Condition Based Maintenance Decision Making with Delay Time Modeling

by

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Abstract

This research targets to develop condition based maintenance models for better maintenance decisions. We address a condition and age based replacement decision problem using the complete history of measured condition observations to minimize long-run average cost or maximize long-run average availability, or both. To estimate the residual lifetime distribution conditional on the history of observed condition information and current age, a delay time model (DTM) based stochastic filtering process (SFP) is used. A long-run average cost model and a long-run average availability model are analyzed in order to develop the theorems necessary for determining the optimum replacement time. A multi-objective decision frontier is proposed that will help maintenance managers deal with trade-offs between the two objectives to minimize the cost and to maximize availability simultaneously. We also proposed models to integrate imperfect maintenance while making replacement decisions. Finally, in order to show the effectiveness of our proposed models, numerical examples are presented.

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I express my gratitude to my family members – specially my parents who inspire me to do the best. My little son Areeb is also a great source of inspiration. Finally, I would like to thank Almighty God for giving me the blessing, guidance and strength throughout my life.

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List of Abbreviation

AD – Alert Delay

CBM - Condition Based Maintenance

DTM - Delay Time Model

FMC - Flexible Manufacturing Cells

HMM - Hidden Markov Model

OTP - Opportunity-Triggered Maintenance

PHM - Proportional Hazard Model

PM - Preventive Maintenance

RMS – Root Mean Square

SFP - Stochastic Filtering Process

1. Introduction

Maintenance decision should be made properly to gain competitive advantage in a business operation. In this research, we target to develop a set of mathematical models that help maintenance managers to make appropriate decisions. In this introductory chapter, we will define our research problem, scope of study, objectives and methodology, and finally ends with a brief description of this thesis organization.

1.1 Problem Statement

In recent decades, with increased level of automation and high capital investment in manufacturing equipment, selecting the best maintenance policy and effective control of maintenance cost are key to success [1]. In order to minimize maintenance cost and avoid consequences incurred by a sudden failure, we can carry out proactive maintenance action, which is called *preventive maintenance* (PM). Usual preventive maintenance is carried out periodically on the basis of maintenance people' s experience or in a fixed time interval.

Among all the PM policies, we can also make maintenance decision based on the condition information of the equipment called *Condition based maintenance* (CBM) to avoid unnecessary maintenance i.e. to execute maintenance activities only when they are needed. CBM recommends implementing maintenance actions based on the current health condition of the equipment which can be assessed with the data obtained from condition monitoring, instead of doing traditional time-based preventive maintenance or failure maintenance. It has the capability to alert the maintenance personnel to repair or replace any component ahead of failure which reduces plant shutdown cost and production interruption. CBM decisions include preventive replacement decisions and/or imperfect preventive maintenance decisions.

CBM provides rapid and significant cost benefits to industries, because it reduces plant shut down cost and production interruption. Therefore, it becomes more popular approach to make maintenance decisions [2]. However, condition monitoring may be a cost intensive procedure in CBM. To leverage the investments in conditioning monitoring, it is very important to select an appropriate method to estimate the lifetime distribution to predict future health condition of the equipment accurately. In CBM decision making, two-phase failure model can play a vital role, where failure process is divided into two phases [3, 4]. In the first phase of this model, the variation of the condition information is not significant and it is a normal variation. However, in the second phase, deterioration of the equipment starts and condition information increases monotonically. As we see a delay in the failure process is an opportunity to take preventive action, two-phase failure model also called a *delay time model* (DTM).

To develop a reliable CBM decision framework, we plan to incorporate the history of condition information and the current age of the equipment. A two-phase failure model called DTM is used in a *stochastic filtering process* (SFP) based prognosis technique to predict the residual life time distribution. DTM is used in the SFP to incorporate the relationship between the residual life at the current monitoring point and the residual life at the previous monitoring point. Figure 1 describes the condition based age replacement policy, where t_i is i^{th} monitoring time and y_i be the condition information collected at t_i. Suppose y_c is the threshold for condition information after which the equipment begins deteriorating rapidly and condition information y_i just exceeds this threshold level. Therefore, at the i^{th} monitoring time it requires to make proper maintenance decision, since unexpected failure cost is higher than the preventive maintenance cost. In this research, at the i^{th} monitoring time, we can predict the residual life distribution of the equipment from the history of condition information and current age of the equipment with the SFP to develop and analyze CBM models for better maintenance decisions.

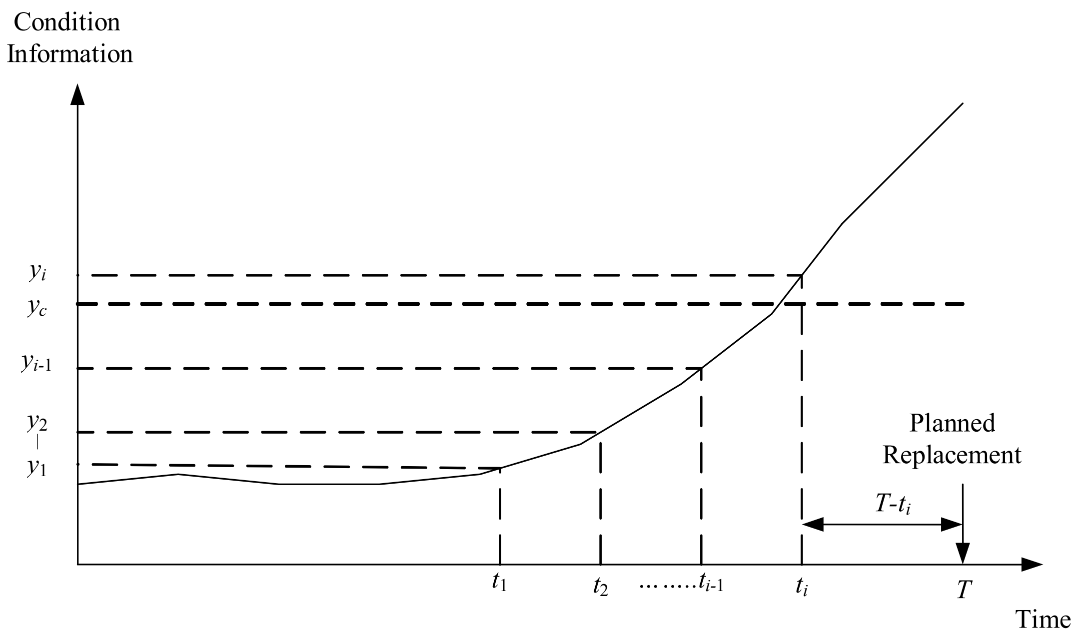

Figure 1.1: Condition based Age Replacement Policy.

In this work, we consider two types of condition based maintenance decisions: preventive replacement and imperfect preventive maintenance. In the preventive replacement, the equipment is replaced with a new one at a scheduled time if the item is not failed before this scheduled time, and the system will be "as good as new". Imperfect preventive maintenance brings the health status of the system somewhere between "as good as new" and "as bad as old".

1.2 Scope and Objective

In the CBM literature, we see that it has received limited attention to develop and analyze DTM based predictive models for maintenance decision making, where the complete history of condition information and current age are used to calculate residual life.

Therefore, our key objectives of this research is to achieve the following two objectives:

- Develop DTM based CBM models for replacement decision.
- Investigate the feasibility of imperfect preventive maintenance while making a replacement decision.

1.3 Methodology

In this research, we will develop DTM based CBM models for replacement decision making by considering multiple objectives such as cost and availability. We will carry out analytical analysis to study the properties of the proposed CBM models to help making replacement decision. From our analysis, we will propose theorems, which will help to determine replacement decisions. To consider multiple objectives simultaneously, a strategy will be developed with a decision frontier that deals with trade-offs between the cost and availability.

Moreover, there is an opportunity to gain competitive advantages by considering the imperfect preventive maintenance and preventive replacement decision simultaneously. Therefore, we will also investigate the feasibility of integrating imperfect preventive maintenance with preventive replacement decision. It is expected that after performing imperfect preventive maintenance, the health condition will be better and this health improvement can be realized with the predicted hazard rate function. This new hazard rate function will be used to make the maintenance decision. Finally, we will develop models to study the feasibility of the imperfect preventive maintenance while making a replacement decision to achieve certain objectives such as net benefit and availability.

1.4 Thesis Organization

We have started this thesis with a background discussion on different types of maintenance policies followed by a problem statement, objectives and methodology used in this research. In Chapter 2, we will discuss background literature. We will discuss two maintenance models and carried out analytical analysis for replacement decision in Chapter 3. In order to show the effectiveness of our proposed model, we will also present a case study with numerical result analysis in this chapter. In Chapter 4, we will discuss models to integrate imperfect maintenance while making replacement decisions. Finally, in Chapter 5, we will discuss conclusion and avenues for future research.

2. Literature Review

In order to find the current state-of-art research in condition based maintenance modeling, we have carried out literature review in three different aspects, namely preventive replacement, imperfect preventive maintenance and integrated CBM decisions.

2.1 Preventive Replacement

Selecting an optimal replacement time is a crucial task for a maintenance manager. To determine the optimum preventive replacement time, an *age-based replacement policy* can be used [5]. The age-based replacement policy is a widely used strategy in maintenance decision making [6-8]. An age-based replacement policy is used to derive expressions for the long run average cost for optimal replacement decision making [9].

Furthermore, researchers have been working to improve their determination of optimum replacement time through the incorporation of condition information [10-12]. We can use a single or multiple objective function in making preventive replacement decision. The optimal replacement time T^* is often determined either by minimizing the long run average cost $C(T)$ or by maximizing the long run average availability $A(T)$. *Proportional hazard model* (PHM) has been used to update the hazard function based on the current monitored condition information and the age of the equipment [11]. Makis and Jardine reported a method to determine the optimum risk level of an equipment for minimizing the total cost per unit time for replacement decision making based on the PHM [13]. PHM is used to determine the remaining lifetime distribution [14]. This remaining lifetime distribution is then used to find the optimum replacement time to maximize the long run average availability. PHM is also used in [15] with an age replacement model, in which the hazard was expressed as a function of condition information and age. According to [16], the PHM uses only the current observation, not all the previous history of the observed health information. A stochastic filtering process (SFP) was proposed in [17] to predict the residual lifetime distribution considering the complete history of observed condition information.

A different prognostic model is proposed in [18] to predict the failure probability from the condition information and the expected reliability after maintenance. It used a genetic algorithm to minimize the risk, where the risk was represented by the trade-off between failure and preventive maintenance. Another two studies also considered genetic algorithm for maintenance decision making with multiple criteria [19-20]. A multi-objective optimization method is discussed in [21] to select an optimal preventive maintenance schedule without considering the health condition of the equipment, whereas [22] suggested considering both cost and reliability simultaneously while making maintenance decision.

2.2 Imperfect Preventive Maintenance

Imperfect preventive maintenance is a maintenance action that can bring the system somewhere, between "as good as new" to "as bad as old" conditions [23]. A detailed review on imperfect maintenance is available in [24]. Two *preventive maintenance* (PM) policies were studied in [25] for repairable systems where in *Policy I*, PM is carried out either at failure or at a predetermined time interval whichever of them comes first and in *Policy II*, PM is always carried out at a predetermined time and minimal repair is carried out in case of failure, whereas *Policy II* is used in [26] to present two imperfect PM models. Jayabalan and Chaudhuri developed an algorithm to find out the number of imperfect maintenance interventions required before each replacement in order to minimize the total cost over a finite time horizon [27]. A new age replacement approach based on imperfect repair with random probability is discussed in [28], where the equipment is replaced at the age *T* and if it fails before the age *T*, it is either perfectly or minimally repaired.

Furthermore, another technique is proposed in [29] for maintenance and replacement decisions, where maintenance cost and acquisition cost functions are increasing with increasing mean downtime, whereas a similar method is proposed in [30] that dealt with cost optimal maintenance and replacement schedule for a new system subject to deterioration considering time value of money in all future costs. A PM model is proposed in [31] for multi-component case with imperfect maintenance which is an extension of the work discussed in [30]. For multi-component system, a confidence limit for optimal replacement time is calculated in [32], where block maintenance is considered.

To demonstrate the effect of PM, a number of techniques were proposed and developed. A concept of virtual age reduction by the PM activity is proposed in [33]. Furthermore, age reduction models are discussed in [34-35], where there is an effective age reduction after imperfect preventive maintenance. The effective age right after the PM is greater than zero and the hazard rate remains as a function of the effective age. Another model for scheduling imperfect preventive maintenance is proposed in [36], where each application of PM reduces the equipment' s effective age. Ben-Daya and Alghamdi proposed an imperfect preventive maintenance model, where they assumed that the reduction in the age of the equipment is proportional to the cost of PM [37].

In order to reflect the effect of PM activities, it has been assumed that PM action reduces the equipment' s effective age. An age based model is proposed in [38] for imperfect PM to determine the optimal number of PMs and the optimal PM' s schedule that minimize total long run expected cost, whereas a maintenance policy proposed in [2], which represents the impact of imperfect maintenance action as the deterioration of the system. Further study has been carried out [39] to investigate effect of imperfect maintenance. They characterized it by random intervention gains and assumed that each maintenance action affects the speed of the system deterioration process. Moreover, additional cost of imperfect preventive maintenance will lower the quality control cost and production cost that ultimately lower the overall long run average cost [40].

Among the many treatments to model imperfect maintenance, improvement factor method is widely used as the maintenance decision can be determined by the system hazard rate or other reliability measure. A model for sequential imperfect maintenance policy for CBM is developed in [41]. They used hybrid hazard rate function where the concept of age reduction factor and hazard rate increase factor are integrated simultaneously, where a reliability-centered predictive maintenance policy is proposed for a continuously monitored system such that when the system reliability reaches to the threshold level, an imperfect preventive maintenance will be performed. A multi-objective optimization model is proposed in [42] to determine optimal preventive maintenance and replacement schedules for a repairable system with increasing failure rate. Routine service is considered as a maintenance action in [43], where it can prolong normal stage of a system. A degradation based maintenance (DBM) model is discussed in [44] to minimize the total cost of an imperfect maintenance. In order to formulate the DBM model, they discussed a relationship between degradation reduction and cost of imperfect maintenance.

A little effort has been made in the above literatures (discussed in this sub-section) to integrate imperfect preventive maintenance while making replacement decision that uses DTM based two-phase failure model to assess the true effect of imperfect maintenance in CBM. Along with the preventive replacement decision, we may also consider imperfect preventive maintenance to improve the health condition of an equipment so that we can postpone/delay the replacement and realize more residual life [45]. This improvement may help us to gain other competitive advantages such reliability, availability, production planning and spare parts inventory etc. However, it is important to develop maintenance models that can evaluate the suitability of such integration in CBM environment by considering all possible benefits and costs.

2.3 Integrated CBM Decision

Maintenance decision has direct effects on production scheduling, quality control and warranty. Previously they have been treated separately. As a result, the effects of PM was ignored and outcome of this decision was not fruitful. If we perform PM, it will incur cost and time. On the other hand, it may reduce the probability of unplanned failure, defective production rate and operating cost etc. Recently researchers are focusing on integrated maintenance models that could handle the above factors simultaneously. To optimize economic production quantity and preventive maintenance level simultaneously a model is proposed in [46], where the effect of preventive maintenance is shown as the reduction of the cost from the nonconforming items and the restoration cost due to shift to the out-of-control state. Later warranty policy is also integrated with the previous decisions in [47]. Combining production scheduling and preventive maintenance planning is considered in [48] to minimize job tardiness, where as in [49] another model is developed that integrates perfect preventive maintenance planning and production scheduling for a single machine. Furthermore, genetic algorithm is used in [50] to solve the integrated model discussed in [49].

Moreover, a model is presented in [51] that integrates the economic production quantity and level of preventive maintenance (PM) simultaneously. It is found from their investigation that the level of PM has impact on the economic production quantity and total cost. Another model is discussed in [52] to calculate optimum economic production quantity (EPQ) for an imperfect process with increasing hazard rate. Imperfect maintenance planning is also integrated in their model. Preventive maintenance increases the percentage of conforming product by an amount proportional to the preventive maintenance level.

The extra cost of imperfect preventive maintenance will lower the quality control cost and production cost that ultimately lower the overall long run average cost [53]. An integrated cost model is proposed in [54] for optimizing the process control and maintenance simultaneously, whereas an age replacement policy is considered in [55] for the optimization of process control. In [56], an integrated model for the joint optimization of production quantity, design of quality control parameters and maintenance level is presented. Another maintenance policy is discussed in [57], where an equipment can be replaced to avoid unnecessary production interruption. The integrated models discussed in this sub-section does not focus on CBM that uses the DTM and the complete history of condition information. It is also important to consider preventive replacement and imperfect preventive maintenance simultaneously while considering production decisions discussed above.

The research work discussed above neither focused on replacement decision making that uses the DTM with the complete history of condition information nor analyzed in detail the long-run average cost model and the long-run average availability model. Formulations of multi-objective condition and age based replacement model, and trade-offs strategies (to help making maintenance decision effectively) between objectives were also not addressed in such paradigm either. In order to overcome these draw backs, in chapter 3, we will develop such models and carry out theoretic analysis to propose theorems that can help maintenance manager to make better decisions. We will also offer trade-off strategies to handle conflicting objectives. We also notice very little effort to develop maintenance model that can investigate the feasibility of integrating imperfect maintenance while making replacement decision in DTM framework. Therefore, in chapter 4, we will propose a new set of maintenance models that can investigate the possibility of such integration.

3. Replacement Decision Analysis[1]

This chapter will introduce DTM based CBM models for replacement decision making. Further analysis will be done on the proposed models to develop replacement strategies.

3.1 Preventive Replacement

Preventive replacement is carried out to avoid higher failure replacement cost, resulting from production loss due to unplanned down time and other logistic delays. Selecting an optimal replacement time is a crucial task for a maintenance manager. An *age-based replacement policy* is widely used [5-7] to determine optimum preventive replacement time. Further, researchers have been working to improve their determination of optimum replacement time through the incorporation of condition information [58-61].

For preventive replacement decisions, single or multiple objective functions may be used. The optimal replacement time, T^*, is often determined either by minimizing the long-run average cost, $C(T)$, or by maximizing the long-run average availability, $A(T)$. The *Proportional Hazard Model*

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[1] *This chapter is partially adopted from Lipi, T. F., Zuo, M. J., Lim, J.-H., Wang, W. (2012) "A Condition Based Replacement Model Using Delay Time Modeling", Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability, Vol. 226, No. 2, pp. 221-233.*

(PHM) has been used to update the hazard function based on information regarding monitored condition information and equipment age [11]. Makis and Jardine [13] used PHM to determine the risk level for equipment that minimizes the total cost per unit of time in replacement decision making. Li *et al.*[14] used the PHM to determine the remaining lifetime distribution. This remaining lifetime distribution was then used to find the replacement time that maximizes the long-run average availability. Scarf [15] used the PHM with an age replacement model, in which the hazard rate was expressed as a function of condition information and age. Wang and Zhang [16] stated that the PHM used only the current observation, not the complete history of observed condition information.

There are other prognostic models available in the literature. For example, Camci [18] proposed a prognostic model for using information to predict the probability of failure and reliability to be expected after maintenance. Genetic algorithm is used to minimize risk, where risk is represented by the trade-off between failure and preventive maintenance. A S*tochastic Filtering Process* (SFP) was used by Wang [17] to predict residual lifetime distribution considering the complete history of equipment condition. A graphical method was proposed for finding replacement time, T^*, that minimizes the long-run average cost, $C(T)$. Wang [63] combined the delay time concept, the *Hidden Markov Model* (HMM), and filtering theory to form a prognosis model for maintenance decision making. Lipi *et al*. [58] developed a preliminary model for age-based replacement decision making that considers the complete history condition information to maximize long-run average availability. They reported a preliminary relationship between the target level of availability and maintenance efficiency. This relationship assists maintenance managers in selecting maintenance crews capable of achieving given target level of long-run average availability. Savsar [59] studied the effects of various maintenance policies on the production rate for flexible manufacturing cells (FMC). To analyze the effects of corrective, preventive and opportunistic maintenance policies on the productivity of FMCs, several analytical models were proposed. It was shown that, the best was the opportunity-triggered maintenance policy (OTP). However, this research did not consider cost in developing maintenance models.

To make efficient replacement decisions, sometimes it is important to consider more than one criterion. Castro *et al*. [19] and Fonseca *et al*. [20] considered multiple criteria for maintenance decision making, and used genetic algorithms to optimize the decision process. Sachdeva *et al*. [21] proposed a multi-objective optimization method which selects an optimal preventive maintenance schedule without considering the condition of the equipment. Castanier *et al*.[63] developed a parametric model for inspecting and maintaining a system that is subject to stochastic gradual deterioration. This model has been used to schedule non-periodic inspections and choose among possible maintenance actions. Finally, a decision framework was proposed to choose the best sequence of maintenance action and to schedule future inspections. Grall *et al*. [64] proposed a model for condition-based inspection/replacement, where the state of the system could be observed through inspection. To assess the performance of different maintenance policies, they developed a cost model. Baxter *et al*. [65] discussed using the stochastic point process for analysis of reparable units. Their model assumes that, after the repair, the virtual age of the unit is a random variable. Dijoux and Gaudoin [66] proposed a new model called alert-delay (AD) to find the dependence between condition-based preventive maintenance time and corrective maintenance time in terms of a warning of the system and a supplementary time. Statistical analysis of this model along with the simulation results with real data set were presented to show the effectiveness of this model.

From the above review of the literature we see that little effort has been made on the analysis of maintenance decision making that uses the complete history of condition information to predict the residual life distribution. To facilitate maintenance decision making, we analyze in detail the long-run average cost model and the long-run average availability model. To minimize the long-run average cost and maximize the long-run average availability, we propose a multi-objective decision frontier that will assist the maintenance managers to deal with trade-offs between objectives. We propose another modified model for replacement decisions, which allows decision makers to obtain at least a minimum level of long run average availability.

Finally, this chapter is organized as follows. Section 3.2 describes a condition and age based replacement policy using the SFP-based model. Section 3.3 formulates a condition and age based replacement model. This model is investigated using a theoretic approach to find the optimal replacement time, T^*. Section 3.4 discusses an availability-based age replacement model, which is also investigated using a theoretic approach to find the optimal replacement time, T^*. Section 3.5 formulates a multi-objective condition and age based replacement model for minimizing long-run average cost, $C(T)$ and maximizing long-run average availability, $A(T)$.

3.2 Replacement Decision Models

Age replacement models can be integrated with residual lifetime distributions conditional on past monitoring history for minimizing long-run average cost or maximizing the long-run average availability. A residual lifetime distribution can be obtained by the SFP technique proposed by

Wang [10]. We use the following two subsections to discuss the age-based replacement policy and SFP.

3.2.1 Age Replacement Policy

The age-based replacement policy preventively replaces equipment when it reaches a predefined age, T, at a cost of C_p ; or correctively replaces if it fails before T at a cost of C_f (see Figure 3.1) [67]. Generally, preventive replacement cost is less than failure replacement cost ($C_p<C_f$) and equipment is assumed to return to the "as good as new" state after preventive replacement or failure replacement.

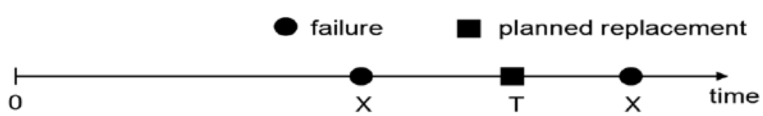

Figure 3.1: Age Replacement Policy

3.2.2 Stochastic Filtering Process (SFP)

We can use the SFP reported in [10] to estimate residual lifetime distribution, $f_i(x_i|Y_i)$, based on conditions recorded from time 0 to time t_i . SFP is used to establish the relationship between all the condition information and the residual lifetime distribution, using *Delay Time Model* (DTM) and conditional probability. According to DTM, the residual delay time at t_i is the residual delay time at t_{i-1} minus the interval between t_i and t_{i-1}, given that the equipment has survived to t_i. We will use the following notations throughout this chapter.

Notations:

t_i : time at monitoring point *i*

x_i : residual lifetime at monitoring time point t_i

y_i : condition information obtained at time t_i

$Y_i = \{y_i, y_{i-1}, ...y_1\}$: history of condition information up to monitoring point *i*

T : scheduled replacement time, to be optimized

T^* : optimum replacement time

T_p : average length of time needed to complete a preventive replacement

T_f : average length of time needed to complete a failure replacement

C_p : cost of each preventive replacement

C_f : cost of each failure replacement

$f_i(y_i|x_i)$: conditional probability density function of y_i given x_i

$f_i(x_i|Y_i)$: probability density function of x_i conditional upon Y_i

$F_i(x_i|Y_i)$: cumulative distribution function of x_i conditional upon Y_i

$R_i(x_i | Y_i)$: reliability function of the residual lifetime, conditional upon Y_i

$h_i(x_i | Y_i)$: hazard function of the residual lifetime, conditional upon Y_i

$h_i^{'}(T - t_i | Y_i)$: derivative of $h_i(T - t_i | Y_i)$

μ_i : mean residual lifetime at monitoring point i

$C(T)$: long-run average cost

$A(T)$: long-run average availability

The probability density function of x_i conditional upon Y_i can be obtained by the following expression in accordance with its conditional probability relationship and DTM:

$$f_i(x_i | Y_i) = \frac{f(y_i | x_i) f_{i-1}(x_i + t_i - t_{i-1} | Y_{i-1})}{\int_0^\infty f(y_i | x_i) f_{i-1}(x_i + t_i - t_{i-1} | Y_{i-1}) dx_i}. \tag{1}$$

To obtain the final expression for conditional residual life, a Weibull distribution with scale parameter, α , and shape parameter, β , is used to represent the lifetime distribution at time zero. The distribution of the data on condition is also assumed to follow the Weibull distribution with scale parameter, ρ , and shape parameter, η. It is noted that other distributions can also be used and the best can be selected by a goodness of fit measure. There is assumed to be a negative correlation between the residual lifetime, x_i , and the condition information, y_i ; that is, the higher the condition information, the shorter the residual lifetime. This relationship is incorporated by the expression as follows: $\rho = \frac{1}{A + B^{-Cx_i}}$.

Using the above distributions and the relationship in Equation (1), we can obtain Equation (2), which describes the conditional residual lifetime distribution:

$$f_i(x_i|Y_i) = \frac{(x_i+t_i)^{\beta-1} e^{-(\alpha(x_i+t_i))^\beta} \prod_{k=1}^{i} \frac{e^{-(y_k(A+Be^{-C(x_i+t_i-t_k)})^{-1})^\eta}}{A+Be^{-C(x_i+t_i-t_k)}}}{\int_0^\infty (z+t_i)^{\beta-1} e^{-(\alpha(z+t_i))^\beta} \prod_{k=1}^{i} \frac{e^{-(y_k(A+Be^{-C(z+t_i-t_k)})^{-1})^\eta}}{A+Be^{-C(z+t_i-t_k)}} dz}. \qquad (2)$$

The model parameters $A, B, C, \alpha, \beta, \eta$ are needed to be estimated from the complete condition information data, $Y_i = \{y_1, y_2, \ldots, y_i\}$. Equation (2) expresses the mathematical model of conditional lifetime distribution derived from the complete history of condition information and current age of the equipment. Therefore, we use this conditional lifetime distribution in this paper to carry out our analysis for condition-based replacement decision.

We can use the above expression for residual life distribution to obtain the following condition-based hazard rate function:

$$h_i(x_i|Y_i) = \frac{(x_i+t_i)^{\beta-1} e^{-(\alpha(x_i+t_i))^\beta} \prod_{k=1}^{i} \frac{e^{-(y_k(A+Be^{-C(x_i+t_i-t_k)})^{-1})^\eta}}{A+Be^{-C(x_i+t_i-t_k)}}}{\int_{x_i}^\infty (z+t_i)^{\beta-1} e^{-(\alpha(z+t_i))^\beta} \prod_{k=1}^{i} \frac{e^{-(y_k(A+Be^{-C(z+t_i-t_k)})^{-1})^\eta}}{A+Be^{-C(z+t_i-t_k)}} dz} \qquad (3)$$

$$= \frac{f_i(x_i)g_i(x_i)}{\int_{x_i}^\infty f_i(x_i)g_i(x_i)dx}$$

where $f_i(x_i) = (x_i + t_i)^{\beta-1} e^{-(\alpha(x_i+t_i))^{\beta}}$ and $g_i(x_i) = \prod_{k=1}^{i} \frac{e^{-(y_k(A+Be^{-C(x_i+t_i-t_k)})^{-1})^{\eta}}}{A+Be^{-C(x_i+t_i-t_k)}}$.

3.3 Condition and Age Based Replacement Model for Minimizing Cost

In the traditional age-based replacement models, the optimum replacement time, T^*, is selected to minimize the expected total cost per unit of time. Two types of cost models, namely an infinite-horizon cost model and a one-replacement-cycle cost model, are available in the literature [68-70]. An infinite-horizon cost model is applicable where the maintenance manager wants to carry out the same replacement strategy over a very long period of time. In this paper we focus on the infinite-horizon cost model. A preliminary model for this purpose has been presented in Lipi *et al.*[71].

In the infinite-horizon cost model, the renewal-reward theorem is invoked to minimize the long-run average cost per unit of time [72]. According to the renewal-reward theorem, the long-run average cost per unit of time is equal to the expected cost during a life cycle divided by the expected length of the life cycle:

$$C(T) = \frac{\text{Expected cost incurred during a cycle}}{\text{Expected length of a cycle}}. \quad (4)$$

According to Coolen-Schrijner and Coolen [68], the long-run average cost per unit of time measured at time zero can be expressed using Equation (5):

$$C(T) = \frac{C_p R(T) + C_f F(T)}{\int_0^T z f(z)dz + TR(T)}. \quad (5)$$

In this research, our objective is to introduce the complete history of condition information in equation (4) that allows condition-based decisions to be made. Our plan is to analyze in detail the resulting models and propose theorems to assist maintenance managers in making replacement decisions.

The long-run average cost per unit of time for the condition and age based replacement strategy can be expressed using Equation (6), in which the current monitoring time is t_i and the history of condition information is Y_i:

$$C(T)=\frac{C_p R_i(T-t_i\left|Y_i\right.)+C_f F_i(T-t_i\left|Y_i\right.)}{t_i+\int\limits_0^{T-t_i} zf_i(z\,|\,Y_i)dz+(T-t_i)(1-\int\limits_0^{T-t_i} f_i(z\left|Y_i\right.)dz)}. \tag{6}$$

In this model residual life time distribution is used to estimate the expected total cost and expected life of the equipment. The residual life time distribution depends on the current age of the equipment and the complete history of the condition information. At each monitoring time, new condition information is added with the previous history of condition information. As a result at each monitoring time, there is a new residual life time distribution that is used in Equation (6) to calculate the long run average cost. Therefore, Equation (6) is updated and approximated along with the new probability distribution at each monitoring point.

After some mathematical manipulation using the denominator of Equation (6) we obtain Equation (7):

$$C(T)=\frac{C_p R_i(T-t_i|Y_i)+C_f F_i(T-t_i|Y_i)}{t_i+\int_0^{T-t_i} R_i(z|Y_i)dz}. \quad (7)$$

Now we investigate Equation (7) using a theoretic approach to determine the optimum replacement time, T^*. To obtain the optimum replacement time, T^*, $\frac{d}{dT}(C(T))=0$ is a necessary and sufficient condition.

Differentiating Equation (7) with respect to T and equating the derivative to zero yields Equation (8):

$$(C_f-C_p)[t_i h_i(T-t_i\,|\,Y_i)+\int_0^{T-t_i}[h_i(T-t_i\,|\,Y_i)-h_i(z\,|\,Y_i)]R_i(z\,|\,Y_i)dz]-C_p=0. \quad (8)$$

The optimum replacement time, T^*, is a solution for Equation (8). Now we have to investigate whether or not Equation (8) has a unique solution. We can do this in two steps. In step one, we check the possible values for the left hand side (LHS) of Equation (8) for the two extreme values of T. In step two, we check whether or not the LHS of Equation (8) increases monotonically. If we find that for the lower and upper extreme ends of T, the LHS of Equation (8) has negative and positive values respectively, and the equation is a monotonically increasing function then the equation has a unique solution and $C(T)$ is a convex function.

3.3.1 The First Step

In this step we check the possible values for the LHS of Equation (8) at its extreme ends. At first we check the lower extreme end (T-t_i= 0) and then we check the upper extreme end (T-t_i= ∞).

At the lower extreme end, $\int_0^{T-t_i} [h_i(T-t_i \mid Y_i) - h_i(z \mid Y_i)]R_i(z \mid Y_i)dz = 0$ and Equation (8) becomes Equation (9):

$$(C_f - C_p)[t_i h_i(T-t_i \mid Y_i)] - C_p = 0. \qquad (9)$$

From Equation (9), we can see that when T-t_i = 0, we have the following three possible cases:

Case 1: If at T=t_i , $(C_f - C_p)[t_i h_i(T-t_i \mid Y_i)] < C_p$, then the LHS of Equation (9) becomes negative (see Figure 3.2). That means that at monitoring point t_i , the slope of $C(T)$ is negative. In this case, we know that the minimum cost, $C(T)$, will be attained at T^* , which is greater than the current point of monitoring time, t_i .

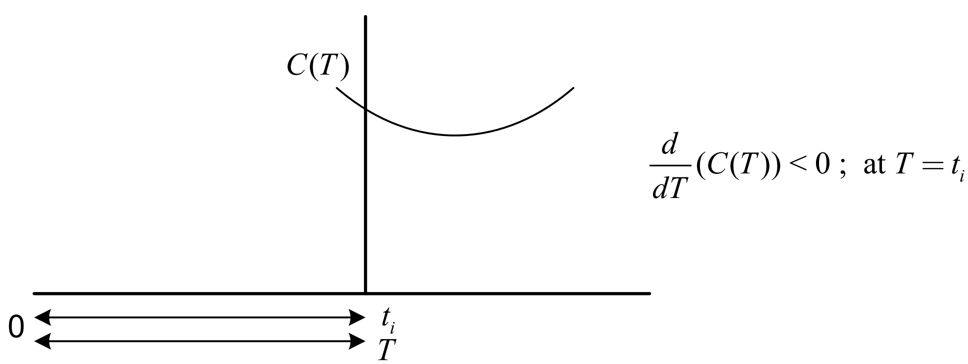

Figure 3.2: The cost function with a negative slope at T= t_i

Case 2: If at T=t_i , $(C_f - C_p)[t_i h_i(T-t_i \mid Y_i)] = C_p$, then the LHS of Equation (9) becomes zero (see Figure 3.3). This means that at monitoring point t_i , the slope of $C(T)$ is zero. In this case the current point of monitoring time, t_i , is the optimum replacement time for obtaining the minimum long-run average cost.

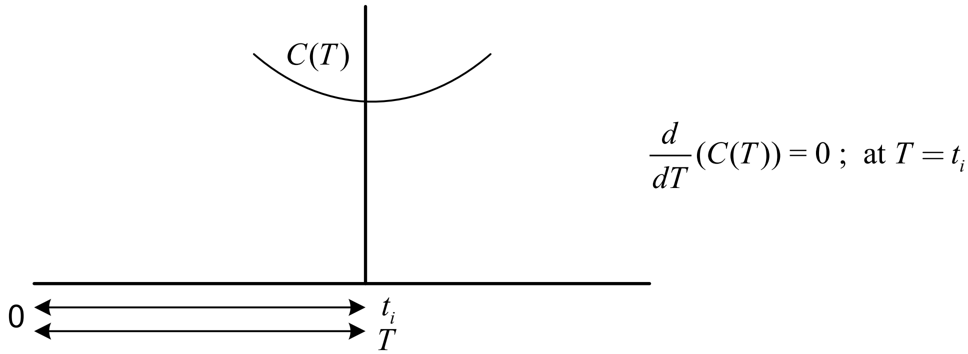

Figure 3.3: The cost function with a slope of zero at $T = t_i$

Case 3: If at T=t_i , $(C_f - C_p)[t_i h_i(T - t_i \mid Y_i)] > C_p$, then the LHS of Equation (9) becomes positive. This means that at the current point t_i , the slope of $C(T)$ is positive; i.e., $C(T)$ is increasing (see Figure 3.4). In this case we should conduct preventive replacement as soon as possible.

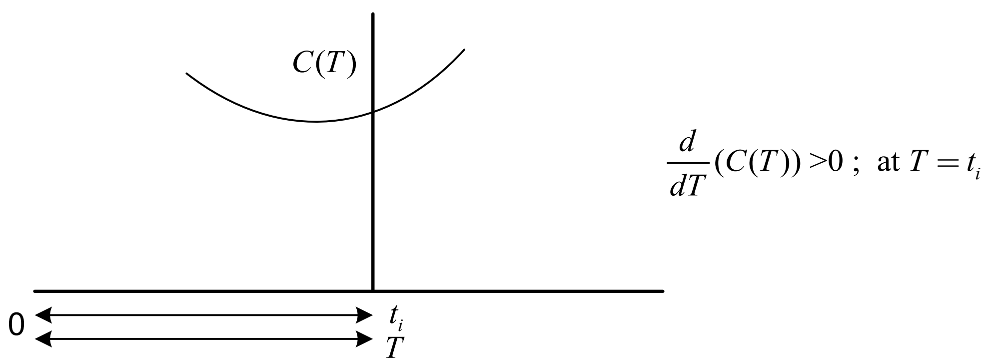

Figure 3.4: The cost function with a positive slope at $T = t_i$

Now, we check the slope of C (T) at the upper extreme end, i.e., $T\text{-}t_i=\infty$:

$$\int_0^{T-t_i} [h_i(T-t_i \mid Y_i) - h_i(z \mid Y_i)] R_i(z \mid Y_i) dz$$

$$= Lim_{T-t_i \to \infty}(h_i(T-t_i \mid Y_i))(Lim_{T-t_i \to \infty} \int_o^{T-t_i} R_i(z \mid Y_i) dz) - Lim_{T-t_i \to \infty}(\int_o^{T-t_i} f_i(z \mid Y_i) dz))$$

$$= Lim_{T-t_i \to \infty}(h_i(T-t_i \mid Y_i))\mu_i - 1).$$

When $h_i(T-t_i \mid Y_i)$ increases strictly in $T\text{-}t_i$, $(Lim_{T-t_i \to \infty}(h_i(T-t_i \mid Y_i))\mu_i - 1 \to \infty$. The LHS of Equation (8) becomes positive at the upper extreme end; i.e., the slope of $C(T)$ at the upper extreme end is positive.

3.3.2 The Second Step

In this step we check whether or not the slope of $C(T)$ is a monotonically increasing function. We denote the slope of $C(T)$ as $Q(T)$. To show that $Q(T)$ is a monotonically increasing function, we differentiate $Q(T)$ with respect to T :

$$\frac{d}{dT}(Q(T)) = (C_f - C_p)[t_i h_i^{'}(T-t_i \mid Y_i) + h_i^{'}(T-t_i \mid Y_i) \int_0^{T-t_i} R_i(z|Y_i) dz]. \tag{10}$$

Since $C_f > C_P$ and $h_i(T-t_i \mid Y_i)$ increases strictly in $T\text{-}t_i$, we have $\frac{d}{dT}(Q(T)) > 0$.

We see that for Case-1, the $C(T)$ function decreases until it reaches to a minimum point, which is the optimal replacement time. Immediately after this optimal replacement time, the $C(T)$ function increases monotonically. For Case-2, at the current time of monitoring, the $C(T)$ function is at its minimum value; immediately after this time point, it will increase monotonically. For Case-3, if we delay replacement, the $C(T)$ function will increase monotonically. From our investigation we see that $C(T)$ is a convex function, hence it has a unique minimum point, which can be obtained by setting its first derivative equal to zero. The above analysis gives us the theorem given below on the uniqueness of the optimal solution for an infinite-horizon cost model.

Theorem 1 : If $C_f > C_P$, $(C_f - C_p)[t_i h_i(T - t_i \mid Y_i)] < C_p$ at the current time of monitoring point, and $h_i(T - t_i \mid Y_i)$ increases strictly in T-t_i, there exists a unique and finite $T^*>t_i$ that minimizes the long-run average cost per unit of time. T^* is the solution for equation (8).

3.4 Condition and Age Based Replacement Model for Maximizing Availability

We can also use the condition and age based replacement model to maximize the long-run average availability of the equipment being monitored. A preliminary model for this purpose has been reported in Lipi *et al.*[58]. This model is useful when availability is a more important measure of the performance of the equipment than is cost. In the following, we further analyze this availability maximization model. Long-run average availability can be expressed as Equation (11):

$$A(T) = \frac{\text{Expected total uptime}}{\text{Expected total uptime} + \text{Expected total downtime}}. \tag{11}$$

Thus, long-run average availability, $A(T)$, can be calculated using Equation (12):

$$A(T)=\frac{\int_0^T zf(z)dz+T(1-\int_0^T f(z)dz)}{\int_0^T (z+T_f)f(z)dz+(T+T_p)(1-\int_0^T f(z)dz)}. \quad (12)$$

To calculate expected total uptime and expected total downtime, we need to use the residual lifetime distribution. The expected total uptime and the expected total downtime can be calculated using the following equations:

$$\text{Expected total uptime} = t_i+\int_0^{T-t_i} zf_i(z|Y_i)dz+(T-t_i)(1-\int_0^{T-t_i} f_i(z|Y_i)dz). \quad (13)$$

$$\text{Expected total downtime} = T_p(1-\int_0^{T-t_i} f_i(z|Y_i)dz)+\int_0^{T-t_i} T_f f_i(z|Y_i)dz. \quad (14)$$

The second and third terms in the RHS of equation (13) are expected additional time the equipment would survive and expectation of equipment survival up to the planned replacement time *T*, respectively. The first and second terms in the RHS of equation (14) are expected time required for planned replacement and expected time required for failure replacement, respectively. Incorporating the expressions of the expected total uptime and the expected total downtime into the Equation (11), we obtain the following long-run average availability:

$$A(T)=\frac{t_i+\int_0^{T-t_i} zf_i(z|Y_i)dz+(T-t_i)(1-\int_0^{T-t_i} f_i(z|Y_i)dz)}{t_i+\int_0^{T-t_i}(z+T_f)f_i(z|Y_i)dz+(T-t_i+T_p)(1-\int_0^{T-t_i} f_i(z|Y_i)dz)}. \quad (15)$$

After some mathematical manipulation of Equation (15), we obtain the following expression of the availability:

$$A(T)=\frac{t_i+\int_0^{T-t_i}R_i(z|Y_i)dz}{t_i+\int_0^{T-t_i}R_i(z|Y_i)dz+T_p+(T_f-T_p)\int_0^{T-t_i}f_i(z|Y_i)dz}. \tag{16}$$

Now we investigate Equation (16) using a theoretic approach to determine the optimum preventive replacement time, T^*. To obtain the replacement time that maximizes the total long-run average availability, $A(T)$, the necessary and sufficient condition is $\frac{d}{dT}(A(T))$=0. Differentiating Equation (16) with respect to T and equating it to zero yields Equation (17):

$$T_p+(T_f-T_p)\int_0^{T-t_i}f_i(z|Y_i)dz-(T_f-T_p)h_i(T-t_i|Y_i)=0 . \tag{17}$$

The optimal T^* can be found by solving Equation (17). We can investigate whether Equation (17) has a unique solution and whether $A(T)$ is a concave function in two steps similar to our analysis in Section 3.3.

3.4.1 The First Step

At the lower extreme end of T; i.e., T-t_i= 0**,** $(T_f-T_p)\int_0^{T-t_i}f_i(z|Y_i)dz=0$ and Equation (17) becomes Equation (18). From Equation (18), we have the following three possible cases:

$$T_p-(T_f-T_p)h_i(T-t_i|Y_i)=0. \tag{18}$$

Case 1: If at T=t_i , $(T_f - T_p)h_i(T - t_i | Y_i) < T_p$, then the LHS of equation (18) becomes positive; i.e., at the monitoring point, t_i , the slope of $A(T)$ is positive (see Figure 3.5). In this case, the optimum replacement time must be later than the current monitoring time.

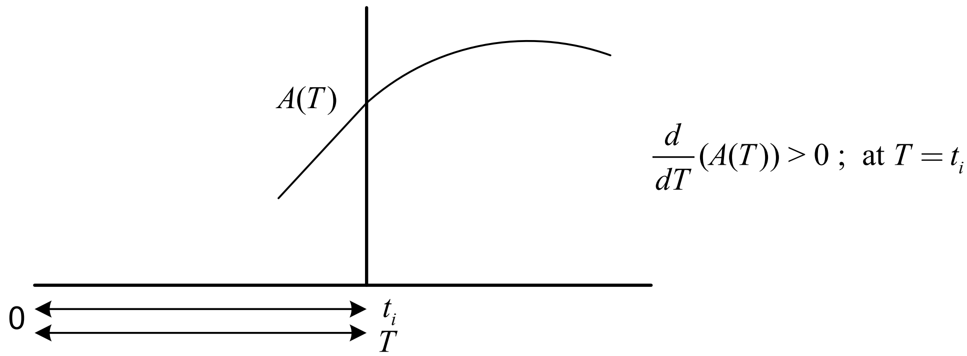

Figure 3.5: The availability function with a positive slope at time $T = t_i$

Case 2: If at T=t_i , $(T_f - T_p)h_i(T - t_i | Y_i) = T_p$, then the LHS of Equation (18) becomes zero (see Figure 3.6); i.e., at the time of monitoring, t_i , the slope of $A(T)$ is zero. In this case, the current time of monitoring is the optimum replacement time for obtaining maximum long-run average availability.

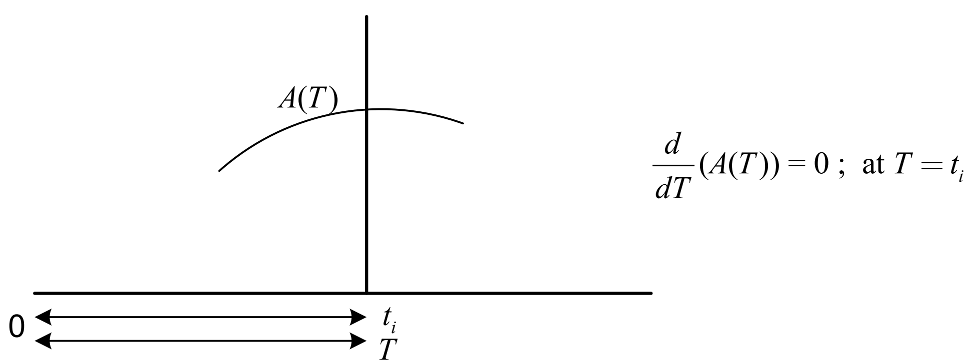

Figure 3.6: The availability function with a slope of zero at $T = t_i$

Case 3: If at T=t_i , $(T_f - T_p)h_i(T - t_i | Y_i) > T_p$, then the LHS of Equation (18) is negative; i.e., at monitoring time, t_i , the slope of $A(T)$ is negative (see Figure 3.7). In this case, the equipment should be replaced immediately.

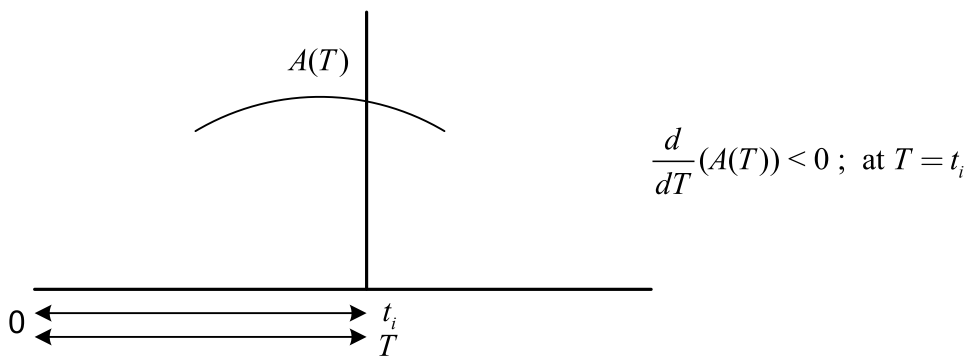

Figure 3.7: The availability function with a negative slope at $T = t_i$

Now, we check the slope of A (T) at the upper extreme end; i.e., T-t_i= ∞ or $T \rightarrow \infty$. At T-t_i=∞ , the LHS of Equation (17) becomes $T_p + (T_f - T_p)\mu_i - (T_f - T_p)Lim_{T-t_i \to \infty} h_i(T - t_i | Y_i)$. When $h_i(T - t_i | Y_i)$ increases strictly in T-t_i , $(Lim_{T-t_i \to \infty}(h_i(T - t_i | Y_i)) \to \infty$; thus, the value of the LHS of Equation (17) becomes negative at the upper extreme end.

3.4.2 The Second Step

We now denote the LHS of Equation (17) as $Z(T)$. To prove that $Z(T)$ decreases monotonically, we take the derivative of $Z(T)$ with respect to T:

$$\frac{d}{dT}(Z(T)) = (T_f - T_p) f_i(T - t_i | Y_i) - (T_f - T_p) h'_i(T - t_i | Y_i).$$

As $T_f > T_p$ and $h_i(T - t_i \mid Y_i)$ increases strictly in T-t_i , we have $\frac{d}{dT}(Z(T)) < 0$.

We see that for Case-1, from the current monitoring time, the $A(T)$ function will increase until it reaches a maximum point, which denotes the optimal replacement time. Immediately after this optimal replacement time, the $A(T)$ function will decrease monotonically. For Case-2, at the current time of monitoring, the $A(T)$ function has reached its maximum value; immediately after this point, it will decrease monotonically. For Case-3, if we delay the replacement, the $A(T)$ function will decrease monotonically. From our investigation we see that $A(T)$ is a concave function, hence it has a unique maximum point, which can be detected from the first derivative of the $A(T)$ function. The above analysis gives us the following theorem on the uniqueness of the optimal solution for an infinite-horizon availability model.

Theorem 2: If $T_f > T_p$, $T_p > (T_f - T_p) h_i(T - t_i \mid Y_i)$ at the current time of monitoring, t_i , and $h_i(T - t_i \mid Y_i)$ increases strictly in T-t_i , there exists a unique and finite $T^* > t_i$ that maximizes long-run average availability. The optimum T^* is the solution for Equation (17).

3.5 Multi-Objective Condition and Age Based Replacement Model

In a real-world maintenance decision, we need to optimize both long-run average cost and long-run average availability, simultaneously. We may not consider weighted average single objective optimization problem to consider both cost and availability as their units are different. Therefore, the following multi-objective optimization problem is formulated:

$$\text{Minimize}: C(T) = \frac{C_p R_i(T - t_i \mid Y_i) + C_f F_i(T - t_i \mid Y_i)}{t_i + \int_0^{T-t_i} R_i(z \mid Y_i) dz}$$

$$\text{Maximize}: A(T) = \frac{t_i + \int_0^{T-t_i} R_i(z \mid Y_i) dz}{t_i + \int_0^{T-t_i} R_i(z \mid Y_i) dz + T_p + (T_f - T_p) \int_0^{T-t_i} f_i(z \mid Y_i) dz}$$

$$\text{Subject to}: T \geq t_i \,.$$

To find the solutions to this multi-objective optimization problem, we can construct a decision frontier to assist the maintenance manager in dealing with trade-offs between cost and availability, while making replacement decisions.

On the other hand, sometimes maintenance managers need to minimize long-run average cost while satisfying at least a minimum level of long-run average availability (ξ), which is referred to as the availability threshold. As a result, we have following modified optimization model:

Minimize: $C(T)$

Subject to: $A(T) \geq \xi$

$T \geq t_i$.

Production managers set a target production quantity required to produce within a specified time period on the basis of customer requirements. This production target is the main criterion that can be used to choose the threshold for $A(T)$. To meet the requirements for a larger production quantity, the maintenance manager is asked to set a higher threshold value for $A(T)$. This restriction allows the maintenance personnel to make maintenance decisions in such a way that the long run average availability of the equipment will be at least the threshold value and the target production level could be achieved within the given time frame.

From the uniqueness of the cost and availability functions observed in Theorems 1 and 2, we see that long-run average cost is a convex function, and long-run average availability is a concave function. To satisfy the constraints in the modified model we may have a range of decision variable, $T \in [T_{\min}, T_{\max}]$, for which we can optimize the objective function. In this case, we can use the following equation to calculate the optimal replacement time (T_{opt}) that minimizes long-run average cost:

$$T_{opt} = \underset{T \in [T_{\min}, T_{\max}]}{\arg\min} \; C(T). \tag{19}$$

3.6 Result Analysis

In order to show the effective of the models proposed in this chapter, we will show some numerical result in this sub-section. These numerical results will show us how our proposed models will be useful to make maintenance decision. We will use the bearing data available in [10]. At first, we need to analyze the properties of hazard rate function.

3.6.1 Properties of Hazard Rate

In condition-based maintenance, the behavior of the hazard rate is a key factor in replacement decision making. It is therefore necessary to investigate the properties of the hazard rate, which is obtained from the complete history of condition information and current age. It is difficult to analyze the properties of the above hazard rate function since it has many components and involves an integration operator. For this reason, we analyzed the hazard rate function using the bearing data in [10] (shown in Figure 3.8) to empirically determine that what parameter combinations, that is, what η and β combinations, cause the hazard rate to increase or strictly increase. This data set has run to failure data for six bearings, and SFP was found to be successful to predict remaining useful life using this data set. Our following empirical study also shows suitability of this data set to show the effectiveness of our proposed methods. The values of the estimated parameters of the hazard rate function are listed as follows: α=0.011, β=1.8729, η=4.5593, A=7.0688, B=27.089 and C=0.0528. Figure 1 shows the total energy of the vibration data in terms of the root mean square (RMS) values, with respect to the number of operating hours for the six bearings. These rms values increase for all bearings as they age, once faults have developed. It is important also to note that, as Figure 1 shows, that in the early stage of the bearings' life, the rms values for the vibration data remain flat but after a certain number of hours of operation, they increase rapidly. These early increases mark the starting point of the defective stage. They also divide the life of a bearing into two stages.

To shed some light on what values for η and β affect the hazard rate, we have divided this analysis into four cases based on different combinations of these parameters. The four cases are: η>1 and β >1, η<1 and β >1, η>1 and β<1, η<1 and β <1. It is found that in all cases $f(x_i)$ decreases while $g(x_i)$ increases and it approaches a constant value when x_i is very large (shown in Figure 3.9 to Figure 3.12). The denominator decreases with x_i and the decreasing rate of the denominator in $h_i(x_i|Y_i)$ dominates the decreasing rate of the $f(x_i)$, which is a part of the numerator. Thus the resultant value for $h_i(x_i|Y_i)$ always increases. From these four cases, it is clear that condition-based hazard rate derived from SFP has an increasing property. It is strictly increasing with increasing slope when η is greater than 1, and increasing slowly when η is less than 1 (shown in

Figure 3.13). As a result, we can conclude that hazard rate increases strictly when η is greater than 1 regardless of whether β >1 or β <1, in this case.

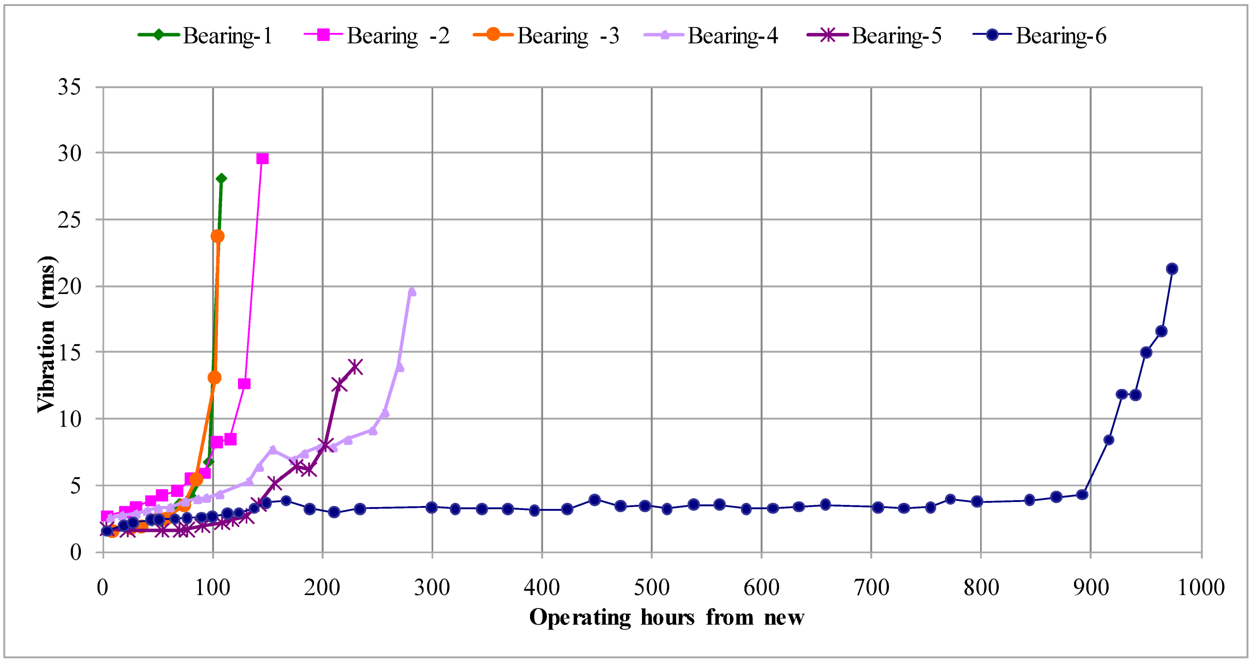

Figure 3.8: Vibration data for six bearings

We used the vibration data for six bearings reported above, to demonstrate the effectiveness of the decision models described in this thesis (see Figure 3.8).

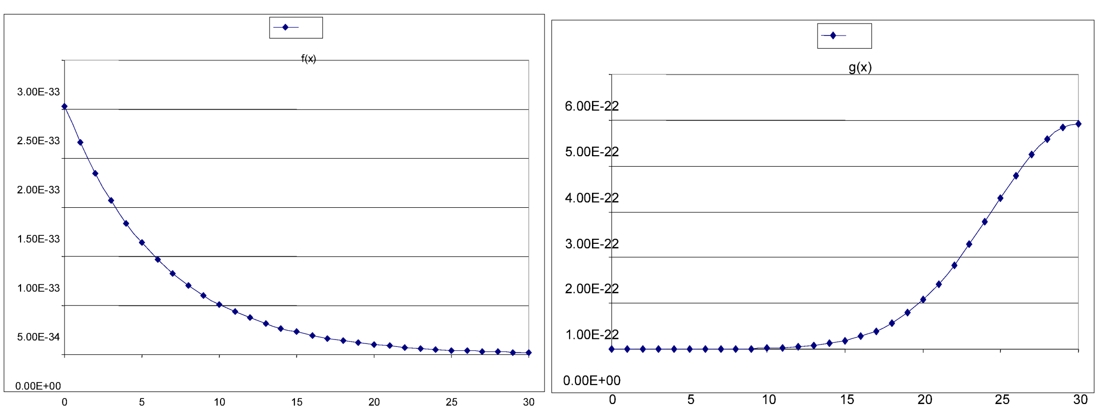

Figure 3.9: Variations of $f(x_i)$ and $g(x_i)$ when η>1 and β >1

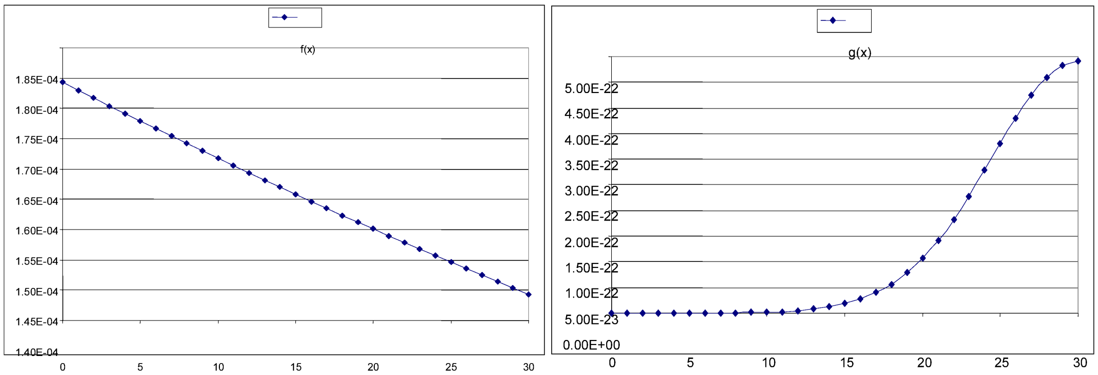

Figure 3.10: Variations of $f(x_i)$ and $g(x_i)$ when $\eta>1$ and $\beta<1$

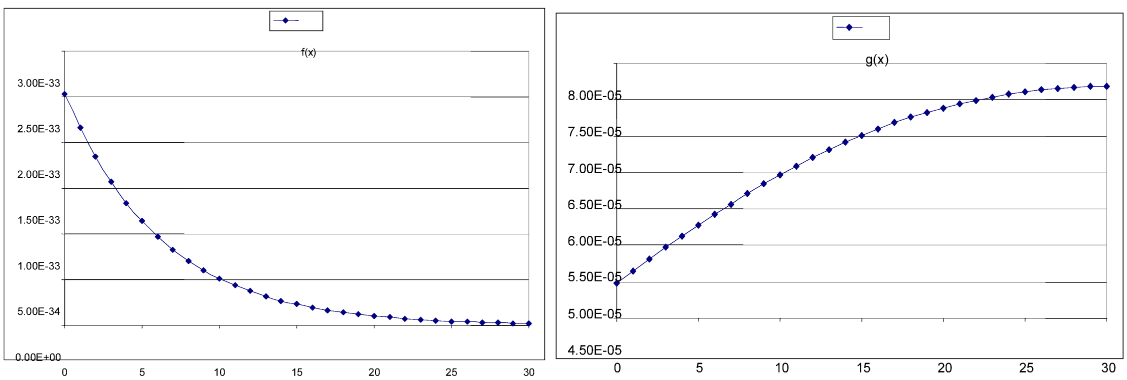

Figure 3.11: Variations of $f(x_i)$ and $g(x_i)$ when $\eta<1$ and $\beta>1$

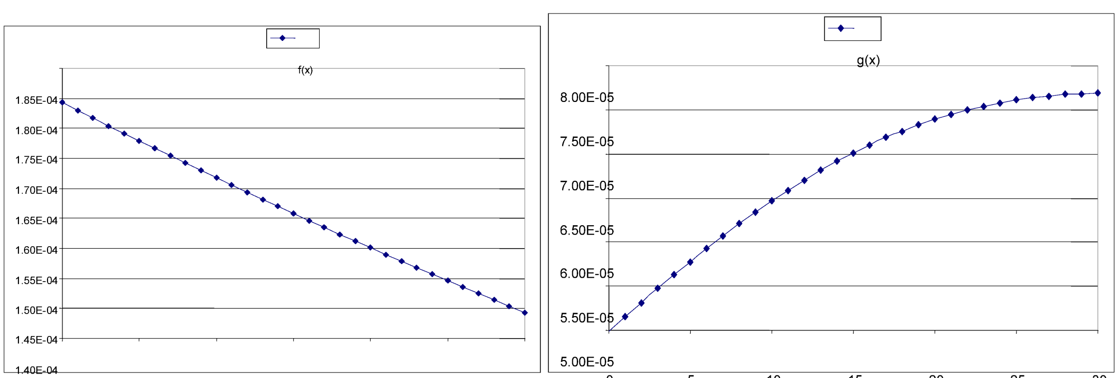

Figure 3.12: Variations of $f(x_i)$ and $g(x_i)$ when $\eta<1$ and $\beta<1$

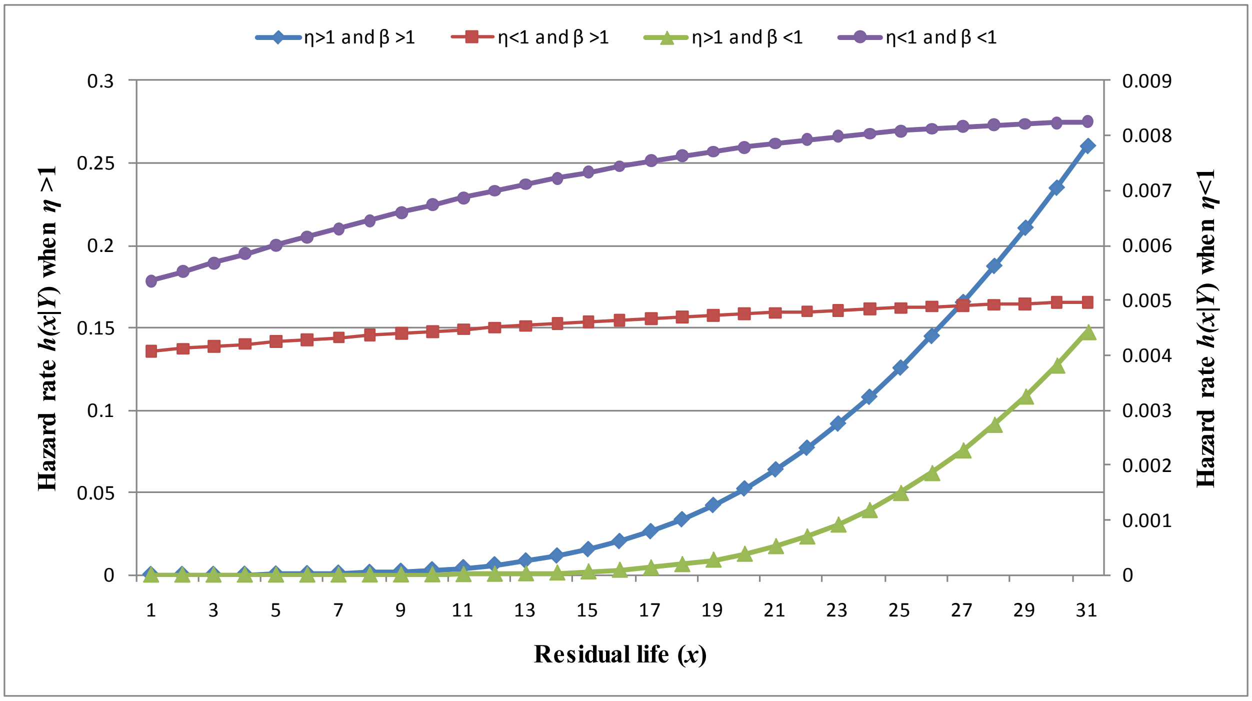

Figure 3.13: Investigation of hazard rate property

3.6.2 Minimizing Long-run Average Cost

With the estimated model parameters and given the condition history outlined in [10], long-run average cost can be calculated using Equation (7), assuming that the cost for preventive replacements (C_p) is \$1000 and the cost for failure replacements (C_f) is \$2000. For the current time of monitoring, the 256th hour, the long-run average cost is plotted as a function of preventive replacement time in Figure 3.14 for bearing 4, from which we can see that the optimum replacement time is the 279th hour if we are to achieve a minimum long-run average cost of 3.63.

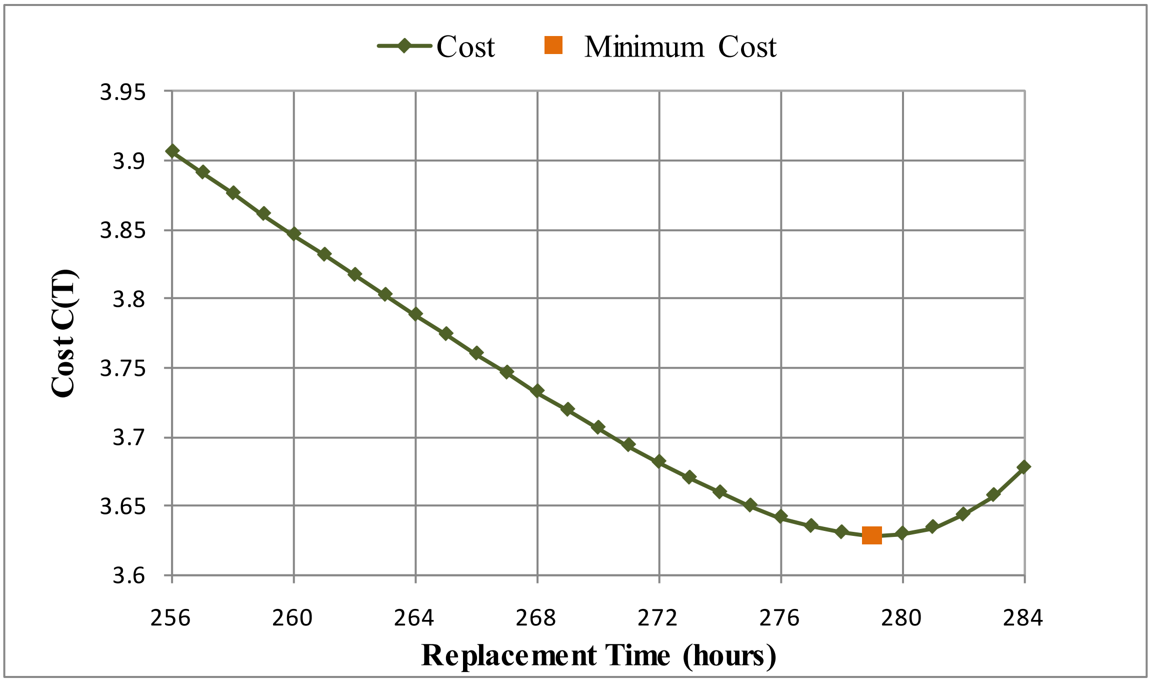

Figure 3.14: Optimum preventive replacement time for minimizing cost

3.6.3 Maximizing Long-run Average Availability

Long-run average availability is calculated using Equation (16), assuming that the average time needed to complete a preventive replacement (T_p) is 5 hours and the average time needed to a complete a failure replacement (T_f) is 50 hours. For the current time of monitoring, the 256th hour, the long-run average availability is plotted as a function of the preventive replacement time in Figure 3.15 for the same bearing 4; from this we can see that the optimum replacement time is the 272nd hour if we are to achieve the maximum availability of 0.981737.

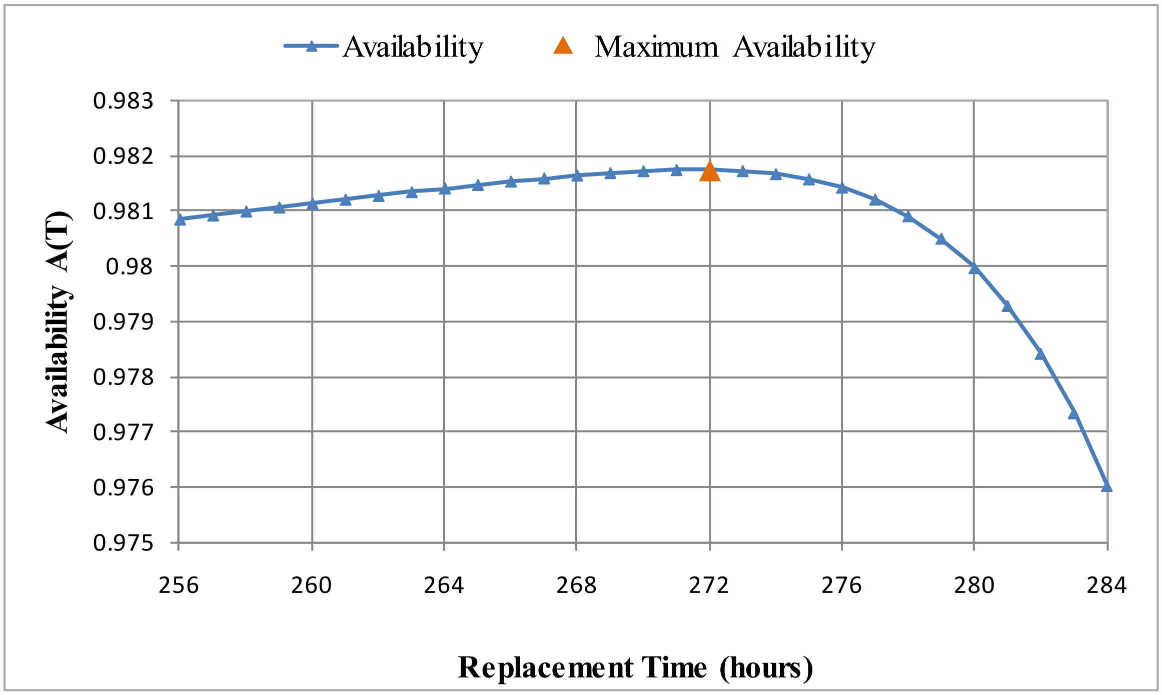

Figure 3.15: Optimum replacement time for maximizing availability

3.6.4 Minimizing $C(T)$ and Maximizing $A(T)$

To determine the optimal preventive replacement time taking into consideration two objectives: minimization of the long-run average cost, $C(T)$, and maximization of the long-run average availability, $A(T)$, we can analyze the two single objective optimization outcomes, where the preventive replacement time at the 272nd hour is optimal when we aim to maximize $A(T)$, and the preventive replacement time at the 279th hour is optimal when we aim to minimize $C(T)$. As a result, an alternative preventive replacement time earlier than the 272nd hour and later than the 279th hour may not be optimal when both objectives are considered (see Figure 3.16). Taking this into consideration, we develop the following decision frontier (illustrated in Figure 3.17) to help the decision maker deal with trade-offs between these two conflicting objectives. In Figure 3.17, the vertical axis corresponds to $C(T)$ and the horizontal axis corresponds to $A(T)$. The numbers plotted represent the preventive replacement times corresponding to the measures of cost and availability. The concept of decision frontier is used when there are multiple objectives in an optimization problem. Usually the multiple objectives compete with one another. Improving the value of one objective function results in the compromise of the other objective functions. In our problem, the two competing objective functions are to maximize the availability function $A(T)$ and to minimize the cost function $C(T)$. The decision frontier given in Figure 3.17 provides the maintenance manager with the optimal options for replacement, depending on his/her desired trade-off between the optimal values of the two objectives. On this frontier, each point reflects the value of the replacement time and its associated long run average cost and long run average availability. The decision maker can easily choose from this frontier the replacement time which best satisfies his/her preference regarding the conflicting objectives.

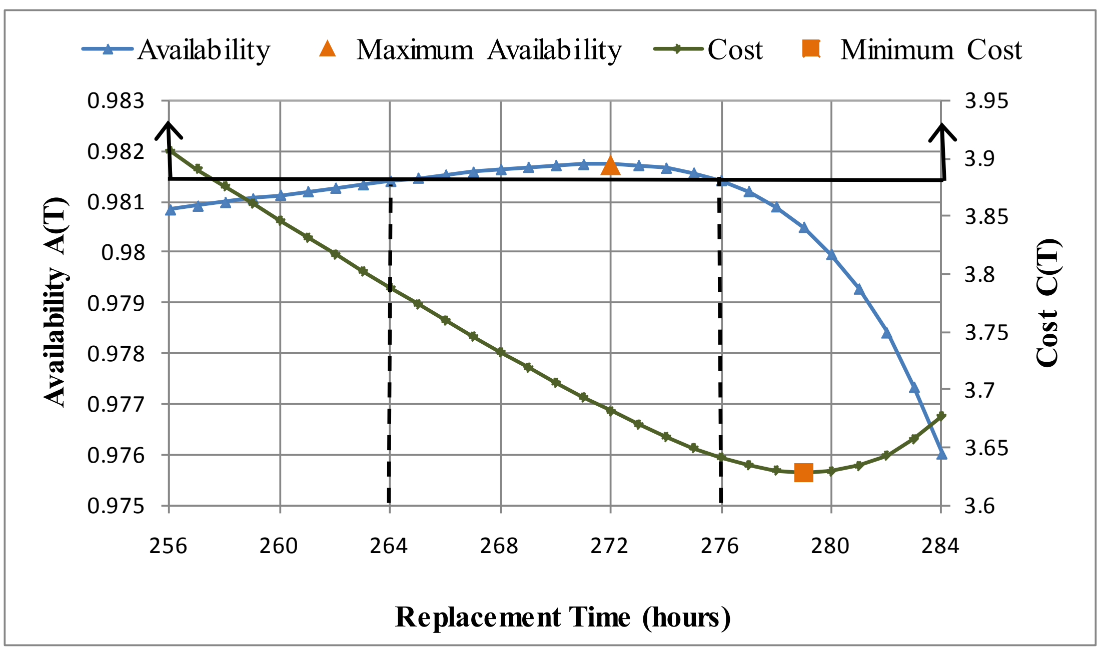

Figure 3.16: Preventive replacement time to minimize $C(T)$ and maximize $A(T)$

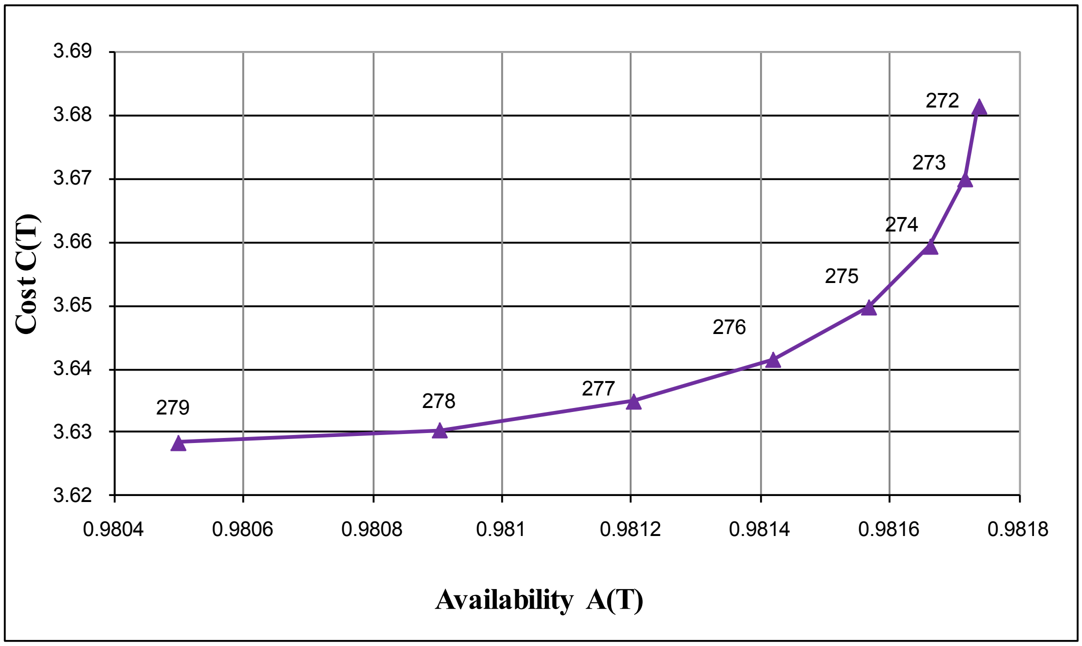

Figure 3.17: Decision frontier for optimizing two objectives

For the modified model, suppose we are restricted to maintaining at least a minimum availability of 0.9815 (shown in Figure 3.16). In other words, the threshold value (ξ) for $A(T)$ is 0.9815; that is, we want to determine a replacement time which will give us a long-run average availability of at least 0.9815. This threshold value is used in Figure 3.16 for graphically determining the replacement time. It gives us a range of replacement times, $T \in [264, 276]$, over which the objective function will be optimized. Using Equation (19), we see that our optimal replacement time is the 276th hour.

The above analysis clearly shows how our proposed DTM based CBM models will help maintenance managers to make replacement decision either for maximizing long run average availability or minimizing long run average cost or both. It also shows us how to handle two conflicting objectives while making maintenance decision. A decision frontier is discussed that deals with trade-offs between the long-run average cost and the long-run average availability. It provides all options relevant to finding the best trade-off between these two conflicting objectives. Another method is also proposed and explained for replacement decision making when the maintenance manager is required to maintain a minimum level of long-run average availability.

3.7 Conclusion

The models developed in this chapter will help maintenance managers select the optimum preventive replacement time for either maximizing long-run average availability or minimizing long-run average cost or trade-off between these two objectives taking into account the complete condition history of the equipment. A comprehensive analytical investigation has been performed and two theorems have been proposed on the nature of the long-run average cost model and the long-run average availability model. However, these models do not consider integration of imperfect maintenance while making replacement decision and did not consider production benefits (revenue), operating costs and maintenance costs simultaneously. In the following chapter, we will develop models to investigate the feasibility of considering imperfect maintenance while making replacement decision by considering above criterion.

4. Integrating Imperfect Maintenance with Replacement Decision

In chapter 3, we have proposed models for replacement decision only. However, imperfect maintenance could be an option to be integrated while making replacement decision. In this chapter, our objective is to develop DTM based CBM models considering such integrations.

Suppose, we are at the current monitoring time (t_i) and we do not have available resources (technical or human) to carry out our replacement action or if we carry out such action at T, it would interrupt our ongoing production or there is a chance that we could realize more residual life by doing imperfect repair at the current time. For the imperfect preventive maintenance, improvement of the health status of the equipment will depend on the history of condition and the current age of the equipment. By performing imperfect preventive maintenance at the i^{th} monitoring time (t_i), we try to push forward the failure or replacement time (Figure 4.1). As a result, the expected residual life $x_i^{'}$ at t_i with the imperfect maintenance will be higher than the expected residual life x_i without imperfect maintenance. It will allow us to have a greater optimum replacement time i.e., we can realize more life from the old equipment. The incremental gain of residual life of the equipment obtained from the imperfect preventive maintenance will affect the long run average availability, expected maintenance cost, expected operating cost and expected total production benefit.

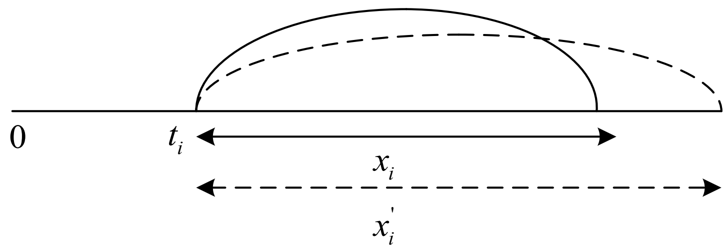

Figure 4.1: Increased residual life with imperfect preventive maintenance

A little effort has been made on considering the imperfect preventive maintenance and preventive replacement decision simultaneously, where the history of condition information is used in the two-phase failure model. However, there is opportunity to gain more advantages by performing the imperfect maintenance, when the equipment starts to deteriorates rapidly. Therefore, it is worthwhile to investigate the feasibility of integrating imperfect preventive maintenance with preventive replacement decision. This research tries to integrate imperfect preventive maintenance with replacement decision in CBM where history of condition information is used through SFP. The purpose of performing imperfect preventive maintenance is to improve the health status, so that we can delay the replacement and realize more residual life. This improvement of health status may allow us to achieve higher long run average availability by delaying the replacement time, maximize production benefit and reduces the operating cost. However, this imperfect preventive maintenance incurs extra cost and time. Therefore, it is important to investigate the feasibility of imperfect preventive maintenance while making a replacement decision to achieve certain objectives.

Here, we propose decision models to integrate condition based imperfect preventive maintenance with replacement decision. These decision models will help us to determine whether we will perform imperfect preventive maintenance or not at current monitoring time and where we should replace to achieve certain objectives. We will use the following notations throughout this paper.

Notations:

t_i : time at monitoring point *i*

x_i : residual lifetime at monitoring time point t_i

y_i : condition information obtained at time t_i

$Y_i = \{y_i, y_{i-1}, ...y_1\}$: history of condition information up to monitoring point *i*

T : scheduled replacement time, to be optimized

T^* : optimum replacement time

T_p : average length of time needed to complete a preventive replacement

T_m : average length of time needed to complete a imperfect preventive maintenance

T_f : average length of time needed to complete a failure replacement

C_p : cost of each preventive replacement

C_m : cost of each imperfect preventive maintenance

C_f : cost of each failure replacement

B_p : production benefit per unit time

$f_i(x_i|Y_i)$: probability density function of x_i conditional upon Y_i

$f_i^{'}(x_i|Y_i)$: probability density function of x_i conditional upon Y_i after imperfect preventive maintenance

$F_i(x_i|Y_i)$: cumulative distribution function of x_i conditional upon Y_i

$F^{'}(x_i|Y_i)$: cumulative distribution function of x_i conditional upon Y_i after imperfect preventive maintenance

$R_i(x_i|Y_i)$: reliability function of the residual lifetime, conditional upon Y_i

$R^{'}(x_i|Y_i)$: reliability function of the residual lifetime, conditional upon Y_i after imperfect preventive maintenance

$h_i(x_i|Y_i)$: hazard function of the residual lifetime, conditional upon Y_i

$h_i^{'}(x_i|Y_i)$: hazard function of the residual lifetime, conditional upon Y_i after imperfect preventive maintenance

μ_i : mean residual lifetime at monitoring point i

$C(T)$: long-run average cost

$A(T)$: long-run average availability

$C^{'}(T)$: long-run average cost after imperfect preventive maintenance

$A^{'}(T)$: long-run average availability after imperfect preventive maintenance

$E_{NB}(T)$: expected net benefit

$E^{'}_{NB}(T)$: expected net benefit after imperfect preventive maintenance

4.1 Model Description

Our goal is to find whether to perform imperfect preventive maintenance at the i^{th} monitoring time (t_i) and to find optimum preventive replacement time (T^*), if we have the history of condition information Y_i up to the i^{th} monitoring time (t_i), where $Y_i = \{y_1, y_2, y_3, ...y_{i-1}, y_i\}$.

We can calculate condition based hazard rate from the history of condition information and current age of the equipment with the SFP [73]. To find out whether imperfect preventive maintenance is needed to perform at the i^{th} monitoring time (t_i) and select optimal preventive replacement time (T^*), the objective functions we are going to use are maximizing long run average availability $A(T)$ and expected net benefit $E_{NB}(T)$. $E_{NB}(T)$ model is used to consider both the cost and benefit in a single model. We can develop CBM models for these objectives considering with and without imperfect maintenance and compare the optimum solutions to find whether it is feasible to perform imperfect preventive maintenance at the i^{th} monitoring time (t_i), while making replacement decision. To investigate the effects of imperfect maintenance, we propose a new model discussed in the following section.

4.2 Modeling of Improvement of Health Status

To predict the health status of the equipment after performing imperfect preventive maintenance, we propose a model to predict the updated hazard rate function. The updated hazard rate function is then used to make the maintenance decision. We expect that after performing imperfect preventive maintenance, the health condition will be better and this improvement can be expressed with the predicted hazard rate function. We propose to combine the concept available in [2,36,74] to model a new hazard rate function, where the effective age after PM will be less than its current original age i.e., there is an effective age reduction after imperfect preventive maintenance and the deterioration of the system will be improved. As a result, we could use an updated condition information associated with the effective age. The effective age right after imperfect preventive maintenance is greater than zero and the hazard rate remains as a function of the effective age and history of condition information (including the updated condition information obtained from the proposed model).

The condition based hazard rate function obtained by the SFP can be expressed with the following equation.

$$h(x_i \left| Y_i \right.) = \frac{(x_i + t_i)^{\beta-1} e^{-(\alpha(x_i+t_i))\beta} \prod_{k=1}^{i} \dfrac{e^{-(y_k(A+Be^{-C(x_i+t_i-t_k)})^{-1})\eta}}{A+Be^{-C(x_i+t_i-t_k)}}}{\int_{x_i}^{\infty} (z+t_i)^{\beta-1} e^{-(\alpha(z+t_i))\beta} \prod_{k=1}^{i} \dfrac{e^{-(y_k(A+Be^{-C(z+t_i-t_k)})^{-1})\eta}}{A+Be^{-C(z+t_i-t_k)}} dz}$$

$$= \frac{f(x_i)g(x_i)}{\int_{x_i}^{\infty} f(x_i)g(x_i)dx}$$

Where $f(x_i) = (x_i + t_i)^{\beta-1} e^{-(\alpha(x_i+t_i))\beta}$ and $g(x_i) = \prod_{k=1}^{i} \frac{e^{-(y_k(A+Be^{-C(x_i+t_i-t_k)})^{-1})^{\eta}}}{A+Be^{-C(x_i+t_i-t_k)}}$

Using the above concept, we propose to develop the following predicted hazard rate function where the effective age of the equipment ($t_i^{'}$) and its associated condition information ($y_i^{'}$) can be deduced by the following two equations, where μ and δ are age reduction factor and degradation improvement factor, respectively.

$$t_i^{'} = t_i(1-\mu)$$
$$y_i^{'} = y_i(1-\delta)$$

We can easily obtain $h^{'}(x_i|Y_i)$ from $h(x_i|Y_i)$ by replacing t_i and y_i with $t_i^{'}$ and $y_i^{'}$, respectively. It is obvious by the definition of age reduction factor that $0 < \mu < 1$, and it represents impact of imperfect maintenance. As the system states after the imperfect maintenance is uncertain, we could assume μ is a random variable [2]. This age reduction factor depends on maintenance cost and real age of the system, and its randomness can be modelled with Beta distribution as it is found to be effective to express maintenance improvement factor. Degradation improvement factor (δ) is $0 < \delta < 1$ and can be estimated from the history of condition information.

4.3 CBM Models

Due to the imperfect preventive maintenance, the improved health status of the equipment is achieved by the expense of extra cost and time associated with the maintenance action. On the other hand, it may provide us some benefits such as increase availability, production benefit and decrease operating cost. Therefore, in this research we use following CBM models for maximizing long run average availability $A(T)$ or maximizing expected net benefit $E_{NB}(T)$ to investigate whether it is worthwhile to perform imperfect preventive maintenance at i^{th} monitoring point, while making replacement decision.

4.3.1 Maximizing the Long Run Average Availability

The purpose of performing imperfect preventive maintenance is to improve the health status, so that we can maximize long run average availability by delaying the replacement time. However, this imperfect preventive maintenance requires time to carry out the maintenance action. Therefore it is required to investigate the feasibility of imperfect preventive maintenance while making a replacement decision to maximize long run average availability.

To develop the decision model for long run average availability with imperfect preventive maintenance, we consider imperfect preventive maintenance time (T_m). In this availability model we use updated hazard rate function found from proposed age reduction model.

$$\text{Long Run Average Availability } A(T) = \frac{\text{Expected total uptime}}{\text{Expected total uptime} + \text{Expected total downtime}}$$

To calculate the expected total uptime and the expected total down time, we need to use the updated residual lifetime distribution. The expected total uptime and the expected total downtime can be calculated with following equations:

Expected total uptime = $t_i + \int_{T_m}^{T-t_i} z f_i'(z|Y_i)dz + (T - T_m - t_i)(1 - \int_0^{T-t_i} f_i'(z|Y_i)dz)$

Expected total down time = $T_m + T_p(1 - \int_0^{T-t_i} f_i'(z|Y_i)dz) + \int_{T_m}^{T-t_i} T_f f_i'(z|Y_i)dz)$

Substituting the expressions of the expected total uptime and the expected total downtime, we obtain the following long run average availability model $A'(T)$ with imperfect preventive maintenance:

$$A'(T) = \frac{t_i + \int_{T_m}^{T-t_i} z f'_i(z|Y_i)dz + (T - T_m - t_i)(1 - \int_0^{T-t_i} f_i'(z|Y_i)dz)}{t_i + T_m + \int_{T_m}^{T-t_i} (z + T_f) f_i'(z|Y_i)dz + (T - T_m - t_i + T_p)(1 - \int_0^{T-t_i} f_i'(z|Y_i)dz)}$$

From this model, it is obvious that whether it is advisable to perform imperfect preventive maintenance at i^{th} monitoring time depends on the imperfect preventive maintenance time (T_m) and the improvement of health status. Long run average availability model $A(T)$ without imperfect preventive maintenance can be calculated by following equation:

$$A(T) = \frac{t_i + \int_0^{T-t_i} z f_i(z|Y_i)dz + (T - t_i)(1 - \int_0^{T-t_i} f_i(z|Y_i)dz)}{t_i + \int_0^{T-t_i} (z + T_f) f_i(z|Y_i)dz + (T - t_i + T_p)(1 - \int_0^{T-t_i} f_i(z|Y_i)dz)}$$

We can calculate the optimum replacement time by maximizing long run average availability for each scenario: with and without imperfect maintenance by the above two equations. Suppose the optimum replacement time without imperfect preventive maintenance T_o and optimum replacement time after the imperfect preventive maintenance $T_o{}'$. Thus the optimum replacement time T^* is one of them for which we can maximize long run average availability. Following equation is used to calculate the optimal replacement time by considering both cases simultaneously:

$$T^* = \underset{T \in \{T_o, T_o^{'}\}}{\arg\max} \left\{ A(T_o), A^{'}(T_o^{'}) \right\}$$

From this above model, it is obvious that whether it is feasible to perform imperfect preventive maintenance or not, depends on the value of imperfect preventive maintenance time (T_m) and the improvement of health status. Without considering this model, it is difficult to make such decisions. One of the main drawbacks of this model is that, it does not consider the associated cost and the benefits of the maintenance action. Therefore, it is required to develop the following model, which considers the maintenance cost along with other costs and benefit.

4.3.2 **Maximizing Expected Net Benefit**

Expected net benefit $E_{NB}(T)$ is the expected profit, which can be calculated by deducting expected cost from the expected production benefit. The purpose of performing imperfect preventive maintenance is to improve the health status, so that we can delay the replacement and realize more residual life. This improvement of health status will allow us to achieve other advantages: minimize cost and maximize production benefit etc. However, this imperfect preventive maintenance incurs extra cost and time. Therefore, it is important to investigate the feasibility of imperfect preventive maintenance while making a replacement decision to maximize expected net benefit. In this model, we have the following three components needed to be considered: expected maintenance cost, expected production benefit and expected operating cost.

4.3.2.1 Expected Maintenance Cost

The expected maintenance cost is the total estimated maintenance cost, which is composed of the imperfect preventive maintenance cost (C_m), failure replacement cost (C_f) and preventive replacement cost (C_p). The last two cost components are uncertain, and depend on the residual life distribution and future maintenance decision. Therefore, the expected maintenance cost with the imperfect preventive maintenance $E'_{MC}(T)$ can be expressed with the following equation if we consider a planned preventive maintenance at time T:

$$E'_{MC}(T) = C_m + C_p R'(T - t_i | Y_i) + C_f F'(T - t_i | Y_i)$$

4.3.2.2 Expected Production Benefit

The expected production benefit is the total estimated benefit we would incur from an equipment during its entire life cycle. The expected production benefit with imperfect preventive maintenance has three components: the total benefit up to i^{th} monitoring time, the additional total benefit after i^{th} monitoring time if the equipment survive up to the planned preventive replacement time (T) and the additional total benefit after i^{th} monitoring time if the equipment fails before the planned preventive replacement time (T). The later two benefit components are uncertain, and depend on the residual life distribution and future maintenance decisions. The expected production benefit with imperfect preventive maintenance $E'_{PB}\left(T\right)$ from the equipment can be expressed with the following equation if we consider a planned preventive maintenance at T:

$$E'_{PB}\left(T\right) \; = B_p t_i + B_p \int_0^{T-t_i} z f'(z \mid Y_i) dz + B_p (T - t_i)(1 - \int_0^{T-t_i} f'(z \middle| Y_i) dz)$$

4.3.2.3 Expected Operating Cost

Expected operating cost is the total estimated operating cost such as fuel cost, power consumption cost etc. to run the machine. By doing imperfect preventive maintenance, health status of the equipment will be improved. This improvement not only reduces the occurrence of failure but also makes the equipment more efficient to operate [36]. Therefore, it is required to develop a model that could reflect the saving of operating cost with the maintenance cost. To develop such model we consider the following assumptions:

(i) Before the equipment reaches to the i^{th} monitoring time, the operating cost per unit time is constant.

(ii) After i^{th} monitoring time, the operating cost will be higher. The incremental amount of total operating cost is proportional to the cumulative failure rate as the equipment starts deteriorating rapidly at this stage. As the equipment gets older, the cumulative failure rate will be higher, and equipment requires higher operating cost.

Suppose C_{oc} is the operating cost per unit time up to i^{th} monitoring time and C_o is the incremental operating cost coefficient, determined by previous data or maintenance personnel experiences. According to the above assumptions, the expected operating cost with imperfect preventive maintenance $E'_{OC}(T)$ at time T can be derived using the following Figure 4.2, where the expected operating cost is the total estimated cost we would incur to operate an equipment during its entire life cycle.

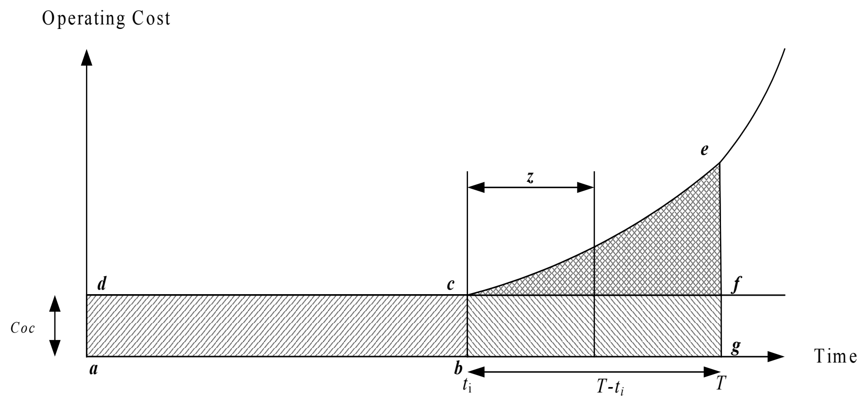

Figure 4.2: Description of operating cost calculation

According to this figure,

area of *abcd*=$t_i C_{oc}$

area of *bgfc*=$(T - t_i) C_{oc}$

area of *cef*=$C_o \int_0^{T-t_i} h'(z|Y_i)dz$

In this situation the expected operating cost with imperfect preventive maintenance $E'_{OC}(T)$ also has three components: the total operating cost up to i^{th} monitoring time, the total operating cost after i^{th} monitoring time if the equipment survive up to the planned preventive replacement time (T) and the total operating cost after i^{th} monitoring time if the equipment fails before the planned preventive replacement time (T) and the last two components are uncertain as they depend on the residual life distribution and future maintenance decision.

The total operating cost up to i^{th} monitoring time is equal to the area of $abcd = t_i C_{oc}$.

Suppose the probability that the equipment will survive up to time T is $R'(T - t_i | Y_i)$, then the total operating cost after the i^{th} monitoring time is equal to the summation of the area under the curve *bgfc* and *cef* times the probability that the equipment will survive up to time T, which can be expressed as follows:

$$\left((T - t_i) C_{oc} + C_o \int_0^{T - t_i} h'(z | Y_i) dz \right) R'(T - t_i | Y_i)$$

The total operating cost after i^{th} monitoring time if the equipment fails (suppose at z), before the planned preventive replacement time (T), can be expressed as follows :

$$\left(z C_{oc} + C_o \int_0^{z} h'(z | Y_i) dz \right) \int_0^{T - t_i} f'(z | Y_i) dz$$

Finally, the expected operating cost $E'_{OC}(T)$ can be obtained by adding all these three cost components as follows:

$$E'_{OC}(T) = t_i C_{oc} + \left((T - t_i) C_{oc} + C_o \int_0^{T-t_i} h'(z|Y_i) dz \right) R'(T - t_i|Y_i) + \left(z C_{oc} + C_o \int_0^{z} h'(z|Y_i) dz \right) \int_0^{T-t_i} f'(z \mid Y_i) dz$$

We know that, $\int_0^{T-t_i} h'(z|Y_i) dz = -\ln R'(T - t_i|Y_i)$

After plugging the above relationship $E'_{OC}(T)$ becomes:

$$E'_{OC}(T) = t_i C_{oc} + \left((T - t_i) C_{oc} - C_o \ln R'(T - t_i|Y_i) \right) R'(T - t_i|Y_i) + \left(z C_{oc} - \ln R'(z|Y_i) \right) \int_0^{T-t_i} f'(z \mid Y_i) dz$$

We expect that by doing imperfect preventive maintenance we can have lower hazard rate. As a result, our operating cost will be less compare to the operating cost without imperfect preventive maintenance for a particular replacement time, because the incremental amount of operating cost calculated by the area under the curve *ecf* will be less. Our above decision model will determine whether it is feasible to perform imperfect preventive maintenance as the imperfect preventive maintenance at current monitoring time incurs extra maintenance cost. The expected net benefit with imperfect preventive maintenance $E'_{NB}(T)$ of the equipment can be expressed with the following equation:

$$E'_{NB}(T) = E'_{PB}(T) - E'_{MC}(T) - E'_{OC}(T)$$

Similarly, we can also calculate the expected net benefit without imperfect preventive maintenance $E_{NB}(T)$ using the original residual life. We can calculate the optimum replacement time by maximizing expected net benefit for each scenario: with and without imperfect maintenance by the above two equations. Suppose the optimum replacement time without imperfect preventive maintenance T_o and optimum replacement time with the imperfect preventive maintenance $T_o^{'}$. Thus the optimum replacement time T^* is one of them for which we can maximize expected net benefit. Following equation is used to calculate the optimal replacement time by considering both cases simultaneously:

$$T^* = \underset{T \in \left\{T_o, T_o^{'}\right\}}{\arg\max} \left\{E_{NB}(T_o), E_{NB}{'}(T_o^{'})\right\}$$

4.4 Conclusion

We have proposed two DTM based CBM models, one is to maximize availability and the other is to maximize net benefit for maintenance decision making. Both models have capability to determine whether it is viable to carry out any imperfect maintenance at the current time and to determine optimum replacement time. The net benefit model considers both revenue and cost to investigate the feasibility of integrating imperfect maintenance and provide us decision by optimizing the net benefit or profit. To make these models more robust, we could also consider other factors such as inventory, quality and production, but this is beyond the scope of this research. In the next chapter, we will summarize our findings and discuss avenues for the future research.

5. Summary & Future Research

This thesis reported a number of maintenance models by taking into account the current age and the complete condition history of the equipment to make better maintenance decisions. These models will help maintenance managers to select the optimum preventive replacement time by optimizing required objectives.

In chapter 3, we have proposed DTM based CBM models for replacement decision making for maximizing the long-run average availability and minimizing the long-run average cost. Apart from the development of maintenance models, a detailed analytical investigation has been performed which studies the nature of the long-run average cost model and the long-run average availability model for replacement decisions. From the observations made during the course of this investigation, two theorems have been proposed for analyzing the properties of the models and calculating the optimum replacement time.

To illustrate the effectiveness of our proposed models, numerical examples have been presented for each scenario in order to show the effectiveness of the proposed methods. Results obtained from the multi-objective optimization problem will assist the maintenance manager to apply his/her preferences. In practical applications, maintenance managers often need to make compromises between cost and availability. If they want to minimize cost, they may choose to live with a lower availability. If they want to maximize the availability, they will have to spend more money. The decision frontier provides all options relevant to finding the best trade-off between these two conflicting objectives. This frontier can be generated ahead of time for the use of the maintenance manager. Another method is also proposed and explained for replacement decision making when the maintenance manager is required to maintain a minimum level of long-run average availability.

However, CBM models discussed in Chapter 3 do not consider imperfect maintenance while making replacement decision. Therefore, in chapter 4, we proposed to integrate imperfect preventive maintenance with replacement decision in CBM, where history of condition information and current age is used through SFP. We proposed to combine the concept given in [2, 36, 74] to model a new hazard rate function, where the effective age after PM will be less than its current original age i.e., there is an effective age reduction after imperfect preventive maintenance and the deterioration of the system will be improved. This improvement of health condition realized from imperfect preventive maintenance may allow us to achieve higher long run average availability by delaying the replacement time. It may also increases revenue and reduces operating cost. However, imperfect preventive maintenance incurs extra cost and time. Therefore, we proposed a number of optimization models to investigate the feasibility of imperfect preventive maintenance while making a replacement decision.

Our proposed models and theorems are limited to apply for CBM decision making, where we could have available history of condition information and the predicted hazard rate is a monotonically increasing function. In the future, these models can be further studied by integrating other relevant objectives and factors such as production, quality, scheduling etc. to see what effects they would have on replacement decisions.

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