University of Alberta

Estimating the Value of Stochastic Irrigation Water Deliveries in Southern Alberta: A Discrete Sequential Stochastic Programming Approach

by

Mohamed Said Gheblawi ©

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ABSTRACT

The main objective of this study is to estimate the expected value of stochastic irrigation water deliveries under trading water rights conditions that may introduce irrigation water shortages. Thus, farmers are faced with added uncertainty in conjunction with other risk sources manifested in output price, spring soil moisture, and precipitation. Risk components of the demand for irrigation water must be identified in order to estimate the water values.

A discrete sequential stochastic programming model is developed and the flow of information is assumed to follow a complete knowledge of the past and present structure. Estimates of the value of stochastic water deliveries resulting from water transfers are obtained through subtracting the values of risk penalties from the deterministic value of water. In addition, an estimate of the risk aversion parameter that captures the decision makers' preferences toward risk is obtained. A sample of twelve farms representing the production-farming units located at the Eastern Irrigation District in southern Alberta is fitted to the empirical models.

The risk aversion parameter estimate that describes the preference of southern Alberta's producers of irrigated crops toward risk is 0.00002, which can be described as mildly risk averse, since the estimated coefficient is positive and relatively close to being risk neutral. The linear non-stochastic model valued water between $47/ha-cm ($576/ac-ft) and $ 132/ha-cm ($1632/ac-ft). The values of risk penalties, for five scenarios of

irrigation water availability ranging from zero to 100 percent, ranged from $41/ha-cm ($503/ac-ft) for the first 5 to 25 percent of irrigation water deliveries to $11/ha-cm ($133/ac-ft) for the remaining 30 to 95 percent of irrigation water deliveries.

The estimates of the marginal value of stochastic irrigation water deliveries reach $128 per ha-cm ($1579/ac-ft) when irrigation water is scarce, compared to $38 per ha-cm ($467/ac-ft) when water is abundantly available. When potato hectares were constrained, the deterministic value of water was $ 19.80 per ha-cm ($ 244.33 per ac-ft) and the value of stochastic irrigation water deliveries declined to $ 12.39 per ha-cm ($ 152.81 per ac-ft).

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1. INTRODUCTION

Crop production in southern Alberta depends on supplemental irrigation to improve yields. The need for supplemental irrigation is due to the lack of sufficient precipitation to supply crop water requirements, as well as the variability of rainfall during the growing season. The Alberta government manages the province's water resources, including river basins, which are the main sources of irrigation water.

In the1800's, Alberta managed its water resources through a system of riparian rights, until the passage of the Northwest Irrigation Act of 1894. Due to increased demand for irrigation water in the early 1900's, in 1931 Alberta passed the *Water Resources Act* that required water licenses for all water uses except for household consumption. The province issued water licenses for agricultural use attached to parcels of land. Licenses under the *Water Resources Act* could only be used for the purpose for which they were issued.

In the latter part of the 20^{th} century, increased competition over water rights due to the increase in demand for water by current and new users necessitated a review of provincial water law and policy. In 1991, the Alberta government started a review of its water law and policy in order to meet present realities and future challenges. The *Water Act* of 1999 aims to manage the province's water resources in more equitable and efficient ways; to ensure the sustainability of the water resources and economic development; and to protect the ecosystem.

The *Water Act* prohibits any inter-basin transfers of water between Alberta's major river basins. However, the Act allows for the transfer of water licenses in areas where the available water is already allocated. It is argued that the trading in water rights will result in water moving from its traditional agricultural uses to other higher value uses, such as urban and industrial (Viney et al., 1996). There is the possibility, perhaps in the future, that senior priorities attached to agricultural parcels of land may be lowered or lost. The growing demand for water, the *Water Act* with its provisions for trading of water rights, global warming and the drought conditions experienced in recent years, are all factors that the typical crop producer in southern Alberta has to consider when making resource allocation decisions. Specifically, these factors are expected to contribute more risk into an already uncertain environment of crop production.

1.1 Problem Statement and Thesis Objectives

In crop production, major sources of risk and uncertainty include weather, insects, plant diseases, and prices. In order to reduce yield variability associated with uncertain and (often) insufficient rainfall, farmers in parts of southern Alberta have invested in supplementary irrigation. However, with the development of formal markets for irrigation water, demand for water to be employed in higher value uses is expected to reduce the availability of the resource to traditional farming and/or raise its cost. Thus, trading of water rights may add to the uncertainties associated with water diversions in terms of restricted and/or variable water supplies and the possibility of escalating water prices.

Economic studies have examined various issues related to Alberta's new water law and policy; including pricing systems (Peacey, 1995; Hatch, 1995); value of water rights (Royer, 1995); efficiency gains from water trade (Lo, 1995; Mahan, 1996, 1997); and farm water demand and risk analysis (Viney et al., 1996). If the assumption that the trading of water rights will increase the variability of irrigation water deliveries is confirmed, then a relevant question will be how this added uncertainty would affect the value of water from the perspective of crop producers in southern Alberta. It is interesting to investigate the impact of risk and uncertainty associated with water as a factor of production on the behaviour of the typical decision maker engaged in the production of irrigated crops in southern Alberta.

The main goal of this study is to estimate the value of stochastic irrigation water deliveries based on data of sample representative agricultural producers of the Eastern Irrigation District (EID). Thus, the scope of this study is not regional, whereby aggregate demands would be estimated. Instead, water demand is estimated for representative producers. Imputed values of water under different water transfer regimes are compared and stochastic demands for irrigation water of the representative farms are estimated in order to achieve this main goal.

Specific objectives of this study are as follows:

1. Development of a deterministic linear programming model capable of replicating production decisions made by sample representative agricultural producers of the EID;
2. Derivation of deterministic irrigation water demand for sample representative farms using this deterministic linear programming model;
3. Development of a non-linear programming model capable of modeling the stochastic and sequential nature of agricultural production and of replicating production decisions made by sample representative agricultural producers of the EID;
4. Estimation of the risk aversion parameter that best reflects the risk posture of the sample representative producers;
5. Derivation of conditional stochastic demands for irrigation water, using shadow prices from the non-linear mathematical programming models;
6. Estimation of risk penalties due to using water as a stochastic input by comparing stochastic to non-stochastic irrigation water demands;
7. Estimation of the value of stochastic irrigation water for the sample representative farms under different water-transfer scenarios, and hence the willingness of farmers to accept for such stochastic water quantities once the water markets exist.

1.2 Thesis Outline

The following chapter includes a survey of economic and mathematical programming literature related to this topic plus it provides a review of the water situation in southern Alberta and the associated legal framework. Chapter Three presents the theoretical background utilized in building the empirical models. Chapter Four contains a presentation of the empirical models and a description of data. Chapter Five presents the results and discusses the findings. Chapter Six summarizes the findings of the study and concludes with recommendations for future research.

2. BACKGROUND AND LITERATURE REVIEW

2.1 Introduction

This chapter presents an overview of the literature related to the topic of this thesis. First, the legislative issues pertaining to the laws managing water resources in Alberta are reviewed with an emphasis on the *Water Act*. The next section describes the water situation in southern Alberta and how it relates to the Eastern Irrigation District, home of the sample of representative farms. The third section of the chapter reviews economic literature pertaining to the water situation in southern Alberta, particularly the studies dealing with pricing, valuation, allocation, and efficiency. The last part of the chapter reviews the mathematical programming literature pertaining to the methodology of discrete sequential stochastic programming (DSSP). DSSP is the tool utilized in this study to estimate the stochastic irrigation water demands.

2.2 Alberta's Water Laws and Policies

Starting in 1991, Alberta undertook a review of its laws and policies governing water resources in the province. The *Water Resources Act* was considered by many to be dated and ill equipped to face the new challenges associated with the management of the province's water resources. The *Water Act* introduces, among many other tools and policies, the possibility of trading water rights between license holders. To understand the historical perspective of the legal environment under which the water resources are managed in Alberta, a review of the *Water Resources Act* and the *Water Act* is presented below.

2.2.1 The *Water Resources Act*[1]

In the late 1800's, the water law of the region that is now Alberta was a system of riparian rights where a landowner claimed the right to use water without any restrictions unless his/her use interfered with downstream users. The increased uses of

[1]Freeman et al. (1993) is the main source of material presented in this section

water-- in particular, the water for substantially more irrigation development-- revealed the inadequacies of the riparian system. A new system of administrative apportionment was introduced in the Northwest Irrigation Act of 1894 and ultimately in Alberta's *Water Resources Act* of 1931. The crown assumed ownership of water resources and consequently water users were required to obtain provincial water licenses for their water consumption.

A provincial water license was usually attached to a parcel of land. Thus, the value of the water right was capitalized into the value of the land and if the land was sold, the license would be passed to the new owners. Another type of license was issued to irrigation projects. These did not change even if the boundaries of the project were to change over time. A water license was acquired either through applying for one or by buying the land or project to which a license was already attached. Licenses under the *Water Resources Act* could only be used for the purpose for which they were issued. Licenses issued earlier had priority to their full amount in times of shortages over later dated licenses. In addition, traditional agricultural uses had priority over potentially higher valued alternative uses (e.g. urban and industrial). In fact, the *Water Resources Act* distinguished between uses of water, and accordingly established a priority list of water use. Domestic use was given the highest priority followed by municipal and irrigation uses. Industrial and waterpower uses were ranked fourth and fifth, respectively. The *Water Resources Act* did not require licenses for domestic water use. Domestic users had riparian rights, which in cases of disputes entitled them to water quantity privileges over other users, while licensed users prevailed over levels of flow of the stream.

The *Water Resources Act* seemed to discourage efficient water use, mainly due to the absence of water "markets." Such water markets are assumed to facilitate the transfer of water rights. The market mechanism, at least theoretically, will encourage current water users to conserve and reallocate excess water units to the highest bidders or to higher valued uses. However, the licensing system under the *Water Resources Act* made it difficult to accommodate new or alternative users in areas where water allocation had reached its limits.

2.2.2 The *Water Act*[2]

The province of Alberta initiated a review of its water management policy and legislation in 1991. The *Water Resources Act* was believed to be primarily a tool for allocating water while the current water management realities and future challenges required changes in policy and legislation to meet water resource management and conservation criteria. The result of this review was the *Water Act* of 1999. This Act states that the rights related to household uses and registrations pertaining to traditional agricultural uses are not subject to transfer provisions, and such rights will always be attached to the land. Household water use does not require licensing and has the highest priority up to a maximum of 1250 cubic meters of water per year per household. In order to provide a fair mechanism for protecting traditional agriculture water uses and minimizing the impact on existing licensed users, the *Water Act* provides a process for registering these uses of water based on the date of their first water use and hence protecting their relative priorities. The applicant is entitled to a maximum of 6250 cubic meters of water per year or the maximum amount specified in an applicable approved water management plan. All new licenses are issued for a specified period of time.

Under the *Water Act*, transferring an allocation of water under a license is allowed provided there are willing traders, i.e., a voluntary process. The government is authorized to monitor the transfer system. This is done through authorizing and reviewing transfers and water management plans as well as having the option of withholding up to ten percent of the water that is being transferred. Trading of water rights can be permanent or temporary, where the transferred allocation of water reverts to the original license holder after a specified period of time. The magnitude and

[2] Source: http://www.qp.gov.ab.ca/Documents/acts/W03.CFM

importance of water transfers remains to be seen and will be subject to market forces. The value of these stochastic water transfers is the concern of this study.

2.3 Southern Alberta's Water Resources and The Eastern Irrigation District[3]

The main surface water source in southern Alberta is the South Saskatchewan River. Its basin consists of four sub-basins: the Red Deer River sub-basin, the Bow River sub-basin, the Oldman River sub-basin, and the South Saskatchewan River sub-basin. Snow packs in the southwestern Alberta Rocky Mountains constitute the important tributaries within these sub-basins. The seasonality and variability of surface water flows, combined with occasional droughts, necessitated the construction of storage reservoirs to provide flow control, irrigation, water quality improvement, and hydroelectric generation. The Bow River sub-basin is estimated to supply about 1850 million cubic meters of water per season while the Oldman River and Red Deer River sub-basins are estimated to contribute 1795 and 500 million cubic meters per season, respectively (Mahan, 1996).

Fifty percent of the water in the South Saskatchewan River basin is "reserved" for use by the province of Saskatchewan. The demand for Alberta's fifty percent of water can be divided into consumptive and non-consumptive uses, which compete for water within and between the sub-basins. The main consumptive user is the agricultural sector, followed by municipal and industrial users. Non-consumptive users include hydroelectric generation and in-stream demands such as wetlands.

The agricultural sector was responsible for approximately 68 percent of total actual gross consumption of water in the South Saskatchewan River basin in 1994. This amount of water was used to irrigate more than 450,000 hectares of farmland. Freeman (1996) notes that irrigation makes up 48.2 percent of total annual license allocations for water in Alberta. This is more than the industrial and municipal uses

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[3]Mahan (1996), Freeman (1996), Royer (1995), Viney et al. (1996), and Underwood McLellan Ltd (1982) are the main sources for the material presented in this section.

combined. Royer (1995) indicates that some of the basins are fully allocated such that existing and new users cannot obtain licenses even if the value of water to them is higher relative to current users.

From an agricultural production perspective, southern Alberta is characterized by a continental, semi arid climate. The main factors affecting the productivity and efficiency of crop production in the region are the relatively short growing season, which is limited to about one-third of a year, and total rainfall, which is usually insufficient for maximum yield realization. Soils of the region are considered comparatively fertile, with adequate nutrient levels to support crop production. The exception is nitrogen, which plays a critical role once water deficiency is alleviated (Underwood McLellan Ltd, 1982).

Southern Alberta farmers have long adopted irrigation systems to enhance the productivity of traditional crops and to reduce the risks associated with dry land farming. The Eastern Irrigation District Act of 1935 enabled the farmers of the district to take over the irrigation system from the Canadian Pacific Railway. It established a democratic and autonomous structure for the district to operate in and made this organization fully responsible for water deliveries. The Eastern Irrigation District (EID) is located within the Bow River sub-basin. The main tributaries to this sub-basin are the Bow and Elbow Rivers. Water in the Bow River sub-basin is used for agricultural, urban, industrial, and hydroelectric purposes. Two other irrigation districts are also located within the Bow River sub-basin: the Bow River Irrigation District and the Western Irrigation District.

Traditional prairie crops dominated the crop mix in the early years of the EID. Grain crops such as wheat, barley, oats, and grass hay were the main cultivated products. Recently, alfalfa has started to dominate the landscape of farmland in the EID, and cash crops such as canola and peas are also gaining popularity among farmers. Soil and climatic restrictions have limited the expansion of specialty crops within the EID.

Alfalfa's recent dominance is attributed to the development of new winter hardy strains and the availability of irrigation water. Alfalfa production caused significant development and expansion of several alfalfa processing enterprises. In general, availability of water led to the expansion in both forage and livestock production. Related to that, the EID operates vast summer grazing lands that are made available to its irrigation members (Viney et al., 1996).

In the 1994 season, actual gross consumption of water at the Bow River sub-basin was approximately 1184 million cubic meters from which the EID consumed approximately 388 million cubic meters, or 32.7 percent (Mahan, 1996). In 1994, approximately 53 percent of the EID's water diversions went to agricultural uses. Return flows and system losses accounted for more than 30 percent of total water diversions. The municipal, industrial, and rural uses totaled about 2 percent while wetland diversions accounted for about 4.5 percent of total water diversions by the EID (Viney et al., 1996).

The water situation in the Bow River sub-basin and around the EID suggests that the possibility of higher water demand in the future that will require some water to be diverted from traditional agricultural uses. The competition between industrial and urban uses over quantities of water used by the agricultural sector would seem to raise the value of water and hence drive it away from use in crop production. Shortages in irrigation water deliveries seem a distinct possibility in the not so distant future. The cost of water is also expected to rise with increased demand. The expected irregularities of water delivery to the farmlands will contribute to the stochastic nature of water as an input employed in the agricultural production process. It is the intention of this study to explore the effects of such uncertainties associated with the water situation in southern Alberta on the value of stochastic irrigation water deliveries.

2.4 Review of Economics Literature Related to The Water Situation in Southern Alberta

There exist a wide range of economic issues and topics related to water use in Alberta. One of these topics involves pricing methods that are available to the government for charging the customers of the resource. There are also allocations issues related to quantities of water "traded" between different sub-basins, although unlawful, and within basins, and related efficiency questions. Valuation of the resource in relation to its use in agriculture and the institutional aspects introduced by new policies and legislation are also issues of interest.

The *Water Act* provides a license transferring mechanism that will allow water reallocation in order to fulfill changing demands. When considering transferring water rights in Alberta in light of the experiences of the United States and Australia, Freeman (1996) concluded that such transferability must adhere to certain basic requirements. Specifically, clearly defined rights and uses outlining the type of use, the place of use, the point of diversion, and the time of use will reduce the potential for conflicts. Linking the proposed transfers to the conditions of the existing water resources as well as continuous inventory of licenses, uses, and stocks will provide the information needed for efficient markets and management. Freeman (1996) also discussed the role of government in ensuring a successful water-trading program and argued that government can play an important role in this process by controlling license issuing and providing a review process to ensure adequate protection of the rights of other consumptive and non-consumptive users. The government can also contribute by monitoring the effects of the transfers on the environment, local and adjacent economies, and on the integrity of the water resource itself.

Several potential negative consequences of transferring water rights can be expected. These include the failure of market mechanisms to accurately value the water rights by either overstating or understating them[4], by altering the way of life in different communities, and perhaps through the production of different externalities. However, since agriculture is the main water user in the region, it is expected that this sector will be the most affected, perhaps to the extent of requiring restructuring of the economies for rural communities in southern Alberta.

Freeman (1996) argued that the inflexibility of Alberta's water management practices under the *Water Resources Act* has locked water resources into marginally lowest-valued use, which is mainly irrigation in agriculture. Freeman also provided a thorough investigation of the American and Australian experiences in transferring water rights, from both the historical and practical perspectives.

Valuation of water resources, especially the portion used in agriculture, has attracted the attention of stakeholders, academics, and various governmental agencies. An example is the study by Royer (1995) where estimates of the value of water rights within agriculture were obtained using a hedonic pricing approach. Royer's model related land sale prices to land attributes as explanatory variables. Eight southern Alberta counties were chosen for the study based on their heavy irrigation water use. Data covered 230 land sales and the hedonic price model had nineteen explanatory variables and a constant. The results suggested that market segmentation does not exist. All other things held constant, the estimate of water rights value-approximately $190 per acre- represents the average difference between land values of farms that have access to irrigation and farms that do not. The value of water rights increases the value of irrigated land by as much as 35 percent over non-irrigated land. This result suggests that water is a very important agricultural input and should be employed efficiently. Tradable water rights are supposed to achieve efficient water use by adding value to a resource mostly thought of as 'free'.

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[4] Taylor et al. (1995, p.248) argued that actual sales prices of water rights may not reflect full social opportunity costs and they "… may vary with participants' knowledge of the market, with financial constraints or distress, or with collaboration of buyers or sellers to overstate or understate actual social costs"

Another group of studies concentrated on issues of water pricing. Peacey (1995) compared a number of pricing schemes under conditions of stochastic supply and demand. These schemes included peak load pricing, interruptible pricing, and priority pricing.

Peak load pricing is assumed to smooth the peaks and valleys of demand for water by charging higher prices at times of high demand and lower prices at periods of low demand. This pricing scheme discourages consumption in peak periods and encourages off-peak consumption. Interruptible pricing is in principle an agreement between the customer and supplier that makes the buyer stop consumption at certain times, or become the first to lose service at times of shortages or peak demand, in return for discounted prices. Priority pricing involves matching the customer's willingness to pay to a priority-based pricing scheme where priority access to the good or service comes with a premium.

Peacey (1995) concluded that optimal peak load pricing would be difficult to implement when demand is stochastic. Consumer expectations of excess capacity and excess demand would affect the effectiveness of interruptible pricing. However, priority pricing provided gains in social welfare over uniform pricing. The matching of the firm demand of fewer consumers with highly reliable services requires less installed capacities, which in return reduces social expenditures and hence improves social welfare. Efficiency gains can also be attributed to the variety of service options available to consumers. Spot markets may require monitoring and may have relatively high transactions costs. Priority pricing combined with verifiable service reliability would reduce such costs substantially and hence improve efficiency. Application of priority pricing and its success seemed to be dependent on the type of market organization adopted, participation of customers, and volume of trade.

Hatch (1995) examined a two-period priority pricing framework and the impact of reliability of service on optimal price and reservoir capacity. The results indicated that moving from the institutionally constrained uniform price and reliability of service contract to multi-priority interruptible service represents a potential Pareto improvement (p.187). When compared to spot pricing, interruptible contracts appeared to be less costly to implement and could realize most of the potential gains from efficient rationing as long as differences in value of reliability to consumers and uses exist over time.

Overall, the water pricing literature suggests the adoption of interruptible service type of schemes. This, in turn, may translate into more random water deliveries to agricultural producers in the future. Such scenarios will contribute to the stochastic nature of the water resource use in agriculture and, therefore, producers have to implement strategies that will deal with such uncertainties. An assessment of the risk penalties for using the stochastic resource is one focus of this study.

Issues pertaining to the allocation of water resources and their efficient use once trading of water rights is introduced are legitimate concerns of planners, users, and society. Several studies have addressed such questions, using mathematical programming models within a general equilibrium framework. For example, Lo (1995) employed a mathematical programming approach to examine the effects of water trade on water allocations and efficiency gains in times of shortage. Since the main water user in Alberta is the agricultural sector, Lo hypothesized that it would be the first to experience the effects of the proposed changes. This was based on the argument that agriculture is a low valued water-user and the higher valued users will attract water away from traditional farming. Four water supply sources and twelve demand sources were modeled in the study. Minimum in-stream flows plus apportionment demand were considered. The objective was to optimize post trade total surplus. The model predicted water allocation under four different scenarios. Scenario 1 assumed the status quo conditions without trading of water rights. Scenario 2 allowed for intra-regional trade. Scenario 3 allowed for interregional trade. Scenarios 2 and 3 held apportionment and in-stream consumption constant from scenario 1. Scenario 4 assumed that apportionment and in-stream quantities were tradable.

The model results suggested that allowing water to be traded would result in water being shifted away from agriculture to other uses. Welfare gains seemed to increase as more water was reallocated from the agricultural sector to the urban and industrial sectors. Assuming a ten percent water shortage, Lo examined the effect of three different institutional regimes. First, when a social planner enforces a market-like allocation, the agricultural sector was found to be a major welfare loser. Second, windfall gains accrue to the government if it was assumed to auction off water licenses every season. Finally, all users and regions gained welfare when private exchange of rights was allowed.

Mahan (1996) expanded on the work of Lo (1995) by using gross inverse demand functions instead of net inverse demand functions to link water users economically. Assuming water trade, Mahan's mathematical programming model maximized total welfare under three alternative scenarios of water allocation. Scenario 1 emulated existing water management practices. The results showed substantial differences in water values between regions and uses, suggesting possible efficiency gains from water trade. Scenario 2 allowed for intra-regional trade (within the sub-basin), while scenario 3 assumed intra- and interregional trade. The results from both scenarios 2 and 3 indicated improvements over the base case in terms of welfare gains from trading water. However, the magnitude of these gains seemed considerably lower than those reported by Lo (1995). The reason for such discrepancies could be attributed to differences in demand functions and aggregation.

Mahan (1997) changed the scenarios of his earlier work by assuming a base case with long-term mean flows and alternate cases that examined drought and surplus regimes. The results indicated that more than ninety percent of the attainable welfare was generated by the existing institutional structure. However, when considering the projected demands for year 2010, larger gains from trade would be generated. This indicated that there is potential for sizable gains because of adopting market institutions to govern water trade. Mahan reported that the choice of the administrative regime would not affect the total welfare in the system. However, under a water auction system the irrigation sector would receive the largest welfare loss, while under a water rights trading regime the irrigation sector would gain welfare. In all regimes considered, it seemed that the government would be the "winner" in terms of the largest sum of welfare.

The preceding types of studies analyzed the welfare effects of trading water rights in a static framework from a regional perspective. However, the scope of this thesis is to analyze the effects of trading water rights on producer welfare through valuation of stochastic water deliveries and from sample representative farms point of view. The next group of studies takes this farm-level approach in examining water use issues.

Kulshreshtha and Tewari (1991) estimated an aggregate derived water demand schedule for the South Saskatchewan River Irrigation District using a single period linear programming framework. The representative farm aggregation approach was used where different single crop representative farms were selected and modeled as profit-maximizing firms subject to resource constraints. Yield responses to joint use of water and fertilizers were incorporated into the model. Summing individual farm's water demand schedules, weighted by relative shares of crops in total irrigated area, yielded the aggregate water demand for the district. Data from a seven-farm-sample were used to optimize the model. One farm had two enterprises: alfalfa and feeder cattle, while the other six were single enterprise units. No specialized feedlot operations were included. On-farm yield data were estimated using crop-water production functions based on the ratio of actual to potential evapotranspiration to explain the ratio of actual to potential yields. A linear relationship between water and fertilizer was used to estimate fertilizer application response at different levels of irrigation.

Variable resource price programming, where the price of water was increased successively until water use becomes uneconomical, was used to obtain stepped derived demand schedules for each farm. The short-run value of water in South Saskatchewan River Irrigation District was estimated to be between \$0.44 and \$ 127.82 (1986 dollars) per acre-foot for different levels of product prices. The long-run value was estimated to be between zero and \$1.59 per acre-foot of water.

A similar study by Viney et al. (1996) used a model with twelve representative EID farms to analyze the effects of changes in water cost and quantity on crop mix. They also estimated the aggregate irrigation demand for the EID. The model maximized farm net income subject to crop rotation, acreage, and crop physical production function constraints. Successive reductions in available irrigation water, up to fifty percent of the full requirements, were imposed. Demand for water was estimated allowing shifts in production away from irrigation to dry land farming. Water demand was estimated as the change in net income per change in water availability converted to dollars per acre-foot.

The results suggested that water reductions and higher water prices would result in increased specialty crop production and a reduction in production of traditional cereals. The farmers' mean-variance trade-off analysis showed that non-diversified specialty crop producing farms are riskier than traditional farms. Specialty crop operations seemed to generate the highest returns. However, high investments and uncertain markets introduced a significant risk element to these enterprises. The main reason for the specialty crop production not expanding was believed to be the possibility of large negative returns. Estimates of water values ranged from \$8 to \$250 per acre-foot.

These last two studies employed mathematical programming techniques in estimating water demands. However, the models adapted for use did not account for the sequential nature of the decision and production processes. In addition, Viney et al. (1996) accounted for only risk associated with revenues. This thesis incorporates other sources of risk related to water availability during the growing season and risk associated with revenues. It also incorporates the sequential component of the production process and the flow of information to the decision maker during the growing season.

In a review of the water situation and outlook in Alberta, Adamowicz and Horbulyk (1996) discussed the nature of the allocation of water under scarcity. They stated that water quantity and quality problems are often caused by inefficiencies on the part of the institutions rather than technological inadequacies. Government water policies, such as tradable water rights, water pricing and others, can define and establish economic instruments to resolve problems of water quantity allocations. Their study provided a review of the research dealing with the water situation in southern Alberta. It appears that trading water rights is preferred over pricing schemes even though the latter may have the potential to offer considerable efficiency improvements through resource reallocation. Issues of in-stream flow requirements, water quality, and rural communities' welfare remain issues open for discussion and research because of water policy reform.

From the historical background and the literature survey of economic research related to water management in Alberta, it appears that the *Water Act* and the implications of introducing water markets to Alberta will have the greatest effect on the agricultural sector. The results from the above reviewed studies also suggest that the trading in water rights will improve welfare for society as a whole. Viney et al. (1996) studied the effects of the legal and policy changes on the agricultural sector via modeling representative farms from the EID. The model utilized portfolio theory to analyze the farmers' decision behavior considering risk in the output market, and consequently estimated the irrigation water demand for the EID. Lo (1995) and Mahan (1996, 1997) utilized mathematical programming models to analyze the water policy impacts on water allocation and efficiency gains especially to the agricultural sector. The reviewed studies appear to stop short of recognizing the different types of risk facing the decision maker in the farming business, other than output price risk. In addition, those mathematical programming studies did not incorporate in their models the sequential nature that characterizes the decision process of agricultural production.

2.5 Review of Related DSSP Literature

Discrete sequential stochastic programming (DSSP) is one of several risk programming tools. DSSP accommodates the sequential nature of farmers' decision process and considers sources of risk and uncertainty other than output price risk; these may include uncertainty related to input availability and magnitude of coefficients of transformation. DSSP appears to posses the potential to analyze in depth the impact of the new water policy reforms on southern Alberta's agricultural sector. The model can be formulated to consider the added uncertainties that farmers will be facing due to these legal and policy changes. These may include uncertainties associated with reduced supplies of irrigation water plus the already existing risk associated with agricultural revenues, spring soil moisture, and precipitation. This section provides a review of the literature related to DSSP and its application to problems in agriculture, which are relevant to this research. However, this review is by no means exhaustive of all published studies in this subject area. Only a handful of representative and relevant DSSP studies are reviewed. Boisvert et al. (1990) and Apland et al. (1993) provide extensive reviews and listings of DSSP studies.

Cocks (1968) developed the general formulation of the DSSP model. DSSP considers the multistage nature of decision processes. It allocates resources to activities within a stage and then optimizes allocation over the next period based on events observed in the first period. The model attaches to some or all the coefficients a modified probability distribution based on past events or actions within the model. In other words, discrete probability distributions can be used to describe the activity coefficients, and/or input-output coefficients, and/or resource supplies.

Rae (1971b) provided one of the first and classical applications of DSSP where he presented an empirical application of solving sequential decision problems under uncertainty. Rae noted the importance of first defining the probability model by isolating the decision dates and then dividing the planning period into a number of stages. The next step is to define the possible random events within each stage, specifying subjective probabilities for the occurrence of each state of nature, and stating the appropriate information structure. The last step is to define activities and constraints for each stage and to specify the objective function.

Rae applied this to a farm unit producing vegetables, modeling the process in three stages. All uncertainties were assumed to be functions of weather conditions and crop market prices. The probability distribution of output price was defined to have two potential 'values', high and low. The weather probability distribution was assigned three levels, good, normal, and bad. The constructed sequential probability model constituted joint probabilities for fifty joint events. Separate budgets were prepared for each crop for each possible state of nature. Requirements of each activity from each resource under all joint outcomes were then specified. A stochastic separable programming model was used to overcome the problem of non-convexity of the utility function. The objective function maximized expected utility formulated as the sum of the joint probabilities times the utility value transformed from net income of sets of separable activities.

Rae's model was first solved deterministically in order to use the management strategy obtained as a benchmark to compare with outcomes of the stochastic solutions. The passive formulation assumed the grower knew future events with certainty. For each joint event, a passive model was solved. A discrete distribution of the fifty activities was converted to an appropriate utility distribution. The difference between the expected net incomes obtained from the deterministic linear model and the stochastic model was interpreted as the amount of money the grower would be willing to pay for perfect information. The stochastic model yielded the optimal strategy to be adopted by the grower. The model also generated distributions of future resource employment and discrete distributions of shadow prices of resources. Rae asserted the importance of efficient use of information received as the planning period progresses. He also noted the need for more states of nature to be formulated at intervals that are more frequent while considering the costs of model complexity, computational limitations, ability to interpret output, and data collection.

A study more closely related to this thesis is by Apland et al. (1980), who investigated the impact of risk aversion on demand for supplementary irrigation of corn using a case study approach. The analytical model utilized a MOTAD-type objective function that maximized expected profits less a cost of bearing risk. Defining activities and resource endowments over twenty-one consecutive periods in the production horizon captured the effects of planting and harvesting dates on yields. The cost coefficient on an irrigation supply activity was parametrically altered to generate the derived demand functions for supplemental irrigation. The impact of risk aversion was investigated by generating demand functions for decision makers with different risk postures (risk neutral, high, and low levels of risk aversion such that the risk aversion parameter takes the values of zero, two, and one, respectively). The results showed that as the marginal risk aversion coefficient was increased from zero, the derived demand functions became more inelastic. Increasing the risk aversion parameter led to a more diversified crop plan and substitution of irrigated for non-irrigated crops. The study's conclusion was that the influence of the manager's risk preference has a marked impact on the firm's demand for irrigation.

DSSP is flexible in adapting to many variants of expressions of the objective function and constraints. Kaiser et al. (1987) used a MOTAD-type objective function to model the participation behavior of Minnesota farmers in commodity programs of 1983 and

1986. The objective function maximized expected net revenues adjusted for risk stated as a linear approximation of the expected value-variance (E-V) model. Risk was measured as the standard deviation of net revenues as estimated by total negative deviations from the mean times a coefficient dependent on the number of joint events.

Kaiser et al. (1987) assumed production and marketing decisions were made in three stages: pre-harvest, harvest, and post-harvest. Production risk was modeled in the second stage and captured through ten discrete states of nature on yield, field time, and harvest field rate. Yield risk was measured by the coefficient of variation based on time-series of yield data.

The first stage was modeled deterministically. The constraining resources of the first two stages were labor, machinery, acreage, and on-farm storage capacity. It was assumed that the producer, at the beginning of stage three, knew the harvest price, but only had probabilistic knowledge of the states of post-harvest price. Discrete price states of nature were defined for the third stage generating one hundred joint events.

Data were for two selected farms engaging in only corn and soybeans production. Provisions were made for farmer participation in the programs by constraining acreage according to program requirements, and applying support and target prices of corn and soybeans.

Kaiser et al. (1987) solved the model by adjusting the risk coefficient parametrically from 0.0 to 2.0 in increments of 0.5 for the non-participation case. The model was resolved for the participation case in order to compare the relative risk efficiencies of participation and non-participation with the associated risk postures. Kaiser et al. (1987) found that participation was E-V dominant to non-participation for all scenarios investigated. However, as risk aversion increased, the potential for reducing risk by participation decreased and the potential for average income enhancement increased.

Several studies utilize DSSP formulations in order to endogenously-determine the values of certain variables. Lambert (1989) modeled the decision problems related to calf retention where production and marketing decisions were presented in four alternative formulations. The first assumed animals are sold following winter-feeding. The second formulation added feeding the animals on rangeland for summer grazing. The third and fourth formulations allowed for optimal feeding and marketing decisions to be solved endogenously. The constraints included the animal performance equations for winter-feeding and summer range, and marketing activities. Prices of animals were allowed to adjust endogenously as the weight of the animals changed.

The objective function of Lambert's (1989) initial basic model was expected return maximization, while the final model minimized total absolute deviations of returns. Although the predominant sales strategy adopted by farmers was to retain animals for fall sale, all models optimized by selling all calves by the end of spring.

Taylor and Young (1995) calculated direct regional foregone agricultural benefits from water transfers by using a DSSP model to estimate regional water demand of a southeastern Colorado county. In evaluating market transfers of water from agricultural to urban uses, agricultural water use becomes society's direct forgone benefits. The analysis of foregone economic benefits is needed to conduct a benefit-cost evaluation from society's perspective. However, the model incorporated agronomic plot data on irrigated crop response into an aggregate regional programming model.

The model used by Taylor and Young (1995) accounted for the sequential nature of crop production decisions and the uncertainties in water supplies and rainfall. The constraints included variations in soil productivity, regional irrigation and precipitation, and site-specific crop production functions. A deterministic linear programming (LP) model could not exactly explain actual agricultural production in the region. However, DSSP primal results reflected, to some extent, the historical crop mix. The dual results were used to derive demand for uncertain water deliveries from which foregone benefits were estimated.

Taylor and Young (1995) adapted Antle's (1983b) view that dynamic models of risk-neutral preferences work better in explaining production risk than the static risk-averse models. The model Taylor et al. used maximized expected regional income over three sequential stages subject to the above-mentioned constraints. Two states of nature for water delivery (adequate and inadequate), and two states of nature for precipitation (dry and wet) were incorporated into the objective function. Analytical production functions relating applied water to yield were used. To obtain the irrigation water requirements, the contribution of effective precipitation under the two precipitation states of nature was subtracted from the total applied water requirements on the production function.

Taylor and Young (1995) derived stochastic demands for irrigation water by parameterizing the constraint on diversions across intervals of ten percent of mean values of diversions in the adequate and inadequate states of nature in order to obtain the shadow prices of diversions under these two states of nature. A cross section of the stochastic demand schedules at the mean level of diversions gave the conditional stochastic demand curves for irrigation water. In other words, these conditional stochastic demands were derived by holding the deliveries in the opposite state of nature constant at the historical mean value.

Conditional stochastic demand curves allowed for the comparison between the stochastic demand curves obtained from DSSP solution and the deterministic demands obtained from the deterministic linear programming model. Such comparison was used to evaluate the risk premium that a farmer would be willing to pay in order to secure irrigation water delivered with certainty. At any level of diversion, the vertical distance between the stochastic and deterministic demands would be the payment that makes the farmer indifferent between certain and uncertain water deliveries. The expected value of uncertain deliveries could be obtained by prorating the value of the deliveries by their probabilities of occurrence.

The area under the stochastic demand curve provided the basis for estimating the forgone benefits of agricultural water. The difference between the areas under the demand curves (value of objective function) before and after water was diverted estimated society's cost of agricultural-to-urban water transfers. To estimate the long run value of water, fixed costs were subtracted from the expected value of water predicted by the model.

Taylor and Young (1995) model showed that water was withdrawn from poorer soils first and thus the forgone benefits of water withdrawn from better soils were relatively higher. The average forgone value of irrigation water was estimated to be US $37 per acre-foot. Although the estimated risk penalty reduced the value of irrigation water by US $6 per acre-foot, it appeared uncertain that its absence nor the added transactions costs would have been the deciding factor in the social benefit of transferring water from agricultural-to-urban uses. The low value of water in irrigated agriculture enabled the market to transfer it to the higher value urban use. Taylor et al.'s (1995) technique of deriving stochastic and conditional stochastic demands for irrigation water is utilized in this study. In addition, their approach of estimating risk penalties is also employed here.

The above background and review of literature introduced issues to be considered when attempting to achieve the goals of this research. DSSP model used in this study improves on previous studies by accounting for the sequential nature of the decision process employed in crop production. It incorporates risk attributed to spring soil moisture, precipitation, irrigation and revenues. Risk preferences of crop producers are also incorporated into the empirical model. The next chapter will introduce the theoretical framework on which the empirical model is based.

3. THEORETICAL BACKGROUND

3.1 Introduction

In the process of achieving the goals of this study, a theoretical foundation is required for building and utilizing the empirical model. Economic theory of the firm provides the theoretical concepts and framework for estimating deterministic and stochastic factor demands while mathematical programming provides a tool for estimating these demand schedules. Therefore, it is beneficial to review these topics in light of the scope of this study and then develop an appropriate theoretical model suitable for supporting the empirical part of the study.

This chapter is organized into two main sections. The first section discusses economic theory of the firm under certainty and uncertainty conditions (Section 3.2). The first part of Section 3.2 presents deterministic theory of the firm while the second part introduces uncertainty to the model of the firm. The second section presents a review of mathematical programming (Section 3.3). It starts by introducing linear programming while the next subsection introduces risk to the model. The last subsection discusses discrete sequential stochastic programming and derivation of stochastic factor demands.

3. 2 Theory of The Firm

Theory of the firm addresses economic questions related to the decision maker's behaviour. Profit maximization and cost minimization are standard assumptions concerning the decision maker's behaviour. The theory also analyzes the questions related to what to produce, how much to produce, and how to produce it. The analysis is usually undertaken from one of two alternative, but consistent, points of view: the output and input perspectives. Deterministic theory of the firm assumes complete knowledge while stochastic theory of the firm assumes uncertainty about prices, technology, input availability, etc. The following two subsections demonstrate the theory of the firm under conditions of certainty and uncertainty in relation to derived deterministic and stochastic factor demands.

3.2.1 Theory of The Firm Under Certainty Conditions

Theory of the firm assumes that decision makers behave as to maximize profits (π). Profits are defined as revenues (R) minus costs (C). Revenue is defined as output price[1] (p), multiplied by quantity of output produced (y). Costs are defined as factor price (w), multiplied by quantity of input employed (x). The firm faces market and technology constraints. If the firm is assumed to be competitive in input and output markets, then the firm exhibits price-taking behaviour. The constraints imposed by technology determine choices that are feasible in producing maximum outputs from given amounts of inputs. A production set defines all combinations of inputs and outputs that comprise a technologically feasible way to produce. The boundary of the production possibilities set (Y) describes the production function f(x), which measures the maximum possible amount of output obtained from a given amount of input(s). Marginal product of a factor of production $\partial f(x)/\partial x$, is defined as the amount of output produced per extra unit of input. Many types of production processes exhibit diminishing marginal products. The value of marginal product (VMP) is the marginal product multiplied by output price. The profit maximization model can be expressed algebraically as:(Varian (1992) p. 26)

Maximize $\pi(p, w) = \max p\, f(x) - w\, x$ (3.1)

The first order conditions are:

$p(\partial f(x)/\partial x) = w$ (3.2)

Equation 3.2 gives the optimal choice of inputs to produce the optimal profit maximizing level of output, which occurs at the point where the VMP of each input equals its price, or at where marginal revenue equals marginal cost. Equation 3.2 can be expressed as: x (p, w), which is the factor demand function stating that input demand is a function of input and output prices subject to technology.

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[1] For simplicity, subscripts indicating multiple inputs and outputs are omitted from notation

3.2.2 Theory of the Firm Under Stochastic Conditions

Departing from the assumption of conditions of certainty leads to the introduction of risk or uncertainty into decision-making. In economic analysis with certainty, it is common to assume the firm is maximizing profit. The profit function can have a well-defined maximum since the firm's prices and output are certain at the time the firm makes its decisions. However, in theory of the firm under uncertainty, the firm is assumed to maximize some objective function that relates the firm's input decisions to its welfare, where input decisions are now related to the probabilities of the different profit levels that are possible ex ante. Therefore, considering uncertainty, the decision maker chooses a production plan that has a set of possible outcomes that will occur with some probability. A random variable, such as yield or output price, is defined by these sets of outcomes or probability distributions. The decision maker is then decides on the choices between alternative probability distributions of outcomes. The decision maker, however, must know what these outcome distributions are before choosing among them. The actual or true distributions are usually not known to the decision maker. However, if rational expectations are assumed, then decision theory concludes that the subjective distributions the decision maker forms correspond to the objective ones, since the manager has an economic incentive to learn about the distributions of outcomes and update his/her information base accordingly. In an agricultural setting, the distinguishing characteristic of farm income is its instability from year to year, while the many possible income outcomes depend on the realization of the prevailing states of nature in a particular year. The problem that faces the decision maker is to rank farm plans based on their income distributions and select the best plan that achieves the goals. This decision making process resembles lotteries.

3.2.2.1 Expected Utility Theory

Maximizing expected utility is a commonly used assumption in economic analysis. However, expected utility theory is rooted in a set of assumed axioms that can be briefly listed as follows:(Binger et al. (1988), pp. 497-500)

- Preferences are complete: For any two outcomes, the decision maker may prefer one to the other or be indifferent;
- Preferences are reflexive: If the decision maker is indifferent between two outcomes, then they must be the same and have to be ranked the same;
- Preferences are transitive: If the decision maker prefers y_1 to y_2 and y_2 to y_3, then y_1 is preferred to y_3;
- Preferences are continuous: The certainty equivalent axiom states that the decision maker is indifferent between getting an outcome with certainty and playing a lottery yielding the certainty equivalent;
- Preferences are substitutable: The lottery can always be substituted for its certainty equivalent in any other lottery;
- Preferences are monotonic: For any two lotteries with the same two outcomes each differ only in probabilities, the lottery that gives higher probability to the most-preferred alternative is preferred to the other lottery.

If the preferences of the decision maker satisfy the above axioms, then numbers $U(y_i)$ can be assigned in association with outcomes y_i, such that if two lotteries are compared, the decision maker will prefer the first lottery to the second if and only if

$$\sum_i \alpha_i U(y_i) > \sum_i \alpha_i' U(y_i) \qquad (3.3)$$

where α_i's are probabilities of occurrence of lotteries. The utility functions that are separable and additive (as in Equation 3.3), and satisfy the listed axioms are called expected utility functions or more commonly as von Neumann-Morgenstern utility functions. Varian (1992) demonstrated a proof for the uniqueness of expected utility up to an affine transformation. Machina (1987) indicated that a von Neumann-Morgenstern utility function could be subjected to monotonic transformation that would make changes in its origin and scale but not its shape.

A risk averting decision maker prefers the expected value of a lottery to the gamble itself, while a risk loving decision maker would prefer the lottery to its expected value.

A risk neutral decision maker is indifferent between the gamble and its expected value. The shape of the utility function indicates the individual's preference toward risk. Risk averse, risk neutral, and risk preferring preferences are characterized by concave, linear, and convex utility functions, respectively.

3.2.2.2 Measures of Risk Preferences

It is useful to have a measure of risk preferences that is invariant to monotonic transformations of the utility function and can be compared between decision makers. The curvature of the expected utility function intuitively represents the degree of risk aversion of the decision maker; the more concave the expected utility function, the more risk averse the decision maker. Similarly, the more convex the expected utility function, the more risk loving the decision maker. The second derivative of a function is a reasonable candidate for measuring risk aversion such that the more concave the utility function is, the more risk averse the decision maker is, and hence the greater is the absolute value of the second derivative of the utility function. However, normalizing the second derivative by the first derivative of the function will yield a measure of risk aversion that is invariant to changes in the expected utility function.

The Pratt- Arrow absolute risk aversion coefficient (λ), which is commonly defined as:

$$\lambda\,(.) = -U''(.)\,/U'(.) \qquad (3.4);$$

where U is the utility, and U′ and U″ are, respectively, the first and second derivatives of U with respect to the argument in U, has been widely used in risk analysis studies which order alternative action choices under conditions of uncertainty. Given a von Neumann-Morgenstern utility function, the value of the corresponding λ (.) will be positive, provided that the derivatives $U'(.) > 0$ and $U''(.) < 0$ hold.

Decreasing absolute risk aversion (DARA) appears to be the most common assumption in empirical research where $\lambda'(.)<0$. Constant absolute risk aversion (CARA) is implied by $\lambda'(.)=0$, while $\lambda'(.)>0$ means increasing absolute risk aversion

(IARA). The implication of CARA is that changes in the decision maker's wealth will not affect the risk premium. IARA suggests increases in wealth will increase the risk premium, while DARA implies reduction in risk premium as wealth increases. It is worth noting that the quadratic utility function exhibits IARA, and hence is typically assumed as a local approximation.

3.2.2.3 Certainty Equivalent and Risk Premium

Certainty equivalent is defined as the amount of outcome received with certainty that will make the decision maker indifferent between that and the original risky prospect. Risk premium is formally defined as the difference between the expected return on the risky action and the certainty equivalent. For a risk-averse decision maker, risk premium is positive which can be considered as the maximum amount the individual is willing to pay in order to eliminate risk. Monotonic transformations of the utility function will not change the indicators of the risk attitude of the individual, namely, the risk premium and the certainty equivalent. However, these risk attitudes are affected by the rate of changes in the curvature of the utility function.

The expected utility of a gamble depends on the probability distribution of the outcomes. The most common example of expected utility function is expected value-variance (E-V) model. The E-V utility function depends on certain summary statistics of the probability distribution, namely, mean and variance. As an example of E-V model, quadratic utility function of wealth yields quadratic expected utility function that is a function of mean and variance of wealth. The quadratic utility function exhibits increasing absolute risk aversion and is a decreasing function of wealth in some ranges.

Freund (1956) developed a farm production model incorporating risk aversion. He used an exponential utility function for income:

$$U= 1-e^{-\lambda(\text{income})} \qquad (3.5)$$

He assumed income is normally distributed. A monotonic transformation of Equation 3.5 yields an expected utility function expressed as:

$E(U) = E(\text{income}) - 1/2\ \lambda\sigma^2_{\text{income}}$ (3.6);

where E is the expected value operator and σ^2 is variance. Equation 3.6 is linear in the mean and variance, and expected utility is equivalent to expected utility of income. Therefore, maximization of expected utility of income is equivalent to maximizing certainty equivalent, which is approximated by expected income minus risk premium. The first term in Equation 3.6 is expected income and the second term is an estimate of risk premium expressed as a function of absolute risk aversion coefficient and variance of income. Thus, these specifications are consistent with formal definitions of certainty equivalent and risk premium.

Applying the certainty equivalent concept to the profit maximization model, again consider Equation 3.1:

Maximize $\pi(p, w) = \max p\, f(x) - w\, x$ (3.1)

If uncertainty is introduced in the form of risk in output price as:

$p = E(p) + \varepsilon$ (3.7);

where ε is normally distributed error term with zero mean and σ^2_ε variance. According to Robison and Barry (1987), expected profit and variance of profit are expressed as:

$E(\pi) = E(p)\, f(x) - wx$ (3.8),

$\sigma^2_\pi = [f(x)]^2\, \sigma^2_\varepsilon$ (3.9)

Since maximizing certainty equivalent (CE) is consistent with profit maximization behaviour, then by substituting Equations 3.8 and 3.9 into Equation 3.1, the optimization model becomes: [2]

Maximize $CE(\pi) = \max E(p)\, f(x) - wx - 1/2\{\lambda[f(x)]^2\sigma^2_\varepsilon\}$ (3.10);

provided $y = f(x)$ (3.11)

[2] The certainty equivalent of profits is based on Robison et al.'s (1987) approximation.

Also, assuming this is a short run analysis and fixed costs are omitted, variable costs (vc) can be expressed as:

$$vc(w,y)= wx \qquad (3.12)$$

Assuming vc (w,0)=0 and vc''(w,y)>0 to insure profit maximization,then Equation 3.10 can be expressed as:

$$\text{Maximize } CE(\pi)=\max E(p)\, y-vc(w,y) -1/2\{\lambda[y]^2\sigma^2_\varepsilon\} \qquad (3.13)$$

The model in Equation 3.13 assumes constant absolute risk aversion (CARA). Comparing Equation 3.13 with 3.1, it is clear that introducing uncertainty into the model, in the form of output price risk, changes the optimization problem and hence the first order conditions will change too. Thus, stochastic input demands are different from deterministic ones. Before moving to derivation of stochastic input demands, some issues related to E-V and risk aversion that are relevant to this study warrant mentioning.

Since E-V analysis requires either an estimate of the risk aversion parameter as when using Freund's (1956) formulation that maximizes certainty equivalent, or a value for the desired income as when using Markowitz's (1959) formulation that minimizes income variance, McCarl et al. (1997) considered Freund's model as better suited to accommodate multi-objective problems. Hazell (1982) and McCarl et al. (1997) discussed the approaches used for specifying risk aversion parameters and listed them as:

- Solve for many possible risk aversion coefficients and derive the efficient frontier;
- Present a derived efficient frontier to a decision maker to pick a point of tangency between the E-V frontier and the utility function that will indicate the parameter value;
- Estimate the risk aversion parameter that minimizes the difference between observed behavior and model prediction;
- Subjectively, elicit risk aversion coefficients from decision makers and fit them into the model;
- Use risk aversion coefficients estimated or used in other studies.

Pope et al. (1983) argued that stochastic demands are appropriate for welfare analysis only if the decision maker's preferences exhibit constant absolute risk aversion. Otherwise, wealth should be included as an argument in the utility function to reflect its effect on welfare. They also argue that farm-level analysis provides less aggregation bias. Pope et al. (1983) argued that although risk aversion behavior is widely supported by econometric evidence, the answer to whether risk aversion is constant, increasing, or decreasing remains ambiguous. Hazell (1982), however, argued that although there is little evidence to suggest that farm incomes are normally distributed, and hence the E-V model may be of limited use, the computational advantages of the model appear to offset against such difficulties. Theoretically preferred utility functions exhibit desirable properties, however such functions appear to have expected values that are difficult to evaluate numerically and the high order polynomials often lead to non-convex solutions.

3.2.2.4 Stochastic Input Demand

If risk is introduced into the decision making process, input demands become stochastic and will be different from deterministic factor demands. When the source of risk is only output price as in the certainty equivalent maximization model in Equation 3.13, then the output level that maximizes Equation 3.13 must satisfy the first order condition:

$\partial CE(\pi) / \partial y = 0$, or:

$$E(p) - vc'(w,y) - \lambda\, y\, \sigma^2_\varepsilon = 0 \qquad (3.14);$$

if marginal cost $vc'(w,y) = \partial vc(w,y)/\partial y > 0$.

From Equation 3.14, it is clear that the firm facing risk will produce less output than the firm operating under conditions of certainty by the quantity $\lambda\, y\, \sigma^2_\varepsilon$ provided that the decision maker exhibits risk averse attitudes. Analogously, the reduction in output may require employment of less input under uncertainty than under certainty conditions.

Just and Pope (1978) introduced risk into technology as in the following production function specification:

$$y=f(x) + h(x)\,\eta \tag{3.15}$$

They assumed that $f'(x)>0$, $f''(x)<0$, $h'(x)\leq 0$, and $\eta\sim(0,\sigma^2_\eta)$. Thus, the expected yield and variance of yield are written as:

$$E(y) = f(x) \tag{3.16}$$

$$\sigma^2_y = [h(x)]^2\,\sigma^2_\eta \tag{3.17}$$

Equation 3.15 assumes that risk in the production function is in this case multiplicative. However, it is possible to assume additive risk. The effect of risk on resultant output supplies and factor demands from both cases will be in the same direction in general and differ only in magnitude due to the differences in function determinants.

The production function in Equation 3.15 satisfies a set of conditions established by Just and Pope, which ensure the characteristics for risky inputs. Note that output is decomposed into deterministic and stochastic elements. Substituting Equation 3.15 into Equation 3.8 results in:

$$\pi = (p+\varepsilon)\,[f(x) + h(x)\eta] - vc(w,y) \tag{3.18}$$

Expected profit becomes:

$$E(\pi) = E\{(p+\varepsilon)[f(x) +(x)\eta] - vc(w,y)\} \qquad (3.19) \text{ or:}$$

$$E(\pi) = E(p)f(x) + \sigma_{py} - vc(w,y) \tag{3.20}$$

According to Mood et al. (1974), if y and p are random and independent, then, $\sigma_{py} = 0$, and the variance of profit will be:

$$\sigma^2_\pi = [E(y)]^2\sigma^2_p + [E(p)]^2\,\sigma^2_y + \sigma^2_p\sigma^2_y \tag{3.21}$$

By substituting Equations 3.20 and 3.21 into Equation 3.10, the certainty equivalent of profit becomes:

$$CE(\pi) = E(p)f(x) - vc(w,y) - \tfrac{1}{2}\,\lambda\,\{[E(y)]^2\,\sigma^2_p + [E(p)]^2\,\sigma^2_y + \sigma^2_p\sigma^2_y\} \tag{3.22}$$

Following Just et al. (1979), Lambert (1990) assumes the stochastic error term, η, in Equation 3.15 to follow a standardized normal distribution. Therefore, variance of yield becomes:

$$\sigma^2_y = [h(x)]^2 \quad (3.23)$$

Lambert (1990) shows the following necessary condition that will characterize the interior solution of maximizing Equation 3.22 subject to the constraints forcing upper and lower bounds on input use:

$$\partial CE(\pi) / \partial x = E(p)f'(x) - w - ½ \lambda \{([E(p)]^2 + \sigma^2_p) h'(x) + 2\sigma^2_p f'(x)f(x)\} = 0 \quad (3.24)$$

The third term in Equation 3.24 is the risk term added to the certain choice of input where input cost equals its VMP. Thus, it is evident that choice of input level under conditions of uncertainty is affected by input and expected output prices, variance of output price, risk aversion parameter, output level, marginal products, and the marginal contribution of the input to the variance of output.

The establishment of a link between the preceding analysis and welfare analysis proceeds by first showing producer surplus as a measure of producer welfare in relation to factor demand. Then a link between risk neutral and stochastic factor demands is forwarded in relation to estimating risk penalties incurred from using risky inputs. The change in producer surplus or change in willingness to pay or accept due to changes in input use is then considered as an estimate of change in producer welfare.

In welfare economics, the measurement of the change in an individual's or a firm's economic well-being due to an event can be assessed in terms of the individual's or the firm's willingness to pay to obtain the event if it is a "good" or to avoid it if it is a "bad". In the case of a firm, in order to assess its welfare under uncertainty, both risk attributes of the technology and risk attitudes of the decision maker must be known.

Theory of the firm under conditions of uncertainty assumes the decision maker is maximizing the expected utility of wealth or profit given a distribution of output conditional on management decisions. In order to maximize expected utility, the decision rule is to choose the input level at which an additional or marginal unit of input gives no higher utility, or equivalently, where the expected marginal utility equals zero. The expected utility function in this case is assumed globally concave in inputs and hence the expected marginal utility curve slopes downward. The area under the expected marginal utility curve measures the firm's welfare in terms of utility.

The concept of producer surplus is used to measure the notion of producer welfare. Producer surplus is defined as revenue minus variable costs of production for the good the producer is selling at the market. A change in producer welfare is equal to the change in producer surplus caused by a change in market equilibrium. In the short run, the behavioural assumption of the firm is maximizing economic profit is equivalent to maximizing producer surplus given the firm's technology, capital stocks, and fixed costs.

Standard economic analysis assuming certainty conditions shows that for a firm to maximize profits, it must employ each input to the point where the additional or marginal benefit of using an additional unit of the input equals the additional or marginal cost of the input. The intersection of the value of the marginal product (VMP) curve of the input and the input supply curve determines the economically efficient or profit maximizing quantity of the input to be used. The area below the VMP curve and above the factor supply curve determines the net benefits to the firm from using the input. However, this area is equivalent to the measure of producer surplus, defined as the difference between revenue and variable input costs.

Analogous to VMP obtained under certainty conditions, each input the firm utilizes in the production process under uncertainty conditions will have an expected value marginal product (EVMP). Optimally, a risk-neutral firm will employ the input up to the point where the EVMP curve is equal to the factor supply curve and the area trapped between these two curves represents the expected producer surplus.

Pope et al. (1979) define a marginally risk-increasing (reducing) input as the input the risk-averse firm uses less (more) of it than the risk-neutral firm. Therefore, a risk-increasing input will exhibit a stochastic value marginal product (SVMP) curve lying to the left of the risk-neutral EVMP curve. The vertical distance between the EVMP curve and the SVMP curve measures the contribution of risk to the marginal value of the input to the firm. When the SVMP curve lies below the EVMP curve, it is an indication of how much risk is effectively causing the marginal value of the input to be less than if the decision maker is risk neutral. The implication is that for a risk averting decision maker to behave as risk neutral, it would require a payment of subsidy equal to the vertical distance between the SVMP curve and the EVMP curve, i.e. the marginal risk premium associated with the input. A positive marginal risk premium indicates a marginally risk-increasing input; and a negative marginal risk premium would indicate a marginally risk-reducing input. The welfare or the expected foregone benefits of the firm using risky inputs can be measured as the area above the factor supply curves and below the SVMP curves, given the SVMP curves are adjusted to take into account the effects of risk on expected utility. To illustrate the application of these concepts to more general cases of input demands, consider Figure 3.1.

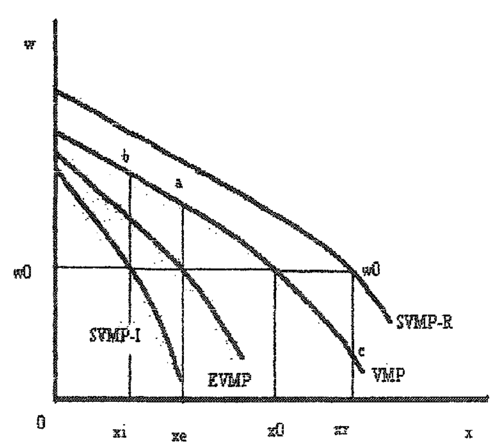

Figure 3.1 Factor Demands, Risk Penalty, and Welfare Change

Under conditions of certainty, the decision maker will employ x_0 of the input where the deterministic factor demand (VMP) equals competitive factor price w_0. The firm faces a perfectly elastic input supply (w_0w_0). In this case, producer's surplus is measured by the area between supply and factor demand under conditions of certainty. If the input is risk increasing and the decision maker is risk neutral, then factor demand is EVMP, and the firm employs x_e of the input. The risk penalty from using a risky input is aw_0, and producer's surplus is the area between w_0w_0 and EVMP. In the case of risk reducing input, an EVMP will lie above VMP and the risk penalty and welfare measures can be deduced as above. If the decision maker is risk averse and the input is risk increasing, then factor demand is SVMP-I, and the firm will employ x_i. The risk penalty from using a risky input is bw_0 and the area between w_0w_0 and SVMP-I measures producer's surplus. If the decision maker is risk averse and the input is risk reducing, then factor demand is SVMP-R, and the firm will employ x_r. The risk reward from using a risk reducing input is w_0c and the area between w_0w_0 and SVMP-R measures producer's surplus.

This study assumes producers are risk averse, however, empirical results will show that the degree of risk aversion is low that the EID producers are considered almost risk neutral. In this case, the difference between EVMP and SVMP will be insignificant. Hence, deterministic and stochastic factor demands (SVMP), assuming risk aversion, are used to estimate risk penalties and/or rewards and changes in producers' welfare or foregone benefits which are obtained from areas under the SVMP curves following Taylor and Young (1995) approach.

Pope et al. (1979) discussed the case of a two factor competitive model employing a Just and Pope-type objective function. They examined the effects of changes in absolute risk aversion, factor prices, and output prices on the characteristics of output supply and derived factor demands and hence on measures of welfare change. They used comparative static methods to determine the conditions under which the desired properties of output supply and factor demand will be attained. They concluded by stating that empirical studies have found evidence of risk reducing inputs. In such cases, risk aversion and marginal increases in it lead to increased factor use.

A production function with multiplicative error term requires econometric estimation in three steps as shown by Just et al. (1979). Lambert (1990) first used a single crop nonlinear programming model that employs the findings of the estimates of a Just and Pope production function. When using a multi-product model, however, Lambert departed from the assumption of multiplicative risk in the production function and adopted a formulation that will be discussed later in this chapter. Taylor and Young (1995) also used analytical production functions relating applied water to yield. For the purposes of this study, data for estimating a Just-Pope production function are not readily available. An objective function formulation similar to Lambert's is used in this study while crop yields are calculated by a water-yield response function. Discussion of these issues is undertaken in the following sections.

Once a theoretical model incorporating risk is developed, the next logical step is to build an empirical model that conforms to theory. One of the many considerations involved in empirical model development is method of estimation. The empirical model is developed to suit either econometric estimation methods or mathematical programming techniques. The following section introduces mathematical programming as an estimation environment suitable for this research.

3.3 Mathematical Programming

Since the price of water is administered and free markets for water do not exist in southern Alberta such that these prices will reflect the actual value of water, then mathematical programming models are more suitable in imputing the residual value of water from farm budgets. In general, econometric estimation and analysis requires a set of assumptions suited for the model to be estimated whether it is a production function, a profit function, or a cost function. Choice of functional form is crucial to the achievement of the objectives of the analysis in terms of what assumptions are maintained and which are tested, and what restrictions to be invoked. Flexible functional forms provide fewer restrictions and hence more hypotheses can be tested.

Econometric estimation requires crucial assumptions about the probability distributions of the variables estimated and tested and the error term(s). Mathematical programs, on the other hand, do not require any explicit assumptions about a model's error term and utilize a set of assumptions about technology that usually conform to economic theory. Data requirements seem to be less demanding in the case of programming methods, however, specific models have their unique data requirements. Formal hypothesis testing is usually not applicable in the case of programming models. Mathematical programming, however, provides a high degree of flexibility in modeling the constraint set since it allows for inequalities. In this study, mathematical programming is employed mainly because of the relative ease of derivation of factor demands from programming models, which is straightforward via parameterization of the right hand side values of the input constraints. The type of data available and the empirical model's suitability to such data also made the choice of mathematical programming easier.

Colby (1989) argued in favour of using of mathematical programming approach in valuing water in irrigated agriculture. Colby's argument was based on the reason that prices farmers pay for irrigation water typically do not vary significantly such that direct estimation of water demand functions based on quantities of water used at different price levels is not possible. Thus, a farm budget approach can be used to estimate the willingness to pay for additional units of water by estimating the contribution to total revenues minus all non-water production costs that would be generated by applying one more unit of water.

The following subsections discuss the background of linear programming and then introduce quadratic programming, which is the basis for risk programming. Then, stochastic programming is introduced and the procedure of incorporating production stages and the structure of information flow are discussed. The last subsection discusses a multi-product version of the Just and Pope-type production function and multistage objective function.

3.3.1 Linear Programming

In the deterministic economic theory of the firm, the decision maker is assumed to operate under conditions of certainty, which implies that input and output prices as well as technology and resource availability are all determined before the production decisions are made. In a linear programming (LP) framework, this implies that activity coefficients, technical coefficients, and resource endowments are all modeled deterministically. Consider the LP model:(Chiang (1984), p.662)

$$\underset{y}{\text{Maximize}} \quad \pi = \sum_{j=1}^{n} c_j y_j \qquad (3.25)$$

$$\text{subject to} \quad \sum_{j=1}^{n} a_{ij} y_j \leq b_i \qquad (i = 1,2,\ldots,m) \qquad (3.26)$$

$$\text{and} \quad y_j \geq 0 \qquad (j = 1,2,\ldots,n) \qquad (3.27);$$

where:

y= a vector of decision variables or output;

c= a vector of objective function coefficients or per unit contribution to profits;

a_{ij} = per unit of output j usage of input i;

and b_i = endowment of input i.

The LP model assumes that c, a, and b are known with certainty (constants) and thus the analysis is deterministic since it operates in a perfect knowledge environment. Other general assumptions of the LP model are:

- The objective function optimizes a set of variables within feasible values;
- Activities and resources are homogeneous across any constraint and can be supplied without any variability;
- Variables exhibit proportionality such that their contributions are assumed constant and independent of variable level;

- The contribution of variables are additive such that no interaction terms are allowed in the objective function and the constraints;
- Decision variables are assumed divisible such that they take any non-negative value;
- At least one right hand side coefficient is not equal to zero.

The objective function (Equation 3.25) is linear and maximizes profits; calculated as the summation of the products of the optimal levels of outputs and their contributions to net returns. The constraint set (Equation 3.26), consisting of linear inequalities, ensures that each resource endowment is greater than the sum of products of its usage by each output times the optimal level of output. The choice variables are constrained to take only positive values (Equation 3.27).

According to Chiang (1984), LP embodies two implicit assumptions regarding technology. It assumes that the production function exhibits constant returns to scale (CRTS) and fixed input ratios (FIR), which is known as Leontief technology of right-angle isoquants. CRTS implies increasing output by any factor requires increasing all inputs by the same factor. FIR implies the composition of inputs producing a certain output is determined in a certain combination and no substitution between inputs is allowed. In other words, if the amount of one input is held constant and all other input increased, the amount of output stays at the level attained by the minimum amount of the fixed input. Thus, one of the main disadvantages of using LP is that it maintains a restriction on technology specification i.e. CRTS.

Since the definition of shadow price is the contribution of an extra unit of a resource to the objective function and the definition of VMP is the value of output produced due to the use of an extra unit of the variable input, therefore, the VMP of an input is estimated by parametrizing the input constraint to obtain its shadow prices. Parameterization is changing the value of the right hand side of the constraint successively until the resource becomes uneconomical. The resultant step function represents the derived deterministic demand for the input.

Violations of any or all of LP assumptions are considered a departure from classical LP. For example, the relaxation of certainty assumption leads to stochastic programming; departure from additive contributions assumption yields a non-linear programming, departure from continuous variables leads to integer programming, and so forth. Introduction of risk into the objective function will be considered first, as in the case of quadratic programming, then risk will be introduced into the coefficients matrix and right hand side values in the form of stochastic programming.

3.3.2 Quadratic Programming

Introduction of uncertainty into the objective function means that the c_j's of Equation 3.25 are no longer deterministic. The decision maker is assumed to know the form of a probability distribution for c_j's with outcomes and associated probabilities of occurrence. Thus, according to expected utility theorem, Equation 3.25 can be expressed in the form of certainty equivalent maximization, which will be consistent with maximization of expected utility of profit.

Robison and Barry (1987) show that if the utility function is characterized by CARA equal to λ, and normally distributed outcomes, g (π), with mean, E (π), and variance, σ_π^2; then the certainty equivalent in Equation 3.13 can be rewritten as:

Maximize $CE(\pi) = E(p)\, y - vc(w,y) - 1/2\lambda\sigma^2_\pi$ (3.28)

A set of linear constraints can be added to Equation 3.28 to form an E-V model:

$$\underset{y}{\text{Maximize}}\quad CE(\pi) = E(p_j)y_j - vc(w_i, y_j) - 1/2\lambda\sigma_\pi^2 \qquad (3.29)$$

$$\text{subject to: } \sum_{j=1}^{n} a_{ij} y_j \le b_i \qquad (i = 1,2,....,m) \qquad (3.30)$$

$$\text{and} \qquad y_j \ge 0 \qquad (j = 1,2,.....,n) \qquad (3.31);$$

where:

CE = certainty equivalent;

π = profits or net returns;

E = expectation operator;

p = output price;

y = quantity of output;

vc = cost of variable inputs;

w = input price;

λ = risk aversion parameter;

σ^2 = variance;

a = transformation coefficient;

and b = resource endowment.

The objective function is no longer linear, because the last term in equation 3.29 is a quadratic function of activities where $\sigma_\pi^2 = y_j^2 \sigma_\varepsilon^2$ and ε is an error term associated with output prices as defined above. For that reason the model belongs to quadratic programming. The main implication of the certainty equivalent formulation is that the variance can be traded off for expected returns at the rate of $\lambda/2$ without affecting the certainty equivalent. The certainty equivalent equation above can be rearranged to become $E(\pi)\text{-}CE(\pi) = \frac{1}{2}\lambda\sigma^2_\pi$. The intuition here is that the risk premium equals Pratt-Arrow's approximation for local risk aversion. This tradeoff is a local approximation and cannot be considered global unless all decision makers are assumed to have CARA functions. A risk-averse decision maker values increment to an outcome at a decreasing rate indicating a concave utility function, which intuitively implies the value of an additional unit of return, is worth less at high-income levels than at low-income levels.

From a historical perspective, the introduction of risk into the objective function was adapted by Markowitz (1959) who expressed the problem of selecting an optimal portfolio of stocks under a budget constraint as a maximization of a function that is quadratic in the value of the stocks. The function was convex with an optimal value under the budget constraint, since the variance-covariance matrix of the outcome was positive semi definite. The Markowitz model measured risk by the variance of outcome of the decisions, and hence, the utility function was a weighted sum of expectations and variance.

Freund (1956) also reported one of the first applications of the E-V model. The objective function traded off expected income for reduced variance such that it maximized expected returns less a risk aversion coefficient times the variance of total income. He remarked that neglecting risk considerations causes deviations of actual plans from those obtained by mathematical programming. E-V formulation has since become a widely used model for ordering choices into efficient and inefficient sets.

Robison and Barry (1987) and later McCarl and Spreen (1997) identified the sufficient conditions justifying the use of E-V model. These conditions include the use of a quadratic utility function and the existence of a normal probability distribution for the possible outcomes (income). Anderson et al. (1977) argued that normality of the probability distribution of income and/or quadratic utility are sufficient conditions for the use of the E-V objective function, but Meyer (1987) has shown that normality or quadratic utility are not necessary. (Apland et al. (1993), p.9)

According to Robison and Barry (1987), the normality assumption is seldom satisfied since many decision variables cannot take negative and positive infinity values nor they are symmetrically distributed. Also, the use of variance as a dispersion index systematically treats the upper and lower deviations from the mean such that the implication is that decision makers are considered as risk averse as well as windfall profit averse. The increasing absolute risk aversion due to the quadratic function assumption and the possibility of the marginal utility becoming negative are questionable traits of the E-V model. Another E-V model requirement is that the risk aversion coefficient has to be measured which may involve the problems associated with estimating the ever-unobservable utility function.

The quadratic utility function provides a reasonable second-order approximation to more desirable functions. Under the E-V decision rule, farmer's preferences among alternative farm plans are based on expected income and associated variance of income. Given risk averse preferences, the iso-utility curves are convex in E-V space, which implies that along every curve, plans with higher variances are preferred only if the expected incomes are also greater, and such compensation must increase at an increasing rate as variances increase. For a 'rational' farmer, this translates to restricting choices to those plans with minimum variances for given expected income levels. The efficient E-V boundary of the set of all feasible farm plans is defined by the efficient E-V pairs for farm plans characterized by minimum variance for an associated expected income level. Farmer's preferences determine the acceptable plan.

Quadratic programming provides a tool to derive the efficient E-V set of farm plans and one of its formulations requires minimizing the variance of income for each possible level of expected income, while retaining feasibility with respect to the available resource constraints. By varying the expected income scalar over its feasible range using parametric procedures, a sequence of solutions can be obtained by increasing expected incomes and variances until the maximum possible total expected income under the resource constraints has been attained. When plotted in E-V space, these pairs constitute the efficient E-V frontier.

There is also some criticism of the approach of measuring utility with the usually imposed simplistic assumption of the existence of a utility function with respect to only one consequence, money or wealth in the current period. The axioms of expected utility theory do not imply that risk preferences can be measured with respect to income received in a single period. However, in Roumasset et al. (1979), several contributors report that utilizing a utility function of one-period money would still provide an approximate description of and/or prescription for decision-making under uncertainty. The choices facing the decision maker usually involve more than one variable. However, Robison and Barry (1987) argued in favor of using the E-V model as an analytical tool rather than a decision theory tool. Strengths of the E-V model include the relative ease of its application to a wide range of problems involving risk, especially through the simplicity of estimating the variance components of the model, and its relative ease in deriving the optimal solutions and performing equilibrium analysis.

Based on the preceding discussion, an E-V-type objective function appears to be a reasonable formulation to be adapted for the purposes of this research. It can accommodate output price risk measured as variance of farmers' income over time. As will be shown below, the E-V objective function is flexible to specifications that dynamics of crop production and multistage aspects can be conveniently modeled as outlined in the DSSP model. This study utilizes a certainty equivalent objective function that has a quadratic risk premium term that justifies the use of E-V model. E-V model is widely used in empirical literature.

3.3.3 Discrete Sequential Stochastic Programming

In the instances where risk is associated with the constraint set, i.e. technical and/or right hand side coefficients in addition to risk in the objective function, discrete sequential stochastic programming provides an analytical tool that allows for modeling a dynamic framework. The basic structure of the sequential stochastic programming model utilizes a formal probability tree framework where the nodes represent decision points and the branches represent alternative possible states of nature. It requires an estimate of a probability distribution for the various values of the uncertain parameters conditional on the events that have occurred. The general formulation of sequential stochastic programming is flexible to the specification of the objective function (E-V, MOTAD, etc.), in addition to inclusion of risk aversion. In each stage of the decision tree, coefficients are dependent on the states of nature, and all types of coefficients are potentially unknown. Their values depend upon the path through the decision tree, given the potential future states of nature and what happened up to that point.

A sequential stochastic programming model, therefore, deviates from the standard deterministic linear programming model by allowing c_j's, a_{ij}'s, and b_i's in Equations 3.25 and 3.26 to be random, and by allowing for the modeling of information flows to the decision maker through the specification of decision stages. Therefore, sequential stochastic programming can accommodate risk in the objective function, the technical coefficients, and the coefficients of the right hand side of the constraints as well as allowing for the inclusion of the dynamics of the decision process based on the information structure.

Rae (1971a), and later Apland et al. (1993), discussed the information structure in relation to the construction of a decision tree for a sequential stochastic programming problem. Two main information structures were identified: complete knowledge of the past (CKP) and complete knowledge of the past and present (CKPP). Combinations of both structures are possible, and therefore, modeling the decision stages in relation to the flow of information is very crucial to the simulation of the decision process. To explain, consider the following example.[3] Suppose that all activities in stage t are selected at the beginning of the stage and the decision maker knows the outcomes of the random events in stages t-i, t-i-1,..,1. The decision maker knows only the probabilities, conditional on known outcomes in prior stages, of outcomes in t-i+1, t-i+2. So, if i=0, then the information structure describes CKPP, while if i=1, CKP is implied. Cases where i>1 describe incomplete knowledge of the past.

To illustrate the general formulation of the sequential stochastic programming model, assume that a decision maker is faced with three decision stages: A, B, and C, as depicted by the decision tree in Figure 3.2.[4] In stage A, the agent makes a decision,

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[3] Apland et al. (1993)

[4] Kaiser et al (1987) and Apland et al (1993)

H_I based on the expectations of several possible future states of nature. Thus, stage A in this case is modeled deterministically. In stage B, either state of nature E_{II1} or E_{II2} occur. Suppose state of nature E_{II1} takes place.

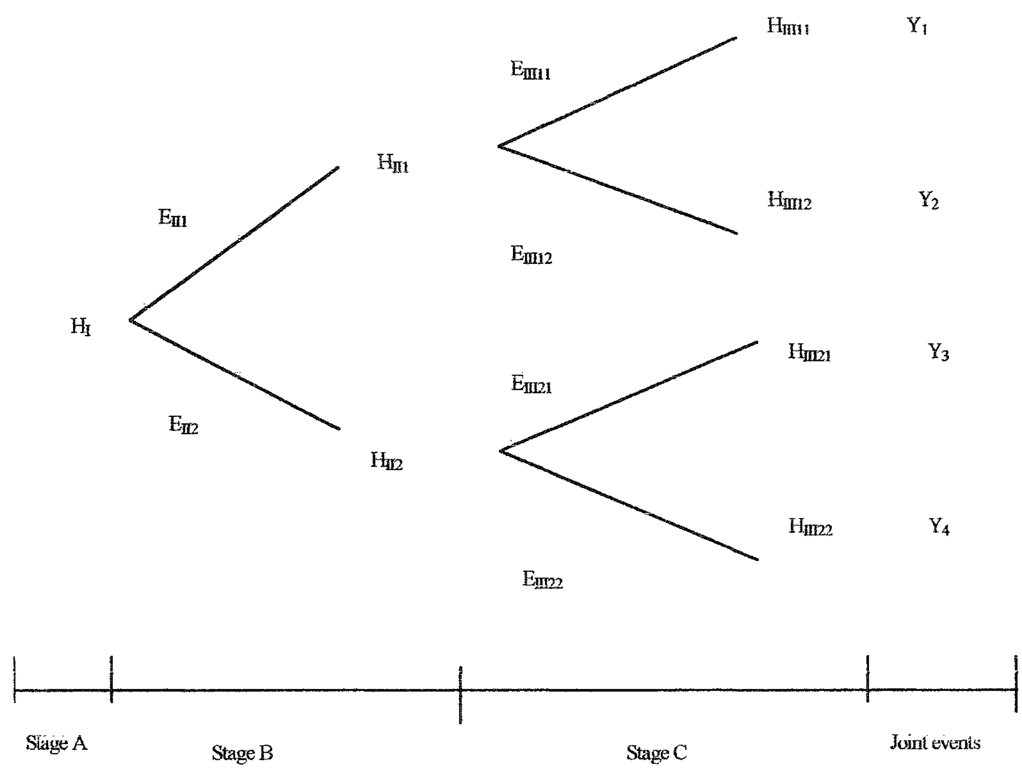

Figure 3.2 Decision Tree of Hypothetical Sequential Stochastic Programming Problem

Then, the agent makes a decision, H_{II1}, conditioned on H_I, the occurrence of E_{II1}, and future uncertain states of nature $E_{III11}$and E_{III12}. In stage C, the agent will observe one of two possible states of nature (E_{III11} and E_{III12}). Assume E_{III11} occurs, the optimal decision variable will be H_{III11}, or joint event Y_1, which is a function of all past decisions (H_I and H_{II1}) and states of nature (E_{II1} and E_{III11}). Since the occurrence of joint events is characterized by a multinomial distribution, one of the joint events Y_1, Y_2, Y_3, or Y_4 will occur in each cycle of the decision process.

According to Antle (1983), for a programming problem to be of a dynamic multi-period sequential type, it must meet specified three criteria. There has to be a sequential dependency of decisions from one stage to the next. In addition, as information becomes available, revision of earlier decisions must be feasible; and as time unfolds, the decision maker is able to use information feedback.

The concept of sequential stochastic programming model, described by the decision tree above, conforms to those criteria by exhibiting divergence of events over time as well as a sequential decision process. To illustrate, consider decisions made in stage A, H_I will affect income and constraints in all states of nature. Decisions made in stage B, H_{II1} or H_{II2}, will depend on the occurrence of either E_{II1} or E_{II2}. However, decisions made in stage C, such as H_{III11}, will only influence income along that branch of the decision tree. Overall, the past determines the state existing at each distinct decision node, while decisions made at each node influence all nodes emanating from that point.

The sequential stochastic programming model is flexible in accommodating different forms of objective functions. In a summary of empirical application of discrete stochastic programming, Apland et al. (1993) list several studies utilizing different types of objective functions; one of such formulations is the E-V-type objective function. To illustrate the use of E-V objective function in a DSSP context, consider Equation 3.28. In the first stage (A), there is one stochastic variable with three possible states of nature. In the second stage (B), there is one random variable with two possible states of nature. The third stage (C) includes one uncertain variable with three possible states of nature. These stochastic elements can potentially be manifested through alternative contribution coefficients in the objective function (c_j), technical coefficients in the constraint set (a_{ij}), and/or right hand side values (b_i). For each of the random elements there exists a probability distribution associating the states of nature with their probabilities of occurrence. The decision tree in Figure 3.3 contains the information structure of the model with eighteen joint events and associated joint probabilities.

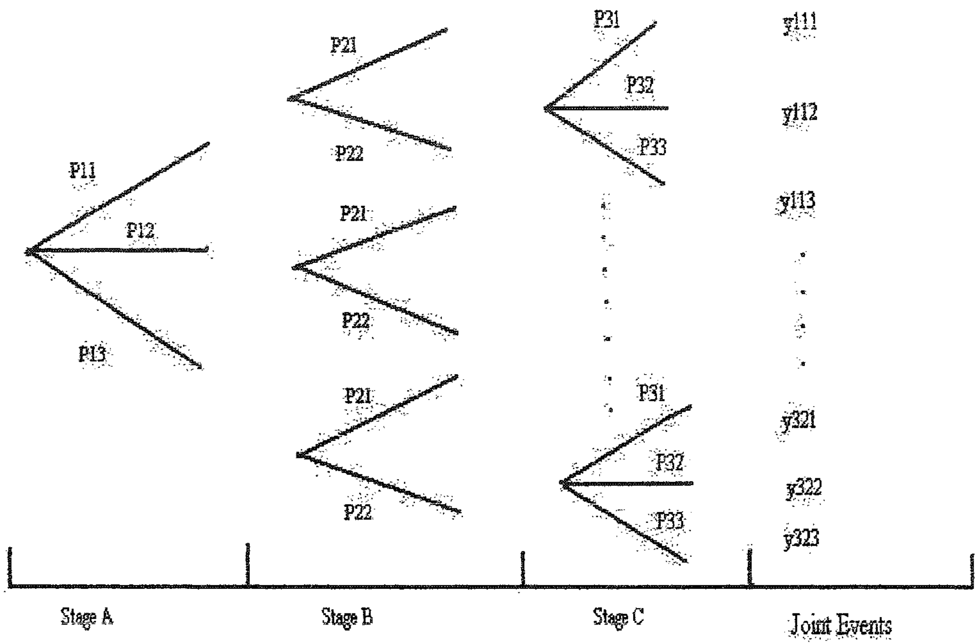

Figure 3.3 A Decision Tree Depicting a Three-Stage DSSP Model

The indexing of joint events shows the realized output such that the first digit is for stage A's three states of nature, the second digit is for stage B's two states of nature, and the third digit is for stage C's three states of nature. For example, joint event y_{212} indicates that in stage A, state of nature two materialized, while in stage B, state of nature one occurred, and in stage C, state of nature two took place. The corresponding joint probability is the product of P_{12}, P_{21}, and P_{32}. Note that probabilities' indexing is set as first digit indicating stage of production and second digit indicating the state of nature occurring in that stage.

Based on the decision tree in Figure 3.3, the objective function in Equation 3.28 can be rewritten as an objective function for a DSSP model with a CKP information structure:

$$\text{Maximize}_{y}\ CE(\pi) = \sum_{i=1}^{3}\sum_{j=1}^{2}\sum_{k=1}^{3} P1_i P2_j P3_k\, y_{ijk}[p - vc(w, y_{ijk})] - 1/2\{\lambda[y_{ijk}]^2 \sigma_\varepsilon^2\} \quad (3.32)$$

$$\text{subject to: } \sum_{j=1}^{n} a_{lj} y_j \le b_l \quad ,\forall l \quad (3.33)$$

$$-dy_i + ey_{ij} \le 0 \quad (3.34)$$

$$-fy_{ij} + gy_{ijk} \le 0 \quad (3.35)$$

$$y_{ijk} \ge 0 \quad (3.36)$$

where:

i, j, k = states of nature in stages A, B, and C respectively;

P1, P2, P3 = probabilities associated with states of nature occurring in stages of production;

y = decision variable or output;

p = expected output price;

vc = per unit of output variable cost of production as a function of input price w and level of output y;

λ = risk aversion parameter;

σ^2_ε = variance covariance of output price;

a = technical coefficients;

b = resources available;

and d, e, f, g = linkage constraints coefficients.

The first term in Equation 3.32 calculates expected income joint probability times the outcome. Note that variable cost is given as an average or per unit term. The second term is the risk premium. However, the summation is over all eighteen joint events, and the sum of all joint probabilities equals one.

The typical constraint set includes equations of transformation, which restrict resource usage not to exceed the value of the right hand side. The technical coefficients and right hand side of such constraints can be stochastic to capture the presence of risk and uncertainty. The number of constraints depends on the number of states of nature in each stage. For instance, in the above example, there will be two constraints, one for each state of nature occurring in stage A, each ensuring resource usage limitation is maintained. For stage B, there will be four constraints since there are two states of nature in stage B conditional on occurrence of one of two states of nature in stage A. In stage C, there will be eighteen constraints.

The net returns can be expressed as constraints, instead of in the objective function, and are usually formulated as the decision variable times the unit contribution to income minus costs. In the above example, there will be eighteen net revenue constraints calculating net returns for each joint event. These constraints in this fashion necessitate the reduction of the first term in the objective function in Equation 3.32 to joint probabilities multiplied by joint net incomes and minus risk premium. DSSP is flexible enough to allow for many forms of constraints to be incorporated into the formulation as long as they conform to the information and stage structures.

An important part of the constraints set is the linkage inequalities. These constraints link decisions of the stages by ensuring the balance of resource transfers, decision variables, accounting matters, among other things. In the above example, typically there will be five linkage constraints; three constraints linking stage B to decisions made in stage A, and two constraints linking stage C to stage B decisions. These constraints follow the branching of the decision tree by linking activities in a later stage branch to the preceding activities. In the above example, there will be a constraint linking each of the three states of nature in stage A to the following two states of nature in stage B. Similarly, another two constraint would link three states of nature in stage C to each of the two states of nature occurring in stage B. The non-negativity constraints ensure decision variables are positive.

The above discussion applies to a DSSP formulation that recognizes a CKP information structure. A CKPP formulation would split the CKP formulation into a number of smaller sub-models depending on the number of states of nature in stage A. In the above example, a CKPP model would involve solving three separate sub-models, i.e. one for each state of nature of stage A. The constraint set, therefore, is modified to the specifics of the problem under consideration.

DSSP limitations are related to model size, data availability, data handling, and modeling time.[5] However, DSSP remains a flexible tool in modeling risk in more than the objective function and the possibility of including a variety of information structures and dynamics of decision making. The restricting assumptions of LP on the objective function and constraints can be relaxed in the case of DSSP such that polynomials can be included in the model. DSSP, therefore, allows for interaction terms and the result is relaxation of the CRTS and FIR restrictions LP imposes on technology. Market and behavioural assumptions are problem specific.

The preceding discussion has established that DSSP is a suitable model to apply in achieving the goals of this study. The model assumes that crop producers maximize certainty equivalent, which is consistent with profit maximization. Producers are assumed to be price takers in input and output markets. The objective function is of E-V type that maximizes certainty equivalent of profits calculated as expected net returns minus risk premium. A risk aversion parameter is required for estimation. The constraint set includes a constraint on irrigation water that through parameterization should provide the shadow prices of the resource that will be used in the construction of the derived stochastic demands for irrigation water. The next chapter develops the empirical model and discusses data issues.

[5] Apland et al. (1993)

4. EMPIRICAL MODEL STRUCTURE AND DATA

4.1 Introduction

The last chapter presented the theoretical basis for deriving deterministic and stochastic factor demands using DSSP. It started by establishing the adequacy of the expected utility theorem in describing producer's preferences under uncertainty. A comparison between deterministic and stochastic factor demands showed the relevance of risk penalties and welfare change due to using risky inputs. The mathematical programming section laid a foundation for the analytical model in terms of its choice over econometric models, and showed how risk programming is handled through DSSP as an appropriate tool for modeling crop production under uncertainty in a dynamic setting.

This chapter starts by a discussion of the analytical deterministic linear programming model required to derive the non-stochastic demand (VMP) for irrigation water. This demand schedule will provide the reference point for measuring the risk penalty for using water as a risky input. Next, an analytical DSSP model suitable for estimating stochastic irrigation water demand of the Eastern Irrigation District is presented by first discussing the structure and relationships of the objective function. The set of constraints is then presented explaining the purpose, structure, and relationships of each individual constraint. The analytical DSSP model is then constructed and estimation issues are discussed. Risk aversion parameters, stochastic irrigation water demands, risk penalty, and producers' surplus are discussed from an estimation perspective. The last section presents data description, sources, and usage.

4.2 Deterministic Model

In order to estimate risk penalties due to using risky inputs, an estimate of the derived deterministic demand for irrigation water is required as a reference point. Varying the right hand side value of a resource constraint, and then plotting the shadow prices of the resource against its quantities derives non-stochastic factor demand. Shadow prices are intuitively interpreted as the opportunity cost of using the resource. Alternatively, it is the amount of money the producer is willing to pay (accept) for an extra unit of the resource to be employed in (withdrawn from) the production process, or it is the contribution of that extra unit of the resource into the objective function. Thus, the shadow price is the value imputed to the resource that has a binding constraint.

The simple deterministic model assumes producers are price takers in input and output markets. Hence, it assumes no risk is associated with prices. Decision makers are assumed to exhibit profit maximization behaviour. Technology is described by the water-yield response function. The response function is a second-degree polynomial with water as the independent variable and crop yield as the dependent variable. The second derivative of the function is assumed positive, and hence the function exhibits diminishing marginal products, which implies that the derived non-stochastic factor demands are negatively sloped.

4.2.1 Objective Function

The objective function of the deterministic model maximizes total net returns of twelve representative EID farms with the possibility of growing eight crops on each:

$$\underset{HD}{\text{Maximize}}\ \ TNR = \sum_{F}\sum_{C}(p_C.AY_C - vc_C)HD_{F,C} \qquad (4.1);$$

where:

TNR= total net returns of the representative farms;

F= farms (1,...,12);

C= crops (1,..., 8);

p= market price of crops in dollars per tonne;

AY= estimated yield using equation 4.4 in tonnes per hectare;

vc= per hectare variable costs of production in dollars;

and HD= decision variable as hectares of crop C grown on farm F.

The objective function is linear and takes per hectare net return for each crop, multiplies it by the decision variable, and then sums total net returns for all farms of the sample. Yields are calculated at average amounts of spring soil moisture, effective irrigation, and precipitation.

4.2.2 Land Constraints

The land constraint inequality ensures that for each farm the total area used in the optimal solution of all crops does not exceed total area of farmland available. Land constraints are expressed as:

$$\sum_{C} HD_{F,C} \leq L_F \quad \forall F \qquad (4.2);$$

where:

HD= decision variable as hectares of crop C grown on farm F;

and L= total land available for each farm.

4.2.3 Crop Rotation Constraints

The rotation constraint restricts the area planted of each crop to a certain ratio of total land available. Farmers set these ratios based mainly on soil fertility and crop disease considerations. Crop rotation constraints are expressed as:

$$HD_{F,C} / L_F \leq R_C \quad \forall F, C \qquad (4.3);$$

where:

HD= decision variable as hectares of crop C grown on farm F;

L= total land available for each farm;

and R= crop rotation requirement for any particular crop.

4.2.4 Irrigation Water Constraint

The irrigation water constraint manages available effective irrigation water such that total consumption does not exceed total available irrigation water and is expressed as:

$$\sum_{F}\sum_{C} HD_{F,C} . EI_C \leq H2O \qquad (4.4);$$

where:

HD= decision variable as hectares of crop C grown on farm F;

EI= effective irrigation in centimeters;

and H2O= total available effective irrigation water.

4.2.5 Deterministic Model

Combining the objective function and the constraints into a deterministic linear model provides the tool to derive non-stochastic irrigation water demand via the parameterization of the water constraint in Equation 4.4. The deterministic model is expressed as follows:

$$\underset{HD}{\text{Maximize}} \quad \sum_{F}\sum_{C}(p_C . AY_C - vc_C) HD_{F,C} \qquad (4.1)$$

$$\text{Subject to} \; : \quad \sum_{C} HD_{F,C} \leq L_F \quad \forall F \qquad (4.2)$$

$$HD_{F,C} / L_F \leq R_C \;\; \forall F, C \qquad (4.3)$$

$$\sum_{F}\sum_{C} HD_{F,C} . EI_C \leq H2O \qquad (4.4)$$

$$HD_{F,C} \geq 0 \;\; \forall F, C \qquad (4.5);$$

where:

F= farm unit (1, ..., 12);

C= crop (1, ..., 8);

p= market price of a crop in dollars per tonne;

AY= estimated crop yield in tonnes per hectare;

vc= variable cost of producing a crop in dollars per hectare;

HD= decision variable or activity level representing hectares of a crop grown on a given farm unit;

L= hectares of land available per farm;

R= crop rotation requirements per crop;

EI= effective irrigation in centimeters;

and H2O = amount of water available for effective irrigation in hectare-centimeters.

Equation 4.5 of the deterministic model insures non-negativity of the decision variable. Estimated yield (AY) is a parameter calculated outside the model based on the water-yield response functions as shown in the data section below.

Parameterizing the right hand side of Equation 4.4 by changing H2O in intervals yields the shadow prices for irrigation water. Plotting these shadow prices against corresponding quantities of irrigation water produces a step function representing the non-stochastic irrigation water demand. Once the deterministic water demand is estimated from the solution to the deterministic model, stochastic water demands need to be estimated from the DSSP model in order to compare the two types of demands for estimating risk penalties.

4.3 Discrete Sequential Stochastic Programming Model

The decision process of a typical crop producer of the EID is assumed to be composed of three stages. Stage A represents the pre-planting and planting operations, stage B represents the growing season operations, and stage C represents harvesting and marketing operations. The decision tree is divided based on crop production and information flow and assumes complete knowledge of the past and present (CKPP). A depiction of the decision tree is conceptually similar to the one shown in Figure 3.3. The CKPP model is solved as three separate sub-models where there is one model for each of the states of nature of spring soil moisture.

Stage A is where the farmer commits the resources to a production plan based on past experience and expected future events and their outcomes. In this case, the farmer will decide on how much land will be committed to each crop. In stage A, the farmer faces the uncertainty of spring soil moisture, but has information about the probabilities of the occurrence of its events. The state of nature that will actually materialize, whether it is dry, normal, or wet, will be determined at the time of pre-planting and planting activities. This implies that the farmer at the time of preparing the soil for planting starts to build a database about spring soil moisture content based on winter snowfall and left over moisture from last fall's rainfall. Thus, the state of nature of spring soil moisture is revealed in the first stage of the production process.

In stage B, effective irrigation water uncertainty is revealed in terms of the adequacy (adequate or inadequate) of the available irrigation water for optimum crop yield. Effective precipitation is modeled in stage C, and deals with the amount of precipitation (high, average, or low) accumulated during the growing season.

4.3.1 Objective Function

The objective function of the DSSP expressed in Equation 3.32 above is with one output produced in a CKP information structure. In order to make this objective function suitable for the purposes of this research, several modifications have to be undertaken. First, it has to be modified to accommodate the multi-product and multiple producer nature of the problem instead of single product and single firm. Secondly, it has to be modified from a CKP to a CKPP information structure. Thirdly, it has to be modified to incorporate production stages specific to the problem.

Lambert (1990) expanded his analysis to accommodate more than one crop in the optimization problem. Lambert (1990) modified Equations 3.20 and 3.21 above to become:

$$E(\pi) = \sum_{c} H_c[E(p_c)\,f_c(x) - vc_c(w,y)] \qquad (4.6),$$

$$\sigma_\pi^2 = \sum_{c} H_c^2\sigma_c^2 + 2\sum_{c}\sum_{c'} H_c H_{c'} \ \mathrm{cov}(\pi_c, \pi_{c'}) \qquad (4.7);$$

where:

E= expectation operator;

π= profit or net return;

H_c= hectares allotted to crop c;

p_c= output price of crop c;

$f_c(x)$= production function determining quantity of output;

vc= variable cost of production;

w= price of variable inputs,

y= quantity of output;

and σ^2 and cov are variance and covariance, respectively.

Substituting (4.6) and (4.7) into (3.32) yields the following DSSP objective function expressed as a non-linear certainty equivalent maximization:

$$\underset{H}{\text{Maximize}}\, CE(\pi) = \sum_{i=1}^{3}\sum_{j=1}^{2}\sum_{k=1}^{3} P1_i P2_j P3_k \sum_{F=1}^{12}\sum_{c=1}^{8} [E(p_c) - vc(w, y_{cijk})][H_{F,C} \cdot y_{cijk}] -$$

$$\lambda / 2 \sum_{c=1}^{8} H'_{F,C} \, cov(\pi_c, \pi_{c'}) H_{F,C} \quad ; \text{for } c > c' \qquad (4.8);$$

where:

CE= certainty equivalent;

π= profit or net return;

i, j, and k= states of nature in stages A, B, and C, respectively;

P1, P2, and P3= probabilities of states of nature i, j, and k, respectively;

F= representative farm;

C= crop;

E= expectation operator;

p= output price;

vc= variable cost of production;

w= price of inputs;

y= level of output;

H= hectares;

λ= risk aversion parameter;

and cov= variance-covariance matrix of profits.

Equation 4.8 assumes there are twelve production units and eight crops as possible enterprises. Note that y_{cijk} measures crop yield per hectare dependent on states of nature occurring in the production stages. The variance-covariance matrix estimates the variability in net revenues for the different crops.

The objective function in this form, therefore, is a maximization of certainty equivalent of profits measured as expected net returns minus risk premium for each joint event for all the crops across all farms. In each cycle of the decision process, a joint event occurs that is characterized by a multinomial distribution from which joint probabilities are obtained. Expected net returns are calculated as the product of net income and joint probabilities of the joint event for all crops and farms combined. For example, y_{5212} indicates yield of the fifth crop such that, at this cycle of the decision process, state of natures 2,1,and 2 occurred in stages A, B, and C, respectively.

The DSSP objective function in Equation 4.8 can be modified to fit the CKPP information structure of the EID representative farms by solving the model for the number of states of nature in the first stage. Therefore, since the spring soil moisture has three states of nature (wet, normal and dry), then the CKPP model will be composed of three separate sub-models, one for each state of nature.

Stages of production are incorporated in the objective function through costs of production. Each stage is assigned a portion of production costs that incur in that stage and attached to the hectares of that stage. These stage production costs are also connected to the constraint set by three constraints calculating total production costs of each stage by multiplying the stage per hectare production costs by hectares assigned to a particular crop at the stage. The objective function of the DSSP model characterized by complete knowledge of the past and present information structure can be written as follows:

$$\underset{HC}{\text{Maximize}}\, CE(\pi) = \sum_{F}\sum_{S1}\sum_{S2}\sum_{S3}(P1_{S1} * P2_{S2} * P3_{S3} *$$

$$\{\sum_{C}(Ep_{C} * TONNES_{F,C,S1,S2,S3}) - CSTA_{F,C,S1} - CSTB_{F,C,S1,S2} - CSTC_{F,C,S1,S2,S3}$$

$$-(\sum_{C}[\lambda/2]*[HC'_{F,C,S1,S2,S3} * \text{cov}(\pi_{C}, \pi_{C'}) * HC_{F,C,S1,S2,S3}])\}) \qquad (4.9);$$

where:

CE= certainty equivalent;

π= profits or net returns;

F= representative farm (1,…,12);

S1= spring soil moisture state of nature (wet, normal, and dry);

S2= effective irrigation state of nature (adequate and inadequate);

S3= precipitation state of nature (high, average, and low);

P1, P2, and P3= probability of states of nature S1, S2, and S3, respectively;

C= crop (1,…,8);

Ep = per tonne expected market price of crops;

TONNES= tonnage of crops produced;

CSTA, CSTB, and CSTC = stages A, B, and C total production costs, respectively;

λ = risk aversion coefficient;

HC= hectares harvested in stage C;

and cov= variance-covariance matrix of net returns.

The CKPP objective function expressed in Equation 4.9 maximizes certainty equivalent of profits given one of the states of nature of spring soil moisture. Certainty equivalent is calculated as expected profits minus risk premium. Expected profits are calculated as the product of multiplying joint probabilities by joint events of revenues. Joint probabilities are the product of multiplying three individual probabilities associated with the joint event. Net revenues are calculated as market price of the crop times the tonnage minus the sum of costs of production at the three stages of production

$(\sum_{C}(Ep_{C} * TONNES_{F,C,S1,S2,S3}) - CSTA_{F,C,S1} - CSTB_{F,C,S1,S2} - CSTC_{F,C,S1,S2,S3})$. Tonnage is calculated through a constraint of yield times hectares harvested of the crop in stage C. Costs of production at each stage are calculated through constraints multiplying hectares assigned for the crop at a given stage of production by per tonne costs for that particular stage. Risk premium is calculated as half the risk aversion coefficient multiplied by the product of the variance-covariance matrix and squared hectares of stage C of a particular crop $(\sum_{C}(\lambda/2)\, HC'_{F,C,S1,S2,S3} \cdot cov_{C} \cdot HC_{F,C,S1,S2,S3})$.

4.3.2 Constraints

The constraints set consists of physical constraints controlling land, rotation, water, and tonnage; accounting constraints calculating total costs; and linkage constraints ensuring the sequential nature of the crop production process. Each state of nature of spring soil moisture has its own sub-model composed of a set of constraints matching the corresponding objective function, and hence the summation over spring soil moisture (S1) that appears in the constraints in fact specifies the given spring soil moisture (SSM) of the particular sub-model.

4.3.2.1 Land Constraints

Land constraints ensure that total land employed by the model is always equal to or less than the total land available i.e. hectares determined by optimal solution will not exceed the area available for each farm. Hectares in land constraint are specified for stage A since that is when the producer decides on how much to produce. Thus, land constraints are expressed as follows:

$$\sum_{C}\sum_{S1} HA_{F,C,S1} \leq L_F \qquad \forall F \qquad (4.10);$$

where:

C= crop (1,…,8);

S1= spring soil moisture state of nature (wet, normal, and dry);

HA= hectares in stage A;

F= representative farm (1,…,12);

and L= area in hectares of land available.

4.3.2.2 Crop Rotation Constraints

As in the case of the deterministic model, crop rotation requirements are based mainly on insect and soil fertility considerations. Specifics of rotation requirements are discussed below in the data section. DSSP crop rotation constraints are set for stage A since that is when farmers make land allotment decisions. Crop rotation constraints are written as follows:

$$\sum_{C}\sum_{S1} HA_{F,C,S1} \leq L_F * R_C \quad \forall F \qquad (4.11);$$

where:

C= crop (1,...,8);

S1= spring soil moisture state of nature (wet, normal, and dry);

HA= hectares in stage A;

F= representative farm (1,...,12);

L= area in hectares of land available;

and R_c= crop rotation requirement.

4.3.2.3 Effective Irrigation Water Constraints

The irrigation water constraints control the distribution of available effective irrigation water among the crops in stage B. Total irrigation water consumption is not to exceed the available irrigation water, given the state of nature of adequate or inadequate irrigation water deliveries. There are two irrigation water constraints, one for the adequate and the other for the inadequate states of nature. Irrigation water constraints are expressed as follows:

$$\sum_{F}\sum_{C}\sum_{S1} HB_{F,C,S1,S2} * EI_{C,S2} \leq H2O_{S2} \quad \forall S2 \qquad (4.12);$$

where:

F= representative farm (1,...,12);

C= crop (1,...,8);

S1= spring soil moisture state of nature (wet, normal, and dry);

HB= hectares of growing crops in stage B;

S2= effective irrigation state of nature (adequate and inadequate);

EI= effective irrigation in centimeters of water for a given crop in a specific state of nature;

and H2O= available irrigation water in a given state of nature.

4.3.2.4 Tonnage Constraints

Tonnage constraints transfer per hectare yield to total tonnage of a certain crop from a particular farm by multiplying hectares in stage C by estimated yield provided the different states of nature. The expression for tonnage constraints is as follows:

$$HC_{F,C,S1,S2,S3} * AY_{C,S1,S2,S3} - TONNES_{F,C,S1,S2,S3} = 0 \quad (4.13);$$

where:

HC= hectares in stage C;

F= representative farm (1,...,12);

C= crop (1,...,8);

S1= spring soil moisture state of nature (wet, normal, and dry);

S2= effective irrigation state of nature (adequate and inadequate);

S3= precipitation state of nature (high, average, and low);

AY= estimated crop yield in tonnes per hectare;

and TONNES= quantity produced from a crop in tonnes.

Tonnages obtained from Equation 4.13 are entered into the objective function to calculate expected net returns. Note here that tonnage and hence expected profits are calculated based on hectares in stage C after resolution of all states of nature and costs of harvesting and marketing are incurred.

4.3.2.5 Total Variable Cost Constraints

Variable costs for each stage of the production process are calculated on a per hectare basis. The cost constraints then convert the per stage costs to total variable costs per crop and farm, which in turn enter the objective function. Costs constraints are expressed as follows:

$$CSTA_{F,C,S1} - (VCA_C * HA_{F,C,S1}) = 0 \quad (4.14)$$

$$CSTB_{F,C,S1,S2} - (VCB_C * HB_{F,C,S1,S2}) = 0 \quad (4.15)$$

$$CSTC_{F,C,S1,S2,S3} - (VCC_C * HC_{F,C,S1,S2,S3}) = 0 \quad (4.16);$$

where:

CSTA, CSTB, and CSTC= total variable costs of production incurred in stages A, B, and C, respectively;

F= representative farm (1,…,12);

C= crop (1,…,8);

S1= spring soil moisture state of nature (wet, normal, and dry);

VCA, VCB, and VCC= per hectare variable costs of stages A, B, and C, respectively;

HA, HB, and HC= hectares in stages A, B, and C, respectively;

S2= effective irrigation state of nature (adequate and inadequate);

and S3= precipitation state of nature (high, average, and low).

4.3.2.6 Linkage Constraints

In order to maintain the sequential nature of the model, stages of production are linked together mathematically by ensuring total hectares in one stage do not exceed the total hectares in an earlier stage. Linkage constraints are imposed on the model in the form of hectares in stage B, of a certain crop under one possible state of nature of effective irrigation, do not exceed total hectares planted of the same crop on the same farm in stage A, given a specific state of nature of spring soil moisture. Similarly, hectares in stage C, under one possible state of nature of effective precipitation, do not exceed hectares in stage B, given a specific state of nature of effective irrigation for the specific crop on a designate farm. Such a formulation follows the structure of the decision tree of CKPP model.

To illustrate activity linkages, consider Figure 4.1. A linkage constraint ensures that total hectares of canola in stage B under an 'adequate' state of nature for effective irrigation will be equal to or less than total hectares of canola in stage A on farm 1, given that the state of nature of spring soil moisture is, for example, 'dry'. Another linkage constraint ensures that total hectares of canola in stage C on farm 1 under, for example, a 'high' state of nature of effective precipitation are no greater than hectares of canola on farm 1 in stage B under either state of nature of effective irrigation.

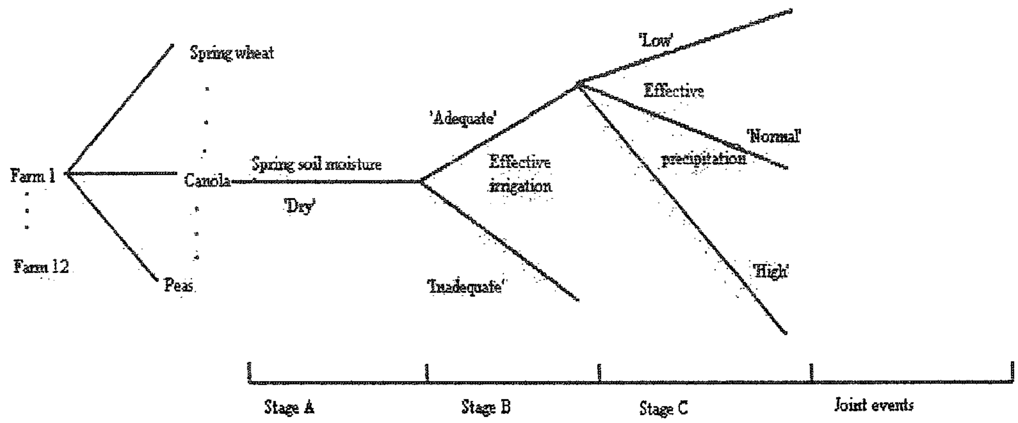

Figure 4.1 Decision Tree Depicting Linkages Between Activities in Different Stages

Conceptually, combining the two types of linkage constraints in effect trails one branch of the decision tree of CKPP model, given specific states of nature in the three stages. Linkage constraints can be expressed as follows:

$$-HA_{F,C,S1} + HB_{F,C,S1,S2} \leq 0 \qquad \forall F,C,S1,S2 \tag{4.17}$$

$$-HB_{F,C,S1,S2} + HC_{F,C,S1,S2,S3} \leq 0 \quad \forall F,C,S1,S2,S3 \tag{4.18}$$

where:

HA, HB, and HC= hectares in stages A, B, and C, respectively;

F= representative farm (1,...,12);

C= crop (1,...,8);

S1= spring soil moisture state of nature (wet, normal, and dry);

S2= effective irrigation state of nature (adequate and inadequate);

and S3= precipitation state of nature (high, average, and low).

4.3.2.7 Non- Negativity Constraints

The last part of the constraint set maintains the non-negativity condition of the decision variables i.e. hectares in each of the stages of the production process. These constraints are written as follows:

$$HA_{F,C,S1}, HB_{F,C,S1,S2}, HC_{F,C,S1,S2,S3} \geq 0 \quad \forall F,C,S1,S2,S3 \qquad (4.19);$$

where:

HA, HB, and HC= hectares in stages A, B, and C, respectively;

F= representative farm (1,…,12);

C= crop (1,…,8);

S1= spring soil moisture state of nature (wet, normal, and dry);

S2= effective irrigation state of nature (adequate and inadequate);

and S3= precipitation state of nature (high, average, and low).

4.3.3 DSSP Model

The analytical DSSP model with three decision stages is based in part on a combination of a theoretical model discussed by Apland and Hauer (1993) and the empirical model utilized by Taylor and Young (1995). Aspects of models from Lambert (1990), Mahan (1997), and Viney et al. (1996) are also incorporated into the current analysis. Each of the three CKPP sub-models corresponding to the three states of nature of spring soil moisture in stage A consists of 576 joint activities since it involves twelve farms, eight crops, two states of nature in stage B, and three states of nature in stage C.

The following is a formulation of a CKPP model, which belongs to the more general DSSP model. The model is a product of combining the objective function and constraint set discussed above.

$$\underset{HA,HB,HC}{\text{Maximize}}\ CE(\pi)=\sum_{F}\sum_{S1}\ \sum_{S2}\sum_{S3}P1_{S1}*P2_{S2}*P3_{S3}*$$

$$\underset{HA,HB,HC}{\text{Maximize}}\ CE(\pi)=\sum_{F}\sum_{S1}\ \sum_{S2}\sum_{S3}P1_{S1}*P2_{S2}*P3_{S3}*$$

$$\{\sum_{C}(Ep_{C}*TONNES_{F,C,S1,S2,S3})-CSTA_{F,C,S1}-CSTB_{F,C,S1,S2}-CSTC_{F,C,S1,S2,S3}$$

$$-\sum_{C}\ [\lambda/2]*[HC'_{F,C,S1,S2,S3}*COV(\pi_{C},\pi_{C'})*HC_{F,C,S1,S2,S3}])\}] \qquad (4.9)$$

Subject to:

$$\text{Land:}\qquad \sum_{C}\sum_{S1}\ HA_{F,C,S1}\leq L_{F}\quad \forall F \qquad (4.10)$$

$$\text{Rotation:}\qquad \sum_{C}\sum_{S1}\ HA_{F,C,S1}\leq L_{F}*R_{C}\quad \forall F \qquad (4.11)$$

$$\text{Water:}\qquad \sum_{F}\sum_{C}\sum_{S1}\ HB_{F,C,S1,S2}*EI_{C,S2}\leq H2O_{S2}\quad \forall S2 \qquad (4.12)$$

$$\text{Tonnage:}\qquad HC_{F,C,S1,S2,S3}*AY_{C,S1,S2,S3}-TONNES_{F,C,S1,S2,S3}=0 \qquad (4.13)$$

$$\text{Costs A:}\qquad CSTA_{F,C,S1}-(VCA_{C}*HA_{F,C,S1})=0 \qquad (4.14)$$

$$\text{Costs B:}\qquad CSTB_{F,C,S1,S2}-(VCB_{C}*HB_{F,C,S1,S2})=0 \qquad (4.15)$$

$$\text{Costs C:}\qquad CSTC_{F,C,S1,S2,S3}-(VCC_{C}*HC_{F,C,S1,S2,S3})=0 \qquad (4.16)$$

$$\text{Link1:}\qquad -HA_{F,C,S1}+HB_{F,C,S1,S2}\leq 0\quad \forall F,C,S1,S2 \qquad (4.17)$$

$$\text{Link2:}\qquad -HB_{F,C,S1,S2}+HC_{F,C,S1,S2,S3}\leq 0\quad \forall F,C,S1,S2,S3 \qquad (4.18)$$

$$HA_{F,C,S1},HB_{F,C,S1,S2},HC_{F,C,S1,S2,S3}\geq 0\quad \forall F,C,S1,S2,S3 \qquad (4.19);$$

where:

CE= certainty equivalent;

π= profits or net returns;

F= representative farm (1,…,12);

S1= spring soil moisture state of nature (wet, normal, and dry);

S2= effective irrigation state of nature (adequate and inadequate);

S3= precipitation state of nature (high, average, and low);

P1, P2, and P3= probability of states of nature S1, S2, and S3, respectively;

C= crop (1,…,8);

Ep = per tonne expected market price of crops;

TONNES= tonnage of crops produced;

CSTA, CSTB, and CSTC = stages A, B, and C total production costs, respectively;

λ = risk aversion coefficient;

HC= hectares harvested in stage C;

cov= variance-covariance matrix of net returns;

HA= hectares in stage A;

L= area in hectares of land available;

R= crop rotation requirement;

HB= hectares in stage B;

EI= effective irrigation in centimeters of water for a given crop in a specific state of nature;

H2O= available irrigation water in a given state of nature;

HC= hectares in stage C;

AY= estimated crop yield in tonnes per hectare;

TONNES= quantity produced from a crop in tonnes;

and VCA, VCB, and VCC= per hectare variable costs of stages A, B, and C, respectively;

The decision variables in the above model are HA, HB, and HC. The model's optimal solution describes the best strategy the producer should follow in order to maximize the certainty equivalent by listing the optimal allocation of land to the different crops in each stage of the production process under the corresponding states of nature.

Based on Apland and Hauer (1993), since the variance-covariance matrix is positive semi-definite, the objective function in Equation 4.9 is concave and a global solution to the model is ensured. The variability and co-variability of expected cash price, Ep_c, is assumed to capture the uncertainties of the output market and hence, price risk is modeled in the objective function. There are marketing strategies available to the farmer other than cash sales such as futures markets and storage. Such marketing options may not be applicable to all the crops included in this study. However, for the purpose of this study, cash prices are used since the objective here is to analyze the policy effects on irrigation water resources rather than assessing the performance of different marketing strategies.

The assumption of independence between output and its price can be seen as unrealistic in the situation of southern Alberta's agriculture. For some crops, e.g. vegetables, that are grown on relatively small acreages in southern Alberta, price and output may be correlated, since the product is mainly marketed locally and output will have a significant impact on prices at times of sizable shortages or surpluses. The independence of output and its price, however, is maintained throughout this study.

The model follows the analytical approach of using experimentally based agronomic production functions of certain crops. It also uses data for a sample of representative farms rather than data for a whole region, which is the approach of aggregate methods. The CKPP model is used in estimating the stochastic and conditional stochastic irrigation water demands, which will be compared to non-stochastic water demands obtained from the solutions to the deterministic model.

4.4 Estimation Approach

In order to estimate the desired irrigation water demands and welfare estimates, algorithms written in GAMS code (Appendix) are solved using GAMS software. Estimates include deterministic irrigation water demand, risk aversion parameters, stochastic irrigation water demands, conditional stochastic water demands, risk penalties, and welfare estimates. According to Apland et al. (1993), since there are three sub-models to the CKPP problem; one sub-model corresponding to each spring soil moisture state of nature, then the optimal strategy for the CKPP problem combines the solutions to the three sub problems, and certainty equivalent of net returns is the sum of the optimal objective function values for the three sub-models. Based on Taylor and Young (1995), values of estimated conditional stochastic irrigation water demands will be prorated by their probabilities of occurrence in order to obtain the expected value of water deliveries under uncertainty.

4.4.1 Deterministic Irrigation Water Demand

Parametrizing the water constraint of the deterministic model in Equation 4.4 yields the shadow prices of the irrigation water resource. Plotting the shadow prices against water quantities depicts the deterministic demand for irrigation water as a factor of production. Shadow prices in this context imply the value of a unit of water to the producer. This value is the amount of money an extra unit of the scarce resource would contribute to the objective function. From another perspective, it is the amount of money the producer is willing to pay in order to obtain that specific extra unit of the resource.

The area under the deterministic irrigation water demand and above the supply curve of the resource provides an estimate of the welfare of the producers under certainty. This area is approximated by the value of the objective function. The deterministic irrigation water demand provides the basis for comparison between the value of irrigation water under conditions of certainty and uncertainty.

4.4.2 Risk Aversion Parameter

One of the parameters of the CKPP model is the risk aversion coefficient. Since there is no readily available estimate that can be applied, the DSSP model is used to estimate the risk posture of the producers. According to McCarl and Spreen (1997), estimates of risk aversion parameters can be obtained such that the difference between observed behavior and the model solutions is minimized. The CKPP model is solved using a range of values for risk aversion coefficients and the parameter that produces the minimum sum of squared deviations of the model's optimum hectares from the sample of representative farm hectares is chosen as the representative risk aversion parameter for farmers' risk preferences. This procedure is applied for the three CKPP sub-models representing the three states of nature of the spring soil moisture.

4.4.3 Stochastic Irrigation Water Demands

Stochastic irrigation water demands are estimated by parametrizing the right hand side values of the water constraints in Equation 4.12. Conceptually, there are three CKPP sub-models, or one for each state of nature of the spring soil moisture. For each sub-model, there are two states of nature for effective irrigation i.e. adequate and inadequate.

Therefore, for each sub-model there exist two irrigation water constraints (Equation 4.12) corresponding to adequate and inadequate states of nature of effective irrigation as follows:

$$\sum_{F}\sum_{C}\sum_{DRY} HB_{F,C,DRY,S2} * EI_{C,S2} \le H2O_{S2} \quad \forall S2 \qquad (4.12a);$$

$$\sum_{F}\sum_{C}\sum_{NORMAL} HB_{F,C,NORMAL,S2} * EI_{C,S2} \le H2O_{S2} \quad \forall S2 \qquad (4.12b);$$

$$\sum_{F}\sum_{C}\sum_{WET} HB_{F,C,WET,S2} * EI_{C,S2} \le H2O_{S2} \quad \forall S2 \qquad (4.12c);$$

For example, Equation 4.12a consists of two constraints as follows:

$$\sum_{F}\sum_{C}\sum_{DRY} HB_{F,C,DRY,ADEQUATE} * EI_{C,ADEQUATE} \le H2O_{ADEQUATE} \qquad (4.12d);$$

$$\sum_{F}\sum_{C}\sum_{DRY} HB_{F,C,DRY,INADEQUATE} * EI_{C,INADEQUATE} \le H2O_{INADEQUATE} \qquad (4.12e);$$

Similarly, Equations 4.12b and 4.12c are split into two constraints each. The result of the parameterization procedure is six stochastic irrigation water demands derived from the three CKPP sub-models. The right hand sides of the two irrigation water constraints for each CKPP sub-model are parametrized simultaneously in order to derive the two stochastic irrigation water demands of the states of nature of effective irrigation. Standard factor demands are drawn in a two-dimensional price-quantity plane. However, stochastic demands derived in this case are shapes expressed in a three- dimensional space with one of the axes for the price per unit of stochastic irrigation water and the other two axes are for the levels of water deliveries under the two states of nature for effective irrigation; adequate and inadequate.

Conditional stochastic demands can be derived by parametrizing the right hand side of one irrigation water constraint, e.g. $H2O_{ADEQUATE}$, while holding the right hand side values of the other irrigation water constraint at a given level, e.g. $H2O_{INADEQUATE}$. This parameterization simulates water shortages by assuming a percentage of full irrigation water deliveries. Such conditional stochastic demands allow for comparisons at equal levels of irrigation diversions. For example, conditional stochastic demands can be derived for fifty percent deliveries of irrigation water under adequate and inadequate states of nature of effective irrigation by parametrizing the right hand side value of one of the irrigation water constraints, e.g. $H2O_{ADEQUATE}$, while holding the value of the right hand side value of the other water constraint, e.g. $H2O_{INADEQUATE}$, at the fifty percent of water deliveries. This process is carried out on the three sub-models. Then, the two conditional stochastic demands from each sub-model are prorated by the corresponding probability of the occurrence of adequate or inadequate effective irrigation. The final step is to aggregate the expected values of water obtained from each sub-model in order to obtain the total expected value of irrigation water at the given probability of occurrence of effective irrigation.

4.4.4 Risk Penalties and Value of Stochastic Irrigation Water Deliveries

The vertical distance between the deterministic and conditional stochastic irrigation water demands produces an estimate of the risk penalty of employing irrigation water as a risk increasing or reducing input. If the deterministic demand lies above the stochastic demand, irrigation water is considered a risk increasing input and the risk penalty intuitively implies the amount of subsidy required in order to make the producer indifferent between the certain and uncertain irrigation water deliveries. If the deterministic demand lies below the stochastic demand, then irrigation water is considered a risk reducing factor of production and the risk reward is equivalent to the amount of tax the producer is willing to pay in order to maintain certain supplies of irrigation water. Risk penalties are measured at given and unified levels of irrigation water diversions across the states of nature of effective irrigation as well as for deterministic deliveries in order to ensure the comparisons are taken at the same benchmarks. Expected value of stochastic irrigation water is measured by prorating the adequate and inadequate deliveries by their respective probabilities of occurrence (Taylor and Young, 1995).

The areas under the stochastic demand schedules measured by the values of the objective functions of the CKPP model estimate the benefits to or welfare of the producers under uncertainty. The sum of the values of the objective functions of the three CKPP sub-models constitutes the total value of the objective function of the CKPP model (Apland et al., 1993). Changes in producers welfare or foregone benefits due to transfer of water to other uses is estimated by comparing the value of the objective function under different regimes of irrigation water availability.

4.5 Data

This section introduces data used in estimating the empirical models. Farm budgeting figures provide cost information about the different crop enterprises. States of nature data and parameters for water-yield response functions are derived from historical information published by Underwood McLellan Ltd. (1982), UMA henceforth. Representative farm sample information is taken from Viney et al. (1996) while crop rotation requirements are based on consultations with field experts. Crop price and yield time series as well as price indexes are gathered from different government publications pertaining to the topic.

4.5.1 Sample Representative Farms of The Eastern Irrigation District

According to Viney et al. (1996), deposits of surface soil material and texture, plus solonetzic soils, limit the continuous production of cereals, oilseeds, and specialty crops in the EID. The rocky and uneven terrain also limits the use of certain types of irrigation systems. Frost risk and limited heat units because of the short growing season limit corn production. Other agronomic constraints include crop diseases, soil fertility, and weed management.

Viney et al. (1996) examined the EID water rolls and defined 'ownership units' based on land parcel location and landowners. Farm ownership distribution revealed that approximately half of the farms in the EID were in the 100 to 300 acre range. Fewer than 20 percent of the farms had more than 500 acres. In terms of irrigated acres, the examination revealed that over half of the EID land is farmed in units of 100 to 500 acres while only 15 percent of the land is farmed in units of greater than 1000 acres. From such results, Viney et al. (1996) defined twelve representative farm sizes with frequency weights reflecting the distribution of farm sizes in relation to total EID acreage. Production patterns for these 12 farm units were assigned based on actual farming practices as reflected by the EID total agricultural output in 1994. The sample representative farms data used in this study are based on the sample used by Viney et al. (1996).

The farms of the EID vary in the area of land available for cropping as well as in the types of crops planted. Some farms seem to diversify more than others in terms of the number of crops planted in a typical year, and some farms seem to be more specialized than others. These specialization and limitation patterns seem to reflect certain constraints on the enterprises, which may include financial, technical, managerial, agronomical, or other constraints.

The crop mix of the representative sample includes irrigated crops (C=1,…,8): spring wheat, soft wheat, barley, canola, alfalfa, pasture, potato and peas, respectively, grown on 12 farms (F=1,…,12) as shown in Table 4.1. Wheat, barley, and canola are characterized by a four-month growing season, May to August, while alfalfa, pasture, potato, and peas have a five-month growing season, May to September (Mahan, 1997). In this study, however, silage and alfalfa seed are eliminated from the sample of crops used in Viney et al. (1996) due to the lack of water-yield response functions of these two crops in UMA (1982) study. In addition, alfalfa seed is usually produced after growing alfalfa for several years then the crop is let into producing seeds while silage is customarily made of barley and fed to on-farm livestock.

Table 4.1 Areas and Crops of Sample Representative Farms (Hectares)

Farm #	Spring Wheat	Soft Wheat	Barley	Canola	Alfalfa	Pasture	Potato	Peas	Total
Farm1	44.51	44.51	113.31	60.70	242.0	159.85	0	0	664.89
Farm2	16.18	16.18	80.93	24.28	113.31	91.05	0	0	341.95
Farm3	16.18	16.18	32.37	24.28	60.70	71.62	0	0	221.36
Farm4	0	0	32.37	40.46	164.30	34.39	0	0	271.54
Farm5	4.04	4.04	24.28	8.09	106.02	36.42	0	0	182.91
Farm6	0	0	28.32	24.28	88.22	48.56	0	0	189.39
Farm7	0	0	16.18	12.14	97.52	32.37	0	0	158.23
Farm8	0	0	24.28	24.28	90.64	0	0	0	139.21
Farm9	0	0	0	20.23	24.28	20.23	8.09	12.14	84.98
Farm10	0	0	0	22.25	28.32	8.09	22.25	22.25	103.19
Farm11	0	0	0	12.14	0	0	21.44	24.28	57.87
Farm12	0	0	0	0	0	57.87	0	0	57.87
Total	80.91	80.91	352.07	273.16	1015.35	560.48	51.79	58.67	2473.34

Cattle enterprises are included in the study by Viney et al. (1996) in the form of calf production but excluded from the analysis of this study due to data deficiencies and the complexity such an enterprise may introduce to the model in terms of added variables and intermediate terms. Therefore, dropping livestock production from the model adds another reason for eliminating barley silage while leaving the barley rotation requirement at the maximum allowable.

Historically, farmers in the EID preferred not to specialize in high return enterprises due to the financial risk represented by the possibility of a negative return. Hence, diversified farming units are the norm among the EID farmers while full farm specialization is a rarity (Viney et al., 1996). Table 4.1 shows the crop mixes of the twelve representative farms along with the areas customarily planted of each crop that are used in this study.

The analysis in this study is analytical in terms of using production functions of crops typical to the region, and data for representative farms. This approach differs from the aggregate regional approach in that it does not take the whole area of the EID into consideration nor do the irrigation water constraints involve the historical water diversions to the EID.

4.5.2 Crop Rotation Requirements[1]

Equations 4.3 and 4.11 represent crop rotation constraints. Specifically, the constraints state that for spring wheat, soft wheat, and canola a maximum of two thirds of available land could be allotted for any of these crops in a given growing season. For barley, the ratio is eighty percent, and it is twenty five percent for each of the remaining crops i.e. alfalfa, pasture, potato, and peas. These crop rotational requirements are based mainly on insect and soil fertility considerations.

4.5.3 Production Costs

Wheat, barley, canola, and alfalfa per acre production costs partitioned into VCA, VCB, and VCC in Equations 4.14-4.16 are obtained from *1997 Costs & Returns Tables For Selected Crops, Irrigated Soils.* Potato and peas data not included in those tables are obtained from *1999 Cropping Alternatives, Selected Cereals, Oilseeds, Forages, and Special Crops: Irrigated Soils.* Budget data for pasture (green feed) are obtained from *1997 Forage Enterprise Costs & Returns Analysis: Irrigated Green feed,* since they are not included in the previous sources.

Production costs figures are transformed to per hectare units. Since the analysis is relevant to the short run, fixed costs such as depreciation are excluded and therefore only variable costs are included in the computations. To minimize trend variability in costs and input prices, 1999 costs are deflated to 1997 dollars using the itemized Farm Input Price Index for western Canada. 1999 crop prices are deflated to 1997 prices

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[1] The details in this section are based on expert opinion provided by Brian Hunt, Rotations Specialist with Alberta Agriculture, Food, and Rural Development.

using the Consumer Price Index of all items for Alberta and substituted in equation 4.1 as p_c. The GAMS program adds up each stage's per hectare variable costs of producing each crop, and then the cost constraints compile total variable costs for the allotted hectares in the given stage. The total variable costs are then entered into the objective function in calculating the net returns.

The cost of seeds and hail and crop insurance are assigned to stage A of the production process. Fuel, machinery and building repairs, utilities, miscellaneous spending, custom work, and special labour costs are divided equally between the three stages of the production process. Costs of interest, paid labour, and unpaid labour related to four-month growing season crops are divided between stages A, B, and C at the ratio 1:3:1, respectively. Costs of interest, paid labour, and unpaid labour related to five-month growing season crops are divided between stages A, B, and C at the ratio 1:2:1, respectively. Transportation and marketing costs are assigned to stage C. Irrigation water related costs such as license, equipment, operating costs, and water rates are excluded in order to impute the residual value of water.

4.5.4 Price and Yield Time Series

The variance-covariance matrix of net revenue of the objective function in the CKPP model, as shown in Equation 4.9, is calculated by first determining the net revenues of crop production for the period 1984-1994. Time series of crop prices and yields are those used by Viney et al. (1996) and listed in various issues of *Agriculture Statistics Yearbook*, except for pasture entries which are taken from *Economics of Milk Production.* The price series is converted to uniform units of dollars per tonne and the yield series is converted to tonnes per hectare. Prices are then expressed in 1997 dollars using the Consumer Price Index mentioned above and then expected price Ep_c is taken as simple average of prices from 1984-94. Output is tested for trend by regressing yields on time and the results show insignificant relationships between yield and time. The 1997 costs of production mentioned above are used in determining net revenues. The variance-covariance matrix of net returns is calculated within the GAMS program as shown in Appendix.

4.5.5 Water-Yield Response Functions

Mahan (1997) and Viney et al. (1996) used empirical water-yield response functions similar to those employed in this study. However, Mahan (1997), Viney et al. (1996), and Kulshreshtha and Tewari (1991) used estimates from UMA (1982), of yield response for given crops. The water-yield function is expressed as:

$AY= f(H_2O, PY, PE)$ (4.20);

where:

AY= estimated crop yield in tonnes per hectare;

H_2O= available moisture in centimeters of water;

PY= potential yield in tonnes per hectare;

and PE= potential evapotranspiration in centimeters of water.

The UMA (1982) study defines potential evapotranspiration (PE) as the amount of evaporation from soil and transpiration from crops, which could occur given sufficient moisture availability. Moisture availability consists of spring soil moisture (SSM), effective precipitation (PRCP), and effective irrigation (EI) all expressed in centimeters of water. PE rates for different crops are estimated at maximum crop yields. However, actual evapotranspiration (AE) is estimated as the sum of SSM, PRCP, and EI.

SSM is defined as the spring soil moisture available at the start of a growing season, which depends on the amount of moisture stored in the soil at the end of the previous season plus winter precipitation. PRCP is defined as total precipitation received during the growing season in the form of rain or snow adjusted by a reduction of ten percent to account for precipitation lost due to deep drainage when soil moisture levels are near optimal. EI is defined as supplemental water to keep the soil moisture at optimal levels and measured as total irrigation adjusted by a reduction of ten percent.

The ratio of actual to potential evapotranspiration is estimated as:

$$(AE/PE)=(SSM+PRCP+EI)/PE \quad (4.21)$$

UMA (1982) showed that actual yield (AY) to potential yield (PY) ratio could be estimated as a function of Equation 4.21 by fitting data to a second-degree polynomial function, such that:

$$[AY/PY]=a_0+a_1\,[(SSM+PRCP+EI)/PE]+a_2\,[(SSM+PRCP+EI)/PE]^2 \quad (4.22);$$

where a_0, a_1, and a_2 are estimable regression coefficients. Since PY can be estimated, then Equation 4.22 was used to estimate crop yields for several crops grown in Alberta based on data from experimental plots and expressed as:

$$AY= PY\,\{a_0+a_1\,[(SSM+PRCP+EI)/PE]+a_2\,[(SSM+PRCP+EI)/PE]^2\} \quad (4.23)$$

Given the values of the estimated parameters of the regression and values of potential yields, varying the amounts of moisture by changing the values of its three parts gives yields of crops under different watering regimes.

Estimates of equation 4.23 are provided in UMA (1982) and are shown in Table 4.2. Note that the intercept estimates are adjusted upward to reflect improvements in yields (Viney et al., 1996). PE is derived from Table 6.1.1 of UMA's (1982) study, while PY data for agroclimate area A_2 are taken from Table 3.4.1 of the same publication. PE for soft wheat is adjusted upward by 25% (Mahan, 1997). Estimated crop yield (AY) in Equations 4.1 and 4.13 is computed as a parameter by GAMS outside the model utilizing the entries from Table 4.2 and values for SSM, PRCP, and EI from the states of nature data below.

Table 4.2. Coefficient Estimates of Crop Water-Yield Response Functions*

Crop	a_0	a_1	a_2	PE(cm)	PY(tonne)
Spring Wheat	-0.191	1.628	-0.557	51	6.7
Soft Wheat	-0.195	1.655	-0.599	63.75	6.7
Barley	-0.199	1.696	-0.644	41	6.4
Canola	-0.199	1.696	-0.644	41	3.5
Alfalfa	-0.097	1.272	-0.313	88	11.2
Pasture	-0.24	1.781	-0.717	75	9.5
Potato	-0.982	2.467	-0.101	58	56
Peas	-0.002	2.498	-1.038	58	2.5

* All estimated coefficients are significant at 95 percent level, UMA (1982)

4.5.6 States of Nature and Probability Distributions

Data pertaining to the states of nature of spring soil moisture (SSM) and effective precipitation (PRCP) and their associated probabilities are collected from UMA (1982), where the South Saskatchewan River Basin is subdivided into five zones. The EID lies in the A_2 agroclimate zone, which is characterized by 1350-1650 heat units per year, 120-140 day growing season, and 250-300 mm moisture deficit. The township of Brooks represents A_2 zone.

4.5.6.1 Spring Soil Moisture (SSM)

UMA (1982) assumes a fall soil water content (FSM) equal to 25 percent of the available soil water range when calculating SSM according to the following formula:

$$SSM=0.177\ (ppn) + FSM \qquad (4.24);$$

where ppn is precipitation for October to April.

Spring soil moisture is modeled in stage A and data are derived from Appendix C of the UMA (1982) study where 'less than' cumulative distribution of SSM is listed for the town of Brooks in millimeters of water. Table 4.3 contains the amounts of moisture existing in topsoil at the time of pre-planting and planting and the associated probabilities.

Table 4.3 Probability Distribution of Spring Soil Moisture

States of Nature of Spring Soil Moisture	Cm	Probability
Wet	11.0	0.342
Normal	8.7	0.324
Dry	8.1	0.334

The probability of SSM is assumed divided equally between its three states of nature implying there is an equal chance for any of the states of nature to occur. The amounts of spring soil moisture from Table 4.3 enter Equation 4.23 while the probabilities of spring soil moisture enter into the objective function (Equation 4.9) as part of determining the probabilities of joint events.

4.5.6.2 Effective Irrigation (EI)

States of nature of effective irrigation and effective precipitation exist for four and five-month growing season crops. Spring wheat, soft wheat, barley, and canola have four-month growing seasons and hence their requirements of irrigation water and rainfall are a function of the time from seeding until before harvesting. On the other hand, pasture, alfalfa, potato, and peas have a five-month growing season and hence the states of nature of effective irrigation and precipitation are different from those for the four-month growing season crops because of the different water requirements partially due to the longer period over which the crops are being in the fields. The probabilities associated with the states of nature of effective irrigation and effective precipitation are expected to differ between the two groups of crops. Hence, these states of nature are modeled as independent such that four-month growing season has its own states of nature of effective irrigation and precipitation, which are independent from those equivalent states of nature for the five-month growing season.

States of nature of effective irrigation are modeled in stage B of the CKPP model. Since there are two growing seasons, the state of nature of effective irrigation is divided into adequate irrigation water deliveries and inadequate deliveries for each of the growing seasons. Probabilities of effective irrigation for each growing season's states of nature add to one. According to UMA (1982), zone A_2 has an annual moisture deficit of 250-300 mm. Therefore, 27.5 cm of effective irrigation is assumed required on average to insure optimum crop yields. Table 4.4 shows the amounts of effective irrigation water and the probabilities associated with the states of nature.

Note that the probability values are assigned arbitrarily since data are not available for shortages in irrigation water, yet. Perhaps after the implementation of the newly adopted system of trading water rights, such data will become readily available.

Table 4.4 Probability Distributions of Effective Irrigation

States of Nature of Effective Irrigation	Four-month Growing Season		Five-month Growing Season	
	cm	Probability	cm	Probability
Adequate	27.5	0.8	28.5	0.8
Inadequate	23.0	0.2	25.0	0.2

However, a lower probability of occurrence of adequate supplies implies a loss of water rights seniority. For example, if the probability of adequate deliveries of irrigation water is equal to 1.0, then the producer is very certain of obtaining the water. But when such probability drops to 0.5, then the producer is less certain of the adequate deliveries, which indicates that other users may have improved their own probabilities of getting the amounts of water to satisfy their demands at the expense of this producer. Effective irrigation values complement SSM in Equation 4.23 when estimating yields of crops; however, the probabilities are used in the objective function of the CKPP model.

4.5.6.3 Effective Precipitation (PRCP)

Effective precipitation is modeled in stage C of the CKPP model. Appendix D of UMA's (1982) study lists the cumulative probabilities of receiving a 'less than' amount of effective precipitation during the growing seasons May to August and May to September in Brooks, Alberta. Therefore, precipitation probabilities add to one for each growing season and independently of the other growing season's precipitation. Table 4.5 shows the amounts of precipitation for each state of nature and the corresponding probabilities.

Table 4.5 Probability Distributions of Effective Precipitation

States of Nature of Effective Precipitation	Four-month Growing Season		Five-month Growing Season	
	cm	Probability	cm	Probability
High	25.0	0.3324	27.5	0.3360
Average	14.4	0.3320	17.0	0.3308
Low	10.1	0.3356	13.2	0.3332

Heywood (1985) reports in his Table 3 and Table 4 values for growing season and non-growing season precipitation closely matching those reported above. In addition, Heywood (1985) states that most producers are somewhere around a 70 to 75 percent level of maximum yield and most operators are either satisfied or limited to these levels. Crop yields derived from equation 4.23 by using the different values of states of nature of spring soil moisture, precipitation, and effective irrigation are consistent with Heywood's statements. Effective precipitation values enter into Equation 4.23 while their probabilities are used in the objective function of the CKPP model.

Data presented in this section are used to solve the deterministic and stochastic models. The next chapter will present results of optimal solutions that are used to validate the models. Then the chapter proceeds by introducing derived deterministic and stochastic irrigation water demands. These demands are the basis for estimating risk penalties due to using irrigation water as a stochastic factor of production. Welfare estimates under conditions of certainty and uncertainty due to changes in irrigation water deliveries will then be presented.

5. RESULTS AND DISCUSSION

5.1 Introduction

The chapter starts by validation of models by construct and by results. Simultaneously, a risk aversion parameter that best represents the risk preferences of the producers is fitted to the DSSP model by applying a range of values and choosing the coefficient that produces the optimal crop mix with minimum squared deviations from sample data.

Deterministic irrigation water demand is then derived from the dual solution to the linear non-stochastic model. The step function relates shadow prices of irrigation water to quantities available. Parameterization of the right hand side values of the irrigation water constraints of the CKPP sub-models yield the derived stochastic irrigation water demands for adequate and inadequate states of nature of effective irrigation. These three dimensional stochastic demands are not suitable for comparison with the derived deterministic water demand for the purposes of estimating the size of risk penalty attributed to using irrigation water as a stochastic input in the crop production process. Conditional stochastic demands are estimated by holding the amount of available irrigation water in the opposite state of nature of effective irrigation constant. Risk penalties are then estimated for intervals of irrigation water availability for five different cross sections of stochastic water demands.

The effects of water transfer on crop production in southern Alberta are then estimated vis-à-vis average foregone benefits due to water diversions away from the EID to other uses. Sensitivity analysis sheds light on the factors that have the potential for changing the results obtained. A GAMS code shown in Appendix below is employed in solving the different models and in report writing.

5.2 Risk Aversion Parameter and Validation of Models

Before presenting and discussing the results of the mathematical programming problem, the empirical models must be validated to ensure reliability of the results and conclusions. The first step is validating the models by construct. Ignizio (1982), McCarl et al. (1997), and Dorward (1999) list many assertions by which a model can be declared valid by construct. One of these assertions is adopted for the purpose of this study. It states that, since the models are consistent with theory and previous work, data are collected from sources using reasonable estimation and accounting procedures, and the initial trial runs yield acceptable results in terms of their consistency with real world values and within anticipated ranges, the models can be declared valid by construct. The next step is validation of the models by results.

McCarl et al. (1997) list many formal and informal tests used to achieve the goal of validating the models by results. However, one way of validating mathematical programming models that is used in the literature, e.g. Taylor et al. (1995) and Brink et al. (1978), is to check for how close the results of the model duplicate real world data. For the purposes of this study, one approach may use absolute deviation of the models' predicted hectares from sample acreages. Another approach, which is applied here, checks for minimum sum of squared deviations of sample hectares from hectares predicted by models' primal solutions provided effective irrigation is set at levels that satisfy all the irrigation water requirements such that the values of spring soil moisture, effective irrigation, and precipitation ensure optimum crop yield.

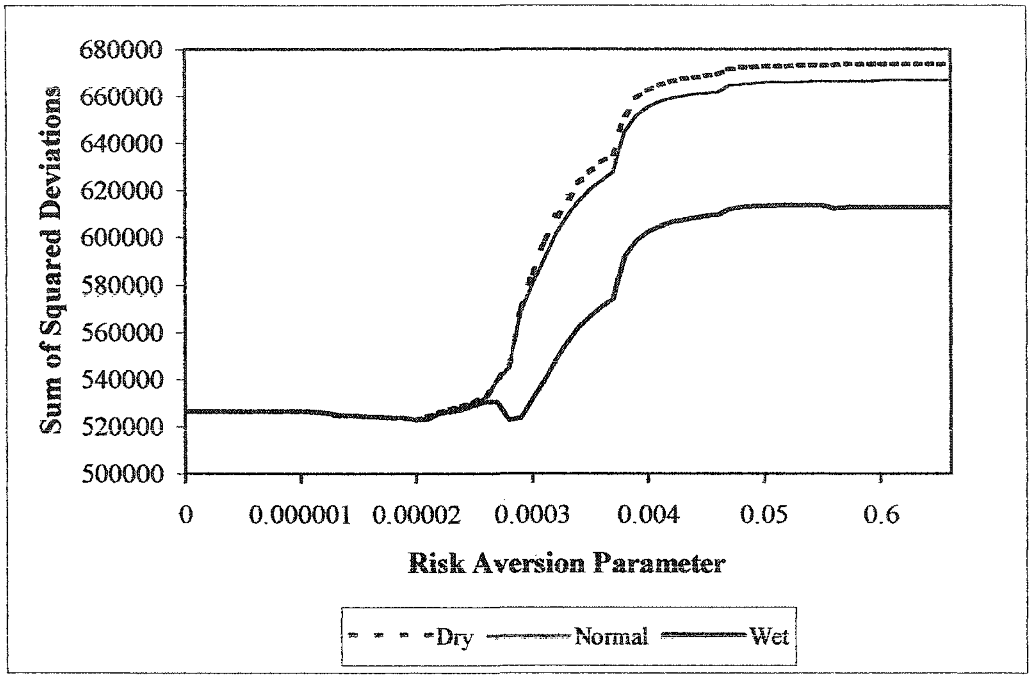
In addition and within the context of model validation, results from the optimal solution, such as shadow prices of resources or opportunity cost of capital, are compared to similar results reported by other related studies or data available. Validation of the DSSP model involves solving three CKPP sub-models corresponding to the three states of nature of spring soil moisture (dry, normal, and wet) while varying the value of risk aversion coefficient.

Estimation of the risk aversion parameter that represents the risk preferences of producers is done simultaneously with the validation of models. A range of risk aversion parameters was used in order to obtain the optimal crop mix that deviates the least from sample hectares and hence the corresponding risk aversion parameter will determine the risk posture of sample producers. 134 risk aversion parameters were applied to the CKPP sub-models ranging from -1.20 to 1.20. Most of these values of risk aversion coefficient fall within ranges reported in similar studies (Brinks et al. (1978) and Raskin et al. (1986)).

Negative values of risk aversion coefficient yield ambiguous results. The risk premium in the objective function is preceded by a minus sign, and hence when multiplied by a negative risk aversion parameter, the result is adding the penalties to the expected net returns rather than subtracting them. As a result, the value of the objective function is greatly inflated. On the other hand, large positive values of the risk aversion coefficient lead to minimal optimal crop mixes and sometimes give relatively smaller values of the objective function. Results using the risk aversion parameter values and their corresponding sum of squared deviations of predicted optimal hectares from sample hectares are presented in Figure 5.1 and summarized in Table 5.1.

The results indicate that a risk aversion parameter of 0.00002 produces the minimum squared deviations estimates of hectares for the 'Dry' and 'Normal' CKPP sub-models, while for the 'Wet' sub-model it is 0.0001. To ensure a common comparison among the three sub-models, 0.00002 is considered as the representative risk aversion coefficient representing the sample producers' preference toward risk. This estimated λ value would be applied to all subsequent model estimation and falls in the almost risk-neutral preferences range of -0.001 to 0.005 mentioned by Raskin et al. (1986).

Figure 5.1 Risk Aversion Parameters and Sum of Squared Deviations of Sample Hectares from Sub-Models' Predictions



In addition, Brinks et al. (1978) reported a value of estimated λ equal to 0.25 for farmers in the U.S. Corn Belt; however, their conclusion was that risk preferences were not important. These values from other studies are mentioned only for purposes of guiding remarks and cannot be compared to the estimates reported here for reasons of differences in units of measurement, locale, time framework, sample characteristics, data specifics, and so forth.

The corresponding crop mixes obtained from solutions to the three CKPP sub-models using λ=0.00002 and predicted hectares of the deterministic model constitute the second part of model validation. Table 5.2 shows a comparison between the optimal crop mixes predicted by the models and sample hectares.

Table 5.1 Risk Aversion Parameters and CKPP Sub-Models Best-Fit Expressed as Sum of Squared Deviations of Sub-Models' Predictions from Sample Hectares

Risk Aversion Parameter	Sum of Squared Deviations of Hectares		
	Spring Soil Moisture State of Nature		
	Dry	Normal	Wet
0.0	526097	526097	526097
0.0000009	526097	526097	526097
0.000006	524388	524381	524355
0.00002	522496	522623	522684
0.00006	528458	528237	527357
0.0001	545745	544888	522525
0.001	635064	627709	573837
0.01	669036	662066	609592
0.1	673157	666244	613978
1.0	673577	666669	612820

The comparison between the optimal hectares and the sample hectares indicates that the deterministic model allocates all the land available to four crops; namely, spring wheat, canola, alfalfa and potato. Four other crops are left out of the optimal solution. The optimal strategy of the CKPP model, resulting from the optimal solutions to the three sub-models, is similar to the deterministic solution plus it introduces soft wheat into the optimal solution plus an insignificant acreage of barley.

Table 5.2 Hectares from Models' Predictions and Sample Representative Farms

Model	Spring wheat	Soft wheat	Barley	Canola	Alfalfa	Pasture	Potato	Peas	Total
CKPP	307	113	1	816	618	0	618	0	2473
Deterministic	421	0	0	816	618	0	618	0	2473
Sample	81	81	352	273	1015	560	52	59	2473

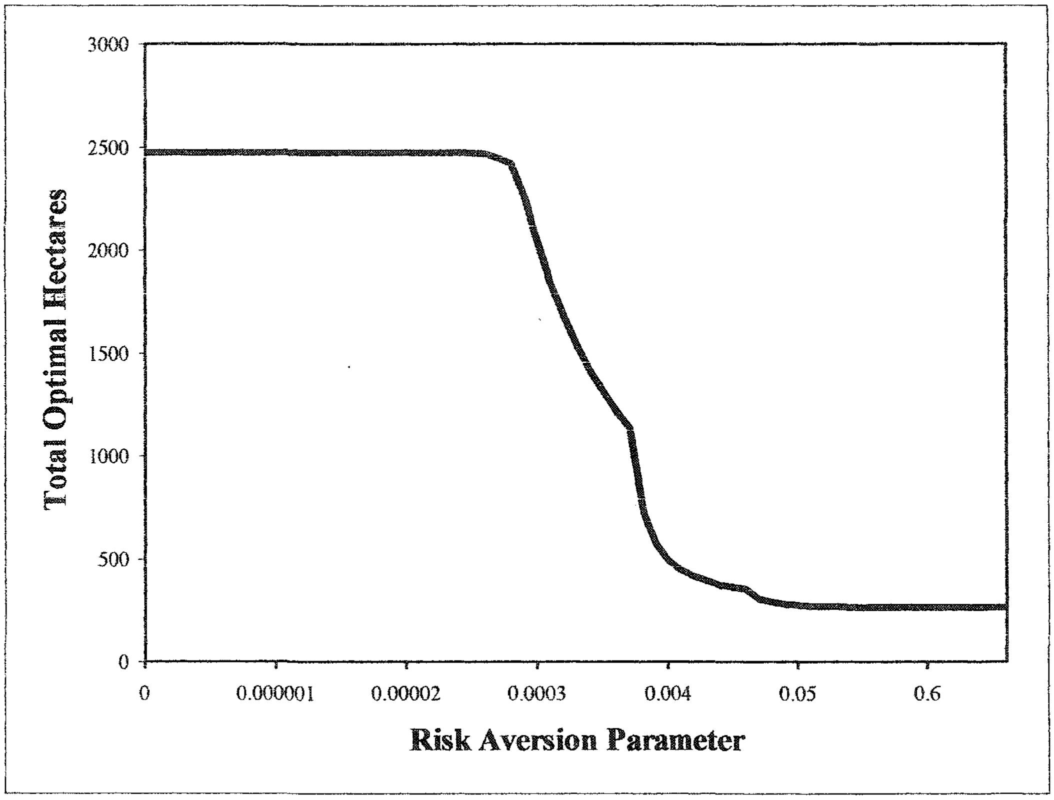
Several observations worth mentioning related to the predicted crop mixes. First, the structure of the sample of representative farms is hypothetical in the sense that the farms are constructed based on the distribution of similar farms in the area of the study, i.e. the EID, by Viney et al. (1996). The sample representative farms comprise other enterprises, such as livestock production, which are not included in this study as mentioned above. Therefore, the absence of such activities may cause the models to yield biased or skewed results in favour of certain enterprises i.e. specification error.

Second, the variation between the deterministic and DSSP predicted hectares are perhaps related to the absence of risk in the deterministic model but considered in the CKPP sub-models. In terms of sum of squared deviations of optimal to sample hectares, the DSSP model seems to emulate the cropping patterns of the EID slightly better than the deterministic model. This is maybe another reason justifying the use of models incorporating price and production risk as opposed to models that are risk neutral or incorporating one type of risk but not both when deriving factor demands.

Third, the agronomic constraints imposed on the models do not coincide with hectares of the representative farms. For example, hectares of both types of wheat are constrained to not exceed thirty-three percent of the available land for each crop. However, there are only four farms in the sample growing wheat with percentages from their total available land ranging only between 2.2 and 7.3 percent. The same can be stated about the other agronomic constraints in relation to crop mix of optimal solution versus sample hectares.

Although the risk aversion parameter of λ=0.00002 provides the best fit of the stochastic model in terms of least squared deviations of optimal from sample hectares, optimal crop mixes obtained from the model's response to the different risk aversion parameter values are interesting and worth mentioning here. In Figure 5.2, the general trend is that total optimal hectares are negatively related to the value of risk aversion parameter while the flat portions of the curve indicate minimum or no change in total predicted hectares. The crop mix, however, changes over the different parts of the curve in Figure 5.2.

Figure 5.2 Risk Aversion Parameter and Total Optimal Hectares



The optimal crop mix when λ=0.00002 includes six crops and all land available is utilized. However, when the producers are assumed risk neutral such that λ=0, the optimal crop portfolio includes four crops only. However, when λ is between zero and less than 0.00002 the optimal crop mix consists of five crops.

Table 5.3 shows the values of risk aversion parameter where the optimal crop mix changes by including or excluding crops from the farm plan. Pasture and peas are included in the optimal crop mix when λ=0.00009 while total land is not fully utilized. As the decision makers adopt a high risk averse posture, total land employed decreases and area allotted for each crop drops significantly as λ approaches 0.4, except for pasture where optimal hectares are positively correlated to the value of risk aversion parameter.

Table 5.3 Risk Aversion Parameter and Optimal Crop mix (Hectares)

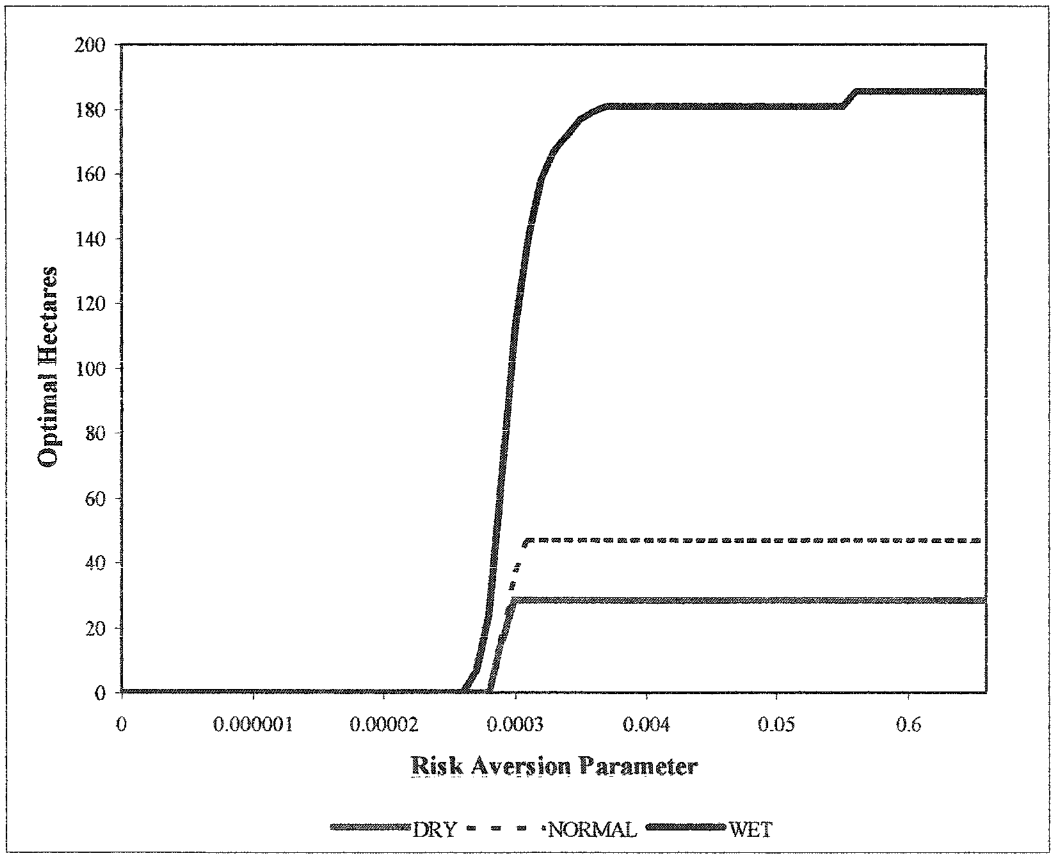
Risk Aversion Parameter	Spring Wheat	Soft Wheat	Barley	Canola	Alfalfa	Pasture	Potato	Peas	Total
0	420.48	0	0	816.24	618.36	0	618.36	0	2473.44
0.000003	415.28	5.21	0	816.24	618.36	0	618.36	0	2473.45
0.00002	306.68	113.28	0.53	816.24	618.36	0	618.36	0	2473.45
0.00009	227.02	155.95	103.3	816.24	519.17	6.85	614.96	1.18	2444.67
0.0001	218.71	152.4	108.64	816.24	498.33	24.1	599.43	0	2417.85
0.0002	177.35	133.09	139.85	806.48	380.14	103.5	513.93	0.92	2255.26
0.4	0.14	0.11	0.15	1.26	0.28	260.95	0.44	0	263.33

Model results in Table 5.3 provide information about the response of individual crops to changes in the value of risk aversion parameter such that crops can be classified into three groups. The first group includes spring wheat, canola, alfalfa, and potato where their response to the increase in the value of risk aversion parameter resembles the shape of total hectares curve depicted in Figure 5.2.

Soft wheat and barley optimal hectares increase with the increase in the value of risk aversion parameter where they peak at λ values of 0.00009 and .0002, respectively. Pasture and peas hectares enter the optimal solution at λ=0.00009. Peas optimal hectares are insignificant in size and peaks and valleys characterize their response to changes in the values of risk aversion parameter. Pasture, however, is employed at an increasing rate as the value of risk aversion parameter is increased up to λ=0.2 where the area planted by pasture levels off at 260.95 hectares. Pasture is apparently the only crop of choice when producers exhibit a relatively higher degree of risk aversion.

The optimal farm plan is an aggregation of three sub-plans derived from the three CKPP sub-models related to spring soil moisture state of nature. The results show that when changing the value of risk aversion parameter, optimal hectares result from adding almost equal amounts of land under the three states of nature of spring soil moisture in the case of all crops except for pasture and peas. In the case of peas, all hectares are planted in the 'wet' state of nature of spring soil moisture. However, for pasture, the area planted under wet conditions is significantly larger than the areas under dry and normal conditions (Figure 5.3), which can be explained in light of the fact that pasture is the crop that is most commonly turned into dry land farming among the sample crops and the product is used as an input in beef production. Thus, when farmers are highly risk averse, they would allocate the land to pasture knowing that it will not be irrigated at the worst case scenario and their losses will be minimal. However, the preference is still in favour of wet soil conditions during planting time, and hence more hectares are allocated to wet spring soil moisture.

Figure 5.3 Optimal Hectares of Pasture under Three States of Nature of Spring Soil Moisture



Thus far, results relating risk aversion parameter values to optimal hectares of crop mixes provided relevant added information about producers' behaviour under uncertainty and therefore enhanced the validity of the CKPP model. Comparisons of dual results of the deterministic and DSSP models to data and results reported in other related studies provide another way of validating the analytical models. The land constraints expressed by equations 4.2 and 4.10 ensure that total hectares allotted to different crops on sample farms do not exceed total area available for the deterministic and stochastic models, respectively. If the constraint becomes binding such that all land is allocated, then the optimal solution will report the value of an extra unit of land expressed as its contribution to the objective function. This land value approximates how much the decision maker is willing to pay for bringing an extra unit of land into production, or willing to accept for renting or leasing it out.

The deterministic value of land is \$397.87 per hectare (\$161 per acre) across all farms and crops. The stochastic value of land ranges from \$174.07 per hectare (\$70.44 per acre) for large farms to \$236.60 per hectare (\$95.75 per acre) for smaller farms. These latter values are comparable to the rent and lease rates reported in *1997 Costs and Returns Tables For Selected Crops*, which averaged \$85.80 per acre for reported irrigated crops.

Kulshreshtha et al. (1991) estimated the short-run value of water in the South Saskatchewan River Irrigation District to be between \$0.44 and \$ 127.82 (1986 dollars) per acre-foot (ac-ft) for different levels of product prices. Viney et al. (1996) estimates of water values ranged from \$8 to \$250 per ac-ft. These estimates are comparable to the deterministic and stochastic water values derived in this study. The deterministic water values range from \$47 per hectare-centimeter (ha-cm) to \$132 per ha-cm (\$576-1632/ac-ft). After excluding risk penalties for using irrigation water as a stochastic input from the deterministic values, the stochastic values of water range from \$38 to \$128 per ha-cm (\$467-1579/ac-ft). It is apparent that estimates from other studies are relatively lower than those obtained from this one, which can be attributed mainly to the over representation of potato, a crop with relatively higher net returns, in the optimal crop mix of this study.

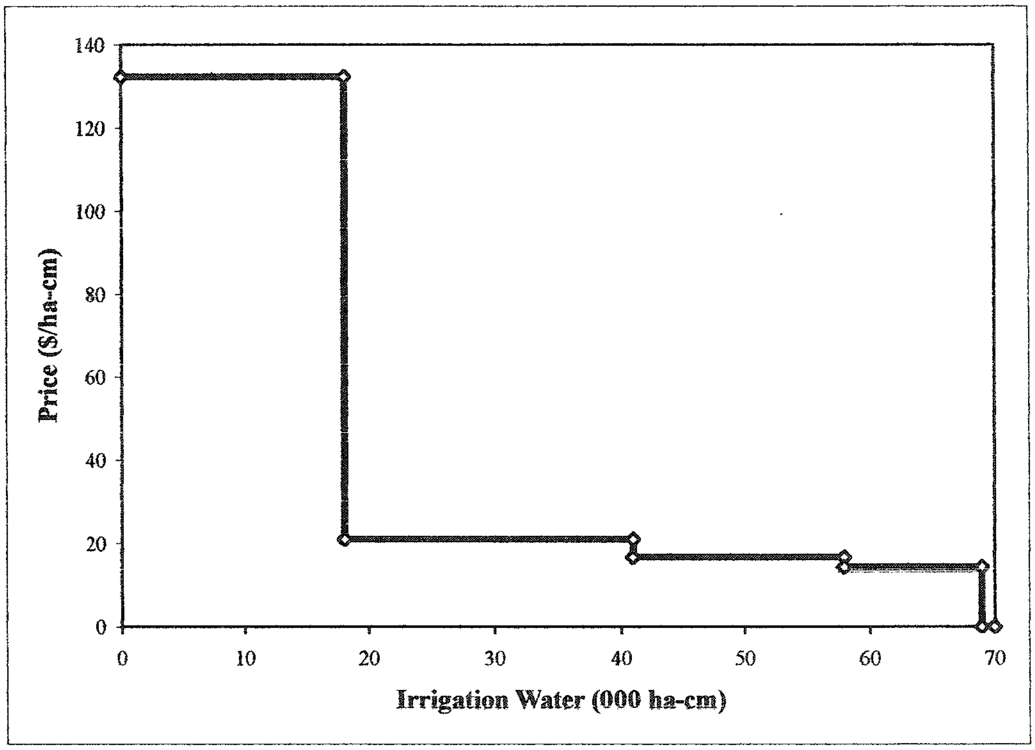
The validation process through comparing obtained results to data available and to similar findings from other studies has demonstrated that the deterministic and DSSP models yield estimates that are realistic, comparable, and hence acceptable. This leads to the conclusion that the deterministic and the CKPP models are valid and hence shadow prices of irrigation water resources can be used to derive the deterministic and stochastic demands for irrigation water. A risk aversion parameter of 0.00002 is assumed a reasonable estimate of the preferences of decision makers towards risk and uncertainty in southern Alberta's agricultural sector utilizing supplemental irrigation in the crop production process, and therefore, it will be used in conjunction with estimating stochastic irrigation water demands.

5.3 Deterministic Irrigation Water Demand

Solutions to the deterministic model yield shadow prices for the right hand side of the irrigation water constraint in Equation 4.4. H2O is parametrically changed by one thousand hectare-centimeters intervals of irrigation water. The constraint assumes net water deliveries to the farm plots while seepage and returns are assumed to happen before this point. In addition, it is assumed that, within a given growing season, spring soil moisture, effective irrigation, and precipitation are random independent variables such that during the same growing season none of these variables affect one another.

According to the estimated step function of the derived deterministic demand for irrigation water depicted in Figure 5.4, the value of water is \$132 per ha-cm or \$1632 per ac-ft for the first 25 percent of irrigation water available. At this stage the optimal crop mix consists of only potato, which makes the value of water relatively high due to the relative high net returns of potato. The first 17,000 units of effective irrigation seem to be critical especially to five-month growing season crops. In an environment characterized by certainty, producers would seem to be willing to pay a premium price for obtaining such extra units of irrigation water. This relatively high value is related to the net returns obtained from such crops as potato that contribute a significantly larger per unit net return into the objective function relative to other crops such as wheat, barley, and canola.

Figure 5.4 Derived Deterministic Demand for Irrigation Water



Once canola enters the optimal crop mix, the marginal value of water drops to $21 per ha-cm ($260/ac-ft). At about 60 percent availability of irrigation water, alfalfa enters the optimal solution and the marginal value of water drops to about $17 per ha-cm ($206/ac-ft). Spring wheat enters the optimal crop mix when the last 20 percent of available irrigation water is employed and the value of water decreases to about $14 per ha-cm ($178/ac-ft).

5.4 Stochastic Irrigation Water Demands

Three CKPP sub-models are solved to estimate stochastic irrigation water demands. For each state of nature of spring soil moisture, dry, normal, and wet, there is a CKPP sub-model. Each model includes two water constraints regulating irrigation water use according to the state of nature of effective irrigation water deliveries: adequate and inadequate (Equations 4.12d-4.12e).

The two water constraints of each sub-model are simultaneously parametrized by changing irrigation water availability by one thousand hectare-centimeters intervals of the two right hand side values ($H2O_{ADEQUATE}$ and $H2O_{INADEQUATE}$) . The resulting shadow prices associated with quantities of irrigation water depict the derived stochastic irrigation water demands from the CKPP sub-models for the given state of nature of spring soil moisture. This procedure is applied to each of the three sub-models (Equations 4.12a-4.12c) and the result is six derived stochastic irrigation water demands that account for risk and uncertainty in revenues, spring soil moisture, effective irrigation, and precipitation.

Figures 5.5, 5.6, and 5.7 depict derived stochastic irrigation water demands for the three states of nature of spring soil moisture (dry, normal, and wet), given the 'adequate' state of nature of effective irrigation. These demand curves are drawn in a three dimensional space such that one axis represents price, a second axis represents quantity of water in the adequate state of nature, and a third axis represents quantity of water in the inadequate state of nature. The general shapes of the graphs of adequate stochastic demands appear to be similar and with all the water in the adequate state of nature is used. However, higher water values are observed when the level of water availability at the opposite inadequate state of nature is approaching its maximum. The main features of these demands are a ridge that is associated with the relatively more profitable potato crop. Then a sudden drop in the value of stochastic irrigation water occurs when alfalfa and canola are included in the optimal crop mix. The plateau beyond the 25 percent level of adequate irrigation water availability represents uniform values for stochastic irrigation water but with a relative slight increase in value as the quantities of the complementary inadequate water become more available.

The value of stochastic irrigation water seems almost identical in the dry and normal states of nature of spring soil moisture. However, in the wet state of nature, the value of stochastic irrigation water appears relatively higher. An intuitive explanation would be that the wet conditions at time of planting reduce risk of having a bad growing season and chances of a good harvest are enhanced and thus irrigation water becomes more valuable and the producer would be willing to pay a higher premium for water in order to ensure the survival of a crop that had a good start.

Figure 5.5 Stochastic Irrigation Water Demand for Dry State of Nature of SSM and Adequate State of Nature of Effective Irrigation

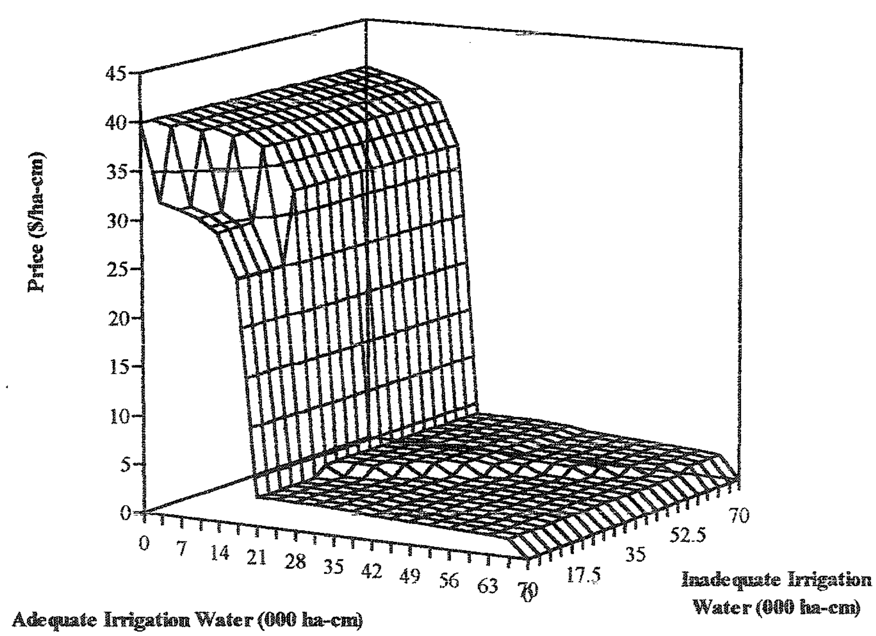


Figure 5.6 Stochastic Irrigation Water Demand for Normal State of Nature of SSM and Adequate State of Nature of Effective Irrigation

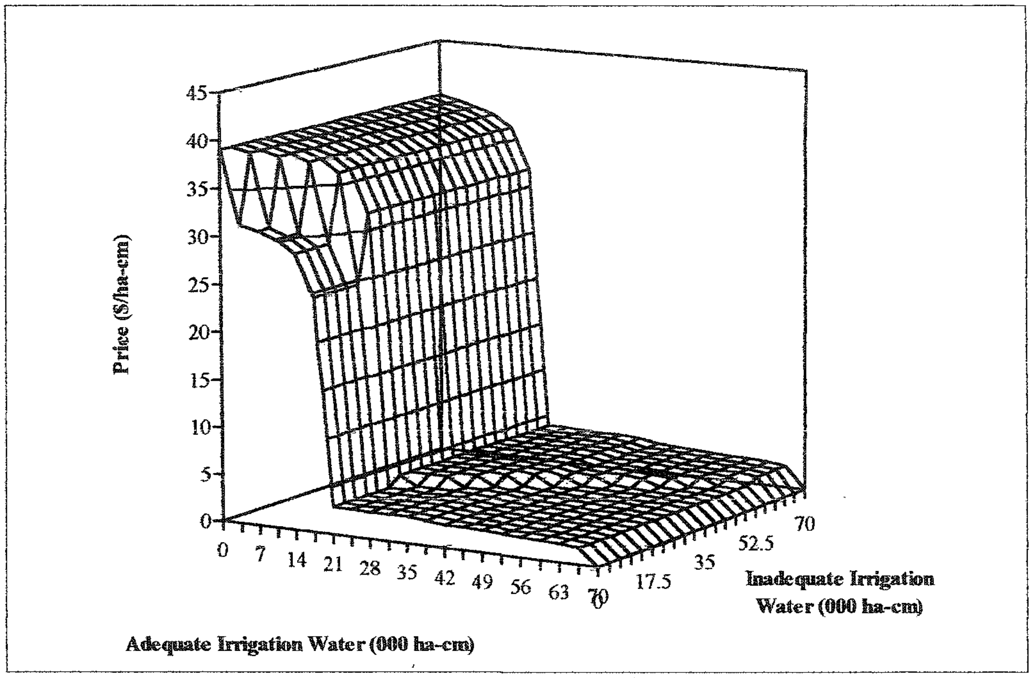
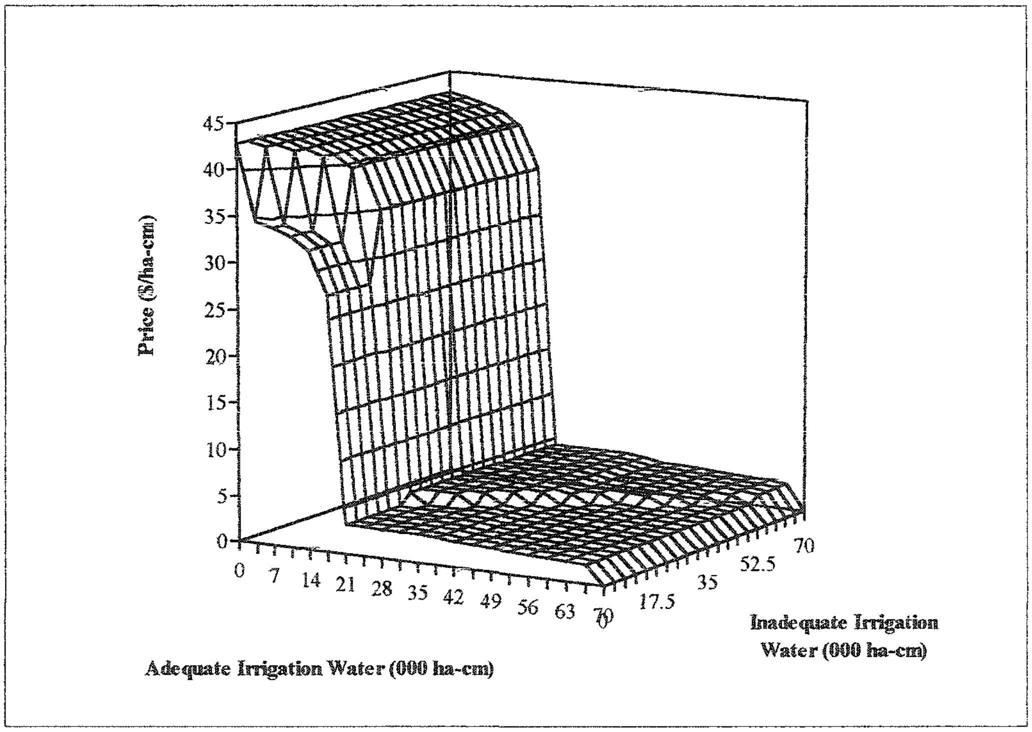


Figure 5.7 Stochastic Irrigation Water Demand for Wet State of Nature of SSM and Adequate State of Nature of Effective Irrigation



The derived stochastic irrigation water demands for the three states of nature of spring soil moisture; dry, normal, and wet (given an inadequate state of nature of effective irrigation) are depicted in Figures 5.8, 5.9, and 5.10. These stochastic demands are complements to those of the adequate state of nature of effective irrigation described above. The value of stochastic inadequate irrigation water is lowest when the spring soil moisture is normal and highest when it is wet although the optimal crop mixes are not significantly different. The inadequate stochastic demands have the same general shape as that of the adequate state of nature ones such that they are composed of a ridge and a plateau. The ridge covers the first 25 percent of available units of inadequate supplies of irrigation water. There is a drop in the value of water in this section when adequate water is expected to be in short supply. Two distinct areas characterize the plateau. The first is associated with inadequate water supplies of up to 50 percent of quantities available while the second is for up to 85 percent of inadequate water supplies. Beyond 85 percent of inadequate water supplies, inadequate irrigation water is not used and has a value equal to zero. Note that when adequate water is in short supply, the value of inadequate irrigation water tends to be zero and appreciates as the quantities of adequate supplies become more abundant. In contrast, the value of adequate irrigation water supplies does not equal to zero under any expected levels of inadequate irrigation water supplies.

Figure 5.8 Stochastic Irrigation Water Demand for Dry State of Nature of SSM and Inadequate State of Nature of Effective Irrigation

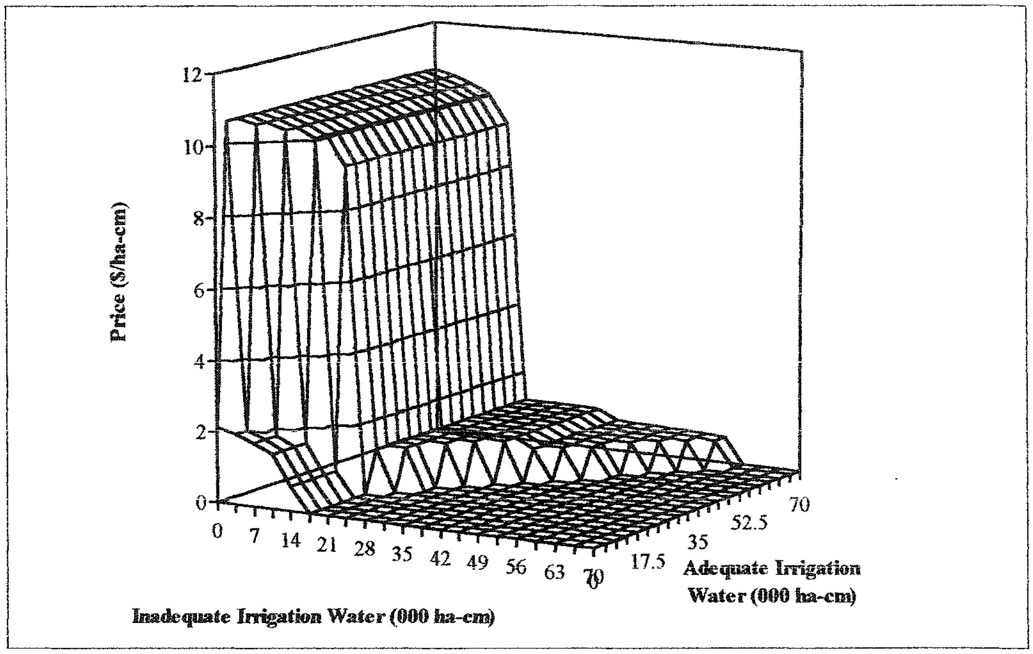


Figure 5.9 Stochastic Irrigation Water Demand for Normal State of Nature of SSM and Inadequate State of Nature of Effective Irrigation

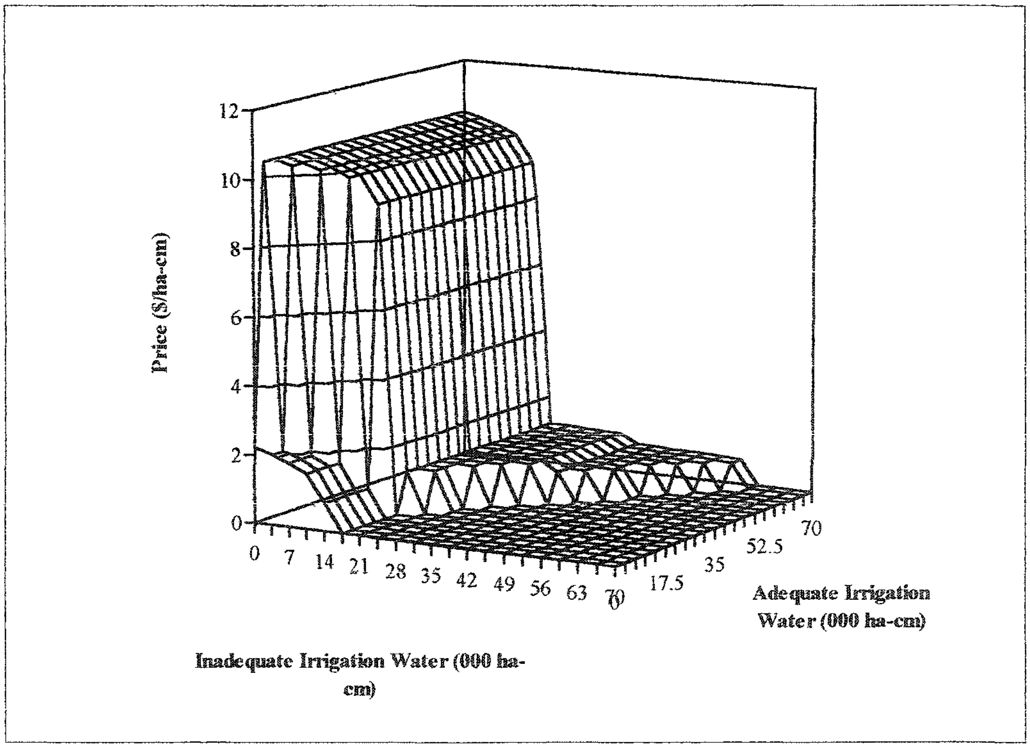
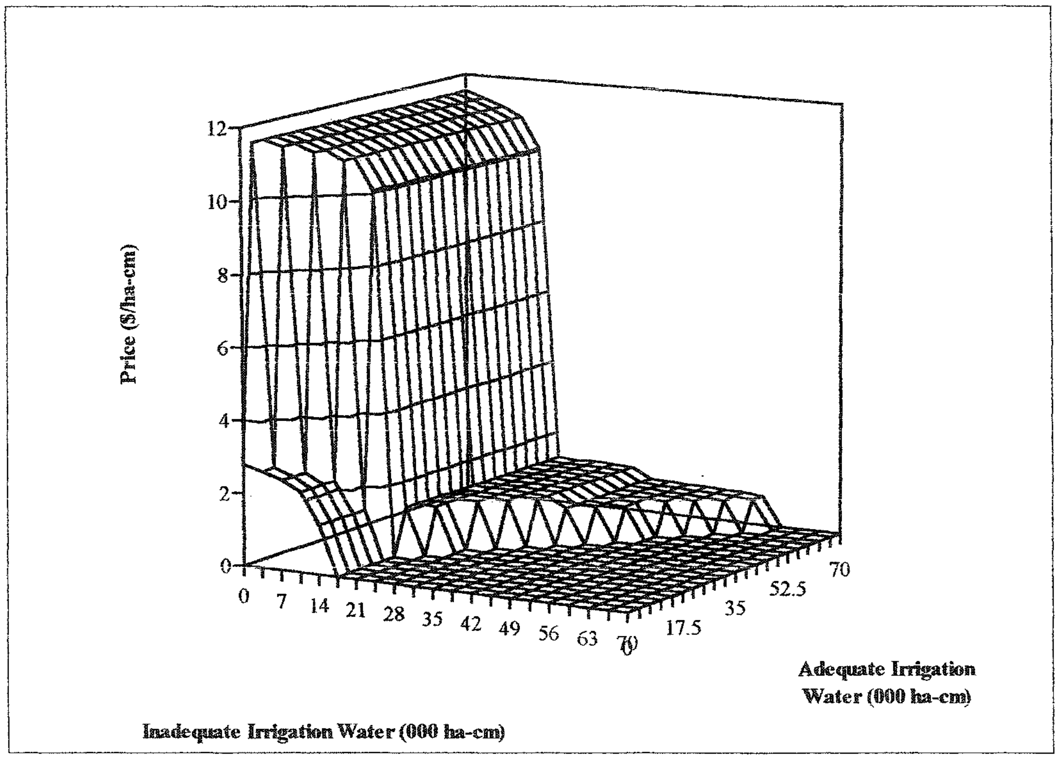


Figure 5.10 Stochastic Irrigation Water Demand for Wet State of Nature of SSM and Inadequate State of Nature of Effective Irrigation



The above estimated stochastic demands are not suitable for comparison with the derived deterministic demand schedule when estimating risk penalties associated with using water as a risky input. Conditional stochastic demands are estimated at predetermined levels of irrigation water deliveries.

5.5 Conditional Stochastic Irrigation Water Demands

In order to estimate risk penalties resulting from using water as a risk increasing or decreasing input, deterministic irrigation water demand is compared to stochastic demand. However, stochastic water demands are multidimensional in terms of the number of price-quantity coordinates, i.e. quantities of irrigation water available under each state of nature associated with each price of the input while the deterministic demand is two-dimensional. Taylor et al. (1995) estimated conditional stochastic demands by holding the quantity of irrigation water available for the opposite state of nature at a selected level. Thus, the estimated conditional stochastic demand is a cross section of the stochastic demand at the predetermined level of water deliveries of the other state of nature.

The conditional stochastic irrigation water demands are then derived by parametrizing the right hand side value of one of the irrigation water constraints of Equations 4.12a – 4.12c (e.g. $H2O_{ADEQUATE}$) and holding the value of the other constraint of the opposite state of nature of effective irrigation (e.g. $H2O_{INADEQUATE}$) at a predetermined value. The resultant step function is a derived stochastic demand for irrigation water for a given state of nature of effective irrigation conditional on the value of the right hand side of the second irrigation water constraint. An infinite number of such conditional stochastic step functions can be estimated of which two define the boundaries given by the values of the right hand side of the constraint of the complementary state of nature of effective irrigation set at zero and 100 percent of available irrigation water.

For the purpose of this study, conditional stochastic demands are estimated while holding the level of water availability in the opposite state of nature at 0, 25, 50, 75, and 100 percent levels. Figures 5.11, 5.12, and 5.13 depict the boundary conditional stochastic irrigation water demands for the adequate state of nature of effective irrigation for each of the three spring-soil-moisture states of nature.

Figure 5.11 Conditional Stochastic Irrigation Water Demands for Dry State of Nature of SSM and Adequate State of Nature of Effective Irrigation

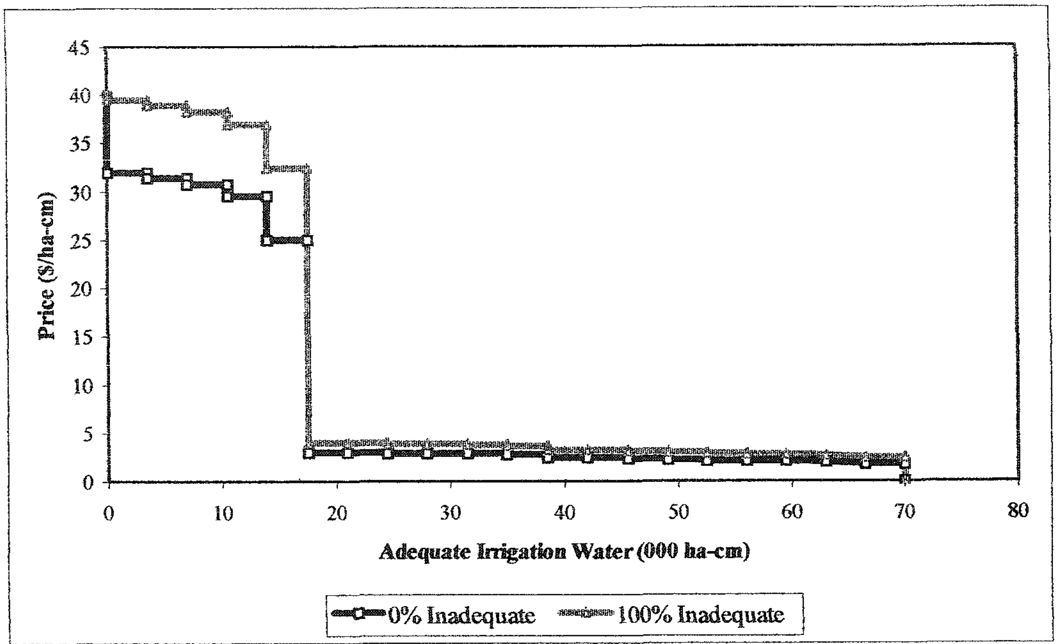


Figure 5.12 Conditional Stochastic Irrigation Water Demands for Normal State of Nature of SSM and Adequate State of Nature of Effective Irrigation

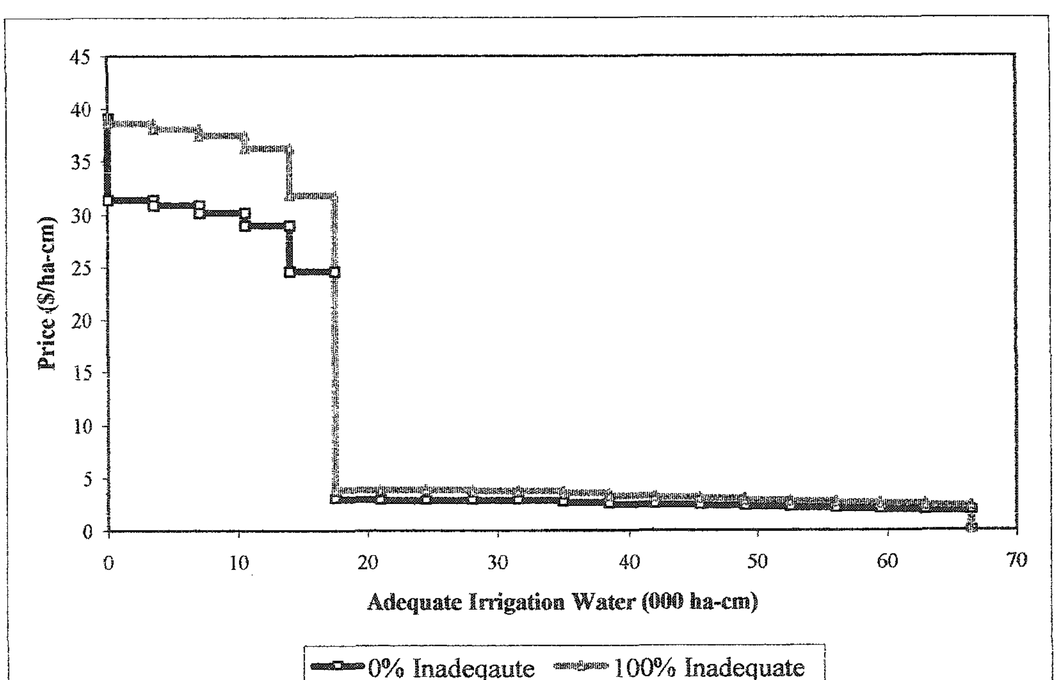
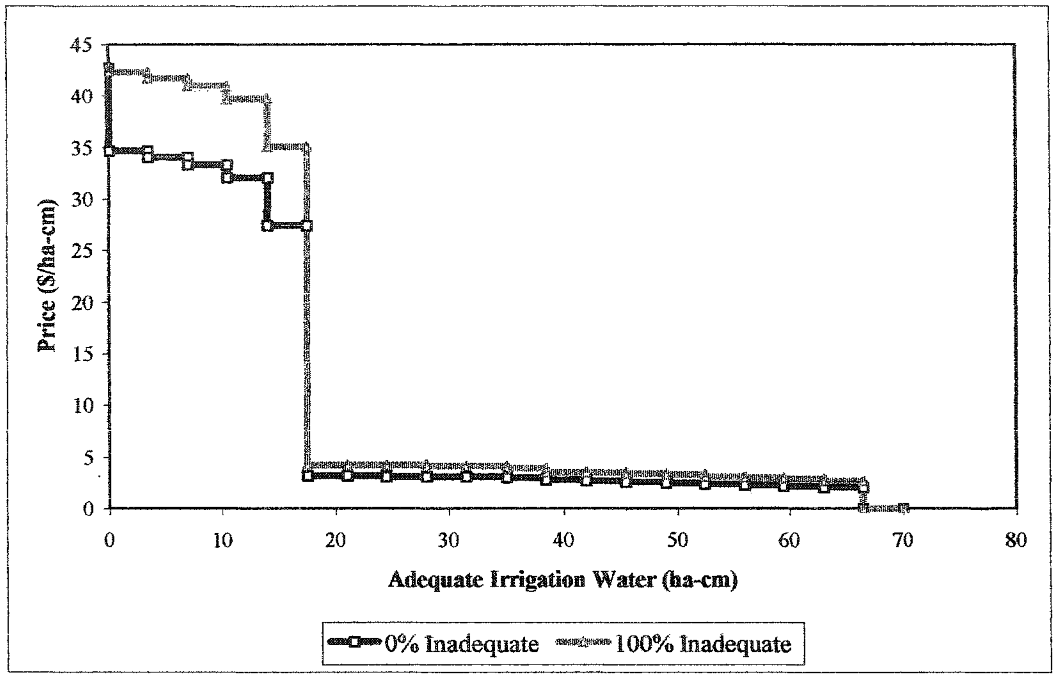


Figure 5.13 Conditional Stochastic Irrigation Water Demands for Wet State of Nature of SSM and Adequate State of Nature of Effective Irrigation



The conditional stochastic demand schedules in Figures 5.11-13 are cross sections of the stochastic demands in Figures 5.5-7 at zero and 100 percent water availability levels of the inadequate or complementary state of nature of effective irrigation. The available adequate irrigation water has always a value greater than zero and all the available supplies are employed in the production process. The value of water in the 'wet' condition is higher than when it is 'normal' or 'dry' when producing potatoes using the first 20,000 ha-cm of available irrigation water. The distance between the two boundary conditional stochastic demands shown in each of these three figures is wider when the quantity of available adequate irrigation water is low and narrows as water becomes abundant. Overall, adequate irrigation water deliveries are higher in value when 100 percent of inadequate water delivery is expected than when it is zero percent.

Figures 5.14, 5.15, and 5.16 depict the conditional stochastic irrigation water demands for the inadequate state of nature of effective irrigation for each of the three spring-soil- moisture states of nature. These demand schedules emphasize the relatively lower value of inadequate irrigation water when compared to adequate deliveries, since adequate deliveries have higher crop yield contributions than inadequate and hence better revenues.

Figure 5.14 Conditional Stochastic Irrigation Water Demands for Dry State of Nature of SSM and Inadequate State of Nature of Effective Irrigation

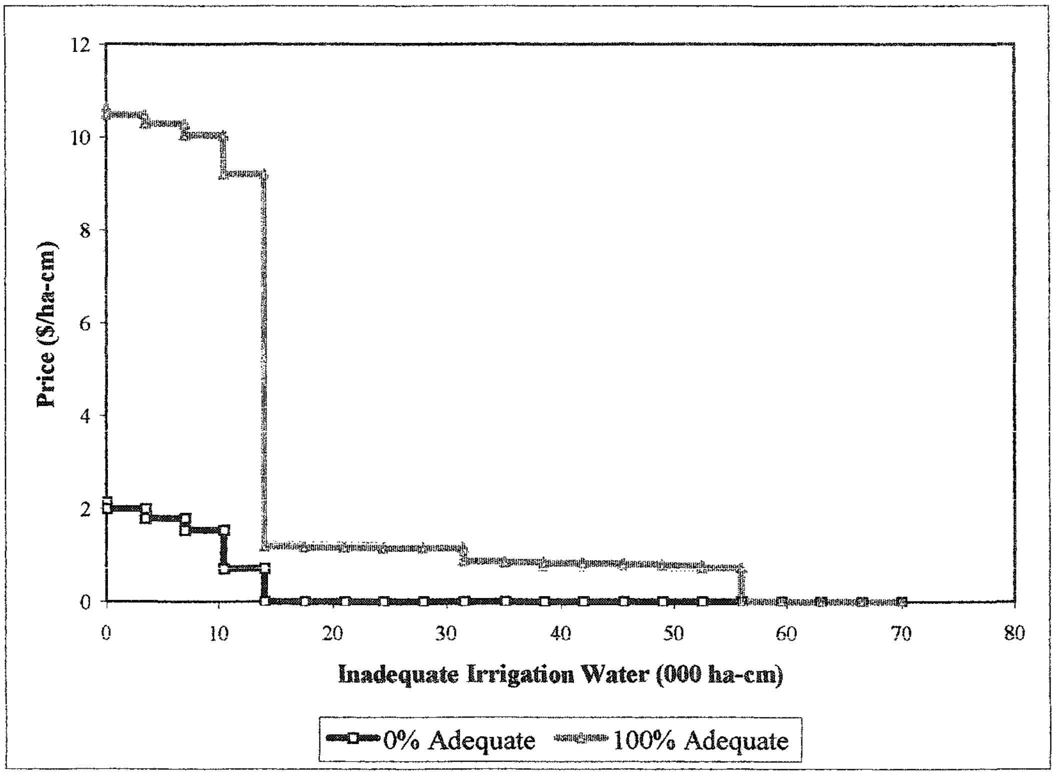


Figure 5.15 Conditional Stochastic Irrigation Water Demands for Normal State of Nature of SSM and Inadequate State of Nature of Effective Irrigation

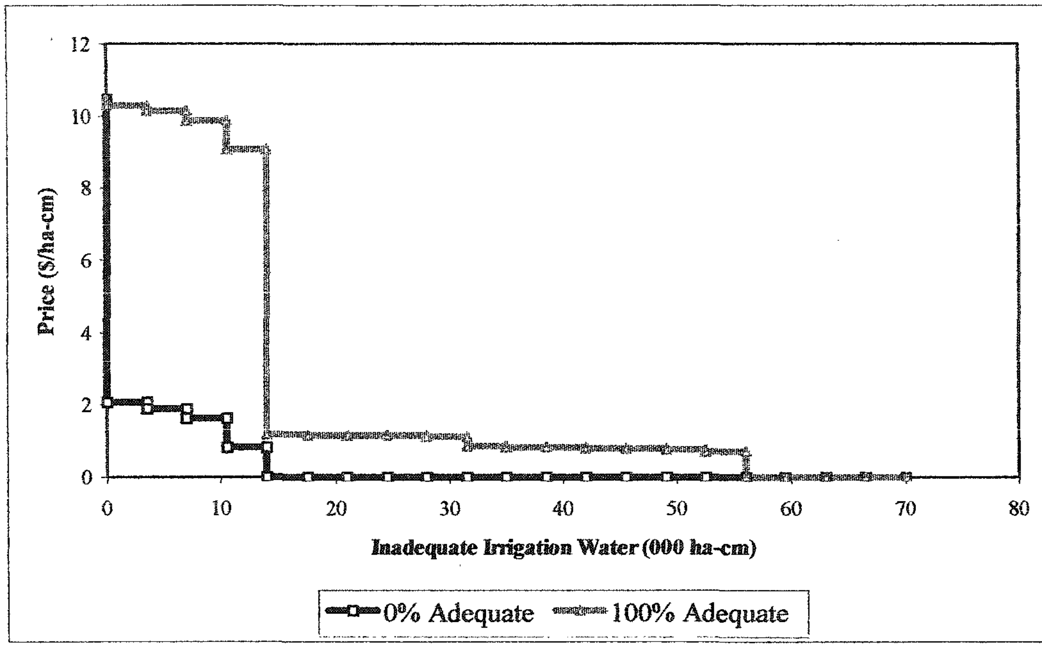
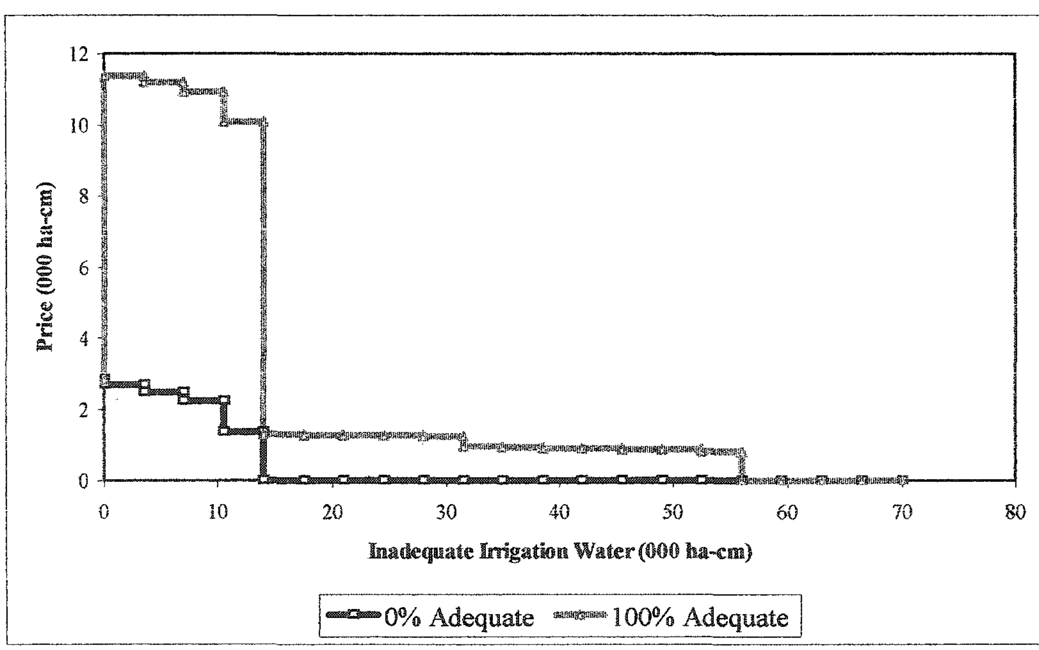


Figure 5.16 Conditional Stochastic Irrigation Water Demands for Wet State of Nature of SSM and Inadequate State of Nature of Effective Irrigation



The value of inadequate supplies is higher in the case of wet conditions of spring soil moisture than when it is dry or normal. About 10,000 hectare-centimeters of inadequate water supplies are not used in the production process when 100 percent of adequate water delivery is expected. However, more than 75 percent of inadequate water remains unused when zero percent of adequate water delivery is expected. For the first 17,500 hectare- centimeters of inadequate water supplies, the value of water is higher when the expected adequate supplies are at 100 percent relative to when it is at zero percent. This complements the situation presented in the case of adequate water supplies conditional on availability of inadequate deliveries. Water is valued higher in either of the states of nature of effective irrigation when water deliveries in the opposite state of nature are expected to be abundant or more certain.

5.6 Risk Penalty

By definition, the vertical distance between the conditional stochastic and deterministic demand schedules measures the amount of penalty attributed to the employment of water as a stochastic input. Equivalently, it is a measure of the payment required in order to make a decision maker indifferent between certain and uncertain deliveries of irrigation water. The price of the deterministic irrigation water deliveries is readily available from the deterministic demand schedule in Figure 5.4.

The optimum CKPP strategy is a combination of the solutions to the three sub-models corresponding to the spring soil moisture states of nature. The expected values of stochastic deliveries of irrigation water are obtained by prorating the values of water by the probabilities of occurrence of adequate and inadequate states of nature of effective irrigation. Thus, the risk penalty is the difference between the value of certain water delivery and the expected value of stochastic deliveries. The values of risk penalties in Table 5.4 are taken at 5 percent intervals of effective irrigation deliveries to the irrigation district, and calculated at equal levels of adequate and inadequate irrigation water set at 0, 25, 50, 75, and 100 percent of the available quantities i.e. conditional stochastic demands.

Table 5.4 Risk Penalties of Using Irrigation Water As A Stochastic Input ($/ha-cm)

Irrigation Water Delivered (%)	Percentages of Adequate and Inadequate Irrigation Water at Which Conditional Stochastic Demands are Derived				
	0%	25%	50%	75%	100%
5	52.57	29.63	29.63	29.63	29.63
10	53.96	31.01	31.01	31.01	31.01
15	55.80	32.83	32.83	32.83	32.83
20	59.34	36.37	36.37	36.37	36.37
25	70.77	52.91	52.18	52.18	52.18
30	13.84	13.84	10.69	10.69	10.69
35	13.89	13.89	10.75	10.75	10.75
40	13.94	13.94	10.82	10.82	10.82
45	14.00	14.00	11.60	10.91	10.91
50	14.10	14.10	11.74	11.20	11.20
55	14.41	14.41	12.14	11.60	11.60
60	10.53	10.53	10.53	8.16	8.16
65	10.67	10.67	10.67	8.85	8.34
70	10.86	10.86	10.86	9.07	8.58
75	11.11	11.11	11.11	9.41	8.92
80	11.36	11.36	11.36	9.68	9.23
85	9.37	9.37	9.37	7.73	7.73
90	9.58	9.58	9.58	9.56	7.97
95	9.99	9.99	9.99	9.99	8.49

The results from the zero percent irrigation water availability conditional stochastic demand are for the case where all the water is assumed withdrawn from the EID and transferred to other users. This scenario is for an extreme case and the resulting risk penalties are relatively larger for the last 25 percent of water transferred away from the district. The results from the other four conditional stochastic demands appear similar in the values of risk penalties across the different ranges of water quantities. These risk penalties for the interval 5 to 25 percent of the quantity of irrigation water delivered to the EID average $40.85 per ha-cm ($503/ac-ft). However, for the remaining 30 to 95 percent of water deliveries the risk penalties average $10.83 per ha-cm ($133/ac-ft).

Risk penalties provided a measure of the amount of risk associated with using irrigation water as a stochastic input in the production process of different crops. In order to gauge the effect of water transfer from the EID on the producers' well being, estimates of stochastic value of water are estimated that should explain how much the producer is willing to accept to sell a unit of irrigation water or how much the producer is willing to pay for an extra unit of water to be employed in the crop production process.

5.7 Value of Stochastic Irrigation Water Deliveries

Producer welfare change estimates are obtained by comparing the values of the objective function under different scenarios of irrigation water availability. The argument is that allowing the transfer of water rights and detaching the water permits from land parcels will induce the movement of water from its traditional marginal use in agriculture to more profitable utilizations such as urban, industrial and commercial demands. Then, traditional farming in southern Alberta may face the situation of severe irrigation water shortages that will force the adoption of decisions that were not considered before by crop producers. On the other hand, the existence of water markets may encourage crop producers to sell their water licenses to the highest bidder. The value of such irrigation water that is characterized by stochastic deliveries from the producer's perspective is what the next section attempts to estimate.

The estimate of the residual value imputed to water is the area under the water demand curve, or equivalently the value of the objective function, and is estimated under different water shortage regimes to examine the change in value under different schemes of water transfer. The difference between this water value and the value of water in its new uses will determine the net welfare effect from society's perspective. From the farmer's point of view, the welfare change depends on the amount the producer receives for trading the water right in relation to the size of the foregone benefit caused by water transfer from crop production. Since there is a risk component attached to the use of water in the crop production process, estimates of non-stochastic and stochastic foregone benefits are obtained in order to analyze the difference in magnitude of value under both circumstances. Average values of irrigation water are calculated as the value of the objective function divided by the quantity of water employed.

In an environment characterized by certainty, average values of water range from \$132 per ha-cm (\$1632/ac-ft) to \$47 per ha-cm (\$576/ac-ft) for zero and 100 percent water delivery regimes, respectively (Table 5.5). Four crops are grown when all water requirements are met. The first 20 percent of water transfer results in spring wheat exiting the optimal solution with a slight increase in the value of water. Alfalfa is next in departing the optimal crop mix while 45 percent of water is withdrawn from the EID. At the 75 percent mark of water transfer, canola is no longer in the product mix and average value of water reaches its maximum. For the last 25 percent of water quantity, only potato is produced with decreasing hectares as water availability declines. However, average value of water stays constant at the maximum level.

If risk and uncertainty are accounted for, then the average values of water are expected to be lower than the values obtained from the deterministic model. This is due to the risk penalty the stochastic model recognizes and eliminates in order to produce the estimates of stochastic demands and hence smaller values for the objective function. Average values of stochastic water deliveries range from \$128 per ha-cm (\$1579/ac-ft) to \$38 per ha-cm (\$467/ac-ft) for zero and 100 percent water delivery regimes, respectively (Table 5.6).

Table 5.5 Deterministic Forecast of Optimal Crop Mix, Net Returns, and Value of Water

Percentage of Irrigation Water Deliveries	Optimal Hectares of								Value of Objective Function ($million)	Value of Water ($/ha-cm)
	Spring wheat	Soft wheat	Barley	Canola	Alfalfa	Pasture	Potato	Peas		
5	0	0	0	0	0	0	123	0	0.46	132.34
10	0	0	0	0	0	0	246	0	0.93	132.34
15	0	0	0	0	0	0	368	0	1.39	132.34
20	0	0	0	0	0	0	491	0	1.85	132.34
25	0	0	0	0	0	0	614	0	2.32	132.34
30	0	0	0	123	0	0	618	0	2.40	114.45
35	0	0	0	250	0	0	618	0	2.48	101.11
40	0	0	0	377	0	0	618	0	2.55	91.10
45	0	0	0	505	0	0	618	0	2.62	83.32
50	0	0	0	632	0	0	618	0	2.70	77.10
55	0	0	0	759	0	0	618	0	2.77	72.01
60	0	0	0	816	68	0	618	0	2.84	67.56
65	0	0	0	816	191	0	618	0	2.90	63.64
70	0	0	0	816	313	0	618	0	2.95	60.29
75	0	0	0	816	436	0	618	0	3.01	57.38
80	0	0	0	816	559	0	618	0	3.07	54.84
85	66	0	0	816	618	0	618	0	3.13	52.53
90	193	0	0	816	618	0	618	0	3.18	50.41
95	320	0	0	816	618	0	618	0	3.23	48.52

Table 5.6 Stochastic Forecast of Optimal Crop Mix, Net Returns, and Value of Water *

Percentage of Irrigation Water Deliveries	Optimal Hectares of								Value of Objective Function ($million)	Value of Water ($/ha-cm)
	Spring wheat	Soft wheat	Barley	Canola	Alfalfa	Pasture	Potato	Peas		
5	0	0	0	0	0	0	140	0	0.45	128.01
10	0	0	0	0	0	0	280	0	0.89	127.03
15	0	0	0	0	0	0	420	0	1.32	125.87
20	0	0	0	0	0	0	560	0	1.74	124.30
25	0	0	0	0	27	0	617	0	2.12	121.03
30	0	0	0	123	36	0	618	0	2.17	103.38
35	0	0	0	250	44	0	618	0	2.21	90.34
40	0	0	0	377	52	0	618	0	2.26	80.54
45	0	0	0	505	59	0	618	0	2.30	72.91
50	0	0	0	632	59	0	618	0	2.34	66.80
55	0	0	0	759	59	0	618	0	2.38	61.77
60	0	0	0	816	104	0	618	0	2.41	57.50
65	0	0	0	816	191	0	618	0	2.45	53.84
70	0	0	0	816	313	0	618	0	2.48	50.68
75	0	0	0	816	436	0	618	0	2.52	47.92
80	73	0	0	816	489	0	618	0	2.55	45.48
85	164	0	0	816	523	0	618	0	2.58	43.31
90	225	45	0	816	544	0	618	0	2.61	41.37
95	277	89	0	816	574	0	618	0	2.63	39.60

*Values of objective function are taken at equal water deliveries for adequate and inadequate water constraints.

Unlike the deterministic model, the stochastic model diversifies production of wheat between spring and soft wheat and introduces barley into the optimal product mix in insignificant amounts at high levels of irrigation water availability. In addition, the stochastic model spreads the production of alfalfa over more units of water- 75 percent of water deliveries, than the deterministic model does- 40 percent of water deliveries (Figures 5.17 and 5.18). Although potato production in both models is identical in area when water is sufficiently available, the stochastic solution suggests more hectares planted when water is in short supply than the deterministic model.

Figure 5.17 Deterministic Optimal Hectares

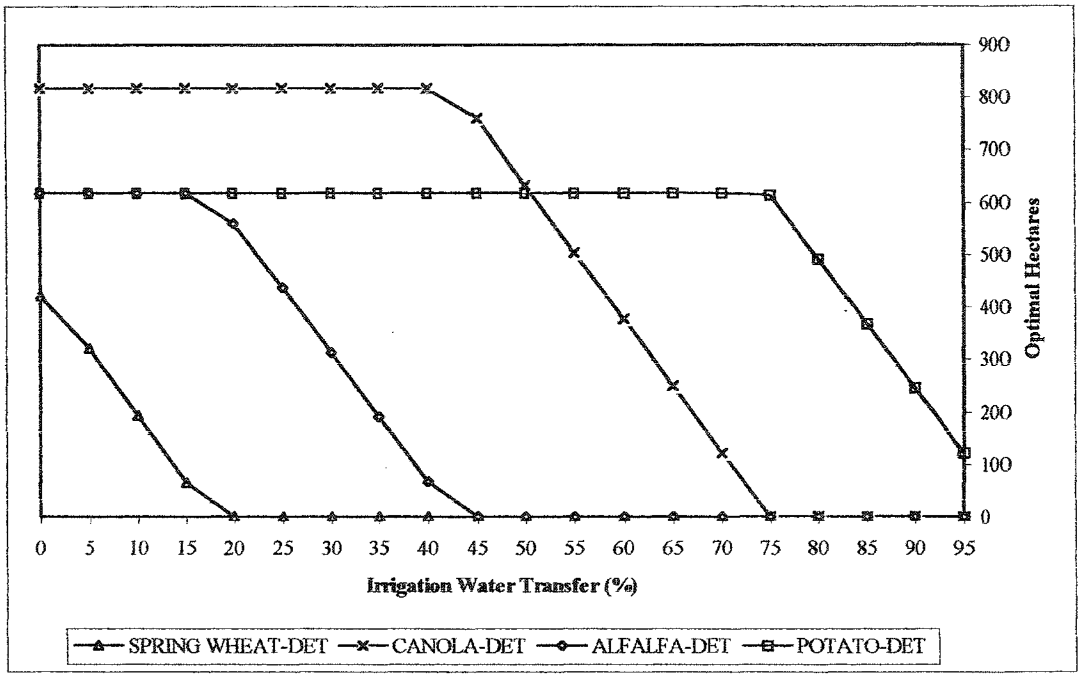
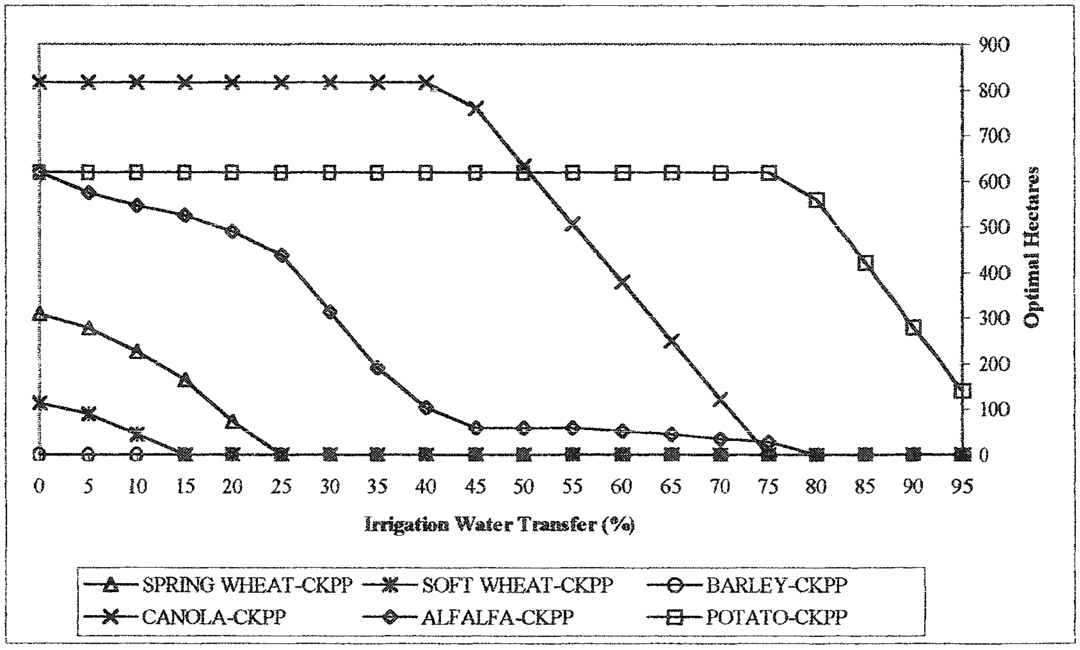


Figure 5.18 Stochastic Optimal Hectares



Diversification of the crop mix obtained from the solution to the stochastic model accounts for the risk and uncertainty associated with the crop production process. The trading of water rights may introduce shortages in irrigation water supply while producers have the opportunity to sell their water rights to the highest bidder. The estimates of average values of stochastic irrigation water deliveries provide an estimate of what the producers are willing to accept for selling their water rights. The positive net effect on farmers' welfare depends on the realization of a market mechanism that will produce a trading value for water licenses that will offset the loss in welfare. If the market fails to achieve a fair value for traded water rights, then crop producers will incur a welfare loss measured by the difference between the water value and market price.

5.8 Sensitivity Analysis

The quantitative characteristics of the empirical model are exhibited by the model's ability to produce valid results. The test for the model's qualitative characteristics is accomplished through sensitivity analysis. The above results can be examined in terms of their sensitivity to relaxation of some assumptions or changes in the values of some variables or parameters and hence test the model for its robustness and stability. The CKPP is utilized to model risk and uncertainty components associated with crop production in southern Alberta. Uncertainty and risk sources are identified as output price, effective irrigation, effective precipitation, and spring soil moisture. Sensitivity analysis examines the effects of changes in the measures of these risk components on the performance of the model in producing valid results as those presented above.

The risk aversion parameter is estimated based on the results generated by the CKPP model. Thus, changes in the values of the risk aversion coefficient are tested for their effect on the results reported. In addition, the potato rotation constraint is examined by further constraining the land allocated to potato to its sample level. Note that all the following sensitivity analyses are performed with 100 percent availability of irrigation water in the adequate and inadequate states of nature of effective irrigation including the base case to which the results are compared. Data pertaining to probabilities and quantities of spring soil moisture and effective precipitation are extracted from UMA (1982).

5.8.1 Risk Aversion Parameter

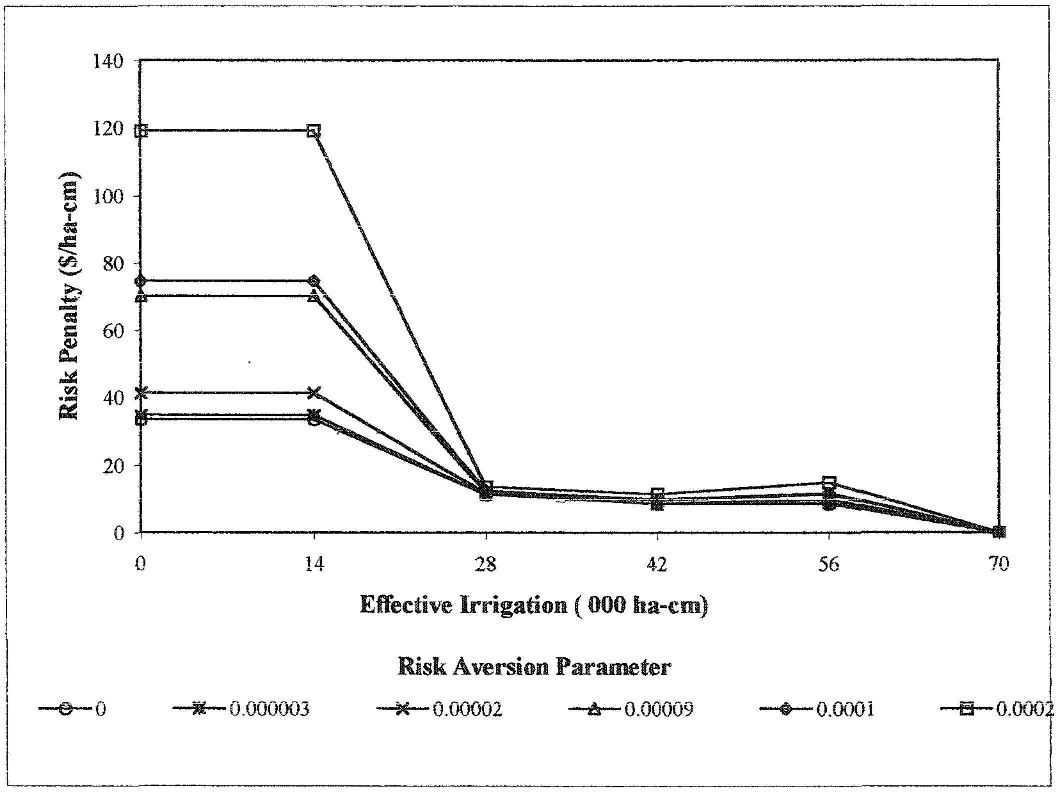
In the preceding sections, it was shown that total optimal hectares, and hence their distribution among individual crops, change as the value of the risk aversion parameter is varied. In this section, the effects of changing the value of risk aversion parameter on risk penalties and value of stochastic irrigation water deliveries are examined.

In general, significant risk penalties are observed with lower availability of irrigation water when changing the value of the risk aversion coefficient as shown in Figure 5.19. When the value of the risk aversion parameter is between risk neutral and λ=0.0002, risk penalties are positive in value indicating that irrigation water is employed as a risk increasing (stochastic) input. Therefore, in order to make the producers indifferent between certain and stochastic water deliveries, they are to be compensated by the amount of the risk penalty. Overall, the results of the risk penalty analysis are sensitive to changes in the value of the risk aversion parameter such that, as producer's preferences move away from risk neutrality toward becoming more risk averse, risk penalties consequently increase. The magnitude of risk penalties in relation to variations in the value of risk aversion parameters is significant when irrigation water is scarce and below 25 percent of normal deliveries. Otherwise, risk penalties are relatively small when water is considered abundantly available especially when shortages are less than 25 percent of regular amounts of irrigation water deliveries.

The values of stochastic irrigation water deliveries tend to decrease as the value of risk aversion parameter increases from zero to 0.0002 (Figure 5.20). The average values range from $40.17 per ha-cm ($495.49/ac-ft) when producers are risk neutral to

$21.96 per ha-cm ($270.87/ac-ft) when λ=0.0002. Note that optimal hectares decrease, as the producers become more risk averse.

Figure 5.19 Risk Aversion Parameter and Risk Penalties



5.8.2 Output Price

Sensitivity of the CKPP model to changes in the price of output is accomplished by employing several scenarios of price change ranging between ± 50 percent of the expected price of spring wheat, soft wheat, barley, canola, alfalfa, and potato at 10 percent intervals. A second group of crops, including pasture and peas, responded to increases in their expected price ranging between 50 and 150 percent at increments of ten percent. Output price change affects the predicted composition of optimal crop mix as shown in Figures 5.21 to 5.28. In these figures, only crops responding to price changes are depicted while crops not affected by the price change are not shown.

Figure 5.20 Risk Aversion Parameter and The Value of Stochastic Irrigation Water Deliveries

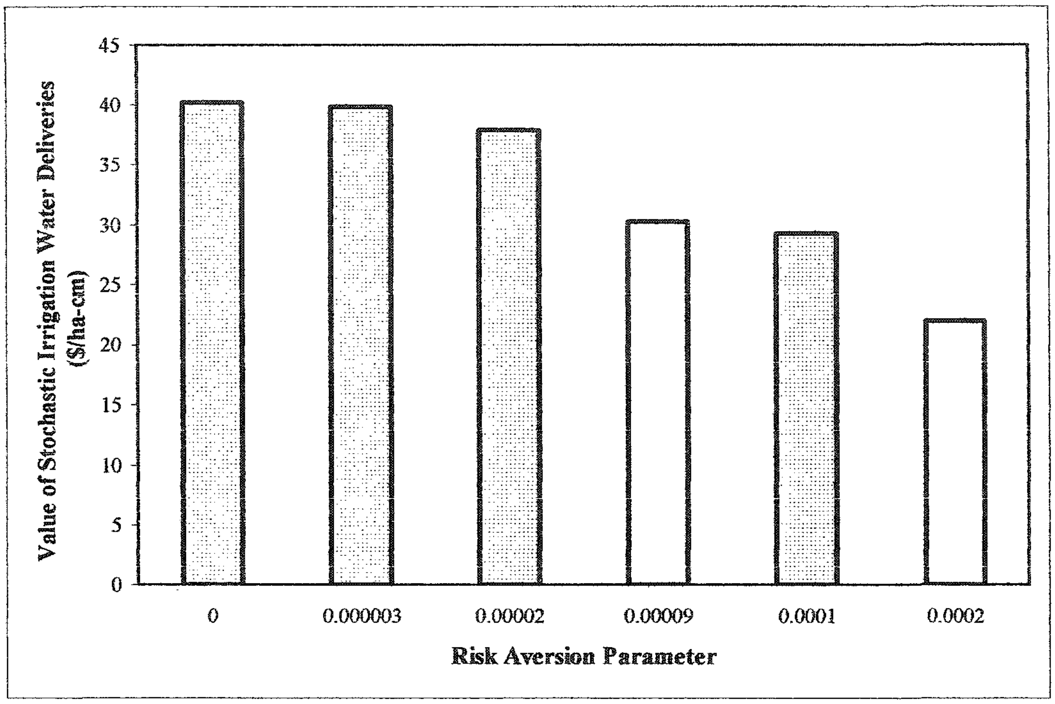
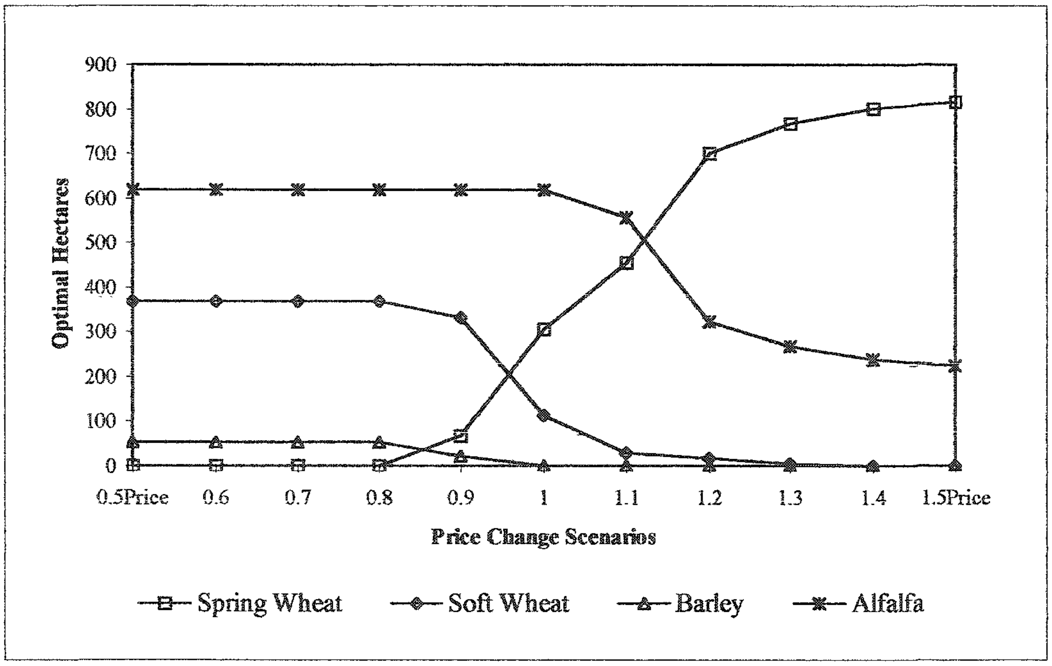


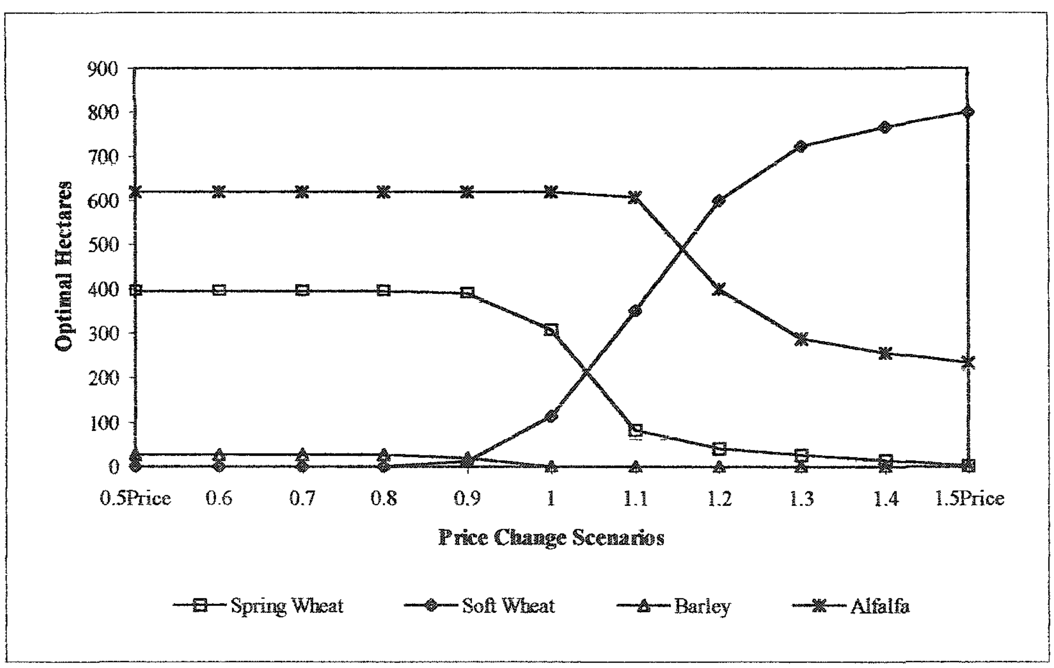
Figure 5.21 Changing Price of Spring Wheat and Optimal Hectares



A 20 percent reduction in the expected price of spring wheat results in the elimination of this crop from the optimal crop mix in favour of additional hectares allotted to barley and soft wheat. On the other hand, a 50 percent price increase of spring wheat will result in more than double its acreage at the expense of areas planted by soft wheat, barley, and alfalfa.

The change in the expected price of soft wheat has similar directional effects as those of spring wheat. However, the model predicts more barley hectares to be planted in the case of reducing the price of spring wheat than in the case of soft wheat price reduction. Alfalfa hectares are almost identical in both cases. However, a 50 percent increase in the price of soft wheat results in almost an eight-fold increase in its predicted optimal hectares.

Figure 5.22 Changing Price of Soft Wheat and Optimal Hectares

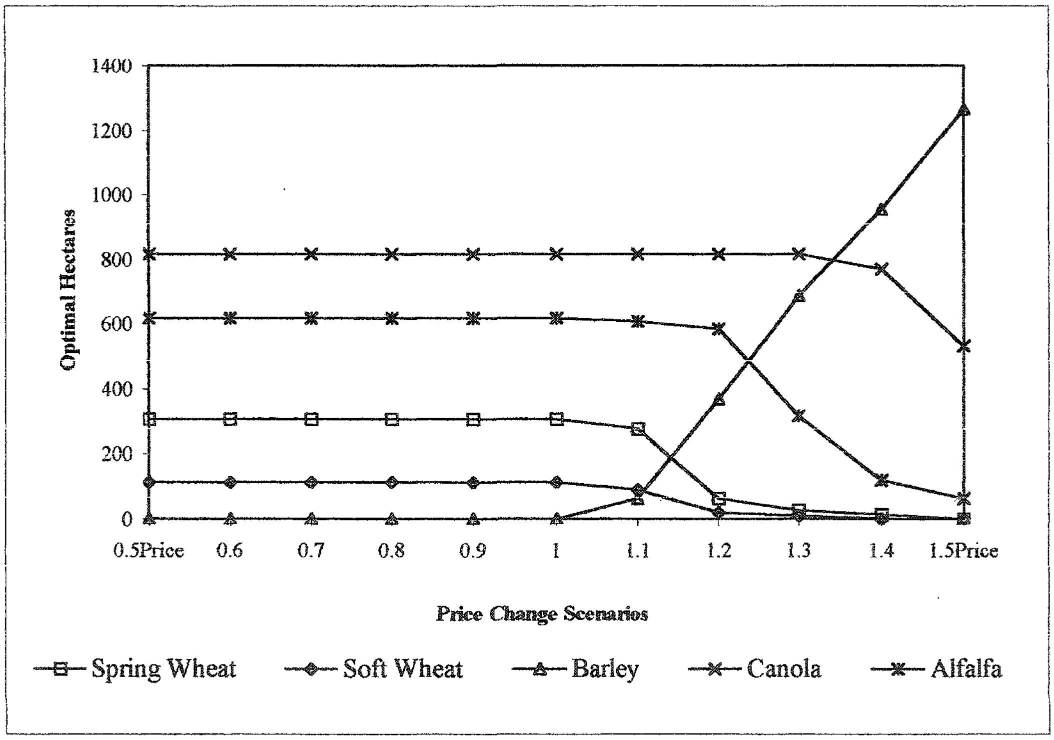


Since barley is not in the optimal crop mix, a reduction in its price does not change the optimal crop portfolio or the associated hectares. However, a 50 percent increase in the expected price of barley results in the elimination of both spring and soft wheat from the optimal crop mix. Simultaneously, alfalfa and canola hectares are reduced by

90 and 35 percent, respectively. Thus far, output price sensitivity reveals, and as expected, that traditional cereals seem to be close substitutes in the crop production process with barley having the highest potential of dominating the predicted crop mix once its expected market price shifts upwards.

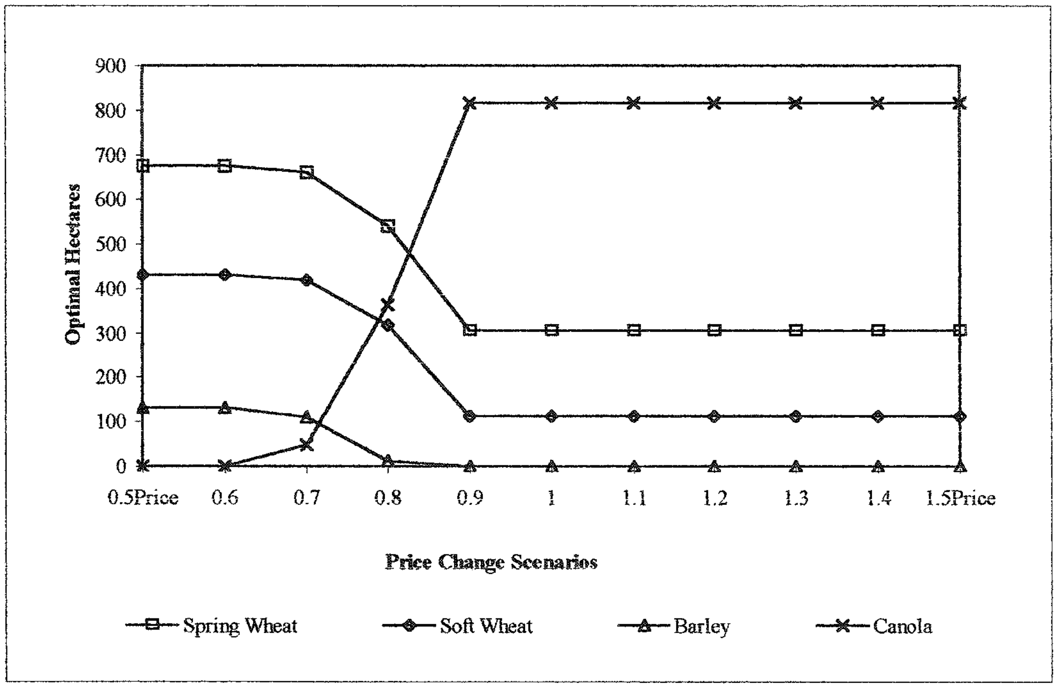
Increases in the price of canola do not change the optimal crop mix or the associated hectares since its agronomic constraint is fully satisfied. However, when the price of canola is reduced by 20 percent, the optimal crop mix starts to change by reducing the hectares of canola in favor of traditional cereals, wheat, and barley. Canola is eliminated from the optimal solution when its price is reduced by 40 percent with soft wheat as the major substitute.

Figure 5.23 Changing Price of Barley and Optimal Hectares



When the expected price of alfalfa is changed, the model gives predictions similar to those of canola case. The optimal crop mix does not change if the price of alfalfa is raised by up to 50 percent since its agronomic constraint is binding. However, when the price is dropped by 40 percent, alfalfa is discarded from the optimal solution and replaced by wheat and barley. The hectares of wheat and barley substituted for alfalfa are less in number than the hectares of the same two crops when substituted for canola when its price is changed. This may suggest that traditional cereals are closer substitutes in production to canola than to alfalfa.

Figure 5.24 Changing Price of Canola and Optimal Hectares



The price change threshold for pasture inclusion into the optimal crop mix is 70 percent above the expected market price. At that point, barley is eliminated and substituted for by pasture. Further increments in the price of pasture result in a reduction of alfalfa and traditional cereals. Pasture is typically the crop that is usually first to be shifted to dryland farming in case crops are competing for irrigation water and is used as feed in the production of beef.

Figure 5.25 Changing Price of Alfalfa and Optimal Hectares

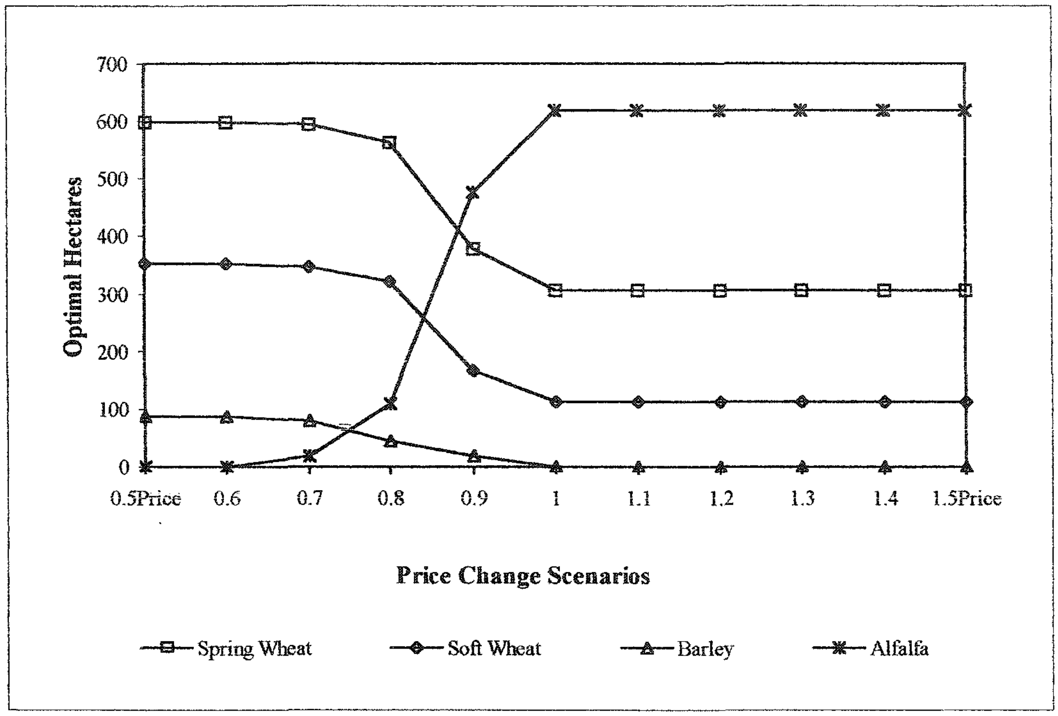
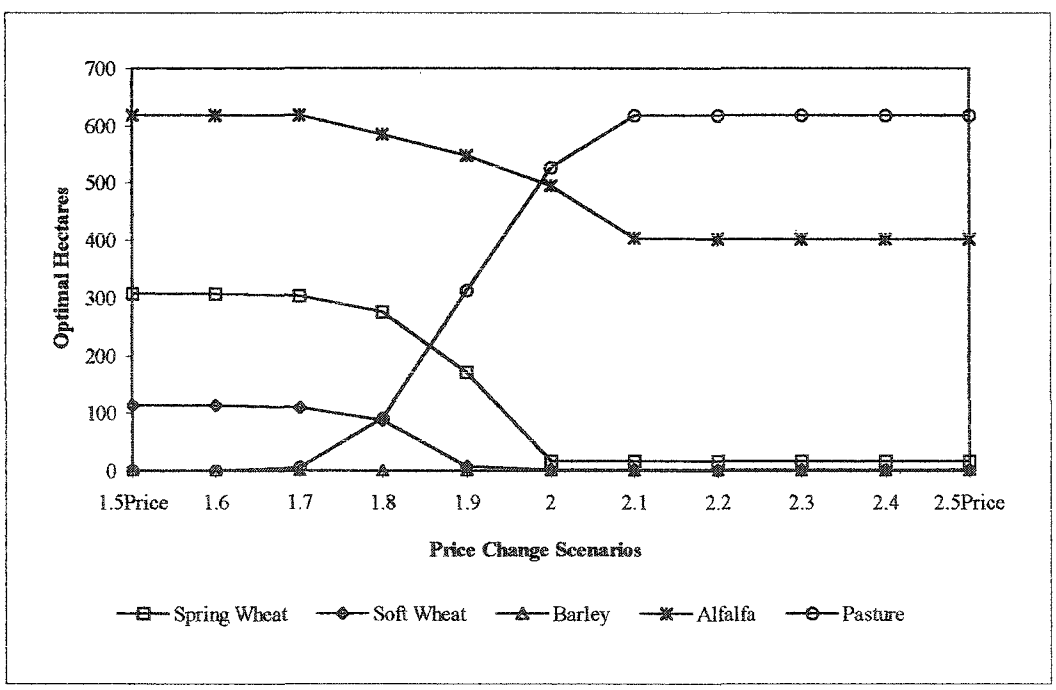
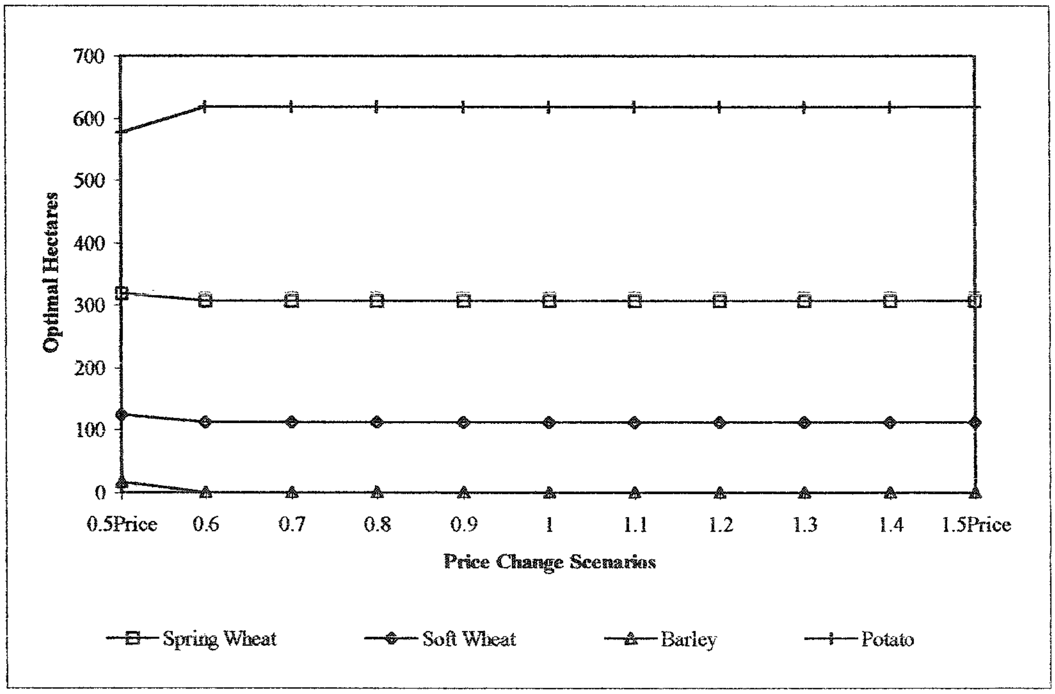


Figure 5.26 Changing Price of Pasture and Optimal Hectares



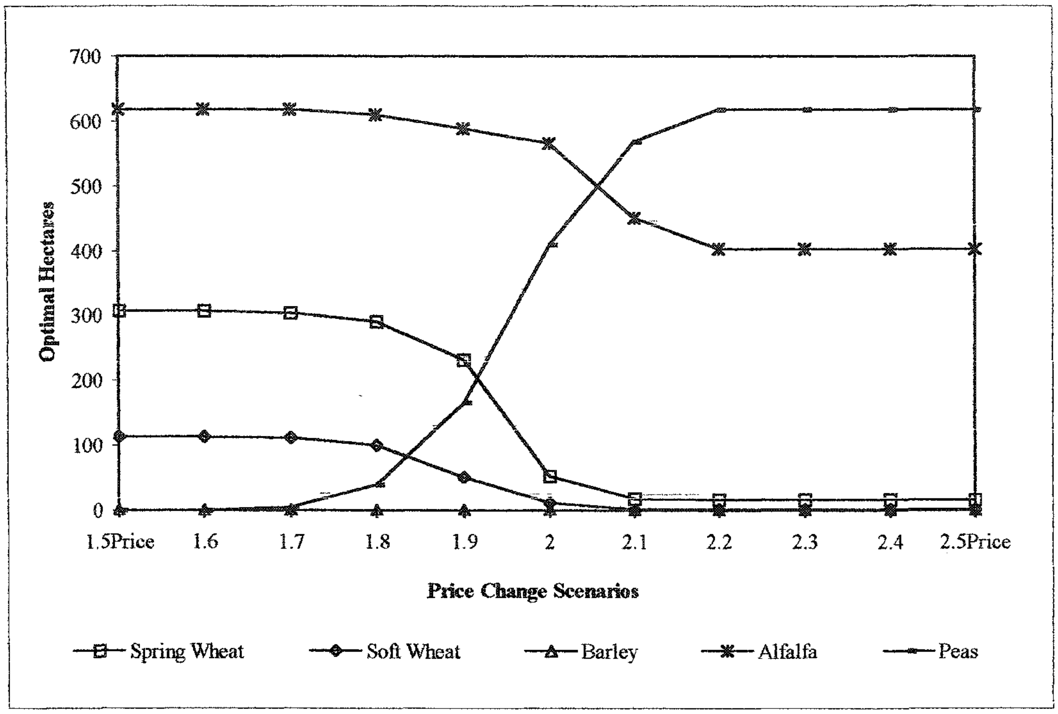
Potato is the dominant crop in the solution to the CKPP model evident by being the last crop to depart the primal solution when irrigation water is parametrically reduced, and by the fact that it is never replaced by any other crop in any of the price change scenarios of all crops. Optimal hectares of potato are not sensitive to the suggested price increases (agronomic constraint fully satisfied), and decline in number only when the expected market price is cut by 50 percent. At that point, traditional cereals substitute for the small number of potato hectares excluded from the optimal crop portfolio.

Figure 5.27 Changing Price of Potato and Optimal Hectares



A 70 percent increase in the expected price of peas insures its inclusion in the optimal crop mix at the expense of traditional cereals' hectares. Beyond that price level, alfalfa is also replaced by peas but at a smaller rate than cereals. In general, peas and pasture are reacting similarly to changes of their own prices.

Figure 5.28 Changing Price of Peas and Optimal Hectares



Turning to the effects of changing output prices on the size of value of stochastic irrigation water deliveries among the first group of crops, which includes spring wheat, soft wheat, barley, canola, alfalfa, and potato, changes in the price of potato are positively and proportionally related to the size of shift in the average value of water (Figure 5.29). Changes in the output price of the crops of the rest of this group have minimal effect on the size of the average value of water. Once again, potato proves to be the dominant crop in the model and the value of water results are highly sensitive to changes in its own expected market price.

The effects of pasture and peas price changes on the size of average value of stochastic irrigation water deliveries are minimal (Figure 5.30). Prices have to be raised by almost 80 percent before any change of average water value can be recorded. Evidently, these two crops are of minimal importance in the optimal crop portfolio of the sample farms.

Figure 5.29 Output Price Change and Average Value of Stochastic Irrigation Water Deliveries (Group One)

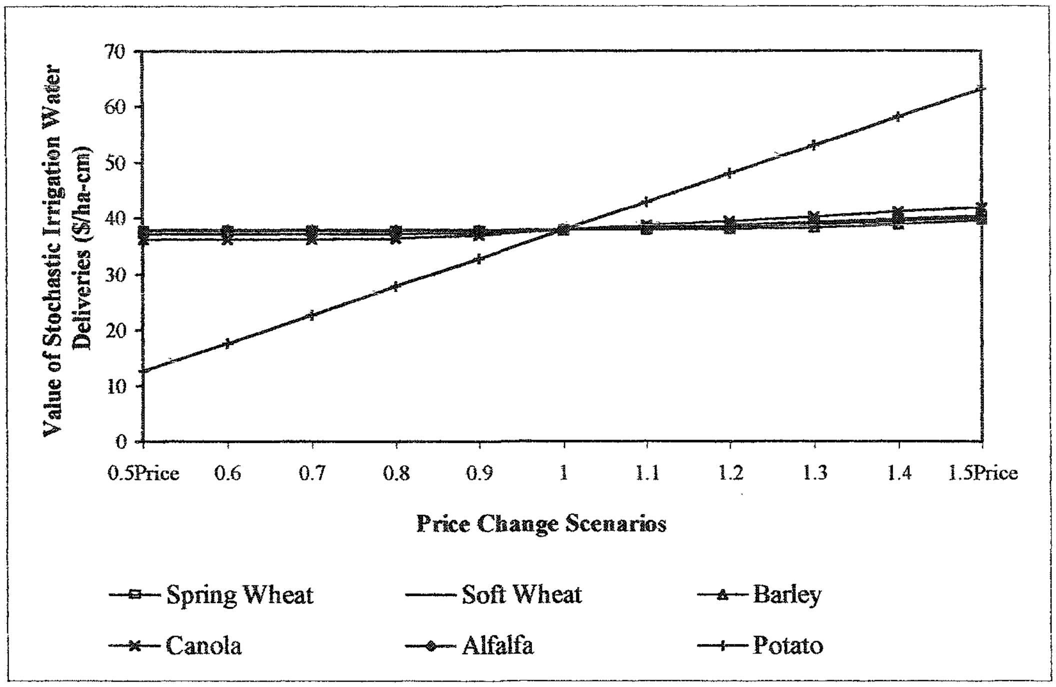
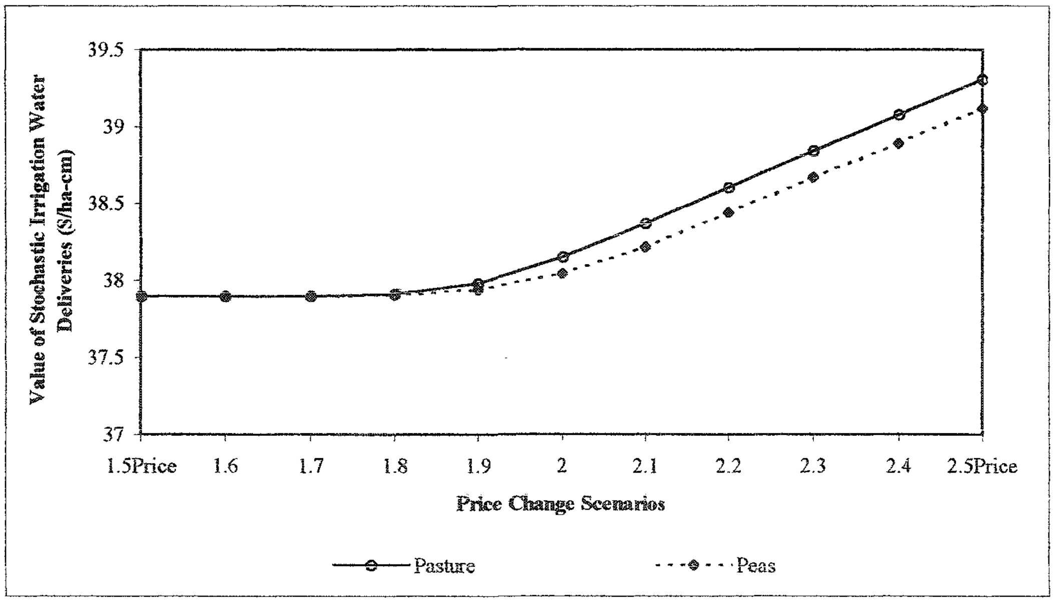


Figure 5.30 Output Price Change and Average Value of Stochastic Irrigation Water Deliveries (Group Two)



5.8.3 Effective Irrigation

Sensitivity analysis of effective irrigation is accomplished through nine scenarios that allow for changes in the probabilities associated with irrigation water deliveries in the adequate and inadequate states of nature as shown in Table 5.7.

Table 5.7 Probabilities Associated with Effective Irrigation Sensitivity Analysis Scenarios

Scenario	P1	P2	P3	P4	P5	P6	P7	P8	P9
Adequate	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
Inadequate	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

Optimal crop mix is insensitive to changes in the probabilities of the states of nature of effective irrigation. However, average value of stochastic irrigation water deliveries decline as the probability of adequate deliveries decreases as shown in Figure 5.31. The increased uncertainty of water deliveries results in a reduction in the value of water. However, the magnitude of such change is small in value since a drop in the probability of adequate water deliveries from 0.9 to 0.1 reduces average water value by only 6 percent.

5.8.4 Effective Precipitation

Effective precipitation is divided into three states of nature: high, average, and low. Crops are divided into two groups based on the length of the growing season: four and five-month growing season crops. Effective precipitation is therefore modeled as two independent seasons for two types of crops with three states of nature each. The scenarios for the sensitivity analysis of effective precipitation are combinations between two sets of probability distributions shown in Tables 5.8 and 5.9.

Figure 5.31 Average Value of Stochastic Irrigation Water Deliveries for Different Effective Irrigation Probability Scenarios

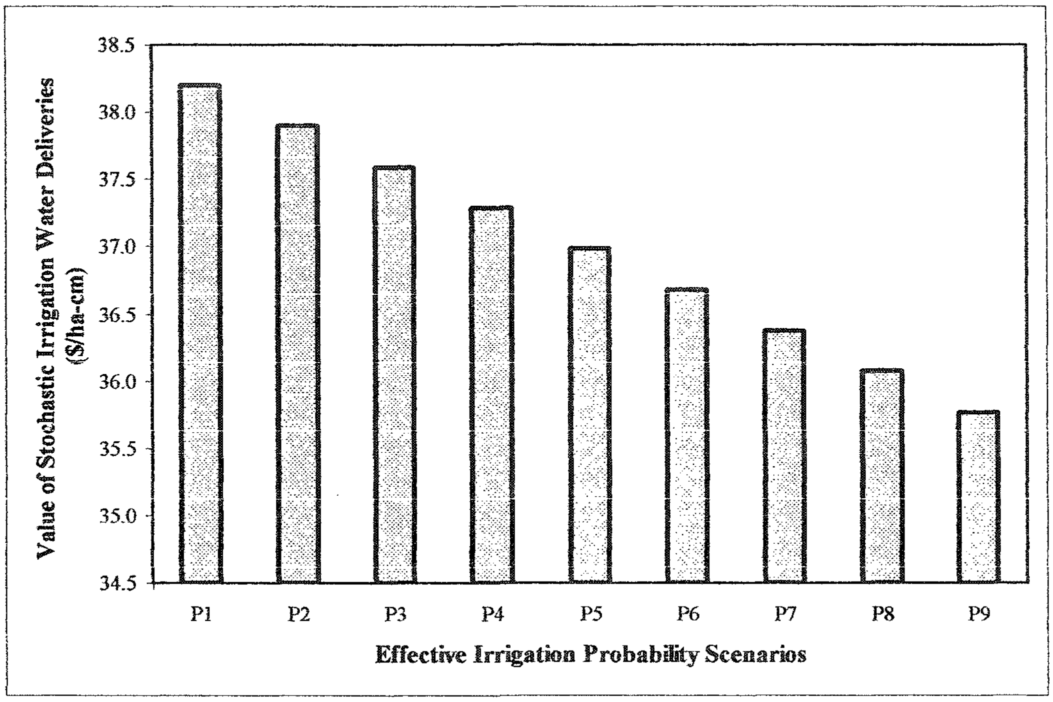


Table 5.8 Sensitivity Analysis Probability Distributions of Effective Precipitation for Four-Month Growing Season Crops.

States of Nature of Effective Precipitation	Distribution							
	A		B		C		D	
	cm	Probability	cm	Probability	cm	Probability	cm	Probability
High	25	0.04	25	0.13	25	0.332	25	0.04
Average	20	0.94	17.5	0.80	14.4	0.332	20	0.49
Low	5	0.02	7.5	0.07	10.1	0.336	12.5	0.47

Table 5.9 Sensitivity Analysis Probability Distributions of Effective Precipitation for Five-Month Growing Season Crops.

States of Nature of Effective Precipitation	Distribution							
	E		F		G		H	
	cm	Probability	cm	Probability	cm	Probability	cm	Probability
High	27.5	0.31	25.5	0.34	27.5	0.336	27.5	0.13
Average	17.5	0.67	15	0.50	17	0.331	20	0.31
Low	7.5	0.02	10	0.16	13.2	0.333	15	0.56

There are sixteen different scenarios for effective precipitation changes e.g. AE, AF, DG, DH. The base case scenario that is applied in this study is combination CG. Results depicted in Figure 5.32 show that only three scenarios yield average values of stochastic irrigation water deliveries less than the base case. It is plausible to conclude that changes in the probability distributions of the states of nature for effective precipitation lead to changes in the estimated average values of water. However, the direction of such a relationship is ambiguous. Relative to the results from the base case, there appears to be room for the average value of water to appreciate based on results of the applied scenarios. However, no specific cause or rule can be identified to detect such a change or explain it.

5.8.5 Spring Soil Moisture

Four scenarios are applied to simulate changes in the values of the states of nature of spring soil moisture: wet, normal, and dry. The proposed probability distributions are shown in Table 5.10. Scenario 3 represents the base case values used in this study where the probabilities of occurring of the three states of nature are close in value. Scenario 1 favors wet and normal conditions while Scenario 2 is skewed toward dry conditions. Scenario 4 emulates high probability of normal state of nature.

Figure 5.32 Average Value of Stochastic Irrigation Water Deliveries for Different Effective Precipitation Probability Scenarios.

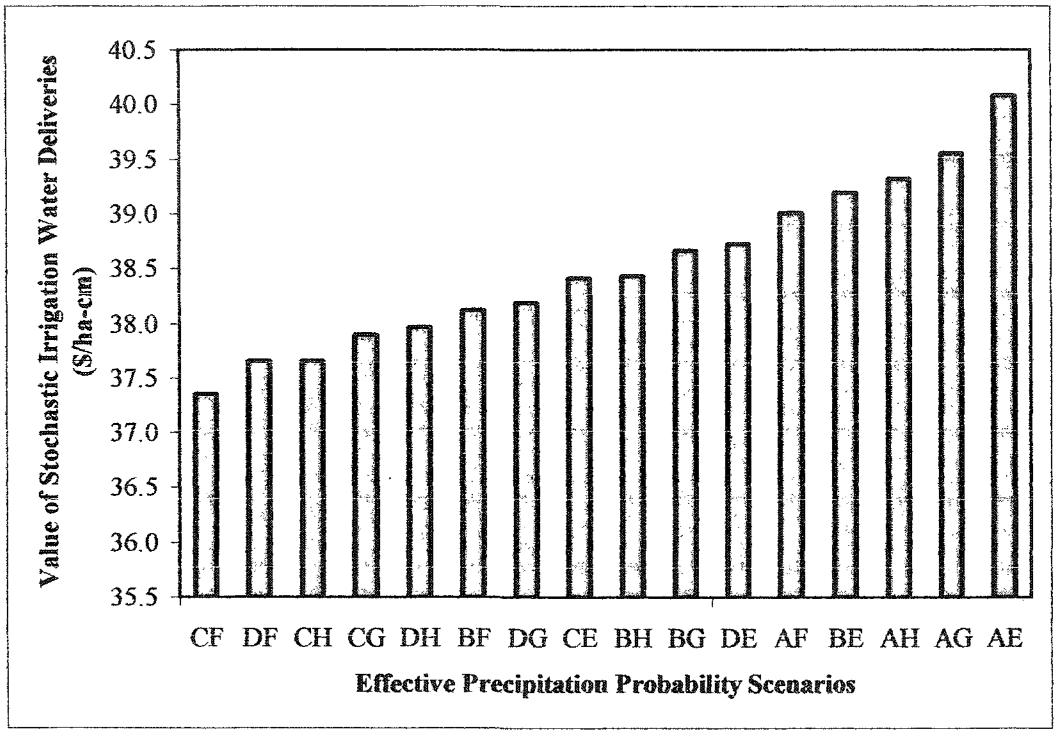
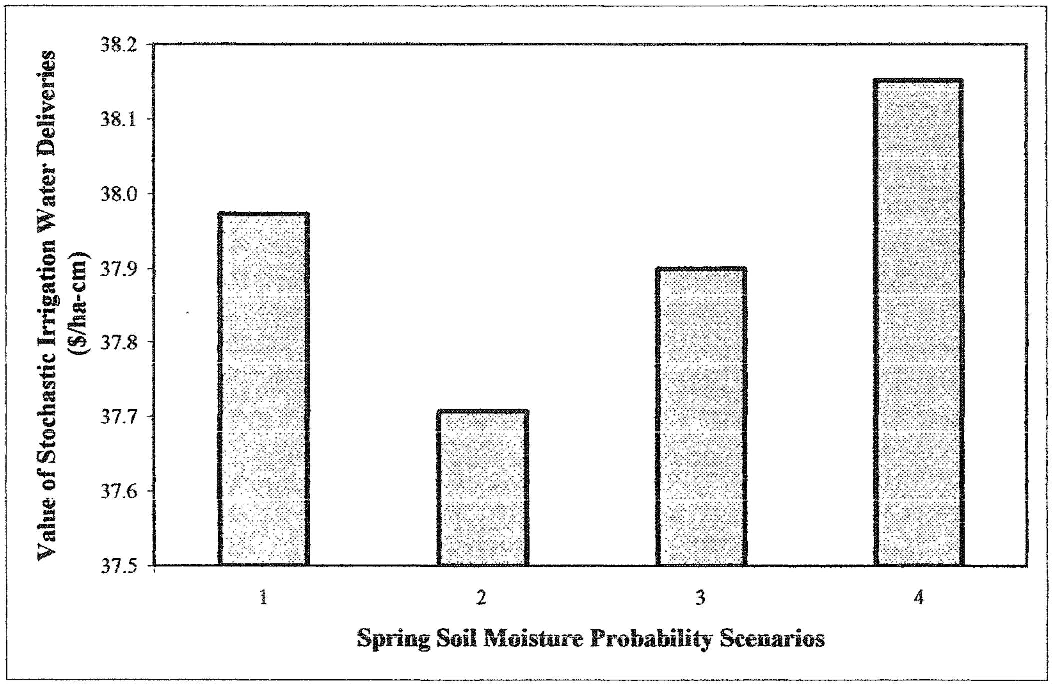


Table 5.10 Sensitivity Analysis Probability Distributions of Spring Soil Moisture

Spring Soil Moisture States of Nature	Scenario							
	1		2		3		4	
	cm	Probability	cm	Probability	cm	Probability	cm	Probability
Wet	11	0.49	11	0.02	11	0.342	11	0.02
Normal	8.5	0.44	9.5	0.47	8.7	0.324	9.5	0.91
Dry	7	0.07	8.5	0.51	8.1	0.334	7	0.07

The results in Figure 5.33 suggest an insignificant change in the average value of stochastic irrigation water deliveries attributed to changes in the probability distributions of spring soil moisture. Only Scenario 2, where probability of wet spring conditions is very low, yields average water values smaller than these of the base case scenario. The high probability of normal spring soil moisture in Scenario 4 provides the largest average values of water, which highlights the importance of planting conditions to the realization of optimal crop yields.

Figure 5.33 Average Value of Stochastic Irrigation Water Deliveries for Different Spring Soil Moisture Probability Scenarios



5.8.6 Constraining Potato Hectares

The market demand for potatoes in southern Alberta is limited. However, the hectares allocated to potato in the optimal crop mix from both the deterministic and stochastic models seem unreasonable in terms of overestimating potato's share of the optimal crop mix. This potentially may cause an upward bias in the estimates of the risk penalties and the values of irrigation water. Constraining the land allocated to the production of potatoes to the hectares of potatoes in the sample farms seems a reasonable exercise in order to estimate unbiased estimates by controlling the over contribution of potato to the model predictions.

By comparing the results from Tables 5.2 and 5.11, it is clear that the constrained models allocate the extra hectares taken from potato production to wheat and barley since their agronomic constraints are not fully satisfied. Thus, the constrained models provide better-diversified optimal crop mixes than the unconstrained ones.

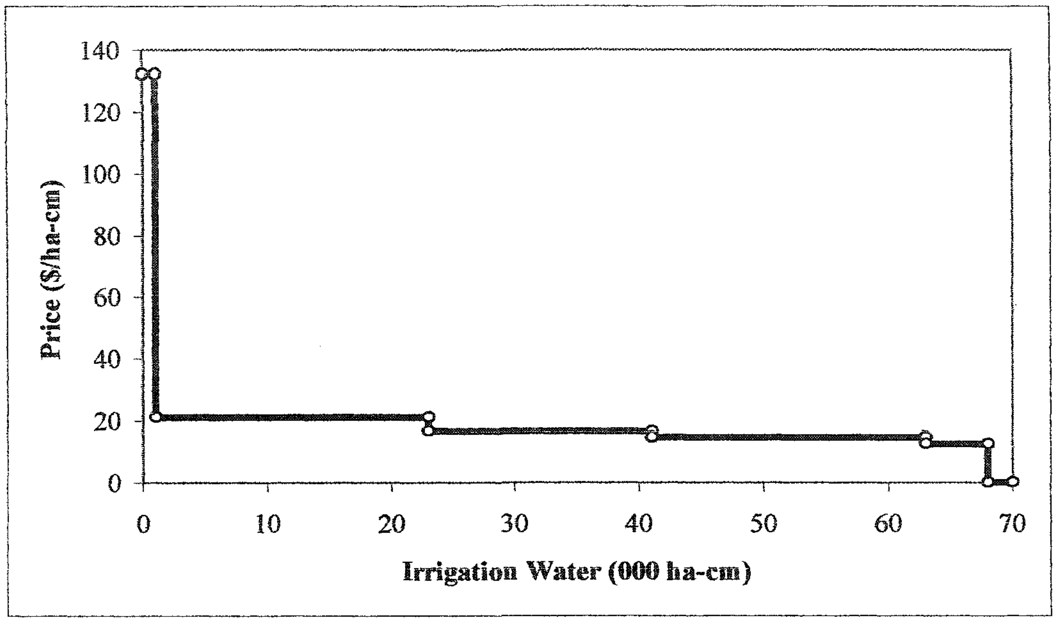
Table 5.11 Sample Hectares and Predictions from Models After Constraining Potato to Sample Hectares

Model	Spring wheat	Soft wheat	Barley	Canola	Alfalfa	Pasture	Potato	Peas	Total
CKPP	640	290	57	816	618	0	52	0	2473
Deterministic	787	200	0	816	618	0	52	0	2473
Sample	81	81	352	273	1015	560	52	59	2473

If the constrained deterministic and stochastic crop mixes are compared, the CKPP solution introduces barley to the crop mix and less spring wheat in favour of extra hectares of soft wheat. All models employ the total available land in the crop production and no hectares are left idle. Overall, the constrained CKPP seems to yield the most diversified crop mix.

By comparing the derived deterministic irrigation water demands in Figures 5.4 and 5.34, it becomes clear that restricting potato production to the sample hectares reduces the jump in water values when most of the irrigation water is withdrawn from the district. The high values for irrigation water are associated with only the last 5 percent of deliveries. When potato hectares are constrained, the average of the marginal values of the remaining 95 percent of the deterministic water deliveries is $ 16.83 per ha-cm ($ 207.58 per ac-ft).

Figure 5.34 Derived Deterministic Demand for Irrigation Water from The Constrained Linear Model



The stochastic irrigation water demands are derived from the three CKPP sub-models. Since the sub-models are restricted to employ only sample hectares of potatoes, the derived stochastic demands are different from those derived from unrestricted sub-models in terms of the absence of the distinct ridge for the last units of water deliveries employed in producing potatoes, and the wider plateau counting for 75 percent of water deliveries utilized in the production of the other crops. Figures 5.35 to 5.40 depict the stochastic irrigation water demand for dry, normal, and wet states of

Figure 5.35 Stochastic Irrigation Water Demand for Dry State of Nature of SSM and Adequate State of Nature of Effective Irrigation (Constrained Potato Hectares)

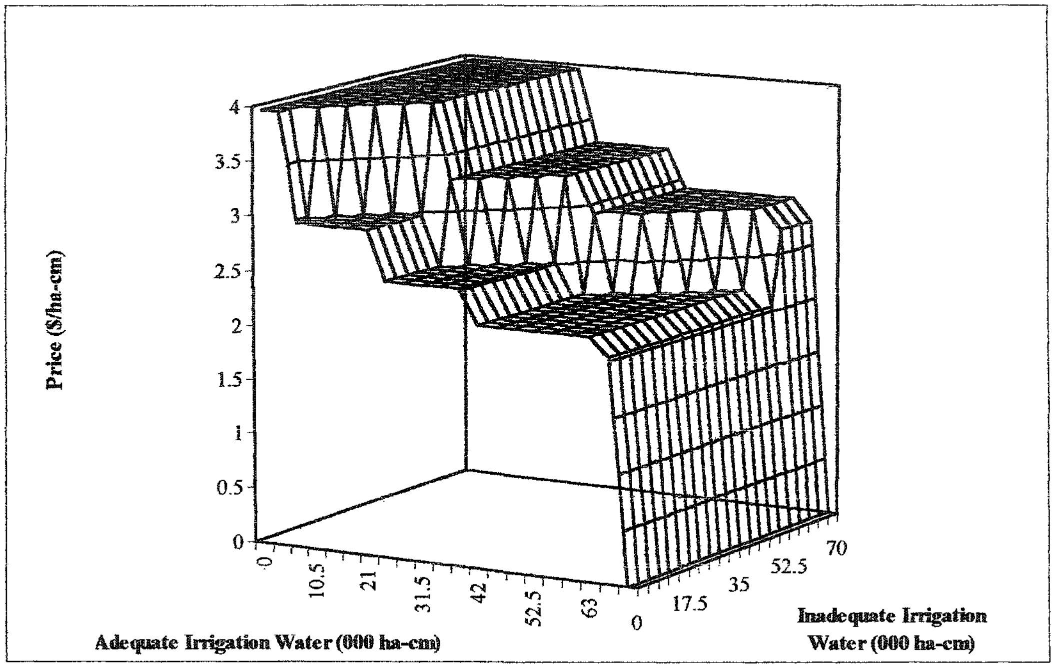


Figure 5.36 Stochastic Irrigation Water Demand for Dry State of Nature of SSM and Inadequate State of Nature of Effective Irrigation (Constrained Potato Hectares)

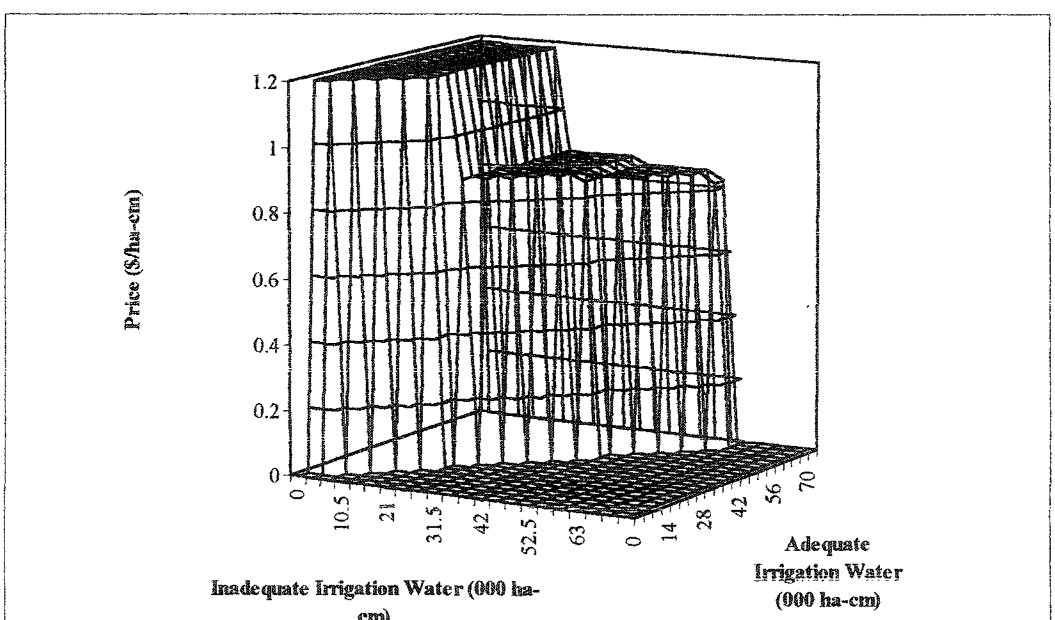


Figure 5.37 Stochastic Irrigation Water Demand for Normal State of Nature of SSM and Adequate State of Nature of Effective Irrigation (Constrained Potato Hectares)

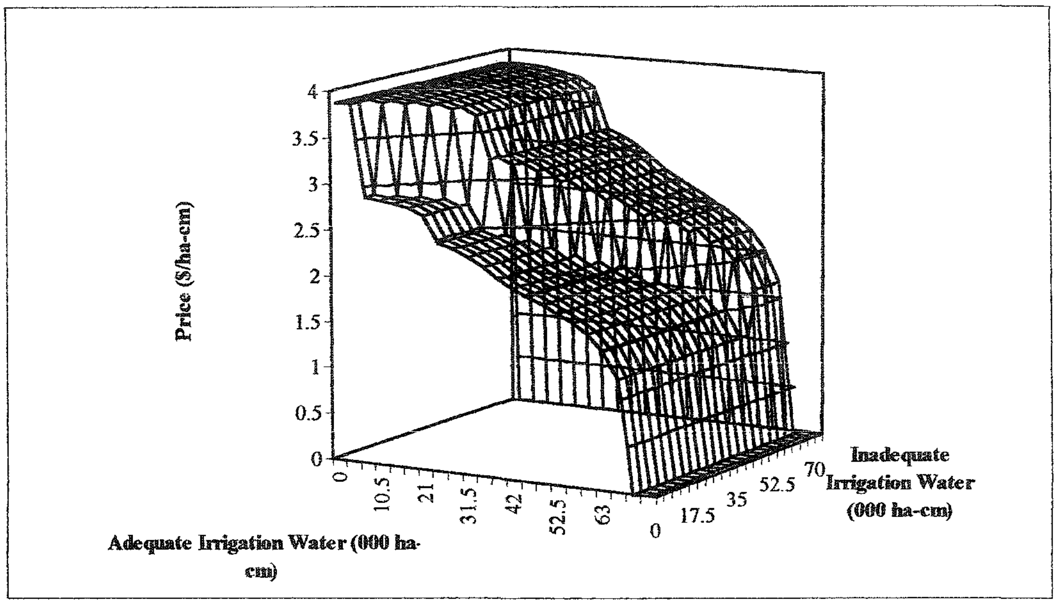


Figure 5.38 Stochastic Irrigation Water Demand for Normal State of Nature of SSM and Inadequate State of Nature of Effective Irrigation (Constrained Potato Hectares)

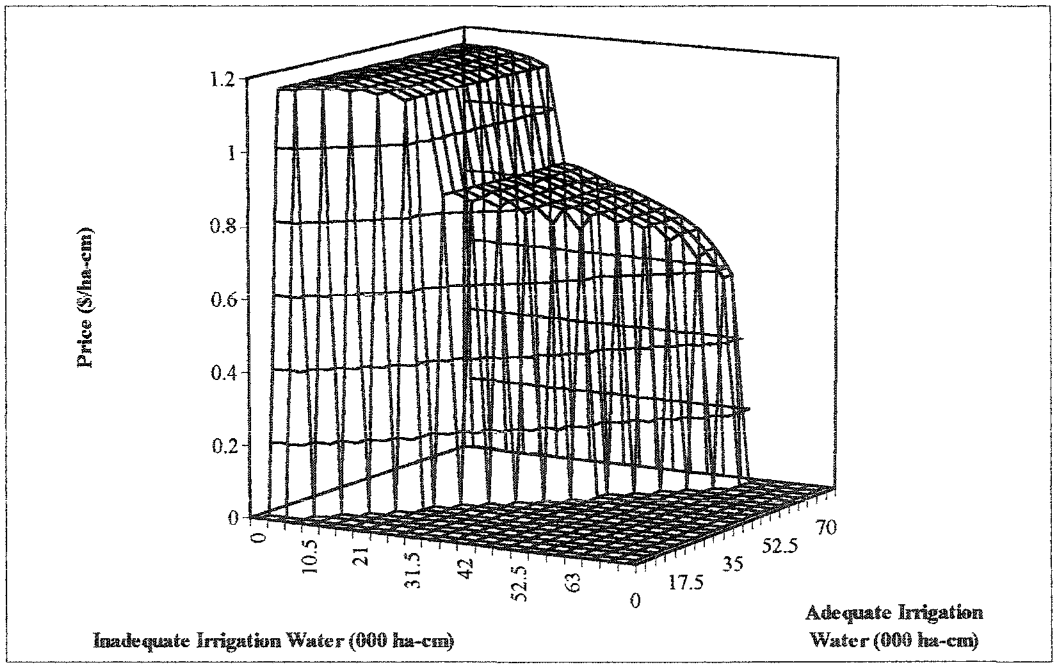


Figure 5.39 Stochastic Irrigation Water Demand for Wet State of Nature of SSM and Adequate State of Nature of Effective Irrigation (Constrained Potato Hectares)

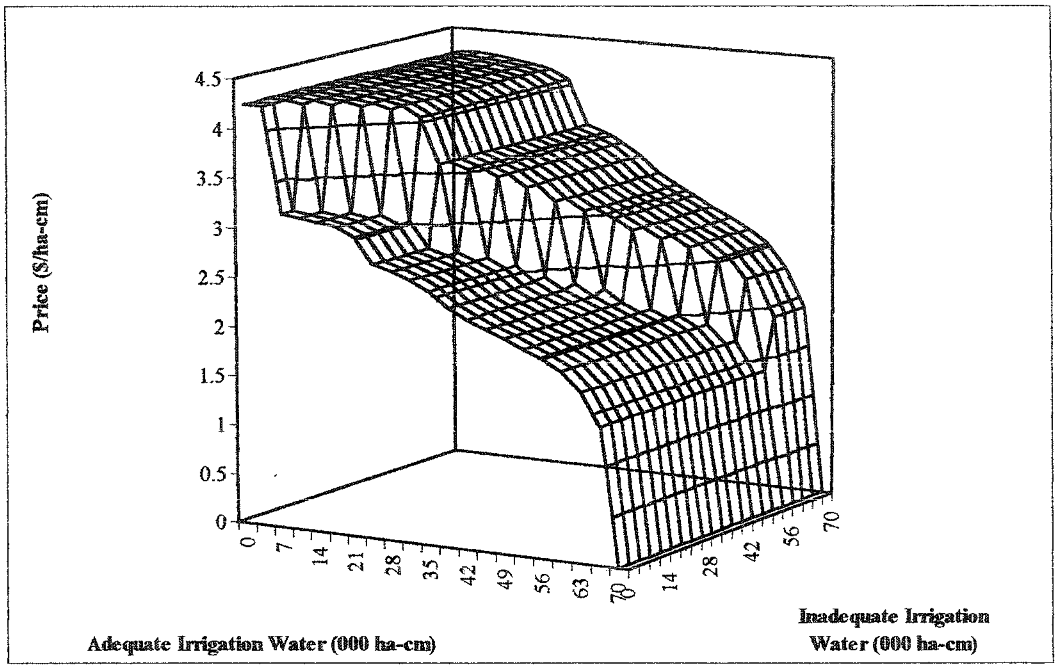
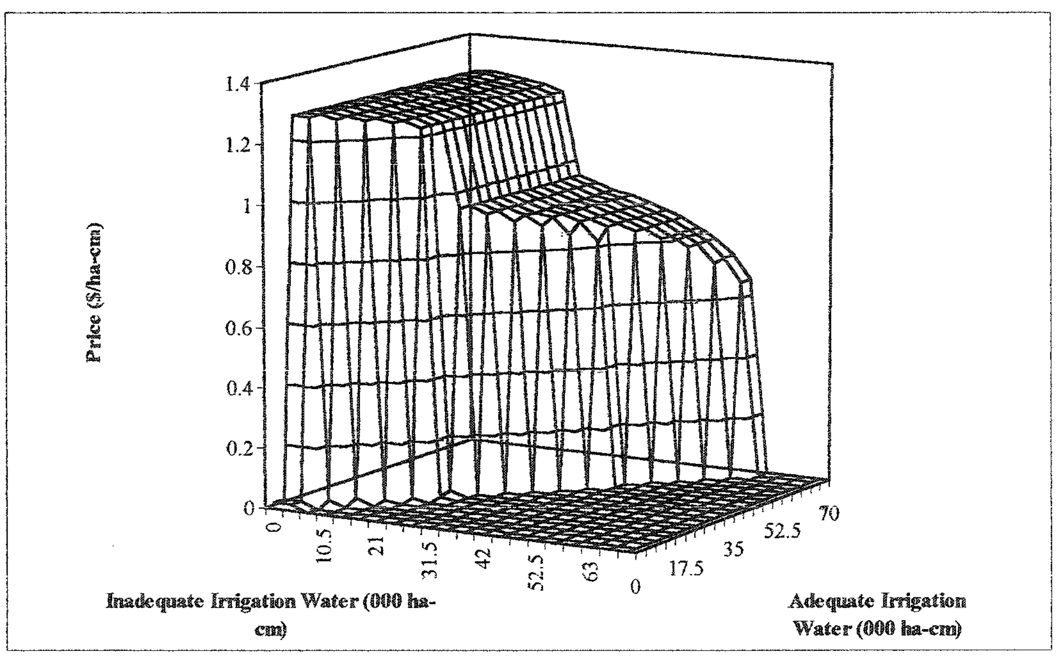


Figure 5.40 Stochastic Irrigation Water Demand for Wet State of Nature of SSM and Inadequate State of Nature of Effective Irrigation (Constrained Potato Hectares)



nature of spring soil moisture and adequate and inadequate states of nature of effective irrigation when potato hectares are restricted to sample areas.

The derived conditional stochastic irrigation water demands when potato hectares are constrained are shown in Figures 5.41 to 5.46. These demand schedules differ from the earlier ones shown in Figures 5.11 to 5.16 when potato hectares are not constrained in that: i) they have a significantly lower choke price, ii) inadequate water deliveries have a value of zero when the expected complement of adequate water delivery is abundant, and iii) the value of inadequate water delivery is zero when the complement adequate water delivery is at zero percent for the first 5 percent of deliveries then increases to around $ 1.2 per ha-cm.

Figure 5.41 Conditional Stochastic Irrigation Water Demands for Dry State of Nature of SSM and Adequate State of Nature of Effective Irrigation (Constrained Potato Hectares)

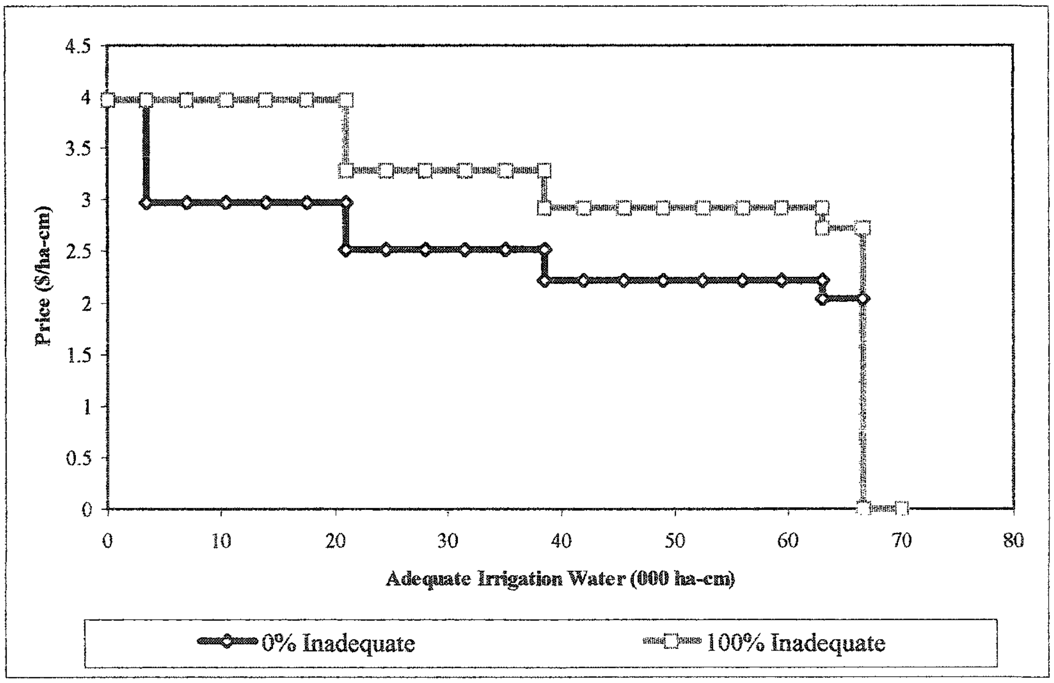


Figure 5.42 Conditional Stochastic Irrigation Water Demands for Dry State of Nature of SSM and Inadequate State of Nature of Effective Irrigation (Constrained Potato Hectares)

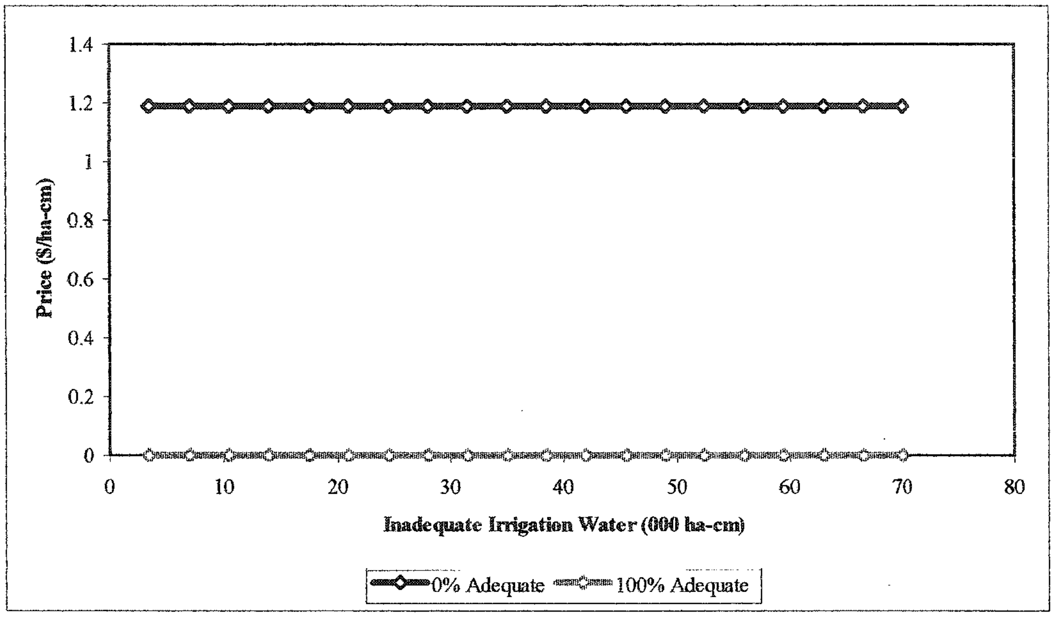


Figure 5.43 Conditional Stochastic Irrigation Water Demands for Normal State of Nature of SSM and Adequate State of Nature of Effective Irrigation (Constrained Potato Hectares)

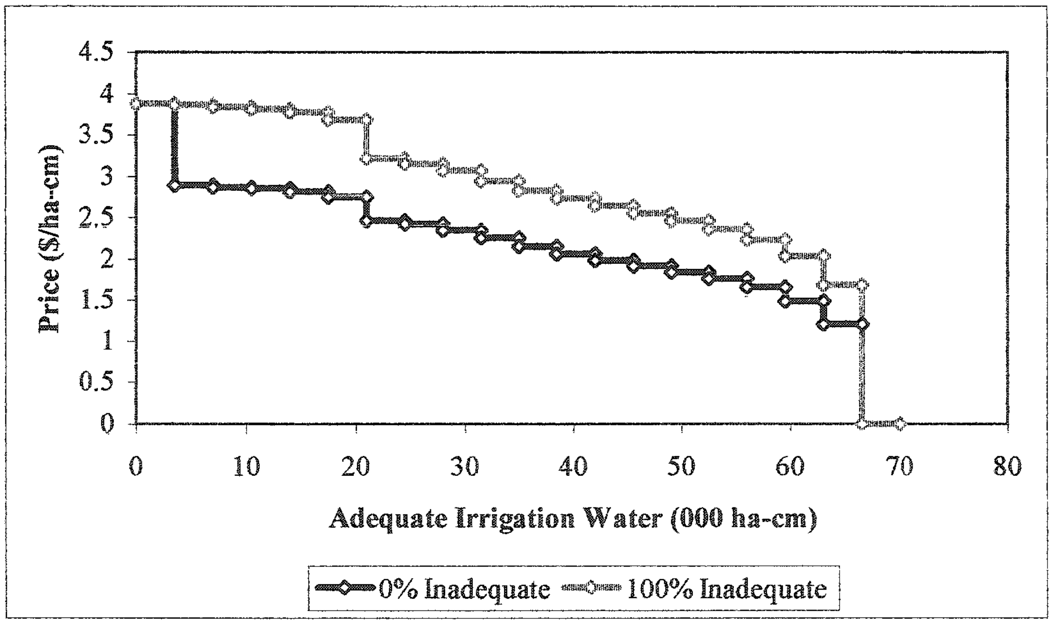


Figure 5.44 Conditional Stochastic Irrigation Water Demands for Normal State of Nature of SSM and Inadequate State of Nature of Effective Irrigation (Constrained Potato Hectares)

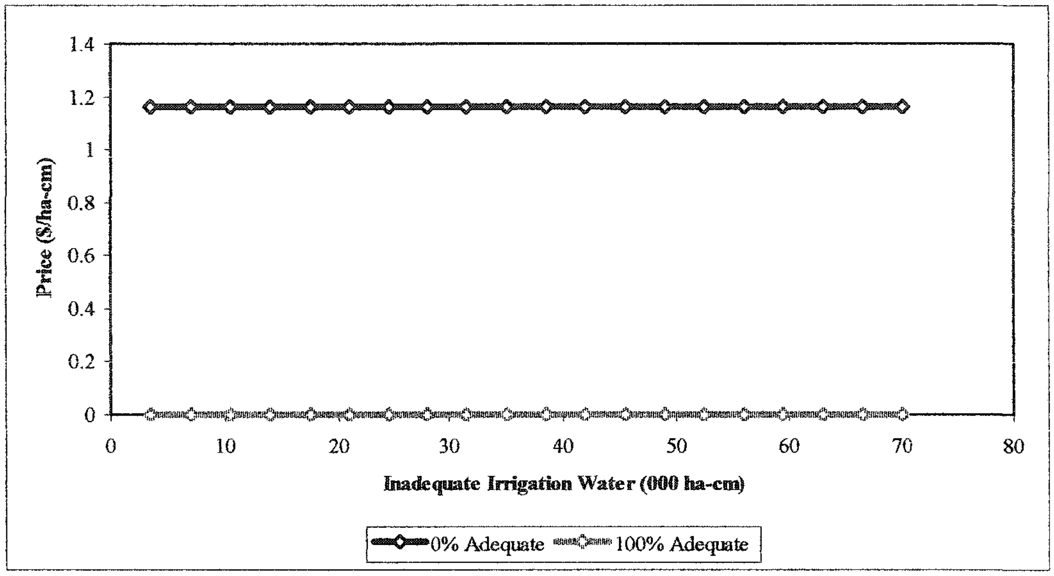


Figure 5.45 Conditional Stochastic Irrigation Water Demands for Wet State of Nature of SSM and Adequate State of Nature of Effective Irrigation (Constrained Potato Hectares)

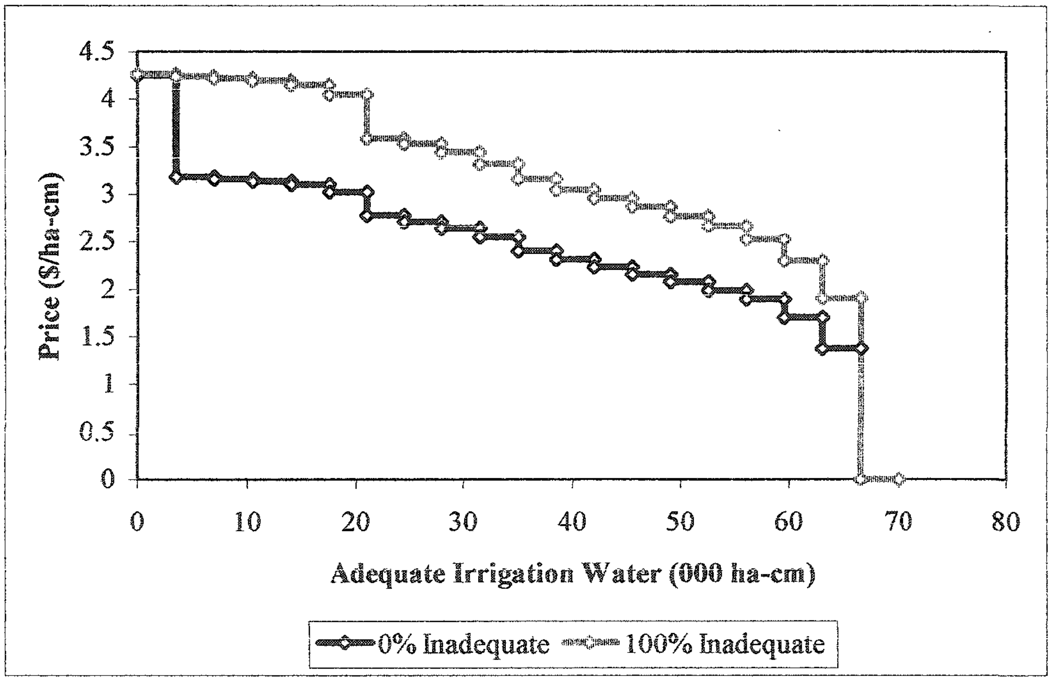
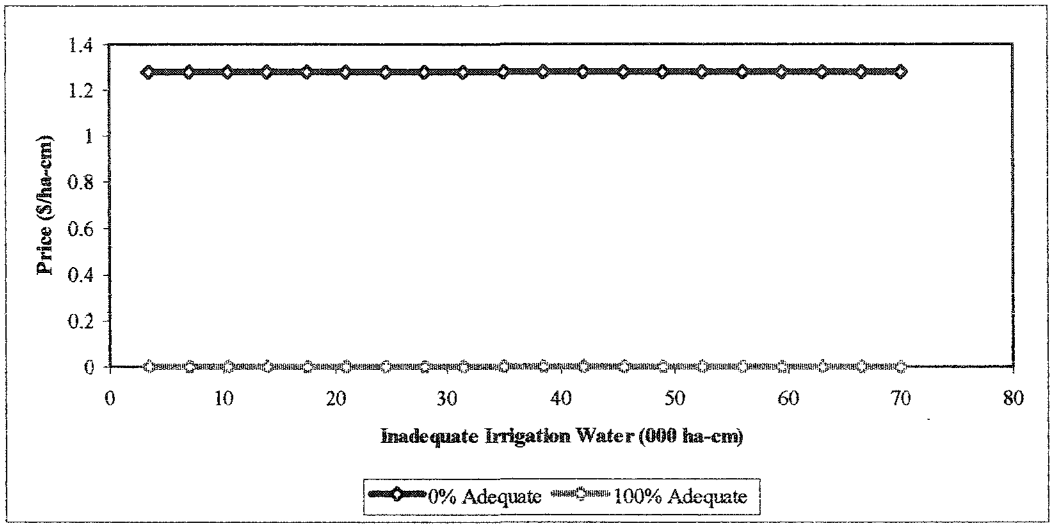


Figure 5.46 Conditional Stochastic Irrigation Water Demands for Wet State of Nature of SSM and Inadequate State of Nature of Effective Irrigation (Constrained Potato Hectares)



The expected values of the derived conditional stochastic demands are obtained by prorating them by their probabilities of occurrence and then compared to the deterministic demand in order to obtain the values of risk penalties associated with employing irrigation water as a stochastic input as shown in Table 5.12.

Risk penalties range from $13.36 to $ 7.39 per ha-cm ($164.79 to $ 91.18 per ac-ft), which are considerably less than the values of risk penalties shown in Table 5.4. The fluctuations in the values of risk penalties reflect the change in the distance between the deterministic and conditional stochastic demands. However, in general, higher risk penalties are associated with smaller available irrigation water for the farmers to employ.

Table 5.12 Risk Penalties of Using Irrigation Water As A Stochastic Input when Potato Hectares Are Constrained ($/ha-cm)

Irrigation Water Delivered (%)	Percentages of Adequate and Inadequate Irrigation Water at Which Conditional Stochastic Demands are Derived				
	0%	25%	50%	75%	100%
5	11.39	11.39	11.39	11.39	11.39
10	13.12	11.42	11.42	11.42	11.42
15	13.15	11.45	11.45	11.45	11.45
20	13.19	11.50	11.50	11.50	11.50
25	13.24	11.57	11.56	11.56	11.56
30	13.36	13.38	11.72	11.72	11.72
35	9.73	9.75	8.59	8.59	8.59
40	9.82	9.84	8.70	8.70	8.70
45	9.93	9.95	8.83	8.83	8.83
50	10.08	10.09	9.03	9.03	9.03
55	10.28	10.30	9.25	9.25	9.25
60	8.48	8.49	8.66	7.51	7.51
65	8.60	8.61	8.81	7.65	7.65
70	8.72	8.73	8.93	7.79	7.79
75	8.84	8.86	9.06	7.95	7.95
80	8.97	8.99	9.19	8.11	8.11
85	9.13	9.15	9.35	8.32	8.32
90	9.42	9.44	9.64	9.70	8.67
95	8.01	8.02	8.22	8.29	7.39

The optimal crop mix varies by the availability of irrigation water and hence the value of water changes accordingly. Table 5.13 shows the deterministic optimal crop mixes and the value of irrigation water under different scenarios of water transfers from the district to other uses. Table 5.14 shows the stochastic crop mixes and the expected value of stochastic water deliveries under different scenarios of water transfers.

Table 5.13 Deterministic Forecast of Optimal Crop Mix, Net Returns, and Value of Water when Potato Hectares Are Constrained

Percentage of Irrigation Water Deliveries	Optimal Hectares of								Value of Objective Function ($million)	Value of Water ($/ha-cm)
	Spring wheat	Soft wheat	Barley	Canola	Alfalfa	Pasture	Potato	Peas		
5	0	0	0	73	0	0	52	0	0.24	68.18
10	0	0	0	200	0	0	52	0	0.31	44.63
15	0	0	0	327	0	0	52	0	0.39	36.77
20	0	0	0	455	0	0	52	0	0.46	32.85
25	0	0	0	582	0	0	52	0	0.53	30.49
30	0	0	0	709	0	0	52	0	0.61	28.92
35	0	0	0	816	20	0	52	0	0.68	27.70
40	0	0	0	816	142	0	52	0	0.74	26.32
45	0	0	0	816	265	0	52	0	0.80	25.25
50	0	0	0	816	388	0	52	0	0.85	24.39
55	0	0	0	816	511	0	52	0	0.91	23.69
60	16	0	0	816	618	0	52	0	0.97	23.08
65	143	0	0	816	618	0	52	0	1.0	22.42
70	270	0	0	816	618	0	52	0	1.1	21.85
75	361	0	0	816	618	0	52	0	1.1	21.36
80	525	0	0	816	618	0	52	0	1.2	20.92
85	652	0	0	816	618	0	52	0	1.2	20.54
90	779	0	0	816	618	0	52	0	1.3	20.21
95	786	120	0	816	618	0	52	0	1.3	19.80

Once potato acreage is constrained to its sample values, the value of the objective function of both the deterministic and stochastic models decline relative to the unconstrained models. The difference between the deterministic and stochastic values of water reflects the average size of the risk penalties associated with the different levels of irrigation water deliveries. However, the average value of risk penalty is about $ 8.06 per ha-cm or $ 99.47 per ac-ft.

Table 5.14 Stochastic Forecast of Optimal Crop Mix, Net Returns, and Value of Water when Potato Hectares Are Constrained*

Percentage of Irrigation Water Deliveries	Optimal Hectares of								Value of Objective Function ($million)	Value of Water ($/ha-cm)
	Spring wheat	Soft wheat	Barley	Canola	Alfalfa	Pasture	Potato	Peas		
5	0	0	0	73	6	0	52	0	0.21	60.74
10	0	0	0	200.	14	0	52	0	0.25	36.42
15	0	0	0	327	22	0	52	0	0.30	28.29
20	0	0	0	455	29	0	52	0	0.34	24.22
25	0	0	.0	582	37	0	522	0	0.38	21.76
30	0	0	0	709	45	0	52	0	0.40	20.10
35	0	0	0	816	72	0	52	0	0.46	18.84
40	0	0	0	816	154	0	52	0	0.50	17.74
45	0	0	0	816	265	0	52	0	0.53	16.86
50	0	0	0	816	388	0	52	0	0.56	16.14
55	23	0	0	816	488	0	52	0	0.60	15.53
60	88	0	0	816	548	0	52	0	0.63	15.00
65	182	17	0	816	564	0	52	0	0.66	14.50
70	265	52	0	816	573	0	52	0	0.69	14.07
75	348	86	0	816	582	0	52	0	0.72	13.68
80	431	121	0	816	591	0	52	0	0.75	13.33
85	513	154	0	816	603	0	52	0	0.77	13.00
90	594	185	0	816	618	0	52	0	0.80	12.70
95	624	249	33	816	618	0	52	0	0.82	12.38

*Evaluated at equal water deliveries for adequate and inadequate water constraints.

The average of the deterministic marginal value of irrigation water when potato hectares are constrained and under all scenarios of water deliveries is $ 27.93 per ha-cm ($ 344.55 per ac-ft). However, the same value decreases considerably when only a small percentage is transferred from the district. In such cases, the average of the deterministic marginal value of irrigation water under all scenarios of water deliveries is $ 19.80 per ha-cm ($ 244.33 per ac-ft).

The average of the stochastic marginal value of irrigation water deliveries when potato hectares are constrained and under all scenarios of water deliveries is $ 19.87 per ha-cm ($ 245.07 per ac-ft). However, if only a small percentage of water available is transferred to other uses, the value of stochastic irrigation water deliveries declines to $ 12.39 per ha-cm ($ 152.81 per ac-ft). These later water values are comparable to those reported in other studies. Kulshreshtha et al. (1991) estimated the short-run value of water in the South Saskatchewan River Irrigation District to be between $0.44 and $ 127.82 (1986 dollars) per acre-foot (ac-ft) for different levels of product prices. Viney et al. (1996) estimates of water values ranged from $8 to $250 per ac-ft. It appears that when the area allocated to potato production is restricted to its hectares in the sample, the models predict reasonable estimates of optimal crop mixes as well as acceptable values of risk penalties and marginal values of deterministic and stochastic water deliveries.

The sensitivity analysis exercise demonstrated the robustness and stability of the CKPP model. Results indicate that the model produces solutions in the directions and magnitudes anticipated. Risk penalties tend to increase as producers become more risk averse, while the value of stochastic irrigation water deliveries tend to decrease as the value of the risk aversion parameter increases. Overall, optimal hectares responses to changes in the expected prices of crops highlighted the relationship between traditional cereal crops in terms of their substitutability in production. Potato, however, remains the dominant cash crop even when its own prices exhibit a downturn. Pasture and peas do not to respond to moderate own price increases. The values of stochastic water deliveries tend to decrease as the probabilities of adequate irrigation deliveries are reduced. Higher water values seem to be positively related to wet spring soil moisture conditions. However, changes in the probability of precipitation have an effect on the value of stochastic water deliveries but the directional relationship seems ambiguous. Constraining potato to its representative sample hectares based on the limited local market for the output yielded more acceptable and comparable estimates of the value of stochastic irrigation water.

5.9 Concluding Remarks

The results of this study show that producers of irrigated crops in southern Alberta have moderately risk-averse preferences and that risk is an important factor in the crop production process in the region. Risk penalty estimates provide evidence of irrigation water being used as a risky or stochastic input. The size of the risk penalty increases as irrigation becomes scarce. The deterministic model estimates of average water values tend to be biased upwards since risk and uncertainty are not accounted for. The stochastic estimates of the average value of irrigation water deliveries show that crop producers of the EID will experience a change in their welfare that is reflected in the price they may be willing to accept for their water licenses. The expected values of water increase proportionally with increased water transfers from the district. Water market performance is therefore of great importance especially in the area of producing clearing prices of water licenses that reflect their true values to all participants and society in general. Long run analysis would consider the fixed costs not accounted for in this study as variable e.g. depreciation of buildings and machinery. In such a case, average foregone benefits that are the surplus attributed to water minus the risk penalty, maybe wiped out if fixed costs are high enough to absorb all the monetary returns. The estimates of the value of stochastic irrigation water when potato is limited to its traditional sample acreage are more realistic and hence policy makers and analysts should consider them when evaluating the issues related to water management.

6. SUMMARY AND CONCLUSION

6.1 Introduction

The South Saskatchewan river basin with its many sub-basins and tributaries runs through southern Alberta and into neighboring Saskatchewan. Agreements between the two provinces govern their shares of consumption of water from the basin. The territorial region that evolved into the province of Alberta in 1905 managed its water resources by riparian rights until the late 1800's. Due to the changing demands for the resource in that era, mainly increased potential demand for irrigation, the Northwest Irrigation Act was enacted in 1894. With the coming of provincehood and the eventual transfer of ownership jurisdiction over national resources to the province in 1930, Alberta passed its *Water Resources Act* of 1931. Under the Act, water use required a water license issued by the government. Irrigation water licenses were attached to land parcels and could not be transferred.

By the 1980's, Alberta realized the need for a change in its laws due to increasing and changing demands for its water resources. Municipal, industrial, recreational, and hydroelectric demands increased over time and the public pressure to maintain the health of ecosystems in the province necessitated a review of the old *Water Resources Act*. In 1999, Alberta passed its new *Water Act* that, in principle, allows for the transfer of water rights from one use to another or among the same uses, but within the same river sub-basin.

The changes in provincial water policy and management are expected to transfer water from its marginal use in agriculture to higher value uses, e.g. municipal and industrial sectors, via government administrated water markets that will facilitate the transfer of water licenses. Several studies investigated various issues related to the effects of the new water policy, including pricing systems, value of water rights, efficiency gains from water trade, and farm water demand and risk analysis.

However, none of those studies have investigated the effects of risk and uncertainty of the value of irrigation water deliveries. Deterministic economic models tend to produce overestimates of the value of water since they do not account for the stochastic components of the crop production process. However, when irrigation water is characterized by stochastic deliveries, in addition to other risk and uncertainty factors, the stochastic water value estimates are expected to be significantly different from estimates with certainty.

Taylor and Young (1995) employed a complete knowledge of the past (CKP) model that does not recognize the risk associated with crop revenues in estimating the foregone benefits due to transferring water from rural to urban use in Colorado. The model utilized in this study follows a complete knowledge of the past and present (CKPP) information structure where spring soil moisture becomes a determinant factor in cropping decisions. In addition, the model introduces risk and uncertainty associated with revenues from crop sales. Thus, the CKPP model is more comprehensive in terms of including more risk and uncertainty sources as well as closely emulating the information flow structure in the decision process of crop production.

The crop production process is characterized by a great deal of uncertainty. Producers take risks when making decisions related to what to produce, where to produce, how to produce, and so on. Part of these uncertainties will be introduced by the possibility of trading water licenses, which may reduce the amount of water available for irrigation and/or contribute to the stochastic nature of its availability. Other uncertainties are related to revenues, soil moisture content during planting time, and rainfall amounts during crops' growing season. These uncertainties are attached to a time framework that is related to the dynamics of crop production and the flow of information producers utilize during the decision making process.

In order to estimate the value of stochastic irrigation water deliveries when irrigation water is in short supply, an estimation of producer's surplus is required. Standard economic theory of the firm stipulates that the area under a factor demand curve and above its supply curve constitutes a measure of producer's surplus. Estimation of factor demands using econometric techniques requires the availability of input prices from competitive markets. Irrigation water rates in southern Alberta are fixed by the government and do not vary by the amount of consumption and hence do not reflect the true value of the resource. An alternative way of estimating factor demands is by utilizing mathematical programming techniques that impute the residual value to water from farm budgets.

Deterministic factor demands derived from solutions to static programming models fail to account for the stochastic portions of demand attributed to risk and the inherent sequential nature of the crop production process. Therefore, an alternative to econometric and static deterministic models has to be developed and employed in order to estimate the value of stochastic irrigation water deliveries, taking into account the sequential stages of the production process that are related to the structure of the flow of information to the decision maker, and also accounting for the sources of risk associated with irrigated crop production.

6.2 Thesis Objectives

The main objective of this study was to estimate the expected value of stochastic irrigation water deliveries under trading water rights conditions that may introduce irrigation water shortages in the future. Thus, farmers are faced with this added uncertainty in conjunction with other risk sources manifest in output price, spring soil moisture, and precipitation. In order to estimate water values while transferring irrigation water to other uses, risk components of the demand for irrigation water must be identified. Therefore, in order to achieve the main objective of this study, estimates of the value of stochastic water deliveries resulting from water transfers require estimates of the expected value of risk penalties resulting from employing such stochastic input. In addition, an estimate of the risk aversion parameter that captures the decision makers' preferences toward risk is needed in order to apply the empirical model.

6.3 Summary of Model

The achievement of the goals of this study requires a development of a mathematical programming model that recognizes the sequential nature of the production process and accounts for risk emanating from its different sources. A discrete sequential stochastic programming (DSSP) model is developed and the flow of information is assumed to follow a complete knowledge of the past and present (CKPP) structure.

The objective function maximizes the certainty equivalent of profits as expected value-variance tradeoff. The tradeoff occurs at a rate related to producers' preferences toward risk measured by the risk aversion parameter. The objective function also accommodates the sequential stages of production i.e. pre-planting and planting, growing, and harvesting and marketing. The sources of risk and uncertainty are identified as revenue that is modeled in the objective function, spring soil moisture modeled in the first stage of the production process, effective irrigation modeled in the second stage, and effective precipitation modeled in the third stage.

Risk related to revenue is measured by a variance-covariance matrix of net returns that is weighted by the number of hectares planted and half the value of the risk aversion parameter to constitute the risk premium portion of the objective function. The expected net returns part of the objective function is weighted by the probabilities of occurrence of states of nature of the other three sources of risk and uncertainty.

Independent discrete probability distributions for spring soil moisture, effective irrigation, and effective precipitation are identified to describe the chances of occurrence of states of nature and the associated amounts of water. Spring soil moisture is assumed to have three states of nature, namely, dry, normal, and wet. Thus, the CKPP model is divided into three respective sub-models. Effective precipitation is divided into three states of nature (low, average, and high) for two growing seasons, namely, four and five-month growing seasons of crops. Effective irrigation is divided into two states of nature (adequate and inadequate) and hence two-irrigation water constraints are modeled that will facilitate the estimation of two complementary stochastic irrigation water demands. Joint probabilities associated with the states of nature weigh the expected net returns of joint event.

The constraint set includes, in addition to the irrigation water constraints, constraints of land, crop rotation, costs, tonnage, and linkage. The later constraints insure the sequencing of activities according to the information structure of CKPP represented in a decision tree.

A sample of representative farms is fitted to the empirical DSSP model. The sample consists of twelve farms representing the production-farming units located at the Eastern Irrigation District (EID) in southern Alberta, and hence the study is not regional in its scope but rather analytical in addition to its use of water-yield response functions. These functions and data pertaining to the probability distributions of the states of nature are obtained from Underwood McLellan Ltd. (UMA,1982). Production costs of eight irrigated crops are obtained from various Alberta government budgeting reports. Time series of prices and yields are gathered from several Alberta *Agriculture Statistics Yearbook* and other publications. Consumer and input price indexes are from Statistics Canada's CANSIM II Database.

Solutions to the deterministic and stochastic models are obtained by running programs written in GAMS code. The parameterization of the water constraint of a deterministic model yields a deterministic derived demand for irrigation water that takes the shape of a downward sloping step function.

Validation of the DSSP model is carried out simultaneously with the estimation of the risk aversion coefficient that best represents the posture of the decision makers towards risk. Stochastic demands for irrigation water are estimated by simultaneously parametrizing the two irrigation water constraints. The result is six stochastic irrigation water demands corresponding to three states of nature of spring soil moisture and two states of nature of effective irrigation. Each stochastic demand is drawn in terms of own price and quantity, and quantity of irrigation water at the complementary state of nature of effective irrigation.

Conditional stochastic irrigation water demands are estimated by parameterizing one water constraint while holding the value of the right hand side of the complementary water constraint at a predetermined level. Conditional stochastic irrigation water demands are weighed by their own probabilities in order to obtain a prorated expected value of water. The distance between deterministic and conditional stochastic demands measures the amount of risk penalty a producer will be willing to pay to ensure water delivery with certainty. These risk penalties are measured at the levels of water deliveries at which a cross section of stochastic demands are taken to produce the conditional stochastic demands.

The area under the stochastic demands defines the value of producers' surplus. It is approximated by the value of the DSSP objective function. This measure of farmers' welfare is free from risk elements that arise from employing water as a stochastic input. The area under the deterministic irrigation water demand measures producers' welfare in a non-stochastic environment. Irrigation shortages may reduce the welfare of farmers vis-à-vis reduced values of the objective function. Hence, comparing the value of the DSSP objective function under two regimes of water deliveries will provide an estimate of welfare change. However, farmers have the opportunity to sell their water licenses in the open market and hence will be compensated for lost revenues accrued from not employing water in crop production. The stochastic demands provide an estimate of how much the crop producers are willing to accept for their water licenses given that such values are without the risk and uncertainty component associated with using water as a stochastic input in crop production.

The difference between the producer surplus estimates obtained from the DSSP and deterministic models is attributed to risk and uncertainty. Dividing the producer surplus estimate by the quantity of water provides an estimate of average value of water. The farmer would lose the value of these units of water if transferred to other uses. Otherwise, these estimates would be the amount of compensation the producer is willing to accept in an open market for foregoing the usage of water in irrigating crops.

6.4 Summary of Empirical Results

Estimation of a risk aversion parameter that describes the preference of southern Alberta's producers of irrigated crops toward risk is the first contribution of this thesis. The decision makers' attitude can be described as mildly risk averse, since the estimated coefficient of 0.00002 is positive and relatively close to being risk neutral. Brinks et al. (1978) reported a value of estimated risk aversion parameter equal to 0.25 for farmers in the U.S. Corn Belt. The comparison of the three states of nature of spring soil moisture shows that producers plant fewer hectares when the soil is dryer than when conditions are wetter, given the same values for risk aversion parameter.

In addition, based on the goodness of fit criterion used to determine the best-fit estimate of risk aversion parameter, i.e. minimum squared sum of deviations (SSD) of predicted hectares from sample hectares are estimated, it appears that deviations are the same for the three CKPP sub-models when the risk aversion parameter is between zero and 0.000002. As the value of the risk aversion parameter increases, SSDs start to rise as well, while the SSD for the 'wet' sub-model is always below those for 'normal' and 'dry' sub-models. This gap in SSDs suggests that farmers are more risk averse as soil moisture content during springtime is lower. This result supports the argument that the value of irrigation water and hence the value of a water license will increase as planting conditions become more favorable and farmers will be planting more hectares since the expected net returns would be higher due to better expected yields. Thus, higher demand on irrigation water is expected in years of suitable planting conditions since there will be more crops on fields to be irrigated in anticipation of relatively abundant crops.

The linear non-stochastic model yielded a step function of derived demand for irrigation water that valued water in average between $47/ha-cm ($576/ac-ft) and $ 132/ha-cm ($1632/ac-ft) depending on whether these water units are first or last to be transferred from agricultural use. However, part of these values is attributed to risk inherent into the crop production process, part of which is using irrigation water as a stochastic input. Kulshreshtha et al. (1991) estimated the short-run value of water in the South Saskatchewan River Irrigation District to be between $0.44 and $ 127.82 (1986 dollars) per acre-foot (ac-ft) for different levels of product prices. Viney et al. (1996) estimates of water values ranged from $8 to $250 per ac-ft. The deterministic water values estimated in this study seem inflated when compared to results from those mentioned studies. A plausible reason for such diversion is that specialization constraints that would force the optimal solution to mimic the sample crop patterns were not used in this study and hence high revenue cash crops, e.g. potato, dominated the optimal crop portfolio and hence the imputed returns to water are relatively higher.

Risk penalties are estimated by the difference between the deterministic and stochastic demand curves. Conditional stochastic demand curves are estimated in order to make such comparisons possible. The second contribution of this thesis is providing estimates of risk penalties associated with using irrigation water as a stochastic input under five scenarios of irrigation water availability ranging from zero to 100 percent. The values of risk penalties, for the five scenarios of irrigation water availability, averaged from $41/ha-cm ($503/ac-ft) for the first 5 to 25 percent of irrigation water deliveries to $11/ha-cm ($133/ac-ft) for the remaining 30 to 95 percent of irrigation water deliveries. These payments would make a crop producer indifferent between stochastic water deliveries and water delivered with certainty.

The estimates of the value of stochastic irrigation water deliveries, which make the third contribution of this thesis, are measured by the area under the stochastic demand curves under different regimes of water transfer divided by the quantity of water delivered. When irrigation water is scarce, the average value of stochastic water deliveries reaches $128 per ha-cm ($1579/ac-ft) compared to $38 per ha-cm ($467/ac-ft) when water is abundantly available. These figures would be an estimate of how much a farmer would accept for selling his water rights that are stochastic in nature.

In a deterministic world, the average value of irrigation water ranges between $132 per ha-cm when water is scarce and $47 per ha-cm when it is abundant. However, the corresponding values of stochastic water deliveries range between $128 and $38 per ha-cm, respectively. Comparing the deterministic average water values to their stochastic counterparts reveals the relatively small average amount of average risk penalty associated with using Alberta's water resources in irrigation. Taylor et al. (1995) reported risk penalty estimates of $US 11/ac-ft at mean water diversions.

Traditional cereals will be the first crops to be abandoned by producers once water starts to migrate towards its alternative uses. Communities specializing in the production of wheat and barley will be affected first due to trade in water rights. Canola and alfalfa will follow while potato will be last to be eliminated from the optimal crop mix.

To check for the effects of changing some of the key elements of the DSSP model on these results, a sensitivity analysis is carried out. The results mentioned above seem robust and significant. Risk penalties seem to increase, as the crop producers become more risk averse expressed as larger positive values of the risk aversion parameter.

However, the value of stochastic irrigation water deliveries decline as the value of the risk aversion parameter increases. A fifty percent increase in the expected price of traditional cereals results in maximum agronomical allowed hectares to be in the optimal crop mix. However, only a 20 percent reduction in their expected prices eliminates the traditional cereals from the primal solution. Canola and alfalfa, on the other hand would be eliminated if their prices drop by 60 percent. Pasture and peas require at least 70 percent increase in their expected prices in order to enter the optimal crop mix. However, Potato optimal hectares would start falling if its expected price drops by more than 60 percent.

The probability of effective irrigation describes the level of water rights seniority. When the probability of obtaining adequate water deliveries is decreased, the average value of stochastic irrigation water deliveries tend to decline too. However, a drop in the probability of adequate water deliveries from 0.9 to 0.1 leads to only 6 percent reduction in average value of water.

Changes in effective precipitation probability distributions appear to have a slightly larger effect on the average value of water than changes in the probabilities of effective irrigation. However, the direction of this effect cannot be determined. Changes in the probability distributions of spring soil moisture appear to have a small effect on the average value of water. However, a higher probability of moist conditions during planting time insures a higher average value of water, which may suggest the importance of a 'good start' for crop production.

Constraining the land that is allocated to potato to its representative sample values resulted in a significant change in the values of the estimates of water value and risk penalties. The average of the deterministic marginal value of irrigation water when potato hectares are constrained and under all scenarios of water deliveries is \$ 27.93 per ha-cm (\$ 344.55 per ac-ft). However, the same value decreases considerably when only a small percentage is transferred from the district. In such cases, the average of the deterministic marginal value of irrigation water under all scenarios of water deliveries is $ 19.80 per ha-cm ($ 244.33 per ac-ft).

Risk penalties range from $13.36 to $ 7.39 per ha-cm ($164.79 to $ 91.18 per ac-ft), which are considerably smaller than the values of risk penalties obtained from the less constrained models. However, the average of the stochastic marginal value of irrigation water deliveries when potato hectares are constrained and under all scenarios of water deliveries is $ 19.87 per ha-cm ($ 245.07 per ac-ft). However, if only a small percentage of water available is transferred to other uses, the value of stochastic irrigation water deliveries declines to $ 12.39 per ha-cm ($ 152.81 per ac-ft).

6.5 Conclusion

Transferring water rights allowed under the *Water Act* may result in reduced and stochastic water deliveries to irrigators in southern Alberta. Estimating the value of stochastic irrigation water deliveries was the main objective of this study. The farm decision makers of the EID seem to be risk averse, but to a lesser extent when the moisture content of the fields is more favorable for planting crops in springtime.

Employing water as a stochastic input in the production of crops comes with a risk penalty that ranges from $11 to $41 per ha-cm depending on the quantities of irrigation water transferred. Average value of water, excluding risk, is estimated at $38 per ha-cm ($467 per acre-foot). These estimates seem higher than expected due to the absence of specialization constraints that would limit the amount of acreage of cash crops as potato in the optimal crop mix, which tend to inflate the value of water estimates. However, when potato hectares are limited to the sample areas, the average value of the stochastic water deliveries equaled to $ 12.39 per ha-cm ($ 152.81 per ac-ft). These later estimates are the ones policy makers and analysts should consider using in their analysis and assessments of the province's water management and policy.

Farmers transferring their water licenses would require an average selling price equal to or higher than these figures in order to mitigate any possible producer welfare loss that may occur if water markets did not function properly in producing a fair equilibrium price.

Drought conditions were observed in many parts of Alberta in the past few years. Many argue that such severe weather conditions are attributed to global warming that causes such noticeable climatic changes. The changes in weather patterns disturb the probability distribution of precipitation and spring soil moisture. The CKPP model predicts higher marginal values for water during 'wet' spring soil conditions relative to 'normal' and 'dry' ones, which reflects the importance of a smooth and evenly distribution of precipitation to farmers.

6.6 Limitations and Directions for Further Research

The main limitation of this research was data availability. The DSSP model is readily expandable to include more activities. Alfalfa seed and silage are two crops that can be added to the potential crop mix if water-yield functions can be accessed. In addition, animal husbandry activities should have been included in the model, and that would have added some complexities in terms of programming intermediate inputs such as silage and pasture. However, data relating animal water consumption and detailed costs of production that can be related to the stages of production time framework were not available. The addition of these activities has the potential of improving the validity of the model and its ability in duplicating actual production patterns.

The sample of representative farms has a built in feature that emphasizes specialization of certain farms in the production of specific crop mixes. However, the constraint set, and especially the crop rotation constraint, is not consistent with these specialization features of the sample farms. Thus, predicted crop mixes appear somewhat different than the sample hectares. Collection of data pertaining to financial, labor, and/or any other constraints may improve the predictive ability of the model.

The representative sample contains 12 farms, however, the reported results are for the whole sample together. This is mainly due to the absence of the above-mentioned constraints recognizing farm unit specialization, which would distinguish one farm from another. In its present form, the model simply satisfies the constraint starting with farm one and down the line until the constraint becomes binding in total. Thus, most of the allotted hectares are concentrated in the farms in top of the sample table.

Once trading of water rights data become available, water deliveries under the adequate and inadequate states of nature can be used in estimating stochastic demands for crops classified by the length of their growing seasons. This can be achieved also by assuming hypothetical scenarios and by modifying the model to accommodate the two groups of crops: four and five-month growing seasons. The model will provide an insight into the differences between the two groups of crops in terms of their demand for irrigation in a stochastic environment as well as to their response to water shortages.

Water-yield response functions had to be adjusted upward to reflect improvements in yields over time. It would be beneficial if new functions are estimated that would capture such technical change and improve on the water-fertilizer relationships that are provided by UMA (1982).

The estimated water values are to be compared to values of water used in alternative sectors. These values need to exclude any risk or uncertainty components attached to them in order to have a fair comparison. Research should be directed towards valuing water in its uses in urban, industrial, and recreational sectors with emphasis on the uncertainty and risk associated with these uses.

Regional analysis can be achieved using this same DSSP model once aggregate data become available. Policy makers, managers, and others can accessibly add as many activities or constraints with minor modifications made to the model, and the primal and dual results should be satisfactory. However, the report writing parts of the GAMS programs, which were another limitation to this study, will need considerable professional improvements to reduce the amount of time needed to debug input files, sort output, and interpret results.

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APPENDIX

GAMS PROGRAMS

```
************************
*DETERMINISTIC MODEL
************************
$OFFLISTING OFFSYMXREF OFFSYMLIST
OPTION LIMROW=0
OPTION LIMCOL=0
OPTION DECIMALS=2;
SETS
C     CROPS
          /1    SPRING WHEAT
           2    SOFT WHEAT
           3    BARLEY
           4    CANOLA
           5    ALFALFA
           6    PASTURE
           7    POTATO
           8    PEAS/
C4(C)  CROPS WITH FOUR MONTH GROWING SEASON
          /1*4/
C5(C)  CROPS WITH FIVE MONTH GROWING SEASON
          /5*8/
I     FACTORS OF PRODUCTION
   /SEDA  SEEDS
    FRTB  FERTILIZERS
    CHMB  CHEMICALS
    CINA  HAIL AND CROP INSURANCE
    MRKC  TRUCKING AND MARKETING
    FULA  FUEL USED IN STAGE A
    FULB  FUEL USED IN STAGE B
    FULC  FUEL USED IN STAGE C
    RPMA  REPAIRS TO MACHINES IN STAGE A
    RPMB  REPAIRS TO MACHINES IN STAGE B
    RPMC  REPAIRS TO MACHINES IN STAGE C
    RPBA  REPAIRS TO BUILDINGS IN STAGE A
    RPBB  REPAIRS TO BUILDINGS IN STAGE B
    RPBC  REPAIRS TO BUILDINGS IN STAGE C
    MCUA  UTILITIES AND MISC. SPENDING IN STAGE A
    MCUB  UTILITIES AND MISC. SPENDING IN STAGE B
    MCUC  UTILITIES AND MISC. SPENDING IN STAGE C
    SPLA  CUSTOM WORK AND SPECIAL LABOUR IN STAGE A
    SPLB  CUSTOM WORK AND SPECIAL LABOUR IN STAGE B
```

```
     SPLC  CUSTOM WORK AND SPECIAL LABOUR IN STAGE C
     INTA  OPERATING INTEREST PAID IN STAGE A
     INTB  OPERATING INTEREST PAID IN STAGE B
     INTC  OPERATING INTEREST PAID IN STAGE C
     PDLA  PAID LABOUR IN STAGE A
     PDLB  PAID LABOUR IN STAGE B
     PDLC  PAID LABOUR IN STAGE C
     UPLA  UNPAID LABOUR IN STAGE A
     UPLB  UNPAID LABOUR IN STAGE B
     UPLC  UNPAID LABOUR IN STAGE C/
VIA(I)  VARIABLE INPUTS USED IN STAGE A
          /SEDA,CINA,FULA,RPMA,RPBA,MCUA,SPLA,INTA,PDLA,UPLA/
VIB(I)  VARIABLE INPUTS USED IN STAGE B
          /FRTB,CHMB,FULB,RPMB,RPBB,MCUB,SPLB,INTB,PDLB,UPLB/
VIC(I)  VARIABLE INPUTS USED IN STAGE C
          /MRKC,FULC,RPMC,RPBC,MCUC,SPLC,INTC,PDLC,UPLC/
F     REPRESENTATIVE FARMS
          /FM1*FM12/
COEF   COEFFICIENTS OF THE WATER-YIELD RESPONSE FUNCTION
          /B0 INTERCEPT
           B1 FIRST DEGREE COEFFICIENT.
           B2 SECOND DEGREE COEFFICIENT.
           PE POTENTIAL EVAPOTRANSPIRATION
           PY POTENTIAL YIELD/
T     TIME INDEX
          /84*94/
VARYIRR  EFFECTIVE IRRIGATION SCENARIOS /1*70/;
TABLE CST(I,C)  COSTS OF PRODUCTION ($ PER HECTARE)
$INCLUDE C:\WATER\DATA\COSTS.TXT
TABLE CY(C,COEF) CROP WATER-YIELD FUNCTIONS' COEFFICIETNS
$INCLUDE C:\WATER\DATA\COEFFICIENTS.TXT
TABLE Y(C,T)  TIME SERIES OF CROP YIELD (TONNES PER HECTARE)
$INCLUDE C:\WATER\DATA\YIELDS.TXT
TABLE P(C,T)  TIME SERIES OF CROP PRICES ($ PER TONNE)
$INCLUDE C:\WATER\DATA\PRICES.TXT
TABLE AH(F,C) ACTUAL PLANTED HECTARES
$INCLUDE C:\WATER\DATA\HECTARES.TXT
PARAMETER
LAND(F)     TOTAL HECTARES OF EACH FARM
                 /FM1  664.898
                  FM2  341.959
                  FM3  221.363
                  FM4  271.544
                  FM5  182.918
                  FM6  189.393
                  FM7  158.232
                  FM8  139.212
                  FM9  84.984
                  FM10 103.195
                  FM11 57.87
```

```
                    FM12 57.87/
DP(C) 1997 CROP PRICES ($ PER TONNE)
$INCLUDE C:\WATER\DATA\DETPRICES.TXT
SCALAR
H2OD      /0/      AMOUNT OF DETERMINISTIC IRRIGATION WATER
SSM    SPRING SOIL MOISTURE (CM)
       / 8.7/
PRCP    EFFECTIVE PRECIPITATION (CM)
       /15.7/;
PARAMETER
EI(C)  EFFECIVE IRRIGATION (CM)
/1*4 27.5
 5*8 28.5/
PARAMETER
VCA(C)          STAGE A VARIABLE COSTS
VCB(C)          STAGE B VARIABLE COSTS
VCC(C)          STAGE C VARIABLE COSTS
TVC(C)          TOTAL VARIABLE COSTS
AY(C)           ESTIMATED YIELD
NR(C)           NET REVENUE;
VCA(C)= SUM(I$VIA(I),CST(I,C));
VCB(C)= SUM(I$VIB(I),CST(I,C));
VCC(C)= SUM(I$VIC(I),CST(I,C));
TVC(C)=VCA(C)+VCB(C)+VCC(C);
AY(C)=(CY(C,'PY')*(CY(C,'B0')+CY(C,'B1')*
    ((SSM+EI(C)+PRCP)/CY(C,'PE'))+CY(C,'B2')*
    ((SSM+EI(C)+PRCP)/CY(C,'PE'))**2));
NR(C)=(DP(C)*AY(C))-VCA(C)-VCB(C)-VCC(C)
DISPLAY VCA,VCB,VCC,TVC,AY,NR;
VARIABLE
ACRESD(F,C)        DETERMINISTIC MODEL HECTARES
TNR                DETERMINISTIC TOTAL NET RETURNS;
POSITIVE VARIABLE
ACRESD(F,C)        DETERMINISTIC DECISION VARIABLE;
EQUATIONS
OBJDET            DETERMINISTIC OBJECTIVE FUNCTION
LNDDET(F)         DETERMINISTIC  LAND CONSTRAINTS
SPWTRTDET(F)      DETERMINISTIC SPRING WHEAT ROTATION CONSTRAINT
SFWTRTDET(F)      DETERMINISTIC SOFT WHEAT ROTATION CONSTRAINT
BRLRTDET(F)       DETERMINISTIC BARLEY ROTATION CONSTRAINT
CNLRTDET(F)       DETERMINISTIC CANOLA ROTATION CONSTRAINT
ALFRTDET(F)       DETERMINISTIC ALFALFA ROTATION CONSTRAINT
PASTRTDET(F)      DETERMINISTIC PASTURE ROTATION CONSTRAINT
POTRTDET(F)       DETERMINISTIC POTATO ROTATION CONSTRAINT
PEASRTDET(F)      DETERMINISTIC PEAS ROTATION CONSTRAINT
WATERDET          EFFECTIVE IRRIGATION WATER CONSTARINT;
OBJDET .. SUM((F,C),((DP(C)*(CY(C,'PY')*(CY(C,'B0')+CY(C,'B1')*
          ((SSM+EI(C)+PRCP)/CY(C,'PE'))+CY(C,'B2')*
          ((SSM+EI(C)+PRCP)/CY(C,'PE'))**2)))-VCA(C)-VCB(C)-
          VCC(C))*ACRESD(F,C))=E=TNR;
```

```
LNDDET(F)..SUM(C,ACRESD(F,C))=L=LAND(F);
SPWTRTDET(F) ..ACRESD(F,'1')=L=LAND(F)*0.33;
SFWTRTDET(F) ..ACRESD(F,'2')=L=LAND(F)*0.33;
BRLRTDET(F) ..ACRESD(F,'3')=L=LAND(F)*0.8;
CNLRTDET(F) ..ACRESD(F,'4')=L=LAND(F)*0.33;
ALFRTDET(F) ..ACRESD(F,'5')=L=LAND(F)*0.25;
PASTRTDET(F) ..ACRESD(F,'6')=L=LAND(F)*0.25;
POTRTDET(F) ..ACRESD(F,'7')=L=LAND(F)*0.25;
PEASRTDET(F) ..ACRESD(F,'8')=L=LAND(F)*0.25;
WATERDET ..SUM((F,C4),ACRESD(F,C4)*EI(C4))+SUM((F,C5),
            ACRESD(F,C5)*EI(C5))=L=H2OD;
MODEL DETERMIN /OBJDET,LNDDET,BRLRTDET,SPWTRTDET,SFWTRTDET,
CNLRTDET,ALFRTDET,PASTRTDET,POTRTDET,PEASRTDET,WATERDET/;
PARAMETERS
OUTPUTA(VARYIRR)       VALUE OF THE OBJECTIVE FUNCTION
OUTPUTB(VARYIRR)       DETERMINISTIC WATER DEMAND
OUTPUTC(*,*)           DETERMINISTIC CROP MIX BY IRRIGATION LEVEL;
OPTION SOLPRINT=OFF
OPTION SYSOUT=OFF
LOOP (VARYIRR,H2OD=(ORD(VARYIRR)*1000);
SOLVE DETERMIN USING LP MAXIMIZING TNR;
OUTPUTA(VARYIRR)=TNR.L;
OUTPUTB(VARYIRR)=WATERDET.M;
OUTPUTC(C,VARYIRR)=SUM(F,ACRESD.L(F,C)));
FILE DETFINAL/DETFINAL.TXT/;
DETFINAL.PC=5;
PUT DETFINAL;
PUT@5'VARYIRR'@15'VALUE OF OBJ. FUN.'/
LOOP((VARYIRR),PUT VARYIRR.TL,OUTPUTA(VARYIRR)/);
PUT @5'VARYIRR'@15'C $/HECTARE-CM'/
LOOP((VARYIRR),PUT VARYIRR.TL,OUTPUTB(VARYIRR)/);
PUT @10'CROP'@20'VARYIRR'@40'HECTARES'/
LOOP((C,VARYIRR),PUT C.TL,PUT VARYIRR.TL, OUTPUTC(C,VARYIRR)/);
PUTCLOSE;
**********************************************************************
*DSSP MODEL WITH CKPP INFORMATION STRUCTURE
*SUB-MODEL FOR 'DRY' STATE OF NATURE OF SPRING SOIL MOISTURE
*NOTE: SIMILAR SUB-MODELS EXIST FOR THE OTHER TWO
*STATES OF NATURE OF SPRING SOIL MOISTURE: 'NORMAL' AND 'WET'
**********************************************************************
*RISK AVERSION PARAMETER ESTIMATION
*******************************************
SETS
S    STAGES OF PRODUCTION
       / A  FIRST STAGE
         B  SECOND STAGE
         C  THIRD STAGE/
S1   STATES OF NATURE OF SPRING SOIL MOISTURE (SSM) IN STAGE A
       /WT WET
        NR NORMAL
```

```
        DR DRY/
S2    STATES OF NATURE OF EFFECTIVE IRRIGATION (EI) IN STAGE B
       /AD ADEQUATE
        IN INADEQUATE/
S3    STATES OF NATURE OF PRECIPITATION (PRCP) IN STAGE C
       /HI4 WET
        AV4 NORMAL
        LO4 DRY
        HI5 WET
        AV5 NORMAL
        LO5 DRY/
RAPS   RISK AVERSION PARAMETER
       /R1*R67/
VARYR      ADEQUATE DELIVERIES SCENARIOS /1*20/
VARYEI     INADEQUATE DELIVERIES SCENARIOS /1*20/;
ALIAS (C,CP);
PARAMETER
EP(C) EXPECTED CROP PRICES ($ PER TONNE)
$INCLUDE C:\WATER\DATA\EPRICES.TXT
TABLE REV(C,T)  REVENUES IN 1997 DOLLARS
$INCLUDE C:\WATER\DATA\REVENUES.TXT
PARAMETER
NREV(C,T)               NET REVENUE MATERIX
MEANNR(C)               MEAN NET REVENUE
VARCOVNR(C,CP)          NET REVENUE VARIANCE COVARIANCE MATRIX;
NREV(C,T)=REV(C,T)-SUM(I$VIA(I),CST(I,C))-
      SUM(I$VIB(I),CST(I,C));
MEANNR(C)=SUM(T,NREV(C,T)/CARD(T));
VARCOVNR(C,CP)=SUM(T,(NREV(C,T)-MEANNR(C))
         *(NREV(CP,T)-MEANNR(CP)))/CARD(T);
DISPLAY NREV,MEANNR,VARCOVNR;
PARAMETER
RISKAVER(RAPS)  RISK AVERSION COEFFICIENTS
/R1  0
R2   0.0000001,
R3   0.0000002,    R4   0.0000003,    R5   0.0000004,    R6   0.0000005,
R7   0.0000006,    R8   0.0000007,    R9   0.0000008,    R10  0.0000009,
R11  0.0000010,    R12  0.0000020,    R13  0.0000030,    R14  0.0000040,
R15  0.0000050,    R16  0.0000060,    R17  0.0000070,    R18  0.0000080,
R19  0.0000090,    R20  0.000010,     R21  0.000020,     R22  0.000030,
R23  0.000040,     R24  0.000050,     R25  0.000060,     R26  0.000070,
R27  0.000080,     R28  0.000090,     R29  0.00010,      R30  0.00020,
R31  0.00030,      R32  0.00040,      R33  0.00050,      R34  0.00060,
R35  0.00070,      R36  0.00080,      R37  0.00090,      R38  0.0010,
R39  0.0020,       R40  0.0030,       R41  0.0040,       R42  0.0050,
R43  0.0060,       R44  0.0070,       R45  0.0080,       R46  0.0090,
R47  0.010,        R48  0.020,        R49  0.030,        R50  0.040,
R51  0.050,        R52  0.060,        R53  0.070,        R54  0.080,
R55  0.090,        R56  0.10,         R57  0.20,         R58  0.30,
R59  0.40,         R60  0.50,         R61  0.60,         R62  0.70,
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R63   0.80,          R64   0.90,          R65   1.00,          R66   1.10,
R67   1.20/
SCALAR
RAC RISK AVERSION PARAMETER /0.00002/
PARAMETER
H2O(S2)  EFFECTIVE IRRIGATION WATER AVAILABLE (HECTARE-CM)
            /AD 0
             IN  0/
PR1(S1)  PROBABILITIES OF SSM STATES OF NATURE IN STAGE A
         /WT 0.342, NR 0.324, DR 0.334 /
PR2(S2)  PROBABILITIES OF EI STATES OF NATURE IN STAGE B
         /AD 0.8, IN 0.2/
PR3(S3)  PROBABILITIES OF PRCP STATES OF NATURE IN STAGE C
         /HI4    0.3324
          AV4    0.3320
          LO4    0.3356
          HI5    0.3360
          AV5    0.3308
          LO5    0.3332/
SSMS(S1)    AMOUNTS OF SPRING SOIL MOISTURE (CM)
         /WT   11.0
          NR   8.7
          DR   8.1/
PRCPS(S3)   AMOUNTS OF EFFECTIVE PRECIPITATION (CM)
         /HI4    25.0
          AV4    14.4
          LO4    10.1
          HI5    27.5
          AV5    17.0
          LO5    13.2/;
TABLE EIS(S2,C)     AMOUNTS OF EFFECTIVE IRRIGATION (CM)
        1*4       5*8
AD      27.5      28.5
IN      23.0      25.0;
PARAMETER
ACTY(C,S1,S2,S3)    ESTIMATED CROP YIELD
SALES(C,S1,S2,S3)   CROP SALES ;
ACTY(C,S1,S2,S3)=
     CY(C,'PY')*(CY(C,'B0')+CY(C,'B1')*
     ((SSMs(S1)+EIs(S2,C)+PRCPs(S3))/CY(C,'PE'))+CY(C,'B2')*
     ((SSMs(S1)+EIs(S2,C)+PRCPs(S3))/CY(C,'PE'))**2);
SALES(C,S1,S2,S3)=EP(C)*ACTY(C,S1,S2,S3);
DISPLAY ACTY,SALES;
VARIABLE
ACRES_A(F,C,S1)              STAGE A HECTARES
ACRES_B(F,C,S1,S2)           STAGE B HECTARES
ACRES_C(F,C,S1,S2,S3)        STAGE C HECTARES
PROFITDR        PROFIT CERTAINTY EQUIVALENT OF CKPP GIVEN SSM IS DR
TONNESDR(F,C,S1,S2,S3) TONNES OF PRODUCTION GIVEN SSM IS DR
CSTADR(F,C,S1)            VARIABLE COST IN STAGE A GIVEN SSM IS DR
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CSTBDR(F,C,S1,S2)        VARIABLE COST IN STAGE B GIVEN SSM IS DR
CSTCDR(F,C,S1,S2,S3)     VARIABLE COST IN STAGE C GIVEN SSM IS DR;
POSITIVE VARIABLES
ACRES_A(F,C,S1)           STAGE A HECTARES
ACRES_B(F,C,S1,S2)        STAGE B HECTARES
ACRES_C(F,C,S1,S2,S3)     STAGE C HECTARES;
EQUATIONS
OBJDR               CKPP MODEL OBJECTIVE FUNCTION GIVEN SSM IS DR
LNDDR(F)            TOTLA LAND CONSTARINT
SPWTRTDR(F)         SPRING WHEAT ROTATION CONSTRAINT CKPP
SFWTRTDR(F)         SOFT WHEAT ROTATION CONSTRAINT CKPP
BRL_ROTDR(F)        BARLEY ROTATION CONSTRAINT CKPP
CNL_ROTDR(F)        CANOLA ROTATION CONSTRAINT CKPP
ALF_ROTDR(F)        ALFALFA ROTATION CONSTRAINT CKPP
PAST_ROTDR(F)       PASTURE ROTATION CONSTRAINT CKPP
POT_ROTDR(F)        POTATO ROTATION CONSTRAINT CKPP
PEAS_ROTDR(F)       PEAS ROTATION CONSTRAINT CKPP
WATERDR(S2)         IRRIGATION WATER AVAILABLE GIVEN DR  AND S24
LINK1DR(F,C,S1,S2)        LINK BETWEEN STAGES A AND B ACTIVITIES
LINK2DR(F,C,S1,S2,S3)     LINK BETWEEN STAGES B AND C ACTIVITIES
VCADR(F,C,S1)             VARIABLE COST IN STAGE A GIVEN SSM IS DR
VCBDR(F,C,S1,S2)             VARIABLE COST IN STAGE B GIVEN SSM IS DR
VCCDR(F,C,S1,S2,S3)          VARIABLE COST IN STAGE C GIVEN SSM IS DR
TONNAGEDR(F,C,S1,S2,S3)        TONNES OF PRODUCTION GIVEN SSM IS DR;
OBJDR..PROFITDR=E=SUM{(F,S1('DR'),S2,S3),PR1('DR')*PR2(S2)*PR3(S3)*[SUM
(C,(EP(C)*TONNESDR(F,C,'DR',S2,S3))-CSTADR(F,C,'DR')-CSTBDR(F,C,'DR',S2)-
CSTCDR(F,C,'DR',S2,S3))-SUM((C,CP),(RAC/2)*ACRES_C(F,C,'DR',S2,S3)*
VARCOVNR(C,CP)*ACRES_C(F,C,'DR',S2,S3))]};
TONNAGEDR(F,C,'DR',S2,S3)..(ACRES_C(F,C,'DR',S2,S3)*ACTY(C,'DR',
                  S2,S3))-TONNESDR(F,C,'DR',S2,S3)=E=0;
VCADR(F,C,'DR')..CSTADR(F,C,'DR')-(VCA(C)*ACRES_A(F,C,'DR'))=E=0;
VCBDR(F,C,'DR',S2)..CSTBDR(F,C,'DR',S2)-(VCB(C)*ACRES_B(F,C,'DR',S2))=E=0;
VCCDR(F,C,'DR',S2,S3)..CSTCDR(F,C,'DR',S2,S3)-
(VCC(C)*ACRES_C(F,C,'DR',S2,S3))=E=0;
LNDDR(F)..SUM((C,S1),ACRES_A(F,C,'DR'))=L=LAND(F);
SPWTRTDR(F) ..SUM(S1,ACRES_A(F,'1','DR'))=L=LAND(F)*0.33;
SFWTRTDR(F) ..SUM(S1,ACRES_A(F,'2','DR'))=L=LAND(F)*0.33;
BRL_ROTDR(F) ..SUM(S1,ACRES_A(F,'3','DR'))=L=LAND(F)*0.8;
CNL_ROTDR(F) ..SUM(S1,ACRES_A(F,'4','DR'))=L=LAND(F)*0.33;
ALF_ROTDR(F) ..SUM(S1,ACRES_A(F,'5','DR'))=L=LAND(F)*0.25;
PAST_ROTDR(F) ..SUM(S1,ACRES_A(F,'6','DR'))=L=LAND(F)*0.25;
POT_ROTDR(F) ..SUM(S1,ACRES_A(F,'7','DR'))=L=LAND(F)*0.25;
PEAS_ROTDR(F) ..SUM(S1,ACRES_A(F,'8','DR'))=L=LAND(F)*0.25;
WATERDR(S2)..SUM((F,C4,S1),ACRES_B(F,C4,'DR',S2)*EIs(S2,C4))+
        SUM((F,C5,S1),ACRES_B(F,C5,'DR',S2)*EIs(S2,C5))=L=H2O(S2);
LINK1DR(F,C,'DR',S2)..-ACRES_A(F,C,'DR')+ACRES_B(F,C,'DR',S2)=L=0;
LINK2DR(F,C,'DR',S2,S3).. -ACRES_B(F,C,'DR',S2)+ACRES_C(F,C,'DR',S2,S3)=L=0;
MODEL CKPPDR /OBJDR,LNDDR,SPWTRTDR,SFWTRTDR,BRL_ROTDR,
CNL_ROTDR,ALF_ROTDR,PAST_ROTDR,POT_ROTDR,PEAS_ROTDR,
WATERDR,LINK1DR, LINK2DR,VCADR,VCBDR,VCCDR,TONNAGEDR/;
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PARAMETER
OUT0DR(*)
OUT1DR(*,*)
OUT2DR(*,*,*)
OUT3DR(*,*,*,*)
OUT4DR(*,*,*,*,*)
OUT5DR(*)
OUT6DR(*,*);
OPTION SOLPRINT=OFF
OPTION SYSOUT=OFF
LOOP (RAPS,RAC=RISKAVER(RAPS);
SOLVE CKPPDR USING NLP MAXIMIZING PROFITDR;
LOOP (VARYR,H2O('AD')=(ORD(VARYR)*3500);
SOLVE CKPPDR USING NLP MAXIMIZING PROFITDR;
LOOP (VARYEI,H2O('IN')=(ORD(VARYEI)*3500);
SOLVE CKPPDR USING NLP MAXIMIZING PROFITDR;
OUT0DR(RAPS)=PROFITDR.L;
OUT1DR(VARYR,VARYEI)=WATERDR.M('AD');
OUT2DR(RAPS,C,'DR')=SUM(F,ACRES_A.L(F,C,'DR'));
OUT3DR(RAPS,C,'DR',S2)=SUM(F,ACRES_B.L(F,C,'DR',S2));
OUT4DR(RAPS,C,'DR',S2,S3)=SUM(F,ACRES_C.L(F,C,'DR',S2,S3));
OUT5DR(RAPS)=SUM((F,C,S1),(AH(F,C)-ACRES_A.L(F,C,'DR'))*(AH(F,C)-
ACRES_A.L(F,C,'DR')));
OUT6DR(VARYR,VARYEI)=WATERDR.M('IN'))));
FILE DRYRAP/DRYRAP.TXT/;
DRYRAP.PC=5;
PUT DRYRAP;
PUT@1'RAPS'@7'VALUE OF OBJ FUNC'/
LOOP((RAPS),PUT RAPS.TL,OUT0DR(RAPS)/);
PUT@1'VARYR'@7'VARYEI'@33'AD $/HECTRARE-CM'/
LOOP((VARYR,VARYEI),PUT VARYR.TL,PUT
VARYEI.TL,OUT1DR(VARYR,VARYEI)/);
PUT@1'VARYR'@7'VARYEI'@33'IN $/HECTRARE-CM'/
LOOP((VARYR,VARYEI),PUT VARYR.TL,PUT
VARYEI.TL,OUT6DR(VARYR,VARYEI)/);
PUT@1'RAPS'@7'C'@15'S1'@35'HECTARES_A'/
LOOP((RAPS,C,S1('DR')),PUT RAPS.TL,PUT C.TL,PUT S1.TL,OUT2DR(RAPS,C,'DR')/);
PUT@1'RAPS'@7'C'@19'S1'@25'S2'@30'HECTARES_B'/
LOOP((RAPS,C,S1('DR'),S2),PUT RAPS.TL,PUT C.TL,PUT S1.TL,PUT
S2.TL,OUT3DR(RAPS,C,'DR',S2)/);
PUT@1'RAPS'@7'C'@19'S1'@25'S2'@30'S3'@35'HECTARES_C'/
LOOP((RAPS,C,S1('DR'),S2,S3),PUT RAPS.TL,PUT C.TL,PUT S1.TL,PUT S2.TL,PUT
S3.TL,OUT4DR(RAPS,C,'DR',S2,S3)/);
PUT@1'RAPS'@7'SUMDEV'/
LOOP((RAPS),PUT RAPS.TL,OUT5DR(RAPS)/);
PUTCLOSE;
************************************************************
*ESTIMATION OF STOCHASTIC IRRIGATION WATER DEMANDS
************************************************************
SETS
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VRYIRR  ADEQUATE DELIVERIES SCENARIOS /1*20/
VRYEI   INADEQUATE DELIVERIES SCENARIOS /1*20/;
SCALAR
RAP          RISK AVERSION PARAMETER
/0.00002/
EQUATIONS
OBJDR1                   CKPP MODEL OBJECTIVE FUNCTION GIVEN SSM IS DR;
OBJDR1..PROFITDR=E=SUM{(F,S1('DR'),S2,S3),PR1('DR')*PR2(S2)*PR3(S3)*
[SUM(C,(EP(C)*TONNESDR(F,C,'DR',S2,S3))-CSTADR(F,C,'DR')-CSTBDR(F,C,'DR',S2)-
CSTCDR(F,C,'DR',S2,S3))-
SUM((C,CP),(RAP/2)*ACRES_C(F,C,'DR',S2,S3)*VARCOVNR(C,CP)*ACRES_C(F,C,'D
R',S2,S3))]};
MODEL CKPPDR1
/OBJDR1,LNDDR,SPWTRTDR,SFWTRTDR,BRL_ROTDR,CNL_ROTDR,ALF_ROTDR,P
AST_ROTDR,POT_ROTDR,PEAS_ROTDR,WATERDR,
LINK1DR, LINK2DR,VCADR,
VCBDR,VCCDR,TONNAGEDR/;
PARAMETER
OU0DR(*,*)
OU1DR(*,*)
OU2DR(*,*,*,*)
OU3DR(*,*,*,*,*)
OU4DR(*,*,*,*,*,*)
OU6DR(*,*);
OPTION SOLPRINT=OFF
OPTION SYSOUT=OFF
LOOP (VRYIRR,H2O('AD')=(ORD(VRYIRR)*3500);
SOLVE CKPPDR1 USING NLP MAXIMIZING PROFITDR;
LOOP (VRYEI,H2O('IN')=(ORD(VRYEI)*3500);
SOLVE CKPPDR1 USING NLP MAXIMIZING PROFITDR;
OU0DR(VRYIRR,VRYEI)=PROFITDR.L;
OU1DR(VRYIRR,VRYEI)=WATERDR.M('AD');
OU2DR(VRYIRR,VRYEI,C,'DR')=SUM(F,ACRES_A.L(F,C,'DR'));
OU3DR(VRYIRR,VRYEI,C,'DR',S2)=SUM(F,ACRES_B.L(F,C,'DR',S2));
OU4DR(VRYIRR,VRYEI,C,'DR',S2,S3)=SUM(F,ACRES_C.L(F,C,'DR',S2,S3));
OU6DR(VRYIRR,VRYEI)=WATERDR.M('IN')));
FILE DRYSWD/DRYSWD.TXT/;
DRYSWD.PC=5;
PUT DRYSWD;
PUT@1'VRYIRR'@7'VRYEI'@33'VALUE OF OBJ FUNC'/
LOOP((VRYIRR,VRYEI),PUT VRYIRR.TL,PUT VRYEI.TL,OU0DR(VRYIRR,VRYEI)/);
PUT@1'VRYIRR'@7'VRYEI'@33'AD $/HECTRARE-CM'/
LOOP((VRYIRR,VRYEI),PUT VRYIRR.TL,PUT VRYEI.TL,OU1DR(VRYIRR,VRYEI)/);
PUT@1'VRYIRR'@7'VRYEI'@33'IN $/HECTRARE-CM'/
LOOP((VRYIRR,VRYEI),PUT VRYIRR.TL,PUT VRYEI.TL,OU6DR(VRYIRR,VRYEI)/);
PUT@1'VRYIRR'@10'VRYEI'@20'C'@25'S1'@30'HECTARES_A'/
LOOP((VRYIRR,VRYEI,C,S1('DR')),PUT VRYIRR.TL,PUT VRYEI.TL,PUT C.TL,PUT
S1.TL,OU2DR(VRYIRR,VRYEI,C,'DR')/);
PUT@1'VRYIRR'@10'VRYEI'@20'C'@25'S1'@30'S2'@35'HECTARES_B'/
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LOOP((VRYIRR,VRYEI,C,S1('DR'),S2),PUT VRYIRR.TL,PUT VRYEI.TL,PUT C.TL,PUT
S1.TL,PUT S2.TL,OU3DR(VRYIRR,VRYEI,C,'DR',S2)/);
PUT@1'VRYIRR'@10'VRYEI'@20'C'@25'S1'@30'S2'@35'S3'@40'HECTARES_C'/
LOOP((VRYIRR,VRYEI,C,S1('DR'),S2,S3),PUT VRYIRR.TL,PUT VRYEI.TL,PUT
C.TL,PUT S1.TL,PUT S2.TL,PUT S3.TL,OU4DR(VRYIRR,VRYEI,C,'DR',S2,S3)/);
PUTCLOSE;
*****************************************************************
*ESTIMATION OF CONDITIONAL STOCHASTIC IRRIGATION WATER
*DEMANDS FOR 'ADEQUATE' SPRING SOIL MOISTURE STATE OF
*NATURE HOLDING 'INADEQUATE' WATER DELIVERIES AT ZERO
*PERCENT OF WATER DELIVERIES
*****************************************************************
SETS
VARY    ADEQUATE DELIVERIES SCENARIOS /1*20/
VARYE   INADEQUATE DELIVERIES SCENARIOS /1/;
EQUATIONS
OBJDR2              CKPP MODEL OBJECTIVE FUNCTION GIVEN SSM IS DR;
OBJDR2..PROFITDR=E=SUM{(F,S1('DR'),S2,S3),PR1('DR')*PR2(S2)*PR3(S3)*[SUM(C,(
EP(C)*TONNESDR(F,C,'DR',S2,S3))-CSTADR(F,C,'DR')-CSTBDR (F,C,'DR',S2)-
CSTCDR(F,C,'DR',S2,S3))-SUM((C,CP),(RAP/2)*ACRES_C(F,C,'DR',S2,S3)*
VARCOVNR(C,CP)*ACRES_C(F,C,'DR',S2,S3))]};
MODEL CKPPDR2
/OBJDR2,LNDDR,SPWTRTDR,SFWTRTDR,BRL_ROTDR,CNL_ROTDR,ALF_ROTDR,P
AST_ROTDR,POT_ROTDR,PEAS_ROTDR,WATERDR,
LINK1DR, LINK2DR,VCADR,
VCBDR,VCCDR,TONNAGEDR/;
PARAMETER
UT0DR(*,*)
UT1DR(*,*)
UT2DR(*,*,*,*)
UT3DR(*,*,*,*,*)
UT4DR(*,*,*,*,*,*)
UT6DR(*,*);
OPTION SOLPRINT=OFF
OPTION SYSOUT=OFF
LOOP (VARY,H2O('AD')=(ORD(VARY)*3500);
SOLVE CKPPDR2 USING NLP MAXIMIZING PROFITDR;
LOOP (VARYE,H2O('IN')=(ORD(VARYE)*0);
SOLVE CKPPDR2 USING NLP MAXIMIZING PROFITDR;
UT0DR(VARY,VARYE)=PROFITDR.L;
UT1DR(VARY,VARYE)=WATERDR.M('AD');
UT2DR(VARY,VARYE,C,'DR')=SUM(F,ACRES_A.L(F,C,'DR'));
UT3DR(VARY,VARYE,C,'DR',S2)=SUM(F,ACRES_B.L(F,C,'DR',S2));
UT4DR(VARY,VARYE,C,'DR',S2,S3)=SUM(F,ACRES_C.L(F,C,'DR',S2,S3));
UT6DR(VARY,VARYE)=WATERDR.M('IN')));
FILE DRAI0/DRAI0.TXT/;
DRAI0.PC=5;
PUT DRAI0;
PUT@1'VARY'@7'VARYE'@33'VALUE OF OBJECTIVE FUNCTION'/
LOOP((VARY,VARYE),PUT VARY.TL,PUT VARYE.TL,UT0DR(VARY,VARYE)/);
```

```
PUT@1'VARY'@7'VARYE'@33'AD $/HECTRARE-CM'/
LOOP((VARY,VARYE),PUT VARY.TL,PUT VARYE.TL,UT1DR(VARY,VARYE)/);
PUT@1'VARY'@7'VARYE'@33'IN $/HECTRARE-CM'/
LOOP((VARY,VARYE),PUT VARY.TL,PUT VARYE.TL,UT6DR(VARY,VARYE)/);
PUT@1'VARY'@10'VARYE'@20'C'@25'S1'@30'HECTARES_A'/
LOOP((VARY,VARYE,C,S1('DR')),PUT VARY.TL,PUT VARYE.TL,PUT C.TL,PUT
S1.TL,UT2DR(VARY,VARYE,C,'DR')/);
PUT@1'VARY'@10'VARYE'@20'C'@25'S1'@30'S2'@35'HECTARES_B'/
LOOP((VARY,VARYE,C,S1('DR'),S2),PUT VARY.TL,PUT VARYE.TL,PUT C.TL,PUT
S1.TL,PUT S2.TL,UT3DR(VARY,VARYE,C,'DR',S2)/);
PUT@1'VARY'@10'VARYE'@20'C'@25'S1'@30'S2'@35'S3'@40'HECTARES_C'/
LOOP((VARY,VARYE,C,S1('DR'),S2,S3),PUT VARY.TL,PUT VARYE.TL,PUT
C.TL,PUT S1.TL,PUT S2.TL,PUT S3.TL,UT4DR(VARY,VARYE,C,'DR',S2,S3)/);
PUTCLOSE;
**************************************************
*SENSITIVITY ANALYSIS OF OUTPUT PRICE CHANGE
**************************************************
SETS
SENS1    SENSETIVITY ANALYSIS SCENARIOS /G1*G11/
VAR   ADEQUATE DELIVERIES SCENARIOS /1*20/
VARI  INADEQUATE DELIVERIES SCENARIOS /1*20/;
PARAMETER
PP1(C)   OUTPUT PRICE CHANGE FOR SENSITIVITY ANALYSIS
/1 0
2 0
3 0
4 0
5 0
6 0
7 0
8 0/
TABLE PC(C,SENS) PRICE CHANGE SCENARIOS
$INCLUDE C:\WATER\DATA\PRICE.TXT
EQUATIONS
OBJDR3            CKPP MODEL OBJECTIVE FUNCTION GIVEN SSM IS DR;
OBJDR3..PROFITDR=E=SUM{(F,S1('DR'),S2,S3),PR1('DR')*PR2(S2)*PR3(S3)*
[SUM(C,(PP(C)*TONNESDR(F,C,'DR',S2,S3))-CSTADR(F,C,'DR')-CSTBDR(F,C,'DR',S2)-
CSTCDR(F,C,'DR',S2,S3))-
SUM((C,CP),(RAP/2)*ACRES_C(F,C,'DR',S2,S3)*
VARCOVNR(C,CP)*ACRES_C(F,C,'DR',S2,S3))]};
MODEL CKPPDR3
/OBJDR3,LNDDR,SPWTRTDR,SFWTRTDR,BRL_ROTDR,CNL_ROTDR,ALF_ROTDR,P
AST_ROTDR,POT_ROTDR,PEAS_ROTDR,WATERDR,
LINK1DR, LINK2DR,VCADR,
VCBDR,VCCDR,TONNAGEDR/;
PARAMETER
OUT0D(*,*,*)
OUT1D(*,*,*)
OUT2D(*,*,*,*,*)
OUT3D(*,*,*)
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OUT4D(*);
OPTION SOLPRINT=OFF
OPTION SYSOUT=OFF
LOOP (SENS1,PP1(C)=EP(C)*PC1(C,SENS1);
SOLVE CKPPDR3 USING NLP MAXIMIZING PROFITDR;
LOOP (VAR,H2O('ad')=(ORD(VAR)*3500);
SOLVE CKPPDR3 USING NLP MAXIMIZING PROFITDR;
LOOP (VARI,H2O('in')=(ORD(VARI)*3500);
SOLVE CKPPDR3 USING NLP MAXIMIZING PROFITDR;
OUT0D(SENS1,VAR,VARI)=PROFITDR.L;
OUT1D(SENS1,VAR,VARI)=WATERDR.M('AD');
OUT2D(SENS1,VAR,VARI,C,'DR')=SUM(F,ACRES_A.L(F,C,'DR'));
OUT3D(SENS1,VAR,VARI)=WATERDR.M('IN');
OUT4D(SENS1)=SUM((F,C,S1),(AH(F,C)-ACRES_A.L(F,C,'DR'))*(AH(F,C)-
ACRES_A.L(F,C,'DR'))))));
FILE DRPRSENS/DRPRSENS.TXT/;
DRPRSENS.PC=5;
PUT DRPRSENS;
PUT@1'SENS1'@5'VAR'@15'VARI'@25'VALUE OF OBJ FUNC'/
LOOP((SENS1,VAR,VARI),PUT SENS1.TL,PUT VAR.TL,PUT
VARI.TL,OUT0D(SENS1,VAR,VARI)/);
PUT@1'SENS1'@5'VAR'@15'VARI'@25'AD $/HECTRARE-CM'/
LOOP((SENS1,VAR,VARI),PUT SENS1.TL,PUT VAR.TL,PUT
VARI.TL,OUT1D(SENS1,VAR,VARI)/);
PUT@1'SENS1'@5'VAR'@15'VARI'@25'IN $/HECTRARE-CM'/
LOOP((SENS1,VAR,VARI),PUT SENS1.TL,PUT VAR.TL,PUT
VARI.TL,OUT3D(SENS1,VAR,VARI)/);
PUT@1'SENS1'@5'VAR'@15'VARI'@25'C'@27'S1'@35'HECTARES_A'/
LOOP((SENS1,VAR,VARI,C,S1('DR')),PUT SENS1.TL,PUT VAR.TL,PUT VARI.TL,PUT
C.TL,PUT S1.TL,OUT2D(SENS1,VAR,VARI,C,'DR')/);
PUT@1'SENS1'@7'SUMDEV'/
LOOP((SENS1),PUT SENS1.TL,OUT4D(SENS1)/);
PUTCLOSE;
**********************************************************************
*SENSITIVITY ANALYSIS OF CHANGES IN PROBABILITY DISTRIBUTIONS *OF
EFFECTIVE IRRIGATION
**********************************************************************
SETS
VARYRR  ADEQUATE DELIVERIES SCENARIOS /1*20/
ARYEI   INADEQUATE DELIVERIES SCENARIOS /1*20/
SENS2    SENSETIVITY ANALYSIS SCENARIOS /P1*P9/;
TABLE PRO(S2,SENS2)  PROBABILITY SCENARIOS
$INCLUDE C:\WATER\DATA\PROBABILITIESA.TXT
EQUATIONS
OBJDR4                  CKPP MODEL OBJECTIVE FUNCTION GIVEN SSM IS NR;
OBJDR4..PROFITDR=E=SUM{(F,S1('DR'),S2,S3),PR1('DR')*PR2(S2)*PR3(S3)*[SUM(C,(
EP(C)*TONNESDR(F,C,'DR',S2,S3))-CSTADR(F,C,'DR')-CSTBDR(F,C,'DR',S2)-
CSTCDR(F,C,'DR',S2,S3))-SUM((C,CP),(RAP/2)*ACRES_C(F,C,'DR',S2,S3)*
VARCOVNR(C,CP)*ACRES_C(F,C,'DR',S2,S3))]};
```

```
MODEL CKPPDR4
/OBJDR4,LNDDR,SPWTRTDR,SFWTRTDR,BRL_ROTDR,CNL_ROTDR,ALF_ROTDR,P
AST_ROTDR,POT_ROTDR,PEAS_ROTDR,WATERDR,
LINK1DR, LINK2DR,VCADR,
VCBDR,VCCDR,TONNAGEDR/;
PARAMETER
OUT0(*,*,*)
OUT1(*,*,*)
OUT6(*,*,*)
OUT2(*,*,*,*,*)
OUT4(*);
OPTION SOLPRINT=OFF
OPTION SYSOUT=OFF
LOOP (SENS2,PR2(S2)=PRO(S2,SENS2);
SOLVE CKPPDR4 USING NLP MAXIMIZING PROFITDR;
LOOP (VARYRR,H2O('ad')=(ORD(VARYRR)*3500);
SOLVE CKPPDR4 USING NLP MAXIMIZING PROFITDR;
LOOP (ARYEI,H2O('in')=(ORD(ARYEI)*3500);
SOLVE CKPPDR4 USING NLP MAXIMIZING PROFITDR;
OUT0(SENS2,VARYRR,ARYEI)=PROFITDR.L;
OUT1(SENS2,VARYRR,ARYEI)=WATERDR.M('AD');
OUT2(SENS2,VARYRR,ARYEI,C,'DR')=SUM(F,ACRES_A.L(F,C,'DR'));
OUT4(SENS2)=SUM((F,C,S1),(AH(F,C)-ACRES_A.L(F,C,'DR'))*(AH(F,C)-
ACRES_A.L(F,C,'DR')));
OUT6(SENS2,VARYRR,ARYEI)=WATERDR.M('IN'))));
FILE DPRBSENS/DPRBSENS.TXT/;
DPRBSENS.PC=5;
PUT DPRBSENS;
PUT@1'SENS2'@7'VARYRR'@15'ARYEI'@20'VALUE OF OBJ FUNC'/
LOOP((SENS2,VARYRR,ARYEI),PUT SENS2.TL,PUT VARYRR.TL,PUT
ARYEI.TL,OUT0(SENS2,VARYRR,ARYEI)/);
PUT@1'SENS2'@7'VARYRR'@15'ARYEI'@20'AD $/HECTRARE-CM'/
LOOP((SENS2,VARYRR,ARYEI),PUT SENS2.TL,PUT VARYRR.TL,PUT
ARYEI.TL,OUT1(SENS2,VARYRR,ARYEI)/);
PUT@1'SENS2'@7'VARYRR'@15'ARYEI'@20'IN $/HECTRARE-CM'/
LOOP((SENS2,VARYRR,ARYEI),PUT SENS2.TL,PUT VARYRR.TL,PUT
ARYEI.TL,OUT6(SENS2,VARYRR,ARYEI)/);
PUT@1'SENS2'@5'VARYRR'@15'ARYEI'@25'C'@27'S1'@35'HECTARES_A'/
LOOP((SENS2,VARYRR,ARYEI,C,S1('DR')),PUT SENS2.TL,PUT VARYRR.TL,PUT
ARYEI.TL,PUT C.TL,PUT S1.TL,OUT2(SENS2,VARYRR,ARYEI,C,'DR')/);
PUT@1'SENS2'@7'SUMDEV'/
LOOP((SENS2),PUT SENS2.TL,OUT4(SENS2)/);
PUTCLOSE;
*********************************************************************
*SENSITIVITY ANALYSIS FOR CHANGES IN PROBABILITY DISTRIBUTIONS OF
*EFFECTIVE PRECIPITATION AND SPRING SOIL MOISTURE ARE
*ACCOMPLISHED BY SUBSTITUTING THE VALUES OF THE DESIRED CHANGES
*IN THE DISTRIBUTIONS AND SOLVING THE CKPP SUB-MODELS. SENSITIVITY
*ANALYSIS FOR CONSTRAINING POTATO HECTARES IS ACCOMPLISHED BY
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*MODEFYING THE ROTATION CONSTRAINT FOR POTATO TO EQUAL THE
*DESIRED VALUE OF THE RIGHT HAND SIDE OF THE CONSTRAINT.
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