

Quantification of Damage and its Effects on the Compressive Strength of an Advanced Ceramic

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Abstract

An understanding of the dynamic failure of damaged ceramics is important in protection applications, where the interaction of the projectile with cracked material is a contributing factor in the overall system performance. In this paper, we investigate the effects of pre-existing internal cracks on the quasi-static and dynamic compressive behavior of an advanced ceramic. We present experiments on a hot-pressed boron carbide in which internal cracks are generated through thermal shocking after which the initial material damage is quantified. Damage characterization was performed via Resonant Ultrasound Spectroscopy (RUS) and high-resolution Computed Tomography (CT). A computational procedure is developed to determine the three-dimensional structure of the internal crack network in the initially damaged material from a series of CT images. The failure and strength of the material is then evaluated experimentally. The uniaxial compressive strength of the predamaged boron carbide samples is determined under both quasistatic and dynamic loading scenarios and this is correlated with the pre-existing crack structure as determined by CT. Damaged samples were found to have average compressive strength of 1.14 GPa in quasistatic loading and 0.68 GPa in dynamic loading compared to 2.98 ± 0.6 GPa and 3.70 ± 0.3 GPa for pristine material, respectively. High speed photography employed during dynamic testing indicates that pre-existing cracks may lead to different failure mechanisms from what is normally seen in pristine material. Ultimately, these insights can be used to design improved materials that are more resistant to dynamic failure.

Keywords: microcracks; thermal shock; compressive fragmentation; brittle failure; experimental mechanics; advanced ceramics;

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Nomenclature

B	initial binary image matrix
<i>E</i>	explanation for the abbreviation
\bar{l}	average measured grain size
M	final binary image matrix
M'	padded final binary image matrix
\bar{s}	average defect size
S	integer image matrix
v_{shear}	shear wave speed
η	defect density
ν	Poisson's ratio
ρ	density
μ_{log}	disk size lognormal distribution mean
μ_n	disk size normal distribution mean
σ_{log}	disk size lognormal distribution standard deviation
σ_n	disk size normal distribution standard deviation

1. Introduction

Designing advanced ceramics for protection applications requires an understanding of failure and fragmentation mechanisms. These mechanisms have been shown to depend on material processing and specifics of the stress-state and strain-rate. Recent work has aimed to understand how the presence of internal fractures prior to loading affects the failure process [1] [2] [3], and the results suggest that pre-existing cracks will have a considerable effect on the material response of advanced ceramics. Studying the mechanical response of damaged material will help us understand the process of damage evolution in advanced ceramics. This paper presents an approach to quantitatively

10 measure the degree of damage within an advanced ceramic and then to relate this to its
11 compressive response.

12 Previous studies of advanced ceramics have focused on characterizing the size,
13 shapes, and types of microstructural features which are believed to serve as fracture
14 initiation points [4] [5] [6] [7]. Studies have also focused on characterizing grain
15 sizes and shapes, as well as material anisotropy [8]. Significant work has also been per-
16 formed to characterize the rate-dependence of compressive strength [9] [10] [11] [12].
17 Under quasistatic compression the most deleterious defects are activated first, leading
18 to crack growth and sample failure [5]. Under dynamic compression, however, the rate
19 of loading is such that additional defects are activated before substantial crack growth
20 occurs from the most deleterious defects (given finite crack speeds), and so dynamic
21 loading leads to the activation of more distributed damage.

22 As experimental work has shown the importance of loading-rate, microstructural
23 features, and microcracks in the failure process, computational models have been de-
24 veloped to try to capture these dependencies [13] [14]. The micro-mechanical mod-
25 eling approach, for example, incorporates pre-existing flaw distributions in the material
26 [15], and defines damage based on an evolving scalar crack density parameter. Similar
27 work by Hu and Ramesh has further extended this type of model to account for flaw
28 orientations and a tensorial damage parameter [16]. Tonge and Ramesh [17] further
29 extended this approach into a full constitutive model with a computational implementa-
30 tion for large-scale simulations of impact events. Other researchers, such as Johnson and
31 Holmquist, have modeled damage and damage evolution through a heuristic approach.
32 The JH2 model defines a set of constants that can be determined by fitting experimental
33 data over a wide range of loading conditions [18]. This approach does not directly
34 incorporate any microstructure or crack statistics, yet can have strong predictive power

35 when parameters are fit to a sufficiently large data set.

36 Motivated by the need to better understand the behavior of damaged materials and
37 provide experimental data for modelling damaged behavior of brittle materials [18]
38 [19] [20], this paper explores the quasistatic and dynamic compressive behavior of pre-
39 cracked boron carbide. To accomplish damaged states, we thermally shock cuboidal
40 specimens to produce internal crack networks that are \sim mm in length scale and of the
41 order of the test specimens. These crack networks are characterized through X-Ray to-
42 mography and Matlab-based image processing and are linked with mechanical proper-
43 ties and mechanical behavior. Other studies have used thermal shocking and mechanical
44 loading to produce internal cracking in specimens within confined test setups [1] [2].
45 In this paper, we use thermal shocking because it allows us to produce porosities of in-
46 terests (\sim 1 to 3%) and crack sizes that are resolvable in X-Ray tomography scans, thus
47 allowing, for the first time, a quantified measure of cracking that can be coupled to me-
48 chanical testing. The use of thermal shocking to produce \sim mm-sized cracked material
49 also yields structural cracking sizes that are similar to those observed during impact into
50 the same material [21], where understanding the impact behavior of this material is an
51 overarching goal of this paper. In undertaking this research, attempts have been made
52 to generate internal cracking through mechanical cycling, but this either resulted in too
53 few and too small of cracks, or chipping at the specimen surface. Altogether, this work
54 contributes to a limited data set in the literature for cracked specimens [1] [2] [22]
55 [23], where the bulk of the work on advanced ceramics has focused on intact materials
56 and resulting fragmentation [4] [21] [24] [25] [26] [27]. Understanding the behavior
57 of materials for intermediate levels of damage is valuable in validating and improving
58 models [16] [15] [17] [18] that can be used to design improved ceramic materials for,
59 for example, ballistic impact, where fracture and fragmentation behaviors are important

60 in their overall performance [28].

61 **2. Material**

62 The material in this study was a hot-pressed boron carbide (Coorstek, Inc.) with
63 a Young's modulus of 430 GPa, a density of 2,510 kg/m³, a Poisson's ratio of 0.16
64 to 0.17, and a fracture toughness of 2.5 MPa \sqrt{m} . These values are provided by the
65 manufacturer. The grain structure is equiaxed with an average grain size of 15 μm .
66 Almost all the grain boundaries have a high misorientation angle ($> 15^\circ$). This material
67 has been used in previous studies by the authors and collaborators, where the focus
68 has been on microstructural characterization [6], compressive strength and failure of
69 intact forms [5] [8], compressive fragmentation [4], and impact fragmentation [21].
70 Additional details are provided in these references.

71 The boron carbide material was received as a tile (conceptualized in 1a with disk-
72 like features in the figure meant to represent the carbonaceous inclusions presented later
73 in b) with dimensions of 305 mm length, 254 mm in width, and 8 mm in thickness.
74 For this study, rectangular prismatic boron carbide samples of approximate dimension
75 $3.5 \times 4.0 \times 5.3\text{mm}$ were machined from these larger tiles with the longest dimension
76 being oriented along the hot-pressing or "through-thickness" (TT) direction. During
77 sample machining, effort was made to minimize sub-surface damage by systematically
78 polishing the edges from $\sim 100 \mu\text{m}$ finish to a $2 \mu\text{m}$ finish following the cutting oper-
79 ation. Examination of X-ray tomography scans (shown later) reveals no clear evidence
80 of sub-surface damage from machining in the cubes under consideration in the current
81 investigation.

82 The inclusions and defects in the microstructure were characterized using a Zeiss
83 optical microscope with an AxioCam MRC camera and a TESCAN MIRA3 field emis-

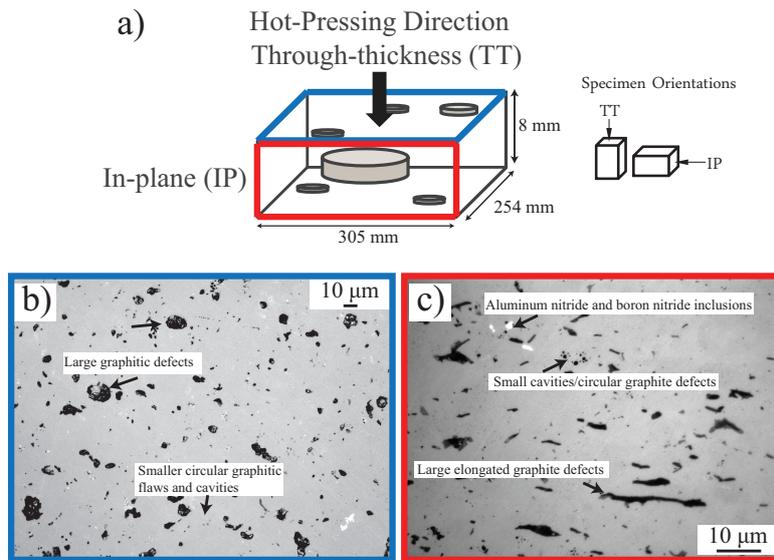


Fig. 1: (a) Conceptualized as-received tile of the hot-pressed boron carbide plate with through-thickness (TT) (in the hot-pressing direction) and in-plane directions (IP) labeled. Optical microscope images of the boron carbide microstructure in the (b) through-thickness (at $10\times$ magnification) and (c) in-plane direction ($100\times$ magnification) with the various types of inclusions and defects. [5]

84 sion Scanning Electron Microscope (SEM) equipped with a fully automated electron
 85 backscatter diffraction (EBSD) analysis system and Energy Dispersive Spectroscopy
 86 (EDS) capabilities. The word “defect” in this paper is used to denote a microstructural
 87 feature that may serve as a failure initiation site and “inclusion” is used to denote a
 88 feature that is not believed to contribute to failure (at least not under the stress states
 89 studied here). The processing-induced inclusions and defects are most easily seen in
 90 optical microscope images such as those shown in Fig. 1b and c. The image on the left
 91 is taken on the TT face, while the image on the right is taken on the IP face (note the
 92 different scale bars). Large, approximately circular, dark features are observed in the
 93 TT images in Fig. 1b. These have been confirmed to be carbonaceous in composition
 94 using EDS. Also highlighted in Fig. 1b are smaller and more circular features. These
 95 are primarily smaller graphitic defects, with other smaller features consisting of cavi-

96 ties/pores (confirmed with SEM/EDS). Brighter phases are also noted in Fig. 1b and
 97 these appear to be primarily comprised of aluminum nitride (AlN) and boron nitride
 98 (BN) (confirmed with SEM/EDS). These inclusions are faceted structures less than 20
 99 μm in size, and are commonly observed at boron carbide grain boundaries.

100 Previous work has shown that graphitic disks in boron carbide are the microstruc-
 101 tural features where fracture typically nucleates during failure in pristine (undamaged)
 102 specimens [5]. Again, these features are shown in the schematic representation of the
 103 as-received tile in Fig. 1a. The orientation, size, and spacing of these defects governs a
 104 variety of length scales that dictate rate dependence of the material strength [29]. These
 105 statistics are summarized in Table 2. The graphitic disk defect size was found to be well
 106 characterized by a lognormal distribution given by μ_{log} and σ_{log} whereas the defect ori-
 107 entation was found to be well characterized by a normal distribution given by μ_n and σ_n
 108 [5].

Material	\bar{l} (μm)	η (#/ m^2)	\bar{s} (μm)	μ_{log} (μm)	σ_{log} (μm)	μ_n	σ_n
PAD Boron Carbide	16.0 ± 2.1	1.41×10^9	4.22 ± 2.54	1.30 ± 0.02	0.53 ± 0.02	$0 \pm 1^\circ$	$20 \pm 1^\circ$

Table I: Microstructure characteristics for pressure aided densification boron carbide, including \bar{l} : average measured grain size, η : defect density measured on the IP face for $s > 0.5\mu\text{m}$, \bar{s} : average defect size, μ_{log} , σ_{log} : disk size lognormal distribution parameters, μ_n , σ_n : disk orientation normal distribution parameters

109

110 For a material that already contains a significant number of flaws, it is not yet known
 111 exactly what effect the presence of these microstructure features will have upon the
 112 evolution of damage in a pre-damaged sample subjected to additional loading. We will
 113 explore this here.

114 **3. Experimental Approach**

115 Studies of damage evolution and the effects of internal cracking on the failure and
116 strength of such materials [1] are limited. We use the approach described in Fig. 2
117 to capture the intermediate state of the material, with the intention of providing insight
118 into dynamic damage evolution.

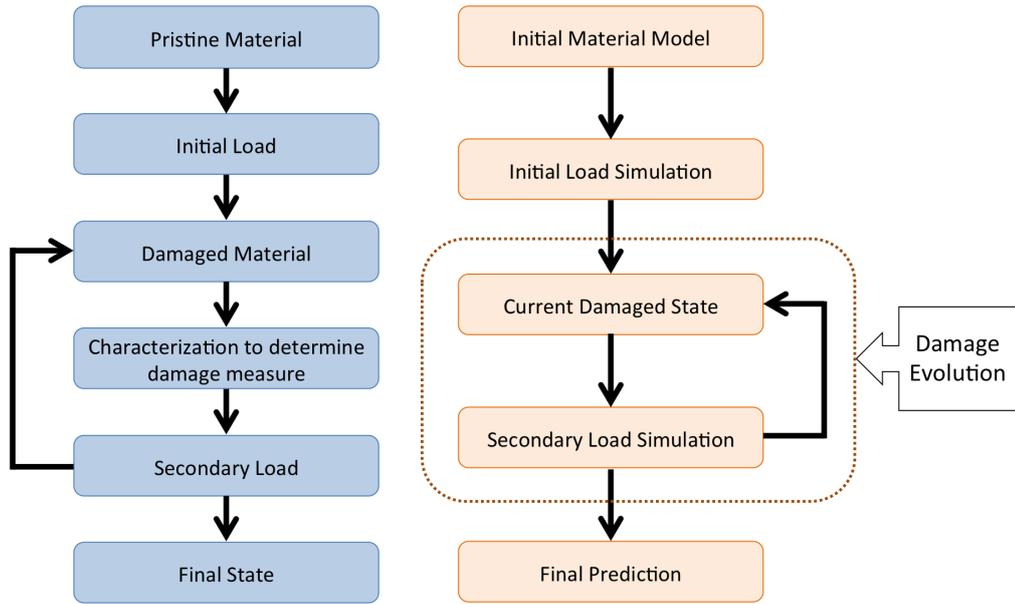


Fig. 2: Left: schematic showing the experimental process developed for this study. Traditional experiments go directly from the *initial load* to the *final state*. Right: schematic showing common computational procedure for simulating loading with damage evolution.

119 We achieve an intermediate damaged state by subjecting samples to an *initial load*
120 that induces internal fracture without actually causing specimen fragmentation. The
121 specimen is then characterized in this damaged state before an additional loading is pro-
122 vided. This process can be repeated until fragmentation occurs and the specimen has lost
123 its load bearing capacity. The *pristine material* in our study is a sample of the boron car-
124 bide material described above. Although the specimen contains a variety of microstruc-

125 tural defects, it is considered to be free of damage until cracks have been induced by the
126 initial (or primary) load. The *primary load* is performed via a thermal shock process
127 described below. This leads to an intermediate *damaged material*. *Characterization* of
128 this material provides a quantifiable *damage measure*. The characterization presented
129 here was carried out via microscale CT to determine the internal crack structure and via
130 resonant ultrasound spectroscopy to examine potential degradation of elastic properties.
131 The secondary loading of this damaged sample is performed either via dynamic com-
132 pressive testing with a Kolsky bar apparatus or through quasistatic compression with an
133 MTS load frame system. The details of these tests are described below. If the secondary
134 loading event does not fragment the sample, the process of characterization and loading
135 can be repeated until a fragmented state is reached.

136 *3.1. Thermal Shock*

137 Material damage was induced via a thermal shock process. Previous studies on
138 heated boron carbide with cooling on the boundary have shown that large thermal gra-
139 dients lead to cracks propagating inward from the material boundary due to large tensile
140 stresses [30]. For example, Chocron et. al. [1] investigated a thermal shock pro-
141 cess through repeated heating and quenching on Pressure Assisted Densification (PAD)
142 boron carbide and found that the extent of the crack propagation depends on the number
143 of thermal cycles.

144 Five samples of boron carbide with approximate dimension $3.5 \times 4 \times 5.3$ mm were
145 heated in a furnace in air. Samples were placed on small alumina tiles which could be
146 easily manipulated with furnace tongs. The alumina tiles were placed on alumina foam
147 blocks in the furnace. Samples were heated for ~ 1 hour to temperatures $550 - 850^\circ\text{C}$.
148 After heating, the alumina tiles were removed from the furnace and tipped over into

149 a beaker of room temperature water thereby immersing the samples. Certain samples
150 were run through two heat and quench cycles to increase the degree of internal cracking.
151 The thermal cycling parameters of the samples can be found in Table 4.1. These are
152 discussed later.

153 *3.2. Resonant Ultrasound Spectroscopy*

154 Pristine and thermally shocked samples were subjected to resonant ultrasound spec-
155 troscopy (RUS) to determine elastic properties [31] [32]. The RUS data was measured
156 using the Magnaflux Quasar RUSpec system at the U.S. Army Research Laboratory in
157 Aberdeen Proving Ground, Maryland. The sample is held in place between two trans-
158 ducers as the system sweeps frequencies, and the resonant frequencies of the sample
159 appear as peaks in the measured spectrum. The samples' dimensions are measured by
160 hand using a micrometer, while densities are measured using the Archimedes' method.
161 This data is input into the RUSpec system as part of the sample information. Using the
162 density, the dimensions and the resonant frequencies, the RUSpec software performs it-
163 erations using the Levenberg - Marquardt algorithm to generate a best fit to the resonant
164 frequencies and thus determine the elastic constants of the sample and the wave speeds
165 which can be found in Table 4.1.

166 *3.3. Microscale Computed Tomography (microCT)*

167 *3.3.1. MicroCT Scanning*

168 The pristine and thermally shocked samples were scanned with a Bruker Skyscan
169 1172 to determine crack characteristics. Scans were performed on the thermally cracked
170 material as well as pristine material to determine a baseline level of damage. Each
171 specimen was mounted on a flat brass spindle using a small foam substrate, and wrapped
172 with parafilm to deter any slight movement during the scan. Single projection images

173 were then examined to determine the proper sensor exposure time as well as tube current
174 and voltage. These were found to be 149 μA and 29 kV with an exposure time of 2, 500
175 ms. A pixel size of 1.34 μm was used to obtain highest resolution of crack morphology
176 while maintaining a reasonable scan time. The scanning results in approximately 2, 000
177 to 4, 000 projection images per specimen depending on the orientation of the sample.

178 3.3.2. 3D Reconstruction

179 The images obtained from CT are then processed to build up a 3D reconstruction
180 of the fractures within the samples. Processing is performed first on the series of 2D
181 images from the scans before a 3D model of the crack morphology can be built. Fig.
182 3 shows the processing procedure for each projection image. The features shown are
183 internal as each projection image corresponds to a cross-section with a different depth
184 in the specimen. We have not shown pristine sample scans for brevity, but they resemble
185 the image seen in Fig. 3a and have similar bright spots corresponding to microstructural
186 features, but lack any visible lines corresponding to internal fracture.

187 The region of interest (ROI) for image processing is defined manually to be close to,
188 but within, the boundary of the material to prevent the edges of the sample from being
189 construed as cracks due to the high contrast at the perimeter. This is done by clicking
190 on points within the image through custom software to create a convex polygon whose
191 lines do not intersect with the boundary of the specimen in the scan. Note that the region
192 of interest shifts slightly from image to image, and so a ROI is defined for the first and
193 last images and a linear interpolation is used between the two limits.

194 In the initial image, the background material is gray with cracks making bright lines
195 on the image. The pixels corresponding to the bright lines of the cracks only make up
196 a small percentage of the image. Background removal (Fig. 3b) is applied whereby

197 for every pixel the mean intensity value for the neighborhood of pixels around it is
198 subtracted from its intensity. This produces a new image with lower intensity than the
199 original image at every pixel. As the bright cracks never make up a significant portion
200 of a neighborhood by percentage of pixels, the resulting image contains low intensity
201 (black) pixels where background (gray) pixels previously existed. Although the bright
202 pixels are not as bright as they were before, they stand out more against the now darker
203 background pixels which have almost zero intensity as can be seen in in Fig. 3b. Simple
204 contrast adjustment of this background image brings the initially bright pixels back up
205 to a high intensity (Fig. 3e). Although this also makes background noise brighter as
206 well, by thresholding most of this can be removed and this results in the binary image
207 in Fig. 3f.

208 In addition to contrast adjustment, gradient imaging enhances sharp contrasts at the
209 sides of cracks. Median filtering is used to reduce some of the noise introduced by the
210 gradient imaging resulting in Fig. 3c. This image is also converted to a binary images
211 with simple thresholding based of a percentile value of the intensity values for the whole
212 ROI. As each of the two processing paths described above has a tendency to miss a few
213 features, the resulting binary images are combined to create a final image.

214 A stack of K binary images output from the 2D processing, each $M \times N$, results
215 in a binary 3D matrix of size $K \times M \times N$. This initial matrix poses the difficulties
216 of (i) large amounts of noise, which appear as pixelized white features in 2D, and (ii)
217 computational difficulties due to the number of indices (i.e., size of the data). A scan for
218 a single specimen can contain $\approx 2,500$ images with each image being $\approx 4,000 \times 4,000$
219 pixels. The corresponding 3D matrix would have 40,000,000,000 indices or voxels
220 (volume-pixels) at full resolution.

221 These difficulties are overcome by scaling down the binary matrix to a new integer

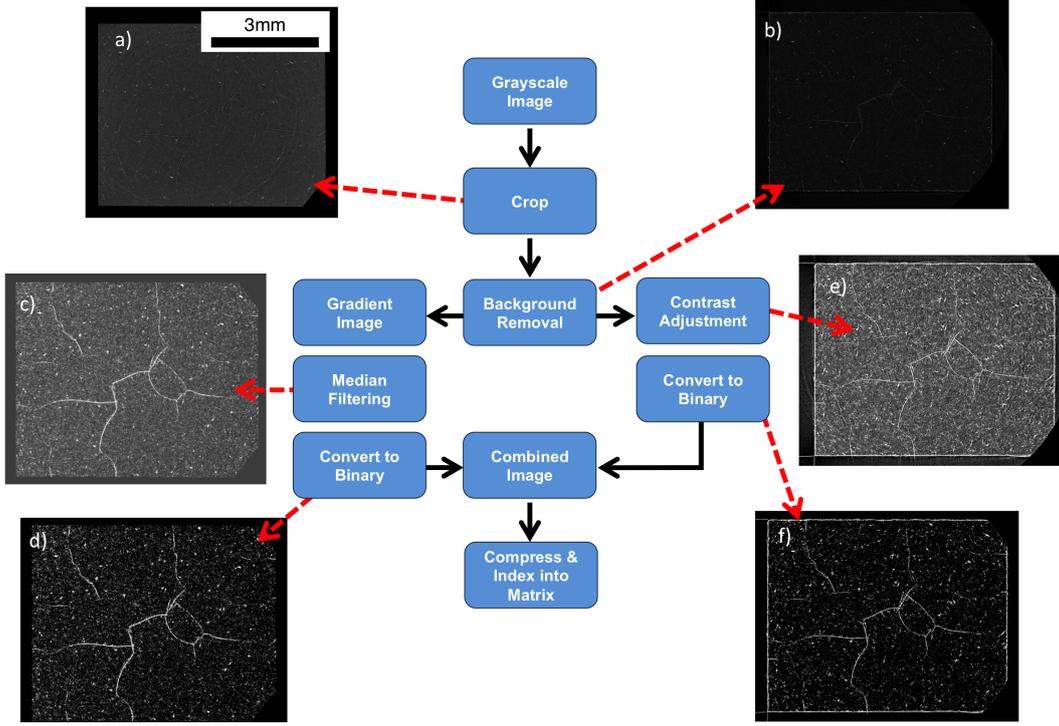


Fig. 3: Image processing flowchart for 2D input image. a) Cropped image with cropping defined within a few microns of the material boundary (see next figure for original image). b) Image after applying a background removal algorithm c) Result of applying median filtering to the gradient image of the result of the background removal step. d) Thresholded binary image from median filtering results e) Contrast adjusted version of background removed image f) Thresholded binary image from contrast adjusted image

222 matrix. For an initial binary matrix \mathbf{B} of size $K \times M \times N$ a new integer matrix, \mathbf{S} of
 223 size $\frac{K}{f} \times \frac{M}{f} \times \frac{N}{f}$ where f is an integer value that defines the scale parameter. For a set of
 224 indices (i, j, k) we define $p_i = f(i - 1) + 1$, $q_j = f(j - 1) + 1$, and $r_k = f(k - 1) + 1$. The
 225 large binary matrix \mathbf{B} is then mapped to the integer matrix \mathbf{S} as

$$\mathbf{S}(i, j, k) = \sum_{p=p_i}^{f_i} \sum_{q=q_j}^{f_j} \sum_{r=r_k}^{f_k} \mathbf{B}(p, q, r) \quad (1)$$

226 The entries in the new matrix \mathbf{S} are in the range of 0 to f^3 . In the resulting integer
 227 matrix regions that correspond to cracks then contain a high integer value whereas re-

228 gions of noise or microstructural defects contain low integer values. A threshold value,
229 ν is defined where $0 < \nu < f^3$ to obtain a final binary matrix, \mathbf{M} from \mathbf{S} .

$$\mathbf{M}(i, j, k) = \begin{cases} 1, & \text{if } \mathbf{S}(i, j, k) > \nu \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

230 By thresholding this integer matrix, noise regions where $0 < \mathbf{S}(i, j, k) < \nu$ can be
231 removed resulting in a binary 3D matrix whose non-zero entries correspond to crack
232 locations. The degree to which the initial matrix should be scaled down and the proper
233 thresholding value would depend on the degree of noise in the initial image and how
234 wide (in terms of pixels) cracks appear. In our case a scale value of $f = 6$ was found
235 to provide good results while also greatly reducing computation time. This reduces the
236 number of matrix entries from ≈ 40 billion to ≈ 185 million.

237 Two different approaches were used to generate 3D STL (Stereolithography) file
238 representations from this binary matrix, \mathbf{M} . An STL file simply contains a list of ver-
239 tices and a list of triangular faces comprised of these vertices. A series of connected
240 triangular faces is referred to as a “mesh.” An STL mesh can either be “two-manifold”
241 or “non-manifold.” Simply put, a two-manifold mesh could be split along edges and
242 laid flat without any faces overlapping. A non-manifold mesh will contain features such
243 as “T” junctions where three or more triangular faces share an edge, faces that share ver-
244 tices but not edges, adjacent faces with opposite normals, or other non-manifold features
245 [33]. When creating a 3D file to represent a fracture surface it may be ideal to represent
246 it as a zero-thickness surface as cracks are not typically considered to have a volume.
247 However, representing any sufficiently complex crack network as a zero thickness sur-
248 face necessitates creating it as a non-manifold mesh as intersecting fracture surfaces
249 will create “T” junctions or non-consistent surface normals.

250 Two reconstruction techniques were pursued. The first method will be referred to as
251 a manifold “volume method” as it creates a manifold mesh where all fracture surfaces
252 have 2 sides and therefor an enclosed volume. The second method will be referred to as
253 a “surface method” as it creates a zero-thickness surface though a non-manifold mesh.

254 3.3.3. Volume Approach

255 The volume method produces a finite thickness surface to represent the crack struc-
256 ture. A penny shaped crack, for example, would be represented as an ellipsoid with
257 this approach. In order to give the cracks two sides, the indices where $\mathbf{M}(i, j, k) = 1$
258 are padded with 1s in all direction to make the cracks thicker resulting in the matrix
259 \mathbf{M}' . After this, the resulting matrix is smoothed using a Gaussian smoothing algorithm.
260 With the cracks thickened, a Marching Cubes (MC) algorithm is used to turn the voxel
261 matrix \mathbf{M}' into a triangulated surface. A detailed explanation of the MC algorithm is
262 contained in the references [34]. The complexity of the resulting mesh is then reduced
263 using a Quadric Edge Collapse decimation algorithm. Finally, the resulting surface is
264 smoothed out using a Poisson surface reconstruction. Renderings of the resulting 3D
265 file can be seen in Fig. 4.

266 3.3.4. Surface Approach

267 The surface method approach produces a zero-thickness surface to represent the
268 crack structure. The final binary matrix, \mathbf{M} is converted to a point cloud, which is a list
269 of XYZ coordinates corresponding to locations of 1s in the matrix. Once a point cloud
270 has been created, the complexity of the point cloud is reduced using a clustered vertex
271 subsampling approach. After this, a local surface normal can be approximated on a per
272 point basis based on the location of nearby points. Once a point cloud with defined
273 point normals has been constructed a 3D Stereolithography file containing a list of faces

274 and vertices can be created. As crack intersections are more feasibly characterized
275 with a zero-thickness surface representation, a segmentation algorithm was developed
276 to segment the branched fracture network into individual surfaces.

277 *3.3.5. 3D Segmentation*

278 **The segmentation of separate cracks from the connected crack network is an unre-**
279 **solved challenge in the literature and an on-going research topic in the authors' groups.**

280 A segmentation algorithm was developed to divide the zero-thickness surface described
281 above into a set of disjoint, approximately planar surfaces. Segmentation begins by
282 removing mesh faces that are expected to branch different crack surfaces. First, a cur-
283 vature parameter is defined over the whole crack surface on a per face basis. Faces
284 with a curvature value that is above a computed threshold are likely located at the junc-
285 tion between two crack surfaces and are therefor removed. After removing this initial
286 set of faces, all faces that share an edge with only two or fewer faces are removed. A
287 breadth-first search (BFS) is then run over the set of remaining faces. This approach
288 considers faces to be individual graph nodes whereby two nodes are adjacent if their
289 corresponding mesh faces share an edge. The BFS generates a spanning forest contain-
290 ing spanning trees where each spanning tree contains the set of faces that comprise an
291 individual crack. For large crack faces further segmentation is performed using face
292 normal parameter that splits large, curved, crack surfaces into smaller groups with more
293 consistent face normals.

294 *3.4. Mechanical Testing*

295 *3.4.1. Quasistatic Testing*

296 Quasistatic compression tests were performed on an MTS Model C43.504 load
297 frame fitted with MTS Model LPS.504 load cells with a maximum load capacity of

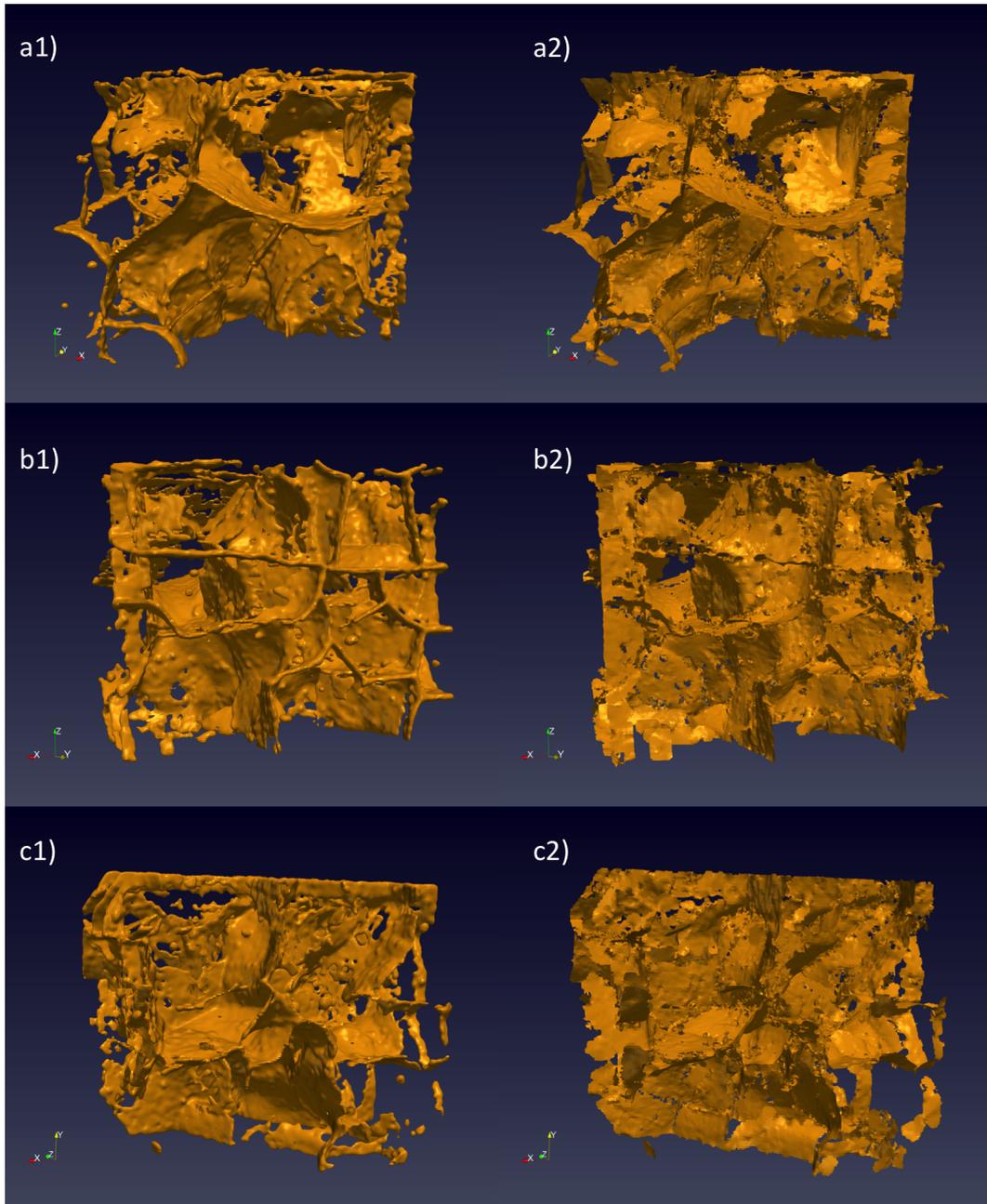


Fig. 4: 3 views of the reconstructions for sample TC69 are shown. The z-axis is the scan direction and the x-axis is the sintering (or TT) direction. The renderings on the left are from the volume reconstruction method (two-manifold). The renderings on the right are from the surface reconstruction (non-manifold). The length in the z-direction is approximately 3.2 mm

298 50 kN. As the boron carbide is harder than the hardened steel platens of the MTS ma-
299 chine, intermediate platens machined from SiC-N or WC with cross-sectional areas
300 larger than that of the boron carbide specimens were used to transmit the compressive
301 load to the boron carbide specimen without damaging the steel platens. A thin sleeve of
302 plastic sheeting was wrapped loosely around the specimen and SiC-N platens to allow
303 for collection of fragments after failure. Compression tests were run with a cross-head
304 displacement rate of $5 \mu\text{m/s}$, resulting in a nominal strain rate of $\approx 10^{-3} /\text{s}$.

305 3.4.2. *Dynamic Testing*

306 Dynamic uniaxial compression was performed via a Kolsky bar apparatus. The Kol-
307 sky bar setup consists of an incident and transmitted bar that sandwich the specimen and
308 a striker bar which impacts the incident bar propagating a stress pulse through it towards
309 the specimen. Kolsky bars have been used extensively to study ceramics including boron
310 carbide and details on experimental design can be readily found in literature [35].

311 The incident and transmitted bars, and projectile all had a diameter of 12.7 mm
312 and are made of maraging steel (VascoMax C-350) with a yield strength of 2.68 GPa,
313 elastic modulus of 200 GPa, Poisson's ratio of 0.29, and a density of $8,100 \text{ kg/m}^3$.
314 Their respective lengths were 1220 mm, 1050 mm and 127 mm. The length of the
315 projectile dictates the width of the stress pulse transmitted through the bars. To protect
316 against damaging the contact surface of the bars, 5 mm platens were placed on both
317 sides of the sample at the interface with the incident and transmitted bars. Platens were
318 made of impedance matched tungsten carbide (LC403, Leech Carbide) jacketed by Ti-
319 6Al-4V sleeves. A small amount of grease was applied at both faces of each platen to
320 reduce friction resulting from the differences in Poisson's ratio. Thin disks of copper
321 and graphite were stacked on each other with a small amount of grease and placed on

322 the impact end of the incident bar to improve the shape of the stress-pulse.

323 A Kirana highspeed camera was used to visualize one of the exposed faces of the
324 cuboidal specimen for imaging the failure. Imaging was performed at a frame rate of 2
325 million frames per second with an exposure time of 500 ns. A diffuse class 4 laser and
326 2 flashbulbs were used to supply sufficient lighting for imaging. A strain gauge placed
327 on the incident bar is used to trigger the camera with a fixed time offset that accounts
328 for the wave speed in the bar to allow for imaging as the stress pulse loads the sample.

329 4. Results & Discussion

330 4.1. Characterization

331 A total of 7 samples were thermally shocked. CT and RUS were performed on 5 of
332 these samples, as well as 3 pristine samples. No cracks are seen in the pristine sample
333 although larger microstructural features can be seen. For all thermally shocked samples
334 a full 3D file of the internal crack structure was constructed. Crack surface areas, wave
335 speed measurements, and Poisson's ratio values from RUS are summarized in Table 4.1.

Sample	Thermal Cycling	Crack Area (mm ²)		Wavespeed (km/s)		ν
		Non-manifold	Two-manifold	Longitudinal	Shear	
TC69	2 Cycles, 850°C	75	57	13.99	8.75	0.179
TC71	1 Cycles, 550°C	28	21	13.88	8.71	0.176
TC72	2 Cycles, 750°C	54	43	13.84	8.72	0.171
TC73	1 Cycles, 750°C	41	33	13.81	8.72	0.169
TC76	2 Cycles, 650°C	40	31	14.02	8.70	0.187
TC52	none	-	-	13.90	8.61	0.189
TC55	none	-	-	13.95	8.61	0.192
TC56	none	-	-	13.80	8.63	0.179

Table II: Thermal history, crack area, wavespeed, and Poisson's ratio (ν) measurements for thermally shocked samples.

336

337 The average longitudinal wave speed value was 13.88 ± 0.08 km/s (intact) and
338 13.91 ± 0.09 km/s (damaged), the average shear wave speed was 8.62 ± 0.01 km/s (in-
339 tact) and 8.72 ± 0.02 km/s (damaged), the average Poisson's ratio was 0.187 ± 0.007
340 (intact) and 0.176 ± 0.007 (damaged). In summary, there is little to a slight increase in
341 longitudinal wave speed, a notable increase in the shear wave, and notable decrease in
342 the Poisson's ratio. The longitudinal wave speeds are slightly higher than the theoretical
343 value 13.6 km/s [36] but close to the reported value experimentally found for sintered
344 boron carbide 14.09 km/s [37]. In some scenarios damage is expected to decrease the
345 longitudinal wave speed and Young's modulus [38]. The data presented here show no
346 decrease in longitudinal wave speed for the damaged material, but rather show a slight
347 increase that is well within the standard deviation of wave speeds seen for intact material
348 and likely not statistically significant. Note that the material studied here has porosity
349 derived for closed cracks at porosity levels of ~ 1 to 2%, whereas the majority of the
350 literature deals with materials that have presumably near-spherical pore shapes. Previ-
351 ous studies on coal have also shown that microfractures do not necessarily decrease the
352 velocity of elastic waves and may even lead to a slight increase [39]. Similar trends
353 have been noted by Phani [40] in porcelain. Given that the Young's modulus and den-
354 sity are relatively unchanged by the introduction of pre-damage, the increase in shear
355 wave speed may be a consequence of the decrease in Poisson's ratio given that they are
356 related by

$$v_{shear} = \sqrt{\frac{E}{2(1 + \nu)\rho}} \quad (3)$$

357 There are numerous papers that demonstrate a decrease in Poisson's ratio for in-
358 creasing porosity [41] [42]. Yu et al. [41] also demonstrate that the ratio of longitudinal

359 and shear wave speed decreases with increasing porosity (as our results indicate). The
360 exact mechanism for these behaviors is outside the scope of this work but is likely re-
361 lated to the size, closure, and orientation of the cracks. In our study, the presence of
362 pre-existing cracks is clearly shown in the CT reconstructions, it is possible that RUS
363 may only be a good indicator of internal damage in advanced ceramics when initial
364 damage levels are significantly higher than seen here.

365 Shown in Fig. 4 is a 3D surface reconstruction showing a branched crack network
366 with cracks that seem to span the whole specimen. The reconstruction program de-
367 veloped for this work outputs STL files that can be viewed in many commercial 3D
368 visualization program (Blender, Meshlab, etc). The orange surfaces show the crack
369 structure and empty space within this structure represents uncracked material. Many
370 of the cracks intersect the sample boundary, consistent with the expectation that cracks
371 will form at the boundary during thermal shock due to high local tensile stresses. It is
372 also clear that straight crack segments are on a similar length scale to the entire speci-
373 men with cracks often being longer than 1 mm. Visual inspection of the left and right
374 sides of Fig. 4 show that the two reconstruction algorithms result in very similar overall
375 crack structures. The surface area values reported in Table 4.1 for the two-manifold
376 surface is actually half the computed surface area as the cracks in this reconstruction
377 have 2 sides. The crack area values in Table 4.1 show that the two-manifold approach
378 consistently records a slightly lower crack area than the non-manifold surface. This is
379 to be expected as the cracks in the non-manifold version often have greater extent than
380 in the two-manifold version. This can be seen, for example, in the bottom right hand
381 corner of Fig. 4c where the small floating crack is clearly larger in Fig. 4c2 than in Fig.
382 4c1.

383 As the crack structure is highly branched, reporting the number of cracks neces-

384 sarily entails defining where one crack ends and another begins. This has implications
385 for many microcrack based damage models which assume that cracks do not intersect.
386 Although the number of cracks and their individual sizes may be difficult to report, it is
387 easy to compute the surface area of all the resulting crack surfaces which may serve as a
388 measure of the damage imparted to the sample. In addition to quantifying the extent of
389 the cracking in terms of surface area, the orientation of crack faces can be characterized
390 even if the crack structure is considered to be one continuous body instead of a group
391 of discrete cracks. Understanding crack orientation is important as it governs how the
392 cracks will interact with the stress field in the body under a specific loading direction
393 [16].

394 *4.2. Strength results & comparisons*

395 Previous studies on this boron carbide tile have found a strength of 2.98 ± 0.60
396 GPa for the quasistatic case and 3.70 ± 0.30 GPa for the dynamic case for the through-
397 thickness direction being tested here [5]. In the same vein, Fig. 5 and Fig. 6 below show
398 a time-series comparison for the visualization of the failure of pristine and predamaged
399 boron carbide with the same Kolsky bar setup.

400 For the intact specimen we see a spanning axial crack forming before the peak stress
401 is achieved, followed by transverse cracking after the peak stress. For the damaged spec-
402 imen large wing cracks can be seen on the photographed surface, likely emanating from
403 cracks already in the the specimen, even before the peak stress is reached. Two notable
404 features can be seen in the fracture and fragmentation of the damaged specimen when
405 compared to the fragmentation of the intact specimen. First, we see large branching
406 cracks spanning the specimen quickly and carving out large fragments during failure in
407 damaged samples. Second, we see large fracture surfaces, likely from the initial crack

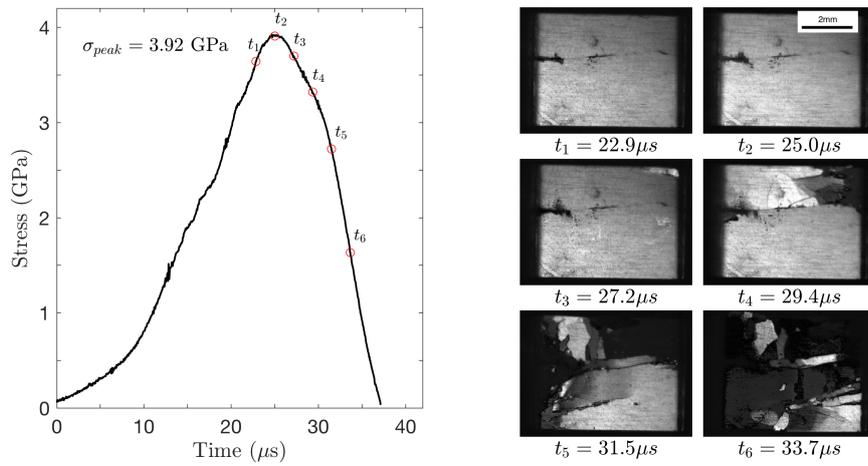


Fig. 5: Stress-time history of dynamic uniaxial compression of pristine boron carbide with time resolved high speed video images [4].

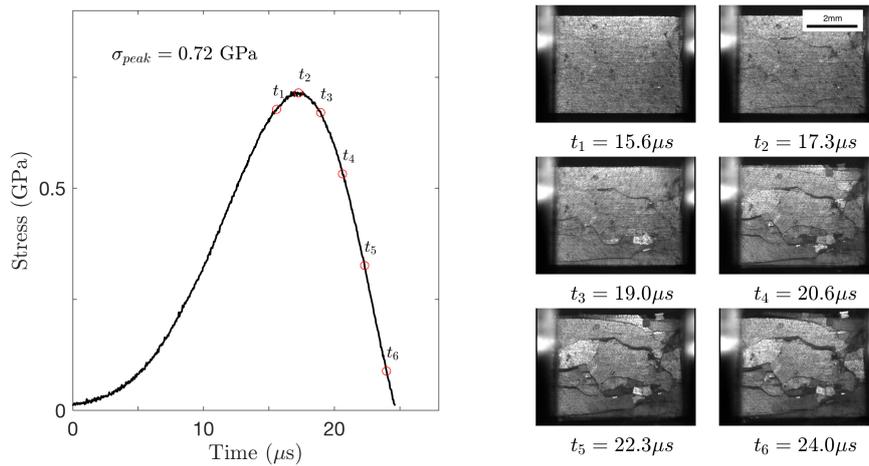


Fig. 6: Stress-time history of dynamic uniaxial compression of thermally damaged boron carbide with time resolved high speed video images. Note the difference in y-axis scale from the previous figure.

408 population on which frictional sliding is occurring. To the authors' knowledge, this is
 409 the first time that such failure observations for cracked materials have been made in
 410 the literature, and this could be important in models describing material failure. If the
 411 initial crack population serves as the initiation points for the generation of fragments, it

412 may indicate that microstructure has a diminished role in the fragmentation of damaged
413 material for the strain rates examined here.

414 The strength results for all quasistatic and dynamic tests from the damaged samples
415 are summarized in Fig. 7. The peak strengths are in the range of 0.64 to 1.55 GPa
416 across all rates. The x-axis gives the total crack area which we will consider now as a
417 quantitative measure of the extent of initial cracking within the sample. In the figure,
418 we plot the range of quasi-static strengths for the undamaged samples as red bars, and
419 the range of dynamic strengths as blue bars with a dot showing the average value [5].
420 This is done for comparison purposes. Damaged samples are plotted as points, with
421 red for quasi-static and blue for dynamic test results. For both dynamic and quasistatic
422 tests, strength was significantly below what we would expect for intact material. It was
423 also found that in general strength decreased with amount of predamage. This is to
424 be expected as the stress intensity factor (SIF) should increase approximately with the
425 size of the cracks. However, significantly more data are required to develop a direct
426 quantitative relationship.

427 To discuss these results, we link with the work of Chocron et al. [1]. In that work,
428 a thermal shock process at 750°C with 2 cycles was applied to boron carbide and the
429 resulting strength of damaged material under confined compression and other multiaxial
430 stress states was examined. The authors found that a Drucker-Prager yield law could
431 be used to describe both damaged and intact material under confined compression. In
432 the case of uniaxial compression with zero additional confining pressure, their model
433 predicts a yield strength of 0.56 GPa for damaged material. For our samples under
434 similar thermal cycling conditions we found a compressive strength of 0.64 GPa in
435 the dynamic loading case and 0.87 to 1.01 GPa in the quasistatic case, which agrees
436 reasonably well with the work of Chocron. Some scatter is to be expected due to the

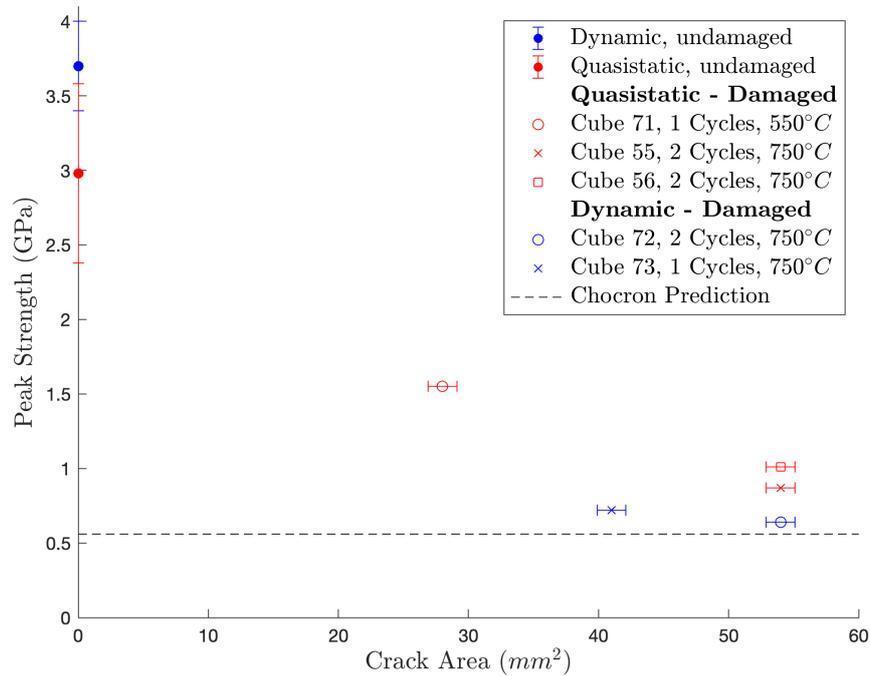


Fig. 7: Strength vs. crack area for quasistatic and dynamic tests on damaged material. The overlapping blue and red shaded regions show the ranges that would be expected for pristine material in the dynamic and quasistatic loading cases.

437 different specimen shape and size used which lead to different boundary conditions
 438 under thermal shock.

439 4.3. Rate effects discussion

440 Perhaps most surprising from the data presented here is the lower strength exhibited
 441 in the dynamic cases as compared to the quasistatic cases. Although small differences
 442 in results would be expected due to differences in test setup and inherent variability
 443 in testing ceramics, dynamic loading rates have generally been shown to increase the
 444 compressive strength of intact ceramics. However, strain rate does not appear to increase
 445 the strength of damaged ceramics, a result in agreement with Chocron et. al. [1].

446 The mechanics behind rate strengthening in ceramics has been well documented
447 [29]. In short, crack growth can only dissipate energy at a finite rate dictated by the
448 speed of elastic waves and crack tips in the material. When energy is input at a rate too
449 high to be dissipated by growing pre-existing cracks or defects the additional energy
450 goes into nucleating more cracks causing a rate strengthening effect. In the case of
451 damaged material where there may be many large cracks already present in the material
452 it is possible that a rate effect is still present but as there are already many available
453 energy pathways, the transition strain rate above which a strong rate effect is seen might
454 be significantly higher than it is for intact material. This, however, would not imply any
455 reason for an inverse rate effect to be present in our samples. Ongoing work is required
456 to confirm this result.

457 **5. Conclusions**

458 Samples of boron carbide were thermally damaged, characterized, and mechanically
459 tested. Strength results for the damaged boron carbide are approximately in agreement
460 with the only other available study on the strength of damaged boron carbide. A com-
461 putational framework for creating a model of a 3D fracture network within a damaged
462 material was developed and used to characterize the fracture network within 5 different
463 specimens. The techniques developed here have the potential to be applied to a variety
464 of other ceramic materials to study damage and crack growth. Future work will study
465 damage evolution by repeatedly computing the crack structure after intermediate levels
466 of damage have been applied.

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- CT can be used to determine 3-dimensional crack structures in ceramics
- Internal cracks can lead to an increase in shear wave speed in ceramics
- Frictional sliding may be more likely to occur in pre-damaged ceramics
- Strain-rate strengthening effects may be diminished for cracked structures

Quantification of Damage and its Effects on the Compressive Strength of an Advanced Ceramic

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Abstract

An understanding of the dynamic failure of damaged ceramics is important in protection applications, where the interaction of the projectile with cracked material is a contributing factor in the overall system performance. In this paper, we investigate the effects of pre-existing internal cracks on the quasi-static and dynamic compressive behavior of an advanced ceramic. We present experiments on a hot-pressed boron carbide in which internal cracks are generated through thermal shocking after which the initial material damage is quantified. Damage characterization was performed via Resonant Ultrasound Spectroscopy (RUS) and high-resolution Computed Tomography (CT). A computational procedure is developed to determine the three-dimensional structure of the internal crack network in the initially damaged material from a series of CT images. The failure and strength of the material is then evaluated experimentally. The uniaxial compressive strength of the predamaged boron carbide samples is determined under both quasistatic and dynamic loading scenarios and this is correlated with the pre-existing crack structure as determined by CT. Damaged samples were found to have average compressive strength of 1.14 GPa in quasistatic loading and 0.68 GPa in dynamic loading compared to 2.98 ± 0.6 GPa and 3.70 ± 0.3 GPa for pristine material, respectively. High speed photography employed during dynamic testing indicates that pre-existing cracks may lead to different failure mechanisms from what is normally seen in pristine material. Ultimately, these insights can be used to design improved materials that are more resistant to dynamic failure.

Keywords: microcracks; thermal shock; compressive fragmentation; brittle failure; experimental mechanics; advanced ceramics;

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Nomenclature

B	initial binary image matrix
E	elastic modulus
\bar{l}	average measured grain size
M	final binary image matrix
M'	padded final binary image matrix
\bar{s}	average defect size
S	integer image matrix
v_{shear}	shear wave speed
η	defect density
ν	Poisson's ratio
ρ	density
μ_{log}	disk size lognormal distribution mean
μ_n	disk size normal distribution mean
σ_{log}	disk size lognormal distribution standard deviation
σ_n	disk size normal distribution standard deviation

1. Introduction

Designing advanced ceramics for protection applications requires an understanding of failure and fragmentation mechanisms. These mechanisms have been shown to depend on material processing and specifics of the stress-state and strain-rate. Recent work has aimed to understand how the presence of internal fractures prior to loading affects the failure process [1] [2] [3], and the results suggest that pre-existing cracks will have a considerable effect on the material response of advanced ceramics. Studying the mechanical response of damaged material will help us understand the process of damage evolution in advanced ceramics. This paper presents an approach to quantitatively

10 measure the degree of damage within an advanced ceramic and then to relate this to its
11 compressive response.

12 Previous studies of advanced ceramics have focused on characterizing the size,
13 shapes, and types of microstructural features which are believed to serve as fracture
14 initiation points [4] [5] [6] [7]. Studies have also focused on characterizing grain
15 sizes and shapes, as well as material anisotropy [8]. Significant work has also been per-
16 formed to characterize the rate-dependence of compressive strength [9] [10] [11] [12].
17 Under quasistatic compression the most deleterious defects are activated first, leading
18 to crack growth and sample failure [5]. Under dynamic compression, however, the rate
19 of loading is such that additional defects are activated before substantial crack growth
20 occurs from the most deleterious defects (given finite crack speeds), and so dynamic
21 loading leads to the activation of more distributed damage.

22 As experimental work has shown the importance of loading-rate, microstructural
23 features, and microcracks in the failure process, computational models have been de-
24 veloped to try to capture these dependencies [13] [14]. The micro-mechanical mod-
25 eling approach, for example, incorporates pre-existing flaw distributions in the material
26 [15], and defines damage based on an evolving scalar crack density parameter. Similar
27 work by Hu and Ramesh has further extended this type of model to account for flaw
28 orientations and a tensorial damage parameter [16]. Tonge and Ramesh [17] further
29 extended this approach into a full constitutive model with a computational implementa-
30 tion for large-scale simulations of impact events. Other researchers, such as Johnson and
31 Holmquist, have modeled damage and damage evolution through a heuristic approach.
32 The JH2 model defines a set of constants that can be determined by fitting experimental
33 data over a wide range of loading conditions [18]. This approach does not directly
34 incorporate any microstructure or crack statistics, yet can have strong predictive power

35 when parameters are fit to a sufficiently large data set.

36 Motivated by the need to better understand the behavior of damaged materials and
37 provide experimental data for modelling damaged behavior of brittle materials [18]
38 [19] [20], this paper explores the quasistatic and dynamic compressive behavior of pre-
39 cracked boron carbide. To accomplish damaged states, we thermally shock cuboidal
40 specimens to produce internal crack networks that are \sim mm in length scale and of the
41 order of the test specimens. These crack networks are characterized through X-Ray to-
42 mography and Matlab-based image processing and are linked with mechanical proper-
43 ties and mechanical behavior. Other studies have used thermal shocking and mechanical
44 loading to produce internal cracking in specimens within confined test setups [1] [2].
45 In this paper, we use thermal shocking because it allows us to produce porosities of in-
46 terests (\sim 1 to 3%) and crack sizes that are resolvable in X-Ray tomography scans, thus
47 allowing, for the first time, a quantified measure of cracking that can be coupled to me-
48 chanical testing. The use of thermal shocking to produce \sim mm-sized cracked material
49 also yields structural cracking sizes that are similar to those observed during impact into
50 the same material [21], where understanding the impact behavior of this material is an
51 overarching goal of this paper. In undertaking this research, attempts have been made
52 to generate internal cracking through mechanical cycling, but this either resulted in too
53 few and too small of cracks, or chipping at the specimen surface. Altogether, this work
54 contributes to a limited data set in the literature for cracked specimens [1] [2] [22]
55 [23], where the bulk of the work on advanced ceramics has focused on intact materials
56 and resulting fragmentation [4] [21] [24] [25] [26] [27]. Understanding the behavior
57 of materials for intermediate levels of damage is valuable in validating and improving
58 models [16] [15] [17] [18] that can be used to design improved ceramic materials for,
59 for example, ballistic impact, where fracture and fragmentation behaviors are important

60 in their overall performance [28].

61 **2. Material**

62 The material in this study was a hot-pressed boron carbide (Coorstek, Inc.) with
63 a Young's modulus of 430 GPa, a density of 2,510 kg/m³, a Poisson's ratio of 0.16
64 to 0.17, and a fracture toughness of 2.5 MPa \sqrt{m} . These values are provided by the
65 manufacturer. The grain structure is equiaxed with an average grain size of 15 μm .
66 Almost all the grain boundaries have a high misorientation angle ($> 15^\circ$). This material
67 has been used in previous studies by the authors and collaborators, where the focus
68 has been on microstructural characterization [6], compressive strength and failure of
69 intact forms [5] [8], compressive fragmentation [4], and impact fragmentation [21].
70 Additional details are provided in these references.

71 The boron carbide material was received as a tile (conceptualized in 1a with disk-
72 like features in the figure meant to represent the carbonaceous inclusions presented later
73 in b) with dimensions of 305 mm length, 254 mm in width, and 8 mm in thickness.
74 For this study, rectangular prismatic boron carbide samples of approximate dimension
75 $3.5 \times 4.0 \times 5.3\text{mm}$ were machined from these larger tiles with the longest dimension
76 being oriented along the hot-pressing or "through-thickness" (TT) direction. During
77 sample machining, effort was made to minimize sub-surface damage by systematically
78 polishing the edges from $\sim 100 \mu\text{m}$ finish to a $2 \mu\text{m}$ finish following the cutting oper-
79 ation. Examination of X-ray tomography scans (shown later) reveals no clear evidence
80 of sub-surface damage from machining in the cubes under consideration in the current
81 investigation.

82 The inclusions and defects in the microstructure were characterized using a Zeiss
83 optical microscope with an AxioCam MRC camera and a TESCAN MIRA3 field emis-

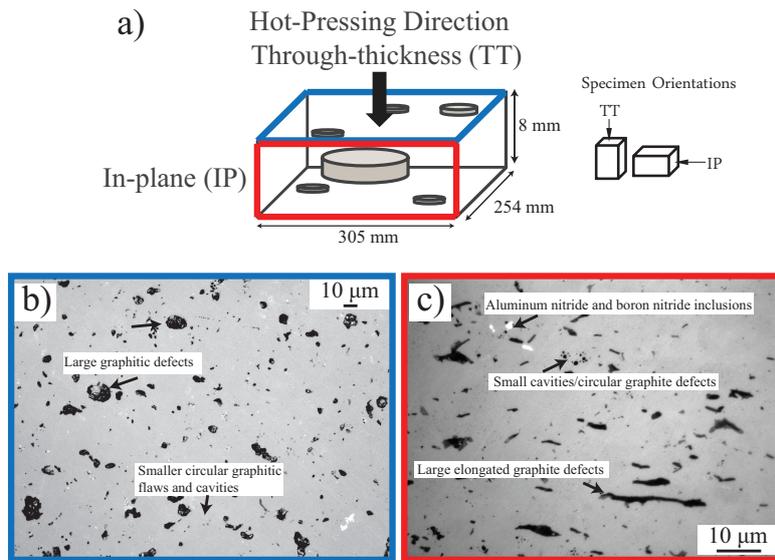


Fig. 1: (a) Conceptualized as-received tile of the hot-pressed boron carbide plate with through-thickness (TT) (in the hot-pressing direction) and in-plane directions (IP) labeled. Optical microscope images of the boron carbide microstructure in the (b) through-thickness (at $10\times$ magnification) and (c) in-plane direction ($100\times$ magnification) with the various types of inclusions and defects. [5]

84 sion Scanning Electron Microscope (SEM) equipped with a fully automated electron
 85 backscatter diffraction (EBSD) analysis system and Energy Dispersive Spectroscopy
 86 (EDS) capabilities. The word “defect” in this paper is used to denote a microstructural
 87 feature that may serve as a failure initiation site and “inclusion” is used to denote a
 88 feature that is not believed to contribute to failure (at least not under the stress states
 89 studied here). The processing-induced inclusions and defects are most easily seen in
 90 optical microscope images such as those shown in Fig. 1b and c. The image on the left
 91 is taken on the TT face, while the image on the right is taken on the IP face (note the
 92 different scale bars). Large, approximately circular, dark features are observed in the
 93 TT images in Fig. 1b. These have been confirmed to be carbonaceous in composition
 94 using EDS. Also highlighted in Fig. 1b are smaller and more circular features. These
 95 are primarily smaller graphitic defects, with other smaller features consisting of cavi-

96 ties/pores (confirmed with SEM/EDS). Brighter phases are also noted in Fig. 1b and
 97 these appear to be primarily comprised of aluminum nitride (AlN) and boron nitride
 98 (BN) (confirmed with SEM/EDS). These inclusions are faceted structures less than 20
 99 μm in size, and are commonly observed at boron carbide grain boundaries.

100 Previous work has shown that graphitic disks in boron carbide are the microstruc-
 101 tural features where fracture typically nucleates during failure in pristine (undamaged)
 102 specimens [5]. Again, these features are shown in the schematic representation of the
 103 as-received tile in Fig. 1a. The orientation, size, and spacing of these defects governs a
 104 variety of length scales that dictate rate dependence of the material strength [29]. These
 105 statistics are summarized in Table 2. The graphitic disk defect size was found to be well
 106 characterized by a lognormal distribution given by μ_{log} and σ_{log} whereas the defect ori-
 107 entation was found to be well characterized by a normal distribution given by μ_n and σ_n
 108 [5].

Material	\bar{l} (μm)	η ($\#/\text{m}^2$)	\bar{s} (μm)	μ_{log} (μm)	σ_{log} (μm)	μ_n	σ_n
PAD Boron Carbide	16.0 ± 2.1	1.41×10^9	4.22 ± 2.54	1.30 ± 0.02	0.53 ± 0.02	$0 \pm 1^\circ$	$20 \pm 1^\circ$

Table I: Microstructure characteristics for pressure aided densification boron carbide, including \bar{l} : average measured grain size, η : defect density measured on the IP face for $s > 0.5\mu\text{m}$, \bar{s} : average defect size, μ_{log} , σ_{log} : disk size lognormal distribution parameters, μ_n , σ_n : disk orientation normal distribution parameters

109

110 For a material that already contains a significant number of flaws, it is not yet known
 111 exactly what effect the presence of these microstructure features will have upon the
 112 evolution of damage in a pre-damaged sample subjected to additional loading. We will
 113 explore this here.

114 **3. Experimental Approach**

115 Studies of damage evolution and the effects of internal cracking on the failure and
 116 strength of such materials [1] are limited. We use the approach described in Fig. 2
 117 to capture the intermediate state of the material, with the intention of providing insight
 118 into dynamic damage evolution.

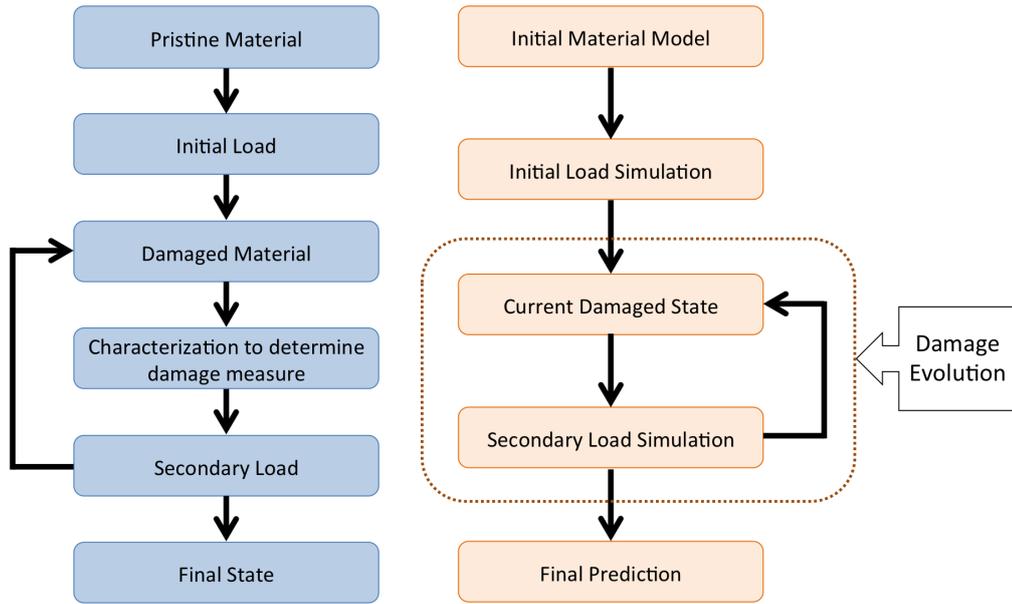


Fig. 2: Left: schematic showing the experimental process developed for this study. Traditional experiments go directly from the *initial load* to the *final state*. Right: schematic showing common computational procedure for simulating loading with damage evolution.

119 We achieve an intermediate damaged state by subjecting samples to an *initial load*
 120 that induces internal fracture without actually causing specimen fragmentation. The
 121 specimen is then characterized in this damaged state before an additional loading is pro-
 122 vided. This process can be repeated until fragmentation occurs and the specimen has lost
 123 its load bearing capacity. The *pristine material* in our study is a sample of the boron car-
 124 bide material described above. Although the specimen contains a variety of microstruc-

125 tural defects, it is considered to be free of damage until cracks have been induced by the
126 initial (or primary) load. The *primary load* is performed via a thermal shock process
127 described below. This leads to an intermediate *damaged material*. *Characterization* of
128 this material provides a quantifiable *damage measure*. The characterization presented
129 here was carried out via microscale CT to determine the internal crack structure and via
130 resonant ultrasound spectroscopy to examine potential degradation of elastic properties.
131 The secondary loading of this damaged sample is performed either via dynamic com-
132 pressive testing with a Kolsky bar apparatus or through quasistatic compression with an
133 MTS load frame system. The details of these tests are described below. If the secondary
134 loading event does not fragment the sample, the process of characterization and loading
135 can be repeated until a fragmented state is reached.

136 *3.1. Thermal Shock*

137 Material damage was induced via a thermal shock process. Previous studies on
138 heated boron carbide with cooling on the boundary have shown that large thermal gra-
139 dients lead to cracks propagating inward from the material boundary due to large tensile
140 stresses [30]. For example, Chocron et. al. [1] investigated a thermal shock pro-
141 cess through repeated heating and quenching on Pressure Assisted Densification (PAD)
142 boron carbide and found that the extent of the crack propagation depends on the number
143 of thermal cycles.

144 Five samples of boron carbide with approximate dimension $3.5 \times 4 \times 5.3$ mm were
145 heated in a furnace in air. Samples were placed on small alumina tiles which could be
146 easily manipulated with furnace tongs. The alumina tiles were placed on alumina foam
147 blocks in the furnace. Samples were heated for ~ 1 hour to temperatures $550 - 850^\circ\text{C}$.
148 After heating, the alumina tiles were removed from the furnace and tipped over into

149 a beaker of room temperature water thereby immersing the samples. Certain samples
150 were run through two heat and quench cycles to increase the degree of internal cracking.
151 The thermal cycling parameters of the samples can be found in Table 4.1. These are
152 discussed later.

153 *3.2. Resonant Ultrasound Spectroscopy*

154 Pristine and thermally shocked samples were subjected to resonant ultrasound spec-
155 troscopy (RUS) to determine elastic properties [31] [32]. The RUS data was measured
156 using the Magnaflux Quasar RUSpec system at the U.S. Army Research Laboratory in
157 Aberdeen Proving Ground, Maryland. The sample is held in place between two trans-
158 ducers as the system sweeps frequencies, and the resonant frequencies of the sample
159 appear as peaks in the measured spectrum. The samples' dimensions are measured by
160 hand using a micrometer, while densities are measured using the Archimedes' method.
161 This data is input into the RUSpec system as part of the sample information. Using the
162 density, the dimensions and the resonant frequencies, the RUSpec software performs it-
163 erations using the Levenberg - Marquardt algorithm to generate a best fit to the resonant
164 frequencies and thus determine the elastic constants of the sample and the wave speeds
165 which can be found in Table 4.1.

166 *3.3. Microscale Computed Tomography (microCT)*

167 *3.3.1. MicroCT Scanning*

168 The pristine and thermally shocked samples were scanned with a Bruker Skyscan
169 1172 to determine crack characteristics. Scans were performed on the thermally cracked
170 material as well as pristine material to determine a baseline level of damage. Each
171 specimen was mounted on a flat brass spindle using a small foam substrate, and wrapped
172 with parafilm to deter any slight movement during the scan. Single projection images

173 were then examined to determine the proper sensor exposure time as well as tube current
174 and voltage. These were found to be 149 μA and 29 kV with an exposure time of 2, 500
175 ms. A pixel size of 1.34 μm was used to obtain highest resolution of crack morphology
176 while maintaining a reasonable scan time. The scanning results in approximately 2, 000
177 to 4, 000 projection images per specimen depending on the orientation of the sample.

178 *3.3.2. 3D Reconstruction*

179 The images obtained from CT are then processed to build up a 3D reconstruction
180 of the fractures within the samples. Processing is performed first on the series of 2D
181 images from the scans before a 3D model of the crack morphology can be built. Fig.
182 3 shows the processing procedure for each projection image. The features shown are
183 internal as each projection image corresponds to a cross-section with a different depth
184 in the specimen. We have not shown pristine sample scans for brevity, but they resemble
185 the image seen in Fig. 3a and have similar bright spots corresponding to microstructural
186 features, but lack any visible lines corresponding to internal fracture.

187 The region of interest (ROI) for image processing is defined manually to be close to,
188 but within, the boundary of the material to prevent the edges of the sample from being
189 construed as cracks due to the high contrast at the perimeter. This is done by clicking
190 on points within the image through custom software to create a convex polygon whose
191 lines do not intersect with the boundary of the specimen in the scan. Note that the region
192 of interest shifts slightly from image to image, and so a ROI is defined for the first and
193 last images and a linear interpolation is used between the two limits.

194 In the initial image, the background material is gray with cracks making bright lines
195 on the image. The pixels corresponding to the bright lines of the cracks only make up
196 a small percentage of the image. Background removal (Fig. 3b) is applied whereby

197 for every pixel the mean intensity value for the neighborhood of pixels around it is
198 subtracted from its intensity. This produces a new image with lower intensity than the
199 original image at every pixel. As the bright cracks never make up a significant portion
200 of a neighborhood by percentage of pixels, the resulting image contains low intensity
201 (black) pixels where background (gray) pixels previously existed. Although the bright
202 pixels are not as bright as they were before, they stand out more against the now darker
203 background pixels which have almost zero intensity as can be seen in in Fig. 3b. Simple
204 contrast adjustment of this background image brings the initially bright pixels back up
205 to a high intensity (Fig. 3e). Although this also makes background noise brighter as
206 well, by thresholding most of this can be removed and this results in the binary image
207 in Fig. 3f.

208 In addition to contrast adjustment, gradient imaging enhances sharp contrasts at the
209 sides of cracks. Median filtering is used to reduce some of the noise introduced by the
210 gradient imaging resulting in Fig. 3c. This image is also converted to a binary images
211 with simple thresholding based of a percentile value of the intensity values for the whole
212 ROI. As each of the two processing paths described above has a tendency to miss a few
213 features, the resulting binary images are combined to create a final image.

214 A stack of K binary images output from the 2D processing, each $M \times N$, results
215 in a binary 3D matrix of size $K \times M \times N$. This initial matrix poses the difficulties
216 of (i) large amounts of noise, which appear as pixelized white features in 2D, and (ii)
217 computational difficulties due to the number of indices (i.e., size of the data). A scan for
218 a single specimen can contain $\approx 2,500$ images with each image being $\approx 4,000 \times 4,000$
219 pixels. The corresponding 3D matrix would have 40,000,000,000 indices or voxels
220 (volume-pixels) at full resolution.

221 These difficulties are overcome by scaling down the binary matrix to a new integer

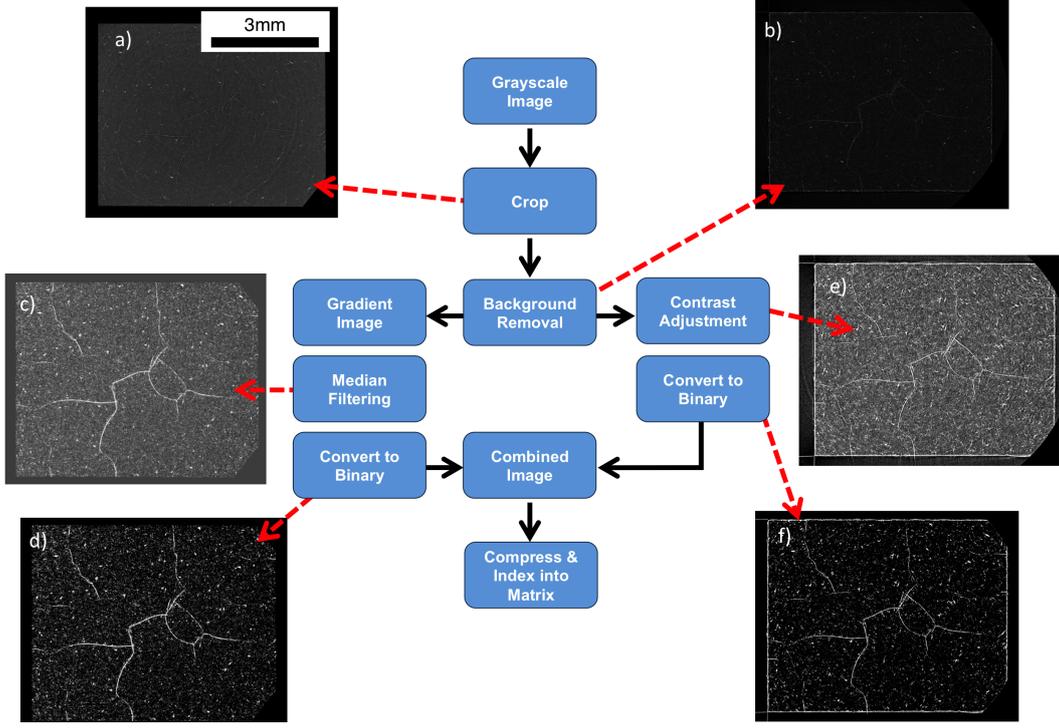


Fig. 3: Image processing flowchart for 2D input image. a) Cropped image with cropping defined within a few microns of the material boundary (see next figure for original image). b) Image after applying a background removal algorithm c) Result of applying median filtering to the gradient image of the result of the background removal step. d) Thresholded binary image from median filtering results e) Contrast adjusted version of background removed image f) Thresholded binary image from contrast adjusted image

222 matrix. For an initial binary matrix \mathbf{B} of size $K \times M \times N$ a new integer matrix, \mathbf{S} of
 223 size $\frac{K}{f} \times \frac{M}{f} \times \frac{N}{f}$ where f is an integer value that defines the scale parameter. For a set of
 224 indices (i, j, k) we define $p_i = f(i - 1) + 1$, $q_j = f(j - 1) + 1$, and $r_k = f(k - 1) + 1$. The
 225 large binary matrix \mathbf{B} is then mapped to the integer matrix \mathbf{S} as

$$\mathbf{S}(i, j, k) = \sum_{p=p_i}^{f_i} \sum_{q=q_j}^{f_j} \sum_{r=r_k}^{f_k} \mathbf{B}(p, q, r) \quad (1)$$

226 The entries in the new matrix \mathbf{S} are in the range of 0 to f^3 . In the resulting integer
 227 matrix regions that correspond to cracks then contain a high integer value whereas re-

228 gions of noise or microstructural defects contain low integer values. A threshold value,
229 ν is defined where $0 < \nu < f^3$ to obtain a final binary matrix, \mathbf{M} from \mathbf{S} .

$$\mathbf{M}(i, j, k) = \begin{cases} 1, & \text{if } \mathbf{S}(i, j, k) > \nu \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

230 By thresholding this integer matrix, noise regions where $0 < \mathbf{S}(i, j, k) < \nu$ can be
231 removed resulting in a binary 3D matrix whose non-zero entries correspond to crack
232 locations. The degree to which the initial matrix should be scaled down and the proper
233 thresholding value would depend on the degree of noise in the initial image and how
234 wide (in terms of pixels) cracks appear. In our case a scale value of $f = 6$ was found
235 to provide good results while also greatly reducing computation time. This reduces the
236 number of matrix entries from ≈ 40 billion to ≈ 185 million.

237 Two different approaches were used to generate 3D STL (Stereolithography) file
238 representations from this binary matrix, \mathbf{M} . An STL file simply contains a list of ver-
239 tices and a list of triangular faces comprised of these vertices. A series of connected
240 triangular faces is referred to as a “mesh.” An STL mesh can either be “two-manifold”
241 or “non-manifold.” Simply put, a two-manifold mesh could be split along edges and
242 laid flat without any faces overlapping. A non-manifold mesh will contain features such
243 as “T” junctions where three or more triangular faces share an edge, faces that share ver-
244 tices but not edges, adjacent faces with opposite normals, or other non-manifold features
245 [33]. When creating a 3D file to represent a fracture surface it may be ideal to represent
246 it as a zero-thickness surface as cracks are not typically considered to have a volume.
247 However, representing any sufficiently complex crack network as a zero thickness sur-
248 face necessitates creating it as a non-manifold mesh as intersecting fracture surfaces
249 will create “T” junctions or non-consistent surface normals.

250 Two reconstruction techniques were pursued. The first method will be referred to as
251 a manifold “volume method” as it creates a manifold mesh where all fracture surfaces
252 have 2 sides and therefor an enclosed volume. The second method will be referred to as
253 a “surface method” as it creates a zero-thickness surface though a non-manifold mesh.

254 3.3.3. Volume Approach

255 The volume method produces a finite thickness surface to represent the crack struc-
256 ture. A penny shaped crack, for example, would be represented as an ellipsoid with
257 this approach. In order to give the cracks two sides, the indices where $\mathbf{M}(i, j, k) = 1$
258 are padded with 1s in all direction to make the cracks thicker resulting in the matrix
259 \mathbf{M}' . After this, the resulting matrix is smoothed using a Gaussian smoothing algorithm.
260 With the cracks thickened, a Marching Cubes (MC) algorithm is used to turn the voxel
261 matrix \mathbf{M}' into a triangulated surface. A detailed explanation of the MC algorithm is
262 contained in the references [34]. The complexity of the resulting mesh is then reduced
263 using a Quadric Edge Collapse decimation algorithm. Finally, the resulting surface is
264 smoothed out using a Poisson surface reconstruction. Renderings of the resulting 3D
265 file can be seen in Fig. 4.

266 3.3.4. Surface Approach

267 The surface method approach produces a zero-thickness surface to represent the
268 crack structure. The final binary matrix, \mathbf{M} is converted to a point cloud, which is a list
269 of XYZ coordinates corresponding to locations of 1s in the matrix. Once a point cloud
270 has been created, the complexity of the point cloud is reduced using a clustered vertex
271 subsampling approach. After this, a local surface normal can be approximated on a per
272 point basis based on the location of nearby points. Once a point cloud with defined
273 point normals has been constructed a 3D Stereolithography file containing a list of faces

274 and vertices can be created. As crack intersections are more feasibly characterized
275 with a zero-thickness surface representation, a segmentation algorithm was developed
276 to segment the branched fracture network into individual surfaces.

277 *3.3.5. 3D Segmentation*

278 The segmentation of separate cracks from the connected crack network is an unre-
279 solved challenge in the literature and an on-going research topic in the authors' groups.
280 A segmentation algorithm was developed to divide the zero-thickness surface described
281 above into a set of disjoint, approximately planar surfaces. Segmentation begins by
282 removing mesh faces that are expected to branch different crack surfaces. First, a cur-
283 vature parameter is defined over the whole crack surface on a per face basis. Faces
284 with a curvature value that is above a computed threshold are likely located at the junc-
285 tion between two crack surfaces and are therefor removed. After removing this initial
286 set of faces, all faces that share an edge with only two or fewer faces are removed. A
287 breadth-first search (BFS) is then run over the set of remaining faces. This approach
288 considers faces to be individual graph nodes whereby two nodes are adjacent if their
289 corresponding mesh faces share an edge. The BFS generates a spanning forest contain-
290 ing spanning trees where each spanning tree contains the set of faces that comprise an
291 individual crack. For large crack faces further segmentation is performed using face
292 normal parameter that splits large, curved, crack surfaces into smaller groups with more
293 consistent face normals.

294 *3.4. Mechanical Testing*

295 *3.4.1. Quasistatic Testing*

296 Quasistatic compression tests were performed on an MTS Model C43.504 load
297 frame fitted with MTS Model LPS.504 load cells with a maximum load capacity of

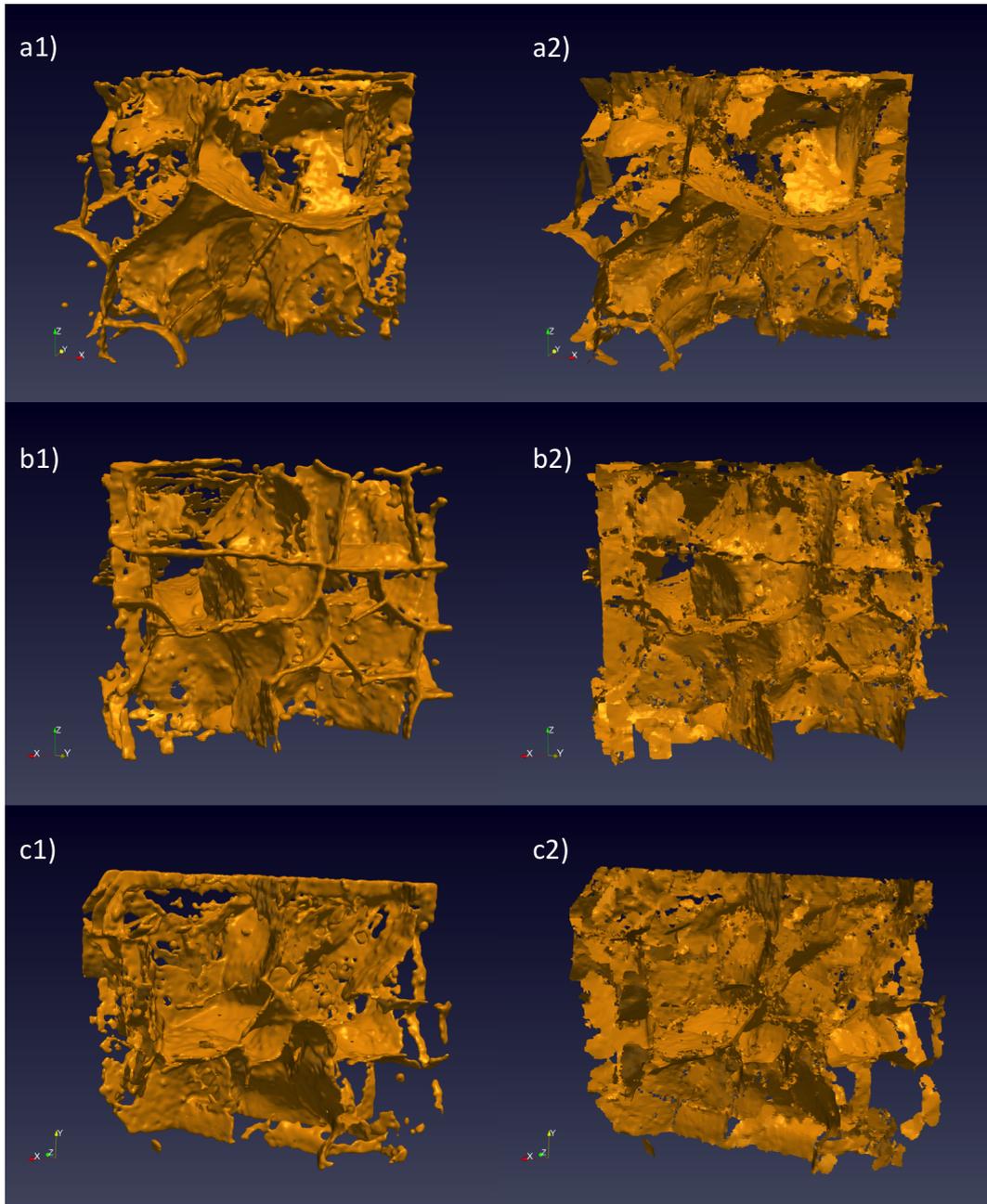


Fig. 4: 3 views of the reconstructions for sample TC69 are shown. The z-axis is the scan direction and the x-axis is the sintering (or TT) direction. The renderings on the left are from the volume reconstruction method (two-manifold). The renderings on the right are from the surface reconstruction (non-manifold). The length in the z-direction is approximately 3.2 mm

298 50 kN. As the boron carbide is harder than the hardened steel platens of the MTS ma-
299 chine, intermediate platens machined from SiC-N or WC with cross-sectional areas
300 larger than that of the boron carbide specimens were used to transmit the compressive
301 load to the boron carbide specimen without damaging the steel platens. A thin sleeve of
302 plastic sheeting was wrapped loosely around the specimen and SiC-N platens to allow
303 for collection of fragments after failure. Compression tests were run with a cross-head
304 displacement rate of $5 \mu\text{m/s}$, resulting in a nominal strain rate of $\approx 10^{-3} /\text{s}$.

305 3.4.2. *Dynamic Testing*

306 Dynamic uniaxial compression was performed via a Kolsky bar apparatus. The Kol-
307 sky bar setup consists of an incident and transmitted bar that sandwich the specimen and
308 a striker bar which impacts the incident bar propagating a stress pulse through it towards
309 the specimen. Kolsky bars have been used extensively to study ceramics including boron
310 carbide and details on experimental design can be readily found in literature [35].

311 The incident and transmitted bars, and projectile all had a diameter of 12.7 mm
312 and are made of maraging steel (VascoMax C-350) with a yield strength of 2.68 GPa,
313 elastic modulus of 200 GPa, Poisson's ratio of 0.29, and a density of $8,100 \text{ kg/m}^3$.
314 Their respective lengths were 1220 mm, 1050 mm and 127 mm. The length of the
315 projectile dictates the width of the stress pulse transmitted through the bars. To protect
316 against damaging the contact surface of the bars, 5 mm platens were placed on both
317 sides of the sample at the interface with the incident and transmitted bars. Platens were
318 made of impedance matched tungsten carbide (LC403, Leech Carbide) jacketed by Ti-
319 6Al-4V sleeves. A small amount of grease was applied at both faces of each platen to
320 reduce friction resulting from the differences in Poisson's ratio. Thin disks of copper
321 and graphite were stacked on each other with a small amount of grease and placed on

322 the impact end of the incident bar to improve the shape of the stress-pulse.

323 A Kirana highspeed camera was used to visualize one of the exposed faces of the
324 cuboidal specimen for imaging the failure. Imaging was performed at a frame rate of 2
325 million frames per second with an exposure time of 500 ns. A diffuse class 4 laser and
326 2 flashbulbs were used to supply sufficient lighting for imaging. A strain gauge placed
327 on the incident bar is used to trigger the camera with a fixed time offset that accounts
328 for the wave speed in the bar to allow for imaging as the stress pulse loads the sample.

329 4. Results & Discussion

330 4.1. Characterization

331 A total of 7 samples were thermally shocked. CT and RUS were performed on 5 of
332 these samples, as well as 3 pristine samples. No cracks are seen in the pristine sample
333 although larger microstructural features can be seen. For all thermally shocked samples
334 a full 3D file of the internal crack structure was constructed. Crack surface areas, wave
335 speed measurements, and Poisson's ratio values from RUS are summarized in Table 4.1.

Sample	Thermal Cycling	Crack Area (mm ²)		Wavespeed (km/s)		ν
		Non-manifold	Two-manifold	Longitudinal	Shear	
TC69	2 Cycles, 850°C	75	57	13.99	8.75	0.179
TC71	1 Cycles, 550°C	28	21	13.88	8.71	0.176
TC72	2 Cycles, 750°C	54	43	13.84	8.72	0.171
TC73	1 Cycles, 750°C	41	33	13.81	8.72	0.169
TC76	2 Cycles, 650°C	40	31	14.02	8.70	0.187
TC52	none	-	-	13.90	8.61	0.189
TC55	none	-	-	13.95	8.61	0.192
TC56	none	-	-	13.80	8.63	0.179

Table II: Thermal history, crack area, wavespeed, and Poisson's ratio (ν) measurements for thermally shocked samples.

336

337 The average longitudinal wave speed value was 13.88 ± 0.08 km/s (intact) and
 338 13.91 ± 0.09 km/s (damaged), the average shear wave speed was 8.62 ± 0.01 km/s (in-
 339 tact) and 8.72 ± 0.02 km/s (damaged), the average Poisson's ratio was 0.187 ± 0.007
 340 (intact) and 0.176 ± 0.007 (damaged). In summary, there is little to a slight increase in
 341 longitudinal wave speed, a notable increase in the shear wave, and notable decrease in
 342 the Poisson's ratio. The longitudinal wave speeds are slightly higher than the theoretical
 343 value 13.6 km/s [36] but close to the reported value experimentally found for sintered
 344 boron carbide 14.09 km/s [37]. In some scenarios damage is expected to decrease the
 345 longitudinal wave speed and Young's modulus [38]. The data presented here show no
 346 decrease in longitudinal wave speed for the damaged material, but rather show a slight
 347 increase that is well within the standard deviation of wave speeds seen for intact material
 348 and likely not statistically significant. Note that the material studied here has porosity
 349 derived for closed cracks at porosity levels of ~ 1 to 2%, whereas the majority of the
 350 literature deals with materials that have presumably near-spherical pore shapes. Previ-
 351 ous studies on coal have also shown that microfractures do not necessarily decrease the
 352 velocity of elastic waves and may even lead to a slight increase [39]. Similar trends
 353 have been noted by Phani [40] in porcelain. Given that the Young's modulus and den-
 354 sity are relatively unchanged by the introduction of pre-damage, the increase in shear
 355 wave speed may be a consequence of the decrease in Poisson's ratio given that they are
 356 related by

$$v_{shear} = \sqrt{\frac{E}{2(1 + \nu)\rho}} \quad (3)$$

357 There are numerous papers that demonstrate a decrease in Poisson's ratio for in-
 358 creasing porosity [41] [42]. Yu et al. [41] also demonstrate that the ratio of longitudinal

359 and shear wave speed decreases with increasing porosity (as our results indicate). The
360 exact mechanism for these behaviors is outside the scope of this work but is likely re-
361 lated to the size, closure, and orientation of the cracks. In our study, the presence of
362 pre-existing cracks is clearly shown in the CT reconstructions, it is possible that RUS
363 may only be a good indicator of internal damage in advanced ceramics when initial
364 damage levels are significantly higher than seen here.

365 Shown in Fig. 4 is a 3D surface reconstruction showing a branched crack network
366 with cracks that seem to span the whole specimen. The reconstruction program de-
367 veloped for this work outputs STL files that can be viewed in many commercial 3D
368 visualization program (Blender, Meshlab, etc). The orange surfaces show the crack
369 structure and empty space within this structure represents uncracked material. Many
370 of the cracks intersect the sample boundary, consistent with the expectation that cracks
371 will form at the boundary during thermal shock due to high local tensile stresses. It is
372 also clear that straight crack segments are on a similar length scale to the entire speci-
373 men with cracks often being longer than 1 mm. Visual inspection of the left and right
374 sides of Fig. 4 show that the two reconstruction algorithms result in very similar overall
375 crack structures. The surface area values reported in Table 4.1 for the two-manifold
376 surface is actually half the computed surface area as the cracks in this reconstruction
377 have 2 sides. The crack area values in Table 4.1 show that the two-manifold approach
378 consistently records a slightly lower crack area than the non-manifold surface. This is
379 to be expected as the cracks in the non-manifold version often have greater extent than
380 in the two-manifold version. This can be seen, for example, in the bottom right hand
381 corner of Fig. 4c where the small floating crack is clearly larger in Fig. 4c2 than in Fig.
382 4c1.

383 As the crack structure is highly branched, reporting the number of cracks neces-

384 sarily entails defining where one crack ends and another begins. This has implications
385 for many microcrack based damage models which assume that cracks do not intersect.
386 Although the number of cracks and their individual sizes may be difficult to report, it is
387 easy to compute the surface area of all the resulting crack surfaces which may serve as a
388 measure of the damage imparted to the sample. In addition to quantifying the extent of
389 the cracking in terms of surface area, the orientation of crack faces can be characterized
390 even if the crack structure is considered to be one continuous body instead of a group
391 of discrete cracks. Understanding crack orientation is important as it governs how the
392 cracks will interact with the stress field in the body under a specific loading direction
393 [16].

394 *4.2. Strength results & comparisons*

395 Previous studies on this boron carbide tile have found a strength of 2.98 ± 0.60
396 GPa for the quasistatic case and 3.70 ± 0.30 GPa for the dynamic case for the through-
397 thickness direction being tested here [5]. In the same vein, Fig. 5 and Fig. 6 below show
398 a time-series comparison for the visualization of the failure of pristine and predamaged
399 boron carbide with the same Kolsky bar setup.

400 For the intact specimen we see a spanning axial crack forming before the peak stress
401 is achieved, followed by transverse cracking after the peak stress. For the damaged spec-
402 imen large wing cracks can be seen on the photographed surface, likely emanating from
403 cracks already in the the specimen, even before the peak stress is reached. Two notable
404 features can be seen in the fracture and fragmentation of the damaged specimen when
405 compared to the fragmentation of the intact specimen. First, we see large branching
406 cracks spanning the specimen quickly and carving out large fragments during failure in
407 damaged samples. Second, we see large fracture surfaces, likely from the initial crack

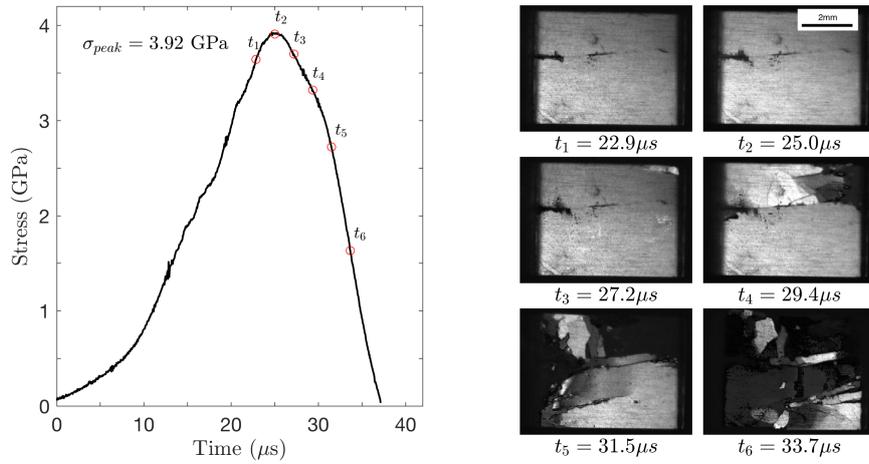


Fig. 5: Stress-time history of dynamic uniaxial compression of pristine boron carbide with time resolved high speed video images [4].

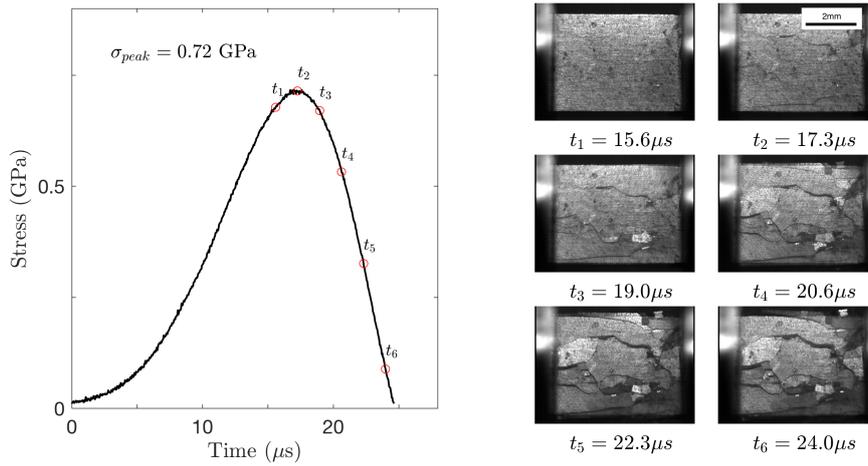


Fig. 6: Stress-time history of dynamic uniaxial compression of thermally damaged boron carbide with time resolved high speed video images. Note the difference in y-axis scale from the previous figure.

408 population on which frictional sliding is occurring. To the authors' knowledge, this is
 409 the first time that such failure observations for cracked materials have been made in
 410 the literature, and this could be important in models describing material failure. If the
 411 initial crack population serves as the initiation points for the generation of fragments, it

412 may indicate that microstructure has a diminished role in the fragmentation of damaged
413 material for the strain rates examined here.

414 The strength results for all quasistatic and dynamic tests from the damaged samples
415 are summarized in Fig. 7. The peak strengths are in the range of 0.64 to 1.55 GPa
416 across all rates. The x-axis gives the total crack area which we will consider now as a
417 quantitative measure of the extent of initial cracking within the sample. In the figure,
418 we plot the range of quasi-static strengths for the undamaged samples as red bars, and
419 the range of dynamic strengths as blue bars with a dot showing the average value [5].
420 This is done for comparison purposes. Damaged samples are plotted as points, with
421 red for quasi-static and blue for dynamic test results. For both dynamic and quasistatic
422 tests, strength was significantly below what we would expect for intact material. It was
423 also found that in general strength decreased with amount of predamage. This is to
424 be expected as the stress intensity factor (SIF) should increase approximately with the
425 size of the cracks. However, significantly more data are required to develop a direct
426 quantitative relationship.

427 To discuss these results, we link with the work of Chocron et al. [1]. In that work,
428 a thermal shock process at 750°C with 2 cycles was applied to boron carbide and the
429 resulting strength of damaged material under confined compression and other multiaxial
430 stress states was examined. The authors found that a Drucker-Prager yield law could
431 be used to describe both damaged and intact material under confined compression. In
432 the case of uniaxial compression with zero additional confining pressure, their model
433 predicts a yield strength of 0.56 GPa for damaged material. For our samples under
434 similar thermal cycling conditions we found a compressive strength of 0.64 GPa in
435 the dynamic loading case and 0.87 to 1.01 GPa in the quasistatic case, which agrees
436 reasonably well with the work of Chocron. Some scatter is to be expected due to the

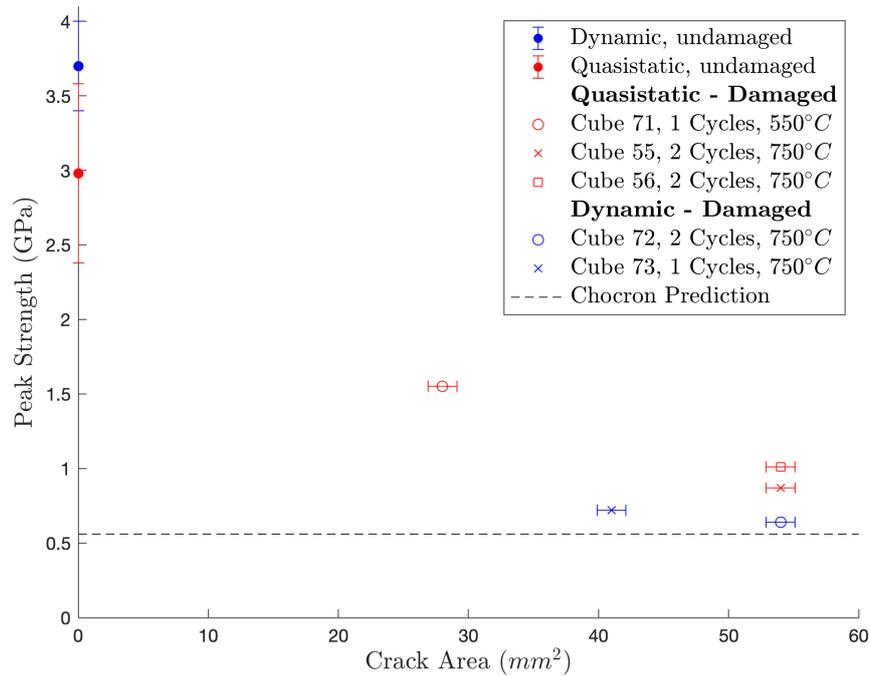


Fig. 7: Strength vs. crack area for quasistatic and dynamic tests on damaged material. The overlapping blue and red shaded regions show the ranges that would be expected for pristine material in the dynamic and quasistatic loading cases.

437 different specimen shape and size used which lead to different boundary conditions
 438 under thermal shock.

439 4.3. Rate effects discussion

440 Perhaps most surprising from the data presented here is the lower strength exhibited
 441 in the dynamic cases as compared to the quasistatic cases. Although small differences
 442 in results would be expected due to differences in test setup and inherent variability
 443 in testing ceramics, dynamic loading rates have generally been shown to increase the
 444 compressive strength of intact ceramics. However, strain rate does not appear to increase
 445 the strength of damaged ceramics, a result in agreement with Chocron et. al. [1].

446 The mechanics behind rate strengthening in ceramics has been well documented
447 [29]. In short, crack growth can only dissipate energy at a finite rate dictated by the
448 speed of elastic waves and crack tips in the material. When energy is input at a rate too
449 high to be dissipated by growing pre-existing cracks or defects the additional energy
450 goes into nucleating more cracks causing a rate strengthening effect. In the case of
451 damaged material where there may be many large cracks already present in the material
452 it is possible that a rate effect is still present but as there are already many available
453 energy pathways, the transition strain rate above which a strong rate effect is seen might
454 be significantly higher than it is for intact material. This, however, would not imply any
455 reason for an inverse rate effect to be present in our samples. Ongoing work is required
456 to confirm this result.

457 **5. Conclusions**

458 Samples of boron carbide were thermally damaged, characterized, and mechanically
459 tested. Strength results for the damaged boron carbide are approximately in agreement
460 with the only other available study on the strength of damaged boron carbide. A com-
461 putational framework for creating a model of a 3D fracture network within a damaged
462 material was developed and used to characterize the fracture network within 5 different
463 specimens. The techniques developed here have the potential to be applied to a variety
464 of other ceramic materials to study damage and crack growth. Future work will study
465 damage evolution by repeatedly computing the crack structure after intermediate levels
466 of damage have been applied.

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