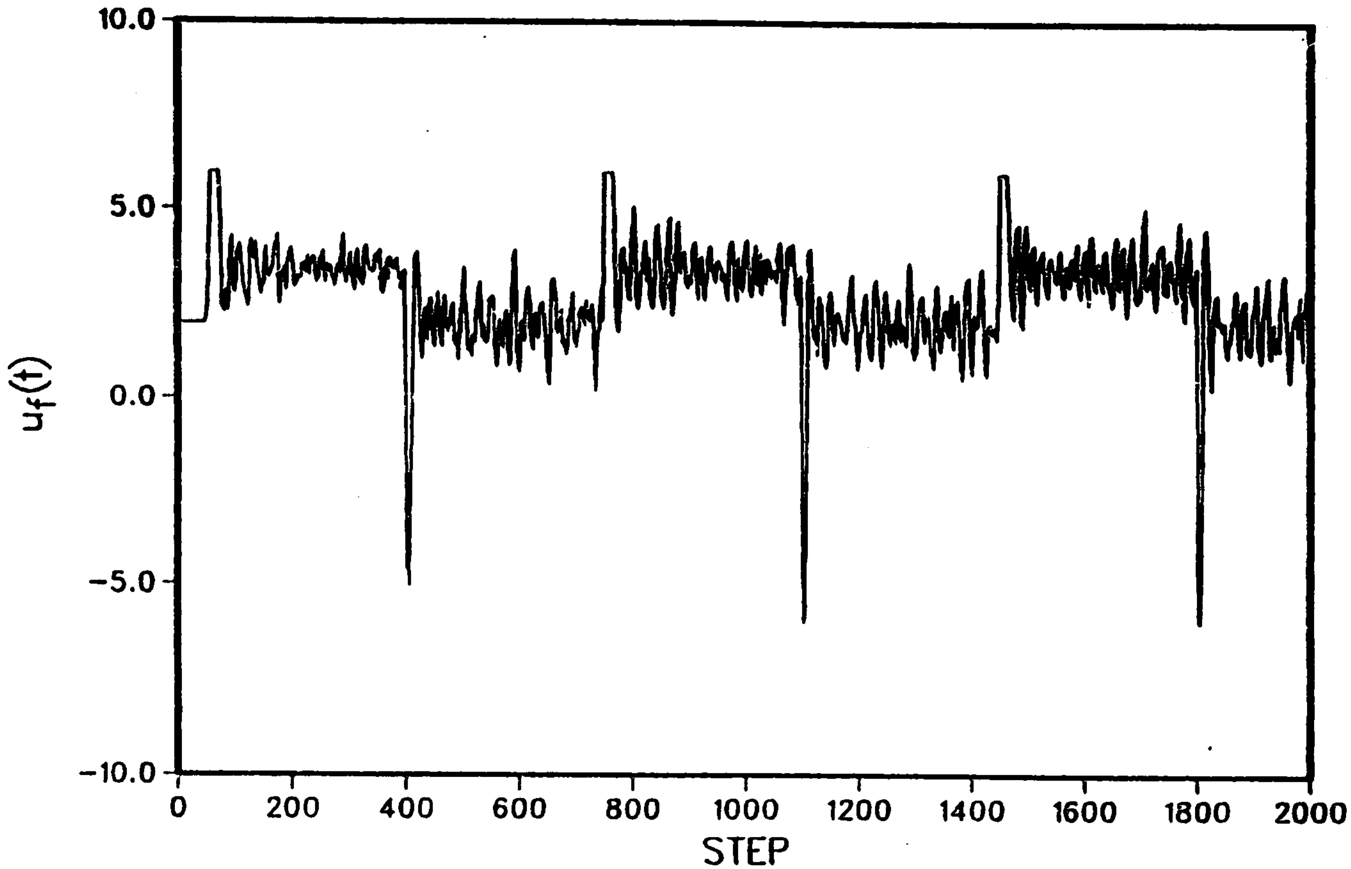

Fig. 5.2.1d Control $u_f(t)$.

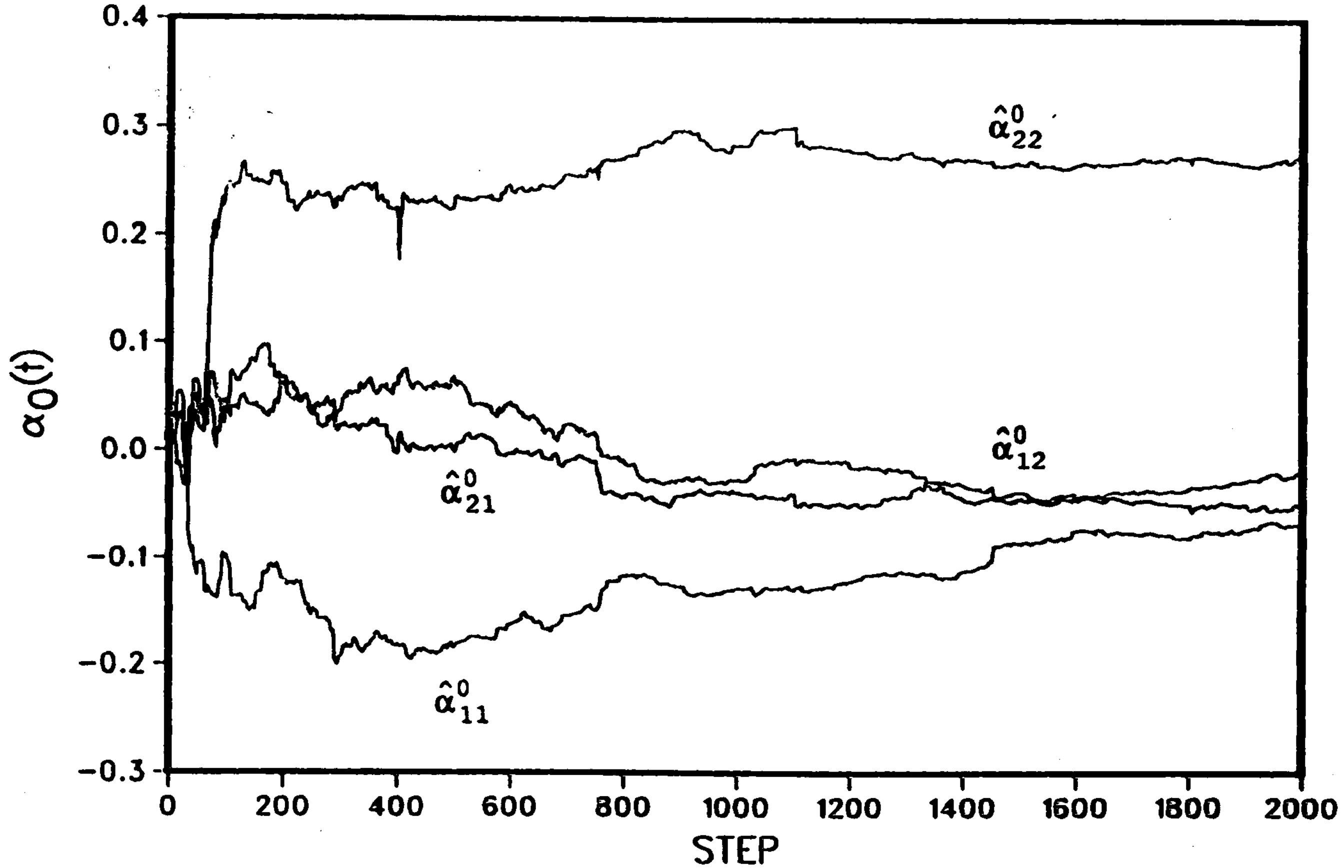

Fig. 5.2.1e Estimation of $\hat{\alpha}_0$.

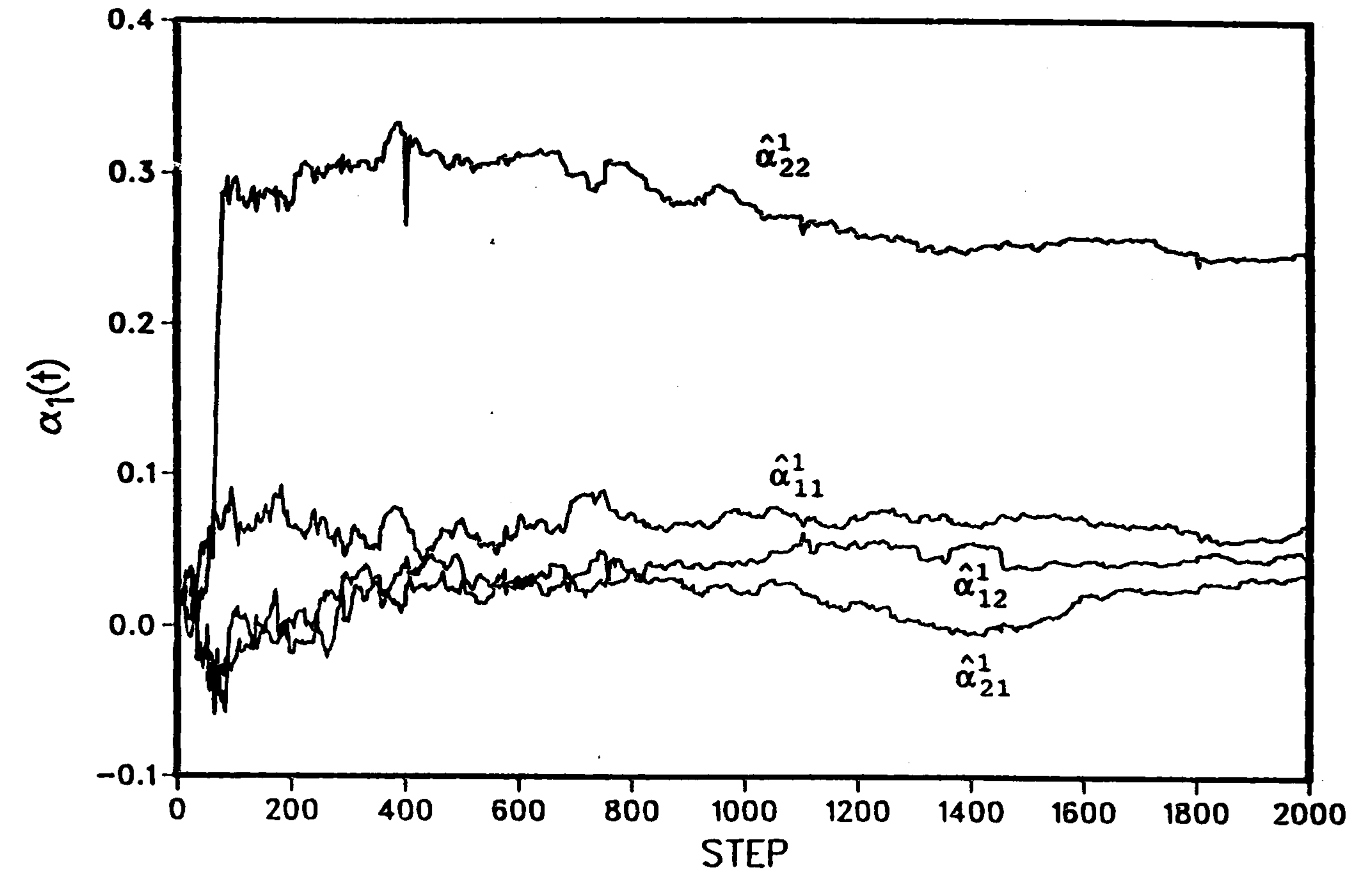

Fig. 5.2.1f Estimation of $\hat{\alpha}_1$.

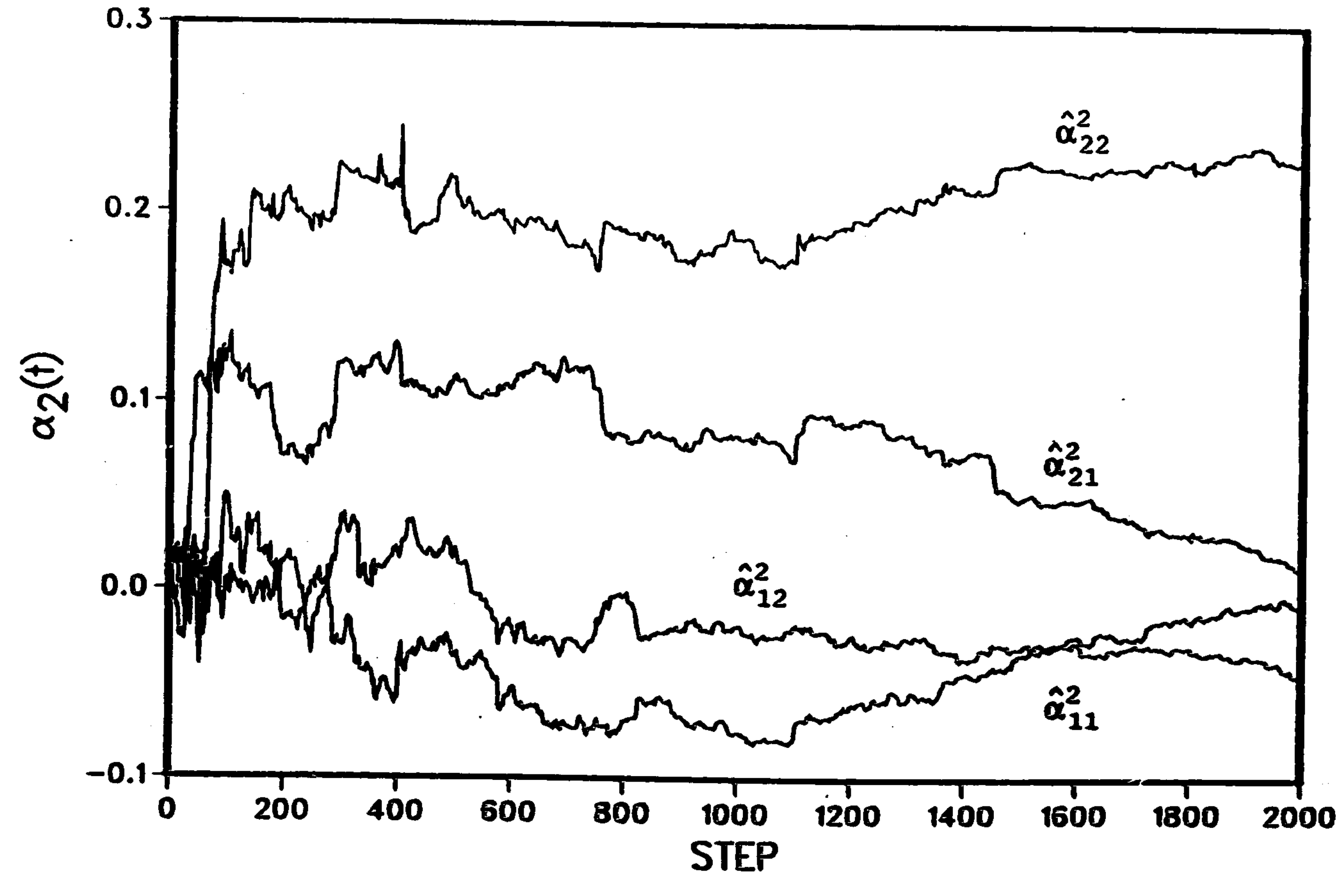

Fig. 5.2.1g Estimation of $\hat{\alpha}_2$.

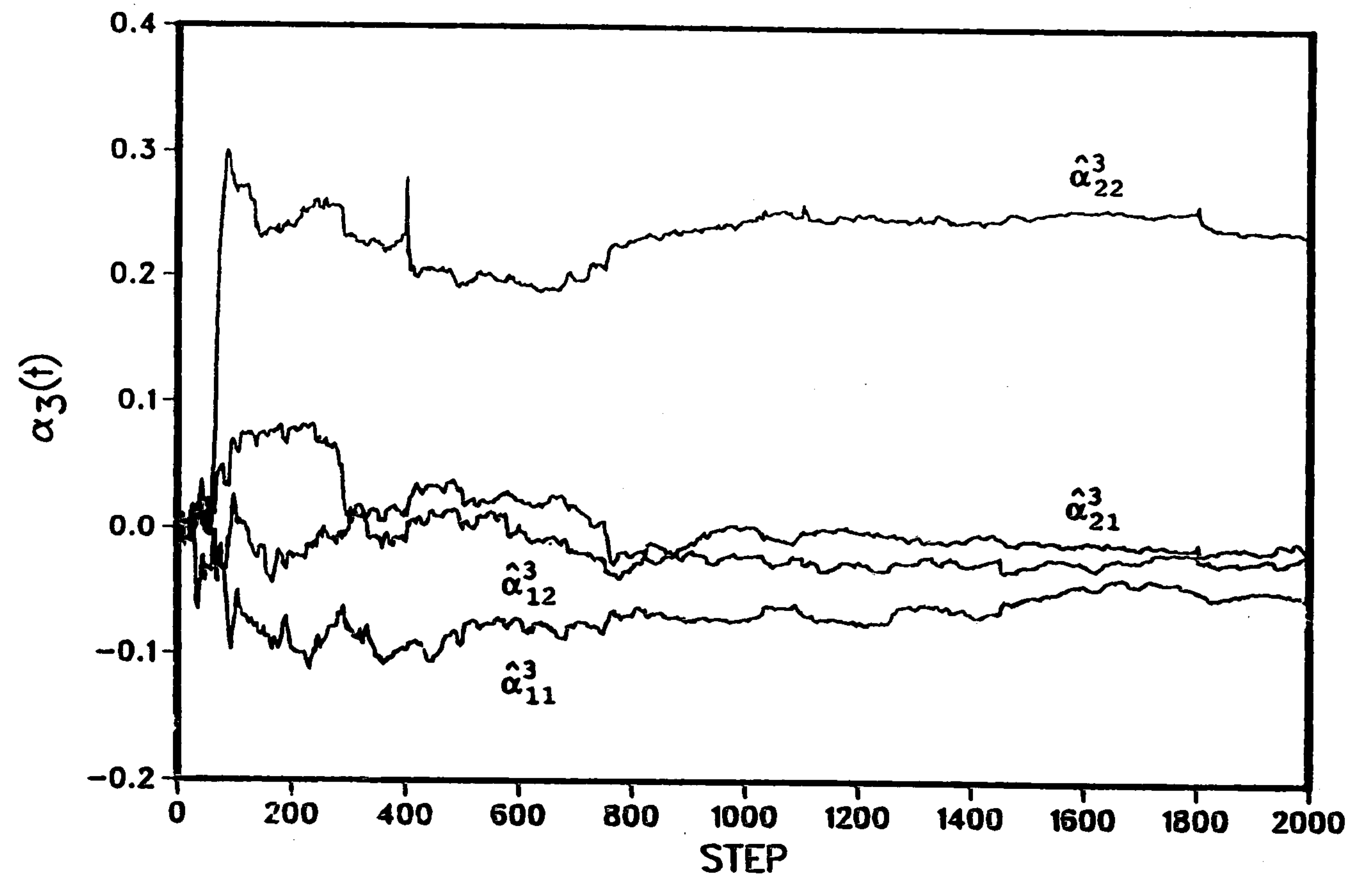

Fig. 5.2.1h Estimation of $\hat{\alpha}_3$.

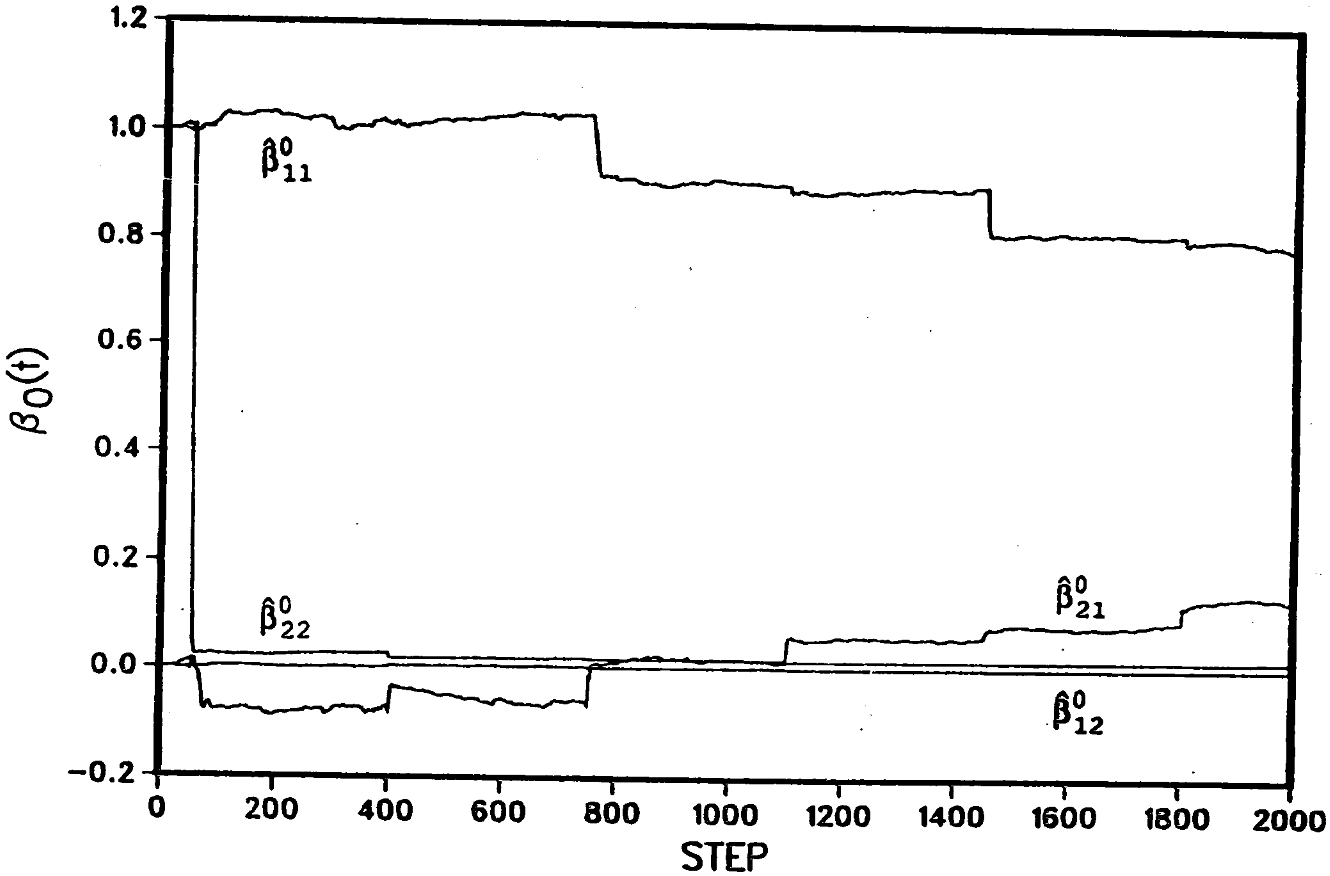

Fig. 5.2.1i Estimation of $\hat{\beta}_0$.

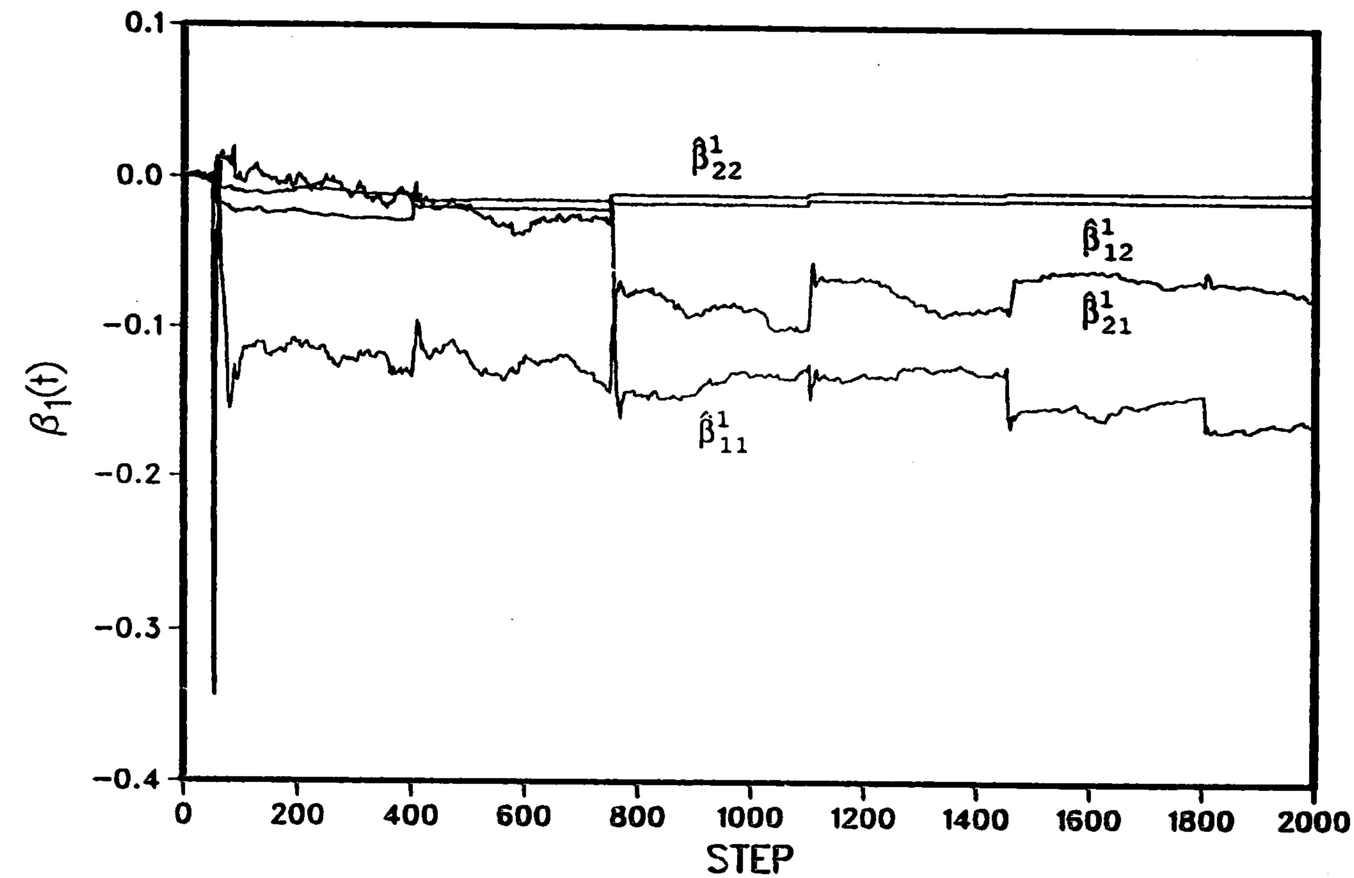

Fig. 5.2.1j Estimation of $\hat{\beta}_1$.

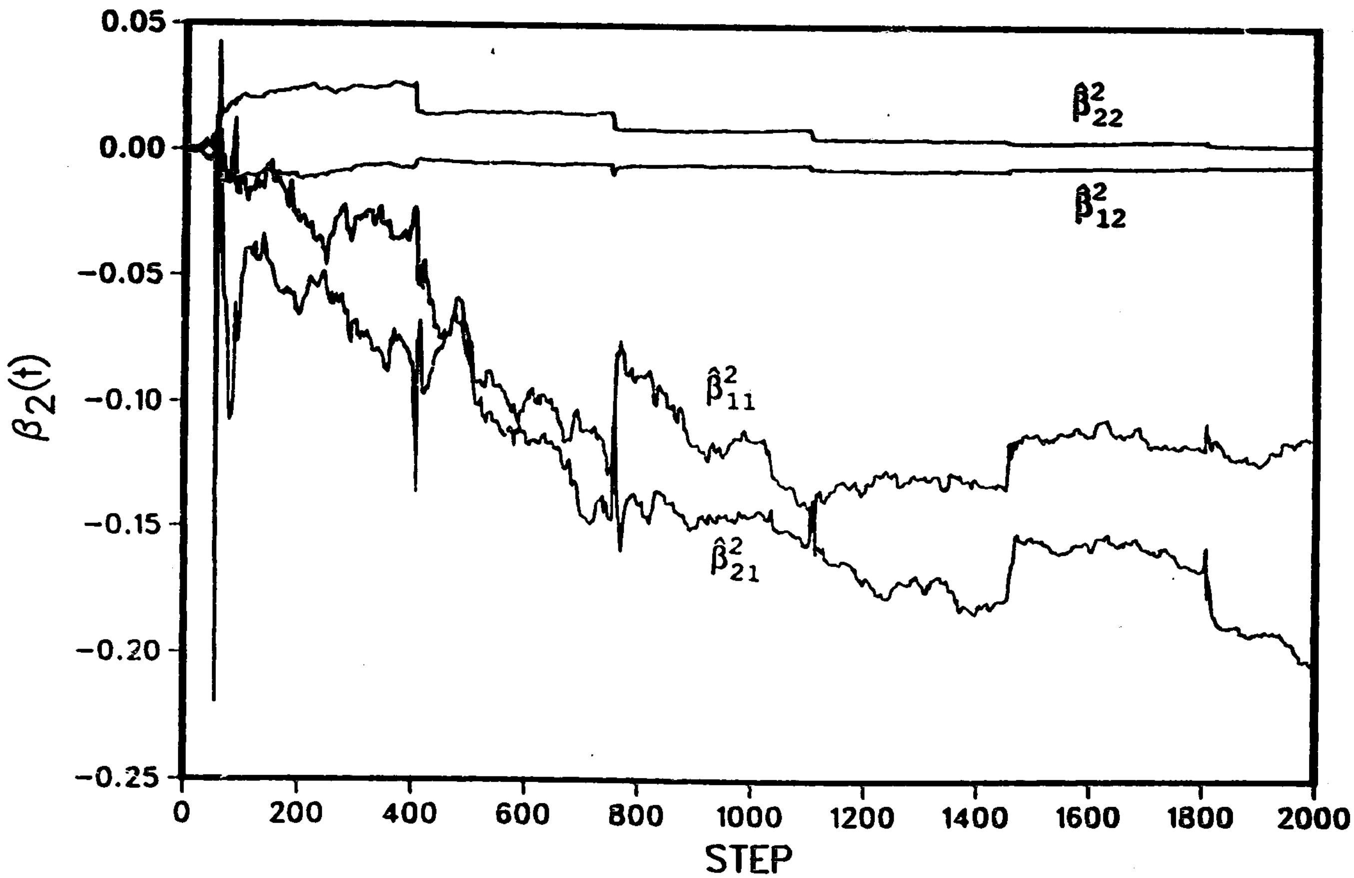

Fig. 5.2.1k Estimation of $\hat{\beta}_2$.

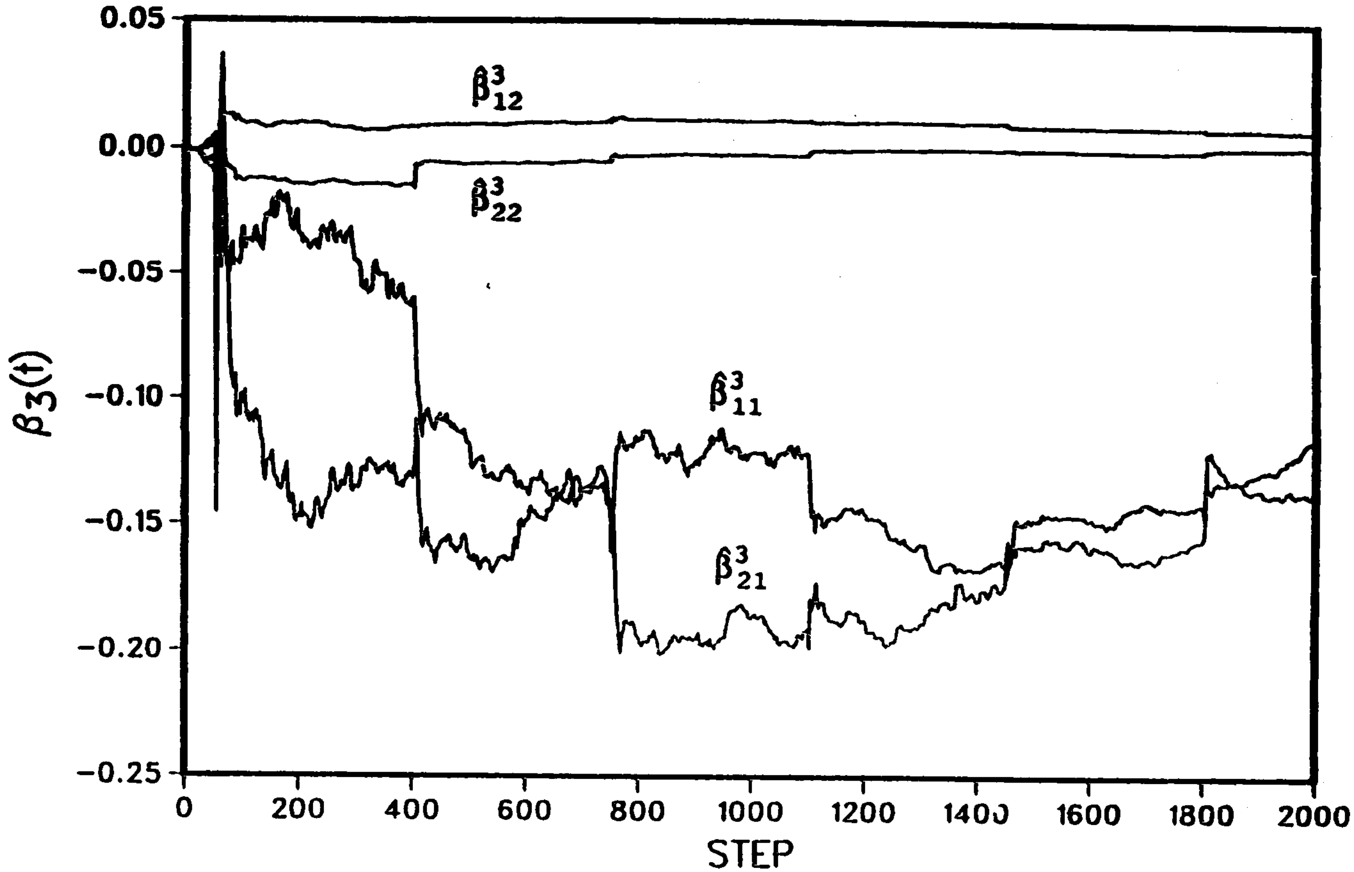

Fig. 5.2.11 Estimation of $\hat{\beta}_3$.

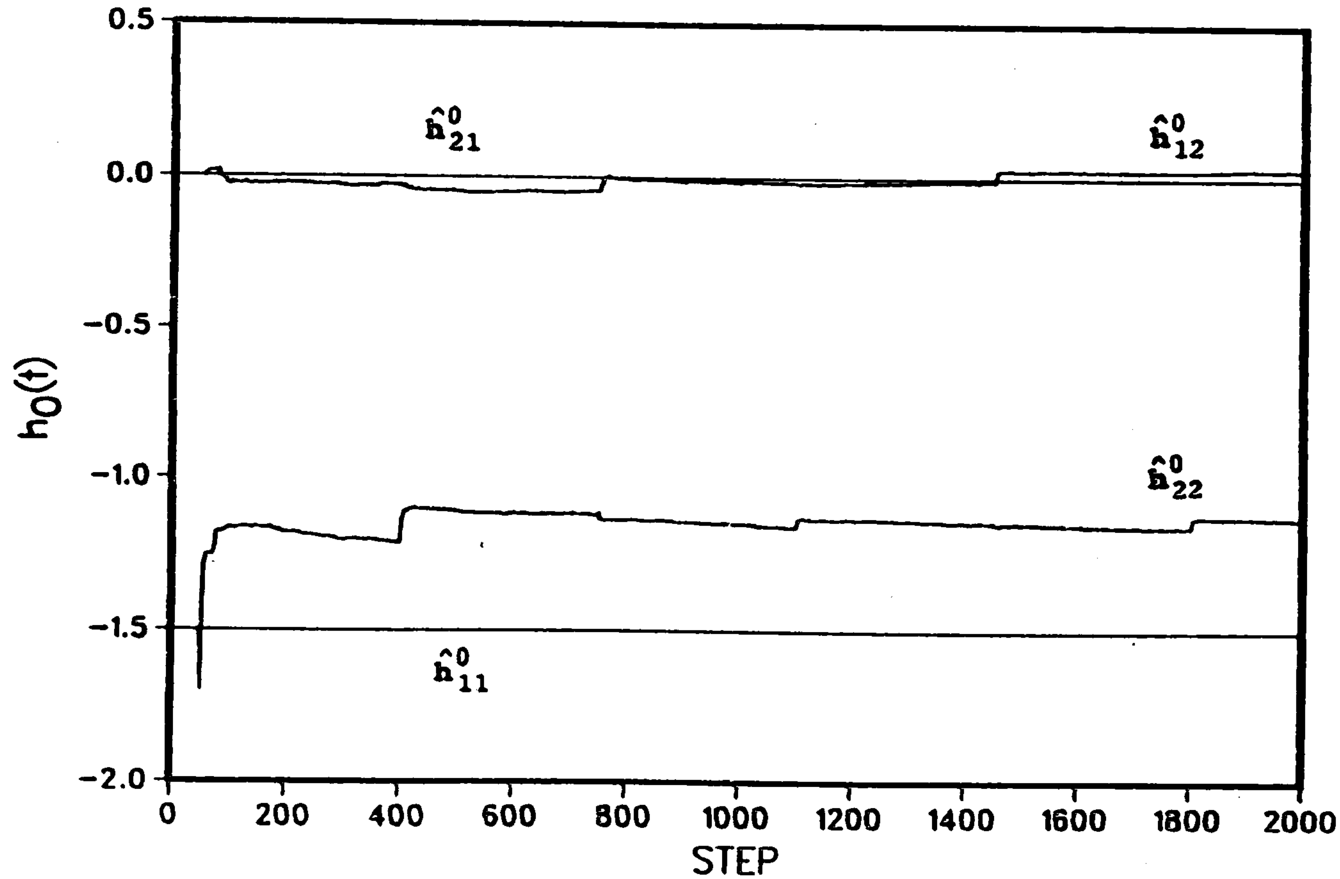

Fig. 5.2.1m Estimation of $\hat{h}_0$.

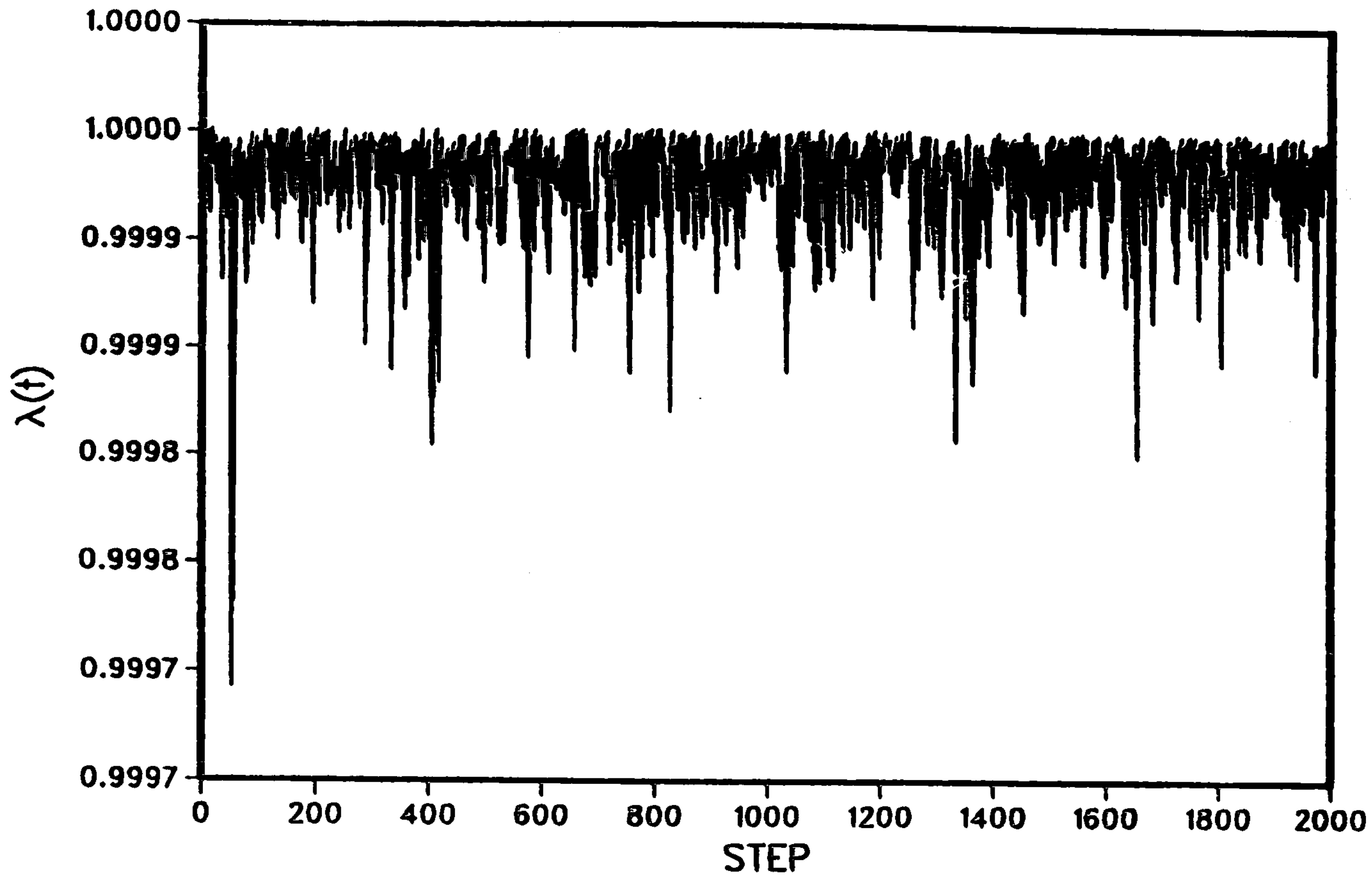

Fig. 5.2.1n Forgetting factor $\lambda(t)$.

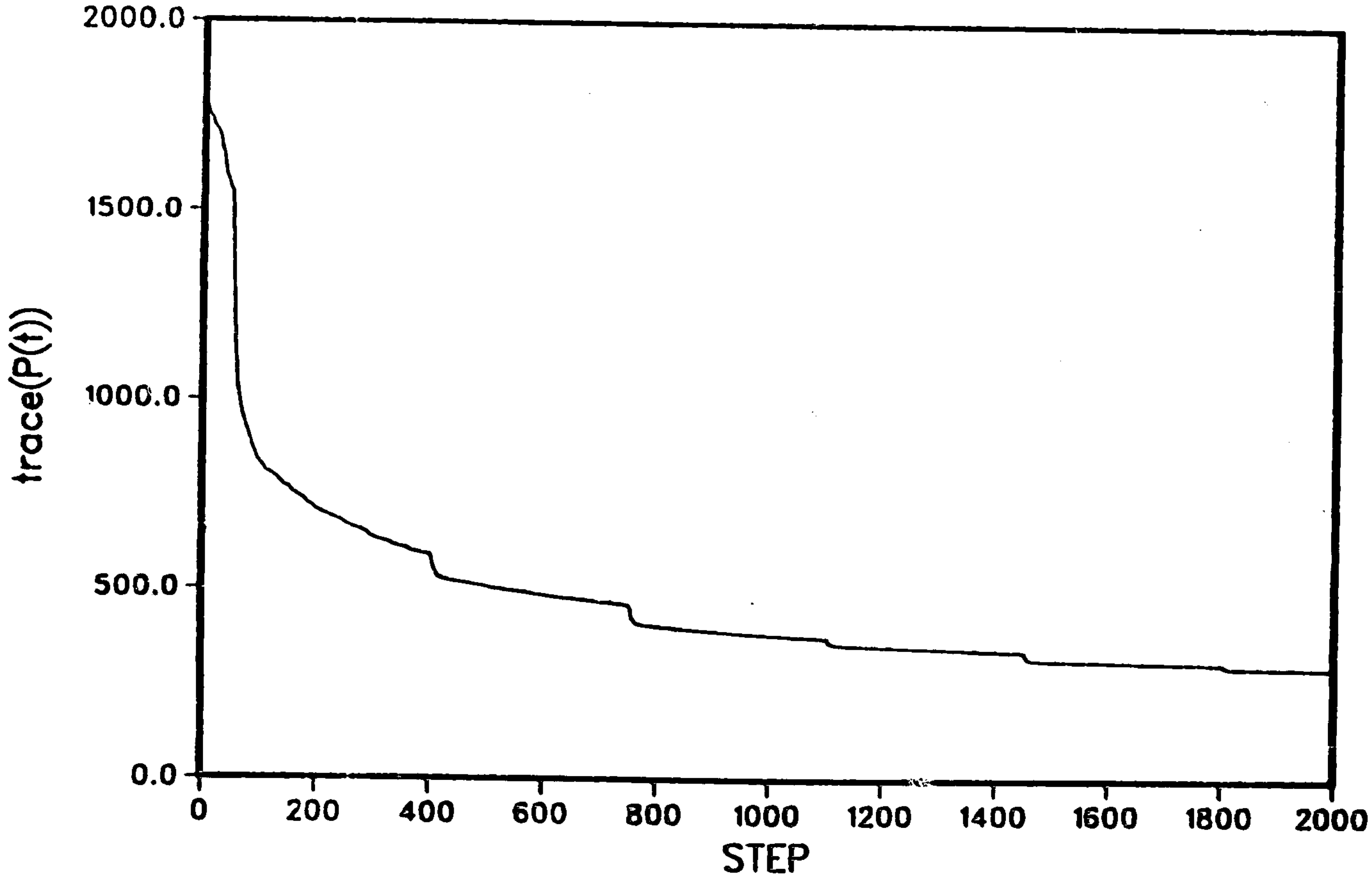

Fig. 5.2.1o Trace of covariance matrix.

5.2.3 Weighting Matrix $Q(q^{-1})$ Without Integration Action

If $Q(q^{-1})$ takes the form of Eqn.(3.3.4), there will be no integration action in the excitation loop. Therefore, a steady-state error in $v_t(t)$ is expected for a step change in $v_r(t)$. Figure 5.2.2 shows the simulation results, where $Q(q^{-1})$=diag(0.1, 0.01). All running conditions are the same as in Figure 5.2.1, i.e., the operating point is at P=0.8 p.u., Q=0.3 p.u., sampling time T=0.05 sec., and the same initial values as given in Section 3.3.5. By comparing the terminal voltage $v_t(t)$ with reference signal $v_r(t)$ in Figure 5.2.2b, one can notice the steady-state error in $v_t(t)$.

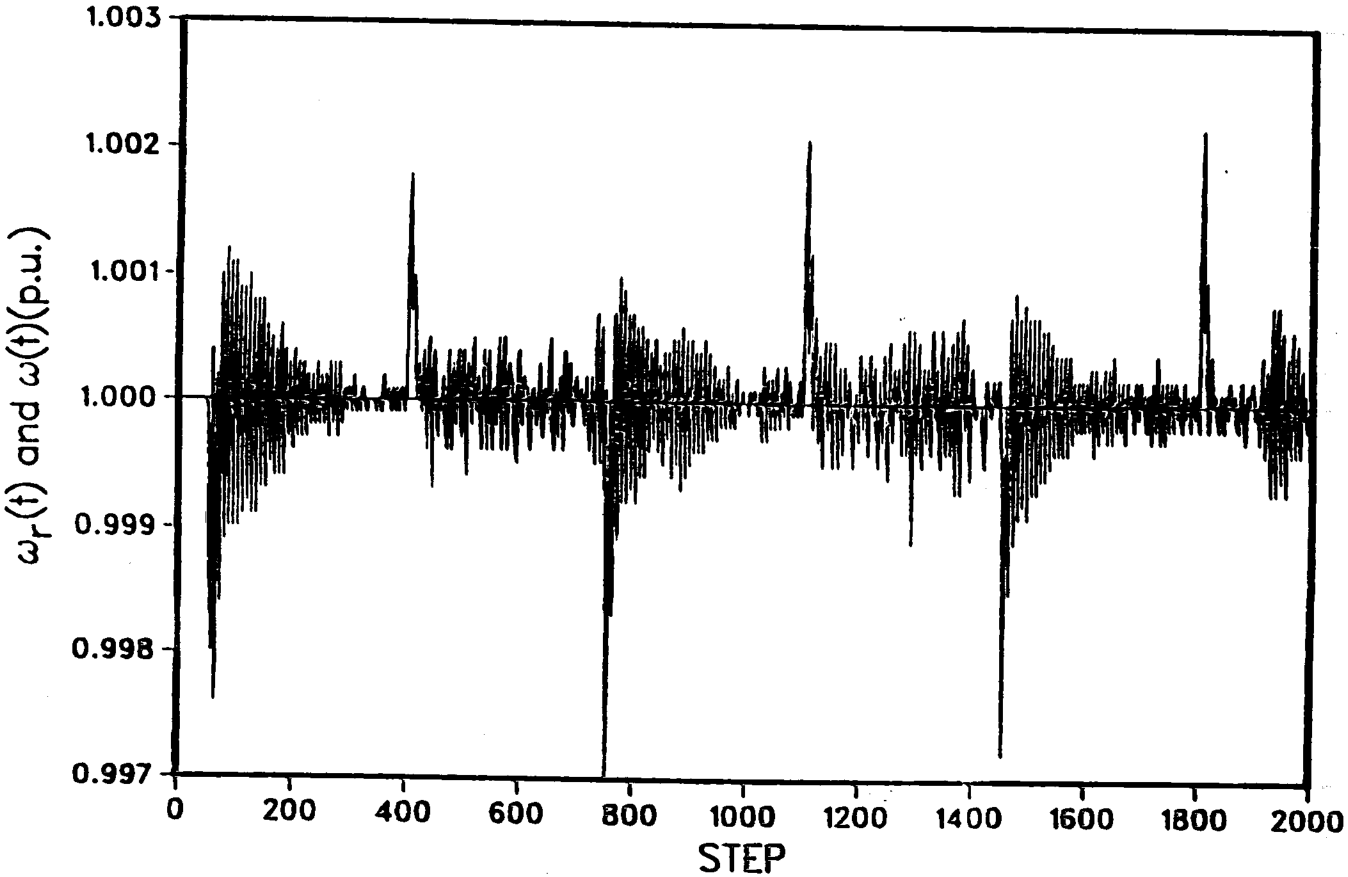

Fig. 5.2.2a Shaft speed reference signal and shaft speed.

Fig. 5.2.2 (a~b) Time responses of the SMIB system using GMV STC without integrator in excitation loop, subject to ±0.1 p.u. step change in $v_r(t)$. Operating point: P=0.8 p.u., Q=0.3 p.u..

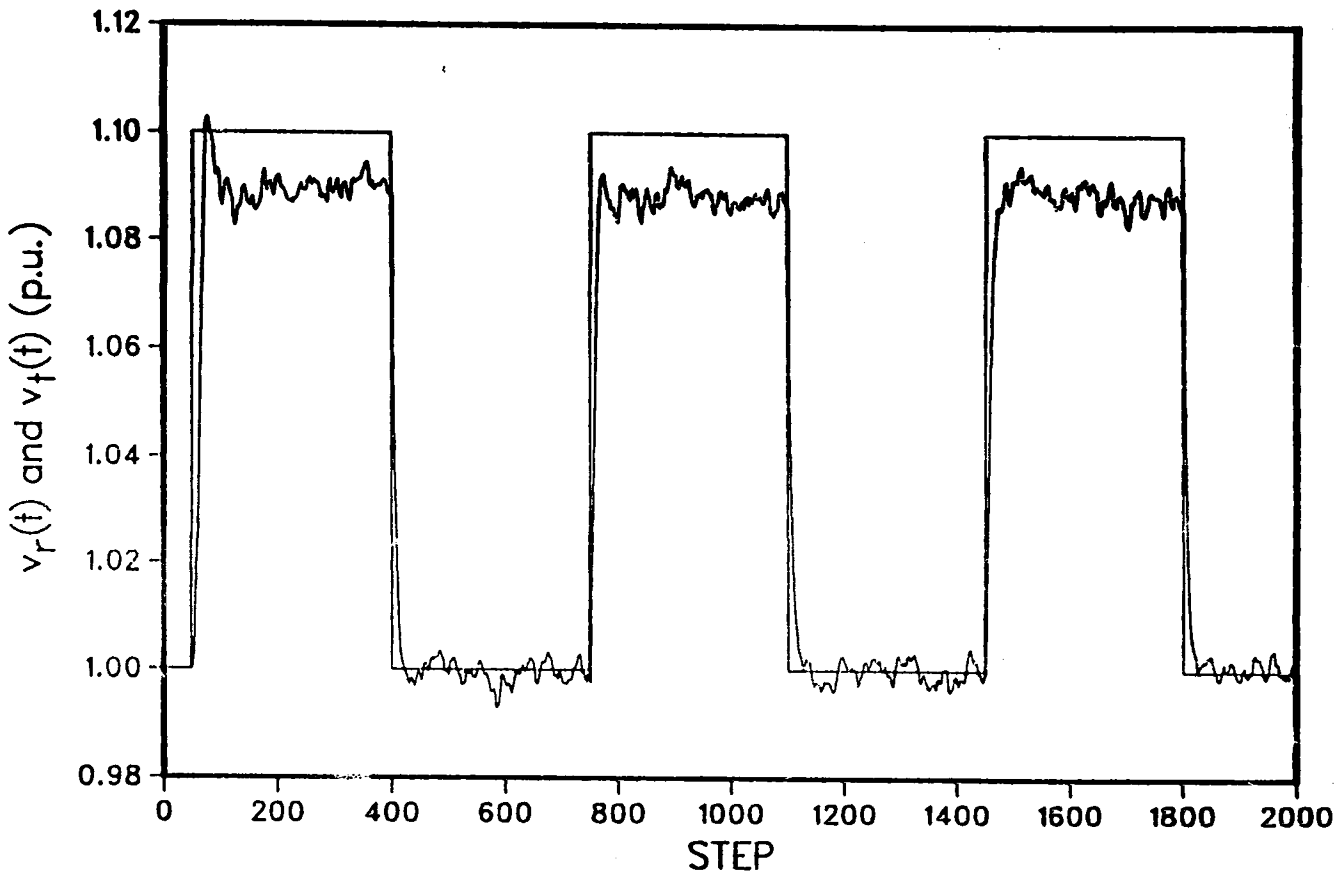

Fig. 5.2.2b Terminal voltage reference signal and terminal voltage.

5.2.4 Comments

In this section, two simulation studies were done. They were used to show the effect on the steady-state error of terminal voltage $v_t(t)$ by choosing different weighting matrix $\mathbf{Q}(q^{-1})$. Figure 5.2.1 shows the control result with equivalent integration action in $\mathbf{Q}(q^{-1})$, while Figure 5.2.2 shows the control result without equivalent integration action in $\mathbf{Q}(q^{-1})$. Since there is no steady-state error existing in terminal voltage $v_t(t)$ for a step change in reference signal $v_r(t)$ in Figure 5.2.1, this type of control action may be beneficial provided that the excitation-loop bandwidth is maintained. A multivariable GMV STC with integral control in terminal voltage loop will be used in the subsequent simulations.

5.3 The Regulating Ability of the GMV Self-Tuning Controller for System Disturbances

5.3.1 Introduction

There is a variety of disturbances existing in a power system. For example, mechanical torque disturbance, transmission line switching, three-phase fault, etc.. The term "system disturbance" is used to represent these disturbances in this thesis. In this section, two simulations will be undertaken to test the regulating ability of the multivariable GMV self-tuning controller of power system. One simulation is subjected to a step change in reference frequency signal $\omega_r(t)$, and another simulation is subjected to a three-phase fault applied at the generator bus. The reason for cataloging the system responses to a step change in $\omega_r(t)$ into system disturbance is that a SMIB system subject to a step change in $\omega_r(t)$ is equivalent to mechanical torque disturbance, as pointed out in Chapter 1.

The simulations in this section are done at operating point P=0.8 p.u., Q=0.3 p.u. as in the previous section. Sampling time is also taken as T=0.05 sec., and applied noise signals are the same as mentioned in Section 5.1.

5.3.2 Disturbance of Step Change in $\omega_r(t)$

It was pointed out in Chapter 1 that the controller (GOV) in the frequency loop works as a power regulator for the SMIB system. Can this characteristic be maintained by the multivariable GMV self-tuning controller? Simulation results shown in Figure 5.3.1 illustrate the control property of the GMV STC for a step disturbance in $\omega_r(t)$.

The time responses shown in Figure 5.3.1 are the records of the SMIB system with multivariable GMV self-tuning controller subject to ±0.01 p.u. step changes in frequency reference signal $\omega_r(t)$. The function of the GMV STC behaving as a power regulator can be seen from Figures 5.3.1a and 5.3.1c, where $\omega(t)$ is maintained at 1 p.u., and mechanical torque T_m shows ±0.09 p.u. step changes corresponding to step changes in $\omega_r(t)$. Terminal voltage $v_t(t)$ in Figure 5.3.1b shows that it is maintained at the level of reference signal $v_r(t)=1.0$ p.u.. Obviously, the self-tuning SMIB system is stable under the disturbance of step changes in $\omega_r(t)$, and it works as a power regulator as expected.

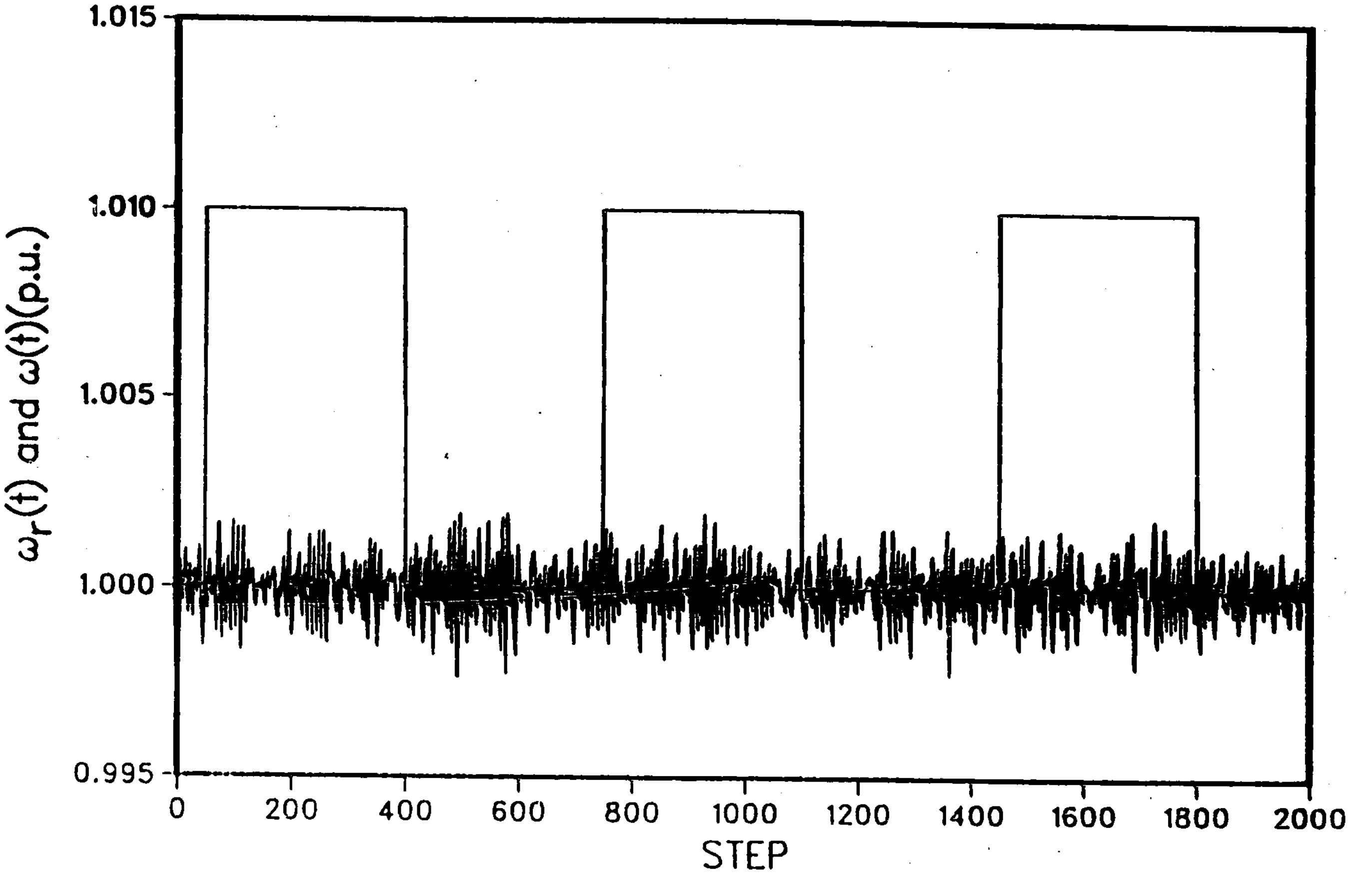

Fig. 5.3.1a $\omega_r(t)$ and $\omega(t)$.

Fig. 5.3.1 (a~c) Time responses of SMIB system with GMV STC subject to ± 0.01 p.u. step change in $\omega_r(t)$ at P=0.8 p.u., Q=0.3 p.u..

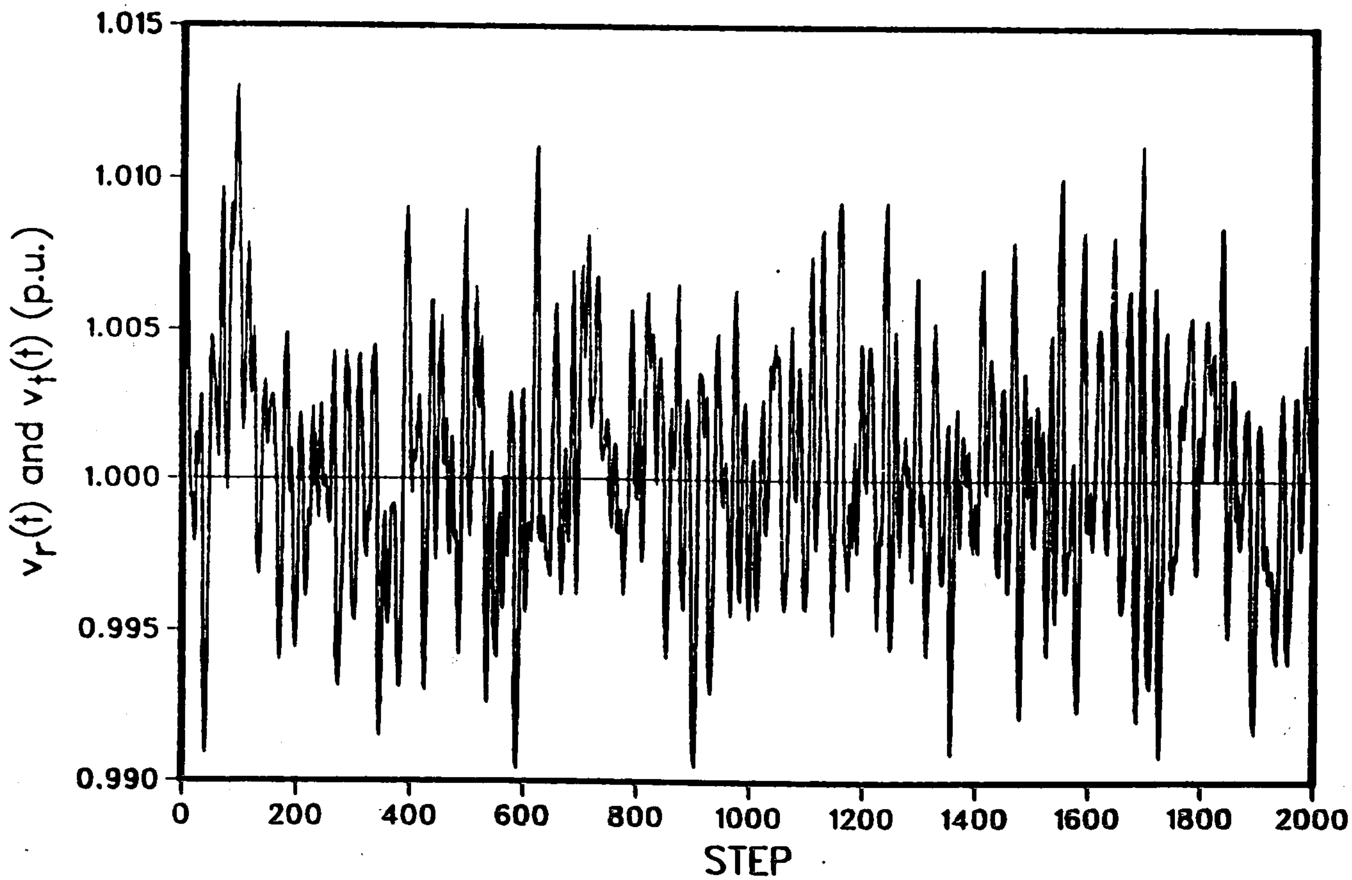

Fig. 5.3.1b Terminal voltage reference signal and terminal voltage.

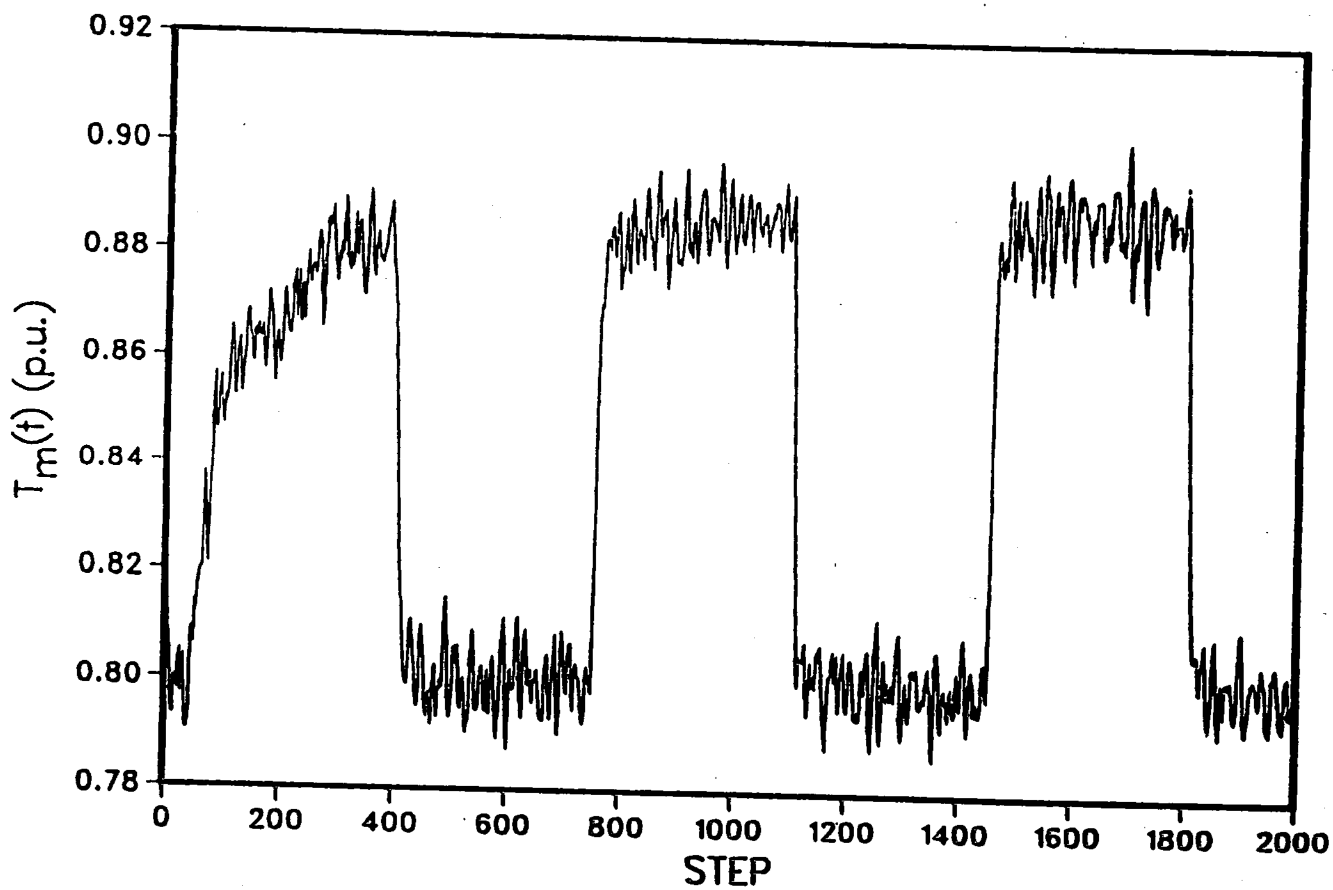

Fig. 5.3.1c Mechanical torque $T_m(t)$.

5.3.3 Three Phase Fault Disturbance

A three phase fault is applied at terminal voltage bus B_1, shown in Figure 5.1.1, in order to simulate the regulating ability of the multivariable GMV self-tuning controller. The fault occurs at sampling step 100 and lasts for 0.1 sec.. The operating point and running conditions are the same as mentioned in Section 5.3.1. Time responses of the three phase fault disturbance are shown in Figure 5.3.2. It shows that a stable control is achieved.

Figure 5.3.3 shows the time responses of $\omega(t)$ and $v_t(t)$ subject to the three-phase fault for the SMIB system with conventional GOV and AVR controllers under the same operating condition as used in Figure 5.3.2. It is evident that the self-tuning control system damped the shaft speed oscillation efficiently by comparing Figure 5.3.2a with Figure 5.3.3a.

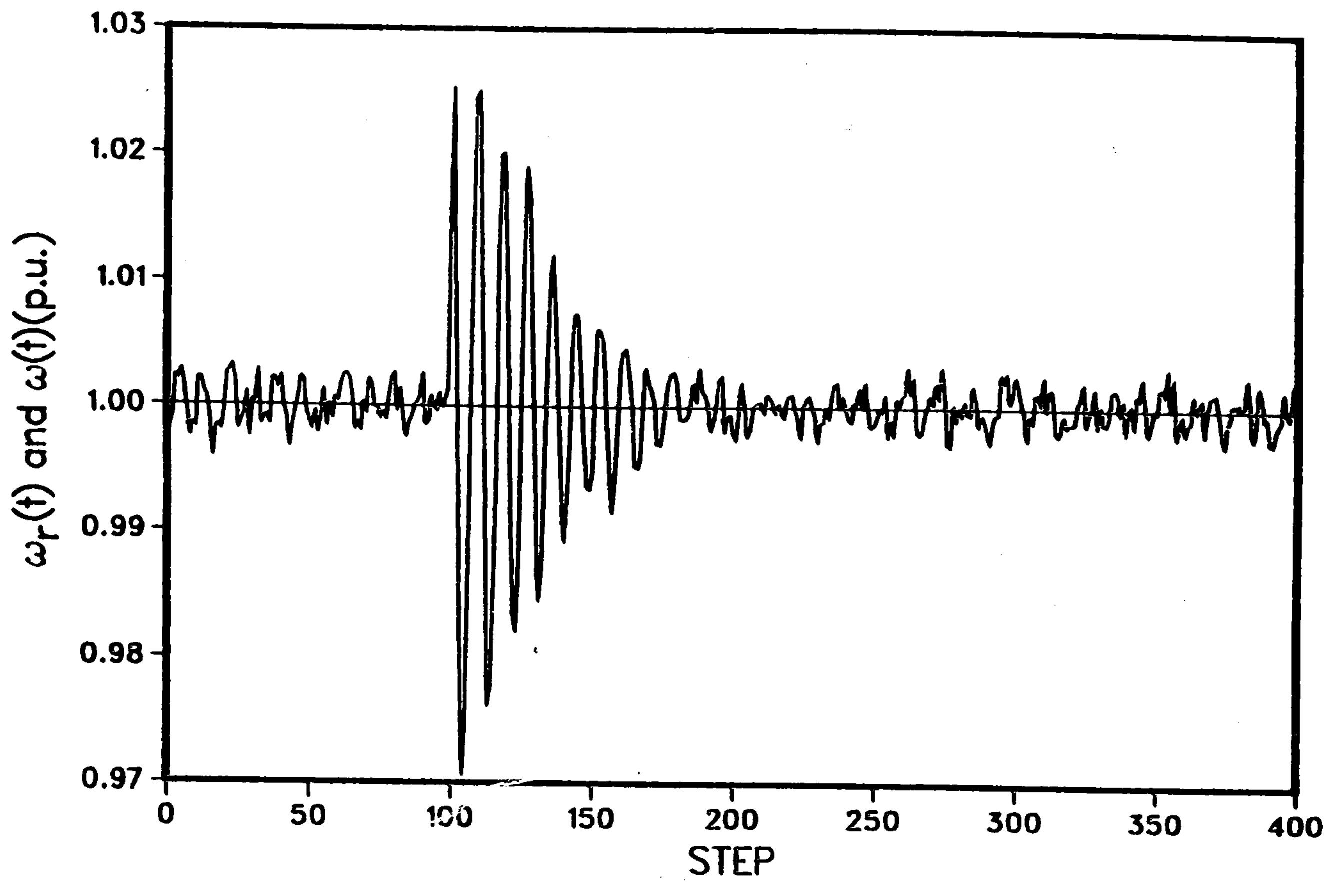

Fig. 5.3.2a $\omega_r(t)$ and $\omega(t)$ for self-tuning system.

Fig. 5.3.2 (a~b) Time responses of the SMIB system using GMV STC, subject to 3-phase fault at P=0.8 p.u., Q=0.3 p.u..

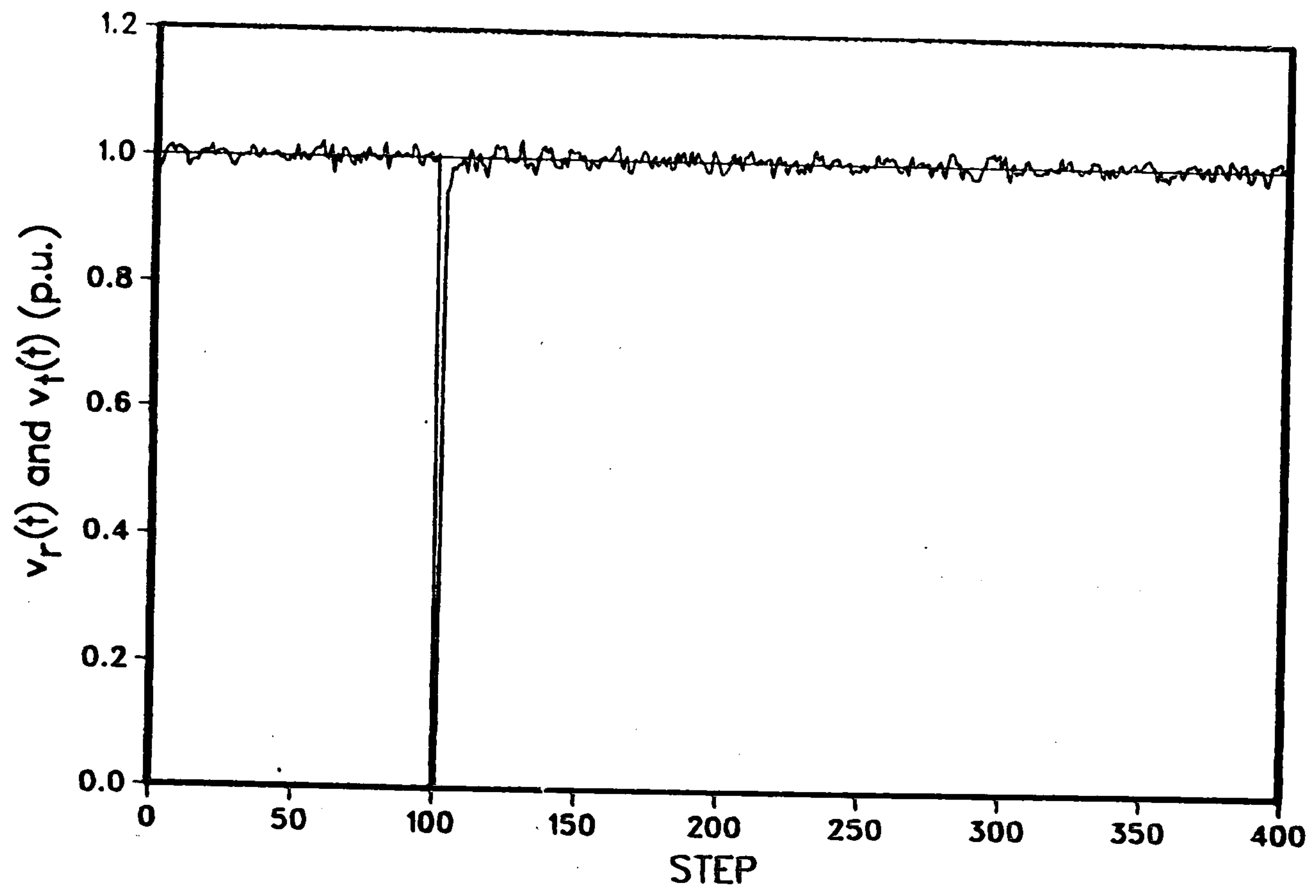

Fig. 5.3.2b $v_r(t)$ and $v_t(t)$ for self-tuning system.

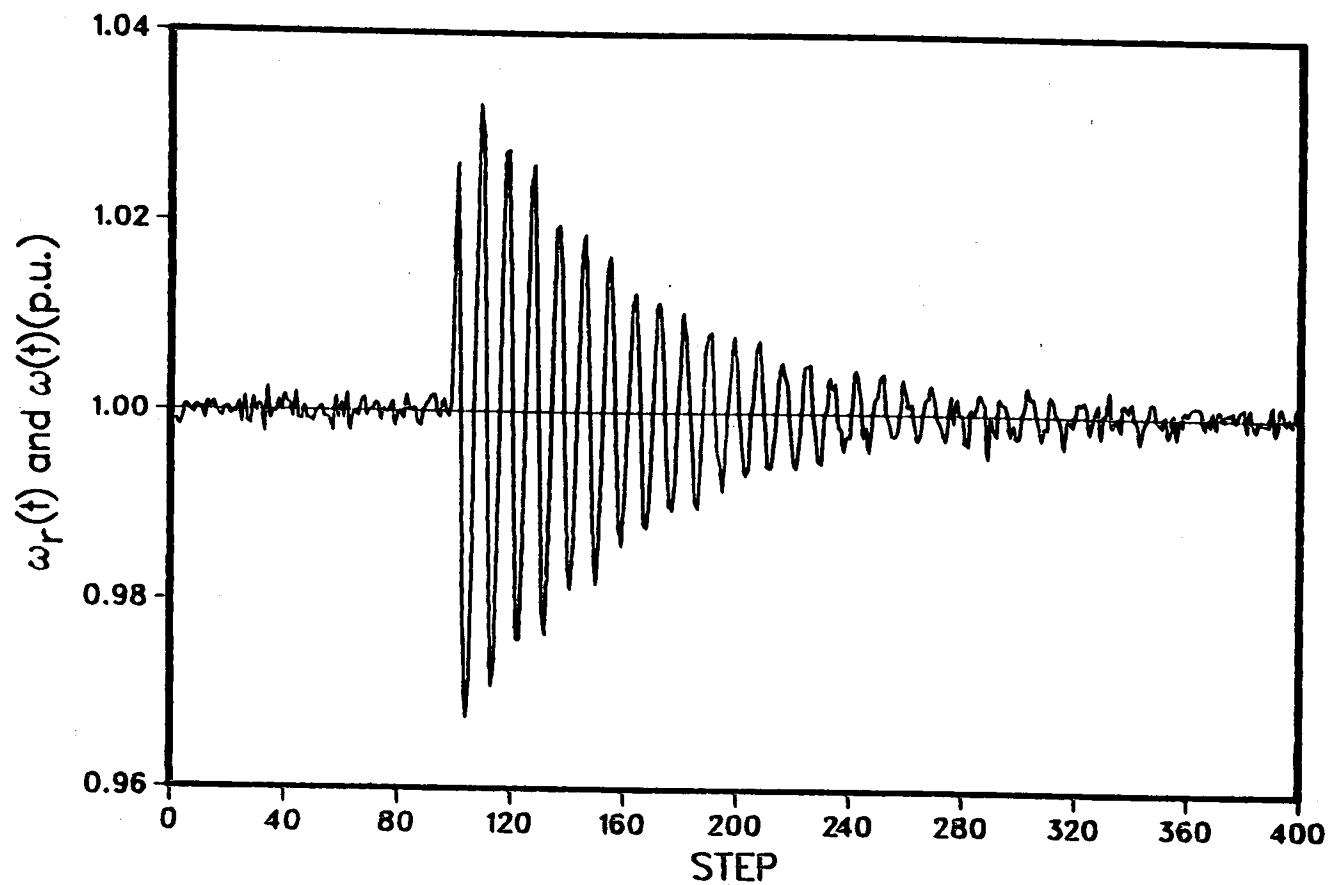

Fig. 5.3.3a $\omega_r(t)$ and $\omega(t)$ for conventional system.

Fig. 5.3.3 (a~b) Time responses of the SMIB system using conventional GOV and AVR, subject to 3-phase fault. Operating point: P=0.8 p.u., Q=0.3 p.u..

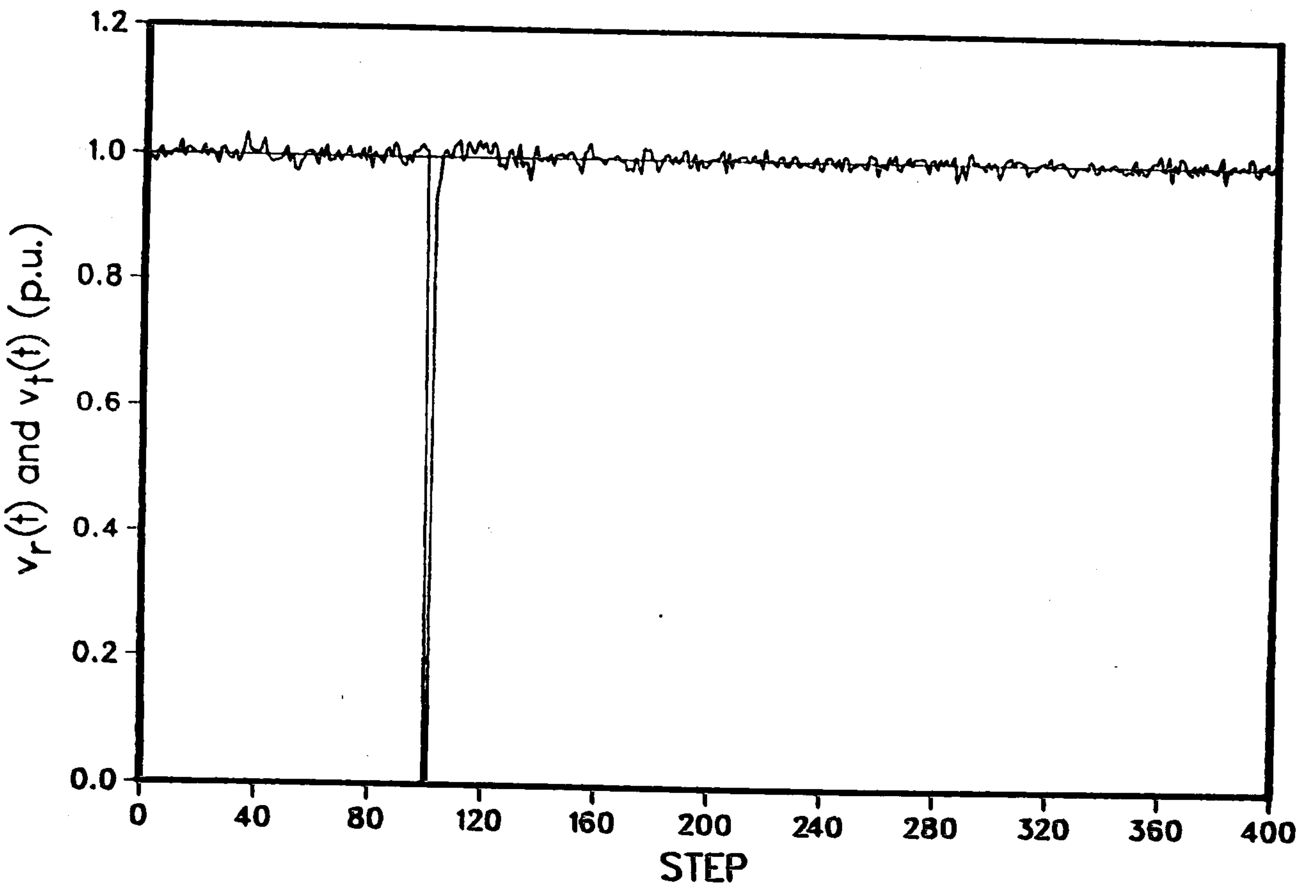

Fig. 5.3.3b $v_r(t)$ and $v_t(t)$ for conventional system.

5.3.4 Comments

In Section 5.3, the SMIB system with multivariable GMV self-tuning controller subject to two kinds of system disturbances was investigated. For the mechanical torque disturbance caused by step changes in reference signal $\omega_r(t)$, the simulation shows that the GMV self-tuning controller can function as a power regulator, and achieves a stable control system. For the three-phase fault disturbance, the self-tuning system achieves not only a stable system, but also a well damped time response of $\omega(t)$. However, the simulations were carried out at one operating point in this section. A wide range of operating points are needed to test the stability of the SMIB system with GMV self-tuning controller. This will be described in the following section.

5.4 Comparison With Conventional Controllers

5.4.1 Introduction

Simulations in previous sections have shown that the multivariable GMV STC with variable forgetting factor can work satisfactorily with the power system at a specific operating point (P=0.8 p.u. and Q=0.3 p.u.) with a noisy background. In this section, simulation studies for the GMV self-tuning system on different operating points subject to different system disturbances will be undertaken. At the same time, a comparison with conventional GOV and AVR for the corresponding running condition will be conducted. All simulations in this section will be done in a deterministic environment.

Before making the comparison studies, it is helpful to investigate the characteristic of the identification in a deterministic environment, i.e., with $\xi_1(t)=\xi_2(t)=0$. As mentioned earlier, the identification can be done in the deterministic case, provided there exists a system disturbance or step change in command signal. A simulation is made to show the identification result in a deterministic case. A 0.1 p.u. step change in reference signal $v_r(t)$ is applied at sampling step 10, and the trajectories of the identified system parameters are recorded. They are shown in

Figure 5.4.1. It is clear that the convergence of these parameters is achieved.

The next section shows a comparison between the GMV STC and the properly-tuned conventional GOV and AVR made on the same SMIB system. The block-diagram of the two systems have been shown in Figures 5.1.2 and 5.1.3, and the parameters of GOV and AVR are listed in Appendix A.

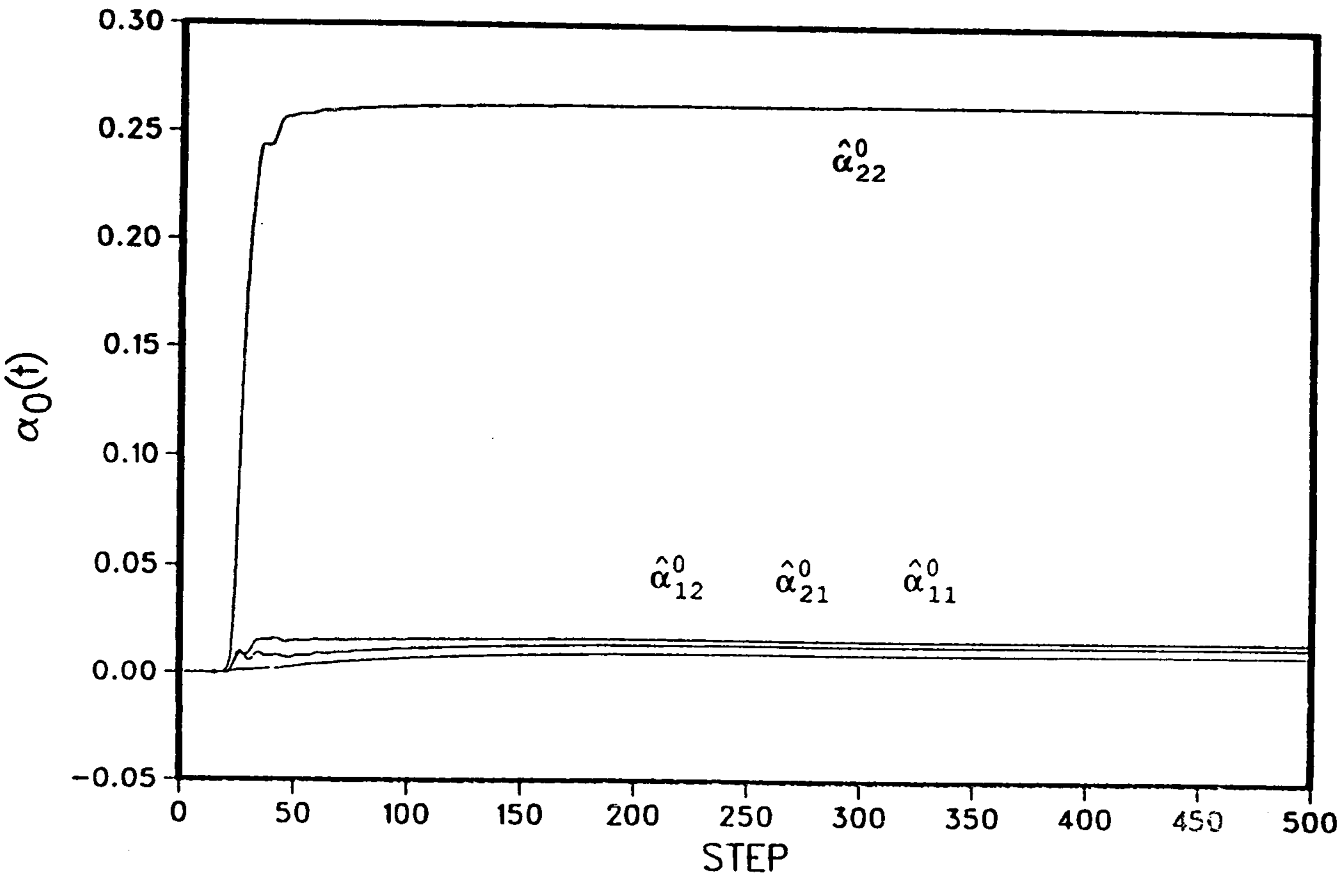

Fig. 5.4.1a Estimation of $\hat{\alpha}_0$.

Fig. 5.4.1 (a~i) Identification in a deterministic environment corresponding to a 0.1 p.u. step change in reference signal $v_r(t)$.

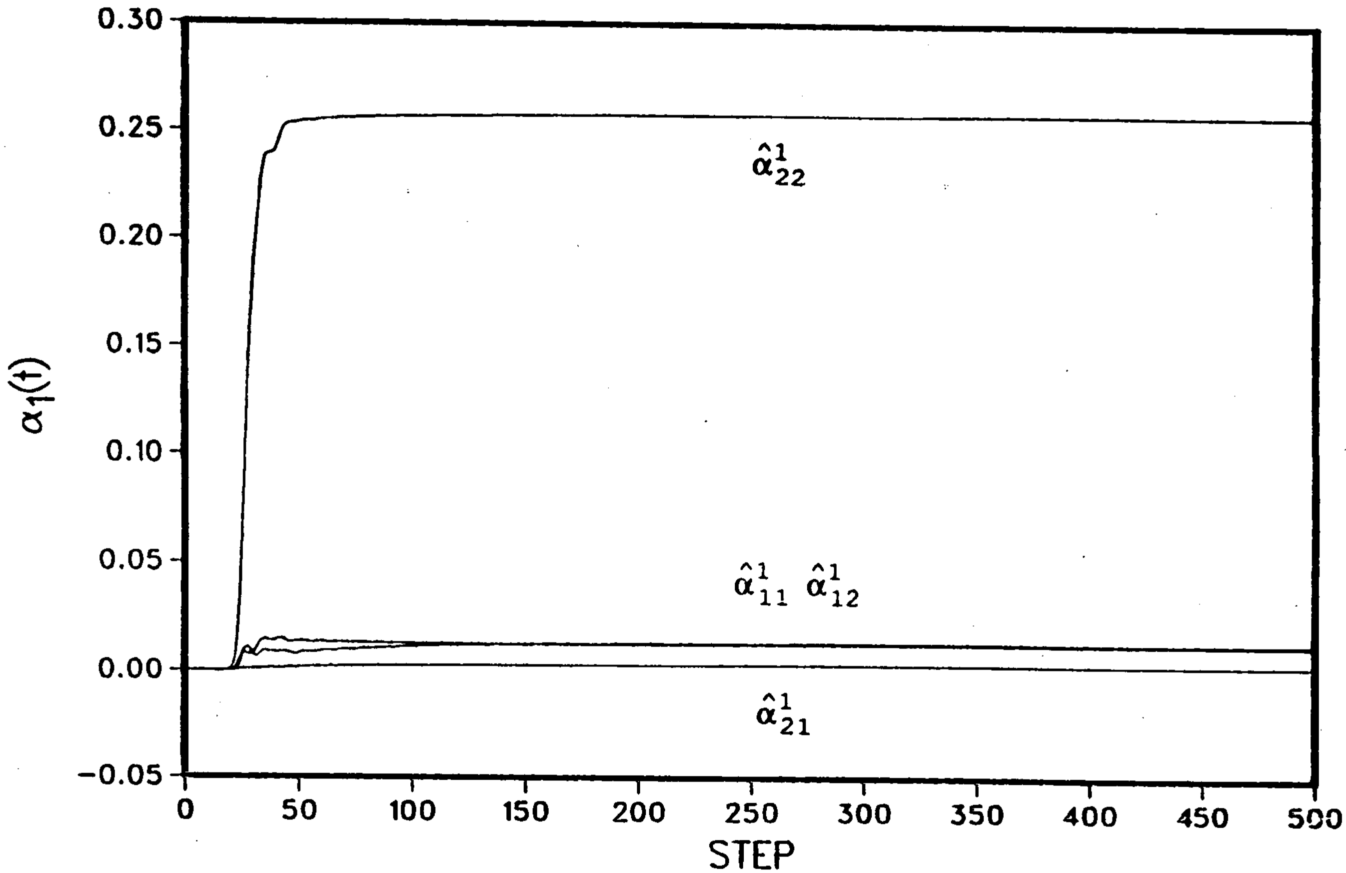

Fig. 5.4.1b Estimation of $\hat{\alpha}_1$.

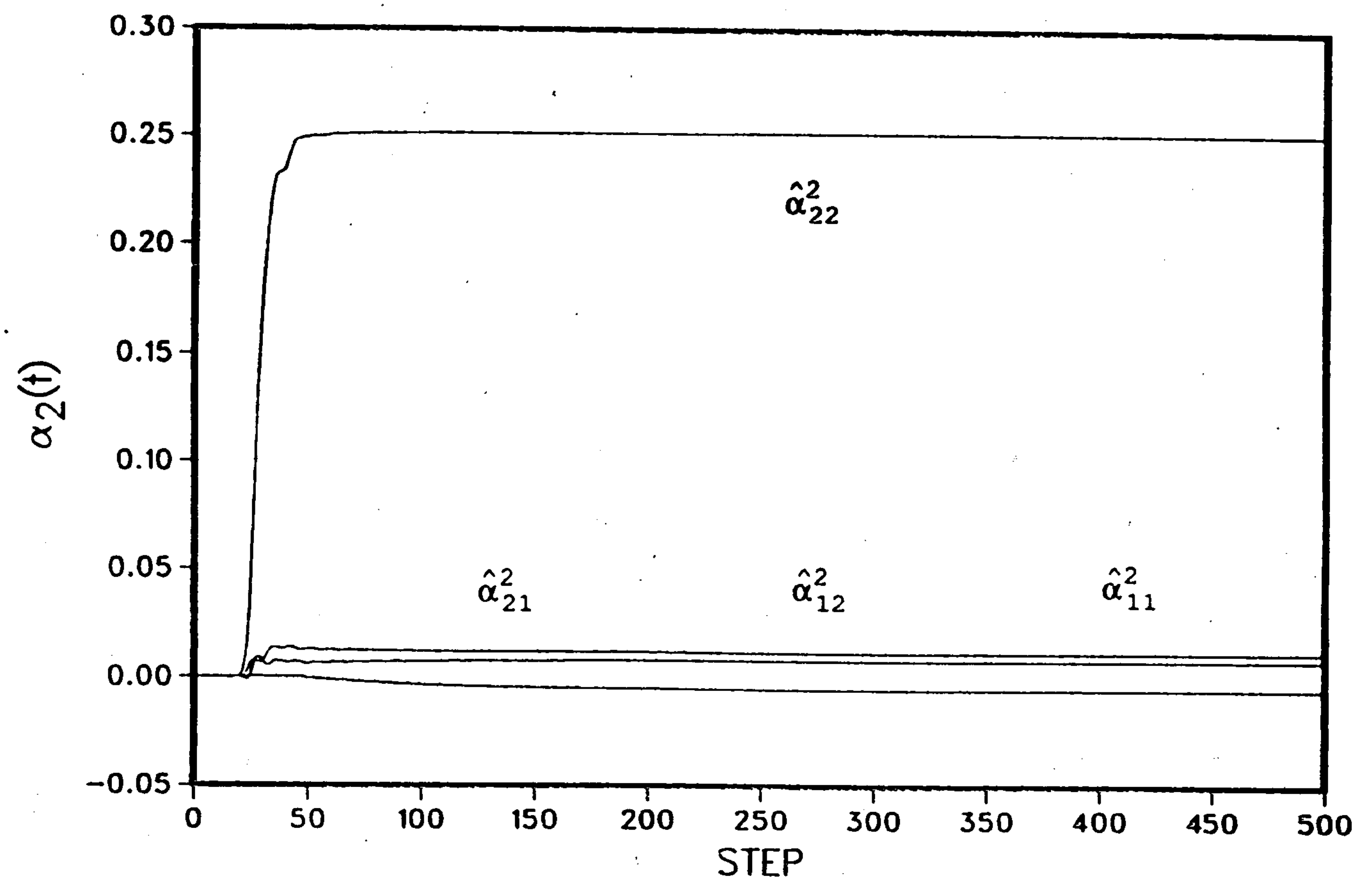

Fig. 5.4.1c Estimation of $\hat{\alpha}_2$.

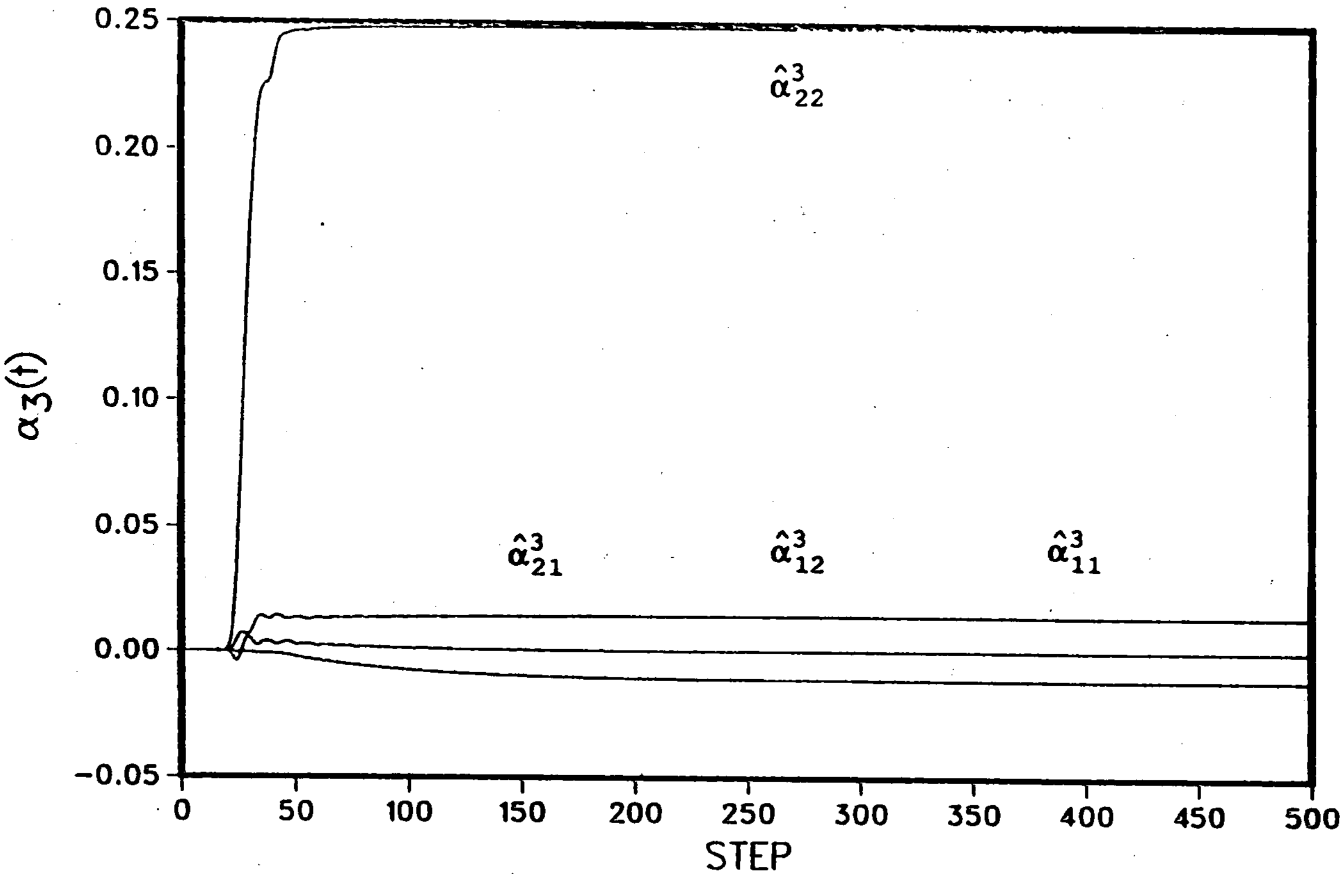

Fig. 5.4.1d Estimation of $\hat{\alpha}_3$.

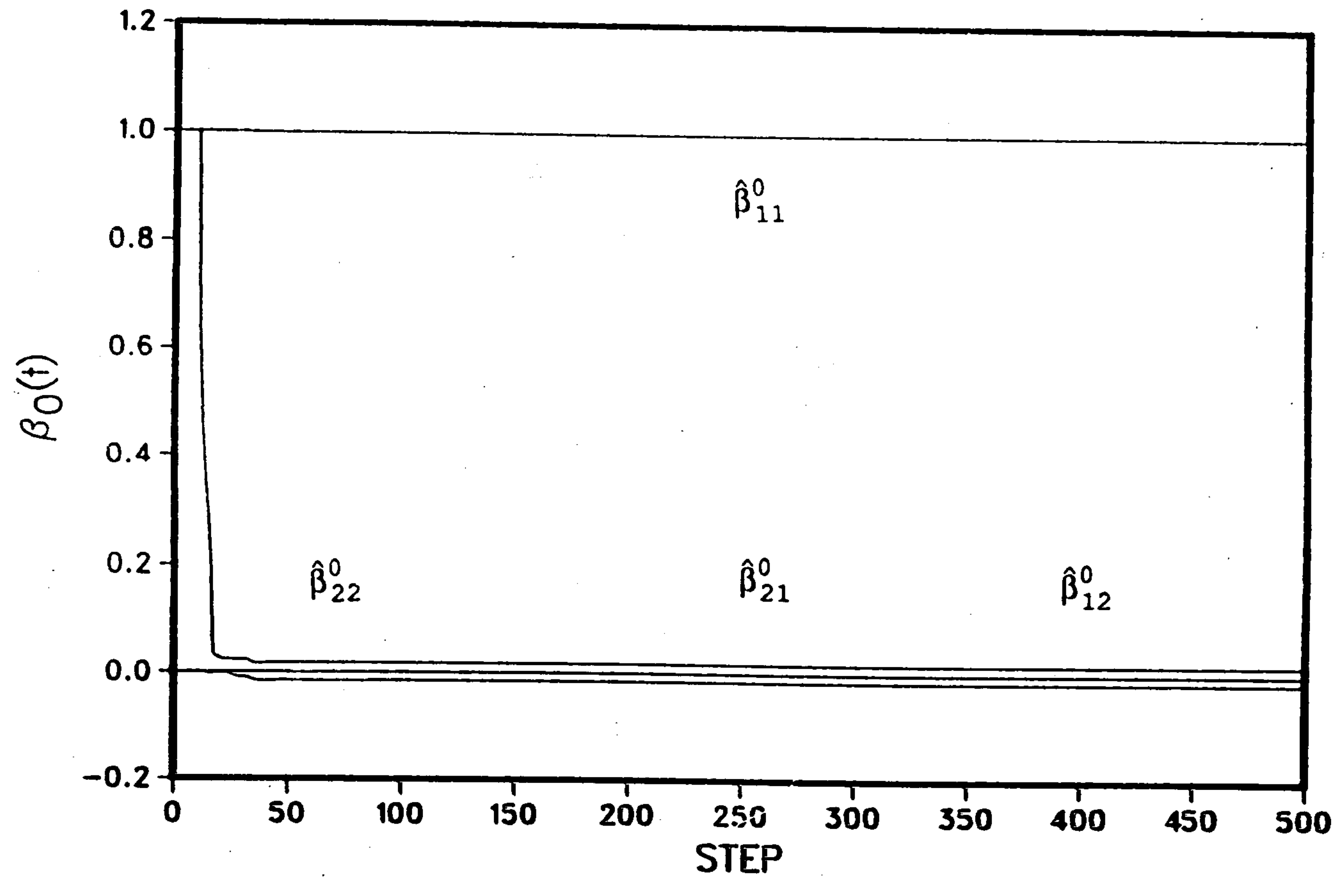

Fig. 5.4.1e Estimation of $\hat{\beta}_0$.

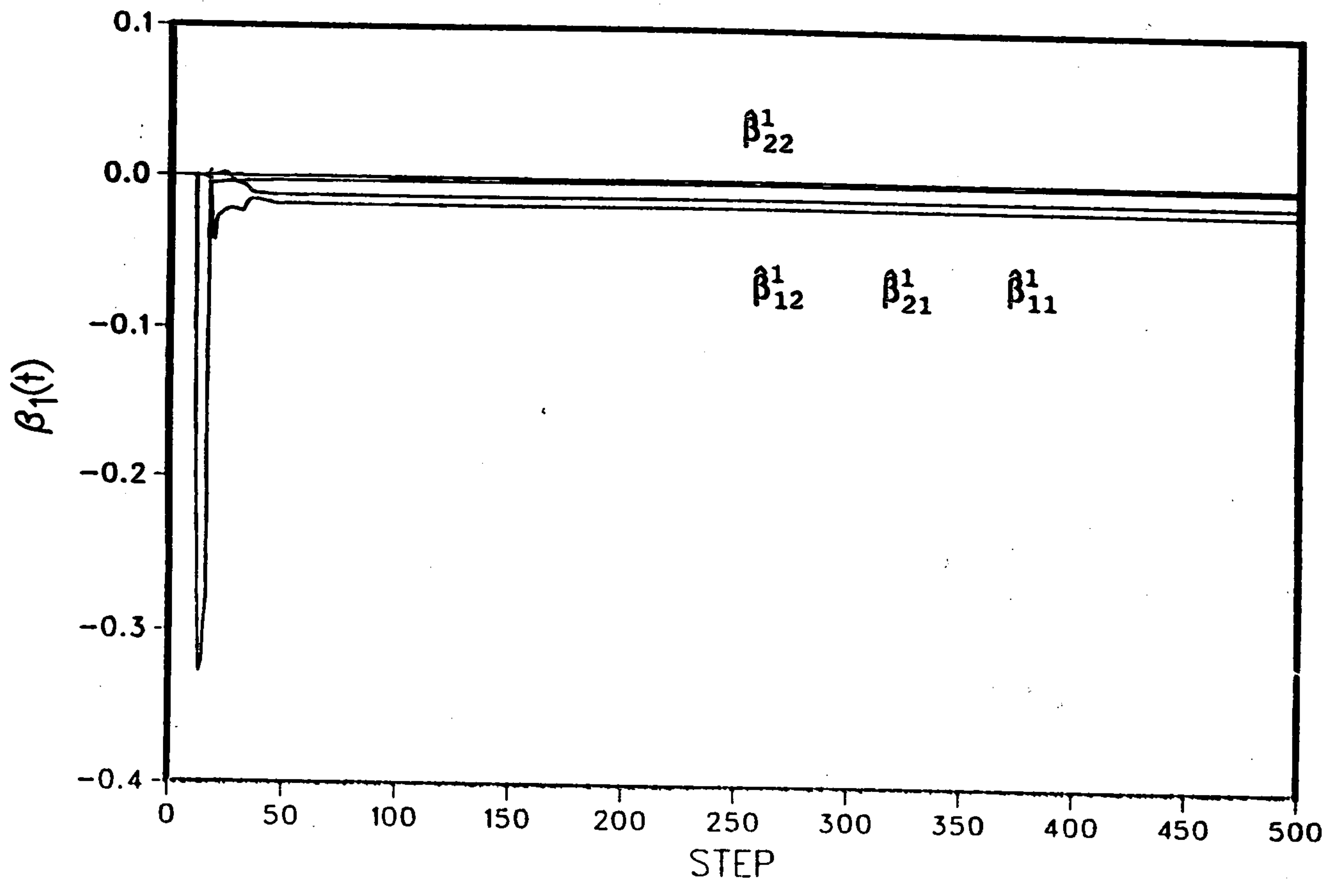

Fig. 5.4.1f Estimation of $\hat{\beta}_1$.

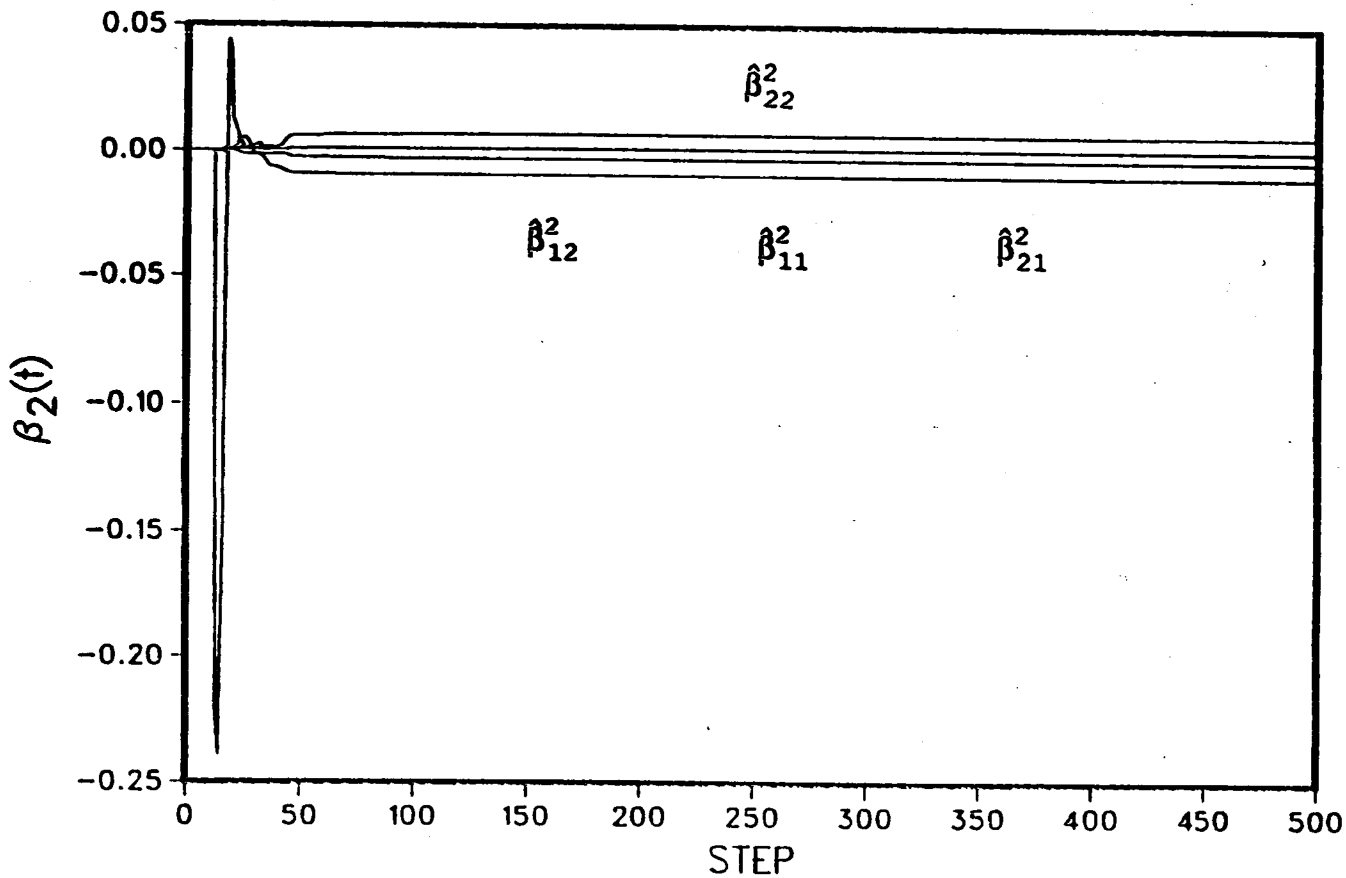

Fig. 5.4.1g Estimation of $\hat{\beta}_2$.

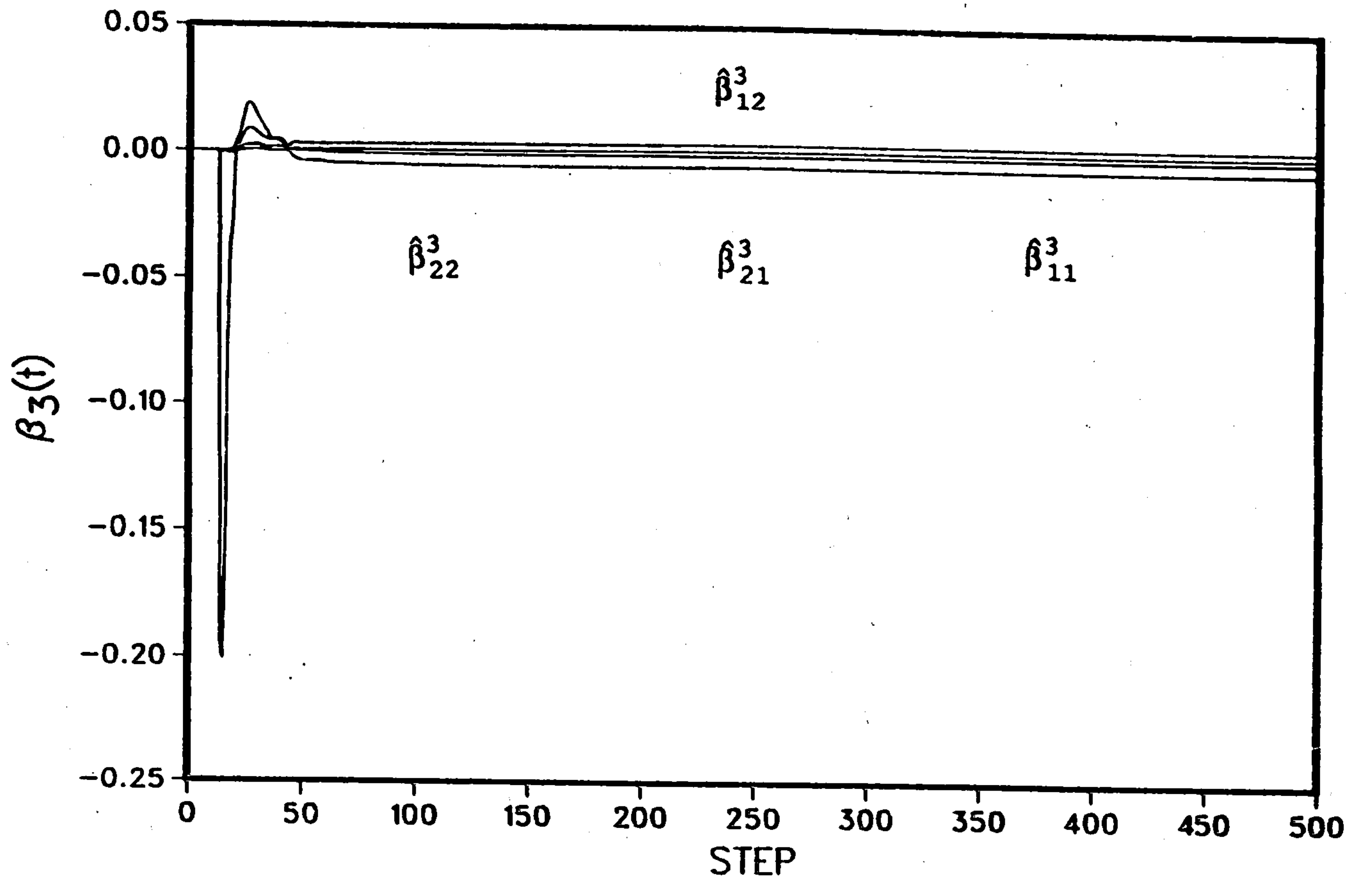

Fig. 5.4.1h Estimation of $\hat{\beta}_3$.

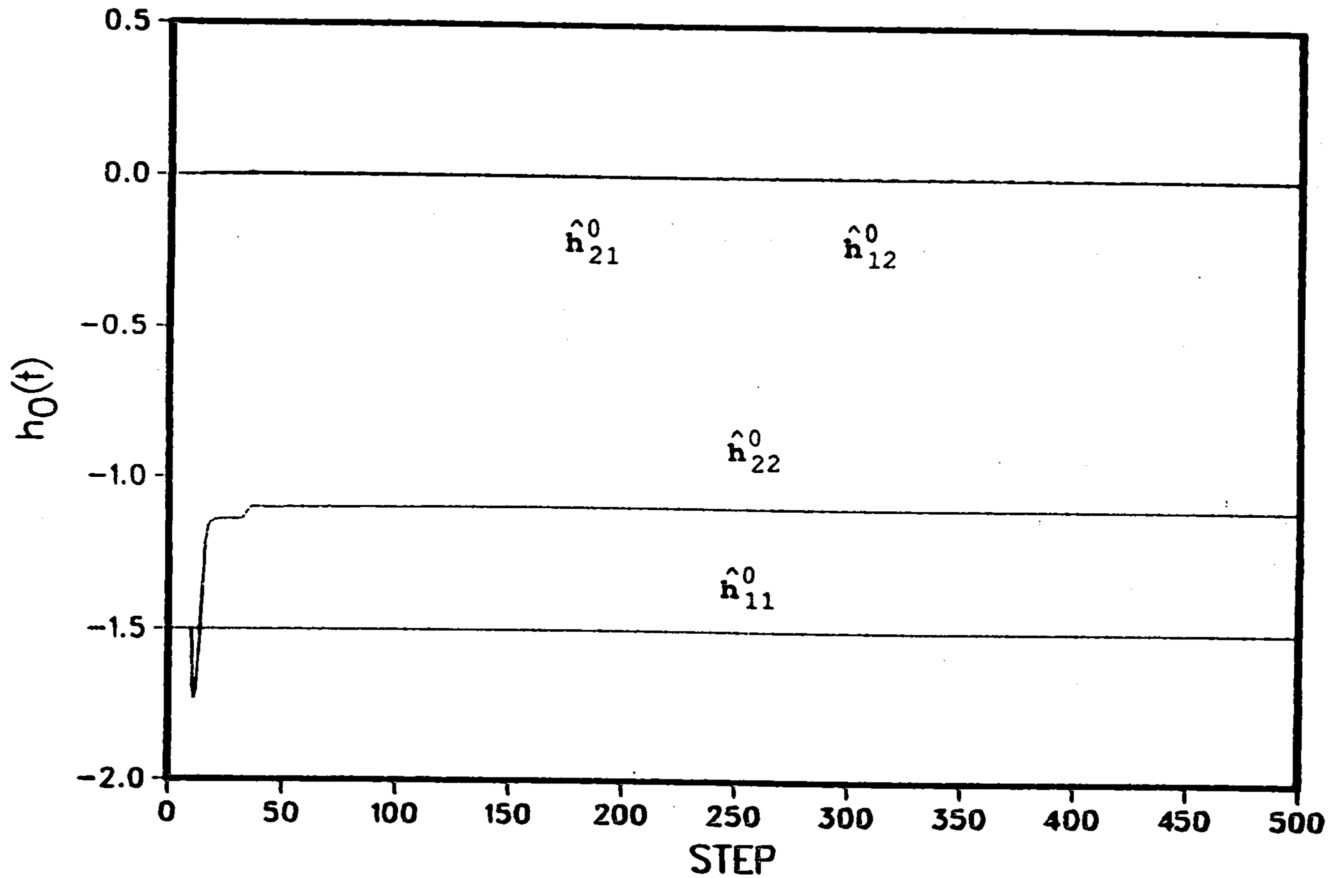

Fig. 5.4.1i Estimation of $\hat{h}_0$.

5.4.2 Comparisons Between GMV STC and Conventional Controller Systems

Simulations on different operating points subject to different system disturbances for the self-tuning power system will be conducted in this subsection. To investigate the benefits of the self-tuning system, simulations for conventional controllers will be also taken in parallel, and comparison will be made for each case.

Time responses shown in Figure 5.4.2 are the simulation results of transmission-line switching. This kind of system disturbance causes a change in system configuration. Breakers b_3 and b_4 shown in Figure 5.1.1 were pulled off at step 100, and re-closed at step 300. The operating point is at P=0.6 p.u., Q=-0.3 p.u.. Figure 5.4.2a shows the time response of the shaft speed $\omega(t)$ of the SMIB system with GMV STC, while Figure 5.4.2c shows the time response of the shaft speed $\omega(t)$ of the SMIB system with conventional GOV and AVR. Comparing the two responses, one can notice that the damping on shaft oscillation is effectively improved in the self-tuning case.

If comparing the steady-state errors of the terminal voltage during the period of 100~300 steps (one of the transmission lines was draped in this period) shown in Figures 5.4.2b and 5.4.2d, one can notice that the error is smaller for the self-tuning system than for the conventional system.

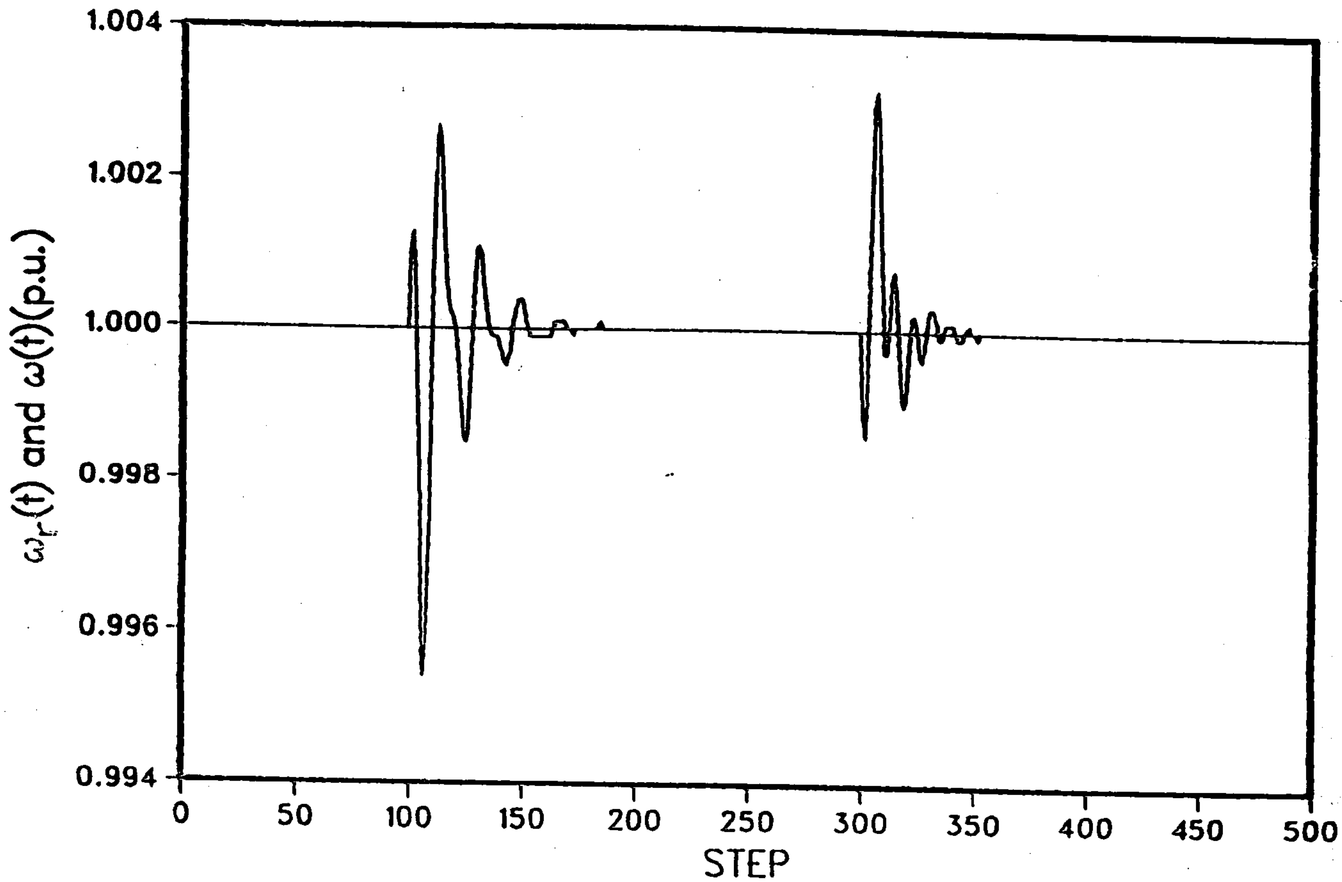

Fig. 5.4.2a $\omega_r(t)$ and $\omega(t)$ for self-tuning system.

Fig. 5.4.2 (a~d) Time responses of SMIB systems using GMV STC and conventional controller subject to transmission line switching. Operating point: P=0.6 p.u., Q=-0.3 p.u..

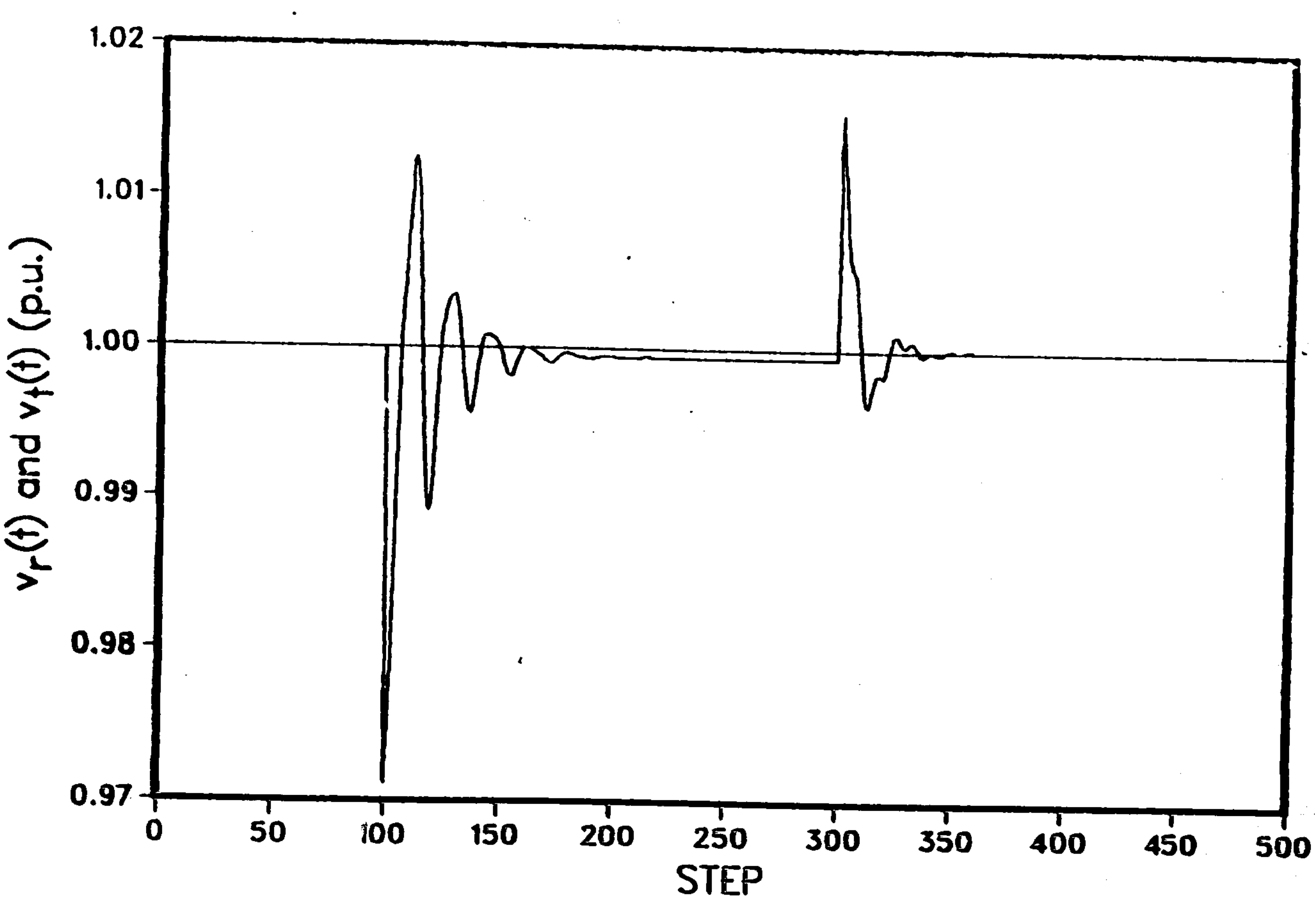

Fig. 5.4.2b $v_r(t)$ and $v_t(t)$ for self-tuning system.

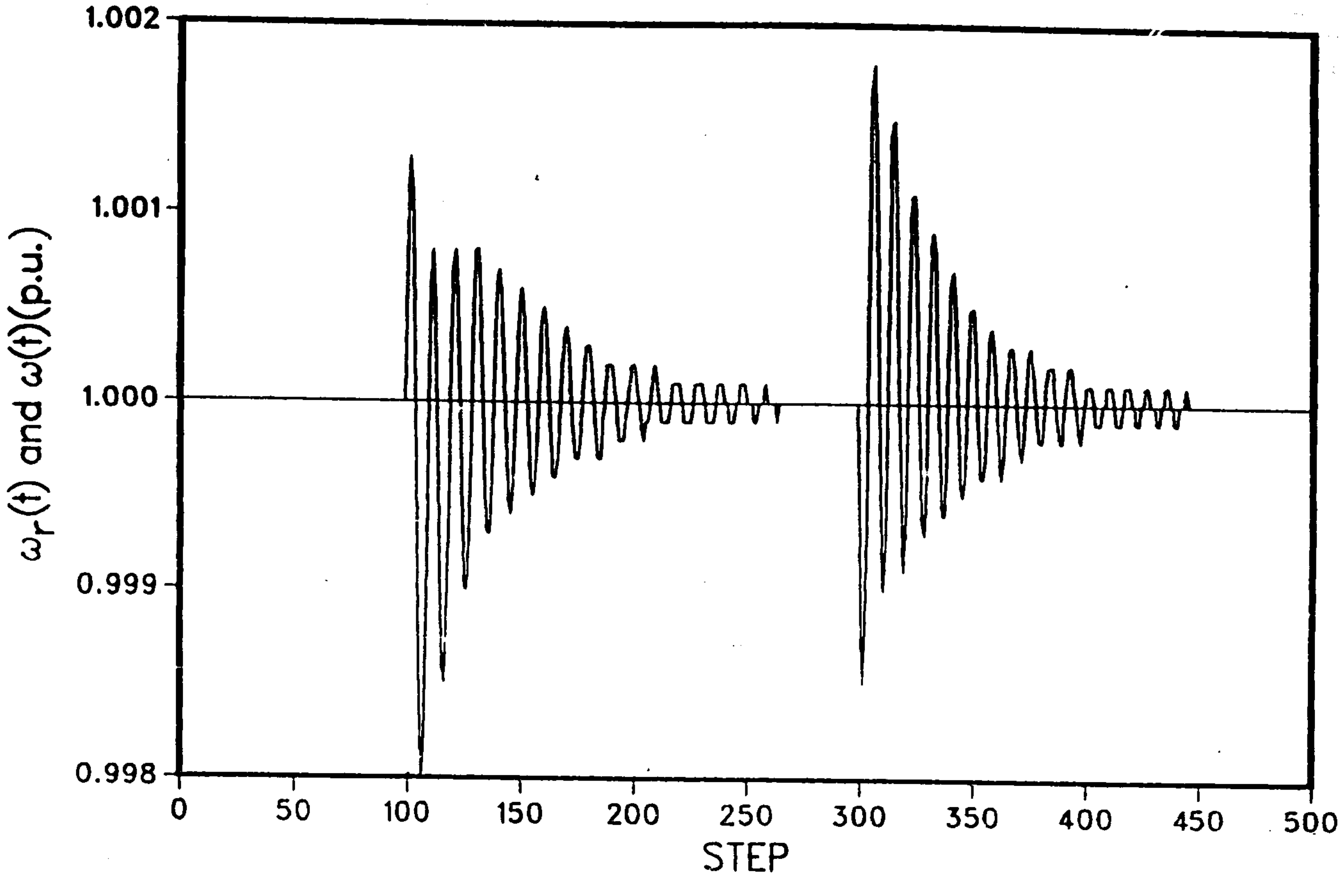

Fig. 5.4.2c $\omega_r(t)$ and $\omega(t)$ for conventional system.

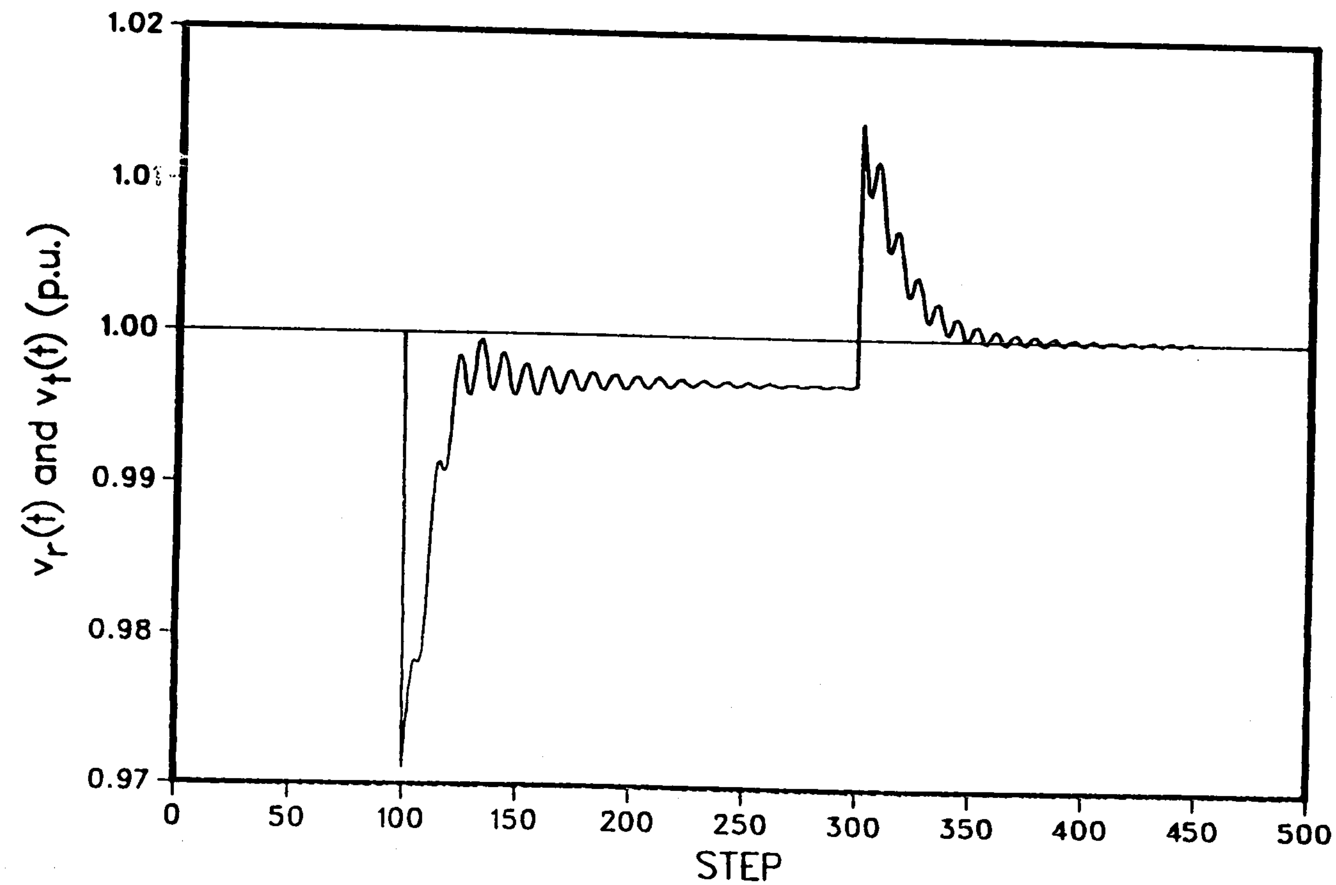

Fig. 5.4.2d $v_r(t)$ and $v_t(t)$ for conventional system.

Figure 5.4.3 shows the time responses for both GMV STC system and conventional system subject to three-phase fault at operating point P=1.0 p.u. and Q=0.5 p.u.. The fault is applied at the terminal voltage bus and lasts for 0.1 sec.. Figure 5.4.3a shows the shaft speed of the GMV STC system and Figure 5.4.3c shows the shaft speed of the conventional GOV and AVR system. The self-tuning system shows improved damping on the oscillation of the generator shaft speed compared to the conventional system.

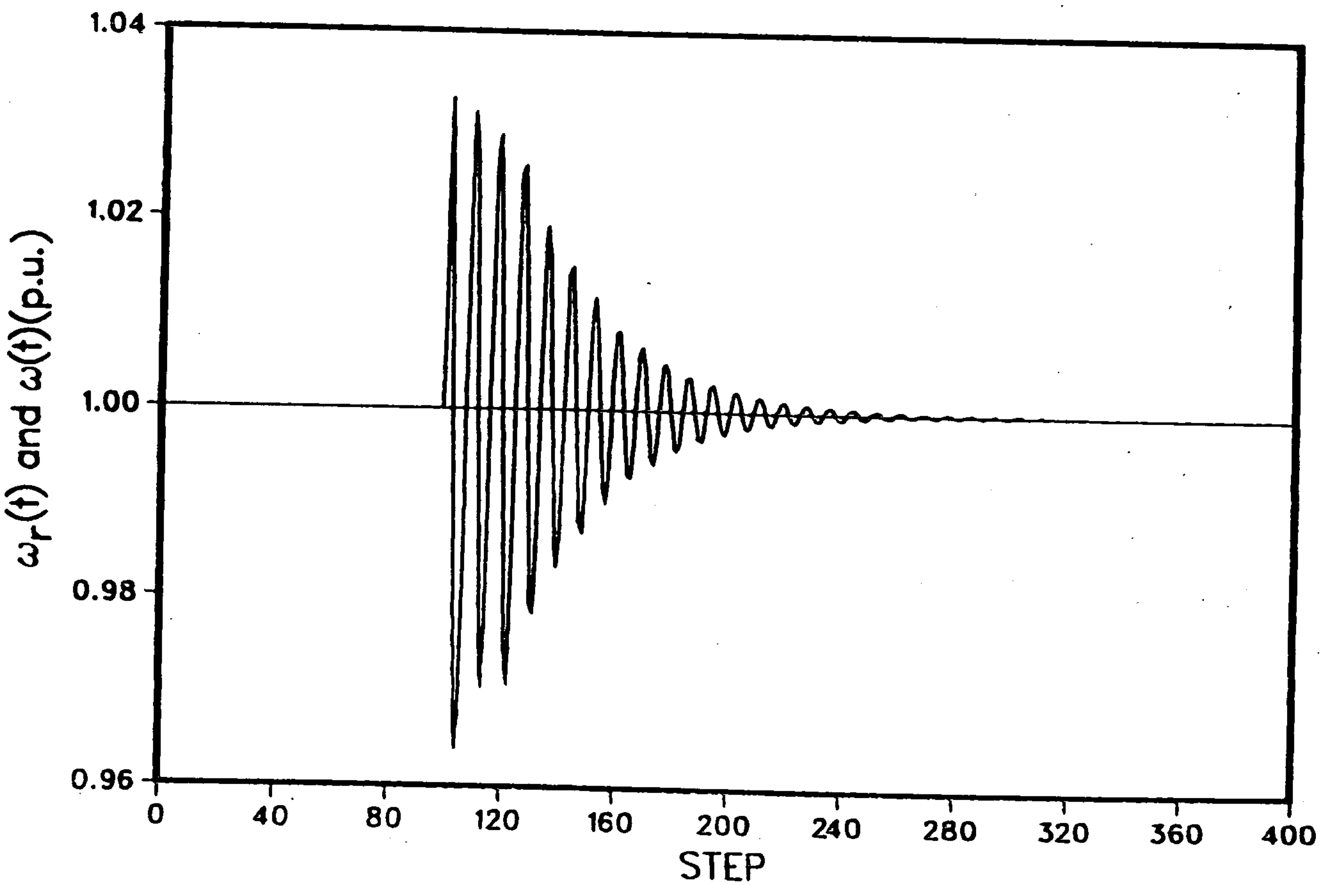

Fig. 5.4.3a $\omega_r(t)$ and $\omega(t)$ for self-tuning system.

Fig. 5.4.3 (a~d) Time responses of SMIB systems using GMV STC and conventional controller subject to 3-phase fault. Operating point: P=1.0 p.u., Q=0.5 p.u..

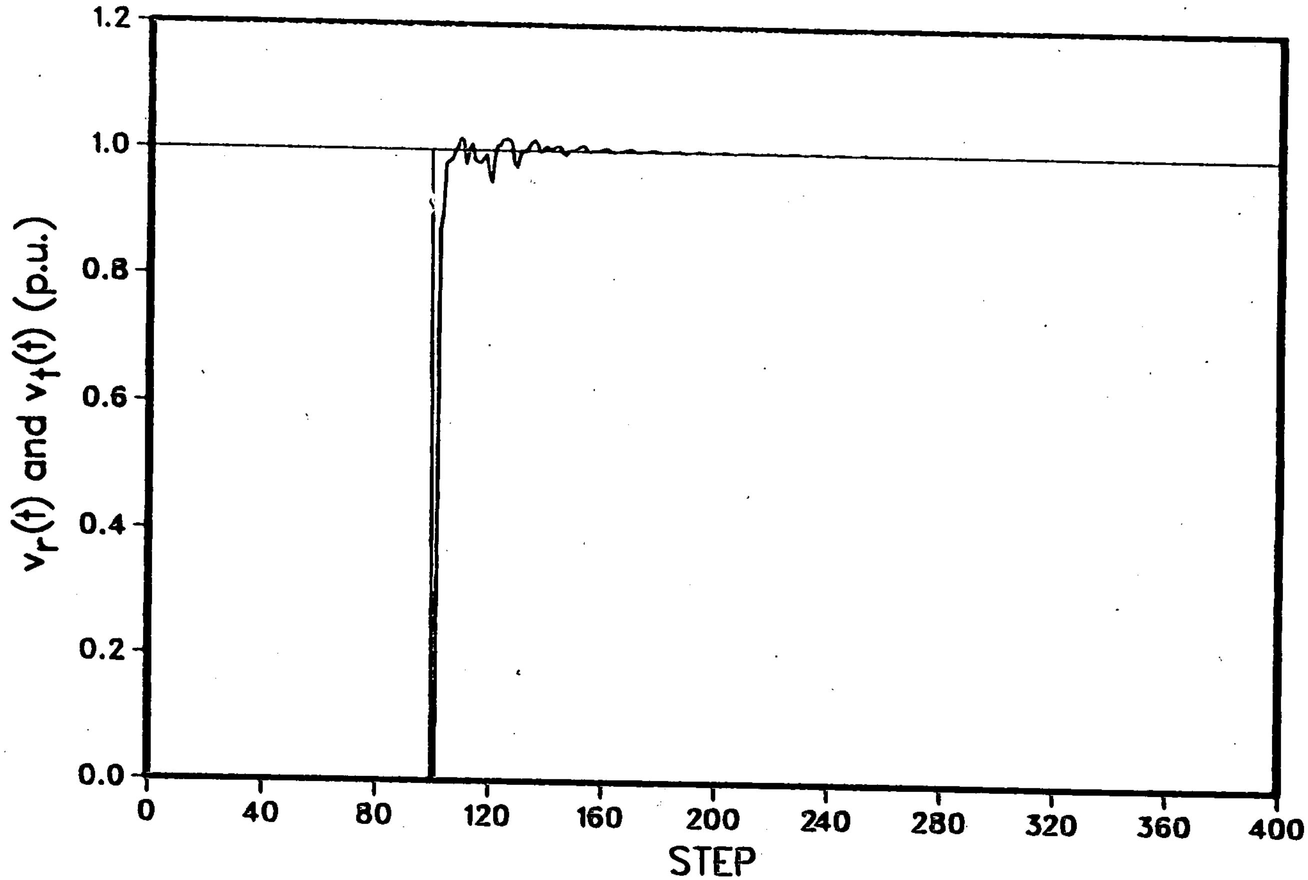

Fig. 5.4.3b $v_r(t)$ and $v_t(t)$ for self-tuning system.

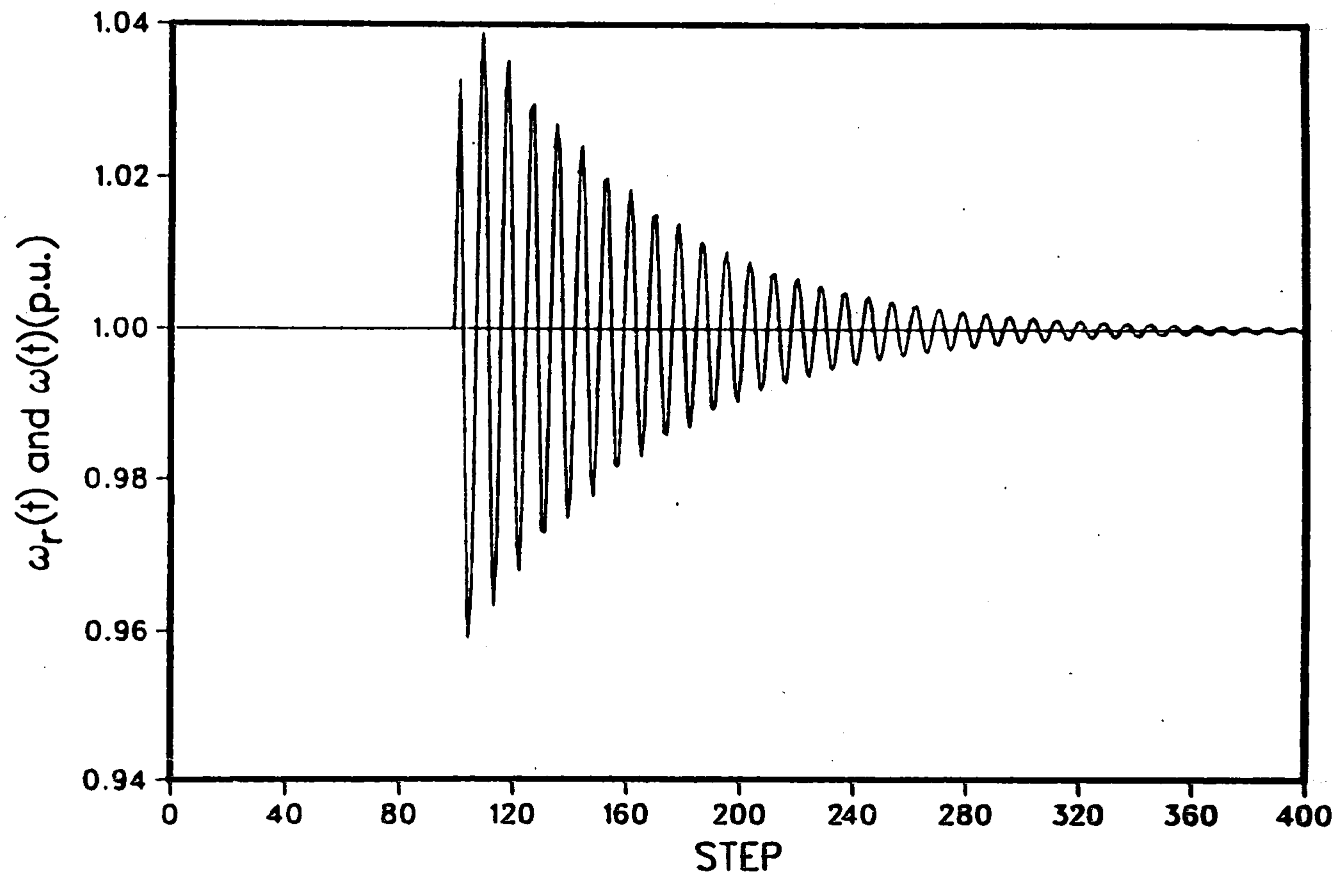

Fig. 5.4.3c $\omega_r(t)$ and $\omega(t)$ for conventional system.

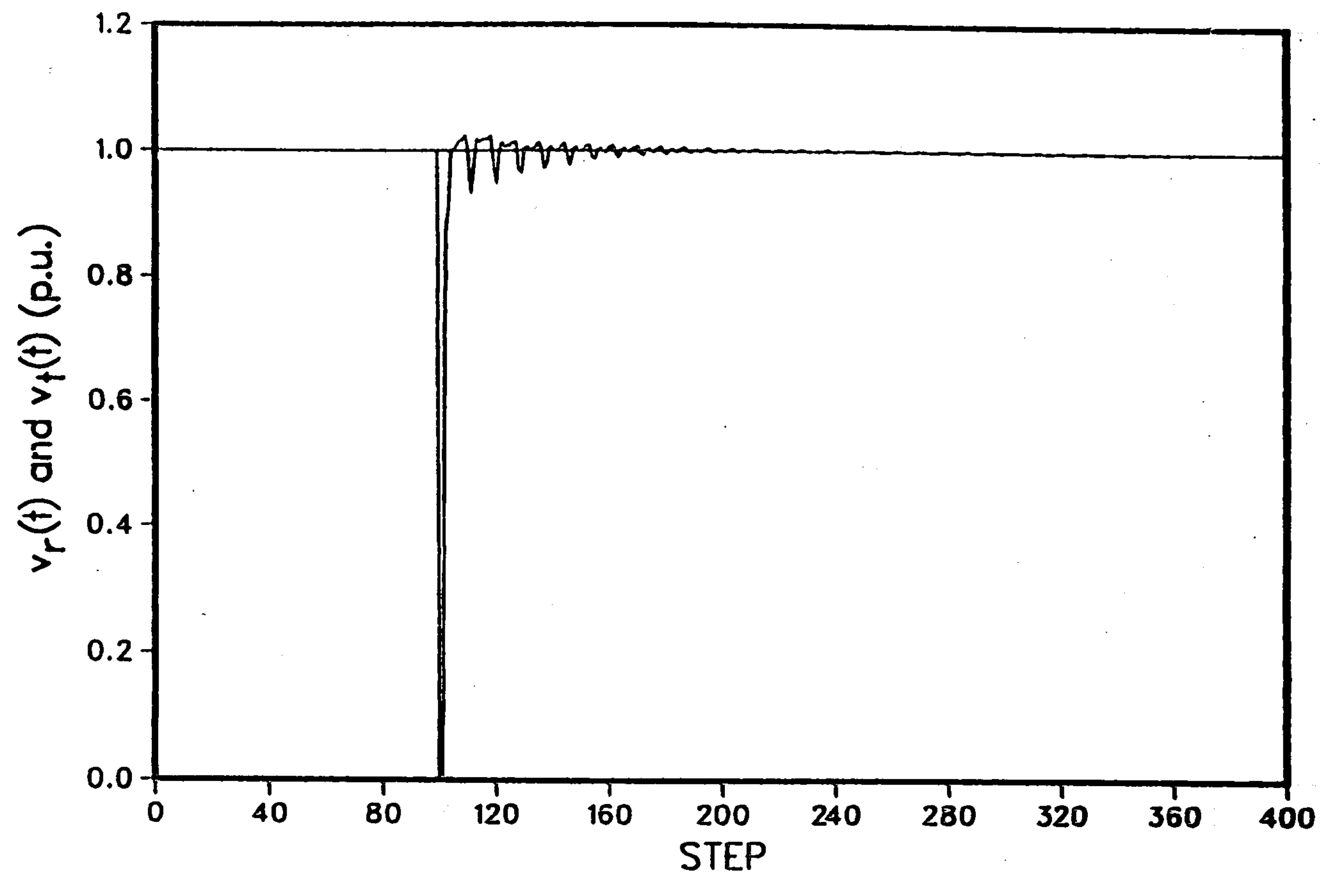

Fig. 5.4.3d $v_r(t)$ and $v_t(t)$ for conventional system.

Finally, the responses of the GMV self-tuning system and conventional system to step electrical power disturbances are simulated at operating point P=0.8 p.u. and Q=-0.3 p.u.. A serial step electrical power disturbance of +0.025, -0.05, +0.1 and -0.075 p.u. was applied at sampling steps 100, 400, 700 and 1000 respectively. The electrical power P_e for both self-tuning and conventional cases are shown in Figures 5.4.4a and 5.4.4d respectively. One can notice that the oscillation in electrical power P_e is reduced as compared with the conventional case. Therefore, as a consequent result, the oscillation of shaft speed for the self-tuning system is also reduced by comparing Figures 5.4.4b with 5.4.4e.

The terminal voltage in Figure 5.4.4c shows that there is no steady-state error for the step electrical power disturbances for the self-tuning system. However, the record shown in Figure 5.4.4f indicates that the steady-state error of the terminal voltage always exists for the conventional system in this test.

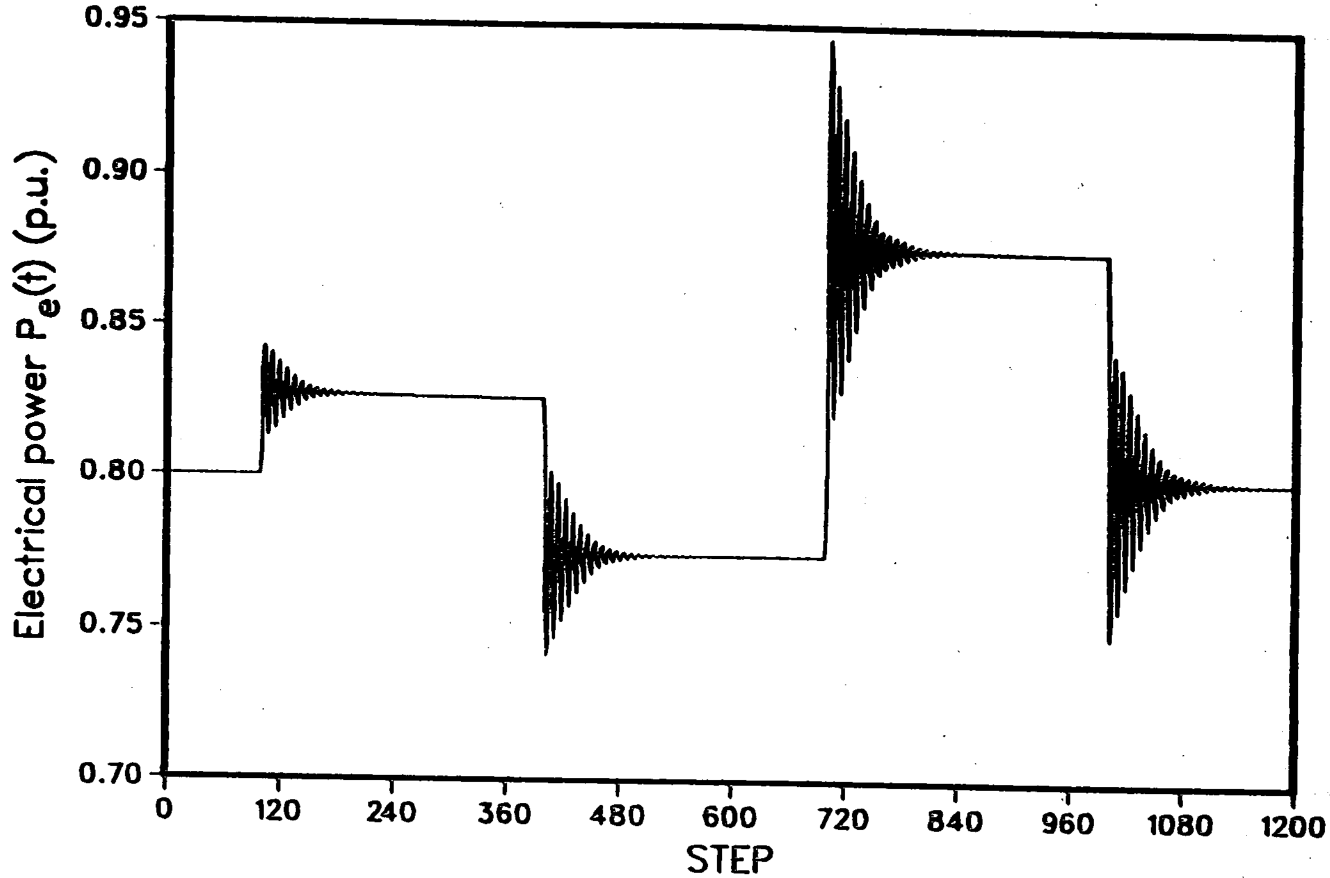

Fig. 5.4.4a Electrical power $P_e(t)$ for self-tuning system.

Fig. 5.4.4 (a~f) Time responses of SMIB systems using GMV STC and conventional controller subject to electrical power disturbance. Operating point: P=0.8 p.u., Q=-0.3 p.u..

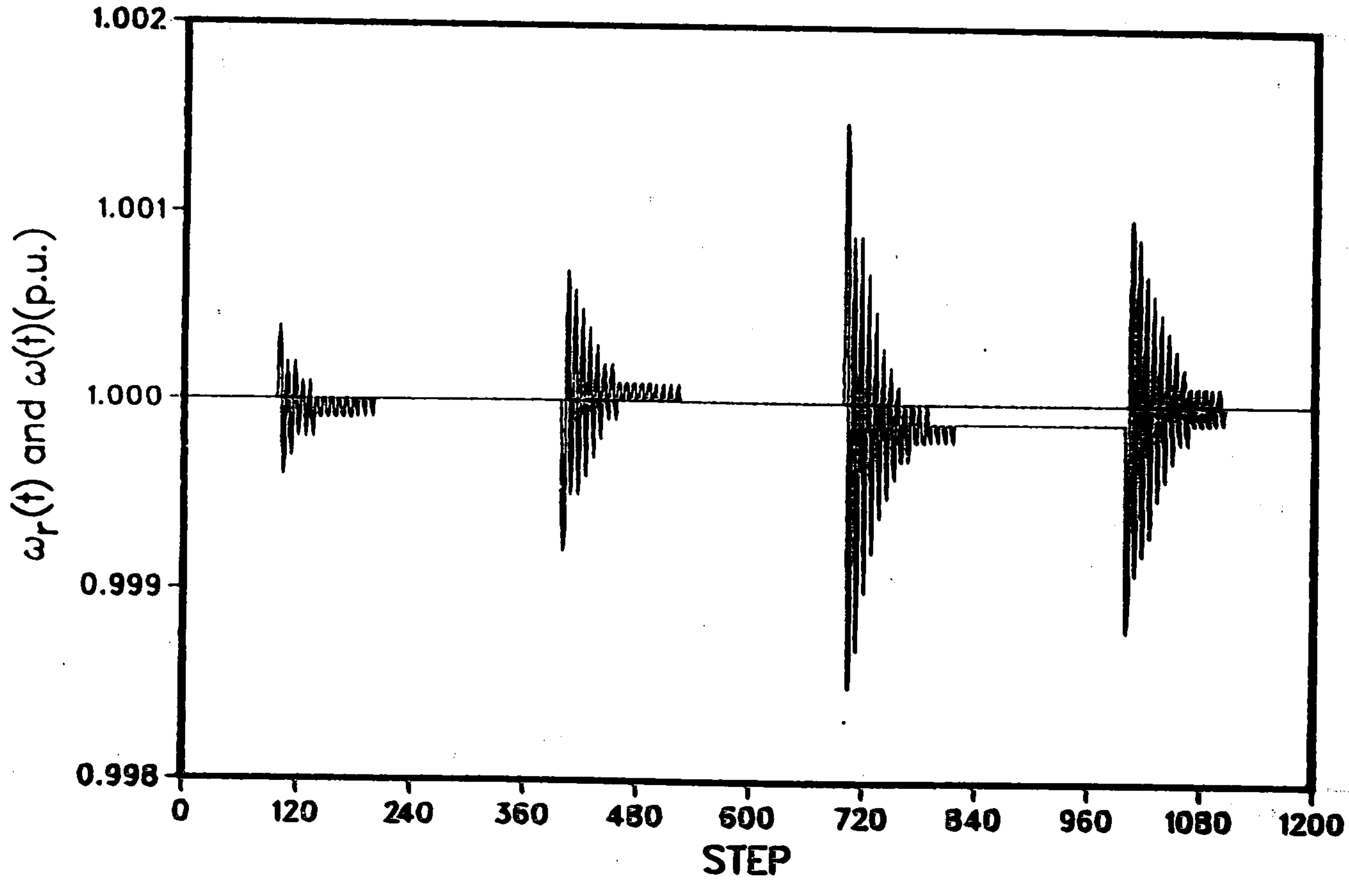

Fig. 5.4.4b $\omega_r(t)$ and $\omega(t)$ for self-tuning system.

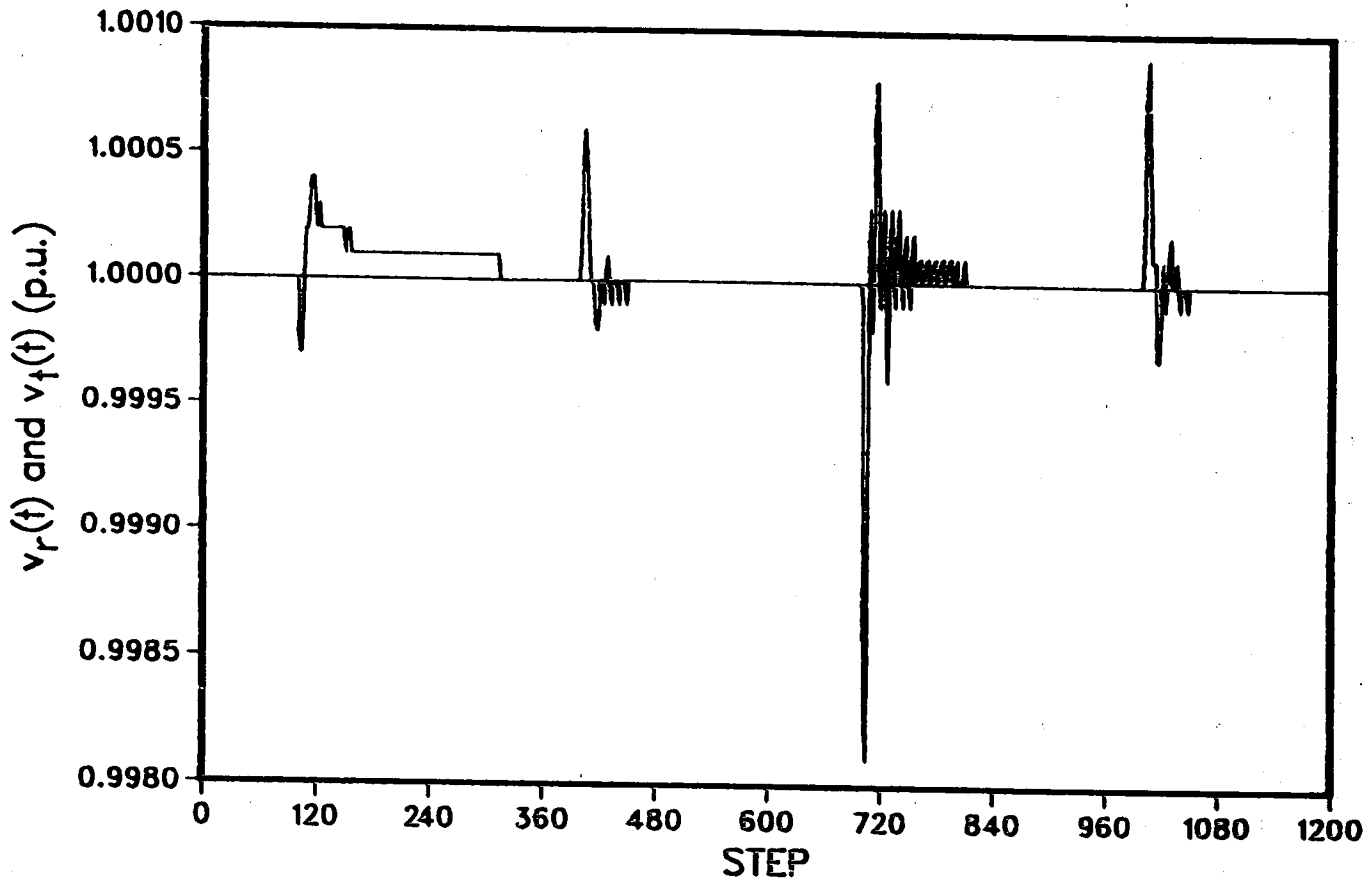

Fig. 5.4.4c $v_r(t)$ and $v_t(t)$ for self-tuning system.

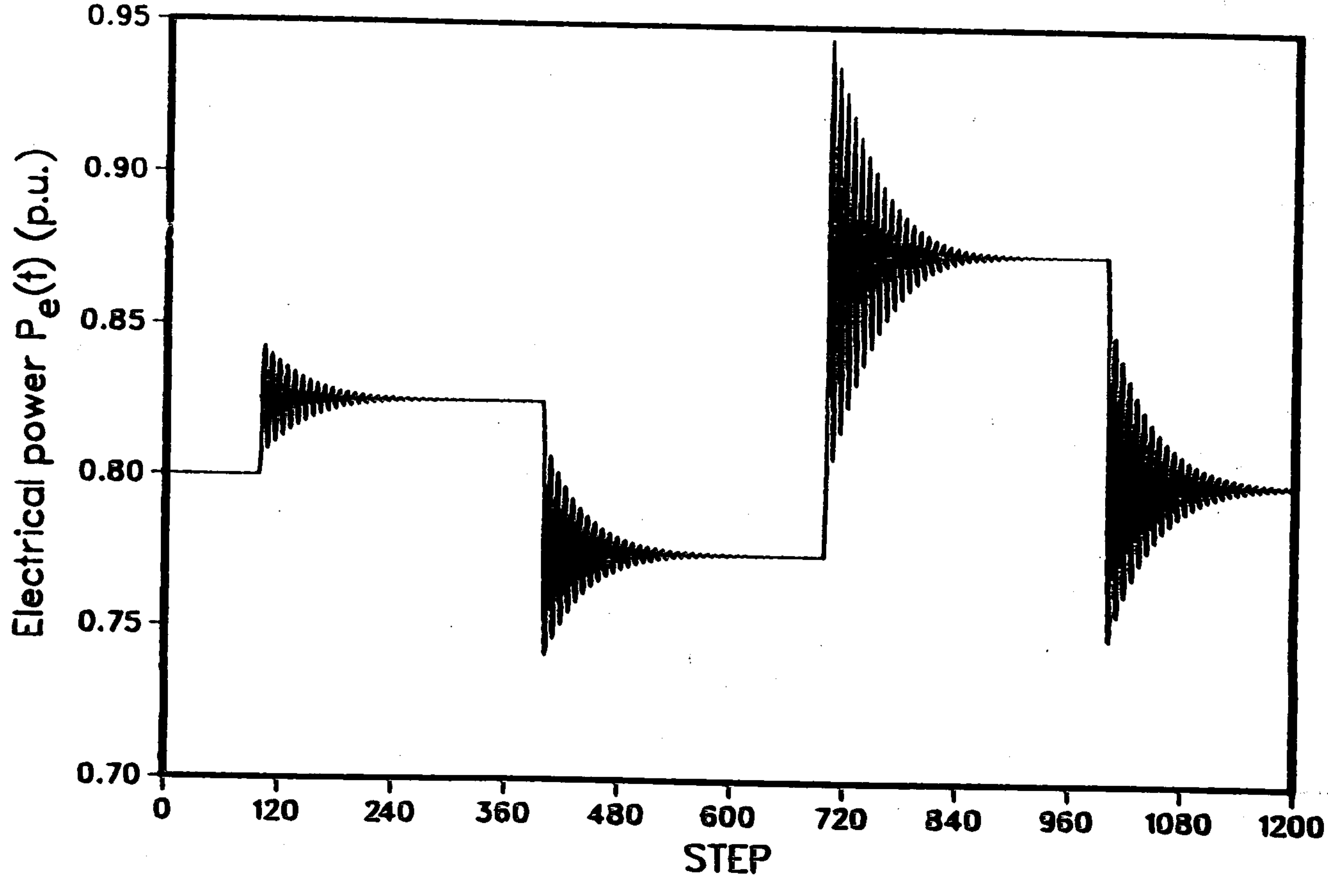

Fig. 5.4.4d Electrical power $P_e(t)$ for conventional system.

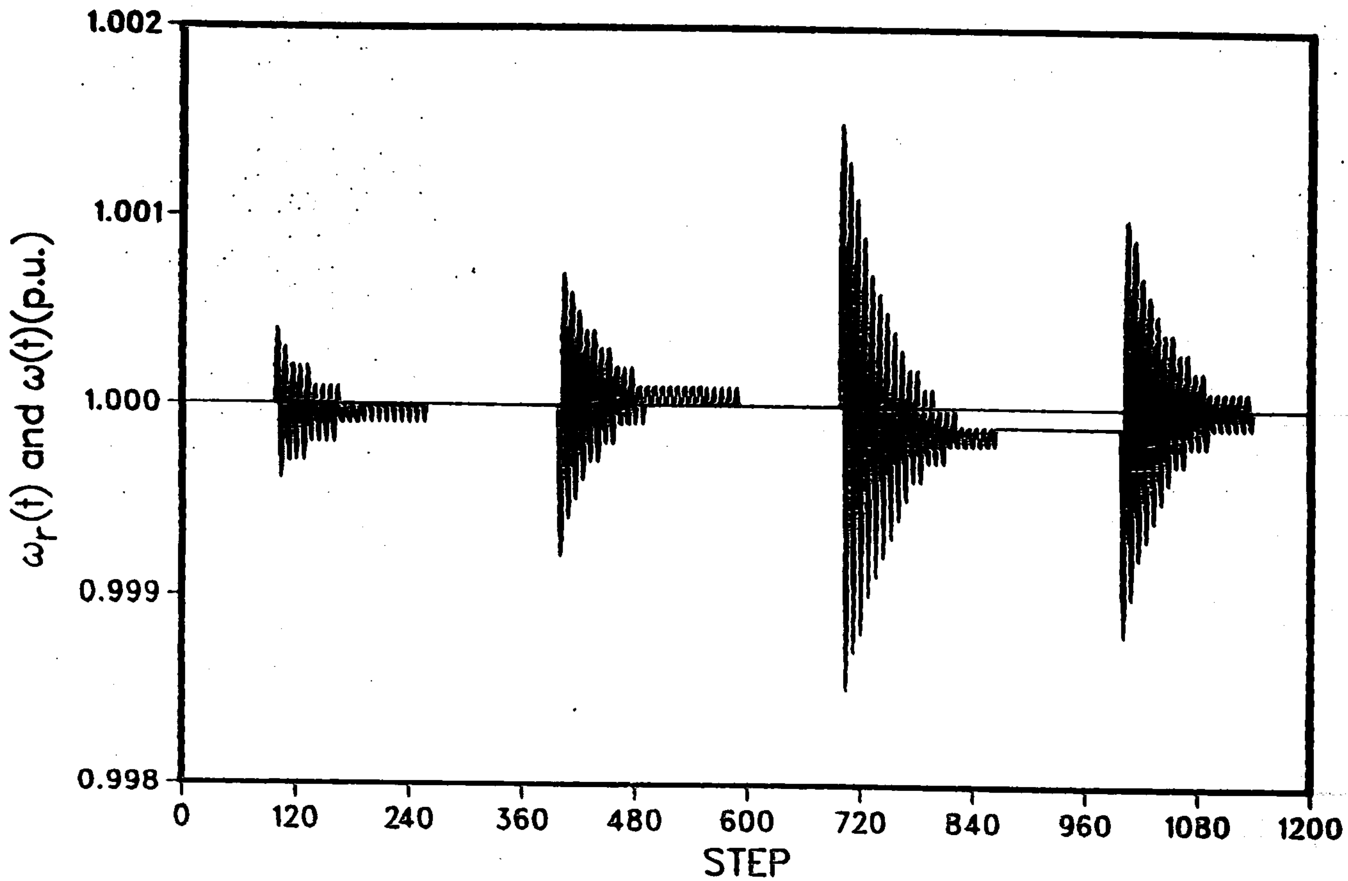

Fig. 5.4.4e $\omega_r(t)$ and $\omega(t)$ for conventional system.

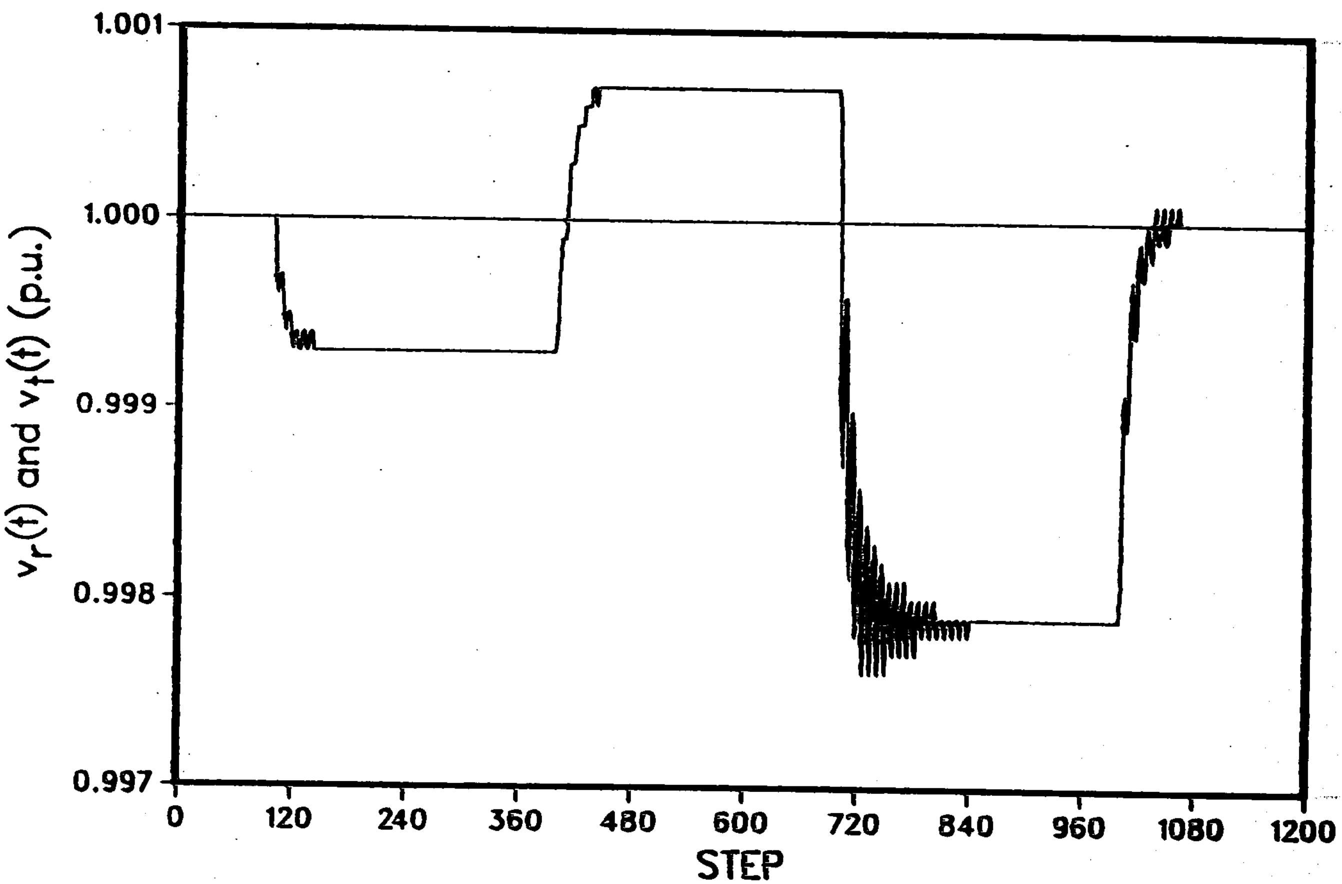

Fig. 5.4.4f $v_r(t)$ and $v_t(t)$ for conventional system.

5.4.3 Comments

Several simulations were made for both GMV self-tuning system and conventional system in this section. The simulations were done over a wide operating range and subject to different system disturbances. Through these simulations and comparisons, it is reasonable to conclude that the SMIB system with multivariable GMV self-tuning controller achieves more accurate control of shaft speed and terminal voltage than the SMIB system with conventional GOV and AVR. As for the damping on shaft oscillations to various disturbances in deterministic case, the simulated GMV STC system has achieved effective improvement over a conventional system.

CHAPTER 6

CONCLUSION AND RECOMMANDATIONS

This chapter puts forward some general conclusions on the results of this thesis and suggests some possible future research work.

6.1 Conclusions

This thesis is entirely devoted to the development of a multivariable GMV self-tuning controller for power systems. The reason for choosing adaptive control strategy for a power system is that a power system is nonlinear, time varying, and works in a stochastic environment.

The main contributions of this thesis can be summarized as follows:

1. The multivariable generalized minimum variance adaptive control theory was applied to a power system, and a practical self-tuning algorithm was derived.

The basic theory on GMV self-tuning control was proposed by Clarke and Gawthrop for single-input/single-output (SISO) systems in 1975, and it was extended to multi-input/multi-output (MIMO) system by Koivo in 1980. This thesis introduces the basic theory of MIMO GMV STC in the first part of Chapter 3, and then develops the STC algorithm for a power system in the second part of Chapter 3.

2. This thesis extends the Recursive Least-Squares (RLS) estimation with variable forgetting factor to the MIMO case. The variable forgetting (VF) factor algorithm was proposed by Fortescue *et al.* in 1981 for the SISO case. The purpose of introducing a VF factor into the RLS is to keep the alertness of the identification through "regulating" the covariance matrix.

3. A major contribution of this project is the development of the proof for the deterministic convergence property of the multivariable GMV self-tuning controller. This result is given in Theorem 4.1 of Section 4.1.

4. It is further proved in this project that for the regulator problem a stochastic system with diagonal interactor matrix of unit time-delay with GMV self-tuning control is equivalent to a deterministic system with a dc offset in the estimated model. This result is given by Lemma 4.2.1 in Section 4.2.

5. Based on Theorem 4.1 and Lemma 4.2.1 the thesis concludes the stability of the power system with

multivariable GMV self-tuning controller under the condition of Assumption 4.1.2.

6. Extensive computer simulation studies on a SMIB system were undertaken to test the derived multivariable STC, and comparisons with conventional, or classical, controllers were made. In the simulation phase a set of nonlinear equations was used to represent turbine-generator system connected to an infinite bus through short transmission lines. All simulations show that the power system with multivariable GMV STC can achieve satisfactory control results, and, in general, offers better control performance than a system with conventional controllers.

6.2 Recommendations for Further Studies

Although this thesis presents extensive studies on the application of multivariable adaptive controllers to a single power system there is still considerable work to be done before considering the installation of such a controller on a real system. It is suggested that further studies be made before proceeding with on-site tests:

First, the behavior of the multivariable GMV STR in a multi-machine system should be investigated. This thesis only investigated the performance of a SMIB system with multivariable GMV self-tuning controller. However, a modern power network is a large system with multi-generators. Therefore, it is important to know what is the behavior of a controller in such a complex circumstance.

Further improvements on the damping property of the self-tuning controller for power systems are required. As can be noted in the simulation studies, the self-tuning system has achieved much improvement of system damping in most cases compared with the conventional system. However, there is only limited improvement on damping the shaft speed oscillation for electrical power disturbance. It will be an interesting project to study how to increase the damping ability of the multivariable STC. It will not only be helpful for power system control, but also offer guidance for other applications.

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APPENDIX A

PARAMETERS OF SIMULATED SMIB SYSTEM

Generator parameters (in p.u.):

$x_d=1.6$ $x'_d=0.32$ $x_q=1.55$ $M=1.5$ $D=1.0$

$F_{hp}=1.0$ $\tau_c=0.1$ $T'_{d0}=5.0$

Transmission line parameters (in p.u.):

$r_e=0.0$ $x_{e1}=x_{e2}=2x_e$ $x_e=0.1$

Limitations on control signals (in p.u.):

$E_{fd} \in [-6.0, 6.0]$ $u_g \in [0, 1.0]$

$u_{pss} \in [-0.12, 0.12]$

GOV parameters:

$k_g=2.5$ $T_g=0.1$

AVR parameters:

$k_e=50$ $T_e=0.05$

APPENDIX B

OPTIMAL K-STEP-AHEAD PREDICTOR OF MIMO SYSTEMS

A k-step-ahead predictor can be obtained from Eqn.(3.2.1) by use the Diaphantine equality,

$$\mathbf{C}(q^{-1}) = \mathbf{A}(q^{-1})\mathbf{E}(q^{-1})+q^{-k}\mathbf{F}(q^{-1}) \tag{B.1}$$

where

$$\mathbf{E}(q^{-1}) = \mathbf{I}+\mathbf{E}_1q^{-1}+\cdots+\mathbf{E}_{k-1}q^{-k+1}$$

$$\mathbf{F}(q^{-1}) = \mathbf{F}_0+\mathbf{F}_1q^{-1}+\cdots+\mathbf{F}_{n-1}q^{-n+1}$$

Introduce the polynomial matrices $\tilde{\mathbf{E}}(q^{-1})$ and $\tilde{\mathbf{F}}(q^{-1})$ given by,

$$\tilde{\mathbf{E}}(q^{-1})\mathbf{F}(q^{-1}) = \tilde{\mathbf{F}}(q^{-1})\mathbf{E}(q^{-1}) \tag{B.2}$$

where $\det\tilde{\mathbf{E}}(q^{-1})=\det\mathbf{E}(q^{-1})$ and $\tilde{\mathbf{E}}(0)=\mathbf{I}$[54]. $\tilde{\mathbf{E}}(q^{-1})$ and $\tilde{\mathbf{F}}(q^{-1})$ always exist but is not unique. In addition, define a polynomial matrix $\tilde{\mathbf{C}}(q^{-1})$

$$\tilde{\mathbf{C}}(q^{-1}) \triangleq \tilde{\mathbf{E}}(q^{-1})\mathbf{A}(q^{-1})+q^{-k}\tilde{\mathbf{F}}(q^{-1}) \tag{B.3}$$

for which,

$$\tilde{\mathbf{C}}(q^{-1})\mathbf{E}(q^{-1}) = \tilde{\mathbf{E}}(q^{-1})\mathbf{C}(q^{-1}) \tag{B.4}$$

Writing Eqn.(3.2.1) as,

$$\mathbf{A}(q^{-1})\mathbf{y}(t+k) = \mathbf{B}(q^{-1})\mathbf{u}(t)+\mathbf{C}(q^{-1})\boldsymbol{\xi}(t+k) \tag{B.5}$$

and pre-multiplying it by $\tilde{\mathbf{E}}(q^{-1})$, it ends up with,

$$\tilde{\mathbf{E}}(q^{-1})\mathbf{A}(q^{-1})\mathbf{y}(t+k)=\tilde{\mathbf{E}}(q^{-1})\mathbf{B}(q^{-1})\mathbf{u}(t)+\tilde{\mathbf{E}}(q^{-1})\mathbf{C}(q^{-1})\boldsymbol{\xi}(t+k)$$

Using Eqns.(B.3) and (B.4), the following relation is obtained,

$$\tilde{\mathbf{C}}(q^{-1})\mathbf{y}(t+k)=\tilde{\mathbf{F}}(q^{-1})\mathbf{y}(t)+\tilde{\mathbf{E}}(q^{-1})\mathbf{B}(q^{-1})\mathbf{u}(t)+\tilde{\mathbf{C}}(q^{-1})\mathbf{E}(q^{-1})\boldsymbol{\xi}(t+k) \tag{B.6}$$

Pre-multiplying by $\tilde{\mathbf{C}}(q^{-1})^{-1}$, the above equation can be expressed as ,

$$\mathbf{y}(t+k)=\tilde{\mathbf{C}}(q^{-1})^{-1}\left[\tilde{\mathbf{F}}(q^{-1})\mathbf{y}(t)+\tilde{\mathbf{E}}(q^{-1})\mathbf{B}(q^{-1})\mathbf{u}(t)\right]+$$

$$+\left[\mathbf{I}+\mathbf{E}_1 q^{-1}+\cdots+\mathbf{E}_{k-1}q^{-k+1}\right]\boldsymbol{\xi}(t+k) \tag{B.7}$$

Therefore, the optimal estimate of $\mathbf{y}(t+k)$ at time t, or k-step-ahead predictor, is,

$$\mathbf{y}^{*}(t+k|t)=\tilde{\mathbf{C}}(q^{-1})^{-1}\left[\tilde{\mathbf{F}}(q^{-1})\mathbf{y}(t)+\tilde{\mathbf{E}}(q^{-1})\mathbf{B}(q^{-1})\mathbf{u}(t)\right] \tag{B.8}$$

If the quantity of $\mathbf{y}^*(t+j|t)$ with $j \le k$ is wanted, a j-step-ahead predictor can be expressed as[71],

$$\mathbf{y}^*(t+j|t)=\tilde{\mathbf{C}}(q^{-1})^{-1}\left[\tilde{\mathbf{F}}_j(q^{-1})\mathbf{y}(t)+\tilde{\mathbf{G}}_j(q^{-1})\mathbf{u}(t+j-k)\right],0<j\le k \quad (B.9)$$

where

$$\tilde{\mathbf{C}}(q^{-1})=\tilde{\mathbf{E}}_j(q^{-1})\mathbf{A}(q^{-1})+q^{-j}\tilde{\mathbf{F}}_j(q^{-1})$$

$$\tilde{\mathbf{G}}_j(q^{-1})=\mathbf{E}_j(q^{-1})\mathbf{B}(q^{-1}) \quad (B.10)$$

This j-step-ahead predictor is required by the form of the cost function, because $\mathbf{y}^*(t+j|t)$ is also needed for $j \le k$.

APPENDIX C

CONCEPT AND RELATED PROPERTIES OF INDUCED NORM

Definition Let $\|\cdot\|$ be a given norm on R^n. Then for each matrix $\mathbf{A} \in R^{n \times n}$, the quantity $\|\mathbf{A}\|$ is defined by

$$\|\mathbf{A}\| = \sup_{\substack{\mathbf{x} \neq 0 \\ \mathbf{x} \in R^n}} \frac{\|\mathbf{A}\mathbf{x}\|}{\|\mathbf{x}\|} \tag{C.1}$$

is called the induced norm of $\mathbf{A}$ corresponding to the vector norm $\|\cdot\|$. ΔΔΔ

Lemma C1 The induced norm of $\mathbf{A} \in R^{n \times n}$ corresponding Euclidean norm of a vector is given by

$$\|\mathbf{A}\|_2 = [\lambda_{max}(\mathbf{A}^T\mathbf{A})]^{\frac{1}{2}} \tag{C.2}$$

where,

$\lambda_{max}(\mathbf{A}^T\mathbf{A})$ =maximum eigenvalue of $\mathbf{A}^T\mathbf{A}$. ΔΔΔ

[Proof] From the definition of the induced norm, the following relationships are true,

$$\|\mathbf{A}\|^2 = \left(\sup_{\mathbf{x}\neq 0}\frac{\|\mathbf{A}\mathbf{x}\|}{\|\mathbf{x}\|}\right)^2$$

$$= \left(\sup_{\mathbf{x}\neq 0}\frac{\mathbf{x}^T\mathbf{A}^T\mathbf{A}\mathbf{x}}{\mathbf{x}^T\mathbf{x}}\right) \tag{C.3}$$

Since $\mathbf{A}^T\mathbf{A}$ is real and symmetric, its eigenvalues are real and non-negative. Let Λ be the diagonal matrix of eigenvalues of $\mathbf{A}^T\mathbf{A}$ so that $\mathbf{A}^T\mathbf{A}\mathbf{X}=\mathbf{X}\Lambda$, where $\mathbf{X}$ is the modal matrix of $\mathbf{A}^T\mathbf{A}$, whose columns comprise the eigenvectors of $\mathbf{A}^T\mathbf{A}$, and it is orthogonal.

Let $\mathbf{x}=\mathbf{X}\mathbf{z}$. Since $\mathbf{X}^T\mathbf{X}=\mathbf{I}$ it follows that

$$\mathbf{X}^T\mathbf{A}^T\mathbf{A}\mathbf{X}=\mathbf{z}^T\Lambda\mathbf{z}$$

Thus Eqn.(C.3) becomes,

$$\|\mathbf{A}\|^2 = \sup_{\mathbf{z}\neq 0}\left(\frac{\mathbf{z}^T\Lambda\mathbf{z}}{\mathbf{z}^T\mathbf{z}}\right)$$

$$= \sup_{\mathbf{z}\neq 0}\left(\frac{\sum_{i=1}^{n}\lambda_i z_i^2}{\sum_{i=1}^{n} z_i^2}\right) \tag{C.4}$$

Defining θ_i as,

$$\theta_i = \frac{z_i^2}{\sum_{j=1}^{n} z_j^2}$$

Eqn.(C.4) can be simplified to,

$$\|\mathbf{A}\|^2 = \sup \sum_{i=1}^{n} \theta_i \lambda_i \tag{C.5}$$

where $\theta_i \geq 0$ and $\sum_{i=1}^{n} \theta_i = 1$.

Choosing the θ_i corresponding to the largest positive λ_i to be unity and all the remainder to be zero maximizes $\sum_{i=1}^{n} \theta_i \lambda_i$. Since all the eigenvalues of $\mathbf{A}^T\mathbf{A}$ are non-negative, the largst positive eigenvalue of $\mathbf{A}^T\mathbf{A}$ is its spectral radius $\lambda_{max}(\mathbf{A}^T\mathbf{A})$. This proves the lemma.

Lemma C2 The following inequality is true for an induced norm of matrix $\mathbf{A}$,

$$\|\mathbf{A}\mathbf{x}\| \leq \|\mathbf{A}\| \, \|\mathbf{x}\| \tag{C.6}$$

ΔΔΔ

[Proof] It can be concluded from the definition of induced norm directly,

$$\|\mathbf{A}\| = \sup_{\mathbf{x} \neq 0} \frac{\|\mathbf{A}\mathbf{x}\|}{\|\mathbf{x}\|}$$

Thus it is true that

$$\|\mathbf{A}\| \geq \frac{\|\mathbf{A}\mathbf{x}\|}{\|\mathbf{x}\|}$$

Then the result of Lemma C.2 follows.

Lemma C3 If $\mathbf{A}=\mathbf{A}^T$, then the induced norm of $\mathbf{A}$ corresponding to Euclidean norm of a vector is,

$$\|\mathbf{A}\| = \lambda_{max}(\mathbf{A}) \tag{C.7}$$

ΔΔΔ

[Proof] If $\mathbf{A}=\mathbf{A}^T$, then

$$\mathbf{A}^T\mathbf{A}=\mathbf{A}^2$$

and the result follows the fact that if λ is an eigenvalue of $\mathbf{A}$, λ^2 will be an eigenvalue of $\mathbf{A}^2$.