UNIVERSITY OF ALBERTA

Evaluation of Mineral Ventures using Modern Financial Methods

BY

Samuel Frimpong

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

Doctor of Philosophy

IN

Mining Engineering

Department of Mining, Metallurgical and Petroleum Engineering

EDMONTON, ALBERTA

FALL, 1992

ISBN 0-315-77406-1

UNIVERSITY OF ALBERTA

RELEASE FORM

NAME OF AUTHOR: Samuel Frimpong

TITLE OF THESIS: Evaluation of Mineral Ventures using Modern Financial Methods

DEGREE FOR WHICH THESIS WAS SUBMITTED: Doctor of Philosophy

YEAR THIS DEGREE GRANTED: Fall 1992

(SIGNED) ____________________

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DATED 21 July 1992

To him who is able ---- the only God our Saviour be glory, majesty, power and authority, through Jesus Christ our Lord, before all ages, now and forevermore! Amen.

Jude 24, 25.

Na deɛ obɛtumi ---- Onyankopon koro a onam yɛn Awurade Yesu Kristo so yɛ yɛn Agyenkwa no, Ono na animuonyam, kɛseyɛ, ahooden, ne tumi nka no mmeresantene seesei ne daa nyinaa! Amen

Yuda 24, 25.

THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled:

Evaluation of Mineral Ventures using Modern Financial Methods

by Samuel Frimpong in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Mining Engineering.

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Dedicated to

Grace and Michael, Mum, Dad and Uncle

for their great help and inspiration.

ABSTRACT

The choice of superior projects or ventures, as well as excellent management of their evolution, are critical to the success of mining organisations. A particularly important part of venture selection and management is connected with the evaluation process. Managers and investors must clearly understand and manage the effects of risk, uncertainty, and various options on venture's value appropriately. Current economic evaluation tools in the mineral industry are inadequate in helping managers and investors to handle these strategic issues.

Advances in the theory of asset pricing have provided solutions to these inadequacies. This thesis project was designed to extend these advances to develop and illustrate a new economic evaluation tool, the derivative asset valuation (DAV) method, to help investors and managers to better select and manage ventures. The DAV method is implemented to examine and quantify various options and their effects on venture's present value, as well as project risks resulting from uncertainties associated with metal price, metal reserve, and expected ore grade.

Graphic illustrations of results show the importance of feasibility study, timing and operating options, and metal price and expected ore grade uncertainties in venture analysis. The effects of multiple-stage feasibility study, feasibility study duration, and metal price volatility on venture's value have been quantified and analysed. Also presented are phase diagrams which delineate metal price boundaries for various expected ore grades and metal reserves at which waiting or abandoning, closing, opening, feasibility study and investment decisions are appropriate.

By using the DAV method, investors and managers can examine and quantify various options available to them in the selection and management of mineral ventures. Also, the effects of risk and uncertainty in the value and viability of a venture can be better understood and controlled using the DAV method. Differences between the present values obtained from the DAV and discounted cash flow (DCF) methods indicate that the latter understates the venture's value when the option to wait to undertake feasibility studies and investment, and operating options have value.

ACKNOWLEDGEMENTS

The author expresses his sincere gratitude and appreciation for guidance, encouragement and support of Drs. Jerry M. Whiting, David G. Laughton, and John D. Whittaker. Dr. Whiting, Chair and Professor of Mining Engineering, provided a good counsel in the selection of courses for the Ph.D. program, supported the author's training and development, and contributed greatly in the selection and completion of the Ph.D. thesis. Dr. Laughton, Assistant Professor of Finance, fostered the author's interest in the relevant finance theory, and made available his knowledge and experience for this study. Dr. Whittaker, Professor of Mechanical Engineering, also raised the author's consciousness to the practical relevance of the study to the industry. Together, they provided a good supervisory team for this study.

The study was made possible, in part, by the financial assistance provided by the University of Alberta/CIDA program, to which the author is greatly indebted.

Special thanks to Ken Benterud, Wade Holst, and Haiying Wang in the Computer Modelling Group under Dr. David Laughton, and Professor W. H. Griffin, Doug Booth and Jacques Gibeau in the Department of Mining, Metallurgical & Petroleum Engineering, for providing assistance to the author in general and specific computer modelling of the problems.

The author is highly indebted to the Placer Dome Inc., and Nerco Con Mine, all of Canada, for the relevant data, and to Messrs. Tony Luteijn and W. A. Trythall of Placer Dome Inc. for time spent with the author on various subjects of the thesis project.

The support of the secretarial staff at the Department of Mining, Metallurgical & Petroleum Engineering, especially Mesdames Key Whiting, Evelyn Waun, Erika Auton, Cindy Heisler, and Louise Bohachyk is highly appreciated. Special thanks also to my colleagues in the Mining Engineering graduate program, Messrs. Chen Xiaoqing, Yuan Pen, and Yong Xie for great companionship.

The author expresses sincere thanks to members of the Sherwood Park Alliance Church, especially the pastoral staff, Ms. Esther Walker, the MacLeans, the Gibbs, the Schultzs, the Seutters, the Dubbeldams, the Paynes, the Thiessens, the Tjostheims, the Cooks, the Hildebrands, the Millingtons, the Johnsons, and the Notdorfts, and the Michener Park Christian Fellowship for their spiritual and material support, which have made the author's stay in Canada an excellent experience.

Above all, the author is highly indebted to his wife, Grace, and son, Michael, for their unfailing support, encouragement and tolerance.

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NOMENCLATURE

The following abbreviations, symbols, subscripts, superscripts, and notations are used in the corresponding chapters[1] in this study:

Chapter 1.0

1 - D	one dimension
2 - D	two dimensions
CAPM	capital asset pricing model
DAV	derivative asset valuation
DCF	discounted cash flow

Chapter 2.0

BCR	benefit cost ratio
GRR	growth rate of return
IRR	internal rate of return
NPV	net present value
PI	profitability index
WGR	wealth growth rate

Chapter 3.0

A	unit cost of producing a pound of metal
a	indicates that the mine is abandoned
c	convenience yield in metal price
CF	cash flow from the mine
CFM	cash inflow to the closed mine
CFO	cash flow from the open mine
dF	instantaneous change in futures price

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1 Where the nomenclature used are same in subsequent chapters they are not repeated.

dH	instantaneous change in the value of mine
dQ	instantaneous change in the metal reserves
dR	instantaneous change in portfolio return
dS	instantaneous change in commodity price
dz	increment to a standard Gaus-Wiener process
F	decision to undertake feasibility study
$F()$	price of futures contract
Fc	feasibility study cost
F_S	first derivative of futures price with respect to metal price
F_{SS}	second derivative of the futures price with respect to metal price
F_t	first derivative of futures price with respect to time
H	value of the mineral venture
HF	value of the mineral venture with ongoing feasibility study
HI	value of the mineral venture with investment
HM	value of the closed mine
HO	value of the open mine
H_Q	first derivative of the mine value with respect to metal reserve
H_S	first derivative of the mine value with respect to metal price
H_{SS}	second derivative of the mine value with respect to metal price
H_t	first derivative of the mine value with respect to time
I	investment cost
Ic	decision to invest under certainty
Iu	decision to invest under uncertainty
j	state of the mine
k	cost of maintaining the mine when it is temporarily closed
$K_{M \to A}$	cost of abondoning the mine if closed
$K_{M \to O}$	cost of opening the mine if closed
$K_{O \to A}$	cost of abondoning the mine if open
$K_{O \to M}$	cost of closing the mine if open
m	indicates that the mine is temporarily closed
n	indicates that the mine is not developed
o	indicates that the mine is open
Q	total metal reserve
q	annual production rate

S	metal price
S_c	critical price for investing in the mine under certainty
$S_{M \to A}$	critical price for abandoning the mine if closed
$S_{M \to O}$	critical price for opening the mine if closed
$S_{O \to M/A}$	critical price for closing or abandoning the mine if open
S_u	critical price for investing or undertaking a feasibility study under uncertainty
V	present value of the mineral venture
W	value of investing under uncertainty
Wc	decision to wait under certainty
Wu	decision to wait under uncertainty
Y	value of either investing or waiting under certainty
Z	value of the mineral venture with feasibility study option
σ	proportional standard deviation in commodity price
μ	expected growth or drift rate in metal price
ϕ	development and operating policy of the mine
ρ	risk-free rate of return
∞	infinity

Chapter 4.0

ADI	alternate direction implicit
E	expected ore grade
Emax	maximum expected ore grade
Emin	minimum expected ore grade
FC_i	feasibility study cost at stage i
FL	feasibility study limit or last feasibility study stage
IC_i	investment cost at stage i
$\overline{H}$	maximum of the mine value with investment and feasibility study options
$\overline{HF}$	mine value with feasibility study option
$\widehat{HF}$	mine value during an ongoing feasibility study
$\overline{HI}$	mine value with investment option
i	feasibility study stage

m	total development period
PICi	present value of investment cost at stage i
prob(E'lE)	probability of E' given E
Q_o	total ore reserve
q_o	annual production rate
r_i	cost of ignorance factor
t	any time in a feasibility study stage
T_i	total duration for feasibility study stage i
var(i)	variance associated with ore grade at stage i
β	unit operating cost

Chapter 5.0

ADE	alternate direction explicit
h	step size in the elliptic direction
k	step size in the parabolic direction
M	number of matrix rows
N	number of matrix columns
ODE	ordinary differential equation
PDE	partial differential equation

Chapter 6.0

OP	operating option
W/OP/F	waiting, operating and feasibility study options
N/O	no options
Diff(WC,IC)_1M	difference in waiting and investing under certainty for 1 Mt reserves
Diff(WC,IC)_2M	difference in waiting and investing under certainty for 2 mt reserves
Diff(CL,OP)_1M	difference between closing and opening for 1 million-ton reserves
Diff(CL,OP)_2M	difference between closing and opening for 2 million-ton reserves

Chapter 7.0

nofeas	no feasibility study is undertaken
w2fww	wait to undertake feasibility study, wait in feasibility phase, wait to invest, but complete only two feasibility study stages
wfww	waiting options before, in, and after feasibility study
wfnw	waiting options before and after feasibility study, but none in the feasibility study phase
wfnn	waiting option before feasibility study, but none in and after the feasibility study phase
1s0_w2fww	similar to w2fww but all the required study is done at once at stage 0
1s0_wfww	similar to wfww but all the required study is done at once at stage 0
1s1_wfww	similar to wfww but all the required study is done at once at stage 1
1s2_wfww	similar to wfww but all the required study is done at once at stage 2
1s3_wfww	similar to wfww but all the required study is done at once at stage 3
ic_novar	investment cost at any stage independent on ore grade uncertainty
nfww	similar to wfww but no option to wait to begin the feasibility study
nfnw	similar to wfnw but no option to wait to begin the feasibility study
nfnn	similar to wfnn but no option to wait to begin the feasibility study
λ	price of risk associated with the mineral commodity
δ	storage cost of mineral commodity

Appendix Section

a, b, c, d, e, f	matrix coefficients
A, C, M	M*N tridiagonal matrices
B, RHS, D, X, P	N*1 matrices
CML	capital market line
$dZ_{S,E}$	instantaneous change in standard Gauss-Wiener process
$E[R_m]$	expected value of market return
$E[R_z]$	expected value of the return on asset z
g	adjustment for the degree of market risk aversion
H_E	first derivative of mine value with respect to ore grade

H_{EE}	second derivative of mine value with respect to ore grade
H_w	first derivative of the mine value with respect to total effort
iu_cost	investment cost
K	delivery price of security
n	sampling rate
N(x)	cumulative distribution function of a standard normal variable
p	factor of variance reduction
$R(Z_o)$	risk-free rate of return
SML	security market line
w_i	total effort at any period i
β_z	sensitivity of the return on asset z to market changes
ΔE	small change in expected ore grade
ΔF	small change in the price of a futures contract
ΔH	small change in the value of a mineral venture
ΔS	small change in metal price
Δt	small change in time
Δw	small change in total effort
Δz	small change in a standard Gauss-Wiener process
ε	random sample from a stadard normal distribution
σ_s	proportional standard deviation in metal price
σ_w	proportional standard deviation in ore grade
$(\psi_i)^2$	total variance associated with ore grade at any period i
$\vdots$	column vector

CHAPTER 1.0

INTRODUCTION

1.1. Background

The twentieth century has been characterized by a tremendous growth in the business sector of the developed and developing economies. The number of business units, their average sizes, and the complexity of their operations have steadily been increasing [Samuelson, Nordhaus and McCallum 1988]. The hunger of the industrialisation process for minerals, the requirements of the world wars, the general developmental goals of world governments, and the aesthetic and material needs of humanity have contributed to the rapid growth of the mineral resource industry.

Industry investors are exposed to a variety of significant risks, because of the continuous flow of new technologies, increased use of capital-intensive equipment, expansion of markets, changing political strategies of host governments about mine ownership, and environmental concerns. The position of the mineral resource industry as a backbone to many economies requires that those connected with its development search for appropriate techniques to support better evaluation of new mining ventures [Myers and Barnett 1985]. This study is part of the process; its focus is mine investment evaluation.

1.1.1. Definition of Mineral Project Evaluation

Capital investment refers to the sequence of decisions that ultimately leads to the acceptance or rejection of spending proposals, along with the subsequent management of the accepted proposals. The entire process comprises the activities of planning, evaluation, selection, implementation, and control, as well as continuous reevaluation and auditing of results [Gentry and O'Neil 1984].

Mineral project evaluation (mine venture analysis, mine investment analysis) consists of an array of analytical and judgemental techniques and processes that can define for an investor the value, viability and uncertainty associated with a project in a given economy [Gentry 1980; Slavich 1982; Stermole 1982; Gocht et al. 1988]. Such an evaluation or analysis also provides the operating management with the technical, operational and economic guidelines for the extraction of the deposit [Frimpong 1988]. Evaluation may also provide part of the basis for decisions about project acquisition, financing, taxation and regulation [Gentry and O'Neil 1984; Sprague and Whittaker 1986; Whiting and Stinnett 1987]. For existing projects, evaluation may provide a means for controlling variances or improving operational standards.

1.1.2. Nature and Scope of Mineral Project Evaluation

Mineral project evaluation is interdisciplinary in nature. In particular, where major projects are concerned, it requires experts from many fields, such as geology, mining, engineering, mineral processing, economics, finance, environmental, and regulatory departments. The decision-making involved combines the vision of the developer, the organising talent of the manager, the analytical ability of the economist, and the technical capability of the engineer, together with the mathematics of finance [Sprague and Whittaker 1986].

The exploration geologist locates geological anomalies and defines potential mineral deposits by establishing deposits' shape, size, grade, depth of cover and many other factors. It involves geological mapping, geophysical prospecting, geochemical surveys, prospect drilling, trenching and/or exploratory underworkings [Payne 1973; Gocht et al. 1988].

A more detailed documented delineation of a potential orebody requires extensive exploratory drilling, sampling and assaying to determine, reliably, ore grade and tonnage, ore mineral and host rock characteristics, structural features, and any other factor which will assist in specifying all the essential geological characteristics of the deposit [Payne 1973; Bruce 1982]. A detailed quantification of the ore reserve tonnage and the associated grade is the next focus of attention. Resulting estimated ore reserves are then classified, according to various levels of uncertainty, into measured, indicated, inferred, paramarginal, submarginal, hypothetical, and speculative categories according to the U.S. Geological Survey classification system [McKelvey 1972; United Nations 1979; U.S. Bureau of Mines 1980; Readdy, Bolin and Mathieson 1982; Buijtor 1983]. The estimated reserves, their respective average grades, and the associated levels of uncertainty, form the basis for the investment analysis.

The type of mining, surface or underground, and the particular mining method to be used are determined by comparing the investor's profit margin using each technically feasible alternative. The type and method of mining are based on the depth, geometry and spatial attitude of the orebody, and the physical properties of the host rock and orebody [Morrison and Russell 1973; Nilson 1982]. Selection of equipment and other machinery, based on the preliminary design, is carefully analysed by the mining engineer. Metallurgical tests on samples from the orebody are carried out to determine a suitable economic method for processing the ore by the mineral processing engineer. Conclusions from such studies form a major part of the basis for initial capital and operating expense estimates.

A "base case" or preferred scenario is selected based on the evaluating team's experience, and the expectation that a particular arrangement of alternatives will provide the best overall solution. Often, this is an extremely important step, because time alone does not allow consideration of all conceivable, or even reasonable, alternatives. This critical phase where judgement plays a very large role is where the methods described later in this report will benefit the decision-making process.

Once a base case is defined, a life-of-mine cash flow analysis is calculated based on the most probable cost values. The "base case" covers development, mining, hoisting, haulage and/or transportation, processing and waste management, and the required support systems. Mineral marketing considerations are very important in determining annual revenue values. However, this aspect is not covered here, because adequate treatment is beyond the scope of this report. The final evaluation must take cognizance of the political and the legal framework, and the environmental concerns associated with the proposed venture. The final product forms the basis for all communications related to raising capital for financing the project [Ballard 1983]. It must, therefore, be a high-quality document which will instill firm confidence in a reading audience with highly-varied backgrounds and viewpoints.

1.2. Rationale for Appropriate Evaluation Tools in the Mineral Industry

Developments since the second World War have stimulated the search for criteria, or systematic decision rules, for project appraisal. Rapid economic growth and technological progress have vastly increased the investment opportunities to firms having limited funds. However, increasing technological complexity and delays encountered in coping with regulatory requirements have increased the lag between decision-making and the benefits of these decisions. The problem has also been compounded by inflation, the increasing size of capital requirements, and an increase in the rate of technical and product obsolescence. Thus, correct project appraisal decisions are crucial [Bromwich 1985]. Mistakes often cannot be recouped, because most capital investments are highly specific, and have little salvage value for alternative uses [Pindyck 1988].

In particular, the mineral industry is extremely capital intensive, and thus investment decisions fix the long-term operating framework and success or failure of the firm. Mineral ventures are also characterized by possible early cost overruns, as some problems are identified only during development, and these severely affect the economic viability of projects [Haldane 1985]. Long preproduction periods, ranging from four to 10 years, is another characteristic of mineral ventures. The significance of these long lead times is amplified when considered in conjunction with the capital intensity of the industry, inflationary trends, high interest rates and volatile markets [Gocht et al. 1988]. Not only do companies commit extremely large capital resources to new mineral ventures, but shareholders and lenders are financially exposed, for long periods, to a variety of uncertainties prior to project start-up, and prior to recovery of their initial investments, even for a projected high yielding venture.

In addition to these, there are a number of other risks associated with mineral ventures. In particular, there are ore reserve, geological and mine development, operating and political risks [Lessard and Graham 1976]. Mineral markets have changed fundamentally. The stochastic nature of output prices, and the inability of producers to sell full output due to changes in supply and demand, also affect project risks significantly. Commodities from mineral ventures often involve intricate contractual arrangements, where performance at specified quality, quantity and cost is required to attain projected profitabilities. Uncertainties are also increasing with the continual depletion of high-grade, near-surface orebodies. Ore grade distribution, orebody dimensions and homogeneity, and other geological and geomechanic characteristics of both the host rock and the orebody, are still complex factors which call for risk-prone judgements, even with the application of geostatistics and well-researched soil and rock mechanics techniques .

Unforeseen governmental actions, e.g., nationalisation, and other changes in the regulations concerning the environment, taxation, currency convertibility and transfer, import duties, exchange rate, are major sources of concern in mineral project evaluation [Pralle 1985]. There is an accelerating trend towards greater participation in mineral projects by host governments throughout the world. Public pressure related to environmental issues, in both developed and developing economies, caused governments to enact environmental legislation in the late 1960's and 70's which affect the economic viability of mineral projects [Parr 1982; Gentry and O'Neil 1984; Croft 1985]. There is an environmental cost associated with the production of a ton of ore today, and it is expected to be much higher in the future. The overall objective is to produce necessary mineral products while minimizing adverse environmental impacts. This involves a systematic and realistic assessment of the tradeoffs involved, and realistic political long-term support of decisions related to mineral venture investments.

The mineral industry also deals with the extraction of nonrenewable resources. Thus, revenues from mining are derived from a complete disposal of a venture's primary asset, the orebody. Under the basic philosophy of the natural heritage theory [Gentry and O'Neil 1984], nonrenewable mineral deposits are regarded as assets created for the benefit of all mankind, and should be mined for the benefit of society as a whole. This has caused many nations, states and provinces to enact tax policies which treat mineral ventures differently, and more severely, than other industries. Also, all mineral ventures have finite lives, determined by the size of the ore deposit and the extraction rate. Therefore, investors must recover their investments and receive an adequate rate of return by the time the reserves are depleted in order to maintain the capability and incentive to make new investments.

These characteristics and problems inherent in mineral ventures demand that the most thorough and sophisticated evaluation be made before it is possible to establish a true measure of the risk and/or return on investment in a mineral project with an acceptable degree of certainty.

1.3. Problem Definition

Current economic evaluation techniques, available to help investors and managers make decisions about the viability, value and uncertainties associated with mineral projects, in the light of the above technical and operational details, are not always adequate. A proper framework for the design of financial analysis methods would be a realistic theory of economic organisation and markets with careful consideration of the dynamics related to the particular venture and its environments [Laughton 1988]. It would also encompass strategic planning of all feasible options that should attract the firm's scarce resources in the short and long-term in order to maximize the market value of the firm [Myers 1984].

Despite major advances in the theory of finance over the past three decades, practical procedures for guiding capital investment have evolved only slowly. Economics in its current state has simplified descriptions of corporations and governments. Missing from these descriptions, however, are many details of the internal structure and behaviour of organisations. Current capital budgeting techniques consider limited descriptions of financial markets, and the effects of varying organisational behaviour in these markets [Laughton 1988]. Agent preferences and endowments, information and belief structures, production and transacting technologies, are all very simple in overall asset pricing models.

Existing economic evaluation techniques in the mineral industry are derived from two past stages of asset pricing theory : (1) the certainty model of evaluation from Fisher (1907) as illustrated in Figure 1, and (2) the one-period asset pricing model.

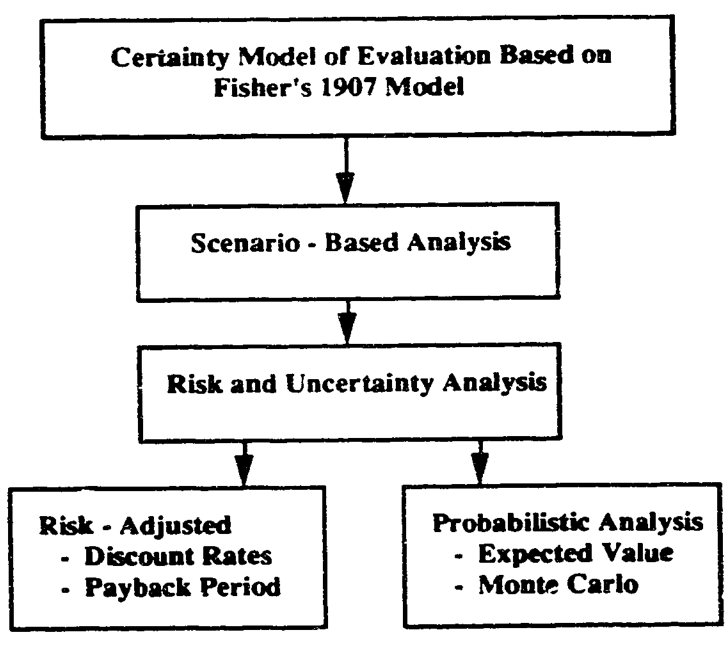

Figure 1 Current Economic Evaluation Models

The standard technique, based on the simplest asset pricing models, which has remained unchanged since it was originally proposed [Dean 1951; Bierman and Schmidt 1960], is derived from a simple adaptation of the Fisher (1907) model of valuation. It is essentially the model often used in mineral venture analysis, as described above. More generally, however, the Fisher model is based on minimal future uncertainty; in other words, it is a model which predicts the future with certainty. Under this technique, the cash flows from an investment project, in each of many scenarios, are discounted at an investor-selected rate, and the resulting net cash flow and net present value are used as data in the project selection process.

To incorporate uncertainty in the valuation process, investors often use a risk-adjusted discount rate or a risk-adjusted payback period. The expected value method and the Monte Carlo method [Hertz 1964] may also be used to estimate expected scenario values using probability distributions of the input variables. Often, analysts take the central tendency of the scenario results from this type of analysis as the most probable return on investment, and the plus/minus variation as a measure of project risk. However, if the discount rate already has a risk premium in its formulation, that is, if it is higher than the rate obtainable from secure investment, such as a government bond, the return is already risk-adjusted. Therefore, the effects of uncertainty show up in two places; at the level of each scenario result and in the spread. The individual impacts of these two effects, as well as their interaction are not clearly stated.

Secondly, even though the spread of scenario results may indicate "something" about the variance of cash flows, risk is related to the contribution of the uncertainties of these cash flows to the uncertainty in the future value of the diversified portfolio of assets held by project shareholders. This contribution to overall uncertainty is best measured by the covariance of the project and portfolio returns, and not by the variance of the cash flows [Cox, Ingersoll, and Ross 1985; Brealey and Myers 1986; Jacoby and Laughton 1990].

The difficult problem of obtaining the appropriate discount rate for a specific project, using this approach led to the development of the *one-period asset pricing model*. Using this approach, the discount rate for a specific project is calculated by applying a one-period asset pricing model, e.g., the *capital asset pricing model (CAPM)*, to estimate a one-period risk premium for a portfolio of assets that has a similar risk structure to the project at hand [Brealey and Myers 1988]. This risk premium is added to the risk-free rate of return to yield the project discount rate. However, every project or venture has its own inherent and external characteristics that make it different even from similar ones, and therefore this approach fails to capture the real discount rate for a specific project.

These methods also fail to recognize the use of new information over time to resolve uncertainties. The problem is that a single-period representation of uncertainty often is inappropriate for the evaluation of projects that occur over multiple periods. The use of a single known discount rate involves an implicit approximation that key future conditions are known and stationary. These conditions include aspects of the economy such as term structure of future interest rates and prices of risk of mineral commodities, and also specifics of the project such as the risk of each of the future net cash flows [Fama 1977]. The risk structure of most projects is not stationary: it evolves as changes occur in the sources and magnitudes of the uncertainties underlying the project, and as this varying uncertainty is filtered through a changing project structure. In particular, it is necessary to consider the period-by-period variation in the structure of risk in order to give appropriate weight to long-term cash flows [Laughton and Jacoby 1991] or to support analysis of operating flexibility [Laughton and Jacoby 1991a].

Also, certain types of mineral project risks and uncertainty, e.g., geological, technical and operational, do not affect the macroeconomic structure of the economy, and therefore should not affect the project discount rate. These types of project risks and uncertainty are unsystematic, i.e., they are peculiar to the project, and should be eliminated through detailed delineation, documentation, and diversification by investors.

Furthermore, these DCF methods are of little help in valuing long-term projects and businesses with opportunities for future management, e.g., investment timing options, feasibility study management, and operational considerations, such as when to close, reopen or abandon a project [Fruhan 1979; Bierman 1980; Alberts and Mctaggart 1984; Myers 1984; Brennan and Schwartz 1985; MacDonald and Siegel 1986; Majd and Pindyck 1987; Frimpong, Laughton and Whiting 1991]. However, all these are very important strategic inputs in the decisions to be considered in a mineral project investment analysis. This is done by defining and incorporating, in the models, the options available to investors throughout the feasibility study, investment, and operational phases of the mineral venture.

As a step towards the solution of these and other pertinent problems faced by organisations, two major advances have been made recently in asset pricing that supersede these one-period models. They represent the third stage in the development of asset pricing theory. They are : (1) continuous time, dynamic arbitrage models of derivative securities [Black and Scholes 1973; Cox and Rubinstein 1985; Hull 1989; Duffie 1989], and (2) full general equilibrium models of asset pricing in continuous time [Merton 1973; Breeden 1978; Cox et al. 1985; Duffie and Huang 1985].

The insights into asset pricing provided by these ideas have had profound effects on the development of many financial markets, such as those for options, futures, and collateralised securities. They give traders the ability to calculate values for complex assets and provide methods for designing hedging positions for firms and other economic agents as they seek to extract value from their financial structures.

The use of these developed theories of finance offers one of the best opportunities to help the mineral industry advance the state-of-the-art methods in mineral venture analysis. It is important, however, to note that most of these ideas have not been explained well enough to engineers and project managers whose duties require project or venture analysis and evaluation.

1.4. Objectives of the Study

The primary objective of this study is to use these new developments in asset pricing theory in a modelling framework, as first introduced by Brennan and Schwartz (1985) and Laughton (1988) to develop a new evaluation tool, the derivative asset valuation method, to:

1. Examine an *array* of decisions about the management of (a) feasibility studies, (b) investment timing option and (c) mine operating options

2. Quantify the *effects of the choice of decision* on the value of a mining project

Also, an analytical review is included of the significant literature published by a selected group of writers and commentators in the field of mineral economics, financial economics and engineering economics. The review is to provide an analysis of the evolution, and problems encountered in the development of these evaluation tools, their application over time, and to identify areas that require further research work.

1.5. Scope and Limits of the Study

This study deals with the economic evaluation of a mineral venture based on the derivative asset valuation (DAV) method [Black and Scholes 1973; Brennan and Schwartz 1985]. It is concerned with *the effective management of a feasibility study program and investment timing in mineral exploration and the development of a mineral venture, producing a single homogeneous commodity, which is priced in a competitive market.*

The value of the mineral venture is modeled using the principle of portfolio replication and the value of a futures contract on the mineral commodity. The approach is to find a self-financing portfolio of simple assets, the cash flows of which replicate, in each possible future, the cash flows in that future from the mine. Analysis of the dynamics of the value of the mineral venture is carried out in one and two dimensions.

In the one-dimensional model, the problem is formulated by examining, thoroughly, the uncertainty associated with the mineral commodity price and the metal reserve and how this uncertainty is expected to be resolved through time. In the two-dimensional model, the ore grade uncertainty, the investment cost and their resolution through feasibility study stages are examined in addition to the mineral commodity price uncertainty. These models are used to estimate the best investment timing, feasibility study management, other operating options, and the resulting value of the mineral venture. The results from the DAV analysis are compared with those from the DCF method.

Generally, the derivative asset valuation method is limited because the explicit analysis rests upon the assumption that the relevant portfolios may be formed by trading in futures contracts in the output commodity, but the general approach can also be used even if the relevant futures markets do not exist. Also, the theory of the derivative asset valuation method requires that it is impossible or extremely difficult for an investor to make arbitrage profits (i.e. riskless profits gained by simultaneously entering into transactions in two or more markets).

The specific models used in this study are also limited because, future volatility of metal price expectations, future price of risks of the mineral commodity, and interest rates are assumed to be known with certainty. This restriction on the process makes the underlying information structure easy to handle.

1.6. Significance of the Study

In current practice, the risk-return trade-off is made implicitly both in the choice of the discount (hurdle) rate, and in the qualitative examination of the distribution of the cash flows and scenario results. The analysis is diffuse and disjointed. Using the methods proposed in this report, risk valuation is performed explicitly and quantitatively. Moreover, it is done at the level of the underlying variables, using uncertainty models of those variables, rather than at the more complex level of the cash flows.

Also, the difficult problem of determining the appropriate discount rate for a specific project is finessed by using, as inputs, the discounting structures (i.e. probabilities, price of risks, and time discount factors) for claims to cash amounts determined by the underlying variables into the models for project cash flows.

The DAV method proposed in this study *demonstrates* how risks associated with projects could be resolved through further investigation, documentation and time. It is also consistent with economic rigour, because it is based on a general equilibrium model that determines asset prices from economic primitives of preferences, technologies, and endowments by analysing the interacting maximizing behaviour of all agents in the economy. In this general equilibrium model, the term structure of future interest rates, price of risk of commodities, specifics of the risks of future cash flows, and other key variables that affect the general economy are treated as stochastic variables.

The proposed method can be used to evaluate mineral projects which have opportunities for future management; for example, investment timing options, feasibility study management, and operating options, such as when to close, reopen or abandon a project. Conventional DCF techniques have no built-in procedures to investigate the appropriate time dynamics of these options to achieve investor objectives.

1.7. Research Methodology

The analysis in this study is based on several methodological procedures. An analytical review of the literature of past and current project evaluation techniques and practices, and current advances in asset pricing theory constitutes the main background for the study.

As a further method of analysis, a model-building approach with specific assumptions is used in the quantitative analysis of the value of the mineral venture. Analysis-of-time dynamics, and the effect of several variables in the model of the mineral venture value, rely to a great extent on existing knowledge in statistics, mathematics, finance, mining engineering and engineering economics. Computer programming (in C Language) is used to translate the mathematical models for experimentation and convenient, detailed analysis for the desired objectives.

The DAV models are tested using data from a copper mine, as a step to make the proposed method of evaluation understandable to potential users. A model developed based on the DCF technique is also tested using the same data. A thorough analysis of the outputs from these two methods provides an insight into how the two methods compare with each other.

1.8. Structure of the Study

Chapter 2 deals with the theory and practice of the state-of-the-art methods currently in use in the mineral industry to evaluate projects, and the theory of the method which forms the basis of the models developed in this thesis. Chapter 3 deals with a preliminary exercise on feasibility study management and the development of a typical mineral venture using the derivative asset valuation method. The value of the mineral venture is modeled based on the principle of portfolio replication and the value of a futures contract on the mineral commodity. The option to wait on feasibility study and investment, and operating options, such as when to close temporarily, reopen or abandon the project, have also been considered. The resolution of mineral project risk as a result of metal price and metal reserve uncertainties is also considered in this model.

In Chapter 4, the value of a mineral venture, with ongoing feasibility study is mathematically modeled to help an investor make the most informed decisions on what magnitude of effort constitutes an optimal feasibility study. Values are formulated for waiting and investing under uncertainty, or undertaking a feasibility study, in multiple stages, to reduce uncertainty associated with the ore grade. The investment cost, at any feasibility stage, is also modeled to incorporate a *cost of ignorance factor*. This cost of ignorance factor is a cost borne by the investor for knowing little about ore grade and reserves. The resolution of mineral project risk, as a result of uncertainties associated with metal price, expected ore grade, and the investment cost, is also provided in these models.

Chapter 5 deals with the solution procedures and the experiments designed to implement them. The solution algorithms for the models in Chapters 3 and 4, and the flow charts of the computer programs used to solve the problems, are presented and described. Chapter 6 deals with the validation results of the models developed in Chapter 3, using real world data from a copper mine. Analyses of these results have been provided to show the merits of the derivative asset valuation method to potential users.

Chapter 7 also presents the validation results of the models of the value of the mineral venture in Chapter 4, using real-world data from a copper mine. Analysis has been provided of the results from various feasibility study strategies, timing options and the state of the economy and their effects on the value of a mineral venture. Chapter 8 deals with the summary, conclusions, and recommendations for further research works arising from this research study. A list of references and appendices have also been provided, at the end of the report, as sources of various citations in this study, and other materials that may be helpful in the use of this document for academic and industrial purposes.

1.9. Relevant Terminology

It may also be helpful to take note of the following terminology to understand their use in subsequent portions of this report. *Arbitrage* is locking in a riskless profit by simultaneously entering into transactions in two or more different markets. A *futures contract* is a trading agreement between two parties to buy or sell an asset at a certain time in the future for a certain price. The party who agrees to buy the asset assumes a *long position* and the one who agrees to sell the asset assumes a *short position*. The date on which this contract matures is termed the *maturity date*. An option gives the holder the right to buy *(call)* or sell *(put)* the underlying asset by a certain date for a specified price. An option that can be exercised at any time up to the maturity date is an *American option*, and it is a *European option* if it can be exercised only at the maturity date.

A *derivative security* or a *contingent claim* is a security whose value depends on the values of other more basic underlying variables. *Derivative asset valuation (DAV)* derives its name from the fact that projects or ventures being evaluated could be derived from other more basic assets by *replication*. A *portfolio* is a collection of securities held by an investor. An *asset* is a property of monetary value to an investor. *Hedging* is used to protect oneself financially especially by buying or selling commodity futures as a protection against loss due to price fluctuation. *Macroeconomics* is the part of economics that deals with whole systems, especially with reference to general levels of output and income and their interrelations.

CHAPTER 2.0

ANALYTICAL SURVEY OF THE LITERATURE

State-of-the-art methods used in the mineral industry to evaluate projects are reviewed. Also, evolution of the theory and some applications of the derivative asset valuation (DAV) method which form the basis of the evaluation models created for this thesis, as well as numerical procedures to solve the resulting partial differential equations, are presented and examined thoroughly. This section will help readers understand a considerable level of detail concerning the strengths and weaknesses of such methods, and will provide a stronger base for understanding the work discussed later in this report. However, intensive study of this chapter is not essential for one to develop a working knowledge of the concepts presented later by the author.

2.1. State-of-the-Art Methods in the Mineral Industry

Current economic evaluation of mineral projects combines quantitative discounted cash flow (DCF) techniques with various sensitivity analyses to provide a basis for judgemental decisions by project analysts and investors. The fundamental objective underlying the DCF techniques is to calculate and compare (a) the internal rate of return (IRR) on initial capital investment, and (b) the net present value (NPV) at a specified discount rate.[1]

A typical venture in the mineral industry is characterized by a large initial capital investment which is put into place over a period of one or more years. In a *discounted cash flow* (DCF) analysis, this investment is mathematically described as specific negative cash flows per year. Once the initial investment is complete, production starts, and new profits after taxes are earned each year. These values, plus any sums excluded from taxation (e.g., depreciation, depletion and sales of capital assets) comprise the annual *positive* cash flows, from start-up to shut-down. Capital replacements and additions/expansions are

[1] See Appendix A.1 for a historical perspective of the DCF techniques.

added algebraically as negative cash flows in the years they occur after start-up. For planning and evaluation purposes, such *negative* cash flows rarely exceed the positive cash flows in absolute values. Therefore, the cash flow stream which is characteristic of mineral ventures rarely has negative values after the period of initial investment and start-up.

Project or venture net present value, *NPV* is the algebraic sum of annual negative and positive cash flows which have been discounted at *any* selected discount rate [Stermole 1982; Gentry and O'Neil 1984]. It is a common practice for evaluations to calculate the NPV of two or more projects using a selected discount rate (i.e., 10 percent), and then to compare the results. The project with the most positive NPV is favoured for development.

Project or venture *IRR* is equal to that discount rate which yields equal negative and positive discounted cash flows at a particular time of evaluation. Therefore, when the cash flow stream of a venture is discounted at its IRR rate, the resulting NPV is zero [Newendorp 1979; Gentry and O'Neil 1982]. *The value maximization rule, using the IRR criterion, is to accept all projects whose IRR's are greater or equal to the investor's minimum acceptable rate of return on investments which bear a comparable level of risk.*

Using either NPV or IRR as a criterion for project evaluation and selection has the objective of recovering all invested capital (initial, replacement, expansion), and also providing a selected rate of return on this capital, calculated over the life of the project. The method is particularly useful in comparing investments which have different cash flow profiles, and which take place at different times. It has mathematical simplicity and logic which are appealing and understandable to engineers and managers, and as a result, it has wide acceptance as a practical aid to investment decision-making.

The *Benefit/Cost Ratio (BCR)*, or the profitability index (PI), is the ratio of the sum of the present value of the benefits to the sum of the present value of all investments and other costs [Quirin 1967]. The value maximization rule, using the BCR or PI criterion, is to accept projects whose BCR or PI is greater or equal to one.

Henry Hoskold in 1877 developed the first mining-related evaluation technique which incorporates the concept of present value [Parks 1950; Gentry and O'Neil 1984]. He assumed that an investor would require a certain return, the *"speculative" or risk-adjusted* return, on his invested capital in a mining property. In addition, he would establish a sinking fund that would yield an amount equivalent to the initial investment to allow him to invest in another property at the end of the project's economic life. This sinking fund is expected to yield the *"safe"* or risk-free rate of return. The decision criterion for this technique is to accept projects with positive net present value and otherwise reject them.

Capen et al. (1970) develop the Growth Rate of Return (GRR). The GRR is calculated by first discounting all negative cash flows (i.e., investment cost) to the project start date using the project reinvestment rate. All positive cash flows are then compounded to some future time also using the project reinvestment rate. Cash flows after this time are discounted back to this time using the reinvestment rate. The GRR is the rate at which the present value of capital investments must grow to equal the sum of the cash flows at this future time. Any project whose GRR is greater than the reinvestment rate is accepted, and it is rejected otherwise.

Berry (1972) also develops the wealth growth of return (WGR) similar to the GRR. The WGR is the compound interest rate which equates the future value of the capital investment with the future value of the cash flows resulting from the project at the termination date of the project. The positive net annual cash flows subsequent to investment are assumed to be reinvested at the firm's reinvestment rate to the project termination date. Capital investments preceding production are discounted to start-up date, using the reinvestment rate. The WGR is then the compound rate at which the cumulative discounted capital must grow in order to equal the future value of the wealth generated by the project. Any project whose WGR is greater than the reinvestment rate is accepted, or it is rejected otherwise.

Derived from the Fisher (1907) certainty model, these DCF techniques for resource evaluation have the following deficiencies: (1) They are based on a theory that predicts the future with certainty; (2) it is difficult to select an appropriate discount rate for a specific project; (3) they fail to recognize the use of new information, over time, to resolve risk and uncertainties associated with projects, and therefore tend to under-value long-lived projects; and (4) they have no built-in procedures to analyse projects which have opportunities for contingent future management, e.g., feasibility study and investment timing, and operating options, such as when to close, reopen, or abandon a project.

Other profitability measures used in the mineral industry, in addition to DCF techniques, are *payback period*, *profit-to-investment ratio*, and *accounting rate of return* [Newendorp 1979; Gentry and O'Neil 1984; Sprague and Whittaker 1986].[2]

[2] Some readers may be interested in reading Appendix A.2 to review the evolution and theory of the capital asset pricing model (CAPM) which is in use for selecting efficient portfolios in the markets and in some industries.

2.2. Advances in Finance Theory

Superior project choice and management are critical to the success of any organisation in the mineral industry. A particularly important part of project or venture evaluation is the analysis of the effects of uncertainty and risk on design, management, and value of the project. In the face of faster and more complicated technological and environmental changes, and more heated and complex international competition, managers and investors should be able to take advantage of superior evaluation methods, which if used well, could help them to better understand and *control the effects of uncertainty and risk* .

Over the past two decades, major advances in asset pricing theory have been made based on a small set of propositions about the structure of financial markets, and the information content of financial market prices. The key proposition is that *evaluation may be carried out, to a good approximation, as if financial markets were competitive and free of transaction barriers. In such a market, different assets which produce the same cash flow results, have the same price. Moreover, in such a market, it is possible to replicate the cash flow results, and thus, the value of a complex asset, such as a proposed or actual mineral venture, by executing a trading strategy in portfolios of simpler assets, such as riskless bonds and metal futures contracts.*

Finally, all asset prices are determined by the risk preferences of investors, as reflected in the markets. Thus, the basic assets that provide information about risk discounting are those that have some direct interaction with future macroeconomic variables. For example, in this application, the basic assets are the metal futures contracts, which are related to the corresponding future prices of the mineral commodity, which are correlated with the state of the economy.

The mathematical representation of the state variables in the underlying asset of the derivative security is based on *stochastic processes*. Any variable whose value changes over time in an uncertain manner is said to follow a stochastic process [Hull 1989]. Examples of stochastic processes include discrete finite state processes, or in continuous time: (1) a diffusion process with continuous sample paths, i.e., the state variables are changing all the time, but the magnitude of those changes are small over a short time period, (2) a compound Poisson process with step function type sample paths, i.e., the state variables do not change over a short time interval, or, with a very small probability, a radical change or "jump" can occur, and (3) a mixture of the above.

The best known results in continuous-time analysis have come from restricting the state variable dynamics to being a diffusion process with a continuous sample path, such as the one used in physics to describe the motion of a particle that is subject to a large number of small molecular shocks [Merton 1975; Hull 1989]. In a similar vein, the price of a stock or commodity is subject to a large number of shocks in the market, and these shocks cause this price to diffuse over time in a manner that is described by a continuous stochastic process.

One important type of diffusion process is the geometric Brownian motion process. The instantaneous change in a variable that follows a geometric Brownian motion process is normal with finite expected value and variance. Also, the derivative of the expected value of the variable with respect to time is proportional to the variable.

2.2.1. Dynamic Arbitrage Models of Derivative Securities

The theory of derivative asset valuation (DAV) is often referred to as "option theory" because of its early application to options on common stock and later applications. Advances in the equilibrium pricing of simple options initiated the development of a general theory of derivative securities.[3]

Black and Scholes (1973) provide the first equilibrium solution to the option pricing problem for simple calls and puts, and suggest that this analysis could provide a basis for a general analysis of contingent claim assets or derivative securities. They show that, under certain conditions, it is possible to create an instantaneously riskless hedge, in each state, by forming a state-dependent portfolio containing stock and European call options. The equilibrium condition that results from setting the return of riskless hedge part to the risk-free rate is a partial differential equation which, with appropriate boundary conditions, can be solved to give the value of the option as a function of the underlying stock price and time.

To derive the option pricing model, they make the following assumptions about the market for the stock and option: (1) There are no transaction costs or differential taxes;

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[3] See Appendix A.3 for a description of the early incomplete equilibrium models of call option pricing.

trading takes place continuously in time; borrowing and short-selling are allowed without restrictions, and with full proceeds available; and borrowing and lending rates are equal; (2) the short-term interest rate is known with certainty; (3) the stock pays no dividend or other distributions during the life of the option; (4) the option can be exercised only at the maturity date; and (5) the stock price follows a geometric Brownian motion through time which produces a log-normal distribution for stock price between any two points in time.

In a subsequent alternative derivation of the Black and Scholes (1973) option pricing formula, *Merton (1973b)* demonstrates that their basic mode of analysis still holds even with a stochastic interest rate, dividend payments by the stock, and the possibility of exercising the option prior to maturity date. He further shows that, as long as the stock price dynamics can be described by a continuous-time diffusion process, the sample path of which is continuous with a probability of one, the Black and Scholes arbitrage technique is valid. *Thorpe (1973)* also shows that dividends and restrictions against the use of proceeds of short-sales do not invalidate the Black and Scholes analysis. *Merton (1977)* refines the Black and Scholes option pricing formula.

Extensions of the Black-Scholes-Merton option pricing model have been made by relaxing some of the underlying assumptions. The Black-Scoles-Merton option pricing is formulated based on the assumptions that (1) there are no transaction costs or differential taxes, (2) the stock price follows a diffusion process with continuous sample path, and (3) the stock volatility is constant.

Ingersoll (1975) modifies the Black and Scholes option pricing model to account for the effect of differential tax rates on capital gains versus ordinary income. He formulates the price of a call option, for the simplest case, where dividends and interest are paid continuously at a given rate, and taxed at different rates, and with zero capital gain taxes.

The effects of transaction costs on the Black and Scholes (1973) option pricing under perfect frictionless markets have also been studied by many, including Leyland (1985), Merton (1990) and Boyle and Vorst (1992). *Leyland (1985)* uses a continuous-time framework, and derives a Black and Scholes type of approximation for the option price in the presence of proportional transaction costs. He constructs a replicating stock-bond portfolio which "almost" replicates the value of the option at maturity.

Merton (1990) sets up the problem in a discrete-time framework and derives the current option value when there are proportional transaction costs on the underlying asset. He constructs a portfolio of the risky asset and riskless bonds that precisely replicates the option value at expiration. His approach makes an allowance for the transaction costs arising from portfolio rebalancing.

Boyle and Vorst (1992) extend Merton's analysis to several periods. They employ a discrete-time framework, and construct the portfolio to replicate a long and short European call option.

Merton and Samuelson (1974) show that the continuous trading solution of the Black and Scholes option pricing model is a valid asymptotic approximation to the discrete-trading solution, provided that the dynamics have continuous sample paths. Under these discrete-trading conditions, the return on the Black and Scholes "no-risk" arbitrage portfolio will have some risk. However, the magnitude of this risk goes to zero as trading interval goes to its continuous limit. However, the Black and Scholes solution is not valid, even in the continuous limit, when the stock price dynamics cannot be represented by a stochastic process with a continuous sample path.

Cox and Ross (1975) demonstrate that a risk-free hedge can still be created, if the stochastic part of the stock price movement has a jump in only one direction with a given amplitude. These two models suggest that the assumption of a continuous sample path for the stock price is not crucial to the analysis.

Merton (1976) examines the most general specification of the stock price movement with both the geometric Brownian motion and the Poisson process. He shows that hedging against both continuous and discrete changes is not possible, and thus, the risk-free hedge is not possible in this case. However, if the jumps are correlated across securities, then the risk associated with the jumps are unsystematic, and it can be minimized by holding a portfolio of hedges. If the equilibrium return on a security is determined by its non-diversifiable risk, then the continuous part of the stock price movement can be hedged using the Black and Scholes technique.

Geske (1979) examines the case in which the volatility of the firm is constant, so that the stock price volatility changes in a systematic way as the stock price rises and falls. *Johnson (1979)* studies the general case in which the instantaneous variance of the stock price follows some stochastic process. However, in order to derive the differential equation that the option price must satisfy, he assumes the existence of an asset with a price that is instantaneously perfectly correlated with the stochastic variance.

Hull and White (1987) examine the problem of a call option on an asset with a price with stochastic volatility. They determine the option price in a series form for the case in which the volatility is correlated with the stock price. They find that the Black and Scholes option price model sometimes overprices options, and that the degree of overpricing increases with increasing maturity date.

2.2.2. Development of a General Equilibrium Framework

The main problem of the dynamic arbitrage equilibrium models is that they are not based on a general equilibrium model that determines asset prices from economic primitives of preferences, technologies, and endowments by analysing the interacting maximizing behaviour of all agents in the economy. They are conditioned by the assumptions put forward to derive them. Unfortunately, full general equilibrium models of asset pricing are not generally usable at this time for resource evaluation, because of the amount of calculations involved. Three types of such models have been proposed, which represent the current best theory of asset pricing.

The first is a set of representative (or single) agent models of which the works by Cox, Ingersoll and Ross (1985) on real interest rates, Brennan and Schwrtz (1982a,b) on regulation and (1984) on the effect of capital structure on managerial behaviour, and Stultz (1987) on real exchange rate are examples. Analyses are carried out as if all agents share the same preferences. Technologies must use malleable perishable goods to produce goods with stochastic returns. These models are consistent with the economic model of human behaviour. They provide a basis for modelling uncertain real interest rates, real exchange rates, and certain inflation assumptions. However, tractable calculations can, at present, be done only in models with restrictive assumptions about preferences and production technologies.

The second set of models are based on consumption and investment decisions under uncertainty [Breeden 1979; Duffie and Zame 1987]. These models are deduced from the portfolio selection behaviour of an arbitrary number of investors who act to maximize the expected wealth of lifetime consumption, and who trade continuously in time. They show that, in equilibrium, investors are compensated for bearing market risk, and for bearing the risk of unfavourable shifts in the investment opportunity set. Furthermore, the expected return on a security with no market risks is not equal to the risk-free rate as given by the single-period capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965) and Mossin (1966).

The last set of models are the pure general equilibrium models in which Arrow-Debreu general equilibria are implemented in economies with traded assets [Duffie and Huang 1985; Huang 1987].

2.2.3. Applications of these Advances in Resource Industry

In the application of the theory of option pricing to evaluate the resource industry, *Black and Scholes (1973)* analyse the value of other contingent claim (derivative) assets. They argue that the position of the stockholders is equivalent to that of the purchaser of a call option, and the bondholders to that of the writer of a call option, and briefly outline the possible applicability of this analysis for many issues in finance and managerial economics.

Tourinho (1979) makes an early attempt to apply the theory to study an ore reserve evaluation. *Myers and Majd (1983)* also makes an early attempt to estimate the value of abandoning an asset, or its salvage value. MacDonald and Siegel (1986) study the optimal timing of investment in an irreversible project where the benefits from the project follow continuous-time stochastic processes. They explore the practical importance of the value of waiting to invest, assuming that investment timing decisions are made by risk-averse investors who hold a well-diversified portfolio. They derive explicit formulas for the value of the optimal time to invest when both the value of the project and the cost of investing are stochastic, and show the dollar value lost by investing in a project at a sub-optimal time.

Paddock, Siegel and Smith (1988) use the option pricing theory to develop an approach to valuing leases for offshore petroleum development. They demonstrate how to integrate an explicit model of equilibrium in the markets for the underlying real asset with option pricing theory to describe the value of a real option. The main weakness of the approach used by Myers and Majd (1983), MacDonald and Siegel (1986) and Paddock, Siegel and Smith (1986) is that they quantify and examine project risk at the level of the project value, instead of the underlying input variables.

Brennan and Schwartz (1985) use the option pricing techniques and stochastic control theory to provide a model for the value of investments in natural resource. They assume that the resource to be exploited is homogeneous and known with certainty, and that costs are also known with certainty. Also, they assume that interest rates, volatility of the metal price, and the convenience yield of the commodity, are constant and known with certainty. They calculate the value of this resource under various operating options, such as when to close, reopen and abandon. They also show the variation in project risk and discount rate due both to depletion of the resource and to stochastic variation in output price.

Laughton (1988) and *Jacoby and Laughton (1991, 1992)* demonstrate the use of a combination of DCF and DAV methods for the valuation of a "now-or-never" oil field development projects. They use the derivative asset valuation (DAV) method to evaluate an oil development project under a complex tax system, and show the deficiencies in the application of the discounted cash flow (DCF) method in such cases, and how they are overcome with the DAV method.

Laughton and Jacoby (1991a) develop a practical method for analysing the investment timing option. Their approach focuses on investments in complex projects which have simple contingent control possibilities that occur in situations defined by simple underlying information models, e.g., the valuation of the rights to explore and develop an oil field. They use this analysis to study the probability of project commitment each year, the value of waiting, and risk characteristics of projects. *Laughton and Jacoby (1991b)* use the derivative asset valuation method to determine the values and the optimal commitment policy for a five-year oilfield development in the U.K. North Sea.

Frimpong, Laughton and Whiting (1991) extend the Brennan and Schwartz (1985) model to consider a simple case of feasibility study management and investment timing in mineral venture development. They show how feasibility study and investment timing options could be used, in addition to operating options (i.e., when to close, reopen, or abandon the mineral project) to maximize the value of the mineral venture.

2.2.4. Numerical Procedures for Solving Option Value Problem

The result of the Black and Scholes option pricing model is a second-order partial differential equation governing the value of the option. Depending on the nature of the boundary conditions which must be satisfied by the value of the derivative security, the partial differential equation may or may not have analytic solution. A simple closed-form solution of this equation exists in the case of a nondividend-paying stock and a stock which pays a continuous dividend proportional to the stock price.

Johnson (1983) and Macmillan (1986) formulate an analytic approximation for an American put on a nondividend-paying stock. Barone-Adesi and Whaley (1987) also apply analytic approximation to other American options.

For other complex cases, numerical methods must be used to solve the resulting partial differential equations since they are not tractable analytically. A number of numerical procedures, including Monte Carlo, binomial lattice, and finite difference methods have been proposed and used to obtain solutions to different cases as discussed below.

Monte Carlo Method: Boyle (1977) uses the Monte Carlo method to value options. This method uses the fact that the distribution of terminal stock prices is determined by the process generating future stock price movements. This process is simulated to generate a series of stock price trajectories. This series determines a set of terminal stock values which can be used to obtain an estimate of the option value. The method is useful in situation where it is difficult to use a more accurate approach. In particular, where returns on the underlying stock are generated by a mixture of stochastic processes or else drawn from empirical distribution. Furthermore, the standard deviation of the results can be estimated for detailed analysis of the results. It provides a fast and flexible method for obtaining approximate answers with confidence limits on the results. One limitation of the Monte Carlo method is that it can only be used for European-style derivative securities.

Binomial Lattice Method: Cox, Ross and Rubinstein (1979) propose the binomial lattice method to value American-style derivative securities. The basic idea is to replace the continuous distribution of stock prices by a two-point discrete distribution over successively smaller time intervals. Convergence to the true option value is obtained by increasing the number of steps. It is simple to implement but it is limited to only one-state variable problem. Boyle (1988) extends the one-state variable binomial lattice method to handle options whose payoffs depend on two underlying variables, even though it is possible that the procedure can be extended to situations involving a higher number of state variables. The lattice then unfolds in several dimensions with probabilities at each node such that each variable has the correct expected growth rate in a risk-neutral world.

Even with only one state variable, Hull (1989) notes that the binomial lattice model may not give the most efficient lattice. Also, in situations where the underlying variable follows a more complicated process than the geometric Brownian motion, a trinomial model [Boyle 1986] may be necessary to provide adequate description. Hull and White (1988) also propose and use the control variate technique to improve the efficiency of lattice binomial approximation.

Finite Difference Methods: Finite difference methods are used to value derivative securities by solving, numerically, the resulting partial differential equations of the derivative securities. These equations are converted into a set of difference equations and solved iteratively. Brennan and Schwartz (1978) use this approach to estimate the value of an option. They show that approximation of an option value by the use of finite difference methods is equivalent to approximating the diffusion process by a "jump" or a step-function process. Courtadon (1982) also proposes another finite difference approximation which improves the Brennan and Schwartz's approach in option valuation. Finite difference methods can be used for all types of derivative securities. They can also be used to value derivative securities with many state variables, but computations and computer time required increase with increasing underlying variables.

All these numerical methods have a dual objective of accuracy and speed of computation. For any given method, greater accuracy can normally be achieved by increasing the computation time. Geske and Shastri (1985) provide a careful compa. ·on of binomial lattice and finite difference methods. They conclude that researchers computing a smaller number of option values may prefer binomial lattice approximation while practitioners in the business of computing a larger number of option values will generally prefer finite difference methods.

2.3. Theoretical Framework of the Study

The study combines the dynamic arbitrage arguments proposed by Black and Scholes (1973) and Merton (1973b, 1977) for valuing options and stochastic control theory, in a continuous-time framework, to formulate the value of a mineral venture.

A full general equilibrium model for the development of a mineral resource venture would have the determinants of metal prices, and of mining costs, interest rates, and inflation as stochastic variables. It is important also to model uncertain geological and technical variables that can affect project viability and profitability. At present, this type of model will be very difficult to implement. However, metal prices and metal reserves, and metal prices and expected ore grade, are treated as stochastic variables, respectively, in the models examined in this study.

The Brennan and Schwartz (1985) model of exhaustible natural resource value is the first important attempt to apply these ideas to value mineral resource ventures. However, by assuming that the metal reserve is certain, they treat the industry as a metal storage industry, the managers of which can decide to sell metal at any time. They do not consider the technical problems the industry faces in ore reserve and grade uncertainties, and the decisions that managers of mineral ventures must make in the light of these uncertainties. Using a simple model, the author shows how uncertainty associated with metal reserves can affect feasibility study and investment decisions in the management of a mineral venture. He then shows how ore grade uncertainty and its resolution, in a multiple-stage feasibility study program, can affect the details of these types of decisions.

2.4. Conclusions

The author reviewed the state-of-the-art methods currently in use in the mineral industry. A thorough review was also carried out on the advances in finance theory which form the basis of this study, their applications, so far, in the industry, and numerical procedures developed to solve the problems based on these ideas.

CHAPTER 3.0

FEASIBILITY STUDY MANAGEMENT AND MINE DEVELOPMENT: A PRELIMINARY EXERCISE

This chapter introduces a one-dimensional model of the value of a typical mineral venture. Equations which determine the value of a mineral venture have been formulated for the following alternative decisions: (1) Waiting or investing under uncertainty; (2) undertaking a feasibility study; (3) waiting or investing under certainty; and (4) the boundary values for these decisions. The value of the mineral venture is calculated using the principle of portfolio replication and the value of a futures contract. The resolution of project risk because of the uncertainty associated with the metal price is also provided in the model.

3.1. Investment Decision Problem

The problem of investing in a mineral venture is choosing a strategy that maximizes the present value of the opportunity. The choice depends on many variables: economic, e.g. metal price, market (competitive or monopolistic), interest rates, indices of real capital and operating costs, the general rate of inflation; technical, e.g. ore grade, mineralogical composition of the ore, ore reserve size, stripping and extraction ratios; and fiscal, e.g., tax regime, currency convertibility, import duties, political stability of the environment [Myers and Barnett 1985; Payne 1973; Morrison and Russel 1973; Ballard 1983; Gentry and O'Neil 1984; Gocht, et al. 1988].

3.1.1. Investment Decision Strategies

Figure 3.1 illustrates the investment decision strategies available to an investor considering investment in a typical mineral venture. At the time of evaluation, NOW, a project evaluator (or investor) could decide to: (1) Conduct a feasibility study (F) to eliminate the uncertainty associated with the amount of reserves and the expected capital and operating costs of the mineral project; (2) to invest in the project without a feasibility study (Iu); and (3) to wait for an appropriate time to make a decision (Wu). The choice of Wu leaves the evaluator with the same decisions at NOW. When alternative Iu is considered, it results in an open mine ready for operation. ***When alternate F is chosen, the uncertainties about the ore reserve and the expected capital and operating costs are eliminated***, and two subsequent choices are made available: (a) To invest under certainty about everything, other than the metal prices (Ic); and (b) to wait under these same conditions (Wc). These subsequent choices result, respectively, in an open mine, or the further opportunity either to invest under certainty, or to wait again, as illustrated in Figure 3.1. For simplicity, it is assumed that: (1) only one commodity is to be produced, that there are no time lags to investment, and that the uncertainty in the cost structure is due only to the uncertainty in the reserve size.

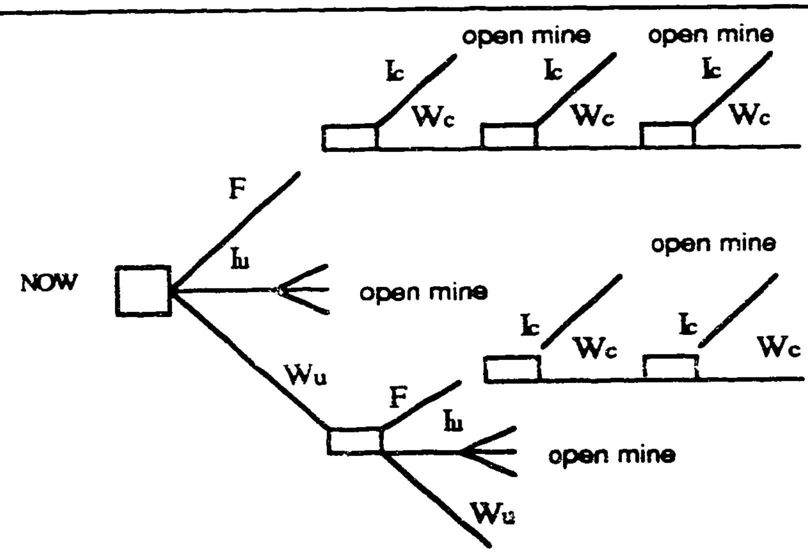

Figure 3.1 Investment Decision Strategies

3.2. Investment Evaluation Model

The investment evaluation model comprises two submodels: (1) The metal futures model; and (2) the mine value model.

3.2.1. Metal Futures Value Model

Consider a typical mining project, which is expected to produce a single homogeneous commodity, whose spot price, S, is determined in a competitive market. S is assumed to follow the following stochastic process:

$$dS = \mu S dt + \sigma S dz \tag{3.1}$$

dz is the increment to a standard Gauss-Wiener process.[1] σ is the instantaneous standard deviation of the spot price; and μ is the local (in time) trend in the metal price [Hull 1989].

It is assumed that the interest rate is known and non-stochastic. It is also assumed that, the convenience yield[2] on the output commodity, and the volatility in the price of the output commodity[3] can be written as functions of the output price and time only. Thus, a relationship can be developed between the spot price and the futures prices of the commodity [Brennan and Schwartz 1985]. Let $F(S, t)$ be the futures price at time t for the delivery of one unit of the commodity at maturity date. The instantaneous change in the futures price is given from Ito's process[4] [Ito 1951; Malliaris and Brock 1982; Hull 1989] by:

$$dF = \left[F_S \mu S + F_t + \frac{1}{2} F_{SS} \sigma^2 S^2\right] dt + F_S \sigma S dz \tag{3.2}$$

When equation (3.1) is substituted into equation (3.2), the result is:

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[1] See Appendix B.1 for a description of the Wiener process.

[2] Convenience yield is the benefit that accrues to the holder of the physical commodity instead of the futures contract on the commodity.

[3] Volatility in the metal price is the proportional standard deviation in the metal price in any future time.

[4] See Appendices B.2 and B.3 for the respective derivations of Ito's lemma and the instanrtaneous change in futures price

$$dF(S,t) = \left(F_t + \frac{1}{2} F_{ss}\, \sigma^2 S^2 \right) dt + F_s\, dS \qquad (3.3)$$

Let dR be the instantaneous return earned by an investor who hedges the exposure of his risk by purchasing one unit of the commodity, and shorts (or sell) $1/F_s$ futures contracts. Because the futures involve no funds outlay initially, the return per dollar of investment including the convenience yield, c, is given by:

$$dR = \frac{dS}{S} + c\,dt - (S\ F_s)^{-1}\, dF \qquad (3.4)$$

When equations (3.1) and (3.2) are substituted into equation (3.4), the result is:

$$dR = (S\ F_s)^{-1} \left[F_s\, cS - \frac{1}{2} F_{ss}\, \sigma^2 S^2 - F_t \right] dt \qquad (3.5)$$

To avoid arbitrage (i.e., riskless profits gained by simultaneously entering into transactions in two or more markets) opportunities, this return must be equal to the riskless return ρdt [Hull 1989], where ρ is the risk-free rate of interest. Thus, the futures price of the commodity satisfies the following partial differential equation:

$$\frac{1}{2} F_{ss}\, \sigma^2 S^2 + F_s\, (\rho - c)\, S + F_t = 0 \qquad (3.6)$$

with the following boundary conditions:

$$F(S,\ T) = S \qquad (3.7)$$

$$F(0,\ t) = 0 \qquad (3.8)$$

$$S^2\, F_{ss}(S,\ t) \rightarrow 0 \quad \text{as } S \rightarrow \infty \qquad (3.9)$$

Equation (3.7) states that the value of the futures contract at maturity date is equal to the current spot price of the metal. Equation (3.8) states that if the spot price is zero, the futures value is zero, and equation (3.9) specifies that at a high spot price the futures value is a linear function of the spot price, consistent with the maturity condition.

3.2.2. General Mine Value Model

Let us consider the mine portion of a mineral venture. The value of the decision to develop and operate the mine, once the ore reserve, Q, is known, depends on the current commodity price, S, the amount of the ore reserve, Q, the time of evaluation, t, the state of the mine, j; and the development and operating policy of the mine, ϕ. Thus, the mine value is written as:

$$H \equiv H(S, Q, t, j; \phi) \tag{3.10}$$

The mine state variable j takes the value o if the mine is open, m if it is closed and being maintained, a if it is abandoned, and n if it is not developed. The instantaneous change in the value of the mine, dH,[5] from Ito's lemma [Ito 1951; Malliaris and Brock 1982], is given by:

$$dH = H_t\,dt + H_Q\,dQ + H_S\,dS + \frac{1}{2} H_{SS} (dS)^2 \tag{3.11}$$

The instantaneous change in the ore reserve is determined by the output rate q, given by:

$$dQ = -\,q\,dt \tag{3.12}$$

and

$$q(j) = \begin{cases} q & j = o \\ 0 & j = m, a, n \end{cases} \tag{3.13}$$

The cash flow, CF, from the mine, is also given by:

---

[5] See Appendix B.4 for a derivation of the instantaneous change in the 1 - D Mine Value model.

$$CF\ (\ S,\ Q,\ t,\ j;\ \phi\ ) = \begin{cases} q\ (\ S\ -\ A\ ) & j = o \\ -\ k & j = m \\ 0 & j = a,\ n \end{cases} \tag{3.14}$$

A is the average cash cost rate for producing at the rate q, and k is the cost for maintaining the mine when it is closed temporarily.

Under the operating policy, ϕ, of the mine, the differential equation governing the mine may be derived by considering dR, the instantaneous return to a portfolio consisting of a long position in the mine, and a short position in H_S/F_S futures contracts, given by:

$$dR = dH + CF\ dt - \left(\frac{H_S}{F_S} \right) dF \tag{3.15}$$

When equations (3.5), (3.11), (3.12) and (3.13) are substituted into equation (3.15), the result is:

$$dR = \left[\frac{1}{2}\sigma^2 S^2 H_{ss} - qH_Q + H_t + CF + S\ (\rho - c)\ H_s\right] dt \tag{3.16}$$

To avoid arbitrage opportunities, equation (3.16) must be equal to the riskless return ρHdt, on the value of the investment. Thus, the value of the mine satisfies the partial differential equation:

$$\frac{1}{2}\sigma^2 S^2 H_{ss} - qH_Q + H_t + CF + S\ (\rho - c)\ H_s - \rho H = 0 \tag{3.17}$$

A key restricting assumption is made that the parameters of this equation, and of all the boundary conditions for it, are independent of time, although possibly still dependent, in the case of the cost data, on the size of the remaining reserves. Under this assumption, equation (3.16) becomes:

$$\frac{1}{2}\sigma^2 S^2 H_{SS} - qH_Q + CF + S\ (\rho - c)\ H_S - \rho\ H = 0 \tag{3.18}$$

Equation (3.18) is the partial differential equation of the mine value. In different states, the value of the mine is obtained by solving a boundary value problem based on this equation.

3.2.2.1. Value of the Open Mine, HO(S, Q)

The value of the open mine depends on the ore reserve, Q, and the rate of production, q. Also, a stream of cash, CFO, flows into the value of the opportunity, as long as the mine remains open. Thus, the value of the open mine satisfies the following differential equation:

$$\frac{1}{2}\sigma^2 S^2 HO_{SS} + S(\rho - c) HO_S - q\, HO_Q - \rho\, HO + CFO = 0 \qquad (3.19)$$

with the following boundary conditions:

$$HO(S, 0) = - K_{O \rightarrow A} \qquad (3.20)$$

$$HO(0, Q) = - K_{O \rightarrow A} \qquad (3.21)$$

$$S^2 HO_{SS}(S, Q) \rightarrow 0 \text{ as } S \rightarrow \infty \qquad (3.22)$$

$$HO(S, Q) = \max(- K_{O \rightarrow A}, HM(S, Q) - K_{O \rightarrow M}) \;\; S \leq S_{O \rightarrow M/A}(Q) \qquad (3.23)$$

$$HO(S, Q) > \max(- K_{O \rightarrow A}, HM(S, Q) - K_{O \rightarrow M}) \;\; S > S_{O \rightarrow M/A}(Q) \qquad (3.24)$$

Equations (3.20) and (3.21) are, respectively, the zero reserve and zero metal price conditions on the value of the open mine. They simply state that the mine will be abandoned in these situations. Equation (3.22) indicates that as the metal price, S, approaches infinity, the second derivative of the open mine value with respect to the metal price goes to zero, consistent with the linear cash flow model [Laughton and Jacoby 1991]. Equation (3.23) provides a cushion against negative cash flows, which could occur at a low metal price. If the price falls below a certain limit, $S_{O \rightarrow M/A}$, the mine should be closed or abandoned. The value of the mine is the maximum of the cost of abandoning the mine, $-K_{O \rightarrow A}$, and the difference between the value of the closed mine and the cost incurred in closing it, $-K_{O \rightarrow M}$. If the price is above this critical price, the value of the mine is greater than the value to be obtained from closing or abandoning the mine, and it is left open. This critical price is determined by the condition in equation (3.24).

3.2.2.2. Value of the Closed Mine. HM(S, Q)

The value of the closed mine depends on the costs of closing and maintaining the mine, -$K_{O \to M}$ and CFM. Because there is no mining activity, q is zero, and hence the term, qH_Q, is zero. Thus, the value of the closed mine also satisfies the following differential equation:

$$\frac{1}{2}\sigma^2 S^2 HM_{SS} + S(\rho - c)HM_S - \rho\, HM + CFM = 0 \tag{3.25}$$

with the following boundary conditions:

$$HM(S, 0) = -K_{M \to A} \tag{3.26}$$

$$HM(0, Q) = -K_{M \to A} \tag{3.27}$$

$$S^2\, HM_{ss}(S, Q) \to 0 \quad \text{as } S \to \infty \tag{3.28}$$

$$HM(S, Q) = HO(S, Q) - K_{M \to O} \qquad S \geq S_{M \to O}(Q) \tag{3.29}$$

$$HM(S, Q) > HO(S, Q) - K_{M \to O} \qquad S < S_{M \to O}(Q) \tag{3.30}$$

$$HM(S, Q) = -K_{M \to A} \qquad S \leq S_{M \to A}(Q) \tag{3.31}$$

$$HM(S, Q) > -K_{M \to A} \qquad S > S_{M \to A}(Q) \tag{3.32}$$

Equations (3.26) and (3.27) are, respectively, the zero reserve and zero metal price conditions on the value of the closed mine. As the metal price approaches infinity, the second derivative of the closed mine value with respect to the metal price goes to zero as illustrated in equation (3.28). If the metal price increases to and beyond a certain critical price, $S_{M \to O}$, the mine will be reopened. The value of the mine will then be equal to the difference between the value of the mine when open, and the cost incurred in opening it, $-K_{M \to O}$, as illustrated in equation (3.29). However, as in equation (3.30), when the price is below the critical price, the value of the closed mine is greater than the value of the open mine, and the mine will remain closed. If the price of the metal falls below a certain critical level, $S_{M \to A}$, the mine is abandoned, as illustrated in equation (3.31). Above this critical price, the value of the closed mine is greater than the abandonment value, and the mine will remain closed as illustrated in equation (3.32).

3.2.2.3. Values of Either Investing or Waiting with Certainty, Y(S, Q)

The values of these decisions (or opportunities) do not depend on CF and q, because there is no mining activity. Thus, Y(S, Q) satisfies the following differential equation:

$$\frac{1}{2}\sigma^2 S^2 Y_{ss} + S(\rho - c) Y_s - \rho Y = 0 \qquad (3.33)$$

with the following boundary conditions:

$$Y(0, Q) = 0 \qquad (3.34)$$

$$S^2 Y_{ss}(S, Q) \to 0 \quad \text{as } S \to \infty \qquad (3.35)$$

$$Y(S, Q) = HO(S, Q) - I \qquad S \geq S_c(Q) \qquad (3.36)$$

$$Y(S, Q) > HO(S, Q) - I \qquad S < S_c(Q) \qquad (3.37)$$

Equation (3.34) is the zero metal price condition for evaluating either decision (or opportunity). As the metal price approaches infinity, the second derivative of the value of investing or waiting under certainty with respect to the metal price goes to zero as illustrated in equation (3.35). When the metal price is greater than a certain critical price, S_c, the investment is made, and the value of the decision is the difference between the value of the open mine and the investment cost as in equation (3.36). When the price is lower than this critical price, the value of the decision to wait is greater than the difference between the value of the open mine and the investment cost, and the investor will choose to wait for an appropriate time to invest as in equation (3.37).

3.2.2.4. Value of the Mineral Project NOW, V(S)

The value of the mineral project at the time of evaluation, V(S), does not incorporate CF and q, because there is no mining activity. Thus, it satisfies the following differential equation:

$$\frac{1}{2}\sigma^2 S^2 V_{ss} + S(\rho - c) V_s - \rho V = 0 \qquad (3.38)$$

with the following boundary conditions:

$$V(0) = 0 \qquad (3.39)$$

$$S^2 V_{ss}(S) \rightarrow 0 \quad \text{as } S \rightarrow \infty \qquad (3.40)$$

$$V(S) = \max\, [W(S),\ Z(S)] \qquad S \in S_u \qquad (3.41)$$

$$V(S) > \max\, [W(S),\ Z(S)] \qquad S \notin S_u \qquad (3.42)$$

$$Z(S) = \int Y(S,\ Q)\ du(Q) - F_c \qquad (3.43)$$

$$W(S) = \int HO(S,\ Q)\ du(Q) - I \qquad (3.44)$$

Equation (3.39) is the zero metal price condition on the present value of the mineral project. As the metal price approaches infinity, the second derivative of the present value of the mineral project with respect to the metal price goes to zero as illustrated in equation (4.40). Within a certain critical metal price region, S_u, the investor will decide either to conduct a feasibility study, F, or to invest without a feasibility study, Iu, as in equation (3.41). Thus, the value of this opportunity is the maximum of the value of the opportunity for the F choice, i.e. Z(S) and W(S), the Iu choice. Outside this critical metal price region, S_u, the value of waiting is greater than the maximum of Z(S) and W(S), and the investor will wait as illustrated in equation (3.42). The value of the mine with the feasibility study and investment options are, respectively, illustrated in equations (3.43) and (3.44). F_C is the feasibility cost, and du(Q) is the probability distribution associated with the ore reserve.

3.4. Conclusions

The value of a typical mineral venture has been modeled, using the DAV method, with metal price and metal reserve quantity as the only dynamic state variables. The equations determining the value of the mine at different states, and the associated boundary conditions are solved, based on Figure 3.1, to maximize the value of the mineral project. The validation of these mine value models, with test results, illustrations and discussions are provided in Chapter 6.

CHAPTER 4.0

DETAILED FEASIBILITY STUDY MANAGEMENT AND MINE DEVELOPMENT

The value of the rights to develop a mineral deposit is modeled in two dimensions (metal price and expected ore grade uncertainties), in this chapter, to help an analyst or investor make the most informed decision on what magnitude of effort constitutes optimal management of a multiple-stage feasibility study program. Values are formulated for waiting, investing under uncertainty, or undertaking a feasibility study to reduce uncertainty associated with expected ore grade. As a consequence, the regions of the state space in which each action is the best choice are also determined. It is assumed that the investor has some diffused knowledge about expected ore grade and reserves at the beginning of the feasibility studies.

The author encountered numerical boundary and discretization problems in the alternate direction implicit algorithm (ADI)[1] for the solution of a model of the feasibility study in which the sampling rate is continuous. These problems were solved by making the feasibility study stages discrete and fixed.

4.1. Optimal Feasibility Before Investment

After investors have identified a potential mineral deposit, one aspect of their development strategy for that prospect may be the collection and analysis of information by means of a feasibility study. The cost of feasibility study, in terms of dollars and time, might be high, but the cost of no feasibility study, or an inadequate one, easily could be higher. Thus, the investor wants to undertake a certain amount of feasibility study to reduce the uncertainties associated with the ore reserve and grade, while at the same time minimizing its cost.

In a typical feasibility study program in the mineral industry, four possible stages may be considered. The first stage is the reconnaissance survey, in which published data on the

[1] See Appendix C.2 for the continuous sampling model and Appendix C.3 for the ADI algorithm and the finite difference equations for this model.

trends and geologic controls of known mineral occurrences, investors' opinions and prejudices, and momentary conditions (e.g., shifts in markets) are the source of data available to investors. *Knowledge about the deposit is very little; therefore, the variance associated with the ore reserve and grad- is very high.* The second stage is the area and target investigations. The object is to form a picture of the geology, and to develop evidence to describe the theory of formation. Some of the methods used are airborne geophysical methods, aerial photography, photogeology, satellite photoimagery, geochemical prospecting and some drilling.

In the third and final feasibility study stages, i.e. the ore delineation, the analysts evaluate a discovered target using exploratory and development drilling. The samples gathered are assayed and examined to determine the mineralization characteristics, rock types and any other factor that will enable the exploration geologists to determine the shape, size, thickness and position of the ore body using geostatistics and geological tools. Mine planning and scheduling, cost estimates and cash flow projections are done based on a comprehensive concept of the total project.

Also, in the final stage, it may be necessary to solve an outstanding technical problem which may cause unacceptable risks in the project value, e.g., major faulting that could endanger underground mine openings or efficient production, an accessory or main mineral that could render the selected processing method inefficient, and solution of market complexities. Pilot plant testing may also be required at this stage.

In this study, it is assumed that, as the analyst or investor progresses through the various feasibility study stages, relatively higher feasibility study costs are expended to carry out the activities at the corresponding stages to result in the reduction of ore grade uncertainty. For example, at feasibility study stage one of this study, the investor increases the feasibility study cost by 300 percent of the cost at stage zero to reduce the ore grade uncertainty. Four stages have been considered in this study.

The problem is to maximize the value of the mineral project with an ongoing feasibility study, subject to the constraints underlying the project, as illustrated in Figure 4.1.

4.2. Decision Strategies Available to Investors

Figure 4.1 illustrates the investment decision strategies with ongoing feasibility study available to an investor considering investment in a typical mineral venture. At the time of evaluation, **NOW**, the investor could decide to: (1) Conduct a feasibility study, **Fu(S, E, var(0))** to reduce the uncertainty associated with the expected ore grade and reserves; (2) invest in the project without the initial feasibility study, **Iu (S, E, var(0))**; and (3) wait for an appropriate time to make a decision, **Wu (S, E, var(0))**.

The choice of **Wu (S, E, var(0))** leaves the investor with the same decisions at **NOW**. If alternate **Iu (S, E, var(0))** is chosen, it results in an open mine. The amount of capital invested in this mine is affected by the extent of the investor's ignorance about the ore deposit, which is based on the magnitude of the variance associated with the expected ore grade and reserves at any feasibility study stage, **var(i) : i = 0, 1, 2, - - - - - -, FL**. **FL** is the feasibility study limit and is also considered as the last feasibility stage in the discrete model, the point at which feasibility study ceases.

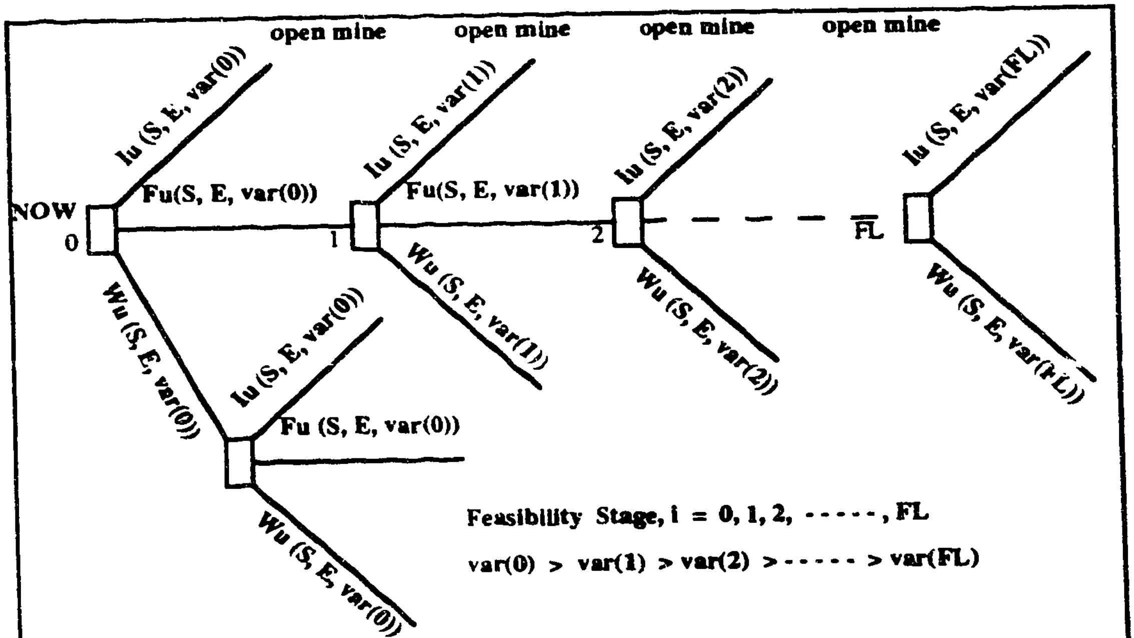

Figure 4.1 Investment Decision Strategies with Ongoing Feasibility Study

When alternate **Fu(S, E, var(0))** is chosen, the amount of uncertainty associated with the expected ore grade and reserves is reduced from **var(0)** to **var(1)** at feasibility study stage 1. At this time, either of the three decisions could be made based on an increased level of knowledge. If the investor chooses to continue the feasibility study, the knowledge level about the expected ore grade and reserves increases as a result of the reduction of the variance associated with the expected ore grade and reserves. At a certain feasibility study stage, **FL**, if investment has not been made, the investor stops the feasibility study, and waits for an appropriate time to invest.

4.3. General Mine Value Model

The value, H, of a mineral venture when the investor is waiting to make a decision either to undertake a feasibility study or to invest depends on: (1) the price of the metal, S; (2) the time of evaluation, t; (3) expected ore grade, E; and (4) the feasibility study stage i. Thus, the value of the mineral venture is written as:

$$H \equiv H\ (S,\ t;\ E,\ i) \qquad (4.1)$$

The instantaneous change in the value of this mineral project[2] [Ito 1951; Malliaris and Brock 1982] is given as:

$$dH = H_s\, dS + H_t\, dt + \frac{1}{2} H_{ss} (dS)^2 \qquad (4.2)$$

$$dS = \mu\ Sdt + \sigma\ SdZ \qquad (4.3)$$

Substituting equation (4.3) into equation (4.2), the instantaneous value of the mine is:

$$dH = H_s (\mu\ Sdt + \sigma\ Sdz) + H_t\ dt + \frac{1}{2} H_{ss} (\mu\ Sdt + \sigma\ Sdz)^2 \qquad (4.4)$$

The instantaneous return to a portfolio with a long position in the mineral venture and a short position in futures contracts, dR, is given by:

$$dR = dH - (H_S / F_S)\ dF \qquad (4.5)$$

---

2 See Appendix B.5 for a derivation of the instantaneous change in the 2 - D mine value model.

Substituting dH and dF from equation (3.2) into equation (4.5), the instantaneous value of the portfolio return is:

$$dH = H_s(\mu\ Sdt + \sigma\ Sdz) + H_t\ dt + \frac{1}{2} H_{ss}(\mu\ Sdt + \sigma\ Sdz)^2$$

$$- (H_S / F_S)\ F_S\ \{S\ ((\mu - \rho) + c)\ dt + \sigma\ S\ dZ\} \qquad (4.6)$$

To avoid arbitrage opportunities, the expected instantaneous return in equation (4.6) must be equal to the riskless return, ρ Hdt. Thus, the value of the mineral venture satisfies the following differential equation:

$$\frac{1}{2}\sigma_S{}^2\ S^2\ H_{SS} + H_t + S\ (\rho - c)\ H_S - \rho\ H = 0 \qquad (4.7)$$

Under the stationarity assumption, equation (4.7) becomes:

$$\frac{1}{2}\sigma_S{}^2\ S^2\ H_{SS} + S\ (\rho - c)\ H_S - \rho\ H = 0 \qquad (4.8)$$

Equation (4.8) is the differential equation model of the value of the mineral venture when the investor is waiting to either undertake a feasibility study or invest.

4.3.1. Boundary Conditions

$$H(0;\ E,\ i) \ = \ 0 \qquad (4.9)$$

$$S^2\ H_{SS}(S;\ E,\ i) \rightarrow 0 \quad \text{as } S \rightarrow \infty \qquad (4.10)$$

$$H(S;\ E,\ i) = \overline{H}(S;\ E,\ i) \qquad S \in S^*(E,i) \qquad (4.11)$$

$$H(S;\ E,\ i) > \overline{H}(S;\ E,\ i) \qquad S \notin S^*(E,i) \qquad (4.12)$$

Equation (4.9) is the zero metal price condition of the value of the mineral venture. As the metal price approaches infinity, the second derivative term in equation (4.7) tends to zero as illustrated in equation (4.10). Within a metal price region defined by $S^*(E, i)$, the investor will decide to undertake a feasibility study or invest, and the value of the venture is the maximum of the value for investing in the venture and undertaking the next stage, if any, of the feasibility study, i.e., $\overline{H}(S; E, i)$. Outside this metal price region, the investor will continue to wait as illustrated in equation (4.12).

The value at the horizon, $\overline{H}(S; E, i)$, at any stage before feasibility study limit, is the maximum of the venture's value with the feasibility study option, $\overline{HF}$, and after investment, $\overline{HI}$, as illustrated in equation (4.13). At the feasibility study limit, the horizon value is equal to the venture's value after investment as illustrated in equation (4.14). The value of the mine when the investor decides to undertake a feasibility study is equal to the difference in the value of the mine with the feasibility study option at the time that stage of study is undertaken, $\widehat{HF}(S, 0; E, i)$, and the feasibility study cost, FC_i, at that corresponding stage as illustrated in equation (4.15).

$$\overline{H}(S; E, i) = \max\left[\, \overline{HF}(S; E, i), \overline{HI}(S; E, i) \,\right] \qquad i < FL \tag{4.13}$$

$$\overline{H}(S; E, i) = \overline{HI}(S; E, i) \qquad i = FL \tag{4.14}$$

$$\overline{HF}(S; E, i) = \widehat{HF}(S, 0; E, i) - FC_i \qquad i < FL \tag{4.15}$$

4.4. Value of the Mine During an Ongoing Feasibility Study

The value, $\widehat{HF}$, of the mineral venture during an ongoing feasibility study at any stage depends on: (1) the metal price, S; (2) the time, t, since feasibility study begun at stage i; (3) expected ore grade, E; and the feasibility study stage, i. Thus, the value of the mine with ongoing feasibility study is given by:

$$\widehat{HF} \equiv \widehat{HF}(S, t; E, i) \qquad S \geq 0;\ E \geq 0;\ 0 \leq t \leq T_i;\ i < FL \tag{4.16}$$

T_i is the total duration of a feasibility study stage i; FL is the last feasibility study stage. Based on equation (4.7), $\widehat{HF}$ follows the following differential equation:

$$\frac{1}{2}\sigma_S^{\,2}\, S^2\, \widehat{HF}_{SS} + \widehat{HF}_t + S\,(\rho - c)\,\widehat{HF}_S - \rho\,\widehat{HF} = 0 \qquad 0< S <\infty;\ 0< t <T_i \tag{4.17}$$

The terminal value of the mineral venture with feasibility study is also given by:

$$\widehat{HF}(S,\ T_i\ ;\ E,\ i) = \int_0^{\infty} H(S;\ E',\ i+1)\ du_i\,(E'|E) \qquad 0 \leq S < \infty \tag{4.18}$$

$du_i\,(E'|E)$ is the probability distribution at the beginning of the feasibility study stage i of the expected ore grade E' at the end of that study stage given E, the expected ore grade at the beginning. The terminal value of the mineral venture in equation (4.18) is solved, numerically, as follows:

$$\widehat{HF}(S,\ T_i\ ;\ E,\ i) = \sum_{E' = E_{min}}^{E_{max}} H(S;\ E',\ i+1) * du_i\,(E'|E) \tag{4.19}$$

E_{max} and E_{min} are the respective maximum expected ore grades.

Other boundary conditions are:

$$\widehat{HF}(0,\ t\ ;\ E,\ i) = 0 \qquad 0 \leq t < T_i \tag{4.20}$$

$$S^2\ \widehat{HF}_{SS}(S,\ t\ ;\ E,\ i) \rightarrow 0 \quad \text{as } S \rightarrow \infty \qquad 0 \leq t < T_i \tag{4.21}$$

The value of the mine during an ongoing feasibility study at any stage is zero, when the metal price is zero as illustrated in equation (4.20). As the metal price approaches infinity, the second derivative term in equation (4.17) tends to zero as illustrated in equation (4.21). Also, note that there are no options.

4.5. Ore Grade Uncertainty and its Resolution

The probability distribution of the expected ore grade E', given E, i.e., $du_i(E'|E)$, is assumed to be lognormally distributed. This is given by:

$$du_i(E' \mid E) = \frac{1}{\sqrt{(2\Pi \ \mathrm{Var}\left[\ln(E' \mid E)\right])}} \exp\left[-\frac{1}{2}\frac{(\ln(E' - E\left[\ln(E') \mid E'\right]))^2}{\mathrm{Var}\left[\ln(E'' \mid E)\right]}\right]\frac{dE'}{E'} \qquad (4.22)$$

A feasibility study in any mineral venture must provide investors and analysts with enough knowledge about the expected ore grade and reserves to enable decision making. It is assumed, in this study, that the resolution of uncertainty about the ore grade with ongoing feasibility study, at various stages, follows the illustration in Figure 4.2. As the amount of feasibility study stages increase, the total variance associated with the ore grade probability distribution reduces.

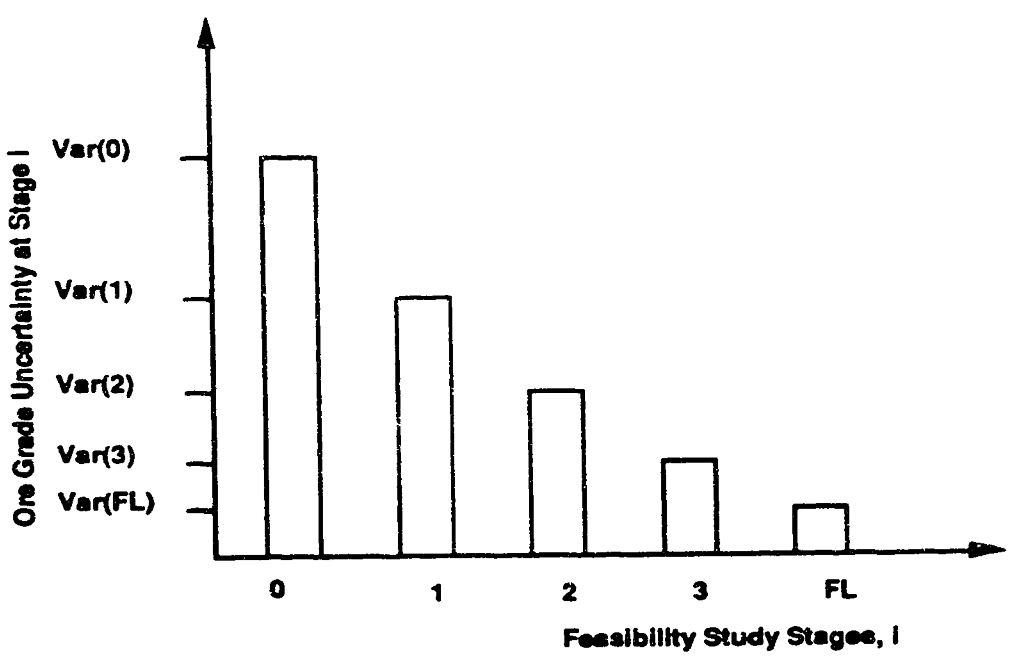

Figure 4.2 Ore Grade Uncertainty Resolution

Beyond a certain critical feasibility study stage, FL, the total variance associated with the ore grade distribution is assymptotic to the residual variance. At this critical point, if investment is not made, the investor should wait for an appropriate investment timing, because additional feasibility study does not add any value to the venture considering the feasibility study cost.

4.6. Feasibility Study Cost Function

The cost of feasibility study at any stage, i, depends on the level of activity at the corresponding stage. It is assumed that the level of activity will increase as the investor progresses through the various feasibility study stages. That is, investors will be induced to spend much money to undertake detailed feasibility study, if initial study results give indication of possible economic deposits. Thus, the cost of feasibility study at any stage i, FC_i, is given by:

$$FC_i = \begin{cases} k_0 & i = 0 \\ k_1 & i = 1 \\ k_2 & i = 2 \\ k_3 & i = 3 \end{cases} \qquad (4.23)$$

In this study, $k_3 = k_2 > k_1 > k_0$.

4.6. Value of Investing in the Mine

The value of investing in the mine, at any time, also depends on: (1) the price of the metal, S; (2) expected ore grade, E; and (3) feasibility study stage i. It is the difference in the value of the open mine, HO(S, E), and the present value of the cost of investing in the mine at the corresponding stage i, PIC_i. Thus, the value of investing in the mine, is given by:

$$\overline{HI}(S;\ E,\ i) = HO(S,\ E) - PIC_i \qquad (4.24)$$

In the formulation of the open mine value for the investment value model, the options to close, reopen or abandon the project have not been considered for simplicity. In this case, once the mine is opened, it is operated until the end of the mine life. Thus, the resulting value of the open mine is given by:

$$HO(S,\ E) = \left(\frac{S\ E\ q_o}{c}\right)\left[1 - \exp\left(\frac{-\ c\ Q_o}{q_o}\right)\right] - \left(\frac{\beta\ q_o}{\rho}\right)\left[1 - \exp\left(\frac{-\ \rho\ Q_o}{q_o}\right)\right] \tag{4.25}$$

β is the unit operating cost for mining and milling ore, and handling waste, and overhead expenses; Q_o and q_o are the respective total ore reserves and annual production rate. All the other variables retain their original definitions.

4.6.1. Investment Cost Model

The investment cost at any time, in or after the feasibility study phase, is illustrated in equation (4.26). In the investment cost model, the investor pays for ignorance about the ore grade and the reserve. At the beginning of the feasibility study, depending on the available information and other conditions, investment could be made at a higher ultimate cost, and this cost reduces as feasibility study progresses through the various stages. Beyond a certain critical feasibility study stage, FL, the feasibility study ceases, if investment has not been made. The investor will wait for an appropriate time to invest, because the cost of doing the feasibility study exceeds the increase in project value. The undiscounted investment cost, IC_i, at feasibility stage i, is given by:

$$IC_i = \begin{cases} IC_0 & i = 0 \\ IC_1 & i = 1 \\ IC_2 & i = 2 \\ IC_3 & i = 3 \\ IC_4 & i = 4 \end{cases} \tag{4.26}$$

$$IC_0 > IC_1 > IC_2 > IC_3 > IC_4 \tag{4.27}$$

$$IC_i = r_i\ IC_4 \tag{4.28}$$

r_i is the cost of ignorance factor associated with investments borne by an investor who decides to invest at stage i. This cost of ignorance factor is high at the beginning, and it reduces as knowledge is gained about the expected ore grade through feasibility studies.

If the development costs are evenly expended discretely at the beginning of each development year, then for a total development period, m, the present value of the investment cost, at any stage i, is also given by:

$$PIC_i = \sum_{t=0}^{m} \frac{IC_i}{m} \exp(-\rho\ t) \tag{4.29}$$

4.7. Phase Dynamics of the Investment Decision Making

Figure 4.3 illustrates the different phases the investment project could be at various metal prices and expected ore grades as the feasibility study stages increase resulting in an increasing knowledge about the expected ore grade. In the wait (W) and dithering (D)[3] regions, no feasibility study is carried out to reduce the uncertainty associated with the expected ore grade. Thus, the expectation of the ore grade probability distribution, E, does not change. However, the price of the metal can either fall or rise to cause the investor to make other decisions. Therefore, upward or downward displacements in mineral venture value can occur in the case of varying metal price, S. At fixed metal price, however, the value of the mineral venture does not increase or decrease in the wait region. Therefore, no displacements can occur in this phase at fixed metal price.

[3] Dithering region is the region in which an investor waits before deciding between two mutually exclusive investment alternatives. Refer to D. G. Laughton (1992) for a discussion on dithering.

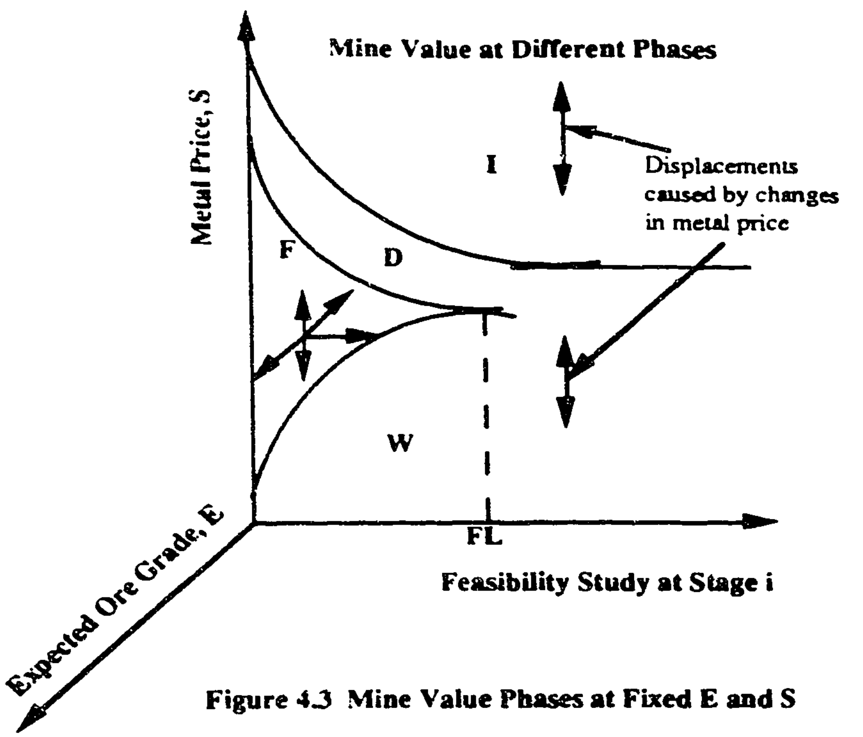

Figure 4.3 Mine Value Phases at Fixed E and S

Beyond a certain critical price, the investor will decide to undertake a feasibility study at the current feasibility study stage (F). At this stage, feasibility study results in decreasing the total variance associated with the expected ore grade and thus, enhancing the knowledge about the venture's viability and profitability. The expected value of the grade probability distribution could increase or decrease. The metal price, S, could also fall or rise. Thus, both vertical and horizontal displacements can occur for any point in the F phase.

Also, in the feasibility study phase, an the investor can decide either to invest, to wait or, if costless waiting is not possible, to abandon the project as a result of an increase level of information (i.e., documentation). Beyond a certain critical feasibility study stage, FL, additional feasibility study does not improve the quality of information about the ore grade or profitability. The value of the mineral venture with ongoing feasibility study beyond this critical point reduces by the increasing cost of feasibility study. Thus, for value maximization, only two phases, I and W, should remain.

In the investment phase (I), it is assumed that feasibility study is completed. Therefore, there is no horizontal movement for any point in this phase. For the case of the mineral venture's value at fixed metal price, no upward nor downward movement occurs, because feasibility study is completed, and the expected value of the ore grade probability distribution remains constant. However, the metal price could rise or fall to affect the value of the mineral venture in this phase, and thus, upward or downward displacements can occur.

4.6. Conclusions

The value of a typical mineral venture has been modeled, in two dimensions (in metal price and expected ore grade uncertainties), to examine the use of detailed feasibility study in mineral venture development. The development cost is also modeled to include the cost of ignorance. The validation, test results, and illustrations of these models, and discussions are provided in Chapter 7.0.

CHAPTER 5.0

SOLUTION PROCEDURE AND EXPERIMENTAL INVESTIGATIONS

The author discusses solution procedures and the experiments designed to implement them. Solution algorithms for the 1 - D and 2 - D problems are described. Also described are flow charts for the computer programs used to solve the problems, the experimental setup, the procedure, and the experimentation process.

5.1. Solution Procedure

Partial and ordinary differential equations are solved, numerically, using finite difference methods, because they are not analytically tractable. The differential equations are converted into a set of difference equations, and solved iteratively. Finite difference methods are approximate in the sense that derivatives at a point are approximated by difference quotients over a small interval; however, the solutions are not approximate in the sense of being crude estimates [Smith 1966].

The total error introduced in the final results is a combination of the discretization and stability errors. By choosing a small discrete interval, the discretization error could be eliminated almost completely. The error due to stability is eliminated by selecting an appropriate algorithm and suitable parameters [Smith 1966].

5.1.1. Solution Algorithm for the 1 - D Problem

Explicit Methods: Many explicit methods have been proposed to solve the 1 - D parabolic problems, e.g. the classic explicit method, Du Fort - Frankel algorithm , Saul'yev methods [Smith 1966; Lapidus and Pinder 1982]. These methods express one unknown pivotal value directly in terms of known pivotal values. They are computationally simple, but have one serious drawback. The step in the elliptic direction, k, should necessarily be very small, because the process is valid only for $0 < k / h^2 < 0.5$. Thus, $k = 0.5\ h^2$, and h, the step in the parabolic direction, must be very small in order to attain reasonable accuracy.

Implicit Methods: Implicit methods require the solution of a set of simultaneous equations to calculate unknown pivotal values. In the backwards implicit formulation [Lapidus and Pinder 1982], three unknown values are expressed in terms of one known value. Crank and Nicholson (1947) also propose and use another implicit method that reduces the total volume of calculation. In their formulation, the second derivative is replaced by the average of the second derivative in the classic explicit method, and the second derivative in the backwards implicit method. The variable-weighted implicit approximation [Lapidus and Pinder 1982] is also formulated using factors θ and $(1 - \theta)$ to weight, respectively, the derivatives in the classic explicit and the backwards implicit methods.

Overall, explicit methods seem to be easier to solve than implicit methods in terms of time and effort spent on the computer. This is because in the explicit methods, only one unknown pivotal value is involved in the use of the finite difference approximations. *However, the higher accuracies, and better stability and convergent properties achieved through the implicit finite difference approximations make them better methods than the explicit ones. In this work, the backward implicit method is used to solve the 1 - D ODEs and PDEs.*[1]

5.1.2. Solution Algorithm for the 2 - D Problem

The explicit finite difference approximations, e.g., Saul'yev II and DuFort-Frankel methods, available for the solution of the 2 - D problems appear attractively simple, but computationally laborious. They are impractical for most problems, because of the conditions for their validity [Smith 1965; Lapidus & Pinder 1982]. The implicit algorithms, e.g. the Crank - Nicholson's and the backwards methods, are valid for all discrete intervals, but they require the solution of (M - 1)(N - 1) pentadiagonal matrix, instead of the tridiagonal matrix in the 1 - D problem, for each step forward in information time, and therefore cannot be solved by a simple recursive process.

[1] See Appendix C.1 for the finite difference equations for the 1 - D mine value model

Alternating Direction Explicit (ADE): Coats and Tehune (1966) develop another 2 - D explicit method called the ADE. This algorithm is unconditionally stable, but only of moderate accuracy. In addition, it has some consistency problems, and must be used with care. Compared to the implicit version of the alternating direction approach, the ADE yields less accurate results, but is faster in computing time. For the same level of truncation error, Coats and Tehune found out that a smaller discrete interval in the parabolic direction must be used in the ADE than with the implicit version (ADI), but the latter requires a more complex algorithm for the solution of the finite difference approximation.

Alternating Direction Implicit (ADI): The most efficient method for rectangular regions is the algorithm proposed by Peaceman and Rachford (1955) and Douglas and Rachford (1956); it is called the alternating direction implicit (ADI) method. This method consists of replacing one of the second order derivatives in the partial differential equation by an implicit difference approximation in terms of the unknown pivotal values, from the kth to the (k - 1)th information time level, and the other second derivative, by an explicit finite difference approximation as described in Appendix C.2.

The 2 - D model is solved using the backward implicit algorithm in the metal price dimension (see Appendix C.1). The lognormal probability distribution associated with the expected ore grade is also calculated using a modified version of the complementary error function [Press et al. 1990].

5.2. Experimental Procedure

Figure 5.1 is a schematic procedure for obtaining the results of an experiment. The process begins with the development of the technical concepts, based on the problem and the objectives of the thesis [see Chapter 1]. The main problem is the inadequacy of existing economic evaluation methods to help investors in assessing the value, viability and uncertainties associated with mining ventures, and the primary objective is to develop a new evaluation tool, based on the derivative asset valuation methods, to help investors in dealing with this problem.

The next stage is the development of the mathematical models. Using the principle of portfolio replication, and the value of futures contracts developed from the derivative asset valuation method, the value of a typical mining project is formulated in 1 - D and 2 - D, using ordinary and partial differential equations. Corresponding initial and boundary conditions are also provided to capture the domain of feasible regions for the various states of an investment project.

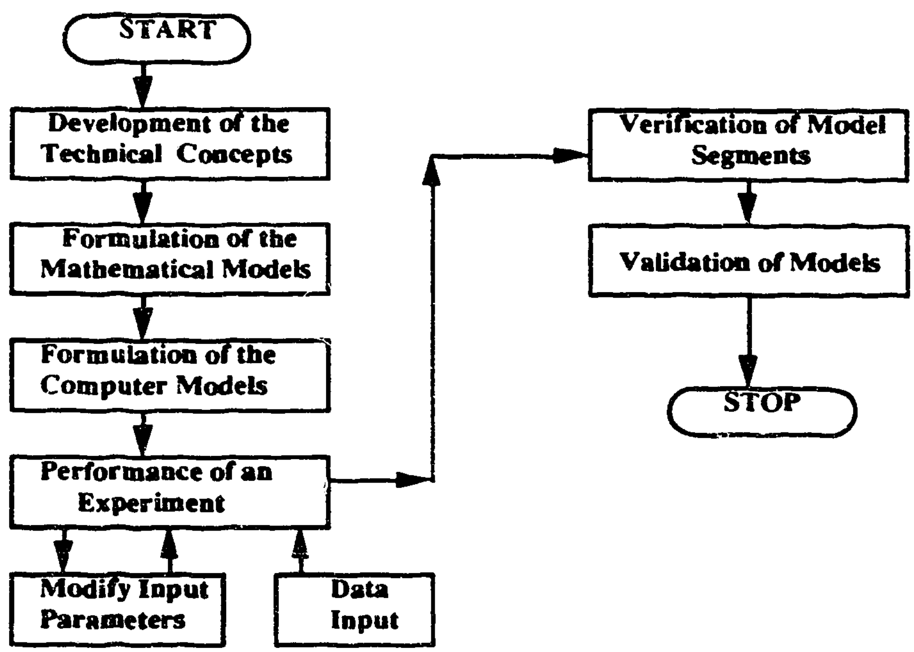

Figure 5.1 Schematic Flow of the Experimental Procedure

For tractability, these equations are discretized using the finite difference methods described under section 5.1 of this chapter. The finite difference representations of the project value are then translated into computer models using the C language. The required data is supplied, and an experiment is conducted. From the results of an experiment, the input data is modified to achieve the desired accuracy and the lowest CPU time. The models are then verified, segment by segment, to ensure that they are producing the required results, and are finally validated using real world data. The analysis was carried out on the RISC/os MIPS Workstation, 4 - 20 H, Faculty of Business, University of Alberta.

5.2.1. Computer Flow Charts and Description of Functions and Routines

The software developed for these solution algorithms are not included in this report. However, anyone interested in them could contact the author, Dr. D. G. Laughton, Room 4-20H, Business Building, University of Alberta, T6G 2R6, or Dr. J. M. Whiting, Room 606, Chemical - Mineral Engineering Building, University of Alberta, T6G 2G6.

Figure 5.2 contains a general layout and interaction of the program files used in solving the 1 - D and 2 - D ordinary and partial differential equations. A shell program called *run* is invoked to, first, make an executable document of all the files in Figure 5.2 using another file called *Makefile*, and then to run this executable document to obtain the program output.

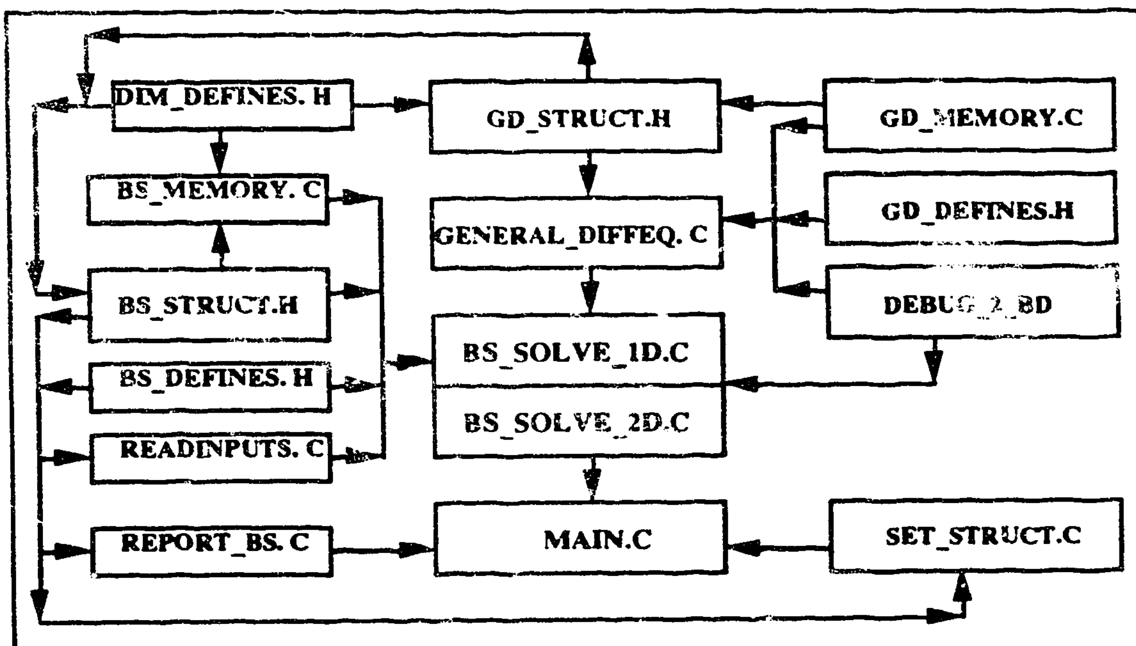

Figure 5.2 General Layout of the Program Files

5.2.1.1. GENERAL_DIFFEQ. C File

This file contains functions and routines for solving the general differential equations using the backward implicit finite difference algorithm for the 1 - D problem, and the backward implicit and the error function for the 2 - D problem. The routines and functions, as illustrated in Figure 5.3, are same in both cases, with internal modifications to deal with each particular problem.

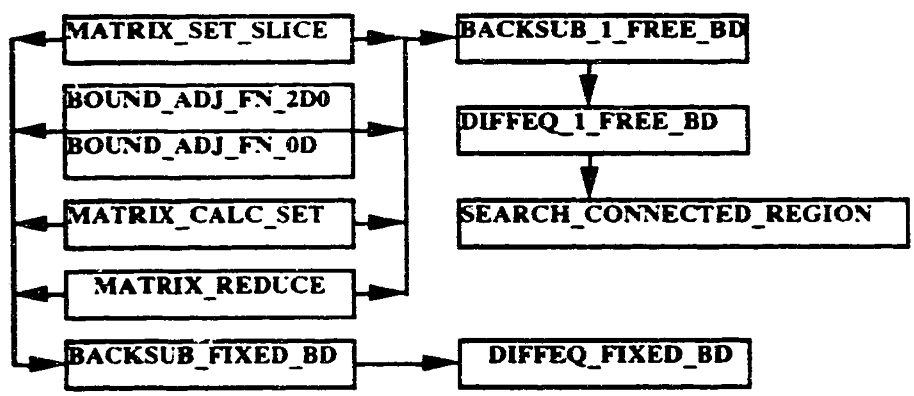

Figure 5.3 Routines and Functions in the GENERAL_DIFFEQ. C File

MATRIX_SET_SLICE: The tridiagonal matrices for the value of the mine when open, closed, or waiting under certainty and uncertainty are set for the 1 - D problem. In the 2 - D problem, the matrices are set for the mine value in the waiting and feasibility study phases; for varying metal price at fixed expected ore grade, and varying expected ore grade at fixed metal price.

BOUND_ADJ_FN_2D0: At infinite metal price for fixed expected ore grade, or infinite expected ore grade for fixed metal price, the second derivative function of the mine value with respect to either of these variables tends to zero. This function is used to adjust the upper matrix row for this condition.

BOUND_ADJ_FN_0D: At zero metal price or expected ore grade, the mine value is zero, and this function is used to adjust the lower matrix row for this condition.

MATRIX_CALC_SET: Reinitializes the matrix formulations using temporary variables which are subsequently used in the MATRIX_REDUCE function. These temporary variables are deinitialized and reused.

MATRIX_REDUCE: The matrix formulations are row-reduced in this function. The direction of row-reduction is provided by the value of DIR which is either -1 (i.e. from top to bottom) or +1 (i.e. from botton. .o top).

BACKSUB_FIXED_BD: Performs the backsubstitution operation on the martrix for a fixed region. The direction of backsubstitution is always opposite to the direction of row-reduction.

DIFFEQ_FIXED_BD: Solves a particular slice using a fixed boundary.

BACKSUB_1_FREE_BD: Performs the backsubstitution operation on the matrix, and finds a single boundary level where the comparison function is supplied by the called routine.

DIFFEQ_1_FREE_BD: Calculates the values of the mine in the open, closed, and waiting under certainty and uncertainty phases, and finds the free boundaries associated with each phase in the 1 - D problem. In the 2 - D problem, the mine values for both waiting and feasibility study phases are calculated with free boundaries at every feasibility study stage.

SEARCH_CONNECTED_REGION: Performs the search for a second boundary in a connected region, and returns TRUE if found. In the 1 - D problem, this routine i to search for the closed mine boundary with the abandonment region, after find losed mine boundary with the open region. In the 2 - D problem, this is usec earch for the boundary between the feasibility study and the waiting phases, after finding the feasibility study phase and the investment phase boundary. The minimum search region is 2, because the routine finds the boundary, and then checks the next position to ensure that it is not the boundary.

INCLUDE FILES: GD_DEFINES. H file contains the definitions and location of the global variables used in the GENERAL_DIFFEQ. C file; GD_STRUCT. H file contains a declaration of all the global variables in the GD_DEFINES. H; and BS_MEMORY. C file contains an allocation of memory for all the variables, declared as arrays, vectors and pointers in the problem structure of the GENERAL_DIFFEQ. C file.

5.2.1.2. BS_SOLVE_1D. C File

This file, as illustrated in Figure 5.4, contains routines and user-supplied functions specifically designed to solve the 1 - D problem.

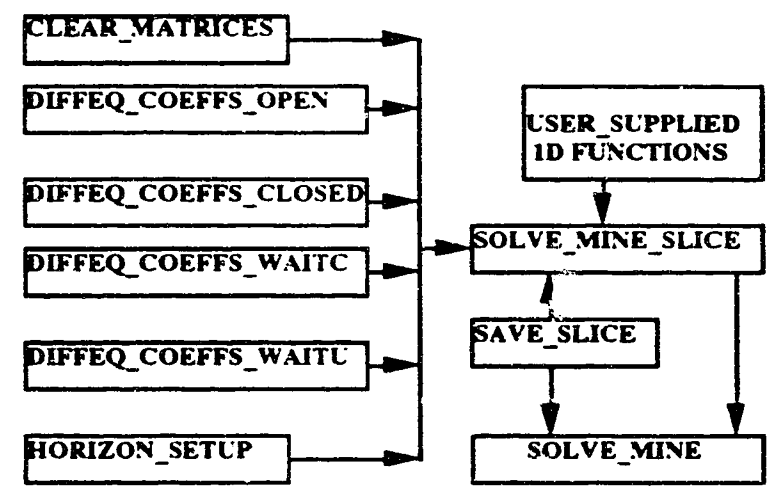

Figure 5.4 Routines and Functions in the BS_SOLVE_1D. C

CLEAR_MATRICES: Initializes the mine values at various lattice points on every slice, boundary values, and output matrices, for the open, closed mines, and waiting under certain and uncertain conditions.

MATRIX COEFFICIENTS ROUTINES: DIFFEQ_COEFFS_OPEN and DIFFEQ_COEFFS_CLOSED provide the values for the differential equation coefficient matrix for the open mine and the closed mine, respectively. DIFFEQ_COEFFS_WAITC and DIFFEQ_COEFFS_WAITU also provide the values for the differential coefficient matrix, respectively, for the mine under certainty and under uncertainty.

HORIZON_SETUP: Initializes the horizon (maximum ore reserve tonnage) values.

SOLVE_MINE_SLICE: Calculates the values of the mine, at each lattice point, when the mine is open, closed, or waiting under certain or uncertain conditions.

SAVE_SLICE: Stores the required information on each slice.

SOLVE_MINE: Controls the single reserve slice stepping procedure, stores samples, and keeps the information useful for the next slice calculation.

USER_SUPPLIED 1 - D FUNCTIONS

FREE BOUNDARY FUNCTIONS: FREE_BD_1_OPEN returns the greater of the open mine value, the closed mine value, and the value of the abandoned mine, in the open region. Boundaries are provided at which the mine will be closed or abandoned, for various reserve levels. FREE_BD_1_CLOSED returns the greater of the closed mine value or the value of the open mine, in the closed region. Boundaries are also provided at which the mine, when closed, will be reopen, for various reserve levels. FREE_BD_2_CLOSED returns the greater of the closed mine value or the value of the abandoned mine, in the closed phase. Boundaries are provided at which the mine will be abandoned when closed, for various reserve levels, are also provided. FREE_BD_1_WAITC returns the greater of the mine values for waiting and investing after the feasibility study. Boundaries are also provided at which the investment will be made. FREE_BD_1_WAITU returns the greater of the mine values for waiting and investing under uncertainty, and the value of the mine for undertaking the feasibility study. Boundaries are provided at which either of these three decisions will be made.

BOUNDARY ADJUSTMENT FUNCTIONS: As the metal price, S, approaches infinity, the second derivative of the mine value with respect to S approaches zero. Also, the mine value is zero when S is zero. BD_ADJ_FN_OPEN, BD_ADJ_FN_CLOSED, BD_ADJ_FN_WAITC and BD_ADJ_FN_WAITU, respectively, are used to adjust the differential equation coefficient matrix of the mine value when open, closed, or waiting under certainty or uncertainty for these two conditions.

5.2.1.3. BS_SOLVE_2D. C File

This file, as illustrated in Figure 5.5, contains routines and functions of the modified 2 - D continuous models.

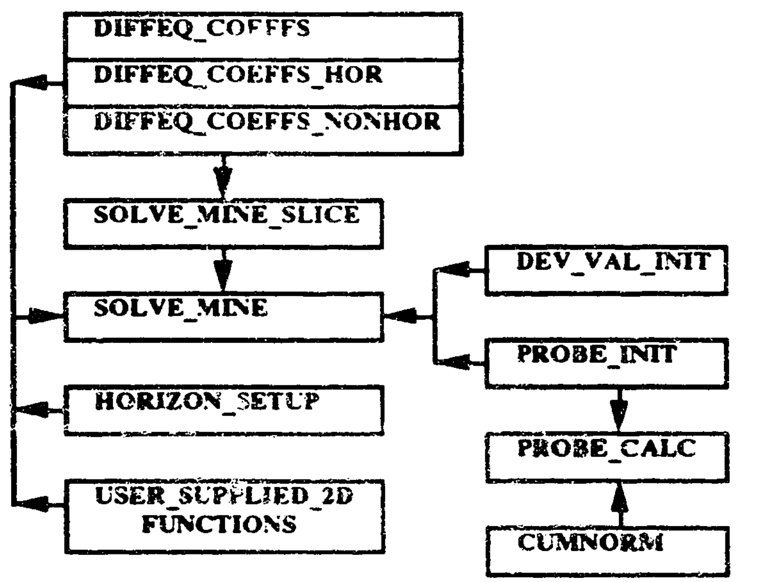

Figure 5.5 Routines and Functions in the BS_SOLVE_2D.C File

MATRIX COEFFICIENT ROUTINES: DIFFEQ_COEFFS initializes all the differential equation coefficients matrices for the mine. DIFFEQ_COEFFS_HOR and DIFFEQ_COEFFS_NONHOR set sampling rate coefficient to zero outside the feasibility study region, and 1.0 in the feasibility study region respectively.

DEV_VAL_INIT: Sets the annual development costs, discounts these annual costs to project evaluation time, and calculates investment value factors for the expected ore grades and metal prices.

PROBE_INIT: Calculates part of the probability distribution associated with the expected ore grade.

CUMNORM: Calculates the cumulative normal probabilities.

PROBE_CALC: Calculates the lognormal probabilities associated with respective expected ore grades.

HORIZON_SETUP: Calculates the investment values for various metal prices and ore grades, and the total value of the mine with feasibility study. The decision to invest is also made.

SOLVE_MINE_SLICE: Calculates the mine values under waiting conditions, and after every feasibility stage, using the finite difference code in the GENERAL_DIFFEQ. C file.

SOLVE_FEAS: Controls the feasibility study stage stepping procedure, and sets the feasibility study - investment boundary, if not set.

USER_SUPPLIED_2D FUNCTIONS

BD_ADJUST_FN: As S or E approaches infinity, the derivative of the mine value with respect to either of these variables approaches zero. Also, the mine value is zero when S or E is zero. This function is used to adjust the differential equation coefficient matrices of the mine value, for these two conditions.

FREE_BD_1: Compares the values from the BACKSUB_1_FREE_BD function with the values of either feasibility and investment, or waiting and investment, to enable the investor to make a decision whether to invest, or wait, when undertaking a feasibility study, and to invest, or do a feasibility study, when waiting.

5.2.1.4. SET_STRUCT. C File

This file contains the routines, SET_FEAS and SET_FEAS_IN, and SET_INPUT, that are used to set up the structures, respectively, for development and exploration, operational, and other input (i.e., lattice, economics and price) variables and parameters for the calculations.

5.2.1.5. READINPUTS. C File

The required data for the calculations carried out in both the 1 - D and 2 - D problems are input by two routines in this file.

5.2.1.6. REPORT_BS. C File

The required output, i.e., boundary and mine values, at each slice for the various phases, for both the 1 - D and 2 - D problems, are produced by two routines, BOUND and REPORT_BS, in this file.

5.2.1.7. MAIN. C File

This file contains the main program that runs the entire programs illustrated in Figure 5.2.

5.2.2. Verification

The programs were verified, segment by segment, using a debugging tool called DEBUG_2_BD to ensure that the respective segments results were accurate. The analytically tractable problems were calculated using EXCEL 3.0 to ensure that the computer results matched the EXCEL results.

5.2.3. Validation

The models are validated using copper mine data, and the results are discussed in chapters 6 and 7.

5.3. Conclusions

The solution procedure, and the experimental investigations for this study have been provided. The computer flow charts, and the experimentation process are also provided.

CHAPTER 6.0

APPLICATION OF THE 1 - D MODEL

The 1 - D model presented in Chapter 3 can be validated using data from an actual copper mine (CUMINE_1D). Results are discussed in this chapter which show the merits of the derivative asset valuation method to the potential user. They are also compared with results from the discounted cash flow technique using the same data.

6.1. Description of Input Data for CUMINE_1D

Input data for CUMINE_1D are classified into ECONOMICS, MINE, LATTICE and REPCON. Variables and parameters in the ECONOMICS file deal with the economic environment of the project. Table 6.1 illustrates the input data in this file.

Table 6.1 Economic Data for CUMINE_1D

Risk-free interest rate (in real terms), ρ	0.03
Convenience yield of metal, c	0.04
Volatility in Metal Price, σ	0.20

The risk-free interest rate is the interest rate (in real terms) on a government treasury bill, which is the safest investment available. The covenience yield of the metal is the benefit that accrues to the investor for holding the physical commodity, instead of a futures contract on the commodity. The volatility in the metal price is the proportional standard deviation of the price at any future time (a measure of the uncertainty associated with the metal price).

Variables and parameters in the MINE file deal with the technical and operating environment of the mineral project. Table 6.2 illustrates the input data in this file.

Table 6.2 Mine Data for CUMINE_1D	
Unit operating cost ($/lb of metal)	0.75
Cost of maintaining mine when temporarily closed ($M)	2.50
Cost of reopening the mine ($M)	5.00
Cost of closing the mine ($M)	25.00
Value of abandoned mine ($M)	0.00
Cost of abandoning the mine ($M)	25.00
Metal production rate (t / year)	100,000
Number of possible mine units	3
Metal reserves (Mt) : 1 of the following 3 types	
Type 1 with probability of occurrence of 0.30	0.00
Type 2 with probability of occurrence of 0.40	1.00
Type 3 with probability of occurrence of 0.30	2.00
Investment costs ($M) for	
Type 1 metal reserves	0.00
Type 2 metal reserves	350.00
Type 3 metal reserves	450.00
Investment cost without a feasibility study ($M)	450.00
Cost of feasibility strategy ($M)	
Type 1	20.00
Type 2	40.00
Type 3	∞

At the beginning of the detailed feasibility study, it is estimated that the outcome could be one of three possible metal reserves, i.e., zero, one, and two million tons with respective probabilities of occurrence of 0.3, 0.4, and 0.3. This means that, with the available information at the beginning of the feasibility study, there is a 30 percent chance of finding no metal reserve; a 40 percent chance of finding one million tons of metal reserve; and a 30 percent chance of finding two million tons of metal reserve. Investment could be made before (i.e. without a feasibility study) or after the feasibility study. The mine, when open, could be temporarily closed, and reopened, or abandoned at a cost to the investor. The abandonment cost includes the costs for disposal of junk materials, reclamation, and compliance with with all final environmental requirements.

The LATTICE parameters deal with the discretization of the ordinary and partial differential equations, and the parameters in the REPCON file are used to control the reporting of the output from the computer analysis.[1]

6.2. Feasibility Study Strategies

In this section, it is assumed that a feasibi :y study can be carried out to completely eliminate all the uncertainties associated with the metal reserves. Three feasibility study strategies are considered as follows:

1. The investor undertakes a feasibility study at the cost of $ 20 million before investing in the mine

2. The investor undertakes a feasibility study at the cost of $ 40 million before investing in the mine

3. The investor undertakes a feasibility study at the co of infinity, i.e., no feasibility study is undertaken, and investment must be made with uncertainty about the quantity of metal reserves

In all these strategies, the investor can wait for an appropriate time to undertake a feasibility study, or to invest. Also, in the operational stage, the mine could be shut down temporarily, reopened, or abandoned, in order to maximize the value of the mineral project.

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[1] See Appendix D.1 for the lattice and report control parameters

6.3. Analysis of the 1 - D Results

The results from the analysis of these strategies, using the derivative asset valuation method, can be compared with those derived by discounted cash flow techniques.[2] In a typical DCF analysis, evaluation of mineral projects is carried out on a "now or never" basis; i.e, if either the calculated return on investment is too low, or the net present value at a specified discount rate is negative, it is rejected. If the project is accepted and developed, it is implicit that it will be operated throughout the term of the cash flow analysis. Options to close, reopen or abandon the project are not amenable considerations.

Figure 6.1 illustrates the *net present value of the mine* under the various feasibility study strategies described above, using the DCF analysis, i.e., with no timing and operating options, and a discount rate of seven percent.

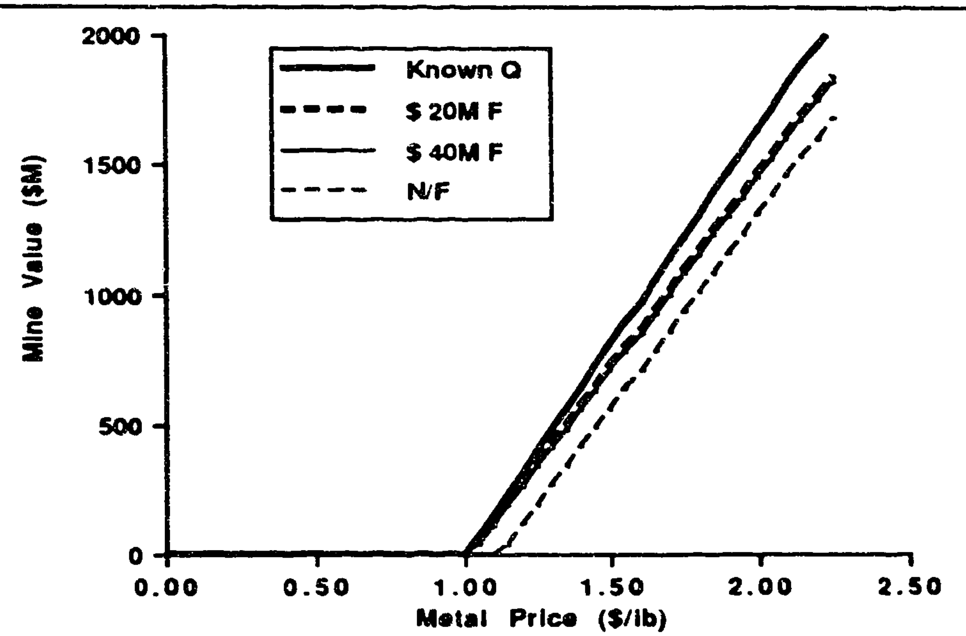

Figure 6.1 Mine Value vs Metal Price (no timing or operating option)

[2] See Appendix E for the program output of the 1 - D DAV model of the mine value.

The first situation (Known Q) is the net present value of the mine at the stage where the investor knows *with certainty* that the reserve is one million tons of metal. The second ($ 20M F) is a situation in which the investor spends $ 20 million on the feasibility study, and in the third ($ 40M F), he spends $ 40 million on the feasibility study. In the last case, (N/F), no feasibility study is undertaken, and the mine must be developed under uncertainty, if it is developed at all. This figure shows that by undertaking a certain amount of feasibility study, the investor can increase the value of the project.

Figure 6.2 illustrates the *value of the feasibility study*. This shows the difference between each of the first three cases and the last case. The feasibility study helps the investor to avoid investing at low prices for a low reserve deposit, and to avoid, at high prices, *the extra investment costs that would be incurred under uncertainty* for a smaller, but profitable reserve. The Known Q case has increasing differential value at high prices, because the expected production time profile is shorter with Known Q, though the expected reserves are the same in all cases at one million tons.

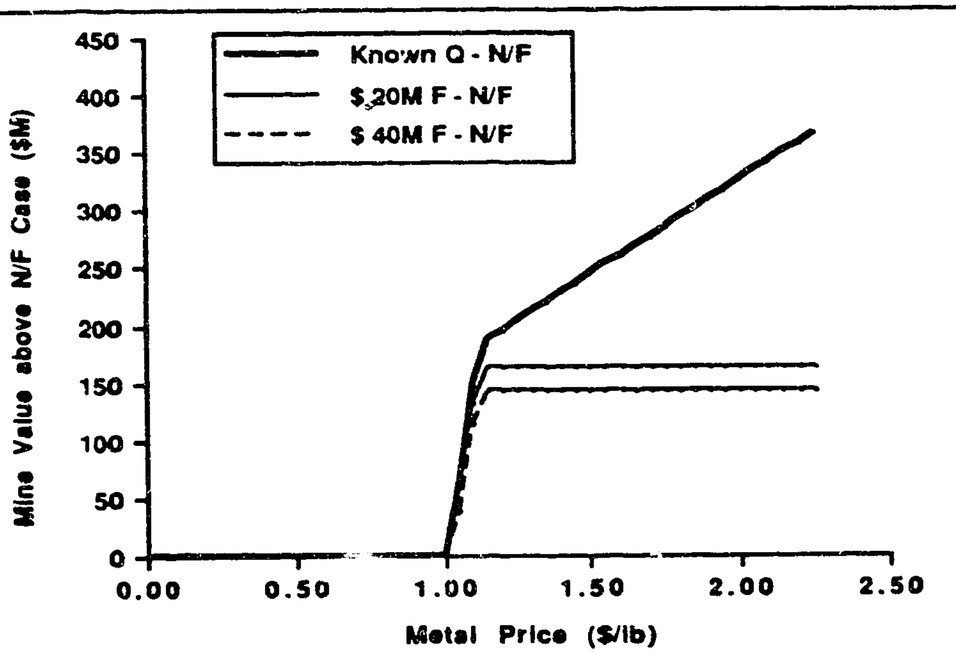

Figure 6.2 Mine Value above N/F Case vs Metal Price (no timing or oprating option)

Figures 6.3 and 6.4 are similar to Figures 6.1 and 6.2, *except that waiting and operating options* are built into the analysis, using the derivative asset valuation method. At high prices, where these extra options have no value, these figures give the same picture, while at lower prices they increase the projected value of the mineral venture. Using the DCF method (in Figures 6.1 and 6.2), the analysis shows that below a price of $ 1.00/lb of metal, the project has no value and would be rejected.

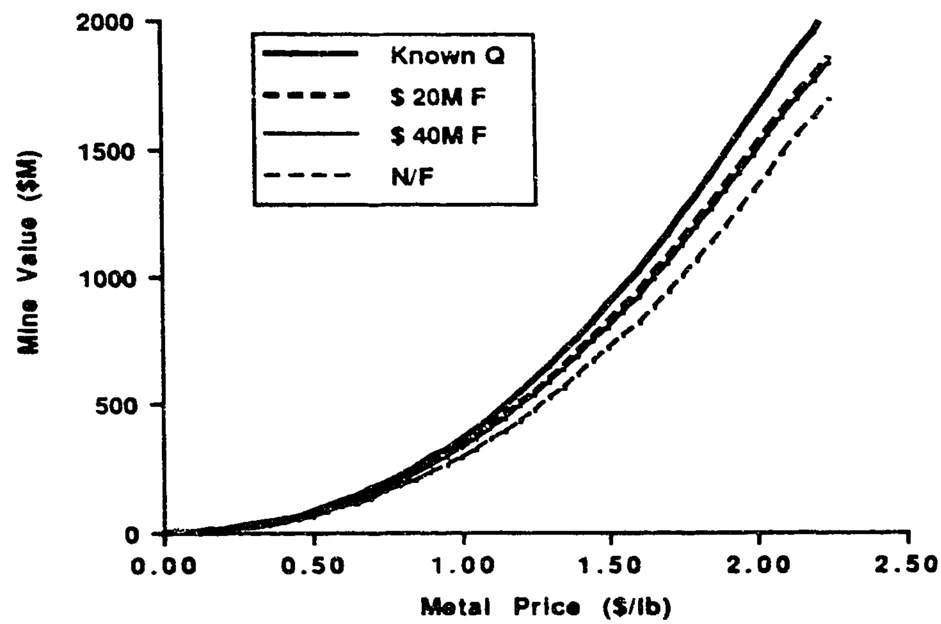

Figure 6.3 Mine Value vs Metal Price (with timing and operating options)

Using the DAV method, however, the project has value, *if the options to wait and operate the mine are available*. In the DAV method, the importance of a feasibility study before investment is shown clearly in Figure 6.4. The differences between the feasibility study cases versus the case with no feasibility study are very significant.

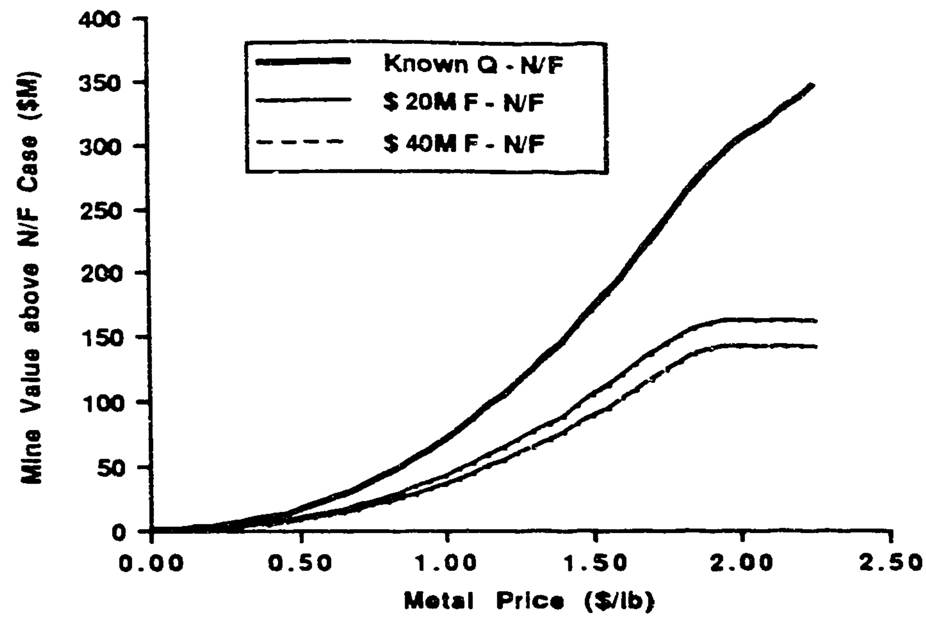

Figure 6.4 Mine Value above N/F Case vs Metal Price (with timing and operating options)

The *effects of timing* the feasibility study and investment, and operating options (OP), using the derivative asset valuation method proposed in this study, are illustrated in Figure 6.5. The first case (W/OP/F) allows the investor to wait (W) for an appropriate time to undertake a feasibility study (F), and then wait (W) to commence any subsequent development. It also allows the investor certain *operating options (OP), such as when to shut down temporarily, to reopen and to abandon when conditions are unfavourable*. The second case (OP/F) is similar to the first, except that it does not allow the timing options before and after the feasibility study. The third case (OP) allows only the operating options to be exercised, if the mine is developed. These three cases are compared with the results of the DCF analysis (N/O) in which no timing and operating options are allowed. The figure shows that the value of the mine using the no option case is the least.

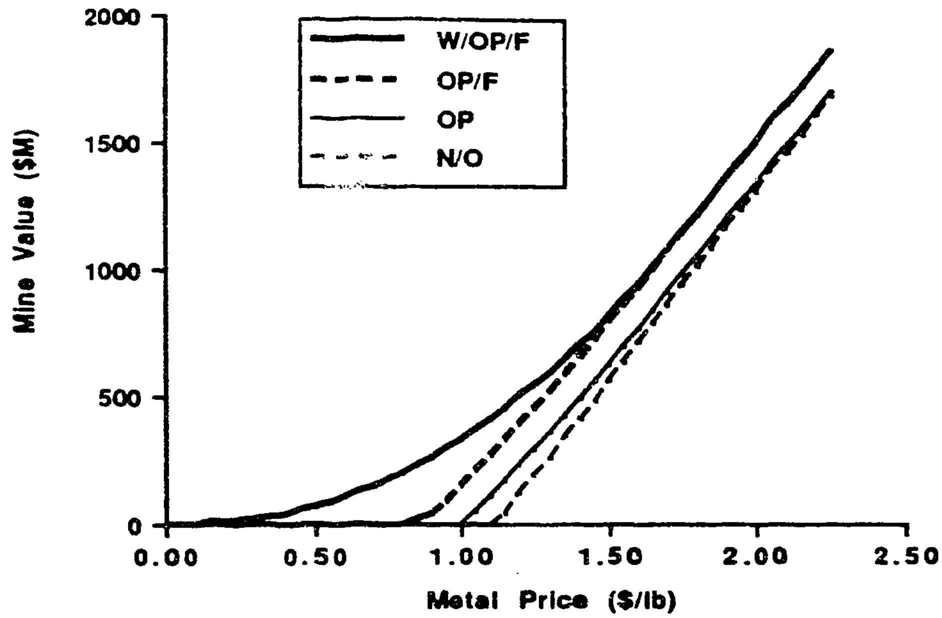

Figure 6.5 Mine Value vs Metal Price
(uncertain reserves with $ 20M or no feasibility study)

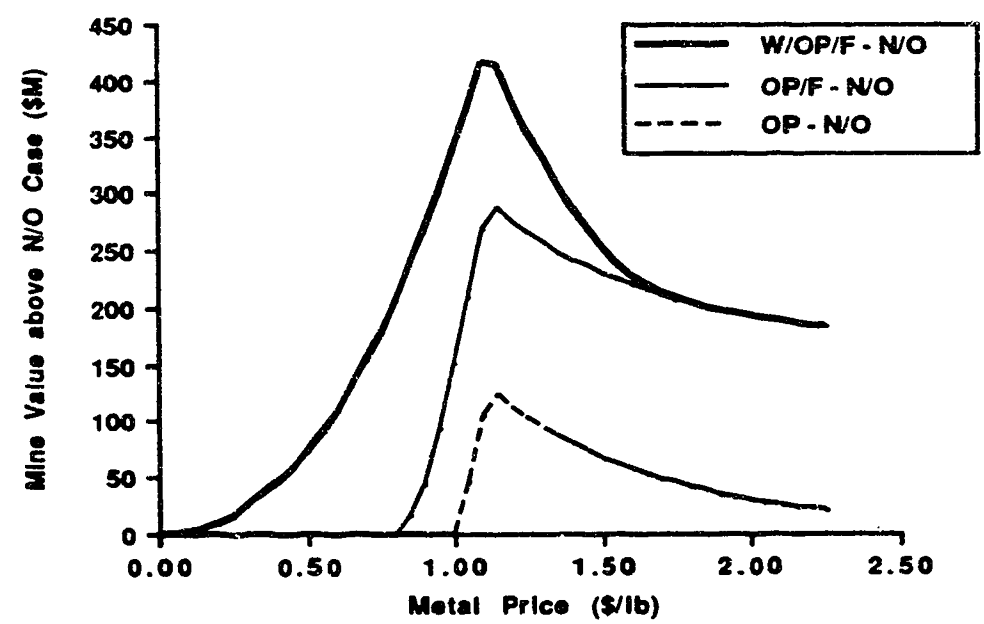

Figure 6.6 Mine Value vs Metal Price
(uncertain reserves with $ 20M or no feasibility study)

Figure 6.6 shows the differences in value of the mineral project for each case versus the no option case. For example, without timing, feasibility study and operating options, about $ 450 million is not captured at a price of $ 1.125/lb of metal, and more marginal but profitable projects could easily be overlooked or rejected.

After the feasibility study, when the investor is certain about the metal reserve, the use of the DAV method is still superior to the DCF method. Figure 6.7 illustrates the value of the mineral venture when it is possible to time the investment and operating options if the mine is developed (W/OP), and when only operating options (OP) exist. The value of the mine derived by the DCF method is plotted as N/O. The figure shows again that the DCF method results are the least of all cases.

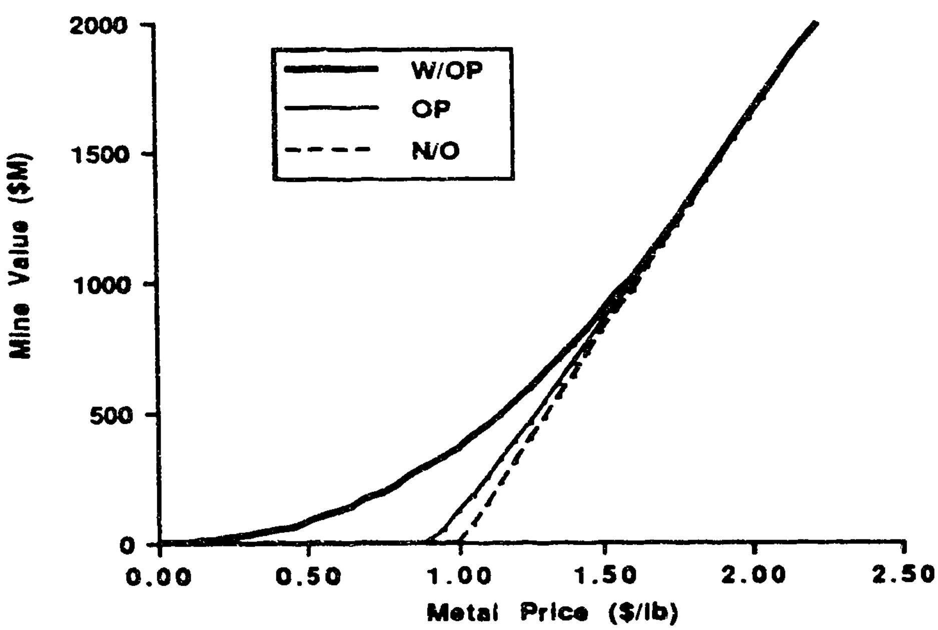

Figure 6.7 Mine Value vs Metal Price (known reserves, Q = 1M tons)

Figure 6.8 focuses on the differences between the first two cases relative to the results of the DCF method. In a situation with high current metal prices, the option to wait, and to close or abandon after opening, are not likely to be chosen, and the value of the option is very small. At about $ 2.125/lb of metal, this value goes to zero.

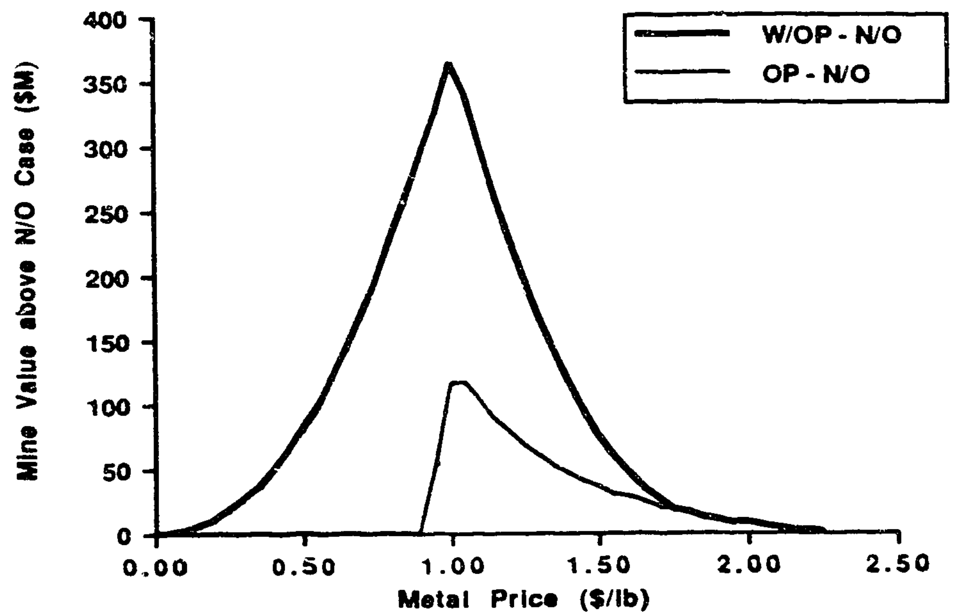

Figure 6.8 Mine Value above N/O Case vs Metal Price (known reserves, Q = 1M tons)

At very low current metal prices, it is unlikely that the mine will be developed at all in the near future, and again the options are not valuable. The options' value can be quite high, however, at medium prices (i.e., between $ 0.25 and $ 1.75/lb of metal) where there is significant uncertainty about the desirability of developing the mine, or if the mine is already opened, closing or abandoning it. Finally, if there is an option to wait, it is chosen, even when investing now would otherwise have positive value. For a mine with operating option, this critical "now or never" investment price (resulting from the DCF analysis) is about $ 1.00/lb of metal, while investment would not take place below a price of $ 1.75/lb, if waiting is possible.

The potential benefit of the feasibility study is again illustrated in Figure 6.9. In this figure, the value of the deposit with uncertain size with no opportunity to undertake a feasibility study is shown. The DAV and DCF methods have been used in evaluating this deposit. Compared with Figure 6.5, it can be seen that the critical investment prices are all shifted upwards, because of the lower value of the mine once developed, and the extra investment cost.

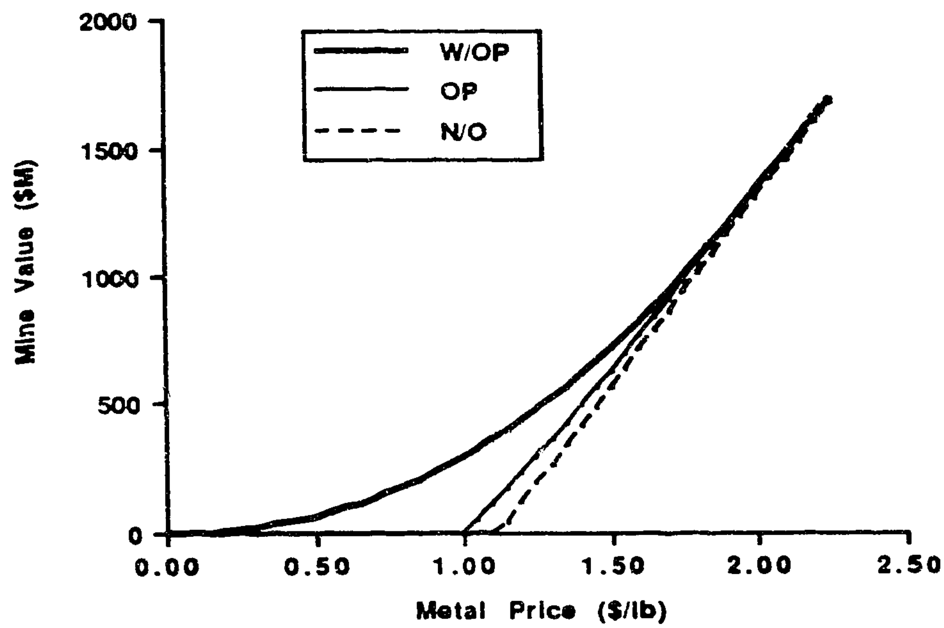

Figure 6.9 Mine Value vs Metal Price (uncertain reserves with no feasibility option)

Figure 6.10 also shows the difference in value of the first two cases (W/OP) and (OP) versus the results of the DCF (N/O). The figure shows again that, in this situation, the DAV method provides additional insight compared to the DCF method.

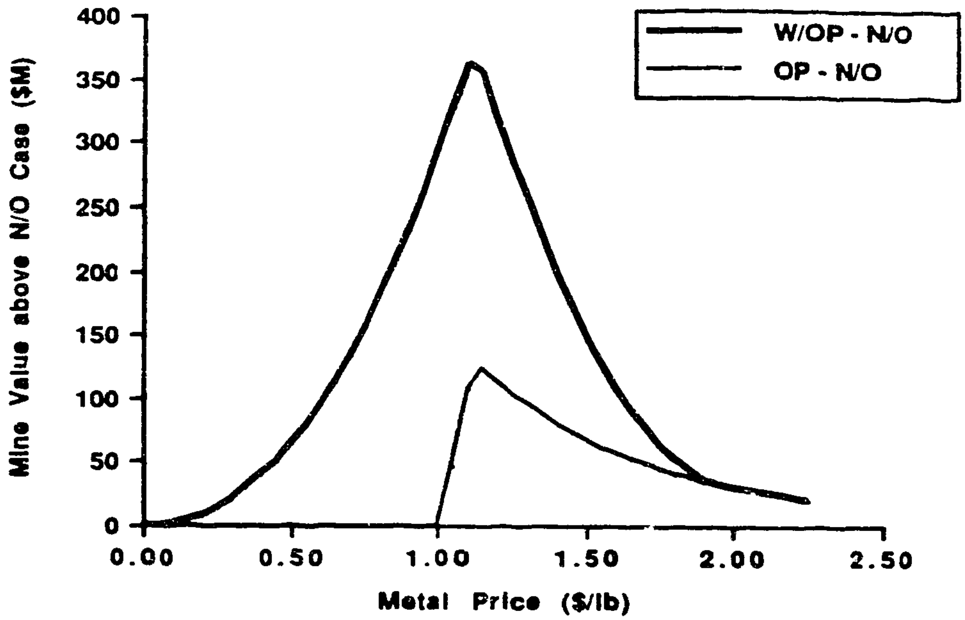

Figure 6.10 Mine Value above N/O Case vs Metal Price (uncertain reserves with no feasibility option)

The value of the options to time the feasibility study and any subsequent development of the mine, and the option to do a feasibility study before developing the mine with changing metal prices are illustrated in Figure 6.11. The first case (Known Q) illustrates the value of waiting to develop the mine once the reserve is known with certainty. The second case (F) illustrates the option to wait to do a feasibility study at the uncertainty stage. The third case (N/F) shows the value of the mineral venture with no feasibility study option, and investment is made under uncertainty. The figure shows that the feasibility study option and the option to wait on development are complementary at lower metal prices, i.e., they reinforce each other, and supplementary at higher metal prices, i.e., they replace each other.

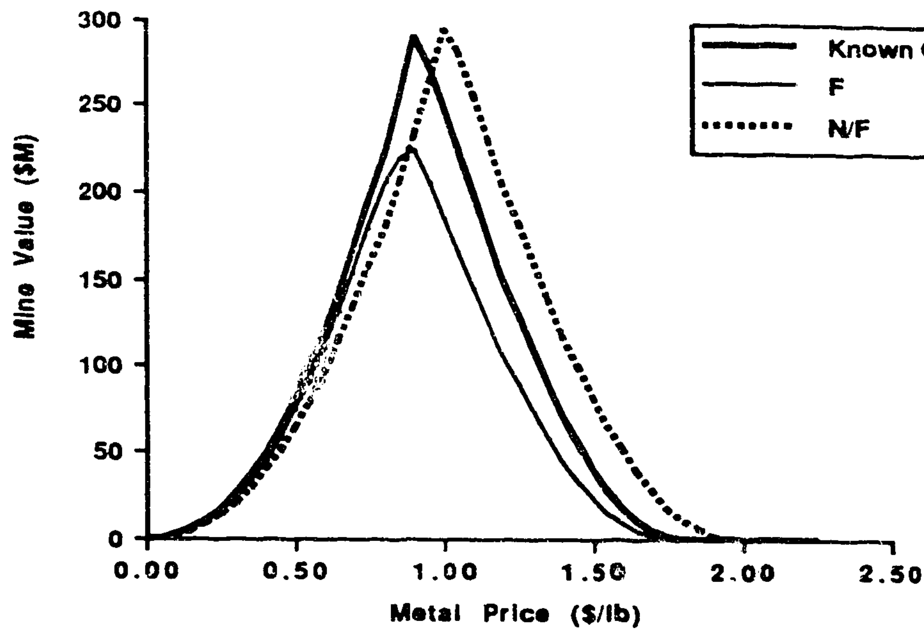

Figure 6.11 Waiting Option Value vs Metal Price

The feasibility study opportunity enhances the waiting option, at low prices, by making the pay-off, for which the organisation is waiting, more valuable. However, above the price the feasibility study would be undertaken, and in the *absence* of the waiting option, the feasibility study substitutes for waiting in helping the investor avoid a less profitable investment. The feasibility study does this by better defining reserves and costs, while waiting allows the evolution of more favourable metal price conditions.

Figure 6.12 illustrates the difference in mine value for waiting and investing under certainty at various metal prices after the feasibility study. The first case, Diff(WC,IC)_1M, illustrates the difference between waiting and investing when the metal reserve is one million tons. The second, Diff(WC,IC)_2M, also illustrates the same idea when the metal reserve is two million tons. The figure shows that, after the feasibility study, the value of waiting to invest in both cases is quite significant. Investment in the two million-ton metal reserves will be made at a lower metal price than in the one million-ton metal reserves, because the concept of economies of scale enables the investor to open the former at a relatively lower metal prices.

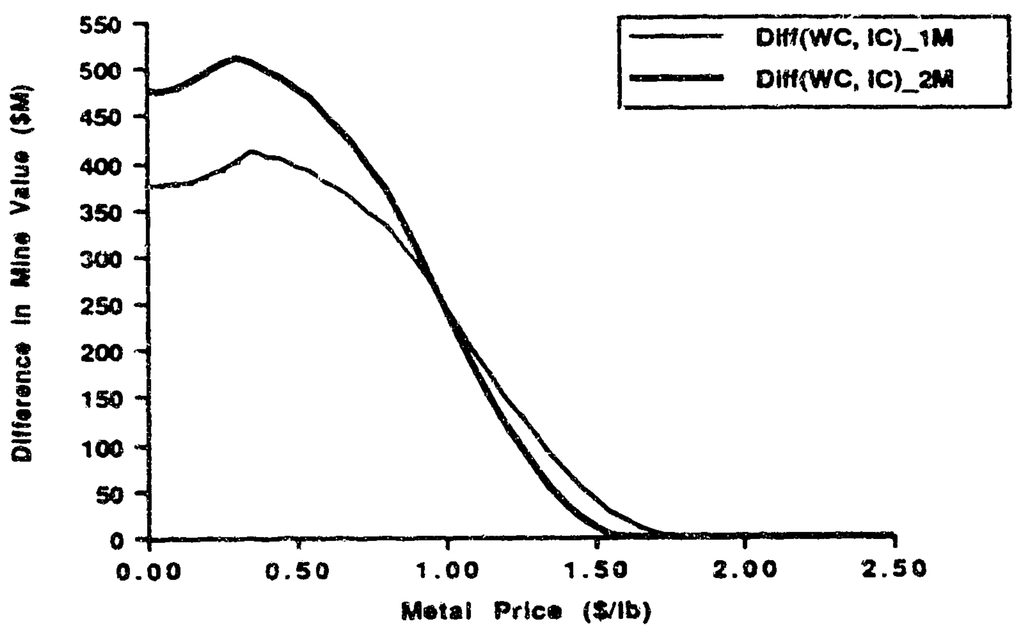

Figure 6.12 Difference in Mine Value for Waiting and Investing under Certainty vs Metal Price

At the operational stage, Figure 6.13 illustrates the boundaries at which certain options can be chosen to maximize the value of the project. The close to open boundaries are higher, because the investor wants to ensure that, by reopening, he will receive enough revenue to cover operating costs, costs of reopening, and an adequate profit. When the mine is open, and metal prices get closer to the unit operating cost, *all things being equal,* the investor will choose to shut down the mine to avoid huge losses. In all cases, the investor is reluctant to abandon the project, because by abandoning, any opportunity is lost to develop the mine at any future period, when profitability improves. Therefore, the abandonment boundaries are all lower.

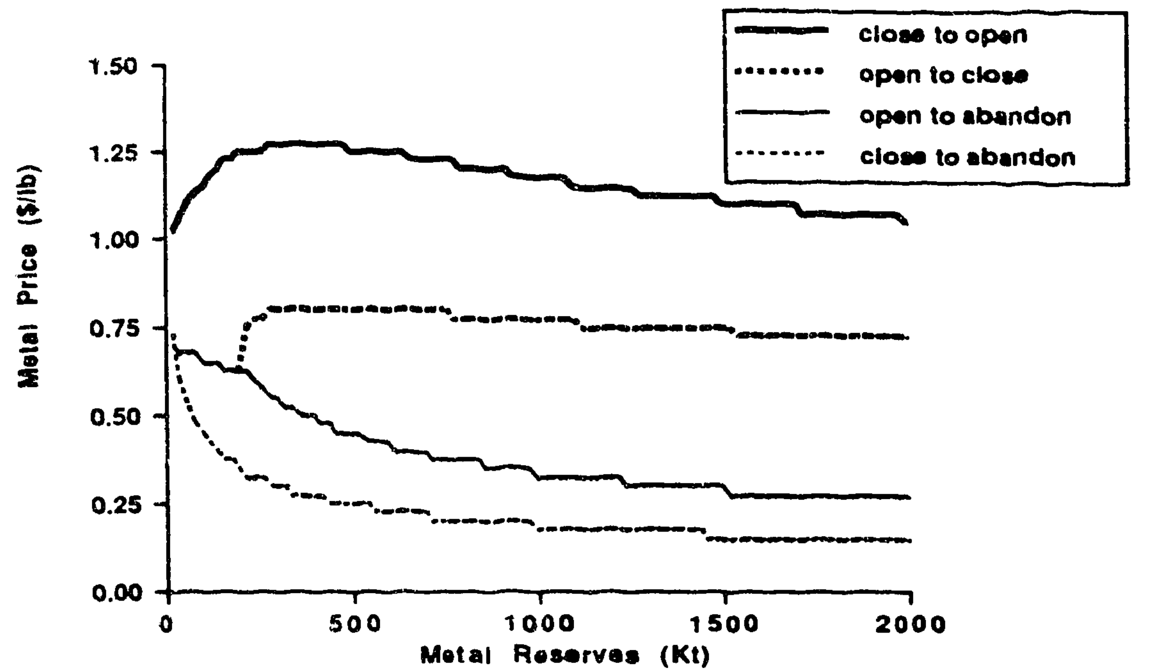

Figure 6.13 Boundary Metal Prices for Various Metal Reserves

At the operational stage, investors can take advantage of the options to shut down temporarily, reopen, or abandon, to maximize the value of the venture. Figure 6.14 illustrates the difference in mine values for a closed and an open mine, in the operational stage, at various metal prices. The first case, Diff(CL, OP)_1M, illustrates the difference between closing and operating the mine when the metal reserve is one million tons, and the second, Diff(CL, OP)_2M, illustrates the same idea for the two million-ton metal reserves. For the typical mineral venture cases being considered, it can be seen from the figure that between metal prices of $ 0.25 and $ 1.05/lb of metal, and $ 0.20 and $ 0.90/lb of metal, it is profitable to shut down, respectively, the one million-ton mine and the two million tons mine temporarily. By so doing, the investor can save up to $ 25 million at metal prices between about $ 0.30 and $ 0.75/lb of metal.

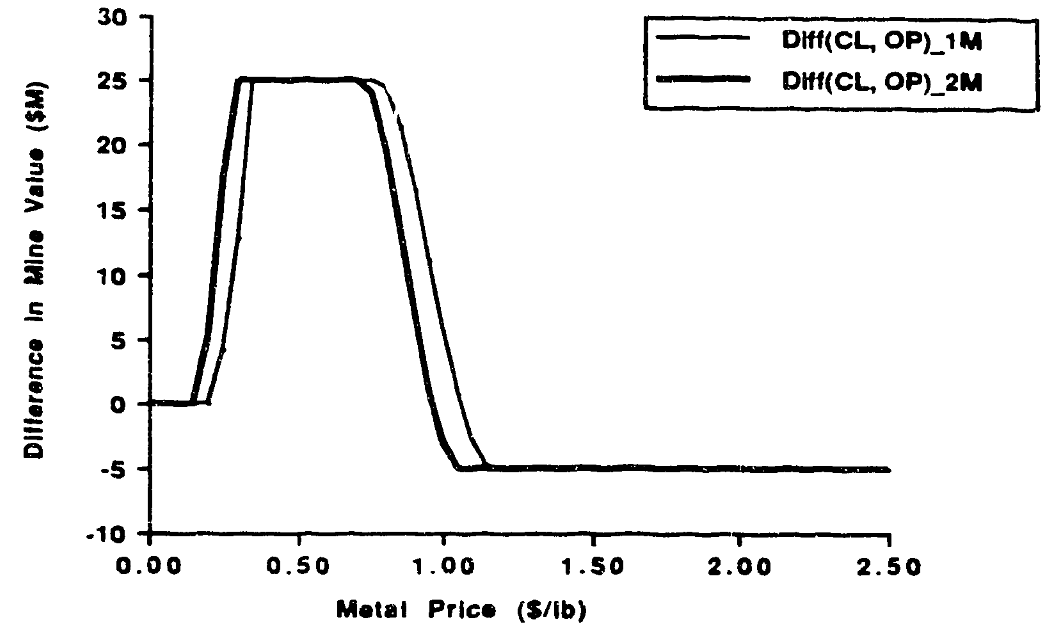

Figure 6.14 Value of Closing when Open vs Metal Price

6.4. Conclusions

In this section, the value of the mineral venture was calculated with the option of being able to use a feasibility study to determine characteristics of a mineral project before making a major development investment. The value of the option to time the feasibility study and mine development was also quantified. The results show that, at low to medium-high metal prices, the value of these options are significant, and should be taken into consideration in mineral project evaluation. Also, at the operational stage, the option to close, reopen, and abandon a project has been shown to contribute significantly to mineral project value.

Unlike the DAV method, the DCF method has no in-built procedures to examine and analyse significant options available to investors, and therefore has the potential to undervalue mineral projects at low to medium-high metal prices.

CHAPTER 7.0

APPLICATION OF THE 2 - D MODEL

This chapter continues the discussions in Chapter 6, and it details the feasibility study strategies and waiting options available to an investor, as well as their value in a mineral venture development. Analysis of the state of the economy is also carried out to show its effects on the value of the mineral venture. A numerical example of the 2 - D model presented in Chapter 4 is considered using data from a copper mine (CUMINE_2D). Results of this example are presented and discussed to show the merits of the derivative asset valuation (DAV) method to the potential users.

7.1. Input Data

The classification of the input data in the 2 - D model is similar to that for the 1 - D model.[1] The economic variables and parameters and their values are illustrated in Table 7.1.

Table 7.1 Economic Data for CUMINE_2D	
Risk-free Interest Rate (in real terms), ρ	0.03
Volatility in Metal Price, σ	0.20
Market Price of Risk of Mineral Commodity, λ	0.40

The market price of risk of the mineral commodity, λ, is the extent to which investors require higher returns to compensate them for bearing the risk associated with the mineral commodity [Hull 1989]. This is related to the convenience yield, c, concept in Chapters 3

[1] See Tables A.3 and A.4 under Appendix D.2 for the respective lattice and report control parameters in the 2 - D mine value model.

and 6 of this report, the risk-free interest rate, ρ, and the volatility in the metal price, σ, by the following equation:

$$\lambda = \frac{1}{\sigma}[c - (\rho + \eta) + \mu] \quad (7.1)$$

η and μ are the respective storage cost and expected growth rate of the commodity price.

Table 7.2 illustrates the feasibility study and mine data used in the 2 - D model.

Table 7.2 Feasibility Study and Mine Data for CUMINE_2D

Total Ore Reserve (Mt)	100.0
Number of Feasibility Stages	4
Development Cost at	
Stage 0 ($M)	550.0
Stage 1 ($M)	450.0
Stage 2 ($M)	425.0
Stage 3 ($M)	415.0
Investment Stage ($M)	405.0
Unit Production Cost ($/lb)	0.50
Total Development Period (years)	3.00
Total Production Period (years)	20.00
Variance associated with the log of Ore Grade at	
Stage 0	3.000
Stage 1	0.113
Stage 2	0.026
Stage 3	0.011
Investment Stage	0.006
Cost of Feasibility Study at	
Stage 0 ($M)	0.05
Stage 1 ($M)	0.20
Stage 2 ($M)	5.00

Stage 3 ($M)	5.00
Feasibility Study time at	
Stage 0 (years)	1.00
Stage 1 (years)	1.00
Stage 2 (years)	1.00
Stage 3 (years)	1.00
Waiting Option at	
Stage 0	True
Stage 1	True
Stage 2	True
Stage 3	True
Investment Stage	True

Four feasibility study stages are considered, and each stage lasts a period of one year. As feasibility study progresses from, say, some careful consideration of a geological anomaly to a detailed, documented, "bankable" report, the variance associated with the expected ore grade reduces from 3.000 at stage 0 to 0.006 at the investment stage, the latter being the expected ore grade residual variance. It is assumed that, at any point during the feasibility study, investment could be made at a cost to the investor, i.e., ***the investor must pay for knowing little*** about the expected ore grade and metal reserves, if he chooses to invest at any time in the feasibility study phase. Stated in another manner, the cost of developing the mine decreases as knowledge is gained about the expected ore grade, metal reserves and the ultimate scope and design of the project. This assumption is based on experience and observation; it serves as the rationale for defining and planning projects through systematic, analytical feasibility studies, where the main objectives are to minimize mistakes and waste and to proceed in the most efficient manner to maximize the net present value of a venture.

Also, the investor has the option to wait at any stage in the feasibility study phase, and at the investment stage, if the waiting option at that stage is ***True***, and if ***False***, no option to wait is allowed at the corresponding stage.

7.2. Feasibility Study Strategies

In this thesis, feasibility study is used to reduce the uncertainty associated with the expected ore grade, and to help investors avoid guesses, undocumented estimates, and over- and under-investment costs which will lead to waste, inefficiencies, and high cost overruns. Fifteen possible feasibility study strategies, classified under four main classes, typical of real-world situations, are used to illustrate the application of the DAV method to help investors in evaluating mineral ventures in specified economic, technical and operational environment. They are:

Multiple-Stage Feasibility Study: In this class, feasibility study strategies are considered in stages. They are carried out in four stages, each lasting a period of one year. This duration can be changed to any period to suit an investor. Eight strategies are considered:

1. *Strategy wfww*: The investor has the option to *wait before and after* a feasibility study program. During the feasibility study, he can also *wait at the end of each stage* before making a decision to embark on the next feasibility study stage. Investment can also be made *any time during* the feasibility study. This normally happens when there are no time constraints on investor's mineral development rights.

2. *Strategy wfnw:* This is similar to wfww, except that *no waiting is allowed in* the feasibility study phase. The required amount of feasibility study must be carried out continuously, stage after stage, for a decision to be made to invest or wait. This is normally the case when management is in a hurry to make a decision on a mineral venture with limited or no information.

3. *Strategy wfnn:* This is similar to wfww, except that there is *no option to wait during and after* the feasibility study. The required feasibility must be undertaken continuously without waiting, and investment must be made immediately, or the concession or mineral development rights must be forfeited.

4. *Strategies nfww, nfnw, and nfnn* are similar, respectively, to wfww, wfnw, and wfnn, except that there *is no option to wait to begin* the feasibility study in the first stage. Feasibility study must begin now or never.

5. *Strategy full_wfww:* This is similar to wfww, except that *all the four stages in the feasibility study phase must be completed before making any decision to invest in the project.* In mineral project development, certain perceived technical, and/or operational problems that might prove to be a fatal (or serious) flaw at any future period, e.g., major faulting, hydrological conditions, slope failure problems, and mineralogical complexes, would warrant this feasibility study strategy. Furthermore, it is quite normal for feasibility studies to go through an exploration, bench scale, pilot plant, and detailed design evolution.

6. *Strategy w2fww*: This is similar to wfww, except that only the first two feasibility study stages are considered.

Single-Stage Feasibility Study: In this class, all the required feasibility study work is carried out in one continuous, time-limited effort. This situation usually arises because the project can be clearly defined, and all useful analyses can be completed without seasonal, regulatory, or money-related disruptions. Five strategies are considered.

7. *Strategies 1s0_wfww, 1s1_wfww, 1s2_wfww, and 1s3_wfww* are similar to wfww, except that all the required feasibility study is carried out at once beginning, respectively, from stages 0, 1, 2, and 3.

6. *Strategy 1s0_w2fww* is similar to w2fww, except that all the required feasibility study is carried at stage 0.

No Feasibility Study: In this class, no feasibility study is undertaken, and investment is made at the cost of $ 550M at stage 0. This situation occurs, though rarely, when investors are convinced that the project could be profitable, irrespective of the little information about the expected ore grade. Only one strategy, *nofeas*, is considered.

Investment Cost Independent of Expected Ore Grade Variance: In this class, the cost of investing at any feasibility study stage does not depend on the knowledge about the expected ore grade. Investment cost at any feasibility study stage is $ 550M irrespective of the expected ore grade uncertainty. This situation does not occur in practice, because information about the expected ore grade and reserves are essential ingredients in mineral venture development cost. One strategy, *ic_novar*, is used to demonstrate the importance of paying attention to the uncertainty associated with the expected ore grade, and the "cost of ignorance" factor in the investment cost.

7.3. Analysis of the 2 - D Results

The results are discussed under two main categories: (1) Effects of feasibility study strategies and their interactions on mineral project value; and (2) effects of different economic parameters on mineral project value.

7.3.1. Effects of Feasibility Study Strategies and their Interactions

This section of the results deals with phase diagrams which delineate appropriate regions in which it is optimal to wait, undertake a feasibility study, or to invest (for the mineral project considered in this section), and mineral project value versus expected ore grade at various metal prices for the above strategies.[2] These diagrams focus on: (1) The importance of a feasibility study before investment; (2) the importance of various waiting options in a mineral project development; (3) the main determinants of the value of a feasibility study in a mineral project development; (4) the value of a multiple-stage feasibility study in a situation of high expected ore grade variances, and low to medium high metal prices; and (5) the effects of feasibility study stage duration on the value of a mineral venture.

7.3.1.1. Strategy wfww: Mineral Project Phase Diagrams

Figures 7.1 to 7.5 illustrate the metal price boundaries at which an investor will undertake a feasibility study if waiting, and invest in the mineral venture if waiting or undertaking a feasibility study for the *strategy wfww*. These figures illustrate the different phases, i.e., waiting, feasibility study, and investment, an investor with the objective of maximizing the value of the mineral venture will be at various expected ore grades and metal prices at different feasibility study stages. The region in which an investor will undertake a feasibility study, described as *feasibility phase* in the figures is largest at stage 0, as illustrated in Figure 7.1, compared to the other cases in Figures 7.2 and 7.3. This is because, at this stage, the uncertainty about the expected ore grade is very high as

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[2] See Appendix F for a typical program output for the 2 - D DAV model of the mine value, i.e., wfww and nfww.

compared to the uncertainties about the expected ore grade in the other stages. Also, the cost of the feasibility study is relatively small.

As the investor progresses through the feasibility stages, this feasibility phase decreases in area, as a result of increasing knowledge about the ore grade and the relatively higher feasibility study costs.

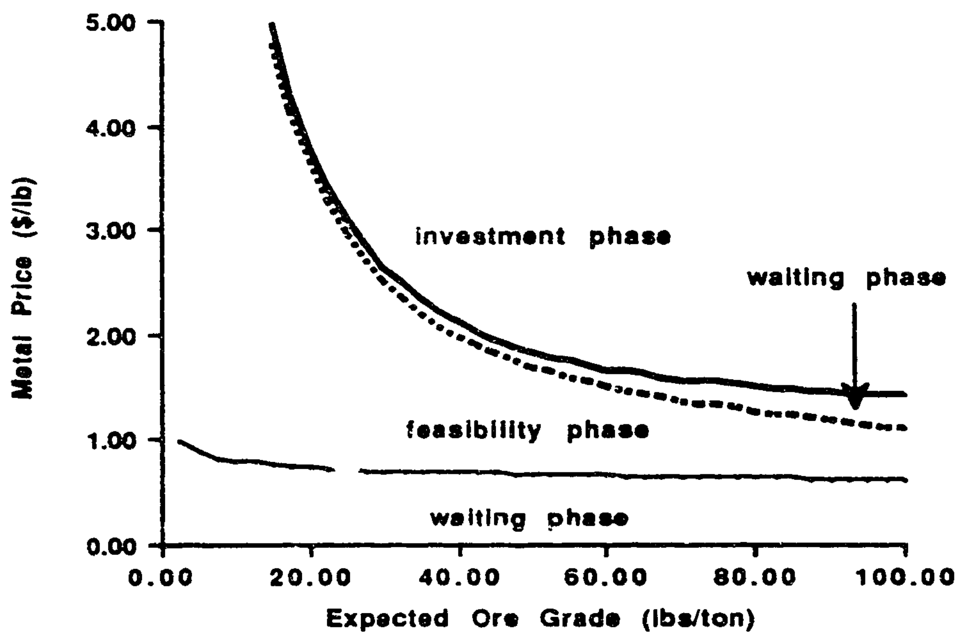

Figure 7.1 Mineral Project Phases at Stage 0 (wfww)

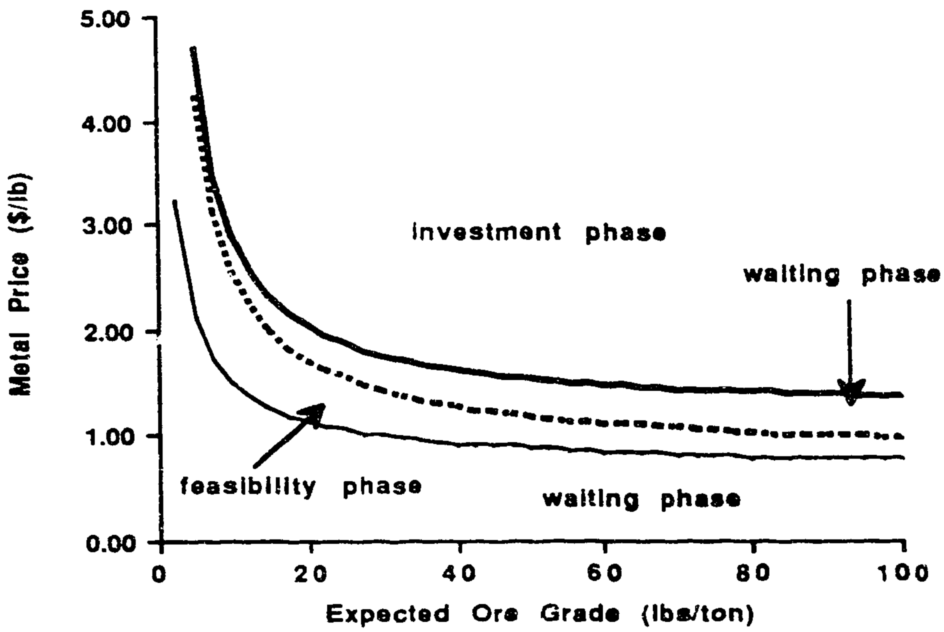

Figure 7.2 Mineral Project Phases at Stage 1 (wfww)

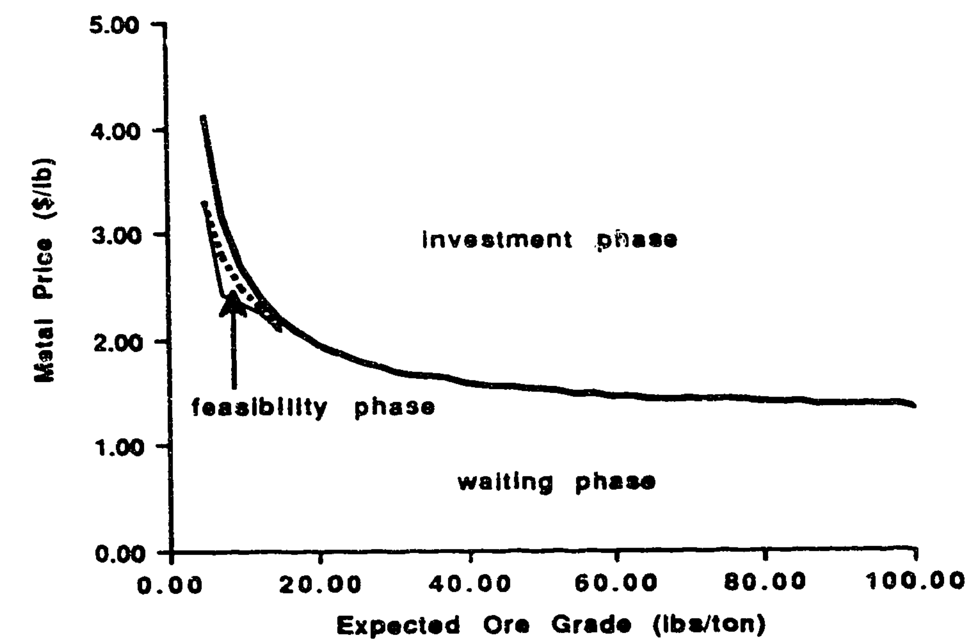

Figure 7.3 Mineral Project Phases at Stage 2 (wfww)

At stage 3, there is no incentive to undertake a feasibility study, as illustrated in Figure 7.4, because of the waiting option and the reduced expected ore grade uncertainty. At stage 0, investment can still take place at very high metal prices. These metal price boundaries for investment also decrease as knowledge is gained about the expected ore grade. Another significant feature of Figures 7.1 and 7.2 is the waiting phase above the feasibility phase. This indicates that, while undertaking a feasibility study at these stages, if the metal prices at various expected ore grades happen to be in this waiting phase, feasibility study should cease, because it is profitable to wait.

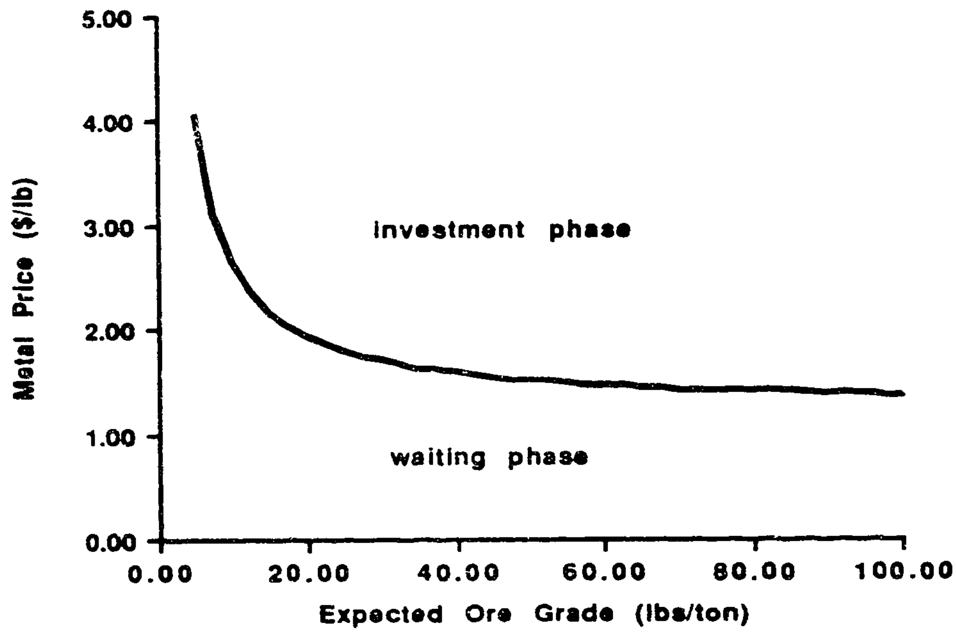

Figure 7.4 Mineral Project Phases at Stage 3 (wfww)

In a situation where there is no option to wait to begin the feasibility study, i.e., where an investor has to begin the feasibility study now, or forfeit his rights to development, but he can wait elsewhere in and after the feasibility study (strategy nfww), all the boundaries are shifted down, as illustrated in Figure 7.6. Thus, without the waiting option to begin the feasibility study, an investor is forced to undertake feasibility study at lower metal prices in order not to lose his mineral development rights.

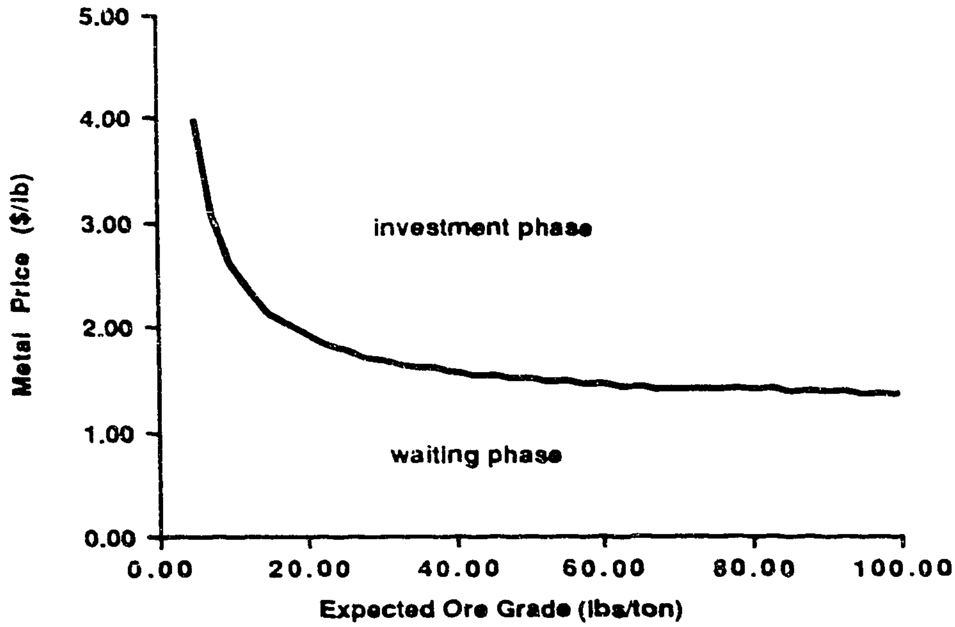

Figure 7.5 Mineral Project Phases at Investment Stage (wfww)

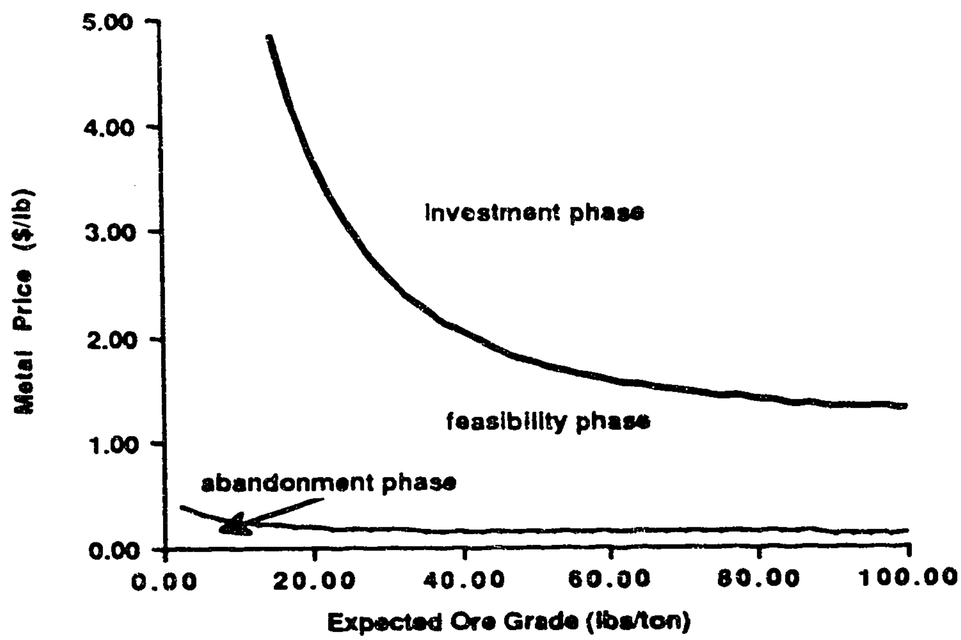

Figure 7.6 Mineral Project Phases at Stage 0 (nfww)

7.3.1.2. Value of the Option to Wait to Begin a Multiple-Stage Feasibility Study Program

The differences in values using strategy wfww versus nfww, at various feasibility study stages, are illustrated in Figures 7.7 to 7.13. Figures 7.7 and 7.8 illustrate this difference at stage 0.

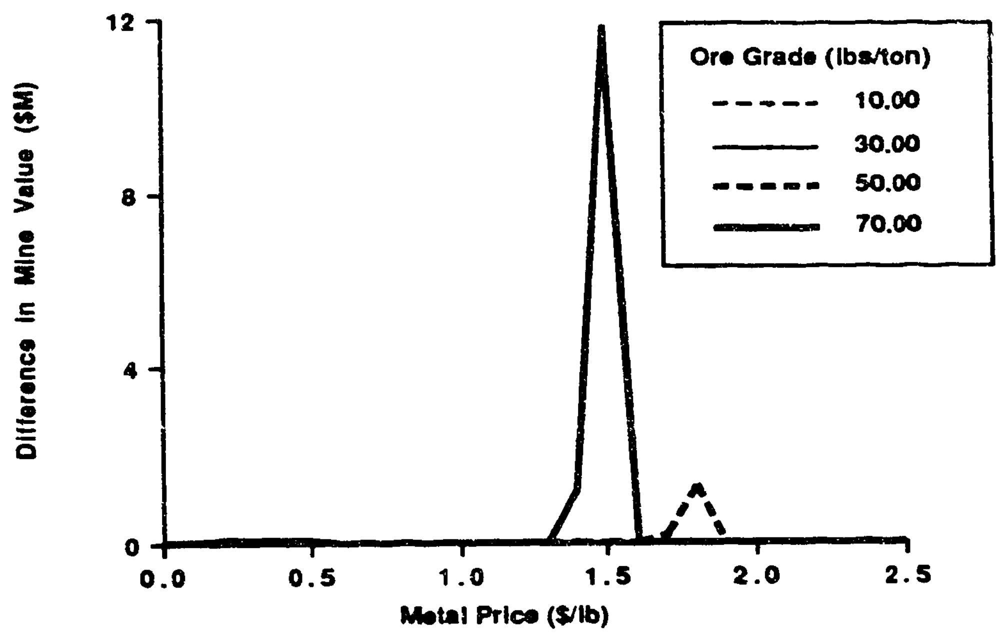

Figure 7.7 Value of the Option to Wait in a Multiple Stage Feasibility Study Program at Stage 0 (wfww)

The first case, 10.00, shows this difference for an expected ore grade of 10.00 lbs/ton of ore at various metal price. The other cases, 30.00, 50.00, and 70.00, also illustrate the same idea for respective ore grades of 30.00, 50.00, and 70.00 lbs/ton of ore. Figure 7.6 shows that without the waiting option to begin the feasibility study, the investor might abandon the project at metal prices between $ 0.00 and $ 0.40 /lb of metal for expected ore grade of 10.00 lbs/ton of ore. Beyond $ 0.40/lb of metal, for the same conditions, the investor is forced to make feasibility and investment decisions based only on current prevailing conditions, which could be detrimental to the viability and profitability of the venture. At stage 0, by abandoning the venture, or making decisions to undertake the feasibility study now or never, the venture could be undervalued by a maximum of $ 12M at a metal price of $ 1.50/lb.

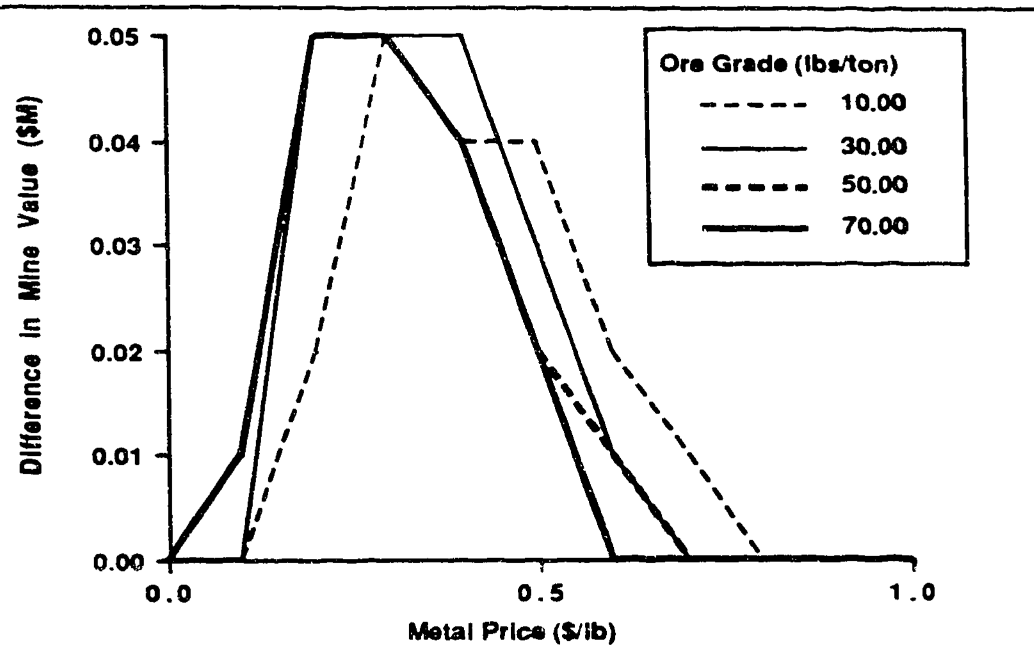

Figure 7.8 Value of the Option to Wait in a Multiple Stage Feasibility Study Program at Stage 0 (wfww)

Because the investor has to make a decision to undertake the feasibility study at the particular stage, or else, abandon the project, he misses the opportunity of being able to use future information in making these decisions at that particular stage. The value of being able to wait to begin the feasibility study at stages 2 and 3 are relatively higher than that at stage 0, partly because of the amount of the feasibility study cost at stages 2 and 3. The option to wait at these stages allows the investor to begin the feasibility study at an appropriate time to avoid the loss that might otherwise be incurred. In this problem, strategy *nfww* could lead to undervaluing the project by a maximum of $ 20M at stage 1 in Figure 7.9, $ 38M at stage 2 in Figure 7.11, $ 40M at stage 3 in Figure 7.12.

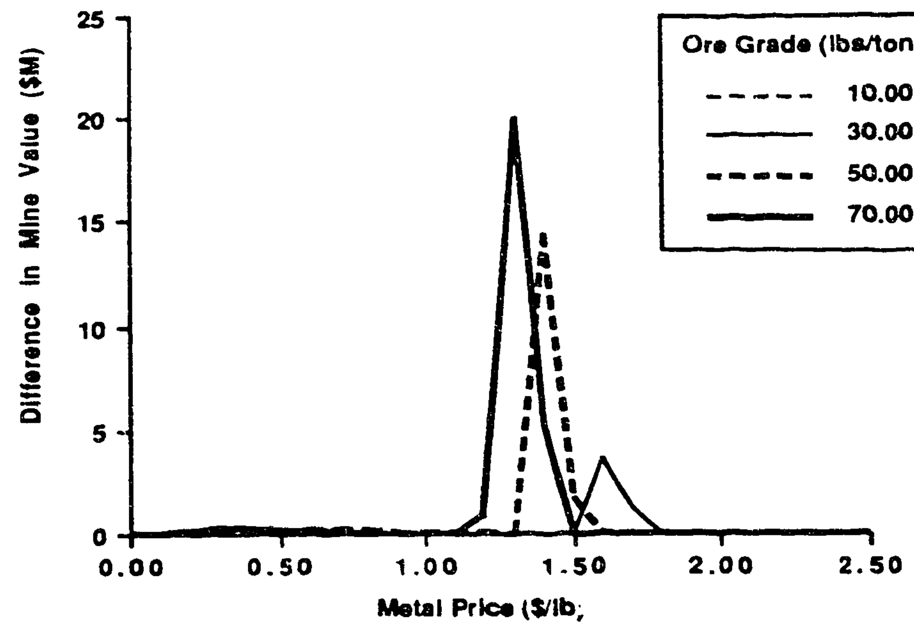

Figure 7.9 Value of the Option to Wait in a Multiple Stage Feasibility Study Program at Stage 1 (wfww)

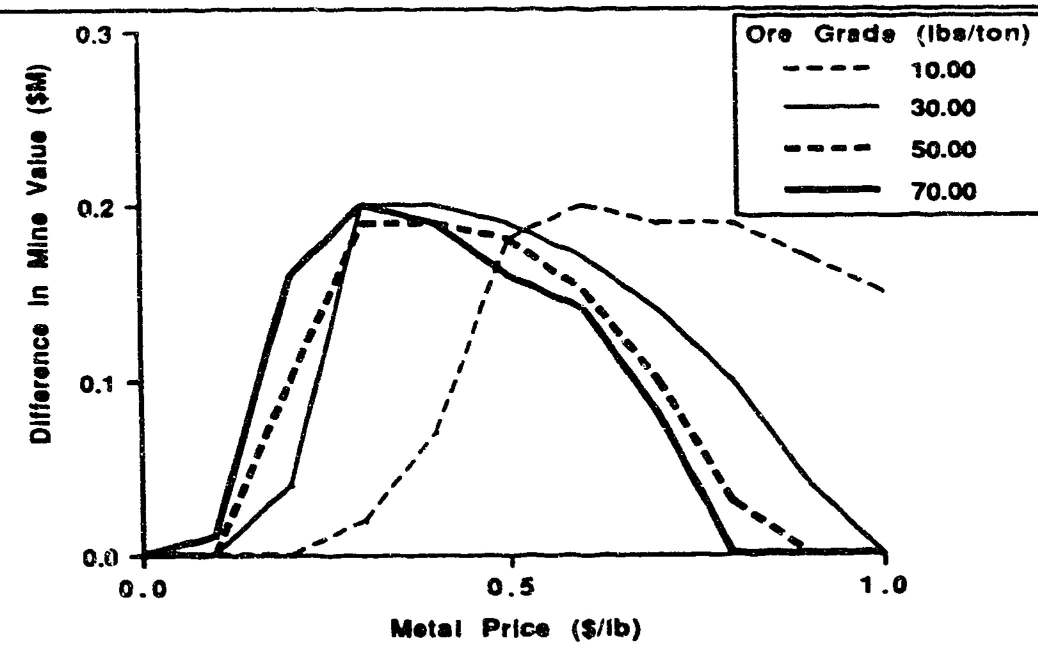

Figure 7.10 Value of the Option to Wait in a Multiple Stage Feasibility Study Program at Stage 1 (wfww)

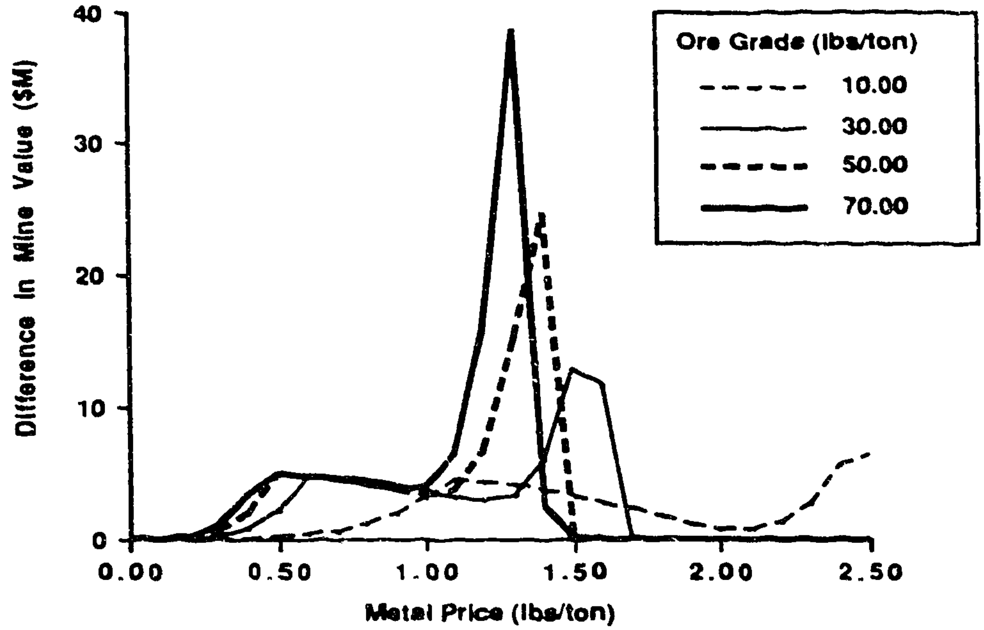

Figure 7.11 Value of the Option to Wait in a Multiple Stage Feasibility Study Program at Stage 2 (wfww)

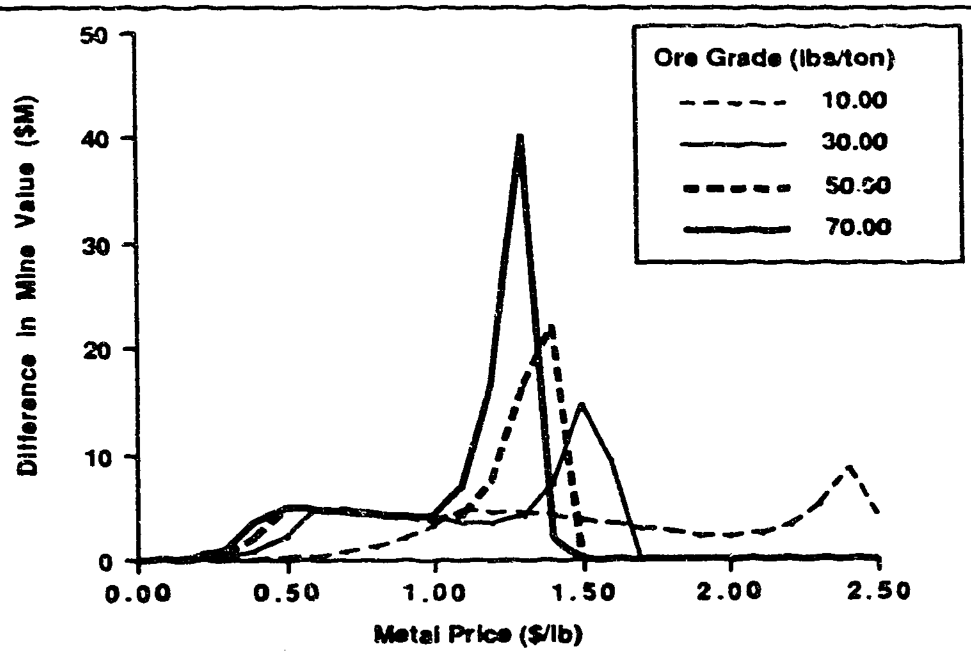

Figure 7.12 Value of the Otion to Wait in a Multiple Stage Feasibility Study Program at Stage 3 (wfww)

At the investment stage, an investor with no option to wait to invest is highly exposed, financially, and the risk of losses is greater because of the investment cost of $ 405M. Strategy *nfww* could lead to undervaluing the project by a maximum of $ 250M at the investment stage as illustrated in Figure 7.13, at a metal price of $ 1.25/lb of metal, and expected ore grade of 70 lbs/ton of ore.

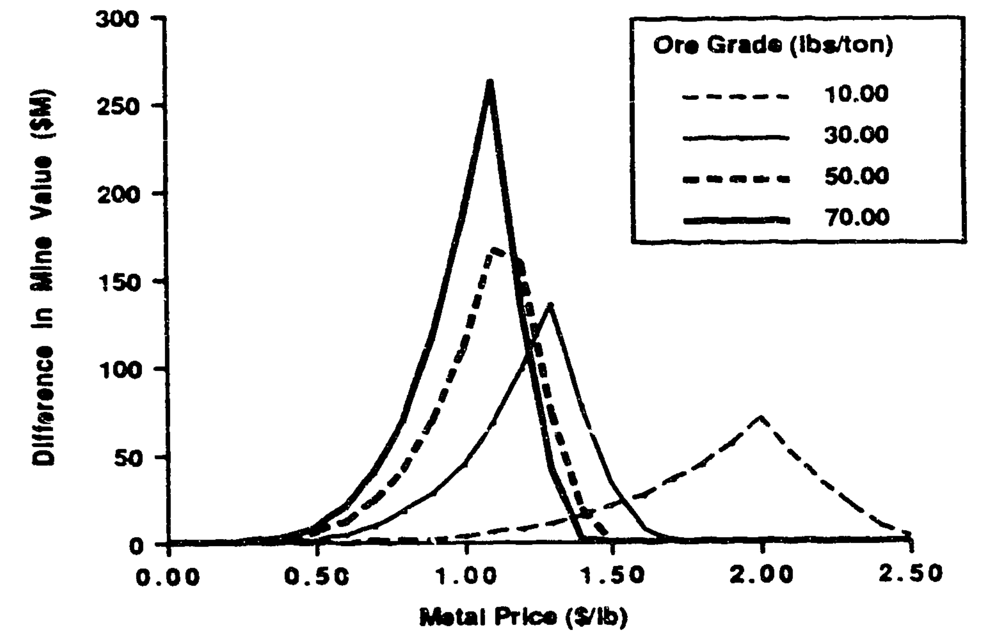

Figure 7.13 Value of the Option to Wait in a Multiple-Stage Feasibility Study Program at the Investment Stage (wfww)

7.3.1.3. Strategy wfnw: Mineral Project Phase Diagrams

Figures 7.14 to 7.18 illustrate the metal price boundaries at which an investor waits to undertake a feasibility study, or invests in the mineral venture if waiting or undertaking a feasibility study for *strategy wfnw*. The *investment, feasibility, waiting, and abandonment phases*, are the respective regions in which decisions about investment, feasibility study, waiting, and abandoning will be made at various metal prices and expected ore grades. As the investor progresses through the feasibility study stages, the feasibility phase in the figures, decreases as a result of increase in knowledge about the expected ore grade. Without the waiting options in all the feasibility study stages in *wfnw*, the metal price boundaries for various decisions in this strategy are lower than those in strategy *wfww*.

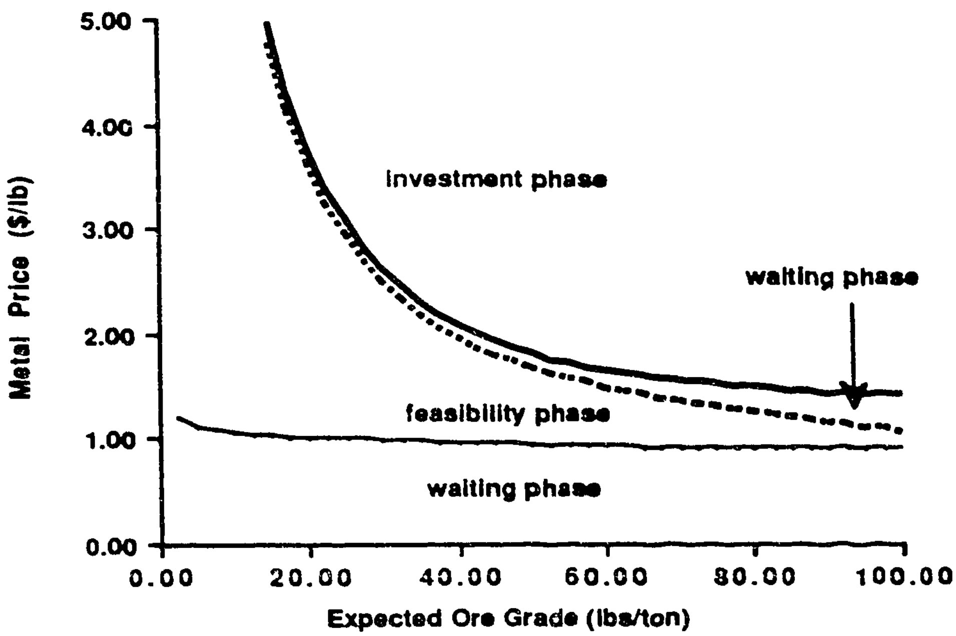

Figure 7.14 Mineral Project Phases at Stage 0 (wfnw)

Without the waiting options in the feasibility study phase, the investor must spend $ 5M to undertake the third stage feasibility study (see Figures 7.4 and 7.17). Also, note that without the option to wait in the feasibility study phase, feasibility study decisions are made at lower metal prices at stages 1, 2, and 3, compared to stage 0, to avoid forfeiture of mineral development rights.

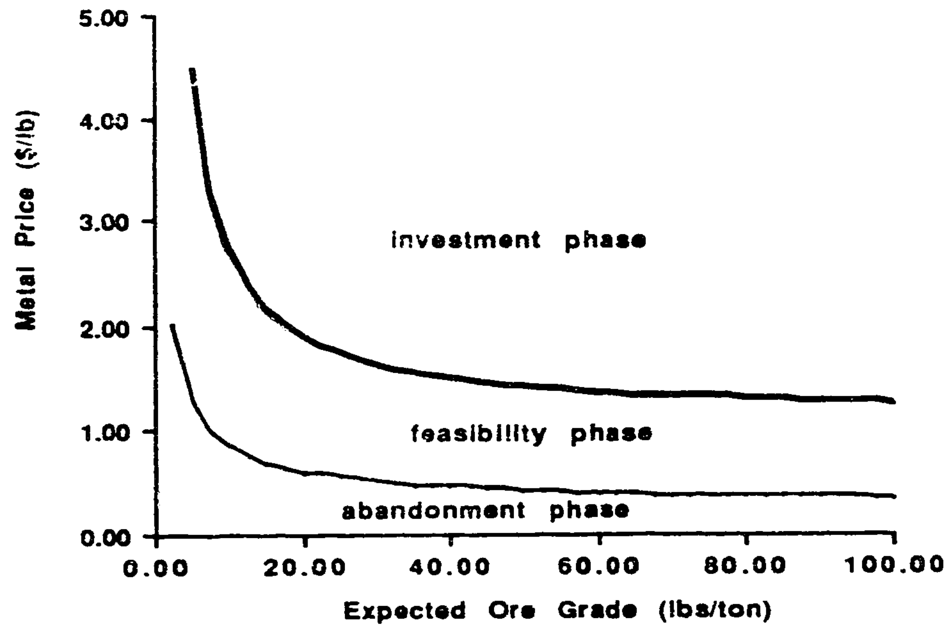

Figure 7.15 Mineral Project Phases at Stage 1 (wfnw)

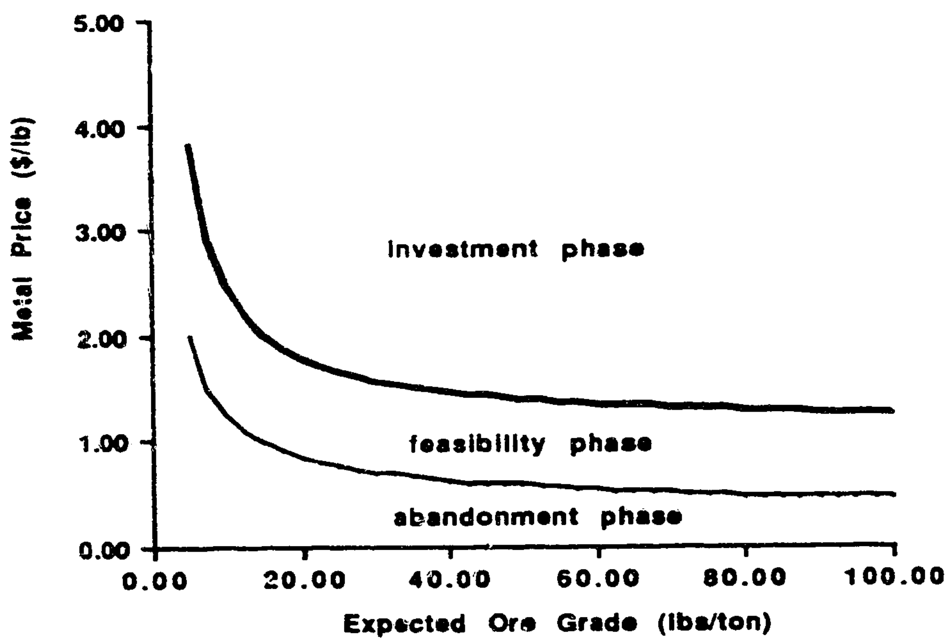

Figure 7.16 Mineral Project Phases at Stage 2 (wfnw)

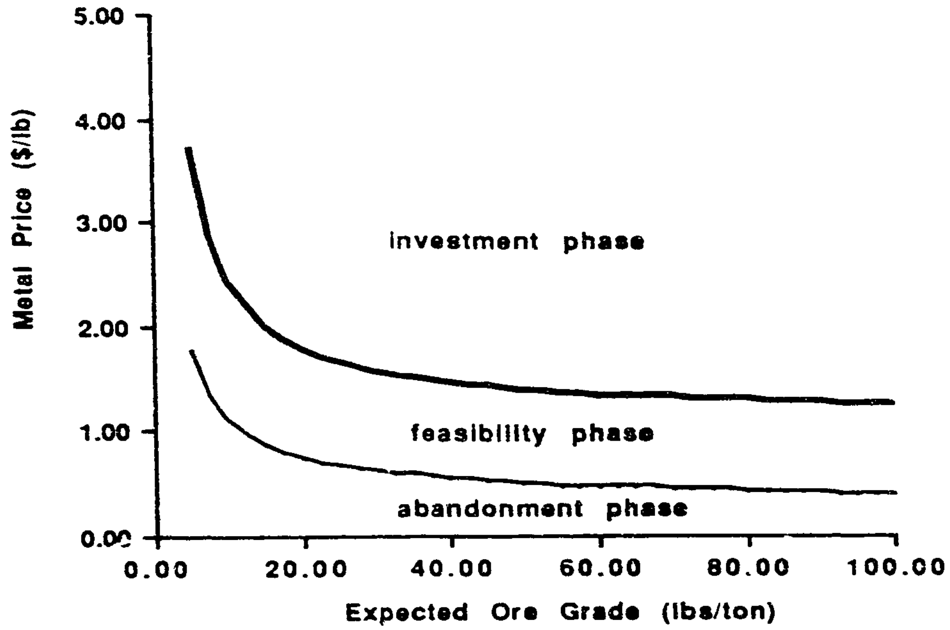

Figure 7.17 Mineral Project Phases at Stage 3 (wfnw)

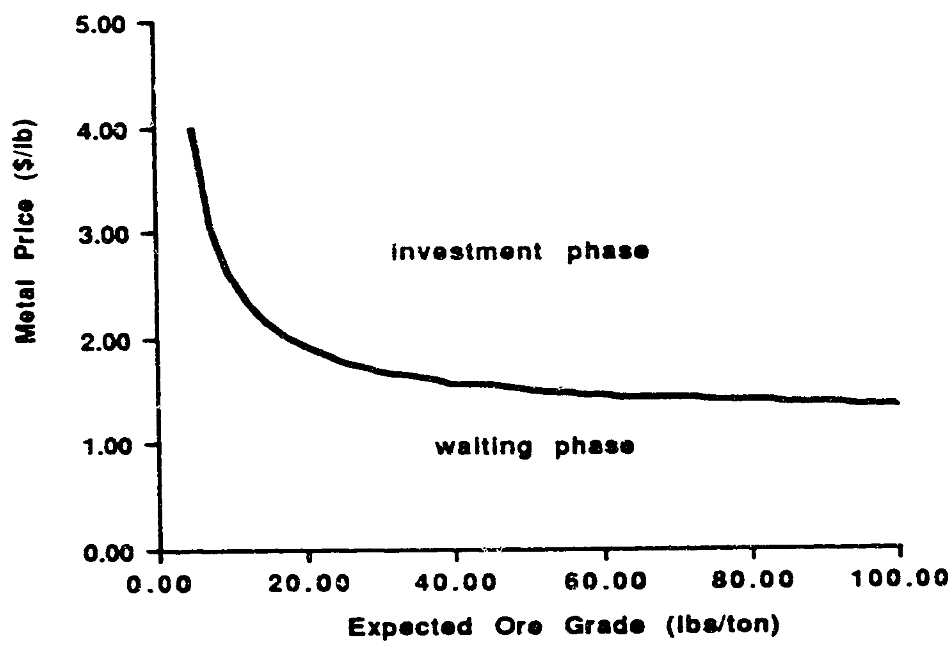

Figure 7.18 Mineral Project Phases at the Investment Stage (wfnw)

Without the option to wait to begin the feasibility study, i.e., strategy *nfnw*, Figure 7.19 shows that the metal price boundaries for the feasibility study and project development at stage 0 are all lower than those for strategy *wfnw*. An investor using strategy *nfnw* must make a decision to undertake a feasibility study now or abandon it. The opportunity to use future information is lost and this affect significantly the value of the mineral venture.

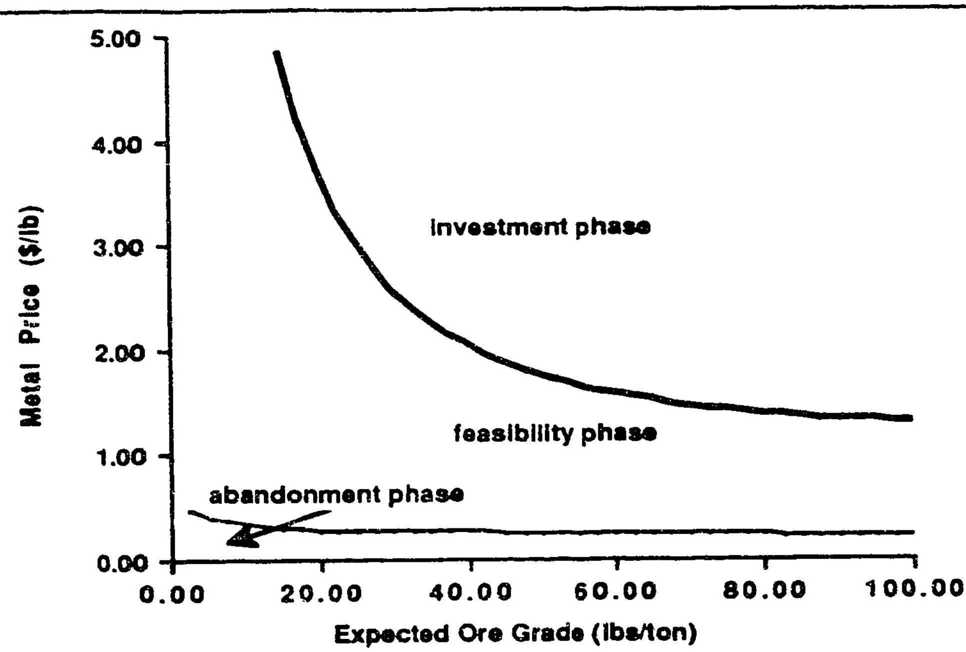

Figure 7.19 Mineral Project Phases at Stage 0 (nfnw)

7.3.1.4. Strategy wfnn: Mineral Project Phase Diagrams

The metal price boundaries for various expected ore grades at which decisions are made to wait, undertake a feasibility study, invest, or abandon a project for the *strategy wfnn* are illustrated in Figures 7.20 to 7.24. In this strategy, the only waiting option available to the investor is at the beginning of the feasibility study. If he decides to undertake the feasibility study, he must complete the required amount of study and invest immediately.

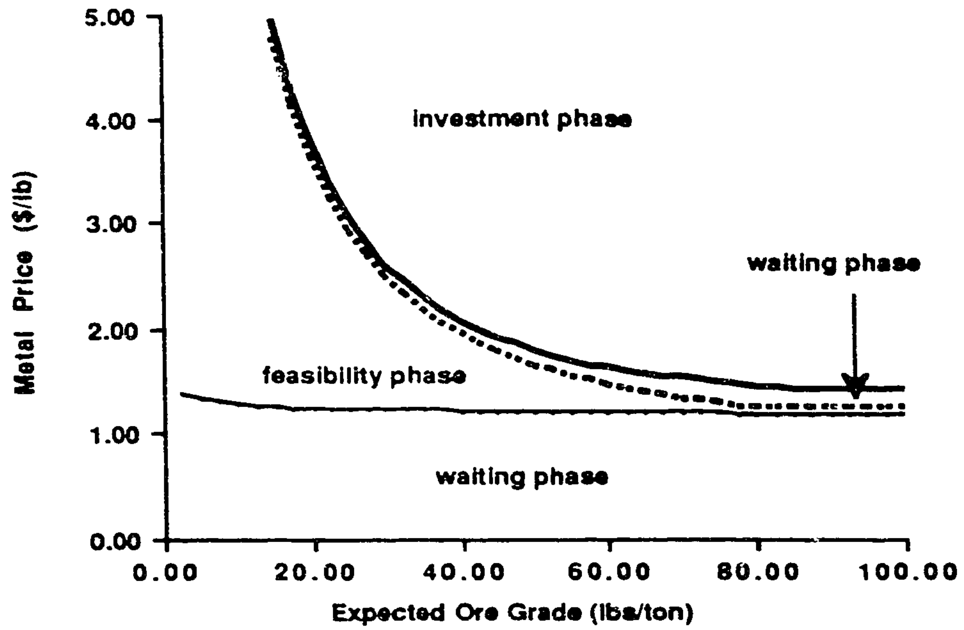

Figure 7.20 Mineral Project Phases at Stage 0 (wfnn)

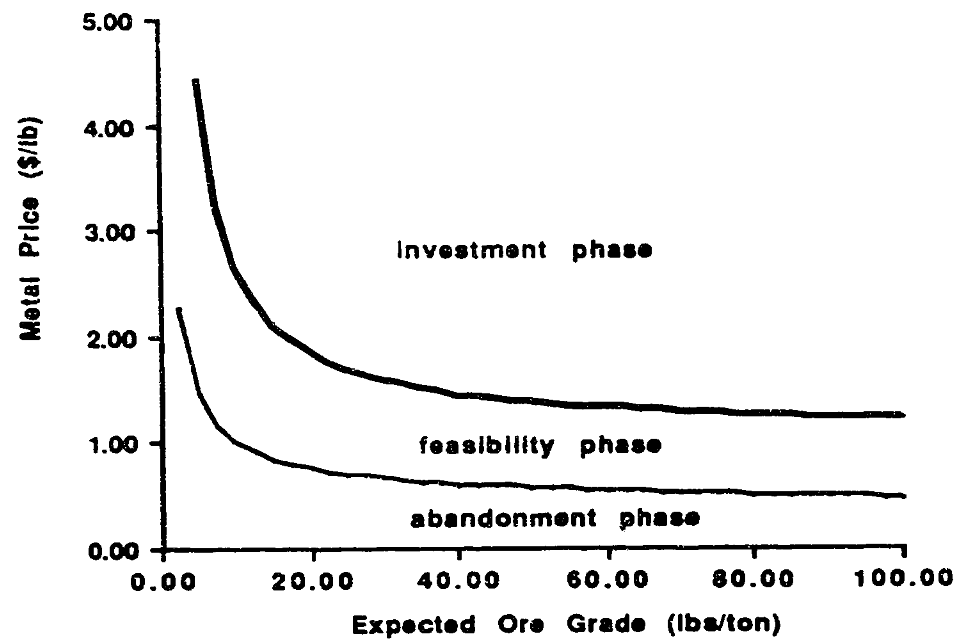

Figure 7.21 Mineral Project Phases at Stage 1 (wfnn)

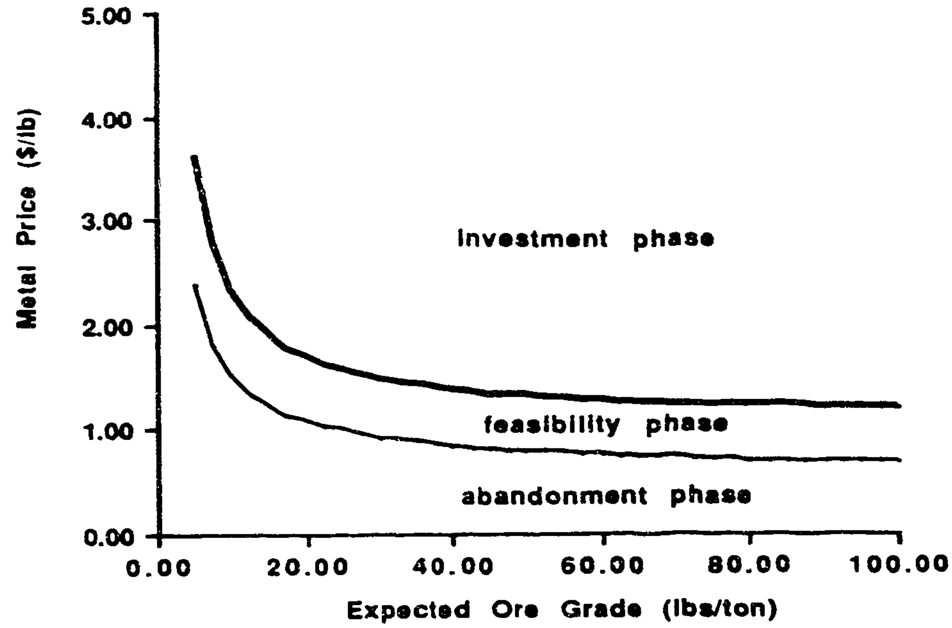

Figure 7.22 Mineral Project Phases at Stage 2 (wfnn)

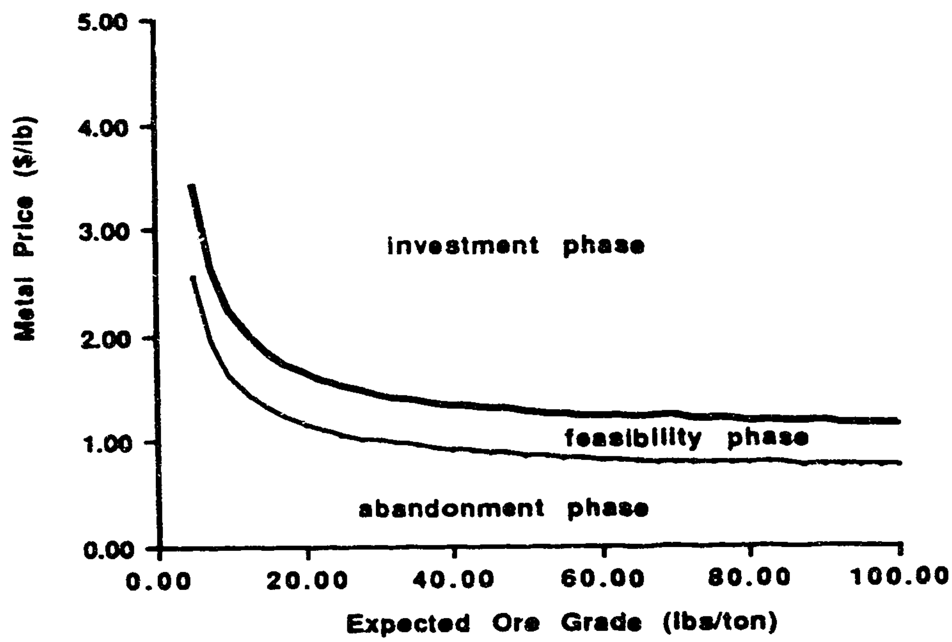

Figure 7.23 Mineral Project Phases at Stage 3 (wfnn)

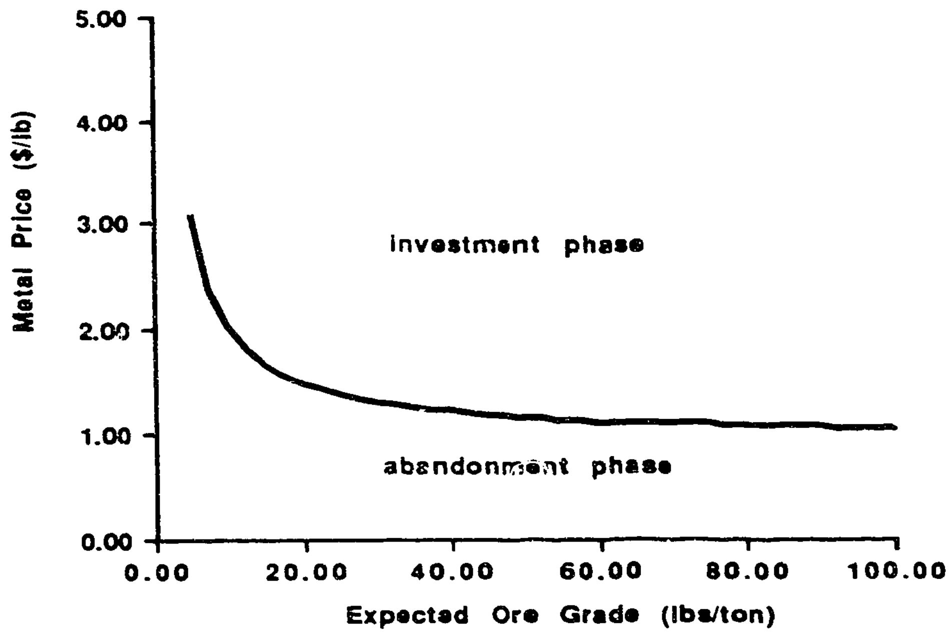

Figure 7.24 Mineral Project Phases at the Investment Stage (wfnn)

In order to make use of the only waiting option available, an investor facing this situation is prepared to begin the feasibility study at relatively higher metal prices as compared to an investor using either strategy *wfww* or *wfnw*.

At feasibility study stages 1, 2, and 3, the only alternate decisions available, apart from feasibility study, are investment and abandonment, and an investor using this strategy is prepared to make feasibility study decisions at lower metal price to avoid abandoning the project or investing prematurely. As expected, the metal price boundaries for strategy *nfnn* are lower than that of Y, as illustrated in Figure 7.25.

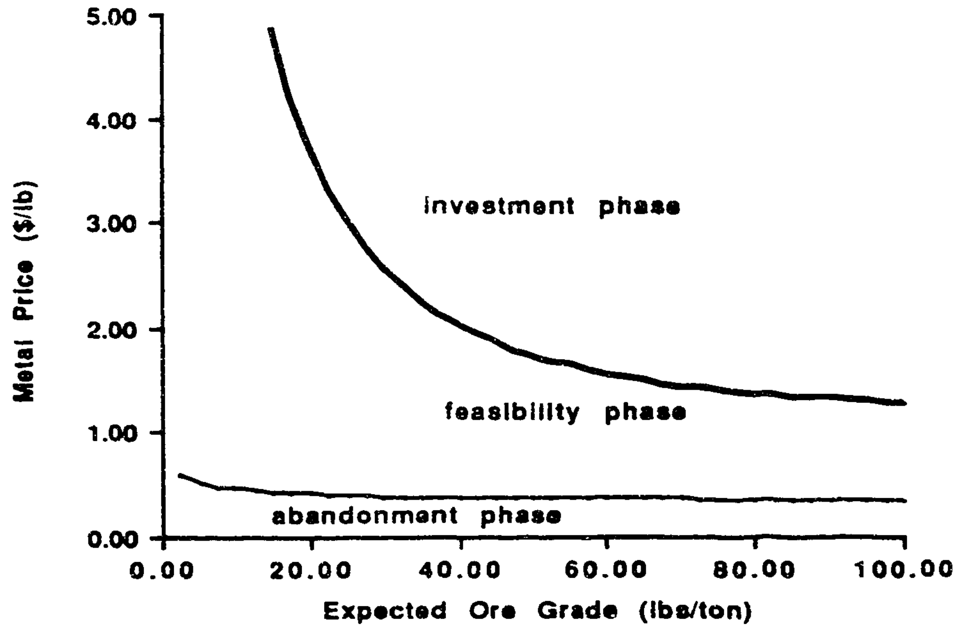

Figure 7.25 Mineral Project Phases at Stage 0 (nfnn)

7.3.1.5. Values of the Options to Wait During and After Feasibility Study

The values of the option to wait *at the end of every stage* in the feasibility study phase, for a multiple-stage feasibility study program, for various expected ore grades are illustrated in Figure 7.26. The first case, 0.50, is the value that an investor could capture when waiting is allowed in the feasibility study phase at a price of $ 0.50/lb of metal. The other cases, 1.00, 1.50, 2.00, and 2.50, are the respective values captured at metal prices of $ 1.00, $ 1.50, $ 2.00, and $ 2.50/lb of metal.

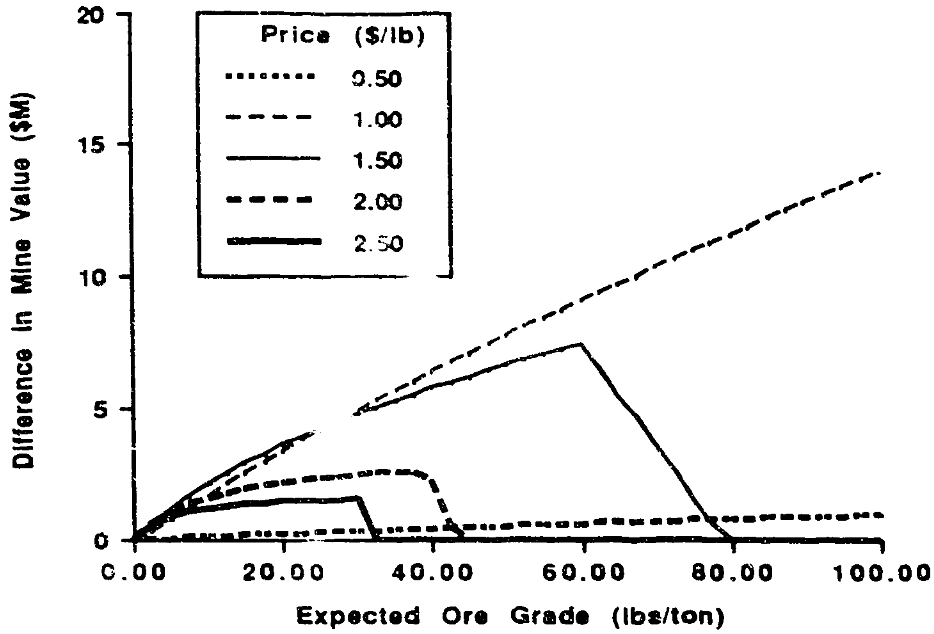

Figure 7.26 Value of all Options to Wait in the Feasibility Study Phase (wfww - winw)

The *value of this option increases* from a price of $ 0.50/lb to a *maximum* of $ 1.00/lb of metal, and then *decreases* with higher metal prices. Within this lower metal price range, the option to wait in the feasibility study phase is attractive, because it allows the investor to avoid unfavourable conditions. As metal price increases, the value of waiting becomes irrelevant, and investors would tend to speed up the feasibility study to make subsequent development decisions.

Figure 7.27 also illustrates the values of the option to wait to invest *after* the required feasibility study is completed, at various expected ore grades. The option to wait on any subsequent development after the feasibility study contributes significantly to the value of the mineral project, because it allows investors time to invest at an appropriate time, instead of hurried decisions after the feasibility study phase. At high metal prices, investors will tend to invest immediately after feasibility study, and the option is very small.

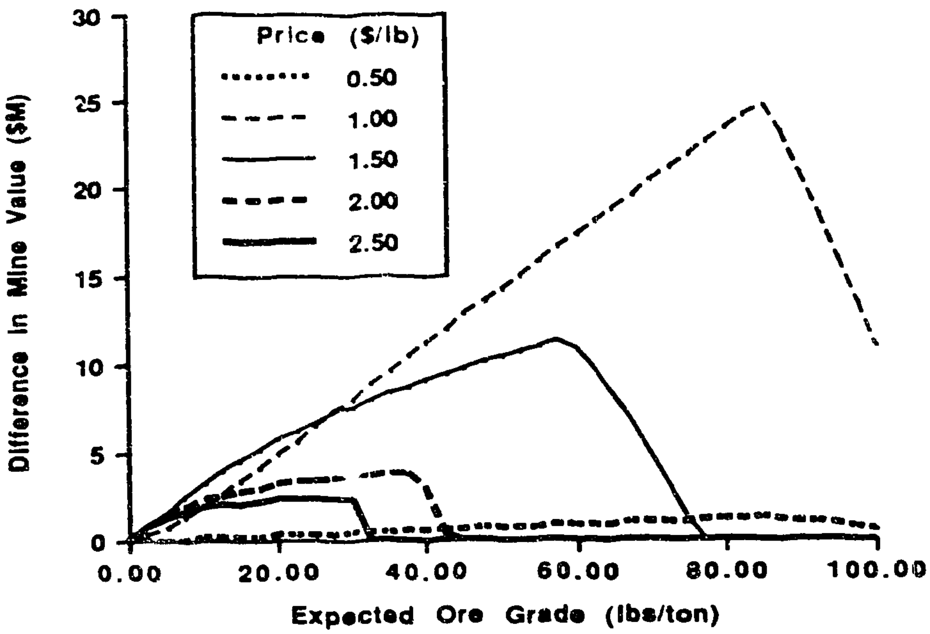

Figure 7.27 Value of the Option to Wait after the Feasibility Study to Invest (wfnw - wfnn)

7.3.1.6. Value of Multiple-Stage Feasibility Study Program

In a mineral development situation, the idea of a multiple-stage feasibility study program, in a situation of high uncertainty about the expected ore grade and metal price, could help an investor avoid unnecessary feasibility study work (time and money), and to enable him to invest at an appropriate time. Figure 7.28 illustrates the differences in values of a multiple-stage feasibility study program at various stages, for various expected ore grades, between strategy *wfww* and *1s0_wfww* at stage 0. The first case, 0.50, is the value gained at a metal price of $ 0.50/lb by considering the feasibility study in stages. The other cases, 1.00, 1.50, 2.00, and 2.50, illustrate the same idea at respective metal prices of $ 1.00, $ 1.50, $ 2.00, and $ 2.50/lb.

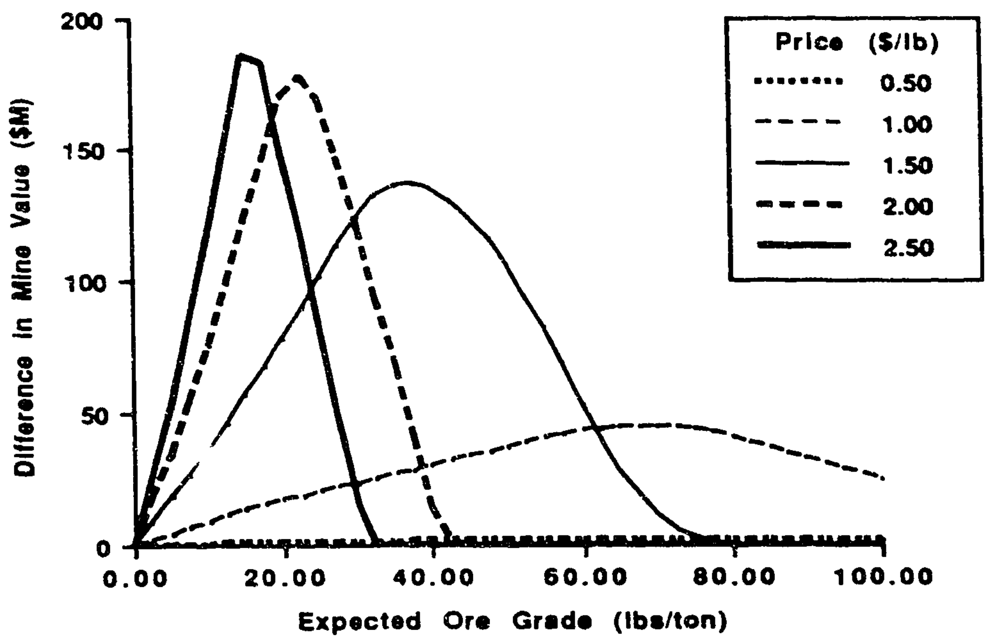

Figure 7.28 Value of a Multiple Stage Feasibility Study Program at Stage 0 (wfww - 1s0_wfww)

If the investor decides to do all the required feasibility study work at stage 0, he forgoes the opportunity of being able to use future available information to guide the program. It is better, under these conditions, to consider the feasibility study in stages, evaluate the mine after every stage, and take advantage of available information to direct the next feasibility study stage. It may be profitable to stop at some point in the feasibility study phase, and to invest, as the strategy *wfww* indicates at stage 3 (refer to Figure 7.4).

At stage 1, when the total variance associated with the expected ore grade has been reduced to 0.113 as a result of the first feasibility study stage, the situation is different as illustrated in Figure 7.29. Considering the feasibility study in stages has some value, but it is small as compared to that in stage 0.

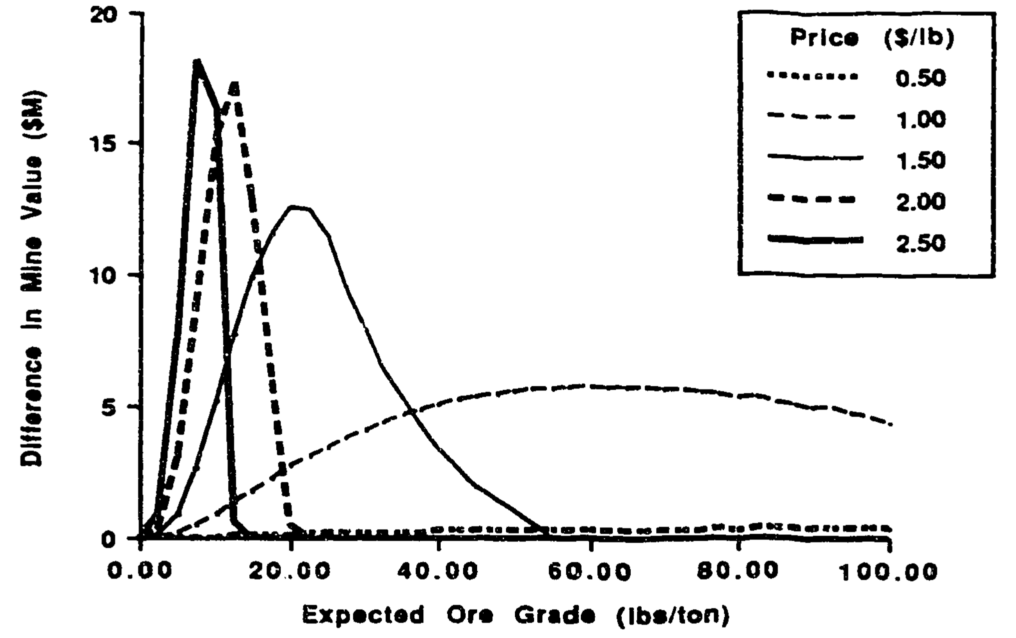

Figure 7.29 Value of a Multiple Stage Feasibility Study Program at Stage 1 (wfww - 1s1_wfww)

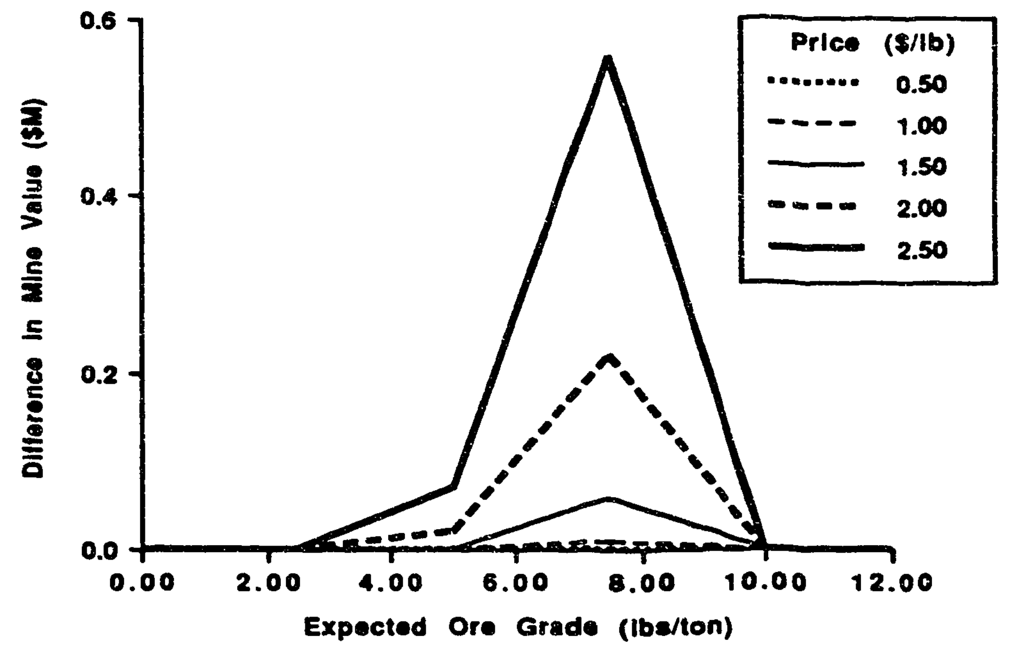

Figure 7.30 Value of a Multiple Stage Feasibility Study Program at Stage 2 (wfww - 1s2_wfww)

This value reduces sharply, as knowledge is gained about the expected ore grade, to a minimum of about $ 0.55M at a metal price of $ 2.50/lb of metal and expected ore grade of 8.00 lbs/ton at stage 2, as illustrated in Figure 7.30, and ceases to be relevant at stage 3. Thus, if information available to quantify the expected ore grade is scanty, and metal prices are within the low to medium-high range (i.e., between $ 0.30/lb and $ 2.5/lb of metal in this case), investors can use a multiple-stage feasibility study to maximize the value of a venture by avoiding unnecessary feasibility study (time and money), and by taking advantage of future available information to plan and undertake subsequent feasibility study programs, if necessary.

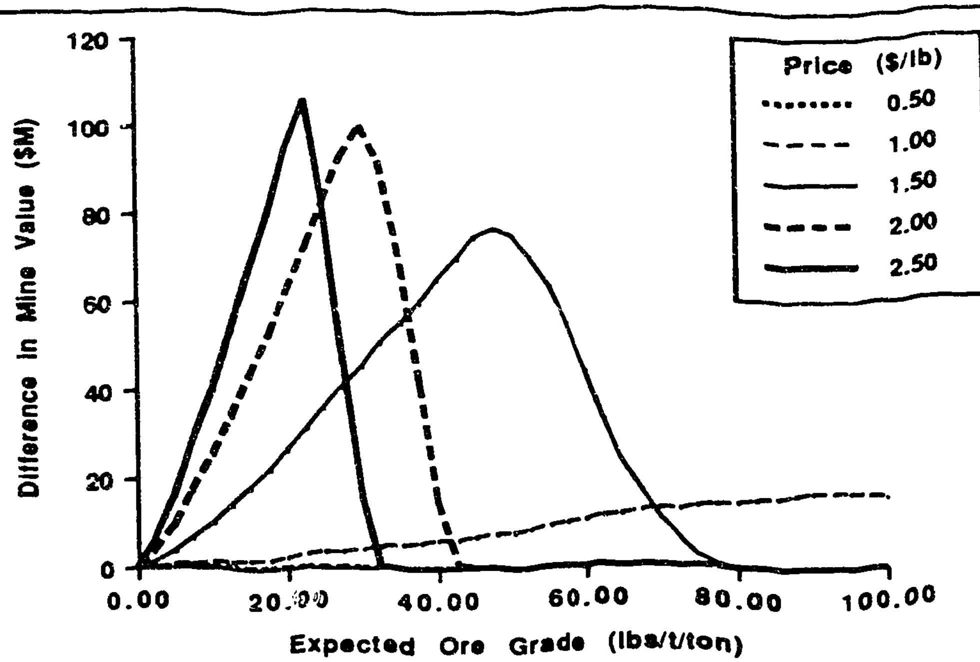

Figure 7.31 Value of a Multiple Stage Feasibility Study Program at Stage 0 (w2fww - 1s0_w2fww)

These figures also prove a significant strategy that has kept many mineral ventures alive and profitable. They show that mineral projects which may not be profitable with a single-stage feasibility study at stage 0 could become profitable using the multiple-stage feasibility study concept. At stage 3, by considering previous feasibility study costs as sunk costs, the value of a mineral venture, using a single-stage feasibility study, is equal to the value using a multiple-stage feasibility study.

Figure 7.31 also illustrates the value of a multiple-stage feasibility study program. This figure shows the difference in mine values using strategies *w2fww* and *1s0_w2fww* at stage 0. Compared to Figure 7.28, there is a reduction in the amount of value captured in the case of the strategies in Figure 7.31. At a metal price of $ 2.50/lb, the maximum amount captured are $ 105M and $ 185M, respectively, in figures 7.28 and 7.31. The main difference is that the required feasibility study in the single-stage strategy in the latter, *1s0_w2fww*, is carried in two years, at the cost of the discounted sum of the costs at stages 0 and 1, while that in the former is carried out in four years at the cost of the total discounted feasibility costs.

7.3.1.7. Value of Optimum Feasibility Study

In certain cases, it is important to note that specific technical, and operational problems may require that a great deal of feasibility study be carried out to avoid foreseeable problems. Nevertheless, investors can overdo feasibility study due to lack of effective planning, control and evaluation of feasibility study programs. An experiment was designed to examine this problem based on the data used in this study. The results are illustrated in Figure 7.32.

This figure shows the value of the mineral venture lost by overdoing the feasibility study at stage 0 using strategy *wfww*. The first case, 0.50, illustrates the value of the mineral venture lost by overdoing the feasibility study at a metal price of $ 0.50/lb of metal for various expected ore grades. The other cases, 1.00, 1.50, 2.00, and 2.50, illustrate the same idea at respective metal prices of $ 1.00, $ 1.50, $ 2.00/lb. By exceeding the limit of feasibility study, capital is wasted, and the optimum investment timing could be missed, resulting in huge losses, especially at high metal prices and high expected ore grades.

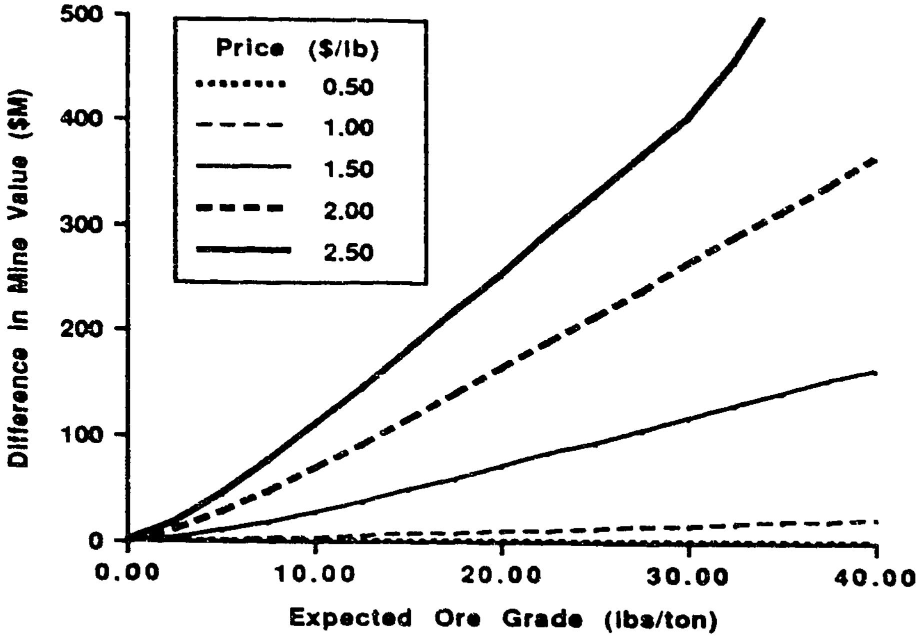

Figure 7.32 Value of Early Termination of the Feasibility Study Program at Stage C (wfww - full_wfww)

7.3.1.8. Determinants of the Feasibility Study Value

Feasibility studies are used to reduce the amount of uncertainty about the expected ore grade and reserves, and to help investors to avoid over- and under-investment cost and, hence, huge cost overruns. An experiment was designed to examine the magnitude of these two driving forces in the feasibility study considered in this study. Figures 7.33 to 7.37 illustrate the results of this experiment.

Figure 7.33 shows the results at feasibility study stage 0 for a metal price of $ 0.50/lb of metal and for various expected ore grades. The first case, *Total Reduction*, is the difference in the mine value using strategy *wfww* versus *nofeas*. It further illustrates the penalty that an investor could pay for not paying attention to the *uncertainty and the costs of ignorance*. The second case, *Uncertainty Reduction*, is the difference in mine value for strategies *ic_novar* versus *nofeas*, and it also shows the penalty an investor could pay for not being able to reduce the *uncertainty* associated with the expected ore grade. The third case, *Cost Reduction*, also illustrates the same idea for strategies *wfww* and *ic_novar*, and again illustrates the penalty an investor could pay for not being able to eliminate the *cost of ignorance* in the development cost.

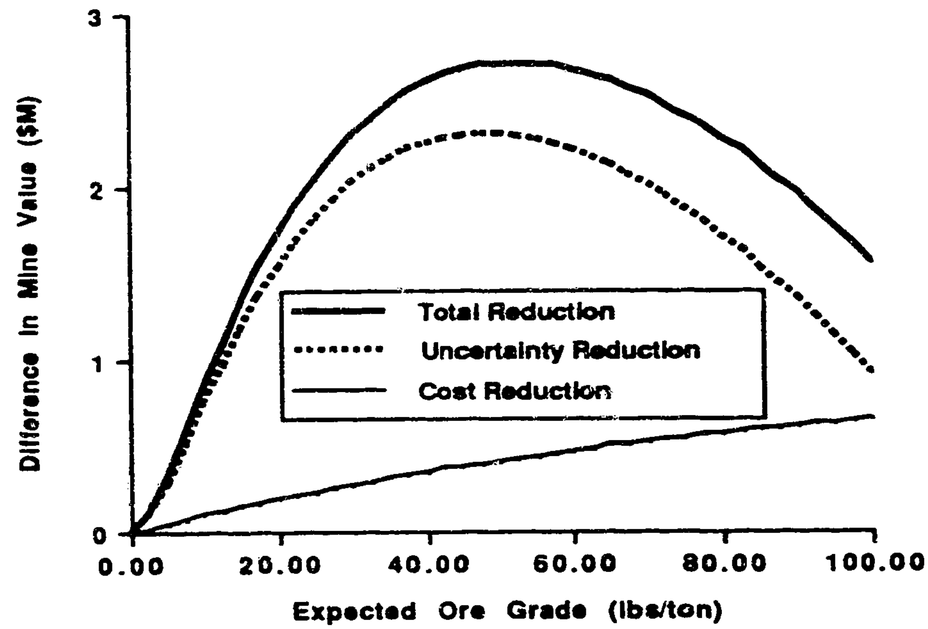

Figure 7.33 Determinants of Feasibility Study Value at Stage 0 and at a Metal Price of $ 0.50/lb

At this low metal price of $ 0.50/lb, it can be seen that the main determinant of the feasibility study value is the uncertainty associated with the expected ore grade. The effect of this uncertainty in the feasibility study value increases sharply to a maximum at about 50.00 lbs/ton of metal, and starts decreasing with increasing expected ore grade. This shows that feasibility study is relevant, at this stage, to reduce uncertainty associated with expected ore grade before considering any mineral project development, especially in the range between 10.00 and 50.00 lbs/ton of metal. The part of the feasibility study value due to the cost of ignorance is relatively small, but it is significant. This part grows steadily with increasing expected ore grade. As the expected ore grade increases, investors are drawn mainly to feasibility study to avoid the cost of ignorance.

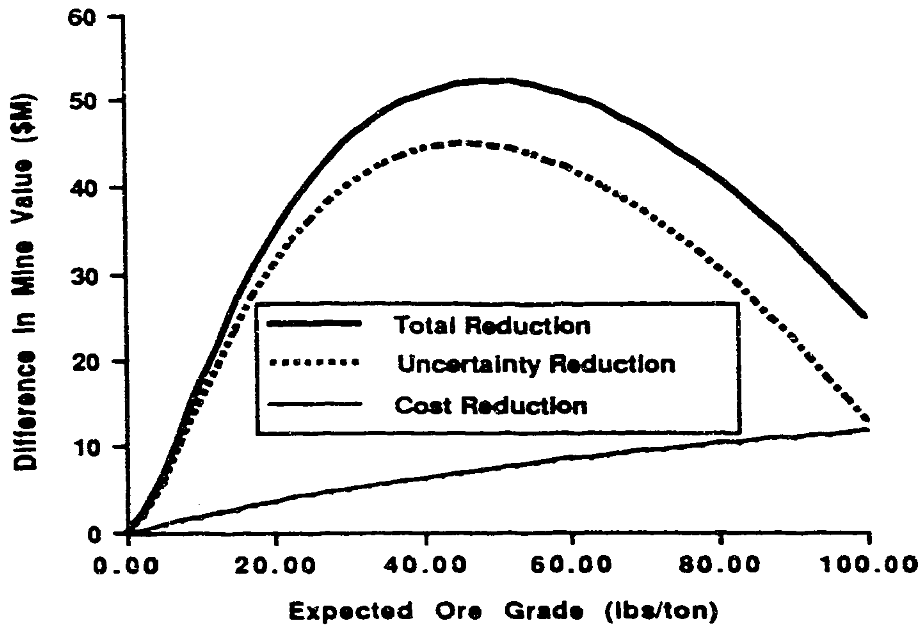

Figure 7.34 Determinants of Feasibility Study Value at Stage 0 and at a Metal Price of $ 1.00/lb

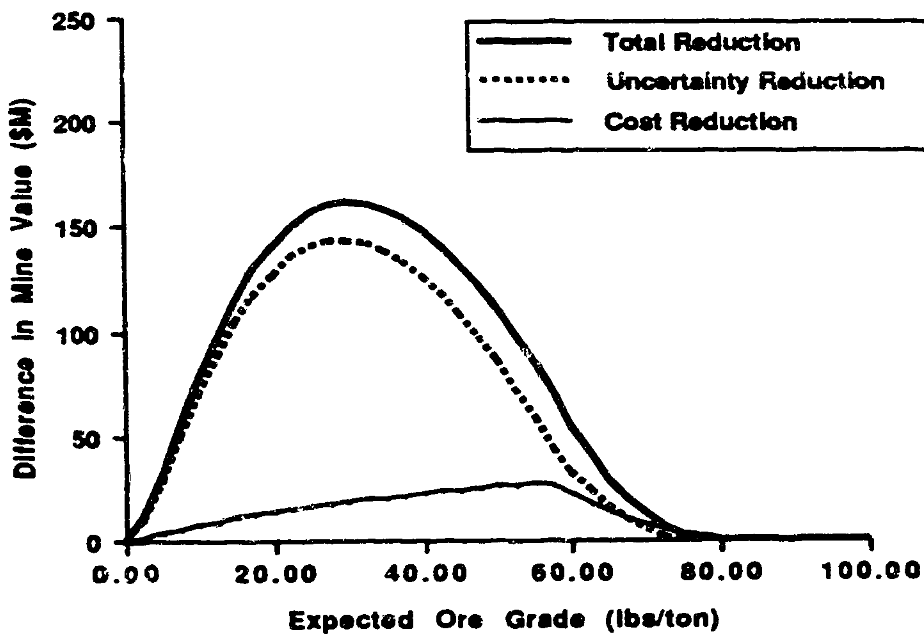

Figure 7.35 Determinants of Feasibility Study Value at Stage 0 and at a Metal Price of $ 1.50/lb

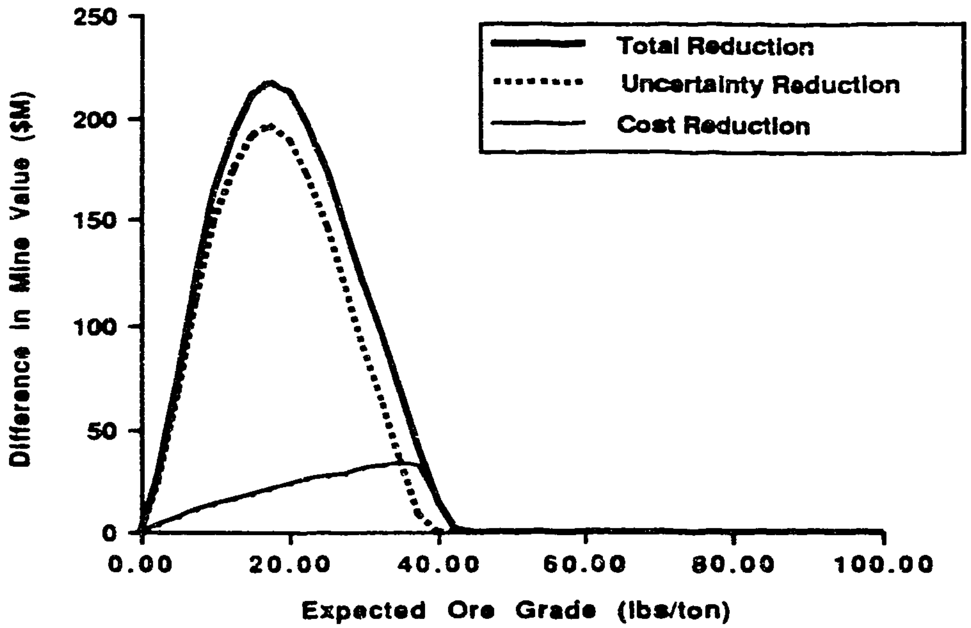

Figure 7.36 Determinants of Feasibility Study Value at Stage 0 and at a Metal Price of $ 2.00/lb

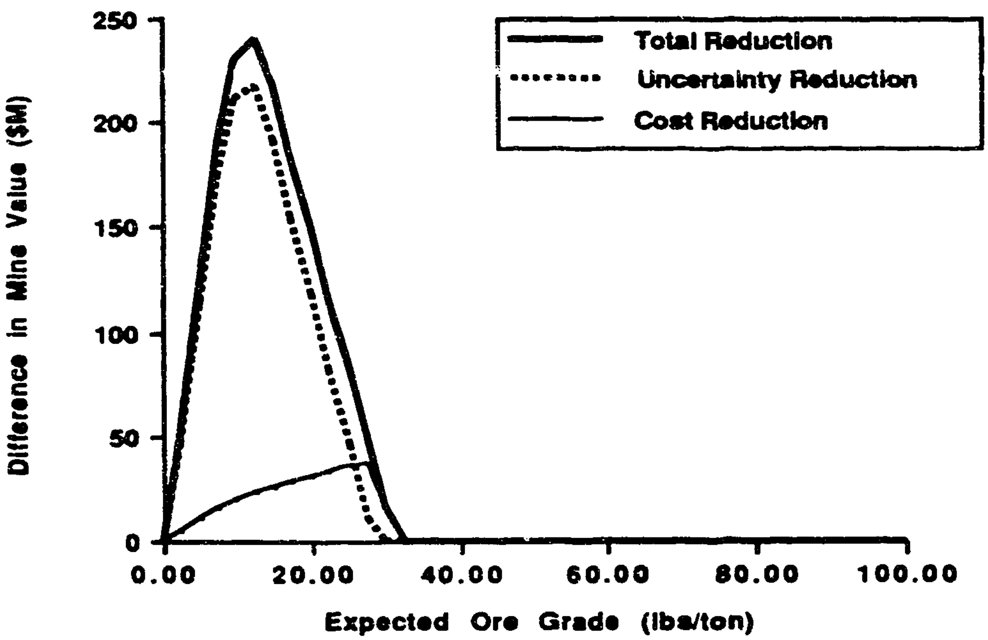

Figure 7.37 Determinants of Feasibility Study Value at Stage 0 and at a Metal Price of $ 2.50/lb

As metal price increases, the critical expected ore grade range in which feasibility study is relevant decreases, but its value in this range increases greatly, as illustrated in Figures 7.34 to 7.37. Thus, feasibility study is required to reduce uncertainty associated with expected ore grade, and to avoid under- or over-investment for low to medium-high expected ore grades, at all metal prices, in order to maximize the value of a mineral venture.

7.3.1.9. Feasibility Study Durations

Another problem that faces investors is the speed at which feasibility study should be carried out to be able to invest at an appropriate time. In carrying out an experiment to examine this problem, changes were made to the feasibility study duration at every stage. The case in Table 7.2 (used in earlier analysis) illustrates a scenario in which the duration of feasibility study at every stage is one year. This duration is changed to two years.

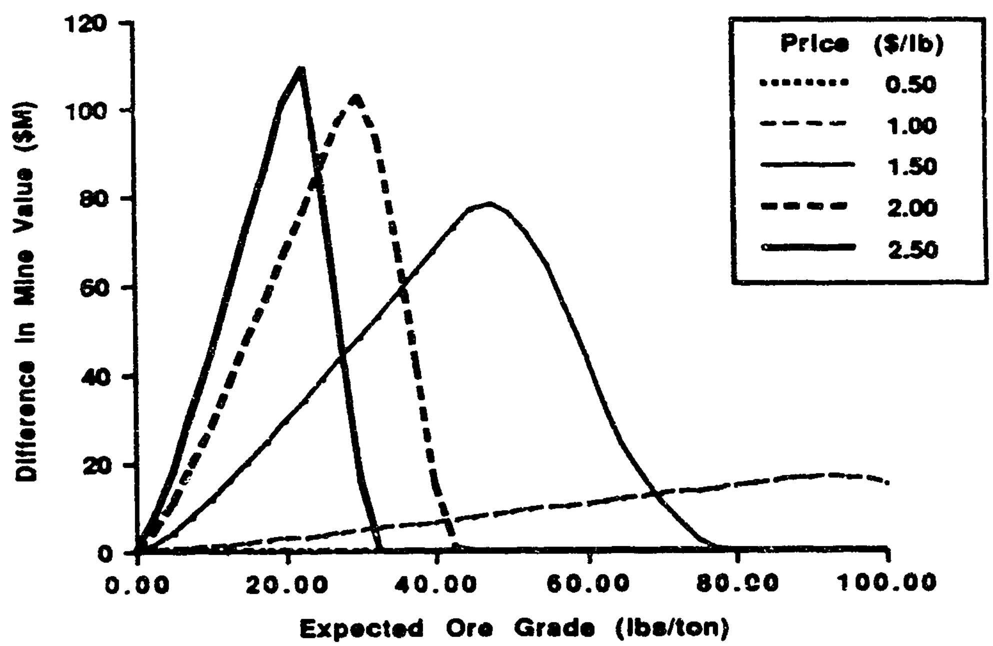

Figure 7.38 Differences in Mine Values using Fast versus Slow Feasibility Study Strategies at Stage 0 (wfww)

Figure 7.38 to 7.40 illustrate the values which can be added by accelerating the feasibility study program at various feasibility study stages. The first case, 0.50, shows very little differences in mine value for using either a fast or slow feasibility study at a metal price of $ 0.50/lb. For a lower metal price, feasibility study program will not be undertaken. The other cases, 1.00, 1.50, 2.00, and 2.50, illustrate differences in mine values, respectively, at metal prices of $ 1.00, $ 1.50, $ 2.00, and $ 2.50/lb of metal. For higher metal prices, and *higher* expected ore grades, it makes no difference whether an investor uses fast or slow feasibility study. This may be due to the fact that these conditions may require a minimal or no feasibility study. Below 70 lbs/ton of ore, and at metal prices above $ 1.00/lb, there is much that can be added with a fast feasibility study program. It may be desirable for investors to hurry up with the program to take advantage of prevailing conditions.

The value of accelerating the study decreases with increasing feasibility study stages. At stage 1, this value is moderately high for high expected ore grades at $ 1.00/lb of metal as illustrated in Figure 7.39.

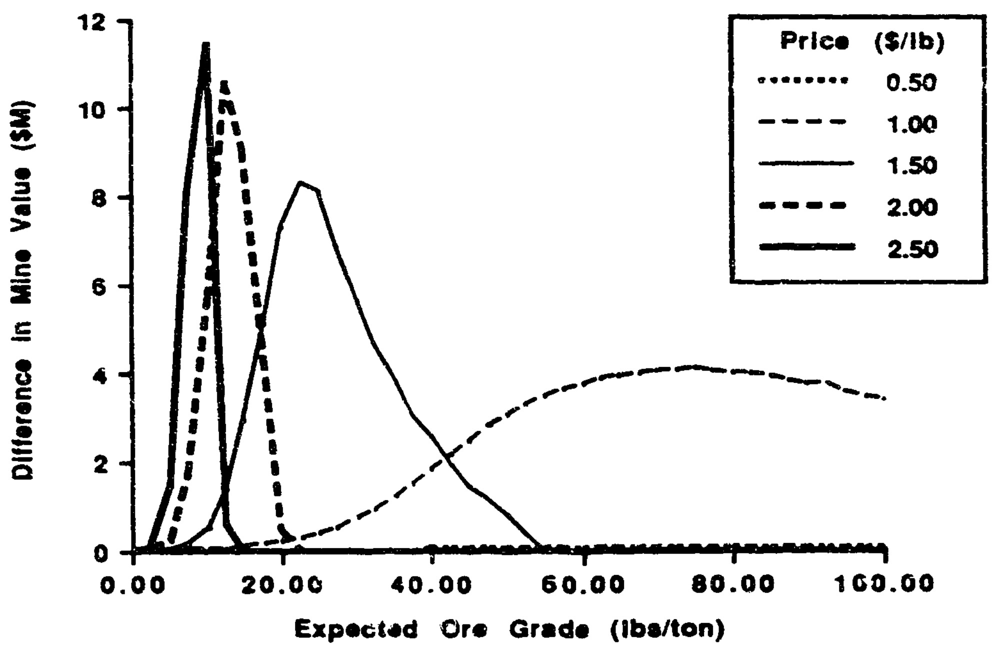

Figure 7.39 Differences in Mine Values using Fast vs Slow Feasibility Study Strategies at Stage 1 (wfww)

This expected ore grades for which a fast program may add value to the project decreases as the metal prices increase above $ 1.00/lb. The amount of uncertainty associated with the expected ore grade and the higher metal price may warrant a minimal or no feasibility study. At stage 2, the value added is very little because only a small amount of feasibility study is required as illustrated in Figures 7.3 and 7.40.

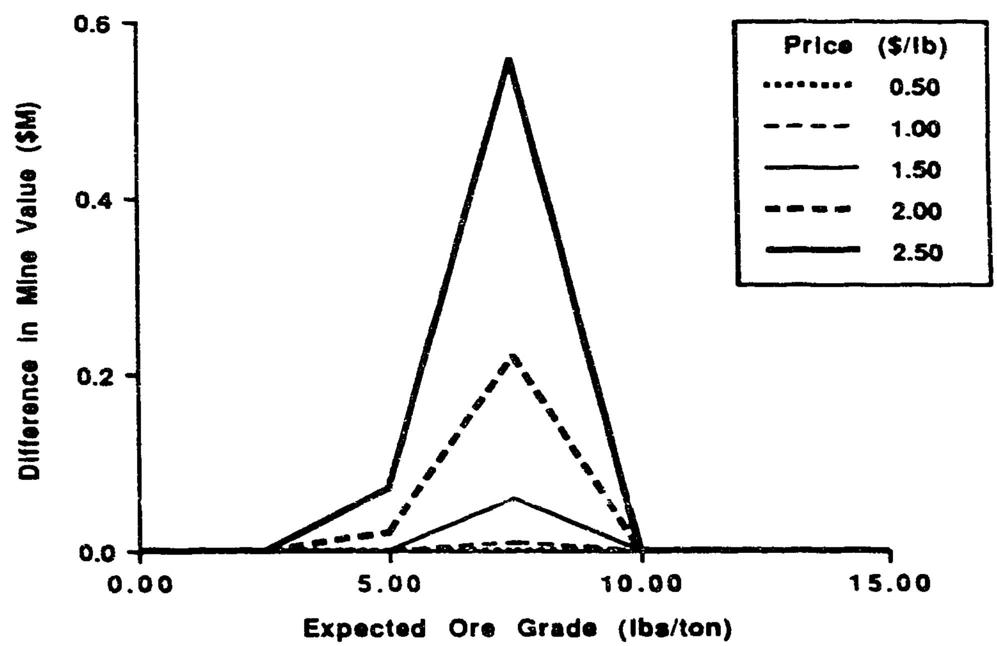

Figure 7.40 Differences in Mine Values using Fast vs Slow Feasibility Study Strategies at Stage 2 (wfww)

7.3.2. Effects of Different Economic Parameters on Projec Value

In the volatile economic environment of most mineral ventures, feasibility study and development should respond adequately to changes that affect mineral project value. The extent of variation in metal prices, *volatility*, must be dealt with in the best quantitative way as possible. Future uncertainties in metal prices is a significant concern in most mineral ventures. An experiment was designed to test the significance of volatility in metal prices in the value of the mineral venture.

Figure 7.41 shows the differences in mine values with proportional standard deviation in metal price of 0.1 and 0.2, respectively, at stage 0. The first case, 0.50, is the difference in mine values for using 0.1 versus 0.2 proportional standard deviations in the metal price at a metal price of $ 0.50/lb of metal and for various expected ore grades. The other cases, 1.00, 1.50, 2.00, and 2.50, also illustrate the same idea, respectively, for metal prices of 1.00, 1.50, 2.00, and $ 2.50/lb of metal.

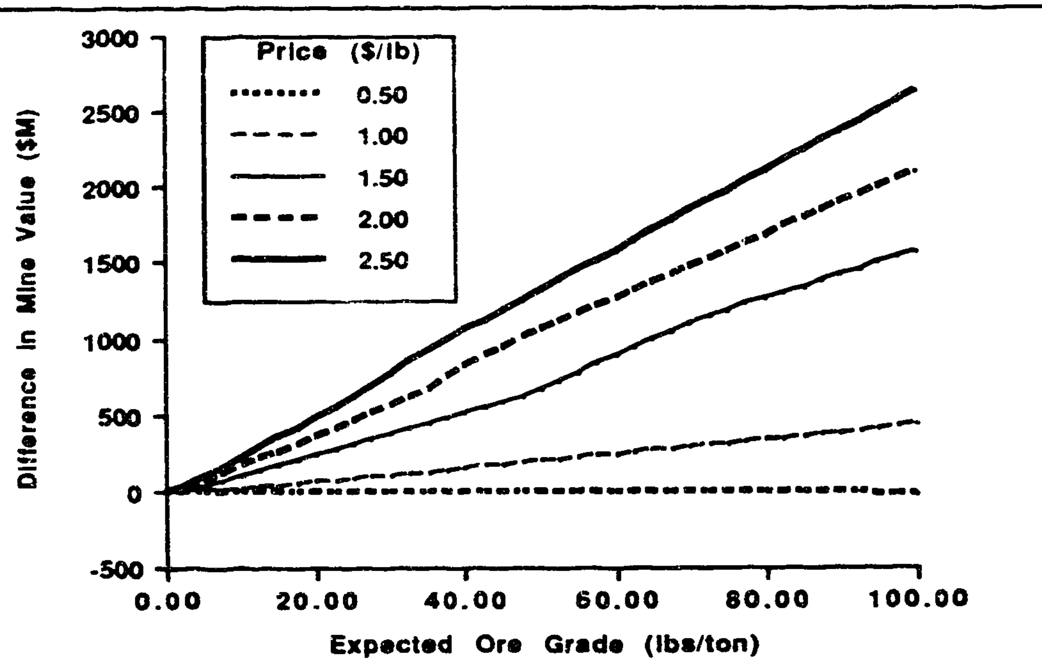

Figure 7.41 Effects of Metal Price Uncertainty on Mine Value at Stage 0 (wfww)

This figure shows that, as uncertainty in metal price decreases, the mineral project value increases. The increase in value accounts for the fact that information is less diffused about the price of metal in the case of a proportional standard deviation of 0.1 than in the case of 0.2. They also illustrate that the effects of metal price uncertainty persist throughout the stage 1 of the study as illustrated in Figure 7.42.

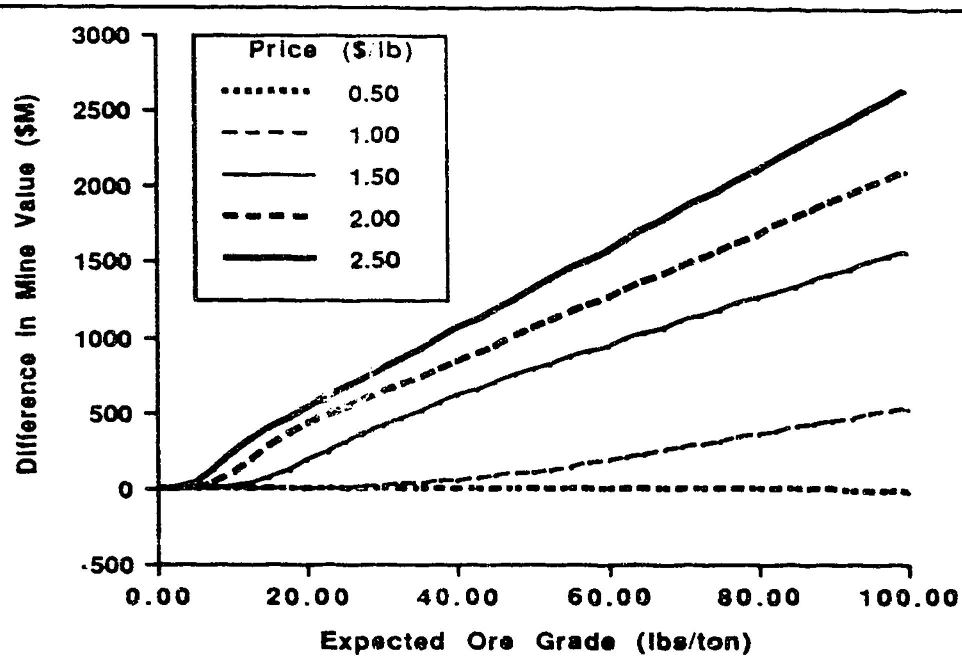

Figure 7.42 Effects of Metal Price Uncertainty on Mine Value at Stage 1 (wfww)

With high uncertainties in metal prices in today's markets, investors in mineral projects will be misled by the results of evaluation tools that do not treat the stochasticity of metal prices rigorously. The DAV method is set up to rigorously deal with metal price uncertainties in mineral project evaluation, and presents one of the most viable economic evaluation methods today.

7.4. Conclusions

All the analyses show how application of the derivative asset valuation method can help to quantify and compare the results of various options which investors must choose in venture feasibility studies and subsequent development decisions. These analyses provide investors with phase diagrams which delineate metal price boundaries for various expected ore grades at which waiting or abandoning, feasibility study, and investment decisions are appropriate. Such diagrams could be used as guides in making many venture-related decisions.

The depth of feasibility study is important, in a situation of high expected ore grade uncertainties and low to medium-high metal prices, before making any subsequent development decisions. Feasibility study is required to reduce this amount of uncertainty, and to help investors avoid either under- or over-investment cost which could lead to high cost overruns.

Implementing the feasibility study in multiple stages, instead of a single-stage study in the beginning, could help investors to maximize the value of the mineral project. This strategy would allow investors to evaluate the project after every stage, and to take advantage of available information to plan and execute the next stage of the program. With this strategy, overdoing the study would be avoided, and investors would be able to make development decisions at the appropriate time. Even if extensive feasibility study must be completed in order to solve foreseeable problems prior to development, it is still better to stage the feasibility studies.

With the DAV method, investors can examine the potential value of a mineral venture with a fast or slow feasibility study program, and to use the appropriate one to maximize the venture's value.

If there are no time constraints on investors' development rights, timing options can be used to maximize the value of mineral projects. The study shows that there is value for being able to wait to undertake a feasibility study and any subsequent development decisions. With these options, investors are able to take advantage of future information to guide the program.

Finally, the analyses show that certain changes in the economy have significant effect on mineral project value, and can be quantified and taken into consideration in project evaluation. With high uncertainties in metal prices in today's markets, investors in mineral projects will be misled by the results of evaluation tools that do not treat this variable rigorously.

The DAV method is set up to rigorously deal with the changes in the economy that affect mineral project value, and presents one of the most viable economic evaluation tools today.

CHAPTER 8.0

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH WORK

This thesis provides an analytical review of the state-of-the-art methods used in the mineral industry to evaluate and assess the economic viability of mineral ventures. Additionally, a review is included of the advances in finance theory that form the basis of the models in this study.

A one-dimensional mathematical model (in metal price) of the value of a typical mineral venture is formulated based on the derivative asset valuation (DAV) method in order to examine the separate values of feasibility study, waiting, and operating options. Results of the DAV method are compared with those from the discounted cash flow (DCF) method.

A two-dimensional mathematical model (in metal price and expected ore grade) of a typical mineral venture, based on the DAV method is included, to quantify the use of various feasibility study strategies, waiting options, and changes in the state of the economy in a mineral venture analysis.

In particular, the following contributions are made to mineral project or venture analysis:

1. *This is the first implementation of the DAV method, in one dimension, to examine the effects of timing of feasibility study and investment on the value of a mineral venture*

2. *This is the first implementation of the DAV method, in two dimensions, to examine the effects of various feasibility study strategies and waiting options on the value of a mineral venture*

This study achieved the objectives set out in Chapter 1.0 to examine an array of decisions about the management of (a) feasibility studies, (b) investment timing options, and (c) mine operating options, and to quantify the effects of a choice of any of these decisions on the value of a mineral project or venture.

From analyses of the *one-dimensional DAV model* of the mineral venture's value, the following conclusions are drawn:

1. The option of being able to use a feasibility study to determine the characteristics of a mineral venture before making development decisions has a significant definable value, using both DCF and DAV methods. A feasibility study enables the investor to clearly understand the financial risk of investing at low metal prices for a low metal reserve. It also allows an investor to avoid the extra investment costs that would be incurred under uncertainty for smaller reserves which are profitable at high metal prices.

2. Carefully selected timing of the feasibility study, and subsequent development decisions, also increase the value of a venture significantly.

3. Options of undertaking a feasibility study, and controlling its timing are complementary at low to medium-high metal prices, i.e., they reinforce each other. At high metal prices, they are supplementary, i.e., they replace one another. At high metal prices, feasibility study takes the place of waiting in helping the investor to avoid an investment which produces low income.

4. In the operational stage, the option of being able to shut the mine down temporarily, reopen, or abandon it when conditions are unfavourable can be used to maximize the venture's value.

5. The metal price boundaries at which decisions about the operating options should be made can be used as guides to ensure efficient financial management of mineral projects or ventures.

6. With no in-built procedures to quantify the values of timing feasibility studies, development decisions, nor operating options, the DCF method always underestimates the value of mineral projects at low to medium-high metal prices. Marginal, but profitable, projects can readily appear less desirable, or be rejected without appropriate consideration. The DAV method provides one of the best supplementary evaluation tools in dealing with deficiencies in the DCF method.

From analyses of the *two-dimensional DAV model* of the value of a mineral venture, the following conclusions are drawn:

1. Phase diagrams which delineate metal price boundaries for various expected ore grades at which waiting or abandoning, feasibility study, and investment decisions are appropriate, could be used as guides in making venture-related decisions.

2. Feasibility study is important, in a situation of high expected ore grade uncertainties and low to medium-high metal prices, before any subsequent development decisions. It is required to reduce uncertainties about expected ore grade, and to help investors to avoid either over- or under-investment costs which can lead to subsequent high venture cost overruns and significantly reduced return on investment.

3. Undertaking the feasibility study in stages, instead of a single-stage study at the beginning, can help investors to maximize the value of a mineral project in all of the feasibility study strategies analysed in this study. Even if extensive feasibility study is required to solve problems prior to development, it is still better to consider the feasibility study in stages. This can help investors to:

    i. Evaluate the project after every stage, and take advantage of the most current information to plan and execute the next stage of the program

    ii. Avoid overdoing the feasibility study

    iii. Make development decisions at the most appropriate times

4. Using the DAV method, investors can examine the projected value of a mineral venture with a short or long feasibility study duration at various stages, and to use the appropriate one to maximize the venture's value.

5. If there are no time constraints on investors' development rights, timing options can also be used to maximize the value of a mineral venture. There is a

quantifiable value associated with being able to wait to undertake a feasibility study and any subsequent development decisions. With such options, investors are able to take advantage of future information and the realisation of correctly projected trends to guide the program.

6. Certain changes in the economy have significant effects on mineral project value. These can be quantified and taken into consideration in project evaluation. With high uncertainties in metal prices in today's markets, various shocks on metal prices in the markets, and the market price of risk on mineral commodities, investors in mineral projects could be misled by the results of evaluation tools that do not treat these variables rigorously. The DAV method is set up to rigorously deal with economic changes that affect the value of a mineral project or venture, and it presents one of the most useful economic evaluation tools today.

There are many recommendations for further research work. The most important ones are the following:

1. Find a solution to the numerical boundary and discretization problems encountered in the use of the alternate direction implicit (ADI) algorithm for the solution of the two-dimensional continuous model (see Appendix C.2)

2. Incorporate operating options in the two-dimensional DAV model of the value of the mineral venture

3. Compare the two-dimensional DAV results with those from the DCF method

4. Incorporate taxes in the problem formulations and analyses of both the one- and two-dimensional models

5. Incorporate a third dimension, ore reserve uncertainty (a significant source of mineral project uncertainty), in the two-dimensional DAV model

6. Continue to formulate additional DAV models which help to quantify options and the effects of various combinations of identifiable variables which are regularly

encountered by mineral venture planners, evaluators and investors. These models can best be created by a thorough exchange of ideas in groups which include experienced engineers, evaluators and investment decision makers.

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Appendix A

EVOLUTION OF PAST STAGES IN ASSET PRICING THEORY

A.1. The Historical Perspective of the DCF Techniques

"The DCF techniques require both the understanding of compound interest and the ability to set out the cash inflows and outflows likely to result from a particular decision to invest. Knowledge of compound interest dates back to the Old Babylonian period (1800 - 1600 B. C.) in Mesopotamia. The earliest applications of DCF were to loans where the cash outlays, and receipts were known, and to life insurance where probabilities could be estimated from historical evidence. Interest tables were first developed by Jean Trenchant (1558) at Lyons, and Simon Stevin (1582) at Antwerp. Trenchant discusses a geometric progression and compound interest, and Stevin also discusses a general rule for finding the most profitable of two ventures. Based on the work carried out by Pierre de Fermat (1601 - 65), Blaise Pascal (1623 - 62) and Christiana Huygens (1629 - 95), and Johann de Witt (1625 - 72) and Edmond Halley (1656 - 1742) combined the chances of death with compound interest to produce the value of a life annuity. The development of life insurance schemes in the 18th century gave rise to actuarial science, under which research work by Charles Ingall (1862), Lt. Col. Oakes (1870), and Herbert Johnson (1881) produced the bond tables.

The DCF techniques were not applied to non-financial investments until the 19th century. Following the work of A. M. Wellington (1887) on the location of railways, Walter O. Pennell (1914) advanced the present value approach. Pennell discusses the decision to either install a new or retain an existing machine, by calculating the interest and sinking fund depreciation on the initial capital investment and then multiplying by an annuity factor to obtain the present worth of the investments. The concept of the annual cost was introduced by Fisher and Grant (1915, 23) and that of the internal rate of return (IRR) and the marginal efficiency were also introduced by Boulding (1936), Keynes (1936) and Samuelson (1937). An early use of the present value criterion was made by the South African Mining Industry Commission (1907 - 8) in an attempt to measure the return on capital invested in the Witwatersrand gold industry. Lehfeldt and Frankel (1923, 35, 67) used the IRR in later investigations of the same problem. The ultimate source of most of the present ideas on DCF techniques is based on the works of Marshall (1907), Bohm - Bawerk (1903), Wickell (1893), and Fisher (1907). Theoretical contributions were made by F. Lutz, J. Heirshleifer, J. H. Lorie, L. J. Savage, F. Modigliani, M. H. Miller, E. Solomon, and J. Dean."

Adapted from Parker, R. H. (1968)

A.2. The Capital Asset Pricing Model (CAPM)

The CAPM, also referred to as the Sharpe-Lintner-Mossin mean-variance equilibrium model of exchange, states that in a well-functioning capital market, the expected risk premium on each investment is proportional to its "beta" [Brealey and Myers 1989]. The "beta" indicates the sensitivity of an investment's return to market movements. The basis of this model is from Markowitz (1959). Markowitz, following Von Neumann and Morgenstern (1953), developed an analysis based on the expected utility (wealth) maximization and proposed a general solution for the portfolio selection problem. Two major advances resulting from Markowitz's work are: (1) Tobin's (1958), Hicks' (1962), and Gordon and Gangolli's (1962) works utilizing the foundations of portfolio theory to draw implications regarding the demand for cash balances, and (2) the general equilibrium models of asset prices by Treynor (1961), Sharpe (1964), Lintner (1965), Mossin (1966) and Fama (1968, 1971).

Tobin divides the investment choice in Markowitz's model into two phases: (1) the choice of a unique optimum combination of risky assets, and (2) a separate choice concerning the allocation of funds between such a combination and a single riskless asset. Hicks uses a similar model to that proposed by Tobin to derive corresponding conclusions about individual investor behaviour. He deals more explicitly with the nature of the conditions which allows the division of investment choice into the two phases described by Tobin. Gordon and Gangolli provide more detailed discussions of this process, including a rigorous proof in the context of a choice among lotteries. Sharpe, Lintner and Mossin extend this model to construct a market equilibrium theory, and produce the security market line (SML) for portfolio performance analysis, plus the capital market line (CML) for efficient portfolio analysis.

The Security Market Line (SML): The SML passes through the market portfolio, with a "beta" of 1, and its intercept with the vertical axis represents the riskless asset return. See Figure A.1. In equilibrium, all traded assets in the markets lie along this line [Brealey and Myers 1989; Sharpe 1964; Jarrow 1988]. Based on certain assumptions [see Jensen 1972; Jarrow 1988], the sensitivity of the return on asset z to changes in the market , β_z, from the CAPM, is given by

$$E\ [R_z] = R\ (Z_0) + \beta_z\ [\ E\ (Rm) - R\ (Z_0)\ ] \qquad \text{(A.1)}$$

E [Rz] is the expected value of the return on the asset z; R (Zo) is the riskfree rate of interest; and E [Rm] is the expected value of the return on the market portfolio (i.e. aggregation of assets traded in the capital market measured by an index, e.g., the Standard and Poor's Composite Index or the New York Stock Exchange Index). β_z for any project is obtained by examining a portfolio of assets that has a similar risk structure to the project. From equation (A.1), the discount rate is obtained for the project. The problem with this approach is that no two projects are similar, and therefore this approach fails to capture the actual discount rate for a project.

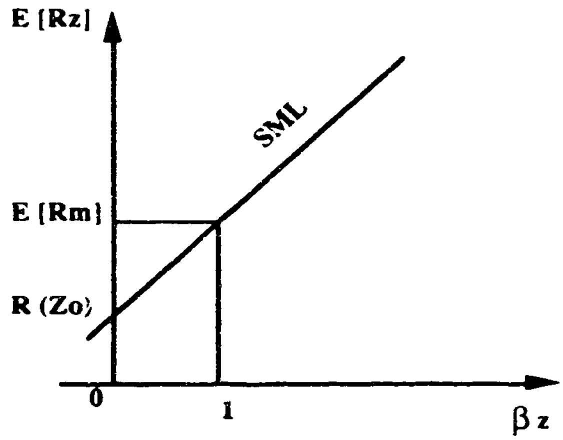

Figure A.1 The Security Market Line

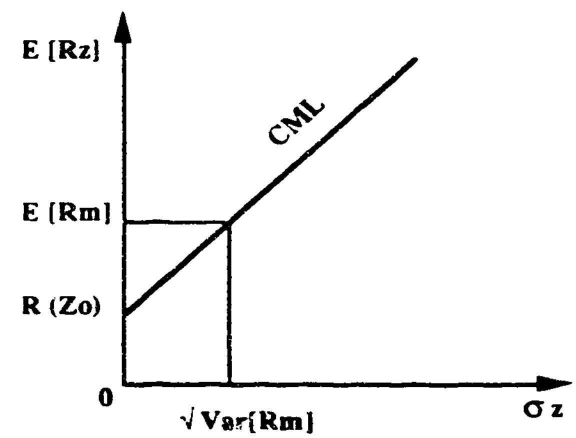

Figure A.2 The Capital Market Line

The Capital Market Line (CML): The CML is used to examine efficient portfolios in capital markets. An efficient portfolio is one which provides maximum expected return for a given level of risk and minimum risk for a given level of expected return [Jarrow 1988]. The return on an efficient portfolio follows the equation of the CML given by

$$E\ [R_z] = R\ (Z_o) + \{(Var\ [R_z])^{0.5}\ /\ (Var\ [R_m])^{0.5}\} * (E\ [R_m] - R\ (Z_o)) \qquad (A.2)$$

$Var[R_m]$ and $Var[R_z]$ are the respective variances of the market portfolio and the asset z.

The graph of the CML equation, in a mean-standard deviation plane of the portfolio is linear, and its intercept represents the riskfree asset, with a standard deviation of zero and expected return of $R(Z_o)$. This line also passes through the market portfolio, as illustrated in Figure A.2. In equilibrium, capital asset prices have adjusted so that the investor, if he follows rational procedures, is able to attain any desired point along the CML. He may obtain a higher expected rate of return on his holdings only by incurring additional risk. In effect, the market presents him with two prices: (1) the price of time or the riskfree rate of interest, $R(Z_0)$, and (2) the price of risk, the additional expected return per unit of risk borne, i.e., the reciprocal of the slope of the CML [Sharpe 1964].

The Alpha Concept: Jensen (1969) develops a technique based on the quantity called alpha, to examine securities in the capital markets. With this concept, the CAPM is used to identify underpriced securities to be purchased and overpriced securities to be sold. The alpha, α_z, on the asset z is defined as

$$\alpha_z = R\ (Z)_{est} - R\ (Z_o) - \beta_z\ [\ E\ (R_m) - R\ (Z_o)\ ] \qquad (A.3)$$

where $R\ (Z)_{est}$ is the estimated expected return on asset z. The evaluation rule is such that if $\alpha_z > 0$, asset is underpriced and should be purchased; if $\alpha_z = 0$, asset is priced correctly; if $\alpha_z < 0$, asset is overpriced and should be sold.

Assets with returns above the SML have positive alpha; those with returns below the SML have negative alpha; and assets with returns on SML have zero alpha. The problem with the alpha concept is that the CAPM requires that $R\ (Z)_{est} = E\ [Rz]$ for all investors. Hence, the alpha in equilibrium, is always zero. However, advantage could be taken of a temporary disequilibrium economics to use this technique to make profits.

The CAPM assumes that investors choose their portfolios according to Markowitz's (1959) mean - variance criterion, and therefore it is subject to all the theoretical objections to this criterion [see Samuelson 1967 and 1970; Borch 1969; Feldstein 1969; Hakansson 1971]. It has also been criticized for the additional assumptions, especially homogeneous expectations, and the single-period nature of the world. While the model predicts that the expected excess return from holding an asset is proportional to its beta, the empirical work of Black, Jensen and Scholes (1972) has demonstrated that low beta assets earn a higher return on average and high beta assets earn a lower return on average than is forecast by the model. Nonetheless, the model is still used because it is an equilibrium model which provides a strong specification of the relationship among asset yields that is easily interpreted, and the empirical evidence suggests that it does explain a significant fraction of the variation in asset returns.

A.3. Incomplete Equilibrium Models of Call Option Pricing

Prior to the Black - Scholes option pricing model, only two assumptions about the statistical process generating the stock price had been offered. Bachelier (1900) suggests an arithmetic Brownian motion process. This assumption leads to unacceptable general equilibrium implications [see Smith 1976]. The other alternative, the geometric Brownian motion process, states that the logarithm of stock prices follows a Wiener process, and is based on four assumptions: (1) The distribution of price ratios is independent of the price level; (2) price ratios are independent; (3) there is no probability that the stock price will become zero; and (4) the variance of the price relatives is infinite.

The Sprenkle Model: Sprenkle (1964) partially removes the first two objections to Bachelier's (1900) formulation. He assumes that stock prices are lognormally distributed, thus explicitly ruling out the possibility of non-positive prices for securities, and removing the associated infinite prices for options. Further, he allows for drift in the random walk, hence positive interest rates and risk aversion. He further assumes that the investor would be willing to pay exactly the expected value of the option price, if he were neutral to risk, and that interest rates are zero. Because the time value is ignored, this model is flawed. The final form of the Sprenkle's model containing a modification for risk, is given by:

$$c = e^{\rho\tau} S\, N\left[\frac{\ln\left(\frac{S}{K}\right) + \left(\mu + \left[\frac{\sigma^2}{2}\right]\tau\right)}{\sigma\sqrt{\tau}}\right] - (1 - g)\, K\, N\left[\frac{\ln\left(\frac{S}{K}\right) + \left(\mu - \left[\frac{\sigma^2}{2}\right]\tau\right)}{\sigma\sqrt{\tau}}\right] \qquad \text{(A.4)}$$

c is the option's price; μ is the expected average rate of growth in the stock price; σ is the instantaneous standard deviation or volatility in stock price; S is the stock price; K is the delivery price; τ is the time to maturity; g is an adjustment for the degree of market risk aversion; and N[x] is the cumulative probability distribution function for a standardized normal variable (i.e., the probability that such variale will be less than x).

The Boness Model: Boness (1964a) allows for time value of money, and thus avoids Sprenkle's error. However, his assumptions ignore the different levels of risk for the stock and the options. He assumes that: (1) The market is competitive in the sense that equilibrium price of all stocks of the same risk class imply the same expected yield on investment; (2) the probability distribution of expected percentage changes in the price of any stock is lognormal; (3) the variance of returns is directly proportional to time; and (4) investors are indifferent to risk. To allow for the time value of money, Boness discounts the derived expected terminal option price to the present, using the expected rate of return on the stock. The solution of the Boness formulation is given by:

$$c = S\,N\left[\frac{\ln\left(\frac{S}{K}\right) + \left(\mu + \left[\frac{\sigma^2}{2}\right]\tau\right)}{\sigma\sqrt{\tau}}\right] - e^{\rho\tau} K\,N\left[\frac{\ln\left(\frac{S}{K}\right) + \left(\mu - \left[\frac{\sigma^2}{2}\right]\tau\right)}{\sigma\sqrt{\tau}}\right] \qquad \text{(A.5)}$$

Boness' fourth assumption would suggest that he uses ρ, the risk-free rate of return, instead of the expected rate of return on the option. However, the fourth assumption also implies that, in equilibrium, the returns on all assets would be equal (i.e., $\mu = g = \rho$ in equilibrium). Hence, he could have used the appropriate riskfree rate of return to avoid the estimation of the expected average rate of growth in the stock price.

The Samuelson Model: Samuelson (1965) assumes that stock prices follow geometric Brownian motion with positive drift, μ, thus allowing for positive interest rates and risk premia. If the option price also grows at the rate i, and further assuming that the terminal stock price distribution is lognormal, the solution to the Samuelson model is also given by:

$$c = e^{(\rho - i)\tau} S\,N\left[\frac{\ln\left(\frac{S}{K}\right) + \left(\mu + \left[\frac{\sigma^2}{2}\right]\tau\right)}{\sigma\sqrt{\tau}}\right] - e^{i\tau} K\,N\left[\frac{\ln\left(\frac{S}{K}\right) + \left(\mu - \left[\frac{\sigma^2}{2}\right]\tau\right)}{\sigma\sqrt{\tau}}\right] \qquad \text{(A.6)}$$

Samuelson also examines the value of an option if the return on the option is greater than the return on the stock (i.e., $i > \mu$). He suggests two situations in which this could occur: (1) If the stock pays a dividend at the rate d, it would be expected that at least $\mu + d = i$; and (2) if the market perceives the option to be more risky than the security, then investors require that $i > \mu$. In the appendix to the Samuelson's paper, Mckean (1965) solves this problem for a perpetual option and lognormally distributed security prices. Samuelson postulates a biased random walk following a geometric Brownian motion. His arguments as to why r and i might be expected to differ are general equilibrium restrictions of Merton

(1973b). He finds that with $i > \mu$ there is a positive probability of premature exercise for the option.

Appendix B

PROCESSES UNDERLYING THE BLACK-SCHOLES' MODEL

B.1. Wiener Process

Wiener process is a particular type of Markov stochastic process. It has been used in physics to describe the motion of a particle that is subject to a large number of small molecular shocks (i.e. the Brownian motion). For a variable, z, that follows a Wiener process, a small change in its value, Δz, as a result of a small interval of time, Δt, has two properties as follows:

1. Δz is related to Δt as given in equation (A.4) below

   $$\Delta z = \varepsilon \sqrt{\Delta t} \qquad (A.7)$$

   where ε is a random sample from a standardized normal distribution.

2. The values of Δz for any two different short intervals of time are independent. Thus, z follows a Markov process.

For continuous time stochastic process, a Wiener process is the limit as $\Delta t \rightarrow 0$ in equation (A.7). Thus, as Δt and $\Delta z \rightarrow 0$, equation (A.4) becomes

$$dz = \varepsilon \sqrt{\ } dt \qquad (A.8)$$

This has a drift rate of zero and a variance rate of 1.0. The drift rate of zero means the expected value of z at any future time is equal to its current value. The variance rate of 1.0 means that the variance of the change in z in a time interval of length T is 1.0 * T. Thus, a generalized Wiener process for a variable x can be defined in terms of dz as follows:

$$dx = a\ dt + b\ dz \qquad (A.9)$$

Thus, in a small time interval, Δt, the change in the value of x, Δx, is given from equations (A.7) and (A.9) as

$$\Delta x = a \, \Delta t + b \, \varepsilon \sqrt{dt} \qquad \text{(A.10)}$$

B.2. Ito's Process

It is a generalized Wiener process whe·· ·he parameters a and b in equation (A.9) are functions of the value of the underlyin· ·anaole x, and time t. Thus, from equation (A.9), Ito's process can be defined as

$$dx = a\,(x, t)\,dt + b\,(x, t)\,dz \qquad \text{(A.11)}$$

Both the drift rate a, and the variance rate b, are liable to change over time.

Consider a continuous function G of a variable x and y. If Δx and Δy are small changes in x and and y, ΔG is the resulting small change in G. Using the Taylor series expansion, ΔG can be expressed as follows

$$\Delta G = \frac{\partial G}{\partial x}\Delta x + \frac{\partial G}{\partial y}\Delta y + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}\Delta x^2 + \frac{\partial^2 G}{\partial x\,\partial y}\Delta x\,\Delta y$$

$$+ \frac{1}{2}\frac{\partial^2 G}{\partial y^2}\Delta y^2 + \cdots \qquad \text{(A.12)}$$

In the limit as Δx and Δy aproach zero, equation (A.12) becomes

$$dG = \frac{\partial G}{\partial x}dx + \frac{\partial G}{\partial y}dy \qquad \text{(A.13)}$$

Now suppose a derivative security is a function of a variable x that follows a general Ito process and that G is a function of x and time t. Expanding ΔG using Taylor series expansion results in the same expression as in equation (A.12) with t instead of y. However, in the limit as Δx and Δt approach zero, the term in x to the second order does not disappear because of the Wiener process it follows. Squaring equation (A.10) we have

$$\Delta x^2 = a^2 \Delta t^2 + b^2 \varepsilon^2 \Delta t + 2ab\varepsilon \Delta t^{3/2} \tag{A.14}$$

Ignoring the higher order terms in t in equation (A.14) results in

$$\Delta x^2 = b^2 \varepsilon^2 \Delta t \tag{A.15}$$

The variance of the standard normal distribution is 1.0. This means that

$$E[\varepsilon^2] - [E(\varepsilon)]^2 = 1 \tag{A.16}$$

But

$$E[\varepsilon] = 0 \quad \Rightarrow \quad E[\varepsilon^2] = 1$$

This implies that as Δt and Δx approach zero, Ito's lemma can be formulated using ΔG as

$$dG = \left[a\frac{\partial G}{\partial x} + \frac{\partial G}{\partial t} + \frac{1}{2} b^2 \frac{\partial^2 G}{\partial x^2} \right] dt + b\frac{\partial G}{\partial x} dz \tag{A.17}$$

B.3. Instantaneous Change in Futures Price

The futures contract is a derivative asset the value of which is derived from the spot price of the commodity, S, and the time to maturity, t. S follows a generalized Wiener process. The value of the futures contract for a small interval of time Δt could be derived using the Taylor series expansion as

$$\Delta F(S, t) = \frac{\partial F}{\partial S}\Delta S + \frac{\partial F}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}(\Delta S)^2 + \frac{1}{2}\frac{\partial^2 F}{\partial t^2}(\Delta t)^2$$

$$+ \frac{\partial^2 F}{\partial S \, \partial t} (\Delta S)(\Delta t) + \cdots + \textbf{higher orders} \qquad (A.18)$$

Substituting the square of ΔS and considering the value of the futures contract in the limit as Δt and ΔS tend to zero, we have

$$dF(S, t) = \frac{\partial F}{\partial S} dS + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S^2} \sigma^2 S^2 \varepsilon^2 dt + \cdots + \textbf{higher orders} \qquad (A.19)$$

Replacing the partial derivatives with respect to S and t by F_S and F_t respectively, and also inputting the value of dS in equation (A.19) the value cf the futures contract is derived as

$$dF(S, t) = \left[F_S \, \mu \, S + F_t + \frac{1}{2} F_{SS} \, \sigma^2 S^2 \right] dt + F_S \, \sigma \, S \, dz \qquad (A.20)$$

B.4. Instantaneous Change in 1 - D Mine Value

The value of the 1 - D mine is derived from the spot price of the commodity, S, the total ore reserve, Q, the time of evaluation, t, the state of the mine, j, and the development and operating policy, Φ. Thus H can be written as

$$H \equiv H(S, Q, t, j; \Phi) \qquad (A.21)$$

The value of the 1 - D mine for a small interval of time at a state j and with an operating policy, Φ, could be derived from the Taylor series expansion as

$$\Delta H(S, Q, t) = \frac{\partial H}{\partial S} \Delta S + \frac{\partial H}{\partial t} \Delta t + \frac{\partial H}{\partial Q} \Delta Q + \frac{1}{2} \frac{\partial^2 H}{\partial S^2} (\Delta S)^2 \; \frac{1}{2} \frac{\partial^2 H}{\partial t^2} (\Delta t)^2$$

$$+ \frac{1}{2} \frac{\partial^2 H}{\partial Q^2} (\Delta Q)^2 + \frac{\partial^2 H}{\partial S \, \partial t} (\Delta S)(\Delta t) + \frac{\partial^2 H}{\partial S \, \partial Q} (\Delta S)(\Delta Q)$$

$$+ \frac{\partial^2 H}{\partial Q \, \partial t} (\Delta Q)(\Delta t) + \cdots + \textbf{higher orders} \qquad (A.22)$$

Substituting the square of ΔS and replacing the partial derivatives of H with respect to S, Q, and t with H_S, H_Q, and H_t, and taking the limit of H as ΔS, ΔQ, and Δt tend to zero, the instantaneous value of the 1 - D mine could be simplified as

$$dH(S, Q, t) = H_S\, dS + H_Q\, dQ + H_t\, dt + \frac{1}{2}\sigma^2 S^2 H_{SS}\, dt \qquad (A.23)$$

B.5. 2 - D Continuous Mine Value Model

The value of a mineral venture with an ongoing feasibility study, H, depends on: (1) The amount of feasibility work, w, carried out. It also depends on: (2) The expected value of the ore grade E, and its variation throughout the ore body; (3) the price of the metal, S; (4) the time of evaluation,t; and (5) the feasibility stage, i. Thus, the mine value is written as:

$$H = H(S, E, w, t, j, i) \qquad (A.24)$$

The indicator j is used to describe the different phases of the mine. It takes the value F in the feasibility phase, I in the investment phase, W in the waiting phase. The instantaneous change in the value of this mineral project[2] [Ito 1951; Malliaris and Brock 1982] is given as:

$$dH = H_t\, dt + H_E\, dE + H_w\, dw + H_S\, dS + \frac{1}{2} H_{SS}(dS)^2 + \frac{1}{2} H_{EE}(dE)^2 \qquad (A.25)$$

$$dw = n dt \qquad (A.26)$$

$$n(j) = \begin{cases} \bar{n} & j = F \\ 0 & j = I, W \end{cases} \qquad (A.27)$$

$$dS = m\, Sdt + s_S\, SdZ_S \qquad (A.28)$$

2 See Appendix B.5 for a derivation of the instantaneous change in the 2 - D mine value model.

$$dE = s_w \; E \; dZ_E \qquad \text{(no drift in E)} \qquad \text{(A.29)}$$

Substituting equations (A.26), (A.28), and (A.29) into equation (A.25), the instantaneous value of the mine is:

$$dH = H_t \, dt + H_E \, E \, \sigma_w \, dZ_E + n \, H_w \, dt + H_S \left(\mu \, Sdt + \sigma_S \, SdZ_S\right) +$$

$$\frac{1}{2} H_{SS} \left(\mu \, Sdt + \sigma_S \, SdZ_S\right)^2 + \frac{1}{2} H_{EE} \left(\sigma_w \, EdZ_E\right)^2 \qquad \text{(A.30)}$$

B.5.1. Grade Variance and its Resolution with Additional Information

A feasibility study in any mineral venture must provide investors and analysts with enough knowledge about the expected ore grade and reserves to enable decision making. It is assumed, in this study, that the resolution of uncertainty about the ore grade with ongoing feasibility study follows the illustration in Figure A.3.

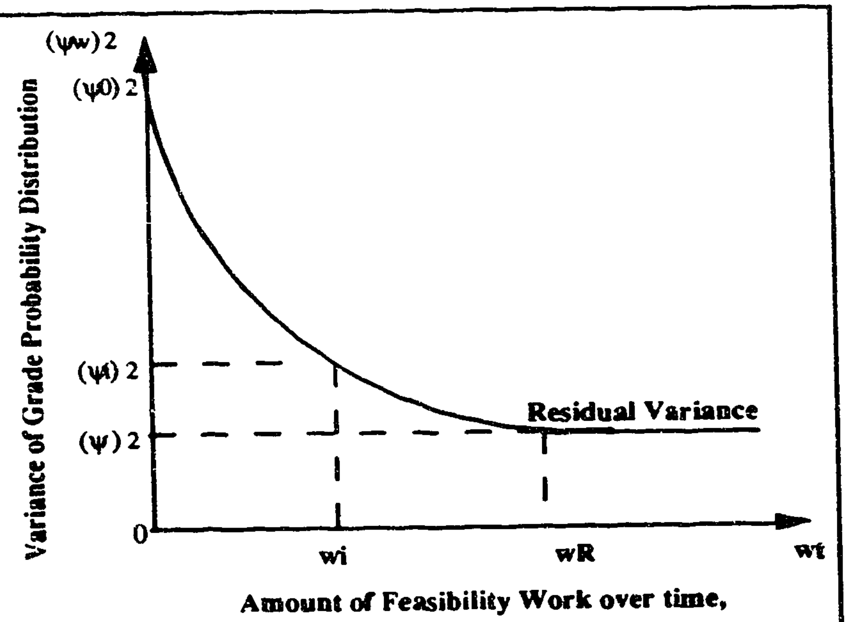

Figure A.3 Grade Variance Resolution with Feasibility Study

As the amount of feasibility work is increased, the total variance associated with the ore grade probability distribution reduces. *Beyond a certain critical information level, w_R, the total variance associated with the ore grade distribution is assymptotic to the residual variance.* At this critical point, if investment is not made, the investor should wait for an appropriate investment timing, because additional feasibility study does not add any meaningful information to the data bank.

Let w be the total feasibility work done at any time of evaluation; σ_w^2 be the rate of change in the variance associated with the grade probability distribution; ψ_w^2 be the total variance associated with grade probability distribution after the feasibility work, w. ψ_w^2 is given by:

$$\psi_w^2 = \frac{\psi_0^2 - \psi_\infty^2}{1 + w p} + \psi_\infty^2 \qquad (A.31)$$

and

$$\sigma_w^2 = -\partial_w \Psi^2(w(t)) * \partial_t w(t) = n p \left(\frac{\psi_0^2 - \psi_\infty^2}{(1 + wp)^2}\right) \equiv \hat{\sigma}^2 n \qquad (A.32)$$

ψ_0^2 is the total variance at the beginning of the feasibility study (see Figure A.3); ψ_∞^2 is the total variance at infinity, i.e. the residual variance; p is a factor of variance reduction with additional information.

B.5.2. Feasibility Cost Function

The cost of gathering information in a feasibility study at any time depends on the rate of drilling (sampling), n per unit time, the average unit cost of drilling the holes, a, the state of the mine, j, and the feasibility study stage, i, and is given by:

$$FC(n, t, j, i) = \begin{cases} \bar{n}\alpha & j = F \\ 0 & j = I, W \end{cases} \qquad (A.33)$$

During the feasibility study, there is no mining activity, and therefore no cash flows into the project value. However, cost is expended in obtaining information about the ore deposit and how to extract it. It is assumed here that the cost of feasibility study at the beginning is zero. Therefore, the instantaneous return to a portfolio with a long position in the mineral venture with ongoing feasibility study, and a short position in futures contracts, dR, is given by:

$$dR = dH - FC\,dt - (H_S / F_S)\,dF \tag{A.34}$$

Substituting dH and dF in equation (A.34), the instantaneous value of the portfolio return is:

$$dR = H_t\,dt + H_E\,E\,\sigma_w\,dZ_E + n\,H_w\,dt + H_S\left(\mu\,Sdt + \sigma_S\,S\,dZ_S\right) +$$

$$\frac{1}{2}H_{SS}\left(\mu\,S\,dt + \sigma_S\,S\,dZ_S\right)^2 + \frac{1}{2}H_{EE}\left(\sigma_w\,E\,dZ_E\right)^2 -$$

$$FCdt - (H_S / F_S)\,F_S\left(S\,((\mu - \rho) + c)\,dt\,\sigma_S\,S\,dZ_S\right) \tag{A.35}$$

The risk factor introduced by the expected value of the grade probability distribution, $H_E\,E\sigma_w\,dZ_E$, does not affect the macro-economic structure. It is unique to the project, and could be reduced, with diversification, to zero. To avoid arbitrage opportunities, the expected instantaneous return must be equal to the riskless return, ρ Hdt. Thus, for a fixed sampling rate, n, the value of the mineral venture, with ongoing mine feasibility study, satisfies the following differential equation:

$$\frac{1}{2}\sigma_S^2\,S^2\,H_{SS} + \frac{1}{2}\sigma_E^2\,E^2\,H_{EE} + H_t + n\,H_w + S\,(\rho - c)\,H_S - FC - \rho\,H = 0 \tag{A.36}$$

The value of the mine with waiting, feasibility study and investment options follows this objective function:

$$\max_{n \in (0, \bar{n})}$$

$$\left[\frac{1}{2}\sigma_S{}^2 S^2 H_{ss} + \frac{1}{2} n\hat{\sigma}_E{}^2 E^2 H_{EE} + H_t + nH_w + S(\rho - c)H_S - FC - \rho H = 0\right] \qquad (A.37)$$

B.6. Instantaneous change in 2 - D Mine Value

The value of the 2 - D mine is derived from the spot price, S, of the commodity, the expected value of the ore grade, E, the amount of the information gathered to date, w, the time of evaluation, t, the state of the mine, j, and the feasibility policy, Φ. Thus H can be written as

$$H \equiv H\,(S,\ E,\ w,\ t,\ j;\ \Phi) \qquad (A.38)$$

The instantaneous value of the of the 2 - D mine for a small change in time, Δt, at state, j, and feasibility study policy, Φ, is given by

$$\Delta H\,(S,\ E,\ w,\ t\,) = \frac{\partial H}{\partial S}\Delta S + \frac{\partial H}{\partial E}\Delta E + \frac{\partial H}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 H}{\partial S^2}(\Delta S\,)^2\ \frac{1}{2}\frac{\partial^2 H}{\partial E^2}(\Delta E\,)^2$$

$$+ \frac{1}{2}\frac{\partial^2 H}{\partial w^2}(\Delta w\,)^2\ \frac{1}{2}\frac{\partial^2 H}{\partial t^2}(\Delta t\,)^2 + \frac{1}{2}\frac{\partial^2 H}{\partial S\,\partial E}(\Delta S)(\Delta E) + \frac{1}{2}\frac{\partial^2 H}{\partial S\,\partial w}(\Delta S)(\Delta w)$$

$$+ \frac{1}{2}\frac{\partial^2 H}{\partial S\,\partial t}(\Delta S)(\Delta t) + \frac{1}{2}\frac{\partial^2 H}{\partial E\,\partial w}(\Delta E)(\Delta w) + \frac{1}{2}\frac{\partial^2 H}{\partial E\,\partial t}(\Delta E)(\Delta t)$$

$$+ \frac{1}{2}\frac{\partial^2 H}{\partial w\,\partial t}(\Delta w)(\Delta t) + \cdots + \text{higher orders} \qquad (A.39)$$

The rate at which the uncertainty associated with the expected ore grade distribution is resolved also follws a Wiener process without drift. Thus, the square of ΔE is given by

$$\Delta E^2 = \sigma^2 E^2 \varepsilon^2 \Delta t \qquad (A.40)$$

By substituting the square of ΔS and ΔE into equation (A.39) and simplifying, the instantaneous value of the 2 - D mine in the limit as ΔS, ΔE, and Δt tend to zero can be derived as

$$dH(S, E, w, t) = H_S\, dS + H_S\, dS + H_E\, dE + H_w\, dw + H_t\, dt + \left(\frac{1}{2}\sigma_S^2 S^2 H_{SS} + \frac{1}{2}\sigma_E^2 E^2 H_{EE}\right) dt \qquad (A.41)$$

APPENDIX C

FINITE DIFFERENCE EQUATIONS

C.1. Finite Difference Equations for 1 - D Mine

The implicit finite difference technique [Lapidus and Pinder 1982; Smith 1965] is used to solve, numerically, the differential equations for the 1 - D problem. It is assumed that the metal price, S, is assumed continuously in the interval [0, Smax], and the ore reserve quantity, Q, is also defined in the interval [0, Qmax]. The intervals [0, Smax] and [0, Qmax] are subdivided respectively into N equal subintervals of length ΔS each and M equal subintervals of length ΔQ each. In the S - Q plane in Figure A.4 below, the mine values are obtained by solving the respective differential equations, slice by slice, for the various points following the indicated direction. The value for every lattice point along the S = 0.0 and Q = 0.0 are known and are each equal to zero. The second derivative of the mine value with respect to S and Q as each tend to Smax and Qmax respectively is also equal to zero.

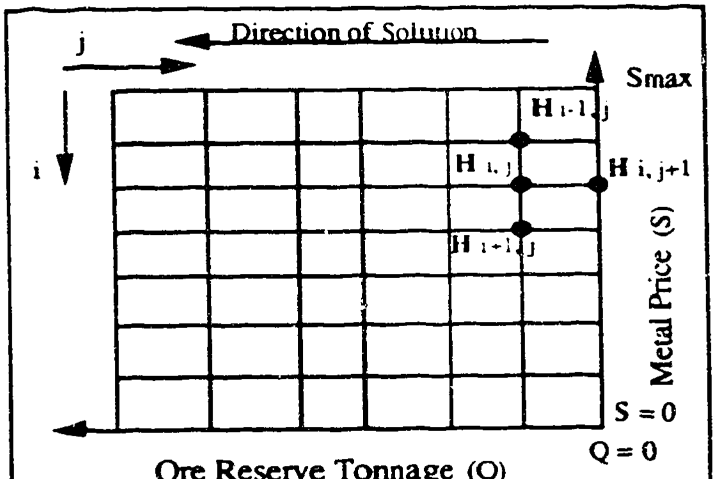

Figure A .4 S - Q Lattice for 1 - D Mine

Writing the differential equation (e.g. of the open mine value) in terms of the four lattice points labelled in Figure A.4, we have the following:

$$\frac{1}{2}\sigma^2 (i\Delta S)^2 \frac{\left[H_{i+1,j} - 2H_{i,j} + H_{i-1,j}\right]}{(\Delta S)^2} + i\Delta S(\rho - c)\frac{\left[H_{i+1,j} - H_{i-1,j}\right]}{2\Delta S}$$

$$+q\frac{\left[H_{i,j-1} - V_{i,j}\right]}{\Delta Q} - \rho H_{i,j} + CFO = 0 \qquad (A.42)$$

By simplifying equation (A.42), and rearranging, we have

$$a\ H_{i-1,j} + b\ H_{i,j} + c\ H_{i+1,j} = k_1 + k_2\ H_{i,j+1} \quad [i = 1 - n;\ j = 1 - m] \qquad (A.43)$$

where

$$a = \frac{1}{2}\sigma_S{}^2 i^2 - \frac{1}{2}(\rho - c) \qquad (A.44)$$

$$b = \left[\sigma_S{}^2 i^2 + \rho + \frac{q}{\Delta Q}\right] \qquad (A.45)$$

$$\bar{c} = \frac{1}{2}\sigma_S{}^2 i^2 + \frac{1}{2}(\rho - c) \qquad (A.46)$$

$$k1 = -\ CF \qquad (A.47)$$

$$k2 = -\ (q/\Delta Q) \qquad (A.48)$$

For any value of j, equation (A.42) constitutes a system of equations in the (n+2) unknowns, i.e. $H_{i,j}$ [i = 0, 1, - - -, n+1]. To complete the system, it is necessary to introduce the two boundary conditions given by $H_{0,j}$ and $H_{n+1,j}$. For this analysis, $H_{0,j} = 0$, and $H_{n+1,j} = 2H_{n,j} - H_{n-1,j}$. Thus, eliminating H0, j and Hn+1, j from equation (A.42) results in equation (A.49).

$$\begin{aligned} b\ H_{1,j} + c\ H_{2,j} &= k_1 + k_2\ H_{1,j+1} = RHS\ (i = 1) \\ a\ H_{i-1,j} + b\ H_{i,j} + c\ H_{i+1,j} &= k_1 + k_2\ H_{i,j+1} = RHS\ (i = 2, - - , n-1) \\ (a - c)\ H_{n-1,j} + (b + 2c)\ H_{n,j} &= k_1 + k_2\ H_{n,j+1} = RHS\ (i = n) \end{aligned} \qquad (A.49)$$

The system of equations in (A.49) is converted into matrix form as

$$AH = RHS \tag{A.50}$$

where

$$A = \begin{bmatrix} b_{11} & c_{12} & 0 & 0 & 0 & 0 \\ a_{21} & b_{22} & c_{23} & 0 & 0 & 0 \\ 0 & a_{32} & b_{33} & c_{34} & 0 & 0 \\ \vdots & \vdots & . & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & a_{n-1,\,n-2} & b_{n-1,\,n-1} & c_{n-1,\,n} \\ 0 & 0 & 0 & 0 & (a-c)_{n,\,n-1} & (b+2c)_{n,\,n} \end{bmatrix} \tag{A.51}$$

$$H = \begin{bmatrix} H_{1,j} + H_{2,j} \\ H_{1,j} + H_{2,j} + H_{3,j} \\ H_{2,j} + H_{3,j} + H_{4,j} \\ \vdots \\ H_{n-2,j} + H_{n-1,j} + H_{n,j} \\ H_{n-1,j} + H_{n,j} \end{bmatrix} \tag{A.52}$$

$$RHS = \begin{bmatrix} k_1 + k_2\ H_{1,\,j+1} \\ k_1 + k_2\ H_{2,\,j+1} \\ k_1 + k_2\ H_{3,\,j+1} \\ \vdots \\ k_1 + k_2\ H_{n-1,\,j+1} \\ k_1 + k_2\ H_{n,\,j+1} \end{bmatrix} \tag{A.53}$$

The Gaussian elimination and backsubstitution techniques [Smith 1965; Lapidus and Pinder 1982; Press et al. 1988] are then used to solve the problem for the unknown values.

C.2. Finite Difference Equations of 2 - D Mine

The value of the mine with ongoing feasibility study is described by equation (4.14). The difference equations are based on the Peaceman and Rachford (1955) and Douglas and Rachford (1956) ADI algorithm for the 2 - D parabolic equation. As described below, the method consists of solving, iteratively, the problem implicitly and explicitly in alternate directions. A typical S - E plane and the various lattice points whose values are required in this plane are illustrated in Figure A.5, and the sequence of the solution along the information time, in each S - E plane begining from the horizon (i.e. k + 1), is also illustrated in Figure A.6.

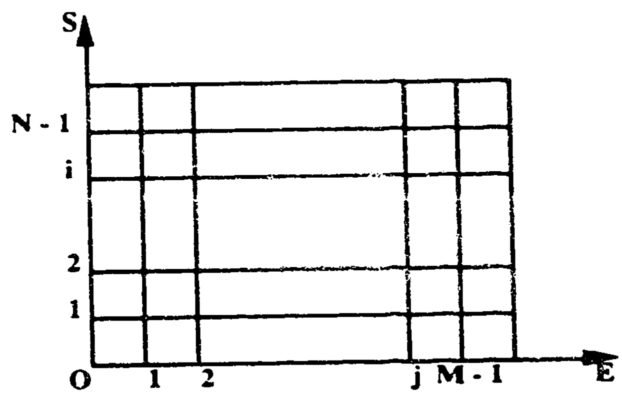

Figure A.5 2 - D lattice Points in the S - E Plane

Alternating Direction Implicit (ADI): The most efficient method for rectangular regions is the algorithm proposed by Peaceman and Rachford (1955) and Douglas and Rachford (1966); it is called the alternating direction implicit (ADI) method. This method consists of replacing one of the second order derivatives in the partial differential equation by an implicit difference approximation in terms of the unknown pivotal values, from the kth to the (k - 1)th information time level, and the other second derivative, by an explicit finite difference approximation as illustrated in Figure A.6. k is an odd integer.

At k+1, the three unknown values along the S slice at k - - H_S+, H_S and H_S- are calculated using the implicit algorithm in S, while the known values of H along the E slice at k+1 - - H_E+, H_E and H_E- form the right-hand side of the matrix formulation.

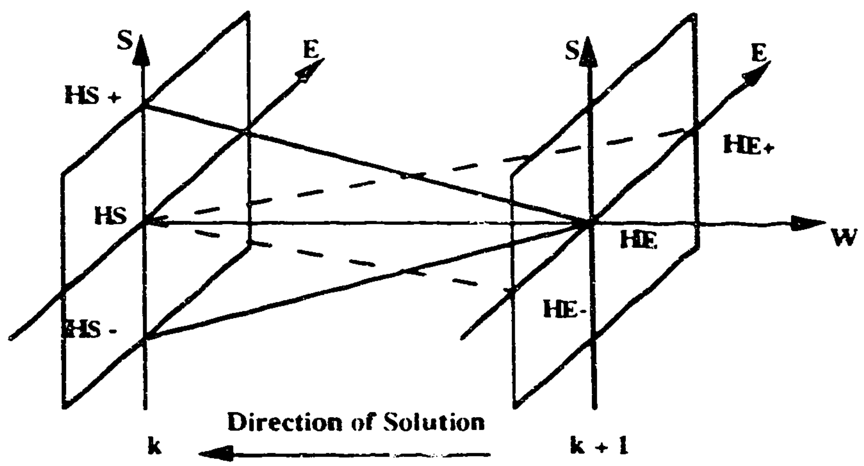

Figure A.6 Solution Sequence - Implicit in S and Explicit in E

The advancement of the solution from the kth to the (k - 1)th information time slice is achieved by replacing H_{EE} by an implicit finite difference approximation, and H_{SS} by an explicit finite difference approximation. The three unknown values at k-1, i.e., H_E+, H_E and H_E- are calculated using the implicit algorithm in E, and the three known values at k, i.e., H_S+, H_S and H_S- form the right hand side of the matrix formulation, as illustrated in Figure A.7. The solution values of the problem are obtained for all the lattice points on the slices in the S - E plane by solving the problem implicitly in S and explicitly in E at odd slices, and implicitly in E and explicitly in S at even slices.

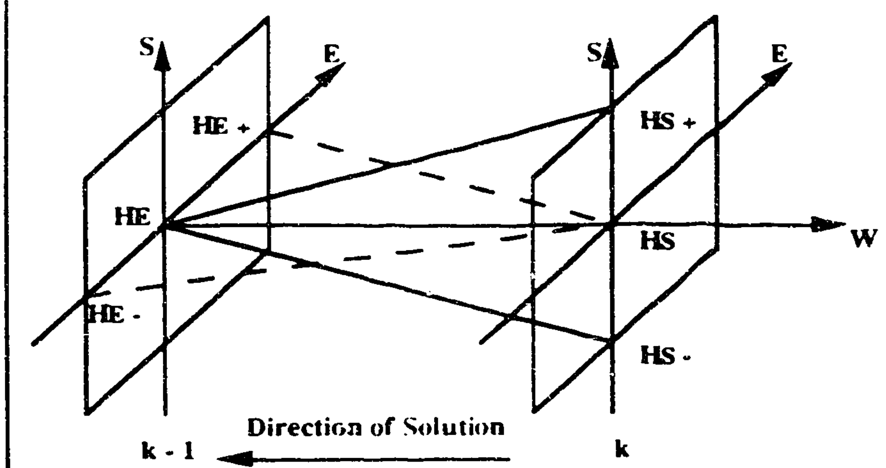

Figure A.7 Solution Sequence - Implicit in E and Explicit in S

The ADI algorithm consists of replacing one of the second order derivatives, e.g. Hss, by an implicit finite difference approximation in terms of the unknown pivotal values of H, from k+1 to k, and the other second order derivative, H_{EE}, by an explicit finite difference approximation. The implicit and explicit order is reversed from k to k - 1.

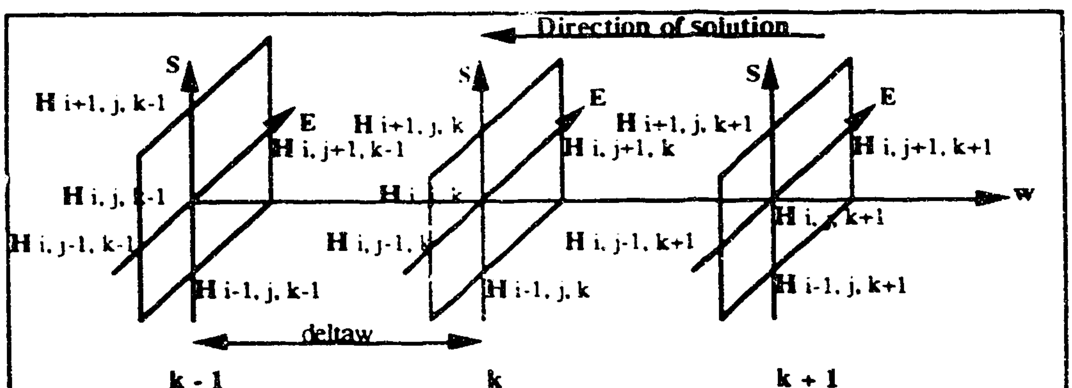

Figure A.8 Sequence of Solution along the Information Time

Finite Difference Equations from k+1 to k

Using Figure A.8, the finite difference equation of the 2 - D continuous model of the mineral venture's value could be derived as follows:

$$H_S = (H_{i+1,\,j,\,k} - H_{i-1,\,j,\,k}) \,/\, 2\,\Delta S \qquad (A.53)$$

$$H_{SS} = (H_{i+1,\,j,\,k} - 2\,H_{i,\,j,\,k} + H_{i-1,\,j,\,k}) \,/\, (\Delta S)^2 \qquad (A.54)$$

$$H_w = (H_{i,\,j,\,k+1} - H_{i,\,j,\,k}) \,/\, \Delta w \qquad (A.55)$$

$$H_{EE} = (H_{i,\,j+1,\,k+1} - 2\,H_{i,\,j,\,k+1} + H_{i,\,j-1,\,k+1}) \,/\, (\Delta E)^2 \qquad (A.56)$$

Substituting equations (A.53) to (A.56) into equation (A.36), we have

$$\frac{1}{2}\,\frac{\sigma_s^2\,(i\,\Delta S)^2}{(\Delta S)^2}\,(H_{i+1,\,j,\,k} - 2\,H_{i,\,j,\,k} + H_{i-1,\,j,\,k}) + \frac{i\Delta S(\rho - c)}{2\Delta S}\,(H_{i+1,\,j,\,k} - H_{i-1,\,j,\,k})$$

$$+ \frac{1}{2}\,\frac{n\,\sigma_w^2\,(j\,\Delta E)^2}{(\Delta E)^2}\,(H_{i,\,j+1,\,k+1} - 2\,H_{i,\,j,\,k+1} + H_{i,\,j-1,\,k+1})$$

$$+ \frac{n}{\Delta w}\,(H_{i,\,j,\,k+1} - H_{i,\,j,\,k}) - n\alpha - \rho\;H_{i,\,j,\,k} = 0 \qquad (A.57)$$

Simplifying equation (A.57), we have

$$a\;H_{i-1,\,j,\,k} + b\;H_{i,\,j,\,k} + c\;H_{i+1,\,j,\,k} + d\;H_{i,\,j-1,\,k+1} + e\;H_{i,\,j,\,k+1}$$

$$+ f\;H_{i,\,j+1,\,k+1} - n\,\alpha \qquad (A.58)$$

$H_{i,\,j-1,\,k+1}$, $H_{i,\,j,\,k+1}$, and $H_{i,\,j+1,\,k+1}$ have been calculated alreadv in the previous slice. Thus,

$$a\;H_{i-1,\,j,\,k} + b\;H_{i,\,j,\,k} + c\;H_{i+1,\,j,\,k} = RHS \qquad (A.59)$$

where

$$RHS = -(d\, H_{i,\, j-1,\, k+1} + e\, H_{i,\, j,\, k+1} + f\, H_{i,\, j+1,\, k+1}) + n\, \alpha \quad (A.60)$$

and the coefficients a, b, c, d, e and f are also given as

$$a = \frac{1}{2} i^2 \sigma_s^2 - \frac{1}{2} i (\rho - c) \quad (A.61)$$

$$b = - i^2 \sigma_s^2 - \frac{n}{\Delta w} - \rho \quad (A.62)$$

$$c = \frac{1}{2} i^2 \sigma_s^2 + \frac{1}{2} i (\rho - c) \quad (A.63)$$

$$d = \frac{1}{2} n\, j^2 \sigma_w^2 \quad (A.64)$$

$$e = \frac{n}{\Delta w} - n\, j^2 \sigma_w^2 \quad (A.65)$$

$$f = \frac{1}{2} n\, j^2 \sigma_w^2 \quad (A.66)$$

For this analysis, $H_{0,\, j,\, k} = 0$, and $H_{n+1,\, j,\, k} = 2H_{n,\, j,\, k} - H_{n-1,\, j,\, k}$, and thus the following equation results

$$\begin{cases} b\, H_{1,\, j,\, k} + c\, H_{2,\, j,\, k} = RHS\, [i = 1] \\ a\, H_{i-1,\, j,\, k} + b\, H_{i,\, j,\, k} + c\, H_{i+1,\, j,\, k} = RHS\, [i = 2, 3, \cdots, n-1] \\ (a - c)\, H_{n-1,\, j,\, k} + (b + 2c)\, H_{i+1,\, j,\, k} = RHS\, [i = n] \end{cases} \quad (A.67)$$

The tridiagonal matrix, implicit in the metal price direction, for equation (A.67), is formulated in a matrix form as

$$CD = E \quad (A.68)$$

where

$$C = \begin{bmatrix} b_{1,1,k} & c_{1,2,k} & 0 & 0 & 0 & 0 \\ a_{2,1,k} & b_{2,2,k} & c_{2,3,k} & 0 & 0 & 0 \\ 0 & a_{3,2,k} & b_{3,3,k} & c_{3,4,k} & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & a_{n-1,n-2,k} & b_{n-1,n-1,k} & c_{n-1,n,k} \\ 0 & 0 & 0 & 0 & (a-c)_{n,n-1,k} & (b+2c)_{n,n,k} \end{bmatrix} \quad (A.69)$$

$$D = \begin{bmatrix} H_{1,1,k} + H_{1,2,k} \\ H_{2,1,k} + H_{2,2,k} + H_{2,3,k} \\ H_{3,2,k} + H_{3,3,k} + H_{3,4,k} \\ \vdots \\ H_{n-1,n-2,k} + H_{n-1,n-1,k} + H_{n-1,n,k} \\ H_{n,n-1,k} + H_{n,n,k} \end{bmatrix} \quad (A.70)$$

$$E = -\begin{bmatrix} e\, H_{1,1,k+1} + f\, H_{1,2,k+1} - \bar{n}\,\alpha \\ d\, H_{2,1,k+1} + e\, H_{2,2,k+1} + f\, H_{2,3,k+1} - \bar{n}\,\alpha \\ d\, H_{3,2,k+1} + e\, H_{3,3,k+1} + f\, H_{3,4,k+1} - \bar{n}\,\alpha \\ \vdots \\ d\, H_{n-1,n-2,k+1} + e\, H_{n-1,n-1,k+1} + f\, H_{n-1,n,k+1} - \bar{n}\,\alpha \\ (d - f)\, H_{n,n-1,k+1} + (e + 2f)\, H_{n,n,k+1} - \bar{n}\,\alpha \end{bmatrix} \quad (A.71)$$

Finite Difference Equations from k to k-1

$$H_s = (H_{i+1,\,j,\,k} - H_{i-1,\,j,\,k}) / 2\,\Delta S \quad (A.72)$$

$$H_{ss} = (H_{i+1,\,j,\,k} - 2\,H_{i,\,j,\,k} + H_{i-1,\,j,\,k}) / (\Delta S)^2 \quad (A.73)$$

$$H_w = (H_{i,\,j,\,k} - H_{i,\,j,\,k-1}) / \Delta w \quad (A.74)$$

$$H_{EE} = (H_{i,\,j+1,\,k-1} - 2\,H_{i,\,j,\,k-1} + H_{i,\,j-1,\,k-1}) / (\Delta E)^2 \quad (A.75)$$

Substituting equations (A.72) to (A.75) into equation (A.36), we have

$$\frac{1}{2}\frac{\sigma_s^{\,2}(i\,\Delta S)^2}{(\Delta S)^2}\left(H_{i+1,j,k} - 2H_{i,j,k} + H_{i-1,j,k}\right) + \frac{i\Delta S(\rho - c)}{2\Delta S}\left(H_{i+1,j,k} - H_{i-1,j,k}\right)$$

$$+\frac{1}{2}\frac{n\,\sigma_w^{\,2}(j\,\Delta E)^2}{(\Delta E)^2}\left(H_{i,j+1,k-1} - 2H_{i,j,k-1} + H_{i,j-1,k-1}\right)$$

$$+\frac{n}{\Delta w}\left(H_{i,j,k} - H_{i,j,k-1}\right) - n\alpha - \rho\, H_{i,j,k-1} = 0 \qquad (A.76)$$

$H_{i+1,j,k}$, $H_{i-1,j,k}$, and $H_{i,j,k}$ have been calculated already in the previous slice. By simplifying equation (A.76), we have

$$d\,H_{i,j-1,k-1} + \bar{e}\,H_{i,j,k-1} + f\,H_{i,j+1,k-1} = RHS \qquad (A.77)$$

where

$$RHS = -\left(a\,H_{i-1,j,k} + \bar{b}\,H_{i,j,k}\,c\,H_{i+1,j,k}\right) + n\alpha \qquad (A.78)$$

a, c, d and f are same as above, and

$$\bar{b} = -\,i^2\sigma_S^{\,2} + n/\Delta w \qquad (A.79)$$

$$\bar{e} = -\,n\,j^2\sigma_w^{\,2} - \rho - n/\Delta w \qquad (A.80)$$

For this analysis, $H_{i,0,k-1} = 0$, and $H_{i,n+1,k-1} = 2H_{i,n,k-1} - H_{i,n-1,k-1}$, and thus the following equation results

$$\begin{cases} \bar{e}\,H_{i,1,k-1} + f\,H_{i,2,k-1} = RHS\,[j=1] \\ d\,H_{i,j-1,k-1} + \bar{e}\,H_{i,j,k-1} + f\,H_{i,j+1,k-1} = RHS\,[j=2,3,\cdots,n-1] \\ (d-f)\,H_{i,n-1,k-1} + (\bar{e}+2f)\,H_{i,n,k-1} = RHS\,[j=n] \end{cases} \qquad (A.81)$$

The tridiagonal matrix, implicit in the expected ore grade, for equation (A.81), is constructed in a matrix form as

$$MX = P \tag{A.82}$$

where

$$M = \begin{bmatrix} \bar{e}_{1,1,k-1} & f_{1,2,k-1} & 0 & 0 & 0 & 0 \\ d_{2,1,k-1} & \bar{e}_{2,2,k-1} & f_{2,3,k-1} & 0 & 0 & 0 \\ 0 & d_{3,2,k-1} & \bar{e}_{3,3,k-1} & f_{3,4,k-1} & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & d_{n-1,n-2,k-1} & \bar{e}_{n-1,n-1,k-1} & f_{n-1,n,k-1} \\ 0 & 0 & 0 & 0 & (d-f)_{n,n-1,k-1} & (\bar{e}+2f)_{n,n,k-1} \end{bmatrix}$$

$$X = \begin{bmatrix} H_{1,1,k-1} + H_{1,2,k-1} \\ H_{2,1,k-1} + H_{2,2,k-1} + H_{2,3,k-1} \\ H_{3,2,k-1} + H_{3,3,k-1} + H_{3,4,k-1} \\ \vdots \\ H_{n-1,n-2,k-1} + H_{n-1,n-1,k-1} + H_{n-1,n,k-1} \\ H_{n,n-1,k-1} + H_{n,n,k-1} \end{bmatrix} \tag{A.84}$$

$$P = -\begin{bmatrix} \bar{b}\, H_{1,1,k} + c\, H_{1,2,k} - \bar{n}\,\alpha \\ a\, H_{2,1,k} + \bar{b}\, H_{2,2,k} + c\, H_{2,3,k} - \bar{n}\,\alpha \\ a\, H_{3,2,k} + \bar{b}\, H_{3,3,k} + c\, H_{3,4,k} - \bar{n}\,\alpha \\ \vdots \\ a\, H_{n-1,n-2,k} + \bar{b}\, H_{n-1,n-1,k} + c\, H_{n-1,n,k} - \bar{n}\,\alpha \\ (a - c)\, H_{n,n-1,k} + (\bar{b} + 2c)\, H_{n,n,k} - \bar{n}\,\alpha \end{bmatrix} \tag{A.85}$$

The tridiagonal matrix, implicit in the expected ore grade, for equation (A.81), is constructed in a matrix form as

$$MX = P \tag{A.82}$$

where

$$M = \begin{bmatrix} \bar{e}_{1,1,k-1} & f_{1,2,k-1} & 0 & 0 & 0 & 0 \\ d_{2,1,k-1} & \bar{e}_{2,2,k-1} & f_{2,3,k-1} & 0 & 0 & 0 \\ 0 & d_{3,2,k-1} & \bar{e}_{3,3,k-1} & f_{3,4,k-1} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & d_{n-1,n-2,k-1} & \bar{e}_{n-1,n-1,k-1} & f_{n-1,n,k-1} \\ 0 & 0 & 0 & 0 & (d-f)_{n,n-1,k-1} & (\bar{e}+2f)_{n,n,k-1} \end{bmatrix}$$

$$X = \begin{bmatrix} H_{1,1,k-1} + H_{1,2,k-1} \\ H_{2,1,k-1} + H_{2,2,k-1} + H_{2,3,k-1} \\ H_{3,2,k-1} + H_{3,3,k-1} + H_{3,4,k-1} \\ \vdots \\ H_{n-1,n-2,k-1} + H_{n-1,n-1,k-1} + H_{n-1,n,k-1} \\ H_{n,n-1,k-1} + H_{n,n,k-1} \end{bmatrix} \tag{A.84}$$

$$P = -\begin{bmatrix} \bar{b}\, H_{1,1,k} + c\, H_{1,2,k} - \bar{n} \\ a\, H_{2,1,k} + \bar{b}\, H_{2,2,k} + c\, H_{2,3,k} - \bar{n}\,\alpha \\ a\, H_{3,2,k} + \bar{b}\, H_{3,3,k} + c\, H_{3,4,k} - \bar{n}\,\alpha \\ \vdots \\ a\, H_{n-1,n-2,k} + \bar{b}\, H_{n-1,n-1,k} + c\, H_{n-1,n,k} - \bar{n}\,\alpha \\ (a - c)\, H_{n,n-1,k} + (\bar{b} + 2c)\, H_{n,n,k} - \bar{n}\,\alpha \end{bmatrix} \tag{A.85}$$

Appendix D

LATTICE AND REPORT CONTROL PARAMETERS

D.1. Lattice and Report Control Parameters of 1 - D Model

Table A.1 Lattice Data for CUMINE_1D

Small Change in Ore Reserve, deltaQ	20.00
Maximum Metal Price, Smax ($/lb)	10.00
Small Change in Metal Price, deltaS	0.05

Table A.2 Report Control Data for CUMINE_1D

Lower Price for Reporting Output, los, ($/lb)	0.00
Higher Price for Reporting Output, his, ($/lb)	10.00
Metal Price Reporting Rate, Srate	0.10
Metal Reserve Reporting Rate, Qrate	100.00
Lower Metal Reserves for Reporting, loQ, (Kt)	0.00
Higher Metal Reserves for Reporting, hiQ, (Kt)	2000.00

D.2. Lattice and Report Control Parameters of 2 - D Model

Table A.3 Lattice Parameters for CUMINE_2D

Small Change in Time, deltat	0.04
Small Change in Metal Price, deltas	0.02
Small Change in Expected Ore Grade, deltae	2.50
Maximum Metal Price, Smax, ($/lb)	5.00
Maximum Expected Ore Grade, Emax, (lbs/ton)	250.0
Numemag	6

Table A.4 Report Control Data for CUMINE_2D

Lower Metal Price for Reporting, los, ($/lb)	0.00
Higher Metal Price for Reporting, his, ($/lb)	5.00
Metal Price Reporting Rate, srate	0.10
Lower Expected Ore Grade for Reporting, loe	0.00
Higher Expected Ore Grade for Reporting, hie	100.0
Expected Ore Grade Reporting Rate, erate	2.50

APPENDIX E

PROGRAM OUTPUT FOR THE 1 - D MODEL

Appendix E.1 Waiting, Feasibility Study, and Investment Values ($M) Under Uncertainty

Feasibility Cost ($M): 20.000000
Probability associated with 1M tons Reserve: 0.100000

Price ($/lb)	Feasibility	Investment	Waiting
0.00	-20.000000	-475.000000	0.000000
0.05	-19.493044	-475.000000	0.498095
0.10	-17.676845	-475.000000	2.282551
0.15	-14.355494	-475.000000	5.545852
0.20	-9.408212	-475.000000	10.406667
0.25	-2.744549	-475.000000	16.953864
0.30	5.708771	-473.908971	25.259439
0.35	16.013798	-471.393103	35.384356
0.40	28.224602	-455.628808	47.381742
0.45	42.389257	-436.559886	61.298830
0.50	58.551151	-414.238254	77.178251
0.55	76.749899	-388.682055	95.058925
0.60	97.022001	-359.890229	114.976715
0.65	119.401345	-327.850446	136.964917
0.70	143.919588	-292.543655	161.054636
0.75	170.606461	-253.842228	187.275082
0.80	199.490012	-211.055459	215.653812
0.85	230.596802	-162.721191	246.216924
0.90	263.952075	-109.633487	278.989221
0.95	299.579892	-52.640485	313.994343
1.00	337.503247	7.583660	351.254881
1.05	377.744171	70.498145	390.792483
1.10	420.323813	135.665595	432.627927
1.15	465.262519	202.729707	476.781204
1.20	512.579894	271.398307	523.278576
1.25	562.294862	341.430368	572.117636
1.30	614.425713	412.625961	623.337355
1.35	668.990155	484.818400	676.948131
1.40	726.005346	557.868057	732.966822
1.45	785.487935	631.657452	791.409787
1.50	847.454094	706.087330	852.292916
1.55	911.919549	781.073506	915.631657
1.60	978.899602	856.544316	981.441046
1.65	1048.340942	932.438538	1049.735731
1.70	1119.961703	1008.703703	1120.529988
1.75	1193.737315	1085.294695	1193.837748
1.80	1269.672610	1162.172610	1269.672610
1.85	1346.803802	1239.303802	1346.803802

1.90	1424.159091	1316.659091	1424.159091
1.95	1501.713105	1394.213105	1501.713105
2.00	1579.443726	1471.943726	1579.443726
2.05	1657.331625	1549.831625	1657.331625
2.10	1735.359872	1627.859872	1735.359872
2.15	1813.513604	1706.013604	1813.513604
2.20	1891.779741	1784.279741	1891.779741
2.25	1970.146752	1862.646752	1970.146752
2.30	2048.604447	1941.104447	2048.604447
2.35	2127.143804	2019.643804	2127.143804
2.40	2205.756819	2098.256819	2205.756819
2.45	2284.436381	2176.936381	2284.436381
2.50	2363.176154	2255.676154	2363.176154

Feasibility Cost ($M): 20.000000

Probability associated with 1M tons Reserve: 0.200000

Price ($/lb)	Feasibility	Investment	Waiting
0.00	-20.000000	-475.000000	0.000000
0.05	-19.503050	-475.000000	0.487614
0.10	-17.722699	-475.000000	2.234519
0.15	-14.466903	-475.000000	5.429151
0.20	-9.617268	-475.000000	10.187680
0.25	-3.085129	-475.000000	16.597104
0.30	5.201342	-472.817942	24.727905
0.35	15.302973	-467.910621	34.639764
0.40	27.272765	-452.372442	46.384689
0.45	41.157844	-433.667728	60.008920
0.50	57.000742	-411.835134	75.554190
0.55	74.840291	-386.884513	93.058601
0.60	94.712271	-358.809479	112.557261
0.65	116.649900	-327.594234	134.082766
0.70	140.684213	-293.217472	157.665565
0.75	166.844352	-255.445551	183.334254
0.80	195.157812	-213.467427	211.115811
0.85	225.650630	-166.382893	241.035784
0.90	258.347551	-114.945377	273.118454
0.95	293.272161	-59.916583	307.386962
1.00	330.447001	-1.903251	343.863428
1.05	369.893667	58.606741	382.569040
1.10	411.632890	121.217799	423.524141
1.15	455.684615	185.606744	466.748301
1.20	502.068060	251.507758	512.260377
1.25	550.801774	318.700840	5[illegible].078569
1.30	601.903690	387.002871	610.220473
1.35	655.391162	456.260617	662.703118

1.40	711.281011	526.345224	717.543008
1.45	769.589558	597.147836	774.756158
1.50	830.332656	668.576099	834.358124
1.55	893.525719	740.551333	896.364028
1.60	959.183748	813.006256	960.788592
1.65	1027.184921	885.883119	1027.646152
1.70	1096.950686	959.132185	1096.950686
1.75	1168.417444	1032.710479	1168.417444
1.80	1241.580747	1106.580747	1241.580747
1.85	1315.710599	1180.710599	1315.710599
1.90	1390.071796	1255.071796	1390.071796
1.95	1464.639644	1329.639644	1464.639644
2.00	1539.392497	1404.392497	1539.392497
2.05	1614.311328	1479.311328	1614.311328
2.10	1689.379374	1554.379374	1689.379374
2.15	1764.581829	1629.581829	1764.581829
2.20	1839.905586	1704.905586	1839.905586
2.25	1915.339017	1780.339017	1915.339017
2.30	1990.871783	1855.871783	1990.871783
2.35	2066.494671	1931.494671	2066.494671
2.40	2142.199457	2007.199457	2142.199457
2.45	2217.978782	2082.978782	2217.978782
2.50	2293.826052	2158.826052	2293.826052

Feasibility Cost ($M): 20.000000

Probability associated with 1M tons Reserve: 0.300000

Price ($/lb)	Feasibility	Investment	Waiting
0.00	-20.000000	-475.000000	0.000000
0.05	-19.513057	-475.000000	0.477437
0.10	-17.768552	-475.000000	2.187884
0.15	-14.578312	-475.000000	5.315842
0.20	-9.826324	-475.000000	9.975058
0.25	-3.425710	-475.000000	16.250715
0.30	4.693914	-471.726913	24.211822
0.35	14.592148	-464.428140	33.916[illegible]
0.40	26.320928	-449.116076	45.416619
0.45	39.926432	-430.775569	58.756507
0.50	55.450333	-409.432015	73.977340
0.55	72.930683	-385.086972	91.116426
0.60	92.402540	-357.728729	110.208141
0.65	113.898456	-327.338022	131.284398
0.70	137.448838	-293.891288	154.375014
0.75	163.082243	-257.048874	179.507986
0.80	190.825611	-215.879395	206.709729
0.85	220.704457	-170.044594	236.005259

0.90	252.743026	-120.257266	267.418349
0.95	286.964429	-67.192680	300.971659
1.00	323.390755	-11.390162	336.686844
1.05	362.043162	46.715338	374.584653
1.10	402.941967	106.770003	414.685002
1.15	446.106710	168.483781	457.007055
1.20	491.556225	231.617208	501.569273
1.25	539.308687	295.971313	548.389478
1.30	589.381666	361.379781	597.484898
1.35	641.792169	427.702834	648.872206
1.40	696.556676	494.822391	702.567564
1.45	753.691181	562.638221	758.586650
1.50	813.211218	631.064868	816.944695
1.55	875.131889	700.029160	877.656509
1.60	939.467895	769.468195	940.736502
1.65	1006.028900	839.327699	1006.198715
1.70	1073.939668	909.560668	1073.939668
1.75	1143.097574	980.126264	1143.097574
1.80	1213.488883	1050.988883	1213.488883
1.85	1284.617397	1122.117397	1284.617397
1.90	1355.984501	1193.484501	1355.984501
1.95	1427.566183	1265.066183	1427.566183
2.00	1499.341268	1336.841268	1499.341268
2.05	1571.291031	1408.791031	1571.291031
2.10	1643.398875	1480.898875	1643.398875
2.15	1715.650053	1553.150053	1715.650053
2.20	1788.031430	1625.531430	1788.031430
2.25	1860.531282	1698.031282	1860.531282
2.30	1933.139119	1770.6391[illegible]	1933.139119
2.35	2005.845539	1843.34553[illegible]	2005.845539
2.40	2078.642094	1916.142094	2078.642094
2.45	2151.521184	1989.021184	2151.521184
2.50	2224.475951	2061.975951	2224.475951

Feasibility Cost ($M): 20.000000

Probability associated with 1M tons Reserve: 0.400000

Price ($/lb)	Feasibility	Investment	Waiting
0.00	-20.000000	-475.000000	0.000000
0.05	-19.523063	-475.000000	0.467323
0.10	-17.814406	-475.000000	2.141536
0.15	-14.689721	-475.000000	5.203233
0.20	-10.035380	-475.000000	9.763750
0.25	-3.766291	-475.000000	15.906465
0.30	4.186485	-470.635883	23.698926
0.35	13.881323	-460.945658	33.198332

0.40	25.369092	-445.859711	44.454527
0.45	38.695019	-427.883410	57.511826
0.50	53.899924	-407.028896	72.410225
0.55	71.021074	-383.289431	89.186242
0.60	90.092810	-356.647979	107.873523
0.65	111.147012	-327.081810	128.503308
0.70	134.213463	-294.565105	151.104778
0.75	159.320134	-258.652197	175.705341
0.80	186.493411	-218.291363	202.330850
0.85	215.758284	-173.706296	231.005792
0.90	247.138501	-125.569156	261.753437
0.95	280.656698	-74.468777	294.595963
1.00	316.334508	-20.877072	329.554568
1.05	354.192658	34.823934	366.649560
1.10	394.251043	92.322207	405.900436
1.15	436.528806	151.360817	447.325950
1.20	481.044391	211.726659	490.944175
1.25	527.815600	273.241785	536.772555
1.30	576.859643	335.756691	584.827952
1.35	628.193175	399.145052	635.126687
1.40	681.832341	463.299558	687.684577
1.45	737.792805	528.128606	742.516971
1.50	796.089780	593.553637	799.638777
1.55	856.738059	659.506986	859.064489
1.60	919.752041	725.930135	920.808214
1.65	984.872880	792.772279	984.883695
1.70	1050.928651	859.989151	1050.928651
1.75	1117.777703	927.542048	1117.777703
1.80	1185.397020	995.397020	1185.397020
1.85	1253.524194	1063.524194	1253.524194
1.90	1321.897205	1131.897205	1321.897205
1.95	1390.492722	1200.492722	1390.492722
2.00	1459.290039	1269.290039	1459.290039
2.05	1528.270734	1338.270734	1528.270734
2.10	1597.418377	1407.418377	1597.418377
2.15	1666.718278	1476.718278	1666.718278
2.20	1736.157275	1546.157275	1736.157275
2.25	1805.723547	1615.723547	1805.723547
2.30	1875.406455	1685.406455	1875.406455
2.35	1945.196406	1755.196406	1945.196406
2.40	2015.084732	1825.084732	2015.084732
2.45	2085.063585	1895.063585	2085.063585
2.50	2155.125849	1965.125849	2155.125849

Feasibility Cost ($M):			20.000000
Probability associated with 1M tons Reserve:			0.500000
Price ($/lb)	Feasibility	Investment	Waiting
0.00	-20.000000	-475.000000	0.000000
0.05	-19.533069	-475.000000	0.457280
0.10	-17.860259	-475.000000	2.095511
0.15	-14.801129	-475.000000	5.091407
0.20	-10.244436	-475.000000	9.553910
0.25	-4.106872	-475.000000	15.564608
0.30	3.679056	-469.544854	23.189595
0.35	13.170498	-457.463177	32.484843
0.40	24.417255	-442.603345	43.499123
0.45	37.463607	-424.991251	56.275798
0.50	52.349515	-404.625776	70.854006
0.55	69.111466	-381.491890	87.269477
0.60	87.783080	-355.567229	105.555137
0.65	108.395568	-326.825598	125.741552
0.70	130.978089	-295.238921	147.857278
0.75	155.558026	-260.255520	171.929132
0.80	182.161211	-220.703332	197.982414
0.85	210.812112	-177.367998	226.041083
0.90	241.533977	-130.881045	256.127907
0.95	274.348967	-81.744875	288.264591
1.00	309.278262	-30.363983	322.471876
1.05	346.342153	22.932530	358.769634
1.10	385.560120	77.874411	397.176941
1.15	426.950902	134.237854	437.712150
1.20	470.532557	191.836109	480.392945
1.25	516.322513	250.512257	525.236395
1.30	564.337619	310.133601	572.258999
1.35	614.594182	370.587269	621.476727
1.40	667.108007	431.776725	672.905059
1.45	721.894428	493.618991	726.559011
1.50	778.968341	556.042406	782.453171
1.55	838.344230	618.984813	840.601723
1.60	900.036188	682.392075	901.018470
1.65	963.716859	746.216859	963.716859
1.70	1027.917633	810.417633	1027.917633
1.75	1092.457832	874.957832	1092.457832
1.80	1157.305157	939.805157	1157.305157
1.85	1222.430991	1004.930991	1222.430991
1.90	1287.809910	1070.309910	1287.809910
1.95	1353.419261	1135.919261	1353.419261
2.00	1419.238810	1201.738810	1419.238810
2.05	1485.250437	1267.750437	1485.250437

2.10	1551.437879	1333.937879	1551.437879
2.15	1617.786503	1400.286503	1617.786503
2.20	1684.283120	1466.783120	1684.283120
2.25	1750.915812	1533.415812	1750.915812
2.30	1817.673791	1600.173791	1817.673791
2.35	1884.547273	1667.047273	1884.547273
2.40	1951.527369	1734.027369	1951.527369
2.45	2018.605987	1801.105987	2018.605987
2.50	2085.775748	1868.275748	2085.775748

Feasibility Cost ($M): 40.000000

Probability associated with 1M tons Reserve: 0.100000

Price ($/lb)	Feasibility	Investment	Waiting
0.00	-40.000000	-475.000000	0.000000
0.05	-39.493044	-475.000000	0.490355
0.10	-37.676845	-475.000000	2.247083
0.15	-34.355494	-475.000000	5.459676
0.20	-29.408212	-475.000000	10.244958
0.25	-22.744549	-475.000000	16.690418
0.30	-14.291229	-473.908971	24.866933
0.35	-3.986202	-471.393103	34.834520
0.40	8.224602	-455.628808	46.645478
0.45	22.389257	-436.559886	60.346309
0.50	38.551151	-414.238254	75.978980
0.55	56.749899	-388.682055	93.581806
0.60	77.022001	-359.890229	113.190094
0.65	99.401345	-327.850446	134.836623
0.70	123.919588	-292.543655	158.552012
0.75	150.606461	-253.842228	184.365018
0.80	179.490012	-211.055459	212.302772
0.85	210.596802	-162.721191	242.390965
0.90	243.952075	-109.633487	274.654014
0.95	279.579892	-52.640485	309.115192
1.00	317.503247	7.583660	345.796740
1.05	357.744171	70.498145	384.719967
1.10	400.323813	135.665595	425.905331
1.15	445.262519	202.729707	469.372511
1.20	492.579894	271.398307	515.140470
1.25	542.294862	341.430368	563.227512
1.30	594.425713	412.625961	613.651330
1.35	[illegible].990155	484.818400	666.429049
1.40	[illegible]	557.868057	721.577267
1.45	765.487935	631.657452	779.112088
1.50	827.454094	706.087330	839.049155

1.55	891.919549	781.073506	901.403677
1.60	958.899602	856.544316	966.190456
1.65	1028.340942	932.438538	1033.423911
1.70	1099.961703	1008.703703	1103.118098
1.75	1173.737315	1085.294695	1175.286730
1.80	1249.672610	1162.172610	1249.943196
1.85	1326.803802	1239.303802	1326.803802
1.90	1404.159091	1316.659091	1404.159091
1.95	1481.713105	1394.213105	1481.713105
2.00	1559.443726	1471.943726	1559.443726
2.05	1637.331625	1549.831625	1637.331625
2.10	1715.359872	1627.859872	1715.359872
2.15	1793.513604	1706.013604	1793.513604
2.20	1871.779741	1784.279741	1871.779741
2.25	1950.146752	1862.646752	1950.146752
2.30	2028.604447	1941.104447	2028.604447
2.35	2107.143804	2019.643804	2107.143804
2.40	2185.756819	2098.256819	2185.756819
2.45	2264.436381	2176.936381	2264.436381
2.50	2343.176154	2255.676154	2343.176154

Feasibility Cost($M): 40.000000

Probability associated with 1M tons Reserve: 0.200000

Price ($/lb)	Feasibility	Investment	Waiting
0.00	-40.000000	-475.000000	0.000000
0.05	-39.503050	-475.000000	0.479229
0.10	-37.722699	-475.000000	2.196094
0.15	-34.466903	-475.000000	5.335790
0.20	-29.617268	-475.000000	10.012490
0.25	-23.085129	-475.000000	16.311696
0.30	-14.798658	-472.817942	24.302678
0.35	-4.697027	-467.910621	34.044090
0.40	7.272765	-452.372442	45.587046
0.45	21.157844	-433.667728	58.976992
0.50	37.000742	-411.835134	74.254942
0.55	54.840291	-386.884513	91.458342
0.60	74.712271	-358.809479	110.621699
0.65	96.649900	-327.594234	131.777046
0.70	120.684213	-293.217472	154.954309
0.75	146.844352	-255.445551	180.181594
0.80	175.157812	-213.467427	207.485412
0.85	205.650630	-166.382893	236.890874
0.90	238.347551	-114.945377	268.421842
0.95	273.272161	-59.916583	302.101062
1.00	310.447001	-1.903251	337.950269

1.05	349.893667	58.606741	375.990290
1.10	391.632890	121.217799	416.241117
1.15	435.684615	185.606744	458.721984
1.20	482.068060	251.507758	503.451423
1.25	530.801774	318.700840	550.447322
1.30	581.903690	387.002871	599.726974
1.35	635.391162	456.260617	651.307114
1.40	691.281011	526.345224	705.203964
1.45	749.589558	597.147836	761.433263
1.50	810.332656	668.576099	820.010299
1.55	873.525719	740.551333	880.949935
1.60	939.183748	813.006256	944.266638
1.65	1007.184921	885.883119	1009.974499
1.70	1076.950686	959.132185	1078.087255
1.75	1148.417444	1032.710479	1148.618310
1.80	1221.580747	1106.580747	1221.580747
1.85	1295.710599	1180.710599	1295.710599
1.90	1370.071796	1255.071796	1370.071796
1.95	1444.639644	1329.639644	1444.639644
2.00	1519.392497	1404.392497	1519.392497
2.05	1594.311328	1479.311328	1594.311328
2.10	1669.379374	1554.379374	1669.379374
2.15	1744.581829	1629.581829	1744.581829
2.20	1819.905586	1704.905586	1819.905586
2.25	1895.339017	1780.339017	1895.339017
2.30	1970.871783	1855.871783	1970.871783
2.35	2046.494671	1931.494671	2046.494671
2.40	2122.199457	2007.199457	2122.199457
2.45	2197.978782	2082.978782	2197.978782
2.50	2273.826052	2158.826052	2273.826052

Feasibility Cost ($M): 40.000000

Probability associated with 1M tons Reserve: 0.300000

Price ($/lb)	Feasibility	Investment	Waiting
0.00	-40.000000	-475.000000	0.000000
0.05	-39.513057	-475.000000	0.468600
0.10	-37.768552	-475.000000	2.147387
0.15	-34.578312	-475.000000	5.217449
0.20	-29.826324	-475.000000	9.790426
0.25	-23.425710	-475.000000	15.949924
0.30	-15.306086	-471.726913	23.763675
0.35	-5.407852	-464.428140	33.289035
0.40	6.320928	-449.116076	44.575983
0.45	19.926432	-430.775569	57.668957
0.50	35.450333	-409.432015	72.608061

0.55	52.930683	-385.086972	89.429912
0.60	72.402540	-357.728729	108.168250
0.65	93.898456	-327.338022	128.854398
0.70	117.448838	-293.891288	151.517619
0.75	143.082243	-257.048874	176.185394
0.80	170.825611	-215.879395	202.883648
0.85	200.704457	-170.044594	231.636935
0.90	232.743026	-120.257266	262.468586
0.95	266.964429	-67.192680	295.400843
1.00	303.390755	-11.390162	330.454961
1.05	342.043162	46.715338	367.651302
1.10	382.941967	106.770003	407.009417
1.15	426.106710	168.483781	448.548112
1.20	471.556225	231.617208	492.285509
1.25	519.308687	295.971313	538.239099
1.30	569.381666	361.379781	586.425790
1.35	621.792169	427.702834	636.861948
1.40	676.556676	494.822391	689.563435
1.45	733.691181	562.638221	744.545639
1.50	793.211218	631.064868	801.823510
1.55	855.131889	700.029160	861.411582
1.60	919.467895	769.468195	923.324001
1.65	986.028900	839.327699	987.574545
1.70	1053.939668	909.560668	1054.176647
1.75	1123.097574	980.126264	1123.097574
1.80	1193.488883	1050.988883	1193.488883
1.85	1264.617397	1122.117397	1264.617397
1.90	1335.984501	1193.484501	1335.984501
1.95	1407.566183	1265.066183	1407.566183
2.00	1479.341268	1336.841268	1479.341268
2.05	1551.291031	1408.791031	1551.291031
2.10	1623.398875	1480.898875	1623.398875
2.15	1695.650053	1553.150053	1695.650053
2.20	1768.031430	1625.531430	1768.031430
2.25	1840.531282	1698.031282	1840.531282
2.30	1913.139119	1770.639119	1913.139119
2.35	1985.845539	1843.345539	1985.845539
2.40	2058.642094	1916.142094	2058.642094
2.45	2131.521184	1989.021184	2131.521184
2.50	2204.475951	2061.975951	2204.475951

Feasibility Cost ($M): **40.000000**

Probability associated with 1M tons Reserve: **0.400000**

Price ($/lb)	Feasibility	Investment	Waiting
0.00	-40.000000	-475.000000	0.000000

0.05	-39.523063	-475.000000	0.458266
0.10	-37.814406	-475.000000	2.100031
0.15	-34.689721	-475.000000	5.102387
0.20	-30.035380	-475.000000	9.574515
0.25	-23.766291	-475.000000	15.598176
0.30	-15.813515	-470.635883	23.239609
0.35	-6.118677	-460.945658	32.554905
0.40	5.369092	-445.859711	43.592939
0.45	18.695019	-427.883410	56.397170
0.50	33.899924	-407.028896	71.006819
0.55	51.021074	-383.289431	87.457694
0.60	70.092810	-356.647979	105.782791
0.65	91.147012	-327.081810	126.012743
0.70	114.213463	-294.565105	148.176167
0.75	139.320134	-258.652197	172.299938
0.80	166.493411	-218.291363	198.409409
0.85	195.758284	-173.706296	226.528593
0.90	227.138501	-125.569156	256.680307
0.95	260.656698	-74.468777	288.886302
1.00	296.334508	-20.877072	323.167363
1.05	334.192658	34.823934	359.543405
1.10	374.251043	92.322207	398.033546
1.15	416.528806	151.360817	438.656179
1.20	461.044391	211.726659	481.429025
1.25	507.815600	273.241785	526.369190
1.30	556.859643	335.756691	573.493209
1.35	608.193175	399.145052	622.817088
1.40	661.832341	463.299558	674.356336
1.45	717.792805	528.128606	728.126006
1.50	776.089780	593.553637	784.140715
1.55	836.738059	659.5[illegible]986	842.414678
1.60	899.752041	725.930135	902.961728
1.65	964.872880	792.772279	965.795340
1.70	1030.928651	859.989151	1030.928651
1.75	1097.777703	927.542048	1097.777703
1.80	1165.397020	995.397020	1165.397020
1.85	1233.524194	1063.524194	1233.524194
1.90	1301.897205	1131.897205	1301.897205
1.95	1370.492722	1200.492722	1370.492722
2.00	1439.290039	1269.290039	1439.290039
2.05	1508.270734	1338.270734	1508.270734
2.10	1577.418377	1407.418377	1577.418377
2.15	1646.718278	1476.718278	1646.718278
2.20	1716.157275	1546.157275	1716.157275
2.25	1785.723547	1615.723547	1785.723547

2.30	1855.406455	1685.406455	1855.406455
2.35	1925.196406	1755.196406	1925.196406
2.40	1995.084732	1825.084732	1995.084732
2.45	2065.063585	1895.063585	2065.063585
2.50	2135.125849	1965.125849	2135.125849

Feasibility Cost ($M):			40.000000
Probability associated with 1M tons Reserve:			0.500000
Price ($/lb)	Feasibility	Investment	Waiting
0.00	-40.000000	-475.000000	0.000000
0.05	-39.533069	-475.000000	0.448037
0.10	-37.860259	-475.000000	2.053157
0.15	-34.801129	-475.000000	4.988499
0.20	-30.244436	-475.000000	9.360806
0.25	-24.106872	-475.000000	15.250015
0.30	-16.320944	-469.544854	22.720886
0.35	-6.829502	-457.463177	31.828257
0.40	4.417255	-442.603345	42.619916
0.45	17.463607	-424.991251	55.138348
0.50	32.349515	-404.625776	69.421899
0.55	49.111466	-381.491890	85.505580
0.60	67.783080	-355.567229	103.421649
0.65	88.395568	-326.825598	123.200054
0.70	110.978089	-295.238921	144.868776
0.75	135.558026	-260.255520	168.454088
0.80	162.161211	-220.703332	193.980779
0.85	190.812112	-177.367998	221.472323
0.90	221.533977	-130.881045	250.951031
0.95	254.348967	-81.744875	282.438166
1.00	289.278262	-30.363983	315.954051
1.05	326.342153	22.932530	351.518155
1.10	365.560120	77.874411	389.149171
1.15	406.950902	134.237854	428.865080
1.20	450.532557	191.836109	470.683207
1.25	496.322513	250.512257	514.620277
1.30	544.337619	310.133601	560.692457
1.35	594.594182	370.587269	608.915393
1.40	647.108007	431.776725	659.304252
1.45	701.894428	493.618991	711.873746
1.50	758.968341	556.042406	766.638169
1.55	818.344230	618.984813	823.611418
1.60	880.036188	682.392075	882.807018
1.65	943.716859	746.216859	944.238142
1.70	1007.917633	810.417633	1007.917633
1.75	1072.457832	874.957832	1072.457832

1.80	1137.305157	939.805157	1137.305157
1.85	1202.430991	1004.930991	1202.430991
1.90	1267.809910	1070.309910	1267.809910
1.95	1333.419261	1135.919261	1333.419261
2.00	1399.238810	1201.738810	1399.238810
2.05	1465.250437	1267.750437	1465.250437
2.10	1531.437879	1333.937879	1531.437879
2.15	1597.786503	1400.286503	1597.786503
2.20	1664.283120	1466.783120	1664.283120
2.25	1730.915812	1533.415812	1730.915812
2.30	1797.673791	1600.173791	1797.673791
2.35	1864.547273	1667.047273	1864.547273
2.40	1931.527369	1734.027369	1931.527369
2.45	1998.605987	1801.105987	1998.605987
2.50	2065.775748	1868.275748	2065.775748

Appendix E.2 Open, Closed, Investment and Waiting Values ($M) Under Certainty

Metal Reserve:1M tons

Price ($/lb)	Open	Closed	Investment	Waiting
0.00	-25.000000	-25.000000	-375.000000	0.000000
0.05	-25.000000	-25.000000	-375.000000	0.516962
0.10	-25.000000	-25.000000	-375.000000	2.369008
0.15	-25.000000	-25.000000	-375.000000	5.755915
0.20	-25.000000	-24.964778	-375.000000	10.800845
0.25	-25.000000	-20.843273	-375.000000	17.596032
0.30	-25.000000	-12.286856	-375.000000	26.216200
0.35	-24.875584	0.124662	-374.875584	36.724623
0.40	-8.885174	16.115173	-358.885174	49.176438
0.45	10.547955	35.548417	-339.452045	63.620669
0.50	33.358627	58.359219	-316.641373	80.101560
0.55	59.520404	84.521142	-290.479596	98.659507
0.60	89.029021	114.029922	-260.970979	119.331731
0.65	121.893342	146.894421	-228.106658	142.152789
0.70	158.130161	183.131436	-191.869839	167.154963
0.75	197.761095	222.762582	-152.238905	194.368570
0.80	241.356509	265.812379	-108.643491	223.822212
0.85	290.940510	312.307034	-59.059490	255.542975
0.90	345.678402	362.273656	-4.321598	289.556600
0.95	404.635612	415.739746	54.635612	325.887623
1.00	467.070571	472.732855	117.070571	364.559494
1.05	532.389548	533.280364	182.389548	405.594675
1.10	600.113391	597.409337	250.113391	449.014736
1.15	669.852671	665.146423	319.852671	494.840423
1.20	741.288857	736.288857	391.288857	543.091728
1.25	814.159896	809.159896	464.159896	593.787949
1.30	888.249051	883.249051	538.249051	646.947737
1.35	963.376183	958.376183	613.376183	702.589148
1.40	1039.390890	1034.390890	689.390890	760.729680
1.45	1116.167067	1111.167067	766.167067	821.386311
1.50	1193.598561	1188.598561	843.598561	884.575532
1.55	1271.595680	1266.595680	921.595680	950.313379
1.60	1350.082376	1345.082376	1000.082376	1018.615455
1.65	1428.993958	1423.993958	1078.993958	1089.496963
1.70	1508.275220	1503.275220	1158.275220	1162.972721
1.75	1587.878911	1582.878911	1237.878911	1239.057186
1.80	1667.764474	1662.764474	1317.764474	1317.764474

1.85	1747.897005	1742.897005	1397.897005	1397.897005
1.90	1828.246387	1823.246387	1478.246387	1478.246387
1.95	1908.786566	1903.786566	1558.786566	1558.786566
2.00	1989.494955	1984.494955	1639.494955	1639.494955
2.05	2070.351922	2065.351922	1720.351922	1720.351922
2.10	2151.340371	2146.340371	1801.340371	1801.340371
2.15	2232.445379	2227.445379	1882.445379	1882.445379
2.20	2313.653896	2308.653896	1963.653896	1963.653896
2.25	2394.954487	2389.954487	2044.954487	2044.954487
2.30	2476.337110	2471.337110	2126.337110	2126.337110
2.35	2557.792936	2552.792936	2207.792936	2207.792936
2.40	2639.314182	2634.314182	2289.314182	2289.314182
2.45	2720.893980	2715.893980	2370.893980	2370.893980
2.50	2802.526256	2797.526256	2452.526256	2452.526256

Metal Reserves: 2M tons

Price	Open	Closed	Investment	Waiting
0.00	-25.000000	-25.000000	-475.000000	0.000000
0.05	-25.000000	-25.000000	-475.000000	0.933863
0.10	-25.000000	-25.000000	-475.000000	4.279482
0.15	-25.000000	-25.000000	-475.000000	10.397741
0.20	-25.000000	-19.851303	-475.000000	19.511127
0.25	-25.000000	-7.534225	-475.000000	31.786257
0.30	-14.089708	10.910614	-464.089708	47.358112
0.35	10.073647	35.074124	-439.926353	66.340996
0.40	39.793310	64.793969	-410.206690	88.834510
0.45	75.017497	100.018365	-374.982503	114.927213
0.50	115.748447	140.74[illegible]53	-334.251553	144.699030
0.55	162.016220	187.017593	-287.983780	178.222932
0.60	213.865543	238.867212	-236.134457	215.566160
0.65	271.348803	296.350800	-178.651197	256.791136
0.70	334.522157	359.524512	-115.477843	301.956177
0.75	404.488959	428.446037	-45.511041	351.116051
0.80	483.593337	503.173294	33.593337	404.322423
0.85	570.264005	583.763666	120.264005	461.624224
0.90	663.237910	670.273553	213.237910	523.067953
0.95	761.510251	762.758120	311.510251	588.697933
1.00	864.272033	861.271164	414.272033	658.556524
1.05	970.865061	965.865061	520.865061	732.684306
1.10	1080.748821	1075.748821	630.748821	811.120240
1.15	1193.475708	1188.475708	743.475708	893.901804

1.20	1308.672218	1303.672218	858.672218	981.065113
1.25	1426.024514	1421.024514	976.024514	1072.645026
1.30	1545.267202	1540.267202	1095.267202	1168.675238
1.35	1666.174538	1661.174538	1216.174538	1269.188364
1.40	1788.553449	1783.553449	1338.553449	1374.216013
1.45	1912.237982	1907.237982	1462.237982	1483.788856
1.50	2037.084813	2032.084813	1587.084813	1597.936683
1.55	2162.969626	2157.969626	1712.969626	1716.688459
1.60	2289.784149	2284.784149	1839.784149	1840.072376
1.65	2417.433717	2412.433717	1967.433717	1967.433717
1.70	2545.835267	2540.835267	2095.835267	2095.835267
1.75	2674.915665	2669.915665	2224.915665	2224.915665
1.80	2804.610313	2799.610313	2354.610313	2354.610313
1.85	2934.861982	2929.861982	2484.861982	2484.861982
1.90	3065.619819	3060.619819	2615.619819	2615.619819
1.95	3196.838522	3191.838522	2746.838522	2746.838522
2.00	3328.477620	3323.477620	2878.477620	2878.477620
2.05	3460.500874	3455.500874	3010.500874	3010.500874
2.10	3592.875757	3587.875757	3142.875757	3142.875757
2.15	3725.573006	3720.573006	3275.573006	3275.573006
2.20	3858.566240	3853.566240	3408.566240	3408.566240
2.25	3991.831624	3986.831624	3541.831624	3541.831624
2.30	4125.347582	4120.347582	3675.347582	3675.347582
2.35	4259.094547	4254.094547	3809.094547	3809.094547
2.40	4393.054739	4388.054739	3943.054739	3943.054739
2.45	4527.211974	4522.211974	4077.211974	4077.211974
2.50	4661.551495	4656.551495	4211.551495	4211.551495

Appendix E.3 Metal Price Boundaries at Different Mine States

O is Open, C is Closed, A is Abandon

Metal Reserves (Kt)	O to C	O to A	C to O	C to A
0	-	-	-	-
20	-	0.700	1.025	0.725
40	-	0.675	1.050	0.600
60	-	0.675	1.100	0.525
80	-	0.675	1.125	0.475
100	-	0.650	1.150	0.450
120	-	0.650	1.175	0.425
140	-	0.650	1.200	0.400
160	-	0.625	1.225	0.375
180	-	0.625	1.225	0.375
200	-	0.625	1.250	0.350
220	0.750	0.625	1.250	0.325
240	0.775	0.600	1.250	0.325
260	0.775	0.575	1.250	0.325
280	0.800	0.550	1.275	0.300
300	0.800	0.550	1.275	0.300
320	0.800	0.525	1.275	0.300
340	0.800	0.525	1.275	0.275
360	0.800	0.500	1.275	0.275
380	0.800	0.500	1.275	0.275
400	0.800	0.475	1.275	0.275
420	0.800	0.475	1.275	0.275
440	0.800	0.475	1.275	0.250
460	0.800	0.450	1.275	0.250
480	0.800	0.450	1.275	0.250
500	0.800	0.450	1.275	0.250
520	0.800	0.450	1.250	0.250
540	0.800	0.425	1.250	0.250
560	0.800	0.425	1.250	0.225
580	0.800	0.425	1.250	0.225
600	0.800	0.425	1.250	0.225
620	0.800	0.400	1.250	0.225
640	0.800	0.400	1.250	0.225
660	0.800	0.400	1.225	0.225
680	0.800	0.400	1.225	0.225
700	0.800	0.400	1.225	0.225
720	0.800	0.375	1.225	0.200
740	0.800	0.375	1.225	0.200
760	0.800	0.375	1.225	0.200
780	0.800	0.375	1.225	0.200

800	0.775	0.375	1.200	0.200
820	0.775	0.375	1.200	0.200
840	0.775	0.350	1.200	0.200
860	0.775	0.350	1.200	0.200
880	0.775	0.350	1.200	0.200
900	0.775	0.350	1.200	0.200
920	0.775	0.350	1.200	0.200
940	0.775	0.350	1.175	0.200
960	0.775	0.350	1.175	0.200
980	0.775	0.350	1.175	0.200
1000	0.775	0.325	1.175	0.175
1020	0.775	0.325	1.175	0.175
1040	0.775	0.325	1.175	0.175
1060	0.775	0.325	1.175	0.175
1080	0.775	0.325	1.175	0.175
1100	0.775	0.325	1.150	0.175
1120	0.775	0.325	1.150	0.175
1140	0.750	0.325	1.150	0.175
1160	0.750	0.325	1.150	0.175
1180	0.750	0.325	1.150	0.175
1200	0.750	0.325	1.150	0.175
1220	0.750	0.300	1.150	0.175
1240	0.750	0.300	1.150	0.175
1260	0.750	0.300	1.150	0.175
1280	0.750	0.300	1.125	0.175
1300	0.750	0.300	1.125	0.175
1320	0.750	0.300	1.125	0.175
1340	0.750	0.300	1.125	0.175
1360	0.750	0.300	1.125	0.175
1380	0.750	0.300	1.125	0.175
1400	0.750	0.300	1.125	0.175
1420	0.750	0.300	1.125	0.175
1440	0.750	0.300	1.125	0.175
1460	0.750	0.300	1.125	0.150
1480	0.750	0.300	1.100	0.150
1500	0.750	0.300	1.100	0.150
1520	0.725	0.275	1.100	0.150
1540	0.725	0.275	1.100	0.150
1560	0.725	0.275	1.100	0.150
1580	0.725	0.275	1.100	0.150
1600	0.725	0.275	1.100	0.150
1620	0.725	0.275	1.100	0.150
1640	0.725	0.275	1.100	0.150
1660	0.725	0.275	1.100	0.150
1680	0.725	0.275	1.100	0.150

1700	0.725	0.275	1.100	0.150
1720	0.725	0.275	1.075	0.150
1740	0.725	0.275	1.075	0.150
1760	0.725	0.275	1.075	0.150
1780	0.725	0.275	1.075	0.150
1800	0.725	0.275	1.075	0.150
1820	0.725	0.275	1.075	0.150
1840	0.725	0.275	1.075	0.150
1860	0.725	0.275	1.075	0.150
1880	0.725	0.275	1.075	0.150
1900	0.725	0.275	1.075	0.150
1920	0.725	0.275	1.075	0.150
1940	0.725	0.275	1.075	0.150
1960	0.725	0.250	1.075	0.150
1980	0.725	0.250	1.075	0.150
2000	0.725	0.250	1.050	0.150

Appendix E.4 Feasibility Study and Investment Boundaries Under Certainty and Uncertainty

1. Boundary Price ($/lb) To invest Under Certainty

Metal Reserves (Mt)	investment
0	-
1	1.800
2	1.625

2. Boundary Price ($/lb) to undertake Feasibility, and Investment under Uncertainty

feasibility cost ($M)	prob of 1M tons	feasibility	investment
100000	0.00	-	1.925
100000	0.05	-	1.925
100000	0.10	-	1.950
100000	0.15	-	1.950
100000	0.20	-	1.950
100000	0.25	-	1.950
100000	0.30	-	1.975
100000	0.35	-	1.975
100000	0.40	-	1.975
100000	0.45	-	2.000
100000	0.50	-	2.000
20	0.00	1.825	-
20	0.05	1.800	-
20	0.10	1.800	-
20	0.15	1.725	-
20	0.20	1.700	-
20	0.25	1.700	-
20	0.30	1.675	-
20	0.35	1.675	-
20	0.40	1.675	-
20	0.45	1.650	-
20	0.50	1.650	-
40	0.00	1.850	-
40	0.05	1.850	-
40	0.10	1.825	-
40	0.15	1.800	-
40	0.20	1.800	-
40	0.25	1.750	-
40	0.30	1.725	-
40	0.35	1.725	-
40	0.40	1.700	-
40	0.45	1.700	-
40	0.50	1.700	-

APPENDIX F

PROGRAM OUTPUT FOR THE 2 - D MODEL

Appendix F.1 Program Output for wfww and nfww

economics data

intrate	0.03
vol	0.2
prisk	0.4
gamma	0
s_target	1.5
bnd_layer	0

feasibility study data

iv0	0
numv	4
staged	2
qo	100
dev_cost[0]	550
dev_cost[1]	450
dev_cost[2]	425
dev_cost[3]	415
dev_cost[4]	405
unit_prod_cost	0.5
dev_time	3
prod_time	20
var[0]	3
var[1]	0.113
var[2]	0.026
var[3]	0.011
var[4]	0.006
stud_time[0]	1
stud_time[1]	1
stud_time[2]	1
stud_time[3]	1
stud_cost[0]	0.05
stud_cost[1]	0.2
stud_cost[2]	5
stud_cost[3]	5
waitopt[0]	1
waitopt[1]	1
waitopt[2]	1
waitopt[3]	1
waitopt[4]	1

lattice data

deltat	0.04
smax	5
deltas	0.02
emax	250
deltae	2.5
numemag	6

report control data

los	0
his	5
srate	0.1
loe	0
hie	100
erate	2.5
numt[0]	25
numt[1]	25
numt[2]	25
numt[3]	25
nums	250
lossamp	0
hissamp	250
ssamprate	5
numssamp	50
nume	100
loesamp	0
hiesamp	40
esamprate	1
numesamp	40

Mine Values in $ M at various Metal Prices and Expected Ore Grades

wfww	var(E)	state	E\S		1.00	1.50	2.00	2.50
opt v	3.000	e	0.0	0.50	0.00	0.00	0.00	0.00
opt v	3.000	e	2.5	0.00	2.31	11.58	28.63	50.78
opt v	3.000	e	5.0	0.11	6.92	33.43	79.26	135.65
opt v	3.000	e	7.5	0.34	12.58	59.68	138.27	232.14
opt v	3.000	e	10.0	0.62	18.86	88.51	201.86	334.61
opt v	3.000	e	12.5	0.94	25.58	119.03	268.27	440.58
opt v	3.000	e	15.0	1.27	32.60	150.70	336.54	548.69
opt v	3.000	e	17.5	1.63	39.84	183.21	406.04	658.15
opt v	3.000	e	20.0	1.99	47.25	216.31	476.39	768.43
opt v	3.000	e	22.5	2.36	54.80	249.86	547.29	879.17
opt v	3.000	e	25.0	2.74	62.44	283.73	618.54	990.11
opt v	3.000	e	27.5	3.12	70.16	317.83	689.99	1101.07
opt v	3.000	e	30.0	3.51	77.94	352.09	761.54	1211.92
opt v	3.000	e	32.5	3.90	85.75	386.45	833.08	1340.06
opt v	3.000	e	35.0	4.29	93.60	420.87	904.55	1484.21
opt v	3.000	e	37.5	4.69	101.47	455.32	975.90	1628.36
opt v	3.000	e	40.0	5.08	109.36	489.76	1047.34	1772.51
opt v	3.000	e	42.5	5.48	117.25	524.17	1130.86	1916.66
opt v	3.000	e	45.0	5.88	125.14	558.54	1227.94	2060.81
opt v	3.000	e	47.5	6.27	133.02	592.84	1325.82	2204.96
opt v	3.000	e	50.0	6.67	140.90	627.06	1423.70	2349.11
opt v	3.000	e	52.5	7.07	148.77	661.19	1521.58	2493.26
opt v	3.000	e	55.0	7.46	156.62	695.21	1619.46	2637.41
opt v	3.000	e	57.5	7.86	164.46	729.13	1717.34	2781.56
opt v	3.000	e	60.0	8.25	172.28	762.94	1815.22	2925.72
opt v	3.000	e	62.5	8.64	180.08	798.53	1913.10	3069.87
opt v	3.000	e	65.0	9.04	187.85	837.24	2010.98	3214.02
opt v	3.000	e	67.5	9.43	195.60	878.97	2108.86	3358.17
opt v	3.000	e	70.0	9.82	203.33	923.00	2206.74	3502.32
opt v	3.000	e	72.5	10.21	211.03	969.13	2304.62	3646.47
opt v	3.000	e	75.0	10.59	218.71	1016.99	2402.50	3790.62
opt v	3.000	e	77.5	10.98	226.36	1066.66	2500.38	3934.77
opt v	3.000	e	80.0	11.37	233.98	1117.60	2598.26	4078.92
opt v	3.000	e	82.5	11.75	241.57	1169.21	2696.14	4223.07
opt v	3.000	e	85.0	12.13	249.13	1220.82	2794.02	4367.22
opt v	3.000	e	87.5	12.51	256.67	1272.43	2891.90	4511.38
opt v	3.000	e	90.0	12.89	264.17	1324.04	2989.78	4655.53
opt v	3.000	e	92.5	13.27	271.65	1375.65	3087.66	4799.68
opt v	3.000	e	95.0	13.64	279.09	1427.26	3185.54	4943.83
opt v	3.000	e	97.5	14.02	286.51	1478.87	3283.42	5087.98
opt v	3.000	e	100.0	14.39	293.89	1530.48	3381.31	5232.13

opt v	0.113 e	0.0	14.76	0.00	0.00	0.00	0.00
opt v	0.113 e	2.5	0.00	0.02	0.13	0.46	1.22
opt v	0.113 e	5.0	0.00	0.51	2.99	10.44	27.21
opt v	0.113 e	7.5	0.03	1.71	9.97	34.47	84.80
opt v	0.113 e	10.0	0.08	3.76	21.91	73.56	169.84
opt v	0.113 e	12.5	0.18	6.65	38.65	125.19	284.48
opt v	0.113 e	15.0	0.33	10.33	59.61	186.19	428.07
opt v	0.113 e	17.5	0.51	14.72	84.16	259.23	572.22
opt v	0.113 e	20.0	0.72	19.72	111.68	346.71	716.38
opt v	0.113 e	22.5	0.97	25.27	141.64	444.09	860.53
opt v	0.113 e	25.0	1.24	31.30	173.60	541.97	1004.68
opt v	0.113 e	27.5	1.54	37.74	207.32	639.85	1148.83
opt v	0.113 e	30.0	1.85	44.57	243.49	737.73	1292.98
opt v	0.113 e	32.5	2.19	51.71	282.01	835.61	1437.13
opt v	0.113 e	35.0	2.54	59.14	322.62	933.49	1581.28
opt v	0.113 e	37.5	2.90	66.81	365.03	1031.37	1725.43
opt v	0.113 e	40.0	3.28	74.71	409.08	1129.25	1869.58
opt v	0.113 e	42.5	3.67	82.81	454.52	1227.13	2013.73
opt v	0.113 e	45.0	4.07	91.08	501.25	1325.01	2157.88
opt v	0.113 e	47.5	4.48	99.52	549.17	1422.89	2302.03
opt v	0.113 e	50.0	4.89	108.09	598.14	1520.77	2446.19
opt v	0.113 e	52.5	5.32	116.80	648.02	1618.65	2590.34
opt v	0.113 e	55.0	5.75	125.62	698.58	1716.53	2734.49
opt v	0.113 e	57.5	6.18	134.55	750.19	1814.41	2878.64
opt v	0.113 e	60.0	6.62	143.57	801.80	1912.29	3022.79
opt v	0.113 e	62.5	7.07	152.69	853.41	2010.17	3166.94
opt v	0.113 e	65.0	7.52	161.88	905.02	2108.06	3311.09
opt v	0.113 e	67.5	7.98	171.15	956.63	2205.94	3455.24
opt v	0.113 e	70.0	8.44	180.48	1008.24	2303.82	3599.39
opt v	0.113 e	72.5	8.90	189.88	1059.85	2401.70	3743.54
opt v	0.113 e	75.0	9.37	199.33	1111.46	2499.58	3887.69
opt v	0.113 e	77.5	9.84	208.84	1163.07	2597.46	4031.85
opt v	0.113 e	80.0	10.32	218.40	1214.68	2695.34	4176.00
opt v	0.113 e	82.5	10.79	228.00	1266.29	2793.22	4320.15
opt v	0.113 e	85.0	11.27	237.65	1317.90	2891.10	4464.30
opt v	0.113 e	87.5	11.75	247.33	1369.51	2988.98	4608.45
opt v	0.113 e	90.0	12.23	257.05	1421.12	3086.86	4752.60
opt v	0.113 e	92.5	12.72	266.81	1472.73	3184.74	4896.75
opt v	0.113 e	95.0	13.21	276.60	1524.33	3282.62	5040.90
opt v	0.113 e	97.5	13.70	286.43	1575.94	3380.50	5185.05
opt v	0.113 e	100.0	14.19	296.32	1627.55	3478.38	5329.20
opt v	0.026 e	0.0	14.68	0.00	0.00	0.00	0.00
opt v	0.026 e	2.5	0.00	0.00	0.00	0.00	0.00
opt v	0.026 e	5.0	0.00	0.37	2.18	7.62	20.10
opt v	0.026 e	7.5	0.02	1.38	8.03	28.05	73.95

opt v	0.026 e	10.0	0.07	3.18	18.52	64.66	170.52
opt v	0.026 e	12.5	0.16	5.83	33.98	118.64	308.19
opt v	0.026 e	15.0	0.29	9.29	54.10	188.87	452.34
opt v	0.026 e	17.5	0.46	13.47	78.46	273.92	596.49
opt v	0.026 e	20.0	0.66	18.30	106.61	370.48	740.64
opt v	0.026 e	22.5	0.90	23.71	138.11	468.36	884.79
opt v	0.026 e	25.0	1.16	29.62	172.58	566.24	1028.95
opt v	0.026 e	27.5	1.45	35.97	209.58	664.12	1173.10
opt v	0.026 e	30.0	1.76	42.72	248.93	762.00	1317.25
opt v	0.026 e	32.5	2.10	49.81	290.22	859.88	1461.40
opt v	0.026 e	35.0	2.44	57.22	333.41	957.76	1605.55
opt v	0.026 e	37.5	2.81	64.88	378.01	1055.64	1749.70
opt v	0.026 e	40.0	3.18	72.80	424.18	1153.52	1893.85
opt v	0.026 e	42.5	3.57	80.94	471.56	1251.40	2038.00
opt v	0.026 e	45.0	3.97	89.26	520.03	1349.28	2182.15
opt v	0.026 e	47.5	4.38	97.74	569.44	1447.16	2326.30
opt v	0.026 e	50.0	4.79	106.40	619.91	1545.04	2470.45
opt v	0.026 e	52.5	5.22	115.21	671.24	1642.92	2614.61
opt v	0.026 e	55.0	5.65	124.11	722.85	1740.80	2758.76
opt v	0.026 e	57.5	6.09	133.17	774.46	1838.68	2902.91
opt v	0.026 e	60.0	6.53	142.31	826.07	1936.56	3047.06
opt v	0.026 e	62.5	6.98	151.56	877.68	2034.44	3191.21
opt v	0.026 e	65.0	7.43	160.87	929.29	2132.32	3335.36
opt v	0.026 e	67.5	7.89	170.31	980.90	2230.20	3479.51
opt v	0.026 e	70.0	8.35	179.75	1032.51	2328.08	3623.66
opt v	0.026 e	72.5	8.82	189.36	1084.12	2425.96	3767.81
opt v	0.026 e	75.0	9.29	198.99	1135.73	2523.84	3911.96
opt v	0.026 e	77.5	9.76	208.62	1187.34	2621.72	4056.11
opt v	0.026 e	80.0	10.23	218.40	1238.95	2719.61	4200.26
opt v	0.026 e	82.5	10.71	228.22	1290.56	2817.49	4344.42
opt v	0.026 e	85.0	11.20	238.03	1342.16	2915.37	4488.57
opt v	0.026 e	87.5	11.68	247.86	1393.77	3013.25	4632.72
opt v	0.026 e	90.0	12.16	257.85	1445.38	3111.13	4776.87
opt v	0.026 e	92.5	12.65	267.84	1496.99	3209.01	4921.02
opt v	0.026 e	95.0	13.14	277.82	1548.60	3306.89	5065.17
opt v	0.026 e	97.5	13.63	287.81	1600.21	3404.77	5209.32
opt v	0.026 e	100.0	14.12	297.86	1651.82	3502.65	5353.47
opt v	0.011 e	0.0	14.61	0.00	0.00	0.00	0.00
opt v	0.011 e	2.5	0.00	0.00	0.00	0.00	0.00
opt v	0.011 e	5.0	0.00	0.39	2.30	8.03	21.18
opt v	0.011 e	7.5	0.02	1.44	8.37	29.21	77.03
opt v	0.011 e	10.0	0.07	3.32	19.34	67.50	178.01
opt v	0.011 e	12.5	0.16	6.06	35.32	123.30	317.90
opt v	0.011 e	15.0	0.30	9.62	56.02	195.59	462.05
opt v	0.011 e	17.5	0.47	13.90	81.01	282.80	606.20

opt v	0.011 e	20.0	0.68	18.84	109.79	380.19	750.35
opt v	0.011 e	22.5	0.92	24.36	141.93	478.07	894.50
opt v	0.011 e	25.0	1.19	30.38	176.98	575.95	1038.65
opt v	0.011 e	27.5	1.49	36.84	214.66	673.83	1182.80
opt v	0.011 e	30.0	1.81	43.69	254.56	771.71	1326.95
opt v	0.011 e	32.5	2.14	50.89	296.47	869.59	1471.11
opt v	0.011 e	35.0	2.50	58.37	340.07	967.47	1615.26
opt v	0.011 e	37.5	2.86	66.14	385.34	1065.35	1759.41
opt v	0.011 e	40.0	3.24	74.13	431.92	1163.23	1903.56
opt v	0.011 e	42.5	3.64	82.34	479.74	1261.11	2047.71
opt v	0.011 e	45.0	4.04	90.74	528.68	1358.99	2191.86
opt v	0.011 e	47.5	4.45	99.31	578.61	1456.87	2336.01
opt v	0.011 e	50.0	4.87	108.02	629.34	1554.75	2480.16
opt v	0.011 e	52.5	5.30	116.88	680.95	1652.63	2624.31
opt v	0.011 e	55.0	5.73	125.88	732.56	1750.51	2768.46
opt v	0.011 e	57.5	6.17	134.93	784.17	1848.39	2912.61
opt v	0.011 e	60.0	6.62	144.18	835.78	1946.27	3056.77
opt v	0.011 e	62.5	7.07	153.43	887.39	2044.15	3200.92
opt v	0.011 e	65.0	7.53	162.86	938.99	2142.03	3345.07
opt v	0.011 e	67.5	7.99	172.30	990.60	2239.91	3489.22
opt v	0.011 e	70.0	8.45	181.85	1042.21	2337.79	3633.37
opt v	0.011 e	72.5	8.92	191.48	1093.82	2435.67	3777.52
opt v	0.011 e	75.0	9.39	201.10	1145.43	2533.55	3921.67
opt v	0.011 e	77.5	9.87	210.84	1197.04	2631.43	4065.82
opt v	0.011 e	80.0	10.34	220.65	1248.65	2729.31	4209.97
opt v	0.011 e	82.5	10.82	230.46	1300.26	2827.19	4354.12
opt v	0.011 e	85.0	11.31	240.28	1351.87	2925.07	4498.27
opt v	0.011 e	87.5	11.79	250.25	1403.48	3022.95	4642.42
opt v	0.011 e	90.0	12.28	260.24	1455.09	3120.83	4786.58
opt v	0.011 e	92.5	12.77	270.23	1506.70	3218.71	4930.73
opt v	0.011 e	95.0	13.26	280.22	1558.31	3316.59	5074.88
opt v	0.011 e	97.5	13.75	290.25	1609.92	3414.47	5219.03
opt v	0.011 e	100.0	14.24	300.41	1661.53	3512.35	5363.18
opt v	0.006 e	0.0	14.74	0.00	0.00	0.00	0.00
opt v	0.006 e	2.5	0.00	0.00	0.00	0.00	0.00
opt v	0.006 e	5.0	0.00	0.42	2.43	8.50	22.42
opt v	0.006 e	7.5	0.02	1.51	8.79	30.69	80.92
opt v	0.006 e	10.0	0.07	3.47	20.19	70.50	185.92
opt v	0.006 e	12.5	0.17	6.30	36.72	128.19	327.61
opt v	0.006 e	15.0	0.31	9.96	58.04	202.61	471.76
opt v	0.006 e	17.5	0.49	14.36	83.66	292.06	615.91
opt v	0.006 e	20.0	0.70	19.41	113.10	389.89	760.06
opt v	0.006 e	22.5	0.95	25.04	145.89	487.77	904.21
opt v	0.006 e	25.0	1.23	31.17	181.60	585.65	1048.36
opt v	0.006 e	27.5	1.53	37.73	219.85	683.53	1192.51

opt v	0.006 e	30.0	1.85	44.69	260.36	781.41	1336.66
opt v	0.006 e	32.5	2.19	51.97	302.77	879.29	1480.81
opt v	0.006 e	35.0	2.55	59.56	347.02	977.18	1624.96
opt v	0.006 e	37.5	2.92	67.40	392.67	1075.06	1769.11
opt v	0.006 e	40.0	3.31	75.47	439.73	1172.94	1913.27
opt v	0.006 e	42.5	3.70	83.76	488.01	1270.82	2057.42
opt v	0.006 e	45.0	4.11	92.23	537.34	1368.70	2201.57
opt v	0.006 e	47.5	4.52	100.88	587.77	1466.58	2345.72
opt v	0.006 e	50.0	4.95	109.68	639.04	1564.46	2489.87
opt v	0.006 e	52.5	5.38	118.59	690.65	1662.34	2634.02
opt v	0.006 e	55.0	5.82	127.64	742.26	1760.22	2778.17
opt v	0.006 e	57.5	6.26	136.81	793.87	1858.10	2922.32
opt v	0.006 e	60.0	6.71	146.06	845.48	1955.98	3066.47
opt v	0.006 e	62.5	7.16	155.41	897.09	2053.86	3210.62
opt v	0.006 e	65.0	7.62	164.85	948.70	2151.74	3354.77
opt v	0.006 e	67.5	8.09	174.33	1000.31	2249.62	3498.92
opt v	0.006 e	70.0	8.55	183.96	1051.92	2347.50	3643.08
opt v	0.006 e	72.5	9.02	193.59	1103.53	2445.38	3787.23
opt v	0.006 e	75.0	9.50	203.28	1155.14	2543.26	3931.38
opt v	0.006 e	77.5	9.97	213.09	1206.75	2641.14	4075.53
opt v	0.006 e	80.0	10.45	222.90	1258.36	2739.02	4219.68
opt v	0.006 e	82.5	10.93	232.71	1309.97	2836.90	4363.83
opt v	0.006 e	85.0	11.42	242.66	1361.58	2934.78	4507.98
opt v	0.006 e	87.5	11.90	252.65	1413.19	3032.66	4652.13
opt v	0.006 e	90.0	12.39	262.63	1464.80	3130.54	4796.28
opt v	0.006 e	92.5	12.88	272.62	1516.41	3228.42	4940.43
opt v	0.006 e	95.0	13.37	282.64	1568.02	3326.30	5084.58
opt v	0.006 e	97.5	13.87	292.80	1619.63	3424.18	5228.74
opt v	0.006 e	100.0	14.36	302.96	1671.24	3522.06	5372.89
			14.86				

nfww	var(E) state	E\S		1.00	1.50	2.00	2.50
hor v	3.000 e	0.0	0.50	0.00	0.00	0.00	0.00
hor v	3.000 e	2.5	0.00	2.31	11.58	28.63	50.78
hor v	3.000 e	5.0	0.07	6.92	33.43	79.26	135.65
hor v	3.000 e	7.5	0.30	12.58	59.68	138.27	232.14
hor v	3.000 e	10.0	0.58	18.86	88.51	201.86	334.61
hor v	3.000 e	12.5	0.90	25.58	119.03	268.27	440.58
hor v	3.000 e	15.0	1.24	32.60	150.70	336.54	548.69
hor v	3.000 e	17.5	1.59	39.84	183.21	406.04	658.15
hor v	3.000 e	20.0	1.95	47.25	216.31	476.39	768.43
hor v	3.000 e	22.5	2.33	54.80	249.86	547.29	879.17
hor v	3.000 e	25.0	2.71	62.44	283.73	618.54	990.11
hor v	3.000 e	27.5	3.09	70.16	317.83	689.99	1101.07

hor v	3.000 e	30.0	3.48	77.94	352.09	761.54	1211.92
hor v	3.000 e	32.5	3.87	85.75	386.45	833.08	1340.06
hor v	3.000 e	35.0	4.26	93.60	420.87	904.55	1484.21
hor v	3.000 e	37.5	4.66	101.47	455.32	975.90	1628.36
hor v	3.000 e	40.0	5.05	109.36	489.76	1047.09	1772.51
hor v	3.000 e	42.5	5.45	117.25	524.17	1130.06	1916.66
hor v	3.000 e	45.0	5.85	125.14	558.54	1227.94	2060.81
hor v	3.000 e	47.5	6.25	133.02	592.84	1325.82	2204.96
hor v	3.000 e	50.0	6.64	140.90	627.06	1423.70	2349.11
hor v	3.000 e	52.5	7.04	148.77	661.19	1521.58	2493.26
hor v	3.000 e	55.0	7.44	156.62	695.21	1619.46	2637.41
hor v	3.000 e	57.5	7.83	164.46	729.13	1717.34	2781.56
hor v	3.000 e	60.0	8.23	172.28	762.94	1815.22	2925.72
hor v	3.000 e	62.5	8.62	180.08	796.62	1913.10	3069.87
hor v	3.000 e	65.0	9.01	187.85	830.18	2010.98	3214.02
hor v	3.000 e	67.5	9.40	195.60	863.61	2108.86	3358.17
hor v	3.000 e	70.0	9.79	203.33	911.16	2206.74	3502.32
hor v	3.000 e	72.5	10.18	211.03	962.77	2304.62	3646.47
hor v	3.000 e	75.0	10.57	218.71	1014.38	2402.50	3790.62
hor v	3.000 e	77.5	10.96	226.36	1065.99	2500.38	3934.77
hor v	3.000 e	80.0	11.34	233.98	1117.60	2598.26	4078.92
hor v	3.000 e	82.5	11.73	241.57	1169.21	2696.14	4223.07
hor v	3.000 e	85.0	12.11	249.13	1220.82	2794.02	4367.22
hor v	3.000 e	87.5	12.49	256.67	1272.43	2891.90	4511.38
hor v	3.000 e	90.0	12.87	264.17	1324.04	2989.78	4655.53
hor v	3.000 e	92.5	13.25	271.65	1375.65	3087.66	4799.68
hor v	3.000 e	95.0	13.62	279.09	1427.26	3185.54	4943.83
hor v	3.000 e	97.5	14.00	286.51	1478.87	3283.42	5087.98
hor v	3.000 e	100.0	14.37	293.89	1530.48	3381.31	5232.13
hor v	0.113 e	0.0	14.74	0.00	0.00	0.00	0.00
hor v	0.113 e	2.5	0.00	0.00	0.00	0.30	1.11
hor v	0.113 e	5.0	0.00	0.32	2.86	10.43	27.21
hor v	0.113 e	7.5	0.00	1.54	9.92	34.47	84.80
hor v	0.113 e	10.0	0.00	3.61	21.91	73.56	169.84
hor v	0.113 e	12.5	0.00	6.53	38.65	125.19	283.92
hor v	0.113 e	15.0	0.13	10.24	59.61	186.19	428.07
hor v	0.113 e	17.5	0.31	14.64	84.16	253.83	572.22
hor v	0.113 e	20.0	0.53	19.67	111.68	346.21	716.38
hor v	0.113 e	22.5	0.78	25.24	141.64	444.09	860.53
hor v	0.113 e	25.0	1.05	31.28	173.60	541.97	1004.68
hor v	0.113 e	27.5	1.35	37.74	207.20	639.85	1148.83
hor v	0.113 e	30.0	1.66	44.57	242.13	737.73	1292.98
hor v	0.113 e	32.5	2.00	51.71	278.18	835.61	1437.13
hor v	0.113 e	35.0	2.35	59.14	315.14	933.49	1581.28
hor v	0.113 e	37.5	2.72	66.81	352.87	1031.37	1725.43

hor v	0.113 e	40.0	3.10	74.71	391.25	1129.25	1869.58
hor v	0.113 e	42.5	3.49	82.81	440.53	1227.13	2013.73
hor v	0.113 e	45.0	3.89	91.08	492.14	1325.01	2157.88
hor v	0.113 e	47.5	4.30	99.52	543.75	1422.89	2302.03
hor v	0.113 e	50.0	4.71	108.09	595.36	1520.77	2446.19
hor v	0.113 e	52.5	5.14	116.80	646.97	1618.65	2590.34
hor v	0.113 e	55.0	5.57	125.62	698.58	1716.53	2734.49
hor v	0.113 e	57.5	6.01	134.55	750.19	1814.41	2878.64
hor v	0.113 e	60.0	6.45	143.57	801.80	1912.29	3022.79
hor v	0.113 e	62.5	6.90	152.69	853.41	2010.17	3166.94
hor v	0.113 e	65.0	7.35	161.88	905.02	2108.06	3311.09
hor v	0.113 e	67.5	7.81	171.15	956.63	2205.94	3455.24
hor v	0.113 e	70.0	8.27	180.48	1008.24	2303.82	3599.39
hor v	0.113 e	72.5	8.74	189.88	1059.85	2401.70	3743.54
hor v	0.113 e	75.0	9.20	199.33	1111.46	2499.58	3887.69
hor v	0.113 e	77.5	9.67	208.84	1163.07	2597.46	4031.85
hor v	0.113 e	80.0	10.15	218.40	1214.68	2695.34	4176.00
hor v	0.113 e	82.5	10.63	228.00	1266.29	2793.22	4320.15
hor v	0.113 e	85.0	11.10	237.65	1317.90	2891.10	4464.30
hor v	0.113 e	87.5	11.59	247.33	1369.51	2988.98	4608.45
hor v	0.113 e	90.0	12.07	257.05	1421.12	3086.86	4752.60
hor v	0.113 e	92.5	12.56	266.81	1472.73	3184.74	4896.75
hor v	0.113 e	95.0	13.04	276.60	1524.33	3282.62	5040.90
hor v	0.113 e	97.5	13.53	286.42	1575.94	3380.50	5185.05
hor v	0.113 e	100.0	14.02	296.27	1627.55	3478.38	5329.20
hor v	0.026 e	0.0	14.52	0.00	0.00	0.00	0.00
hor v	0.026 e	2.5	0.00	0.00	0.00	0.00	0.00
hor v	0.026 e	5.0	0.00	0.00	0.00	3.57	17.52
hor v	0.026 e	7.5	0.00	0.00	3.93	26.02	73.95
hor v	0.026 e	10.0	0.00	0.00	15.18	63.93	164.04
hor v	0.026 e	12.5	0.00	1.25	31.28	115.51	308.19
hor v	0.026 e	15.0	0.00	4.84	51.88	177.54	452.34
hor v	0.026 e	17.5	0.00	9.17	76.34	272.60	596.49
hor v	0.026 e	20.0	0.00	14.14	103.98	370.48	740.64
hor v	0.026 e	22.5	0.00	19.68	134.23	468.36	884.79
hor v	0.026 e	25.0	0.00	25.71	166.59	566.24	1028.95
hor v	0.026 e	27.5	0.00	32.18	200.64	664.12	1173.10
hor v	0.026 e	30.0	0.00	39.02	236.07	762.00	1317.25
hor v	0.026 e	32.5	0.00	46.20	272.60	859.88	1461.40
hor v	0.026 e	35.0	0.00	53.67	310.05	957.76	1605.55
hor v	0.026 e	37.5	0.00	61.40	361.58	1055.64	1749.70
hor v	0.026 e	40.0	0.00	69.36	413.19	1153.52	1893.85
[illegible]	0.026 e	42.5	0.00	77.52	464.80	1251.40	2038.00
hor v	0.026 e	45.0	0.00	85.85	516.41	1349.28	2182.15
hor v	0.026 e	47.5	0.00	94.35	568.02	1447.16	2326.30

hor v	0.026 e	50.0	0.00	102.99	619.63	1545.04	2470.45
hor v	0.026 e	52.5	0.32	111.76	671.24	1642.92	2614.61
hor v	0.026 e	55.0	0.75	120.65	722.85	1740.80	2758.76
hor v	0.026 e	57.5	1.19	129.64	774.46	1838.68	2902.91
hor v	0.026 e	60.0	1.64	138.73	826.07	1936.56	3047.06
hor v	0.026 e	62.5	2.09	147.91	877.68	2034.44	3191.21
hor v	0.026 e	65.0	2.55	157.17	929.29	2132.32	3335.36
hor v	0.026 e	67.5	3.01	166.50	980.90	2230.20	3479.51
hor v	0.026 e	70.0	3.47	175.90	1032.51	2328.08	3623.66
hor v	0.026 e	72.5	3.94	185.36	1084.12	2425.96	3767.81
hor v	0.026 e	75.0	4.41	194.87	1135.73	2523.84	3911.96
hor v	0.026 e	77.5	4.89	204.44	1187.34	2621.72	4056.11
hor v	0.026 e	80.0	5.36	214.05	1238.95	2719.61	4200.26
hor v	0.026 e	82.5	5.84	223.71	1290.56	2817.49	4344.42
hor v	0.026 e	85.0	6.32	233.41	1342.16	2915.37	4488.57
hor v	0.026 e	87.5	6.81	243.14	1393.77	3013.25	4632.72
hor v	0.026 e	90.0	7.29	252.92	1445.38	3111.13	4776.87
hor v	0.026 e	92.5	7.78	262.72	1496.99	3209.01	4921.02
hor v	0.026 e	95.0	8.27	272.56	1548.60	3306.89	5065.17
hor v	0.026 e	97.5	8.76	282.42	1600.21	3404.77	5209.32
hor v	0.026 e	100.0	9.26	292.32	1651.82	3502.65	5353.47
hor v	0.011 e	0.0	9.75	0.00	0.00	0.00	0.00
hor v	0.011 e	2.5	0.00	0.00	0.00	0.00	0.00
hor v	0.011 e	5.0	0.00	0.00	0.00	3.51	17.41
hor v	0.011 e	7.5	0.00	0.00	3.91	25.98	74.33
hor v	0.011 e	10.0	0.00	0.00	15.55	65.36	173.75
hor v	0.011 e	12.5	0.00	1.40	32.17	118.66	317.90
hor v	0.011 e	15.0	0.00	5.07	53.24	184.43	462.05
hor v	0.011 e	17.5	0.00	9.48	78.13	282.31	606.20
hor v	0.011 e	20.0	0.00	14.53	106.22	380.19	750.35
hor v	0.011 e	22.5	0.00	20.15	136.92	478.07	894.50
hor v	0.011 e	25.0	0.00	26.28	169.70	575.95	1038.65
hor v	0.011 e	27.5	0.00	32.84	204.16	673.83	1182.80
hor v	0.011 e	30.0	0.00	39.77	239.94	771.71	1326.95
hor v	0.011 e	32.5	0.00	47.04	276.81	869.59	1471.11
hor v	0.011 e	35.0	0.00	54.59	319.68	967.47	1615.26
hor v	0.011 e	37.5	0.00	62.40	371.29	1065.35	1759.41
hor v	0.011 e	40.0	0.00	70.43	422.90	1163.23	1903.56
hor v	0.011 e	42.5	0.00	78.66	474.51	1261.11	2047.71
hor v	0.011 e	45.0	0.00	87.07	526.12	1358.99	2191.86
hor v	0.011 e	47.5	0.00	95.64	577.73	1456.87	2336.01
hor v	0.011 e	50.0	0.00	104.35	629.34	1554.75	2480.16
hor v	0.011 e	52.5	0.39	113.19	680.95	1652.63	2624.31
hor v	0.011 e	55.0	0.82	122.13	732.56	1750.51	2768.46
hor v	0.011 e	57.5	1.27	131.19	784.17	1848.39	2912.61

hor v	0.011 e	60.0	1.72	140.33	835.78	1946.27	3056.77
hor v	0.011 e	62.5	2.17	149.56	887.39	2044.15	3200.92
hor v	0.011 e	65.0	2.63	158.87	938.99	2142.03	3345.07
hor v	0.011 e	67.5	3.09	168.24	990.60	2239.91	3489.22
hor v	0.011 e	70.0	3.56	177.68	1042.21	2337.79	3633.37
hor v	0.011 e	72.5	4.03	187.18	1093.82	2435.67	3777.52
hor v	0.011 e	75.0	4.50	196.74	1145.43	2533.55	3921.67
hor v	0.011 e	77.5	4.98	206.34	1197.04	2631.43	4065.82
hor v	0.011 e	80.0	5.46	216.00	1248.65	2729.31	4209.97
hor v	0.011 e	82.5	5.94	225.69	1300.26	2827.19	4354.12
hor v	0.011 e	85.0	6.42	235.42	1351.87	2925.07	4498.27
hor v	0.011 e	87.5	6.91	245.19	1403.48	3022.95	4642.42
hor v	0.011 e	90.0	7.40	255.00	1455.09	3120.83	4786.58
hor v	0.011 e	92.5	7.89	264.84	1506.70	3218.71	4930.73
hor v	0.011 e	95.0	8.38	274.70	1558.31	3316.59	5074.88
hor v	0.011 e	97.5	8.87	284.60	1609.92	3414.47	5219.03
hor v	0.011 e	100.0	9.37	294.52	1661.53	3512.35	5363.18
hor v	0.006 e	0.0	9.87	0.00	0.00	0.00	0.00
hor v	0.006 e	2.5	0.00	0.00	0.00	0.00	0.00
hor v	0.006 e	5.0	0.00	0.00	0.00	0.00	0.00
hor v	0.006 e	7.5	0.00	0.00	0.00	0.00	39.30
hor v	0.006 e	10.0	0.00	0.00	0.00	0.00	183.46
hor v	0.006 e	12.5	0.00	0.00	0.00	96.25	327.61
hor v	0.006 e	15.0	0.00	0.00	0.00	194.13	471.76
hor v	0.006 e	17.5	0.00	0.00	0.00	292.01	615.91
hor v	0.006 e	20.0	0.00	0.00	19.73	389.89	760.06
hor v	0.006 e	22.5	0.00	0.00	71.34	487.77	904.21
hor v	0.006 e	25.0	0.00	0.00	122.95	585.65	1048.36
hor v	0.006 e	27.5	0.00	0.00	174.56	683.53	1192.51
hor v	0.006 e	30.0	0.00	0.00	226.17	781.41	1336.66
hor v	0.006 e	32.5	0.00	0.00	277.78	879.29	1480.81
hor v	0.006 e	35.0	0.00	0.00	329.39	977.18	1624.96
hor v	0.006 e	37.5	0.00	0.00	381.00	1075.06	1769.11
hor v	0.006 e	40.0	0.00	0.00	432.61	1172.94	1913.27
hor v	0.006 e	42.5	0.00	0.00	484.22	1270.82	2057.42
hor v	0.006 e	45.0	0.00	0.00	535.83	1368.70	2201.57
hor v	0.006 e	47.5	0.00	0.00	587.43	1466.58	2345.72
hor v	0.006 e	50.0	0.00	0.00	639.04	1564.46	2489.87
hor v	0.006 e	52.5	0.00	0.00	690.65	1662.34	2634.02
hor v	0.006 e	55.0	0.00	0.00	742.26	1760.22	2778.17
hor v	0.006 e	57.5	0.00	0.00	793.87	1858.10	2922.32
hor v	0.006 e	60.0	0.00	0.00	845.48	1955.98	3066.47
hor v	0.006 e	62.5	0.00	0.00	897.09	2053.86	3210.62
hor v	0.006 e	65.0	0.00	0.00	948.70	2151.74	3354.77
hor v	0.006 e	67.5	0.00	0.00	1000.31	2249.62	3498.92

hor v	0.006 e	70.0	0.00	0.00	1051.92	2347.50	3643.08
hor v	0.006 e	72.5	0.00	0.00	1103.53	2445.38	3787.23
hor v	0.006 e	75.0	0.00	0.00	1155.14	2543.26	3931.38
hor v	0.006 e	77.5	0.00	0.00	1206.75	2641.14	4075.53
hor v	0.006 e	80.0	0.00	0.00	1258.36	2739.02	4219.68
hor v	0.006 e	82.5	0.00	0.00	1309.97	2836.90	4363.83
hor v	0.006 e	85.0	0.00	0.00	1361.58	2934.78	4507.98
hor v	0.006 e	87.5	0.00	0.00	1413.19	3032.66	4652.13
hor v	0.006 e	90.0	0.00	0.00	1464.80	3130.54	4796.28
hor v	0.006 e	92.5	0.00	0.00	1516.41	3228.42	4940.43
hor v	0.006 e	95.0	0.00	0.00	1568.02	3326.30	5084.58
hor v	0.006 e	97.5	0.00	0.00	1619.63	3424.18	5228.74
hor v	0.006 e	100.0	0.00	0.00	1671.24	3522.06	5372.89
			0.00				

Appendix F.2 Feasibility Study, Waiting, and Investment Boundaries for wfww and nfww

wfww

var(E)	Ore	Grade	feasibility	Metal Price Boundaries: waiting	Metal Price Boundaries: investment
3.000	e	0.0	-	-	-
3.000	e	2.5	0.98	-	-
3.000	e	5.0	0.88	-	-
3.000	e	7.5	0.82	-	-
3.000	e	10.0	0.80	-	-
3.000	e	12.5	0.78	-	-
3.000	e	15.0	0.76	4.78	4.98
3.000	e	17.5	0.74	4.14	4.32
3.000	e	20.0	0.74	3.66	3.80
3.000	e	22.5	0.72	3.28	3.42
3.000	e	25.0	0.72	2.96	3.10
3.000	e	27.5	0.70	2.72	2.86
3.000	e	30.0	0.70	2.52	2.64
3.000	e	32.5	0.70	2.34	2.48
3.000	e	35.0	0.68	2.20	2.34
3.000	e	37.5	0.68	2.08	2.22
3.000	e	40.0	0.68	1.98	2.12
3.000	e	42.5	0.68	1.90	2.02
3.000	e	45.0	0.68	1.82	1.94
3.000	e	47.5	0.66	1.74	1.88
3.000	e	50.0	0.66	1.68	1.82
3.000	e	52.5	0.66	1.64	1.78
3.000	e	55.0	0.66	1.58	1.74
3.000	e	57.5	0.66	1.54	1.70
3.000	e	60.0	0.66	1.50	1.66
3.000	e	62.5	0.64	1.46	1.64
3.000	e	65.0	0.64	1.42	1.62
3.000	e	67.5	0.64	1.40	1.58
3.000	e	70.0	0.64	1.36	1.56
3.000	e	72.5	0.64	1.34	1.54
3.000	e	75.0	0.64	1.32	1.54
3.000	e	77.5	0.64	1.30	1.52
3.000	e	80.0	0.64	1.26	1.50
3.000	e	82.5	0.64	1.24	1.48
3.000	e	85.0	0.64	1.22	1.48
3.000	e	87.5	0.62	1.20	1.46
3.000	e	90.0	0.62	1.18	1.46
3.000	e	92.5	0.62	1.16	1.44

3.000	e	95.0	0.62	1.14	1.44
3.000	e	97.5	0.62	1.12	1.44
3.000	e	100.0	0.62	1.10	1.42
0.113	e	0.0	-	-	-
0.113	e	2.5	3.22	-	-
0.113	e	5.0	2.12	4.24	4.70
0.113	e	7.5	1.72	3.12	3.48
0.113	e	10.0	1.50	2.56	2.90
0.113	e	12.5	1.36	2.22	2.54
0.113	e	15.0	1.26	2.00	2.32
0.113	e	17.5	1.18	1.82	2.16
0.113	e	20.0	1.14	1.70	2.04
0.113	e	22.5	1.08	1.62	1.94
0.113	e	25.0	1.06	1.54	1.86
0.113	e	27.5	1.02	1.48	1.80
0.113	e	30.0	1.00	1.42	1.76
0.113	e	32.5	0.98	1.38	1.72
0.113	e	35.0	0.96	1.34	1.68
0.113	e	37.5	0.94	1.30	1.64
0.113	e	40.0	0.92	1.28	1.62
0.113	e	42.5	0.92	1.24	1.60
0.113	e	45.0	0.90	1.22	1.58
0.113	e	47.5	0.90	1.20	1.56
0.113	e	50.0	0.88	1.18	1.54
0.113	e	52.5	0.88	1.16	1.52
0.113	e	55.0	0.86	1.14	1.50
0.113	e	57.5	0.86	1.14	1.50
0.113	e	60.0	0.84	1.12	1.48
0.113	e	62.5	0.84	1.10	1.48
0.113	e	65.0	0.84	1.10	1.46
0.113	e	67.5	0.84	1.08	1.46
0.113	e	70.0	0.82	1.08	1.44
0.113	e	72.5	0.82	1.06	1.44
0.113	e	75.0	0.82	1.06	1.44
0.113	e	77.5	0.82	1.04	1.42
0.113	e	80.0	0.80	1.04	1.42
0.113	e	82.5	0.80	1.02	1.42
0.113	e	85.0	0.80	1.02	1.40
0.113	e	87.5	0.80	1.02	1.40
0.113	e	90.0	0.80	1.00	1.40
0.113	e	92.5	0.78	1.00	1.40
0.113	e	95.0	0.78	1.00	1.38
0.113	e	97.5	0.78	0.98	1.38
0.113	e	100.0	0.78	0.98	1.38
0.026	e	0.0	-	-	-

0.026	e	2.5	-	-	-
0.026	e	5.0	3.26	3.30	4.12
0.026	e	7.5	2.46	2.50	3.16
0.026	e	10.0	-	-	2.68
0.026	e	12.5	-	-	2.38
0.026	e	15.0	-	-	2.18
0.026	e	17.5	-	-	2.06
0.026	e	20.0	-	-	1.94
0.026	e	22.5	-	-	1.86
0.026	e	25.0	-	-	1.80
0.026	e	27.5	-	-	1.76
0.026	e	30.0	-	-	1.70
0.026	e	32.5	-	-	1.66
0.026	e	35.0	-	-	1.64
0.026	e	37.5	-	-	1.62
0.026	e	40.0	-	-	1.58
0.026	e	42.5	-	-	1.56
0.026	e	45.0	-	-	1.54
0.026	e	47.5	-	-	1.52
0.026	e	50.0	-	-	1.52
0.026	e	52.5	-	-	1.50
0.026	e	55.0	-	-	1.48
0.026	e	57.5	-	-	1.48
0.026	e	60.0	-	-	1.46
0.026	e	62.5	-	-	1.46
0.026	e	65.0	-	-	1.44
0.026	e	67.5	-	-	1.44
0.026	e	70.0	-	-	1.44
0.026	e	72.5	-	-	1.42
0.026	e	75.0	-	-	1.42
0.026	e	77.5	-	-	1.42
0.026	e	80.0	-	-	1.40
0.026	e	82.5	-	-	1.40
0.026	e	85.0	-	-	1.40
0.026	e	87.5	-	-	1.38
0.026	e	90.0	-	-	1.38
0.026	e	92.5	-	-	1.38
0.026	e	95.0	-	-	1.38
0.026	e	97.5	-	-	1.38
0.026	e	100.0	-	-	1.36
0.011	e	0.0	-	-	
0.011	e	2.5	-	-	
0.011	e	5.0	-	-	4.06
0.011	e	7.5	-	-	3.10
0.011	e	10.0	-	-	2.64

0.011	e	12.5	-	-	2.36
0.011	e	15.0	-	-	2.16
0.011	e	17.5	-	-	2.04
0.011	e	20.0	-	-	1.94
0.011	e	22.5	-	-	1.86
0.011	e	25.0	-	-	1.78
0.011	e	27.5	-	-	1.74
0.011	e	30.0	-	-	1.70
0.011	e	32.5	-	-	1.66
0.011	e	35.0	-	-	1.62
0.011	e	37.5	-	-	1.60
0.011	e	40.0	-	-	1.58
0.011	e	42.5	-	-	1.56
0.011	e	45.0	-	-	1.54
0.011	e	47.5	-	-	1.52
0.011	e	50.0	-	-	1.50
0.011	e	52.5	-	-	1.50
0.011	e	55.0	-	-	1.48
0.011	e	57.5	-	-	1.46
0.011	e	60.0	-	-	1.46
0.011	e	62.5	-	-	1.46
0.011	e	65.0	-	-	1.44
0.011	e	67.5	-	-	1.44
0.011	e	70.0	-	-	1.42
0.011	e	72.5	-	-	1.42
0.011	e	75.0	-	-	1.42
0.011	e	77.5	-	-	1.40
0.011	e	80.0	-	-	1.40
0.011	e	82.5	-	-	1.40
0.011	e	85.0	-	-	1.40
0.011	e	87.5	-	-	1.38
0.011	e	90.0	-	-	1.38
0.011	e	92.5	-	-	1.38
0.011	e	95.0	-	-	1.38
0.011	e	97.5	-	-	1.36
0.011	e	100.0	-	-	1.36
0.006	e	0.0	-	-	
0.006	e	2.5	-	-	
0.006	e	5.0	-	-	3.98
0.006	e	7.5	-	-	3.06
0.006	e	10.0	-	-	2.60
0.006	e	12.5	-	-	2.32
0.006	e	15.0	-	-	2.14
0.006	e	17.5	-	-	2.02
0.006	e	20.0	-	-	1.92

0.006	e	22.5	-	-	1.84
0.006	e	25.0	-	-	1.78
0.006	e	27.5	-	-	1.72
0.006	e	30.0	-	-	1.68
0.006	e	32.5	-	-	1.64
0.006	e	35.0	-	-	1.62
0.006	e	37.5	-	-	1.60
0.006	e	40.0	-	-	1.56
0.006	e	42.5	-	-	1.54
0.006	e	45.0	-	-	1.54
0.006	e	47.5	-	-	1.52
0.006	e	50.0	-	-	1.50
0.006	e	52.5	-	-	1.48
0.006	e	55.0	-	-	1.48
0.006	e	57.5	-	-	1.46
0.006	e	60.0	-	-	1.46
0.006	e	62.5	-	-	1.44
0.006	e	65.0	-	-	1.44
0.006	e	67.5	-	-	1.42
0.006	e	70.0	-	-	1.42
0.006	e	72.5	-	-	1.42
0.006	e	75.0	-	-	1.40
0.006	e	77.5	-	-	1.40
0.006	e	80.0	-	-	1.40
0.006	e	82.5	-	-	1.40
0.006	e	85.0	-	-	1.38
0.006	e	87.5	-	-	1.38
0.006	e	90.0	-	-	1.38
0.006	e	92.5	-	-	1.38
0.006	e	95.0	-	-	1.36
0.006	e	97.5	-	-	1.36
0.006	e	100.0	-	-	1.36

nfww		Metal Price Boundaries		
var(e)	Ore	Grade	feasibility	investment
3.000	e	0.0	-	-
3.000	e	2.5	0.40	-
3.000	e	5.0	0.32	-
3.000	e	7.5	0.26	-
3.000	e	10.0	0.24	-
3.000	e	12.5	0.22	-
3.000	e	15.0	0.22	4.86
3.000	e	17.5	0.20	4.22
3.000	e	20.0	0.20	3.72

3.000	e	22.5	0.18	3.34
3.000	e	25.0	0.18	3.02
3.000	e	27.5	0.18	2.78
3.000	e	30.0	0.18	2.58
3.000	e	32.5	0.18	2.40
3.000	e	35.0	0.16	2.26
3.000	e	37.5	0.16	2.14
3.000	e	40.0	0.16	2.04
3.000	e	42.5	0.16	1.96
3.000	e	45.0	0.16	1.88
3.000	e	47.5	0.16	1.80
3.000	e	50.0	0.16	1.76
3.000	e	52.5	0.14	1.70
3.000	e	55.0	0.14	1.66
3.000	e	57.5	0.14	1.62
3.000	e	60.0	0.14	1.58
3.000	e	62.5	0.14	1.54
3.000	e	65.0	0.14	1.52
3.000	e	67.5	0.14	1.50
3.000	e	70.0	0.14	1.48
3.000	e	72.5	0.14	1.46
3.000	e	75.0	0.14	1.44
3.000	e	77.5	0.14	1.42
3.000	e	80.0	0.14	1.40
3.000	e	82.5	0.14	1.38
3.000	e	85.0	0.14	1.36
3.000	e	87.5	0.14	1.36
3.000	e	90.0	0.12	1.34
3.000	e	92.5	0.12	1.34
3.000	e	95.0	0.12	1.32
3.000	e	97.5	0.12	1.32
3.000	e	100.0	0.12	1.30
0.113	e	0.0	-	-
0.113	e	2.5	1.62	-
0.113	e	5.0	0.80	4.48
0.113	e	7.5	0.60	3.32
0.113	e	10.0	0.50	2.74
0.113	e	12.5	0.44	2.40
0.113	e	15.0	0.40	2.18
0.113	e	17.5	0.36	2.02
0.113	e	20.0	0.34	1.90
0.113	e	22.5	0.32	1.80
0.113	e	25.0	0.30	1.74
0.113	e	27.5	0.28	1.68
0.113	e	30.0	0.28	1.62

0.113	e	32.5	0.26	1.58
0.113	e	35.0	0.26	1.56
0.113	e	37.5	0.26	1.52
0.113	e	40.0	0.24	1.50
0.113	e	42.5	0.24	1.48
0.113	e	45.0	0.24	1.46
0.113	e	47.5	0.24	1.44
0.113	e	50.0	0.22	1.42
0.113	e	52.5	0.22	1.40
0.113	e	55.0	0.22	1.38
0.113	e	57.5	0.22	1.38
0.113	e	60.0	0.22	1.36
0.113	e	62.5	0.20	1.36
0.113	e	65.0	0.20	1.34
0.113	e	67.5	0.20	1.34
0.113	e	70.0	0.20	1.32
0.113	e	72.5	0.20	1.32
0.113	e	75.0	0.20	1.32
0.113	e	77.5	0.20	1.30
0.113	e	80.0	0.20	1.30
0.113	e	82.5	0.18	1.30
0.113	e	85.0	0.18	1.28
0.113	e	87.5	0.18	1.28
0.113	e	90.0	0.18	1.28
0.113	e	92.5	0.18	1.28
0.113	e	95.0	0.18	1.26
0.113	e	97.5	0.18	1.26
0.113	e	100.0	0.18	1.26
0.026	e	0.0	-	-
0.026	e	2.5	-	-
0.026	e	5.0	1.76	3.84
0.026	e	7.5	1.30	2.94
0.026	e	10.0	1.08	2.46
0.026	e	12.5	0.94	2.18
0.026	e	15.0	0.84	2.00
0.026	e	17.5	0.78	1.88
0.026	e	20.0	0.72	1.78
0.026	e	22.5	0.68	1.70
0.026	e	25.0	0.64	1.64
0.026	e	27.5	0.62	1.60
0.026	e	30.0	0.60	1.56
0.026	e	32.5	0.58	1.52
0.026	e	35.0	0.56	1.50
0.026	e	37.5	0.54	1.46
0.026	e	40.0	0.52	1.44

0.026	e	42.5	0.52	1.42
0.026	e	45.0	0.50	1.40
0.026	e	47.5	0.50	1.38
0.026	e	50.0	0.48	1.38
0.026	e	52.5	0.48	1.36
0.026	e	55.0	0.46	1.36
0.026	e	57.5	0.46	1.34
0.026	e	60.0	0.46	1.34
0.026	e	62.5	0.44	1.32
0.026	e	65.0	0.44	1.32
0.026	e	67.5	0.44	1.30
0.026	e	70.0	0.42	1.30
0.026	e	72.5	0.42	1.30
0.026	e	75.0	0.42	1.28
0.026	e	77.5	0.42	1.28
0.026	e	80.0	0.40	1.28
0.026	e	82.5	0.40	1.28
0.026	e	85.0	0.40	1.26
0.026	e	87.5	0.40	1.26
0.026	e	90.0	0.40	1.26
0.026	e	92.5	0.38	1.26
0.026	e	95.0	0.38	1.24
0.026	e	97.5	0.38	1.24
0.026	e	100.0	0.38	1.24
0.011	e	0.0	-	-
0.011	e	2.5	-	-
0.011	e	5.0	1.76	3.70
0.011	e	7.5	1.30	2.84
0.011	e	10.0	1.08	2.42
0.011	e	12.5	0.94	2.16
0.011	e	15.0	0.84	1.98
0.011	e	17.5	0.78	1.86
0.011	e	20.0	0.72	1.76
0.011	e	22.5	0.68	1.68
0.011	e	25.0	0.64	1.62
0.011	e	27.5	0.62	1.58
0.011	e	30.0	0.60	1.54
0.011	e	32.5	0.58	1.50
0.011	e	35.0	0.56	1.48
0.011	e	37.5	0.54	1.46
0.011	e	40.0	0.52	1.44
0.011	e	42.5	0.52	1.42
0.011	e	45.0	0.50	1.40
0.011	e	47.5	0.50	1.38
0.011	e	50.0	0.48	1.36

0.011	e	52.5	0.48	1.36
0.011	e	55.0	0.46	1.34
0.011	e	57.5	0.46	1.34
0.011	e	60.0	0.46	1.32
0.011	e	62.5	0.44	1.32
0.011	e	65.0	0.44	1.30
0.011	e	67.5	0.44	1.30
0.011	e	70.0	0.42	1.30
0.011	e	72.5	0.42	1.28
0.011	e	75.0	0.42	1.28
0.011	e	77.5	0.42	1.28
0.011	e	80.0	0.40	1.28
0.011	e	82.5	0.40	1.26
0.011	e	85.0	0.40	1.26
0.011	e	87.5	0.40	1.26
0.011	e	90.0	0.40	1.26
0.011	e	92.5	0.38	1.24
0.011	e	95.0	0.38	1.24
0.011	e	97.5	0.38	1.24
0.011	e	100.0	0.38	1.24
0.006	e	0.0	-	-
0.006	e	2.5	-	-
0.006	e	5.0	-	3.06
0.006	e	7.5	-	2.34
0.006	e	10.0	-	2.00
0.006	e	12.5	-	1.78
0.006	e	15.0	-	1.64
0.006	e	17.5	-	1.54
0.006	e	20.0	-	1.46
0.006	e	22.5	-	1.40
0.006	e	25.0	-	1.36
0.006	e	27.5	-	1.32
0.006	e	30.0	-	1.28
0.006	e	32.5	-	1.26
0.006	e	35.0	-	1.24
0.006	e	37.5	-	1.22
0.006	e	40.0	-	1.20
0.006	e	42.5	-	1.18
0.006	e	45.0	-	1.16
0.006	e	47.5	-	1.16
0.006	e	50.0	-	1.14
0.006	e	52.5	-	1.14
0.006	e	55.0	-	1.12
0.006	e	57.5	-	1.12
0.006	e	60.0	-	1.10

0.006	e	62.5	-	1.10
0.006	e	65.0	-	1.10
0.006	e	67.5	-	1.08
0.006	e	70.0	-	1.08
0.006	e	72.5	-	1.08
0.006	e	75.0	-	1.08
0.006	e	77.5	-	1.06
0.006	e	80.0	-	1.06
0.006	e	82.5	-	1.06
0.006	e	85.0	-	1.06
0.006	e	87.5	-	1.06
0.006	e	90.0	-	1.06
0.006	e	92.5	-	1.04
0.006	e	95.0	-	1.04
0.006	e	97.5	-	1.04
0.006	e	100.0	-	1.04